JEE Main 2013 Paper 1 (B.E./B.Tech) Apr 22 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2013, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 22 April 2013, Single slot. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. If α and β are roots of the equation $x^2 + px + \frac{3p}{4} = 0$, such that $|\alpha - \beta| = \sqrt{10}$, then p belongs to the set: [4 marks]

(A) $\{2, -5\}$

(B) $\{-3, 2\}$

(C) $\{-2, 5\}$

(D) $\{3, -5\}$

Correct Answer: (C) $\{-2, 5\}$

Solution:

Concept:

- For a monic quadratic with roots α, β, Vieta's relations give their sum and product without solving for the roots separately.
- For real coefficients, the squared modulus of the root difference equals the absolute value of the discriminant: $|\alpha - \beta|^2 = |p^2 - 3p|$. This covers both real and conjugate roots.

Step 1: Express the root difference using Vieta The sum is $\alpha + \beta = -p$ and the product is $\alpha\beta = 3p/4$. Hence

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = p^2 - 3p.$$

Since the given modulus is $\sqrt{10}$, taking absolute values gives $|p^2 - 3p| = 10$. We must consider both signs, rather than assuming the roots are real at the outset.

Step 2: Solve both possible discriminant equations If $p^2 - 3p = 10$, then

$$p^2 - 3p - 10 = (p - 5)(p + 2) = 0,$$

so $p = 5$ or $p = -2$. The other possibility, $p^2 - 3p = -10$, gives $p^2 - 3p + 10 = 0$, whose discriminant is $9 - 40 = -31$; it yields no real parameter value. At each of the retained values the original discriminant is exactly 10, so the required root separation is indeed $\sqrt{10}$. Of the listed sets, only the third contains both values; the first, second and fourth each include a value that fails the discriminant test.

Step 3: Conclusion The parameter values are $p \in \{-2, 5\}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the absolute value of the discriminant when the separation is given as a complex modulus; then check both signs before selecting the set.

2. If a complex number z satisfies the equation $x + \sqrt{2}|z+1| + i = 0$, then $|z|$ is equal to: [4 marks]

(A) 2

(B) $\sqrt{3}$

(C) $\sqrt{5}$

(D) 1

Correct Answer: The printed equation uses an undefined x distinct from z, so it does not determine |z|.

Solution:

Concept:

- A complex equality requires its real and imaginary parts to agree separately. A modulus such as $|z+1|$ is always a nonnegative real number.
- The printed equation contains two different symbols, x and z. Without a relation between them, an equation involving both cannot generally determine $|z|$.

Step 1: Retain the two printed variables Write $z = a + ib$, where a, b are real. Then $|z+1| = \sqrt{(a+1)^2 + b^2}$, and the stated equation becomes

$$x = -\sqrt{2}\sqrt{(a+1)^2 + b^2} - i.$$

This determines x in terms of z, rather than determining z. No condition in the question identifies x with z or fixes its value. In particular, it is not valid to replace the first printed variable by another one merely to produce a numerical answer.

Step 2: Test whether the modulus is determined If x may be complex, take $z = 0$. The choice $x = -\sqrt{2} - i$ satisfies the equation and gives $|z| = 0$. Alternatively, $z = 1$ and $x = -2\sqrt{2} - i$ also satisfy it and give $|z| = 1$. Thus even two different moduli occur, and indeed any z can be accommodated by the displayed choice of x. If x is instead intended to be real, the imaginary part of the left side is 1, so no solution exists. Neither reading singles out one of the four printed values. The missing relationship is essential, not a computational detail.

Step 3: Conclusion As printed, $|z|$ is not determined: no unique option follows from the equation containing the separate variable x.

Quick Tip to Solve: Before separating a complex equation into real and imaginary parts, check that every variable is defined and that its type is specified.

3. The number of ways in which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any question, is: [4 marks]

(A) ${}^{30}C_7$

(B) ${}^{21}C_8$

(C) ${}^{21}C_7$

(D) ${}^{30}C_8$

Correct Answer: (C) ${}^{21}C_7$

Solution:

Concept:

- Treat the questions as eight distinct labelled positions. An allocation is a vector of nonnegative integer marks satisfying the stated total and lower bounds.
- The number of nonnegative integer solutions of $y_1 + \cdots + y_k = N$ is $\binom{N+k-1}{k-1}$, obtained by placing $k-1$ separators among N stars.

Step 1: Remove the compulsory minimum allocation Let m_j be the integer allocation to question j, for $j = 1, \ldots, 8$. The conditions are

$$m_1 + \cdots + m_8 = 30, \qquad m_j \geq 2.$$

Assign the compulsory two to each question first. This uses $8 \times 2 = 16$ from the total. On putting $y_j = m_j - 2$, every new variable is nonnegative and the remaining equation is

$$y_1 + \cdots + y_8 = 30 - 16 = 14.$$

The translation is reversible: each admissible y-vector produces exactly one original allocation by adding two to each component.

Step 2: Count the remaining distributions Arrange 14 identical stars and seven separators in a row. The stars before the first separator count y_1, those between successive separators count the intermediate variables, and those after the last count y_8. Adjacent separators are allowed because a question may receive no extra allocation. There are $14 + 7 = 21$ positions, and choosing the seven separator positions gives

$$\binom{21}{7}.$$

The alternatives using 30 fail to remove the compulsory allocation. Choosing eight separator positions would create nine groups, rather than the eight required questions.

Step 3: Conclusion The required number of allocations is $\binom{21}{7}$. Hence, the correct option is (C).

Quick Tip to Solve: For a distribution with a common lower bound, subtract that bound from each variable before applying stars and bars.

4. Given sum of the first n terms of an A.P. is $2n + 3n^2$. Another A.P. is formed with the same first term and double of the common difference, the sum of n terms of the new A.P. is: [4 marks]

(A) $n + 4n^2$

(B) $6n^2 - n$

(C) $n^2 + 4n$

(D) $3n + 2n^2$

Correct Answer: (B) $6n^2 - n$

Solution:

Concept:

- The first term of a sequence with partial sums S_n is S_1, and its general term is $S_n - S_{n-1}$.
- For an arithmetic progression with first term a and common difference d, the sum is $S_n = na + \frac{n(n-1)}{2}d$. Doubling the difference leaves the first term unchanged.

Step 1: Recover the original first term and difference The given partial sum is $S_n = 2n + 3n^2$. Therefore $a = S_1 = 2 + 3 = 5$. For the general term, subtract consecutive sums:

$$\begin{aligned} a_n &= (2n + 3n^2) - \{2(n-1) + 3(n-1)^2\} \\ &= 6n - 1. \end{aligned}$$

The difference between successive terms is consequently $d = 6$. As a direct check, $S_2 = 16$, so the second term is $16 - 5 = 11$, which differs from 5 by 6.

Step 2: Apply the changed common difference The new progression has first term 5 and common difference $2d = 12$. Its partial sum is

$$\begin{aligned} S'_n &= 5n + \frac{n(n-1)}{2}(12) \\ &= 5n + 6n(n-1) \\ &= 6n^2 - n. \end{aligned}$$

This does not simply double the original sum, because the first term is retained rather than doubled. At $n = 1$, the result gives 5 as required; at $n = 2$, it gives 22, the sum of the new first two terms 5 and 17. The first and third options also start at 5 but give an incorrect difference, while the fourth reproduces the wrong quadratic coefficient.

Step 3: Conclusion The new sum is $6n^2 - n$. Hence, the correct option is (B).

Quick Tip to Solve: For a quadratic partial sum, twice its quadratic coefficient is the common difference; its value at n=1 gives the first term.

5. The sum $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\cdots$ up to 11 terms is: [4 marks]

(A) $\frac{7}{2}$

(B) $\frac{11}{4}$

(C) $\frac{11}{2}$

(D) $\frac{60}{11}$

Correct Answer: (C) $\frac{11}{2}$

Solution:

Concept:

- The sum of the first k squares is $1^2+\cdots+k^2=k(k+1)(2k+1)/6$. This identity converts the variable denominator to a product.
- A term proportional to $1/[k(k+1)]$ telescopes because $1/[k(k+1)]=1/k-1/(k+1)$. Consecutive terms then cancel.

Step 1: Write the general term in a cancellable form The numerator of the kth term is $2k+1$, while its denominator is the sum of squares up to k. Thus

$$\begin{aligned} t_k &= \frac{2k+1}{k(k+1)(2k+1)/6} \\ &= \frac{6}{k(k+1)} \\ &= 6\left(\frac{1}{k}-\frac{1}{k+1}\right). \end{aligned}$$

For example, $k=1$ gives 3, agreeing with the first displayed term, and $k=2$ gives 1, agreeing with $5/(1+4)$. These checks confirm both the numerator pattern and the indexing.

Step 2: Sum all eleven terms and retain the endpoints Using the difference form, the required finite sum is

$$\begin{aligned} \sum_{k=1}^{11} t_k &= 6\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{11}-\frac{1}{12}\right)\right] \\ &= 6\left(1-\frac{1}{12}\right)=\frac{11}{2}. \end{aligned}$$

The last denominator is 12, not 11, since the negative part of the eleventh difference is $-1/(11+1)$. The result is less than the infinite limiting sum 6 and greater than the first two terms' sum 4, which also rules out the first two options. The fourth value does not equal the endpoint expression.

Step 3: Conclusion The sum of the eleven terms is $11/2$. Hence, the correct option is (C).

Quick Tip to Solve: When the numerator is 2k+1 over a sum of k squares, cancel that factor first; the remaining reciprocal product telescopes immediately.

6. If the 7th term in the binomial expansion of $\left(\frac{3}{\sqrt[3]{84}} + \sqrt{3}\ln x\right)^9$, $x > 0$, is equal to 729, then x can be: [4 marks]

(A) e^2

(B) e

(C) $e/2$

(D) $2e$

Correct Answer: (B) e

Solution:

Concept:

- In $(a+b)^n$, the $(r+1)$th term is $T_{r+1} = \binom{n}{r} a^{n-r} b^r$. The seventh term therefore uses $r = 6$, not 7.
- The natural logarithm requires $x > 0$. An even power equation for $\ln x$ can yield two signs, so both must be checked before comparing the listed choices.

Step 1: Evaluate the coefficient of the seventh term Here $n = 9$, $a = 3/\sqrt[3]{84}$ and $b = \sqrt{3}\ln x$. Therefore

$$\begin{aligned} T_7 &= \binom{9}{6}\left(\frac{3}{\sqrt[3]{84}}\right)^3 (\sqrt{3}\ln x)^6 \\ &= 84 \cdot \frac{27}{84} \cdot 27(\ln x)^6 \\ &= 729(\ln x)^6. \end{aligned}$$

We used $\binom{9}{6} = 84$, the cube of the cube root in the denominator is 84, and $(\sqrt{3})^6 = 27$. Each factor matters; in particular the logarithm is raised to the sixth power, not multiplied by 6.

Step 2: Solve the logarithmic equation and test the choices The stated value of the term is 729, so

$$729(\ln x)^6 = 729 \quad \Longrightarrow \quad (\ln x)^6 = 1.$$

Since the logarithm is real, $\ln x = 1$ or $\ln x = -1$. Hence $x = e$ or $x = e^{-1}$, both of which satisfy $x > 0$. The question asks what x can be, rather than requiring every possible value to be listed. Only e occurs among the alternatives. For e^2, the logarithm is 2 and its sixth power is 64; for $e/2$ and $2e$, the logarithms are $1 - \ln 2$ and $1 + \ln 2$, neither of which is 1 or -1.

Step 3: Conclusion The possible values are e and e^{-1}; the listed valid value is e. Hence, the correct option is (B).

Quick Tip to Solve: Use r=6 for a seventh binomial term, and keep both logarithm signs when the resulting exponent is even.

7. The number of solutions of the equation $\sin^{-1} x = 2\tan^{-1} x$ (in principal values) is: [4 marks]

(A) 1

(B) 4

(C) 2

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- The real principal value $\sin^{-1} x$ is defined for $-1 \le x \le 1$ and lies in $[-\pi/2, \pi/2]$.
- The identity $\sin(2\tan^{-1} x) = 2x/(1+x^2)$ follows from the double-angle formula. Taking sine gives necessary candidate roots, which must be checked in the original principal-value equation.

Step 1: Reduce the equation to algebraic candidates The arcsine domain first restricts x to $[-1, 1]$. Take the sine of both sides of the given equality:

$$x = \sin(2\tan^{-1} x) = \frac{2x}{1+x^2}.$$

The denominator is strictly positive for every real x, so multiplication introduces no zero-denominator issue. Rearranging gives

$$x(1+x^2) = 2x \quad \Longrightarrow \quad x(x^2-1) = 0.$$

Thus the only possible real roots are $-1, 0, 1$. Do not divide through by x, because that would discard the legitimate candidate 0.

Step 2: Check the principal values for every candidate At $x = 0$, both sides of the original equality are 0. At $x = 1$, the left side is $\pi/2$ and the right side is $2(\pi/4) = \pi/2$. At $x = -1$, they are $-\pi/2$ and $2(-\pi/4) = -\pi/2$. All three candidates satisfy the equation. No other root can exist, because every solution must have satisfied the factored necessary equation. Moreover, on the allowed interval $2\tan^{-1} x$ remains in $[-\pi/2, \pi/2]$, consistent with the principal arcsine range. The choices 1 and 2 omit valid endpoint or zero roots, and 4 exceeds the complete candidate count.

Step 3: Conclusion The solution set is $\{-1, 0, 1\}$, containing three values. Hence, the correct option is (D). The supplied key lists a single solution, but direct principal-value substitution verifies all three roots, so the printed equation gives (D).

Quick Tip to Solve: After applying sine to an inverse-trigonometric equation, verify every algebraic candidate in the original principal ranges, including endpoints.

8. Statement-1: The number of common solutions of the trigonometric equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is two. Statement-2: The number of solutions of the equation $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, \pi]$ is two. [4 marks]

(A) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is false; Statement-2 is true.

(D) Statement-1 is true; Statement-2 is false.

Correct Answer: (B) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1.

Solution:

Concept:

- Use $\cos 2\theta = 1 - 2\sin^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$ to reduce both equations to polynomials in $s = \sin\theta$.
- A common solution must satisfy both equations; a solution count for only one equation does not by itself prove the common count.

Step 1: Check Statement-1 The first equation becomes $4s^2 - 1 = 0$, so $s = \pm\frac{1}{2}$. The second becomes

$$2s^2 + 3s - 2 = (2s - 1)(s + 2) = 0.$$

Since a sine lies between -1 and 1, only $s = \frac{1}{2}$ is admissible in the second equation. It also satisfies the first. In $[0, 2\pi]$, the common solutions are exactly $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$, so Statement-1 is true.

Step 2: Check Statement-2 and the explanation On $[0, \pi]$, the equation $\sin\theta = \frac{1}{2}$ has the same two solutions, so Statement-2 is true. However, Statement-2 alone says nothing about the first equation or the excluded interval $(\pi, 2\pi]$. The reduction of both equations is needed to establish their intersection; the second statement is therefore not a correct explanation of the first.

Step 3: Conclusion Both statements are true, but Statement-2 is not a correct explanation for Statement-1. Hence, the correct option is (B).

Quick Tip to Solve: For common trigonometric roots, intersect the admissible sine values before counting the angles.

9. If the x-intercept of some line L is double as that of the line, $3x + 4y = 12$ and the y-intercept of L is half as that of the same line, then the slope of L is: [4 marks]

(A) -3

(B) $-3/8$

(C) $-3/2$

(D) $-3/16$

Correct Answer: (D) $-3/16$

Solution:

Concept:

- The intercept form is $x/a + y/b = 1$, where a and b are the signed intercepts on the coordinate axes.
- The slope of a line with nonzero intercepts is $m = -b/a$, obtained by writing its equation as $y = b - (b/a)x$.

Step 1: Find the reference intercepts For $3x + 4y = 12$, setting $y = 0$ gives $x = 4$, while setting $x = 0$ gives $y = 3$. Thus its intercepts are 4 and 3 respectively. The new line doubles only the horizontal intercept and halves only the vertical intercept, so its intercepts are $a = 8$ and $b = \frac{3}{2}$.

Step 2: Calculate the new slope Insert these two intercepts in the line equation:

$$\frac{x}{8} + \frac{y}{3/2} = 1 \quad \Longrightarrow \quad y = \frac{3}{2} - \frac{3}{16}x.$$

Therefore its slope is $-\frac{3}{16}$. It is one quarter of the original slope $-\frac{3}{4}$: doubling the run and halving the rise reduces their ratio by a factor of four. None of the other negative values has that required ratio.

Step 3: Conclusion The slope of L is $-\frac{3}{16}$. Hence, the correct option is (D).

Quick Tip to Solve: With intercept changes, compute the new slope as minus vertical intercept divided by horizontal intercept.

10. The acute angle between two lines such that the direction cosines l, m, n, of each of them satisfy the equations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$ is: [4 marks]

(A) 15°

(B) 30°

(C) 60°

(D) 45°

Correct Answer: (C) 60°

Solution:

Concept:

- Direction cosines obey $l^2 + m^2 + n^2 = 1$. Multiplying all three direction ratios by a nonzero constant describes the same line.

- The acute angle between lines with direction vectors $\mathbf{u}, \mathbf{v}$ satisfies $\cos\alpha = |\mathbf{u}\cdot\mathbf{v}|/(|\mathbf{u}||\mathbf{v}|)$.

Step 1: Determine the two directions From $l+m+n=0$, put $n=-(l+m)$ in the second equation. Then

$$l^2+m^2-(l+m)^2=-2lm=0.$$

Hence either $l=0$ or $m=0$. If $l=0$, then $m=-n$, giving direction ratios $(0,1,-1)$. If $m=0$, then $l=-n$, giving $(1,0,-1)$. These exhaust the possibilities; all three cosines cannot be zero.

Step 2: Apply the dot product Take $\mathbf{u}=(0,1,-1)$ and $\mathbf{v}=(1,0,-1)$. Their dot product is 1 and each has magnitude $\sqrt{2}$. Consequently

$$\cos\alpha=\frac{1}{\sqrt{2}\sqrt{2}}=\frac{1}{2}, \qquad \alpha=60^\circ.$$

Reversing either direction only reverses the signed cosine; taking the absolute value preserves the acute angle. The other offered angles have different cosines.

Step 3: Conclusion The acute angle between the lines is 60°. Hence, the correct option is (C).

Quick Tip to Solve: Eliminate one direction cosine first; the quadratic constraint often factors into the two line directions.

11. If a circle C passing through $(4,0)$ touches the circle $x^2+y^2+4x-6y-12=0$ externally at a point $(1,-1)$, then the radius of the circle C is: [4 marks]

(A) 5

(B) $2\sqrt{5}$

(C) 4

(D) $\sqrt{57}$

Correct Answer: (A) 5

Solution:

Concept:

- The centres of externally tangent circles and their point of contact are collinear, with the contact point between the centres.
- Complete squares to find the given circle centre and radius; the new radius is the distance of its centre from the contact point.

Step 1: Parametrise the unknown centre The given circle is $(x+2)^2+(y-3)^2=25$, with centre $O=(-2,3)$ and radius 5. Its contact point is $P=(1,-1)$, and $\overrightarrow{OP}=(3,-4)$. For external tangency the unknown centre H lies beyond P, away from O. Write

$$H=(1+3t,-1-4t), \quad t>0, \qquad r=HP=5t.$$

Step 2: Use the second point on the circle The point $A=(4,0)$ lies on the unknown circle, so

$HA^2 = r^2$. Thus

$$(3t-3)^2 + (-1-4t)^2 = 25t^2.$$

Expanding gives $25t^2 - 10t + 10 = 25t^2$, hence $t = 1$ and $r = 5$. The centre is $(4, -5)$: it is indeed 5 units from both P and A, while its distance from O is 10, equal to the sum of radii. The alternative radii fail this passage-through-point condition.

Step 3: Conclusion The radius of circle C is 5. Hence, the correct option is (A).

Quick Tip to Solve: For external tangency, place the new centre on the ray from the old centre through the contact point.

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12. Statement-1: The line $x - 2y = 2$ meets the parabola, $y^2 + 2x = 0$ only at the point $(-2, -2)$. Statement-2: The line $y = mx - \frac{1}{2m}$, $m \neq 0$, is tangent to the parabola, $y^2 = -2x$ at the point $\left(-\frac{1}{2m^2}, -\frac{1}{m}\right)$. [4 marks]

(A) Statement-1 is true; Statement- 2 is false

(B) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

(C) Statement-1 is false; Statement-2 is true

(D) Statement-1 a true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

Correct Answer: (B) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

Solution:

Concept:

- An intersection is a tangency when substituting a line into a conic gives a repeated root.
- For $y^2 = 4ax$, a nonvertical tangent of slope $m \neq 0$ is $y = mx + a/m$, with contact point $(a/m^2, 2a/m)$. Here $a = -\frac{1}{2}$.

Step 1: Verify Statement-1 directly The line gives $x = 2 + 2y$. Substituting in the parabola produces

$$y^2 + 2(2+2y) = 0 \quad \Longrightarrow \quad (y+2)^2 = 0.$$

The only intersection has $y = -2$ and $x = -2$. Hence the line meets the parabola only at $(-2, -2)$, and Statement-1 is true.

Step 2: Verify Statement-2 With $a = -\frac{1}{2}$, the tangent formula becomes $y = mx - \frac{1}{2m}$. Its point of contact is $\left(-\frac{1}{2m^2}, -\frac{1}{m}\right)$, exactly as stated. Alternatively, substitution gives $(my + 1)^2 = 0$, again a repeated root. Thus Statement-2 is true.

Step 3: Connect the two statements Rewrite the line of Statement-1 as $y = \frac{1}{2}x - 1$. It is precisely the tangent in Statement-2 with $m = \frac{1}{2}$, whose contact point is $(-2, -2)$. Therefore the general tangent statement establishes the special unique-intersection claim, so it is a correct explanation.

Step 4: Conclusion Both statements are true, and Statement-2 correctly explains Statement-1. Hence, the correct option is (B).

Quick Tip to Solve: Compare a proposed line with the slope form of the tangent before solving a quadratic intersection.

13. Let the equations of two ellipses be

$$E_1 : \frac{x^2}{3} + \frac{y^2}{2} = 1 \quad \textbf{and} \quad E_2 : \frac{x^2}{16} + \frac{y^2}{b^2} = 1.$$

If the product of their eccentricities is $\frac{1}{2}$, then the length of the minor axis of ellipse E_2 is: [4 marks]

(A) 8

(B) 9

(C) 4

(D) 2

Correct Answer: The printed question permits minor-axis length 4 (C) when b squared is 4, or 8 (A) when b squared is 64; it does not specify which axis is major.

Solution:

Concept:

- For an ellipse with semimajor axis a and semiminor axis c, the eccentricity is $e = \sqrt{1 - c^2/a^2}$. The larger denominator determines the major axis.
- The length of the minor axis is twice the smaller semiaxis. The symbol b alone does not impose $b < 4$ in the displayed equation.

Step 1: Find the second eccentricity For E_1, the squared semiaxes are 3 and 2, so $e_1 = \sqrt{1 - 2/3} = 1/\sqrt{3}$. From $e_1 e_2 = \frac{1}{2}$, we obtain $e_2 = \sqrt{3}/2$, and hence the squared ratio of the smaller to larger semiaxis of E_2 is $1 - e_2^2 = 1/4$.

Step 2: Examine both possible major axes If $b^2 < 16$, then $b^2/16 = 1/4$, so $b^2 = 4$. The minor axis is then $2\sqrt{4} = 4$, corresponding to option C. If $b^2 > 16$, instead $16/b^2 = 1/4$, so $b^2 = 64$. The minor axis is then $2\sqrt{16} = 8$, corresponding to option A. Both ellipses are nondegenerate and both satisfy the exact stated eccentricity product. Neither 9 nor 2 can be the minor-axis length under these conditions.

Step 3: Conclusion With the usual intended assumption $b^2 < 16$, the minor axis is 4 (option C). As printed without that restriction, the value 8 (option A) is also possible, so there is no unique option.

Quick Tip to Solve: Always identify the larger ellipse denominator before applying the eccentricity formula; do not infer an axis ordering from its letter.

14. The statement p →(q →p) is equivalent to [4 marks]

(A) p →q

(B) p →(p ∨q)

(C) p →(p →q)

(D) p →(p ∧q)

Correct Answer: (B) p →(p ∨q)

Solution:

Concept:

- An implication $r \to s$ is equivalent to $\neg r \vee s$. It fails only when its antecedent is true and consequent false.
- Logical equivalence means agreement for every truth assignment, not merely for one chosen value of p or q.

Step 1: Simplify the given implication Apply the implication rule first to the inner expression: $q \to p \equiv \neg q \vee p$. Then

$$p \to (q \to p) \equiv \neg p \vee \neg q \vee p \equiv \text{True}.$$

The terms $\neg p \vee p$ already form a tautology, independently of the value of q. Thus we must select a tautology among the options.

