

CBSE Class 12 Physics Question Paper 2026 with Solution PDF-Set 55/1/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) Question paper is divided into **FIVE** sections – Sections **A, B, C, D** and **E**.
- (iii) In **Section A** : Question numbers **1** to **16** are Multiple Choice (MCQ) type questions. Each question carries **1** mark.
- (iv) In **Section B** : Question numbers **17** to **21** are Very Short Answer (VSA) type questions. Each question carries **2** marks.
- (v) In **Section C** : Question numbers **22** to **28** are Short Answer (SA) type questions. Each question carries **3** marks.
- (vi) In **Section D** : Question numbers **29** & **30** are Case Study-Based questions. Each question carries **4** marks.
- (vii) In **Section E** : Question numbers **31** to **33** are Long Answer (LA) type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section **A**.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculator is **NOT** allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Section A (16 marks)

1. Four independent waves are expressed as:

(i) $y_1 = A_1 \sin \omega t$ (ii) $y_2 = A_2 \sin 2\omega t$

(iii) $y_3 = A_3 \cos \omega t$ (iv) $y_4 = A_4 \sin(\omega t + \pi/3)$

The interference between two of these waves is possible in:

[1 mark]

- (A) (i) and (iii) only
(B) (iii) and (iv) only
(C) (i), (iii) and (iv) only
(D) All of them

Correct Answer: (C) (i), (iii) and (iv) only

Solution:

Concept:

- Sustained interference requires waves of the same frequency with a constant phase difference.
- A sine and a cosine of the same angular frequency differ only in phase: $\cos \omega t = \sin(\omega t + \pi/2)$.

Step 1: Compare the frequencies and phases Waves (i), (iii) and (iv) all have angular frequency ω . Their phase constants are respectively 0, $\pi/2$ and $\pi/3$, so any pair selected from these has a constant phase difference. Unequal amplitudes do not prevent interference; they affect the contrast of maxima and minima.

Step 2: Exclude the wave with a different frequency Wave (ii) has angular frequency 2ω , so its phase difference from any of the other waves changes with time. It cannot give the sustained interference intended here. Options (A) and (B) each omit another eligible pair, while (D) wrongly includes wave (ii).

Step 3: Conclusion Any two waves selected from (i), (iii) and (iv) can produce sustained interference. Hence, the correct option is (C).

Quick Tip to Solve: Rewrite cosine as a phase-shifted sine, then compare the coefficients of time before comparing phase constants.

2. An electromagnetic wave passes from vacuum into a dielectric medium with relative electrical permittivity $(3/2)$ and relative magnetic permeability $(8/3)$. Then, its:

[1 mark]

- (A) wavelength is doubled and frequency remains unchanged.
(B) wavelength is doubled and frequency is halved.
(C) wavelength is halved and frequency remains unchanged.
(D) wavelength and frequency both will remain unchanged.

Correct Answer: (C) wavelength is halved and frequency remains unchanged.

Solution:

Concept:

- The wave speed in a medium is $v = c/\sqrt{\epsilon_r \mu_r}$, where c is its speed in vacuum and ϵ_r, μ_r are the relative permittivity and permeability.
- Frequency f remains unchanged at a stationary boundary, and wavelength is given by $\lambda = v/f$.

Step 1: Calculate the speed in the medium Substitute both relative material constants; permeability must not be assumed to be unity here:

$$\epsilon_r \mu_r = \frac{3}{2} \times \frac{8}{3} = 4, \quad v = \frac{c}{\sqrt{4}} = \frac{c}{2}.$$

Step 2: Compare the wavelength and frequency The incident wave fixes the oscillation frequency at the interface, so $f_{\text{medium}} = f_{\text{vacuum}}$. Therefore,

$$\frac{\lambda_{\text{medium}}}{\lambda_{\text{vacuum}}} = \frac{v}{c} = \frac{1}{2}.$$

Options (A) and (B) incorrectly double the wavelength; (B) also changes the frequency. Option (D) misses the reduction in wave speed and wavelength.

Step 3: Conclusion The wavelength is halved and the frequency remains unchanged. Hence, the correct option is (C).

Quick Tip to Solve: Multiply the two relative constants before taking their square root; the wavelength changes in the same ratio as the speed.

3. In an unbiased p-n junction, at equilibrium, which of the following statements is true? [1 mark]

- (A) Diffusion current is zero but drift current exists.
- (B) Diffusion current exists but drift current is zero.
- (C) Diffusion and drift currents are equal and opposite.
- (D) Both the diffusion and drift currents exist but are unequal.

Correct Answer: (C) Diffusion and drift currents are equal and opposite.

Solution:

Concept:

- Carrier concentration gradients produce diffusion current across a p-n junction, even without an external bias.
- The built-in electric field of the depletion region produces an opposing drift current. At thermal equilibrium the net current is zero.

Step 1: Identify the two current mechanisms Majority electrons diffuse from the n-side to the p-side, and majority holes diffuse from the p-side to the n-side. The depletion-region field drives charge carriers in the opposite sense, producing drift current. The absence of an applied voltage does not remove either mechanism.

Step 2: Apply the equilibrium condition Charge cannot accumulate continuously at equilibrium, so the total current must vanish. With one direction chosen as positive,

$$I_{\text{diffusion}} + I_{\text{drift}} = 0.$$

Thus the two currents have equal magnitudes and opposite directions. Options (A) and (B) incorrectly eliminate one current; the unequal currents in (D) would produce a nonzero net current and violate equilibrium.

Step 3: Conclusion Diffusion and drift currents are equal in magnitude and opposite in direction.

Hence, the correct option is (C).

Quick Tip to Solve: For an unbiased junction at equilibrium, set the sum of drift and diffusion currents to zero; do not set each current to zero.

4. A conducting wire connects two charged metallic spheres A and B of radii r_1 and r_2 respectively. The distance between the spheres is very large compared to their radii. The ratio of electric fields, (E_A/E_B) at the surfaces of spheres A and B will be: [1 mark]

- (A) $\frac{r_1}{r_2}$
(B) $\frac{r_2}{r_1}$
(C) $\frac{r_1^2}{r_2^2}$
(D) $\frac{r_2^2}{r_1^2}$

Correct Answer: (B) $\frac{r_2}{r_1}$

Solution:

Concept:

- Conductors joined by a conducting wire attain equal potentials in electrostatic equilibrium.
- For an isolated conducting sphere, $V = kQ/r$ and $E = kQ/r^2 = V/r$, where Q is its charge, r its radius and $k = 1/(4\pi\epsilon_0)$.

Step 1: Use equal potentials to compare the charges Since the separation is much larger than the radii, each sphere can be treated as an isolated sphere when calculating its surface potential. On connecting the spheres, charge redistributes until

$$\frac{kQ_A}{r_1} = \frac{kQ_B}{r_2}, \quad \frac{Q_A}{Q_B} = \frac{r_1}{r_2}.$$

Step 2: Calculate the surface-field ratio The surface field depends on both charge and radius:

$$\frac{E_A}{E_B} = \frac{Q_A}{Q_B} \frac{r_2^2}{r_1^2} = \frac{r_1}{r_2} \frac{r_2^2}{r_1^2} = \frac{r_2}{r_1}.$$

Option (A) confuses the charge ratio with the field ratio. Options (C) and (D) introduce squared-radius ratios without accounting correctly for charge redistribution. The smaller sphere has the larger field at the same potential.

Step 3: Conclusion $E_A/E_B = r_2/r_1$. Hence, the correct option is (B).

Quick Tip to Solve: For connected, widely separated spheres, use their common potential and write $E = V/r$ to obtain the inverse-radius ratio directly.

5. The electric potential for various points in x - y plane is given by $V = 1.0x^2 - 2.0y^2$, where V is in volts and x, y are in metres. The angle that the electric field at point $(2.0 \text{ m}, 1.0 \text{ m})$ makes with the positive x -axis is: [1 mark]

- (A) 45°
- (B) 90°
- (C) 135°
- (D) 315°

Correct Answer: (C) 135°

Solution:

Concept:

- Electric field is the negative gradient of potential: $\vec{E} = -\nabla V$. Thus $E_x = -\partial V/\partial x$ and $E_y = -\partial V/\partial y$.
- The signs of both field components determine its quadrant; $\tan \theta = E_y/E_x$ alone does not distinguish opposite directions.

Step 1: Differentiate the potential and evaluate the components For $V = 1.0x^2 - 2.0y^2$, differentiation with respect to each coordinate gives

$$E_x = -2.0x, \quad E_y = 4.0y.$$

At $(x, y) = (2.0 \text{ m}, 1.0 \text{ m})$, the components are $E_x = -4.0 \text{ V m}^{-1}$ and $E_y = +4.0 \text{ V m}^{-1}$.

Step 2: Locate the direction in the correct quadrant The negative horizontal component and positive vertical component place the field in the second quadrant. Equal component magnitudes give a reference angle $\tan^{-1}(1) = 45^\circ$, so the direction measured anticlockwise from the positive x -axis is $180^\circ - 45^\circ$. Option (A) has the wrong horizontal sign, (B) would require a zero horizontal component, and (D) points in the opposite quadrant.

Step 3: Conclusion The electric field makes an angle of 135° with the positive x -axis. Hence, the correct option is (C).

Quick Tip to Solve: Apply the minus sign in $\vec{E} = -\nabla V$ first, then use the signs of E_x and E_y to choose the quadrant.

6. A current of 1.5 A is maintained in a copper wire of length 1 m with area of cross-section $1.7 \times 10^{-7} \text{ m}^2$. The magnitude of electric field in the wire is [$\rho_{\text{Cu}} = 1.7 \times 10^{-8} \Omega \text{ m}$]: [1 mark]

- (A) 0.15 V/m
- (B) 0.30 V/m
- (C) 1.5 V/m
- (D) 3.0 V/m

Correct Answer: (A) 0.15 V/m

Solution:

Concept:

- Microscopic Ohm's law gives $E = \rho J$, where E is electric-field magnitude, ρ is resistivity and J is current density.

- For a uniform wire of cross-sectional area A carrying current I , $J = I/A$, so $E = \rho I/A$.

Step 1: Substitute the printed resistivity and area The paper gives $\rho = 1.7 \times 10^{-8} \Omega \text{ m}$, whereas the area is $A = 1.7 \times 10^{-7} \text{ m}^2$. Their exponents differ by one:

$$E = \frac{(1.7 \times 10^{-8})(1.5)}{1.7 \times 10^{-7}} = 1.5 \times 10^{-1} \text{ V m}^{-1}.$$

Step 2: Check the result using the wire resistance With $L = 1 \text{ m}$, $R = \rho L/A = 0.10 \Omega$, so the voltage drop is $IR = 0.15 \text{ V}$. Dividing by the length again gives the same field. Option (C) loses the factor 10^{-1} from the exponent ratio; (B) and (D) introduce an unsupported factor of two. The length cancels in $E = IR/L$.

Step 3: Conclusion The electric field has magnitude 0.15 V m^{-1} . Hence, the correct option is (A).

Quick Tip to Solve: Cancel the identical 1.7 factors first and retain $10^{-8}/10^{-7} = 10^{-1}$; this prevents the factor-of-ten error.

7. Light from a small object in air falls on a spherical glass surface ($n = 1.5$) of radius of curvature R . A real image of the object will be formed if the object distance u is related to R as: [1 mark]

- (A) $u < \frac{R}{2}$
 (B) $\frac{R}{2} < u < R$
 (C) $R < u < 2R$
 (D) $u > 2R$

Correct Answer: (D) $u > 2R$

Solution:

Concept:

- Refraction at a spherical surface obeys $n_2/v - n_1/u_s = (n_2 - n_1)/R$, where u_s and v are signed object and image distances.
- For a real object in air on the incident side, $u_s = -u$, with $u > 0$ its distance magnitude as used in the options. A real image in glass requires $v > 0$.

Step 1: Substitute the refractive indices with the proper signs Take the positive direction along the incident light. For the converging air-to-glass surface, the centre of curvature is in glass and $R > 0$. With $n_1 = 1$, $n_2 = 1.5$ and $u_s = -u$,

$$\frac{1.5}{v} + \frac{1}{u} = \frac{0.5}{R}, \quad \frac{1.5}{v} = \frac{0.5}{R} - \frac{1}{u}.$$

Step 2: Impose the condition for a real image A finite real image requires the right-hand side to be positive:

$$\frac{0.5}{R} > \frac{1}{u} \implies u > 2R.$$

At $u = 2R$, the refracted rays are parallel and the image is at infinity. Every range in (A), (B) and (C) has $u < 2R$, making $v < 0$ and giving a virtual image instead.

Step 3: Conclusion A real image is formed when the object-distance magnitude satisfies $u > 2R$. Hence, the correct option is (D).

Quick Tip to Solve: Treat the u in the positive-distance options as a magnitude: substitute the signed object coordinate $-u$ into the refraction formula.

8. The ratio of the potential energy to the kinetic energy of an electron in n^{th} orbit of Bohr model of hydrogen atom is: [1 mark]

- (A) $-\frac{1}{2}$
- (B) $\frac{1}{2}$
- (C) 2
- (D) -2

Correct Answer: (D) -2

Solution:

Concept:

- In a Bohr orbit of radius r , the attractive Coulomb force supplies the centripetal force: $mv^2/r = ke^2/r^2$, where $k = 1/(4\pi\epsilon_0)$.
- The electron has kinetic energy $K = mv^2/2$ and electrostatic potential energy $U = -ke^2/r$, taking zero potential energy at infinite separation.

Step 1: Express the kinetic energy in terms of radius Multiply the force equation by $r/2$ to obtain

$$K = \frac{1}{2}mv^2 = \frac{ke^2}{2r}.$$

This is positive, whereas the potential energy of the bound electron-proton pair is negative.

Step 2: Form the requested ratio and compare the options Divide potential energy by kinetic energy in the same orbit:

$$\frac{U}{K} = \frac{-ke^2/r}{ke^2/(2r)} = -2.$$

The radius cancels, so the ratio is independent of the orbit number n . The value $-1/2$ in (A) is the inverse ratio K/U ; the positive values in (B) and (C) ignore the negative potential energy.

Step 3: Conclusion The potential-energy to kinetic-energy ratio is $U/K = -2$ for every Bohr orbit. Hence, the correct option is (D).

Quick Tip to Solve: For a Coulomb-bound Bohr orbit, use $U = -2K$; check which energy is in the numerator before choosing a ratio.

