

CBSE Class 12 Mathematics Question Paper 2026 with Solution PDF-Set 65-5-2

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38** questions. **All questions are compulsory.**
- (ii) This question paper is divided into **five** Sections – **A, B, C, D and E.**
- (iii) In **Section A**, Questions no. **1 to 18** are Multiple Choice Questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B**, Questions no. **21 to 25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C**, Questions no. **26 to 31** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D**, Questions no. **32 to 35** are Long Answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E**, Questions no. **36 to 38** are case study based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (ix) Use of calculator is not allowed.

Section A (20 marks)

This section comprises Multiple Choice Questions (MCQs) of 1 mark each.

1. If $A = \begin{pmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix}$ and $A + A' = I$, then θ is equal to:

[1 mark]

- (A) 0
- (B) $\frac{\pi}{6}$
- (C) $\frac{\pi}{3}$
- (D) π

Correct Answer: (B) $\frac{\pi}{6}$

Solution:

Concept:

- The transpose interchanges rows and columns: $(A')_{ij} = A_{ji}$.
- Equality of matrices requires equality of corresponding entries; I has diagonal entries 1 and off-diagonal entries 0.

Step 1: Compute the transpose and sum Changing rows into columns gives

$$A' = \begin{pmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{pmatrix}, \quad A + A' = \begin{pmatrix} 2 \sin \theta & 0 \\ 0 & 2 \sin \theta \end{pmatrix}.$$

The opposite off-diagonal terms cancel.

Step 2: Compare with the identity and test the options The diagonal condition is $2 \sin \theta = 1$, so $\sin \theta = 1/2$. At the listed angles $0, \pi/6, \pi/3, \pi$, the sine values are respectively $0, 1/2, \sqrt{3}/2, 0$. Thus only the second listed angle satisfies the required matrix equality. Other angles can satisfy the equation, but they are not among the printed choices.

Step 3: Conclusion $\theta = \frac{\pi}{6}$ among the given choices. Hence, the correct option is (B).

Quick Tip to Solve: Add a matrix to its transpose first: here the off-diagonal terms cancel, leaving only the sine condition.

2. If A is a symmetric matrix, then for any matrix B of order same as A , BAB' is a/an:

[1 mark]

- (A) Skew symmetric matrix
- (B) Identity matrix
- (C) Symmetric matrix
- (D) Null matrix

Correct Answer: (C) Symmetric matrix

Solution:

Concept:

- A symmetric matrix satisfies $A' = A$.
- Transposition reverses product order: $(XYZ)' = Z'Y'X'$, and $(B')' = B$.

Step 1: Transpose the given product Let $M = BAB'$. All matrices are square of the same order, so the products are defined. Transpose each factor in reverse order:

$$M' = (BAB')' = (B')'A'B' = BAB'.$$

Here we used the given symmetry of A .

Step 2: Identify the guaranteed property Since $M' = M$, the product is symmetric for every allowed B . Identity and null matrices are possible in special cases but are not forced: for instance, $A = 2I$ and $B = I$ give $M = 2I$, which is neither identity nor null. This example is also not skew symmetric, since $M' \neq -M$.

Step 3: Conclusion BAB' is a symmetric matrix. Hence, the correct option is (C).

Quick Tip to Solve: For a product shaped like BAB' , transpose in reverse order and substitute $A' = A$.

3. Let $A = \begin{pmatrix} 0 & -3 & 4 \\ 1 & 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 0 & 1 \\ 2 & 4 & 0 \end{pmatrix}$. If $A + B + C = O$, then matrix C is:

[1 mark]

(A) $\begin{pmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{pmatrix}$

(B) $\begin{pmatrix} 3 & 3 & 5 \\ -3 & -4 & -2 \end{pmatrix}$

(C) $\begin{pmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{pmatrix}$

(D) $\begin{pmatrix} -3 & -3 & -5 \\ 3 & 4 & 2 \end{pmatrix}$

Correct Answer: (C) $\begin{pmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{pmatrix}$

Solution:

Concept:

- Matrices of the same order are added entry by entry.
- The additive inverse of a matrix M is $-M$, formed by changing every entry sign.

Step 1: Add the two printed matrices Use the entry -3 in the first row, second column of A . The entrywise addition is

$$A + B = \begin{pmatrix} 0 - 3 & -3 + 0 & 4 + 1 \\ 1 + 2 & 0 + 4 & 2 + 0 \end{pmatrix} = \begin{pmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{pmatrix}.$$

All six entries must be computed before selecting an option.

Step 2: Take the additive inverse and compare The equation $A + B + C = O$ gives

$$C = -(A + B) = \begin{pmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{pmatrix}.$$

Adding this matrix to $A + B$ makes every entry zero. Option (A) retains the positive 5 and the second row signs; (B) fails to negate the 5; (D) has incorrect first two entries and second row signs.

Step 3: Conclusion $C = \begin{pmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{pmatrix}$. Hence, the correct option is (C).

Quick Tip to Solve: For $A + B + C = O$, find $A + B$ and negate every entry, including zeros and negative entries.

4. For the inverse trigonometric functions, which of the following Principal Value Branch is **not** correctly defined? [1 mark]

(A) $\tan^{-1} : \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(B) $\sec^{-1} : \mathbb{R} - (-1, 1) \rightarrow [0, \pi] - \left\{\frac{\pi}{2}\right\}$

(C) $\cot^{-1} : \mathbb{R} \rightarrow (0, \pi)$

(D) $\operatorname{cosec}^{-1} : \mathbb{R} - (-1, 1) \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Correct Answer: (D) $\operatorname{cosec}^{-1} : \mathbb{R} - (-1, 1) \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Solution:

Concept:

- An inverse trigonometric principal branch uses a one-to-one restriction of the original function.
- The principal range of $\operatorname{cosec}^{-1} x$ is $[-\pi/2, \pi/2] \setminus \{0\}$, since cosecant is undefined at zero.

Step 1: Check the first three principal branches The tangent inverse maps $\mathbb{R}$ into $(-\pi/2, \pi/2)$, with both endpoints excluded. The secant inverse has domain $\mathbb{R} \setminus (-1, 1)$ and range $[0, \pi] \setminus \{\pi/2\}$, exactly as option (B) prints. The cotangent inverse maps $\mathbb{R}$ into $(0, \pi)$. Thus options (A), (B) and (C) correctly specify the standard branches.

Step 2: Find the missing exclusion in the last branch Option (D) includes 0 in its stated range. However, $\operatorname{cosec} 0 = 1/\sin 0$ is undefined, and there is no real input whose inverse cosecant equals zero. The two endpoint angles are allowed, giving cosecant values -1 and 1 ; only the middle value 0 must be excluded.

Step 3: Conclusion Option (D) is incorrectly defined because its range must exclude 0 . Hence, the correct option is (D).

Quick Tip to Solve: Check undefined angles inside each stated inverse range: inverse secant excludes $\pi/2$, and inverse cosecant excludes 0 .

5. If $f(x) = \begin{cases} \frac{x^2 - 4x - 5}{x + 1}, & x \neq -1 \\ k, & x = -1 \end{cases}$ is continuous at $x = -1$, then the value of k is: [1 mark]

(A) Any real value

(B) 6

(C) -1

(D) -6

Correct Answer: (D) -6

Solution:

Concept:

- Continuity at $x = a$ requires $\lim_{x \rightarrow a} f(x) = f(a)$.
- A common factor may be cancelled when computing a limit through points where that factor is

Quick Tip to Solve: Factor the numerator before substituting at a removable discontinuity, then set the separate value equal to the limit.

nonzero.

Step 1: Factor and simplify near the specified point The numerator factors as $x^2 - 4x - 5 = (x - 5)(x + 1)$. For $x \neq -1$,

$$f(x) = \frac{(x - 5)(x + 1)}{x + 1} = x - 5.$$

This cancellation concerns values near -1 , and does not substitute into the original zero denominator.

Step 2: Apply the continuity condition Both the left-hand and right-hand limits equal the value of the polynomial expression at -1 :

$$\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} (x - 5) = -1 - 5 = -6.$$

Since the separately defined value is $f(-1) = k$, continuity fixes one unique value. Thus an arbitrary real number, $+6$, and -1 cannot make the function continuous.

Step 3: Conclusion $k = -6$. Hence, the correct option is (D).

Quick Tip to Solve: Factor the numerator before substituting at a removable discontinuity, then set the separate value equal to the limit.

6. If A is a non-singular matrix, then which of the following is *not* true?

[1 mark]

- (A) $\text{adj } A$ is singular
- (B) $(\text{adj } A)^{-1} = \text{adj}(A^{-1})$
- (C) $|A| \neq 0$
- (D) A^{-1} exists

Correct Answer: (A) $\text{adj } A$ is singular

Solution:

Concept:

- A non-singular square matrix has $|A| \neq 0$, so A^{-1} exists.
- The adjoint identity $A \text{adj } A = |A|I$ gives $\text{adj } A = |A|A^{-1}$.

Step 1: Determine whether the adjoint is invertible Because $|A| \neq 0$ and A^{-1} is invertible, their scalar product $\text{adj } A = |A|A^{-1}$ is also invertible. In particular,

$$(\text{adj } A)^{-1} = \frac{1}{|A|}A.$$

Thus the assertion that the adjoint is singular is false.

Step 2: Verify the remaining printed statements Applying the same identity to A^{-1} , and using $|A^{-1}| = 1/|A|$, gives

$$\text{adj}(A^{-1}) = |A^{-1}|(A^{-1})^{-1} = \frac{1}{|A|}A.$$

This equals the expression above, so option (B) is true. Options (C) and (D) are precisely the determinant and inverse conditions for a non-singular matrix, hence are true as well.

Quick Tip to Solve: For non-singular A , replace its adjoint by $|A|A^{-1}$; it immediately shows that the adjoint is invertible.

Step 3: Conclusion The false statement is that $\text{adj } A$ is singular. Hence, the correct option is (A).

Quick Tip to Solve: For non-singular A , replace its adjoint by $|A|A^{-1}$; it immediately shows that the adjoint is invertible.

7. Absolute minimum value of $f(x) = (x - 2)^2 + 5$ in the interval $[-3, 2]$ is:

[1 mark]

- (A) -3
- (B) 2
- (C) 5
- (D) 30

Correct Answer: (C) 5

Solution:

Concept:

- A square is nonnegative: $(x - 2)^2 \geq 0$ for every real x .
- An absolute minimum is the least function value attained in the specified interval, including endpoints.

Step 1: Find a lower bound and check attainment For the given function,

$$f(x) = (x - 2)^2 + 5 \geq 5.$$

Equality occurs exactly when $x - 2 = 0$, namely at $x = 2$. The interval $[-3, 2]$ includes this endpoint, so the lower bound is actually attained there.

Step 2: Distinguish the minimum value from its location The endpoint values are $f(-3) = (-5)^2 + 5 = 30$ and $f(2) = 5$. Also $f'(x) = 2(x - 2) \leq 0$ throughout the interval, confirming that the function decreases towards its right endpoint. Option (B) gives the location of the minimum, not its value; option (A) is the left endpoint; option (D) gives the left endpoint function value.

Step 3: Conclusion The absolute minimum value is 5 , attained at $x = 2$. Hence, the correct option is (C).

Quick Tip to Solve: For $(x - a)^2 + c$, test whether a belongs to the interval; if it does, the minimum value is c .

8. If the area of $\triangle ABC$ with vertices $A(3, 1)$, $B(-2, 1)$ and $C(0, k)$ is 5 sq. units, then values of k are: [1 mark]

- (A) 3, 1
- (B) $-1, 3$
- (C) $-1, 2$
- (D) 0, 2

Correct Answer: (B) $-1, 3$

Solution:

Concept:

- The triangle area is half the product of base length and perpendicular height.
- For a horizontal base on $y = 1$, a point with ordinate k has perpendicular distance $|k - 1|$ from the base line.

