

CBSE Class 12 Physics Question Paper 2026 with Solution PDF-Set 55/3/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1 to 16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** – Questions no. **17 to 21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** – Questions no. **22 to 28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** – Questions no. **29 and 30** are Case Study-based questions. Each question carries **4** marks.
- (vii) In **Section E** – Questions no. **31 to 33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is **not** allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. A particle of mass m and charge q starts from rest and moves in an electric field $\vec{E} = E_0\hat{i}$. After travelling a distance x in the field along the x -axis, the kinetic energy of the particle will be: [1 mark]

- (A) qE_0x^2
- (B) qE_0x
- (C) q^2E_0x
- (D) $q^2E_0^2x$

Correct Answer: (B) qE_0x

Solution:

Concept:

- A charge q in a uniform electric field $\vec{E} = E_0\hat{i}$ experiences the constant electric force $\vec{F} = qE_0\hat{i}$.
- The work-energy theorem gives $K_f - K_i = W$, where W is the work done by the net force and K_i, K_f are the initial and final kinetic energies.

Step 1: Calculate the electric work For the stated displacement x along the field axis, the force is constant, so its work is

$$W = \int_0^x qE_0 dx' = qE_0x.$$

The displacement has the direction of the motion caused by the electric force, so the work gained as kinetic energy is positive.

Step 2: Apply the initial condition and check the options The particle starts from rest, hence $K_i = 0$ and the entire electric work becomes kinetic energy. The mass affects the resulting speed, but it cancels from the expression for kinetic energy gained over a fixed displacement. Option (A) has an extra factor of distance, option (C) an extra factor of charge, and option (D) extra factors of both charge and field; none has the required dependence or energy dimensions.

Step 3: Conclusion The kinetic energy is $K_f = qE_0x$. Hence, the correct option is (B).

Quick Tip to Solve: For a charge released from rest in a uniform field, calculate force times displacement before trying to find its acceleration or speed.

2. A square loop of side 50 cm is placed in a uniform magnetic field of 3.0 T acting perpendicular to the plane of the loop. If the loop is rotated through an angle of 90° in 0.3 s, the value of emf induced in the loop would be: [1 mark]

- (A) 0.25 V
- (B) 0.50 V
- (C) 0.75 V
- (D) 1.0 V

Correct Answer: No printed option is correct: the average induced emf has magnitude 2.5 V.

Solution:

Concept:

- Magnetic flux through one plane loop is $\Phi = BA \cos \theta$, where B is the magnetic field, A the loop area, and θ the angle between the field and the area normal.
- Faraday's law gives the magnitude of the average induced emf as $|\varepsilon_{av}| = |\Phi_f - \Phi_i|/\Delta t$, where Δt is the rotation time.

Step 1: Find the initial and final flux The side of the square is 50 cm = 0.50 m, giving

$$A = (0.50)^2 = 0.25 \text{ m}^2.$$

Initially the field is perpendicular to the loop's plane, so $\theta_i = 0^\circ$. After the usual quarter-turn that changes the loop's orientation to the field, $\theta_f = 90^\circ$. Thus

$$\Phi_i = 3.0(0.25) = 0.75 \text{ Wb}, \quad \Phi_f = 0.$$

Step 2: Evaluate the average emf and compare every option The stated time is 0.3 s, so

$$|\varepsilon_{av}| = \frac{0.75}{0.3} = 2.5 \text{ V}.$$

This differs from (A) 0.25 V, (B) 0.50 V, (C) 0.75 V, and (D) 1.0 V. The number 0.75 represents the change of flux in webers, not the emf in volts. The given total rotation and duration determine an average emf under this standard interpretation; an instantaneous emf would require the time dependence of the rotation. The printed options therefore do not supply the calculated answer.

Step 3: Conclusion The magnitude of the average induced emf is 2.5 V. None of the printed options (A)–(D) is correct for the stated data.

Quick Tip to Solve: Square the side after converting centimetres to metres, then divide the flux change by the rotation time; flux alone is not an emf.

3. A plane electromagnetic wave travels through a medium and the magnetic field associated with it is given by $B = 5 \times 10^{-8} \sin(3 \times 10^{10} t - 150 x)$ T, where x is in metres and t is in seconds. The velocity of the wave is: [1 mark]

- (A) $2.0 \times 10^8 \text{ m s}^{-1}$
- (B) $4.5 \times 10^7 \text{ m s}^{-1}$
- (C) $3.5 \times 10^7 \text{ m s}^{-1}$
- (D) $2.5 \times 10^8 \text{ m s}^{-1}$

Correct Answer: (A) $2.0 \times 10^8 \text{ m s}^{-1}$

Solution:

Concept:

- A plane wave of the form $B = B_0 \sin(\omega t - kx)$ travels in the positive x -direction; B_0 is its magnetic amplitude, ω its angular frequency, and k its wave number.
- The wave speed is $v = \omega/k$, obtained by following a point of constant phase $\omega t - kx$.

Step 1: Identify the two phase coefficients Compare the given argument $3 \times 10^{10} t - 150x$ with $\omega t - kx$. This gives

$$\omega = 3 \times 10^{10} \text{ rad s}^{-1}, \quad k = 150 \text{ rad m}^{-1}.$$

The coefficient $5 \times 10^{-8} \text{ T}$ is the amplitude, so it is not used to calculate the speed.

Step 2: Divide the angular frequency by the wave number A fixed phase satisfies $\omega t - kx = \text{constant}$, which gives $dx/dt = \omega/k$. Substitution yields

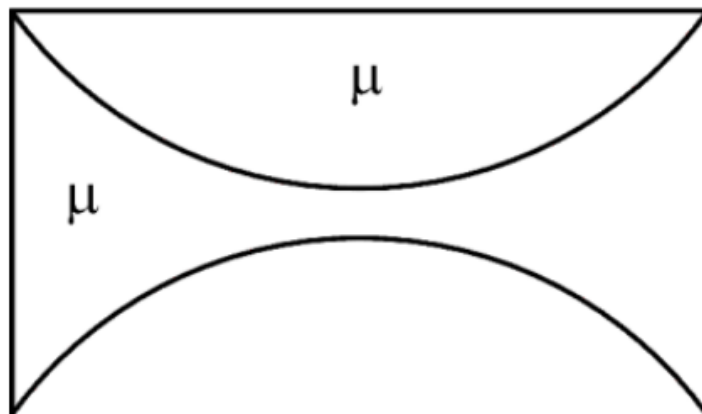
$$v = \frac{3 \times 10^{10}}{150} = 2.0 \times 10^8 \text{ m s}^{-1}.$$

Only (A) matches this ratio. The values in (B), (C), and (D) do not satisfy $kv = \omega$. Since the wave is in a medium, its speed need not equal the vacuum value $3.0 \times 10^8 \text{ m s}^{-1}$.

Step 3: Conclusion The velocity is $2.0 \times 10^8 \text{ m s}^{-1}$ along the positive x -direction. Hence, the correct option is (A).

Quick Tip to Solve: In a travelling-wave sine argument, divide the coefficient of time by the magnitude of the coefficient of position; ignore the amplitude.

4. A thin plano-convex lens and a thin equi-concave lens are kept coaxially in contact as shown in the figure. Assuming both the lenses are made of glass of refractive index μ , and R is the radius of curvature of each curved surface, the focal length of the combination is: [1 mark]



- (A) $\frac{R}{\mu - 1}$
 (B) $-\frac{R}{\mu - 1}$
 (C) $\frac{2R}{\mu - 1}$
 (D) $-\frac{2R}{\mu - 1}$

Correct Answer: (B) $-\frac{R}{\mu - 1}$

Solution:

Concept:

- The lens-maker formula in air is $P = 1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where P is lens power, f the focal length, and R_1, R_2 the signed surface radii.
- The powers of thin coaxial lenses in contact add: $1/F = P_1 + P_2$, where F is the equivalent focal length. Convex lenses have positive power and concave lenses negative power.

Step 1: Write the powers of the two lenses The plane face of the plano-convex lens has zero curvature. Its single convex face of radius magnitude R therefore gives

$$P_1 = \frac{\mu - 1}{R}.$$

For the equi-concave lens, the two surfaces have signed radii $-R$ and $+R$, respectively, along the direction of light. Hence

$$P_2 = (\mu - 1) \left(-\frac{1}{R} - \frac{1}{R} \right) = -\frac{2(\mu - 1)}{R}.$$

Step 2: Add the powers and check the sign Since the lenses are thin and in contact,

$$\frac{1}{F} = \frac{\mu - 1}{R} - \frac{2(\mu - 1)}{R} = -\frac{\mu - 1}{R}.$$

The stronger concave power makes the combination diverging, excluding the positive answers (A) and (C). The two powers partly cancel, leaving one concave-surface contribution; option (D) incorrectly doubles the magnitude of the resulting focal length.

Step 3: Conclusion The combination has focal length $F = -R/(\mu - 1)$. Hence, the correct option is (B).

Quick Tip to Solve: Add lens powers, not focal lengths: one convex surface contributes $+(\mu - 1)/R$, while the two concave surfaces contribute twice the negative amount.

5. The phase difference between the two superimposing waves that give rise to a bright spot in a Young's double-slit experiment is (n is an integer): [1 mark]

- (A) $2n\pi$
- (B) $2n\pi + \frac{\pi}{4}$
- (C) $2n\pi + \frac{\pi}{2}$
- (D) $2n\pi + \pi$

Correct Answer: (A) $2n\pi$

Solution:

Concept:

- A bright interference fringe is an intensity maximum produced by constructive interference of the two coherent waves.
- For component intensities I_1, I_2 and phase difference ϕ , the resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$.

Step 1: Apply the condition for maximum intensity The two component intensities are fixed, so the resultant intensity is largest when $\cos \phi = 1$. The corresponding phase differences are

$$\phi = 2n\pi, \quad n \in \mathbb{Z}.$$

Physically, the arriving waves are in phase: their displacement maxima and minima reinforce one another.

Step 2: Test the other phase differences Adding a whole number of full cycles $2n\pi$ does not change the interference. The extra $\pi/4$ in (B) gives $\cos \phi = 1/\sqrt{2}$, so the intensity is below its maximum. The extra $\pi/2$ in (C) gives $\cos \phi = 0$, also below the maximum. The extra π in (D) gives $\cos \phi = -1$, corresponding to an intensity minimum. Thus only (A) is the bright-fringe condition.

Step 3: Conclusion The required phase difference is $2n\pi$, with n an integer. Hence, the correct option is (A).

Quick Tip to Solve: Reduce a proposed phase difference modulo 2π : a bright-fringe maximum leaves zero phase, while a minimum leaves π .

6. A telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. The magnifying power and the length of the telescope tube will be respectively: [1 mark]

- (A) 24, 150 cm
- (B) 42, 138 cm
- (C) 24, 138 cm
- (D) 42, 150 cm

Correct Answer: (A) 24, 150 cm

Solution:

Concept:

- For an astronomical telescope in normal adjustment, the final image is at infinity and the magnitude of angular magnification is $|M| = f_o/f_e$, where f_o, f_e are the objective and eyepiece focal lengths.
- In normal adjustment the objective's image lies in the eyepiece's focal plane, giving lens separation $L = f_o + f_e$.

Step 1: Calculate the magnifying power Use the standard normal-adjustment setting, since no alternative final-image distance is specified. With $f_o = 144$ cm and $f_e = 6.0$ cm,

$$|M| = \frac{144}{6.0} = 24.$$

The signed angular magnification is -24 , indicating an inverted image; the options give magnifying power as its positive magnitude.

Step 2: Find the tube length and compare the pairs The two converging lenses have their adjacent focal planes in common, so their focal lengths add:

$$L = 144 + 6.0 = 150 \text{ cm.}$$

Option (C) uses the difference $144 - 6 = 138$ cm, which is not the length of this telescope in normal adjustment. Options (B) and (D) both give 42, which is not the ratio of the supplied focal lengths. Only (A) has both values correct.

Step 3: Conclusion The magnifying power is 24 and the tube length is 150 cm. Hence, the correct option is (A).

Quick Tip to Solve: For a normally adjusted astronomical telescope, divide the focal lengths for magnifying power and add them for the tube length.

7. In which of the following, total internal reflection does not occur?

[1 mark]

- (A) Twinkling of stars
- (B) Brilliance of diamonds
- (C) Optical fibre
- (D) Reflecting prism

Correct Answer: (A) Twinkling of stars

Solution:

Concept:

- Total internal reflection occurs when light travels from an optically denser medium to a rarer one and its angle of incidence exceeds the critical angle.
- Refraction through atmospheric layers with varying refractive indices changes the direction and received intensity of starlight; this produces the twinkling of stars.

Step 1: Identify the cause of stellar twinkling A star appears nearly as a point source. Its light passes through moving layers of air whose densities and refractive indices fluctuate. The resulting changing refraction alters the light reaching the observer, making the apparent brightness fluctuate. This phenomenon is atmospheric refraction, so total internal reflection is not its cause.

Step 2: Check the three total-reflection applications In a diamond, the high refractive index makes the critical angle small, allowing repeated total internal reflections that contribute to its brilliance. An optical fibre guides light by total internal reflection at its higher-index core and lower-index cladding boundary. A reflecting prism is designed so that internal incidence at the reflecting face exceeds the critical angle. Therefore (B), (C), and (D) all involve total internal reflection under their operating conditions.

Step 3: Conclusion Twinkling of stars is caused by atmospheric refraction rather than total internal reflection. Hence, the correct option is (A).

Quick Tip to Solve: Associate stellar twinkling with changing atmospheric refraction; associate fibres, diamonds, and reflecting prisms with a critical-angle condition.

