

CBSE Class 12 Physics Question Paper 2026 with Solution PDF-Set 55/2/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. All questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – Sections **A, B, C, D** and **E**.
- (iii) In **Section A** : Question numbers **1** to **16** are Multiple Choice (MCQ) type questions. Each question carries 1 mark.
- (iv) In **Section B** : Question numbers **17** to **21** are Very Short Answer (VSA) type questions. Each question carries 2 marks.
- (v) In **Section C** : Question numbers **22** to **28** are Short Answer (SA) type questions. Each question carries 3 marks.
- (vi) In **Section D** : Question numbers **29** & **30** are Case Study-Based questions. Each question carries 4 marks.
- (vii) In **Section E** : Question numbers **31** to **33** are Long Answer (LA) type questions. Each question carries 5 marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculator is NOT allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

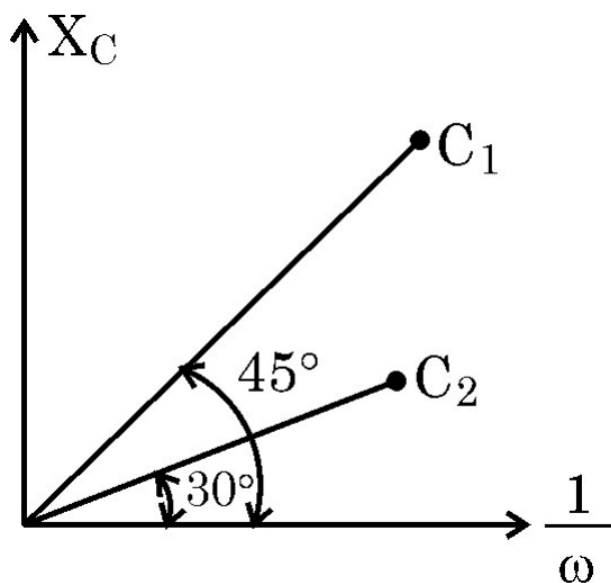
$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Section A (16 marks)

1. The figure shows the variation of capacitive reactance (X_C) of two ideal capacitors of capacitances C_1 and C_2 with the reciprocal of angular frequency ($1/\omega$) of ac source. The value of C_1/C_2 is: [1 mark]



- (A) $\frac{1}{2}$
- (B) 2
- (C) $\sqrt{3}$
- (D) $\frac{1}{\sqrt{3}}$

Correct Answer: (D) $\frac{1}{\sqrt{3}}$

Solution:

Concept:

- Capacitive reactance is $X_C = 1/(\omega C)$, where C is capacitance and ω is angular frequency.
- The slope s of an X_C versus $1/\omega$ graph is $1/C$. A steeper line therefore means a smaller capacitance.

Step 1: Read the two inclinations The lines for C_1 and C_2 make 45° and 30° , respectively, with the horizontal axis. On these common axes,

$$\frac{s_1}{s_2} = \frac{\tan 45^\circ}{\tan 30^\circ} = \sqrt{3}.$$

Step 2: Invert the slope ratio Since $s_1 = 1/C_1$ and $s_2 = 1/C_2$,

$$\frac{C_1}{C_2} = \frac{s_2}{s_1} = \frac{1}{\sqrt{3}}.$$

Option (C) fails to invert the slope ratio. Option (B) incorrectly makes $C_1 > C_2$; option (A) does not match the tangent ratio.

Step 3: Conclusion $C_1/C_2 = 1/\sqrt{3}$. Hence, the correct option is (D).

Quick Tip to Solve: Write slope = $1/C$ first, then invert the ratio of the tangents of the two inclinations.

2. The magnetic flux ϕ (in Wb) linked with a coil is related to time t (in s) as $\phi = 5At^2 + Bt - 2C$. The SI units of A and B are respectively: [1 mark]

- (A) Wb s^2 , Wb s
- (B) Wb s^{-1} , Wb
- (C) Wb s^{-2} , Wb s^{-1}
- (D) Wb s^{-1} , Wb s^{-2}

Correct Answer: (C) Wb s^{-2} , Wb s^{-1}

Solution:

Concept:

- Dimensional homogeneity requires every term added in a physical equation to have the same dimensions and units.
- Magnetic flux ϕ has SI unit weber (Wb); time t has unit second (s). Numerical factors such as 5 and 2 are dimensionless.

Step 1: Determine the unit of the quadratic coefficient In $\phi = 5At^2 + Bt - 2C$, the term $5At^2$ must have unit Wb. If square brackets denote SI units, then

$$[A] \text{ s}^2 = \text{Wb} \Rightarrow [A] = \text{Wb s}^{-2}.$$

Dividing by t^2 , rather than multiplying by it, fixes the negative exponent.

Step 2: Determine the unit of the linear coefficient The term Bt must also have unit Wb, giving

$$[B] \text{ s} = \text{Wb} \Rightarrow [B] = \text{Wb s}^{-1}.$$

Option (A) uses positive time powers. Option (B) assigns each coefficient too little inverse-time dependence, while option (D) interchanges the required units. The constant C itself has unit Wb.

Step 3: Conclusion The SI units of A and B are $\boxed{\text{Wb s}^{-2}, \text{Wb s}^{-1}}$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: For any flux term proportional to t^k , assign its coefficient the unit Wb s^{-k} .

3. A galvanometer of resistance $27\ \Omega$ is converted into an ammeter of range (0-10 mA) using a resistance of $3\ \Omega$. The galvanometer will show full scale deflection for a current of about: [1 mark]

- (A) 10 mA
- (B) 100 mA
- (C) 1 mA
- (D) 3 mA

Correct Answer: (C) 1 mA

Solution:

Concept:

- An ammeter is made by connecting a low resistance shunt S in parallel with a galvanometer of resistance G .
- At full scale, the total current is $I = I_g + I_s$, and equal potential differences across the parallel branches give $I_g G = I_s S$.

Step 1: Express the full-scale current sharing Here $G = 27\ \Omega$, $S = 3\ \Omega$ and the maximum ammeter reading is $I = 10\ \text{mA}$. The galvanometer receives only part of this total current. Substituting $I_s = I - I_g$ into the parallel-branch relation gives

$$I_g G = (I - I_g) S \Rightarrow I_g (G + S) = I S.$$

Step 2: Calculate the galvanometer current Therefore,

$$I_g = \frac{IS}{G + S} = \frac{10 \times 3}{27 + 3} \text{ mA} = 1 \text{ mA}.$$

The shunt carries the remaining 9 mA, so both branches have a potential difference of 27 mV. Option (A) confuses total current with galvanometer current; (B) exceeds the total current. The value in (D) fails the equal-voltage condition.

Step 3: Conclusion The galvanometer gives full-scale deflection at 1 mA. Hence, the correct option is (C).

Quick Tip to Solve: The galvanometer receives the fraction $S/(G + S)$ of the total current; here that fraction is $3/30 = 1/10$.

4. Two metal spheres of radii r_1 and r_2 ($r_2 > r_1$) having charges q_1 and q_2 respectively kept in air, are brought in contact. Which of the following statements is NOT correct? [1 mark]

- (A) The total charge of the two spheres is conserved.
(B) Both spheres attain the same potential.
(C) The final potential of the system equals $\frac{1}{4\pi\epsilon_0} \frac{(q_1 + q_2)}{(r_1 + r_2)}$.
(D) The final potential of the system equals $\frac{1}{4\pi\epsilon_0} \frac{(q_1 + q_2)(r_1 + r_2)}{r_1 r_2}$.

Correct Answer: (D) The final potential of the system equals $\frac{1}{4\pi\epsilon_0} \frac{(q_1 + q_2)(r_1 + r_2)}{r_1 r_2}$.

Solution:

Concept:

- When conductors are connected, charge flows until their potentials become equal. Total charge is conserved if no charge is supplied or lost.
- In the standard spherical-capacitance model, a sphere of radius r has capacitance $4\pi\epsilon_0 r$, where ϵ_0 is the permittivity of free space.

Step 1: Apply charge conservation and equal potential Let the final charges be q'_1 and q'_2 , and let the common potential be V . Charge conservation gives $q'_1 + q'_2 = q_1 + q_2$. Thus statement (A) is correct. At electrostatic equilibrium there is no potential difference driving further charge flow, so statement (B) is also correct.

Step 2: Calculate the common potential Using the spherical-capacitance formula for the two spheres,

$$q'_1 = 4\pi\epsilon_0 r_1 V, \quad q'_2 = 4\pi\epsilon_0 r_2 V.$$

Adding these equations and using the conserved total charge gives

$$V = \frac{1}{4\pi\epsilon_0} \frac{q_1 + q_2}{r_1 + r_2}.$$

This is statement (C). Statement (D) instead multiplies the total charge by $1/r_1 + 1/r_2$; it does not use the combined capacitance $4\pi\epsilon_0(r_1 + r_2)$.

Step 3: Conclusion Statement (D) is the incorrect expression for the final potential. Hence, the correct option is (D).

Quick Tip to Solve: For connected spheres in this model, add their capacitances and divide the conserved total charge by that sum.

5. A person is standing facing geographic north and a vertical wire in front of him is carrying a current in downward direction. The direction of magnetic field at a point on the right side of wire is: [1 mark]

- (A) South
- (B) East
- (C) West
- (D) North

Correct Answer: (A) South

Solution:

Concept:

- Magnetic field lines around a straight current-carrying wire are concentric circles in planes perpendicular to the wire.
- By the right-hand thumb rule, the thumb points along conventional current and the curled fingers give the magnetic field direction.

Step 1: Identify the observation point A person facing geographic north has east on his right. Thus the requested point lies east of the vertical wire. The current points vertically downward, so the field circulates clockwise when viewed from above.

Step 2: Apply the right-hand thumb rule Point the right thumb downward. At the eastern side of the wire, the curled fingers point south. Equivalently, choose east, north and upward as $\hat{x}, \hat{y}, \hat{z}$; then the field direction follows

$$(-\hat{z}) \times \hat{x} = -\hat{y},$$

which is south. East and west are radial directions at this point, whereas the field is tangential. North would be obtained if the current were upward or if the point lay west of the wire.

Step 3: Conclusion The magnetic field at the point on the right side of the wire is directed South. Hence, the correct option is (A).

Quick Tip to Solve: First convert the observer's right to east; then use a downward right thumb to obtain the southward field at that point.

6. Which one of the following materials has magnetic susceptibility χ in the range $-1 \leq \chi < 0$? [1 mark]

- (A) Aluminium
- (B) Copper chloride
- (C) Copper
- (D) Nickel

Correct Answer: (C) Copper

Solution:

Concept:

- Magnetic susceptibility χ relates magnetisation M to the applied magnetic field intensity H through $M = \chi H$.
- Diamagnetic materials have negative susceptibility: their induced magnetisation opposes the

applied field. Paramagnetic and ferromagnetic materials have positive susceptibility.

Step 1: Identify the magnetic class from the sign The interval $-1 \leq \chi < 0$ contains negative values and therefore identifies diamagnetic behaviour. Ordinary diamagnets have a small negative susceptibility. The lower endpoint -1 corresponds to the ideal limiting case of perfect diamagnetism, rather than the susceptibility of every diamagnetic material.

Step 2: Compare the four listed materials Copper is diamagnetic, so its small negative susceptibility lies in the stated interval. Aluminium and copper chloride are paramagnetic and have positive susceptibilities. Nickel is ferromagnetic and has a large positive susceptibility under the usual conditions considered here. Thus the negative sign excludes each of these three alternatives.

Step 3: Conclusion Copper has susceptibility in the given negative range. Hence, the correct option is (C).

Quick Tip to Solve: Use the sign before comparing materials: negative χ means diamagnetic, and copper is the diamagnet in this list.

7. The electric field and the magnetic field components of an electromagnetic wave travelling in vacuum are represented as $E = m \sin(\alpha z - \beta t)$ and $B = n \sin(\alpha z - \beta t)$, where m , n , α and β are constants characterising the wave. Then: [1 mark]

- (A) $\frac{m}{n} = c, \frac{\beta}{\alpha} \neq c$
(B) $\frac{n}{m} = \frac{\alpha}{\beta} = c$
(C) $\frac{\beta}{\alpha} = c, \frac{m}{n} \neq c$
(D) $\frac{m}{n} = \frac{\beta}{\alpha} = c$

Correct Answer: (D) $\frac{m}{n} = \frac{\beta}{\alpha} = c$

Solution:

Concept:

- For an electromagnetic wave in vacuum, the electric and magnetic field amplitudes satisfy $E_0/B_0 = c$, where c is the speed of light.
- A wave with phase $kz - \omega t$ has propagation speed $v = \omega/k$; k is the wave number and ω is the angular frequency.

Step 1: Compare the field amplitudes In the given expressions, m and n are the electric and magnetic field amplitudes. Hence the vacuum-wave relation gives

$$\frac{m}{n} = \frac{E_0}{B_0} = c.$$

It follows that $n/m = 1/c$, not c .

Step 2: Read the wave speed from its phase Comparing $\alpha z - \beta t$ with $kz - \omega t$ identifies $\alpha = k$ and $\beta = \omega$. A constant-phase point obeys $\alpha dz - \beta dt = 0$, so

$$v = \frac{dz}{dt} = \frac{\beta}{\alpha} = c.$$

Option (A) incorrectly denies this phase-speed relation, while option (C) incorrectly denies the amplitude relation. Option (B) sets both inverse-speed ratios equal to a speed, so it also fails.

Step 3: Conclusion Both relations hold together: $m/n = \beta/\alpha = c$. Hence, the correct option is (D).

Quick Tip to Solve: Read the time coefficient over the distance coefficient for speed, then match it to the electric-to-magnetic amplitude ratio.

8. Two identical circular loops A and B of metal wire are arranged on a table close to each other. Loop A is connected to a battery and the current in it increases with time. In response, the loop B : [1 mark]

- (A) remains stationary
- (B) is attracted by loop A
- (C) is repelled by loop A
- (D) rotates about its centre of mass which is fixed

Correct Answer: (C) is repelled by loop A

Solution:

Concept:

- Faraday's law gives the induced emf $\mathcal{E}_B = -d\Phi_B/dt$, where Φ_B is the magnetic flux through loop B .
- Lenz's law makes the induced current oppose the change in flux. Parallel currents attract; antiparallel currents repel.

Step 1: Determine the induced current The loops lie side by side in the same plane. Suppose the current in A is anticlockwise when viewed from above. Its magnetic field through the neighbouring loop B , which lies outside A , points into the table. As this inward flux increases, B develops an anticlockwise current to create an opposing outward field. Reversing the current in A reverses both directions without changing the interaction.

