

JEE Main 2013 Paper 1 (B.E./B.Tech) Apr 9 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 360

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2013, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 9 April 2013, Single slot. The total duration is 3 hours.
- (ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.
- (iii) Each multiple choice question (MCQ) has four options, of which only one is correct.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. The values of a for which one root of the equation $x^2 - (a + 1)x + a^2 + a - 8 = 0$ exceeds 2 and the other is lesser than 2, are given by: [4 marks]

(1) $3 < a < 10$

(2) $a \geq 10$

(3) $-2 < a < 3$

(4) $a \leq -2$

Correct Answer: (3) $-2 < a < 3$

Solution:

Concept:

- For a monic quadratic $f(x)$ with two distinct real roots, its value is negative precisely between the roots.
- Thus having one root strictly below 2 and the other strictly above 2 is equivalent to $f(2) < 0$. Strict inequalities exclude a root equal to 2.

Step 1: Evaluate the polynomial at the separating point Set $f(x) = x^2 - (a + 1)x + a^2 + a - 8$. The leading coefficient is positive for every real parameter a , so the parabola opens upwards. Substituting the point that must lie between the two roots gives

$$f(2) = 4 - 2(a + 1) + a^2 + a - 8 = a^2 - a - 6.$$

A negative value here guarantees two real intersections, one on either side of 2: the polynomial tends to positive infinity in both directions. No separate discriminant restriction is needed.

Step 2: Solve the strict sign inequality Factor the expression at 2:

$$f(2) = (a - 3)(a + 2).$$

This product is negative exactly when its two factors have opposite signs, which happens between the ordered zeros -2 and 3 . Hence $-2 < a < 3$. At either endpoint, $f(2) = 0$, so one root is 2 and the demand for strict separation fails. For $a \leq -2$ or $a \geq 3$, the product is nonnegative; thus none of the ranges in (1), (2), or (4) meets the requirement throughout.

Step 3: Conclusion The required parameter range is $-2 < a < 3$. Hence, the correct option is (3).

Quick Tip to Solve: To place a fixed number strictly between the roots of a monic quadratic, substitute that number and require a negative value.

2. If $Z_1 \neq 0$ and Z_2 are complex numbers such that Z_2/Z_1 is purely imaginary, then $\left| \frac{2Z_1 + 3Z_2}{2Z_1 - 3Z_2} \right|$ is equal to: [4 marks]

- (1) 2
- (2) 5
- (3) 3
- (4) 1

Correct Answer: (4) 1

Solution:

Concept:

- A purely imaginary quotient can be written as $Z_2/Z_1 = it$, where t is real.
- The modulus of $u + iv$ is $\sqrt{u^2 + v^2}$; conjugate complex numbers have equal moduli. Also $|z/w| = |z|/|w|$ for $w \neq 0$.

Step 1: Divide by the nonzero complex number Since $Z_1 \neq 0$, divide both numerator and denominator by Z_1 . With $Z_2/Z_1 = it$, the given expression becomes

$$\left| \frac{2 + 3it}{2 - 3it} \right|.$$

The denominator never vanishes for real t , because its real part is 2. This also verifies that the original expression is defined under the stated hypothesis.

Step 2: Compare the numerator and denominator moduli The two complex numbers $2 + 3it$ and $2 - 3it$ are conjugates. Consequently,

$$\left| \frac{2 + 3it}{2 - 3it} \right| = \frac{\sqrt{4 + 9t^2}}{\sqrt{4 + 9t^2}} = 1.$$

The result does not depend on the size or sign of the imaginary quotient. It remains 1 even if the zero purely imaginary value is allowed. The quotient itself need not be real; it is its modulus that is required. Values 2, 5, and 3 in (1), (2), and (3) are impossible because equal positive moduli must have ratio 1.

Step 3: Conclusion The modulus is $\boxed{1}$. Hence, the correct option is (4).

Quick Tip to Solve: When a complex quotient has conjugate numerator and denominator, its modulus is immediately 1; do not confuse the quotient with its modulus.

3. A committee of 4 persons is to be formed from 2 ladies, 2 old men and 4 young men such that it includes at least 1 lady, at least 1 old man and at most 2 young men. Then the total number of ways in which this committee can be formed is [4 marks]

- (1) 40
- (2) 41
- (3) 16
- (4) 32

Correct Answer: (2) 41

Solution:

Concept:

- An unordered committee is counted with combinations: $\binom{n}{r}$ chooses r persons from n .
- Divide the committees into disjoint cases according to their number of young men. Both a lady and an old man must appear in every case.

Step 1: Count the cases with zero or one young man With no young men, all four members must be the two ladies and the two old men. There is $\binom{2}{2}\binom{2}{2} = 1$ such committee. With exactly one young man, the remaining three members must include both categories. The possible numbers of ladies and old men are (1, 2) and (2, 1). Their total is

$$\binom{4}{1} \left[\binom{2}{1}\binom{2}{2} + \binom{2}{2}\binom{2}{1} \right] = 4(2 + 2) = 16.$$

The persons within each category are distinct, but ordering the four selected members does not produce a new committee.

Step 2: Count two young men and add the disjoint cases With exactly two young men, there are two places left, so one must be filled by a lady and the other by an old man. This gives

$$\binom{4}{2} \binom{2}{1} \binom{2}{1} = 6 \cdot 2 \cdot 2 = 24.$$

There can be no further cases because at most two young men are permitted. Thus the required total is

$$1 + 16 + 24 = 41.$$

The value 40 in (1) omits the committee with no young men, 16 in (3) counts only the one-young-man case, and 32 in (4) is not the exhaustive sum of the permitted cases.

Step 3: Conclusion The total number of committees is 41. Hence, the correct option is (2).

Quick Tip to Solve: For committee restrictions, classify by the restricted category first; include its zero-member case whenever the wording permits it.

4. Let $a_1, a_2, a_3, \dots$ be an A.P. such that $\frac{a_1+a_2+\dots+a_p}{a_1+a_2+a_3+\dots+a_q} = \frac{p^3}{q^3}$; $p \neq q$. Then $\frac{a_6}{a_{21}}$ is equal to : [4 marks]

- (1) 41/11
- (2) 31/121
- (3) 11/41
- (4) 121/1861

Correct Answer: The printed condition does not determine a unique ratio.

Solution:

Concept:

- For an arithmetic progression with first term A and common difference d , $a_n = A + (n - 1)d$ and $S_n = n[2A + (n - 1)d]/2$.
- The printed equation is $S_p/S_q = p^3/q^3$ for unspecified distinct positive integers p, q . A conclusion must follow from this actual condition, without changing the progression or the exponent.

Step 1: Derive what the printed condition implies Put $b = 2A - d$, so $S_n = n(b + nd)/2$. Since the displayed sum ratio is defined, $S_q \neq 0$. The condition gives

$$q^2(b + pd) = p^2(b + qd).$$

Factoring and using $p \neq q$ yields

$$(p + q)b + pqd = 0.$$

The case $d = 0$ would force $b = 0$, making the sums zero and the ratio undefined, so $d \neq 0$. Hence

$$\frac{a_6}{a_{21}} = \frac{b + 11d}{b + 41d} = \frac{11(p + q) - pq}{41(p + q) - pq},$$

whenever the requested denominator is nonzero. This expression depends on the unspecified indices.

Step 2: Demonstrate nonuniqueness without altering the question For $p = 1, q = 2, A = 1, d = 6$, the progression has $S_1/S_2 = 1/8 = p^3/q^3$, but $a_6/a_{21} = 31/121$. For $p = 2, q = 3, A = -1, d = 10$, its sums are $S_2 = 8$ and $S_3 = 27$, so the same printed condition holds, while $a_6/a_{21} = 49/199$. Thus two admissible progressions give different answers. If the relation were instead required for every distinct pair of positive indices, it would be incompatible with any nonzero arithmetic progression, since its partial sum is quadratic rather than cubic. Neither reading justifies a unique listed option.

Step 3: Conclusion As printed, the question has no uniquely determined answer. In particular, the keyed value 121/1861 is not forced by the stated arithmetic-progression condition; no single option can be selected consistently.

Quick Tip to Solve: Substitute the actual partial-sum formula before using a ratio shortcut. A claimed constant answer must survive two admissible choices of the unspecified indices.

5. The sum of the series $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots$ up to 10 terms is:

[4 marks]

- (1) $18/11$
- (2) $22/13$
- (3) $20/11$
- (4) $16/9$

Correct Answer: (3) $20/11$

Solution:

Concept:

- The sum of the first n natural numbers is $n(n+1)/2$. The first term 1 is also the reciprocal of the first such sum.
- The decomposition $1/[n(n+1)] = 1/n - 1/(n+1)$ makes the consecutive terms telescope.

Step 1: Express each term using its index The denominator of the n th term is $1 + 2 + \dots + n$, not the square of n . Therefore its reciprocal is

$$t_n = \frac{1}{n(n+1)/2} = \frac{2}{n(n+1)} = 2 \left(\frac{1}{n} - \frac{1}{n+1} \right).$$

For $n = 1$ this is 1, and for $n = 2$ it is $1/3$, agreeing with the displayed series. There are exactly ten indexed terms, ending with the reciprocal of $1 + 2 + \dots + 10$.

Step 2: Cancel the intermediate fractions Adding the decomposed terms gives

$$\begin{aligned} S_{10} &= 2 \left[\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \dots + \left(\frac{1}{10} - \frac{1}{11}\right) \right] \\ &= 2 \left(1 - \frac{1}{11} \right) = \frac{20}{11}. \end{aligned}$$

Every intermediate reciprocal cancels once with the opposite sign. The final denominator is 11 because decomposing the tenth term introduces $1/(10+1)$. The other proposed fractions are distinct from $20/11$, so none can represent this exact cancellation; approximating the separate terms is unnecessary.

Step 3: Conclusion The required sum is $\boxed{20/11}$. Hence, the correct option is (3).

Quick Tip to Solve: When a denominator is a triangular number, replace its reciprocal by $2(1/n - 1/(n+1))$; retain the last uncanceled denominator $n+1$.

6. The ratio of the coefficient of x^{15} to the term independent of x in the expansion of $\left(x^2 + \frac{2}{x}\right)^{15}$ is: [4 marks]

- (1) 7 : 16
- (2) 7 : 64
- (3) 1 : 4
- (4) 1 : 32

Correct Answer: (4) 1 : 32

Solution:

Concept:

- In $(u + v)^{15}$, choosing v from r factors gives the term $\binom{15}{r} u^{15-r} v^r$, with $0 \leq r \leq 15$.
- The exponent of x determines which term supplies the required coefficient. Binomial symmetry gives $\binom{15}{5} = \binom{15}{10}$.

Step 1: Find the term with the required power The general term of the stated expansion is

$$T_{r+1} = \binom{15}{r} (x^2)^{15-r} (2/x)^r = \binom{15}{r} 2^r x^{30-3r}.$$

For the coefficient of x^{15} , solve $30 - 3r = 15$, which gives $r = 5$. The coefficient, excluding the power of x itself, is consequently $\binom{15}{5} 2^5$. The exponent falls by 3 whenever r increases by 1 because selecting $2/x$ replaces a factor x^2 .

Step 2: Find the constant term and form the ratio The term independent of x has exponent zero. Thus $30 - 3r = 0$ gives $r = 10$, and the constant term is $\binom{15}{10} 2^{10}$. Both indices lie in the permitted range of the binomial expansion. Dividing the first coefficient by this term gives

$$\frac{\binom{15}{5} 2^5}{\binom{15}{10} 2^{10}} = 2^{-5} = \frac{1}{32}.$$

The binomial coefficients cancel exactly, so the uncanceled factor is solely the five-power difference of 2. The ratios 7 : 16, 7 : 64, and 1 : 4 in the other options do not equal this value.

Step 3: Conclusion The required ratio is $\boxed{1 : 32}$. Hence, the correct option is (4).

Quick Tip to Solve: Find the exponent as a function of the selection index before finding coefficients; then check whether complementary indices cancel the binomial factors.

7. A value of x for which $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1}x)$, is :

[4 marks]

- (1) $-1/2$
- (2) 1
- (3) 0
- (4) $1/2$

Correct Answer: (1) $-1/2$

Solution:

Concept:

- For principal arccotangent, $0 < \cot^{-1} t < \pi$, so its sine is positive:

$$\sin(\cot^{-1} t) = \frac{1}{\sqrt{1+t^2}}.$$

- Principal arctangent lies between $-\pi/2$ and $\pi/2$, so its cosine is positive:

$$\cos(\tan^{-1} x) = \frac{1}{\sqrt{1+x^2}}.$$

Both identities hold for every real argument.

Step 1: Convert the inverse functions to algebra Let $\theta = \cot^{-1}(1+x)$. Since $\cot \theta = 1+x$, the identity $1 + \cot^2 \theta = \csc^2 \theta$, together with positive sine in its principal range, gives

$$\sin \theta = \frac{1}{\sqrt{1+(1+x)^2}}.$$

Similarly, for $\phi = \tan^{-1} x$, the identity $1 + \tan^2 \phi = \sec^2 \phi$ and positive principal-range cosine give

$$\cos \phi = \frac{1}{\sqrt{1+x^2}}.$$

Keeping the signs is essential when replacing inverse trigonometric expressions.

Step 2: Equate the positive expressions and check the options The denominators are positive, so equality is equivalent to

$$1 + (1+x)^2 = 1 + x^2.$$

Expanding cancels the quadratic terms, leaving $1 + 2x = 0$, and hence $x = -1/2$. Substitution gives $1 + x = 1/2$, so both original sides equal $2/\sqrt{5}$, confirming that no extraneous root has appeared. The other values 1 , 0 , and $1/2$ do not satisfy $1 + 2x = 0$, so their corresponding positive denominators cannot be equal.

Step 3: Conclusion The required value is $x = -1/2$. Hence, the correct option is (1).

Quick Tip to Solve: Use the positive square-root identities for sine of principal arccotangent and cosine of principal arctangent; then compare their denominators.

8. A light ray emerging from the point source placed at $P(1, 3)$ is reflected at a point Q in the axis of x . If the reflected ray passes through the point $R(6, 7)$, then the abscissa of Q is : [4 marks]

- (1) 1
- (2) 3
- (3) $7/2$
- (4) $5/2$

Correct Answer: (4) $5/2$

Solution:

Concept:

- Reflection in the x -axis reverses the sign of the ordinate: $(x, y) \mapsto (x, -y)$. A reflected optical path is straightened by reflecting either endpoint in the mirror.
- The point-slope equation through (x_1, y_1) with slope m is $y - y_1 = m(x - x_1)$. The reflection point lies on the mirror, so its ordinate is zero.

Step 1: Reflect the source in the mirror The mirror is the x -axis. Reflecting the source $P(1, 3)$ gives the image $P'(1, -3)$. By equality of the angles of incidence and reflection, the actual broken path PQR has its reflection point Q on the straight line joining P' to R . Both physical endpoints are above the mirror, while the image is below it, so this line crosses the mirror between the image and R .

Step 2: Find the intercept of the straightened ray The slope of $P'R$ is $(7 + 3)/(6 - 1) = 2$. Its equation is therefore

$$y + 3 = 2(x - 1), \quad y = 2x - 5.$$

At Q , set $y = 0$, giving $2x = 5$ and $x = 5/2$. The incident slope is $(0 - 3)/(5/2 - 1) = -2$, while the outgoing slope is $(7 - 0)/(6 - 5/2) = 2$. Thus their angles with the horizontal have equal magnitudes, independently verifying the reflection law. The other proposed abscissae do not satisfy $2x - 5 = 0$; using the original source and receiver without reflecting would describe an unrelated straight segment.

Step 3: Conclusion The abscissa is $\boxed{5/2}$. Hence, the correct option is (4).

Quick Tip to Solve: For reflection from a coordinate axis, reflect one endpoint first and intersect the resulting straight line with that axis.

9. If the three lines $x - 3y = p$, $ax + 2y = q$ and $ax + y = r$ form a right-angled triangle, then [4 marks]

(1) $a^2 - 9a + 18 = 0$

(2) $a^2 - 6a - 12 = 0$

(3) $a^2 - 6a - 18 = 0$

(4) $a^2 - 9a + 12 = 0$

Correct Answer: (1) $a^2 - 9a + 18 = 0$

Solution:

Concept:

- Two nonvertical lines of finite slopes m_1, m_2 are perpendicular exactly when $m_1 m_2 = -1$. A right-angled triangle must have one perpendicular pair among its three side-lines.
- The slope of $Ax + By = C$, for $B \neq 0$, is $-A/B$; the constant changes the intercept, not the slope.

Step 1: List the slopes of the three sides Rearrange the equations as $y = x/3 - p/3$, $y = -ax/2 + q/2$, and $y = -ax + r$. Thus

$$m_1 = \frac{1}{3}, \quad m_2 = -\frac{a}{2}, \quad m_3 = -a.$$

All three slopes are finite for every real a . Hence there is no separate vertical-line case to add. The constants p, q, r locate the sides; their assumed formation of a triangle excludes concurrent or otherwise degenerate arrangements.

Step 2: Test each possible perpendicular pair For the first and second lines, $m_1 m_2 = -a/6 = -1$, yielding $a = 6$. For the first and third, $m_1 m_3 = -a/3 = -1$, yielding $a = 3$. The remaining pair would require $m_2 m_3 = a^2/2 = -1$, impossible for real a . Thus the possible parameter values are exactly 3 and 6, so

$$(a - 3)(a - 6) = 0 \implies a^2 - 9a + 18 = 0.$$

Substitution shows that the alternatives with constant terms $-12, -18$, or $+12$ do not vanish at both required values. The condition is a statement about the directions of the sides, and does not require calculating the vertices.

Step 3: Conclusion $a^2 - 9a + 18 = 0$. Hence, the correct option is (1).

Quick Tip to Solve: With three side-lines, test all three slope products; retain the real parameter values before forming their polynomial.

10. If each of the lines $5x + 8y = 13$ and $4x - y = 3$ contains a diameter of the circle $x^2 + y^2 - 2(a^2 - 7a + 11)x - 2(a^2 - 6a + 6)y + b^3 + 1 = 0$, then [4 marks]

- (1) $a = 5$ and $b \notin (-1, 1)$
- (2) $a = 1$ and $b \notin (-1, 1)$
- (3) $a = 2$ and $b \notin (-\infty, 1)$
- (4) $a = 5$ and $b \in (-\infty, 1)$

Correct Answer: (4) $a = 5$ and $b \in (-\infty, 1)$

Solution:

Concept:

- Every diameter passes through the centre, so two distinct lines containing diameters intersect at the centre of the circle.
- For $x^2 + y^2 - 2hx - 2ky + d = 0$, the centre is (h, k) , and the squared radius is $h^2 + k^2 - d > 0$ for a nondegenerate real circle.

Step 1: Determine the centre and the parameter From $4x - y = 3$, obtain $y = 4x - 3$. Substitution into the other diameter line gives $5x + 8(4x - 3) = 13$, hence $37x = 37$. The centre is $(1, 1)$. Comparing this with the circle coefficients yields

$$a^2 - 7a + 11 = 1, \quad a^2 - 6a + 6 = 1.$$

The first equation factors as $(a - 2)(a - 5) = 0$; the second as $(a - 1)(a - 5) = 0$. Their common solution is $a = 5$. Choosing $a = 1$ or $a = 2$ satisfies only one coordinate condition and therefore cannot give the required centre.

Step 2: Impose the condition for a real radius With $a = 5$, complete the squares in the circle equation:

$$(x - 1)^2 + (y - 1)^2 = 1 - b^3.$$

Thus a genuine circle requires $1 - b^3 > 0$. Since the real cubic function is strictly increasing, this is exactly $b < 1$, or $b \in (-\infty, 1)$. Equality would collapse the circle to a point, which has no diameter of positive length. The condition $b \notin (-1, 1)$ incorrectly discards valid negative and small positive values and includes invalid values above 1; it is not the radius condition. Both the centre and positivity must be checked, rather than selecting an option from the value of a alone.

Step 3: Conclusion $a = 5, b \in (-\infty, 1)$. Hence, the correct option is (4).

Quick Tip to Solve: Intersect the diameter lines to get the centre, then enforce a strictly positive squared radius.

11. Statement-1: The slope of the tangent at any point P on a parabola, whose axis is the axis of x and vertex is at the origin, is inversely proportional to the ordinate of the point P . **Statement-2:** The system of parabolas $y^2 = 4ax$ satisfies a differential equation of degree 1 and order 1. [4 marks]

- (1) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
 (2) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
 (3) Statement-1 is true; Statement-2 is false.
 (4) Statement-1 is false; Statement-2 is true.

Correct Answer: (2) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

Solution:

Concept:

- For a fixed parabola $y^2 = 4ax$, implicit differentiation gives $2y \, dy/dx = 4a$, so its tangent slope is $2a/y$.
- The order of a differential equation is the highest derivative present; its degree is the power of that derivative when the equation is polynomial in derivatives.

Step 1: Evaluate Statement-1 directly Take the nondegenerate parabola $y^2 = 4ax$, with its vertex at the origin and axis along the x -axis. Differentiating at a point where $y \neq 0$ gives

$$\frac{dy}{dx} = \frac{2a}{y}.$$

For this fixed parabola, $2a$ is constant, so the slope is inversely proportional to the ordinate. Statement-1 is true. At the vertex the tangent is vertical and the ordinary finite slope is undefined; the inverse dependence describes all points with a finite tangent slope and its limiting vertical behaviour.

Step 2: Evaluate Statement-2 by eliminating the parameter To obtain a differential equation for the family, differentiate once and eliminate a . From $4a = y^2/x$ and $2yy' = 4a$, we obtain

$$2xyy' - y^2 = 0.$$

It contains only the first derivative, to the first power, so its order and degree are both 1. Statement-2 is therefore true. The polynomial form also avoids division by x when specifying the differential relation.

Step 3: Decide whether the second statement explains the first Knowing only that an equation has order 1 and degree 1 does not specify its slope as a function of the ordinate. Many unrelated families have first-order, first-degree equations with completely different slope laws. Statement-1 follows from the particular differentiated equation $2yy' = 4a$, rather than merely its order and degree. Thus the two statements are true, but the second does not supply the explanation claimed in option (1); options (3) and (4) incorrectly declare one of the statements false.

Step 4: Conclusion Both statements are true, but Statement-2 is not the correct explanation of Statement-1. Hence, the correct option is (2).

Quick Tip to Solve: In statement questions, distinguish an equation's classification from the specific algebraic relation that explains the result.

12. Equation of the line passing through the points of intersection of the parabola $x^2 = 8y$ and the ellipse $\frac{x^2}{3} + y^2 = 1$ is

[4 marks]

- (1) $y - 3 = 0$
- (2) $y + 3 = 0$
- (3) $3y + 1 = 0$
- (4) $3y - 1 = 0$

Correct Answer: (4) $3y - 1 = 0$

Solution:

Concept:

- At a common point of two conics, both equations hold simultaneously. Substitution eliminates one coordinate.
- The real parabola $x^2 = 8y$ satisfies $y \geq 0$; this sign condition must be applied after solving the algebraic elimination equation.

Step 1: Eliminate the square of the abscissa At an intersection, substitute $x^2 = 8y$ into the ellipse equation. This yields

$$\frac{8y}{3} + y^2 = 1, \quad 3y^2 + 8y - 3 = 0.$$

The quadratic factors as

$$(3y - 1)(y + 3) = 0.$$

The possible ordinates furnished by elimination are consequently $y = 1/3$ and $y = -3$. These are algebraic candidates only; they must still produce real coordinates in the original parabola.

Step 2: Retain the real common points and join them For $y = -3$, the parabola would require $x^2 = -24$, impossible for a real point. This candidate must be discarded. For $y = 1/3$, we obtain $x^2 = 8/3$, giving the two real points

$$\left(\sqrt{\frac{8}{3}}, \frac{1}{3}\right), \quad \left(-\sqrt{\frac{8}{3}}, \frac{1}{3}\right).$$

They have distinct abscissae and the same ordinate, so their joining line is horizontal: $y = 1/3$, or $3y - 1 = 0$. The option $y + 3 = 0$ corresponds to the rejected negative ordinate, while $y - 3 = 0$ and $3y + 1 = 0$ do not pass through either actual intersection. Checking the original equation prevents a spurious root introduced by eliminating x^2 from being treated as a real intersection.

Step 3: Conclusion The required line is $3y - 1 = 0$. Hence, the correct option is (4).

Quick Tip to Solve: After eliminating a squared coordinate, check its nonnegativity; an algebraic root need not represent a real intersection.

13. If a and c are positive real numbers and the ellipse $\frac{x^2}{4c^2} + \frac{y^2}{c^2} = 1$ has four distinct points in common with the circle $x^2 + y^2 = 9a^2$, then [4 marks]

(1) $9ac - 9a^2 - 2c^2 < 0$

(2) $6ac + 9a^2 - 2c^2 < 0$

(3) $9ac - 9a^2 - 2c^2 > 0$

(4) $6ac + 9a^2 - 2c^2 > 0$

Correct Answer: (3) $9ac - 9a^2 - 2c^2 > 0$

Solution:

Concept:

- The ellipse has semiaxes $2c$ and c , while the concentric circle has radius $3a$. Four distinct intersections require the radius to lie strictly between these semiaxes.
- A factorisation can express two strict bounds as one inequality, provided the signs and the endpoint exclusions are preserved.

