

CBSE Class 12 Physics Question Paper 2026 with Solution PDF-Set 55/2/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. All questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – Sections **A, B, C, D** and **E**.
- (iii) In **Section A** : Question numbers **1 to 16** are Multiple Choice (MCQ) type questions. Each question carries 1 mark.
- (iv) In **Section B** : Question numbers **17 to 21** are Very Short Answer (VSA) type questions. Each question carries 2 marks.
- (v) In **Section C** : Question numbers **22 to 28** are Short Answer (SA) type questions. Each question carries 3 marks.
- (vi) In **Section D** : Question numbers **29 & 30** are Case Study-Based questions. Each question carries 4 marks.
- (vii) In **Section E** : Question numbers **31 to 33** are Long Answer (LA) type questions. Each question carries 5 marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculator is NOT allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Section A (16 marks)

1. Three point charges $2q$, $-2q$ and q are kept at the vertices of an equilateral triangle of side l . The potential energy of the system is: [1 mark]

- (A) zero
(B) $\frac{-2q^2}{\pi\epsilon_0 l}$
(C) $\frac{q^2}{2\pi\epsilon_0 l}$
(D) $\frac{-q^2}{\pi\epsilon_0 l}$

Correct Answer: (D) $\frac{-q^2}{\pi\epsilon_0 l}$

Solution:

Concept:

- The electrostatic potential energy of point charges is the sum of the energies of all distinct pairs: $U = \sum_{i < j} kq_i q_j / r_{ij}$, where $k = 1/(4\pi\epsilon_0)$.
- A pair of like charges contributes positive energy and a pair of unlike charges contributes negative energy. Each pair is counted once.

Step 1: List the three interacting pairs All separations equal the side l . The products for the pairs $(2q, -2q)$, $(-2q, q)$ and $(q, 2q)$ are $-4q^2$, $-2q^2$ and $+2q^2$, respectively.

Step 2: Add the pair energies The last two contributions cancel, leaving the attractive interaction of the first pair:

$$U = \frac{k}{l}(-4q^2 - 2q^2 + 2q^2) = -\frac{4kq^2}{l} = -\frac{q^2}{\pi\epsilon_0 l}.$$

Thus the energy is not zero or positive, excluding (A) and (C). Option (B) would count the surviving pair contribution twice.

Step 3: Conclusion The system has potential energy $U = -q^2/(\pi\epsilon_0 l)$. Hence, the correct option is (D).

Quick Tip to Solve: For three charges at equal separations, add the three signed charge products first; count each pair only once.

2. Two metal spheres of radii r_1 and r_2 ($r_2 > r_1$) having charges q_1 and q_2 respectively kept in air, are brought in contact. Which of the following statements is NOT correct? [1 mark]

- (A) The total charge of the two spheres is conserved.
(B) Both spheres attain the same potential.
(C) The final potential of the system equals $\frac{1}{4\pi\epsilon_0} \frac{(q_1 + q_2)}{(r_1 + r_2)}$.
(D) The final potential of the system equals $\frac{1}{4\pi\epsilon_0} \frac{(q_1 + q_2)(r_1 + r_2)}{r_1 r_2}$.

Correct Answer: (D) The final potential of the system equals $\frac{1}{4\pi\epsilon_0} \frac{(q_1 + q_2)(r_1 + r_2)}{r_1 r_2}$.

Solution:

Concept:

- Charge is conserved when isolated conducting spheres exchange charge. At electrostatic equilibrium, connected conductors have equal potential.
- In the usual charged-sphere model, a sphere of radius r has capacitance $C = 4\pi\epsilon_0 r$, and its charge is $Q = CV$.

Step 1: Apply conservation and equal potential Let the final charges be Q_1 and Q_2 , and let their common potential be V . Then $Q_1 + Q_2 = q_1 + q_2$, so statement (A) is correct. Charge transfer stops when the potentials are equal, making statement (B) correct as well.

Step 2: Calculate the common potential Using the standard individual-sphere capacitances assumed in this question,

$$\begin{aligned} q_1 + q_2 &= (C_1 + C_2)V \\ &= 4\pi\epsilon_0(r_1 + r_2)V, \\ V &= \frac{1}{4\pi\epsilon_0} \frac{q_1 + q_2}{r_1 + r_2}. \end{aligned}$$

This verifies (C). The expression in (D) incorrectly uses $1/r_1 + 1/r_2$ as the radius factor; connected sphere capacitances must be added before taking the reciprocal.

Step 3: Conclusion Statement (D) gives the incorrect final potential. Hence, the correct option is (D).

Quick Tip to Solve: For spheres sharing a common potential, add their capacitances: the denominator of V contains $r_1 + r_2$.

3. The maximum kinetic energy of the electrons emitted from a photosensitive surface depends on: [1 mark]

- (A) work function of the surface ϕ_0 only.
- (B) frequency of the incident radiation ν only.
- (C) intensity of the incident radiation I only.
- (D) Both ϕ_0 and ν .

Correct Answer: (D) Both ϕ_0 and ν .

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi_0$, where $h\nu$ is the incident photon energy and ϕ_0 is the work function of the surface.
- The work function is the minimum energy needed to liberate an electron. Photoemission requires $h\nu \geq \phi_0$.

Step 1: Identify the two energy contributions A photon transfers energy $h\nu$ to an electron. After the electron spends ϕ_0 in escaping from the surface, the largest possible remaining kinetic energy is $h\nu - \phi_0$. Increasing frequency increases this maximum, while increasing the work function at fixed frequency decreases it.

Step 2: Check each proposed dependence Option (A) omits the photon frequency and (B) omits the material's work function, so neither is complete. At a fixed frequency above threshold, increasing intensity increases the photon arrival rate and hence the number of emitted electrons, not their maximum kinetic energy. Thus (C) also fails; the equation contains both ϕ_0 and ν , as stated in (D).

Step 3: Conclusion The maximum kinetic energy depends on both the work function ϕ_0 and the incident frequency ν . Hence, the correct option is (D).

Quick Tip to Solve: Write $K_{\max} = h\nu - \phi_0$ first: frequency sets photon energy, while intensity controls the photon arrival rate.

4. In Bohr model of hydrogen atom, for large values of n , the distance between the consecutive orbits is proportional to: [1 mark]

- (A) $\sqrt{n}$
- (B) n
- (C) n^2
- (D) n^3

Correct Answer: (B) n

Solution:

Concept:

- In the Bohr model of hydrogen, the radius of the n th orbit is $r_n = a_0 n^2$, where a_0 is the Bohr radius and n is the principal quantum number.
- The distance between consecutive circular orbits is the difference of their radii, $\Delta r = r_{n+1} - r_n$, rather than either orbit's radius itself.

Step 1: Subtract consecutive orbit radii Substituting the radii of the n th and $(n + 1)$ th orbits gives

$$\Delta r = a_0[(n + 1)^2 - n^2] = a_0(2n + 1).$$

The n^2 terms cancel, so a spacing proportional to n^2 , as in option (C), would confuse radius with the difference of radii.

Step 2: Take the large quantum-number limit For large n , the constant 1 is negligible compared with $2n$. Therefore $\Delta r \simeq 2a_0n$, which grows linearly with n . Neither the square-root dependence in (A) nor the cubic dependence in (D) follows from the radius difference.

Step 3: Conclusion The distance between consecutive Bohr orbits is proportional to n for large n . Hence, the correct option is (B).

Quick Tip to Solve: When a question asks for spacing between orbits, calculate $(n + 1)^2 - n^2$; do not copy the n^2 radius law.

5. A straight long wire lying along y -axis, carries a current of 1 A along $-y$ direction. The magnetic field due to the conductor at a point (50 cm, 0, 0) will point along: [1 mark]

- (A) z -axis
- (B) $-z$ -axis
- (C) x -axis
- (D) $-x$ -axis

Correct Answer: (A) z -axis

Solution:

Concept:

- The magnetic field of a straight current-carrying wire forms circles about the wire. Its direction follows the right-hand thumb rule, with the thumb along the current.
- Equivalently, the Biot–Savart law gives the field direction as $d\ell \times \hat{r}$, where $d\ell$ points with the current and $\hat{r}$ points from a wire element toward the observation point.

Step 1: Locate the point relative to the wire The wire lies on the y -axis and the current points along $-\hat{y}$. The point (50 cm, 0, 0) is on the positive x -axis, so its perpendicular displacement from the wire is along $+\hat{x}$.

Step 2: Apply the vector direction rule Using $\hat{x} \times \hat{y} = \hat{z}$, the field direction is

$$(-\hat{y}) \times \hat{x} = -(-\hat{z}) = +\hat{z}.$$

Option (B) reverses this direction. Options (C) and (D) give radial directions, whereas the field must be tangent to its circular field line and perpendicular to the positive x -direction here. The current magnitude and distance determine field strength, not this direction.

Step 3: Conclusion The magnetic field points along the positive z -axis. Hence, the correct option is (A).

Quick Tip to Solve: Write current direction cross perpendicular displacement: here $(-\hat{y}) \times \hat{x} = +\hat{z}$.

6. Which of the following materials has positive and small value of magnetic susceptibility? [1 mark]

- (A) Cu
- (B) Al
- (C) Bi
- (D) Ni

Correct Answer: (B) Al

Solution:

Concept:

- Magnetic susceptibility χ_m relates the magnetisation M to the applied magnetic field intensity H : $M = \chi_m H$.
- Paramagnetic materials have a small positive susceptibility. Diamagnetic materials have a small negative susceptibility, while ferromagnetic materials have a large positive response under ordinary conditions.

Step 1: Interpret the sign and magnitude A positive susceptibility means that magnetisation is along the applied field. The additional condition that the value is small identifies weak attraction and hence paramagnetism, rather than the strong response associated with ferromagnetism.

Step 2: Classify the listed materials Aluminium (Al) is paramagnetic and satisfies both conditions. Copper (Cu) and bismuth (Bi) are diamagnetic, so (A) and (C) have the wrong sign. Nickel (Ni) is ferromagnetic at ordinary temperature; although its response is positive, it is not the small paramagnetic response required here, eliminating (D).

Step 3: Conclusion Aluminium has a small positive magnetic susceptibility. Hence, the correct option is (B).

Quick Tip to Solve: Use both clues: the positive sign removes Cu and Bi, and the small magnitude separates Al from ferromagnetic Ni.

7. A galvanometer of resistance $27\ \Omega$ is converted into an ammeter of range (0-10 mA) using a resistance of $3\ \Omega$. The galvanometer will show full scale deflection for a current of about: [1 mark]

- (A) 10 mA
- (B) 100 mA
- (C) 1 mA
- (D) 3 mA

Correct Answer: (C) 1 mA

Solution:

Concept:

- To convert a galvanometer into an ammeter, a low shunt resistance S is connected in parallel with its resistance G , so most of the total current bypasses the galvanometer.
- Equal voltages across parallel branches give $I_g G = (I - I_g)S$, where I is the ammeter's full-scale current and I_g is the galvanometer's full-scale current.

Step 1: Apply the parallel-branch voltage relation Here $G = 27\ \Omega$, $S = 3\ \Omega$ and $I = 10\ \text{mA}$. The

galvanometer carries I_g , while the shunt carries $I - I_g$. Rearranging their equal-voltage equation gives

$$I_g(G + S) = IS, \quad I_g = \frac{IS}{G + S}.$$

Step 2: Substitute and check the division of current

$$I_g = \frac{(10 \text{ mA})(3 \Omega)}{27 \Omega + 3 \Omega} = 1 \text{ mA}.$$

The shunt then carries 9 mA; each branch has a voltage of 27 mV. Option (A) confuses total ammeter current with galvanometer current, (B) exceeds the total current, and (D) would give unequal voltages across the two parallel resistances.

Step 3: Conclusion The galvanometer shows full-scale deflection at $I_g = 1 \text{ mA}$. Hence, the correct option is (C).

Quick Tip to Solve: The galvanometer receives the fraction $S/(G + S) = 3/30$ of the total current; the lower resistance carries more current.

8. The magnetic flux ϕ (in Wb) linked with a coil is related to time t (in s) as $\phi = 5At^2 + Bt - 2C$. The SI units of A and B are respectively: [1 mark]

- (A) Wb s^2 , Wb s
- (B) Wb s^{-1} , Wb
- (C) Wb s^{-2} , Wb s^{-1}
- (D) Wb s^{-1} , Wb s^{-2}

Correct Answer: (C) Wb s^{-2} , Wb s^{-1}

Solution:

Concept:

- Dimensional homogeneity requires every term added in an equation to have the same dimensions. Thus each term in $\phi = 5At^2 + Bt - 2C$ must have the unit of magnetic flux, the weber (Wb).
- The numerical factors 5 and 2 are dimensionless, while the SI unit of time t is the second (s).

Step 1: Determine the unit of the quadratic coefficient Since At^2 is a magnetic flux, the unit of A must cancel the two powers of time:

$$[A] = \frac{[\phi]}{[t]^2} = \text{Wb s}^{-2}.$$

A positive power s^2 , as in option (A), would instead give the product At^2 an extra factor s^4 .

Step 2: Determine the unit of the linear coefficient Similarly, Bt must also have the unit weber:

$$[B] = \frac{[\phi]}{[t]} = \text{Wb s}^{-1}.$$

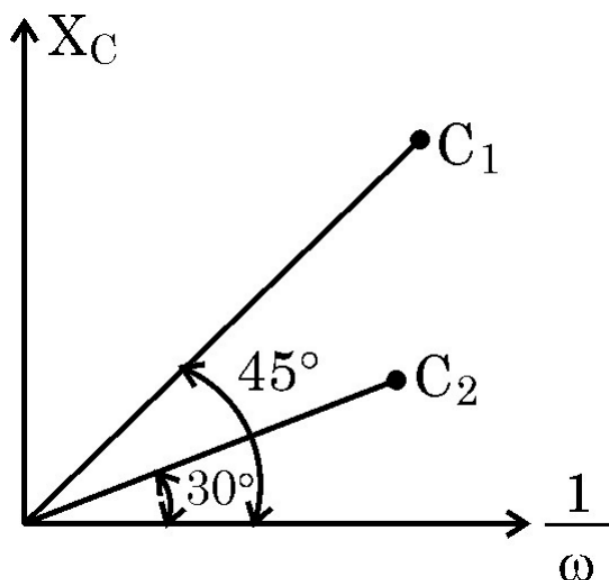
Option (B) supplies too few inverse powers of time for both coefficients, while option (D) interchanges their required powers. The constant C itself has the unit weber and does not alter this comparison.