Step 2: Compare the forces on the facing portions Although both currents circulate in the same sense, they flow in opposite directions along the nearest portions of the two loops. Their repulsion is stronger than the attraction between the more distant portions. Thus a net force acts away from A . The changing flux rules out remaining stationary, and the planar symmetry gives no torque to produce the rotation described in option (D).

Step 3: Conclusion Loop B is repelled by loop A . Hence, the correct option is (C).

Quick Tip to Solve: For coplanar neighbouring loops, compare the currents in the facing wire portions; do not apply the coaxial-loop picture.

9. In Bohr model of hydrogen atom, an electron is moving in an orbit, with total energy -3.4 eV. Its angular momentum will be: [1 mark]

(A) 1.05×10^{-34} J s

(B) 6.63×10^{-34} J s

(C) 3.31×10^{-34} J s

(D) 2.1×10^{-34} J s

Correct Answer: (D) 2.1×10^{-34} J s

Solution:

Concept:

- In hydrogen, the energy of the n -th Bohr orbit is $E_n = -13.6/n^2$ eV, where n is a positive integer.
- Bohr's angular-momentum condition is $L_n = nh/(2\pi) = n\hbar$, where $h = 6.63 \times 10^{-34}$ J s.

Step 1: Identify the orbit from its energy Equating the given total energy to the hydrogen energy formula gives

$$-\frac{13.6}{n^2} = -3.4 \Rightarrow n^2 = 4 \Rightarrow n = 2.$$

The negative energy describes a bound electron; it does not make the quantum number or the angular-momentum magnitude negative.

Step 2: Apply angular-momentum quantisation Substituting the second-orbit quantum number,

$$L_2 = \frac{2(6.63 \times 10^{-34})}{2\pi} = 2.11 \times 10^{-34} \text{ J s.}$$

Option (A) is approximately $\hbar$, the first-orbit value. Option (B) is h , which omits the required factor $n/(2\pi)$. Option (C) is approximately $h/2$, which also fails Bohr's condition for $n = 2$.

Step 3: Conclusion The angular momentum is 2.1×10^{-34} J s. Hence, the correct option is (D).

Quick Tip to Solve: First use the energy ratio $13.6/|E_n|$ to find n^2 ; then multiply $\hbar$, not h , by n .

10. According to Bohr model of hydrogen atom, the time period of an electron in its n^{th} orbit is proportional to: [1 mark]

(A) n

(B) n^2

(C) n^3

(D) n^4

Correct Answer: (C) n^3

Solution:

Concept:

- For a hydrogen atom in Bohr's model, the orbit radius varies as $r_n \propto n^2$ and the electron speed as $v_n \propto 1/n$.
- The revolution period is the orbit circumference divided by the speed: $T_n = 2\pi r_n/v_n$.

Step 1: Combine the radius and speed dependences Writing $r_n = r_1 n^2$ and $v_n = v_1/n$, where the

subscript 1 denotes the first orbit, gives

$$T_n = \frac{2\pi r_1 n^2}{v_1/n} = \left(\frac{2\pi r_1}{v_1}\right) n^3.$$

The factor in parentheses is the constant first-orbit period T_1 .

Step 2: Check the proposed powers The circumference increases by n^2 , while the speed decreases by $1/n$; the slower motion therefore supplies one further factor of n to the period. Option (B) accounts only for the radius change. Option (A) misses that change, and option (D) would introduce an extra, unsupported factor of n .

Step 3: Conclusion The time period obeys $T_n \propto n^3$. Hence, the correct option is (C).

Quick Tip to Solve: Remember the Bohr scalings together: $r \sim n^2$, $v \sim n^{-1}$, so $T \sim r/v \sim n^3$.

11. The equivalent mass of a photon of wavelength 100 nm is:

[1 mark]

- (A) 2.21×10^{-35} kg
- (B) 6.63×10^{-33} kg
- (C) 1.11×10^{-35} kg
- (D) 3.31×10^{-33} kg

Correct Answer: (A) 2.21×10^{-35} kg

Solution:

Concept:

- A photon has energy $E = hc/\lambda$, where h is Planck's constant, c is the speed of light and λ is its wavelength.
- Its energy-equivalent mass is $m_{\text{eq}} = E/c^2 = h/(\lambda c)$. This is distinct from its rest mass, which is zero.

Step 1: Convert the wavelength to SI units Since $1 \text{ nm} = 10^{-9} \text{ m}$, the stated wavelength is

$$\lambda = 100 \times 10^{-9} \text{ m} = 1.00 \times 10^{-7} \text{ m}.$$

Use $h = 6.63 \times 10^{-34} \text{ J s}$ and $c = 3.00 \times 10^8 \text{ m s}^{-1}$.

Step 2: Calculate the equivalent mass Substitution in the energy-equivalence relation gives

$$m_{\text{eq}} = \frac{6.63 \times 10^{-34}}{(1.00 \times 10^{-7})(3.00 \times 10^8)} = 2.21 \times 10^{-35} \text{ kg}.$$

The denominator is $30 \text{ m}^2 \text{ s}^{-1}$, fixing both the coefficient and exponent. Options (B) and (D) are too large, while (C) is approximately half this value; there is no factor of two in $E = hc/\lambda$.

Step 3: Conclusion The photon's equivalent mass is $2.21 \times 10^{-35} \text{ kg}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $m_{\text{eq}} = h/(\lambda c)$ directly, and convert nanometres before combining the powers of ten.

12. A point charge of $5\ \mu\text{C}$ is located at $(1\ \text{cm}, 1\ \text{cm})$ in x - y plane. A second charge of $6\ \mu\text{C}$ is brought from infinity to point $(-3\ \text{cm}, 4\ \text{cm})$. The work done in the process is: [1 mark]

- (A) $0.54\ \text{J}$
- (B) $5.4\ \text{J}$
- (C) $-0.54\ \text{J}$
- (D) $10.8\ \text{J}$

Correct Answer: (B) $5.4\ \text{J}$

Solution:

Concept:

- The potential due to a point charge q_1 at distance r is $V = kq_1/r$, where $k = 9 \times 10^9\ \text{N m}^2\text{C}^{-2}$.
- Work done by an external agent in bringing q_2 slowly from infinity is $W_{\text{ext}} = q_2V = kq_1q_2/r$, equal to the increase in electrostatic potential energy.

Step 1: Calculate the distance between the charges Use the difference between the two position coordinates, rather than either point's distance from the origin:

$$r = \sqrt{(-3 - 1)^2 + (4 - 1)^2}\ \text{cm} = 5\ \text{cm} = 0.05\ \text{m}.$$

The coordinate differences form a 3-4-5 right triangle.

Step 2: Find the increase in potential energy With $q_1 = 5 \times 10^{-6}\ \text{C}$ and $q_2 = 6 \times 10^{-6}\ \text{C}$,

$$W_{\text{ext}} = \frac{(9 \times 10^9)(5 \times 10^{-6})(6 \times 10^{-6})}{0.05} = 5.4\ \text{J}.$$

Both charges are positive, so bringing them closer requires positive external work against repulsion. Options (A) and (C) have the wrong magnitude, and (C) also has the wrong sign. Option (D) doubles the interaction energy unnecessarily: one pair contributes kq_1q_2/r only once.

Step 3: Conclusion The work required to bring the second charge from infinity is $+5.4\ \text{J}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the separation from coordinate differences first; the 3-4-5 triangle here makes $r = 0.05\ \text{m}$.

13. Assertion (A): Two electric heaters of power P_1 and P_2 ($> P_1$) are joined in series across a dc source of voltage V . The power consumed by the combination will be less than that consumed by P_1 when connected across the same source.

Reason (R): The power consumed by an electric device when connected to a dc source of voltage V is proportional to its resistance. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- For a resistor across a fixed voltage V , the power is $P = VI = V^2/R$, where R is its resistance.
- Series resistances add: $R_s = R_1 + R_2$. The larger total resistance draws less current from the same voltage source.

Step 1: Evaluate the assertion Let the heater resistances be R_1 and R_2 . Since each individual power is specified for the same voltage, $R_1 = V^2/P_1$ and $R_2 = V^2/P_2$. In series,

$$P_s = \frac{V^2}{R_1 + R_2} = \frac{P_1 P_2}{P_1 + P_2}.$$

Thus $P_s/P_1 = P_2/(P_1 + P_2) < 1$. The combination consumes less power than heater 1 alone, so Assertion (A) is true.

Step 2: Evaluate the reason At a fixed voltage, $P = V^2/R$ is inversely proportional to resistance, not directly proportional to it. Therefore Reason (R) is false. The relation $P = I^2 R$ implies direct proportionality only when current is held fixed, which is not the condition here.

Step 3: Check the explanatory relation The assertion follows because adding the second resistance increases the denominator of V^2/R . The false proportionality stated in the reason cannot explain it. This rules out both options that make the reason true and also rules out option (D), which incorrectly calls the assertion false.

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Before comparing heater powers, identify what is fixed: use V^2/R for a fixed voltage and $I^2 R$ for a fixed current.

14. Assertion (A): Light added to light can produce darkness.

Reason (R): When two coherent light waves interfere, there is darkness at position of destructive interference. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Concept:

- The superposition principle adds the electric-field amplitudes of light waves; intensity is proportional to the square of the resultant amplitude.
- For coherent waves of equal intensity I_0 , the resultant intensity is $I = 4I_0 \cos^2(\phi/2)$, where ϕ is their constant phase difference at the observation point.

Step 1: Evaluate the assertion Two coherent light waves of equal amplitude and the same polarisation can arrive in opposite phases. Their electric fields then cancel, making the resultant amplitude zero. Thus adding light to light can produce a dark point, so Assertion (A) is true. This is possible because fields, rather than individual intensities, are added.

Step 2: Evaluate the reason At destructive interference, $\phi = (2m + 1)\pi$, where m is an integer. For the usual equal-amplitude case,

$$I = 4I_0 \cos^2 \left(\frac{(2m + 1)\pi}{2} \right) = 0.$$

Hence Reason (R) is true in this standard interference setting. Equal amplitudes are required for complete darkness; unequal amplitudes leave a nonzero minimum.

Step 3: Check whether the reason explains the assertion Destructive interference is precisely the mechanism by which the two incident light fields cancel and produce darkness. The reason therefore explains the assertion, ruling out option (B). Options (C) and (D) incorrectly treat the reason as false. Energy is redistributed to brighter regions of the interference pattern, so cancellation at a point does not violate energy conservation.

Step 4: Conclusion Both statements are true, and the reason correctly explains the assertion. Hence, the correct option is (A).

Quick Tip to Solve: To test whether two light waves can give darkness, check opposite phase and equal field amplitudes; add the fields before squaring.

15. Assertion (A): On forward biasing a p-n junction diode, the height of the barrier potential increases.

Reason (R): In forward biasing of a p-n junction diode, the direction of the applied voltage is in the same direction as the built-in potential. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

Concept:

- The built-in electric field in a p-n junction points from the positively charged donor ions on the n-side towards the negatively charged acceptor ions on the p-side.
- Forward bias connects the p-side to the positive terminal and the n-side to the negative terminal. The applied field opposes the junction field, reducing the effective barrier from V_b to approximately $V_b - V_F$, where V_F is the forward voltage.

Step 1: Judge the assertion Forward bias reduces the barrier potential and narrows the depletion region, so majority carriers cross the junction more easily. The assertion says that the barrier height increases; therefore Assertion (A) is false.

Step 2: Judge the reason The externally applied field is directed from the p-side to the n-side, opposite to the built-in field. Thus the applied voltage opposes the built-in potential barrier rather than reinforcing it. Reason (R), which claims the same direction, is false.

Step 3: Check the explanatory relationship Since both statements are false, R cannot explain A. Options (A) and (B) require both statements to be true, while option (C) requires a true assertion; none fits this junction behaviour.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: Connect p to positive for forward bias, then remember that the applied field opposes the barrier and lowers it.

16. Assertion (A): On increasing the intensity of incident light of frequency $\nu (> \nu_0)$ on a photosensitive surface, the photocurrent increases.

Reason (R): The stopping potential for a photosensitive surface increases with increase of frequency $\nu (> \nu_0)$ of incident light. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

Solution:

Concept:

- At a fixed frequency ν , each photon has energy $h\nu$. Increasing light intensity increases the number of photons incident per second and hence the rate of photoelectron emission, provided $\nu > \nu_0$.
- Einstein's photoelectric equation is $eV_s = K_{\max} = h\nu - \phi$, where V_s is the magnitude of the stopping potential and $\phi = h\nu_0$ is the work function of the surface.

Step 1: Judge the assertion For the same photosensitive surface and a fixed collecting voltage, higher intensity at the given frequency supplies more photons per second. More photoelectrons are emitted and collected per second, so the photocurrent increases. The assertion is true.

Step 2: Judge the reason The work function is fixed for the surface. From $V_s = (h/e)\nu - \phi/e$, increasing the incident frequency increases the stopping potential. The reason is also true.

Step 3: Test whether the reason explains the assertion The assertion concerns the number of emitted electrons and a change of intensity, whereas the reason concerns their maximum kinetic energy and a change of frequency. Increasing intensity at fixed frequency does not increase V_s . Therefore the true reason does not explain the true assertion. Option (A) incorrectly connects the two effects; options (C) and (D) incorrectly reject a true statement.

Step 4: Conclusion Both statements are true, but the reason is not the correct explanation of the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Separate the two controls: intensity changes photoelectron number, while frequency changes maximum kinetic energy and stopping potential.

Section B (10 marks)

17. Find ratio (λ_a/λ_p) of the de Broglie wavelength λ_a and λ_p associated respectively with an alpha particle and a proton,

(i) if they are moving with the same kinetic energy.

(ii) just after they are accelerated through the same potential difference.

[2 marks]

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is the momentum. For a non-relativistic particle of mass m and kinetic energy K , $p = \sqrt{2mK}$.
- An alpha particle has approximately $m_a = 4m_p$ and charge $q_a = 2e$, whereas a proton has mass m_p and charge $q_p = e$. Acceleration from rest through a potential difference V gives kinetic energy $K = qV$.

Step 1: Compare wavelengths at the same kinetic energy (i) With $K_a = K_p = K$, use $\lambda = h/\sqrt{2mK}$. Planck's constant and the common kinetic energy cancel in the ratio:

$$\frac{\lambda_a}{\lambda_p} = \sqrt{\frac{m_p K_p}{m_a K_a}} = \sqrt{\frac{m_p}{4m_p}} = \frac{1}{2}.$$

At equal kinetic energy, the heavier alpha particle has twice the proton's momentum, so its wavelength is smaller.

