

CBSE Class 12 Physics Question Paper 2026 with Solution PDF-Set 55/2/2



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. All questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – Sections **A, B, C, D** and **E**.
- (iii) In **Section A** : Question numbers **1** to **16** are Multiple Choice (MCQ) type questions. Each question carries 1 mark.
- (iv) In **Section B** : Question numbers **17** to **21** are Very Short Answer (VSA) type questions. Each question carries 2 marks.
- (v) In **Section C** : Question numbers **22** to **28** are Short Answer (SA) type questions. Each question carries 3 marks.
- (vi) In **Section D** : Question numbers **29** & **30** are Case Study-Based questions. Each question carries 4 marks.
- (vii) In **Section E** : Question numbers **31** to **33** are Long Answer (LA) type questions. Each question carries 5 marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculator is NOT allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Section A (16 marks)

1. Three point charges $+q$, $-2q$ and $+q$ are placed along x -axis at points $x = -1$ m, $x = 0$ m and $x = 2$ m respectively. The potential energy of the system is: [1 mark]

- (A) $-\frac{q^2}{\pi\epsilon_0}$
(B) $-\frac{2q^2}{3\pi\epsilon_0}$
(C) $\frac{q^2}{4\pi\epsilon_0}$
(D) $-\frac{5q^2}{4\pi\epsilon_0}$

Correct Answer: (B) $-\frac{2q^2}{3\pi\epsilon_0}$

Solution:

Concept:

- The electrostatic potential energy of point charges is the sum over distinct pairs: $U = \sum_{i < j} kq_i q_j / r_{ij}$, where $k = 1/(4\pi\epsilon_0)$ and r_{ij} is their separation.
- Unlike charges contribute negative energy and like charges contribute positive energy. Each pair must be counted once, including the two outer charges.

Step 1: Identify all three pairs Label the charges from left to right as $q_1 = +q$, $q_2 = -2q$ and $q_3 = +q$. Their separations are $r_{12} = 1$ m, $r_{23} = 2$ m and $r_{13} = 3$ m. The outer separation is the sum of the two adjacent separations.

Step 2: Add the pair energies with their signs The two unlike-charge terms are negative, while the outer like-charge term is positive:

$$\begin{aligned} U &= kq^2 \left(-\frac{2}{1\text{ m}} - \frac{2}{2\text{ m}} + \frac{1}{3\text{ m}} \right) \\ &= -\frac{8kq^2}{3\text{ m}}. \end{aligned}$$

With the distances inserted in SI units as in the printed options, the coefficient is $-8/3$ times kq^2 . Options (A), (C) and (D) instead have coefficients -4 , $+1$ and -5 , respectively, so none matches the sum.

Step 3: Conclusion The energy matches $-2q^2/(3\pi\epsilon_0)$ in the printed SI notation. Hence, the correct option is (B).

Quick Tip to Solve: For three collinear charges, list all three separations first; the outer pair here contributes the easily missed positive term $kq^2/(3\text{ m})$.

2. A photosensitive surface is illuminated by radiations of wavelength λ_1 , λ_2 ($> \lambda_1$) and λ_3 one by one and photoemission is observed in each case. λ_1 and λ_2 lie in UV range and λ_3 in visible range. If V_1 , V_2 and V_3 are stopping potentials in these cases respectively, then: [1 mark]

(A) $V_1 = V_2 = V_3$

(B) $V_2 > V_1 > V_3$

(C) $V_1 > V_2 > V_3$

(D) $V_3 > V_1 > V_2$

Correct Answer: (C) $V_1 > V_2 > V_3$

Solution:

Concept:

- Einstein's photoelectric equation gives $eV_s = K_{\max} = hc/\lambda - \phi$, where V_s is the stopping potential, e is the elementary charge and ϕ is the work function.
- For the same photosensitive surface, ϕ is fixed; a shorter incident wavelength gives greater photon energy and therefore a larger stopping potential.

Step 1: Order the incident wavelengths The question states $\lambda_2 > \lambda_1$. Both are ultraviolet wavelengths, whereas λ_3 is visible and hence longer than either ultraviolet wavelength. Thus $\lambda_1 < \lambda_2 < \lambda_3$. Photoemission occurs for all three, so each photon energy exceeds the same work function.

Step 2: Compare the maximum kinetic energies Taking reciprocals reverses the wavelength order. Multiplication by hc , subtraction of the same ϕ , and division by positive e preserve the resulting energy order:

$$\frac{hc}{\lambda_1} > \frac{hc}{\lambda_2} > \frac{hc}{\lambda_3} \implies V_1 > V_2 > V_3.$$

Option (A) wrongly assumes equal photon energies. Option (B) reverses the ordering of the two ultraviolet cases, while (D) incorrectly gives the longest-wavelength visible radiation the largest stopping potential.

Step 3: Conclusion The stopping potentials satisfy $V_1 > V_2 > V_3$. Hence, the correct option is (C).

Quick Tip to Solve: On one surface, rank stopping potentials by the inverse wavelength; place visible light below ultraviolet light in the stopping-potential order.

3. In Bohr model of hydrogen atom, the value of potential energy of an electron in n^{th} orbit varies with n as:

[1 mark]

- (A) $\frac{1}{n^2}$
- (B) $\frac{1}{n}$
- (C) n
- (D) n^2

Correct Answer: (A) $\frac{1}{n^2}$

Solution:

Concept:

- In the Bohr model of hydrogen, the orbital radius is $r_n = a_0 n^2$, where a_0 is the ground-state radius and n is the principal quantum number.
- The electron-proton electrostatic potential energy, with zero at infinite separation, is $U_n = -e^2/(4\pi\epsilon_0 r_n)$. The negative sign represents the attractive interaction.

Step 1: Substitute the Bohr radius dependence Inserting $r_n = a_0 n^2$ into the Coulomb potential-energy expression gives

$$U_n = -\frac{e^2}{4\pi\epsilon_0 a_0} \frac{1}{n^2}.$$

All factors preceding $1/n^2$ are independent of the orbit number. Thus the dependence is inverse-square, even though the energy itself is negative.

Step 2: Distinguish energy from radius Equivalently, $U_n = -27.2/n^2$ eV, so increasing n makes the potential energy less negative and reduces its magnitude. Option (B) misses one power of n ; (C) predicts a linear dependence; and (D) is the dependence of orbital radius, not potential energy.

Step 3: Conclusion The potential energy varies as $1/n^2$, with a fixed negative proportionality constant. Hence, the correct option is (A).

Quick Tip to Solve: Recall $r_n \propto n^2$, then use $U \propto -1/r$; do not confuse the orbital-radius dependence with the potential-energy dependence.

4. Two metal spheres of radii r_1 and r_2 ($r_2 > r_1$) having charges q_1 and q_2 respectively kept in air, are brought in contact. Which of the following statements is NOT correct? [1 mark]

- (A) The total charge of the two spheres is conserved.
(B) Both spheres attain the same potential.
(C) The final potential of the system equals $\frac{1}{4\pi\epsilon_0} \frac{(q_1 + q_2)}{(r_1 + r_2)}$.
(D) The final potential of the system equals $\frac{1}{4\pi\epsilon_0} \frac{(q_1 + q_2)(r_1 + r_2)}{r_1 r_2}$.

Correct Answer: (D) The final potential of the system equals $\frac{1}{4\pi\epsilon_0} \frac{(q_1 + q_2)(r_1 + r_2)}{r_1 r_2}$.

Solution:

Concept:

- Contact between conducting spheres permits charge transfer until their potentials become equal. In the absence of earthing, the total charge $Q = q_1 + q_2$ is conserved.
- In the standard spherical-capacitance model used here, $C_i = 4\pi\epsilon_0 r_i$. At a common potential V , the charges satisfy $q'_i = C_i V$, where primes denote final charges.

Step 1: Apply equilibrium and charge conservation Charge flows only until there is no potential difference between the spheres. Consequently, statements (A) and (B) are correct. The final charges need not be equal because the radii differ; instead $q'_1/r_1 = q'_2/r_2$.

Step 2: Calculate the common potential Adding the two final charges gives

$$q_1 + q_2 = q'_1 + q'_2 = 4\pi\epsilon_0(r_1 + r_2)V,$$
$$V = \frac{1}{4\pi\epsilon_0} \frac{q_1 + q_2}{r_1 + r_2}.$$

This verifies statement (C). Statement (D) uses the factor $(r_1 + r_2)/(r_1 r_2) = 1/r_1 + 1/r_2$, whereas the common-potential calculation requires $1/(r_1 + r_2)$. It is therefore not the general expression for the final potential.

Step 3: Conclusion The statement that is NOT correct is (D). Hence, the correct option is (D).

Quick Tip to Solve: For spheres sharing a potential, add their capacitances first: $V = Q/(C_1 + C_2)$. Also check the word NOT before choosing the option.

5. In Bohr model of hydrogen atom, an electron makes a transition from $n = 4$ state to $n = 1$ state and a photon of frequency ν is emitted. The frequency of photon emitted when an electron makes a transition from $n = 4$ state to $n = 2$ state in the same model is: [1 mark]

- (A) $\frac{\nu}{3}$
(B) $\frac{\nu}{4}$
(C) $\frac{\nu}{5}$
(D) $\frac{\nu}{6}$

Correct Answer: (C) $\frac{\nu}{5}$

Solution:

Concept:

- Hydrogen energy levels obey $E_n = -13.6/n^2$ eV. The emitted photon's energy is the positive difference $h\nu = E_i - E_f$, where h is Planck's constant.
- For a transition from n_i to a lower n_f , the frequency is proportional to $1/n_f^2 - 1/n_i^2$. Taking a frequency ratio cancels the common energy scale and h .

Step 1: Write the two energy differences Let ν' be the frequency for the $4 \rightarrow 2$ transition. The given $4 \rightarrow 1$ transition and the required transition satisfy

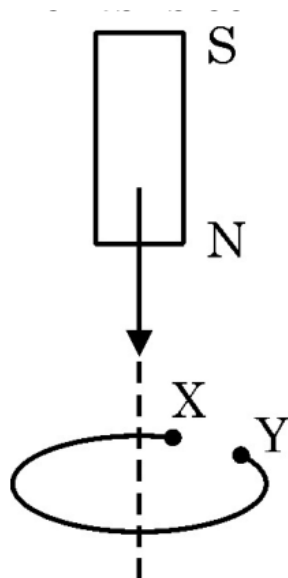
$$h\nu = (13.6 \text{ eV}) \left(1 - \frac{1}{16} \right) = (13.6 \text{ eV}) \frac{15}{16},$$
$$h\nu' = (13.6 \text{ eV}) \left(\frac{1}{4} - \frac{1}{16} \right) = (13.6 \text{ eV}) \frac{3}{16}.$$

Step 2: Compare the frequencies Dividing the second equation by the first gives $\nu'/\nu = (3/16)/(15/16) = 1/5$. The level spacing depends on reciprocal squares, so halving the final orbit number does not give a simple linear energy change. Options (A), (B) and (D), which give ratios $1/3$, $1/4$ and $1/6$, do not match the energy-difference ratio.

Step 3: Conclusion The photon emitted in the $4 \rightarrow 2$ transition has frequency $\nu/5$. Hence, the correct option is (C).

Quick Tip to Solve: When two transitions begin at the same level, divide their reciprocal-square differences; here the common denominator 16 cancels immediately.

6. Figure shows a magnet dropped through a small loop with a small cut. Which of the following statements is correct? [1 mark]



- (A) The speed of the falling magnet increases as it approaches the loop and starts decreasing as it crosses the loop.
- (B) Acceleration of magnet increases as it approaches the loop and starts decreasing as it crosses the loop.
- (C) Speed of magnet remains uniform as it moves through the loop.
- (D) Acceleration of the magnet remains uniform as it moves through the loop.

Correct Answer: (D) Acceleration of the magnet remains uniform as it moves through the loop.

Solution:

Concept:

- A changing magnetic flux produces an induced emf according to Faraday's law, $\mathcal{E} = -d\Phi_B/dt$. A sustained current around a loop also requires a closed conducting path.
- Lenz's-law magnetic braking arises from the magnetic field of an induced current. A cut interrupts that current path, so the ideal open loop does not exert the usual braking force on the magnet.

Step 1: Recognise the interrupted circuit As the magnet falls, its magnetic flux through the loop changes. An emf develops, but the gap between X and Y makes the loop an open circuit. Therefore no circulating induced current flows around it in the ideal circuit model, and no opposing magnetic braking force is produced.

Step 2: Apply the equation of motion Neglecting air resistance, the magnet is acted on by its weight, so $ma = mg$ and $a = g$ downward both before and after crossing the loop. Its speed continues to increase during the fall; it does not decrease after crossing as (A) claims or remain constant as (C) claims. Option (B) incorrectly assigns a changing acceleration despite the absence of the loop-current force.

Step 3: Conclusion The magnet falls with uniform downward acceleration g . Hence, the correct option is (D).

Quick Tip to Solve: Check whether the loop is closed before applying magnetic braking: an induced emf can exist across a cut even when no circulating current flows.

7. The expressions of magnetic fields associated with four electromagnetic waves are given below:

I. $B_1 = (4 \times 10^{-6} \text{ T}) \sin[0.7 \times 10^3 x + 1.4 \times 10^{11} t]$

II. $B_2 = (2 \times 10^{-7} \text{ T}) \sin[0.6 \times 10^3 x + 1.5 \times 10^{11} t]$

III. $B_3 = (3 \times 10^{-5} \text{ T}) \sin[0.5 \times 10^3 x + 1.5 \times 10^{11} t]$

IV. $B_4 = (5 \times 10^{-4} \text{ T}) \sin[0.2 \times 10^4 x + 4.8 \times 10^{11} t]$

Which wave is travelling in free space?

[1 mark]

(A) I

(B) II

(C) III

(D) IV

Correct Answer: (C) III

Solution:

Concept:

- For a sinusoidal wave $B = B_0 \sin(kx + \omega t)$, the speed is $v = \omega/k$, where k is the coefficient of position and ω is the coefficient of time.
- An electromagnetic wave in free space has speed $c = 3 \times 10^8 \text{ m s}^{-1}$. The amplitude B_0 does not determine this speed; the plus sign in the phase indicates propagation along negative x .

