

CBSE Class 12 Mathematics Question Paper 2026 with Solution PDF-Set 65-3-1

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38** questions. All questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D and E**.
- (iii) In **Section A**, Questions no. **1 to 18** are multiple choice questions (MCQs) and questions number **19 and 20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B**, Questions no. **21 to 25** are very short answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C**, Questions no. **26 to 31** are short answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D**, Questions no. **32 to 35** are long answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E**, Questions no. **36 to 38** are case study based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 3 questions in Section E.
- (ix) Use of calculator is not allowed.

Section A (20 marks)

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The domain of $f(x) = \cos^{-1}(2x - 5)$ is :

[1 mark]

- (A) $[-1, 1]$
- (B) $[4, 6]$
- (C) $[-7, -3]$
- (D) $[2, 3]$

Correct Answer: (D) $[2, 3]$

Solution:

Concept:

- The real function $\cos^{-1} t$ is defined precisely for $-1 \leq t \leq 1$.
- The domain is the set of permitted input values x , whereas the range of the principal inverse cosine is $[0, \pi]$.

Step 1: Impose the inverse-cosine restriction The complete argument is $2x - 5$, so the necessary and sufficient condition is

$$-1 \leq 2x - 5 \leq 1.$$

The interval $[-1, 1]$ applies to this argument, not directly to x .

Step 2: Solve the compound inequality Add 5 throughout and divide by the positive number 2:

$$4 \leq 2x \leq 6 \implies 2 \leq x \leq 3.$$

Both endpoints are allowed because inverse cosine is defined at -1 and 1 . Options (A), (B) and (C) do not satisfy this condition throughout their intervals.

Step 3: Conclusion The domain is $[2, 3]$. Hence, the correct option is (D).

Quick Tip to Solve: Apply the interval restriction to the whole argument before solving for the input.

2. If $A^2 = 4A + 3I$ and $A^{-1} = xA + yI$, then the value of $(x + y)$ is :

[1 mark]

- (A) -1
- (B) 1
- (C) $\frac{5}{3}$
- (D) 7

Correct Answer: (A) -1

Solution:

Concept:

- For an invertible matrix A , $AA^{-1} = I$, where I is the identity of the same order.
- A polynomial relation in A can be rearranged to exhibit a matrix whose product with A is I .

Quick Tip to Solve: Factor the polynomial as A times a linear expression, then divide by its identity coefficient.

Step 1: Rearrange the given relation From $A^2 = 4A + 3I$, obtain

$$A^2 - 4A = 3I, \quad A(A - 4I) = 3I.$$

Since 3 is nonzero, division gives an explicit inverse:

$$A \left(\frac{1}{3}A - \frac{4}{3}I \right) = I.$$

Step 2: Identify the coefficients Thus $A^{-1} = \frac{1}{3}A - \frac{4}{3}I$. Comparing with the stated form gives $x = \frac{1}{3}$ and $y = -\frac{4}{3}$, and therefore

$$x + y = \frac{1}{3} - \frac{4}{3} = -1.$$

The other choices $1, \frac{5}{3}, 7$ do not equal this sum; the negative coefficient of I must be retained.

Step 3: Conclusion $x + y = -1$. Hence, the correct option is (A).

Quick Tip to Solve: Factor the polynomial as A times a linear expression, then divide by its identity coefficient.

3. If A and B are skew-symmetric matrices of same order, then $AB' + BA'$ is a/an :

[1 mark]

- (A) symmetric matrix
- (B) skew-symmetric matrix
- (C) null matrix
- (D) identity matrix

Correct Answer: (A) symmetric matrix

Solution:

Concept:

- A matrix M is symmetric if $M' = M$, and skew-symmetric if $M' = -M$.
- Transposition reverses multiplication: $(XY)' = Y'X'$.

Step 1: Take the transpose of the expression Put $M = AB' + BA'$. Since a double transpose returns the original matrix,

$$M' = (AB')' + (BA')' = BA' + AB'.$$

Step 2: Compare with the original matrix Addition of matrices is commutative, so $M' = AB' + BA' = M$. Hence the expression is symmetric. Using skew symmetry also gives $M = -(AB + BA)$, with the same result. It need not be zero or the identity: for $A = B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, it equals $2I$, which is not skew-symmetric.

Step 3: Conclusion $AB' + BA'$ is a symmetric matrix. Hence, the correct option is (A).

Quick Tip to Solve: Transpose the full sum and reverse each product; do not assume that a product inherits skew symmetry.

4. If $\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} A = \begin{pmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{pmatrix}$, then order of A must be :

[1 mark]

- (A) 3×1
- (B) 1×3
- (C) 1×1
- (D) 3×3

Correct Answer: (B) 1×3

Solution:

Concept:

- The product of an $m \times n$ matrix and an $n \times p$ matrix has order $m \times p$.
- Matrix multiplication is defined only when the inner dimensions agree.

Step 1: Use the inner dimensions The displayed column matrix has three rows and one column, hence order 3×1 . If A has order $r \times s$, multiplication on its left by this column is possible only if $r = 1$. Therefore A must have one row.

Step 2: Use the product dimensions The resulting matrix has order 3×3 , so $s = 3$. This can also be checked using $A = \begin{pmatrix} -1 & 2 & 1 \end{pmatrix}$: multiplying it by the column $(4, 1, 3)'$ gives exactly the displayed entries. A 1×1 matrix would yield a single column, while the 3×1 and 3×3 options cannot be multiplied here.

Step 3: Conclusion The order of A is 1×3 . Hence, the correct option is (B).

Quick Tip to Solve: Use the inner dimensions to find the row count and the product columns to find the column count.

5. If a square matrix A is such that $A^2 = A$ and $(I - A)^3 = xA + I$, then value of x must be : [1 mark]

- (A) 7
- (B) 5
- (C) -7
- (D) -1

Correct Answer: (D) -1

Solution:

Concept:

- A matrix satisfying $A^2 = A$ is idempotent; consequently $A^3 = A$.
- The identity I commutes with A , allowing the binomial expansion of $(I - A)^3$.

Step 1: Expand the matrix cube Because $IA = AI = A$,

$$(I - A)^3 = I - 3A + 3A^2 - A^3.$$

From $A^2 = A$, multiplying by A gives $A^3 = A^2 = A$.

Quick Tip to Solve: For an idempotent matrix, replace every positive power by A before combining coefficients.

Step 2: Reduce and compare the coefficient Substituting these relations,

$$(I - A)^3 = I - 3A + 3A - A = I - A.$$

Thus the expression has the stated form $xA + I$ with coefficient $x = -1$. The values 7, 5, -7 do not reproduce the reduced polynomial; treating A^2 and A^3 as separate powers would miss the idempotence condition.

Step 3: Conclusion $x = -1$. Hence, the correct option is (D).

Quick Tip to Solve: For an idempotent matrix, replace every positive power by A before combining coefficients.

6. If $B(\text{adj } B) = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$, then the value of $\det(B^{-1})$ is :

[1 mark]

- (A) $\frac{1}{3}$
- (B) $\frac{1}{9}$
- (C) 3
- (D) 9

Correct Answer: (C) 3

Solution:

Concept:

- For a square matrix B , $B \text{adj } B = \det(B)I$.
- If $\det(B) \neq 0$, then $\det(B^{-1}) = 1/\det(B)$.

Step 1: Read the determinant from the adjoint identity The given diagonal matrix is $\frac{1}{3}I_3$. Comparing it directly with $B \text{adj } B = \det(B)I_3$ gives

$$\det(B) = \frac{1}{3}.$$

This nonzero value confirms that the inverse exists.

Step 2: Apply the reciprocal rule Taking the determinant of $BB^{-1} = I$ yields $\det(B)\det(B^{-1}) = 1$. Therefore

$$\det(B^{-1}) = \frac{1}{1/3} = 3.$$

Option (A) is the determinant of B itself. Neither $1/9$ nor 9 follows the reciprocal rule; cubing a diagonal entry here would instead compute the determinant of the given product, not directly that of B .

Step 3: Conclusion $\det(B^{-1}) = 3$. Hence, the correct option is (C).

Quick Tip to Solve: Read $\det B$ from the scalar in $B \text{adj } B$; take its reciprocal only after that comparison.

7. The value of k for which the function $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ k(x+1), & x = 0 \end{cases}$ is a continuous function, is :

[1 mark]

- (A) $\frac{1}{4}$
 (B) 2
 (C) $\frac{1}{2}$
 (D) 0

Correct Answer: (D) 0

Solution:

Concept:

- Continuity at $x = 0$ requires $\lim_{x \rightarrow 0} f(x) = f(0)$.
- The sine function is bounded: $-1 \leq \sin t \leq 1$ for every real t .

Step 1: Find the limit of the nonzero branch Although $\sin(1/x)$ oscillates as $x \rightarrow 0$, its magnitude is at most 1. Hence

$$-x^2 \leq x^2 \sin(1/x) \leq x^2.$$

Both bounding functions tend to zero, so the squeeze theorem gives $\lim_{x \rightarrow 0} x^2 \sin(1/x) = 0$.

Step 2: Match the value at the joining point The branch specified for $x = 0$ gives $f(0) = k(0+1) = k$. Thus continuity requires $k = 0$. Away from zero, the nonzero branch is a composition and product of continuous functions, so this condition suffices for continuity everywhere. The three nonzero choices leave a mismatch at the origin.

Step 3: Conclusion $k = 0$. Hence, the correct option is (D).

Quick Tip to Solve: Bound the oscillating sine by 1, then compare its damped limit with the assigned value at zero.

8. If $\sin^{-1} x = y$, then $\frac{dy}{dx}$ is :

[1 mark]

- (A) $\cos^{-1} x$
 (B) $\cos y$
 (C) $\frac{1}{1-x^2}$
 (D) $\sec y$

Correct Answer: (D) $\sec y$

Solution:

Concept:

- If $y = \sin^{-1} x$, then $\sin y = x$ with principal values $-\pi/2 \leq y \leq \pi/2$.
- Implicit differentiation gives $\cos y \, dy/dx = 1$ for $-1 < x < 1$.

Step 1: Differentiate the inverse relation Rewrite the relation as $x = \sin y$. Differentiating with

Quick Tip to Solve: Convert the inverse relation to x equals $\sin y$; implicit differentiation immediately gives $\sec y$.

respect to x , including the chain-rule factor, gives

$$1 = \cos y \frac{dy}{dx}, \quad \frac{dy}{dx} = \frac{1}{\cos y} = \sec y.$$

Step 2: Check the alternatives On the interior of the principal range, $\cos y = \sqrt{1-x^2} > 0$, so the derivative also equals $1/\sqrt{1-x^2}$. Printed option (C), however, is $1/(1-x^2)$, which generally differs. Option (B) gives dx/dy , while option (A) is another inverse trigonometric function and not the required derivative. The reciprocal cosine form is therefore the matching choice.

Step 3: Conclusion $\boxed{dy/dx = \sec y}$ for $-1 < x < 1$. Hence, the correct option is (D).

Quick Tip to Solve: Convert the inverse relation to x equals $\sin y$; implicit differentiation immediately gives $\sec y$.

9. The rate of change of volume of a sphere with respect to its diameter, when its radius is 5 cm, is : [1 mark]

- (A) $400\pi \text{ cm}^3/\text{cm}$
- (B) $100\pi \text{ cm}^3/\text{cm}$
- (C) $50\pi \text{ cm}^3/\text{cm}$
- (D) $25\pi \text{ cm}^3/\text{cm}$

Correct Answer: (C) $50\pi \text{ cm}^3/\text{cm}$

Solution:

Concept:

- The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.
- The diameter is $d = 2r$; hence $dr/dd = 1/2$. The required rate is dV/dd .

Step 1: Express the volume using diameter Since $r = d/2$,

$$V = \frac{4}{3}\pi \left(\frac{d}{2}\right)^3 = \frac{\pi d^3}{6}.$$

The differentiation variable is the diameter, so differentiating directly with respect to radius would give twice the required result.

Step 2: Differentiate and substitute At $r = 5$ cm, the diameter is $d = 10$ cm. Thus

$$\frac{dV}{dd} = \frac{\pi d^2}{2} = \frac{\pi(10)^2}{2} = 50\pi \text{ cm}^3/\text{cm}.$$

Option (B) is the rate with respect to radius; options (A) and (D) do not follow the diameter formula.

Step 3: Conclusion The required rate is $\boxed{50\pi \text{ cm}^3/\text{cm}}$. Hence, the correct option is (C).

Quick Tip to Solve: For differentiation with respect to diameter, multiply dV/dr by $1/2$.