Step 2: Include charge for acceleration through the same voltage (ii) After acceleration through the same potential difference from rest, the kinetic energies are $K_a = 2eV$ and $K_p = eV$. They are not equal because the alpha particle carries twice the charge. Therefore

$$\frac{\lambda_a}{\lambda_p} = \sqrt{\frac{m_p q_p V}{m_a q_a V}} = \sqrt{\frac{m_p e}{(4m_p)(2e)}} = \frac{1}{\sqrt{8}}.$$

Step 3: Conclusion The required ratios are (i) $\lambda_a/\lambda_p = 1/2$ and (ii) $\lambda_a/\lambda_p = 1/(2\sqrt{2})$.

Quick Tip to Solve: At equal kinetic energy compare $m^{-1/2}$; at equal accelerating voltage compare $(mq)^{-1/2}$, including the alpha particle's double charge.

18. Write two points of difference between intrinsic and extrinsic semiconductors.

[2 marks]

Solution:

Concept:

- An intrinsic semiconductor is a pure semiconductor in which charge carriers are produced by thermal excitation. Every electron released into the conduction band leaves a hole in the valence band.
- An extrinsic semiconductor is obtained by doping a pure semiconductor with a small, controlled quantity of donor or acceptor impurity. Doping makes one kind of mobile charge carrier more numerous than the other.

Step 1: Compare purity and preparation **First difference:** An intrinsic semiconductor is pure silicon

or germanium, with no deliberately added impurity. An extrinsic semiconductor contains a small amount of suitable impurity: for example, pentavalent phosphorus added to silicon gives n-type material, while trivalent boron gives p-type material.

Step 2: Compare electron and hole concentrations **Second difference:** In an intrinsic semiconductor the electron concentration n_e and hole concentration n_h are equal: $n_e = n_h$. Neither type is a majority carrier. In an extrinsic semiconductor the concentrations are unequal: $n_e > n_h$ for n-type and $n_h > n_e$ for p-type. Thus electrons are the majority carriers in n-type material, whereas holes are the majority carriers in p-type material. The other kind remains present as minority carriers.

Step 3: Conclusion Intrinsic semiconductors are pure and have equal electron and hole concentrations; extrinsic semiconductors are doped and have unequal concentrations, with a distinct majority carrier type.

Quick Tip to Solve: Give differences along two matching headings: purity and carrier concentration. Do not claim that an n-type semiconductor contains no holes.

19(a). A 5 cm long pencil is placed along the principal axis of a concave mirror of focal length 20 cm such that its nearest end is at a distance of 25 cm from the mirror. Calculate the length of the image of the pencil. [2 marks]

Solution:

Concept:

- The mirror formula is $1/f = 1/v + 1/u$, where f , u and v are the signed focal length, object distance and image distance, measured from the pole.
- For a pencil along the principal axis, its ends have different object distances. Find the image of each end separately; the axial image length is the absolute difference between the two image coordinates.

Step 1: Assign signed distances to the two ends Use the Cartesian sign convention, taking the direction of incident light as positive. The concave mirror has $f = -20$ cm. The near end is at $u_1 = -25$ cm, while the far end is $25 + 5 = 30$ cm in front of the mirror, so $u_2 = -30$ cm. Both ends lie between the focus and the centre of curvature.

Step 2: Locate the image of the near end Rearranging the mirror formula gives $1/v_1 = 1/f - 1/u_1$. Substitution yields

$$\frac{1}{v_1} = -\frac{1}{20} + \frac{1}{25} = -\frac{1}{100} \text{ cm}^{-1},$$
$$v_1 = -100 \text{ cm}.$$

The negative sign places this real image in front of the mirror.

Step 3: Locate the image of the far end Apply the same formula to the second end:

$$\frac{1}{v_2} = -\frac{1}{20} + \frac{1}{30} = -\frac{1}{60} \text{ cm}^{-1},$$
$$v_2 = -60 \text{ cm}.$$

Both image ends are on the same side of the mirror. Their separation is therefore found by subtracting their distances, rather than adding them. The far object end forms the nearer image end.

Step 4: Conclusion The length of the pencil's image is

$$L' = |v_1 - v_2| = |-100 - (-60)| \text{ cm} = \boxed{40 \text{ cm}}.$$

Quick Tip to Solve: For an object lying along the axis, image its two ends separately. The transverse magnification $-v/u$ cannot be applied to the whole axial length.

OR

19(b). In a Young's double-slit experiment, a beam of light consisting of two wavelengths 500 nm and 600 nm is used. The interference fringes are observed at a screen placed 1.8 m away from the plane of slits (slit separation 0.3 mm). Calculate the least distance from the central maximum where the bright fringes due to both the wavelengths coincide. [2 marks]

Solution:

Concept:

- In Young's double-slit experiment, the position of the bright fringe of order m is $y_m = m\lambda D/d$, where λ is the wavelength, D the screen distance and d the slit separation.
- Bright fringes of two wavelengths coincide when their path differences are equal: $m\lambda_1 = n\lambda_2$. The least nonzero distance uses the smallest positive integer orders m, n .

Step 1: Find the first coincident bright-fringe orders For $\lambda_1 = 500 \text{ nm}$ and $\lambda_2 = 600 \text{ nm}$, the coincidence condition gives

$$500m = 600n \Rightarrow 5m = 6n.$$

Since 5 and 6 have no common factor, the smallest positive solution is $m = 6$, $n = 5$. Thus the sixth bright fringe of the shorter wavelength meets the fifth bright fringe of the longer wavelength. The central maximum corresponds to $m = n = 0$ and is excluded when seeking the next coincidence.

Step 2: Substitute the screen distance and slit separation Convert the data to SI units: $D = 1.8 \text{ m}$, $d = 0.3 \times 10^{-3} \text{ m}$ and $\lambda_1 = 500 \times 10^{-9} \text{ m}$. The required position is

$$\begin{aligned} y &= \frac{6(500 \times 10^{-9})(1.8)}{0.3 \times 10^{-3}} \text{ m} \\ &= 1.8 \times 10^{-2} \text{ m}. \end{aligned}$$

Using $n = 5$ and 600 nm gives the same value, checking the coincidence condition.

Step 3: Conclusion The least nonzero distance from the central maximum is $\boxed{1.8 \times 10^{-2} \text{ m} = 1.8 \text{ cm}}$, on either side of the centre.

Quick Tip to Solve: Reduce the wavelength ratio $500 : 600 = 5 : 6$; the first coincident fringe orders are the reverse ratio, $6 : 5$.

20. (a) Why is manganin used in wire-wound standard resistors? Calculate the temperature at which the resistance of a wire increases by 25% of its resistance at room temperature 27.5°C . The temperature coefficient of resistance of the material of the wire is 0.004°C^{-1} .

(b) How does the resistivity of a semiconductor depend on temperature? Show it on a plot. [2 marks]

Solution:

Concept:

- A standard resistor should maintain a nearly constant resistance as temperature changes. Manganin has a very small temperature coefficient of resistance and a high resistivity.
- With a constant temperature coefficient α , the resistance relative to its value R_0 at reference temperature T_0 is $R_T = R_0[1 + \alpha(T - T_0)]$.
- A semiconductor has a negative temperature coefficient of resistivity: increasing temperature generates more mobile charge carriers, so its resistivity ρ falls.

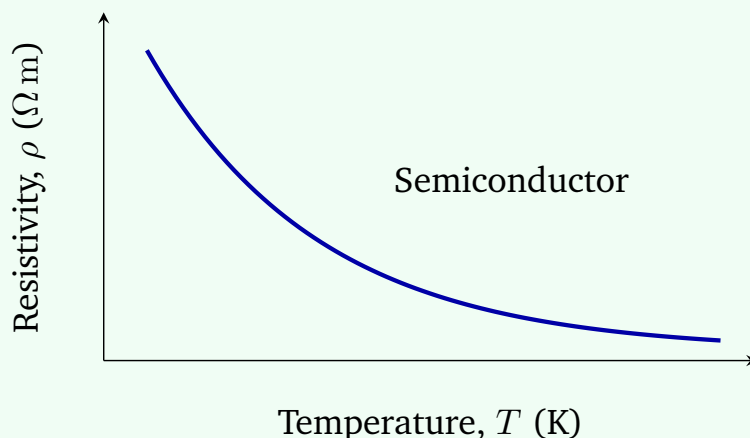
Step 1: Explain the choice of manganin in part (a) Manganin's very small temperature coefficient keeps a standard resistor's resistance stable. Its high resistivity gives the required resistance with a practical length of wire.

Step 2: Calculate the wire temperature in part (a) At room temperature, $T_0 = 27.5^\circ\text{C}$, let the resistance be R_0 . A 25% increase means $R_T = 1.25R_0$. Therefore,

$$\begin{aligned}1.25 &= 1 + 0.004(T - 27.5), \\T - 27.5 &= \frac{0.25}{0.004} = 62.5^\circ\text{C}, \\T &= 27.5 + 62.5 = 90^\circ\text{C}.\end{aligned}$$

Here 62.5°C is the temperature rise above room temperature, rather than the final temperature.

Step 3: Describe and plot the semiconductor behaviour in part (b) Increasing temperature generates more electron-hole pairs, increasing the conductivity and decreasing the resistivity. The usual ρ -versus- T curve falls nonlinearly. The plot below is qualitative.



Qualitative resistivity-temperature graph for a semiconductor

Step 4: Conclusion Manganin is used for its nearly temperature-independent resistance and high resistivity. The wire reaches 90°C . A semiconductor's resistivity decreases with increasing temperature.

Quick Tip to Solve: First find the rise $\Delta T = (\Delta R/R_0)/\alpha$, then add 27.5°C to obtain the final temperature.

21. A ray of light is incident at an angle $2A$ on one of the faces of a glass prism of refracting angle A . After refraction from this face, the ray strikes the opposite face which is silvered. The reflected ray from this face retraces on its path. Trace the ray diagram and find the relation between the refractive index of material of prism and the angle of prism. [2 marks]

Solution:

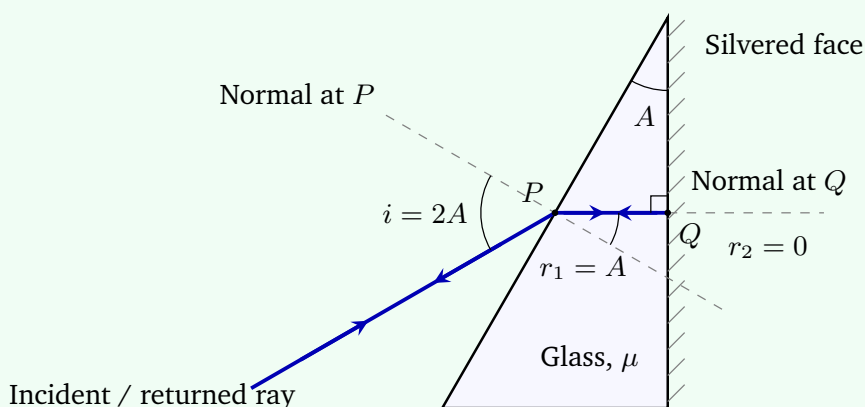
Concept:

- At the first face, Snell's law for light entering glass from air gives $\sin i = \mu \sin r_1$, where μ is the refractive index of glass and the angles are measured from the normal.
- For a prism of angle A , the internal angles with the two face normals satisfy $r_1 + r_2 = A$. A ray reflected by a plane mirror retraces its incident path only at normal incidence.

Step 1: Trace the ray and apply the retracing condition Let the ray enter the first face at P and reach the silvered face at Q . To return along QP , the ray must strike the silvered face normally, so $r_2 = 0$. Thus the prism relation gives

$$r_1 + r_2 = A \Rightarrow r_1 = A.$$

The incident angle at P is $i = 2A$. On reflection, the ray travels back along QP and then emerges from P along the original incident path in the reverse direction, by reversibility of light.



Ray path in the prism; paired arrows show the forward and returning rays on the same line.

Step 2: Apply Snell's law at the first face With the refractive index of air taken as unity, substitute $i = 2A$ and $r_1 = A$:

$$\begin{aligned} \sin 2A &= \mu \sin A, \\ \mu &= \frac{\sin 2A}{\sin A} = \frac{2 \sin A \cos A}{\sin A}. \end{aligned}$$

The angle A is the nonzero refracting angle of the prism, so cancelling $\sin A$ is valid. The relation follows from the normal incidence at the silvered face, not from minimum deviation through a prism.

Step 3: Conclusion The ray is normal to the silvered face and retraces its entire path after reflection. The required relation is $\boxed{\mu = 2 \cos A}$.

Quick Tip to Solve: When a ray retraces after reflection, first set its angle with the mirror normal to zero; then use $r_1 + r_2 = A$.

Section C (21 marks)

22(a). (i) Define mutual inductance of a pair of coils. Write its SI unit.

(ii) A long solenoid of radius R and length L has n turns per unit length. A circular loop of radius r ($< R$) is placed inside at the centre of the solenoid such that its axis coincides with the axis of the solenoid. Obtain the mutual inductance of the solenoid and the loop. [3 marks]

Solution:

Concept:

- Mutual inductance M is the flux linkage of one coil per unit current in the other coil: $N_2\Phi_{21} = MI_1$, for fixed geometry and a linear magnetic medium. Here Φ_{21} is the flux through each turn of coil 2 caused by current I_1 in coil 1.
- Inside a long air-core solenoid, away from its ends, the field is approximately uniform: $B = \mu_0 nI$, where n is the number of turns per unit length and μ_0 is the permeability of free space.

Step 1: (i) Define mutual inductance and its unit The mutual inductance of a pair of coils is numerically equal to the magnetic flux linked with one coil when unit current flows in the other. Equivalently, for fixed coils, the induced emf in coil 2 is

$$\varepsilon_2 = -M \frac{dI_1}{dt}.$$

Thus, the pair has mutual inductance of one henry if a current changing at 1 A s^{-1} in one coil induces an emf of magnitude 1 V in the other. Its SI unit is the henry:

$$1 \text{ H} = 1 \text{ Wb A}^{-1} = 1 \text{ V s A}^{-1}.$$

Step 2: (ii) Find the flux through the inner loop Let a current I flow through the solenoid. Since the circular loop is at its centre, use the long-solenoid approximation and neglect end effects. The loop's axis coincides with the solenoid's axis, so its area vector is parallel to the magnetic field. As $r < R$, the entire loop lies inside the field region. Its area is πr^2 , giving

$$\Phi = B(\pi r^2) \cos 0^\circ = \mu_0 n I \pi r^2.$$

There is only one turn in the circular loop, so its flux linkage equals Φ ; no additional factor of the solenoid's total number of turns is needed.