Step 2: Test the proposed equivalents For option B,

$$p \to (p \vee q) \equiv \neg p \vee p \vee q \equiv \text{True}.$$

The other three options all fail for p true and q false. Option A is directly $p \to q$; option C reduces to $\neg p \vee \neg p \vee q$, also $p \to q$; option D reduces to $\neg p \vee (p \wedge q)$, which has the same truth values. They therefore cannot match a statement that is always true.

Step 3: Conclusion The equivalent statement is $p \to (p \vee q)$. Hence, the correct option is (B).

Quick Tip to Solve: Reduce nested implications to disjunctions; a complementary pair immediately identifies a tautology.

15. Mean of 5 observations is 7 . If four of these observations are 6, 7, 8, 10 and one is missing then the variance of all the five observations is [4 marks]

(A) 4

(B) 6

(C) 8

(D) 2

Correct Answer: (A) 4

Solution:

Concept:

- The arithmetic mean is the sum of all observations divided by their number. Here the five observations must have total $5 \times 7 = 35$.
- The variance of the complete data set is $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$. The divisor is n, not the sample-estimation divisor $n-1$.

Step 1: Recover the missing observation Let the missing observation be u. Use the given mean before calculating the dispersion:

$$\frac{6+7+8+10+u}{5} = 7.$$

The four known observations add to 31, hence

$$u = 35 - 31 = 4.$$

Thus the full data set is $6, 7, 8, 10, 4$. Its total is 35, so its mean is indeed the stated value 7.

Step 2: Calculate the squared deviations Subtract the mean from each observation, retaining the negative deviations before squaring:

$$6-7=-1, \quad 7-7=0, \quad 8-7=1, \quad 10-7=3, \quad 4-7=-3.$$

Therefore

$$\sum(x_i-7)^2 = 1+0+1+9+9 = 20, \qquad \sigma^2 = \frac{20}{5} = 4.$$

As a separate check, the sum of squares is $36+49+64+100+16 = 265$, and $265/5 - 7^2 = 53-49 = 4$. The values 6 and 8 do not equal this mean square deviation. The value 2 is the standard deviation, namely $\sqrt{4}$, rather than the requested variance.

Step 3: Conclusion The variance of all five observations is $\boxed{4}$. Hence, the correct option is (A). The supplied answer key gives the value 2, but that is the standard deviation; the printed question asks for variance, which the calculation proves is 4.

Quick Tip to Solve: First recover any missing observation from the given mean; only then average the squared deviations. Do not confuse variance with its square root.

16. If two vertices of an equilateral triangle are $A(-a,0)$ and $B(a,0)$, $a>0$, and the third vertex C lies above the x-axis, then the equation of the circumcircle of $\triangle ABC$ is [4 marks]

(A) $3x^2+3y^2-2\sqrt{3}ay=3a^2$

(B) $3x^2+3y^2-2ay=3a^2$

(C) $x^2+y^2-2ay=a^2$

(D) $x^2+y^2-\sqrt{3}ay=a^2$

Correct Answer: (A) $3x^2+3y^2-2\sqrt{3}ay=3a^2$

Solution:

Concept:

- The altitude of an equilateral triangle of side s is $\sqrt{3}s/2$, and its circumcentre lies on each perpendicular bisector.
- A circle through $(-a,0)$ and $(a,0)$ has its centre on the y-axis. Its equation can be written $x^2+y^2+Fy-a^2=0$.

Step 1: Find the third vertex The distance between the given vertices is $AB=2a$. Since the triangle is equilateral and C is above the axis, its altitude has positive height

$$h=\frac{\sqrt{3}}{2}(2a)=\sqrt{3}a.$$

The altitude passes through the midpoint $(0,0)$, giving $C=(0,\sqrt{3}a)$. The negative height would describe the reflected triangle below the axis and must be excluded.

Step 2: Determine the circle coefficient For a general circle $x^2+y^2+Dx+Fy+G=0$, substitution of $(a,0)$ and $(-a,0)$ gives $D=0$ and $G=-a^2$. Now substitute C:

$$3a^2+F\sqrt{3}a-a^2=0.$$

Because $a>0$, division by $\sqrt{3}a$ is valid and yields

$$F=-\frac{2a}{\sqrt{3}}.$$

Thus the circle is

$$x^2+y^2-\frac{2a}{\sqrt{3}}y=a^2.$$

Multiplying every term by 3 gives $3x^2+3y^2-2\sqrt{3}ay=3a^2$. Its centre is $(0,a/\sqrt{3})$, as expected for an equilateral triangle. Each other option has a different coefficient of y, so fails when C is substituted.

Step 3: Conclusion The circumcircle is $\boxed{3x^2+3y^2-2\sqrt{3}ay=3a^2}$. Hence, the correct option is (A).

Quick Tip to Solve: Use symmetry to remove the linear x term, then substitute the third vertex to fix the remaining coefficient.

17. Let $R = \{(3,3),(5,5),(9,9),(12,12),(5,12),(3,9),(3,12),(3,5)\}$ be a relation on the set $A = \{3,5,9,12\}$. Then R is: [4 marks]

(A) reflexive, symmetric but not transitive

(B) symmetric, transitive but not reflexive

(C) an equivalence relation

(D) reflexive, transitive but not symmetric

Correct Answer: (D) reflexive, transitive but not symmetric

Solution:

Concept:

- A relation on A is reflexive when every (u,u), $u \in A$, belongs to it; it is symmetric when every pair has its reverse.
- Transitivity requires $(u,w) \in R$ whenever $(u,v) \in R$ and $(v,w) \in R$. An equivalence relation must satisfy all three properties.

Step 1: Check reflexivity and symmetry The set has four elements $3,5,9,12$. The relation explicitly contains $(3,3),(5,5),(9,9),(12,12)$, so every element is related to itself and reflexivity holds. However, $(3,5) \in R$ while $(5,3) \notin R$. This one counterexample proves that symmetry fails. Similarly, the reverse of $(5,12)$ is absent. Therefore every option claiming symmetry, including an equivalence relation, is already excluded.

Step 2: Check every nontrivial chain Diagonal pairs do not create any new requirement: composing (u,u) with (u,v), or (u,v) with (v,v), returns the existing pair. Among the off-diagonal pairs $(5,12),(3,9),(3,12),(3,5)$, the only chain with two non-diagonal links is

$$3 \longrightarrow 5 \longrightarrow 12.$$

It requires $(3,12)$, which is present. The elements 9 and 12 lead only to themselves, so no further nontrivial chain is possible. Hence the transitivity condition holds for all possible intermediate elements, not just for the example chain.

Step 3: Conclusion The relation is reflexive and transitive but not symmetric. Hence, the correct option is (D).

Quick Tip to Solve: After handling the diagonal pairs, test transitivity only on chains made from the off-diagonal pairs; one missing reverse pair settles symmetry.

18. If p, q, r are three real numbers satisfying the matrix equation

$$\begin{bmatrix} p & q & r \end{bmatrix} \begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 \end{bmatrix},$$

then $2p + q - r$ equals [4 marks]

(A) -3

(B) -1

(C) 4

(D) 2

Correct Answer: (A) -3

Solution:

Concept:

- In a row-vector product, each output entry is the dot product of the row vector with the corresponding column of the matrix.
- The matrix equation gives three simultaneous linear equations in p, q, r; solve these before evaluating the requested linear combination.

Step 1: Expand the matrix equation The first column is $(3, 3, 2)^T$, the second is $(4, 2, 0)^T$, and the third is $(1, 3, 2)^T$. Matching the three entries of the product with $[3\ 0\ 1]$ gives

$$\begin{aligned} 3p + 3q + 2r &= 3, \\ 4p + 2q &= 0, \\ p + 3q + 2r &= 1. \end{aligned}$$

It is essential to multiply by columns: treating the rows as the coefficients would produce a different system and would not represent the printed row-vector equation.

Step 2: Solve and evaluate the expression Subtract the third equation from the first. The q and r terms cancel, leaving

$$2p = 2, \qquad p = 1.$$

The second equation then yields

$$4 + 2q = 0, \qquad q = -2.$$

Substitute these values in the third equation:

$$1 - 6 + 2r = 1, \qquad r = 3.$$

Check the first equation: $3 - 6 + 6 = 3$, so all three output entries agree with the source equation. Finally,

$$2p + q - r = 2(1) + (-2) - 3 = -3.$$

Thus the printed value -3, rather than -1, 4, or 2, is obtained. The signs of both q and the subtracted r matter in this last step.

Step 3: Conclusion The required value is $\boxed{-3}$. Hence, the correct option is (A).

Quick Tip to Solve: For a row-vector product, read the matrix column by column. Subtract equations with matching coefficients before attempting full elimination.

19. If the system of linear equations

$$x_1 + 2x_2 + 3x_3 = 6,$$
$$x_1 + 3x_2 + 5x_3 = 9,$$
$$2x_1 + 5x_2 + ax_3 = b$$

is consistent and has an infinite number of solutions, then: [4 marks]

(A) $a = 8$, b can be any real number

(B) $b = 15$, a can be any real number

(C) $a \in \mathbb{R} \setminus \{8\}$ and $b \in \mathbb{R} \setminus \{15\}$

(D) $a = 8,\ b = 15$

Correct Answer: (D) $a = 8,\ b = 15$

Solution:

Concept:

- Three linear equations in three unknowns have infinitely many solutions when the coefficient and augmented matrices have the same rank, smaller than three.
- If two independent equations are retained, a third equation must be their linear combination, including its right-hand side, to leave a free variable.

Step 1: Eliminate the first two variables The first two equations are independent because their coefficient triples $(1, 2, 3)$ and $(1, 3, 5)$ are not proportional. Add them to obtain

$$2x_1 + 5x_2 + 8x_3 = 15.$$

The printed third equation has exactly the same coefficients of x_1 and x_2. Subtracting the sum from it gives the reduced condition

$$(a - 8)x_3 = b - 15.$$

This single relation identifies when the third equation supplies a new restriction and when it is redundant.

Step 2: Enforce a free variable and consistency If $a \neq 8$, this relation fixes x_3 uniquely. Subtracting the original first equation from the second then fixes x_2, and the first fixes x_1, so the solution is unique. If $a = 8$ but $b \neq 15$, the reduced condition is $0 = b - 15$, a contradiction. Hence infinitely many solutions require both $a = 8$ and $b = 15$. Conversely these values make the third equation the sum of the other two. Let $x_3 = t$; the first two give

$$x_2 = 3 - 2t, \qquad x_1 = t.$$

Every real t gives a solution, proving sufficiency. Neither parameter can therefore be arbitrary as the other options suggest.

Step 3: Conclusion The required parameters are $\boxed{a = 8,\ b = 15}$. Hence, the correct option is (D).

Quick Tip to Solve: Compare a candidate dependent equation with a simple sum of the independent equations; the constants must obey the same combination.

20. Let $f(x) = -1 + |x - 2|$ and $g(x) = 1 - |x|$; then the set of all points where $f \circ g$ is discontinuous is [4 marks]

(A) $\{0, 2\}$

(B) $\{0, 1, 2\}$

(C) $\{0\}$

(D) an empty set

Correct Answer: (D) an empty set

Solution:

Concept:

- The absolute-value function is continuous at every real number, including the point where its graph changes slope.
- A composition of continuous functions is continuous wherever the inner function takes values in the domain of the outer function. Here both domains are all real numbers.

Step 1: Check the functions and substitute The function $g(x) = 1-|x|$ is continuous for all real x, since subtraction of a continuous function from a constant preserves continuity. Similarly $f(u) = -1+|u-2|$ is continuous for every real input u. The sharp corners in their graphs are failures of differentiability, not failures of continuity. Direct substitution into the composition gives

$$(f \circ g)(x) = -1 + |(1 - |x|) - 2| = -1 + | - 1 - |x||.$$

Step 2: Simplify the absolute value and inspect the domain For every real x, $|x| \geq 0$, so $-1 - |x| < 0$. Its absolute value is therefore $1 + |x|$, without any need to split into extra intervals. Consequently

$$(f \circ g)(x) = -1 + (1 + |x|) = |x| \qquad (x \in \mathbb{R}).$$

This expression has no denominator, radical restriction, or discontinuous piece. At zero its left and right limits are both zero and equal its value. At the other suggested points 1 and 2, it is also continuous. The finite sets in options (A), (B), and (C) thus confuse corners or shifts with discontinuities; none is a set of discontinuity points.

Step 3: Conclusion The composition is continuous on $\mathbb{R}$, so its discontinuity set is $\boxed{\varnothing}$. Hence, the correct option is (D).

Quick Tip to Solve: Simplify the composition before considering special points. An absolute-value corner may destroy a derivative while leaving continuity intact.

21. For $a > 0$, $t \in (0, \pi/2)$, let $x = \sqrt{a^{\sin^{-1} t}}$ and $y = \sqrt{a^{\cos^{-1} t}}$. Then $1 + \left(\frac{dy}{dx}\right)^2$ equals [4 marks]

(A) x^2/y^2

(B) y^2/x^2

(C) $(x^2 + y^2)/y^2$

(D) $(x^2 + y^2)/x^2$

Correct Answer: (D) $(x^2 + y^2)/x^2$

Solution:

Concept:

- For real inverse trigonometric functions, $\sin^{-1} t + \cos^{-1} t = \pi/2$ on their common domain. This identity can turn a pair of parametric equations into one implicit equation.
- At a regular point of a parametric curve, $dy/dx = (dy/dt)/(dx/dt)$. Positive powers of a positive base make both x and y positive.

Step 1: Eliminate the parameter The square roots give $x = a^{(\sin^{-1} t)/2}$ and $y = a^{(\cos^{-1} t)/2}$. Multiplying them and adding exponents yields

$$xy = a^{(\sin^{-1} t + \cos^{-1} t)/2} = a^{\pi/4}.$$

Thus, on the real regular portion of the curve, xy is a positive constant independent of t. The real inverse functions require $|t| \leq 1$; the given positive interval must be intersected with that domain when interpreting real coordinates.

Step 2: Differentiate and compare the options Implicit differentiation of $xy = a^{\pi/4}$, with fixed a, gives

$$y + x\frac{dy}{dx} = 0, \qquad \frac{dy}{dx} = -\frac{y}{x}.$$

Therefore

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{y^2}{x^2} = \frac{x^2 + y^2}{x^2}.$$

The denominator is x^2, not y^2, because solving the product derivative divides by x. Option (B) omits the added 1, option (A) reverses the squared ratio, and option (C) is the reciprocal-ratio version. The differentiation applies where the parameter describes a regular curve, in particular $a \neq 1$ and $0 < t < 1$. If $a = 1$, both coordinates are constant and a slope along that degenerate parameterization is undefined; the stated formula is the regular-curve result.

Step 3: Conclusion For the regular real curve, the expression is $\boxed{(x^2 + y^2)/x^2}$. Hence, the correct option is (D).

Quick Tip to Solve: When inverse-sine and inverse-cosine exponents occur together, multiply the coordinates first: their exponents have a constant sum.

22. Statement-1: The function $x^2(e^x + e^{-x})$ is increasing for all $x > 0$. Statement-2: The functions x^2e^x and x^2e^{-x} are increasing for all $x > 0$ and the sum of two increasing functions in any interval (a, b) is an increasing function in (a, b). [4 marks]

(A) Statement-1 is false; Statement-2 is true

(B) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(C) Statement-1 is true; Statement-2 is false

(D) Statement-1is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

Correct Answer: (C) Statement-1 is true; Statement-2 is false

Solution:

Concept:

- A differentiable function is increasing on an interval when its derivative is positive throughout that interval. Each summand must be tested on the entire interval before applying the sum rule.
- The exponential satisfies $e^x > e^{-x} > 0$ for $x > 0$. Use the product rule to differentiate $x^2(e^x + e^{-x})$.

Step 1: Test Statement-1 directly Let $f(x) = x^2(e^x + e^{-x})$. Differentiating both factors gives

$$f\prime(x) = 2x(e^x + e^{-x}) + x^2(e^x - e^{-x}).$$

For every $x > 0$, the first term is positive since both exponentials and x are positive. The second term is also positive because $e^x > e^{-x}$. Thus $f\prime(x) > 0$ throughout the required interval and Statement-1 is true.

Step 2: Test the individual functions in Statement-2 The derivatives of the claimed summands are

$$\frac{d}{dx}(x^2e^x) = xe^x(x + 2), \qquad \frac{d}{dx}(x^2e^{-x}) = xe^{-x}(2 - x).$$

The first is positive for all positive x. The second is positive only for $0 < x < 2$, zero at 2, and negative for $x > 2$. For example, at $x = 3$ it equals $-3e^{-3} < 0$. Hence the claim that both functions increase for all positive x is false. The sum rule itself is valid, but cannot rescue this false premise. Options (A), (B) and (D) therefore have the wrong truth values.

Step 3: Conclusion Statement-1 is true and Statement-2 is false. Hence, the correct option is (C).

Quick Tip to Solve: Check each summand on the full stated interval; a positive increasing sum need not have every summand increasing.

23. The maximum area of a right angled triangle with hypotenuse h is: [4 marks]

(A) $\frac{h^2}{2\sqrt{2}}$

(B) $\frac{h^2}{2}$

(C) $\frac{h^2}{\sqrt{2}}$

(D) $\frac{h^2}{4}$

Correct Answer: (D) $\frac{h^2}{4}$

Solution:

Concept:

- If the perpendicular sides are $a, b > 0$, Pythagoras gives $a^2 + b^2 = h^2$, while the area is $A = ab/2$.
- The inequality $(a - b)^2 \geq 0$ gives $2ab \leq a^2 + b^2$, with equality exactly when $a = b$.

Step 1: Express the area using the two legs Take a and b as the legs meeting at the right angle. The hypotenuse is fixed, so

$$a^2 + b^2 = h^2, \qquad A = \frac{1}{2}ab.$$

The area depends on the product of the legs, rather than on their sum. As the triangle becomes very thin, one leg approaches zero and the area approaches zero, so an equal-sided right triangle is the natural candidate for the maximum.

Step 2: Bound the product and verify equality Apply the nonnegative-square inequality without changing the hypotenuse constraint:

$$0 \leq (a - b)^2 = a^2 + b^2 - 2ab = h^2 - 2ab.$$

Consequently

$$ab \leq \frac{h^2}{2}, \qquad A \leq \frac{h^2}{4}.$$

The bound is attained by $a = b = h/\sqrt{2}$, which indeed satisfies Pythagoras and forms a nondegenerate right triangle. Its area is $\frac{1}{2}(h/\sqrt{2})^2 = h^2/4$. Thus this is a maximum, not merely an upper estimate. Each of the other proposed values exceeds this bound and is impossible for a right triangle with the prescribed hypotenuse.

Step 3: Conclusion The maximum area is $\boxed{h^2/4}$, attained by an isosceles right triangle. Hence, the correct option is (D).

Quick Tip to Solve: For a fixed hypotenuse, use $2ab \leq a^2 + b^2$; equality immediately gives both the largest area and the optimal legs.

24. If $\int \frac{x^2 - x + 1}{x^2 + 1} e^{\cot^{-1} x}\, dx = A(x)\, e^{\cot^{-1} x} + C$**, then** $A(x)$ **is equal to** [4 marks]

(A) $-x$

(B) x

(C) $\sqrt{1 - x}$

(D) $\sqrt{1 + x}$

Correct Answer: (B) x

Solution:

Concept:

- The inverse cotangent derivative on its usual continuous real branch is $\frac{d}{dx} \cot^{-1} x = -\frac{1}{1 + x^2}$.
- If an antiderivative is $A(x)e^{g(x)}$, its derivative is $e^{g(x)}[A\prime(x) + A(x)g\prime(x)]$. Compare this expression with the integrand.

Step 1: Differentiate the proposed antiderivative form Set $g(x) = \cot^{-1} x$. Differentiation gives

$$\frac{d}{dx}\{A(x)e^{g(x)}\} = e^{g(x)} \left(A\prime(x) - \frac{A(x)}{1 + x^2} \right).$$

Since $e^{g(x)}$ is nonzero, comparison with the given integrand requires

$$(1 + x^2)A\prime(x) - A(x) = x^2 - x + 1.$$

This identity lets us test the options without performing integration by parts or introducing another integral.

Step 2: Test the linear candidate and rule out the others For $A(x) = x$, its derivative is 1, and the left side becomes

$$(1 + x^2) \cdot 1 - x = x^2 - x + 1,$$

exactly as required. Equivalently,

$$\frac{d}{dx} \left(x e^{\cot^{-1} x} \right) = \frac{x^2 - x + 1}{x^2 + 1} e^{\cot^{-1} x}.$$

The candidate $-x$ produces the negative numerator instead. At $x = 0$, the candidates $\sqrt{1 - x}$ and $\sqrt{1 + x}$ give $(1 + x^2)A\prime - A = -3/2$ and $-1/2$, respectively, whereas the required value is 1. Hence neither square-root option can satisfy the identity even locally near zero.

Step 3: Conclusion An antiderivative is $xe^{\cot^{-1} x} + C$, so the listed coefficient is $\boxed{A(x) = x}$. Hence, the correct option is (B).

Quick Tip to Solve: When the integral already suggests an exponential product, differentiate the candidate coefficient first and compare its numerator.

25. The integral $\int_{7\pi/4}^{7\pi/3} \sqrt{\tan^2 x}\, dx$ is equal to: [4 marks]

(A) $\log(2\sqrt{2})$

(B) $\log 2$

(C) $2\log 2$

(D) $\log\sqrt{2}$

Correct Answer: (A) $\log(2\sqrt{2})$

Solution:

Concept:

- The principal square root satisfies $\sqrt{\tan^2 x} = |\tan x|$, not necessarily $\tan x$. Its sign must be checked before integrating.
- An antiderivative of $\tan x$ is $-\log|\cos x|$. Split the interval where $\tan x$ changes sign.

Step 1: Locate the sign change inside the interval The interval runs from $7\pi/4$ to $7\pi/3$ and contains 2π. There is no pole of tangent in this interval: the neighbouring poles are $3\pi/2$ and $5\pi/2$, outside it. Tangent is negative on $(7\pi/4, 2\pi)$ and positive on $(2\pi, 7\pi/3)$. Thus the integrand is continuous, and

$$I = -\int_{7\pi/4}^{2\pi} \tan x\, dx + \int_{2\pi}^{7\pi/3} \tan x\, dx.$$

Ignoring the absolute value would incorrectly subtract the first positive contribution.

Step 2: Evaluate the two nonnegative contributions Using $\cos(7\pi/4) = 1/\sqrt{2}$, $\cos(2\pi) = 1$, and $\cos(7\pi/3) = 1/2$, we obtain

$$\begin{aligned} I &= [\log|\cos x|]_{7\pi/4}^{2\pi} + [-\log|\cos x|]_{2\pi}^{7\pi/3} \\ &= \log 1 - \log(1/\sqrt{2}) - \log(1/2) + \log 1 \\ &= \log\sqrt{2} + \log 2 = \log(2\sqrt{2}). \end{aligned}$$

Option (B) includes only the second contribution, while (D) includes only the first. Option (C) would require the first contribution to equal the second; they are $\frac{1}{2}\log 2$ and $\log 2$, respectively.

Step 3: Conclusion The integral is $\boxed{\log(2\sqrt{2})}$. The printed limits give this value, rather than the supplied keyed value $\log\sqrt{2}$, which omits the contribution above 2π. Hence, the correct option is (A).

Quick Tip to Solve: Always replace $\sqrt{u^2}$ by $|u|$, then split at the zeros of u that lie between the limits.

26. The area of the region (in sq. units), in the first quadrant bounded by the parabola $y = 9x^2$ and the lines $x = 0$, $y = 1$ and $y = 4$, is [4 marks]

(A) 7/9

(B) 14/3

(C) 7/3

(D) 14/9

Correct Answer: (D) 14/9

Solution:

Concept:

- For horizontal strips, the area is $\int_{y_1}^{y_2} (x_{\text{right}} - x_{\text{left}})\, dy$; here the vertical range is $1 \le y \le 4$.
- The first-quadrant branch of $y = 9x^2$ is $x = \sqrt{y}/3$. The other boundary is the vertical axis $x = 0$.

Step 1: Identify the width of a horizontal strip The horizontal lines $y = 1$ and $y = 4$ bound the region below and above. At a height y between these values, the left edge is $x = 0$ and the right edge is the positive branch of the parabola:

$$9x^2 = y \quad \Longrightarrow \quad x = \frac{\sqrt{y}}{3}.$$

We use the positive root because the region is in the first quadrant. Therefore a strip of thickness dy has area $\frac{\sqrt{y}}{3}\, dy$. Vertical strips would require a division into cases, whereas horizontal strips give a single integral.

Step 2: Integrate over the stated vertical limits The required area is

$$A = \int_1^4 \left(\frac{\sqrt{y}}{3} - 0 \right) dy = \frac{1}{3} \left[\frac{2}{3} y^{3/2} \right]_1^4$$
$$= \frac{2}{9}(4^{3/2} - 1) = \frac{2}{9}(8 - 1) = \frac{14}{9}.$$

No factor of two is appropriate: only the first-quadrant region is requested. As a check, its width increases from $1/3$ to $2/3$ over a height of 3, so its area must lie between 1 and 2. The alternatives $7/9$, $14/3$ and $7/3$ violate these elementary bounds, while $14/9$ lies within them.

