

CBSE Class 12 Mathematics Question Paper 2026 with Solution PDF-Set 65-1-3

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 80

General Instructions

- (i) This Question paper contains **38** questions. **All questions are compulsory.**
- (ii) Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.
- (iii) In Section **A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) and Question Number **19 & 20** are Assertion-Reason based questions of **1** mark each.
- (iv) In Section **B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In Section **C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In Section **D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.
- (vii) In Section **E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in few questions in all the Sections except Section – **A**.
- (ix) Use of calculator is **NOT** allowed.

Section A (20 marks)

Question 1 to 20 are multiple choice questions of 1 mark each.

1. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then the value of $|\vec{a} \cdot \vec{b}|$ is

[1 mark]

- (A) $6\sqrt{3}$
- (B) $8\sqrt{3}$
- (C) $12\sqrt{3}$
- (D) $3\sqrt{12}$

Correct Answer: (C) $12\sqrt{3}$

Solution:

Concept:

- For vectors making angle θ , $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta$.
- The identity $(\vec{a} \cdot \vec{b})^2 + |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2$ removes the unknown angle.

Step 1: Apply the identity The product of the magnitudes is $8 \times 3 = 24$. Therefore

$$|\vec{a} \cdot \vec{b}|^2 = 24^2 - 12^2 = 576 - 144 = 432.$$

The given cross product fixes the sine of the angle, but the absolute value makes the acute or obtuse choice irrelevant.

Step 2: Take the positive square root Since a magnitude cannot be negative,

$$|\vec{a} \cdot \vec{b}| = \sqrt{432} = 12\sqrt{3}.$$

The alternatives $6\sqrt{3}$, $8\sqrt{3}$ and $3\sqrt{12} = 6\sqrt{3}$ do not square to 432, so none satisfies the identity.

Step 3: Conclusion $|\vec{a} \cdot \vec{b}| = 12\sqrt{3}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the dot-cross identity directly whenever both magnitudes and one product magnitude are known.

2. The length of perpendicular drawn from the point $(3, 4, 2)$ on the line $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$ is

[1 mark]

- (A) 2
- (B) 9
- (C) 5
- (D) $\sqrt{29}$

Correct Answer: (C) 5

Solution:

Concept:

- A line with direction ratios $(0, 0, 1)$ through the origin is the z -axis.
- The shortest distance to the z -axis is $\sqrt{x^2 + y^2}$; the foot keeps the same z -coordinate.

Quick Tip to Solve: For distance to a coordinate axis, omit that axis coordinate and square the other two.

Step 1: Interpret the line The symmetric notation with zero direction ratios means $x = 0$, $y = 0$, while z is unrestricted. Write a general point on the line as $Q = (0, 0, t)$. For $P = (3, 4, 2)$,

$$PQ^2 = 3^2 + 4^2 + (2 - t)^2 = 25 + (2 - t)^2.$$

Step 2: Minimise the distance The square term is non-negative and vanishes at $t = 2$. Thus the perpendicular foot is $(0, 0, 2)$ and the length is $\sqrt{25} = 5$. The value 2 is the height coordinate, 9 is merely 3^2 , and $\sqrt{29}$ is the distance to the origin, not to the line.

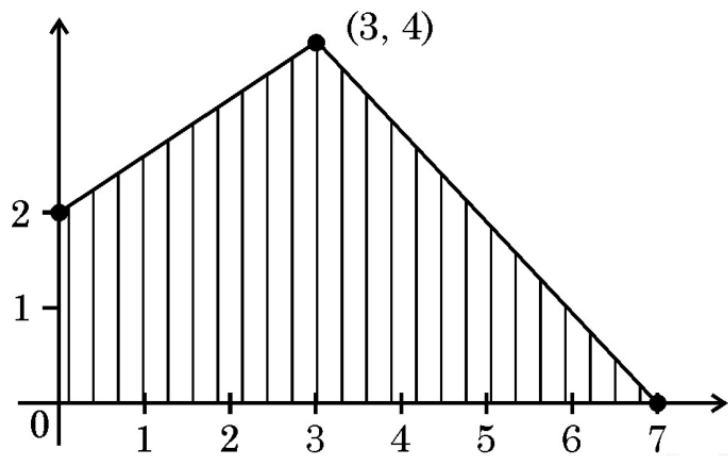
Step 3: Conclusion The perpendicular distance is 5. Hence, the correct option is (C).

Quick Tip to Solve: For distance to a coordinate axis, omit that axis coordinate and square the other two.

3. The feasible region of a linear programming problem with objective function $Z = 5x + 7y$ is shown below:

The maximum value of Z - minimum value of Z is

[1 mark]



- (A) 8
- (B) 29
- (C) 35
- (D) 43

Correct Answer: (D) 43

Solution:

Concept:

- A linear objective function attains its extrema on a bounded polygonal feasible region at corner points.
- Evaluate $Z = 5x + 7y$ at every vertex, including the origin when it belongs to the shaded region.

Step 1: Read the vertices and evaluate The shaded region has vertices $(0, 0)$, $(0, 2)$, $(3, 4)$ and $(7, 0)$.

Quick Tip to Solve: Include every corner of the shaded region, especially the origin, before subtracting extrema.

Their objective values are

$$\begin{aligned} Z(0, 0) &= 0, & Z(0, 2) &= 14, \\ Z(3, 4) &= 15 + 28 = 43, & Z(7, 0) &= 35. \end{aligned}$$

Step 2: Compare maximum and minimum The maximum is 43 at (3, 4), whereas the minimum is 0 at (0, 0). The required difference is therefore $43 - 0 = 43$. Option 8 subtracts 35 from 43, option 29 subtracts 14 from 43, and option 35 uses another vertex value; none uses the actual minimum.

Step 3: Conclusion The required difference is 43. Hence, the correct option is (D).

Quick Tip to Solve: Include every corner of the shaded region, especially the origin, before subtracting extrema.

4. The degree of an objective function of a linear programming problem is

[1 mark]

- (A) 0
- (B) 1
- (C) 2
- (D) Any natural number

Correct Answer: (B) 1

Solution:

Concept:

- The objective of a linear programming problem has the form $Z = ax + by$, where a, b are constants.
- The degree of a polynomial is the highest total exponent of its variable terms with nonzero coefficients.

Step 1: Identify the powers In a linear objective each variable appears only to the first power. A term such as ax has degree 1, as does by ; their sum is therefore a polynomial of degree 1 in the usual nonconstant linear programming objective.

Step 2: Check the alternatives Degree 0 would describe a constant objective rather than the standard linear objective. Degree 2 would involve quadratic terms such as x^2 or xy , which are excluded from linear programming. The degree is fixed by linearity, so it cannot be any arbitrary natural number.

Step 3: Conclusion The degree is 1. Hence, the correct option is (B).

Quick Tip to Solve: Recognise a linear objective by first powers of variables and the absence of products of variables.

5. If $\sin^{-1} x + \pi = y$, then

[1 mark]

- (A) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
(B) $-\frac{3\pi}{2} \leq y \leq -\frac{\pi}{2}$
(C) $\frac{\pi}{2} \leq y \leq \frac{3\pi}{2}$
(D) $0 \leq y \leq \pi$

Correct Answer: (C) $\frac{\pi}{2} \leq y \leq \frac{3\pi}{2}$

Solution:

Concept:

- The principal range of $\sin^{-1} x$ is $[-\pi/2, \pi/2]$, for $-1 \leq x \leq 1$.
- Adding a constant to a function shifts both endpoints of its range by that constant.

Step 1: Start from the principal range By the definition of inverse sine,

$$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}.$$

These endpoints are attained at $x = -1$ and $x = 1$, respectively, so both must remain included in the resulting interval.

Step 2: Translate the interval Add π to all three terms and use $y = \sin^{-1} x + \pi$:

$$\frac{\pi}{2} \leq y \leq \frac{3\pi}{2}.$$

Option A gives the unshifted range, option B shifts in the wrong direction, and option D misses the upper endpoint $3\pi/2$; consequently only option C gives the full range.

Step 3: Conclusion $\frac{\pi}{2} \leq y \leq \frac{3\pi}{2}$. Hence, the correct option is (C).

Quick Tip to Solve: For a constant added outside an inverse trigonometric function, shift both principal-range endpoints.

6. If $A = [a_{ij}]_{3 \times 3}$ is a scalar matrix, then which of the following must be true?

[1 mark]

- (A) A must be a symmetric matrix.
(B) A must be a skew-symmetric matrix.
(C) A must be an identity matrix.
(D) A must be a null matrix.

Correct Answer: (A) A must be a symmetric matrix.

Solution:

Concept:

- A scalar matrix is $A = \lambda I_3$: all off-diagonal entries are zero and every diagonal entry equals λ .

Quick Tip to Solve: Test a claim about every scalar matrix with the simple example $2I$ after checking the definition.

- A symmetric matrix satisfies $A^T = A$, whereas a skew-symmetric matrix satisfies $A^T = -A$.

Step 1: Take the transpose Transposing λI_3 leaves its equal diagonal entries and zero off-diagonal entries unchanged. Thus

$$A^T = (\lambda I_3)^T = \lambda I_3 = A.$$

Every scalar matrix is therefore symmetric, irrespective of the scalar value.

Step 2: Test the other mandatory claims For example, $A = 2I_3$ is a scalar matrix but is neither skew-symmetric, the identity matrix, nor the null matrix. Skew-symmetry would require $\lambda = -\lambda$, hence $\lambda = 0$. Identity and null matrices require the special choices 1 and 0, so options B, C and D need not hold.

Step 3: Conclusion A must be symmetric. Hence, the correct option is (A).

Quick Tip to Solve: Test a claim about every scalar matrix with the simple example $2I$ after checking the definition.

7. Which of the following properties is/are true for two matrices of suitable orders?

- (i) $(A + B)' = A' + B'$
- (ii) $(A - B)' = B' - A'$
- (iii) $(AB)' = A'B'$
- (iv) $(kAB)' = kB'A'$ (k is a scalar)

[1 mark]

- (A) (i) only
- (B) (i), (ii) and (iii)
- (C) (i) and (ii)
- (D) (i) and (iv)

Correct Answer: (D) (i) and (iv)

Solution:

Concept:

- Transposition preserves addition and subtraction: $(A \pm B)^T = A^T \pm B^T$.
- Transposition reverses the order of a product: $(AB)^T = B^T A^T$, and a scalar factor is unchanged.

Step 1: Judge addition and subtraction Statement (i) is the standard addition rule, hence true. For subtraction the correct identity is $(A - B)^T = A^T - B^T$, not $B^T - A^T$. Thus (ii) is false in general; choosing unequal matrices immediately distinguishes the two expressions.

Step 2: Judge the product rules The transpose of a product reverses the factor order, so (iii), $(AB)^T = A^T B^T$, is false in general. Including a scalar gives $(kAB)^T = k B^T A^T$, exactly statement (iv). Hence the true statements are (i) and (iv); options A, B and C either omit (iv) or include false statements.

Step 3: Conclusion Only (i) and (iv) are true. Hence, the correct option is (D).

Quick Tip to Solve: On transposing a product, reverse the matrix order but keep any scalar multiplier unchanged.

8. If $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$, then

[1 mark]

- (A) $\Delta_1 = 2\Delta_2$
(B) $\Delta_2 = -2\Delta_1$
(C) $\Delta_1 = \Delta_2$
(D) $\Delta_2 = -\Delta_1$

Correct Answer: (B) $\Delta_2 = -2\Delta_1$

Solution:

Concept:

- The determinant of a diagonal matrix is the product of its diagonal entries.
- Exchanging two rows changes the sign of a determinant; a triangular determinant is also the product of diagonal entries.

Step 1: Evaluate the first determinant The first determinant is diagonal, so

$$\Delta_1 = 1 \times 2 \times 3 = 6.$$

The symbols in the printed question denote determinant values, which are scalars, so compare the evaluated numbers rather than the entries of the displayed arrays.

Step 2: Evaluate and compare the second Expanding the second determinant along its last row gives

$$\Delta_2 = 6 \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} = 6(0 - 2) = -12.$$

Thus $\Delta_2 = -2\Delta_1$. Option A would require $6 = -24$, option C would require $6 = -12$, and option D would require $-12 = -6$, so all three fail.