9. Welders wear special glass goggles or face masks with glass windows to protect their eyes from:

[1 mark]

- (A) Infrared rays
- (B) Ultraviolet rays
- (C) X-rays
- (D) Microwaves

Correct Answer: (B) Ultraviolet rays

Solution:

Concept:

- An electric welding arc is a strong source of ultraviolet radiation as well as visible and infrared radiation.
- Ultraviolet radiation can damage the surface tissues of the eye; suitable protective glass filters this radiation before it reaches the eyes.

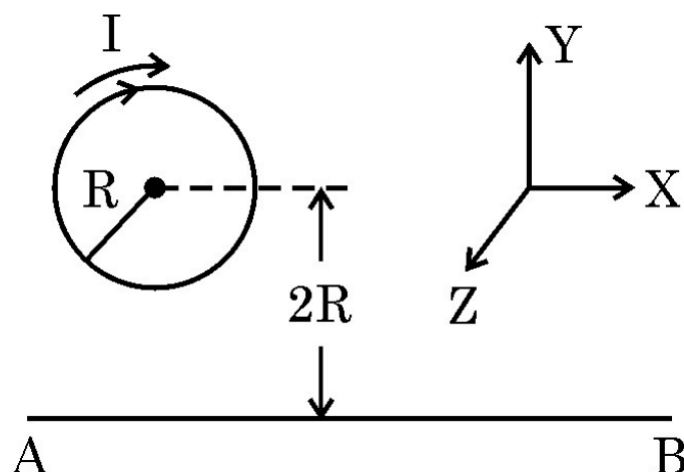
Step 1: Identify the radiation associated with the welding arc The very hot electric arc emits intense ultraviolet rays. These rays are invisible, so reducing visible glare alone does not establish that the eyes are adequately protected. The special glass used in welding goggles or face shields provides the relevant filtering.

Step 2: Compare the listed alternatives Infrared radiation is also present and is associated with heating, but ultraviolet radiation is the standard eye-damaging radiation targeted by this question. X-rays and microwaves are not the characteristic radiation for which the stated welding-eye protection is used. Thus (A), (C) and (D) do not identify the intended radiation.

Step 3: Conclusion Welders use the special protective glass to protect their eyes from ultraviolet rays. Hence, the correct option is (B).

Quick Tip to Solve: Link an electric welding arc with intense ultraviolet emission when identifying the purpose of special protective goggles.

10. A circular loop has radius R and carries current I as shown in figure. In order that the net magnetic field at the centre of the loop is zero, the current in wire AB should have magnitude: [1 mark]



- (A) $2\pi I$, along $+X$ -axis
- (B) $2\pi I$, along $-X$ -axis
- (C) πI , along $+X$ -axis
- (D) πI , along $-X$ -axis

Correct Answer: (A) $2\pi I$, along $+X$ -axis

Solution:

Concept:

- A circular loop of radius R carrying current I produces a central magnetic field of magnitude $B_{\text{loop}} = \mu_0 I / (2R)$.
- A long straight wire carrying current I_w produces $B_{\text{wire}} = \mu_0 I_w / (2\pi d)$ at perpendicular distance d . The right-hand rule gives each field's direction.

Step 1: Find the wire current from the field magnitudes The figure labels the perpendicular distance from the loop's centre to wire AB as $d = 2R$. Equal opposing fields are needed for zero resultant:

$$\frac{\mu_0 I}{2R} = \frac{\mu_0 I_w}{2\pi(2R)} \Rightarrow I_w = 2\pi I.$$

Using R instead of the labelled distance $2R$ would incorrectly give πI , so (C) and (D) have the wrong magnitude.

Step 2: Apply the right-hand rule to select the direction The loop current is clockwise as viewed on the page, so its field at the centre points into the page. A wire current from A to B, along $+X$, produces a field out of the page at the loop centre above the wire. This opposes the loop field. Reversing the wire current would make the two fields reinforce, so (B) fails the direction test.

Step 3: Conclusion The wire must carry current $2\pi I$ from A to B, along the $+X$ -axis. Hence, the correct option is (A).

Quick Tip to Solve: Read the centre-to-wire distance as $2R$, then check magnitude and field direction separately with the right-hand rule.

11. The phenomenon of interference is shown by:

[1 mark]

- (A) longitudinal mechanical wave only
- (B) transverse mechanical wave only
- (C) electromagnetic waves only
- (D) all these waves

Correct Answer: (D) all these waves

Solution:

Concept:

- Interference is the redistribution of wave intensity when waves superpose; reinforcement or cancellation depends on their relative phase.
- The superposition principle applies to longitudinal and transverse mechanical waves as well as electromagnetic waves. A stable interference pattern requires suitable coherence.

Step 1: Check the mechanical wave alternatives Longitudinal waves, such as sound waves, can reinforce or cancel where they overlap. Transverse mechanical waves, such as waves on a stretched string, can also superpose constructively or destructively. Thus both listed mechanical wave types can show interference; neither exclusive claim in (A) or (B) is correct.

Step 2: Include electromagnetic waves Light is an electromagnetic wave and exhibits interference, as in Young's double-slit experiment. However, interference is not restricted to electromagnetic waves, so the word "only" makes (C) incorrect. The phenomenon is a general consequence of superposition under appropriate conditions, rather than a test for a particular type of wave.

Step 3: Conclusion Longitudinal mechanical waves, transverse mechanical waves and electromagnetic waves can all show interference. Hence, the correct option is (D).

Quick Tip to Solve: Interference tests superposition, while polarisation tests transverse character; do not confuse the two phenomena.

12. A series LCR circuit with $R = 3\ \Omega$, $X_C = 4\ \Omega$, $X_L = 8\ \Omega$ is connected to a 220 V, 50 Hz ac source. The power factor for the circuit is:

[1 mark]

- (A) 0.30
- (B) 0.45
- (C) 0.50
- (D) 0.60

Correct Answer: (D) 0.60

Solution:

Concept:

- In a series LCR circuit, the net reactance is $X = X_L - X_C$, where X_L and X_C are the inductive and capacitive reactances.
- The impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and the power factor is $\cos \phi = R/Z$, with ϕ the phase difference between voltage and current.

Step 1: Calculate the net reactance and impedance The printed values are $R = 3\ \Omega$, $X_C = 4\ \Omega$ and

$X_L = 8 \Omega$. Thus

$$X = 8 - 4 = 4 \Omega,$$

$$Z = \sqrt{3^2 + 4^2} = 5 \Omega.$$

Because the net reactance is positive, the circuit is inductive and the current lags the source voltage.

Step 2: Find the ratio of resistance to impedance Substitute in the power-factor formula:

$$\cos \phi = \frac{R}{Z} = \frac{3}{5} = 0.60.$$

The source voltage is not needed for this ratio, and the stated reactances already incorporate the frequency. Options (A), (B) and (C) do not equal R/Z ; adding reactances or using $R/(R + X)$ would not give the impedance triangle.

Step 3: Conclusion The power factor is 0.60, with lagging current. Hence, the correct option is (D).

Quick Tip to Solve: Subtract the two reactances first; the resulting 3-4-5 impedance triangle gives $\cos \phi = 3/5$ immediately.

13. For question numbers 13 to 16, two statements are given – one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer to these questions from options (A), (B), (C) and (D) as given below:

Reason (A): The mass of a nucleus is less than the sum of the masses of the constituent nucleons.

Reason (R): Energy is absorbed when the nucleons are bound together to form a nucleus. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- The mass defect of a bound nucleus is $\Delta m = Zm_p + Nm_n - M$, where Z and N are its proton and neutron numbers, and M is its nuclear mass.
- Binding energy is $B = \Delta mc^2$. It is released in forming a bound nucleus from free nucleons and must be supplied to separate that nucleus into its nucleons.

Step 1: Evaluate statement A Treat the first statement, labelled A, as the assertion in the given answer-code scheme. A bound nucleus has lower rest energy than its separated constituent nucleons. Consequently $\Delta m > 0$, so $M < Zm_p + Nm_n$. The assertion concerning its mass is true.

Step 2: Evaluate statement R When initially free nucleons bind to form a nucleus, the decrease in rest energy is released as binding energy. Energy is absorbed in the reverse process of breaking the nucleus apart. The reason incorrectly says that energy is absorbed during formation, so it is false.

Step 3: Check the explanation and answer codes A false reason cannot explain the true mass-defect statement. Options (A) and (B) require a true reason, while (D) requires a false assertion; each

conflicts with the evaluations above. The appropriate code is the one for a true assertion and a false reason.

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Formation of a bound nucleus releases energy; separating it absorbs energy. Use that direction check to decide the reason.

14. Assertion (A): All atoms have a net magnetic moment.

Reason (R): A current loop does not always behave as a magnetic dipole.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false.

Solution:

Concept:

- An atom's magnetic moment is the vector sum of contributions from its electrons' orbital motion and spin; these contributions can cancel.
- A planar current-carrying loop has magnetic dipole moment $\mu = IA$, where I is the current and A is the oriented area vector.

Step 1: Evaluate the assertion about all atoms In some atoms the electron magnetic moments cancel, leaving zero net permanent magnetic moment. Atoms with closed electronic shells provide familiar examples, whereas unpaired electrons can give other atoms a non-zero moment. Thus the claim that all atoms have a net magnetic moment is false.

Step 2: Evaluate the reason about a current loop A current-carrying loop has a magnetic dipole moment with direction given by the right-hand rule. Its torque in a uniform magnetic field is $\tau = \mu \times B$, the standard magnetic-dipole behaviour. Therefore the statement that a current loop does not always behave as a magnetic dipole is false in this context.

Step 3: Test the explanatory link and options The atomic cancellation in the assertion is separate from the magnetic-dipole character of an individual current loop. Since both statements are false, the reason cannot provide a true explanation. Options (A) and (B) require both statements to be true, while (C) requires a true assertion, so none fits.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish an individual electron's magnetic moment from the vector sum for the atom; paired contributions can cancel completely.

15. Reason (A): In Bohr model of hydrogen atom, the energy levels are discrete and quantised.

Reason (R): In a hydrogen atom, the electrostatic force on the electron provides the necessary centripetal force to it to revolve around the nucleus. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- In Bohr's model, angular momentum is quantised: $m_e v_n r_n = n\hbar$, where $n = 1, 2, 3, \dots$, m_e is the electron mass and $\hbar = h/(2\pi)$.
- The Coulomb attraction provides centripetal force: $m_e v^2/r = e^2/(4\pi\epsilon_0 r^2)$. This classical force equation alone does not select discrete orbits.

Step 1: Judge statement A The first statement serves as the assertion, although the English paper labels it Reason (A). It is true: the allowed hydrogen energies are $E_n = -13.6/n^2$ eV, with integral n . Thus the energies have separate allowed values rather than a continuous range.

Step 2: Judge statement R The reason is also true. The attractive electrostatic force between the proton and electron is directed towards the nucleus and supplies the centripetal force required for the circular orbit in Bohr's model.

Step 3: Test whether R explains A Coulomb's law and circular motion also apply classically, where orbital radii and energies can vary continuously. Discrete energies require the additional angular-momentum quantisation postulate. Therefore R does not explain A: option (A) wrongly asserts this link, while (C) and (D) wrongly declare a true statement false.

Step 4: Conclusion Both A and R are true, but R is not the correct explanation of A. Hence, the correct option is (B).

Quick Tip to Solve: For Bohr-model assertion questions, distinguish the force that maintains an orbit from the quantisation condition that selects allowed orbits.

16. Assertion (A): If accelerated electrons are passed through a narrow slit, a diffraction pattern is observed.

Reason (R): Electrons behave as both particles and waves.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- A moving electron has de Broglie wavelength $\lambda = h/p$, where h is Planck's constant and p is its momentum.
- Diffraction is the spreading and interference of a wave after passage through an aperture; it becomes observable when the slit is sufficiently narrow relative to the wavelength and the observation geometry.

Step 1: Judge the assertion The assertion is true. A beam of accelerated electrons can produce a diffraction pattern after passing through a suitably narrow slit. Although each electron is detected at a localised point, the accumulated distribution has maxima and minima characteristic of wave diffraction.

Step 2: Judge the reason The reason is true. Electrons show particle properties, such as localised detection and definite charge, and wave properties described by their de Broglie wavelength. Acceleration changes their momentum and hence their wavelength; it does not eliminate their wave behaviour.

Step 3: Establish the explanatory link The wave aspect of electron behaviour accounts directly for the diffraction pattern stated in A. Thus R explains A. Option (B) misses this connection, while options (C) and (D) incorrectly deny the truth of the reason or of both statements.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: When an electron experiment shows diffraction or interference, connect the observation directly to its de Broglie wave nature.

Section B (10 marks)

17. What is the order of magnitude of drift velocity of electrons in a conductor? Deduce the relation between the current flowing through a conductor and drift velocity of electrons in it. [2 marks]

Solution:

Concept:

- Drift velocity is the small average velocity acquired by free electrons under an applied electric field, superposed on their random thermal motion.
- Electric current is the rate of flow of charge, $I = \Delta Q / \Delta t$. Let n be the number of free electrons per unit volume, A the conductor's cross-sectional area and e the magnitude of electron charge.

Step 1: State the typical drift-speed scale The usual order of magnitude of electron drift speed in a metallic conductor is 10^{-4} m s^{-1} . This is a typical value, not a universal constant: it depends on the current density and free-electron density. The net electron drift is opposite to the applied electric field, whereas conventional current is along the field.

Step 2: Count the electrons crossing a section Let the magnitude of drift velocity be v_d . In a time interval Δt , electrons drift through the distance $v_d \Delta t$. Thus the volume whose free electrons cross a chosen section is $Av_d \Delta t$, and the corresponding number and charge magnitude are

$$N = nAv_d \Delta t, \quad \Delta Q = Ne = neAv_d \Delta t.$$

Step 3: Relate current to drift speed Dividing the charge crossing the section by the elapsed time gives

$$I = \frac{\Delta Q}{\Delta t} = neAv_d.$$

This equation relates magnitudes. In vector form, the current density is $\mathbf{J} = -nev_d$, because electrons carry negative charge.

Step 4: Conclusion The typical drift speed is of order 10^{-4} m s^{-1} , and the required magnitude relation is $I = neAv_d$.

Quick Tip to Solve: Start with the swept volume $Av_d \Delta t$; multiply by electron density and charge, then divide by Δt to obtain current.

