JEE Main 2013 Paper 1 (B.E./B.Tech) Apr 25 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2013, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 25 April 2013, Single slot. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. If p and q are non-zero real numbers and $\alpha^3 + \beta^3 = -p$, $\alpha\beta = q$, then a quadratic equation whose roots are $\frac{\alpha^2}{\beta}, \frac{\beta^2}{\alpha}$ is: [4 marks]

(1) $px^2 - qx + p^2 = 0$

(2) $qx^2 + px + q^2 = 0$

(3) $px^2 + qx + p^2 = 0$

(4) $qx^2 - px + q^2 = 0$

Correct Answer: (2) $qx^2 + px + q^2 = 0$

Solution:

Concept:

- A quadratic with roots u, v is $x^2 - (u+v)x + uv = 0$, up to multiplication by a non-zero constant.
- Combine fractions before using the supplied symmetric expressions. Since $q = \alpha\beta \neq 0$, both denominators in the proposed roots are non-zero.

Step 1: Calculate the sum of the new roots Put $u = \alpha^2/\beta$ and $v = \beta^2/\alpha$. Giving the fractions the common denominator $\alpha\beta$ produces exactly the given sum of cubes:

$$u + v = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{-p}{q}.$$

No individual values of α or β are needed; the information about their symmetric combinations is sufficient.

Step 2: Calculate their product The powers in the numerator cancel one power of each denominator:

$$uv = \frac{\alpha^2\beta^2}{\alpha\beta} = \alpha\beta = q.$$

In particular, the product is q, not q^2; the latter appears only after clearing the leading denominator of the polynomial.

Step 3: Form the quadratic and compare the options Substitute these two results into the root-coefficient form:

$$x^2 + \frac{p}{q}x + q = 0.$$

Multiplication by non-zero q gives

$$qx^2 + px + q^2 = 0.$$

Option (4) has the opposite sum of roots. Options (1) and (3) use the wrong leading and constant coefficients and do not encode the required sum and product in general.

Step 4: Conclusion The required equation is $\boxed{qx^2 + px + q^2 = 0}$. Hence, the correct option is (2).

Quick Tip to Solve: For transformed roots, find their sum and product directly; here the common denominator immediately exposes the given sum of cubes.

2. Let z satisfy $|z| = 1$ and $z = 1 - \bar{z}$. Statement 1: z is a real number. Statement 2: Principal argument of z is $\frac{\pi}{3}$. [4 marks]

(A) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.

(B) Statement 1 is false; Statement 2 is true.

(C) Statement 1 is true, Statement 2 is false.

(D) Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

Correct Answer: No option is universally valid: Statement 1 is false, while Statement 2 holds only for the root with positive imaginary part.

Solution:

Concept:

- If $z = x + iy$, its conjugate is $\bar{z} = x - iy$; adding a number and its conjugate gives twice its real part.
- The condition $|z| = 1$ gives $x^2 + y^2 = 1$. The principal argument depends on the sign of the imaginary part as well as the real part.

Step 1: Determine the real part Rewrite the given equation as $z + \bar{z} = 1$. With $z = x + iy$, this becomes $2x = 1$, so $x = 1/2$. The imaginary terms cancel: the equation does not restrict their sign. In particular, it does not say that z lies in the upper half-plane.

Step 2: Find both permitted complex numbers Use the unit-modulus condition:

$$\frac{1}{4} + y^2 = 1, \qquad y = \pm\frac{\sqrt{3}}{2}.$$

Therefore

$$z = \frac{1}{2} + \frac{\sqrt{3}}{2}i \quad \text{or} \quad z = \frac{1}{2} - \frac{\sqrt{3}}{2}i.$$

Both roots have nonzero imaginary parts, so Statement 1 is false for every allowed value of z.

Step 3: Evaluate the principal arguments and options The upper root has principal argument $\pi/3$, while the lower root has principal argument $-\pi/3$. Both are within the principal range. Thus Statement 2 is not guaranteed by the hypotheses. Option (B) describes the upper root only; it fails for the equally valid lower root. Options (A), (C) and (D) each assert that Statement 1 is true and are therefore false for both roots. Consequently none of the four choices describes all solutions of the question as stated. Selecting (B) would require an additional positive-imaginary-part condition, which is absent.

Step 4: Conclusion $\boxed{z = \frac{1}{2} \pm i\frac{\sqrt{3}}{2}}$, with principal arguments $\pm\pi/3$. Statement 1 is false and Statement 2 is not necessarily true; no listed option is universally correct. Option (B) applies only to the upper root.

Quick Tip to Solve: After using a modulus equation, retain both square-root signs and locate each root before deciding its principal argument.

3. 5-digit numbers are to be formed using 2, 3, 5, 7, 9 without repeating the digits. If p be the number of such numbers that exceed 20000 and q be the number of those that lie between 30000 and 90000, then $p : q$ is: [4 marks]

(1) 6 : 5

(2) 3 : 2

(3) 4 : 3

(4) 5 : 3

Correct Answer: (4) 5 : 3

Solution:

Concept:

- With five distinct digits and no repetitions, fixing the leading digit leaves $4!$ orders for the remaining places.
- To count numbers in a numerical interval, first determine which leading digits satisfy its bounds.

Step 1: Count the numbers exceeding 20000 The available digits are $2, 3, 5, 7, 9$, and all five must occur in each five-digit number. Every arrangement exceeds 20000: a number beginning with 2 has four further non-zero digits, and numbers beginning with larger digits plainly exceed the bound. Thus

$$p = 5! = 120.$$

There is no special exclusion for the leading digit 2; even the smallest arrangement is greater than 20000.

Step 2: Count the numbers between the second pair of bounds A number between 30000 and 90000 must begin with 3, 5 or 7. A number beginning with 2 is too small. Every number beginning with 9 is greater than 90000 because its remaining digits are non-zero. For each of the three permitted first digits, arrange the four remaining distinct digits in $4!$ ways:

$$q = 3 \times 4! = 72.$$

The endpoints themselves cannot be formed from the supplied digits, so strict or inclusive endpoints give the same count.

Step 3: Simplify and compare the ratio Cancel the common factor $4!$ before calculating:

$$p : q = (5 \times 4!) : (3 \times 4!) = 5 : 3.$$

The alternatives 6:5, 3:2 and 4:3 would require a different count of allowed leading digits; none matches five choices for the first count and three choices for the second.

Step 4: Conclusion $\boxed{p : q = 5 : 3}$. Hence, the correct option is (4).

Quick Tip to Solve: Count allowed first digits before arranging the tail; the common factorial cancels in the ratio.

4. Given a sequence of 4 numbers, first three of which are in G.P. and the last three are in A.P. with common difference six. If first and last terms of this sequence are equal, then the last term is [4 marks]

(1) 16

(2) 8

(3) 4

(4) 2

Correct Answer: (2) 8

Solution:

Concept:

- Three consecutive terms a, b, c of a geometric progression satisfy $b^2 = ac$.
- The final three terms form an arithmetic progression with common difference 6, so they can be expressed using the last term.

Step 1: Express all four terms using the last term Let the final term be t. The first term equals the final term by the condition in the question. Moving backwards through the last three terms of the arithmetic progression subtracts 6 each time. Hence the complete sequence is

$$t, \quad t-12, \quad t-6, \quad t.$$

This representation enforces both the common difference and the equality of the endpoints before the geometric condition is used.

Step 2: Apply the geometric progression condition The first three terms are consecutive terms of a geometric progression, so the square of the middle term equals the product of its neighbours:

$$(t-12)^2 = t(t-6).$$

Expanding both sides and cancelling t^2 gives

$$t^2 - 24t + 144 = t^2 - 6t, \qquad 18t = 144.$$

Therefore $t = 8$. The equation is linear after cancellation, so it supplies just one possible final term.

Step 3: Check both progression requirements For $t = 8$, the sequence becomes $8, -4, 2, 8$. The ratios of the first three terms are $-4/8 = 2/(-4) = -1/2$. The differences in the last three terms are $2-(-4) = 8-2 = 6$, and the endpoints agree. The other offered final terms 16, 4 and 2 fail the equation $18t = 144$.

Step 4: Conclusion The last term is $\boxed{8}$. Hence, the correct option is (2).

Quick Tip to Solve: Use the arithmetic progression to write all terms in one variable, then apply the middle-term square test to the geometric progression.

5. The value of $1^2 + 3^2 + 5^2 + \ldots + 25^2$ is : [4 marks]

(1) 2925

(2) 1469

(3) 1728

(4) 1456

Correct Answer: (1) 2925

Solution:

Concept:

- The first n positive odd integers are $2k-1$, for $k = 1, \ldots, n$.
- Use $\sum k = n(n+1)/2$ and $\sum k^2 = n(n+1)(2n+1)/6$ to sum their squares.

Step 1: Identify the number of terms The last odd number is 25. Equating it to $2n-1$ gives

$$2n - 1 = 25, \qquad n = 13.$$

Thus there are thirteen terms, including both 1^2 and 25^2. Counting only twelve intervals between the odd numbers would give an incorrect upper limit.

Step 2: Expand the odd-square sum Write the required sum as

$$S = \sum_{k=1}^{13} (2k-1)^2 = 4\sum_{k=1}^{13} k^2 - 4\sum_{k=1}^{13} k + 13.$$

The familiar sums give

$$\sum_{k=1}^{13} k = 91, \qquad \sum_{k=1}^{13} k^2 = \frac{13 \cdot 14 \cdot 27}{6} = 819.$$

The final constant contribution is 13 because the constant 1 occurs once in each squared expression.

Step 3: Evaluate and check the result Substitution now gives

$$S = 4(819) - 4(91) + 13 = 3276 - 364 + 13 = 2925.$$

Equivalently, the sum of the first n odd squares is $n(4n^2-1)/3$, which at $n = 13$ gives $13 \cdot 675/3 = 2925$. Both calculations agree. Therefore 1469, 1728 and 1456 do not equal the sum of all thirteen specified squares.

Step 4: Conclusion $\boxed{1^2 + 3^2 + \cdots + 25^2 = 2925}$. Hence, the correct option is (1).

Quick Tip to Solve: Find the term count from the last odd number: $25 = 2n - 1$ means $n = 13$, then use $n(4n^2 - 1)/3$.

6. If for positive integers $r > 1$, $n > 2$, the coefficients of the $(3r)$th and $(r+2)$th powers of x in the expansion of $(1+x)^{2n}$ are equal, then n is equal to: [4 marks]

(1) $2r+1$

(2) $2r-1$

(3) $3r$

(4) $r+1$

Correct Answer: (1) $2r+1$

Solution:

Concept:

- The coefficient of x^k in $(1+x)^{2n}$ is $\binom{2n}{k}$.
- Within a binomial row, equal coefficients have either identical indices or complementary indices whose sum is $2n$.

Step 1: Translate the equality of coefficients The two powers named in the question yield

$$\binom{2n}{3r} = \binom{2n}{r+2}.$$

In a fixed binomial row the coefficients increase strictly up to the middle and then decrease symmetrically. Therefore two equal entries either occupy the same position or are mirror positions. This accounts for both possibilities instead of assuming the complementary case without checking.

Step 2: Exclude equal indices If the indices were identical,

$$3r = r+2 \quad \Longrightarrow \quad r = 1.$$

This contradicts the given strict inequality $r > 1$. Thus the two coefficients must be at distinct, complementary positions in the row. The restriction on r is essential; without it the equality of coefficients at the same position would give no equation determining n.

Step 3: Use the complementary indices For complementary positions the indices add to the upper index:

$$3r + (r+2) = 2n.$$

Hence

$$2n = 4r+2, \qquad n = 2r+1.$$

This also satisfies the position range: $3r \le 4r+2 = 2n$ and $r+2 \le 2n$ for the allowed positive integers. The alternatives $2r-1$, $3r$ and $r+1$ do not follow from the required complementary-index sum.

Step 4: Conclusion $\boxed{n = 2r+1}$. Hence, the correct option is (1).

Quick Tip to Solve: When two binomial coefficients in one row are equal, check equal indices first; if excluded, add their indices to get the row number.

7. Let $A = \{\theta : \sin\theta = \tan\theta\}$ and $B = \{\theta : \cos\theta = 1\}$ be two sets. Then: [4 marks]

(1) $A = B$

(2) $A \not\subset B$

(3) $B \not\subset A$

(4) $A \subset B$ and $B - A \neq \varnothing$

Correct Answer: (2) $A \not\subset B$

Solution:

Concept:

- The equation involving $\tan\theta$ is defined only when $\cos\theta \neq 0$; this domain restriction must be retained.
- Set inclusion is checked by comparing all solutions, or disproved by one angle belonging to one set but not the other.

Step 1: Solve the equation defining the first set On the permitted domain, replace $\tan\theta$ by $\sin\theta/\cos\theta$ and multiply by the non-zero denominator:

$$\sin\theta\cos\theta = \sin\theta, \qquad \sin\theta(\cos\theta - 1) = 0.$$

Thus either $\sin\theta = 0$ or $\cos\theta = 1$. The first condition gives every $\theta = k\pi$, $k \in \mathbb{Z}$. The second gives only even multiples of π, already contained in the first family. All these angles satisfy the original tangent-domain restriction.

Step 2: Find and compare the two sets We obtain

$$A = \{k\pi : k \in \mathbb{Z}\}, \qquad B = \{2k\pi : k \in \mathbb{Z}\}.$$

Hence every element of B is an element of A, but the converse fails: $\pi \in A$ while $\cos\pi = -1$, so $\pi \notin B$. Therefore B is a proper subset of A, and A is not a subset of B.

Step 3: Test each printed relation Option (1) is false because the witness π separates the sets. Option (2) correctly states that A is not contained in B. Option (3) is false because B is contained in A. Option (4) requires the opposite inclusion; additionally $B - A$ is empty, rather than non-empty. The negation sign in the printed second option is decisive.

Step 4: Conclusion $\boxed{A \not\subset B}$, with $B \subset A$. Hence, the correct option is (2).

Quick Tip to Solve: Never divide a trigonometric equation by sine without checking its zeros; here the odd multiples of pi establish the strict inclusion.

8. If the image of point $P(2,3)$ in a line L is $Q(4,5)$, then the image of point $R(0,0)$ in the same line is: [4 marks]

(A) $(2,2)$

(B) $(4,5)$

(C) $(3,4)$

(D) $(7,7)$

Correct Answer: (D) $(7,7)$

Solution:

Concept:

- A reflecting line is the perpendicular bisector of the segment joining a point and its image. Every point on that line is equidistant from the two points.
- For a line $ax + by + c = 0$, the image of (x_0, y_0) is $(x_0 - 2ad, y_0 - 2bd)$, where $d = (ax_0 + by_0 + c)/(a^2 + b^2)$.

Step 1: Find the mirror line The midpoint of $P(2,3)$ and $Q(4,5)$ is $M(3,4)$, and the slope of PQ is $(5-3)/(4-2) = 1$. The mirror therefore has slope -1 and passes through M. Hence its equation is

$$y - 4 = -(x - 3), \qquad x + y - 7 = 0.$$

This choice also places P and Q on opposite sides of the mirror at the same perpendicular distance.

Step 2: Reflect the origin and check the candidates For $R = (0,0)$, use $a = b = 1$, $c = -7$. Then $d = -7/2$, so

$$R' = (0 - 2(-7/2), 0 - 2(-7/2)) = (7,7).$$

As a geometric check, the midpoint of the origin and $(7,7)$ is $(7/2, 7/2)$, which satisfies $x + y = 7$; their joining line has slope 1 and is perpendicular to the mirror. The other three candidates do not have their midpoint with the origin on this mirror and therefore cannot be images of R.

Step 3: Conclusion The image of $R(0,0)$ is $\boxed{(7,7)}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the perpendicular bisector of the given point-image pair first; its equation determines every other reflection uniquely.

9. Let $x \in (0,1)$. The set of all x such that $\sin^{-1} x > \cos^{-1} x$, is the interval: [4 marks]

(A) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

(B) $\left(\frac{1}{\sqrt{2}}, 1\right)$

(C) $(0,1)$

(D) $\left(0, \frac{\sqrt{3}}{2}\right)$

Correct Answer: (B) $\left(\frac{1}{\sqrt{2}}, 1\right)$

Solution:

Concept:

- For $x \in [-1,1]$, the principal inverse functions satisfy $\sin^{-1} x + \cos^{-1} x = \pi/2$. On $(0,1)$, both principal angles lie between 0 and $\pi/2$.
- The inverse sine is strictly increasing on $[-1,1]$. Thus comparing inverse sines preserves the order of their arguments. A strict inequality excludes a point where the two angles are equal.

Step 1: Reduce the inequality to one inverse function Substitute $\cos^{-1} x = \pi/2 - \sin^{-1} x$ into the printed inequality. This gives

$$\sin^{-1} x > \frac{\pi}{2} - \sin^{-1} x \quad \Longleftrightarrow \quad 2\sin^{-1} x > \frac{\pi}{2}.$$

Therefore $\sin^{-1} x > \pi/4$. There is no extra factor of x multiplying the inverse sine: the printed expression compares the two inverse trigonometric functions directly.

Step 2: Use monotonicity and intersect with the domain Since $\pi/4 = \sin^{-1}(1/\sqrt{2})$, monotonicity gives $x > 1/\sqrt{2}$. Combining this with the stated domain $0 < x < 1$, we obtain

$$x \in \left(\frac{1}{\sqrt{2}}, 1\right).$$

The boundary $x = 1/\sqrt{2}$ gives equality of the two angles and is excluded. Option (A) lies below that boundary; (C) includes many values below it; and (D) also contains such values and stops before 1. For instance, at a small positive x the inverse sine is near zero while inverse cosine is near $\pi/2$, so none of those wider or lower intervals can work.

Step 3: Conclusion The required interval is $\boxed{(1/\sqrt{2}, 1)}$. Hence, the correct option is (B).

Quick Tip to Solve: When inverse sine and inverse cosine have the same argument, replace one by the complementary angle before solving the inequality.

10. Statement 1: The only circle having radius $\sqrt{10}$ and a diameter along line $2x + y = 5$ is $x^2 + y^2 - 6x + 2y = 0$. Statement 2: $2x + y = 5$ is a normal to the circle $x^2 + y^2 - 6x + 2y = 0$. [4 marks]

(A) Statement 1 is false; Statement 2 is true.

(B) Statement 1 is true; Statement 2 is true; Statement 2 is a correct explanation for Statement 1.

(C) Statement 1 is true; Statement 2 is false.

(D) Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

Correct Answer: (A) Statement 1 is false; Statement 2 is true.

Solution:

Concept:

- A diameter lies on a line through the centre of a circle. Specifying its supporting line and a radius does not fix the position of the centre along that line.
- A normal at a point of a circle passes through the centre. Hence a line through the centre meets the circle at endpoints of a diameter and is a normal at either endpoint.

Step 1: Test the asserted uniqueness in Statement 1 Complete the squares in the given equation:

$$(x - 3)^2 + (y + 1)^2 = 10.$$

Its centre is $(3, -1)$, its radius is $\sqrt{10}$, and its centre satisfies $2(3) - 1 = 5$. So this is indeed one suitable circle. However, any centre (h, k) with $2h + k = 5$ gives another circle $(x - h)^2 + (y - k)^2 = 10$ with the same radius and a diameter along the same line. For example, $(0, 5)$ gives $x^2 + (y - 5)^2 = 10$, a distinct circle. Thus the word the word only makes Statement 1 false.

Step 2: Test Statement 2 independently The centre $(3, -1)$ lies on $2x + y = 5$, as already checked. Consequently this line is a radial line through the centre and is perpendicular to the tangent at either of its intersection points with the circle. It is therefore a normal to the given circle, so Statement 2 is true. The truth of this normal property cannot establish the false uniqueness assertion. Options (B), (C), and (D) all assert that Statement 1 is true and are excluded by the explicit counterexample above.

Step 3: Conclusion Statement 1 is false and Statement 2 is true. Hence, the correct option is (A).

Quick Tip to Solve: Watch for the word only: a fixed radius with the centre anywhere on a given line still gives infinitely many circles.

11. If a circle of unit radius is divided into two parts by an arc of another circle subtending an angle 60° on the circumference of the first circle, then the radius of the arc is: [4 marks]

(1) $\sqrt{3}$

(2) $\frac{1}{2}$

(3) 1

(4) None of these

Correct Answer: The angle fixes the common chord but does not determine the radius of the second circle.

Solution:

Concept:

- An angle inscribed in a circle depends on its chord and the radius of that circle. If a chord subtends an inscribed angle θ in a circle of radius R, its length is $2R\sin\theta$.
- A fixed chord belongs to infinitely many circles: their centres lie on its perpendicular bisector. The radius also depends on the distance of the centre from the chord, which the given angle does not specify.

Step 1: Find what the stated angle actually determines Let A and B be the two intersection points of the circles, and let C be the point on the circumference of the first circle where the stated angle is measured. The first circle has unit radius and $\angle ACB = 60^\circ$. The extended sine rule in triangle ACB gives

$$AB = 2(1)\sin 60^\circ = \sqrt{3}.$$

This determines the common chord AB. It does not specify an angle at the centre of the second circle or at any point of its circumference. Transferring the given angle to that second circle would add a condition absent from the question.

Step 2: Exhibit distinct radii compatible with the same chord Choose coordinates $A = (-\sqrt{3}/2, 0)$, $B = (\sqrt{3}/2, 0)$. A unit circle through A and B can have centre $O = (0, 1/2)$. Every circle centred at $(0, t)$ through the same endpoints has radius

$$r = \sqrt{t^2 + \frac{3}{4}}.$$

For example, $t = -1/2$ gives a second circle of radius 1, distinct from the first, whereas $t = -3/2$ gives radius $\sqrt{3}$. Both intersect the unit circle at A and B, and an arc inside the unit circle divides its disk into two parts. The original inscribed angle remains 60° because A, B and the first circle have not changed. Thus options (1) and (3) can both occur. Radius $1/2$ cannot span a chord of length $\sqrt{3}$, but excluding it does not choose between the other admissible values.

Step 3: Conclusion The information given does not determine a unique radius: both $\boxed{1}$ and $\boxed{\sqrt{3}}$, among infinitely many values, are compatible. Consequently no single numerical option is forced by the stated geometry.

Quick Tip to Solve: For intersecting circles, distinguish the angle measured on the first circle from the angle on the second; a common chord alone does not fix both radii.

12. A point on the ellipse, $4x^2 + 9y^2 = 36$, where the normal is parallel to the line, $4x - 2y - 5 = 0$, is: [4 marks]

(A) $\left(\frac{9}{5}, \frac{8}{5}\right)$
(B) $\left(\frac{8}{5}, -\frac{9}{5}\right)$
(C) $\left(-\frac{9}{5}, \frac{8}{5}\right)$
(D) $\left(\frac{8}{5}, \frac{9}{5}\right)$

Correct Answer: (A) $\left(\frac{9}{5}, \frac{8}{5}\right)$

Solution:

Concept:

- Implicit differentiation of $4x^2 + 9y^2 = 36$ gives the tangent slope $dy/dx = -4x/(9y)$ at a regular point with $y \neq 0$.
- For a nonhorizontal and nonvertical tangent, the normal slope is the negative reciprocal, $m_n = 9y/(4x)$. Parallel lines have equal slopes.

Step 1: Impose the required normal direction The given line is $y = 2x - 5/2$, so its slope is 2. Set the normal slope equal to 2:

$$\frac{9y}{4x} = 2 \quad \Longrightarrow \quad y = \frac{8}{9}x.$$

Neither an endpoint on the x-axis nor an endpoint on the y-axis has a finite normal slope of 2, so no omitted zero-denominator case supplies another candidate here.

Step 2: Intersect with the ellipse and compare the options Substitute $y = 8x/9$ into the ellipse:

$$4x^2 + 9\left(\frac{8x}{9}\right)^2 = 36, \qquad \frac{100}{9}x^2 = 36.$$

Hence $x = \pm 9/5$, with corresponding $y = \pm 8/5$ of the same sign. Of these two antipodal points, the printed options contain $(9/5, 8/5)$. It satisfies the equation because $4(81/25) + 9(64/25) = 36$, and its normal slope is 2. Option (C) is also on the ellipse but has normal slope -2. Options (B) and (D) interchange the coordinate magnitudes and do not even lie on this ellipse: their substituted value is $985/25$, rather than 36.

Step 3: Conclusion The listed point is $\boxed{(9/5, 8/5)}$. Hence, the correct option is (A). This differs from the sheet key: the printed coordinates with negative x give normal slope -2, whereas the required slope is 2.