10. $\int \frac{dx}{2^x + 2^{-x}}$ is equal to :

[1 mark]

- (A) $\tan^{-1}(2^x) + C$
(B) $\tan^{-1}(2^{-x}) + C$
(C) $\frac{\tan^{-1}(2^x)}{\log 2} + C$
(D) $(\log 2) \tan^{-1}(2^x) + C$

Correct Answer: (C) $\frac{\tan^{-1}(2^x)}{\log 2} + C$

Solution:

Concept:

- Multiply numerator and denominator by 2^x to expose a standard substitution.
- For $t = 2^x$, $dt = (\log 2)2^x dx$; also $\int dt/(1 + t^2) = \tan^{-1} t + C$.

Step 1: Transform the integrand Write the integral as

$$I = \int \frac{2^x dx}{1 + 2^{2x}}.$$

Set $t = 2^x$, so $2^x dx = dt/\log 2$. The factor $\log 2$ is essential because the exponential has base 2.

Step 2: Integrate and check the options The substitution gives

$$I = \frac{1}{\log 2} \int \frac{dt}{1 + t^2} = \frac{\tan^{-1} t}{\log 2} + C.$$

Replacing t by 2^x gives option (C). Differentiating (A) introduces an extra factor $\log 2$; (B) has a negative derivative, and (D) introduces the factor $(\log 2)^2$.

Step 3: Conclusion $I = \frac{\tan^{-1}(2^x)}{\log 2} + C$. Hence, the correct option is (C).

Quick Tip to Solve: Multiply by $2^x/2^x$, then remember to divide by $\log 2$.

11. $\int_{-1}^1 (1 - |x|) dx$ is equal to :

[1 mark]

(A) $2 \int_0^1 (1 + x) dx$

(B) $2 \int_{-1}^0 (1 + x) dx$

(C) 0

(D) $2 \int_{-1}^0 (1 - x) dx$

Correct Answer: (B) $2 \int_{-1}^0 (1 + x) dx$

Solution:

Concept:

- The function $1 - |x|$ is even, so its integrals over $[-1, 0]$ and $[0, 1]$ are equal.
- On $[-1, 0]$, $|x| = -x$, so $1 - |x| = 1 + x$.

Step 1: Use the negative half of the interval Let $I = \int_{-1}^1 (1 - |x|) dx$. By evenness and the expression of the absolute value for negative x ,

$$I = 2 \int_{-1}^0 (1 - |x|) dx = 2 \int_{-1}^0 (1 + x) dx.$$

This is the expression in option (B).

Step 2: Verify the numerical values Indeed,

$$I = 2 \left[x + \frac{x^2}{2} \right]_{-1}^0 = 1.$$

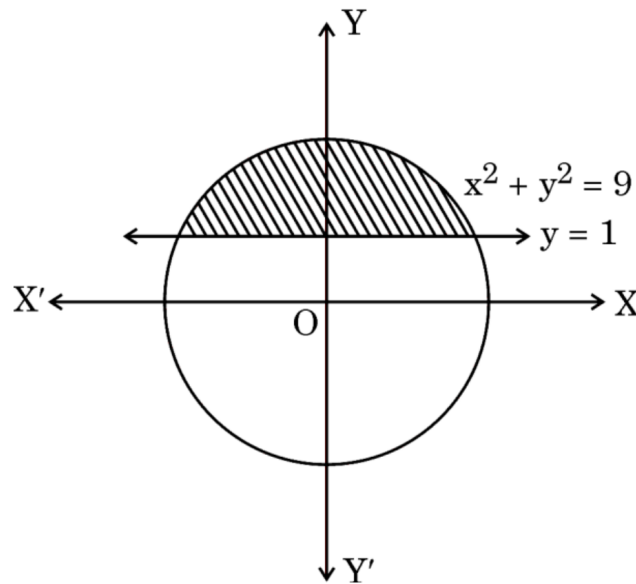
Option (A) equals 3, option (C) equals 0, and option (D) equals 3; none of those can equal the given integral. On the positive interval, the corresponding correct integrand would be $1 - x$.

Step 3: Conclusion $2 \int_{-1}^0 (1 + x) dx$. Hence, the correct option is (B).

Quick Tip to Solve: For an absolute value integral, write $|x| = -x$ before using the negative interval.

12. The area of the shaded region of the circle given below is equal to :

[1 mark]



- (A) $\int_1^3 \sqrt{9-y^2} dy$
(B) $2 \int_1^3 \sqrt{9-y^2} dy$
(C) $\int_0^3 \sqrt{9-x^2} dx$
(D) $2 \int_0^3 \sqrt{9-x^2} dx$

Correct Answer: (B) $2 \int_1^3 \sqrt{9-y^2} dy$

Solution:

Concept:

- The circle is $x^2 + y^2 = 9$, and the shaded cap lies above $y = 1$.
- A horizontal strip has area $(x_{\text{right}} - x_{\text{left}}) dy$.

Step 1: Determine the strip width Solving the circle equation gives $x = \pm\sqrt{9-y^2}$. The full horizontal width is therefore $2\sqrt{9-y^2}$, accounting for both sides of the vertical axis.

Step 2: Integrate over the shaded heights The cap extends from the chord $y = 1$ to the top of the circle, $y = 3$. Hence

$$A = 2 \int_1^3 \sqrt{9-y^2} dy.$$

Option (A) gives only half the cap. Options (C) and (D) give a quarter-circle and a semicircle, respectively, so neither matches the shading.

Step 3: Conclusion The shaded area is $\boxed{2 \int_1^3 \sqrt{9-y^2} dy}$. Hence, the correct option is (B).

Quick Tip to Solve: Use horizontal strips: double the positive x value and integrate from $y = 1$ to 3.

13. $\frac{dy}{dx} = F(x, y)$ will be a homogeneous differential equation for which of the following functions ?

(i) $F(x, y) = 3x + 2y$

(ii) $F(x, y) = \sin \frac{y}{x} + \log y - \log x$

(iii) $F(x, y) = e^{y/x} + 1$

(iv) $F(x, y) = \sqrt{x^2 + y^2} - y$

[1 mark]

(A) (i) and (ii)

(B) (i), (ii) and (iii)

(C) (ii), (iii) and (iv)

(D) (ii) and (iii)

Correct Answer: (D) (ii) and (iii)

Solution:

Concept:

- The equation $dy/dx = F(x, y)$ is homogeneous when F depends only on y/x .
- Equivalently, $F(tx, ty) = F(x, y)$ for admissible positive t : F must have degree zero.

Step 1: Check functions (ii) and (iii) In (ii), $\log y - \log x = \log(y/x)$ on the domain where both logarithms are defined. Thus

$$F(x, y) = \sin(y/x) + \log(y/x),$$

which depends only on y/x . Function (iii), $e^{y/x} + 1$, also depends only on this ratio. Both give homogeneous differential equations.

Step 2: Reject functions (i) and (iv) For (i), $F(tx, ty) = t(3x + 2y)$, so the value generally changes with t . For (iv) and positive t ,

$$F(tx, ty) = t(\sqrt{x^2 + y^2} - y),$$

which also has degree one rather than zero. Options (A), (B), and (C) each include at least one of these invalid functions.

Step 3: Conclusion Only (ii) and (iii) satisfy the condition. Hence, the correct option is (D).

Quick Tip to Solve: For $dy/dx = F$, test degree zero; merely being a homogeneous function of degree one is insufficient.

14. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ?

[1 mark]

- (A) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$
- (B) $|\vec{a} + \vec{b}| \geq |\vec{a}| + |\vec{b}|$
- (C) $|\vec{a} - \vec{b}| = |\vec{a}| - |\vec{b}|$
- (D) $|\vec{a} \times \vec{b}| \geq |\vec{a}||\vec{b}|$

Correct Answer: (A) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$

Solution:

Concept:

- For nonzero vectors making angle θ , $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$.
- The bounds $-1 \leq \cos \theta \leq 1$ imply $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$, including zero vectors.

Step 1: Establish the dot-product bound Since the product of magnitudes is nonnegative and $\cos \theta \leq 1$,

$$\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|.$$

If either vector is zero, both sides equal zero. Thus option (A) holds for every pair, without any requirement on their directions.

Step 2: Test the other statements Choose opposite unit vectors for (B): the left side is 0, whereas the right side is 2. The same choice disproves (C), whose sides become 2 and 0. Choose parallel unit vectors for (D): the cross-product magnitude is 0, but the product of magnitudes is 1. These counterexamples rule out all three alternatives.

Step 3: Conclusion The always true statement is $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$. Hence, the correct option is (A).

Quick Tip to Solve: Test universal vector claims using parallel and opposite unit vectors before doing lengthy calculations.

15. If $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 198$ and $|\vec{a}| = 10|\vec{b}|$, then :

[1 mark]

- (A) $|\vec{a}| = \sqrt{2}$
- (B) $|\vec{b}| = \sqrt{2}$
- (C) $|\vec{b}| = 10\sqrt{2}$
- (D) $|\vec{a}| = \frac{10}{\sqrt{2}}$

Correct Answer: (B) $|\vec{b}| = \sqrt{2}$

Solution:

Concept:

- The identity $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$ follows from symmetry of the dot product.
- Vector magnitudes are nonnegative, so the positive square root is used.

Quick Tip to Solve: Expand the dot product first; the mixed terms cancel even when the vectors are not perpendicular.

Step 1: Expand and cancel the mixed terms Using $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$,

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2 = 198.$$

There is no need to determine the angle between the two vectors because the mixed terms cancel.

Step 2: Use the magnitude ratio Put $|\vec{b}| = s$, so $|\vec{a}| = 10s$. Then

$$100s^2 - s^2 = 198, \quad 99s^2 = 198, \quad s^2 = 2.$$

Thus $|\vec{b}| = \sqrt{2}$ and $|\vec{a}| = 10\sqrt{2}$. These values match (B); (A), (C), and (D) conflict with either the given ratio or the difference of squares.

Step 3: Conclusion $|\vec{b}| = \sqrt{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Expand the dot product first; the mixed terms cancel even when the vectors are not perpendicular.

16. If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of lines L_1 and L_2 respectively and θ is the acute angle between them, then : [1 mark]

(A) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

(B) $\sin \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

(C) $\tan \theta = \frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$

(D) $\cos \theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$

Correct Answer: (D) $\cos \theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$

Solution:

Concept:

- Direction cosines give unit direction vectors $\hat{u}_1 = (l_1, m_1, n_1)$ and $\hat{u}_2 = (l_2, m_2, n_2)$.
- The acute angle between lines is independent of which direction is chosen along either line.

Step 1: Take the dot product of unit directions If ϕ is the angle between the chosen unit directions, then

$$\cos \phi = \hat{u}_1 \cdot \hat{u}_2 = l_1 l_2 + m_1 m_2 + n_1 n_2.$$

Their magnitudes are both one, since the sum of the squares of the direction cosines of each line equals one.

Step 2: Convert to the acute angle between lines Reversing a direction leaves its line unchanged but changes the sign of the dot product. Therefore, for the acute angle θ ,

$$\cos \theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|.$$

Option (A) can be negative for a reversed direction. The dot product gives a cosine, not the sine in (B), and the component ratios in (C) do not define the angle and may be undefined.

Quick Tip to Solve: When the question asks for the acute angle between lines, take the absolute value of the unit-direction dot product.

Step 3: Conclusion $\cos \theta = \frac{|l_1 l_2 + m_1 m_2 + n_1 n_2|}{\sqrt{l_1^2 + m_1^2 + n_1^2} \sqrt{l_2^2 + m_2^2 + n_2^2}}$. Hence, the correct option is (D).

Quick Tip to Solve: When the question asks for the acute angle between lines, take the absolute value of the unit-direction dot product.

17. Direction ratios of lines l_1 and l_2 are $\langle 12, -3, 9 \rangle$ and $\langle 4, q, -p \rangle$ respectively. The values of p and q for which l_1 and l_2 are parallel are respectively : [1 mark]

- (A) $-1, 3$
- (B) $3, 1$
- (C) $-3, -1$
- (D) $-1, -3$

Correct Answer: (C) $-3, -1$

Solution:

Concept:

- Parallel lines have proportional direction ratios: $(a_1, b_1, c_1) = k(a_2, b_2, c_2)$ for a nonzero scalar k .
- The order and signs of all three coordinates must agree with the same scale factor.

