

CBSE Class 12 Mathematics Question Paper 2026 with Solution PDF-Set 65-3-2

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D and E**.
- (iii) In **Section A**, Questions no. **1 to 18** are multiple choice questions (MCQs) and questions number **19 and 20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B**, Questions no. **21 to 25** are very short answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C**, Questions no. **26 to 31** are short answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D**, Questions no. **32 to 35** are long answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E**, Questions no. **36 to 38** are case study based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 3 questions in Section E.
- (ix) Use of calculator is not allowed.

Section A (20 marks)

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $\tan^{-1} x = y$, then $\frac{dy}{dx}$ is equal to :

[1 mark]

- (A) $(\sec^{-1} x)^2$
- (B) $\sec^2 y$
- (C) $\frac{1}{\sqrt{1+x^2}}$
- (D) $\cos^2 y$

Correct Answer: (D) $\cos^2 y$

Solution:

Concept:

- The inverse relation $y = \tan^{-1} x$ gives $x = \tan y$, with $-\pi/2 < y < \pi/2$.
- Implicit differentiation uses the chain rule: $\frac{d}{dx} \tan y = \sec^2 y \frac{dy}{dx}$.

Step 1: Differentiate the inverse relation Differentiate $x = \tan y$ with respect to x , remembering that y is a function of x :

$$1 = \sec^2 y \frac{dy}{dx}.$$

Step 2: Compare the possible derivatives Dividing by $\sec^2 y$ gives $\frac{dy}{dx} = \cos^2 y = \frac{1}{1+x^2}$. Option (B) omits the reciprocal. Option (C) has a square root and is not this derivative. The inverse-secant expression in (A) is unrelated to differentiating the inverse tangent.

Step 3: Conclusion $\frac{dy}{dx} = \cos^2 y$. Hence, the correct option is (D).

Quick Tip to Solve: Convert $y = \tan^{-1} x$ to $x = \tan y$; the derivative with respect to x is the reciprocal of $\sec^2 y$.

2. The rate of change of volume of a sphere with respect to its diameter, when its radius is 5 cm, is :

[1 mark]

- (A) $400\pi \text{ cm}^3/\text{cm}$
- (B) $100\pi \text{ cm}^3/\text{cm}$
- (C) $50\pi \text{ cm}^3/\text{cm}$
- (D) $25\pi \text{ cm}^3/\text{cm}$

Correct Answer: (C) $50\pi \text{ cm}^3/\text{cm}$

Solution:

Concept:

- For a sphere of radius r , the volume is $V = \frac{4}{3}\pi r^3$.

Quick Tip to Solve: When differentiating sphere volume with respect to diameter, halve $4\pi r^2$ before substituting the radius.

- The diameter is $d = 2r$, so $\frac{dr}{dd} = \frac{1}{2}$ and $\frac{dV}{dd} = \frac{dV}{dr} \frac{dr}{dd}$.

Step 1: Differentiate with respect to diameter The required independent variable is the diameter, not the radius. Apply the chain rule:

$$\frac{dV}{dd} = 4\pi r^2 \cdot \frac{1}{2} = 2\pi r^2.$$

Step 2: Insert the given radius At $r = 5$ cm, the diameter is 10 cm, and

$$\left. \frac{dV}{dd} \right|_{r=5} = 2\pi(5)^2 = 50\pi \text{ cm}^3/\text{cm}.$$

Option (B) gives the rate with respect to radius, while (A) also wrongly substitutes the diameter for the radius. Option (D) loses an additional factor of two.

Step 3: Conclusion The required rate is $50\pi \text{ cm}^3/\text{cm}$. Hence, the correct option is (C).

Quick Tip to Solve: When differentiating sphere volume with respect to diameter, halve $4\pi r^2$ before substituting the radius.

3. $\int \frac{dx}{\sqrt{e^{-2x} - 1}}$ is equal to :

[1 mark]

- (A) $\sin^{-1} e^{-x} + C$
 (B) $\log|e^{-x} + \sqrt{e^{-2x} - 1}| + C$
 (C) $\sin^{-1} e^x + C$
 (D) $\log|e^{-x} - \sqrt{e^{-2x} - 1}| + C$

Correct Answer: (C) $\sin^{-1} e^x + C$

Solution:

Concept:

- For real values of the integrand, $e^{-2x} - 1 > 0$, so $x < 0$.
- The standard integral is $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$, where $|u| < 1$.

Step 1: Rewrite the square root Since $e^x > 0$, factor the radicand without changing signs:

$$\sqrt{e^{-2x} - 1} = e^{-x} \sqrt{1 - e^{2x}}.$$

Thus the integral becomes $\int \frac{e^x dx}{\sqrt{1 - e^{2x}}}$.

Step 2: Use an exponential substitution Put $u = e^x$, so $du = e^x dx$. Then

$$I = \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C.$$

Quick Tip to Solve: For $\sqrt{e^{-2x} - 1}$, factor out e^{-x} first; the remaining numerator $e^x dx$ matches $d(e^x)$.

Option (A) is not real on $x < 0$. Put $t = e^{-x}$. The logarithmic options have derivatives

$$\text{Option (B): } -\frac{t}{\sqrt{t^2 - 1}},$$

$$\text{Option (D): } \frac{t}{\sqrt{t^2 - 1}}.$$

Neither equals the required integrand $1/\sqrt{t^2 - 1}$.

Step 3: Conclusion $I = \sin^{-1}(e^x) + C$. Hence, the correct option is (C).

Quick Tip to Solve: For $\sqrt{e^{-2x} - 1}$, factor out e^{-x} first; the remaining numerator $e^x dx$ matches $d(e^x)$.

4. $\int_{-1}^1 (1 - |x|) dx$ is equal to :

[1 mark]

(A) $2 \int_0^1 (1 + x) dx$

(B) $2 \int_{-1}^0 (1 + x) dx$

(C) 0

(D) $2 \int_{-1}^0 (1 - x) dx$

Correct Answer: (B) $2 \int_{-1}^0 (1 + x) dx$

Solution:

Concept:

- The modulus satisfies $|x| = -x$ for $x \leq 0$ and $|x| = x$ for $x \geq 0$.
- For an even function f , integrals over the two halves of $[-a, a]$ are equal.

Step 1: Use symmetry and remove the modulus The function $1 - |x|$ is even. On $[-1, 0]$, it equals $1 + x$, so

$$\int_{-1}^1 (1 - |x|) dx = 2 \int_{-1}^0 (1 + x) dx.$$

Step 2: Check the competing expressions The displayed expression evaluates to

$$2 \left[x + \frac{x^2}{2} \right]_{-1}^0 = 1.$$

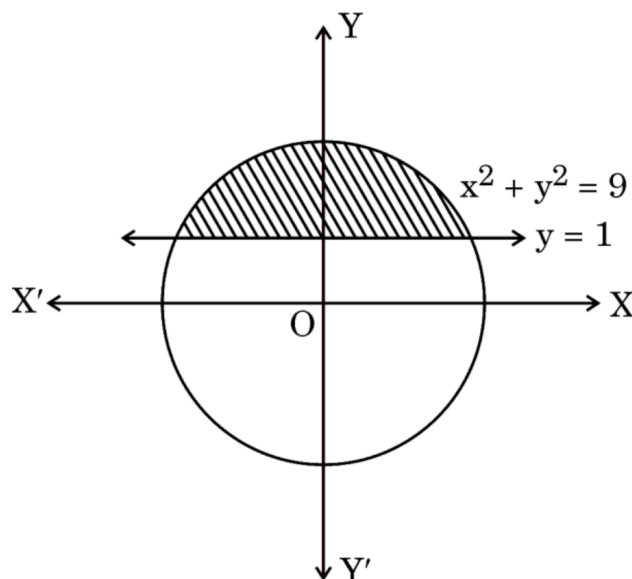
Option (A) equals $2(1 + 1/2) = 3$, and option (D) also equals 3, because it uses the wrong modulus sign on the negative interval. Option (C) is zero, but the positive areas on the two sides add rather than cancel.

Step 3: Conclusion The required expression is $2 \int_{-1}^0 (1 + x) dx$. Hence, the correct option is (B).

Quick Tip to Solve: For a modulus integral, split at zero or exploit evenness; on the negative half-axis replace $|x|$ by $-x$.

5. The area of the shaded region of the circle given below is equal to :

[1 mark]



- (A) $\int_1^3 \sqrt{9 - y^2} dy$
 (B) $2 \int_1^3 \sqrt{9 - y^2} dy$
 (C) $\int_0^3 \sqrt{9 - x^2} dx$
 (D) $2 \int_0^3 \sqrt{9 - x^2} dx$

Correct Answer: (B) $2 \int_1^3 \sqrt{9 - y^2} dy$

Solution:

Concept:

- The circle in the figure is $x^2 + y^2 = 9$, so at height y , its two boundary points have $x = \pm\sqrt{9 - y^2}$.
- An area computed using horizontal strips is $A = \int (x_{\text{right}} - x_{\text{left}}) dy$.

Step 1: Identify the strip width and limits The shaded cap lies above $y = 1$ and below the top of the circle at $y = 3$. Each horizontal strip extends across both sides of the vertical axis, with width

$$\sqrt{9 - y^2} - (-\sqrt{9 - y^2}) = 2\sqrt{9 - y^2}.$$

Step 2: Form the area integral and compare Integrating these widths from $y = 1$ to $y = 3$ gives

$$A = 2 \int_1^3 \sqrt{9 - y^2} dy.$$

Option (A) counts only half of the shaded cap. Option (C) is a quarter-circle area, while (D) is a semicircle area; neither uses the lower boundary $y = 1$ shown for the shaded region.

Quick Tip to Solve: For a circular cap above a horizontal line, use horizontal strips and subtract the left boundary from the right boundary.

Step 3: Conclusion The shaded area is $2 \int_1^3 \sqrt{9 - y^2} dy$. Hence, the correct option is (B).

Quick Tip to Solve: For a circular cap above a horizontal line, use horizontal strips and subtract the left boundary from the right boundary.

6. $\frac{dy}{dx} = F(x, y)$ will be a homogeneous differential equation for which of the following functions ?

(i) $F(x, y) = 3x + 2y$

(ii) $F(x, y) = \sin \frac{y}{x} + \log y - \log x$

(iii) $F(x, y) = e^{y/x} + 1$

(iv) $F(x, y) = \sqrt{x^2 + y^2} - y$

[1 mark]

(A) (i) and (ii)

(B) (i), (ii) and (iii)

(C) $\frac{5}{3}$

(D) 7

Correct Answer: Only (ii) and (iii); none of the printed options is defensible.

Solution:

Concept:

- The equation $dy/dx = F(x, y)$ is homogeneous when F is homogeneous of degree zero: $F(tx, ty) = F(x, y)$ for admissible scaling.
- Equivalently, the right-hand side must be expressible as a function of the ratio y/x alone.

Step 1: Test the functions containing ratios On its logarithmic domain $x > 0, y > 0$, expression (ii) can be written as

$$F(x, y) = \sin(y/x) + \log(y/x).$$

It depends only on y/x . Expression (iii), $e^{y/x} + 1$, also depends only on this ratio. Both therefore give homogeneous differential equations.

Step 2: Test the remaining functions and the printed choices Under positive scaling, (i) becomes $3tx + 2ty = t(3x + 2y)$, and (iv) becomes $\sqrt{t^2x^2 + t^2y^2} - ty = t(\sqrt{x^2 + y^2} - y)$. These are degree-one functions, not degree-zero functions, so neither qualifies. Options (A) and (B) incorrectly include (i). The printed options (C) and (D) are the numerical values $5/3$ and 7 , respectively; they do not name any selection of the listed functions. Thus none of the four printed choices expresses the required combination.

Step 3: Conclusion Only functions (ii) and (iii) give homogeneous differential equations. None of the printed options is defensible.

Quick Tip to Solve: For $dy/dx = F(x, y)$, check degree zero, not merely that all terms share some degree; degree one does not qualify.

7. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ?

[1 mark]

- (A) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$
(B) $|\vec{a} + \vec{b}| \geq |\vec{a}| + |\vec{b}|$
(C) $|\vec{a} - \vec{b}| = |\vec{a}| - |\vec{b}|$
(D) $|\vec{a} \times \vec{b}| \geq |\vec{a}||\vec{b}|$

Correct Answer: (A) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$

Solution:

Concept:

- For nonzero vectors making an angle θ , $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$, with $-1 \leq \cos \theta \leq 1$.
- The triangle inequality gives $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$; the cross-product magnitude is $|\vec{a}||\vec{b}| \sin \theta$.

Step 1: Bound the dot product Since $\cos \theta \leq 1$, multiplying by the nonnegative product of magnitudes yields

$$\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|.$$

If either vector is zero, both sides vanish, so the inequality remains valid.

Step 2: Reject claims that are not universal Take perpendicular unit vectors. Their sum and difference both have magnitude $\sqrt{2}$. Thus (B) would require $\sqrt{2} \geq 2$, and (C) would require $\sqrt{2} = 0$, both false. For parallel unit vectors, the cross product is zero, whereas the product of magnitudes is one; hence (D) is also false in general.

Step 3: Conclusion $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$ always holds. Hence, the correct option is (A).

Quick Tip to Solve: Test universal vector claims with perpendicular or parallel unit vectors; these quickly expose reversed inequalities and false equalities.