8. Radiation of wavelength 200 nm is incident on a photosensitive surface of work function 4.2 eV. The kinetic energy of fastest photoelectrons emitted from this surface will be close to: [1 mark]

- (A) 3.5 eV
- (B) 3.0 eV
- (C) 2.5 eV
- (D) 2.0 eV

Correct Answer: (D) 2.0 eV

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = hc/\lambda - \phi$, where ϕ is the work function and $K_{\max}$ is the maximum kinetic energy.
- The photon energy in electron volts is approximately $E_\gamma = 1240/\lambda$ when the wavelength λ is expressed in nanometres.

Step 1: Find the energy of an incident photon For the printed wavelength $\lambda = 200 \text{ nm}$,

$$E_{\gamma} = \frac{1240}{200} = 6.20 \text{ eV}.$$

This exceeds the work function 4.2 eV , so photoelectrons can be emitted.

Step 2: Subtract the energy needed to escape Each fastest electron uses the work-function energy to leave the surface; the remaining energy becomes its kinetic energy:

$$K_{\max} = 6.20 - 4.2 = 2.0 \text{ eV}.$$

The values 3.5 , 3.0 and 2.5 eV in options (A), (B) and (C) all exceed this available excess energy. Using the given physical constants instead of the rounded value of hc gives essentially the same nearest option.

Step 3: Conclusion The kinetic energy of the fastest photoelectrons is close to 2.0 eV . Hence, the correct option is (D).

Quick Tip to Solve: For wavelengths in nanometres, calculate $1240/\lambda$ in eV first, then subtract the work function in eV.

9. A proton and an alpha particle have equal momentum. The ratio of their kinetic energies (E_p/E_{α}) and de Broglie wavelengths associated with them ($\lambda_p/\lambda_{\alpha}$) respectively are: [1 mark]

- (A) 2, 1
- (B) 1, 2
- (C) 4, 1
- (D) 1, 4

Correct Answer: (C) 4, 1

Solution:

Concept:

- For a nonrelativistic particle of mass m and momentum p , its kinetic energy is $E = p^2/(2m)$.
- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant. An alpha particle has approximately four times the proton's mass: $m_{\alpha} = 4m_p$.

Step 1: Compare the kinetic energies at equal momentum Since $p_p = p_{\alpha} = p$, the common factor $p^2/2$ cancels. Kinetic energy is therefore inversely proportional to mass:

$$\frac{E_p}{E_{\alpha}} = \frac{p^2/(2m_p)}{p^2/(2m_{\alpha})} = \frac{m_{\alpha}}{m_p} = 4.$$

Equal momentum does not imply equal kinetic energy. The lighter proton has the greater kinetic energy.

Step 2: Compare the de Broglie wavelengths The wavelength depends directly on momentum through its reciprocal, so

$$\frac{\lambda_p}{\lambda_{\alpha}} = \frac{h/p_p}{h/p_{\alpha}} = \frac{p_{\alpha}}{p_p} = 1.$$

Option (A) incorrectly uses an energy ratio of 2, while options (B) and (D) give unequal wavelengths

despite the equal momenta. The alpha particle's charge does not enter either comparison here.

Step 3: Conclusion The ratios E_p/E_α and λ_p/λ_α are 4 and 1, respectively. Hence, the correct option is (C).

Quick Tip to Solve: At equal momentum, use $E \propto 1/m$ but $\lambda = h/p$: reverse the mass ratio for energies and keep wavelengths equal.

10. If r_1 and r_2 are the radii of atomic nuclei of mass numbers 64 and 27 respectively, then the value of $\left(\frac{r_1}{r_2}\right)$ is: [1 mark]

- (A) 1
- (B) $\frac{4}{3}$
- (C) $\frac{3}{4}$
- (D) $\frac{27}{64}$

Correct Answer: (B) $\frac{4}{3}$

Solution:

Concept:

- The empirical nuclear-radius relation is $r = r_0 A^{1/3}$, where A is the mass number and r_0 is nearly the same constant for all nuclei.
- Nuclear volume is proportional to mass number, so nuclear radius is proportional to its cube root, not to mass number itself.

Step 1: Form the radius ratio The nucleus of mass number 64 has radius r_1 , and that of mass number 27 has radius r_2 . Applying the same radius constant to both gives

$$\frac{r_1}{r_2} = \frac{r_0(64)^{1/3}}{r_0(27)^{1/3}} = \left(\frac{64}{27}\right)^{1/3}.$$

Step 2: Evaluate the cube roots and check the order Since $64 = 4^3$ and $27 = 3^3$,

$$\frac{r_1}{r_2} = \frac{4}{3}.$$

The ratio must exceed 1 because the first nucleus is larger. Thus (A) incorrectly assumes equal radii, (C) reverses the required ratio, and (D) uses the inverted mass-number ratio without taking a cube root.

Step 3: Conclusion The required radius ratio is $r_1/r_2 = 4/3$. Hence, the correct option is (B).

Quick Tip to Solve: For nuclear radius ratios, cancel r_0 , take the cube roots of the mass numbers, and check that the heavier nucleus has the larger radius.

11. When the forward bias voltage in a semiconductor diode is changed from 0.8 V to 1.0 V, the forward current changes by 2.0 mA. The forward bias resistance of the diode will be: [1 mark]

- (A) 200 Ω
- (B) 175 Ω
- (C) 100 Ω
- (D) 125 Ω

Correct Answer: (C) 100 Ω

Solution:

Concept:

- The forward resistance inferred from changes in voltage and current is the dynamic resistance, $r_f = \Delta V / \Delta I$.
- Here ΔV is the change in forward voltage and ΔI is the corresponding change in current. Convert milliamperes to amperes to obtain resistance in ohms.

Step 1: Calculate the voltage and current changes The printed voltage changes from 0.8 V to 1.0 V. Hence,

$$\Delta V = 1.0 - 0.8 = 0.2 \text{ V}, \quad \Delta I = 2.0 \text{ mA} = 2.0 \times 10^{-3} \text{ A}.$$

The stated current is already a change, so no further subtraction is needed for it.

Step 2: Divide the voltage change by the current change Over the given interval, the forward dynamic resistance is

$$r_f = \frac{0.2}{2.0 \times 10^{-3}} = 100 \Omega.$$

The other values, 200, 175 and 125 Ω , do not equal this ratio. Using either endpoint voltage instead of ΔV would be incorrect: the question supplies changes for a nonlinear diode, rather than a complete voltage-current operating point.

Step 3: Conclusion The forward bias resistance over the stated interval is 100 Ω . Hence, the correct option is (C).

Quick Tip to Solve: When a diode question gives changes, use $\Delta V / \Delta I$; remembering $1 \text{ V} / 1 \text{ mA} = 1 \text{ k}\Omega$ prevents a factor-of-1000 error.

12. The process named ‘minority carrier injection’ in a p-n junction diode occurs during: [1 mark]

- (A) forward biasing
- (B) reverse biasing
- (C) no biasing at low temperature
- (D) no biasing at high temperature

Correct Answer: (A) forward biasing

Solution:

Concept:

- Forward bias lowers the potential barrier of a p-n junction and allows majority carriers to diffuse across the junction.
- Electrons are majority carriers in the n-region but minority carriers in the p-region; holes are

majority carriers in the p-region but minority carriers in the n-region.

Step 1: Identify how minority carriers are injected Under forward bias, electrons cross from the n-side into the p-side and holes cross from the p-side into the n-side. After crossing, these carriers are minority carriers in the receiving region, increasing the minority-carrier concentration near the junction. This process is called minority carrier injection.

Step 2: Compare the other bias conditions Reverse bias raises the barrier and suppresses majority-carrier diffusion; its small reverse current arises mainly from thermally generated minority carriers being swept across the junction. Hence option (B) does not describe injection. With no applied bias, equilibrium diffusion and drift balance. Changing the temperature can change thermal carrier generation, but neither unbiased condition in (C) or (D) is the forward-bias injection process.

Step 3: Conclusion Minority carrier injection in a p-n junction diode occurs during forward biasing. Hence, the correct option is (A).

Quick Tip to Solve: Follow each carrier across the junction: an n-side majority electron becomes a p-side minority electron when forward bias injects it.

13. Assertion (A): In a Wheatstone bridge circuit, if we interchange the position of the cell and the galvanometer, the balance condition $\frac{P}{Q} = \frac{R}{S}$ remains unchanged.

Reason (R): $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{Q}{S} = \frac{P}{R}$ so balance condition remains same.

Select the correct answer from the codes (A), (B), (C) and (D) given below:

[1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- A Wheatstone bridge is balanced when the galvanometer terminals are at equal potential, so no current flows through it.
- For bridge resistances P, Q, R, S , the usual balance relation $P/Q = R/S$ is equivalent to the product relation $PS = QR$.

Step 1: Evaluate the Assertion Interchanging the cell and galvanometer connects the source across the former detector diagonal. The two series branches then consist of P, R and Q, S . Equal potentials at their junctions require equal voltage-divider fractions:

$$\frac{P}{P+R} = \frac{Q}{Q+S} \Rightarrow PS = QR.$$

This is the same product relation as before the interchange. Thus Assertion (A) is true.

Step 2: Evaluate the Reason Cross-multiply the original balance condition and divide by RS :

$$\frac{P}{Q} = \frac{R}{S} \Rightarrow PS = QR \Rightarrow \frac{P}{R} = \frac{Q}{S}.$$

This is exactly the printed relation $Q/S = P/R$, so Reason (R) is true.

Step 3: Test the explanatory connection The Reason gives the ratio condition for the rearranged series branches, and that condition is algebraically equivalent to the original one. It therefore explains why the balance condition remains unchanged. Option (B) misses this connection, while (C) and (D) incorrectly label at least one true statement as false.

Step 4: Conclusion Both Assertion (A) and Reason (R) are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: Reduce both Wheatstone balance ratios to $PS = QR$; the unchanged cross-product reveals why swapping the two diagonals preserves balance.

14. Assertion (A): The cylindrical soft iron core in a moving coil galvanometer only makes the magnetic field radial and does not affect the strength of the magnetic field.

Reason (R): In a moving coil galvanometer, the plane of the coil is always perpendicular to the magnetic field.

Select the correct answer from the codes (A), (B), (C) and (D) given below:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false.

Solution:

Concept:

- The high magnetic permeability of a soft iron core increases the magnetic field strength in a moving-coil galvanometer; the core and curved pole pieces also produce a radial field.
- The magnetic torque is $\tau = NIAB \sin \theta$, where N is the number of turns, I the current, A the coil area, B the field strength, and θ the angle between the coil's normal and the field.

Step 1: Evaluate the Assertion The cylindrical soft iron core helps make the field radial and also strengthens it because soft iron has high permeability. The claim that it does not affect the magnetic field strength is therefore incorrect. Assertion (A) is false.

Step 2: Evaluate the Reason In the radial field, the plane of the rotating coil remains parallel to the local magnetic field at its active sides. The coil's normal is perpendicular to the field, giving $\theta = 90^\circ$ and $\tau = NIAB$. The Reason wrongly says that the plane is perpendicular to the field, confusing the plane with its normal. Hence Reason (R) is false.

Step 3: Test the explanatory connection Since both statements are false, the Reason cannot explain the Assertion. Options (A) and (B) incorrectly regard both statements as true, and option (C) incorrectly

accepts the Assertion. A plane perpendicular to the field would instead give a parallel normal and zero torque.

Step 4: Conclusion Both the Assertion and the Reason are false. Hence, the correct option is (D).

Quick Tip to Solve: For a radial-field galvanometer, remember two facts together: the iron core strengthens the field, and the coil's plane stays parallel to it.

15. Assertion (A): Photoelectric current depends upon the intensity of the incident radiation.

Reason (R): Stopping potential is independent of the intensity of the incident radiation.

Select the correct answer from the codes (A), (B), (C) and (D) given below:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- For incident frequency above the threshold frequency, increasing intensity increases the number of photons incident per second and hence the number of emitted photoelectrons per second.
- Einstein's photoelectric equation is $eV_s = K_{\max} = h\nu - \phi$, where V_s is the stopping potential, ν the frequency and ϕ the work function.

Step 1: Evaluate the assertion At fixed frequency and fixed collecting conditions, stronger illumination supplies more photons per second. More electrons are emitted and collected per second, so the photoelectric current increases. Thus, Assertion (A) is true.

Step 2: Evaluate the reason At fixed frequency, the energy $h\nu$ of each photon remains unchanged when intensity changes. Therefore, $K_{\max}$ and the stopping potential remain unchanged. Reason (R) is also true.

Step 3: Check the explanatory connection The reason concerns the maximum energy of an individual photoelectron; the assertion concerns the number of electrons collected per second. Independence of stopping potential from intensity does not explain the increase in electron count. Thus, (A) is excluded by the missing explanatory link, while (C) and (D) incorrectly classify true statements as false.

Step 4: Conclusion Both the assertion and the reason are true, but the reason does not correctly explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Separate photon number from photon energy: intensity controls photocurrent, whereas frequency controls the stopping potential for a given metal.

16. Assertion (A): Nuclear forces are always attractive.

Reason (R): The nuclear force between protons and neutrons in a nucleus is a weak force.

Select the correct answer from the codes (A), (B), (C) and (D) given below:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false.

Solution:

Concept:

- The nuclear force between nucleons is a very strong, short-range force. It provides the binding that holds protons and neutrons together in a nucleus.
- The nucleon interaction is attractive over typical nuclear separations, but becomes strongly repulsive at very small separations. This repulsive core prevents nucleons from collapsing into one point.