Step 1: Find the common scale factor The first components give $12/4 = 3$. Thus the second direction vector must be one third of the first, giving

$$\langle 4, q, -p \rangle = \frac{1}{3} \langle 12, -3, 9 \rangle = \langle 4, -1, 3 \rangle.$$

Step 2: Check the signs and options Comparison gives $q = -1$ and $-p = 3$, so $p = -3$. The required order is (p, q) . Option (A) gives a wrong third component, option (B) has the wrong signs, and option (D) has neither the required second nor third component. Only (C) makes all three ratios equal.

Step 3: Conclusion $p = -3, q = -1$. Hence, the correct option is (C).

Quick Tip to Solve: Use the known first coordinate to set the scale factor, then remember that the third coordinate is $-p$, not p .

18. If E and F are two independent events such that $P(E) = \frac{3}{10}$, $P(E \cup F) = \frac{1}{2}$, then $P(E|F) - P(F|E)$ is equal to : [1 mark]

- (A) $\frac{2}{7}$
- (B) $\frac{3}{35}$
- (C) $\frac{1}{70}$
- (D) $\frac{1}{7}$

Correct Answer: (C) $\frac{1}{70}$

Solution:

Concept:

- For independent events, $P(E \cap F) = P(E)P(F)$, and conditioning does not change either probability.
- The addition rule is $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

Step 1: Find the probability of F Put $P(F) = p$. Independence and the given union probability yield

$$\frac{1}{2} = \frac{3}{10} + p - \frac{3}{10}p = \frac{3}{10} + \frac{7}{10}p.$$

Hence $\frac{7}{10}p = \frac{1}{5}$, giving $p = \frac{2}{7}$.

Step 2: Evaluate the conditional difference Both probabilities are positive, so the conditional probabilities exist. Independence gives $P(E | F) = \frac{3}{10}$ and $P(F | E) = \frac{2}{7}$. Therefore

$$P(E | F) - P(F | E) = \frac{3}{10} - \frac{2}{7} = \frac{21 - 20}{70} = \frac{1}{70}.$$

Option (A) is merely $P(F)$; (B) and (D) do not equal the calculated difference.

Step 3: Conclusion The required probability difference is $\frac{1}{70}$. Hence, the correct option is (C).

Quick Tip to Solve: For independent events replace conditional probabilities by their unconditional values, after finding the missing probability from the union.

19. Assertion (A): One of the particular solutions of the differential equation $\frac{dy}{dx} = e^{x+y}$ can be $e^x + e^{-y} = -2$.

Reason (R): $e^x + e^{-y} = C$ is the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (D) Assertion (A) is false, but Reason (R) is true.

Solution:

Concept:

- For real x, y , both e^x and e^{-y} are strictly positive.
- Separation gives $e^{-y} dy = e^x dx$; a valid particular solution must have a nonempty real domain.

Step 1: Judge Assertion (A) The proposed equation is $e^x + e^{-y} = -2$. Its left-hand side is strictly positive for every real pair (x, y) , whereas its right-hand side is negative. It cannot define a real-valued solution on any interval. Thus Assertion (A) is false.

Step 2: Judge Reason (R) Integrating the separated equation gives

$$-e^{-y} = e^x + C_1, \quad e^x + e^{-y} = C.$$

Differentiating this relation yields $e^x - e^{-y}y' = 0$, hence $y' = e^{x+y}$, as required. Thus Reason (R) is true on the real domains where the relation defines y .

Step 3: Assess the explanation The integration constant cannot be chosen so that the implicit equation has no real points. For a real solution the equality requires $C > e^x > 0$ throughout its domain. Consequently the true general-solution statement does not make the asserted negative value admissible. The truth values are false for A and true for R.

Step 4: Conclusion The assertion is false, but the reason is true. Hence, the correct option is (D).

Quick Tip to Solve: Before substituting an arbitrary integration constant, check whether the resulting implicit equation has real solutions.

20. Assertion (A): The vectors $\vec{a}$ and $(-2\vec{a})$, where $\vec{a} \neq \vec{0}$, are collinear vectors.

Reason (R): $\vec{a} \cdot (-2\vec{a}) = 0$.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- Two nonzero vectors are collinear if one is a scalar multiple of the other.
- The dot product satisfies $\vec{a} \cdot \vec{a} = |\vec{a}|^2$, which is positive for $\vec{a} \neq \vec{0}$.

Step 1: Judge Assertion (A) The vector $-2\vec{a}$ is a scalar multiple of $\vec{a}$. Since $\vec{a} \neq \vec{0}$, both vectors are nonzero and lie along the same line, with opposite directions. Thus Assertion (A) is true.

Step 2: Judge Reason (R) By linearity of the dot product,

$$\vec{a} \cdot (-2\vec{a}) = -2(\vec{a} \cdot \vec{a}) = -2|\vec{a}|^2 < 0.$$

Therefore this dot product is not zero, and Reason (R) is false.

Step 3: Assess the explanation Collinearity follows from scalar proportionality. A zero dot product instead characterises perpendicular nonzero vectors. The angle here is π , and $\cos \pi = -1$, which also confirms the negative dot product. Since R is false, it cannot explain the true assertion.

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: A negative scalar reverses a vector along the same line; use $-2|\vec{a}|^2$ to test the dot-product claim.

Section B (16 marks)

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. If $x = t + \frac{1}{t}$ **and** $y = t - \frac{1}{t}$, **find** $\frac{dy}{dx}$ **at** $t = 2$.

[2 marks]

Solution:

Concept:

- For $x = x(t)$, $y = y(t)$, the chain rule gives $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ provided $dx/dt \neq 0$.
- The derivative of t^{-1} is $-t^{-2}$; the parameter here must be nonzero.

Step 1: Differentiate both parametric equations Write the reciprocal as t^{-1} . Differentiating each

Quick Tip to Solve: Differentiate both coordinates with respect to t before substituting, and divide the y derivative by the x derivative.

equation with respect to the same parameter gives

$$\frac{dx}{dt} = \frac{d}{dt}(t + t^{-1}) = 1 - \frac{1}{t^2}, \quad \frac{dy}{dt} = \frac{d}{dt}(t - t^{-1}) = 1 + \frac{1}{t^2}.$$

The plus sign in the second derivative follows because the reciprocal term in y already has a minus sign.

Step 2: Apply the chain rule at the given parameter At $t = 2$, the two derivatives are

$$\left. \frac{dx}{dt} \right|_{t=2} = 1 - \frac{1}{4} = \frac{3}{4}, \quad \left. \frac{dy}{dt} \right|_{t=2} = 1 + \frac{1}{4} = \frac{5}{4}.$$

Since $3/4 \neq 0$, division is valid. Thus

$$\left. \frac{dy}{dx} \right|_{t=2} = \frac{5/4}{3/4} = \frac{5}{3}.$$

Equivalently, the derivative for admissible parameters is $(t^2 + 1)/(t^2 - 1)$, which gives the same value at 2.

Step 3: Conclusion $\left. \frac{dy}{dx} \right|_{t=2} = \frac{5}{3}.$

Quick Tip to Solve: Differentiate both coordinates with respect to t before substituting, and divide the y derivative by the x derivative.

22. Find the sub-interval(s) of $(0, \frac{\pi}{2})$ in which $f(x) = \tan x - 4x$ is increasing.

[2 marks]

Solution:

Concept:

- A differentiable function increases on any interval where $f'(x) > 0$.
- Here $\frac{d}{dx} \tan x = \sec^2 x$; on $(0, \pi/2)$, cosine is positive and strictly decreasing.

Step 1: Find the derivative and its sign condition Differentiating gives

$$f'(x) = \sec^2 x - 4.$$

Thus we require $\sec^2 x > 4$. Since $\cos x > 0$ on the specified domain, this is equivalent to $1/\cos^2 x > 4$, or $\cos^2 x < 1/4$, and hence $\cos x < 1/2$. Positivity of cosine is essential when taking the square root.

Step 2: Locate the interval within the prescribed domain Cosine decreases from 1 towards 0 on $(0, \pi/2)$, and $\cos(\pi/3) = 1/2$. Consequently $f'(x) > 0$ precisely when $\pi/3 < x < \pi/2$, while it is negative for $0 < x < \pi/3$. At $x = \pi/3$, the derivative is zero, so this is the boundary separating decrease from increase. The right endpoint $\pi/2$ is excluded because it is outside the given domain and tangent is undefined there.

Step 3: Conclusion The interval of increase, expressed with strict derivative positivity, is $(\frac{\pi}{3}, \frac{\pi}{2})$.

Quick Tip to Solve: On the first quadrant convert $\sec^2 x > 4$ to $\cos x < 1/2$; the critical angle is $\pi/3$.

23(a). Find the value of $\sin[\cot^{-1} \sqrt{2} (\cos(\tan^{-1} 1))]$.

[2 marks]

Solution:

Concept:

- The principal value of $\tan^{-1} 1$ is $\pi/4$, which lies in $(-\pi/2, \pi/2)$.
- For positive arguments the principal $\cot^{-1}$ value lies in $(0, \pi/2)$; in particular $\cot^{-1} 1 = \pi/4$.

Step 1: Evaluate the inner inverse tangent and cosine Let $\theta = \tan^{-1} 1$. Since $\tan(\pi/4) = 1$ and $\pi/4$ belongs to the principal range, $\theta = \pi/4$. Hence

$$\cos(\tan^{-1} 1) = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}.$$

The argument supplied to the inverse cotangent is the product $\sqrt{2} \cos(\tan^{-1} 1)$, which therefore simplifies to $\sqrt{2} \cdot 1/\sqrt{2} = 1$.

Step 2: Apply inverse cotangent and then sine After simplifying the entire argument, the required expression becomes

$$\sin \left[\cot^{-1} \left(\sqrt{2} \cdot \frac{1}{\sqrt{2}} \right) \right] = \sin(\cot^{-1} 1).$$

The principal angle whose cotangent is 1 is $\pi/4$. Therefore the last expression is $\sin(\pi/4) = 1/\sqrt{2}$. Taking the principal inverse value avoids introducing additional angles differing by multiples of π , which would change the final sine.

Step 3: Conclusion The required value is $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

Quick Tip to Solve: Simplify the full product inside $\cot^{-1}$ first: $\sqrt{2} \cos(\tan^{-1} 1) = 1$.

OR

23(b). A relation R on $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (3, 3), (1, 2)\}$. Is R a symmetric relation? Justify. Write the smallest relation set R_1 such that $R \cup R_1$ becomes an equivalence relation on the set $\{1, 2, 3\}$.

[2 marks]

Solution:

Concept:

- A relation is symmetric when $(a, b) \in R$ implies $(b, a) \in R$.
- An equivalence relation is reflexive, symmetric and transitive. Reflexivity requires (a, a) for every element of the underlying set.

Step 1: Test symmetry The pair $(1, 2)$ belongs to R , but its reverse $(2, 1)$ does not. Thus the defining condition for symmetry fails. The diagonal pairs $(1, 1)$ and $(3, 3)$ are their own reverses and do not remove this failure.

Step 2: Complete the relation minimally Reflexivity forces us to add $(2, 2)$, and symmetry forces us to

Quick Tip to Solve: Add missing diagonal pairs first, then reverse pairs; finally check whether transitivity forces anything further.

add (2, 1). With these additions, the relation is

$$R \cup R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}.$$

Its equivalence classes are $\{1, 2\}$ and $\{3\}$. Every pair within each class is present, so any composable pair of relations has its required transitive pair present. No pairs connecting 3 with 1 or 2 are required. Both added pairs are mandatory, so no smaller set can work.

Step 3: Conclusion R is not symmetric. The smallest set is $R_1 = \{(2, 2), (2, 1)\}$.

Quick Tip to Solve: Add missing diagonal pairs first, then reverse pairs; finally check whether transitivity forces anything further.

24. If for two unit vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + 2\vec{b}| = |2\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$. [2 marks]

Solution:

Concept:

- For vectors $\vec{u}, \vec{v}$, $|\vec{u} + \vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 + 2\vec{u} \cdot \vec{v}$.
- For two unit vectors making angle θ , $\vec{a} \cdot \vec{b} = \cos \theta$, with $0 \leq \theta \leq \pi$.

Step 1: Square and expand the equal magnitudes Both magnitudes are nonnegative, so squaring preserves their equality. Since $|\vec{a}| = |\vec{b}| = 1$,

$$\begin{aligned} |\vec{a} + 2\vec{b}|^2 &= 1 + 4 + 4\vec{a} \cdot \vec{b}, \\ |2\vec{a} - \vec{b}|^2 &= 4 + 1 - 4\vec{a} \cdot \vec{b}. \end{aligned}$$

The opposite signs of the mixed terms arise from the plus and minus signs in the two vectors.