18. A wire of length L is bent round into (i) a square coil having N turns and (ii) a circular coil having N turns. The coil in both cases is free to turn about a vertical axis coinciding with the plane of the coil, in a uniform, horizontal magnetic field and carry the same currents. Find the ratio of the maximum value of the torque acting on the square coil to that on the circular coil. [2 marks]

Solution:

Concept:

- The torque on a plane coil is $\tau = NIAB \sin \theta$, where N is the number of turns, I the current, A the area of one turn, B the magnetic field and θ the angle between the coil's normal and the field.
- Maximum torque is $\tau_{\max} = NIAB$. With equal N , I and B , the torque ratio equals the ratio of the enclosed areas per turn.

Step 1: Find the area of each square turn The total wire length L is shared among N identical turns,

so each turn has perimeter L/N . If the square side is s , then

$$4s = \frac{L}{N}, \quad s = \frac{L}{4N}, \quad A_s = s^2 = \frac{L^2}{16N^2}.$$

Step 2: Find the area of each circular turn If the circular turn has radius r , its circumference is also L/N . Therefore

$$2\pi r = \frac{L}{N}, \quad r = \frac{L}{2\pi N}, \quad A_c = \pi r^2 = \frac{L^2}{4\pi N^2}.$$

Step 3: Compare the maximum torques The coil can turn about the given vertical axis to make its normal perpendicular to the horizontal field, so $\sin \theta = 1$ is attainable. Cancelling the common factors gives

$$\frac{\tau_{s,\max}}{\tau_{c,\max}} = \frac{NIBA_s}{NIBA_c} = \frac{L^2/(16N^2)}{L^2/(4\pi N^2)} = \frac{\pi}{4}.$$

The circular turn encloses more area for the same perimeter, which is consistent with its larger torque.

Step 4: Conclusion The ratio of maximum square-coil torque to maximum circular-coil torque is $\pi : 4$, or $\tau_{s,\max}/\tau_{c,\max} = \pi/4$.

Quick Tip to Solve: Divide the total wire length by the number of turns before finding either area; equal current, field and turns then leave only an area ratio.

19(a). A beam of light consisting of two wavelengths 400 nm and 600 nm is used to illuminate a single slit of width 1 mm. Find the least distance of the point from the central maximum where the dark fringes due to both wavelengths coincide on the screen placed 1.5 m from the slit. [2 marks]

Solution:

Concept:

- For single-slit diffraction, dark fringes obey $a \sin \theta = m\lambda$, where a is slit width, λ the wavelength and $m = 1, 2, 3, \dots$ the order of a minimum on either side.
- For a distant screen and small angle, the distance from the centre is $y_m \simeq m\lambda D/a$, where D is the slit-to-screen distance. Coincident minima must have the same y .

Step 1: Find the lowest coincident orders Let the orders for $\lambda_1 = 400$ nm and $\lambda_2 = 600$ nm be m_1 and m_2 . The same slit and screen give

$$m_1\lambda_1 = m_2\lambda_2 \Rightarrow 400m_1 = 600m_2 \Rightarrow 2m_1 = 3m_2.$$

The smallest positive integers are $m_1 = 3$, $m_2 = 2$. Zero order is excluded because the centre is a maximum, not a dark fringe.

Step 2: Calculate the distance from the centre Convert the slit width to $a = 1 \times 10^{-3}$ m and use $D = 1.5$ m. The third minimum for the shorter wavelength is located at

$$y \simeq \frac{3(400 \times 10^{-9})(1.5)}{1 \times 10^{-3}} = 1.8 \times 10^{-3} \text{ m}.$$

Substituting $m_2 = 2$ and 600×10^{-9} m gives the same value. Also $\sin \theta = m_1\lambda_1/a = 1.2 \times 10^{-3}$, confirming the small-angle approximation.

Step 3: Conclusion The least distance is $\boxed{1.8 \text{ mm}}$ on either side of the central maximum, where the third dark fringe for 400 nm meets the second for 600 nm.

Quick Tip to Solve: Reduce 400 : 600 to 2 : 3, then reverse this ratio to get the first coincident dark orders 3 : 2; exclude the central maximum.

OR

19(b). In a Young's double-slit experimental set-up with slit separation 0.6 mm a beam of light consisting of two wavelengths 440 nm and 660 nm is used to obtain interference pattern on a screen kept 1.5 m in front of the slits. Find the least distance of the point from the central maximum where the bright fringes due to both the wavelengths coincide. [2 marks]

Solution:

Concept:

- In Young's double-slit experiment, a bright fringe of order n occurs at $y_n = n\lambda D/d$, where D is the screen distance and d is the slit separation.
- Bright fringes of two wavelengths coincide when their path differences agree: $n_1\lambda_1 = n_2\lambda_2$. The first coincidence away from the central maximum uses the smallest positive integer orders.

Step 1: Find the smallest coincident fringe orders Let n_1 correspond to 440 nm and n_2 to 660 nm. At a common bright fringe,

$$n_1(440) = n_2(660) \Rightarrow \frac{n_1}{n_2} = \frac{3}{2}.$$

The smallest nonzero orders are therefore $n_1 = 3$ and $n_2 = 2$. The central maximum has both orders zero; the requested least distance refers to the next common bright fringe.

Step 2: Calculate its distance from the centre Convert the slit separation to SI units: $d = 0.6 \text{ mm} = 6.0 \times 10^{-4} \text{ m}$. Using $D = 1.5 \text{ m}$,

$$y = \frac{3(440 \times 10^{-9})(1.5)}{6.0 \times 10^{-4}} = 3.3 \times 10^{-3} \text{ m}.$$

Using the second order of 660 nm gives the same value, because $3(440) = 2(660) = 1320 \text{ nm}$.

Step 3: Conclusion The least nonzero distance is $\boxed{3.3 \text{ mm}}$, on either side of the central maximum. The third bright fringe of 440 nm coincides with the second of 660 nm.

Quick Tip to Solve: Reduce the wavelength ratio 440 : 660 to 2 : 3; reverse it to obtain the first coincident bright-fringe orders 3 : 2.

20. In an electron microscope, accelerated electrons have wavelength of 0.011 nm. Calculate the voltage through which electrons were accelerated to attain this wavelength.

(Take $e = 1.6 \times 10^{-19}$ C, $m_e = 9 \times 10^{-31}$ kg, $h = 6.6 \times 10^{-34}$ J.s)

[2 marks]

Solution:

Concept:

- The de Broglie wavelength of an electron is $\lambda = h/p$, where h is Planck's constant and p is its momentum.
- An electron accelerated from rest through a potential difference V gains kinetic energy $eV = p^2/(2m_e)$. Thus, in the usual nonrelativistic treatment, $\lambda = h/\sqrt{2m_e eV}$.

Step 1: Express the accelerating voltage in terms of wavelength Substitute $p = h/\lambda$ into the kinetic-energy equation and rearrange:

$$eV = \frac{h^2}{2m_e \lambda^2} \Rightarrow V = \frac{h^2}{2m_e e \lambda^2}.$$

Convert the given wavelength before substitution:

$$\lambda = 0.011 \text{ nm} = 0.011 \times 10^{-9} \text{ m} = 1.1 \times 10^{-11} \text{ m}.$$

Step 2: Substitute the printed constants Use $m_e = 9 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, and $h = 6.6 \times 10^{-34}$ J s:

$$\begin{aligned} V &= \frac{(6.6 \times 10^{-34})^2}{2(9 \times 10^{-31})(1.6 \times 10^{-19})(1.1 \times 10^{-11})^2} \\ &= \frac{4.356 \times 10^{-67}}{3.4848 \times 10^{-71}} = 1.25 \times 10^4 \text{ V}. \end{aligned}$$

This is the magnitude of the accelerating potential difference; the electron gains energy as it moves toward the higher electric potential.

Step 3: Conclusion The required accelerating voltage is $V = 1.25 \times 10^4 \text{ V} = 12.5 \text{ kV}$, using the standard nonrelativistic de Broglie relation.

Quick Tip to Solve: Convert 0.011 nm to 1.1×10^{-11} m first, then square the entire wavelength in $V = h^2/(2m_e e \lambda^2)$.

21. Suppose a nucleus with mass number $A = 240$ and B.E. per nucleon = 7.6 MeV, breaks into two nuclei, each of mass number $A = 120$ with B.E. per nucleon = 8.5 MeV. Calculate the energy released in the process.

[2 marks]

Solution:

Concept:

- Total nuclear binding energy equals the mass number times the binding energy per nucleon: $B = A(B/A)$. Here A is the number of nucleons.
- When the products have a greater total binding energy than the original nucleus, their combined rest energy is lower. The energy released is $Q = B_{\text{products}} - B_{\text{parent}}$.

Step 1: Calculate the binding energy of the parent nucleus The original nucleus contains 240 nucleons

with a binding energy of 7.6 MeV per nucleon. Therefore,

$$B_{\text{parent}} = 240 \times 7.6 = 1824 \text{ MeV}.$$

The supplied 7.6 MeV is an energy per nucleon, so it must be multiplied by the mass number before comparing whole nuclei.

Step 2: Find the increase in total binding energy Each daughter nucleus contains 120 nucleons and has total binding energy $120 \times 8.5 = 1020 \text{ MeV}$. There are two daughters, so

$$B_{\text{products}} = 2 \times 120 \times 8.5 = 2040 \text{ MeV},$$
$$Q = 2040 - 1824 = 216 \text{ MeV}.$$

The nucleon count is conserved, since $2 \times 120 = 240$. Their average binding increases by $8.5 - 7.6 = 0.9 \text{ MeV}$ per nucleon, which independently gives $240 \times 0.9 = 216 \text{ MeV}$.

Step 3: Conclusion The energy released in the fission of one parent nucleus is 216 MeV. The positive release follows from the increased binding of the two daughter nuclei.

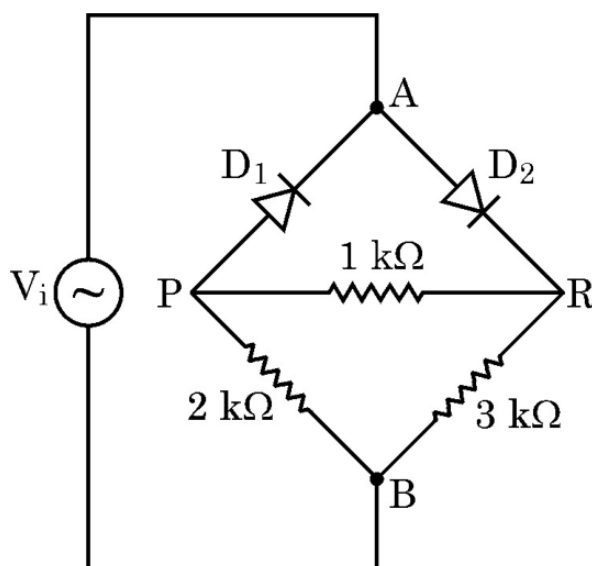
Quick Tip to Solve: When all 240 nucleons acquire the same increase in binding energy per nucleon, multiply 240 by $8.5 - 7.6$; do not count only one daughter.

Section C (21 marks)

22. An ac voltage $V_i = 12 \sin(100\pi t)$ V is applied between points A and B in a network of two ideal diodes and three resistors as shown in figure.

During the positive half-cycle of the input voltage V_i supplied to the network:

- Identify which of the two diodes will conduct and why?
 - Redraw an equivalent circuit diagram to show the flow of current.
 - Calculate the output voltage drops V_o across the three resistors when the input voltage attains its peak value.
- [3 marks]



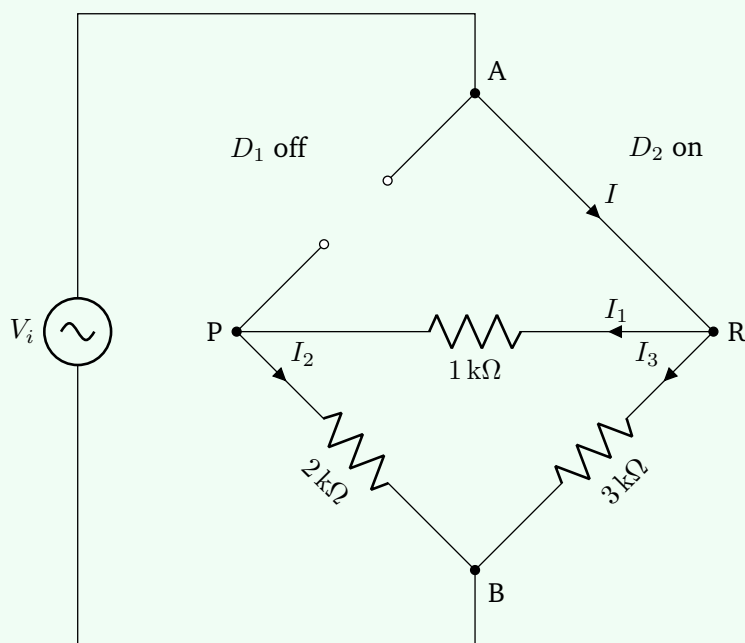
Solution:

Concept:

- An ideal diode is a short circuit when forward biased and an open circuit when reverse biased. Forward bias means that the anode is at a higher potential than the cathode.
- In a conducting resistor branch, Ohm's law gives the drop $V = IR$. Series resistors carry the same current, while parallel branches have the same end-to-end voltage.

Step 1: Identify the conducting diode during the positive half-cycle For $V_i > 0$, point A is positive relative to B. In the printed circuit, D_2 has its anode at A and cathode at R, so it is forward biased and conducts. Its ideal voltage drop is zero, giving $V_R = V_A$. Diode D_1 has its cathode at A and anode at P. The resistor divider between R and B places P below A in potential, so D_1 is reverse biased and does not conduct.

Step 2: Replace the diodes by their equivalent connections Replace D_2 by a wire and D_1 by an open gap. Current flows from A through the conducting D_2 to R, then divides into two paths: directly through $3\text{ k}\Omega$ to B, and through $1\text{ k}\Omega$ from R to P followed by $2\text{ k}\Omega$ from P to B. Thus the two parallel branches are $3\text{ k}\Omega$ and $(1 + 2)\text{ k}\Omega$. The arrows in the equivalent circuit show conventional current during the positive half-cycle.



Positive-half-cycle equivalent circuit. The open gap represents D_1 ; the A–R wire represents conducting D_2 .