Step 3: Conclusion The SI units of A and B are respectively Wb s^{-2} and Wb s^{-1} . Hence, the correct

option is (C).

Quick Tip to Solve: For each polynomial term in flux, divide the unit weber by the same power of seconds that multiplies its coefficient.

9. The figure shows the variation of capacitive reactance (X_C) of two ideal capacitors of capacitances C_1 and C_2 with the reciprocal of angular frequency ($1/\omega$) of ac source. The value of C_1/C_2 is: [1 mark]



- (A) $\frac{1}{2}$
- (B) 2
- (C) $\sqrt{3}$
- (D) $\frac{1}{\sqrt{3}}$

Correct Answer: (D) $\frac{1}{\sqrt{3}}$

Solution:

Concept:

- The capacitive reactance of an ideal capacitor is $X_C = 1/(\omega C)$, where ω is the angular frequency and C is the capacitance.
- On a plot of X_C against $1/\omega$, the straight-line slope is $m = 1/C$. Therefore a steeper line represents a smaller capacitance.

Step 1: Read both slopes from the graph The line labelled C_1 makes an angle 45° with the horizontal axis; the line labelled C_2 makes an angle 30° with that same axis. Since both use the same axes and scale, their slope ratio is

$$\frac{m_1}{m_2} = \frac{\tan 45^\circ}{\tan 30^\circ} = \sqrt{3}.$$

Step 2: Invert the slope ratio to obtain the capacitance ratio Substitute $m_1 = 1/C_1$ and $m_2 = 1/C_2$:

$$\frac{C_1}{C_2} = \frac{m_2}{m_1} = \frac{\tan 30^\circ}{\tan 45^\circ} = \frac{1}{\sqrt{3}}.$$

Option (C) is the slope ratio itself and incorrectly treats capacitance as proportional to slope. Options (A) and (B) do not equal the ratio of the tangents; the angles themselves must not be divided.

Step 3: Conclusion The required ratio is $C_1/C_2 = 1/\sqrt{3}$. Hence, the correct option is (D).

Quick Tip to Solve: On an X_C versus $1/\omega$ graph, capacitance is the reciprocal of slope: invert the tangent ratio before selecting the option.

10. A magnet held vertically, with its north pole down, is dropped along the axis of a closed solenoid placed vertically on a table. If the observer looks down from the top, then: [1 mark]

- (A) the induced current will flow in the anticlockwise direction.
- (B) the induced current will flow in the clockwise direction.
- (C) no induced current will flow in the solenoid.
- (D) the magnet will fall with a constant velocity.

Correct Answer: (A) the induced current will flow in the anticlockwise direction.

Solution:

Concept:

- Lenz's law states that the induced current produces a magnetic field opposing the change in magnetic flux which causes it.
- By the right-hand grip rule, an anticlockwise current viewed from a solenoid's end makes that end a north pole; a clockwise current makes it a south pole.

Step 1: Identify the pole induced at the upper end As the magnet is dropped towards the solenoid, its north pole approaches the upper end. The downward magnetic flux through the solenoid increases. To oppose this increase, the induced magnetic field is upward, so the upper end must behave as a north pole and repel the approaching north pole.

Step 2: Apply the specified viewing direction The observer looks down at this upper end. To make the upper end a north pole, the current must circulate anticlockwise as seen by that observer. Clockwise current would make the upper face south and assist the increasing flux, so (B) fails. The changing flux in a closed solenoid produces current, excluding (C). Magnetic opposition does not require a constant falling velocity, excluding (D). This direction refers to the approaching or entering stage described; if the magnet later passes through and moves away, the flux change reverses and so does the induced current.

Step 3: Conclusion As the north pole approaches the solenoid, the induced current is anticlockwise when viewed from above. Hence, the correct option is (A).

Quick Tip to Solve: First make the near face repel the approaching pole, then use the end-on rule: anticlockwise current makes that face north.

11. An electromagnetic wave is propagating along x -axis. At any instant, the phase difference (in radian) between the electric field ($\vec{E}$) and the magnetic field ($\vec{B}$) associated with the wave is: [1 mark]

- (A) zero
- (B) $\frac{\pi}{4}$
- (C) $\frac{\pi}{2}$
- (D) π

Correct Answer: (A) zero

Solution:

Concept:

- In a travelling electromagnetic wave, the electric field $\vec{E}$ and magnetic field $\vec{B}$ oscillate in phase: their maxima, minima and zeros occur together at a given position.
- The two field vectors are perpendicular to each other and to the direction of propagation. This geometrical perpendicularity does not represent a time-phase difference.

Step 1: Compare the oscillating field expressions For propagation along the positive x -axis, choose the electric and magnetic fields along y and z respectively. They may be written as

$$\begin{aligned}\vec{E}(x, t) &= E_0 \cos(kx - \omega t) \hat{y}, \\ \vec{B}(x, t) &= B_0 \cos(kx - \omega t) \hat{z}.\end{aligned}$$

Here k is the wave number and ω the angular frequency. Both expressions contain the same phase argument.

Step 2: Distinguish phase from direction Subtracting the two phase arguments gives zero. Option (C), $\pi/2$, confuses the right angle between the field directions with a phase lag. Options (B) and (D) would imply that the oscillations reach their positive maxima at different times, contrary to the in-phase relation of the two fields.

Step 3: Conclusion The phase difference between the electric and magnetic fields is 0 rad. Hence, the correct option is (A).

Quick Tip to Solve: Keep two facts separate: the electric and magnetic vectors are perpendicular in space, but their oscillations are in phase in time.

12. In Bohr model of hydrogen atom, the electron makes a transition from $n = 5$ to $n = 1$ state. As a result, a photon of wavelength λ is emitted in the process. The wavelength of the photon emitted when an electron makes a transition from energy level $n = 5$ to $n = 2$ will be: [1 mark]

- (A) $\frac{8}{7}\lambda$
(B) $\frac{24}{7}\lambda$
(C) $\frac{16}{7}\lambda$
(D) $\frac{32}{7}\lambda$

Correct Answer: (D) $\frac{32}{7}\lambda$

Solution:

Concept:

- The hydrogen spectrum obeys the Rydberg formula $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where n_i and n_f are the initial and final principal quantum numbers and R_H is the Rydberg constant.
- Photon energy is $E = hc/\lambda$. A smaller transition energy therefore produces a longer wavelength, so wavelength ratios are the inverse of energy ratios.

Step 1: Express the wavelength of the transition to the ground state For the printed transition from $n_i = 5$ to $n_f = 1$, the emitted wavelength is λ . Hence

$$\frac{1}{\lambda} = R_H \left(1 - \frac{1}{25} \right) = \frac{24R_H}{25}.$$

Step 2: Compare the transition ending at the second level Let its wavelength be λ' . For $5 \rightarrow 2$,

$$\frac{1}{\lambda'} = R_H \left(\frac{1}{4} - \frac{1}{25} \right) = \frac{21R_H}{100}.$$

Dividing the reciprocal wavelengths in the correct order gives

$$\frac{\lambda'}{\lambda} = \frac{24/25}{21/100} = \frac{96}{21} = \frac{32}{7}.$$

The factor exceeds unity because the transition to $n = 2$ releases less energy. Options (A), (B) and (C) do not equal this ratio; using unsquared level numbers or reversing the ratio gives an incorrect result.

Step 3: Conclusion The wavelength for the transition $5 \rightarrow 2$ is $\lambda' = \frac{32}{7}\lambda$. Hence, the correct option is (D).

Quick Tip to Solve: Calculate both Rydberg brackets and divide the first by the second for the wavelength ratio; the common Rydberg constant cancels.

13. Assertion (A): On increasing the intensity of incident light of frequency $\nu (> \nu_0)$ on a photosensitive surface, the photocurrent increases.

Reason (R): The stopping potential for a photosensitive surface increases with increase of frequency $\nu (> \nu_0)$ of incident light. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

Solution:

Concept:

- Above the threshold frequency ν_0 , increased light intensity at fixed frequency means more photons strike the surface per second and more photoelectrons are emitted per second.
- Einstein's photoelectric equation is $eV_s = h\nu - \phi_0 = h(\nu - \nu_0)$, where V_s is the stopping potential, e the elementary charge and ϕ_0 the work function.

Step 1: Evaluate the assertion At fixed $\nu > \nu_0$, each photon has sufficient energy to eject an electron. Increasing intensity increases the photon arrival rate, hence the photoelectron emission rate. Under the same collecting conditions this increases the photocurrent. Assertion (A) is true.

Step 2: Evaluate the reason For the same photosensitive surface, ϕ_0 remains fixed. Therefore

$$V_s = \frac{h}{e}(\nu - \nu_0)$$

increases with frequency ν . The reason correctly states this frequency dependence, so Reason (R) is also true.

Step 3: Test whether the reason explains the assertion The assertion concerns the number of emitted electrons and hence the current; the reason concerns their maximum kinetic energy and hence the stopping potential. Increased intensity at fixed frequency changes the number of photons, not their individual energy. Thus the reason does not explain why the photocurrent increases. Option (A) incorrectly asserts an explanatory link; options (C) and (D) incorrectly declare one or both statements false.

Step 4: Conclusion Both the assertion and the reason are true, but the reason does not explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: For photoelectric questions, associate intensity with electron count and photocurrent, and frequency with maximum kinetic energy and stopping potential.

14. Assertion (A): On forward biasing a p-n junction diode, the height of the barrier potential increases.

Reason (R): In forward biasing of a p-n junction diode, the direction of the applied voltage is in the same direction as the built-in potential. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

Concept:

- The depletion region of a p-n junction contains fixed positive ions on the n-side and fixed negative ions on the p-side. Its built-in electric field points from n to p.
- Forward bias connects the p-side to the positive terminal and the n-side to the negative terminal. The applied field opposes the built-in field, lowering the effective junction barrier.

Step 1: Evaluate the assertion If V_{bi} is the equilibrium barrier potential and V_F the applied forward voltage, the effective barrier is reduced approximately to

$$V_{\text{barrier}} = V_{bi} - V_F.$$

The depletion layer narrows and majority carriers cross the junction more easily. Therefore the statement that the barrier height increases is false.

Step 2: Evaluate the reason The applied forward-bias field points from the positively connected p-side towards the negatively connected n-side. This is opposite to the built-in field, which points from n to p. Equivalently, the applied voltage opposes the built-in barrier potential. Thus the reason's claim that the two act in the same direction is also false.

Step 3: Check the explanatory relation and code Neither statement describes forward bias correctly, so the reason cannot explain the assertion. The barrier would increase under reverse bias, when the applied and built-in fields reinforce each other. Options (A) and (B) require both statements to be true, and option (C) requires a true assertion; all three therefore fail.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: Remember the forward connection, positive to p and negative to n: its field opposes the junction field and lowers the barrier.

15. Assertion (A): Light added to light can produce darkness.

Reason (R): When two coherent light waves interfere, there is darkness at position of destructive interference. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Concept:

- The superposition principle adds the electric-field displacements of the two light waves; their intensities do not simply add at an interference minimum.
- For coherent waves, $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$, where ϕ is their phase difference and I_1, I_2 are the individual intensities.

Step 1: Evaluate the assertion Two coherent light waves of equal amplitude can arrive in opposite phase at a point. Their electric fields then cancel, so the resultant intensity is zero. Thus adding light to light can produce local darkness, and Assertion (A) is true.

Step 2: Evaluate the reason Destructive interference occurs for $\phi = (2n + 1)\pi$, with integer n . For equal individual intensities $I_1 = I_2 = I_0$,

$$I_{\min} = 2I_0 - 2I_0 = 0.$$

This gives the dark positions of the interference pattern, so Reason (R) is true. Equal amplitudes give complete darkness; unequal amplitudes give a nonzero minimum.

Step 3: Check the explanatory link The destructive interference named in R is precisely the mechanism that produces the darkness in A. Option (B) wrongly denies this link; (C) and (D) wrongly reject the true statements. Energy is redistributed to brighter regions, so local darkness does not violate energy conservation.

Step 4: Conclusion Both statements are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: At a dark fringe, add the fields with their relative phase; equal opposite fields cancel even though both sources emit light.

16. Assertion (A): Two electric heaters of power P_1 and P_2 ($> P_1$) are joined in series across a dc source of voltage V . The power consumed by the combination will be less than that consumed by P_1 when connected across the same source.

Reason (R): The power consumed by an electric device when connected to a source of voltage V is proportional to its resistance. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- For a resistive heater, the power is $P = VI = I^2R = V^2/R$, where V is its potential difference, I its current and R its resistance.
- Resistances add in series: $R_s = R_1 + R_2$. At a fixed supply voltage, increasing the total resistance reduces the total power.

Step 1: Evaluate the assertion Let the two heater resistances be R_1 and R_2 . The first heater alone consumes V^2/R_1 across the stated source, whereas the series combination consumes

$$P_s = \frac{V^2}{R_1 + R_2} < \frac{V^2}{R_1}.$$

Both resistances are positive, so Assertion (A) is true. If P_1, P_2 denote the individual powers at voltage V , this also gives $P_s = P_1 P_2 / (P_1 + P_2) < P_1$.

Step 2: Evaluate the reason With the device connected to a fixed voltage V , its power is V^2/R , which is inversely proportional to its resistance. Thus Reason (R), claiming direct proportionality, is false. The expression I^2R implies direct proportionality only when the current is held fixed.

Step 3: Check the explanatory link The false reason cannot explain the true assertion. Options (A) and (B) incorrectly treat R as true, and option (D) incorrectly treats A as false. The reduction in series power follows from the increased total resistance at fixed voltage.

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: First identify what the source fixes: use V^2/R for fixed voltage and I^2R for fixed current before comparing powers.

Section B (10 marks)

17. Write two points of difference between intrinsic and extrinsic semiconductors.

[2 marks]

Solution:

Concept:

- An intrinsic semiconductor is a pure semiconductor whose mobile carriers are produced thermally as electron-hole pairs.
- An extrinsic semiconductor is formed by controlled doping, which changes the relative electron and hole concentrations. Let n be the free-electron concentration and p the hole concentration.

Step 1: Compare composition **Intrinsic:** The semiconductor is pure, with no deliberately added impurity; pure silicon or germanium is an example.

Extrinsic: A small controlled amount of a suitable impurity is added to the pure semiconductor. Pentavalent donor atoms produce an n-type material, whereas trivalent acceptor atoms produce a p-type material. Thus the first distinction is the absence or presence of intentional doping.

Step 2: Compare charge-carrier concentrations **Intrinsic:** Each thermally generated free electron leaves a hole, so their concentrations are equal: $n = p = n_i$, where n_i is the intrinsic carrier concentration.