Step 3: Relate the flux linkage to current By definition, $M = \Phi/I$. Substitution gives

$$M = \frac{\mu_0 n I \pi r^2}{I} = \mu_0 n \pi r^2.$$

If the solenoid has $N_s = nL$ turns in all, the same result is $M = \mu_0 N_s \pi r^2 / L$. Thus the stated turn density already accounts for the length. The relevant flux area is the smaller loop's area, not the solenoid's cross-sectional area.

Step 4: Conclusion Mutual inductance is flux linkage per unit current in the other coil, with SI unit

henry (H). For the given coaxial solenoid and single loop,

$$M = \mu_0 n \pi r^2.$$

Quick Tip to Solve: Find the field using the solenoid, then multiply by the inner loop's area πr^2 . With n specified per unit length, do not multiply by nL again.

OR

22(b). Two long straight parallel conductors A and B carrying steady currents I_a and I_b in the same direction are separated by a distance d . Deduce the expressions for the force acting on length L of conductor B due to conductor A and show it in figure. Write the expression for the force acting on length L of conductor A due to conductor B and show that it follows Newton's third law. [3 marks]

Solution:

Concept:

- The magnetic field a distance d from a long straight wire carrying current I has magnitude $B = \mu_0 I / (2\pi d)$. Its direction follows the right-hand thumb rule.
- A straight length L carrying current I in a magnetic field experiences $\vec{F} = I\vec{L} \times \vec{B}$, where $\vec{L}$ points along the current. Here the wire and the field are perpendicular, so $F = ILB$.

Step 1: Find the field of conductor A at conductor B Take A on the left and B on the right, with both currents directed upwards in the plane of the page. At every point of conductor B , the perpendicular distance from A is d . The magnetic field due to A therefore has magnitude

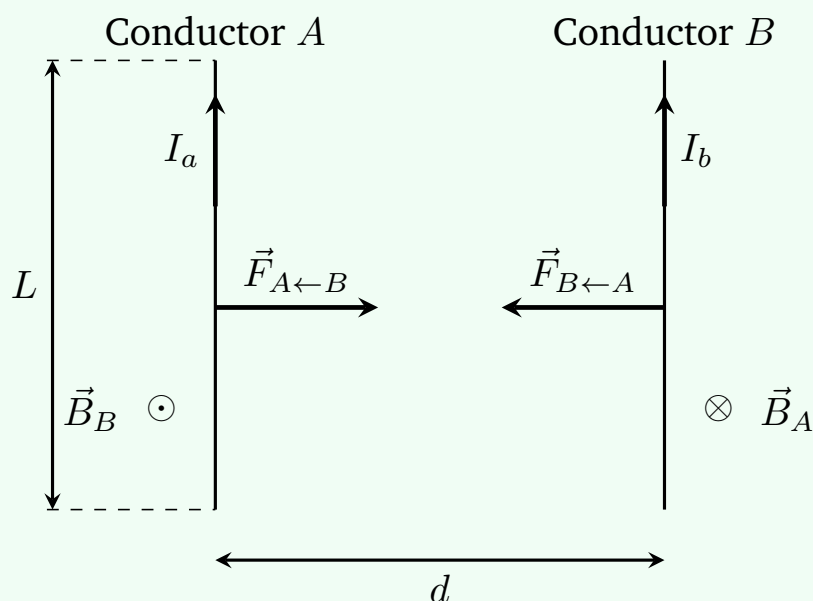
$$B_A = \frac{\mu_0 I_a}{2\pi d}.$$

By the right-hand thumb rule this field is directed into the page at B . The cross in the figure denotes a field into the page, while the dot denotes a field out of the page.

Step 2: Deduce the force on length L of conductor B The field $\vec{B}_A$ is perpendicular to the current in B . Consequently, the force on its length L has magnitude

$$F_{B \leftarrow A} = I_b L B_A \sin 90^\circ = \frac{\mu_0 I_a I_b L}{2\pi d}.$$

The direction of $I_b \vec{L} \times \vec{B}_A$ is to the left, towards A . Thus B is attracted towards A . The arrows in the figure show the forces on equal lengths of the two wires.



Parallel upward currents give attractive forces; $\odot$ is out of the page and $\otimes$ is into the page.

Step 3: Find the reciprocal force and compare the vectors At A, conductor B produces a field of magnitude $B_B = \mu_0 I_b / (2\pi d)$, directed out of the page. Hence, the force on length L of A is to the right, towards B, with magnitude

$$F_{A \leftarrow B} = I_a L B_B = \frac{\mu_0 I_a I_b L}{2\pi d}.$$

If $\hat{x}$ points from A to B, the two forces are

$$\begin{aligned}\vec{F}_{B \leftarrow A} &= -\frac{\mu_0 I_a I_b L}{2\pi d} \hat{x}, \\ \vec{F}_{A \leftarrow B} &= +\frac{\mu_0 I_a I_b L}{2\pi d} \hat{x}.\end{aligned}$$

They act on different conductors, have equal magnitudes and opposite directions, as required by Newton's third law.

Step 4: Conclusion The conductors attract each other. Each length L experiences force $\mu_0 I_a I_b L / (2\pi d)$, and $\vec{F}_{A \leftarrow B} = -\vec{F}_{B \leftarrow A}$, verifying Newton's third law.

Quick Tip to Solve: Use the current in the source wire to find B , then multiply by the other wire's IL . Equal-direction currents give forces towards each other.

23. Derive an expression for the magnetic field $\vec{B}$, due to a circular coil of N turns, each of radius r carrying current I , at a distance x from the centre along its axis. [3 marks]

Solution:

Concept:

- Biot–Savart law gives the field of a current element: $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{s}}{s^2}$, where $\vec{s}$ joins the element to the observation point and $\hat{s}$ is its unit vector.
- Superposition and circular symmetry cancel the components perpendicular to a coil's axis. The axial components add. For N closely wound identical turns, the total field is N times the field of one turn.

Step 1: Set up the geometry of one turn Let O be the centre of a circular turn of radius r , and let P lie on its axis at $OP = x$. For a current element $d\vec{\ell}$ at a point A of the turn, $OA = r$ and $OA \perp OP$. The distance of every element from P is therefore

$$AP = s = \sqrt{r^2 + x^2}.$$

The tangent $d\vec{\ell}$ is perpendicular to both the radius OA and the axis OP . Thus it is perpendicular to $\vec{s}$, which lies in the plane containing these two directions. Biot–Savart law then gives

$$dB = \frac{\mu_0 I d\ell}{4\pi s^2}.$$

Step 2: Resolve the field along the axis Let θ be the acute angle between the line AP and the axis for $x > 0$. The geometry gives $\sin \theta = r/s$. Since $d\vec{B}$ is perpendicular to AP and lies in the plane AOP , its component along the axis has magnitude

$$dB_{\text{ax}} = dB \sin \theta = \frac{\mu_0 I r d\ell}{4\pi s^3}.$$

The element diametrically opposite A produces an equal field whose component perpendicular to the axis is opposite to that from A . These components cancel pairwise, while the axial components reinforce. Hence the resultant field is entirely axial.

Step 3: Integrate around the turn and include all N turns The quantities r and s are constant around the turn, and $\oint d\ell = 2\pi r$. Therefore, for one turn,

$$\begin{aligned} B_1 &= \oint dB_{\text{ax}} = \frac{\mu_0 I r}{4\pi s^3} \oint d\ell \\ &= \frac{\mu_0 I r}{4\pi (r^2 + x^2)^{3/2}} (2\pi r) = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}. \end{aligned}$$

All N turns carry the same current in the same sense, so their fields add: $B = NB_1$. Curling the fingers of the right hand with the current makes the thumb point along the resultant axial field. Denote this direction by the unit vector $\hat{n}$. At $x = 0$, the result reduces to $B = \mu_0 NI/(2r)$, the field at the centre.

Step 4: Conclusion The magnetic field at an axial distance x is

$$\vec{B}(x) = \frac{\mu_0 NI r^2}{2(r^2 + x^2)^{3/2}} \hat{n}.$$

It is directed along the coil's axis, with its sense determined by the right-hand rule.

Quick Tip to Solve: Every element is the same distance $\sqrt{r^2 + x^2}$ from an axial point. Cancel transverse components first, then integrate only $dB r / \sqrt{r^2 + x^2}$.

24. (a) Consider the following nuclides: ${}^{12}_6\text{C}$, ${}^{198}_{80}\text{Hg}$, ${}^{14}_6\text{C}$, ${}^{197}_{79}\text{Au}$. Group them into isotopes and isotones.
 (b) How does the size of a nucleus depend on its mass number A ? Hence prove that the density of nucleus is a constant, independent of A , for all nuclei. [3 marks]

Solution:

Concept:

- For a nuclide A_ZX , Z is the proton number, A is the nucleon number and the neutron number is $N = A - Z$.
- Isotopes have the same Z but different A ; isotones have the same N but different Z .
- Nuclear radius obeys $R = R_0 A^{1/3}$, where $R_0 \simeq 1.2 \times 10^{-15}$ m. Nuclear mass is approximately Am_n , with m_n the average nucleon mass.

Step 1: (a) Count the neutrons and group the nuclides Write the proton and neutron numbers explicitly before comparing the nuclides:

Nuclide	Z	$N = A - Z$
${}^{12}_6\text{C}$	6	$12 - 6 = 6$
${}^{198}_{80}\text{Hg}$	80	$198 - 80 = 118$
${}^{14}_6\text{C}$	6	$14 - 6 = 8$
${}^{197}_{79}\text{Au}$	79	$197 - 79 = 118$

The two carbon nuclides have equal proton numbers but different neutron numbers, so they form the isotope pair. Mercury-198 and gold-197 have different proton numbers but both contain 118 neutrons, so they form the isotone pair.

Step 2: (b) Express nuclear size and volume in terms of mass number The radius, and hence also the diameter, is proportional to the cube root of the mass number. Treating a nucleus as approximately spherical gives

$$R = R_0 A^{1/3}, \quad V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} R_0^3 A.$$

Thus increasing the number of nucleons increases the nuclear volume in the same proportion. It does not increase the radius in direct proportion to A .

Step 3: (b) Divide nuclear mass by volume Taking the proton and neutron masses as approximately equal and neglecting the small binding-energy correction, the nuclear mass is $M \simeq Am_n$. Therefore,

$$\rho = \frac{M}{V} = \frac{Am_n}{(4\pi/3)R_0^3 A} = \frac{3m_n}{4\pi R_0^3}.$$

The factor A cancels. Since m_n and R_0 are constants, nuclear density is approximately the same for

all nuclei. Using $m_n \simeq 1.67 \times 10^{-27} \text{ kg}$ gives $\rho \simeq 2.3 \times 10^{17} \text{ kg m}^{-3}$.

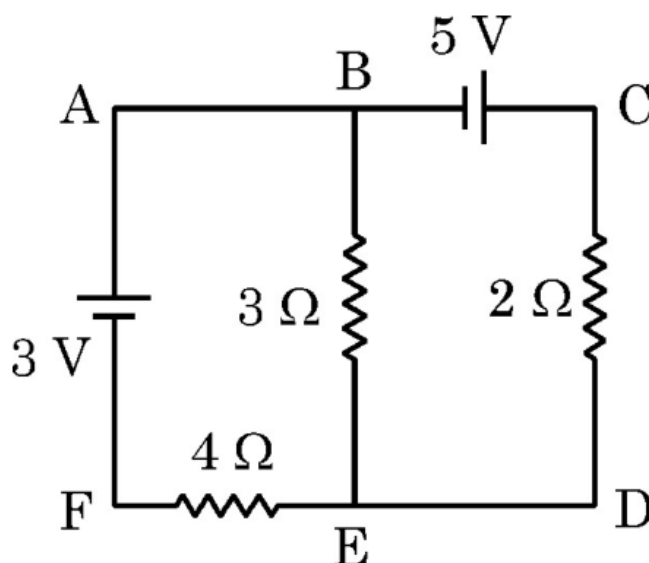
Step 4: Conclusion Isotopes: ${}^{12}_6\text{C}$ and ${}^{14}_6\text{C}$. Isotones: ${}^{198}_{80}\text{Hg}$ and ${}^{197}_{79}\text{Au}$. Since $R \propto A^{1/3}$, $\rho = 3m_n/(4\pi R_0^3)$ is independent of A in the standard nuclear-size model.

Quick Tip to Solve: Calculate $A - Z$ to find isotones; for the density proof, cube the radius relation before substituting the nuclear mass.

25. (a) Explain the statement: “Current is a scalar although we represent current with an arrow”.

(b) Use Kirchhoff’s rules to find the current through 3Ω resistor in the circuit shown in the figure:

[3 marks]



Solution:

Concept:

- Electric current $I = dQ/dt$ is the signed rate of charge flow through a chosen cross-section. It adds algebraically and is a scalar; current density $\vec{J}$ is a vector.
- Kirchhoff’s junction rule expresses charge conservation: total current entering a junction equals total current leaving it.
- Kirchhoff’s loop rule expresses energy conservation: the algebraic sum of potential changes around a closed loop is zero. Across a resistor in the assumed current direction, the change is $-IR$; across a cell from negative to positive, it is $+\mathcal{E}$.

Step 1: (a) Explain the meaning of the current arrow The arrow identifies the chosen sense of conventional charge flow through a wire. A current opposite to that reference direction is represented by a negative number. The currents in different branches combine by algebraic addition at a junction, irrespective of the angles between the wires; they do not obey the parallelogram law of vector addition. Thus an arrow alone does not make current a vector. More formally, $I = \int \vec{J} \cdot d\vec{A}$ is a scalar flux through the oriented cross-section.

Step 2: (b) Assign currents and read the battery polarities Let I_1 flow from B to E through 3Ω , and let I_2 flow from C to D through 2Ω . The junction rule then makes the current in the left branch $I_1 + I_2$, flowing from E through F to A and B. These are assumed directions and may yield a negative current. In the printed figure, the long plate of the 3V cell is toward A, and the long plate of the 5V cell is

toward C. Hence traversal F to A gives a 3 V rise, and B to C gives a 5 V rise.

Step 3: (b) Apply Kirchhoff's loop rule Traverse the left loop E-F-A-B-E. With currents measured in amperes and resistances in ohms,

$$-4(I_1 + I_2) + 3 - 3I_1 = 0 \Rightarrow 7I_1 + 4I_2 = 3.$$

Traverse the right loop B-C-D-E-B. The final crossing E to B is opposite to the assumed I_1 , so its potential change is positive:

$$5 - 2I_2 + 3I_1 = 0 \Rightarrow -3I_1 + 2I_2 = 5.$$

Doubling the second equation and subtracting it from the first eliminates I_2 :

$$(7I_1 + 4I_2) - (-6I_1 + 4I_2) = 3 - 10,$$

$$13I_1 = -7,$$

$$I_1 = -\frac{7}{13} \text{ A}.$$

The negative sign reverses the assumed B-to-E direction. As a consistency check, $I_2 = 22/13 \text{ A}$ and the left-branch current is $15/13 \text{ A}$, satisfying the junction rule at E.

