

CBSE Class 12 Physics Question Paper 2026 with Solution PDF-Set 55/3/2



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. All questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D and E**.
- (iii) In **Section A** – Questions no. **1 to 16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** – Questions no. **17 to 21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** – Questions no. **22 to 28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** – Questions no. **29 and 30** are Case Study-based questions. Each question carries **4** marks.
- (vii) In **Section E** – Questions no. **31 to 33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is not allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. The phase difference between the two superimposing waves that give rise to a bright spot in a Young's double-slit experiment is (n is an integer): [1 mark]

- (A) $2n\pi$
- (B) $2n\pi + \frac{\pi}{4}$
- (C) $2n\pi + \frac{\pi}{2}$
- (D) $2n\pi + \pi$

Correct Answer: (A) $2n\pi$

Solution:

Concept:

- Constructive interference produces a bright fringe when the two coherent waves reach the point in the same phase.
- For phase difference ϕ , the intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$, where I_1 and I_2 are the individual wave intensities.

Step 1: Apply the maximum-intensity condition At a bright fringe, the interference term must be largest. Therefore $\cos \phi = 1$. Since cosine repeats after every full cycle, this requires an integral number of complete phase cycles: $\phi = 2n\pi$, where n is an integer. Equivalently, the path difference is $n\lambda$.

Step 2: Check the remaining phase offsets The extra phase offsets in (B), (C) and (D) are respectively $\pi/4$, $\pi/2$ and π . Their cosine values are $1/\sqrt{2}$, 0 and -1 , so none gives maximum intensity. The last gives destructive interference, whereas the other two give intermediate intensities.

Step 3: Conclusion The required phase difference is $2n\pi$. Hence, the correct option is (A).

Quick Tip to Solve: For a bright fringe, reduce the phase modulo 2π : the remainder must be zero.

2. If r_1 and r_2 are the radii of atomic nuclei of mass numbers 64 and 27 respectively, then the value of $\left(\frac{r_1}{r_2}\right)$ is: [1 mark]

- (A) 1
- (B) $\frac{4}{3}$
- (C) $\frac{3}{4}$
- (D) $\frac{27}{64}$

Correct Answer: (B) $\frac{4}{3}$

Solution:

Concept:

- The nuclear radius follows $R = R_0 A^{1/3}$, where A is the mass number and R_0 is approximately the same constant for all nuclei.
- Nuclear volume is proportional to the number of nucleons; hence radius depends on the cube root

of mass number, not directly on mass number.

Step 1: Form the ratio of the two radii For the nuclei in the stated order, $r_1 = R_0(64)^{1/3}$ and $r_2 = R_0(27)^{1/3}$. The common constant cancels when the radii are divided:

$$\frac{r_1}{r_2} = \left(\frac{64}{27}\right)^{1/3} = \frac{4}{3}.$$

Step 2: Check the physical order and other options The first nucleus has the larger mass number, so its radius must be larger. Thus the ratio cannot be 1, $3/4$ or $27/64$. In particular, $3/4$ reverses the requested order, while $27/64$ both reverses the mass-number ratio and omits the cube root.

Step 3: Conclusion $r_1/r_2 = 4/3$. Hence, the correct option is (B).

Quick Tip to Solve: Recognise $64 = 4^3$ and $27 = 3^3$, then take the cube roots before dividing.

3. An equiconvex lens of focal length 15 cm is cut into two equal plano-convex halves. The focal length of each half will be: [1 mark]

- (A) 30 cm
- (B) 20 cm
- (C) 15 cm
- (D) 7.5 cm

Correct Answer: (A) 30 cm

Solution:

Concept:

- For a thin lens in air, the lens-maker formula is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the refractive index and R_1, R_2 are signed surface radii.
- An equiconvex lens has two equal curved-surface contributions to its power. Cutting it into plano-convex halves replaces one curved surface of each half by a plane surface, whose radius is infinite.

Step 1: Compare the original and remaining curvatures Let the magnitude of each original surface radius be R . Taking $R_1 = R$ and $R_2 = -R$, the original lens has $1/f = 2(\mu - 1)/R$. A plano-convex half has only one curved surface, so $1/f' = (\mu - 1)/R = (1/2)(1/f)$.

Step 2: Use the reciprocal relation between power and focal length Halving the power doubles the focal length: $f' = 2f = 2 \times 15$ cm. A value of 15 cm would wrongly preserve the power despite removing one curved-surface contribution. The value 7.5 cm would double the power, and 20 cm does not satisfy the derived factor of two.

Step 3: Conclusion The focal length of each plano-convex half is 30 cm. Hence, the correct option is (A).

Quick Tip to Solve: For equal curved surfaces, replacing one by a plane halves lens power and doubles focal length.

4. An electric dipole $\vec{P} = 2qa\hat{i}$ is placed in x - y plane, centred at the origin. Let E_1 and E_2 be the magnitudes of electric field at a point distant r ($\gg a$) from its centre, along its axis and on its equatorial plane, respectively. Then the value of $\left(\frac{E_1}{E_2}\right)$ is: [1 mark]

- (A) $\frac{1}{4}$
 (B) $\frac{1}{2}$
 (C) 2
 (D) 4

Correct Answer: (C) 2

Solution:

Concept:

- For a short electric dipole of moment magnitude $p = 2qa$, the axial field at distance $r \gg a$ has magnitude $E_{\text{axial}} = (1/4\pi\epsilon_0)(2p/r^3)$.
- At the same distance on the equatorial plane, the field magnitude is $E_{\text{equatorial}} = (1/4\pi\epsilon_0)(p/r^3)$. Its direction is opposite to the dipole moment.

Step 1: Use the far-field condition in the question The printed condition $r \gg a$ permits the short-dipole expressions. Since $\vec{P} = 2qa\hat{i}$, the axial points lie along the x -axis; equatorial points lie in the perpendicular plane through the origin. Only the magnitudes are requested, so the opposite equatorial direction introduces no minus sign into the ratio.

Step 2: Divide fields at the same radial distance The dipole moment, inverse-cube factor and electrostatic constant cancel:

$$\frac{E_1}{E_2} = \frac{(1/4\pi\epsilon_0)(2p/r^3)}{(1/4\pi\epsilon_0)(p/r^3)} = 2.$$

The value $1/2$ reverses the fields. The values 4 and $1/4$ incorrectly square the factor; the question asks for electric field, not a quantity proportional to its square.

Step 3: Conclusion $E_1/E_2 = 2$. Hence, the correct option is (C).

Quick Tip to Solve: At equal large distances, remember the dipole field coefficients: axial 2, equatorial 1.

5. A plano-convex lens, made of glass ($n = 1.5$) has a curved surface whose radius is 40 cm. The image formed of an object is of the same size as that of the object. The distance of the object from the lens is: [1 mark]

- (A) 40 cm
- (B) 80 cm
- (C) 160 cm
- (D) 200 cm

Correct Answer: (C) 160 cm

Solution:

Concept:

- For a thin plano-convex lens in air, the lens-maker formula gives $1/f = (n - 1)/R$, where n is the glass refractive index and R is the magnitude of the curved surface radius.
- The Cartesian lens formula is $1/f = 1/v - 1/u$, with magnification $m = v/u$. A real object has a real inverted image of equal size at the $2f$ positions.

Step 1: Determine the focal length from the curvature The plane surface has infinite radius and contributes zero to the lens power. With $n = 1.5$ and $R = 40$ cm,

$$\frac{1}{f} = \frac{1.5 - 1}{40 \text{ cm}}, \quad f = 80 \text{ cm}.$$

Step 2: Apply the equal-size image condition An inverted image of the same size has $m = -1$, so $v = -u$. The lens formula becomes $1/f = -2/u$, or $u = -2f = -160$ cm. Thus the object distance is the positive magnitude of u . A same-size upright image would require $v = u$, inconsistent with a finite focal length. The choices 40 cm and 80 cm confuse the required distance with R and f ; 200 cm does not satisfy $|m| = 1$.

Step 3: Conclusion The object is 160 cm from the lens. Hence, the correct option is (C).

Quick Tip to Solve: First compute $f = R/(n - 1)$; an equal-size real image then requires an object distance $2f$.

6. Radiation of wavelength 200 nm is incident on a photosensitive surface of work function 4.2 eV. The kinetic energy of fastest photoelectrons emitted from this surface will be close to: [1 mark]

- (A) 3.5 eV
- (B) 3.0 eV
- (C) 2.5 eV
- (D) 2.0 eV

Correct Answer: (D) 2.0 eV

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where λ is the incident wavelength and ϕ is the surface work function.
- Using $hc \simeq 1240 \text{ eV nm}$ gives photon energy directly in electronvolts when the wavelength is expressed in nanometres. The fastest photoelectrons have the maximum kinetic energy.

Step 1: Calculate the incident photon energy For $\lambda = 200 \text{ nm}$,

$$E_{\gamma} = \frac{1240 \text{ eV nm}}{200 \text{ nm}} = 6.2 \text{ eV}.$$

This exceeds the work function 4.2 eV , so photoelectron emission is energetically possible.

Step 2: Deduct the energy required to escape the surface The maximum remaining energy is $K_{\max} = (6.2 - 4.2) \text{ eV}$. The other proposed kinetic energies, 3.5 , 3.0 and 2.5 eV , would require photon energies of 7.7 , 7.2 and 6.7 eV , respectively, after adding the work function. Each exceeds the actual photon energy and therefore violates the energy balance.

Step 3: Conclusion The fastest photoelectrons have $K_{\max} \simeq 2.0 \text{ eV}$. Hence, the correct option is (D).

Quick Tip to Solve: With wavelength in nanometres, use $1240/\lambda$ for photon energy in eV, then subtract the work function.

7. A metallic rod of 1 m length is rotated with a frequency of 40 rev/s , with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius 1 m , about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field of 0.5 T parallel to the axis is present in the region. The value of emf induced between the centre and the metallic ring is close to: [1 mark]

- (A) 20 V
- (B) 32 V
- (C) 40 V
- (D) 63 V

Correct Answer: (D) 63 V

Solution:

Concept:

- A radial rod rotating in a uniform magnetic field perpendicular to its plane has motional emf $\varepsilon = \frac{1}{2} B \omega L^2$ between its ends, where L is its length and ω its angular speed.
- Rotation frequency ν in revolutions per second is converted by $\omega = 2\pi\nu$. The local speed of the rod increases from zero at its hinged centre to ωL at the rim.

Step 1: Account for the speed variation along the rod At distance r from the centre, a short rod element dr moves with speed $v = \omega r$. The magnetic force separates charges along the rod, giving $d\varepsilon = B \omega r dr$. Integrating from the centre to the ring,

$$\varepsilon = \int_0^L B \omega r dr = \frac{1}{2} B \omega L^2.$$

Step 2: Substitute the stated field and rotation frequency Here $B = 0.5 \text{ T}$, $L = 1 \text{ m}$ and $\omega = 2\pi \times 40 = 80\pi \text{ rad s}^{-1}$. Hence

$$\varepsilon = \frac{1}{2} \times 0.5 \times 80\pi \times 1^2 \text{ V} = 20\pi \text{ V} \simeq 62.8 \text{ V}.$$

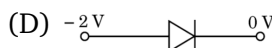
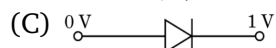
The value 20 V omits the factor π from this result. Neither 32 V nor 40 V is close to the computed emf. Using one common speed for the entire rod would also miss the required integration factor.

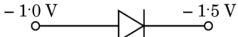
Step 3: Conclusion The induced emf is close to 63 V . Hence, the correct option is (D).

Quick Tip to Solve: For a rod rotating about one end, combine $\omega = 2\pi\nu$ with the half-factor to obtain $\varepsilon = \pi B\nu L^2$.

8. Which of the following p-n junction diodes is forward biased?

[1 mark]



Correct Answer: (B) -1.0 V  -1.5 V

Solution:

Concept:

- A p-n junction is forward biased when its p-side (anode) is at a higher potential than its n-side (cathode): $V_p - V_n > 0$.
- The vertical bar of the diode symbol identifies the cathode. In all four printed circuits the p-side is on the left and the n-side is on the right.

Step 1: Compare the terminal potentials Subtract the right-hand potential from the left-hand potential for each diode:

$$(A) : V_p - V_n = 5 - 10 = -5\text{ V},$$

$$(B) : V_p - V_n = -1.0 - (-1.5) = +0.5\text{ V},$$

$$(C) : V_p - V_n = 0 - 1 = -1\text{ V},$$

$$(D) : V_p - V_n = -2 - 0 = -2\text{ V}.$$

Step 2: Apply the forward-bias condition Only circuit (B) gives a positive anode-to-cathode voltage. Although both terminals there have negative potentials, -1.0 V is higher than -1.5 V . Circuits (A), (C) and (D) all place the p-side at a lower potential, so each is reverse biased. Absolute positivity of a terminal is not the deciding condition.

Step 3: Conclusion The diode in circuit (B) is forward biased. Hence, the correct option is (B).

Quick Tip to Solve: Locate the cathode bar, then calculate anode potential minus cathode potential; a positive difference means forward bias even if both potentials are negative.

9. When the forward bias voltage in a semiconductor diode is changed from 0.8 V to 1.0 V, the forward current changes by 2.0 mA. The forward bias resistance of the diode will be: [1 mark]

- (A) 200 Ω
- (B) 175 Ω
- (C) 100 Ω
- (D) 125 Ω

Correct Answer: (C) 100 Ω

Solution:

Concept:

- The forward dynamic resistance of a diode over a small operating interval is $r_f = \Delta V / \Delta I$, where ΔV and ΔI are the corresponding voltage and current changes.
- Convert current to amperes before using Ohm's unit: 1 mA = 10^{-3} A and 1 Ω = 1 V A $^{-1}$.

Step 1: Find the changes in voltage and current The voltage rises from 0.8 V to 1.0 V, while the given current change is already a difference, not an absolute current:

$$\Delta V = 1.0 - 0.8 = 0.2 \text{ V}, \quad \Delta I = 2.0 \times 10^{-3} \text{ A}.$$

Step 2: Calculate the forward resistance Use the ratio of changes because a diode has a nonlinear current-voltage characteristic:

$$r_f = \frac{0.2}{2.0 \times 10^{-3}} = 100 \Omega.$$

Neither endpoint voltage should be divided by the current change. The other printed values, 200 Ω , 175 Ω and 125 Ω , do not equal the measured voltage change divided by the current change.

Step 3: Conclusion The forward bias resistance is 100 Ω . Hence, the correct option is (C).