8. If $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 198$ and $|\vec{a}| = 10|\vec{b}|$, then :

[1 mark]

- (A) $|\vec{a}| = \sqrt{2}$
(B) $|\vec{b}| = \sqrt{2}$
(C) $|\vec{b}| = 10\sqrt{2}$
(D) $|\vec{a}| = \frac{10}{\sqrt{2}}$

Correct Answer: (B) $|\vec{b}| = \sqrt{2}$

Solution:

Concept:

- Dot-product distributivity and symmetry imply $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$.
- Vector magnitudes are nonnegative, so the positive square root is taken when recovering a magnitude from its square.

Quick Tip to Solve: Expand the dot product as a difference of squared magnitudes before inserting the factor ten; that factor becomes one hundred when squared.

Step 1: Expand and cancel the mixed products Using $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$, we obtain

$$|\vec{a}|^2 - |\vec{b}|^2 = 198.$$

Let $|\vec{b}| = b \geq 0$. The given relation becomes $|\vec{a}| = 10b$.

Step 2: Solve for the two magnitudes Substitute the magnitude relation and simplify:

$$100b^2 - b^2 = 198, \quad 99b^2 = 198, \quad b^2 = 2.$$

Hence $b = \sqrt{2}$ and $|\vec{a}| = 10\sqrt{2}$. Options (A) and (C) attach a magnitude to the wrong vector, and (D) gives $10/\sqrt{2}$, which does not equal the required magnitude of $\vec{a}$.

Step 3: Conclusion $|\vec{b}| = \sqrt{2}$ and $|\vec{a}| = 10\sqrt{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Expand the dot product as a difference of squared magnitudes before inserting the factor ten; that factor becomes one hundred when squared.

9. If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of lines L_1 and L_2 respectively and θ is the acute angle between them, then : [1 mark]

(A) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

(B) $\sin \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

(C) $\tan \theta = \frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$

(D) $\cos \theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$

Correct Answer: (D) $\cos \theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$

Solution:

Concept:

- Direction cosines form a unit direction vector: $\mathbf{u} = (l_1, m_1, n_1)$ and $\mathbf{v} = (l_2, m_2, n_2)$, with $|\mathbf{u}| = |\mathbf{v}| = 1$.
- The dot product gives the cosine of the angle between chosen directions. The acute angle between lines uses its absolute value.

Step 1: Apply the dot-product formula If ϕ is the angle between the chosen unit vectors, then

$$\cos \phi = \mathbf{u} \cdot \mathbf{v} = l_1 l_2 + m_1 m_2 + n_1 n_2.$$

Reversing the direction of one line reverses the sign of this dot product without changing the line.

Step 2: Choose the acute angle For the acute angle θ , the cosine is positive, so $\cos \theta = |\cos \phi|$. Option (A) omits the necessary absolute value and can give a negative cosine. Option (B) uses sine instead of cosine, while the sum of ratios in (C) is not an angle formula and may even be undefined.

Step 3: Conclusion $\cos \theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$. Hence, the correct option is (D).

Quick Tip to Solve: For the acute angle between lines, take the absolute value of the dot product of their unit direction vectors.

10. Direction ratios of lines l_1 and l_2 respectively are $\langle 1, -2, 3 \rangle$ and $\langle -2, p, -6 \rangle$. The value of p for which $l_1 \parallel l_2$ is : [1 mark]

- (A) -4
- (B) 4
- (C) -10
- (D) 10

Correct Answer: (B) 4

Solution:

Concept:

- Two lines are parallel when their nonzero direction vectors are scalar multiples of one another.
- If $(a_2, b_2, c_2) = k(a_1, b_1, c_1)$, the same nonzero multiplier k must work for all three coordinates.

Step 1: Find the common multiplier Compare the first and third coordinates of the two direction vectors:

$$k = \frac{-2}{1} = -2, \quad \frac{-6}{3} = -2.$$

Both comparisons give the same multiplier, so the second vector must be -2 times the first.

Step 2: Check the second coordinate The second coordinates must obey

$$p = (-2)(-2) = 4.$$

This yields $(-2, 4, -6) = -2(1, -2, 3)$. The negative multiplier means opposite chosen directions along parallel lines. The values $-4, -10, 10$ do not give the required multiplier in the second coordinate, so the other options fail.

Step 3: Conclusion The required value is $p = 4$. Hence, the correct option is (B).

Quick Tip to Solve: Use two known coordinate ratios to find the multiplier, then apply that multiplier to the unknown coordinate.

11. If E and F are two independent events such that $P(E) = \frac{3}{10}$, $P(E \cup F) = \frac{1}{2}$, then $P(E|F) - P(F|E)$ is equal to : [1 mark]

- (A) $\frac{2}{7}$
(B) $\frac{3}{35}$
(C) $\frac{1}{70}$
(D) $\frac{1}{7}$

Correct Answer: (C) $\frac{1}{70}$

Solution:

Concept:

- For independent events, $P(E \cap F) = P(E)P(F)$, and conditioning on either event does not change the other event's probability.
- The addition rule is $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

Step 1: Calculate the probability of the second event Put $P(F) = p$. Independence and the addition rule give

$$\begin{aligned}\frac{1}{2} &= \frac{3}{10} + p - \frac{3}{10}p, \\ \frac{1}{5} &= \frac{7}{10}p, \\ p &= \frac{2}{7}.\end{aligned}$$

Both event probabilities are positive, so the conditional probabilities are defined.

Step 2: Use independence in the conditional probabilities We have $P(E | F) = P(E) = 3/10$ and $P(F | E) = P(F) = 2/7$. Thus

$$P(E | F) - P(F | E) = \frac{3}{10} - \frac{2}{7} = \frac{21 - 20}{70}.$$

Option (A) is merely $P(F)$, and option (B) is $P(E \cap F) = 3/35$; neither is the requested difference. Option (D) also disagrees with the subtraction.

Step 3: Conclusion The required difference is $\frac{1}{70}$. Hence, the correct option is (C).

Quick Tip to Solve: For independent events, first replace each conditional probability by the corresponding unconditional probability.

12. The domain of $\sin^{-1}(1 - 2x)$ is :

[1 mark]

- (A) $[-1, 1]$
- (B) $[-1, 3]$
- (C) $[-2, 2]$
- (D) $[0, 1]$

Correct Answer: (D) $[0, 1]$

Solution:

Concept:

- The real-valued function $\sin^{-1} t$ is defined exactly for $-1 \leq t \leq 1$.
- For a composite inverse-sine function, impose this restriction on its entire argument, not directly on x .

Step 1: Restrict the argument of inverse sine The argument here is $1 - 2x$, so we require

$$-1 \leq 1 - 2x \leq 1.$$

Subtracting 1 from every part gives $-2 \leq -2x \leq 0$.

Step 2: Solve and check the inequalities Division by -2 reverses both inequality signs:

$$1 \geq x \geq 0.$$

At $x = 0$ and $x = 1$, the arguments are 1 and -1 , respectively, and inverse sine is defined at both. Each other proposed interval contains $x = -1$, giving argument 3, which lies outside the real domain of inverse sine.

Step 3: Conclusion The domain is $[0, 1]$. Hence, the correct option is (D).

Quick Tip to Solve: Write the double inequality for the full argument; reverse the inequality signs when dividing by a negative number.

13. If $A^2 = 4A + 3I$ and $A^{-1} = xA + yI$, then the value of $(x + y)$ is :

[1 mark]

- (A) -1
- (B) 1
- (C) $\frac{5}{3}$
- (D) 7

Correct Answer: (A) -1

Solution:

Concept:

- If A is invertible, multiplying a polynomial identity in A by A^{-1} reduces every positive power by one.
- The identity matrix satisfies $IA = AI = A$. An expression C is the inverse of A when $AC = CA = I$.

Quick Tip to Solve: Rearrange the polynomial to isolate a multiple of I , then factor out A to read off its inverse.

Step 1: Rearrange the given matrix identity From $A^2 = 4A + 3I$, we obtain

$$A^2 - 4A = 3I, \quad A \left(\frac{A - 4I}{3} \right) = I.$$

Since $(A - 4I)/3$ is a polynomial in A , it commutes with A , and its product with A on the other side is also I .

Step 2: Express the inverse in the required form Therefore

$$A^{-1} = \frac{1}{3}A - \frac{4}{3}I.$$

In the required polynomial form, take $x = 1/3$ and $y = -4/3$, giving $x + y = (1 - 4)/3$. Option (C) results from incorrectly using a plus sign before $4I$; options (B) and (D) do not match this sum.

Step 3: Conclusion $x + y = -1$. Hence, the correct option is (A).

Quick Tip to Solve: Rearrange the polynomial to isolate a multiple of I , then factor out A to read off its inverse.

14. If A and B are skew-symmetric matrices of same order, then $AB' + BA'$ is a/an : [1 mark]

- (A) symmetric matrix
- (B) skew-symmetric matrix
- (C) null matrix
- (D) identity matrix

Correct Answer: (A) symmetric matrix

Solution:

Concept:

- For real matrices, $(XY)' = Y'X'$ and $(X + Y)' = X' + Y'$. The prime denotes transpose.
- A matrix M is symmetric if $M' = M$, and skew-symmetric if $M' = -M$.

Step 1: Take the transpose of the entire expression Let $M = AB' + BA'$. Applying the transpose rule to each product gives

$$\begin{aligned} M' &= (AB')' + (BA')' \\ &= BA' + AB'. \end{aligned}$$

Transposing reverses the order of multiplication in each product.

Step 2: Compare the transpose with the original Matrix addition is commutative, so $M' = AB' + BA' = M$. Hence the expression is necessarily symmetric. It need not be null, identity or skew-symmetric: taking $A = B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ gives $AB' + BA' = 2I$, which excludes all three competing descriptions as general conclusions.

Step 3: Conclusion The matrix $AB' + BA'$ is symmetric. Hence, the correct option is (A).

Quick Tip to Solve: Transpose the full sum and reverse the order in each product; two terms that interchange prove symmetry immediately.

15. If a matrix B is such that $B \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{pmatrix}$, then the order of matrix B is : [1 mark]

- (A) 1×3
- (B) 3×1
- (C) 3×3
- (D) 1×1

Correct Answer: (B) 3×1

Solution:

Concept:

- For matrix multiplication, $(m \times n)(n \times p)$ produces an $m \times p$ matrix.
- The number of columns of the first factor must equal the number of rows of the second factor.

Step 1: Use the inner-dimension condition The row matrix $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$ has order 1×3 . If B has order $m \times n$, its product with this row matrix is defined only when $n = 1$. The resulting product then has order $m \times 3$.

Step 2: Match the product dimensions The displayed right-hand side has three rows and three columns, so $m = 3$. Thus dimensional compatibility requires a 3×1 matrix. Options (A) and (C) have three columns and cannot multiply a 1×3 matrix on the left. Option (D) would produce only one row. Strictly, the displayed entries are inconsistent: an outer product with $(1, 2, 3)$ makes each row proportional to $(1, 2, 3)$, whereas $(0, 1, 1)$ is not. The order question nevertheless has the stated unique dimensional choice.

Step 3: Conclusion The required order by matrix multiplication is 3×1 . Hence, the correct option is (B).

Quick Tip to Solve: For an order question, match the inner dimensions first and then read the number of rows from the product.

16. If a square matrix A is such that $A^2 = A$ and $(I - A)^3 = xA + I$, then value of x must be : [1 mark]

- (A) 7
- (B) 5
- (C) -7
- (D) -1

Correct Answer: (D) -1

Solution:

Concept:

- An idempotent matrix satisfies $A^2 = A$. Consequently $A^3 = A^2A = A$.
- The identity matrix commutes with A , so the binomial expansion applies to $(I - A)^3$.

Quick Tip to Solve: For an idempotent matrix, every positive power equals the matrix itself; reduce powers before combining coefficients.

Step 1: Expand the cube Use $I^2 = I$ and $IA = AI = A$ to obtain

$$(I - A)^3 = I - 3A + 3A^2 - A^3.$$

The alternating signs arise from the negative term $-A$.

Step 2: Reduce powers using idempotence Substituting $A^2 = A$ and $A^3 = A$ gives

$$(I - A)^3 = I - 3A + 3A - A = I - A.$$

The coefficient of A in this identity is -1 , matching $xA + I$. Values $7, 5, -7$ do not give this identity for a general idempotent matrix; for example, $A = I$ immediately rules them out. If $A = 0$, the equality alone does not determine x , but -1 is the choice that works for every idempotent matrix.

Step 3: Conclusion The identity has $x = -1$. Hence, the correct option is (D).

Quick Tip to Solve: For an idempotent matrix, every positive power equals the matrix itself; reduce powers before combining coefficients.

17. If $A(\text{adj } A) = \begin{pmatrix} 2026 & 0 & 0 \\ 0 & 2026 & 0 \\ 0 & 0 & 2026 \end{pmatrix}$, then the value of $|\text{adj } A|$ is equal to : [1 mark]

- (A) 2026
- (B) $(2026)^{-1}$
- (C) $(2026)^{-2}$
- (D) $(2026)^2$

Correct Answer: (D) $(2026)^2$

Solution:

Concept:

- For a square matrix A , $A \text{adj } A = |A|I$, where $|A|$ is its determinant and I is the identity matrix of the same order.
- For a matrix of order n , $|\text{adj } A| = |A|^{n-1}$.