Step 3: Conclusion The area is $\boxed{14/9}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: When the bounding lines specify heights, solve the curve for x and integrate horizontal widths with respect to y.

27. Consider the differential equation:

$$\frac{dy}{dx} = \frac{y^3}{2(xy^2 - x^2)}.$$

Statement-1: The substitution $z = y^2$ transforms the above equation into a first order homogenous differential equation. Statement-2: The solution of this differential equation is $y^2e^{-y^2/x} = C$. [4 marks]

(A) Both statements are false

(B) Statement-1 is true and statement- 2 is false

(C) Statement-1 is false and statement-2 is true

(D) Both statements are true

Correct Answer: (D) Both statements are true

Solution:

Concept:

- With $z = y^2$, the chain rule gives $dz/dx = 2y\,dy/dx$. A first-order equation is homogeneous if its right side depends only on z/x.
- For $dz/dx = F(z/x)$, put $v = z/x$, so $z = vx$ and $dz/dx = v + x\,dv/dx$.

Step 1: Apply the substitution in Statement-1 On the domain of the given equation, $x \neq 0$ and $y^2 \neq x$. Multiplying by $2y$ and putting $z = y^2$ yields

$$\frac{dz}{dx} = \frac{y^4}{xy^2 - x^2} = \frac{z^2}{xz - x^2} = \frac{(z/x)^2}{z/x - 1}.$$

The right side is a function of z/x alone, so the resulting first-order equation is homogeneous. Statement-1 is true.

Step 2: Integrate and check Statement-2 For a nonzero solution, let $v = z/x$. Then

$$v + x\frac{dv}{dx} = \frac{v^2}{v - 1}, \qquad x\frac{dv}{dx} = \frac{v}{v - 1}.$$

Separate variables and integrate:

$$\int \left(1 - \frac{1}{v}\right) dv = \int \frac{dx}{x}, \qquad v - \log|v| = \log|x| + K.$$

Since $|vx| = |z|$, this gives $v - \log|z| = K$, or $ze^{-z/x} = C$ after absorbing constants and signs. Substituting back gives $y^2e^{-y^2/x} = C$. Direct differentiation also confirms it:

$$\frac{d}{dx}(y^2e^{-y^2/x}) = e^{-y^2/x}\left[2yy'\left(1 - \frac{y^2}{x}\right) + \frac{y^4}{x^2}\right] = 0$$

when the given derivative is inserted. The solution $y = 0$, omitted during division by v, is included by $C = 0$. Thus Statement-2 is true as well, excluding all alternatives that declare either statement false.

Step 3: Conclusion Both statements are true; the implicit solution is $\boxed{y^2e^{-y^2/x} = C}$ on intervals where the differential equation is defined. Hence, the correct option is (D).

Quick Tip to Solve: After using $z = y^2$, divide numerator and denominator by x^2 to recognize the homogeneous form immediately.

28. If $\hat{a}, \hat{b}, \hat{c}$ are unit vectors satisfying $\hat{a} - \sqrt{3}\hat{b} + \hat{c} = 0$, then the angle between the vectors $\hat{a}$ and $\hat{c}$ is [4 marks]

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{2}$

Correct Answer: (B) $\frac{\pi}{3}$

Solution:

Concept:

- For unit vectors $\hat{a}$ and $\hat{c}$, $\hat{a} \cdot \hat{c} = \cos\theta$, where $0 \le \theta \le \pi$ is their angle.
- Take the squared magnitude of a vector equality using $|\mathbf{u} + \mathbf{v}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 + 2\mathbf{u} \cdot \mathbf{v}$.

Step 1: Isolate the sum of the two relevant vectors The given relation can be rearranged without changing signs as

$$\hat{a} + \hat{c} = \sqrt{3}\,\hat{b}.$$

Because $\hat{b}$ is a unit vector, the magnitude of the right side is $\sqrt{3}$, so its squared magnitude is 3. Taking a magnitude is useful here because the direction of $\hat{b}$ is not requested; its unit length is the only further information needed.

Step 2: Expand the squared magnitude and solve for the angle Let θ be the angle between $\hat{a}$ and $\hat{c}$. Expanding the left side gives

$$\begin{aligned} |\hat{a} + \hat{c}|^2 &= \hat{a} \cdot \hat{a} + \hat{c} \cdot \hat{c} + 2\hat{a} \cdot \hat{c} \\ &= 1 + 1 + 2\cos\theta = 3. \end{aligned}$$

Hence $\cos\theta = 1/2$, and the angle between the vectors is $\theta = \pi/3$. There is no supplementary-angle ambiguity because $\cos\theta$ is positive and the vector angle lies in $[0, \pi]$. For comparison, $\pi/4$, $\pi/6$ and $\pi/2$ have cosines $1/\sqrt{2}$, $\sqrt{3}/2$ and 0, none equal to $1/2$. These eliminate the other three printed options directly.

Step 3: Conclusion The angle between $\hat{a}$ and $\hat{c}$ is $\boxed{\pi/3}$. Hence, the correct option is (B).

Quick Tip to Solve: For a sum of unit vectors, square its length to turn a vector equation into one scalar equation for the cosine.

29. Let Q be the foot of perpendicular from the origin to the plane $4x - 3y + z + 13 = 0$ and R be a point $(-1, -6)$ on the plane. Then length QR is: [4 marks]

(A) $\sqrt{14}$

(B) $\sqrt{\frac{19}{2}}$

(C) $3\sqrt{\frac{7}{2}}$

(D) $\frac{3}{\sqrt{2}}$

Correct Answer: No listed answer follows from the incomplete three-dimensional point.

Solution:

Concept:

- The perpendicular from the origin to a plane is parallel to its normal vector. For the plane $4x - 3y + z + 13 = 0$, a normal is $\mathbf{n} = (4, -3, 1)$.
- A point in three-dimensional Cartesian space requires three coordinates. The distance between two specified points is the square root of the sum of the squared coordinate differences.

Step 1: Find the perpendicular foot Write $Q = t(4, -3, 1)$, since OQ is normal to the plane. Substitute this point into the plane equation:

$$16t + 9t + t + 13 = 0 \quad \Longrightarrow \quad t = -\frac{1}{2}.$$

Therefore

$$Q = \left(-2, \frac{3}{2}, -\frac{1}{2}\right), \qquad OQ^2 = \frac{13}{2}.$$

The dot-product condition fixes this foot independently of the incomplete point R.

Step 2: Examine the specified point The question gives $R = (-1, -6)$, a pair rather than a three-coordinate point. Thus its literal notation does not specify a three-dimensional point. Even interpreting the two entries as $x = -1$ and $y = -6$, and using the stated condition that R lies on the plane, does not produce a listed answer. That conditional interpretation gives

$$4(-1) - 3(-6) + z + 13 = 0 \quad \Longrightarrow \quad z = -27.$$

It would then yield

$$QR^2 = 1 + \left(-\frac{15}{2}\right)^2 + \left(-\frac{53}{2}\right)^2 = \frac{1519}{2}.$$

The squared printed choices are $14, 19/2, 63/2, 9/2$, all different. This calculation tests a possible reading; it does not supply a replacement coordinate in the question.

Step 3: Conclusion The given pair does not specify the stated three-dimensional point, and the natural conditional reading also gives none of the four choices. Hence no unique listed answer is defensible as written.

Quick Tip to Solve: For a plane-distance problem, find the normal foot first and check that every alleged three-dimensional point has three coordinates and satisfies the plane.

30. Given two independent events, if the probability that exactly one of them occurs is (26)/(49) and the probability that none of them occurs is (15)/(49), then the probability of more probable of the two events is [4 marks]

(A) 4/7

(B) 6/7

(C) 3/7

(D) 5/7

Correct Answer: (A) 4/7

Solution:

Concept:

- For independent events E and F with probabilities p and q, the probability of both is pq. The probability of neither is (1-p)(1-q).
- Exactly one event occurs through two mutually exclusive possibilities: E without F, or F without E. Thus its probability is p(1-q)+q(1-p).

Step 1: Find the probability of both events The outcomes neither, exactly one, and both form a complete partition of the sample space. Their probabilities sum to one, so

$$pq = 1 - \frac{26}{49} - \frac{15}{49} = \frac{8}{49}.$$

This step uses the given probabilities directly and avoids treating exactly one as the probability of at least one.

Step 2: Determine the sum of the probabilities The expression for exactly one event is $p + q - 2pq$, because the intersection is counted in both individual probabilities and must be removed twice. Therefore

$$p + q = \frac{26}{49} + 2\frac{8}{49} = \frac{42}{49} = \frac{6}{7}.$$

As a check, $1 - (p + q) + pq = 1 - 6/7 + 8/49 = 15/49$, as required.

Step 3: Solve for the individual probabilities The numbers p and q are the two roots of the quadratic with their sum and product:

$$t^2 - \frac{6}{7}t + \frac{8}{49} = 0 \quad \Longrightarrow \quad (7t - 4)(7t - 2) = 0.$$

Hence the probabilities are $4/7$ and $2/7$. The more probable event has the larger of these values. The choices $6/7$ and $3/7$ do not equal the larger root; $5/7$ is not a root either.

Step 4: Conclusion The more probable event has probability $\boxed{4/7}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the three disjoint outcomes neither, exactly one, and both to recover the product before forming the quadratic.

Physics (120 marks)

31. The dimensions of angular momentum, latent heat and capacitance are, respectively [4 marks]

(A) $ML^2TA^2,\ L^2T^{-2},\ M^{-1}L^{-2}T^2$

(B) $ML^2T^{-2},\ L^2T^2,\ M^{-1}L^{-2}T^4A^2$

(C) $ML^2T^{-1},\ L^2T^{-2},\ ML^2TA^2$

(D) $ML^2T^{-1},\ L^2T^{-2},\ M^{-1}L^{-2}T^4A^2$

Correct Answer: (D) $ML^2T^{-1},\ L^2T^{-2},\ M^{-1}L^{-2}T^4A^2$

Solution:

Concept:

- Angular momentum is the moment of linear momentum, so its dimensions are length times mass times velocity. The base dimensions used are mass M, length L, time T and electric current A.
- Specific latent heat is energy per unit mass. Capacitance is charge divided by potential difference, while potential difference is energy divided by charge.

Step 1: Find the mechanical dimensions For angular momentum, $\boldsymbol{L} = \boldsymbol{r} \times m\boldsymbol{v}$, so the dimensional product is

$$[L] = L\,M\,(LT^{-1}) = ML^2T^{-1}.$$

A cross product changes direction but does not change the product of dimensions. For latent heat in this context we use the specific latent heat in $Q = m\ell$. Since heat is energy,

$$[\ell] = \frac{ML^2T^{-2}}{M} = L^2T^{-2}.$$

It is essential to divide by mass; the dimensions of total heat alone would retain a mass factor.

Step 2: Express capacitance in base dimensions Electric charge has dimensions $[q] = AT$, because charge is current multiplied by time. Potential difference is energy per unit charge, so

$$[V] = \frac{ML^2T^{-2}}{AT} = ML^2T^{-3}A^{-1}.$$

Using $C = q/V$, we obtain

$$[C] = \frac{AT}{ML^2T^{-3}A^{-1}} = M^{-1}L^{-2}T^4A^2.$$

Thus current appears squared, and time has power four, rather than power two.

Step 3: Compare the full ordered triple Only the fourth choice contains all three results in the required order. The first choice gives incorrect angular-momentum and capacitance dimensions; the second has the wrong time powers in the mechanical quantities. Although the third begins with the correct angular momentum and latent heat, its capacitance has the wrong mass, length and time powers.

Step 4: Conclusion The dimensions are $ML^2T^{-1},\ L^2T^{-2},\ M^{-1}L^{-2}T^4A^2$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: For capacitance use charge squared divided by energy; this immediately gives the time power four and current power two.

32. A ball projected from ground at an angle of 45∘ just clears a wall in front. If point of projection is 4 m from the foot of wall and ball strikes the ground at a distance of 6 m on the other side of the wall, the height of the wall is [4 marks]

(A) 4.4 m

(B) 2.4 m

(C) 3.6 m

(D) 1.6 m

Correct Answer: (B) 2.4 m

Solution:

Concept:

- For a projectile returning to its launch level, the range is $R = u^2 \sin(2\theta)/g$, where u is launch speed and theta is the launch angle.
- The trajectory is $y = x \tan\theta - gx^2/(2u^2 \cos^2\theta)$. At 45 degrees this reduces to $y = x - x^2/R$.

Step 1: Identify the full horizontal range The wall is $4\,\text{m}$ from the launch point, and the landing point is another $6\,\text{m}$ beyond the wall. Both are horizontal distances measured from the foot of the wall along the ground. Therefore

$$R = 4 + 6 = 10\,\text{m}.$$

Since the launch angle is 45°, $\sin 90^\circ = 1$, and consequently $u^2/g = R = 10\,\text{m}$. No numerical value of g is needed.

Step 2: Evaluate the trajectory at the wall At this angle, $\tan 45^\circ = 1$ and $2\cos^2 45^\circ = 1$. Substitute these values in the trajectory equation:

$$y = x - \frac{gx^2}{u^2} = x - \frac{x^2}{R}.$$

At the wall the horizontal coordinate is $x = 4\,\text{m}$. Hence

$$y = 4 - \frac{4^2}{10} = 4 - 1.6 = 2.4\,\text{m}.$$

The phrase just clears means that the top of the wall coincides with the trajectory at that position.

Step 3: Check the geometric consistency The highest point occurs halfway through the range, at $x = 5\,\text{m}$, and has height $R/4 = 2.5\,\text{m}$. The wall at x=4 lies just before this point and its height 2.4 m is slightly smaller, which is consistent. The alternatives 4.4 m and 3.6 m exceed the maximum possible height. The value 1.6 m is the downward quadratic correction, not the resulting height.

Step 4: Conclusion The height of the wall is $\boxed{2.4\,\text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: At 45 degrees use y=x(R-x)/R; the range is the sum of the distances on the two sides of the wall.

33. Two blocks of mass M1= 20 kg and M2= 12 kg are connected by a metal rod of mass 8 kg. The system is pulled vertically up by applying a force of 480 N as shown. The tension at the mid-point of the rod is: [4 marks]

(A) 144 N

(B) 96 N

(C) 240 N

(D) 192 N

Correct Answer: (D) 192 N

Solution:

Concept:

- Treat the two blocks and the connecting rod as one system to find their common vertical acceleration. Internal rod tensions then cancel.
- To find tension at a cut in a massive rod, isolate everything below the cut. The tension supports and accelerates that lower subsystem.

Step 1: Find the effective upward acceleration The total mass of the system is

$$M = 20 + 12 + 8 = 40\,\text{kg}.$$

Taking upward as positive, Newton's second law for the entire system gives

$$480 - 40g = 40a \quad \Longrightarrow \quad g + a = \frac{480}{40} = 12\,\text{m}\,\text{s}^{-2}.$$

This effective quantity is all that is required. Thus the result does not depend on choosing g as 9.8 or 10 in separate intermediate calculations.

Step 2: Cut the rod at its midpoint The figure shows the 20 kg block at the top and the 12 kg block at the bottom. Below the midpoint of the uniform 8 kg connecting rod are the lower block and half the

rod. Their combined mass is

$$m_{\text{lower}} = 12 + \frac{8}{2} = 16\,\text{kg}.$$

On this subsystem the upward force is the tension T at the cut; its downward weight is $16g$. Both the block and the lower half of the rod have the same acceleration as the complete system.

Step 3: Apply the subsystem force equation Newton's law for that lower subsystem is

$$T - 16g = 16a.$$

Therefore

$$T = 16(g + a) = 16 \times 12 = 192\,\text{N}.$$

Using only the lower block would omit the lower half of the massive rod and would incorrectly give 144 N. A value of 96 N corresponds to applying the effective acceleration to the rod mass alone; 240 N does not describe the mass below the midpoint. Tension is not constant along a rod with appreciable mass.

Step 4: Conclusion The tension at the midpoint is $\boxed{192\,\text{N}}$. Hence, the correct option is (D).

Quick Tip to Solve: At any cut of a massive connector, multiply the mass below that cut by g+a; first find g+a from the complete system.

34. A body starts from rest on a long inclined plane of slope 45°. The coefficient of friction between the body and the plane varies as $\mu = 0.3x$, where x is distance travelled down the plane. The body will have maximum speed (for $g = 10\,\text{m/s}^2$) when $x =$ [4 marks]

(A) 9.8 m

(B) 27 m

(C) 12 m

(D) 3.33 m

Correct Answer: (D) 3.33 m

Solution:

Concept:

- Down an incline of angle theta, the gravitational component is $mg\sin\theta$, while the normal reaction is $mg\cos\theta$. Kinetic friction opposes the downward motion with magnitude $\mu mg\cos\theta$.
- As long as the body is moving downward, increasing speed requires positive acceleration. The maximum speed occurs where acceleration changes from positive to negative.

Step 1: Write the position dependent acceleration Take the positive direction down the plane. The normal reaction has no component along this direction, but fixes the friction magnitude. Newton's second law gives

$$ma = mg\sin 45^\circ - \mu mg\cos 45^\circ.$$

Substitute $\mu = 0.3x$ and $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$:

$$a(x) = \frac{g}{\sqrt{2}}(1 - 0.3x).$$

The mass cancels, and the force along the plane decreases continuously as the distance increases.

Step 2: Locate the change of acceleration sign Initially x=0 and the acceleration is positive, so the body begins gaining speed. Set the acceleration equal to zero:

$$1 - 0.3x = 0 \quad \Longrightarrow \quad x = \frac{1}{0.3} = \frac{10}{3}\,\text{m}.$$

Before this point acceleration is positive; immediately after it acceleration is negative while the body is still moving downwards. Hence the speed has a maximum here, rather than a minimum.

Step 3: Distinguish maximum speed from stopping distance The relation $a = v\,dv/dx$ gives

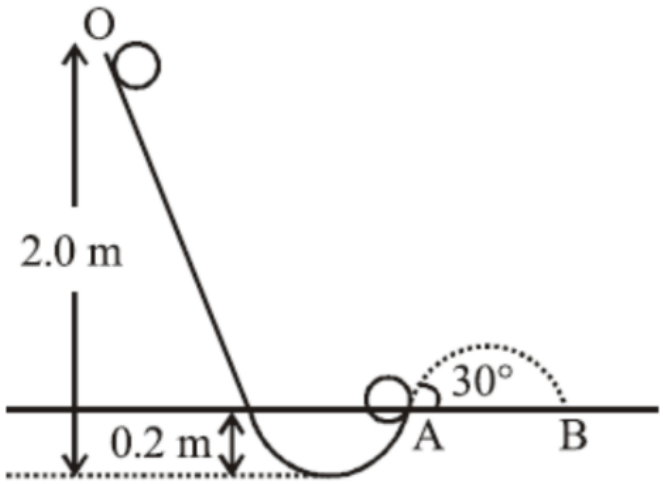
$$v^2 = \sqrt{2}g(x - 0.15x^2)$$

after integrating from rest. At x=10/3 the bracket is positive, confirming the body is moving and the sign-change test applies. The later zero of v is at x=20/3, which is a different position. None of 9.8 m, 27 m or 12 m satisfies the force-balance condition $\mu = \tan 45^\circ = 1$. The supplied value of g affects the speed, but cancels from the position of its maximum.

Step 4: Conclusion The maximum speed occurs at $x = 10/3\,\text{m} \simeq \boxed{3.33\,\text{m}}$. Hence, the correct option is (D).

Quick Tip to Solve: For friction increasing with distance, set mu equal to tan(theta) to find maximum speed, not the point where the body stops.

35. A tennis ball (treated as a hollow spherical shell) starting from O rolls down a hill. At point A the ball becomes airborne, leaving at an angle of 30° with the horizontal. The ball strikes the ground at B. What is the distance AB? The moment of inertia of a spherical shell of mass m and radius R about its diameter is $\frac{2}{3}mR^2$. [4 marks]



(A) 1.87 m

(B) 2.08 m

(C) 1.57 m

(D) 1.77 m

Correct Answer: (A) 1.87 m

Solution:

Concept:

- For rolling without slipping, $v = \omega R$, and total kinetic energy is $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. For a hollow spherical shell $I = \frac{2}{3}mR^2$.
- After leaving the track, the centre of mass follows projectile motion. For equal launch and landing levels, the horizontal range is $v^2 \sin(2\theta)/g$.

Step 1: Read the vertical drop to the launch point The figure gives O at 2.0 m above the bottom reference level. The horizontal level through A and B is 0.2 m above that reference. The loss of height from O to A is therefore

$$h = 2.0 - 0.2 = 1.8\,\text{m}.$$

The lower point of the curved track is not the launch point. Mechanical energy conservation depends only on this net vertical drop, not on the curved length travelled along the hill.

Step 2: Include rotational kinetic energy The ball starts from rest and rolls without slipping until it leaves the track. Using the given moment of inertia and $\omega = v/R$, conservation of energy gives

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\frac{v^2}{R^2} = \frac{5}{6}mv^2.$$

Hence

$$v^2 = \frac{6}{5}gh = \frac{6}{5}g(1.8) = 2.16g.$$

Treating the ball as a particle sliding without rotation would overestimate its launch speed; the rotational term must be retained.

Step 3: Find the airborne range The figure shows that A and B are on the same horizontal level and the departure angle is 30°. Thus

$$AB = \frac{v^2 \sin 60^\circ}{g} = 2.16\frac{\sqrt{3}}{2} = 1.08\sqrt{3}\,\text{m} \simeq 1.87\,\text{m}.$$

The ball continues to spin in flight, but gravity determines the translational projectile motion, and the spin does not alter this range in the stated model. The other choices do not follow from both the 1.8 m drop and the shell inertia.

Step 4: Conclusion The distance is $\boxed{AB \simeq 1.87\,\text{m}}$. The printed figure therefore gives option (A), rather than the supplied keyed value of 2.08 m, which would result from using a 2.0 m drop without subtracting the launch height. Hence, the correct option is (A).

Quick Tip to Solve: First subtract the launch height from the start height, then include rotational energy before applying the equal-level projectile range formula.

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36. The change in the value of acceleration of earth towards sun, when the moon comes from the position of solar eclipse to the position on the other side of earth in line with sun is: (mass of the moon $= 7.36 \times 10^{22}$ kg, radius of the moon's orbit $= 3.8 \times 10^8$ m). [4 marks]

(1) $6.73 \times 10^{-5}\,\text{m/s}^2$

(2) $6.73 \times 10^{-3}\,\text{m/s}^2$

(3) $6.73 \times 10^{-2}\,\text{m/s}^2$

(4) $6.73 \times 10^{-4}\,\text{m/s}^2$

Correct Answer: (1) $6.73 \times 10^{-5}\,\text{m/s}^2$

Solution:

Concept:

- The gravitational acceleration of the earth produced by the moon has magnitude $a_m = GM_m/r^2$, where r is their separation. Acceleration does not depend on the mass of the body being accelerated.
- Take the direction from earth towards the sun as positive. Moving the moon from one side of the earth to the opposite side reverses its acceleration contribution. The solar contribution stays the same for this comparison.

Step 1: Identify the two lunar contributions At solar eclipse the moon is between earth and sun, so its attraction accelerates earth towards the sun. The total component is $a_\odot + a_m$. On the opposite side, the lunar attraction points away from the sun, giving $a_\odot - a_m$. Thus the new component minus the old one is $-2a_m$. The question asks for the amount of change, so compare the positive magnitude $2a_m$, rather than using the acceleration of the moon towards earth.

Step 2: Calculate the magnitude of the change Using $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$,

$$|\Delta a| = \frac{2GM_m}{r^2} = \frac{2(6.67 \times 10^{-11})(7.36 \times 10^{22})}{(3.8 \times 10^8)^2} \simeq 6.8 \times 10^{-5}\,\mathrm{m\,s^{-2}}.$$

The listed value 6.73×10^{-5} agrees to the precision of the supplied astronomical data and constants. The remaining values are respectively a hundred, a thousand and ten times larger; none has the correct scale. The factor two is essential because the attraction reverses instead of vanishing.

Step 3: Conclusion The acceleration towards the sun decreases by approximately $6.73 \times 10^{-5}\,\mathrm{m\,s^{-2}}$. Hence, the correct option is (1).

Quick Tip to Solve: When a force of unchanged magnitude reverses direction, its contribution changes by twice its magnitude; subtract the signed components before substituting values.

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37. A uniform wire (Young's modulus $2 \times 10^{11}\,\mathrm{N\,m^{-2}}$) is subjected to longitudinal tensile stress of $5 \times 10^7\,\mathrm{N\,m^{-2}}$. If the overall volume change in the wire is 0.02%, the fractional decrease in the radius of the wire is close to: [4 marks]

(1) 1.0×10^{-4}

(2) 1.5×10^{-4}

(3) 0.25×10^{-4}

(4) 5×10^{-4}

Correct Answer: (3) 0.25×10^{-4}

Solution:

Concept:

- Young's modulus relates longitudinal stress and strain: $Y = \sigma/\epsilon_l$, with $\epsilon_l = \Delta l/l$. A tensile stress increases the length but usually decreases the transverse dimensions.
- For a cylindrical wire $V = \pi r^2 l$. To first order in small strains, $\Delta V/V = 2\Delta r/r + \Delta l/l$. Products and squares of strains can be neglected here.