Step 1: Calculate the four wave speeds Read ω and k from each phase, retaining their powers of ten:

$$\begin{aligned}v_{\text{I}} &= \frac{1.4 \times 10^{11}}{0.7 \times 10^3} = 2.0 \times 10^8 \text{ m s}^{-1}, \\v_{\text{II}} &= \frac{1.5 \times 10^{11}}{0.6 \times 10^3} = 2.5 \times 10^8 \text{ m s}^{-1}, \\v_{\text{III}} &= \frac{1.5 \times 10^{11}}{0.5 \times 10^3} = 3.0 \times 10^8 \text{ m s}^{-1}, \\v_{\text{IV}} &= \frac{4.8 \times 10^{11}}{0.2 \times 10^4} = 2.4 \times 10^8 \text{ m s}^{-1}.\end{aligned}$$

Step 2: Compare with the speed in free space Only wave III gives $\omega/k = c$. Waves I, II and IV have smaller ratios and therefore do not satisfy the free-space speed condition. Their different magnetic amplitudes do not change this comparison. All four phases have the same propagation-direction sign, so direction also cannot distinguish the options.

Step 3: Conclusion Wave III travels at $3 \times 10^8 \text{ m s}^{-1}$, the speed of electromagnetic waves in free space. Hence, the correct option is (C).

Quick Tip to Solve: Divide the time coefficient by the position coefficient in each phase; for wave IV, retain 10^4 instead of the 10^3 used in the other three.

8. Which of the following substance has relative magnetic permeability $\mu_r \gg 1$?

[1 mark]

- (A) Aluminium
- (B) Copper
- (C) Lead
- (D) Nickel

Correct Answer: (D) Nickel

Solution:

Concept:

- Relative magnetic permeability is $\mu_r = \mu/\mu_0 = 1 + \chi_m$, where χ_m is magnetic susceptibility.
- Ferromagnetic materials have a large positive susceptibility and hence $\mu_r \gg 1$. Paramagnetic and diamagnetic materials have permeability close to unity.

Step 1: Identify the required magnetic behaviour The condition $\mu_r \gg 1$ means much greater than unity, not merely slightly greater. It characterises a ferromagnetic substance whose magnetic domains can align strongly in an applied field.

Step 2: Compare all four substances Nickel is ferromagnetic under ordinary conditions. Aluminium is paramagnetic, so its μ_r is only slightly greater than unity. Copper and lead are diamagnetic, so their μ_r values are slightly less than unity. Therefore the other three substances do not satisfy the stated condition.

Step 3: Conclusion Nickel has $\mu_r \gg 1$. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish $\mu_r > 1$ from $\mu_r \gg 1$: the latter points to a ferromagnet such as nickel.

9. A straight conductor lies along x -axis and carries a current of 2 A along $+x$ direction. The magnetic field at a point $(0, 40 \text{ cm}, 0)$ due to 1 cm length of conductor centered at the origin points along:

[1 mark]

- (A) y -axis
- (B) $-y$ -axis
- (C) z -axis
- (D) $-z$ -axis

Correct Answer: (C) z -axis

Solution:

Concept:

- The Biot–Savart law gives $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3}$, where $d\vec{\ell}$ follows the current and $\vec{r}$ points from the element to the observation point.
- The field is perpendicular to both vectors; its sense follows the right-hand rule for their cross product.

Step 1: Set the current and displacement directions The current is along $+x$, so $d\vec{\ell} = dx \hat{i}$. For an element at $(x, 0, 0)$ and the observation point $(0, 0.40 \text{ m}, 0)$, the displacement is $\vec{r} = -x\hat{i} + 0.40\hat{j}$, in SI units.

Step 2: Apply the cross product The component along the wire contributes zero to the cross product,

leaving

$$d\vec{\ell} \times \vec{r} = 0.40 dx (\hat{i} \times \hat{j}) = 0.40 dx \hat{k}.$$

Every element of the stated length therefore contributes along $+z$. The field cannot be along either y direction, and $-z$ would require reversal of the current or the observation side. The current and length affect the magnitude, not this direction.

Step 3: Conclusion The magnetic field points along the positive z -axis. Hence, the correct option is (C).

Quick Tip to Solve: For current along $+x$ and a point on $+y$, use $\hat{i} \times \hat{j} = \hat{k}$ immediately.

10. A galvanometer of resistance 27Ω is converted into an ammeter of range (0-10 mA) using a resistance of 3Ω . The galvanometer will show full scale deflection for a current of about: [1 mark]

- (A) 10 mA
- (B) 100 mA
- (C) 1 mA
- (D) 3 mA

Correct Answer: (C) 1 mA

Solution:

Concept:

- A galvanometer becomes an ammeter by connecting a low shunt resistance in parallel, so most of the total current bypasses the galvanometer.
- At full scale, equal voltages across the parallel branches give $I_g G = (I - I_g)S$, where G is galvanometer resistance, S is shunt resistance and I is the ammeter range.

Step 1: Apply the equal-voltage condition Here $G = 27 \Omega$, $S = 3 \Omega$ and the total full-scale current is $I = 10$ mA. Only the part I_g flows through the galvanometer; the shunt carries $I - I_g$.

$$27I_g = 3(10 - I_g),$$

with both currents expressed in milliamperes.

Step 2: Solve for the galvanometer current Rearranging gives

$$I_g = \frac{IS}{G + S} = \frac{10 \times 3}{27 + 3} \text{ mA} = 1 \text{ mA}.$$

The shunt carries 9 mA, as expected for the lower resistance. The 10 mA option confuses total and galvanometer currents; 100 mA exceeds the total, and 3 mA fails the equal-voltage condition.

Step 3: Conclusion The galvanometer shows full-scale deflection at 1 mA. Hence, the correct option is (C).

Quick Tip to Solve: For a parallel shunt, use $I_g/I = S/(G + S)$; here the galvanometer carries one tenth of the total current.

11. The magnetic flux ϕ (in Wb) linked with a coil is related to time t (in s) as $\phi = 5At^2 + Bt - 2C$. The SI units of A and B are respectively: [1 mark]

- (A) Wb s^2 , Wb s
- (B) Wb s^{-1} , Wb
- (C) Wb s^{-2} , Wb s^{-1}
- (D) Wb s^{-1} , Wb s^{-2}

Correct Answer: (C) Wb s^{-2} , Wb s^{-1}

Solution:

Concept:

- Dimensional homogeneity requires every term added in an equation to have the same units as the quantity on the other side.
- Magnetic flux has SI unit weber (Wb), and time has unit second (s); numerical factors such as 5 and 2 are dimensionless.

Step 1: Determine the units of each coefficient Since $5At^2$ must have units of flux, $[A] \text{ s}^2 = \text{Wb}$. Similarly, Bt must have flux units, so $[B] \text{ s} = \text{Wb}$. Thus

$$[A] = \text{Wb s}^{-2}, \quad [B] = \text{Wb s}^{-1}.$$

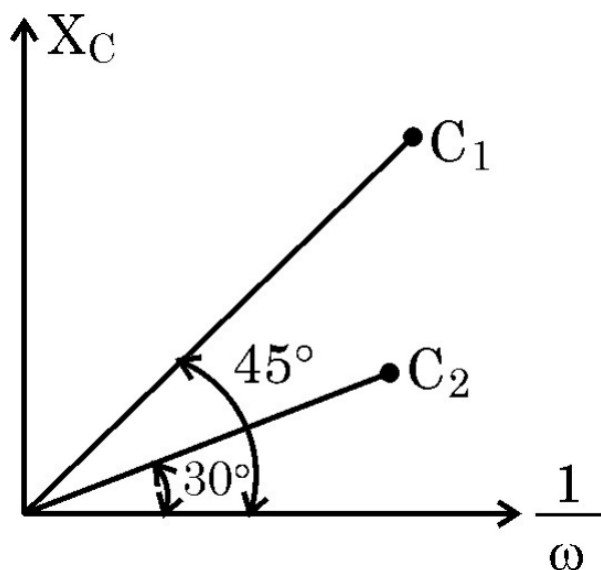
The constant term also implies $[C] = \text{Wb}$, consistent with the equation.

Step 2: Check the order and reject the alternatives The coefficient of the quadratic time term needs two inverse powers of seconds; the coefficient of the linear term needs one. Option (A) uses positive powers, option (B) does not cancel the time factors, and option (D) reverses the required units. Only option (C) makes both terms fluxes in the stated order.

Step 3: Conclusion The SI units of A and B are Wb s^{-2} and Wb s^{-1} , respectively. Hence, the correct option is (C).

Quick Tip to Solve: For a coefficient multiplying t^n , divide the units of the final quantity by s^n .

12. The figure shows the variation of capacitive reactance (X_C) of two ideal capacitors of capacitances C_1 and C_2 with the reciprocal of angular frequency ($1/\omega$) of ac source. The value of C_1/C_2 is: [1 mark]



- (A) $\frac{1}{2}$
 (B) 2
 (C) $\sqrt{3}$
 (D) $\frac{1}{\sqrt{3}}$

Correct Answer: (D) $\frac{1}{\sqrt{3}}$

Solution:

Concept:

- Capacitive reactance is $X_C = 1/(\omega C)$, where ω is angular frequency and C is capacitance.
- A graph of X_C against $1/\omega$ is a straight line with slope $1/C$. With common axis scales, the ratio of slopes equals the ratio of the tangents of their inclinations.

Step 1: Read the inclinations from the graph The line labelled C_1 makes 45° with the horizontal axis, and the line labelled C_2 makes 30° . Both angles are measured from the horizontal; 45° is not the angle between the two lines.

$$\frac{m_1}{m_2} = \frac{\tan 45^\circ}{\tan 30^\circ} = \sqrt{3}.$$

Step 2: Convert the slope ratio into a capacitance ratio Since $m_1 = 1/C_1$ and $m_2 = 1/C_2$, the desired ratio is the inverse of the slope ratio:

$$\frac{C_1}{C_2} = \frac{m_2}{m_1} = \frac{\tan 30^\circ}{\tan 45^\circ} = \frac{1}{\sqrt{3}}.$$

The steeper line must represent the smaller capacitance. Thus 2 and $\sqrt{3}$ cannot be the required ratio, while $1/2$ does not match the tangent ratio.

Step 3: Conclusion The capacitance ratio is $C_1/C_2 = 1/\sqrt{3}$. Hence, the correct option is (D).

Quick Tip to Solve: In an X_C versus $1/\omega$ graph, invert the slope ratio to obtain the capacitance ratio.

13. Assertion (A): Light added to light can produce darkness.

Reason (R): When two coherent light waves interfere, there is darkness at position of destructive interference. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Concept:

- The superposition principle adds the electric-field amplitudes of light waves, whereas observed intensity is proportional to the square of the resultant amplitude.
- For two coherent waves, $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$, where δ is their phase difference. Equal intensities and an odd multiple of π give zero resultant intensity.

Step 1: Evaluate the assertion Light added to light can produce darkness because equal-amplitude waves arriving in opposite phase cancel their electric fields. For $I_1 = I_2 = I_0$ and $\delta = \pi$,

$$I = 2I_0 + 2I_0 \cos \pi = 0.$$

Thus Assertion (A) is true. The word “can” refers to suitable coherent waves and suitable positions.

Step 2: Evaluate the reason Reason (R) describes destructive interference, which produces dark positions when the interfering amplitudes are equal. This is the standard dark-fringe condition; unequal amplitudes give a nonzero minimum. In the stated interference context, Reason (R) is true.

Step 3: Test the explanatory link Destructive interference is precisely the mechanism by which two light waves can give darkness at a point. Therefore the reason explains the assertion, excluding option (B). Options (C) and (D) wrongly treat at least one statement as false. Energy is redistributed into brighter regions, not destroyed.

Step 4: Conclusion Both statements are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: For “light plus light gives darkness”, add field amplitudes first and check for equal amplitudes in opposite phase.

14. Assertion (A): Two electric heaters of power P_1 and P_2 ($> P_1$) are joined in series across a dc source of voltage V . The power consumed by the combination will be less than that consumed by P_1 when connected across the same source.

Reason (R): The power consumed by an electric device when connected to a dc source of voltage V is proportional to its resistance. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- For a resistive device across a fixed voltage V , the electrical power is $P = VI = V^2/R$, where R is its resistance.
- Resistances add in series: $R_s = R_1 + R_2$. Increasing the total resistance at fixed voltage reduces the power drawn from the source.

Step 1: Evaluate the assertion Treating the heater resistances as fixed, their individual powers at voltage V give $R_1 = V^2/P_1$ and $R_2 = V^2/P_2$. In series across the same source,

$$P_s = \frac{V^2}{R_1 + R_2} = \frac{P_1 P_2}{P_1 + P_2}.$$

Since $P_1 + P_2 > P_2$, we have $P_s < P_1$. Equivalently, $R_1 + R_2 > R_1$ directly proves this result. Assertion (A) is true.

Step 2: Evaluate the reason At the fixed source voltage specified, $P = V^2/R$ makes power inversely proportional to resistance. Reason (R) claims direct proportionality and is false. The relation $P = I^2 R$ implies direct proportionality only when current, rather than voltage, is held constant.

Step 3: Select the assertion–reason relation The assertion is true and the reason is false, so the reason cannot explain the assertion. Options (A) and (B) incorrectly call the reason true, while option (D) incorrectly calls the assertion false.

Step 4: Conclusion The series combination consumes $P_s = P_1 P_2 / (P_1 + P_2) < P_1$. The assertion is true but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: First identify what is fixed: use $P = V^2/R$ for the same voltage source and $P = I^2 R$ for the same current.

15. Assertion (A): On increasing the intensity of incident light of frequency $\nu (> \nu_0)$ on a photosensitive surface, the photocurrent increases.

Reason (R): The stopping potential for a photosensitive surface increases with increase of frequency $\nu (> \nu_0)$ of incident light. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

Solution:

Concept:

- At a fixed frequency above the threshold frequency, increasing intensity increases the number of photons incident per second and hence the rate of photoelectron emission.
- Einstein's photoelectric equation is $eV_s = h\nu - \phi$, where V_s is the stopping potential, e is the elementary charge, h is Planck's constant and ϕ is the work function.

Step 1: Evaluate the assertion Since $\nu > \nu_0$, the incident photons can eject electrons. Raising the intensity at this fixed frequency supplies more photons per second, producing more photoelectrons per second. Under the same collection conditions the photocurrent therefore increases. Assertion (A) is true.

Step 2: Evaluate the reason The work function is fixed for the given surface. From $V_s = (h\nu - \phi)/e$, a higher frequency produces a larger maximum electron kinetic energy and a larger stopping potential. Reason (R) is also true.

