

CBSE Class 12 Physics Question Paper 2026 with Solution PDF-Set 55/4/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1 to 16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** – Questions no. **17 to 21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** – Questions no. **22 to 28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** – Questions no. **29 and 30** are case study-based questions. Each question carries **4** marks.
- (vii) In **Section E** – Questions no. **31 to 33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is **not** allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. Radiation of wavelength 331 nm irradiates the following metals:

Metal	Work Function (eV)
Na	1.92
K	2.15
Ca	3.20
Mo	4.17

Which of the following statements is correct?

[1 mark]

- (A) Only Na and K show photoelectric emission.
- (B) Only Mo will not show photoelectric emission.
- (C) All of the given metals show photoelectric emission.
- (D) None of them show photoelectric emission.

Correct Answer: (B) Only Mo will not show photoelectric emission.

Solution:

Concept:

- A photon of wavelength λ has energy $E = hc/\lambda$, where h is Planck's constant and c is the speed of light.
- Photoelectric emission occurs when the photon energy is at least the metal's work function ϕ . The excess energy becomes the maximum kinetic energy of an emitted electron.

Step 1: Calculate the incident photon energy Convert the wavelength to metres and the energy from joules to electronvolts:

$$E = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{(331 \times 10^{-9})(1.6 \times 10^{-19})} \text{ eV} \simeq 3.76 \text{ eV}.$$

Step 2: Compare with all four work functions The photon energy exceeds 1.92 eV, 2.15 eV and 3.20 eV, so Na, K and Ca emit photoelectrons. It is below 4.17 eV, so Mo does not. Option (A) wrongly excludes Ca; option (C) wrongly includes Mo; option (D) wrongly excludes all three emitting metals.

Step 3: Conclusion Only Mo will not show photoelectric emission. Hence, the correct option is (B).

Quick Tip to Solve: First calculate one photon energy in eV, then compare it directly with every tabulated work function; do not compare wavelength with energy.

2. The ratio of amplitude of electric field to the amplitude of the magnetic field associated with an electromagnetic wave propagating in glass ($n = 1.5$) is: [1 mark]

- (A) $3 \times 10^8 \text{ m s}^{-1}$
- (B) $2 \times 10^8 \text{ m s}^{-1}$
- (C) $3 \cdot 3 \times 10^{-9} \text{ m s}^{-1}$
- (D) $5 \times 10^{-9} \text{ m s}^{-1}$

Correct Answer: (B) $2 \times 10^8 \text{ m s}^{-1}$

Solution:

Concept:

- For an electromagnetic wave, the ratio of the electric-field amplitude E_0 to the magnetic-field amplitude B_0 equals the wave speed v in its medium: $E_0/B_0 = v$.
- The refractive index n is defined by $n = c/v$, where c is the speed of light in vacuum.

Step 1: Determine the speed in glass With $n = 1.5$, the wave travels more slowly than in vacuum:

$$v = \frac{c}{n} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m s}^{-1}.$$

Step 2: Apply the amplitude relation Substitute the medium's wave speed in $E_0/B_0 = v$. Using $3 \times 10^8 \text{ m s}^{-1}$ as in (A) would ignore the glass. The small numerical values in (C) and (D) resemble reciprocal speeds, which belong to B_0/E_0 , not the ratio requested; their printed speed units do not repair that inversion.

Step 3: Conclusion The amplitude ratio is $E_0/B_0 = 2 \times 10^8 \text{ m s}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: For an electromagnetic wave in a medium of index n , replace the vacuum relation $E_0/B_0 = c$ with $E_0/B_0 = c/n$.

3. An ac voltage is given as $v = 14 \sin(314t)$ V. The average and the effective value of the voltage (in V) over a cycle are respectively: [1 mark]

- (A) 14 and 7
- (B) 10 and 14
- (C) 0 and 10
- (D) 10 and 0

Correct Answer: (C) 0 and 10

Solution:

Concept:

- A sinusoidal voltage $v = V_0 \sin \omega t$ has peak value V_0 . Its positive and negative half-cycles give equal and opposite contributions to its average over a complete cycle.
- The effective voltage is the root mean square value $V_{\text{rms}} = V_0/\sqrt{2}$, because the mean of $\sin^2 \omega t$ over one cycle is $1/2$.

Step 1: Average the voltage over one complete cycle The given peak voltage is $V_0 = 14\text{V}$. Over

$T = 2\pi/314$, the two signed half-cycle areas cancel:

$$V_{av} = \frac{1}{T} \int_0^T 14 \sin(314t) dt = 0.$$

Step 2: Calculate the effective voltage Squaring makes both half-cycles contribute positively, so

$$V_{rms} = \frac{14}{\sqrt{2}} \text{ V} \simeq 9.9 \text{ V} \simeq 10 \text{ V}.$$

Options (A), (B) and (D) all give a nonzero full-cycle average and are therefore incorrect. In particular, the zero signed average does not imply zero heating effect or zero rms voltage.

Step 3: Conclusion The average and effective values are 0 V and approximately 10 V, respectively. Hence, the correct option is (C).

Quick Tip to Solve: For a pure sine wave over a full cycle, write the signed average as zero immediately; divide the amplitude by $\sqrt{2}$ only for the effective value.

4. In a reversed-biased p-n junction diode, the applied voltage mostly drops across:

[1 mark]

- (A) p-region only
- (B) n-region only
- (C) depletion region
- (D) the diode

Correct Answer: (C) depletion region

Solution:

Concept:

- The depletion region of a p-n junction contains fixed ionised donors and acceptors but very few mobile charge carriers, so it has a much greater resistance than the neutral p- and n-regions.
- Reverse bias pulls the majority carriers away from the junction, widening the depletion layer and increasing its potential barrier.

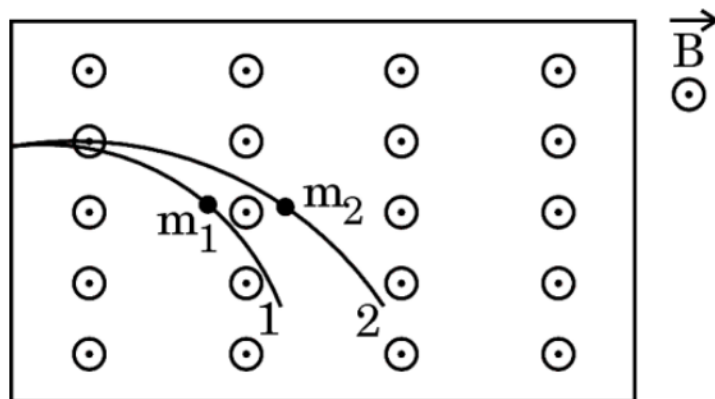
Step 1: Identify the region of greatest resistance Mobile majority carriers allow the bulk p- and n-regions to conduct relatively easily. The depleted junction has very few such carriers and supports the strong electric field associated with the applied reverse voltage.

Step 2: Locate the main voltage drop Consequently, nearly all the reverse-bias potential difference appears across the depletion layer, with only small drops in the neutral regions. Thus (A) and (B) identify the wrong regions. Option (D), the diode as a whole, does not identify the particular internal region across which the voltage mostly drops.

Step 3: Conclusion The applied reverse voltage mostly drops across the depletion region. Hence, the correct option is (C).

Quick Tip to Solve: In reverse bias, look for the carrier-poor, high-resistance depletion layer when locating the main voltage drop inside a p-n diode.

5. Two particles of masses m_1 and m_2 having charges q_1 and q_2 respectively are projected with the same velocity in a region of uniform magnetic field $\vec{B}$ pointing vertically upward. If they describe circular paths as shown in the figure, one may conclude that: [1 mark]



- (A) $\frac{m_1}{m_2} > \frac{q_1}{q_2}$
 (B) $\frac{m_1}{m_2} > \frac{q_2}{q_1}$
 (C) $\frac{m_1}{m_2} < \frac{q_1}{q_2}$
 (D) $\frac{m_1}{m_2} < \frac{q_2}{q_1}$

Correct Answer: (C) $\frac{m_1}{m_2} < \frac{q_1}{q_2}$

Solution:

Concept:

- A charged particle moving perpendicular to a uniform magnetic field follows a circular path because the magnetic force provides centripetal force: $|q|vB = mv^2/r$.
- The radius is $r = mv/(|q|B)$. For equal speeds in the same field, it is proportional to the mass-to-charge-magnitude ratio; the direction of bending fixes the charge sign.

Step 1: Compare the radii in the figure Path 1 bends more sharply than path 2, so $r_1 < r_2$. Both particles have the same initial velocity and bend in the same sense in the same magnetic field, implying that their charges have the same sign. Therefore $q_1/q_2 = |q_1|/|q_2| > 0$.

Step 2: Relate the radii to the mass and charge ratios Cancel the common speed and field magnitude:

$$\frac{r_1}{r_2} = \frac{m_1 |q_2|}{m_2 |q_1|} < 1 \implies \frac{m_1}{m_2} < \frac{|q_1|}{|q_2|} = \frac{q_1}{q_2}.$$

Option (A) reverses this inequality. Options (B) and (D) use the reciprocal charge ratio, which does not follow when the radius relation is rearranged.

Step 3: Conclusion The figure implies $\frac{m_1}{m_2} < \frac{q_1}{q_2}$. Hence, the correct option is (C).

Quick Tip to Solve: Identify the more sharply curved path first, then use $r \propto m/|q|$; a smaller radius means a smaller mass-to-charge-magnitude ratio.

6. Paschen series in spectrum of hydrogen atom lies in:

[1 mark]

- (A) infrared region
- (B) ultraviolet region
- (C) visible region
- (D) partly in ultraviolet region and partly in visible region

Correct Answer: (A) infrared region

Solution:

Concept:

- The Paschen series consists of hydrogen emission transitions from an initial level $n_i = 4, 5, 6, \dots$ to the final level $n_f = 3$.
- The Rydberg formula gives $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where λ is the emitted wavelength and $R_H \simeq 1.097 \times 10^7 \text{ m}^{-1}$.

Step 1: Find the shortest Paschen wavelength The largest photon energy occurs at the series limit $n_i \rightarrow \infty$, which gives

$$\lambda_{\min} = \frac{9}{R_H} \simeq 8.20 \times 10^{-7} \text{ m} = 820 \text{ nm}.$$

Every other Paschen line has a longer wavelength than this limit.

Step 2: Identify the spectral region Even 820 nm lies beyond the red end of the visible spectrum, so all Paschen lines lie in the infrared. Ultraviolet emission characterises the Lyman series; the prominent visible hydrogen lines belong to the Balmer series. Thus the visible, ultraviolet and mixed visible-ultraviolet alternatives do not describe Paschen emission.

Step 3: Conclusion The Paschen series lies in the infrared region. Hence, the correct option is (A).

Quick Tip to Solve: Recall the final levels in order: Lyman ends at 1, Balmer at 2, and Paschen at 3; Paschen begins the infrared hydrogen series.

7. The kinetic energy of a charged particle is increased to four times of its initial value. The de Broglie wavelength associated with the particle will:

[1 mark]

- (A) increase by 100% of its initial value.
- (B) increase by 50% of its initial value.
- (C) decrease by 25% of its initial value.
- (D) decrease by 50% of its initial value.

Correct Answer: (D) decrease by 50% of its initial value.

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is the particle's momentum.
- For nonrelativistic motion, kinetic energy $K = p^2/(2m)$, so $\lambda = h/\sqrt{2mK}$. For a fixed particle mass m , wavelength is inversely proportional to $\sqrt{K}$.

Step 1: Compare the initial and final wavelengths Write $K_f = 4K_i$, keeping the mass unchanged.

The ratio is

$$\frac{\lambda_f}{\lambda_i} = \sqrt{\frac{K_i}{K_f}} = \sqrt{\frac{1}{4}} = \frac{1}{2}.$$

Thus the final wavelength is half the initial wavelength.

Step 2: Calculate the percentage decrease Measure the change relative to the initial value:

$$\frac{\lambda_i - \lambda_f}{\lambda_i} \times 100\% = \left(1 - \frac{1}{2}\right) 100\% = 50\%.$$

The wavelength decreases when kinetic energy increases, ruling out (A) and (B). Option (C) does not apply the inverse square-root relation and gives the wrong reduction.

Step 3: Conclusion The de Broglie wavelength decreases by 50% of its initial value. Hence, the correct option is (D).

Quick Tip to Solve: Multiply kinetic energy by a factor a , then divide the wavelength by $\sqrt{a}$; calculate the percentage change from the original wavelength.

8. The resistivity ρ of a metal increases with rise in temperature because:

[1 mark]

- (A) only relaxation time τ of electrons decreases with temperature.
- (B) only number of electrons per unit volume 'n' increases appreciably.
- (C) τ decreases with temperature but 'n' does not change appreciably.
- (D) τ decreases with temperature and 'n' increases.

Correct Answer: (C) τ decreases with temperature but 'n' does not change appreciably.

Solution:

Concept:

- The resistivity of a metal is $\rho = m/(ne^2\tau)$, where m and e are the electron mass and charge magnitude, n is the free-electron number density, and τ is the mean relaxation time.
- In an ordinary metal, heating increases lattice vibrations and electron scattering, while the free-electron density changes negligibly.

Step 1: Identify the temperature-dependent factor Stronger lattice vibrations cause more frequent collisions. The mean time τ between successive collisions therefore decreases. Since n is approximately constant, $\rho \propto 1/\tau$, so the resistivity increases.

Step 2: Compare the statements about both quantities Option (C) explicitly states both relevant facts: τ decreases and n does not change appreciably. Option (A) mentions the decrease of τ , but omits the essential qualification about n , making (C) the complete description. Options (B) and (D) incorrectly attribute an appreciable increase of carrier density to heating; an increase of n alone would decrease resistivity.

Step 3: Conclusion The relaxation time decreases while the free-electron density remains approximately constant. Hence, the correct option is (C).

Quick Tip to Solve: Write $\rho = m/(ne^2\tau)$: for metals, hold n nearly fixed and track the decrease in τ .

9. Two coils are placed closed to each other. The mutual inductance of the pair of coils depends upon the: [1 mark]

- (A) rate at which currents change in the two coils.
- (B) relative position and orientation of the coils.
- (C) currents in the two coils.
- (D) value of voltage induced in one coil due to change in value of current in the other coil.

Correct Answer: (B) relative position and orientation of the coils.

Solution:

Concept:

- Mutual inductance M relates the flux linkage of one coil to the current in the other: $N_2\Phi_{21} = MI_1$, where N_2 is the number of turns in the second coil.
- For fixed coils in a linear magnetic medium, M depends on geometry, relative placement and the permeability of the medium.