Step 1: Evaluate the assertion The word “always” makes the assertion false. Although the nuclear force is attractive over the separation range responsible for nuclear binding, it is repulsive at extremely small nucleon separations, roughly below 0.5 fm.

Step 2: Evaluate the reason The force binding a proton and a neutron is the strong nuclear force, not the weak interaction. The weak interaction is involved in processes such as beta decay. Therefore, Reason (R) is false.

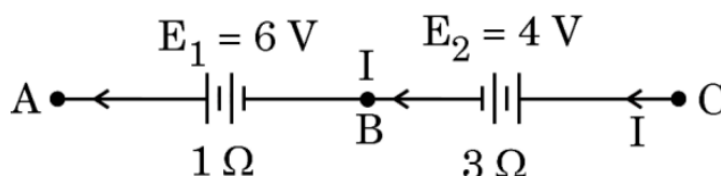
Step 3: Apply the assertion-reason codes Since neither statement is true, there is no valid explanatory connection between them. Options (A) and (B) require both statements to be true, and option (C) requires the assertion to be true; all three are excluded.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: For a statement that nuclear forces are “always attractive”, remember the repulsive core at very small separation; nuclear binding itself is due to the strong force.

Section B (10 marks)

17(a). In the given figure, a steady current I flows through the circuit when points A and C are connected by a wire of negligible resistance. Find the potential difference between points B and C . [2 marks]



Solution:

Concept:

- Kirchhoff's loop rule states that the algebraic sum of potential changes around a closed loop is zero. Crossing a cell from its negative to positive terminal gives a rise equal to its emf.
- A resistance produces a potential drop Ir along the current. For a cell being charged, current enters its positive terminal and its terminal potential difference is $E + Ir$, where E is its emf and r its internal resistance.

Step 1: Find the current from the opposing emfs The longer cell plates show that the 6 V cell has its positive terminal towards A , whereas the 4 V cell has its positive terminal towards C . Thus the emfs oppose each other. Since the connecting wire has negligible resistance, $V_A = V_C$; the only resistance in the closed loop is $1 + 3 = 4\ \Omega$. Along the indicated current, from C through B to A , Kirchhoff's rule gives

$$6 - 4 - I(1 + 3) = 0, \quad I = \frac{6 - 4}{4} = 0.5\text{ A}.$$

Step 2: Calculate the potential difference across the second cell Current enters the positive terminal of the 4 V cell at C , so this cell is being charged. Traversing from C to B crosses its emf from positive to negative and also follows the current through its internal resistance. Therefore,

$$V_B - V_C = -E_2 - Ir_2 = -4 - (0.5)(3) = -5.5\text{ V}.$$

As a check, the delivering 6 V cell has $V_A - V_B = 6 - (0.5)(1) = 5.5\text{ V}$, consistent with $V_A = V_C$.

Step 3: Conclusion The potential difference has magnitude 5.5 V , with C at the higher potential. Explicitly, $V_C - V_B = 5.5\text{ V}$, or $V_B - V_C = -5.5\text{ V}$.

Quick Tip to Solve: Check the long plates before adding emfs. Here the weaker cell is charged, so use $E_2 + Ir_2$ for its terminal voltage.

17(b). A battery of emf 21 V and internal resistance $3\ \Omega$ is connected to a resistor. If the current in the circuit is 3 A, find:

(i) the resistance of the resistor.

(ii) the terminal voltage of the battery.

[2 marks]

Solution:

Concept:

- For a battery delivering current through an external resistor, the emf satisfies $E = I(R + r)$, where R is the external resistance and r is the battery's internal resistance.
- The terminal voltage is the potential difference available across the external resistor: $V = IR = E - Ir$. The quantity Ir is the voltage drop inside the battery.

Step 1: Determine the external resistance The external resistor and the internal resistance carry the same current, so their resistances add. The given emf is $E = 21\text{ V}$, the internal resistance is $r = 3\ \Omega$, and the current is $I = 3\text{ A}$. Hence the total circuit resistance is

$$R + r = \frac{E}{I} = \frac{21}{3} = 7\ \Omega.$$

Subtract the internal resistance to obtain the resistance asked for in part (i):

$$R = 7 - 3 = 4\ \Omega.$$

The total resistance $7\ \Omega$ must not be mistaken for the external resistor alone.

Step 2: Find the battery's terminal voltage The battery supplies current, so its terminal voltage is smaller than its emf by the internal voltage drop. For part (ii),

$$V = E - Ir = 21 - (3)(3) = 12\text{ V}.$$

This agrees with the potential difference across the external resistor, $IR = (3)(4) = 12\text{ V}$. Thus the emf is shared as 12 V across the resistor and 9 V inside the battery.

Step 3: Conclusion (i) The resistance of the external resistor is $4\ \Omega$. (ii) The terminal voltage of the battery is 12 V.

Quick Tip to Solve: First compute E/I , then subtract the internal resistance. Check the terminal voltage using both $E - Ir$ and IR .

18. Explain why diffraction of sound is more common in daily experience than that of light. [2 marks]

Solution:

Concept:

- Diffraction is the spreading of waves as they pass through an opening or around the edges of an obstacle. It is appreciable when the opening or obstacle size is comparable to the wavelength.
- For an aperture of width a , the characteristic angular spread increases with the ratio λ/a , where λ is the wavelength. A much smaller wavelength gives much less spreading for the same aperture.

Step 1: Compare sound wavelengths with everyday objects Audible sound has wavelengths ranging

approximately from a few centimetres to several metres in air. These lengths are comparable to everyday openings and obstacles such as doors, windows and walls. Sound therefore spreads appreciably around their edges and into the geometrical shadow region; for example, we can hear a person speaking beyond a doorway even without a direct line of sight.

Step 2: Compare light wavelengths with the same objects Visible light has wavelengths of approximately 4×10^{-7} to 7×10^{-7} m. These are extremely small compared with ordinary doorways and obstacles, so λ/a is tiny and the diffraction spread is generally too small to notice in daily life. Light therefore produces comparatively sharp shadows. To observe conspicuous diffraction of light, narrow slits or very small obstacles are usually required.

Step 3: Conclusion Diffraction of sound is more commonly noticed because its wavelength is comparable to everyday openings and obstacles. Visible light has a much smaller wavelength, so its diffraction by the same objects is usually negligible to the eye.

Quick Tip to Solve: Compare wavelength with obstacle size: for the same doorway, sound has a much larger λ/a and therefore spreads through a much larger angle.

19. Explain the terms mass defect and binding energy. How are they related?

[2 marks]

Solution:

Concept:

- A bound nucleus has less rest mass than the sum of the rest masses of its free constituent protons and neutrons. This difference is the mass defect.
- Einstein's mass-energy relation is $E = mc^2$, where c is the speed of light in vacuum. The energy corresponding to the mass defect is the nuclear binding energy.

Step 1: Define the mass defect Consider a nucleus with mass number A , atomic number Z and nuclear mass M_N . It contains Z protons and $A - Z$ neutrons. If m_p and m_n are their individual rest masses, the mass defect is

$$\Delta m = Zm_p + (A - Z)m_n - M_N.$$

Here M_N is the mass of the nucleus, excluding atomic electrons. The positive difference represents the decrease in rest mass when the separated nucleons form the bound nucleus.

Step 2: Define binding energy and relate it to the mass defect Binding energy is the minimum energy required to separate a nucleus completely into its constituent free protons and neutrons. Equivalently, the same energy is released when these free nucleons combine to form the nucleus. The loss of rest mass on formation accounts for this released energy through mass-energy equivalence.

Step 3: Conclusion Mass defect is the shortfall in the nuclear mass compared with the total mass of its free nucleons. Binding energy is the energy needed to separate those nucleons, and the relation is $B = \Delta m c^2$. When Δm is in kg and c in m s^{-1} , B is in J.

Quick Tip to Solve: Subtract the nuclear mass from the free-nucleon mass sum; multiply this positive mass defect by c^2 to obtain the binding energy.

20. A particle of mass M at rest splits up into two particles of masses m_1 and m_2 having non-zero velocities. Calculate the ratio of the de Broglie wavelengths associated with the two particles.

[2 marks]

Solution:

Concept:

- In an isolated splitting process, total linear momentum is conserved. The parent particle is at rest, so its initial momentum is zero.
- The de Broglie wavelength of a particle is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of the particle's momentum.

Step 1: Apply momentum conservation Let $\vec{p}_1$ and $\vec{p}_2$ be the momenta of the two particles just after the parent splits. With no external impulse during the splitting,

$$\vec{p}_1 + \vec{p}_2 = \vec{0} \Rightarrow \vec{p}_1 = -\vec{p}_2.$$

Thus the products move in opposite directions with equal non-zero momentum magnitudes: $p_1 = p_2$. In the non-relativistic notation this is $m_1 v_1 = m_2 v_2$, so unequal masses generally imply unequal speeds, not unequal momenta.

Step 2: Compare the matter wavelengths Use the magnitude of momentum in each de Broglie relation:

$$\lambda_1 = \frac{h}{p_1}, \quad \lambda_2 = \frac{h}{p_2}, \quad \frac{\lambda_1}{\lambda_2} = \frac{p_2}{p_1}.$$

The equality of the momentum magnitudes makes this ratio independent of M , m_1 and m_2 . Opposite momentum directions do not introduce a minus sign into the wavelength, which is a positive scalar.

Step 3: Conclusion The two particles have equal de Broglie wavelengths: $\lambda_1 : \lambda_2 = 1 : 1$.

Quick Tip to Solve: For a parent at rest producing exactly two particles, first equate the momentum magnitudes; their de Broglie wavelengths then follow directly from h/p .

21. How are charge carriers created in an intrinsic semiconductor? Explain.

[2 marks]

Solution:

Concept:

- An intrinsic semiconductor is a pure semiconductor without intentional impurity doping. Its mobile charge carriers are conduction electrons and holes.
- Thermal excitation creates an electron-hole pair: an electron gains sufficient energy to reach the conduction band and leaves a vacant state, called a hole, in the valence band.

Step 1: Explain the generation of an electron-hole pair In a pure silicon or germanium crystal, valence electrons participate in covalent bonds. At absolute zero, the valence band is full and the conduction band is empty, so there are no thermally generated mobile carriers. At a finite temperature, some electrons gain enough thermal energy to overcome the energy gap E_g . A covalent bond is thereby broken and its excited electron becomes free to move through the crystal in the conduction band.

Step 2: Identify the hole and the carrier concentrations The missing valence electron leaves a hole that behaves as a mobile positive charge. A neighbouring valence electron can fill this vacancy, leaving another vacancy behind; successive such transfers correspond to motion of the hole. Hence both the

conduction electron and the hole can carry current when an electric field is applied. Each excitation creates one electron and one hole, so in an intrinsic semiconductor their number densities are equal:

$$n_e = n_h = n_i,$$

where n_i is the intrinsic carrier concentration. Thermal generation and electron-hole recombination balance at thermal equilibrium.

Step 3: Conclusion Charge carriers in an intrinsic semiconductor are created when thermal energy releases valence electrons from covalent bonds into the conduction band. Equal numbers of mobile electrons and holes are formed as electron-hole pairs.

Quick Tip to Solve: Explain carrier creation as a pair: every electron promoted to the conduction band leaves exactly one hole in the valence band, giving $n_e = n_h$.

Section C (21 marks)

22. (a) Establish the relation between drift velocity of electrons (v_d) and electric current (I) in a conductor.

(b) How is v_d affected when the length of the conductor is doubled, keeping the voltage applied across the conductor constant? [3 marks]

Solution:

Concept:

- Drift speed v_d is the magnitude of the electrons' average velocity under an electric field. Conventional current flows opposite to electron drift.
- Current is charge transferred per unit time: $I = \Delta Q / \Delta t$. With electron number density n , a volume $\mathcal{V}$ contains $n\mathcal{V}$ free electrons.
- For a uniform conductor, $v_d = eE\tau/m_e$ and $E = V/L$, where e is the magnitude of electronic charge, τ is the relaxation time, m_e is the electron mass, V is the applied voltage and L is the conductor length.

Step 1: (a) Count the electrons crossing a section In a uniform conductor of cross-sectional area A , electrons drift a distance $v_d\Delta t$ in time Δt . The net transfer across a section is therefore due to the electrons in a cylinder of length $v_d\Delta t$. Its volume and number of electrons are

$$\mathcal{V} = Av_d\Delta t, \quad N = nAv_d\Delta t.$$

Random thermal motion contributes no net current; directed drift transfers net charge.

Step 2: (a) Express the current in terms of drift speed Each electron carries charge of magnitude e , so the magnitude of the charge transported is $\Delta Q = Ne$. Therefore,

$$I = \frac{\Delta Q}{\Delta t} = \frac{nAv_d\Delta t e}{\Delta t} = neAv_d.$$

In vector form the current density is $\vec{J} = -ne\vec{v}_d$; the minus sign expresses the opposite directions of conventional current and electron drift.

Step 3: (b) Compare drift speeds at fixed voltage For the same conductor material at unchanged

temperature, τ remains unchanged. Substituting $E = V/L$ into the drift-speed formula gives

$$v_d = \frac{e\tau V}{m_e L}.$$

Doubling the length while keeping V fixed reduces the electric field to $E' = V/(2L) = E/2$. Consequently,

$$v'_d = \frac{e\tau V}{m_e(2L)} = \frac{v_d}{2}.$$

Step 4: Conclusion (a) The relation is $I = neAv_d$ for magnitudes; electron drift is opposite to conventional current. (b) At constant applied voltage, doubling the conductor length halves the drift speed: $v'_d = v_d/2$.

