

JEE Main 2013 Paper 1 (B.E./B.Tech) Apr 23 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 360

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2013, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 23 April 2013, Single slot. The total duration is 3 hours.
- (ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.
- (iii) Each multiple choice question (MCQ) has four options, of which only one is correct.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

1. The least integral value α of x such that $\frac{x-5}{x^2+5x-14} > 0$, satisfies:

[4 marks]

(A) $\alpha^2 + 3\alpha - 4 = 0$

(B) $\alpha^2 - 5\alpha + 4 = 0$

(C) $\alpha^2 - 7\alpha + 6 = 0$

(D) $\alpha^2 + 5\alpha - 6 = 0$

Correct Answer: (D) $\alpha^2 + 5\alpha - 6 = 0$

Solution:

Concept:

- The sign of a rational function changes at a simple zero or simple pole. The denominator must be nonzero throughout its domain.
- Factoring the quadratic identifies all critical points, after which the strict inequality is checked on each intervening interval.

Step 1: Factor and order the critical values The denominator factors as $x^2 + 5x - 14 = (x+7)(x-2)$. Hence the expression is

$$(x-5)/((x+7)(x-2)).$$

The ordered critical values are $-7, 2, 5$. The first two are excluded poles; 5 is a zero excluded by the strict inequality. These exclusions must be made before taking an integral value.

Step 2: Construct the sign intervals For $x < -7$, the numerator is negative and the denominator positive, so the ratio is negative. On $(-7, 2)$, both numerator and denominator are negative, giving a positive ratio. On $(2, 5)$, the ratio is negative, and on $(5, \infty)$, it is positive. Thus

$$x \in (-7, 2) \cup (5, \infty).$$

The least permitted integer is therefore $\alpha = -6$, rather than the excluded endpoint -7 .

Step 3: Test the proposed equations Substitution of -6 gives $36 - 18 - 4 = 14$ in (A), $36 + 30 + 4 = 70$ in (B), and $36 + 42 + 6 = 84$ in (C). For (D),

$$\alpha^2 + 5\alpha - 6 = 36 - 30 - 6 = 0.$$

Only the fourth equation is satisfied.

Step 4: Conclusion The least integral value is $\boxed{-6}$, which satisfies $\alpha^2 + 5\alpha - 6 = 0$. Hence, the correct option is (D). The supplied key selects the first alternative, but the printed inequality admits -6 as its least integer and $(-6)^2 + 3(-6) - 4 = 14$, so the literal question proves (D).

Quick Tip to Solve: Place the numerator zero and both denominator zeros on one number line; exclude poles before choosing the least integer.

2. Let $a = \operatorname{Im} \left(\frac{1+z^2}{2iz} \right)$, where z is any non-zero complex number. The set $A = \{a : |z| = 1 \text{ and } z \neq \pm 1\}$ is equal to [4 marks]

- (A) $(-1, 1)$
- (B) $[-1, 1]$
- (C) $[0, 1)$
- (D) $(-1, 0]$

Correct Answer: (A) $(-1, 1)$

Solution:

Concept:

- For a complex number on the unit circle, write $z = e^{i\theta} = \cos \theta + i \sin \theta$, where θ is real. Then $z^{-1} = e^{-i\theta}$, and $z + z^{-1} = 2 \cos \theta$.
- The imaginary part of $u + iv$, with real u, v , is v . In particular $1/i = -i$, so multiplication by $1/i$ changes a real quantity into a purely imaginary quantity.

Step 1: Simplify the expression on the unit circle Since $z \neq 0$, divide both terms in the numerator by z before taking the imaginary part. This avoids expanding a quotient in Cartesian coordinates. We obtain

$$\frac{1 + z^2}{2iz} = \frac{z + z^{-1}}{2i} = \frac{2 \cos \theta}{2i} = -i \cos \theta.$$

Thus $a = -\cos \theta$, which is real. The minus sign comes from $1/i = -i$; it must not be lost when extracting the imaginary part.

Step 2: Apply the exclusions and determine the entire range On the unit circle, $\cos \theta$ takes every value from -1 to 1 . The endpoint $a = -1$ requires $\cos \theta = 1$, hence $z = 1$; the endpoint $a = 1$ requires $\cos \theta = -1$, hence $z = -1$. Both points are explicitly excluded. Every interior value remains attainable: for any $-1 < a < 1$, choose $\theta = \arccos(-a)$, which lies strictly between 0 and π . This gives an allowed z . Therefore the range is an open interval. Option (B) wrongly includes its endpoints. Options (C) and (D) each omit half the attainable values; for example $a = 1/2$ and $a = -1/2$ are both possible.

Step 3: Conclusion The required set is $A = (-1, 1)$. Hence, the correct option is (A).

Quick Tip to Solve: On $|z| = 1$, replace $1/z$ by $\bar{z}$. Then check which excluded points produce the endpoints of the range.

3. The sum of the series: $(2)^2 + 2(4)^2 + 3(6)^2 + \dots$ up to 10 terms is

[4 marks]

- (A) 11300
- (B) 11200
- (C) 12100
- (D) 12300

Correct Answer: (C) 12100

Solution:

Concept:

- The terms shown have a multiplier k and an even number $2k$ being squared, so the k -th term is $k(2k)^2 = 4k^3$, for $k = 1, 2, \dots, 10$.
- The sum of the first n cubes is $\sum_{k=1}^n k^3 = \left[\frac{n(n+1)}{2} \right]^2$. The bracket is the sum of the first n positive integers.

Step 1: Express the displayed pattern with a summation The first term is $1 \times 2^2 = 4$, the second is $2 \times 4^2 = 32$, and the third is $3 \times 6^2 = 108$. Thus the number outside the square and half the number inside it increase together. This is not the sum of the squares of even integers alone, because the successive multipliers $1, 2, 3, \dots$ must also be retained. Consequently,

$$S_{10} = \sum_{k=1}^{10} k(2k)^2 = 4 \sum_{k=1}^{10} k^3.$$

Ten terms means the final index is $k = 10$, with final term $10 \times 20^2 = 4000$.

Step 2: Evaluate the sum of cubes and compare the options Insert $n = 10$ into the cube-sum identity, square the whole triangular number, and only then multiply by four:

$$\sum_{k=1}^{10} k^3 = \left(\frac{10 \cdot 11}{2} \right)^2 = 55^2 = 3025,$$

$$S_{10} = 4 \cdot 3025 = 12100.$$

An equivalent arithmetic check is $4[10 \cdot 11/2]^2 = (10 \cdot 11)^2 = 110^2$. It produces the same integer without a decimal approximation. The other offered values, 11300, 11200 and 12300, do not equal this exact sum, so they are excluded rather than chosen by the approximate size of the last term.

Step 3: Conclusion The sum of the first ten terms is 12100. Hence, the correct option is (C).

Quick Tip to Solve: Combine the multiplier k with $(2k)^2$ first: it becomes $4k^3$, making the cube-sum formula immediately applicable.

4. If $a_1, a_2, a_3, \dots, a_n, \dots$ are in A.P. such that $a_4 - a_7 + a_{10} = m$, then the sum of first 13 terms of this A.P. is [4 marks]

- (A) 10 m
- (B) 12 m
- (C) 13 m
- (D) 15 m

Correct Answer: (C) 13 m

Solution:

Concept:

- In an arithmetic progression with first term a and common difference d , the n -th term is $a_n = a + (n - 1)d$.
- The sum of the first n terms is $S_n = \frac{n}{2}[2a + (n - 1)d]$. When n is odd, this equals n times the middle term.

Step 1: Identify the information contained in the given relation Write all three terms using the same first term and common difference. Their positions are important: $a_4 = a + 3d$, $a_7 = a + 6d$, and $a_{10} = a + 9d$. Therefore,

$$a_4 - a_7 + a_{10} = (a + 3d) - (a + 6d) + (a + 9d) = a + 6d.$$

The expression equals the seventh term itself, because the fourth and tenth terms are equally spaced about the seventh term. The hypothesis thus states $a_7 = a + 6d = m$. It determines the middle term needed for the sum, even though a and d are not determined separately.

Step 2: Use the seventh term to calculate the sum There are thirteen terms, so the middle term is the seventh. Apply the sum formula directly:

$$S_{13} = \frac{13}{2}[2a + 12d] = 13(a + 6d) = 13m.$$

As a separate check, pair a_1 with a_{13} , a_2 with a_{12} , and continue through a_6 with a_8 . Each of these six pairs sums to $2a_7 = 2m$; the unpaired middle term contributes another m . Hence the total is $6(2m) + m = 13m$. The coefficients 10, 12, and 15 in the other options do not match the thirteen equal middle-term contributions.

Step 3: Conclusion The required sum is $S_{13} = 13m$. Hence, the correct option is (C).

Quick Tip to Solve: For an odd number of A.P. terms, find the middle term first; the sum is the number of terms multiplied by that middle term.

5. The sum of the rational terms in the binomial expansion of $(2^{1/2} + 3^{1/5})^{10}$ is:

[4 marks]

- (A) 25
- (B) 32
- (C) 9
- (D) 41

Correct Answer: (D) 41

Solution:

Concept:

- The general term of $(a + b)^{10}$ is $T_{k+1} = \binom{10}{k} a^{10-k} b^k$, for integers $0 \leq k \leq 10$.
- A positive product of powers of distinct primes is rational only when each prime exponent is integral.

Step 1: Write the general term with prime powers Choose $a = 2^{1/2}$ and $b = 3^{1/5}$. Then

$$T_{k+1} = \binom{10}{k} 2^{(10-k)/2} 3^{k/5}.$$

The binomial coefficient is an integer, so the possible irrationality comes from the prime powers. Since 2 and 3 are distinct primes, an irrational power of one cannot be cancelled by a power of the other.

Step 2: Impose both integer-exponent conditions For the power of 2 to be integral, $10 - k$ must be even, so k is even. For the power of 3 to be integral, k must be divisible by 5. Among $0, 1, \dots, 10$, their common solutions are $k = 0$ and $k = 10$. The intermediate candidate $k = 5$ leaves the half-integral power $2^{5/2}$, so it is not a rational term.

Step 3: Add the two rational terms The endpoint terms are

$$T_1 = 2^5 = 32, \quad T_{11} = 3^2 = 9.$$

Their sum is $32 + 9 = 41$. The alternatives 32 and 9 each retain only one endpoint term; 25 is not the sum of the rational terms. There are no additional terms meeting both exponent conditions.

Step 4: Conclusion The sum of the rational terms is 41. Hence, the correct option is (D).

Quick Tip to Solve: For radicals of different primes, require each exponent to be integral separately; here k must be even and divisible by five.

6. The number of solutions of the equation $\sin 2x - 2 \cos x + 4 \sin x = 4$ in the interval $[0, 5\pi]$ is [4 marks]

- (A) 3
- (B) 5
- (C) 4
- (D) 6

Correct Answer: (A) 3

Solution:

Concept:

- The double-angle identity $\sin 2x = 2 \sin x \cos x$ converts the equation into an expression that can be factored by grouping.
- For real x , $-1 \leq \cos x \leq 1$, so $\cos x + 2$ is always positive. The equation $\sin x = 1$ has solutions $x = \pi/2 + 2k\pi$, where k is an integer.

Step 1: Factor the equation without losing or adding solutions Move the constant on the right to the left and substitute the double-angle identity:

$$2 \sin x \cos x - 2 \cos x + 4 \sin x - 4 = 0.$$

Group the first two terms and the last two terms. Both groups contain $\sin x - 1$, giving

$$2 \cos x (\sin x - 1) + 4 (\sin x - 1) = 0,$$

$$2 (\sin x - 1) (\cos x + 2) = 0.$$

The second factor cannot vanish for any real x , since it lies in $[1, 3]$. Thus all solutions, and only the solutions, come from $\sin x = 1$. No division by a potentially zero sine or cosine has been made.

Step 2: Count the permissible values of the integer parameter Use the complete sine solution and impose both endpoints of the stated closed interval:

$$0 \leq \frac{\pi}{2} + 2k\pi \leq 5\pi \iff -\frac{1}{4} \leq k \leq \frac{9}{4}.$$

The only integers in this range are 0, 1, 2, yielding $x = \pi/2, 5\pi/2, 9\pi/2$. Neither endpoint 0 nor 5π has sine equal to one, so there is no additional endpoint solution. Each listed angle satisfies the factored equation and hence the original equation. There are exactly three solutions, ruling out the offered counts five, four and six.

Step 3: Conclusion There are 3 solutions in $[0, 5\pi]$. Hence, the correct option is (A).

Quick Tip to Solve: Factor by $\sin x - 1$ before counting roots; the apparent second branch $\cos x = -2$ contributes no real solutions.

7. If two lines L_1 and L_2 in space, are defined by

$$L_1 = \{x = \sqrt{\lambda}y + (\sqrt{\lambda} - 1), \quad z = (\sqrt{\lambda} - 1)y + \sqrt{\lambda}\},$$

$$L_2 = \{x = \sqrt{\mu}y + (1 - \sqrt{\mu}), \quad z = (1 - \sqrt{\mu})y + \sqrt{\mu}\}$$

then L_1 is perpendicular to L_2 , for all nonnegative reals λ and μ , such that:

[4 marks]

(A) $\sqrt{\lambda} + \sqrt{\mu} = 1$

(B) $\lambda \neq \mu$

(C) $\lambda + \mu = 0$

(D) $\lambda = \mu$

Correct Answer: (C) $\lambda + \mu = 0$

Solution:

Concept:

- Taking a free parameter in a Cartesian line description gives a direction vector from its coefficients.
- Two directions are perpendicular precisely when their dot product is zero; nonnegative square roots cannot cancel each other unless both vanish.

Step 1: Extract direction vectors Put $a = \sqrt{\lambda}$ and $b = \sqrt{\mu}$, with $a, b \geq 0$. Using y as the parameter, the first line has direction $\mathbf{d}_1 = (a, 1, a - 1)$, and the second has direction $\mathbf{d}_2 = (b, 1, 1 - b)$. The constant terms only locate the lines and do not enter the angle between their directions.

Step 2: Apply the zero-dot-product condition The dot product simplifies to

$$\begin{aligned}\mathbf{d}_1 \cdot \mathbf{d}_2 &= ab + 1 + (a - 1)(1 - b) \\ &= ab + 1 + a - 1 - ab + b \\ &= a + b.\end{aligned}$$

Thus perpendicularity requires $\sqrt{\lambda} + \sqrt{\mu} = 0$. Since both summands are nonnegative, this holds exactly when $\lambda = \mu = 0$, or equivalently $\lambda + \mu = 0$.

Step 3: Check the alternatives Option (A) makes the dot product 1, not zero. Merely taking unequal parameters in (B) does not make it zero. Equality in (D) includes positive equal parameters, for example $\lambda = \mu = 1$, whose dot product is 2. Option (C) forces both parameters to vanish and gives direction vectors $(0, 1, -1)$ and $(0, 1, 1)$, which are indeed orthogonal.

Step 4: Conclusion The necessary and sufficient condition is $\lambda + \mu = 0$. Hence, the correct option is (C). The supplied key selects the equality-only alternative, but equality alone permits positive equal parameters whose dot product is nonzero; the printed equations require (C).

Quick Tip to Solve: Read direction ratios from the coefficient of the free parameter, then simplify the dot product before considering the square roots.

8. Let θ_1 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_2 = 0$ and θ_2 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_3 = 0$, where c_1, c_2, c_3 are any real numbers. Statement-1: If c_2 and c_3 are proportional, then $\theta_1 = \theta_2$. Statement-2: $\theta_1 = \theta_2$ for all c_2 and c_3 .

[4 marks]

- (A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement-1.
 (B) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation of Statement-1.
 (C) Statement-1 is false; Statement-2 is true.
 (D) Statement-1 is true; Statement-2 is false.

Correct Answer: (A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement-1.

Solution:

Concept:

- The direction of a line $Ax + By + C = 0$ depends on A, B , while changing C translates the line parallel to itself.
- For slopes m_1, m_2 , the acute angle satisfies $\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$.

Step 1: Compare the directions Both angles use the first line with slope $-2/3$. The second line in each pair has slope $1/5$, irrespective of whether its constant is c_2 or c_3 . In particular, neither constant can change the direction. Therefore translating either of these lines never changes the angle it makes with the fixed first line.

Step 2: Check both statements and their connection For either pair, the common acute angle is determined by

$$\tan \theta = \left| \frac{1/5 + 2/3}{1 - 2/15} \right| = \frac{13/15}{13/15} = 1.$$

Thus both angles are 45° under the acute-angle convention; the same equality holds if the corresponding obtuse angles are selected. Statement-2 is true for every choice of the constants. The proportional case of Statement-1 is a special case of that universal result, so Statement-1 is true and Statement-2 explains it. The alternative saying both are true without explanation fails, as do the alternatives making either statement false.

Step 3: Conclusion Both statements are true, and Statement-2 correctly explains Statement-1. Hence, the correct option is (A).

Quick Tip to Solve: Constants translate a straight line; compare the coefficients of x, y before doing any angle calculation.

9. If the circle $x^2 + y^2 - 6x - 8y + (25 - a^2) = 0$ touches the axis of x , then a equals.

[4 marks]

- (A) 0
- (B) ± 4
- (C) ± 2
- (D) ± 3

Correct Answer: (B) ± 4

Solution:

Concept:

- Complete squares to put a circle in the form $(x - h)^2 + (y - k)^2 = r^2$, with centre (h, k) and radius $r \geq 0$.
- A circle tangent to the x -axis has radius equal to the perpendicular distance of its centre from that axis, namely $|k|$.

Step 1: Identify the centre and radius Group the terms involving each coordinate and add the completing-square constants:

$$x^2 - 6x + y^2 - 8y + 25 - a^2 = 0.$$

Since $x^2 - 6x = (x - 3)^2 - 9$ and $y^2 - 8y = (y - 4)^2 - 16$, the equation becomes

$$(x - 3)^2 + (y - 4)^2 = a^2.$$

The centre is therefore $(3, 4)$, and the radius is $\sqrt{a^2} = |a|$. The absolute value matters because a radius cannot be negative even when the parameter is negative.

Step 2: Apply the tangency condition The distance from $(3, 4)$ to the x -axis is 4. Tangency requires $|a| = 4$, giving $a = 4$ or $a = -4$. As a direct check, setting $y = 0$ in the completed-square equation gives $(x - 3)^2 + 16 = a^2$. Exactly one intersection occurs only when $a^2 = 16$, at $(3, 0)$. The value 0 gives a point circle away from the axis; the choices ± 2 and ± 3 give radii smaller than the centre-axis distance and cannot touch it.

Step 3: Conclusion The required parameter values are $a = \pm 4$. Hence, the correct option is (B).

Quick Tip to Solve: For axis tangency, use the coordinate perpendicular to the axis: centre height for the x -axis, centre abscissa for the y -axis.

10. The point of intersection of the normals to the parabola $y^2 = 4x$ at the ends of its latus rectum is: [4 marks]

- (A) (0, 2)
- (B) (3, 0)
- (C) (0, 3)
- (D) (2, 0)

Correct Answer: (B) (3, 0)

Solution:

Concept:

- For $y^2 = 4ax$, the latus rectum lies on $x = a$ and has endpoints $(a, 2a)$, $(a, -2a)$.
- Implicit differentiation gives the tangent slope; the normal slope is its negative reciprocal at an ordinary point.

Step 1: Find the endpoints and normal slopes Here $a = 1$, so the latus-rectum endpoints are $P = (1, 2)$ and $Q = (1, -2)$. Differentiate the equation:

$$2y \frac{dy}{dx} = 4, \quad \frac{dy}{dx} = \frac{2}{y}.$$

At P the tangent slope is 1 and hence the normal slope is -1 . At Q the tangent slope is -1 and hence the normal slope is 1. The two normals are symmetric about the axis of the parabola, which is the x -axis.

Step 2: Intersect the two normal equations The normal through P is

$$y - 2 = -(x - 1), \quad y = 3 - x.$$

The normal through Q is

$$y + 2 = x - 1, \quad y = x - 3.$$

Equating the two values of y gives $3 - x = x - 3$, so $x = 3$ and then $y = 0$. This point satisfies both normal equations. The candidates (0, 2) and (0, 3) are off the symmetry axis and do not satisfy both equations. Although (2, 0) lies on the axis, it gives $0 \neq 3 - 2$ in the upper-endpoint normal, so it also fails.

Step 3: Conclusion The normals meet at (3, 0). Hence, the correct option is (B).

Quick Tip to Solve: The normals at opposite latus-rectum endpoints meet on the parabola axis; symmetry supplies a quick check on the second coordinate.

11. A tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ meets the x -axis at P and the y -axis at Q . Lines PR and QR are drawn such that $OPRQ$ is a rectangle (where O is the origin). Then R lies on: [4 marks]

(A) $\frac{4}{x^2} + \frac{2}{y^2} = 1$

(B) $\frac{2}{x^2} - \frac{4}{y^2} = 1$

(C) $\frac{2}{x^2} + \frac{4}{y^2} = 1$

(D) $\frac{4}{x^2} - \frac{2}{y^2} = 1$

Correct Answer: (D) $\frac{4}{x^2} - \frac{2}{y^2} = 1$

Solution:

Concept:

- A nonvertical tangent $y = mx + c$ to $x^2/a^2 - y^2/b^2 = 1$ satisfies $c^2 = a^2m^2 - b^2$.
- The opposite vertex of a rectangle with adjacent vertices on the coordinate axes has coordinates equal to those two intercepts.

Step 1: Write the tangent condition Let the tangent be $y = mx + c$. Substitution into the given hyperbola gives

$$(2 - 4m^2)x^2 - 8mcx - 4c^2 - 8 = 0.$$

For tangency, its discriminant vanishes, which simplifies to $c^2 = 4m^2 - 2$. Equivalently this is the standard tangent condition with $a^2 = 4, b^2 = 2$. A genuine rectangle with the stated two finite intercepts uses nonzero m, c ; the vertical tangent limiting case does not supply such a pair of intercepts.

Step 2: Express the opposite vertex and eliminate the slope The intercepts are $P = (-c/m, 0)$ and $Q = (0, c)$. Thus write the opposite rectangle vertex as $R = (X, Y) = (-c/m, c)$. It follows that

$$\frac{4}{X^2} - \frac{2}{Y^2} = \frac{4m^2 - 2}{c^2} = 1.$$

Hence the required locus is $4/X^2 - 2/Y^2 = 1$. The plus-sign alternatives discard the hyperbola tangent condition, and the alternative with 2 and 4 interchanged assigns the semiaxis coefficients to the wrong intercepts. Replacing the temporary coordinates X, Y by x, y gives the printed fourth expression.

Step 3: Conclusion The vertex R lies on $\frac{4}{x^2} - \frac{2}{y^2} = 1$. Hence, the correct option is (D).

Quick Tip to Solve: For tangent-intercept rectangles, use the intercepts as the opposite vertex coordinates, then divide the tangent condition by c^2 .

12. For integers m and n , both greater than 1, consider the following three statements: P : m divides n ; Q : m divides n^2 ; R : m is prime. Then: [4 marks]

(A) $(Q \wedge R) \rightarrow P$

(B) $(P \wedge Q) \rightarrow R$

(C) $Q \rightarrow R$

(D) $Q \rightarrow P$

Correct Answer: (A) $(Q \wedge R) \rightarrow P$

Solution:

Concept:

- For a prime p , Euclid's lemma says that $p \mid uv$ implies $p \mid u$ or $p \mid v$.
- An implication is universally valid only when no allowed integers make its premise true and conclusion false; one counterexample disproves it.

Step 1: Test the implication involving a prime Suppose both Q and R hold. Thus $m \mid n^2$ and m is prime. Apply prime divisibility to $n^2 = n \cdot n$:

$$m \mid n \cdot n \implies m \mid n.$$

Both factors are the same, so either conclusion of Euclid's lemma is exactly P . Consequently $(Q \wedge R) \rightarrow P$ always holds. The prime hypothesis is the crucial restriction; a composite divisor of a square need not divide its square root.

Step 2: Rule out each competing implication For $(P \wedge Q) \rightarrow R$, take $m = 4, n = 4$. Then 4 divides both 4 and 16, but 4 is not prime, so that implication fails. The same pair disproves $Q \rightarrow R$: divisibility of a square does not force its divisor to be prime. Finally take $m = 4, n = 2$. Now $4 \mid 2^2$, so Q is true, while $4 \nmid 2$, so P is false. This disproves $Q \rightarrow P$. All chosen integers exceed 1, as required by the question.

Step 3: Conclusion The valid implication is $(Q \wedge R) \rightarrow P$. Hence, the correct option is (A).