Step 1: Identify what changes the magnetic coupling Moving or rotating one coil changes the fraction of the magnetic flux produced by it that links the other coil. Therefore the relative position and orientation of the coils determine their mutual inductance.

Step 2: Distinguish inductance from induced voltage The induced emf is $\varepsilon_2 = -M dI_1/dt$. The current and its rate of change affect flux linkage and induced emf, respectively, but do not set the geometrical coefficient M . Thus (A) and (C) confuse operating conditions with a property of the coil pair. The voltage in (D) is an effect of M and the current-change rate, rather than an independent cause of M .

Step 3: Conclusion Mutual inductance depends on the relative position and orientation of the coils. Hence, the correct option is (B).

Quick Tip to Solve: Separate the coil-pair property M from the changing-current factor in $\varepsilon = -M dI/dt$.

10. Which of the following statements is true about mobility of charge carriers in a metal? [1 mark]

- (A) Mobility increases with increase in applied electric field.
- (B) Mobility decreases with increase in temperature.
- (C) Mobility is independent of the mass of the charge carrier.
- (D) Mobility increases with increase in temperature.

Correct Answer: (B) Mobility decreases with increase in temperature.

Solution:

Concept:

- Mobility is the drift-speed magnitude per unit electric field: $\mu = v_d/E$, where v_d is the drift speed and E is the applied electric-field magnitude.
- The relaxation-time model gives $v_d = eE\tau/m$, hence $\mu = e\tau/m$, with e the carrier charge magnitude, m its mass and τ its mean relaxation time.

Step 1: Apply the temperature dependence in a metal Heating the metal increases lattice vibrations. Electrons undergo more frequent collisions, so their relaxation time τ decreases. Since μ is proportional to τ , the mobility decreases as temperature increases.

Step 2: Check the other proposed dependences At a fixed temperature in the usual Ohmic regime, increasing E increases v_d in the same proportion and leaves v_d/E unchanged, so (A) is incorrect. The expression $e\tau/m$ explicitly contains the carrier mass, ruling out (C). Option (D) reverses the temperature trend and would require fewer collisions on heating.

Step 3: Conclusion Mobility of charge carriers in a metal decreases with increasing temperature. Hence, the correct option is (B).

Quick Tip to Solve: For a heated metal, use the chain: more lattice vibrations, smaller τ , smaller $\mu = e\tau/m$.

11. A galvanometer of resistance G is converted into a voltmeter of range (0 - V) by connecting a resistor of $250\ \Omega$ with it. If resistor of $250\ \Omega$ is replaced by another resistor of $900\ \Omega$, its range becomes (0 - 3 V). The resistance G of the galvanometer is: [1 mark]

- (A) $150\ \Omega$
- (B) $125\ \Omega$
- (C) $100\ \Omega$
- (D) $75\ \Omega$

Correct Answer: (D) $75\ \Omega$

Solution:

Concept:

- A galvanometer becomes a voltmeter by adding a large resistance R in series. Its full-scale voltage is $V_{\max} = I_g(G + R)$, where G is the galvanometer resistance.
- The same galvanometer has the same full-scale current I_g for both ranges, so taking the ratio eliminates this unknown current.

Step 1: Write the two full-scale voltage equations For the first series resistance, $V = I_g(G + 250\ \Omega)$. Replacing it by $900\ \Omega$ makes the range three times as large:

$$3V = I_g(G + 900\ \Omega).$$

Both the external resistor and the internal galvanometer resistance contribute to the total series resistance.

Step 2: Eliminate the current and solve for resistance Dividing the second equation by the first gives

$$\begin{aligned} 3 &= \frac{G + 900\ \Omega}{G + 250\ \Omega}, \\ 3G + 750\ \Omega &= G + 900\ \Omega, \\ 2G &= 150\ \Omega. \end{aligned}$$

As a check, the resulting total resistances are $325\ \Omega$ and $975\ \Omega$, whose ratio is exactly three. The values in (A), (B) and (C) do not satisfy this ratio.

Step 3: Conclusion The galvanometer resistance is $G = 75\ \Omega$. Hence, the correct option is (D).

Quick Tip to Solve: For a changed voltmeter range, compare $G + R$ at fixed I_g ; the added resistor alone does not determine the ratio.

12. Photons of frequency ν are incident on the surfaces of two metals A and B of threshold frequencies $\frac{\nu}{2}$ and $\frac{\nu}{3}$. The ratio of maximum kinetic energy of electrons emitted from metal A to that from metal B is: [1 mark]

- (A) $\frac{1}{3}$
 (B) $\frac{3}{4}$
 (C) $\frac{2}{3}$
 (D) $\frac{3}{2}$

Correct Answer: (B) $\frac{3}{4}$

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where $h\nu$ is the incident photon energy and ϕ is the metal's work function.
- If the threshold frequency is ν_0 , then $\phi = h\nu_0$, so $K_{\max} = h(\nu - \nu_0)$.

Step 1: Find the maximum kinetic energy for each metal Both metals receive photons of the same frequency ν , above both thresholds. Subtract the appropriate threshold energy separately:

$$K_A = h\left(\nu - \frac{\nu}{2}\right) = \frac{h\nu}{2},$$

$$K_B = h\left(\nu - \frac{\nu}{3}\right) = \frac{2h\nu}{3}.$$

Step 2: Form the requested ratio in the given order Divide the energy for metal A by that for metal B:

$$\frac{K_A}{K_B} = \frac{h\nu/2}{2h\nu/3} = \frac{1}{2} \times \frac{3}{2}.$$

The quantities h and ν cancel. Metal A has a higher work function, so its emitted electrons must have a smaller maximum kinetic energy. Option (D), which exceeds unity, fails this check; (A) and (C) do not equal the ratio obtained after subtracting the work functions.

Step 3: Conclusion The required ratio is $K_A/K_B = 3/4$. Hence, the correct option is (B).

Quick Tip to Solve: Subtract each threshold frequency from the common incident frequency before taking the ratio; do not divide the thresholds.

13. Assertion (A): When a convex lens made of glass is immersed in water, its converging power increases.

Reason (R): The focal length of a lens depends only on the radii of curvature of its two faces. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false.

Solution:

Concept:

- For a thin lens in a surrounding medium, the lens-maker relation is $P = 1/f = (n_g/n_m - 1)(1/R_1 - 1/R_2)$, where n_g and n_m are the glass and medium refractive indices.
- Lens power P therefore depends on both the surface radii R_1, R_2 and the refractive index of the lens relative to its surroundings.

Step 1: Evaluate the Assertion Water has a larger refractive index than air and a smaller one than ordinary lens glass. On immersion, $n_g/n_m - 1$ decreases while the surface curvatures remain fixed. The positive power of the convex lens decreases and its focal length increases. The Assertion, which says the converging power increases, is false.

Step 2: Evaluate the Reason The lens-maker relation contains the relative refractive index as well as the radii. Changing the surrounding medium changes the focal length without changing either radius. Thus the word “only” makes the Reason false.

Step 3: Check the explanation and answer code A false Reason cannot explain the claimed increase of power. Both statements are false, which is explicitly the combination assigned to (D) in this paper. Options (A) and (B) require both statements to be true; option (C) requires a true Assertion.

Step 4: Conclusion Both the Assertion and the Reason are false. Hence, the correct option is (D).

Quick Tip to Solve: On immersing a lens, change its relative refractive index n_g/n_m in the lens-maker relation; its curvatures stay fixed.

14. Assertion (A): The conductivity of an n-type semiconductor is higher than that of a p-type semiconductor at a given temperature.

Reason (R): The electrons being in the conduction band in n-type semiconductor are more mobile than the holes in the valence band in p-type semiconductor. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- Semiconductor conductivity is $\sigma = e(n\mu_e + p\mu_h)$, where n, p are electron and hole concentrations and μ_e, μ_h are their mobilities.
- In an n-type semiconductor electrons are the majority carriers; in a p-type semiconductor holes are the majority carriers. In the usual comparison for the same material, $\mu_e > \mu_h$.

Step 1: Evaluate the Assertion The standard comparison assumes comparable doping and hence equal majority-carrier concentration N at the given temperature. Neglecting the small minority-carrier contribution gives $\sigma_n \simeq eN\mu_e$ and $\sigma_p \simeq eN\mu_h$. Since electrons have greater mobility, the n-type sample has higher conductivity. Thus the Assertion is true under the intended comparison; temperature alone would not fix conductivity for arbitrarily different doping levels.

Step 2: Evaluate the Reason Electrons in the conduction band respond to an electric field with a greater drift speed than holes for the usual semiconductor comparison. Equivalently, their mobility is greater. The Reason correctly states this property and is true.

Step 3: Test the explanatory link At equal carrier concentration, conductivity is proportional to the majority-carrier mobility. The mobility difference in the Reason therefore directly explains the conductivity difference in the Assertion. Option (B) wrongly denies that link, while (C) and (D) wrongly classify the Reason as false.

Step 4: Conclusion For the standard comparison at equal majority-carrier density, both statements are true and the Reason correctly explains the Assertion. Hence, the correct option is (A).

Quick Tip to Solve: Use $\sigma \simeq eN\mu$: compare mobilities only after holding the majority-carrier concentration fixed.

15. Assertion (A): The work done, in taking a unit charge around a closed loop of an electric circuit involving cells and resistors in the loop, is zero.

Reason (R): The potential at a point depends on the location of the point in the loop. After completing one round, the charge comes back to the point of start. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- Kirchhoff's loop rule states that the algebraic sum of potential changes around a closed circuit loop is zero: $\sum \Delta V = 0$.
- Electric potential is single-valued at each point. The net electrostatic work on a charge q between two points is $W = q(V_i - V_f)$.

Step 1: Evaluate the assertion A complete circuit traversal ends at its starting point, so the initial and final potentials are equal. For unit charge, the algebraic work around the loop is therefore zero. The potential rises across cells balance the potential drops across resistors. This does not mean that a resistor dissipates no energy; the cells supply the energy lost in the resistors. Assertion (A) is true.

Step 2: Evaluate the reason In the steady circuit considered here, each position has a definite potential. Returning to the original position returns the charge to the same potential, giving $V_f - V_i = 0$. Both statements in Reason (R) are therefore true.

Step 3: Test the explanatory connection The return to the same potential is precisely why the algebraic potential change, and hence the net work per unit charge, vanishes around a closed loop. Thus R directly explains A. Option (B) incorrectly denies this connection, while options (C) and (D) incorrectly treat a true statement as false.

Step 4: Conclusion Both Assertion (A) and Reason (R) are true, and R correctly explains A. Hence, the correct option is (A).

Quick Tip to Solve: For a closed loop, compare the final and initial potentials before adding individual cell rises and resistor drops.

16. Assertion (A): When a ferromagnetic substance is heated to high temperature it becomes paramagnetic in nature.

Reason (R): The disappearance of magnetisation of a ferromagnet is abrupt and not gradual. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- Ferromagnetic order is associated with alignment of atomic magnetic moments. Thermal agitation increasingly disrupts this alignment as temperature rises.
- Above the Curie temperature T_C , a ferromagnet loses spontaneous magnetisation and behaves as a paramagnet; T_C is characteristic of the material.

Step 1: Evaluate the assertion Heating a ferromagnetic substance sufficiently, beyond its Curie temperature, destroys its spontaneous ferromagnetic order. It then exhibits paramagnetic behaviour. The phrase high temperature refers to this temperature range, so Assertion (A) is true.

Step 2: Evaluate the reason As temperature increases, thermal motion progressively disrupts the alignment of atomic magnetic moments. The spontaneous magnetisation decreases gradually and vanishes at the Curie temperature. The stated claim that its disappearance is abrupt and not gradual is therefore false in the standard description of this transition.

Step 3: Select the appropriate truth combination Since R is false, it cannot explain the true assertion. Options (A) and (B) both require R to be true, so neither applies; option (D) incorrectly declares A false as well. The required combination is a true assertion with a false reason.

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish the Curie temperature where spontaneous magnetisation vanishes from the gradual decrease in magnetisation on approaching it.

Section B (10 marks)

17. The hole concentration in an intrinsic semiconductor is $5 \times 10^8 \text{ m}^{-3}$. When it is doped with certain impurity, the electron concentration becomes $4 \times 10^{12} \text{ m}^{-3}$. Find the new value of the hole concentration. Also identify the type of new semiconductor formed after doping. [2 marks]

Solution:

Concept:

- In an intrinsic semiconductor, the electron concentration n_i equals the hole concentration p_i : $n_i = p_i$.
- At a fixed temperature and thermal equilibrium, the mass-action law is $np = n_i^2$, where n and p are the electron and hole concentrations after doping.
- An n-type semiconductor has electrons as majority carriers; a p-type semiconductor has holes as majority carriers.

Step 1: Find the intrinsic carrier product The printed intrinsic hole concentration is $p_i = 5 \times 10^8 \text{ m}^{-3}$. Intrinsic electrons and holes occur in equal numbers, so n_i has the same value. At the unchanged temperature, the carrier product is

$$n_i^2 = (5 \times 10^8)^2 = 25 \times 10^{16} \text{ m}^{-6}.$$

Step 2: Calculate the hole concentration after doping Substitute the new electron concentration $n = 4 \times 10^{12} \text{ m}^{-3}$ into the mass-action law. Doping changes the two concentrations, but their equilibrium product remains fixed at the given temperature:

$$\begin{aligned} p &= \frac{n_i^2}{n} \\ &= \frac{25 \times 10^{16}}{4 \times 10^{12}} \text{ m}^{-3} \\ &= 6.25 \times 10^4 \text{ m}^{-3}. \end{aligned}$$

Step 3: Identify the majority carrier The new electron concentration is much greater than the new hole concentration: $4 \times 10^{12} \gg 6.25 \times 10^4$. Electrons are therefore the majority carriers and holes are minority carriers. This is consistent with adding a donor impurity, which raises the electron concentration above its intrinsic value.

Step 4: Conclusion The new hole concentration is $6.25 \times 10^4 \text{ m}^{-3}$, and the doped material is an n-type semiconductor.

Quick Tip to Solve: Square the intrinsic concentration before dividing by the new electron concentration; identify the type from the larger final carrier concentration.

18. An electric dipole consists of two point charges $+1\ \mu\text{C}$ and $-1\ \mu\text{C}$, held 10 cm apart. It is subjected to a uniform electric field of 100 N/C. Calculate the amount of work done in turning the dipole from its position of stable equilibrium to the position of unstable equilibrium, in the field. [2 marks]

Solution:

Concept:

- The electric dipole moment has magnitude $p = qd$, where q is the magnitude of either charge and d is the full separation between the charges.
- A dipole at angle θ to a uniform electric field has potential energy $U = -pE \cos \theta$. Stable equilibrium is at $\theta = 0$, and unstable equilibrium is at $\theta = \pi$.
- The work supplied by an external agent in turning the dipole slowly, with no change in kinetic energy, is $W_{\text{ext}} = U_f - U_i$.