Quick Tip to Solve: When a diode question gives changes, subtract the endpoint voltages and use $\Delta V / \Delta I$; convert milliamperes to amperes first.

10. A plane electromagnetic wave travels through a medium and the magnetic field associated with it is given by $B = 5 \times 10^{-8} \sin(3 \times 10^{10} t - 150 x)$ T, where x is in metres and t is in seconds. The velocity of the wave is: [1 mark]

- (A) $2.0 \times 10^8 \text{ m s}^{-1}$
- (B) $4.5 \times 10^7 \text{ m s}^{-1}$
- (C) $3.5 \times 10^7 \text{ m s}^{-1}$
- (D) $2.5 \times 10^8 \text{ m s}^{-1}$

Correct Answer: (A) $2.0 \times 10^8 \text{ m s}^{-1}$

Solution:

Concept:

- A travelling wave written as $B = B_0 \sin(\omega t - kx)$ has angular frequency ω , wave number k , and phase speed $v = \omega/k$.
- A fixed phase obeys $\omega t - kx = \text{constant}$. Differentiating gives $dx/dt = \omega/k > 0$, so this sign combination represents propagation along the positive x-direction.

Step 1: Identify the coefficients in the phase Comparing the printed argument $3 \times 10^{10}t - 150x$ with $\omega t - kx$, we obtain

$$\omega = 3 \times 10^{10} \text{ rad s}^{-1}, \quad k = 150 \text{ rad m}^{-1}.$$

The coefficient $5 \times 10^{-8} \text{ T}$ is the magnetic-field amplitude; it does not determine the speed.

Step 2: Divide angular frequency by wave number The radian factors cancel in their ratio:

$$v = \frac{3 \times 10^{10}}{150} = 2.0 \times 10^8 \text{ m s}^{-1}.$$

Options (B), (C) and (D) do not equal this ratio. No extra factor of 2π is required because both ω and k are angular quantities. Since the wave is in a medium, its speed need not equal the vacuum value.

Step 3: Conclusion The wave travels at $2.0 \times 10^8 \text{ m s}^{-1}$ along the positive x-direction. Hence, the correct option is (A).

Quick Tip to Solve: For a wave given as a sine of time and position, divide the coefficient of t by the magnitude of the coefficient of x ; ignore the amplitude.

11. The velocity of the photoelectrons emitted from a surface depends upon the:

[1 mark]

- (A) Wavelength of incident radiation only
- (B) Intensity of incident radiation only
- (C) Work function of the surface only
- (D) Both wavelength of the incident radiation and work function of the surface

Correct Answer: (D) Both wavelength of the incident radiation and work function of the surface

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = hc/\lambda - \phi$, where h is Planck's constant, c is light speed, λ is incident wavelength and ϕ is the surface work function.
- The maximum kinetic energy is $K_{\max} = \frac{1}{2}m_e v_{\max}^2$, where m_e is electron mass and $v_{\max}$ is the maximum emitted photoelectron speed.

Step 1: Express the photoelectron speed For incident photons above the threshold energy, the photon supplies the work function and the remaining energy can appear as kinetic energy:

$$v_{\max} = \sqrt{\frac{2}{m_e} \left(\frac{hc}{\lambda} - \phi \right)}.$$

Thus the speed depends on the photon energy through wavelength and on the energy needed to remove an electron from the surface.

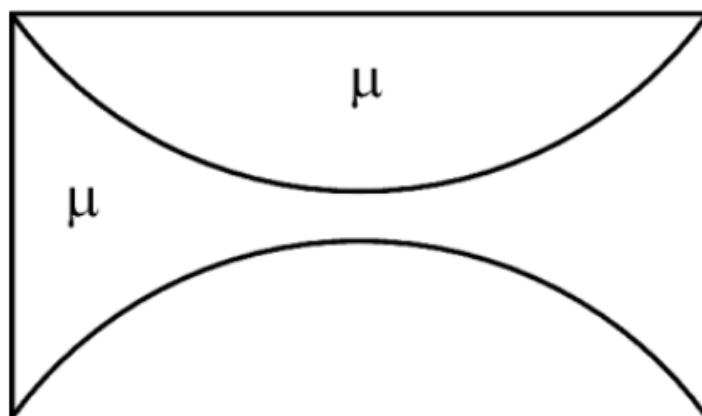
Step 2: Test the listed dependencies Option (A) omits the work function, while option (C) omits wavelength. At fixed wavelength, increasing intensity supplies more photons per second and increases

the emission rate; it does not increase the energy per photon or the maximum photoelectron speed. Thus option (B) is incorrect. Emitted electrons may have a range of speeds, but their maximum speed follows the displayed energy relation.

Step 3: Conclusion Photoelectron speed depends on both the wavelength of the incident radiation and the surface work function. Hence, the correct option is (D).

Quick Tip to Solve: Use $hc/\lambda - \phi$ for photoelectron energy: wavelength sets energy supplied, work function sets energy spent, and intensity sets the photon arrival rate.

12. A thin plano-convex lens and a thin equi-concave lens are kept coaxially in contact as shown in the figure. Assuming both the lenses are made of glass of refractive index μ , and R is the radius of curvature of each curved surface, the focal length of the combination is: [1 mark]



- (A) $\frac{R}{\mu - 1}$
 (B) $-\frac{R}{\mu - 1}$
 (C) $\frac{2R}{\mu - 1}$
 (D) $-\frac{2R}{\mu - 1}$

Correct Answer: (B) $-\frac{R}{\mu - 1}$

Solution:

Concept:

- For a thin lens in air, the lens-maker formula gives its power as $P = 1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the glass refractive index and R_1, R_2 are signed surface radii.
- Thin lenses in contact have additive powers: $1/F = 1/f_1 + 1/f_2$, with F the equivalent focal length. A plane surface has infinite radius and contributes zero curvature.

Step 1: Find the power of each lens The plano-convex lens has one plane surface and one convex surface of radius magnitude R , so its positive power is

$$P_1 = \frac{\mu - 1}{R}.$$

For the equi-concave lens, the Cartesian radii along the direction of incident light are $R_1 = -R$ and

$R_2 = +R$. Therefore,

$$P_2 = (\mu - 1) \left(-\frac{1}{R} - \frac{1}{R} \right) = -\frac{2(\mu - 1)}{R}.$$

Step 2: Add the powers and take the reciprocal Since the lenses are coaxial and in contact,

$$\frac{1}{F} = P_1 + P_2 = \frac{\mu - 1}{R} - \frac{2(\mu - 1)}{R} = -\frac{\mu - 1}{R}.$$

The combination is diverging because the concave lens has twice the magnitude of the convex lens's power. This rules out the positive focal lengths in (A) and (C); the magnitude in (D) is twice the reciprocal of the calculated net power.

Step 3: Conclusion The focal length of the combination is $F = -\frac{R}{\mu - 1}$. Hence, the correct option is (B).

Quick Tip to Solve: Count signed curved surfaces: the plano-convex lens supplies one positive contribution and the equi-concave lens supplies two negative contributions; add powers before inverting.

13. Assertion (A): Photoelectric current depends upon the intensity of the incident radiation.

Reason (R): Stopping potential is independent of the intensity of the incident radiation.

Select the correct answer from the codes (A), (B), (C) and (D) given below:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- At a fixed incident frequency above threshold, increasing intensity increases the number of incident photons per second and hence the photoelectron emission rate. Collected photoelectric current is $I = e\dot{N}$, where $\dot{N}$ is the number of electrons collected per second.
- The stopping potential V_s satisfies $eV_s = K_{\max} = h\nu - \phi$, where ν is incident frequency and ϕ is the surface work function. It measures maximum photoelectron energy.

Step 1: Evaluate the assertion With the frequency, illuminated surface and collection conditions fixed, stronger illumination supplies more photons per second. More electrons are emitted and collected per second, so the photoelectric current increases. The assertion that photoelectric current depends on incident intensity is true.

Step 2: Evaluate the reason At fixed frequency, the energy of each photon is unchanged when intensity changes. Therefore $K_{\max} = h\nu - \phi$, and thus the stopping potential, remain unchanged. The reason is also true.

Step 3: Check whether the reason explains the assertion The reason describes the energy of the fastest photoelectrons, whereas the assertion concerns how many electrons are collected per second. Current changes because the photon arrival rate changes; intensity-independent stopping potential does not supply that explanation. Therefore (A) is ruled out, while (C) and (D) incorrectly classify the truth of the statements.

Step 4: Conclusion Both the assertion and the reason are true, but the reason is not the correct explanation of the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Separate photon number from photon energy: intensity controls photoelectric current, while frequency and work function determine stopping potential.

14. Assertion (A): Nuclear forces are always attractive.

Reason (R): The nuclear force between protons and neutrons in a nucleus is a weak force.

Select the correct answer from the codes (A), (B), (C) and (D) given below:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false.

Solution:

Concept:

- The nuclear force between nucleons is strongly attractive at typical nuclear separations but becomes strongly repulsive at very small separations. Its character depends on separation, so it is not always attractive.
- The force binding protons and neutrons inside a nucleus is a strong, short-range nuclear interaction. The weak interaction is a distinct interaction associated with processes such as beta decay.

Step 1: Evaluate the assertion Nuclear attraction binds nucleons at separations of order 1 fm, but the force has a repulsive core at sufficiently smaller separations. This short-distance repulsion prevents nucleons from collapsing into one another. The word “always” therefore makes the assertion false.

Step 2: Evaluate the reason The proton-neutron binding force is strong within its short effective range. Its short range must not be confused with weakness: the weak force and the strong nuclear force are different interactions. Therefore the claim that this binding force is a weak force is false.

Step 3: Check the explanatory relation and answer code Since both statements are false, the reason

cannot provide a valid explanation of the assertion. Options (A) and (B) both require two true statements, and option (C) requires a true assertion; none matches this evaluation.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: Test the word “always” against the repulsive nuclear core, and remember that a short-range nuclear binding force is strong rather than weak.

15. Assertion (A): The cylindrical soft iron core in a moving coil galvanometer only makes the magnetic field radial and does not affect the strength of the magnetic field.

Reason (R): In a moving coil galvanometer, the plane of the coil is always perpendicular to the magnetic field.

Select the correct answer from the codes (A), (B), (C) and (D) given below:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false.

Solution:

Concept:

- A soft iron core has high magnetic permeability. It strengthens the magnetic field and, together with the curved pole pieces, makes the field radial.
- The torque on a current-carrying coil is $\tau = NIAB \sin \theta$, where θ is the angle between the coil's normal and the magnetic field.

Step 1: Evaluate the assertion The core increases the magnetic field strength inside the coil; its role is not limited to making the field radial. The word “only” and the claim that the field strength is unaffected therefore make Assertion (A) false.

Step 2: Evaluate the reason In the radial field of a moving coil galvanometer, the field is parallel to the plane of the coil at every deflection. It is the normal to the coil that is perpendicular to the field. Thus $\theta = 90^\circ$ and $\tau = NIAB$. Reason (R), which calls the plane perpendicular to the field, is false.

Step 3: Check the explanation and the codes Since both statements are false, the reason cannot explain the assertion. Codes (A) and (B) require both statements to be true, while code (C) requires a true assertion; none describes this case.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: In a radial field, distinguish the coil's plane from its normal: the plane is parallel to the field, so the normal makes 90° with it.

16. Assertion (A): In a Wheatstone bridge circuit, if we interchange the position of the cell and the galvanometer, the balance condition $\frac{P}{Q} = \frac{R}{S}$ remains unchanged.

Reason (R): $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{Q}{S} = \frac{P}{R}$ so balance condition remains same.

Select the correct answer from the codes (A), (B), (C) and (D) given below:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- A Wheatstone bridge is balanced when its galvanometer terminals have equal potentials, so the galvanometer current is zero.
- For arm resistances P, Q, R, S , the balance relation is $P/Q = R/S$, equivalently $PS = QR$. The resistances are positive and nonzero.

Step 1: Evaluate the assertion Interchanging the supply and the galvanometer interchanges the two diagonals used for them. At balance, the resistance ratios associated with the new supply branches become P/R and Q/S . Equality of these ratios gives $PS = QR$, the original balance condition. The assertion is true.

Step 2: Evaluate the reason Cross-multiply the original relation and then divide by RS :

$$\frac{P}{Q} = \frac{R}{S} \implies PS = QR \implies \frac{P}{R} = \frac{Q}{S}.$$

This is exactly the equivalent ratio relation stated in the reason. The reason is therefore true.

Step 3: Test the explanatory connection The ratio relation in the reason is the balance relation after the interchange, and its algebraic equivalence proves that no new condition is imposed. Thus the reason explains the assertion. Code (B) incorrectly denies this connection, and codes (C) and (D) incorrectly treat at least one statement as false.

Step 4: Conclusion Both statements are true, and the reason correctly explains the assertion. Hence, the correct option is (A).

Quick Tip to Solve: Reduce each proposed Wheatstone balance ratio to a product relation: both arrangements here give $PS = QR$.

Section B (10 marks)

17. Calculate the de Broglie wavelength of a neutron having kinetic energy of 150 eV. [2 marks]

Solution:

Concept:

- The de Broglie wavelength of a particle of momentum p is $\lambda = h/p$, where h is Planck's constant.
- For a nonrelativistic neutron of mass m_n and kinetic energy K , $K = p^2/(2m_n)$; hence $\lambda = h/\sqrt{2m_n K}$.

Step 1: Convert the kinetic energy to SI units Use $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$. The electron-volt is an energy unit and can be used for a neutral particle such as a neutron.

$$K = 150 \times 1.6 \times 10^{-19} = 2.40 \times 10^{-17} \text{ J}.$$

This energy is much smaller than the neutron's rest energy, so the nonrelativistic relation between momentum and kinetic energy is applicable.

Step 2: Calculate the momentum and wavelength Use the supplied values $m_n = 1.675 \times 10^{-27} \text{ kg}$ and $h = 6.63 \times 10^{-34} \text{ J s}$. First find the momentum:

$$\begin{aligned} p &= \sqrt{2 \times 1.675 \times 10^{-27} \times 2.40 \times 10^{-17}} \\ &= \sqrt{8.04 \times 10^{-44}} \\ &\approx 2.835 \times 10^{-22} \text{ kg m s}^{-1}. \end{aligned}$$

Now substitute in the de Broglie relation:

$$\lambda = \frac{6.63 \times 10^{-34}}{2.835 \times 10^{-22}} \approx 2.338 \times 10^{-12} \text{ m}.$$

Step 3: Conclusion The neutron's de Broglie wavelength is $\lambda \approx 2.34 \times 10^{-12} \text{ m}$, or 2.34 pm.