Step 3: Conclusion $\Delta_2 = -2\Delta_1$. Hence, the correct option is (B).

Quick Tip to Solve: For these sparse determinants, evaluate the diagonal determinant first and expand the other along its last row.

9. One of the values of x for which $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is [1 mark]

- (A) 0
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{2}$

Correct Answer: (B) $\frac{\pi}{4}$

Solution:

Concept:

- For a two-by-two determinant, $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.
- The identity $2 \sin x \cos x = \sin 2x$ converts the determinant into a trigonometric equation.

Step 1: Evaluate the determinant The determinant equals $\cos x \sin x - \sin x(-\cos x) = 2 \sin x \cos x = \sin 2x$. Thus the required equation is $\sin 2x = 1$. The negative sign in the lower-left entry is essential: the two products add.

Step 2: Check the given values At $x = \pi/4$, $\sin 2x = \sin(\pi/2) = 1$. The values $x = 0$ and $x = \pi/2$ give zero, while $x = \pi/3$ gives $\sqrt{3}/2$, so these three options do not satisfy the equation.

Step 3: Conclusion One such value is $x = \pi/4$. Hence, the correct option is (B).

Quick Tip to Solve: Expand the determinant first; the negative lower-left entry makes it $\sin 2x$.

10. If A and B are symmetric matrices of same order, then $(AB - BA)$ is a [1 mark]

- (A) Zero matrix
(B) Identity matrix
(C) Symmetric matrix
(D) Skew symmetric matrix

Correct Answer: (D) Skew symmetric matrix

Solution:

Concept:

- A matrix A is symmetric when $A^T = A$, where T denotes transpose.
- The transpose reverses a product: $(AB)^T = B^T A^T$. A matrix M is skew symmetric when $M^T = -M$.

Step 1: Transpose the difference Write $M = AB - BA$. Since both given matrices are symmetric,

$$M^T = (AB)^T - (BA)^T = B^T A^T - A^T B^T = BA - AB.$$

Equal order ensures these products and their difference are defined.

Quick Tip to Solve: Transposing a product reverses its factors; apply this before using symmetry.

Step 2: Identify the matrix type Thus $M^T = -(AB - BA) = -M$, which is the defining property of a skew symmetric matrix. The difference need not be zero because symmetric matrices need not commute. It cannot generally be an identity matrix or a symmetric matrix; a zero difference is only a special case.

Step 3: Conclusion $AB - BA$ is skew symmetric. Hence, the correct option is (D).

Quick Tip to Solve: Transposing a product reverses its factors; apply this before using symmetry.

11. The least value of $f(x) = e^{-x}$ in $[0, 3]$ is

[1 mark]

- (A) e^{-3}
- (B) -1
- (C) 1
- (D) $-e^3$

Correct Answer: (A) e^{-3}

Solution:

Concept:

- If $f'(x) < 0$ throughout an interval, f decreases there.
- A decreasing continuous function on $[a, b]$ attains its least value at the right endpoint b .

Step 1: Determine monotonicity For $f(x) = e^{-x}$, differentiation gives $f'(x) = -e^{-x}$. The exponential is positive for every real x , hence $f'(x) < 0$ throughout $[0, 3]$. Therefore the function is strictly decreasing.

Step 2: Compare endpoint values and options The endpoint values are $f(0) = 1$ and $f(3) = e^{-3}$, with $e^{-3} < 1$. The negative options -1 and $-e^3$ are impossible because the function is positive. The value 1 is the greatest value on the interval, not the least.

Step 3: Conclusion The least value is e^{-3} , attained at $x = 3$. Hence, the correct option is (A).

Quick Tip to Solve: For e^{-x} , increasing x decreases the value, so use the right endpoint.

12. If $\int \frac{3ax}{b^2 + c^2x^2} dx = A \log |b^2 + c^2x^2| + K$, then the value of A is

[1 mark]

- (A) $3a$
- (B) $\frac{3a}{2b^2}$
- (C) $\frac{3a}{b^2c^2}$
- (D) $\frac{3a}{2c^2}$

Correct Answer: (D) $\frac{3a}{2c^2}$

Solution:

Concept:

- For $u = u(x)$, $\int u'(x)/u(x) dx = \log |u(x)| + K$.
- Here a, b, c are constants, and differentiating $b^2 + c^2x^2$ gives $2c^2x$.

Step 1: Choose the substitution Put $u = b^2 + c^2x^2$, so $du = 2c^2x dx$. Therefore $3ax dx = \frac{3a}{2c^2} du$. The coefficient depends on the derivative of the denominator, which eliminates the constant term b^2 .

Step 2: Integrate and compare coefficients The integral becomes

$$\frac{3a}{2c^2} \int \frac{du}{u} = \frac{3a}{2c^2} \log |b^2 + c^2x^2| + K.$$

Comparing the coefficient of the given logarithm determines A . The coefficients in (A), (B) and (C) fail to compensate for the derivative factor $2c^2$.

Step 3: Conclusion $A = \frac{3a}{2c^2}$. Hence, the correct option is (D).

Quick Tip to Solve: Differentiate the expression inside the logarithm to identify its missing multiplier.

13. The value of $\int_{-1}^1 \frac{x^3}{x^2 + 2|x| + 1} dx$ is

[1 mark]

- (A) 0
- (B) $\log 2$
- (C) $2 \log 2$
- (D) $\frac{1}{2} \log 2$

Correct Answer: (A) 0

Solution:

Concept:

- A function g is odd when $g(-x) = -g(x)$.
- For a continuous odd function, $\int_{-a}^a g(x) dx = 0$, since equal opposite contributions cancel.

Step 1: Check the parity of the integrand Let $g(x) = x^3/(x^2 + 2|x| + 1)$. Replacing x by $-x$ changes the numerator to $-x^3$, while the denominator stays unchanged. Thus $g(-x) = -g(x)$. Also, the

Quick Tip to Solve: Test parity before integrating: $|x|$ is even and x^3 is odd.

denominator is $(|x| + 1)^2 > 0$, so the integrand is continuous on the entire interval.

Step 2: Apply the symmetric-limit property The limits -1 and 1 are symmetric about zero. Therefore the integral over the negative half cancels the integral over the positive half. The positive logarithmic values in options (B), (C) and (D) cannot describe this cancellation.

Step 3: Conclusion $\int_{-1}^1 \frac{x^3}{x^2 + 2|x| + 1} dx = 0$. Hence, the correct option is (A).

Quick Tip to Solve: Test parity before integrating: $|x|$ is even and x^3 is odd.

14. The area bounded by the curve $y = x|x|$, x -axis and the ordinates $x = -1$ and $x = 1$ is given by [1 mark]

- (A) 0
- (B) $\frac{1}{3}$
- (C) $\frac{2}{3}$
- (D) $\frac{4}{3}$

Correct Answer: (C) $\frac{2}{3}$

Solution:

Concept:

- Geometric area between $y = f(x)$ and the x -axis is $\int |f(x)| dx$.
- For $y = x|x|$, the curve is $-x^2$ for $x < 0$ and x^2 for $x \geq 0$.

Step 1: Use positive areas on both sides The curve lies below the axis on $[-1, 0]$ and above it on $[0, 1]$. Its distance from the axis is $|x|x| = x^2$. Hence the total bounded area is $\int_{-1}^0 x^2 dx + \int_0^1 x^2 dx$.

Step 2: Evaluate the two equal contributions Each contribution equals $1/3$, giving $2 \int_0^1 x^2 dx = 2[x^3/3]_0^1 = 2/3$. The option zero is the signed integral, not geometric area. The value $1/3$ includes only half the region, and $4/3$ counts the region twice.

Step 3: Conclusion The bounded area is $\boxed{2/3 \text{ square units}}$. Hence, the correct option is (C).

Quick Tip to Solve: When a curve crosses the axis, integrate its absolute value to obtain total area.

15. The integrating factor of differential equation $R \frac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is [1 mark]

- (A) $e^{\int \frac{P}{Q} dy}$
(B) $e^{\int P dy}$
(C) $e^{\int \frac{P}{R} dy}$
(D) $e^{\int \frac{P}{R} dx}$

Correct Answer: (C) $e^{\int \frac{P}{R} dy}$

Solution:

Concept:

- A linear equation in dependent variable x has form $dx/dy + p(y)x = q(y)$.
- Its integrating factor is $e^{\int p(y) dy}$, integrating with respect to the independent variable y .

Step 1: Normalize the differential equation On an interval where $R(y) \neq 0$, divide the entire equation by R :

$$\frac{dx}{dy} + \frac{P}{R}x = \frac{Q}{R}.$$

Thus the coefficient of x after normalization is $p(y) = P/R$.

Step 2: Form the integrating factor Using the standard linear-equation rule gives $IF = e^{\int (P/R) dy}$. Option (A) incorrectly divides by Q ; option (B) omits division by R ; option (D) integrates with respect to x instead of y . The coefficient on the right side does not determine the integrating factor.

Step 3: Conclusion $IF = e^{\int \frac{P}{R} dy}$. Hence, the correct option is (C).

Quick Tip to Solve: First make the coefficient of dx/dy equal to one; then integrate the coefficient of x in y .

16. The order and degree of the differential equation $\frac{d}{dx}(y')^3 + (y')^3 = 1$, respectively, are (where $y' = \frac{dy}{dx}$) [1 mark]

- (A) 1, 3
(B) 2, 1
(C) 3, 1
(D) 3, 2

Correct Answer: (B) 2, 1

Solution:

Concept:

- The order is the order of the highest derivative present in a differential equation.
- For an equation polynomial in its derivatives, the degree is the power of its highest-order derivative.

Step 1: Expand the printed derivative The chain rule gives $\frac{d}{dx}(y')^3 = 3(y')^2 y''$. Hence the printed

Quick Tip to Solve: Expand an outer differentiation before identifying the highest derivative and its power.

equation becomes

$$3(y')^2 y'' + (y')^3 - 1 = 0.$$

The highest derivative present is y'' , so its order is two.

Step 2: Identify the degree The expanded equation is polynomial in the derivatives, and y'' occurs to the first power. Thus the degree is one. The cube applies to y' , not to the highest derivative after differentiation. Option (A) misses the second derivative, while (C) and (D) mistake an exponent for derivative order.

Step 3: Conclusion The order and degree are $\boxed{2, 1}$, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Expand an outer differentiation before identifying the highest derivative and its power.

17. The value of p for which vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is [1 mark]

- (A) 0
- (B) 1
- (C) $\frac{5}{2}$
- (D) $-\frac{5}{2}$

Correct Answer: (C) $\frac{5}{2}$

Solution:

Concept:

- For nonzero vectors, perpendicularity is equivalent to a zero dot product: $\vec{a} \cdot \vec{b} = 0$.
- The dot product multiplies corresponding Cartesian components and adds the products.

Step 1: Form the dot product Here $\vec{a} = (1, 2, 3)$ and $\vec{b} = (2, -p, 1)$. Their dot product is

$$\vec{a} \cdot \vec{b} = 1(2) + 2(-p) + 3(1) = 5 - 2p.$$

The minus sign belongs to the second component of the second vector.

Step 2: Apply perpendicularity and check options The required condition gives $5 - 2p = 0$, so $2p = 5$. For options (A), (B), (C) and (D), the dot products are respectively 5, 3, 0, 10. Thus only the third option makes the dot product vanish; both vectors remain nonzero.

Step 3: Conclusion $p = \frac{5}{2}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the signed components first; perpendicular vectors require their dot product, not their cross product, to vanish.

18. The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is [1 mark]

- (A) 8
- (B) -8
- (C) 2
- (D) $\frac{5}{2}$

Correct Answer: (A) 8

Solution:

Concept:

- Three points are collinear when their displacement vectors from one common point are proportional.
- For position vectors $\vec{a}, \vec{b}, \vec{c}$, compare $\vec{b} - \vec{a}$ with $\vec{c} - \vec{a}$.