Step 3: Check the explanatory connection The assertion concerns the number of electrons emitted per second when intensity changes. The reason concerns the maximum energy per electron when frequency changes. The reason does not explain the assertion, so option (A) fails. Options (C) and (D) incorrectly classify at least one true statement as false.

Step 4: Conclusion Both statements are true, but the reason does not explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Separate photon number from photon energy: intensity controls photocurrent, while frequency controls stopping potential.

16. Assertion (A): On forward biasing a p-n junction diode, the height of the barrier potential increases.

Reason (R): In forward biasing of a p-n junction diode, the direction of the applied voltage is in the same direction as the built-in potential. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

Concept:

- A junction's built-in electric field points from the positively charged donor ions on the n side towards the negatively charged acceptor ions on the p side.
- Forward bias connects the p side to the positive terminal and the n side to the negative terminal. Its electric field opposes the built-in field and lowers the potential barrier.

Step 1: Evaluate the assertion If V_b is the equilibrium barrier height and V_F is the applied forward voltage, the effective barrier is reduced to approximately $V_b - V_F$. The depletion layer also narrows, allowing majority carriers to cross more easily. The assertion that the barrier height increases is false.

Step 2: Evaluate the reason The applied forward bias acts in opposition to the built-in barrier: its field points from p to n, whereas the built-in field points from n to p. The claim that the applied voltage acts in the same direction as the built-in potential is therefore false.

Step 3: Check the explanatory connection Since both statements are false, the reason cannot explain the assertion. Options (A) and (B) require both statements to be true, and option (C) requires a true assertion; none describes this junction correctly.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: Remember the forward-bias connection p-positive, n-negative: it lowers the barrier and narrows the depletion layer.

Section B (10 marks)

17(a). A 5 cm long pencil is placed along the principal axis of a concave mirror of focal length 20 cm such that its nearest end is at a distance of 25 cm from the mirror. Calculate the length of the image of the pencil. [2 marks]

Solution:

Concept:

- The mirror formula is $1/f = 1/v + 1/u$, where f , u and v are the signed focal, object and image distances measured from the pole.
- For a pencil along the principal axis, find the image position of each endpoint separately. The image length is the absolute separation of these two image positions.

Step 1: Assign signed distances to the endpoints Take the incident light direction as positive in the Cartesian convention. The concave mirror has $f = -20$ cm. The near end is 25 cm in front of the mirror and the far end is $25 + 5 = 30$ cm in front, giving $u_1 = -25$ cm and $u_2 = -30$ cm.

Step 2: Apply the mirror formula to each endpoint For the near end,

$$\frac{1}{v_1} = \frac{1}{-20} - \frac{1}{-25} = -\frac{1}{100} \text{ cm}^{-1}, \quad v_1 = -100 \text{ cm}.$$

For the far end,

$$\frac{1}{v_2} = \frac{1}{-20} - \frac{1}{-30} = -\frac{1}{60} \text{ cm}^{-1}, \quad v_2 = -60 \text{ cm}.$$

Both image endpoints lie in front of the mirror. The endpoint nearer the focus has its image farther from the mirror. Since the pencil lies entirely beyond the focus, its image extends continuously between these positions.

Step 3: Conclusion The length of the image is

$$L = |v_1 - v_2| = |-100 - (-60)| = \boxed{40 \text{ cm}}.$$

Quick Tip to Solve: For an object along the axis, subtract the two endpoint image distances; transverse magnification cannot be applied to its full axial length.

17(b). In a Young's double-slit experiment, a beam of light consisting of two wavelengths 500 nm and 600 nm is used. The interference fringes are observed at a screen placed 1.8 m away from the plane of slits (slit separation 0.3 mm). Calculate the least distance from the central maximum where the bright fringes due to both the wavelengths coincide. [2 marks]

Solution:

Concept:

- In Young's double-slit experiment the m th bright fringe is at $y_m = m\lambda D/d$, where λ is the wavelength, D the screen distance and d the slit separation.
- Bright fringes of two wavelengths coincide when they have the same path difference: $m\lambda_1 = n\lambda_2$, with integer orders m and n .

Step 1: Find the smallest nonzero coincident orders The central maximum corresponds to $m = n = 0$. For the nearest other coincidence, use $\lambda_1 = 500$ nm and $\lambda_2 = 600$ nm:

$$500m = 600n \Rightarrow 5m = 6n.$$

Since 5 and 6 have no common factor, the smallest positive integers satisfying this relation are $m = 6$ and $n = 5$. Thus the sixth bright fringe of 500 nm coincides with the fifth bright fringe of 600 nm.

Step 2: Calculate the common position Convert all lengths to SI units: $D = 1.8$ m, $d = 0.3 \times 10^{-3}$ m and $\lambda_1 = 500 \times 10^{-9}$ m. Then

$$y = \frac{6(500 \times 10^{-9})(1.8)}{0.3 \times 10^{-3}} = 1.8 \times 10^{-2} \text{ m}.$$

As a check, using the fifth order of the second wavelength gives the same numerator because $5 \times 600 = 6 \times 500$. Smaller positive orders cannot give a common maximum.

Step 3: Conclusion The least nonzero distance from the central maximum is 1.8 cm, on either side of the central maximum.

Quick Tip to Solve: Reduce the wavelength ratio $500 : 600 = 5 : 6$; the first coincident bright-fringe orders are the reversed pair 6 : 5.

18. Calculate the temperature at which the resistance of a conductor becomes 20% more than its resistance at 27°C . The value of the temperature coefficient of resistance of the material of conductor is $2.0 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$. [2 marks]

Solution:

Concept:

- In the linear temperature model, $R_T = R_{27}[1 + \alpha(T - 27)]$, where R_{27} is the resistance at the given reference temperature 27°C , R_T is the resistance at temperature T , and α is the temperature coefficient.
- A resistance increase of 20% means $(R_T - R_{27})/R_{27} = 20/100 = 0.20$. A temperature difference has the same numerical value in degrees Celsius and kelvin.

Step 1: Express the required fractional increase The final resistance must be $R_T = 1.20R_{27}$. Substitute

this into the temperature relation, using the given coefficient as constant in the stated linear model:

$$1.20R_{27} = R_{27}[1 + \alpha(T - 27)].$$

Dividing by R_{27} and subtracting 1 removes the unknown initial resistance. Thus

$$0.20 = \alpha(T - 27).$$

Step 2: Calculate the temperature rise Insert $\alpha = 2.0 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$:

$$T - 27 = \frac{0.20}{2.0 \times 10^{-4}} = 1000 \text{ }^\circ\text{C}.$$

This is the required rise above the initial temperature, so it must be added to $27 \text{ }^\circ\text{C}$. The positive coefficient makes the required temperature higher than the starting value.

Step 3: Conclusion The required temperature is

$$T = 27 + 1000 = \boxed{1027 \text{ }^\circ\text{C}}.$$

Quick Tip to Solve: Use $\Delta T = 0.20/\alpha$ for a 20% increase, then add the initial $27 \text{ }^\circ\text{C}$; the rise alone is not the final temperature.

19. A ray of light is incident on the face of a triangular prism of refracting angle 60° and it just suffers total internal reflection at the other face. Find the angle of incidence for the ray if the refractive index of the material of the prism is $\sqrt{2}$. [2 marks]

Solution:

Concept:

- At the limiting condition for total internal reflection from glass to air, the internal angle of incidence equals the critical angle C , where $\sin C = 1/\mu$ and μ is the refractive index of glass relative to air.
- Inside a prism, $r_1 + r_2 = A$, where A is the prism angle and r_1, r_2 are measured from the normals to the two faces. At the first face, Snell's law gives $\sin i = \mu \sin r_1$.

Step 1: Determine the limiting angle at the second face The phrase “just suffers total internal reflection” specifies the critical-angle limit at the second face. Since the prism is surrounded by air,

$$\sin C = \frac{1}{\sqrt{2}}, \quad C = 45^\circ.$$

Thus $r_2 = C$. The prism relation then fixes the refraction angle at the first face:

$$r_1 = A - r_2 = 60^\circ - 45^\circ = 15^\circ.$$

Step 2: Apply refraction at the first face The incident ray travels from air into the prism, so the

refractive index multiplies the sine of the internal angle, not that of the external angle. Hence

$$\begin{aligned}\sin i &= \sqrt{2} \sin 15^\circ \\ &= \sqrt{2} \left(\frac{\sqrt{6} - \sqrt{2}}{4} \right) = \frac{\sqrt{3} - 1}{2}.\end{aligned}$$

Taking the inverse sine gives the angle measured from the normal to the first face.

Step 3: Conclusion The required angle of incidence at the limiting condition is

$$i = \sin^{-1} \left(\frac{\sqrt{3} - 1}{2} \right) \simeq 21.47^\circ.$$

Quick Tip to Solve: First set the second-face angle equal to the critical angle, then use $r_1 = A - C$ before applying Snell's law at entry.

20. Write two points of difference between intrinsic and extrinsic semiconductors.

[2 marks]

Solution:

Concept:

- An intrinsic semiconductor is a pure semiconductor. Its mobile electrons and holes are generated in pairs by thermal excitation.
- Doping means adding a controlled, small amount of suitable impurity to a semiconductor. The resulting extrinsic semiconductor has majority and minority charge carriers.

Step 1: Compare their composition **First difference: purity.** An intrinsic semiconductor is pure silicon or germanium, with no intentionally added dopant. An extrinsic semiconductor is doped with a suitable impurity: a pentavalent donor produces an n-type semiconductor, while a trivalent acceptor produces a p-type semiconductor. Thus the distinction concerns controlled doping, not a change of the host material from silicon or germanium.

Step 2: Compare the carrier concentrations **Second difference: electron and hole populations.** In an intrinsic semiconductor, the electron concentration n_e equals the hole concentration n_h :

$$n_e = n_h = n_i,$$

where n_i is the intrinsic carrier concentration. In an extrinsic semiconductor, these concentrations are unequal: $n_e > n_h$ for n-type and $n_h > n_e$ for p-type. Both kinds of carriers are present, but the dopant determines which kind is the majority carrier.

Step 3: Conclusion Intrinsic semiconductors are undoped and have equal electron and hole concentrations. Extrinsic semiconductors are doped and have unequal concentrations, with electrons dominant in n-type and holes dominant in p-type material.

Quick Tip to Solve: Give two matched comparisons: pure versus doped, then $n_e = n_h$ versus unequal carrier concentrations; name the majority carrier for each doped type.

21. Find the ratio (λ_α/λ_p) of the de Broglie wavelengths λ_α and λ_p associated respectively with an alpha particle and a proton:

(i) if they are moving with the same kinetic energy.

(ii) just after they are accelerated through the same potential difference.

[2 marks]

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is momentum. For a non-relativistic particle of mass m and kinetic energy K , $p = \sqrt{2mK}$.
- A particle accelerated from rest through a potential difference V gains kinetic energy $K = qV$, with q its charge magnitude. For an alpha particle, $m_\alpha \simeq 4m_p$ and $q_\alpha = 2e$, whereas a proton has charge $q_p = e$.

Step 1: Compare wavelengths at equal kinetic energy (i) Let both particles have kinetic energy K . Substituting the momentum in the wavelength formula gives

$$\lambda = \frac{h}{\sqrt{2mK}}.$$

When the ratio is taken, the common factors h , 2, and K cancel:

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p}{m_\alpha}} = \sqrt{\frac{m_p}{4m_p}}.$$

Thus the heavier particle has the shorter wavelength; equal kinetic energy does not imply equal momentum.

Step 2: Include the charge for equal accelerating potential (ii) For acceleration from rest through the same potential difference V , use $K = qV$. The alpha particle acquires twice the proton's kinetic energy because it carries twice its charge. Therefore

$$\lambda = \frac{h}{\sqrt{2mqV}},$$

and the wavelength ratio becomes

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p q_p}{m_\alpha q_\alpha}} = \sqrt{\frac{m_p e}{(4m_p)(2e)}}.$$

Both the mass factor and the charge factor must be retained in this case.

Step 3: Conclusion The required dimensionless ratios are

$$\boxed{\text{(i) } \frac{\lambda_\alpha}{\lambda_p} = \frac{1}{2}, \quad \text{(ii) } \frac{\lambda_\alpha}{\lambda_p} = \frac{1}{2\sqrt{2}}.}$$

Quick Tip to Solve: At equal kinetic energy compare $1/\sqrt{m}$; at equal accelerating potential compare $1/\sqrt{mq}$, since the alpha particle has charge $2e$.

Section C (21 marks)

22(a). (i) Define mutual inductance of a pair of coils. Write its SI unit.

(ii) A long solenoid of radius R and length L has n turns per unit length. A circular loop of radius r ($< R$) is placed inside at the centre of the solenoid such that its axis coincides with the axis of the solenoid. Obtain the mutual inductance of the solenoid and the loop. [3 marks]

Solution:

Concept:

- Mutual inductance M relates the magnetic flux linkage of one coil to the current in the other: $N_2\Phi_{21} = MI_1$. Here Φ_{21} is the flux through one turn of coil 2 due to current I_1 in coil 1, and N_2 is its number of turns.
- Near the centre of a long air-core solenoid, the magnetic field is uniform and axial: $B = \mu_0 nI$, where n is the number of turns per unit length and μ_0 is the permeability of free space.
- For a uniform field perpendicular to the plane of a loop, the flux is $\Phi = BA$, where A is the loop area. A single circular loop has one turn.

Step 1: Define mutual inductance and state its unit (i) The mutual inductance of a pair of coils is the magnetic flux linkage with one coil per unit current flowing in the other, for a fixed arrangement of the coils:

$$M = \frac{N_2\Phi_{21}}{I_1}.$$

Equivalently, the induced emf in the second coil is $\varepsilon_2 = -M dI_1/dt$. Its SI unit is the henry, symbol H:

$$1 \text{ H} = 1 \text{ Wb A}^{-1} = 1 \text{ V s A}^{-1}.$$

The pair has mutual inductance one henry if a current changing at 1 A s^{-1} in one coil induces an emf of magnitude 1 V in the other.

Step 2: Calculate the field and linked flux (ii) Let current I flow through the solenoid. Its total turns are $N = nL$, so its central field is

$$B = \mu_0 \frac{N}{L} I = \mu_0 nI.$$

The loop's axis coincides with the solenoid's axis, so the field is parallel to the loop's area vector. Since $r < R$, the whole loop lies inside the nearly uniform field. Only the loop area πr^2 contributes to its flux:

$$\Phi = B\pi r^2 = \mu_0 nI\pi r^2.$$

Using πR^2 would incorrectly count field passing outside the loop.