Step 1: Calculate the dipole moment in SI units The two charges have equal magnitude $q = 1 \times 10^{-6}\ \text{C}$. Their full separation is $d = 10\ \text{cm} = 0.10\ \text{m}$; it is not necessary to double this stated distance. Thus

$$p = qd = (1 \times 10^{-6})(0.10) = 1.0 \times 10^{-7}\ \text{C m}.$$

Step 2: Evaluate the energy change between the equilibria In stable equilibrium the dipole is parallel to the field, so its energy is $U_i = -pE$. In unstable equilibrium it is antiparallel, giving $U_f = +pE$. Using $E = 100\ \text{N C}^{-1}$, the required external work is

$$\begin{aligned} W_{\text{ext}} &= (+pE) - (-pE) = 2pE \\ &= 2(1.0 \times 10^{-7})(100) \\ &= 2.0 \times 10^{-5}\ \text{J}. \end{aligned}$$

This work is positive because the rotation raises the potential energy. The electric field itself does negative work of the same magnitude during the rotation.

Step 3: Conclusion The work required to turn the dipole from stable to unstable equilibrium is $2.0 \times 10^{-5}\ \text{J}$ ($20\ \mu\text{J}$).

Quick Tip to Solve: A stable-to-unstable rotation changes the energy from $-pE$ to $+pE$, so use $2pE$ and the full charge separation in $p = qd$.

19(a). State Huygens principle. How did Huygens justify the absence of the backwave on a spherical wavefront? [2 marks]

Solution:

Concept:

- A wavefront is a surface of constant phase. The direction of propagation at a point is along the normal to the wavefront.
- Huygens' construction obtains a later wavefront from the envelope of secondary wavelets emitted by all points of an earlier wavefront.

Step 1: State Huygens' principle Every point on a given wavefront acts as a source of secondary wavelets. These wavelets spread through the medium with the same speed as the original wave. After a time interval Δt , each has travelled a distance $v\Delta t$, where v is the wave speed. The common

forward envelope, or surface tangent to all these secondary wavelets, gives the new position of the wavefront.

Step 2: Explain the absence of the backwave A purely geometrical construction with complete spherical wavelets would also allow a backward envelope. To exclude it, Huygens assumed that the amplitude of secondary wavelets is maximum in the forward direction and zero in the backward direction. For an outward-moving spherical wavefront, forward means along the outward radial normal, while backward means towards the centre. Since the backward secondary disturbance has zero amplitude under this assumption, the backward envelope does not represent a propagating wave. Thus only the outward wavefront is retained.

Step 3: Conclusion Each point of a wavefront emits secondary wavelets whose forward envelope is the next wavefront. Huygens excluded the backwave by assuming zero secondary-wave amplitude in the backward direction.

Quick Tip to Solve: State both parts of Huygens' construction, then identify the zero-backward-amplitude assumption that excludes the inward spherical envelope.

OR

19(b). In a single-slit diffraction experiment, light of wavelength λ illuminates the slit of width 'a'. The diffraction pattern is observed on a screen kept at a distance D from the slits.

(i) Depict variation of intensity in the fringe pattern with the angular position of the fringes.

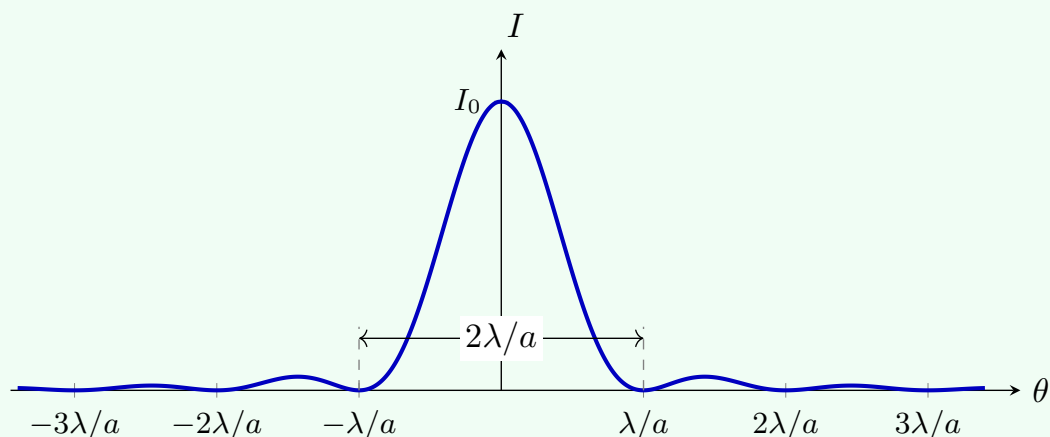
(ii) How is the linear width of central maximum affected when separation between the slit and the screen is decreased? [2 marks]

Solution:

Concept:

- For a slit of width a illuminated by wavelength λ , interference between wavelets from the slit gives $I = I_0(\sin \beta / \beta)^2$, where $\beta = \pi a \sin \theta / \lambda$, θ is the angle from the central direction and I_0 is the central intensity.
- Dark fringes occur at $a \sin \theta = m\lambda$, for $m = \pm 1, \pm 2, \dots$. For small angles, the first minima are at $\theta = \pm \lambda / a$, and positions on a screen at distance D obey $y \simeq D\theta$.

Step 1: (i) Sketch the angular intensity distribution The central maximum at $\theta = 0$ is the brightest. On both sides, weaker secondary maxima alternate with zero-intensity minima, with peak intensities decreasing away from the centre. In the small-angle approximation, the central maximum has angular width $2\lambda/a$, twice the width between consecutive minima on either side, as shown.



Single-slit diffraction intensity versus angle, in the small-angle approximation.

Step 2: (ii) Relate the screen distance to the linear width The first minima on the screen have coordinates $y_1 \simeq D\lambda/a$ and $-y_1$. Therefore, the linear width of the central maximum is

$$w = 2y_1 = \frac{2D\lambda}{a}.$$

With λ and a fixed, $w \propto D$. Thus $w'/w = D'/D$ for a changed distance D' , within the usual far-field diffraction conditions.

Step 3: Conclusion (i) The graph shows the broad central peak and weaker secondary peaks. (ii) Decreasing D decreases the linear width $2D\lambda/a$ in the same proportion; the angular width remains unchanged.

Quick Tip to Solve: Locate the first minima at $\pm\lambda/a$ before drawing the central peak; multiply their angular separation by D to find the linear width.

20. A convex lens of refractive index 1.5 has a focal length of 20 cm in air. Find its nature and focal length when it is immersed in a transparent liquid of refractive index 1.25. [2 marks]

Solution:

Concept:

- The lens-maker's formula for a thin lens in a surrounding medium is $1/f = (n_\ell/n_m - 1)(1/R_1 - 1/R_2)$, where n_ℓ and n_m are the absolute refractive indices of the lens and medium, respectively, and R_1, R_2 are its signed radii of curvature.
- Immersion changes the relative refractive index but leaves the lens surfaces unchanged. A convex lens remains converging when $n_\ell > n_m$; a positive focal length denotes this converging nature.

Step 1: Express the common curvature factor Let $K = 1/R_1 - 1/R_2$. In air, $n_m \simeq 1$ and the convex lens has $f_{\text{air}} = +20$ cm. Hence,

$$\frac{1}{f_{\text{air}}} = (1.5 - 1)K = 0.5K.$$

In the liquid, use the index of the lens relative to the liquid, $1.5/1.25 = 1.2$, rather than its absolute index. Therefore,

$$\frac{1}{f_{\text{liquid}}} = (1.2 - 1)K = 0.2K.$$

Step 2: Take the ratio and identify the nature Dividing the two lens-maker equations eliminates the unchanged factor K :

$$\frac{f_{\text{air}}}{f_{\text{liquid}}} = \frac{0.2}{0.5} = 0.4,$$
$$f_{\text{liquid}} = \frac{20}{0.4} \text{ cm} = +50 \text{ cm}.$$

Since the lens index 1.5 still exceeds the liquid index 1.25, the refractive power is positive. The smaller index contrast weakens the convergence, consistent with the larger positive focal length.

Step 3: Conclusion In the liquid, the lens remains **converging** and has focal length $f = +50 \text{ cm}$.

Quick Tip to Solve: For the same lens in two media, divide the lens-maker equations: the curvature cancels, and focal length varies inversely with $n_{\ell}/n_m - 1$.

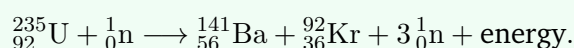
21. Differentiate between nuclear fission and nuclear fusion, giving one example for each. [2 marks]

Solution:

Concept:

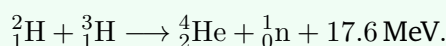
- Nuclear fission splits a heavy nucleus into lighter nuclei, whereas nuclear fusion combines light nuclei to form a heavier nucleus. These energy-releasing processes move the products towards greater binding energy per nucleon.
- The released energy comes from a reduction in total rest mass, $Q = \Delta mc^2$, where Δm is the mass defect of the reaction and c is the speed of light. Total mass number and total charge are conserved in each nuclear equation.

Step 1: Describe nuclear fission and give an example In fission, a heavy nucleus breaks into two medium-mass nuclei, usually with the emission of neutrons and energy. For example, absorption of a slow neutron by uranium-235 can produce the following fission channel:



Here, the mass numbers balance as $235 + 1 = 141 + 92 + 3$, and the atomic numbers balance as $92 = 56 + 36$. The emitted neutrons can initiate further fissions, making a chain reaction possible.

Step 2: Describe nuclear fusion and give an example In fusion, two light nuclei combine to form a heavier nucleus with release of energy. Very high temperatures are needed for appreciable thermonuclear fusion because the positively charged nuclei must approach closely despite their electric repulsion. A standard example is deuterium-tritium fusion:



The mass numbers balance as $2 + 3 = 4 + 1$, and the atomic numbers as $1 + 1 = 2 + 0$. Thus, the light reactants combine into helium, with a neutron also emitted.

Step 3: Conclusion **Fission** is the splitting of a heavy nucleus, illustrated by neutron-induced uranium-

235 fission. **Fusion** is the combining of light nuclei, illustrated by deuterium-tritium fusion to form helium and a neutron. Both examples release nuclear energy.

Quick Tip to Solve: Distinguish the processes by the initial nuclei: one heavy nucleus splits in fission; two light nuclei join in fusion. Check both total mass number and charge in each example.

Section C (21 marks)

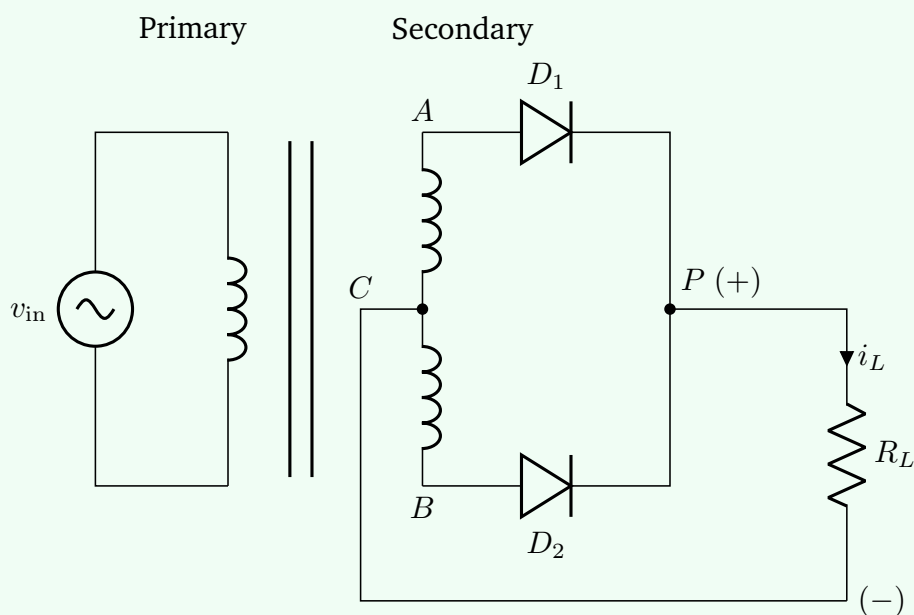
22. With the help of a circuit diagram, explain the working of a full wave rectifier. Depict the input and output waveforms. [3 marks]

Solution:

Concept:

- A p-n junction diode conducts appreciably in forward bias and blocks current in reverse bias. Ideal diodes are assumed in the waveforms.
- In a centre-tapped transformer, the two equal secondary halves have opposite instantaneous voltages relative to their common centre tap C . A full-wave rectifier uses both half-cycles to send current through the load in one direction.

Step 1: Connect the full-wave rectifier Connect secondary ends A and B to the anodes of diodes D_1 and D_2 , respectively. Join their cathodes at P . Connect the load resistance R_L between P and centre tap C . Apply the alternating input to the transformer primary. The output voltage is measured across R_L , with P positive relative to C .



Centre-tapped full-wave rectifier; both cathodes meet at P .

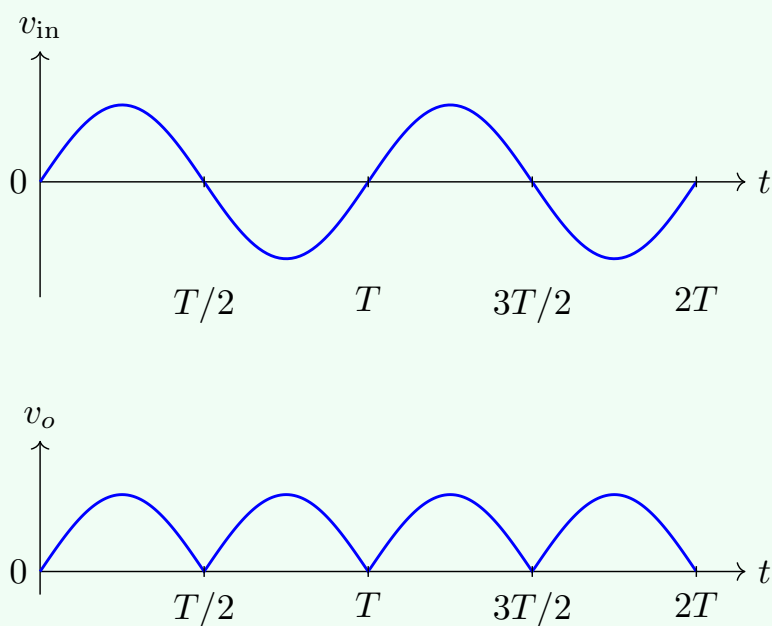
Step 2: Follow the current during both half-cycles When A is positive relative to C , B is negative: D_1 is forward biased and D_2 is reverse biased. Current follows $A \rightarrow D_1 \rightarrow P \rightarrow R_L \rightarrow C$, returning through the upper secondary half.