Step 1: Identify the determinant and the order The displayed matrix is $2026I_3$. Comparing it with $A \text{adj } A = |A|I_3$ gives $|A| = 2026$, and the order of A is 3.

Step 2: Apply the adjoint determinant formula Substitute $n = 3$ and $|A| = 2026$:

$$|\text{adj } A| = |A|^{3-1} = (2026)^2.$$

Option (A) gives $|A|$ itself. Options (B) and (C) use reciprocal powers, whereas the adjoint determinant requires the positive power $n - 1 = 2$.

Step 3: Conclusion $|\text{adj } A| = (2026)^2$. Hence, the correct option is (D).

Quick Tip to Solve: Read the matrix order before using the adjoint formula: a third-order matrix requires the square of its determinant.

18. The value of k for which the function $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ k(x+1), & x = 0 \end{cases}$ is a continuous function, is :

[1 mark]

- (A) $\frac{1}{4}$
- (B) 2
- (C) $\frac{1}{2}$
- (D) 0

Correct Answer: (D) 0

Solution:

Concept:

- Continuity at $x = 0$ requires $\lim_{x \rightarrow 0} f(x) = f(0)$.
- The sine function is bounded: $|\sin t| \leq 1$. The squeeze theorem therefore controls an oscillatory sine multiplied by a factor tending to zero.

Step 1: Evaluate the limit at the joining point For every $x \neq 0$,

$$-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2.$$

Both bounding functions tend to zero as $x \rightarrow 0$. Hence the two-sided limit of $f(x)$ at zero is 0, even though $\sin(1/x)$ itself has no limit there.

Step 2: Match the limit with the defined value The second branch applies at zero, so $f(0) = k(0+1) = k$. Continuity requires $k = 0$. Away from zero, $x^2 \sin(1/x)$ is already continuous. Each nonzero value in (A), (B) and (C) would make $f(0)$ different from its limit.

Step 3: Conclusion $k = 0$. Hence, the correct option is (D).

Quick Tip to Solve: Bound the oscillating factor by one, find the limit of the product, and only then substitute into the branch defining the value at zero.

19. Assertion (A): The vectors $\vec{a}$ and $(-2\vec{a})$, where $\vec{a} \neq \vec{0}$, are collinear vectors.

Reason (R): $\vec{a} \cdot (-2\vec{a}) = 0$.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- Two nonzero vectors are collinear when one is a scalar multiple of the other; a negative multiplier reverses direction.
- The dot product satisfies $\vec{a} \cdot \vec{a} = |\vec{a}|^2$, which is positive for a nonzero vector.

Step 1: Evaluate the assertion The second vector is -2 times the first. They are parallel in opposite directions and hence collinear. Thus Assertion (A) is true; collinearity does not require the same direction.

Step 2: Evaluate the reason By the scalar-multiplication property of the dot product,

$$\vec{a} \cdot (-2\vec{a}) = -2(\vec{a} \cdot \vec{a}) = -2|\vec{a}|^2 < 0,$$

since $\vec{a} \neq \vec{0}$. Therefore this dot product cannot be zero, and Reason (R) is false.

Step 3: Check the explanatory connection The assertion follows from scalar proportionality, not from a zero dot product. A zero dot product would indicate perpendicular nonzero vectors. Since R is false, it cannot explain A; this eliminates (A) and (B), while the true assertion eliminates (D).

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Use scalar multiples to test collinearity and dot products to test perpendicularity; a negative multiple remains collinear.

20. Assertion (A): One of the particular solutions of the differential equation $\frac{dy}{dx} = e^{x+y}$ can be $e^x + e^{-y} = -2$.

Reason (R): $e^x + e^{-y} = C$ is the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (D) Assertion (A) is false, but Reason (R) is true.

Solution:

Concept:

- A separable differential equation can be integrated after collecting the variables: $e^{-y} dy = e^x dx$.
- A particular real solution must satisfy both the integrated equation and the range restrictions of its functions; every real exponential is strictly positive.

Step 1: Evaluate the assertion using positivity For real x and y , both e^x and e^{-y} are positive. Their sum cannot be -2 , so the stated equation represents no real solution curve. Assertion (A) is false.

Step 2: Derive and evaluate the reason Separate and integrate the given equation:

$$\begin{aligned}e^{-y} dy &= e^x dx, \\ -e^{-y} &= e^x + C_1, \\ e^x + e^{-y} &= C.\end{aligned}$$

This is its general implicit solution, so Reason (R) is true. Its real members satisfy $C > 0$ and $e^x < C$, giving $y = -\ln(C - e^x)$ on their domains.

Step 3: Check whether the reason supports the assertion An integration constant cannot be chosen to produce an impossible real equation. Thus the true general form does not justify setting $C = -2$. Options (A), (B) and (C) all incorrectly declare the assertion true.

Step 4: Conclusion The assertion is false, but the reason is true. Hence, the correct option is (D).

Quick Tip to Solve: Before choosing a constant in an exponential solution, check positivity: a sum of real exponentials can never equal a negative number.

Section B (10 marks)

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21(a). Find the value of $\sin[\cot^{-1} \sqrt{2} (\cos(\tan^{-1} 1))]$.

[2 marks]

Solution:

Concept:

- Principal inverse trigonometric values satisfy $\tan^{-1} 1 = \pi/4$ and $\cot^{-1} 1 = \pi/4$; the principal range of $\cot^{-1}$ is $(0, \pi)$.
- Evaluate a nested trigonometric expression from the innermost function outward. In the given expression the argument of $\cot^{-1}$ is $\sqrt{2} \cos(\tan^{-1} 1)$.

Step 1: Evaluate the inner inverse tangent and cosine Let $\theta = \tan^{-1} 1$. Its principal value lies in $(-\pi/2, \pi/2)$, and the angle in this interval with tangent equal to one is $\pi/4$. Thus

$$\cos(\tan^{-1} 1) = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}.$$

The cosine is positive because this principal angle lies in the first quadrant.

Step 2: Simplify the cotangent argument and evaluate the sine Multiplying the preceding value by $\sqrt{2}$ makes the argument of the inverse cotangent equal to one. Hence

$$\sqrt{2} \cos(\tan^{-1} 1) = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1,$$

$$\cot^{-1}(1) = \frac{\pi}{4},$$

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.$$

The principal inverse-cotangent value is again in the first quadrant; no additional multiple of π is introduced.

Step 3: Conclusion The required value is $\boxed{\frac{1}{\sqrt{2}}}$, equivalently $\frac{\sqrt{2}}{2}$.

Quick Tip to Solve: Work from the inside outward: the factor $\sqrt{2}$ cancels $\cos(\pi/4)$, leaving the familiar angle $\cot^{-1} 1$.

21(b). A relation R on $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (3, 3), (1, 2)\}$. Is R a symmetric relation? Justify. Write the smallest relation set R_1 such that $R \cup R_1$ becomes an equivalence relation on the set $\{1, 2, 3\}$. [2 marks]

Solution:

Concept:

- A relation is symmetric if $(u, v) \in R$ always implies $(v, u) \in R$.
- An equivalence relation must be reflexive, symmetric and transitive on the given set. Reflexivity requires every diagonal pair (u, u) .

Step 1: Test symmetry of the given relation The pair $(1, 2)$ belongs to R , but its reverse $(2, 1)$ does not. Thus the defining condition for symmetry fails, even though the diagonal pairs reverse to themselves. Therefore R is not symmetric.

Step 2: Add the pairs forced by equivalence On $A = \{1, 2, 3\}$, reflexivity requires $(1, 1), (2, 2), (3, 3)$. Only $(2, 2)$ is missing. Symmetry also forces the missing reverse pair $(2, 1)$. Adding these gives

$$S = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}.$$

This relation contains every ordered pair within the class $\{1, 2\}$, and the diagonal pair for the separate class $\{3\}$. Consequently it is reflexive and symmetric. It is transitive because any chain of related elements stays within one of these classes, where its endpoint pair is already present.

Step 3: Verify that the added relation is smallest Both added pairs are necessary: omitting $(2, 2)$ breaks reflexivity, and omitting $(2, 1)$ breaks symmetry. Since these two additions already give an equivalence relation, no additional pair involving 3 and another element is required.

Step 4: Conclusion R is not symmetric. The smallest required relation is $R_1 = \{(2, 2), (2, 1)\}$.

Quick Tip to Solve: Add missing diagonal and reversed pairs first; then use the classes $\{1, 2\}$ and $\{3\}$ to check whether transitivity needs anything more.

22. If for two unit vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + 2\vec{b}| = |2\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$. [2 marks]

Solution:

Concept:

- A unit vector has magnitude one, so $\vec{a} \cdot \vec{a} = \vec{b} \cdot \vec{b} = 1$.
- For the angle θ between two nonzero vectors, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$. Also $|\vec{u}|^2 = \vec{u} \cdot \vec{u}$.

Step 1: Square the given magnitude equation Both sides are nonnegative magnitudes, so squaring preserves their equality. Expand each square with the dot product:

$$\begin{aligned} |\vec{a} + 2\vec{b}|^2 &= |\vec{a}|^2 + 4|\vec{b}|^2 + 4\vec{a} \cdot \vec{b}, \\ |2\vec{a} - \vec{b}|^2 &= 4|\vec{a}|^2 + |\vec{b}|^2 - 4\vec{a} \cdot \vec{b}. \end{aligned}$$

Quick Tip to Solve: Square both magnitudes and compare cross terms: the equal unit-vector terms cancel immediately, forcing the dot product to vanish.

The minus sign in the second vector makes its cross term negative.

Step 2: Use the unit magnitudes to obtain the angle Substituting $|\vec{a}| = |\vec{b}| = 1$ into the equality gives

$$\begin{aligned}5 + 4\vec{a} \cdot \vec{b} &= 5 - 4\vec{a} \cdot \vec{b}, \\8\vec{a} \cdot \vec{b} &= 0, \\\cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = 0.\end{aligned}$$

The angle between vectors is chosen in $[0, \pi]$, so the unique angle satisfying this condition is $\pi/2$. As a check, perpendicular unit vectors make both original magnitudes equal to $\sqrt{5}$.

Step 3: Conclusion The angle between the vectors is 90° , or $\pi/2$ radians.

Quick Tip to Solve: Square both magnitudes and compare cross terms: the equal unit-vector terms cancel immediately, forcing the dot product to vanish.

23(a). If the lines $\frac{x-3}{1} = \frac{1-y}{1} = \frac{z+2}{p}$ and $\frac{2-x}{3} = \frac{y+1}{5} = \frac{z+56}{2p}$ are perpendicular to each other, then find the value(s) of p . [2 marks]

Solution:

Concept:

- Direction ratios are the coefficients of the parameter in the parametric equations of a line; a reversed numerator changes the corresponding sign.
- Two directions (a_1, b_1, c_1) and (a_2, b_2, c_2) are perpendicular when their scalar product $a_1a_2 + b_1b_2 + c_1c_2$ is zero.

Step 1: Identify the signed direction ratios Equate the first line's ratios to t . Then $x = 3 + t$, $y = 1 - t$, and $z = -2 + pt$, so its direction ratios are $(1, -1, p)$. Similarly, equating the second line's ratios to s gives $x = 2 - 3s$, $y = -1 + 5s$, and $z = -56 + 2ps$. Thus its direction ratios are $(-3, 5, 2p)$. The constant coordinates locate the lines but do not determine their directions.

Step 2: Apply the perpendicularity condition Set the scalar product of the two direction vectors equal to zero:

$$\begin{aligned}(1)(-3) + (-1)(5) + p(2p) &= 0, \\-3 - 5 + 2p^2 &= 0, \\p^2 &= 4.\end{aligned}$$

Both square roots are admissible, since neither makes the printed denominators involving p zero.

Step 3: Conclusion The required values are $p = 2$ or $p = -2$.

Quick Tip to Solve: Read $1 - y$ as giving direction ratio -1 and $2 - x$ as giving -3 ; ignoring these signs changes the perpendicularity equation.

23(b). Find the vector equation of a line passing through the origin and perpendicular to both the lines $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and $\vec{r} = \mu(\hat{i} - \hat{j} + \hat{k})$. [2 marks]

Solution:

Concept:

- In $\vec{r} = \vec{a} + \lambda\vec{b}$, the vector $\vec{b}$ gives the direction of the line, while $\vec{a}$ locates a point on it.
- The cross product of two non-parallel direction vectors is perpendicular to both. A line through the origin with direction $\vec{d}$ has equation $\vec{r} = t\vec{d}$, where $t \in \mathbb{R}$.

Step 1: Calculate a common perpendicular direction The given direction vectors are $\vec{b}_1 = 3\hat{i} + 4\hat{j} + 2\hat{k}$ and $\vec{b}_2 = \hat{i} - \hat{j} + \hat{k}$. Their cross product is

$$\begin{aligned}\vec{d} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 1 & -1 & 1 \end{vmatrix} \\ &= \hat{i}(4 + 2) - \hat{j}(3 - 2) + \hat{k}(-3 - 4) \\ &= 6\hat{i} - \hat{j} - 7\hat{k}.\end{aligned}$$

This is nonzero, so it defines the required direction.

Step 2: Use the origin and verify perpendicularity The point vector of the origin is $\vec{0}$, so use $\vec{r} = \vec{0} + t\vec{d}$. To check the direction, compute

$$\vec{d} \cdot \vec{b}_1 = 18 - 4 - 14 = 0, \quad \vec{d} \cdot \vec{b}_2 = 6 + 1 - 7 = 0.$$

Thus the proposed line has zero scalar product with each given direction, and setting $t = 0$ places it at the origin.