Step 1: Find displacement vectors Let the points be $A = (-1, -1, 2)$, $B = (2, m, 5)$ and $C = (3, 11, 6)$. Subtracting the coordinates of A gives

$$\overrightarrow{AB} = (3, m + 1, 3), \quad \overrightarrow{AC} = (4, 12, 4).$$

Step 2: Use proportional components The first and third components fix the common ratio as $3/4$. Hence $(m + 1)/12 = 3/4$, which gives $m + 1 = 9$. The other printed choices $-8, 2, 5/2$ give middle components $-7, 3, 7/2$, respectively, and none equals the required value 9.

Step 3: Conclusion $m = 8$. Hence, the correct option is (A).

Quick Tip to Solve: Subtract one common point from the other two before comparing component ratios; position vectors themselves need not be parallel.

19. Assertion (A): In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$.

Reason (R): For any two events A and B , $P(A|B) = \frac{P(A \cup B)}{P(B)}$. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true and Reason (R) is false.
- (D) Assertion (A) is false and Reason (R) is true.

Correct Answer: (C) Assertion (A) is true and Reason (R) is false.

Solution:

Concept:

- Conditional probability restricts the sample space to the conditioning event.
- For $P(B) > 0$, the correct formula is $P(A | B) = P(A \cap B)/P(B)$.

Step 1: Judge the assertion Let $A = \{2, 3, 5\}$ be the prime outcomes and $B = \{1, 3, 5\}$ the odd outcomes. Among the three equally likely odd outcomes, precisely 3 and 5 are prime. Therefore $P(A | B) = 2/3$, and the assertion is true.

Step 2: Judge the printed reason The printed reason has $A \cup B$ in its numerator. A union includes outcomes outside the conditioning event and does not count only favourable outcomes within B . Here $A \cup B = \{1, 2, 3, 5\}$, so the printed ratio is $(4/6)/(3/6) = 4/3$. It cannot be a probability, and the reason is false.

Step 3: Determine the explanation relationship The assertion follows from the intersection formula, not the union formula actually printed in the reason. Thus the reason cannot explain the assertion. Options (A) and (B) wrongly treat the reason as true; option (D) wrongly treats the assertion as false.

Step 4: Conclusion Assertion is true and Reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Check the set-operation symbol in the reason: conditional probability uses an intersection, never a union.

20. Assertion (A): Lines given by $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$.

Reason (R): Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are perpendicular to each other if $\vec{b}_1 \cdot \vec{b}_2 = 0$.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true and Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is true.

Correct Answer: (D) Assertion (A) is false and Reason (R) is true.

Solution:

Concept:

- Direction ratios of a parametric line are the coefficients of its parameter in x, y, z .
- Perpendicular directions have zero scalar product.

Step 1: Judge the assertion Taking $y = t$ in the first line gives $(x, y, z) = (q, 0, s) + t(p, 1, r)$. Similarly the second line has direction vector $(p', 1, r')$. Their dot product is $pp' + 1 + rr'$. Perpendicularity requires $pp' + rr' = -1$. The asserted value 1 instead makes the dot product 2, so the assertion is false.

Step 2: Judge the reason For nonzero direction vectors $\vec{b}_1$ and $\vec{b}_2$, $\vec{b}_1 \cdot \vec{b}_2 = |\vec{b}_1||\vec{b}_2|\cos\theta$. A zero dot product therefore gives $\theta = \pi/2$, which is precisely perpendicularity of their directions. The reason states this standard criterion and is true.

Step 3: Choose the assertion-reason code The reason correctly supplies the test, but applying it contradicts the assertion because of the extra product $1 \cdot 1$ from the y -components. Options (A), (B) and (C) all require a true assertion, so each is excluded.

Step 4: Conclusion Assertion is false and Reason is true. Hence, the correct option is (D).

Quick Tip to Solve: When y is the parameter, remember its direction ratio is 1; its contribution changes the condition to $pp' + rr' = -1$.

Section B (10 marks)

This section comprises Very Short Answer (VSA) type questions of 2 marks each.

21(a). Simplify: $\tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right)$, $0 < x < \frac{\pi}{4}$.

[2 marks]

Solution:

Concept:

- The tangent subtraction identity is $\tan(\alpha - \beta) = (\tan \alpha - \tan \beta)/(1 + \tan \alpha \tan \beta)$.
- The principal value of $\tan^{-1} t$ lies in $(-\pi/2, \pi/2)$.

Quick Tip to Solve: After recognising a tangent identity, check the resulting angle is in the inverse tangent principal range before cancelling the functions.

Step 1: Rewrite the argument The restriction $0 < x < \pi/4$ gives $0 < 2x < \pi/2$, so $\cos 2x > 0$ and division by it is valid. Consequently

$$\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x} = \frac{1 - \tan 2x}{1 + \tan 2x} = \tan\left(\frac{\pi}{4} - 2x\right).$$

The denominator is positive since both sine and cosine are positive in this interval.

Step 2: Verify the principal-value range Multiplying the given interval by two gives $0 < 2x < \pi/2$. Subtracting from $\pi/4$ yields

$$-\frac{\pi}{4} < \frac{\pi}{4} - 2x < \frac{\pi}{4}.$$

This whole interval lies inside the principal range $(-\pi/2, \pi/2)$ of inverse tangent. Thus no addition or subtraction of π is needed when applying $\tan^{-1}$ to the rewritten tangent.

Step 3: Conclusion

$$\tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right) = \frac{\pi}{4} - 2x.$$

Quick Tip to Solve: After recognising a tangent identity, check the resulting angle is in the inverse tangent principal range before cancelling the functions.

OR

21(b). Evaluate: $\tan\left[\sin^{-1} 1 - \cos^{-1}\left(-\frac{1}{2}\right)\right].$

[2 marks]

Solution:

Concept:

- The principal range of $\sin^{-1}$ is $[-\pi/2, \pi/2]$, while that of $\cos^{-1}$ is $[0, \pi]$.
- Tangent is an odd function: $\tan(-\theta) = -\tan \theta$.

Step 1: Evaluate the inverse trigonometric angles Since $\sin(\pi/2) = 1$ and $\pi/2$ lies in the principal range of inverse sine, $\sin^{-1} 1 = \pi/2$. For inverse cosine, the argument is $-1/2$. The angle $2\pi/3$ lies in $[0, \pi]$ and has cosine $-1/2$, so $\cos^{-1}(-1/2) = 2\pi/3$. Choosing $\pi/3$ would give positive cosine and is therefore incorrect.

Step 2: Subtract the angles and apply tangent The angle inside the printed outer tangent is

$$\frac{\pi}{2} - \frac{2\pi}{3} = \frac{3\pi - 4\pi}{6} = -\frac{\pi}{6}.$$

Its cosine is nonzero, so the tangent is defined. Using oddness and the standard value $\tan(\pi/6) = 1/\sqrt{3}$, we obtain

$$\tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}.$$

The negative sign agrees with the angle being in the fourth quadrant.

Quick Tip to Solve: Evaluate each inverse function in its own principal range first, then subtract the angles and apply the printed outer function.

Step 3: Conclusion

$$\tan \left[\sin^{-1} 1 - \cos^{-1} \left(-\frac{1}{2} \right) \right] = -\frac{1}{\sqrt{3}}.$$

Quick Tip to Solve: Evaluate each inverse function in its own principal range first, then subtract the angles and apply the printed outer function.

22(a). Check whether function $f(x)$ defined as $f(x) = \begin{cases} \frac{|x-3|}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ is continuous at $x = 3$ or not. [2 marks]

Solution:**Concept:**

- A function is continuous at $x = a$ when its left-hand limit, right-hand limit and value $f(a)$ are all equal.
- For $x < 3$, the number $x - 3$ is negative, so $|x - 3| = -(x - 3)$. Use the branch corresponding to the side from which the limit is taken.

Step 1: Calculate the left-hand limit For every $x < 3$, we can cancel $x - 3$, since it is nonzero on this branch:

$$f(x) = \frac{-(x-3)}{2(x-3)} = -\frac{1}{2}.$$

Therefore $\lim_{x \rightarrow 3^-} f(x) = -\frac{1}{2}$. The first formula need not be defined at 3 itself to calculate this one-sided limit.

Step 2: Calculate the right-hand limit and the function value The branch for $x \geq 3$ is a polynomial divided by a nonzero constant. Hence

$$\lim_{x \rightarrow 3^+} f(x) = \frac{3-6}{6} = -\frac{1}{2}, \quad f(3) = \frac{3-6}{6} = -\frac{1}{2}.$$

Both one-sided limits agree, so the two-sided limit exists and is equal to the assigned value at the joining point.

Step 3: Conclusion $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3) = -\frac{1}{2}$. Thus f is continuous at $x = 3$.

Quick Tip to Solve: Replace the absolute value using the sign on the relevant branch before taking a limit; then check the actual assigned value separately.

22(b). If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

[2 marks]

Solution:**Concept:**

- Treat y as a function of x . The chain rule gives $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$.
- The product rule gives $\frac{d}{dx}(xy) = y + x\frac{dy}{dx}$; collect all terms containing the derivative before substituting the coordinates.

Step 1: Differentiate and isolate the derivative Differentiating the given relation with respect to x , we obtain

$$\sqrt{3}(2x + 2yy') = 4(y + xy').$$

Thus $(2\sqrt{3}y - 4x)y' = 4y - 2\sqrt{3}x$, and, wherever the coefficient in the denominator is nonzero,

$$\frac{dy}{dx} = \frac{4y - 2\sqrt{3}x}{2\sqrt{3}y - 4x}.$$

Here y' denotes dy/dx .

Step 2: Evaluate at the specified point At $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$, the numerator is $2\sqrt{3} - \sqrt{3} = \sqrt{3}$, while the denominator is $3 - 2 = 1$. Therefore the derivative is well defined at this point. Substitution gives

$$\left.\frac{dy}{dx}\right|_{(1/2, \sqrt{3}/2)} = \frac{\sqrt{3}}{1}.$$

The point also satisfies the original equation: both sides equal $\sqrt{3}$.

Step 3: Conclusion The required derivative at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is $\boxed{\sqrt{3}}$.

Quick Tip to Solve: Keep the product-rule term y when differentiating xy ; omitting it changes both the slope formula and the answer.

23(a). Simplify: $\sin^{-1}\left(\sqrt{\frac{1 + \cos 2x}{2}}\right)$, $0 < x < \frac{\pi}{2}$.

[2 marks]

Solution:**Concept:**

- The double-angle identity $1 + \cos 2x = 2\cos^2 x$ converts the radicand into a square.
- The square root satisfies $\sqrt{u^2} = |u|$. The principal range of $\sin^{-1}$ is $[-\pi/2, \pi/2]$, so an angle must belong to that range before applying inverse cancellation.

Quick Tip to Solve: After using a half-angle identity, remove the absolute value only after checking the quadrant, and then check the inverse-function range.

Step 1: Simplify the square root using the given interval Using the double-angle identity,

$$\sqrt{\frac{1 + \cos 2x}{2}} = \sqrt{\cos^2 x} = |\cos x|.$$

Since $0 < x < \pi/2$, x lies in the first quadrant and $\cos x > 0$. Thus the square root is $\cos x$, and the expression becomes $\sin^{-1}(\cos x)$.

Step 2: Choose the angle in the principal inverse-sine range The complementary-angle identity gives $\cos x = \sin(\pi/2 - x)$. The hypothesis also gives $0 < \pi/2 - x < \pi/2$, which is inside the principal range of inverse sine. Hence

$$\sin^{-1}(\cos x) = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - x\right)\right) = \frac{\pi}{2} - x.$$

The interval condition justifies this cancellation; inverse sine cannot cancel sine for arbitrary angles without a range check.

Step 3: Conclusion The simplified expression is $\boxed{\frac{\pi}{2} - x}$, for $0 < x < \frac{\pi}{2}$.

Quick Tip to Solve: After using a half-angle identity, remove the absolute value only after checking the quadrant, and then check the inverse-function range.

OR

23(b). Evaluate: $\cos[\sin^{-1}(-1) - \tan^{-1}(-\sqrt{3})]$.

[2 marks]

Solution:

Concept:

- The principal range of $\sin^{-1}$ is $[-\pi/2, \pi/2]$, so $\sin^{-1}(-1) = -\pi/2$.
- The principal range of $\tan^{-1}$ is $(-\pi/2, \pi/2)$; tangent is odd, and $\tan(\pi/3) = \sqrt{3}$.