Quick Tip to Solve: For an ellipse given implicitly, differentiate first and use the negative reciprocal for the normal; do not confuse it with the tangent slope.

13. Consider the system of equations: $x + ay = 0$, $y + az = 0$ and $z + ax = 0$. Then the set of all real values of a for which the system has a unique solution is: [4 marks]

(A) $\mathbb{R} \setminus \{1\}$

(B) $\mathbb{R} \setminus \{-1\}$

(C) $\{1, -1\}$

(D) $\{1, 0, -1\}$

Correct Answer: (B) $\mathbb{R} \setminus \{-1\}$

Solution:

Concept:

- A homogeneous square linear system always has the zero solution. It has exactly one solution if and only if its coefficient determinant is nonzero.
- For real a, $1 + a^3 = (a + 1)(a^2 - a + 1)$, and $a^2 - a + 1 = (a - 1/2)^2 + 3/4$ is strictly positive.

Step 1: Compute the coefficient determinant In the variable order (x, y, z), the coefficient matrix and its determinant are

$$A = \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{pmatrix}, \qquad \det A = 1 - a(0 - a^2) = 1 + a^3.$$

Thus the system has a unique solution precisely when $a^3 \neq -1$. The plus sign on the cubic term is essential: the off-diagonal product is a three-cycle and enters the determinant with a positive sign.

Step 2: Identify the excluded real value and check it Factor the determinant as $(a + 1)(a^2 - a + 1)$. The second factor is positive for every real a, so the only singular value is $a = -1$. At that value the equations become $x = y$, $y = z$, and $z = x$, admitting all triples (t, t, t); this explicitly confirms nonuniqueness. For every other real a the determinant is nonzero and only $(0, 0, 0)$ remains. Option (A) wrongly excludes 1, where the determinant is 2. Options (C) and (D) give finite sets, whereas values such as a=2 also give a nonzero determinant and a unique solution.

Step 3: Conclusion The required parameter set is $\boxed{\mathbb{R} \setminus \{-1\}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a homogeneous system, test the determinant rather than solving for all variables; singularity is the only obstruction to uniqueness.

14. Let p and q be any two logical statements and $r : p \to (\sim p \vee q)$. If r has a truth value F, then the truth values of p and q are respectively: [4 marks]

(A) F, F

(B) T, T

(C) T, F

(D) F, T

Correct Answer: (C) T, F

Solution:

Concept:

- An implication $A \to B$ is false only when its antecedent A is true and its consequent B is false. In every other case the implication is true.
- A disjunction $A \vee B$ is false only when both components are false. Negation reverses a truth value: if p is true, $\sim p$ is false.

Step 1: Use the false implication condition Here the antecedent is p and the consequent is $\sim p \vee q$. Since the whole statement r is false, p must be true and $\sim p \vee q$ must be false. In particular, neither row with p false can be the answer: a false antecedent makes an implication true regardless of its consequent. This already eliminates options (A) and (D).

Step 2: Force the disjunction to be false and verify The disjunction $\sim p \vee q$ can be false only if $\sim p = F$ and $q = F$. The first condition agrees with $p = T$, so the required ordered pair is (T, F). Substitute directly: $p = T$ gives $\sim p = F$, and with $q = F$ the consequent becomes $F \vee F = F$; hence $r = T \to F = F$, as required. In option (B), q is true, making the consequent true and r true. Equivalently, using $p \to s \equiv \sim p \vee s$, the statement simplifies to $\sim p \vee q$; its unique false truth-table row is again p true and q false.

Step 3: Conclusion The truth values of p and q are respectively $\boxed{T, F}$. Hence, the correct option is (C).

Quick Tip to Solve: To make an implication false, first force its antecedent true; then make every term in an OR consequent false.

15. In a set of $2n$ observations, half of them are equal to a and the remaining half are equal to $-a$. If the standard deviation of all the observations is 2, then the value of $|a|$ is: [4 marks]

(A) 2

(B) $\sqrt{2}$

(C) 4

(D) $2\sqrt{2}$

Correct Answer: (A) 2

Solution:

Concept:

- For equally weighted observations, the arithmetic mean is $\bar{x} = \frac{1}{N}\sum x_i$, and population variance is $\sigma^2 = \frac{1}{N}\sum(x_i - \bar{x})^2$.
- Standard deviation is the non-negative square root of variance. Thus $\sqrt{a^2} = |a|$, irrespective of whether the repeated observation a is positive or negative.

Step 1: Find the mean using the equal frequencies There are n observations equal to a and n equal to $-a$, so each positive contribution to the sum is balanced by a negative one. Consequently,

$$\bar{x} = \frac{na + n(-a)}{2n} = 0.$$

The total number of observations is $2n$; this denominator is used because the question gives the standard deviation of the complete set of observations, rather than an unbiased estimate of a sampled population.

Step 2: Calculate the variance and impose the given deviation Every observation lies at the same distance $|a|$ from the mean. Squaring the deviations therefore gives a^2 for all $2n$ observations. Hence

$$\sigma^2 = \frac{n(a-0)^2 + n(-a-0)^2}{2n} = a^2.$$

Taking the non-negative square root yields $\sigma = |a|$. Since the stated standard deviation is 2, it follows that $|a| = 2$. The frequency n cancels, as expected: repeating every value equally often changes neither the spread nor its standard deviation. The alternatives $\sqrt{2}$, 4, and $2\sqrt{2}$ would instead give variances 2, 16, and 8, whereas the given deviation requires variance 4.

Step 3: Conclusion The required value is $\boxed{|a| = 2}$. Hence, the correct option is (A).

Quick Tip to Solve: For an equally frequent pair of values a and $-a$, the mean is zero and the standard deviation is immediately $|a|$; do not confuse it with the variance a^2.

16. A common tangent to the conics $x^2 = 6y$ and $2x^2 - 4y^2 = 9$ is: [4 marks]

(A) $x - y = \dfrac{3}{2}$

(B) $x + y = 1$

(C) $x + y = \dfrac{9}{2}$

(D) $x - y = 1$

Correct Answer: (A) $x - y = \dfrac{3}{2}$

Solution:

Concept:

- A line is tangent to a conic when substitution of its equation gives a quadratic with a repeated root; its discriminant must vanish.
- For the parabola $x^2 = 4ay$, the line $y = mx + c$ is tangent when $c = -am^2$. Here $4a = 6$, so $a = 3/2$.

Step 1: Identify which listed lines touch the parabola All the proposed lines have slope $m = 1$ or $m = -1$, so $m^2 = 1$. Substituting $y = mx + c$ into $x^2 = 6y$ gives

$$x^2 - 6mx - 6c = 0.$$

A repeated intersection requires $36m^2 + 24c = 0$, hence $c = -3/2$. Option (A) is $y = x - 3/2$, with exactly this intercept. Options (B), (C) and (D) have intercepts 1, $9/2$, and -1, respectively, and therefore are not tangent to the parabola. This already singles out (A), but the second conic must also be checked.

Step 2: Verify tangency to the hyperbola directly Insert $y = x - 3/2$ into the second equation. Then

$$\begin{aligned} 2x^2 - 4(x - 3/2)^2 &= 9, \\ -2x^2 + 12x - 18 &= 0, \\ (x - 3)^2 &= 0. \end{aligned}$$

There is one repeated intersection, $x = 3$, giving $y = 3/2$. The same line meets the parabola according to

$$x^2 = 6(x - 3/2) \quad \Longrightarrow \quad (x - 3)^2 = 0.$$

Thus it touches both conics at $(3, 3/2)$. This explicit double-root check establishes genuine tangency rather than merely showing that the line passes through a common point. None of the other listed equations passes the necessary parabola test.

Step 3: Conclusion A common tangent is $\boxed{x - y = 3/2}$. Hence, the correct option is (A).

Quick Tip to Solve: Test candidate lines against the simpler conic first. For a common tangent, a repeated-root intersection must hold for both curves, not just for one.

17. Let $S = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} : a_{ij} \in \{0,1,2\},\ a_{11} = a_{22} \right\}$. Then the number of non-singular matrices in the set S is: [4 marks]

(A) 27

(B) 24

(C) 10

(D) 20

Correct Answer: (D) 20

Solution:

Concept:

- A square matrix is non-singular exactly when its determinant is nonzero. For $M = \begin{pmatrix} a & b \\ c & a \end{pmatrix}$, the determinant is $a^2 - bc$.
- Each entry belongs to $\{0,1,2\}$. Count ordered pairs (b,c), because exchanging the off-diagonal entries generally gives a different matrix.

Step 1: Fix the common diagonal entry The condition $a_{11} = a_{22}$ leaves three independent choices: the common diagonal value a, the upper-right entry b, and the lower-left entry c. There are $3^3 = 27$ matrices before imposing non-singularity. For each of the three diagonal values there are $3 \times 3 = 9$ ordered off-diagonal pairs. We exclude precisely those satisfying $bc = a^2$, rather than trying to list every matrix individually.

Step 2: Count admissible off-diagonal pairs in each case If $a = 0$, non-singularity requires $bc \neq 0$. Both b and c must be in $\{1,2\}$, producing $2 \times 2 = 4$ matrices. If $a = 1$, the only forbidden pair is $(1,1)$, because it is the only pair with product 1. There are therefore $9 - 1 = 8$ matrices. If $a = 2$, the only forbidden pair is $(2,2)$, the sole pair with product 4; this gives another 8 matrices. Thus

$$N = 4 + 8 + 8 = 20.$$

As a cross-check, there are five singular matrices for $a = 0$, one for $a = 1$, and one for $a = 2$; subtracting these seven from 27 also gives 20. Option (A) counts all matrices, including singular ones. Options (B) and (C) do not match the complete case count.

Step 3: Conclusion The number of non-singular matrices is $\boxed{20}$. Hence, the correct option is (D).

Quick Tip to Solve: When the diagonal entries coincide, use $\det M = a^2 - bc$ and split by the diagonal value. Remember that (b,c) is an ordered pair.

18. Consider the function $f(x) = \lfloor x \rfloor + |1 - x|$, $-1 \le x \le 3$, where $\lfloor x \rfloor$ is the greatest integer function.

Statement 1: f is not continuous at $x = 0, 1, 2, 3$.

Statement 2:

$$f(x) = \begin{cases} -x, & -1 \le x < 0, \\ 1 - x, & 0 \le x < 1, \\ 1 + x, & 1 \le x < 2, \\ 2 + x, & 2 \le x \le 3. \end{cases}$$

[4 marks]

(A) Statement 1 is true; Statement 2 is false.

(B) Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

(C) Statement 1 is true; Statement 2 is true; Statement 2 is a correct explanation for Statement 1.

(D) Statement 1 is false; Statement 2 is true.

Correct Answer: (A) Statement 1 is true; Statement 2 is false.

Solution:

Concept:

- The greatest-integer function $\lfloor x \rfloor$ is constant on each interval $[k, k + 1)$ and jumps at integers.
- Continuity at an interior point requires both one-sided limits to equal the function value; at an endpoint only the limit from within the domain is required.

Step 1: Derive the actual pieces from the defining formula For $-1 \le x < 1$, the absolute value is $|1 - x| = 1 - x$; for $x \ge 1$, it equals $x - 1$. Combining this with the integer part gives

$$f(x) = \begin{cases} -x, & -1 \le x < 0, \\ 1 - x, & 0 \le x < 1, \\ x, & 1 \le x < 2, \\ 1 + x, & 2 \le x < 3, \\ 5, & x = 3. \end{cases}$$

In particular, on $[1, 2)$ the correct expression is x, not $1 + x$, and on $[2, 3)$ it is $1 + x$, not $2 + x$. At $x = 3$, the floor is 3, so $f(3) = 3 + 2 = 5$. Thus the piecewise formula in Statement 2 is false.

Step 2: Check each continuity claim independently At $x = 0$, the left limit is 0 and the value is 1. At $x = 1$, the left limit is 0 and the value is 1. At $x = 2$, the left limit is 2 and the value is 3. Finally, at the right endpoint 3, the limit from within the domain is 4, whereas $f(3) = 5$. Each discrepancy establishes a discontinuity, including the endpoint discontinuity at 3. Statement 1 is therefore true. The absolute-value term is continuous everywhere, so it cannot remove a unit jump in the floor function. Options (B), (C) and (D) all declare Statement 2 true, contradicting the pieces obtained directly from the original definition.

Step 3: Conclusion Statement 1 is true and Statement 2 is false. Hence, the correct option is (A).

Quick Tip to Solve: Always derive the pieces from the original function before assessing a proposed formula. At the last integer endpoint, compare its value with the left-hand limit.

19. A spherical balloon is being inflated at the rate of $35\,\text{cm}^3/\text{min}$. The rate of increase in the surface area (in cm^2/min) of the balloon when its diameter is $14\,\text{cm}$ is: [4 marks]

(A) 10

(B) $\sqrt{10}$

(C) 100

(D) $10\sqrt{10}$

Correct Answer: (A) 10

Solution:

Concept:

- For a sphere of radius r, its volume is $V = \frac{4}{3}\pi r^3$ and its surface area is $A = 4\pi r^2$.
- The chain rule connects their time rates through the changing radius: $dA/dt = (dA/dr)(dr/dt)$ and $dV/dt = (dV/dr)(dr/dt)$.

Step 1: Find the instantaneous radius and its rate The diameter is $14\,\text{cm}$, so the instantaneous radius is $r = 7\,\text{cm}$. The inflation rate refers to volume: $dV/dt = 35\,\text{cm}^3/\text{min}$, not to radius or surface area. Differentiate the volume formula with respect to time:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

At the stated instant,

$$\frac{dr}{dt} = \frac{35}{4\pi(7)^2} = \frac{5}{28\pi}\,\text{cm/min}.$$

The positive sign agrees with an expanding balloon.

Step 2: Convert the radius rate to an area rate Differentiate the surface-area formula using the same instantaneous radius:

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}.$$

Substitution gives

$$\frac{dA}{dt} = 8\pi(7)\frac{5}{28\pi} = 10\,\text{cm}^2/\text{min}.$$

Equivalently, eliminate the radius rate before substitution:

$$\frac{dA}{dt} = \frac{8\pi r}{4\pi r^2}\frac{dV}{dt} = \frac{2}{r}\frac{dV}{dt}.$$

This again gives $(2/7)35 = 10$. The cancellation of π is exact, so no approximate value of π is needed. Options (B), (C), and (D) give different area rates and cannot satisfy this relation; in particular, a square root is not introduced by either differentiation.

Step 3: Conclusion The surface area increases at $\boxed{10\,\text{cm}^2/\text{min}}$. Hence, the correct option is (A).

Quick Tip to Solve: For any expanding sphere, use $dA/dt = (2/r)(dV/dt)$. Convert a given diameter to radius before applying this short relation.

20. Let $f(1) = -2$ and $f'(x) \geq 4.2$ for $1 \leq x \leq 6$. The possible value of $f(6)$ lies in the interval: [4 marks]

(A) $[15, 19)$

(B) $(-\infty, 12)$

(C) $[12, 15)$

(D) $[19, \infty)$

Correct Answer: (D) $[19, \infty)$

Solution:

Concept:

- The mean value theorem gives $f(b) - f(a) = f\prime(c)(b - a)$ for some $c \in (a, b)$, when the function is continuous on the closed interval and differentiable inside it.
- A lower bound on the derivative gives a lower bound on the average rate of increase, so $f\prime(x) \geq m$ implies $f(b) \geq f(a) + m(b - a)$.

Step 1: Apply the derivative bound over the full interval The starting value is $f(1) = -2$, and the interval length is $6 - 1 = 5$, not 6. The derivative is at least 4.2 throughout the interval. The mean value theorem therefore gives a point $c \in (1, 6)$ for which

$$\frac{f(6) - f(1)}{6 - 1} = f\prime(c) \geq 4.2.$$

Multiplying by the positive number 5 preserves the inequality and yields

$$f(6) + 2 \geq 21.$$

Thus $f(6) \geq 19$. Differentiability as assumed by the stated derivative condition supplies the regularity needed for this standard bound.

Step 2: Check the endpoint and the extent of the possible interval The lower endpoint is attainable: take the straight-line function

$$f(x) = -2 + 4.2(x - 1).$$

It has derivative exactly 4.2 and gives $f(6) = 19$. There is no specified upper bound on the derivative; for any $L \geq 19$, the function $f(x) = -2 + ((L + 2)/5)(x - 1)$ has derivative at least 4.2 and satisfies $f(6) = L$. Hence the possible values fill $[19, \infty)$. Options (A) and (C) lie below 19, and option (B) lies below 12; every one violates the minimum increase of 21 from the initial value. The closed bracket at 19 is essential because equality is allowed in the derivative hypothesis.

Step 3: Conclusion The possible value satisfies $\boxed{f(6) \in [19, \infty)}$. Hence, the correct option is (D).

Quick Tip to Solve: Multiply the derivative bound by the interval length and add the initial value. Keep the endpoint closed when the derivative inequality permits equality.

21. If an equation of a tangent to the curve, $y = \cos(x + y)$, $-1 \le x \le 1 + \pi$, is $x + 2y = k$ then k is equal to : [4 marks]

(A) 1

(B) 2

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

Correct Answer: (D) $\frac{\pi}{2}$

Solution:

Concept:

- For a curve defined implicitly, differentiate both sides with respect to x, remembering that y depends on x. The chain rule gives $d(x + y)/dx = 1 + y'$.
- The tangent line $x + 2y = k$ has slope $-1/2$. At its point of contact, the curve's derivative must equal this slope and the point must also satisfy the original curve.

Step 1: Impose the tangent's slope on the curve Differentiating $y = \cos(x + y)$ implicitly gives

$$y' = -\sin(x + y)(1 + y').$$

Write $s = \sin(x + y)$. Rearranging yields $y'(1 + s) = -s$. At a point where the required tangent exists, its finite slope is $y' = -1/2$, so substitution into this equation gives

$$-\frac{1}{2}(1 + s) = -s \quad \Longrightarrow \quad s = 1.$$

This value makes $1 + s = 2 \neq 0$, so the implicit derivative is well defined there. Consequently, $x + y = \pi/2 + 2m\pi$, where m is an integer.

Step 2: Locate the point of contact in the given interval At all these possible arguments the cosine vanishes. The defining equation therefore forces $y = 0$, and hence

$$x = \frac{\pi}{2} + 2m\pi.$$

The interval $-1 \le x \le 1 + \pi$ admits only $m = 0$: $m = -1$ gives $-3\pi/2 < -1$, while $m = 1$ gives $5\pi/2 > 1 + \pi$; further integers are still outside. Thus the contact point is $(\pi/2, 0)$. It satisfies the curve since $\cos(\pi/2) = 0$, and its derivative is indeed $-1/2$. Substituting this point in the tangent gives $k = x + 2y = \pi/2$. The other proposed constants 1, 2, and $\pi/4$ would give parallel lines that do not pass through this required contact point.

Step 3: Conclusion The tangent is $x + 2y = \pi/2$, so $\boxed{k = \pi/2}$. Hence, the correct option is (D).

Quick Tip to Solve: Match the tangent's slope before finding the contact point. Then use both the original implicit equation and the stated interval to choose the admissible point.

22. If the integral $\int \frac{\cos 8x + 1}{\cot 2x - \tan 2x}\,dx = A\cos 8x + k$, where k is an arbitrary constant, then A is equal to: [4 marks]

(A) $-\frac{1}{16}$
(B) $\frac{1}{16}$
(C) $\frac{1}{8}$
(D) $-\frac{1}{8}$

Correct Answer: (A) $-\frac{1}{16}$

Solution:

Concept:

- Use the double-angle identities $1+\cos 8x = 2\cos^2 4x$ and $\cot u - \tan u = 2\cot 2u$. The antiderivative is determined on any interval where the original integrand is defined.
- The chain rule gives $\frac{d}{dx}\cos 8x = -8\sin 8x$; the coefficient of the resulting sine therefore determines A.

Step 1: Simplify the denominator Combine the two quotients over a common denominator rather than integrating them separately.

$$\cot 2x - \tan 2x = \frac{\cos^2 2x - \sin^2 2x}{\sin 2x \cos 2x} = \frac{2\cos 4x}{\sin 4x}.$$

These identities are applied only where the printed denominator and both original trigonometric quotients exist.

Step 2: Reduce the integrand to one sine The numerator has precisely the factor needed to cancel one cosine. Thus, on each admissible interval,

$$\frac{1+\cos 8x}{\cot 2x - \tan 2x} = \frac{2\cos^2 4x \sin 4x}{2\cos 4x} = \sin 4x \cos 4x = \frac{1}{2}\sin 8x.$$

The factor one half comes from the product-to-double-angle identity and must be retained.

Step 3: Integrate and compare coefficients Now the integral is immediate:

$$\int \frac{1}{2}\sin 8x\,dx = -\frac{1}{16}\cos 8x + k.$$

Differentiating the proposed expression gives $(-1/16)(-8)\sin 8x = \frac{1}{2}\sin 8x$, confirming both the negative sign and the scale. A positive coefficient has the wrong derivative sign, while a coefficient of magnitude $1/8$ gives twice the required integrand.

Step 4: Conclusion Comparing with $A\cos 8x + k$, we obtain $\boxed{A = -1/16}$. Hence, the correct option is (A).

Quick Tip to Solve: Reduce the quotient before integrating; after obtaining $\frac{1}{2}\sin 8x$, divide by the inner derivative 8 and retain the negative cosine sign.

23. For $0 \le x \le \frac{\pi}{2}$, the value of $\int_0^{\sin^2 x} \sin^{-1}(\sqrt{t})\,dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t})\,dt$ equals: [4 marks]

(A) $\frac{\pi}{4}$

(B) 0

(C) 1

(D) $-\frac{\pi}{4}$

Correct Answer: (A) $\frac{\pi}{4}$

Solution:

Concept:

- For a continuous integrand, the fundamental theorem of calculus differentiates an integral with variable upper limit; the chain rule supplies the derivative of that limit.
- On $[0, \pi/2]$, $\sqrt{\sin^2 x} = \sin x$ and $\sqrt{\cos^2 x} = \cos x$, so the principal inverse trigonometric functions both return x.

Step 1: Show the expression is constant Denote the sum by $F(x)$. For an interior point of the interval,

$$F\prime(x) = \sin^{-1}(\sin x)\sin 2x - \cos^{-1}(\cos x)\sin 2x = x\sin 2x - x\sin 2x = 0.$$

The two upper limits vary in opposite directions, and the inverse functions have identical values on the specified interval. Thus F is constant; continuity also includes the endpoints.

Step 2: Evaluate the constant at zero At $x = 0$, the first integral vanishes and the second has upper limit 1. Substitute $t = u^2$, so that $dt = 2u\,du$:

$$F(0) = \int_0^1 \cos^{-1}\sqrt{t}\,dt = \int_0^1 2u\cos^{-1} u\,du.$$

Integration by parts gives

$$F(0) = [u^2\cos^{-1} u]_0^1 + \int_0^1 \frac{u^2}{\sqrt{1-u^2}}\,du.$$

The boundary term is zero. With $u = \sin v$, the remaining integral is $\int_0^{\pi/2} \sin^2 v\,dv = \pi/4$.

Step 3: Select the matching option The sum is independent of x and equals the endpoint value just calculated. Both integrands are nonnegative, so a negative result is impossible; the endpoint integral is strictly positive, excluding zero. Its exact evaluation is $\pi/4$, rather than 1.

Step 4: Conclusion The required value is $\boxed{\pi/4}$ for the whole stated interval. Hence, the correct option is (A).

Quick Tip to Solve: For complementary upper limits $\sin^2 x$ and $\cos^2 x$, differentiate the sum first; cancellation often reduces the calculation to one endpoint integral.

24. Let $f : [-2, 3] \to [0, \infty)$ be a continuous function such that $f(1 - x) = f(x)$ for all $x \in [-2, 3]$. If R_1 is the numerical value of the area of the region bounded by $y = f(x)$, $x = -2$, $x = 3$ and the axis of x, and $R_2 = \int_{-2}^{3} xf(x)\,dx$, then: [4 marks]

(A) $3R_1 = 2R_2$

(B) $2R_1 = 3R_2$

(C) $R_1 = R_2$

(D) $R_1 = 2R_2$

Correct Answer: (D) $R_1 = 2R_2$

Solution:

Concept:

- Because $f \geq 0$, its area under the graph is the ordinary integral $R_1 = \int_{-2}^{3} f(x)\,dx$, without absolute-value splitting.
- The substitution $u = 1 - x$ reflects the interval $[-2, 3]$ about its midpoint $x = 1/2$; the symmetry condition is $f(1 - u) = f(u)$.