Step 3: Conclusion The required vector equation is $\boxed{\vec{r} = t(6\hat{i} - \hat{j} - 7\hat{k}), t \in \mathbb{R}}$.

Quick Tip to Solve: Use only the coefficients of the parameters in the cross product; the constant vector of the first line is not its direction vector.

24. If $x = e^{t+\frac{1}{t}}$ and $y = e^{t-\frac{1}{t}}$, then find $\frac{dy}{dx}$ at $t = -2$. [2 marks]

Solution:

Concept:

- For functions $x = x(t)$ and $y = y(t)$, the derivative is $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, provided $dx/dt \neq 0$.
- Apply the chain rule $\frac{d}{dt}e^{u(t)} = e^{u(t)}u'(t)$ and the reciprocal derivative $\frac{d}{dt}(1/t) = -1/t^2$.

Step 1: Differentiate both parametric expressions Differentiate the exponents as well as the exponential functions. Since the reciprocal terms have opposite signs, their derivative contributions also have

Quick Tip to Solve: Subtract the exponents before substituting: $(t - 1/t) - (t + 1/t) = -2/t$, which equals 1 at $t = -2$.

opposite signs:

$$\frac{dx}{dt} = e^{t+1/t} \left(1 - \frac{1}{t^2}\right), \quad \frac{dy}{dt} = e^{t-1/t} \left(1 + \frac{1}{t^2}\right).$$

At $t = -2$, $dx/dt = \frac{3}{4}e^{-5/2} \neq 0$, so the required quotient is defined.

Step 2: Form the quotient and evaluate it Divide the derivatives, subtracting the exponential powers:

$$\frac{dy}{dx} = e^{-2/t} \frac{1 + 1/t^2}{1 - 1/t^2} = e^{-2/t} \frac{t^2 + 1}{t^2 - 1}.$$

Now $-2/t = 1$ and $t^2 = 4$ at the stated parameter value. The exponential factor is therefore e , while the rational factor is $(4 + 1)/(4 - 1) = 5/3$.

Step 3: Conclusion Hence $\boxed{\frac{dy}{dx} \Big|_{t=-2} = \frac{5e}{3}}.$

Quick Tip to Solve: Subtract the exponents before substituting: $(t - 1/t) - (t + 1/t) = -2/t$, which equals 1 at $t = -2$.

25. Find the sub-interval of $(0, \infty)$ in which $f(x) = x^2 e^{-x}$ is increasing.

[2 marks]

Solution:

Concept:

- A differentiable function is strictly increasing on an interval where its derivative is positive. To find the interval, determine where the sign of $f'(x)$ changes.
- Use the product rule $(uv)' = u'v + uv'$ and $\frac{d}{dx}e^{-x} = -e^{-x}$. An exponential is positive for every real input.

Step 1: Differentiate and factor the expression In $f(x) = x^2 e^{-x}$, both factors depend on x , so differentiate them using the product rule:

$$\begin{aligned} f'(x) &= 2xe^{-x} + x^2(-e^{-x}) \\ &= xe^{-x}(2 - x). \end{aligned}$$

Factoring makes the sign analysis simpler than comparing the two original derivative terms separately.

Step 2: Determine the sign on the given domain The domain under consideration is $(0, \infty)$. Here $x > 0$ and $e^{-x} > 0$, so only $2 - x$ determines the derivative's sign. Thus

$$f'(x) > 0 \iff 2 - x > 0 \iff x < 2.$$

Combining this with $x > 0$ gives $0 < x < 2$. At $x = 2$, the derivative vanishes, and for $x > 2$ it is negative, so the function decreases after that stationary point.

Step 3: Conclusion The required interval of increase is $\boxed{(0, 2)}$.

Quick Tip to Solve: Factor $f'(x)$ as $xe^{-x}(2 - x)$; on the positive axis the first two factors cannot change sign, so inspect only $2 - x$.

Section C (18 marks)

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26(a). Find : $\int \frac{\cos x}{(2 + \sin x)(4 + \sin x)} dx$

[3 marks]

Solution:

Concept:

- When a rational expression in $\sin x$ is multiplied by $\cos x dx$, use $u = \sin x$ and $du = \cos x dx$ to reduce the integral to a rational function of u .
- Distinct linear denominator factors can be separated by partial fractions. Integrate each resulting term using $\int \frac{du}{u+a} = \ln |u+a| + C$, where C is an arbitrary constant.

Step 1: Substitute for the sine function Denote the required integral by I . Put $u = \sin x$, so that the entire numerator differential is $du = \cos x dx$. The integral becomes

$$I = \int \frac{du}{(u+2)(u+4)}.$$

This substitution leaves two distinct linear factors in the denominator, which is the standard setting for partial fractions.

Step 2: Determine the partial-fraction constants Write

$$\frac{1}{(u+2)(u+4)} = \frac{A}{u+2} + \frac{B}{u+4}.$$

After multiplication by $(u+2)(u+4)$, the identity is $1 = A(u+4) + B(u+2)$. Substituting $u = -2$ gives $A = 1/2$, and substituting $u = -4$ gives $B = -1/2$. These substitutions determine the coefficients in the polynomial identity; they do not restrict the original variable x .

Step 3: Integrate the separated terms Apply the logarithmic integration formula to each term:

$$\begin{aligned} I &= \frac{1}{2} \int \frac{du}{u+2} - \frac{1}{2} \int \frac{du}{u+4} \\ &= \frac{1}{2} \ln |u+2| - \frac{1}{2} \ln |u+4| + C. \end{aligned}$$

Finally replace u by $\sin x$ and combine the logarithms. For real x , both $2 + \sin x$ and $4 + \sin x$ are positive, so the absolute-value signs may be omitted.

Step 4: Conclusion

$$\int \frac{\cos x}{(2 + \sin x)(4 + \sin x)} dx = \frac{1}{2} \ln \left(\frac{2 + \sin x}{4 + \sin x} \right) + C.$$

Quick Tip to Solve: The factors differ by 2, so use $\frac{1}{(u+2)(u+4)} = \frac{1}{2} \left(\frac{1}{u+2} - \frac{1}{u+4} \right)$ directly after substitution.

26(b). Find : $\int \frac{x+3}{x^2+4x+5} dx$

[3 marks]

Solution:**Concept:**

- For a quadratic denominator, split the numerator into a multiple of its derivative and a constant. The derivative term integrates by $\int f'(x)/f(x) dx = \ln |f(x)| + C$.
- Completing the square gives the remaining integral the standard form $\int du/(1+u^2) = \tan^{-1} u + C$, where u is the shifted variable.

Step 1: Match the derivative of the denominator Write $I = \int (x+3)/(x^2+4x+5) dx$. The denominator has derivative $2x+4$, so express the numerator as

$$x+3 = \frac{1}{2}(2x+4) + 1.$$

Therefore linearity of integration gives

$$I = \frac{1}{2} \int \frac{2x+4}{x^2+4x+5} dx + \int \frac{dx}{x^2+4x+5}.$$

This split keeps the extra constant term; omitting it would miss the inverse-tangent contribution.

Step 2: Integrate the two standard forms The first integral is logarithmic. For the second, complete the square:

$$x^2+4x+5 = (x+2)^2+1.$$

With $u = x+2$, we have $du = dx$, hence

$$\begin{aligned} \frac{1}{2} \int \frac{2x+4}{x^2+4x+5} dx &= \frac{1}{2} \ln(x^2+4x+5), \\ \int \frac{dx}{(x+2)^2+1} &= \tan^{-1}(x+2). \end{aligned}$$

The logarithm needs no absolute-value sign because $(x+2)^2+1 > 0$ for every real x . As a check, the derivatives of these terms combine to $(x+2+1)/(x^2+4x+5)$, the given integrand.

Step 3: Conclusion

$$\int \frac{x+3}{x^2+4x+5} dx = \frac{1}{2} \ln(x^2+4x+5) + \tan^{-1}(x+2) + C.$$

Here C is an arbitrary real constant.

Quick Tip to Solve: First write $x+3 = (x+2)+1$: the $x+2$ part gives half a logarithm, while the remaining 1 gives an inverse tangent after completing the square.

27(a). Find the general solution of the differential equation $(x^2 + y^2) dy = xy dx$.

[3 marks]

Solution:

Concept:

- A first-order differential equation is homogeneous if its derivative is a function of a ratio of the variables. Treating x as a function of y gives $dx/dy = x/y + y/x$.
- For a homogeneous equation in dx/dy , put $x = vy$, where v depends on y , so that $dx/dy = v + y dv/dy$. The resulting equation can be separated and integrated.

Step 1: Substitute the ratio of the variables On a region where $xy \neq 0$, rewrite the given equation as

$$\frac{dx}{dy} = \frac{x^2 + y^2}{xy} = \frac{x}{y} + \frac{y}{x}.$$

Put $x = vy$, so $x/y = v$. Applying the product rule and substituting gives

$$v + y \frac{dv}{dy} = v + \frac{1}{v}.$$

The common v terms cancel. Multiplying the remaining equation by v and dividing by y separates the variables:

$$v dv = \frac{dy}{y}.$$

Step 2: Integrate and restore the original variables Integrate both sides to obtain

$$\frac{v^2}{2} = \ln |y| + C_1.$$

Since $v = x/y$, multiplying by 2 and absorbing $2C_1$ into a new arbitrary constant C gives

$$\frac{x^2}{y^2} - 2 \ln |y| = C, \quad y \neq 0.$$

To check, differentiation with respect to x , followed by multiplication by $y^3/2$, yields

$$xy - (x^2 + y^2) \frac{dy}{dx} = 0,$$

exactly the original equation. The original equation also admits $y = 0$, which was excluded when dividing by y .

Step 3: Conclusion The general nonzero solution is

$$\boxed{\frac{x^2}{y^2} - 2 \ln |y| = C} \quad \text{or} \quad \boxed{x^2 = y^2(2 \ln |y| + C)}.$$

The constant solution $y = 0$ is also admitted.

Quick Tip to Solve: Use $x = vy$ here: in $dx/dy = x/y + y/x$, the two v terms cancel immediately, leaving $v dv = dy/y$.

27(b). Find the particular solution of the differential equation $\frac{dy}{dx} - 3y \cot x = \sin 2x$, given that $y = 2$ when $x = \frac{\pi}{2}$. [3 marks]

Solution:

Concept:

- A linear differential equation $dy/dx + P(x)y = Q(x)$ has integrating factor $IF = e^{\int P(x) dx}$. Multiplication by this factor makes the left side $d(y IF)/dx$.
- Use $\sin 2x = 2 \sin x \cos x$ and $d(\csc x)/dx = -\cos x / \sin^2 x$. The initial value then determines the arbitrary constant.

Step 1: Find and apply the integrating factor Comparing with the linear form gives $P(x) = -3 \cot x$ and $Q(x) = \sin 2x$. On the interval $0 < x < \pi$, which contains the initial point $\pi/2$, the sine is positive, and

$$IF = e^{-3 \int \cot x dx} = e^{-3 \ln(\sin x)} = \csc^3 x.$$

Multiply the differential equation by $\csc^3 x$. By the product rule,

$$\frac{d}{dx}(y \csc^3 x) = \sin 2x \csc^3 x = \frac{2 \cos x}{\sin^2 x}.$$

Step 2: Integrate and impose the initial value Substitute $u = \sin x$, so $du = \cos x dx$, on the right side:

$$\begin{aligned} y \csc^3 x &= 2 \int u^{-2} du \\ &= -\frac{2}{u} + C = -2 \csc x + C. \end{aligned}$$

Thus the solution family is $y = -2 \sin^2 x + C \sin^3 x$. At $x = \pi/2$, the sine equals 1, and the given value $y = 2$ gives

$$2 = -2 + C, \quad C = 4.$$

For verification, differentiating the resulting expression gives $y' = -4 \sin x \cos x + 12 \sin^2 x \cos x$. Subtracting $3y \cot x$ leaves $2 \sin x \cos x = \sin 2x$, as required.

Step 3: Conclusion The required particular solution is

$$\boxed{y = 4 \sin^3 x - 2 \sin^2 x}, \quad 0 < x < \pi.$$

It satisfies both the differential equation and $y(\pi/2) = 2$.

Quick Tip to Solve: After multiplying by $\csc^3 x$, simplify $\sin 2x$ immediately; the right side becomes $-2 d(\csc x)/dx$, making the integration direct.

28. If $(\sin x)^y = y^{\cos x}$, then find $\frac{dy}{dx}$.

[3 marks]

Solution:

Concept:

- Logarithmic differentiation brings a variable exponent down as a multiplier: $\ln(a^b) = b \ln a$, for $a > 0$. Work on a differentiable real branch with $\sin x > 0$ and $y > 0$.
- In implicit differentiation y is a function of x , so $d(\ln y)/dx = y'/y$, where $y' = dy/dx$. Use the product rule on both sides after taking logarithms.

Step 1: Take logarithms of the given equation The equation contains variable exponents on both sides. Taking natural logarithms of $(\sin x)^y = y^{\cos x}$ gives

$$y \ln(\sin x) = \cos x \ln y.$$

This converts the powers to products and avoids treating y as a constant. Both y and $\ln y$ must be differentiated through their dependence on x .