Extrinsic: Under the usual doping-dominated conditions, the concentrations are unequal. In n-type material, $n > p$, so electrons are majority carriers and holes are minority carriers. In p-type material, $p > n$, so holes are majority carriers and electrons are minority carriers. The solid remains electrically neutral despite unequal mobile-carrier concentrations.

Step 3: Conclusion The two differences are: intrinsic material is pure and has $n = p$; extrinsic material is intentionally doped and has majority and minority carriers, with $n > p$ for n-type or $p > n$ for p-type.

Quick Tip to Solve: Give two matched comparisons: purity versus doping, then equal carrier concentrations versus majority and minority carriers.

18. Find ratio (λ_a/λ_p) of the de Broglie wavelength λ_a and λ_p associated respectively with an alpha particle and a proton,

(i) if they are moving with the same kinetic energy.

(ii) just after they are accelerated through the same potential difference.

[2 marks]

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$. For nonrelativistic kinetic energy $K = p^2/(2m)$, this becomes $\lambda = h/\sqrt{2mK}$, where h is Planck's constant and m is the particle mass.
- A particle of charge magnitude q , accelerated from rest through a potential difference V , gains kinetic energy $K = qV$. For an alpha particle, $m_\alpha \simeq 4m_p$ and $q_\alpha = 2e$, while $q_p = e$.

Step 1: Compare wavelengths at equal kinetic energy For part (i), $K_\alpha = K_p = K$. Cancel the

common factors h and $2K$ in the wavelength ratio:

$$\frac{\lambda_a}{\lambda_p} = \sqrt{\frac{m_p K}{m_\alpha K}} = \sqrt{\frac{m_p}{4m_p}} = \frac{1}{2}.$$

The heavier alpha particle has twice the proton's momentum at the same kinetic energy, so its wavelength is half as large.

Step 2: Compare wavelengths after equal accelerating voltage For part (ii), take the acceleration from rest as usual. The particles gain different kinetic energies because their charges differ: $K_\alpha = 2eV$ and $K_p = eV$. Therefore,

$$\frac{\lambda_a}{\lambda_p} = \sqrt{\frac{m_p q_p V}{m_\alpha q_\alpha V}} = \sqrt{\frac{m_p e}{(4m_p)(2e)}} = \frac{1}{\sqrt{8}}.$$

Here both the greater mass and the greater gained energy increase the alpha particle's momentum. Equal accelerating voltage therefore does not mean equal kinetic energy.

Step 3: Conclusion For equal kinetic energy, the ratio is $\lambda_a/\lambda_p = 1/2$. For equal accelerating potential difference, the ratio is $\lambda_a/\lambda_p = 1/(2\sqrt{2})$.

Quick Tip to Solve: At equal kinetic energy compare $\sqrt{m}$; at equal accelerating voltage compare $\sqrt{mq}$, remembering that an alpha particle has charge $2e$.

19. A ray of light in air is incident at angle $\angle i$ on a face of an equilateral glass prism and is refracted through the prism. As $\angle i$ is varied, it is observed that the ray undergoes minimum deviation, when $\angle i$ is three-fourth of the angle of the prism. Calculate the speed of light in the prism. [2 marks]

Solution:

Concept:

- At minimum deviation through a prism in air, the ray path is symmetric: $i = e$ and $r_1 = r_2 = A/2$, where A is the prism angle and r_1, r_2 are the internal refraction angles.
- Snell's law at the first face gives $\mu = \sin i / \sin r_1$. The speed of light in glass is $v = c/\mu$, taking the refractive index of air as unity and $c = 3 \times 10^8 \text{ m s}^{-1}$.

Step 1: Determine the angles at minimum deviation The prism is equilateral, so its refracting angle is $A = 60^\circ$. The specified angle of incidence at minimum deviation is three-fourths of this angle:

$$i = \frac{3}{4}A = \frac{3}{4} \times 60^\circ = 45^\circ.$$

By symmetry of the minimum-deviation path, the angle of refraction at the first face is $r_1 = A/2 = 30^\circ$. These angles are measured from the normals to the corresponding faces.

Step 2: Calculate the refractive index Applying Snell's law from air to glass gives

$$\mu = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}.$$

The index is greater than unity, consistent with light travelling more slowly in glass than in air. Equivalently, the minimum deviation is $2i - A = 30^\circ$.

Step 3: Convert refractive index to speed Divide the vacuum speed by the glass refractive index:

$$v = \frac{c}{\mu} = \frac{3 \times 10^8}{\sqrt{2}} \simeq 2.12 \times 10^8 \text{ m s}^{-1}.$$

The inverse dependence on μ is essential; multiplying by μ would incorrectly predict a speed exceeding c .

Step 4: Conclusion The speed of light in the prism is $v = 2.12 \times 10^8 \text{ m s}^{-1}$.

Quick Tip to Solve: At minimum deviation, immediately use $r = A/2$; here $i = 3A/4$, so Snell's law gives the index without first finding the deviation.

20. A heating element using nichrome is connected to a 220 V supply. Initially it draws a current of 2.9 A. After some time, the current attains a steady value of 2.5 A. Find the steady temperature of the heating element if the room temperature is 27°C . The temperature coefficient of resistance of nichrome is $1.7 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$. [2 marks]

Solution:

Concept:

- At a fixed potential difference V , Ohm's law gives $R = V/I$, where I is the current through the element. A decrease in current therefore indicates an increase in resistance.
- The linear temperature relation is $R_T = R_{27}[1 + \alpha(T - 27)]$, where R_{27} is the resistance at room temperature, T is the steady temperature in degrees Celsius and α is the temperature coefficient of resistance.

Step 1: Compare the initial and steady resistances Initially the element is at room temperature, 27°C . The same supply voltage acts at both temperatures, so

$$\begin{aligned} R_{27} &= \frac{220}{2.9} = 75.86 \, \Omega, \\ R_T &= \frac{220}{2.5} = 88.0 \, \Omega, \\ \frac{R_T}{R_{27}} &= \frac{2.9}{2.5} = 1.16. \end{aligned}$$

Using the current ratio avoids rounding either resistance before calculating the temperature rise.

Step 2: Calculate the temperature rise Substitute the resistance ratio and the given coefficient into the temperature relation:

$$\begin{aligned} 1.16 &= 1 + (1.7 \times 10^{-4})(T - 27), \\ T - 27 &= \frac{0.16}{1.7 \times 10^{-4}} = 941.18 \, ^\circ\text{C}, \\ T &= 27 + 941.18 = 968.18 \, ^\circ\text{C}. \end{aligned}$$

The positive temperature rise is consistent with nichrome's positive temperature coefficient and the observed reduction in current.

Step 3: Conclusion The steady temperature of the nichrome element is approximately 968°C .

Quick Tip to Solve: For a constant-voltage heater, use $R_T/R_{27} = I_{27}/I_T$; then add room temperature to the calculated temperature rise.

21(a). A 5 cm long pencil is placed along the principal axis of a concave mirror of focal length 20 cm such that its nearest end is at a distance of 25 cm from the mirror. Calculate the length of the image of the pencil. [2 marks]

Solution:

Concept:

- For a spherical mirror, $1/f = 1/v + 1/u$, where f , u and v are the focal length, object distance and image distance with Cartesian signs.
- A pencil lying along the principal axis has its ends at different object distances. Find the image of each end separately; their separation is the image length.

Step 1: Locate the image of the nearer end Take the pole as the origin and the direction of incident light as positive. For the concave mirror, $f = -20$ cm. The nearer end is at $u_1 = -25$ cm, so

$$\frac{1}{v_1} = \frac{1}{f} - \frac{1}{u_1} = -\frac{1}{20} + \frac{1}{25} = -\frac{1}{100},$$
$$v_1 = -100 \text{ cm.}$$

Thus the image of this end is real and lies in front of the mirror.

Step 2: Locate the other image endpoint and subtract The other end is $25 + 5 = 30$ cm from the pole, giving $u_2 = -30$ cm. Applying the mirror formula again,

$$\frac{1}{v_2} = -\frac{1}{20} + \frac{1}{30} = -\frac{1}{60},$$
$$v_2 = -60 \text{ cm,}$$
$$L' = |v_1 - v_2| = |-100 - (-60)| = 40 \text{ cm.}$$

Both image endpoints are on the same side of the mirror, so their distances are subtracted. A single transverse magnification cannot be applied to a pencil extending along the axis.

Step 3: Conclusion The length of the image of the pencil is 40 cm, extending from 60 cm to 100 cm in front of the mirror.

Quick Tip to Solve: For an axial object, apply the mirror formula to both ends and take the absolute difference of the two image distances.

21(b). In a Young's double-slit experiment, a beam of light consisting of two wavelengths 500 nm and 600 nm is used. The interference fringes are observed at a screen placed 1.8 m away from the plane of slits (slit separation 0.3 mm). Calculate the least distance from the central maximum where the bright fringes due to both the wavelengths coincide. [2 marks]

Solution:

Concept:

- In Young's double-slit experiment, the position of the m th bright fringe is $y_m = m\lambda D/d$, where λ is the wavelength, D the screen distance and d the slit separation.
- Bright fringes for two wavelengths coincide when their path differences agree: $m\lambda_1 = n\lambda_2$. The nearest noncentral coincidence uses the smallest positive integer orders m, n .

Step 1: Find the smallest coincident fringe orders Let m be the order for 500 nm and n the order for 600 nm. Equating their fringe positions gives

$$m(500) = n(600) \Rightarrow 5m = 6n.$$

The smallest positive integers satisfying this equation are $m = 6$ and $n = 5$. Thus the sixth bright fringe of the shorter wavelength coincides with the fifth bright fringe of the longer wavelength. The order-zero coincidence is the central maximum itself and is excluded when finding the first coincidence away from it.

Step 2: Calculate the common fringe position Convert all lengths to metres: $\lambda_1 = 500 \times 10^{-9}$ m, $d = 0.3 \times 10^{-3}$ m and $D = 1.8$ m. Then

$$\begin{aligned} y &= \frac{6(500 \times 10^{-9})(1.8)}{0.3 \times 10^{-3}} \\ &= 1.8 \times 10^{-2} \text{ m} = 1.8 \text{ cm}. \end{aligned}$$

Using $n = 5$ and $\lambda_2 = 600$ nm gives the same position, since both products $m\lambda_1$ and $n\lambda_2$ equal 3000 nm.

Step 3: Conclusion The least nonzero distance from the central maximum at which the bright fringes coincide is 1.8 cm, on either side of the centre.

Quick Tip to Solve: Reduce the wavelength ratio 500 : 600 to 5 : 6; the first coincident fringe orders are the reversed ratio, 6 : 5.

Section C (21 marks)

22. (a) Consider the following nuclides: ${}^{12}_6\text{C}$, ${}^{198}_{80}\text{Hg}$, ${}^{14}_6\text{C}$, ${}^{197}_{79}\text{Au}$. Group them into isotopes and isotones.
 (b) How does the size of a nucleus depend on its mass number A ? Hence prove that the density of nucleus is a constant, independent of A , for all nuclei. [3 marks]

Solution:

Concept:

- A nuclide A_ZX has atomic number Z , mass number A and neutron number $N = A - Z$. Isotopes have the same Z but different A ; isotones have the same N but different Z .
- The nuclear radius follows $R = R_0 A^{1/3}$, where R_0 is approximately 1.2×10^{-15} m. Nuclear mass is approximately $M = Am_N$, where m_N is the mean nucleon mass.
- Treating the nucleus as spherical, its volume is $V = 4\pi R^3/3$, and its average density is $\rho = M/V$.

Step 1: Classify the nuclides in part (a) Calculate the neutron numbers to distinguish equality of atomic number from equality of neutron number:

Nuclide	Z	A	$N = A - Z$
${}^{12}_6\text{C}$	6	12	6
${}^{198}_{80}\text{Hg}$	80	198	118
${}^{14}_6\text{C}$	6	14	8
${}^{197}_{79}\text{Au}$	79	197	118

The two carbon nuclides have the same atomic number, $Z = 6$, but different mass numbers, so they are isotopes. Mercury-198 and gold-197 have equal neutron numbers, $N = 118$, and different atomic numbers, so they are isotones.

Step 2: Express the nuclear size and volume in part (b) The empirical radius law gives the required dependence of nuclear size on mass number:

$$R = R_0 A^{1/3} \Rightarrow R \propto A^{1/3}.$$

Hence the volume is

$$V = \frac{4}{3}\pi(R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A.$$

Thus nuclear volume increases directly in proportion to the number of nucleons, even though radius grows only as its cube root.

Step 3: Derive the nuclear density Using $M \simeq Am_N$, divide mass by volume:

$$\rho = \frac{Am_N}{(4/3)\pi R_0^3 A} = \frac{3m_N}{4\pi R_0^3}.$$

The factor A cancels. Since m_N and R_0 are approximately the same for all nuclei, the density is independent of mass number within this standard nuclear model. Small mass differences and binding-energy corrections are neglected in $M \simeq Am_N$.

Step 4: Conclusion The isotopes are ${}^{12}_6\text{C}$ and ${}^{14}_6\text{C}$; the isotones are ${}^{198}_{80}\text{Hg}$ and ${}^{197}_{79}\text{Au}$. Nuclear radius varies as $A^{1/3}$, giving the approximately constant density $\rho = 3m_N/(4\pi R_0^3)$, independent of A .

Quick Tip to Solve: Compare Z for isotopes and $A - Z$ for isotones. For density, cube the radius law first: both nuclear mass and volume then contain the same factor A .

23. (a) An electric field $\vec{E} = E_0\hat{i}$ exists in a region of space. Draw three equipotential surfaces in the region.