Step 1: Compute the base and perpendicular height The vertices $A(3, 1)$ and $B(-2, 1)$ have the same ordinate, so AB is horizontal with length $|3 - (-2)| = 5$. The third vertex is $C(0, k)$, whose distance from the line $y = 1$ is $|k - 1|$. Hence

$$\text{Area} = \frac{1}{2}(5)|k - 1|.$$

The absolute value allows the vertex to lie above or below the base.

Step 2: Solve both cases and test the choices The area condition gives

$$\frac{5}{2}|k - 1| = 5 \Rightarrow |k - 1| = 2.$$

Thus $k - 1 = 2$ or $k - 1 = -2$, giving $k = 3$ or $k = -1$. The alternative value 1 would give zero area, 2 would give area $5/2$, and 0 would also give area $5/2$; these values cannot replace either required ordinate.

Step 3: Conclusion The values are $k = -1, 3$. Hence, the correct option is (B).

Quick Tip to Solve: When two triangle vertices share an ordinate, use their horizontal separation as the base and an absolute ordinate difference as height.

9. Derivative of $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$, with respect to x is:

[1 mark]

- (A) -1
- (B) 1
- (C) $\frac{\pi}{4}$
- (D) $-\frac{\pi}{4}$

Correct Answer: (A) -1

Solution:

Concept:

- Use $\sin x + \cos x = \sqrt{2} \cos(\pi/4 - x)$.
- The principal inverse cosine returns angles in $[0, \pi]$.

Step 1: Check the inverse branch Put $t = \pi/4 - x$. The given interval implies $0 < t < \pi/2$, so $\cos^{-1}(\cos t) = t$. Thus the expression is exactly $\pi/4 - x$, with no branch change inside the stated interval.

Step 2: Differentiate and test the options The derivative is $d(\pi/4 - x)/dx = 0 - 1 = -1$. Option (B) has the opposite sign and would follow from reversing the angle incorrectly. Options (C) and (D) give constant angles rather than the slope of the simplified linear function.

Step 3: Conclusion The derivative is $\boxed{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: First place the angle inside the principal inverse-cosine interval, then differentiate.

10. $\int \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} dx$ is equal to :

[1 mark]

- (A) $\log |\sec^2 x| + C$
- (B) $\log |\cos x| + C$
- (C) $\log |1 + \cos 2x| + C$
- (D) $\log |\sec x| + C$

Correct Answer: The integrand is $|\tan x|$; without a stated interval no single listed primitive is generally valid. (D) works for nonnegative tangent and (B) for nonpositive tangent.

Solution:

Concept:

- Use $1 - \cos 2x = 2 \sin^2 x$ and $1 + \cos 2x = 2 \cos^2 x$.
- The nonnegative square root obeys $\sqrt{u^2} = |u|$.

Step 1: Simplify while preserving the sign Where $\cos x \neq 0$, the integrand equals $\sqrt{\tan^2 x} = |\tan x|$. Therefore on intervals with positive tangent its primitive is $\log |\sec x| + C$, whereas on intervals with negative tangent its primitive is $\log |\cos x| + C$.

Step 2: Differentiate all four choices The derivatives of (A), (B), (C), and (D) are respectively $2 \tan x$,

Quick Tip to Solve: Retain the absolute value when simplifying the square root of a squared trigonometric function.

$-\tan x$, $-2\tan x$, and $\tan x$. None agrees with $|\tan x|$ throughout the unrestricted printed domain. Option (D) is the conventional intended choice on intervals with nonnegative tangent; (B) works on intervals with nonpositive tangent.

Step 3: Conclusion As printed, no single option is valid throughout the domain. The intended option is (D) when $\tan x \geq 0$.

Quick Tip to Solve: Retain the absolute value when simplifying the square root of a squared trigonometric function.

11. The value of $\int_0^1 \frac{1}{3x-4} dx$ is :

[1 mark]

- (A) $\frac{1}{3} \log 4$
- (B) $-\frac{1}{3} \log 4$
- (C) $\log(-4)$
- (D) $\log 4$

Correct Answer: (B) $-\frac{1}{3} \log 4$

Solution:

Concept:

- Use $\int dx/(ax+b) = \log|ax+b|/a + C$.
- The definite integral is the primitive at the upper endpoint minus that at the lower endpoint.

Step 1: Integrate the rational function The denominator $3x-4$ never vanishes on $[0, 1]$; indeed it is negative throughout. Substitution $u = 3x-4$, $du = 3 dx$, gives the real primitive $\frac{1}{3} \log|3x-4|$.

Step 2: Evaluate and compare

$$I = \left[\frac{1}{3} \log|3x-4| \right]_0^1 = \frac{1}{3} (\log 1 - \log 4) = -\frac{1}{3} \log 4.$$

Options (A) and (D) are positive, contradicting the negative integrand. Option (C) uses the non-real quantity $\log(-4)$ instead of the logarithm of the absolute value and misses the factor $1/3$.

Step 3: Conclusion $-\frac{1}{3} \log 4$. Hence, the correct option is (B).

Quick Tip to Solve: The sign of the integrand over the whole interval quickly rules out positive proposed values.

12. The general solution for the differential equation $\frac{dy}{dx} = e^{3x-y}$ is:

[1 mark]

- (A) $3e^y = e^{3x} + C$
- (B) $\log(3x - y) = C$
- (C) $e^{3x-y} = C$
- (D) $-e^y + 3e^{3x} = C$

Correct Answer: (A) $3e^y = e^{3x} + C$

Solution:

Concept:

- Factor $e^{3x-y} = e^{3x}e^{-y}$ to separate the variables.
- The integral of e^{3x} is $e^{3x}/3$.

Step 1: Separate and integrate Multiplying by $e^y dx$ gives $e^y dy = e^{3x} dx$. Hence $e^y = \frac{1}{3}e^{3x} + C_1$. Multiplication by 3 gives $3e^y = e^{3x} + C$, with $C = 3C_1$ still arbitrary.

Step 2: Verify each candidate Differentiating the obtained relation gives $3e^y y' = 3e^{3x}$, recovering $y' = e^{3x-y}$. Options (B) and (C) instead make $3x - y$ constant and force $y' = 3$. Differentiating (D) gives $y' = 9e^{3x-y}$, so its numerical coefficient is incorrect.

Step 3: Conclusion $3e^y = e^{3x} + C$. Hence, the correct option is (A).

Quick Tip to Solve: Differentiate a proposed implicit solution to check the coefficient of its exponential term.

13. The order and degree of the differential equation $\frac{d^2y}{dx^2} = 1 - \left(\frac{d^3y}{dx^3}\right)^2$ is:

[1 mark]

- (A) Order = 3, Degree = 3
- (B) Order = 2, Degree = 2
- (C) Order = 3, Degree = 2
- (D) Order = 2, Degree = 1

Correct Answer: (C) Order = 3, Degree = 2

Solution:

Concept:

- Order means the order of the highest derivative present.
- Degree means the power of the highest-order derivative in a polynomial equation in derivatives.

Step 1: Identify the highest derivative The printed equation is $y'' = 1 - (y''')^2$. It contains derivatives of orders 2 and 3. Thus its order is 3, even though the second derivative is isolated on the left.

Step 2: Read the degree after rearranging Write the equation as

$$(y''')^2 + y'' - 1 = 0.$$

It is already polynomial in the derivatives. The third derivative has exponent 2, so the degree is 2. Option (A) gives the wrong degree; (B) and (D) overlook the third derivative and therefore give the

Quick Tip to Solve: Find the highest derivative anywhere in the equation before reading its exponent.

wrong order.

Step 3: Conclusion Order = 3, Degree = 2. Hence, the correct option is (C).

Quick Tip to Solve: Find the highest derivative anywhere in the equation before reading its exponent.

14. An ant is observed crawling on a sheet of paper along a straight line given by equation $y = 2x - 4$. Area of the surface covered by the ant bounded by y -axis, x -axis and $x = 1$ is: [1 mark]

- (A) 1 sq. unit
- (B) 3 sq. units
- (C) 2 sq. units
- (D) 4 sq. units

Correct Answer: (B) 3 sq. units

Solution:

Concept:

- Geometric area is the integral of the positive vertical height.
- The y -axis and $x = 1$ give integration limits 0 and 1.

Step 1: Determine the vertical height On $0 \leq x \leq 1$, the line $y = 2x - 4$ lies below the horizontal axis, from $y = -4$ to $y = -2$. Thus the height from the line to the axis is $0 - (2x - 4) = 4 - 2x$, rather than the negative ordinate itself.

Step 2: Compute and check the area

$$A = \int_0^1 (4 - 2x) dx = [4x - x^2]_0^1 = 4 - 1 = 3.$$

As a check, the region is a trapezium with parallel sides of lengths 4 and 2 separated by distance 1, giving $\frac{1}{2}(4 + 2) \times 1 = 3$. None of the other printed values equals this positive area.

Step 3: Conclusion The area is 3 square units. Hence, the correct option is (B).

Quick Tip to Solve: For a curve below the horizontal axis, integrate zero minus its ordinate to obtain positive area.

15. If position vector $\vec{p}$ of a point $(24, n)$ is such that $|\vec{p}| = 25$, then the value of n is:

[1 mark]

- (A) ± 49
- (B) ± 5
- (C) ± 1
- (D) ± 7

Correct Answer: (D) ± 7

Solution:

Concept:

- The position vector of (a, b) is $a\hat{i} + b\hat{j}$.
- Its magnitude is $\sqrt{a^2 + b^2}$.

Step 1: Apply the magnitude formula The vector is $\vec{p} = 24\hat{i} + n\hat{j}$. Hence $\sqrt{24^2 + n^2} = 25$, or $576 + n^2 = 625$. Squaring is valid since both sides of the magnitude equation are nonnegative.

Step 2: Solve and compare the choices Thus $n^2 = 625 - 576 = 49$, so $n = \pm 7$. Both signs give $\sqrt{576 + 49} = 25$. Option (A) confuses the square of the coordinate with the coordinate itself. The values in (B) and (C) give squared magnitudes 601 and 577, respectively, and therefore fail the given condition.

Step 3: Conclusion $n = \pm 7$. Hence, the correct option is (D).

Quick Tip to Solve: Use the 7-24-25 right-triangle triple and keep both signs because magnitude squares the coordinate.

16. The corner points of the feasible region determined by the system of linear constraints are $(0, 0)$, $(0, 40)$, $(20, 40)$, $(60, 20)$ and $(60, 0)$. If the objective function of an LPP is $Z = 4x + 3y$, then the maximum value is:

[1 mark]

- (A) 200
- (B) 300
- (C) 240
- (D) 120

Correct Answer: (B) 300

Solution:

Concept:

- A linear objective on a bounded polygonal feasible region attains its maximum at a corner.
- Evaluate $Z = 4x + 3y$ at every listed corner.

Quick Tip to Solve: Evaluate the objective at every corner, including those away from the coordinate axes.

Step 1: Calculate the corner values

$$\begin{aligned}Z(0, 0) &= 0, \\Z(0, 40) &= 120, \\Z(20, 40) &= 80 + 120 = 200, \\Z(60, 20) &= 240 + 60 = 300, \\Z(60, 0) &= 240.\end{aligned}$$

Each vertex is tested with the same objective coefficients, covering all candidates for the maximum.

Step 2: Compare all candidate values The greatest of 0, 120, 200, 300, 240 is 300, reached at (60, 20). The other printed answers 200, 240 and 120 are feasible corner values, but each is smaller. Maximizing just one coordinate would not justify the objective maximum, since both coordinates contribute.

Step 3: Conclusion The maximum is 300, attained at (60, 20). Hence, the correct option is (B).

Quick Tip to Solve: Evaluate the objective at every corner, including those away from the coordinate axes.

17. If $3P(A) = P(B) = \frac{3}{5}$ and $P(A|B) = \frac{2}{3}$, then $P(A \cup B)$ is:

[1 mark]

- (A) $\frac{3}{5}$
- (B) $\frac{1}{5}$
- (C) $\frac{2}{15}$
- (D) $\frac{2}{5}$

Correct Answer: (D) $\frac{2}{5}$

Solution:

Concept:

- Conditional probability satisfies $P(A | B) = P(A \cap B)/P(B)$, when $P(B) > 0$.
- The addition rule is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Step 1: Read the individual probabilities The factor 3 multiplies only $P(A)$. Thus $P(A) = 1/5$ and $P(B) = 3/5$; treating both probabilities as $3/5$ would misread the given equality.