Step 2: Differentiate both products Since $d[\ln(\sin x)]/dx = \cot x$, the product rule yields

$$y' \ln(\sin x) + y \cot x = -\sin x \ln y + \frac{\cos x}{y} y'.$$

In particular, differentiating $\cos x$ contributes a negative sign, while differentiating $\ln y$ contributes the factor y'/y .

Step 3: Collect the derivative terms Bring the two terms containing y' to the left and all the other terms to the right:

$$y' \left[\ln(\sin x) - \frac{\cos x}{y} \right] = -\sin x \ln y - y \cot x.$$

Multiply by $-y$ to remove the fraction within the coefficient:

$$y' [\cos x - y \ln(\sin x)] = y \sin x \ln y + y^2 \cot x.$$

Divide by the coefficient where it is nonzero. This condition is needed because at a zero coefficient the differentiated relation does not determine a unique finite slope by division.

Step 4: Conclusion

$$\boxed{\frac{dy}{dx} = \frac{y \sin x \ln y + y^2 \cot x}{\cos x - y \ln(\sin x)}}$$

on the stated real branch, wherever the denominator is nonzero.

Quick Tip to Solve: After taking logarithms, apply the product rule on both sides and collect every y' term before dividing; this preserves the minus sign from $d(\cos x)/dx$.

29. A survey was conducted on the patients who have undergone knee replacement surgeries. It was found that Robotic Knee replacement surgeries have 90% success rate.

On a particular day, robotic surgery was performed on three patients, A , B and C , one after the other. Assuming that the success and failure of each surgery is independent of each other, find the probability that :

- (i) exactly one surgery is successful,
(ii) at most two surgeries are successful.

[3 marks]

Solution:

Concept:

- For independent surgeries, the probability of a specified sequence is the product of the individual success or failure probabilities.
- If X counts successes in n independent trials with success probability p , then $P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$.

Step 1: Identify the probabilities Each surgery succeeds with probability $p = 90/100 = 0.9$, and fails with probability $q = 1 - p = 0.1$. There are three independent surgeries, so X , the number of successful surgeries among patients A, B, C , has binomial parameters $n = 3$ and $p = 0.9$. Independence allows us to multiply probabilities even though the surgeries occur one after another.

Step 2: Calculate the probability of exactly one success In part (i), the successful patient may be A, B , or C . The mutually exclusive patterns are $SFF, F SF, FFS$, where S denotes success and F failure. Each pattern has probability $(0.9)(0.1)^2$, so

$$P(X = 1) = \binom{3}{1} (0.9)(0.1)^2 = 3(0.9)(0.01) = 0.027.$$

The factor 3 is essential because the question does not specify which patient's surgery succeeds.

Step 3: Use the complement for at most two successes In part (ii), at most two means zero, one or two successes. The only excluded outcome is success in all three surgeries. Thus

$$P(X \leq 2) = 1 - P(X = 3) = 1 - (0.9)^3 = 1 - 0.729 = 0.271.$$

As a check, $P(X = 0) = 0.001$ and $P(X = 2) = 3(0.9)^2(0.1) = 0.243$; adding these to $P(X = 1) = 0.027$ also gives 0.271.

Step 4: Conclusion (i) The probability of exactly one successful surgery is $\boxed{0.027}$. (ii) The probability of at most two successful surgeries is $\boxed{0.271}$.

Quick Tip to Solve: For three trials, calculate "at most two successes" as one minus the probability of all three successes; do not confuse it with exactly two.

30(a). Find : $\int \frac{dx}{x^{1/2} + x^{1/3}}$

[3 marks]

Solution:

Concept:

- Fractional powers with denominators 2 and 3 become integer powers under $x = t^6$, since 6 is their least common multiple.
- Polynomial division reduces a rational integrand to powers of t and a logarithmic term: $\int (t + 1)^{-1} dt = \ln |t + 1| + C$.

Step 1: Remove the fractional exponents Let $I = \int \frac{dx}{x^{1/2} + x^{1/3}}$. On the real domain of the original integrand, $x > 0$: the square root excludes negative x , and the denominator is zero at $x = 0$. Put $t = x^{1/6} > 0$. Then

$$x = t^6, \quad dx = 6t^5 dt, \quad x^{1/2} = t^3, \quad x^{1/3} = t^2.$$

Substituting both the denominator and the differential gives

$$I = \int \frac{6t^5}{t^3 + t^2} dt = 6 \int \frac{t^3}{t + 1} dt.$$

Step 2: Divide the polynomial before integrating The numerator has higher degree than the denominator, so use polynomial division. The identity

$$t^3 = (t + 1)(t^2 - t + 1) - 1$$

gives $\frac{t^3}{t + 1} = t^2 - t + 1 - \frac{1}{t + 1}$. Hence

$$\begin{aligned} I &= 6 \int \left(t^2 - t + 1 - \frac{1}{t + 1} \right) dt \\ &= 2t^3 - 3t^2 + 6t - 6 \ln |t + 1| + C. \end{aligned}$$

The minus sign on the logarithm comes from the remainder -1 , not from the substitution.

Step 3: Return to the original variable Replace t, t^2, t^3 by $x^{1/6}, x^{1/3}, x^{1/2}$, respectively. Since $t > 0$, we have $t + 1 > 0$, so the absolute value in the logarithm can be omitted. The arbitrary constant absorbs all constants produced by the integration.

Step 4: Conclusion

$$\boxed{I = 2x^{1/2} - 3x^{1/3} + 6x^{1/6} - 6 \ln(1 + x^{1/6}) + C}, \quad x > 0.$$

Quick Tip to Solve: Choose the sixth root to remove both fractional powers at once, and divide t^3 by $t + 1$ before integrating.

30(b). Find : $\int \tan^{-1}\left(\frac{1-x}{1+x}\right) dx$

[3 marks]

Solution:

Concept:

- Integration by parts gives $\int u dv = uv - \int v du$; inverse trigonometric functions are convenient choices for u .
- The chain rule gives $\frac{d}{dx} \tan^{-1} z = \frac{z'}{1+z^2}$, and $\int \frac{x}{1+x^2} dx = \frac{1}{2} \ln(1+x^2) + C$.

Step 1: Differentiate the inverse tangent carefully Set $u = \tan^{-1}\left(\frac{1-x}{1+x}\right)$, with $x \neq -1$. For its inner function $z = (1-x)/(1+x)$, the quotient rule gives

$$z' = \frac{-(1+x) - (1-x)}{(1+x)^2} = \frac{-2}{(1+x)^2}.$$

Therefore

$$\begin{aligned} \frac{du}{dx} &= \frac{-2/(1+x)^2}{1 + (1-x)^2/(1+x)^2} \\ &= \frac{-2}{(1+x)^2 + (1-x)^2} = -\frac{1}{1+x^2}. \end{aligned}$$

Differentiating the printed expression directly avoids assuming an inverse-tangent identity outside its valid branch.

Step 2: Apply integration by parts Take $dv = dx$, so $v = x$ and $du = -dx/(1+x^2)$. If I denotes the required integral, then

$$I = x \tan^{-1}\left(\frac{1-x}{1+x}\right) + \int \frac{x}{1+x^2} dx.$$

The remaining integral has its numerator proportional to the derivative of its denominator. With $s = 1+x^2$ and $ds = 2x dx$, it is $\frac{1}{2} \ln(1+x^2)$, up to an additive constant.

Step 3: Verify the antiderivative Differentiating the first term produces the original inverse tangent minus $x/(1+x^2)$. Differentiating the logarithmic term produces $+x/(1+x^2)$, so these rational terms cancel. The result is valid on either interval separated by $x = -1$; the integration constant may be chosen independently on each interval.

Step 4: Conclusion

$$I = x \tan^{-1}\left(\frac{1-x}{1+x}\right) + \frac{1}{2} \ln(1+x^2) + C, \quad x \neq -1.$$

Quick Tip to Solve: Differentiate the inverse tangent as printed: its derivative simplifies to $-1/(1+x^2)$, making integration by parts immediate without a branch restriction.

31. Evaluate : $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{2^x + 1} dx$

[3 marks]

Solution:

Concept:

- On symmetric limits, replacing x by $-x$ preserves the integral: $\int_{-a}^a f(x) dx = \int_{-a}^a f(-x) dx$.
- The numerator $\cos^2 x$ is even, and the complementary denominator identity is $\frac{1}{1+2^x} + \frac{1}{1+2^{-x}} = 1$.
- The double-angle identity $\cos^2 x = (1 + \cos 2x)/2$ evaluates the remaining trigonometric integral.

Step 1: Reflect the variable about zero Write the required integral as

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1 + 2^x} dx.$$

Under $x = -t$, the bounds reverse and $dx = -dt$; the two reversals cancel. Since $\cos^2(-t) = \cos^2 t$, renaming the dummy variable gives

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1 + 2^{-x}} dx = \int_{-\pi/2}^{\pi/2} \frac{2^x \cos^2 x}{1 + 2^x} dx.$$

Multiplying the second integrand's numerator and denominator by 2^x puts it over the same denominator as the original.

Step 2: Add the two forms of the integral Adding the original and reflected expressions cancels the exponential denominator:

$$\begin{aligned} 2I &= \int_{-\pi/2}^{\pi/2} \cos^2 x \left(\frac{1}{1 + 2^x} + \frac{2^x}{1 + 2^x} \right) dx \\ &= \int_{-\pi/2}^{\pi/2} \cos^2 x dx. \end{aligned}$$

The original integrand is not itself even. It is the sum of its values at x and $-x$ that has simplified to $\cos^2 x$.

Step 3: Evaluate the elementary integral Use the double-angle identity and substitute the limits:

$$\begin{aligned} 2I &= \left[\frac{x}{2} + \frac{\sin 2x}{4} \right]_{-\pi/2}^{\pi/2} \\ &= \frac{\pi}{4} - \left(-\frac{\pi}{4} \right) = \frac{\pi}{2}. \end{aligned}$$

Both sine terms vanish because $\sin \pi = \sin(-\pi) = 0$. The displayed quantity is $2I$, so a final division by two is necessary.

Step 4: Conclusion The required definite integral is $I = \frac{\pi}{4}$.

Quick Tip to Solve: On symmetric limits with an even numerator over $1 + 2^x$, add the reflected integral: the weights $1/(1 + 2^x)$ and $1/(1 + 2^{-x})$ sum to one.

Section D (20 marks)

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32(a). On the inauguration day of a new showroom, a lucky draw was organized and some vouchers of Rs 1,000 and Rs 500 were given to the lucky draw winners.

A total of 60 vouchers were given on the day. The number of Rs 1,000 vouchers added to 3 times the number of Rs 500 vouchers, gives 100. Express the given information as a system of linear equations in two variables. Hence, find the number of vouchers of each type by matrix method. [5 marks]

Solution:

Concept:

- Translate the two conditions into linear equations by assigning one variable to the number of vouchers of each denomination.
- A system can be written as $AX = B$, where A is its coefficient matrix, X is the column of unknowns and B is the column of constants.

- If $\det A \neq 0$, then the unique solution is $X = A^{-1}B$. For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, its inverse is $\frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Step 1: Define the variables and form the equations Let x be the number of Rs 1,000 vouchers and y the number of Rs 500 vouchers. These variables count vouchers; they do not represent their monetary values. Since a total of 60 vouchers were distributed,

$$x + y = 60.$$

The other statement adds the number of Rs 1,000 vouchers to three times the number of Rs 500 vouchers. Hence,

$$x + 3y = 100.$$

These are the required two linear equations in the two unknown counts.

Step 2: Express the system in matrix form Keep the unknowns in the same order, x followed by y , in both equations. The coefficient of each variable becomes the corresponding matrix entry:

$$\underbrace{\begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}}_A \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_X = \underbrace{\begin{pmatrix} 60 \\ 100 \end{pmatrix}}_B.$$

Thus the system has the form $AX = B$.

Step 3: Find the inverse of the coefficient matrix First check that the inverse exists:

$$\det A = 1(3) - 1(1) = 2 \neq 0.$$

Interchanging the diagonal entries and changing the signs of the other two entries gives the adjugate.

Quick Tip to Solve: Use voucher counts as the unknowns, not rupee amounts. Keep their order fixed in the coefficient matrix so the first and second entries of the solution identify the correct denominations.

Therefore,

$$\text{adj } A = \begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}, \quad A^{-1} = \frac{1}{2} \begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}.$$

The nonzero determinant also shows that the voucher counts are uniquely determined by the given conditions.

Step 4: Multiply by the inverse and check the counts Premultiply $AX = B$ by A^{-1} . Since $A^{-1}A = I$, we obtain

$$\begin{aligned} X &= A^{-1}B \\ &= \frac{1}{2} \begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 60 \\ 100 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 180 - 100 \\ -60 + 100 \end{pmatrix} = \begin{pmatrix} 40 \\ 20 \end{pmatrix}. \end{aligned}$$

Both entries are nonnegative integers, as voucher counts must be. Substitution checks both original conditions: $40 + 20 = 60$ and $40 + 3(20) = 100$.

Step 5: Conclusion The equations are $x + y = 60$ and $x + 3y = 100$. By the matrix method, there were 40 vouchers of Rs 1,000 and 20 vouchers of Rs 500.

Quick Tip to Solve: Use voucher counts as the unknowns, not rupee amounts. Keep their order fixed in the coefficient matrix so the first and second entries of the solution identify the correct denominations.