Quick Tip to Solve: Convert 150 eV to joules before using SI constants, and use the neutron mass; the familiar electron wavelength shortcut does not apply.

18. Explain, with the help of suitable diagram, the two important processes that occur during the formation of a p-n junction. [2 marks]

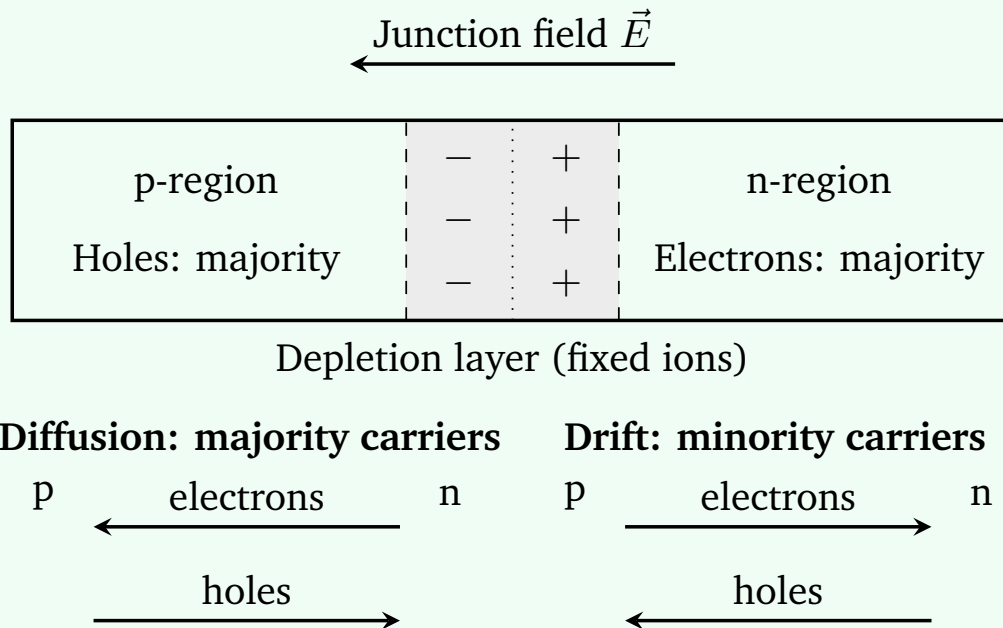
Solution:

Concept:

- Diffusion is the motion of charge carriers from a region of higher concentration to one of lower concentration, caused by the concentration gradient.
- Drift is the motion of charge carriers under an electric field. Electrons move opposite to the field and holes move along it.

Step 1: Explain diffusion and the depletion layer When a p-n junction forms, majority electrons diffuse from the n-region to the p-region, while majority holes diffuse from the p-region to the n-region. They recombine near the junction. This leaves immobile negative acceptor ions on the

p-side and positive donor ions on the n-side. The region near the junction becomes depleted of mobile majority carriers and is called the depletion layer. These fixed ions establish an electric field directed from the n-side to the p-side and a potential barrier opposing further majority-carrier diffusion.



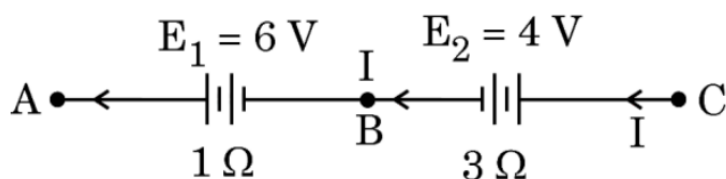
Depletion-layer charges, junction field, and carrier-motion directions

Step 2: Explain drift and equilibrium The junction field drives minority electrons from the p-side to the n-side and minority holes from the n-side to the p-side. This produces a drift current opposite to the conventional diffusion current. As the barrier develops, diffusion decreases. At thermal equilibrium, with no external bias, the magnitudes of the drift and diffusion currents are equal; their sum is zero. Carrier motion continues even though the net current vanishes.

Step 3: Conclusion The two processes are **diffusion due to the concentration gradient** and **drift due to the junction electric field**. At equilibrium their opposing currents balance.

Quick Tip to Solve: Draw negative fixed ions on the p-side and positive fixed ions on the n-side first; the field points from n to p, fixing all drift directions.

19(a). In the given figure, a steady current I flows through the circuit when points A and C are connected by a wire of negligible resistance. Find the potential difference between points B and C . [2 marks]



Solution:

Concept:

- Kirchhoff's loop rule states that the algebraic sum of potential changes around a closed loop is zero. A wire of negligible resistance gives $V_A = V_C$.
- A cell delivering current has terminal voltage $E - Ir$; a cell being charged has terminal voltage $E + Ir$, where E is its emf and r its internal resistance.

Step 1: Identify the cell polarities and find the current The longer plate is the positive terminal. The 6 V cell has its positive terminal towards A , whereas the 4 V cell has its positive terminal towards C . Thus the emfs oppose each other. The larger emf drives current in the shown direction $C \rightarrow B \rightarrow A$, returning through the connecting wire. The internal resistances add:

$$6 - 4 - I(1 + 3) = 0,$$

$$I = \frac{6 - 4}{1 + 3} = 0.50 \text{ A}.$$

Step 2: Calculate the potential difference across the second cell Current enters the positive terminal at C of the 4 V cell, so this cell is being charged. Moving from C to B , both its emf and its internal resistance give potential drops:

$$V_B - V_C = -4 - I(3) = -4 - (0.50)(3) = -5.50 \text{ V}.$$

As a check, the 6 V cell is delivering current, giving $V_A - V_B = 6 - (0.50)(1) = 5.50 \text{ V}$. Since $V_A = V_C$, this agrees with the result above.

Step 3: Conclusion The potential difference has magnitude 5.5 V , with C at the higher potential. Equivalently, $V_B - V_C = -5.5 \text{ V}$.

Quick Tip to Solve: Check whether current enters a cell's long plate: if it does, the cell is charging and its terminal voltage is $E + Ir$.

19(b). A battery of emf 21 V and internal resistance $3\ \Omega$ is connected to a resistor. If the current in the circuit is 3 A, find:

(i) the resistance of the resistor.

(ii) the terminal voltage of the battery.

[2 marks]

Solution:

Concept:

- A battery supplying current has its internal resistance r in series with the external resistor R . Hence its emf is $E = I(R + r)$, where I is the circuit current.
- The terminal voltage of a discharging battery is $V = E - Ir$. It also equals the potential difference IR across the external resistor.

Step 1: Calculate the external resistance The given values are $E = 21\text{ V}$, $r = 3\ \Omega$, and $I = 3\text{ A}$. First find the total series resistance from the emf and current, then subtract the battery's internal resistance:

$$R + r = \frac{E}{I} = \frac{21}{3} = 7\ \Omega,$$

$$R = 7 - 3 = 4\ \Omega.$$

Using E/I alone would give the total resistance, rather than the resistance of the connected resistor asked for in part (i).

Step 2: Calculate the terminal voltage Since the battery supplies current, the internal voltage drop is subtracted from the emf:

$$V = E - Ir = 21 - (3)(3) = 12\text{ V}.$$

As a consistency check, the voltage across the external resistor is $IR = (3)(4) = 12\text{ V}$, equal to the terminal voltage.

Step 3: Conclusion (i) The resistor has resistance $R = 4\ \Omega$. (ii) The terminal voltage of the battery is $V = 12\text{ V}$.

Quick Tip to Solve: Calculate $R = E/I - r$; then check the terminal voltage using both $E - Ir$ and IR .

20. Draw a labelled ray diagram of an astronomical telescope when final image is formed at infinity. Write the expression for its magnifying power.

[2 marks]

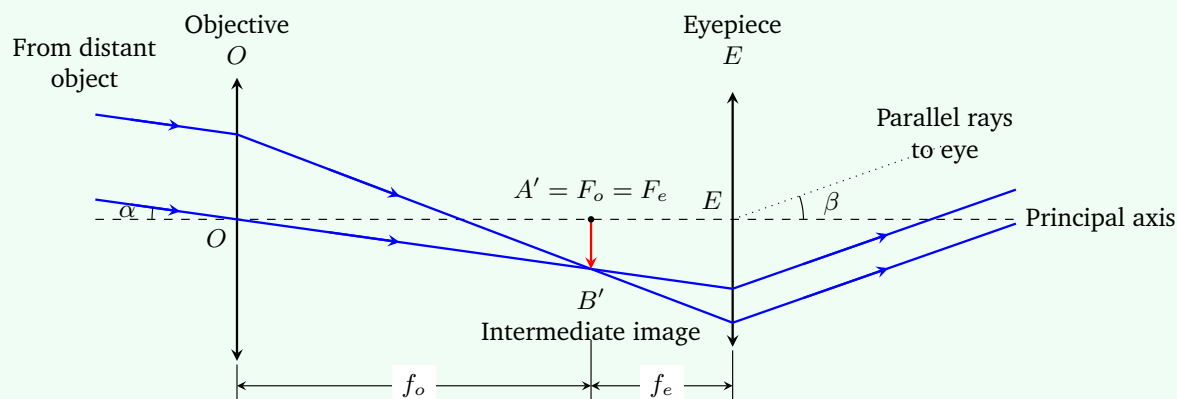
Solution:

Concept:

- In normal adjustment an astronomical telescope forms its final image at infinity. The real image formed by the objective lies in the first focal plane of the eyepiece.
- Angular magnifying power is $M = \beta/\alpha$, where α is the small angle subtended by the distant object at the unaided eye and β is the angle subtended by the final image through the telescope.

Step 1: Draw the telescope in normal adjustment The objective O , of focal length f_o , receives parallel rays from a distant object and forms a real inverted intermediate image $A'B'$. Place the convex eyepiece E , of focal length f_e , so that F_o and F_e coincide at A' . The lens separation is therefore $f_o + f_e$. Rays from B' emerge parallel after refraction at the eyepiece, placing the final image at infinity. The

dotted line through E is parallel to the emergent rays and shows their angle to the axis.



Astronomical telescope in normal adjustment; $A'B'$ is the inverted intermediate image.

Step 2: Express the angular magnifying power If the intermediate image has height of magnitude h , the paraxial geometry gives $|\alpha| \simeq h/f_o$ and $|\beta| \simeq h/f_e$. Hence the magnitude of the magnifying power is

$$|M| = \frac{|\beta|}{|\alpha|} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}.$$

The final image is inverted relative to the distant object, so the signed angular magnifying power carries a negative sign.

Step 3: Conclusion For the final image at infinity, $M = -f_o/f_e$, with magnitude $|M| = f_o/f_e$. The objective and eyepiece are separated by $f_o + f_e$.

Quick Tip to Solve: Make the objective's image plane coincide with the eyepiece's first focal plane; the emerging rays must then be parallel.

21. Explain the terms mass defect and binding energy. How are they related?

[2 marks]

Solution:

Concept:

- A bound nucleus has a smaller rest mass than the sum of the rest masses of its free constituent protons and neutrons. Their difference is called the mass defect.
- Einstein's mass-energy relation converts this mass difference into the binding energy: $E_b = \Delta m c^2$, where c is the speed of light in vacuum.

Step 1: Define the mass defect Consider a nucleus with atomic number Z , mass number A , and nuclear mass M . It contains Z protons and $A - Z$ neutrons. If m_p and m_n are the masses of a free proton and neutron respectively, its mass defect is

$$\Delta m = Zm_p + (A - Z)m_n - M.$$

Here M is the mass of the nucleus alone, not the mass of the neutral atom including its electrons. For a bound nucleus the sum of the free nucleon masses exceeds M , so Δm is positive.

Step 2: Define binding energy and relate it to mass defect The binding energy of a nucleus is the energy required to separate it completely into its constituent free protons and neutrons. The same amount of energy is released when these free nucleons combine to form the nucleus. This reduction

in the system's energy corresponds to its reduction in rest mass:

$$E_b = \Delta m c^2.$$

When Δm is expressed in kilograms and c in m s^{-1} , the binding energy is obtained in joules. It is the total binding energy of the nucleus, rather than binding energy per nucleon.

Step 3: Conclusion Mass defect is the deficit of nuclear mass relative to the total mass of its free nucleons. Binding energy is the energy needed to separate the nucleus into those nucleons, related by

$$E_b = \Delta m c^2.$$

Quick Tip to Solve: Write mass defect as free nucleon mass minus nuclear mass, then multiply by c^2 to find the total binding energy.

Section C (29 marks)

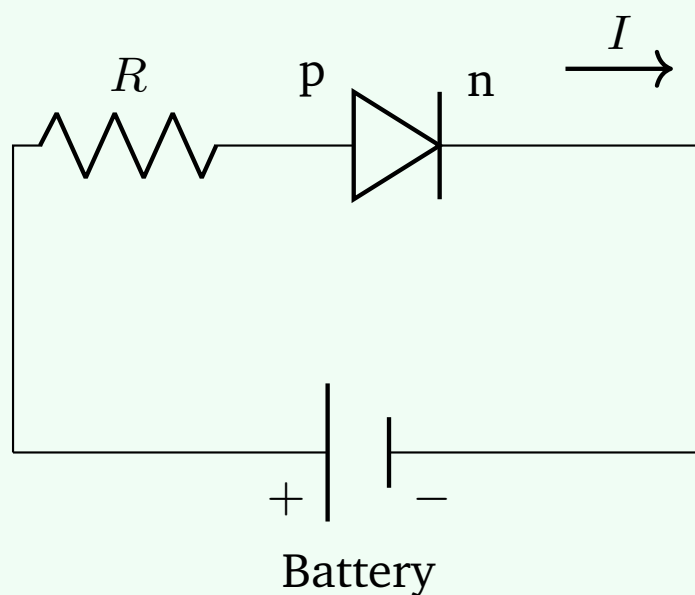
22. With the help of circuit diagrams, briefly explain the forward biasing and the reverse biasing of a p-n junction diode. [3 marks]

Solution:

Concept:

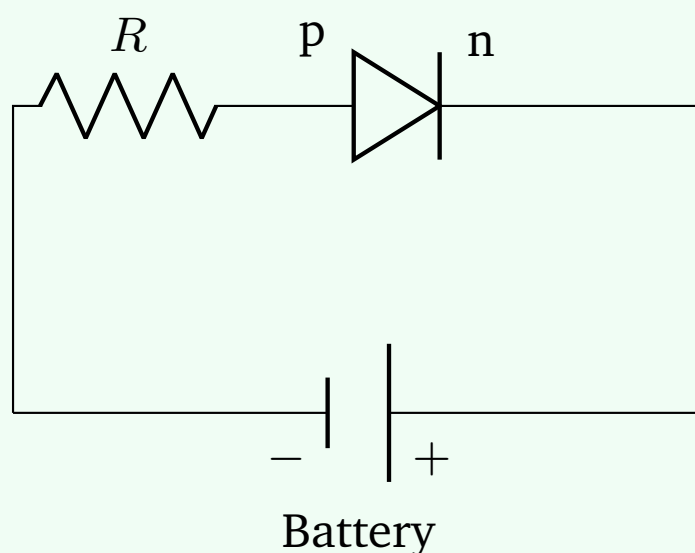
- Biasing means applying an external potential difference across a p-n junction. The p-side is the anode and the n-side is the cathode.
- The depletion region contains immobile ions and produces a potential barrier. The applied voltage can reduce or increase this barrier, thereby controlling the junction current.