Step 1: Reflect the weighted integral Write $R_2 = \int_{-2}^{3} xf(x)\,dx$. Under $u = 1 - x$, the endpoints become 3 and -2, and $dx = -du$. Reversing those limits gives

$$R_2 = \int_{-2}^{3} (1 - u)f(1 - u)\,du = \int_{-2}^{3} (1 - u)f(u)\,du.$$

The second equality uses precisely the given symmetry, not an assumption that the function is even about zero.

Step 2: Add the reflected and original forms The integration variable is a dummy variable, so add the two expressions term by term:

$$2R_2 = \int_{-2}^{3} \big[u + (1 - u)\big] f(u)\,du = \int_{-2}^{3} f(u)\,du = R_1.$$

Hence the weighted mean coordinate, whenever the area is nonzero, is $R_2/R_1 = 1/2$, as expected for symmetry about the midpoint.

Step 3: Compare the stated relations The relation that always follows is $R_1 = 2R_2$. The other displayed ratios do not follow from reflection. For example, the admissible function $f(x) = 1$ has $R_1 = 5$ and $R_2 = [x^2/2]_{-2}^{3} = 5/2$; it satisfies the fourth relation and disproves the other three as general claims. The zero function also satisfies the derived identity, without requiring division by its zero area.

Step 4: Conclusion The symmetry gives $\boxed{R_1 = 2R_2}$. Hence, the correct option is (D).

Quick Tip to Solve: For $f(a + b - x) = f(x)$, pair the weighted integral with its reflected form to obtain $\int_a^b xf(x)\,dx = \frac{a+b}{2} \int_a^b f(x)\,dx$.

25. The equation of the curve passing through the origin and satisfying the differential equation $(1+x^2)\frac{dy}{dx}+2xy=4x^2$ is [4 marks]

(A) $(1+x^2)y=x^3$

(B) $3(1+x^2)y=2x^3$

(C) $(1+x^2)y=3x^3$

(D) $3(1+x^2)y=4x^3$

Correct Answer: (D) $3(1+x^2)y=4x^3$

Solution:

Concept:

- The product rule identifies $(1+x^2)y\prime+2xy$ as $\frac{d}{dx}[(1+x^2)y]$. Equivalently the linear equation has integrating factor $1+x^2$.
- The curve passes through the origin, so its integration constant must be determined from $y(0)=0$, after integrating the differential equation.

Step 1: Recognise an exact product derivative The coefficient of y is the derivative of the coefficient of $y\prime$. Therefore

$$\frac{d}{dx}[(1+x^2)y]=(1+x^2)\frac{dy}{dx}+2xy=4x^2.$$

This avoids an unnecessary division; if the standard linear-equation method is used, its integrating factor is $e^{\int 2x/(1+x^2)dx}=1+x^2$, the same multiplier.

Step 2: Integrate and impose the initial point Integrating both sides with respect to x yields

$$(1+x^2)y=\frac{4}{3}x^3+C.$$

At the origin, the left side is $(1+0)0=0$, and the cubic term vanishes, giving $C=0$. Multiplying by 3 then gives $3(1+x^2)y=4x^3$. The factor $1+x^2$ is positive for real x, so this relation defines the required real curve everywhere.

Step 3: Verify and reject the other coefficients Dividing the resulting equation by 3 and differentiating recovers the exact original equation, and substitution of $x=y=0$ checks the initial point. The other alternatives instead have $(1+x^2)y$ equal to x^3, $(2/3)x^3$, or $3x^3$; their derivatives give $3x^2$, $2x^2$, or $9x^2$, respectively, instead of the required $4x^2$.

Step 4: Conclusion The required curve is $\boxed{3(1+x^2)y=4x^3}$. Hence, the correct option is (D).

Quick Tip to Solve: Before computing an integrating factor, check whether the coefficient of y is the derivative of the coefficient of $y\prime$; that exposes a product derivative immediately.

26. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$, and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 30°, then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ equals: [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{3\sqrt{3}}{2}$

(C) 3

(D) $\frac{3}{2}$

Correct Answer: (D) $\frac{3}{2}$

Solution:

Concept:

- The identity $|\vec{c} - \vec{a}|^2 = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a} \cdot \vec{c}$ converts the distance condition into a scalar equation.
- The magnitude of a cross product is $|\vec{u} \times \vec{v}| = |\vec{u}|\,|\vec{v}| \sin\theta$, where θ is the angle between the two vectors.

Step 1: Determine the magnitude of c Put $r = |\vec{c}|$, so that $r \geq 0$. From $\vec{a} = (2, 1, -2)$, $|\vec{a}|^2 = 4+1+4 = 9$, and the given dot product is $\vec{a} \cdot \vec{c} = r$. Squaring the distance condition gives

$$8 = r^2 + 9 - 2r, \qquad (r-1)^2 = 0.$$

Consequently $|\vec{c}| = 1$; no individual components of $\vec{c}$ are needed.

Step 2: Calculate the first cross product With $\vec{b} = (1, 1, 0)$, direct component calculation gives

$$\vec{a} \times \vec{b} = (1 \cdot 0 - (-2) \cdot 1,\ (-2) \cdot 1 - 2 \cdot 0,\ 2 \cdot 1 - 1 \cdot 1) = (2, -2, 1).$$

Its magnitude is $\sqrt{4+4+1} = 3$. As a check, its dot products with both $\vec{a}$ and $\vec{b}$ vanish, as required for a cross product.

Step 3: Use the specified angle The angle between this cross product and $\vec{c}$ is explicitly 30°, so

$$|(\vec{a} \times \vec{b}) \times \vec{c}| = 3 \cdot 1 \cdot \sin 30^\circ = \frac{3}{2}.$$

Using cosine would give the tempting $3\sqrt{3}/2$, but cosine belongs to a dot product. Omitting the first cross-product magnitude gives $1/2$, while omitting the sine gives 3.

Step 4: Conclusion The required magnitude is $\boxed{3/2}$. Hence, the correct option is (D).

Quick Tip to Solve: Find $|\vec{c}|$ by squaring the distance condition, then apply the cross-product magnitude formula; solving for all three components is unnecessary.

27. Let $A(-3,2)$ and $B(-2,1)$ be the vertices of a triangle ABC. If the centroid of this triangle lies on the line $3x+4y+2=0$, then the vertex C lies on the line: [4 marks]

(A) $4x+3y+5=0$

(B) $3x+4y+3=0$

(C) $4x+3y+3=0$

(D) $3x+4y+5=0$

Correct Answer: (B) $3x+4y+3=0$

Solution:

Concept:

- The centroid of a triangle is the coordinate-wise average of its three vertices: $G=((x_A+x_B+x_C)/3,(y_A+y_B+y_C)/3)$.
- A point lies on a given line exactly when its coordinates satisfy that line equation; substitute the centroid into its locus to find the locus of the unknown vertex.

Step 1: Express the centroid using the unknown vertex Let $C=(h,k)$. Adding the given vertex coordinates separately gives $x_A+x_B=-3-2=-5$ and $y_A+y_B=2+1=3$. Therefore

$$G=\left(\frac{h-5}{3},\frac{k+3}{3}\right).$$

Both coordinates are divided by the same factor 3 because there are three vertices; the centroid is not the midpoint of AB.

Step 2: Impose the line condition The specified line contains G, hence

$$3\left(\frac{h-5}{3}\right)+4\left(\frac{k+3}{3}\right)+2=0.$$

Multiplying the entire equation by 3, including its constant term, gives

$$3(h-5)+4(k+3)+6=0.$$

Collecting constants yields $-15+12+6=3$, so the vertex obeys $3h+4k+3=0$.

Step 3: Identify the vertex locus and check alternatives Replace the dummy vertex coordinates (h,k) by (x,y) to get $3x+4y+3=0$. The coefficients 3 and 4 retain their original order; exchanging them cannot result from a coordinate-wise average. The constant 5 in the last alternative would arise from an incorrect constant calculation. Substituting any point on the derived line into the displayed centroid formula verifies that its centroid lies on the original line, so the condition is sufficient as well as necessary for nondegenerate triangles.

Step 4: Conclusion The vertex C lies on $\boxed{3x+4y+3=0}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the unknown vertex as (h,k), average all three vertices, and multiply the centroid line equation by 3 before collecting constants.

28. Let ABC be a triangle with vertices at points $A(2,3,5)$, $B(-1,3,2)$ and $C(\lambda,5,\mu)$ in three dimensional space. If the median through A is equally inclined with the axes, then (λ,μ) is equal to: [4 marks]

(A) $(10,7)$

(B) $(7,5)$

(C) $(7,10)$

(D) $(5,7)$

Correct Answer: (C) $(7,10)$

Solution:

Concept:

- The median from A joins A to the midpoint of BC. The midpoint coordinates are obtained by averaging the corresponding endpoint coordinates.
- A line equally inclined with the three positive coordinate axes has equal direction cosines. Hence a direction vector along that inclination has equal components.

Step 1: Form the direction vector of the median Let M be the midpoint of BC. Then

$$M = \left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right).$$

Subtracting $A = (2,3,5)$ gives

$$\overrightarrow{AM} = \left(\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}\right).$$

In particular, the middle component is fixed at 1 and is positive, fixing the common sign when the direction cosines are equal.

Step 2: Equate the three direction ratios All components share the same positive normalising magnitude when converted into direction cosines. Equal inclinations therefore require

$$\frac{\lambda-5}{2} = 1 = \frac{\mu-8}{2}.$$

Thus $\lambda - 5 = 2$ and $\mu - 8 = 2$, giving $\lambda = 7$ and $\mu = 10$. This uses equality of angles, rather than merely equality of their absolute cosines.

Step 3: Verify the pair against the geometric condition For $(\lambda,\mu) = (7,10)$, $C = (7,5,10)$, so $M = (3,4,6)$ and $\overrightarrow{AM} = (1,1,1)$. Its direction cosines are all $1/\sqrt{3}$, confirming equal inclinations. The other options give median direction vectors $(5/2,1,-1/2)$, $(1,1,-3/2)$, or $(0,1,-1/2)$, none of which has equal direction cosines. The median must be found using the midpoint, not the direction of side AC.

Step 4: Conclusion The required pair is $\boxed{(\lambda,\mu) = (7,10)}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the midpoint of the opposite side before forming the median direction; its known middle component then determines the equal direction ratios.

29. The equation of a plane through the line of intersection of the planes $x + 2y = 3$, $y - 2z + 1 = 0$, and perpendicular to the first plane is: [4 marks]

(A) $2x - y - 10z = 9$

(B) $2x - y + 7z = 11$

(C) $2x - y + 10z = 11$

(D) $2x - y - 9z = 10$

Correct Answer: (C) $2x - y + 10z = 11$

Solution:

Concept:

- Every plane through the intersection of two nonparallel planes $P_1 = 0$ and $P_2 = 0$ can be written as $P_1 + \lambda P_2 = 0$, apart from the second plane itself.
- Two planes are perpendicular when their normal vectors have zero scalar product. A plane $ax + by + cz + d = 0$ has normal vector (a, b, c).

Step 1: Form the family of planes Take $P_1 = x + 2y - 3$ and $P_2 = y - 2z + 1$. Every point on the common line satisfies both equations, so it satisfies their linear combination. Thus write

$$x + 2y - 3 + \lambda(y - 2z + 1) = 0.$$

Collecting coefficients gives

$$x + (2 + \lambda)y - 2\lambda z + (\lambda - 3) = 0.$$

The normal to this candidate plane is $(1, 2 + \lambda, -2\lambda)$.

Step 2: Impose perpendicularity The first plane has normal $(1, 2, 0)$. Set the dot product of the normals equal to zero:

$$1 + 2(2 + \lambda) = 0.$$

Hence $5 + 2\lambda = 0$, or $\lambda = -5/2$. Substitute this value in the family and multiply by two:

$$2x - y + 10z - 11 = 0.$$

The second plane alone is not another solution, because its normal $(0, 1, -2)$ has nonzero dot product with $(1, 2, 0)$.

Step 3: Check the printed alternatives The resulting normal $(2, -1, 10)$ has dot product $2 - 2 = 0$ with the first normal, and the equation was constructed to contain the intersection line. Options (A), (B) and (D) have different coefficients of z or constants; despite their normals being perpendicular to the first normal, they fail the common-line condition. Both requirements are needed.

Step 4: Conclusion The required plane is $\boxed{2x - y + 10z = 11}$. Hence, the correct option is (C).

Quick Tip to Solve: Build the family first, then use the normal-vector dot product; perpendicularity alone does not establish that a plane contains the required line.

30. If the events A and B are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$ and $P(B) = \frac{1-x}{4}$, then the set of possible values of x lies in the interval: [4 marks]

(A) $[0, 1]$

(B) $\left[\frac{1}{3}, \frac{2}{3}\right]$

(C) $\left[-\frac{1}{3}, \frac{5}{9}\right]$

(D) $\left[-\frac{7}{9}, \frac{4}{9}\right]$

Correct Answer: (C) $\left[-\frac{1}{3}, \frac{5}{9}\right]$

Solution:

Concept:

- For mutually exclusive events, $P(A \cup B) = P(A) + P(B)$, since $P(A \cap B) = 0$.
- Every probability lies between zero and one. Therefore nonnegative individual probabilities and a sum no greater than one constrain the parameter.

Step 1: Use the individual probabilities Nonnegativity gives

$$\frac{3x+1}{3} \geq 0 \quad \Rightarrow \quad x \geq -\frac{1}{3},$$

$$\frac{1-x}{4} \geq 0 \quad \Rightarrow \quad x \leq 1.$$

The upper bounds $P(A) \leq 1$ and $P(B) \leq 1$ separately give $x \leq 2/3$ and $x \geq -3$. We must also use mutual exclusivity; these separate bounds do not finish the problem.

Step 2: Bound the probability of the union Since the events are disjoint,

$$P(A \cup B) = \frac{3x+1}{3} + \frac{1-x}{4} = \frac{9x+7}{12}.$$

A union cannot have probability greater than one, so

$$\frac{9x+7}{12} \leq 1 \quad \Rightarrow \quad x \leq \frac{5}{9}.$$

Combining all bounds leaves $-1/3 \leq x \leq 5/9$. Nonnegative summands make the lower bound on the union automatic.

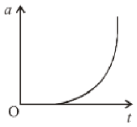
Step 3: Verify the endpoints and exclude the other intervals At $x = -1/3$, the two probabilities are 0 and $1/3$; at $x = 5/9$, they are $8/9$ and $1/9$. These are permitted, so both endpoints are included. Every intermediate value gives two nonnegative probabilities with sum at most one. Option (A) permits a sum greater than one near its upper endpoint; (B) omits allowable negative values and extends too far; (D) permits negative $P(A)$.

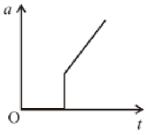
Step 4: Conclusion The possible values form $\boxed{[-1/3,\ 5/9]}$. Hence, the correct option is (C).

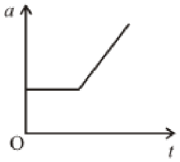
Quick Tip to Solve: For mutually exclusive events, impose the bound on the sum as well as on each probability; the union often supplies the decisive endpoint.

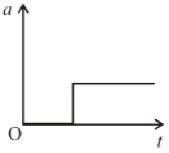
Physics (120 marks)

31. A block is placed on a rough horizontal plane. A time dependent horizontal force F = kt acts on the block, where k is a positive constant. The acceleration - time graph of the block is : [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- Static friction adjusts to balance an applied force up to its limiting value $f_{s,\max} = \mu_s mg$, so a block initially at rest has zero acceleration before slipping.
- Once the block moves, kinetic friction is approximately constant, $f_k = \mu_k mg$, with $\mu_k < \mu_s$ in the standard rough-surface model. Newton's law then gives $a = (kt - f_k)/m$.

Step 1: Identify the initial interval at rest Let m be the block mass. While $kt \le \mu_s mg$, static friction equals kt in magnitude and acts oppositely. The horizontal resultant is zero, hence $a = 0$. The limiting instant is

$$t_s = \frac{\mu_s mg}{k}.$$

Thus the graph must initially coincide with the time axis, rather than have a positive intercept or increase immediately.

Step 2: Find the acceleration after slipping begins Just after the applied force exceeds limiting static friction, the resisting force becomes $\mu_k mg$. Consequently

$$a(t) = \frac{k}{m}t - \mu_k g, \qquad t > t_s.$$

The graph is a straight line with positive slope k/m. At the transition its right-hand value is

$$a(t_s^+) = (\mu_s - \mu_k)g > 0.$$

The drop from limiting static to kinetic friction produces the upward jump shown in the second graph.

Step 3: Compare the four graphs Option (A) curves upward and would require a changing acceleration slope, inconsistent with a linearly increasing applied force and constant kinetic friction. Option (C) starts with nonzero acceleration while the block should remain at rest. Option (D) becomes horizontal

after the jump, although $F = kt$ continues increasing. Only option (B) has the initial zero interval followed by a jump and a rising straight segment.

Step 4: Conclusion The acceleration is initially zero, then jumps upward and increases linearly with time. Hence, the correct option is (B).

Quick Tip to Solve: Separate sticking from sliding: static friction balances the force, whereas constant kinetic friction makes acceleration follow the applied force linearly.

32. The maximum range of a bullet fired from a toy pistol mounted on a car at rest is $R_0 = 40\,\mathrm{m}$. What will be the acute angle of inclination of the pistol for maximum range when the car is moving in the direction of firing with uniform velocity $v = 20\,\mathrm{m\,s^{-1}}$ on a horizontal surface? $(g = 10\,\mathrm{m\,s^{-2}})$ [4 marks]

(A) 30°

(B) 60°

(C) 75°

(D) 45°

Correct Answer: (B) 60°

Solution:

Concept:

- For a projectile launched and received at the same level, maximum range with a stationary launcher is $R_0 = u^2/g$, where u is its speed relative to the launcher.
- Horizontal velocities add under Galilean transformation. A car moving along the firing direction gives ground-frame components $u_x = v + u\cos\theta$ and $u_y = u\sin\theta$.

Step 1: Recover the muzzle speed The car is initially at rest in the range measurement, so

$$u = \sqrt{gR_0} = \sqrt{10 \times 40} = 20\,\mathrm{m\,s^{-1}}.$$

This equals the given speed v of the moving car. The vertical component is unchanged by the car's horizontal motion, while the horizontal component gains v. The relevant range is measured along the ground.

Step 2: Express the ground range The time to return to the launch level is $T = 2u\sin\theta/g$. Multiplying by the constant ground-frame horizontal velocity gives

$$R(\theta) = \frac{2u}{g}\sin\theta\,(v + u\cos\theta).$$

For $v = u$, this is proportional to $\sin\theta(1 + \cos\theta)$. This differs from the stationary expression proportional to $\sin 2\theta$.

Step 3: Maximize and check the alternatives Differentiation yields

$$\frac{dR}{d\theta} = \frac{2u}{g}\left(v\cos\theta + u\cos 2\theta\right).$$

With $v = u$, put $c = \cos\theta$ to obtain

$$2c^2 + c - 1 = (2c - 1)(c + 1) = 0.$$

For an acute angle $0 < c < 1$, only $c = 1/2$ is admissible, giving $\theta = 60^\circ$. The derivative is positive near zero and negative near 90°, so this point is the maximum. The other listed acute angles do not make the derivative vanish.

Step 4: Conclusion The pistol should be inclined at $\boxed{60^\circ}$ for maximum ground range. Hence, the correct option is (B).

Quick Tip to Solve: Add the car's horizontal speed before maximizing range; the familiar stationary-launch angle does not apply to the moving launcher.

33. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be most likely proportional to [4 marks]

(A) v^4

(B) v^2

(C) v

(D) v^3

Correct Answer: (D) v^3

Solution:

Concept:

- Kinetic energy of a moving mass is $K = \frac{1}{2}mv^2$. Wind speed therefore determines the energy per unit mass through a factor v^2.
- Power is energy transferred per unit time. A blade assembly intercepts an air mass per second $\dot{m} = \rho A v$, where ρ is air density and A is its effective swept area.

Step 1: Determine the mass intercepted per second In a short interval Δt, the wind moves a distance $v\Delta t$. The volume crossing the fixed swept area A is therefore $Av\Delta t$, and its mass is $\rho Av\Delta t$. Hence

$$\frac{\Delta m}{\Delta t} = \rho A v.$$

Faster wind delivers more air per second; ignoring this factor would incorrectly lead to a square-law answer.

Step 2: Convert kinetic energy flow into electrical power The kinetic energy of this intercepted air is

$$\Delta K = \frac{1}{2}(\rho A v \Delta t)v^2.$$

Divide by Δt to get the available wind power:

$$P_{\text{wind}} = \frac{1}{2}\rho A v^3.$$

If a fixed fraction η becomes electrical energy, then

$$P_{\text{electric}} = \frac{1}{2}\eta\rho A v^3.$$

The stated fixed conversion fraction, blade area and air density do not change the speed exponent.

Step 3: Compare the proposed speed dependences The factor v^2 from kinetic energy per unit mass combines with the factor v from mass flow. Thus the power scales as v^3, so doubling speed multiplies power by eight in this model. Option (B) omits the mass-flow factor; option (C) omits the speed dependence of energy per unit mass; option (A) includes an extra speed factor not present in either quantity.

Step 4: Conclusion The electrical output is proportional to $\boxed{v^3}$. Hence, the correct option is (D).

Quick Tip to Solve: For wind-power scaling, multiply energy per kilogram by kilograms passing per second: the two speed factors are squared and linear, respectively.

---

34. A ring of mass M and radius R is rotating about its axis with angular velocity ω. Two identical bodies each of mass m are now gently attached at the two ends of a diameter of the ring. Because of this, the kinetic energy loss will be: [4 marks]

(A) $\frac{m(M+2m)}{M}\omega^2R^2$

(B) $\frac{Mm}{M+m}\omega^2R^2$

(C) $\frac{Mm}{M+2m}\omega^2R^2$

(D) $\frac{(M+m)M}{M+2m}\omega^2R^2$

Correct Answer: (C) $\frac{Mm}{M+2m}\omega^2R^2$

Solution:

Concept:

- With no external torque about the rotation axis, angular momentum is conserved, even though attachment of initially nonrotating masses is an inelastic process.
- The moment of inertia of a thin ring about its central axis is MR^2; a point mass m at radius R contributes mR^2. Rotational kinetic energy is $K = \frac{1}{2}I\omega^2$.

Step 1: Write the initial and final moments of inertia Initially only the ring rotates, so

$$I_i = MR^2, \qquad L_i = MR^2\omega.$$

Each attached body lies at distance R from the axis. Consequently

$$I_f = MR^2 + 2mR^2 = (M + 2m)R^2.$$

The fact that the bodies are at opposite ends does not cancel their moments of inertia; both positive contributions add.

Step 2: Find the reduced angular velocity Conserving angular momentum about the axis gives

$$MR^2\omega = (M + 2m)R^2\omega_f,$$

$$\omega_f = \frac{M}{M + 2m}\omega.$$

The ring slows because its moment of inertia increases. Kinetic energy is not conserved during attachment, so its initial and final values must be calculated separately rather than equated.

Step 3: Subtract final energy from initial energy The two energies are

$$K_i = \frac{1}{2}MR^2\omega^2, \qquad K_f = \frac{M^2R^2\omega^2}{2(M + 2m)}.$$

Therefore

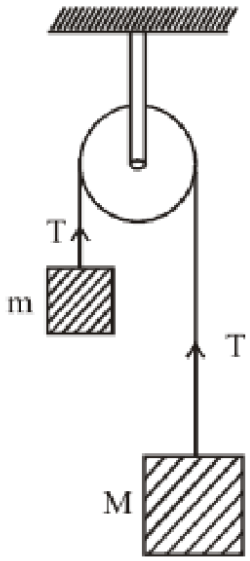
$$\Delta K = \frac{1}{2}MR^2\omega^2\left(1 - \frac{M}{M + 2m}\right) = \frac{Mm}{M + 2m}\omega^2R^2.$$

This is positive and tends to zero as $m \to 0$. Option (A) grows without bound for very large attached mass, while a loss cannot exceed the initial energy. Options (B) and (D) also disagree with the conserved-angular-momentum result; (D) does not even vanish when $m \to 0$.