Quick Tip to Solve: Whenever divisibility of a square is given, distinguish prime divisors from composite ones; $4 \mid 2^2$ but $4 \nmid 2$ is a useful check.

13. If the median and the range of four numbers $\{x, y, 2x + y, x - y\}$, where $0 < y < x < 2y$, are 10 and 28 respectively, then the mean of the numbers is: [4 marks]

- (A) 18
(B) 10
(C) 5
(D) 14

Correct Answer: (D) 14

Solution:

Concept:

- The median of four ordered observations is the mean of the middle two, and the range is the largest minus the smallest.
- The arithmetic mean uses all four observations with their multiplicities: add them before dividing by 4.

Step 1: Order the four observations From $0 < y < x < 2y$, we have $x - y > 0$, $x - y < y$, and $y < x$. Also $2x + y > x$. Thus their increasing order is

$$x - y < y < x < 2x + y.$$

The middle observations are y, x , so the median condition gives

$$(x + y)/2 = 10, \quad x + y = 20.$$

The extreme observations are $x - y$ and $2x + y$; hence the range condition gives

$$(2x + y) - (x - y) = 28, \quad x + 2y = 28.$$

Sorting first avoids choosing the wrong pair for the median.

Step 2: Solve for the observations and calculate the mean Subtracting $x + y = 20$ from $x + 2y = 28$ yields $y = 8$, and then $x = 12$. These values satisfy $0 < 8 < 12 < 16$, so they respect all the stated inequalities. The observations are 12, 8, 32, 4, whose ordered median is $(8 + 12)/2 = 10$ and whose range is $32 - 4 = 28$, confirming both conditions. Their mean is

$$\frac{x + y + (2x + y) + (x - y)}{4} = \frac{4x + y}{4} = \frac{56}{4} = 14.$$

The alternative 10 confuses median with mean; 18 and 5 are inconsistent with the verified total of 56.

Step 3: Conclusion The arithmetic mean of the four numbers is 14. Hence, the correct option is (D).

Quick Tip to Solve: Use the inequalities to sort symbolic observations first; only then form equations for the median and range.

14. If the extremities of the base of an isosceles triangle are the points $(2a, 0)$ and $(0, a)$ and the equation of one of the sides is $x = 2a$, then the area of the triangle, in square units, is: [4 marks]

- (A) $\frac{5}{4}a^2$
(B) $\frac{5}{2}a^2$
(C) $\frac{25a^2}{4}$
(D) $5a^2$

Correct Answer: (B) $\frac{5}{2}a^2$

Solution:

Concept:

- The two sides joining the apex to the base endpoints of an isosceles triangle have equal lengths.
- For a vertical side, triangle area is half its length times the horizontal distance of the opposite vertex from its line.

Step 1: Locate the apex using equal sides Let the base endpoints be $A = (2a, 0)$, $B = (0, a)$. Since the side $x = 2a$ contains A , its other endpoint, the apex, is $C = (2a, t)$. The isosceles condition is $CA = CB$, so squared distances give

$$t^2 = (2a)^2 + (t - a)^2.$$

Expanding and cancelling t^2 yields

$$2at = 5a^2, \quad t = \frac{5a}{2}.$$

A nondegenerate triangle has $a \neq 0$, so dividing by a here is legitimate. The formula for area ultimately involves a^2 , making it independent of the sign of a .

Step 2: Use the vertical side as an area base The length AC is $5|a|/2$, and the perpendicular distance of $B = (0, a)$ from the vertical line $x = 2a$ is $2|a|$. Therefore

$$\text{Area} = \frac{1}{2} \cdot \frac{5|a|}{2} \cdot 2|a| = \frac{5}{2}a^2.$$

The determinant formula gives the same result: $\frac{1}{2}|2at| = |at|$. The option $5a^2/4$ loses a factor of 2, $5a^2$ omits the triangle factor of one-half, and $25a^2/4$ is the square of a side length rather than the required area.

Step 3: Conclusion The area is $\frac{5}{2}a^2$ square units. Hence, the correct option is (B).

Quick Tip to Solve: Use a horizontal or vertical side as the area base when available; its perpendicular height is then just a coordinate difference.

15. On the sides AB, BC, CA of a $\triangle ABC$, 3, 4, 5 distinct points (excluding vertices A, B, C) are respectively chosen. The number of triangles that can be constructed using these chosen points as vertices are [4 marks]

- (A) 210
(B) 205
(C) 215
(D) 220

Correct Answer: (B) 205

Solution:

Concept:

- A triangle is determined by an unordered selection of three non-collinear points. For n distinct points, the number of unrestricted selections is $\binom{n}{3}$.
- Three points on one straight side are collinear and form no triangle. Subtract these selections separately for each side; vertices of the original triangle are excluded.

Step 1: Count all selections of three points The three sides contain $3 + 4 + 5 = 12$ chosen points. None is an original vertex, so no point is counted on two sides. Ignoring collinearity for the moment gives

$$\binom{12}{3} = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 220.$$

We use combinations because permuting three selected vertices does not produce a different triangle.

Step 2: Remove the selections that lie on a side The invalid selections contained in sides AB, BC, CA number $\binom{3}{3}, \binom{4}{3}, \binom{5}{3}$, respectively. Thus their total is

$$1 + 4 + 10 = 15.$$

These groups do not overlap because the chosen points exclude the original vertices. A line distinct from the three sides meets the boundary of a non-degenerate triangle in at most two points, so no other triple of the selected boundary points is collinear. Therefore

$$N = 220 - 15 = 205.$$

Option (D) counts the degenerate triples as triangles. Options (A) and (C) subtract only part of the required fifteen invalid selections; neither removes all three side contributions.

Step 3: Conclusion The number of triangles is $\boxed{205}$. Hence, the correct option is (B).

Quick Tip to Solve: For points placed on a triangle boundary, first choose any three from the total, then subtract the triples wholly on each side.

16. Let $R = \{(x, y) : x, y \in N \text{ and } x^2 - 4xy + 3y^2 = 0\}$, where N is the set of all natural numbers. Then the relation R is: [4 marks]

- (A) reflexive but neither symmetric nor transitive.
- (B) symmetric and transitive.
- (C) reflexive and symmetric.
- (D) reflexive and transitive.

Correct Answer: (A) reflexive but neither symmetric nor transitive.

Solution:

Concept:

- A relation is reflexive when every pair (x, x) belongs to it, symmetric when reversal preserves membership, and transitive when two linked pairs imply the third.
- Factor the defining polynomial to describe membership exactly before testing the three properties.

Step 1: Describe the related pairs The equation factors as

$$x^2 - 4xy + 3y^2 = (x - y)(x - 3y) = 0.$$

Therefore xRy means either $x = y$ or $x = 3y$. For every natural number x , the first condition holds for (x, x) , so the relation is reflexive. This does not by itself imply symmetry or transitivity.

Step 2: Test symmetry with a counterexample Take $x = 3, y = 1$. The pair $(3, 1)$ belongs to the relation because $3 = 3 \cdot 1$. Its reversal $(1, 3)$ satisfies neither $1 = 3$ nor $1 = 3 \cdot 3$, so it does not belong. One such counterexample disproves symmetry on the full set of natural numbers.

Step 3: Test transitivity and classify the relation Take the pairs $(9, 3)$ and $(3, 1)$, both of which belong because the first coordinate is three times the second. However, $(9, 1)$ belongs neither to the equality branch nor to the three-times branch. Thus transitivity fails. Options (B) and (C) wrongly assert symmetry, and (D) wrongly asserts transitivity; (A) has all three properties correctly stated.

Step 4: Conclusion The relation is reflexive but neither symmetric nor transitive. Hence, the correct option is (A). The supplied key selects the reflexive-and-transitive alternative, but the printed relation contains $(9, 3)$ and $(3, 1)$ while excluding $(9, 1)$, disproving transitivity and proving (A).

Quick Tip to Solve: After factoring into $x=y$ or $x=3y$, use 3,1 to test reversal and the chain 9,3,1 to test transitivity.

17. Let A , other than I or $-I$, be a 2×2 real matrix such that $A^2 = I$, I being the unit matrix. Let $\text{Tr}(A)$ be the sum of diagonal elements of A .

Statement-1: $\text{Tr}(A) = 0$.

Statement-2: $\det(A) = -1$.

[4 marks]

(A) Statement-1 is true; Statement-2 is false

(B) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(C) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(D) Statement-1 is false; Statement-2 is true

Correct Answer: (C) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Solution:

Concept:

- For a real matrix satisfying $A^2 = I$, each eigenvalue satisfies $r^2 = 1$. The polynomial $r^2 - 1$ has distinct roots, so the matrix is diagonalizable.
- For a two-by-two matrix, the characteristic identity is $A^2 - \text{Tr}(A)A + \det(A)I = 0$.

Step 1: Find the determinant of the permitted matrix The only possible eigenvalues are $+1$ and -1 . If both were $+1$, diagonalizability would imply $A = I$; if both were -1 , it would imply $A = -I$. Both are explicitly excluded. Therefore the two eigenvalues must be 1 and -1 , so their product is $\det A = -1$. Statement-2 is true, and their sum already gives trace zero.

Step 2: Derive the trace from Statement-2 Insert $A^2 = I$ and $\det A = -1$ in the characteristic identity:

$$I - \text{Tr}(A)A - I = 0.$$

Hence $\text{Tr}(A)A = 0$. The matrix A is invertible, since $A^{-1} = A$; multiplication by its inverse gives $\text{Tr}(A) = 0$. Statement-1 is therefore true, and Statement-2 together with the given involutory identity explains it.

Step 3: Match the logical choice Option (A) wrongly declares the determinant statement false. Option (D) wrongly declares the trace statement false. Both statements are true, and the characteristic identity explicitly establishes the explanatory link, so the no-explanation choice (B) is excluded. Option (C) captures both the truth and the explanation.

Step 4: Conclusion Both statements are true, and Statement-2 correctly explains Statement-1 using $A^2 = I$. Hence, the correct option is (C). The supplied key selects the no-explanation alternative; however, the characteristic identity together with the given $A^2 = I$ and Statement-2 directly forces trace zero, proving the explanatory choice (C).

Quick Tip to Solve: For a nontrivial two-by-two involution, the eigenvalues are $+1$ and -1 ; determinant and trace then follow immediately.

18. Statement-1: The system of linear equations

$$x + (\sin \alpha)y + (\cos \alpha)z = 0,$$

$$x + (\cos \alpha)y + (\sin \alpha)z = 0,$$

$$x - (\sin \alpha)y - (\cos \alpha)z = 0$$

has a non-trivial solution for only one value of α lying in the interval $(0, \frac{\pi}{2})$.

Statement-2: The equation in α

$$\begin{vmatrix} \cos \alpha & \sin \alpha & \cos \alpha \\ \sin \alpha & \cos \alpha & \sin \alpha \\ \cos \alpha & -\sin \alpha & -\cos \alpha \end{vmatrix} = 0$$

has only one solution lying in the interval $(0, \frac{\pi}{2})$.

[4 marks]

(A) Statement-1 is true, Statement-2 is true, Statement-2 is not correct explanation for Statement-1.

(B) Statement-1 is true, Statement-2 is true, Statement-2 is correct explanation for Statement-1.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false, Statement-2 is true.

Correct Answer: (B) Statement-1 is true, Statement-2 is true, Statement-2 is correct explanation for Statement-1.

Solution:

Concept:

- A homogeneous square linear system has a nontrivial solution if and only if its coefficient determinant vanishes.
- Multiplying a column of a determinant by a nonzero factor multiplies the determinant by that factor without changing its zeros.

Step 1: Compute the system determinant Write $s = \sin \alpha$ and $c = \cos \alpha$. The coefficient matrix is

$$M = \begin{pmatrix} 1 & s & c \\ 1 & c & s \\ 1 & -s & -c \end{pmatrix}.$$

Expanding along the first column gives

$$\det M = 2(s^2 - c^2) = -2 \cos 2\alpha.$$

It vanishes when $2\alpha = \pi/2$ in the specified interval, namely only at $\alpha = \pi/4$. Thus Statement-1 is true.

Step 2: Relate the second determinant to the first The determinant in Statement-2 has first column $(c, s, c)^T$, second column $(s, c, -s)^T$, and third column $(c, s, -c)^T$. Direct expansion gives

$$D_2 = 2c(s^2 - c^2) = c \det M.$$

This direct identity is the relevant link. On $(0, \pi/2)$, $c > 0$, so the two determinants have exactly the

same zeros.

Step 3: Assess the explanatory relation Statement-2 therefore has exactly one solution, $\pi/4$. Because its determinant is the system determinant multiplied by the strictly nonzero factor $\cos \alpha$, this uniqueness gives precisely the uniqueness needed in Statement-1. Both statements are true and the second explains the first. Choices (C) and (D) incorrectly make a statement false, while (A) misses the determinant relation.

Step 4: Conclusion Both statements are true; their determinant equations are equivalent on the given interval. Hence, the correct option is (B). The supplied key selects the second-statement-false alternative, but the printed second determinant is $\cos(\alpha)$ times the system determinant, with $\cos(\alpha) > 0$ on the given interval; it has the same unique zero and proves (B).

Quick Tip to Solve: Compare determinant zeros only after checking that the extra factor $\cos(\alpha)$ cannot vanish in the stated open interval.

19. $S = \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{13} + \tan^{-1} \frac{1}{21} + \tan^{-1} \frac{1}{31}$, then $\tan S$ is equal to [4 marks]

- (A) $\frac{1}{2}$
- (B) $\frac{3}{7}$
- (C) $\frac{5}{7}$
- (D) $\frac{6}{7}$

Correct Answer: (C) $\frac{5}{7}$

Solution:

Concept:

- The tangent subtraction identity gives $\tan(A - B) = (\tan A - \tan B)/(1 + \tan A \tan B)$. With positive integers this turns the given inverse tangents into consecutive angle differences.
- For $n \geq 1$, both $\tan^{-1}(n+1)$ and $\tan^{-1} n$ lie in $(0, \pi/2)$, and their positive difference also lies in that interval. Thus the principal inverse tangent introduces no extra multiple of π .

Step 1: Express each term as a consecutive difference For a positive integer n ,

$$\tan(\tan^{-1}(n+1) - \tan^{-1} n) = \frac{(n+1) - n}{1 + n(n+1)} = \frac{1}{n^2 + n + 1}.$$

The principal-angle observation therefore proves

$$\tan^{-1} \frac{1}{n^2 + n + 1} = \tan^{-1}(n+1) - \tan^{-1} n.$$

The printed denominators 3, 7, 13, 21, 31 are precisely $n^2 + n + 1$ for $n = 1, 2, 3, 4, 5$; there are five terms.

Step 2: Cancel the intermediate angles and evaluate the tangent Expanding the five differences makes every intermediate angle cancel:

$$\begin{aligned} S &= (\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) \\ &\quad + (\tan^{-1} 4 - \tan^{-1} 3) + (\tan^{-1} 5 - \tan^{-1} 4) \\ &\quad + (\tan^{-1} 6 - \tan^{-1} 5) \\ &= \tan^{-1} 6 - \frac{\pi}{4}. \end{aligned}$$

Hence

$$\tan S = \frac{6-1}{1+6} = \frac{5}{7}.$$

The other proposed values $1/2, 3/7, 6/7$ differ from this exact tangent-subtraction result; in particular, using six rather than five in the numerator omits the subtraction of $\tan(\pi/4) = 1$.

Step 3: Conclusion We obtain $\tan S = 5/7$. Hence, the correct option is (C).

Quick Tip to Solve: Recognise denominators of the form $n(n+1)+1$: their inverse tangents telescope as differences of consecutive integer inverse tangents.

20. Let f be a composite function of x defined by

$$f(u) = \frac{1}{u^2 + u - 2}, \quad u(x) = \frac{1}{x-1}.$$

Then the number of points x where f is discontinuous is:

[4 marks]

- (A) 4
- (B) 3
- (C) 2
- (D) 1

Correct Answer: (B) 3

Solution:

Concept:

- A composite function is defined only where the inner function exists and its output belongs to the domain of the outer function.
- For a rational function, zeros of the denominator are excluded, including holes retained from the original composite domain.

Step 1: Find the forbidden values of the inner variable The outer denominator factors as

$$u^2 + u - 2 = (u+2)(u-1).$$

Thus the outer function is undefined at $u = -2$ and $u = 1$. The inner function $u(x) = 1/(x-1)$ is itself undefined at $x = 1$, so that input must already be excluded before substitution.

Step 2: Pull the restrictions back to x The equation $1/(x-1) = 1$ gives $x = 2$. The equation $1/(x-1) = -2$ gives $x-1 = -1/2$, hence $x = 1/2$. These are distinct from the inner-function exclusion $x = 1$. Away from these three values, both rational functions are defined and continuous, hence their composite is continuous.

Step 3: Check the simplified form without losing the hole Substitution yields

$$\begin{aligned} F(x) &= \frac{(x-1)^2}{1+(x-1)-2(x-1)^2} \\ &= \frac{(x-1)^2}{(2x-1)(2-x)}. \end{aligned}$$

Its original domain still excludes 1 even though the simplified formula has a finite limit there. There are two poles and one removable hole, giving three excluded discontinuity points. Options 4, 2 and 1 miscount these restrictions.

Step 4: Conclusion The three points are $x = 1/2, 1, 2$. Hence, the correct option is (B).

Quick Tip to Solve: List the inner-function pole first, then solve for each forbidden outer-function input; simplification must not erase the original hole.

21. If $f(x) = \sin(\sin x)$ and $f'(x) + \tan x f'(x) + g(x) = 0$, then $g(x)$ is

[4 marks]

- (A) $\cos^2 x \cos(\sin x)$
- (B) $\sin^2 x \cos(\cos x)$
- (C) $\sin^2 x \sin(\cos x)$
- (D) $\cos^2 x \sin(\sin x)$

Correct Answer: (D) $\cos^2 x \sin(\sin x)$

Solution:

Concept:

- The chain rule differentiates $\sin v(x)$ as $\cos v(x) v'(x)$. Apply it first to the inner function $v(x) = \sin x$.
- The second derivative of a product requires the product rule. Keep the factors $\cos(\sin x)$ and $\cos x$ separate so that the cancellation with $\tan x f'(x)$ is visible.

Step 1: Differentiate twice using the chain and product rules For $f(x) = \sin(\sin x)$, the first derivative is

$$f'(x) = \cos(\sin x) \cos x.$$

Differentiating its first factor gives $-\sin(\sin x) \cos x$, whereas differentiating its second factor gives $-\sin x$. Therefore

$$\begin{aligned} f''(x) &= [-\sin(\sin x) \cos x] \cos x \\ &\quad + \cos(\sin x)(-\sin x) \\ &= -\cos^2 x \sin(\sin x) - \sin x \cos(\sin x). \end{aligned}$$

The square of cosine comes from two occurrences of the derivative of the inner sine.

Step 2: Use the given relation to isolate the unknown function Where $\tan x$ is defined, multiply the first derivative by it:

$$\tan x f'(x) = \frac{\sin x}{\cos x} \cos(\sin x) \cos x = \sin x \cos(\sin x).$$

Adding this to the second derivative cancels the final term exactly. The stated equation becomes

$$-\cos^2 x \sin(\sin x) + g(x) = 0.$$

Thus $g(x) = \cos^2 x \sin(\sin x)$. Option (A) retains the cosine of the inner sine rather than its sine. Options (B) and (C) use both the wrong squared outer factor and the wrong inner argument; neither can arise from differentiating $\sin(\sin x)$.

Step 3: Conclusion The required function is $g(x) = \cos^2 x \sin(\sin x)$, on the domain of the given relation. Hence, the correct option is (D).

Quick Tip to Solve: When a relation contains $f''(x) + \tan x f'(x)$, write both derivatives before expanding: the first-derivative product often cancels a term.

22. If the curves $\frac{x^2}{\alpha} + \frac{y^2}{4} = 1$ and $y^3 = 16x$ intersect at right angles, then a value of α is : [4 marks]

- (A) 2
- (B) 4/3
- (C) 1/2
- (D) 3/4

Correct Answer: (B) 4/3

Solution:

Concept:

- Implicit differentiation gives the tangent slope of a curve at a point by treating y as a function of x .
- Two finite nonzero tangent slopes m_1, m_2 represent perpendicular tangents exactly when $m_1 m_2 = -1$. Both curve equations must hold at their common point.

Step 1: Differentiate the two curve equations For the first curve, differentiation with respect to x gives

$$\frac{2x}{\alpha} + \frac{y}{2} \frac{dy}{dx} = 0, \quad m_1 = -\frac{4x}{\alpha y}.$$

For the second curve, the same operation gives

$$3y^2 \frac{dy}{dx} = 16, \quad m_2 = \frac{16}{3y^2}.$$

At an intersection, $y = 0$ would imply $x = 0$ from the cubic, but the origin does not satisfy the first curve. Thus $y \neq 0$, and the displayed slopes are well defined. The cubic also implies $x \neq 0$.

Step 2: Apply perpendicularity and substitute the cubic relation Multiplying the slopes and imposing a right angle yields

$$-\frac{64x}{3\alpha y^3} = -1 \implies 64x = 3\alpha y^3.$$

Substitute $y^3 = 16x$, and cancel the nonzero x :

$$64x = 48\alpha x, \quad \alpha = \frac{64}{48} = \frac{4}{3}.$$

This value permits real intersections. Indeed, inserting $x = y^3/16$ into the first curve gives $3y^6/1024 + y^2/4 = 1$; its left side increases continuously from zero to values greater than one for positive y , so a common point exists. At that point the product of slopes is exactly -1 . Values $2, 1/2, 3/4$ fail the necessary equation $64 = 48\alpha$, excluding the other options.

Step 3: Conclusion The required value is $\alpha = \frac{4}{3}$. Hence, the correct option is (B).

Quick Tip to Solve: Multiply the implicit slopes before solving for an intersection; the relation $y^3 = 16x$ cancels the coordinates and leaves only α .

23. The cost of running a bus from A to B is Rs. $(av + b/v)$ where v km/h is the average speed of the bus. When the bus travels at 30 km/h, the cost comes out to be Rs. 75 while at 40 km/h, it is Rs. 65. Then the most economical speed (in km/h) of the bus is: [4 marks]

- (A) 45
- (B) 50
- (C) 60
- (D) 40

Correct Answer: (C) 60

Solution:

Concept:

- For positive speed, minimize $C(v) = av + b/v$ by setting $C'(v) = 0$ and checking that the stationary point is a minimum.
- The two known costs determine the constants a and b through simultaneous linear equations.

Step 1: Determine the cost constants The data give $30a + b/30 = 75$ and $40a + b/40 = 65$. Multiplying by the corresponding speeds yields

$$900a + b = 2250, \quad 1600a + b = 2600.$$

Subtraction gives $700a = 350$, hence $a = 1/2$. Substituting back gives $b = 2250 - 450 = 1800$. Both constants are positive, which is needed for a finite positive minimum.

Step 2: Find and classify the stationary speed Thus

$$C(v) = v/2 + 1800/v, \quad C'(v) = 1/2 - 1800/v^2.$$

Setting the derivative to zero yields $v^2 = 3600$. Speed is positive, so $v = 60$ km/h. Moreover $C''(v) = 3600/v^3 > 0$ for all positive speeds, proving this point is a strict minimum. Cost diverges at both ends of the positive-speed domain, so the minimum is global.

Step 3: Compare the choices The optimum cost is $C(60) = 30 + 30 = 60$ rupees. For the other listed speeds, $C(45) = 62.5$, $C(50) = 61$, and $C(40) = 65$ rupees, each larger. The equality of the two cost contributions at 60 also agrees with the positive-term arithmetic-geometric mean inequality.

Step 4: Conclusion The most economical speed is 60 km/h. Hence, the correct option is (C).

Quick Tip to Solve: For $av + b/v$ with positive constants, the minimizing speed is $\sqrt{b/a}$, where the two contributions to cost are equal.

24. If a curve passes through the point $(2, \frac{7}{2})$ and has slope $(1 - \frac{1}{x^2})$ at any point (x, y) on it, then the ordinate of the point on the curve whose abscissa is -2 is: [4 marks]

- (A) $-\frac{3}{2}$
- (B) $\frac{3}{2}$
- (C) $\frac{5}{2}$
- (D) $-\frac{5}{2}$

Correct Answer: (A) $-\frac{3}{2}$

Solution:

Concept:

- The derivative dy/dx gives the slope; integrating it supplies the family of curves with that slope.
- Use the specified point to determine the integration constant before evaluating another ordinate.