Step 2: Determine the angle Equating the expressions gives $5 + 4\vec{a} \cdot \vec{b} = 5 - 4\vec{a} \cdot \vec{b}$, hence $8\vec{a} \cdot \vec{b} = 0$. Therefore $\cos \theta = 0$. The only angle between vectors in $[0, \pi]$ satisfying this is $\theta = \pi/2$. At this angle both original magnitudes are $\sqrt{5}$, which also checks the condition.

Step 3: Conclusion The angle between the unit vectors is 90° .

Quick Tip to Solve: Square a magnitude equality and compare mixed dot-product terms before attempting to find the angle.

25(a). If the lines $\frac{x-3}{1} = \frac{1-y}{1} = \frac{z+2}{p}$ and $\frac{2-x}{3} = \frac{y+1}{5} = \frac{z+56}{2p}$ are perpendicular to each other, then find the value(s) of p . [2 marks]

Solution:

Concept:

- Write all coordinate numerators as coordinate minus constant before reading the direction ratios.
- Two lines with nonzero direction vectors (a, b, c) and (d, e, f) are perpendicular when $ad + be + cf = 0$.

Quick Tip to Solve: Reverse the sign of a direction ratio whenever its numerator is written as a constant minus a coordinate.

0.

Step 1: Read the direction vectors with their signs Writing the first line parametrically gives $x = 3 + t$, $y = 1 - t$, $z = -2 + pt$. Its direction vector is therefore $(1, -1, p)$. For the second line, put the common ratio equal to s : $x = 2 - 3s$, $y = -1 + 5s$, $z = -56 + 2ps$. Its direction vector is $(-3, 5, 2p)$. The constants specifying points on the lines do not enter the perpendicularity condition.

Step 2: Apply the zero dot-product condition Using these direction vectors,

$$\begin{aligned}(1)(-3) + (-1)(5) + p(2p) &= 0, \\ -3 - 5 + 2p^2 &= 0, \\ p^2 &= 4.\end{aligned}$$

Both signs of the square root must be retained. For either root the vectors remain nonzero and their dot product equals zero, so both satisfy the required direction condition.

Step 3: Conclusion The required values are $p = 2$ or $p = -2$.

Quick Tip to Solve: Reverse the sign of a direction ratio whenever its numerator is written as a constant minus a coordinate.

OR

25(b). Find the vector equation of a line passing through the origin and perpendicular to both the lines $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and $\vec{r} = \mu(\hat{i} - \hat{j} + \hat{k})$. [2 marks]

Solution:

Concept:

- A line through the origin has vector equation $\vec{r} = t\vec{d}$, where $\vec{d}$ is a nonzero direction vector.
- The cross product of two nonparallel direction vectors is perpendicular to each of them. Position vectors do not determine their directions.

Step 1: Identify the two directions The coefficients of the parameters give $\vec{a} = 3\hat{i} + 4\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$. The vector $2\hat{i} - \hat{j} + 2\hat{k}$ locates a point on the first line; it is not used in finding the common perpendicular direction.

Step 2: Compute and check a common perpendicular direction Calculate

$$\begin{aligned}\vec{a} \times \vec{b} &= (4 \cdot 1 - 2(-1))\hat{i} + (2 \cdot 1 - 3 \cdot 1)\hat{j} \\ &\quad + (3(-1) - 4 \cdot 1)\hat{k} \\ &= 6\hat{i} - \hat{j} - 7\hat{k}.\end{aligned}$$

This vector is nonzero. Its dot products with the two given directions are $18 - 4 - 14 = 0$ and $6 + 1 - 7 = 0$, respectively, confirming perpendicularity to both.

Step 3: Use the required point The requested line passes through the origin, so its fixed position vector is the zero vector. Multiplying the common perpendicular direction by any real parameter gives

Quick Tip to Solve: Use the coefficients of the line parameters in the cross product, and then insert the required passing point.

every point on the required line. A nonzero scalar multiple, including the negative of this direction, gives the same line.

Step 4: Conclusion The vector equation is $\vec{r} = t(6\hat{i} - \hat{j} - 7\hat{k}), t \in \mathbb{R}$.

Quick Tip to Solve: Use the coefficients of the line parameters in the cross product, and then insert the required passing point.

26(a). Find : $\int \frac{dx}{x^{1/2} + x^{1/3}}$

[3 marks]

Solution:

Concept:

- Use the least common multiple of the root denominators to remove fractional powers: put $t = x^{1/6}$, so $x = t^6$.
- Polynomial division reduces a rational integrand to powers of the substitution variable and a logarithmic term. Here the real integrand is defined for $x > 0$.

Step 1: Remove the fractional powers For $x > 0$, let $t = x^{1/6} > 0$. Then $dx = 6t^5 dt$, $x^{1/2} = t^3$ and $x^{1/3} = t^2$. Thus

$$I = \int \frac{6t^5}{t^3 + t^2} dt = 6 \int \frac{t^3}{t+1} dt.$$

Cancelling t^2 is valid since $t > 0$. Choosing a sixth root has turned both powers into integer powers in one substitution.

Step 2: Divide and integrate term by term The division identity $t^3 = (t+1)(t^2 - t + 1) - 1$ gives

$$\frac{t^3}{t+1} = t^2 - t + 1 - \frac{1}{t+1}.$$

Consequently,

$$\begin{aligned} I &= 6 \int \left(t^2 - t + 1 - \frac{1}{t+1} \right) dt \\ &= 2t^3 - 3t^2 + 6t - 6 \ln|t+1| + C. \end{aligned}$$

Each polynomial term is integrated by the power rule, and the final reciprocal contributes a logarithm.

Step 3: Return to x and verify Substituting $t = x^{1/6}$ gives powers $t^3 = x^{1/2}$ and $t^2 = x^{1/3}$. Since $t+1 > 0$, the absolute-value signs in the logarithm may be omitted. As a check, the derivative with respect to t of the antiderivative is $6t^3/(t+1)$. Multiplication by $dt/dx = 1/(6t^5)$ gives $1/[t^2(t+1)] = 1/(x^{1/2} + x^{1/3})$, exactly the original integrand.

Step 4: Conclusion

$$\int \frac{dx}{x^{1/2} + x^{1/3}} = 2x^{1/2} - 3x^{1/3} + 6x^{1/6} - 6 \ln(1 + x^{1/6}) + C$$

Quick Tip to Solve: For square and cube roots in one denominator, the sixth-root substitution converts the integral to rational form.

26(b). Find : $\int \tan^{-1}\left(\frac{1-x}{1+x}\right) dx$

[3 marks]

Solution:

Concept:

- Integration by parts: $\int u dv = uv - \int v du$. Choose the inverse trigonometric factor as u , since its derivative is rational.
- The chain rule gives $\frac{d}{dx} \tan^{-1} z = \frac{z'}{1+z^2}$. The given integrand is defined on intervals excluding $x = -1$.

Step 1: Differentiate the inverse tangent directly Put $u = \tan^{-1}\left(\frac{1-x}{1+x}\right)$. For $z = (1-x)/(1+x)$, the quotient rule gives $z' = -2/(1+x)^2$. Therefore

$$\begin{aligned} \frac{du}{dx} &= \frac{-2/(1+x)^2}{1 + (1-x)^2/(1+x)^2} \\ &= \frac{-2}{(1+x)^2 + (1-x)^2} = -\frac{1}{1+x^2}. \end{aligned}$$

This calculation does not require an inverse-tangent addition formula and therefore avoids changing the principal branch.

Step 2: Apply integration by parts Take $dv = dx$, so $v = x$. Writing the desired integral as I , obtain

$$I = x \tan^{-1}\left(\frac{1-x}{1+x}\right) + \int \frac{x}{1+x^2} dx.$$

In the remaining integral substitute $t = 1+x^2$, so $dt = 2x dx$. It becomes $\frac{1}{2} \int dt/t = \frac{1}{2} \ln(1+x^2)$, since $1+x^2 > 0$.

Step 3: Verify the antiderivative Differentiating the product contributes the original inverse tangent and $-x/(1+x^2)$. Differentiating the logarithmic term contributes $+x/(1+x^2)$. These rational terms cancel, leaving exactly the printed integrand on each interval of its domain.

Step 4: Conclusion

$$\int \tan^{-1}\left(\frac{1-x}{1+x}\right) dx = x \tan^{-1}\left(\frac{1-x}{1+x}\right) + \frac{1}{2} \ln(1+x^2) + C.$$

Quick Tip to Solve: Differentiate the inverse tangent directly before integrating by parts; this avoids any branch correction in a tangent-subtraction identity.

27. Evaluate : $\int_0^\pi \frac{\sin^{2026} x}{\sin^{2026} x + \cos^{2026} x} dx$

[3 marks]

Solution:

Concept:

- For an integrable function, reflection gives $\int_0^a f(x) dx = \int_0^a f(a-x) dx$.
- Even powers remove sign changes: $(-y)^{2026} = y^{2026}$. Also $\sin(\pi - x) = \sin x$ and $\cos(\pi - x) = -\cos x$.

Step 1: Reduce the printed interval by symmetry Let $f(x) = \frac{\sin^{2026} x}{\sin^{2026} x + \cos^{2026} x}$ and $I = \int_0^\pi f(x) dx$. The denominator never vanishes because sine and cosine cannot both be zero. Since the exponent is even, $f(\pi - x) = f(x)$. In the integral over $[\pi/2, \pi]$, substitute $t = \pi - x$. Reversing the resulting limits shows that this half contributes the same amount as the first half. Hence $I = 2J$, where $J = \int_0^{\pi/2} f(x) dx$.

Step 2: Pair complementary angles In J , make the substitution $t = \pi/2 - x$. Sine and cosine interchange, giving

$$J = \int_0^{\pi/2} \frac{\cos^{2026} t}{\cos^{2026} t + \sin^{2026} t} dt.$$

Rename the dummy variable t as x and add this expression to the original definition of J . Their common denominator equals the sum of their numerators, so

$$2J = \int_0^{\pi/2} 1 dx = \frac{\pi}{2}.$$

Thus $J = \pi/4$.

Step 3: Return to the full interval The complementary-angle pairing applies on the half interval. The printed question extends to π , so its integral is twice that half-interval value: $I = 2(\pi/4)$. The large even exponent changes the shape of the integrand, but not these symmetry relations.

Step 4: Conclusion

$$\boxed{\int_0^\pi \frac{\sin^{2026} x}{\sin^{2026} x + \cos^{2026} x} dx = \frac{\pi}{2}.$$

Quick Tip to Solve: First halve the interval using reflection about $\pi/2$, then exchange sine and cosine on the interval from zero to $\pi/2$.

Section C (27 marks)

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

28(a). Find : $\int \frac{\cos x}{(2 + \sin x)(4 + \sin x)} dx$

[3 marks]

Solution:

Concept:

- If an integrand contains $\cos x dx$ and rational expressions in $\sin x$, use $t = \sin x$, so $dt = \cos x dx$.
- For two distinct linear factors, write $\frac{1}{(t+a)(t+b)} = \frac{A}{t+a} + \frac{B}{t+b}$, and determine the constants by comparing coefficients.

Step 1: Convert to a rational integral Let $t = \sin x$. The differential $dt = \cos x dx$ exactly matches the numerator, so the required integral becomes

$$I = \int \frac{dt}{(t+2)(t+4)}.$$

There is no additional trigonometric factor to remove. The two denominator factors are distinct and linear, making partial fractions suitable.

Step 2: Determine the partial fractions Set $1 = A(t+4) + B(t+2)$. Comparison of the coefficient of t and the constant term gives $A + B = 0$ and $4A + 2B = 1$. Therefore $A = 1/2$ and $B = -1/2$. Consequently

$$I = \frac{1}{2} \int \frac{dt}{t+2} - \frac{1}{2} \int \frac{dt}{t+4}.$$

Keeping the minus sign is essential; a sum would not reconstruct the original constant numerator.

Step 3: Integrate and substitute back Using $\int dt/(t+a) = \ln|t+a| + C$, we obtain $I = \frac{1}{2} \ln|t+2| - \frac{1}{2} \ln|t+4| + C$. Replace t by $\sin x$. Since $-1 \leq \sin x \leq 1$, both $2 + \sin x$ and $4 + \sin x$ are positive. Thus their logarithms can be combined without absolute values. Differentiation gives $\frac{1}{2} \cos x [1/(2 + \sin x) - 1/(4 + \sin x)]$, which simplifies to the given integrand.

Step 4: Conclusion

$$I = \frac{1}{2} \ln \left(\frac{2 + \sin x}{4 + \sin x} \right) + C.$$

Quick Tip to Solve: Use sine as the substitution variable; the gap between the two linear constants immediately gives the coefficient one-half.