Step 1: Find the longitudinal strain The applied stress and Young's modulus have the same units, so their ratio is dimensionless.

$$\epsilon_l = \frac{\sigma}{Y} = \frac{5 \times 10^7}{2 \times 10^{11}} = 2.5 \times 10^{-4}.$$

The volume increase is $0.02\% = 0.02/100 = 2.0 \times 10^{-4}$, which is smaller than the longitudinal increase. Therefore the cross-sectional area must decrease; treating volume strain as longitudinal strain would miss the contraction.

Step 2: Extract the radial contraction Let $f = -\Delta r/r$ denote the positive fractional decrease in radius. Substitute the signed radial change into the small-strain volume relation:

$$2.0 \times 10^{-4} = 2.5 \times 10^{-4} - 2f.$$

Consequently,

$$f = \frac{(2.5 - 2.0) \times 10^{-4}}{2} = 0.25 \times 10^{-4}.$$

The factor two comes from the squared radius in the area, not from a second wire. A value 1.0×10^{-4} would predict a much smaller volume increase, while 1.5×10^{-4} or 5×10^{-4} would even predict a volume decrease for the stated longitudinal strain.

Step 3: Conclusion The fractional decrease in the radius is $0.25 \times 10^{-4} = 2.5 \times 10^{-5}$. Hence, the correct option is (3).

Quick Tip to Solve: For a thin cylindrical wire use volume strain = longitudinal strain + twice radial strain, keeping a negative sign for radial contraction.

38. Air of density $1.2\,\mathrm{kg\,m^{-3}}$ is blowing across the horizontal wings of an aeroplane in such a way that its speeds above and below the wings are $150\,\mathrm{m\,s^{-1}}$ and $100\,\mathrm{m\,s^{-1}}$, respectively. The pressure difference between the upper and lower sides of the wings, is: [4 marks]

(1) $60\,\mathrm{N\,m^{-2}}$

(2) $180\,\mathrm{N\,m^{-2}}$

(3) $7500\,\mathrm{N\,m^{-2}}$

(4) $12500\,\mathrm{N\,m^{-2}}$

Correct Answer: (3) $7500\,\mathrm{N\,m^{-2}}$

Solution:

Concept:

- For steady ideal flow at the same height, Bernoulli's equation is $P + \frac{1}{2}\rho v^2 = \text{constant}$. The gravitational term cancels between the upper and lower surfaces of a horizontal wing.
- The faster stream has lower static pressure. Thus the air above the wing has lower pressure and the positive lifting pressure difference is $P_{\text{lower}} - P_{\text{upper}}$.

Step 1: Relate the upper and lower pressures Use the supplied air density $\rho = 1.2\,\mathrm{kg\,m^{-3}}$, upper speed $v_u = 150\,\mathrm{m\,s^{-1}}$, and lower speed $v_l = 100\,\mathrm{m\,s^{-1}}$. Applying Bernoulli's relation gives

$$P_l + \frac{1}{2}\rho v_l^2 = P_u + \frac{1}{2}\rho v_u^2.$$

Rearranging, $P_l - P_u = \frac{1}{2}\rho(v_u^2 - v_l^2)$. This is a pressure difference, so the atmospheric reference pressure is unnecessary. The direction is consistent with upward lift.

Step 2: Evaluate the speed-square difference First square the speeds and then subtract; squaring their difference would be a different operation.

$$v_u^2 - v_l^2 = 150^2 - 100^2 = 22500 - 10000 = 12500\,\mathrm{m^2\,s^{-2}}.$$

Hence

$$P_l - P_u = \frac{1}{2}(1.2)(12500) = 7500\,\mathrm{N\,m^{-2}}.$$

The units follow from density times speed squared. The value 12500 is only the numerical speed-square difference and omits the density factor. Neither 60 nor 180 can follow from the Bernoulli dynamic-pressure difference with the stated speeds.

Step 3: Conclusion The lower pressure exceeds the upper pressure by $7500\,\mathrm{N\,m^{-2}}$. Hence, the correct option is (3).

Quick Tip to Solve: At equal heights subtract the squares of the two speeds, then multiply by half the density; do not square the speed difference.

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39. Given that $1\,\mathrm{g}$ of water in liquid phase has volume $1\,\mathrm{cm^3}$ and in vapour phase $1671\,\mathrm{cm^3}$ at atmospheric pressure and the latent heat of vaporization of water is $2256\,\mathrm{J/g}$; the change in the internal energy in joules for $1\,\mathrm{g}$ of water at $373\,\mathrm{K}$ when it changes from liquid phase to vapour phase at the same temperature is: [4 marks]

(1) 2256

(2) 167

(3) 2089

(4) 1

Correct Answer: (3) 2089

Solution:

Concept:

- With work done by the system taken as positive, the first law is $Q = \Delta U + W$. At constant external pressure the expansion work is $W = P\Delta V$.
- The latent heat supplies the total heat absorbed during vaporization. At a phase transition the temperature may remain constant while intermolecular potential energy, and hence internal energy, changes.

Step 1: Calculate heat and expansion work For $1\,\mathrm{g}$, the heat absorbed is $Q = mL = (1)(2256) = 2256\,\mathrm{J}$. The volume change is the final volume minus the initial volume, not the final volume alone:

$$\Delta V = (1671 - 1)\,\mathrm{cm^3} = 1670 \times 10^{-6}\,\mathrm{m^3}.$$

Using the atmospheric-pressure approximation $P = 10^5\,\mathrm{Pa}$, the water does positive work on its surroundings:

$$W = P\Delta V = (10^5)(1670 \times 10^{-6}) = 167\,\mathrm{J}.$$

Step 2: Separate the energy stored from the work done The absorbed heat has two destinations: increasing internal energy and pushing back the atmosphere. Therefore

$$\Delta U = Q - W = 2256 - 167 = 2089\,\mathrm{J}.$$

Choosing $2256\,\mathrm{J}$ would incorrectly count all the latent heat as stored internal energy. Choosing $167\,\mathrm{J}$ gives only the expansion work. The value $1\,\mathrm{J}$ has no energy-balance basis. Constant temperature

does not imply zero internal-energy change for liquid-to-vapour conversion: the ideal-gas result that internal energy depends only on temperature cannot be applied across these two different phases.

Step 3: Conclusion The increase in internal energy is $2089\,\mathrm{J}$, using the pressure approximation implicit in the options. Hence, the correct option is (3).

Quick Tip to Solve: During boiling at constant pressure subtract the expansion work from the latent heat to obtain the internal-energy increase.

40. An ideal gas at atmospheric pressure is adiabatically compressed so that its density becomes 32 times of its initial value. If the final pressure of gas is 128 atmospheres, the value of γ of the gas is: [4 marks]

(1) 1.5

(2) 1.4

(3) 1.3

(4) 1.6

Correct Answer: (2) 1.4

Solution:

Concept:

- For the adiabatic ideal-gas relation used here, $PV^{\gamma} = \text{constant}$, where $\gamma = C_P/C_V$. Pressure must be compared using absolute pressures.
- For a fixed mass m, density is $\rho = m/V$. Thus multiplying density by a factor divides the volume by the same factor, and $P \propto \rho^{\gamma}$.

Step 1: Express the volume ratio from the density ratio The final density is thirty-two times the initial density while the amount of gas remains unchanged. Consequently,

$$\frac{\rho_f}{\rho_i} = 32, \qquad \frac{V_i}{V_f} = 32.$$

The gas starts at one atmosphere and ends at 128 atmospheres, so $P_f/P_i = 128$. Compression raises both pressure and density; using $V_f/V_i = 32$ instead would reverse the physical change and lead to an inadmissible negative exponent.

Step 2: Solve for the heat-capacity ratio Apply the adiabatic relation between the two equilibrium states:

$$\frac{P_f}{P_i} = \left(\frac{V_i}{V_f}\right)^{\gamma}, \qquad 128 = 32^{\gamma}.$$

Since $128 = 2^7$ and $32 = 2^5$, equating powers gives

$$2^7 = 2^{5\gamma}, \qquad \gamma = \frac{7}{5} = 1.4.$$

This is dimensionless, as a ratio of heat capacities must be. Substituting 1.5, 1.3 or 1.6 yields powers $2^{7.5}$, $2^{6.5}$ or 2^8, respectively, rather than the required 2^7. No conversion from atmospheres to pascals is

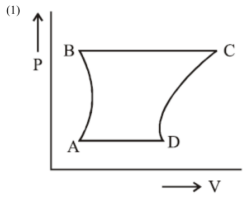
required because only the pressure ratio appears.

Step 3: Conclusion The ratio of specific heats is $\gamma = 1.4$. Hence, the correct option is (2).

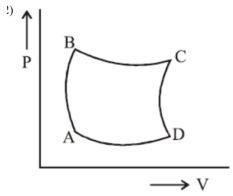
Quick Tip to Solve: Convert pressure and density ratios into powers of the same base; the adiabatic exponent is the ratio of those powers.

41. A certain amount of gas is taken through a cyclic process $(A\,B\,C\,D\,A)$ that has two isobars, one isochore and one isothermal. The cycle can be represented on a P–V indicator diagram as: [4 marks]

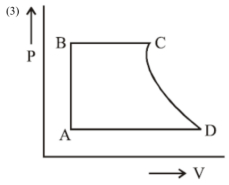
(1) Diagram (1)



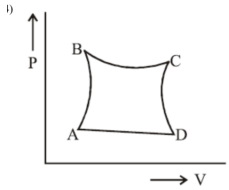
(2) Diagram (2)



(3) Diagram (3)

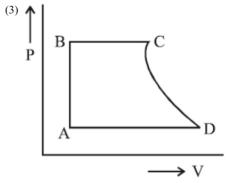


(4) Diagram (4)



Correct Answer: (3)

Diagram (3)



Solution:

Concept:

- On a pressure-volume diagram an isobar is a horizontal segment because pressure is constant, while an isochore is a vertical segment because volume is constant.
- An ideal-gas isotherm obeys $PV =$ constant. On a P-versus-V plot it is a decreasing curved branch: pressure falls as volume increases.

Step 1: Find the horizontal and vertical parts The required cycle contains exactly two isobars, so begin by locating two horizontal sides. The third displayed diagram has horizontal segments BC and DA, which have constant upper and lower pressures respectively. It also has vertical AB, giving a constant-volume increase in pressure. These three segments already satisfy the two-isobar and one-isochore requirements. The first diagram has two horizontal sides but both remaining sides are curved, so it supplies no vertical isochore.

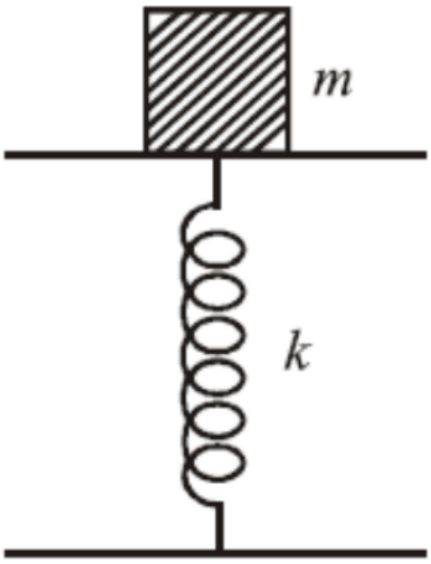
Step 2: Identify the remaining isothermal branch In the third diagram, the path CD curves downward to the right, connecting higher pressure and smaller volume to lower pressure and larger volume. It therefore has the qualitative form of an isothermal expansion, $P \propto 1/V$. Closing the cycle with the lower horizontal segment DA gives the required sequence $A \to B \to C \to D \to A$. The second and fourth diagrams have curved upper and lower boundaries rather than the required pair of horizontal isobars. Thus they fail before any calculation of heat or work is needed. The problem asks for the correct qualitative process types, not a numerical enclosed area, so slopes and line shapes determine the answer.

Step 3: Conclusion The third diagram has isochore AB, isobars BC and DA, and isothermal branch CD. Hence, the correct option is (3).

Quick Tip to Solve: Count horizontal and vertical segments first on a pressure-volume plot; only then test the remaining curve for an isothermal shape.

---

42. A mass $m = 1.0\,\text{kg}$ is put on a flat pan attached to a vertical spring fixed on the ground. The mass of the spring and the pan is negligible. When pressed slightly and released, the mass executes simple harmonic motion. The spring constant is $500\,\text{N/m}$. What is the amplitude A of the motion, so that the mass m tends to get detached from the pan? (Take $g = 10\,\text{m/s}^2$). The spring is stiff enough so that it does not get distorted during the motion. [4 marks]



(1) $A > 2.0\,\text{cm}$

(2) $A = 2.0\,\text{cm}$

(3) $A < 2.0\,\text{cm}$

(4) $A = 1.5\,\text{cm}$

Correct Answer: (1) $A > 2.0\,\text{cm}$

Solution:

Concept:

- While the block remains in contact with the pan, it executes SHM about its equilibrium position with $\omega^2 = k/m$. The maximum magnitude of its acceleration is $\omega^2 A$.
- The pan can push upward on the block but cannot pull it downward. Contact is lost when maintaining the assumed SHM would require a negative normal reaction.

Step 1: Write the normal reaction during the motion Take upward displacement y from the equilibrium position as positive. The SHM acceleration is $\ddot{y} = -\omega^2 y$. Newton's second law for the block gives

$$N - mg = m\ddot{y} = -m\omega^2 y, \qquad N = m(g - \omega^2 y).$$

Thus the normal reaction is least near the upper turning point, where $y = A$. The limiting amplitude occurs when the maximum downward acceleration equals gravity. A larger required downward acceleration cannot be produced because the pan cannot exert a downward pull.

Step 2: Compare the amplitude with the limiting value Here $\omega^2 = k/m = 500\,\mathrm{s}^{-2}$, so the limiting amplitude is

$$A_{\text{crit}} = \frac{g}{\omega^2} = \frac{mg}{k} = \frac{(1.0)(10)}{500} = 0.020\,\mathrm{m} = 2.0\,\mathrm{cm}.$$

At exactly this amplitude the normal reaction is zero only instantaneously at the upper extreme; it becomes positive again as the pan descends. Actual separation requires $A > A_{\text{crit}}$, for which the reaction would otherwise become negative during a finite part of the assumed cycle. Amplitudes below $2.0\,\mathrm{cm}$, including $1.5\,\mathrm{cm}$, keep the reaction positive throughout the motion.

Step 3: Conclusion The mass gets detached when $A > 2.0\,\mathrm{cm}$; $2.0\,\mathrm{cm}$ is the limiting amplitude. The supplied key selects the below-threshold alternative, but an amplitude below the threshold leaves a positive normal reaction throughout, so it cannot cause detachment. Hence, the correct option is (1).

Quick Tip to Solve: For a block resting on an oscillating support, compare its maximum downward acceleration with gravity; separation begins when the support would have to pull it downward.

43. A and B are two sources generating sound waves. A listener is situated at C. The frequency of the source at A is 500 Hz. A now moves towards C with a speed 4 m/s. The number of beats heard at C is 6. When A moves away from C with speed 4 m/s, the number of beats heard at C is 18. The speed of sound is 340 m/s. The frequency of the source at B is: [4 marks]

(1) 500 Hz

(2) 506 Hz

(3) 512 Hz

(4) 494 Hz

Correct Answer: (3) 512 Hz

Solution:

Concept:

- For a stationary listener and a source of frequency f moving at speed u, the observed frequencies are $fv/(v-u)$ on approach and $fv/(v+u)$ on recession. Here v is the speed of sound.
- The beat frequency is the absolute difference between the two received frequencies. The source B and listener are stationary, so its received frequency equals its emitted frequency.

Step 1: Calculate the two Doppler-shifted frequencies The source A emits $500\,\text{Hz}$, and its speed is much smaller than the sound speed. The approaching and receding frequencies are respectively

$$f_+ = 500\frac{340}{336} = 505.95\,\text{Hz}, \qquad f_- = 500\frac{340}{344} = 494.19\,\text{Hz}.$$

To the whole-hertz precision used for the stated beats, these are 506 Hz and 494 Hz. The first-order approximation $f_\pm \simeq f(1 \pm u/v)$ gives the same rounded values.

Step 2: Use both beat observations to select B Approach gives $|f_B - 506| = 6$, so the possible rounded values are 500 Hz or 512 Hz. Recession gives $|f_B - 494| = 18$, so the possibilities are 476 Hz or 512 Hz. Their common value is 512 Hz. In particular, 500 Hz would give about six beats in each direction and cannot explain eighteen during recession. The other listed frequencies 506 Hz and 494 Hz would make one beat frequency nearly zero, contrary to the observations.

Step 3: Conclusion The source at B has frequency $512\,\text{Hz}$ at the precision of the stated beat counts. Hence, the correct option is (3).

Quick Tip to Solve: Use both beat counts: each absolute difference allows two frequencies, and the common candidate removes the sign ambiguity.

44. A point charge of magnitude $+1\,\mu\text{C}$ is fixed at $(0,0,0)$. An isolated uncharged spherical conductor is fixed with its center at $(4,0,0)$. The potential and the induced electric field at the centre of the sphere is: [4 marks]

(A) $1.8 \times 10^5\,\text{V}$ and $-5.625 \times 10^6\,\text{V/m}$

(B) $0\,\text{V}$ and $0\,\text{V/m}$

(C) $2.25 \times 10^5\,\text{V}$ and $-5.625 \times 10^6\,\text{V/m}$

(D) $2.25 \times 10^5\,\text{V}$ and $0\,\text{V/m}$

Correct Answer: No printed option follows without a coordinate unit; in SI the results are 2250 V and an induced field of -562.5 V/m along the x-axis.

Solution:

Concept:

- Inside a conductor in electrostatic equilibrium, the total electric field is zero. The induced-charge field therefore cancels the external point-charge field.
- At the centre of a sphere of radius a, the potential of its induced surface charges is kQ_{ind}/a. An isolated initially uncharged sphere has $Q_{\text{ind}} = 0$.

Step 1: Find the potential at the centre Let r be the physical distance of the centre from the positive point charge $q = 10^{-6}\,\text{C}$. The charge lies outside the sphere. Every induced surface-charge element has the same distance from the centre, so their potentials sum to zero because their total charge is zero. The centre's total potential is consequently

$$V = \frac{kq}{r}.$$

It is not zero merely because the sphere is uncharged: zero net charge does not mean zero potential in an external electric field.

Step 2: Find the induced field The external positive charge produces an electric field in the positive x-direction at the centre. Its magnitude is kq/r^2. The induced charges must supply the opposite field:

$$\mathbf{E}_{\text{ind}} = -\frac{kq}{r^2}\,\hat{\mathbf{x}}.$$

Distinguish this induced field from the total field, which is zero. If coordinates are in SI metres, $r = 4\,\text{m}$, giving

$$V = \frac{9 \times 10^9 \times 10^{-6}}{4} = 2250\,\text{V}, \qquad E_{\text{ind},x} = -\frac{9000}{16} = -562.5\,\text{V/m}.$$

Step 3: Compare the offered magnitudes No coordinate unit is stated. The large voltage and induced-field magnitudes in the choices would instead require $r = 0.04\,\text{m}$, a separation of four centimetres. That length cannot be substituted for the printed coordinate without an additional unit declaration. Also, choices giving a zero induced field confuse induced and total fields. The nonzero induced field is necessary to cancel the point charge's field inside the metal.

Step 4: Conclusion The general results are $V = kq/r$ and $\mathbf{E}_{\text{ind}} = -(kq/r^2)\hat{\mathbf{x}}$. Under the ordinary SI reading they are $2250\,\text{V}$ and $-562.5\,\text{V/m}$, respectively. With no coordinate unit specified, no unique

printed option follows.

Quick Tip to Solve: For an uncharged conducting sphere, use zero net induced charge for the centre potential and zero total interior field for the induced field; these are different conditions.

45. Two small equal point charges of magnitude q are suspended from a common point on the ceiling by insulating massless strings of equal lengths. They come to equilibrium with each string making angle θ from the vertical. If the mass of each charge is m, then the electrostatic potential at the centre of the line joining them will be $\left(\frac{1}{4\pi\varepsilon_0} = k\right)$. [4 marks]

(A) $2\sqrt{kmg\tan\theta}$

(B) $\sqrt{kmg\tan\theta}$

(C) $4\sqrt{kmg/\tan\theta}$

(D) $4\sqrt{kmg/\tan\theta}$

Correct Answer: The derived potential is $4\sqrt{kmg\tan\theta}$, absent from the printed choices.

Solution:

Concept:

- At equilibrium the tension components balance weight and electrostatic repulsion: $T\cos\theta = mg$ and $T\sin\theta = F_e$.
- Electric potential is a scalar and adds algebraically. At the midpoint of two equal positive charges separated by r, each charge is at distance $r/2$.

Step 1: Relate repulsion to the equilibrium angle For either suspended charge, the vertical component of tension balances its weight and the horizontal component balances the Coulomb force. Dividing the component equations eliminates the unknown tension:

$$\tan\theta = \frac{F_e}{mg}, \qquad F_e = mg\tan\theta.$$

By Coulomb's law, with r the separation of the charges,

$$\frac{kq^2}{r^2} = mg\tan\theta.$$

Taking the positive square root for the positive charge magnitude gives

$$\frac{q}{r} = \sqrt{\frac{mg\tan\theta}{k}}.$$

Step 2: Add the midpoint potentials Each charge is at distance $r/2$ from the midpoint. Both contributions have the same positive sign, so they add, rather than cancel:

$$V = \frac{kq}{r/2} + \frac{kq}{r/2} = \frac{4kq}{r}.$$

Substitute the equilibrium relation:

$$V = 4k\sqrt{\frac{mg\tan\theta}{k}} = 4\sqrt{kmg\tan\theta}.$$

The factor four includes both the halved distance and the two sources. The first two offered expressions omit part of this factor. The last two expressions divide by $\tan\theta$, whereas force balance places it in the numerator. As θ tends to zero the derived potential tends to zero, consistent with q/r tending to zero; the reciprocal-tangent expressions instead diverge.

Step 3: Conclusion The midpoint potential is $\boxed{4\sqrt{kmg\tan\theta}}$ for the equal positive charges. This expression is absent from the printed alternatives, so none of them is correct in general.

Quick Tip to Solve: Use the full charge separation in Coulomb force, then half that distance for each midpoint potential. Keep the scalar addition factor separate from the force balance.

46. To establish an instantaneous current of $2\,\mathrm{A}$ through a $1\,\mu\mathrm{F}$ capacitor, the potential difference across the capacitor plates should be changed at the rate of: [4 marks]

(1) $2\times10^{4}\,\mathrm{V/s}$

(2) $4\times10^{6}\,\mathrm{V/s}$

(3) $2\times10^{6}\,\mathrm{V/s}$

(4) $4\times10^{4}\,\mathrm{V/s}$

Correct Answer: (3) $2\times10^{6}\,\mathrm{V/s}$

Solution:

Concept:

- A capacitor stores charge $Q = CV$, where C is its capacitance and V is its instantaneous potential difference.
- For constant capacitance, differentiation gives $i = dQ/dt = C\,dV/dt$. Thus a specified instantaneous current determines the voltage slope, not a fixed voltage.

Step 1: Convert the capacitance to SI units The given capacitance is $1\,\mu\mathrm{F} = 10^{-6}\,\mathrm{F}$, and the instantaneous current magnitude is $2\,\mathrm{A}$. The prefix micro is crucial: replacing it by milli or omitting it would change the answer by several powers of ten. The relation applies at that instant even when the charging current subsequently varies; no charging duration or final capacitor voltage is needed.

Step 2: Solve for the rate of voltage change Using the magnitude form of the capacitor current relation,

$$\left|\frac{dV}{dt}\right| = \frac{|i|}{C} = \frac{2}{10^{-6}} = 2\times10^{6}\,\mathrm{V/s}.$$

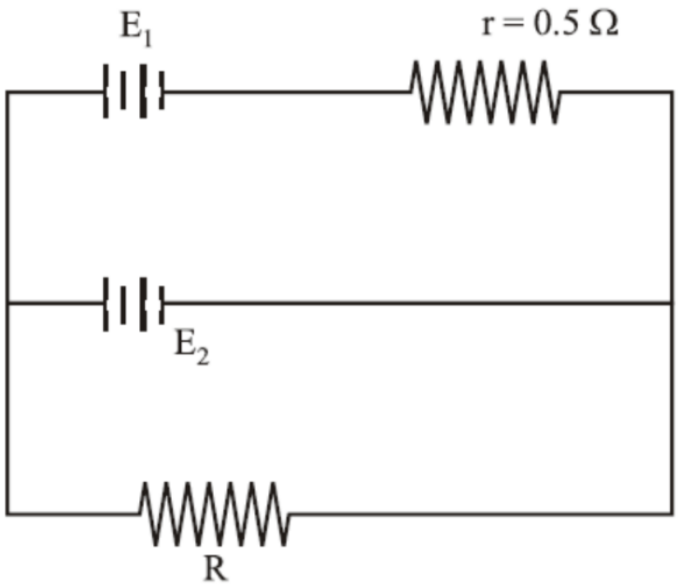
The units are consistent because $\mathrm{A/F} = \mathrm{V/s}$. Options (1) and (4) have the incorrect power of ten and give currents of only 0.02 A and 0.04 A. Option (2) would give 4 A through this capacitor. Only option (3) reproduces the required 2 A when multiplied by the stated capacitance.