Quick Tip to Solve: Use the swept volume $Av_d\Delta t$ to derive the current relation. For a length change at fixed voltage, first use $E = V/L$, then $v_d \propto E$.

23(a). A circular coil of 30 turns and radius 8.0 cm carrying a current of 6 A is suspended vertically in a uniform horizontal magnetic field of 1.0 T. The field lines make an angle of 30° with the plane of the coil. Calculate the magnitude of the external torque that must be applied to prevent the coil from turning. What would happen if the circular coil is replaced by a planar coil of irregular shape that encloses the same area, keeping other parameters unchanged? [3 marks]

Solution:

Concept:

- The magnetic moment of a planar coil is $\mu = NIA\hat{n}$, where N is the number of turns, I is current, A is the area of each turn and $\hat{n}$ is its unit normal.
- In a uniform magnetic field, the torque is $\tau_B = \mu \times B$, so its magnitude is $NIAB \sin \theta$. Here θ is the angle between the normal to the coil and the field, not the angle with its plane.
- To prevent rotation, the external torque must balance the magnetic torque: $\tau_{\text{ext}} = -\tau_B$.

Step 1: Find the area and the angle with the normal Convert the radius to SI units before calculating the area of one turn:

$$r = 8.0 \text{ cm} = 0.080 \text{ m}, \quad A = \pi r^2 = \pi(0.080)^2 = 0.0064\pi \text{ m}^2.$$

The field makes 30° with the plane. As the normal is perpendicular to the plane, the corresponding angle for the torque formula is $\theta = 90^\circ - 30^\circ = 60^\circ$.

Step 2: Balance the magnetic torque Use $N = 30$, $I = 6 \text{ A}$, $B = 1.0 \text{ T}$ and the area found above:

$$\begin{aligned} \tau_{\text{ext}} = \tau_B &= NIAB \sin 60^\circ \\ &= 30 \times 6 \times 0.0064\pi \times 1.0 \times \frac{\sqrt{3}}{2} \\ &\approx 3.13 \text{ N m}. \end{aligned}$$

The applied torque acts opposite to the turning tendency produced by the field; this makes the net torque zero at the stated orientation.

Step 3: Examine the change of shape For any planar closed coil in a uniform field, the magnetic

moment depends on its enclosed area, current and number of turns. It does not depend on whether its outline is circular or irregular. Since the replacement has the same area and all other parameters, including orientation, remain unchanged, both the magnetic moment and magnetic torque remain unchanged. The same balancing external torque is therefore needed.

Step 4: Conclusion The required external torque has magnitude 3.13 N m (approximately 3.1 N m), opposite to the magnetic torque. Replacing the coil by a planar irregular coil of equal area leaves this requirement unchanged.

Quick Tip to Solve: When the angle is given with the coil's plane, use $\tau = NIAB \cos \alpha$. Equal-area planar coils have equal torque at the same orientation in a uniform field.

OR

23(b). An alpha particle (mass $6.4 \times 10^{-27} \text{ kg}$ and charge $3.2 \times 10^{-19} \text{ C}$) having 8.0 MeV energy, enters a region of a uniform magnetic field of 0.5 T . If the field is directed perpendicular to the velocity of the particle, find the radius of the circular path described by the particle. Mention the condition under which the particle in this region (i) describes a helical path, and (ii) goes straight undeviated.

[3 marks]

Solution:

Concept:

- The magnetic force is $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. It is perpendicular to velocity and does no work, so the particle's speed and kinetic energy remain constant.
- For entry perpendicular to the field, $qvB = mv^2/r$. With kinetic energy $K = mv^2/2$, the circular radius is $r = \sqrt{2mK}/(|q|B)$, where m is mass and q is charge.
- Resolve oblique motion into $v_{\parallel}$ along the field and $v_{\perp}$ perpendicular to it: the parallel component remains unchanged, while the perpendicular component produces circular motion.

Step 1: Convert the energy and obtain the speed Using $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$, the given kinetic energy is

$$K = 8.0 \times 10^6 \times 1.6 \times 10^{-19} = 1.28 \times 10^{-12} \text{ J}.$$

Hence, with the given alpha-particle mass,

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.28 \times 10^{-12})}{6.4 \times 10^{-27}}} = 2.0 \times 10^7 \text{ m s}^{-1}.$$

Step 2: Apply the centripetal-force condition Because the velocity is perpendicular to the magnetic field, the magnetic force supplies the entire centripetal force. Cancel one factor of v in $qvB = mv^2/r$ and substitute:

$$\begin{aligned} r &= \frac{mv}{qB} \\ &= \frac{(6.4 \times 10^{-27})(2.0 \times 10^7)}{(3.2 \times 10^{-19})(0.5)} \\ &= 0.80 \text{ m}. \end{aligned}$$

The radius is positive; the sign of charge determines the sense of rotation rather than the magnitude of the radius.

Step 3: State the conditions for the other paths (i) A helical path occurs when both $v_{\parallel} \neq 0$ and $v_{\perp} \neq 0$: the entry direction is neither parallel, antiparallel nor perpendicular to the field. Circular motion across the field combines with uniform motion along it, producing a helix with its axis along the field.

(ii) For straight undeviated motion, the moving particle must enter parallel or antiparallel to the field. Then $v_{\perp} = 0$, so $\mathbf{v} \times \mathbf{B} = 0$ and the magnetic force vanishes.

Step 4: Conclusion The circular-path radius is $r = 0.80$ m. Both nonzero parallel and perpendicular velocity components give a helix; velocity parallel or antiparallel to the field gives straight undeviated motion.

Quick Tip to Solve: Convert MeV to joules before using $r = \sqrt{2mK}/(|q|B)$. To identify the path, first check whether each velocity component along and across the field is zero.

24. (a) Discuss the behaviour of an inductor connected to (i) a dc source, and (ii) a high frequency ac source.

(b) What is the phase relation between current and voltage in an ideal inductor connected to an ac source? Draw a phasor diagram for the circuit. [3 marks]

Solution:

Concept:

- Self-induction opposes a change in current: the induced emf is $\varepsilon = -L di/dt$, where L is the inductance. The applied voltage across an ideal inductor is $v = L di/dt$.
- Inductive reactance is $X_L = \omega L = 2\pi fL$, where f is frequency and ω is angular frequency. It measures an ideal inductor's opposition to sinusoidal alternating current.

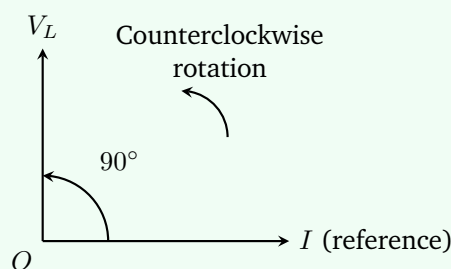
Step 1: Explain the response to direct current (a)(i) At switch-on, self-induced emf opposes the rise of current. For steady dc, $di/dt = 0$, so the induced emf vanishes; also $f = 0$ gives $X_L = 0$. An ideal inductor therefore acts as a short circuit for steady dc. A practical coil has winding resistance, which limits its steady current. Strictly, a nonzero constant voltage across a perfectly ideal inductor gives a continuously rising current; the short-circuit description applies when the current is steady.

Step 2: Explain the response to high-frequency alternating current (a)(ii) For high-frequency ac, $X_L = 2\pi fL$ is large. At a fixed applied rms voltage, the ideal-inductor current is $I_{\text{rms}} = V_{\text{rms}}/X_L$, so the current is small. The inductor strongly opposes high-frequency ac because a faster change in current produces a larger opposing induced emf.

Step 3: Establish and draw the phase relation (b) Take the sinusoidal current as the reference, $i = I_0 \sin \omega t$. Differentiating gives

$$\begin{aligned} v &= L \frac{di}{dt} = \omega L I_0 \cos \omega t \\ &= V_0 \sin \left(\omega t + \frac{\pi}{2} \right), \quad V_0 = \omega L I_0. \end{aligned}$$

The voltage leads current by one quarter-cycle. For counterclockwise phasor rotation, place current to the right and voltage upward, ahead by 90° .



Ideal inductor: voltage leads current by 90° .

Step 4: Conclusion An inductor opposes changes in current: it offers zero inductive opposition to steady dc but large reactance to high-frequency ac. In an ideal ac inductor, current lags voltage by $\pi/2$ radians, or 90° .

Quick Tip to Solve: Use $X_L = 2\pi fL$ for the frequency comparison. For the phasor sketch, put current along the reference axis and voltage 90° ahead in the direction of rotation.

25. (a) Depict the variation of electric field ($\vec{E}$) and magnetic field ($\vec{B}$) with respect to the direction of propagation of an electromagnetic wave. Write their two important characteristics.

(b) Show that $\frac{1}{\sqrt{\epsilon_0\mu_0}}$ gives the velocity of an electromagnetic wave in free space.

[3 marks]

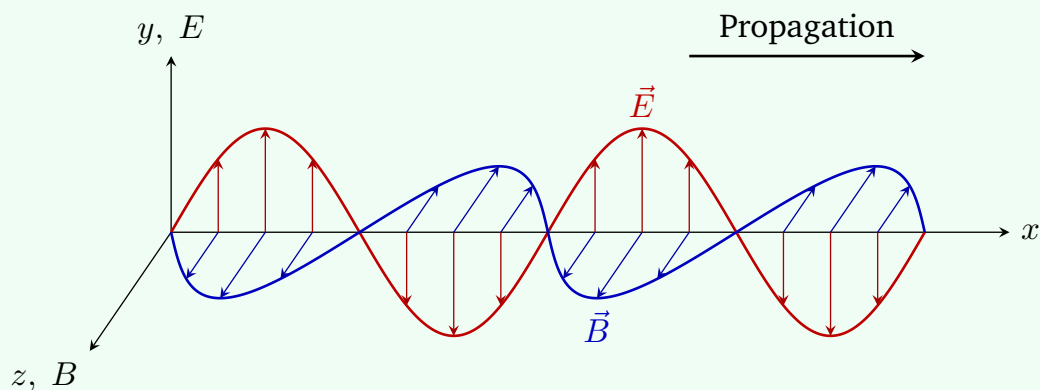
Solution:

Concept:

- In a plane electromagnetic wave, the electric field $\vec{E}$ and magnetic field $\vec{B}$ are transverse to the propagation direction, which is along $\vec{E} \times \vec{B}$.
- For propagation along $+x$, one may write $\vec{E} = E_0 \sin(kx - \omega t)\hat{y}$ and $\vec{B} = B_0 \sin(kx - \omega t)\hat{z}$, where k is the wave number and ω is the angular frequency.
- Free-space permittivity ϵ_0 and permeability μ_0 determine the speed c ; the field amplitudes obey $E_0 = cB_0$.

Step 1: (a) Depict the fields and state two characteristics The sketch shows the fields at a fixed instant, with distance x measured along the direction of propagation. Electric-field oscillations lie in the xy plane and magnetic-field oscillations lie in the xz plane. The two field scales are shown separately; equal drawn heights do not imply equal physical amplitudes.

First, $\vec{E}$, $\vec{B}$ and the propagation direction are mutually perpendicular, so the wave is transverse. Second, the two fields oscillate in the same phase: their zeros, maxima and minima occur at the same positions and times. Their frequencies and wavelengths are therefore equal.



A plane electromagnetic wave at a fixed instant; field amplitudes use separate scales.

Step 2: (b) Obtain the speed from the wave equation Maxwell's equations in free space give the electric-field wave equation

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}.$$

For the sinusoidal wave, the two derivatives are $-k^2 E$ and $-\omega^2 E$, respectively. Hence

$$k^2 = \mu_0 \epsilon_0 \omega^2, \quad \left(\frac{\omega}{k}\right)^2 = \frac{1}{\mu_0 \epsilon_0}.$$

Since the wave speed is ω/k , the positive square root gives the required speed.

Step 3: Check the dimensions and numerical value In SI units, $[\epsilon_0] = \text{C}^2 \text{N}^{-1} \text{m}^{-2}$ and $[\mu_0] = \text{N A}^{-2} = \text{N s}^2 \text{C}^{-2}$. Thus $[\epsilon_0 \mu_0] = \text{s}^2 \text{m}^{-2}$, and its inverse square root has the units m s^{-1} of velocity. Substitution gives

$$\frac{1}{\sqrt{(8.85 \times 10^{-12})(4\pi \times 10^{-7})}} \simeq 3.00 \times 10^8 \text{ m s}^{-1}.$$

Step 4: Conclusion The fields are mutually perpendicular and transverse, and they oscillate in phase. In free space the wave travels along $\vec{E} \times \vec{B}$ with speed $c = 1/\sqrt{\epsilon_0 \mu_0} \simeq 3.00 \times 10^8 \text{ m s}^{-1}$.

Quick Tip to Solve: Draw both sinusoidal fields with zeros at the same positions, in perpendicular planes; use $\vec{E} \times \vec{B}$ to check the propagation arrow.

26. A ray of light is travelling through a rectangular glass slab (refractive index $\frac{3}{2}$) and is incident on the horizontal glass-air surface at the critical angle for the two media. The slab is then brought in contact with water (refractive index $\frac{4}{3}$) such that a thin horizontal layer of water is formed on the surface of the slab. Find the angle at which the ray will emerge into air from the water-air surface.

[3 marks]

Solution:

Concept:

- Snell's law at a plane boundary is $n_1 \sin i = n_2 \sin r$, where the angles are measured from the normal and n_1, n_2 are refractive indices.
- At the critical angle C for a denser-to-rarer boundary, the refracted ray makes 90° with the normal:
 $n_{\text{dense}} \sin C = n_{\text{rare}}.$

- Parallel horizontal interfaces have parallel normals, so the angle of the ray in the water is also its incidence angle at the water-air boundary.