Step 3: Divide the loop's flux linkage by the current The loop has a single turn, so its flux linkage equals Φ . Substitution into the defining relation gives

$$M = \frac{\Phi}{I} = \frac{\mu_0 nI\pi r^2}{I}.$$

The current cancels, as expected for fixed geometry in air. The specified length L is already accounted for in the turn density $n = N/L$; no additional factor of L is required.

Step 4: Conclusion Mutual inductance is flux linkage in one coil per unit current in the other; its SI

unit is the henry. For the given solenoid and single loop,

$$M = \mu_0 n \pi r^2.$$

Quick Tip to Solve: Use the area of the smaller loop, πr^2 , and remember that n already means turns per unit length; do not multiply the final expression by L .

OR

22(b). Two long straight parallel conductors A and B carrying steady currents I_a and I_b in the same direction are separated by a distance d . Deduce the expressions for the force acting on length L of conductor B due to conductor A and show it in figure. Write the expression for the force acting on length L of conductor A due to conductor B and show that it follows Newton's third law. [3 marks]

Solution:

Concept:

- A long straight conductor carrying current I produces magnetic field $B = \mu_0 I / (2\pi r)$ at perpendicular distance r in air or vacuum; μ_0 is the permeability of free space.
- The magnetic force on a straight current-carrying length is $\vec{F} = I \vec{L} \times \vec{B}$, where $\vec{L}$ points along the current. Thus $F = ILB$ when the conductor and field are perpendicular.

Step 1: Find the field of conductor A at conductor B Draw both currents upward, with A to the left of B . Every point on B is a perpendicular distance d from the long conductor A , so the field there is uniform along the selected length:

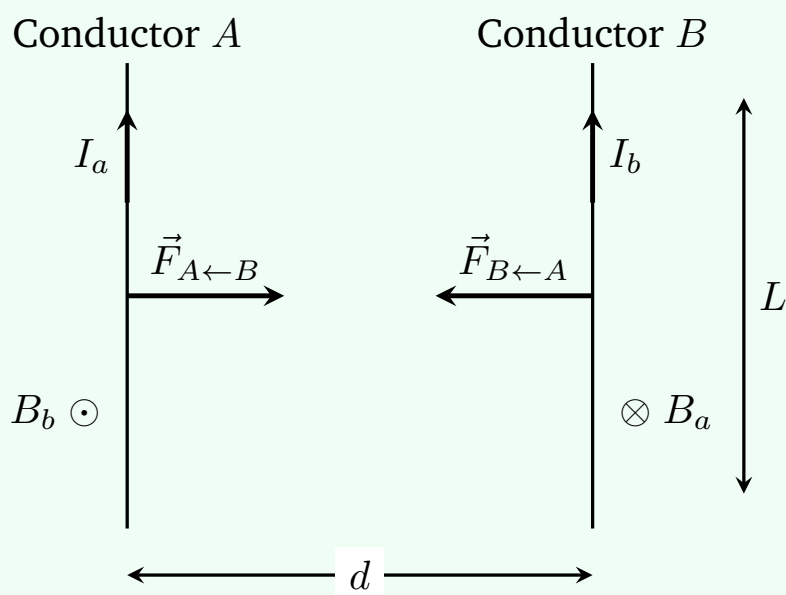
$$B_a = \frac{\mu_0 I_a}{2\pi d}.$$

By the right-hand thumb rule, this field is directed into the plane of the figure at B .

Step 2: Determine the force on length L of B The current I_b is perpendicular to $\vec{B}_a$. Therefore the magnitude of the force due to A on length L of B is

$$F_{B \leftarrow A} = I_b L B_a = \frac{\mu_0 I_a I_b L}{2\pi d}.$$

The cross product of the upward current direction and the inward field points left, towards conductor A . The interaction is attractive, as shown by the force arrows in the figure.



Parallel upward currents attract; $\otimes$ means into the page and $\odot$ means out of it.

Step 3: Find the reciprocal force and compare vectors At conductor A, the field due to B is $B_b = \mu_0 I_b / (2\pi d)$, directed out of the plane. Hence the force on the same length L of A is

$$F_{A \leftarrow B} = I_a L B_b = \frac{\mu_0 I_a I_b L}{2\pi d}.$$

Its direction is right, towards B. Taking $\hat{x}$ from A to B, the two force vectors are

$$\vec{F}_{A \leftarrow B} = \frac{\mu_0 I_a I_b L}{2\pi d} \hat{x}, \quad \vec{F}_{B \leftarrow A} = -\frac{\mu_0 I_a I_b L}{2\pi d} \hat{x}.$$

They act on different conductors and have equal magnitudes and opposite directions.

Step 4: Conclusion The two conductors attract with force $F = \mu_0 I_a I_b L / (2\pi d)$ on equal lengths L . Since $\vec{F}_{A \leftarrow B} = -\vec{F}_{B \leftarrow A}$, these forces obey Newton's third law.

Quick Tip to Solve: First find the field made by the other wire, then use ILB . Parallel currents attract, so each force arrow must point towards the other conductor.

23. Calculate the de Broglie wavelength associated with an electron revolving in the second excited state of hydrogen atom. The ground state energy of the hydrogen atom is -13.6 eV. Take $m_e = 9 \times 10^{-31}$ kg, $h = 6.6 \times 10^{-34}$ J s. [3 marks]

Solution:

Concept:

- In hydrogen, the energy of the orbit with principal quantum number n is $E_n = -13.6/n^2$ eV. The ground state is $n = 1$; the second excited state is $n = 3$.
- For an electron in a Coulomb orbit, its potential energy is $U = -2K$, so its total energy is $E = K + U = -K$. The positive kinetic energy K is therefore the magnitude of the negative total energy.
- The de Broglie relation is $\lambda = h/p = h/\sqrt{2m_e K}$, where p is the electron momentum, h is Planck's

constant and m_e is its mass. Use $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.

Step 1: Identify the orbit and its kinetic energy The first excited state is $n = 2$, so the second excited state corresponds to $n = 3$, not $n = 2$. Its total energy and kinetic energy are

$$E_3 = -\frac{13.6}{3^2} = -1.511 \text{ eV}, \quad K = -E_3 = 1.511 \text{ eV}.$$

The negative sign of E_3 expresses that the electron is bound to the proton; it does not make the electron's kinetic energy negative.

Step 2: Convert the kinetic energy and find momentum Since the given mass and Planck's constant are in SI units, convert the energy to joules before substituting:

$$K = \frac{13.6}{9}(1.6 \times 10^{-19}) = 2.418 \times 10^{-19} \text{ J}.$$

Using the mass specified in the question gives

$$\begin{aligned} p &= \sqrt{2m_e K} \\ &= \sqrt{2(9 \times 10^{-31})(2.418 \times 10^{-19})} \\ &= 6.597 \times 10^{-25} \text{ kg m s}^{-1}. \end{aligned}$$

Step 3: Apply the de Broglie relation Divide the supplied value of h by the momentum:

$$\lambda = \frac{6.6 \times 10^{-34}}{6.597 \times 10^{-25}} = 1.0005 \times 10^{-9} \text{ m}.$$

The units reduce to metres because $\text{J s} = \text{kg m}^2 \text{s}^{-1}$. The orbit number enters through the electron's kinetic energy; the energy released in a transition is not needed here.

Step 4: Conclusion The electron's de Broglie wavelength in the second excited state is approximately $1.0 \times 10^{-9} \text{ m} = 1.0 \text{ nm}$.

Quick Tip to Solve: Convert the phrase second excited state to $n = 3$, then use $K = 13.6/n^2$ in eV and convert it to joules before taking the square root.

24. Name the electromagnetic waves which are used:

- (i) as a diagnostic tool in medicine
- (ii) in remote switches for TV sets
- (iii) in water purifiers

Also write their wavelength range.

[3 marks]

Solution:

Concept:

- Electromagnetic waves are classified by wavelength λ or frequency ν , related in vacuum by $c = \nu\lambda$. Their interaction with matter determines their practical uses.
- The boundaries of neighbouring spectral bands are approximate. State a wavelength interval for the named band, with consistent units: $1 \text{ nm} = 10^{-9} \text{ m}$ and $1 \text{ mm} = 10^{-3} \text{ m}$.

Step 1: Identify the waves used in medical diagnosis (i) **X-rays** are used as a diagnostic tool, for

example in radiographs of bones. They penetrate soft tissues more readily than bones, so different absorption produces image contrast. Their approximate wavelength range is

$$10^{-3} \text{ nm to } 1 \text{ nm, or } 10^{-12} \text{ m to } 10^{-9} \text{ m.}$$

These wavelengths are shorter than those of ultraviolet radiation.

Step 2: Identify the waves used in television remotes (ii) **Infrared waves** carry the coded signal from a remote control to the receiver in a television set. The wavelength of the signal lies beyond the red end of the visible spectrum. The infrared band has the approximate range

$$700 \text{ nm to } 1 \text{ mm, or } 7 \times 10^{-7} \text{ m to } 10^{-3} \text{ m.}$$

The remote uses a selected infrared wavelength; it need not emit the entire band.

Step 3: Identify the waves used in water purifiers (iii) **Ultraviolet waves** are used to disinfect water because suitable ultraviolet radiation inactivates microorganisms. Their approximate wavelength range is

$$1 \text{ nm to } 400 \text{ nm, or } 10^{-9} \text{ m to } 4 \times 10^{-7} \text{ m.}$$

Water disinfection uses a suitable part of this band. Ultraviolet wavelengths are shorter than visible wavelengths and longer than X-ray wavelengths in this classification.

Step 4: Conclusion (i) X-rays: 10^{-12} – 10^{-9} m; (ii) infrared: 7×10^{-7} – 10^{-3} m; (iii) ultraviolet: 10^{-9} – 4×10^{-7} m. These are approximate spectral-band ranges.

Quick Tip to Solve: Link the use to the effect: X-rays give absorption contrast, infrared carries remote-control signals, and ultraviolet disinfects water. Convert every endpoint to metres to check the powers of ten.

25. (i) Draw electric field lines and equipotential surfaces for a system of two equal and opposite point charges separated by some distance.

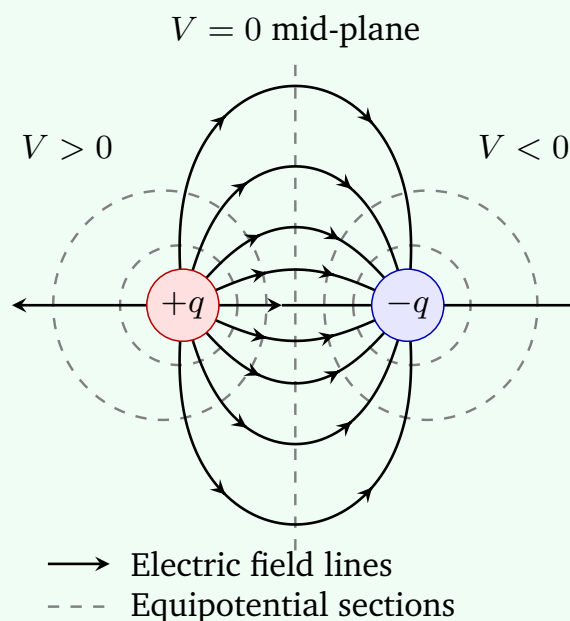
(ii) Why electric field $\vec{E}$ at a point on an equipotential surface must be perpendicular to the surface at that point? [3 marks]

Solution:

Concept:

- A pair of equal and opposite charges separated by a distance forms an electric dipole. Field lines leave the positive charge and enter the negative charge; their tangent gives the direction of $\vec{E}$.
- The dipole potential is $V = (q/4\pi\epsilon_0)(1/r_+ - 1/r_-)$, where r_+ and r_- are the distances from the positive and negative charges. An equipotential surface has constant V .
- For an infinitesimal displacement, the potential change is $dV = -\vec{E} \cdot d\vec{\ell}$. Consequently the component of electric field tangent to an equipotential surface is zero.

Step 1: (i) Draw the dipole field and equipotentials The solid arrowed lines in the figure run from $+q$ to $-q$. The dashed curves are plane sections of equipotential surfaces, which cut the field lines at right angles. Near either charge the surfaces are nearly spherical; farther away they are distorted by the other charge. Rotating the dashed curves about the horizontal dipole axis gives the corresponding three-dimensional surfaces. The perpendicular bisector is a plane with $r_+ = r_-$, so $V = 0$ everywhere on it. This zero potential does not mean that the electric field vanishes there.



Dipole field lines and sections of equipotential surfaces in a plane through the charges.

Step 2: (ii) Prove that the field is normal to the surface Take any small nonzero displacement $d\vec{\ell}$ along an equipotential surface at a point where $E \neq 0$. Because its endpoints have the same potential, $dV = 0$. If θ is the angle between $\vec{E}$ and this tangential displacement, then

$$0 = dV = -\vec{E} \cdot d\vec{\ell} = -E d\ell \cos \theta.$$

Hence $\cos \theta = 0$ and $\theta = 90^\circ$. This holds for every tangent direction at the point, so $\vec{E}$ must point along the surface normal. Equivalently, a tangential field would do nonzero work on a test charge moved along the surface, contradicting the equality of potential.

Step 3: Conclusion The dipole field lines point from $+q$ to $-q$ and cross equipotential surfaces normally. The electric field has no tangential component on an equipotential surface because $dV = -\vec{E} \cdot d\vec{\ell} = 0$ for every displacement along it.

Quick Tip to Solve: Draw arrows from the positive charge to the negative charge, then draw dashed equipotentials at right angles; include the zero-potential mid-plane between the charges.

- 26. (a) Consider the following nuclides:** ${}^{12}_6\text{C}$, ${}^{198}_{80}\text{Hg}$, ${}^{14}_6\text{C}$, ${}^{197}_{79}\text{Au}$. **Group them into isotopes and isotones.**
(b) How does the size of a nucleus depend on its mass number A ? Hence prove that the density of nucleus is a constant, independent of A , for all nuclei. [3 marks]

Solution:

Concept:

- In a nuclide ${}^A_Z\text{X}$, Z is the proton number, A is the total number of nucleons, and the neutron number is $n = A - Z$.
- Isotopes have the same Z but different A ; isotones have the same neutron number but different Z .
- Nuclear radius follows $R = R_0 A^{1/3}$, where $R_0 \simeq 1.2 \times 10^{-15} \text{ m}$. Nuclear mass is approximately Am_N , with m_N the mean nucleon mass.