Step 3: Calculate the peak current and the three voltage drops At the positive peak, $V_i = 12\text{ V}$. The series branch and the single-resistor branch each have resistance $3\text{ k}\Omega$, so

$$I_1 = I_2 = \frac{12}{(1 + 2) \times 10^3} = 4\text{ mA}, \quad I_3 = \frac{12}{3 \times 10^3} = 4\text{ mA}.$$

Apply $V = IR$ to each resistor:

$$V_R - V_P = (4 \times 10^{-3})(1 \times 10^3) = 4\text{ V},$$

$$V_P - V_B = (4 \times 10^{-3})(2 \times 10^3) = 8\text{ V},$$

$$V_R - V_B = (4 \times 10^{-3})(3 \times 10^3) = 12\text{ V}.$$

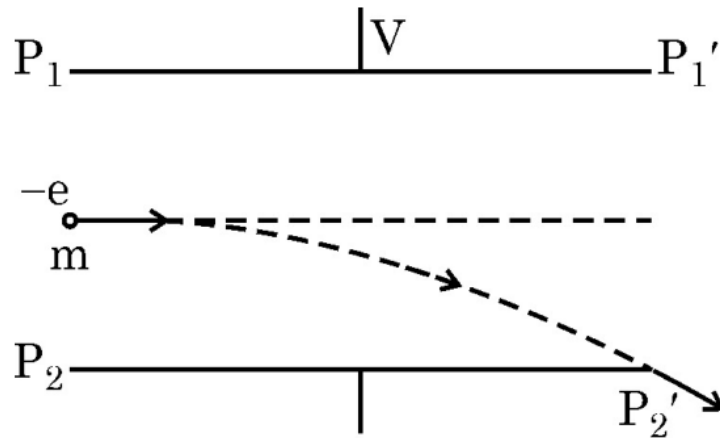
Setting $V_B = 0$ gives $V_A = V_R = 12\text{ V}$ and $V_P = 8\text{ V}$, confirming the reverse bias of D_1 . The source supplies $I = I_1 + I_3 = 8\text{ mA}$ at the peak.

Step 4: Conclusion D_2 conducts; D_1 is off. At the positive peak, the voltage drops across $1\text{ k}\Omega$, $2\text{ k}\Omega$, and $3\text{ k}\Omega$ are respectively 4 V , 8 V , 12 V , with the polarities specified above.

Quick Tip to Solve: First replace the on diode by a wire and the off diode by a gap; then recognize $1\text{ k}\Omega + 2\text{ k}\Omega$ in parallel with $3\text{ k}\Omega$.

23. Figure shows a narrow beam of electrons entering with a velocity of 3×10^7 m/s, symmetrically through the space between two parallel horizontal plates P_1P_1' and P_2P_2' kept 2 cm apart.

If each plate is 3 cm long, calculate the potential difference V applied between the plates so that the beam just strikes the end P_2' . [3 marks]



Solution:

Concept:

- Between parallel plates separated by d , the uniform electric field has magnitude $E = V/d$, where V is the potential difference magnitude; edge effects are neglected.
- An electron of mass m_e and charge $-e$ has acceleration magnitude $a = eE/m_e$, opposite to the electric field. Its horizontal velocity remains constant.
- With zero initial vertical velocity, the vertical displacement is $y = \frac{1}{2}at^2$; the transit time through plate length L is $t = L/u$.

Step 1: Identify the required displacement and transit time The beam enters midway between the plates. To strike the lower plate at its far end P_2' , it must fall through half the plate separation while travelling the full horizontal length. Thus

$$d = 2 \times 10^{-2} \text{ m}, \quad y = d/2 = 1.0 \times 10^{-2} \text{ m}, \\ L = 3 \times 10^{-2} \text{ m}, \quad u = 3 \times 10^7 \text{ m s}^{-1}.$$

The electric force is vertical, so the horizontal motion is uniform:

$$t = \frac{L}{u} = \frac{3 \times 10^{-2}}{3 \times 10^7} = 1.0 \times 10^{-9} \text{ s}.$$

Step 2: Calculate the acceleration and plate voltage Taking downward displacement as positive, the required acceleration is

$$a = \frac{2y}{t^2} = \frac{2(1.0 \times 10^{-2})}{(1.0 \times 10^{-9})^2} = 2.0 \times 10^{16} \text{ m s}^{-2}.$$

Use $m_e = 9.1 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ C. Equating electric force to mass times acceleration gives

$$E = \frac{m_e a}{e} = \frac{(9.1 \times 10^{-31})(2.0 \times 10^{16})}{1.6 \times 10^{-19}} = 1.1375 \times 10^5 \text{ V m}^{-1}, \\ V = Ed = (1.1375 \times 10^5)(2.0 \times 10^{-2}) = 2.275 \times 10^3 \text{ V}.$$

To accelerate a negative electron downward, the electric field must point upward. Hence the lower

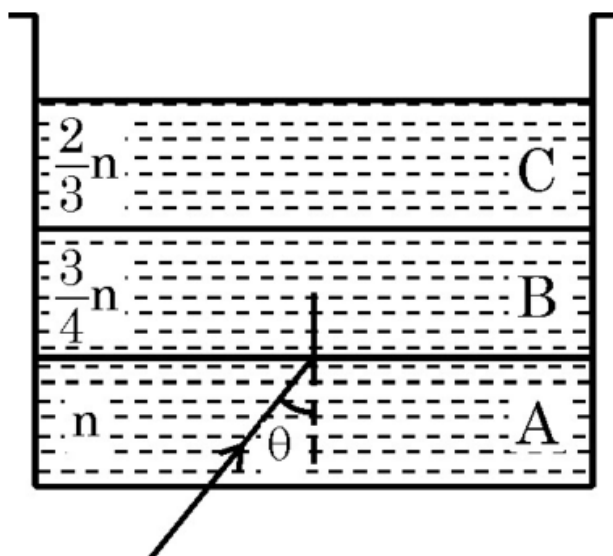
plate must be at the higher potential.

Step 3: Conclusion The required potential difference is $V \approx 2.28 \times 10^3 \text{ V}$, or 2.28 kV, with $P_2 P'_2$ positive relative to $P_1 P'_1$.

Quick Tip to Solve: Use half the plate gap for the electron's deflection, but the full gap in $E = V/d$; the horizontal transit time is always L/u .

24. (a) State the two conditions under which total internal reflection occurs.

(b) A transparent container contains layers of three immiscible transparent liquids A, B and C of refractive indices n , $\frac{3}{4}n$ and $\frac{2}{3}n$, respectively. A laser beam is incident at the interface between A and B at an angle θ as shown in figure. Prove that the beam does not enter region C at all for $\sin \theta \geq \frac{2}{3}$. [3 marks]



Solution:

Concept:

- Total internal reflection occurs only for light travelling from an optically denser medium to an optically rarer medium, with incidence angle greater than the critical angle.
- Snell's law is $n_1 \sin i = n_2 \sin r$, with angles measured from the normal. At the critical angle i_c , $\sin i_c = n_2/n_1$ for $n_1 > n_2$.
- For parallel interfaces, their normals are parallel. A transmitted ray therefore meets the second interface at the same angle to the normal that it has inside the intermediate liquid.

Step 1: State the two conditions in part (a) First, light must be incident from a medium of higher refractive index towards one of lower refractive index. Second, its angle of incidence must exceed the critical angle for that pair of media: $i > i_c$. At $i = i_c$, the refracted ray grazes the interface; this limiting case is distinct from total internal reflection.

Step 2: Apply Snell's law to the printed liquid indices In part (b), the indices are $n_A = n$, $n_B = 3n/4$, and $n_C = 2n/3$. First consider a ray that enters B, and let its angle in B be r . At the A–B interface,

$$n \sin \theta = \frac{3n}{4} \sin r \quad \Rightarrow \quad \sin r = \frac{4}{3} \sin \theta.$$

Because the liquid interfaces are parallel, its incidence angle at B–C is also r . The critical angle there obeys

$$\sin i_c = \frac{n_C}{n_B} = \frac{2n/3}{3n/4} = \frac{8}{9}.$$

Thus, for $2/3 < \sin \theta < 3/4$, one has $\sin r > 8/9$, so $r > i_c$ and the ray undergoes total internal reflection at B–C.

Step 3: Check the equality case and larger incidence angles When $\sin \theta = 2/3$, Snell's law gives $\sin r = 8/9$, so the refracted ray at B–C is along the interface at 90° to its normal. It does not propagate into the interior of C. This is the critical, grazing case. For completeness, when $\sin \theta > 3/4$, the A–B interface itself gives total internal reflection because its critical-angle sine is $n_B/n_A = 3/4$. The ray then never enters B and cannot reach C. At $\sin \theta = 3/4$, it grazes the A–B interface and likewise cannot reach the upper interface.

Step 4: Conclusion Therefore $\sin \theta \geq 2/3$ prevents the beam from entering the interior of region C. Equality gives a grazing ray at B–C; greater values give total internal reflection at B–C or already at A–B.

Quick Tip to Solve: For parallel layers, track $n \sin i$. Check equality separately: the critical ray runs along the boundary, while total internal reflection requires a strictly larger angle.

25(a). (a) Using Gauss's law, deduce an expression for electric field at a point due to a uniformly charged infinite plane thin sheet.

(b) Two large thin plane sheets, each having surface charge density σ , are held close and parallel to each other in air. What is the net electric field at a point (i) inside and (ii) outside, the sheets?

[3 marks]

Solution:

Concept:

- Gauss's law states $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the charge enclosed by a closed Gaussian surface and ϵ_0 is the permittivity of free space.
- Symmetry makes the field of a uniform infinite plane sheet perpendicular to the sheet, with equal magnitude on its two sides. Fields due to different sheets add vectorially by superposition.

Step 1: Choose a Gaussian pillbox for part (a) Let the sheet have uniform positive surface charge density σ . Choose a short cylindrical Gaussian surface crossing the sheet, with its axis normal to the sheet and its two plane end faces each of area A , parallel to it. The sheet charge enclosed by this cylinder is $Q_{\text{enc}} = \sigma A$. By translational and rotational symmetry there is no component of electric field parallel to the sheet. The field points outward on both sides of a positive sheet and has the same magnitude E at the two end faces, taken at equal distances from the sheet.

Step 2: Evaluate the flux and obtain the field The field is parallel to the curved cylindrical surface, so its flux through that surface is zero. At each end face, $\vec{E}$ is along its outward area vector, giving flux EA . Gauss's law therefore gives

$$EA + EA = \frac{\sigma A}{\epsilon_0} \Rightarrow E = \frac{\sigma}{2\epsilon_0}.$$

The area cancels and the result contains no distance from the sheet. Thus the field is uniform on

either side of an ideal infinite sheet. For negative charge density the magnitude is $|\sigma|/(2\epsilon_0)$ and its direction is towards the sheet.

Step 3: Superpose the fields of the two sheets in part (b) Both sheets carry the same signed surface charge density σ . Treat them as infinite sheets at points away from their edges; in air take the permittivity as ϵ_0 . Each sheet produces field magnitude $|\sigma|/(2\epsilon_0)$. **(i) Inside, between the sheets:** the two equal fields point in opposite directions, so

$$E_{\text{inside}} = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0.$$

(ii) Outside the sheets: the fields point in the same direction on either exterior side and add. For $\sigma > 0$,

$$E_{\text{outside}} = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0},$$

normal to and away from the pair. For negative sheets the direction reverses towards the pair.

Step 4: Conclusion A single infinite sheet produces $E = |\sigma|/(2\epsilon_0)$, normal to its surface. For two sheets with identical charge density, $E_{\text{inside}} = 0$ and $E_{\text{outside}} = |\sigma|/\epsilon_0$; the exterior field is outward for positive sheets and inward for negative sheets.

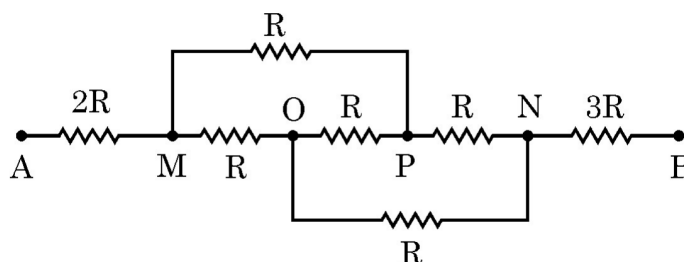
Quick Tip to Solve: Count flux through both pillbox faces to retain the factor of two. For two sheets, draw the field directions first: like charges cancel between the sheets and reinforce outside.

OR

25(b). (a) Obtain the condition of balance of a Wheatstone bridge.

(b) Find net resistance of the network of resistors connected between A and B, as shown in figure.

[3 marks]



Solution:

Concept:

- A Wheatstone bridge is balanced when the two junctions joined by its galvanometer are at the same potential, so the galvanometer current is zero.
- Ohm's law gives the potential drop IR . Series resistances add, while for parallel branches, $1/R_{\text{eq}} = 1/R_a + 1/R_b$.

Step 1: Set up the balanced bridge For a general bridge, use the junction arrangement in the given network. Let the four arm resistances along MO , ON , MP , PN be R_1 , R_2 , R_3 , R_4 , respectively, with a galvanometer joining O and P . Apply a potential difference between M and N . At balance $I_g = 0$, so $V_O = V_P$. Let I_1 flow through the entire branch MON , and I_2 through the entire branch MPN .

Step 2: Equate the corresponding potential drops Since O and P are equipotential, the drops from M to these junctions are equal. Likewise, the drops from these junctions to N are equal. Thus

$$I_1 R_1 = I_2 R_3, \quad I_1 R_2 = I_2 R_4.$$

Dividing these equations eliminates the branch currents:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}, \quad R_1 R_4 = R_2 R_3.$$

This is the condition of balance: the resistance ratios in the two arms must be equal.

Step 3: Identify the balanced part of the given network Here $MO = ON = MP = PN = R$, so $R_{MO}/R_{ON} = R_{MP}/R_{PN} = 1$. The bridge between M and N is therefore balanced. The central resistor OP , also of resistance R , has zero potential difference and carries no current; it can be omitted when calculating the equivalent resistance. The remaining paths MON and MPN each contain two resistors R in series.

Step 4: Combine the inner branches and outer resistors The two $2R$ branches are in parallel between M and N :

$$R_{MN} = \frac{(2R)(2R)}{2R + 2R} = R.$$

The resistor $AM = 2R$ and the resistor $NB = 3R$ carry the total current and are in series with this equivalent resistance. Hence

$$R_{AB} = 2R + R_{MN} + 3R = 2R + R + 3R.$$

Step 5: Conclusion The Wheatstone bridge balances when $R_1/R_2 = R_3/R_4$. For the given network, the equivalent resistance between A and B is $R_{AB} = 6R$.