Step 4: Conclusion Current is a scalar because it combines algebraically; its arrow denotes a reference sense of charge flow. The current through 3Ω is $7/13 \text{ A} \simeq 0.54 \text{ A}$, directed upward from E to B.

Quick Tip to Solve: Mark each battery's long plate as positive before writing loops; retain a negative calculated current and reverse its assumed arrow at the end.

26. (a) Draw electric field lines and equipotential surfaces for a system of two equal positive charges separated by some distance.

(b) Using the drawing in (a) above, explain that electric field need not be a constant over each point of an equipotential surface. [3 marks]

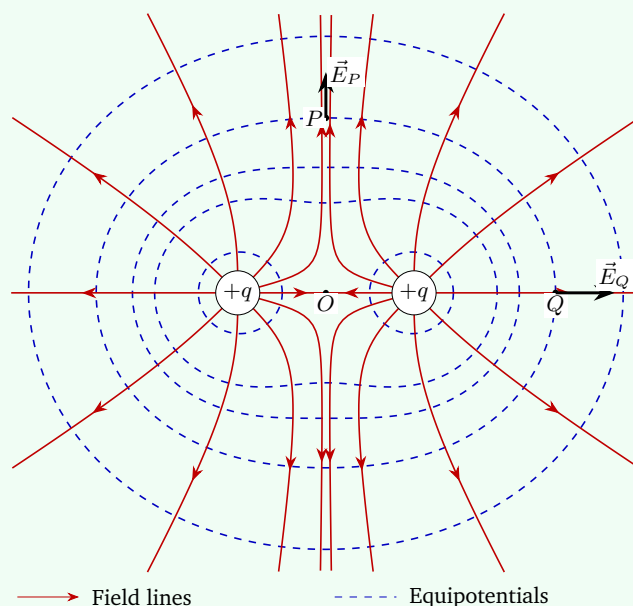
Solution:

Concept:

- For equal positive point charges $+q$, superposition gives $V = kq(1/r_1 + 1/r_2)$, where $k = 1/(4\pi\epsilon_0)$ and r_1, r_2 are distances from the charges.
- An equipotential surface has the same potential at every point. Electric field lines cross it normally, because $\vec{E} = -\nabla V$.
- The field magnitude is the normal rate of decrease of potential: $E = |dV/dn|$. This need not be the same at different points on one equipotential.

Step 1: (a) Draw the field lines and equipotentials The solid arrowed lines below point away from the two positive charges. Between the charges they bend away from the central region; no field line joins the positive charges. At the midpoint O , their fields cancel. The dashed curves are plane sections through equipotential surfaces. Rotating these sections about the line joining the charges gives the three-dimensional surfaces. Close to each charge, the surfaces are nearly spherical around that charge. Far away, they surround both charges and approach spheres centred at O . At every regular intersection, a field line is normal to an equipotential. The marked points P and Q lie on the

same dashed equipotential.



Plane section for two equal positive charges. The dashed contour through P, Q has constant potential, while the local fields differ.

Step 2: (b) Distinguish constant potential from constant field Along the dashed surface through P and Q , $dV = 0$ for a tangential displacement. Hence the tangential field component vanishes; this does not constrain the normal potential gradient to have the same value everywhere on the surface. The field at P points upward, whereas at Q it points to the right, so even its direction is different. Its magnitude can also differ: the spacing between nearby equipotentials for a fixed potential change is smaller near Q than near P . Since

$$E \simeq \frac{|\Delta V|}{\Delta n},$$

a smaller normal spacing Δn means a larger field. Thus $E_Q > E_P$ in the illustrated example although $V_Q = V_P$. The condition of equal potential concerns variation along one surface; the field strength concerns variation normal to it.

Step 3: Conclusion The diagram shows outward field lines and perpendicular equipotential surfaces for two equal positive charges. Constant potential on a surface does not imply a constant electric field over that surface: both its direction and its magnitude can vary.

Quick Tip to Solve: Read field strength from the normal spacing of neighbouring equipotentials for equal potential intervals, not from the value of one equipotential alone.

27. Name the electromagnetic waves which are (i) also known as 'heat waves', (ii) used in LASIK eye surgery, (iii) used to destroy cancer cells. Also write their wavelength range. [3 marks]

Solution:

Concept:

- Electromagnetic radiation is classified by wavelength λ . Its photon energy is $E = hc/\lambda$, where h is Planck's constant and c is the speed of light.
- The infrared, ultraviolet and gamma-ray regions have different characteristic applications. Wavelength boundaries are approximate; adjacent named regions can overlap.

Step 1: Identify the heat waves (i) **Infrared waves** are called heat waves. Hot bodies emit them strongly, and their absorption increases molecular motion and heats the absorbing material. Their wavelength range is approximately

$$700 \text{ nm to } 1 \text{ mm} \quad (7 \times 10^{-7} \text{ m to } 10^{-3} \text{ m}).$$

Step 2: Identify the radiation used in LASIK (ii) **Ultraviolet radiation** from an excimer laser is used to reshape the cornea in LASIK eye surgery. A commonly used laser wavelength is 193 nm, within the ultraviolet band. The band extends approximately from 400 nm down to 0.6 nm, often rounded in spectrum tables to

$$1 \text{ nm to } 400 \text{ nm} \quad (10^{-9} \text{ m to } 4 \times 10^{-7} \text{ m}).$$

Step 3: Identify the radiation used against cancer cells (iii) **Gamma rays** are used to destroy cancer cells in radiotherapy. Their high-energy photons cause ionisation and damage the cells. The broadly quoted gamma-ray wavelengths run from about 10^{-10} m down to 10^{-14} m and still shorter wavelengths. In simplified, non-overlapping spectrum tables the gamma-ray region is shown as $\lambda < 10^{-12}$ m; the broader physical range overlaps the X-ray region.

Step 4: Conclusion (i) Infrared: approximately 700 nm to 1 mm. (ii) Ultraviolet: approximately 0.6 nm to 400 nm. (iii) Gamma rays: approximately 10^{-10} m down to less than 10^{-14} m.

Quick Tip to Solve: Link the applications in order: heating–infrared, corneal reshaping–ultraviolet, cancer treatment–gamma rays. Check that the characteristic wavelengths decrease along this sequence.

28. (a) A particle of charge q and mass m is accelerated through a potential difference V . Deduce an expression for the de Broglie wavelength associated with the particle.

(b) Plot a graph for a particle of charge q and mass m showing variation of de Broglie wavelength λ with $\frac{1}{\sqrt{V}}$, where V is the potential difference through which the particle is accelerated. [3 marks]

Solution:

Concept:

- A particle of momentum p has de Broglie wavelength $\lambda = h/p$, where h is Planck's constant.

- For a particle accelerated from rest, the electric work becomes kinetic energy: $K = qV$, taking q as the magnitude of its charge and V as the accelerating potential difference. For non-relativistic motion, $K = p^2/(2m)$.

Step 1: Obtain the momentum after acceleration (a) Let the final speed of the particle be v . Neglecting its initial kinetic energy and any energy loss, conservation of energy gives

$$qV = \frac{1}{2}mv^2.$$

Since $p = mv$, multiply the energy equation by $2m$ and take the positive square root to obtain the momentum magnitude:

$$p^2 = 2mqV, \quad p = \sqrt{2mqV}.$$

This uses the usual non-relativistic approximation; the gained kinetic energy must be small compared with the rest energy mc^2 .

Step 2: Substitute the momentum into the de Broglie relation Combining the matter-wave relation with the momentum just found,

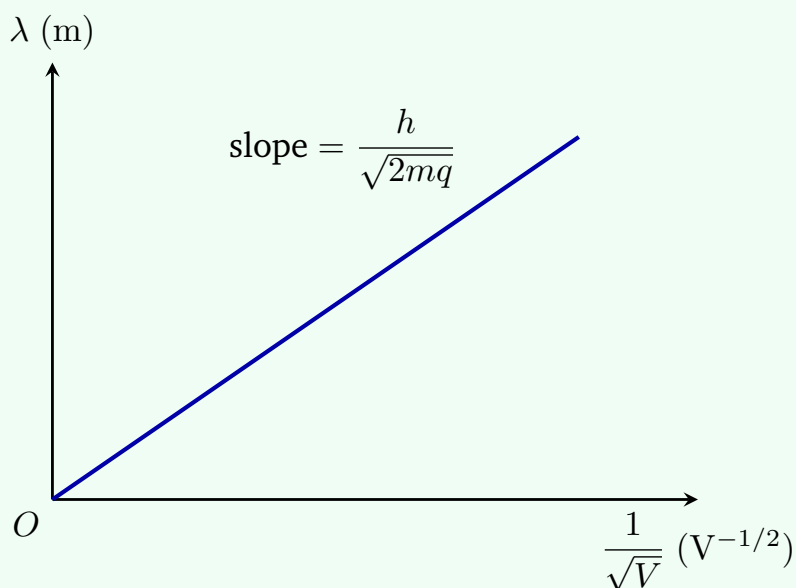
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}} = \frac{h}{\sqrt{2mq}} \frac{1}{\sqrt{V}}.$$

Thus, for fixed mass and charge, increasing the accelerating voltage increases the momentum and decreases the wavelength. If q is instead written as a signed charge, replace it in this expression by $|q|$.

Step 3: Plot wavelength against the inverse square root of voltage (b) The printed question asks for λ on the vertical axis and $1/\sqrt{V}$ on the horizontal axis. Put $y = \lambda$ and $x = 1/\sqrt{V}$. Then

$$y = kx, \quad k = \frac{h}{\sqrt{2mq}} > 0.$$

Therefore the graph is a straight line with positive slope $h/\sqrt{2mq}$ and zero intercept in this approximation. The axes are labelled with λ in metres and $1/\sqrt{V}$ in $V^{-1/2}$. The origin shows the mathematical extrapolation; a finite accelerating voltage corresponds to a positive value of $1/\sqrt{V}$.



Variation of de Broglie wavelength with the inverse square root of accelerating voltage.

Step 4: Conclusion $\lambda = \frac{h}{\sqrt{2mqV}}$, for a particle accelerated from rest in the non-relativistic regime. The graph of λ versus $1/\sqrt{V}$ is a straight line through the origin with slope $h/\sqrt{2mq}$.

Quick Tip to Solve: Rewrite the derived wavelength as a constant times the exact horizontal-axis quantity, $1/\sqrt{V}$; its coefficient immediately gives the straight-line slope.

Section D (8 marks)

Question numbers 29 and 30 are case study based questions. Read the following paragraphs and answer the questions that follow.

29. A p-type or n-type semiconductor can be converted into a p-n junction by doping it with suitable impurity. The motion of majority charge carriers causes diffusion current across the junction while the barrier electric field causes motion of minority carriers for drift current. In case of unbiased diode, the diffusion and drift currents are equal. This equilibrium is disturbed by the biasing batteries. Diodes, therefore, allow currents in one direction. This property of diode is used in making rectifiers.

[4 marks]

29(i). Silicon is doped with which of the following to obtain p-type semiconductor?

[1 mark]

- (A) Phosphorus
- (B) Arsenic
- (C) Boron
- (D) Antimony

Correct Answer: (C) Boron

Solution:

Concept:

- Silicon is tetravalent: each silicon atom supplies four valence electrons to form four covalent bonds with neighbouring atoms.
- A trivalent impurity acts as an acceptor and creates holes, producing a p-type semiconductor. A pentavalent impurity acts as a donor and produces n-type material.

Step 1: Identify the impurity that creates a hole Boron has three valence electrons. When a boron atom replaces silicon in the crystal, one of the four covalent bonds lacks an electron. This deficiency gives rise to a mobile hole. With boron doping, holes are the majority charge carriers, so the material is p-type.

Step 2: Check the other three impurities Phosphorus, arsenic and antimony each have five valence electrons. After four electrons participate in covalent bonding, the fifth is readily donated to the crystal. Each therefore increases the free-electron concentration and produces n-type silicon. Consequently options (A), (B) and (D) cannot produce the required p-type semiconductor.

Step 3: Conclusion Silicon must be doped with the trivalent acceptor **boron**. Hence, the correct option is (C).

Quick Tip to Solve: For silicon, compare impurity valency with four: three gives a hole and p-type material; five gives an extra electron and n-type material.

29(ii). A semiconductor has an electron concentration of $5 \times 10^{22} \text{ m}^{-3}$. The concentration of holes is (given $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$): [1 mark]

- (A) $5 \times 10^{22} \text{ m}^{-3}$
- (B) $1.5 \times 10^{16} \text{ m}^{-3}$
- (C) $9 \times 10^8 \text{ m}^{-3}$
- (D) $4.5 \times 10^9 \text{ m}^{-3}$

Correct Answer: (D) $4.5 \times 10^9 \text{ m}^{-3}$

Solution:

Concept:

- At thermal equilibrium, the mass-action law is $n_e n_h = n_i^2$, where n_e is electron concentration, n_h is hole concentration and n_i is the intrinsic carrier concentration at the same temperature.
- Hence the minority-hole concentration is $n_h = n_i^2 / n_e$. Carrier concentrations have units of m^{-3} .

Step 1: Substitute the given concentrations The question gives $n_e = 5 \times 10^{22} \text{ m}^{-3}$ and $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$. Squaring the intrinsic concentration and dividing by the electron concentration gives

$$\begin{aligned} n_h &= \frac{(1.5 \times 10^{16})^2}{5 \times 10^{22}} \text{ m}^{-3} \\ &= \frac{2.25}{5} \times 10^{10} \text{ m}^{-3} \\ &= 4.5 \times 10^9 \text{ m}^{-3}. \end{aligned}$$

Step 2: Check the result against the options The hole concentration is much smaller than the electron concentration, as expected for this n-type sample. Option (A) incorrectly equates the two concentrations. Options (B) and (C) do not satisfy $n_e n_h = n_i^2$. For option (D), the product is $(5 \times 10^{22})(4.5 \times 10^9) = 2.25 \times 10^{32} \text{ m}^{-6}$, exactly equal to n_i^2 .

Step 3: Conclusion The hole concentration is $n_h = 4.5 \times 10^9 \text{ m}^{-3}$. Hence, the correct option is (D).

Quick Tip to Solve: Square both the coefficient and the power of ten in n_i before dividing: $(1.5 \times 10^{16})^2 = 2.25 \times 10^{32}$.