OR

32(b). Given that $P = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, $Q = \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix}$ and $R = \begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix}$, find a matrix S such that $PQ - RS$ is a null matrix. [5 marks]

Solution:

Concept:

- The null matrix has every entry zero. Thus $PQ - RS = O$ is equivalent to $RS = PQ$, where P, Q, R are the given matrices and S is unknown.
- Matrix multiplication is generally not commutative. If R is invertible, premultiplication by R^{-1} gives $S = R^{-1}(PQ)$; the order of the factors must be preserved.
- For $R = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $ad - bc \neq 0$, $R^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Step 1: Rearrange the given matrix equation The requirement that $PQ - RS$ be a null matrix means that the corresponding entries of PQ and RS must be equal. Hence,

$$RS = PQ.$$

We first evaluate the known product PQ , then remove the factor R on the left using its inverse. All the given matrices are of order 2×2 , so these products are defined.

Quick Tip to Solve: In $RS = PQ$, cancel the left factor by multiplying on the left: $S = R^{-1}PQ$. Here $\det R = 1$, so its inverse is simply its adjugate.

Step 2: Calculate the product of the first two matrices Multiply each row of P by each column of Q . In particular, retain the minus sign in the first row of P :

$$\begin{aligned} PQ &= \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 2(5) - 1(7) & 2(2) - 1(4) \\ 3(5) + 4(7) & 3(2) + 4(4) \end{pmatrix} \\ &= \begin{pmatrix} 3 & 0 \\ 43 & 22 \end{pmatrix}. \end{aligned}$$

Step 3: Determine the inverse of R For the given matrix R , its determinant is

$$\det R = 2(8) - 5(3) = 16 - 15 = 1.$$

Since this determinant is nonzero, R is invertible. The inverse formula gives

$$R^{-1} = \frac{1}{1} \begin{pmatrix} 8 & -5 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 8 & -5 \\ -3 & 2 \end{pmatrix}.$$

Consequently the equation determines exactly one matrix S .

Step 4: Premultiply by the inverse Multiplying both sides of $RS = PQ$ on the left by R^{-1} , we have $R^{-1}RS = R^{-1}PQ$. Therefore,

$$\begin{aligned} S &= \begin{pmatrix} 8 & -5 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 43 & 22 \end{pmatrix} \\ &= \begin{pmatrix} 24 - 215 & 0 - 110 \\ -9 + 86 & 0 + 44 \end{pmatrix} \\ &= \begin{pmatrix} -191 & -110 \\ 77 & 44 \end{pmatrix}. \end{aligned}$$

Right multiplication by R^{-1} would not cancel the left factor R , which is why the order used above is essential.

Step 5: Verify the required null matrix Substitute the calculated entries directly:

$$\begin{aligned} RS &= \begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix} \begin{pmatrix} -191 & -110 \\ 77 & 44 \end{pmatrix} \\ &= \begin{pmatrix} -382 + 385 & -220 + 220 \\ -573 + 616 & -330 + 352 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 0 \\ 43 & 22 \end{pmatrix} = PQ. \end{aligned}$$

Thus their difference has all four entries equal to zero, as required.

Quick Tip to Solve: In $RS = PQ$, cancel the left factor by multiplying on the left: $S = R^{-1}PQ$. Here $\det R = 1$, so its inverse is simply its adjugate.

Step 6: Conclusion The required matrix is $S = \begin{pmatrix} -191 & -110 \\ 77 & 44 \end{pmatrix}$, for which $PQ - RS = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

Quick Tip to Solve: In $RS = PQ$, cancel the left factor by multiplying on the left: $S = R^{-1}PQ$. Here $\det R = 1$, so its inverse is simply its adjugate.

33(a). Represent the equations of lines l_1 and l_2 in vector form and check whether they are intersecting or not.

$$l_1 : \frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$$

$$l_2 : \frac{x+1}{-1} = \frac{2-y}{-2} = \frac{z-5}{5}$$

[5 marks]

Solution:

Concept:

- A line through the point with position vector $\vec{a}$, parallel to a nonzero vector $\vec{b}$, has vector equation $\vec{r} = \vec{a} + t\vec{b}$, where t is a real parameter.
- Two lines intersect only when their three coordinate equations hold simultaneously for some values of their independent parameters. A match in only two coordinates is insufficient.

Step 1: Parameterise the first line Set each ratio in l_1 equal to λ . Then

$$x = -3 - 3\lambda, \quad y = 1 + \lambda, \quad z = 5 + 5\lambda.$$

Thus l_1 passes through $(-3, 1, 5)$ and has direction vector $(-3, 1, 5)$. Its vector equation is

$$\vec{r} = -3\hat{i} + \hat{j} + 5\hat{k} + \lambda(-3\hat{i} + \hat{j} + 5\hat{k}).$$

Step 2: Parameterise the second line carefully Set the ratios in l_2 equal to μ . In its middle ratio the numerator is $2 - y$, so

$$\frac{2-y}{-2} = \mu \implies y = 2 + 2\mu.$$

The other coordinates are $x = -1 - \mu$ and $z = 5 + 5\mu$. Therefore

$$\vec{r} = -\hat{i} + 2\hat{j} + 5\hat{k} + \mu(-\hat{i} + 2\hat{j} + 5\hat{k}).$$

Notice that the middle direction ratio is positive because both signs in the original middle ratio reverse.

Step 3: Equate all three coordinates A common point must satisfy

$$-3 - 3\lambda = -1 - \mu,$$

$$1 + \lambda = 2 + 2\mu,$$

$$5 + 5\lambda = 5 + 5\mu.$$

Quick Tip to Solve: Rewrite $(2 - y)/(-2)$ as $(y - 2)/2$ before reading direction ratios. Use the identical z -coordinates to equate parameters, then check both remaining coordinates.

The third equation gives $\lambda = \mu$. Substituting into the first gives $-2\lambda = 2$, hence $\lambda = \mu = -1$. The second equation then reads $0 = 0$, so the candidate satisfies all three equations. Substitution into either line gives the coordinates $(0, 0, 0)$.

Step 4: Conclusion The vector forms are obtained in Steps 1 and 2. The lines intersect at the origin $(0, 0, 0)$, corresponding to $\lambda = \mu = -1$.

Quick Tip to Solve: Rewrite $(2 - y)/(-2)$ as $(y - 2)/2$ before reading direction ratios. Use the identical z -coordinates to equate parameters, then check both remaining coordinates.

OR

33(b). Opposite sides of a square are along the lines :

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Find the area of the square if direction ratios of other pair of opposite sides of the square are given by $\langle -3, 6, p \rangle$. Also, find the value of p . [5 marks]

Solution:

Concept:

- For parallel lines $\vec{r} = \vec{a} + \lambda\vec{d}$ and $\vec{r} = \vec{b} + \mu\vec{d}$, their perpendicular separation is $h = |(\vec{b} - \vec{a}) \times \vec{d}|/|\vec{d}|$. Opposite sides of a square are separated by its side length, so its area is h^2 .
- Adjacent sides are perpendicular: their direction vectors have zero dot product. They must also lie in the same plane as both opposite sides; perpendicularity alone does not guarantee this compatibility.

Step 1: Identify the parallel lines and their displacement From the equations, take

$$\vec{a} = (1, 2, -4), \quad \vec{b} = (3, 3, -5), \quad \vec{d} = (2, 3, 6).$$

Both lines share the same direction vector. Their base-point displacement is

$$\vec{c} = \vec{b} - \vec{a} = (2, 1, -1), \quad |\vec{d}|^2 = 4 + 9 + 36 = 49.$$

The vector $\vec{c}$ need not itself be a square's side: the two base points are arbitrary points on the opposite-side lines.

Step 2: Calculate the perpendicular separation Evaluate the cross product:

$$\vec{c} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}.$$

Consequently,

$$h^2 = \frac{9^2 + (-14)^2 + 4^2}{49} = \frac{293}{49}.$$

Quick Tip to Solve: Square the parallel-line distance directly to find area. After using the dot product for perpendicularity, check that the proposed joining direction lies in the plane of the two lines.

Thus the area determined by the two parallel lines alone is $293/49$ square units. As a check, the projection formula gives $|\vec{c}|^2 - (\vec{c} \cdot \vec{d})^2/|\vec{d}|^2 = 6 - 1/49$, the same value.

Step 3: Apply the perpendicularity condition Let the prescribed direction of the other sides be $\vec{e} = (-3, 6, p)$. Adjacent sides of a square require

$$\vec{d} \cdot \vec{e} = 0 \implies 2(-3) + 3(6) + 6p = 0.$$

Hence $12 + 6p = 0$, which forces $p = -2$. The resulting direction $(-3, 6, -2)$ is perpendicular to $\vec{d}$.

Step 4: Check compatibility of all the printed data The preceding calculations give the usual requested numerical results, but the prescribed other-side direction must also lie in the plane containing the parallel lines. A normal to that plane is $\vec{n} = \vec{c} \times \vec{d} = (9, -14, 4)$. For the forced value $p = -2$,

$$\vec{n} \cdot \vec{e} = 9(-3) - 14(6) + 4(-2) = -119 \neq 0.$$

Therefore this direction leaves the plane of the two given lines and cannot be a side joining them. Equivalently, the actual perpendicular connecting direction is proportional to

$$\vec{c} - \frac{\vec{c} \cdot \vec{d}}{|\vec{d}|^2} \vec{d} = \frac{1}{49}(96, 46, -55),$$

which is not parallel to $(-3, 6, -2)$. The printed direction condition is consequently incompatible with a square on these lines.

Step 5: Conclusion The standard distance and perpendicularity calculations yield

$$\boxed{\text{area} = 293/49 \text{ square units, } p = -2}.$$

Strictly, no square satisfies all the printed conditions: the area is the square area from the two lines alone, while the prescribed other-side direction is incompatible with their plane.

Quick Tip to Solve: Square the parallel-line distance directly to find area. After using the dot product for perpendicularity, check that the proposed joining direction lies in the plane of the two lines.

34. Show that $f : \mathbb{R}_+ \rightarrow [-5, \infty)$ given by $f(x) = 4x^2 + 4x - 5$ is both one-one and onto, where $\mathbb{R}_+ = [0, \infty)$. Also, find $p \in \mathbb{R}_+$ such that $f(p) = 3$. [5 marks]

Solution:

Concept:

- A function is one-one if $f(a) = f(b)$ implies $a = b$ for every pair a, b in its domain.
- A function is onto if every element y of its codomain has at least one pre-image x in its domain satisfying $f(x) = y$.
- Here the domain is $[0, \infty)$, so a root obtained while solving $f(x) = y$ is admissible only when it is nonnegative.

Step 1: Check that the output lies in the stated codomain For every $x \geq 0$, both $4x^2$ and $4x$ are

Quick Tip to Solve: Rewrite this quadratic as $f(x) = (2x + 1)^2 - 6$. On $x \geq 0$, use the positive square root to construct the pre-image and check its nonnegativity.

nonnegative. Hence

$$f(x) = 4x^2 + 4x - 5 \geq -5.$$

Thus the given formula defines a function from $\mathbb{R}_+ = [0, \infty)$ to $[-5, \infty)$. The lower endpoint is attained at $x = 0$, since $f(0) = -5$.

Step 2: Prove that equal outputs require equal inputs Let $a, b \in [0, \infty)$ and suppose $f(a) = f(b)$. Then

$$\begin{aligned}4a^2 + 4a - 5 &= 4b^2 + 4b - 5, \\a^2 - b^2 + a - b &= 0, \\(a - b)(a + b + 1) &= 0.\end{aligned}$$

Because $a, b \geq 0$, we have $a + b + 1 \geq 1 > 0$. Therefore the second factor cannot vanish, and the equality forces $a - b = 0$, that is, $a = b$. This establishes the required injectivity on the specified domain.

Step 3: Construct a pre-image of every codomain element Take an arbitrary $y \in [-5, \infty)$. To solve $f(x) = y$, complete the square:

$$\begin{aligned}4x^2 + 4x - 5 &= y, \\(2x + 1)^2 &= y + 6.\end{aligned}$$

Since $y \geq -5$, we have $y + 6 \geq 1$. Choose

$$x = \frac{\sqrt{y + 6} - 1}{2} \geq 0.$$

This choice belongs to the domain, and direct substitution gives

$$f(x) = (2x + 1)^2 - 6 = (\sqrt{y + 6})^2 - 6 = y.$$

Thus every y in the full stated codomain, including $y = -5$, has a valid pre-image. This establishes surjectivity.

Step 4: Solve for the requested input The condition $f(p) = 3$ gives

$$\begin{aligned}4p^2 + 4p - 5 &= 3, \\p^2 + p - 2 &= 0, \\(p + 2)(p - 1) &= 0.\end{aligned}$$

The algebraic candidates are $p = -2$ and $p = 1$. The value -2 is excluded because $p \in \mathbb{R}_+$. The remaining value satisfies $f(1) = 4 + 4 - 5 = 3$.

Step 5: Conclusion The function $f : [0, \infty) \rightarrow [-5, \infty)$ is both one-one and onto, hence bijective. The required value is $\boxed{p = 1}$.

Quick Tip to Solve: Rewrite this quadratic as $f(x) = (2x + 1)^2 - 6$. On $x \geq 0$, use the positive square root to construct the pre-image and check its nonnegativity.