Step 1: (a) Compare proton and neutron numbers Subtract the atomic number from the mass number for each nuclide:

Nuclide	A	Z	$n = A - Z$
${}^{12}_6\text{C}$	12	6	6
${}^{198}_{80}\text{Hg}$	198	80	118
${}^{14}_6\text{C}$	14	6	8
${}^{197}_{79}\text{Au}$	197	79	118

Both carbon nuclei have six protons, but their mass numbers are different; hence they form the isotope pair. Mercury and gold have different proton numbers, but each contains 118 neutrons; hence they form the isotone pair. Equal or nearly equal mass numbers alone do not identify isotones.

Step 2: (b) Express nuclear volume in terms of mass number The empirical radius law states that the nuclear radius grows as the cube root of A , rather than in direct proportion to A . Approximating a nucleus as a sphere gives

$$R = R_0 A^{1/3}, \quad V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} R_0^3 A.$$

Thus nuclear volume is proportional to the number of nucleons.

Step 3: Divide nuclear mass by nuclear volume Neglecting the small proton–neutron mass difference and binding-energy corrections, the nuclear mass is $M \simeq A m_N$. Therefore

$$\rho = \frac{M}{V} = \frac{A m_N}{(4\pi/3) R_0^3 A} = \frac{3 m_N}{4\pi R_0^3}.$$

The factor A cancels because adding nucleons increases mass and volume in the same proportion. With $m_N \simeq 1.67 \times 10^{-27} \text{ kg}$, this gives $\rho \simeq 2.3 \times 10^{17} \text{ kg m}^{-3}$. The common nuclear density is an approximation within the empirical radius law.

Step 4: Conclusion Isotopes: ${}^{12}_6\text{C}$ and ${}^{14}_6\text{C}$. Isotones: ${}^{198}_{80}\text{Hg}$ and ${}^{197}_{79}\text{Au}$. Since $R = R_0 A^{1/3}$, the density $\rho = 3 m_N / (4\pi R_0^3)$ is independent of A in this model.

Quick Tip to Solve: Compute $A - Z$ before grouping isotones; for nuclear density, cube $R_0 A^{1/3}$ first so that the cancelling factor A is explicit.

27. Derive an expression for the magnetic field $\vec{B}$, due to a circular coil of N turns, each of radius r carrying current I , at a distance x from the centre along its axis. [3 marks]

Solution:

Concept:

- Biot–Savart law gives $d\vec{B} = (\mu_0 I / 4\pi) (d\vec{\ell} \times \hat{s}) / s^2$, where $d\vec{\ell}$ points along the current and s joins the current element to the observation point.
- For a circular coil, transverse magnetic-field components from diametrically opposite elements cancel on the axis. Their axial components add; the field of N closely wound identical turns is N times the single-turn field.

Step 1: Set up the axial geometry Let O be the coil centre and P a point on its axis with $OP = x$. Take the axial unit vector $\hat{n}$ along the thumb direction given by the right-hand grip rule for the current. Every current element lies at radius r from O , so its distance from P is the same:

$$s = \sqrt{r^2 + x^2}.$$

The tangent element $d\vec{\ell}$ is perpendicular both to its radius and to the coil axis. Hence it is perpendicular to s , which lies in the plane containing that radius and the axis.

Step 2: Resolve the field of a current element The sine factor in Biot–Savart law is therefore unity, giving

$$dB = \frac{\mu_0 I d\ell}{4\pi(r^2 + x^2)}.$$

Since $d\vec{B}$ is perpendicular to s , the angle θ between $d\vec{B}$ and the coil axis satisfies $\cos \theta = r/s$. Thus the axial component is

$$dB_{\parallel} = dB \frac{r}{s} = \frac{\mu_0 I r d\ell}{4\pi(r^2 + x^2)^{3/2}}.$$

For each element, the diametrically opposite element produces an equal and opposite transverse component at P . Only the axial components survive after summing around the complete circle.

Step 3: Integrate around the coil and include every turn The quantities I , r and x are constant in the integration, and the total length of one turn is $\oint d\ell = 2\pi r$. Therefore

$$\begin{aligned} B_{\text{one turn}} &= \frac{\mu_0 I r}{4\pi(r^2 + x^2)^{3/2}} \oint d\ell \\ &= \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}, \\ \vec{B}(P) &= \frac{\mu_0 N I r^2}{2(r^2 + x^2)^{3/2}} \hat{n}. \end{aligned}$$

All turns carry current in the same sense, so their axial fields add. As a check, setting $x = 0$ gives the familiar centre field $\mu_0 N I / (2r)$.

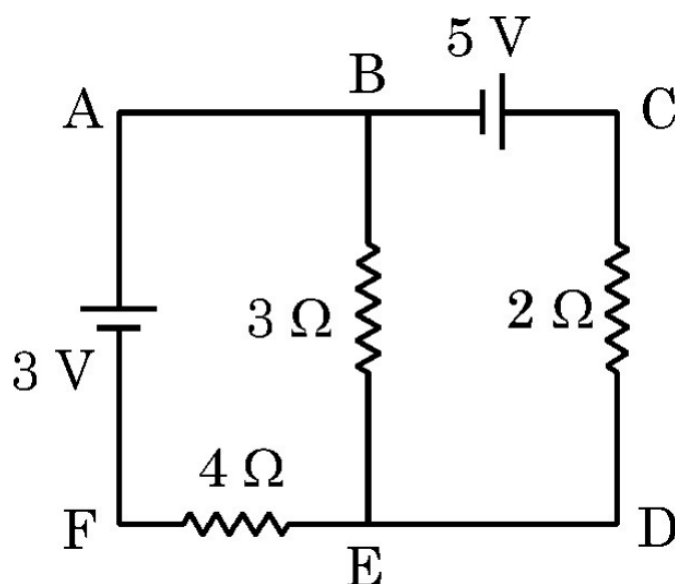
Step 4: Conclusion At axial distance x , $\vec{B} = \frac{\mu_0 N I r^2}{2(r^2 + x^2)^{3/2}} \hat{n}$. The direction is along the coil axis, fixed by the right-hand grip rule: curled fingers follow the current and the thumb gives $\hat{n}$.

Quick Tip to Solve: On a circular coil's axis, first use $s^2 = r^2 + x^2$, then multiply each element's field by r/s ; omitting this projection gives the wrong power of distance.

28. (a) Explain the statement: “Current is a scalar although we represent current with an arrow”.

(b) Use Kirchhoff’s rules to find the current through $3\ \Omega$ resistor in the circuit shown in the figure:

[3 marks]



Solution:

Concept:

- Electric current $I = dq/dt$ is a signed scalar. Its reference arrow specifies the sense in which charge crosses a conductor; currents at a junction add algebraically, not by vector addition.
- Kirchhoff’s junction rule expresses charge conservation: the total current entering a junction equals the total leaving it.
- Kirchhoff’s loop rule expresses energy conservation: $\sum \Delta V = 0$ around a closed loop. Across a cell from negative to positive terminal the potential rises by its emf; along a resistor’s current it falls by IR .

Step 1: Explain why the current arrow does not make current a vector A current arrow indicates the chosen direction of conventional charge flow along the wire. It is a reference for the sign of I , not a spatial vector to be resolved into components. If currents enter a junction through wires meeting at any angle, their magnitudes still add algebraically. For example, incoming currents I_a and I_b give outgoing current $I_a + I_b$, independently of the angle between the wires. Hence current does not obey the vector addition law. Current density is a vector, but current itself is a scalar.

Step 2: Assign branch currents and read the cell polarities Let I_1 flow from B to E through $3\ \Omega$, I_2 from C to D through $2\ \Omega$, and I_3 from E to F through $4\ \Omega$. The longer plate of the $3\ \text{V}$ cell is toward A, while that of the $5\ \text{V}$ cell is toward C. Applying the junction rule at E gives

$$I_3 = I_1 + I_2.$$

Any negative current obtained below means that its actual direction is opposite to the assigned direction.

Step 3: Apply the loop rule to the left and right loops Traversing F–A–B–E–F crosses the left cell from negative to positive and both resistors along their assigned currents:

$$3 - 3I_1 - 4I_3 = 0.$$

Traversing B–C–D–E–B crosses the right cell from negative to positive, the $2\ \Omega$ resistor along I_2 , and the $3\ \Omega$ resistor opposite I_1 :

$$5 - 2I_2 + 3I_1 = 0.$$

Thus $I_2 = (5 + 3I_1)/2$ and $I_3 = (3 - 3I_1)/4$, with currents measured in amperes.

Step 4: Solve the junction equation and check the direction Substituting the loop results in $I_3 = I_1 + I_2$ gives

$$\begin{aligned}\frac{3 - 3I_1}{4} &= I_1 + \frac{5 + 3I_1}{2}, \\ 3 - 3I_1 &= 4I_1 + 10 + 6I_1, \\ 13I_1 &= -7.\end{aligned}$$

The resulting branch currents are $I_1 = -7/13\text{ A}$, $I_2 = 22/13\text{ A}$, and $I_3 = 15/13\text{ A}$. They satisfy the junction rule because $-7/13 + 22/13 = 15/13$. The negative sign on I_1 reverses its assumed B-to-E direction.

Step 5: Conclusion Current is a scalar because it follows algebraic addition, with the arrow serving only as a sign convention. The current through the $3\ \Omega$ resistor is $7/13\text{ A} \approx 0.54\text{ A}$, directed upward from E to B.

Quick Tip to Solve: Read each cell's longer plate first, choose branch arrows freely, and retain any negative current until you state its actual direction.

Section D (8 marks)

Question numbers **29** and **30** are case study based questions. Read the following paragraphs and answer the questions that follow.

29. In an experiment with convex lens of focal length f , the screen is fixed at a distance D from the object. A student slowly moves the lens away from the object towards the screen and finds that she is able to form sharp image of the object for two positions of the lens. The distance between these two positions of the lens is d . [4 marks]

29(i). The value of d is: [1 mark]

- (A) $\sqrt{D(D - 4f)}$
- (B) $\sqrt{D(D - 2f)}$
- (C) $2\sqrt{Df}$
- (D) $\sqrt{D(D - f)}$

Correct Answer: (A) $\sqrt{D(D - 4f)}$

Solution:

Concept:

- For a real object and real image formed by a convex lens, using positive distance magnitudes u and v , the lens formula is $1/f = 1/u + 1/v$.
- The fixed object-screen separation is $D = u + v$. The two lens positions interchange u and v , and their separation is $d = |v - u|$.

Step 1: Express the product of the two distances Combining the lens formula with the fixed separation

gives

$$\frac{1}{f} = \frac{u+v}{uv} = \frac{D}{uv}, \quad uv = fD.$$

Thus the sum and product of the two positive distances are known without solving separately for either distance.

Step 2: Calculate the separation of the lens positions Since the two positions interchange the object and image distances,

$$d^2 = (v-u)^2 = (u+v)^2 - 4uv = D^2 - 4fD.$$

The coefficient of fD is therefore four; options (B) and (D) have incorrect coefficients. Option (C) instead gives $d^2 = 4fD$, which omits the D^2 term and does not follow from the difference of the two distances.

Step 3: Conclusion The required distance is $d = \sqrt{D(D-4f)}$. Hence, the correct option is (A).

Quick Tip to Solve: Remember the pair $u+v = D$ and $uv = fD$; then obtain the displacement from $(v-u)^2 = (u+v)^2 - 4uv$.

29(ii). Compared to the size of the object, the images formed in the two positions of the lens are respectively: [1 mark]

- (A) reduced, enlarged
- (B) reduced, reduced
- (C) enlarged, enlarged
- (D) enlarged, reduced

Correct Answer: (D) enlarged, reduced

Solution:

Concept:

- For a real image formed by a convex lens, the magnitude of transverse magnification is $|m| = v/u$, where u and v are positive object and image distances.
- The displacement method gives two conjugate lens positions: the values of u and v interchange, so the two magnification magnitudes are reciprocals.

Step 1: Examine the first position reached from the object The lens moves from the object toward the screen. At the first position giving a sharp image, the lens is closer to the object: $u < v$. Therefore $|m_1| = v/u > 1$, and this image is enlarged. A position with the object inside the focal length gives a virtual image and cannot form a sharp image on the screen.

Step 2: Compare the second position and the options At the second sharp-image position, the lens is nearer the screen and the conjugate distances have interchanged. Hence $|m_2| = u/v < 1$, and this image is reduced. Both real images are inverted, but their sizes differ. Option (A) reverses the stated order of motion; options (B) and (C) cannot satisfy the reciprocal relation $|m_1 m_2| = 1$ for two distinct positions.

Step 3: Conclusion The images are respectively enlarged, reduced. Hence, the correct option is (D).

Quick Tip to Solve: Follow the lens from object to screen: its first sharp image has the longer image distance and is enlarged; the second is reduced.

29(iii). If the distance between object and screen is 80.00 cm and the lens forms sharp images at two positions separated by 20.00 cm, the focal length of convex lens is: [1 mark]

- (A) 15.50 cm
- (B) 18.75 cm
- (C) 20.50 cm
- (D) 22.75 cm

Correct Answer: (B) 18.75 cm

Solution:

Concept:

- In the convex-lens displacement method, $d^2 = D^2 - 4Df$, where D is the object-screen distance, d the separation between lens positions, and f the focal length.
- Rearranging gives $f = (D^2 - d^2)/(4D)$. All distances must use the same unit; centimetres then give the focal length in centimetres.

Step 1: Substitute the measured distances Here $D = 80.00$ cm and $d = 20.00$ cm. Therefore

$$f = \frac{(80.00)^2 - (20.00)^2}{4(80.00)} \text{ cm} = \frac{6400 - 400}{320} \text{ cm}.$$

The numerator has units of square centimetres and the denominator has units of centimetres, so the final quantity is a length.

Step 2: Evaluate and check the conjugate distances The quotient is $6000/320 = 18.75$. As an independent check, the two positive distances are $(D - d)/2 = 30.00$ cm and $(D + d)/2 = 50.00$ cm. Their product divided by their sum gives $30.00 \times 50.00/80.00 = 18.75$ cm. The other listed focal lengths do not satisfy these measured conjugate distances; values above $D/4 = 20.00$ cm would not allow two sharp real-image positions at all.

Step 3: Conclusion The focal length is 18.75 cm. Hence, the correct option is (B).

Quick Tip to Solve: For a quick numerical check, use $f = D/4 - d^2/(4D) = 20.00 - 1.25 = 18.75$ cm.