Quick Tip to Solve: Compare the two arm ratios between M and N before combining resistors: equal ratios make O and P equipotential, eliminating the central branch.

26. A parallel plate capacitor of capacitance C is charged to V volt by a battery. After sometime the battery is disconnected and the distance between the plates is doubled. A slab of dielectric constant $k = 1.8$ is then introduced to completely fill the space between the plates. How will the following be affected?

- The capacitance of the capacitor.
- The electric field between the plates of the capacitor.
- The energy stored in the capacitor.

Justify your answer in each case.

[3 marks]

Solution:

Concept:

- For plate area A , separation d , and dielectric constant k , a fully filled parallel plate capacitor has capacitance $C = k\epsilon_0 A/d$, neglecting edge effects.
- Once the battery is disconnected, charge Q on the isolated capacitor stays constant. The field is $E = Q/(k\epsilon_0 A)$, and its stored energy is $U = Q^2/(2C)$.

Step 1: Fix the conserved quantity and initial values Let the initial separation be d , with air between the plates. Before disconnecting the battery,

$$C = \frac{\epsilon_0 A}{d}, \quad Q = CV, \quad E_0 = \frac{V}{d}, \quad U_0 = \frac{1}{2} CV^2.$$

After disconnection there is no conducting path to remove or supply charge, so $Q' = Q$. The potential difference is free to change; it must not be held at V .

Step 2: Calculate the final capacitance The final separation is $2d$, and the dielectric of constant 1.8 fills this entire gap. Thus

$$C' = \frac{1.8\epsilon_0 A}{2d} = 0.9C.$$

Doubling the gap alone would halve the capacitance; inserting the dielectric then multiplies that value by 1.8. Therefore the final capacitance decreases by 10% relative to its initial value.

Step 3: Calculate the final electric field The free surface charge density Q/A is unchanged. For fixed charge and area, increasing the separation alone does not change the field of ideal parallel plates. Dielectric polarisation reduces the field by the factor k :

$$E' = \frac{Q}{1.8\epsilon_0 A} = \frac{E_0}{1.8} = \frac{5}{9} E_0 = \frac{5V}{9d}.$$

As a check, $V' = Q/C' = 10V/9$, so $V'/(2d) = 5V/(9d)$, consistent with this field.

Step 4: Calculate the final stored energy Use the fixed-charge form of the energy expression:

$$U' = \frac{Q^2}{2C'} = \frac{Q^2}{2(0.9C)} = \frac{10}{9} U_0 = \frac{5}{9} CV^2.$$

The final energy is approximately 11.1% greater than the initial energy. There is no contradiction with constant charge: external work in separating the plates changes the energy, and dielectric insertion subsequently lowers it from the value reached just after separation.

Step 5: Conclusion Relative to the initial capacitor, the final values are

$$C' = 0.9C, \quad E' = \frac{5}{9} E_0, \quad U' = \frac{10}{9} U_0 = \frac{5}{9} CV^2,$$

where $E_0 = V/d$ and $U_0 = CV^2/2$.

Quick Tip to Solve: Write $Q = CV = \text{constant}$ as soon as the battery is disconnected; then use $C'/C = k/2$, $E'/E_0 = 1/k$, and $U'/U_0 = C/C'$.

27. (a) Draw the V-I characteristics of silicon diode.

(b) Explain the following terms:

(i) minority carrier injection in forward bias

(ii) breakdown voltage in reverse bias

[3 marks]

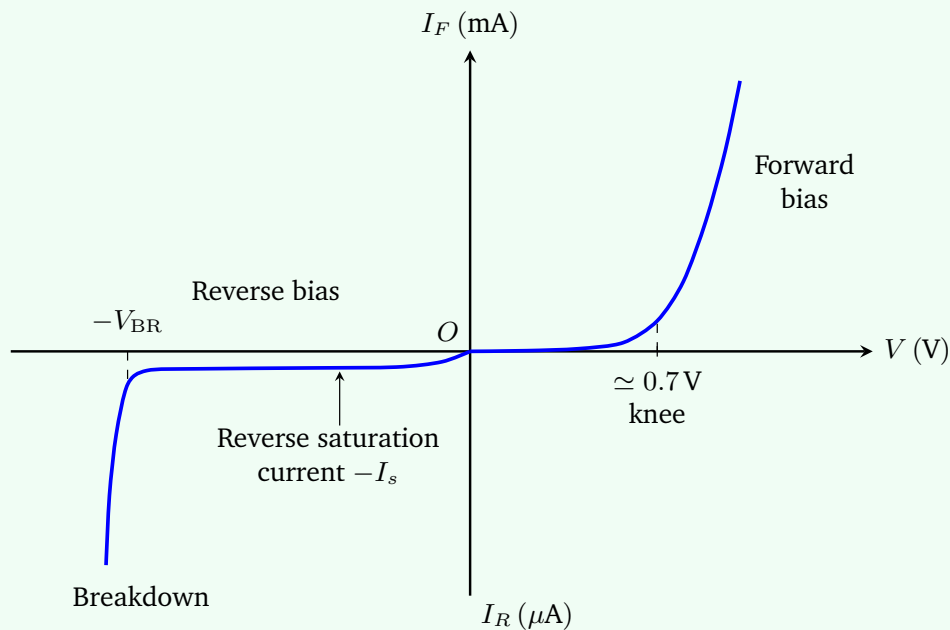
Solution:

Concept:

- Forward bias lowers the junction potential barrier and narrows the depletion layer, allowing majority carriers to diffuse across the junction.
- A silicon diode has an approximate forward knee voltage of 0.7 V. Reverse current remains small until the magnitude of the reverse voltage reaches the breakdown value V_{BR} .

Step 1: Sketch and interpret the diode characteristic Take voltage V across the diode on the horizontal axis and conventional diode current I on the vertical axis. Positive voltage denotes forward bias. The forward current is very small initially and rises steeply around the silicon knee voltage, approximately 0.7 V at room temperature. This is a practical knee, not an abrupt mathematical switch.

Under reverse bias, both voltage and current are negative in this convention. A small, nearly constant reverse saturation current I_s , associated with minority carriers, flows before breakdown. At $V = -V_{BR}$, the magnitude of the reverse current rises sharply. The sketch uses different forward and reverse current scales to make the small reverse current visible.



Silicon diode characteristic; the forward and reverse current scales differ.

Step 2: Explain minority carrier injection in forward bias Applying forward bias connects the p side to the positive terminal and the n side to the negative terminal. The reduced barrier allows electrons, which are majority carriers in the n region, to cross into the p region. There they are minority carriers. Similarly, holes diffuse from the p region into the n region and become minority carriers there. This transfer of majority carriers across the junction, producing an excess minority carrier concentration in

the opposite region near the depletion layer, is called minority carrier injection. The injected carriers subsequently diffuse and recombine.

Step 3: Explain breakdown voltage in reverse bias Breakdown voltage V_{BR} is the magnitude of the reverse-bias voltage at which the junction starts conducting a large reverse current, with only a small further increase in reverse voltage. The strong electric field causes breakdown by the Zener or avalanche mechanism, depending on the junction. It is shown on the negative voltage side as $-V_{BR}$. An external resistance must limit this current to avoid excessive heating and damage.

Step 4: Conclusion The silicon diode characteristic has a forward knee near 0.7 V, a small reverse saturation current, and a steep reverse-current rise at breakdown. Forward injection makes carriers minorities after they enter the opposite region; V_{BR} specifies the onset of large reverse conduction.

Quick Tip to Solve: In the graph, label the 0.7 V silicon knee and $-V_{BR}$. For injection, name each carrier's starting region and explain why it becomes a minority carrier after crossing.

28. (a) Why does one prefer to view the image formed at infinity than that formed at near point in microscope/telescope?

(b) Consider lenses L_1 , L_2 and L_3 as specified in the following table. Which of them will you select as objective and eyepiece for constructing best possible (i) telescope (ii) compound microscope? Give reason for your answer.

Lens	L_1	L_2	L_3
Power	6 D	3 D	10 D
Aperture	1 cm	8 cm	1 cm

[3 marks]

Solution:

Concept:

- A normal eye views an image at infinity with relaxed ciliary muscles; an image at the near point requires maximum accommodation.
- Lens power and focal length obey $P = 1/f$, with P in dioptres and f in metres. For a telescope in normal adjustment, the magnitude of angular magnification is $M = f_o/f_e$.
- For a compound microscope, $M \simeq LD/(f_o f_e)$, where L is tube length and D the least distance of distinct vision. Both lenses should have short focal lengths.

Step 1: Explain comfortable viewing at infinity For part (a), an image at infinity sends parallel rays to the eye. A normal eye focuses them without accommodation, keeping its ciliary muscles relaxed. Viewing a near-point image requires strong accommodation and causes greater strain during prolonged observation. Normal adjustment is therefore preferred for comfortable viewing.

Step 2: Convert the three powers into focal lengths Using the positive powers of the converging

lenses gives

$$f_1 = \frac{1}{6} \text{ m} = 16.7 \text{ cm},$$
$$f_2 = \frac{1}{3} \text{ m} = 33.3 \text{ cm},$$
$$f_3 = \frac{1}{10} \text{ m} = 10.0 \text{ cm}.$$

Thus L_2 has the longest focal length and the largest aperture, while L_3 has the shortest focal length.

Step 3: Select the telescope lenses For part (b)(i), choose L_2 as objective and L_3 as eyepiece. Their focal-length ratio is the largest available, giving $M = 33.3/10.0 \simeq 3.33$. The objective's 8 cm aperture collects more light from a distant object and gives better diffraction-limited resolving power than the 1 cm apertures.

Step 4: Select the compound-microscope lenses For part (b)(ii), choose L_3 as objective and L_1 as eyepiece. They have the two shortest focal lengths, with the shortest assigned to the objective for strong initial linear magnification. Both have 1 cm apertures. The longer focal length of L_2 would reduce magnification for the same tube length.

Step 5: Conclusion An image at infinity is preferred because the eye remains relaxed. For the telescope use objective L_2 and eyepiece L_3 ; for the compound microscope use objective L_3 and eyepiece L_1 .

Quick Tip to Solve: Rank focal lengths by reversing the power order: the telescope takes the longest objective, while the microscope takes the shortest objective.

Section D (8 marks)

29. A galvanometer is used to detect or/and measure small currents in an electrical circuit. It essentially works on the fact that a current-carrying coil experiences a deflecting torque when placed in a magnetic field. This deflection in the coil can be measured and it is related to the current flowing in the coil, the number of turns in the coil, area of the coil and the magnetic field. A hair spring attached to the coil provides a counter torque and helps in measuring the deflection. A galvanometer can be converted to an ammeter or a voltmeter of desired range by using suitable resistances. [4 marks]

29(i). The torque on the coil remains constant irrespective of the coil's orientation during rotation due to: [1 mark]

- (A) use of soft iron core which increases the magnetic field.
- (B) radial magnetic field
- (C) hair spring which provides the counter torque
- (D) eddy current in the iron core which causes damping.

Correct Answer: (B) radial magnetic field

Solution:

Concept:

- The magnetic torque on a coil is $\tau = NIAB \sin \alpha$, where N is its number of turns, I its current, A its area and α the angle between its normal and the field B .
- In a radial magnetic field, the field stays parallel to the coil's plane as the coil turns, so its normal remains perpendicular to the field.

Step 1: Apply the radial-field condition The radial geometry maintains $\alpha = 90^\circ$ throughout the permitted rotation. Hence $\sin \alpha = 1$ and $\tau = NIAB$. For a fixed current, this deflecting torque is independent of the coil's angular position.

Step 2: Distinguish the other effects The stronger field mentioned in (A) increases torque but field strength alone does not remove its angular dependence. The hair spring in (C) supplies a restoring torque proportional to deflection. Eddy-current damping in (D) suppresses oscillations; it does not establish the angle-independent magnetic torque.

Step 3: Conclusion The radial magnetic field keeps the deflecting torque independent of orientation. Hence, the correct option is (B).

Quick Tip to Solve: In a radial-field galvanometer, set the angle between the coil normal and the field to 90° , even as the coil rotates.

29(ii). The best way to increase current sensitivity of a galvanometer is by:

[1 mark]

- (A) increasing number of turns of the coil
- (B) increasing area of coil and magnetic field strength
- (C) decreasing area of coil and magnetic field strength
- (D) increasing torsional constant of the hair spring

Correct Answer: Both (A) and (B) increase current sensitivity; the printed question supplies no quantitative criterion that uniquely makes one the best.

Solution:

Concept:

- At equilibrium in a radial-field galvanometer, $NIAB = k\theta$, where k is the torsional constant of the spring and θ is the deflection.
- Current sensitivity is deflection per unit current: $S_I = \theta/I = NAB/k$. It increases with N , A or B , and decreases with k .

Step 1: Test the two proposed increases In (A), increasing the number of turns N , with the other parameters fixed, increases S_I . In (B), increasing both the coil area A and field strength B also increases S_I . Option (B) is a plausible intended choice because it increases two numerator factors, but the sizes of all proposed changes are unspecified. A sufficiently large increase in N could give a greater improvement than small increases in A and B .

Step 2: Test the remaining choices and the wording Option (C) decreases A and B , so it reduces sensitivity. Option (D) increases k , so it also reduces sensitivity. Coil resistance does not appear in $S_I = NAB/k$; an increase in resistance is therefore not a valid reason to reject (A) for current sensitivity. The word “best” needs a specified comparison or design constraint that is absent from the printed question.

Step 3: Conclusion Both (A) and (B) increase current sensitivity. Option (B) is a plausible intended answer, but the printed data do not establish a unique best option; the item is ambiguous as written.

Quick Tip to Solve: Write $S_I = NAB/k$ before comparing changes, and do not confuse current sensitivity with voltage sensitivity, which also depends on resistance.

29(iii)(a). A moving coil galvanometer has a coil with area of cross-section $4.0 \times 10^{-3} \text{ m}^2$ and number of turns 50. The coil is rotating in a magnetic field of 0.25 T. The torque acting on the coil when a current of 5 A passes through it is:

[1 mark]

- (A) 1.0 N m
- (B) 2.0 N m
- (C) 0.50 N m
- (D) 0.25 N m

Correct Answer: (D) 0.25 N m

Solution:

Concept:

- A moving-coil galvanometer uses a radial magnetic field, giving a deflecting torque $\tau = NIAB$, independent of the coil's orientation.
- Here N denotes the number of turns, I the coil current, A the area of one turn and B the field

strength; the SI unit of torque is N m.