Step 1: Connect the diode in forward bias Connect the p-side to the positive terminal and the n-side to the negative terminal of the battery, as shown. The series resistor limits the current and protects the diode. The applied electric field opposes the junction's built-in field, reducing the potential barrier and narrowing the depletion layer. Majority carriers are pushed towards the junction: holes from the p-region and electrons from the n-region cross it more readily. Consequently, an appreciable forward current flows when the applied voltage is sufficient. Conventional current through the diode is from p to n, and its forward resistance is relatively low.



Forward bias: p to positive and n to negative; conventional current flows from p to n.

Step 2: Reverse the battery connections Connect the p-side to the negative terminal and the n-side to the positive terminal. The applied field now supports the built-in junction field, increasing the potential barrier and widening the depletion region. Majority carriers move away from the junction, so their flow across it is strongly suppressed. A very small reverse current, due to thermally generated minority carriers, can still flow from n to p. Thus, below breakdown, the diode offers a very high reverse resistance. The statement that a reverse-biased diode blocks current is an approximation that neglects this small leakage current.



Reverse bias: p to negative and n to positive; only a small reverse leakage current flows below breakdown.

Step 3: Conclusion Forward bias connects p to positive and n to negative, lowering the barrier and allowing appreciable current. Reverse bias connects p to negative and n to positive, raising the barrier and allowing only a small minority-carrier current below breakdown.

Quick Tip to Solve: Check the battery terminal connected to p first: p-positive means forward bias; p-negative means reverse bias. The cathode bar in the diode symbol identifies the n-side.

23. Define the term ‘critical angle’. A small bulb is placed at the bottom of a tank containing a liquid of refractive index μ at a depth H . It is observed that the light emerges from a circular area of radius r of the surface. Obtain the expression for r in terms of H and μ . [3 marks]

Solution:

Concept:

- The critical angle C is the angle of incidence in an optically denser medium for which the angle of refraction in the rarer medium is 90° . It is measured from the normal to the interface.
- Snell’s law at a liquid-air interface gives $\mu \sin C = 1$, taking the refractive index of air as unity. For incidence greater than C , total internal reflection occurs.
- A ray from a source at depth H , incident at horizontal distance x from the point vertically above the source, has $\tan i = x/H$, where i is its angle with the surface normal.

Step 1: Identify the limiting ray that can emerge Light rays reaching the surface with incidence angles smaller than the critical angle emerge into air. The limiting ray has incidence angle C and is refracted along the surface. Rays farther from the point vertically above the bulb have larger angles of incidence and undergo total internal reflection. Since the bulb emits in all azimuthal directions and the depth is the same in every vertical cross-section, the limiting points form a circle on the horizontal surface.

Step 2: Relate the radius to the critical angle Consider the right triangle formed by the bulb, the point vertically above it, and a point on the boundary circle. The vertical side is H , the horizontal side is the radius r , and the angle between the limiting ray and the vertical is C . Thus,

$$\tan C = \frac{r}{H}, \quad r = H \tan C.$$

The angle at the bulb equals the incidence angle because the vertical through the bulb and the surface normal at the point of incidence are parallel.

Step 3: Eliminate the critical angle using refraction At the critical angle, Snell’s law becomes

$$\mu \sin C = \sin 90^\circ = 1.$$

Therefore, for $\mu > 1$,

$$\begin{aligned} \sin C &= \frac{1}{\mu}, \\ \cos C &= \sqrt{1 - \frac{1}{\mu^2}} = \frac{\sqrt{\mu^2 - 1}}{\mu}, \\ \tan C &= \frac{\sin C}{\cos C} = \frac{1}{\sqrt{\mu^2 - 1}}. \end{aligned}$$

Step 4: Conclusion The radius of the circular area from which light emerges is

$$r = \frac{H}{\sqrt{\mu^2 - 1}}.$$

The critical angle is the incidence angle in the liquid corresponding to grazing refraction into air. The circle's edge is set by this limiting angle.

Quick Tip to Solve: First use the geometry $r = H \tan C$; then use $\sin C = 1/\mu$. Replacing $\tan C$ directly by $1/\mu$ would give an incorrect radius.

24(a). A circular coil of 30 turns and radius 8.0 cm carrying a current of 6 A is suspended vertically in a uniform horizontal magnetic field of 1.0 T. The field lines make an angle of 30° with the plane of the coil. Calculate the magnitude of the external torque that must be applied to prevent the coil from turning. What would happen if the circular coil is replaced by a planar coil of irregular shape that encloses the same area, keeping other parameters unchanged? [3 marks]

Solution:

Concept:

- A planar coil of N turns, area A per turn and current I has magnetic moment $\vec{m} = NI\vec{A}$, directed normal to its plane by the right-hand rule.
- In a uniform magnetic field, $\vec{\tau}_B = \vec{m} \times \vec{B}$, so $\tau_B = NIAB \sin \theta$, where θ is the angle between the coil's normal and the field, rather than the angle with its plane.
- To prevent rotation, the applied external torque must balance the magnetic torque: $\vec{\tau}_{\text{ext}} = -\vec{\tau}_B$. For a planar coil in a uniform field, the torque depends on its area and orientation, not the detailed shape of its boundary.

Step 1: Determine the area and the angle to the normal The radius must first be expressed in SI units. The field makes 30° with the plane, so its angle with a suitable normal is $90^\circ - 30^\circ = 60^\circ$. The opposite normal gives the same torque magnitude. Hence,

$$\begin{aligned}a &= 8.0 \text{ cm} = 0.080 \text{ m}, \\A &= \pi a^2 = \pi(0.080)^2 \text{ m}^2, \\\sin \theta &= \sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}.\end{aligned}$$

Step 2: Evaluate the magnetic torque to be balanced Substitute $N = 30$, $I = 6 \text{ A}$ and $B = 1.0 \text{ T}$ into the torque formula:

$$\begin{aligned}\tau_B &= 30 \times 6 \times \pi(0.080)^2 \times 1.0 \times \frac{\sqrt{3}}{2}, \\\approx &3.13 \text{ N m}.\end{aligned}$$

This magnetic torque tends to rotate the coil so that its magnetic moment aligns with the field. The

external torque must have the same magnitude and the opposite rotational sense to keep the coil stationary.

Step 3: Compare a coil of irregular shape For the replacement planar coil, the number of turns, enclosed area, current, orientation and field are all unchanged. Its magnetic moment $NI\vec{A}$ is therefore unchanged. In a uniform field the torque on each complete loop is determined by that moment, even though forces on individual portions of the wire may differ with the shape. Consequently, the external torque required for balance has the same magnitude as before.

Step 4: Conclusion The required external torque has magnitude 3.13 N m (approximately 3.1 N m) and acts opposite to the magnetic torque. Replacing the circle by a planar irregular coil of the same area leaves the required balancing torque unchanged.

Quick Tip to Solve: When the angle is stated with the plane of a coil, use $NIAB \cos \alpha$. Use the sine form only for an angle measured from the normal.

OR

24(b). An alpha particle (mass $6.4 \times 10^{-27} \text{ kg}$ and charge $3.2 \times 10^{-19} \text{ C}$) having 8.0 MeV energy, enters a region of a uniform magnetic field of 0.5 T . If the field is directed perpendicular to the velocity of the particle, find the radius of the circular path described by the particle. Mention the condition under which the particle in this region (i) describes a helical path, and (ii) goes straight undeviated.

[3 marks]

Solution:

Concept:

- The magnetic force on a charge q moving with velocity $\vec{v}$ is $\vec{F} = q(\vec{v} \times \vec{B})$. Its magnitude is $qvB \sin \theta$, where θ is the angle between the velocity and field.
- For perpendicular entry, magnetic force supplies centripetal force: $qvB = mv^2/r$. With kinetic energy $K = mv^2/2$, the radius is $r = \sqrt{2mK}/(|q|B)$.
- A magnetic field does no work. The velocity component parallel to the field stays constant, while the perpendicular component produces circular motion.

Step 1: Convert the kinetic energy into joules The given 8.0 MeV is kinetic energy. Using $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$,

$$K = 8.0 \times 10^6 \times 1.6 \times 10^{-19} = 1.28 \times 10^{-12} \text{ J}.$$

The field is perpendicular to the initial velocity, so the full speed contributes to the circular motion.

Step 2: Calculate the radius of the circular path Substitute $m = 6.4 \times 10^{-27} \text{ kg}$, $q = 3.2 \times 10^{-19} \text{ C}$ and $B = 0.5 \text{ T}$:

$$\begin{aligned} r &= \frac{\sqrt{2(6.4 \times 10^{-27})(1.28 \times 10^{-12})}}{(3.2 \times 10^{-19})(0.5)} \\ &= \frac{1.28 \times 10^{-19}}{1.60 \times 10^{-19}} \text{ m} = 0.80 \text{ m}. \end{aligned}$$

The magnetic force continually changes the direction of motion without changing its speed or kinetic energy.

Step 3: State the conditions for the other trajectories (i) A helical path occurs when both $v_{\parallel} = v \cos \theta$ and $v_{\perp} = v \sin \theta$ are nonzero: the velocity is neither parallel nor perpendicular to $\vec{B}$. Circular motion

perpendicular to the field combines with uniform motion along it.

(ii) The particle goes straight undeviated when its velocity is parallel or antiparallel to the field, so $\theta = 0^\circ$ or 180° . Then $\vec{v} \times \vec{B} = 0$, and the magnetic force vanishes.

Step 4: Conclusion The radius is $r = 0.80 \text{ m}$. Nonzero parallel and perpendicular velocity components give a helix; motion parallel or antiparallel to the field is straight and undeviated.

Quick Tip to Solve: Convert MeV to joules first, then use $r = \sqrt{2mK}/(|q|B)$ directly. For path questions, split the velocity into components parallel and perpendicular to the field.

25. A series combination of circuit elements X and Y is connected across an ac source. It is found that the voltage is ahead of current in phase by $\frac{\pi}{4}$ radian. When element Y is replaced by element Z , the current leads the voltage by $\frac{\pi}{4}$ radian.

(a) Identify the elements X , Y and Z .

(b) What will the phase angle and power factor for the circuit be if X , Y and Z were connected in series across the same ac source? What can you say about the current that flows in the circuit in this case?

[3 marks]

Solution:

Concept:

- In a resistor, voltage and current are in phase. In an ideal inductor, voltage leads current by $\pi/2$; in a capacitor, current leads voltage by $\pi/2$.
- For a series circuit, the voltage-current phase angle obeys $\tan \phi = (X_L - X_C)/R$, where R is resistance, $X_L = \omega L$ and $X_C = 1/(\omega C)$ are the reactances at angular frequency ω .

Step 1: Identify the resistor and inductor (a) A phase difference of $\pi/4$, intermediate between zero and $\pi/2$, requires resistance together with reactance. The element common to both combinations is therefore the resistor: $X = R$.

With X and Y , voltage leads current, so the circuit is inductive and Y is an inductor L . For this series RL combination,

$$\tan\left(\frac{\pi}{4}\right) = \frac{X_L}{R} = 1 \Rightarrow X_L = R.$$

Step 2: Identify the capacitor With X and Z , current leads voltage, which is the phase relation of a series RC circuit. Hence the element Z is a capacitor C . Taking the phase of voltage relative to current gives $\phi = -\pi/4$, so

$$\tan\left(-\frac{\pi}{4}\right) = -\frac{X_C}{R} = -1 \Rightarrow X_C = R.$$

Thus the given phase angles establish $X_L = X_C$ at the frequency of the same ac source.

Step 3: Find the phase and current at resonance (b) When the resistor, inductor and capacitor are connected in series, their inductive and capacitive reactances cancel:

$$\tan \phi = \frac{X_L - X_C}{R} = 0, \quad \phi = 0, \quad \cos \phi = 1.$$

The circuit is at series resonance, with minimum impedance $Z_{\text{imp}} = R$. Its current is in phase with the

source voltage and has the maximum rms value, for this fixed source voltage and resistance:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}.$$

Step 4: Conclusion X is a resistor, Y an inductor and Z a capacitor. Together they form a resonant series RLC circuit: $\phi = 0, \cos \phi = 1$, and the current is maximum and in phase with the applied voltage.

Quick Tip to Solve: A phase angle of 45° in a series RL or RC circuit means reactance equals resistance. Equal inductive and capacitive reactances cancel when both are present.

26. (a) Depict the variation of electric field ($\vec{E}$) and magnetic field ($\vec{B}$) with respect to the direction of propagation of an electromagnetic wave. Write their two important characteristics.