Step 1: Integrate the given slope The slope equation is

$$\frac{dy}{dx} = 1 - x^{-2}.$$

Integration gives

$$y = x + x^{-1} + C, \quad x \neq 0,$$

since the derivative of x^{-1} is $-x^{-2}$. The plus sign on the reciprocal term is important: using a minus sign would give the wrong slope.

Step 2: Use the supplied point At the given point $(2, 7/2)$,

$$7/2 = 2 + 1/2 + C = 5/2 + C.$$

Therefore $C = 1$, and the curve described by the question is $y = x + 1/x + 1$. This constant must be retained when computing the ordinate at a negative abscissa; the slope alone does not determine a unique vertical translate.

Step 3: Evaluate at the new abscissa and compare Putting $x = -2$ gives

$$y = -2 - 1/2 + 1 = -3/2.$$

This agrees only with option (A). The positive alternatives have the wrong sign. The value $-5/2$ in (D) corresponds to omitting the determined additive constant. Differentiating $x + 1/x + 1$ checks that the supplied slope is reproduced wherever $x \neq 0$.

Step 4: Conclusion The required ordinate is $\boxed{-3/2}$. Hence, the correct option is (A).

Quick Tip to Solve: Integrate the slope first and use the point immediately; forgetting the constant here produces the tempting $-5/2$ alternative.

25. The integral $\int \frac{x \, dx}{2 - x^2 + \sqrt{2 - x^2}}$ equals:

[4 marks]

- (A) $\log |1 + \sqrt{2 + x^2}| + c$
(B) $-\log |1 + \sqrt{2 - x^2}| + c$
(C) $-x \log |1 - \sqrt{2 - x^2}| + c$
(D) $x \log |1 - \sqrt{2 + x^2}| + c$

Correct Answer: (B) $-\log |1 + \sqrt{2 - x^2}| + c$

Solution:

Concept:

- The radical $\sqrt{2 - x^2}$ suggests substituting it directly, because its differential contains $x \, dx$.
- After substitution, factor the denominator before cancelling; then integrate the resulting reciprocal linear term.

Step 1: Make the radical the new variable Let $u = \sqrt{2 - x^2}$. On any interval where the integrand is defined, $u > 0$ and $2u \, du = -2x \, dx$, giving $x \, dx = -u \, du$. Also $2 - x^2 = u^2$. Hence

$$\int \frac{x \, dx}{2 - x^2 + \sqrt{2 - x^2}} = \int \frac{-u \, du}{u^2 + u}.$$

The real domain is $-\sqrt{2} < x < \sqrt{2}$, since the denominator vanishes at the two endpoints.

Step 2: Cancel and integrate Factor $u^2 + u = u(u + 1)$. Since $u > 0$, cancellation is valid, giving

$$-\int \frac{du}{u + 1} = -\log |u + 1| + C.$$

Replacing u returns

$$-\log |1 + \sqrt{2 - x^2}| + C.$$

There is no extra factor of x outside the logarithm; x was absorbed into the differential.

Step 3: Check by differentiation and exclude alternatives Differentiation gives

$$\frac{d}{dx}\{-\log(1+u)\} = \frac{x}{u(1+u)},$$

exactly the printed integrand. Option (A) has the wrong radical and sign structure. The external x in options (C) and (D) generates an additional logarithmic term under the product rule, so neither can have the required derivative.

Step 4: Conclusion The antiderivative is $-\log|1 + \sqrt{2-x^2}| + C$. Hence, the correct option is (B).

Quick Tip to Solve: With $x dx$ above a denominator built from $2-x^2$ and its square root, set $u = \sqrt{2-x^2}$ so the factor u cancels.

26. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^{2x}} dx$ is:

[4 marks]

- (A) π
- (B) $\frac{\pi}{2}$
- (C) 4π
- (D) $\frac{\pi}{4}$

Correct Answer: (D) $\frac{\pi}{4}$

Solution:

Concept:

- For a symmetric interval, substitution $x \mapsto -x$ preserves the integral after reversing the limits.
- The complementary weights satisfy $1/(1+2^{2x}) + 1/(1+2^{-2x}) = 1$, while $\sin^2 x$ is even.

Step 1: Reflect the variable Denote the required integral by I . Changing x to its negative gives

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^{-2x}} dx.$$

The numerator remains unchanged because $\sin^2(-x) = \sin^2 x$. This reflection is useful even though the original weighted integrand itself is not even.

Step 2: Add the two equivalent forms Put $a = 2^{2x} > 0$. Then

$$\frac{1}{1+a} + \frac{1}{1+1/a} = \frac{1}{1+a} + \frac{a}{1+a} = 1.$$

Adding the original and reflected integrals therefore yields

$$2I = \int_{-\pi/2}^{\pi/2} \sin^2 x dx.$$

Thus the exponential denominator contributes a net factor of one-half over the symmetric interval,

without needing an antiderivative of its individual terms.

Step 3: Evaluate and match the alternatives Using $\sin^2 x = (1 - \cos 2x)/2$,

$$2I = \left[\frac{x}{2} - \frac{\sin 2x}{4} \right]_{-\pi/2}^{\pi/2} = \frac{\pi}{2}.$$

Hence $I = \pi/4$. Options π , $\pi/2$ and 4π do not equal this value; $\pi/2$ is the value of twice the required integral, not of I itself.

Step 4: Conclusion The integral equals $\boxed{\pi/4}$. Hence, the correct option is (D).

Quick Tip to Solve: On symmetric limits, pair the weights at x and $-x$; reciprocal exponential powers often make those weights add to one.

27. The area under the curve $y = |\cos x - \sin x|$, $0 \leq x \leq \frac{\pi}{2}$, and above x -axis is:

[4 marks]

- (A) $2\sqrt{2}$
- (B) $2\sqrt{2} - 2$
- (C) $2\sqrt{2} + 2$
- (D) 0

Correct Answer: (B) $2\sqrt{2} - 2$

Solution:

Concept:

- Area above the x -axis is the integral of a nonnegative ordinate; an absolute value must be resolved where its interior changes sign.
- The equality $\cos x = \sin x$ on $[0, \pi/2]$ occurs at $x = \pi/4$, dividing the integration interval.

Step 1: Locate the sign change On $[0, \pi/4]$, $\cos x \geq \sin x$, so the ordinate equals $\cos x - \sin x$. On $[\pi/4, \pi/2]$, it equals $\sin x - \cos x$. Therefore

$$A = \int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x) dx.$$

This split avoids cancelling equal positive areas on the two sides of the zero.

Step 2: Evaluate both positive contributions The first contribution is $[\sin x + \cos x]_0^{\pi/4} = \sqrt{2} - 1$. The second is $[-\cos x - \sin x]_{\pi/4}^{\pi/2} = \sqrt{2} - 1$. Adding them gives

$$A = 2(\sqrt{2} - 1) = 2\sqrt{2} - 2.$$

Reflection around $\pi/4$ provides a check that the two contributions should be equal.

Step 3: Rule out the other values The value zero in (D) would result from integrating the signed expression without its absolute value. Option (A) drops the endpoint contribution of 2, and (C) adds that contribution instead of subtracting it. The positive result in (B) is also less than the rectangle

area $\pi/2$, consistent with the ordinate never exceeding 1.

Step 4: Conclusion The required area is $2\sqrt{2} - 2$ square units. Hence, the correct option is (B).

Quick Tip to Solve: Split an absolute-value integral at every zero of its interior; here symmetry lets you double the area from 0 to $\pi/4$.

28. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, then the value of α for which the vectors $\vec{u} = (\alpha - 2)\vec{a} + \vec{b}$ and $\vec{v} = (2 + 3\alpha)\vec{a} - 3\vec{b}$ are collinear is: [4 marks]

- (A) $\frac{3}{2}$
- (B) $\frac{2}{3}$
- (C) $-\frac{3}{2}$
- (D) $-\frac{2}{3}$

Correct Answer: (B) $\frac{2}{3}$

Solution:

Concept:

- Non-collinear vectors are linearly independent, so corresponding coefficients must match in any equality between their linear combinations.
- For two nonzero collinear vectors, one is a scalar multiple of the other.

Step 1: Introduce the scalar multiplier Write $\vec{v} = k\vec{u}$. Comparing the coefficients of the independent vectors $\vec{a}$ and $\vec{b}$ gives

$$2 + 3\alpha = k(\alpha - 2), \quad -3 = k.$$

The second equation fixes the multiplier immediately because the coefficient of $\vec{b}$ in $\vec{u}$ is 1. Neither combination is the zero vector, since its $\vec{b}$ coefficient is nonzero.

Step 2: Solve the remaining equation Use $k = -3$ in the first equation:

$$2 + 3\alpha = -3(\alpha - 2) = -3\alpha + 6.$$

Hence $6\alpha = 4$, giving $\alpha = 2/3$. At this value,

$$\vec{u} = -\frac{4}{3}\vec{a} + \vec{b}, \quad \vec{v} = 4\vec{a} - 3\vec{b} = -3\vec{u},$$

directly verifying collinearity.

Step 3: Check the other choices The coefficient condition can also be written $6\alpha - 4 = 0$. The values $3/2, -3/2, -2/3$ in (A), (C), and (D) give 5, -13 , and -8 , respectively, instead of zero. Only (B) preserves the same multiplier for both independent-vector components.

Step 4: Conclusion The required parameter is $\alpha = 2/3$. Hence, the correct option is (B).

Quick Tip to Solve: Compare the simple $\vec{b}$ coefficients first: they fix the scalar multiplier at -3 before you solve for α .

29. If the projections of a line segment on the x , y and z -axes in 3-dimensional space are 2, 3 and 6 respectively, then the length of the line segment is: [4 marks]

- (A) 12
- (B) 7
- (C) 9
- (D) 6

Correct Answer: (B) 7

Solution:

Concept:

- The projections of a directed line segment on three mutually perpendicular coordinate axes are its Cartesian components. Their signs specify orientation; their squares determine the length.
- For components $\Delta x, \Delta y, \Delta z$, the three-dimensional distance formula is $L^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$.

Step 1: Identify the three perpendicular components The given projection lengths are 2, 3, 6. They are perpendicular components of one segment, rather than three segments arranged end to end. First combine the horizontal components: the projection in the xy -plane has squared length $2^2 + 3^2 = 13$. The remaining component, of length 6, is perpendicular to that plane.

Step 2: Apply the spatial distance formula Apply Pythagoras once more to the planar projection and the vertical component:

$$L = \sqrt{13 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7.$$

The positive root is required because a segment length cannot be negative. Adding the projection lengths would give 11 and would incorrectly treat perpendicular directions as collinear.

Step 3: Check the alternatives Option (B) gives $L^2 = 49$, exactly equal to the sum of the three squared projections. The values 12, 9 and 6 in options (A), (C) and (D) give squared lengths 144, 81 and 36, respectively, none equal to 49. A length of 6 would also leave no room for the two nonzero perpendicular components.

Step 4: Conclusion The length of the line segment is $\boxed{7}$. Hence, the correct option is (B).

Quick Tip to Solve: For projections on mutually perpendicular axes, square each projection, add, and take the positive square root; do not add the projections themselves.

30. A, B, C try to hit a target simultaneously but independently. Their respective probabilities of hitting the target are $\frac{3}{4}, \frac{1}{2}, \frac{5}{8}$. The probability that the target is hit by A or B but not by C is [4 marks]

- (A) 21/64
(B) 7/8
(C) 7/32
(D) 9/64

Correct Answer: (A) 21/64

Solution:

Concept:

- For independent trials, the probability of a specified combination of results is the product of their individual probabilities.
- The inclusive event $A \cup B$ means at least one of A and B hits. Its probability is $1 - P(A^c)P(B^c)$, which includes the possibility that both hit.

Step 1: Find the probability that A or B hits The failure probabilities are $P(A^c) = 1/4$ and $P(B^c) = 1/2$. Independence therefore gives

$$P(A^c \cap B^c) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}.$$

Taking the complement,

$$P(A \cup B) = 1 - \frac{1}{8} = \frac{7}{8}.$$

This is not an exclusive-or event: a simultaneous hit by A and B is allowed.

Step 2: Require C to miss the target C misses with probability $P(C^c) = 1 - 5/8 = 3/8$. Since C acts independently of A and B, the required event has probability

$$P((A \cup B) \cap C^c) = \frac{7}{8} \cdot \frac{3}{8} = \frac{21}{64}.$$

Alternatively, the disjoint cases A only, B only and both A and B give $9/64 + 3/64 + 9/64 = 21/64$.

Step 3: Compare the listed probabilities Option (A) equals the calculated probability. The value 7/8 in (B) omits the condition that C misses. The value 9/64 in (D) counts only one of the disjoint successful cases, rather than their union. The remaining value 7/32 in (C) also fails the product calculation and is not the stated event.

Step 4: Conclusion The required probability is $\boxed{21/64}$. Hence, the correct option is (A).

Quick Tip to Solve: For independent shooters, compute at least one hit using one minus all misses, then multiply by the required miss probability of the excluded shooter.

Physics (120 marks)

31. If the time period t of the oscillation of a drop of liquid of density d , radius r , vibrating under surface tension s is given by the formula $t = \sqrt{r^{2b} s^c d^{a/2}}$. It is observed that the time period is directly proportional to $\sqrt{\frac{d}{s}}$. The value of b should therefore be: [4 marks]

- (A) $\frac{3}{4}$
- (B) $\sqrt{3}$
- (C) $\frac{3}{2}$
- (D) $\frac{2}{3}$

Correct Answer: (C) $\frac{3}{2}$

Solution:

Concept:

- The dimensions of density are $[d] = ML^{-3}$, while surface tension is force per unit length, so $[s] = MT^{-2}$. Radius has dimension $[r] = L$.
- The stated time dependence $t \propto \sqrt{d/s}$ fixes the mass and time factors. The remaining power of radius must remove the residual length dimension.

Step 1: Use the stated density and surface-tension dependence The expression under the square root is $r^{2b} s^c d^{a/2}$. Taking its square root gives $t = r^b s^{c/2} d^{a/4}$, apart from a dimensionless coefficient. The given proportionality to $d^{1/2} s^{-1/2}$ therefore requires $a/4 = 1/2$, or $a = 2$, and $c/2 = -1/2$, or $c = -1$.

Step 2: Balance the power of length It follows that $t \propto r^b \sqrt{d/s}$. Now

$$\left[\frac{d}{s}\right] = \frac{ML^{-3}}{MT^{-2}} = L^{-3}T^2, \quad \left[\sqrt{d/s}\right] = L^{-3/2}T.$$

Thus the full right-hand side has dimension $L^{b-3/2}T$. A time period has no length dimension, so $b - 3/2 = 0$, giving $b = 3/2$.

Step 3: Test the proposed exponents With $b = 3/2$, the dimensions are exactly T , as required. The exponent $3/4$ leaves $L^{-3/4}$, and $2/3$ leaves $L^{-5/6}$; both are dimensionally incorrect. The value $\sqrt{3}$ also fails to cancel the length exponent because it is not $3/2$.

Step 4: Conclusion The required exponent is $b = 3/2$. Hence, the correct option is (C).

Quick Tip to Solve: For liquid-drop oscillations, first note that $\sqrt{d/s}$ has dimensions $T/L^{3/2}$; multiplying by $r^{3/2}$ immediately supplies a time.

32. A 70 kg man leaps vertically into the air from a crouching position. To take the leap the man pushes the ground with a constant force F to raise himself. The center of gravity rises by 0.5 m before he leaps. After the leap the c.g. rises by another 1 m. The maximum power delivered by the muscles is: (Take $g = 10 \text{ m s}^{-2}$) [4 marks]

- (A) 6.26×10^3 Watts at the start
- (B) 6.26×10^3 Watts at take off
- (C) 6.26×10^4 Watts at the start
- (D) 6.26×10^4 Watts at take off

Correct Answer: No option equals total muscular mechanical power; (B) equals the net kinetic-energy rate only.

Solution:

Concept:

- The take-off speed follows from the free-flight rise: $v^2 = 2gh$. During the push, constant acceleration gives $v^2 = 2a\Delta y$.
- The upward supporting force obeys $F - mg = ma$. Muscle work supplies both kinetic energy and gravitational potential energy, so its instantaneous rate is Fv ; the rate of kinetic-energy increase alone is $(F - mg)v$.

Step 1: Calculate the speed and upward force The additional airborne rise is $h = 1 \text{ m}$, so

$$v = \sqrt{2(10)(1)} = \sqrt{20} \text{ m s}^{-1}.$$

During the 0.5 m pushing displacement the acceleration is

$$a = \frac{v^2}{2(0.5)} = 20 \text{ m s}^{-2}.$$

Hence the upward force is

$$F = 70(20 + 10) = 2100 \text{ N}.$$

Step 2: Distinguish muscle power from net kinetic power The speed increases throughout the push, so any constant-force power is largest just before take-off and zero at the start. The mechanical power supplied by the muscles is

$$P_{\text{muscles, max}} = Fv = 2100\sqrt{20} = 9.39 \times 10^3 \text{ W}.$$

The net translational kinetic-energy rate instead is

$$P_{K, \text{max}} = mav = 1400\sqrt{20} = 6.26 \times 10^3 \text{ W}.$$

Their difference is mgv , the rate of increase of gravitational potential energy.

Step 3: Assess the answer choices Options (A) and (C) cannot describe maximum power at the start because the initial speed is zero. Option (D) has an incorrect magnitude. Option (B) equals the maximum net kinetic-energy rate, but excludes the simultaneous increase in gravitational potential

energy. It is the intended choice if power means net accelerating power; it is not the literal total mechanical power delivered by the muscles.

Step 4: Conclusion The intended net-power answer is (B), $6.26 \times 10^3 \text{ W}$ at take-off. For the literal muscular mechanical power, the result is $9.39 \times 10^3 \text{ W}$ at take-off, which is absent from the options. Thus the wording does not give a single physically correct listed answer.

Quick Tip to Solve: In a jump, separate the supporting force $m(a + g)$ from the net force ma : their powers differ by the rate at which gravitational potential energy increases.

33. A boy of mass 20 kg is standing on a 80 kg free to move long cart. There is negligible friction between cart and ground. Initially, the boy is standing 25 m from a wall. If he walks 10 m on the cart towards the wall, then the final distance of the boy from the wall will be [4 marks]

- (A) 15 m
- (B) 12.5 m
- (C) 15.5 m
- (D) 17 m

Correct Answer: (D) 17 m

Solution:

Concept:

- With negligible external horizontal force, the horizontal position of the combined boy-cart centre of mass remains fixed.
- The boy walks 10 m relative to the cart. His displacement relative to the ground is the relative walking displacement plus the cart displacement.

Step 1: Define ground-frame displacements Take the direction towards the wall as positive. Let the cart displacement relative to the ground be x . The boy moves 10 m relative to the cart, so his ground displacement is $x + 10$. The cart recoils away from the wall, so the expected sign of x is negative. The relative walking distance cannot simply be subtracted from the initial wall distance.

Step 2: Apply centre-of-mass conservation The total mass-weighted horizontal displacement is zero:

$$20(x + 10) + 80x = 0.$$

Thus

$$100x + 200 = 0, \quad x = -2 \text{ m}.$$

The boy therefore moves

$$\Delta x_{\text{boy}} = 10 - 2 = 8 \text{ m}$$

towards the wall relative to the ground. The cart moves 2 m away from the wall, exactly balancing the boy's centre-of-mass contribution.

Step 3: Calculate and check the final distance The boy was initially 25 m from the wall. After

approaching it by 8 m, his remaining distance is

$$d_f = 25 - 8 = 17 \text{ m.}$$

Option (A), 15 m, would require a stationary cart. The values 12.5 m and 15.5 m in (B) and (C) do not satisfy the mass-weighted displacement equation together with the 10 m relative walk.

Step 4: Conclusion The final distance from the wall is 17 m. Hence, the correct option is (D).

Quick Tip to Solve: For walking on a free cart, the walker's ground displacement is the relative walking distance multiplied by cart mass divided by total mass.

34. A projectile of mass M is fired so that the horizontal range is 4 km. At the highest point the projectile explodes in two parts of masses $M/4$ and $3M/4$ respectively and the heavier part starts falling down vertically with zero initial speed. The horizontal range (distance from point of firing) of the lighter part is [4 marks]

- (A) 16 km
- (B) 1 km
- (C) 10 km
- (D) 2 km

Correct Answer: (C) 10 km

Solution:

Concept:

- At the highest point of a projectile trajectory the vertical velocity is zero, while the horizontal velocity remains unchanged.
- During an instantaneous explosion the external impulse is negligible, so momentum is conserved. Afterwards both fragments have the same vertical fall time if both start with zero vertical velocity.

Step 1: Locate the explosion and compare fall times Let the original horizontal speed be u_x , and let T be the time from the highest point to the ground. The original trajectory is symmetric, so its total horizontal range is $2u_x T = 4 \text{ km}$. Thus the explosion occurs $u_x T = 2 \text{ km}$ from the firing point. The heavy fragment is initially at rest, so its vertical momentum is zero; conservation of vertical momentum makes the lighter fragment's initial vertical velocity zero as well.

Step 2: Conserve horizontal momentum If the lighter fragment has horizontal speed v , the heavy fragment contributes no momentum immediately after the explosion. Therefore

$$Mu_x = \frac{M}{4}v + \frac{3M}{4}(0), \quad v = 4u_x.$$

Both fragments fall from the original maximum height with zero initial vertical velocity, so the lighter one remains airborne for time T . Its additional horizontal travel is

$$vT = 4u_x T = 8 \text{ km.}$$

Step 3: Measure the range from the firing point Add the 2 km already covered before the explosion:

$$R_{\text{light}} = 2 + 8 = 10 \text{ km.}$$

Option (A), 16 km, incorrectly multiplies the entire original range by four rather than only the remaining travel. Options (B) and (D), 1 km and 2 km, fail momentum conservation and omit the lighter fragment's substantial forward motion after the explosion.

Step 4: Conclusion The lighter fragment lands 10 km from the firing point. Hence, the correct option is (C).

Quick Tip to Solve: At an apex explosion, first separate the already-covered half-range from the remaining half; momentum changes only the fragment's subsequent horizontal travel.

35. A particle of mass 2 kg is moving such that at time t , its position, in meter, is given by $\vec{r}(t) = 5\hat{i} - 2t^2\hat{j}$. The angular momentum of the particle at $t = 2 \text{ s}$ about the origin in $\text{kg m}^{-2} \text{ s}^{-1}$ is: [4 marks]

- (A) $-80\hat{k}$
- (B) $(10\hat{i} - 16\hat{j})$
- (C) $-40\hat{k}$
- (D) $40\hat{k}$

Correct Answer: (A) $-80\hat{k}$

Solution:

Concept:

- Linear momentum is $\mathbf{p} = m\mathbf{v}$, with velocity obtained by differentiating the position vector.
- Angular momentum about the origin is the vector cross product $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. For motion in the xy -plane, $L_z = xp_y - yp_x$.

Step 1: Differentiate the position vector The supplied position law is

$$\mathbf{r}(t) = 5\hat{i} - 2t^2\hat{j}.$$

Differentiating each Cartesian component with respect to time gives

$$\mathbf{v}(t) = -4t\hat{j}.$$

The x coordinate is constant, so there is no x component of velocity or momentum. At $t = 2 \text{ s}$,

$$\mathbf{r} = 5\hat{i} - 8\hat{j} \text{ m}, \quad \mathbf{v} = -8\hat{j} \text{ m s}^{-1}.$$

Step 2: Form the momentum and cross product For mass $m = 2 \text{ kg}$,

$$\mathbf{p} = 2(-8\hat{j}) = -16\hat{j} \text{ kg m s}^{-1}.$$

Consequently,

$$\mathbf{L} = (5\hat{\mathbf{i}} - 8\hat{\mathbf{j}}) \times (-16\hat{\mathbf{j}}) = -80\hat{\mathbf{k}} \text{ kg m}^2 \text{ s}^{-1}.$$

The parallel $\hat{\mathbf{j}}$ terms contribute zero, and $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$ fixes the negative sign.

Step 3: Check the alternatives and units Option (A) has both the correct magnitude and direction. Option (B) is an in-plane vector and cannot be the angular momentum for this planar motion. Option (C) omits the factor of two from the mass, while (D) also has the wrong sign. Angular momentum has units $\text{kg m}^2 \text{ s}^{-1}$; the negative power of metre in the question's unit string is dimensionally inconsistent.

Step 4: Conclusion The angular momentum is $-80\hat{\mathbf{k}} \text{ kg m}^2 \text{ s}^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: For planar motion with constant x coordinate, use $L_z = mxv_y$; the y coordinate contributes nothing when $v_x = 0$.