Step 1: Find both principal angles Let $\alpha = \sin^{-1}(-1)$ and $\beta = \tan^{-1}(-\sqrt{3})$. Since $\sin(-\pi/2) = -1$, the inverse-sine angle is $\alpha = -\pi/2$. Also $\tan(-\pi/3) = -\sqrt{3}$, and $-\pi/3$ lies in the inverse-tangent principal range. Therefore $\beta = -\pi/3$. Using a positive angle for this negative argument would give an incorrect sign in the final angle.

Step 2: Subtract the angles and evaluate cosine The expression is $\cos(\alpha - \beta)$. Substituting the principal values and keeping the two minus signs distinct gives

$$\alpha - \beta = -\frac{\pi}{2} - \left(-\frac{\pi}{3}\right) = -\frac{\pi}{6}.$$

Cosine is even, so $\cos(-\pi/6) = \cos(\pi/6) = \sqrt{3}/2$. This also places the angle in the fourth quadrant, where cosine is positive.

Step 3: Conclusion The required value is $\boxed{\frac{\sqrt{3}}{2}}$.

Quick Tip to Solve: Put a negative inverse-tangent value in parentheses before subtracting it; subtraction then becomes addition.

24. Using vectors, find the area of $\triangle ABC$ with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$. [2 marks]

Solution:

Concept:

- Two side vectors from the same vertex are $\vec{AB} = B - A$ and $\vec{AC} = C - A$.
- The magnitude $|\vec{AB} \times \vec{AC}|$ is the area of their parallelogram. The triangle has half this area.

Step 1: Form the two side vectors Subtract the coordinates of $A(1, 2, 3)$ from the other vertices:

$$\vec{AB} = (1, -3, 1), \quad \vec{AC} = (3, 3, -4).$$

Both vectors begin at A , which makes the cross-product area formula apply directly. Their coordinates are directed differences, so the negative components must be retained.

Step 2: Compute the cross product and its magnitude Using the determinant expansion,

$$\begin{aligned}\vec{AB} \times \vec{AC} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix} \\ &= 9\hat{i} + 7\hat{j} + 12\hat{k}.\end{aligned}$$

For example, the middle coefficient is $-[1(-4) - 1(3)] = 7$. Consequently,

$$|\vec{AB} \times \vec{AC}| = \sqrt{9^2 + 7^2 + 12^2} = \sqrt{274}.$$

This is the parallelogram area, so it must still be divided by two for the triangle.

Step 3: Conclusion The area of $\triangle ABC$ is $\boxed{\frac{\sqrt{274}}{2}}$ square units.

Quick Tip to Solve: Use side vectors from one vertex, watch the minus sign on the middle determinant term, and halve the cross-product magnitude.

25. Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths. [2 marks]

Solution:

Concept:

- For adjacent sides represented by $\vec{a}$ and $\vec{b}$, the diagonal from their common initial point has vector $\vec{a} + \vec{b}$. The other diagonal, directed from the end of $\vec{a}$ to the end of $\vec{b}$, has vector $\vec{b} - \vec{a}$.
- The length of a vector $x\hat{i} + y\hat{j} + z\hat{k}$ is its magnitude $\sqrt{x^2 + y^2 + z^2}$. Reversing a diagonal changes its vector sign but leaves its length unchanged.

Step 1: Obtain the diagonal vectors Let the adjacent sides be $\vec{OA} = \vec{a}$ and $\vec{OB} = \vec{b}$, and let C be the

Quick Tip to Solve: Add the adjacent-side vectors for one diagonal and subtract them for the other; square the components only after finding these vectors.

opposite vertex. Using the parallelogram law and vector subtraction,

$$\begin{aligned}\vec{OC} &= \vec{a} + \vec{b} \\ &= (3 + 1)\hat{i} - 2\hat{j} + (2 + 2)\hat{k} \\ &= 4\hat{i} - 2\hat{j} + 4\hat{k}, \\ \vec{AB} &= \vec{b} - \vec{a} \\ &= (1 - 3)\hat{i} + (0 + 2)\hat{j} + (2 - 2)\hat{k} \\ &= -2\hat{i} + 2\hat{j}.\end{aligned}$$

Step 2: Calculate the two lengths Apply the three-dimensional magnitude formula to each diagonal. The missing $\hat{k}$ -component of $\vec{AB}$ is zero:

$$\begin{aligned}|\vec{OC}| &= \sqrt{4^2 + (-2)^2 + 4^2} = \sqrt{36}, \\ |\vec{AB}| &= \sqrt{(-2)^2 + 2^2 + 0^2} = \sqrt{8}.\end{aligned}$$

The opposite direction on the second diagonal gives $2\hat{i} - 2\hat{j}$, which represents the same geometric diagonal with reverse orientation.

Step 3: Conclusion The diagonal vectors are $4\hat{i} - 2\hat{j} + 4\hat{k}$ and $-2\hat{i} + 2\hat{j}$, with lengths 6 and $2\sqrt{2}$, respectively.

Quick Tip to Solve: Add the adjacent-side vectors for one diagonal and subtract them for the other; square the components only after finding these vectors.

Section C (18 marks)

This section comprises Short Answer (SA) type questions of 3 marks each.

26. Evaluate: $\int_0^1 \log(1 + x^2) dx$.

[3 marks]

Solution:

Concept:

- Integration by parts gives $\int u dv = uv - \int v du$. Choose the logarithmic factor as u , because its derivative is rational.
- Use $\frac{x^2}{1 + x^2} = 1 - \frac{1}{1 + x^2}$ and $\int \frac{dx}{1 + x^2} = \tan^{-1} x + C$. Here $\log$ denotes the natural logarithm.

Step 1: Apply integration by parts Put $I = \int_0^1 \log(1 + x^2) dx$. Set $u = \log(1 + x^2)$ and $dv = dx$, so that $du = \frac{2x}{1 + x^2} dx$ and $v = x$. The integrand and its derivative are continuous throughout the interval, so the definite integration-by-parts formula applies:

$$I = [x \log(1 + x^2)]_0^1 - \int_0^1 \frac{2x^2}{1 + x^2} dx.$$

Quick Tip to Solve: After integrating the logarithm by parts, replace x squared over one plus x squared by one minus the reciprocal denominator.

At the upper endpoint the boundary term is $\log 2$; at the lower endpoint it is zero.

Step 2: Simplify the remaining integral Write the numerator as $x^2 = (1 + x^2) - 1$. This avoids further integration by parts and gives

$$\begin{aligned} I &= \log 2 - 2 \int_0^1 \left(1 - \frac{1}{1+x^2} \right) dx \\ &= \log 2 - 2 \left[x - \tan^{-1} x \right]_0^1. \end{aligned}$$

Since $\tan^{-1} 1 = \pi/4$ and $\tan^{-1} 0 = 0$, the bracket equals $1 - \pi/4$. Angles in this antiderivative are measured in radians. As a check, the integrand is nonnegative on $[0, 1]$, and the resulting value is positive, as required.

Step 3: Conclusion

$$\int_0^1 \log(1+x^2) dx = \log 2 - 2 + \frac{\pi}{2}.$$

Quick Tip to Solve: After integrating the logarithm by parts, replace x squared over one plus x squared by one minus the reciprocal denominator.

27(a). Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball. [3 marks]

Solution:

Concept:

- The law of total probability is $P(R) = P(E_1)P(R | E_1) + P(E_2)P(R | E_2)$, where E_1, E_2 are mutually exclusive, exhaustive bag-selection events and R is the event of drawing red.
- For a fair die all six faces are equally likely. In a random draw from a bag, the probability of red equals the number of red balls divided by the total number of balls in that bag.

Step 1: Find the probabilities of selecting the bags Let E_1 denote selection of bag I and E_2 denote selection of bag II. A number less than 3 is either 1 or 2; hence E_1 contains two die outcomes. The remaining four outcomes 3, 4, 5, 6 select bag II. Therefore

$$P(E_1) = \frac{2}{6} = \frac{1}{3}, \quad P(E_2) = \frac{4}{6} = \frac{2}{3}.$$

Exactly one bag is selected on every throw, so these two events form a partition of the experiment.

Step 2: Find the conditional red-ball probabilities Bag I contains $3 + 4 = 7$ balls, of which 3 are red. Bag II contains $8 + 6 = 14$ balls, of which 8 are red. Thus

$$P(R | E_1) = \frac{3}{7}, \quad P(R | E_2) = \frac{8}{14} = \frac{4}{7}.$$

Quick Tip to Solve: First count the die faces selecting each bag; use these probabilities to weight the red fractions, rather than averaging the two fractions equally.

The different bag totals must be used in these conditional probabilities; the balls in both bags are not pooled before the draw.

Step 3: Combine the two mutually exclusive routes A red ball can be drawn through bag I or through bag II. Their joint-event probabilities add, giving

$$\begin{aligned} P(R) &= \frac{1}{3} \cdot \frac{3}{7} + \frac{2}{3} \cdot \frac{4}{7} \\ &= \frac{3}{21} + \frac{8}{21} = \frac{11}{21}. \end{aligned}$$

This is a weighted average of the red fractions, with bag II receiving twice the weight of bag I.

Step 4: Conclusion The probability that the drawn ball is red is $\boxed{\frac{11}{21}}$.

Quick Tip to Solve: First count the die faces selecting each bag; use these probabilities to weight the red fractions, rather than averaging the two fractions equally.

OR

27(b). The probability of simultaneous occurrence of atleast one of the two events X and Y is a . If the probability that exactly one of the events X, Y occurs is b , prove that $P(X') + P(Y') = 2 - 2a + b$.

[3 marks]

Solution:

Concept:

- At least one of X, Y occurring is the union $X \cup Y$. Exactly one occurring is the disjoint union $(X \cap Y') \cup (X' \cap Y)$.
- Use inclusion-exclusion, $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$, and the complement rule $P(X') = 1 - P(X)$.

Step 1: Express the intersection using the two given probabilities Let $c = P(X \cap Y)$, the probability that both events occur. The event that at least one occurs splits into three disjoint possibilities: only X , only Y , or both. The first two together have probability b , while all three together have probability a . Consequently

$$a = b + c, \quad c = a - b.$$

Here the phrase at least one represents the union; it includes the case where both occur, whereas exactly one excludes that case.

Step 2: Obtain the sum of the event probabilities Apply inclusion-exclusion to the union specified in the question:

$$\begin{aligned} a &= P(X) + P(Y) - c, \\ P(X) + P(Y) &= a + c \\ &= a + (a - b) = 2a - b. \end{aligned}$$

No independence assumption is needed. The intersection is determined directly by subtracting the

Quick Tip to Solve: The difference between at least one and exactly one is the both-events probability: first write the intersection as a minus b .

exactly-one probability from the at-least-one probability.

Step 3: Apply the complement rule Subtract the sum just obtained from 2:

$$\begin{aligned}P(X^c) + P(Y^c) &= (1 - P(X)) + (1 - P(Y)) \\&= 2 - (P(X) + P(Y)) \\&= 2 - (2a - b).\end{aligned}$$

This also agrees with a disjoint-event count: neither event occurs with probability $1 - a$ and contributes to both complements, while the exactly-one event contributes to just one complement.

Step 4: Conclusion

$$P(X^c) + P(Y^c) = 2 - 2a + b.$$

Hence proved.

Quick Tip to Solve: The difference between at least one and exactly one is the both-events probability: first write the intersection as a minus b.

28(a). Find: $\int \sqrt{\frac{x+2}{x-2}} dx.$

[3 marks]

Solution:

Concept:

- The identity $x + 2 = (x - 2) + 4$ helps rewrite the irrational integrand in terms of $\sqrt{x^2 - 4}$. Its real domain consists of the intervals $x > 2$ and $x < -2$.
- Use $\frac{d}{dx} \sqrt{x^2 - a^2} = \frac{x}{\sqrt{x^2 - a^2}}$ and $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln |x + \sqrt{x^2 - a^2}| + C$, on intervals where these expressions are defined.

Step 1: Rewrite the integrand on the positive interval For $x > 2$, both $x - 2$ and $x + 2$ are positive. Multiplying inside the square root gives

$$\sqrt{\frac{x+2}{x-2}} = \frac{x+2}{\sqrt{(x+2)(x-2)}} = \frac{x}{\sqrt{x^2-4}} + \frac{2}{\sqrt{x^2-4}}.$$

Splitting the numerator this way produces one derivative of a square root and one standard logarithmic integral.