(b) Two point charges $-q$ and $+q$ are located at points $(-a, 0, 0)$ and $(a, 0, 0)$ respectively. Find the electrostatic potential at point $(x, 0, 0)$ where $x \gg a$. [3 marks]

Solution:

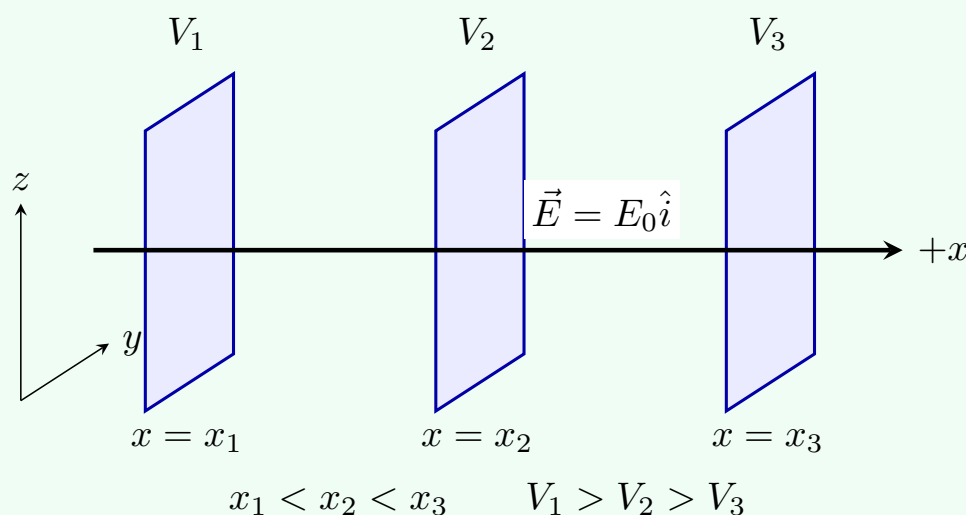
Concept:

- An equipotential surface has the same potential at every point. Since $dV = -\vec{E} \cdot d\vec{l}$, the electric field is normal to such a surface.
- The potential of a point charge Q , with zero at infinity, is $V = Q/(4\pi\epsilon_0 d)$, where d is the distance from the charge. Potentials add algebraically.
- A pair of charges $-q, +q$ separated by $2a$ has dipole moment $\vec{p} = 2qa\hat{i}$, directed from the negative charge to the positive charge.

Step 1: (a) Determine and draw the equipotential planes The given field is uniform and points along the positive x -axis. Thus

$$\frac{dV}{dx} = -E_0, \quad V(x) = V_0 - E_0x.$$

There is no dependence on y or z . Consequently, each plane $x = \text{constant}$, parallel to the yz -plane, is an equipotential surface. Draw any three parallel planes perpendicular to the field, as shown. If $x_1 < x_2 < x_3$ and $E_0 > 0$, their potentials obey $V_1 > V_2 > V_3$. Equal plane spacing corresponds to equal potential differences in this uniform field.



Three equipotential planes perpendicular to the uniform field; the shaded parallelograms represent portions of infinite planes.

Step 2: (b) Add the potentials of the two charges The point $P(x, 0, 0)$ lies on the positive axial side, with $x \gg a$. Its distances from $+q$ at $x = a$ and $-q$ at $x = -a$ are $x - a$ and $x + a$, respectively.

Keeping the signs of the charges,

$$\begin{aligned} V(P) &= \frac{1}{4\pi\epsilon_0} \left(\frac{q}{x-a} - \frac{q}{x+a} \right) \\ &= \frac{1}{4\pi\epsilon_0} \frac{2qa}{x^2 - a^2}. \end{aligned}$$

This is positive because the observation point is closer to the positive charge.

Step 3: Apply the distant-point approximation Factor the denominator as $x^2[1 - (a/x)^2]$. Since $a/x \ll 1$, neglect $(a/x)^2$ compared with unity. The leading potential therefore varies as $1/x^2$; the $1/x$ contributions of the two equal and opposite charges cancel. The dipole moment magnitude is $p = 2qa$.

Step 4: Conclusion (a) Three parallel yz -planes, $x = x_1, x_2, x_3$, are equipotential surfaces. (b) With zero potential at infinity,

$$V(x) \simeq \frac{1}{4\pi\epsilon_0} \frac{2qa}{x^2} = \frac{p}{4\pi\epsilon_0 x^2}, \quad x \gg a.$$

Quick Tip to Solve: Draw equipotential surfaces perpendicular to the field; for the dipole, subtract the two signed potentials before using the far-distance approximation.

24. Name the electromagnetic waves which are used for:

- (i) detection of fractures in bones
- (ii) physiotherapy
- (iii) radar systems

Also write their wavelength range.

[3 marks]

Solution:

Concept:

- The electromagnetic spectrum is divided into regions according to wavelength and frequency, related in vacuum by $c = \nu\lambda$. Here c is the speed of light, ν is frequency and λ is wavelength.
- The use of a radiation band depends on its interaction with matter: penetration and absorption contrast for imaging, heating for physiotherapy, and reflection with directional transmission for radar.

Step 1: (i) Identify the radiation for bone-fracture detection **X-rays** are used in radiography to detect fractures. Bones absorb X-rays more strongly than the surrounding soft tissues, producing contrast in the recorded image. A break in the normal bone structure can therefore be identified. The approximate X-ray wavelength range is

$$10^{-13} \text{ m to } 10^{-8} \text{ m.}$$

These short wavelengths correspond to high-frequency electromagnetic radiation.

Step 2: (ii) Identify the radiation used in physiotherapy **Infrared waves** are used for heat treatment in physiotherapy. Their absorption produces warming of the treated region; they are therefore often called heat waves. The approximate infrared wavelength range is

$$7 \times 10^{-7} \text{ m to } 10^{-3} \text{ m,}$$

that is, about 700 nm to 1 mm, extending from just beyond the red end of visible light.

Step 3: (iii) Identify the radiation used in radar systems Microwaves are used in radar. They can be transmitted as a directional beam and reflected by distant objects such as aircraft. The delay between the transmitted pulse and its returning echo is used to determine the object's distance. The approximate microwave wavelength range is

$$10^{-3} \text{ m to } 10^{-1} \text{ m,}$$

or 1 mm to 0.1 m. These are conventional spectrum ranges; neighbouring bands do not have perfectly sharp physical boundaries.

Step 4: Conclusion (i) Bone-fracture detection: **X-rays**, approximately 10^{-13} to 10^{-8} m. (ii) Physiotherapy: **infrared**, approximately 7×10^{-7} to 10^{-3} m. (iii) Radar: **microwaves**, approximately 10^{-3} to 10^{-1} m.

Quick Tip to Solve: Link each use to its physical effect: X-rays give bone contrast, infrared gives heat, and microwaves give radar echoes; then write all wavelength limits in metres.

25(a). (i) Define mutual inductance of a pair of coils. Write its SI unit.

(ii) A long solenoid of radius R and length L has n turns per unit length. A circular loop of radius r ($< R$) is placed inside at the centre of the solenoid such that its axis coincides with the axis of the solenoid. Obtain the mutual inductance of the solenoid and the loop. [3 marks]

Solution:

Concept:

- Mutual inductance M relates the magnetic flux linkage of one coil to the current in the other: $N_2\Phi_{21} = MI_1$. Here N_2 is the number of turns in the receiving coil and Φ_{21} is the flux through each turn due to current I_1 .
- For a long air-core solenoid carrying current I , the field near its centre is uniform and axial: $B = \mu_0 nI$, where n is the number of turns per unit length and μ_0 is the permeability of free space.
- Magnetic flux through a plane loop of area A in a uniform field is $\Phi = BA \cos \theta$, where θ is the angle between the field and the loop's area vector.

Step 1: (i) Define mutual inductance and its unit The mutual inductance of a pair of coils is the flux linkage with one coil per unit current in the other coil, for fixed geometry and a linear magnetic medium:

$$M = \frac{N_2\Phi_{21}}{I_1}.$$

Equivalently, it is the magnitude of the induced emf in one coil when the current in the other changes at unit rate, since $\varepsilon_2 = -M dI_1/dt$ for consistent reference orientations. Its SI unit is the **henry**, symbol H:

$$1 \text{ H} = 1 \text{ Wb A}^{-1} = 1 \text{ V s A}^{-1}.$$

Step 2: (ii) Calculate the flux through the circular loop Let the solenoid carry current I . Since the loop is at the centre of a long solenoid, end effects are negligible and the magnetic field at the loop is $B = \mu_0 nI$. The loop's axis coincides with the solenoid axis, so its area vector is parallel to the field and $\cos \theta = 1$. The entire loop lies inside the solenoid because $r < R$. The area pierced by the field for

this single-turn loop is therefore πr^2 , giving

$$\Phi = B\pi r^2 = \mu_0 n I \pi r^2.$$

The larger area πR^2 must not be used: flux outside the boundary of the smaller loop does not link it.

Step 3: Relate the linked flux to the current For the single circular loop, the flux linkage is just Φ . Substitution into $M = \Phi/I$ cancels the current. If the solenoid has N turns over its length L , then $n = N/L$, so the same expression can also be written using N/L . The result is independent of R as long as the loop stays completely within the uniform-field region.

Step 4: Conclusion Mutual inductance is the flux linkage of one coil per unit current in the other; its SI unit is the **henry**. The mutual inductance of the long solenoid and the coaxial single loop is

$$M = \mu_0 n \pi r^2 = \frac{\mu_0 N \pi r^2}{L}.$$

Quick Tip to Solve: Use the field from the long solenoid and the area of the smaller receiving loop; because n already means turns per unit length, do not multiply by L again.

OR

25(b). Two long straight parallel conductors A and B carrying steady currents I_a and I_b in the same direction are separated by a distance d . Deduce the expressions for the force acting on length L of conductor B due to conductor A and show it in figure. Write the expression for the force acting on length L of conductor A due to conductor B and show that it follows Newton's third law. [3 marks]

Solution:

Concept:

- A long straight conductor carrying current I produces a magnetic field of magnitude $B = \mu_0 I / (2\pi r)$ at perpendicular distance r , where μ_0 is the permeability of free space.
- A straight length L carrying current in a magnetic field experiences $\vec{F} = I \vec{L} \times \vec{B}$, with $\vec{L}$ along the conventional current. Thus $F = ILB$ when the current is perpendicular to the field.

Step 1: Find the magnetic field of conductor A at B Take both currents vertically upwards, with conductor A to the left of conductor B . Their perpendicular separation is d . By the right-hand thumb rule, the magnetic field produced by A at B is directed into the plane of the paper. Its magnitude is

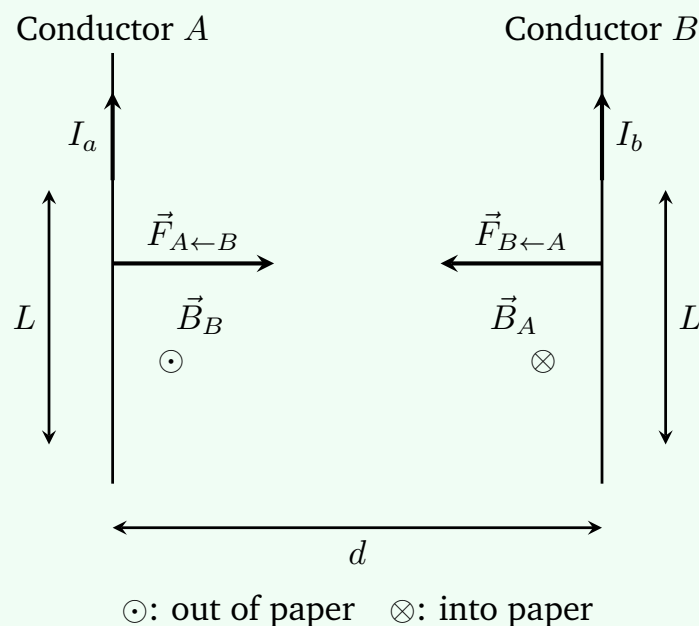
$$B_A = \frac{\mu_0 I_a}{2\pi d}.$$

This field is perpendicular to the current in B , and is approximately uniform over the chosen length L for sufficiently long conductors.

Step 2: Calculate the force on B due to A The force on length L of conductor B is

$$F_{B \leftarrow A} = I_b L B_A = \frac{\mu_0 I_a I_b L}{2\pi d}.$$

The cross product of the upward current direction and the field into the paper points to the left. Hence the force on B is towards A : currents flowing in the same direction attract.



Equal lengths of parallel conductors attract when their currents have the same direction.

Step 3: Calculate the reciprocal force and compare vectors At conductor A, the field produced by B has magnitude $B_B = \mu_0 I_b / (2\pi d)$ and points out of the paper. The upward current in A therefore experiences a force to the right, towards B. On the same length L,

$$F_{A \leftarrow B} = I_a L B_B = \frac{\mu_0 I_a I_b L}{2\pi d}.$$

If $\hat{x}$ points from A to B, the two forces are

$$\begin{aligned}\vec{F}_{B \leftarrow A} &= -\frac{\mu_0 I_a I_b L}{2\pi d} \hat{x}, \\ \vec{F}_{A \leftarrow B} &= +\frac{\mu_0 I_a I_b L}{2\pi d} \hat{x}.\end{aligned}$$

They act on different conductors, with equal magnitudes and opposite directions, as required by Newton's third law.

Step 4: Conclusion Each length L experiences an attractive force of magnitude $\mu_0 I_a I_b L / (2\pi d)$. Thus $\vec{F}_{A \leftarrow B} = -\vec{F}_{B \leftarrow A}$, consistent with Newton's third law.

Quick Tip to Solve: First find the field of the other wire; then use $I\vec{L} \times \vec{B}$. Parallel currents attract, so each force arrow must point towards the other wire.

26. (a) State Bohr's second postulate and mention its significance.

(b) Prove that, in Bohr model of hydrogen atom as principal quantum number n becomes large, the energy levels get closer and closer. [3 marks]

Solution:

Concept:

- Bohr's quantisation postulate permits only those stationary orbits for which the electron's angular momentum is an integral multiple of $\hbar = h/(2\pi)$, where h is Planck's constant.
- For hydrogen, the energy of the orbit with principal quantum number n is $E_n = -13.6/n^2$ eV, with zero energy assigned to the separated electron and proton.

Step 1: State Bohr's second postulate and its significance An electron can revolve around the nucleus only in an orbit in which its angular momentum is

$$L = m_e v_n r_n = n \frac{h}{2\pi}, \quad n = 1, 2, 3, \dots,$$

where m_e is the electron mass, v_n its orbital speed and r_n its orbital radius. Thus the angular momentum cannot take arbitrary continuous values. The postulate selects discrete allowed orbits and, together with the Coulomb-force equation, gives discrete radii and energies. Transitions between these energy levels account for the discrete line spectrum of hydrogen.

Step 2: Find the separation between successive energy levels The energies of two neighbouring levels are $E_n = -13.6/n^2$ eV and $E_{n+1} = -13.6/(n+1)^2$ eV. The upper level is less negative, so the positive energy separation is

$$\begin{aligned} \Delta E_n &= E_{n+1} - E_n \\ &= 13.6 \left[\frac{1}{n^2} - \frac{1}{(n+1)^2} \right] \text{ eV} \\ &= 13.6 \frac{2n+1}{n^2(n+1)^2} \text{ eV}. \end{aligned}$$

The subtraction is between neighbouring levels; the magnitude of one level's energy alone is not the level separation.