36. A body of mass ' m ' is tied to one end of a spring and whirled round in a horizontal plane with a constant angular velocity. The elongation in the spring is 1 cm. If the angular velocity is doubled, the elongation in the spring is 5 cm. The original length of the spring is [4 marks]

- (A) 15 cm
- (B) 12 cm
- (C) 16 cm
- (D) 10 cm

Correct Answer: (A) 15 cm

Solution:

Concept:

- For uniform circular motion, the required inward force is $m\omega^2 r$, where r is the distance of the body from the fixed end.
- Hooke's law gives the tension kx , where k is the spring constant and x its extension. The radius is the stretched length $L + x$, not the extension alone.

Step 1: Write the first force balance Let the natural length be L cm. The mass and spring constant are unchanged in the two experiments. At angular speed ω , the extension is 1 cm, so the radius is $L + 1$ cm. Converting both lengths to metres gives

$$k(0.01) = m\omega^2 \frac{L + 1}{100}.$$

The spring tension itself supplies the centripetal force; no extra outward force is needed in this inertial-frame balance.

Step 2: Use the doubled angular speed The new speed is 2ω , and the new extension is 5 cm. Thus

$$k(0.05) = 4m\omega^2 \frac{L + 5}{100}.$$

Divide this equation by the first to remove k , m , and ω :

$$5 = 4 \frac{L + 5}{L + 1}.$$

Hence $5L + 5 = 4L + 20$, giving $L = 15$. A check is that the radii become 16 cm and 20 cm: multiplying the speed-squared ratio 4 by the radius ratio 20/16 gives the observed force ratio 5. Natural lengths 12, 16 and 10 cm do not satisfy this ratio.

Step 3: Conclusion The original length of the spring is 15 cm. Hence, the correct option is (A).

Quick Tip to Solve: Always use natural length plus extension as the circular radius; doubling angular speed multiplies ω^2 by four.

37. A copper wire of length 1.0 m and a steel wire of length 0.5 m having equal cross-sectional areas are joined end to end. The composite wire is stretched by a certain load which stretches the copper wire by 1 mm. If the Young's moduli of copper and steel are respectively $1.0 \times 10^{11} \text{ N m}^{-2}$ and $2.0 \times 10^{11} \text{ N m}^{-2}$, the total extension of the composite wire is [4 marks]

- (A) 1.75 mm
- (B) 2.0 mm
- (C) 1.50 mm
- (D) 1.25 mm

Correct Answer: (D) 1.25 mm

Solution:

Concept:

- Young's modulus is $Y = FL/(A\Delta L)$, so the extension of a wire is $\Delta L = FL/(AY)$. Here F is the tensile force and A the cross-sectional area.
- Wires joined end to end carry the same tensile force in equilibrium. Their extensions add, whereas their lengths and Young's moduli must be used separately.

Step 1: Compare the extensions Write subscripts c and s for copper and steel. Since the wires have equal cross-sectional areas and carry the same load,

$$\frac{\Delta L_s}{\Delta L_c} = \frac{L_s/Y_s}{L_c/Y_c} = \frac{L_s Y_c}{L_c Y_s}.$$

The common force and area cancel. This is why the magnitude of the applied load need not be known. A shorter and stiffer wire contributes a smaller extension.

Step 2: Calculate the total extension Use $L_s = 0.5 \text{ m}$, $L_c = 1.0 \text{ m}$, $Y_c = 1.0 \times 10^{11} \text{ N m}^{-2}$, and $Y_s = 2.0 \times 10^{11} \text{ N m}^{-2}$. Then

$$\frac{\Delta L_s}{\Delta L_c} = \frac{0.5 \times 1.0}{1.0 \times 2.0} = \frac{1}{4}.$$

The copper extends by 1 mm, so the steel extends by 0.25 mm. Therefore

$$\Delta L_{\text{total}} = 1.00 + 0.25 = 1.25 \text{ mm.}$$

The 2.0 mm option would treat both extensions as equal. The 1.50 mm option neglects the stiffness difference, and 1.75 mm likewise fails to use both the length and modulus ratios.

Step 3: Conclusion The composite wire extends by 1.25 mm. Hence, the correct option is (D).

Quick Tip to Solve: For wires in series under the same force and area, compare L/Y first and then add the individual extensions.

38. Wax is coated on the inner wall of a capillary tube and the tube is then dipped in water. Then, compared to the unwaxed capillary, the angle of contact θ and the height h upto which water rises change. These changes are : [4 marks]

- (A) θ increases and h also increases
- (B) θ decreases and h also decreases
- (C) θ increases and h decreases
- (D) θ decreases and h increases

Correct Answer: (C) θ increases and h decreases

Solution:

Concept:

- Capillary rise follows $h = 2T \cos \theta / (\rho g r)$, where T is surface tension, r the tube radius, ρ liquid density, and θ the angle of contact measured through the liquid.
- Water wets clean glass, giving a small contact angle. Wax is comparatively non-wetting, so coating the wall with wax increases the water-solid contact angle.

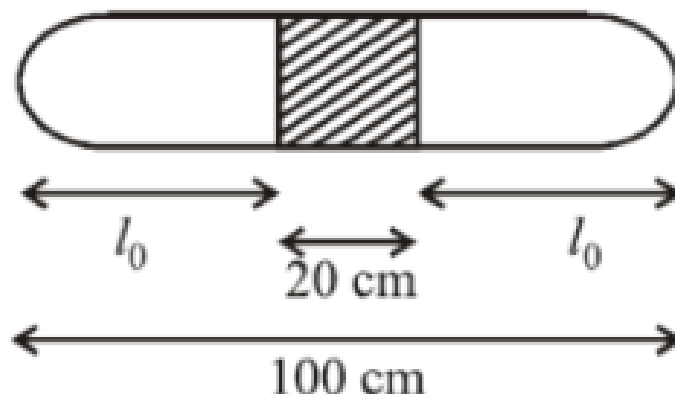
Step 1: Identify the effect of the coating The liquid is still water, but the solid surface touching it changes from glass to wax. Adhesion of water to this surface becomes weaker relative to cohesion within water. Consequently the water does not spread as readily along the wall, and the contact angle θ increases. The alternatives predicting a decrease of θ , namely (B) and (D), therefore have the wrong direction for the surface effect.

Step 2: Apply the capillary-rise relation With the same water and tube dimensions, the factor $2T/(\rho g r)$ is unchanged, and the signed height is proportional to $\cos \theta$. Increasing θ reduces $\cos \theta$, so the height of rise decreases. If the coating produces an obtuse contact angle, $\cos \theta$ is negative and the rise becomes a capillary depression; this is still a decrease in the signed height. Thus option (A), which predicts that both angle and rise increase, contradicts the capillary formula. Option (C) gives both changes consistently. The change is a surface-wetting effect, not an increase in density or gravitational acceleration.

Step 3: Conclusion The contact angle θ increases and the capillary rise h decreases. Hence, the correct option is (C).

Quick Tip to Solve: For a change in wall material, decide wetting first; then use the sign and size of $\cos \theta$ to predict capillary rise.

39. A thin tube sealed at both ends is 100 cm long. It lies horizontally, the middle 20 cm containing mercury and two equal ends containing air at standard atmospheric pressure. If the tube is now turned to a vertical position, by what amount will the mercury be displaced ? (Given : cross-section of the tube can be assumed to be uniform) [4 marks]



- (A) 2.95 cm
- (B) 5.18 cm
- (C) 8.65 cm
- (D) 0.0 cm

Correct Answer: (B) 5.18 cm

Solution:

Concept:

- The sealed air columns obey Boyle's law $PV = \text{constant}$ when compared at the same temperature. Uniform cross-section makes each volume proportional to its column length.
- Across a vertical mercury column of height 20 cm, the pressure difference is 20 cm of mercury. Standard atmospheric pressure is 76 cm of mercury.

Step 1: Express the final air pressures Initially each air column has length $(100 - 20)/2 = 40$ cm and pressure 76 cm of mercury. On making the tube vertical, the mercury moves downward by x cm. The lower air column is compressed to $40 - x$, while the upper one expands to $40 + x$. Boyle's law separately for the two sealed quantities gives

$$P_{\text{lower}} = \frac{76 \times 40}{40 - x}, \quad P_{\text{upper}} = \frac{76 \times 40}{40 + x}.$$

Pressures are expressed throughout in centimetres of mercury.

Step 2: Balance the mercury column Hydrostatic equilibrium requires $P_{\text{lower}} - P_{\text{upper}} = 20$, so

$$3040 \left(\frac{1}{40 - x} - \frac{1}{40 + x} \right) = 20.$$

Combining the fractions gives

$$\frac{6080x}{1600 - x^2} = 20 \Rightarrow x^2 + 304x - 1600 = 0.$$

The physical root is

$$x = -152 + \sqrt{24704} \simeq 5.18 \text{ cm.}$$

The other root would make a column length negative and is inadmissible. Zero displacement cannot supply the necessary pressure difference; 2.95 cm and 8.65 cm do not satisfy the pressure-balance equation.

Step 3: Conclusion The mercury is displaced downward by approximately 5.18 cm. Hence, the correct option is (B).

Quick Tip to Solve: Use two separate Boyle-law equations for the trapped air, then equate their pressure difference to the mercury-column height.

40. The ratio of the coefficient of volume expansion of a glass container to that of a viscous liquid kept inside the container is 1 : 4. What fraction of the inner volume of the container should the liquid occupy so that the volume of the remaining vacant space will be same at all temperatures ? [4 marks]

- (A) 2 : 5
- (B) 1 : 4
- (C) 1 : 64
- (D) 1 : 8

Correct Answer: (B) 1 : 4

Solution:

Concept:

- For a small temperature change ΔT , a volume changes by $\Delta V = \beta V \Delta T$, where β is its coefficient of volume expansion.
- The cavity of a glass container expands with the same volume-expansion coefficient as the glass. The vacant volume is cavity volume minus liquid volume.

Step 1: Write the vacant volume after heating Let the initial inner volume be V , and let the liquid initially occupy the fraction f , so its volume is fV . Denote the glass and liquid coefficients by β_g and β_l . At a temperature increase ΔT , the cavity and liquid volumes are respectively $V(1 + \beta_g \Delta T)$ and $fV(1 + \beta_l \Delta T)$. Subtracting them yields

$$V_{\text{vacant}} = V(1 - f) + V(\beta_g - f\beta_l)\Delta T.$$

Step 2: Set the temperature-dependent change to zero For the remaining empty space to have the same volume at every temperature within the usual constant-coefficient expansion model, the coefficient of ΔT must vanish:

$$\beta_g - f\beta_l = 0, \quad f = \frac{\beta_g}{\beta_l}.$$

The given ratio is $\beta_g : \beta_l = 1 : 4$, hence $f = 1/4$. Equal absolute increases of cavity and liquid volume then cancel. The alternative fractions $2/5$, $1/64$, and $1/8$ leave a nonzero temperature coefficient. Cubing the given ratio is inappropriate because it already concerns volume expansion.

Step 3: Conclusion The liquid must occupy one quarter of the initial inner volume, i.e. the ratio is 1 : 4. Hence, the correct option is (B).

Quick Tip to Solve: An unchanged empty volume requires equal absolute expansions of the container cavity and its liquid, not equal fractional expansions.

41. 500 g of water and 100 g of ice at 0°C are in a calorimeter whose water equivalent is 40 g. 10 g of steam at 100°C is added to it. Then water in the calorimeter is : (Latent heat of ice = 80 cal/g, Latent heat of steam = 540 cal/g) [4 marks]

- (A) 580 g
- (B) 590 g
- (C) 600 g
- (D) 610 g

Correct Answer: (B) 590 g

Solution:

Concept:

- Condensing steam releases latent heat mL_v . The resulting water also releases $mc\Delta T$ as it cools to the final temperature.
- An ice-water mixture remains at 0°C while both phases coexist. Energy supplied at this stage melts ice according to $Q = mL_f$.

Step 1: Check whether all the ice melts If 10 g of steam condenses at 100°C and its condensate cools to 0°C , the heat released is

$$Q = 10(540) + 10(1)(100) = 6400 \text{ cal.}$$

Melting the entire initial 100 g of ice would need $100 \times 80 = 8000 \text{ cal}$. Because the available heat is less, some ice remains and the final temperature really is 0°C , validating the cooling temperature used. The calorimeter starts and ends at that same temperature, so its water equivalent 40 g absorbs no net sensible heat.

Step 2: Find the actual liquid-water mass The amount of ice that melts is

$$m_{\text{melted}} = \frac{6400}{80} = 80 \text{ g.}$$

The liquid water consists of the original 500 g, the newly melted 80 g, and the 10 g of condensed steam:

$$m_{\text{water}} = 500 + 80 + 10 = 590 \text{ g.}$$

There are still $100 - 80 = 20 \text{ g}$ of ice. The total mass is $590 + 20 = 610 \text{ g}$, as required by conservation. The 580 g choice omits the condensate; 600 g and 610 g count too much material as liquid, with 610 g incorrectly treating all the ice as melted.

Step 3: Conclusion The liquid water in the calorimeter has mass 590 g, with 20 g of ice remaining.

Hence, the correct option is (B).

Quick Tip to Solve: First compare steam heat with the heat needed to melt all the ice; include condensed steam when counting the final liquid water.

42. This question has Statement-1 and Statement-2. Of the four choices given after the Statements, choose the one that best describes the two Statements. Statement 1: The internal energy of a perfect gas is entirely kinetic and depends only on absolute temperature of the gas and not on its pressure or volume. Statement 2: A perfect gas is heated keeping pressure constant and later at constant volume. For the same amount of heat the temperature of the gas at constant pressure is lower than that at constant volume

[4 marks]

- (A) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1.
(B) Statement-1 is true, Statement-2 is false.
(C) Statement-1 is true, Statement-2 is true but Statement-2 is not the correct explanation of Statement-1.
(D) Statement-1 is false, Statement-2 is true.

Correct Answer: (C) Statement-1 is true, Statement-2 is true but Statement-2 is not the correct explanation of Statement-1.

Solution:

Concept:

- For a fixed quantity of an ideal gas, the internal energy is molecular kinetic energy and depends on absolute temperature alone.
- For equal supplied heat, $Q = nC\Delta T$; an ideal gas has $C_p - C_v = R > 0$.

Step 1: Assess the internal-energy statement An ideal gas has no intermolecular potential-energy contribution in its standard model. Its internal energy is the total molecular kinetic energy, including the accessible molecular degrees of freedom. For a fixed amount and composition it is a function of T , not a separate function of pressure or volume. For a gas with fixed f degrees of freedom, $U = (f/2)nRT$. Statement-1 is true.

Step 2: Compare heating at constant pressure and volume For identical initial temperatures and the same heat input Q ,

$$\Delta T_p = \frac{Q}{nC_p}, \quad \Delta T_v = \frac{Q}{nC_v}.$$

Since $C_p > C_v$, the temperature rise is smaller at constant pressure. At constant pressure some of the heat supports expansion work, whereas at constant volume no boundary work is done. This establishes the intended comparison in Statement-2, which is true.

Step 3: Determine whether the second explains the first The heat-capacity comparison describes the distribution of added heat between internal energy and expansion work. It does not establish the microscopic absence of intermolecular potential energy or the temperature-only dependence of U . Rather, that ideal-gas internal-energy property is used in deriving the heat-capacity relation. Thus Statement-2 is not the explanation of Statement-1. Choices (B) and (D) wrongly make one statement false; (A) reverses the explanatory logic.

Step 4: Conclusion Both statements are true, but Statement-2 is not the correct explanation of Statement-1. Hence, the correct option is (C).

Quick Tip to Solve: True heat-capacity statements need not explain a microscopic energy statement; distinguish the consequence of a model from its reason.

43. Bob of a simple pendulum of length l is made of iron. The pendulum is oscillating over a horizontal coil carrying direct current. If the time period of the pendulum is T then: [4 marks]

- (A) $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is smaller than in air alone.
- (B) $T = 2\pi\sqrt{\frac{l}{g}}$ and damping is larger than in air alone.
- (C) $T > 2\pi\sqrt{\frac{l}{g}}$ and damping is smaller than in air alone.
- (D) $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is larger than in air alone.

Correct Answer: (D) $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is larger than in air alone.

Solution:

Concept:

- A simple pendulum has period $T_0 = 2\pi\sqrt{l/g}$, where l is its length and g is gravitational acceleration. An additional downward restoring force makes its small-oscillation period shorter.
- Iron is attracted towards the stronger magnetic field near a current-carrying coil. Motion of a conducting iron bob through a non-uniform magnetic field also produces eddy currents, whose effects oppose the motion by Lenz's law.

Step 1: Identify the additional attraction The horizontal coil below the pendulum produces a non-uniform magnetic field. The iron bob becomes magnetised and is attracted towards the coil, irrespective of which face of the coil is its north pole. Thus the force towards the lower region supplements gravity. For small displacements this increases the restoring action towards the lowest position. If its equivalent downward contribution is denoted by F_m , then the corresponding effective acceleration exceeds g .

Step 2: Compare the period and damping Writing the small-oscillation result in terms of the increased restoring acceleration gives

$$T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}} < 2\pi\sqrt{\frac{l}{g}}, \quad g_{\text{eff}} > g.$$

The magnetic flux through conducting portions of the moving bob varies as it oscillates. The induced eddy currents dissipate mechanical energy as heat and give extra damping in addition to air resistance. A direct current in the coil does not prevent this effect: the bob itself moves through a spatially varying field. Thus neither unchanged period nor reduced damping is appropriate. Options (A) and

(C) incorrectly reduce damping, while (B) incorrectly leaves the period unchanged.

Step 3: Conclusion The period decreases and damping is greater than in air alone: $T < 2\pi\sqrt{l/g}$. Hence, the correct option is (D).

Quick Tip to Solve: Treat magnetic attraction and eddy-current damping separately: attraction changes the restoring force, while induced currents remove oscillation energy.

44. A sonometer wire of length 114 cm is fixed at both the ends. Where should the two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1 : 3 : 4 ? [4 marks]

- (A) At 36 cm and 84 cm from one end
- (B) At 24 cm and 72 cm from one end
- (C) At 48 cm and 96 cm from one end
- (D) At 72 cm and 96 cm from one end

Correct Answer: (D) At 72 cm and 96 cm from one end

Solution:

Concept:

- For a stretched wire fixed at both ends, the fundamental frequency is $f = (1/2L)\sqrt{\tau/\mu}$, where L is vibrating length, τ is tension and μ is mass per unit length.
- The three segments belong to the same wire under the same tension. Their wave speed is therefore common, so their fundamental frequencies are inversely proportional to their lengths.

Step 1: Convert the frequency ratio into lengths Let the three frequencies be $f_0, 3f_0, 4f_0$. Because fL is constant for each segment, the corresponding lengths satisfy

$$L_1 : L_2 : L_3 = 1 : \frac{1}{3} : \frac{1}{4} = 12 : 4 : 3.$$

It is important to invert the frequency ratio: the lowest frequency requires the longest segment, not the shortest. Denote the common length unit in this ratio by x .

Step 2: Find the bridge positions and test alternatives The total length is 114 cm, giving

$$12x + 4x + 3x = 114, \quad x = 6 \text{ cm.}$$

Hence the required segment lengths are 72 cm, 24 cm and 18 cm. Placing the bridges at 72 cm and 96 cm from one end makes the intervals 0–72, 72–96 and 96–114 cm, with precisely those lengths. The inverse lengths are in ratio 1 : 3 : 4. Option (A) makes lengths 36, 48 and 30 cm; option (B) makes 24, 48 and 42 cm; option (C) makes 48, 48 and 18 cm. None has the required inverse-length ratio, and (C) even gives two equal frequencies.

Step 3: Conclusion The bridges should be at 72 cm and 96 cm from one end. Hence, the correct option is (D).

Quick Tip to Solve: First invert the frequency ratio and clear denominators. Bridge coordinates are cumulative segment lengths, rather than the three individual lengths.

45. Consider a finite insulated, uncharged conductor placed near a finite positively charged conductor. The uncharged body must have a potential [4 marks]

- (A) less than the charged conductor and more than at infinity
- (B) more than the charged conductor and less than at infinity
- (C) more than the charged conductor and more than at infinity
- (D) less than the charged conductor and less than at infinity

Correct Answer: (A) less than the charged conductor and more than at infinity

Solution:

Concept:

- In electrostatic equilibrium each conductor is an equipotential, although the surface charge on a neutral conductor can redistribute by induction. Neutrality specifies zero net charge, not zero electric potential.
- Take electric potential at infinity to be zero. A finite positively charged conductor is a positive-potential source; an isolated neutral conductor acquires an intermediate potential through electrostatic induction.

Step 1: Separate induced charges from net charge The nearby positive conductor attracts negative charge to the facing surface of the initially uncharged conductor. An equal positive induced charge appears elsewhere, leaving its total charge zero. These induced charges make the electric field inside the conductor vanish and bring its whole surface to a common potential. It would be incorrect to identify the conductor with earth: insulation gives no conducting path to an infinite reservoir and does not fix its potential at zero.

Step 2: Place its potential between the source and infinity A useful precise description uses the capacitance coefficients of the two finite conductors. If V_c is the potential of the charged conductor and V_u that of the neutral one, its net charge obeys

$$0 = Q_u = C_{uu}V_u + C_{uc}V_c, \quad V_u = \frac{-C_{uc}}{C_{uu}}V_c.$$

Here $C_{uu} > 0$, $C_{uc} < 0$, and $0 < -C_{uc}/C_{uu} < 1$ for finite separated conductors in otherwise empty space. Thus $0 < V_u < V_c$. Physically the induced neutral body samples the positive potential of the source while remaining at a lower potential than that source. Options (B) and (C) put it above the positive source; (B) and (D) also put it below zero. Both directions contradict this ordering.

Step 3: Conclusion The neutral conductor has potential less than that of the charged conductor and greater than that at infinity. Hence, the correct option is (A).

Quick Tip to Solve: An insulated neutral conductor need not have zero potential. Zero potential is a grounding condition; neutrality is a condition on total charge.

46. A liquid drop having 6 excess electrons is kept stationary under a uniform electric field of 25.5 kV m^{-1} . The density of liquid is $1.26 \times 10^3 \text{ kg m}^{-3}$. The radius of the drop is (neglect buoyancy).

[4 marks]

- (A) $4.3 \times 10^{-7} \text{ m}$
(B) $7.8 \times 10^{-7} \text{ m}$
(C) $0.078 \times 10^{-7} \text{ m}$
(D) $3.4 \times 10^{-7} \text{ m}$

Correct Answer: (B) $7.8 \times 10^{-7} \text{ m}$

Solution:

Concept:

- A stationary charged drop has zero resultant force. Neglecting buoyancy, the magnitude of the upward electric force balances its weight: $|q|E = mg$.
- For a spherical drop of density ρ and radius r , $m = (4/3)\pi r^3 \rho$. Six excess electrons give charge magnitude $|q| = 6e$, where $e = 1.60 \times 10^{-19} \text{ C}$.

Step 1: Write the balance with consistent units The electric field magnitude is $E = 25.5 \times 10^3 \text{ V m}^{-1}$, and the liquid density is $\rho = 1.26 \times 10^3 \text{ kg m}^{-3}$. The drop is negatively charged, so the field must point downwards for its electric force to point upwards. Only the magnitude is needed to find the size. Balancing the forces gives

$$6eE = \frac{4}{3}\pi r^3 \rho g, \quad r^3 = \frac{18eE}{4\pi \rho g}.$$

Step 2: Calculate the radius and compare the options Using $g = 9.8 \text{ m s}^{-2}$, the electric force is

$$6eE = 6(1.60 \times 10^{-19})(25.5 \times 10^3) = 2.448 \times 10^{-14} \text{ N}.$$

Therefore

$$r^3 = \frac{3(2.448 \times 10^{-14})}{4\pi(1.26 \times 10^3)(9.8)} \simeq 4.73 \times 10^{-19} \text{ m}^3, \\ r \simeq 7.79 \times 10^{-7} \text{ m}.$$

The required cube root matters: the force depends on volume, hence on the cube of radius. Values $4.3 \times 10^{-7} \text{ m}$ and $3.4 \times 10^{-7} \text{ m}$ give too small a weight, while $0.078 \times 10^{-7} \text{ m}$ is smaller by a factor of 100 and its weight by a factor of a million.

Step 3: Conclusion The radius is approximately $7.8 \times 10^{-7} \text{ m}$. Hence, the correct option is (B).

Quick Tip to Solve: Balance force before substituting numbers, convert kV/m to V/m, and take the cube root only after calculating the volume factor.