29(iii). During forward biasing of a p-n junction diode, the:

[1 mark]

- (A) current is mainly due to drifting of majority carriers.
- (B) current is mainly due to drifting of minority carriers.
- (C) diffusion and drift currents are equal.
- (D) current is of the order of 1 A.

Correct Answer: No option is correct as a general statement as printed: forward current is mainly due to diffusion of majority carriers; option (A) incorrectly says drifting.

Solution:

Concept:

- Diffusion current results from carrier concentration gradients across a p-n junction. Electrons from the n-side and holes from the p-side cross the junction as majority carriers.
- The junction electric field produces drift current, chiefly from minority carriers. In an unbiased

junction, diffusion and drift currents have equal magnitudes and opposite directions.

Step 1: Determine the effect of forward bias Forward bias lowers the potential barrier and reduces the depletion width. More majority carriers can cross the junction, so the diffusion current rises substantially while the minority-carrier drift current remains comparatively small. The net forward current is therefore dominated by **diffusion of majority carriers**.

Step 2: Test every printed option (A) names majority carriers but incorrectly attributes the junction current to their *drifting*; the relevant mechanism is diffusion. (B) describes minority-carrier drift, which does not dominate forward conduction. (C) describes the zero-net-current equilibrium of an unbiased junction and therefore fails under forward bias. (D) cannot be inferred: the forward current depends on the applied voltage, circuit resistance and diode, and has no universal value or order of 1 A.

Step 3: Conclusion **There is no correct option as printed.** The correct physical statement is that forward current is mainly due to **diffusion of majority carriers**. Option (A) would express that statement if its word “drifting” were replaced by “diffusion”.

Quick Tip to Solve: For junction-current questions, pair majority carriers with diffusion and minority carriers with drift; equal opposing currents describe an unbiased junction.

29(iv)(a). The threshold voltage for silicon diode is about:

[1 mark]

- (A) 0.2 V
- (B) 0.5 V
- (C) 0.7 V
- (D) 1.5 V

Correct Answer: (C) 0.7 V

Solution:

Concept:

- The threshold or knee voltage is the approximate forward voltage beyond which a diode's forward current rises rapidly.
- For a silicon p-n junction diode at ordinary temperature, the standard knee-voltage value is about 0.7 V.

Step 1: Identify the material and bias condition The question specifies silicon. Its threshold refers to forward bias: the applied voltage reduces the junction barrier sufficiently for a substantial majority-carrier diffusion current. It is an approximate operating value, rather than a sharp switch at an exact voltage.

Step 2: Compare the given voltages The listed 0.7 V matches the usual silicon knee voltage. The values 0.2 V and 0.5 V are below this standard value, while 1.5 V is well above it. A diode may conduct at voltages other than the knee, but those voltages are not its quoted threshold.

Step 3: Conclusion The threshold voltage is approximately 0.7 V. Hence, the correct option is (C).

Quick Tip to Solve: Recall the usual forward knees: silicon about 0.7 V, germanium about 0.3 V; first identify the material.

29(iv)(b). When we dope Ge with a pentavalent element, four of its electrons bond with four germanium neighbours but fifth electron remains weakly bound. The ionisation energy for this electron is about: [1 mark]

- (A) 0.01 eV
- (B) 0.05 eV
- (C) 0.1 eV
- (D) 0.15 eV

Correct Answer: (A) 0.01 eV

Solution:

Concept:

- A pentavalent dopant in germanium forms four covalent bonds and leaves one weakly bound donor electron.
- Donor ionisation requires the small energy difference between the donor level and the conduction band, about 0.01 eV in germanium.

Step 1: Identify the electron that is released The fifth electron is not needed for the four covalent bonds. Its weak binding allows it to enter the conduction band with a small energy input, leaving an immobile positive donor ion. The energy asked for is this donor ionisation energy, not the energy needed to break a germanium covalent bond.

Step 2: Select the germanium donor energy The standard approximate value for a donor electron in Ge is 0.01 eV. The value 0.05 eV is the usual donor ionisation scale quoted for silicon, not germanium. The remaining values, 0.1 eV and 0.15 eV, are also too large for the stated weakly bound donor in Ge.

Step 3: Conclusion The ionisation energy is about 0.01 eV. Hence, the correct option is (A).

Quick Tip to Solve: For the fifth electron of a pentavalent donor, recall Ge: 0.01 eV and Si: 0.05 eV; these are not band-gap energies.

30. In an experiment with convex lens of focal length f , the screen is fixed at a distance D from the object. A student slowly moves the lens away from the object towards the screen and finds that she is able to form sharp image of the object for two positions of the lens. The distance between these two positions of the lens is d . [4 marks]

30(i). The value of d is: [1 mark]

- (A) $\sqrt{D(D - 4f)}$
- (B) $\sqrt{D(D - 2f)}$
- (C) $2\sqrt{Df}$
- (D) $\sqrt{D(D - f)}$

Correct Answer: (A) $\sqrt{D(D - 4f)}$

Solution:

Concept:

- Let x and y be the positive object-lens and lens-screen distances. For a sharp real image, $1/f = 1/x + 1/y$, where $f > 0$ is the convex lens's focal length.
- The fixed object-screen separation is $D = x + y$; interchanging x and y gives the second lens position.

Step 1: Express the two lens positions Since $y = D - x$, the lens formula gives

$$\begin{aligned} xy &= f(x + y) = fD, \\ x(D - x) &= fD, \\ x^2 - Dx + fD &= 0. \end{aligned}$$

The two roots are the distances of the two possible lens positions from the object:

$$x_{\pm} = \frac{D \pm \sqrt{D^2 - 4fD}}{2}.$$

Step 2: Subtract the positions and check the options Their separation is

$$d = x_+ - x_- = \sqrt{D^2 - 4fD} = \sqrt{D(D - 4f)}.$$

Two distinct positions require $D > 4f$. At $D = 4f$, the positions merge and $d = 0$; each of the expressions in (B), (C) and (D) remains nonzero there, so those expressions fail this physical check.

Step 3: Conclusion The lens displacement is $d = \sqrt{D(D - 4f)}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $x + y = D$, $xy = fD$ and $d = |x - y|$; then $d^2 = (x + y)^2 - 4xy = D^2 - 4fD$ immediately.

30(ii). Compared to the size of the object, the images formed in the two positions of the lens are respectively: [1 mark]

- (A) reduced, enlarged
- (B) reduced, reduced
- (C) enlarged, enlarged
- (D) enlarged, reduced

Correct Answer: (D) enlarged, reduced

Solution:

Concept:

- For a real image formed by a convex lens, the magnitude of linear magnification is $|m| = y/x$, where x and y are positive object and image distances.
- With a fixed object and screen, the two sharp-image positions exchange the object and image distances; their magnification magnitudes are reciprocal.

Step 1: Examine the first sharp-image position The lens moves from the object towards the screen, so it meets the sharp-image position nearer the object first. Here the object-lens distance x is smaller than the lens-screen distance y . Hence $|m_1| = y/x > 1$, and this image is enlarged. Both images are real and inverted, but the question asks only for their sizes.

Step 2: Examine the second position and compare the order In the second position, nearer the screen, the distances exchange roles. Thus $|m_2| = x/y < 1$, and the image is reduced. This rules out (A), which reverses the order. Options (B) and (C) incorrectly give the same size category twice; reciprocal magnifications at two distinct positions require one above unity and the other below unity.

Step 3: Conclusion In the order encountered, the images are enlarged and then reduced. Hence, the correct option is (D).

Quick Tip to Solve: Follow the stated direction of lens motion: near the object gives an enlarged image; near the screen gives a reduced image.

30(iii). If the distance between object and screen is 80.00 cm and the lens forms sharp images at two positions separated by 20.00 cm, the focal length of convex lens is: [1 mark]

- (A) 15.50 cm
- (B) 18.75 cm
- (C) 20.50 cm
- (D) 22.75 cm

Correct Answer: (B) 18.75 cm

Solution:

Concept:

- In the lens-displacement method, $f = (D^2 - d^2)/(4D)$, where D is the object-screen distance and d is the separation of the two lens positions.
- The two positive conjugate distances are $(D - d)/2$ and $(D + d)/2$; the focal length also satisfies $f = xy/(x + y)$.

Step 1: Substitute the measured distances Keep both measurements in centimetres: $D = 80.00$ cm

and $d = 20.00$ cm. Then

$$\begin{aligned} f &= \frac{(80.00)^2 - (20.00)^2}{4(80.00)} \text{ cm} \\ &= \frac{6400 - 400}{320} \text{ cm} = 18.75 \text{ cm.} \end{aligned}$$

Step 2: Check with the conjugate distances The object and image distances in one position are $x = (80 - 20)/2 = 30$ cm and $y = (80 + 20)/2 = 50$ cm. They give $xy/(x + y) = 1500/80 = 18.75$ cm, confirming the result. Only (B) matches this value. Options (C) and (D) also violate $f < D/4 = 20$ cm, required for two distinct positions; (A) does not reproduce the measured displacement.

Step 3: Conclusion The focal length is 18.75 cm. Hence, the correct option is (B).

Quick Tip to Solve: Factor the numerator: $f = (D - d)(D + d)/(4D)$. Here this is $60 \times 100/320$ cm.

30(iv)(a). Consider a convex lens of focal length 15 cm. For which of the following values of object-screen distance, two positions of the object can be found to obtain sharp image on the screen?

[1 mark]

- (A) 45 cm
- (B) 50 cm
- (C) 55 cm
- (D) 65 cm

Correct Answer: (D) 65 cm

Solution:

Concept:

- In the displacement experiment, the separation of the two sharp-image lens positions satisfies $d^2 = D(D - 4f)$, where D is the fixed object-screen distance and $f > 0$ is the focal length.
- Two distinct real positions require $D > 4f$. For $D = 4f$, the positions merge into one, with object and image each at distance $2f$ from the lens.

Step 1: Apply the two-position condition The passage fixes the object and screen and moves the lens; the phrase “two positions of the object” in this sub-question is read in that stated displacement-method context. For $f = 15$ cm, the limiting object-screen separation is

$$4f = 4(15) = 60 \text{ cm.}$$

Thus the chosen separation must exceed 60 cm to give two distinct sharp-image positions.

Step 2: Test all four distances The values 45 cm, 50 cm and 55 cm are all below 60 cm; for each, $D(D - 4f) < 0$, so no real displacement-method positions exist. For $D = 65$ cm, the quantity is $65(65 - 60) = 325 \text{ cm}^2 > 0$, giving two distinct positions.

Step 3: Conclusion The required object-screen distance is 65 cm. Hence, the correct option is (D).

Quick Tip to Solve: Multiply the focal length by four and choose a separation strictly greater than that value; equality gives only one position.

30(iv)(b). A thin convex lens of focal length 10 cm and another thin lens of focal length f are placed coaxially in contact. If the power of their combination is $\frac{10}{3}$ D, the value of f is: [1 mark]

- (A) –15 cm
- (B) –10 cm
- (C) –20 cm
- (D) –30 cm

Correct Answer: (A) –15 cm

Solution:

Concept:

- For thin lenses in contact, the equivalent power is $P = P_1 + P_2$; powers add algebraically.
- Lens power is $P = 1/f$ in dioptres when f is in metres. A convex lens has positive power and a concave lens has negative power.

Step 1: Find the first and second lens powers The convex lens has focal length $f_1 = +10 \text{ cm} = +0.10 \text{ m}$, so $P_1 = 1/0.10 = +10 \text{ D}$. The printed combination power is $P = 10/3 \text{ D}$. Therefore,

$$P_2 = P - P_1 = \frac{10}{3} - 10 = -\frac{20}{3} \text{ D}.$$

Its negative sign shows that the second lens is diverging; it reduces the converging power of the first lens.

Step 2: Convert the remaining power to focal length Taking the reciprocal in metres gives

$$f = \frac{1}{P_2} = -\frac{3}{20} \text{ m} = -0.15 \text{ m} = -15 \text{ cm}.$$

Check the other choices by adding their powers to +10 D: focal lengths –10 cm, –20 cm and –30 cm give total powers 0 D, 5 D and $20/3 \text{ D}$, respectively. None equals the specified $10/3 \text{ D}$.

Step 3: Conclusion The second lens has focal length $f = -15 \text{ cm}$ and is concave. Hence, the correct option is (A).

Quick Tip to Solve: Convert centimetres to metres before taking reciprocals, and subtract the known lens power from the total before finding the unknown focal length.

Section E (15 marks)

31(a). Derive an expression for the capacitance of a parallel plate capacitor of plate area A and plate separation d with air present between the plates.

(b) Two air-filled capacitors of capacitances C_1 and C_2 are connected in parallel with a dc battery. After the capacitors are fully charged, a slab of dielectric constant K is inserted between the plates of each capacitor. How will the (i) charge on each capacitor and (ii) energy stored in the capacitor affected after the slab is introduced. [5 marks]

Solution:

Concept:

- Capacitance is $C = Q/V$, where Q is the magnitude of charge on either plate and V is the potential difference between the plates.
- A large uniformly charged sheet produces electric field $\sigma/(2\epsilon_0)$ on each side. Here σ is surface charge density and ϵ_0 is the permittivity of free space; air has approximately this permittivity.
- A dielectric completely filling the space between the plates multiplies capacitance by its dielectric constant K . For a capacitor connected to an ideal dc battery, V remains fixed and the stored energy is $U = \frac{1}{2}CV^2$.

Step 1: (a) Find the electric field between the plates Consider two parallel conducting plates, each of area A , carrying equal and opposite charges $+Q$ and $-Q$. Their separation d is small compared with their lateral dimensions, so edge effects are neglected and the field in the air gap is uniform. The magnitude of surface charge density is $\sigma = Q/A$. Between the plates, the fields of the positive and negative plates point in the same direction, from the positive plate towards the negative plate, and add:

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}.$$

Outside the pair, the two fields oppose each other and cancel in this ideal approximation.

Step 2: (a) Relate potential difference to charge The magnitude of the potential difference across the uniform air gap is the electric field multiplied by the separation:

$$V = Ed = \frac{Qd}{\epsilon_0 A}.$$

Using the definition of capacitance,

$$C = \frac{Q}{V} = \frac{Q}{Qd/(\epsilon_0 A)} = \frac{\epsilon_0 A}{d}.$$

Thus capacitance depends on the plate geometry and the intervening medium. The charge cancels because increasing Q increases V in the same proportion.