Step 2: Apply the conditional and addition formulae Using the supplied numbers gives

$$P(A \cap B) = \frac{2}{3} \cdot \frac{3}{5} = \frac{2}{5}, \quad P(A \cup B) = \frac{1}{5} + \frac{3}{5} - \frac{2}{5} = \frac{2}{5}.$$

Options (A), (B) and (C) give different numbers and do not follow this substitution. There is a consistency issue in the printed data: the calculated intersection exceeds $P(A) = 1/5$, so no actual events can satisfy all the premises. Nevertheless, the intended formal calculation has a listed result.

Step 3: Conclusion The intended numerical answer is 2/5, option (D).

Quick Tip to Solve: First divide the equality $3P(A) = 3/5$ by 3; then compute the intersection before using the union rule.

18. If vectors $\vec{a} = 3\hat{i} + 2\hat{j} + \lambda\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$, represent the two strips of the Red Cross sign placed outside a doctor's clinic, then the value of λ is: [1 mark]

- (A) 1
- (B) $\frac{5}{2}$
- (C) $\frac{2}{5}$
- (D) 0

Correct Answer: (C) $\frac{2}{5}$

Solution:

Concept:

- The two strips of a Red Cross sign are perpendicular.
- For nonzero perpendicular vectors, $\vec{a} \cdot \vec{b} = 0$; multiply corresponding components and add.

Step 1: Translate the geometry Because the strips meet at a right angle, their direction vectors must have zero scalar product. Their magnitudes need not be equal: it is their angle that determines the condition.

Step 2: Substitute the components

$$\vec{a} \cdot \vec{b} = 3(2) + 2(-4) + \lambda(5) = 6 - 8 + 5\lambda.$$

Equating this to zero gives $5\lambda = 2$. Option (A) would give a dot product of 3, option (B) gives $21/2$, and option (D) gives -2 ; each is nonzero.

Step 3: Conclusion $\lambda = 2/5$. Hence the correct option is (C).

Quick Tip to Solve: Use the dot product for perpendicular directions and retain the negative sign of the second vector's j-component.

19. Assertion (A): A relation R on the set $\{1, 2, 3\}$ defined as $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$ is an equivalence relation.

Reason (R): A relation that is reflexive, symmetric and transitive is an equivalence relation. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- An equivalence relation is reflexive, symmetric and transitive.
- Reflexivity requires every (x, x) ; symmetry reverses pairs; transitivity combines (x, y) and (y, z) .

Step 1: Judge the assertion All three diagonal pairs $(1, 1), (2, 2), (3, 3)$ occur, so the relation is reflexive. The only distinct-element pair $(1, 2)$ has its reverse $(2, 1)$, so it is symmetric. Within $\{1, 2\}$ every ordered pair occurs, while 3 relates only to itself. Every possible two-pair chain therefore has its required endpoint pair; the relation is transitive. Assertion (A) is true.

Step 2: Judge the reason Reason (R) states precisely the definition of an equivalence relation. It is therefore true.

Step 3: Check the explanatory link The three properties verified for this particular relation are exactly the criteria in the reason. Thus the reason explains why this relation is an equivalence relation. Option (B) incorrectly denies this link, and options (C) and (D) incorrectly declare one statement false.

Step 4: Conclusion Both statements are true and the reason correctly explains the assertion. The correct option is (A).

Quick Tip to Solve: Partition this set into $\{1, 2\}$ and $\{3\}$: complete pairing within each block immediately verifies transitivity.

20. Assertion (A): Consider a Linear Programming Problem with minimise $Z = x + 2y$ subject to constraints $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$ which gives minimum Z at infinitely many points. The corner points of feasible region are $(0, 3)$ and $(6, 0)$.

Reason (R): If two corner points produce the same minimum value of the objective function, then every point on the line segment joining the points will give the same minimum value. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- The feasible region is the intersection of all the given half-planes, including $x, y \geq 0$.
- A linear objective has the same value throughout a feasible segment whose endpoints have the same objective value.

Step 1: Judge the assertion From $x + 2y \geq 6$, every feasible point has $Z \geq 6$. On the line $x + 2y = 6$ in the first quadrant, write $y = 3 - x/2$, with $0 \leq x \leq 6$. Then $2x + y = 3 + 3x/2 \geq 3$. Thus the entire segment from $(0, 3)$ to $(6, 0)$ is feasible and attains 6. The other constraint does not cut off any part of this segment. These endpoints are the two corner points, so the assertion is true.

Step 2: Judge the reason For endpoints P, Q with the same minimum objective value 6, any convex combination $(1-t)P + tQ$, $0 \leq t \leq 1$, remains feasible. Linearity gives $Z((1-t)P + tQ) = (1-t)6 + t6 = 6$. Hence the reason is true.

Step 3: Check whether the reason explains the assertion The corner values are $Z(0, 3) = 6$ and $Z(6, 0) = 6$. Applying the reason to these two distinct feasible corners gives infinitely many minimizing points on their joining segment. This directly explains the assertion, ruling out (B); neither statement is false, ruling out (C) and (D).

Step 4: Conclusion The minimum is $Z = 6$ on the whole segment joining $(0, 3)$ and $(6, 0)$. Correct option: (A).

Quick Tip to Solve: When a constraint is the objective lower bound itself, check whether its boundary segment satisfies the remaining constraints.

Section B (10 marks)

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Evaluate $\sin \left[\cos^{-1} \cos \left(\frac{7\pi}{6} \right) \right]$.

[2 marks]

Solution:

Concept:

- The principal range of $\cos^{-1}$ is $[0, \pi]$.
- For $\theta \in [0, \pi]$, $\sin \theta \geq 0$ and $\sin \theta = \sqrt{1 - \cos^2 \theta}$.

Step 1: Evaluate the inner cosine The angle $7\pi/6 = \pi + \pi/6$ lies in the third quadrant, where cosine is negative. Hence

$$\cos \left(\frac{7\pi}{6} \right) = -\cos \left(\frac{\pi}{6} \right) = -\frac{\sqrt{3}}{2}.$$

It is essential to evaluate this cosine before taking its inverse, because the supplied angle is outside the principal range.

Step 2: Choose the principal inverse-cosine value Let $\theta = \cos^{-1}(-\sqrt{3}/2)$. In $[0, \pi]$, the angle with this cosine is $5\pi/6$, since $\cos(\pi - \pi/6) = -\sqrt{3}/2$. Thus $\theta = 5\pi/6$, not $7\pi/6$.

Step 3: Evaluate the outer sine

$$\sin \theta = \sin \left(\frac{5\pi}{6} \right) = \sin \left(\frac{\pi}{6} \right) = \frac{1}{2}.$$

The positive sign also follows from the principal range: the sine of any angle returned by $\cos^{-1}$ is nonnegative.

Step 4: Conclusion

$$\sin \left[\cos^{-1} \cos \left(\frac{7\pi}{6} \right) \right] = \frac{1}{2}.$$

Quick Tip to Solve: Do not cancel cosine with inverse cosine outside $[0, \pi]$; first reflect the angle into that principal range.

22(a). Three honey bees were found flying along the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 4\hat{j} - 2\hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{k}$ respectively. Find the value of λ such that the path for $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$. [2 marks]

Solution:

Concept:

- Perpendicular vectors have zero scalar product.
- The scalar product is distributive: $(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = \vec{a} \cdot \vec{c} + \lambda(\vec{b} \cdot \vec{c})$.

Quick Tip to Solve: Compute $\vec{a} \cdot \vec{c}$ and $\vec{b} \cdot \vec{c}$ separately; here they are 8 and -4, respectively.

Step 1: Form the required direction vector Using the components given in the question,

$$\vec{a} + \lambda \vec{b} = 2\hat{i} + (-3 + 4\lambda)\hat{j} + (1 - 2\lambda)\hat{k}.$$

The coefficient of $\hat{i}$ is unchanged because $\vec{b}$ has zero i-component. Also $\vec{c} = 3\hat{i} + 0\hat{j} + 2\hat{k}$, so its j-component contributes nothing to the dot product.

Step 2: Impose perpendicularity

$$\begin{aligned}(\vec{a} + \lambda \vec{b}) \cdot \vec{c} &= 2(3) + (-3 + 4\lambda)(0) + (1 - 2\lambda)(2) \\ &= 8 - 4\lambda.\end{aligned}$$

The required right angle gives $8 - 4\lambda = 0$, hence $\lambda = 2$.

Step 3: Check the obtained value At $\lambda = 2$, the direction is $2\hat{i} + 5\hat{j} - 3\hat{k}$, which is nonzero. Its dot product with $3\hat{i} + 2\hat{k}$ is $6 - 6 = 0$, confirming perpendicularity.

Step 4: Conclusion The required scalar is $\boxed{\lambda = 2}$.

Quick Tip to Solve: Compute $\vec{a} \cdot \vec{c}$ and $\vec{b} \cdot \vec{c}$ separately; here they are 8 and -4, respectively.

OR

22(b). If A, B and C be three non-collinear points such that $\vec{AB} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{AC} = 2\hat{i} - 3\hat{j}$, then find the area of $\triangle ABC$. [2 marks]

Solution:

Concept:

- The magnitude $|\vec{AB} \times \vec{AC}|$ is the area of the parallelogram on these two adjacent sides.
- The triangle has half that area: $\text{Area}(ABC) = \frac{1}{2}|\vec{AB} \times \vec{AC}|$.

Step 1: Calculate the vector product The sides originate at the same vertex A, so the area formula applies directly. Insert the missing k-component of $\vec{AC}$ as zero:

$$\begin{aligned}\vec{AB} \times \vec{AC} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -3 & 0 \end{vmatrix} \\ &= \hat{i}(0 - 3) - \hat{j}(0 + 2) + \hat{k}(-3 - 4) \\ &= -3\hat{i} - 2\hat{j} - 7\hat{k}.\end{aligned}$$

In particular, the middle cofactor carries a minus sign.

Step 2: Take the magnitude and halve it The three unit vectors are mutually perpendicular, so

$$|\vec{AB} \times \vec{AC}| = \sqrt{(-3)^2 + (-2)^2 + (-7)^2} = \sqrt{62}.$$

This is the parallelogram area. The diagonal divides that parallelogram into two equal triangles, so

Quick Tip to Solve: Use two side vectors from the same vertex; take the cross-product magnitude and remember the triangle factor 1/2.

the requested triangle area is half this value. The nonzero cross product is consistent with the given non-collinearity.

Step 3: Conclusion

$$\text{Area}(\triangle ABC) = \frac{\sqrt{62}}{2} \text{ square units}.$$

Quick Tip to Solve: Use two side vectors from the same vertex; take the cross-product magnitude and remember the triangle factor $1/2$.

23. Determine the interval(s) in which $f(x) = 5x^{3/2} - 3x^{5/2}$, $x > 0$ is increasing.

[2 marks]

Solution:

Concept:

- A differentiable function is increasing where its derivative is positive. Here the stated domain is $x > 0$, so every factor $\sqrt{x}$ is positive.
- The power rule gives $\frac{d}{dx}x^n = nx^{n-1}$; factorisation makes the sign of the derivative easy to determine.

Step 1: Differentiate and factor For $f(x) = 5x^{3/2} - 3x^{5/2}$, differentiate each term:

$$f'(x) = \frac{15}{2}x^{1/2} - \frac{15}{2}x^{3/2} = \frac{15}{2}\sqrt{x}(1 - x).$$

The common factor $\frac{15}{2}\sqrt{x}$ is strictly positive throughout the given domain.

Step 2: Determine the sign on the domain Thus $f'(x) > 0$ is equivalent to $1 - x > 0$. Combining $x < 1$ with $x > 0$ gives $0 < x < 1$. At $x = 1$, the derivative is zero, while for $x > 1$, it is negative. Hence the sign changes from positive to negative at the stationary point; zero is excluded because it is outside the stated domain.