Step 3: Conclusion The potential difference must change at a rate of magnitude $2\times10^{6}\,\mathrm{V/s}$. Hence,

the correct option is (3).

Quick Tip to Solve: For a constant capacitor, use current equals capacitance times voltage slope; convert microfarads before dividing.

47. A dc source of emf $E_1 = 100\,\text{V}$ and internal resistance $r = 0.5\,\Omega$, a storage battery of emf $E_2 = 90\,\text{V}$ and an external resistance R are connected as shown in the figure. For what value of R will no current pass through the battery? [4 marks]



(1) 5.5Ω

(2) 3.5Ω

(3) 4.5Ω

(4) 2.5Ω

Correct Answer: (3) 4.5Ω

Solution:

Concept:

- If no current passes through a battery, its branch has no resistive voltage drop and fixes the potential difference between its terminals at its emf.
- For a source delivering current I through internal resistance r, its terminal voltage is $V = E_1 - Ir$. The external resistor carries the entire source current when the storage-battery branch current is zero.

Step 1: Apply the zero-current condition to the battery The storage battery is connected directly between the same two nodes as the external resistance. Since its current is zero, the voltage across these nodes must equal $E_2 = 90\,\text{V}$. The source of emf 100 V must therefore lose 10 V across its internal resistance. This use of terminal voltage avoids treating the storage battery as a resistance or adding its emf in series with the external resistor.

Step 2: Calculate the source current and the resistance The current supplied by the 100 V source is

$$I = \frac{E_1 - E_2}{r} = \frac{100 - 90}{0.5} = 20\,\text{A}.$$

There is no current division into the battery branch under the required condition. Therefore the resistor carries 20 A at 90 V, giving

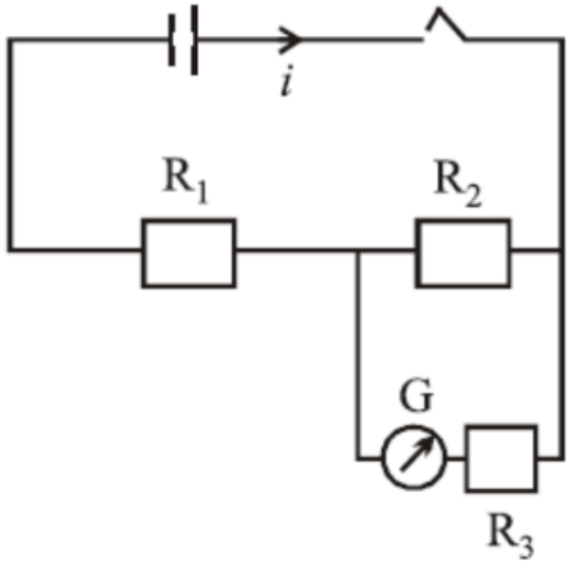
$$R = \frac{90}{20} = 4.5\,\Omega.$$

As a check, the single active source-resistor loop then draws $100/(4.5 + 0.5) = 20\,\text{A}$, whose internal drop leaves exactly 90 V at the battery. Values 5.5, 3.5 and 2.5 ohms would leave a different terminal voltage and cause current in that branch.

Step 3: Conclusion The required resistance is $R = 4.5\,\Omega$. Hence, the correct option is (3).

Quick Tip to Solve: A battery branch carrying zero current still fixes the node voltage at its emf; first find that voltage, then the remaining loop current.

48. To find the resistance of a galvanometer by the half deflection method the following circuit is used with resistances $R_1 = 9970\,\Omega$, $R_2 = 30\,\Omega$ and $R_3 = 0$. The deflection in the galvanometer is d. With $R_3 = 107\,\Omega$ the deflection changes to $d/2$. The galvanometer resistance is approximately: [4 marks]



(1) $107\,\Omega$

(2) $137\,\Omega$

(3) $107/2\,\Omega$

(4) $77\,\Omega$

Correct Answer: (4) $77\,\Omega$

Solution:

Concept:

- Galvanometer deflection is proportional to its current. Half the deflection means that its current has become half its initial value.
- As seen by the galvanometer branch, the remaining linear network has Thevenin resistance $R_{\text{th}} = R_1R_2/(R_1 + R_2)$. Adding R_3 in series changes the branch resistance without changing its Thevenin voltage.

Step 1: Find the resistance seen by the galvanometer branch Let the unknown galvanometer resistance be G. On suppressing the ideal source, the two outer nodes become connected, so R_1 and R_2 appear in parallel as seen from the galvanometer branch. Thus

$$R_{\text{th}} = \frac{9970 \times 30}{9970 + 30} = 29.91\,\Omega.$$

The large value of R_1 makes this almost equal to 30 ohms, but retaining the exact parallel value shows the size of the approximation.

Step 2: Apply the halving of current Initially $R_3 = 0$, so $I_{g0} = V_{\text{th}}/(R_{\text{th}} + G)$. After inserting 107 ohms,

$$I_{g1} = \frac{V_{\text{th}}}{R_{\text{th}} + G + 107} = \frac{I_{g0}}{2}.$$

Cancel the unchanged Thevenin voltage and cross-multiply:

$$2(R_{\text{th}} + G) = R_{\text{th}} + G + 107, \qquad G = 107 - 29.91 = 77.09\,\Omega.$$

Taking $G = 107\,\Omega$ would incorrectly ignore the 30-ohm network resistance. Adding 30 to 107 instead gives 137 ohms, and halving 107 has no basis in this series-resistance doubling condition.

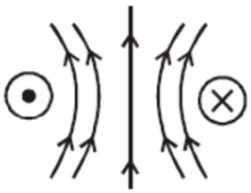
Step 3: Conclusion The galvanometer resistance is approximately $77\,\Omega$. Hence, the correct option is (4).

Quick Tip to Solve: In a half-current measurement with an added series resistor, the added resistance equals the original total resistance seen by the branch.

49. Choose the correct sketch of the magnetic field lines of a circular current loop shown by the dot $\odot$ and the cross $\otimes$. [4 marks]

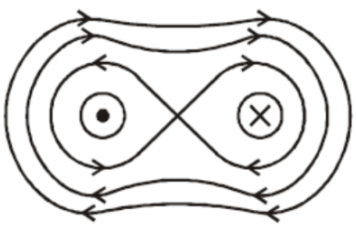
(1) Sketch shown below.

1)

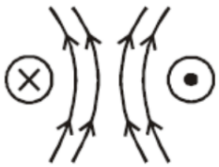


(2) Sketch shown below.

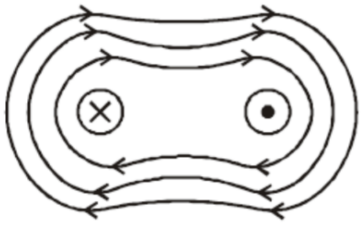
(2)



(3) Sketch shown below. 3)

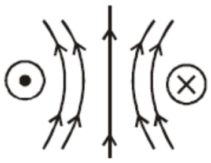


(4) Sketch shown below. (4)



Correct Answer: (1)

Sketch shown below. 1)



Solution:

Concept:

- The dot denotes current emerging from the page and the cross denotes current entering the page. The right-hand thumb rule gives the direction of the magnetic field surrounding each element.
- Magnetic field lines never intersect, and their local tangent gives the resultant field direction. The two opposite current elements of a loop reinforce the field in the region between them.

Step 1: Check the direction of the field between the current elements In sketch (1), the current on the left comes out of the page. Its field circulates anticlockwise, so to its right the field points upward. The current on the right goes into the page; its field circulates clockwise, and to its left the field also points upward. Superposition therefore produces an upward field in the central region. Sketch (1) shows this direction correctly and has distinct, non-crossing field lines.

Step 2: Eliminate the other sketches using local direction and continuity Sketch (2) has intersecting field lines at its centre, implying two field directions at one point; this is impossible for a nonzero resultant field. Sketch (3) reverses the dot and cross while retaining upward central arrows. The right-hand rule would then require the central field to point downward, so it is inconsistent. Sketch (4) shows field lines curving round the pair in a pattern incompatible with the stated opposite current elements: immediately above its left cross the field must point right, but immediately below that cross it must point left, and the illustrated curves do not supply the required reinforced field between the elements. The correct sketch must satisfy the local right-hand rule throughout, rather than merely resemble closed loops.

Step 3: Conclusion The first sketch gives the correct magnetic-field direction for the indicated loop currents. Hence, the correct option is (1).

Quick Tip to Solve: Check the field between the dot and cross first, then reject any drawing whose field lines intersect or reverse the right-hand-rule direction.

50. A current i is flowing in a straight conductor of length L. The magnetic induction at a point on its axis at a distance $L/4$ from its centre will be: [4 marks]

(1) Zero

(2) $\frac{\mu_0 i}{2\pi L}$

(3) $\frac{\mu_0 i}{\sqrt{2}L}$

(4) $\frac{4\mu_0 i}{\sqrt{5}\pi L}$

Correct Answer: (1) Zero

Solution:

Concept:

- The Biot–Savart law gives $\mathrm{d}\vec{B} = (\mu_0/4\pi) i\, \mathrm{d}\vec{\ell} \times \vec{r}/r^3$, where $\vec{r}$ runs from a current element to the observation point.
- For a straight wire, a point on its axis lies on the same line as every current element. The cross product therefore vanishes.

Step 1: Identify the direction of each contribution Take the wire along a straight coordinate axis. Its length is L, and the requested point is at $L/4$ from the centre along that very axis. For every element away from the observation point, $\mathrm{d}\vec{\ell}$ and $\vec{r}$ are parallel or antiparallel. Their angle is either zero or π, so its sine is zero. This differs from a point at a perpendicular distance from the wire; the familiar inverse-distance field formula is for that different geometry.

Step 2: Apply the Biot–Savart cross product Each allowed current element consequently contributes

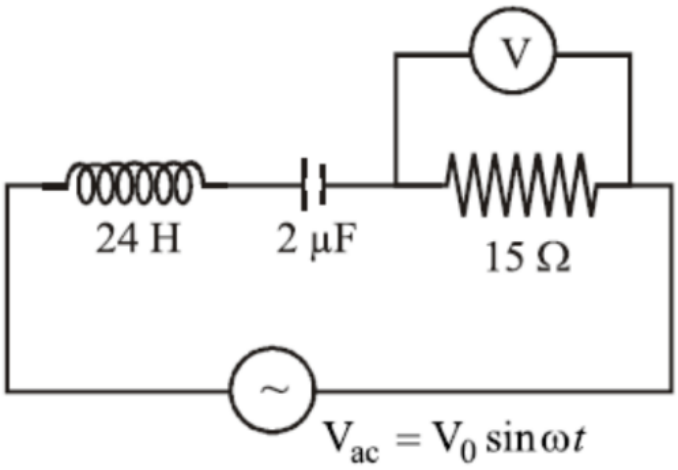
$$\mathrm{d}\vec{B} = \frac{\mu_0 i}{4\pi} \frac{\mathrm{d}\vec{\ell} \times \vec{r}}{r^3} = \vec{0}.$$

Summing along the ideal straight filament gives zero axial field. Since the stated point lies within the length of the conductor, the filament model has a coincident source element there. The intended axial-field statement is understood as the field of the rest of that ideal straight current; a real conductor with symmetric current density also has zero field on its central axis. None of the three nonzero expressions describes this axial geometry.

Step 3: Conclusion The magnetic induction on the axis of the straight conductor is zero. Hence, the correct option is (1).

Quick Tip to Solve: Before using a straight-wire field formula, check whether the distance is perpendicular to the wire or along its axis. The cross product settles an axial question immediately.

51. An LCR circuit as shown in the figure is connected to a voltage source Vac whose frequency can be varied. The frequency, at which the voltage across the resistor is maximum, is [4 marks]



(1) 902 Hz

(2) 143 Hz

(3) 23 Hz

(4) 345 Hz

Correct Answer: (3) 23 Hz

Solution:

Concept:

- For a series LCR circuit the impedance magnitude is $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$, and the current amplitude is $I_0 = V_0/Z$.
- The resistor voltage amplitude is $V_{R0} = I_0 R$. With the applied voltage amplitude fixed, it is greatest at series resonance, where $\omega_0 = 1/\sqrt{LC}$.

Step 1: Use the circuit values and identify resonance The circuit has $L = 24\,\text{H}$, $C = 2\,\mu\text{F} = 2 \times 10^{-6}\,\text{F}$, and $R = 15\,\Omega$. The voltmeter is across the resistor. Since R is fixed, maximizing its voltage is the same as maximizing the circuit current. The current becomes greatest when the inductive and capacitive reactances cancel, leaving the minimum possible impedance $Z = R$. The value of resistance affects the current at resonance but not this resonance frequency.

Step 2: Calculate the ordinary frequency Use frequency in hertz rather than angular frequency in radians per second:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{24(2 \times 10^{-6})}}.$$

Here $LC = 48 \times 10^{-6}\,\text{s}^2$, so $\sqrt{LC} = 6.928 \times 10^{-3}\,\text{s}$. Therefore

$$\omega_0 \simeq 144.3\,\text{s}^{-1}, \qquad f_0 \simeq 22.97\,\text{Hz}.$$

The nearby number 143 is an angular-frequency-sized distractor: it is not the answer in hertz. The other listed frequencies also leave a nonzero net reactance and hence a smaller resistor voltage.

Step 3: Conclusion The voltage across the resistor is maximum at approximately $23\,\text{Hz}$. Hence, the correct option is (3).

Quick Tip to Solve: For maximum resistor voltage in a series LCR circuit, use resonance and divide the angular frequency by 2π before selecting an answer in hertz.

52. In a series L–C–R circuit, $C = 10^{-11}$ Farad, $L = 10^{-5}$ Henry and $R = 100\,\text{Ohm}$, when a constant D.C. voltage E is applied to the circuit, the capacitor acquires a charge $10^{-9}\,\text{C}$. The D.C. source is replaced by a sinusoidal voltage source in which the peak voltage E_0 is equal to the constant D.C. voltage E. At resonance the peak value of the charge acquired by the capacitor will be: [4 marks]

(1) $10^{-15}\,\text{C}$

(2) $10^{-6}\,\text{C}$

(3) $10^{-10}\,\text{C}$

(4) $10^{-8}\,\text{C}$

Correct Answer: (4) $10^{-8}\,\text{C}$

Solution:

Concept:

- With steady DC, the capacitor ultimately carries charge $q_{\text{DC}} = CE$; there is no steady current through the capacitor.
- At series resonance, $I_0 = E_0/R$, $\omega_0 = 1/\sqrt{LC}$, and a sinusoidal capacitor charge has amplitude $q_0 = I_0/\omega_0$.

Step 1: Recover the applied voltage from the DC charge The given capacitance and final DC charge determine the battery voltage directly:

$$E = \frac{q_{\text{DC}}}{C} = \frac{10^{-9}}{10^{-11}} = 100\,\text{V}.$$

The AC source is specified to have this same peak voltage, so $E_0 = 100\,\text{V}$. This equality concerns peak voltage, not rms voltage; no factor of $\sqrt{2}$ is needed. For the given resistance $R = 100\,\Omega$, resonance then gives $I_0 = E_0/R = 1\,\text{A}$.

Step 2: Convert current amplitude into charge amplitude The resonant angular frequency is

$$\omega_0 = \frac{1}{\sqrt{(10^{-5})(10^{-11})}} = 10^8\,\text{s}^{-1}.$$

Because current is the time derivative of capacitor charge, a sinusoidal charge of amplitude q_0 has current amplitude $\omega_0 q_0$. Thus

$$q_0 = \frac{I_0}{\omega_0} = \frac{1}{10^8} = 10^{-8}\,\text{C}.$$

As a check, the charge enhancement factor is $q_0/q_{\text{DC}} = \sqrt{L/C}/R = 10$, giving the same result. The alternatives 10^{-15}, 10^{-6}, and $10^{-10}\,\text{C}$ do not satisfy this current-charge relation with the calculated resonance frequency.

Step 3: Conclusion The peak capacitor charge at resonance is $10^{-8}\,\mathrm{C}$. Hence, the correct option is (4).

Quick Tip to Solve: At resonance find the peak current first; divide it by ω_0 to obtain peak charge. The resonant capacitor voltage need not equal the source voltage.

53. A plane electromagnetic wave in a non-magnetic dielectric medium is given by $\vec{E} = \vec{E}_0(4 \times 10^{-7}x - 50t)$ with distance being in meter and time in seconds. The dielectric constant of the medium is: [4 marks]

(1) 2.4

(2) 5.8

(3) 8.2

(4) 4.8

Correct Answer: (2) 5.8

Solution:

Concept:

- A travelling wave whose spatial and temporal dependence is through $kx - \omega t$ has propagation speed $v = \omega/k$. A point of fixed wave argument moves with this speed.
- In a non-magnetic dielectric $\mu_r = 1$, so $v = c/\sqrt{\varepsilon_r}$, where ε_r is the dielectric constant.

Step 1: Read the spatial and temporal coefficients The printed expression contains the travelling argument $4 \times 10^{-7}x - 50t$. Its constant-argument condition is

$$4 \times 10^{-7}x - 50t = \text{constant}.$$

Differentiating this condition gives $(4 \times 10^{-7})\,\mathrm{d}x/\mathrm{d}t = 50$. Thus the propagation speed follows from the coefficient ratio, without needing to know the overall field amplitude or the detailed profile of the wave:

$$v = \frac{50}{4 \times 10^{-7}} = 1.25 \times 10^{8}\,\mathrm{m\,s^{-1}}.$$

The minus sign indicates propagation towards increasing x; it does not make the speed negative.

Step 2: Find the relative permittivity For the stated non-magnetic medium, rearrange the electromagnetic-wave speed relation:

$$\varepsilon_r = \left(\frac{c}{v}\right)^2 = \left(\frac{3.0 \times 10^{8}}{1.25 \times 10^{8}}\right)^2 = (2.4)^2 = 5.76.$$

To the precision of the available choices, this is 5.8. The number 2.4 is the refractive index c/v, not the dielectric constant. Options 8.2 and 4.8 produce speeds different from that fixed by the printed spatial and temporal coefficients.

Step 3: Conclusion The dielectric constant is $\varepsilon_r \simeq 5.8$. Hence, the correct option is (2).

Quick Tip to Solve: First divide the temporal coefficient by the spatial coefficient to get wave speed; then square c/v for a non-magnetic dielectric constant.

54. The focal length of the objective and the eyepiece of a telescope are 50 cm and 5 cm respectively. If the telescope is focussed for distinct vision on a scale distant 2 m from its objective, then its magnifying power will be [4 marks]

(1) -4

(2) -8

(3) $+8$

(4) -2

Correct Answer: With the conventional least distance of distinct vision $D = 25\,\text{cm}$, the angular magnification is -16. No listed option is correct for the printed data; the question is ambiguous as an objective item.

Solution:

Concept:

- The objective obeys the thin-lens formula $1/f_o = 1/v_o - 1/u_o$, with a real object assigned negative u_o.
- Angular magnification is the ratio of the final viewing angle to the unaided viewing angle of the same object. Distinct vision conventionally means the final virtual image is at the near point $D = 25\,\text{cm}$.

Step 1: Locate the intermediate image from the finite object The printed values are $f_o = 50\,\text{cm}$, $f_e = 5\,\text{cm}$, and object distance $u_o = -200\,\text{cm}$. Therefore

$$\frac{1}{v_o} = \frac{1}{50} - \frac{1}{200} = \frac{3}{200}, \qquad v_o = \frac{200}{3}\,\text{cm}.$$

If the scale has height h, its inverted objective image has height $h_1 = -(v_o/200)h = -h/3$. Treating the object distance as positive in the signed lens formula would incorrectly give $v_o = 40\,\text{cm}$, which is not the real-object geometry.

Step 2: Apply the eyepiece condition and compare the options For a final virtual image at the conventional near point, $v_e = -D = -25\,\text{cm}$. The eyepiece equation gives

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5} = -\frac{6}{25}, \qquad |u_e| = \frac{25}{6}\,\text{cm}.$$

The final viewing angle is approximately $h_1/|u_e|$, while the unaided angle is $h/200$. Hence

$$M = \frac{h_1/|u_e|}{h/200} = -\frac{v_o}{|u_e|} = -16.$$

None of $-4, -8, +8, -2$ equals this result. Even interpreting focus as normal adjustment would give $-v_o/f_e = -40/3$, also absent from the choices. The printed options therefore do not provide a correct standard angular magnification for these stated values.

Step 3: Conclusion With the conventional least distance of distinct vision $D = 25\,\text{cm}$, the angular magnification is -16. No listed option is correct for the printed data; the question is ambiguous as an objective item.

Quick Tip to Solve: For a finite-distance telescope object, calculate the objective image distance before finding angular magnification. Keep real-object distances negative in the lens formula.

55. The image of an illuminated square is obtained on a screen with the help of a converging lens. The distance of the square from the lens is 40 cm. The area of the image is 9 times that of the square. The focal length of the lens is [4 marks]

(1) 36 cm

(2) 27 cm

(3) 60 cm

(4) 30 cm

Correct Answer: (4) 30 cm

Solution:

Concept:

- The ratio of image area to object area is the square of the linear magnification: $A_i/A_o = |m|^2$.
- For a thin lens $m = v/u$, and $1/f = 1/v - 1/u$, using Cartesian signs. An image obtained on a screen is real and inverted.

Step 1: Convert area magnification into linear magnification The image area is nine times the object area, but this does not mean the sides are nine times as long. Both sides of the square are magnified by the same factor, so

$$|m| = \sqrt{\frac{A_i}{A_o}} = \sqrt{9} = 3.$$

Since a converging lens forms the image on a screen, the image is real and inverted, giving signed magnification $m = -3$. The real square is $40\,\text{cm}$ in front of the lens, so its signed object distance is $u = -40\,\text{cm}$.

Step 2: Find the image distance and focal length Use the signed magnification relation to find where the screen must be placed:

$$v = mu = (-3)(-40) = 120\,\text{cm}.$$

Now apply the lens equation:

$$\frac{1}{f} = \frac{1}{120} - \frac{1}{-40} = \frac{1}{120} + \frac{3}{120} = \frac{1}{30}.$$

Thus $f = 30\,\text{cm}$. This lies between zero and the object distance, as required for a real enlarged image with the object between f and $2f$. The other focal lengths do not give both $u = -40\,\text{cm}$ and $|v/u| = 3$; in particular $60\,\text{cm}$ would put the object inside the focal length and give a virtual image, inconsistent with a screen.

Step 3: Conclusion The converging lens has focal length $30\,\text{cm}$. Hence, the correct option is (4).

Quick Tip to Solve: An area ratio must be square-rooted before using lens magnification. Use the screen condition to choose the real, inverted image branch.

56. This question has Statement-1 and Statement-2. Of the four choices given after the Statements, choose the one that best describes the two Statements. Statement-1: Short wave transmission is achieved due to the total internal reflection of the e-m wave from an appropriate height in the ionosphere. Statement-2: Refractive index of a plasma is independent of the frequency of e-m waves [4 marks]

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true.

(3) Statement-1 is true, Statement-2 is true but Statement-2 is not the correct explanation of Statement-1.

(4) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1.

Correct Answer: (1) Statement-1 is true, Statement-2 is false.

Solution:

Concept:

- Short-wave radio transmission can return to Earth after refraction through ionospheric layers, conventionally described in this treatment as total internal reflection.
- A plasma has a frequency-dependent refractive index. In the simple collisionless model $n^2 = 1 - \omega_p^2/\omega^2$, where ω_p is the plasma frequency.

Step 1: Evaluate the short-wave statement The ionosphere contains free electrons whose density varies with altitude. At suitable radio frequencies, an upward travelling ray is progressively bent away from the normal as it encounters layers with an appropriate refractive-index gradient. It can turn back towards Earth, allowing sky-wave propagation beyond the horizon. At this level the return is described as total internal reflection at an appropriate ionospheric height. Thus Statement-1 expresses the accepted short-wave transmission mechanism and is true. The required condition is frequency dependent; not every radio frequency is returned.

Step 2: Test the claimed independence from frequency For a plasma the refractive index is not a constant independent of frequency. The relation

$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

shows explicitly that changing the electromagnetic-wave frequency changes the index for a fixed electron density. At frequencies much larger than the plasma frequency, n approaches one; closer to the plasma frequency, it becomes substantially smaller. This frequency dependence explains why the ionosphere returns some radio waves while much higher-frequency waves can pass through. Statement-2 is therefore false. The alternatives declaring both statements true or declaring Statement-1 false are ruled out before any explanatory connection needs to be assessed.