Step 1: Identify the values in SI units The given quantities are $N = 50$, $A = 4.0 \times 10^{-3} \text{ m}^2$, $B = 0.25 \text{ T}$ and $I = 5 \text{ A}$. The current is printed in amperes, not milliamperes, and the area is already in square metres, so neither needs a unit conversion.

Step 2: Calculate the magnetic torque Substitute directly into the radial-field torque formula:

$$\begin{aligned}\tau &= 50 \times 5 \times (4.0 \times 10^{-3}) \times 0.25 \\ &= 0.25 \text{ N m.}\end{aligned}$$

Options (A), (B) and (C) give 1.0, 2.0 and 0.50 N m, respectively; all exceed the product of the four specified quantities. No additional factor of two is needed for the two sides of the coil, because $NIAB$ already gives the torque of the full force couple.

Step 3: Conclusion The torque on the coil is 0.25 N m . Hence, the correct option is (D).

Quick Tip to Solve: Multiply $NIA = 50 \times 5 \times 0.004 = 1$ first; the numerical torque then equals the stated field strength, 0.25.

OR

29(iii)(b). A galvanometer coil has a resistance of 15Ω and the meter shows full scale deflection for a current of 3 mA . The value of resistance required to convert it into a voltmeter of range $(0\text{-}12 \text{ V})$ is: [1 mark]

- (A) 4015Ω
- (B) 3985Ω
- (C) 415Ω
- (D) 385Ω

Correct Answer: (B) 3985Ω

Solution:

Concept:

- To convert a galvanometer into a voltmeter, connect a high resistance R in series, so the full-scale current I_g flows through the galvanometer and the added resistance.
- For galvanometer resistance G and desired full-scale voltage V , Ohm's law gives $V = I_g(G + R)$, hence $R = V/I_g - G$.

Step 1: Find the required total resistance The galvanometer has $G = 15 \Omega$ and full-scale current $I_g = 3 \text{ mA} = 3 \times 10^{-3} \text{ A}$. At $V = 12 \text{ V}$, the complete voltmeter must have resistance

$$R_{\text{total}} = \frac{V}{I_g} = \frac{12}{3 \times 10^{-3}} = 4000 \Omega.$$

Step 2: Subtract the coil resistance The existing coil already supplies 15Ω of this total. Therefore the additional series resistance is

$$R = 4000 - 15 = 3985 \Omega.$$

Option (A) incorrectly adds G to the required total. Options (C) and (D) are near 400Ω , reflecting a factor-of-ten error; they cannot restrict the current to 3 mA at 12 V .

Step 3: Conclusion Connect $3985\ \Omega$ in series with the galvanometer. Hence, the correct option is (B).

Quick Tip to Solve: For a voltmeter, V/I_g is the total resistance: always subtract the galvanometer's own resistance to find the added resistor.

29(iv). A galvanometer with coil of resistance $20\ \Omega$ shows full scale deflection for a current of $5\ \text{mA}$. To convert it into an ammeter of range $(0-10\ \text{A})$, a resistance of: [1 mark]

- (A) $0.05\ \Omega$ should be connected in series with it
- (B) $0.05\ \Omega$ should be connected in parallel with it
- (C) $0.01\ \Omega$ should be connected in parallel with it
- (D) $0.01\ \Omega$ should be connected in series with it

Correct Answer: (C) $0.01\ \Omega$ should be connected in parallel with it

Solution:

Concept:

- A galvanometer becomes an ammeter by connecting a low shunt resistance S in parallel, diverting most of the measured current away from the delicate coil.
- The parallel branches have equal potential differences: $I_g G = (I - I_g)S$. Here G is coil resistance, I_g its full-scale current and I the desired ammeter range.

Step 1: Find the current through the shunt Convert the coil current to amperes: $I_g = 5\ \text{mA} = 0.005\ \text{A}$. At the desired full-scale current $I = 10\ \text{A}$, the shunt must carry

$$I_s = I - I_g = 10 - 0.005 = 9.995\ \text{A}.$$

The coil and shunt each have the voltage $I_g G = 0.005 \times 20 = 0.100\ \text{V}$ across them.

Step 2: Calculate the parallel resistance Use the equal-voltage condition to obtain

$$S = \frac{I_g G}{I - I_g} = \frac{0.100}{9.995} = 0.010005 \dots \Omega \simeq 0.01\ \Omega.$$

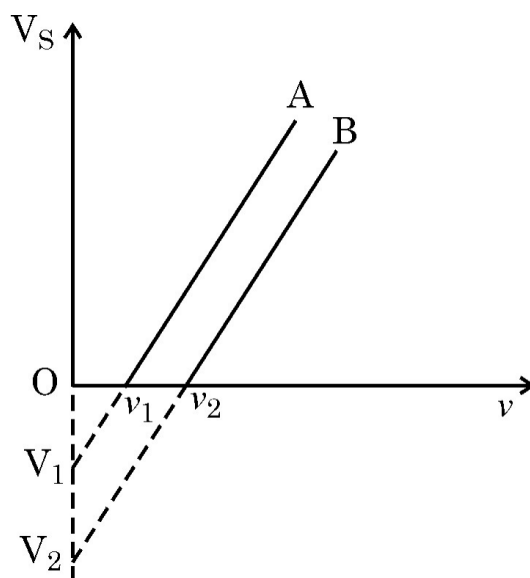
Options (A) and (D) specify series connections, which cannot divert current from the coil. Option (B) uses a parallel connection but its $0.05\ \Omega$ value would carry only $0.100/0.05 = 2\ \text{A}$ when the galvanometer reaches full scale, so it does not give the stated range.

Step 3: Conclusion A resistance of approximately $0.01\ \Omega$ must be connected in parallel with the galvanometer. Hence, the correct option is (C).

Quick Tip to Solve: For an ammeter use a parallel shunt; when $I \gg I_g$, $S \simeq I_g G/I$ gives a quick check of its value.

30. A researcher performs an experiment on photo-electric effect using two metals A and B with unknown work functions. She illuminates the surfaces of A and B with monochromatic radiation of various frequencies and records the value of corresponding stopping potentials (V_s). The graph shows the variation of stopping potential (V_s) with the frequency of incident radiation (ν) for metals A and B. [4 marks]

30(i). From the graph, the work functions of A and B are (h is Planck's constant and e value of charge on an electron): [1 mark]



- (A) ν_1 and ν_2
- (B) V_1 and V_2
- (C) $h\nu_1$ and $h\nu_2$
- (D) $\frac{h\nu_1}{e}$ and $\frac{h\nu_2}{e}$

Correct Answer: (C) $h\nu_1$ and $h\nu_2$

Solution:

Concept:

- Einstein's photoelectric equation is $eV_s = h\nu - \phi$, where V_s is the stopping potential, e is the magnitude of the electron charge and ϕ is the metal's work function.
- At the threshold frequency ν_0 , the maximum kinetic energy vanishes, so $\phi = h\nu_0$.

Step 1: Read the threshold frequencies The horizontal intercept of each line is the frequency at which its stopping potential is zero. The graph therefore gives threshold frequencies ν_1 for A and ν_2 for B. The negative vertical intercepts instead represent $-\phi/e$.

Step 2: Convert the thresholds into work functions Applying $\phi = h\nu_0$ separately gives

$$\phi_A = h\nu_1, \quad \phi_B = h\nu_2.$$

Option (A) gives frequencies, not energies. Option (B) gives potentials, and dividing the threshold

energies by e , as in (D), also gives potentials. Only (C) has the required energy values.

Step 3: Conclusion The work functions of A and B are $h\nu_1$ and $h\nu_2$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Read the horizontal intercept of a stopping-potential graph and multiply it by h to obtain the work function.

30(ii). For radiation of frequency $\nu > \nu_2$ incident on the surfaces of A and B, the maximum kinetic energy of ejected electron is: [1 mark]

- (A) greater for metal A because it has a smaller work function.
- (B) greater for metal B because it has a larger work function.
- (C) greater for metal B because it has higher threshold frequency.
- (D) the same for both metal A and metal B because it is independent of work functions of metals.

Correct Answer: (A) greater for metal A because it has a smaller work function.

Solution:

Concept:

- The maximum photoelectron kinetic energy is $K_{\max} = h\nu - \phi$, where $h\nu$ is the incident photon energy and ϕ is the work function.
- The work function is $\phi = h\nu_0$, with ν_0 the threshold frequency. A larger threshold leaves less kinetic energy at a given incident frequency.

Step 1: Compare the two work functions The graph shows $\nu_1 < \nu_2$. Thus $\phi_A = h\nu_1 < \phi_B = h\nu_2$. Because the incident frequency exceeds ν_2 , both metals emit photoelectrons.

Step 2: Compare energies for the same incident light The photon energy $h\nu$ is identical for the two surfaces. Subtracting the two photoelectric equations gives

$$K_{A,\max} - K_{B,\max} = h(\nu_2 - \nu_1) > 0.$$

Therefore A leaves more energy for the emitted electron. Options (B) and (C) reverse the effect of increasing work function or threshold frequency. Option (D) is false because the work function is explicitly subtracted from the photon energy.

Step 3: Conclusion The maximum kinetic energy is greater for metal A because A has the smaller work function. Hence, the correct option is (A).

Quick Tip to Solve: For the same incident frequency, the metal with the smaller horizontal intercept gives the faster photoelectrons.

30(iii). If the intensity of the incident radiation for both metals A and B, is doubled keeping its frequency constant, then: [1 mark]

- (A) the slope of the parallel lines will increase.
- (B) the slope of the parallel lines will decrease.
- (C) the threshold frequencies for both A and B will decrease.
- (D) the slope of the parallel lines will not change but more electrons will be emitted per second.

Correct Answer: (D) the slope of the parallel lines will not change but more electrons will be emitted per second.

Solution:

Concept:

- Einstein's equation gives $V_s = (h/e)\nu - \phi/e$, so the slope of a stopping-potential versus frequency graph is h/e . Here ϕ is the work function and e is the positive magnitude of electron charge.
- At fixed frequency, greater intensity means more incident photons per second, while each photon still has energy $h\nu$.

Step 1: Check which graph quantities can change Doubling intensity changes neither h/e nor either metal's work function. Thus both slopes remain unchanged, ruling out (A) and (B). The threshold frequency $\nu_0 = \phi/h$ also remains unchanged, ruling out (C).

Step 2: Determine the change in electron emission At an emitting frequency, twice the light intensity supplies twice as many photons per second to a fixed illuminated area. More photoelectrons are therefore emitted per second. Their maximum kinetic energy $h\nu - \phi$, and hence their stopping potential, stay the same because the photon energy has not changed.

Step 3: Conclusion The slopes of the parallel lines do not change, but more electrons are emitted per second. Hence, the correct option is (D).

Quick Tip to Solve: Separate photon number from photon energy: intensity controls emission rate, while frequency controls maximum kinetic energy.

30(iv)(a). The threshold frequency for a metal surface is ν_0 . If the radiation of frequency $3\nu_0$ illuminates the surface, the maximum kinetic energy (KE) of photoelectrons is E_1 . If the frequency were increased to $6\nu_0$, the maximum KE of the photoelectrons becomes E_2 . Then $\frac{E_1}{E_2}$ equals: [1 mark]

- (A) 1/3
- (B) 1/2
- (C) 2/5
- (D) 3/4

Correct Answer: (C) 2/5

Solution:

Concept:

- A metal of threshold frequency ν_0 has work function $\phi = h\nu_0$, where h is Planck's constant.
- Einstein's equation is $K_{\max} = h\nu - \phi = h(\nu - \nu_0)$. The work function must be subtracted before comparing kinetic energies.

Step 1: Calculate the two maximum energies Both incident frequencies exceed the threshold, so

emission occurs in both cases. The metal is unchanged and has the same work function in both experiments:

$$E_1 = h(3\nu_0 - \nu_0) = 2h\nu_0,$$

$$E_2 = h(6\nu_0 - \nu_0) = 5h\nu_0.$$

Step 2: Form the requested ratio Divide the first energy by the second and cancel the common factor:

$$\frac{E_1}{E_2} = \frac{2h\nu_0}{5h\nu_0} = \frac{2}{5}.$$

Option (B), $1/2$, is the ratio of photon energies and incorrectly ignores the work function. Options (A) and (D) likewise do not equal the ratio of the excess energies above threshold.

Step 3: Conclusion The ratio is $E_1/E_2 = 2/5$. Hence, the correct option is (C).

Quick Tip to Solve: For incident frequencies $a\nu_0$ and $b\nu_0$, take the kinetic-energy ratio as $(a - 1)/(b - 1)$.

OR

30(iv)(b). Let m be the slope of the graph line for metal B. If e is the value of electron charge, then Planck's constant ' h ' is given by: [1 mark]

- (A) me
- (B) $\frac{1}{me}$
- (C) $\frac{m}{e}$
- (D) $\frac{e}{m}$

Correct Answer: (A) me

Solution:

Concept:

- The stopping potential satisfies $eV_s = h\nu - \phi$, where ϕ is the metal's work function and e is the magnitude of electron charge.
- In a straight-line relation $y = mx + c$, the slope m is the coefficient of the horizontal-axis variable x .

Step 1: Identify the graph slope Divide Einstein's equation by e to write stopping potential as a function of incident frequency:

$$V_s = \frac{h}{e}\nu - \frac{\phi}{e}.$$

Since the graph plots V_s vertically and ν horizontally, its slope for metal B is $m = h/e$. The work function changes the intercept but not this slope.

Step 2: Isolate Planck's constant Multiplying $m = h/e$ by e gives $h = me$. This also has the correct units: slope has units Vs , so me has units $CVs = Js$. The reciprocal or quotient expressions in (B),

(C) and (D) neither rearrange the slope equation correctly nor give these units.

Step 3: Conclusion Planck's constant is $h = me$. Hence, the correct option is (A).

Quick Tip to Solve: For a stopping-potential versus frequency graph, multiply its slope by the electron charge magnitude to obtain h .

Section E (15 marks)

31(a). (a) Using the relation for refraction at a curved spherical surface, derive the expression for lens maker's formula.

(b) Three lenses L_1 , L_2 and L_3 , each of focal length 40 cm, are placed coaxially. The distance between L_1 and L_2 and between L_2 and L_3 are 120 cm and 20 cm respectively. An object is kept at a distance of 80 cm to the left of lens L_1 . Find the distance of the final image formed from the object. [5 marks]

Solution:

Concept:

- For paraxial refraction from refractive index n_1 to n_2 at a spherical surface, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$. Here u , v and R are signed distances from the surface pole.
- A thin lens in air obeys $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. Take incident light from left to right; distances measured to the right are positive and those to the left are negative.
- In a sequence of lenses, the image formed by one lens is the object for the next. Parallel incident rays form an image in the focal plane of a converging lens.