Step 1: Find the strict radius bounds Write a point on the ellipse as $x = 2c \cos t$, $y = c \sin t$. Its squared distance from the origin is

$$x^2 + y^2 = c^2 + 3c^2 \cos^2 t.$$

Hence that distance ranges from c to $2c$. For a circle of radius $3a$ to meet the ellipse at four distinct points, it must cut away from the axes, requiring

$$c < 3a < 2c.$$

At either endpoint, two pairs merge into only two axis points. Outside this interval there are no real intersections, so strict inequalities are essential.

Step 2: Convert the interval into the offered quadratic form The preceding bounds say $3a - c > 0$ and $2c - 3a > 0$. Multiplying gives

$$(3a - c)(2c - 3a) > 0.$$

Expanding the product produces

$$9ac - 9a^2 - 2c^2 > 0.$$

Conversely, since $a, c > 0$, both factors cannot be negative simultaneously: that would require $3a < c$ and $3a > 2c$. Thus this positive-product condition is exactly equivalent to the desired interval, and not merely a necessary consequence. Option (1) reverses the sign and describes the outside region. The expressions in (2) and (4) do not factor into the two intersection bounds. For example $a = c$ gives no four intersections, yet the expression in (4) is positive, showing why that alternative is insufficient.

Step 3: Conclusion $9ac - 9a^2 - 2c^2 > 0$. Hence, the correct option is (3).

Quick Tip to Solve: For concentric circle-ellipse intersections, compare the circle radius with both semiaxes and exclude tangency endpoints.

14. The value of $\lim_{x \rightarrow 0} \frac{1}{x} \left[\tan^{-1} \left(\frac{x+1}{2x+1} \right) - \frac{\pi}{4} \right]$ is

[4 marks]

- (1) 1
- (2) $-\frac{1}{2}$
- (3) 2
- (4) 0

Correct Answer: (2) $-\frac{1}{2}$

Solution:

Concept:

- For a differentiable function f , the limit $\lim_{x \rightarrow 0} [f(x) - f(0)]/x$ equals $f'(0)$.
- The chain rule gives $d(\tan^{-1} u)/dx = u'/(1 + u^2)$. The rational inner function is differentiable wherever its denominator is nonzero.

Step 1: Recognise the difference quotient Define $u(x) = (x+1)/(2x+1)$ and $f(x) = \tan^{-1} u(x)$, using the usual principal inverse tangent. Near zero, the denominator $2x+1$ is nonzero. Also $u(0) = 1$, so $f(0) = \tan^{-1}(1) = \pi/4$. Consequently the printed limit is exactly

$$\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x} = f'(0).$$

This identification also confirms that the subtraction in the numerator removes the constant term; the whole bracket, rather than just the inverse tangent, is divided by x .

Step 2: Differentiate and evaluate at zero The quotient rule gives

$$u'(x) = \frac{(2x+1) - 2(x+1)}{(2x+1)^2} = -\frac{1}{(2x+1)^2}.$$

Applying the chain rule,

$$f'(x) = \frac{-1/(2x+1)^2}{1 + ((x+1)/(2x+1))^2} = -\frac{1}{(2x+1)^2 + (x+1)^2}.$$

At zero, the denominator is $1 + 1 = 2$, giving $f'(0) = -1/2$. The negative sign is consistent with the inner ratio decreasing near zero; it rules out the positive alternatives 1 and 2. The derivative is nonzero, which rules out option (4). Continuity of the denominator at zero ensures that both one-sided limits give the same value.

Step 3: Conclusion The limit is $\boxed{-\frac{1}{2}}$. Hence, the correct option is (2).

Quick Tip to Solve: A limit with a subtracted function value and division by the increment is often a derivative in disguise.

15. Statement-1: The statement $A \rightarrow (B \rightarrow A)$ is equivalent to $A \rightarrow (A \vee B)$. **Statement-2:** The statement $\sim [(A \wedge B) \rightarrow (\sim A \vee B)]$ is a tautology. [4 marks]

- (1) Statement- 1 is false; Statement- 2 is true
(2) Statement-1 is true; Statement-2 is true; Statement- 2 is not correct explanation for Statement-1
(3) Statement-1 is true; Statement- 2 is false
(4) Statement-1 is true; Statement-2 is true; Statement- 2 is the correct explanation for Statement-1

Correct Answer: (3) Statement-1 is true; Statement- 2 is false

Solution:

Concept:

- An implication satisfies $P \rightarrow Q \equiv \sim P \vee Q$. It is false only when its antecedent is true and its consequent is false.
- Two compound statements are equivalent when their truth values agree for every assignment of their component statements. A tautology is always true; a contradiction is always false.

Step 1: Simplify both sides of Statement-1 Apply the implication rule to the inner implication first, and then to the outer one:

$$A \rightarrow (B \rightarrow A) \equiv \sim A \vee (\sim B \vee A) \equiv (\sim A \vee A) \vee \sim B.$$

The pair $\sim A \vee A$ is always true, so this entire expression is true for all choices of A, B . Similarly,

$$A \rightarrow (A \vee B) \equiv \sim A \vee A \vee B$$

is always true. Since both expressions have the same truth value in every case, they are equivalent. Thus Statement-1 is true.

Step 2: Evaluate the negated implication in Statement-2 The implication inside the brackets becomes

$$(A \wedge B) \rightarrow (\sim A \vee B) \equiv \sim (A \wedge B) \vee \sim A \vee B.$$

By De Morgan's law this is $\sim A \vee \sim B \vee \sim A \vee B$. Because $\sim B \vee B$ is always true, the bracketed implication is a tautology. Its negation is consequently always false. The negated statement printed in Statement-2 is a contradiction, so Statement-2 is false. This rules out both options asserting that the two statements are true, as well as the option reversing their truth values.

Step 3: Conclusion Statement-1 is true and Statement-2 is false. Hence, the correct option is (3).

Quick Tip to Solve: Before calling a statement a tautology, check whether a negation stands outside the whole bracket: it reverses every truth value.

16. The mean of a data set consisting of 20 observations is 40 . If one observation 53 was wrongly recorded as 33 , then the correct mean will be [4 marks]

- (1) 41
- (2) 49
- (3) 40.5
- (4) 42.5

Correct Answer: (1) 41

Solution:

Concept:

- The arithmetic mean of n observations is $\bar{x} = \frac{\sum x_i}{n}$. Therefore the total represented by a reported mean is $n\bar{x}$.
- When one observation is corrected, replace its recorded value in the total: corrected total = recorded total - incorrect value + actual value. The observation count remains unchanged.

Step 1: Recover the recorded total The reported data set contains 20 observations and has mean 40. Its total, including the incorrectly recorded entry, is therefore

$$S_{\text{recorded}} = 20 \times 40 = 800.$$

The entry contributing to this total is 33, whereas its actual value is 53. Thus the original total is too small, rather than too large. There is no extra observation to add: the same entry is being replaced. In particular, the divisor must stay 20; changing it would describe a different data set.

Step 2: Replace the wrong entry and divide by the count Subtract the value that was used and add the value that should have been used:

$$S_{\text{correct}} = 800 - 33 + 53 = 820.$$

Hence the corrected arithmetic mean is

$$\bar{x}_{\text{correct}} = \frac{820}{20} = 41.$$

As a direct check, the correction increases the total by $53 - 33 = 20$. Spreading this increase over 20 observations increases the mean by exactly 1, from 40 to 41. Option (2), 49, would require an increase of 180 in the total; option (3), 40.5, would require an increase of 10; and option (4), 42.5, would require an increase of 50. None matches the actual correction of 20.

Step 3: Conclusion The correct mean is 41. Hence, the correct option is (1).

Quick Tip to Solve: For a single corrected entry, update the mean directly by adding $(\text{actual} - \text{recorded})/n$ to the old mean.

17. The matrix $A^2 + 4A - 5I$, where I is the identity matrix and $A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$, equals: [4 marks]

(1) $4 \begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix}$

(2) $4 \begin{pmatrix} 0 & -1 \\ 2 & 2 \end{pmatrix}$

(3) $32 \begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix}$

(4) $32 \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

Correct Answer: (1) $4 \begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix}$

Solution:

Concept:

- Matrix multiplication uses row-by-column products. Thus $(A^2)_{ij} = \sum_k A_{ik}A_{kj}$; squaring a matrix does not mean squaring its individual entries.
- The identity matrix I has ones on its diagonal and zeros elsewhere. Subtracting $5I$ therefore subtracts 5 only from the diagonal entries.

Step 1: Multiply the matrix by itself Using the printed matrix, compute all four entries explicitly:

$$A^2 = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} = \begin{pmatrix} 1+8 & 2-6 \\ 4-12 & 8+9 \end{pmatrix} = \begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix}.$$

The negative signs in the off-diagonal entries arise from multiplying by the entry -3 . Keeping these products separate prevents an elementwise-squaring error. For example, the lower-left entry is $4(1) + (-3)(4) = -8$, not 16.

Step 2: Combine the three matrix terms First add $4A$, and then subtract the identity term:

$$\begin{aligned} A^2 + 4A - 5I &= \begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix} + \begin{pmatrix} 4 & 8 \\ 16 & -12 \end{pmatrix} - \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \\ &= \begin{pmatrix} 8 & 4 \\ 8 & 0 \end{pmatrix} = 4 \begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix}. \end{aligned}$$

This matches option (1) entry by entry. Option (2) has a zero upper-left entry and a negative upper-right entry, both incompatible with the calculation. Option (3) has the right inner pattern but its multiplier 32 makes the nonzero entries eight times too large. Option (4) also has an incorrect multiplier and does not preserve the calculated entry ratios.

Step 3: Conclusion $A^2 + 4A - 5I = 4 \begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix}$. Hence, the correct option is (1).

Quick Tip to Solve: Compute the off-diagonal entries first: the identity term cannot change them, so they quickly distinguish the proposed matrix options.

18. If a, b, c are sides of a scalene triangle, then the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is: [4 marks]

- (1) non - negative
- (2) negative
- (3) positive
- (4) non-positive

Correct Answer: The determinant is strictly negative, so both (2) negative and (4) non-positive are true; there is no unique literal choice.

Solution:

Concept:

- A third-order determinant can be expanded along any row, with alternating cofactor signs. Here the order of the last two rows matters for the sign.
- For real numbers, $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$. This is strictly positive unless all three numbers are equal.

Step 1: Expand and factor the determinant Let the required determinant be D . Expansion along the first row gives

$$\begin{aligned} D &= a(bc - a^2) - b(b^2 - ac) + c(ab - c^2) \\ &= 3abc - a^3 - b^3 - c^3. \end{aligned}$$

Now use the sum-of-cubes factorisation

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca).$$

Thus D is the negative of this product. The minus sign is essential: a different cyclic row order would produce the opposite determinant, so it must not be inferred from a memorised pattern without checking the displayed rows.

Step 2: Determine the sign from the side lengths Write the result in a form whose factors have known signs:

$$D = -\frac{a + b + c}{2} [(a - b)^2 + (b - c)^2 + (c - a)^2].$$

All triangle side lengths are positive, so $a + b + c > 0$. A scalene triangle has distinct sides; in particular they cannot all be equal. Hence the sum of squared differences is strictly positive, and the overall minus sign makes $D < 0$. The positive and non-negative options have the wrong sign. Option (2), negative, is the strongest classification. However, any negative number is also non-positive, so option (4) is literally true as well. The alternatives overlap: the strictness of the result does not make the weaker true inequality false. Thus the printed alternatives do not provide a unique mathematically true choice.

Step 3: Conclusion The determinant is strictly negative. Option (2) states the strongest result, but option (4) is also true; the printed choices have no unique literal answer.

Quick Tip to Solve: Factor cyclic three-variable determinants into a side sum and squared differences; the squared differences reveal when equality is possible.

19. Let $A = \{1, 2, 3, 4\}$ and $R : A \rightarrow A$ be the relation defined by $R = \{(1, 1), (2, 3), (3, 4), (4, 2)\}$. The correct statement is [4 marks]

- (1) R does not have an inverse
- (2) R is not a one to one function
- (3) R is an onto function
- (4) R is not a function

Correct Answer: (3) R is an onto function

Solution:

Concept:

- A relation from A to A is a function if each member of its domain has exactly one image. It is one-one when different inputs have different images.
- A function is onto when its range equals its declared codomain. A one-one and onto function is bijective and has an inverse function.

Step 1: Check that the relation is a one-one function Read each ordered pair as an input-output assignment:

$$1 \mapsto 1, \quad 2 \mapsto 3, \quad 3 \mapsto 4, \quad 4 \mapsto 2.$$

Every element of $A = \{1, 2, 3, 4\}$ occurs once as a first coordinate. No input has two images and no input is missing, so R is a function. Its four output values are 1, 3, 4 and 2, with no repetition. Therefore distinct inputs have distinct outputs, and the function is one-one. This directly contradicts options (4) and (2), respectively; neither a reordered output list nor an output different from its input prevents a relation from being a function.

Step 2: Compare the range with the codomain The set of actual output values is

$$\text{Range}(R) = \{1, 3, 4, 2\} = \{1, 2, 3, 4\} = A.$$

Every member of the codomain is attained, so the function is onto. It is consequently bijective. Reversing the ordered pairs produces the inverse function

$$R^{-1} = \{(1, 1), (3, 2), (4, 3), (2, 4)\}.$$

Each element of A again appears exactly once as a first coordinate, confirming that this inverse is well defined. Thus option (1), which denies an inverse, is false. Option (3), asserting that R is an onto function, is the only correct statement among those listed.

Step 3: Conclusion R is an onto function (and is also one-one). Hence, the correct option is (3).

Quick Tip to Solve: For a finite self-map with distinct outputs, sort the outputs and compare them with the codomain; this checks onto immediately.

20. Let $f(x) = \frac{x^2 - x}{x^2 + 2x}$, $x \neq 0, -2$. Then $\frac{d}{dx}[f^{-1}(x)]$ (wherever it is defined) is equal to: [4 marks]

(1) $-\frac{1}{(1-x)^2}$

(2) $\frac{3}{(1-x)^2}$

(3) $\frac{1}{(1-x)^2}$

(4) $-\frac{3}{(1-x)^2}$

Correct Answer: (2) $\frac{3}{(1-x)^2}$

Solution:

Concept:

- The notation f^{-1} denotes the inverse function, not the reciprocal $1/f$. Find it by writing $y = f(t)$ and solving for t .
- Common factors may be cancelled only while retaining the original domain restrictions. The quotient rule gives $(u/v)' = (u'v - uv')/v^2$.

Step 1: Simplify and solve for the inverse For the permitted inputs $t \neq 0, -2$, cancel the factor t :

$$y = \frac{t^2 - t}{t^2 + 2t} = \frac{t - 1}{t + 2}.$$

Rearrange without interchanging an inverse and a reciprocal:

$$yt + 2y = t - 1, \quad t(y - 1) = -1 - 2y, \quad t = \frac{1 + 2y}{1 - y}.$$

Therefore $f^{-1}(y) = (1 + 2y)/(1 - y)$ on the range of f . The value $y = 1$ is impossible for the original fraction, and $y = -1/2$ would give the excluded input $t = 0$. These restrictions do not change the derivative formula, but they specify where this inverse exists.

Step 2: Differentiate the inverse expression Replace the inverse input by x and apply the quotient rule:

$$\begin{aligned} \frac{d}{dx}f^{-1}(x) &= \frac{2(1-x) - (1+2x)(-1)}{(1-x)^2} \\ &= \frac{2 - 2x + 1 + 2x}{(1-x)^2} = \frac{3}{(1-x)^2}. \end{aligned}$$

As a sign check, the simplified original function has derivative $3/(t+2)^2 > 0$; its inverse must also have positive derivative wherever defined. This rules out both negative options (1) and (4). The numerator 3 comes from both quotient-rule terms, so option (3), with numerator 1, omits part of the differentiation. The result holds for $x \neq 1, -1/2$, the domain of the inverse.

Step 3: Conclusion $\frac{d}{dx}[f^{-1}(x)] = \frac{3}{(1-x)^2}$, wherever the inverse is defined. Hence, the correct option is (2).

Quick Tip to Solve: For an inverse derivative, solve for the inverse first and keep domain holes inherited from any cancelled factor.

21. Statement-1: The equation $x \log x = 2 - x$ is satisfied by at least one value of x lying between 1 and 2. **Statement-2:** The function $f(x) = x \log x$ is an increasing function in $[1, 2]$, $g(x) = 2 - x$ is a decreasing function in $[1, 2]$, and the graphs represented by these functions intersect at a point in $[1, 2]$. [4 marks]

- (1) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
 (2) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
 (3) Statement-1 is false; Statement-2 is true.
 (4) Statement-1 is true; Statement-2 is false.

Correct Answer: (1) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Solution:

Concept:

- If a continuous function has opposite signs at the two ends of an interval, the intermediate value theorem guarantees a zero strictly between them.
- The equation $f(x) = g(x)$ describes an intersection of their graphs. Increasing f and decreasing g can intersect at most once; existence must also be checked using their endpoint values.

Step 1: Check the monotonicity and endpoint positions For $f(x) = x \log x$ on $[1, 2]$, differentiation gives

$$f'(x) = 1 + \log x > 0,$$

with the standard natural-log convention. Also $g(x) = 2 - x$ has derivative $g'(x) = -1 < 0$. Thus the stated directions of variation are correct. At the left endpoint, $f(1) = 0 < g(1) = 1$. At the right endpoint, $f(2) = 2 \log 2 > g(2) = 0$. The first graph therefore starts below the second and ends above it. Their continuity prevents them from changing that relative position without meeting.

Step 2: Use the intersection to evaluate both statements Define the continuous difference

$$h(x) = f(x) - g(x) = x \log x + x - 2.$$

Its endpoint values satisfy $h(1) = -1 < 0$ and $h(2) = 2 \log 2 > 0$. Hence there is some $c \in (1, 2)$ with $h(c) = 0$, or $c \log c = 2 - c$. This proves Statement-1. The preceding monotonicity and intersection checks prove every claim in Statement-2, and the intersection is exactly the existence assertion in Statement-1. Thus Statement-2 correctly explains Statement-1. Option (2) incorrectly denies this explanation, and options (3) and (4) incorrectly declare one of the verified statements false. In fact $h'(x) = 2 + \log x > 0$, so this intersection is unique, although only existence is asked.

Step 3: Conclusion Both statements are true, and Statement-2 correctly explains Statement-1. Hence, the correct option is (1).

Quick Tip to Solve: For an intersection claim, subtract the functions and check endpoint signs; opposite monotonicity alone proves uniqueness, not existence.

22. If the surface area of a sphere of radius r is increasing uniformly at the rate $8 \text{ cm}^2/\text{s}$, then the rate of change of its volume is: [4 marks]

- (1) constant
- (2) proportional to $\sqrt{r}$
- (3) proportional to r^2
- (4) proportional to r

Correct Answer: (4) proportional to r

Solution:

Concept:

- The surface area and volume of a sphere of instantaneous radius r are $S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$. Both depend on time through the same radius.
- The chain rule relates the required volume rate to the given area rate: $dV/dt = (dV/dr)(dr/dt)$. Uniform increase of surface area does not imply uniform increase of radius.

Step 1: Differentiate the surface area Let t denote time in seconds and r be measured in centimetres. The stated condition is $dS/dt = 8 \text{ cm}^2/\text{s}$. Differentiating with respect to time gives

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt} = 8, \quad \frac{dr}{dt} = \frac{1}{\pi r}.$$

Thus the radius increases more slowly as the sphere becomes larger. The radius rate itself is inversely proportional to r , which must not be confused with the requested volume rate.

Step 2: Calculate the volume rate Differentiate the volume formula and substitute the radius rate already obtained:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi r^2 \frac{1}{\pi r} = 4r \text{ cm}^3/\text{s}.$$

The factors π cancel and one power of r remains. Equivalently, $dV/dS = (4\pi r^2)/(8\pi r) = r/2$, which gives the same result when multiplied by the given area rate. The alternatives constant, square-root dependence and quadratic dependence all disagree with this linear expression.

Step 3: Conclusion The volume increases at the rate $4r \text{ cm}^3/\text{s}$, proportional to the instantaneous radius r . Hence, the correct option is (4).

Quick Tip to Solve: For a sphere, use $dV/dS = r/2$ to convert a given surface-area rate directly into the volume rate.

23. If $\int \frac{dx}{x+x^7} = p(x)$, then $\int \frac{x^6}{x+x^7} dx$ is equal to:

[4 marks]

- (1) $\ln|x| - p(x) + c$
- (2) $\ln|x| + p(x) + c$
- (3) $x - p(x) + c$
- (4) $x + p(x) + c$

Correct Answer: (1) $\ln|x| - p(x) + c$

Solution:

Concept:

- If $p(x)$ is an antiderivative of $1/(x+x^7)$, then $p'(x) = 1/(x+x^7)$ wherever the denominator is nonzero.
- Linearity of integration allows fractions with a common denominator to be combined before integrating. Since $x+x^7 = x(1+x^6)$, the real domain excludes $x=0$.

Step 1: Combine the two integrands Write $I = \int x^6/(x+x^7) dx$. Adding the integrand defining p gives a simple rational identity:

$$\frac{x^6}{x+x^7} + \frac{1}{x+x^7} = \frac{x^6+1}{x(1+x^6)} = \frac{1}{x}.$$

The cancellation is valid for real $x \neq 0$, since $1+x^6$ is strictly positive. This avoids an unnecessary substitution or an explicit formula for $p(x)$.

Step 2: Integrate and verify the result Integrate the identity on either interval $x > 0$ or $x < 0$:

$$I + p(x) = \ln|x| + c, \quad I = \ln|x| - p(x) + c.$$

The arbitrary constants in the two indefinite integrals are absorbed into the single constant c . Differentiate to check:

$$\frac{d}{dx}(\ln|x| - p(x)) = \frac{1}{x} - \frac{1}{x+x^7} = \frac{x^6}{x+x^7}.$$

The expression with a plus sign before $p(x)$ instead adds the unwanted fraction. Replacing $\ln|x|$ by x produces derivative 1 instead of $1/x$, so neither remaining alternative works.

Step 3: Conclusion The required integral is $\ln|x| - p(x) + c$ on each interval avoiding zero. Hence, the correct option is (1).

Quick Tip to Solve: When one integral is given as a function, add or subtract its integrand from the unknown integrand before attempting integration.

24. If $x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$, then $\frac{d^2y}{dx^2}$ is equal to:

[4 marks]

- (1) y
- (2) $\sqrt{1+y^2}$
- (3) $\frac{x}{\sqrt{1+y^2}}$
- (4) y^2

Correct Answer: (1) y

Solution:

Concept:

- The fundamental theorem of calculus gives $d(\int_0^y f(t) dt)/dy = f(y)$ for a continuous integrand.
- For an invertible differentiable relation, $dy/dx = 1/(dx/dy)$. A second derivative with respect to x requires the chain-rule factor dy/dx , not merely differentiation with respect to y .

Step 1: Differentiate the integral with respect to its upper limit The integrand $(1+t^2)^{-1/2}$ is continuous and positive for every real t . Therefore the relation defines a strictly increasing function x of y , and its derivative never vanishes:

$$\frac{dx}{dy} = \frac{1}{\sqrt{1+y^2}}, \quad \frac{dy}{dx} = \sqrt{1+y^2}.$$

The expression in the second option is thus the first derivative, rather than the second derivative requested. No integration of the original expression is necessary.

Step 2: Apply the chain rule once more Differentiate the first derivative as a function of y , remembering that y depends on x :

$$\frac{d^2y}{dx^2} = \frac{d}{dy} \sqrt{1+y^2} \frac{dy}{dx} = \frac{y}{\sqrt{1+y^2}} \sqrt{1+y^2} = y.$$

The denominator and the first-derivative factor cancel exactly. As a separate check, the integral gives $x = \operatorname{arsinh} y$, or $y = \sinh x$, whose second derivative is itself. The result holds for positive, zero and negative y . A square y^2 would lose the sign for negative y , and the expression involving x does not follow from the chain rule.

Step 3: Conclusion $\frac{d^2y}{dx^2} = y$. Hence, the correct option is (1).

Quick Tip to Solve: For an integral with variable upper limit, differentiate first; for the inverse second derivative, multiply the derivative with respect to y by dy/dx .

25. The area bounded by the curve $y = \ln(x)$ and the lines $y = 0$, $y = \ln(3)$ and $x = 0$ is equal to: [4 marks]

- (1) 3
- (2) $3 \ln(3) - 2$
- (3) $3 \ln(3) + 2$
- (4) 2

Correct Answer: (4) 2

Solution:

Concept:

- The curve $y = \ln x$, defined for $x > 0$, can be rewritten as $x = e^y$. This form is convenient when the boundaries are horizontal lines.
- The area of a horizontal strip is $(x_{\text{right}} - x_{\text{left}}) dy$. The given line $x = 0$ forms the left boundary and the exponential curve forms the right boundary.

Step 1: Identify the strip limits and width The lower and upper edges of the region are $y = 0$ and $y = \ln 3$. For each height in this interval, the strip begins at the vertical axis $x = 0$ and ends at $x = e^y$. Thus its width is e^y , including the part between $x = 0$ and $x = 1$. At the bottom the curve has $x = 1$, and at the top it has $x = 3$. Integrating $\ln x$ from 1 to 3 alone would describe a different region under the logarithm, not the entire region specified here.