Step 2: Integrate the two terms Writing I for the required integral,

$$\begin{aligned}I &= \int \frac{x dx}{\sqrt{x^2-4}} + 2 \int \frac{dx}{\sqrt{x^2-4}} \\&= \sqrt{x^2-4} + 2 \ln |x + \sqrt{x^2-4}| + C.\end{aligned}$$

To verify the logarithmic term, put $s = \sqrt{x^2 - 4}$. Its derivative is $(1 + x/s)/(x + s) = 1/s$; hence differentiating the displayed expression gives $(x + 2)/s$, as required for $x > 2$.

Quick Tip to Solve: Rationalize the square-root ratio to expose the numerator $x + 2$, and check its sign before removing square roots.

Step 3: Account for the other real interval For $x < -2$, the square root in the question is still positive, whereas $x + 2 < 0$. Therefore $\sqrt{(x+2)/(x-2)} = -(x+2)/\sqrt{x^2-4}$, and the primitive on this interval is the negative of the expression obtained above. The integration constant may be chosen independently on the two disconnected intervals.

Step 4: Conclusion For $x > 2$, the integral is $\sqrt{x^2-4} + 2 \ln |x + \sqrt{x^2-4}| + C$. For $x < -2$, it is $-\sqrt{x^2-4} - 2 \ln |x + \sqrt{x^2-4}| + C$.

Quick Tip to Solve: Rationalize the square-root ratio to expose the numerator $x + 2$, and check its sign before removing square roots.

OR

28(b). Find: $\int \frac{x^2}{(x^2+9)(x^2+16)} dx$.

[3 marks]

Solution:

Concept:

- For a product of distinct quadratic factors, use partial fractions with constant numerators when the rational expression is even in x .
- The standard formula $\int dx/(x^2 + a^2) = a^{-1} \tan^{-1}(x/a) + C$ applies for positive a . Here the quadratic factors correspond to $a = 3$ and $a = 4$.

Step 1: Determine the partial fractions Let

$$\frac{x^2}{(x^2+9)(x^2+16)} = \frac{A}{x^2+9} + \frac{B}{x^2+16}.$$

Multiplying by the denominator yields $x^2 = A(x^2+16) + B(x^2+9)$. Comparing coefficients of x^2 and the constant terms gives $A + B = 1$ and $16A + 9B = 0$. Substituting $B = 1 - A$ gives $7A = -9$, so $A = -9/7$ and $B = 16/7$.

Step 2: Integrate each quadratic term The integral now becomes

$$\begin{aligned} I &= -\frac{9}{7} \int \frac{dx}{x^2+9} + \frac{16}{7} \int \frac{dx}{x^2+16} \\ &= -\frac{9}{7} \cdot \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + \frac{16}{7} \cdot \frac{1}{4} \tan^{-1}\left(\frac{x}{4}\right) + C. \end{aligned}$$

Each inverse tangent therefore carries the additional reciprocal of the square root of its quadratic constant.

Step 3: Check the coefficients by differentiation The derivative of $-\frac{3}{7} \tan^{-1}(x/3)$ is $-9/[7(x^2+9)]$, while that of $\frac{4}{7} \tan^{-1}(x/4)$ is $16/[7(x^2+16)]$. Combining these gives numerator $[-9(x^2+16) + 16(x^2+9)]/7 = x^2$, confirming the original integrand.

Quick Tip to Solve: After partial fractions, divide each coefficient by 3 or 4 when integrating the corresponding quadratic denominator.

Step 4: Conclusion

$$\int \frac{x^2 dx}{(x^2 + 9)(x^2 + 16)} = \frac{4}{7} \tan^{-1} \left(\frac{x}{4} \right) - \frac{3}{7} \tan^{-1} \left(\frac{x}{3} \right) + C.$$

Quick Tip to Solve: After partial fractions, divide each coefficient by 3 or 4 when integrating the corresponding quadratic denominator.

29. If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \cos 2x}$ **and** $I_2 = \int_{-1/2}^{1/2} |x| dx$, **then show that** $I_1 - 4I_2 = 0$.

[3 marks]

Solution:**Concept:**

- The double-angle identity $1 + \cos 2x = 2 \cos^2 x$ reduces the first integral to an integral of $\sec^2 x$.
- The function $|x|$ is even. Thus $\int_{-a}^a |x| dx = 2 \int_0^a x dx$; the absolute value must be accounted for before using an antiderivative.

Step 1: Evaluate the trigonometric integral Using the double-angle identity,

$$\begin{aligned} I_1 &= \int_{-\pi/4}^{\pi/4} \frac{dx}{2 \cos^2 x} \\ &= \frac{1}{2} [\tan x]_{-\pi/4}^{\pi/4} \\ &= \frac{1}{2} (1 - (-1)) = 1. \end{aligned}$$

The denominator stays positive throughout this interval, so no singularity interrupts the evaluation. The two endpoint tangent values have opposite signs; subtracting the lower value therefore adds one.

Step 2: Evaluate the absolute-value integral On $[-1/2, 0]$, $|x| = -x$, and on $[0, 1/2]$, $|x| = x$. These two portions have equal area by symmetry. Consequently,

$$\begin{aligned} I_2 &= 2 \int_0^{1/2} x dx \\ &= 2 \left[\frac{x^2}{2} \right]_0^{1/2} \\ &= 2 \cdot \frac{1}{8} = \frac{1}{4}. \end{aligned}$$

Using x over the entire symmetric interval would instead give zero and would ignore the absolute value in the integrand.

Step 3: Compare the two evaluated quantities Substitute the separately calculated values into the requested expression:

$$I_1 - 4I_2 = 1 - 4 \left(\frac{1}{4} \right) = 1 - 1 = 0.$$

Quick Tip to Solve: Replace $1 + \cos 2x$ by $2 \cos^2 x$, and use evenness to evaluate the absolute-value integral on its positive half.

Equivalently, the value of the first integral equals four times the area represented by the second integral, which establishes the stated relationship.

Step 4: Conclusion $I_1 = 1$, $I_2 = \frac{1}{4}$, and hence $I_1 - 4I_2 = 0$.

Quick Tip to Solve: Replace $1 + \cos 2x$ by $2 \cos^2 x$, and use evenness to evaluate the absolute-value integral on its positive half.

30(a). Find the general solution of the differential equation: $(y^2 - x^2) dx = 2xy dy$.

[3 marks]

Solution:

Concept:

- A homogeneous differential equation with terms of the same degree can be solved using $y = vx$, where $v = y/x$ and $dy/dx = v + x dv/dx$.
- After this substitution, separate the variables and integrate. A resulting implicit relation can be checked by differentiation against the original differential equation.

Step 1: Introduce the homogeneous substitution For $x \neq 0$ and $y \neq 0$, rewrite the given equation as

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}.$$

The right-hand side depends only on y/x , which confirms homogeneity. Put $y = vx$, so that $dy/dx = v + x dv/dx$. Substitution gives

$$v + x \frac{dv}{dx} = \frac{v^2 - 1}{2v}.$$

Step 2: Separate and integrate Moving v to the right side gives

$$x \frac{dv}{dx} = -\frac{v^2 + 1}{2v}, \quad \frac{2v}{v^2 + 1} dv = -\frac{dx}{x}.$$

Integrating both sides,

$$\ln(v^2 + 1) = -\ln|x| + C_1.$$

Since $v^2 + 1 > 0$, the logarithm on the left needs no absolute-value signs. Combining logarithms and exponentiating shows that $|x|(v^2 + 1)$ is constant on any interval where x has fixed sign.

Step 3: Return to the original variables and verify Absorb the fixed sign of x into an arbitrary constant C , giving $x(v^2 + 1) = C$. Substituting $v = y/x$ and multiplying by x yields $x^2 + y^2 = Cx$. For a direct check, differentiate this relation:

$$2x dx + 2y dy = C dx.$$

Multiplication by x , followed by $Cx = x^2 + y^2$, gives $2xy dy = (y^2 - x^2) dx$, exactly the equation in the question. The implicit form also avoids retaining the quotients used during the substitution.

Step 4: Conclusion The general solution is $x^2 + y^2 = Cx$, where C is an arbitrary constant.

Quick Tip to Solve: With $y = vx$, simplify $(v^2 - 1)/(2v) - v$ carefully: it becomes $-(v^2 + 1)/(2v)$.

30(b). Find the particular solution of the differential equation $(1 + e^{2x}) dy + (1 + y^2)e^x dx = 0$, given that $y(1) = 0$. [3 marks]

Solution:

Concept:

- Separate the variables by putting all factors in y with dy and all factors in x with dx .
- Use $\int dt/(1 + t^2) = \tan^{-1} t + C$. The substitution $t = e^x$ gives $dt = e^x dx$ and $e^{2x} = t^2$.

Step 1: Separate and integrate The factors $1 + y^2$ and $1 + e^{2x}$ are strictly positive for real variables. Divide by their product to get

$$\frac{dy}{1 + y^2} = -\frac{e^x dx}{1 + e^{2x}}.$$

Integrate both sides. On the right put $t = e^x$, so the numerator is exactly dt :

$$\tan^{-1} y = -\int \frac{dt}{1 + t^2} = -\tan^{-1}(e^x) + C.$$

Thus the general integrated relation is $\tan^{-1} y + \tan^{-1}(e^x) = C$, where C is a real constant. No constant real solution is lost in the separation, since $1 + y^2$ never vanishes.

Step 2: Use the initial value Substitute $x = 1$, $y = 0$ in the integrated relation to obtain $C = \tan^{-1} e$. Therefore

$$\tan^{-1} y = \tan^{-1} e - \tan^{-1}(e^x).$$

Taking the tangent and applying $\tan(A - B) = (\tan A - \tan B)/(1 + \tan A \tan B)$ gives

$$y = \frac{e - e^x}{1 + e^{x+1}}.$$

The denominator is positive for every real x ; the difference of the two inverse tangents lies in $(-\pi/2, \pi/2)$. Thus the explicit expression agrees with the inverse-tangent relation, and at $x = 1$ it gives $y = 0$.

Step 3: Conclusion The particular solution is $y = \frac{e - e^x}{1 + e^{x+1}}$, equivalently $\tan^{-1} y + \tan^{-1}(e^x) = \tan^{-1} e$.

Quick Tip to Solve: Recognise the inverse-tangent integral by substituting $t = e^x$, and determine the integration constant before taking the tangent.

31. Solve the following linear programming problem graphically:

Minimize $Z = 13x - 15y$

Subject to constraints

$$x + y \leq 7,$$

$$2x - 3y + 6 \geq 0,$$

$$x \geq 0, y \geq 0.$$

[3 marks]

Solution:

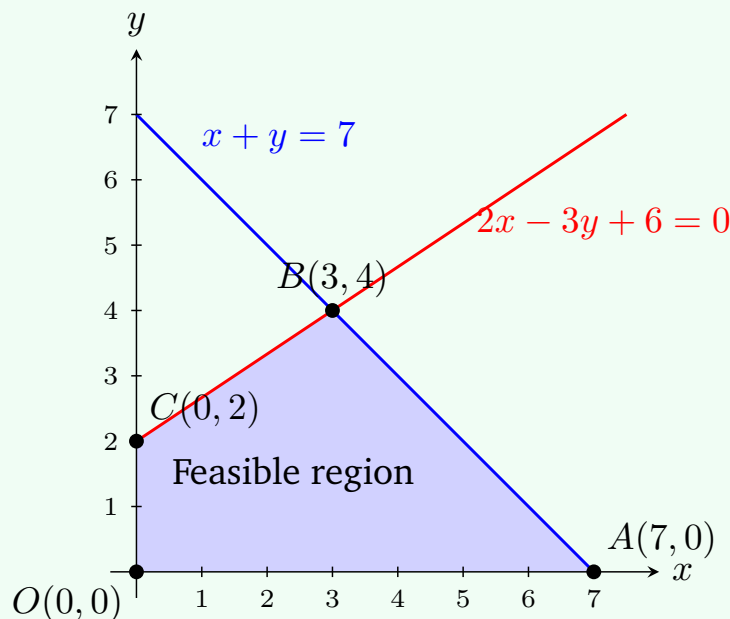
Concept:

- Plot each boundary line and retain the half-plane satisfying its inequality. Non-negativity restricts the feasible region to the first quadrant.
- A linear objective on a nonempty bounded polygon attains its minimum at a vertex. Evaluate $Z = 13x - 15y$ at every feasible vertex.