Step 3: Evaluate the large- n behaviour For $n \gg 1$, $2n+1 \simeq 2n$ and $(n+1)^2 \simeq n^2$. Hence

$$\Delta E_n \simeq \frac{27.2}{n^3} \text{ eV} \quad \longrightarrow \quad 0 \quad \text{as } n \longrightarrow \infty.$$

Meanwhile E_n approaches zero from below. Therefore many high- n bound levels lie within a small energy interval just below the ionisation limit.

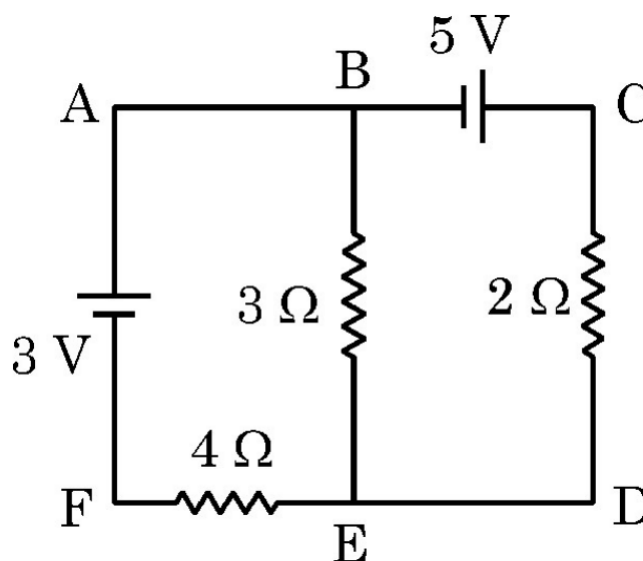
Step 4: Conclusion Bohr's second postulate is $m_e v_n r_n = nh/(2\pi)$, which quantises the allowed orbits and energies. The neighbouring-level spacing decreases as $1/n^3$ for large n , so the energy levels become closer and closer.

Quick Tip to Solve: To prove that levels crowd together, calculate $E_{n+1} - E_n$ and take large n ; do not use $E_n \rightarrow 0$ alone as the proof of their spacing.

27. (a) Explain the statement: “Current is a scalar although we represent current with an arrow”.

(b) Use Kirchhoff’s rules to find the current through $3\ \Omega$ resistor in the circuit shown in the figure:

[3 marks]



Solution:

Concept:

- Electric current is the rate of charge flow, $I = dq/dt$. It is a signed scalar; its arrow selects the reference direction of conventional charge flow. Current density $\vec{J}$, in contrast, is a vector.
- Kirchhoff’s junction rule follows from charge conservation: total current entering a junction equals total current leaving it. Kirchhoff’s loop rule follows from energy conservation: the algebraic sum of potential changes around a closed loop is zero.

Step 1: Explain why the current arrow does not make current a vector A quantity is a vector only if it obeys vector addition and transforms as a vector. Currents meeting at a junction add algebraically, independent of the angles between the wires; they do not combine by the parallelogram law. An arrow on a wire simply indicates which direction is chosen as positive. A negative calculated current means that the actual flow is opposite to this reference direction. Therefore current is a scalar despite the use of an arrow.

Step 2: Assign branch currents and apply the junction rule Let I flow upwards from E to B through the $3\ \Omega$ resistor. Take I_L along $B \rightarrow A \rightarrow F \rightarrow E$ in the left branch and I_R along $B \rightarrow C \rightarrow D \rightarrow E$ in the right branch. At junction B ,

$$I = I_L + I_R.$$

The long plate of the $3\ \text{V}$ cell is towards A ; the long plate of the $5\ \text{V}$ cell is towards C . These polarities determine the emf signs in the loop equations.

Step 3: Write the two independent loop equations Traverse the left loop $B \rightarrow A \rightarrow F \rightarrow E \rightarrow B$. Crossing the $3\ \text{V}$ cell from positive to negative gives a potential drop, and both resistors are traversed along their assumed currents:

$$-3 - 4I_L - 3I = 0 \quad \Rightarrow \quad 4I_L + 3I = -3.$$

Traverse the right loop $B \rightarrow C \rightarrow D \rightarrow E \rightarrow B$. The 5 V cell is crossed from negative to positive:

$$5 - 2I_R - 3I = 0 \Rightarrow 2I_R + 3I = 5.$$

Here resistances are in ohms, currents in amperes, and every term in the loop equations is a potential change in volts.

Step 4: Eliminate the outer-branch currents From the two loop equations,

$$I_L = \frac{-3 - 3I}{4}, \quad I_R = \frac{5 - 3I}{2}.$$

Substituting into the junction rule gives

$$\begin{aligned} I &= \frac{-3 - 3I}{4} + \frac{5 - 3I}{2}, \\ 4I &= -3 - 3I + 10 - 6I, \\ 13I &= 7. \end{aligned}$$

The solution is positive for the upward reference direction. As a consistency check, $I_L = -15/13$ A and $I_R = 22/13$ A, whose algebraic sum is $7/13$ A. The negative left-branch value reverses only that branch's assumed direction.

Step 5: Conclusion (a) Current is a signed scalar because it adds algebraically; the arrow specifies a reference direction of flow. (b) The current through the $3\ \Omega$ resistor is $I = 7/13$ A $\simeq 0.538$ A, directed upwards from E to B .

Quick Tip to Solve: Mark the long cell plate as positive before writing loops. Keep every assumed branch direction fixed; a negative result then identifies a reversed current without rewriting the circuit.

28. Derive an expression for the magnetic field $\vec{B}$, due to a circular coil of N turns, each of radius r carrying current I , at a distance x from the centre along its axis. [3 marks]

Solution:

Concept:

- Biot–Savart law gives the magnetic field of a current element: $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{R}}{R^3}$, where $\vec{R}$ points from the element to the observation point and μ_0 is the permeability of free space.
- On the axis of a circular loop, symmetry cancels the components perpendicular to the axis. The axial components add, and the fields of N identical closely wound turns add by superposition.

Step 1: Set up the loop geometry Let O be the centre of a circular turn of radius r , and let P lie on its axis at distance x from O . Every element of the loop is at the same distance

$$R = \sqrt{r^2 + x^2}$$

from P . The tangent element $d\vec{l}$ is perpendicular to both the radial direction in the loop and the axis, so it is perpendicular to $\vec{R}$. Thus the sine factor in Biot–Savart law is unity and

$$dB = \frac{\mu_0 I dl}{4\pi(r^2 + x^2)}.$$

Step 2: Resolve the field along the axis For two diametrically opposite elements, the transverse components of $d\vec{B}$ are equal and opposite, while the axial components have the same direction. If θ is the angle between $\vec{R}$ and the axis, then $\sin \theta = r/R$. Because $d\vec{B}$ is perpendicular to $\vec{R}$, its component along the axis is

$$dB_{\text{axis}} = dB \sin \theta = \frac{\mu_0 I r dl}{4\pi(r^2 + x^2)^{3/2}}.$$

Step 3: Integrate around each turn and add the turns The quantities I , r and x are constant for the integration. Using the circumference $\oint dl = 2\pi r$, the field of one turn is

$$B_1 = \frac{\mu_0 I r}{4\pi(r^2 + x^2)^{3/2}} \oint dl = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}.$$

All N closely wound turns carry current in the same sense, so their axial fields add: $B = NB_1$. Choose $\hat{n}$ along the axial direction given by the right-hand grip rule, with curled fingers following the current. The field has this direction. At the centre, substituting $x = 0$ recovers $\mu_0 NI/(2r)$, which checks the result.

Step 4: Conclusion The magnetic field at the axial point is

$$\vec{B} = \frac{\mu_0 N I r^2}{2(r^2 + x^2)^{3/2}} \hat{n}.$$

Its SI unit is tesla; its direction along the axis is given by the right-hand grip rule.

Quick Tip to Solve: Multiply the Biot–Savart magnitude by the axial projection factor $r/\sqrt{r^2 + x^2}$ before integrating; then check the result at $x = 0$.

Section D (8 marks)

Question numbers 29 and 30 are case study based questions. Read the following paragraphs and answer the questions that follow.

29. A p-type or n-type semiconductor can be converted into a p-n junction by doping it with suitable impurity. The motion of majority charge carriers causes diffusion current across the junction while the barrier electric field causes motion of minority carriers for drift current. In case of unbiased diode, the diffusion and drift currents are equal. This equilibrium is disturbed by the biasing batteries. Diodes, therefore, allow currents in one direction. This property of diode is used in making rectifiers.

[4 marks]

29(i). Silicon is doped with which of the following to obtain p-type semiconductor?

[1 mark]

- (A) Phosphorus
- (B) Arsenic
- (C) Boron
- (D) Antimony

Correct Answer: (C) Boron

Solution:

Concept:

- Silicon is tetravalent: each silicon atom normally forms four covalent bonds with its neighbours in the crystal.
- A trivalent acceptor impurity creates a hole because it supplies only three valence electrons for these bonds. Holes are the majority carriers in a p-type semiconductor.

Step 1: Identify the acceptor dopant Boron has three valence electrons. When a boron atom replaces a silicon atom, one bond lacks an electron. The resulting hole can move through the lattice as neighbouring electrons fill it, so boron doping produces p-type silicon. The doped crystal as a whole remains electrically neutral.

Step 2: Check the remaining elements Phosphorus, arsenic and antimony are all pentavalent. Each can form four covalent bonds and has an additional, weakly bound electron available for conduction. They are donor impurities and produce n-type silicon, so options (A), (B) and (D) do not meet the required p-type condition.

Step 3: Conclusion Silicon must be doped with the trivalent acceptor boron. Hence, the correct option is (C).

Quick Tip to Solve: For silicon, compare dopant valency with four: three gives holes (p-type), while five gives extra electrons (n-type).

29(ii). A semiconductor has an electron concentration of $5 \times 10^{22} \text{ m}^{-3}$. The concentration of holes is (given $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$): [1 mark]

(A) $5 \times 10^{22} \text{ m}^{-3}$

(B) $1.5 \times 10^6 \text{ m}^{-3}$

(C) $9 \times 10^8 \text{ m}^{-3}$

(D) $4.5 \times 10^9 \text{ m}^{-3}$

Correct Answer: (D) $4.5 \times 10^9 \text{ m}^{-3}$

Solution:

Concept:

- At thermal equilibrium, electron concentration n and hole concentration p obey the mass-action law $np = n_i^2$, where n_i is the intrinsic carrier concentration at the same temperature.
- Increasing the majority electron concentration decreases the minority hole concentration; $n = p = n_i$ applies to an intrinsic semiconductor, not to this doped sample.

Step 1: Rearrange the mass-action relation The given electron concentration is $n = 5 \times 10^{22} \text{ m}^{-3}$, and $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$. The unknown is the hole concentration, so

$$p = \frac{n_i^2}{n}.$$

Step 2: Substitute and check the options Square both the coefficient and the power of ten:

$$\begin{aligned} p &= \frac{(1.5 \times 10^{16})^2}{5 \times 10^{22}} \text{ m}^{-3} \\ &= \frac{2.25}{5} \times 10^{10} \text{ m}^{-3} \\ &= 4.5 \times 10^9 \text{ m}^{-3}. \end{aligned}$$

Option (A) simply repeats the electron concentration. The printed values in options (B) and (C), $1.5 \times 10^6 \text{ m}^{-3}$ and $9 \times 10^8 \text{ m}^{-3}$, do not satisfy $np = n_i^2$. The calculated value is much smaller than n , as expected for minority holes in n-type material.

Step 3: Conclusion The hole concentration is $4.5 \times 10^9 \text{ m}^{-3}$. Hence, the correct option is (D).

Quick Tip to Solve: Square n_i before dividing by n , then check that holes are much less numerous when the electron concentration is far above n_i .

29(iii). During forward biasing of a p-n junction diode, the:

[1 mark]

- (A) current is mainly due to drifting of majority carriers.
- (B) current is mainly due to drifting of minority carriers.
- (C) diffusion and drift currents are equal.
- (D) current is of the order of 1 A.

Correct Answer: No printed option is fully correct: forward junction current is mainly majority-carrier diffusion; (A) is apparently intended but incorrectly says drifting.

Solution:

Concept:

- Diffusion is carrier transport caused by a concentration gradient, whereas drift is transport caused by an electric field. These mechanisms must be distinguished at a p-n junction.
- Forward bias lowers the junction barrier and permits increased diffusion of majority carriers across the junction; minority-carrier drift gives the small opposing reverse saturation current.

Step 1: Determine the forward-current mechanism Forward bias connects the p-side to the positive terminal and the n-side to the negative terminal. The reduced barrier allows holes from the p-side and electrons from the n-side to diffuse across the junction. The large increase is in the majority-carrier diffusion current, so it exceeds the opposing minority-carrier drift current. Drift can occur in the bulk semiconductor, but it is not the junction mechanism being contrasted here.

Step 2: Assess every printed alternative Option (A) names the appropriate majority carriers but incorrectly calls their junction transport drifting. It would state the standard result if the word were diffusion. Option (B) describes the minority-carrier drift associated mainly with reverse current. Option (C) is the equilibrium relation for an unbiased diode; forward bias produces a net current. Option (D) is not a general property: current depends on the applied voltage, diode and external circuit, and need not be of order 1 A. Therefore none of the options is fully correct as printed.

Step 3: Conclusion The physically correct statement is that forward current is mainly due to **diffusion of majority carriers**. No printed option states this correctly. Option (A) is apparently intended, with “drifting” replaced by “diffusion”, but the printed question has no fully correct option.

Quick Tip to Solve: At the junction, associate forward bias with majority-carrier diffusion and reverse current with minority-carrier drift; do not confuse carrier type with transport mechanism.

29(iv)(a). The threshold voltage for silicon diode is about:

[1 mark]

- (A) 0.2 V
- (B) 0.5 V
- (C) 0.7 V
- (D) 1.5 V

Correct Answer: (C) 0.7 V

Solution:

Concept:

- The threshold or knee voltage is the approximate forward voltage beyond which the current through a diode increases rapidly.
- In the usual room-temperature model, a silicon p-n junction diode has a threshold of approximately

0.7 V; a germanium diode has a smaller threshold, about 0.3 V.

Step 1: Match the material to its knee voltage The question specifies silicon. Its forward characteristic has the familiar knee near 0.7 V, after which a small increase in voltage produces a comparatively large increase in forward current. This is an approximate characteristic value, not an abrupt universal switching voltage.