Step 3: (b)(i) Apply the fixed battery voltage to each capacitor Let the battery voltage be V . The parallel connection places this same potential difference across both capacitors. The battery is not stated to be disconnected, so it remains connected during insertion. With the slabs completely filling the respective gaps, the new capacitances are

$$C'_1 = KC_1, \quad C'_2 = KC_2.$$

Initially, $Q_1 = C_1V$ and $Q_2 = C_2V$. After insertion,

$$Q'_1 = KC_1V = KQ_1, \quad Q'_2 = KC_2V = KQ_2.$$

Therefore, the magnitude of charge on each capacitor becomes K times its initial value. The battery supplies additional charges $(K - 1)Q_1$ and $(K - 1)Q_2$ to maintain its voltage.

Step 4: (b)(ii) Calculate the change in stored energy Because the voltage stays constant, use the energy formula in terms of capacitance and voltage. For either capacitor, labelled $j = 1, 2$,

$$U_j = \frac{1}{2}C_jV^2, \quad U'_j = \frac{1}{2}C'_jV^2 = \frac{1}{2}KC_jV^2 = KU_j.$$

Each capacitor therefore stores K times its original energy; the increase is $(K - 1)U_j$. The total energy of the pair also increases by the same factor, since

$$U'_{\text{total}} = \frac{1}{2}K(C_1 + C_2)V^2 = KU_{\text{total}}.$$

Step 5: Conclusion (a) The air-filled parallel plate capacitor has $C = \epsilon_0 A/d$, neglecting edge effects. (b) With the battery connected and each gap filled by dielectric, $Q'_1 = KQ_1$, $Q'_2 = KQ_2$ and $U'_1 = KU_1$, $U'_2 = KU_2$.

Quick Tip to Solve: First identify what stays fixed: a connected battery fixes voltage. Then use $Q = CV$ and $U = \frac{1}{2}CV^2$; both scale directly with the new capacitance.

OR

31(b). (a) An electric field $\vec{E}$ is established across the ends of a cylindrical conductor of length L and area of cross-section A . Discuss how electrons attain an average velocity, independent of time. Hence, obtain a relation between current in the conductor and this 'average velocity' of electrons.

(b) (i) This 'average velocity' is found to be few mm/s for currents in range of a few amperes. How then is current established almost the instant a circuit is closed?

(ii) Two copper wires having their radii in the ratio of 3 : 2 are connected in series across a battery. Find the ratio of the drift velocities of electrons in the wires. [5 marks]

Solution:

Concept:

- Drift velocity is the small average velocity acquired by conduction electrons in an electric field. For electron charge $-e$, mass m , and mean relaxation time τ , $\vec{v}_d = -e\tau\vec{E}/m$.
- The magnitude of current is $I = neAv_d$, where n is the number of free electrons per unit volume, A is the cross-sectional area, and v_d is the magnitude of drift velocity. Conventional current is opposite to electron drift.

- Wires in series carry the same steady current. For wires of the same material, equal carrier density gives $v_d \propto 1/A \propto 1/r^2$, where r is wire radius.

Step 1: (a) Explain how a constant average drift develops Without an applied electric field, conduction electrons move randomly because of thermal motion. Their velocities have no preferred direction, so the average velocity and net current are zero. An applied field exerts force $\vec{F} = -e\vec{E}$ on each electron, giving acceleration

$$\vec{a} = -\frac{e}{m}\vec{E}.$$

Between successive collisions with lattice ions and other scattering centres, an electron gains a velocity component opposite to the field. Collisions randomise its motion and prevent this directed velocity from increasing indefinitely. Let τ be the mean relaxation time characterising these collisions. Averaging over the many electrons, whose random thermal velocities cancel, gives the steady drift velocity

$$\vec{v}_d = \vec{a}\tau = -\frac{e\tau}{m}\vec{E}.$$

For a constant field and unchanged conductor temperature, τ is constant. Thus the average drift velocity, after a brief transient, is independent of time even though individual electrons continually accelerate and scatter.

Step 2: (a) Count the charge crossing a section Let $v_d = |\vec{v}_d|$. In time Δt , the net directed motion carries electrons through a distance $v_d\Delta t$. The electrons contributing to the net flow across a cross-section of area A occupy a volume $Av_d\Delta t$. If there are n free electrons per unit volume, their number is $nAv_d\Delta t$, and the magnitude of charge crossing is

$$\Delta Q = neAv_d\Delta t.$$

Therefore,

$$I = \frac{\Delta Q}{\Delta t} = neAv_d.$$

Equivalently, the current density is $\vec{J} = -ne\vec{v}_d$, so conventional current flows along $\vec{E}$. For a voltage V across length L , $E = V/L$, giving the consistent form $I = ne^2\tau AV/(mL)$.

Step 3: (b)(i) Distinguish signal propagation from electron drift Free electrons already exist throughout all the conducting parts of the circuit. When the switch closes, the electromagnetic disturbance establishing the electric field propagates around the circuit at a speed of the order of the speed of light, determined by the circuit and its surrounding medium. Electrons at each location begin drifting when this disturbance reaches them; they do not have to travel from the battery to the other end of the wire first. The field propagation time in an ordinary laboratory circuit is extremely short, so current appears almost immediately even when individual electrons drift at only a few mm s^{-1} . This response has a finite propagation time; it is not literally instantaneous.

Step 4: (b)(ii) Compare the drift velocities in the series wires Label the wires so that $r_1 : r_2 = 3 : 2$. Since both wires are copper, take their free-electron number density n to be the same. In series they

carry the same steady current I . Hence,

$$neA_1v_{d1} = neA_2v_{d2}, \quad \frac{v_{d1}}{v_{d2}} = \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}.$$

The wider wire needs a smaller drift speed because it has more charge carriers available across its cross-section to carry the same current.

Step 5: Conclusion (a) Repeated collisions establish $\vec{v}_d = -e\tau\vec{E}/m$, and the current magnitude is $I = neAv_d$. (b)(i) The circuit responds rapidly because the electric field propagates rapidly to electrons already present throughout it. (b)(ii) For radii 3 : 2, the drift-speed ratio is $v_{d1} : v_{d2} = 4 : 9$.

Quick Tip to Solve: For the two copper wires in series, write $I = ne\pi r^2 v_d$ for each and cancel the common I , n , and e . Invert the squared radius ratio, not the radius ratio itself.

32(a). A series combination of L , C and R is connected to an a.c. source. Using a phasor diagram, derive an expression for the impedance of the circuit and phase difference between V and I .

(b) Under what conditions the

(i) impedance of the circuit is minimum?

(ii) Wattless current flows in the circuit?

[5 marks]

Solution:

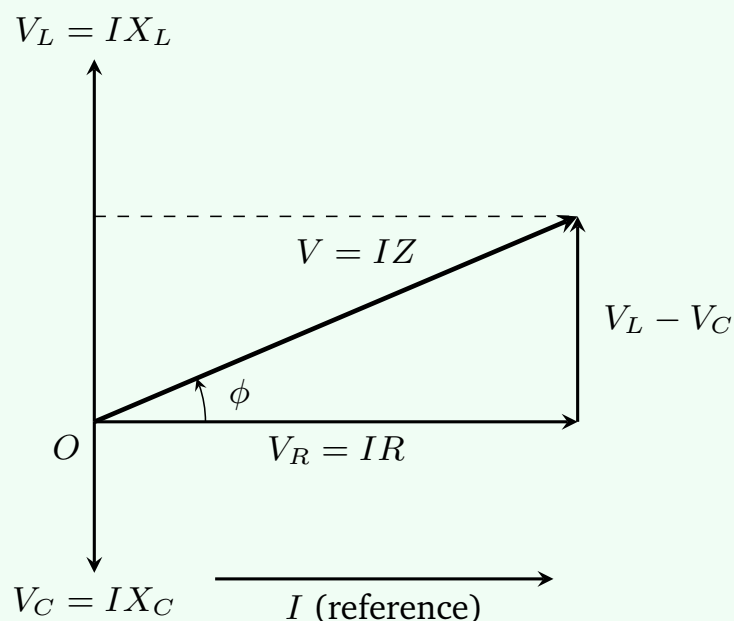
Concept:

- In a series circuit the same current flows through R , L and C . At angular frequency ω , the inductive and capacitive reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$, respectively.
- Taking current as the reference phasor, the resistor voltage is in phase with it; the inductor voltage leads it by 90° , and the capacitor voltage lags it by 90° .
- The impedance is $Z = V/I$, where V and I are rms values. Average a.c. power is $P_{av} = VI \cos \phi = I^2 R$, where ϕ is the phase of source voltage relative to current.

Step 1: (a) Construct the voltage phasors Assume an ideal inductor and capacitor, with all resistance represented by R . Take the common current phasor along the positive horizontal direction. The rms voltage magnitudes are

$$V_R = IR, \quad V_L = IX_L, \quad V_C = IX_C.$$

Draw V_R horizontally, V_L vertically upward and V_C vertically downward. Since V_L and V_C are opposite phasors, their resultant is the signed vertical component $V_L - V_C$. Add this to V_R to obtain the source-voltage phasor V . The diagram shows the inductive case $V_L > V_C$; the vertical resultant reverses for a capacitive circuit.



Series LCR voltage phasors for $X_L > X_C$; current is the horizontal reference.

Step 2: (a) Derive the impedance from the right triangle The resistive and net reactive voltage components are perpendicular, so Pythagoras gives

$$\begin{aligned} V^2 &= V_R^2 + (V_L - V_C)^2 \\ &= I^2 R^2 + I^2 (X_L - X_C)^2. \end{aligned}$$

Dividing by I^2 and taking the positive square root yields

$$Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

Impedance and both reactances are measured in ohms. The opposing reactive voltages must be subtracted before combining with the resistor voltage.

Step 3: (a) Obtain the phase difference and its sign In the voltage triangle, the angle from the current direction to the source-voltage phasor is ϕ . Thus, for $R > 0$,

$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}, \quad \phi = \tan^{-1} \left(\frac{\omega L - 1/(\omega C)}{R} \right).$$

If $X_L > X_C$, then $\phi > 0$: voltage leads current, or current lags voltage. If $X_L < X_C$, then $\phi < 0$: current leads voltage. When $X_L = X_C$, voltage and current are in phase. Equivalently, for a source $v = V_m \sin \omega t$, the current is $i = I_m \sin(\omega t - \phi)$.

Step 4: (b)(i) Find the condition for minimum impedance For fixed R , the squared reactive term in Z^2 is non-negative. It vanishes when the inductive and capacitive reactances are equal:

$$X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}.$$

This is series resonance, occurring at $f_0 = 1/(2\pi\sqrt{LC})$. Here $Z_{\min} = R$, $\phi = 0$, and the current is maximum for a fixed source voltage.

Step 5: (b)(ii) State when the current is wattless Wattless current means that current flows while the average power consumed over a cycle is zero. For nonzero rms voltage and current,

$$P_{av} = VI \cos \phi = 0 \Rightarrow \cos \phi = 0 \Rightarrow |\phi| = 90^\circ.$$

The circuit must therefore be purely reactive, with zero effective resistance: an ideal purely inductive or purely capacitive circuit is the standard example. In a series LC circuit, the same condition holds when $R = 0$ and $X_L \neq X_C$. Energy is stored and returned to the source, with no net consumption. If $R > 0$, then $P_{av} = I^2 R > 0$ for nonzero current, so the total current cannot be wattless. Resonance in a resistive circuit is not a wattless condition; its power factor is unity.

Step 6: Conclusion $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$ and $\tan \phi = (\omega L - 1/(\omega C))/R$, with positive ϕ denoting current lag. At resonance, $\omega_0 = 1/\sqrt{LC}$ and $Z_{min} = R$. Wattless current requires zero resistance and a phase difference of 90° , so the circuit is purely reactive.

Quick Tip to Solve: Draw current and V_R horizontally, then subtract the opposing V_L and V_C phasors. Resonance gives $\phi = 0$; wattless current requires $|\phi| = 90^\circ$.

OR

32(b). (i) With the help of a labelled diagram, explain the principle, construction and working of an a.c. generator.

(ii) Deduce an expression for the induced emf in the coil of the generator.

(iii) If T is the time period of the rotation of the coil, at what values of T in a cycle, the emf generated is maximum? [5 marks]

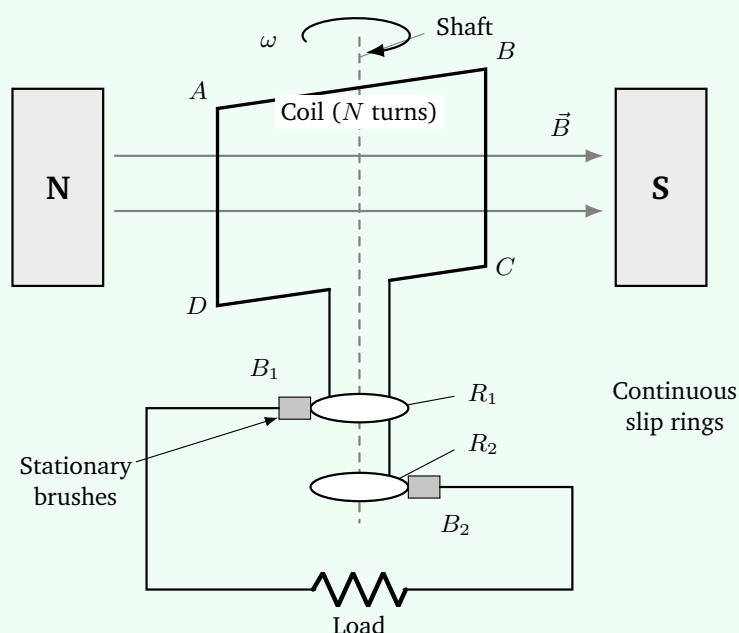
Solution:

Concept:

- An a.c. generator converts mechanical energy into electrical energy by electromagnetic induction. Faraday's law gives $\varepsilon = -d(N\Phi)/dt$, where $N\Phi$ is the magnetic flux linked with the whole coil.
- For a coil of area A in a uniform magnetic field B , the flux through one turn is $\Phi = BA \cos \theta$. Here θ is the angle between the field and the normal to the coil.
- For uniform rotation, $\omega = 2\pi/T$. Choosing the coil normal parallel to the field at $t = 0$ gives $\theta = \omega t$.

Step 1: (i) State the principle and describe the construction When a coil rotates in a magnetic field, the magnetic flux linked with it changes and an emf is induced. Its direction opposes the change producing it, as expressed by the negative sign in Faraday's law (Lenz's law).

A rectangular armature coil $ABCD$, having N turns of insulated copper wire, is mounted on a shaft between the pole pieces of a strong magnet. The shaft is perpendicular to the magnetic field. The two free ends of the coil are connected separately to two continuous metal slip rings R_1 and R_2 , insulated from each other and from the shaft. The coil and rings rotate together. Two stationary carbon brushes B_1 and B_2 maintain sliding contact with the rings and connect the rotating coil to an external load.