During the next half-cycle, B is positive and A is negative relative to C . Now D_2 conducts and D_1 blocks. Current follows $B \rightarrow D_2 \rightarrow P \rightarrow R_L \rightarrow C$, returning through the lower secondary half. Thus the load current always flows from P towards C .

Step 3: Compare the input and output waveforms Let the input frequency be f , and let the voltage of secondary end A relative to C be $V_m \sin \omega t$, where $\omega = 2\pi f$. The opposite secondary end supplies the other half-cycle, so the ideal output is

$$v_o = V_m |\sin \omega t|, \quad i_L = \frac{V_m}{R_L} |\sin \omega t|.$$

The input alternates in sign whereas every output pulse is positive. There are two output pulses in one input period $T = 1/f$; hence the ripple frequency is $2f$. With no smoothing capacitor, the output is pulsating direct voltage.



Alternating input and full-wave output; amplitudes use separate scales.

Step 4: Conclusion The two diodes conduct alternately and rectify both half-cycles. The load receives current in a fixed direction, giving a pulsating DC output of frequency $2f$.

Quick Tip to Solve: Trace each conducting diode to the same load terminal: the load-current direction must stay unchanged when the secondary polarity reverses.

23. (a) Explain how the dual aspect of matter is evident in the de Broglie relation.

(b) Radiation of wavelength λ is incident on a photosensitive surface. Find the de Broglie wavelength of electrons emitted from the surface. Assume that the work function of the surface is negligible.

[3 marks]

Solution:

Concept:

- The de Broglie relation is $\lambda_{dB} = h/p$, where h is Planck's constant and p is the particle's momentum. Wavelength describes wave behaviour, whereas momentum is a particle property.
- Einstein's photoelectric equation gives $K_{\max} = hc/\lambda - \phi$, where λ is the incident radiation wavelength, c is the speed of light and ϕ is the work function.
- For a non-relativistic electron of mass m_e , $K = p^2/(2m_e)$; this connects the photoelectron's kinetic energy to its de Broglie wavelength.

Step 1: Explain the dual aspect in part (a) In $\lambda_{\text{dB}} = h/p$, the same material particle is characterised by both a wavelength and a momentum. Thus the relation links its wave aspect, represented by λ_{dB} , with its particle aspect, represented by p . For motion at speeds much smaller than c , $p = m_e v$. Electron diffraction reveals this associated wave nature, while a localised electron detection reveals particle behaviour. The relation does not assign the incident light wavelength to the electron: each wavelength belongs to its own physical object.

Step 2: Find the photoelectron energy in part (b) Each incident photon has energy $E_\gamma = hc/\lambda$. Neglecting the work function as instructed, the available photon energy becomes the emitted electron's kinetic energy. For the maximum-energy photoelectron,

$$K_{\text{max}} = \frac{hc}{\lambda}, \quad p_{\text{max}} = \sqrt{2m_e K_{\text{max}}} = \sqrt{\frac{2m_e hc}{\lambda}}.$$

This is the usual Class 12 non-relativistic treatment. Photoelectrons that lose energy before emerging can have lower kinetic energies.

Step 3: Apply the de Broglie relation Substitute the momentum into the matter-wave relation:

$$\begin{aligned} \lambda_{\text{dB}} &= \frac{h}{\sqrt{2m_e hc/\lambda}} \\ &= \sqrt{\frac{h\lambda}{2m_e c}}. \end{aligned}$$

Since wavelength decreases as momentum increases, this expression corresponds to the shortest emitted-electron wavelength if the photoelectrons have a range of energies; it is the wavelength in the ideal no-energy-loss model used here.

Step 4: Conclusion (a) The de Broglie relation links the wave property λ_{dB} with the particle property p , expressing matter's dual nature. (b) The required photoelectron wavelength is $\lambda_{\text{dB}} = \sqrt{h\lambda/(2m_e c)}$ in the stated idealisation.

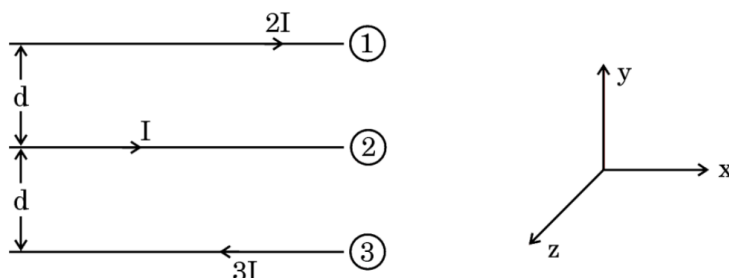
Quick Tip to Solve: First convert photon wavelength to electron kinetic energy using hc/λ ; then use $h/\sqrt{2m_e K}$, keeping the two wavelengths distinct.

24(a). The figure given below shows three straight long parallel conductors (1), (2) and (3) kept in x-y plane, carrying currents $2I$, I and $3I$ respectively as shown in figure.

Find the magnitude and direction of:

(i) net magnetic field at a point on conductor (1) and

(ii) net magnetic force acting on unit length of conductor (1), due to conductors (2) and (3). [3 marks]



Solution:

Concept:

- A long straight conductor carrying current I_s produces magnetic field $B = \mu_0 I_s / (2\pi r)$ at perpendicular distance r . Its direction follows the right-hand rule; μ_0 is the permeability of free space.
- Magnetic fields add vectorially. The force per unit length on a wire is $\vec{F}/L = I_1 \hat{\ell} \times \vec{B}_{\text{ext}}$, where $\hat{\ell}$ points along its current and the external field excludes its own field.

Step 1: Read the separations and current directions Wire (1) carries $2I$ along $+x$. Wire (2), a distance d below it, carries I along $+x$. Wire (3), a distance $2d$ below wire (1), carries $3I$ along $-x$.

Step 2: Add the magnetic fields for part (i) At a point on wire (1), the displacement from either source wire is along $+y$. Since $\hat{x} \times \hat{y} = \hat{z}$, the field of wire (2) is along $+z$, while reversing the source current in wire (3) reverses its field to $-z$. Thus

$$\begin{aligned}\vec{B}_2 &= \frac{\mu_0 I}{2\pi d} \hat{z}, \\ \vec{B}_3 &= -\frac{\mu_0 (3I)}{2\pi (2d)} \hat{z}, \\ \vec{B}_{\text{ext}} &= \left(\frac{\mu_0 I}{2\pi d} - \frac{3\mu_0 I}{4\pi d} \right) \hat{z} = -\frac{\mu_0 I}{4\pi d} \hat{z}.\end{aligned}$$

Although wire (3) is twice as far away, its current is three times larger, so its magnetic field dominates.

Step 3: Evaluate the force per unit length for part (ii) The current direction in wire (1) is $\hat{\ell} = \hat{x}$. Using $\hat{x} \times (-\hat{z}) = \hat{y}$,

$$\frac{\vec{F}_1}{L} = 2I \hat{x} \times \left(-\frac{\mu_0 I}{4\pi d} \hat{z} \right) = \frac{\mu_0 I^2}{2\pi d} \hat{y}.$$

As a direction check, wire (2) attracts wire (1) downwards because their currents are parallel, while wire (3) repels it upwards. The latter force is larger since the ratio of current to separation is $3I/(2d) > I/d$.

Step 4: Conclusion (i) The net magnetic field has magnitude $\mu_0 I / (4\pi d)$ along $-z$, perpendicular to the x - y plane. (ii) The net force per unit length has magnitude $\mu_0 I^2 / (2\pi d)$ along $+y$, upwards and away from the lower conductors. The SI units are tesla and N m^{-1} , respectively.

Quick Tip to Solve: Use d for wire (2) and $2d$ for wire (3), and assign field signs before adding; verify the final force direction using attraction and repulsion.

OR

24(b). A rectangular loop of sides l and b and resistance 'R' is kept in a region in which the magnetic field varies as $B = B_0 \sin \omega t$.

(i) Derive expression for the emf induced in the loop.

(ii) Find the effective value of current that flows in the loop.

[3 marks]

Solution:

Concept:

- Magnetic flux through a plane loop of area A in a uniform field is $\Phi_B = BA \cos \theta$, where θ is the angle between the field and the area normal.
- Faraday's law gives the induced emf as $\varepsilon = -d\Phi_B/dt$. The minus sign expresses Lenz's law: the induced current opposes the change in flux.
- For a purely resistive loop, $i = \varepsilon/R$. Its effective current is the root-mean-square value, $I_{\text{rms}} = \sqrt{\langle i^2 \rangle}$.

Step 1: (i) Express the magnetic flux The loop has area $A = lb$. Assume that the uniform field is perpendicular to its plane and choose the positive area normal along the positive field direction. Then

$$\Phi_B(t) = B(t)lb = B_0 lb \sin \omega t.$$

The area remains constant, so only the changing field changes the flux.

Step 2: (i) Differentiate the flux to obtain the emf Apply Faraday's law to the single loop:

$$\varepsilon(t) = -\frac{d}{dt}(B_0 lb \sin \omega t) = -B_0 lb \omega \cos \omega t.$$

Thus the peak emf is $\varepsilon_0 = B_0 lb \omega$. The sign refers to the positive circulation chosen by the area normal.

Step 3: (ii) Calculate the effective current Neglecting self-inductance, apply Ohm's law to the resistive loop:

$$i(t) = -\frac{B_0 lb \omega}{R} \cos \omega t.$$

Over one complete cycle, $\langle \cos^2 \omega t \rangle = 1/2$. Therefore

$$I_{\text{rms}}^2 = \frac{B_0^2 l^2 b^2 \omega^2}{R^2} \langle \cos^2 \omega t \rangle = \frac{B_0^2 l^2 b^2 \omega^2}{2R^2}.$$

This is the steady current giving the same average heating; the cycle-average alternating current is zero.

Step 4: Conclusion For the perpendicular-field, resistive-loop case,

$$\varepsilon(t) = -B_0 lb\omega \cos \omega t, \quad I_{\text{rms}} = \frac{B_0 lb\omega}{\sqrt{2}R}.$$

For a fixed general orientation, multiply the signed emf by $\cos \theta$ and the effective current by $|\cos \theta|$.

Quick Tip to Solve: Differentiate the sine field first to obtain the peak emf $B_0 lb\omega$; then divide by $R\sqrt{2}$ to obtain the effective current.

25. What are X-rays? How are they produced? Give two uses of X-rays.

[3 marks]

Solution:

Concept:

- X-rays are high-frequency, short-wavelength electromagnetic waves. Like other electromagnetic waves, they consist of mutually perpendicular oscillating electric and magnetic fields and travel in vacuum at speed c .
- An X-ray photon has energy $E = h\nu = hc/\lambda$, where h is Planck's constant, ν is frequency and λ is wavelength. Their short wavelength gives them high photon energy and substantial penetrating power.

Step 1: Identify the nature of X-rays X-rays form the high-energy part of the electromagnetic spectrum beyond ultraviolet radiation, with wavelengths generally much shorter than visible light. They are electrically neutral radiation, not a stream of electrons. Their penetration through matter depends on the photon energy and on the nature and thickness of the material.

Step 2: Explain their production in an X-ray tube A heated cathode emits electrons by thermionic emission in an evacuated tube. A large potential difference accelerates these electrons towards a heavy-metal target, commonly tungsten, at the anode. On striking the target, the fast electrons undergo rapid deceleration; a fraction of their kinetic energy is emitted as X-ray photons, while most becomes heat. This is the standard production mechanism of the continuous X-ray spectrum. Characteristic X-rays can also arise when an incident electron removes an inner-shell electron of a target atom and another atomic electron fills the vacancy.

Step 3: State two uses and their physical basis **1. Medical radiography:** X-rays are used to detect bone fractures. Bone and soft tissue absorb X-rays by different amounts, creating contrast in the recorded image.

2. Crystal-structure analysis: X-ray diffraction is used to study atomic arrangements in crystals. X-ray wavelengths are comparable to interatomic spacings, so the diffraction pattern provides information about the crystal lattice.

Step 4: Conclusion X-rays are short-wavelength electromagnetic radiation, produced in an X-ray tube mainly by the rapid deceleration of high-speed electrons at a metal target. Two uses are detecting bone fractures and determining crystal structure by diffraction.

Quick Tip to Solve: Link each use to a property: differential absorption explains bone images, while wavelengths comparable to atomic spacing explain crystal diffraction.

26. (a) Distinguish between isotopes and isobars, giving one example for each.

(b) Derive the relation between atomic mass unit (u) and electron volt (eV).

[3 marks]

Solution:

Concept:

- Atomic number Z counts protons; mass number A counts protons plus neutrons. The neutron number is $N = A - Z$.
- One atomic mass unit, 1 u, is one-twelfth of the mass of a neutral carbon-12 atom at rest in its ground state. Numerically, $1 \text{ u} \simeq 1.6605 \times 10^{-27} \text{ kg}$.
- Einstein's mass-energy relation is $E = mc^2$. An electron volt is the energy gained by an electron through a potential difference of 1 V, so $1 \text{ eV} \simeq 1.6022 \times 10^{-19} \text{ J}$.

Step 1: (a) Distinguish isotopes from isobars Isotopes are atoms of the same element with the same atomic number Z but different mass numbers A . They have equal proton numbers and different neutron numbers. For example, ${}^1_1\text{H}$ and ${}^2_1\text{H}$ both have one proton, but contain zero and one neutron respectively.

Isobars are atoms of different elements with the same mass number A but different atomic numbers Z . For example, ${}^{14}_6\text{C}$ and ${}^{14}_7\text{N}$ each contain fourteen nucleons; their proton numbers are six and seven, and their neutron numbers are eight and seven. Thus isotopes share Z , whereas isobars share A .

Step 2: (b) Find the energy equivalent of one atomic mass unit Substitute the mass of one atomic mass unit into $E = mc^2$. Using $c \simeq 2.9979 \times 10^8 \text{ m s}^{-1}$,

$$\begin{aligned} E &= (1.6605 \times 10^{-27})(2.9979 \times 10^8)^2 \text{ J} \\ &\simeq 1.4924 \times 10^{-10} \text{ J}. \end{aligned}$$

This converts a mass into its equivalent rest energy; the atomic mass unit itself remains a unit of mass.

Step 3: (b) Convert joules into electron volts Divide by the number of joules in one electron volt:

$$\begin{aligned} E &= \frac{1.4924 \times 10^{-10}}{1.6022 \times 10^{-19}} \text{ eV} \\ &\simeq 9.315 \times 10^8 \text{ eV} \\ &= 931.5 \text{ MeV}. \end{aligned}$$

The conversion from electron volts to mega-electron volts uses $1 \text{ MeV} = 10^6 \text{ eV}$. Hence a mass defect measured in u can be converted into binding energy in MeV by multiplying its numerical value by 931.5.