Step 2: Eliminate the other values The standard silicon knee value is not 0.2 V or 0.5 V, so options (A) and (B) are not the expected textbook estimate. The value 1.5 V in option (D) is also far above the usual threshold for a single silicon p-n junction. Although operating voltage varies with current and temperature, the requested approximate value is the standard silicon value.

Step 3: Conclusion The threshold voltage of a silicon diode is approximately 0.7 V . Hence, the correct option is (C).

Quick Tip to Solve: Memorise the material pair for ordinary p-n diodes: silicon is about 0.7 V, whereas germanium is about 0.3 V.

OR

29(iv)(b). When we dope Ge with a pentavalent element, four of its electrons bond with four germanium neighbours but fifth electron remains weakly bound. The ionisation energy for this electron is about: [1 mark]

- (A) 0.01 eV
- (B) 0.05 eV
- (C) 0.1 eV
- (D) 0.15 eV

Correct Answer: (A) 0.01 eV

Solution:

Concept:

- A pentavalent impurity in tetravalent germanium forms four covalent bonds and retains a fifth, weakly bound donor electron.
- The donor level lies just below the conduction band. The small donor ionisation energy in germanium is approximately 0.01 eV, so room-temperature thermal energy can readily release the extra electron.

Step 1: Identify the energy transition The question asks for the energy needed to free the donor's fifth electron, not to break a normal covalent bond or excite an electron across the full forbidden gap. This fifth electron is only weakly bound because the four covalent bonds are already complete. It needs only the small donor-to-conduction-band energy.

Step 2: Use the germanium donor value For pentavalent impurities in germanium, the standard approximate ionisation energy is 0.01 eV. The value 0.05 eV is the familiar larger donor-energy estimate

for silicon, so option (B) confuses the host materials. The energies 0.1 eV and 0.15 eV in options (C) and (D) are also too large for the usual shallow donor in germanium.

Step 3: Conclusion The fifth electron requires approximately 0.01 eV for ionisation. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish the donor ionisation energy from the band gap: the weakly bound extra electron in germanium needs only about 0.01 eV.

30. In an experiment with convex lens of focal length f , the screen is fixed at a distance D from the object. A student slowly moves the lens away from the object towards the screen and finds that she is able to form sharp image of the object for two positions of the lens. The distance between these two positions of the lens is d . [4 marks]

30(i). The value of d is: [1 mark]

(A) $\sqrt{D(D - 4f)}$

(B) $\sqrt{D(D - 2f)}$

(C) $2\sqrt{Df}$

(D) $\sqrt{D(D - f)}$

Correct Answer: (A) $\sqrt{D(D - 4f)}$

Solution:

Concept:

- For a convex lens forming a real image, let x and y be the positive object and image distances. The lens formula becomes $1/f = 1/x + 1/y$.
- The fixed object-screen separation is $D = x + y$. The two sharp-image positions interchange x and y , so their separation is $d = |y - x|$.

Step 1: Express the product of the distances Both the object and the real image lie on opposite sides of the lens. Using their positive distance magnitudes avoids mixing Cartesian signs. Substituting their sum in the lens formula gives

$$\frac{1}{f} = \frac{x+y}{xy} = \frac{D}{xy} \Rightarrow xy = fD.$$

Step 2: Evaluate the displacement between the positions Exchanging object and image distances gives the second lens position. Therefore, square the difference and use the known sum and product:

$$d^2 = (y - x)^2 = (x + y)^2 - 4xy = D^2 - 4fD.$$

Options (B) and (D) use incorrect coefficients of fD . Option (C) gives $d^2 = 4fD$, omitting the D^2 term and the subtraction. Only the expression in (A) also vanishes at the limiting separation $D = 4f$.

Step 3: Conclusion The distance between the two lens positions is $d = \sqrt{D(D - 4f)}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $x + y = D$, $xy = fD$ and $(x - y)^2 = (x + y)^2 - 4xy$ to obtain the displacement without solving a quadratic.

30(ii). Compared to the size of the object, the images formed in the two positions of the lens are respectively: [1 mark]

- (A) reduced, enlarged
- (B) reduced, reduced
- (C) enlarged, enlarged
- (D) enlarged, reduced

Correct Answer: (D) enlarged, reduced

Solution:

Concept:

- For a real image formed by a convex lens, the magnitude of magnification is $|m| = y/x$, where x and y are the positive object and image distances.
- For two distinct sharp-image positions at fixed object-screen distance D , the object and image distances interchange. Their magnification magnitudes are therefore reciprocal.

Step 1: Examine the first sharp-image position The lens moves from the object towards the screen. It first reaches the position nearer the object, where $x < D/2$ and $y = D - x > D/2$. Thus $y > x$, giving $|m| > 1$. The first sharp image is enlarged, even though it is inverted like the other real image.

Step 2: Examine the second sharp-image position At the second position the lens lies nearer the screen. The distances exchange, so now $x > y$ and $|m| < 1$; this image is reduced. Option (A) reverses the specified direction of movement. Options (B) and (C) incorrectly give the same size category twice, whereas reciprocal unequal magnifications put one image above and the other below unit magnification.

Step 3: Conclusion The images formed, in the order encountered, are enlarged and reduced respectively. Hence, the correct option is (D).

Quick Tip to Solve: A lens nearer the object gives the enlarged image; nearer the screen it gives the reduced image. Use the stated direction of travel to order them.

30(iii). If the distance between object and screen is 80.00 cm and the lens forms sharp images at two positions separated by 20.00 cm, the focal length of convex lens is: [1 mark]

- (A) 15.50 cm
- (B) 18.75 cm
- (C) 20.50 cm
- (D) 22.75 cm

Correct Answer: (B) 18.75 cm

Solution:

Concept:

- In the convex-lens displacement method, $d^2 = D^2 - 4Df$, where D is the fixed object-screen distance, d is the separation of the two sharp-image positions, and f is the focal length.
- Solving for the focal length gives $f = (D^2 - d^2)/(4D)$. All lengths must be expressed in the same unit.

Step 1: Substitute the measured distances Here $D = 80.00$ cm and $d = 20.00$ cm. Keeping centimetres

throughout gives

$$f = \frac{(80.00)^2 - (20.00)^2}{4(80.00)} \text{ cm} = \frac{6400 - 400}{320} \text{ cm}.$$

Step 2: Evaluate and check the candidate values The numerator is 6000 cm^2 , and division by 320 cm gives 18.75 cm. As a check, the two positive conjugate distances are $(D - d)/2 = 30.00 \text{ cm}$ and $(D + d)/2 = 50.00 \text{ cm}$; their product divided by their sum again gives $1500/80 = 18.75 \text{ cm}$. Option (A) does not reproduce the measured displacement, while (C) and (D) exceed $D/4 = 20.00 \text{ cm}$ and cannot give two sharp-image positions.

Step 3: Conclusion The focal length of the convex lens is $f = 18.75 \text{ cm}$. Hence, the correct option is (B).

Quick Tip to Solve: First calculate $D - d$ and $D + d$; their product is $D^2 - d^2$, so here $f = (60 \times 100)/(4 \times 80) \text{ cm}$.

30(iv)(a). Consider a convex lens of focal length 15 cm. For which of the following values of object-screen distance, two positions of the object can be found to obtain sharp image on the screen?

[1 mark]

- (A) 45 cm
- (B) 50 cm
- (C) 55 cm
- (D) 65 cm

Correct Answer: (D) 65 cm

Solution:

Concept:

- For fixed object-screen separation D , the two conjugate lens positions have displacement $d = \sqrt{D(D - 4f)}$, where f is the convex-lens focal length.
- Two distinct positions require $d > 0$, hence $D > 4f$. At $D = 4f$, the two positions merge into a single position with object and image distances both $2f$.

Step 1: Determine the threshold separation Using the displacement experiment described in the passage, the focal length $f = 15 \text{ cm}$ gives

$$4f = 4(15 \text{ cm}) = 60 \text{ cm}.$$

Therefore, the object-screen distance must exceed 60 cm for two distinct sharp-image positions. Merely requiring a real square root would include the coincident-position case, so the strict inequality matters.

Step 2: Test the four listed separations Options (A), (B) and (C) give $D = 45, 50$ and 55 cm , respectively. Each is below 60 cm, making $D(D - 4f)$ negative, so none admits the two positions. The remaining choice has $D = 65 \text{ cm}$, for which $D(D - 4f) = 65(5) = 325 \text{ cm}^2 > 0$, so the positions are real and distinct.

Step 3: Conclusion The required object-screen distance is 65 cm. Hence, the correct option is (D).

Quick Tip to Solve: Compare every proposed separation directly with $4f = 60 \text{ cm}$; choose a value strictly above it when two distinct positions are required.

30(iv)(b). A thin convex lens of focal length 10 cm and another thin lens of focal length f are placed coaxially in contact. If the power of their combination is $\frac{10}{3}$ D, the value of f is: [1 mark]

- (A) –15 cm
- (B) –10 cm
- (C) –20 cm
- (D) –30 cm

Correct Answer: (A) –15 cm

Solution:

Concept:

- The power of a thin lens is $P = 1/f$, with f in metres and P in dioptres. A converging lens has positive power and a diverging lens negative power.
- For coaxial thin lenses in contact, the equivalent power is the algebraic sum: $P = P_1 + P_2$.

Step 1: Find the unknown lens power The given convex lens has $f_1 = 10 \text{ cm} = 0.10 \text{ m}$, so $P_1 = 1/0.10 = 10 \text{ D}$. Using the printed combination power $10/3 \text{ D}$,

$$P_2 = P - P_1 = \frac{10}{3} - 10 = -\frac{20}{3} \text{ D}.$$

The negative power is required because adding this lens reduces the initial positive power.

Step 2: Convert the power to focal length and compare Taking the reciprocal in SI units gives

$$f = \frac{1}{P_2} = -\frac{3}{20} \text{ m} = -0.15 \text{ m} = -15 \text{ cm}.$$

Thus the second lens is concave. For comparison, the focal lengths in (B), (C) and (D) would give combination powers 0, 5 and $20/3 \text{ D}$, respectively; none equals the stated $10/3 \text{ D}$.

Step 3: Conclusion The focal length of the second lens is $f = -15 \text{ cm}$. Hence, the correct option is (A).

Quick Tip to Solve: Subtract the known +10 D from the total power before taking a reciprocal; convert metres to centimetres only at the end.

Section E (15 marks)

31(a). What are coherent sources? Why are they necessary for observing stable interference pattern? Draw a graph showing the variation of intensity of light with the position on the screen in Young's double-slit experiment.

(b) Find the intensity of light at a point on the screen when two interfering waves of the same intensity (I_0) have a path difference of (i) $\frac{\lambda}{4}$ and (ii) $\frac{\lambda}{3}$. [5 marks]

Solution:

Concept:

- Coherent sources emit waves of the same frequency with a constant phase difference. For light, a common polarization is also needed for interference with full visibility.
- The resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$, where I_1, I_2 are the separate intensities and $\phi = 2\pi\Delta/\lambda$ is the phase difference due to path difference Δ .
- For equal intensities $I_1 = I_2 = I_0$, the interference law becomes $I = 2I_0(1 + \cos \phi) = 4I_0 \cos^2(\phi/2)$. The wavelength is λ .

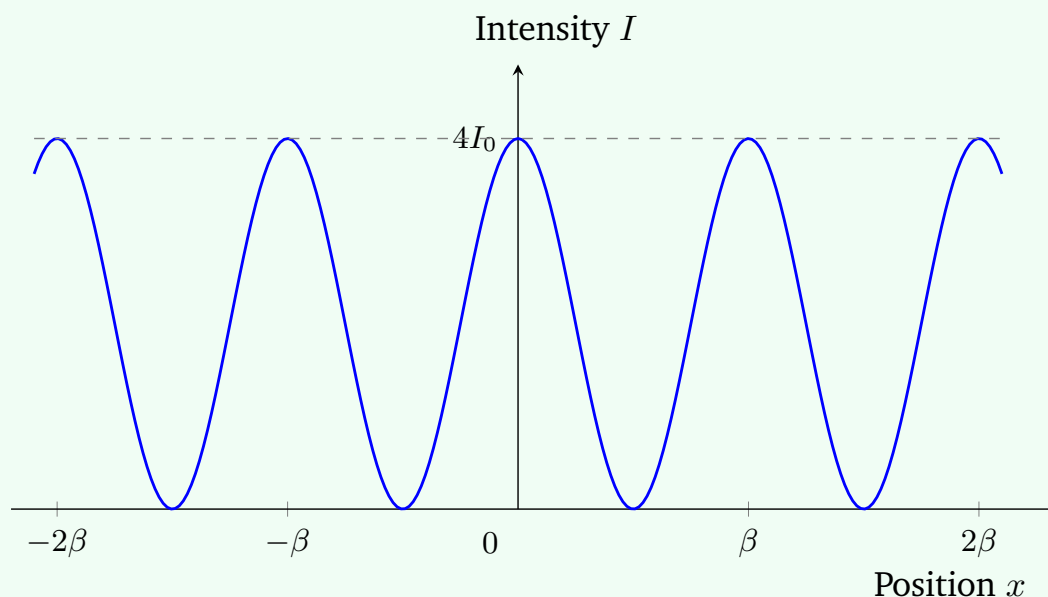
Step 1: Define coherent sources for part (a) Two sources are coherent if they emit waves of the same frequency and maintain a fixed phase difference with time. Their phases need not be equal: a nonzero but constant difference also gives coherence. In Young's experiment, the two narrow slits are illuminated by the same monochromatic source, so each slit produces a wave derived from that common source and the required phase relationship is maintained.

Step 2: Explain why the fringe pattern remains stable At a fixed point on the screen, the geometrical path difference remains fixed. With coherent sources, the total phase difference at that point is therefore constant with time, so its resultant intensity also stays fixed. Constructive interference produces bright fringes and destructive interference produces dark fringes at definite positions. If the relative phase fluctuates rapidly, these positions fluctuate and the time-averaged interference term vanishes. The eye then observes approximately $I_1 + I_2$, rather than a stationary set of bright and dark fringes. Thus a fixed phase relationship is essential for a stable, observable pattern.