Step 4: Conclusion The kinetic energy loss is $\boxed{\frac{Mm}{M + 2m}\omega^2R^2}$. Hence, the correct option is (C).

Quick Tip to Solve: When masses attach to a freely rotating body, conserve angular momentum first and then calculate the energy loss; do not conserve rotational energy.

35. Two blocks of masses m and M are connected by means of a metal wire of cross-sectional area A passing over a frictionless fixed pulley as shown in the figure. The system is then released. If $M = 2m$, then the stress produced in the wire is: [4 marks]



(A) $\frac{2mg}{3A}$

(B) $\frac{4mg}{3A}$

(C) $\frac{mg}{A}$

(D) $\frac{3mg}{4A}$

Correct Answer: (B) $\frac{4mg}{3A}$

Solution:

Concept:

- Tensile stress is the tension divided by the wire cross-sectional area: $\sigma = T/A$. The tension must be obtained from the accelerating system.
- For an ideal light wire over a frictionless fixed pulley, tension has the same magnitude on both sides and the two blocks have equal acceleration magnitudes. Apply $F = ma$ separately to each mass.

Step 1: Apply Newton's law to the two blocks The heavier block has mass $M = 2m$, so it descends while the lighter block rises. Taking each direction of motion as positive,

$$T - mg = ma,$$

$$2mg - T = 2ma.$$

The weights do not balance because the blocks accelerate; neither mg nor $2mg$ can simply be used as the tension.

Step 2: Find the acceleration and tension Adding the equations cancels the internal tension:

$$mg = 3ma \quad \Rightarrow \quad a = \frac{g}{3}.$$

Substituting into the lighter block equation gives

$$T = mg + ma = mg + \frac{mg}{3} = \frac{4mg}{3}.$$

As a check, the heavier block equation also gives $T = 2mg - 2m(g/3) = 4mg/3$. The tension correctly lies between the two weights.

Step 3: Calculate the stress and compare the options The wire has cross-sectional area A, so

$$\sigma = \frac{T}{A} = \frac{4mg}{3A}.$$

The units are force per area, as required for stress. Option (A) is too small and would not even support the lighter mass's upward acceleration. Option (C) assumes equilibrium of the lighter mass. Option (D) is also below mg/A, inconsistent with $T > mg$. Only option (B) agrees with both block equations.

Step 4: Conclusion The stress in the wire is $\boxed{4mg/(3A)}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the common tension from the accelerating Atwood system, then divide by wire area; stress is not the difference of the two weights divided by area.

36. In an experiment, a small steel ball falls through a liquid at a constant speed of 10 cm/s. If the steel ball is pulled upward with a force equal to twice its effective weight, how fast will it move upward ? [4 marks]

(A) 5 cm/s

(B) Zero

(C) 10 cm/s

(D) 20 cm/s

Correct Answer: (C) 10 cm/s

Solution:

Concept:

- The effective weight is the gravitational force minus buoyancy: $W_{\text{eff}} = mg - F_b$, directed downward.
- For a small sphere moving slowly through a viscous liquid, Stokes drag is proportional to speed: $F_d = 6\pi\eta r v$. The drag always opposes the motion.

Step 1: Use the original terminal fall The given constant downward speed means that the acceleration is zero. Thus the upward viscous resistance balances the effective weight. With $v_0 = 10\,\text{cm/s}$,

$$6\pi\eta r v_0 = W_{\text{eff}}.$$

Buoyancy is already included in the effective weight; it must not be subtracted a second time.

Step 2: Balance forces during upward motion The applied upward force is $2W_{\text{eff}}$. During upward motion the effective weight and the viscous resistance both act downward. At the new steady speed u,

$$2W_{\text{eff}} - W_{\text{eff}} - 6\pi\eta r u = 0.$$

Hence $6\pi\eta r u = W_{\text{eff}} = 6\pi\eta r v_0$, giving $u = v_0$. The twofold applied force does not produce twice the speed because one effective weight is needed simply to oppose gravity after buoyancy. Zero speed cannot balance this net upward force; half speed gives insufficient drag and twice the speed gives excessive drag.

Step 3: Conclusion The steady upward speed is $10\,\text{cm/s}$. Hence, the correct option is (C).

Quick Tip to Solve: When reversing terminal motion, reverse the drag direction too. Subtract the effective weight from the upward pull before comparing drag magnitudes.

---

37. A mass of 50 g of water in a closed vessel, with surroundings at a constant temperature takes 2 minutes to cool from 30°C to 25°C. A mass of 100 g of another liquid in an identical vessel with identical surroundings takes the same time to cool from 30°C to 25°C. The specific heat of the liquid is: (The water equivalent of the vessel is 30 g.) [4 marks]

(A) $2.0\,\text{kcal/kg}$

(B) $7\,\text{kcal/kg}$

(C) $3\,\text{kcal/kg}$

(D) $0.5\,\text{kcal/kg}$

Correct Answer: (D) $0.5\,\text{kcal/kg}$

Solution:

Concept:

- Under identical surface and surrounding conditions the cooling power is the same function of temperature for both vessels.
- The total heat capacity is the sum of the liquid and vessel capacities: $C = mc + C_v$. A water equivalent of 30 g means $C_v = 30c_w$.

Step 1: Compare the cooling histories Newton cooling can be written $C\,dT/dt = -hA(T - T_s)$. Integrating between the same two temperatures gives

$$t = \frac{C}{hA}\ln\left(\frac{30 - T_s}{25 - T_s}\right).$$

Both vessels are identical and have the same surroundings, initial and final temperatures, and cooling time. Every factor except C is therefore identical, so their total heat capacities must be equal. There is no need to know the actual surrounding temperature.

Step 2: Equate capacities and solve Using grams consistently, the water vessel has heat capacity

$50c_w + 30c_w = 80c_w$. The other vessel has heat capacity $100c + 30c_w$. Thus

$$100c + 30c_w = 80c_w, \qquad c = \frac{50}{100}c_w = 0.5c_w.$$

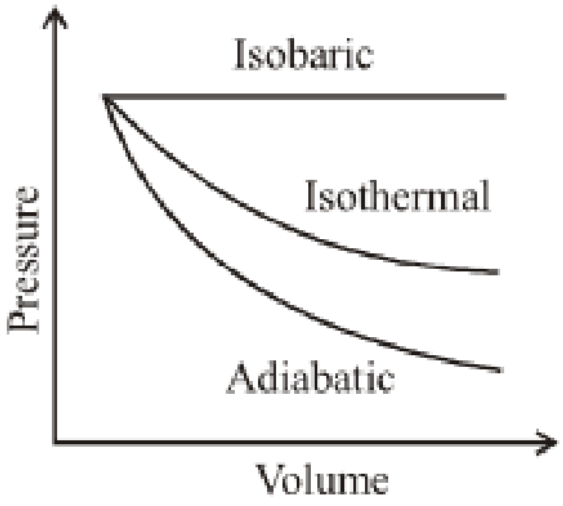
Since water has specific heat $1\,\text{kcal}\,\text{kg}^{-1}\,\text{K}^{-1}$, the liquid has half this value. The vessel contribution cancels because the two vessels are identical; treating 80 g as only the liquid capacity would incorrectly include the same vessel on just one side. The larger values in the other three options would give a longer cooling time.

Step 3: Conclusion The specific heat is $0.5\,\text{kcal}\,\text{kg}^{-1}\,\text{K}^{-1}$, corresponding to the printed value $0.5\,\text{kcal/kg}$. Hence, the correct option is (D).

Quick Tip to Solve: For identical vessels cooling through the same temperature interval in the same time, equate total heat capacities; the common vessel capacity then cancels.

38. A sample of gas expands from V_1 to V_2. In which of the following, the work done will be greatest?

[4 marks]



(A) Same in all processes

(B) Isobaric process

(C) Isothermal process

(D) Adiabatic process

Correct Answer: (B) Isobaric process

Solution:

Concept:

- The work done by a gas in a quasistatic expansion is $W = \int_{V_1}^{V_2} P\,dV$, the area below its pressure-volume curve.
- The plotted processes begin at the same initial state. Over the expansion interval the isobaric curve lies above the isothermal curve, which lies above the adiabatic curve.

Step 1: Compare pressure at each intermediate volume For the horizontal isobaric path, pressure stays at its initial value P_1. In the isothermal expansion of an ideal gas, $P = P_1V_1/V$, which decreases as volume increases. For an adiabatic expansion, $P = P_1(V_1/V)^\gamma$ with $\gamma > 1$, so pressure falls faster. Accordingly, for $V_1 < V \leq V_2$,

$$P_{\text{isobaric}} > P_{\text{isothermal}} > P_{\text{adiabatic}}.$$

This is also directly visible in the supplied graph.

Step 2: Compare the areas under the curves All three integrals cover the same positive volume interval. Integrating the pressure ordering therefore preserves it:

$$W_{\text{isobaric}} > W_{\text{isothermal}} > W_{\text{adiabatic}}.$$

The isobaric work is the full rectangle $P_1(V_2 - V_1)$; the other paths exclude areas below that horizontal line. The work cannot be the same in all processes, because work depends on the complete path in the pressure-volume plane rather than only the two volume values. The isothermal and adiabatic options have strictly smaller areas than the horizontal path shown.

Step 3: Conclusion The greatest work is done in the isobaric process. Hence, the correct option is (B).

Quick Tip to Solve: Compare areas on a pressure-volume graph: the uppermost expansion path between the same volume limits gives the largest positive work.

39. In the isothermal expansion of 10 g of gas from volume V to $2V$ the work done by the gas is 575 J. What is the root mean square speed of the molecules of the gas at that temperature? [4 marks]

(A) 398 m/s

(B) 520 m/s

(C) 499 m/s

(D) 532 m/s

Correct Answer: (C) 499 m/s

Solution:

Concept:

- For an ideal gas undergoing isothermal expansion, $W = nRT\ln(V_f/V_i)$. The given expansion has $V_f/V_i = 2$.
- Kinetic theory gives $v_{\text{rms}}^2 = 3RT/M_{\text{mol}}$. Since the sample mass is $m = nM_{\text{mol}}$, this becomes $nRT = mv_{\text{rms}}^2/3$.

Step 1: Eliminate the gas identity and temperature The gas species and its molar mass are not supplied, so it is useful to eliminate them. Substitute the kinetic-theory expression for nRT into the work equation:

$$W = \frac{mv_{\text{rms}}^2}{3}\ln 2.$$

Solving for the speed gives

$$v_{\text{rms}} = \sqrt{\frac{3W}{m \ln 2}}.$$

This uses the total mass of the sample, not the mass of a single molecule. The isothermal condition ensures that the molecular rms speed remains constant throughout the expansion.

Step 2: Insert the numerical data and compare Convert the sample mass to SI units: $m = 10\,\text{g} = 0.010\,\text{kg}$. Using $W = 575\,\text{J}$ and $\ln 2 \simeq 0.693$,

$$v_{\text{rms}} = \sqrt{\frac{3(575)}{0.010(0.693)}} \simeq \sqrt{2.49 \times 10^5} \simeq 499\,\text{m/s}.$$

The square-root argument has units $\text{J/kg} = \text{m}^2/\text{s}^2$, as required. The alternatives 398, 520 and 532 m/s do not reproduce the given work when inserted into $mv^2 \ln 2/3$. In particular, retaining grams instead of kilograms would give an incorrect speed scale.

Step 3: Conclusion The root mean square molecular speed is approximately $499\,\text{m/s}$. Hence, the correct option is (C).

Quick Tip to Solve: For isothermal work questions with sample mass but no gas identity, replace nRT by $mv_{\text{rms}}^2/3$ before substituting numbers.

40. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. When the cylinder is given a downward push and released, it starts oscillating vertically with a small amplitude. The time period T of the oscillations of the cylinder will be: [4 marks]

(A) Smaller than $2\pi \left[\frac{M}{k + A\sigma g}\right]^{1/2}$

(B) $2\pi\sqrt{\frac{M}{k}}$

(C) Larger than $2\pi \left[\frac{M}{k + A\sigma g}\right]^{1/2}$

(D) $2\pi \left[\frac{M}{k + A\sigma g}\right]^{1/2}$

Correct Answer: (D) $2\pi \left[\frac{M}{k + A\sigma g}\right]^{1/2}$

Solution:

Concept:

- For a vertical displacement x, a spring changes its restoring force by kx, where k is its force constant.
- A cylinder displaced downward by x displaces an additional volume Ax of liquid. Its buoyant force increases upward by $\sigma g Ax$.

Step 1: Measure displacement from equilibrium Choose downward displacement x as positive and let the equilibrium spring extension be e_0. At equilibrium the spring force and buoyancy balance the weight:

$$ke_0 + \sigma g A\frac{L}{2} = Mg.$$

During a small downward displacement, the extension becomes $e_0 + x$ and the submerged length becomes $L/2 + x$. Small amplitude is essential: the bottom remains submerged and the top remains above the surface, so the submerged volume changes linearly with x.

Step 2: Find the effective stiffness and period The net downward force after displacement is

$$F = Mg - k(e_0 + x) - \sigma g A(L/2 + x).$$

Subtracting the equilibrium balance cancels the constant terms and leaves

$$F = -(k + A\sigma g)x.$$

Hence $M\ddot{x} = -(k + A\sigma g)x$, an SHM equation with $\omega^2 = (k + A\sigma g)/M$. The additional buoyant restoring force acts with the spring, so their stiffnesses add. The spring-only expression in option (B) neglects buoyancy; options (A) and (C) incorrectly state inequalities relative to the exact ideal-model result.

Step 3: Conclusion The time period is $T = 2\pi\sqrt{\dfrac{M}{k + A\sigma g}}$. The printed ideal-model equation gives equality, so the sheet key selecting the smaller-than alternative is inconsistent with the stated forces. Hence, the correct option is (D).

Quick Tip to Solve: For a floating or partially immersed body, differentiate buoyancy with respect to displacement. Add this restoring stiffness to the spring constant.

41. In a transverse wave the distance between a crest and neighbouring trough at the same instant is 4.0 cm and the distance between a crest and trough at the same place is 1.0 cm. The next crest appears at the same place after a time interval of 0.4 s. The maximum speed of the vibrating particles in the medium is [4 marks]

(A) $\frac{3\pi}{2}$ cm/s

(B) $\frac{5\pi}{2}$ cm/s

(C) $\frac{\pi}{2}$ cm/s

(D) 2π cm/s

Correct Answer: (B) $\frac{5\pi}{2}$ cm/s

Solution:

Concept:

- The transverse displacement of a medium particle is sinusoidal: $y = A\sin(\omega t + \phi)$, so its maximum speed is $v_{\max} = A\omega$.
- The crest-to-trough displacement at one position is $2A$. The time between successive crests at that position is the period T, with $\omega = 2\pi/T$.

Step 1: Identify amplitude, wavelength and period The spatial separation of a crest and its neighbouring trough at the same instant is half a wavelength. Thus $\lambda/2 = 4.0\,\text{cm}$, giving $\lambda = 8.0\,\text{cm}$. This distance concerns the direction of wave propagation. By contrast, the crest-to-trough distance at a fixed place describes the full transverse excursion: $2A = 1.0\,\text{cm}$, so $A = 0.5\,\text{cm}$. The next crest arrives after $0.4\,\text{s}$, hence $T = 0.4\,\text{s}$.

Step 2: Calculate the particle speed amplitude Differentiating the particle displacement gives $v_y = A\omega\cos(\omega t + \phi)$. Its largest magnitude is therefore

$$v_{\max} = \frac{2\pi A}{T} = \frac{2\pi(0.5)}{0.4} = \frac{5\pi}{2}\,\text{cm/s}.$$

The wavelength would be used to find the propagation speed λ/T, not this transverse particle speed. Using 1.0 cm as the amplitude would double the result, while using the 4.0 cm separation would confuse a longitudinal position interval with transverse motion. The other listed multiples of π do not follow from the measured amplitude and period.

Step 3: Conclusion The maximum speed of a vibrating particle is $\frac{5\pi}{2}$ cm/s. Hence, the correct option is (B).

Quick Tip to Solve: Separate the two distances: crest-to-trough along the wave gives half a wavelength, while the vertical excursion gives twice the amplitude.

42. This question has Statement-1 and Statement-2. Of the four choices given after the Statements, choose the one that best describes the two Statements. Statement 1: No work is required to be done to move a test charge between any two points on an equipotential surface. Statement 2 : Electric lines of force at the equipotential surfaces are mutually perpendicular to each other [4 marks]

(A) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.

(B) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

(C) Statement 1 is true, Statement 2 is false.

(D) Statement 1 is false, Statement 2 is true.

Correct Answer: (C) Statement 1 is true, Statement 2 is false.

Solution:

Concept:

- An equipotential surface has the same electric potential at every point; the work associated with moving a test charge is $W = q\Delta V$.
- The electric field is normal to an equipotential surface. This relates a field line to the surface; it does not make different field lines mutually perpendicular.

Step 1: Test Statement 1 using potential difference Let the initial and final positions lie on one equipotential surface. Their potentials satisfy $V_i = V_f$, so

$$\Delta V = V_f - V_i = 0, \qquad W_{\text{ext}} = q(V_f - V_i) = 0.$$

The work done by the electrostatic field is its negative and is also zero. Thus no work against the electrostatic force is required for the stated displacement, and Statement 1 is true. This conclusion is independent of the shape of the surface and of the particular two points chosen.

Step 2: Read Statement 2 literally The statement says that the electric lines of force at equipotential surfaces are mutually perpendicular to each other. That is false: the correct perpendicularity is between the electric field and the equipotential surface. For example, in a uniform electric field, all field lines are parallel to one another while the equipotential planes are perpendicular to them. Field lines cannot cross at right angles, since the field at an ordinary point has a unique direction. Therefore Statement 2 does not state a true explanation; it is itself false. Options (A) and (B) both incorrectly call it true, while option (D) also incorrectly calls Statement 1 false.

Step 3: Conclusion Statement 1 is true and Statement 2 is false. Hence, the correct option is (C).

Quick Tip to Solve: Check what the word perpendicular connects: electric field lines are perpendicular to equipotential surfaces, rather than perpendicular to one another.

43. The surface charge density of a thin charged disc of radius R is σ. The value of the electric field at the centre of the disc is $\frac{\sigma}{2\varepsilon_0}$. With respect to the field at the centre, the electric field along the axis at a distance R from the centre of the disc: [4 marks]

(1) reduces by 70.7%

(2) reduces by 29.3%

(3) reduces by 9.7%

(4) reduces by 14.6%

Correct Answer: (1) reduces by 70.7%

Solution:

Concept:

- On the axis of a uniformly charged thin disc of radius R, the field at distance $z > 0$ is $E(z) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$.
- The reference field given in the question is the limiting field just outside the centre: $E_0 = \sigma/(2\varepsilon_0)$. A percentage reduction is measured relative to this initial field.

Step 1: Substitute the specified axial distance Use the radius both as the size of the disc and as the observation distance. Setting $z = R$ in the axial-field expression gives

$$E(R) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{R}{\sqrt{R^2 + R^2}}\right) = E_0\left(1 - \frac{1}{\sqrt{2}}\right).$$

Thus the field remains directed along the axis, but its magnitude is only about $0.2929E_0$. The radius cancels because the observation distance is specified as a fixed multiple of that radius.

Step 2: Calculate the reduction rather than the remaining fraction The decrease in field is the initial field minus the final field:

$$\Delta E = E_0 - E(R) = \frac{E_0}{\sqrt{2}}.$$

Hence the percentage decrease is

$$100\frac{\Delta E}{E_0} = \frac{100}{\sqrt{2}}\% = 70.71\%.$$

The tempting value 29.3% is the fraction of field left, not the percentage lost. Neither 9.7% nor 14.6% equals the decrease obtained from the disc formula. All comparisons use the supplied central field, so no separate value of the charge density is needed.

Step 3: Conclusion The field is reduced by approximately 70.7%. Hence, the correct option is (1).

Quick Tip to Solve: First find E/E_0; for a percentage decrease use $100(1 - E/E_0)$, rather than reporting the remaining percentage.

44. The gravitational field in a region is given by: $\vec{E} = (5\,\mathrm{N/kg})\hat{\imath} + (12\,\mathrm{N/kg})\hat{\jmath}$. If the potential at the origin is taken to be zero, then the ratio of the potential at the points $(12\,\mathrm{m}, 0)$ and $(0, 5\,\mathrm{m})$ is: [4 marks]

(1) Zero

(2) 1

(3) $\frac{144}{25}$

(4) $\frac{25}{144}$

Correct Answer: (2) 1

Solution:

Concept:

- Gravitational potential is potential energy per unit mass. Its relation to the gravitational field is $\vec{E} = -\nabla V$, or $dV = -\vec{E} \cdot d\vec{r}$.
- For a spatially constant field with components E_x, E_y, the potential difference from the origin is $V(x, y) - V(0, 0) = -E_x x - E_y y$. The sign is negative along the field.

Step 1: Integrate the uniform field The given components are constants, so the line integral is independent of the path. With the prescribed reference $V(0, 0) = 0$,

$$V(x, y) = -\int_{(0,0)}^{(x,y)} (5\,dx + 12\,dy) = -5x - 12y.$$

Here the coordinates are in metres and the potential is in $\mathrm{J/kg}$. The magnitude $13\,\mathrm{N/kg}$ of the field is not to be multiplied by an arbitrary coordinate: it is the dot product with displacement that determines the potential change.

Step 2: Evaluate the two potentials and compare them At the first point only the displacement parallel to the x-axis contributes, giving

$$V(12, 0) = -5(12) = -60\,\mathrm{J/kg}.$$

At the second point only the displacement parallel to the y-axis contributes, giving

$$V(0, 5) = -12(5) = -60\,\mathrm{J/kg}.$$

Therefore their ratio, in the stated order, is $(-60)/(-60) = 1$. The ratio is not zero because neither potential is zero. The squared-coordinate ratios $144/25$ and $25/144$ would ignore the unequal field components and have no basis in the potential integral.

Step 3: Conclusion Both specified points have potential $-60\,\mathrm{J/kg}$, so their potential ratio is 1. Hence, the correct option is (2).

Quick Tip to Solve: In a constant field compare $E_x x + E_y y$ at the points; equal dot products identify points on the same equipotential.

45. A parallel plate capacitor having a separation between the plates d, plate area A and material with dielectric constant K has capacitance C_0. Now one-third of the material is replaced by another material with dielectric constant $2K$, so that effectively there are two capacitors, one with area $\frac{1}{3}A$, dielectric constant $2K$ and another with area $\frac{2}{3}A$, dielectric constant K. If the capacitance of this new capacitor is C, then C/C_0 is: [4 marks]

(1) 1
(2) $\frac{4}{3}$
(3) $\frac{2}{3}$
(4) $\frac{1}{3}$

Correct Answer: (2) $\frac{4}{3}$

Solution:

Concept:

- A parallel-plate capacitor completely filled with dielectric constant K has capacitance $C = K\varepsilon_0 A/d$, neglecting edge effects.
- Side-by-side dielectric regions bridge the same pair of plates, so their potential difference is common. Their capacitances therefore add in parallel, rather than through the reciprocal rule for series capacitors.

Step 1: Write the original capacitance and identify the combination Before replacement the entire plate area has dielectric constant K, so

$$C_0 = \frac{K\varepsilon_0 A}{d}.$$

The replacement divides the plate area, not the separation. One region covers one-third of the area and the other covers two-thirds, while both span the full distance d. Since each region is connected to the same upper and lower conducting plates, both experience exactly the same voltage. The total charge is the sum of the charges stored over the two regions.

Step 2: Add the capacitances of the two regions The first region has twice the dielectric constant and one-third the area:

$$C_1 = \frac{2K\varepsilon_0(A/3)}{d} = \frac{2}{3}C_0.$$

The second region retains dielectric constant K over the remaining area:

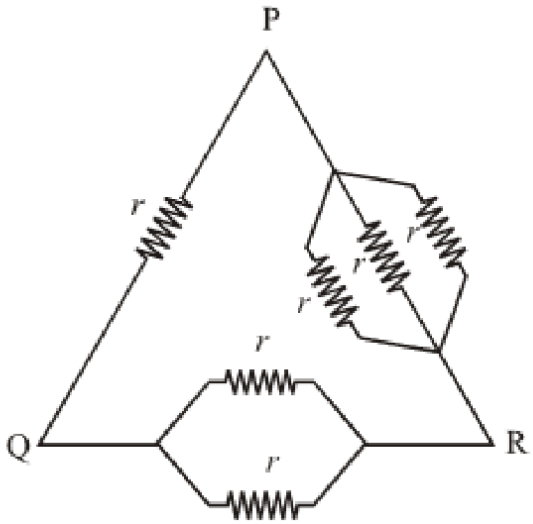
$$C_2 = \frac{K\varepsilon_0(2A/3)}{d} = \frac{2}{3}C_0.$$

Thus $C = C_1 + C_2 = 4C_0/3$. The unchanged value 1 is excluded because increasing the dielectric constant in a nonzero region increases capacitance. The value $2/3$ describes either region alone; $1/3$ neither adds both regions nor includes the doubled dielectric constant.