(b) Show that $\frac{1}{\sqrt{\epsilon_0 \mu_0}}$ gives the velocity of an electromagnetic wave in free space. [3 marks]

Solution:

Concept:

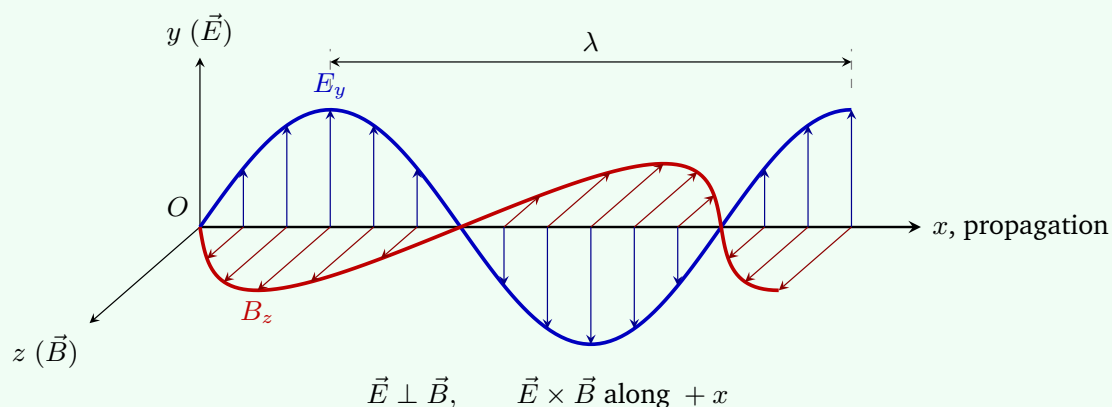
- In a plane electromagnetic wave, $\vec{E}$ and $\vec{B}$ are mutually perpendicular and both are perpendicular to the propagation direction $\vec{E} \times \vec{B}$.
- In free space, the fields oscillate in phase, with amplitudes related by $E_0 = cB_0$, where c is the wave speed.
- Maxwell's equations give the wave equation $\partial^2 E / \partial x^2 = \mu_0 \epsilon_0 \partial^2 E / \partial t^2$, where ϵ_0 and μ_0 are the permittivity and permeability of free space.

Step 1: Depict the two fields and state their characteristics (a) Choose propagation along $+x$, electric field along y , and magnetic field along z . A suitable representation is

$$\begin{aligned}\vec{E}(x, t) &= E_0 \sin(kx - \omega t) \hat{y}, \\ \vec{B}(x, t) &= B_0 \sin(kx - \omega t) \hat{z}.\end{aligned}$$

Here k is the wave number and ω the angular frequency. The diagram shows their spatial variation at one instant, with separate amplitude scales for the two fields.

Two important characteristics are: (i) the wave is transverse, with $\vec{E} \perp \vec{B}$ and both perpendicular to propagation; (ii) the fields oscillate in phase with the same frequency and wavelength, reaching their maxima and zeros at the same positions and times. The relation $E_0/B_0 = c$ also holds.



A plane electromagnetic wave travelling along $+x$, shown in perspective at a fixed instant. Field amplitudes use separate scales.

Step 2: Obtain the free-space wave speed (b) For these field directions, Faraday's law and the Maxwell-Ampere law in free space give

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}, \quad -\frac{\partial B_z}{\partial x} = \mu_0 \epsilon_0 \frac{\partial E_y}{\partial t}.$$

Differentiate the first relation with respect to x , then use the time derivative of the second relation:

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial}{\partial t} \left(\frac{\partial B_z}{\partial x} \right) = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2}.$$

The standard wave equation for speed v has coefficient $1/v^2$ multiplying the second time derivative. Therefore

$$\frac{1}{v^2} = \mu_0 \epsilon_0 \Rightarrow v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

Step 3: Verify the numerical value and dimensions Using $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ and $\mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2}$,

$$v = \frac{1}{\sqrt{(8.854 \times 10^{-12})(4\pi \times 10^{-7})}} \simeq 3.0 \times 10^8 \text{ m s}^{-1}.$$

The units of $\mu_0 \epsilon_0$ are $\text{s}^2 \text{ m}^{-2}$, since $\text{C} = \text{A s}$. Its inverse square root therefore has the units of speed, and its value equals the speed of light in free space.

Step 4: Conclusion The electric and magnetic fields are transverse, mutually perpendicular and in phase. Their free-space propagation speed is $c = 1/\sqrt{\epsilon_0 \mu_0} \simeq 3.0 \times 10^8 \text{ m s}^{-1}$.

Quick Tip to Solve: Draw the electric and magnetic crests at the same propagation positions, and use $\vec{E} \times \vec{B}$ to check the travel direction. Compare the wave-equation coefficient with $1/v^2$ to obtain the speed.

27. Differentiate between nuclear fission and nuclear fusion. Give one example for each with nuclear reaction. [3 marks]

Solution:

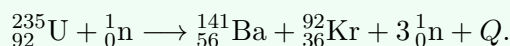
Concept:

- Nuclear fission splits a heavy nucleus into lighter nuclei, whereas nuclear fusion combines light nuclei to form a heavier nucleus.
- Energy is released when the products have greater total binding energy. The corresponding decrease in rest mass gives $Q = \Delta mc^2$, where Δm is the mass defect of the reaction.

Step 1: Distinguish the two processes In fission, a heavy nucleus such as uranium breaks into two medium-mass fragments, usually after absorbing a neutron. Additional neutrons are released and can cause further fissions, making a chain reaction possible. Fission does not require the reacting nuclei to overcome a mutual electrostatic barrier because the incident neutron is uncharged.

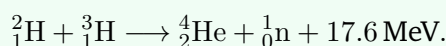
In fusion, two light nuclei join to make a heavier nucleus. Since both nuclei are positively charged, they repel each other. Very high temperature is needed in thermonuclear fusion to bring them sufficiently close for the attractive nuclear force to act. Fusion powers stars, while controlled fission is used in conventional nuclear reactors.

Step 2: Give a balanced fission example One possible neutron-induced fission channel of uranium-235 is



The mass numbers balance as $235 + 1 = 141 + 92 + 3 = 236$, and the atomic numbers balance as $92 = 56 + 36$. Approximately 200 MeV is released per fission; the exact energy depends on the fission products.

Step 3: Give a balanced fusion example Deuterium and tritium can fuse according to



Here the mass numbers balance as $2 + 3 = 4 + 1$, while the atomic numbers balance as $1 + 1 = 2 + 0$. The helium nucleus is more tightly bound, so the total rest mass decreases and the released energy appears mainly as kinetic energy of the products.

Step 4: Conclusion Fission is the splitting of a heavy nucleus, illustrated by neutron-induced uranium-235 fission. Fusion is the combination of light nuclei, illustrated by deuterium-tritium fusion. Both release nuclear binding energy, but their starting nuclei and initiation conditions differ.

Quick Tip to Solve: For each nuclear equation, check mass number and atomic number separately on both sides; fission splits a heavy nucleus, while fusion joins light nuclei.

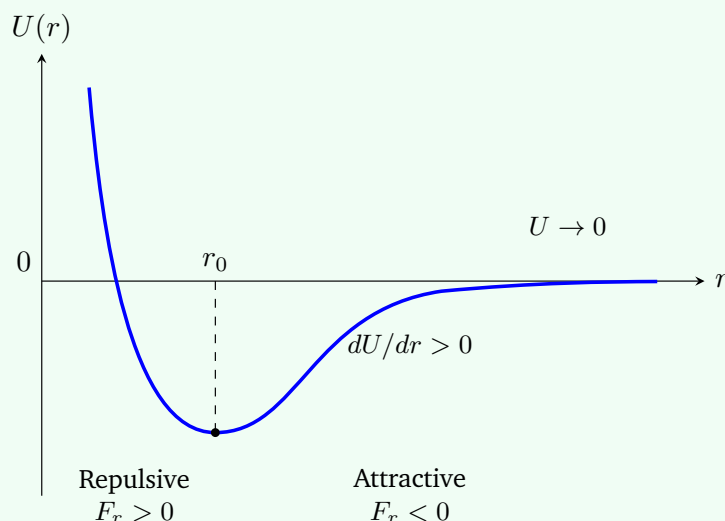
28. Draw a plot showing the variation of potential energy of a pair of nucleons as a function of their separation (r). Using this plot, show that the nuclear force is (i) attractive for $r > r_0$, and (ii) repulsive for $r < r_0$. [3 marks]

Solution:

Concept:

- For a central conservative force, the radial component is $F_r = -dU/dr$, where $U(r)$ is the potential energy and r is the separation of the nucleons.
- A positive F_r tends to increase the separation and is repulsive; a negative F_r tends to decrease the separation and is attractive. At the minimum $r = r_0$, $dU/dr = 0$.

Step 1: Sketch the potential-energy well Choose $U = 0$ for widely separated nucleons. At very small separation the potential rises steeply to positive values, representing the strong repulsive core. As separation increases, the potential falls to a negative minimum at r_0 , then rises towards zero. The negative well represents a bound pair. Beyond a few femtometres, the potential and its slope become practically zero, reflecting the short range of the nuclear force. The following graph is schematic.



Schematic nucleon potential: repulsive core, attractive well and short-range tail.

Step 2: Read the slope for separations greater than the minimum To the right of the minimum, within the range of the nuclear interaction, the graph slopes upward as r increases. Therefore,

$$r > r_0 : \quad \frac{dU}{dr} > 0 \quad \Rightarrow \quad F_r = -\frac{dU}{dr} < 0.$$

The force points towards decreasing separation, pulling the nucleons towards each other. At large separation the slope approaches zero, so the attraction becomes negligible.

Step 3: Read the slope for separations less than the minimum To the left of r_0 , the potential decreases steeply as the separation increases. Thus,

$$r < r_0 : \quad \frac{dU}{dr} < 0 \quad \Rightarrow \quad F_r = -\frac{dU}{dr} > 0.$$

The force points towards increasing separation and pushes the nucleons apart. At $r = r_0$, the horizontal tangent gives zero net force; small displacements on either side produce a restoring force towards this stable equilibrium separation.

Step 4: Conclusion The potential-energy minimum at r_0 , together with $F_r = -dU/dr$, shows that the

nuclear force is attractive for $r > r_0$ within its range, and repulsive for $r < r_0$. It becomes negligible at sufficiently large separation.

Quick Tip to Solve: Determine attraction or repulsion from the slope of the potential curve, not from the sign of the potential energy: force has the opposite sign to the slope.

29. A charged particle $+q$ in an electric field $\vec{E}$ experiences a force in the direction of the electric field. As a result, its kinetic energy changes. Similarly, the charged particle also experiences a force when it moves in a magnetic field $\vec{B}$. But this magnetic force is perpendicular to both velocity $\vec{v}$ of the charged particle and the magnetic field $\vec{B}$, so it cannot change the kinetic energy of the charged particle. Consider two charged particles 1 and 2 of masses m and $\frac{m}{2}$ having charges $-q$ and $+2q$ respectively. They are accelerated from rest through the same potential difference V and acquire kinetic energy K_1 and K_2 . Then they enter in a region of uniform magnetic field $\vec{B}$ perpendicular to their velocities. [4 marks]

29(i). The ratio of their kinetic energies $\left(\frac{K_1}{K_2}\right)$ is: [1 mark]

- (A) $\frac{1}{2}$
- (B) $\frac{1}{4}$
- (C) 4
- (D) 1

Correct Answer: (A) $\frac{1}{2}$

Solution:

Concept:

- A particle accelerated from rest through an accelerating potential difference of magnitude V gains kinetic energy $K = |Q|V$, where Q is its charge.
- The charge magnitude determines the gained kinetic energy. The mass determines the resulting speed, and a subsequent magnetic force does no work.

Step 1: Calculate the two energy gains The first particle has charge magnitude q , while the second has charge magnitude $2q$. Since both start from rest and are accelerated through the same potential-difference magnitude,

$$K_1 = qV, \quad K_2 = 2qV.$$

The negative sign of the first charge determines its acceleration direction, not a negative kinetic energy.

Step 2: Form the ratio and check the alternatives Cancelling the common accelerating potential gives

$$\frac{K_1}{K_2} = \frac{qV}{2qV} = \frac{1}{2}.$$

The mass ratio must not be inserted into this energy ratio, so $1/4$ is unjustified. Equal accelerating potentials give equal energies only for equal charge magnitudes, ruling out 1. The value 4 is also inconsistent with the second particle gaining twice the first particle's energy.

Step 3: Conclusion $K_1/K_2 = 1/2$. Hence, the correct option is (A).

Quick Tip to Solve: For particles accelerated from rest through the same potential, write $K_1/K_2 = |Q_1|/|Q_2|$ immediately; the masses cancel out of the energy comparison.

29(ii). The ratio of the radii of the circular paths described by them $\left(\frac{r_1}{r_2}\right)$ is: [1 mark]

- (A) $\frac{1}{\sqrt{2}}$
- (B) $\sqrt{2}$
- (C) $\frac{1}{2}$
- (D) 2

Correct Answer: (D) 2

Solution:

Concept:

- For perpendicular motion in a uniform magnetic field, $|Q|vB = Mv^2/r$, giving $r = Mv/(|Q|B)$, where M is the particle mass.
- Acceleration from rest gives $Mv^2/2 = |Q|V$. Combining the two relations gives $r = \sqrt{2MV/|Q|}/B$.

Step 1: Express the radius in terms of mass and charge Substitute $v = \sqrt{2|Q|V/M}$ into the magnetic-radius formula:

$$r = \frac{M}{|Q|B} \sqrt{\frac{2|Q|V}{M}} = \frac{1}{B} \sqrt{\frac{2MV}{|Q|}}.$$

Thus, for the same accelerating potential and magnetic field, the radius is proportional to $\sqrt{M/|Q|}$.

Step 2: Substitute the two masses and charge magnitudes With $M_1 = m$, $M_2 = m/2$, $|Q_1| = q$ and $|Q_2| = 2q$,

$$\frac{r_1}{r_2} = \sqrt{\frac{M_1|Q_2|}{M_2|Q_1|}} = \sqrt{\frac{m(2q)}{(m/2)q}} = \sqrt{4} = 2.$$

The value $\sqrt{2}$ would account for only one of the two changes in mass and charge. The values $1/\sqrt{2}$ and $1/2$ incorrectly make the second particle's radius larger. The charge signs reverse the senses of curvature but do not make a radius negative.

Step 3: Conclusion $r_1/r_2 = 2$. Hence, the correct option is (D).

Quick Tip to Solve: After acceleration through a common potential, use $r \propto \sqrt{M/|Q|}$; do not assume the two particles enter the magnetic field at equal speeds.

29(iii). Suppose particles 1 and 2 enter the magnetic field $\vec{B} = B_0\hat{k}$ with velocities $\vec{v}_1 = v_1\hat{i}$ and $\vec{v}_2 = v_2\hat{i}$. Then: [1 mark]

- (A) both particles revolve clockwise
- (B) both particles revolve anticlockwise
- (C) particle 1 revolves clockwise while particle 2 revolves anticlockwise
- (D) particle 1 revolves anticlockwise while particle 2 revolves clockwise

Correct Answer: (D) particle 1 revolves anticlockwise while particle 2 revolves clockwise

Solution:

Concept:

- The magnetic force is $\vec{F} = Q(\vec{v} \times \vec{B})$. Its direction reverses when the sign of the charge reverses.
- In a right-handed coordinate system, $\hat{i} \times \hat{k} = -\hat{j}$. Clockwise and anticlockwise motion in the xy -plane are described as viewed from the positive z -axis towards the origin.