Step 2: Integrate the horizontal-strip width The required geometric area is positive and is

$$A = \int_0^{\ln 3} (e^y - 0) dy = [e^y]_0^{\ln 3} = 3 - 1 = 2.$$

This calculation includes all strips between the stated horizontal lines. For comparison, the integral $\int_1^3 \ln x dx = 3 \ln 3 - 2$ is the area below the logarithmic curve between 1 and 3, which explains the tempting second option. The first option forgets the lower endpoint contribution, and the plus expression $3 \ln 3 + 2$ is not the integral of the stated strip width. No approximation to $\ln 3$ is needed.

Step 3: Conclusion The bounded area is 2 square units. Hence, the correct option is (4).

Quick Tip to Solve: When a logarithmic curve is bounded by two horizontal lines and the vertical axis, rewrite it as $x = e^y$ and integrate with respect to y .

26. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector of the type $\vec{b} + \lambda\vec{c}$ for some scalar λ , whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$, is: [4 marks]

- (1) $2\hat{i} + \hat{j} + 5\hat{k}$
- (2) $2\hat{i} + 3\hat{j} - 3\hat{k}$
- (3) $2\hat{i} - \hat{j} + 5\hat{k}$
- (4) $2\hat{i} + 3\hat{j} + 3\hat{k}$

Correct Answer: (2) $2\hat{i} + 3\hat{j} - 3\hat{k}$

Solution:

Concept:

- The magnitude of the scalar projection of $\vec{v}$ on a nonzero vector $\vec{a}$ is $|\vec{v} \cdot \vec{a}|/|\vec{a}|$. The absolute value allows either sign of the projection.
- For $\vec{v} = \vec{b} + \lambda\vec{c}$, linearity gives $\vec{v} \cdot \vec{a} = \vec{b} \cdot \vec{a} + \lambda(\vec{c} \cdot \vec{a})$. Its components must also retain this same value of λ .

Step 1: Form the projection equation Use the given components to evaluate

$$|\vec{a}| = \sqrt{4 + 1 + 1} = \sqrt{6}, \quad \vec{b} \cdot \vec{a} = 2 - 2 - 1 = -1, \quad \vec{c} \cdot \vec{a} = 2 - 1 - 2 = -1.$$

Consequently $\vec{v} \cdot \vec{a} = -1 - \lambda$. The required projection magnitude implies

$$\frac{|-1 - \lambda|}{\sqrt{6}} = \sqrt{\frac{2}{3}}, \quad |1 + \lambda| = 2.$$

Both signs must be retained: $\lambda = 1$ or $\lambda = -3$.

Step 2: Compare the resulting vectors with the options The general vector has components

$$\vec{b} + \lambda\vec{c} = (1 + \lambda)\hat{i} + (2 + \lambda)\hat{j} + (-1 - 2\lambda)\hat{k}.$$

For $\lambda = 1$, it is $2\hat{i} + 3\hat{j} - 3\hat{k}$; for $\lambda = -3$, it is $-2\hat{i} - \hat{j} + 5\hat{k}$. Only the first appears among the printed alternatives. Each alternative has first component 2, which already forces $\lambda = 1$; its other components must then be 3 and -3. This rules out every other option even before checking its projection. Directly, the selected vector has dot product $4 - 3 - 3 = -2$ with $\vec{a}$, giving the requested magnitude $2/\sqrt{6} = \sqrt{2/3}$.

Step 3: Conclusion A vector satisfying both conditions is $2\hat{i} + 3\hat{j} - 3\hat{k}$. Hence, the correct option is (2).

Quick Tip to Solve: Use both the projection condition and the component form $\vec{b} + \lambda\vec{c}$; satisfying the projection alone does not establish that an option belongs to the required family.

27. A vector $\vec{n}$ is inclined to the x -axis at 45° , to the y -axis at 60° and at an acute angle to the z -axis. If $\vec{n}$ is a normal to a plane passing through the point $(\sqrt{2}, -1, 1)$, then the equation of the plane is: [4 marks]

- (1) $4\sqrt{2}x + 7y + z - 2$
- (2) $2x + y + 2z = 2\sqrt{2} + 1$
- (3) $3\sqrt{2}x - 4y - 3z = 7$
- (4) $\sqrt{2}x - y - z = 2$

Correct Answer: No printed option matches the plane required by the stated acute direction angles: $\sqrt{2}x + y + z = 2$.

Solution:

Concept:

- The direction cosines l, m, n of a vector obey $l^2 + m^2 + n^2 = 1$. An acute angle with an axis requires the corresponding direction cosine to be positive.
- A plane through (x_0, y_0, z_0) with normal proportional to (A, B, C) has equation $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$.

Step 1: Determine the normal from the stated angles The first two direction cosines are $l = \cos 45^\circ = 1/\sqrt{2}$ and $m = \cos 60^\circ = 1/2$. The identity for direction cosines gives

$$n^2 = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$$

The angle with the positive z -axis is acute, so $n = 1/2$, rather than $-1/2$. Therefore a normal is proportional to $(\sqrt{2}, 1, 1)$. Reversing all three components would describe the same plane, but reversing only the last two would describe a different normal.

Step 2: Find the plane and test every printed choice Insert the stated point $(\sqrt{2}, -1, 1)$ into the point-normal equation:

$$\sqrt{2}(x - \sqrt{2}) + (y + 1) + (z - 1) = 0, \quad \sqrt{2}x + y + z = 2.$$

None of the printed alternatives equals this equation. The first is an expression without an equality and has coefficients not proportional to the required normal. The second and third likewise have incompatible coefficient ratios; the third passes the given point but has the wrong normal. The fourth passes through the point, but its normal $(\sqrt{2}, -1, -1)$ makes obtuse angles with both the positive y - and z -axes. It therefore violates the stated angles. Passing through the point alone is insufficient to choose it.

Step 3: Conclusion The literal conditions give $\boxed{\sqrt{2}x + y + z = 2}$. No printed option satisfies all the conditions, so the question has no correct listed option.

Quick Tip to Solve: After deriving a plane, compare both its normal direction and passage through the stated point; one of these checks alone can accept a distractor.

28. The vector $((\hat{i} \times \vec{a}) \cdot \vec{b})\hat{i} + ((\hat{j} \times \vec{a}) \cdot \vec{b})\hat{j} + ((\hat{k} \times \vec{a}) \cdot \vec{b})\hat{k}$ is equal to:

[4 marks]

- (1) $\vec{b} \times \vec{a}$
- (2) $\vec{a}$
- (3) $\vec{a} \times \vec{b}$
- (4) $\vec{b}$

Correct Answer: (3) $\vec{a} \times \vec{b}$

Solution:

Concept:

- The scalar triple product is unchanged by a cyclic permutation: $(\vec{u} \times \vec{v}) \cdot \vec{w} = \vec{u} \cdot (\vec{v} \times \vec{w})$. Interchanging only two vectors reverses its sign.
- A vector $\vec{c}$ is reconstructed from its components in the orthonormal Cartesian basis as $(\hat{i} \cdot \vec{c})\hat{i} + (\hat{j} \cdot \vec{c})\hat{j} + (\hat{k} \cdot \vec{c})\hat{k}$.

Step 1: Recognise each scalar coefficient Apply the cyclic scalar-triple-product identity separately to the three coefficients:

$$\begin{aligned}(\hat{i} \times \vec{a}) \cdot \vec{b} &= \hat{i} \cdot (\vec{a} \times \vec{b}), \\(\hat{j} \times \vec{a}) \cdot \vec{b} &= \hat{j} \cdot (\vec{a} \times \vec{b}), \\(\hat{k} \times \vec{a}) \cdot \vec{b} &= \hat{k} \cdot (\vec{a} \times \vec{b}).\end{aligned}$$

These are precisely the Cartesian components of $\vec{a} \times \vec{b}$. The order is cyclic, so no minus sign is introduced.

Step 2: Reconstruct the vector and check the sign Set $\vec{c} = \vec{a} \times \vec{b}$. The given expression becomes

$$(\hat{i} \cdot \vec{c})\hat{i} + (\hat{j} \cdot \vec{c})\hat{j} + (\hat{k} \cdot \vec{c})\hat{k} = \vec{c}.$$

As a sign check, take $\vec{a} = \hat{i}$ and $\vec{b} = \hat{j}$. Only the $\hat{k}$ coefficient survives, and it is $(\hat{k} \times \hat{i}) \cdot \hat{j} = 1$; the result is $\hat{k}$, agreeing with $\vec{a} \times \vec{b}$. The reversed product $\vec{b} \times \vec{a}$ would give $-\hat{k}$, while $\vec{a}$ and $\vec{b}$ would give the wrong directions. Thus all other choices fail even this simple independent test.

Step 3: Conclusion The vector is $\vec{a} \times \vec{b}$. Hence, the correct option is (3).

Quick Tip to Solve: When a sum contains three triple products multiplying $\hat{i}, \hat{j}, \hat{k}$, use cyclic permutation to recognise the components of a single cross product.

29. If the lines $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z+1}{3}$ and $\frac{x+2}{2} = \frac{y-k}{3} = \frac{z}{4}$ are coplanar, then the value of k is: [4 marks]

- (1) $\frac{11}{2}$
 (2) $-\frac{11}{2}$
 (3) $\frac{9}{2}$
 (4) $-\frac{9}{2}$

Correct Answer: (1) $\frac{11}{2}$

Solution:

Concept:

- Two non-parallel lines are coplanar when the vector joining one point on each line has zero scalar triple product with their direction vectors.
- For direction vectors $\mathbf{a}$ and $\mathbf{b}$, the condition is $(\mathbf{B} - \mathbf{A}) \cdot (\mathbf{a} \times \mathbf{b}) = 0$. It excludes skew lines without requiring us to find the plane first.

Step 1: Identify points and direction vectors The first line passes through $A = (-1, 1, -1)$ and has direction $\mathbf{a} = (2, 1, 3)$. The second passes through $B = (-2, k, 0)$ and has direction $\mathbf{b} = (2, 3, 4)$. Therefore $\mathbf{B} - \mathbf{A} = (-1, k-1, 1)$. The signs in the symmetric equations determine these points: $x+1=0$ gives $x=-1$, not $+1$.

Step 2: Impose the scalar triple product condition Compute the normal to the two directions:

$$\mathbf{a} \times \mathbf{b} = (4 - 9, 6 - 8, 6 - 2) = (-5, -2, 4).$$

It is nonzero, so the lines are not parallel. Coplanarity now gives

$$(-1)(-5) + (k-1)(-2) + (1)(4) = 0.$$

$$5 - 2k + 2 + 4 = 0 \implies k = \frac{11}{2}.$$

Step 3: Check the other choices The coplanarity expression is $11 - 2k$. For $k = -11/2, 9/2, -9/2$, it equals 22, 2, 20, respectively, so none vanishes. As a direct check for $k = 11/2$, the parameters on the two lines can be chosen as $t = -3$ and $s = -5/2$, giving their common point $(-7, -2, -10)$. Thus the condition actually produces intersecting lines in one plane.

Step 4: Conclusion The required value is $k = \frac{11}{2}$. Hence, the correct option is (1).

Quick Tip to Solve: Read points from the zero numerators first, then use one scalar triple product; this avoids solving three simultaneous line equations.

30. The probability of a man hitting a target is $2/5$. He fires at the target k times (k , a given number). Then the minimum k , so that the probability of hitting the target at least once is more than $7/10$, is: [4 marks]

- (1) 3
- (2) 5
- (3) 2
- (4) 4

Correct Answer: (1) 3

Solution:

Concept:

- For independent shots with hit probability $p = 2/5$, each miss has probability $1 - p = 3/5$.
- The complement of at least one hit is no hits at all. Hence $P(\text{at least one hit}) = 1 - (3/5)^k$, where k is a positive integer.

Step 1: Express the event through its complement It is unnecessary to add the probabilities of exactly one, two and more hits separately. The only excluded outcome is a miss on every shot. Independence allows multiplication:

$$P(\text{all misses}) = \underbrace{\frac{3}{5} \frac{3}{5} \cdots \frac{3}{5}}_{k \text{ factors}} = \left(\frac{3}{5}\right)^k.$$

Thus the required inequality is

$$1 - \left(\frac{3}{5}\right)^k > \frac{7}{10}, \quad \text{or} \quad \left(\frac{3}{5}\right)^k < \frac{3}{10}.$$

Step 2: Find the first integer satisfying the inequality For one shot the hit probability is 0.4, which is too small. For two shots,

$$1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25} = 0.64 < 0.7.$$

For three shots,

$$1 - \left(\frac{3}{5}\right)^3 = 1 - \frac{27}{125} = \frac{98}{125} = 0.784 > 0.7.$$

The miss probability decreases strictly as k increases because $0 < 3/5 < 1$. Consequently, once the threshold is crossed at three shots, all larger integers also qualify.

Step 3: Compare the listed integers Option (3), namely two shots, fails the required threshold. Options (2) and (4), namely five and four shots, give probabilities above the threshold but are not the minimum. The question asks for the least number of trials, so satisfying the inequality alone is insufficient; the preceding integer must be checked as well.

Step 4: Conclusion The minimum number of shots is $k = 3$. Hence, the correct option is (1).

Quick Tip to Solve: For an at-least-once event, calculate one minus the all-fail probability and test the two integers on either side of the threshold.

Physics (120 marks)

31. From the following, the quantity (constructed from the basic constants of nature), that has the dimensions, as well as correct order of magnitude, vis-a-vis typical atomic size, is: [4 marks]

- (1) $\frac{e^2}{4\pi\epsilon_0 mc^2}$
- (2) $\frac{4\pi\epsilon_0 e^2}{me^2}$
- (3) $\frac{me^2}{4\pi\epsilon_0 b^2}$
- (4) $\frac{4\pi\epsilon_0 mc^2}{e^2}$

Correct Answer: None of the printed alternatives is a defensible atomic-size length; options (2) and (3) contain defective expressions.

Solution:

Concept:

- A proposed atomic size must first have the dimensions of length and then have a magnitude of roughly 10^{-10} m. Passing the dimensional test alone is insufficient.
- Coulomb's law gives $[e^2/(4\pi\epsilon_0)] = [\text{energy}] [\text{length}]$, while mc^2 is an energy. Here e is the elementary charge, m the electron mass and c the speed of light.

Step 1: Test the dimensions and scale of option (1) Dividing the Coulomb energy-length combination by the rest energy gives a length:

$$r_e = \frac{e^2}{4\pi\epsilon_0 mc^2}.$$

With the usual electron constants, its value is

$$r_e = \frac{(8.99 \times 10^9)(1.60 \times 10^{-19})^2}{(9.11 \times 10^{-31})(3.00 \times 10^8)^2} \simeq 2.81 \times 10^{-15} \text{ m}.$$

This is the classical electron radius, approximately five orders of magnitude below an atomic radius. Thus option (1) passes the length test but fails the required atomic-size test.

Step 2: Check the other literal printed alternatives In option (2), the two occurrences of e^2 cancel, leaving $4\pi\epsilon_0/m$. This is not a length: in SI base quantities,

$$[\epsilon_0/m] = \text{C}^2 \text{ s}^2 \text{ kg}^{-2} \text{ m}^{-3}.$$

Option (3) contains b^2 , although no constant b is defined in the question. It therefore does not specify a calculable quantity constructed from the stated basic constants. Its symbol cannot be silently replaced by a different constant. Option (4) is exactly the reciprocal of option (1), so its dimension is inverse length and it cannot represent an atomic radius.

Step 3: Identify the atomic scale without changing an option For comparison, the Bohr-radius combination is

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{me^2} \simeq 5.29 \times 10^{-11} \text{ m},$$

where $\hbar = h/(2\pi)$ is the reduced Planck constant. It has both the required length dimension and an atomic magnitude. However, the printed option (2) has e^2 , not $\hbar^2$, in its numerator. The correct comparison formula does not authorize changing that printed expression.

Step 4: Conclusion As printed, no unique correct option can be selected. Option (1) is a much smaller length, option (4) is an inverse length, option (2) is not a length, and option (3) uses an undefined constant. The atomic-scale combination $4\pi\epsilon_0\hbar^2/(me^2)$ is not among the literal alternatives.

Quick Tip to Solve: Apply the dimensional test first, then the size test: a length near 10^{-15} m is not an atomic radius near 10^{-10} m.

32. Two springs of force constants 300 N/m (Spring A) and 400 N/m (Spring B) are joined together in series. The combination is compressed by 8.75 cm. The ratio of energy stored in A and B is E_A/E_B . Then E_A/E_B is equal to: [4 marks]

- (1) $\frac{4}{3}$
 (2) $\frac{16}{9}$
 (3) $\frac{3}{4}$
 (4) $\frac{9}{16}$

Correct Answer: (1) $\frac{4}{3}$

Solution:

Concept:

- Springs joined in series carry the same compressive force F . Hooke's law gives $x_A = F/k_A$ and $x_B = F/k_B$.
- The elastic energy of a spring is $E = \frac{1}{2}kx^2 = F^2/(2k)$. At a common force the softer spring therefore stores more energy.

Step 1: Use the common force The force transmitted through the junction is the same for both springs in static compression. Their compressions obey

$$\frac{x_A}{x_B} = \frac{k_B}{k_A} = \frac{400}{300} = \frac{4}{3}.$$

The total compression is 0.0875 m, so $x_A + x_B = 0.0875$ m. This total is shared unequally; assigning the same compression to both would describe springs in parallel instead.

Step 2: Calculate the energy ratio directly Using the equal force rather than equal compression,

$$\frac{E_A}{E_B} = \frac{F^2/(2k_A)}{F^2/(2k_B)} = \frac{k_B}{k_A} = \frac{400}{300} = \frac{4}{3}.$$

The common force and the factor of two cancel, explaining why the ratio does not depend on the stated total compression.

Step 3: Verify using the numerical compressions The equivalent force constant is

$$k_{\text{eq}} = \frac{300 \times 400}{300 + 400} = \frac{1200}{7} \text{ N/m.}$$

Thus $F = k_{\text{eq}}(0.0875) = 15 \text{ N}$, giving $x_A = 0.050 \text{ m}$ and $x_B = 0.0375 \text{ m}$. The energies are $E_A = 0.375 \text{ J}$ and $E_B = 0.28125 \text{ J}$, again in ratio $4/3$. The reciprocal $3/4$ reverses the springs, while $16/9$ and $9/16$ incorrectly square the ratio without including the different spring constants.

Step 4: Conclusion The ratio is $E_A/E_B = 4/3$. Hence, the correct option is (1).

Quick Tip to Solve: For series springs use the same force and write energy as $F^2/(2k)$; for parallel springs use the same displacement instead.

33. A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. The angular speed of the door just after the bullet embeds into it will be: [4 marks]

- (1) 6.25 rad/s
- (2) 0.625 rad/s
- (3) 3.35 rad/s
- (4) 0.335 rad/s

Correct Answer: (2) 0.625 rad/s

Solution:

Concept:

- Angular momentum about the hinge axis is conserved during the short impact because the hinge force has zero moment about that axis.
- A uniform door of mass M and width a has moment of inertia $Ma^2/3$ about its vertical edge. The embedded bullet contributes $m(a/2)^2$. Kinetic energy is not conserved in embedding.

Step 1: Calculate the incoming angular momentum The bullet hits the centre of the door, at perpendicular distance $r = a/2 = 0.50 \text{ m}$ from the hinge. Convert its mass before substitution:

$$m = 10 \text{ g} = 0.010 \text{ kg.}$$

The incoming momentum is normal to the door, so

$$L_i = mvr = (0.010)(500)(0.50) = 2.50 \text{ kg m}^2/\text{s.}$$

The initially stationary door contributes no angular momentum.

Step 2: Include both contributions to the final inertia The combined system rotates about the fixed

hinge immediately after impact. Its inertia is

$$I = \frac{Ma^2}{3} + mr^2 = \frac{12(1.0)^2}{3} + 0.010(0.50)^2 = 4.0025 \text{ kg m}^2.$$

Angular momentum conservation gives

$$I\omega = L_i, \quad \omega = \frac{2.50}{4.0025} = 0.6246 \text{ rad/s}.$$

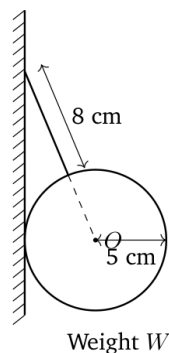
Rounding to the precision of the choices gives 0.625 rad/s. Neglecting the tiny bullet inertia also gives this value to the stated precision.

Step 3: Check the physical scale and alternatives The hinge can exert an external impulse, so conserving the total linear momentum of the bullet and door would be inappropriate. The large energy loss in embedding also excludes kinetic-energy conservation. A value 6.25 rad/s is ten times too large; the other two listed speeds do not satisfy $I\omega = 2.50 \text{ kg m}^2/\text{s}$. The selected speed is consistent with the substantial inertia of the door.

Step 4: Conclusion The angular speed just after impact is approximately 0.625 rad/s. Hence, the correct option is (2).

Quick Tip to Solve: During a hinged-body collision conserve angular momentum about the hinge, and include the embedded particle in the final moment of inertia.

34. A uniform sphere of weight W and radius 5 cm is being held by a string as shown in the figure. The tension in the string will be: [4 marks]



- (1) $\frac{12W}{5}$
- (2) $\frac{5W}{12}$
- (3) $\frac{13W}{5}$
- (4) $\frac{13W}{12}$

Correct Answer: (4) $\frac{13W}{12}$

Solution:

Concept:

- In equilibrium the resultant force and torque on the sphere vanish. Its weight acts through its centre, and the normal reaction of the smooth wall also passes through the centre.
- The tension must therefore act along a line through the centre as well. Resolve it vertically to balance W ; the wall reaction is horizontal.

Step 1: Construct the triangle from the figure The exposed string length is 8 cm, and its extension from the sphere surface to the centre adds the radius 5 cm. Thus the distance from the wall attachment point to the centre is

$$AO = 8 + 5 = 13 \text{ cm.}$$

The centre is one radius, 5 cm, horizontally away from the vertical wall. The vertical side of the right triangle is therefore

$$\sqrt{13^2 - 5^2} = \sqrt{144} = 12 \text{ cm.}$$

Step 2: Resolve the tension vertically Let θ be the angle of the string to the vertical. The triangle gives $\cos \theta = 12/13$ and $\sin \theta = 5/13$. Since the wall normal has no vertical component, vertical force balance requires

$$T \cos \theta = W, \quad T \frac{12}{13} = W.$$

Consequently,

$$T = \frac{13W}{12}.$$

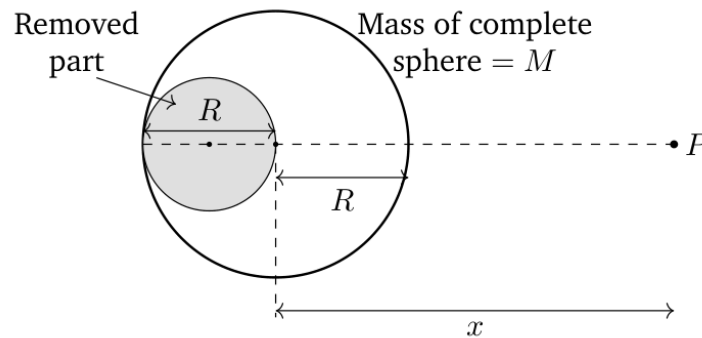
The horizontal balance then gives the normal reaction $N = T(5/13) = 5W/12$, a useful consistency check.

Step 3: Distinguish the possible ratios Tension must be greater than the weight because only its vertical component supports the sphere. This eliminates $5W/12$ immediately; that expression is the wall reaction, not the tension. Using the horizontal fraction $5/13$ in the weight balance would incorrectly give $13W/5$. The value $12W/5$ confuses the two legs of the triangle. Correctly using its vertical-to-hypotenuse ratio selects the remaining choice.

Step 4: Conclusion The tension is $T = \frac{13W}{12}$. Hence, the correct option is (4).

Quick Tip to Solve: Extend the string to the centre before forming the right triangle: the hypotenuse is string length plus radius, here $8 + 5 = 13$ cm.

35. The gravitational field, due to the 'left over part' of a uniform sphere (from which a part as shown has been 'removed out'), at a very far off point P , located as shown, would be (nearly): [4 marks]



- (1) $\frac{5}{6} \frac{GM}{x^2}$
 (2) $\frac{8}{9} \frac{GM}{x^2}$
 (3) $\frac{7}{8} \frac{GM}{x^2}$
 (4) $\frac{6}{7} \frac{GM}{x^2}$

Correct Answer: (3) $\frac{7}{8} \frac{GM}{x^2}$

Solution:

Concept:

- For a uniform sphere, mass is proportional to volume, hence to the cube of radius. The removed mass can be subtracted using superposition.
- At a point very far from the body, its leading gravitational field is $GM_{\text{remaining}}/x^2$. Differences between the distances to the original centre and the cavity centre affect only higher-order corrections.

Step 1: Determine the removed mass from the figure The large sphere has radius R and mass M . In the figure the removed sphere has diameter R , so its radius is $R/2$. Both regions have the same original uniform density ρ . Therefore

$$\frac{M_{\text{removed}}}{M} = \frac{(4\pi/3)\rho(R/2)^3}{(4\pi/3)\rho R^3} = \frac{1}{8}.$$

The remaining mass is

$$M_{\text{remaining}} = M - \frac{M}{8} = \frac{7M}{8}.$$

Step 2: Use the far-field approximation The point P lies a distance x to the right of the original centre, with $x \gg R$. The removed sphere centre lies $R/2$ to the left of that centre. By superposition, the exact external field magnitude along the common line is

$$g = \frac{GM}{x^2} - \frac{G(M/8)}{(x + R/2)^2}.$$

Since $R/(2x)$ is very small, the denominator of the second term can be replaced by x^2 to leading order.