Step 3: Conclusion The ratio is $\boxed{C/C_0 = 4/3}$. Hence, the correct option is (2).

Quick Tip to Solve: When dielectric regions divide plate area, add K_iA_i; when they divide the gap thickness, use the series rule instead.

46. Six equal resistances are connected between points P, Q and R as shown in figure. Then net resistance will be maximum between: [4 marks]



(1) P and R

(2) P and Q

(3) Q and R

(4) Any two points

Correct Answer: (2) P and Q

Solution:

Concept:

- Resistors joining the same two nodes are in parallel. For n identical resistors of resistance r, their parallel equivalent is r/n.
- In a triangle with one unconnected external terminal, the equivalent between the other two terminals is one side in parallel with the series sum of the remaining two sides. Ideal connecting wires form single equipotential nodes.

Step 1: Reduce the three sides of the triangle The diagram contains one resistor on the P-Q side, two parallel resistors on the Q-R side and three parallel resistors on the P-R side. The short connecting wires at each end of each cluster join the resistor ends to the same two vertices. Hence the reduced triangle has

$$R_{PQ}^{\text{side}} = r, \qquad R_{QR}^{\text{side}} = \frac{r}{2}, \qquad R_{PR}^{\text{side}} = \frac{r}{3}.$$

The six resistors have now all been included: one plus two plus three. The apparent lengths of the drawn branches do not change their resistances.

Step 2: Compute each terminal-pair resistance For P and Q, the alternate path passes through R and has resistance $r/3 + r/2 = 5r/6$, so

$$R_{PQ} = r \parallel \frac{5r}{6} = \frac{5r}{11}.$$

For P and R, the alternate path has resistance $r + r/2 = 3r/2$, giving

$$R_{PR} = \frac{r}{3} \parallel \frac{3r}{2} = \frac{3r}{11}.$$

Similarly,

$$R_{QR} = \frac{r}{2} \parallel \left(r + \frac{r}{3}\right) = \frac{4r}{11}.$$

Thus $5r/11 > 4r/11 > 3r/11$. P-R and Q-R give smaller values, while the unequal reduced sides rule out the claim that all pairs have the same resistance.

Step 3: Conclusion The maximum equivalent resistance is $5r/11$, measured between P and Q. Hence, the correct option is (2).

Quick Tip to Solve: Reduce each parallel cluster to one triangle side before comparing terminal pairs; count all paths, including the path through the third vertex.

47. The earth's magnetic field lines resemble that of a dipole at the centre of the earth. If the magnetic moment of this dipole is close to $8 \times 10^{22}\,\mathrm{A\,m^2}$, the value of earth's magnetic field near the equator is close to (radius of the earth $= 6.4 \times 10^{6}\,\mathrm{m}$): [4 marks]

(1) 0.6 Gauss

(2) 1.2 Gauss

(3) 1.8 Gauss

(4) 0.32 Gauss

Correct Answer: (4) 0.32 Gauss

Solution:

Concept:

- The magnitude of the magnetic field on the equatorial line of a magnetic dipole of moment m is $B_{\text{eq}} = \mu_0 m/(4\pi R^3)$, at distance R from its centre.
- In SI units $\mu_0/(4\pi) = 10^{-7}$. Convert the resulting tesla value using $1\,\mathrm{G} = 10^{-4}\,\mathrm{T}$. The axial field is twice the equatorial field and must not be used here.

Step 1: Substitute the dipole moment and Earth radius The equator lies on the equatorial plane of the assumed central dipole, and the observation point is at distance equal to the Earth radius. Therefore

$$B_{\text{eq}} = 10^{-7}\frac{8 \times 10^{22}}{(6.4 \times 10^{6})^3}\,\mathrm{T}.$$

The denominator must be cubed as a whole:

$$(6.4 \times 10^{6})^3 = 262.144 \times 10^{18} = 2.62144 \times 10^{20}\,\mathrm{m^3}.$$

The moment is given in $\mathrm{A\,m^2}$, exactly the unit required by the dipole formula, so no extra geometric factor or loop area is needed.

Step 2: Convert the computed field to gauss and select the nearest value Combining the powers of ten and numerical factors gives

$$B_{\text{eq}} = \frac{8 \times 10^{15}}{2.62144 \times 10^{20}}\,\text{T} = 3.05 \times 10^{-5}\,\text{T}.$$

In gauss this is

$$B_{\text{eq}} = \frac{3.05 \times 10^{-5}}{10^{-4}}\,\text{G} = 0.305\,\text{G}.$$

The closest listed value is $0.32\,\text{G}$. Approximately $0.6\,\text{G}$ would result from incorrectly using the axial dipole field. The values $1.2\,\text{G}$ and $1.8\,\text{G}$ are still larger and do not follow from the supplied moment and radius.

Step 3: Conclusion The equatorial field is approximately $0.305\,\text{G}$, closest to $0.32\,\text{G}$. Hence, the correct option is (4).

Quick Tip to Solve: For an Earth-dipole estimate, choose the equatorial formula first, cube the radius, and convert tesla to gauss only at the end.

---

48. One of the two small circular coils, (none of them having any self-inductance) is suspended with a V-shaped copper wire, with plane horizontal. The other coil is placed just below the first one with plane horizontal. Both the coils are connected in series with a dc supply. The coils are found to attract each other with a force. Which one of the following statements is incorrect? [4 marks]

(1) Both the coils carry currents in the same direction.

(2) Coils will attract each other, even if the supply is an ac source.

(3) Force is proportional to d^{-1}.

(4) Force is proportional to d^{-2}.

Correct Answer: The printed question does not define d or specify a separation regime; its force-distance alternatives do not determine a unique incorrect statement.

Solution:

Concept:

- Coaxial loops attract when their magnetic moments are parallel. Reversing both currents simultaneously reverses both moments and preserves attraction.
- A force-distance power law depends on the coil geometry and separation regime. The force between complete circular loops is not generally the inverse-distance law for two long straight parallel wires.

Step 1: Check the statements about current direction The observed attraction corresponds to currents circulating in the same sense when viewed from the same side, so statement (1) is consistent with the observation. The loops are in series, and their self-inductance is neglected. Replacing the dc supply with ac therefore reverses the current in both loops together. The magnetic interaction is proportional to the product of their currents, which becomes $i(t)^2$. The force remains attractive during both half-cycles and vanishes at the instants when the current is zero. Thus statement (2) is also

consistent, although the magnitude of the force varies with time.

Step 2: Test whether either printed distance law is universal The symbol d is not defined in the question. Even if it denotes the axial separation, no relation between that separation and the coil radii is supplied. For example, at separations large compared with both radii, each small coil acts as a magnetic dipole. If their moments are m_1 and m_2, the axial field and attractive force magnitude are

$$B(d) = \frac{\mu_0 m_1}{2\pi d^3}, \qquad F = m_2 \left|\frac{dB}{dd}\right| = \frac{3\mu_0 m_1 m_2}{2\pi d^4}.$$

In this physically allowed limit, neither d^{-1} nor d^{-2} is the correct dependence. An inverse-distance approximation can arise for nearby locally parallel wire segments, but the printed wording supplies no such restriction. Therefore the alternatives cannot establish a unique incorrect statement for the given coils.

Step 3: Conclusion The question as printed is ambiguous. The supplied answer key selects option (4), but a universal d^{-1} dependence in option (3) is also unjustified without an additional geometrical assumption. No unique incorrect option follows from the stated information.

Quick Tip to Solve: Before using a magnetic force power law, identify the geometry and compare the separation with the coil radius; straight-wire and dipole limits have different powers.

49. A metal sample carrying a current along X-axis with density J_x is subjected to a magnetic field B_z (along z-axis). The electric field E_y developed along Y-axis is directly proportional to J_x as well as B_z. The constant of proportionality has SI unit: [4 marks]

(1) $\frac{\mathrm{m}^2}{\mathrm{A}}$

(2) $\frac{\mathrm{m}^3}{\mathrm{A\,s}}$

(3) $\frac{\mathrm{m}^2}{\mathrm{A\,s}}$

(4) $\frac{\mathrm{A\,s}}{\mathrm{m}^3}$

Correct Answer: (2) $\frac{\mathrm{m}^3}{\mathrm{A\,s}}$

Solution:

Concept:

- The stated proportionality can be written $E_y = R_H J_x B_z$, defining the coefficient R_H. Its unit is obtained by dividing the electric-field unit by the units of current density and magnetic field.
- Current density is current per cross-sectional area, with unit $\mathrm{A\,m^{-2}}$. The Lorentz-force relation implies that E/B has the dimensions of speed.

Step 1: Express the required unit as a quotient The transverse field is the Hall electric field, arising as charge carriers are deflected by the magnetic field until electric and magnetic forces balance

transversely. For the unit calculation only the stated proportionality is required:

$$[R_H] = \frac{[E_y]}{[J_x][B_z]}.$$

Since $[E_y] = \mathrm{V\,m^{-1}}$, $[J_x] = \mathrm{A\,m^{-2}}$, and $[B_z] = \mathrm{T} = \mathrm{V\,s\,m^{-2}}$, all the needed units can be reduced using volts, amperes, metres and seconds.

Step 2: Cancel the common units and check the alternatives Substitution and cancellation give

$$[R_H] = \frac{\mathrm{V\,m^{-1}}}{(\mathrm{A\,m^{-2}})(\mathrm{V\,s\,m^{-2}})} = \mathrm{m^3\,A^{-1}\,s^{-1}}.$$

Equivalently, E/B is a speed, so dividing $\mathrm{m/s}$ by $\mathrm{A/m^2}$ gives the same result immediately. The unit can also be written $\mathrm{m^3/C}$ because $\mathrm{C} = \mathrm{A\,s}$. The first and third options contain the wrong power of length, and the fourth is the reciprocal of the required coefficient. No material-specific carrier density is needed to determine its unit.

Step 3: Conclusion The SI unit is $\boxed{\mathrm{m^3/(A\,s)}}$. Hence, the correct option is (2).

Quick Tip to Solve: Use E/B as a speed; dividing it by current density quickly gives $\mathrm{m^3/(A\,s)}$ without expanding tesla into base units.

50. When resonance is produced in a series LCR circuit, then which of the following is not correct?

[4 marks]

(A) Current in the circuit is in phase with the applied voltage.

(B) Inductive and capacitive reactances are equal.

(C) If R is reduced, the voltage across capacitor will increase.

(D) Impedance of the circuit is maximum.

Correct Answer: (D) Impedance of the circuit is maximum.

Solution:

Concept:

- For a series LCR circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_L = \omega L$ and $X_C = 1/(\omega C)$. The current is $I = V/Z$.
- At resonance, the inductive and capacitive reactances cancel. The circuit behaves resistively and the current and source voltage have zero phase difference.

Step 1: Apply the resonance condition At the resonant angular frequency, $X_L = X_C$. Substituting this equality gives

$$Z_{\text{res}} = \sqrt{R^2 + 0} = R.$$

Since the squared reactance term cannot be negative, this is the minimum possible impedance for the given resistance. Consequently the current is maximum, rather than the impedance being maximum. This makes the statement in option (D) the candidate for the incorrect statement.

Step 2: Check the remaining statements The phase relation satisfies $\tan\phi = (X_L - X_C)/R = 0$, so option (A) is correct. Equality of the two reactances is precisely the condition for resonance, so option (B) is correct. At fixed source voltage and the same resonant frequency, the capacitor voltage is

$$V_C = IX_C = \frac{VX_C}{R}.$$

Reducing the resistance therefore increases the current and the voltage across the capacitor. Option (C) is also correct; the capacitor voltage can exceed the source voltage because the inductor voltage opposes it in phase.

Step 3: Conclusion The incorrect statement is that the impedance is maximum. At resonance it is minimum and equals R. Hence, the correct option is (D).

Quick Tip to Solve: In series resonance remember the paired facts: impedance is minimum, while current and power factor are maximum.

51. A series LR circuit is connected to an ac source of frequency ω and the inductive reactance is equal to $2R$. A capacitance of capacitive reactance equal to R is added in series with L and R. The ratio of the new power factor to the old one is: [4 marks]

(A) $\sqrt{\frac{2}{3}}$

(B) $\sqrt{\frac{2}{5}}$

(C) $\sqrt{\frac{3}{2}}$

(D) $\sqrt{\frac{5}{2}}$

Correct Answer: (D) $\sqrt{\frac{5}{2}}$

Solution:

Concept:

- The power factor of a series circuit containing resistance and reactance is $\cos\phi = R/Z$. The resistive component determines the real power.
- Inductive and capacitive reactances have opposite signs, so their net contribution is $X_L - X_C$, not their sum.

Step 1: Calculate the original power factor Initially the circuit contains only L and R, with inductive reactance $X_L = 2R$. Its impedance and power factor are therefore

$$Z_{\text{old}} = \sqrt{R^2 + (2R)^2} = \sqrt{5}R, \qquad p_{\text{old}} = \frac{R}{\sqrt{5}R} = \frac{1}{\sqrt{5}}.$$

The magnitude of the applied voltage is not needed because power factor is determined by the phase relation. The source frequency stays unchanged when the capacitor is inserted.

Step 2: Include the capacitor and form the requested ratio The added capacitor has $X_C = R$. Thus

the net reactance becomes $2R - R = R$, and the circuit remains inductive. The new quantities are

$$Z_{\text{new}} = \sqrt{R^2 + R^2} = \sqrt{2}R, \qquad p_{\text{new}} = \frac{1}{\sqrt{2}}.$$

Hence

$$\frac{p_{\text{new}}}{p_{\text{old}}} = \frac{1/\sqrt{2}}{1/\sqrt{5}} = \sqrt{\frac{5}{2}}.$$

The ratio exceeds one, as expected because partial reactance cancellation improves the power factor. Options (A) and (B) incorrectly give a decrease; option (C) does not correspond to either impedance.

Step 3: Conclusion The ratio of the new power factor to the old one is $\sqrt{5/2}$. Hence, the correct option is (D).

Quick Tip to Solve: For unchanged resistance, the ratio of power factors equals the inverse ratio of impedances; subtract the two reactances before squaring.

52. A printed page is pressed by a glass of water. The refractive index of the glass and water is 1.5 and 1.33, respectively. If the thickness of the bottom of glass is 1 cm and depth of water is 5 cm, how much the page will appear to be shifted if viewed from the top ? [4 marks]

(1) 1.033 cm

(2) 3.581 cm

(3) 1.3533 cm

(4) 1.90 cm

Correct Answer: No listed option equals the calculated upward shift, approximately 1.574 cm.

Solution:

Concept:

- For near-normal viewing from air through a layer of thickness t and refractive index μ, its apparent thickness is t/μ.
- For several plane parallel transparent layers, the apparent depths add: $d_{\text{apparent}} = \sum t_i/\mu_i$. The upward shift is real depth minus apparent depth.

Step 1: Calculate the apparent position of the page The page lies below a glass bottom of thickness $1\,\text{cm}$ and a water column of depth $5\,\text{cm}$. Both interfaces are viewed from above, so both layers contribute to the displacement. The actual depth below the top water surface is

$$d_{\text{real}} = 1 + 5 = 6\,\text{cm}.$$

Using the printed refractive indices,

$$d_{\text{apparent}} = \frac{1}{1.5} + \frac{5}{1.33} = 0.666667 + 3.759398 = 4.426065\,\text{cm}.$$

The distances are measured from the same upper reference surface, which is essential when subtracting them.

Step 2: Compare the upward shift with the listed answers The apparent upward displacement is consequently

$$\Delta d = 6 - 4.426065 = 1.573935\,\text{cm}.$$

Equivalently, each layer contributes $t(1 - 1/\mu)$, giving $0.333333 + 1.240602\,\text{cm}$. Neither rounding the result nor using the usual approximation $\mu_{\text{water}} = 4/3$, which gives $1.583333\,\text{cm}$, yields any listed value. In particular the keyed value $1.3533\,\text{cm}$ in option (3) is inconsistent with the printed data. Options (1), (2) and (4) also disagree with the sum of the two layer shifts.

Step 3: Conclusion The page appears shifted upward by approximately $1.574\,\text{cm}$. No printed option is defensible for these data; the answer-key choice (3) does not match the literal question.

Quick Tip to Solve: Compute the shift of each transparent layer separately as $t(1 - 1/\mu)$, then add; do not average their refractive indices.

---

53. A thin glass plate of thickness is $\frac{2500}{3}\lambda$ (λ is wavelength of light used) and refractive index $\mu = 1.5$ is inserted between one of the slits and the screen in Young's double slit experiment. At a point on the screen equidistant from the slits, the ratio of the intensities before and after the introduction of the glass plate is: [4 marks]

(1) 2 : 1

(2) 1 : 4

(3) 4 : 1

(4) 4 : 3

Correct Answer: (3) 4 : 1

Solution:

Concept:

- Introducing a plate of thickness t and refractive index μ in one path adds optical path difference $\delta = (\mu - 1)t$.
- For the equal coherent slit amplitudes of the usual Young experiment, $I = 4I_s \cos^2(\pi\delta/\lambda)$, where I_s is the intensity from one slit.

Step 1: Find the phase change introduced by the plate Before inserting the plate, the selected screen point is equidistant from the slits. Its geometrical path difference is zero, so it is a constructive-interference point with $I_{\text{before}} = 4I_s$. With the given plate, the additional optical path is

$$\delta = (1.5 - 1)\frac{2500}{3}\lambda = \frac{1250}{3}\lambda.$$

A whole number of wavelengths does not alter the interference intensity. Here $1250/3 = 416 + 2/3$, so the effective residual optical path is $2\lambda/3$.

Step 2: Evaluate the changed intensity and its ratio The phase difference is $\phi = 2\pi\delta/\lambda$; modulo 2π, it is $4\pi/3$. Therefore

$$I_{\text{after}} = 2I_s(1 + \cos\phi) = 2I_s\left(1 - \frac{1}{2}\right) = I_s.$$

Equivalently $\cos^2(1250\pi/3) = 1/4$ in the cosine-squared formula. Thus

$$I_{\text{before}} : I_{\text{after}} = 4I_s : I_s = 4 : 1.$$

Option (2) reverses the requested order. Options (1) and (4) would require different residual phase differences and cannot arise from the stated plate thickness.

Step 3: Conclusion The ratio of intensities before and after insertion is $4 : 1$. Hence, the correct option is (3).

Quick Tip to Solve: Reduce the added optical path modulo one wavelength before calculating the phase; the large integer part does not affect intensity.

---

54. The source that illuminates the double slit in 'double - slit interference experiment' emits two distinct monochromatic waves of wavelength 500 nm and 600 nm, each of them producing its own pattern on the screen. At the central point of the pattern when path difference is zero, maxima of both the patterns coincide and the resulting interference pattern is most distinct at the region of zero path difference. But as one moves out of this central region, the two fringe systems are gradually out of step such that maximum due to on wavelength coincides with the minimum due to the other and the combined fringe system becomes completely indistinct. This may happen when path difference in nm is [4 marks]

(1) 2000

(2) 3000

(3) 1000

(4) 1500

Correct Answer: (4) 1500

Solution:

Concept:

- For wavelength λ, a path difference $\delta = m\lambda$ gives a maximum and $\delta = (m + 1/2)\lambda$ gives a minimum.
- The two distinct wavelengths produce separate fringe systems. A maximum of one falling on a minimum of the other makes the combined fringes indistinct in the stated experiment.

Step 1: Test coincidence of a maximum and a minimum For the two wavelengths, inspect the dimensionless orders $\delta/500$ and $\delta/600$, with all lengths in nm. At the candidate $\delta = 1500\,\text{nm}$, these are

$$\frac{1500}{500} = 3, \qquad \frac{1500}{600} = \frac{5}{2}.$$

The 500 nm radiation therefore produces its third-order maximum, while the 600 nm radiation produces a minimum. Their bright and dark contributions occur at the same screen location, satisfying the condition stated in the question.

Step 2: Check that the other proposed paths fail At $2000\,\text{nm}$, the orders are 4 and $10/3$; the latter is neither an integer nor a half-integer, so a maximum does not coincide with a minimum. At $3000\,\text{nm}$,

the orders are 6 and 5, which are both integers: maxima coincide and the pattern is strong rather than indistinct. At $1000\,\text{nm}$, the orders are 2 and $5/3$, again failing the maximum-minimum condition. Only the fourth proposed value gives the required integer/half-integer pairing. This comparison avoids confusing a common maximum with loss of contrast.

Step 3: Conclusion The specified condition occurs at a path difference of $1500\,\text{nm}$. Hence, the correct option is (4).

Quick Tip to Solve: Divide each proposed path difference by both wavelengths: look for an integer paired with a half-integer, rather than two integers.

---

55. This question has Statement-1 and Statements2. Of the four choices given after the Statements, choose the one that best describes the two Statements. Statement-1: Out of radio waves and microwaves, the radio waves undergo more diffraction. Statement-2: Radio waves have greater frequency compared to microwaves. [4 marks]

(1) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1.

(2) Statement-1 is false, Statement-2 is true.

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is true, Statement-2 is true but Statement-2 is not the correct explanation of Statement-1.

Correct Answer: (3) Statement-1 is true, Statement-2 is false.

Solution:

Concept:

- Diffraction becomes appreciable when a wave encounters an aperture or obstacle whose size is comparable with its wavelength. For a fixed opening, angular spreading grows with wavelength.
- All electromagnetic waves in vacuum obey $c = f\lambda$. Radio waves have longer wavelengths and therefore lower frequencies than microwaves in the comparison intended here.

Step 1: Evaluate Statement-1 using wavelength Radio waves have much longer wavelengths than microwaves. For the same obstacle or opening, the dimensionless ratio of wavelength to obstacle size is therefore larger for radio waves. Their bending into the geometrical shadow and their spreading are more pronounced. For example, the single-slit scale $\sin\theta \sim \lambda/a$, where a is the opening width, shows why increasing wavelength increases the angular spread. Statement-1 is consequently true; it reflects the relative diffraction of these waves, rather than a claim that every possible aperture gives an identical amount of diffraction.

Step 2: Evaluate Statement-2 and select the consistent choice Since c is the same for electromagnetic radiation in vacuum, frequency is inversely proportional to wavelength:

$$f = \frac{c}{\lambda}.$$

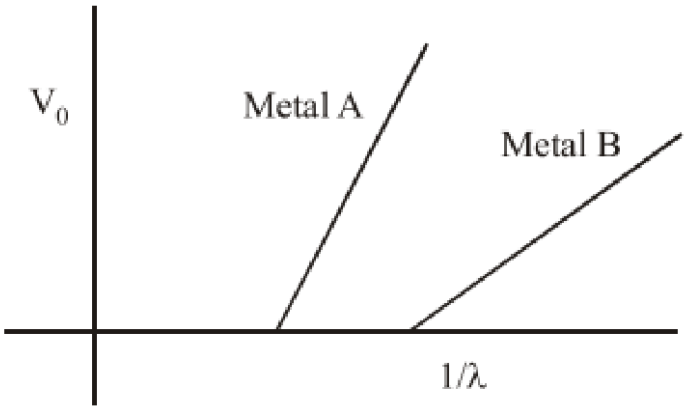
The larger radio wavelength means a smaller frequency, not a greater frequency. Statement-2 is false. Hence it cannot explain Statement-1. Options (1) and (4) both incorrectly call Statement-2 true, while option (2) incorrectly calls Statement-1 false and Statement-2 true. The remaining choice correctly

combines a true first statement with a false second statement.

Step 3: Conclusion Statement-1 is true and Statement-2 is false. Hence, the correct option is (3).

Quick Tip to Solve: When comparing diffraction, first compare wavelengths; use $f\lambda = c$ only afterward to translate the ordering into frequencies.

56. In an experiment on photoelectric effect, a student plots stopping potential V_0 against reciprocal of the wavelength λ of the incident light for two different metals A and B. These are shown in the figure. Looking at the graphs, you can most appropriately say that: [4 marks]



(1) Work function of metal B is greater than that of metal A.