Step 1: Determine the initial force on each particle Both velocities point along $+\hat{i}$, and the magnetic field points along $+\hat{k}$. Taking v_1, v_2, B_0 as positive magnitudes,

$$\begin{aligned}\vec{F}_1 &= (-q)(v_1\hat{i} \times B_0\hat{k}) = +qv_1B_0\hat{j}, \\ \vec{F}_2 &= (2q)(v_2\hat{i} \times B_0\hat{k}) = -2qv_2B_0\hat{j}.\end{aligned}$$

The negative particle initially bends towards $+y$, while the positive particle bends towards $-y$.

Step 2: Relate the initial bending to the sense of rotation Viewed from $+z$, a particle moving to the right that bends upward turns anticlockwise; one moving to the right that bends downward turns clockwise. Thus opposite signs give opposite senses, ruling out options (A) and (B). Option (C) reverses the charge-dependent forces. The unequal speed magnitudes affect the radii, but do not change the senses of rotation.

Step 3: Conclusion Particle 1 revolves anticlockwise and particle 2 revolves clockwise, viewed from $+z$. Hence, the correct option is (D).

Quick Tip to Solve: First find $\vec{v} \times \vec{B}$ for a positive charge, then reverse the force for a negative charge; use the initial bend to identify the sense of revolution.

29(iv)(a). If period of revolution for particle 1 is 4 s, then for particle 2, the period will be: [1 mark]

- (A) 1 s
- (B) 2 s
- (C) 4 s
- (D) 8 s

Correct Answer: (A) 1 s

Solution:

Concept:

- For a particle of mass M and charge magnitude $|Q|$ moving perpendicular to a uniform magnetic field B , magnetic force provides the centripetal force.
- The period is $T = 2\pi M/(|Q|B)$; it depends on the mass-to-charge ratio, not on the speed. The sign of charge changes the sense of rotation only.

Step 1: Obtain the ratio of periods Using $|Q|vB = Mv^2/r$ and $T = 2\pi r/v$, the speed cancels. Both particles experience the same field, so

$$\frac{T_2}{T_1} = \frac{m_2}{m_1} \frac{|Q_1|}{|Q_2|} = \frac{m/2}{m} \frac{q}{2q} = \frac{1}{4}.$$

Step 2: Substitute the given period The second particle has half the mass and twice the charge magnitude, and each change reduces its period by a factor of two:

$$T_2 = \frac{4\text{ s}}{4} = 1\text{ s}.$$

The value 2 s neglects one of these changes; 4 s incorrectly assumes equal periods, and 8 s gives an increase despite the smaller mass-to-charge ratio.

Step 3: Conclusion The period of particle 2 is 1 s. Hence, the correct option is (A).

Quick Tip to Solve: In the same magnetic field, compare $m/|Q|$ directly; the accelerating voltage is unnecessary for finding the period ratio.

OR

29(iv)(b). If the value of momentum for particles 1 and 2 are p_1 and p_2 , then:

[1 mark]

- (A) $p_1 = \frac{p_2}{2}$
- (B) $p_1 = p_2$
- (C) $p_1 = 2p_2$
- (D) $p_1 = 4p_2$

Correct Answer: (B) $p_1 = p_2$

Solution:

Concept:

- Acceleration from rest through a potential difference of magnitude V gives a particle kinetic energy $K = |Q|V$, where $|Q|$ is its charge magnitude.
- Non-relativistic momentum magnitude satisfies $p = \sqrt{2MK}$, where M is the mass. A magnetic field does no work and therefore preserves this magnitude.

Step 1: Find the kinetic energies The charges are $-q$ and $+2q$, so the energy gains are positive and proportional to their magnitudes:

$$K_1 = qV, \quad K_2 = 2qV.$$

Opposite charge signs determine opposite directions of electric acceleration; they do not make a kinetic energy negative.

Step 2: Compare the momentum magnitudes Particle 2 has twice the kinetic energy but half the mass. Consequently,

$$\frac{p_1}{p_2} = \sqrt{\frac{mK_1}{(m/2)K_2}} = \sqrt{\frac{mqV}{(m/2)(2qV)}} = 1.$$

Thus neither the factor 1/2 in (A) nor the factors 2 and 4 in (C) and (D) survive: the products of mass

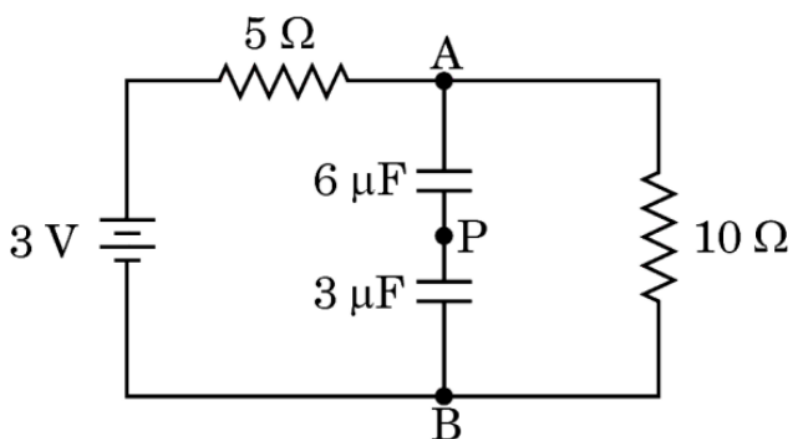
and kinetic energy are equal. This comparison concerns magnitudes, not the momentum vectors as they turn in the field.

Step 3: Conclusion The two momentum magnitudes are equal: $p_1 = p_2$. Hence, the correct option is (B).

Quick Tip to Solve: For acceleration through the same voltage, compare $M|Q|$ inside the square root; here halving the mass exactly cancels doubling the charge.

30. Capacitors are manufactured with certain standard capacitances and working voltages. However, these standard values may not be the ones that are actually needed in a particular application. Two or more capacitors can be grouped in series or in parallel to achieve desired capacitance and voltage. When connected in series, the total capacitance decreases while the voltage rating increases, whereas in parallel connections, the total capacitance increases and maintains the same voltage rating. A capacitor stores energy in the electric field between its plates and stored energy is proportional to the square of the voltage and capacitance $U = \frac{1}{2}CV^2$, where symbols have their usual meanings. Two capacitors, one of $3\ \mu\text{F}$ and the other of $6\ \mu\text{F}$, are connected in series in the circuit as shown in the figure, for a long time.

[4 marks]



30(i). The total capacitance of the circuit is:

[1 mark]

- (A) $6\ \mu\text{F}$
- (B) $3\ \mu\text{F}$
- (C) $9\ \mu\text{F}$
- (D) $2\ \mu\text{F}$

Correct Answer: (D) $2\ \mu\text{F}$

Solution:

Concept:

- Capacitors connected in series carry equal charge magnitudes, while their potential differences add.

- The equivalent capacitance C_{eq} of two series capacitors is given by $1/C_{\text{eq}} = 1/C_1 + 1/C_2$.

Step 1: Identify the capacitor combination The $6\ \mu\text{F}$ and $3\ \mu\text{F}$ capacitors lie consecutively in the branch between A and B , with their common junction at P . They therefore form a series pair. The resistor in the other branch does not change the series-capacitance rule for this pair.

Step 2: Calculate the equivalent capacitance Substitute the two capacitances in the product-over-sum form:

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{6 \times 3}{6 + 3} \mu\text{F} = 2\ \mu\text{F}.$$

This is smaller than either individual capacitance, as required for a series pair. The value $9\ \mu\text{F}$ would apply to a parallel connection; $6\ \mu\text{F}$ and $3\ \mu\text{F}$ simply reproduce individual capacitor values.

Step 3: Conclusion The total capacitance is $2\ \mu\text{F}$. Hence, the correct option is (D).

Quick Tip to Solve: Use product over sum for two capacitors in series, then check that the answer is below the smaller capacitance, here $3\ \mu\text{F}$.

30(ii). The current in the $10\ \Omega$ resistor is:

[1 mark]

- (A) $0.3\ \text{A}$
- (B) $0.6\ \text{A}$
- (C) $0.2\ \text{A}$
- (D) 0

Correct Answer: (C) $0.2\ \text{A}$

Solution:

Concept:

- After a long time with a steady DC source, an ideal capacitor carries no steady current; its branch behaves as an open circuit.
- Ohm's law gives $I = V/R$, where V is the potential difference across the resistance R . Series resistances add.

Step 1: Trace the conducting path at steady state No steady current flows through the central capacitor branch. The outer loop remains complete through the $3\ \text{V}$ battery, the $5\ \Omega$ resistor and the $10\ \Omega$ resistor. These two resistors therefore carry the same current, with total resistance

$$R = 5 + 10 = 15\ \Omega.$$

Step 2: Apply Ohm's law to the outer loop The current in the $10\ \Omega$ resistor is

$$I = \frac{3\ \text{V}}{15\ \Omega} = 0.2\ \text{A}.$$

The value $0.3\ \text{A}$ ignores the $5\ \Omega$ resistor, while $0.6\ \text{A}$ ignores the $10\ \Omega$ resistor. Zero current applies only to the capacitor branch; the outer resistive loop is still closed.

Step 3: Conclusion The current through the $10\ \Omega$ resistor is $0.2\ \text{A}$. Hence, the correct option is (C).

Quick Tip to Solve: At long times, open only the capacitor branch mentally and trace the remaining closed loop before adding the resistances.

30(iii). The potential difference between point A and B is:

[1 mark]

- (A) 2 V
- (B) 0.3 V
- (C) 0.2 V
- (D) 3 V

Correct Answer: (A) 2 V

Solution:

Concept:

- In DC steady state the capacitor branch draws no current, so the source current flows through the two resistors in series.
- Points connected to the same two nodes have the same potential difference. Here $V_{AB} = V_A - V_B$ equals the voltage across the $10\ \Omega$ resistor.

Step 1: Find the steady current The outer loop contains a 3 V battery and resistances $5\ \Omega$ and $10\ \Omega$. Since the central capacitor branch is open to steady current,

$$I = \frac{3}{5 + 10} \text{ A} = 0.2 \text{ A}.$$

Step 2: Calculate the voltage between the indicated nodes The $10\ \Omega$ resistor is connected directly between A and B . The positive battery terminal feeds A through the $5\ \Omega$ resistor, so A is at the higher potential:

$$V_{AB} = I(10\ \Omega) = 0.2 \times 10 \text{ V} = 2 \text{ V}.$$

The remaining 1 V is dropped across $5\ \Omega$, so V_{AB} cannot be the full 3 V supply. Neither 0.3 V nor 0.2 V satisfies Ohm's law for the $10\ \Omega$ resistor at this current.

Step 3: Conclusion The potential difference is $V_A - V_B = 2 \text{ V}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the voltage divider directly: the voltage across AB is $3 \times 10 / (5 + 10) = 2 \text{ V}$, since the capacitors draw no steady current.

30(iv)(a). The value of charge on the plates of the $6\ \mu\text{F}$ capacitor is:

[1 mark]

- (A) $6\ \mu\text{C}$
- (B) $4\ \mu\text{C}$
- (C) $12\ \mu\text{C}$
- (D) $8\ \mu\text{C}$

Correct Answer: (B) $4\ \mu\text{C}$

Solution:

Concept:

- Series capacitors carry equal charge magnitudes Q , with $Q = C_{\text{eq}} V_{AB}$. Here C_{eq} is their equivalent capacitance.
- At DC steady state, the capacitors draw no current. Their total voltage equals the voltage across the parallel $10\ \Omega$ resistor.

Step 1: Find the voltage across the capacitor pair The steady current flows through the $5\ \Omega$ and $10\ \Omega$ resistors in series:

$$I = \frac{3}{5 + 10}\text{ A} = 0.2\text{ A}, \quad V_{AB} = 0.2 \times 10\text{ V} = 2\text{ V}.$$

Step 2: Calculate the common series charge The series capacitance is $C_{\text{eq}} = 6 \times 3 / (6 + 3) = 2\ \mu\text{F}$. Hence the charge magnitude on each capacitor is

$$Q = C_{\text{eq}} V_{AB} = (2\ \mu\text{F})(2\text{ V}) = 4\ \mu\text{C}.$$

Using the full battery voltage would give the incorrect $6\ \mu\text{C}$; using $6\ \mu\text{F}$ across the entire 2 V would give the incorrect $12\ \mu\text{C}$. The value $8\ \mu\text{C}$ incorrectly adds the equal plate-charge magnitudes.

Step 3: Conclusion The plates of the $6\ \mu\text{F}$ capacitor carry $+4\ \mu\text{C}$ and $-4\ \mu\text{C}$, respectively. Hence, the correct option is (B).

Quick Tip to Solve: First find the voltage across the whole series pair; multiply it by the series capacitance to obtain the charge on either capacitor.

OR

30(iv)(b). The wire between two capacitors is cut at point P . The current in the circuit will: [1 mark]

- (A) increase
- (B) decrease
- (C) remain the same
- (D) first increase then become stable

Correct Answer: (C) remain the same

Solution:

Concept:

- Ideal capacitors carry no current after a long time on a steady DC supply. Their branch is already an open circuit for steady current.
- The source current is set by the remaining conducting loop, using $I = V/R$, where V is the battery voltage and R is the loop resistance.

Step 1: Find the current before cutting the wire Initially, the capacitors have reached steady state, so the central branch carries zero current. Only the outer loop, containing the $5\ \Omega$ and $10\ \Omega$ resistors, conducts:

$$I_{\text{before}} = \frac{3}{5 + 10}\text{ A} = 0.2\text{ A}.$$

Step 2: Check which conducting path is interrupted Cutting the wire at P breaks the capacitor branch, which already carried no current. The battery and both resistors remain connected in the same outer

loop, so

$$I_{\text{after}} = \frac{3}{15} \text{ A} = 0.2 \text{ A}.$$

Neither the supply voltage nor the resistance of this loop changes. Thus there is no increase, decrease, or initial rise in its current in the ideal circuit considered here.

Step 3: Conclusion The current remains the same, at 0.2 A. Hence, the correct option is (C).

Quick Tip to Solve: Before analysing a cut, mark the branches that already carry zero current; cutting such a branch leaves the steady resistive current unchanged.

Section E (15 marks)

31(a). (i) A light bulb and an open coil inductor are connected in series across an ac source of variable frequency. How will the glow of the bulb be affected when:

(I) an iron bar is inserted inside the coil, and

(II) the frequency of the source is decreased?