Hence

$$g \simeq \frac{GM}{x^2} - \frac{GM}{8x^2} = \frac{7GM}{8x^2}.$$

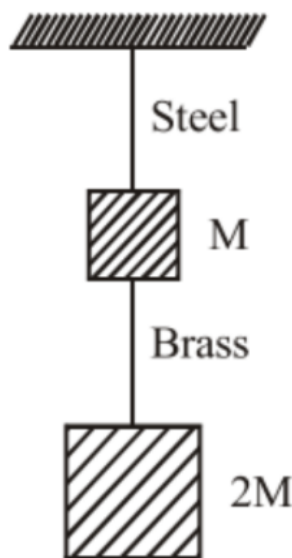
The field points towards the remaining body, to the left at P .

Step 3: Check the alternatives The factor must equal the remaining fraction of mass in this far-field limit. Removing half the radius removes one eighth of the volume, not one half or one quarter. The factors $5/6$, $8/9$, and $6/7$ would imply different removed volume fractions and therefore do not follow from the pictured cavity. The displaced centre affects the next correction but cannot change the leading coefficient $7/8$.

Step 4: Conclusion The field magnitude is nearly $\frac{7}{8} \frac{GM}{x^2}$, directed towards the sphere. Hence, the correct option is (3).

Quick Tip to Solve: For a distant field, first subtract the cavity mass using the cube of its radius ratio; centre shifts matter only in higher-order terms.

36. If the ratio of lengths, radii and Young's moduli of steel and brass wires in the figure are a , b and c respectively, then the corresponding ratio of increase in their lengths is: [4 marks]



- (1) $\frac{3c}{2ab^2}$
- (2) $\frac{2a^2c}{b}$
- (3) $\frac{3a}{2b^2c}$
- (4) $\frac{2ac}{b^2}$

Correct Answer: (3) $\frac{3a}{2b^2c}$

Solution:

Concept:

- For a uniform wire in its elastic range, its extension is $\Delta L = FL/(AY)$, where F is the tensile force, L is the original length, $A = \pi r^2$ is its cross-sectional area and Y is its Young modulus.
- The tension in each segment of a vertical arrangement equals the total weight supported below that segment; it is not necessarily the same in both wires.

Step 1: Find the force in each wire The lower brass wire supports only the lower mass $2M$, so its tension is $F_b = 2Mg$. The upper steel wire supports both the intermediate mass M and the lower mass $2M$, so its tension is $F_s = 3Mg$. Thus $F_s/F_b = 3/2$. The mass of the wires is neglected, as in the given arrangement; assigning equal tensions would miss the load attached between the wires.

Step 2: Form the extension ratio Write the extension formula separately for steel and brass and divide:

$$\frac{\Delta L_s}{\Delta L_b} = \frac{F_s L_s r_b^2 Y_b}{F_b L_b r_s^2 Y_s}.$$

The stated ratios mean $L_s/L_b = a$, $r_s/r_b = b$ and $Y_s/Y_b = c$. The area ratio introduces the square of the radius ratio, while the Young-modulus ratio enters reciprocally because a stiffer wire extends less.

Step 3: Substitute and compare the choices Consequently,

$$\frac{\Delta L_s}{\Delta L_b} = \frac{3}{2} a \frac{1}{b^2} \frac{1}{c} = \frac{3a}{2b^2c}.$$

Option (1) incorrectly reverses the length and modulus dependence. Option (2) has a squared length ratio and does not have the required inverse-square radius factor. Option (4) also puts the modulus ratio in the numerator and uses the wrong load factor.

Step 4: Conclusion The ratio of the increases in length of steel and brass is $\boxed{3a/(2b^2c)}$. Hence, the correct option is (3).

Quick Tip to Solve: First find the total suspended load below each wire; then use extension proportional to $FL/(r^2Y)$.

37. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements. Statement-1: A capillary is dipped in a liquid and liquid rises to a height h in it. As the temperature of the liquid is raised, the height h increases (if the density of the liquid and the angle of contact remain the same). Statement-2: Surface tension of a liquid decreases with the rise in its temperature. [4 marks]

- (1) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation for Statement-1.
(2) Statement-1 is false, Statement-2 is true.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation for Statement-1.

Correct Answer: (2) Statement-1 is false, Statement-2 is true.

Solution:

Concept:

- For capillary rise in a cylindrical tube, $h = 2\gamma \cos \theta / (\rho g r)$, where γ is surface tension, θ is contact angle, ρ is density and r is the tube radius.
- Surface tension ordinarily decreases as the temperature of a liquid increases. The stated fixed density and contact angle isolate this temperature effect.

Step 1: Test Statement-1 using the rise formula The tube radius and gravitational acceleration do not change. The question also fixes the density and contact angle. Therefore all factors in the capillary-rise formula except γ are constant, giving

$$h \propto \gamma.$$

If temperature increases and surface tension decreases, the upward surface-tension force becomes smaller. A smaller column weight can then be supported, so the equilibrium rise decreases rather than increases. Statement-1 is false.

Step 2: Test Statement-2 independently At higher temperature the thermal motion of molecules becomes stronger and the cohesive effect responsible for surface tension becomes weaker. Surface tension consequently decreases as the liquid is heated, approaching zero near the critical temperature. This is precisely the trend stated in Statement-2, so Statement-2 is true. The reasoning uses the fixed-density and fixed-contact-angle conditions in the question rather than adding their usual temperature changes.

Step 3: Select the truth-value combination We have obtained the combination false for Statement-1 and true for Statement-2. Option (2) states exactly this combination. Both options (1) and (4) declare the first statement true, so neither can be correct irrespective of the explanation clause. Option (3) makes the second statement false, in opposition to the established temperature dependence of surface tension.

Step 4: Conclusion Statement-1 is false and Statement-2 is true. Hence, the correct option is (2).

Quick Tip to Solve: When density and contact angle are fixed, capillary rise follows surface tension directly: heating reduces both.

38. On a linear temperature scale Y, water freezes at -160°Y and boils at -50°Y . On this Y scale, a temperature of 340 K would be read as: (water freezes at 273 K and boils at 373 K) [4 marks]

- (1) -73.7°Y
- (2) -233.7°Y
- (3) -86.3°Y
- (4) -106.3°Y

Correct Answer: (3) -86.3°Y

Solution:

Concept:

- Any two linear temperature scales are related by an affine expression $Y = mK + b$; two calibration points determine both constants.
- Corresponding fractions of the interval between the freezing and boiling points are equal, even when the numerical readings are negative.

Step 1: Find the scale interval and gradient Water freezes at 273 K, where the Y reading is -160 , and boils at 373 K, where the reading is -50 . Thus the Kelvin interval is 100 K, while the Y interval is

$$(-50) - (-160) = 110.$$

The scale therefore rises by $110/100 = 1.1$ Y units for every kelvin. The gradient is positive: a warmer sample must have a less negative Y reading.

Step 2: Measure the rise from the freezing point The required temperature is 340 K, which exceeds the stated freezing point by

$$340 - 273 = 67 \text{ K}.$$

The corresponding increase on the Y scale is $1.1 \times 67 = 73.7$ units. Add this increase to the freezing-point reading rather than using it as the final reading:

$$Y = -160 + 73.7 = -86.3.$$

The offset matters because zero on the Y scale is not the freezing point.

Step 3: Check the interpolation and alternatives The temperature 340 K lies between the two calibration temperatures; therefore its Y reading must lie between -160 and -50 . The computed value does so. Option (2), -233.7 , even lies outside this interval and reverses the temperature trend. Options (1) and (4) do not satisfy the interpolation ratio

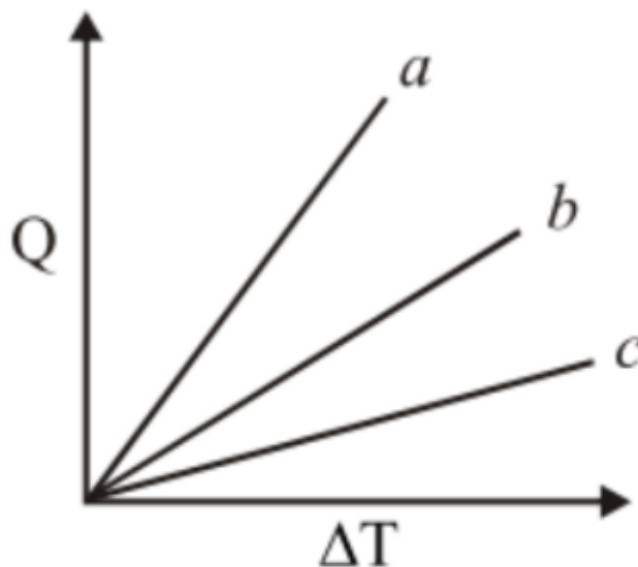
$$\frac{Y + 160}{110} = \frac{340 - 273}{100} = 0.67.$$

Only option (3) gives the required fraction.

Step 4: Conclusion The reading is -86.3°Y . Hence, the correct option is (3).

Quick Tip to Solve: Use the fraction of the freezing-to-boiling interval; retain the negative calibration readings when adding the offset.

39. Figure shows the variation in temperature (ΔT) with the amount of heat supplied (Q) in an isobaric process corresponding to a monoatomic (M), diatomic (D) and a polyatomic (P) gas. The initial state of all the gases are the same and the scales for the two axes coincide. Ignoring vibrational degrees of freedom, the lines a , b and c respectively correspond to: [4 marks]



- (1) P, M and D
- (2) M, D and P
- (3) P, D and M
- (4) D, M and P

Correct Answer: (3) P, D and M

Solution:

Concept:

- For isobaric heating, $Q = nC_p\Delta T$; on a graph with Q vertical and ΔT horizontal, the slope equals nC_p .
- Ignoring vibrational modes, equipartition gives $C_v = fR/2$ and $C_p = (f + 2)R/2$, with $f = 3, 5, 6$ for monoatomic, diatomic and non-linear polyatomic gases respectively.

Step 1: Identify the plotted slope correctly In the figure the vertical axis is Q and the horizontal axis is ΔT . Therefore the relevant slope is $Q/\Delta T$, not its reciprocal. Since all gases start from the same thermodynamic state, their initial pressure, volume and temperature agree, and $PV = nRT$ gives equal mole numbers. Their graph slopes can consequently be ranked by C_p alone.

Step 2: Compare the heat capacities With vibrations neglected, a monoatomic gas has only three translational degrees of freedom and $C_p = 5R/2$. A diatomic gas has three translational and two rotational degrees of freedom, giving $C_p = 7R/2$. For the polyatomic gas the usual non-linear rigid-molecule model has three rotational degrees of freedom as well, giving $C_p = 4R$. Thus

$$C_{p,P} > C_{p,D} > C_{p,M}.$$

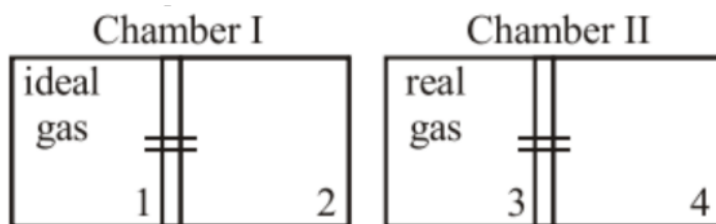
The polyatomic sample needs the most heat for the same temperature rise.

Step 3: Match the lines and reject the other orders Line a is steepest, line b has intermediate slope and line c is shallowest. They therefore represent P, D and M respectively. Option (1) incorrectly puts M ahead of D in the last two positions. Option (2) would make M require the greatest heat, while option (4) makes D steeper than P. Both contradict the heat-capacity ordering read using the actual axes.

Step 4: Conclusion The lines a, b, c correspond to $[P, D, M]$, respectively. Hence, the correct option is (3). This result differs from the supplied key, which reverses the heat-capacity ordering: the printed vertical axis is Q , so option (3) follows from the graph.

Quick Tip to Solve: Check which quantity is vertical before ranking slopes: a Q -against- ΔT graph has slope equal to heat capacity.

40. There are two identical chambers, completely thermally insulated from surroundings. Both chambers have a partition wall dividing the chambers in two compartments. Compartment 1 is filled with an ideal gas and compartment 3 is filled with a real gas. Compartments 2 and 4 are vacuum. A small hole (orifice) is made in the partition walls and the gases are allowed to expand in vacuum. Statement-1: No change in the temperature of the gas takes place when ideal gas expands in vacuum. However, the temperature of real gas goes down (cooling) when it expands in vacuum. Statement-2: The internal energy of an ideal gas is only kinetic. The internal energy of a real gas is kinetic as well as potential. [4 marks]



- (1) Statement-1 is false and Statement-2 is true
- (2) Statement-1 and Statement-2 both are true. Statement-2 is the correct explanation of Statement-1
- (3) Statement-1 is true and Statement-2 is false
- (4) Statement-1 and Statement-2 both are true. Statement-2 is not correct explanation of Statement-1

Correct Answer: (1) Statement-1 is false and Statement-2 is true

Solution:

Concept:

- For an insulated free expansion into vacuum, heat exchange is zero and external work is zero, so the first law gives $\Delta U = Q - W = 0$.
- An ideal gas has no intermolecular potential-energy contribution: $U = U(T)$. In a real gas, molecular interactions also contribute potential energy.

Step 1: Apply the first law to both chambers The walls are thermally insulating, so no heat enters

either chamber. The gases expand against vacuum, whose opposing pressure is zero; hence the boundary work against the surroundings is also zero. Both expansions therefore preserve the total internal energy:

$$Q = 0, \quad W = 0, \quad \Delta U = 0.$$

This is free expansion in an isolated chamber, not a reversible expansion at fixed pressure or a throttling experiment.

Step 2: Evaluate the ideal-gas behaviour For an ideal gas, internal energy consists of molecular kinetic energy and depends only on temperature. With a fixed amount of gas and $\Delta U = 0$, its temperature cannot change. For example, $\Delta U = nC_v\Delta T$ immediately gives $\Delta T = 0$. Thus the ideal-gas part of Statement-1 agrees with the kinetic-only description in Statement-2.

Step 3: Test whether real-gas cooling is universal For a real gas, internal energy contains both kinetic and intermolecular potential energy, as Statement-2 says. However, this distinction does not imply that every real gas must cool. If attraction dominates, separation increases potential energy and can reduce kinetic energy; if repulsion dominates, the potential energy can instead decrease and the gas can warm. The question specifies no particular gas, equation of state or interaction regime. More generally, for a fixed amount of gas,

$$dU = C_V dT + \left[T \left(\frac{\partial P}{\partial T} \right)_V - P \right] dV.$$

Along equal-internal-energy equilibrium states, the temperature change therefore depends on the sign of the square-bracketed term. It is zero for an ideal gas but is not constrained to be positive for every real gas. Thus the claim that a real gas necessarily cools makes Statement-1 false, even though its ideal-gas clause is correct. Statement-2 remains true. Options (2) and (4) wrongly declare Statement-1 true; option (3) also wrongly declares Statement-2 false.

Step 4: Conclusion Statement-1 is false because real-gas cooling is not universal; Statement-2 is true because real gases have intermolecular potential energy as well as kinetic energy. Hence, the correct option is (1).

Quick Tip to Solve: Free expansion fixes total internal energy. It fixes an ideal gas temperature, but a real gas temperature change needs information about its interactions.

41. Two simple pendulums of length 1 m and 4 m respectively are both given small displacement in the same direction at the same instant. They will be again in phase after the shorter pendulum has completed number of oscillations equal to

[4 marks]

- (1) 2
- (2) 7
- (3) 5
- (4) 3

Correct Answer: (1) 2

Solution:

Concept:

- For small oscillations of a simple pendulum, $T = 2\pi\sqrt{L/g}$, where L is its length and g is gravitational acceleration.
- Two oscillations initially in phase return to the same phase when their accumulated phase difference first becomes 2π , or one complete relative cycle.

Step 1: Find the ratio of the periods Let the shorter pendulum have period T_s and the longer one period T_l . Their lengths are 1 m and 4 m, and they experience the same gravity. Hence

$$\frac{T_l}{T_s} = \sqrt{\frac{4}{1}} = 2, \quad T_l = 2T_s.$$

The shorter pendulum thus makes two oscillations in the time the longer one makes one, rather than four oscillations; period depends on the square root of length.

Step 2: Impose the first return to equal phase The pendulums begin with the same phase, since they are displaced in the same direction simultaneously. Their angular frequencies satisfy $\omega_l = \omega_s/2$. The phase difference at time t is

$$\Delta\phi = (\omega_s - \omega_l)t = \frac{\omega_s t}{2}.$$

The first positive time giving equal phases modulo 2π satisfies $\omega_s t/2 = 2\pi$, so $t = 2T_s = T_l$. At that instant both return to their respective original phase.

Step 3: Count the shorter pendulum oscillations The shorter pendulum has completed

$$N_s = \frac{t}{T_s} = 2$$

oscillations. Option (1) is therefore the first recurrence. The counts 3, 5 and 7 in the remaining choices are all odd; at each such instant the shorter pendulum has returned to its starting phase but the longer pendulum has completed a half-integer number of cycles and is in the opposite phase. Equality of positions at an intermediate instant alone would not establish equality of phase.

Step 4: Conclusion The shorter pendulum completes $\boxed{2}$ oscillations before they are again in phase. Hence, the correct option is (1).

Quick Tip to Solve: Convert the length ratio to a period ratio by taking its square root, then find the first common phase recurrence.

42. When two sound waves travel in the same direction in a medium, the displacements of a particle located at x at time t are given by:

$$y_1 = 0.05 \cos(0.50\pi x - 100\pi t), \quad y_2 = 0.05 \cos(0.46\pi x - 92\pi t).$$

Here y_1, y_2 and x are in metres and t is in seconds. The speed of sound in the medium is: [4 marks]

- (1) 92 m/s
- (2) 200 m/s
- (3) 100 m/s
- (4) 332 m/s

Correct Answer: (2) 200 m/s

Solution:

Concept:

- A progressive wave travelling in the positive x direction has the form $y = A \cos(kx - \omega t + \phi)$, where k is angular wave number and ω is angular frequency.
- A point of constant phase satisfies $kx - \omega t = \text{constant}$, so the phase speed is $v = \omega/k$. Amplitude does not enter this speed.

Step 1: Read the coefficients of the first wave Comparing $y_1 = 0.05 \cos(0.50\pi x - 100\pi t)$ with the standard form gives

$$k_1 = 0.50\pi \text{ rad m}^{-1}, \quad \omega_1 = 100\pi \text{ rad s}^{-1}.$$

The coefficient of time is angular frequency, not the speed itself. Dividing by the angular wave number removes the common radian factor and leaves metres per second.

Step 2: Compute and verify the medium speed For the first wave,

$$v_1 = \frac{100\pi}{0.50\pi} = 200 \text{ m s}^{-1}.$$

Independently, the second wave has $k_2 = 0.46\pi$ and $\omega_2 = 92\pi$, and therefore

$$v_2 = \frac{92\pi}{0.46\pi} = 200 \text{ m s}^{-1}.$$

Both waves consequently propagate at the same speed in the medium, even though their angular frequencies and wavelengths differ.

Step 3: Distinguish speed from other wave quantities Option (1), 92 m/s, uses only the numerical time coefficient of the second wave and omits the wave-number division. Option (3), 100 m/s, similarly confuses the first time coefficient with a velocity. The familiar air-speed value 332 m/s in option (4) cannot replace the value determined by the supplied equations. One may also use the beat-envelope speed: the ratio $(100\pi - 92\pi)/(0.50\pi - 0.46\pi)$ again equals 200 m/s.

Step 4: Conclusion The speed of sound in the medium is 200 m/s. Hence, the correct option is (2).

Quick Tip to Solve: For a wave written with $kx - \omega t$, divide the time coefficient by the space coefficient; the common π cancels.

43. An engine approaches a hill with a constant speed. When it is at a distance of 0.9 km, it blows a whistle whose echo is heard by the driver after 5 seconds. If the speed of sound in air is 330 m/s, then the speed of the engine is: [4 marks]

- (1) 32 m/s
- (2) 27.5 m/s
- (3) 60 m/s
- (4) 30 m/s

Correct Answer: (4) 30 m/s

Solution:

Concept:

- An echo is a sound pulse travelling to a reflecting surface and back to the moving listener. Use distances in the ground frame, where sound travels at speed $v = 330$ m/s.
- If the initial hill distance is d , engine speed is u , and total echo time is t , the two sound-path lengths sum to $2d - ut$.

Step 1: Write the sound path length The engine initially stands $d = 0.9$ km = 900 m from the hill. In the full 5 s, it advances $5u$ towards the hill. The outward sound path is 900 m; when the reflected pulse meets the driver, the return path is $900 - 5u$. Thus the sound does not cover 1800 m: the listener has moved towards the reflection point.

Step 2: Solve for the engine speed The sound covers the sum of these distances at its ground-frame speed throughout the journey:

$$330(5) = 900 + (900 - 5u).$$

Therefore $5u = 1800 - 1650 = 150$, giving

$$u = 30 \text{ m/s}.$$

A direct check gives outward time $900/330$ and return distance 750 m, whose travel time $750/330$ makes the total exactly 5 s. The other candidate speeds would give echo times $1800/(330 + u)$, respectively about 4.97, 5.03 and 4.62 s, instead of the stated value.

Step 3: Conclusion The engine approaches the hill at 30 m/s. Hence, the correct option is (4).

Quick Tip to Solve: For an approaching engine hearing its own echo, use $t = 2d/(v + u)$; the return distance is shortened by the engine motion.

44. Two point dipoles of dipole moment $\vec{p}_1$ and $\vec{p}_2$ are at a distance x from each other and $\vec{p}_1 \parallel \vec{p}_2$. The force between the dipoles is: [4 marks]

(1) $\frac{1}{4\pi\epsilon_0} \frac{4p_1p_2}{x^4}$

(2) $\frac{1}{4\pi\epsilon_0} \frac{3p_1p_2}{x^3}$

(3) $\frac{1}{4\pi\epsilon_0} \frac{6p_1p_2}{x^4}$

(4) $\frac{1}{4\pi\epsilon_0} \frac{8p_1p_2}{x^4}$

Correct Answer: Parallel dipole moments alone do not specify their orientation relative to the separation; the axial result is option (3).

Solution:

Concept:

- The energy of a point dipole in an external field is $U = -\vec{p} \cdot \vec{E}$; a force is obtained from the spatial gradient of this energy.
- Parallel dipole moments do not specify whether their common direction is along or perpendicular to the line joining their centres. Dipole fields depend on this additional angle.

Step 1: Evaluate the axial arrangement Write $k = 1/(4\pi\epsilon_0)$. If the two parallel moments lie along their separation, the first dipole produces the axial field $E = 2kp_1/x^3$ along the second moment. Consequently

$$U_{\text{axial}} = -\frac{2kp_1p_2}{x^3}, \quad F_x = -\frac{dU}{dx} = -\frac{6kp_1p_2}{x^4}.$$

The magnitude is $6kp_1p_2/x^4$, corresponding to option (3); the negative radial sign indicates attraction.

Step 2: Check whether parallel moments determine this result For a different arrangement also satisfying $\vec{p}_1 \parallel \vec{p}_2$, put the separation perpendicular to both moments. The equatorial field is opposite to the moments, so

$$U_{\text{equatorial}} = \frac{kp_1p_2}{x^3}, \quad F_x = \frac{3kp_1p_2}{x^4}.$$

This is repulsive and has a different magnitude. Thus the printed parallelism condition permits at least two different forces. Options (1) and (4) give neither of these standard limiting cases; (2) has an x^{-3} dependence, characteristic of the interaction energy rather than the force. The stated data therefore cannot select one force uniquely.

Step 3: Conclusion The question as printed is under-specified: parallel moments alone do not determine the force. For the additional axial condition the magnitude is $6p_1p_2/(4\pi\epsilon_0x^4)$, option (3). The supplied key points to (2), but its x^{-3} expression cannot be the force between fixed point dipoles.

Quick Tip to Solve: Check both the dipole directions and the line joining them. The interaction energy varies as x^{-3} , while the force varies as x^{-4} .

45. Two balls of same mass and carrying equal charge are hung from a fixed support of length l . At electrostatic equilibrium, assuming that angles made by each thread is small, the separation, x between the balls is proportional to: [4 marks]

- (1) l
- (2) l^2
- (3) $l^{2/3}$
- (4) $l^{1/3}$

Correct Answer: (4) $l^{1/3}$

Solution:

Concept:

- For each suspended ball at equilibrium, the horizontal tension component balances the Coulomb repulsion and the vertical component balances its weight.
- With equal thread lengths l and equal small deflections θ , symmetry gives $x = 2l \sin \theta$, and $\sin \theta \simeq \tan \theta \simeq \theta$.

Step 1: Balance the forces on one ball Let each mass be m , each charge q , and each tension T . The mutual electric force has magnitude $F_e = kq^2/x^2$, with $k = 1/(4\pi\epsilon_0)$. The force equations are

$$T \sin \theta = \frac{kq^2}{x^2}, \quad T \cos \theta = mg.$$

Their ratio eliminates the unknown tension:

$$\tan \theta = \frac{kq^2}{mgx^2}.$$

The force is evaluated at the full ball-to-ball distance x , not at $x/2$.

Step 2: Insert the small-angle geometry The displacement of either ball from the vertical is $x/2$, so at small angles $\tan \theta \simeq x/(2l)$. Substitute this in the force ratio:

$$\frac{x}{2l} = \frac{kq^2}{mgx^2} \implies x^3 = \frac{2kq^2}{mg}l.$$

When mass and charge are held fixed, the factor $2kq^2/(mg)$ is constant. Taking a cube root gives $x \propto l^{1/3}$. The linear, square and two-thirds-power options overlook the inverse-square decrease of the repulsion as the separation grows.