Step 4: Conclusion Isotopes have equal Z and different A , for example ${}^1_1\text{H}$ and ${}^2_1\text{H}$; isobars have equal A and different Z , for example ${}^{14}_6\text{C}$ and ${}^{14}_7\text{N}$. The mass-energy conversion is

$$(1 \text{ u})c^2 \simeq 9.315 \times 10^8 \text{ eV} = 931.5 \text{ MeV},$$

or equivalently $1 \text{ u} \simeq 931.5 \text{ MeV}/c^2$.

Quick Tip to Solve: Check which nuclear number stays fixed: Z for isotopes, A for isobars. In the mass-energy conversion, retain c^2 so that mass and energy units are not confused.

27. (a) 'Current is a scalar quantity, although we represent current with an arrow.' Explain.

(b) Derive the balance condition for a Wheatstone Bridge.

[3 marks]

Solution:

Concept:

- Electric current is the signed rate of charge flow, $I = dQ/dt$. Currents add algebraically, whereas vectors combine by vector addition.
- Ohm's law gives a potential drop $V = IR$. A Wheatstone bridge is balanced when the galvanometer current is zero and its two terminals have equal potential.

Step 1: (a) Explain the meaning of the current arrow The arrow indicates the chosen direction of conventional positive-charge flow through a conductor. It fixes the sign of the scalar current; current in the opposite sense is negative. It does not make current a vector. At a junction, charge conservation gives the algebraic relation $\sum I_{\text{in}} = \sum I_{\text{out}}$, regardless of the angles between the wires. The currents are not resolved into spatial components or added by the parallelogram law. Current density $\vec{J}$ is a vector, but current through a chosen surface is the scalar $I = \int \vec{J} \cdot d\vec{A}$.

Step 2: (b) Define the four arms of the bridge Let the two branches between battery terminals A and C be ABC and ADC . Put resistances P, Q in arms AB, BC , and R, S in arms AD, DC , respectively. Connect the galvanometer between B and D . At balance $I_g = 0$, so $V_B = V_D$. The same current I_1 flows through P, Q , and another current I_2 flows through R, S , since no current leaves either midpoint through the galvanometer.

Step 3: Equate the potential drops and divide The equal midpoint potentials give equal drops from A to each midpoint and from each midpoint to C :

$$\begin{aligned} V_A - V_B &= V_A - V_D \Rightarrow I_1 P = I_2 R, \\ V_B - V_C &= V_D - V_C \Rightarrow I_1 Q = I_2 S. \end{aligned}$$

Dividing the first equation by the second eliminates both branch currents:

$$\frac{I_1 P}{I_1 Q} = \frac{I_2 R}{I_2 S}.$$

Step 4: Conclusion (a) Current is a scalar because its arrow specifies a flow convention and currents obey algebraic addition. (b) With the arms defined above, the Wheatstone-bridge balance condition is

$$\boxed{P/Q = R/S}, \text{ or } PS = QR.$$

Quick Tip to Solve: At bridge balance first write equal midpoint potentials; divide the two equal-drop equations to eliminate the unknown currents.

28. Two coils, one of radius 0.5 cm having 10 turns and the other of radius 5 cm having 50 turns are placed coaxially in air such that their centres are coincident. Calculate:

- (a) the magnetic flux through the smaller coil if the larger coil carries a current of 3 A, and
(b) the mutual inductance of the two coils.

[3 marks]

Solution:

Concept:

- The field at the centre of a circular coil is $B = \mu_0 NI / (2R)$, where N is its number of turns, I its current and R its radius.
- For a nearly uniform field perpendicular to a turn of area A , its flux is $\phi = BA$. For a coil of n turns, total flux linkage is $n\phi = MI$, which defines mutual inductance M .

Step 1: Convert the radii and identify the field-producing coil The larger coil produces the field. Its radius, turns and current are $R = 5 \times 10^{-2}$ m, $N = 50$ and $I = 3$ A. The receiving smaller coil has $r = 5 \times 10^{-3}$ m and $n = 10$. Because $r/R = 0.1$, use the standard approximation that the field at the common centre is uniform across the smaller coil. Their common axis is normal to both coil planes, so the flux factor is $\cos 0^\circ = 1$.

Step 2: (a) Calculate the field and the linked flux In air take $\mu_0 = 4\pi \times 10^{-7}$ H m⁻¹. Then

$$B = \frac{(4\pi \times 10^{-7})(50)(3)}{2(5 \times 10^{-2})} = 6\pi \times 10^{-4} \text{ T.}$$

The area of each smaller turn is $A = \pi r^2 = 2.5\pi \times 10^{-5}$ m². Thus the flux through each turn is

$$\phi = BA = 15\pi^2 \times 10^{-9} \text{ Wb} \simeq 1.48 \times 10^{-7} \text{ Wb.}$$

The total flux linked with the ten-turn smaller coil is

$$n\phi = 10(1.48 \times 10^{-7}) = 1.48 \times 10^{-6} \text{ Wb-turn.}$$

Stating both quantities distinguishes the flux through one turn from the total linkage of the coil.

Step 3: (b) Divide the total linkage by the source current Mutual inductance includes the receiving coil's number of turns. Therefore

$$M = \frac{n\phi}{I} = \frac{1.48 \times 10^{-6}}{3} \simeq 4.93 \times 10^{-7} \text{ H.}$$

Equivalently, substitution before rounding gives $M = \mu_0 N n \pi r^2 / (2R)$. The current cancels, as expected for air coils with fixed geometry.

Step 4: Conclusion (a) Flux through each smaller-coil turn is $1.48 \times 10^{-7} \text{ Wb}$; total linked flux of its ten turns is $1.48 \times 10^{-6} \text{ Wb-turn}$. (b) The mutual inductance is $4.93 \times 10^{-7} \text{ H} = 0.493 \mu\text{H}$.

Quick Tip to Solve: Use the large coil's radius in the magnetic-field formula, the small coil's area in the flux, and multiply by receiving turns before finding mutual inductance.

Section D (8 marks)

Questions number 29 and 30 are Case Study-based questions. Read the following paragraphs and answer the questions that follow.

29. An astronomical telescope consists of two converging lenses. One of them of large aperture and large focal length is called objective lens and the other one, of smaller focal length and smaller aperture is called the eyepiece. It is used to see distant objects which are not seen clearly with naked eyes. The image formed by the objective lens acts as an object for the eyepiece and the final image produced by the eyepiece is magnified. [4 marks]

29(i). The images formed by the objective lens and the eyepiece are respectively: [1 mark]

- (A) virtual, real
- (B) real, virtual
- (C) virtual, virtual
- (D) real, real

Correct Answer: (B) real, virtual

Solution:

Concept:

- A converging objective forms a real image of a distant object near its focal plane, where the refracted rays actually meet.
- The eyepiece acts as a magnifier of this intermediate image and provides a virtual final image, at infinity in normal adjustment or at a finite near point when adjusted accordingly.

Step 1: Identify the objective image The rays from a distant astronomical object enter the objective nearly parallel. The objective converges them to a real, inverted intermediate image. Thus options (A) and (C), which describe the objective image as virtual, are incorrect.

Step 2: Identify the eyepiece image The intermediate image is placed at the eyepiece's focal plane for normal adjustment, or just inside its focal length for a finite virtual final image. The eyepiece sends rays into the observer's eye without forming a real final image on the outgoing side. Therefore the final image is virtual, eliminating option (D).

Step 3: Conclusion The objective and eyepiece form real and virtual images, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Follow the two stages in order: the objective makes a real intermediate image, then the eyepiece magnifies it for viewing.

29(ii). The magnification produced by the telescope does not depend upon the:

[1 mark]

- (A) colour of light
- (B) focal length of objective lens
- (C) focal length of eyepiece
- (D) apertures of objective lens and eyepiece

Correct Answer: (D) apertures of objective lens and eyepiece

Solution:

Concept:

- In normal adjustment, the magnitude of a telescope's angular magnifying power is $M = f_o/f_e$, where f_o and f_e are the objective and eyepiece focal lengths.
- The lens apertures determine the admitted light and influence resolution, but do not enter the angular magnification formula.

Step 1: Identify the variables controlling magnification The expression $M = f_o/f_e$ explicitly contains both focal lengths. Increasing f_o increases magnification, whereas increasing f_e decreases it. Hence neither (B) nor (C) is a quantity on which magnification does not depend. In refracting lenses, focal lengths can depend on colour through dispersion; a general independence from colour is not guaranteed, so (A) is not the intended general statement.

Step 2: Separate aperture effects from image enlargement Changing the diameters of the objective and eyepiece while keeping their focal lengths fixed changes how much light passes through them. It does not change the ratio of viewing angles defining the telescope's magnification. Thus the apertures are absent from the magnifying-power expression, unlike the focal lengths.

Step 3: Conclusion The magnification does not depend upon the apertures of the objective lens and eyepiece. Hence, the correct option is (D).

Quick Tip to Solve: Write $M = f_o/f_e$ first: focal lengths control magnification, while aperture mainly controls light collection and resolution.

29(iii). Which of the following statements is not correct for this telescope?

[1 mark]

- (A) The focal length of objective lens (f_o) is larger than the focal length of eyepiece (f_e).
- (B) Its magnifying power can be increased by increasing the focal length of objective lens (f_o).
- (C) The distance between two lenses is more than ($f_o + f_e$).
- (D) The magnifying power can be decreased by increasing the focal length of eyepiece.

Correct Answer: (C) The distance between two lenses is more than ($f_o + f_e$).

Solution:

Concept:

- In normal adjustment an astronomical telescope has lens separation $L = f_o + f_e$ and angular magnifying power of magnitude $M = f_o/f_e$.
- For a final virtual image at the near point, the intermediate image lies inside the eyepiece's focal length, so the separation is less than $f_o + f_e$.

Step 1: Test the statements about focal lengths and magnification The objective has a longer focal length than the eyepiece, so (A) agrees with the telescope described. Since $M = f_o/f_e$, increasing f_o

increases magnifying power and increasing f_e decreases it when the other quantity is fixed. Thus (B) and (D) are also correct statements.

Step 2: Test the statement about lens separation For a distant object, the objective image forms at distance f_o behind it. In normal adjustment this intermediate image must be f_e in front of the eyepiece, giving $L = f_o + f_e$, not a larger distance. When the final virtual image is brought to the near point, the eyepiece is moved closer to the intermediate image and $L < f_o + f_e$. Therefore the claim in (C) fails for the usual viewing adjustments.

Step 3: Conclusion The incorrect statement is that the distance between the lenses is more than $f_o + f_e$. Hence, the correct option is (C).

Quick Tip to Solve: For normal telescope adjustment, add the focal lengths for tube length and divide them for magnifying power; note that this question asks for the incorrect statement.

29(iv)(a). An astronomical telescope has objective lens and eyepiece of focal lengths 80 cm and 4 cm respectively. To view the image in normal adjustment, the lenses must be separated by a distance of: [1 mark]

- (A) 84 cm
- (B) 76 cm
- (C) 20 cm
- (D) 320 cm

Correct Answer: (A) 84 cm

Solution:

Concept:

- Normal adjustment means the final image is at infinity, so the light emerging from the eyepiece is parallel.
- The real objective image must lie at the eyepiece's first focal plane. For two converging lenses, the tube length is consequently $L = f_o + f_e$.

Step 1: Locate the intermediate image between the lenses For the distant object, the objective forms its image 80 cm behind itself. To produce parallel emergent rays, the eyepiece must lie another 4 cm beyond this image, with the intermediate image at its front focus. Both distances contribute positively to the separation.

Step 2: Add the two focal lengths and check the alternatives Substituting the given focal lengths gives

$$L = f_o + f_e = 80 + 4 = 84 \text{ cm.}$$

The difference 76 cm is inappropriate for this telescope with two converging lenses. The value 20 is the dimensionless focal-length ratio and gives magnifying power, not a length. The product 320 cm^2 also cannot represent the required lens separation, so (B), (C) and (D) do not apply.

Step 3: Conclusion The lenses must be separated by 84 cm in normal adjustment. Hence, the correct option is (A).

Quick Tip to Solve: In normal adjustment of a two-converging-lens telescope, add the focal lengths for separation; their ratio gives magnification.

29(iv)(b). Consider the telescope described in question (iv) (a). Its magnifying power in normal adjustment will be: [1 mark]

- (A) 320
- (B) 84
- (C) 76
- (D) 20

Correct Answer: (D) 20

Solution:

Concept:

- In normal adjustment, an astronomical telescope forms the final image at infinity; the intermediate image is at the common focal plane of its lenses.
- The magnitude of angular magnifying power is $M = f_o/f_e$, where f_o and f_e are the focal lengths of the objective and eyepiece.

Step 1: Identify the focal lengths Part (iv)(a) gives $f_o = 80$ cm and $f_e = 4$ cm. Both focal lengths are in the same unit, so their ratio is dimensionless. Normal adjustment requires the ratio formula without a least-distance-of-distinct-vision correction.

Step 2: Calculate the angular magnification Substituting the focal lengths gives

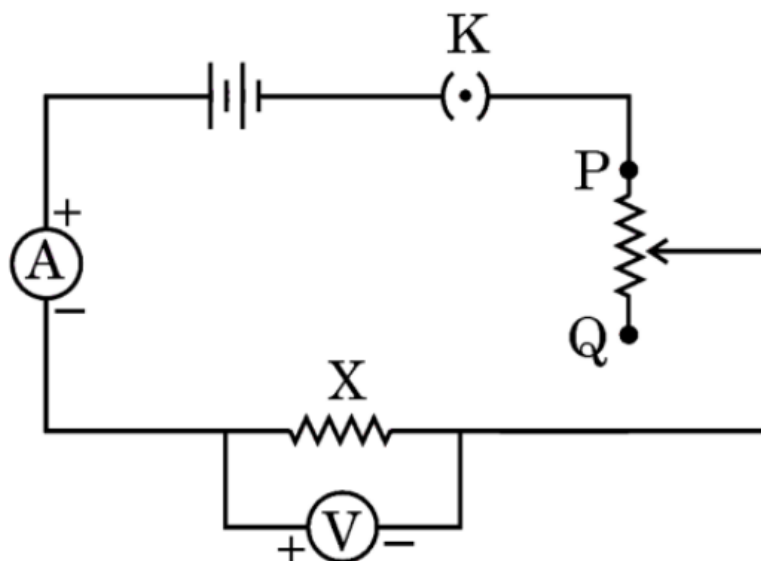
$$|M| = \frac{f_o}{f_e} = \frac{80}{4} = 20.$$

The signed magnification is -20 , indicating inversion; the positive options quote its magnitude. The values 84 and 76 come from adding or subtracting focal lengths and represent lengths, while 320 is their product; none is the required ratio.