Step 3: Conclusion The function is increasing on $(0, 1)$.

Quick Tip to Solve: Factor out the positive square-root factor before solving the derivative inequality; retain the stated domain $x > 0$.

24. Find the angle between the following pair of lines:

$$\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6} \text{ and } \frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{-2}.$$

[2 marks]

Solution:

Concept:

- For lines with direction vectors $\vec{a}$ and $\vec{b}$, their smaller angle θ satisfies $\cos \theta = \frac{|\vec{a} \cdot \vec{b}|}{|\vec{a}||\vec{b}|}$.

Quick Tip to Solve: Normalize both reversed z-numerators first; reading negative denominators directly would give incorrect direction vectors.

- Write each symmetric equation with numerators $x - x_0, y - y_0, z - z_0$ before reading its direction ratios. Reversing a numerator also reverses its denominator.

Step 1: Read the correct direction ratios The third ratios are $\frac{1-z}{-6} = \frac{z-1}{6}$ and $\frac{6-z}{-2} = \frac{z-6}{2}$. Therefore direction vectors for the two lines are $\vec{a} = (3, 2, 6)$ and $\vec{b} = (1, 2, 2)$. The points through which the lines pass do not affect their angle.

Step 2: Apply the dot-product formula Compute

$$\vec{a} \cdot \vec{b} = 3(1) + 2(2) + 6(2) = 19, \quad |\vec{a}| = \sqrt{9 + 4 + 36} = 7, \quad |\vec{b}| = \sqrt{1 + 4 + 4} = 3.$$

Hence $\cos \theta = \frac{19}{7 \cdot 3} = \frac{19}{21}$. The absolute value selects the smaller angle between the undirected lines, and here the dot product is already positive.

Step 3: Conclusion The angle between the lines is $\theta = \cos^{-1}\left(\frac{19}{21}\right)$.

Quick Tip to Solve: Normalize both reversed z-numerators first; reading negative denominators directly would give incorrect direction vectors.

25(a). Differentiate x^x with respect to $x \log x$.

[2 marks]

Solution:

Concept:

- Logarithmic differentiation applies to $y = x^x$, with $x > 0$: taking the natural logarithm turns the variable power into the product $\log y = x \log x$.
- If $t = x \log x$, differentiation with respect to t uses $\frac{dy}{dt} = \frac{dy/dx}{dt/dx}$ wherever $dt/dx \neq 0$.

Step 1: Differentiate the variable-power function Let $y = x^x$. Taking logarithms and differentiating with respect to x gives

$$\log y = x \log x, \quad \frac{1}{y} \frac{dy}{dx} = \log x + 1.$$

Thus $\frac{dy}{dx} = x^x(1 + \log x)$, rather than the ordinary constant-exponent power-rule expression.

Step 2: Change the differentiation variable Put $t = x \log x$. Then $\frac{dt}{dx} = 1 + \log x$, so

$$\frac{dy}{dt} = \frac{x^x(1 + \log x)}{1 + \log x} = x^x$$

wherever the quotient is defined. Moreover, the identity $y = e^{x \log x} = e^t$ directly gives $dy/dt = e^t$; this also justifies the result at $x = e^{-1}$, where the quotient form has both numerator and denominator zero.

Step 3: Conclusion $\frac{d(x^x)}{d(x \log x)} = x^x$.

Quick Tip to Solve: Rewrite x^x as $e^{x \log x}$: the requested differentiation variable is already its exponent.

25(b). If $y = P \cos(ux) + Q \sin(ux)$, then show that $\frac{d^2y}{dx^2} + u^2y = 0$.

[2 marks]

Solution:**Concept:**

- Treat P, Q, u as constants. The chain rule gives $\frac{d}{dx} \cos(ux) = -u \sin(ux)$ and $\frac{d}{dx} \sin(ux) = u \cos(ux)$.
- Two differentiations of either sine or cosine with constant angular factor u multiply the original function by $-u^2$.

Step 1: Find the first derivative For $y = P \cos(ux) + Q \sin(ux)$, apply the chain rule separately to the two summands:

$$\frac{dy}{dx} = -Pu \sin(ux) + Qu \cos(ux).$$

Each term acquires one factor of u from differentiating the inner expression ux ; the cosine term also acquires a minus sign.

Step 2: Find the second derivative and substitute Differentiate again, keeping all constants outside the derivatives:

$$\begin{aligned} \frac{d^2y}{dx^2} &= -Pu^2 \cos(ux) - Qu^2 \sin(ux) \\ &= -u^2 [P \cos(ux) + Q \sin(ux)] \\ &= -u^2 y. \end{aligned}$$

Adding u^2y to the two sides proves the required identity. Both coefficients P and Q can be arbitrary, because the same multiplier occurs for each summand.

Step 3: Conclusion Hence $\boxed{\frac{d^2y}{dx^2} + u^2y = 0}$, as required.

Quick Tip to Solve: Account for an inner factor u at each differentiation; two chain-rule factors produce u^2 .

Section C (18 marks)

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. A thin metallic wire in the shape of a circular ring has its enclosed area increasing at a uniform rate when heated. Show that the rate of change of circumference varies inversely as the radius. [3 marks]

Solution:**Concept:**

- For a circular ring of radius r , the enclosed area is $A = \pi r^2$ and the circumference is $C = 2\pi r$. As the ring is heated, all three quantities depend on time t .
- An increasing area at a uniform rate means $dA/dt = k$, where $k > 0$ is constant. The chain rule relates this prescribed rate to dr/dt and dC/dt .

Quick Tip to Solve: Eliminate dr/dt between the area and circumference derivatives; the common factor 2π cancels immediately.

Step 1: Express the radius rate using the area rate Let $r = r(t) > 0$ be the instantaneous radius. Differentiate the area formula with respect to time, remembering that radius varies:

$$\frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = 2\pi r \frac{dr}{dt}.$$

Since the area increases at the constant rate k , we obtain

$$k = 2\pi r \frac{dr}{dt}, \quad \frac{dr}{dt} = \frac{k}{2\pi r}.$$

Thus the same fixed area increase requires a smaller increase of radius when the ring is larger.

Step 2: Differentiate the circumference and eliminate the radius rate Using $C = 2\pi r$, differentiate with respect to the same time variable:

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt} = 2\pi \left(\frac{k}{2\pi r} \right) = \frac{k}{r}.$$

Hence the product $r \, dC/dt = k$ is constant. This is precisely the defining form of inverse proportionality: doubling the instantaneous radius halves the circumference growth rate, provided the area growth rate remains fixed. The relation concerns the rate of change of circumference, rather than circumference itself, which remains directly proportional to radius.

Step 3: Conclusion $\boxed{\frac{dC}{dt} = \frac{k}{r}}$, where $k = dA/dt$ is constant; therefore $\boxed{\frac{dC}{dt} \propto \frac{1}{r}}$.

Quick Tip to Solve: Eliminate dr/dt between the area and circumference derivatives; the common factor 2π cancels immediately.

27. Solve the following Linear Programming Problem graphically:

Maximize $Z = \frac{2x}{5} + \frac{3y}{10}$ **subject to constraints**

$$2x + y \leq 1000$$

$$x + y \leq 800$$

$$x, y \geq 0.$$

[3 marks]

Solution:

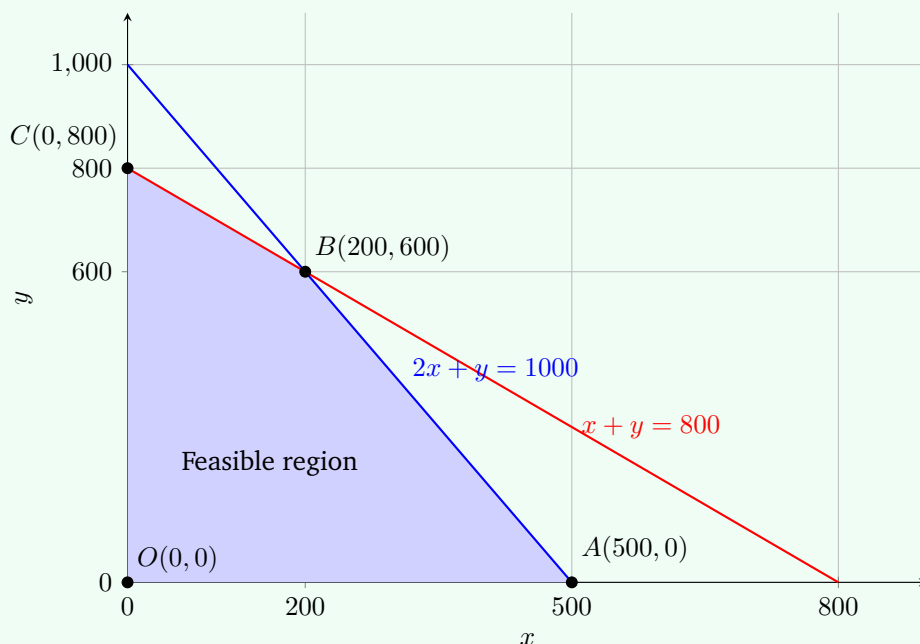
Concept:

- The feasible region is the common part of all constraint half-planes. Here it lies in the first quadrant because $x \geq 0$ and $y \geq 0$.
- For a bounded nonempty polygonal feasible region, a linear objective attains its maximum at a corner point. Evaluate $Z = \frac{2}{5}x + \frac{3}{10}y$ at every vertex.

Step 1: Draw the constraint lines and identify the feasible side The line $2x + y = 1000$ has intercepts $(500, 0)$ and $(0, 1000)$. The line $x + y = 800$ has intercepts $(800, 0)$ and $(0, 800)$. Draw both lines with these intercepts. The origin satisfies both inequalities, so the required side of each line is the side containing the origin. Intersect these half-planes with the first quadrant; the common shaded region

Quick Tip to Solve: Clear fractions by maximizing $4x + 3y$; its maximizing vertex is unchanged, and divide its value by 10.

in the graph is the feasible region.



Feasible region and its corner points

Step 2: Find all corner points The boundary on the x -axis ends at $A = (500, 0)$, while that on the y -axis ends at $C = (0, 800)$. At the intersection of the two oblique boundaries, subtracting $x + y = 800$ from $2x + y = 1000$ gives $x = 200$, and hence $y = 600$. Thus the remaining vertex is $B = (200, 600)$, and the fourth vertex is $O = (0, 0)$. Each of these satisfies every constraint.

Step 3: Evaluate the objective at each vertex Using the corner-point method,

$$\begin{aligned} Z(O) &= 0, \\ Z(A) &= \frac{2}{5}(500) = 200, \\ Z(B) &= \frac{2}{5}(200) + \frac{3}{10}(600) = 80 + 180 = 260, \\ Z(C) &= \frac{3}{10}(800) = 240. \end{aligned}$$

The region is bounded, so these four comparisons are sufficient. The value at the intersection exceeds the values at both axis vertices and the origin.

Step 4: Conclusion The maximum value is $Z_{\max} = 260$, attained at $x = 200, y = 600$.

Quick Tip to Solve: Clear fractions by maximizing $4x + 3y$; its maximizing vertex is unchanged, and divide its value by 10.

28. Solve the differential equation $y dx + (x - y^3) dy = 0$.

[3 marks]

Solution:

Concept:

- Treat x as a function of y . A linear equation $\frac{dx}{dy} + P(y)x = Q(y)$ has integrating factor $IF = e^{\int P(y) dy}$.
- Multiplication by the integrating factor makes the left side an exact product derivative, which can then be integrated with respect to y .

Step 1: Express the equation in linear form The independent variable can conveniently be taken as y , since the coefficient of dy contains x only to its first power. From the printed equation,

$$y \frac{dx}{dy} + x - y^3 = 0.$$

On an interval where $y \neq 0$, division by y gives

$$\frac{dx}{dy} + \frac{1}{y}x = y^2.$$

Thus $P(y) = 1/y$ and $Q(y) = y^2$. Taking y , rather than x , as the independent variable is what makes the equation linear.