35. Solve the following Linear Programming Problem graphically :

Maximise $Z = 12x + 18y$

subject to the constraints

$$x + y \leq 1200$$

$$x - 2y \geq 0$$

$$x + 3y \geq 600$$

$$x \geq 0, y \geq 0$$

[5 marks]

Solution:

Concept:

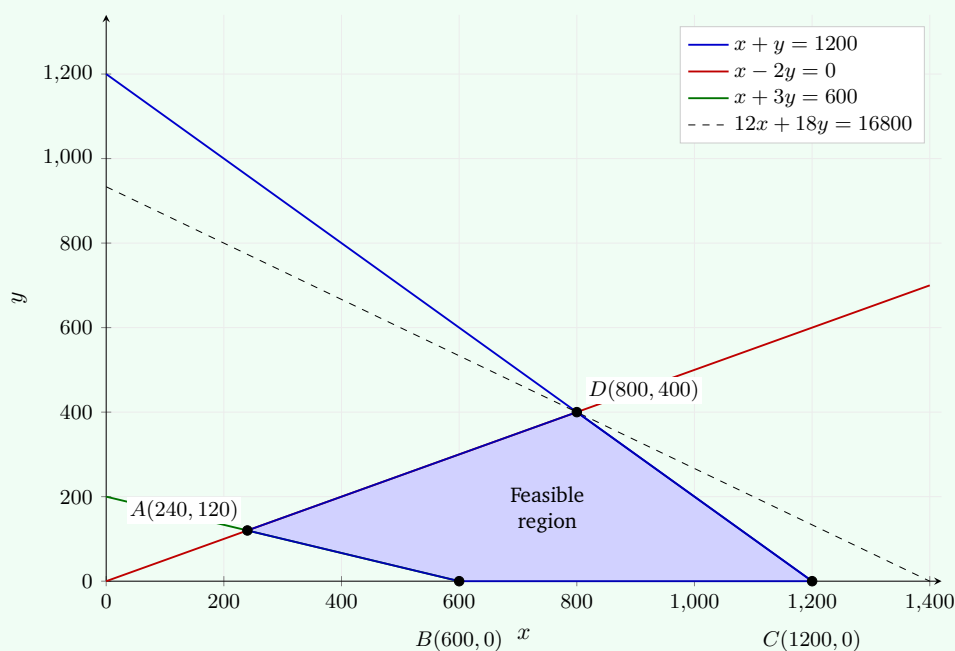
- Each linear inequality represents a closed half-plane. Its boundary is obtained by replacing the inequality sign with equality.
- The feasible region is the intersection of all the required half-planes, including $x \geq 0$ and $y \geq 0$.
- For a nonempty bounded feasible polygon, a linear objective function attains its maximum at a corner point. Evaluate $Z = 12x + 18y$ at every feasible vertex.

Step 1: Plot the boundary lines and choose their half-planes Draw the following three boundary lines using their intercepts or two convenient points:

Boundary	First point	Second point
$x + y = 1200$	(1200, 0)	(0, 1200)
$x - 2y = 0$	(0, 0)	(800, 400)
$x + 3y = 600$	(600, 0)	(0, 200)

The inequality $x + y \leq 1200$ selects the side containing the origin. The constraint $x - 2y \geq 0$ is equivalent to $y \leq x/2$, so select the side below that line. For $x + 3y \geq 600$, select the side away from the origin. Finally restrict the drawing to the first quadrant. Their common region is the shaded quadrilateral $ABCD$ shown below; every boundary is included because all inequalities are non-strict.

Quick Tip to Solve: Find the feasible polygon before evaluating Z . Here $x = 2y$ intersects the two oblique boundaries at (240, 120) and (800, 400); the other two vertices lie on the x -axis.



The shaded quadrilateral $ABCD$ is the feasible region; the dashed objective line touches it at D .

Step 2: Calculate all feasible vertices At A , the lines $x = 2y$ and $x + 3y = 600$ meet:

$$2y + 3y = 600 \Rightarrow y = 120, \quad x = 240.$$

On the x -axis, put $y = 0$. The constraints then require $600 \leq x \leq 1200$, giving $B = (600, 0)$ and $C = (1200, 0)$. At D , solve $x = 2y$ together with $x + y = 1200$:

$$2y + y = 1200 \Rightarrow y = 400, \quad x = 800.$$

These four points satisfy every constraint. The intersection of $x + y = 1200$ and $x + 3y = 600$ gives $y = -300$, outside the first quadrant, and is rejected. No point on the y -axis is feasible: $x = 0$ and $x \geq 2y$, with $y \geq 0$, force $y = 0$, which violates $x + 3y \geq 600$. Thus no further feasible vertices have been omitted.

Step 3: Compare the objective function at the corner points The region is nonempty and bounded by $x \geq 0$, $y \geq 0$ and $x + y \leq 1200$. The corner-point method therefore applies. The objective values are

Vertex	(x, y)	$Z = 12x + 18y$
A	$(240, 120)$	$2880 + 2160 = 5040$
B	$(600, 0)$	7200
C	$(1200, 0)$	14400
D	$(800, 400)$	$9600 + 7200 = 16800$

Comparing these values identifies D as the maximizing corner. The dashed objective line $12x + 18y = 16800$ passes through D and supports the feasible region there. Its slope $-2/3$ differs from the slopes of the two edges meeting at D , so the maximum is attained at that single vertex.

Step 4: Conclusion The maximum value is $Z_{\max} = 16800$, attained at $x = 800, y = 400$.

Quick Tip to Solve: Find the feasible polygon before evaluating Z . Here $x = 2y$ intersects the two oblique boundaries at $(240, 120)$ and $(800, 400)$; the other two vertices lie on the x -axis.

Section E (12 marks)

This section comprises 3 case study based questions of 4 marks each.

36. A survey was conducted to find out the success rate of students who qualified the entrance examination by dropping a year after class XII.

As per the data collected, 40% students appearing in the examination were dropouts and the remaining students were regular students of class XII.

Of the dropouts, 5% qualify the examination while 10% of the regular students qualify the examination.

Based on the above information, answer the following questions.

[4 marks]

36(i). Find the probability that a student selected at random is a regular student.

[1 mark]

Solution:

Concept:

- Let D denote a dropout and R a regular student. These two categories are mutually exclusive and together include every student appearing in the examination.
- For complementary events, $P(R) = 1 - P(D)$. A percentage is converted to a probability by dividing by 100.

Step 1: Identify the relevant population The student is chosen at random from all students appearing in the examination. The survey states that 40% of this population consists of dropouts, so $P(D) = 40/100 = 0.40$. The qualification rates describe a different event and are not needed to decide whether the selected student is regular.

Step 2: Use the complement of the dropout event All remaining students are regular students of class XII. Hence the regular proportion is $100\% - 40\% = 60\%$, giving

$$P(R) = 1 - P(D) = 1 - 0.40 = 0.60.$$

This also verifies that the two category probabilities add to unity.

Step 3: Conclusion The required probability is $0.60 = \frac{3}{5}$.

Quick Tip to Solve: For a selection from all students, use the category percentages first; qualification rates do not affect the regular-student proportion.

36(ii). A student is selected at random from a group of dropout students. What is the probability that the student will not qualify the examination ?

[1 mark]

Solution:

Concept:

- Let D denote a dropout and Q denote qualification in the examination. Selection from the dropout group means that D is already given.
- Within a fixed group, qualification and non-qualification are complementary: $P(Q^c \mid D) =$

Quick Tip to Solve: When the student is selected from the dropout group, complement the dropout qualification rate directly; do not multiply by the dropout share.

$$1 - P(Q | D).$$

Step 1: Read the qualification rate within the selected group The sample space here consists only of dropout students. According to the survey, 5% of dropouts qualify, so $P(Q | D) = 5/100 = 0.05$. The overall dropout proportion 0.40 is not multiplied into this conditional probability because the selection has already been restricted to dropouts.

Step 2: Calculate the complementary probability A dropout either qualifies or does not qualify. Their conditional probabilities therefore sum to 1. Thus

$$P(Q^c | D) = 1 - 0.05 = 0.95.$$

Equivalently, out of every 100 dropout students, 95 do not qualify.

Step 3: Conclusion The required probability is $0.95 = \frac{19}{20}$.

Quick Tip to Solve: When the student is selected from the dropout group, complement the dropout qualification rate directly; do not multiply by the dropout share.

36(iii)(a). A student selected at random qualified the examination. Find the probability that student is not a dropout. [2 marks]

Solution:

Concept:

- Let R denote a regular student, D a dropout and Q qualification. Since these categories exhaust the students, not being a dropout means being regular.
- The multiplication rule gives $P(R \cap Q) = P(R)P(Q | R)$, while total probability gives $P(Q) = P(R)P(Q | R) + P(D)P(Q | D)$.
- Bayes theorem reverses the conditioning: $P(R | Q) = P(R \cap Q)/P(Q)$.

Step 1: Calculate the probability of qualification The survey gives $P(D) = 0.40$, $P(R) = 0.60$, $P(Q | D) = 0.05$ and $P(Q | R) = 0.10$. Qualification can occur in either of the two disjoint categories, so

$$\begin{aligned} P(Q) &= (0.40)(0.05) + (0.60)(0.10) \\ &= 0.02 + 0.06 = 0.08. \end{aligned}$$

The regular-and-qualified probability is $P(R \cap Q) = 0.60 \times 0.10 = 0.06$.

Step 2: Condition on the information that the student qualified The selected student is known to have qualified, so the denominator must be the overall qualifying probability rather than the total regular-student proportion. Therefore

$$P(R | Q) = \frac{P(R \cap Q)}{P(Q)} = \frac{0.06}{0.08} = \frac{3}{4}.$$

For a frequency check, among 1000 students, 20 qualified dropouts and 60 qualified regular students give 80 qualifiers in all; $60/80 = 3/4$ are regular.

Quick Tip to Solve: For a student known to have qualified, divide the regular-and-qualified probability by the total qualification probability from both groups.

Step 3: Conclusion The probability that the qualified student is not a dropout is $\boxed{\frac{3}{4} = 0.75}$.

Quick Tip to Solve: For a student known to have qualified, divide the regular-and-qualified probability by the total qualification probability from both groups.

OR

36(iii)(b). A student selected at random did not qualify the examination. Find the probability that the student was a regular student. [2 marks]

Solution:

Concept:

- Let R denote a regular student, D a dropout student and F failure to qualify. The two student categories form a partition, so $P(F) = P(R)P(F | R) + P(D)P(F | D)$.
- Bayes' theorem reverses the conditioning: $P(R | F) = \frac{P(R)P(F | R)}{P(F)}$. The denominator must count all students who fail to qualify.

Step 1: Identify the prior and conditional probabilities The survey states that 40% are dropouts and the rest are regular students. Hence $P(D) = 0.4$ and $P(R) = 0.6$. Since 5% of dropouts and 10% of regular students qualify, the complementary probabilities are

$$P(F | D) = 0.95, \quad P(F | R) = 0.90.$$

Step 2: Restrict the sample space to students who did not qualify A student can fail to qualify through either category. Their disjoint contributions are

$$P(R \cap F) = 0.6(0.90) = 0.54, \quad P(D \cap F) = 0.4(0.95) = 0.38.$$

Therefore $P(F) = 0.54 + 0.38 = 0.92$. Among these unsuccessful students, the required proportion belonging to the regular group is $0.54/0.92$. The value 0.90 alone would condition on being regular, which is the reverse of the condition in the question.

Step 3: Conclusion The probability that the student was regular, given that the student did not qualify, is

$$\boxed{P(R | F) = \frac{0.54}{0.92} = \frac{27}{46}}.$$

Quick Tip to Solve: Imagine 100 students: 54 regular students and 38 dropouts fail, so use 54 out of the total 92 failures.

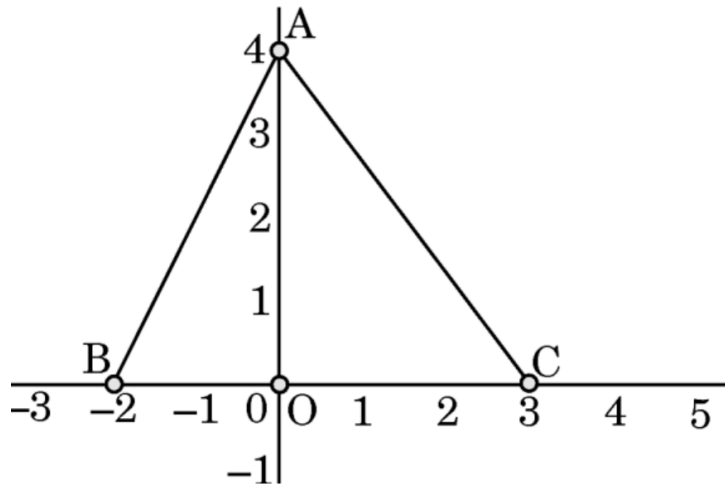
37. There is a triangular park in the society. The park is divided into two sections as shown in the figure.

In the region OAC, children are allowed to play games like cricket, football, while in the region AOB, activities which involve running are not allowed.

The vertices of the triangular park ABC are $A(0, 4)$, $B(-2, 0)$ and $C(3, 0)$.

Based on the above information, answer the following questions :

[4 marks]



37(i). Write the equation of the boundary line AB of the park.