Step 1: Find the original incidence angle in glass Let the glass-air critical angle be C_g . With $n_g = 3/2$ and $n_a = 1$,

$$\sin C_g = \frac{n_a}{n_g} = \frac{2}{3}, \quad C_g \simeq 41.8^\circ.$$

Adding a water layer changes the medium beyond the glass surface, but the incident ray inside the glass still makes this same angle with the normal.

Step 2: Apply Snell's law at the glass-water boundary Let r_w be the angle of the transmitted ray in water. Using $n_w = 4/3$,

$$\begin{aligned} n_g \sin C_g &= n_w \sin r_w, \\ \frac{3}{2} \left(\frac{2}{3} \right) &= \frac{4}{3} \sin r_w, \\ \sin r_w &= \frac{3}{4}. \end{aligned}$$

Thus $r_w \simeq 48.6^\circ$. Transmission into water is possible because this sine is less than one.

Step 3: Apply Snell's law at the water-air boundary The water-air surface is parallel to the glass-water surface; therefore the incidence angle at the upper surface is r_w . If e is the emergence angle in air,

$$n_w \sin r_w = n_a \sin e \quad \Rightarrow \quad \frac{4}{3} \left(\frac{3}{4} \right) = \sin e = 1.$$

The ray in water is exactly at the water-air critical angle, since its sine is $n_a/n_w = 3/4$. This is the limiting refraction condition, with the emergent direction along the surface.

Step 4: Conclusion The ray emerges at 90° to the normal of the water-air surface, that is, it grazes the horizontal surface (0° to the surface).

Quick Tip to Solve: Across parallel layers, $n \sin \theta$ remains constant: directly equate $n_g \sin C_g$ to $n_a \sin e$ to see why the water layer does not change the final angle.

27. Differentiate between nuclear fission and nuclear fusion. Give one example for each with nuclear reaction. [3 marks]

Solution:

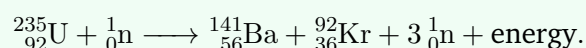
Concept:

- Nuclear fission splits a heavy nucleus into two medium-mass nuclei; nuclear fusion combines light nuclei to form a heavier nucleus.
- Energy is released when the products have greater total binding energy than the reactants. The corresponding decrease Δm in total rest mass gives $Q = \Delta mc^2$, where c is the speed of light.
- In each nuclear reaction, total mass number A (nucleon number) and atomic number Z (charge number) are conserved, although total rest mass need not remain unchanged.

Step 1: Compare the two nuclear processes In fission, a heavy nucleus such as uranium splits into lighter fragments, generally emitting neutrons. Neutron-induced fission can sustain a chain reaction because the emitted neutrons may cause further fissions; such a reaction can be controlled in a nuclear reactor.

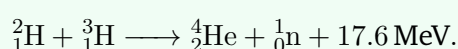
In fusion, two light nuclei combine to produce a more tightly bound nucleus. Thermonuclear fusion requires very high temperature and suitable density and confinement, so that nuclei can overcome their electrostatic repulsion with the aid of quantum tunnelling. Fusion powers stars. Fusion of light nuclei generally releases more energy per unit mass of fuel than fission of heavy nuclei.

Step 2: Give a balanced example of fission Absorption of a neutron by uranium-235 can produce barium-141, krypton-92 and three neutrons:



The mass numbers balance as $235 + 1 = 141 + 92 + 3 = 236$, and the charge numbers balance as $92 = 56 + 36$. The liberated neutrons explain the possibility of a chain reaction.

Step 3: Give a balanced example of fusion A deuterium nucleus and a tritium nucleus can fuse to produce helium-4 and a neutron:



Here $2 + 3 = 4 + 1 = 5$ for mass number and $1 + 1 = 2 + 0$ for charge number. The products have lower total rest mass; this mass defect supplies the released energy, mainly as kinetic energy of the products.

Step 4: Conclusion Fission is the splitting of a heavy nucleus, exemplified by neutron-induced uranium-235 fission. Fusion is the combining of light nuclei, exemplified by deuterium-tritium fusion. Both can release energy by increasing binding energy per nucleon.

Quick Tip to Solve: Identify splitting versus combining first, then check both the upper mass numbers and lower charge numbers on the two sides of each nuclear equation.

28. With the help of circuit diagrams, briefly explain the forward biasing and the reverse biasing of a p-n junction diode. [3 marks]

Solution:

Concept:

- A p-n junction has a depletion layer and a built-in potential barrier. Biasing means applying an external dc voltage across its p and n regions.
- Forward bias opposes the junction barrier and favours majority-carrier motion across the junction; reverse bias strengthens the barrier and suppresses that motion.

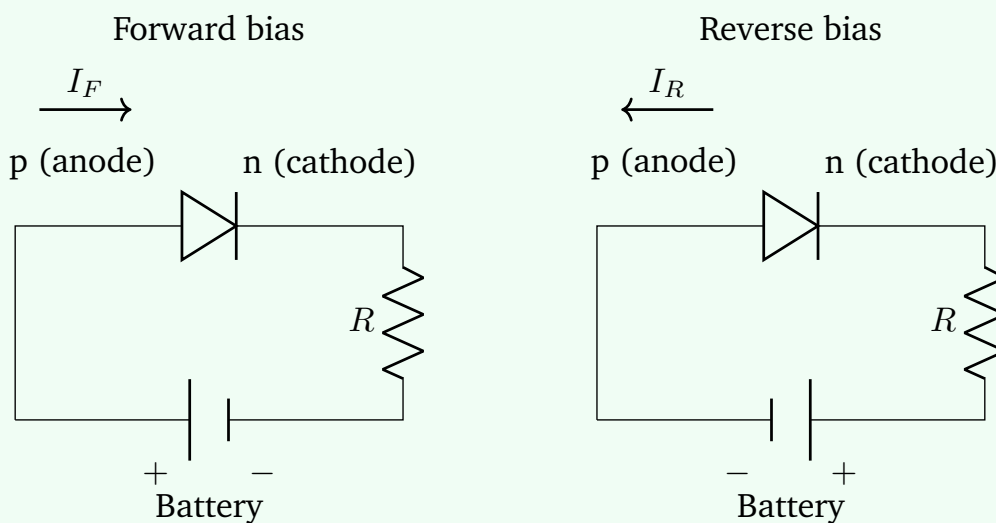
Step 1: Connect and explain forward bias Connect the p side (anode) to the positive terminal of a battery and the n side (cathode) to its negative terminal, with a resistor R to limit the current, as shown. The applied electric field opposes the built-in electric field of the depletion layer. The potential barrier is reduced and the depletion layer becomes narrower.

Majority carriers (holes in p and electrons in n) are pushed towards the junction and cross it more easily. The diode offers low resistance once appreciable forward conduction begins. The conventional forward current is from p to n.

Step 2: Reverse the battery connections Connect the p side to the negative terminal and the n side to the positive terminal of the battery. The applied electric field now supports the built-in junction field. Majority carriers are pulled away from the junction, increasing the depletion-layer width and

the potential barrier.

The junction offers very high resistance, almost blocking majority-carrier current. A very small reverse saturation current due to thermally generated minority carriers still flows from n to p. This applies below breakdown; at breakdown the reverse current rises sharply.



Forward bias (left) and reverse bias (right). The long battery plate is positive; R limits current.

Step 3: Conclusion In forward bias, p is connected to + and n to –, reducing the barrier and allowing appreciable current. In reverse bias, p is connected to – and n to +, widening the depletion layer and allowing only a small leakage current below breakdown.

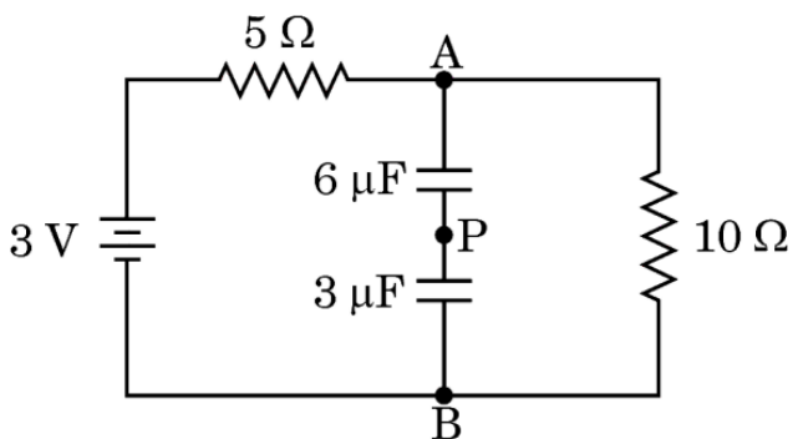
Quick Tip to Solve: First mark the diode's p side and n side. A positive battery terminal on p means forward bias; reversing both battery connections gives reverse bias.

Section D (8 marks)

Questions number 29 and 30 are Case Study-based questions. Read the following paragraphs and answer the questions that follow.

29. Capacitors are manufactured with certain standard capacitances and working voltages. However, these standard values may not be the ones that are actually needed in a particular application. Two or more capacitors can be grouped in series or in parallel to achieve desired capacitance and voltage. When connected in series, the total capacitance decreases while the voltage rating increases, whereas in parallel connections, the total capacitance increases and maintains the same voltage rating. A capacitor stores energy in the electric field between its plates and stored energy is proportional to the square of the voltage and capacitance $U = \frac{1}{2}CV^2$, where symbols have their usual meanings.

Two capacitors, one of $3\ \mu\text{F}$ and the other of $6\ \mu\text{F}$, are connected in series in the circuit as shown in the figure, for a long time. [4 marks]



29(i). The total capacitance of the circuit is:

[1 mark]

- (A) $6\ \mu\text{F}$
- (B) $3\ \mu\text{F}$
- (C) $9\ \mu\text{F}$
- (D) $2\ \mu\text{F}$

Correct Answer: (D) $2\ \mu\text{F}$

Solution:

Concept:

- Series capacitors carry equal charge magnitudes; their voltages add, so $C_{\text{eq}}^{-1} = C_1^{-1} + C_2^{-1}$.
- For capacitances C_1 and C_2 , $C_{\text{eq}} = C_1 C_2 / (C_1 + C_2)$, smaller than either one.

Step 1: Identify the capacitor connection The $6\ \mu\text{F}$ and $3\ \mu\text{F}$ capacitors meet at P , with no branch at that junction. Hence they are in series; the resistors do not change their equivalent capacitance.

Step 2: Calculate and check the equivalent Keeping both capacitances in the same units,

$$C_{\text{eq}} = \frac{6 \times 3}{6 + 3} \mu\text{F} = \frac{18}{9} \mu\text{F} = 2 \mu\text{F}.$$

The value is below $3\ \mu\text{F}$, as required. Option (C), $9\ \mu\text{F}$, would be the parallel sum. Options (A) and (B) merely repeat an individual capacitance and do not combine both series capacitors.

Step 3: Conclusion The total capacitance is $2\ \mu\text{F}$. Hence, the correct option is (D).

Quick Tip to Solve: For these two series capacitors, use product divided by sum and check that the result is less than $3\ \mu\text{F}$.

29(ii). The current in the $10\ \Omega$ resistor is:

[1 mark]

- (A) $0.3\ \text{A}$
- (B) $0.6\ \text{A}$
- (C) $0.2\ \text{A}$
- (D) 0

Correct Answer: (C) $0.2\ \text{A}$

Solution:

Concept:

- After a long time on a dc supply, an ideal capacitor is fully charged and its steady current is zero.
- Ohm's law gives $I = V/R$ for supply voltage V and total loop resistance R .

Step 1: Identify the conducting loop The capacitor branch is open in steady state. The separate resistive loop remains closed: the $3\ \text{V}$ battery drives current through $5\ \Omega$ and $10\ \Omega$ in series.

Step 2: Apply Ohm's law The two resistors carry the same current, so

$$I_{10} = \frac{3}{5 + 10}\ \text{A} = 0.2\ \text{A}.$$

Option (A) ignores the $5\ \Omega$ resistor; (B) ignores the $10\ \Omega$ resistor. Option (D) wrongly treats the open capacitor branch as breaking the separate resistive loop.

Step 3: Conclusion The current in the $10\ \Omega$ resistor is $0.2\ \text{A}$. Hence, the correct option is (C).

Quick Tip to Solve: Open the capacitor branch, then trace the intact loop through both resistors.

29(iii). The potential difference between point A and B is:

[1 mark]

- (A) $2\ \text{V}$
- (B) $0.3\ \text{V}$
- (C) $0.2\ \text{V}$
- (D) $3\ \text{V}$

Correct Answer: (A) $2\ \text{V}$

Solution:

Concept:

- In dc steady state, no current flows through the capacitor branch.
- Components across the same two nodes have the same voltage; Ohm's law gives $V = IR$.

Step 1: Identify the conducting loop With the capacitor branch open, the battery drives current through $5\ \Omega$ and $10\ \Omega$ in series. The total loop resistance is $15\ \Omega$.

Step 2: Find the voltage across the two nodes Points A and B are the terminals of the $10\ \Omega$ resistor, so

$$V_{AB} = \frac{3}{5 + 10} \times 10\text{ V} = 2\text{ V}.$$

Option (D) ignores the drop across $5\ \Omega$. Option (C) mistakes the numerical current for a voltage; (B) also fails Ohm's law for the actual loop.

Step 3: Conclusion The potential difference between A and B is 2 V . Hence, the correct option is (A).