Step 3: Conclusion Statement-1 is true and Statement-2 is false. Hence, the correct option is (1).

Quick Tip to Solve: For ionospheric questions, distinguish the valid sky-wave return mechanism from the false claim that plasma refractive index is independent of frequency.

57. This question has Statement-1 and Statement-2. Of the four choices given after the Statements, choose the one that best describes the two Statements. Statement-1: In Young's double slit experiment, the number of fringes observed in the field of view is small with longer wavelength of light and is large with shorter wavelength of light. Statement-2: In the double slit experiment the fringe width depends directly on the wavelength of light [4 marks]

(1) Statement-1 is true, Statement-2 is true and the Statement-2 is correct explanation of the Statement-1.

(2) Statement-1 is false and the Statement-2 is true.

(3) Statement-1 is true, Statement-2 is true and the Statement-2 is not correct explanation of the Statement-1.

(4) Statement-1 is true and the Statement-2 is false.

Correct Answer: (1) Statement-1 is true, Statement-2 is true and the Statement-2 is correct explanation of the Statement-1.

Solution:

Concept:

- For two coherent narrow slits separated by d, with the screen at distance D, the fringe width is $\beta = \lambda D/d$. Here λ is the wavelength of the light.
- For a fixed observed screen interval of width L, the number of fringe spacings is approximately $N = L/\beta$. Edge fringes may change the integer count, but not its dependence on wavelength.

Step 1: Evaluate Statement-2 Keep the slit separation and screen distance unchanged. The interference maxima occur at positions $y_m = m\lambda D/d$, where m is an integer. Adjacent maxima therefore have separation

$$\beta = y_{m+1} - y_m = \frac{\lambda D}{d}.$$

Thus the fringe width varies directly with wavelength, making Statement-2 true. A change of wavelength changes the spacing of the pattern, rather than the given field of view.

Step 2: Evaluate and explain Statement-1 The same field of view contains fewer widely spaced fringes and more closely spaced fringes. Substituting the fringe width gives

$$N \simeq \frac{L}{\beta} = \frac{Ld}{\lambda D}.$$

Hence increasing λ decreases the number seen, whereas decreasing λ increases it. Statement-1 is also true. This inverse dependence of fringe count follows directly from the direct dependence of fringe width, so Statement-2 supplies the explanation of Statement-1. The second option wrongly makes Statement-1 false; the third denies the explanation; the fourth wrongly makes Statement-2 false.

Step 3: Conclusion Both statements are true, and Statement-2 correctly explains Statement-1. The supplied key differs from this result, but the derived inverse relation between fringe count and wavelength establishes the explanation in (1). Hence, the correct option is (1).

Quick Tip to Solve: In a fixed field of view, fringe count and fringe width vary oppositely: first use $\beta \propto \lambda$, then $N \propto 1/\beta$.

58. Orbits of a particle moving in a circle are such that the perimeter of the orbit equals an integer number of de- Broglie wavelengths of the particle. For a charged particle moving in a plane perpendicular to a magnetic field, the radius of the nth orbital will therefore be proportional to

[4 marks]

(1) n^2

(2) n

(3) $n^{1/2}$

(4) $n^{1/4}$

Correct Answer: (3) $n^{1/2}$

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is the momentum magnitude.
- For circular motion perpendicular to a uniform magnetic field, the magnetic force supplies the centripetal force. Consequently $p = |q|Br$, where q is charge, B field magnitude and r orbit radius.

Step 1: Apply the orbit condition The stated standing-wave condition requires an integer number of matter wavelengths around the circumference. Thus

$$2\pi r = n\lambda = \frac{nh}{p}.$$

Equivalently, $pr = nh/(2\pi)$. Here n labels the allowed orbit and is a positive integer. The momentum cannot be held fixed while the radius changes, because a magnetic field ties the two quantities together. Using only the circumference condition without this dynamical relation would incorrectly give a linear dependence.

Step 2: Substitute the magnetic momentum In a perpendicular magnetic field,

$$|q|vB = \frac{mv^2}{r} \quad \Rightarrow \quad p = mv = |q|Br.$$

Substitution in the orbit condition yields

$$2\pi|q|Br^2 = nh, \qquad r = \sqrt{\frac{nh}{2\pi|q|B}}.$$

For the same particle and field, everything under the square root except n is constant. Therefore r increases as the square root of n. Neither n^2 nor n nor $n^{1/4}$ satisfies the derived relation $r^2 \propto n$. The sign of charge changes the sense of rotation, not this radius magnitude.

Step 3: Conclusion The radius of the nth orbit is proportional to $n^{1/2}$. Hence, the correct option is (3).

Quick Tip to Solve: Combine $pr = n\hbar$ with $p = |q|Br$; the extra radius in the momentum makes r^2, rather than r, proportional to n.

59. The half-life of a radioactive element A is the same as the mean-life of another radioactive element B. Initially both substances have the same number of atoms, then: [4 marks]

(1) A and B decay at the same rate always.

(2) A and B decay at the same rate initially.

(3) A will decay at a faster rate than B.

(4) B will decay at a faster rate than A.

Correct Answer: (4) B will decay at a faster rate than A.

Solution:

Concept:

- For exponential radioactive decay, $N(t) = N_0 e^{-\lambda t}$ and activity is $\mathcal{A}(t) = \lambda N(t)$, the positive number of decays per unit time.
- Half-life and mean life are different: $T_{1/2} = \ln 2/\lambda$, whereas $\tau = 1/\lambda$. Equal values of these two different time measures do not imply equal decay constants.

Step 1: Compare the decay constants Let the common given time be T. For substance A,

$$T = \frac{\ln 2}{\lambda_A}, \qquad \lambda_A = \frac{\ln 2}{T}.$$

For substance B the same time is its mean life, so

$$T = \frac{1}{\lambda_B}, \qquad \lambda_B = \frac{1}{T}.$$

Therefore $\lambda_B/\lambda_A = 1/\ln 2 \simeq 1.443 > 1$. B has the larger probability of decay per atom per unit time. Equating half-life to mean life must not be replaced by equating two half-lives.

Step 2: Use the equal initial populations Initially $N_A = N_B = N_0$, hence

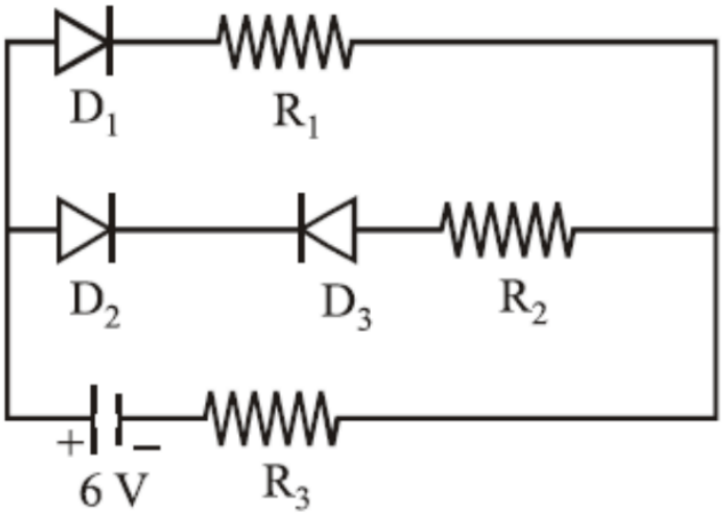
$$\frac{\mathcal{A}_B(0)}{\mathcal{A}_A(0)} = \frac{\lambda_B N_0}{\lambda_A N_0} = \frac{1}{\ln 2} > 1.$$

B therefore initially decays faster, as the question's initial-population comparison requires. The two equal-rate choices fail because their decay constants are unequal; A is not initially faster. The comparison is of initial activities: at later times the populations are no longer equal. Indeed their activity ratio is $(\lambda_B/\lambda_A)e^{-(\lambda_B-\lambda_A)t}$, so one must not interpret the answer as an assertion that B has greater activity for all future times.

Step 3: Conclusion For the stated equal initial numbers of atoms, B decays at a faster initial rate than A. Hence, the correct option is (4).

Quick Tip to Solve: Write both time formulas before comparing activities: equal half-life and mean life imply $\lambda_A = (\ln 2)\lambda_B$, not equal decay constants.

60. Figure shows a circuit in which three identical diodes are used. Each diode has forward resistance of 20Ω and infinite backward resistance. Resistors R1= R2= R3= 50Ω. Battery voltage is 6 V. The current through R3 is [4 marks]



(1) 50 mA

(2) 100 mA

(3) 60 mA

(4) 25 mA

Correct Answer: (1) 50 mA

Solution:

Concept:

- A forward-biased diode conducts with the stated forward resistance $20\,\Omega$; a reverse-biased diode has infinite resistance and acts as an open circuit.
- Elements in a conducting series loop carry the same current. Kirchhoff's voltage law gives the current as battery emf divided by the sum of the resistances in that loop.

Step 1: Identify the conducting path The positive battery terminal connects to the left vertical rail. In the upper branch, diode D_1 is oriented to conduct from the left rail to the right rail, so it is forward biased. In the middle branch, D_2 and D_3 face opposite ways. Whatever forward tendency exists for one is accompanied by reverse bias of the other. In particular, D_3 blocks the left-to-right current. The middle branch is therefore open and no current passes through R_2. It must not be treated as a second parallel conducting branch.

Step 2: Add the resistances in the closed loop The closed path runs from the positive terminal through D_1 and R_1, then returns through R_3 to the negative terminal. Its resistance is

$$R_{\text{loop}} = 20 + 50 + 50 = 120\,\Omega.$$

The current through every part of this loop, including R_3, is

$$I_{R_3} = \frac{6}{120} = 0.050\,\text{A} = 50\,\text{mA}.$$

As a voltage check, the diode drops $1\,\mathrm{V}$, and each of R_1, R_3 drops $2.5\,\mathrm{V}$, giving $6\,\mathrm{V}$ in total. The larger alternatives neglect a series resistance or incorrectly permit current in the blocked middle branch; $25\,\mathrm{mA}$ would require twice the actual loop resistance.

Step 3: Conclusion The current through R_3 is $50\,\mathrm{mA}$. Hence, the correct option is (1).

Quick Tip to Solve: Determine diode directions before reducing the resistor network. Two oppositely directed series diodes block their entire branch when reverse resistance is infinite.

Chemistry (120 marks)

61. The density of $3\,\mathrm{M}$ solution of sodium chloride is $1.252\,\mathrm{g\,mL^{-1}}$. The molality of the solution will be: (molar mass, $\mathrm{NaCl} = 585\,\mathrm{g\,mol^{-1}}$) [4 marks]

(1) $260\,\mathrm{m}$

(2) $2.18\,\mathrm{m}$

(3) $2.79\,\mathrm{m}$

(4) $3.00\,\mathrm{m}$

Correct Answer: No physically meaningful molality follows from the printed data; none of the options is defensible.

Solution:

Concept:

- Molarity is moles of solute per litre of solution, whereas molality is moles of solute per kilogram of solvent. Density converts the chosen solution volume into its total mass.
- The solvent mass must be positive and equals solution mass minus solute mass. A negative result signals mutually inconsistent printed data rather than a physically meaningful negative concentration.

Step 1: Calculate masses from the literal data Take $1.000\,\mathrm{L}$ of the stated solution. The molarity gives

$$n_{\mathrm{NaCl}} = 3.000\,\mathrm{mol}.$$

Its total mass follows from the density:

$$m_{\mathrm{solution}} = 1000 \times 1.252 = 1252\,\mathrm{g}.$$

The question explicitly supplies the molar mass $585\,\mathrm{g\,mol^{-1}}$. Using that printed value, rather than silently replacing it with another number, gives

$$m_{\mathrm{solute}} = 3 \times 585 = 1755\,\mathrm{g}.$$

The solute alone would thus weigh more than the entire solution.

Step 2: Test whether molality can be defined The implied solvent mass is

$$m_{\mathrm{solvent}} = 1252 - 1755 = -503\,\mathrm{g}.$$

This is impossible for a solution. Formally substituting it would give $3/(-0.503) \simeq -5.96\,\mathrm{mol\,kg^{-1}}$, but that negative value is not a physical molality and is not an offered option. Every listed option is positive, so none follows from the literal premises. Even substituting the familiar chemical molar mass would change the supplied question rather than solve it as printed. The answer therefore cannot be selected by assuming an omitted decimal point in the data or in an option; those changes are not authorized by the printed statement.

Step 3: Conclusion The printed data imply a negative solvent mass and therefore no physically meaningful molality. None of the four options is defensible for the question as printed.

Quick Tip to Solve: Before dividing by solvent mass, check that solute mass is smaller than solution mass. This simple positivity test detects inconsistent concentration data immediately.

62. The wave number of the first emission line in the Balmer series of H-Spectrum is: (R = Rydberg constant): [4 marks]

(1) $\frac{5}{36}R$
(2) $\frac{9}{400}R$
(3) $\frac{7}{6}R$
(4) $\frac{3}{4}R$

Correct Answer: (1) $\frac{5}{36}R$

Solution:

Concept:

- The Balmer series consists of hydrogen emission transitions ending at principal quantum number $n_f = 2$. Its first line is the transition from the next higher level, $n_i = 3$.
- The Rydberg formula for wave number is $\widetilde{\nu} = 1/\lambda = R(1/n_f^2 - 1/n_i^2)$. Wave number has inverse-length units, so the dimensionless bracket multiplies R.

Step 1: Identify the two levels An emission line is produced when an electron loses energy by falling to a lower level. All Balmer lines end at $n_f = 2$. The first member has the smallest allowed jump in this series, namely $n_i = 3$ to $n_f = 2$; it is not a transition ending at $n = 1$. The latter would belong to the Lyman series. Using the series limit $n_i \to \infty$ would also describe a different spectral feature rather than the first line.

Step 2: Evaluate the Rydberg expression Substitute the identified quantum numbers:

$$\widetilde{\nu} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right).$$

Taking a common denominator,

$$\widetilde{\nu} = R\frac{9-4}{36} = \frac{5}{36}R.$$

The result is positive as required for an emitted photon and has the same units as R. The factor $9/400$ comes from a different pair of levels; $7/6$ is even larger than unity and cannot arise from this bracket. The factor $3/4$ corresponds to the $2 \to 1$ Lyman transition, not to Balmer emission. Thus none of the

other listed values represents the specified first line.

Step 3: Conclusion The required wave number is $\widetilde{\nu} = \frac{5}{36}R$. Hence, the correct option is (1).

Quick Tip to Solve: For the first line of a series ending at n, use the transition $(n+1) \to n$; for Balmer this is $3 \to 2$.

63. The order of increasing sizes of atomic radii among the elements O, S, Se and As is: [4 marks]

(1) $\mathrm{As} < \mathrm{S} < \mathrm{O} < \mathrm{Se}$

(2) $\mathrm{Se} < \mathrm{S} < \mathrm{As} < \mathrm{O}$

(3) $\mathrm{O} < \mathrm{S} < \mathrm{As} < \mathrm{Se}$

(4) $\mathrm{O} < \mathrm{S} < \mathrm{Se} < \mathrm{As}$

Correct Answer: (4) $\mathrm{O} < \mathrm{S} < \mathrm{Se} < \mathrm{As}$

Solution:

Concept:

- Atomic radius generally increases down a group because an additional occupied electron shell is introduced and shielding increases.
- Across a period from left to right, atomic radius generally decreases because the nuclear charge rises while the added electrons enter the same principal shell. The resulting effective nuclear attraction pulls the outer electrons closer.

Step 1: Compare the group-16 elements Oxygen, sulfur and selenium all lie in group 16. Their outer electrons occupy the second, third and fourth principal shells, respectively. The increase in shell number and shielding down the group outweighs the rise in nuclear charge, giving

$$r(\mathrm{O}) < r(\mathrm{S}) < r(\mathrm{Se}).$$

Thus any offered order placing oxygen above sulfur or selenium in size is immediately excluded. Comparing electron counts alone would miss the additional shells and would not correctly predict the trend.

Step 2: Place arsenic relative to selenium Arsenic and selenium are both in period 4, with arsenic in group 15 immediately to the left of selenium in group 16. Their valence electrons occupy the same principal shell. Selenium has the larger nuclear charge and, across this short interval, its valence electrons experience a stronger effective attraction. Hence selenium is smaller than arsenic:

$$r(\mathrm{Se}) < r(\mathrm{As}).$$

Combining this with the group-16 comparison yields

$$r(\mathrm{O}) < r(\mathrm{S}) < r(\mathrm{Se}) < r(\mathrm{As}).$$

The third option reverses the final neighboring-period comparison, while the first two also contradict the established oxygen-to-sulfur trend. The ordering is therefore fixed by both down-group and

across-period trends.

Step 3: Conclusion The supplied key differs from this result, but As lies to the left of Se in period 4 and is larger, so the printed order in (4) follows the periodic trend. The increasing order is $O < S < Se < As$. Hence, the correct option is (4).

Quick Tip to Solve: First order O, S and Se down their common group, then insert As using its position immediately to the left of Se in the same period.

64. The solubility order for alkali metal fluoride in water is [4 marks]

(1) $LiF < RbF < KF < NaF$

(2) $RbF < KF < NaF < LiF$

(3) $LiF > NaF > KF > RbF$

(4) $LiF < NaF < KF < RbF$

Correct Answer: (4) $LiF < NaF < KF < RbF$

Solution:

Concept:

- Dissolution of an ionic solid involves separating its ions from the lattice and hydrating the separated ions. The observed solubility depends on the balance of these contributions to the Gibbs energy, not on ion size alone.
- For alkali-metal fluorides, the small fluoride ion gives lithium fluoride an especially strong lattice. The observed aqueous solubility increases in the sequence LiF, NaF, KF, RbF.

Step 1: Compare the lithium salt with the others Lithium has the smallest cation in this series. Its attraction to the small fluoride ion is consequently very strong, making the lattice difficult to break. Although Li^+ is strongly hydrated, that hydration does not compensate sufficiently for the lattice stability of LiF. Thus lithium fluoride is the least soluble member, eliminating the orders that put it first in decreasing solubility or last in increasing solubility.

Step 2: Apply the fluoride trend As the cation becomes larger from sodium through potassium to rubidium, the lattice is weakened and these fluorides dissolve more readily. The relevant net trend is therefore

$$LiF < NaF < KF < RbF.$$

This is an order of increasing solubility. Option (1) incorrectly places sodium fluoride above both potassium and rubidium fluorides; (2) makes lithium fluoride the most soluble, while (3) reverses the correct trend.

Step 3: Conclusion The increasing aqueous solubility is $LiF < NaF < KF < RbF$. Hence, the correct option is (4).

Quick Tip to Solve: For alkali-metal fluorides, remember the unusually low solubility of LiF; do not apply the hydroxide or carbonate trends indiscriminately.

65. Bond order normally gives idea of stability of a molecular species. All the molecules viz. H_2, Li_2 and B_2 have the same bond order yet they are not equally stable. Their stability order is: [4 marks]

(1) $H_2 > B_2 > Li_2$

(2) $Li_2 > H_2 > B_2$

(3) $Li_2 > B_2 > H_2$

(4) None of these

Correct Answer: (1) $H_2 > B_2 > Li_2$

Solution:

Concept:

- Bond order is $\frac{1}{2}(N_b - N_a)$, where N_b and N_a are the numbers of electrons in bonding and antibonding molecular orbitals. It predicts bond multiplicity but is not an absolute bond-energy scale across different elements.
- Bond stability also depends on orbital size, overlap and internuclear separation. Greater bond dissociation energy means a stronger, more stable bond against dissociation into atoms.

Step 1: Identify what equal bond order means Each of H_2, Li_2 and B_2 has bond order one, so all possess a net single bond in the molecular-orbital description. This equality does not mean that their bonding orbitals have equal energies or that the bonds require equal energies to break. Hydrogen uses compact 1s orbitals; lithium and boron use valence orbitals of the second shell, so the overlap and internuclear distances differ.

Step 2: Compare the strengths of the actual bonds The compact hydrogen orbitals overlap very effectively, giving the strongest bond of these three species. The boron molecule is more strongly bound than the lithium molecule; lithium has a much longer, weaker bond formed from diffuse 2s orbitals. Representative dissociation energies are about 436, 290 and $100\,\mathrm{kJ\,mol^{-1}}$ for hydrogen, boron and lithium respectively. Thus

$$H_2 > B_2 > Li_2.$$

The two options placing lithium first confuse bond order with bond strength. Since this sequence is listed, None of these is also excluded.

Step 3: Conclusion The stability order is $H_2 > B_2 > Li_2$. Hence, the correct option is (1). This follows from the bond strengths of the printed molecules, even though the supplied answer key selects None of these.

Quick Tip to Solve: Use bond order to compare closely related species; across different elements also examine orbital overlap, bond length and dissociation energy.

66. Given the following reaction energy changes (in kJ):

$\mathrm{Li(s)} \longrightarrow \mathrm{Li(g)}$	161
$\mathrm{Li(g)} \longrightarrow \mathrm{Li^+(g)}$	520
$\frac{1}{2}\mathrm{F_2(g)} \longrightarrow \mathrm{F(g)}$	77
$\mathrm{F(g)} + \mathrm{e^-} \longrightarrow \mathrm{F^-(g)}$	**Electron gain enthalpy**
$\mathrm{Li^+(g)} + \mathrm{F^-(g)} \longrightarrow \mathrm{LiF(s)}$	-1047
$\mathrm{Li(s)} + \frac{1}{2}\mathrm{F_2(g)} \longrightarrow \mathrm{LiF(s)}$	-617

Based on the data provided, the value of electron gain enthalpy of fluorine would be: [4 marks]

(1) $-300\,\mathrm{kJ\,mol^{-1}}$

(2) $-350\,\mathrm{kJ\,mol^{-1}}$

(3) $-328\,\mathrm{kJ\,mol^{-1}}$

(4) $-228\,\mathrm{kJ\,mol^{-1}}$

Correct Answer: (3) $-328\,\mathrm{kJ\,mol^{-1}}$

Solution:

Concept:

- Hess's law states that the enthalpy change for an overall reaction equals the sum of the enthalpy changes of any sequence of steps producing that reaction.
- In the Born-Haber cycle, the formation of an ionic solid is resolved into atomisation, ionisation, electron gain and lattice formation. Electron gain enthalpy is signed; an exothermic electron attachment has a negative value.

Step 1: Write the enthalpy balance Let the unknown electron gain enthalpy of fluorine be $x\,\mathrm{kJ\,mol^{-1}}$. Sublime lithium, ionise it, dissociate half a mole of fluorine molecules, attach an electron to fluorine and form the solid lattice. Adding these steps cancels all gaseous intermediates and the electron. The net process is exactly $\mathrm{Li(s)} + \frac{1}{2}\mathrm{F_2(g)} \longrightarrow \mathrm{LiF(s)}$, whose enthalpy change is given as $-617\,\mathrm{kJ\,mol^{-1}}$.

Step 2: Substitute the signed energy changes The lattice term supplied is lattice formation, so it is already negative and must not be reversed:

$$\begin{aligned} -617 &= 161 + 520 + 77 + x - 1047, \\ -617 &= x - 289, \\ x &= -328\,\mathrm{kJ\,mol^{-1}}. \end{aligned}$$

As a check, the energy input before electron attachment is $758\,\mathrm{kJ\,mol^{-1}}$. Electron attachment and lattice formation release $328 + 1047 = 1375\,\mathrm{kJ\,mol^{-1}}$, leaving the net release of $617\,\mathrm{kJ\,mol^{-1}}$. The other proposed values fail this Hess-law sum.

Step 3: Conclusion The electron gain enthalpy of fluorine is $-328\,\mathrm{kJ\,mol^{-1}}$. Hence, the correct option is (3).

Quick Tip to Solve: Keep lattice formation negative in a Born-Haber sum; lattice dissociation is the opposite process and has the opposite sign.

67. The reaction $X \longrightarrow Y$ is an exothermic reaction. Activation energy of the reaction for X into Y is $150\,kJ\,mol^{-1}$. Enthalpy of reaction is $135\,kJ\,mol^{-1}$. The activation energy for the reverse reaction, $Y \longrightarrow X$, will be: [4 marks]

(1) $280\,kJ\,mol^{-1}$

(2) $285\,kJ\,mol^{-1}$

(3) $270\,kJ\,mol^{-1}$

(4) $15\,kJ\,mol^{-1}$

Correct Answer: (2) $285\,kJ\,mol^{-1}$

Solution:

Concept:

- Activation energy is the energy difference between the transition state and the reactants of the chosen direction: $E_{a,f} = E^{\ddagger} - E_X$ and $E_{a,r} = E^{\ddagger} - E_Y$.
- The reaction enthalpy is $\Delta H = E_Y - E_X = E_{a,f} - E_{a,r}$. An exothermic reaction has $\Delta H < 0$, so its reverse activation energy is larger than its forward activation energy.