Step 1: (a) Apply refraction at the first lens surface Consider a thin lens of refractive index μ , surrounded by air. Let the signed radii of its first and second surfaces be R_1 and R_2 , respectively. For a biconvex lens with light travelling to the right, $R_1 > 0$ and $R_2 < 0$; the derivation itself applies to other shapes with their signed radii. Let an object be at signed distance u . If the second surface were absent, refraction at the first surface, from air into the lens material, would form an intermediate image at v_1 . Thus,

$$\frac{\mu}{v_1} - \frac{1}{u} = \frac{\mu - 1}{R_1}.$$

Step 2: Apply refraction at the second surface and add The intermediate image acts as the object for the second surface. Since the lens is thin, the separation between its two poles is neglected, so the signed object distance for this surface is v_1 . On refraction from the lens material into air, the final image is at v :

$$\frac{1}{v} - \frac{\mu}{v_1} = \frac{1 - \mu}{R_2}.$$

Adding the two surface equations eliminates the intermediate-image term:

$$\begin{aligned} \frac{1}{v} - \frac{1}{u} &= \frac{\mu - 1}{R_1} + \frac{1 - \mu}{R_2} \\ &= (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right). \end{aligned}$$

For an object at infinity, $1/u \rightarrow 0$ and the image forms at $v = f$. Therefore the lens maker's formula is

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

This form assumes a thin lens in air and rays close to the principal axis.

Step 3: (b) Locate the image formed by the first lens Take the position of L_1 as $x = 0$, with all positions in centimetres. Then L_2 is at $x = 120$, L_3 is at $x = 140$, and the object is at $x = -80$. Using the given focal lengths as $f = +40$ cm, the first lens has $u_1 = -80$ cm:

$$\frac{1}{v_1} = \frac{1}{f} + \frac{1}{u_1} = \frac{1}{40} - \frac{1}{80} = \frac{1}{80},$$

$$v_1 = +80 \text{ cm}.$$

Thus the first real image is at $x = 80$ cm, between the first and second lenses.

Step 4: Determine the rays emerging from the second lens The first image lies $120 - 80 = 40$ cm to the left of L_2 . Hence its object distance is $u_2 = -40$ cm, exactly the front focal distance:

$$\frac{1}{v_2} = \frac{1}{40} + \frac{1}{-40} = 0.$$

The second lens therefore sends out parallel rays; its image is at infinity. These rays remain parallel while travelling the 20 cm from L_2 to L_3 .

Step 5: Locate the final image and measure from the original object The third lens receives parallel rays and brings them to its rear focal plane, so $v_3 = +40$ cm. The final image coordinate and the separation from the original object are

$$x_{\text{image}} = 120 + 20 + 40 = 180 \text{ cm},$$

$$d = x_{\text{image}} - x_{\text{object}} = 180 - (-80) = 260 \text{ cm}.$$

Step 6: Conclusion (a) For a thin lens in air, $\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$, with signed radii. (b) The final real image is 40 cm to the right of L_3 , and its distance from the original object is $\boxed{260 \text{ cm} = 2.60 \text{ m}}$.

Quick Tip to Solve: Track all positions from L_1 : the first image at 80 cm lies at the front focus of L_2 . Its parallel output makes the last image one focal length beyond L_3 ; include the initial 80 cm when measuring from the object.

31(b). (a) Draw a ray diagram to show the image formation by a concave mirror when the object is kept between its focus and the centre of curvature. Using this diagram, derive the mirror formula.

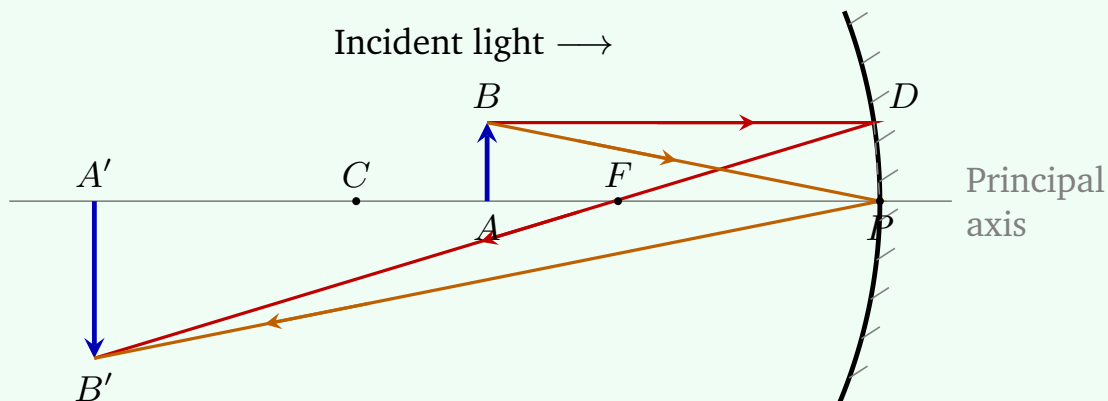
(b) A concave mirror produces a two times magnified virtual image of an object kept 10 cm in front of it. Calculate the focal length of the mirror. [5 marks]

Solution:

Concept:

- A concave spherical mirror brings a paraxial ray parallel to its principal axis through its principal focus F . For a small aperture, $PF = PC/2$, where P is the pole and C is the centre of curvature.
- Reflection at the pole obeys $i = r$. Similar right triangles in the ray diagram relate the positive height and distance magnitudes; Cartesian signs are introduced after deriving that geometrical relation.
- In the new Cartesian convention, distances measured opposite to incident light are negative. Mirror magnification is $m = h_i/h_o = -v/u$, with u , v and f denoting signed object distance, image distance and focal length.

Step 1: (a) Construct the image using two rays Place the upright object AB between F and C . From its tip B , draw a ray parallel to the principal axis to the mirror at D ; after reflection it passes through F . Draw another ray from B to the pole P , and reflect it with equal angles on the two sides of the principal axis. The reflected rays intersect at B' ; its perpendicular to the axis gives A' . Thus the construction locates the image beyond C , below the axis. The diagram uses the paraxial approximation: the transverse height of D is AB , and the small axial displacement of D from the pole's transverse plane is neglected.



Paraxial construction: the object is between F and C , and the real inverted image is beyond C .

Step 2: (a) Use the triangles formed by the pole ray Write the positive lengths as $AP = U$, $A'P = V$, $PF = L$, $AB = h$ and $A'B' = H$. The image height magnitude is $H > 0$, although its signed height is negative. The right triangles ABP and $A'B'P$ have equal angles at P , by the law of reflection. Hence they are similar and

$$\frac{H}{h} = \frac{V}{U}.$$

Step 3: (a) Use the focal ray and derive the formula For the parallel ray, the small-aperture approximation gives $DP \simeq h$, with DP treated as perpendicular to the axis. The right triangles DPF

and $A'B'F$ are similar. Since A' lies beyond F , $A'F = V - L$. Therefore

$$\frac{H}{h} = \frac{V - L}{L}.$$

Equating the two height ratios and rearranging gives

$$\begin{aligned}\frac{V}{U} &= \frac{V - L}{L}, \\ VL &= UV - UL, \\ L(U + V) &= UV, \\ \frac{1}{L} &= \frac{1}{U} + \frac{1}{V}.\end{aligned}$$

Incident light travels from left to right. In this diagram the object, real image and focus all lie to the left of P , so $u = -U$, $v = -V$ and $f = -L$. Multiplying the magnitude relation by -1 converts it to the Cartesian mirror relation. The derivation assumes paraxial rays; it does not assume that all rays of a large-aperture spherical mirror meet at exactly one point.

Step 4: (b) Determine the signed image distance The object is in front of the mirror, so $u = -10$ cm. A virtual image of a real object in a concave mirror is erect. Therefore two times magnified means $m = +2$, not -2 . Using mirror magnification,

$$+2 = -\frac{v}{-10 \text{ cm}} \Rightarrow v = +20 \text{ cm}.$$

The positive image distance places this virtual image behind the mirror, as required by the question.

Step 5: (b) Substitute in the mirror relation Keep the signs of both distances while taking their reciprocals:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{-10 \text{ cm}} + \frac{1}{20 \text{ cm}} = -\frac{1}{20 \text{ cm}}.$$

The negative reciprocal places the focus in front of the mirror, consistent with a concave mirror. The object distance magnitude is smaller than the focal-length magnitude, which also agrees with production of an erect, enlarged virtual image.

Step 6: Conclusion (a) For an object between F and C , the image is real, inverted and enlarged, beyond C . The paraxial mirror formula is $\boxed{1/f = 1/u + 1/v}$. (b) The mirror has $\boxed{f = -20 \text{ cm}}$, a focal-length magnitude of 20 cm.

Quick Tip to Solve: For an enlarged virtual image in a concave mirror, take magnification as positive first. With $m = +2$, $v = -2u$; this immediately checks that the image is behind the mirror before substituting in the mirror formula.

32(a). (a) State Faraday's law of electromagnetic induction.

(b) Derive an expression for the self-inductance of an air-filled long solenoid of length l and cross-sectional area A having N turns.

(c) A conducting rod of length 50 cm, with one end pivoted, is rotated with angular speed of 60 rpm in a uniform magnetic field of 4.0 mT directed perpendicular to the plane of rotation of rod. Find the emf induced in the rod. [5 marks]

Solution:

Concept:

- Magnetic flux through a turn is $\Phi_B = \int \vec{B} \cdot d\vec{A}$. Faraday's law relates induced emf to the rate of change of flux linkage: $\mathcal{E} = -d(N\Phi_B)/dt$, where N is the number of turns.
- For a long air-filled solenoid carrying current I , its nearly uniform internal field is $B = \mu_0 n I$, with turn density $n = N/l$. Self-inductance L is defined by $N\Phi_B = LI$.
- In a rotating rod, the speed varies with distance r from the pivot: $v = \omega r$. A small radial element dr therefore contributes motional emf $d\mathcal{E} = B\omega r dr$ when the field is perpendicular to the rotation plane.

Step 1: (a) State Faraday's law Whenever the magnetic flux linked with a circuit changes, an emf is induced in it. The magnitude of the induced emf equals the time rate of change of its magnetic flux linkage. For a coil of N turns, each linked by flux Φ_B ,

$$\mathcal{E} = -\frac{d(N\Phi_B)}{dt} = -N \frac{d\Phi_B}{dt}.$$

The second equality assumes a fixed number of turns. The negative sign represents Lenz's law: the induced emf drives a current, if the circuit is closed, whose magnetic effect opposes the change in flux that produces it.

Step 2: (b) Find the flux linkage of the solenoid Let a current I flow through the solenoid. Since it is long, end effects are negligible, and its internal magnetic field is approximately uniform and parallel to its axis:

$$B = \mu_0 \frac{N}{l} I.$$

The plane of each turn is perpendicular to the field, so the flux through one turn of cross-sectional area A is

$$\Phi_B = BA = \frac{\mu_0 N I A}{l}.$$

All N turns link approximately the same flux. Hence the total flux linkage is

$$N\Phi_B = \frac{\mu_0 N^2 A}{l} I.$$

Step 3: (b) Apply the definition of self-inductance Comparing the flux linkage with $N\Phi_B = LI$ gives

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 A}{l}.$$

Here μ_0 is the permeability of free space, which is also used for air to a very good approximation. For fixed solenoid geometry, Faraday's law consequently gives the self-induced emf as

$$\mathcal{E}_{\text{self}} = -\frac{d(LI)}{dt} = -L \frac{dI}{dt}.$$

Thus the inductance depends on the solenoid's geometry and permeability, and its SI unit is the henry.

Step 4: (c) Convert the data and account for the varying speed Use R for the rod length to distinguish it from the solenoid length l . The given quantities become

$$\begin{aligned} R &= 50 \text{ cm} = 0.50 \text{ m}, \\ B &= 4.0 \text{ mT} = 4.0 \times 10^{-3} \text{ T}, \\ f &= \frac{60}{60} = 1 \text{ s}^{-1}, \\ \omega &= 2\pi f = 2\pi \text{ rad s}^{-1}. \end{aligned}$$

A small element at distance r from the pivot moves tangentially with speed ωr . Since the magnetic field is perpendicular to the plane, $\vec{v} \times \vec{B}$ is along the rod. Each element contributes emf of magnitude $B\omega r dr$. The pivot is stationary, whereas the outer end moves fastest; therefore a single tip speed cannot be applied to the entire rod.

Step 5: (c) Integrate along the rod and substitute Add the contributions from the pivot to the free end:

$$\begin{aligned} |\mathcal{E}| &= \int_0^R B\omega r dr = \frac{1}{2} B\omega R^2, \\ |\mathcal{E}| &= \frac{1}{2} (4.0 \times 10^{-3}) (2\pi) (0.50)^2 \text{ V} \\ &= \pi \times 10^{-3} \text{ V}. \end{aligned}$$

The rotation sense and the field's into-page or out-of-page direction are not specified, so the numerical result is the magnitude of the emf between the rod's ends.

Step 6: Conclusion Faraday's law is $\mathcal{E} = -d(N\Phi_B)/dt$. The air-filled long solenoid has self-inductance $L = \mu_0 N^2 A/l$. The rotating rod develops emf of magnitude $|\mathcal{E}| = 3.14 \times 10^{-3} \text{ V} = 3.14 \text{ mV}$, approximately 3.1 mV to two significant figures.

Quick Tip to Solve: Multiply the flux through one solenoid turn by N before finding L . For a rod rotating about one end, use $\mathcal{E} = B\omega R^2/2$, and convert rpm to radians per second with $\omega = 2\pi(\text{rpm})/60$.

OR

32(b). (a) Draw a labelled diagram of a step-up transformer. State the principle on which it works and obtain the ratio of secondary voltage to primary voltage in terms of number of turns and currents in the two coils.