47. A parallel plate capacitor of area 60 cm^2 and separation 3 mm is charged initially to $90 \mu\text{C}$. If the medium between the plate gets slightly conducting and the plate loses the charge initially at the rate of $2.5 \times 10^{-8} \text{ C/s}$, then what is the magnetic field between the plates? [4 marks]

- (A) $2.5 \times 10^{-8} \text{ T}$
 (B) $2.0 \times 10^{-7} \text{ T}$
 (C) $1.63 \times 10^{-11} \text{ T}$
 (D) Zero

Correct Answer: (D) Zero

Solution:

Concept:

- Maxwell's displacement current is $I_d = \epsilon_0 d\Phi_E/dt$. It enters the Ampere–Maxwell law together with the actual conduction current, with algebraic signs fixed by a common surface normal.
- In an isolated leaky capacitor, charge conduction through the dielectric reduces the plate charge. For a uniform field between the plates, the conduction current and displacement current are equal in magnitude and opposite in direction.

Step 1: Relate leakage to the changing electric field Let Q be the positive charge on one plate, and choose the surface normal along the electric field from that plate to the negative plate. With plate area A ,

$$E = \frac{Q}{\epsilon_0 A}, \quad \Phi_E = EA = \frac{Q}{\epsilon_0}.$$

The leakage conduction current flows along this normal and has magnitude $I_c = 2.5 \times 10^{-8} \text{ A}$. Because this current drains the positive plate, $dQ/dt = -I_c$. Consequently $I_d = \epsilon_0 d\Phi_E/dt = dQ/dt = -I_c$.

Step 2: Add the two contributions before finding the magnetic field The current responsible for magnetic circulation is the total in the Ampere–Maxwell law:

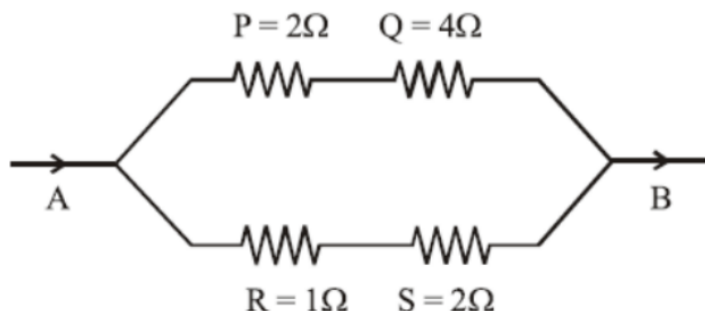
$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0(I_c + I_d) = \mu_0(I_c - I_c) = 0.$$

The same cancellation applies locally to corresponding portions of a uniform parallel-plate region, so no circulating magnetic field is produced between the plates in the idealised situation. Using the leakage current alone would neglect the decreasing electric flux and would predict a spurious field. The initial charge, area and plate separation are not needed once this cancellation has been recognised. All three nonzero numerical alternatives (A), (B) and (C) omit the cancellation between the two signed current contributions.

Step 3: Conclusion The net current in the Ampere–Maxwell law is zero, so the magnetic field between the plates is zero. Hence, the correct option is (D).

Quick Tip to Solve: For a discharging isolated capacitor with a conducting dielectric, include both leakage current and displacement current; they cancel rather than add.

48. Which of the four resistances P , Q , R and S generate the greatest amount of heat when a current flows from A to B? [4 marks]



- (A) Q
- (B) S
- (C) P
- (D) R

Correct Answer: (B) S

Solution:

Concept:

- Joule heating over a fixed time is proportional to power $P = I^2 R$. Series elements carry the same current, while parallel branches have the same potential difference.
- The upper branch has resistance $2 + 4 = 6\ \Omega$; the lower branch has $1 + 2 = 3\ \Omega$.

Step 1: Find the branch currents Let V be the potential difference between A and B. The current through the series pair P,Q is $I_u = V/6$, whereas that through R,S is $I_l = V/3 = 2I_u$. Therefore one cannot compare the four resistances alone: the smaller-resistance branch receives twice as much current, and heating depends on the square of that current.

Step 2: Calculate all four powers Using the printed resistances,

$$P_P = (V/6)^2(2) = V^2/18,$$

$$P_Q = (V/6)^2(4) = V^2/9,$$

$$P_R = (V/3)^2(1) = V^2/9,$$

$$P_S = (V/3)^2(2) = 2V^2/9.$$

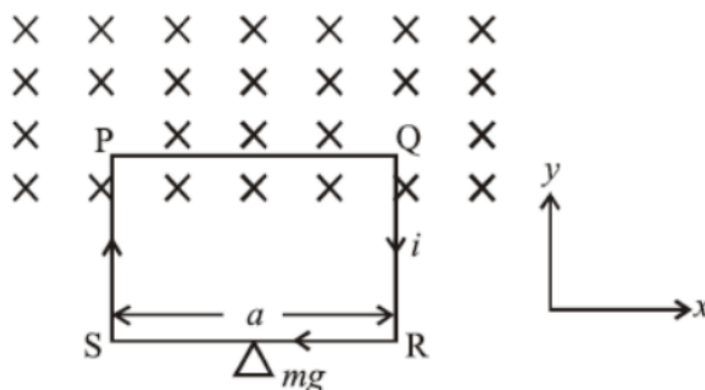
Thus the relative powers are $1 : 2 : 2 : 4$. For any common duration the generated heats have exactly the same ratio.

Step 3: Select the largest heating element S dissipates twice the power of either Q or R and four times that of P. Q has the greatest individual resistance but lies in the low-current branch, so option (A) is not correct. P and R likewise dissipate less than S. The conclusion is independent of the actual applied voltage or total current because all powers contain the same common scale factor.

Step 4: Conclusion Resistance S generates the greatest heat. Hence, the correct option is (B).

Quick Tip to Solve: Find branch currents before applying $I^2 R$; the lower branch has twice the current, which quadruples its heating factor.

49. A rectangular loop of wire, supporting a mass m , hangs with one end in a uniform magnetic field $\vec{B}$ pointing out of the plane of the paper. A clockwise current is set up such that $i > mg/Ba$, where a is the width of the loop. Then: [4 marks]



- (A) The weight rises due to a vertical force caused by the magnetic field and work is done on the system.
 (B) The weight do not rise due to vertical force caused by the magnetic field and work is done on the system.
 (C) The weight rises due to a vertical force caused by the magnetic field but no work is done on the system.
 (D) The weight rises due to a vertical force caused by the magnetic field and work is extracted from the magnetic field.

Correct Answer: The written outward field gives downward force, whereas the inward crosses give upward force and the keyed answer (A).

Solution:

Concept:

- The magnetic force on a straight current segment is $\mathbf{F} = i\mathbf{l} \times \mathbf{B}$. Clockwise current on the upper horizontal segment PQ runs from left to right.
- Crosses conventionally denote magnetic field into the page; the printed sentence instead specifies a field out of the page, so the two supplied directions conflict.

Step 1: Apply the field direction stated in words Take x rightward, y upward, and z out of the page. On PQ the current points along $+\hat{x}$. The stated outward field is $+B\hat{z}$, hence

$$\mathbf{F}_{PQ} = iaB(\hat{x} \times \hat{z}) = -iaB\hat{y}.$$

This is downward and adds to gravity. The inequality $i > mg/(Ba)$ increases that downward magnetic force; it cannot make the mass rise. Thus the rising claims in (A), (C), and (D) do not follow from the written field direction.

Step 2: Check the direction shown by the crosses If the conventional crosses in the figure are used instead, $\mathbf{B} = -B\hat{z}$, so $\mathbf{F}_{PQ} = +iaB\hat{y}$. Then $iaB > mg$ gives an upward net force. In that interpretation the mass rises and positive mechanical work is done on it, matching (A). The electrical supply provides the energy; the magnetic field is not an energy source. Consequently (C) and (D) are unsuitable even in that interpretation.

Step 3: Explain the answer-key conflict The supplied key selects (A), which corresponds to the inward

field drawn by the crosses. The literal outward-field sentence gives no rise, corresponding to the no-rise portion of (B); its general work wording does not remove the contradictory field directions. Since the figure and sentence specify opposite forces, one unique physical conclusion cannot be justified from all the printed data together. Neither source direction has been silently replaced.

Step 4: Conclusion The printed directions are inconsistent: the outward field stated in words gives a downward force, while the inward crosses give an upward force and the keyed option (A). Therefore the question has no unique consistent answer as printed; (A) follows only from the field direction shown by the crosses.

Quick Tip to Solve: Use the right-hand rule on the horizontal segment inside the field; check whether crosses mean inward before interpreting an upward-force threshold.

50. A particle of charge $16 \times 10^{-16} \text{ C}$ moving with velocity 10 m s^{-1} along x -axis enters a region where magnetic field of induction $\vec{B}$ is along the y -axis and an electric field of magnitude 10^4 V m^{-1} is along the negative z -axis. If the charged particle continues moving along x -axis, the magnitude of $\vec{B}$ is: [4 marks]

- (A) $16 \times 10^3 \text{ Wb m}^{-2}$
- (B) $2 \times 10^3 \text{ Wb m}^{-2}$
- (C) $1 \times 10^3 \text{ Wb m}^{-2}$
- (D) $4 \times 10^3 \text{ Wb m}^{-2}$

Correct Answer: (C) $1 \times 10^3 \text{ Wb m}^{-2}$

Solution:

Concept:

- A charged particle in electric and magnetic fields experiences the Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. Here q is the particle charge and $\vec{v}$ is its velocity.
- For undeflected motion in perpendicular fields, the electric and magnetic forces cancel. Their magnitudes therefore obey $qE = qvB$, or $B = E/v$.

Step 1: Determine the force directions The particle travels along positive x , while the magnetic field points along positive y . The right-hand rule gives $\vec{v} \times \vec{B}$ along positive z . The electric field points along negative z , so the two forces on the positive charge oppose one another. Since the path remains along the original axis, the transverse resultant must vanish. Merely having perpendicular fields is insufficient; their force magnitudes must also agree.

Step 2: Balance and calculate Equate the magnitudes and cancel the common charge:

$$qE = qvB \implies B = \frac{10^4}{10} = 10^3 \text{ T}.$$

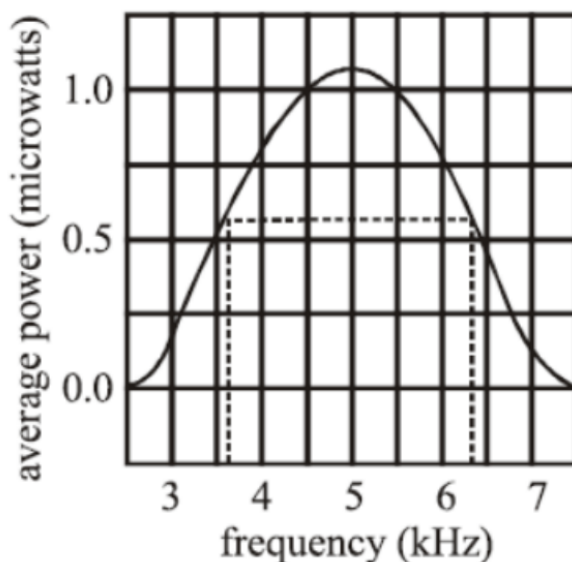
The given charge $16 \times 10^{-16} \text{ C}$ cancels and does not multiply the result. Since $1 \text{ T} = 1 \text{ Wb m}^{-2}$, this equals the value in (C). Options (A), (B) and (D) would give magnetic forces respectively sixteen, two and four times the required balancing force and would deflect the particle.

Step 3: Conclusion The required magnetic induction is $1 \times 10^3 \text{ Wb m}^{-2}$. Hence, the correct option is

(C).

Quick Tip to Solve: For a velocity selector, verify that the forces oppose before using $B = E/v$; the charge then cancels.

51. The plot given below is of the average power delivered to an LRC circuit versus frequency. The quality factor of the circuit is: [4 marks]



- (A) 5.0
- (B) 2.0
- (C) 2.5
- (D) 0.4

Correct Answer: (B) 2.0

Solution:

Concept:

- The quality factor of a series LRC resonance curve is $Q = f_0/(f_2 - f_1)$, where f_0 is the resonant frequency and f_1, f_2 are its half-power frequencies.
- The half-power frequencies are read at half the maximum average power, not at half the maximum current amplitude. Their separation is the bandwidth.

Step 1: Read the resonance and half-power points The maximum of the power curve occurs at approximately 5.0 kHz, so $f_0 \simeq 5.0$ kHz. The dotted horizontal line marks half the peak power. Its two intersections with the curve project down near 3.6 kHz and 6.4 kHz. These are approximate graphical readings from the indicated dotted guides; the graph does not mark exact numerical values at those feet. Use the half-power guides rather than the points where power approaches zero, since those endpoints do not define the resonance bandwidth.

Step 2: Calculate the dimensionless ratio The graphical bandwidth and quality factor are approxi-

mately

$$\Delta f \simeq 6.4 - 3.6 = 2.8 \text{ kHz}, \quad Q \simeq \frac{5.0}{2.8} \simeq 1.8.$$

Among the supplied choices, 2.0 is the nearest and is the intended answer (B). The schematic plot does not support claiming an exact bandwidth of 2.5 kHz. Both frequencies are expressed in the same units, which cancel in the ratio. Option (A) confuses the resonant-frequency number with the dimensionless ratio; (C) is substantially above the graphical estimate; and (D) is nearer the reciprocal ratio $\Delta f/f_0$. A narrower resonance at fixed central frequency would have a larger quality factor, consistent with the definition.

Step 3: Conclusion The graph gives Q approximately 1.8; the nearest and intended listed value is 2.0. Hence, the correct option is (B).

Quick Tip to Solve: Read the two half-power intersections first, subtract their frequencies, then divide the peak frequency by that bandwidth.

52. Select the correct statement from the following:

[4 marks]

- (A) Electromagnetic waves cannot travel in vacuum.
- (B) Electromagnetic waves are longitudinal waves.
- (C) Electromagnetic waves are produced by charges moving with uniform velocity.
- (D) Electromagnetic waves carry both energy and momentum as they propagate through space.

Correct Answer: (D) Electromagnetic waves carry both energy and momentum as they propagate through space.

Solution:

Concept:

- An electromagnetic wave consists of time-varying electric and magnetic fields that sustain one another and can propagate through vacuum at speed c .
- Radiation transports energy and momentum. For energy U travelling in vacuum the associated momentum magnitude is U/c ; absorption therefore produces radiation pressure.

Step 1: Check propagation and field orientation Option (A) is false because no material medium is required: sunlight crosses nearly empty interplanetary space before reaching Earth. In a plane electromagnetic wave the electric field, magnetic field and propagation direction are mutually perpendicular. The fields oscillate transversely, so option (B), which calls the wave longitudinal, is also false. Electric and magnetic fields can exist around a uniformly moving charge without that charge emitting radiation merely because its velocity is nonzero.

Step 2: Identify radiation and transport Emission of electromagnetic radiation is associated with accelerated charges or changing current distributions. A charge in uniform rectilinear motion does not radiate in an inertial frame, so (C) is not the correct production statement. By contrast, the wave carries energy flux described by the Poynting vector $\vec{S} = \vec{E} \times \vec{B}/\mu_0$. It also transports momentum, whose transfer to a surface explains radiation pressure. Thus both claims in (D), energy transport and momentum transport, hold simultaneously; momentum transport does not require the wave to carry

matter along with it.

Step 3: Conclusion Electromagnetic waves carry both energy and momentum as they propagate through space. Hence, the correct option is (D).

Quick Tip to Solve: Test electromagnetic-wave statements against three facts: vacuum propagation, transverse fields and radiation from accelerated charges.

53. This question has Statement-1 and Statement-2. Of the four choices given after the Statements, choose the one that best describes the two Statements. Statement 1: Very large size telescopes are reflecting telescopes instead of refracting telescopes. Statement 2: It is easier to provide mechanical support to large size mirrors than large size lenses. [4 marks]

- (A) Statement-1 is true and Statement-2 is false.
(B) Statement-1 is false and Statement-2 is true.
(C) Statement-1 and statement-2 are true and Statement-2 is correct explanation for statement-1.
(D) Statements-1 and statement-2 are true and Statement-2 is not the correct explanation for statement-1.

Correct Answer: (C) Statement-1 and statement-2 are true and Statement-2 is correct explanation for statement-1.

Solution:

Concept:

- A reflecting telescope forms its primary image with a concave mirror, whereas a refracting telescope uses a large objective lens.
- A mirror needs only its reflecting face optically accessible. It can therefore be supported from behind, unlike a transmitting lens whose aperture must remain clear.

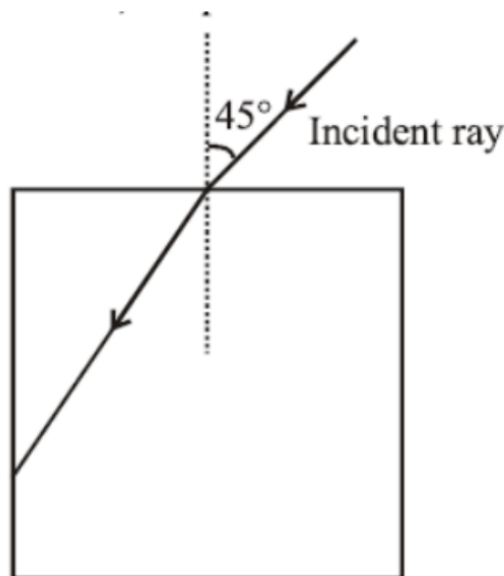
Step 1: Evaluate the telescope statement Very large astronomical telescopes use reflecting primary mirrors. A large collecting aperture gathers faint light and improves angular resolution, but a large glass lens is heavy and can sag if supported only around its edge. It must also transmit light through its full thickness. A mirror is therefore the practical choice for very large apertures. Statement-1 is true; option (B), which declares it false, cannot be selected.

Step 2: Test the support statement and causal link A mirror can rest on a distributed support system behind its reflecting surface. Such supports reduce deformation without blocking the incoming beam at the useful reflecting face. A lens must transmit the beam, so opaque supports across its rear face would obstruct the light and cannot be used in the same manner. Statement-2 is consequently true. It directly explains a major advantage that makes large reflecting telescopes feasible. Mirrors also avoid chromatic aberration, but the presence of additional advantages does not invalidate the stated mechanical explanation. Option (A) wrongly rejects Statement-2; option (D) accepts both statements but wrongly denies their explanatory connection.

Step 3: Conclusion Both statements are true, and Statement-2 correctly explains Statement-1. Hence, the correct option is (C).

Quick Tip to Solve: For large telescope objectives, ask where mechanical supports can be placed without blocking the optical path.

54. A light ray falls on a square glass slab as shown in the diagram. The index of refraction of the glass, if total internal reflection is to occur at the vertical face, is equal to: [4 marks]



- (A) $\frac{\sqrt{2}+1}{2}$
- (B) $\sqrt{\frac{5}{2}}$
- (C) $\frac{3}{2}$
- (D) $\sqrt{\frac{3}{2}}$

Correct Answer: (D) $\sqrt{\frac{3}{2}}$

Solution:

Concept:

- Snell's law at the horizontal air-glass interface gives $\sin 45^\circ = n \sin r$, with r measured from the vertical normal inside the glass.
- At the perpendicular vertical face the incidence angle is $90^\circ - r$. The critical angle for glass to air satisfies $\sin C = 1/n$, and strict total internal reflection requires incidence greater than C .

Step 1: Relate the two incidence angles The figure shows incidence at 45° to the normal of the top face. Thus

$$\sin r = \frac{1}{\sqrt{2}n}.$$

The normal to the vertical face is horizontal, so the incidence there is the complementary angle $i = 90^\circ - r$. At the critical threshold $i = C$, giving $n \cos r = 1$. This complement is essential: using r itself as the second incidence angle would apply Snell's law to the wrong normal.

Step 2: Find the threshold and distinguish strict reflection Squaring the threshold equation gives

$$n^2 \left(1 - \frac{1}{2n^2} \right) = 1 \implies n^2 = \frac{3}{2}.$$

Therefore $n_{\text{crit}} = \sqrt{3/2}$, option (D). Strict total internal reflection occurs for $n > \sqrt{3/2}$, while equality

is the limiting critical case. The printed alternatives (B), $\sqrt{5/2}$, and (C), $3/2$, are both above this threshold and therefore also permit total internal reflection. Alternative (A), $(\sqrt{2} + 1)/2$, is below it and does not. Thus the wording does not specify a unique actual index: the intended choice is the critical limiting index, not every index capable of total internal reflection.

Step 3: Conclusion The limiting critical index is $\sqrt{3/2}$, the intended option (D). Literally, strict total internal reflection requires $n > \sqrt{3/2}$, so (B) and (C) also satisfy that condition; the stated question has this ambiguity.

Quick Tip to Solve: On perpendicular faces the refracted angle and next incidence angle are complementary; separate the critical threshold from the strict inequality.

55. n identical waves each of intensity I_0 interfere with each other. The ratio of maximum intensities if the interference is (i) coherent and (ii) incoherent is: [4 marks]

- (A) n^2
- (B) $\frac{1}{n}$
- (C) $\frac{1}{n^2}$
- (D) n

Correct Answer: (D) n

Solution:

Concept:

- For identical waves, intensity is proportional to the square of amplitude: $I_0 = kA_0^2$, where A_0 is one wave's amplitude.
- Coherent waves can have a fixed phase relation and add amplitudes constructively. For mutually incoherent waves, time-averaged interference terms vanish and intensities add.

Step 1: Add the coherent amplitudes at a maximum At a constructive maximum all n coherent waves have the same phase. Their resultant amplitude is $A_{\max} = nA_0$, so

$$I_{\text{coh}} = k(nA_0)^2 = n^2 I_0.$$

This is a maximum intensity at the point of constructive interference, not an increase in the total energy supplied by the sources. Interference redistributes the energy spatially; bright regions are balanced by reduced intensity elsewhere.

Step 2: Add incoherent intensities and form the ratio For incoherent waves the relative phases fluctuate. The cross terms in the squared resultant amplitude average to zero during observation. Therefore

$$I_{\text{inc}} = I_0 + I_0 + \cdots + I_0 = nI_0, \quad \frac{I_{\text{coh}}}{I_{\text{inc}}} = \frac{n^2 I_0}{nI_0} = n.$$

Option (A) gives the coherent enhancement relative to a single wave, rather than the requested ratio. Options (B) and (C) reverse the enhancement or compare the wrong quantities. For $n = 2$, the result is also checked by the familiar values $4I_0$ and $2I_0$, whose ratio is 2.

Step 3: Conclusion The ratio of coherent maximum intensity to incoherent intensity is n . Hence, the correct option is (D).

Quick Tip to Solve: Coherent maxima add amplitudes before squaring; incoherent waves add intensities directly. Form the requested ratio only afterwards.

56. Electrons are accelerated through a potential difference V and protons are accelerated through a potential difference $4V$. The de-Broglie wavelengths are λ_e and λ_p for electrons and protons respectively. The ratio of $\frac{\lambda_e}{\lambda_p}$ is given by: (given m_e is mass of electron and m_p is mass of proton).

[4 marks]

(A) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$

(B) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_e}{m_p}}$

(C) $\frac{\lambda_e}{\lambda_p} = \frac{1}{2} \sqrt{\frac{m_e}{m_p}}$

(D) $\frac{\lambda_e}{\lambda_p} = 2 \sqrt{\frac{m_p}{m_e}}$

Correct Answer: (D) $\frac{\lambda_e}{\lambda_p} = 2 \sqrt{\frac{m_p}{m_e}}$

Solution:

Concept:

- A particle accelerated from rest through potential difference U gains kinetic energy $|q|U$, neglecting relativistic corrections.
- Using $p = \sqrt{2mK}$ and $\lambda = h/p$, its wavelength is $\lambda = h/\sqrt{2m|q|U}$. Electron and proton charges have equal magnitude e .

Step 1: Write both momenta from their energies The electron gains energy eV , whereas the proton gains $e(4V) = 4eV$. The sign of the electron charge determines which direction accelerates it, but the kinetic-energy gain uses the magnitude of the potential drop. Thus

$$p_e = \sqrt{2m_e eV}, \quad p_p = \sqrt{2m_p e(4V)}.$$

The proton's larger accelerating potential must be included along with its larger mass; comparing masses alone misses a factor.

Step 2: Take the inverse momentum ratio Because wavelength is inversely proportional to momentum,

$$\frac{\lambda_e}{\lambda_p} = \frac{p_p}{p_e} = \sqrt{\frac{2m_p e(4V)}{2m_e eV}} = 2 \sqrt{\frac{m_p}{m_e}}.$$

The Planck constant and equal charge magnitude cancel. Option (A) omits the square root of the factor 4 in the voltage; (B) additionally reverses the mass ratio. Option (C) reverses both factors and corresponds to the reciprocal wavelength ratio. The electron wavelength should be much larger because its momentum is much smaller, which provides a useful qualitative check.