Step 3: Conclusion The separation is proportional to $l^{1/3}$. Hence, the correct option is (4).

Quick Tip to Solve: Eliminate tension first, then use $\theta \simeq x/(2l)$. The inverse-square Coulomb force makes the final relation cubic in x .

46. In a metre bridge experiment null point is obtained at 40 cm from one end of the wire when resistance X is balanced against another resistance Y . If $X < Y$, then the new position of the null point from the same end, if one decides to balance a resistance of $3X$ against Y , will be close to:

[4 marks]

- (1) 80 cm
- (2) 75 cm
- (3) 67 cm
- (4) 50 cm

Correct Answer: (3) 67 cm

Solution:

Concept:

- At the balance point of a metre bridge, the ratio of the two gap resistances equals the ratio of the two uniform wire-segment lengths.
- For balance length l measured from the end adjacent to X , $X/Y = l/(100 - l)$, with lengths measured in centimetres.

Step 1: Determine the original resistance ratio The null point divides the wire into 40 cm and 60 cm segments. The given condition $X < Y$ places X opposite the shorter segment, so the ratio is

$$\frac{X}{Y} = \frac{40}{60} = \frac{2}{3}.$$

Using the inverse ratio would contradict $X < Y$. The balance principle uses the potential drops along the wire, and does not depend on the battery voltage or galvanometer resistance at null.

Step 2: Find the new balance length Replacing X by $3X$ triples the resistance ratio:

$$\frac{3X}{Y} = 3 \left(\frac{2}{3} \right) = 2.$$

If the new balance length from the same end is l , then

$$\frac{l}{100 - l} = 2 \implies 3l = 200 \implies l = 66.67 \text{ cm}.$$

It moves beyond the midpoint because the resistance on that side is now larger than Y . For comparison, the other proposed lengths 80, 75 and 50 cm would give ratios 4, 3 and 1, respectively, rather than 2.

Step 3: Conclusion The new balance point is approximately 67 cm from the same end. Hence, the correct option is (3).

Quick Tip to Solve: Multiply the resistance ratio, not the balance length: $l/(100 - l)$ is proportional to the gap-resistance ratio.

47. A letter 'A' is constructed of a uniform wire with resistance $1.0\ \Omega$ per cm. The sides of the letter are 20 cm and the cross piece in the middle is 10 cm long. The apex angle is 60° . The resistance between the ends of the legs is close to:

[4 marks]

- (1) $50.0\ \Omega$
- (2) $10\ \Omega$
- (3) $36.7\ \Omega$
- (4) $26.7\ \Omega$

Correct Answer: (4) $26.7\ \Omega$

Solution:

Concept:

- The resistance of a uniform wire is its length multiplied by its resistance per unit length. Here each centimetre contributes $1.0\ \Omega$.
- Two paths connected between the same pair of junctions are in parallel; an unbranched segment carrying the full terminal current is in series with that parallel combination.

Step 1: Identify the junctions and wire resistances Call the two feet L and R , the apex A , and the cross-piece endpoints P and Q . Each side is 20 cm, with the cross piece joining the midpoints; thus LP , PA , AQ and QR are each 10 cm long and each has resistance $10\ \Omega$. The 10 cm cross piece PQ also has resistance $10\ \Omega$. The stated 60° apex is consistent with this midpoint geometry.

Step 2: Reduce the two paths between the midpoints Between P and Q , current can travel directly along the cross piece, of resistance $10\ \Omega$, or through $P \rightarrow A \rightarrow Q$, of series resistance $20\ \Omega$. Hence

$$R_{PQ} = \frac{10 \times 20}{10 + 20} = \frac{20}{3}\ \Omega.$$

The two lower half-legs are in series with this network, giving

$$R_{LR} = 10 + \frac{20}{3} + 10 = \frac{80}{3}\ \Omega \simeq 26.7\ \Omega.$$

Adding every wire in series would give $50\ \Omega$ and miss the branch. Taking only the cross piece ignores the two lower legs; the $36.7\ \Omega$ value likewise does not follow the actual junction connections.

Step 3: Conclusion The equivalent resistance between the feet is approximately $26.7\ \Omega$. Hence, the correct option is (4).

Quick Tip to Solve: Mark the two cross-piece junctions first: the cross piece is parallel to the two upper half-legs, while the lower half-legs stay in series.

48. A shunt of resistance $1\ \Omega$ is connected across a galvanometer of $120\ \Omega$ resistance. A current of 5.5 ampere gives full scale deflection in the galvanometer. The current that will give full scale deflection in the absence of the shunt is nearly:

[4 marks]

- (1) $5.5\ \text{A}$
- (2) $0.5\ \text{A}$
- (3) $0.004\ \text{A}$
- (4) $0.045\ \text{A}$

Correct Answer: (4) $0.045\ \text{A}$

Solution:

Concept:

- A shunt is a low resistance connected in parallel with a galvanometer. Parallel branches have the same potential difference, so most of the total current bypasses the high-resistance coil.
- The coil current required for full-scale deflection, I_g , is a property of the galvanometer. Removing a shunt changes the total current needed, not this coil threshold.

Step 1: Write the current-division equation Let I_g flow through the $G = 120\ \Omega$ coil and I_s through the $S = 1\ \Omega$ shunt at full scale. Equal branch voltages give

$$I_g G = I_s S \implies I_s = 120 I_g.$$

With the shunt attached, the quoted full-scale current is the total input $I = 5.5\ \text{A}$, which divides between the two paths. Kirchhoff's junction rule then gives $I = I_g + I_s = 121 I_g$.

Step 2: Calculate the unshunted full-scale current Solving for the coil current gives

$$I_g = \frac{5.5}{121} = 0.04545 \dots \text{A}.$$

Without the shunt, all supplied current must pass through the coil. Therefore a total current of about $0.045\ \text{A}$, or $45\ \text{mA}$, now causes full-scale deflection. Substitution checks the branch currents: $I_s \simeq 5.4545\ \text{A}$ and their sum is $5.5\ \text{A}$. Option (1) fails to account for the bypass current; (2) and (3) give neither the required division factor nor equal parallel-branch voltages at the stated total current.

Step 3: Conclusion The full-scale current without the shunt is nearly $0.045\ \text{A}$. Hence, the correct option is (4).

Quick Tip to Solve: For a galvanometer of resistance G shunted by S , use $I_g = IS/(G + S)$; never put the shunt and coil in series.

49. A uniform electric field $\vec{E}$ exists between the plates of a charged condenser. A charged particle enters the space between the plates and perpendicular to $\vec{E}$. The path of the particle between the plates is a: [4 marks]

- (1) straight line
- (2) hyperbola
- (3) parabola
- (4) circle

Correct Answer: (3) parabola

Solution:

Concept:

- A particle of charge q and mass m experiences a constant force $q\vec{E}$ in a uniform electric field; its acceleration is $\vec{a} = q\vec{E}/m$.
- Initial motion perpendicular to a constant acceleration combines uniform velocity in one direction with uniformly accelerated motion in the other, just as in projectile motion.

Step 1: Resolve the motion into perpendicular components Choose the x -axis along the entering velocity v_0 , and the y -axis along the field $\vec{E}$. Take the entry point as the origin. Initially $v_x = v_0$ and $v_y = 0$. The field produces no force in the x -direction, so $a_x = 0$ and $x = v_0 t$. Along the field, $a_y = qE/m$ is constant, giving

$$y = \frac{qE}{2m} t^2.$$

The sign of the charge decides on which side of the initial straight trajectory the particle moves.

Step 2: Eliminate time and identify the curve Using $t = x/v_0$ in the transverse displacement gives

$$y = \frac{qE}{2mv_0^2} x^2.$$

This quadratic relation is the equation of a parabola. A straight path would require no transverse acceleration. A circular path requires a force whose direction continually turns to remain normal to the velocity, unlike this fixed electric force. A hyperbola does not satisfy the derived displacement relation. The uniform-field result applies while the particle is between the plates, neglecting edge fields; it then leaves with the velocity acquired at the exit.

Step 3: Conclusion The trajectory between the plates is a parabola. Hence, the correct option is (3).

Quick Tip to Solve: Eliminate time between $x = v_0 t$ and $y = at^2/2$. A constant transverse electric force produces a parabolic path.

50. An electric current is flowing through a circular coil of radius R . The ratio of the magnetic field at the centre of the coil and that at a distance $2\sqrt{2}R$ from the centre of the coil and on its axis is:

[4 marks]

- (1) $2\sqrt{2}$
- (2) 27
- (3) 36
- (4) 8

Correct Answer: (2) 27

Solution:

Concept:

- The field on the axis of a circular coil is $B(x) = \mu_0 N I R^2 / [2(R^2 + x^2)^{3/2}]$, where N is the number of turns and I the current.
- At the centre, $x = 0$, so $B_0 = \mu_0 N I / (2R)$. The same current, radius and number of turns occur in both fields and cancel in their ratio.

Step 1: Write the field ratio Compare the centre with the specified axial point, rather than comparing two distances by an inverse-square law. A loop is an extended source, so that shortcut would not apply at these distances.

$$\frac{B_0}{B(x)} = \frac{(R^2 + x^2)^{3/2}}{R^3} = \left(1 + \frac{x^2}{R^2}\right)^{3/2}.$$

Step 2: Substitute the axial distance The distance is $x = 2\sqrt{2}R$, hence $x^2 = 8R^2$. Therefore

$$\frac{B_0}{B(x)} = (1 + 8)^{3/2} = 9^{3/2} = 27.$$

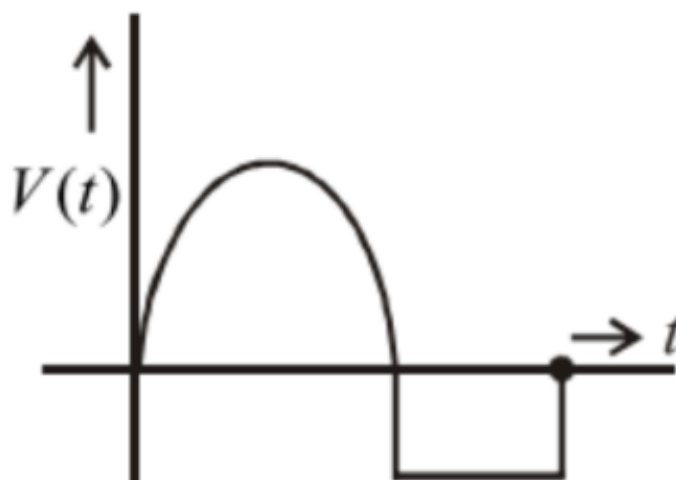
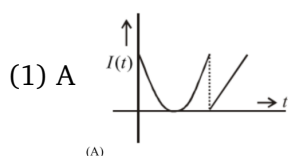
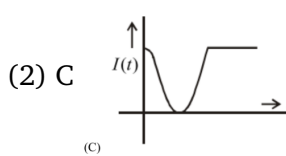
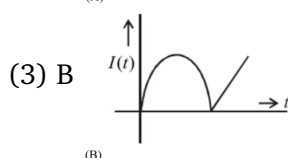
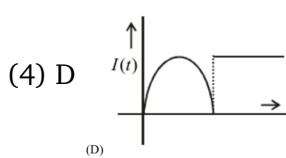
The field at the axial point is thus one twenty-seventh of the central field. The result is dimensionless and exceeds unity, as required because the axial field decreases away from the centre. The values $2\sqrt{2}$, 36 and 8 do not include the correct $3/2$ power of $1 + x^2/R^2$.

Step 3: Conclusion The required ratio is 27 : 1. Hence, the correct option is (2).

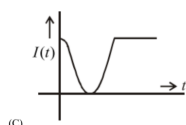
Quick Tip to Solve: For a circular loop, first form $B_0/B(x) = (1 + x^2/R^2)^{3/2}$; only then insert the distance.

51. Two coils, X and Y , are kept in close vicinity of each other. When a varying current, $I(t)$, flows through coil X , the induced emf $V(t)$ in coil Y varies in the manner shown here. The variation of $I(t)$ with time can then be represented by the graph labelled as graph:

[4 marks]



Correct Answer: (2) C



Solution:

Concept:

- For fixed coil positions and constant mutual inductance M , Faraday's law gives $V(t) = -M dI/dt$ for a consistent terminal polarity.
- The sign of emf fixes the sign of the current slope; its magnitude fixes the slope magnitude. A finite jump in emf can change slope but cannot itself cause a jump in current.

Step 1: Translate emf into the current slope For fixed mutual inductance,

$$\frac{dI}{dt} = -\frac{V(t)}{M}.$$

During the initial positive hump of emf, the current decreases. Its downward slope is small initially, becomes steepest where the emf is greatest, then becomes small again as the emf approaches zero. The current need not start at zero: the integration constant is its initial value. Thus the initial branch must fall rather than rise from zero. This excludes graphs B and D, whose first branches initially rise.

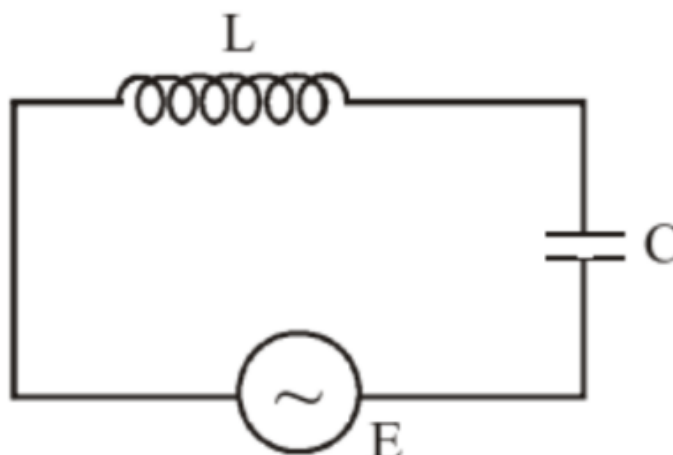
Step 2: Follow the later intervals and compare the graphs During the subsequent interval of constant

negative emf, dI/dt is constant and positive, so the current rises along a straight segment. After the emf returns to zero, $dI/dt = 0$ and the current remains constant. Graph C shows the required sequence: a decreasing curved branch, an increasing segment, and then a horizontal final segment. The joining corners in the schematic represent the changes of slope. Graph A has a finite drop in current at the junction and then continues rising instead of ending in a plateau; a finite step in emf does not produce a discontinuous current. Graphs B and D have the wrong initial sense of variation. Therefore C is the appropriate schematic current graph.

Step 3: Conclusion The required current variation is graph C. Hence, the correct option is (2). The supplied key selects graph A, but its current discontinuity and final rising segment do not follow the displayed emf.

Quick Tip to Solve: Compare the sign of the entire emf interval with the sign of the current slope; do not match the visual shapes of the two curves.

52. In the circuit shown here, the voltage across E and C are respectively 300 V and 400 V. The voltage E of the ac source is: [4 marks]



- (1) 400 Volt
- (2) 500 Volt
- (3) 100 Volt
- (4) 700 Volt

Correct Answer: No listed option follows the literal printed question.

Solution:

Concept:

- In an ideal series LC circuit, the inductor and capacitor voltages are opposite in phase: $V_s = |V_L - V_C|$. They cannot be added as in-phase scalar voltages.
- The diagram labels the inductor L , the capacitor C , and the alternating source E . A stated voltage across E is therefore already the source voltage.

Step 1: Read the labels literally The text specifies that the voltages across E and C are respectively

300 V and 400 V, then asks for the voltage E of the source. In the diagram E is the source, not the inductor. On the literal wording the requested source voltage has already been specified as 300 V. None of the four choices, 400 V, 500 V, 100 V and 700 V, equals 300 V. The capacitor value cannot change a voltage that is explicitly supplied for the source itself.

Step 2: Explain the conventional circuit calculation If the first E were L , the familiar intended calculation would instead be

$$V_s = |300 - 400| \text{ V} = 100 \text{ V}.$$

This would select option (3), because inductive and capacitive voltages are antiphase. The 700 V choice adds their magnitudes, 500 V combines perpendicular phasors, and 400 V takes only the capacitor voltage. However, that calculation substitutes a different component label into the statement. It is a conditional interpretation, not a consequence of the literal printed wording. The printed component labels and given values must remain visible so the discrepancy can be assessed.

Step 3: Conclusion As printed, E is given as 300 V and no option matches. If the first E was intended to mean L , the result would be 100 V, option (3). The literal question has no defensible listed answer.

Quick Tip to Solve: Identify which component each voltage belongs to before using phasor addition; the source label E and inductor label L are different.

53. Photons of an electromagnetic radiation has an energy 11keV each. To which region of electromagnetic spectrum does it belong ?

[4 marks]

- (1) X-ray region
- (2) Ultra violet region
- (3) Infrared region
- (4) Visible region

Correct Answer: (1) X-ray region

Solution:

Concept:

- A photon has energy $E = h\nu = hc/\lambda$. Higher photon energy corresponds to higher frequency and shorter wavelength.
- The useful conversion is $hc \simeq 1240 \text{ eV nm}$. X-rays have subnanometre wavelengths, whereas visible and infrared photons have much lower energies.

Step 1: Convert the given photon energy Each photon carries $11 \text{ keV} = 1.1 \times 10^4 \text{ eV}$. The prefix kilo multiplies by one thousand; treating the energy as 11 eV would move the radiation to a very different spectral region. Using the photon relation,

$$\lambda = \frac{hc}{E} = \frac{1240}{11000} \text{ nm} \simeq 0.113 \text{ nm}.$$

This is $1.13 \times 10^{-10} \text{ m}$, approximately one angstrom.

Step 2: Identify the spectral band A wavelength near 0.1 nm lies in the X-ray region. It is far shorter

than visible wavelengths, approximately 400–700 nm. Infrared wavelengths are longer still and their photon energies are below visible photon energies. Ultraviolet lies beyond violet toward shorter wavelengths, but an 11 keV photon is well beyond the ordinary ultraviolet band and is classified here as an X-ray. The energy scale, not the intensity of the beam or the number of photons, determines this identification. Thus only the X-ray choice matches both the calculated wavelength and the keV energy scale.

Step 3: Conclusion The radiation belongs to the X-ray region. Hence, the correct option is (1).

Quick Tip to Solve: Photon energies of a few keV normally indicate X-rays; confirm by $\lambda(\text{nm}) = 1240/E(\text{eV})$.

54. Light is incident from a medium into air at two possible angles of incidence (A) 20° and (B) 40° . In the medium light travels 3.0 cm in 0.2 ns. The ray will: [4 marks]

- (1) suffer total internal reflection in both cases (A) and (B)
- (2) suffer total internal reflection in case (B) only
- (3) have partial reflection and partial transmission in case (B)
- (4) have 100% transmission in case (A)

Correct Answer: (2) suffer total internal reflection in case (B) only

Solution:

Concept:

- The refractive index is $n = c/v$, where v is light speed in the medium and c its speed in vacuum.
- For incidence from a medium of index n into air, $\sin i_c = 1/n$. Total internal reflection occurs only for incidence angles greater than i_c .

Step 1: Calculate the index and critical angle Use consistent SI units for the supplied distance and time:

$$v = \frac{3.0 \times 10^{-2}}{0.2 \times 10^{-9}} = 1.5 \times 10^8 \text{ m s}^{-1}.$$

Hence $n = (3.0 \times 10^8)/(1.5 \times 10^8) = 2$, and

$$\sin i_c = \frac{1}{2}, \quad i_c = 30^\circ.$$

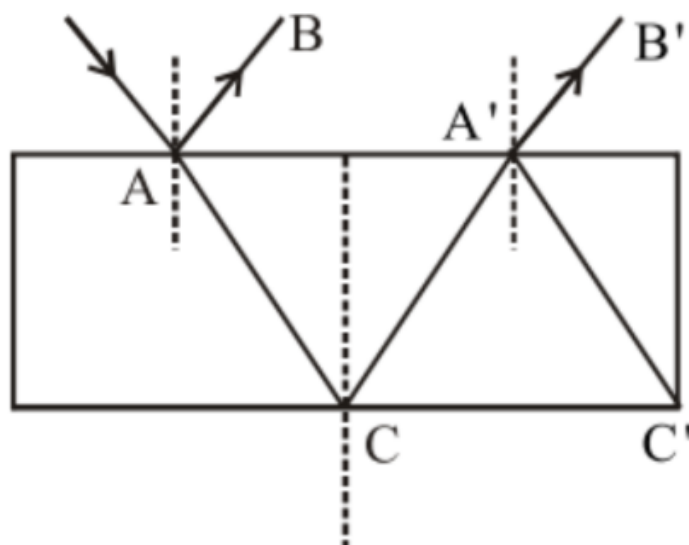
The ray travels from the optically denser medium toward air, so the critical-angle criterion applies.

Step 2: Test each incidence angle For case (A), $20^\circ < 30^\circ$, so a refracted ray emerges into air and a reflected part generally remains in the medium. For case (B), $40^\circ > 30^\circ$, so there is total internal reflection and no propagating transmitted ray into air in the ideal ray-optics description. Therefore reflection cannot be total in both cases. The choice asserting partial transmission in case (B) contradicts the critical angle, while the choice asserting complete transmission in case (A) neglects the ordinarily reflected fraction. Only the statement that case (B) alone undergoes total internal reflection satisfies both comparisons.

Step 3: Conclusion Total internal reflection occurs in case (B) only. Hence, the correct option is (2).

Quick Tip to Solve: Find the critical angle from the measured speed first, then compare each incidence angle separately with it.

55. A ray of light of intensity I is incident on a parallel glass slab at point A as shown in diagram. It undergoes partial reflection and refraction. At each reflection, 25% of incident energy is reflected. The rays AB and $A'B'$ undergo interference. The ratio of $I_{\max}$ and $I_{\min}$ is: [4 marks]



- (1) 49 : 1
- (2) 7 : 1
- (3) 4 : 1
- (4) 8 : 1

Correct Answer: (1) 49 : 1

Solution:

Concept:

- At each encounter with the slab boundary, the intensity reflection fraction is $r = 1/4$, and the transmitted fraction is $t = 3/4$, neglecting absorption.
- Two coherent beams of intensities I_1, I_2 give $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$.

Step 1: Find the two emerging intensities The beam AB is reflected directly at A , so $I_1 = I/4$. The beam that emerges along $A'B'$ is transmitted into the slab at A , reflected once at C , then transmitted out at A' . Its intensity is therefore

$$I_2 = I \left(\frac{3}{4} \right) \left(\frac{1}{4} \right) \left(\frac{3}{4} \right) = \frac{9I}{64}.$$

Each factor refers to the intensity arriving at that particular interface, not to the original beam intensity.

Step 2: Use amplitudes to form the contrast The amplitude ratio is $\sqrt{I_1} : \sqrt{I_2} = (1/2) : (3/8) = 4 : 3$. Consequently

$$\frac{I_{\max}}{I_{\min}} = \frac{(4 + 3)^2}{(4 - 3)^2} = 49.$$

Reflection phase changes affect the locations of bright and dark fringes but not these attainable maximum and minimum values. The value 7 is the ratio of the summed and differenced amplitudes

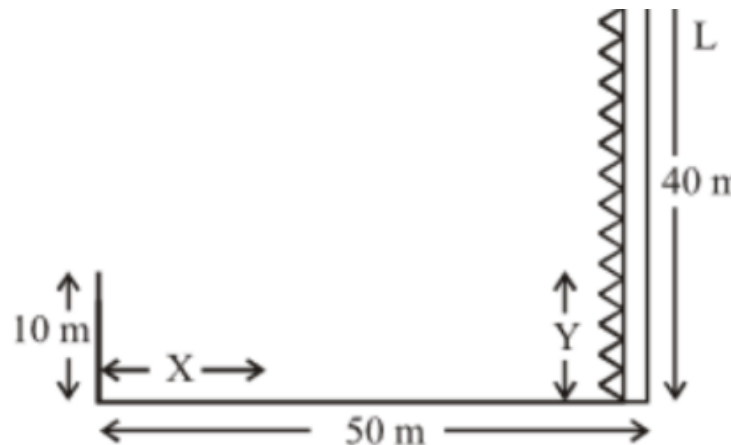
before squaring. The other proposed values do not use both transmission losses and the square-root relation between intensity and amplitude. The nonzero minimum is expected because the two beam intensities are unequal.

Step 3: Conclusion The required intensity ratio is 49 : 1. Hence, the correct option is (1).

Quick Tip to Solve: Track every transmission and reflection first; interference combines amplitudes, so take square roots before forming the maximum-to-minimum ratio.

56. A person lives in a high-rise building on the bank of a river 50 m wide. Across the river is a well lit tower of height 40 m. When the person, who is at a height of 10 m, looks through a polarizer at an appropriate angle at light of the tower reflecting from the river surface, he notes that intensity of light coming from distance X from his building is the least and this corresponds to the light coming from light bulbs at height Y on the tower. The values of X and Y are respectively close to (refractive index of water $\simeq 4/3$):

[4 marks]



- (1) 25 m, 10 m
- (2) 13 m, 27 m
- (3) 22 m, 13 m
- (4) 17 m, 20 m

Correct Answer: (2) 13 m, 27 m

Solution:

Concept:

- At Brewster incidence from air onto water, $\tan i_B = n_{\text{water}}/n_{\text{air}} = 4/3$. The reflected beam is fully plane-polarized and can be extinguished by a suitably oriented polarizer.
- The incidence and reflection angles are measured from the vertical normal to the horizontal water surface, and these two angles are equal.