Quick Tip to Solve: Use the resistor voltage divider: $V_{AB} = 3 \times 10 / (5 + 10)\text{ V}$.

29(iv)(a). The value of charge on the plates of the $6\ \mu\text{F}$ capacitor is:

[1 mark]

- (A) $6\ \mu\text{C}$
- (B) $4\ \mu\text{C}$
- (C) $12\ \mu\text{C}$
- (D) $8\ \mu\text{C}$

Correct Answer: (B) $4\ \mu\text{C}$

Solution:

Concept:

- Series capacitors carry equal charge magnitudes: $Q = C_{\text{eq}} V_{AB}$.
- At dc steady state the capacitor branch is open, so the resistors determine V_{AB} by voltage division.

Step 1: Obtain the voltage and capacitance Across the series capacitors, the $10\ \Omega$ resistor sets $V_{AB} = 3 \times 10 / (5 + 10) = 2\text{ V}$. Their equivalent capacitance is $6 \times 3 / (6 + 3) = 2\ \mu\text{F}$.

Step 2: Calculate the common series charge Each capacitor carries the same charge magnitude:

$$Q = (2\ \mu\text{F})(2\text{ V}) = 4\ \mu\text{C}.$$

Option (A) uses the full battery voltage across the pair; (C) wrongly puts all 2 V across $6\ \mu\text{F}$. Option (D) doubles the common charge instead of finding the charge on either plate.

Step 3: Conclusion Each plate has charge magnitude $4\ \mu\text{C}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the pair's voltage and equivalent capacitance; both capacitors carry equal charge.

29(iv)(b). The wire between two capacitors is cut at point P . The current in the circuit will: [1 mark]

- (A) increase
- (B) decrease
- (C) remain the same
- (D) first increase then become stable

Correct Answer: (C) remain the same

Solution:

Concept:

- In dc steady state, the current through an ideal capacitor is zero.
- Cutting a branch already carrying zero current does not change the remaining resistive circuit.

Step 1: Examine the branch before cutting No current flows through P after a long time. The battery still drives current through the separate loop containing the two resistors.

Step 2: Compare the circuit after cutting Cutting at P changes neither the battery nor the resistive loop, so

$$I_{\text{before}} = I_{\text{after}} = \frac{3}{5 + 10} \text{ A} = 0.2 \text{ A}.$$

The unchanged voltage and resistance rule out increases (A), (D) and a decrease (B).

Step 3: Conclusion The circuit current remains the same. Hence, the correct option is (C).

Quick Tip to Solve: Check the cut wire first: it already carries zero steady current.

30. A charged particle $+q$ in an electric field $\vec{E}$ experiences a force in the direction of the electric field. As a result, its kinetic energy changes. Similarly, the charged particle also experiences a force when it moves in a magnetic field $\vec{B}$. But this magnetic force is perpendicular to both velocity $\vec{v}$ of the charged particle and the magnetic field $\vec{B}$, so it cannot change the kinetic energy of the charged particle. Consider two charged particles 1 and 2 of masses m and $\frac{m}{2}$ having charges $-q$ and $+2q$ respectively. They are accelerated from rest through the same potential difference V and acquire kinetic energy K_1 and K_2 . Then they enter in a region of uniform magnetic field $\vec{B}$ perpendicular to their velocities. [4 marks]

30(i). The ratio of their kinetic energies $\left(\frac{K_1}{K_2}\right)$ is: [1 mark]

- (A) $\frac{1}{2}$
- (B) $\frac{1}{4}$
- (C) 4
- (D) 1

Correct Answer: (A) $\frac{1}{2}$

Solution:

Concept:

- A particle accelerated from rest through a potential difference of magnitude V gains kinetic energy $K = |Q|V$, where Q is its charge.
- The magnetic force is perpendicular to velocity, so it does no work and leaves this kinetic energy unchanged.

Step 1: Calculate the energy gained by each particle The charges are $-q$ and $+2q$, with magnitudes q and $2q$. Since both particles are accelerated from rest, their kinetic energies are positive:

$$K_1 = qV, \quad K_2 = 2qV.$$

The negative sign of the first charge determines its acceleration direction, not a negative kinetic energy.

Step 2: Compare the energies and test the options Dividing the energies cancels the common potential difference:

$$\frac{K_1}{K_2} = \frac{qV}{2qV} = \frac{1}{2}.$$

Mass does not enter the electric work. Thus $1/4$ cannot follow from the mass ratio; 4 reverses and changes the energy ratio, while 1 would require equal charge magnitudes. These exclude (B), (C) and (D).

Step 3: Conclusion The kinetic-energy ratio is $K_1/K_2 = 1/2$. Hence, the correct option is (A).

Quick Tip to Solve: For acceleration from rest through the same potential magnitude, compare charge magnitudes directly: $K_1/K_2 = |Q_1|/|Q_2|$.

30(ii). The ratio of the radii of the circular paths described by them $\left(\frac{r_1}{r_2}\right)$ is:

[1 mark]

- (A) $\frac{1}{\sqrt{2}}$
- (B) $\sqrt{2}$
- (C) $\frac{1}{2}$
- (D) 2

Correct Answer: (D) 2

Solution:

Concept:

- For velocity perpendicular to a uniform magnetic field of magnitude B , the magnetic force provides centripetal force: $|Q|vB = Mv^2/r$, so $r = Mv/(|Q|B)$.
- Acceleration from rest through potential magnitude V gives $Mv^2/2 = |Q|V$. Combining the two relations gives $r = \sqrt{2MV/|Q|B}$.

Step 1: Substitute the two masses and charge magnitudes For particle 1, $M_1 = m$ and $|Q_1| = q$. For particle 2, $M_2 = m/2$ and $|Q_2| = 2q$. The common V and B cancel in the ratio:

$$\frac{r_1}{r_2} = \sqrt{\frac{M_1/|Q_1|}{M_2/|Q_2|}} = \sqrt{\frac{m/q}{(m/2)/(2q)}}.$$

Step 2: Simplify and check the alternatives The second mass-to-charge ratio is $m/(4q)$, one quarter of the first. Therefore,

$$\frac{r_1}{r_2} = \sqrt{4} = 2.$$

The factor lies inside a square root. Values $1/\sqrt{2}$ and $\sqrt{2}$ miss one of the mass or charge factors; $1/2$ is the reversed radius ratio. Thus (A), (B) and (C) fail. The charge signs change the bending directions, not the positive radii.

Step 3: Conclusion Particle 1 describes a circle of twice the radius of particle 2: $r_1/r_2 = 2$. Hence, the correct option is (D).

Quick Tip to Solve: For particles accelerated through equal potential differences, use $r \propto \sqrt{M/|Q|}$; their speeds need not be equal.

30(iii). Suppose particles 1 and 2 enter the magnetic field $\vec{B} = B_0\hat{k}$ with velocities $\vec{v}_1 = v_1\hat{i}$ and $\vec{v}_2 = v_2\hat{i}$. Then: [1 mark]

- (A) both particles revolve clockwise
- (B) both particles revolve anticlockwise
- (C) particle 1 revolves clockwise while particle 2 revolves anticlockwise
- (D) particle 1 revolves anticlockwise while particle 2 revolves clockwise

Correct Answer: (D) particle 1 revolves anticlockwise while particle 2 revolves clockwise

Solution:

Concept:

- The magnetic force is $\vec{F} = Q(\vec{v} \times \vec{B})$, with the signed charge Q . Here $\hat{i} \times \hat{k} = -\hat{j}$.
- Clockwise and anticlockwise senses are read in the xy -plane as viewed from the positive z -axis toward the origin; $+\hat{k}$ then points toward the observer.

Step 1: Find the initial force on each particle Both velocities point along $+\hat{i}$ and the field is $B_0\hat{k}$, so the cross product points along $-\hat{j}$. Using $Q_1 = -q$ and $Q_2 = +2q$,

$$\vec{F}_1 = +qv_1B_0\hat{j}, \quad \vec{F}_2 = -2qv_2B_0\hat{j}.$$

Thus particle 1 initially bends upward, whereas particle 2 bends downward.

Step 2: Identify the senses of revolution A velocity initially to the right that turns upward rotates anticlockwise. A velocity initially to the right that turns downward rotates clockwise. Hence opposite charge signs rule out the common senses in (A) and (B). Option (C) reverses the force directions; (D) agrees with both. The unequal speed magnitudes affect the radii, not these senses of turning.

Step 3: Conclusion Particle 1 revolves anticlockwise and particle 2 clockwise in the stated view. Hence, the correct option is (D).

Quick Tip to Solve: First calculate $\hat{i} \times \hat{k} = -\hat{j}$; then reverse that direction only for the negative charge before deciding the turning sense.

30(iv)(a). If period of revolution for particle 1 is 4 s, then for particle 2, the period will be: [1 mark]

- (A) 1 s
- (B) 2 s
- (C) 4 s
- (D) 8 s

Correct Answer: (A) 1 s

Solution:

Concept:

- The period of a charged particle in a uniform perpendicular magnetic field is $T = 2\pi M/(|Q|B)$, where M is mass and $|Q|$ is charge magnitude.
- This follows from $T = 2\pi r/v$ and $r = Mv/(|Q|B)$; speed cancels, so at a fixed field the period depends only on $M/|Q|$.

Step 1: Compare the mass-to-charge ratios Substitute $M_1 = m$, $|Q_1| = q$, $M_2 = m/2$ and $|Q_2| = 2q$.

The common field and 2π cancel:

$$\frac{T_2}{T_1} = \frac{M_2/|Q_2|}{M_1/|Q_1|} = \frac{(m/2)/(2q)}{m/q} = \frac{1}{4}.$$

Step 2: Use the given period and check the choices With $T_1 = 4$ s,

$$T_2 = \frac{1}{4} \times 4 \text{ s} = 1 \text{ s}.$$

Halving the mass and doubling the charge each halves the period, so both effects must be included. The value 2 s misses one effect; 4 s ignores both, and 8 s gives an increase despite the decreased mass-to-charge ratio. Thus (B), (C) and (D) are excluded.

Step 3: Conclusion The period of revolution of particle 2 is 1 s. Hence, the correct option is (A).

Quick Tip to Solve: In the same magnetic field, compare $M/|Q|$ directly: half the mass and twice the charge make the period one quarter as large.

OR

30(iv)(b). If the value of momentum for particles 1 and 2 are p_1 and p_2 , then:

[1 mark]

- (A) $p_1 = \frac{p_2}{2}$
- (B) $p_1 = p_2$
- (C) $p_1 = 2p_2$
- (D) $p_1 = 4p_2$

Correct Answer: (B) $p_1 = p_2$

Solution:

Concept:

- For non-relativistic motion, kinetic energy and momentum magnitude are related by $K = p^2/(2M)$, where M is the particle's mass.
- A particle accelerated from rest through potential magnitude V gains $K = |Q|V$. Therefore its momentum magnitude is $p = \sqrt{2M|Q|V}$.

Step 1: Calculate the mass-charge products The charge sign does not enter the kinetic energy or momentum magnitude. For the two particles,

$$M_1|Q_1| = mq, \quad M_2|Q_2| = \frac{m}{2}(2q) = mq.$$

The smaller mass of particle 2 exactly compensates for its larger charge magnitude.

Step 2: Compare the momentum magnitudes Both particles cross the same potential magnitude, so

$$\frac{p_1}{p_2} = \sqrt{\frac{M_1|Q_1|V}{M_2|Q_2|V}} = \sqrt{\frac{mq}{mq}} = 1.$$

Ratios 1/2, 2 and 4 in (A), (C) and (D) are inconsistent with these equal products. The magnetic force subsequently changes momentum direction, but not its magnitude, since it does no work. The comparison here concerns the stated values of momentum, not equality of the two momentum vectors

at every instant.

Step 3: Conclusion The momentum magnitudes are equal: $p_1 = p_2 = \sqrt{2mqV}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal accelerating potential differences, compare the products $M|Q|$: equal products give equal momentum magnitudes even when masses differ.

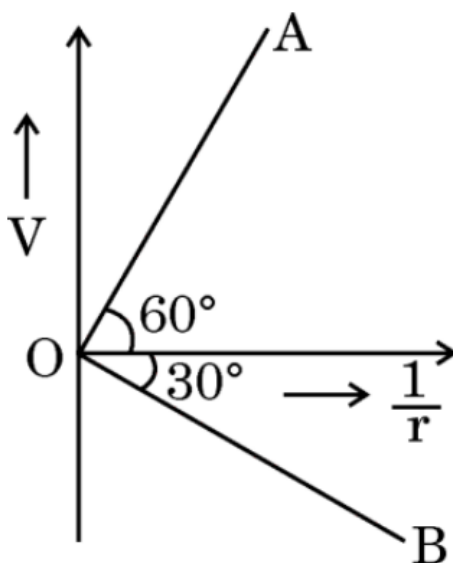
Section E (15 marks)

31(a). (i) In the figure, OA and OB show the variation of electric potential V at a point due to two point charges Q_1 and Q_2 with $\frac{1}{r}$ respectively. Here r represents the distance of the point from the two point charges.

(I) Identify the nature of the two charges Q_1 and Q_2 .

(II) What is the value of $\left(\frac{Q_1}{Q_2}\right)$? Justify your answer.