Step 2: Find the integrating factor and integrate The integrating factor is $e^{\int dy/y} = e^{\log|y|} = |y|$. On either interval of fixed sign, a nonzero constant multiple is equally valid, so use $IF = y$. Multiplying the linear equation by y ,

$$y \frac{dx}{dy} + x = y^3.$$

By the product rule, this is $\frac{d}{dy}(xy) = y^3$. Integrating both sides with respect to y ,

$$xy = \int y^3 dy = \frac{y^4}{4} + C,$$

where C is an arbitrary real constant.

Step 3: Verify the implicit relation in the original equation Differentiate the resulting relation without dividing by y :

$$d(xy) - d\left(\frac{y^4}{4}\right) = y dx + x dy - y^3 dy = 0.$$

This is exactly the original differential equation, so the implicit form retains the required relation. The constant solution $y = 0$ is also contained in the relation with $C = 0$, whereas the explicit expression for x obtained by division is used only where $y \neq 0$.

Step 4: Conclusion The general solution is $xy - \frac{y^4}{4} = C$. For $y \neq 0$, it may also be written $x = \frac{y^3}{4} + \frac{C}{y}$.

Quick Tip to Solve: Before dividing, notice that $y dx + x dy = d(xy)$; the original equation directly becomes $d(xy) = y^3 dy$.

29(a). Find: $\int \frac{x - \sin x}{1 - \cos x} dx$.

[3 marks]

Solution:

Concept:

- Use the half-angle identities $1 - \cos x = 2 \sin^2(x/2)$ and $\sin x = 2 \sin(x/2) \cos(x/2)$. They give $1/(1 - \cos x) = \frac{1}{2} \csc^2(x/2)$.
- Integration by parts is $\int u dv = uv - \int v du$. Also $\frac{d}{dx} \cot(x/2) = -\frac{1}{2} \csc^2(x/2)$.

Step 1: Split the integrand and simplify with half angles Let I denote the required integral. On an interval where $1 - \cos x \neq 0$,

$$\begin{aligned} I &= \int \frac{x}{1 - \cos x} dx - \int \frac{\sin x}{1 - \cos x} dx \\ &= \frac{1}{2} \int x \csc^2(x/2) dx - \int \cot(x/2) dx. \end{aligned}$$

The second equality follows by cancelling one factor of $\sin(x/2)$ in the sine quotient. Keep the two terms separate so that the cancellation arising in integration by parts remains visible.

Step 2: Apply integration by parts to the first term Take $u = x$ and $dv = \frac{1}{2} \csc^2(x/2) dx$. Then $du = dx$ and $v = -\cot(x/2)$; the factor $1/2$ is essential because the angle is $x/2$. Hence

$$\frac{1}{2} \int x \csc^2(x/2) dx = -x \cot(x/2) + \int \cot(x/2) dx.$$

Substitution into the expression for I makes the two integrals of $\cot(x/2)$ cancel.

Step 3: Check the resulting antiderivative Differentiating the remaining expression gives

$$\begin{aligned} \frac{d}{dx} [-x \cot(x/2)] &= -\cot(x/2) + \frac{x}{2} \csc^2(x/2) \\ &= -\frac{\sin x}{1 - \cos x} + \frac{x}{1 - \cos x} \\ &= \frac{x - \sin x}{1 - \cos x}. \end{aligned}$$

This checks both the sign and the half-angle chain-rule factor. The identity is valid on every interval avoiding $x = 2k\pi$, where the printed denominator vanishes; the integration constant is arbitrary on each such interval.

Step 4: Conclusion

$$\boxed{\int \frac{x - \sin x}{1 - \cos x} dx = -x \cot \frac{x}{2} + C.}$$

Quick Tip to Solve: Look for a product derivative: differentiating $-x \cot(x/2)$ creates exactly the two terms in this integrand.

29(b). Evaluate: $\int_0^2 \frac{1}{\sqrt{x^2 + 2x + 3}} dx$.

[3 marks]

Solution:

Concept:

- Completing the square converts the quadratic to $x^2 + 2x + 3 = (x + 1)^2 + 2$, which is strictly positive throughout the integration interval.
- The standard antiderivative is $\int du/\sqrt{u^2 + c^2} = \ln|u + \sqrt{u^2 + c^2}| + C$, where $c > 0$. A definite integral is the upper endpoint value minus the lower endpoint value.

Step 1: Complete the square and change variable Let the required integral be I . We have

$$I = \int_0^2 \frac{dx}{\sqrt{(x+1)^2 + 2}}.$$

Set $u = x + 1$, so $du = dx$. The lower limit $x = 0$ becomes $u = 1$, while the upper limit $x = 2$ becomes $u = 3$. Thus

$$I = \int_1^3 \frac{du}{\sqrt{u^2 + (\sqrt{2})^2}}.$$

Changing the limits at the same time as the variable avoids mixing an antiderivative in u with endpoints in x .

Step 2: Apply the logarithmic antiderivative Using the standard formula with $c = \sqrt{2}$,

$$\begin{aligned} I &= \left[\ln \left| u + \sqrt{u^2 + 2} \right| \right]_1^3 \\ &= \ln(3 + \sqrt{11}) - \ln(1 + \sqrt{3}) \\ &= \ln \left(\frac{3 + \sqrt{11}}{1 + \sqrt{3}} \right). \end{aligned}$$

The logarithm arguments are positive because $u \geq 1$, so the absolute-value signs can be removed at both endpoints. The numerator exceeds the denominator, giving a positive logarithm, consistent with integrating a positive function over $[0, 2]$.

Step 3: Conclusion The required value is $\boxed{\ln \left(\frac{3 + \sqrt{11}}{1 + \sqrt{3}} \right)}$.

Quick Tip to Solve: Complete the square first, and immediately change both limits when substituting $u = x + 1$.

30(a). A die is rolled. Consider events: $A = \{1, 2, 5\}$, $B = \{3, 5\}$, $C = \{2, 3, 4, 5\}$ **and hence find:**

(i) $P(A|C)$ and $P(C|A)$

(ii) $P(A \cap B|C)$ and $P(A \cup B|C)$

[3 marks]

Solution:

Concept:

- For events E, F with $P(F) > 0$, conditional probability is $P(E | F) = P(E \cap F)/P(F)$. The conditioning event supplies the denominator.
- A fair die has six equally likely outcomes, so $P(E) = n(E)/6$. Consequently, conditional probabilities can be computed by counting favourable outcomes within the conditioning set.

Step 1: Find the intersections for part (i) The sample space is $S = \{1, 2, 3, 4, 5, 6\}$. The given events have sizes $n(A) = 3$ and $n(C) = 4$, and

$$A \cap C = \{2, 5\}, \quad n(A \cap C) = 2.$$

Therefore,

$$P(A | C) = \frac{2/6}{4/6} = \frac{1}{2}, \quad P(C | A) = \frac{2/6}{3/6} = \frac{2}{3}.$$

Although the intersection in the two numerators is identical, their denominators differ. Given C , only its four outcomes remain possible; given A , only its three outcomes remain possible.

Step 2: Find the compound events for part (ii) First, $A \cap B = \{5\}$, and this outcome belongs to C . Hence

$$P(A \cap B | C) = \frac{P(A \cap B \cap C)}{P(C)} = \frac{1/6}{4/6} = \frac{1}{4}.$$

Next, form the union before restricting it to C :

$$A \cup B = \{1, 2, 3, 5\}, \quad (A \cup B) \cap C = \{2, 3, 5\}.$$

Thus

$$P(A \cup B | C) = \frac{3/6}{4/6} = \frac{3}{4}.$$

Outcome 1 is in the union but is excluded by the condition C ; outcome 4 is in C but does not lie in the union.

Step 3: Conclusion (i) $P(A | C) = \frac{1}{2}$, $P(C | A) = \frac{2}{3}$. (ii) $P(A \cap B | C) = \frac{1}{4}$, $P(A \cup B | C) = \frac{3}{4}$.

Quick Tip to Solve: List the conditioning set first; count only outcomes inside it in every conditional-probability numerator.

30(b). A box contains 6 cards numbered 1 to 6. A student is asked to pick up two cards, one by one after replacement and note down the numbers on the cards. Let A be the event of getting sum of the numbers on two cards as 10, and B , the event of a number other than 4 on the first card selected. Find $P(A \text{ and } B)$ and find whether the events A and B are independent events or not. [3 marks]

Solution:

Concept:

- With replacement, each card selection has six equally likely possibilities independently. The sample space consists of $6 \times 6 = 36$ equally likely ordered pairs.
- Events A and B are independent exactly when $P(A \cap B) = P(A)P(B)$. Independence of the two selections does not automatically imply independence of events defined using them.

Step 1: List the outcomes in the required intersection Write an outcome as (i, j) , where i is the first card number and j is the second. Order matters because B restricts the first card. To obtain sum 10, both numbers must lie between 1 and 6, giving precisely

$$A = \{(4, 6), (5, 5), (6, 4)\}.$$

The event B requires $i \neq 4$. It excludes $(4, 6)$, but keeps $(6, 4)$, since a 4 on the second card is allowed. Therefore

$$A \cap B = \{(5, 5), (6, 4)\}, \quad P(A \cap B) = \frac{2}{36} = \frac{1}{18}.$$

Replacement also makes the repeated outcome $(5, 5)$ possible.

Step 2: Compute the marginal probabilities and test independence The three outcomes in A give

$$P(A) = \frac{3}{36} = \frac{1}{12}.$$

For B , the first number can be any of 1, 2, 3, 5, 6, while the second can be any of the six numbers. Hence B contains $5 \times 6 = 30$ outcomes, and

$$P(B) = \frac{30}{36} = \frac{5}{6}.$$

Compare the product with the intersection probability:

$$P(A)P(B) = \frac{1}{12} \cdot \frac{5}{6} = \frac{5}{72}, \quad P(A \cap B) = \frac{1}{18} = \frac{4}{72}.$$

These values differ. Knowing that the sum is 10 changes the proportion of outcomes whose first number is not 4, so the independence criterion fails.

Step 3: Conclusion $P(A \text{ and } B) = \frac{1}{18}$, and A and B are not independent.

Quick Tip to Solve: Use ordered pairs and remove only the pair with first entry 4; a second entry 4 is permitted.

31(a). Let three toys A , B and C be placed in the same straight line. If the position vectors of A , B and C are $55\hat{i} - 2\hat{j}$, $5\hat{i} + 8\hat{j}$ and $a\hat{i} - 52\hat{j}$ respectively, find the value of ' a '. [3 marks]

Solution:

Concept:

- The displacement vector from P to Q is $\overrightarrow{PQ} = \vec{q} - \vec{p}$, where $\vec{p}, \vec{q}$ are their position vectors.
- Three points are collinear when two displacement vectors from the same point are scalar multiples. In two dimensions, this is equivalently the vanishing determinant of their component pairs.

Step 1: Calculate two displacement vectors from A The position vectors are

$$\vec{a}_A = 55\hat{i} - 2\hat{j}, \quad \vec{a}_B = 5\hat{i} + 8\hat{j}, \quad \vec{a}_C = a\hat{i} - 52\hat{j}.$$

Here a is the unknown scalar coordinate, not a displacement vector. Subtract the position vector of A from those of B and C :

$$\begin{aligned}\overrightarrow{AB} &= (5 - 55)\hat{i} + (8 + 2)\hat{j} = -50\hat{i} + 10\hat{j}, \\ \overrightarrow{AC} &= (a - 55)\hat{i} + (-52 + 2)\hat{j} = (a - 55)\hat{i} - 50\hat{j}.\end{aligned}$$

Using a common starting point lets us impose parallelism directly.