Step 3: Obtain and plot the intensity distribution Take identical narrow slits with equal individual intensity I_0 , separation d , and screen distance D , with $D \gg d$. For a point at transverse displacement x from the central bright fringe, the small-angle approximation gives $\Delta \simeq dx/D$. Hence

$$\begin{aligned}\phi &= \frac{2\pi dx}{\lambda D}, \\ I(x) &= 2I_0 \left[1 + \cos \left(\frac{2\pi dx}{\lambda D} \right) \right] \\ &= 4I_0 \cos^2 \left(\frac{\pi x}{\beta} \right), \quad \beta = \frac{\lambda D}{d}.\end{aligned}$$

The graph below is the ideal interference distribution, neglecting the finite slit-width diffraction envelope. Maxima occur at $x = n\beta$, where $I_{\max} = 4I_0$; minima occur at $x = (n + \frac{1}{2})\beta$, where $I_{\min} = 0$, for integer n . Adjacent bright fringes and adjacent dark fringes each have the same separation β . The central fringe at $x = 0$ is bright.



Ideal Young's double-slit intensity pattern for equal slit intensities; $\beta = \lambda D/d$.

Step 4: Evaluate part (b)(i) for a path difference of one-quarter wavelength Convert path difference into phase difference before using the intensity formula:

$$\Delta = \frac{\lambda}{4}, \quad \phi = \frac{2\pi}{\lambda} \frac{\lambda}{4} = \frac{\pi}{2},$$

$$I = 2I_0 \left(1 + \cos \frac{\pi}{2} \right) = 2I_0(1 + 0) = 2I_0.$$

The interference term is zero at this phase difference, so the resultant intensity equals the sum of the two separate intensities.

Step 5: Evaluate part (b)(ii) for a path difference of one-third wavelength Here the phase difference is greater than $\pi/2$, so the interference term is negative:

$$\Delta = \frac{\lambda}{3}, \quad \phi = \frac{2\pi}{\lambda} \frac{\lambda}{3} = \frac{2\pi}{3},$$

$$I = 2I_0 \left(1 + \cos \frac{2\pi}{3} \right) = 2I_0 \left(1 - \frac{1}{2} \right) = I_0.$$

This is partial destructive interference; complete cancellation would instead require an odd multiple of π as the phase difference.

Step 6: Conclusion Coherent sources have equal frequency and a constant phase difference, keeping the interference fringes stationary. For equal slit intensities, the ideal Young's pattern varies periodically from 0 to $4I_0$. The required intensities are $2I_0$ for $\Delta = \lambda/4$ and I_0 for $\Delta = \lambda/3$.

Quick Tip to Solve: First use $\phi = 2\pi\Delta/\lambda$, then substitute in $I = 2I_0(1 + \cos \phi)$. The maximum is $4I_0$, while I_0 denotes the intensity from just one slit.

31(b). (a) Draw a labelled ray diagram of a refracting telescope when it forms image of a distant object at infinity. Derive expression for its magnifying power.

(b) (i) In a telescope the objective has much larger aperture than the eyepiece. Why?

(ii) Write two advantages of reflecting telescope over refracting telescope.

[5 marks]

Solution:

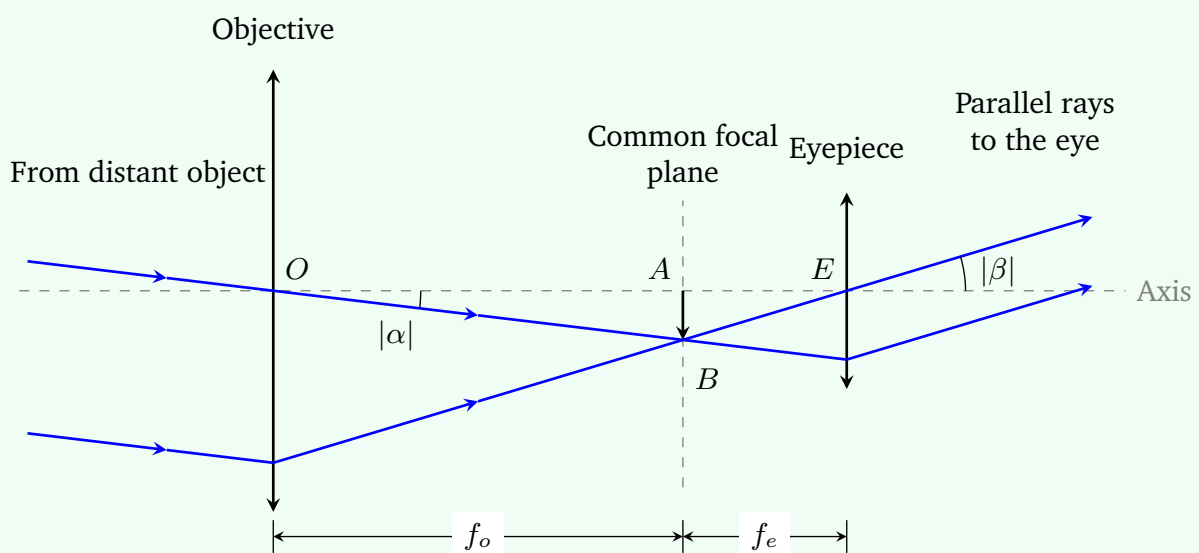
Concept:

- In normal adjustment, the objective forms a real image of a distant object in its focal plane. This plane coincides with the first focal plane of the eyepiece, so the final image is at infinity.
- Angular magnifying power is $M = \beta/\alpha$, where α is the small angle subtended by the distant object at the unaided eye and β is the angle subtended by its final image at the eye using the telescope.
- For small angles, $\tan \theta \simeq \theta$. A circular objective of diameter D collects light in proportion to D^2 , while its diffraction-limited angular resolution improves as D increases.

Step 1: (a) Construct the telescope in normal adjustment Let the objective and eyepiece have positive focal lengths f_o and f_e , respectively, with $f_o > f_e$. Parallel rays from one point of the distant object enter the objective. The objective converges them to B , forming a real, inverted intermediate image AB in its second focal plane. Place the eyepiece so that AB lies in its first focal plane, as shown. After refraction through the eyepiece, the rays from B become parallel; hence the final virtual image is at infinity and the eye observes it without accommodation. The lens separation is

$$OE = f_o + f_e.$$

The dashed vertical line shows the common focal plane. The ray through the centre of either thin lens proceeds without deviation at that lens; the arrows show light travelling from the objective towards the eyepiece.



Refracting telescope in normal adjustment; $A = F_o = F_e$, and the emergent rays are parallel.

Step 2: (a) Derive the angular magnifying power Write the height of the intermediate image as $AB = h$. The object is very distant, so the small angular size of the object at the objective is essentially the same as its angular size at the unaided eye. From the objective-side triangle and the eyepiece-side

triangle,

$$\begin{aligned} |\alpha| &\simeq \tan |\alpha| = \frac{h}{f_o}, \\ |\beta| &\simeq \tan |\beta| = \frac{h}{f_e}, \\ |M| &= \frac{|\beta|}{|\alpha|} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}. \end{aligned}$$

The objective inverts the intermediate image and the eyepiece does not erect it. Thus the final image is inverted relative to the distant object. With signed angular magnification, the angles therefore have opposite signs and

$$M = -\frac{f_o}{f_e}.$$

The diagram labels the magnitudes of the two small angles; the minus sign expresses inversion, not a negative image size.

Step 3: (b)(i) Explain the larger objective aperture A distant astronomical source sends very little light into a small area. The objective is the light-collecting element, so a larger aperture intercepts more light and allows faint objects to be seen more clearly. For the same incident intensity, the collected light is proportional to $\pi D^2/4$. A larger objective also reduces the diffraction limit,

$$\theta_{\min} \simeq \frac{1.22\lambda}{D},$$

improving the ability to distinguish closely spaced distant objects. The eyepiece receives the beam already gathered by the objective, so it can have a smaller aperture while allowing the viewing beam to pass.

Step 4: (b)(ii) State two advantages of a reflecting telescope First, its mirror objective is free from chromatic aberration: reflection does not disperse the light into wavelength-dependent focal positions, so different colours have the same focus. A refracting objective requires correction for this effect. Second, a mirror can be supported from its back, making a much larger objective practical without the distortion associated with supporting a large lens only at its rim. The resulting larger aperture provides greater light-gathering ability and resolving power.

Step 5: Conclusion For a refracting telescope in normal adjustment, the focal planes coincide, the final image is at infinity, and $M = -f_o/f_e$ (magnitude f_o/f_e); its length is $f_o + f_e$. The larger objective gathers more light and improves resolution. Reflecting telescopes avoid chromatic aberration and permit larger, back-supported objectives.

Quick Tip to Solve: Put the intermediate image at the common focal plane before drawing the emerging parallel rays; divide the objective focal length by the eyepiece focal length, and use a minus sign for inversion.

32(a). Derive an expression for the capacitance of a parallel plate capacitor of plate area A and plate separation d with air present between the plates.

(b) Two air-filled capacitors of capacitances C_1 and C_2 are connected in parallel with a dc battery. After the capacitors are fully charged, a slab of dielectric constant K is inserted between the plates of each capacitor. How will the (i) charge on each capacitor and (ii) energy stored in the capacitor be affected after the slab is introduced.

[5 marks]

Solution:

Concept:

- Capacitance is $C = Q/V$, where Q is the magnitude of charge on either plate and V is the potential difference between the plates.
- A large plane sheet of surface charge density σ produces a field of magnitude $\sigma/(2\epsilon_0)$ on either side. The fields of two oppositely charged parallel plates add between them.
- A dielectric of relative permittivity K , completely filling the space between fixed plates, changes the capacitance from C to KC . A connected ideal dc battery maintains a constant potential difference.
- The electrostatic energy stored in a capacitor is $U = \frac{1}{2}CV^2$. Use this form when the voltage is held constant.

Step 1: (a) Find the field between the plates Consider two large parallel conducting plates, each of area A , carrying charges $+Q$ and $-Q$. Their separation d is small compared with their lateral dimensions, so edge effects can be neglected. Treat air as having permittivity ϵ_0 . The uniform surface charge density has magnitude $\sigma = Q/A$. Between the plates, the field of the positive plate points away from it and the field of the negative plate points towards it, so both fields have the same direction. Hence,

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}.$$

Outside the ideal large-plate arrangement, the two fields cancel. The potential difference is determined by the uniform field in the air gap.

Step 2: Relate the potential difference to the charge The magnitude of potential difference across a uniform field over distance d is $V = Ed$. Therefore,

$$V = \frac{Qd}{\epsilon_0 A}, \quad C = \frac{Q}{V} = \frac{Q}{Qd/(\epsilon_0 A)} = \frac{\epsilon_0 A}{d}.$$

Thus the capacitance is proportional to plate area and inversely proportional to separation. It depends on the geometry and the medium, rather than on the particular charge supplied to the plates.

Step 3: (b) Identify what stays constant during insertion Let the battery voltage be V . Since the capacitors are connected in parallel, each has potential difference V . The question does not disconnect the battery, so each continues to have this same voltage after the dielectric is inserted. Taking the inserted slab to fill the gap between each pair of plates, the new capacitances are

$$C'_1 = KC_1, \quad C'_2 = KC_2.$$

Polarisation of the dielectric reduces the field associated with a fixed free charge; the connected battery supplies additional free charge to maintain the original voltage.

Step 4: (b)(i) Calculate the new charge on each capacitor Initially the magnitudes of the charges are

$Q_1 = C_1V$ and $Q_2 = C_2V$. With the increased capacitances and unchanged voltage,

$$Q'_1 = C'_1V = KC_1V = KQ_1,$$

$$Q'_2 = C'_2V = KC_2V = KQ_2.$$

The magnitude on either plate of each capacitor increases by a factor K , with opposite signs retained on its two plates. The extra charge supplied to capacitor i is $(K - 1)C_iV$.

Step 5: (b)(ii) Calculate the change in stored energy Before insertion, $U_1 = \frac{1}{2}C_1V^2$ and $U_2 = \frac{1}{2}C_2V^2$. Since voltage remains fixed,

$$U'_1 = \frac{1}{2}C'_1V^2 = \frac{1}{2}KC_1V^2 = KU_1,$$

$$U'_2 = \frac{1}{2}C'_2V^2 = \frac{1}{2}KC_2V^2 = KU_2.$$

Hence the stored energy in each capacitor, and their total stored energy, increase by the same factor K . A decrease by the factor $1/K$ would apply to an isolated capacitor with fixed charge, whereas here charge flows through the battery connection during insertion.

Step 6: Conclusion For the air-filled parallel plate capacitor, $C = \epsilon_0 A/d$. With the battery connected and the dielectric filling the gap, each capacitor has $Q'_i = KQ_i$ and $U'_i = KU_i$, for $i = 1, 2$.

Quick Tip to Solve: First check whether the battery remains connected: here V is fixed, so both $Q = CV$ and $U = \frac{1}{2}CV^2$ scale directly with the new capacitance KC .

OR

32(b). (a) An electric field $\vec{E}$ is established across the ends of a cylindrical conductor of length L and area of cross-section A . Discuss how electrons attain an average velocity, independent of time. Hence, obtain a relation between current in the conductor and this 'average velocity' of electrons.

(b) (i) This 'average velocity' is found to be few mm/s for currents in range of a few amperes. How then is current established almost the instant a circuit is closed?

(ii) Two copper wires having their radii in the ratio of 3 : 2 are connected in series across a battery. Find the ratio of the drift velocities of the electrons in the wires. [5 marks]

Solution:

Concept:

- Drift velocity $\vec{v}_d$ is the average velocity acquired by conduction electrons in an applied electric field, superposed on their random thermal motion. An electron has charge $-e$, with $e > 0$.
- In the relaxation-time model, $\vec{v}_d = -e\tau\vec{E}/m$, where m is the electron mass and τ is the mean time between successive momentum-randomising collisions.
- Current magnitude is the charge crossing a cross-section per unit time: $I = neAv_d$, where n is the number of free electrons per unit volume, A is cross-sectional area and $v_d = |\vec{v}_d|$.
- Components in series carry the same current. For wires of the same material at the same conditions, n is the same, so their drift speeds are inversely proportional to cross-sectional areas.

Step 1: (a) Explain acceleration between collisions Without an electric field, conduction electrons move randomly and their vector average velocity is zero. On applying $\vec{E}$ along the conductor, every

electron experiences the force $\vec{F} = -e\vec{E}$ and acceleration

$$\vec{a} = -\frac{e\vec{E}}{m}.$$

During a free flight of duration t after a collision, an electron with initial random velocity $\vec{u}$ has velocity $\vec{v} = \vec{u} - e\vec{E}t/m$. Collisions with the lattice and imperfections repeatedly randomise its motion, preventing an indefinitely growing directed velocity. Individual electron velocities fluctuate, but their average can be steady.