(ii) Two point charges $-2 \mu\text{C}$ and $5 \mu\text{C}$ are placed at $(-30 \text{ cm}, 0)$ and $(30 \text{ cm}, 0)$ respectively in an external electric field $\vec{E} = \frac{A}{x^2} \hat{i}$, where $A = 9 \times 10^5 \text{ N m}^2 \text{ C}^{-1}$. Find the electrostatic potential energy of this configuration. [5 marks]



Solution:

Concept:

- For a point charge Q , the potential is $V = kQ/r$, with $k = 1/(4\pi\epsilon_0)$. Therefore, the signed slope of a graph of V against $1/r$ equals kQ .
- The electric field and potential satisfy $E_x = -dV_{\text{ext}}/dx$. The potential reference must be fixed before calculating the energy of charges in an external field.
- For two charges, the configuration energy is $U = q_1 V_{\text{ext}}(x_1) + q_2 V_{\text{ext}}(x_2) + kq_1 q_2 / r_{12}$, where r_{12} is their separation. The mutual interaction is counted once.

Step 1: (i)(I) Identify the signs from the graph Line OA , representing Q_1 , rises above the horizontal axis as $1/r$ increases, so its slope is positive. Since k is positive, Q_1 is positive. Line OB , representing Q_2 , falls below the axis and has negative slope, so Q_2 is negative. The signs follow from the potentials,

not simply from the steepness of the lines.

Step 2: (i)(II) Compare the signed slopes The line for Q_1 makes an angle 60° above the positive horizontal axis. The line for Q_2 makes an angle 30° below that axis. Thus the second slope is negative:

$$\frac{Q_1}{Q_2} = \frac{kQ_1}{kQ_2} = \frac{\tan 60^\circ}{\tan(-30^\circ)} = \frac{\sqrt{3}}{-1/\sqrt{3}} = -3.$$

Hence the magnitude of Q_1 is three times that of Q_2 , and the charges have opposite signs.

Step 3: (ii) Determine the external potential The printed field is $\vec{E} = (A/x^2)\hat{i}$. Integrating its relation to potential gives

$$\frac{dV_{\text{ext}}}{dx} = -\frac{A}{x^2}, \quad V_{\text{ext}}(x) = \frac{A}{x} + C.$$

Taking the external potential to tend to zero as $|x|$ tends to infinity fixes $C = 0$ on each side of the singular point $x = 0$. At the specified positions, $x_1 = -0.30$ m and $x_2 = +0.30$ m, so

$$\begin{aligned} V_{\text{ext}}(x_1) &= \frac{9 \times 10^5}{-0.30} = -3.0 \times 10^6 \text{ V}, \\ V_{\text{ext}}(x_2) &= \frac{9 \times 10^5}{0.30} = +3.0 \times 10^6 \text{ V}. \end{aligned}$$

The coordinate sign must be retained in A/x , even though the field contains x^2 .

Step 4: Add the external and mutual energies The interaction with the prescribed external field is

$$\begin{aligned} U_{\text{ext}} &= (-2 \times 10^{-6})(-3.0 \times 10^6) + (5 \times 10^{-6})(3.0 \times 10^6) \\ &= 6 + 15 = 21 \text{ J}. \end{aligned}$$

The two charges are separated by $r_{12} = 0.30 - (-0.30) = 0.60$ m. Their mutual potential energy is negative because their signs are opposite:

$$U_{12} = \frac{(9 \times 10^9)(-2 \times 10^{-6})(5 \times 10^{-6})}{0.60} = -0.15 \text{ J}.$$

The required energy of the two-charge configuration includes both of these contributions; the fixed external-field source's own energy is not part of this calculation.

Step 5: Conclusion (i) Q_1 is positive, Q_2 is negative, and $Q_1/Q_2 = -3$. (ii) With zero external potential at infinity, the electrostatic potential energy is $U = 21 - 0.15 = 20.85$ J.

Quick Tip to Solve: Use signed slopes for the charge ratio. For the energy, first integrate the printed field, retain the sign of each coordinate in the potential, and count the mutual interaction only once.

31(b). (i) Two infinitely long straight wires having linear charge densities $-\lambda$ and 3λ are held vertically parallel to each other, distance r apart in free space. Find the nature and magnitude of the force/length exerted by one wire on the other.

(ii) A small hollow conducting sphere of radius r_1 is given a charge Q . It is surrounded by a concentric conducting spherical shell of inner radius r_2 and outer radius r_3 , having charge $-3q$.

If a point charge $2q$ were kept at the centre, find:

(I) the electric flux through a concentric spherical Gaussian surface of radius x for (1) $x < r_1$, and (2) $r_1 < x < r_2$.

(II) electric field at a point distant x from the centre for (1) $x > r_3$, and (2) $r_1 < x < r_2$.

(III) surface charge density on the inner surface of (1) sphere, and (2) shell.

[5 marks]

Solution:

Concept:

- An infinite straight line with linear charge density λ_s produces a field of magnitude $|\lambda_s|/(2\pi\epsilon_0 r)$ at perpendicular distance r . Force per unit length on another line is its charge per length times this field.
- Gauss's law gives $\Phi_E = Q_{\text{encl}}/\epsilon_0$. For a spherically symmetric charge distribution, $\vec{E} = Q_{\text{encl}}\hat{r}/(4\pi\epsilon_0 x^2)$, where $\hat{r}$ points radially outward.
- In electrostatic equilibrium the field inside conducting material is zero. A Gaussian surface within the metal therefore encloses zero net charge; this fixes the induced inner-surface charge.
- Each isolated conductor retains its specified net charge. Central placement makes the induced charges uniform, with surface density $\sigma = Q_s/(4\pi R^2)$ on a spherical surface of radius R .

Step 1: (i) Calculate the force per unit length The wire with density $-\lambda$ produces a field of magnitude $\lambda/(2\pi\epsilon_0 r)$ at the other wire. An element of length L of the second wire carries charge $3\lambda L$, so

$$\frac{F}{L} = 3\lambda \frac{\lambda}{2\pi\epsilon_0 r} = \frac{3\lambda^2}{2\pi\epsilon_0 r}.$$

The field of the negative wire points towards it. Therefore the positive wire is pulled towards the negative wire: the force is attractive. The force on the other wire has the same magnitude per unit length and the opposite direction.

Step 2: (ii) Find the induced surface charges The central charge is $2q$. A Gaussian surface in the small sphere's conducting material must enclose zero charge, requiring its inner surface to carry $-2q$. Since the small sphere's total charge is Q , its outer surface carries $Q + 2q$.

Similarly, a Gaussian surface in the surrounding shell's metal encloses the central charge, the entire small sphere and the shell's inner-surface charge $Q_{2,\text{in}}$. Thus

$$2q + Q + Q_{2,\text{in}} = 0, \quad Q_{2,\text{in}} = -(Q + 2q).$$

The shell has net charge $-3q$, as printed. Hence its outer surface carries

$$Q_{2,\text{out}} = -3q - [-(Q + 2q)] = Q - q.$$

The capital Q and the lower-case q are distinct quantities and must not be identified.

Step 3: (ii)(I) Apply Gauss's law to the two regions For $0 < x < r_1$, the concentric Gaussian sphere

encloses only the central point charge. The conductor's surface charges lie outside it:

$$\Phi_E(x < r_1) = \frac{2q}{\epsilon_0}.$$

For $r_1 < x < r_2$, the surface encloses both the central charge $2q$ and the small conductor's net charge Q . Therefore

$$\Phi_E(r_1 < x < r_2) = \frac{Q + 2q}{\epsilon_0}.$$

It is the net enclosed charge that determines flux, independently of the radius of each Gaussian sphere within its region.

Step 4: (ii)(II) Determine the electric fields Spherical symmetry makes the field radial and constant in magnitude on each concentric Gaussian sphere. For $x > r_3$, all charges are enclosed, with total $2q + Q - 3q = Q - q$. Using $E_r 4\pi x^2 = Q_{\text{encl}}/\epsilon_0$,

$$\vec{E}(x > r_3) = \frac{Q - q}{4\pi\epsilon_0 x^2} \hat{r}.$$

In the gap between the conductors, the enclosed charge is $Q + 2q$, giving

$$\vec{E}(r_1 < x < r_2) = \frac{Q + 2q}{4\pi\epsilon_0 x^2} \hat{r}.$$

Each expression is a signed radial field: it is outward for a positive numerator, inward for a negative numerator, and zero if that numerator vanishes.

Step 5: (ii)(III) Divide each inner-surface charge by its area Using the given radius r_1 for the small hollow sphere and r_2 for the shell's inner surface, the corresponding uniform charge densities are

$$\sigma_{1,\text{in}} = -\frac{2q}{4\pi r_1^2}, \quad \sigma_{2,\text{in}} = -\frac{Q + 2q}{4\pi r_2^2}.$$

These are inner-surface densities; using the net charge Q or $-3q$ directly would ignore electrostatic induction and give incorrect results.

Step 6: Conclusion (i) The wires attract with $F/L = 3\lambda^2/(2\pi\epsilon_0 r)$. (ii)(I) The fluxes are $2q/\epsilon_0$ and $(Q + 2q)/\epsilon_0$, respectively. (II) The fields are $(Q - q)\hat{r}/(4\pi\epsilon_0 x^2)$ outside the shell and $(Q + 2q)\hat{r}/(4\pi\epsilon_0 x^2)$ in the gap. (III) The inner-surface densities are $-2q/(4\pi r_1^2)$ and $-(Q + 2q)/(4\pi r_2^2)$, respectively.

Quick Tip to Solve: Start at the central charge and move outward: make the enclosed charge zero inside each metal layer, then use each conductor's fixed net charge to obtain its outer-surface charge.

32(a). (i) A light bulb and an open coil inductor are connected in series across an ac source of variable frequency. How will the glow of the bulb be affected when:

(I) an iron bar is inserted inside the coil, and

(II) the frequency of the source is decreased?

Justify your answers. Assume that in each above case other factors remain unchanged.

(ii) An ac voltage $V = 280 \sin(100\pi t)$ volt is connected across a series LCR circuit in which $R = 400 \Omega$, $L = \frac{5}{\pi}$ H and $C = \frac{50}{\pi} \mu\text{F}$. Taking $\sqrt{2} = 1.4$, calculate:

(I) impedance of the circuit.

(II) rms value of current that flows in the circuit.

(III) power factor of the circuit.

[5 marks]

Solution:

Concept:

- An inductor has reactance $X_L = \omega L = 2\pi fL$, where f is the ac frequency and L is its self-inductance. At fixed source rms voltage, increasing the impedance decreases the current.
- For a bulb of resistance R_b , the electrical power producing its glow is $P_b = I_{\text{rms}}^2 R_b$. The ideal series bulb-inductor circuit has impedance $\sqrt{R_b^2 + X_L^2}$.
- In a series LCR circuit, $X_C = 1/(\omega C)$ and $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where C is capacitance and R is resistance. The rms current is $I_{\text{rms}} = V_{\text{rms}}/Z$, and the power factor is $\cos \phi = R/Z$.

Step 1: (i)(I) Insert the iron bar Iron has a much higher magnetic permeability than air, so inserting it increases the magnetic flux linkage per unit current and hence increases the coil's self-inductance L . At unchanged frequency, $X_L = 2\pi fL$ increases. Thus the series impedance increases, the rms current falls, and the power $I_{\text{rms}}^2 R_b$ delivered to the bulb decreases. The bulb therefore glows dimmer.

Step 2: (i)(II) Decrease the source frequency With the coil and all other factors unchanged, decreasing f reduces $X_L = 2\pi fL$. The impedance of the bulb-inductor combination decreases, allowing a larger rms current at the same applied rms voltage. The bulb receives more power and glows brighter. This part concerns the bulb and inductor circuit, which contains no capacitor.

Step 3: (ii)(I) Calculate the reactances and impedance Comparing the applied voltage with $V = V_0 \sin \omega t$, its peak value is $V_0 = 280 \text{ V}$, and $\omega = 100\pi \text{ rad s}^{-1}$. Convert the printed capacitance to farads before substituting:

$$C = \frac{50}{\pi} \times 10^{-6} \text{ F}.$$

The two reactances are

$$X_L = (100\pi) \frac{5}{\pi} = 500 \Omega,$$

$$X_C = \frac{1}{(100\pi)(50 \times 10^{-6}/\pi)} = 200 \Omega.$$

Inductive and capacitive reactances oppose each other, so their net value is $X_L - X_C = 300 \Omega$. Resistance and net reactance combine in quadrature:

$$Z = \sqrt{400^2 + 300^2} = 500 \Omega.$$

Step 4: (ii)(II) Find the rms current The coefficient 280 in the sine function is the peak voltage, so

first use the given approximation $\sqrt{2} = 1.4$ to find the rms value:

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{280}{1.4} = 200 \text{ V}.$$

Applying Ohm's law for the series ac circuit then gives

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{200}{500} = 0.40 \text{ A}.$$

Step 5: (ii)(III) Calculate the power factor The resistance is the in-phase component of the impedance, giving

$$\cos \phi = \frac{R}{Z} = \frac{400}{500} = 0.80.$$

Since $X_L > X_C$, the circuit is net inductive: its current lags the applied voltage. Hence the power factor is lagging; it has no unit.

Step 6: Conclusion (i) The bulb becomes **dimmer** on inserting the iron bar and **brighter** on decreasing the frequency. (ii) The impedance is 500Ω , the rms current is 0.40 A , and the power factor is 0.80 lagging.

Quick Tip to Solve: Read V_0 and ω directly from the sine function, convert microfarads to farads, and subtract the reactances before forming Z . Here R , net reactance and Z form a $400 : 300 : 500$ triangle.

OR

32(b). (i) State Lenz's law and explain that it follows the law of conservation of energy.

(ii) Write the dimensional formula for self-inductance.