Justify your answers. Assume that in each above case other factors remain unchanged.

(ii) An ac voltage $V = 280 \sin(100\pi t)$ volt is connected across a series LCR circuit in which $R = 400 \, \Omega$, $L = \frac{5}{\pi} \text{ H}$ and $C = \frac{50}{\pi} \, \mu\text{F}$. Taking $\sqrt{2} = 1.4$, calculate:

(I) impedance of the circuit.

(II) rms value of current that flows in the circuit.

(III) power factor of the circuit.

[5 marks]

Solution:

Concept:

- Inductive reactance is $X_L = \omega L = 2\pi fL$, where L is inductance and f is the source frequency. For a bulb and an inductor in series, the current is $I_{\text{rms}} = V_{\text{rms}} / \sqrt{R_{\text{tot}}^2 + X_L^2}$.
- The bulb's brightness depends on its average electrical power $P_b = I_{\text{rms}}^2 R_b$, where R_b is the bulb resistance. Compare the current while keeping the other factors unchanged, as specified.
- In a series LCR circuit, $X_C = 1/(\omega C)$, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and the power factor is $\cos \phi = R/Z$. For a sinusoidal voltage of peak value V_0 , $V_{\text{rms}} = V_0/\sqrt{2}$.

Step 1: (i)(I) Examine the effect of the iron bar Inserting an iron bar increases the magnetic permeability within the coil and hence increases its self-inductance L . At unchanged frequency, $X_L = 2\pi fL$ increases. The series impedance therefore increases, reducing the current through both the coil and the bulb. Since the bulb's power is proportional to the square of this current, its glow becomes dimmer.

Step 2: (i)(II) Examine the effect of reducing frequency In this separate change, the inductance and supply rms voltage remain unchanged. Decreasing f decreases $X_L = 2\pi fL$. The series impedance decreases, so the rms current increases. The bulb dissipates greater power and glows brighter. This reasoning concerns the bulb-inductor circuit in part (i), which contains no capacitor.

Step 3: (ii)(I) Calculate the reactances and impedance Comparing the applied voltage with $V = V_0 \sin \omega t$ gives $V_0 = 280 \text{ V}$ and $\omega = 100\pi \text{ rad s}^{-1}$. Convert the printed capacitance to farads before

finding its reactance:

$$C = \frac{50}{\pi} \times 10^{-6} \text{ F},$$
$$X_L = (100\pi) \frac{5}{\pi} = 500 \Omega,$$
$$X_C = \frac{1}{(100\pi)(50 \times 10^{-6}/\pi)} = 200 \Omega.$$

The net reactance is $X_L - X_C = 300 \Omega$, which is inductive. Therefore the impedance magnitude is

$$Z = \sqrt{400^2 + 300^2} = \sqrt{250000} = 500 \Omega.$$

Step 4: (ii)(II) Use the rms voltage to find the rms current The coefficient 280 is the peak voltage, so it must first be converted to the rms value. Using the approximation specified in the question,

$$V_{\text{rms}} = \frac{280}{1.4} = 200 \text{ V}.$$

Ohm's law for the complete series circuit then gives

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{200}{500} = 0.40 \text{ A}.$$

Step 5: (ii)(III) Calculate the power factor Resistance is the part of the impedance responsible for average power dissipation, so

$$\cos \phi = \frac{R}{Z} = \frac{400}{500} = 0.80.$$

Since $X_L > X_C$, the current lags the applied voltage; this is a lagging power factor. As a check, $\tan \phi = (X_L - X_C)/R = 300/400$, consistent with the same impedance triangle.

Step 6: Conclusion (i) Inserting the iron bar makes the bulb dimmer; decreasing the frequency makes it brighter. (ii) The impedance is 500Ω , the rms current is 0.40 A , and the power factor is 0.80 lagging.

Quick Tip to Solve: Read the coefficient of t as ω , convert microfarads to farads, and subtract X_C from X_L before using the impedance triangle.

31(b). (i) State Lenz's law and explain that it follows the law of conservation of energy.

(ii) Write the dimensional formula for self-inductance.

The current in a coil changes from 8.0 A to 2.0 A in 0.6 s. If an average emf induced in the coil is 50 V, calculate the self-inductance of the coil. [5 marks]

Solution:

Concept:

- Lenz's law states that the induced emf, and hence the induced current in a closed circuit, has a direction such that its magnetic effect opposes the change in magnetic flux that produces it. It is represented by the negative sign in $\varepsilon = -N d\Phi_B/dt$.
- Conservation of energy requires the electrical energy generated by induction to come from work done by an external agent or from energy already stored in the system. Here N is the number of turns and Φ_B is the magnetic flux through one turn.
- For a coil of constant self-inductance L , the average induced emf is $\bar{\varepsilon} = -L \Delta I/\Delta t$. Thus $L = |\bar{\varepsilon}| \Delta t/|\Delta I|$, with SI unit henry.

Step 1: (i) State the law and identify what is opposed The induced current is directed so that the magnetic field it produces opposes the change in the linked magnetic flux. It opposes an increase of flux by producing an opposing flux, and opposes a decrease by producing flux in the original direction. The opposition is to the change in flux, so the induced field is not necessarily opposite to the original field in every situation.

Step 2: (i) Explain consistency with conservation of energy Consider the north pole of a magnet approaching a closed conducting coil. The magnetic flux through the coil increases. By Lenz's law, the face of the coil toward the magnet behaves as a north pole, repelling the approaching magnet. To maintain the motion, an external agent must do work against this opposing force. That mechanical work supplies the induced electrical energy, which can be dissipated as heat in the coil or temporarily stored in its magnetic field.

When the magnet is withdrawn, the coil's near face becomes a south pole and attracts it, again opposing the motion and requiring external work. If the induced current instead aided the flux change, the approaching magnet would be pulled in more strongly while electrical energy was also generated without an energy supply. Such a self-reinforcing production of energy would violate conservation of energy. The opposing direction prescribed by Lenz's law prevents this violation.

Step 3: (ii) Derive the dimensional formula of self-inductance From $\varepsilon = -L dI/dt$, self-inductance has dimensions of emf multiplied by time and divided by current. Voltage is energy per unit charge, so

$$[\varepsilon] = \frac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}].$$

Therefore,

$$[L] = \frac{[\varepsilon][T]}{[A]} = [ML^2T^{-2}A^{-2}].$$

In these dimensional brackets, M, L, T, A represent mass, length, time and electric current respectively; outside the brackets, L denotes the coil's self-inductance.

Step 4: (ii) Calculate the self-inductance of the coil The current falls from 8.0 A to 2.0 A, giving

$$\Delta I = 2.0 - 8.0 = -6.0 \text{ A},$$

$$\frac{\Delta I}{\Delta t} = \frac{-6.0}{0.6} = -10 \text{ A s}^{-1}.$$

Use the magnitude of the average emf to obtain the positive inductance:

$$L = \frac{50}{10} = 5.0 \text{ H}.$$

The negative current-change rate indicates that the induced emf acts to sustain the original current, opposing its decrease as required by Lenz's law; it does not make the inductance negative.

Step 5: Conclusion Lenz's law gives an induced current that opposes the change in magnetic flux, ensuring that induced electrical energy has an energy source. The dimensional formula of self-inductance is $[ML^2T^{-2}A^{-2}]$, and the coil's self-inductance is 5.0 H.

Quick Tip to Solve: For a falling current, use $|\Delta I|$ to calculate the positive inductance; use the minus sign in Lenz's law separately to describe the induced emf's direction.

32(a). (i) A point object is kept in front of a convex spherical surface of radius of curvature R . Draw the ray diagram to show the formation of image and derive the relation between the object and image distance (u and v) in terms of refractive index n of the medium and R .

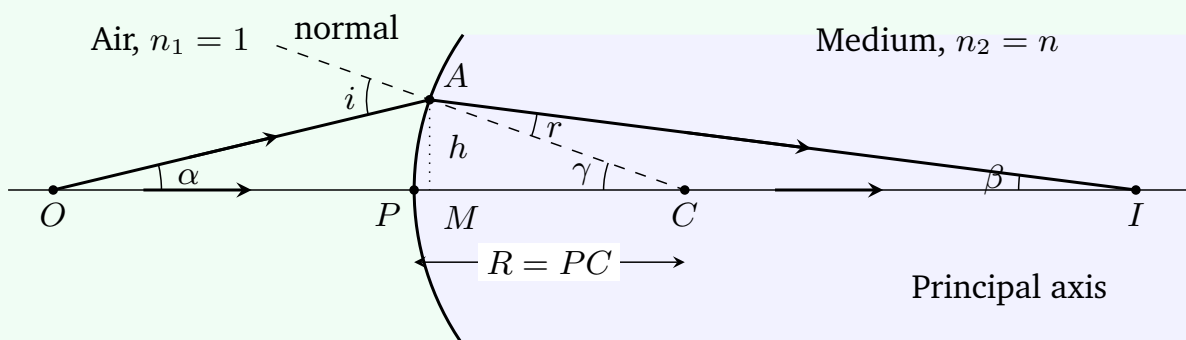
(ii) A convex lens of focal length of 20 cm is used to form the image of an object placed 30 cm away from the lens. Find the position and nature of the image formed. [5 marks]

Solution:

Concept:

- For paraxial refraction from air into a medium of refractive index n , Snell's law is $\sin i = n \sin r$, or $i \simeq nr$ for small angles measured in radians.
- Use the Cartesian sign convention: measure all axial distances from the pole or optical centre, taking the direction of incident light as positive. A real object on the left has $u < 0$; the convex surface shown has $R > 0$.
- A thin lens obeys $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, and its transverse magnification is $m = \frac{v}{u}$. Here f , u and v are signed focal, object and image distances.

Step 1: (i) Construct the refracted rays Let P be the pole of the convex spherical interface, C its centre of curvature and O a point object in air. The medium of index $n > 1$ lies on the right. Draw a ray OA incident near the pole; the radius AC is the normal at A . The ray bends towards this normal and meets the principal axis at I . The axial ray OP is normally incident and continues undeviated to I . Thus the two rays locate the image. The sketch shows the real-image configuration used for the derivation; angles are enlarged for clarity.



Paraxial refraction at a convex spherical interface; AC is the normal, and the axial ray is undeviated.

Step 2: Relate the angles using the ray geometry Drop AM perpendicular to the principal axis and put $AM = h$. Let the positive acute angles that OA , AI and AC make with the axis be α , β and γ , respectively. The angle of incidence is measured between AO and the backward extension of the normal, so $i = \alpha + \gamma$. The refracted ray lies between the forward normal and the horizontal, hence $r = \gamma - \beta$. Applying the small-angle form of Snell's law,

$$\begin{aligned}\alpha + \gamma &= n(\gamma - \beta), \\ \alpha + n\beta &= (n - 1)\gamma.\end{aligned}$$

Step 3: Express the angles in terms of signed distances For a narrow paraxial pencil of rays, the sagitta PM is negligible compared with axial distances. Consequently, using positive lengths OP , PI and PC ,

$$\alpha \simeq \frac{h}{OP}, \quad \beta \simeq \frac{h}{PI}, \quad \gamma \simeq \frac{h}{PC}.$$

Substitution and cancellation of the nonzero height h give

$$\frac{1}{OP} + \frac{n}{PI} = \frac{n - 1}{PC}.$$

In the Cartesian convention, $OP = -u$, $PI = v$ and $PC = R$ for this diagram. Replacing these positive lengths by the corresponding signed distances gives the required spherical-interface relation. The air index has been taken as unity; with incident index n_1 and refracted index n_2 , the same derivation yields $n_2/v - n_1/u = (n_2 - n_1)/R$.

Step 4: (ii) Substitute the lens data The convex lens has $f = +20$ cm. Since the real object is 30 cm in front of it, $u = -30$ cm. Therefore,

$$\begin{aligned}\frac{1}{v} &= \frac{1}{f} + \frac{1}{u} = \frac{1}{20} - \frac{1}{30} \\ &= \frac{1}{60} \text{ cm}^{-1}.\end{aligned}$$

To determine orientation and size, evaluate

$$m = \frac{v}{u} = \frac{60}{-30} = -2.$$

A positive image distance places the image on the side opposite the object. Negative magnification indicates inversion, and its magnitude gives the factor by which the image height exceeds the object height. This also agrees with the ray rule for an object between F and $2F$: its image lies beyond $2F$.

Step 5: Conclusion (i) For refraction from air into the medium, $\frac{n}{v} - \frac{1}{u} = \frac{n-1}{R}$, with Cartesian signed distances and paraxial rays. (ii) $v = +60 \text{ cm}$: the image is formed 60 cm from the lens on the opposite side, and is real, inverted and twice the object's size.

Quick Tip to Solve: Keep the two formulas distinct: the spherical surface has n/v , while the lens has $1/v$. For part (ii), first write $u = -30 \text{ cm}$; using a positive object distance gives the wrong image position.

OR

32(b). (i) Two thin converging lenses of focal length f_1 and f_2 are placed coaxially in contact. Derive expression for the focal length of the combination.

(ii) A beam of coherent light of wavelength 550 nm is incident normal to the plane of a pair of two slits S_1 and S_2 each of width $1.2 \times 10^{-6} \text{ m}$ separated by 1.1 mm. Dark and bright fringes are observed on a screen 2.2 m away from the plane of the slits.

Calculate:

(I) fringe width.

(II) distance of the second dark fringe from the central maximum.

(III) what will happen when the entire apparatus is immersed in water.

[5 marks]

Solution:

Concept:

- For a thin lens, the Cartesian sign convention gives $1/v - 1/u = 1/f$, where u and v are signed object and image distances and f is focal length.
- For coherent slits separated by d , observed on a distant screen at distance D , the interference fringe width is $\beta = \lambda D/d$. The n th dark fringe lies at $y_n = (n - 1/2)\beta$ from the central bright fringe.
- In a medium of refractive index μ , light's frequency is unchanged, whereas its speed and wavelength become c/μ and λ/μ . Fixed slit and screen distances then give $\beta_{\text{medium}} = \beta/\mu$.

Step 1: (i) Apply the lens formula to each lens Let an axial object at signed distance u from the first lens form an intermediate image at signed distance v_1 . For the first converging lens,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}.$$

The intermediate image serves as the object for the second lens. Because the lenses are thin and in contact, their optical centres may be treated as coincident. Thus the signed object distance for the second lens is v_1 ; this is a virtual object when the first lens would focus the incident rays beyond it. If the final image is at v ,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}.$$

Step 2: (i) Obtain the equivalent focal length Add the two equations so that the intermediate-image term cancels:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

An equivalent single lens of focal length F produces the same final image from the same object, so

$1/v - 1/u = 1/F$. Therefore,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}, \quad F = \frac{f_1 f_2}{f_1 + f_2}.$$

Both focal lengths are positive, so the combination is converging and its focal length is smaller than either individual focal length.