29(iv)(a). Consider a convex lens of focal length 15 cm. For which of the following values of object-screen distance, two positions of the object can be found to obtain sharp image on the screen?

[1 mark]

- (A) 45 cm
- (B) 50 cm
- (C) 55 cm
- (D) 65 cm

Correct Answer: (D) 65 cm

Solution:

Concept:

- In the displacement method, two distinct positions of a convex lens produce sharp real images only if $D > 4f$, where D is the fixed object-screen separation and f the focal length.
- The lens-position separation satisfies $d^2 = D(D - 4f)$. A positive d^2 gives two positions; $D = 4f$ gives one coincident position, while $D < 4f$ gives no real-image position on that screen.

Step 1: Apply the displacement condition The printed phrase about two positions of the object is read in the context of the experiment, where the object and screen are fixed and the lens is moved. For the given focal length,

$$4f = 4(15 \text{ cm}) = 60 \text{ cm}.$$

Thus two distinct lens positions require an object-screen separation greater than 60 cm, not merely equal to it.

Step 2: Check every offered separation The values 45 cm, 50 cm, and 55 cm are all below 60 cm; each makes $D(D - 4f)$ negative and is inadmissible. The remaining value 65 cm is above the threshold and gives $d^2 = 65(65 - 60) = 325 \text{ cm}^2 > 0$, so the two positions are distinct.

Step 3: Conclusion The required object-screen distance is 65 cm. Hence, the correct option is (D).

Quick Tip to Solve: Compute $4f$ before checking the options: select a distance strictly above this threshold when two distinct lens positions are required.

29(iv)(b). A thin convex lens of focal length 10 cm and another thin lens of focal length f are placed coaxially in contact. If the power of their combination is $\frac{10}{3}$ D, the value of f is: [1 mark]

- (A) –15 cm
- (B) –10 cm
- (C) –20 cm
- (D) –30 cm

Correct Answer: (A) –15 cm

Solution:

Concept:

- For two thin lenses placed coaxially in contact, their powers add: $P = P_1 + P_2$. Power in dioptres is the reciprocal of focal length in metres.
- A convex lens has positive focal length and power; a concave lens has negative focal length and power. The unknown lens must retain its algebraic sign in the calculation.

Step 1: Convert the known focal length to power The given convex lens has $f_1 = +10 \text{ cm} = +0.10 \text{ m}$, and hence

$$P_1 = \frac{1}{0.10} = +10 \text{ D}.$$

The printed combined power is $P = 10/3 \text{ D}$, which is smaller than the power of this convex lens alone.

Step 2: Determine the second lens and check the options Using the addition rule,

$$P_2 = P - P_1 = \frac{10}{3} - 10 = -\frac{20}{3} \text{ D},$$

$$f = \frac{1}{P_2} = -\frac{3}{20} \text{ m} = -15 \text{ cm}.$$

The negative sign identifies a concave lens. Substitution of the other offered focal lengths, –10 cm, –20 cm, and –30 cm, gives combined powers 0 D, 5 D, and $20/3 \text{ D}$, respectively. None equals the printed combined power.

Step 3: Conclusion The unknown lens has focal length $f = -15 \text{ cm}$ and is concave. Hence, the correct option is (A).

Quick Tip to Solve: Convert centimetres to metres before taking reciprocals; since the combination is weaker than the convex lens alone, the second power must be negative.

30. A p-type or n-type semiconductor can be converted into a p-n junction by doping it with suitable impurity. The motion of majority charge carriers causes diffusion current across the junction while the barrier electric field causes motion of minority carriers for drift current. In case of unbiased diode, the diffusion and drift currents are equal. This equilibrium is disturbed by the biasing batteries. Diodes, therefore, allow currents in one direction. This property of diode is used in making rectifiers.

[4 marks]

30(i). Silicon is doped with which of the following to obtain p-type semiconductor?

[1 mark]

- (A) Phosphorus
- (B) Arsenic
- (C) Boron
- (D) Antimony

Correct Answer: (C) Boron

Solution:

Concept:

- Pure silicon is tetravalent: each silicon atom normally forms four covalent bonds with its neighbours.
- A trivalent impurity acts as an acceptor, creating holes as majority carriers and producing a p-type semiconductor.

Step 1: Identify the required dopant valency To obtain p-type silicon, replace a silicon atom by an impurity with three valence electrons. One bond then lacks an electron; the corresponding vacancy behaves as a mobile hole. Boron has three valence electrons and provides this acceptor behaviour.

Step 2: Compare all the listed elements Phosphorus, arsenic and antimony each have five valence electrons. Their extra electron is readily donated to the conduction band, so these impurities make silicon n-type. Thus options (A), (B) and (D) provide the opposite type of doping to that requested.

Step 3: Conclusion Silicon must be doped with boron to obtain a p-type semiconductor. Hence, the correct option is (C).

Quick Tip to Solve: For silicon, remember: three valence electrons create a hole; five supply an extra electron.

30(ii). A semiconductor has an electron concentration of $5 \times 10^{22} \text{ m}^{-3}$. The concentration of holes is (given $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$):

[1 mark]

- (A) $5 \times 10^{22} \text{ m}^{-3}$
- (B) $1.5 \times 10^6 \text{ m}^{-3}$
- (C) $9 \times 10^8 \text{ m}^{-3}$
- (D) $4.5 \times 10^9 \text{ m}^{-3}$

Correct Answer: (D) $4.5 \times 10^9 \text{ m}^{-3}$

Solution:

Concept:

- At thermal equilibrium, the electron and hole concentrations satisfy the mass-action law $np = n_i^2$, where n_i is the intrinsic carrier concentration at the same temperature.
- In an n-type sample, increasing the majority electron concentration reduces the equilibrium

minority hole concentration; the two concentrations need not be equal.

Step 1: Express the hole concentration The given electron concentration is $n = 5 \times 10^{22} \text{ m}^{-3}$, while $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$. Rearrange the equilibrium relation to obtain

$$p = \frac{n_i^2}{n}.$$

Step 2: Substitute and compare the choices Squaring the intrinsic concentration squares both the coefficient and the power of ten:

$$p = \frac{(1.5 \times 10^{16})^2}{5 \times 10^{22}} = \frac{2.25}{5} \times 10^{10} = 4.5 \times 10^9 \text{ m}^{-3}.$$

Option (A) incorrectly equates holes with the majority electrons. Options (B) and (C) do not satisfy $np = n_i^2$; only option (D) does. The result is much smaller than n , as expected for minority holes.

Step 3: Conclusion The hole concentration is $4.5 \times 10^9 \text{ m}^{-3}$. Hence, the correct option is (D).

Quick Tip to Solve: Square the entire intrinsic concentration before dividing by the given electron concentration.

30(iii). During forward biasing of a p-n junction diode, the:

[1 mark]

- (A) current is mainly due to drifting of majority carriers.
- (B) current is mainly due to drifting of minority carriers.
- (C) diffusion and drift currents are equal.
- (D) current is of the order of 1 A.

Correct Answer: No printed option correctly describes the forward current across the junction; it is mainly diffusion of majority carriers.

Solution:

Concept:

- Diffusion is carrier motion caused by a concentration gradient. Drift is carrier motion caused by an electric field; these are distinct transport mechanisms.
- Forward bias lowers the junction barrier and increases majority-carrier diffusion. The opposing minority-carrier drift current remains comparatively small.

Step 1: Identify the dominant forward-current mechanism A forward voltage reduces the potential barrier, allowing many more majority electrons and holes to cross the depletion region by diffusion. Consequently, diffusion exceeds drift and there is a net forward current. The passage itself distinguishes majority-carrier diffusion from minority-carrier drift.

Step 2: Test the printed alternatives Option (A) identifies majority carriers but incorrectly calls the main junction transport drifting; the expected word is diffusion. Option (B) describes the minority-carrier drift contribution, which is not the dominant forward current. Option (C) describes the unbiased equilibrium, not forward bias. Option (D) cannot be a general statement: the current depends on the applied voltage, diode and external circuit, so forward bias alone does not imply a current of 1 A.

Step 3: Conclusion The forward current across a p-n junction is mainly due to diffusion of majority

carriers. None of the printed options states this correctly; option (A) would be correct if “drifting” were replaced by “diffusion”.

Quick Tip to Solve: Separate the carrier type from the transport mechanism: forward junction current is majority-carrier diffusion.

30(iv)(a). The threshold voltage for silicon diode is about:

[1 mark]

(A) 0.2 V

(B) 0.5 V

(C) 0.7 V

(D) 1.5 V

Correct Answer: (C) 0.7 V

Solution:

Concept:

- The threshold or knee voltage is the approximate forward voltage beyond which a diode's forward current rises rapidly.
- In the standard room-temperature approximation, this voltage is about 0.7 V for a silicon diode, compared with about 0.3 V for germanium.

Step 1: Identify the semiconductor material The question specifies silicon. In the usual forward characteristic of a silicon p-n junction, appreciable current begins around the knee at 0.7 V. This is an approximate characteristic value; it is not a perfectly sharp switching voltage.

Step 2: Match the standard threshold to the choices Of the listed values, 0.7 V matches the silicon threshold. The lower values 0.2 V and 0.5 V are not the standard silicon knee voltage, while 1.5 V is too high for that approximation. Thus none of (A), (B) or (D) gives the requested textbook value.

Step 3: Conclusion The threshold voltage of a silicon diode is approximately 0.7 V. Hence, the correct option is (C).

Quick Tip to Solve: Associate silicon with a 0.7 V knee and germanium with a 0.3 V knee.

30(iv)(b). When we dope Ge with a pentavalent element, four of its electrons bond with four germanium neighbours but fifth electron remains weakly bound. The ionisation energy for this electron is about: [1 mark]

- (A) 0.01 eV
- (B) 0.05 eV
- (C) 0.1 eV
- (D) 0.15 eV

Correct Answer: (A) 0.01 eV

Solution:

Concept:

- A pentavalent donor impurity in germanium uses four valence electrons to form covalent bonds and retains a weakly bound fifth electron.
- The donor level lies just below the conduction-band edge. Donor ionisation requires only this small energy separation, approximately 0.01 eV in germanium.

Step 1: Identify the electron being liberated The fifth electron is an extra donor electron, not one of the four electrons participating in covalent bonds. Its weak binding makes its ionisation energy much smaller than the host semiconductor's full band gap, so a small amount of thermal energy can free it.

Step 2: Compare the donor energy values For a pentavalent impurity in germanium, the textbook donor ionisation energy is about 0.01 eV. The 0.05 eV scale is associated with donor electrons in silicon, rather than the requested germanium approximation. The remaining choices 0.1 eV and 0.15 eV also exceed the stated shallow-donor energy scale for germanium.

Step 3: Conclusion The fifth electron requires approximately 0.01 eV for ionisation. Hence, the correct option is (A).

Quick Tip to Solve: For shallow donor electrons, remember the approximate energies: Ge 0.01 eV, Si 0.05 eV.

Section E (15 marks)

31(a). A series combination of L , C and R is connected to an a.c. source. Using a phasor diagram, derive an expression for the impedance of the circuit and phase difference between V and I .

(b) Under what conditions the

(i) impedance of the circuit is minimum?

(ii) Wattless current flows in the circuit?

[5 marks]

Solution:

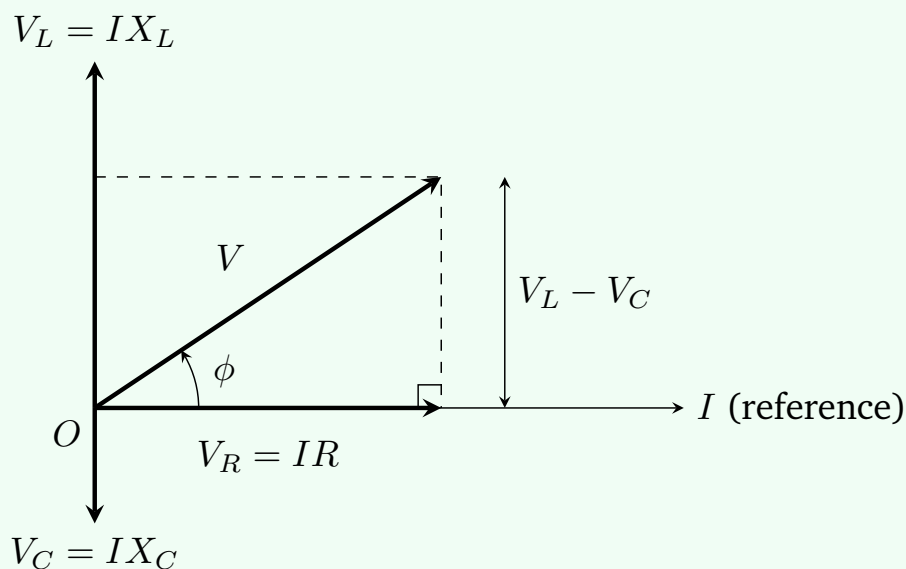
Concept:

- In a series LCR circuit the same alternating current flows through all components. The inductive and capacitive reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$, where ω is the source's angular frequency.
- Taking current as the reference phasor, the resistor voltage is in phase with current, the inductor voltage leads it by 90° , and the capacitor voltage lags it by 90° . Voltages therefore add vectorially.
- The impedance magnitude is $Z = V/I$, using rms voltage V and rms current I . Average power is $P_{av} = VI \cos \phi = I^2 R$, where ϕ is the phase of source voltage relative to current.

Step 1: (a) Draw the component voltage phasors Let the current phasor lie along the positive horizontal direction. The component voltage magnitudes are

$$V_R = IR, \quad V_L = IX_L = I\omega L, \quad V_C = IX_C = \frac{I}{\omega C}.$$

The phasors V_L and V_C point in opposite vertical directions, so their resultant is $V_L - V_C$. The diagram shows the inductive case $V_L > V_C$. The source voltage $\vec{V}$ is the diagonal obtained by adding $\vec{V}_R$ to this vertical resultant. For the capacitive case, the resultant and $\vec{V}$ lie below the horizontal direction.



Series LCR phasors for $X_L > X_C$; the source voltage makes angle ϕ with the current.

Step 2: Derive the impedance from the voltage triangle Kirchhoff's voltage law applies to the instantaneous voltages, and hence to their phasor sum. Since V_R is perpendicular to $V_L - V_C$, the

right triangle gives

$$\begin{aligned} V^2 &= V_R^2 + (V_L - V_C)^2 \\ &= (IR)^2 + \left[I \left(\omega L - \frac{1}{\omega C} \right) \right]^2. \end{aligned}$$

Dividing by I^2 and taking the positive square root gives

$$Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}.$$

Thus resistance and net reactance combine in quadrature; adding their magnitudes directly would ignore their different phases. The impedance is measured in Ω .