Step 1: Locate the reflecting point The observer is 10 m above the water, and the horizontal distance

to the reflecting point is X. Thus the outgoing reflected ray makes an angle i_B to the vertical satisfying

$$\tan i_B = \frac{X}{10} = \frac{4}{3}, \quad X = \frac{40}{3} \text{ m} \simeq 13.3 \text{ m}.$$

The tangent is horizontal distance divided by vertical height, because the angle is measured from the normal rather than from the river surface.

Step 2: Find the corresponding tower height The horizontal distance from that point to the tower is $50 - X = 110/3$ m. Equal incidence and reflection angles give

$$\frac{50 - X}{Y} = \frac{4}{3}, \quad Y = \frac{3}{4} \left(\frac{110}{3} \right) = 27.5 \text{ m}.$$

This height is below the 40 m tower top, so the required bulb can lie on the tower. Rounding to the offered approximate values gives 13 m and 27 m. The other pairs do not simultaneously satisfy the Brewster-angle tangent on both sides of the reflection point; equal heights or equal horizontal distances are not required.

Step 3: Conclusion $X \simeq 13.3$ m and $Y \simeq 27.5$ m, close to 13 m and 27 m. Hence, the correct option is (2).

Quick Tip to Solve: For a horizontal reflecting surface, use horizontal distance divided by height as $\tan i_B$, and apply the same ratio to both ray segments.

57. In the Bohr model an electron moves in a circular orbit around the proton. Considering the orbiting electron to be a circular current loop, the magnetic moment of the hydrogen atom, when the electron is in n th excited state, is [4 marks]

- (1) $\frac{e}{2m} \frac{n^2 h}{2\pi}$
- (2) $\frac{e}{m} \frac{nh}{2\pi}$
- (3) $\frac{e}{2m} \frac{nh}{2\pi}$
- (4) $\frac{e}{m} \frac{n^2 h}{2\pi}$

Correct Answer: No exact option for the literal n th excited state; Option (3) is the expression for principal quantum number n .

Solution:

Concept:

- A charge moving periodically constitutes a current of magnitude $I = e/T$, where e is the magnitude of electronic charge and T is the orbital period.
- The magnetic moment magnitude is current times loop area, $\mu = I\pi r^2$, and orbital angular momentum is $L = mvr$. The electron magnetic moment points opposite to its angular momentum.

Step 1: Relate the moment to angular momentum For an electron moving at speed v in a circular

orbit of radius r , the time for one revolution is $T = 2\pi r/v$. Therefore

$$I = \frac{ev}{2\pi r}, \quad \mu = I\pi r^2 = \frac{evr}{2} = \frac{e}{2m}L.$$

The radius cancels into angular momentum; it is not appropriate to infer a quadratic dependence simply because the area contains r^2 .

Step 2: Apply the Bohr quantisation condition If the principal quantum number of the orbit is N , Bohr quantisation gives $L = N\hbar = Nh/(2\pi)$. Consequently

$$\mu = \frac{e}{2m} \frac{Nh}{2\pi}.$$

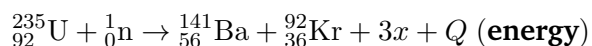
Thus the moment is linear in the principal quantum number. Options (1) and (4) incorrectly introduce its square, and option (2) misses the factor one-half in the orbital gyromagnetic ratio.

Step 3: Distinguish the orbit number from the excitation number The ground state has $N = 1$; the first excited state has $N = 2$. Hence a literal n th excited state has $N = n + 1$, giving $\mu = (e/2m)(n + 1)h/(2\pi)$. None of the listed expressions states that result. If n denotes the orbit number instead, the expression printed in option (3) is obtained.

Step 4: Conclusion For principal quantum number n , $\mu = (e/2m)nh/(2\pi)$, corresponding to option (3). The printed phrase n th excited state instead gives $\mu = (e/2m)(n + 1)h/(2\pi)$, for which no option is exact.

Quick Tip to Solve: First decide whether the symbol counts principal orbits or excited states; the latter count starts one below the principal quantum number.

58. When Uranium is bombarded with neutrons, it undergoes fission. The fission reaction can be written as:



where three particles named x are produced and energy Q is released. What is the name of the particle x ? [4 marks]

- (1) electron
- (2) α -particle
- (3) neutron
- (4) neutrino

Correct Answer: (3) neutron

Solution:

Concept:

- In a nuclear reaction, total mass number A and total atomic number Z are conserved separately. Mass number counts protons plus neutrons; atomic number counts protons.
- A neutron is represented by ${}_0^1\text{n}$: it has mass number one and charge number zero. The released energy comes from a decrease in rest mass, not from a failure of nucleon accounting.

Step 1: Balance the mass numbers The incident system consists of uranium-235 and one neutron, so its total mass number is $235 + 1 = 236$. The two heavy products contribute $141 + 92 = 233$. If each of the three identical particles x has mass number A_x , conservation requires

$$236 = 233 + 3A_x \Rightarrow A_x = 1.$$

Three nucleons therefore remain outside the two stated fragments.

Step 2: Balance the charge numbers Uranium contributes atomic number 92, and the incident neutron contributes zero. The charge numbers of the fragments are $56 + 36 = 92$. Thus

$$92 = 56 + 36 + 3Z_x \Rightarrow Z_x = 0.$$

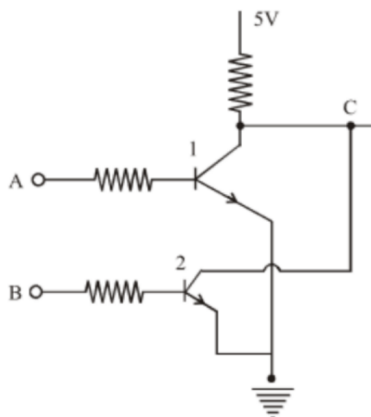
Each emitted particle has zero nuclear charge. Combining this with mass number one identifies each particle uniquely as a neutron.

Step 3: Reject the other proposed particles An electron and a neutrino both have mass number zero and cannot supply the missing three nucleons, irrespective of their different electric charges. An alpha particle has mass number four and charge number two; three alpha particles would contribute twelve mass units and six positive charges, contradicting both balances. The energy term Q denotes energy released; it is not another particle carrying missing mass number.

Step 4: Conclusion The emitted particle is $x = {}^1_0\text{n}$, a neutron. Hence, the correct option is (3).

Quick Tip to Solve: Find the missing mass number and charge number separately: a remainder of (1,0) identifies a neutron without needing nuclear masses.

59. Consider two npn transistors as shown in figure. If 0 Volts corresponds to false and 5 Volts correspond to true then the output at C corresponds to [4 marks]



- (1) A NAND B
- (2) A OR B
- (3) A AND B
- (4) A NOR B

Correct Answer: (4) A NOR B

Solution:

Concept:

- An npn transistor used as a switch conducts from collector to emitter when its base input is high relative to the grounded emitter; a low base input switches it off.
- A pull-up resistor holds an output near the positive supply when no transistor provides a conducting path to ground. Any conducting parallel branch pulls that output low.

Step 1: Read the connections in the circuit The two collectors join the output node C , which is connected through a resistor to the 5 V supply. Both emitters return to ground. The crossing in the drawing has a bridge, so it does not introduce an additional connection between the collector link and the emitter return. Inputs A and B independently control the two bases through their resistors. The two switching paths from output to ground are therefore in parallel.

Step 2: Test all input combinations When $A = 0$ and $B = 0$, both transistors are off. No conducting transistor discharges the output node, and the pull-up resistor gives $C = 1$. With $A = 1, B = 0$, the upper transistor conducts and pulls C low. With $A = 0, B = 1$, the lower transistor does the same. With both inputs high, both conduct and the output is again low. Thus the output sequence for inputs 00, 01, 10, 11 is 1, 0, 0, 0.

Step 3: Identify the Boolean operation An OR gate would give 0, 1, 1, 1; the circuit gives its complement. An AND gate would be high only for 11, while a NAND gate would remain high for 01 and 10. Neither matches the parallel transistor pull-down action. The observed operation is

$$C = \overline{A + B},$$

where the overbar denotes logical negation and the plus denotes OR. The supplied key selects NAND, but the printed parallel collector-emitter paths give NOR; NAND would require series pull-down paths.

Step 4: Conclusion The output is A NOR B . The supplied NAND key conflicts with the parallel pull-down paths, which independently invert OR. Hence, the correct option is (4).

Quick Tip to Solve: Parallel npn pull-down switches make the output low when either input is high: OR at the switches becomes NOR at the output.

60. If a carrier wave $c(t) = A \sin \omega_c t$ is amplitude modulated by a modulator signal $m(t) = A \sin \omega_m t$, then the equation of modulated signal $[C_m(t)]$ and its modulation index are respectively [4 marks]

(1) $C_m(t) = A(1 + \sin \omega_m t) \sin \omega_c t$ and 2

(2) $C_m(t) = A(1 + \sin \omega_m t) \sin \omega_m t$ and 1

(3) $C_m(t) = A(1 + \sin \omega_m t) \sin \omega_c t$ and 1

(4) $C_m(t) = A(1 + \sin \omega_c t) \sin \omega_m t$ and 2

Correct Answer: (3) $C_m(t) = A(1 + \sin \omega_m t) \sin \omega_c t$ and 1

Solution:

Concept:

- For sinusoidal amplitude modulation, the carrier amplitude is varied by the message while the rapid oscillation remains at carrier angular frequency ω_c .
- The conventional AM expression is $s(t) = A_c[1 + \mu \sin(\omega_m t)] \sin(\omega_c t)$, where $\mu = A_m/A_c$ is the modulation index and A_m, A_c are the message and carrier amplitudes.

Step 1: Determine the modulation index The given carrier is $c(t) = A \sin \omega_c t$, so its amplitude is $A_c = A$. The message is $m(t) = A \sin \omega_m t$, with the same amplitude $A_m = A$. Hence

$$\mu = \frac{A_m}{A_c} = \frac{A}{A} = 1.$$

The modulation is 100 percent; the equality of amplitudes does not make the modulation index equal to two.

Step 2: Insert the message into the envelope The instantaneous carrier amplitude is the sum of its unmodulated amplitude and the sinusoidal variation, $A + A \sin \omega_m t$. Multiplying this envelope by the carrier oscillation gives

$$C_m(t) = A(1 + \sin \omega_m t) \sin \omega_c t.$$

The envelope varies between zero and $2A$. Its maximum value $2A$ must not be confused with the dimensionless modulation index, which compares the two original amplitudes.

Step 3: Check the candidate expressions Option (1) has the correct waveform but an incorrect index of two. In option (2), the final sine oscillates at the message frequency rather than the carrier frequency, so it cannot be the required modulated carrier. Option (4) interchanges the roles of ω_c and ω_m in the envelope and oscillatory factor and also gives the wrong index. Option (3) alone gives both the proper envelope and the unit index.

Step 4: Conclusion $C_m(t) = A(1 + \sin \omega_m t) \sin \omega_c t$, with modulation index $\mu = 1$. Hence, the correct option is (3).

Quick Tip to Solve: Keep the carrier sine outside the amplitude bracket; compute the index from the original amplitudes, not the maximum envelope.

61. In an atom how many orbital(s) will have the quantum numbers: $n = 3$, $l = 2$ and $m_l = +2$?

[4 marks]

- (1) 5
- (2) 3
- (3) 1
- (4) 7

Correct Answer: (3) 1

Solution:

Concept:

- An atomic spatial orbital is specified by three quantum numbers: principal number n , azimuthal number l , and magnetic number m_l .
- For a fixed l , allowed magnetic quantum numbers are $-l, -l + 1, \dots, l$, giving $2l + 1$ orbitals only when m_l has not already been fixed.

Step 1: Identify the subshell The principal quantum number $n = 3$ specifies the third shell. For this shell, l can be 0, 1 or 2, so the supplied $l = 2$ is allowed and identifies the $3d$ subshell. This is a valid combination, rather than an excluded orbital such as $n = 2, l = 2$. The shell number alone would not specify a unique orbital because several subshells and magnetic states belong to the shell.

Step 2: Apply the magnetic-number restriction For $l = 2$, the five possible magnetic numbers are

$$m_l = -2, -1, 0, +1, +2.$$

If only $n = 3, l = 2$ had been supplied, all five spatial orbitals would satisfy those two conditions. Here the question additionally fixes $m_l = +2$. Exactly one member of the five-state list obeys that restriction, so the number of allowed spatial orbitals is one.

Step 3: Separate orbitals from electron spin states An orbital can accommodate two electrons with opposite spin projections, but those spin possibilities do not create two different spatial orbitals. The question asks for orbitals, not electrons. Option (1), five, counts the entire d subshell and overlooks the given magnetic number. Three is the number of orbitals in a p subshell, and seven is the number in an f subshell; neither matches the specified complete set of three spatial quantum numbers.

Step 4: Conclusion The specified $n = 3, l = 2, m_l = +2$ describes exactly one orbital. Hence, the correct option is (3).

Quick Tip to Solve: Three fixed spatial quantum numbers specify one orbital; use $2l + 1$ only when the magnetic quantum number is free.

62. Electron gain enthalpy with negative sign of fluorine is less than that of chlorine due to [4 marks]

- (1) High ionization enthalpy of fluorine
- (2) Smaller size of chlorine atom
- (3) Smaller size of fluorine atom
- (4) Bigger size of $2p$ orbital of fluorine

Correct Answer: (3) Smaller size of fluorine atom

Solution:

Concept:

- Electron gain enthalpy is the enthalpy change when an isolated gaseous atom accepts an electron to form a gaseous anion. A negative value means that the process releases energy.
- The energy released depends on both nuclear attraction and electron-electron repulsion. Compact orbitals can produce enough repulsion to reverse a simple size-based expectation.

Step 1: Locate the incoming electron Fluorine has valence configuration $2s^2 2p^5$, and the added electron enters its compact $2p$ subshell. Chlorine has valence configuration $3s^2 3p^5$, so its added electron enters a more spatially extended $3p$ subshell. In each case the result is a filled valence p subshell; the difference is the space available to accommodate the added electron.

Step 2: Compare repulsion in the two subshells The small size of fluorine crowds the existing electrons into a compact region. An incoming electron therefore encounters relatively strong electron-electron repulsion in the $2p$ orbital. In chlorine, the larger $3p$ region spreads electron density out and reduces this repulsive penalty. The greater repulsion in fluorine offsets part of the attractive energy gain, so its electron addition releases less energy than chlorine electron addition.

Step 3: Interpret the options and the negative sign The statement compares the magnitude of the energy released, commonly expressed as electron gain enthalpy with the negative sign removed. Chlorine has the more negative electron gain enthalpy; fluorine has the smaller release in magnitude. Ionization enthalpy concerns removing an electron and is not the direct explanation. Chlorine is larger rather than smaller than fluorine, and fluorine has a compact rather than a bigger $2p$ orbital. Only the smaller fluorine size supplies the required repulsion argument.

Step 4: Conclusion The exception is due to the smaller size of the fluorine atom and the resulting strong repulsion in its compact $2p$ subshell. Hence, the correct option is (3).

Quick Tip to Solve: For the fluorine-chlorine exception, compare crowding in $2p$ and $3p$; chlorine accepts an electron with a greater energy release.

63. Which one of the following molecules is polar?

[4 marks]

- (1) XeF_4
- (2) IF_5
- (3) SbF_5
- (4) CF_4

Correct Answer: (2) IF_5

Solution:

Concept:

- A molecule is polar when the vector sum of its bond dipole moments is nonzero. Polar individual bonds do not by themselves establish molecular polarity.
- VSEPR geometry and the spatial arrangement of identical ligands determine whether bond dipoles cancel. Lone pairs may lower symmetry, but the actual molecular geometry must be checked.

Step 1: Determine the geometry of iodine pentafluoride In IF_5 , iodine has seven valence electrons. Five form single bonds to fluorine, leaving one lone pair on the central atom. There are six electron domains, with octahedral electron-domain arrangement. One domain is a lone pair, so the molecular shape is square pyramidal. The four fluorine atoms in the square base have cancelling lateral dipole components; the axial bond has no identical bond opposite it to cancel its dipole.

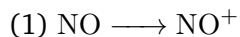
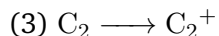
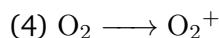
Step 2: Check cancellation in the other molecules XeF_4 has four bonding pairs and two lone pairs, giving an octahedral electron-domain arrangement with the lone pairs opposite each other. Its square planar molecular geometry makes the four Xe–F bond dipoles cancel. SbF_5 is trigonal bipyramidal: three equatorial bond dipoles cancel in their plane, and the two axial dipoles cancel each other. CF_4 is a regular tetrahedron with four identical bonds whose dipole vectors sum to zero.

Step 3: Use net dipole rather than bond polarity All four candidates contain bonds between atoms with different electronegativities. The distinction is therefore their molecular symmetry, not whether they possess polar bonds. Only square pyramidal IF_5 has an uncompensated axial component, so it has a permanent net molecular dipole. The symmetric planar, bipyramidal and tetrahedral arrangements of the other candidates are nonpolar.

Step 4: Conclusion IF_5 is polar because its square pyramidal geometry prevents complete cancellation of its bond dipoles. Hence, the correct option is (2).

Quick Tip to Solve: Count lone pairs first, identify the molecular shape, then add bond dipoles as vectors rather than counting polar bonds.

64. In which of the following ionization processes the bond energy has increased and also the magnetic behaviour has changed from paramagnetic to diamagnetic? [4 marks]



Correct Answer: (1) $\text{NO} \longrightarrow \text{NO}^+$

Solution:

Concept:

- The bond order is $(N_b - N_a)/2$, where N_b and N_a are the numbers of electrons in bonding and antibonding molecular orbitals. For the same pair of atoms, increased bond order generally means a stronger bond and increased bond energy.
- A species with an unpaired electron is paramagnetic; if all occupied orbitals have paired electrons, it is diamagnetic. Ionization must therefore remove the unpaired electron from an antibonding orbital to satisfy both conditions here.

Step 1: Examine the ionization of nitric oxide NO has eleven valence electrons. Its highest occupied antibonding π^* orbital contains one unpaired electron, making it paramagnetic. Removing this electron leaves ten valence electrons and no unpaired electron in NO^+ . Its bond order changes as follows:

$$\text{BO}(\text{NO}) = \frac{8 - 3}{2} = 2.5, \quad \text{BO}(\text{NO}^+) = \frac{8 - 2}{2} = 3.$$

Thus the bond becomes stronger while the ion becomes diamagnetic.

Step 2: Check the other ionizations against both requirements In N_2 , the bond order is 3 and all electrons are paired. Ionization removes a bonding electron, giving N_2^+ with bond order 2.5 and one unpaired electron; both changes oppose the required result. C_2 is also initially diamagnetic, and removal of a bonding electron decreases its bond order from 2 to 1.5. In O_2 , removing an antibonding electron increases the bond order from 2 to 2.5, but O_2^+ retains one unpaired electron and is still paramagnetic. Hence increased bond energy alone does not establish the correct choice.

Step 3: Conclusion Only $\text{NO} \longrightarrow \text{NO}^+$ increases bond order from 2.5 to 3 and changes paramagnetic to diamagnetic behaviour. Hence, the correct option is (1).

Quick Tip to Solve: Check both conditions separately: removing an antibonding electron strengthens the bond, but the ion is diamagnetic only when no unpaired electron remains.

65. By how many folds the temperature of a gas would increase when the root mean square velocity of the gas molecules in a container of fixed volume is increased from $5 \times 10^4 \text{ cm s}^{-1}$ to $10 \times 10^4 \text{ cm s}^{-1}$?

[4 marks]

- (1) Two
- (2) Three
- (3) Six
- (4) Four

Correct Answer: (4) Four

Solution:

Concept:

- The root mean square speed of an ideal gas is $v_{\text{rms}} = \sqrt{3RT/M}$, where T is absolute temperature in kelvin, R is the molar gas constant and M is molar mass.
- For the same gas, R and M remain constant, so $v_{\text{rms}}^2 \propto T$. A fold change compares the final temperature with the initial temperature, rather than comparing their difference with the initial value.

Step 1: Form the ratio of the molecular speeds Both speeds use the same units, so no conversion is necessary in their ratio. Let the initial and final temperatures be T_1 and T_2 . Squaring the kinetic-theory formula gives $v_1^2 = 3RT_1/M$ and $v_2^2 = 3RT_2/M$. Dividing eliminates the molar mass and the gas constant:

$$\frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^2.$$

The given fixed volume does not alter this molecular-speed relation; heating at fixed volume changes the pressure as well.

Step 2: Evaluate the temperature factor The final speed is twice the initial speed:

$$\frac{v_2}{v_1} = \frac{10 \times 10^4}{5 \times 10^4} = 2.$$

Therefore,

$$\frac{T_2}{T_1} = 2^2 = 4.$$

The gas reaches four times its original absolute temperature. The options Two and Six do not follow the square dependence. Three is the fractional rise $(T_2 - T_1)/T_1$, rather than the requested final-to-initial fold factor. Temperature ratios must use kelvin, even though no absolute numerical temperature is needed here.

Step 3: Conclusion The final absolute temperature is four times the initial temperature: $T_2 = 4T_1$. Hence, the correct option is (4).

Quick Tip to Solve: When an rms speed is multiplied by a factor k , multiply the absolute temperature by k^2 ; doubling speed requires four times the temperature.

66. Given: (I) $\text{H}_2(\text{g}) + \frac{1}{2} \text{O}_2(\text{g}) \longrightarrow \text{H}_2\text{O}(\text{l}), \Delta H_{298\text{K}}^\circ = -285.9 \text{ kJ mol}^{-1}$; (II) $\text{H}_2(\text{g}) + \frac{1}{2} \text{O}_2(\text{g}) \longrightarrow \text{H}_2\text{O}(\text{g}), \Delta H_{298\text{K}}^\circ = -241.8 \text{ kJ mol}^{-1}$. **The molar enthalpy of vapourisation of water will be:** [4 marks]

(1) $241.8 \text{ kJ mol}^{-1}$

(2) 22.0 kJ mol^{-1}

(3) 44.1 kJ mol^{-1}

(4) $527.7 \text{ kJ mol}^{-1}$

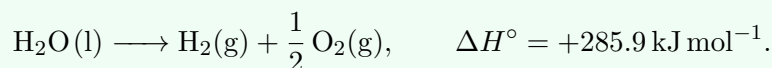
Correct Answer: (3) 44.1 kJ mol^{-1}

Solution:

Concept:

- Enthalpy is a state function. Hess's law allows reactions to be added or subtracted, with the corresponding enthalpy changes treated in the same way.
- Molar enthalpy of vapourisation corresponds to $\text{H}_2\text{O}(\text{l}) \longrightarrow \text{H}_2\text{O}(\text{g})$. The gas is at higher enthalpy than the liquid at the same temperature, so this phase change requires a positive enthalpy change.

Step 1: Reverse the liquid-water formation reaction To obtain vapourisation, liquid water must be a reactant and gaseous water the product. Reverse reaction (I), changing the sign of its enthalpy:



Keep reaction (II) in its given direction, with $\Delta H^\circ = -241.8 \text{ kJ mol}^{-1}$. Both reactions form or consume exactly one mole of water, so no additional stoichiometric scaling is needed.

Step 2: Add the reactions and their enthalpies Adding reversed (I) to (II) cancels $\text{H}_2(\text{g})$ and $\frac{1}{2}\text{O}_2(\text{g})$ on opposite sides, leaving only $\text{H}_2\text{O}(\text{l}) \longrightarrow \text{H}_2\text{O}(\text{g})$. Thus,

$$\begin{aligned} \Delta H_{\text{vap}}^\circ &= \Delta H_f^\circ(\text{H}_2\text{O}(\text{g})) - \Delta H_f^\circ(\text{H}_2\text{O}(\text{l})) \\ &= -241.8 - (-285.9) = +44.1 \text{ kJ mol}^{-1}. \end{aligned}$$

The positive sign agrees with heat absorption during vapourisation. The value 241.8 is the magnitude of a formation enthalpy, 22.0 would incorrectly halve a one-mole result, and 527.7 adds magnitudes instead of subtracting the two state enthalpies.

Step 3: Conclusion The molar enthalpy of vapourisation of water is $+44.1 \text{ kJ mol}^{-1}$ at the specified temperature. The supplied key disagrees, but the printed values give $-241.8 - (-285.9) = +44.1 \text{ kJ mol}^{-1}$. Hence, the correct option is (3).

Quick Tip to Solve: For liquid-to-gas conversion, subtract the liquid formation enthalpy from the gas formation enthalpy and check that the answer is positive.

67. In reaction $A + 2B \rightleftharpoons 2C + D$, initial concentration of B was 1.5 times of [A], but at equilibrium the concentrations of A and B became equal. The equilibrium constant for the reaction is: [4 marks]

- (1) 8
- (2) 4
- (3) 12
- (4) 6

Correct Answer: (2) 4

Solution:

Concept:

- For $A + 2B \rightleftharpoons 2C + D$, the concentration equilibrium constant is $K_c = [C]^2[D]/([A][B]^2)$. Stoichiometric coefficients become exponents in this expression.
- Take the initial reactant concentrations as a and $1.5a$, with products initially absent in the stated reactant mixture. If the extent per unit volume is x , the changes are $-x, -2x, +2x, +x$, respectively.