Step 3: Conclusion The wavelength ratio is $\lambda_e/\lambda_p = 2\sqrt{m_p/m_e}$. Hence, the correct option is (D).

Quick Tip to Solve: Write $\lambda \propto 1/\sqrt{mU}$; the factor four in voltage becomes a factor two after the square root.

57. In the Bohr's model of hydrogen-like atom the force between the nucleus and the electron is modified as $F = \frac{e^2}{4\pi\epsilon_0} \left(\frac{1}{r^2} + \frac{\beta}{r^3} \right)$, where β is a constant. For this atom, the radius of the n^{th} orbit in terms of the Bohr radius $\left(a_0 = \frac{\epsilon_0 h^2}{m\pi e^2} \right)$ is: [4 marks]

- (A) $r_n = a_0 n - \beta$
 (B) $r_n = a_0 n^2 + \beta$
 (C) $r_n = a_0 n^2 - \beta$
 (D) $r_n = a_0 n + \beta$

Correct Answer: (C) $r_n = a_0 n^2 - \beta$

Solution:

Concept:

- For a circular orbit, the attractive central force supplies mv^2/r . Bohr quantisation imposes $mvr = n\hbar$, where $\hbar = h/(2\pi)$.
- The given force is $F = ke^2(1/r^2 + \beta/r^3)$, with $k = 1/(4\pi\epsilon_0)$. Since the two terms have equal dimensions, β has the dimension of length.

Step 1: Eliminate the orbital speed Angular-momentum quantisation gives $v = n\hbar/(mr)$. Substituting into the centripetal-force condition yields

$$\frac{n^2 \hbar^2}{mr^3} = ke^2 \left(\frac{1}{r^2} + \frac{\beta}{r^3} \right).$$

This substitution retains the r^{-3} correction rather than treating it as an independent constant force. Multiply through by r^3 , which is positive for a physical circular orbit, to obtain $n^2 \hbar^2 / m = ke^2(r + \beta)$.

Step 2: Recognise the Bohr radius and solve Rearranging gives

$$r + \beta = \frac{n^2 \hbar^2}{mke^2} = n^2 \frac{\epsilon_0 h^2}{m\pi e^2} = n^2 a_0, \quad r_n = a_0 n^2 - \beta.$$

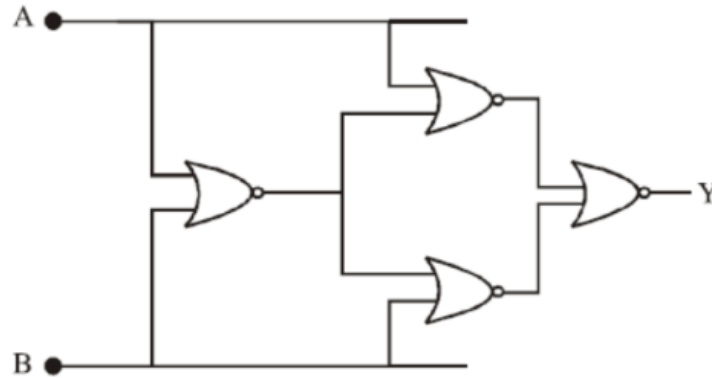
The physical orbit requires the resulting radius to be positive. With $\beta = 0$, the ordinary Bohr radius law $r_n = a_0 n^2$ is recovered. Options (A) and (D) have a linear dependence on n , contradicting the quantisation result. Option (B) gives the wrong sign: the extra attraction for positive β reduces, rather than increases, the radius at a fixed angular momentum.

Step 3: Conclusion The allowed radius is $r_n = a_0 n^2 - \beta$, for positive physical radius. Hence, the correct option is (C).

Quick Tip to Solve: Eliminate speed using angular momentum first; multiplying the force equation by r^3 makes the correction a simple shift.

58. A system of four gates is set up as shown. The 'truth table' corresponding to this system is :

[4 marks]



(A)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(B)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

(C)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(D)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Correct Answer: (A)

Solution:

Concept:

- A NOR gate outputs the negation of the OR of its inputs: $\text{NOR}(a, b) = \overline{a + b}$, where $+$ denotes Boolean OR.

- The first gate output is $X = \overline{A + B}$. The upper and lower intermediate outputs are $U = \overline{A + X}$ and $L = \overline{B + X}$; the last NOR gate gives $Y = \overline{U + L}$.

Step 1: Evaluate equal input pairs All four symbols have an inversion bubble at the output, so every gate must be treated as NOR, not ordinary OR. If $A = B = 0$, the first output is $X = 1$. Each middle gate then receives a 1 and returns 0, so the final gate returns $Y = 1$. If $A = B = 1$, then $X = 0$, but each middle gate still returns 0 because its other input is 1. The final result is again $Y = 1$. Equal inputs therefore give a high output.

Step 2: Evaluate unequal pairs and compare tables For $(A, B) = (0, 1)$, $X = 0$, $U = 1$, $L = 0$, hence $Y = 0$. For $(A, B) = (1, 0)$, $X = 0$, $U = 0$, $L = 1$, hence again $Y = 0$. In input order 00, 01, 10, 11, the output column is consequently 1, 0, 0, 1: the XNOR or equivalence function. This exactly matches table (A). Table (B) fails for equal inputs; (C) wrongly gives 1 for input 10; and (D) wrongly gives 1 for input 01 and 0 for input 11. Explicit gate-by-gate evaluation avoids mistaking this four-NOR construction for XOR.

Step 3: Conclusion The truth-table output is 1, 0, 0, 1 for 00, 01, 10, 11. Hence, the correct option is (A).

Quick Tip to Solve: Label the first NOR output and evaluate all four input pairs; the final inversion turns this NOR construction into XNOR.

59. Which of the following statement is NOT correct?

[4 marks]

- (A) Ground wave signals are more stable than the sky wave signals.
- (B) The critical frequency of an ionospheric layer is the highest frequency that will be reflected back by the layer when it is vertically incident.
- (C) Electromagnetic waves of frequencies higher than about 30 MHz cannot penetrate the ionosphere.
- (D) Sky wave signals in the broadcast frequency range are stronger at night than in the day time.

Correct Answer: (C) Electromagnetic waves of frequencies higher than about 30 MHz cannot penetrate the ionosphere.

Solution:

Concept:

- The ionosphere can return suitable radio waves to Earth; frequencies above its applicable limiting value usually pass through it instead of being returned.
- The critical frequency is the highest frequency returned at vertical incidence for a given ionospheric layer. Radio-wave propagation also depends on incidence angle and the state of ionisation.

Step 1: Identify the reversed penetration claim Option (C) says electromagnetic waves above about 30 MHz cannot penetrate the ionosphere. This reverses the usual high-frequency behaviour. Such waves generally penetrate the ionosphere and continue into space instead of undergoing the sky-wave return useful for long-distance high-frequency broadcasting. This is one reason satellite communication uses sufficiently high carrier frequencies. Therefore (C) is the statement sought by the negative wording, NOT correct.

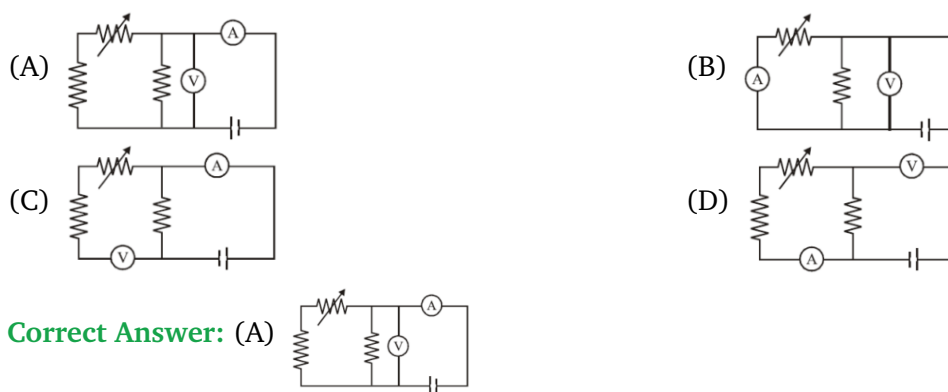
Step 2: Check the other propagation statements Ground waves follow the Earth's surface and are ordinarily more stable than sky waves, which vary with ionospheric conditions and can suffer fading; hence (A) is correct. Option (B) states the definition of critical frequency at vertical incidence and is correct. In the broadcast frequency range, daytime ionisation of the lower D region causes greater absorption. At night this absorption decreases, allowing stronger sky-wave reception over long distances, so (D) is also correct in the stated context. The question concerns standard propagation regimes; it does not claim that every frequency and every path behaves identically.

Step 3: Conclusion The false statement is the claim that frequencies above about 30 MHz cannot penetrate the ionosphere. Hence, the correct option is (C).

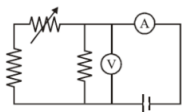
Quick Tip to Solve: For a NOT-correct propagation question, distinguish reflection back to Earth from transmission through the ionosphere into space.

60. Correct set up to verify Ohm's law is

[4 marks]



Correct Answer: (A)



Solution:

Concept:

- To determine a resistance by Ohm's law, measure its voltage with a high-resistance voltmeter connected in parallel and its current with a low-resistance ammeter in series.
- A rheostat varies the current. The ratio V/I represents only the resistor or network across which the voltage is measured and through which the measured current flows.

Step 1: Trace the meters in the proposed circuits In (A), the voltmeter is connected between the top and bottom rails, while the ammeter lies in the supply lead. It therefore reads the total current supplied to the parallel resistive network, and the voltmeter reads the voltage across that complete network. This is the intended series-ammeter, parallel-voltmeter arrangement. In (B), the ammeter lies only in the left-hand rheostat branch, although the voltmeter spans the full parallel network. Its current is not the total network current. Thus (B) does not measure matching voltage and total current for the network.

Step 2: Separate the intended answer from an individual-resistor measurement In (C), the voltmeter is inserted in a resistive branch rather than directly across the measured load; in (D), it is in the supply path. Such series placement of a high-resistance voltmeter prevents the intended ordinary current measurement. Hence the intended diagram is (A). There is, however, a topology qualification: (A)

contains two resistive branches, so its ammeter does not read the current through the central resistor alone. It tests the equivalent network at a fixed rheostat setting, not an isolated central resistance while that parallel rheostat is varied. For the usual experiment on a single specified resistor, the rheostat should instead be in series with that resistor and the ammeter.

Step 3: Conclusion Option (A) is the intended correct meter arrangement for the whole resistive network. Its ammeter measures total network current, not solely the central-resistor current. Hence, the intended correct option is (A).

Quick Tip to Solve: Trace the actual wires: the ammeter current and voltmeter voltage must refer to the same load, whether a single resistor or an entire network.

Chemistry (120 marks)

61. Number of atoms in the following samples of substances is largest in

[4 marks]

- (A) 4.0 g of hydrogen
- (B) 71.0 g of chlorine
- (C) 127.0 g of iodine
- (D) 48.0 g of magnesium

Correct Answer: (A) 4.0 g of hydrogen

Solution:

Concept:

- One mole of atoms contains Avogadro's number, N_A , of atoms. For an elemental sample, the amount of atoms is its mass divided by its atomic molar mass.
- Hydrogen, chlorine and iodine occur as diatomic molecules, but the question asks for atoms. Multiplying the molecular amount by two gives the same result as using the atomic molar mass directly.

Step 1: Convert each sample into moles of atoms Use the usual atomic molar masses 1, 35.5, 127 and 24 g mol^{-1} for hydrogen, chlorine, iodine and magnesium, respectively. The amounts of atoms are

$$\begin{aligned} n_{\text{H}} &= \frac{4.0}{1} = 4.0 \text{ mol}, & n_{\text{Cl}} &= \frac{71.0}{35.5} = 2.0 \text{ mol}, \\ n_{\text{I}} &= \frac{127.0}{127} = 1.0 \text{ mol}, & n_{\text{Mg}} &= \frac{48.0}{24} = 2.0 \text{ mol}. \end{aligned}$$

Step 2: Compare the atom counts on the same basis Multiply each amount by the same constant N_A . The four atom counts, in option order, are $4N_A$, $2N_A$, N_A and $2N_A$. Thus hydrogen supplies twice as many atoms as either chlorine or magnesium, and four times as many as iodine. Using molecular masses would first give two moles of hydrogen molecules, one mole of chlorine molecules and half a mole of iodine molecules; each molecule contributes two atoms, reproducing these results. Neither the largest mass nor the largest number of molecular moles alone is the comparison requested.

Step 3: Conclusion The 4.0 g hydrogen sample contains $4N_A$ atoms, the largest of the four counts.

Hence, the correct option is (A).

Quick Tip to Solve: For the number of atoms in elemental samples, compare mass divided by atomic molar mass; diatomic molecularity then needs no separate correction.

62. The de Broglie wavelength of a car of mass 1000 kg and velocity 36 km/hr is

[4 marks]

(A) $6.626 \times 10^{-34} \text{ m}$

(B) $6.626 \times 10^{-38} \text{ m}$

(C) $6.626 \times 10^{-31} \text{ m}$

(D) $6.626 \times 10^{-30} \text{ m}$

Correct Answer: (B) $6.626 \times 10^{-38} \text{ m}$

Solution:

Concept:

- A body of momentum p has de Broglie wavelength $\lambda = h/p$, where $h = 6.626 \times 10^{-34} \text{ J s}$.
- For a nonrelativistic car, momentum is $p = mv$. Mass and velocity must be converted to SI units before substitution.

Step 1: Convert the car speed and calculate momentum The speed 36 km/hr is

$$v = 36 \times \frac{1000}{3600} = 10 \text{ m s}^{-1}.$$

With $m = 1000 \text{ kg}$, the momentum is

$$p = 1000 \times 10 = 10^4 \text{ kg m s}^{-1}.$$

The speed is vastly smaller than the speed of light, so the nonrelativistic momentum is appropriate. Dividing by 3.6, rather than multiplying by it, is essential when converting kilometres per hour to metres per second.

Step 2: Divide Planck constant by momentum Now substitute the SI momentum into the matter-wave relation:

$$\lambda = \frac{6.626 \times 10^{-34}}{10^4} = 6.626 \times 10^{-38} \text{ m}.$$

The units reduce because $\text{J s} = \text{kg m}^2 \text{ s}^{-1}$, so dividing by momentum leaves metres. Option (A) merely repeats Planck's numerical exponent and ignores momentum. Options (C) and (D) are many orders of magnitude too large. A heavy moving car has a huge momentum compared with an electron in a typical diffraction experiment, so its exceedingly small wavelength is physically reasonable and wave effects are unobservable on ordinary scales.

Step 3: Conclusion The car's de Broglie wavelength is $6.626 \times 10^{-38} \text{ m}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert 36 km/hr to 10 m/s first; dividing 10^{-34} by 10^4 gives 10^{-38} , not 10^{-30} .

63. Which is the correct order of second ionization potential of C, N, O and F in the following?

[4 marks]

(A) $O > N > F > C$

(B) $O > F > N > C$

(C) $F > O > N > C$

(D) $C > N > O > F$

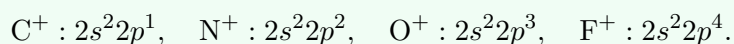
Correct Answer: (B) $O > F > N > C$

Solution:

Concept:

- The second ionisation potential removes an electron from a singly charged ion, $X^+ \rightarrow X^{2+} + e^-$; its ordering must use the configurations of the ions, not neutral atoms.
- A half-filled $2p^3$ subshell has extra exchange stability. Removing an electron from that arrangement requires unusually high energy. Effective nuclear attraction also generally increases across a period.

Step 1: Write the singly charged ion configurations Ignoring the unchanged $1s^2$ core, the relevant configurations are



The second electron is removed from these ions. In particular, oxygen has already lost one electron from neutral $2p^4$, leaving a stable half-filled $2p^3$ ion. This is a different situation from the familiar first-ionisation anomaly between neutral nitrogen and oxygen.

Step 2: Combine the stability exception with the periodic trend Removing an electron from O^+ breaks its half-filled subshell, making oxygen's second ionisation potential especially high. Removing one from F^+ instead produces the half-filled F^{2+} configuration, so fluorine falls below oxygen despite its higher nuclear charge. Fluorine still exceeds nitrogen and carbon because of its larger effective nuclear attraction; nitrogen exceeds carbon in the ordinary across-period comparison. Thus the order is $O > F > N > C$. Option (A) incorrectly puts nitrogen above fluorine; (C) misses oxygen's half-filled-ion stability; and (D) reverses the main periodic trend.

Step 3: Conclusion The correct second-ionisation order is $O > F > N > C$. Hence, the correct option is (B).

Quick Tip to Solve: For second ionisation, first remove one electron on paper; oxygen then becomes the half-filled-shell exception.

64. The shape of IF_6^- is:

[4 marks]

- (A) Trigonally distorted octahedron
- (B) Pyramidal
- (C) Octahedral
- (D) Square antiprism

Correct Answer: (A) Trigonally distorted octahedron

Solution:

Concept:

- VSEPR theory counts electron pairs around the central atom. Bonding pairs determine the positions of the attached atoms; a lone pair also repels those bonds and can distort the molecular shape.
- For a monovalent ligand such as fluorine, the steric-number count is $(V + M + A - C)/2$, where V is the central atom valence-electron count, M is the number of ligands, and A, C are the negative and positive charges.

Step 1: Count the electron pairs at iodine Iodine is a group-17 element and contributes seven valence electrons. Six fluorine atoms form six single bonds, while the negative charge contributes one additional electron. The count around iodine is therefore

$$\text{SN} = \frac{7 + 6 + 1}{2} = 7.$$

Six pairs are bond pairs and the remaining pair is nonbonding. Thus the species is described as AX_6E , rather than the lone-pair-free AX_6 case.

Step 2: Relate the lone pair to the observed shape The six fluorine atoms surround iodine in a distorted octahedral arrangement. Repulsion by the nonbonding electron pair prevents a perfectly regular octahedron; the conventional description for this ion is a trigonally distorted octahedron. A pyramidal arrangement would not account for the six ligand positions. A square antiprism requires eight ligand positions, so it cannot describe six fluorine atoms. The unqualified octahedral option misses the distortion associated with the lone pair.

Step 3: Conclusion IF_6^- has six bond pairs and one lone pair at iodine and is trigonally distorted octahedral. Hence, the correct option is (A).

Quick Tip to Solve: Include the negative charge when counting valence electrons: IF_6^- has a lone pair, unlike a regular six-coordinate AX_6 species.

65. Bond distance in HF is 9.17×10^{-11} m. Dipole moment of HF is 6.104×10^{-30} C m. The percentage ionic character in HF will be: (electron charge = 1.60×10^{-19} C) [4 marks]

- (A) 61.0%
- (B) 38.0%
- (C) 35.5%
- (D) 41.5%

Correct Answer: (D) 41.5%

Solution:

Concept:

- The dipole moment is the product of separated charge and bond distance: $\mu = qr$. A hypothetical fully ionic bond has charges of magnitude e and therefore $\mu_{\text{ionic}} = er$.
- The percentage ionic character compares the measured molecular dipole moment with this fully ionic value: $100\mu_{\text{obs}}/\mu_{\text{ionic}}$. Both dipole moments must be expressed in the same units.

Step 1: Calculate the fully ionic dipole moment Use the given charge and the bond distance without changing either power of ten. The result is

$$\begin{aligned}\mu_{\text{ionic}} &= (1.60 \times 10^{-19})(9.17 \times 10^{-11}) \\ &= 14.672 \times 10^{-30} \text{ C m} \\ &= 1.4672 \times 10^{-29} \text{ C m.}\end{aligned}$$

This corresponds to complete transfer of one electron over the stated internuclear separation, rather than the actual partial charge in the bond.

Step 2: Compare the measured dipole moment The observed dipole moment is smaller, so the ionic character must be below one hundred percent. Substitution gives

$$\% \text{ ionic character} = \frac{6.104 \times 10^{-30}}{14.672 \times 10^{-30}} \times 100 = 41.60\% \text{ approximately.}$$

Within the precision and rounding of the listed values, this is the 41.5% choice. The alternatives 61.0%, 38.0% and 35.5% do not give the observed-to-fully-ionic ratio; no inversion of that ratio is justified.

Step 3: Conclusion The ionic character is approximately 41.6%, corresponding to the listed value 41.5%. Hence, the correct option is (D).

Quick Tip to Solve: First calculate er in C m, then divide the measured dipole moment by it; the powers of ten cancel before the percentage is taken.

66. The correct order of viscosity of the following liquids will be

[4 marks]

- (A) Water < methyl alcohol < dimethyl ether < glycerol
(B) methyl alcohol < glycerol < water < dimethyl ether
(C) dimethyl ether < methyl alcohol < water < glycerol
(D) glycerol < dimethyl ether < water < methyl alcohol

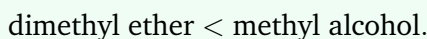
Correct Answer: (C) dimethyl ether < methyl alcohol < water < glycerol

Solution:

Concept:

- Viscosity is resistance to flow. Strong intermolecular attractions hinder the relative movement of molecules and generally increase viscosity when the liquids are compared under similar conditions.
- Hydrogen bonding depends on both suitable donor groups, such as O–H, and acceptor lone pairs. The number of such groups and the extent of the intermolecular network are central to this comparison.

Step 1: Compare the ether and the monohydric alcohol Dimethyl ether, CH_3OCH_3 , has an oxygen atom with lone pairs but no O–H bond. Its pure liquid cannot make the extensive self hydrogen bonding possible in an alcohol. Methyl alcohol, CH_3OH , contains one hydroxyl group, so molecules associate through hydrogen bonds. This gives the first part of the increasing-viscosity order:



The mere presence of oxygen in an ether does not provide a hydrogen-bond donor.

Step 2: Place water and glycerol in the sequence Water can form a more extensively connected hydrogen-bond network than methyl alcohol, whose methyl group limits that connectivity. Glycerol, $\text{HOCH}_2\text{CH}(\text{OH})\text{CH}_2\text{OH}$, has three hydroxyl groups and many opportunities for intermolecular hydrogen bonding; its larger molecular framework also resists flow. It is the most viscous of the four. Thus water follows methyl alcohol and glycerol comes last. Option (A) incorrectly puts ether above both water and alcohol; (B) puts ether last, and (D) puts glycerol first. Each reverses the key intermolecular-force comparison.

Step 3: Conclusion The increasing order is dimethyl ether < methyl alcohol < water < glycerol. Hence, the correct option is (C).

Quick Tip to Solve: For this set, start with the ether lacking an O–H donor, then compare the extent of hydrogen bonding; the three hydroxyl groups put glycerol last.

67. Which of the following statements/relationships is not correct in thermodynamic changes ?

[4 marks]

- (A) $\Delta U = 0$ (isothermal reversible expansion of a gas)
(B) $w = -nRT \ln(V_2/V_1)$ (isothermal reversible expansion of an ideal gas)
(C) $w = nRT \ln(V_2/V_1)$ (isothermal reversible expansion of an ideal gas)
(D) For a system of constant volume heat involved directly changes to internal energy.

Correct Answer: (C) $w = nRT \ln(V_2/V_1)$ (isothermal reversible expansion of an ideal gas)

Solution:

Concept:

- Using the chemistry sign convention, $\Delta U = q + w$, and pressure-volume work is $w = -\int P_{\text{ext}} dV$. Expansion therefore gives negative work on the system.
- For an ideal gas undergoing a reversible isothermal change, $P = nRT/V$, while its internal energy depends only on temperature. At constant volume, pressure-volume work vanishes.

Step 1: Integrate the work during expansion In a reversible process the external pressure differs infinitesimally from the gas pressure, so use $P_{\text{ext}} = nRT/V$ in the work integral. Because the temperature is constant,

$$\begin{aligned} w &= -\int_{V_1}^{V_2} \frac{nRT}{V} dV \\ &= -nRT \ln\left(\frac{V_2}{V_1}\right). \end{aligned}$$

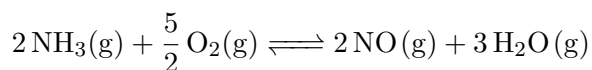
During expansion $V_2 > V_1$, so the logarithm is positive and $w < 0$. The positive sign in option (C) would instead make expansion work positive, contrary to this convention.