28(b). Find : $\int \frac{x+3}{x^2+4x+5} dx$

[3 marks]

Solution:**Concept:**

- For a quadratic denominator $D(x)$, split the numerator into a multiple of $D'(x)$ plus a constant.
- The standard forms are $\int D'(x)/D(x) dx = \ln |D(x)| + C$ and $\int du/(1+u^2) = \tan^{-1} u + C$.

Step 1: Separate the derivative of the denominator Let $D(x) = x^2 + 4x + 5$, so $D'(x) = 2x + 4$. Express the numerator as

$$x + 3 = \frac{1}{2}(2x + 4) + 1.$$

Therefore the required integral separates into

$$I = \frac{1}{2} \int \frac{2x + 4}{x^2 + 4x + 5} dx + \int \frac{dx}{x^2 + 4x + 5}.$$

The residual constant is one, because the first part contributes $x + 2$, not $x + 3$.

Step 2: Integrate the logarithmic part In the first integral substitute $v = x^2 + 4x + 5$; its differential is $dv = (2x + 4) dx$. This gives $\frac{1}{2} \ln |v|$. Complete the square to see that $x^2 + 4x + 5 = (x + 2)^2 + 1 > 0$ for every real x , so the absolute value can be omitted.

Step 3: Complete the square in the remaining part The second integral is $\int dx/[(x + 2)^2 + 1]$. With $u = x + 2$ and $du = dx$, it becomes $\int du/(u^2 + 1) = \tan^{-1} u$. Thus the two contributions are a logarithm and an inverse tangent. As a check, their derivatives combine to

$$(x + 2)/(x^2 + 4x + 5) + 1/(x^2 + 4x + 5) = (x + 3)/(x^2 + 4x + 5),$$

exactly the original rational function.

Step 4: Conclusion

$$\int \frac{x+3}{x^2+4x+5} dx = \frac{1}{2} \ln(x^2+4x+5) + \tan^{-1}(x+2) + C.$$

Quick Tip to Solve: Split $x+3$ as one-half of the denominator derivative plus one; complete the square only for the remaining constant numerator.

29(a). Find the general solution of the differential equation $2x^2 \frac{dy}{dx} = y^2 + 2xy$.

[3 marks]

Solution:

Concept:

- A homogeneous differential equation has the form $dy/dx = F(y/x)$. Substitute $y = vx$, where v is a function of x .
- The product rule gives $dy/dx = v + x dv/dx$. After substitution, separate the variables and integrate; check any solution excluded by division.

Step 1: Recognise the homogeneous form For $x \neq 0$, divide the given equation by $2x^2$:

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{y}{x} \right)^2 + \frac{y}{x}.$$

The right-hand side depends only on y/x , so the substitution $y = vx$ reduces the equation to one in v and x .

Step 2: Substitute and separate variables Using $dy/dx = v + x dv/dx$, we obtain

$$v + x \frac{dv}{dx} = \frac{v^2}{2} + v, \quad x \frac{dv}{dx} = \frac{v^2}{2}.$$

For $v \neq 0$, this becomes

$$\frac{dv}{v^2} = \frac{dx}{2x}.$$

The terms linear in v cancel, which is the key simplification.

Step 3: Integrate and restore the original variables Integrating both sides gives

$$-\frac{1}{v} = \frac{1}{2} \ln |x| + C_0.$$

Since $v = y/x$, this is

$$-\frac{2x}{y} = \ln |x| + 2C_0.$$

Relabelling the arbitrary constant gives $2x/y + \ln |x| = C$. The division by v^2 excluded $v = 0$; substituting $y = 0$ into the original equation confirms that it too is a solution. Each expression is considered on an interval avoiding $x = 0$ and any zero of its denominator.

Step 4: Conclusion The general nonzero solution is

$$y = \frac{2x}{C - \ln |x|} \quad (x \neq 0).$$

The additional solution is $y = 0$.

Quick Tip to Solve: Use $y = vx$: the linear v terms cancel before you separate variables. Always recover the zero solution after dividing by v^2 .

29(b). Find a particular solution of the differential equation $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$, given that $y = 0$ when $x = 0$. [3 marks]

Solution:

Concept:

- For a separable equation, collect all factors involving y with dy and all factors involving x with dx .
- The substitution $t = e^y$ gives $dt = e^y dy$. The initial condition determines the integration constant and the relevant interval.

Step 1: Separate the variables The given differential equation is

$$(x+1)\frac{dy}{dx} = 2e^{-y} - 1 = \frac{2 - e^y}{e^y}.$$

Hence, where the denominators are nonzero,

$$\frac{e^y}{2 - e^y} dy = \frac{dx}{x+1}.$$

At the prescribed initial point $(0, 0)$, we have $2 - e^0 = 1$, so this separation is valid in a neighbourhood of the initial point.

Step 2: Integrate using an exponential substitution Put $t = e^y$, so that $dt = e^y dy$. Then

$$\int \frac{dt}{2-t} = \int \frac{dx}{x+1}, \quad -\ln|2-t| = \ln|x+1| + C.$$

Returning to y ,

$$\ln|(2 - e^y)(x+1)| = -C.$$

On the interval containing $x = 0$, the product is positive initially and cannot change sign while this solution is defined. Therefore write $(2 - e^y)(x+1) = K$.

Step 3: Apply the initial condition and check At $x = 0$, $y = 0$, we get $K = (2 - 1)(1) = 1$. Thus

$$e^y = 2 - \frac{1}{x+1} = \frac{2x+1}{x+1}.$$

The interval containing zero on which the logarithm is real is $x > -1/2$. As a check, differentiation gives

$$y' = \frac{2}{2x+1} - \frac{1}{x+1} = \frac{1}{(2x+1)(x+1)}.$$

Thus $(x+1)y' = 1/(2x+1) = 2e^{-y} - 1$, and the initial value is satisfied.

Step 4: Conclusion The required particular solution is

$$\boxed{y = \ln\left(\frac{2x+1}{x+1}\right)}, \quad x > -\frac{1}{2}.$$

Quick Tip to Solve: Multiply the separated numerator by e^y ; then $t = e^y$ makes the integral a simple logarithm.

30. If $(\sin x)^y = y^{\cos x}$, then find $\frac{dy}{dx}$.

[3 marks]

Solution:

Concept:

- Logarithmic differentiation simplifies variable powers: take logarithms before differentiating.
- When y depends on x , use both the product rule and $d(\ln y)/dx = y'/y$. For real logarithms, work where $\sin x > 0$ and $y > 0$.

Step 1: Take natural logarithms From $(\sin x)^y = y^{\cos x}$, take logarithms on the real domain where both bases are positive:

$$y \ln(\sin x) = \cos x \ln y.$$

Both y and the exponent $\cos x$ vary with x ; neither may be treated as a constant. Logarithms change the variable powers into products that can be differentiated directly.

Step 2: Differentiate both sides implicitly Let $y' = dy/dx$. Applying the product rule to each side and the chain rule to the logarithms,

$$y' \ln(\sin x) + y \cot x = -\sin x \ln y + \frac{\cos x}{y} y'.$$

Here $d \ln(\sin x)/dx = \cos x / \sin x = \cot x$, while the derivative of $\cos x$ accounts for the negative sign in the first term on the right.

Step 3: Collect the derivative terms and simplify Move both terms containing y' to the same side:

$$\left(\frac{\cos x}{y} - \ln(\sin x) \right) y' = y \cot x + \sin x \ln y.$$

Multiply numerator and denominator by y when solving for y' . This removes the fraction within the denominator. The resulting formula applies at points of a differentiable branch for which $\cos x - y \ln(\sin x) \neq 0$; where that coefficient vanishes, division cannot be used to determine the derivative.

Step 4: Conclusion

$$\boxed{\frac{dy}{dx} = \frac{y^2 \cot x + y \sin x \ln y}{\cos x - y \ln(\sin x)}}.$$

Quick Tip to Solve: After taking logs, underline both products: forgetting either product-rule term changes the numerator or denominator.

31. A survey was conducted on the patients who have undergone knee replacement surgeries. It was found that Robotic Knee replacement surgeries have 90% success rate.

On a particular day, robotic surgery was performed on three patients, A , B and C , one after the other. Assuming that the success and failure of each surgery is independent of each other, find the probability that :

- (i) exactly one surgery is successful,
- (ii) at most two surgeries are successful.

[3 marks]

Solution:

Concept:

- For each surgery, success probability is $p = 0.9$, and failure probability is $q = 1 - p = 0.1$. Independence permits multiplication of probabilities.
- For three independent trials with the same success probability, $P(X = r) = \binom{3}{r} p^r q^{3-r}$, where X counts successful surgeries.

Step 1: Identify the independent trial outcomes Write S for success and F for failure, recording outcomes in the order A, B, C . Since each surgery succeeds with probability 0.9 and fails with probability 0.1, any specified outcome has probability equal to the product of its three factors. Different outcome sequences are mutually exclusive, so their probabilities may be added.

Step 2: Part (i): exactly one successful surgery The possible sequences are SFF , FSF and FFS . Each has probability $0.9(0.1)^2$. Hence

$$P(X = 1) = \binom{3}{1} (0.9)(0.1)^2 = 3(0.9)(0.01) = 0.027.$$

The factor of three is necessary because any one of the three patients can have the sole successful surgery.

Step 3: Part (ii): at most two successful surgeries At most two means zero, one or two successes; its only excluded possibility is that all three surgeries succeed. Therefore the complement method gives

$$P(X \leq 2) = 1 - P(X = 3) = 1 - (0.9)^3 = 1 - 0.729 = 0.271.$$

We can also check by adding the allowed probabilities:

$$P(X = 0) = (0.1)^3 = 0.001,$$

$$P(X = 1) = 0.027,$$

$$P(X = 2) = 3(0.9)^2(0.1) = 0.243.$$

Their sum is $0.001 + 0.027 + 0.243 = 0.271$, agreeing with the complement calculation.

Step 4: Conclusion The required probabilities are (i) 0.027 and (ii) 0.271.

Quick Tip to Solve: For at most two successes among three trials, subtract the probability of three successes from one; do not omit zero successes.

32. Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = \frac{x}{\sqrt{1+x^2}}$ is one-one but not onto. [5 marks]

Solution:

Concept:

- A function is one-one when equal images imply equal inputs. Strict increase throughout the domain guarantees this property.
- A function is onto when its range equals its codomain. Here the codomain is the whole real line $\mathbb{R}$.
- The denominator $\sqrt{1+x^2}$ is positive for every real input, so differentiation and comparison are valid everywhere.

Step 1: Differentiate the given function Write $f(x) = x(1+x^2)^{-1/2}$. Using the product rule and chain rule,

$$\begin{aligned} f'(x) &= (1+x^2)^{-1/2} - x^2(1+x^2)^{-3/2} \\ &= \frac{1+x^2-x^2}{(1+x^2)^{3/2}} \\ &= \frac{1}{(1+x^2)^{3/2}} > 0. \end{aligned}$$

The inequality holds for every real x , since $1+x^2$ is always positive. Thus there are no exceptional inputs at which the derivative is undefined or nonpositive.

Step 2: Prove injectivity on the real line Let $a < b$ be two real numbers. The function is continuous on $[a, b]$ and differentiable on (a, b) . By the mean value theorem, there exists $c \in (a, b)$ such that

$$f(b) - f(a) = f'(c)(b - a) > 0.$$

Therefore $f(a) < f(b)$. Distinct inputs have distinct images, so f is strictly increasing and one-one. Equivalently, if $f(a) = f(b)$, neither $a < b$ nor $b < a$ is possible, forcing $a = b$.

Step 3: Find the range and examine surjectivity Since $x^2 < 1+x^2$, taking positive square roots gives $|x| < \sqrt{1+x^2}$. Consequently,

$$|f(x)| = \frac{|x|}{\sqrt{1+x^2}} < 1.$$

Every image belongs to $(-1, 1)$, and neither endpoint is attained. Conversely, let $-1 < y < 1$. The real number

$$x = \frac{y}{\sqrt{1-y^2}}$$

is defined because $1-y^2 > 0$. For this input,

$$1+x^2 = \frac{1}{1-y^2}, \quad f(x) = \frac{y/\sqrt{1-y^2}}{1/\sqrt{1-y^2}} = y.$$

Thus every value strictly between -1 and 1 occurs, proving that the range is exactly $(-1, 1)$. This is smaller than the specified codomain $\mathbb{R}$. In particular, 1 is a real codomain element with no preimage, so the onto condition fails.