(2) For light of certain wavelength falling on both metal, maximum kinetic energy of electrons emitted from A will be greater than those emitted from B.

(3) Work function of metal A is greater than that of metal B.

(4) Students data is not correct.

Correct Answer: (4) Students data is not correct.

Solution:

Concept:

- Einstein's photoelectric equation is $eV_0 = hc/\lambda - \Phi$, where Φ is the metal work function and V_0 is stopping potential.
- A plot of V_0 against $1/\lambda$ must be a straight line with slope hc/e. This slope is independent of the metal; its work function changes only the intercept.

Step 1: Derive the universal slope for both metals Rearrange the photoelectric equation with horizontal variable $x = 1/\lambda$:

$$V_0 = \frac{hc}{e}x - \frac{\Phi}{e}.$$

Changing from metal A to metal B changes the intercept, but cannot change the gradient. The constants h, c and electron charge magnitude e are universal. Both metals use the same axes, so their slopes are directly comparable.

Step 2: Compare the required behaviour with the displayed lines The line for A is visibly steeper than the line for B. These cannot both represent correct stopping potentials against reciprocal wavelength. Their horizontal intercepts appear to suggest a work-function ordering, making option (1) tempting, while the drawn heights may suggest option (2). Option (3) reverses that apparent intercept ordering. However, these deductions accept graphs that already violate the universal slope condition. The most appropriate conclusion is that the data are inconsistent; a different metal cannot give a different slope.

Step 3: Conclusion The two lines must have the same slope hc/e, whereas the figure gives different slopes. The student's data are not correct. Hence, the correct option is (4).

Quick Tip to Solve: Before reading work functions from intercepts, check the slope: a V_0 versus $1/\lambda$ graph has the universal gradient hc/e.

---

57. A copper ball of radius 1 cm and work function 4.47 eV is irradiated with ultraviolet radiation of wavelength 2500 Å. The effect of irradiation results in the emission of electrons from the ball. Further the ball will acquire charge and due to this there will be a finite value of the potential on the ball. The charge acquired by the ball is [4 marks]

(1) $5.5 \times 10^{-13}\,\mathrm{C}$

(2) $7.5 \times 10^{-13}\,\mathrm{C}$

(3) $4.5 \times 10^{-12}\,\mathrm{C}$

(4) $2.5 \times 10^{-11}\,\mathrm{C}$

Correct Answer: (1) $5.5 \times 10^{-13}\,\mathrm{C}$

Solution:

Concept:

- Einstein's photoelectric equation gives the maximum emitted kinetic energy: $K_{\max} = hc/\lambda - \phi$, where ϕ is the work function.
- An isolated conducting sphere has capacitance $C = 4\pi\varepsilon_0 R$. Electron emission stops when its positive surface potential satisfies $eV = K_{\max}$; then $Q = CV$.

Step 1: Calculate the available photoelectron energy The wavelength is $2500\,\text{Å} = 250\,\mathrm{nm}$. Using $hc \simeq 1240\,\mathrm{eV\,nm}$, the incident photon energy is

$$E_\gamma = \frac{1240}{250} = 4.96\,\mathrm{eV}.$$

Subtract the work function rather than equating the full photon energy to an electrical potential energy:

$$K_{\max} = 4.96 - 4.47 = 0.49\,\mathrm{eV}.$$

This is the greatest energy available for escaping the charged ball.

Step 2: Find the limiting positive charge Loss of electrons makes the copper ball positive. Further emitted electrons are attracted back, and steady charging ends at $V = 0.49\,\mathrm{V}$. With $R = 0.01\,\mathrm{m}$ and

$1/(4\pi\varepsilon_0) = 9.0 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$,

$$Q = \frac{RV}{9.0 \times 10^9} = \frac{0.01(0.49)}{9.0 \times 10^9} = 5.44 \times 10^{-13}\,\mathrm{C}.$$

The small difference from the printed value is rounding. The other choices require a larger stopping potential than the photons can produce, and so fail the energy balance.

Step 3: Conclusion The acquired charge is positive and approximately $5.5 \times 10^{-13}\,\mathrm{C}$. Hence, the correct option is (1).

Quick Tip to Solve: Find the excess photon energy in eV first: its numerical value directly gives the limiting sphere potential in volts.

58. A 12.5eV electron beam is used to bombard gaseous hydrogen at room temperature. It will emit [4 marks]

(1) 2 lines in the Lyman series and 1 line in the Balmar series

(2) 3 lines in the Lyman series

(3) 1 line in the Lyman series and 2 lines in the Balmar series

(4) 3 lines in the Balmer series

Correct Answer: (1) 2 lines in the Lyman series and 1 line in the Balmar series

Solution:

Concept:

- Hydrogen energy levels obey $E_n = -13.6/n^2\,\mathrm{eV}$. At room temperature nearly all atoms are in the ground state $n = 1$.
- An incident electron can excite an atom only when its energy is at least the required level difference. Transitions ending at $n = 1$ form the Lyman series; those ending at $n = 2$ form the Balmer series.

Step 1: Determine the highest accessible level The threshold excitation energies out of the ground state are

$$\Delta E_{1\to2} = 13.6\left(1 - \frac{1}{4}\right) = 10.2\,\mathrm{eV},$$

$$\Delta E_{1\to3} = 13.6\left(1 - \frac{1}{9}\right) = 12.09\,\mathrm{eV},$$

and

$$\Delta E_{1\to4} = 13.6\left(1 - \frac{1}{16}\right) = 12.75\,\mathrm{eV}.$$

Thus a 12.5 eV beam can populate levels 2 and 3 but cannot reach level 4. The electron need not lose all its energy in the collision; any unused energy remains kinetic energy.

Step 2: Count the allowed emission lines Atoms excited to level 3 may decay directly by $3 \to 1$, or in a cascade $3 \to 2 \to 1$. Atoms excited directly to level 2 also give $2 \to 1$. Consequently the distinct downward transitions are $3 \to 1$, $2 \to 1$, and $3 \to 2$. The first two end at level 1 and are Lyman lines. Only the last ends at level 2 and is a Balmer line. Counting all three as Lyman lines ignores their

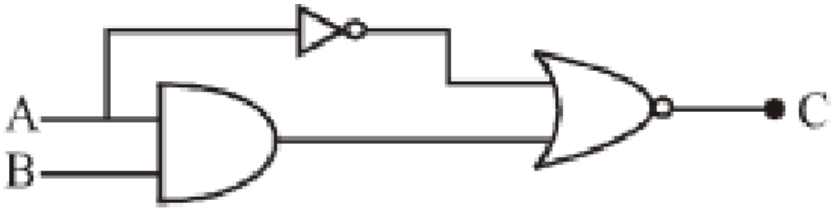
different final levels; neither two nor three Balmer lines are possible with a highest populated level of 3.

Step 3: Conclusion The gas emits two Lyman lines and one Balmer line (spelled Balmar in the printed option). Hence, the correct option is (1).

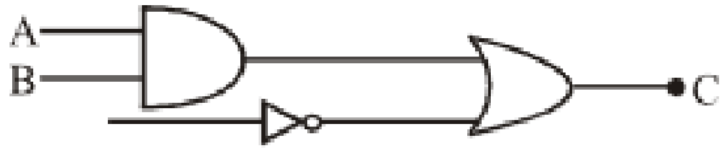
Quick Tip to Solve: Find the largest reachable principal quantum number before using the line-count formula; then classify each line by its final level.

59. Which of the following circuits correctly represents the following truth table? [4 marks]

A	B	C
0	0	0
0	1	0
1	0	1
1	1	0

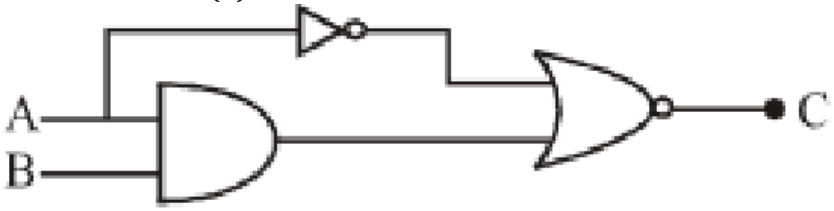
(1) 

(2) 

(3) 

(4) 

Correct Answer: (1)



Solution:

Concept:

- The truth table gives output 1 only for $A = 1, B = 0$. Its Boolean expression is therefore $C = A\overline{B}$.
- For checking a composite circuit, write an expression at each gate and apply De Morgan's law: $\overline{X + Y} = \overline{X}\,\overline{Y}$.

Step 1: Read the required output condition In the four printed rows the output sequence is $0, 0, 1, 0$ for input pairs $(0, 0), (0, 1), (1, 0), (1, 1)$. The only successful input pair requires A to be true and B to be false. Hence the target product term is

$$C = A\overline{B}.$$

A circuit need not contain a visibly separate NOT gate on B to implement this term: equivalent expressions can be formed with a final inverted gate.

Step 2: Simplify option 1 and eliminate the alternatives In option 1 the upper branch inverts A, the lower branch forms AB, and the final gate is NOR. Therefore

$$C = \overline{\overline{A} + AB} = A\,\overline{AB} = A(\overline{A} + \overline{B}) = A\overline{B}.$$

This reproduces all four rows exactly. Option 2 gives $(A + B)B = B$, so it wrongly produces 1 for inputs $(0, 1)$. Option 3 gives $AB + \overline{B} = A + \overline{B}$; its output is already 1 at $(0, 0)$, unlike the table. Option 4 is NAND, $\overline{AB}$, which also gives 1 at $(0, 0)$. These counterexamples reject every other circuit without relying on the appearance of the gates.

Step 3: Conclusion Option 1 implements $C = A\overline{B}$, exactly matching the given truth table. Hence, the correct option is (1).

Quick Tip to Solve: Turn each circuit into a Boolean expression; one conflicting input row is sufficient to reject an incorrect option.

60. Which of the following modulated signal has the best noise-tolerance ? [4 marks]

(1) Long-wave

(2) Short-wave

(3) Medium-wave

(4) Amplitude-modulated

Correct Answer: No unique option is defensible: the choices mix carrier-wave bands with a modulation method and omit frequency modulation.

Solution:

Concept:

- Noise tolerance describes how faithfully a receiver recovers the message in the presence of unwanted electrical fluctuations. It depends on the modulation method, receiver and signal-to-noise ratio, rather than on a carrier-band name alone.

- In amplitude modulation the message is carried by changes of carrier amplitude. Additive amplitude noise therefore directly corrupts the same quantity that conveys the information. Frequency modulation can suppress much amplitude noise by limiting before frequency detection.

Step 1: Identify what each choice specifies Long-wave, short-wave and medium-wave describe ranges of carrier wavelength or frequency. They do not, by themselves, specify whether the transmitted signal is amplitude-modulated, frequency-modulated or modulated in some other way. For example, changing the carrier frequency of an amplitude-modulated transmitter does not change the basic fact that both the wanted message and amplitude noise affect its envelope. Propagation and external-noise conditions can also differ between bands, so a universal ranking needs additional operating conditions.

Step 2: Check whether a modulation comparison is possible The fourth choice, amplitude-modulated, is a modulation type rather than a wavelength band. It can overlap the first three descriptions: a carrier in a named band can itself be amplitude-modulated. Thus the four choices are neither mutually exclusive modulation schemes nor signals with stated comparable noise conditions. In the usual comparison of AM with FM, FM has better rejection of amplitude noise because amplitude limiting removes fluctuations before the frequency-sensitive detector recovers the message. However, frequency modulation is absent from the printed choices. Selecting one of the three bands would silently assume a transmitter, receiver and noise environment that the question does not specify; selecting AM would supply no comparison against another stated modulation method.

Step 3: Conclusion The printed choices do not determine a unique correct option. Frequency modulation is the usual answer to the AM-versus-FM noise-tolerance comparison, but it is not among these choices; no option can be selected uniquely from the information given.

Quick Tip to Solve: Compare modulation methods under comparable noise conditions; carrier wavelength labels alone cannot establish which signal has the best noise tolerance.

Chemistry (120 marks)

61. 6 litres of an alkene require 27 litres of oxygen at constant temperature and pressure for complete combustion. The alkene is [4 marks]

(1) Ethene

(2) Propene

(3) 1-Butene

(4) 2-Butene

Correct Answer: (2) Propene

Solution:

Concept:

- An acyclic alkene with one carbon-carbon double bond has molecular formula C_nH_{2n}. Complete combustion converts its carbon to CO_2 and hydrogen to H_2O.
- At the same temperature and pressure, gas volumes are proportional to mole numbers, so the oxygen-to-alkene volume ratio equals the corresponding stoichiometric ratio.

Step 1: Balance combustion for a general alkene The n carbon atoms produce n carbon dioxide molecules, and the 2n hydrogen atoms produce n water molecules. The oxygen requirement is therefore 3n oxygen atoms, or $3n/2$ oxygen molecules per alkene molecule:

$$\mathrm{C}_n\mathrm{H}_{2n} + \frac{3n}{2}\mathrm{O}_2 \longrightarrow n\mathrm{CO}_2 + n\mathrm{H}_2\mathrm{O}.$$

This balances carbon, hydrogen and oxygen independently; the volume of water vapour is not needed to find the oxygen consumption.

Step 2: Use the measured volume ratio The printed volumes give

$$\frac{V_{\mathrm{O}_2}}{V_{\text{alkene}}} = \frac{27}{6} = \frac{9}{2} = \frac{3n}{2}, \qquad n = 3.$$

The alkene must therefore be $\mathrm{C_3H_6}$, propene. A direct check gives $2\,\mathrm{C_3H_6} + 9\,\mathrm{O_2} \longrightarrow 6\,\mathrm{CO_2} + 6\,\mathrm{H_2O}$, so 6 litres of propene require 27 litres of oxygen. Ethene would require only 18 litres; either butene isomer would require 36 litres, because both have formula $\mathrm{C_4H_8}$. The double-bond position cannot change the complete combustion requirement. The supplied key selects (4), but 2-butene requires 36 litres of oxygen for the printed 6 litres and is inconsistent with the stated data.

Step 3: Conclusion The printed ratio $27/6 = 3n/2$ gives $n = 3$, so the alkene is propene. The supplied key selects 2-butene, but that alkene would require 36 litres of oxygen, contradicting the stated 27 litres. Hence, the correct option is (2).

Quick Tip to Solve: For an alkene, divide the oxygen volume by alkene volume and multiply by $2/3$ to obtain the number of carbon atoms.

---

62. How many grams of methyl alcohol should be added to 10 litre tank of water to prevent its freezing at 268 K? (K_f for water is $1.86\,\mathrm{K\,kg\,mol^{-1}}$.) [4 marks]

(1) 880.07 g

(2) 899.04 g

(3) 886.02 g

(4) 868.06 g

Correct Answer: (3) 886.02 g

Solution:

Concept:

- For a dilute non-electrolyte solution, $\Delta T_f = K_f m$, where m is the molality in moles of solute per kilogram of solvent. Methanol has van't Hoff factor $i = 1$.
- Use the mass of water as solvent, not the volume or mass of the final solution. A 10 litre tank of water has approximately 10 kg of solvent using water density $1\,\mathrm{kg\,L^{-1}}$.

Step 1: Find the required molality The normal freezing point of water is $273.15\,\mathrm{K}$. For an onset of

freezing at $268\,\mathrm{K}$, the required depression is

$$\Delta T_f = 273.15 - 268 = 5.15\,\mathrm{K}.$$

Thus

$$m = \frac{5.15}{1.86} = 2.7688\,\mathrm{mol\,kg^{-1}}.$$

Methanol is molecular and does not dissociate into ions, so no factor of two or other electrolyte correction enters this calculation.

Step 2: Convert the required moles into grams The solvent mass is 10 kg, so the required methanol amount and mass are

$$n = 2.7688(10) = 27.688\,\mathrm{mol},$$

$$w = nM = 27.688(32.0) = 886.02\,\mathrm{g}.$$

At this boundary the freezing point is lowered to 268 K; adding at least this amount prevents freezing above that temperature in the ideal colligative-property model. The other three printed masses do not reproduce the calculated molality: in particular 868.06 g gives a depression of about 5.046 K rather than 5.15 K. Using a rounded initial freezing point of 273 K instead would give about 860 g, also not the keyed option. The supplied key selects (4), but the printed temperatures and K_f, with the standard freezing point $273.15\,\mathrm{K}$, give (3).

Step 3: Conclusion The required methyl alcohol mass is $32.0 \times 10 \times 5.15/1.86 = 886.02\,\mathrm{g}$. The supplied key selects 868.06 g, but that mass does not produce the required 5.15 K depression for the printed temperatures. Hence, the correct option is (3).

Quick Tip to Solve: Convert litres of water into kilograms of solvent before applying molality, and keep the 0.15 K in the water freezing point when the options are close.

63. 10 mL of 2(M) NaOH solution is added to 200 mL of 0.5 (M) of NaOH solution. What is the final concentration ? [4 marks]

(1) 0.57(M)

(2) 5.7(M)

(3) 11.4(M)

(4) 1.14(M)

Correct Answer: (1) 0.57(M)

Solution:

Concept:

- Molarity is moles of solute per litre of solution: $M = n/V$. On mixing two solutions of the same solute without reaction, their solute amounts add.
- For additive solution volumes, the final concentration is the volume-weighted mean: $M_f = (M_1V_1 + M_2V_2)/(V_1 + V_2)$.

Step 1: Compute the two amounts of sodium hydroxide The concentrated portion contains

$$n_1 = (2.0\,\text{mol}\,\text{L}^{-1})(0.010\,\text{L}) = 0.020\,\text{mol}.$$

The larger, more dilute portion contains

$$n_2 = (0.5\,\text{mol}\,\text{L}^{-1})(0.200\,\text{L}) = 0.100\,\text{mol}.$$

These are solutions of the same substance, so there is no neutralisation or other stoichiometric subtraction. The total solute amount is $n_f = 0.020 + 0.100 = 0.120\,\text{mol}$.

Step 2: Divide by the total solution volume Using the usual additive-volume assumption for this mixture,

$$V_f = 10 + 200 = 210\,\text{mL} = 0.210\,\text{L},$$

$$M_f = \frac{0.120}{0.210} = 0.571428\ldots\,\text{mol}\,\text{L}^{-1}.$$

Rounded to the precision of the options this is 0.57 M. A useful independent check is that a weighted mean must lie between 0.5 M and 2 M, and should be much nearer 0.5 M because the 200 mL portion dominates. Options 5.7 M and 11.4 M exceed both starting concentrations. Although 1.14 M lies between the endpoints, it fails conservation of moles and greatly overweights the small 10 mL portion.

Step 3: Conclusion The final sodium hydroxide concentration is $0.57\,\text{M}$. Hence, the correct option is (1).

Quick Tip to Solve: Keep both volumes in the same unit in the weighted-mean formula; the result should lie close to the concentration of the much larger portion.

---

64. Given (A) $n = 5,\ m_\ell = +1$; (B) $n = 2,\ \ell = 1,\ m_\ell = -1,\ m_s = -1/2$. The maximum number of electron(s) in an atom that can have the quantum numbers as given in (A) and (B) are respectively:

[4 marks]

(1) 25 and 1

(2) 8 and 1

(3) 2 and 4

(4) 4 and 1

Correct Answer: (2) 8 and 1

Solution:

Concept:

- An electron is specified by four quantum numbers: n, ℓ, m_ℓ and m_s. For a fixed principal quantum number, $\ell = 0, 1, \ldots, n-1$.
- For each subshell, $m_\ell = -\ell, \ldots, +\ell$. Each allowed orbital holds two electrons, one with each spin projection $m_s = \pm 1/2$, by the Pauli exclusion principle.

Step 1: Count the orbitals allowed in (A) Here $n = 5$ permits $\ell = 0, 1, 2, 3, 4$, but the additional

requirement $m_\ell = +1$ rules out $\ell = 0$. Each of the four remaining values $\ell = 1, 2, 3, 4$ has exactly one orbital with this magnetic quantum number. The spin is unspecified, so each of those four orbitals accommodates two electrons. Therefore the maximum number is

$$N_A = 4 \times 2 = 8.$$

Counting every orbital of the fifth shell would give 25 orbitals, but those do not all have the specified magnetic quantum number.

Step 2: Apply all four restrictions in (B) For $n = 2, \ell = 1$, the electron belongs to the $2p$ subshell. Fixing $m_\ell = -1$ chooses one particular orbital, and fixing $m_s = -1/2$ selects just one of its two possible spin states. Two electrons cannot share this entire set of four quantum numbers. Thus

$$N_B = 1.$$

The pair must be 8 and 1. The alternatives with 25 or 4 as the first count fail the orbital count; the alternative with 4 as the second count violates the complete specification of a single state.

Step 3: Conclusion The maximum populations are 8 and 1, respectively. Hence, the correct option is (2).

Quick Tip to Solve: Fix the magnetic quantum number first, count the subshells that contain it, and multiply by two only if spin is not fixed.

65. The catenation tendency of C, Si and Ge is in the order $\mathrm{Ge} < \mathrm{Si} < \mathrm{C}$. The bond energies (in $\mathrm{kJ\,mol^{-1}}$) of $\mathrm{C{-}C}$, $\mathrm{Si{-}Si}$ and $\mathrm{Ge{-}Ge}$ bonds are respectively: [4 marks]

(1) 348, 297, 260

(2) 297, 348, 260

(3) 348, 260, 297

(4) 260, 297, 348

Correct Answer: (1) 348, 297, 260

Solution:

Concept:

- Catenation is the ability of an element to form chains by bonding to other atoms of the same element. Strong element–element bonds favour stable chains.
- Down Group 14, increasing atomic size leads to poorer orbital overlap and weaker single bonds. Thus the homonuclear bond strength and catenation tendency decrease from carbon to silicon to germanium.

Step 1: Translate the catenation order into bond strengths The given order is $\mathrm{Ge} < \mathrm{Si} < \mathrm{C}$. In the same sequence the homonuclear bond becomes stronger, so the required energies must satisfy

$$D(\mathrm{C{-}C}) > D(\mathrm{Si{-}Si}) > D(\mathrm{Ge{-}Ge}).$$

Carbon forms strong chains because its smaller orbitals overlap effectively. Silicon and germanium have more diffuse orbitals and weaker overlap. The question is about the bonds between identical atoms, so it is these bond energies, rather than bonds to oxygen or another atom, that we compare.

Step 2: Assign the three numerical energies Among the supplied values the largest is 348, the middle value is 297 and the smallest is 260, all in $\mathrm{kJ\,mol^{-1}}$. Assigning them in decreasing order to carbon, silicon and germanium gives

$$D(\mathrm{C{-}C}) = 348, \quad D(\mathrm{Si{-}Si}) = 297, \quad D(\mathrm{Ge{-}Ge}) = 260.$$

Option (2) would make silicon stronger than carbon; option (3) would make germanium stronger than silicon; option (4) would reverse the overall carbon-to-germanium trend. Each contradicts the stated order and the orbital-overlap explanation.

Step 3: Conclusion The respective bond energies are $348, 297, 260\,\mathrm{kJ\,mol^{-1}}$. Hence, the correct option is (1).

Quick Tip to Solve: For these Group 14 single bonds, stronger self-bonding means greater catenation; put the largest energy beside carbon.

66. Which one of the following cannot function as an oxidising agent? [4 marks]

(1) $\mathrm{I^-}$

(2) $\mathrm{S(s)}$

(3) $\mathrm{NO_3{}^-(aq)}$

(4) $\mathrm{Cr_2O_7{}^{2-}}$

Correct Answer: (1) $\mathrm{I^-}$

Solution:

Concept:

- An oxidising agent accepts electrons and is itself reduced. Its relevant element must be able to attain a lower oxidation state in the reaction.
- For the ordinary chemistry of these species, iodide already has iodine in its lowest usual oxidation state, -1. Sulfur at zero, nitrogen at $+5$, and chromium at $+6$ can be reduced.

Step 1: Test the iodide ion In $\mathrm{I^-}$, iodine has oxidation state -1. It does not normally accept another electron to form a stable iodine species with an oxidation state below -1. Instead it loses electrons and acts as a reducing agent:

$$2\,\mathrm{I^-} \longrightarrow \mathrm{I_2} + 2\,\mathrm{e^-}.$$

The electrons appear on the product side, showing that this is oxidation of iodide. Hence iodide cannot perform the electron-accepting role asked about in the question.