Step 2: Obtain the steady average velocity Average over the many conduction electrons. The random velocities cancel, $\langle \vec{u} \rangle = 0$. In the standard relaxation-time description, the mean effective free-flight time is τ . Therefore,

$$\vec{v}_d = -\frac{e\tau}{m}\vec{E}, \quad v_d = \frac{eE\tau}{m}.$$

For a constant applied field and fixed temperature, E and τ are constant; thus the steady drift velocity is independent of elapsed time after the brief initial transient. Acceleration due to the field is balanced, on average, by loss of directed momentum in collisions. Electron drift is opposite to $\vec{E}$.

Step 3: Relate electron drift to current In a short time interval Δt , the net drift displacement is $v_d\Delta t$. The electrons contributing to net charge transport across a cross-section occupy an effective cylinder of volume $Av_d\Delta t$. Their number is $nAv_d\Delta t$, and the transported charge magnitude is

$$\Delta Q = neAv_d\Delta t.$$

Dividing by the time interval gives

$$I = \frac{\Delta Q}{\Delta t} = neAv_d.$$

Conventional current flows opposite to the electron drift, along the electric field. In vector form, $\vec{J} = -ne\vec{v}_d = (ne^2\tau/m)\vec{E}$. If the applied potential difference across the length L is V , then $E = V/L$ and $I = ne^2\tau AV/(mL)$.

Step 4: (b)(i) Explain why current appears almost immediately Free electrons are already present throughout every wire in the circuit. Closing the switch establishes an electromagnetic disturbance which propagates around the circuit at a very high speed, of the order of the speed of light in the surrounding medium. As the local electric field is established, the electrons already present at that location begin to drift. Electrons do not need to travel all the way from the battery to a device before the device carries current. Thus the small drift speed of a few mm s^{-1} is compatible with an almost immediate response in a circuit of ordinary size. The propagation time is finite, so the establishment of current is not literally instantaneous.

Step 5: (b)(ii) Compare the drift speeds in the copper wires Let wire 1 have radius r_1 and wire 2 have radius r_2 , with $r_1 : r_2 = 3 : 2$. The series connection makes their currents equal, and both copper wires have the same free-electron number density n . Using $A = \pi r^2$,

$$neA_1v_{d1} = neA_2v_{d2}, \quad \frac{v_{d1}}{v_{d2}} = \frac{A_2}{A_1} = \frac{r_2^2}{r_1^2} = \frac{2^2}{3^2} = \frac{4}{9}.$$

The thicker wire therefore has the smaller drift speed: its larger cross-section contains more charge carriers per unit length, so a smaller average displacement per second transports the same charge per second.

Step 6: Conclusion The steady electron drift velocity is $\boxed{\vec{v}_d = -e\tau\vec{E}/m}$, and the current magnitude is

$I = neAv_d$. Current begins almost immediately because the electric-field disturbance travels rapidly to electrons already throughout the circuit. For radii 3 : 2, the drift-speed ratio in that order is $v_{d1} : v_{d2} = 4 : 9$.

Quick Tip to Solve: In series, first set the two currents equal. For the same material, $v_d \propto 1/A \propto 1/r^2$, so square the radius ratio and invert it; distinguish this drift speed from the much faster field propagation.

33(a). A series combination of L , C and R is connected to an a.c. source. Using a phasor diagram, derive an expression for the impedance of the circuit and phase difference between V and I .

(b) Under what conditions the

(i) impedance of the circuit is minimum?

(ii) Wattless current flows in the circuit?

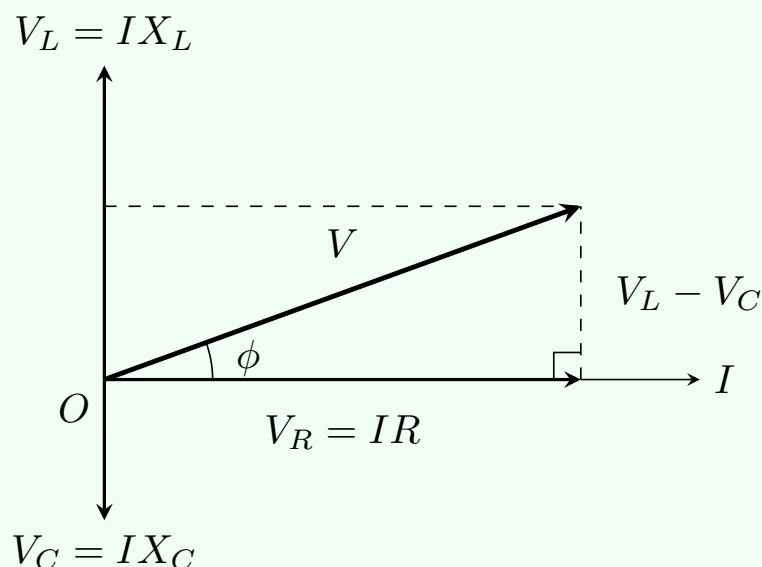
[5 marks]

Solution:

Concept:

- A series LCR circuit has the same sinusoidal current through all three elements. At angular frequency ω , inductive reactance is $X_L = \omega L$ and capacitive reactance is $X_C = 1/(\omega C)$, both measured in Ω .
- The resistor voltage is in phase with current, the inductor voltage leads current by 90° , and the capacitor voltage lags current by 90° . Alternating voltages must therefore be added as phasors.
- Impedance is $Z = V/I$, using rms voltage V and rms current I . Average power is $P = VI \cos \phi = I^2 R$, where ϕ is the phase of the source voltage relative to current.

Step 1: (a) Construct the voltage phasors Choose the common current I as the horizontal reference. Draw $V_R = IR$ horizontally, $V_L = IX_L$ vertically upwards and $V_C = IX_C$ vertically downwards. Since the inductive and capacitive voltages are opposite, their resultant is the signed vertical component $V_L - V_C$. Add this to V_R to obtain the source-voltage phasor V . The diagram shows the case $X_L > X_C$; its resultant lies above the current axis. The same construction gives a downward resultant when $X_C > X_L$.



Voltage phasors for $X_L > X_C$, with current as reference; the dotted sides form the voltage triangle.

Step 2: (a) Derive the impedance from the right triangle The resistive and net reactive voltage components are perpendicular. By Pythagoras,

$$\begin{aligned} V^2 &= V_R^2 + (V_L - V_C)^2 \\ &= (IR)^2 + (IX_L - IX_C)^2 \\ &= I^2 [R^2 + (X_L - X_C)^2]. \end{aligned}$$

Dividing by I^2 and taking the positive square root gives

$$Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

Thus the circuit current is $I = V/Z$. The same derivation works with peak voltages and peak current, provided the convention is used consistently.

Step 3: (a) Obtain the phase difference and its sign Taking the ratio of the vertical to horizontal sides of the voltage triangle,

$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \frac{\omega L - 1/(\omega C)}{R}.$$

For $R > 0$, therefore,

$$\phi = \tan^{-1} \left(\frac{\omega L - 1/(\omega C)}{R} \right), \quad \cos \phi = \frac{R}{Z}.$$

If $X_L > X_C$, $\phi > 0$: voltage leads current, so current lags voltage. If $X_C > X_L$, $\phi < 0$: current leads voltage. If the two reactances are equal, voltage and current are in phase.

Step 4: (b)(i) Find the condition for minimum impedance For fixed R , the square $(X_L - X_C)^2$ is

non-negative and is smallest when it is zero. Hence minimum impedance occurs at series resonance:

$$X_L = X_C, \quad \omega_0 L = \frac{1}{\omega_0 C}, \quad \omega_0 = \frac{1}{\sqrt{LC}}.$$

Equivalently, the source frequency must be $f_0 = 1/(2\pi\sqrt{LC})$. Then $Z_{\min} = R$, $\phi = 0$, and for a fixed applied voltage the current is maximum, $I_{\max} = V/R$. The inductor and capacitor voltages cancel as phasors even though each may be nonzero.

Step 5: (b)(ii) State the condition for wattless current Wattless current means a nonzero alternating current with zero average power consumed over a complete cycle. Since $P = VI \cos \phi$, this requires $\cos \phi = 0$, or a phase difference of 90° in magnitude. For an ideal series circuit, this means zero resistance and purely reactive, nonzero impedance: $R = 0$ with $X_L \neq X_C$. A pure ideal inductor or a pure ideal capacitor is a standard example. Energy is stored in the magnetic or electric field during part of the cycle and returned to the source during another part, so no net energy is dissipated. With any positive resistance and nonzero current, $I^2 R > 0$, so the current is not completely wattless. The ideal zero-resistance resonance case is excluded because $Z = 0$ would not permit a finite sinusoidal steady-state current from a nonzero ideal voltage source.

Step 6: Conclusion The series impedance is $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$, and the voltage-current phase angle obeys $\tan \phi = (\omega L - 1/(\omega C))/R$. Impedance is minimum, $Z = R$, at $\omega_0 = 1/\sqrt{LC}$. Nonzero wattless current requires $|\phi| = 90^\circ$: an ideal purely reactive circuit with $R = 0$ and finite nonzero reactance.

Quick Tip to Solve: Use current as the horizontal reference: subtract the two vertical reactive voltages before applying Pythagoras. Resonance gives zero phase difference; wattless current gives a phase difference of 90° .

OR

33(b). (i) With the help of a labelled diagram, explain the principle, construction and working of an a.c. generator.

(ii) Deduce an expression for the induced emf in the coil of the generator.

(iii) If T is the time period of the rotation of the coil, at what values of T in a cycle, the emf generated is maximum? [5 marks]

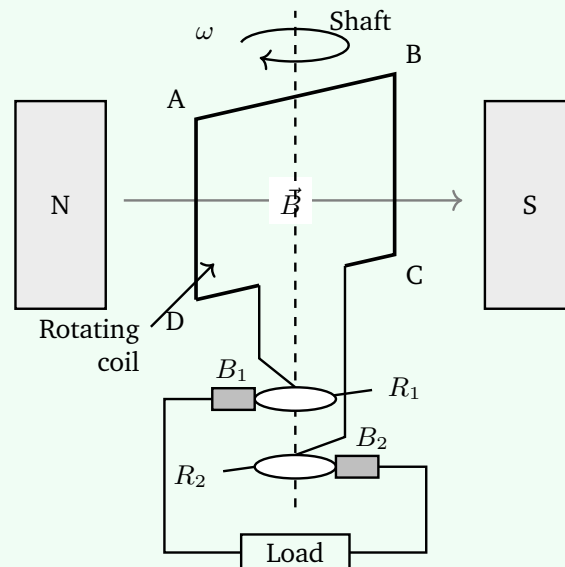
Solution:

Concept:

- Faraday's law: the induced emf is $e = -d(N\Phi)/dt$, where $N\Phi$ is the magnetic flux linkage of a coil of N turns. The minus sign expresses Lenz's law.
- For a plane coil of area A in a uniform field B , the flux per turn is $\Phi = BA \cos \theta$, where θ is the angle between the coil's area normal and the magnetic field.
- Uniform rotation has angular speed $\omega = 2\pi/T$, where T is the rotation period. An a.c. generator converts mechanical energy into electrical energy.

Step 1: (i) State the principle and describe the construction The generator works on electromagnetic induction: rotating a coil in a magnetic field changes its flux linkage and induces an emf across its ends. A rectangular armature coil $ABCD$, containing N insulated turns, is mounted on a shaft between the north and south poles of a strong magnet. The shaft is perpendicular to the magnetic field and is driven by an external mechanical source.

The two ends of the coil connect separately to two continuous metallic slip rings R_1 and R_2 . These rings rotate with the coil but are insulated from each other and from the shaft. Stationary carbon brushes B_1 and B_2 press against the respective rings and connect the rotating coil to the external load. The labelled schematic shows the coil, field poles, shaft, rings, brushes and load.



R_1, R_2 : slip rings B_1, B_2 : carbon brushes

Schematic a.c. generator: each coil end joins its own continuous slip ring and stationary brush.

Step 2: (i) Explain the working during a revolution As the shaft turns, the angle between the coil's area normal and the field changes continuously. The two active sides of the coil cut the field lines, producing induced emfs which add around the coil. When the load is connected, current flows through one ring and brush, the load, and the other brush and ring.

After each half revolution the motion of each active side relative to the field reverses. The induced emf and the current through the external load therefore reverse direction. Continuous slip rings maintain contact with the same coil ends throughout rotation, so the output is alternating. Its direction can be found using Fleming's right-hand rule. The driving source supplies mechanical work against the opposing electromagnetic torque, in accordance with energy conservation.

Step 3: (ii) Differentiate the magnetic flux linkage Choose $t = 0$ when the coil's area normal is parallel to $\vec{B}$, so that the flux linkage is maximum. For uniform rotation, $\theta = \omega t$, and hence

$$N\Phi = NBA \cos(\omega t).$$

Applying Faraday's law gives

$$\begin{aligned} e(t) &= -\frac{d}{dt}[NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t). \end{aligned}$$

Thus the emf varies sinusoidally, with peak value $e_0 = NBA\omega$. Equivalently,

$$e(t) = \frac{2\pi NBA}{T} \sin\left(\frac{2\pi t}{T}\right).$$

Here A is the area of each turn; N is dimensionless. With B in tesla, A in square metres and ω in radians per second, e is in volts. Reversing the chosen terminal polarity changes the overall sign, but does not change the peak magnitude.

Step 4: (iii) Locate the peak emf values within one cycle In the phase convention just chosen, the magnitude is greatest when $|\sin(\omega t)| = 1$. During $0 \leq t < T$, this occurs at

$$\omega t = \frac{\pi}{2}, \frac{3\pi}{2} \implies t = \frac{T}{4}, \frac{3T}{4}.$$

At $T/4$, $e = +e_0$, the positive maximum. At $3T/4$, $e = -e_0$, the negative peak; its magnitude is also maximum. At both positions the coil's plane is parallel to the field, its flux is zero, and the rate of change of flux has its greatest magnitude. At $t = 0, T/2, T$, the flux has an extreme value and the emf is zero. An arbitrary different starting angle shifts all these times by the corresponding phase offset.

Step 5: Conclusion The rotating coil and slip rings produce an alternating emf $e = NBA\omega \sin(\omega t)$, with $e_0 = NBA\omega$. Starting from maximum flux at $t = 0$, the emf has maximum magnitude at $t = T/4, 3T/4$; these are respectively the positive and negative peaks.

Quick Tip to Solve: Start the timing from maximum flux: emf is zero there and reaches its first peak a quarter-turn later. Slip rings preserve the alternating output, and maximum emf occurs when the coil's plane is parallel to the field.