Step 4: Conclusion The function $f(x) = \frac{x}{\sqrt{1+x^2}}$ is one-one because it is strictly increasing. Its range

Quick Tip to Solve: A positive derivative establishes injectivity, while comparing the numerator with the denominator immediately shows why values outside the interval cannot occur.

is $(-1, 1)$, so it is not onto $\mathbb{R}$.

Quick Tip to Solve: A positive derivative establishes injectivity, while comparing the numerator with the denominator immediately shows why values outside the interval cannot occur.

33. Solve the following Linear Programming Problem graphically :

Maximise $Z = 600x + 400y$

subject to the constraints

$$x + 2y \leq 12$$

$$4x + 5y \geq 20$$

$$2x + y \leq 12$$

$$x, y \geq 0$$

[5 marks]

Solution:

Concept:

- Each linear inequality determines a closed half-plane. Their intersection with $x \geq 0$ and $y \geq 0$ is the feasible region.
- For a nonempty bounded polygonal feasible region, a linear objective attains its maximum at a vertex. Evaluate $Z = 600x + 400y$ at every vertex.

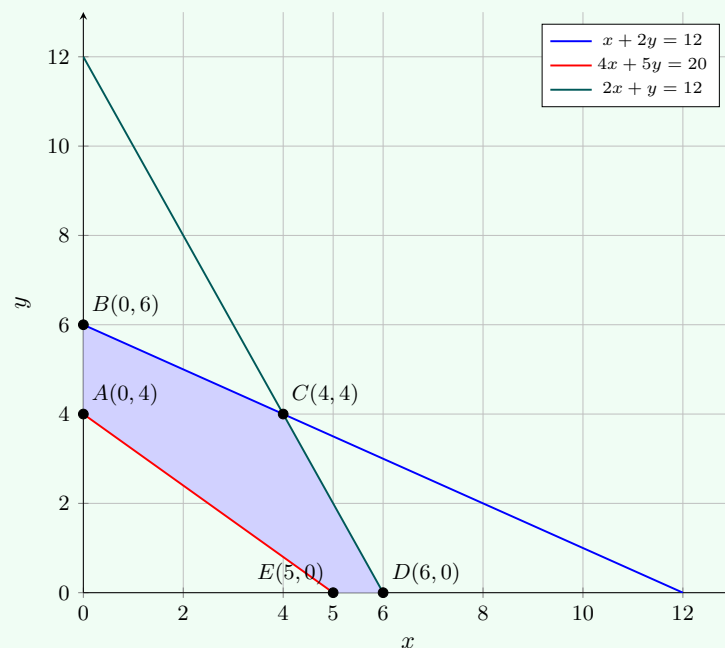
Step 1: Plot the three boundary lines Replace the inequalities temporarily by equalities. The line $x + 2y = 12$ passes through $(12, 0)$ and $(0, 6)$. The line $4x + 5y = 20$ passes through $(5, 0)$ and $(0, 4)$. The line $2x + y = 12$ passes through $(6, 0)$ and $(0, 12)$. The origin satisfies the first and third inequalities, so retain their half-planes towards the origin. It does not satisfy $4x + 5y \geq 20$, so retain the half-plane away from the origin for that constraint. Also retain only the first quadrant. Draw each boundary as a solid line, since equality is allowed.

Step 2: Identify the feasible polygon and its vertices On the y -axis the inequalities give $4 \leq y \leq 6$, producing $A(0, 4)$ and $B(0, 6)$. On the x -axis they give $5 \leq x \leq 6$, producing $D(6, 0)$ and $E(5, 0)$. The remaining vertex is the intersection of the two upper boundaries:

$$x + 2y = 12, \quad 2x + y = 12 \quad \implies \quad x = y = 4.$$

Thus $C = (4, 4)$, and it satisfies the lower constraint because $4(4) + 5(4) = 36 \geq 20$. The intersection of $x + 2y = 12$ with $4x + 5y = 20$ has $x = -20/3$, while the intersection of $2x + y = 12$ with $4x + 5y = 20$ has $y = -4/3$. These lie outside the first quadrant and are excluded. The shaded pentagon $ABCDE$ is nonempty and bounded.

Quick Tip to Solve: Shade towards the origin for the two upper constraints and away from it for the lower constraint; then test all five corners of the resulting pentagon.



The shaded pentagon $ABCDE$ is the feasible region in the first quadrant.

Step 3: Evaluate the objective at every corner Substitute each feasible vertex in the objective function:

Vertex (x, y)	$Z = 600x + 400y$
$A(0, 4)$	1600
$B(0, 6)$	2400
$C(4, 4)$	4000
$D(6, 0)$	3600
$E(5, 0)$	3000

The corner-point theorem applies because the feasible polygon is bounded. The largest entry is 4000, attained at C . At this point the first and third constraints hold with equality, and the second holds as $36 \geq 20$; nonnegativity also holds. The objective has unequal values at all the vertices adjacent to C , so the maximum is attained uniquely there.

Step 4: Conclusion The maximum value is $Z_{\max} = 4000$, attained at $x = 4, y = 4$.

Quick Tip to Solve: Shade towards the origin for the two upper constraints and away from it for the lower constraint; then test all five corners of the resulting pentagon.

34(a). On the inauguration day of a new showroom, a lucky draw was organized and some vouchers of Rs 1,000 and Rs 500 were given to the lucky draw winners.

A total of 60 vouchers were given on the day. The number of Rs 1,000 vouchers added to 3 times the number of Rs 500 vouchers, gives 100. Express the given information as a system of linear equations in two variables. Hence, find the number of vouchers of each type by matrix method. [5 marks]

Solution:

Concept:

- Define the unknowns as numbers of vouchers, keeping the two denominations in the same order throughout the equations and matrix calculation.
- A system of two linear equations can be written as $AX = B$, where A is the coefficient matrix, X is the column of unknowns and B is the column of constants.
- If $|A| \neq 0$, the unique solution is $X = A^{-1}B$, with $A^{-1} = \frac{1}{|A|} \text{adj } A$.

Step 1: Define the variables and form the equations Let x be the number of Rs 1,000 vouchers and y the number of Rs 500 vouchers. Since the total number of vouchers is 60, the first equation is $x + y = 60$. The second statement says that the number of Rs 1,000 vouchers, added to three times the number of Rs 500 vouchers, is 100. Thus,

$$\begin{aligned}x + y &= 60, \\x + 3y &= 100.\end{aligned}$$

The denominations identify the voucher types; they are not coefficients in these equations because the statements concern numbers of vouchers, rather than their total monetary value.

Step 2: Write the matrix equation and find the inverse The required matrix form is

$$\underbrace{\begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}}_A \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_X = \underbrace{\begin{pmatrix} 60 \\ 100 \end{pmatrix}}_B.$$

The determinant is $|A| = 1(3) - 1(1) = 2 \neq 0$, so the inverse exists and the solution is unique. Interchanging the diagonal entries and changing the signs of the off-diagonal entries gives

$$\text{adj } A = \begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}, \quad A^{-1} = \frac{1}{2} \begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}.$$

Step 3: Multiply by the inverse and verify the counts Multiplying $AX = B$ on the left by A^{-1} , we obtain

$$\begin{aligned}X &= A^{-1}B = \frac{1}{2} \begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 60 \\ 100 \end{pmatrix} \\&= \frac{1}{2} \begin{pmatrix} 180 - 100 \\ -60 + 100 \end{pmatrix} = \begin{pmatrix} 40 \\ 20 \end{pmatrix}.\end{aligned}$$

Both entries are non-negative integers, as required for counts. They satisfy the total-count condition, $40 + 20 = 60$, and also the second condition, $40 + 3(20) = 100$. Thus the matrix result satisfies both

Quick Tip to Solve: Keep the columns ordered as Rs 1,000 then Rs 500, and check both original count equations after multiplying by the inverse.

original statements with the denominations assigned in the correct order.

Step 4: Conclusion The system is $x + y = 60$, $x + 3y = 100$. There were $\boxed{40}$ vouchers of Rs 1,000 and $\boxed{20}$ vouchers of Rs 500.

Quick Tip to Solve: Keep the columns ordered as Rs 1,000 then Rs 500, and check both original count equations after multiplying by the inverse.

OR

34(b). Given that $P = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, $Q = \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix}$ and $R = \begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix}$, find a matrix S such that $PQ - RS$ is a null matrix. [5 marks]

Solution:

Concept:

- A null matrix has all entries equal to zero. Thus $PQ - RS = O$ is equivalent to $RS = PQ$.
- Matrix multiplication uses the dot product of a row of the first matrix with a column of the second matrix. The order of multiplication must be preserved.
- If $|R| \neq 0$, multiply $RS = PQ$ on the left by R^{-1} to get the unique solution $S = R^{-1}PQ$.

Step 1: Translate the null-matrix condition and calculate the product The given condition is $PQ - RS = O$, where O is the 2×2 null matrix. Moving RS to the other side gives $RS = PQ$. First calculate the right-hand side using the given matrices:

$$\begin{aligned} PQ &= \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 2(5) - 7 & 2(2) - 4 \\ 3(5) + 4(7) & 3(2) + 4(4) \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 43 & 22 \end{pmatrix}. \end{aligned}$$

In particular, the top-right entry is zero, while the bottom-left entry is 43. Retaining these positions is essential for the subsequent matrix product.

Step 2: Establish that the inverse of R exists For $R = \begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix}$,

$$|R| = 2(8) - 5(3) = 16 - 15 = 1 \neq 0.$$

Hence R is invertible. By the inverse formula for a 2×2 matrix,

$$R^{-1} = \frac{1}{1} \begin{pmatrix} 8 & -5 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 8 & -5 \\ -3 & 2 \end{pmatrix}.$$

We must multiply on the left, because S occurs to the right of R in RS . Indeed, $R^{-1}RS = R^{-1}PQ$ gives $IS = R^{-1}PQ$. Right multiplication would not isolate S , since matrix multiplication is generally not commutative.

Quick Tip to Solve: In $RS=PQ$, cancel R by multiplying on the left by its inverse; then check the result by calculating RS entry by entry.

Step 3: Calculate S and check the original condition Therefore,

$$\begin{aligned} S &= R^{-1}PQ \\ &= \begin{pmatrix} 8 & -5 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 43 & 22 \end{pmatrix} \\ &= \begin{pmatrix} 24 - 215 & -110 \\ -9 + 86 & 44 \end{pmatrix} = \begin{pmatrix} -191 & -110 \\ 77 & 44 \end{pmatrix}. \end{aligned}$$

As a direct check,

$$RS = \begin{pmatrix} -382 + 385 & -220 + 220 \\ -573 + 616 & -330 + 352 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 43 & 22 \end{pmatrix} = PQ.$$

Consequently every entry of $PQ - RS$ is zero. The nonzero determinant of R also ensures that no other matrix S satisfies the condition.

Step 4: Conclusion The required matrix is $S = \begin{pmatrix} -191 & -110 \\ 77 & 44 \end{pmatrix}$, for which $PQ - RS = O$.

Quick Tip to Solve: In $RS=PQ$, cancel R by multiplying on the left by its inverse; then check the result by calculating RS entry by entry.

Section D (5 marks)

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

35(a). Represent the equations of lines l_1 and l_2 in vector form and check whether they are intersecting or not.

$$l_1 : \frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$$

$$l_2 : \frac{x+1}{-1} = \frac{2-y}{-2} = \frac{z-5}{5}$$

[5 marks]

Solution:

Concept:

- A line through the point with position vector $\vec{a}$, parallel to a nonzero vector $\vec{b}$, has equation $\vec{r} = \vec{a} + t\vec{b}$, where t is a real parameter.
- Two lines intersect if their coordinate equations have a common solution for their two independent parameters. Equality must hold in all three coordinates, not merely in two.
- The sign in a symmetric equation must be retained: $\frac{2-y}{-2} = \frac{y-2}{2}$, so the second line has positive y -direction ratio 2.

Step 1: Convert the first line into vector form Put each ratio in the equation of l_1 equal to λ . Then

$$x = -3 - 3\lambda, \quad y = 1 + \lambda, \quad z = 5 + 5\lambda.$$

Quick Tip to Solve: Use the identical z -coordinate expressions first to obtain equal parameters; then verify the remaining coordinate after solving for them.