Step 2: Check the remaining relationships Option (B) contains the correct negative sign. In the ideal-gas treatment implicit in the isothermal gas comparison, constant temperature gives $\Delta U = nC_V \Delta T = 0$, as in (A). At fixed volume, with only pressure-volume work considered, $dV = 0$ and hence $w = 0$; the first law then gives $q_V = \Delta U$, which is the statement in (D). Thus the sign error in (C) is the incorrect relationship, rather than the zero internal-energy change or the constant-volume heat relation.

Step 3: Conclusion The correct work expression is $w = -nRT \ln(V_2/V_1)$; option (C) has the opposite sign. Hence, the correct option is (C).

Quick Tip to Solve: Check the sign before integrating: an expanding gas does work on its surroundings, so work on the system is negative in the chemistry convention.

68. (1) $\text{N}_2(\text{g}) + 3 \text{H}_2(\text{g}) \rightleftharpoons 2 \text{NH}_3(\text{g}), K_1$; **(2)** $\text{N}_2(\text{g}) + \text{O}_2(\text{g}) \rightleftharpoons 2 \text{NO}(\text{g}), K_2$; **(3)** $\text{H}_2(\text{g}) + \frac{1}{2} \text{O}_2(\text{g}) \rightleftharpoons \text{H}_2\text{O}(\text{g}), K_3$. **The equation for the equilibrium constant of the reaction**



(K_4) in terms of K_1, K_2 and K_3 is:

[4 marks]

(A) $\frac{K_1 K_2}{K_3}$

(B) $\frac{K_1 K_3^2}{K_2}$

(C) $K_1 K_2 K_3$

(D) $\frac{K_2 K_3^3}{K_1}$

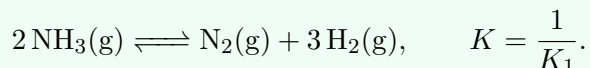
Correct Answer: (D) $\frac{K_2 K_3^3}{K_1}$

Solution:

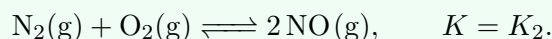
Concept:

- When an equilibrium reaction is reversed, its equilibrium constant becomes the reciprocal of the original constant. Multiplying all reaction coefficients by a number raises the constant to that power.
- Adding balanced equilibrium equations multiplies their constants. Choose the direction and multiples so that intermediate species cancel and the target stoichiometric coefficients remain.

Step 1: Reverse the ammonia-formation equation The target consumes two moles of ammonia, so reverse reaction (1):

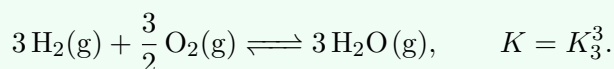


Keep reaction (2) in its forward direction to form two moles of nitric oxide:

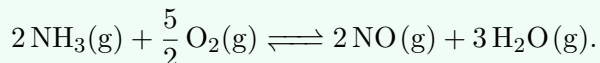


These two equations cancel nitrogen, but leave three moles of hydrogen on the product side.

Step 2: Remove hydrogen and obtain the target reaction Multiply reaction (3) by three, giving



Add all three equations. The nitrogen and hydrogen cancel, while the oxygen coefficients add to $1 + 3/2 = 5/2$. The net equation is exactly



Therefore the net constant is $K_2 K_3^3 / K_1$. Option (A) reverses the wrong constant; (B) has the wrong directions and water exponent; (C) neither reverses ammonia formation nor cubes the water-formation constant.

Step 3: Conclusion

$$K_4 = \frac{K_2 K_3^3}{K_1}.$$

Hence, the correct option is (D).

Quick Tip to Solve: Reverse reaction (1), retain reaction (2), and triple reaction (3); those same operations give the reciprocal, first power and third power of their constants.

69. What is the pH of a 10^{-4} M OH^- solution at 330 K, if K_w at 330 K is $10^{-13.6}$?

[4 marks]

- (A) 4
(B) 9.0
(C) 10
(D) 9.6

Correct Answer: (D) 9.6

Solution:**Concept:**

- The ionic product of water is $K_w = [\text{H}^+][\text{OH}^-]$. Its value depends on temperature, so the familiar $\text{pH} + \text{pOH} = 14$ must not be imposed at every temperature.
- Define $\text{pH} = -\log_{10}[\text{H}^+]$, $\text{pOH} = -\log_{10}[\text{OH}^-]$ and $\text{p}K_w = -\log_{10} K_w$. Their relation is $\text{pH} + \text{pOH} = \text{p}K_w$.

Step 1: Find pOH from the given hydroxide concentration The question specifies the hydroxide-ion concentration directly as 10^{-4} M. Therefore

$$\text{pOH} = -\log_{10}(10^{-4}) = 4.$$

This value is not the pH, because it was calculated from hydroxide rather than hydrogen ions. At the stated temperature 330 K, the given ionic product gives

$$\text{p}K_w = -\log_{10}(10^{-13.6}) = 13.6.$$

It is this temperature-specific number that must be used.

Step 2: Calculate and check the pH Apply the logarithmic relation:

$$\text{pH} = \text{p}K_w - \text{pOH} = 13.6 - 4 = 9.6.$$

A direct hydrogen-ion calculation gives the same value:

$$[\text{H}^+] = \frac{10^{-13.6}}{10^{-4}} = 10^{-9.6} \text{ M}.$$

Taking its negative logarithm again gives pH equal to 9.6.

Option (A) is the pOH, whereas (C) uses the inappropriate room-temperature value $\text{p}K_w = 14$. Option

(B) is not the difference of the supplied logarithms. Since the neutral pH here is $13.6/2 = 6.8$, the computed value correctly describes a basic solution.

Step 3: Conclusion The solution has $\text{pH} = 9.6$ at 330 K. Hence, the correct option is (D).

Quick Tip to Solve: Write $\text{pH} = \text{p}K_w - \text{pOH}$ first; replace 14 with the supplied $\text{p}K_w$ whenever the temperature changes.

70. The numbers of protons, electrons and neutrons in a molecule of heavy water are respectively [4 marks]

- (A) 8, 10, 11
- (B) 10, 10, 10
- (C) 10, 11, 10
- (D) 11, 10, 10

Correct Answer: (B) 10, 10, 10

Solution:

Concept:

- Heavy water is D_2O , where deuterium ${}^2_1\text{H}$ replaces ordinary hydrogen. Each deuterium nucleus has one proton and one neutron, and its neutral atom has one electron.
- The usual oxygen isotope in this count is ${}^{16}_8\text{O}$, containing eight protons, eight electrons and eight neutrons. Isotopes change neutron numbers while retaining the same atomic number.

Step 1: Count the particles from deuterium There are two deuterium atoms in one molecule of heavy water. Their combined contribution is therefore

$$N_p(\text{two D}) = 2 \times 1 = 2,$$

$$N_e(\text{two D}) = 2 \times 1 = 2,$$

$$N_n(\text{two D}) = 2 \times (2 - 1) = 2.$$

The extra mass of deuterium is due to its neutron. It does not give an extra proton or electron, since its atomic number remains one.

Step 2: Add the oxygen contribution and compare the options One oxygen atom contributes eight of each particle in this isotope. Addition gives

$$N_p = 2 + 8 = 10,$$

$$N_e = 2 + 8 = 10,$$

$$N_n = 2 + 8 = 10.$$

Bond formation redistributes electrons but does not remove them from a neutral molecule, so the electron total must equal the proton total. Options (A), (C) and (D) have unequal proton and electron counts and cannot represent neutral heavy water. Option (A) also fails to count the two deuterium protons. The equal totals in (B) satisfy both the isotope composition and neutrality.

Step 3: Conclusion The respective numbers of protons, electrons and neutrons are 10, 10, 10. Hence, the correct option is (B).

Quick Tip to Solve: Use mass number minus atomic number for neutrons, and check neutrality: D_2O must contain as many electrons as protons.

71. Equimolar solutions of the following compounds are prepared separately in water. Which will have the lowest pH value ? [4 marks]

- (A) $BeCl_2$
- (B) $SrCl_2$
- (C) $CaCl_2$
- (D) $MgCl_2$

Correct Answer: (A) $BeCl_2$

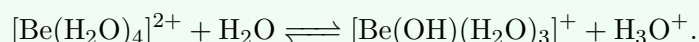
Solution:

Concept:

- All four salts contain chloride, the conjugate base of the strong acid HCl ; chloride does not cause appreciable basic hydrolysis in water.
- The acidity therefore depends mainly on the hydrated metal cation. At the same positive charge, a smaller cation has greater charge density and polarises coordinated water more strongly.

Step 1: Compare the metal ions The cations are Be^{2+} , Mg^{2+} , Ca^{2+} and Sr^{2+} . All have charge +2, so their ionic sizes decide the relative polarising power. Ionic radius increases down Group 2. Thus the beryllium ion is the smallest, and it attracts electron density from the oxygen atoms of surrounding water molecules most strongly. Comparing charge alone would not distinguish these four ions.

Step 2: Relate polarisation to hydrogen ion release Strong polarisation weakens an $O-H$ bond in a coordinated water molecule, making proton release easier. A schematic first hydrolysis step is



The enhanced formation of H_3O^+ lowers pH. Magnesium chloride undergoes much weaker acidic hydrolysis, while calcium and strontium chlorides are still less effective in acidifying water. Equal salt concentrations remove concentration as the reason for the difference; it is the nature of the cation that matters. Consequently options B, C and D cannot give the lowest pH among these salts.

Step 3: Conclusion $BeCl_2$ produces the most acidic equimolar solution because Be^{2+} has the greatest polarising power. Hence, the correct option is (A).

Quick Tip to Solve: For chlorides of Group 2 metals at equal concentration, compare cation size: the smallest divalent ion gives the strongest hydrolysis and the lowest pH.

72. Monocarboxylic acids are functional isomers of

[4 marks]

- (A) Ethers
- (B) Amines
- (C) Esters
- (D) Alcohols

Correct Answer: (C) Esters

Solution:

Concept:

- Functional isomers have the same molecular formula but different functional groups. A change in the functional group must not change the total numbers of carbon, hydrogen and oxygen atoms.
- A saturated open-chain monocarboxylic acid has the group —COOH , whereas an ester has —COO— ; both belong to the general formula $\text{C}_n\text{H}_{2n}\text{O}_2$.

Step 1: Construct an acid-ester pair Take ethanoic acid and methyl methanoate as a direct example:



Both have molecular formula $\text{C}_2\text{H}_4\text{O}_2$. In the acid, the carbonyl carbon is attached to an OH group; in the ester, the oxygen is attached to another carbon group. Their identical atom counts and different functional groups establish functional isomerism. An ester is not merely another arrangement within the same acid group; its characteristic bonding pattern has changed.

Step 2: Exclude the remaining functional groups Ethers contain the linkage R—O—R' and a simple saturated acyclic ether has only one oxygen atom, with general formula $\text{C}_n\text{H}_{2n+2}\text{O}$. A simple monohydric alcohol also has one oxygen atom and this same general formula, although its group is —OH . Neither matches the two oxygens of the acid family. Amines contain nitrogen and therefore cannot share the molecular formula of an acid containing only carbon, hydrogen and oxygen. Thus options A and D fail the oxygen-count test, and option B fails the element-composition test. Esters alone provide the required alternative among the choices.

Step 3: Conclusion Monocarboxylic acids are functional isomers of esters, as illustrated by CH_3COOH and HCOOCH_3 . Hence, the correct option is (C).

Quick Tip to Solve: Check the molecular formula before comparing functional groups: acids and esters share two oxygen atoms, while the simple ether-alcohol pair shares one.

73. In a face centred cubic lattice, atoms of A form the corner points and atoms of B form the face centred points. If two atoms of A are missing from the corner points, the formula of the ionic compound is:

[4 marks]

(A) AB_3

(B) AB_4

(C) A_2B_5

(D) AB_2

Correct Answer: (B) AB_4

Solution:

Concept:

- An atom at a cubic unit-cell corner is shared by eight cells, so its contribution to one cell is $1/8$. A face-centred atom is shared by two cells, so its contribution is $1/2$.
- The chemical formula follows the simplest whole-number ratio of effective atom counts in one unit cell, rather than the number of visible sites in its drawing.

Step 1: Count the remaining A contribution There are eight corner positions in a cube. Two corner atoms of A are absent, so only six corner positions remain occupied. Each occupied corner still contributes only one eighth of its atom to the chosen unit cell. Therefore

$$N_A = (8 - 2) \times \frac{1}{8} = \frac{6}{8} = \frac{3}{4}.$$

Removing two corner-site atoms does not subtract two whole atoms from a single cell: the corner-sharing factor must be applied first. The vacancies do not change the contribution of the six surviving sites.

Step 2: Count B and simplify the ratio All six faces have a B atom at the centre. The question does not remove any of these atoms, so

$$N_B = 6 \times \frac{1}{2} = 3.$$

The composition is consequently

$$A : B = \frac{3}{4} : 3 = 3 : 12 = 1 : 4.$$

The formula is obtained by dividing both counts by $3/4$. The ordinary fully occupied corner-and-face arrangement would give $1 : 3$, but that would ignore the two vacancies, so AB_3 is incorrect. Neither A_2B_5 nor AB_2 has the ratio calculated from the stated occupancies. No body-centred atom is specified, so none may be included in the count.

Step 3: Conclusion The effective unit-cell counts are $N_A = 3/4$ and $N_B = 3$, giving $A : B = 1 : 4$. The formula is AB_4 . Hence, the correct option is (B).

Quick Tip to Solve: For vacancy problems, count occupied sites first, multiply by their sharing fractions, and only then reduce the resulting atom ratio.

74. Vapour pressure of pure benzene is 119 torr and that of toluene is 37.0 torr at the same temperature.

Mole fraction of toluene in vapour phase which is in equilibrium with a solution of benzene and toluene having a mole fraction of toluene 0.50, will be [4 marks]

- (A) 0.137
- (B) 0.237
- (C) 0.435
- (D) 0.205

Correct Answer: (B) 0.237

Solution:

Concept:

- For an ideal liquid mixture, Raoult's law gives the partial pressure $p_i = x_i p_i^\circ$, where x_i is the liquid mole fraction and p_i° is the pure-liquid vapour pressure.
- Dalton's law gives the vapour mole fraction $y_i = p_i/P$, with P equal to the sum of all partial pressures. Liquid and vapour mole fractions need not be equal.

Step 1: Calculate the two partial pressures The liquid mole fraction of toluene is $x_T = 0.50$. Since the solution contains only benzene and toluene, the liquid mole fraction of benzene is $x_B = 1 - 0.50 = 0.50$. Hence

$$p_T = 0.50 \times 37.0 = 18.5 \text{ torr},$$

$$p_B = 0.50 \times 119 = 59.5 \text{ torr}.$$

The more volatile benzene has the larger partial pressure even though the two liquids have equal mole fractions. The same pressure unit is used for both components, so no pressure conversion is needed.

Step 2: Divide by the total pressure The equilibrium total vapour pressure is

$$P = 18.5 + 59.5 = 78.0 \text{ torr}.$$

Therefore the required vapour mole fraction is

$$y_T = \frac{18.5}{78.0} = 0.23718 \dots \simeq 0.237.$$

This dimensionless fraction is smaller than the liquid value of 0.50 because toluene is the less volatile component. The denominator must include both partial pressures; using just the pure benzene pressure or the pure toluene pressure would not give a vapour mole fraction. Options A, C and D do not equal the ratio 18.5/78.0, whereas B does.

Step 3: Conclusion The mole fraction of toluene in the equilibrium vapour is approximately 0.237. Hence, the correct option is (B).

Quick Tip to Solve: Apply Raoult first and Dalton second: multiply each liquid mole fraction by its pure vapour pressure, then divide the required partial pressure by their sum.

75. Given:

$$E_{\frac{1}{2}\text{Cl}_2/\text{Cl}^-}^\circ = 1.36 \text{ V}, \quad E_{\text{Cr}^{3+}/\text{Cr}}^\circ = -0.74 \text{ V},$$
$$E_{\text{Cr}_2\text{O}_7^{2-}/\text{Cr}^{3+}}^\circ = 1.33 \text{ V}, \quad E_{\text{MnO}_4^-/\text{Mn}^{2+}}^\circ = 1.51 \text{ V}.$$

The correct order of reducing power of the species (Cr , Cr^{3+} , Mn^{2+} and Cl^-) will be:

[4 marks]

- (A) $\text{Mn}^{2+} < \text{Cl}^- < \text{Cr}^{3+} < \text{Cr}$
(B) $\text{Mn}^{2+} < \text{Cl}^{3+} < \text{Cl}^- < \text{Cr}$
(C) $\text{Cr}^{3+} < \text{Cl}^- < \text{Mn}^{2+} < \text{Cr}$
(D) $\text{Cr}^{3+} < \text{Cl}^- < \text{Cr} < \text{Mn}^{2+}$

Correct Answer: (A) $\text{Mn}^{2+} < \text{Cl}^- < \text{Cr}^{3+} < \text{Cr}$

Solution:

Concept:

- A reducing agent donates electrons and is itself oxidised. For a redox couple written as a reduction, the species on the reduced side is the corresponding reducing agent.
- The standard oxidation potential of that reduced species is the negative of the listed standard reduction potential. A larger oxidation potential means greater reducing power under the specified standard conditions.

Step 1: Identify the reduced member of each couple The four supplied reduction couples contain the reducing species Cl^- , Cr , Cr^{3+} and Mn^{2+} , respectively. For example, $\text{Cr}^{3+} + 3\text{e}^- \longrightarrow \text{Cr}$ has reduction potential -0.74 V , so the reverse oxidation of metallic chromium has potential $+0.74 \text{ V}$. In the dichromate couple, Cr^{3+} is the reduced member and can be compared as an electron donor by reversing that couple. Do not compare the ionic charges directly: charge alone does not determine the oxidation tendency.

Step 2: Reverse and order the potentials Reversing the supplied couples gives the following standard oxidation potentials for the named species:

$$\begin{aligned}\text{Mn}^{2+} &: -1.51 \text{ V}, \\ \text{Cl}^- &: -1.36 \text{ V}, \\ \text{Cr}^{3+} &: -1.33 \text{ V}, \\ \text{Cr} &: +0.74 \text{ V}.\end{aligned}$$

Increasing numerical value therefore gives increasing reducing strength in this same order. Potentials are intensive quantities: they are not multiplied by the number of electrons in a balanced half-reaction. Option B contains Cl^{3+} , which is not one of the four reducing species listed in the question and has no supplied couple. Even interpreting that entry as chromium(III) would place it below chloride, contrary to the potentials. Option C puts manganese(II) too high, and D incorrectly makes manganese(II) strongest.

Step 3: Conclusion The increasing order is $\text{Mn}^{2+} < \text{Cl}^- < \text{Cr}^{3+} < \text{Cr}$. Hence, the correct option is (A).

Quick Tip to Solve: To compare reducing agents from reduction-potential data, identify each reduced species and reverse the potential sign; the largest oxidation potential gives the strongest donor.

76. The rate constant of a zero order reaction is $2.0 \times 10^{-2} \text{ mol L}^{-1} \text{ s}^{-1}$. If the concentration of the reactant after 25 seconds is 0.5 M, what is the initial concentration? [4 marks]

- (A) 0.5 M
- (B) 1.25 M
- (C) 12.5 M
- (D) 1.0 M

Correct Answer: (D) 1.0 M

Solution:

Concept:

- For a zero-order reaction, reactant concentration decreases linearly: $[A]_t = [A]_0 - kt$, where $[A]_0$ is the initial concentration and $[A]_t$ is its value after time t .
- The zero-order constant has concentration-per-time units. Thus kt is the concentration consumed, not a fraction of the reactant or a logarithmic change.

Step 1: Find the concentration consumed Use the supplied values $k = 2.0 \times 10^{-2} \text{ mol L}^{-1} \text{ s}^{-1}$ and $t = 25 \text{ s}$. Their product is

$$kt = (2.0 \times 10^{-2})(25) = 0.50 \text{ mol L}^{-1}.$$

The seconds cancel, leaving the same concentration unit as the stated 0.5 M. This quantity is how much reactant has disappeared during the 25 seconds. It must be added to the amount remaining in order to reconstruct the initial concentration; subtracting it would reverse the physical meaning of consumption.

Step 2: Recover and check the initial concentration Rearrange the integrated law before substituting:

$$[A]_0 = [A]_t + kt = 0.50 + 0.50 = 1.0 \text{ mol L}^{-1}.$$

As a check, inserting $[A]_0 = 1.0 \text{ M}$ into the rate law gives $[A]_{25} = 1.0 - 0.50 = 0.50 \text{ M}$, exactly the stated final value. Option A gives the final concentration rather than the initial one; it would predict zero reactant after 25 seconds. Option B would predict 0.75 M, and option C would predict 12.0 M, so both fail the direct substitution check. A first-order exponential expression is inappropriate because the reaction is explicitly zero order.

Step 3: Conclusion The initial concentration is $1.0 \text{ mol L}^{-1} = 1.0 \text{ M}$. Hence, the correct option is (D).

Quick Tip to Solve: For zero order, draw the mental balance: initial concentration equals concentration remaining plus kt consumed. The answer must exceed the remaining concentration.

77. Smoke is an example of

[4 marks]

- (A) Solid dispersed in solid
- (B) Gas dispersed in liquid
- (C) Solid dispersed in gas
- (D) Gas dispersed in solid

Correct Answer: (C) Solid dispersed in gas

Solution:

Concept:

- A colloid consists of a dispersed phase distributed through a continuous dispersion medium. Its classification depends on the physical states of these two components.
- An aerosol has a gas as its dispersion medium. A solid aerosol contains fine solid particles, while a liquid aerosol contains tiny liquid droplets.

Step 1: Identify the particles and the medium Smoke contains very fine solid particles, such as soot, carried through air. The soot particles form the dispersed phase; air is the continuous dispersion medium. Thus the first state in the description must be solid and the second must be gas. The word smoke refers to the suspended particles rather than merely to the gaseous products of combustion. Those gases alone would form a molecular gas mixture, not the solid-particle colloid being classified here.

Step 2: Compare all four classifications Option C, solid dispersed in gas, matches the particle and medium states exactly. Option A would require a solid continuous medium, as in a solid sol, whereas smoke moves through air. Option B would mean gas bubbles within a liquid, a foam-like system, rather than solid particles suspended in a gas. Option D describes gas trapped in a solid medium, as in a solid foam; there is no solid continuous matrix in smoke. The classification also distinguishes smoke from fog: fog contains liquid water droplets dispersed in air and is therefore a liquid aerosol. Both have air as the medium, but their dispersed phases differ. No chemical reaction or charge measurement is needed to choose between the listed state combinations; identifying which material is particulate and which is continuous settles the answer.

Step 3: Conclusion Smoke is a solid aerosol: solid particles are dispersed in a gaseous medium. Hence, the correct option is (C).

Quick Tip to Solve: Name the dispersed particles first and the continuous medium second: smoke is soot in air (solid in gas), while fog is water droplets in air (liquid in gas).

78. Which has trigonal bipyramidal shape?

[4 marks]

- (A) XeOF_4
- (B) XeO_3
- (C) XeO_3F_2
- (D) XeOF_2

Correct Answer: (C) XeO_3F_2

Solution:

Concept:

- VSEPR theory counts electron domains at the central atom. A multiple bond occupies one domain, while a lone pair also occupies one domain.
- Five bonding domains and no central lone pair give a trigonal bipyramidal molecular shape, denoted AX_5 . Xenon contributes eight valence electrons.

Step 1: Count domains in the candidate with five ligands In XeO_3F_2 , xenon is bonded to three oxygen atoms and two fluorine atoms. The three xenon–oxygen multiple bonds and two xenon–fluorine single bonds give five bonding domains. The usual expanded-valence Lewis structure has no lone pair on xenon; the three oxygen atoms lie equatorially and the two fluorine atoms occupy axial positions. Consequently both its electron-domain geometry and molecular shape are trigonal bipyramidal.

Step 2: Eliminate the other oxyfluorides XeOF_4 has five bonded atoms and one xenon lone pair: its six-domain arrangement is octahedral and its molecular shape is square pyramidal. XeO_3 has three bonding domains and one lone pair, giving a trigonal pyramidal shape. XeOF_2 has three bonded atoms and two lone pairs; the five-domain arrangement yields a T-shaped molecule. Thus the mere presence of five electron domains, as in the last compound, does not by itself guarantee a trigonal bipyramidal molecular shape.

Step 3: Conclusion XeO_3F_2 is the AX_5 species with trigonal bipyramidal shape. Hence, the correct option is (C).

Quick Tip to Solve: Count bonded atoms and central lone pairs separately: five domains can give AX_5 , AX_4E or AX_3E_2 , with different molecular shapes.