Step 1: Plot the constraints and shade their intersection The line $x + y = 7$ passes through $(7, 0)$ and $(0, 7)$; its allowed half-plane is below the line. The other constraint is

$$2x - 3y + 6 \geq 0 \iff y \leq \frac{2}{3}x + 2.$$

Plot its boundary through $(0, 2)$ and $(3, 4)$, retaining the side containing the origin. Together with $x \geq 0$ and $y \geq 0$, the common region is the shaded quadrilateral $OABC$ in the diagram. Both boundary lines are included because the inequalities are non-strict.



The common feasible region OABC in the first quadrant

Step 2: Find the vertices and evaluate the objective The two oblique boundaries meet where $y = 7 - x$. Substituting in $2x - 3y + 6 = 0$ gives

$$2x - 3(7 - x) + 6 = 0 \Rightarrow 5x = 15 \Rightarrow x = 3, \quad y = 4.$$

Quick Tip to Solve: Rewrite the second inequality as $y \leq (2/3)x + 2$ before shading; then compare the objective at every corner, including both axis corners.

Hence the feasible vertices are $O(0, 0)$, $A(7, 0)$, $B(3, 4)$ and $C(0, 2)$. Their objective values are

Vertex	$Z = 13x - 15y$
$(0, 0)$	0
$(7, 0)$	91
$(3, 4)$	$39 - 60 = -21$
$(0, 2)$	-30

The feasible polygon is closed and bounded, so comparing these four values is sufficient. The smallest value occurs at C . The objective-line slope $13/15$ differs from both oblique-edge slopes, and the adjacent axis edge also has a different slope, so the minimum is unique.

Step 3: Conclusion The minimum value is $Z_{\min} = -30$, attained at $(x, y) = (0, 2)$.

Quick Tip to Solve: Rewrite the second inequality as $y \leq (2/3)x + 2$ before shading; then compare the objective at every corner, including both axis corners.

Section D (20 marks)

This section comprises Long Answer (LA) type questions of 5 marks each.

32. Show that line AB passing through points $A(0, 4, 1)$, $B(2, 3, -1)$ and the line CD passing through points $C(4, 5, 0)$, $D(2, 6, 2)$ are parallel. Also, find distance between them. [5 marks]

Solution:

Concept:

- A direction vector of the line through points P, Q is $\overrightarrow{PQ} = \mathbf{q} - \mathbf{p}$. Two nonzero scalar-multiple direction vectors describe parallel directions.
- The distance between parallel lines is $h = |\overrightarrow{AC} \times \mathbf{d}|/|\mathbf{d}|$, where A, C lie on the two lines and $\mathbf{d}$ is their common direction vector. This is the height of the parallelogram formed by the joining vector and the direction vector.

Step 1: Calculate and compare direction vectors Subtract the initial point's coordinates from the terminal point's coordinates:

$$\begin{aligned}\overrightarrow{AB} &= (2 - 0, 3 - 4, -1 - 1) = (2, -1, -2), \\ \overrightarrow{CD} &= (2 - 4, 6 - 5, 2 - 0) = (-2, 1, 2).\end{aligned}$$

Since $\overrightarrow{CD} = -\overrightarrow{AB}$, these nonzero vectors have the same line direction. The minus sign reverses orientation but does not alter the infinite line's direction. Thus the two lines have parallel directions, with common direction vector $\mathbf{d} = (2, -1, -2)$.

Step 2: Find a joining vector and the direction magnitude Choose A on the first line and C on the second. Then

$$\overrightarrow{AC} = (4 - 0, 5 - 4, 0 - 1) = (4, 1, -1), \quad |\mathbf{d}| = \sqrt{2^2 + (-1)^2 + (-2)^2} = 3.$$

Quick Tip to Solve: For parallel lines, use any joining vector and divide its cross-product magnitude with the common direction vector by the magnitude of that direction vector.

The segment AC need not be perpendicular to the lines, so its length alone is not their distance. The cross product with $\mathbf{d}$ discards its parallel component. Dividing that cross-product magnitude by $|\mathbf{d}|$ gives exactly the length of the perpendicular component.

Step 3: Evaluate the cross product and separation Expanding the determinant with its alternating signs gives

$$\begin{aligned}\overrightarrow{AC} \times \mathbf{d} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & -1 \\ 2 & -1 & -2 \end{vmatrix} \\ &= (-2 - 1)\hat{i} - (-8 + 2)\hat{j} + (-4 - 2)\hat{k} \\ &= -3\hat{i} + 6\hat{j} - 6\hat{k}.\end{aligned}$$

Therefore $|\overrightarrow{AC} \times \mathbf{d}| = \sqrt{9 + 36 + 36} = 9$, and the perpendicular separation is $h = 9/3$. The nonzero cross product confirms that the parallel lines are distinct, rather than coincident.

Step 4: Conclusion The lines AB and CD are parallel, and the distance between them is 3 units.

Quick Tip to Solve: For parallel lines, use any joining vector and divide its cross-product magnitude with the common direction vector by the magnitude of that direction vector.

33(a). A relation R is defined on $\mathbb{Z}$, the set of integers, as

$$R = \{(x, y) : |x - y| \text{ is divisible by a prime number } p, x, y \in \mathbb{Z}\}.$$

Check whether R is an equivalence relation or not.

[5 marks]

Solution:

Concept:

- A relation on $\mathbb{Z}$ is an equivalence relation when it is reflexive, symmetric and transitive. Each property must hold for all integers involved.
- For the given prime number p , divisibility means $p \mid n$ if $n = pk$ for some integer k . In particular, $p \mid 0$, since $0 = p \cdot 0$.
- Divisibility of $|x - y|$ by p is equivalent to divisibility of $x - y$ by p : changing the sign of an integer changes only the sign of its integer multiplier.

Step 1: Verify reflexivity Let x be any integer. Its difference from itself satisfies

$$|x - x| = 0 = p \cdot 0.$$

Thus the given prime p divides $|x - x|$. By the definition of R , we have $(x, x) \in R$. Since this argument applies to every $x \in \mathbb{Z}$, the relation is reflexive. Zero is divisible by every nonzero integer; it need not itself be prime for this argument to apply.

Step 2: Verify symmetry Let $x, y \in \mathbb{Z}$ and assume $(x, y) \in R$. Then p divides $|x - y|$. However,

$$|y - x| = |-(x - y)| = |x - y|.$$

Quick Tip to Solve: Replace absolute-difference divisibility with signed-difference divisibility, and add the two signed equations to prove transitivity for the same given prime.

The same prime p therefore divides $|y - x|$, so $(y, x) \in R$. This verifies symmetry for every pair in the relation, without any assumption about which of the two integers is larger.

Step 3: Verify transitivity using signed differences Let $x, y, z \in \mathbb{Z}$, with $(x, y) \in R$ and $(y, z) \in R$. For the given prime p , both differences are divisible by p . Equivalently, there exist integers m, n such that

$$x - y = pm, \quad y - z = pn.$$

Adding these signed equations yields

$$x - z = (x - y) + (y - z) = p(m + n).$$

Since $m + n$ is an integer, $p \mid (x - z)$, and consequently $p \mid |x - z|$. Hence $(x, z) \in R$, proving transitivity. Using signed differences avoids incorrectly asserting that absolute differences always add: $|x - z| = |x - y| + |y - z|$ need not hold.

Step 4: Conclusion For the given prime number p , R is reflexive, symmetric and transitive. Therefore R is an equivalence relation on $\mathbb{Z}$.

Quick Tip to Solve: Replace absolute-difference divisibility with signed-difference divisibility, and add the two signed equations to prove transitivity for the same given prime.

OR

33(b). A function $f : \mathbb{R} - \left\{\frac{3}{5}\right\} \rightarrow \mathbb{R} - \left\{\frac{3}{5}\right\}$ is defined as $f(x) = \frac{3x+2}{5x-3}$. Show that f is one-one and onto. [5 marks]

Solution:

Concept:

- A function is one-one if equal images force equal preimages: $f(a) = f(b) \Rightarrow a = b$.
- It is onto if every element y of the stated codomain has a preimage x in the domain.
- For a rational function, the denominator must be nonzero, and a proposed preimage must obey the domain restriction.

Step 1: Check that the function maps into the given codomain Put $D = \mathbb{R} \setminus \{3/5\}$. For $x \in D$, $5x - 3 \neq 0$, so $f(x)$ exists. Also,

$$f(x) - \frac{3}{5} = \frac{5(3x+2) - 3(5x-3)}{5(5x-3)} = \frac{19}{5(5x-3)} \neq 0.$$

Thus no allowed input has image $3/5$, and the prescribed codomain is respected. This check matters because proving injectivity alone would not verify that the stated mapping is well-defined.

Step 2: Prove that equal images imply equal inputs Let $a, b \in D$ and suppose $f(a) = f(b)$. Both

Quick Tip to Solve: After solving for the preimage, check that it cannot equal the excluded value; here the constant numerator 19 makes that check immediate.

denominators are nonzero, so cross-multiplication is valid:

$$\begin{aligned}(3a + 2)(5b - 3) &= (3b + 2)(5a - 3), \\ 15ab - 9a + 10b - 6 &= 15ab - 9b + 10a - 6, \\ 19b &= 19a.\end{aligned}$$

Dividing by the nonzero constant 19 gives $a = b$. Since the inputs were arbitrary, f is one-one on its entire domain.

Step 3: Construct a preimage for every allowed output Take an arbitrary $y \in D$. Solving $y = f(x)$ for x gives

$$\begin{aligned}y(5x - 3) &= 3x + 2, \\ (5y - 3)x &= 3y + 2, \\ x &= \frac{3y + 2}{5y - 3}.\end{aligned}$$

The denominator is nonzero because $y \neq 3/5$. Moreover,

$$x - \frac{3}{5} = \frac{19}{5(5y - 3)} \neq 0,$$

so this real preimage belongs to the domain. To verify rather than merely propose it, substitute:

$$f(x) = \frac{(19y)/(5y - 3)}{19/(5y - 3)} = y.$$

Hence every element of the codomain is attained and f is onto.

Step 4: Conclusion The function f is both one-one and onto, hence bijective. Its inverse has the same formula: $f^{-1}(y) = (3y + 2)/(5y - 3)$, for $y \neq 3/5$.

Quick Tip to Solve: After solving for the preimage, check that it cannot equal the excluded value; here the constant numerator 19 makes that check immediate.

34(a). If $A = \begin{pmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{pmatrix}$, find A^{-1} and use it to solve the following system of equations:
 $-2y + z = 7$, $2x - y - z = 8$, $x - 2y = 10$. [5 marks]

Solution:

Concept:

- A square matrix A is invertible when $|A| \neq 0$, and $A^{-1} = \text{adj } A / |A|$.
- The cofactor C_{ij} is $(-1)^{i+j}$ times the corresponding minor, and the adjugate is the transpose of the cofactor matrix.
- The coefficient matrix must be read from the equations in the chosen variable order. Here it is A^T , so $(A^T)^{-1} = (A^{-1})^T$.

Quick Tip to Solve: Compare the equation coefficients with both A and its transpose before multiplying by the inverse; here the equations require the transpose of the computed inverse.

Step 1: Evaluate the determinant and cofactors Expanding along the first row,

$$\begin{aligned}|A| &= -2 \begin{vmatrix} -2 & -2 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} -2 & -1 \\ 1 & -1 \end{vmatrix} \\ &= -2(2) + 3 = -1.\end{aligned}$$

Thus the inverse exists. Computing all nine signed minors gives the cofactor matrix

$$C = \begin{pmatrix} -2 & -2 & 3 \\ -1 & -1 & 2 \\ -3 & -2 & 4 \end{pmatrix}.$$

For example, $C_{12} = -((-2)(0) - (-2)(1)) = -2$, while $C_{31} = 2(-2) - 1(-1) = -3$. The alternating signs must be applied before transposing.