Step 1: Use the equal equilibrium reactant concentrations The equilibrium concentrations can be written without choosing any numerical starting concentration:

$$\begin{aligned}[A] &= a - x, & [B] &= 1.5a - 2x, \\ [C] &= 2x, & [D] &= x.\end{aligned}$$

The condition in the question gives

$$a - x = 1.5a - 2x \implies x = 0.5a.$$

Both reactants remain at positive concentration, so this extent is physically permissible. Substituting it gives $[A] = [B] = a/2$, $[C] = a$, and $[D] = a/2$.

Step 2: Substitute all four concentrations into the equilibrium expression Use the powers from the balanced equation, rather than just a product-to-reactant concentration ratio:

$$K_c = \frac{a^2(a/2)}{(a/2)(a/2)^2} = \frac{a^3/2}{a^3/8} = 4.$$

The unknown a cancels because the total stoichiometric powers are three on both sides of the expression. No absolute starting concentration is needed. The options 8, 12 and 6 fail this substitution; 8, in particular, can arise from overlooking the factor $a/2$ for D in the numerator. Equality of A and B alone does not mean that C and D are also equal.

Step 3: Conclusion The concentration equilibrium constant for the given reaction is $K_c = 4$. Hence, the correct option is (2).

Quick Tip to Solve: Write the stoichiometric concentration changes first; equal final reactant concentrations determine the extent, after which every factor in K_c is fixed.

68. Solid $\text{Ba}(\text{NO}_3)_2$ is gradually dissolved in a $1.0 \times 10^{-4} \text{ M Na}_2\text{CO}_3$ solution. At which concentration of Ba^{2+} , precipitate of BaCO_3 begins to form? (K_{sp} for $\text{BaCO}_3 = 5.1 \times 10^{-9}$) [4 marks]

- (1) $5.1 \times 10^{-5} \text{ M}$
- (2) $7.1 \times 10^{-8} \text{ M}$
- (3) $4.1 \times 10^{-5} \text{ M}$
- (4) $8.1 \times 10^{-7} \text{ M}$

Correct Answer: (1) $5.1 \times 10^{-5} \text{ M}$

Solution:

Concept:

- For the sparingly soluble salt BaCO_3 , the dissolution equilibrium is $\text{BaCO}_3(\text{s}) \rightleftharpoons \text{Ba}^{2+}(\text{aq}) + \text{CO}_3^{2-}(\text{aq})$, with $K_{\text{sp}} = [\text{Ba}^{2+}][\text{CO}_3^{2-}]$.
- Precipitation begins at the saturation threshold, where the ionic product equals K_{sp} ; below it the solution is unsaturated. Before precipitation, dissolved sodium carbonate supplies one carbonate ion per formula unit.

Step 1: Identify the carbonate concentration before precipitation The soluble electrolyte supplies carbonate according to $\text{Na}_2\text{CO}_3 \longrightarrow 2\text{Na}^+ + \text{CO}_3^{2-}$. Thus the carbonate concentration to use in the solubility-product calculation is

$$[\text{CO}_3^{2-}] = 1.0 \times 10^{-4} \text{ M}.$$

The factor of two applies to sodium ions, which do not appear in the solubility product. Similarly, nitrate ions supplied by barium nitrate are spectator ions. At the instant precipitation starts, carbonate removal into the solid is infinitesimal, so the given initial carbonate concentration is used.

Step 2: Calculate the barium-ion threshold At saturation,

$$[\text{Ba}^{2+}]_{\text{threshold}} = \frac{K_{\text{sp}}}{[\text{CO}_3^{2-}]} = \frac{5.1 \times 10^{-9}}{1.0 \times 10^{-4}} = 5.1 \times 10^{-5} \text{ M}.$$

Checking gives $(5.1 \times 10^{-5})(1.0 \times 10^{-4}) = 5.1 \times 10^{-9}$, exactly the given solubility product. Every other offered barium concentration is smaller than this threshold and therefore produces an ionic product below K_{sp} . Taking $\sqrt{K_{\text{sp}}}$ would apply to dissolution into pure water with equal ion concentrations, not to a solution already containing carbonate.

Step 3: Conclusion Barium carbonate begins to precipitate when $[\text{Ba}^{2+}] = 5.1 \times 10^{-5} \text{ M}$. Hence, the correct option is (1).

Quick Tip to Solve: With one ion already present, divide K_{sp} by its concentration; do not use the pure-water solubility formula $s = \sqrt{K_{\text{sp}}}$.

69. Given: $X\text{Na}_2\text{HAsO}_3 + Y\text{NaBrO}_3 + Z\text{HCl} \longrightarrow \text{NaBr} + \text{H}_3\text{AsO}_4 + \text{NaCl}$. The values of X, Y and Z in the above redox reaction are, respectively: [4 marks]

(1) 2, 1, 2

(2) 2, 1, 3

(3) 3, 1, 6

(4) 3, 1, 4

Correct Answer: (3) 3, 1, 6

Solution:

Concept:

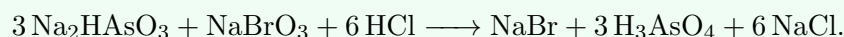
- The total electrons lost in oxidation must equal the electrons gained in reduction. Oxidation numbers identify the atom ratio before hydrogen, oxygen and spectator ions are balanced.
- In HAsO_3^{2-} , arsenic is +3, while in H_3AsO_4 it is +5. Bromine is +5 in BrO_3^- and -1 in Br^- . The reaction therefore couples a two-electron oxidation with a six-electron reduction.

Step 1: Balance the electron transfer to find X and Y For arsenic, $1 + x + 3(-2) = -2$ gives $x = +3$ in the reactant; $3(+1) + x + 4(-2) = 0$ gives $x = +5$ in the product. Each As atom loses two electrons. For bromine, $x + 3(-2) = -1$ gives +5 initially, so reduction to -1 gains six electrons. Consequently three arsenic atoms are needed per bromine atom:

$$X : Y = 3 : 1.$$

Using the smallest whole-number coefficients gives $X = 3$ and $Y = 1$, and the corresponding product coefficients are three for arsenic acid and one for sodium bromide.

Step 2: Balance hydrochloric acid and verify every element The three arsenite units supply three hydrogen atoms, while three arsenic acid molecules contain nine. Six additional hydrogen atoms are supplied by 6HCl , so $Z = 6$. The complete balanced equation is



The sodium count is seven on each side, chlorine is six, arsenic is three, bromine is one, hydrogen is nine, and oxygen is twelve. Options with $X=2$ violate electron balance. The choice (3,1,4) balances the electron ratio but supplies too few hydrogen and chlorine atoms.

Step 3: Conclusion The required smallest coefficients are $X = 3$, $Y = 1$, $Z = 6$. Hence, the correct option is (3).

Quick Tip to Solve: First match the two-electron As oxidation to the six-electron Br reduction; then use hydrogen balance to obtain the HCl coefficient.

70. Sodium carbonate cannot be used in place of $(\text{NH}_4)_2\text{CO}_3$ for the identification of Ca^{2+} , Ba^{2+} and Sr^{2+} ions (in group V) during mixture analysis because: [4 marks]

- (1) Mg^{2+} ions will also be precipitated.
- (2) Concentration of CO_3^{2-} ions is very low.
- (3) Sodium ions will react with acid radicals.
- (4) Na^+ ions will interfere with the detection of Ca^{2+} , Ba^{2+} , Sr^{2+} ions.

Correct Answer: (1) Mg^{2+} ions will also be precipitated.

Solution:

Concept:

- Group V analysis separates Ba^{2+} , Sr^{2+} and Ca^{2+} as their carbonates using ammonium carbonate in an ammoniacal medium containing ammonium chloride.
- Selectivity depends on controlling the carbonate-ion concentration. A salt precipitates when its ionic product exceeds its solubility product; a reagent giving excessive carbonate can also precipitate magnesium and spoil the separation.

Step 1: Explain why ammonium carbonate provides a selective reagent The desired reactions have the form $\text{M}^{2+} + \text{CO}_3^{2-} \longrightarrow \text{MCO}_3(\text{s})$, where M represents calcium, strontium or barium. The ammonium carbonate reagent is used with the ammonium-ion/ammonia medium, which controls the available carbonate through the carbonate–hydrogen carbonate acid-base equilibrium and the ammonium-ion environment. This gives conditions under which the group V carbonates precipitate while magnesium stays in the filtrate for later identification. The medium and reagent together are essential to the group separation; simply providing any soluble carbonate is insufficient.

Step 2: Compare sodium carbonate and eliminate the other explanations Sodium carbonate supplies carbonate readily without introducing the ammonium-ion control required for the separation. The higher effective carbonate availability causes Mg^{2+} to precipitate as magnesium carbonate as well. Magnesium would then accompany the intended group V precipitate, making identification unreliable. The carbonate concentration is therefore not too low, eliminating option (2). Reaction of sodium ions with acid radicals is not the reason, eliminating option (3). Sodium ions themselves do not directly mask the identification of these alkaline-earth cations, so option (4) also misses the actual interference. The interfering species in the precipitate is magnesium.

Step 3: Conclusion Sodium carbonate is unsuitable because Mg^{2+} ions would also precipitate with the group V carbonates. Hence, the correct option is (1).

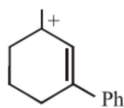
Quick Tip to Solve: For a group reagent, ask which ion must remain in the filtrate: excess carbonate brings magnesium into the calcium–strontium–barium precipitate.

71. Which one of the following is most stable ?

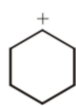
[4 marks]

Which one of the following is most stable ?

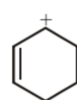
(1)



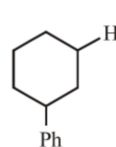
(2)



(3)



(4)



(1) Structure (1) in the figure

(2) Structure (2) in the figure

(3) Structure (3) in the figure

(4) Structure (4) in the figure

Correct Answer: (1) Structure (1) in the figure

Solution:

Concept:

- A carbocation is stabilised when its electron deficiency is spread over several atoms by resonance, rather than confined to one carbon.
- Alkyl groups stabilise a positively charged carbon by the electron-releasing inductive effect and hyperconjugation; phenyl conjugation can provide additional resonance stabilisation.

Step 1: Locate the conjugated positive centre In structure (1), the positively charged ring carbon is adjacent to the carbon-carbon double bond. It is therefore allylic. Moving the double-bond electrons towards that centre transfers the positive charge to the other end of the allylic unit, the carbon bearing the phenyl group. This resonance contributor is benzylic and can delocalise the charge further into the phenyl ring.

Step 2: Compare the other structures Structure (2) is an ordinary cyclohexyl carbocation, so it lacks the adjacent double bond needed for allylic delocalisation. Structure (3) is allylic, but lacks the additional phenyl conjugation available to structure (1). Structure (4) has a saturated ring between its indicated substituents and does not possess the conjugated allylic-benzylic system of (1). The positive centre in (1) also has favourable alkyl substitution. Thus neither a simple inductive comparison nor merely counting ring atoms captures its extra stabilisation.

Step 3: Conclusion Structure (1) has the greatest stabilisation through the conjugated allylic and benzylic resonance system. Hence, the correct option is (1).

Quick Tip to Solve: For competing carbocations, draw where the positive charge moves after shifting an adjacent double bond; check whether it can then enter a phenyl ring.

72. Rate of dehydration of alcohols follows the order

[4 marks]

(1) $2^\circ > 1^\circ > \text{CH}_3\text{OH} > 3^\circ$

(2) $3^\circ > 2^\circ > 1^\circ > \text{CH}_3\text{OH}$

(3) $2^\circ > 3^\circ > 1^\circ > \text{CH}_3\text{OH}$

(4) $\text{CH}_3\text{OH} > 1^\circ > 2^\circ > 3^\circ$

Correct Answer: (2) $3^\circ > 2^\circ > 1^\circ > \text{CH}_3\text{OH}$

Solution:

Concept:

- Dehydration removes water from an alcohol to form an alkene, usually by heating with a strong acid.
- Protonation converts the hydroxyl group into a better leaving group. More stable carbocation character lowers the barrier to elimination, giving the usual reactivity order tertiary, secondary, primary.

Step 1: Relate substitution to the intermediate An unprotonated hydroxyl group is a poor leaving group. In acid it becomes an oxonium group, from which water can leave. Secondary and tertiary alcohols can undergo elimination with substantial carbocation formation. The tertiary carbocation is stabilised by three alkyl groups, whereas a secondary one has only two. Hyperconjugation and the electron-releasing inductive effect therefore favour faster dehydration of the tertiary alcohol.

Step 2: Arrange all four cases Primary alcohols dehydrate less readily, commonly through a concerted elimination rather than a freely formed primary carbocation. Methanol has no adjacent carbon bearing a hydrogen and cannot undergo the ordinary intramolecular dehydration to an alkene; it is consequently placed last in the listed order. The sequence is therefore $3^\circ > 2^\circ > 1^\circ > \text{CH}_3\text{OH}$. Options (1) and (3) incorrectly put a secondary alcohol ahead of a tertiary one. Option (4) reverses the trend and incorrectly puts methanol first.

Step 3: Conclusion The rate follows $3^\circ > 2^\circ > 1^\circ > \text{CH}_3\text{OH}$. Hence, the correct option is (2).

Quick Tip to Solve: For acid dehydration, first compare carbocation stability; also check that an adjacent carbon exists so that alkene formation is possible.

73. Which of the following compounds is not expected to show Lassaignes' test for nitrogen ?

[4 marks]

- (1) Propanenitrile
- (2) Hydroxylamine hydrochloride
- (3) Nitromethane
- (4) Ethanamine

Correct Answer: (2) Hydroxylamine hydrochloride

Solution:

Concept:

- In Lassaigne's nitrogen test, sodium fusion converts carbon and nitrogen from the organic compound into ionic sodium cyanide, NaCN .
- The cyanide ion subsequently forms ferrocyanide with ferrous ions; oxidation and reaction with ferric ions yield the characteristic Prussian-blue product. Carbon as well as nitrogen must be available for cyanide formation.

Step 1: Inspect the composition of each compound Propanenitrile, $\text{CH}_3\text{CH}_2\text{CN}$, contains both carbon and nitrogen. Nitromethane, CH_3NO_2 , also contains both elements, and so does ethanamine, $\text{CH}_3\text{CH}_2\text{NH}_2$. Under suitable sodium-fusion conditions, these substances can supply the carbon and nitrogen needed to produce cyanide. The functional group need not be a nitrile originally: sodium fusion is intended to convert covalently bound elements into detectable ionic forms.

Step 2: Identify the missing requirement Hydroxylamine hydrochloride is $\text{NH}_2\text{OH} \cdot \text{HCl}$. Although it contains nitrogen, it has no carbon atom. Fusion of this compound alone therefore cannot furnish the cyanide ion required for the standard nitrogen test. The absence of carbon, rather than absence of nitrogen, is the decisive point. The three carbon-containing options cannot be selected for this reason; the test is not simply a direct colour reaction of every nitrogen-containing molecule.

Step 3: Conclusion Hydroxylamine hydrochloride is not expected to give the ordinary Lassaigne nitrogen test because it contains no carbon. Hence, the correct option is (2).

Quick Tip to Solve: Before applying the sodium-fusion nitrogen test, check for both C and N in the formula; nitrogen alone does not supply cyanide.

74. The addition of HI in the presence of peroxide catalyst does not follow anti-Markovnikov's rule because: [4 marks]

- (1) HI is a strong reducing agent.
- (2) The H–I bond is too strong to be broken homolytically.
- (3) An I atom combines with an H atom to give back HI.
- (4) An iodine atom is not reactive enough to add across a double bond.

Correct Answer: (4) An iodine atom is not reactive enough to add across a double bond.

Solution:

Concept:

- The peroxide effect is a radical-chain addition to an alkene. A halogen radical must first add to the double bond, forming a new carbon-halogen bond and a carbon-centred radical.
- For a sustained chain, the propagation steps must be energetically favourable. The familiar anti-Markovnikov peroxide effect operates with hydrogen bromide, not with every hydrogen halide.

Step 1: Examine the iodine-radical addition step The proposed chain would require $I^\bullet$ to add across the alkene double bond. This step exchanges part of the carbon-carbon double-bond bonding for a carbon-iodine bond and generates an alkyl radical. The carbon-iodine bond is relatively weak, so the radical addition is energetically unfavourable and readily reversible. Iodine radicals therefore do not add effectively enough to sustain the required chain.

Step 2: Reject the alternative explanations The statement that HI is a reducing agent does not specify the failed propagation step. The H–I bond is actually relatively weak, so saying that it is too strong to undergo homolysis is incorrect. Combination of free hydrogen and iodine atoms may terminate radicals, but it is not the decisive general reason that the peroxide-driven chain cannot proceed. The key distinction is the unfavourable addition of iodine to the double bond, which option (4) expresses.

Step 3: Conclusion The iodine atom is not reactive enough to add effectively across the double bond and maintain the anti-Markovnikov radical chain. Hence, the correct option is (4).

Quick Tip to Solve: For the peroxide effect, test both propagation steps energetically; producing radicals alone does not guarantee a chain addition.

75. An element having an atomic radius of 0.14 nm crystallizes in an fcc unit cell. What is the length of a side of the cell ? [4 marks]

- (1) 0.56 nm
- (2) 0.24 nm
- (3) 0.96 nm
- (4) 0.4 nm

Correct Answer: (4) 0.4 nm

Solution:

Concept:

- In a face-centred cubic unit cell, nearest-neighbour atoms touch along a face diagonal, not along the cell edge.
- For edge length a and atomic radius r , the face diagonal is $\sqrt{2}a$, and the touching sequence along it spans four radii: $\sqrt{2}a = 4r$.

Step 1: Write the contact relation Choose one square face of the unit cell. Its two corner atoms and its face-centred atom have collinear centres along a diagonal. From one corner centre to the face-centre is $2r$, and from the face-centre to the opposite corner centre is another $2r$. Thus the entire face diagonal has length $4r$. By the Pythagorean theorem for a square face, that same diagonal is $a\sqrt{2}$. Equating them gives $a = 2\sqrt{2}r$.

Step 2: Substitute and compare with the choices Using the radius in nanometres avoids any unnecessary unit conversion:

$$a = 2\sqrt{2}(0.14 \text{ nm}) = 0.28\sqrt{2} \text{ nm} \simeq 0.396 \text{ nm}.$$

To the precision of the options, this is 0.4 nm. The value 0.56 nm is the face diagonal $4r$, not the edge. The other proposed lengths, 0.24 nm and 0.96 nm, fail the fcc contact condition. In particular, an edge shorter than the diameter would contradict the actual nearest-neighbour geometry.

Step 3: Conclusion The side length is approximately 0.4 nm. Hence, the correct option is (4).

Quick Tip to Solve: Remember the direction of contact: simple cubic uses the edge, bcc the body diagonal, and fcc the face diagonal.

76. 12 g of a nonvolatile solute dissolved in 108 g of water produces the relative lowering of vapour pressure of 0.1. The molecular mass of the solute is [4 marks]

- (1) 80
- (2) 60
- (3) 20
- (4) 40

Correct Answer: (3) 20

Solution:

Concept:

- Raoult's law for an ideal solution with a nonvolatile solute gives relative lowering $(p^0 - p)/p^0 = x_2$, where x_2 is the solute mole fraction.
- For the usual dilute-solution treatment, $x_2 \simeq n_2/n_1 = (w_2/M_2)/(w_1/M_1)$. The distinction between this approximation and the exact mole fraction matters for the printed data.

Step 1: Apply the conventional dilute-solution relation The solvent is water, so its molar mass is 18 g mol^{-1} and the number of solvent moles is

$$n_1 = \frac{108}{18} = 6 \text{ mol.}$$

Writing the unknown solute molar mass as M_2 , the standard dilute-solution substitution gives

$$0.1 \simeq \frac{12/M_2}{6} = \frac{2}{M_2}, \quad M_2 \simeq 20 \text{ g mol}^{-1}.$$

This is the value offered by option (3); the values 80, 60 and 40 would instead predict approximate relative lowerings 0.025, 0.033 and 0.05.

Step 2: Check the approximation against the exact equation If the given relative lowering is treated as exact, the solute moles also belong in the denominator:

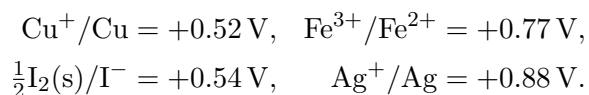
$$0.1 = \frac{12/M_2}{6 + 12/M_2} = \frac{12}{6M_2 + 12}.$$

This gives $M_2 = 18 \text{ g mol}^{-1}$, which is absent from the choices. Thus 20 is the answer obtained using the conventional dilute-solution approximation implicit in this MCQ; it is not the exact Raoult-law value for a lowering of 0.1. Stating this distinction avoids silently equating mole fraction with the solute-to-solvent mole ratio.

Step 3: Conclusion Using the conventional dilute-solution approximation, the molecular mass is 20 (molar mass 20 g mol^{-1}). Hence, the correct option is (3). The exact mole-fraction equation would give 18 g mol^{-1} , which is not listed.

Quick Tip to Solve: Check whether a vapour-pressure problem uses the dilute approximation n_2/n_1 or the exact fraction $n_2/(n_1 + n_2)$.

77. Electrode potentials E° are given below:



Based on the above potentials, the strongest oxidizing agent will be:

[4 marks]

- (1) Cu^+
- (2) Fe^{3+}
- (3) Ag^+
- (4) I_2

Correct Answer: (3) Ag^+

Solution:

Concept:

- An oxidising agent accepts electrons and is itself reduced. A more positive standard reduction potential means a greater tendency for that reduction under standard conditions.
- To rank oxidising agents, compare the oxidised species on the left of the reduction half-reactions, keeping every potential in the reduction convention given.

Step 1: Identify the electron acceptors The listed couples represent reduction of Cu^+ to copper, Fe^{3+} to Fe^{2+} , iodine to iodide, and Ag^+ to silver. The candidate oxidants are therefore exactly the species in the options. Their standard reduction potentials are respectively +0.52, +0.77, +0.54, and +0.88 V. No sign reversal is required because all are already written as reduction couples.

Step 2: Compare the numerical potentials The potentials increase in the order

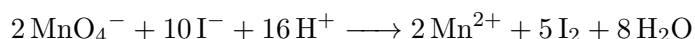
$$+0.52 < +0.54 < +0.77 < +0.88 \text{ V}.$$

Hence the oxidising strengths increase as $\text{Cu}^+ < \text{I}_2 < \text{Fe}^{3+} < \text{Ag}^+$. Silver ions have the greatest electron-accepting tendency among these species. Copper(I), iodine and iron(III) are all weaker oxidants because their given reduction potentials are lower. The coefficient $\frac{1}{2}$ written before iodine does not change its electrode potential: a potential is an intensive quantity, so it is not multiplied when a half-reaction is scaled.

Step 3: Conclusion The strongest oxidising agent is Ag^+ , with $E^\circ = +0.88 \text{ V}$. Hence, the correct option is (3).

Quick Tip to Solve: For strongest oxidant, select the oxidised member of the couple with the most positive reduction potential; do not reverse the signs.

78. The instantaneous rate of disappearance of MnO_4^- ion in the following reaction is $4.56 \times 10^{-3} \text{ M s}^{-1}$.



The rate of appearance of I_2 is:

[4 marks]

(1) $4.56 \times 10^{-4} \text{ M s}^{-1}$

(2) $1.14 \times 10^{-2} \text{ M s}^{-1}$

(3) $1.14 \times 10^{-3} \text{ M s}^{-1}$

(4) $5.7 \times 10^{-3} \text{ M s}^{-1}$

Correct Answer: (2) $1.14 \times 10^{-2} \text{ M s}^{-1}$

Solution:

Concept:

- The rate of a reaction is the rate of consumption or formation of a species divided by its stoichiometric coefficient. It has the same value whichever species is used.
- For the given balanced reaction, $r = -\frac{1}{2} \frac{d[\text{MnO}_4^-]}{dt} = \frac{1}{5} \frac{d[\text{I}_2]}{dt}$. Concentrations are measured in molarity and time in seconds.

Step 1: Write the positive disappearance rate The permanganate concentration decreases, so its derivative is negative. The stated positive rate is $-d[\text{MnO}_4^-]/dt = 4.56 \times 10^{-3} \text{ M s}^{-1}$. Dividing by the coefficient 2 gives the common reaction rate:

$$r = \frac{4.56 \times 10^{-3}}{2} = 2.28 \times 10^{-3} \text{ M s}^{-1}.$$

Step 2: Use the iodine coefficient Two moles of permanganate disappear for every five moles of iodine formed. Therefore iodine appears faster, by the factor $5/2$, rather than at the same numerical rate.

$$\frac{d[\text{I}_2]}{dt} = 5r = \frac{5}{2}(4.56 \times 10^{-3}) = 1.14 \times 10^{-2} \text{ M s}^{-1}.$$

This proportionality applies to instantaneous rates as well as average rates, because it follows directly from the balanced reaction.

Step 3: Check the options Option (1) is ten times smaller than the given disappearance rate, whereas iodine must form faster. Option (3) is one tenth of the calculated value. Option (4) uses a factor $5/4$, which is not the ratio of the printed stoichiometric coefficients. Only (2) uses the required factor $5/2$, with the same concentration-per-time units.

Step 4: Conclusion The rate of appearance of iodine is $1.14 \times 10^{-2} \text{ M s}^{-1}$. Hence, the correct option is (2).

Quick Tip to Solve: For two species in one reaction, take the ratio of their coefficients first; attach signs only when writing concentration derivatives.