(b) The ratio of the number of turns in the primary to the secondary of an ideal transformer is 1 : 5. If 5 kW power at 200 V is supplied to the primary, find (i) current in the primary, and (ii) output voltage. [5 marks]

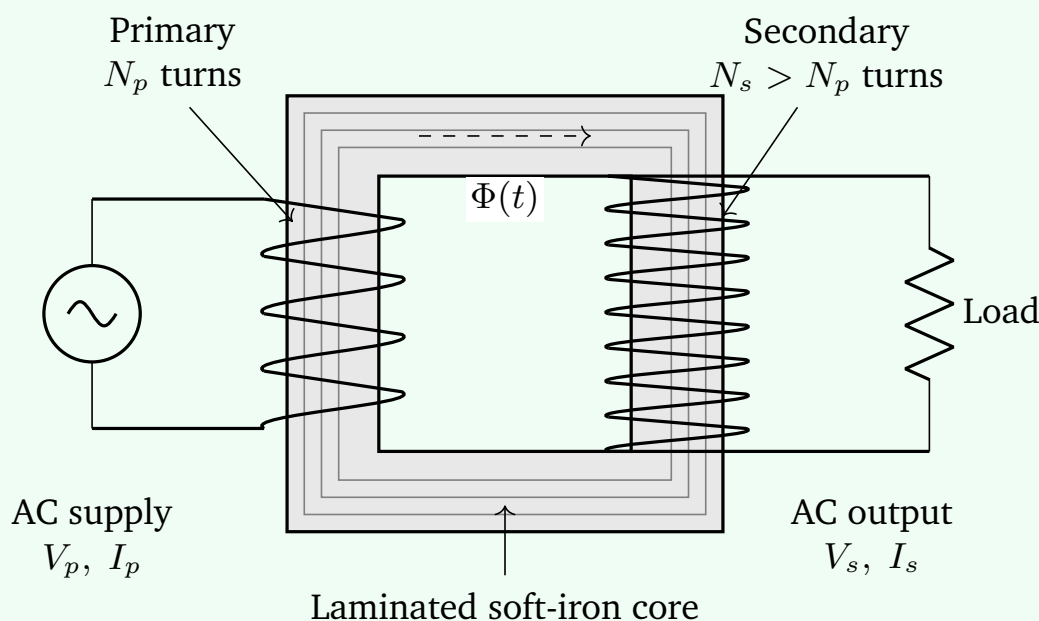
Solution:

Concept:

- A transformer works on mutual induction: alternating current in the primary produces a changing magnetic flux that links the secondary and induces an emf in it.
- Faraday's law gives the induced emf in an N -turn coil as $e = -N d\Phi/dt$, where Φ is the magnetic flux through each turn. For an ideal transformer, the same flux links both coils.

- For an ideal transformer, $V_s/V_p = N_s/N_p$ and $V_p I_p = V_s I_s$, using rms voltages and currents for the usual resistive-load case. The subscripts p and s refer to the primary and secondary.

Step 1: (a) Draw the step-up transformer and explain its operation The primary coil has N_p turns and is connected to an AC supply. The secondary has more turns, $N_s > N_p$, and supplies the load. Both insulated windings are wound on a common laminated soft-iron core, which provides a low-reluctance path for the alternating magnetic flux. Laminations reduce eddy-current losses. The changing primary current sets up a changing core flux, which induces an emf in the secondary by mutual induction. A steady DC supply cannot sustain this action because it does not produce continuously changing flux. The two windings are electrically insulated from one another; energy is transferred through their magnetic coupling.



Step-up transformer: a common laminated core links the primary and secondary, with $N_s > N_p$.

Step 2: (a) Derive the voltage and turns relation Neglect flux leakage, winding resistance and core losses. Let $\Phi(t)$ be the common instantaneous magnetic flux through each turn. Faraday's law, with the same reference orientation for both windings, gives

$$e_p = -N_p \frac{d\Phi}{dt},$$

$$e_s = -N_s \frac{d\Phi}{dt}.$$

Dividing these equations cancels the common rate of change of flux. Thus the induced-emf magnitudes, and therefore their rms values, are in the turns ratio. In the ideal transformer the terminal-voltage magnitudes equal the corresponding induced-emf magnitudes, so

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

Since $N_s > N_p$, the secondary voltage exceeds the primary voltage. The AC frequency remains unchanged.

Step 3: (a) Relate the voltage ratio to the currents An ideal transformer has no power loss. For the

standard resistive-load treatment, conservation of power gives

$$V_p I_p = V_s I_s.$$

Dividing by $V_p I_s$ and combining with the turns relation gives

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}.$$

Thus a step-up transformer raises voltage while lowering current. It does not increase the output power. Equivalently, $I_s/I_p = N_p/N_s$; the current ratio is inverse to the turns ratio.

Step 4: (b)(i) Calculate the primary current The supplied power is $P_p = 5 \text{ kW} = 5000 \text{ W}$, and the primary voltage is $V_p = 200 \text{ V}$. Using the ideal-transformer power relation intended in this numerical,

$$I_p = \frac{P_p}{V_p} = \frac{5000}{200} = 25 \text{ A}.$$

Step 5: (b)(ii) Calculate the output voltage The given turns ratio is $N_p : N_s = 1 : 5$, so $N_s/N_p = 5$. Therefore,

$$V_s = V_p \frac{N_s}{N_p} = 200 \times 5 = 1000 \text{ V}.$$

As a consistency check, the ideal secondary current would be $I_s = 25/5 = 5 \text{ A}$, giving output power $1000 \times 5 = 5000 \text{ W}$, equal to the input power.

Step 6: Conclusion The step-up transformer works on mutual induction and has $N_s > N_p$, with

$$\boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}}.$$

For the given ideal transformer, (i) the primary current is $\boxed{25 \text{ A}}$, and (ii) the output voltage is $\boxed{1000 \text{ V}}$.

Quick Tip to Solve: Convert 5 kW to watts before using $I_p = P_p/V_p$. With primary-to-secondary turns 1 : 5, multiply the primary voltage by 5 and divide the primary current by 5.

- 33(a).** (a) An electric dipole consists of two point charges q and $-q$ separated by a distance $2a$. Derive an expression for the electric field $\vec{E}$ due to this dipole at a point distant r from the centre of the dipole on the equatorial plane. Write the expression for the electric field at a far off point, i.e. $r \gg a$.
- (b) A dipole is placed in x - y plane such that charges q and $-q$ are located at $x = a$ and $x = b$ respectively. There exists an electric field $\vec{E} = 2\hat{i} \text{ N/C}$ in the region. Calculate the force $\vec{F}$ and torque $\vec{\tau}$ experienced by the dipole. [5 marks]

Solution:

Concept:

- The electric dipole moment is $\vec{p} = q\vec{d}$, where $\vec{d}$ points from the negative charge to the positive charge. For separation $2a$, its magnitude is $p = 2aq$.
- Electric fields add vectorially. The field of a point charge at displacement $\vec{R}$ from it is $\vec{E} = kq\vec{R}/R^3$, where $k = 1/(4\pi\epsilon_0)$.

- An external field exerts force $q\vec{E}$ on charge q . In a uniform field the net dipole force is zero, while the torque is $\vec{\tau} = \vec{p} \times \vec{E}$.

Step 1: (a) Set up the equatorial geometry Place the dipole centre at the origin, with $+q$ at $(a, 0)$ and $-q$ at $(-a, 0)$, so that $\vec{p} = 2aq\hat{i}$. By rotational symmetry about the dipole axis, an arbitrary point of the equatorial plane may be represented by $P = (0, r)$. Its distance from either charge is

$$s = \sqrt{r^2 + a^2}.$$

The two fields have equal magnitudes kq/s^2 . Their components perpendicular to the dipole axis cancel, whereas their axial components both point from the positive charge towards the negative charge.

Step 2: Add the two electric-field vectors The displacement vectors from the charges to P are $-a\hat{i} + r\hat{j}$ and $a\hat{i} + r\hat{j}$. Keeping the negative sign of the second charge explicitly gives

$$\begin{aligned}\vec{E}_+ &= \frac{kq(-a\hat{i} + r\hat{j})}{(r^2 + a^2)^{3/2}}, \\ \vec{E}_- &= \frac{-kq(a\hat{i} + r\hat{j})}{(r^2 + a^2)^{3/2}}, \\ \vec{E}_{\text{eq}} &= \frac{-2kaq\hat{i}}{(r^2 + a^2)^{3/2}} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{(r^2 + a^2)^{3/2}}.\end{aligned}$$

Thus the equatorial field is antiparallel to the dipole moment. The minus sign specifies this direction, rather than a negative field magnitude.

Step 3: Obtain the far-field expression Factor the denominator as $r^3(1 + a^2/r^2)^{3/2}$. For $r \gg a$, the ratio a^2/r^2 is negligible, so

$$\vec{E}_{\text{eq}} \simeq -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{r^3}.$$

The field therefore decreases as the inverse cube of distance in this limit. The distance r here is measured from the centre, not from either individual charge.

Step 4: (b) Calculate the resultant external force The printed external field is $\vec{E} = 2\hat{i}$ N/C, which is independent of position. Consequently the two charges experience equal and opposite forces even when a and b differ:

$$\begin{aligned}\vec{F}_+ &= q(2\hat{i} \text{ N/C}), \\ \vec{F}_- &= -q(2\hat{i} \text{ N/C}), \\ \vec{F}_{\text{net}} &= \vec{0}.\end{aligned}$$

No factor of a or b enters these forces because the stated field is uniform.

Step 5: Calculate the torque for the stated locations Taking the stated locations $x = a$ and $x = b$ in their usual intended sense as points on the x -axis, the dipole moment in this part is $\vec{p} = q(a - b)\hat{i}$. It is parallel or antiparallel to the applied field. Hence

$$\vec{\tau} = \vec{p} \times \vec{E} = q(a - b)\hat{i} \times (2\hat{i} \text{ N/C}) = \vec{0}.$$

Both forces act along the dipole axis, so they have no turning effect. Strictly, the printed statement gives only the x -coordinates: if different unspecified y -coordinates were allowed, the force would still vanish but the torque would instead be $\vec{\tau} = -2q(y_+ - y_-)\hat{k}$, in SI units. The zero-torque result uses the intended common y -coordinate.

Step 6: Conclusion (a) $\vec{E}_{\text{eq}} = -\frac{\vec{p}}{4\pi\epsilon_0(r^2 + a^2)^{3/2}}$, and for $r \gg a$, $\vec{E}_{\text{eq}} \simeq -\frac{\vec{p}}{4\pi\epsilon_0 r^3}$. In both cases the field points opposite to $\vec{p}$. (b) For the intended placement on the x -axis, $\vec{F} = \vec{0}$ N and $\vec{\tau} = \vec{0}$ N m.

Quick Tip to Solve: On the equatorial plane, cancel the transverse components first. For the second part, read the printed field carefully: a constant field gives zero net force, and an aligned dipole also has zero torque.

OR

33(b). (a) Two cells of emf E_1 and E_2 with internal resistances r_1 and r_2 respectively, are connected in parallel by connecting their positive terminals together and negative terminals together. Deduce an expression for equivalent emf and equivalent internal resistance of the combination.

(b) A parallel combination, as stated in (a) above, of two cells of emfs E and $3E$ and internal resistances R each is connected across a resistance $2R$. Find the current that flows through resistance $2R$. [5 marks]

Solution:

Concept:

- For a cell supplying current I , its terminal potential difference is $V = E - Ir$, where E is its emf and r is its internal resistance. A negative supplied current means that the cell is being charged.
- Cells connected in parallel between the same pair of terminals have the same terminal voltage V . Kirchhoff's junction rule gives the external current as $I = I_1 + I_2$.
- An equivalent cell obeys $V = E_{\text{eq}} - Ir_{\text{eq}}$. With a load resistance R_L , the current is $I = E_{\text{eq}}/(R_L + r_{\text{eq}})$.

Step 1: (a) Write the current supplied by each cell Let A be the common positive terminal and B the common negative terminal. Denote their potential difference by $V = V_A - V_B$. Take I_1 and I_2 as currents delivered by the two cells towards A , and I as the current entering the external circuit. The terminal-voltage equations are

$$\begin{aligned} V &= E_1 - I_1 r_1, & I_1 &= \frac{E_1 - V}{r_1}, \\ V &= E_2 - I_2 r_2, & I_2 &= \frac{E_2 - V}{r_2}. \end{aligned}$$

These signed equations remain valid even if the higher-emf cell drives current into the lower-emf cell.

Step 2: Combine the currents and identify the equivalent cell Applying the junction rule at A gives

$$\begin{aligned} I &= \frac{E_1}{r_1} + \frac{E_2}{r_2} - V \left(\frac{1}{r_1} + \frac{1}{r_2} \right), \\ V &= \frac{E_1/r_1 + E_2/r_2}{1/r_1 + 1/r_2} - I \frac{1}{1/r_1 + 1/r_2}. \end{aligned}$$

Comparing this with $V = E_{\text{eq}} - Ir_{\text{eq}}$ yields

$$\begin{aligned} E_{\text{eq}} &= \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}, \\ r_{\text{eq}} &= \frac{r_1 r_2}{r_1 + r_2}. \end{aligned}$$

The equivalent emf is the open-circuit terminal voltage, found by setting the external current I to zero. The individual cell currents need not then vanish if the emfs differ. For like terminals joined, the emf terms add with the signs shown above.

Step 3: (b) Substitute the given emfs and internal resistances Here $E_1 = E$, $E_2 = 3E$ and $r_1 = r_2 = R$. Thus

$$E_{\text{eq}} = \frac{ER + 3ER}{R + R} = 2E,$$

$$r_{\text{eq}} = \frac{R^2}{2R} = \frac{R}{2}.$$

Equal internal resistances give the arithmetic mean of the two emfs, while the equivalent internal resistance is half of either resistance. The external resistance $2R$ is in series with this equivalent internal resistance.

Step 4: Calculate the load current and verify the result Applying Ohm's law to the equivalent complete circuit,

$$I = \frac{2E}{2R + R/2} = \frac{4E}{5R}.$$

The load voltage is $V = I(2R) = 8E/5$. Therefore the signed currents delivered by the two cells are

$$I_1 = \frac{E - 8E/5}{R} = -\frac{3E}{5R},$$

$$I_2 = \frac{3E - 8E/5}{R} = \frac{7E}{5R}.$$

Their sum is $4E/(5R)$, agreeing with the load current. The negative value of I_1 simply means that the lower-emf cell receives current; it does not invalidate the parallel combination.

Step 5: Conclusion (a) $E_{\text{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$ and $r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2}$. (b) The current through the external resistance $2R$ is $I = \frac{4E}{5R}$, in amperes when E is in volts and R in ohms, flowing from the common positive terminal to the common negative terminal through the load.

Quick Tip to Solve: For two parallel cells with equal internal resistance, average their emfs and halve their internal resistance, then add the load resistance before calculating the current.