Step 3: Obtain the phase difference and its direction If ϕ is the angle made by the source voltage with the current reference, the same triangle yields, for $R > 0$,

$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}, \quad \phi = \tan^{-1} \left(\frac{\omega L - 1/(\omega C)}{R} \right).$$

With this sign convention, $i = I_0 \sin \omega t$ corresponds to $v = V_0 \sin(\omega t + \phi)$. If $X_L > X_C$, $\phi > 0$: voltage leads current, so current lags. If $X_C > X_L$, $\phi < 0$: current leads voltage. If the two reactances are equal, voltage and current are in phase.

Step 4: (b)(i) Find the condition for minimum impedance For fixed resistance, the nonnegative term $(X_L - X_C)^2$ is smallest when it vanishes. Hence the series circuit has minimum impedance at resonance:

$$X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}.$$

Equivalently, $f_0 = 1/(2\pi\sqrt{LC})$. At this frequency $Z_{\min} = R$ and $\phi = 0$. For a finite positive R , the current is maximum and the power factor is unity.

Step 5: (b)(ii) State when a nonzero current is wattless Wattless current means that current flows while the average power consumed over a complete cycle is zero. For nonzero rms voltage and current,

$$P_{av} = VI \cos \phi = 0 \Rightarrow \cos \phi = 0, \quad |\phi| = 90^\circ.$$

This occurs in a purely reactive ideal circuit, with no resistance: $R = 0$. A pure inductor or a pure capacitor is the standard example; energy stored during part of a cycle is returned to the source during another part. An ideal series LC combination is also wattless when $R = 0$ and $X_L \neq X_C$, so its net reactance is nonzero. A circuit with finite resistance and nonzero current cannot be exactly wattless because $P_{av} = I^2 R > 0$. Resonance in a resistive LCR circuit is therefore not a wattless condition.

Step 6: Conclusion $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$, and the signed voltage-current phase angle satisfies $\tan \phi = (\omega L - 1/(\omega C))/R$. Minimum impedance is R at $\omega_0 = 1/\sqrt{LC}$. Nonzero wattless current requires a purely reactive circuit, with $R = 0$ and voltage and current in phase quadrature.

Quick Tip to Solve: Use current as the horizontal reference: subtract V_C from V_L vertically, then apply Pythagoras for Z and the tangent ratio for ϕ . At resonance the phase angle is zero; for wattless current its magnitude is 90° .

31(b). (i) With the help of a labelled diagram, explain the principle, construction and working of an a.c. generator.

(ii) Deduce an expression for the induced emf in the coil of the generator.

(iii) If T is the time period of the rotation of the coil, at what values of T in a cycle, the emf generated is maximum? [5 marks]

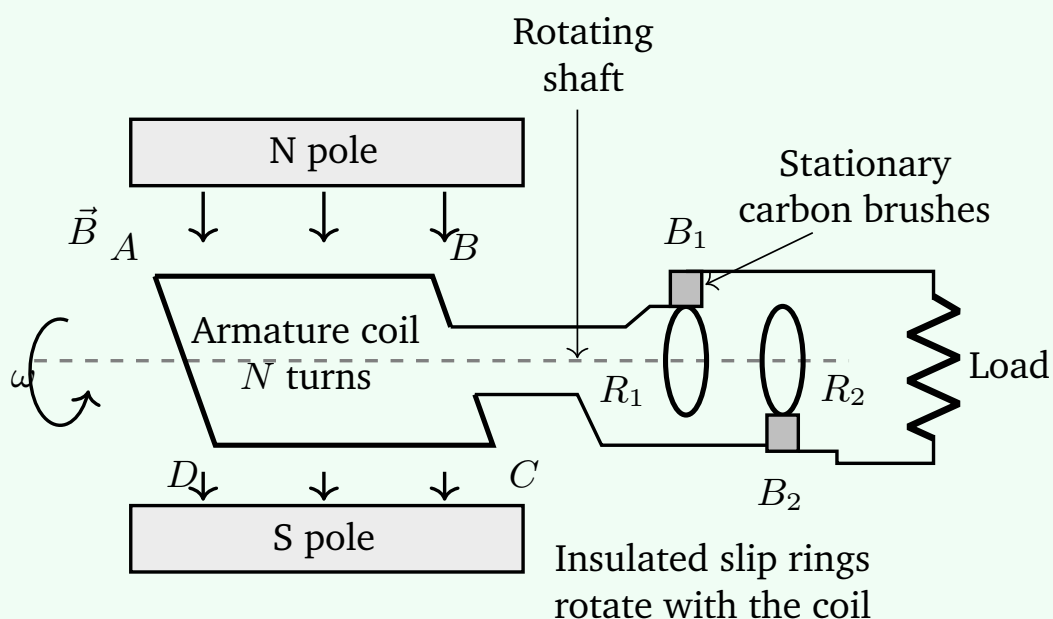
Solution:

Concept:

- Faraday's law gives the induced emf as $e = -d(N\Phi)/dt$, where $N\Phi$ is the magnetic flux linkage of an N -turn coil. Mechanical rotation changes this linkage and converts mechanical energy into electrical energy.
- For a coil of area A in a uniform field of magnitude B , the flux per turn is $\Phi = BA \cos \theta$, where θ is the angle between the field and the coil's area normal.
- Uniform rotation has angular speed $\omega = 2\pi/T$, where T is the period. The initial orientation fixes the phase of the alternating emf.

Step 1: (i) State the principle and identify the parts An a.c. generator works on electromagnetic induction: an emf is induced whenever the magnetic flux linked with a coil changes. A rectangular armature coil $ABCD$, with N turns of insulated wire, rotates between the north and south poles of a strong magnet. A shaft driven by an external mechanical source rotates the coil about an axis perpendicular to the magnetic field.

The two ends of the coil connect separately to two complete metal slip rings R_1 and R_2 . These rings are insulated from one another and from the shaft, and rotate with the coil. Stationary carbon brushes B_1 and B_2 press against the respective rings and connect the rotating coil to the external load. The drawing is schematic; the two rings share the shaft's axis.



Schematic a.c. generator: rotating armature, coaxial slip rings and stationary brushes connected to a load.

Step 2: (i) Explain how alternating emf is produced As the armature rotates, the angle between its normal and the magnetic field changes continuously. Hence its flux linkage changes, producing an

emf. With the external circuit closed, current passes through the slip rings, brushes and load. After each half revolution, the motion of each active side relative to the field reverses, and so does the induced emf. Each brush remains in contact with the same complete slip ring, so the output polarity reverses every half cycle and the output is alternating. The driving source supplies the mechanical work that becomes electrical energy.

Step 3: (ii) Write the instantaneous flux linkage Let $t = 0$ be the instant when the coil's normal is parallel to the field; its plane is then perpendicular to the field and its positive flux is maximum. For constant angular speed, $\theta = \omega t$. Therefore the flux per turn and the total flux linkage are

$$\Phi = BA \cos \omega t, \quad N\Phi = NBA \cos \omega t.$$

The quantities N , B and A remain constant during rotation; the changing factor is the orientation.

Step 4: (ii) Differentiate to obtain the induced emf Applying Faraday's law to the complete coil gives

$$\begin{aligned} e(t) &= -\frac{d}{dt}(NBA \cos \omega t) \\ &= NBA\omega \sin \omega t \\ &= e_0 \sin\left(\frac{2\pi t}{T}\right), \quad e_0 = NBA\omega = \frac{2\pi NBA}{T}. \end{aligned}$$

Here e_0 is the peak emf, measured in volts. The minus sign in Faraday's law expresses Lenz's law; the displayed sine sign uses the chosen positive coil normal and terminal convention. The emf is zero when the flux is at either extreme, because its instantaneous rate of change is then zero.

Step 5: (iii) Find the peak instants in one cycle For the time origin chosen above, the emf has maximum magnitude when

$$|\sin \omega t| = 1 \quad \Rightarrow \quad \omega t = \frac{\pi}{2}, \frac{3\pi}{2}.$$

Using $\omega = 2\pi/T$, these instants are $t = T/4$ and $t = 3T/4$. At these positions the coil's plane is parallel to the field, so the flux is zero but its rate of change has maximum magnitude. More precisely, $e = +e_0$ at $T/4$, whereas $e = -e_0$ at $3T/4$; the latter is the negative peak. Thus if "maximum" refers only to the signed positive value, it occurs at $T/4$. A different initial coil orientation would shift these instants by the corresponding phase.

Step 6: Conclusion The a.c. generator uses a rotating coil, two insulated slip rings and stationary brushes to produce $e = NBA\omega \sin \omega t$, with peak magnitude $e_0 = NBA\omega$. Starting from maximum positive flux, the two magnitude peaks occur at $t = T/4$ and $3T/4$, with opposite polarities; the positive maximum is at $T/4$.

Quick Tip to Solve: Set the time origin before using a sine expression: maximum flux gives zero emf, while zero flux gives peak emf. With maximum positive flux at the start, the peaks occur at one-quarter and three-quarters of a revolution.

32(a). What are coherent sources? Why they are necessary for observing stable interference pattern? Draw a graph showing the variation of intensity of light with the position on the screen in Young's double-slit experiment.

(b) Find the intensity of light at a point on the screen when two interfering waves of the same intensity (I_0) have a path difference of

(i) $\frac{\lambda}{4}$ and (ii) $\frac{\lambda}{3}$.

[5 marks]

Solution:

Concept:

- Coherent sources emit waves of the same frequency with a constant phase difference. Their fixed phase relation allows a stationary interference pattern.
- For waves with individual intensities I_1 and I_2 , the resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$, where ϕ is their phase difference at the observation point.
- A path difference Δ gives $\phi = 2\pi\Delta/\lambda$, where λ is the wavelength. For equal incident intensities I_0 , $I = 4I_0 \cos^2(\phi/2)$.

Step 1: (a) Define coherence and explain its necessity Two sources are coherent if the light waves they emit have the same frequency and maintain a constant phase difference with time. The phase difference need not be zero; it must remain fixed. In Young's experiment the two slits are illuminated by one source so that they act as coherent secondary sources.

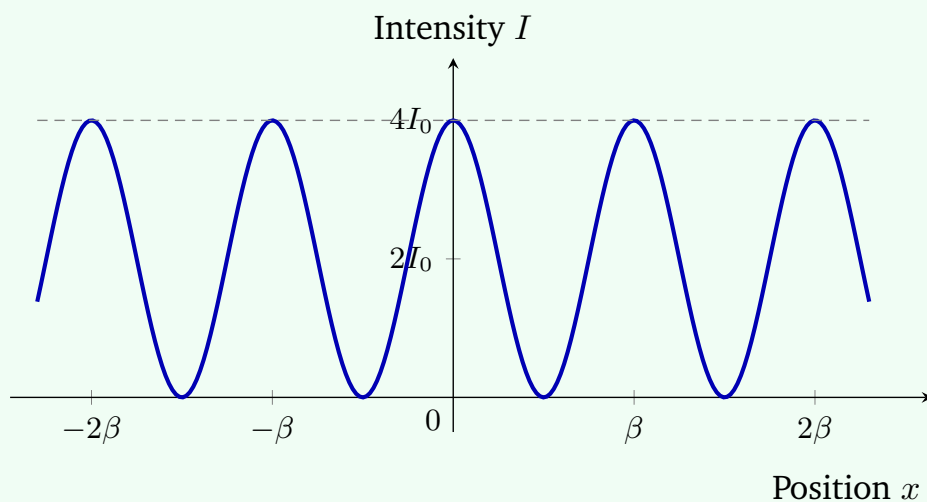
At any fixed point on the screen, the geometrical path difference is fixed. If the source phase difference also stays fixed, the total phase difference there is constant. Hence constructive interference always occurs at the same bright-fringe positions and destructive interference at the same dark-fringe positions.

With independent incoherent sources, the relative phase changes rapidly and randomly. Bright and dark positions fluctuate too rapidly to give a stable pattern, and the time average of the interference term is zero. The observed average intensity is then $I_1 + I_2$, with no sustained fringe contrast.

Step 2: (a) Obtain and plot intensity against screen position Take equally illuminated narrow slits separated by d , a screen at distance D with $D \gg d$, and coordinate x measured from the central bright fringe. In the small-angle region, the path difference is $\Delta \simeq dx/D$. The usual ideal interference graph neglects the slow single-slit diffraction envelope.

$$\begin{aligned} I(x) &= 2I_0 \left[1 + \cos \left(\frac{2\pi dx}{\lambda D} \right) \right] \\ &= 4I_0 \cos^2 \left(\frac{\pi x}{\beta} \right), \quad \beta = \frac{\lambda D}{d}. \end{aligned}$$

Here β is the fringe width. The intensity is $4I_0$ at $x = n\beta$, including the central maximum at $x = 0$, and zero at $x = (n + \frac{1}{2})\beta$, where n is any integer. Successive bright fringes and successive dark fringes are equally spaced by β .



Ideal YDSE intensity versus position for equal slit intensities; $\beta = \lambda D/d$.

Step 3: (b) Express intensity in terms of the given path difference Each of the two interfering waves has intensity I_0 . Substituting $I_1 = I_2 = I_0$ into the interference law gives

$$\begin{aligned} I &= I_0 + I_0 + 2\sqrt{I_0^2} \cos \phi \\ &= 2I_0(1 + \cos \phi), \quad \phi = \frac{2\pi\Delta}{\lambda}. \end{aligned}$$

Step 4: (b)(i) Substitute the path difference of one-quarter wavelength For $\Delta = \lambda/4$, the waves arrive in phase quadrature:

$$\phi = \frac{2\pi}{\lambda} \frac{\lambda}{4} = \frac{\pi}{2}.$$

Thus the interference term vanishes at this point:

$$I = 2I_0 \left(1 + \cos \frac{\pi}{2}\right) = 2I_0(1 + 0) = 2I_0.$$

Step 5: (b)(ii) Substitute the path difference of one-third wavelength For $\Delta = \lambda/3$, the phase difference and intensity are

$$\begin{aligned} \phi &= \frac{2\pi}{\lambda} \frac{\lambda}{3} = \frac{2\pi}{3}, \\ I &= 2I_0 \left(1 + \cos \frac{2\pi}{3}\right) \\ &= 2I_0 \left(1 - \frac{1}{2}\right) = I_0. \end{aligned}$$

Step 6: Conclusion Coherent sources have the same frequency and a constant phase difference, ensuring fixed bright and dark fringes. For equal slit intensities the ideal YDSE graph varies periodically between 0 and $4I_0$, with fringe width $\beta = \lambda D/d$. The required intensities are $2I_0$ for $\Delta = \lambda/4$ and I_0 for $\Delta = \lambda/3$.