Step 3: (ii)(I) Calculate the fringe width Convert the wavelength and the slit separation to SI units: $\lambda = 550 \times 10^{-9} \text{ m}$, $d = 1.1 \times 10^{-3} \text{ m}$, and $D = 2.2 \text{ m}$. The given width of each slit, $1.2 \times 10^{-6} \text{ m}$, affects the diffraction envelope; interference spacing uses the separation between the two slits.

$$\beta = \frac{\lambda D}{d} = \frac{550 \times 10^{-9} \times 2.2}{1.1 \times 10^{-3}} = 1.1 \times 10^{-3} \text{ m} = 1.1 \text{ mm}.$$

Step 4: (ii)(II) Locate the second dark fringe Destructive interference occurs when the path difference is $(2n - 1)\lambda/2$. The first dark fringe is at $\beta/2$; hence the second is at $3\beta/2$, on either side of the centre:

$$|y_2| = \frac{3\beta}{2} = \frac{3}{2}(1.1 \text{ mm}) = 1.65 \text{ mm}.$$

Step 5: (ii)(III) Examine immersion in water The apparatus dimensions stay fixed, while the wavelength in water decreases to λ/μ_w . Thus every fringe distance measured from the centre decreases in the same ratio:

$$\beta_w = \frac{1.1}{\mu_w} \text{ mm}, \quad |y_{2,w}| = \frac{1.65}{\mu_w} \text{ mm}.$$

The fringes move closer together and the central maximum stays at the symmetry centre. Using the usual value $\mu_w = 4/3$, these become $\beta_w = 0.825 \text{ mm}$ and $|y_{2,w}| = 1.2375 \text{ mm}$, respectively.

Step 6: Conclusion The lenses have equivalent focal length $F = f_1 f_2 / (f_1 + f_2)$. The fringe width in air is 1.1 mm , and the second dark fringe is 1.65 mm from the central maximum. Immersion in water contracts the fringe pattern by the factor $1/\mu_w$, giving 0.825 mm spacing if $\mu_w = 4/3$.

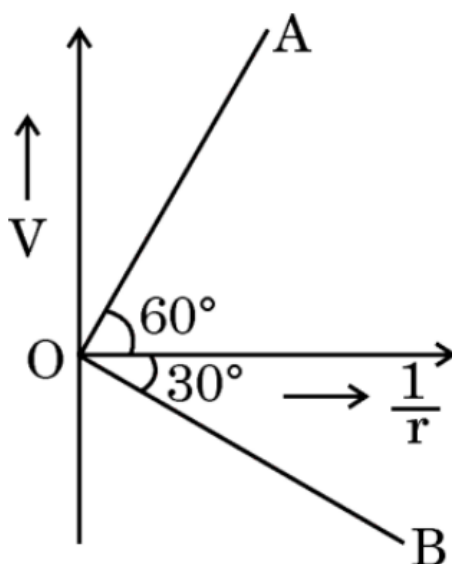
Quick Tip to Solve: Add the two lens equations to eliminate the intermediate image. For the interference calculation, use slit separation as d , and count dark fringes at $\beta/2, 3\beta/2, 5\beta/2, \dots$

33(a). (i) In the figure, OA and OB show the variation of electric potential V at a point due to two point charges Q_1 and Q_2 with $\frac{1}{r}$ respectively. Here r represents the distance of the point from the two point charges.

(I) Identify the nature of the two charges Q_1 and Q_2 .

(II) What is the value of $\left(\frac{Q_1}{Q_2}\right)$? Justify your answer.

(ii) Two point charges $-2 \mu\text{C}$ and $5 \mu\text{C}$ are placed at $(-30 \text{ cm}, 0)$ and $(30 \text{ cm}, 0)$ respectively in an external electric field $\vec{E} = \frac{A}{x^2} \hat{i}$, where $A = 9 \times 10^5 \text{ N m}^2 \text{ C}^{-1}$. Find the electrostatic potential energy of this configuration. [5 marks]



Solution:

Concept:

- A point charge Q produces potential $V = kQ/r$, taking zero at infinity, with $k = 1/(4\pi\epsilon_0)$. Hence the signed slope of a graph of V against $1/r$ is kQ .
- Along the x -axis, the external electric field and its potential are related by $E_x = -dV_{\text{ext}}/dx$. Potential must be obtained by integration when the field varies with position.
- The energy of two charges in an imposed external potential is $U = q_1 V_{\text{ext}}(x_1) + q_2 V_{\text{ext}}(x_2) + kq_1 q_2 / r_{12}$. The mutual interaction is included once, and the self-energies of the point charges are excluded.

Step 1: (i)(I) Determine the signs of the charges For a positive distance r , the sign of the potential is the sign of the source charge. Line OA , representing Q_1 , has a positive slope and lies above the horizontal axis. Therefore Q_1 is positive. Line OB , representing Q_2 , has a negative slope and lies below that axis, so Q_2 is negative.

Step 2: (i)(II) Compare the signed slopes On the same graph, the proportionality between charge and slope gives

$$\frac{Q_1}{Q_2} = \frac{m_{OA}}{m_{OB}} = \frac{\tan 60^\circ}{\tan(-30^\circ)} = \frac{\sqrt{3}}{-1/\sqrt{3}} = -3.$$

The angle below the positive horizontal axis must carry a negative sign. Thus the magnitude of Q_1 is three times that of Q_2 , and their signs are opposite.

Step 3: (ii) Integrate the external field The printed field is $E_x = A/x^2$. Therefore

$$\frac{dV_{\text{ext}}}{dx} = -\frac{A}{x^2}, \quad V_{\text{ext}}(x) = \frac{A}{x} + C.$$

Use the conventional reference $V_{\text{ext}} \rightarrow 0$ at infinity on each side of the singular point $x = 0$, so $C = 0$. The two charge coordinates are $x_1 = -0.30$ m and $x_2 = +0.30$ m. Hence

$$\begin{aligned} V_{\text{ext}}(x_1) &= \frac{9 \times 10^5}{-0.30} = -3.0 \times 10^6 \text{ V}, \\ V_{\text{ext}}(x_2) &= \frac{9 \times 10^5}{+0.30} = +3.0 \times 10^6 \text{ V}. \end{aligned}$$

The coordinate x_1 remains negative in the potential even though the field contains x^2 .

Step 4: (ii) Calculate the energy in the external field Substitute the charges in coulombs and the external potentials at their respective positions:

$$\begin{aligned} U_{\text{ext}} &= q_1 V_{\text{ext}}(x_1) + q_2 V_{\text{ext}}(x_2) \\ &= (-2 \times 10^{-6})(-3 \times 10^6) + (5 \times 10^{-6})(3 \times 10^6) \\ &= 6 + 15 = 21 \text{ J}. \end{aligned}$$

Each term is positive: the negative charge is at negative potential and the positive charge is at positive potential.

Step 5: (ii) Include the interaction between the charges Their separation is $r_{12} = 0.30 - (-0.30) = 0.60$ m. Their mutual energy is negative because the two charges attract:

$$U_{12} = \frac{kq_1q_2}{r_{12}} = \frac{(9 \times 10^9)(-2 \times 10^{-6})(5 \times 10^{-6})}{0.60} = -0.15 \text{ J}.$$

Adding this once to the external-field contribution gives

$$U = U_{\text{ext}} + U_{12} = 21 - 0.15 = 20.85 \text{ J}.$$

Step 6: Conclusion Q_1 is positive, Q_2 is negative, and $Q_1/Q_2 = -3$. With the conventional zero of external potential at infinity, the electrostatic potential energy of the two-charge configuration is 20.85 J.

Quick Tip to Solve: Read charge ratios from signed slopes. In the energy calculation, retain the sign of x in $V = A/x$, and add the mutual pair energy only once.

33(b). (i) Two infinitely long straight wires having linear charge densities $-\lambda$ and 3λ are held vertically parallel to each other, distance r apart in free space. Find the nature and magnitude of the force/length exerted by one wire on the other.

(ii) A small hollow conducting sphere of radius r_1 is given a charge Q . It is surrounded by a concentric conducting spherical shell of inner radius r_2 and outer radius r_3 , having charge $-3q$.

If a point charge $2q$ were kept at the centre, find:

(I) the electric flux through a concentric spherical Gaussian surface of radius x for (1) $x < r_1$, and (2) $r_1 < x < r_2$.

(II) electric field at a point distant x from the centre for (1) $x > r_3$, and (2) $r_1 < x < r_2$.

(III) surface charge density on the inner surface of (1) sphere, and (2) shell.

[5 marks]

Solution:

Concept:

- Gauss's law is $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the algebraic enclosed charge and ϵ_0 is the permittivity of free space.
- An infinite line with linear charge density λ' produces field magnitude $E = |\lambda'|/(2\pi\epsilon_0 r)$ at perpendicular distance r . The force per unit length on a second line is its charge per unit length times the external field.
- In electrostatic equilibrium the field inside conducting material vanishes. Thus the charge induced on a cavity's inner surface cancels the total charge inside the cavity; the remaining conductor charge lies on its outer surface.
- Concentric spherical conductors and a central point charge preserve spherical symmetry. Hence $\vec{E} = Q_{\text{enc}}\hat{r}/(4\pi\epsilon_0 x^2)$, and a uniformly charged spherical surface of radius a has density $\sigma = Q_s/(4\pi a^2)$. Here $\hat{r}$ points radially outward.

Step 1: (i) Obtain the field of one infinitely long charged wire Choose a cylindrical Gaussian surface of radius r and length L around the wire with charge density 3λ . The field is perpendicular to the curved surface, and there is no flux through the flat end faces. Therefore, in magnitude,

$$E(2\pi rL) = \frac{3|\lambda|L}{\epsilon_0}, \quad E = \frac{3|\lambda|}{2\pi\epsilon_0 r}.$$

This is the external field acting on the other wire; a wire's own field is not used to calculate the force exerted by the other wire.

Step 2: (i) Calculate force per unit length and its nature A length L of the second wire carries charge $-\lambda L$. Multiplying its charge magnitude by the field gives

$$\frac{F}{L} = |\lambda|E = \frac{3\lambda^2}{2\pi\epsilon_0 r}.$$

The charge densities have opposite signs, so each wire is pulled toward the other. The forces on corresponding lengths are equal in magnitude and opposite in direction. The SI unit of force per unit length is N m^{-1} .

Step 3: (ii) Determine the charges induced on the conducting surfaces Keep the two symbols distinct: the small sphere has total charge Q , the surrounding shell has total charge $-3q$, and the central point charge is $2q$. A Gaussian surface within the metal of the small sphere must enclose zero net

charge. Consequently its inner surface acquires $-2q$, while charge conservation puts $Q + 2q$ on its outer surface.

A Gaussian surface inside the metal of the surrounding shell must also enclose zero charge. The central charge together with the whole small sphere contributes $2q + Q$. Thus the shell's inner surface carries $-(Q + 2q)$. Its outer surface carries the remaining charge,

$$Q_{\text{shell, out}} = -3q - [-(Q + 2q)] = Q - q.$$

These assignments preserve the specified total charge of each isolated conductor and make the field vanish in each conducting material.

Step 4: (ii)(I) Calculate the flux in the two specified regions For $0 < x < r_1$, the Gaussian sphere lies in the central cavity and encloses only the point charge $2q$. Gauss's law gives

$$\Phi_{x < r_1} = \frac{2q}{\epsilon_0}.$$

For $r_1 < x < r_2$, it encloses both the central point charge and the entire small sphere. The small sphere's net charge remains Q , even though it is redistributed between its surfaces. Hence

$$\Phi_{r_1 < x < r_2} = \frac{Q + 2q}{\epsilon_0}.$$

Charges on the surrounding shell are outside these Gaussian surfaces and do not change either enclosed charge. Flux is an algebraic quantity, with the outward normal used for its sign.

Step 5: (ii)(II) Calculate the electric fields using spherical symmetry For $x > r_3$, the total enclosed charge is the sum of the point charge and the charges of both conductors:

$$Q_{\text{enc}} = 2q + Q - 3q = Q - q.$$

Since the radial field has constant magnitude on a concentric Gaussian sphere,

$$\vec{E}(x) = \frac{Q - q}{4\pi\epsilon_0 x^2} \hat{r}, \quad x > r_3.$$

For $r_1 < x < r_2$, the enclosed charge is instead $Q + 2q$, giving

$$\vec{E}(x) = \frac{Q + 2q}{4\pi\epsilon_0 x^2} \hat{r}, \quad r_1 < x < r_2.$$

A positive coefficient means outward field and a negative coefficient means inward field; its absolute value gives the magnitude. In particular the exterior field vanishes if $Q = q$.

Step 6: (ii)(III) Find the inner surface charge densities The charge is uniform on each spherical surface because the point charge is at the common centre. Divide the corresponding inner surface charge by its area. For the hollow sphere of radius r_1 ,

$$\sigma_{\text{sphere, in}} = \frac{-2q}{4\pi r_1^2}.$$

For the surrounding shell, whose inner radius is r_2 ,

$$\sigma_{\text{shell, in}} = -\frac{Q + 2q}{4\pi r_2^2}.$$

Use the inner radius r_2 , not the outer radius r_3 , for the requested shell density. The shell's net charge $-3q$ determines the charge left on its outer surface; its inner charge is fixed by the zero field condition in its metal.

Step 7: Conclusion (i) The wires attract with $F/L = 3\lambda^2/(2\pi\epsilon_0 r)$.

(ii)(I) The fluxes are $2q/\epsilon_0$ for $x < r_1$, and $(Q + 2q)/\epsilon_0$ for $r_1 < x < r_2$.

(II) The fields are $(Q - q)\hat{r}/(4\pi\epsilon_0 x^2)$ outside the shell, and $(Q + 2q)\hat{r}/(4\pi\epsilon_0 x^2)$ between the sphere and shell.

(III) The inner surface densities are $-2q/(4\pi r_1^2)$ on the sphere and $-(Q + 2q)/(4\pi r_2^2)$ on the shell.

Quick Tip to Solve: First assign each inner surface the negative of the charge in its cavity, then use the conductor's given net charge to find its outer surface charge. Keep Q and q distinct.