Step 2: Impose collinearity and check the value Since the toys lie on one straight line, let $\overrightarrow{AC} = k\overrightarrow{AB}$ for a scalar k . Comparing the $\hat{j}$ components gives

$$-50 = 10k \implies k = -5.$$

Comparing the $\hat{i}$ components then gives

$$a - 55 = (-5)(-50) = 250 \implies a = 305.$$

As a check, this value makes $\overrightarrow{AC} = 250\hat{i} - 50\hat{j} = -5\overrightarrow{AB}$. The negative multiplier indicates opposite directions from A , which is fully consistent with collinearity; the points need not occur in the order A, B, C along the line.

Step 3: Conclusion The required value is $\boxed{a = 305}$.

Quick Tip to Solve: Find the scalar multiplier from the known $\hat{j}$ components, then apply it to the $\hat{i}$ components.

OR

31(b). If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then prove that $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 9$. [3 marks]

Solution:

Concept:

- For any vector $\vec{u}$, $|\vec{u}|^2 = \vec{u} \cdot \vec{u} \geq 0$. A squared magnitude cannot be negative.
- A unit vector has squared magnitude 1. The identity $|\vec{u} - \vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2\vec{u} \cdot \vec{v}$ expands a squared

Quick Tip to Solve: Expand the squared magnitude of the sum of the three unit vectors to control all three dot products simultaneously.

difference.

Step 1: Expand the sum of squared differences Put $S = |\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$. Since all three vectors are unit vectors, each squared difference contributes two unit squared magnitudes. Thus

$$\begin{aligned} S &= (2 - 2\vec{a} \cdot \vec{b}) + (2 - 2\vec{b} \cdot \vec{c}) + (2 - 2\vec{c} \cdot \vec{a}) \\ &= 6 - 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}). \end{aligned}$$

We need a lower bound for the sum of the three dot products. Bounding each difference separately would only give the weaker bound 12.

Step 2: Use the squared magnitude of the vector sum Expanding the squared magnitude of the sum gives

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) \geq 0.$$

Therefore the sum of the dot products is at least $-3/2$. Combining the two expansions also gives the exact identity

$$S = 9 - |\vec{a} + \vec{b} + \vec{c}|^2.$$

The term subtracted from 9 is nonnegative. Equality holds precisely when $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, which is possible for coplanar unit vectors separated by angles of 120° . Hence the proposed upper bound is sharp.

Step 3: Conclusion Therefore $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 9$, as required.

Quick Tip to Solve: Expand the squared magnitude of the sum of the three unit vectors to control all three dot products simultaneously.

Section D (20 marks)

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. Find cost (per kg) of each fertilizer A, B and C that the farmer needs to buy, such that 1 kg each of fertilizer A and C added to 2 kg of B costs him Rs. 400. Also, cost of each kg of fertilizer B and C added together is equal to cost of 1 kg of fertilizer A. However, cost of 3 kg of fertilizer B added to Rs. 200 is the same as cost of 1 kg of fertilizer A and C together. Use matrix method to find the solution. [5 marks]

Solution:

Concept:

- A system of three linear equations can be written as $MX = D$, where M is the coefficient matrix, X the column of unknowns and D the constant column.
- If $|M| \neq 0$, the inverse exists and the unique solution is $X = M^{-1}D$, with $M^{-1} = \text{adj } M / |M|$.

Step 1: Translate the three cost conditions Let x, y, z be the costs, in rupees per kilogram, of fertilizers A, B, C, respectively. The first purchase costs $x + 2y + z = 400$. The second statement says $y + z = x$.

Quick Tip to Solve: Keep the unknown order A, B, C fixed in every matrix row, and check the obtained prices in all three original cost statements.

The third says $3y + 200 = x + z$. Moving the unknown terms to the left gives

$$x + 2y + z = 400, \quad x - y - z = 0, \quad x - 3y + z = 200.$$

In particular, the fixed amount 200 is a cost added to $3y$, rather than a quantity of fertilizer.

Step 2: Form the matrix equation and check invertibility These equations have the matrix form

$$\underbrace{\begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & -1 \\ 1 & -3 & 1 \end{pmatrix}}_M \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_X = \underbrace{\begin{pmatrix} 400 \\ 0 \\ 200 \end{pmatrix}}_D.$$

Expanding the determinant along the first row,

$$|M| = 1(-1 - 3) - 2(1 + 1) + 1(-3 + 1) = -4 - 4 - 2 = -10.$$

Since this determinant is nonzero, all three costs are determined uniquely and the inverse matrix method applies.

Step 3: Find the inverse and multiply by the constant column Computing the cofactors by deleting the relevant row and column, with alternating signs, gives the cofactor matrix

$$C = \begin{pmatrix} -4 & -2 & -2 \\ -5 & 0 & 5 \\ -1 & 2 & -3 \end{pmatrix}.$$

Its transpose is the adjugate. Consequently,

$$M^{-1} = -\frac{1}{10} \begin{pmatrix} -4 & -5 & -1 \\ -2 & 0 & 2 \\ -2 & 5 & -3 \end{pmatrix}.$$

Multiplying by D , we get

$$\begin{aligned} X &= -\frac{1}{10} \begin{pmatrix} -4 & -5 & -1 \\ -2 & 0 & 2 \\ -2 & 5 & -3 \end{pmatrix} \begin{pmatrix} 400 \\ 0 \\ 200 \end{pmatrix} \\ &= -\frac{1}{10} \begin{pmatrix} -1800 \\ -400 \\ -1400 \end{pmatrix} = \begin{pmatrix} 180 \\ 40 \\ 140 \end{pmatrix}. \end{aligned}$$

Step 4: Verify each purchase condition Substituting these costs gives $180 + 2(40) + 140 = 400$. Also $40 + 140 = 180$, as required by the second condition. Finally, $3(40) + 200 = 320$ and $180 + 140 = 320$, so the third condition is satisfied as well. Each cost is positive and represents the price of one kilogram of the corresponding fertilizer.

Step 5: Conclusion The costs per kilogram are $A : \text{Rs. } 180, \quad B : \text{Rs. } 40, \quad C : \text{Rs. } 140$.

Quick Tip to Solve: Keep the unknown order A, B, C fixed in every matrix row, and check the obtained prices in all three original cost statements.

33(a). Find the foot of the perpendicular from the point $(0, 2, 3)$ on the line $\frac{-x-3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9}$ and hence find the length of the perpendicular. [5 marks]

Solution:

Concept:

- A line in symmetric form gives a fixed point and a direction vector. Simplify each printed fraction before identifying these quantities.
- If H is the foot from P to a line of direction $\vec{b}$, then H lies on the line and $\overrightarrow{PH} \cdot \vec{b} = 0$.
- The perpendicular length is $PH = \sqrt{(x_H - x_P)^2 + (y_H - y_P)^2 + (z_H - z_P)^2}$.

Step 1: Simplify the printed equation The negative signs cancel in the first two fractions, while the third simplifies by cancelling a factor of three. Hence

$$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = t.$$

A point on the line is $A = (-3, 1, -4)$, and its direction vector is $\vec{b} = (5, 2, 3)$. Therefore any point on this line has the form

$$H = (-3 + 5t, 1 + 2t, -4 + 3t).$$

The foot must have this form because it lies on the given line. The parameter is determined by the perpendicularity condition, rather than chosen arbitrarily.

Step 2: Find the foot using the scalar product For the given point $P = (0, 2, 3)$, the vector from P to the prospective foot is

$$\overrightarrow{PH} = (-3 + 5t, -1 + 2t, -7 + 3t).$$

It must be perpendicular to the direction $(5, 2, 3)$. Thus

$$\begin{aligned} 5(-3 + 5t) + 2(-1 + 2t) + 3(-7 + 3t) &= 0, \\ -15 + 25t - 2 + 4t - 21 + 9t &= 0, \\ 38t - 38 &= 0. \end{aligned}$$

It follows that $t = 1$, giving $H = (2, 3, -1)$. In the original printed equation, all three ratios at this point equal one; this independently verifies that the computed foot belongs to the line.

Step 3: Calculate the distance and verify perpendicularity The displacement from the given point to the foot is now $\overrightarrow{PH} = (2, 1, -4)$. Its scalar product with the line direction is $2(5) + 1(2) - 4(3) = 0$, confirming perpendicularity. The distance is

$$PH = \sqrt{(2-0)^2 + (3-2)^2 + (-1-3)^2} = \sqrt{4+1+16} = \sqrt{21}.$$

Since the connecting segment is perpendicular to the line, this is the required perpendicular length. For any other point on the line, the corresponding segment from P is the hypotenuse of a right triangle with PH as one side, and is consequently no shorter.

Step 4: Conclusion The foot is $(2, 3, -1)$, and the perpendicular length is $\sqrt{21}$ units.

Quick Tip to Solve: Simplify the symmetric fractions first, then use a single dot-product equation to determine the foot parameter.

33(b). Find the value of p if the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (p\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ is $\frac{3}{\sqrt{2}}$ units. [5 marks]

Solution:

Concept:

- For lines with fixed points $\vec{a}_1, \vec{a}_2$ and non-parallel directions $\vec{b}_1, \vec{b}_2$, $d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$.
- The cross product gives a vector perpendicular to both lines. The shortest distance is the magnitude of the component of the joining vector along this common normal.
- An absolute-value equation $|u| = c$, with $c > 0$, has the two possibilities $u = c$ and $u = -c$.

Step 1: Identify the points and directions Reading the fixed terms and coefficients of the parameters gives

$$\begin{aligned}\vec{a}_1 &= (1, 2, 1), & \vec{b}_1 &= (1, -1, 1), \\ \vec{a}_2 &= (p, -1, -1), & \vec{b}_2 &= (2, 1, 2).\end{aligned}$$

Thus the joining vector between these chosen points is $\vec{a}_2 - \vec{a}_1 = (p-1, -3, -2)$. The directions are not proportional, so the common-normal formula applies with a nonzero denominator.

Step 2: Compute the common normal The cross product of the directions is

$$\begin{aligned}\vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} \\ &= (-2-1)\hat{i} - (2-2)\hat{j} + (1+2)\hat{k} \\ &= -3\hat{i} + 3\hat{k}.\end{aligned}$$

Its magnitude is $\sqrt{(-3)^2 + 0^2 + 3^2} = 3\sqrt{2}$. Its scalar products with the two directions are $-3 + 3 = 0$ and $-6 + 6 = 0$, confirming the common normal.

Step 3: Use the prescribed shortest distance The scalar triple product in the numerator is

$$\begin{aligned}(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= (p-1, -3, -2) \cdot (-3, 0, 3) \\ &= -3(p-1) - 6 = -3p - 3.\end{aligned}$$

Consequently

$$d = \frac{|-3p-3|}{3\sqrt{2}} = \frac{|p+1|}{\sqrt{2}}.$$

Equating this to the given $3/\sqrt{2}$ gives $|p+1| = 3$. Both signs must be retained: $p+1 = 3$ gives $p = 2$, whereas $p+1 = -3$ gives $p = -4$. For either value the numerator has magnitude nine, giving $9/(3\sqrt{2}) = 3/\sqrt{2}$. The positive distance confirms that both choices give skew lines.

Step 4: Conclusion The possible values are $p = 2$ or $p = -4$.

Quick Tip to Solve: Keep the absolute value in the shortest-distance formula until the last step; dropping it loses one valid value of the parameter.