The current in a coil changes from 8.0 A to 2.0 A in 0.6 s . If an average emf induced in the coil is 50 V , calculate the self-inductance of the coil. [5 marks]

Solution:

Concept:

- Lenz's law gives the direction of induced current: its magnetic effect opposes the change in magnetic flux that produces it. For a coil of N turns, Faraday's law is $\varepsilon = -N d\Phi_B/dt$; its minus sign expresses Lenz's law.
- The induced emf in a coil of constant self-inductance L is $\varepsilon = -L dI/dt$, where I is the coil current. For an average emf, $|\varepsilon_{\text{av}}| = L|\Delta I|/\Delta t$.
- Conservation of energy requires the electrical energy produced by induction to come from work done by an external agent or from energy already stored in the system.

Step 1: (i) State Lenz's law The direction of the induced current in a closed circuit is such that the magnetic field associated with it opposes the change in magnetic flux that induces the current. The opposition is to the *change* in flux. Thus an increasing flux is opposed by an induced field in the opposite direction, while a decreasing flux is opposed by an induced field in the same direction as the original flux.

Step 2: Relate the law to conservation of energy Consider pushing the north pole of a magnet towards a closed conducting coil. The magnetic flux through the coil increases. According to Lenz's law, the approaching face of the coil behaves as a north pole and repels the magnet. An external agent must

therefore do work against this magnetic force to maintain the motion. The supplied mechanical energy becomes electrical energy, which may be dissipated as heat in the coil's resistance.

If instead the induced current assisted the flux change, the coil would attract the approaching magnet, increasing its speed and producing still more current. Both the magnet's kinetic energy and the heat produced in the coil would increase without a corresponding energy supply. That would violate conservation of energy. Similarly, on withdrawing the magnet, the induced current produces an attraction that opposes withdrawal, again requiring external work. Hence the opposing direction prescribed by Lenz's law is consistent with, and follows from, conservation of energy.

Step 3: (ii) Derive the dimensions of self-inductance From the magnitude relation $|\varepsilon| = L|dI/dt|$,

$$[L] = \frac{[\varepsilon][t]}{[I]}.$$

Electric potential difference is work per charge. Work has dimensions ML^2T^{-2} , and charge has dimensions AT , where A denotes the electric-current dimension. Thus

$$\begin{aligned} [\varepsilon] &= ML^2T^{-3}A^{-1}, \\ [L] &= ML^2T^{-2}A^{-2}. \end{aligned}$$

The L inside a dimensional formula denotes length; the italic L outside the formula denotes the coil's self-inductance. Its SI unit is the henry, with $1H = 1VsA^{-1}$.

Step 4: Calculate the self-inductance of the coil The current falls from $8.0A$ to $2.0A$ in $0.6s$. Its average rate of change is

$$\frac{\Delta I}{\Delta t} = \frac{2.0 - 8.0}{0.6} = -10As^{-1}.$$

Use the magnitude of this rate with the given average emf magnitude $50V$:

$$L = \frac{|\varepsilon_{av}|}{|\Delta I|/\Delta t} = \frac{50}{10} = 5.0H.$$

The negative current-change rate means the induced emf acts to sustain the original current and oppose its decrease; it does not make the self-inductance negative.

Step 5: Conclusion Lenz's law requires the induced current to oppose the change that produces it, so the energy obtained by induction has a corresponding source. The dimensional formula of self-inductance is $ML^2T^{-2}A^{-2}$, and the coil's self-inductance is $5.0H$.

Quick Tip to Solve: Use $L = |\varepsilon_{av}|\Delta t/|I_f - I_i|$ for a finite current change. Take the magnitude to calculate L , then use Lenz's law separately to explain the emf direction.

33(a). (i) A point object is kept in front of a convex spherical surface of radius of curvature R . Draw the ray diagram to show the formation of image and derive the relation between the object and image distance (u and v) in terms of refractive index n of the medium and R .

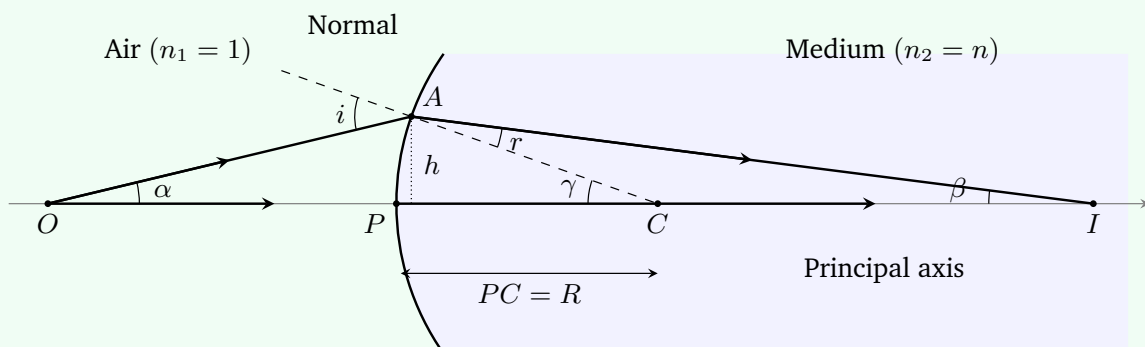
(ii) A convex lens of focal length of 20 cm is used to form the image of an object placed 30 cm away from the lens. Find the position and nature of the image formed. [5 marks]

Solution:

Concept:

- For refraction from air into a medium of refractive index n , Snell's law is $\sin i = n \sin r$, where i and r are measured from the normal. For paraxial rays, the angles are small in radians, so $i \simeq nr$.
- Use the Cartesian sign convention: measure distances from the pole or optical centre, taking the direction of incident light as positive. Thus a real object on the left has $u < 0$, and a convex refracting surface whose centre is on the right has $R > 0$.
- For a thin lens, $1/f = 1/v - 1/u$, where f , v and u are the signed focal length, image distance and object distance. The transverse magnification is $m = v/u$.

Step 1: (i) Construct the ray diagram Take the point object O in air and a convex spherical surface separating air from the medium of index n . Let P be the pole and C the centre of curvature. The ray OP , incident normally, travels undeviated along the principal axis. A paraxial ray OA , striking the surface at height h , bends towards the normal AC and meets the axial ray at the image I . The diagram shows a real-image configuration; the final signed formula also applies when the image is virtual.



Refraction from air into a medium at a convex spherical surface (schematic).

Step 2: (i) Relate the angles using the geometry Let α , β and γ be the small positive angles that OA , AI and the normal AC , respectively, make with the principal axis in the shown configuration. From the geometry at A ,

$$i = \alpha + \gamma, \quad r = \gamma - \beta.$$

The surface has a small aperture, so the axial displacement of A from P is negligible. With OP , PI and PC denoting positive lengths, the small-angle approximation gives

$$\alpha \simeq \frac{h}{OP}, \quad \beta \simeq \frac{h}{PI}, \quad \gamma \simeq \frac{h}{PC}.$$

Step 3: (i) Apply Snell's law and introduce signed distances Substituting the geometrical relations

into $i \simeq nr$,

$$\begin{aligned}\alpha + \gamma &= n(\gamma - \beta), \\ \alpha + n\beta &= (n - 1)\gamma, \\ \frac{1}{OP} + \frac{n}{PI} &= \frac{n - 1}{PC}.\end{aligned}$$

In the diagram, $OP = -u$, $PI = v$ and $PC = R$. Consequently, the required relation for refraction from air into the medium is

$$\boxed{\frac{n}{v} - \frac{1}{u} = \frac{n - 1}{R}}.$$

This derivation assumes paraxial rays and a spherical refracting surface. If the incident medium has refractive index n_1 and the second medium has index n_2 , the corresponding form is $n_2/v - n_1/u = (n_2 - n_1)/R$.

Step 4: (ii) Substitute into the thin-lens formula The convex lens has $f = +20$ cm, and the real object is 30 cm in front of it, so $u = -30$ cm. Hence,

$$\begin{aligned}\frac{1}{v} &= \frac{1}{f} + \frac{1}{u} \\ &= \frac{1}{20} - \frac{1}{30} \\ &= \frac{1}{60} \text{ cm}^{-1}, \quad v = +60 \text{ cm}.\end{aligned}$$

The positive image distance places the image on the opposite side of the lens from the object. Its magnification is

$$m = \frac{v}{u} = \frac{60}{-30} = -2.$$

A negative magnification means that the image is inverted; its magnitude shows that the image is twice the object's size. The refracted rays actually meet, so the image is real.

Step 5: Conclusion (i) For refraction from air at the convex spherical surface, $\boxed{n/v - 1/u = (n - 1)/R}$.
(ii) The image forms $\boxed{60 \text{ cm}}$ from the lens on the opposite side. It is real, inverted and enlarged twofold, with $m = -2$.

Quick Tip to Solve: Write $u = -30$ cm before substituting. An object between $f = 20$ cm and $2f = 40$ cm must give a real, enlarged image beyond $2f$, which checks the result $v = 60$ cm.

33(b). (i) Two thin converging lenses of focal length f_1 and f_2 are placed coaxially in contact. Derive expression for the focal length of the combination.

(ii) A beam of coherent light of wavelength 550 nm is incident normal to the plane of a pair of two slits S_1 and S_2 each of width 1.2×10^{-6} m separated by 1.1 mm. Dark and bright fringes are observed on a screen 2.2 m away from the plane of the slits.

Calculate:

(I) fringe width.

(II) distance of the second dark fringe from the central maximum.

(III) what will happen when the entire apparatus is immersed in water.

[5 marks]

Solution:

Concept:

- For a thin lens, the Cartesian sign convention gives $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, where u and v are signed object and image distances and f is the focal length. For lenses in contact, the image formed by the first lens acts as the object for the second.
- In Young's double-slit experiment, the fringe width is $\beta = \lambda D/d$, where λ is the wavelength in the medium, D the slit-to-screen distance and d the separation of slit centres. The n th dark fringe lies at $|y_n| = (n - \frac{1}{2})\beta$, with $n = 1, 2, \dots$
- On entering a medium of refractive index μ , the light frequency remains unchanged, while its speed and wavelength become c/μ and λ/μ , respectively. The geometrical distances remain unchanged on immersion.

Step 1: (i) Apply the lens formula to the first lens Let an object be at signed distance u from the first converging lens. Suppose that this lens alone would form its image at signed distance v_1 . In the thin-lens and paraxial approximation,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}.$$

Since the two lenses are coaxial and in contact, their separation can be neglected. Thus the same point at distance v_1 is the object point for the second lens. If it lies on the outgoing side, it is a virtual object, so its signed object distance is still v_1 , not $-v_1$.

Step 2: (i) Combine the two lens equations If the second lens forms the final image at signed distance v , its lens equation is

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}.$$

Adding this to the equation for the first lens eliminates the intermediate image distance:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

An equivalent single lens of focal length F must produce the same final image for the same object, so $1/v - 1/u = 1/F$. Consequently,

$$\boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}}, \quad \boxed{F = \frac{f_1 f_2}{f_1 + f_2}}.$$

Both focal lengths are positive for converging lenses, hence $F > 0$ and F is smaller than either

individual focal length. The combination is therefore more strongly converging.

Step 3: (ii)(I) Calculate the fringe width Convert the given wavelength and slit separation to SI units:

$$\lambda = 550 \times 10^{-9} \text{ m}, \quad d = 1.1 \times 10^{-3} \text{ m}, \quad D = 2.2 \text{ m}.$$

For a point at transverse distance y near the screen centre, the path difference is approximately dy/D . Adjacent bright fringes differ in path difference by λ , giving

$$\begin{aligned} \beta &= \frac{\lambda D}{d} = \frac{550 \times 10^{-9} \times 2.2}{1.1 \times 10^{-3}} \text{ m} \\ &= 1.1 \times 10^{-3} \text{ m} = \boxed{1.1 \text{ mm}}. \end{aligned}$$

The given individual slit width, $1.2 \times 10^{-6} \text{ m}$, controls the diffraction envelope; it is not the separation d used to calculate the interference fringe spacing.

Step 4: (ii)(II) Locate the second dark fringe Destructive interference requires a path difference $(2n - 1)\lambda/2$. The first dark fringe is at $\beta/2$ from the central maximum, and the second is at $3\beta/2$. Thus

$$|y_2| = \frac{3\beta}{2} = \frac{3}{2}(1.1 \text{ mm}) = \boxed{1.65 \text{ mm}}.$$

There is a corresponding dark fringe on each side of the central maximum, at signed positions $y = \pm 1.65 \text{ mm}$; the requested distance is positive.

Step 5: (ii)(III) Describe immersion in water Let the refractive index of water relative to air be μ_w . With the whole apparatus immersed uniformly, the wavelength becomes $\lambda_w = \lambda/\mu_w$, while D and d remain the same. Hence

$$\beta_w = \frac{\beta}{\mu_w}, \quad |y_{2,w}| = \frac{|y_2|}{\mu_w}.$$

The fringes move closer together, and the pattern contracts symmetrically about the central bright fringe. The central maximum remains at the same position because the two paths to it remain equal. If the usual approximation $\mu_w = 4/3$ is used, the new fringe width is 0.825 mm and the second dark fringe is 1.2375 mm , approximately 1.24 mm , from the centre.

Step 6: Conclusion (i) The combination has focal length $F = f_1 f_2 / (f_1 + f_2)$. (ii) The fringe width is 1.1 mm , and the second dark fringe is 1.65 mm from the central maximum. In water, all fringe distances from the centre decrease by the factor $1/\mu_w$, while the central maximum stays fixed.

Quick Tip to Solve: For lenses in contact, add reciprocal focal lengths. For these fringes, use the slit separation 1.1 mm , not the individual slit width, and remember that the second dark fringe is at $3\beta/2$, not 2β .