[1 mark]

Solution:

Concept:

- The slope of the line through (x_1, y_1) and (x_2, y_2) is $m = (y_2 - y_1)/(x_2 - x_1)$, provided the denominator is nonzero.
- A line with slope m through (x_1, y_1) has equation $y - y_1 = m(x - x_1)$.

Step 1: Calculate the slope of AB The boundary joins $A(0, 4)$ to $B(-2, 0)$. Therefore

$$m_{AB} = \frac{4 - 0}{0 - (-2)} = 2.$$

Step 2: Use the point A to write the equation Substitute the slope and the coordinates of A in point-slope form:

$$y - 4 = 2(x - 0).$$

As a check, substituting $x = -2$ gives $y = 0$, so the equation also passes through B, as required. The boundary segment itself has $-2 \leq x \leq 0$.

Step 3: Conclusion The equation of the boundary line is $y = 2x + 4$, equivalently $2x - y + 4 = 0$.

Quick Tip to Solve: Since A lies on the y-axis, its ordinate immediately gives the intercept 4; only the slope remains to be found.

37(ii). Write the equation of the boundary line AC of the park.

[1 mark]

Solution:

Concept:

- The slope of a line through distinct points (x_1, y_1) and (x_2, y_2) is $m = (y_2 - y_1)/(x_2 - x_1)$, when $x_2 \neq x_1$.
- Point-slope form $y - y_1 = m(x - x_1)$ determines a line from its slope and a point on it.

Step 1: Find the slope of AC The coordinates of the two endpoints are $A(0, 4)$ and $C(3, 0)$. Hence

$$m_{AC} = \frac{0 - 4}{3 - 0} = -\frac{4}{3}.$$

The negative sign agrees with the fall in height as we move rightwards from A to C.

Step 2: Substitute the coordinates of A Using the point $A(0, 4)$ gives

$$y - 4 = -\frac{4}{3}(x - 0).$$

Multiplying by 3 and rearranging gives $4x + 3y = 12$. Substituting $C(3, 0)$ confirms the equation; the boundary segment has $0 \leq x \leq 3$.

Step 3: Conclusion The required boundary line is $4x + 3y = 12$, or $y = 4 - \frac{4}{3}x$.

Quick Tip to Solve: The intercepts are 3 and 4, so the intercept form $x/3 + y/4 = 1$ gives the same equation immediately.

37(iii)(a). Using integration, find the area of region OAC, in which children are allowed to play cricket, football.

[2 marks]

Solution:

Concept:

- The area under a non-negative curve $y = f(x)$, between $x = a$ and $x = b$, is $\int_a^b f(x) dx$.
- For region OAC, the upper boundary is AC and the lower boundary is the x -axis. The vertices are $O(0, 0)$, $A(0, 4)$ and $C(3, 0)$.

Step 1: Express the boundary AC as a function Using the two intercepts, the equation of AC is

$$\frac{x}{3} + \frac{y}{4} = 1, \quad y = 4 - \frac{4x}{3}.$$

This expression is non-negative throughout $0 \leq x \leq 3$, so the definite integral gives the geometric area directly. A vertical strip has height $4 - 4x/3$ and width dx .

Step 2: Integrate over the horizontal extent of OAC The left endpoint is the origin and the right

Quick Tip to Solve: For OAC, integrate the line AC from $x = 0$ to $x = 3$; check the integral using half the product of intercepts, $\frac{1}{2}(3)(4) = 6$.

endpoint is C. Therefore

$$\begin{aligned}\text{Area}(OAC) &= \int_0^3 \left(4 - \frac{4x}{3}\right) dx \\ &= \left[4x - \frac{2x^2}{3}\right]_0^3 \\ &= (12 - 6) - 0 = 6.\end{aligned}$$

The region lies entirely above the horizontal axis, so no splitting or absolute-value correction is required.

Step 3: Conclusion The area of region OAC, where children may play cricket and football, is 6 square units.

Quick Tip to Solve: For OAC, integrate the line AC from $x = 0$ to $x = 3$; check the integral using half the product of intercepts, $\frac{1}{2}(3)(4) = 6$.

OR

37(iii)(b). Using integration, find the area of region AOB.

[2 marks]

Solution:

Concept:

- The area between an upper curve $y = f(x)$ and the x -axis is $\int_a^b f(x) dx$, provided $f(x) \geq 0$ on the interval.
- Region AOB has vertices $A(0, 4)$, $O(0, 0)$ and $B(-2, 0)$. Its upper boundary is the straight line AB.

Step 1: Find the height of a vertical strip The slope of AB is $(4 - 0)/(0 - (-2)) = 2$, and its vertical intercept is 4. Thus

$$y = 2x + 4.$$

For $-2 \leq x \leq 0$, this line lies above the x -axis. A strip through the region consequently has height $(2x + 4) - 0$, measured from the horizontal axis to AB.

Step 2: Integrate from B to the vertical axis Use increasing limits, starting at $x = -2$ and ending at $x = 0$:

$$\begin{aligned}\text{Area}(AOB) &= \int_{-2}^0 (2x + 4) dx \\ &= [x^2 + 4x]_{-2}^0 \\ &= 0 - (4 - 8) = 4.\end{aligned}$$

Although the region lies to the left of the vertical axis, its strip heights are positive. Negative horizontal coordinates therefore do not imply a negative area.

Step 3: Conclusion Using integration, the area of region AOB is 4 square units.

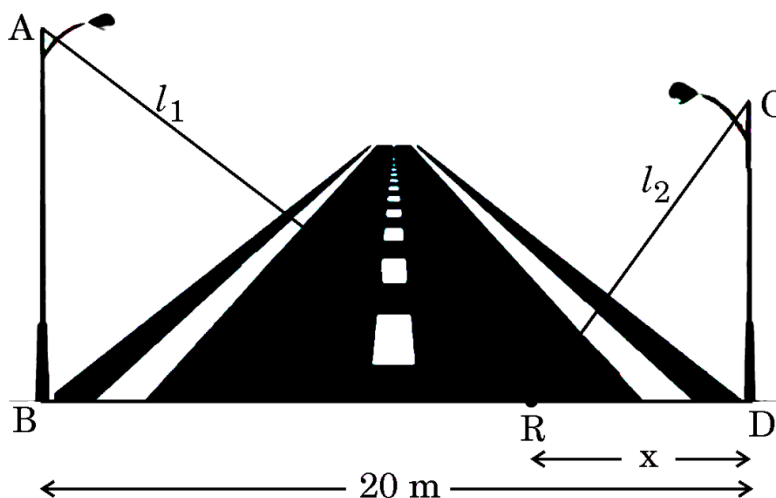
Quick Tip to Solve: On the left of the vertical axis, retain the negative limits -2 and 0 in that order; the positive strip height $2x + 4$ gives a positive area.

38. Two vertical light poles of height 22 m and 16 m stand on the opposite sides of a 20 m wide road as shown below in the figure.

Two ladders of length l_1 and l_2 are placed from a common point R on the road at a distance of x m from the smaller pole.

Based on the above information, answer the following questions :

[4 marks]



38(i). Express $p(x) = l_1 + l_2$ in terms of x .

[1 mark]

Solution:

Concept:

- A vertical pole, the horizontal road and its ladder form a right triangle. By Pythagoras, the ladder length is the square root of the sum of the squares of the other two sides.
- The road is 20 m wide. If R is x m from the smaller pole, it is $(20 - x)$ m from the taller pole, with $0 \leq x \leq 20$.

Step 1: Express the two ladder lengths The ladder l_1 reaches the 22 m pole and l_2 reaches the 16 m pole. Taking positive square roots because lengths are positive,

$$l_1 = \sqrt{(20 - x)^2 + 484}, \quad l_2 = \sqrt{x^2 + 256}.$$

Step 2: Add the lengths to form the requested function The given function is $p(x) = l_1 + l_2$. Substitute the two hypotenuse lengths directly; the square roots remain because this function adds lengths:

$$p(x) = \sqrt{(20 - x)^2 + 484} + \sqrt{x^2 + 256}.$$

Step 3: Conclusion $p(x) = \sqrt{(20 - x)^2 + 484} + \sqrt{x^2 + 256}$, for $0 \leq x \leq 20$, with lengths measured in metres.

Quick Tip to Solve: Since x is measured from the smaller pole, pair x^2 with 16^2 , and $(20 - x)^2$ with 22^2 .

38(ii). Find $p'(x)$.

[1 mark]

Solution:

Concept:

- Pythagoras' theorem gives each ladder length from the pole height and its horizontal distance from R .
- The chain rule gives $\frac{d}{dx} \sqrt{u(x)} = \frac{u'(x)}{2\sqrt{u(x)}}$, where $u(x) > 0$.

Step 1: Express the two lengths Since x is measured from the smaller pole, the horizontal distances from R to the smaller and taller poles are x and $20 - x$, respectively. Thus

$$p(x) = \sqrt{(20 - x)^2 + 484} + \sqrt{x^2 + 256}, \quad 0 \leq x \leq 20.$$

Step 2: Apply the chain rule to both terms The derivative of $(20 - x)^2$ is $-2(20 - x)$, whereas the derivative of x^2 is $2x$. Therefore,

$$p'(x) = \frac{-2(20 - x)}{2\sqrt{(20 - x)^2 + 484}} + \frac{2x}{2\sqrt{x^2 + 256}}.$$

The negative sign in the first term reflects that the horizontal distance to the taller pole decreases as x increases.

Step 3: Conclusion

$$p'(x) = \frac{x - 20}{\sqrt{(20 - x)^2 + 484}} + \frac{x}{\sqrt{x^2 + 256}}.$$

Quick Tip to Solve: Attach 22^2 to $(20 - x)^2$, and remember the inner derivative -1 when differentiating the taller-pole term.

38(iii)(a). Find the value of x for which $l_1^2 + l_2^2$ is minimum.

[2 marks]

Solution:

Concept:

- For a ladder reaching a vertical pole, Pythagoras' theorem gives its squared length as the sum of the squared pole height and squared horizontal distance.
- An interior stationary point of a twice differentiable function $S(x)$ satisfies $S'(x) = 0$; if $S''(x) > 0$ throughout the interval, it is the unique minimum.

Step 1: Form the sum of the squared lengths The point R is x m from the 16 m pole, so it is $(20 - x)$ m from the 22 m pole. Hence,

$$\begin{aligned} S(x) &= l_1^2 + l_2^2 \\ &= 22^2 + (20 - x)^2 + 16^2 + x^2 \\ &= 2x^2 - 40x + 1140, \quad 0 \leq x \leq 20. \end{aligned}$$

Quick Tip to Solve: For the sum of squared ladder lengths, the pole heights contribute only constants; minimize $x^2 + (20 - x)^2$.

We minimize this sum directly, because the question asks for the sum of squares rather than the sum of ladder lengths.

Step 2: Locate and verify the minimum Differentiating the quadratic gives

$$S'(x) = 4x - 40, \quad S''(x) = 4 > 0.$$

The stationary condition is $4x - 40 = 0$, giving $x = 10$, which lies inside the permitted interval. To confirm the minimum over the entire road, write

$$S(x) = 2(x - 10)^2 + 940 \geq 940.$$

Equality occurs precisely at $x = 10$; both road edges give a larger value.

Step 3: Conclusion The sum $l_1^2 + l_2^2$ is minimum when $x = 10$ m, so the common foot of the ladders is midway between the poles.

Quick Tip to Solve: For the sum of squared ladder lengths, the pole heights contribute only constants; minimize $x^2 + (20 - x)^2$.

OR

38(iii)(b). If the 22 m long pole is also replaced by a 16 m long pole, at what distance from either pole should the ladders be kept so that the sum of squares of lengths of ladders needed to reach the top of the pole is minimum ?

[2 marks]

Solution:

Concept:

- Pythagoras' theorem gives $l^2 = h^2 + d^2$, where h is the pole height and d is the horizontal distance of the ladder's foot from its base.
- To minimize a differentiable function on an interval, locate its stationary point and verify that it gives the least value. A positive second derivative identifies a strict local minimum.

Step 1: Write the objective for equal pole heights After replacement both poles have height 16 m, and their bases remain 20 m apart. Let the common foot of the ladders be x m from one pole and $(20 - x)$ m from the other, with $0 \leq x \leq 20$. The sum of their squared lengths is

$$\begin{aligned} S(x) &= (16^2 + x^2) + (16^2 + (20 - x)^2) \\ &= 2x^2 - 40x + 912. \end{aligned}$$

Step 2: Minimize the quadratic on the road Differentiate with respect to x :

$$S'(x) = 4x - 40, \quad S''(x) = 4 > 0.$$

Quick Tip to Solve: With equal pole heights, symmetry suggests the midpoint; confirm it by minimizing $x^2 + (20 - x)^2$, whose derivative is $4x - 40$.

Setting $S'(x) = 0$ gives $x = 10$. This is an allowed position between the poles. Also,

$$S(x) = 2(x - 10)^2 + 712 \geq 712.$$

Thus the stationary point is the unique absolute minimum on the road, since the squared term is nonnegative and vanishes only there. The other horizontal distance is $20 - 10 = 10$ m as well.

Step 3: Conclusion The ladders should have their common foot at the midpoint of the road, 10 m from either pole.

Quick Tip to Solve: With equal pole heights, symmetry suggests the midpoint; confirm it by minimizing $x^2 + (20 - x)^2$, whose derivative is $4x - 40$.