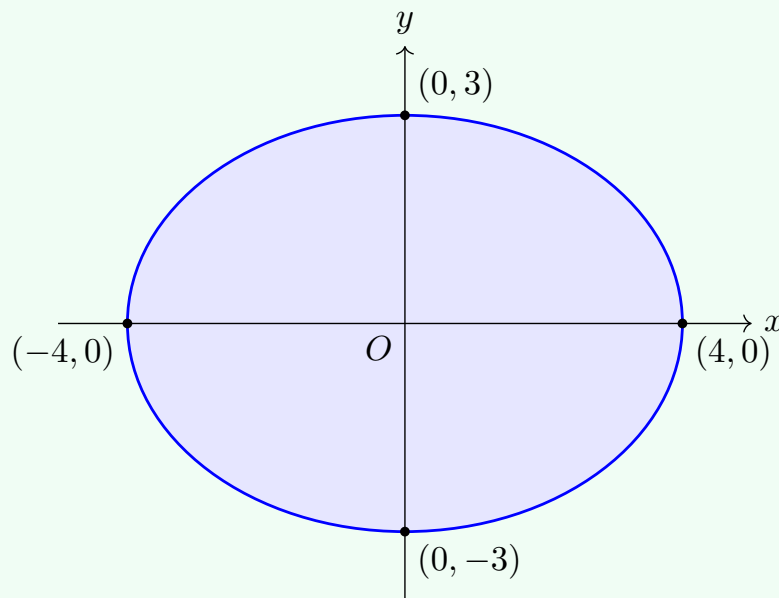
34. Sketch the curve described by $\{(x, y) : 9x^2 + 16y^2 = 144\}$ and find the area of the region enclosed by it, using integration. [5 marks]

Solution:

Concept:

- The standard ellipse $x^2/a^2 + y^2/b^2 = 1$ has centre at the origin and intercepts $(\pm a, 0)$ and $(0, \pm b)$.
- The area under a positive curve is $\int y \, dx$. Symmetry about both axes gives total enclosed area equal to four times the first-quadrant area.
- Substitute $x = a \sin \theta$ to evaluate an integral containing $\sqrt{a^2 - x^2}$; use $\cos^2 \theta = (1 + \cos 2\theta)/2$.

Step 1: Identify and sketch the curve Dividing the equation by 144, we obtain $x^2/16 + y^2/9 = 1$. It represents an ellipse centred at the origin. Its semi-major axis is 4, along the horizontal axis, and its semi-minor axis is 3, along the vertical axis. Setting $y = 0$ gives $x = \pm 4$; setting $x = 0$ gives $y = \pm 3$. Plot these four intercepts and join them with a smooth ellipse, symmetric about both coordinate axes, as shown below.



Ellipse with semi-axes 4 and 3

Step 2: Set up the area integral Solving for the positive branch gives

$$y = \frac{3}{4}\sqrt{16 - x^2}, \quad 0 \leq x \leq 4.$$

In the first quadrant, a vertical strip has height y and infinitesimal width dx . The region bounded by this arc and the coordinate axes therefore has area $\int_0^4 y \, dx$. The signs of x and y do not change the given equation, so all four quadrant regions have the same area. Thus

$$A = 4 \int_0^4 \frac{3}{4}\sqrt{16 - x^2} \, dx = 3 \int_0^4 \sqrt{16 - x^2} \, dx.$$

Step 3: Evaluate using integration Put $x = 4 \sin \theta$, so $dx = 4 \cos \theta \, d\theta$. The limits become 0 and $\pi/2$.

Quick Tip to Solve: Use symmetry to integrate only the first-quadrant branch, and remember to multiply that area by four.

Since $\cos \theta \geq 0$ throughout this interval, the square root is $4 \cos \theta$. Consequently,

$$\begin{aligned} A &= 48 \int_0^{\pi/2} \cos^2 \theta \, d\theta \\ &= 24 \int_0^{\pi/2} (1 + \cos 2\theta) \, d\theta \\ &= 24 \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\pi/2}. \end{aligned}$$

The sine term is zero at both endpoints, leaving $24(\pi/2)$. This evaluation uses the integral directly, rather than assuming the ellipse-area formula.

Step 4: Conclusion The requested sketch is an ellipse with intercepts $(\pm 4, 0)$ and $(0, \pm 3)$. Its enclosed area is 12π square units.

Quick Tip to Solve: Use symmetry to integrate only the first-quadrant branch, and remember to multiply that area by four.

35(a). If $y\sqrt{x^2+1} = \log(\sqrt{x^2+1}-x)$, then show that $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$.

[5 marks]

Solution:

Concept:

- For differentiable functions u, v , the product rule is $(uv)' = u'v + uv'$.
- The chain rule gives $\frac{d}{dx} \log u = u'/u$ for $u > 0$, with logarithm understood as the natural logarithm.
- Writing $s = \sqrt{x^2+1}$ gives $s' = x/s$, while $(s-x)(s+x) = 1$ shows that $s-x$ is positive for every real x .

Step 1: Check the logarithm and prepare the differentiation Let $s = \sqrt{x^2+1}$. Since $x^2+1 > x^2$, we have $s > |x|$, and therefore $s-x > 0$ for every real x . Thus the logarithm in the question is defined and differentiable. The given relation can be written as $ys = \log(s-x)$, where both y and s depend on x . From the chain rule,

$$\frac{ds}{dx} = \frac{1}{2\sqrt{x^2+1}} \cdot 2x = \frac{x}{s}.$$

We must apply the product rule on the left; treating y as a constant would lose the required derivative term.

Step 2: Differentiate both sides Using the product rule on ys and the chain rule on the logarithm gives

$$s \frac{dy}{dx} + y \frac{x}{s} = \frac{\frac{x}{s} - 1}{s-x}.$$

To simplify the right side, put its numerator over the common denominator s :

$$\frac{\frac{x}{s} - 1}{s-x} = \frac{x-s}{s(s-x)} = -\frac{1}{s}.$$

Quick Tip to Solve: In the logarithmic derivative, rewrite $x/\sqrt{x^2+1} - 1$ as $-(\sqrt{x^2+1}-x)/\sqrt{x^2+1}$ to cancel immediately.

The cancellation is valid because $s - x > 0$. In particular, the derivative of the logarithm is negative; retaining this sign is essential to the required identity.

Step 3: Rearrange into the required form The differentiated relation now reads

$$s \frac{dy}{dx} + \frac{xy}{s} = -\frac{1}{s}.$$

Multiplying every term by s , which is strictly positive, gives

$$s^2 \frac{dy}{dx} + xy = -1.$$

Substitute $s^2 = x^2 + 1$, and move the constant to the left. This yields precisely the combination of the derivative, the product xy , and the constant requested in the question. The calculation holds on the whole real domain of the original relation.

Step 4: Conclusion Hence $\boxed{(x^2 + 1) \frac{dy}{dx} + xy + 1 = 0}$, as required.

Quick Tip to Solve: In the logarithmic derivative, rewrite $x/\sqrt{x^2 + 1} - 1$ as $-(\sqrt{x^2 + 1} - x)/\sqrt{x^2 + 1}$ to cancel immediately.

OR

35(b). Find the differential of $x^{\cot x} + \frac{2x^2 - 3}{2x^2 - x + 2}$ with respect to x .

[5 marks]

Solution:

Concept:

- Logarithmic differentiation is suitable for $u(x)^{v(x)}$: taking natural logarithms turns the variable exponent into a product. For the real logarithmic calculation here, take $x > 0$ and $\sin x \neq 0$.
- The product rule is $(fg)' = f'g + fg'$, and $\frac{d}{dx} \cot x = -\csc^2 x$, while $\frac{d}{dx} \log x = 1/x$.
- The quotient rule gives $\left(\frac{N}{D}\right)' = \frac{N'D - ND'}{D^2}$. For a differentiable function y , its differential is $dy = y' dx$.

Step 1: Separate the two terms and take logarithms Let $y = u + v$, where

$$u = x^{\cot x}, \quad v = \frac{2x^2 - 3}{2x^2 - x + 2}.$$

Both the base and the exponent depend on x , so take natural logarithms: $\log u = (\cot x) \log x$.

Step 2: Differentiate the logarithmic equation Applying the chain rule on the left and the product rule on the right yields

$$\frac{u'}{u} = -\csc^2 x \log x + \frac{\cot x}{x}.$$

Quick Tip to Solve: For a variable base raised to a variable exponent, take logarithms first; keep the quotient-rule subtraction bracket intact until both products are expanded.

Multiplying by $u = x^{\cot x}$, we obtain

$$u' = x^{\cot x} \left(\frac{\cot x}{x} - \log x \csc^2 x \right).$$

The negative sign comes from differentiating $\cot x$.

Step 3: Apply the quotient rule to the rational term Set $N = 2x^2 - 3$ and $D = 2x^2 - x + 2$. Then $N' = 4x$ and $D' = 4x - 1$, so

$$v' = \frac{4x(2x^2 - x + 2) - (2x^2 - 3)(4x - 1)}{(2x^2 - x + 2)^2}.$$

Expand the numerator carefully:

$$\begin{aligned} 4x(2x^2 - x + 2) &= 8x^3 - 4x^2 + 8x, \\ (2x^2 - 3)(4x - 1) &= 8x^3 - 2x^2 - 12x + 3. \end{aligned}$$

Subtracting the second polynomial from the first gives $-2x^2 + 20x - 3$.

Step 4: Combine derivatives and form the differential Add u' and v' by the sum rule, then multiply by dx to express dy .

Step 5: Conclusion The required differential is

$$dy = \left[x^{\cot x} \left(\frac{\cot x}{x} - \log x \csc^2 x \right) + \frac{-2x^2 + 20x - 3}{(2x^2 - x + 2)^2} \right] dx.$$

Thus dy/dx is the expression inside the square brackets.

Quick Tip to Solve: For a variable base raised to a variable exponent, take logarithms first; keep the quotient-rule subtraction bracket intact until both products are expanded.

Section E (12 marks)

This section comprises 3 Case Study based questions of 4 marks each.

36. There are three types of vaccines A_1, A_2, A_3 , available in the market to protect the population of the country from spread of certain infection. According to a survey conducted, it was found that 25% of the population was given Vaccine A_1 , 35% of the population was given Vaccine A_2 and 40% of the population was given Vaccine A_3 . The survey also stated that the probabilities that Vaccines A_1, A_2 and A_3 would protect against the infection were 60%, 55% and 50% respectively.

Based on the above information, answer the following questions:

Find the probability that:

[4 marks]

36(i). The person taking vaccine A_2 will get infected.

[1 mark]

Solution:

Concept:

- For a person already known to have received vaccine A_2 , the relevant probability is conditional on A_2 , rather than on a random vaccine choice.
- Protection and infection are complementary outcomes in this model: if I denotes infection, then $P(I | A_2) = 1 - P(I^c | A_2)$.

Step 1: Identify the given conditional protection probability The survey states that vaccine A_2 protects against infection with probability 55%. Hence $P(I^c | A_2) = 0.55$. The wording already restricts attention to a person taking A_2 , so the population fraction 35% receiving that vaccine is not multiplied here.

Step 2: Take the complement within the vaccinated group Within the group receiving A_2 , the probabilities of protection and infection add to one. Therefore

$$P(I | A_2) = 1 - 0.55 = 0.45.$$

Multiplying by 0.35 would instead compute the joint probability that a randomly selected person both received A_2 and became infected, which answers a different question.

Step 3: Conclusion The probability that the person taking vaccine A_2 gets infected is $0.45 = 45\%$.

Quick Tip to Solve: When the vaccine is already specified, subtract its protection rate from one without multiplying by its population share.

36(ii). If a person is chosen randomly, he/she will be protected from the infection.

[1 mark]

Solution:

Concept:

- Let E denote protection from infection. The vaccine events A_1, A_2, A_3 are mutually exclusive and exhaustive.
- Total probability gives $P(E) = \sum_{i=1}^3 P(A_i)P(E | A_i)$; each protection rate must be weighted by its vaccine's population share.

Quick Tip to Solve: Multiply each protection rate by its population share before adding the three contributions.

Step 1: Record the three vaccine groups The survey gives $P(A_1) = 0.25$, $P(A_2) = 0.35$, and $P(A_3) = 0.40$. Their conditional protection probabilities are 0.60, 0.55, 0.50, respectively. Since the groups partition the population, add their contributions.

Step 2: Apply total probability

$$\begin{aligned} P(E) &= (0.25)(0.60) + (0.35)(0.55) + (0.40)(0.50) \\ &= 0.15 + 0.1925 + 0.20 = 0.5425. \end{aligned}$$

Simply averaging the protection rates would ignore the unequal vaccine-group sizes.

Step 3: Conclusion The probability of protection is $\boxed{0.5425 = \frac{217}{400}}$.

Quick Tip to Solve: Multiply each protection rate by its population share before adding the three contributions.