Step 1: Assign the sign of the reaction enthalpy The reaction $X \longrightarrow Y$ is explicitly stated to be exothermic. Therefore the quoted enthalpy magnitude corresponds to

$$\Delta H = -135\,kJ\,mol^{-1}.$$

Product Y lies below reactant X in energy by this amount. The transition state lies $150\,kJ\,mol^{-1}$ above X. Starting from the lower product therefore requires a larger rise to reach the same transition state.

Step 2: Calculate the reverse barrier Rearrange the relation between the barriers:

$$\begin{aligned} E_{a,r} &= E_{a,f} - \Delta H, \\ &= 150 - (-135), \\ &= 285\,kJ\,mol^{-1}. \end{aligned}$$

The result exceeds the forward barrier by $135\,kJ\,mol^{-1}$, exactly as the energy ordering requires. Subtracting the enthalpy magnitude to obtain 15 would incorrectly put an exothermic product above its reactant. The values 280 and 270 do not satisfy the given energy difference.

Step 3: Conclusion The reverse activation energy is $285\,kJ\,mol^{-1}$. Hence, the correct option is (2).

Quick Tip to Solve: For an exothermic forward reaction, add the heat released to its activation energy to obtain the reverse barrier.

68. Which one of the following arrangements represents the correct order of solubilities of sparingly soluble salts Hg_2Cl_2, $Cr_2(SO_4)_3$, $BaSO_4$ and $CrCl_3$ respectively? [4 marks]

(1) $BaSO_4 > Hg_2Cl_2 > Cr_2(SO_4)_3 > CrCl_3$

(2) $BaSO_4 > Hg_2Cl_2 > CrCl_3 > Cr_2(SO_4)_3$

(3) $BaSO_4 > CrCl_3 > Hg_2Cl_2 > Cr_2(SO_4)_3$

(4) $Hg_2Cl_2 > BaSO_4 > CrCl_3 > Cr_2(SO_4)_3$

Correct Answer: No unique listed solubility order is justified by the given salts and information.

Solution:

Concept:

- Molar solubility s depends on a salt-specific equilibrium constant and on its dissolution stoichiometry. Different salts cannot be ranked by stoichiometry alone.
- The solubility product is the product of dissolved-ion activities, approximated by concentration powers for an ideal dilute solution. A comparison using equal K_{sp} requires that equality to be explicitly supplied.

Step 1: Write the formal solubility relations If each salt were treated as a sparingly soluble ideal electrolyte with molar solubility s in pure water, its stoichiometry would give

$$\begin{aligned} BaSO_4: &\quad K_{sp} = s^2, \\ Hg_2Cl_2: &\quad K_{sp} = s(2s)^2 = 4s^3, \\ CrCl_3: &\quad K_{sp} = s(3s)^3 = 27s^4, \\ Cr_2(SO_4)_3: &\quad K_{sp} = (2s)^2(3s)^3 = 108s^5. \end{aligned}$$

The K_{sp} symbol in each line denotes that particular salt's constant; these constants are not automatically equal. Taking different roots of unspecified different constants cannot establish a ranking.

Step 2: Check the chemical classification and choices Barium sulfate and mercurous chloride are familiar sparingly soluble salts. Chromium(III) chloride and chromium(III) sulfate, however, are water-soluble salts, so calling all four sparingly soluble does not support the proposed comparison. In particular, each displayed ranking places barium sulfate above chromium(III) chloride, which contradicts their ordinary aqueous-solubility behaviour. One cannot repair this by silently assuming all four K_{sp} values are equal: that is an extra quantitative premise, and it would still require a defined ideal-equilibrium model. Neither the salts' names nor their numbers of ions provide the missing constants.

Step 3: Conclusion As stated, the question supplies neither the necessary solubility data nor a valid common-K_{sp} premise, and the printed orders conflict with the ordinary solubility of the chromium salts. No unique listed option is defensible.

Quick Tip to Solve: Before ranking solubilities from ionic stoichiometry, check whether the salts really are sparingly soluble and whether their equilibrium constants are given or explicitly equal.

69. NaOH is a strong base. What will be pH of 5.0×10^{-2} M NaOH solution? ($\log 2 = 0.3$) [4 marks]

(1) 14.00

(2) 13.70

(3) 13.00

(4) 12.70

Correct Answer: (4) 12.70

Solution:

Concept:

- Sodium hydroxide is a strong base and dissociates essentially completely in dilute aqueous solution: $\mathrm{NaOH} \longrightarrow \mathrm{Na^+} + \mathrm{OH^-}$. One mole of solute produces one mole of hydroxide ions.
- At the usual reference temperature of $25^\circ\mathrm{C}$, $\mathrm{pOH} = -\log_{10}[\mathrm{OH^-}]$ and $\mathrm{pH} + \mathrm{pOH} = 14$. Concentrations are expressed relative to $1\,\mathrm{mol\,L^{-1}}$ in this elementary calculation.

Step 1: Determine the hydroxide concentration There is one hydroxide ion per formula unit of sodium hydroxide. Complete dissociation therefore gives

$$[\mathrm{OH^-}] = 5.0 \times 10^{-2}\,\mathrm{mol\,L^{-1}} = 0.050\,\mathrm{mol\,L^{-1}}.$$

This is overwhelmingly larger than the hydroxide contribution from water, so the latter can be neglected. We do not use a weak-base square-root formula because sodium hydroxide is already fully ionised.

Step 2: Evaluate pOH and convert to pH Using the supplied $\log 2 = 0.3$, we have $\log 5 = \log(10/2) = 0.7$. Hence

$$\begin{aligned} \mathrm{pOH} &= -\log(5.0 \times 10^{-2}) \\ &= 2 - 0.7 = 1.30, \\ \mathrm{pH} &= 14.00 - 1.30 = 12.70. \end{aligned}$$

A pH of 14 would correspond to a much greater hydroxide concentration, about 1 M. Values 13.70 and 13.00 also imply concentrations different from 0.050 M and fail the logarithmic calculation.

Step 3: Conclusion The pH is 12.70. Hence, the correct option is (4).

Quick Tip to Solve: Calculate pOH first for a strong base, then subtract it from 14; use log 5 = 1 - log 2 when only log 2 is supplied.

70. Values of dissociation constant, K_a, are given as follows:

Acid	K_a
HCN	6.2×10^{-10}
HF	7.2×10^{-4}
HNO_2	4.0×10^{-4}

Correct order of increasing base strength of the bases CN^-, F^- and $NO_2{}^-$ will be: [4 marks]

(1) $F^- < CN^- < NO_2{}^-$

(2) $NO_2{}^- < CN^- < F^-$

(3) $F^- < NO_2{}^- < CN^-$

(4) $NO_2{}^- < F^- < CN^-$

Correct Answer: (3) $F^- < NO_2{}^- < CN^-$

Solution:

Concept:

- For a conjugate acid-base pair in water, $K_aK_b = K_w$. At a fixed temperature K_w is common to all pairs, so the stronger acid has the weaker conjugate base.
- The conjugate bases of HCN, HF and HNO_2 are CN^-, F^- and $NO_2{}^-$, respectively. Compare the acid constants before reversing the order for their bases.

Step 1: Rank the given acids The supplied dissociation constants are

$$K_a(\text{HF}) = 7.2 \times 10^{-4}, \quad K_a(\text{HNO}_2) = 4.0 \times 10^{-4}, \quad K_a(\text{HCN}) = 6.2 \times 10^{-10}.$$

The first two share an exponent, so their coefficients establish that HF is stronger than nitrous acid. Hydrogen cyanide has a far smaller constant and is much weaker than either. Thus decreasing acid strength is $\text{HF} > \text{HNO}_2 > \text{HCN}$.

Step 2: Reverse the order for the conjugate bases Because $K_b = K_w/K_a$, the largest acid constant produces the smallest base constant. At 25°C, approximate base constants illustrate the reversal:

$$K_b(F^-) = 1.39 \times 10^{-11},$$
$$K_b(NO_2{}^-) = 2.50 \times 10^{-11},$$
$$K_b(CN^-) = 1.61 \times 10^{-5}.$$

Therefore the increasing base-strength order is fluoride, nitrite, cyanide. Options (1) and (2) incorrectly place cyanide before a much weaker base, and (4) incorrectly places nitrite below fluoride.

Step 3: Conclusion The increasing base-strength order is $F^- < NO_2{}^- < CN^-$. Hence, the correct option is (3).

Quick Tip to Solve: When comparing conjugate bases, rank their parent acids by Ka and then reverse that ranking; smaller Ka means larger Kb.

71. For which of the following compounds Kjeldahl method can be used to determine the percentage of Nitrogen? [4 marks]

(1) Nitrobenzene

(2) Pyridine

(3) Alanine

(4) Diazomethane

Correct Answer: (3) Alanine

Solution:

Concept:

- Kjeldahl estimation converts suitable organic nitrogen into ammonium sulfate by digestion with concentrated H_2SO_4. The ammonia released after adding excess alkali is measured to determine nitrogen content.
- The ordinary method does not quantitatively convert nitrogen in nitro groups, diazo groups or aromatic heterocyclic rings into ammonium sulfate. The chemical environment of nitrogen must therefore be checked before applying it.

Step 1: Identify the nitrogen that can be converted Alanine is an amino acid, $CH_3CH(NH_2)COOH$. Its nitrogen is present in an amino group, which is converted into ammonium sulfate during digestion. Excess alkali then releases ammonia:

$$(NH_4)_2SO_4 + 2\,NaOH \longrightarrow Na_2SO_4 + 2\,NH_3 + 2\,H_2O.$$

The amount of this ammonia gives the nitrogen in the original sample.

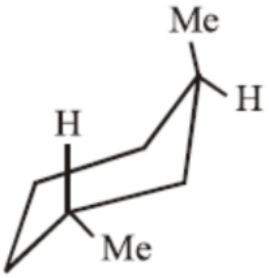
Step 2: Exclude the other nitrogen compounds Nitrobenzene contains a nitro group, $-NO_2$, rather than an amino group. Pyridine has nitrogen as a member of an aromatic ring. Diazomethane contains diazo nitrogen. These forms are not quantitatively recovered as ammonia in the ordinary Kjeldahl procedure. Merely containing nitrogen is not enough for a compound to be suitable; reliable conversion into ammonium ions is essential.

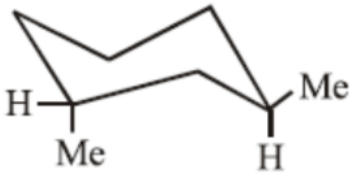
Step 3: Conclusion Alanine has nitrogen in the required amino form and is suitable for Kjeldahl estimation. Hence, the correct option is (3).

Quick Tip to Solve: Check the nitrogen functional group first: amino nitrogen is suitable, whereas nitro, diazo and aromatic ring nitrogen are excluded in the ordinary Kjeldahl method.

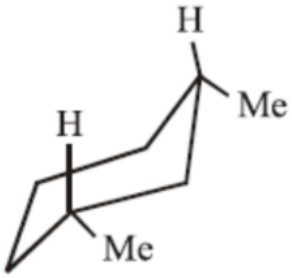
72. Arrange in the correct order of stability (decreasing order) for the following molecules: (I)

[4 marks]

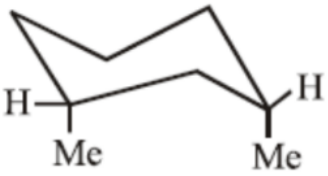




(II)



(III)



(IV)

(1) $(I) > (II) > (III) > (IV)$

(2) $(IV) > (III) > (II) \approx (I)$

(3) $(I) > (II) \approx (III) > (IV)$

(4) $(III) > (I) \approx (II) > (IV)$

Correct Answer: (4) $(III) > (I) \approx (II) > (IV)$

Solution:

Concept:

- In a cyclohexane chair, an axial methyl group experiences unfavourable 1,3-diaxial interactions with axial groups on the same side of the ring. An equatorial methyl group avoids most of this steric crowding.
- For chairs containing the same substituents, compare the number of axial methyl groups. A diequatorial arrangement is generally more stable than an axial-equatorial arrangement, which is more stable than a diaxial arrangement.

Step 1: Read the methyl positions in each chair In structure (I), the upper methyl group is axial and the lower methyl group is equatorial. In (II), one methyl group is again axial and the other is equatorial. In (III), both methyl groups point outward along equatorial bonds, whereas in (IV), both methyl groups occupy axial positions. The axial hydrogens shown in the figures help identify these directions; a downward bond is not automatically equatorial.

Step 2: Compare the steric penalties Structure (III) has no axial methyl group and therefore the smallest 1,3-diaxial penalty. Structures (I) and (II) each have one axial methyl group, so their main steric penalties are approximately equal. Structure (IV) has two axial methyl groups and suffers the greatest crowding. Thus the decreasing stability order is

$$(III) > (I) \approx (II) > (IV).$$

Options that put (IV) first reverse the effect of axial crowding; options that put (I) above (III) ignore the advantage of two equatorial methyl groups.

Step 3: Conclusion The diequatorial chair (III) is most stable, the mixed chairs (I) and (II) are approximately equal, and diaxial (IV) is least stable. Hence, the correct option is (4).

Quick Tip to Solve: Count axial methyl groups in each drawing before ranking: zero is best, one is intermediate, and two is worst for these chairs.

73. A major component of Borsch reagent is obtained by reacting hydrazine hydrate with which of the following ? [4 marks]

(1) 1-Chloro-2-nitrobenzene

(2) 1-Chloro-2,6-dinitrobenzene

(3) 1-Chloro-2,4-dinitrobenzene

(4) 1-Chloro-2,4,6-trinitrobenzene

Correct Answer: (3) 1-Chloro-2,4-dinitrobenzene

Solution:

Concept:

- The active carbonyl-detecting component of the reagent is 2,4-dinitrophenylhydrazine, commonly abbreviated 2,4-DNPH. It reacts with aldehydes and ketones to form coloured hydrazone precipitates.
- Hydrazine hydrate replaces activated aromatic chlorine by a hydrazino group, $-NHNH_2$. Nitro groups ortho and para to chlorine stabilise the intermediate in nucleophilic aromatic substitution.

Step 1: Identify the required aromatic precursor The product must have a hydrazino group at carbon 1 and nitro groups at carbons 2 and 4. Therefore the corresponding chloro precursor is 1-chloro-2,4-dinitrobenzene, which is the structure in option (3). Hydrazine replaces chlorine while retaining both nitro groups and their positions on the benzene ring.

Step 2: Match each option to its substitution pattern The transformation can be represented as

$$2,4-(NO_2)_2C_6H_3Cl + NH_2NH_2 \longrightarrow 2,4-(NO_2)_2C_6H_3NHNH_2 + HCl.$$

Option (1) has only one nitro group and would give an ortho-nitrophenylhydrazine. Option (2) has nitro groups at 2 and 6, producing the wrong positional isomer. Option (4) has three nitro groups and would retain an extra nitro group relative to the required 2,4 pattern. None supplies the required 2,4-dinitrophenyl group except (3).

Step 3: Conclusion The precursor is 1-chloro-2,4-dinitrobenzene, whose chlorine is replaced by the hydrazino group to give 2,4-DNPH. Hence, the correct option is (3).

Quick Tip to Solve: Work backward from 2,4-DNPH: replace only its hydrazino group by chlorine and keep the two nitro groups at positions 2 and 4.

74. Clemmensen reduction of a ketone is carried out in the presence of [4 marks]

(1) $LiAlH_4$

(2) $Zn-Hg$ with HCl

(3) Glycol with KOH

(4) H_2 with Pt as catalyst

Correct Answer: (2) $Zn-Hg$ with HCl

Solution:

Concept:

- Clemmensen reduction converts the carbonyl group of an aldehyde or ketone into a methylene group using zinc amalgam and concentrated hydrochloric acid. It is a reduction in a strongly acidic medium.
- The carbon skeleton is retained: for a ketone the transformation is $R-CO-R \longrightarrow R-CH_2-R$. Reduction to an alcohol is a different transformation and uses different reagents.

Step 1: Select the characteristic reducing system Zinc amalgam, written $Zn-Hg$, together with HCl, is the characteristic Clemmensen reagent. The amalgamated zinc surface reduces the carbonyl group under acidic conditions. In symbolic form,

$$R-CO-R \xrightarrow{Zn-Hg/HCl} R-CH_2-R.$$

Thus the oxygen of the ketone is removed and two hydrogen atoms occupy its position at the carbonyl carbon.

Step 2: Distinguish the alternative reagents Lithium aluminium hydride normally reduces a ketone to a secondary alcohol, so it does not identify Clemmensen reduction. Glycol with potassium hydroxide is not the zinc-amalgam acidic reagent; it should not be confused with the basic conditions of Wolff-Kishner reduction, which require hydrazine. Hydrogen with platinum denotes catalytic hydrogenation and is also a different reagent system. The acid-base character and named reagent must both agree

before identifying the reaction.

Step 3: Conclusion Clemmensen reduction of a ketone is carried out with zinc amalgam and hydrochloric acid. Hence, the correct option is (2).

Quick Tip to Solve: Remember the pair: Clemmensen uses Zn-Hg/HCl in acid; Wolff-Kishner uses hydrazine and strong base.

75. Which of the following would not give 2-phenylbutane as the major product in a Friedel-Crafts alkylation reaction ? [4 marks]

(1) 1-Butene + HF

(2) 2-Butanol + H_2SO_4

(3) Butanoyl chloride + $AlCl_3$, then Zn, HCl

(4) Butyl chloride + $AlCl_3$

Correct Answer: (3) Butanoyl chloride + $AlCl_3$, then Zn, HCl

Solution:

Concept:

- Friedel-Crafts alkylation introduces an alkyl group through a carbocation-like electrophile. A primary butyl electrophile can rearrange to the more stable secondary butyl electrophile before attack on benzene.
- Friedel-Crafts acylation instead uses an acylium ion, which does not undergo ordinary carbocation rearrangement. Subsequent reduction removes carbonyl oxygen without shifting the carbon skeleton.

Step 1: Follow the alkylating routes Protonation of 1-butene by HF gives a secondary butyl electrophile, and acid treatment of 2-butanol also generates a secondary butyl electrophile. Alkylation of benzene then produces $C_6H_5CH(CH_3)CH_2CH_3$, 2-phenylbutane. With butyl chloride and $AlCl_3$, rearrangement of the initially primary electrophile to the secondary form similarly favours the sec-butylbenzene product.

Step 2: Track the acylation-reduction route Butanoyl chloride with $AlCl_3$ gives the unrearranged aryl ketone $C_6H_5COCH_2CH_2CH_3$. Reduction with zinc and hydrochloric acid converts its carbonyl group to methylene:

$$C_6H_5COCH_2CH_2CH_3 \xrightarrow{Zn/HCl} C_6H_5CH_2CH_2CH_2CH_3.$$

This is 1-phenylbutane, not 2-phenylbutane. In the acylation route the benzene ring remains attached to the terminal carbon of the four-carbon chain throughout; reducing the carbonyl does not create a secondary attachment point.

Step 3: Conclusion Butanoyl chloride followed by carbonyl reduction gives 1-phenylbutane and therefore does not give 2-phenylbutane as the major product. Hence, the correct option is (3).

Quick Tip to Solve: To install an unrearranged straight chain, acylate first and reduce afterward; ordinary alkylation can rearrange the carbocation.

76. Which one of the following statements about packing in solids is incorrect ? [4 marks]

(1) Coordination number in bcc mode of packing is 8

(2) Coordination number in hep mode of packing is 12

(3) Void space in hcp mode of packing is 32%

(4) Void space in ccp mode of packing is 26%

Correct Answer: (3) Void space in hcp mode of packing is 32%

Solution:

Concept:

- Packing efficiency is the fraction of crystal volume occupied by the constituent spheres. Void percentage equals 100 minus the packing percentage. Coordination number counts the nearest neighbours of one sphere.
- Both hexagonal close packing (hcp) and cubic close packing (ccp) have coordination number 12 and packing efficiency approximately 74 percent. Body-centred cubic packing has coordination number 8 and lower packing efficiency.

Step 1: Check the coordination-number statements In a bcc lattice, the body-centred atom has the eight corner atoms as its nearest neighbours, so statement (1) is correct. In an hcp structure, a sphere touches six neighbours in its own layer, three in the layer above and three below. Its coordination number is therefore

$$6 + 3 + 3 = 12,$$

agreeing with statement (2), where the printed abbreviation refers to hexagonal close packing.

Step 2: Compute the unoccupied fraction for close packing The occupied fraction for either hcp or ccp is

$$\eta = \frac{\pi}{3\sqrt{2}} \approx 0.7405.$$

Hence their void fraction is

$$1 - \eta \approx 0.2595,$$

or about 26 percent. The claim that hcp has 32 percent void space is incorrect, whereas the claim of 26 percent void space for ccp is correct. A void space near 32 percent belongs to bcc packing, whose packing efficiency is about 68 percent, and should not be assigned to hcp.

Step 3: Conclusion The incorrect statement is that hcp packing has 32 percent void space; its actual void space is approximately 26 percent. Hence, the correct option is (3).

Quick Tip to Solve: Keep the pairs together: hcp and ccp have 74 percent packing and 26 percent voids; bcc has about 68 percent packing and 32 percent voids.

77. A molecule M associates in a given solvent according to the equation $M \rightleftharpoons (M)_n$. For a certain concentration of M, the van't Hoff factor was found to be 0.9 and the fraction of associated molecules was 0.2. The value of n is: [4 marks]

(1) 3

(2) 5

(3) 2

(4) 4

Correct Answer: (3) 2

Solution:

Concept:

- The van't Hoff factor i is the ratio of the number of solute particles present after association to the number initially present. Association reduces particle number and therefore gives $i < 1$.
- If a fraction α of initially monomeric molecules associates into aggregates containing n monomers each, the remaining particle fraction is $i = (1-\alpha) + \alpha/n$. The associated fraction counts monomers consumed, not aggregates formed.

Step 1: Count monomers and aggregates separately Take one mole of monomer units initially. Since $\alpha = 0.2$, the amount of unassociated monomer is $1 - 0.2 = 0.8$ mol. The associated 0.2 mol of monomer units forms $0.2/n$ mol of aggregates because each aggregate consumes n monomers. Thus the total number of moles of dissolved particles is

$$0.8 + \frac{0.2}{n}.$$

Solvent molecules are not included in this solute particle count.

Step 2: Use the measured van't Hoff factor The initial solute particle amount was one mole, so the measured factor gives

$$0.9 = 0.8 + \frac{0.2}{n}.$$

Subtracting 0.8 yields

$$0.1 = \frac{0.2}{n}, \qquad n = 2.$$

The alternatives can also be checked directly: $n = 3, 4, 5$ would give particle factors approximately $0.8667, 0.85, 0.84$, respectively, all below 0.9. For dimers, the 0.2 mol consumed produces 0.1 mol of dimers, leaving a total of 0.9 mol of particles as required.

Step 3: Conclusion The molecules associate as dimers, so $n = 2$. Hence, the correct option is (3).

Quick Tip to Solve: For association, write particles left as unassociated monomers plus aggregates: $1-\alpha+\alpha/n$. Do not count all associated monomer units as separate particles.

78. Flocculation value of $BaCl_2$ is much less than that of KCl for sol A and flocculation value of Na_2SO_4 is much less than that of NaBr for sol B. The correct statement among the following is: [4 marks]

(1) Both the sols A and B are negatively charged

(2) Sol A is positively charged and Sol B is negatively charged

(3) Both the sols A and B are positively charged

(4) Sol A is negatively charged and sol B is positively charged

Correct Answer: (4) Sol A is negatively charged and sol B is positively charged

Solution:

Concept:

- The flocculation value is the minimum concentration of an electrolyte needed to coagulate a sol. A smaller value means greater coagulating power, rather than weaker action.
- According to the Hardy–Schulze rule, the ion with charge opposite to the sol is the effective coagulating ion. Its coagulating power increases sharply with its valency.

Step 1: Identify the effective ion for sol A Both electrolytes contain chloride ions, so their important difference is the cation: Ba^{2+} versus K^{+}. The much smaller flocculation value of $BaCl_2$ shows that the divalent barium ion coagulates A more efficiently than the monovalent potassium ion. Thus cations are the counter-ions for A, meaning that A carries a negative charge. The common chloride anion cannot explain the observed advantage of barium chloride.

Step 2: Identify the effective ion for sol B Both electrolytes contain sodium ions. Their contrasting coagulating powers must therefore be associated with SO_4^{2-} and Br^{-}. Sulphate has the larger negative charge and its salt has the lower flocculation value. Hence anions are the counter-ions for B, and B is positively charged. Combining the two conclusions rules out the statements that both sols have the same charge and the statement with their charges reversed.

Step 3: Conclusion Sol A is negatively charged, whereas sol B is positively charged. This charge assignment is the reverse of that given in the supplied answer key. Hence, the correct option is (4).

Quick Tip to Solve: Compare salts sharing one ion: the ion whose valency changes identifies the counter-ion, and the sol has the opposite sign.