Step 2: Form the adjugate and inverse Transposing C and dividing by the determinant,

$$\text{adj } A = \begin{pmatrix} -2 & -1 & -3 \\ -2 & -1 & -2 \\ 3 & 2 & 4 \end{pmatrix}, \quad A^{-1} = \begin{pmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ -3 & -2 & -4 \end{pmatrix}.$$

Multiplication provides a useful independent check: the first row of A produces $(1, 0, 0)$, the second produces $(0, 1, 0)$, and the third produces $(0, 0, 1)$ when multiplied by this inverse. Hence $AA^{-1} = I$.

Step 3: Write and solve the printed system using the inverse With $X = (x, y, z)^T$ and $B = (7, 8, 10)^T$, the printed equations give

$$\begin{pmatrix} 0 & -2 & 1 \\ 2 & -1 & -1 \\ 1 & -2 & 0 \end{pmatrix} X = B.$$

The coefficient matrix is precisely A^T . Taking the transpose of the inverse already found therefore yields

$$\begin{aligned}X &= (A^{-1})^T B \\ &= \begin{pmatrix} 2 & 2 & -3 \\ 1 & 1 & -2 \\ 3 & 2 & -4 \end{pmatrix} \begin{pmatrix} 7 \\ 8 \\ 10 \end{pmatrix} \\ &= \begin{pmatrix} 14 + 16 - 30 \\ 7 + 8 - 20 \\ 21 + 16 - 40 \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \\ -3 \end{pmatrix}.\end{aligned}$$

Check each equation directly: $-2(-5) + (-3) = 7$, $2(0) - (-5) - (-3) = 8$, and $0 - 2(-5) = 10$. All three are satisfied, and invertibility ensures uniqueness.

Step 4: Conclusion $A^{-1} = \begin{pmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ -3 & -2 & -4 \end{pmatrix}$. The unique solution is $\boxed{x = 0, y = -5, z = -3}$.

Quick Tip to Solve: Compare the equation coefficients with both A and its transpose before multiplying by the inverse; here the equations require the transpose of the computed inverse.

34(b). If $\begin{pmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{pmatrix}$ is a singular matrix, then find all values of x where $x \in \left[0, \frac{\pi}{2}\right]$. [5 marks]

Solution:

Concept:

- A square matrix is singular if and only if its determinant is zero. This condition reduces the given matrix problem to an equation in x .
- The identities $1 - \cos 2x = 2 \sin^2 x$ and $\sin 3x = 3 \sin x - 4 \sin^3 x$ express the equation using a single trigonometric quantity.
- For $0 \leq x \leq \pi/2$, $0 \leq \sin x \leq 1$, and sine is strictly increasing. Thus each admissible value of $\sin x$ gives exactly one value of x in the required interval.

Step 1: Apply the singularity condition Let the given matrix be A . Expanding its determinant along the first row, with cofactor signs $+$, $-$, $+$, gives

$$\begin{aligned} |A| &= 3(8 - 7 \cos 2x) + (-14 + 11 \cos 2x) + \sin 3x(-49 + 44) \\ &= 10 - 10 \cos 2x - 5 \sin 3x. \end{aligned}$$

Here the second contribution is positive times its minor because the entry is -1 and its cofactor sign is negative. Since A is singular, $|A| = 0$. Dividing by 5, we obtain

$$2(1 - \cos 2x) = \sin 3x.$$

Step 2: Convert to a polynomial in sine Substituting the double-angle and triple-angle identities yields

$$4 \sin^2 x = 3 \sin x - 4 \sin^3 x.$$

Put $s = \sin x$. Moving every term to one side and factorising,

$$\begin{aligned} 4s^3 + 4s^2 - 3s &= 0, \\ s(4s^2 + 4s - 3) &= 0, \\ s(2s - 1)(2s + 3) &= 0. \end{aligned}$$

We have kept the factor s ; dividing by sine at the start would lose a possible endpoint solution.

Step 3: Enforce the interval and verify the candidates The algebraic possibilities are $s = 0$, $s = 1/2$ and $s = -3/2$. The last is impossible for a real sine value. On the specified interval, $\sin x = 0$ occurs at $x = 0$, while $\sin x = 1/2$ occurs at $x = \pi/6$. No supplementary-angle solution lies in this interval. Directly, at $x = 0$ the determinant is $10 - 10(1) - 5(0) = 0$. At $x = \pi/6$, it is $10 - 10(1/2) - 5(1) = 0$. Thus both admissible candidates satisfy the original singularity condition, and the factorisation exhausts all possibilities.

Step 4: Conclusion All the required values are $x = 0$ or $x = \frac{\pi}{6}$.

Quick Tip to Solve: Factor out $\sin x$ instead of dividing by it: this preserves the endpoint $x = 0$. Then reject polynomial roots outside the allowed sine range.

35. If $x = a(\sin t - t \cos t)$ and $y = b(\cos t + t \sin t)$, then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

[5 marks]

Solution:

Concept:

- For a curve specified parametrically by $x = x(t)$, $y = y(t)$, its derivative is $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ wherever $dx/dt \neq 0$.
- To find the second derivative, differentiate the first derivative with respect to t and divide again by dx/dt : $\frac{d^2y}{dx^2} = \frac{d}{dt}(dy/dx) / \frac{dx}{dt}$.
- Use the product rule for $t \cos t$ and $t \sin t$, treating a and b as constants. Also, $\frac{d}{dt} \cot t = -\csc^2 t$.

Step 1: Differentiate the two parametric expressions From $x = a(\sin t - t \cos t)$, the product rule gives

$$\begin{aligned}\frac{dx}{dt} &= a [\cos t - (\cos t - t \sin t)] \\ &= at \sin t.\end{aligned}$$

Similarly, from $y = b(\cos t + t \sin t)$,

$$\begin{aligned}\frac{dy}{dt} &= b [-\sin t + (\sin t + t \cos t)] \\ &= bt \cos t.\end{aligned}$$

In each expression the derivative of the isolated trigonometric term cancels one part of the product-rule derivative, leaving the term containing t .

Step 2: Obtain the first derivative using the chain rule At a regular parameter value, where $at \sin t \neq 0$, the chain rule gives

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{bt \cos t}{at \sin t} = \frac{b}{a} \cot t.$$

The common factor t cancels. Eliminating the parameter is unnecessary.

Step 3: Differentiate the slope and convert back to x First differentiate the slope with respect to t , keeping b/a constant:

$$\frac{d}{dt} \left(\frac{dy}{dx} \right) = \frac{b}{a} \frac{d}{dt} (\cot t) = -\frac{b}{a} \csc^2 t.$$

The derivative with respect to x requires a further division by $dx/dt = at \sin t$. Hence

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{-(b/a) \csc^2 t}{at \sin t} \\ &= -\frac{b}{a^2 t \sin^3 t}.\end{aligned}$$

The extra division by $at \sin t$ supplies the second factor of a and the third factor of sine. These formulas apply where $dx/dt \neq 0$.

Step 4: Conclusion $\frac{dy}{dx} = \frac{b}{a} \cot t$ and $\frac{d^2y}{dx^2} = -\frac{b}{a^2 t \sin^3 t}$, at regular parameter values $at \sin t \neq 0$.

Quick Tip to Solve: After differentiating the slope with respect to t , divide by dx/dt once more; this is what supplies the extra factor $at \sin t$ in the second derivative.

Section E (12 marks)

This section comprises of 3 case study based questions of 4 marks each.

36(i). Roundabouts are often made on busy roads to ease the traffic and avoid red lights. One such round-about is made such that equation representing its boundary is given by $C_1 : x^2 + y^2 = 64$. There is a circular pond with a fountain in the middle of the roundabout whose equation is given by $C_2 : x^2 + y^2 = 4$.

Represent the given equations C_1 and C_2 with the help of a diagram.

[1 mark]

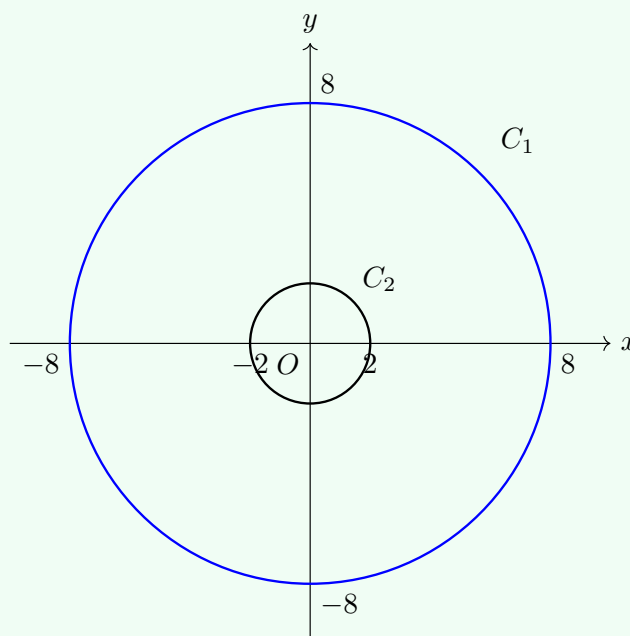
Solution:

Concept:

- A circle $x^2 + y^2 = r^2$ has centre $(0, 0)$ and radius r .
- Circles with the same centre and different radii are concentric.

Step 1: Identify the two radii Comparing $C_1 : x^2 + y^2 = 64$ with the standard equation gives radius $\sqrt{64} = 8$. Likewise, $C_2 : x^2 + y^2 = 4$ has radius $\sqrt{4} = 2$. Both equations have no linear terms, so both centres are the origin.

Step 2: Locate and draw the boundaries The outer circle meets the coordinate axes at $(\pm 8, 0)$ and $(0, \pm 8)$, while the pond boundary meets them at $(\pm 2, 0)$ and $(0, \pm 2)$. Draw both circles about the same origin with the inner radius one-quarter of the outer radius.



Concentric roundabout and pond boundaries

Step 3: Conclusion The diagram represents concentric circles C_1 and C_2 with common centre $O(0, 0)$ and radii 8 and 2, respectively.

Quick Tip to Solve: Read each radius by taking the square root of the constant on the right.

36(ii). Express y as a function of x , ($y = f(x)$), for both C_1 and C_2 .

[1 mark]

Solution:

Concept:

- Isolate y^2 and take both square roots to describe the upper and lower semicircles.
- A real square root requires its radicand to be non-negative.

Step 1: Express the outer boundary For C_1 , rearranging gives $y^2 = 64 - x^2$. Thus its upper semicircle is $y = \sqrt{64 - x^2}$ and its lower semicircle is $y = -\sqrt{64 - x^2}$, with $-8 \leq x \leq 8$.

Step 2: Express the pond boundary For C_2 , we have $y^2 = 4 - x^2$. The corresponding upper and lower branches are $y = \sqrt{4 - x^2}$ and $y = -\sqrt{4 - x^2}$, with $-2 \leq x \leq 2$. Each branch is a function of x ; the complete circle consists of both branches.

Step 3: Conclusion $C_1 : y = \pm\sqrt{64 - x^2}, x \in [-8, 8]$; $C_2 : y = \pm\sqrt{4 - x^2}, x \in [-2, 2]$.

Quick Tip to Solve: Keep both signs for a complete circle; use the positive branch for area above the axis.

36(iii)(a). Using integration find the area of region covered by the roundabout.

[2 marks]

Solution:

Concept:

- The area under a non-negative curve $y = f(x)$ is $\int_a^b f(x) dx$.
- A circle is symmetric about both axes, so its enclosed area is four times its first-quadrant area.

Step 1: Set up the definite integral The roundabout is bounded by $x^2 + y^2 = 64$. Its upper boundary is $y = \sqrt{64 - x^2}$. In the first quadrant, x runs from 0 to 8. Hence the area enclosed by this boundary is

$$A = 4 \int_0^8 \sqrt{64 - x^2} dx.$$

This integral measures the entire disk inside the stated boundary.