36(iii)(a). The person was given Vaccine A_1 , given that the randomly chosen person is infected.

[2 marks]

Solution:

Concept:

- Let I denote infection. Protection and infection are complementary outcomes, so $P(I | A_i) = 1 - P(I^c | A_i)$.
- Bayes' theorem is $P(A_1 | I) = P(A_1)P(I | A_1)/P(I)$. The denominator includes infected people from every vaccine group.

Step 1: Find the conditional infection probabilities Using the stated protection rates, $P(I | A_1) = 1 - 0.60 = 0.40$, $P(I | A_2) = 1 - 0.55 = 0.45$, and $P(I | A_3) = 1 - 0.50 = 0.50$. The probability of belonging to the first group and being infected is $P(A_1 \cap I) = (0.25)(0.40) = 0.10$.

Step 2: Calculate the overall infection probability As the vaccine groups are mutually exclusive and exhaustive,

$$\begin{aligned} P(I) &= (0.25)(0.40) + (0.35)(0.45) + (0.40)(0.50) \\ &= 0.10 + 0.1575 + 0.20 = 0.4575. \end{aligned}$$

Step 3: Condition on the observed infection Restricting the sample space to infected people gives

$$P(A_1 | I) = \frac{P(A_1 \cap I)}{P(I)} = \frac{0.10}{0.4575} = \frac{1000}{4575} = \frac{40}{183}.$$

This differs from $P(I | A_1)$: the question specifies infection first and asks which vaccine was received.

Step 4: Conclusion The required probability is $\boxed{\frac{40}{183}}$.

Quick Tip to Solve: In Bayes' theorem, use the probability of the given event, infection here, as the denominator.

36(iii)(b). The person was given Vaccine A_3 , given that the randomly chosen person is not infected.

[2 marks]

Solution:

Concept:

- Let E denote not being infected, equivalently protection in the stated survey. Let A_3 denote receiving Vaccine A_3 .
- Bayes' theorem gives $P(A_3 | E) = P(A_3)P(E | A_3)/P(E)$, where $P(E)$ is obtained by total probability over all vaccine groups.

Step 1: Find the joint probability for Vaccine A3 and protection The survey states $P(A_3) = 0.40$ and $P(E | A_3) = 0.50$. Hence $P(A_3 \cap E) = (0.40)(0.50) = 0.20$. This joint probability refers to the entire population; the required conditional probability refers only to people not infected.

Step 2: Find the probability of not being infected Weight each vaccine's protection rate by its population share:

$$\begin{aligned} P(E) &= P(A_1)P(E | A_1) + P(A_2)P(E | A_2) \\ &\quad + P(A_3)P(E | A_3) \\ &= (0.25)(0.60) + (0.35)(0.55) + (0.40)(0.50) \\ &= 0.15 + 0.1925 + 0.20 = 0.5425. \end{aligned}$$

Step 3: Apply Bayes' theorem Restricting attention to the protected group,

$$P(A_3 | E) = \frac{0.20}{0.5425} = \frac{2000}{5425} = \frac{80}{217}.$$

The conditioning reverses the direction of the supplied probability $P(E | A_3)$, so using 0.50 directly would answer a different question.

Step 4: Conclusion The required probability is $\boxed{\frac{80}{217}}$.

Quick Tip to Solve: Use the protected contribution of Vaccine A3 as numerator and total protection as denominator.

37. A company produces cylindrical tumblers, open from the top. Since they want uniformity in the product, they fix the surface area of the tumblers produced.

Based on the above information, answer the following questions:

If for a tumbler, V is its volume, h the height and r the radius of the circular base, then: [4 marks]

37(i). Differentiate its volume with respect to radius of the base, where the surface area is constant. [2 marks]

Solution:

Concept:

- For an open cylindrical tumbler, the fixed surface area is $S = \pi r^2 + 2\pi r h$, where $r > 0$ is the base radius and $h > 0$ is its height. There is one circular base and no top.
- The volume is $V = \pi r^2 h$. Since S is fixed, h varies with r ; express h in terms of r before differentiating.

Step 1: Use the fixed surface area Let the constant surface area be S . Then

$$2\pi r h = S - \pi r^2, \quad h = \frac{S - \pi r^2}{2\pi r}.$$

This expression includes both the base and the curved surface, and is valid for positive dimensions.

Step 2: Express the volume and differentiate Substitution into the volume gives

$$V = \pi r^2 \left(\frac{S - \pi r^2}{2\pi r} \right) = \frac{Sr}{2} - \frac{\pi r^3}{2}.$$

Treating S as constant,

$$\frac{dV}{dr} = \frac{S}{2} - \frac{3\pi r^2}{2}.$$

Equivalently, substituting $S = \pi r^2 + 2\pi r h$ gives $\frac{dV}{dr} = \pi r(h - r)$.

Step 3: Conclusion Thus, at constant surface area, $\frac{dV}{dr} = \frac{S - 3\pi r^2}{2} = \pi r(h - r)$.

Quick Tip to Solve: For an open tumbler use one base, then eliminate the changing height before differentiating its volume.

37(ii). If the company wants to maximize the volume of each tumbler, then establish a relation between its height and the radius of the base. [2 marks]

Solution:

Concept:

- For fixed surface area S , an open cylindrical tumbler has $S = \pi r^2 + 2\pi r h$ and $V = \pi r^2 h$, with $r, h > 0$.
- An interior maximum satisfies $\frac{dV}{dr} = 0$; a negative second derivative verifies that the stationary volume is a maximum.

Quick Tip to Solve: Keep the missing top in mind: the open cylinder has maximum volume at height equal to radius.

Step 1: Find the stationary radius Using the area constraint,

$$h = \frac{S - \pi r^2}{2\pi r}, \quad V = \frac{Sr}{2} - \frac{\pi r^3}{2}.$$

Hence

$$\frac{dV}{dr} = \frac{S - 3\pi r^2}{2}.$$

At a stationary value, $S = 3\pi r^2$.

Step 2: Obtain the height and verify the maximum Compare this with $S = \pi r^2 + 2\pi rh$:

$$3\pi r^2 = \pi r^2 + 2\pi rh \implies 2\pi r^2 = 2\pi rh.$$

Since $r > 0$, division gives $h = r$. Also,

$$\frac{d^2V}{dr^2} = -3\pi r < 0.$$

Thus the stationary value is a maximum. In the physical range $0 < r < \sqrt{S/\pi}$, the volume tends to zero at both boundaries and the unique stationary radius lies inside, so this is the maximum possible volume.

Step 3: Conclusion The volume is greatest when $h = r$: the height equals the radius of the circular base.

Quick Tip to Solve: Keep the missing top in mind: the open cylinder has maximum volume at height equal to radius.

38. A school wants the students of class XII to do a project on ‘Sustainability’ keeping the world environment in mind. They select the student participants on the basis of an essay writing competition. 7 students out of 80 are selected for the project and are categorized into two sets such that:

Girl students belong to Set $A = \{G_1, G_2, G_3, G_4\}$,

Boy students belong to Set $B = \{B_1, B_2, B_3\}$.

Based on the above information, answer the following questions:

[4 marks]

38(i). How many relations are possible from Set $A \rightarrow$ Set B ?

[1 mark]

Solution:

Concept:

- A relation from A to B is any subset of the Cartesian product $A \times B$.
- If a finite set contains n elements, it has 2^n subsets: each element can independently be included or excluded.

Step 1: Count the possible ordered pairs The given sets contain $|A| = 4$ girls and $|B| = 3$ boys. Each element of A can be paired with each element of B , so

$$|A \times B| = 4 \times 3 = 12.$$

Quick Tip to Solve: Count relations as subsets of the Cartesian product: use 2 raised to the product of the set sizes.

The selection of seven students fixes these two set sizes; the original total of eighty students does not enter this count.

Step 2: Count all subsets of the product For every one of these twelve ordered pairs, a relation may either include it or omit it. These independent choices give

$$2^{12} = 4096$$

relations, including the empty relation and the entire Cartesian product. There is no requirement that a relation assign exactly one boy to each girl; that restriction applies to functions.

Step 3: Conclusion The number of relations from A to B is 4096.

Quick Tip to Solve: Count relations as subsets of the Cartesian product: use 2 raised to the product of the set sizes.

38(ii). Let R be a relation from $A \rightarrow B$ such that $R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$. Is R an injective function? Justify your answer. [1 mark]

Solution:

Concept:

- A function from A to B assigns exactly one image in B to each element of A .
- An injective function satisfies $f(x) = f(y) \Rightarrow x = y$: distinct inputs must have distinct images.

Step 1: Check whether the relation is a function The given relation contains both (G_1, B_1) and (G_1, B_2) . Since $B_1 \neq B_2$, the same input G_1 is assigned two different images. Thus the uniqueness condition for a function fails.

Step 2: Check the injectivity requirement Injectivity is a property of a function, so a relation that is not a function cannot be an injective function. Further, the pairs (G_2, B_2) and (G_3, B_2) show that two distinct inputs share an image; this also conflicts with the distinct-image requirement.

Step 3: Conclusion R is not an injective function; in fact, it is not a function because G_1 has two distinct images.

Quick Tip to Solve: First check repeated first coordinates for conflicting images; test injectivity only after establishing that the relation is a function.

38(iii)(a). Let the relation R from $A \rightarrow A$ be such that

$$R = \{(x, y), x, y \in A, x \text{ and } y \text{ are students from the same colony in the city}\}.$$

Verify if R is an equivalence relation.

[2 marks]

Solution:

Concept:

- A relation on A is an equivalence relation if it is reflexive, symmetric and transitive.
- Here xRy means that students x and y belong to the same colony. The properties must hold for all applicable students in A , irrespective of which colonies they belong to.

Step 1: Verify reflexivity Take any student $x \in A$. Student x belongs to the same colony as himself or herself. Therefore $(x, x) \in R$ for every $x \in A$, so R is reflexive.

Step 2: Verify symmetry Let $x, y \in A$ and suppose $(x, y) \in R$. Then x and y are students from the same colony. This also means that y and x are from the same colony. Hence $(y, x) \in R$, and R is symmetric.

Step 3: Verify transitivity Let $x, y, z \in A$, with $(x, y) \in R$ and $(y, z) \in R$. Thus x belongs to the same colony as y , and y belongs to the same colony as z . Consequently x and z belong to that same colony, giving $(x, z) \in R$. Therefore R is transitive.

Step 4: Conclusion Since R is reflexive, symmetric and transitive, it is an equivalence relation on A .

Quick Tip to Solve: For a relation defined by belonging to the same category, verify self-membership, reversal and chaining in that order.

OR

38(iii)(b). Verify if any function $f : B \rightarrow A$ is bijective. Give reason to support your answer. [2 marks]

Solution:

Concept:

- A bijection is a function that is both injective (one-one) and surjective (onto).
- For finite sets, a bijection between domain and codomain requires equal cardinalities. Surjectivity requires every codomain element to occur as an image.

Step 1: Count the elements of the two sets From the case study, the domain of f is $B = \{B_1, B_2, B_3\}$, containing three students, and its codomain is $A = \{G_1, G_2, G_3, G_4\}$, containing four students. Thus

$$|B| = 3, \quad |A| = 4.$$

Each domain element has exactly one image under a function, so the three elements of B can produce at most three distinct images in A .

Step 2: Test whether surjectivity is possible For $f : B \rightarrow A$ to be onto, all four elements of A would

Quick Tip to Solve: Compare finite set sizes first: a smaller domain cannot cover every element of a larger codomain, so a bijection is impossible.

have to be images. But $|f(B)| \leq 3 < 4 = |A|$, so at least one element of A has no preimage. This holds even if the three inputs have distinct images; repeated images only reduce the size of the image set further. Hence every function from B to A fails to be surjective.

Step 3: Conclusion No function $f : B \rightarrow A$ can be bijective, because a three-element domain cannot map onto a four-element codomain.

Quick Tip to Solve: Compare finite set sizes first: a smaller domain cannot cover every element of a larger codomain, so a bijection is impossible.