Step 3: Conclusion The magnifying power in normal adjustment is 20. Hence, the correct option is (D).

Quick Tip to Solve: For normal adjustment, divide the objective focal length by the eyepiece focal length; their sum gives the telescope length instead.

30. A student sets up the circuit as shown in the figure to find the value of unknown resistance X and records a set of readings of the voltmeter and the ammeter by using the rheostat. [4 marks]



30(i). If resistor X were made of manganin and readings for V and I are taken without switching off the circuit, the graph between V and I will be as: [1 mark]



Correct Answer: (A)

Solution:

Concept:

- Ohm's law gives $V = IX$, where X is the resistance, V the potential difference across it and I the current through it.
- Manganin has a very small temperature coefficient of resistance, so its resistance remains nearly constant even when continuous current causes moderate heating.

Step 1: Assess the effect of keeping the circuit on Leaving the circuit switched on can heat the resistor. For manganin, the resulting change in resistance is negligible in this experiment, so the ratio $V/I = X$ stays nearly constant across the readings.

Step 2: Match the axes and shape In the printed options, I is on the vertical axis and V on the horizontal axis. Thus

$$I = \frac{1}{X}V.$$

A constant X gives a straight line through the origin with positive slope $1/X$, as in (A). Options (B), (C) and (D) are curved and therefore imply a varying resistance. In particular, the flattening in (C) would describe a substantial increase in resistance on heating, which manganin largely avoids.

Step 3: Conclusion The graph is approximately a straight line through the origin. Hence, the correct option is (A).

Quick Tip to Solve: Read the axes first: for this printed I -versus- V graph, the constant slope is $1/X$, not X .

30(ii). Error in the value of X obtained from different sets of voltmeter and ammeter readings, is: [1 mark]

- (A) due to error in voltmeter reading only.
- (B) due to error in ammeter reading only.
- (C) equal to the sum of error in voltmeter reading and error in ammeter reading.
- (D) equal to error in voltmeter reading divided by the error in ammeter reading.

Correct Answer: (C) equal to the sum of error in voltmeter reading and error in ammeter reading.

Solution:

Concept:

- The measured resistance is $X = V/I$; uncertainties in both the voltmeter reading V and the ammeter reading I affect the result.
- For a quotient, the maximum fractional errors add: $\Delta X/X = \Delta V/V + \Delta I/I$, to first order in small uncertainties.

Step 1: Apply the quotient error rule The largest possible inferred resistance occurs when the voltage is overestimated and the current underestimated. Expanding $(V + \Delta V)/(I - \Delta I)$ for small errors gives

$$\frac{\Delta X}{X} = \frac{\Delta V}{V} + \frac{\Delta I}{I}.$$

Equivalently, the percentage error in X is the sum of the percentage errors in V and I .

Step 2: Interpret and compare the printed choices Option (C) expresses this standard sum rule, with “error” understood as fractional or percentage error. Absolute errors measured in volts and amperes cannot literally be added. Options (A) and (B) each omit one source of uncertainty. Option (D) incorrectly divides the uncertainties merely because the measured quantities themselves form a quotient.

Step 3: Conclusion The maximum fractional (or percentage) errors in the voltmeter and ammeter readings add. With this standard interpretation, the correct option is (C).

Quick Tip to Solve: For $X = V/I$, convert both meter uncertainties to fractions or percentages before adding them; never add volts to amperes.

30(iii). If the movable end of rheostat is moved towards P, then:

[1 mark]

- (A) reading in ammeter decreases and reading in voltmeter increases.
- (B) readings in both voltmeter and ammeter increase.
- (C) reading in ammeter increases and reading in voltmeter decreases.
- (D) readings in both voltmeter and ammeter decrease.

Correct Answer: (B) readings in both voltmeter and ammeter increase.

Solution:

Concept:

- A rheostat used with one fixed terminal and its slider inserts only the resistance between that fixed terminal and the slider into the circuit.
- With a fixed supply, reducing series resistance increases the current. Across the fixed unknown resistance X , the voltmeter measures $V_X = IX$.

Step 1: Identify the active rheostat length In the printed circuit, terminal P is connected to the supply through the key, and the slider is connected to the resistor circuit. Terminal Q is not connected externally. Moving the slider towards P shortens the resistive length between P and the slider, so the inserted rheostat resistance decreases.

Step 2: Track both meter readings Since the total series resistance decreases, the circuit current rises and the ammeter reading increases. The resistance X is unchanged, so the larger current produces a larger potential difference $V_X = IX$; the voltmeter reading also rises. Options (A) and (C) give opposite changes although V_X is proportional to current. Option (D) would apply to increasing the active rheostat length, the reverse of the specified motion.

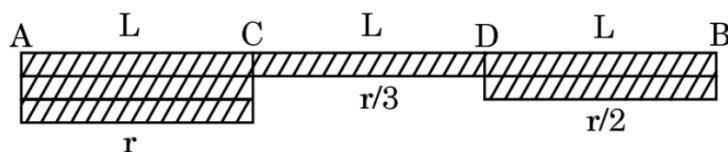
Step 3: Conclusion The readings of both the ammeter and the voltmeter increase. Hence, the correct option is (B).

Quick Tip to Solve: Trace the connected fixed terminal before moving a rheostat slider: here only the length from P to the slider sets its resistance.

30(iv)(a). Suppose the unknown resistance X is replaced by a wire made of the same metal. This wire consists of three parts, of the same length L but has radii r , $r/3$ and $r/2$ as shown in the figure.

For a particular setting of the rheostat, let v_1 , v_2 and v_3 be the value of drift velocities in parts AC, CD and DB. Then:

[1 mark]



(A) $v_1 > v_2 > v_3$

(B) $v_2 > v_3 > v_1$

(C) $v_3 > v_2 > v_1$

(D) $v_1 = v_2 = v_3$

Correct Answer: (B) $v_2 > v_3 > v_1$

Solution:

Concept:

- In a steady current through a continuous wire, the same current I flows through every cross-section; charge does not accumulate at the junctions.
- The magnitude of drift velocity is $v_d = I/(neA)$, where n is the free-electron number density, e the electron charge magnitude and A the cross-sectional area.

Step 1: Compare the cross-sectional areas All three parts are made of the same metal, so they have the same n . Their radii in the printed figure are r , $r/3$ and $r/2$ for AC, CD and DB respectively. Thus

$$A_1 = \pi r^2, \quad A_2 = \frac{\pi r^2}{9}, \quad A_3 = \frac{\pi r^2}{4}.$$

Step 2: Use current continuity to compare speeds Since I , n and e are common, drift speed varies inversely with area:

$$v_1 : v_2 : v_3 = \frac{1}{A_1} : \frac{1}{A_2} : \frac{1}{A_3} = 1 : 9 : 4.$$

The narrowest part CD therefore has the greatest drift speed. Option (A) wrongly makes the widest part fastest; (C) reverses the order of the two narrower parts. Option (D) confuses equal current with equal drift speed when the areas differ. The equal lengths do not affect this comparison.

Step 3: Conclusion The drift speeds satisfy $v_2 > v_3 > v_1$. Hence, the correct option is (B).

Quick Tip to Solve: For successive parts of one metal wire, square each radius and invert: radii $r, r/3, r/2$ give drift-speed ratio $1 : 9 : 4$.

30(iv)(b). Consider the same wire, as shown in figure in question (iv) (a) connected in place of X. For a particular setting of rheostat, let E_1 , E_2 and E_3 be the value of electric fields in part AC, CD and DB. Then: [1 mark]

- (A) $E_1 = E_2 = E_3$
 (B) $E_3 > E_2 > E_1$
 (C) $E_2 > E_3 > E_1$
 (D) $E_1 > E_2 > E_3$

Correct Answer: (C) $E_2 > E_3 > E_1$

Solution:

Concept:

- Current continuity gives the same steady current I in all three successive parts of the wire, but the current density is $J = I/A$, where A is the cross-sectional area.
- The microscopic form of Ohm's law is $E = \rho J$, where ρ is resistivity. For the same metal at the same temperature, ρ is common, so $E = \rho I/A$.

Step 1: Find the areas in the printed wire The radii of AC, CD and DB are respectively r , $r/3$ and $r/2$. Therefore their cross-sectional areas are

$$A_1 = \pi r^2, \quad A_2 = \frac{\pi r^2}{9}, \quad A_3 = \frac{\pi r^2}{4}.$$

The common length does not imply equal resistance, because resistance also depends inversely on area.

Step 2: Compare the electric fields For common current and resistivity, the field varies inversely with cross-sectional area:

$$E_1 : E_2 : E_3 = \frac{1}{A_1} : \frac{1}{A_2} : \frac{1}{A_3} = 1 : 9 : 4.$$

The narrowest part CD needs the largest field to carry the same current. Option (A) confuses a common current with a common field. Option (B) makes the intermediate-width part DB have the largest field, and (D) makes the widest part AC have the largest field; both contradict $E \propto 1/A$.

Step 3: Conclusion The electric fields satisfy $E_2 > E_3 > E_1$. Hence, the correct option is (C).

Quick Tip to Solve: In a stepped wire of one metal, both drift speed and electric field vary as $1/r^2$; the smallest radius gives the largest value.

Section E (15 marks)

31(a). (i) Define the terms (I) resonant frequency, and (II) power factor of a series LCR circuit. For what value of the power factor will the power dissipated in the circuit be maximum?

(ii) An inductor of $\frac{5}{\pi}$ H, a capacitor of $\frac{50}{\pi}$ μ F and a resistor of 400Ω are connected in series across an ac voltage $v = 140 \sin(100\pi t)$ V. Calculate:

(I) impedance of the circuit, and

(II) rms value of current that flows in the circuit.

(Take $\sqrt{2} = 1.4$)

[5 marks]

Solution:

Concept:

- In a series LCR circuit, the inductive and capacitive reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$, where ω is the angular frequency of the supply.
- The impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. The resistor voltage is in phase with current, while the net reactive voltage is perpendicular to it in the phasor diagram.
- The power factor is $\cos \phi = R/Z$, where ϕ is the phase difference between supply voltage and current. The average power is $P = V_{\text{rms}} I_{\text{rms}} \cos \phi$.
- For sinusoidal voltage and current, $V_{\text{rms}} = V_0/\sqrt{2}$ and $I_{\text{rms}} = V_{\text{rms}}/Z$, where V_0 is the voltage amplitude.

Step 1: Define the resonant frequency for part (i)(I) The resonant frequency is the frequency of the applied ac supply at which the inductive and capacitive reactances are equal. The net reactance then vanishes, the impedance has its minimum value R , and the current amplitude is maximum for a fixed supply voltage. Setting the two reactances equal gives

$$\begin{aligned}\omega_0 L &= \frac{1}{\omega_0 C}, \\ \omega_0 &= \frac{1}{\sqrt{LC}}, \quad f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}.\end{aligned}$$

Here f_0 is the resonant frequency in hertz, L is the inductance and C is the capacitance.

Step 2: Define the power factor for part (i)(II) The power factor is the cosine of the phase difference between the applied voltage and the current. Equivalently, it is the ratio of average power to the product of rms voltage and rms current:

$$\cos \phi = \frac{P}{V_{\text{rms}} I_{\text{rms}}} = \frac{R}{Z}.$$

Its maximum value is unity, reached when $\phi = 0$, so voltage and current are in phase. For a fixed rms supply voltage and resistance,

$$P = \frac{V_{\text{rms}}^2 R}{R^2 + (X_L - X_C)^2}.$$

Thus the power dissipated is maximum when $X_L = X_C$, which gives $Z = R$ and $\cos \phi = 1$. Ideal inductors and capacitors store and return energy; the average dissipation occurs in the resistance.

Step 3: Extract the supply parameters and calculate reactances Compare $v = 140 \sin(100\pi t)$ V with

$v = V_0 \sin \omega t$. Therefore $V_0 = 140 \text{ V}$ and $\omega = 100\pi \text{ rad s}^{-1}$. The printed component values are

$$L = \frac{5}{\pi} \text{ H}, \quad C = \frac{50}{\pi} \times 10^{-6} \text{ F}, \quad R = 400 \Omega.$$

Use angular frequency directly in the reactance formulae:

$$X_L = 100\pi \times \frac{5}{\pi} = 500 \Omega,$$
$$X_C = \frac{1}{100\pi \times (50/\pi) \times 10^{-6}} = 200 \Omega.$$

The net reactance is $X_L - X_C = 300 \Omega$. It is positive, so this operating circuit is inductive and is not at resonance.

Step 4: Find the impedance for part (ii)(I) Inductive and capacitive reactances oppose each other, whereas resistance and net reactance combine in quadrature. Hence their magnitudes must not all be added arithmetically:

$$Z = \sqrt{400^2 + (500 - 200)^2}$$
$$= \sqrt{160000 + 90000} = 500 \Omega.$$

The operating power factor is consequently $R/Z = 400/500 = 0.8$, consistent with a nonzero reactive component.

Step 5: Find the rms current for part (ii)(II) The coefficient 140 is the peak voltage, not the rms voltage. Using the supplied approximation $\sqrt{2} = 1.4$, first calculate

$$V_{\text{rms}} = \frac{140}{1.4} = 100 \text{ V}.$$

Ohm's law for the complete ac circuit then gives

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{100}{500} = 0.20 \text{ A}.$$

As a check, the peak current is $I_0 = 140/500 = 0.28 \text{ A}$; dividing it by 1.4 also gives the same rms value.

Step 6: Conclusion The resonant frequency is $f_0 = 1/(2\pi\sqrt{LC})$, where $X_L = X_C$. The power factor is $\cos \phi = R/Z$; maximum power dissipation occurs at $\cos \phi = 1$. For the given supply, the circuit impedance is 500Ω and its rms current is 0.20 A .

Quick Tip to Solve: Cancel the printed π factors before evaluating the reactances. The resistance and net reactance form a 400–300–500 triangle; then convert the 140 V peak supply to its 100 V rms value.

31(b). (i) Draw a labelled diagram of a step-up transformer. Obtain the ratio of secondary voltage to primary voltage in terms of number of turns in the two coils.