Step 2: Check that the other species can accept electrons Elemental sulfur can be reduced from zero to -2, for example $\mathrm{S} + 2\,\mathrm{e^-} \longrightarrow \mathrm{S^{2-}}$. In nitrate, nitrogen is $+5$ and can fall to lower states; a familiar acidic reduction is $\mathrm{NO_3{}^- + 4\,H^+ + 3\,e^- \longrightarrow NO + 2\,H_2O}$. In dichromate, chromium is $+6$ and can

become $+3$:

$$Cr_2O_7^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O.$$

Each of these three reactions consumes electrons. Therefore sulfur, nitrate and dichromate can function as oxidising agents under suitable conditions, excluding options (2), (3) and (4).

Step 3: Conclusion I^- cannot function as an oxidising agent in this comparison. Hence, the correct option is (1).

Quick Tip to Solve: Ask which species has no ordinary lower oxidation state available: that species cannot be the electron acceptor.

67. In which of the following sets, all the given species are isostructural? [4 marks]

(1) CO_2, NO_2, ClO_2, SiO_2

(2) $PCl_3, AlCl_3, BCl_3, SbCl_3$

(3) BF_3, NF_3, PF_3, AlF_3

(4) $BF_4^-, CCl_4, NH_4^+, PCl_4^+$

Correct Answer: (4) $BF_4^-, CCl_4, NH_4^+, PCl_4^+$

Solution:

Concept:

- Isostructural species have the same molecular geometry. VSEPR predicts geometry from the number of bonding domains and lone pairs around the central atom.
- Four bonding domains with no lone pair give tetrahedral geometry. Three bonding domains plus one lone pair (four electron domains in total) give a trigonal pyramid, while three bonding domains without a lone pair give a trigonal plane.

Step 1: Identify the common shape of the fourth set In BF_4^-, boron makes four B–F bonds and has no central lone pair. Carbon in CCl_4 also has four bonds and no lone pair. In NH_4^+, nitrogen forms four bonds and the positive charge removes the lone pair of ammonia from the electron count. Phosphorus in PCl_4^+ likewise has four bonding pairs and no lone pair. Thus each central atom has four electron domains, all bonding, and each ion or molecule is tetrahedral. Charges do not by themselves determine the shape; the complete central valence-electron count does.

Step 2: Exclude the sets with mixed geometries The first set includes linear CO_2 and bent NO_2 and ClO_2, so it cannot be uniformly isostructural. In the second set, PCl_3 and $SbCl_3$ are pyramidal, whereas the simple monomeric BCl_3 and $AlCl_3$ structures are planar. The third set again mixes planar BF_3 with pyramidal NF_3 and PF_3; that already rules it out regardless of the solid-state structure of aluminium fluoride. Only the fourth set has the same molecular shape throughout.

Step 3: Conclusion BF_4^-, CCl_4, NH_4^+ and PCl_4^+ are all tetrahedral. Hence, the correct option is (4).

Quick Tip to Solve: Count central lone pairs as well as attached atoms: four attached atoms give a tetrahedron here because every central atom has zero lone pairs.

68. The internuclear distances in $O-O$ bonds for O_2^+, O_2, O_2^- and O_2^{2-}, respectively, are: [4 marks]

(1) 1.30 Å, 1.49 Å, 1.12 Å, 1.21 Å

(2) 1.49 Å, 1.21 Å, 1.12 Å, 1.30 Å

(3) 1.21 Å, 1.12 Å, 1.49 Å, 1.30 Å

(4) 1.12 Å, 1.21 Å, 1.30 Å, 1.49 Å

Correct Answer: (4) 1.12 Å, 1.21 Å, 1.30 Å, 1.49 Å

Solution:

Concept:

- Bond order is $(N_b - N_a)/2$, where N_b and N_a are the numbers of bonding and antibonding electrons. Higher bond order normally means a shorter bond for the same two atoms.
- The electron removed from O_2 or added to it occupies an antibonding π^* orbital. Removing it increases bond order; adding it decreases bond order.

Step 1: Calculate the relative bond orders Neutral oxygen has bond order 2. Removing one antibonding electron gives O_2^+, so the bond order becomes $2 + 1/2 = 2.5$. Adding one electron gives O_2^-, with bond order $2 - 1/2 = 1.5$, and adding two gives peroxide O_2^{2-}, with bond order $2 - 1 = 1$. Therefore

$$\mathrm{BO}(O_2^+) > \mathrm{BO}(O_2) > \mathrm{BO}(O_2^-) > \mathrm{BO}(O_2^{2-}).$$

The sequence is not obtained by simply comparing net charge: it depends on which molecular orbital gains or loses electrons.

Step 2: Match shorter bonds to higher bond orders The internuclear separation must increase as these bond orders decrease. Put the listed lengths in increasing order, $1.12 < 1.21 < 1.30 < 1.49$, and assign them successively to the four species in the question. This gives

$$O_2^+ : 1.12, \quad O_2 : 1.21, \quad O_2^- : 1.30, \quad O_2^{2-} : 1.49$$

in angstroms. The other three options each violate at least one of these inequalities; for example, they would make the peroxide bond shorter than a higher-order bond or give the cation a longer bond than neutral oxygen.

Step 3: Conclusion The distances are $1.12, 1.21, 1.30, 1.49$ Å, respectively. Hence, the correct option is (4).

Quick Tip to Solve: For the oxygen series, each added electron enters an antibonding orbital: bond order falls by one-half and the bond becomes longer.

69. The structure of which of the following chloro species can be explained on the basis of dsp^2 hybridization? [4 marks]

(1) $PdCl_4^{2-}$

(2) $FeCl_4^{2-}$

(3) $CoCl_4^{2-}$

(4) $NiCl_4^{2-}$

Correct Answer: (1) $PdCl_4^{2-}$

Solution:

Concept:

- In the valence-bond description, dsp^2 hybridisation uses one d, one s and two p orbitals to form four coplanar bonds with square-planar geometry.
- A four-coordinate complex can instead be tetrahedral with sp^3 hybridisation. The metal ion, its d-electron count, and the ligand-field splitting determine which geometry is favoured.

Step 1: Identify the square-planar palladium complex Each listed species has four chloride ligands and an overall charge of -2, so the central metal has oxidation state $+2$. Palladium is a 4d transition metal and Pd^{2+} has a $4d^8$ configuration. The relatively large splitting for this 4d metal favours the square-planar arrangement of $[PdCl_4]^{2-}$. In the valence-bond model, pairing leaves a suitable d orbital available to combine with one s and two p orbitals, giving four dsp^2 hybrid orbitals directed in a plane.

Step 2: Distinguish the other tetrachloro complexes Iron(II) is d^6, cobalt(II) is d^7, and nickel(II) is d^8, but chloride is a weak-field ligand for these 3d metal ions. Their tetrahedral tetrachloro complexes are described by sp^3, not dsp^2, hybridisation. In particular, one must not select $[NiCl_4]^{2-}$ merely because nickel(II), like palladium(II), has eight d electrons: the 3d and 4d metals behave differently with chloride. All four species are four-coordinate, so coordination number alone cannot distinguish the answer; the square-planar geometry does.

Step 3: Conclusion $[PdCl_4]^{2-}$ has the square-planar structure represented by dsp^2 hybridisation. Hence, the correct option is (1).

Quick Tip to Solve: Do not equate every four-coordinate complex with one hybridisation: palladium(II) chloride is square planar, while nickel(II) chloride is tetrahedral.

70. Which one of the following is the wrong assumption of kinetic theory of gases? [4 marks]

(1) Momentum and energy always remain conserved.

(2) Pressure is the result of elastic collision of molecules with the container's wall.

(3) Molecules are separated by great distances compared to their sizes.

(4) All the molecules move in straight line between collision and with same velocity.

Correct Answer: (4) All the molecules move in straight line between collision and with same velocity.

Solution:

Concept:

- In the ideal-gas model, molecules move in straight lines at constant velocity between collisions because intermolecular forces are neglected during that interval.
- The molecules have a distribution of speeds and directions. Elastic collisions conserve total momentum and kinetic energy, but can change the velocity of each colliding molecule.

Step 1: Separate straight-line motion from equal velocities The straight-line part of the fourth statement is an ideal-gas assumption: until the next collision there is no resultant force, so a molecule retains its velocity. The additional assertion that all molecules have the same velocity is false. At a given temperature the molecules have a range of speeds and random directions, as described by the Maxwell distribution. Temperature fixes an average kinetic energy,

$$\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT,$$

rather than requiring every molecule to have exactly the same kinetic energy or velocity. Even two molecules with the same speed may have different velocity vectors.

Step 2: Check the remaining assumptions Molecular collisions are treated as perfectly elastic, so total momentum and total kinetic energy are conserved in a collision; the first statement expresses that collision assumption. Molecules striking a wall transfer momentum to it, and the time rate of momentum transfer gives force and hence pressure, so the second statement is correct. The molecular dimensions are negligible relative to the mean separations in the dilute ideal-gas model, supporting the third statement. None of these statements requires individual velocities to remain equal. Thus the false clause is specifically the claim of a common velocity in the fourth option.

Step 3: Conclusion The assumption that all molecules have the same velocity is wrong. Hence, the correct option is (4).

Quick Tip to Solve: Keep the two ideas separate: each molecule has constant velocity between collisions, but different molecules do not have one common velocity.

71. In which of the following exothermic reactions, the heat liberated per mole is the highest ?

[4 marks]

(A) $CaO + H_2O \longrightarrow Ca(OH)_2$

(B) $SrO + H_2O \longrightarrow Sr(OH)_2$

(C) $BaO + H_2O \longrightarrow Ba(OH)_2$

(D) $MgO + H_2O \longrightarrow Mg(OH)_2$

Correct Answer: (C) $BaO + H_2O \longrightarrow Ba(OH)_2$

Solution:

Concept:

- Hydration of a basic oxide converts it into the corresponding hydroxide: $MO(s) + H_2O(l) \longrightarrow M(OH)_2(s)$. Heat liberated is the magnitude of the negative reaction enthalpy.
- For the alkaline-earth oxides listed, the exothermicity of reaction with water increases down the group. The decreasing stability of the oxide lattice relative to the hydroxide is relevant; this is not simply the hydration of isolated gaseous ions.

Step 1: Identify the common reaction Each option represents the same stoichiometric conversion of one mole of an alkaline-earth oxide into one mole of hydroxide. Therefore compare the reaction enthalpies per mole of oxide directly, without multiplying by any stoichiometric factor. All four metals form divalent ions, so the formal oxidation state does not distinguish the answers.

Step 2: Apply the group trend The order of increasing heat release for the listed oxides is

$$MgO < CaO < SrO < BaO.$$

Magnesium oxide reacts much less vigorously with water than the heavier oxides. Barium oxide consequently gives the largest magnitude of heat liberation. Options (A) and (B) are intermediate members of this trend, while (D) is its least exothermic member; none exceeds barium oxide.

Step 3: Conclusion The reaction $BaO + H_2O \longrightarrow Ba(OH)_2$ liberates the most heat per mole. This follows the oxide-to-hydroxide exothermicity trend, so (C) is retained rather than the supplied key's calcium-oxide reaction. Hence, the correct option is (C).

Quick Tip to Solve: Compare complete oxide-to-hydroxide reaction enthalpies down Group 2; do not replace this reaction with hydration of a gaseous metal ion.

72. Given that: (i) $\Delta_f H^\circ$ of N_2O is $82\,kJ\,mol^{-1}$; (ii) bond energies of $N{\equiv}N$, $N{=}N$, $O{=}O$ and $N{=}O$ are $946, 418, 498$ and $607\,kJ\,mol^{-1}$, respectively. The resonance energy of N_2O is: [4 marks]

(A) $-88\,kJ$

(B) $-66\,kJ$

(C) $-62\,kJ$

(D) $-44\,kJ$

Correct Answer: (A) $-88\,kJ$

Solution:

Concept:

- Hess's law gives an estimated reaction enthalpy as total bond energy of bonds broken minus that of bonds formed.
- Resonance stabilisation lowers the observed enthalpy below the hypothetical localised-bond value. Here the signed resonance energy is $H_{\text{actual}} - H_{\text{localised}}$.

Step 1: Calculate the localised-bond formation enthalpy Form one mole of N_2O from $N_2 + \frac{1}{2}\,O_2$. Breaking the elemental bonds requires

$$946 + \tfrac{1}{2}(498) = 1195\,kJ\,mol^{-1}.$$

The localised $N{=}N{=}O$ description contains one $N{=}N$ and one $N{=}O$ bond, whose formation releases

$$418 + 607 = 1025\,kJ\,mol^{-1}.$$

Thus the hypothetical formation enthalpy is $1195 - 1025 = 170\,kJ\,mol^{-1}$.

Step 2: Compare the hypothetical and actual states The measured formation enthalpy is only $82\,kJ\,mol^{-1}$. Since both states are compared with the same elements, their difference is the resonance contribution:

$$E_{\text{res}} = 82 - 170 = -88\,kJ\,mol^{-1}.$$

The negative sign expresses lowering of energy by delocalisation. Neither -66, -62 nor $-44\,kJ$ equals this Hess-law difference; halving it would incorrectly change the one-mole basis.

Step 3: Conclusion The signed resonance energy for one mole of N_2O is $-88\,kJ$. Hence, the correct option is (A).

Quick Tip to Solve: Use half an oxygen molecule for formation of one mole of nitrous oxide, then subtract the localised estimate from the measured enthalpy.

73. The ratio K_p/K_c for the reaction $\mathrm{CO(g)} + \frac{1}{2}\,\mathrm{O_2(g)} \rightleftharpoons \mathrm{CO_2(g)}$ is: [4 marks]

(A) $\dfrac{1}{\sqrt{RT}}$

(B) $(RT)^{1/2}$

(C) RT

(D) 1

Correct Answer: (A) $\dfrac{1}{\sqrt{RT}}$

Solution:

Concept:

- For an ideal gaseous equilibrium, $K_p = K_c(RT)^{\Delta n_g}$, using consistent pressure, concentration and gas-constant units.
- The exponent Δn_g is the sum of gaseous product coefficients minus the sum of gaseous reactant coefficients. Fractional coefficients must be retained.

Step 1: Count the gaseous coefficients The written reaction is $\mathrm{CO(g)} + \frac{1}{2}\,\mathrm{O_2(g)} \rightleftharpoons \mathrm{CO_2(g)}$. There is one mole of gaseous product and $1 + \frac{1}{2} = \frac{3}{2}$ moles of gaseous reactants on its stoichiometric basis. Therefore

$$\Delta n_g = 1 - \left(1 + \tfrac{1}{2}\right) = -\tfrac{1}{2}.$$

Oxygen contributes one-half, not one, to this count; the equilibrium has not been doubled.

Step 2: Substitute the negative exponent Dividing the standard relation by K_c gives

$$\frac{K_p}{K_c} = (RT)^{-1/2} = \frac{1}{\sqrt{RT}}.$$

A positive square-root factor, option (B), would correspond to the reversed reaction. Option (C) would require a gaseous mole change of $+1$, while option (D) would require zero change. Neither occurs in the printed equation. Although doubling the equation gives an integer exponent, it also squares both equilibrium constants and does not answer the ratio asked here.

Step 3: Conclusion $K_p/K_c = 1/\sqrt{RT}$. Hence, the correct option is (A).

Quick Tip to Solve: Write product-minus-reactant gaseous coefficients before inserting RT; a negative half exponent means the square root is in the denominator.

74. What would be the pH of a solution obtained by mixing $5\,\mathrm{g}$ of acetic acid and $7.5\,\mathrm{g}$ of sodium acetate and making the volume equal to $500\,\mathrm{mL}$? $(K_a = 1.75 \times 10^{-5},\ pK_a = 4.76)$ [4 marks]

(A) $\mathrm{pH} = 4.70$

(B) $\mathrm{pH} < 4.70$

(C) pH of solution will be equal to pH of acetic acid

(D) $4.76 < \mathrm{pH} < 5.0$

Correct Answer: (D) $4.76 < \mathrm{pH} < 5.0$

Solution:

Concept:

- A mixture of a weak acid and its conjugate-base salt is an acidic buffer. Its pH is $\mathrm{pH} = pK_a + \log_{10}([\mathrm{CH_3COO^-}]/[\mathrm{CH_3COOH}])$.
- For acid and salt in the same final volume, the concentration ratio equals their mole ratio. Convert the given masses to moles because the molar masses are different.

Step 1: Calculate the acid and salt amounts Using molar masses $60\,\mathrm{g\,mol^{-1}}$ for acetic acid and $82\,\mathrm{g\,mol^{-1}}$ for sodium acetate,

$$n_{\text{acid}} = \frac{5}{60} = 0.08333\,\mathrm{mol}, \qquad n_{\text{salt}} = \frac{7.5}{82} = 0.09146\,\mathrm{mol}.$$

The common volume is $0.500\,\mathrm{L}$, so it cancels from their concentration ratio. The ratio is $0.09146/0.08333 = 1.0976$.

Step 2: Evaluate the buffer pH and compare the options Apply the Henderson–Hasselbalch equation:

$$\mathrm{pH} = 4.76 + \log_{10}(1.0976) \simeq 4.76 + 0.0404 = 4.80.$$

The salt is present in slightly greater molar amount, so the pH lies slightly above pK_a. Thus it is neither 4.70 nor below 4.70, eliminating (A) and (B). Acetate supplies a substantial conjugate-base concentration, so the solution does not have the pH of acetic acid alone, eliminating (C).

Step 3: Conclusion The buffer pH is approximately 4.80, satisfying $4.76 < \mathrm{pH} < 5.0$. Hence, the correct option is (D).

Quick Tip to Solve: Compare salt and acid moles first: slightly more acetate than acetic acid places the buffer pH just above pKa.

75. Which one of the following arrangements represents the correct order of the proton affinity of the given species [4 marks]

(A) $I^- < F^- < HS^- < NH_2^-$

(B) $HS^- < NH_2^- < F^- < I^-$

(C) $F^- < I^- < NH_2^- < HS^-$

(D) $NH_2^- < HS^- < I^- < F^-$

Correct Answer: (A) $I^- < F^- < HS^- < NH_2^-$

Solution:

Concept:

- Proton affinity describes the tendency of a species to accept H^+. A stronger base has greater proton affinity.
- The relevant conjugate acids are HI, HF, H_2S and NH_3. A species whose conjugate acid more readily loses its proton has less tendency to bind that proton.

Step 1: Compare iodide with fluoride HI is a much stronger acid than HF. Its conjugate base I^- is correspondingly weaker than F^-. The large iodide ion holds an added proton less strongly than the compact fluoride ion. Hence the first comparison is $I^- < F^-$ in proton affinity. This immediately contradicts (C), and the position of iodide at the high-affinity end also rules out (B).

Step 2: Place hydrosulfide and amide H_2S is a weaker acid than HF, so HS^- accepts a proton more strongly than F^-. Ammonia is an extremely weak acid compared with either of these acids; its conjugate base NH_2^- is therefore the strongest proton acceptor listed. Combining the comparisons gives

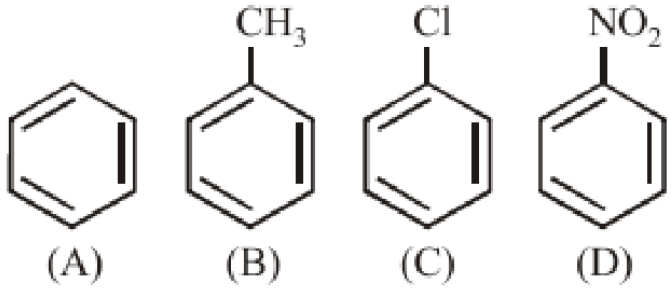
$$I^- < F^- < HS^- < NH_2^-.$$

Option (D) reverses the role of amide by making it the weakest proton acceptor and cannot be correct.

Step 3: Conclusion The increasing order is $I^- < F^- < HS^- < NH_2^-$. Hence, the correct option is (A).

Quick Tip to Solve: Write each anion with its conjugate acid, then reverse the acid-strength order to obtain the proton-affinity order.

76. Given the above compounds, the correct order of reactivity in electrophilic substitution reactions will be: [4 marks]



(A) B > A > C > D

(B) D > C > B > A

(C) A > B > C > D

(D) B > C > A > D

Correct Answer: (A) B > A > C > D

Solution:

Concept:

- Electrophilic substitution is faster when a substituent increases electron density in the benzene ring and stabilises the positively charged sigma complex.
- The pictured compounds are benzene (A), toluene (B), chlorobenzene (C) and nitrobenzene (D). A methyl group activates, chlorine deactivates overall, and a nitro group strongly deactivates the ring.

Step 1: Compare toluene with benzene The CH_3 group in B donates electron density by its inductive effect and hyperconjugation. This stabilises the intermediate formed when the ring attacks an electrophile. Toluene is consequently more reactive than unsubstituted benzene, so $B > A$. Option (C), which puts A above B, is inconsistent with this activating effect.

Step 2: Compare the deactivated rings Chlorine in C withdraws electron density by a strong negative inductive effect. Its resonance donation controls ortho/para orientation but does not make chlorobenzene more reactive than benzene overall. The NO_2 group in D withdraws by both inductive and resonance effects and strongly destabilises the cationic intermediate. Therefore $A > C > D$. Combining this with $B > A$ gives

$$B > A > C > D.$$

Option (B) incorrectly makes nitrobenzene fastest, while option (D) incorrectly places chlorobenzene above benzene.

Step 3: Conclusion The reactivity order is $B > A > C > D$. Hence, the correct option is (A).

Quick Tip to Solve: Keep orientation and rate separate: chlorine directs ortho/para but deactivates, whereas methyl both directs ortho/para and activates.

77. Copper crystallises in fcc with a unit length of 361 pm. What is the radius of copper atom ?

[4 marks]

(A) 157 pm

(B) 128 pm

(C) 108 pm

(D) 181 pm

Correct Answer: (B) 128 pm

Solution:

Concept:

- In a face-centred cubic unit cell, nearest-neighbour atoms touch along a face diagonal rather than an edge.
- For cube edge a, the face diagonal is $\sqrt{2}a$. It spans four atomic radii, giving $4r = \sqrt{2}a$, or $r = a/(2\sqrt{2})$.

Step 1: Use the correct line of contact A face diagonal passes through a corner atom, the face-centred atom and the opposite corner atom. The distances between their centres total $2r + 2r = 4r$. Equating this to the geometrical face diagonal gives

$$4r = \sqrt{2}a.$$

The body diagonal is not the contact direction in an fcc lattice, so the bcc expression $\sqrt{3}a/4$ must not be used. Likewise, $a = 2r$ would describe simple-cubic edge contact, not fcc packing.

Step 2: Substitute the printed edge length For $a = 361\,\text{pm}$,

$$r = \frac{\sqrt{2}(361)}{4}\,\text{pm} = \frac{361}{2\sqrt{2}}\,\text{pm} \simeq 127.6\,\text{pm}.$$

Rounding to the whole-picometre options gives $128\,\text{pm}$. The unit remains picometres because the geometrical factor is dimensionless. Option (A), about $157\,\text{pm}$, corresponds to using the bcc contact relation; option (D), about $181\,\text{pm}$, is half the edge; and (C) does not satisfy face-diagonal contact.

Step 3: Conclusion The copper atomic radius is approximately $128\,\text{pm}$. Hence, the correct option is (B).

Quick Tip to Solve: For fcc remember that four radii lie on a face diagonal; use a divided by twice the square root of two.

78. The Gibbs energy for the decomposition of Al_2O_3 at 500 C is as follows:

$$\frac{2}{3}Al_2O_3 \longrightarrow \frac{4}{3}Al + O_2, \qquad \Delta_r G = +940\,\text{kJ mol}^{-1}.$$

The potential difference needed for the electrolytic reduction of aluminium oxide at $500^\circ C$ should be at least: [4 marks]

(A) 4.5 V

(B) 3.0 V

(C) 5.0 V

(D) 2.5 V

Correct Answer: (D) 2.5 V

Solution:

Concept:

- The minimum reversible electrical work needed for a non-spontaneous reaction equals its positive Gibbs energy change. For the written stoichiometric reaction, $\Delta_r G = nFE_{\text{min}}$, where n is the number of moles of electrons and $F \approx 96500\,\text{C mol}^{-1}$.
- The value of n must refer to exactly the same reaction as the quoted Gibbs energy. Aluminium changes from oxidation state +3 to zero, so every mole of aluminium requires three moles of electrons.