Quick Tip to Solve: Convert a path fraction Δ/λ into phase by multiplying by 2π , then use $I = 2I_0(1 + \cos \phi)$. Check that the result lies between 0 and $4I_0$.

32(b). (a) Draw a labelled ray diagram of a refracting telescope when it forms image of a distant object at infinity. Derive expression for its magnifying power.

(b) (i) In a telescope the objective has much larger aperture than the eye piece. Why?

(ii) Write two advantages of reflecting telescope over refracting telescope.

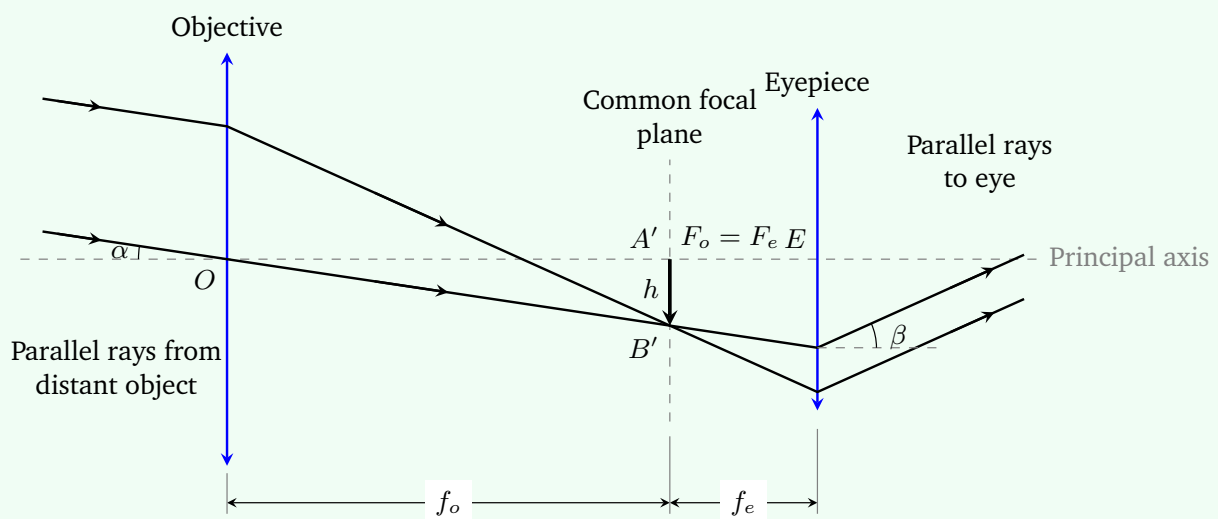
[5 marks]

Solution:

Concept:

- A refracting astronomical telescope uses a convex objective of long focal length f_o and a convex eyepiece of short focal length f_e . The objective forms a real, inverted image of a distant object in its focal plane.
- In normal adjustment, that intermediate image lies in the first focal plane of the eyepiece, so the final image is at infinity. The lens separation is $L = f_o + f_e$.
- Angular magnifying power is the ratio of the angle subtended by the final image at the eye to that subtended by the distant object at the unaided eye. For paraxial rays, $\tan \theta \simeq \theta$, with angles in radians.

Step 1: (a) Set up the telescope in normal adjustment The objective O receives a parallel bundle of rays from a point on the distant object. It brings these rays to the image point B' , below the principal axis; $A'B'$ is the real, inverted intermediate image. Place the eyepiece E so that the objective's second focus F_o coincides with the eyepiece's first focus F_e . Since B' lies in this common focal plane, the eyepiece produces a parallel emergent bundle. The observer therefore sees the final enlarged, inverted image at infinity, with a relaxed eye. In the diagram, all arrows show light travelling from left to right, and the two emergent rays are parallel.



Refracting telescope in normal adjustment: $F_o = F_e$, $A'B' = h$, and the final image is at infinity. Angles α and β denote magnitudes.

Step 2: (a) Derive the angular magnification Let the magnitude of the intermediate image height be $h = A'B'$. Let α and β be the positive magnitudes of the small angles made by the incident and emergent bundles with the principal axis. The objective and eyepiece triangles give

$$\tan \alpha = \frac{h}{f_o}, \quad \tan \beta = \frac{h}{f_e}.$$

Therefore, for small angles,

$$|M| = \frac{\beta}{\alpha} \simeq \frac{\tan \beta}{\tan \alpha} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}.$$

The final image is inverted relative to the original object, so with the usual signed convention the magnifying power is $M = -f_o/f_e$. The negative sign indicates inversion; when magnifying power is quoted as a magnitude, it is f_o/f_e . Thus a long-focal-length objective and a short-focal-length eyepiece give high angular magnification.

Step 3: (b)(i) Explain the larger objective aperture Distant astronomical objects send very little light into the telescope. A large objective intercepts more of that light: its light-gathering area is $\pi D_o^2/4$, where D_o is its diameter. This makes faint objects observable. A larger diameter also reduces the diffraction-limited angular separation, $\theta_{\min} \simeq 1.22\lambda/D_o$, so nearby details can be resolved more clearly. The eyepiece mainly magnifies the intermediate image and passes the collected beam to the eye; it does not need the same large collecting aperture.

Step 4: (b)(ii) State two advantages of a reflecting telescope 1. **No chromatic aberration:** its objective is a mirror, and reflection does not disperse light into different focal lengths for different colours as a refracting objective does.

2. **A much larger objective can be constructed:** a mirror can be supported from behind, whereas a lens must be supported at its rim and transmit light through its bulk. Large mirrors can therefore be built and supported more easily, providing greater light collection and resolving power.

Step 5: Conclusion For a refracting telescope in normal adjustment, the final image is at infinity and

$$M = -\frac{f_o}{f_e}, \quad |M| = \frac{f_o}{f_e}, \quad L = f_o + f_e.$$

The large objective collects more light and improves resolution. Reflecting telescopes avoid chromatic aberration and allow much larger, rear-supported objectives.

Quick Tip to Solve: Put the objective image at the eyepiece focus first; then divide the two small-angle relations, so the common image height cancels. Use the minus sign to record inversion.

33(a). Derive an expression for the capacitance of a parallel plate capacitor of plate area A and plate separation d with air present between the plates.

(b) Two air-filled capacitors of capacitances C_1 and C_2 are connected in parallel with a dc battery. After the capacitors are fully charged, a slab of dielectric constant K is inserted between the plates of each capacitor. How will the (i) charge on each capacitor and (ii) energy stored in the capacitor be affected after the slab is introduced. [5 marks]

Solution:

Concept:

- Capacitance is $C = Q/V$, where Q is the magnitude of charge on either plate and V is the potential difference between the plates.
- A large uniformly charged plane sheet produces a field $\sigma/(2\epsilon_0)$ on either side, where $\sigma = Q/A$ is surface charge density. For air, the permittivity is approximately ϵ_0 .
- A dielectric completely filling the plate gap changes the capacitance to $C' = KC$. A connected ideal dc battery maintains constant voltage, and the stored electrostatic energy is $U = \frac{1}{2}CV^2$.

Step 1: (a) Find the electric field between the plates Consider two conducting parallel plates, each of area A , carrying charges $+Q$ and $-Q$, with separation d . Take their linear dimensions to be much larger than d , so edge effects can be neglected and the surface charge density is uniform. Between the plates, the fields of the positive and negative sheets point in the same direction, from the positive plate towards the negative plate. Thus they add:

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}.$$

Outside the plates, their fields oppose and cancel in this ideal approximation.

Step 2: (a) Obtain the potential difference and capacitance The field in the air gap is uniform. Therefore the magnitude of the potential difference between the plates is

$$V = Ed = \frac{Qd}{\epsilon_0 A}.$$

Use the definition of capacitance to eliminate the charge:

$$C = \frac{Q}{V} = \frac{Q}{Qd/(\epsilon_0 A)} = \frac{\epsilon_0 A}{d}.$$

The charge cancels, showing that the capacitance depends on plate geometry and the intervening medium, rather than the amount of charge supplied.

Step 3: (b)(i) Compare the charges with the battery connected Let the battery voltage be V . Since the capacitors are connected in parallel, each has this same voltage. The battery remains connected during insertion, as no disconnection is specified. With the dielectric filling each plate gap, the new capacitances are $C'_1 = KC_1$ and $C'_2 = KC_2$. Initially $Q_1 = C_1 V$ and $Q_2 = C_2 V$; after insertion,

$$Q'_1 = KC_1 V = KQ_1, \quad Q'_2 = KC_2 V = KQ_2.$$

Extra charge flows from the battery onto each capacitor to maintain the fixed potential difference. For capacitor i , the additional charge is $(K - 1)C_i V$, where $i = 1, 2$.

Step 4: (b)(ii) Compare the stored energies at fixed voltage Before insertion the energies are $U_1 = \frac{1}{2}C_1 V^2$ and $U_2 = \frac{1}{2}C_2 V^2$. After inserting the dielectric, use the new capacitance and the unchanged battery voltage:

$$U'_1 = \frac{1}{2}(KC_1)V^2 = KU_1, \\ U'_2 = \frac{1}{2}(KC_2)V^2 = KU_2.$$

The total stored energy also increases by the factor K . Charge is not fixed in this connected arrangement, so applying the isolated-capacitor result would be inappropriate.

Step 5: Conclusion For the air-filled parallel plate capacitor, $C = \epsilon_0 A/d$. With the battery connected and the dielectric filling each gap, the charge on each capacitor and its stored energy both become K times their initial values: $Q'_i = KQ_i$, $U'_i = KU_i$, for $i = 1, 2$.

Quick Tip to Solve: First decide what stays fixed: a connected battery fixes voltage. Then use $Q = CV$ and $U = \frac{1}{2}CV^2$; both scale directly with the capacitance.

33(b). (a) An electric field $\vec{E}$ is established across the ends of a cylindrical conductor of length L and area of cross-section A . Discuss how electrons attain an average velocity, independent of time. Hence, obtain a relation between current in the conductor and this 'average velocity' of electrons.

(b) (i) This 'average velocity' is found to be few mm/s for currents in range of a few amperes. How then is current established almost the instant a circuit is closed?

(ii) Two copper wires having their radii in the ratio of 3 : 2 are connected in series across a battery. Find the ratio of the drift velocities of the electrons in the wires. [5 marks]

Solution:

Concept:

- Conduction electrons have random thermal motion with zero average velocity in the absence of an electric field. An applied field produces a small directed mean velocity called drift velocity.
- For electron charge $-e$, mass m and mean relaxation time τ , the steady drift velocity is $\vec{v}_d = -e\tau\vec{E}/m$. Here e is the positive magnitude of electron charge.
- If n is the number of free electrons per unit volume, the magnitude of current through cross-sectional area A is $I = neAv_d$, where $v_d = |\vec{v}_d|$.

Step 1: (a) Explain how a steady average velocity develops The free electrons already move rapidly and randomly within the conductor. Without an applied field, their velocities cancel on averaging, so there is no net current. When the field $\vec{E}$ is established, each electron experiences the force and acceleration

$$\vec{F} = -e\vec{E}, \quad \vec{a} = -\frac{e\vec{E}}{m}.$$

An electron accelerates between collisions with the lattice. Collisions continually randomise its motion and remove the systematic momentum gained from the field. If τ is the average relaxation time, averaging over the electrons gives

$$\vec{v}_d = \vec{a}\tau = -\frac{e\tau}{m}\vec{E}.$$

Thus the drift is opposite to the electric field. For a steady field and fixed temperature, τ is constant. After a very short initial transient, momentum gained from the field balances momentum lost through collisions, so the average drift velocity is independent of elapsed time. Individual electrons still accelerate between collisions; a constant average does not mean each electron has constant velocity.

Step 2: (a) Relate current to drift velocity During a short interval Δt , the net directed displacement is $v_d\Delta t$. The electrons contributing net charge across a cross-section occupy a cylinder of volume $Av_d\Delta t$. Their number and the magnitude of charge transported are

$$N = nAv_d\Delta t, \quad \Delta Q = Ne = neAv_d\Delta t.$$

Consequently,

$$I = \frac{\Delta Q}{\Delta t} = neAv_d.$$

Conventional current is opposite to electron drift and along the field. In vector form the current density is $\vec{J} = -ne\vec{v}_d$. If the potential difference across length L is V , then $E = V/L$ and the same relation gives $I = ne^2A\tau V/(mL)$.

Step 3: (b)(i) Distinguish drift speed from signal propagation A metal wire contains conduction electrons throughout its length before the switch is closed. Closing the circuit sends an electromagnetic

disturbance through the circuit at a very high speed, typically of the order of the speed of light, as determined by the surrounding medium and circuit geometry. When the field reaches each region, electrons already there begin to acquire a drift velocity. Current therefore does not require an electron to travel from the battery through the whole circuit. The propagation delay over an ordinary circuit is extremely small, although not exactly zero, so the current is established almost immediately even when the electron drift speed is only a few mm/s.

Step 4: (b)(ii) Compare the drift speeds in the two wires Denote the radii by r_1 and r_2 , with $r_1 : r_2 = 3 : 2$. Since both wires are copper, take the same free-electron number density n . Their series connection requires the same steady current through both wires. Using $A = \pi r^2$,

$$ne\pi r_1^2 v_{d1} = ne\pi r_2^2 v_{d2}.$$

Cancelling the common factors gives

$$\frac{v_{d1}}{v_{d2}} = \frac{r_2^2}{r_1^2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}.$$

The thicker wire has a greater number of charge carriers crossing per unit drift displacement, so it needs a smaller drift speed for the same current.

Step 5: Conclusion The steady electron drift velocity is $\vec{v}_d = -e\tau \vec{E}/m$, and the current magnitude is $I = neAv_d$. Rapid propagation of the electric field makes current appear almost immediately throughout the circuit. For radii in the ratio 3 : 2, the corresponding drift speeds are in the ratio $v_{d1} : v_{d2} = 4 : 9$.

Quick Tip to Solve: For series wires of the same metal, keep I and n fixed: v_d varies as $1/A$, hence as $1/r^2$. Reverse the radius ratio and then square it.