79. When a small amount of KMnO_4 is added to concentrated H_2SO_4 , a green oily compound is obtained which is highly explosive in nature. Compound may be: [4 marks]

- (A) MnSO_4
- (B) Mn_2O_7
- (C) MnO_2
- (D) Mn_2O_3

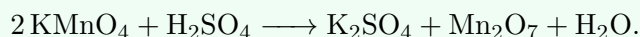
Correct Answer: (B) Mn_2O_7

Solution:

Concept:

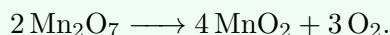
- Concentrated H_2SO_4 acts as a strongly dehydrating acid and converts permanganate into the acid anhydride of permanganic acid.
- For Mn_2O_7 , charge neutrality gives $2x + 7(-2) = 0$, so manganese remains in oxidation state +7.

Step 1: Write the formation reaction Permanganate first gives the corresponding strongly acidic manganese(VII) species in the acid medium. Removal of water produces manganese(VII) oxide. The net reaction can be represented as



The equation conserves potassium, manganese, sulfur, hydrogen and oxygen. The product Mn_2O_7 is the green, oily liquid identified by the question, rather than a lower manganese oxide.

Step 2: Use the colour and instability to eliminate alternatives Mn_2O_7 is highly unstable and explosive because it can decompose to the more stable manganese(IV) oxide while liberating oxygen:



MnSO_4 contains manganese(II) and is not the green oily explosive anhydride. MnO_2 is a dark solid with manganese(IV), and Mn_2O_3 is a solid manganese(III) oxide. Neither lower oxide matches the oxidation state or the distinctive physical description.

Step 3: Conclusion The green oily explosive compound is manganese(VII) oxide, Mn_2O_7 . Hence, the correct option is (B).

Quick Tip to Solve: A green oily explosive product from permanganate and concentrated sulfuric acid signals the +7 oxide Mn_2O_7 ; lower manganese oxides are solids.

80. Identify incorrect statement:

[4 marks]

- (A) Cu_2O is colourless.
- (B) Copper (I) compounds are colourless except when colour results from charge transfer.
- (C) Copper (I) compounds are diamagnetic.
- (D) Cu_2S is black.

Correct Answer: (A) Cu_2O is colourless.

Solution:

Concept:

- A copper atom has configuration $[\text{Ar}]3d^{10}4s^1$; the Cu^+ ion therefore has a filled $3d^{10}$ subshell.
- A filled d subshell has no unpaired electrons and no ordinary intra- d transition, but charge-transfer and solid-state electronic transitions can still produce colour.

Step 1: Test the allegedly colourless oxide Copper(I) oxide, Cu_2O , is characteristically red or brick-red. Therefore the unqualified statement that it is colourless is false. One must not infer the colour of every copper(I) solid solely from the absence of partially filled d orbitals. The electronic structure of an extended solid and charge-transfer processes can absorb visible light even when the isolated metal-ion configuration is d^{10} .

Step 2: Check the remaining three statements The qualified statement in (B) allows colour arising from charge transfer and so correctly expresses the usual rule for copper(I) compounds. In the elementary electron-configuration treatment, both $3d$ and the remaining occupied shells of Cu^+ contain paired electrons; hence its compounds are described as diamagnetic, consistent with (C). Copper(I) sulfide, Cu_2S , is black, so (D) is also correct. The question asks for an incorrect statement, rather than for the most general statement about colour, making the observed red colour of Cu_2O decisive.

Step 3: Conclusion Cu_2O is red, not colourless. Hence, the incorrect statement and correct option is (A).

Quick Tip to Solve: For d^{10} ions, absence of $d-d$ colour does not exclude charge-transfer or solid-state colour; remember red Cu_2O and black Cu_2S .

81. The magnetic moment of the complex anion $[\text{Cr}(\text{NO})(\text{NH}_3)(\text{CN})_4]^{2-}$ is:

[4 marks]

- (A) 5.91 BM
- (B) 3.87 BM
- (C) 1.73 BM
- (D) 2.82 BM

Correct Answer: (C) 1.73 BM

Solution:

Concept:

- In the usual nitrosyl-complex electron-counting convention, the linear nitrosyl ligand is NO^+ ; ammonia is neutral and each cyanide ligand is CN^- .
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)} \text{ BM}$, where n is the number of unpaired electrons. Strong-field ligands favour low-spin filling.

Step 1: Calculate the chromium oxidation state and d count Let the chromium oxidation state be x . Applying charge balance to the printed anion gives

$$x + (+1) + 0 + 4(-1) = -2, \quad x = +1.$$

Chromium is in group 6, so chromium(I) has a formal $d^{6-1} = d^5$ configuration. Treating nitrosyl as neutral instead would give chromium(II), but that omits the standard NO^+ description of the strongly bound nitrosyl ligand and changes the electronic count.

Step 2: Fill the strong-field orbitals and compute the moment The six-coordinate complex contains the strong-field cyanide and nitrosyl ligands. The low-spin d^5 configuration is $t_{2g}^5 e_g^0$, leaving one unpaired electron. Thus

$$\mu = \sqrt{1(1+2)} = \sqrt{3} \simeq 1.73 \text{ BM}.$$

The values 5.91 and 3.87 BM would indicate five and three unpaired electrons, respectively, rather than low-spin d^5 . The value 2.82 BM indicates two unpaired electrons and follows the inappropriate neutral-NO, low-spin d^4 count; it does not follow the nitrosyl charge balance above.

Step 3: Conclusion The standard NO^+ assignment gives one unpaired electron and $\mu = 1.73 \text{ BM}$. Hence, the correct option is (C). The supplied key gives 2.82 BM, but its two-unpaired-electron count conflicts with the printed nitrosyl complex under the usual NO^+ convention.

Quick Tip to Solve: Account for the charge of nitrosyl before finding the metal d count: replacing NO^+ by neutral NO changes chromium(I), d^5 , into chromium(II), d^4 .

82. Identify the incorrect statement:

[4 marks]

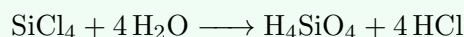
- (A) In $(\text{Si}_3\text{O}_9)^{6-}$, tetrahedral SiO_4 units share two oxygen atoms.
(B) Trialkylchlorosilane on hydrolysis gives R_3SiOH .
(C) SiCl_4 undergoes hydrolysis to give H_4SiO_4 .
(D) $(\text{Si}_3\text{O}_9)^{6-}$ has cyclic structure.

Correct Answer: No unique incorrect statement on the literal hydrolysis reading; the keyed choice refers to subsequent condensation.

Solution:**Concept:**

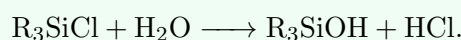
- A cyclic trisilicate contains three SiO_4 tetrahedra, with each tetrahedron sharing two oxygen atoms; its composition is $(\text{Si}_3\text{O}_9)^{6-}$.
- Hydrolysis replaces a silicon–chlorine bond with a silicon–hydroxyl bond. Silanols may subsequently condense to form a silicon–oxygen–silicon linkage; hydrolysis and condensation are distinct steps.

Step 1: Verify the cyclic silicate and silicon tetrachloride statements Each tetrahedron in the three-membered silicate ring shares two corner oxygen atoms with neighbouring tetrahedra. Counting shared atoms once gives $3(4) - 3 = 9$ oxygen atoms. With three $+4$ silicon atoms and nine -2 oxygen atoms the charge is -6 . Thus (A) and (D) correctly describe the cyclic ion. The formal hydrolysis equation

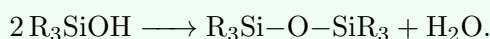


also validates (C), with silicic acid able to undergo further condensation.

Step 2: Distinguish the literal hydrolysis product from its condensation product For a trialkylchlorosilane, hydrolysis itself is represented by



Therefore (B) is true as a statement about the first hydrolysis product. The resulting silanol can condense:



The final condensed product is a disiloxane. Calling (B) incorrect requires interpreting the question as asking for that final product after condensation, a condition the wording does not state. On the literal hydrolysis reading there is no unique incorrect statement among the four options.

Step 3: Conclusion The item is ambiguous as worded: all four statements are valid when hydrolysis is distinguished from subsequent condensation. The supplied key selects (B), corresponding to the final disiloxane product, but R_3SiOH is the hydrolysis intermediate, so a unique incorrect option is not justified by the literal wording.

Quick Tip to Solve: Write hydrolysis and condensation as separate balanced equations before judging a silanol product; R_3SiOH can be an intermediate even when a disiloxane is ultimately isolated.

83. The order of reactivity of the given haloalkanes towards nucleophile is:

[4 marks]

- (A) $\text{RI} > \text{RBr} > \text{RCl}$
- (B) $\text{RCl} > \text{RBr} > \text{RI}$
- (C) $\text{RBr} > \text{RCl} > \text{RI}$
- (D) $\text{RBr} > \text{RI} > \text{RCl}$

Correct Answer: (A) $\text{RI} > \text{RBr} > \text{RCl}$

Solution:

Concept:

- For otherwise comparable haloalkanes, nucleophilic substitution requires cleavage of the carbon–halogen bond; a better leaving group normally increases reactivity.
- The carbon–halogen bond weakens down the halogen group, while leaving-group ability follows $\text{I}^- > \text{Br}^- > \text{Cl}^-$.

Step 1: Compare bond cleavage for the same alkyl group The carbon–iodine bond is longer and weaker than the carbon–bromine bond, which is weaker than the carbon–chlorine bond. Consequently the carbon–halogen bond is most readily broken in RI and least readily broken in RCl, when the alkyl group and reaction conditions are held comparable. Iodide is also a larger, more polarizable and weaker base than bromide or chloride, so it accommodates the departing negative charge most readily.

Step 2: Rank the substrates and reject the reversed orders Combining easier bond cleavage with better leaving-group ability gives



Option (B) reverses the trend by putting the strongest carbon–chlorine bond first. Option (C) incorrectly puts iodide last, and option (D) incorrectly puts bromide above iodide. Although the rate also depends on whether the process is $\text{S}_{\text{N}}1$ or $\text{S}_{\text{N}}2$, substrate structure, solvent and nucleophile, these options compare the halogen while keeping the common alkyl group R unchanged, so the standard leaving-group trend decides the answer.

Step 3: Conclusion The order is $\text{RI} > \text{RBr} > \text{RCl}$. Hence, the correct option is (A).

Quick Tip to Solve: For a fixed alkyl group, rank the leaving group first: iodide leaves more readily than bromide, which leaves more readily than chloride.

84. The reaction of phenol with benzoyl chloride to give phenyl benzoate is known as:

[4 marks]

- (A) Claisen reaction
- (B) Schotten-Baumann reaction
- (C) Reimer-Tiemann reaction
- (D) Gatterman-Koch reaction

Correct Answer: (B) Schotten-Baumann reaction

Solution:

Concept:

- Phenols form esters on acylation with an acid chloride; benzoyl chloride supplies the benzoyl group $\text{C}_6\text{H}_5\text{CO}$.
- The Schotten–Baumann reaction is acylation or benzylation in the presence of aqueous alkali, which removes the acid formed and promotes reaction.

Step 1: Identify the bond formed in benzylation Phenol reacts through oxygen with benzoyl chloride to give the ester phenyl benzoate. Its net transformation can be written



In alkaline medium phenoxide is the effective oxygen nucleophile, and the base neutralizes hydrogen chloride. The new bond is between the phenolic oxygen and the benzoyl carbonyl carbon; this is benzylation, rather than substitution of a formyl group onto the aromatic ring.

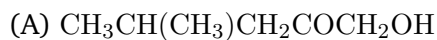
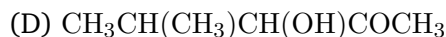
Step 2: Match the reaction name and eliminate the distractors Benzylation of phenol with benzoyl chloride under alkaline conditions is called the Schotten–Baumann reaction. Claisen condensation involves ester enolates and formation of a carbon–carbon bond, not the esterification shown here. The Reimer–Tiemann reaction introduces a formyl group into phenol using chloroform and alkali. The Gattermann–Koch reaction is aromatic formylation using carbon monoxide and hydrogen chloride with suitable Lewis-acid catalysts. Neither formylation produces phenyl benzoate, so only the benzylation name matches the reactants and product.

Step 3: Conclusion The conversion of phenol and benzoyl chloride into phenyl benzoate is the Schotten–Baumann reaction. Hence, the correct option is (B).

Quick Tip to Solve: Phenol plus benzoyl chloride gives an O-benzoylated ester: connect this reagent pair with Schotten–Baumann, rather than the phenol formylation reactions.

85. Which of the following is the product of aldol condensation?

[4 marks]



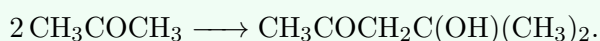
Correct Answer: (B) $(\text{CH}_3)_2\text{C}(\text{OH})\text{CH}_2\text{COCH}_3$

Solution:

Concept:

- An aldehyde or ketone containing an alpha hydrogen can form an enolate ion. Its alpha carbon attacks the carbonyl carbon of another molecule, making a new carbon-carbon bond.
- The initial aldol product is a beta-hydroxy aldehyde or beta-hydroxy ketone: the hydroxyl group is on the beta carbon relative to the carbonyl. Heating may then dehydrate this product to an alpha,beta-unsaturated carbonyl compound.

Step 1: Identify the diagnostic hydroxyl position Number the carbonyl carbon as carbon 1, the adjacent carbon as alpha, and the next carbon as beta. In an aldol addition, the attacking enolate supplies the carbonyl that remains, whereas the attacked carbonyl becomes the alcohol. Consequently, the new alcohol is separated from the surviving carbonyl carbon by one carbon atom. For acetone self-condensation the initial product is



This is diacetone alcohol, a beta-hydroxy ketone.

Step 2: Test all four structures Option (B), $(\text{CH}_3)_2\text{C}(\text{OH})\text{CH}_2\text{COCH}_3$, has exactly this beta-hydroxy ketone arrangement: the CH_2 is alpha and the carbon bearing OH is beta. In (A), COCH_2OH places the hydroxyl on an alpha carbon. Option (D) likewise places OH on the carbon directly adjacent to the carbonyl. In (C), $\text{HOCH}_2\text{CH}_2\text{CH}_2\text{COCH}_3$, the hydroxyl-bearing carbon is gamma to the carbonyl. Thus only (B) has the characteristic aldol connectivity; the wording uses condensation for the aldol process including its initial addition product.

Step 3: Conclusion The beta-hydroxy ketone $(\text{CH}_3)_2\text{C}(\text{OH})\text{CH}_2\text{COCH}_3$, diacetone alcohol, is the aldol product shown. Hence, the correct option is (B).

Quick Tip to Solve: Locate the carbonyl first and count outward: an aldol addition product has OH on the beta carbon, not the alpha or gamma carbon.

86. The order of basicity of amines in gaseous state is

[4 marks]

(A) $1^\circ > 2^\circ > 3^\circ > \text{NH}_3$

(B) $3^\circ > 2^\circ > \text{NH}_3 > 1^\circ$

(C) $3^\circ > 2^\circ > 1^\circ > \text{NH}_3$

(D) $\text{NH}_3 > 1^\circ > 2^\circ > 3^\circ$

Correct Answer: (C) $3^\circ > 2^\circ > 1^\circ > \text{NH}_3$

Solution:

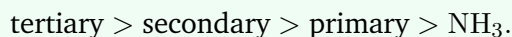
Concept:

- A base accepts a proton through an available lone pair. An amine nitrogen donates its lone pair to H^+ , forming an ammonium ion.
- Alkyl groups release electron density through the positive inductive effect. In the gaseous state there is no aqueous hydration contribution to reverse the simple alkyl-substitution trend.

Step 1: Compare electron donation to nitrogen Write the protonation of a trialkylamine as



Alkyl groups increase electron density at nitrogen and help stabilize the protonated ion. A tertiary amine has three electron-releasing alkyl groups, a secondary amine has two, and a primary amine has one; ammonia has none. For comparable alkyl groups in the gas phase this makes the lone pair progressively more effective at binding a proton as substitution increases. The corresponding basicity order is



Step 2: Use the specified medium and reject the alternatives The condition gaseous state matters. In water the hydration of the conjugate acid and steric access of solvent can compete with the inductive effect, so one should not transfer an aqueous basicity order to this question. Option (A) incorrectly puts the primary amine above the tertiary amine. Option (B) correctly begins with tertiary and secondary amines but incorrectly places ammonia above the primary amine. Option (D) makes ammonia strongest, opposite to the electron-donating effect of alkyl substitution. Only option (C) follows the gas-phase trend throughout.

Step 3: Conclusion For comparable alkylamines in the gaseous state, $3^\circ > 2^\circ > 1^\circ > \text{NH}_3$. Hence, the correct option is (C).

Quick Tip to Solve: Underline gaseous state before comparing amines: count the electron-releasing alkyl groups and leave aqueous solvation effects out.

87. Which of the following polymer is a polyamide ?

[4 marks]

- (A) Terylene
- (B) Nylon
- (C) Rubber
- (D) Vulcanised rubber

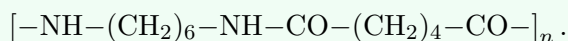
Correct Answer: (B) Nylon

Solution:

Concept:

- A polyamide contains repeated amide linkages, $-\text{CO}-\text{NH}-$, in its main polymer chain. Classification depends on the linkage, rather than whether the material is elastic or fibrous.
- Nylon is a family of synthetic polyamides. Nylon-6,6 forms from hexamethylenediamine and adipic acid; nylon-6 forms from caprolactam.

Step 1: Identify the linkage in nylon In nylon-6,6, the diamine reacts with the dicarboxylic acid by condensation, eliminating water as amide bonds form. Its repeating segment can be written as



Each junction between the acid-derived and amine-derived portions includes the $-\text{CO}-\text{NH}-$ grouping. Because this linkage repeats along the backbone, nylon is a polyamide. The presence of both nitrogen and a carbonyl in the same linkage is the decisive structural feature.

Step 2: Distinguish the remaining polymer classes Terylene is made from ethylene glycol and terephthalic acid. Its repeating linkage is an ester, $-\text{CO}-\text{O}-$, so it is a polyester rather than a polyamide. Natural rubber is cis-1,4-polyisoprene, formed from isoprene units and having a hydrocarbon backbone with residual carbon-carbon double bonds; it does not contain repeated amide bonds. Vulcanised rubber is rubber whose chains have been joined by sulphur crosslinks. Sulphur crosslinking improves mechanical properties but does not introduce a repeating $-\text{CO}-\text{NH}-$ backbone. Thus neither rubber option satisfies the defining linkage criterion.

Step 3: Conclusion Nylon contains repeating amide linkages in its main chain and is therefore a polyamide. Hence, the correct option is (B).

Quick Tip to Solve: Classify condensation polymers by the backbone bond: $\text{CO}-\text{NH}$ means polyamide, whereas $\text{CO}-\text{O}$ means polyester.

88. H_1 -Receptor antagonists is a term associated with

[4 marks]

- (A) Antiseptics
- (B) Antihistamins
- (C) Antacids
- (D) Analgesics

Correct Answer: (B) Antihistamins

Solution:

Concept:

- Histamine acts at specific receptors and contributes to allergic symptoms such as sneezing, itching and a runny nose. A receptor antagonist blocks the response produced by the natural messenger.
- Antihistamine drugs used against allergic reactions act at histamine H_1 receptors. The receptor subtype is important: gastric acid secretion is associated with histamine H_2 receptors.

Step 1: Connect the receptor with its messenger The designation H_1 identifies a histamine receptor subtype. When histamine binds to these receptors, it can produce characteristic allergic responses. An H_1 -receptor antagonist prevents the receptor-mediated action of histamine and therefore reduces such symptoms. These drugs are termed antihistamines. Examples encountered in everyday-life chemistry include chlorpheniramine and brompheniramine. The description concerns competition or blockade at a biological receptor, rather than direct chemical destruction of histamine.

Step 2: Separate antihistamines from the other drug classes Antiseptics inhibit or kill microorganisms on living tissue, so their defining action is antimicrobial rather than H_1 -receptor blockade. Antacids counter excessive stomach acidity, classically by neutralising acid; drugs that suppress histamine-stimulated gastric acid secretion target H_2 , not H_1 , receptors. Analgesics relieve pain and are classified by that action, not by blocking histamine H_1 receptors. Therefore the receptor name directly selects the antihistamine class in option (B), even though the printed option spells the word Antihistamins.

Step 3: Conclusion H_1 -receptor antagonists are antihistamines used to reduce allergic responses. Hence, the correct option is (B).

Quick Tip to Solve: Associate H_1 with allergy and H_2 with gastric acid secretion; this prevents confusing antihistamines with drugs used for acidity.

89. Glycosidic linkage is actually an

[4 marks]

- (A) Carbonyl bond
- (B) Ether bond
- (C) Ester bond
- (D) Amide bond

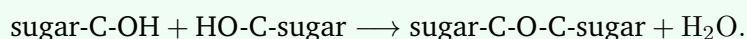
Correct Answer: (B) Ether bond

Solution:

Concept:

- A glycosidic linkage joins sugar units through an oxygen atom. In the usual disaccharides, the bridge has the carbon–oxygen–carbon connectivity C–O–C.
- An ether has oxygen bonded to two carbon groups, R–O–R'. This differs from an ester or amide, which includes a carbonyl group adjacent to oxygen or nitrogen.

Step 1: Examine the bridge between the sugars Formation of a typical disaccharide involves condensation between hydroxyl groups of monosaccharide units, with elimination of water. One hydroxyl is associated with the anomeric carbon; an oxygen bridge then connects that carbon to a carbon of the other sugar. A schematic representation is



The linking oxygen has a carbon atom on each side. For the simple functional-group classification asked here, this is an ether linkage. The precise carbon positions distinguish individual glycosidic bonds but do not change the oxygen-bridge classification.

Step 2: Compare the functional groups in the options A carbonyl bond in option (A) is C=O, a double bond to oxygen; it is not the single-bonded oxygen bridge joining the sugars. An ester in option (C) has –CO–O–, requiring an adjacent carbonyl carbon. An amide in option (D) has –CO–NH–, involving nitrogen and a carbonyl. Neither group describes the ordinary glycosidic bridge between carbohydrate units. Option (B), ether bond, gives the matching C–O–C connectivity. Hydrolysis breaks this linkage and regenerates the monosaccharide units.

Step 3: Conclusion The usual oxygen glycosidic linkage between sugar units is an ether-type C–O–C bond. Hence, the correct option is (B).

Quick Tip to Solve: Look at the connecting atom: C–O–C indicates an ether-type sugar bridge; an ester would also need a neighbouring carbonyl.

90. Natural glucose is termed D-glucose because

[4 marks]

- (A) –OH on the second carbon is on the right side in Fischer projection.
- (B) –OH on the sixth carbon is on the right side in Fischer projection.
- (C) –OH on the fifth carbon is on the right side in Fischer projection.
- (D) It is dextrorotatory.

Correct Answer: (C) –OH on the fifth carbon is on the right side in Fischer projection.

Solution:

Concept:

- The D/L designation specifies a carbohydrate configuration relative to glyceraldehyde. In a Fischer projection, inspect the chiral carbon farthest from the carbonyl group.
- D and L are configurational labels, whereas plus and minus describe the experimentally measured direction of optical rotation. The two systems must not be identified with each other.

Step 1: Locate the reference stereocentre in glucose Open-chain glucose is an aldohexose, so its aldehyde carbon is numbered carbon 1. Its stereogenic centres are carbons 2, 3, 4 and 5, while carbon 6 is the terminal CH_2OH carbon. Therefore carbon 5 is the chiral centre farthest from the carbonyl group. With the aldehyde group placed at the top of the standard Fischer projection, natural glucose has the hydroxyl group at carbon 5 on the right. This relative configuration matches D-glyceraldehyde and establishes the designation D-glucose.

Step 2: Reject rotation and the wrong carbon positions The hydroxyl at carbon 2 in option (A) does not determine the D/L family: changing another stereocentre can give a different sugar while retaining the D configuration. Carbon 6 in option (B) is not a stereogenic centre in open-chain glucose because it is a CH_2OH group, so its hydroxyl cannot define D/L. Option (D) refers to dextrorotation, represented by a plus sign. Natural glucose is indeed dextrorotatory, but that observation is not the reason for the letter D; D sugars can be either dextrorotatory or levorotatory. Only option (C) identifies the required configurational reference centre and its hydroxyl orientation.

Step 3: Conclusion Natural glucose is D-glucose because its carbon-5 hydroxyl group is on the right in the standard Fischer projection. Hence, the correct option is (C).

Quick Tip to Solve: For any sugar, find the chiral carbon farthest from the carbonyl: right-hand OH means D, independent of the measured sign of rotation.