79. In Goldschmidt aluminothermic process which of the following reducing agents is used: [4 marks]

(1) calcium

(2) coke

(3) Al-powder

(4) sodium

Correct Answer: (3) Al-powder

Solution:

Concept:

- Aluminothermy is reduction of a metal oxide by aluminium. Aluminium is oxidised to the very stable oxide Al_2O_3, while the metal ion in the original oxide is reduced.
- The formation of aluminium oxide releases a large amount of heat. Powdered aluminium supplies a large contact area so the strongly exothermic reaction proceeds rapidly after ignition.

Step 1: Recognise the reducing material The name aluminothermic identifies aluminium as the substance that removes oxygen from the metal oxide. In the Goldschmidt process it is used as finely divided aluminium powder, mixed intimately with the oxide. Aluminium has a strong affinity for oxygen, and the stable oxide it forms provides the thermodynamic driving force. It is a reagent, not merely a catalyst: its oxidation state rises from zero to +3.

Step 2: Check the process with a representative reaction The familiar thermite example is

$$Fe_2O_3 + 2\,Al \longrightarrow Al_2O_3 + 2\,Fe.$$

Iron(III) is reduced to metallic iron and aluminium loses electrons, so aluminium is the reducing agent. The heat released can produce molten metal, which explains the welding application of the reaction. Coke belongs to carbothermic reduction, while calcium and sodium are used for other metallurgical reductions; none defines Goldschmidt aluminothermy. The word powder specifies the practical physical form of the aluminium.

Step 3: Conclusion The reducing agent is aluminium powder. Hence, the correct option is (3).

Quick Tip to Solve: In aluminothermy, write the oxygen transfer to aluminium first; the aluminium becomes alumina and the other metal is liberated.

80. Oxidation state of sulphur in anions SO_3^{2-}, $S_2O_4^{2-}$ and $S_2O_6^{2-}$ increases in the orders: [4 marks]

(1) $S_2O_6^{2-} < S_2O_4^{2-} < SO_3^{2-}$

(2) $SO_3^{2-} < S_2O_4^{2-} < S_2O_6^{2-}$

(3) $S_2O_4^{2-} < SO_3^{2-} < S_2O_6^{2-}$

(4) $S_2O_4^{2-} < S_2O_6^{2-} < SO_3^{2-}$

Correct Answer: (3) $S_2O_4^{2-} < SO_3^{2-} < S_2O_6^{2-}$

Solution:

Concept:

- The sum of oxidation numbers in a polyatomic ion equals its overall charge. Oxygen has oxidation number -2 in each of these oxyanions; these are not peroxides.
- When an ion contains two sulphur atoms, divide their combined oxidation-number contribution by two to obtain the oxidation state per sulphur atom.

Step 1: Calculate each sulphur oxidation state Let the oxidation state of sulphur be x, treating each species separately. For sulphite,

$$x + 3(-2) = -2 \quad \Rightarrow \quad x = +4.$$

For dithionite,

$$2x + 4(-2) = -2 \quad \Rightarrow \quad x = +3.$$

For dithionate,

$$2x + 6(-2) = -2 \quad \Rightarrow \quad x = +5.$$

The factor two in the last two equations is essential because their formula units contain two sulphur atoms.

Step 2: Place the ions in increasing order The calculated values increase as $+3 < +4 < +5$. Attach each value to its own ion before comparing the offered sequences: dithionite comes first, sulphite comes next, and dithionate comes last. The first choice incorrectly makes dithionate lowest; the second interchanges the first two values; the fourth places sulphite above dithionate. Counting oxygen atoms alone without also counting sulphur atoms would give the wrong comparison.

Step 3: Conclusion The order is $S_2O_4^{2-} < SO_3^{2-} < S_2O_6^{2-}$. Hence, the correct option is (3).

Quick Tip to Solve: For an oxyanion, solve the charge-balance equation per central atom; do not compare the total contribution from two atoms with that from one.

81. XeO_4 molecule is tetrahedral having: [4 marks]

(1) Two $p\pi - d\pi$ bonds

(2) One $p\pi - d\pi$ bonds

(3) Four $p\pi - d\pi$ bonds

(4) Three $p\pi - d\pi$ bonds

Correct Answer: (3) Four $p\pi - d\pi$ bonds

Solution:

Concept:

- In the conventional bonding description used by this question, XeO_4 has four equivalent Xe=O bonds. Each double bond consists of one sigma component and one pi component.
- The four sigma bonds define a tetrahedral arrangement around xenon. The pi components are described in the traditional model as $p\pi - d\pi$ bonding between oxygen and xenon.

Step 1: Count the equivalent xenon–oxygen linkages Xenon tetroxide contains one central xenon atom surrounded by four oxygen atoms. Its conventional Lewis representation has a double bond from xenon to each oxygen and no stereochemically active lone pair on xenon. Thus the electron-domain arrangement of the four bonding directions is tetrahedral. The geometry counts four equivalent bond directions; it does not imply that only one or two oxygens participate in multiple bonding.

Step 2: Separate sigma and pi components Every conventional Xe=O double bond contributes one sigma bond along the internuclear axis and one pi bond. Four such double bonds therefore give four sigma components and four pi components. In the terminology of the offered choices these are four $p\pi - d\pi$ bonds. The alternatives with one, two or three pi bonds would arbitrarily leave some of the four equivalent oxygen linkages without the indicated pi component and do not match this conventional description. The answer follows the historical orbital model explicitly used in the question; the tetrahedral shape follows from the four bonding domains.

Step 3: Conclusion In the stated conventional model, XeO_4 has four $p\pi - d\pi$ bonds. Hence, the correct option is (3).

Quick Tip to Solve: When a question counts pi bonds in a conventional oxo structure, count one pi component for each central-atom–oxygen double bond.

82. Which of the following is diamagnetic? [4 marks]

(1) $[Fe(CN)_6]^{3-}$

(2) $[Co(ox)_3]^{3-}$

(3) $[FeF_6]^{3-}$

(4) $[CoF_6]^{3-}$

Correct Answer: (2) $[Co(ox)_3]^{3-}$

Solution:

Concept:

- A complex is diamagnetic only when all its electrons are paired. Determine the metal oxidation state and d-electron count before filling the split d orbitals.
- For an octahedral complex the lower orbitals are t_{2g} and the upper orbitals are e_g. Whether electrons pair depends on the competition between ligand-field splitting and pairing energy.

Step 1: Identify the cobalt(III) oxalato complex Each oxalato ligand, ox, is $C_2O_4^{2-}$. Three such ligands contribute -6; an overall charge of -3 therefore means cobalt is +3. Cobalt(III) is $3d^6$. In the tris(oxalato)cobaltate(III) complex the splitting is sufficient for the low-spin arrangement

$$t_{2g}^6 e_g^0.$$

All six d electrons are paired, giving no net spin contribution and hence diamagnetism.

Step 2: Eliminate the remaining complexes Both iron complexes contain Fe^{3+}, which is d^5. Strong-field cyanide gives low-spin t_{2g}^5, but this still leaves one unpaired electron, so it is paramagnetic. Weak-field fluoride gives high-spin d^5 with five unpaired electrons. In $[CoF_6]^{3-}$, cobalt(III) is again d^6, but fluoride produces the high-spin arrangement $t_{2g}^4 e_g^2$, with four unpaired electrons. Thus neither a strong ligand alone nor an even d-electron count alone establishes diamagnetism; actual orbital occupancy decides it.

Step 3: Conclusion $[Co(ox)_3]^{3-}$ has all electrons paired and is diamagnetic. Hence, the correct option is (2).

Quick Tip to Solve: Low-spin d5 still has one unpaired electron; low-spin d6 is fully paired. Always fill the orbitals before assigning magnetism.

83. Which of the following statements is incorrect? [4 marks]

(1) Fe^{2+} ion also gives blood red colour with SCN^- ion.

(2) Fe^{3+} ion also gives blood red colour with SCN^- ion.

(3) On passing H_2S into Na_2ZnO_2 solution a white ppt of ZnS is formed.

(4) Cupric ion reacts with excess of ammonia solution to give deep blue colour of $[Cu(NH_3)_4]^{2+}$ ion.

Correct Answer: (1) Fe^{2+} ion also gives blood red colour with SCN^- ion.

Solution:

Concept:

- Qualitative tests depend on the oxidation state of an ion as well as its element. The blood-red thiocyanato complex is characteristic of iron(III), not iron(II).
- Precipitation of a sparingly soluble sulphide and formation of a coloured coordination complex are distinct observations. Test every offered statement because the question asks for the incorrect one.

Step 1: Check the thiocyanate statements Iron(III) reacts with thiocyanate to form intensely blood-red thiocyanato species, commonly represented by

$$Fe^{3+} + SCN^- \rightleftharpoons [FeSCN]^{2+}.$$

Iron(II) does not directly give this characteristic blood-red test. If an iron(II) sample produces red colour after oxidation, the responding species is the iron(III) formed, not an intrinsic iron(II) thiocyanate test. Therefore the first statement is incorrect while the second expresses the accepted ferric-ion test.

Step 2: Verify the other observations Hydrogen sulphide converts zincate into white zinc sulphide; the observation can be summarised as

$$Na_2ZnO_2 + 2\,H_2S \longrightarrow ZnS + Na_2S + 2\,H_2O.$$

The precipitate is white, so the third statement is consistent with zinc chemistry. Excess ammonia coordinates to cupric ion, producing the characteristic deep-blue ammine complex represented by $[Cu(NH_3)_4]^{2+}$; water ligands may be included in a fuller hydrated formula. Thus the fourth statement is also correct. Only the claim assigning the blood-red thiocyanate test to ferrous ion fails.

Step 3: Conclusion The statement about Fe^{2+} giving a blood-red colour with SCN^- is incorrect. Hence, the correct option is (1).

Quick Tip to Solve: For the blood-red thiocyanate test, remember ferric iron: Fe(III). Distinguish it from reactions of ferrous Fe(II).

84. The Wurtz-Fittig reaction involves condensation of: [4 marks]

(1) two molecules of aryl halides

(2) one molecule of each of aryl-halide and alkyl-halide

(3) one molecule of each of aryl-halide and phenol

(4) two molecules of aralkyl-halides

Correct Answer: (2) one molecule of each of aryl-halide and alkyl-halide

Solution:

Concept:

- The Wurtz–Fittig reaction couples an aryl halide with an alkyl halide using sodium metal in dry ether. A new carbon–carbon bond joins the aryl group to the alkyl group.
- The related names distinguish the coupling partners: Wurtz uses alkyl halides, Fittig uses aryl halides, and Wurtz–Fittig uses one of each.

Step 1: Write the mixed coupling reaction Let Ar denote an aryl group, R an alkyl group and X a halogen. The characteristic net reaction is

$$\mathrm{Ar{-}X + R{-}X + 2\,Na \longrightarrow Ar{-}R + 2\,NaX.}$$

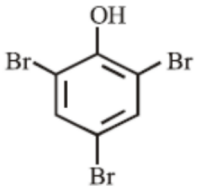
It is carried out in dry ether. The product is an alkyl-substituted aromatic compound because a carbon of the aromatic group becomes bonded to the carbon originally bearing halogen in the alkyl halide. For example, bromobenzene and methyl bromide can give toluene under these conditions.

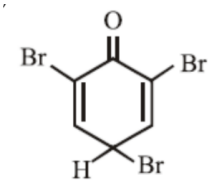
Step 2: Distinguish the other proposed reactant pairs Coupling two aryl-halide molecules is described as the Fittig reaction and gives a biaryl rather than the mixed aryl–alkyl product. Phenol has a hydroxyl group and is not the aryl-halide partner required in this named reaction; its reaction with sodium is primarily acid–metal chemistry. Two aralkyl halides have their halogen on side-chain carbon, so they do not supply the required pair consisting of an aryl halide and an alkyl halide. The mixed reactant pair in the second statement is the defining combination, regardless of possible side reactions in practical preparations.

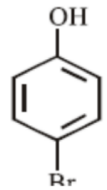
Step 3: Conclusion The reaction couples one molecule of an aryl halide with one molecule of an alkyl halide. Hence, the correct option is (2).

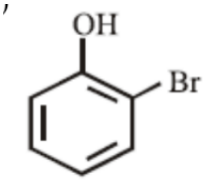
Quick Tip to Solve: Use the names as a pairing guide: Wurtz is alkyl–alkyl, Fittig is aryl–aryl, and Wurtz–Fittig is aryl–alkyl.

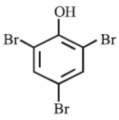
85. What is the structure of the major product when phenol is treated with bromine water? [4 marks]

(1) 

(2) 

(3) 

(4) 

Correct Answer: (1) 

Solution:

Concept:

- The hydroxyl group activates the benzene ring by resonance donation of an oxygen lone pair, increasing electron density especially at the two ortho positions and the para position.
- Bromine water is aqueous Br_2. Phenol reacts readily under these conditions without a Lewis-acid catalyst, and substitution can occur at all three activated positions.

Step 1: Identify the positions activated by the hydroxyl group Number the carbon bearing OH as carbon 1. Donation from oxygen stabilises the intermediate formed when an electrophile attacks carbon 2, 4 or 6. These are the two ortho positions and the para position. The strong activating effect of phenol is why bromine water reacts much more readily with it than with benzene; adding $FeBr_3$ is unnecessary.

Step 2: Apply the aqueous bromination conditions With bromine water the reaction does not stop mainly at one substituted position. Hydrogen at each of positions 2, 4 and 6 is replaced by bromine, giving a white precipitate of 2,4,6-tribromophenol. The overall substitution is

$$C_6H_5OH + 3\,Br_2 \longrightarrow C_6H_2Br_3OH + 3\,HBr.$$

The phenolic hydroxyl group remains attached to the aromatic ring.

Step 3: Compare the four structures The first structure retains the aromatic ring and OH group and places Br at both ortho positions and at the para position, exactly as required. The third and fourth structures are only monobrominated phenols, so they do not represent the principal product under aqueous bromination conditions. The second has a carbonyl group and a non-aromatic ring; that is not the substitution product asked for here.

Step 4: Conclusion The major product is 2,4,6-tribromophenol, the first structure. Hence, the correct option is (1).

Quick Tip to Solve: For phenol with bromine water, look for bromine at both ortho positions and the para position while the phenolic OH remains unchanged.

86. Amongst the following alcohols which would react fastest with conc. HCl and $ZnCl_2$? [4 marks]

(1) pentanol

(2) 2-methyl butanol

(3) 2-pentanol

(4) 2-methyl butan-2-ol

Correct Answer: (4) 2-methyl butan-2-ol

Solution:

Concept:

- Concentrated HCl with anhydrous $ZnCl_2$ is Lucas reagent. The Lewis acid activates the alcoholic hydroxyl group so that it can be displaced by chloride.
- For comparable simple alcohols at room temperature, the usual Lucas-test reactivity order is tertiary greater than secondary greater than primary; the tertiary carbocation is stabilised most strongly.

Step 1: Classify the carbon bearing the hydroxyl group In pentanol and 2-methyl butanol, the hydroxyl-bearing carbon is primary in the listed alcohols. In 2-pentanol it is bonded to two carbon groups and one hydrogen, so this alcohol is secondary. In 2-methylbutan-2-ol, carbon 2 bears both the hydroxyl group and three carbon substituents: two methyl groups and one ethyl group. It is therefore a tertiary alcohol.

Step 2: Relate the rate to the substitution mechanism Coordination of oxygen with $ZnCl_2$, together with protonation in concentrated acid, makes departure of the hydroxyl-derived leaving group easier. A tertiary alcohol then forms a relatively stable tertiary carbocation, which is rapidly captured by chloride. The reaction can be represented generally as

$$R_3COH + HCl \xrightarrow{ZnCl_2} R_3CCl + H_2O.$$

Alkyl groups stabilise the positive charge through inductive donation and hyperconjugation.

Step 3: Eliminate the slower choices The two primary alcohols cannot form comparably stable carbocations and react much more slowly under the standard test conditions. The secondary alcohol reacts faster than a primary one, but its secondary carbocation is less stabilised than the tertiary intermediate from 2-methylbutan-2-ol. Merely having a branched carbon elsewhere in the chain does not make the alcohol tertiary; classification must be made at the carbon directly bearing the hydroxyl group.

Step 4: Conclusion 2-Methylbutan-2-ol reacts fastest with concentrated HCl and $ZnCl_2$. Hence, the correct option is (4).

Quick Tip to Solve: In a Lucas-reagent question, classify the carbon attached directly to OH; a tertiary alcohol is normally the fastest of primary, secondary and tertiary choices.

87. In Williamson synthesis of mixed ether having a primary and a tertiary alkyl group if tertiary halide is used, then [4 marks]

(1) Rate of reaction will be slow due to slow cleavage of carbon-halogen bond.

(2) Alkene will be the main product.

(3) Simple ether will form instead of mixed ether.

(4) Expected mixed ether will be formed.

Correct Answer: (2) Alkene will be the main product.

Solution:

Concept:

- Williamson ether synthesis normally couples an alkoxide ion with an unhindered alkyl halide by the S_N2 mechanism.
- A tertiary alkyl halide is sterically hindered toward backside substitution. With a strongly basic alkoxide, removal of a beta hydrogen and elimination of halide favour alkene formation.

Step 1: Identify the unsuitable partner The desired ether contains a primary alkyl group and a tertiary alkyl group. If the tertiary group is supplied as the alkyl halide, the carbon bonded to halogen has three carbon substituents around it. The alkoxide nucleophile cannot approach that crowded carbon effectively for the single-step backside displacement required in Williamson synthesis.

Step 2: Follow the competing elimination reaction An alkoxide is both a nucleophile and a strong base. Instead of displacing halide at the tertiary carbon, it abstracts a hydrogen from a neighbouring carbon. The carbon-hydrogen bond electrons form a double bond as halide leaves in an $E2$ elimination. For example, a tert-butyl halide with an alkoxide gives isobutene:

$$(CH_3)_3CBr + RO^- \longrightarrow (CH_3)_2C{=}CH_2 + ROH + Br^-.$$

This example illustrates the general reason that an alkene, rather than the planned ether, dominates.

Step 3: Select the product and reject the alternatives The difficulty is not simply a slow cleavage of the carbon-halogen bond followed by useful ether formation. The strong base opens a different reaction pathway. Nor does the reaction automatically rearrange the reagents into a simple ether: elimination consumes the tertiary halide. To prepare the intended mixed ether successfully, one would normally reverse the roles, using a tertiary alkoxide and a primary alkyl halide so that substitution occurs at the accessible primary carbon.

Step 4: Conclusion When the tertiary alkyl group is introduced as the halide, an alkene is the main product. Hence, the correct option is (2).

Quick Tip to Solve: For an unsymmetrical Williamson ether, place the less hindered group in the alkyl halide; a tertiary halide with alkoxide usually eliminates.

88. The polymer used for optical lenses is [4 marks]

(1) polypropylene

(2) polyvinyl chloride

(3) polythene

(4) polymethyl methacrylate

Correct Answer: (4) polymethyl methacrylate

Solution:

Concept:

- An optical lens material must transmit visible light well and maintain a clear, smooth surface. Transparency is a key property distinguishing the appropriate polymer.
- Polymethyl methacrylate, abbreviated PMMA, is a transparent acrylic polymer used as an optical material and as an alternative to glass in suitable applications.

Step 1: Match the required use with the polymer property The question asks for the polymer used for optical lenses, so the decisive requirement is optical clarity rather than simply flexibility, toughness or low cost. A useful lens material must allow light to pass with little scattering and be capable of being shaped into a stable optical surface. PMMA is well known for these properties.

Step 2: Identify the acrylic polymer Polymethyl methacrylate is formed by addition polymerisation of methyl methacrylate. The carbon-carbon double bond opens during polymerisation while the ester substituent remains on the chain:

$$n\,CH_2{=}C(CH_3)COOCH_3 \longrightarrow \left[-CH_2-C(CH_3)(COOCH_3)-\right]_n.$$

Its transparent, rigid acrylic character is consistent with use in optical components. The ester groups are part of the substituents, so this is not a condensation polymerisation with elimination of a small molecule.

Step 3: Distinguish the other listed polymers Polypropylene and polythene are polyolefins widely used for containers, films and other general-purpose plastic articles; they are not the standard answer for optical lenses in this comparison. Polyvinyl chloride is commonly associated with pipes, coverings and related products. Although additives and processing can modify polymer properties, the named optical-use polymer among these four choices is PMMA. The choice follows its characteristic transparency, rather than assuming all plastic materials behave alike.

Step 4: Conclusion The polymer used for optical lenses is polymethyl methacrylate (PMMA). Hence, the correct option is (4).

Quick Tip to Solve: Associate PMMA with transparent acrylic optical materials; distinguish it from the common packaging and construction plastics listed alongside it.

89. Which of the following statements about aspirin is not true? [4 marks]

(1) It is effective in relieving pain

(2) It is a neurologically active drug

(3) It has antiblood clotting action

(4) It belongs to narcotic analgesics

Correct Answer: (4) It belongs to narcotic analgesics

Solution:

Concept:

- Analgesics relieve pain. Aspirin is a non-narcotic analgesic, in contrast to opioid or narcotic analgesics such as morphine.
- Aspirin also has antipyretic and anti-inflammatory actions and inhibits platelet aggregation, accounting for its antiblood-clotting action.

Step 1: Read the negative wording carefully The required choice is the statement that is not true, so a genuine property of aspirin must be rejected as the answer. Aspirin is acetylsalicylic acid and is a familiar pain-relieving medicine. The statement that it is effective in relieving pain is therefore true; it cannot be the requested false statement.

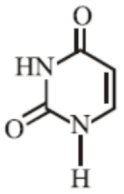
Step 2: Check the other two functional descriptions Aspirin is included among neurologically active drugs in the textbook grouping that includes analgesics. This broad classification does not make it a narcotic. It also reduces platelet aggregation by inhibiting cyclooxygenase-dependent formation of thromboxane. Consequently, the listed antiblood-clotting action is a true property. This action concerns the tendency of platelets to aggregate, rather than aspirin being an opioid painkiller.

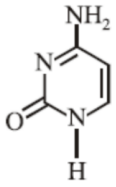
Step 3: Identify the incorrect classification The classification of an analgesic depends on its nature and mode of action, not merely on whether it reduces pain. Aspirin belongs to the non-narcotic class. Narcotic analgesics such as morphine act through opioid pathways and have a different characteristic profile. Thus the fourth statement directly assigns aspirin to the wrong category. None of the other statements negates its recognised analgesic, broad neurologically active, or antiplatelet properties. Keeping the classification separate from the therapeutic effect prevents confusing all pain-relieving drugs with narcotics.

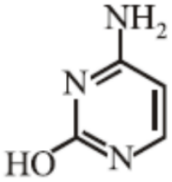
Step 4: Conclusion The false statement is that aspirin belongs to narcotic analgesics; it is a non-narcotic analgesic. Hence, the correct option is (4).

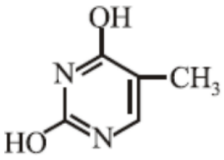
Quick Tip to Solve: Underline "not true" first, then separate the fact that a drug relieves pain from whether it is classified as narcotic or non-narcotic.

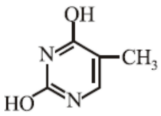
90. Which of the following structures represents thymine ? [4 marks]

(1) 

(2) 

(3) 

(4) 

Correct Answer: (4) 

Solution:

Concept:

- Thymine is 5-methyluracil, a pyrimidine base. Its identifying features are a six-membered ring with nitrogen at positions 1 and 3 and a methyl substituent at position 5.
- Keto-enol tautomerism shifts hydrogen atoms and double bonds without changing the molecular formula or the carbon-nitrogen skeleton. A drawn enol tautomer can therefore represent thymine even though the common biological form is predominantly keto.

Step 1: Identify the pyrimidine skeleton Inspect each six-membered ring and locate its two nitrogen atoms. The first three structures have no methyl group on the ring. The fourth has a CH_3 substituent on the carbon adjacent to the upper hydroxyl-bearing carbon. Numbering the pyrimidine ring from nitrogen gives this methyl-bearing carbon position 5, the distinctive substitution of thymine.

Step 2: Recognise the tautomer shown The usual keto representation of thymine has carbonyl groups at positions 2 and 4. The fourth drawing instead shows hydroxyl groups at those positions with shifted double bonds and unprotonated ring nitrogens: it is the corresponding enol representation. Proton transfers from the oxygen-hydrogen groups to ring nitrogens, accompanied by shifting of the pi bonds, give the familiar 5-methylpyrimidine-2,4-dione form. No methyl group is added or removed in this tautomerisation.

Step 3: Compare the distractors The first structure is the keto form of uracil and lacks the position-5 methyl group. The second is a cytosine-type keto structure with an amino group in place of one oxygen functionality. The third is its corresponding hydroxyl or enol representation and likewise retains an amino group and lacks the methyl substituent. Only the fourth retains the two oxygen functionalities and the 5-methyl group needed for thymine. Its less familiar tautomeric appearance does not change that identification.

Step 4: Conclusion The fourth structure represents the enol tautomer of thymine (5-methyluracil). Hence, the correct option is (4).

Quick Tip to Solve: Identify thymine by the 5-methyl group on the uracil skeleton; allow for a shift between keto and enol drawings before rejecting a structure.