79. The migration of dispersion medium under the influence of an electric potential is called [4 marks]

- (1) Cataphoresis
- (2) Electroosmosis
- (3) Electrophoresis
- (4) Sedimentation

Correct Answer: (2) Electroosmosis

Solution:

Concept:

- The dispersion medium is the continuous phase of a colloidal system; colloidal particles form the dispersed phase. These are different parts of the system.
- An applied electric field can move charged colloidal particles or, when particles are held fixed, cause the surrounding medium to flow. The name of the effect depends on which phase moves.

Step 1: Identify the phase named in the question The question explicitly asks about migration of the dispersion medium. This is the liquid or other continuous medium surrounding the colloidal particles. It does not ask for migration of the dispersed particles themselves. The distinction is decisive: both phases interact electrically at the interface, but their observed motion is described by different terms.

Step 2: Apply the definition of electroosmosis Electroosmosis is the movement of the dispersion medium relative to an immobilised dispersed phase under an applied electric potential. When the colloidal particles cannot move freely, the medium can move instead, owing to the charged interfacial region. Thus the process named in the question is electroosmosis. The definition does not require choosing a particular direction of migration here.

Step 3: Distinguish the other processes Cataphoresis is conventionally used for migration of colloidal particles toward the cathode, while electrophoresis describes migration of charged colloidal particles in an electric field more generally. Both concern the dispersed phase rather than the medium asked about. Sedimentation is settling under gravity or an applied centrifugal field; it is not the electrically driven migration specified in the question. Hence (1), (3) and (4) describe different behaviour.

Step 4: Conclusion Migration of the dispersion medium under an electric potential is electroosmosis. Hence, the correct option is (2).

Quick Tip to Solve: Underline the moving phase: particles suggest electrophoresis, whereas movement of the medium relative to fixed particles suggests electroosmosis.

80. Calcination is the process in which

[4 marks]

- (1) Ore is heated above its melting point to expel H_2O or CO_2 or SO_2 .
- (2) Ore is heated below its melting point to expel volatile impurities.
- (3) Ore is heated above its melting point to remove S, As and Sb as SO_2 , As_2O_3 and Sb_2O_3 , respectively.
- (4) Ore is heated below its melting point to expel H_2O or CO_2 .

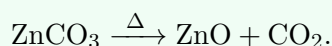
Correct Answer: (4) Ore is heated below its melting point to expel H_2O or CO_2 .

Solution:

Concept:

- Calcination is strong heating of an ore below its melting point in the absence of air or with only a limited supply of air. The ore remains solid while volatile components are removed.
- Hydrated ores can lose H_2O , and carbonate ores can decompose with evolution of CO_2 . The resulting solid is commonly an oxide that can undergo further extraction.

Step 1: Identify the thermal process In calcination the objective is decomposition or dehydration of the ore, rather than melting it. Heating below the melting point allows water of hydration or carbon dioxide from a carbonate to leave while a solid residue remains. For example,



A hydrated oxide similarly loses its combined water on heating.

Step 2: Match the exact description Option (4) gives both distinguishing features together: the ore is heated below its melting point, and the substances expelled are H_2O or CO_2 . This is the standard metallurgical description of calcination. No oxidation by a free supply of air is required to bring about these decomposition reactions.

Step 3: Exclude the competing descriptions Options (1) and (3) both state that the temperature is above the melting point, contradicting calcination. Option (3) also describes oxidative removal of sulphur, arsenic and antimony, which is associated with roasting in air rather than ordinary calcination. Option (2) mentions unspecified volatile impurities and is less precise; it does not identify the characteristic water or carbon dioxide removed from hydrated and carbonate ores. The specifically correct textbook description is (4).

Step 4: Conclusion Calcination heats the ore below its melting point to expel H_2O or CO_2 . Hence, the correct option is (4).

Quick Tip to Solve: Remember the pair: carbonate or hydrated ore, below melting point. Distinguish this decomposition from roasting sulphide ores in air.

81. Trigonal bipyramidal geometry is shown by

[4 marks]

- (1) XeOF_2
- (2) XeO_3F_2
- (3) FXeOSO_2F
- (4) $[\text{XeF}_8]^{2-}$

Correct Answer: (2) XeO_3F_2

Solution:

Concept:

- VSEPR theory counts regions of bonding and lone-pair electron density around a central atom. A multiple bond counts as one region when determining the arrangement of atoms.
- A trigonal bipyramidal molecule has five bonding directions and no central lone pair: three equatorial positions and two axial positions. Its electron-domain and molecular geometries then coincide.

Step 1: Count the regions in xenon trioxide difluoride In XeO_3F_2 , xenon is bonded to three oxygen atoms and two fluorine atoms. A Lewis description uses three $\text{Xe}=\text{O}$ bonds and two $\text{Xe}-\text{F}$ bonds. These consume all eight xenon valence electrons in bonding, so there is no lone pair left on xenon. Although a $\text{Xe}=\text{O}$ bond contains more than one shared electron pair, it supplies just one direction in the VSEPR geometry.

Step 2: Assign the geometry There are consequently five bonding regions and no lone-pair region around xenon. The corresponding AX_5 arrangement is trigonal bipyramidal. The oxygen multiple bonds occupy the three equatorial positions, where their stronger electron-density repulsions can be accommodated, and the two fluorine atoms occupy axial positions. The molecular shape is therefore the geometry stated in the question.

Step 3: Check the remaining formulae XeOF_2 has three bonded atoms and two lone pairs on xenon: its five-domain electron arrangement is trigonal bipyramidal, but the molecular shape is T-shaped. In FXeOSO_2F , xenon is bonded to fluorine and oxygen and retains lone pairs, so it is not an AX_5 molecule. $[\text{XeF}_8]^{2-}$ has eight attached fluorine atoms, incompatible with a five-coordinate trigonal bipyramid. These eliminate (1), (3) and (4).

Step 4: Conclusion XeO_3F_2 has trigonal bipyramidal molecular geometry. Hence, the correct option is (2).

Quick Tip to Solve: Count bonded atoms and central lone pairs separately: a trigonal bipyramidal electron arrangement alone does not guarantee that molecular shape.

82. Potassium dichromate when heated with concentrated sulphuric acid and a soluble chloride, gives brown-red vapours of

[4 marks]

- (1) CrO_3
- (2) CrCl_3
- (3) CrO_2Cl_2
- (4) Cr_2O_3

Correct Answer: (3) CrO_2Cl_2

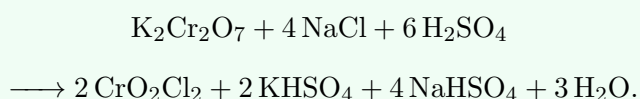
Solution:

Concept:

- The chromyl chloride test detects chloride by heating a suitable soluble chloride with potassium dichromate and concentrated sulphuric acid.
- The volatile brown-red product is chromyl chloride, CrO_2Cl_2 . Chromium remains in the +6 oxidation state in this oxochloride.

Step 1: Identify the reaction conditions Potassium dichromate in concentrated sulphuric acid supplies chromium(VI) under strongly acidic, dehydrating conditions. Chloride from a soluble salt combines with the chromium-oxygen species to form the volatile oxochloride. The distinctive observation is brown-red vapour, rather than a precipitated chromium salt or oxide.

Step 2: Write a representative balanced equation Using sodium chloride as the soluble chloride, the reaction may be written



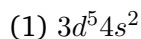
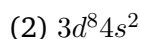
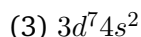
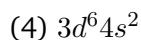
This accounts for two chromium atoms, four chloride ions and the water formed in the acidic medium. Each product molecule contains one chromium, two oxygen and two chlorine atoms.

Step 3: Verify the formula and reject the alternatives For CrO_2Cl_2 , the oxygen contribution is -4 and the chlorine contribution is -2 , so neutral chromyl chloride contains chromium in oxidation state $+6$, consistent with dichromate. CrO_3 is a chromium(VI) oxide, but it is not the chloride-containing vapour of this test. CrCl_3 and Cr_2O_3 contain chromium(III), and neither is the characteristic volatile brown-red chromyl chloride. Thus only (3) matches both composition and observation.

Step 4: Conclusion The brown-red vapours are chromyl chloride, CrO_2Cl_2 . Hence, the correct option is (3).

Quick Tip to Solve: Link the words dichromate, concentrated sulphuric acid and chloride with the chromyl chloride test; the volatile product is an oxochloride.

83. The element with which of the following outer electron configuration may exhibit the largest number of oxidation states in its compounds [4 marks]



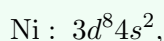
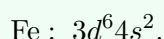
Correct Answer: (1) $3d^5 4s^2$

Solution:

Concept:

- Transition elements can exhibit variable oxidation states because their outer ns and nearby (n-1)d electrons have comparable energies and can participate in bonding.
- Across the early first transition series, the number of available valence electrons rises toward manganese. Manganese then shows the widest familiar range of oxidation states among the listed elements.

Step 1: Identify the elements from their configurations The configurations correspond respectively to manganese, nickel, cobalt and iron:



All are transition elements, so merely having an incompletely filled d subshell is not sufficient to distinguish the answer.

Step 2: Compare their range of oxidation states Manganese has seven outer electrons available across its 3d and 4s subshells. It exhibits the familiar oxidation states +2, +3, +4, +5, +6 and +7 in different compounds. Examples include MnCl_2 , MnO_2 , K_2MnO_4 and KMnO_4 , illustrating several widely separated values. The highest state +7 uses all seven outer electrons in the formal oxidation-state count.

Step 3: Eliminate the later transition elements Nickel, cobalt and iron most commonly display smaller sets dominated by +2 and +3, although additional states can occur under suitable conditions. Moving beyond manganese does not make all the additional d electrons equally available for high oxidation states; increasing effective nuclear attraction makes their removal less favourable. Therefore the largest number in this standard comparison belongs to manganese rather than to the configuration with the most d electrons.

Step 4: Conclusion The configuration $3d^5 4s^2$, belonging to manganese, shows the largest range among the choices. Hence, the correct option is (1).

Quick Tip to Solve: For the widest oxidation-state range in the first transition series, look near manganese, rather than simply choosing the largest d-electron count.

84. Type of isomerism which exists between $[\text{Pd}(\text{C}_6\text{H}_5)_2(\text{SCN})_2]$ and $[\text{Pd}(\text{C}_6\text{H}_5)_2(\text{NCS})_2]$ is: [4 marks]

- (1) Linkage isomerism
- (2) Coordination isomerism
- (3) Ionisation isomerism
- (4) Solvate isomerism

Correct Answer: (1) Linkage isomerism

Solution:

Concept:

- An ambidentate ligand can bind to a metal through either of two different donor atoms. Thiocyanate has possible sulphur and nitrogen donor sites.
- Linkage isomers retain the same overall composition and metal centre but differ in the donor atom through which an ambidentate ligand is attached.

Step 1: Identify the unchanged parts Both printed complexes contain the same palladium centre, two phenyl groups and two thiocyanate-derived ligands. Thus their overall constituent atoms do not change. The difference is written specifically as SCN in one coordination formula and NCS in the other, so the donor end of the same ligand must be examined.

Step 2: Compare the ligand attachments In the first complex, SCN^- denotes sulphur-bonded thiocyanato coordination. In the second, NCS^- denotes nitrogen-bonded coordination. These are two ways for the same ambidentate ion to attach to palladium. Changing from a metal–S bond to a metal–N bond changes the connectivity at the metal without exchanging the ligand for a chemically different ion. This is precisely linkage isomerism.

Step 3: Exclude the other isomerism types Coordination isomerism requires redistribution of ligands between complex cations and complex anions; that is not what is shown. Ionisation isomerism exchanges an inner-sphere ligand with a counter-ion, producing different ions in solution, but no such exchange is present here. Solvate isomerism involves solvent molecules moving into or out of the coordination sphere; neither printed formula changes its solvent content. Therefore (2), (3) and (4) do not describe the displayed difference.

Step 4: Conclusion The complexes differ in sulphur versus nitrogen attachment of thiocyanate, so they are linkage isomers. Hence, the correct option is (1).

Quick Tip to Solve: When SCN and NCS are interchanged while the rest of a complex remains the same, inspect the donor atom: S-bound versus N-bound means linkage isomerism.

85. For the reaction $\text{C}_6\text{H}_5\text{CH}_2\text{CH}=\text{CH}_2 + \text{HCl} \longrightarrow \text{X}$, **X** is:

[4 marks]

- (1) $\text{C}_6\text{H}_5\text{CH}_2\text{CH}(\text{Cl})\text{CH}_3$
- (2) $\text{C}_6\text{H}_5\text{CH}_2\text{CH}_2\text{CH}_2\text{Cl}$
- (3) $\text{C}_6\text{H}_5\text{CH}(\text{Cl})\text{CH}_2\text{CH}_3$
- (4) $o\text{-ClC}_6\text{H}_4\text{CH}_2\text{CH}=\text{CH}_2$

Correct Answer: (3) $\text{C}_6\text{H}_5\text{CH}(\text{Cl})\text{CH}_2\text{CH}_3$

Solution:

Concept:

- Addition of HCl to an alkene proceeds by protonation of the double bond followed by capture of the carbocation by chloride.
- A carbocation may undergo a 1,2-hydride shift if the resulting cation is more stable. A benzylic cation is stabilised by resonance with the benzene ring.

Step 1: Protonate the terminal double bond The starting compound is allylbenzene, $\text{C}_6\text{H}_5\text{CH}_2\text{CH}=\text{CH}_2$, whose double bond is in the side chain. Proton addition to its terminal carbon gives the secondary cation $\text{C}_6\text{H}_5\text{CH}_2\text{CH}^+\text{CH}_3$. Protonation in the other direction would give a less stable primary terminal cation, so it does not determine the major product. Chloride is the nucleophile; this reaction does not require substitution of a hydrogen of the aromatic ring.

Step 2: Shift hydride to the benzylic carbon A hydride migrates from the adjacent benzylic CH_2 group to the positively charged carbon. The positive charge therefore moves onto the carbon directly attached to the ring:



Both cations are secondary, but only the latter is benzylic and resonance-stabilised. The carbon skeleton remains a three-carbon chain attached to the same phenyl group.

Step 3: Capture the rearranged cation Chloride attacks the benzylic cation to give $\text{C}_6\text{H}_5\text{CH}(\text{Cl})\text{CH}_2\text{CH}_3$. Option (1) is the unrearranged addition product and overlooks the favourable hydride shift. Option (2) places chlorine on the terminal carbon, contrary to the cation pathway. Option (4) chlorinates the aromatic ring while retaining the alkene, which is not the result of electrophilic addition of hydrogen chloride.

Step 4: Conclusion The major product is $\text{C}_6\text{H}_5\text{CH}(\text{Cl})\text{CH}_2\text{CH}_3$, with chlorine on the benzylic carbon. Hence, the correct option is (3).

Quick Tip to Solve: After the first protonation, check whether a single hydride shift places the positive charge next to a benzene ring before adding chloride.

86. Aryl fluoride may be prepared from arene diazonium chloride using

[4 marks]

- (1) HBF_4/Δ
- (2) $\text{HBF}_4/\text{NaNO}_2, \text{Cu}, \Delta$
- (3) CuF/HF
- (4) Cu/HF

Correct Answer: (1) HBF_4/Δ

Solution:

Concept:

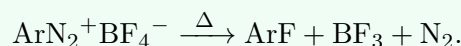
- The Balz–Schiemann reaction replaces the diazonium group of an arene diazonium salt with fluorine.
- Fluoroboric acid, HBF_4 , first converts the diazonium chloride into the corresponding diazonium tetrafluoroborate; heating this salt releases nitrogen.

Step 1: Form the tetrafluoroborate salt Let Ar represent the aromatic group. Treating the arene diazonium chloride with fluoroboric acid produces the tetrafluoroborate salt:



The role of this first operation is counter-ion exchange. It provides the tetrafluoroborate ion required for the subsequent thermal replacement; it does not make fluorine substitute the ring by ordinary nucleophilic displacement of chloride.

Step 2: Heat the salt to release the aryl fluoride On heating, the diazonium tetrafluoroborate decomposes to the aryl fluoride:



Evolution of stable nitrogen gas accompanies replacement of the diazonium group. Thus the listed pair HBF_4 and heat gives the required transformation directly, with the aromatic carbon framework preserved.

Step 3: Compare the competing reagent choices The option containing NaNO_2 and copper introduces a different diazonium substitution pathway rather than the specified thermal fluoroborate decomposition. Fluorination here is not the ordinary copper(I)-halide Sandmeyer method used for chloride or bromide, so CuF/HF is not the appropriate choice. Copper with hydrofluoric acid likewise does not supply the required diazonium tetrafluoroborate intermediate. These comparisons identify the named preparation unambiguously.

Step 4: Conclusion Use HBF_4 , followed by heating: this is the Balz–Schiemann preparation of aryl fluoride. Hence, the correct option is (1).

Quick Tip to Solve: For a diazonium group replaced by fluorine, look for tetrafluoroborate formation followed by heating, rather than a copper halide.

87. An ether (A), $C_5H_{12}O$, when heated with excess of hot concentrated HI produced two alkyl halides which when treated with NaOH yielded compounds (B) and (C). Oxidation of (B) and (C) gave a propanone and an ethanoic acid respectively. The IUPAC name of the ether (A) is: [4 marks]

- (1) 2-ethoxypropane
- (2) ethoxypropane
- (3) methoxybutane
- (4) 2-methoxybutane

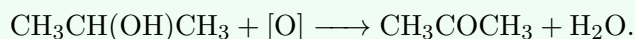
Correct Answer: (1) 2-ethoxypropane

Solution:

Concept:

- Hot concentrated HI cleaves an ether; with excess reagent both alkyl portions ultimately appear as the corresponding alkyl iodides.
- Aqueous NaOH converts alkyl iodides to alcohols. Oxidation of a secondary alcohol gives a ketone, whereas a primary alcohol can give a carboxylic acid.

Step 1: Identify the alcohol giving propanone Propanone is CH_3COCH_3 . Its alcohol precursor is propan-2-ol, $CH_3CH(OH)CH_3$, since oxidation changes the secondary alcohol group to a carbonyl group without changing the carbon skeleton:



Consequently one alkyl iodide formed by cleavage must be 2-iodopropane, and one side of the original ether must be an isopropyl group.

Step 2: Identify the alcohol giving ethanoic acid Ethanoic acid is obtained by oxidation of ethanol:



Thus the other cleavage fragment is iodoethane and the second alkyl group of the ether is ethyl. Combining the ethyl and isopropyl groups across oxygen gives $CH_3CH_2OCH(CH_3)_2$. This contains five carbon atoms and twelve hydrogen atoms, as required by $C_5H_{12}O$.

Step 3: Name and check the reconstructed ether Take propane as the longer parent chain. The oxygen-linked ethyl group is an ethoxy substituent on carbon 2, so the IUPAC name is 2-ethoxypropane. Ethoxypropane without this locant represents the straight-chain alternative and would yield propan-1-ol rather than propan-2-ol. The methoxybutanes give methyl and four-carbon cleavage fragments, which cannot furnish the stated three-carbon ketone together with the two-carbon acid.

Step 4: Conclusion The ether is $CH_3CH_2OCH(CH_3)_2$, named 2-ethoxypropane. Hence, the correct option is (1).

Quick Tip to Solve: Work backwards from the oxidation products to their alcohols, then join the two corresponding alkyl groups through oxygen.

88. Formaldehyde can be distinguished from acetaldehyde by the use of

[4 marks]

- (1) Schiff's reagent
- (2) Tollen's reagent
- (3) I_2 /Alkali
- (4) Fehling's solution

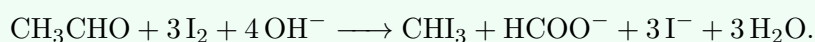
Correct Answer: (3) I_2 /Alkali

Solution:

Concept:

- The iodoform test uses iodine in alkali and is positive for a methyl ketone group, $CH_3-C(=O)-R$ (where R is an alkyl or aryl group), and for acetaldehyde.
- Acetaldehyde, CH_3CHO , gives yellow iodoform, CHI_3 , whereas formaldehyde, $HCHO$, lacks the required methyl group.

Step 1: Apply iodine and alkali to acetaldehyde Acetaldehyde has three hydrogens on the methyl carbon next to its carbonyl group. Under alkaline conditions these are replaced by iodine, after which cleavage gives yellow iodoform and formate. The overall balanced ionic equation is



The yellow precipitate is the positive observation; it makes the test useful even without isolating the other reaction products.

Step 2: Compare the response of formaldehyde Formaldehyde has the structure $H-C(=O)-H$. There is no methyl carbon adjacent to the carbonyl group and hence no group that can become CI_3 and then CHI_3 . It therefore does not give the yellow iodoform precipitate. The different observations for the two aldehydes allow iodine and alkali to distinguish them.

Step 3: Reject tests shared by both aldehydes Schiff's reagent responds to the aldehyde functional group and is positive for both compounds. Tollens' reagent is reduced by both aldehydes, producing metallic silver, so a positive result does not identify one of them. Both are aliphatic aldehydes and also reduce Fehling's solution to the characteristic precipitate. These three tests establish aldehydic behaviour but do not provide the required distinction between formaldehyde and acetaldehyde.

Step 4: Conclusion Use I_2 in alkali: acetaldehyde gives yellow CHI_3 , while formaldehyde does not. Hence, the correct option is (3).

Quick Tip to Solve: For formaldehyde versus acetaldehyde, inspect the methyl group beside the carbonyl; only acetaldehyde supplies it for the iodoform test.

89. If a polythene sample contains two monodisperse fractions in the ratio 2 : 3 with degree of polymerization 100 and 200, respectively, then its weight average molecular weight will be [4 marks]

- (1) 4900
- (2) 4600
- (3) 4300
- (4) 5200

Correct Answer: (1) 4900

Solution:

Concept:

- The repeat unit of polythene is $-\text{CH}_2-\text{CH}_2-$, of relative mass 28. A degree of polymerisation n therefore corresponds to molecular mass $28n$ in the repeat-unit approximation.
- For numbers of chains N_i of molecular mass M_i , the weight-average molecular mass is $\overline{M}_w = \frac{\sum_i N_i M_i^2}{\sum_i N_i M_i}$.

Step 1: Calculate the two chain molecular masses The first monodisperse fraction has degree of polymerisation 100, so every chain in it has $M_1 = 100 \times 28 = 2800$. The second has degree 200 and hence $M_2 = 200 \times 28 = 5600$. The mass of the ethene repeat unit is used, because addition polymerisation retains all atoms of the monomer. End-group corrections are not included in the stated degree-of-polymerisation treatment.

Step 2: Insert the fraction ratio into the weighted formula Take relative chain numbers $N_1 : N_2 = 2 : 3$; any common multiplying factor cancels. Then

$$\begin{aligned}\overline{M}_w &= \frac{2(2800)^2 + 3(5600)^2}{2(2800) + 3(5600)} \\ &= \frac{109760000}{22400} = 4900.\end{aligned}$$

The square in the numerator gives heavier chains a larger contribution than they receive in a simple chain-number average.

Step 3: Check the weighting and the alternatives The corresponding mass contributions are $2 \times 2800 = 5600$ and $3 \times 5600 = 16800$, giving mass fractions $1/4$ and $3/4$. A separate check is $\overline{M}_w = (1/4)2800 + (3/4)5600 = 4900$. It lies between 2800 and 5600, and exceeds the number average $(2 \times 2800 + 3 \times 5600)/5 = 4480$, as expected for unequal chain masses. The other three numerical options do not follow the weight-average expression.

Step 4: Conclusion The weight-average molecular weight is 4900, corresponding to $\overline{M}_w = 4900 \text{ g mol}^{-1}$. Hence, the correct option is (1).

Quick Tip to Solve: For a chain-number ratio use squared molecular masses in the numerator; alternatively convert the number ratio to mass fractions first.

90. Which of the following enzyme converts starch into maltose?

[4 marks]

- (1) Diastase
- (2) Maltase
- (3) Zymase
- (4) Invertase

Correct Answer: (1) Diastase

Solution:

Concept:

- Enzymes are biological catalysts whose activity is specific to a substrate and a particular chemical transformation.
- Diastase is the enzyme preparation associated with hydrolysis of starch to maltose. Maltose is a disaccharide containing two glucose units.

Step 1: Identify the substrate and required product The substrate named in the question is starch, a polysaccharide built from glucose units. The demanded product is maltose, not free glucose or ethanol. This distinction fixes the hydrolytic stage being asked about: breakdown of the large starch polymer to the smaller disaccharide. It does not ask for complete conversion of every glycosidic bond to individual glucose molecules.

Step 2: Match the conversion with diastase Diastase catalyses the hydrolysis of starch to maltose, using water to cleave glycosidic linkages. In the usual textbook representation, starch is acted upon by diastase to produce maltose. Because an enzyme accelerates a particular reaction without changing the nature of its required substrate, the enzyme must be selected by this specific substrate–product pair rather than merely by its association with carbohydrates.

Step 3: Distinguish the remaining enzymes Maltase acts on maltose and hydrolyses it further:



It therefore belongs to the next stage, not the conversion from starch. Zymase is associated with alcoholic fermentation of glucose to ethanol and carbon dioxide, so its substrate and products differ from those asked for. Invertase hydrolyses sucrose to glucose and fructose; sucrose is a different disaccharide. Only diastase matches both starch as the starting material and maltose as the required product.

Step 4: Conclusion Diastase converts starch into maltose. Hence, the correct option is (1).

Quick Tip to Solve: Memorise the sequence starch –diastase–> maltose –maltase–> glucose; the enzyme name must match the starting substrate.