A.c. generator: separate continuous slip rings connect the rotating coil to stationary brushes and the load (schematic).

Step 2: (i) Explain how alternating emf is produced A turbine or another mechanical drive turns the shaft at angular speed ω . The active opposite sides of the coil cut magnetic field lines in opposite directions; the induced emfs act around the coil in the same sense and add. Their direction can be found by Fleming's right-hand rule.

After every half-rotation, each active side moves in the opposite direction relative to the field, so the induced emf reverses. The continuous slip rings preserve each coil end's connection to its own brush, allowing this reversal to appear across the external load. With a closed resistive load, alternating current therefore flows. The mechanical drive supplies the electrical energy delivered to the load.

Step 3: (ii) Differentiate the flux linkage Let A be the area of each turn, B the field strength and N the number of turns. Take $t = 0$ when the coil normal is parallel to the field, so that the flux linkage is initially maximum. At time t , the coil normal makes angle ωt with the field. Thus

$$N\Phi(t) = NBA \cos(\omega t).$$

Applying Faraday's law gives

$$\begin{aligned} \varepsilon(t) &= -\frac{d}{dt}[NBA \cos(\omega t)] \\ &= NBA\omega \sin(\omega t) \\ &= \varepsilon_0 \sin\left(\frac{2\pi t}{T}\right), \end{aligned}$$

where $\varepsilon_0 = NBA\omega = 2\pi NBA/T$ is the peak emf. The sinusoidal variation explicitly shows that the polarity reverses every half-cycle.

Step 4: (iii) Locate the peak emf in one cycle The magnitude of the emf is greatest when $|\sin(2\pi t/T)| = 1$. During $0 \leq t < T$,

$$\frac{2\pi t}{T} = \frac{\pi}{2}, \frac{3\pi}{2} \implies t = \frac{T}{4}, \frac{3T}{4}.$$

At $T/4$, the emf is $+\varepsilon_0$, its positive maximum; at $3T/4$, it is $-\varepsilon_0$, its negative peak. At both instants

the coil's plane is parallel to the field, the flux is zero, and its rate of change is greatest. These times use the stated initial orientation; changing the starting orientation shifts the phase.

Step 5: Conclusion The a.c. generator produces $\varepsilon = NBA\omega \sin(\omega t)$ using a rotating coil, slip rings and brushes. With initially maximum flux, the emf has maximum magnitude at $t = T/4$ and $3T/4$; the positive maximum alone occurs at $T/4$.

Quick Tip to Solve: Start the cycle with maximum flux: emf is then zero. A quarter-turn later the flux is zero but the emf is a peak; the next peak has opposite sign after another half-turn.

33(a). What are coherent sources? Why are they necessary for observing stable interference pattern? Draw a graph showing the variation of intensity of light with the position on the screen in Young's double-slit experiment.

(b) Find the intensity of light at a point on the screen when two interfering waves of the same intensity (I_0) have a path difference of

(i) $\frac{\lambda}{4}$ and (ii) $\frac{\lambda}{3}$.

[5 marks]

Solution:

Concept:

- Coherent sources emit waves of the same frequency with a constant phase difference. This fixes the relative phase at each point of an interference pattern.
- For two waves with intensities I_1 and I_2 , the resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$, where ϕ is their phase difference at the observation point.
- A path difference Δ corresponds to a phase difference $\phi = 2\pi\Delta/\lambda$, where λ is the wavelength. For equal incident intensities I_0 , $I = 4I_0 \cos^2(\phi/2)$.

Step 1: (a) Define coherence and explain why the pattern stays steady Two sources are coherent if they produce waves of the same frequency and maintain a constant phase difference. The constant phase difference need not be zero. In Young's experiment, the two slits are illuminated by the same monochromatic source, so the light reaching them has a definite phase relationship.

At a fixed point on the screen, the geometrical path difference is fixed. Coherence therefore keeps the total phase difference at that point fixed in time. Constructive interference continues at the same bright-fringe positions, while destructive interference continues at the same dark-fringe positions.

With independent incoherent sources, the relative phase changes rapidly and randomly. Maxima and minima then shift rapidly; over the observation time, the interference term averages to zero. The observed intensity becomes $I_1 + I_2$, so no stable bright and dark fringe pattern is visible.

Step 2: (a) Obtain the intensity variation and draw the graph Consider the usual Young's experiment with identical slits contributing equal intensity I_0 . Let d be the slit separation, D the distance of the screen from the slits, and x the displacement from the central bright fringe. For $D \gg d$ and small observation angles,

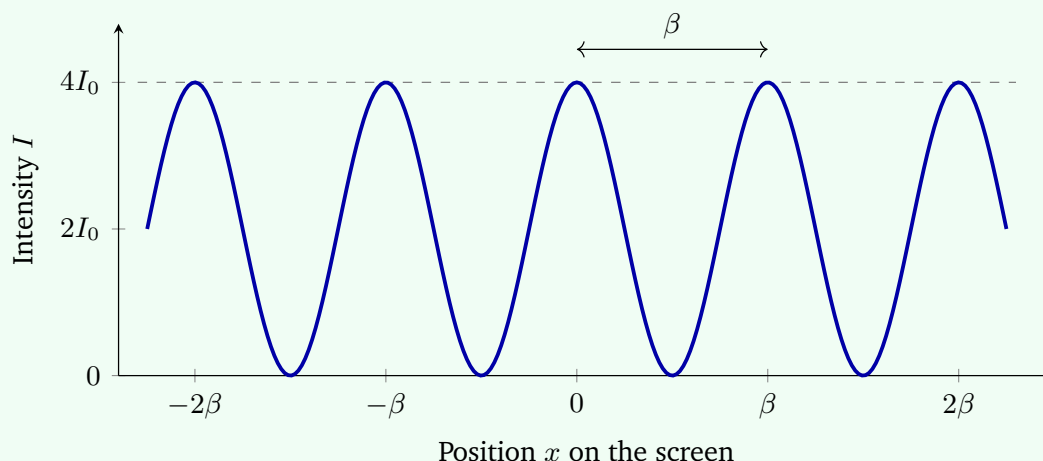
$$\Delta \simeq \frac{dx}{D}, \quad \phi = \frac{2\pi dx}{\lambda D}.$$

Substituting in the interference formula gives

$$I(x) = 2I_0 \left[1 + \cos \left(\frac{2\pi dx}{\lambda D} \right) \right]$$

$$= 4I_0 \cos^2 \left(\frac{\pi x}{\beta} \right), \quad \beta = \frac{\lambda D}{d}.$$

Here β is the fringe width. Bright fringes occur at $x = n\beta$, with $I_{\max} = 4I_0$; dark fringes occur at $x = (n + \frac{1}{2})\beta$, with $I_{\min} = 0$, where n is any integer. The central maximum is at $x = 0$, and successive maxima are equally spaced. The graph below represents this ideal equal-intensity interference pattern, neglecting the single-slit diffraction envelope.



Ideal Young's interference pattern for equal slit intensities; $\beta = \lambda D/d$.

Step 3: (b)(i) Convert the path difference to phase for one-quarter wavelength Use the intensity of each wave, I_0 , in the equal-intensity expression. For $\Delta = \lambda/4$,

$$\phi = \frac{2\pi}{\lambda} \frac{\lambda}{4} = \frac{\pi}{2},$$

$$I = 2I_0(1 + \cos \phi)$$

$$= 2I_0 \left(1 + \cos \frac{\pi}{2} \right) = 2I_0.$$

The interference term is zero at this phase difference, so the intensity equals the sum of the separate intensities at this point, even though the waves remain coherent.

Step 4: (b)(ii) Convert the path difference to phase for one-third wavelength For $\Delta = \lambda/3$, the corresponding phase difference is $2\pi/3$, whose cosine is negative:

$$\phi = \frac{2\pi}{\lambda} \frac{\lambda}{3} = \frac{2\pi}{3},$$

$$I = 2I_0 \left(1 + \cos \frac{2\pi}{3} \right)$$

$$= 2I_0 \left(1 - \frac{1}{2} \right) = I_0.$$

This is partial destructive interference: the intensity is below $2I_0$, but it is not zero because the phase difference is not an odd multiple of π .

Step 5: Conclusion Coherent sources have the same frequency and a constant phase difference,

keeping bright and dark fringes fixed. For equal slit intensities, the ideal Young's interference graph varies periodically between 0 and $4I_0$, with fringe width $\beta = \lambda D/d$. The requested intensities are $I = 2I_0$ for $\Delta = \lambda/4$ and $I = I_0$ for $\Delta = \lambda/3$.

Quick Tip to Solve: First convert the path difference to phase using $\phi = 2\pi\Delta/\lambda$. Then use $I = 2I_0(1 + \cos \phi)$; remember that I_0 is the intensity of each wave, while the maximum is $4I_0$.

OR

33(b). (a) Draw a labelled ray diagram of a refracting telescope when it forms image of a distant object at infinity. Derive expression for its magnifying power.

(b) (i) In a telescope the objective has much larger aperture than the eye piece. Why?

(ii) Write two advantages of reflecting telescope over refracting telescope.

[5 marks]

Solution:

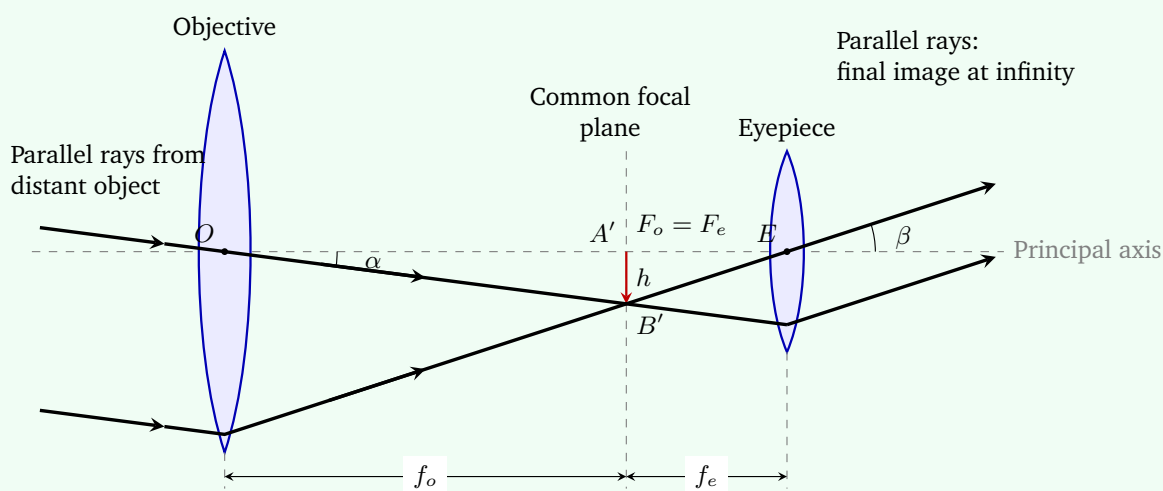
Concept:

- A refracting astronomical telescope uses a converging objective of focal length f_o and a converging eyepiece of focal length f_e , with $f_o > f_e$.
- In normal adjustment, the real image formed by the objective lies in the front focal plane of the eyepiece. The eyepiece then sends out parallel rays, giving a final image at infinity.
- Angular magnifying power is the ratio of the angle subtended at the eye by the final image to that subtended by the distant object at the unaided eye. For paraxial rays, $\tan \theta \simeq \theta$ in radians.
- Light collected by an objective of diameter D is proportional to its area, $\pi D^2/4$. Its diffraction-limited angular resolution improves as $\theta_{\min} \simeq 1.22\lambda/D$ decreases.

Step 1: (a) Set up the telescope in normal adjustment Let O and E be the optical centres of the objective and eyepiece. Parallel rays from one point of a distant object are brought to a focus at B' by the objective. The objective forms a real, inverted, diminished intermediate image $A'B'$ in its focal plane. Position the eyepiece so that this plane is also its front focal plane: F_o coincides with F_e at A' . Thus the lens separation is

$$OE = f_o + f_e.$$

Rays from B' emerge parallel after refraction by the eyepiece. The final image is therefore at infinity and remains inverted relative to the original object. In the diagram, α and β denote the positive magnitudes of the small object and image angles; the rays show their opposite orientations.



Refracting telescope in normal adjustment: the common focal plane contains $A'B'$, and the emerging rays are parallel.

Step 2: (a) Derive the angular magnifying power Write the magnitude of the intermediate image height as $h = A'B'$. The ray through the optical centre O of the objective is undeviated. From the triangle $OA'B'$,

$$\tan \alpha = \frac{h}{f_o}, \quad \alpha \simeq \frac{h}{f_o}.$$

The ray from B' through the optical centre E of the eyepiece also passes undeviated. Its direction is parallel to the other emerging rays. From the triangle $EA'B'$,

$$\tan \beta = \frac{h}{f_e}, \quad \beta \simeq \frac{h}{f_e}.$$

Taking the ratio cancels the intermediate image height:

$$|M| = \frac{\beta}{\alpha} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}.$$

Since the final image is inverted, the signed magnifying power is $M = -f_o/f_e$. A long-focal-length objective and a short-focal-length eyepiece give a large angular magnification.

Step 3: (b)(i) Explain the larger objective aperture The objective receives light from a very distant and often faint object. Its large aperture intercepts more light, allowing faint celestial objects to be observed. A larger objective diameter also reduces diffraction spreading and increases resolving power, so closely spaced stars or fine details can be distinguished. The eyepiece receives the converging beam already collected by the objective and magnifies the intermediate image; it therefore needs a much smaller aperture.

Step 4: (b)(ii) Give two advantages of a reflecting telescope **1. No chromatic aberration from the objective:** The objective is a mirror. Reflection does not separate colours by wavelength-dependent refraction, so all colours have the same focal length, avoiding the colour fringes produced by a simple objective lens.

2. Larger practical aperture: A mirror can be supported from its back, whereas a transmitting objective lens must be supported at its edge. Thus a large mirror can be made and supported more easily, giving greater light-collecting and resolving power at a practical cost.

Step 5: Conclusion In normal adjustment, $M = -f_o/f_e$, with magnitude f_o/f_e , and $OE = f_o + f_e$. The large objective gathers more light and improves resolution. A reflecting telescope avoids chromatic

aberration of the objective and permits a larger, readily supported aperture.

Quick Tip to Solve: Put the objective's rear focus at the eyepiece's front focus for a final image at infinity. Use the same intermediate height h in both angle triangles; it cancels to give f_o/f_e .