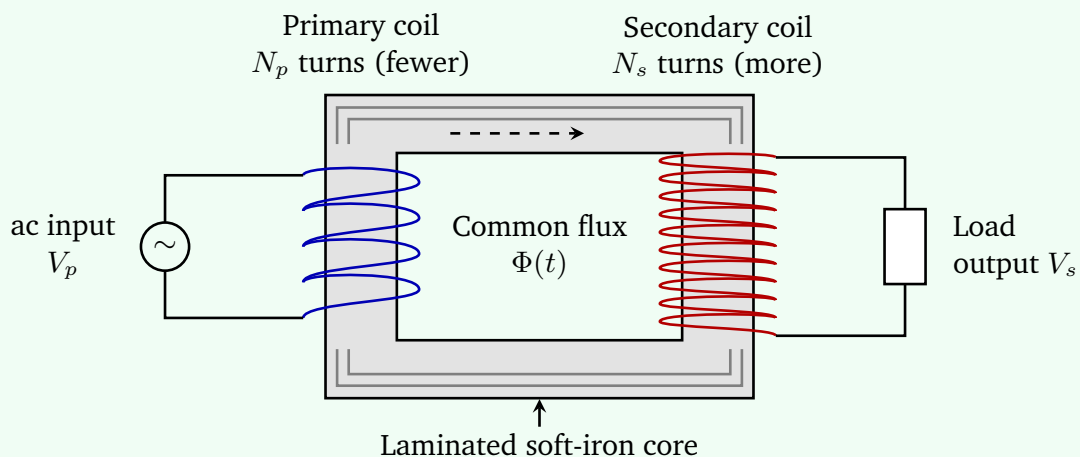
(ii) The number of turns in the primary and the secondary coil of an ideal transformer are 100 and 5000 respectively. If 3.3 kW power is supplied to the transformer at 220 V, find (I) current in the primary coil, and (II) output voltage. [5 marks]

Solution:

Concept:

- A transformer works on mutual induction: an alternating current in the primary produces a changing magnetic flux in a common iron core, inducing an emf in the secondary.
- Faraday's law gives $e = -N d\Phi/dt$, where N is the coil's number of turns and Φ is the magnetic flux through each turn. In an ideal transformer, both coils link the same flux.
- For an ideal transformer, the rms voltage ratio is $V_s/V_p = N_s/N_p$. A step-up transformer has $N_s > N_p$. With no power loss, $P_{in} = P_{out} = V_p I_p = V_s I_s$ for the usual resistive-load numerical.

Step 1: (i) Construction and labelled diagram The primary coil has fewer turns N_p , while the secondary coil has more turns N_s . Both insulated coils are wound on a common laminated soft-iron core. The primary is connected to an ac supply and the secondary supplies the load. The core provides a path for the changing magnetic flux linking the two coils; lamination reduces eddy-current losses in a practical transformer. The sketch shows the difference in winding counts schematically.



Step-up transformer: fewer primary turns and more secondary turns on a common laminated soft-iron core.

Step 2: (i) Derive the voltage ratio using the common flux Let $\Phi(t)$ be the instantaneous flux through each turn. Neglecting flux leakage, the same flux links each primary and secondary turn. With the same winding-reference convention, Faraday's law gives

$$e_p = -N_p \frac{d\Phi}{dt}, \quad e_s = -N_s \frac{d\Phi}{dt}.$$

Therefore, the induced-emf magnitudes have the ratio $|e_s|/|e_p| = N_s/N_p$ whenever the flux is changing. The rms emfs obey the same ratio. For sinusoidal flux $\Phi = \Phi_m \sin \omega t$, this also follows explicitly from

$$E_p = \frac{N_p \omega \Phi_m}{\sqrt{2}}, \quad E_s = \frac{N_s \omega \Phi_m}{\sqrt{2}}.$$

In the ideal transformer, winding resistance and leakage are negligible, so the primary applied rms

voltage equals the primary induced-emf magnitude and the secondary terminal rms voltage equals its induced emf. Hence $V_s/V_p = E_s/E_p = N_s/N_p$. Since $N_s > N_p$, the secondary voltage is greater than the primary voltage. The increased voltage is accompanied by a smaller available secondary current, consistent with energy conservation.

Step 3: (ii)(I) Calculate the current in the primary coil The given input power is $P_{\text{in}} = 3.3 \text{ kW} = 3300 \text{ W}$, and the primary rms voltage is $V_p = 220 \text{ V}$. Using the ideal-transformer power relation intended in this numerical,

$$I_p = \frac{P_{\text{in}}}{V_p} = \frac{3300}{220} \text{ A} = 15 \text{ A}.$$

The factor of 1000 in converting kilowatts to watts is necessary before dividing by the voltage.

Step 4: (ii)(II) Calculate the output voltage and check the result Here $N_p = 100$ and $N_s = 5000$, giving a turns ratio of $N_s/N_p = 50$. Thus

$$V_s = V_p \frac{N_s}{N_p} = 220 \times \frac{5000}{100} \text{ V} = 11000 \text{ V}.$$

As a check, the corresponding ideal secondary current is $I_s = I_p/50 = 0.30 \text{ A}$, so $V_s I_s = 11000 \times 0.30 = 3300 \text{ W}$, equal to the supplied power. The voltage rises by the same factor by which the current falls.

Step 5: Conclusion The transformer is step-up because $N_s > N_p$, and its voltage ratio is $V_s/V_p = N_s/N_p$. For the given ideal transformer, (I) $I_p = 15 \text{ A}$ and (II) $V_s = 11000 \text{ V} = 11 \text{ kV}$.

Quick Tip to Solve: First form the turns ratio $5000/100 = 50$: multiply the input voltage by 50, but use $3300/220$ directly for the primary current.

32(a). (i) Explain the following statements giving reason:

(I) An equipotential surface through a point is normal to the electric field at that point.

(II) When a dielectric is placed in an external electric field, the electric field inside the dielectric is less than that outside it.

(III) The potential difference between the plates of a charged parallel plate capacitor decreases when its plates are brought closer.

(ii) Obtain an expression for the work done to dissociate the system of three charges q , $-4q$ and $2q$ placed at the vertices A, B and C respectively of an equilateral triangle of side 'a'. [5 marks]

Solution:

Concept:

- Along an infinitesimal displacement $d\vec{l}$, the potential change is $dV = -\vec{E} \cdot d\vec{l}$. An equipotential surface has constant potential.
- Polarisation of a dielectric produces bound charges whose electric field opposes the applied field, reducing the resultant field inside it.
- For a parallel plate capacitor, $C = \epsilon_0 A/d$ and $V = Q/C$, where A is the plate area, d is their

separation and Q is the charge on either plate in magnitude.

- The electrostatic potential energy of point charges is the sum over distinct pairs, $U = \sum_{i < j} k q_i q_j / r_{ij}$, where $k = 1/(4\pi\epsilon_0)$. Quasistatic external work is the change in potential energy.

Step 1: (i)(I) Relate the field to a tangent displacement Take any small displacement $d\vec{l}$ along the equipotential surface through the point. Since both ends have the same potential,

$$dV = 0 \Rightarrow \vec{E} \cdot d\vec{l} = 0.$$

Thus the electric field has no component along any tangent to the surface. Wherever the field is nonzero, it must therefore be normal to the equipotential surface. Otherwise a test charge displaced along the surface would experience nonzero work, contradicting the constant potential.

Step 2: (i)(II) Account for the induced field in the dielectric In the applied field $\vec{E}_0$, the positive and negative charges within the dielectric molecules shift slightly in opposite directions, or existing molecular dipoles tend to align. This polarisation produces bound charges at its surfaces. Their field $\vec{E}_p$ opposes the applied field inside the dielectric, so

$$\vec{E} = \vec{E}_0 + \vec{E}_p, \quad E = E_0 - E_p < E_0.$$

For the usual uniformly filled parallel plate arrangement, a linear dielectric of relative permittivity $K > 1$ gives $E = E_0/K$. The reduction is due to polarisation, not free conduction through the dielectric.

Step 3: (i)(III) Keep the capacitor charge fixed The statement concerns a charged capacitor isolated from a voltage supply, so its charge Q remains constant. Bringing the plates closer decreases d , which increases the capacitance:

$$C = \frac{\epsilon_0 A}{d}, \quad V = \frac{Q}{C} = \frac{Qd}{\epsilon_0 A}.$$

Hence the potential difference decreases in direct proportion to separation. Equivalently, the ideal field $Q/(\epsilon_0 A)$ stays fixed while $V = Ed$ decreases. If a battery remained connected, it would instead maintain the potential difference and change the charge.

Step 4: (ii) Add the energies of the three charge pairs Every pair of vertices is separated by a . With $q_A = q$, $q_B = -4q$ and $q_C = 2q$, count the pairs AB, BC and CA once each:

$$\begin{aligned} U_i &= \frac{k}{a} [q(-4q) + (-4q)(2q) + (2q)q] \\ &= \frac{kq^2}{a} (-4 - 8 + 2) = -\frac{10kq^2}{a}. \end{aligned}$$

The two unlike-charge pairs contribute negative energy; the like-charge pair contributes positive energy. During complete dissociation all three mutual separations become infinite, making every pair energy zero: $U_f = 0$. For a slow separation with no change in kinetic energy,

$$W_{\text{ext}} = U_f - U_i = 0 - \left(-\frac{10kq^2}{a} \right).$$

Step 5: Conclusion The field is normal to an equipotential surface, polarisation reduces the dielectric's internal field, and decreasing the separation of an isolated charged capacitor lowers its voltage. The external work required to dissociate the three charges is

$$W_{\text{ext}} = \frac{10kq^2}{a} = \frac{10q^2}{4\pi\epsilon_0 a}.$$

It is positive; the electrostatic field does the corresponding negative work.

Quick Tip to Solve: For three charges, list AB, BC and CA exactly once and keep the charge signs. Dissociation work is minus the initial total potential energy, since the final energy is zero.

OR

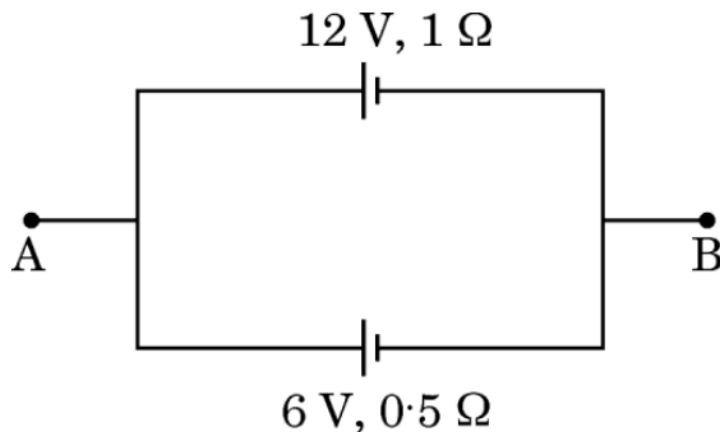
32(b). (i) Answer the following giving reason:

(I) The electron drift speed is estimated to be only a few mm/s for currents in the range of a few amperes. How, then, is the current established almost the instant a circuit is closed?

(II) A low voltage supply from which one needs high currents must have very low internal resistance. Why?

(III) The assertion that $V = IR$ is a statement of Ohm's law is not true. Why?

(ii) Two cells of emfs 12 V and 6 V are connected in parallel as shown in the figure. Their internal resistances are $1\ \Omega$ and $0.5\ \Omega$ respectively. Calculate the emf and internal resistance of the equivalent cell between points A and B. [5 marks]



Solution:

Concept:

- Electron drift speed is the small average velocity due to an electric field. The electromagnetic signal establishing that field propagates through a circuit far faster than individual electrons drift.
- A cell with emf $\mathcal{E}$ and internal resistance r supplies terminal voltage $V = \mathcal{E} - Ir$ when delivering current I ; its internal heating rate is I^2r .
- Ohm's law requires $V \propto I$ with constant resistance when temperature and other physical conditions remain fixed. Merely defining $R = V/I$ does not establish this proportionality.

- For cells connected in parallel with like poles together, Kirchhoff's laws give

$$\mathcal{E}_{\text{eq}} = \frac{\mathcal{E}_1/r_1 + \mathcal{E}_2/r_2}{1/r_1 + 1/r_2}$$

and

$$r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2}.$$

Step 1: (i)(I) Distinguish drift from signal propagation Conduction electrons already exist throughout the metallic circuit. When the switch closes, an electromagnetic disturbance travels along the circuit at a speed comparable to the speed of light, depending on its surrounding medium and geometry. It establishes the electric field at distant portions after a very short propagation delay. The electrons already present there then acquire directed drift motion locally. Thus current does not wait for an electron to travel all the way from the battery through the circuit. A drift speed of a few mm s^{-1} is compatible with the almost immediate establishment of current in an ordinary small circuit; the establishment is very rapid, not literally instantaneous.

Step 2: (i)(II) Limit the internal voltage drop For a source delivering current, the useful terminal voltage is

$$V = \mathcal{E} - Ir.$$

If the required current I is large while the source voltage is small, even a moderate internal resistance would produce an appreciable drop Ir , leaving too little voltage across the external load. Therefore r must be very small, so that $Ir \ll \mathcal{E}$. This also limits the power $I^2 r$ converted into heat inside the source and allows efficient delivery of a large current.

Step 3: (i)(III) State the essential restriction in Ohm's law The relation $V = IR$, by itself, can be written at an operating point simply by defining the resistance as $R = V/I$. Such a ratio may change when the applied voltage or current changes, as it does for a non-ohmic element. Ohm's law makes the additional physical assertion that, at fixed temperature and other physical conditions, the current through an ohmic conductor is directly proportional to the potential difference across it. Thus V/I remains constant over its ohmic range, and the V -versus- I graph is a straight line through the origin. This constancy, absent from the bare equation, is the law's essential content.

Step 4: (ii) Obtain the emf from the open-circuit voltage The longer plate of each cell symbol faces A, so both positive terminals are connected to A. Let $V = V_A - V_B$, and define the current delivered by each branch towards A as I_1 and I_2 . The branch relations are

$$I_1 = \frac{\mathcal{E}_1 - V}{r_1}, \quad I_2 = \frac{\mathcal{E}_2 - V}{r_2}.$$

With no external load, their sum is zero. Therefore

$$\begin{aligned} \frac{\mathcal{E}_1 - V}{r_1} + \frac{\mathcal{E}_2 - V}{r_2} &= 0, \\ \mathcal{E}_{\text{eq}} = V &= \frac{\mathcal{E}_1/r_1 + \mathcal{E}_2/r_2}{1/r_1 + 1/r_2} \\ &= \frac{12/1 + 6/0.5}{1/1 + 1/0.5} \text{ V} = \frac{24}{3} \text{ V}. \end{aligned}$$

The unequal emfs cause an internal circulating current even with the output open. Here the branch currents are $(12 - 8)/1 = 4 \text{ A}$ and $(6 - 8)/0.5 = -4 \text{ A}$, consistently summing to zero.