Thus a point on this line is $(-3, 1, 5)$, and its direction vector is $(-3, 1, 5)$. Writing the three coordinate equations together gives

$$l_1: \vec{r} = -3\hat{i} + \hat{j} + 5\hat{k} + \lambda(-3\hat{i} + \hat{j} + 5\hat{k}).$$

Step 2: Convert the second line into vector form Independently set the three ratios for l_2 equal to μ . In particular, $2 - y = -2\mu$, which gives $y = 2 + 2\mu$. Hence

$$x = -1 - \mu, \quad y = 2 + 2\mu, \quad z = 5 + 5\mu,$$

and the required vector equation is

$$l_2: \vec{r} = -\hat{i} + 2\hat{j} + 5\hat{k} + \mu(-\hat{i} + 2\hat{j} + 5\hat{k}).$$

The two direction vectors are not proportional: the ratios of their first and second components would be 3 and 1/2. The lines therefore are not parallel, but intersection still requires checking all coordinates.

Step 3: Solve and verify the common-point equations At a common point the corresponding coordinates must satisfy

$$-3 - 3\lambda = -1 - \mu,$$

$$1 + \lambda = 2 + 2\mu,$$

$$5 + 5\lambda = 5 + 5\mu.$$

The third equation gives $\lambda = \mu$. Substitution in the first equation gives $-3 - 3\lambda = -1 - \lambda$, so $-2\lambda = 2$ and $\lambda = \mu = -1$. For these parameters the second equation reads $0 = 0$, so it too is satisfied. Substituting into the first line gives $(-3 + 3, 1 - 1, 5 - 5) = (0, 0, 0)$; the second line also gives $(-1 + 1, 2 - 2, 5 - 5) = (0, 0, 0)$.

Step 4: Conclusion The vector equations are $\vec{r} = -3\hat{i} + \hat{j} + 5\hat{k} + \lambda(-3\hat{i} + \hat{j} + 5\hat{k})$ and $\vec{r} = -\hat{i} + 2\hat{j} + 5\hat{k} + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$. The lines intersect at the origin $(0, 0, 0)$.

Quick Tip to Solve: Use the identical z-coordinate expressions first to obtain equal parameters; then verify the remaining coordinate after solving for them.

35(b). Opposite sides of a square are along the lines :

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Find the area of the square if direction ratios of other pair of opposite sides of the square are given by $\langle -3, 6, p \rangle$. Also, find the value of p . [5 marks]

Solution:

Concept:

- Opposite sides of a square are parallel; its side length is the perpendicular distance between the lines containing either pair of opposite sides.
- For parallel lines through position vectors $\vec{a}_1, \vec{a}_2$ with common direction vector $\vec{b}$, their distance is $s = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$. The area of the square is s^2 .
- Adjacent sides of a square are perpendicular, so their direction vectors have zero scalar product: $\vec{b} \cdot \vec{c} = 0$.

Step 1: Identify the two parallel supporting lines From the given equations, take

$$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k},$$

$$\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k},$$

$$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}.$$

Both lines have the same nonzero direction vector $\vec{b}$, so they are parallel. Their connecting displacement is

$$\vec{d} = \vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}, \quad |\vec{b}| = \sqrt{4 + 9 + 36} = 7.$$

The length of $\vec{d}$ itself is not the side of the square, because this connecting segment need not be perpendicular to the two lines. Its perpendicular component supplies the required separation.

Step 2: Calculate the perpendicular distance and square area Compute the cross product explicitly:

$$\begin{aligned} \vec{d} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} \\ &= (6 + 3)\hat{i} - (12 + 2)\hat{j} + (6 - 2)\hat{k} \\ &= 9\hat{i} - 14\hat{j} + 4\hat{k}. \end{aligned}$$

Consequently $|\vec{d} \times \vec{b}|^2 = 81 + 196 + 16 = 293$, and the square's side length is $s = \sqrt{293}/7$. Squaring the distance gives its area $s^2 = 293/49$. As an independent arithmetic check, $\vec{d} \cdot \vec{b} = 4 + 3 - 6 = 1$ and

Quick Tip to Solve: Square the parallel-line distance formula directly to find the area; use a zero dot product separately to find the missing direction ratio.

$|\vec{d}|^2 = 6$, so the perpendicular component has squared length $6 - 1/49 = 293/49$, in agreement with the cross-product calculation.

Step 3: Determine the missing direction ratio The other pair of opposite sides is perpendicular to the given pair. Its direction vector is $\vec{c} = -3\hat{i} + 6\hat{j} + p\hat{k}$. Therefore

$$\begin{aligned}\vec{b} \cdot \vec{c} &= 0, \\ 2(-3) + 3(6) + 6p &= 0, \\ 12 + 6p &= 0, \quad p = -2.\end{aligned}$$

Substitution checks the required right angle directly: $-6 + 18 - 12 = 0$. Reversing a direction vector would describe the same direction and does not change this perpendicularity condition.

Step 4: Conclusion The area of the square is $\frac{293}{49}$ square units, and the missing direction ratio is $p = -2$.

Quick Tip to Solve: Square the parallel-line distance formula directly to find the area; use a zero dot product separately to find the missing direction ratio.

Section E (12 marks)

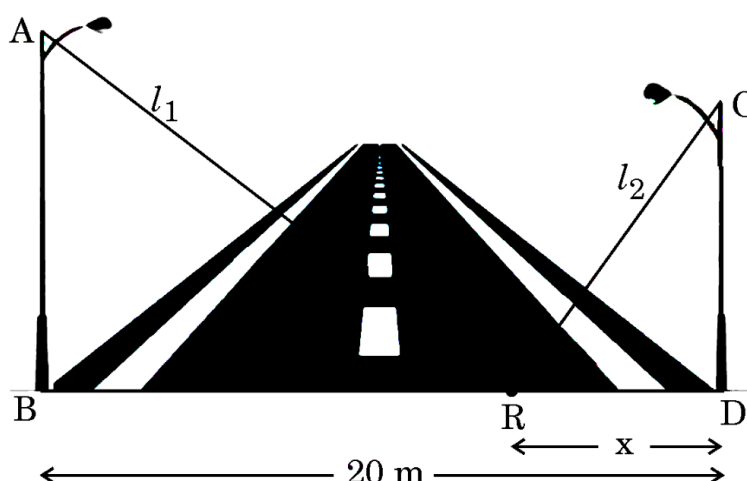
This section comprises 3 case study based questions of 4 marks each.

36. Two vertical light poles of height 22 m and 16 m stand on the opposite sides of a 20 m wide road as shown below in the figure.

Two ladders of length l_1 and l_2 are placed from a common point R on the road at a distance of x m from the smaller pole.

Based on the above information, answer the following questions :

[4 marks]



36(i). Express $p(x) = l_1 + l_2$ in terms of x .

[1 mark]

Solution:

Concept:

- Pythagoras theorem gives the length of a ladder as the square root of the sum of the squared vertical height and horizontal distance.
- The point R is x metres from the smaller pole, so the horizontal distances are $RD = x$ and $BR = 20 - x$, with $0 \leq x \leq 20$.

Step 1: Find each ladder length The vertical poles and the road form right angles. In triangle ABR , the pole height is $AB = 22$, while $BR = 20 - x$. In triangle CDR , the height is $CD = 16$ and $RD = x$. Hence

$$l_1 = \sqrt{22^2 + (20 - x)^2}, \quad l_2 = \sqrt{16^2 + x^2}.$$

Both square roots are positive because they represent lengths.

Step 2: Form the requested sum The printed definition is $p(x) = l_1 + l_2$. Add the two lengths themselves:

$$p(x) = \sqrt{484 + (20 - x)^2} + \sqrt{256 + x^2}.$$

The heights are fixed, while changing x changes both horizontal distances. This expression is defined throughout the road interval.

Step 3: Conclusion $p(x) = \sqrt{484 + (20 - x)^2} + \sqrt{256 + x^2}, \quad 0 \leq x \leq 20.$

Quick Tip to Solve: Assign x to the 16 m pole first; the other horizontal distance is $20 - x$.

36(ii). Find $p'(x)$.

[1 mark]

Solution:

Concept:

- For a differentiable positive function $u(x)$, the chain rule gives $\frac{d}{dx}\sqrt{u(x)} = \frac{u'(x)}{2\sqrt{u(x)}}$.
- Use the sum of lengths from part (i): $p(x) = \sqrt{484 + (20 - x)^2} + \sqrt{256 + x^2}$.

Step 1: Differentiate the first ladder length Let $u(x) = 484 + (20 - x)^2$. Since $\frac{d}{dx}(20 - x) = -1$,

$$u'(x) = -2(20 - x) = 2(x - 20).$$

Therefore

$$\frac{d}{dx}\sqrt{484 + (20 - x)^2} = \frac{x - 20}{\sqrt{484 + (20 - x)^2}}.$$

The negative sign expresses that this ladder becomes shorter as R moves towards the taller pole.

Step 2: Differentiate the second ladder length and add For $v(x) = 256 + x^2$, we have $v'(x) = 2x$, giving

$$\frac{d}{dx}\sqrt{256 + x^2} = \frac{x}{\sqrt{256 + x^2}}.$$

Differentiate the two summands separately and add the results. The constant pole heights contribute no derivative.

Step 3: Conclusion

$$p'(x) = \frac{x - 20}{\sqrt{484 + (20 - x)^2}} + \frac{x}{\sqrt{256 + x^2}}.$$

Quick Tip to Solve: Keep the factor -1 from differentiating 20-x; it controls the sign of the first term.

36(iii)(a). Find the value of x for which $l_1^2 + l_2^2$ is minimum.

[2 marks]

Solution:

Concept:

- To minimize the sum of squares, define a separate objective $S(x) = l_1^2 + l_2^2$; it differs from the sum of lengths $p(x)$ in part (i).
- An interior stationary point satisfies $S'(x) = 0$. A positive second derivative confirms a minimum.

Step 1: Express the squared-length objective By Pythagoras theorem, $l_1^2 = 22^2 + (20 - x)^2$ and $l_2^2 = 16^2 + x^2$. Therefore

$$\begin{aligned} S(x) &= 484 + (20 - x)^2 + 256 + x^2 \\ &= 2x^2 - 40x + 1140. \end{aligned}$$

The valid interval is $0 \leq x \leq 20$, since the foot of the ladders lies on the road between the poles. The constant vertical contributions affect the minimum value, but do not affect where it occurs.

Quick Tip to Solve: For the sum of squared lengths, constant pole heights drop out of the derivative; balance x and $20-x$.

Step 2: Locate and verify the minimum Differentiating gives

$$S'(x) = 4x - 40.$$

The stationary equation $4x - 40 = 0$ gives $x = 10$, which lies inside the allowed interval. Furthermore, $S''(x) = 4 > 0$ throughout that interval. To verify the global nature of the minimum, complete the square:

$$S(x) = 2(x - 10)^2 + 940 \geq 940.$$

Equality holds only when $x = 10$. The endpoints instead give $S(0) = S(20) = 1140$, so neither endpoint improves upon the interior value.

Step 3: Conclusion The sum $l_1^2 + l_2^2$ is minimum at $x = 10 \text{ m}$, the midpoint of the road; its minimum value is 940 m^2 .

Quick Tip to Solve: For the sum of squared lengths, constant pole heights drop out of the derivative; balance x and $20-x$.

OR

36(iii)(b). If the 22 m long pole is also replaced by a 16 m long pole, at what distance from either pole should the ladders be kept so that the sum of squares of lengths of ladders needed to reach the top of the pole is minimum ? [2 marks]

Solution:

Concept:

- By Pythagoras' theorem, a ladder reaching height h from horizontal distance d has squared length $h^2 + d^2$.
- An interior stationary point of a differentiable function is a strict minimum when its second derivative is positive. The distance between the pole bases remains 20 m.

Step 1: Express the sum of squared ladder lengths After the replacement, both poles have height 16 m. Let the common foot of the ladders be x m from one pole, so that it is $(20 - x)$ m from the other, with $0 \leq x \leq 20$. Thus

$$S(x) = l_1^2 + l_2^2 = (16^2 + x^2) + [16^2 + (20 - x)^2] = 2x^2 - 40x + 912.$$

The pole heights contribute constants; the location is determined by the two horizontal distances.

Step 2: Differentiate and verify the minimum

$$S'(x) = 4x - 40 = 0 \implies x = 10, \quad S''(x) = 4 > 0.$$

This stationary point lies inside the permitted interval. More explicitly, $S(x) = 2(x - 10)^2 + 712$, so every other permissible position increases the sum. This also verifies the global minimum, including the two endpoints.

Quick Tip to Solve: When both pole heights are equal, minimize the sum of squared horizontal distances by placing the ladders at the midpoint.

Step 3: Conclusion The ladders should be kept at the midpoint of the road, $\boxed{10\text{ m}}$ from either pole.