Step 2: Evaluate by trigonometric substitution Put $x = 8 \sin \theta$, so $dx = 8 \cos \theta d\theta$ and $\sqrt{64 - x^2} = 8 \cos \theta$. The limits become 0 and $\pi/2$. Therefore

$$\begin{aligned} A &= 256 \int_0^{\pi/2} \cos^2 \theta d\theta \\ &= 128 \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\pi/2} \\ &= 64\pi. \end{aligned}$$

Both endpoint sine terms vanish. Symmetry accounts for all four quadrants, so no further factor is needed.

Step 3: Conclusion The area covered by the roundabout is $\boxed{64\pi}$ square units.

Quick Tip to Solve: Use four times the first-quadrant integral, with the radius as the upper limit.

36(iii)(b). Using integration, find the area of region covered by circular pond.

[2 marks]

Solution:

Concept:

- The area under a non-negative curve $y = f(x)$ is $\int_a^b f(x) dx$.
- A circle is symmetric about both axes, so its enclosed area is four times its first-quadrant area.

Step 1: Set up the definite integral The circular pond is bounded by $x^2 + y^2 = 4$. Its upper boundary is $y = \sqrt{4 - x^2}$. In the first quadrant, x runs from 0 to 2. Hence the area enclosed by this boundary is

$$A = 4 \int_0^2 \sqrt{4 - x^2} dx.$$

This integral measures the entire disk inside the stated boundary.

Step 2: Evaluate by trigonometric substitution Put $x = 2 \sin \theta$, so $dx = 2 \cos \theta d\theta$ and $\sqrt{4 - x^2} = 2 \cos \theta$. The limits become 0 and $\pi/2$. Therefore

$$\begin{aligned} A &= 16 \int_0^{\pi/2} \cos^2 \theta d\theta \\ &= 8 \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\pi/2} \\ &= 4\pi. \end{aligned}$$

Both endpoint sine terms vanish. Symmetry accounts for all four quadrants, so no further factor is needed.

Step 3: Conclusion The area covered by the circular pond is $\boxed{4\pi}$ square units.

Quick Tip to Solve: Use four times the first-quadrant integral, with the radius as the upper limit.

37(i). An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹300 per subscriber for unlimited free deliveries. The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹1, ten subscribers will discontinue. Assume that the company increased the annual fee by ₹ x .

How many subscribers will discontinue after an increase of ₹ x in annual fee?

[1 mark]

Solution:

Concept:

- The predicted loss is proportional to the increase in annual fee.
- For each increase of one rupee, ten subscribers discontinue.

Step 1: Identify the rate of subscriber loss The stated prediction gives a loss rate of 10 subscribers per rupee of fee increase. The variable x measures only the increase in fee, so the rate is applied to x , rather than to the original fee of 300 rupees.

Quick Tip to Solve: Multiply the increase alone by ten; do not apply the loss rate to the full new fee.

Step 2: Scale the loss to the stated increase An increase of x rupees therefore gives $10 \times x = 10x$ subscribers discontinuing. As a check, $x = 0$ gives no predicted loss, and $x = 1$ gives the specified loss of ten subscribers. The remaining subscriber count would be $5000 - 10x$.

Step 3: Conclusion $10x$ subscribers will discontinue.

Quick Tip to Solve: Multiply the increase alone by ten; do not apply the loss rate to the full new fee.

37(ii). If $R(x)$ denotes the total revenue collected after the increase of ₹ x in subscription fee, express $R(x)$ as a function of x . [1 mark]

Solution:

Concept:

- Annual revenue equals annual fee per subscriber multiplied by the remaining number of subscribers.
- After an increase of x rupees, the fee is $300 + x$ and the predicted loss is $10x$.

Step 1: Find the two revenue factors The original annual fee is 300 rupees, so the revised annual fee is $300 + x$ rupees per subscriber. Starting with 5000 subscribers and subtracting the predicted $10x$ departures leaves $5000 - 10x$ paying subscribers.

Step 2: Form and expand the revenue function Multiply the fee by the number who remain:

$$\begin{aligned} R(x) &= (300 + x)(5000 - 10x) \\ &= 1500000 + 5000x - 3000x - 10x^2 \\ &= 1500000 + 2000x - 10x^2. \end{aligned}$$

The revenue is measured in rupees per year. The physical model for a fee increase has $0 \leq x \leq 500$, so the predicted remaining subscriber count is non-negative.

Step 3: Conclusion $R(x) = (300 + x)(5000 - 10x) = 1500000 + 2000x - 10x^2$ rupees per year.

Quick Tip to Solve: Update both factors: the fee increases while the number of paying subscribers decreases.

37(iii)(a). Find the value of x for which $R(x)$ is maximum. [2 marks]

Solution:

Concept:

- Total revenue is subscription fee multiplied by the number of subscribers: $R(x) = (300 + x)(5000 - 10x)$.
- At an interior maximum, $R'(x) = 0$; a negative second derivative confirms a maximum.

Step 1: Express and differentiate the revenue An increase of x rupees leaves $5000 - 10x$ subscribers, each paying $300 + x$ rupees. Thus

$$R(x) = 1500000 + 2000x - 10x^2, \quad R'(x) = 2000 - 20x.$$

Quick Tip to Solve: Expand the revenue product first; the maximum occurs at the zero of its linear derivative.

The feasible fee increase satisfies $0 \leq x \leq 500$, since the number of subscribers cannot be negative.

Step 2: Locate and verify the maximum Setting the derivative equal to zero gives

$$2000 - 20x = 0 \implies x = 100.$$

Also, $R''(x) = -20 < 0$, so the revenue curve is concave down and this stationary point gives its maximum. The value 100 lies in the feasible interval. Equivalently, $R(x) = 1600000 - 10(x - 100)^2$, which is largest when the squared term vanishes.

Step 3: Conclusion The revenue is maximum for $x = 100$, that is, an increase of Rs 100 in the annual subscription fee.

Quick Tip to Solve: Expand the revenue product first; the maximum occurs at the zero of its linear derivative.

OR

37(iii)(b). Find the sub-intervals of $(0, 5000)$ in which $R(x)$ is increasing and decreasing. [2 marks]

Solution:

Concept:

- The revenue function is $R(x) = (300 + x)(5000 - 10x)$.
- A differentiable function increases where its derivative is positive and decreases where its derivative is negative.

Step 1: Differentiate the revenue function After a fee increase of x rupees, the fee is $300 + x$ rupees and the subscriber count is $5000 - 10x$. Therefore

$$R(x) = 1500000 + 2000x - 10x^2, \quad R'(x) = 2000 - 20x = 20(100 - x).$$

The only stationary point is $x = 100$, which splits the interval specified in the question.

Step 2: Determine the sign on each sub-interval For $0 < x < 100$, the factor $100 - x$ is positive, so $R'(x) > 0$. For $100 < x < 5000$, this factor is negative, so $R'(x) < 0$. At $x = 100$, the derivative is zero, giving the transition between these two behaviours. These are the monotonicity intervals of the polynomial on the printed interval $(0, 5000)$. In the subscription model itself, nonnegative subscriber numbers restrict the physically feasible increase to $0 \leq x \leq 500$.

Step 3: Conclusion $R(x)$ increases on $(0, 100)$ and decreases on $(100, 5000)$.

Quick Tip to Solve: Factor the derivative as $20(100-x)$; its sign changes only at $x=100$.

38(i). In an online jackpot, there is one first prize of ₹3,00,000, two second prizes of ₹2,00,000 each and three third prizes of ₹50,000 each. A total of 1,00,000 jackpot tickets each costing ₹100 were sold thereby raising a fund of ₹1,00,00,000. Rohan bought one ticket.

What are the possible amounts, the person can win?

[1 mark]

Solution:

Concept:

- A prize amount is the gross amount awarded, without subtracting the ticket price.
- A ticket may receive one of the stated prizes or no prize.

Step 1: Read the distinct prize values The first prize is Rs 3, 00, 000, each second prize is Rs 2, 00, 000, and each third prize is Rs 50, 000. Repeated prizes of the same value do not create additional distinct amounts.

Step 2: Include the outcome with no prize Only $1 + 2 + 3 = 6$ tickets receive prizes among 1, 00, 000 tickets. Thus a ticket can also receive no prize, represented by an amount of 0. The Rs 100 ticket cost affects net gain, but the question asks for amounts won.

Step 3: Conclusion The possible amounts are Rs 0, Rs 50, 000, Rs 2, 00, 000, Rs 3, 00, 000.

Quick Tip to Solve: List each distinct prize once, and include zero for the no-prize outcome.

38(ii)(a). What is the probability that the person wins atleast ₹2,00,000?

[2 marks]

Solution:

Concept:

- For equally likely tickets, $P(E) = \frac{\text{number of favourable tickets}}{\text{total number of tickets}}$.
- At least a given amount includes that amount as well as every larger amount.

Step 1: Identify the qualifying prizes The event is winning at least Rs 2, 00, 000. The one first prize of Rs 3, 00, 000 qualifies, and both second prizes of Rs 2, 00, 000 qualify. The three third prizes of Rs 50, 000 do not qualify, since each is below the threshold. Thus the favourable count is $1 + 2 = 3$.

Step 2: Divide by the total number of tickets There are 1, 00, 000 equally likely tickets in the draw, and Rohan owns one ticket. The three qualifying prizes correspond to three distinct winning tickets, so their events are mutually exclusive for his ticket. Hence

$$P(E) = \frac{1}{100000} + \frac{2}{100000} = \frac{3}{100000}.$$

The purchase price and total collected fund do not enter this ratio, because the event concerns the prize amount rather than net profit.

Step 3: Conclusion The required probability is $\frac{3}{100000}$.

Quick Tip to Solve: Count the first prize and both second prizes; at least includes equality.

38(ii)(b). What is the probability that the person does not win any amount?

[2 marks]

Solution:

Concept:

- The complement rule is $P(E^c) = 1 - P(E)$.
- There are $1 + 2 + 3 = 6$ prize-winning tickets out of 100000 equally likely tickets.

Step 1: Count the tickets receiving some prize One ticket receives the first prize, two tickets receive second prizes, and three tickets receive third prizes. Thus 6 distinct tickets receive a nonzero amount. Any one of these prizes counts as winning some amount, irrespective of how large the prize is. Therefore

$$P(\text{some prize}) = \frac{6}{100000}.$$

Step 2: Use the complementary event Every ticket either wins one of the prizes or wins no amount. These outcomes are mutually exclusive and exhaustive, so the probability of receiving nothing is

$$P(\text{no prize}) = 1 - \frac{6}{100000} = \frac{99994}{100000} = \frac{49997}{50000}.$$

Direct counting gives the same result: $100000 - 6 = 99994$ tickets receive no prize. The ticket price is a purchase expense, not an amount won, and does not alter which tickets fall in this event.

Step 3: Conclusion The probability of winning no amount is $\boxed{\frac{49997}{50000}}$.

Quick Tip to Solve: For no prize, subtract the total of all six winning tickets from the ticket count.

38(iii). In another jackpot, Rohan also bought a ticket having a prize money of ₹5,00,000. The chances of winning the jackpot are 1 in 1,00,000. Find the probability that on exactly one of tickets he wins the jackpot.

[1 mark]

Solution:

Concept:

- For independent jackpots, multiply probabilities of outcomes in the two separate draws.
- Exactly one win means first draw won and second lost, or first lost and second won; these two cases are disjoint.

Step 1: Find the win probabilities Let A denote receiving a prize on the first ticket and B receiving the prize on the second ticket. The first draw has 6 winning tickets, so $P(A) = 6/100000$. The second draw has the given probability $P(B) = 1/100000$. The two jackpot draws are independent.

Quick Tip to Solve: Add win-lose and lose-win, using six winning tickets in the first jackpot.

Step 2: Add the two exclusive cases Exactly one ticket wins on $A \cap B^c$ or $A^c \cap B$. Thus

$$\begin{aligned} P(\text{exactly one win}) &= P(A)[1 - P(B)] + [1 - P(A)]P(B) \\ &= \frac{6(99999) + 99994}{100000^2} \\ &= \frac{699988}{10000000000}. \end{aligned}$$

Counting either win alone would also count the outcome in which both tickets win, which must be excluded.

Step 3: Conclusion The required probability is

$$\boxed{\frac{174997}{2500000000}}.$$

Quick Tip to Solve: Add win-lose and lose-win, using six winning tickets in the first jackpot.