Step 5: (ii) Determine the equivalent internal resistance For an external load drawing total current $I = I_1 + I_2$, adding the branch equations and rearranging gives

$$V = \mathcal{E}_{\text{eq}} - I \left(\frac{1}{1/r_1 + 1/r_2} \right).$$

Comparing this expression with the terminal equation of one cell shows that the internal resistances combine in parallel:

$$r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2} = \frac{1 \times 0.5}{1 + 0.5} \Omega = \frac{1}{3} \Omega.$$

Step 6: Conclusion Rapid field propagation explains the prompt current, a small internal resistance preserves the useful low terminal voltage at high current, and Ohm's law additionally requires constant V/I . The equivalent cell is

$$\mathcal{E}_{\text{eq}} = 8\text{V}, \quad r_{\text{eq}} = \frac{1}{3} \Omega \approx 0.33 \Omega,$$

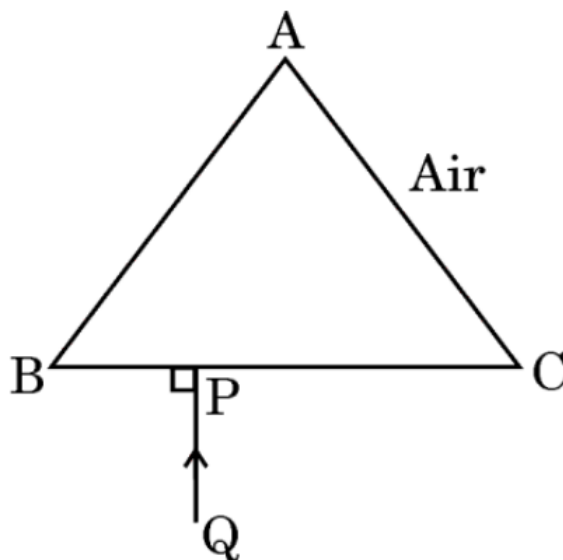
with A as its positive terminal and B as its negative terminal.

Quick Tip to Solve: Read the long plates first to determine cell polarity. For like poles in parallel, weight each emf by its conductance $1/r$; unequal emfs must not simply be averaged.

33(a). (i) Define refractive index of a medium in terms of speed of light.

(ii) Derive the relation for the refractive index (μ) of a prism in terms of angle of minimum deviation (δ_m) and angle of prism (A).

(iii) A ray of light QP is incident normally on the face BC of a triangular prism ABC of refractive index 1.5 kept in air, as shown in the figure. Trace the path of the ray as it passes through the prism and give relevant explanation. [5 marks]



Solution:

Concept:

- The absolute refractive index is $\mu = c/v$, where c is the speed of light in vacuum and v is its speed in the medium.

- For a prism in air, Snell's law gives $\mu = \sin i / \sin r_1$. The internal refraction angles satisfy $r_1 + r_2 = A$, and the deviation is $\delta = i + e - A$.
- At minimum deviation the light path through a prism is symmetric: $i = e$ and $r_1 = r_2$. For light going from the prism into air, total internal reflection occurs only when the internal incidence angle exceeds $\theta_c = \sin^{-1}(1/\mu)$.

Step 1: (i) Define refractive index using the speed of light The absolute refractive index of a medium is the ratio of the speed of light in vacuum to the speed of light in that medium:

$$\mu = \frac{c}{v}.$$

Since it is a ratio of two speeds, refractive index has no unit. A larger refractive index corresponds to a smaller speed of light in the medium.

Step 2: (ii) Relate the prism angles and the deviation Let i and e be the angles of incidence and emergence in air, and r_1 and r_2 the internal angles at the first and second refracting faces. All four angles are measured from the corresponding face normals. Geometry of the prism, whose refracting angle is A , gives

$$r_1 + r_2 = A.$$

The ray turns through $i - r_1$ at the first face and through $e - r_2$ at the second face. Hence the total deviation is

$$\begin{aligned}\delta &= (i - r_1) + (e - r_2) \\ &= i + e - (r_1 + r_2) = i + e - A.\end{aligned}$$

Step 3: (ii) Apply the minimum-deviation condition At minimum deviation, reversibility and symmetry of the ray path give equal external angles and equal internal angles. Therefore,

$$\begin{aligned}i &= e, & r_1 &= r_2 = \frac{A}{2}, \\ \delta_m &= 2i - A & \Rightarrow & i = \frac{A + \delta_m}{2}.\end{aligned}$$

Applying Snell's law at the first face, taking the refractive index of air as unity, gives

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}.$$

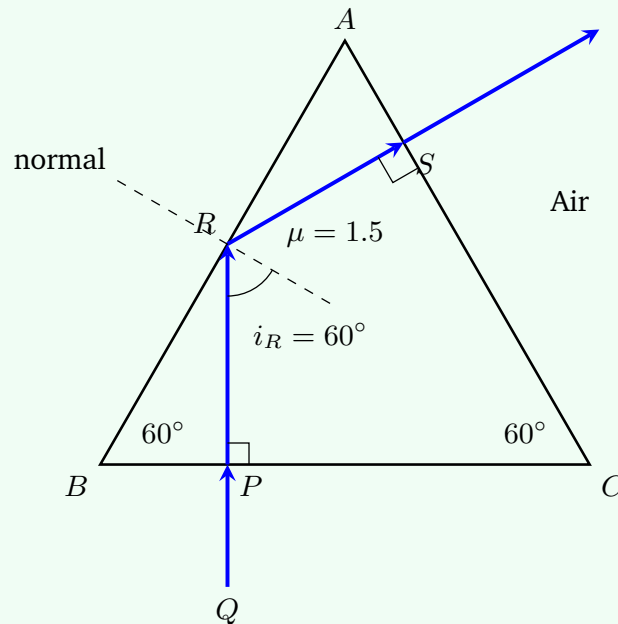
This relation uses the minimum deviation, not the deviation for an arbitrary incident ray.

Step 4: (iii) Trace entry at BC and calculate the critical angle The incident ray QP is normal to BC , so its incidence angle is zero. Snell's law gives zero refraction angle: the ray enters the prism along the same straight line and reaches the face AB at a point R . Its speed changes on entry even though its direction does not. For the prism-to-air boundary,

$$\sin \theta_c = \frac{1}{1.5} = \frac{2}{3}, \quad \theta_c \simeq 41.8^\circ.$$

Step 5: (iii) Identify the geometric information needed at AB The printed figure gives no angle values or equal-side markings. Thus a numerical incidence angle at AB , and consequently a unique complete ray path, cannot be deduced from the stated data alone. If $\beta = \angle ABC$, the vertical internal ray has incidence angle $i_R = \beta$ at AB . It is totally internally reflected when $\beta > 41.8^\circ$; when $\beta < 41.8^\circ$, it refracts out through AB , with $\sin e_R = 1.5 \sin \beta$. At equality the refracted ray grazes AB .

Step 6: (iii) Draw the ray path for the equilateral-prism interpretation If ABC is intended to be equilateral, explicitly assume $A = B = C = 60^\circ$. Then the ray strikes AB at $i_R = 60^\circ > \theta_c$, so it undergoes total internal reflection at R . The reflected ray also makes 60° with the normal to AB ; it travels toward AC , at 30° above BC . Since AC is inclined at 60° to BC , this ray is normal to AC . It reaches AC at S and emerges without changing direction. The diagram shows this conditional trace; the assumed angles are not additional data printed in the question.



Conditional ray trace for an equilateral prism: normal entry at P , total internal reflection at R , and normal emergence at S .

Step 7: Conclusion (i) $\mu = c/v$. (ii) $\mu = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}$. (iii) Entry at BC is undeviated. For an equilateral prism, the path is $Q \rightarrow P \rightarrow R \rightarrow S$: total internal reflection at AB , followed by normal emergence through AC . Without the missing prism-angle specification, that complete path is conditional.

Quick Tip to Solve: Always compare the angle to the face normal with θ_c , not the angle to the face. Check the prism's angle data before assuming an equilateral-prism ray path.

33(b). (i) What is the difference between a ray and a wavefront?

(ii) A plane wave is incident on a reflecting surface. Using Huygens principle, show how it is reflected from the surface. Hence, verify the law of reflection.

(iii) Depict refraction of a plane wave by a convex lens.

[5 marks]

Solution:

Concept:

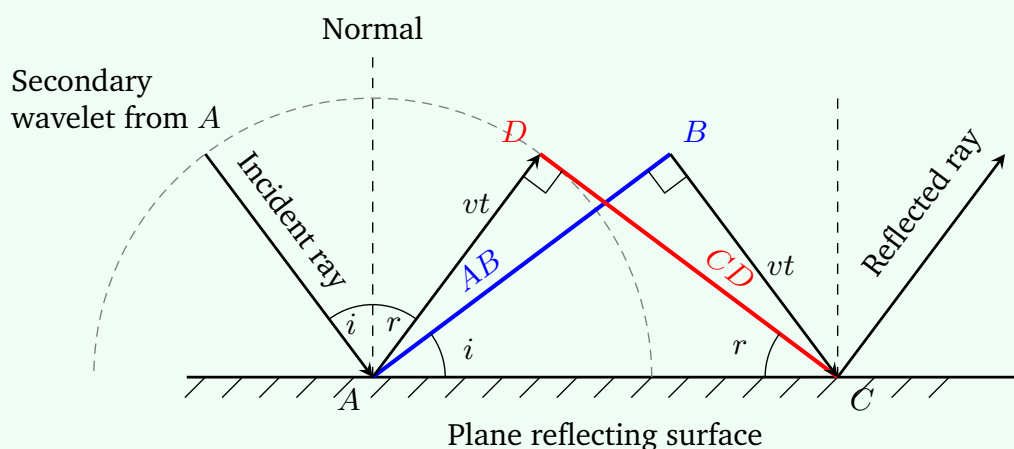
- A wavefront is the locus of all points vibrating in the same phase at a given instant. In an isotropic medium, a ray gives the direction of light propagation and is normal to the wavefront.
- Huygens' principle: each point on a wavefront acts as a source of secondary wavelets. The forward envelope of these wavelets gives the new wavefront after a time interval.
- In reflection the incident and reflected waves travel in the same medium with the same speed v . In a time t , each travels a distance vt .
- A convex lens introduces a greater optical delay near its thicker centre and converts an incident plane wavefront into a converging spherical wavefront, in the paraxial approximation.

Step 1: (i) Distinguish a ray from a wavefront A **ray** is an imaginary directed line representing the direction in which light energy travels. A **wavefront** is a surface joining points of equal phase; it may be plane, spherical or cylindrical. Thus a ray specifies a direction, whereas a wavefront specifies a surface of constant phase. In a homogeneous isotropic medium, rays are perpendicular to the wavefronts. For example, a parallel beam has parallel rays and plane wavefronts normal to those rays.

Step 2: (ii) Construct the reflected wavefront Let AC lie on a plane reflecting surface, and let AB be an incident plane wavefront. Point A reaches the surface first. In the time t taken by the disturbance at B to reach C along the incident ray, the secondary reflected wavelet from A grows to radius AD :

$$BC = vt, \quad AD = vt.$$

Draw the tangent CD from C to this wavelet. It is also the common tangent to the reflected secondary wavelets emitted by the intervening points of the surface, because those points are reached successively by the incident wave. Hence CD is the reflected plane wavefront. The incident rays are normal to AB , while the reflected rays are normal to CD ; in particular, $AD \perp CD$.



Huygens' construction: AB is incident, CD is reflected, and $BC = AD = vt$.

Step 3: (ii) Verify the law of reflection Let i and r be the angles made by the incident and reflected

rays with the normal to the surface. The angle between two lines equals the angle between their corresponding perpendiculars, so

$$\angle BAC = i, \quad \angle DCA = r.$$

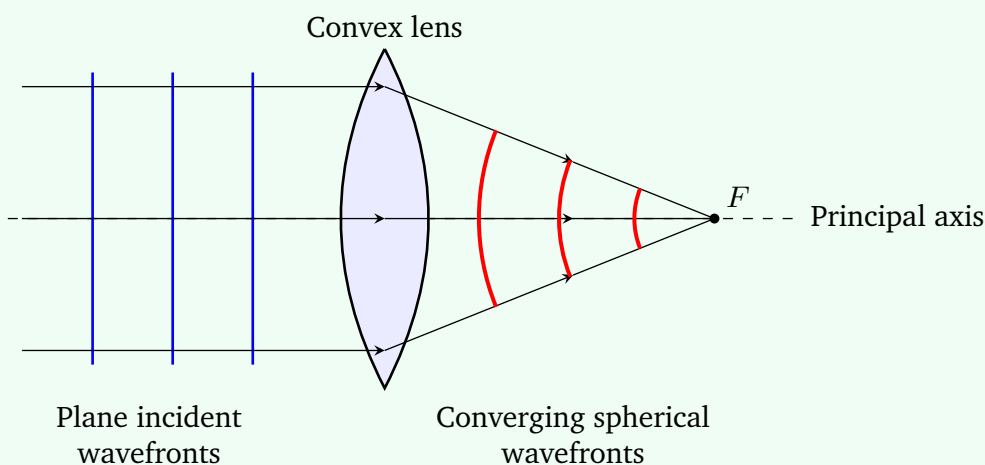
Since $AB \perp BC$ and $AD \perp DC$, the triangles ABC and ADC are right-angled and have the common hypotenuse AC . Therefore,

$$\sin i = \frac{BC}{AC} = \frac{vt}{AC},$$

$$\sin r = \frac{AD}{AC} = \frac{vt}{AC}.$$

Thus $\sin i = \sin r$, giving $i = r$ for the physical angles between 0° and 90° . The construction lies in the plane containing the incident ray and the surface normal; the reflected ray also lies in this plane. This verifies both statements of the law of reflection.

Step 4: (iii) Depict the wavefront after a convex lens Let plane wavefronts fall normally on a thin convex lens with its principal axis parallel to the incident rays. Light travels more slowly in the lens material than in air. The central part of each wavefront crosses more glass than the edges, so it suffers a greater delay. After emergence the wavefront is curved, with its concave side facing the principal focus F . In the paraxial approximation the emergent wavefronts are portions of spheres centred at F . Their normals are converging rays, which meet at F . The figure shows cross-sections of the plane incident wavefronts and the converging spherical wavefronts; the arrows show the propagation direction.



A convex lens converts plane wavefronts into spherical wavefronts converging towards F .

Step 5: Conclusion A ray indicates the direction of propagation and is normal to a wavefront, which joins points of equal phase. Huygens' construction gives a reflected plane wavefront and proves $i = r$, with the incident ray, reflected ray and normal coplanar. A convex lens converts an axial plane wavefront into a spherical wavefront converging to its principal focus.

Quick Tip to Solve: In the reflection construction, compare the equal distances $BC = AD = vt$ using the common hypotenuse AC . For the lens sketch, draw converging rays first and make each emergent wavefront perpendicular to them.