Quick Tip to Solve: When both pole heights are equal, minimize the sum of squared horizontal distances by placing the ladders at the midpoint.

37. A survey was conducted to find out the success rate of students who qualified the entrance examination by dropping a year after class XII.

As per the data collected, 40% students appearing in the examination were dropouts and the remaining students were regular students of class XII.

Of the dropouts, 5% qualify the examination while 10% of the regular students qualify the examination. Based on the above information, answer the following questions. [4 marks]

37(i). Find the probability that a student selected at random is a regular student. [1 mark]

Solution:

Concept:

- Complementary events are mutually exclusive and exhaustive, so their probabilities add to 1.
- A percentage $r\%$ corresponds to probability $r/100$.

Step 1: Identify the two groups Let D be the event that the selected student is a dropout and R the event that the student is a regular class XII student. The survey divides all candidates into these two groups, so $R = D^c$. We are given $P(D) = 40/100 = 0.40$.

Step 2: Take the complementary probability Therefore,

$$P(R) = 1 - P(D) = 1 - 0.40 = 0.60.$$

The qualification percentages are unnecessary here because selection is from all candidates, without any condition about qualifying.

Step 3: Conclusion The probability that the selected student is a regular student is $\boxed{0.60 = \frac{3}{5}}$.

Quick Tip to Solve: For selection from all candidates, subtract the dropout proportion from one; do not use qualification rates.

37(ii). A student is selected at random from a group of dropout students. What is the probability that the student will not qualify the examination ? [1 mark]

Solution:

Concept:

- When selection is restricted to a group, probabilities are conditional on belonging to that group.
- Within a fixed group, qualifying and not qualifying are complementary events: $P(Q^c | D) = 1 - P(Q | D)$.

Quick Tip to Solve: When the student is already chosen from dropouts, complement the dropout qualification rate directly.

Step 1: Use the dropout group as the sample space Let D denote being a dropout and Q denote qualifying the examination. The student is selected from dropout students only. Hence the relevant given probability is $P(Q | D) = 5/100 = 0.05$, rather than the overall dropout probability 0.40.

Step 2: Find the probability of not qualifying Within this group, every student either qualifies or does not qualify, and these outcomes cannot occur together. Therefore,

$$P(Q^c | D) = 1 - 0.05 = 0.95.$$

Multiplying by 0.40 would instead give the joint probability of being a dropout and not qualifying among all candidates, which is a different event.

Step 3: Conclusion The required probability is $0.95 = \frac{19}{20}$.

Quick Tip to Solve: When the student is already chosen from dropouts, complement the dropout qualification rate directly.

37(iii)(a). A student selected at random qualified the examination. Find the probability that student is not a dropout. [2 marks]

Solution:

Concept:

- Let D denote a dropout, R a regular student and Q qualification. These two student categories partition the sample space, so $P(R) = 1 - P(D)$.
- Bayes' theorem gives $P(R | Q) = \frac{P(R)P(Q | R)}{P(D)P(Q | D) + P(R)P(Q | R)}$.

Step 1: Record the category and success probabilities The survey gives $P(D) = 0.40$, so $P(R) = 0.60$. The qualification probabilities within the categories are $P(Q | D) = 0.05$ and $P(Q | R) = 0.10$. A student who is not a dropout is a regular student in this survey.

Step 2: Find the total probability of qualifying Since a qualified student must belong to exactly one of these two categories, the total probability is

$$P(Q) = 0.40(0.05) + 0.60(0.10) = 0.02 + 0.06 = 0.08.$$

Of this total, the probability of being both regular and qualified is $P(R \cap Q) = 0.06$. Conditioning on qualification restricts the sample space to the students represented by the probability 0.08.

Step 3: Apply the required conditional probability Consequently,

$$P(R | Q) = \frac{P(R \cap Q)}{P(Q)} = \frac{0.06}{0.08} = \frac{3}{4}.$$

Step 4: Conclusion The probability that the qualified student is not a dropout is $\frac{3}{4}$.

Quick Tip to Solve: Weight each category's qualification rate by its population share before applying Bayes' theorem.

37(iii)(b). A student selected at random did not qualify the examination. Find the probability that the student was a regular student. [2 marks]

Solution:

Concept:

- Let R denote a regular student, D a dropout and Q' failure to qualify. The categories R, D are mutually exclusive and exhaustive.
- Complementary conditional probabilities satisfy $P(Q' | R) = 1 - P(Q | R)$; Bayes' theorem reverses the conditioning to obtain $P(R | Q')$.

Step 1: Find the failure rates for both categories The category probabilities are $P(D) = 0.40$ and $P(R) = 0.60$. Since 5% of dropouts qualify and 10% of regular students qualify,

$$P(Q' | D) = 0.95, \quad P(Q' | R) = 0.90.$$

These are failure rates within the respective categories, rather than probabilities of category membership among unsuccessful students.

Step 2: Combine the probabilities of not qualifying By the law of total probability,

$$P(Q') = 0.40(0.95) + 0.60(0.90) = 0.38 + 0.54 = 0.92.$$

The probability of being both a regular student and unsuccessful is $P(R \cap Q') = 0.60(0.90) = 0.54$.

Step 3: Condition on the given failure The required probability is

$$P(R | Q') = \frac{P(R \cap Q')}{P(Q')} = \frac{0.54}{0.92} = \frac{27}{46}.$$

The denominator includes unsuccessful students from both categories, because the selected student's category is not initially known.

Step 4: Conclusion The probability that the student who did not qualify was regular is $\boxed{\frac{27}{46}}$.

Quick Tip to Solve: For a student known to have failed, complement both success rates before forming the Bayes denominator.

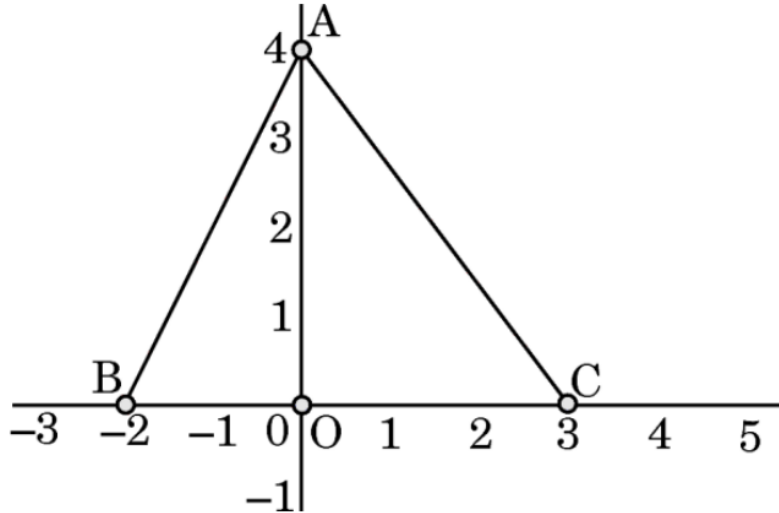
38. There is a triangular park in the society. The park is divided into two sections as shown in the figure.

In the region OAC, children are allowed to play games like cricket, football, while in the region AOB, activities which involve running are not allowed.

The vertices of the triangular park ABC are $A(0, 4)$, $B(-2, 0)$ and $C(3, 0)$.

Based on the above information, answer the following questions :

[4 marks]



38(i). Write the equation of the boundary line AB of the park.

[1 mark]

Solution:

Concept:

- The slope of a line through (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$, provided $x_2 \neq x_1$.
- A line through (x_1, y_1) with slope m has point-slope equation $y - y_1 = m(x - x_1)$.

Step 1: Calculate the slope of the boundary The endpoints are $A(0, 4)$ and $B(-2, 0)$. Hence

$$m_{AB} = \frac{0 - 4}{-2 - 0} = 2.$$

Both coordinate differences are negative, so the slope is positive.

Step 2: Use point A to obtain the line Substituting $A(0, 4)$ into the point-slope equation gives $y - 4 = 2(x - 0)$, or $y = 2x + 4$. As a check, putting $x = -2$ gives $y = 0$, so the equation also passes through B.

Step 3: Conclusion The boundary line AB is $y = 2x + 4$, equivalently $2x - y + 4 = 0$.

Quick Tip to Solve: The point A lies on the y-axis, so its ordinate directly gives the intercept 4 after finding the slope.

38(ii). Write the equation of the boundary line AC of the park.

[1 mark]

Solution:

Concept:

- The slope of a line through (x_1, y_1) and (x_2, y_2) is $m = (y_2 - y_1)/(x_2 - x_1)$, provided $x_2 \neq x_1$.
- A line with slope m through (x_1, y_1) has equation $y - y_1 = m(x - x_1)$.

Step 1: Calculate the slope of AC The given endpoints are $A(0, 4)$ and $C(3, 0)$. Therefore,

$$m_{AC} = \frac{0 - 4}{3 - 0} = -\frac{4}{3}.$$

The negative slope agrees with the boundary descending from A to C as the horizontal coordinate increases.

Step 2: Apply the point-slope equation Using point A gives $y - 4 = -\frac{4}{3}(x - 0)$. Multiplying by 3 and collecting the variable terms gives $3y - 12 = -4x$. As a check, both endpoints satisfy $4x + 3y = 12$: at A the left side is 12, and at C it is also 12.

Step 3: Conclusion The equation of the boundary line AC is $4x + 3y = 12$, equivalently $y = 4 - \frac{4}{3}x$.

Quick Tip to Solve: With intercepts 3 and 4, use $x/3 + y/4 = 1$ and multiply throughout by 12.

38(iii)(a). Using integration, find the area of region OAC, in which children are allowed to play cricket, football.

[2 marks]

Solution:

Concept:

- The area between an upper curve $y = f(x)$ and a lower curve $y = g(x)$, for $a \leq x \leq b$, is $\int_a^b [f(x) - g(x)] dx$.
- Here O is the origin, OA lies on the y-axis and OC lies on the x-axis. The sloping boundary AC joins $A(0, 4)$ and $C(3, 0)$.

Step 1: Identify the boundaries and limits The slope of AC is $(0 - 4)/(3 - 0) = -4/3$. Since its y-intercept is 4, its equation is $y = 4 - \frac{4}{3}x$. For a vertical strip in OAC, the lower boundary is $y = 0$ and the upper boundary is AC. The limits are $x = 0$ and $x = 3$. The strip height $4 - \frac{4}{3}x$ is non-negative on this interval.

Step 2: Integrate the strip area Summing the vertical strip areas gives

$$\begin{aligned}\text{Area}(OAC) &= \int_0^3 \left(4 - \frac{4}{3}x\right) dx \\ &= \left[4x - \frac{2}{3}x^2\right]_0^3 \\ &= 12 - 6 = 6.\end{aligned}$$

The triangle check gives $\frac{1}{2} \times OC \times OA = \frac{1}{2}(3)(4) = 6$.

Quick Tip to Solve: Use $x=0$ to $x=3$ for OAC; integrate the AC line above the x-axis over that interval.

Step 3: Conclusion The area of region OAC is 6 square units.

Quick Tip to Solve: Use $x=0$ to $x=3$ for OAC; integrate the AC line above the x-axis over that interval.

OR

38(iii)(b). Using integration, find the area of region AOB.

[2 marks]

Solution:

Concept:

- The area of a region between $y = f(x)$ and the x-axis is $\int_a^b f(x) dx$ when $f(x) \geq 0$ on $[a, b]$.
- The boundary AB is a straight line through $A(0, 4)$ and $B(-2, 0)$. The limits must follow the x-coordinates in increasing order.

Step 1: Determine the line and the region The slope of AB is $(4 - 0)/(0 - (-2)) = 2$. Passing through A, the line is $y - 4 = 2x$, or $y = 2x + 4$. Region AOB has OB on the x-axis and OA on the y-axis. Hence, for $-2 \leq x \leq 0$, a vertical strip has its lower edge at $y = 0$ and its upper edge on AB. Its non-negative height is $2x + 4$.

Step 2: Evaluate the area integral Integrating the strip height over the interval gives

$$\begin{aligned}\text{Area}(AOB) &= \int_{-2}^0 (2x + 4) dx \\ &= [x^2 + 4x]_{-2}^0 \\ &= 0 - (4 - 8) = 4.\end{aligned}$$

The triangle check gives $\frac{1}{2} \times OB \times OA = \frac{1}{2}(2)(4) = 4$.

Step 3: Conclusion The area of region AOB is 4 square units.

Quick Tip to Solve: For the left-hand region, integrate $2x+4$ from -2 to 0 ; negative x does not mean negative strip height.