Step 1: Count electrons for the written reaction The reaction produces $4/3$ mole of aluminium. Its cathodic reduction is therefore

$$\frac{4}{3}Al^{3+} + 4e^- \longrightarrow \frac{4}{3}Al.$$

Thus $n = 4$, not 3 or 6. Equivalently, oxidation of two oxide ions gives one molecule of oxygen and releases four electrons, confirming the same count. Scaling the reaction without scaling its Gibbs energy would change the calculated voltage incorrectly.

Step 2: Convert energy to voltage Convert kilojoules to joules before dividing by charge:

$$E_{\text{min}} = \frac{940 \times 10^3}{4 \times 96500} = 2.435\,\text{V}.$$

A positive applied voltage supplies the required energy; its magnitude is used here. Of the listed choices, 2.5 V is the closest threshold above the reversible value. The values 3.0, 4.5 and 5.0 V also exceed that value but are unnecessarily large for the requested minimum. Actual electrolysis can require extra voltage because of resistance and overpotentials, which are not specified in this thermodynamic question.

Step 3: Conclusion The reversible threshold is about $2.44\,\text{V}$, represented by $2.5\,\text{V}$ in the choices. Hence, the correct option is (D). The supplied answer key lists 3.0 V, but the electron count in the printed fractional reaction gives the smaller threshold above, so (D) follows from the printed data.

Quick Tip to Solve: Count electrons from the fractional reaction exactly as written before using Gibbs energy divided by nF.

79. A solution of copper sulphate ($CuSO_4$) is electrolysed for 10 minutes with a current of 1.5 amperes. The mass of copper deposited at the cathode (at. mass of Cu = 63 u) is: [4 marks]

(A) 0.3892 g

(B) 0.2938 g

(C) 0.2398 g

(D) 0.3928 g

Correct Answer: (B) 0.2938 g

Solution:

Concept:

- Faraday's law gives deposited mass $m = MQ/(nF)$, where M is molar mass, $Q = It$ is charge and n is the electrons required per metal ion.
- Copper sulphate contains Cu^{2+}; at the cathode, $Cu^{2+} + 2\,e^- \longrightarrow Cu$. Thus one mole of copper requires two faradays, not one.

Step 1: Calculate the delivered charge The current is constant at $1.5\,A = 1.5\,C\,s^{-1}$. The duration must be in seconds:

$$t = 10 \times 60 = 600\,s, \qquad Q = 1.5 \times 600 = 900\,C.$$

This is the total electron charge available for reducing copper ions. The calculation assumes the stated electrolysis deposits copper with the usual unit current efficiency, since no competing process or efficiency correction is supplied.

Step 2: Use the copper equivalent mass The given atomic mass corresponds to $M = 63\,g\,mol^{-1}$. Using $F = 96500\,C\,mol^{-1}$,

$$m = \frac{63 \times 900}{2 \times 96500}\,g = 0.29378\,g \approx 0.2938\,g.$$

As a check, the amount of electrons is $900/96500$ mol and the amount of copper is half that value. Multiplying that amount by 63 reproduces the mass. Using a one-electron reduction would double the result, while using minutes as seconds would reduce it by a factor of sixty. None of the other three listed masses follows from the stated charge and copper valency.

Step 3: Conclusion The deposited mass is $0.2938\,g$. Hence, the correct option is (B).

Quick Tip to Solve: For copper sulphate, divide the charge in faradays by two before multiplying by the molar mass of copper.

80. A radioactive isotope having a half-life period of 3 days was received after 12 days. If 3 g of the isotope is left in the container, what would be the initial mass of the isotope? [4 marks]

(A) 12 g

(B) 36 g

(C) 48 g

(D) 24 g

Correct Answer: (C) 48 g

Solution:

Concept:

- Radioactive decay follows first-order kinetics. The surviving amount after time t is $m = m_0(1/2)^{t/t_{1/2}}$, where m_0 is the initial mass of the radioactive isotope.
- Each completed half-life halves the amount of the undecayed isotope. When working backwards from a measured surviving mass, multiply by two for every elapsed half-life.

Step 1: Count the half-lives The elapsed time is 12 days and the half-life is 3 days. Their units already agree, so

$$N = \frac{12}{3} = 4.$$

There are four successive halvings, not three. The corresponding surviving fraction is

$$(1/2)^4 = 1/16.$$

The question concerns the mass of the radioactive isotope still present, rather than the combined mass of isotope and decay products in a sealed container.

Step 2: Recover the initial isotope mass Substitute the measured surviving mass $m = 3\,\mathrm{g}$:

$$3 = \frac{m_0}{16}, \qquad m_0 = 3 \times 16 = 48\,\mathrm{g}.$$

Check by running the process forward: after 3, 6, 9 and 12 days, the isotope masses would be 24, 12, 6 and 3 g respectively. Thus the final observation is reproduced exactly. A starting mass of 12 g would leave only 0.75 g after four half-lives, 36 g would leave 2.25 g and 24 g would leave 1.5 g. These alternatives do not match the stated remaining mass; adding a fixed amount per half-life would also be inconsistent with exponential decay.

Step 3: Conclusion The initial mass of the isotope was $48\,\mathrm{g}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the number of half-lives as elapsed time divided by half-life, then multiply the surviving mass by the corresponding power of two.

81. In which of the following octahedral complex species will the magnitude of Δ_o be maximum?

[4 marks]

(A) $[Co(H_2O)_6]^{2+}$

(B) $[Co(CN)_6]^{3-}$

(C) $[Co(C_2O_4)_3]^{3-}$

(D) $[Co(NH_3)_6]^{3+}$

Correct Answer: (B) $[Co(CN)_6]^{3-}$

Solution:

Concept:

- For an octahedral complex, Δ_o is the energy separation between the lower t_{2g} and higher e_g orbital sets. Its size depends on the metal, oxidation state and ligand field strength.
- For the ligands in these choices, the relevant spectrochemical ordering is ${C_2O_4}^{2-} < H_2O < NH_3 < CN^-$. Cyanide gives a particularly strong ligand field.

Step 1: Identify the oxidation states Water and ammonia are neutral, so the water complex contains cobalt(II), whereas the ammonia complex contains cobalt(III). In $[Co(CN)_6]^{3-}$, six cyanide ions contribute charge -6; hence cobalt has oxidation state +3. Three oxalate ligands likewise contribute -6 in $[Co(C_2O_4)_3]^{3-}$, giving cobalt(III). Thus cyanide, oxalate and ammonia can be directly compared at the same metal oxidation state.

Step 2: Compare the ligand fields Among those cobalt(III) complexes, cyanide is the strongest-field ligand and therefore produces the largest octahedral splitting. Ammonia gives substantial splitting but lies below cyanide in the spectrochemical series; oxalate is weaker still. The aqua complex additionally has the lower cobalt oxidation state, which generally reduces metal-ligand interaction and splitting. There is no need to count unpaired electrons or calculate a numerical crystal-field stabilization energy: those are consequences of the splitting, whereas the question asks which complex has the largest orbital separation. The cyanide complex therefore dominates the available comparisons consistently.

Step 3: Conclusion $[Co(CN)_6]^{3-}$ has the maximum Δ_o. Hence, the correct option is (B).

Quick Tip to Solve: Compare oxidation states first; among complexes of the same cobalt ion, cyanide produces a stronger field than ammonia, water or oxalate.

82. In nucleophilic substitution reaction, order of halogens as incoming (attacking) nucleophile is: $I^- > Br^- > Cl^-$. The order of halogens as departing nucleophile should be: [4 marks]

(A) $Br^- > I^- > Cl^-$

(B) $I^- > Br^- > Cl^-$

(C) $Cl^- > Br^- > I^-$

(D) $Cl^- > I^- > Br^-$

Correct Answer: (B) $I^- > Br^- > Cl^-$

Solution:

Concept:

- A leaving group departs with the electron pair of the bond being broken. A good leaving group can stabilize the negative charge after departure, so weak bases generally leave more readily than strong bases.
- For halide ions, leaving-group ability follows $I^- > Br^- > Cl^-$. The larger iodide ion disperses negative charge well and the carbon-iodine bond is weaker than the corresponding carbon-bromine or carbon-chlorine bond.

Step 1: Interpret the departing species The phrase departing nucleophile refers to the halide leaving group. It is not a request to reverse the stated incoming-nucleophile series automatically. Incoming nucleophilicity measures attack on an electrophilic carbon and can depend strongly on solvent. Leaving-group ability instead measures the ease of carbon-halogen bond cleavage and stability of the departing ion. These are distinct comparisons, even when their orders coincide under the conditions implicit in the question.

Step 2: Rank the halide leaving groups Iodide is the weakest base of the three halide ions and is most readily accommodated as a departing anion. Bromide is intermediate, and chloride is the least effective leaving group among these three. Therefore

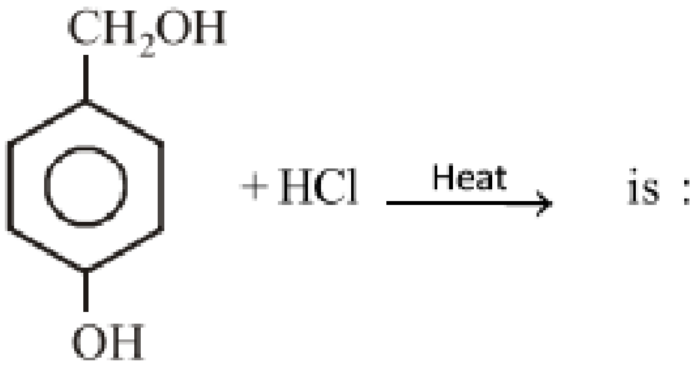
$$I^- > Br^- > Cl^-.$$

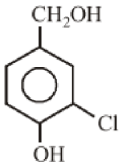
Option (A) incorrectly places bromide ahead of iodide. Option (C) reverses the full series and would rank chloride as the best leaving group, contrary to both bond-strength and basicity considerations. Option (D) also incorrectly puts chloride first and bromide last. The printed second option alone matches the standard leaving-group sequence.

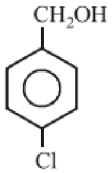
Step 3: Conclusion The departing-group order is $I^- > Br^- > Cl^-$. Hence, the correct option is (B).

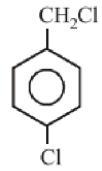
Quick Tip to Solve: For a departing halide, use weak basicity and ease of carbon-halogen bond cleavage rather than mechanically reversing nucleophilicity.

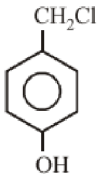
83. The major product in the following reaction is: [4 marks]

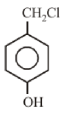


(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- Hydrogen chloride converts a benzylic alcohol to the corresponding benzyl chloride by replacing the hydroxyl group on the side chain. Protonation makes water a good leaving group.
- The hydroxyl group directly bonded to an aromatic ring is phenolic. Its carbon-oxygen bond has partial double-bond character from resonance and is not replaced by chloride through the ordinary alcohol substitution pathway.

Step 1: Locate the two different hydroxyl groups The starting molecule has a CH_2OH substituent and a phenolic OH substituent in para positions. The former is attached through a saturated benzylic carbon, whereas the latter is attached directly to the aromatic ring. Although both groups contain an oxygen-hydrogen bond, their substitution behaviour with hydrochloric acid is different. Treating them as two identical alcoholic groups would incorrectly replace both oxygen-containing groups.

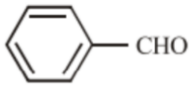
Step 2: Apply hydrochloric acid selectively The benzylic hydroxyl oxygen is protonated, and loss or displacement of water followed by chloride capture converts $-CH_2OH$ to $-CH_2Cl$. The phenolic hydroxyl remains in place. Hence the product is para-(chloromethyl)phenol. Option (A) instead introduces chlorine onto the ring, but hydrochloric acid is not an electrophilic chlorinating reagent. Option (B) replaces the phenolic hydroxyl while retaining the benzylic alcohol, which reverses the

expected selectivity. Option (C) replaces both groups and also incorrectly converts the aryl hydroxyl to aryl chloride. Only option (D) changes the benzylic group and preserves the phenolic group.

Step 3: Conclusion The product is $HO-C_6H_4-CH_2Cl$ with the substituents para to each other. Hence, the correct option is (D). The supplied answer key selects the dichloro product, but that product would also replace a phenolic hydroxyl with chlorine; the printed HCl treatment instead gives (D).

Quick Tip to Solve: Distinguish a benzylic alcohol from a phenol: HCl replaces the side-chain hydroxyl while the ring-bound hydroxyl remains.

84. Cannizaro's reaction is not given by: [4 marks]

(A) 

(B) 

(C) CH_3CHO

(D) $HCHO$

Correct Answer: (C) CH_3CHO

Solution:

Concept:

- The Cannizzaro reaction is disproportionation of an aldehyde without an alpha hydrogen in concentrated alkali. One molecule is reduced to an alcohol and another is oxidized to a carboxylate.
- An aldehyde with an alpha hydrogen can form an enolate and generally undergoes aldol chemistry instead. The alpha carbon is the carbon directly adjacent to the aldehyde carbon, not the aldehyde carbon itself.

Step 1: Inspect the aldehydes without alpha hydrogen Option (A) is benzaldehyde. The aromatic carbon attached to the CHO group has no hydrogen available as an alpha hydrogen for enolate formation. Option (B) has a cyclohexane carbon bonded to the aldehyde group, a methyl group and two ring carbons; that alpha carbon therefore has no hydrogen. Formaldehyde in option (D) has no alpha carbon at all. All three satisfy the structural condition for Cannizzaro disproportionation under suitable strongly alkaline conditions.

Step 2: Identify the aldehyde with alpha hydrogen Acetaldehyde, CH_3CHO, contains a methyl carbon directly next to its carbonyl carbon, and that methyl group supplies three alpha hydrogens. Base can remove one of these hydrogens to generate the enolate intermediate responsible for aldol addition. Thus acetaldehyde is the exception requested by the negative wording. The hydrogen attached directly to the aldehyde carbon must not be counted as an alpha hydrogen: every aldehyde has that aldehydic hydrogen, including the three compounds that do undergo Cannizzaro reaction. This distinction rules out selecting formaldehyde merely because its formula contains hydrogen.

Step 3: Conclusion CH_3CHO does not give the usual Cannizzaro reaction. Hence, the correct option is (C).

Quick Tip to Solve: Check the carbon adjacent to CHO for a hydrogen; do not mistake the aldehydic hydrogen for an alpha hydrogen.

85. Phenol on heating with $CHCl_3$ and NaOH gives salicylaldehyde. The reaction is called: [4 marks]

(1) Reimer–Tiemann reaction

(2) Claisen reaction

(3) Cannizzaro's reaction

(4) Hell–Volhard–Zelinsky reaction

Correct Answer: (1) Reimer–Tiemann reaction

Solution:

Concept:

- The Reimer–Tiemann reaction introduces a formyl group, $-CHO$, into phenol using chloroform and aqueous alkali, followed by acidic work-up.
- In alkaline solution phenol forms phenoxide, which activates its ortho and para positions. Chloroform supplies the reactive dichlorocarbene intermediate, $:CCl_2$.

Step 1: Identify the transformation The starting material is phenol, whereas salicylaldehyde is 2-hydroxybenzaldehyde. Thus the transformation replaces an ortho ring hydrogen by a formyl group while retaining the phenolic hydroxyl group. The name salicylaldehyde locates the $-CHO$ group adjacent to $-OH$. This is formylation of the aromatic ring, rather than oxidation of the hydroxyl group.

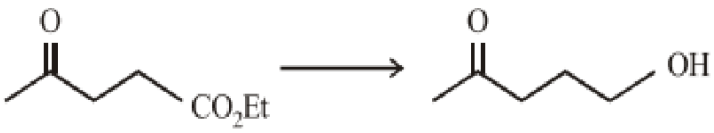
Step 2: Match the reagents and product Chloroform reacts with strong base to generate dichlorocarbene. The activated phenoxide ring reacts predominantly at the ortho position; subsequent hydrolysis gives the aldehyde group. The characteristic combination phenol, $CHCl_3$, and $NaOH$, producing salicylaldehyde, identifies the Reimer–Tiemann reaction.

Step 3: Exclude the other named reactions Claisen condensation forms a carbon–carbon bond between ester-derived species and does not describe this phenol formylation. The Cannizzaro reaction is disproportionation of an aldehyde without an alpha hydrogen into an alcohol and a carboxylate. The Hell–Volhard–Zelinsky reaction halogenates the alpha position of a carboxylic acid. None gives the stated reagent-to-product correspondence.

Step 4: Conclusion The formation of salicylaldehyde from phenol with chloroform and alkali is the Reimer–Tiemann reaction. Hence, the correct option is (1).

Quick Tip to Solve: Associate phenol + $CHCl_3/NaOH$ with ortho formylation: salicylaldehyde signals Reimer–Tiemann.

86. Which of the following reagent(s) used for the conversion? [4 marks]



(1) glycol/$LiAlH_4/H_3O^+$

(2) glycol/NaH/H_3O^+

(3) $LiAlH_4$

(4) $NaBH_4$

Correct Answer: (1) glycol/$LiAlH_4/H_3O^+$

Solution:

Concept:

- $LiAlH_4$ reduces both esters and ketones. An ester gives a primary alcohol, whereas a ketone gives a secondary alcohol.
- Ethylene glycol protects a ketone as a cyclic ketal. Ketals resist hydride reduction and can be hydrolysed back to the ketone by aqueous acid.

Step 1: Identify the group that must be changed The reaction figure contains a ketone at one end of the chain and an ethyl ester at the other. In the product, the ester group has become a terminal primary alcohol, but the original ketone carbonyl is still present. Therefore the required sequence must reduce the ester selectively while preserving the ketone functionality. Simply applying a strong reducing agent to both groups would give the wrong product.

Step 2: Protect, reduce, and restore the ketone First use glycol to convert the ketone into its cyclic ketal under the usual acid-catalysed protection conditions. The ester remains available for reduction. Treatment with $LiAlH_4$ then converts the ester group into $-CH_2OH$; the protected ketone is unaffected by this hydride reagent. Finally, H_3O^+ hydrolyses the ketal, regenerating the ketone seen in the product.

Step 3: Compare all the reagent choices Option (1) supplies this protection–reduction–deprotection sequence. Sodium hydride in option (2) is mainly a strong base and does not reduce an ester into the primary alcohol here. Unprotected $LiAlH_4$, option (3), would also reduce the ketone. $NaBH_4$, option (4), normally reduces the ketone and leaves the ester unreduced, the reverse of the required selectivity.

Step 4: Conclusion Use glycol, then $LiAlH_4$, then H_3O^+ to obtain the indicated keto alcohol. Hence, the correct option is (1).

Quick Tip to Solve: When an ester must be reduced while a ketone survives, protect the ketone as a ketal before using $LiAlH_4$.

87. Carbylamine forms from aliphatic or aromatic primary amine via which of the following intermediates?

[4 marks]

(1) Carbanion

(2) Carbene

(3) Carbocation

(4) Carbon radical

Correct Answer: (2) Carbene

Solution:

Concept:

- The carbylamine reaction converts a primary amine, RNH_2, into an isocyanide, RNC, using chloroform and strong alkali.
- Base-induced alpha elimination from chloroform produces dichlorocarbene, $:CCl_2$, a neutral electron-deficient species with a divalent carbon.

Step 1: Generate the reactive species The base first removes the relatively acidic hydrogen of chloroform, producing the trichloromethyl anion. Loss of chloride from that anion gives dichlorocarbene. The two stages can be represented as follows:

$$CHCl_3 + OH^- \longrightarrow CCl_3^- + H_2O$$

$$CCl_3^- \longrightarrow :CCl_2 + Cl^-$$

The intermediate that reacts with the primary amine is therefore a carbene.

Step 2: Connect carbene formation to carbylamine The nitrogen lone pair of the primary amine attacks the electron-deficient carbon of dichlorocarbene. Further proton transfers and base-promoted elimination of hydrogen chloride lead to an isocyanide, with connectivity $R{-}N{\equiv}C$. This explains why the same test works for both aliphatic and aromatic primary amines: both possess the required primary amino group.

Step 3: Distinguish the listed intermediates A carbanion does occur briefly in generating dichlorocarbene, but the characteristic intermediate through which the amine is converted into carbylamine is dichlorocarbene itself. A carbocation would be a positively charged carbon species; a radical would have an unpaired electron. Neither describes the neutral divalent $:CCl_2$ intermediate produced by this alpha-elimination route.

Step 4: Conclusion The characteristic intermediate is dichlorocarbene, $:CCl_2$. Hence, the correct option is (2).

Quick Tip to Solve: In the carbylamine test, remember the chloroform-to-$:CCl_2$ alpha elimination before the amine attacks.

88. Which of the following statement is not correct? [4 marks]

(1) Amylopectin is a branched polymer of α-glucose.

(2) Cellulose is a linear polymer of β-glucose.

(3) Glycogen is the food reserve of plants.

(4) All proteins are polymers of α-amino acids.

Correct Answer: (3) Glycogen is the food reserve of plants.

Solution:

Concept:

- Polysaccharides differ in their monomers, glycosidic linkages, branching and biological roles. Amylopectin and glycogen contain alpha-linked glucose units.
- Plants mainly store carbohydrate as starch, whereas animals store it as glycogen. Proteins are polypeptides built from alpha-amino acids.

Step 1: Check the glucose polymers Amylopectin is the branched component of starch. Its alpha-glucose units are joined mainly by alpha-(1 to 4) glycosidic bonds, with alpha-(1 to 6) bonds at branching points. Thus statement (1) is correct. Cellulose consists of unbranched chains of beta-glucose joined through beta-(1 to 4) linkages. Its linear chains form the structural material of plant cell walls, so statement (2) is also correct.

Step 2: Check the biological food reserve Glycogen is a highly branched glucose polymer that serves as the major carbohydrate reserve in animals, particularly in liver and muscle. It is also found as a storage polysaccharide in fungi. The usual food reserve of plants is starch, comprising amylose and amylopectin. Therefore assigning glycogen as the food reserve of plants is the incorrect statement.

Step 3: Check the protein statement and the negative demand Proteins are composed of alpha-amino acid residues linked by peptide bonds between amino and carboxyl groups. Statement (4) is therefore correct in the biochemical context of this question. Since the question asks for the statement that is not correct, the three true statements must be excluded and the plant-storage claim selected.

Step 4: Conclusion Glycogen is principally an animal storage polysaccharide; plants store carbohydrate as starch. Hence, the incorrect statement and the correct option is (3).

Quick Tip to Solve: Pair starch with plant storage, glycogen with animal storage, and cellulose with plant structure before checking the option wording.

89. Bakelite is obtained from phenol by reacting with [4 marks]

(1) Acetal

(2) CH_3CHO

(3) HCHO

(4) Chlorobenzene

Correct Answer: (3) HCHO

Solution:

Concept:

- Bakelite is a phenol–formaldehyde condensation polymer. Formaldehyde has the molecular formula $HCHO$.
- Phenol has reactive ortho and para positions, allowing formaldehyde-derived methylene bridges to join aromatic rings into a cross-linked network.

Step 1: Identify the comonomer The name phenol–formaldehyde resin identifies the two monomers: phenol and formaldehyde. Formaldehyde is methanal, $HCHO$, the simplest aldehyde. On reaction with phenol under suitable acidic or basic conditions, it first gives hydroxymethyl derivatives at positions activated by the phenolic hydroxyl group.

Step 2: Explain the formation of the polymer Further condensation links phenolic rings through methylene, $-CH_2-$, bridges with elimination of water. The initial products can form relatively linear resins; continued reaction and curing yield the extensively cross-linked material called Bakelite. Because bonds connect chains into a network, Bakelite is a thermosetting polymer and cannot be softened and remoulded repeatedly by heating.

Step 3: Match and eliminate the options Option (3), $HCHO$, is formaldehyde and is the required comonomer. CH_3CHO, option (2), is acetaldehyde, not the formaldehyde used to define Bakelite. An acetal is a functional derivative rather than the stated aldehyde monomer. Chlorobenzene lacks the aldehyde group needed for the phenol–formaldehyde condensation. These alternatives do not give the standard Bakelite preparation.

Step 4: Conclusion Phenol reacts with formaldehyde, $HCHO$, to form Bakelite. Hence, the correct option is (3).

Quick Tip to Solve: Remember Bakelite as the phenol–formaldehyde resin: the one-carbon aldehyde $HCHO$ supplies its methylene bridges.

90. Among the following vitamins the one whose deficiency causes rickets (bone deficiency) is

[4 marks]

(1) Vitamin A

(2) Vitamin B

(3) Vitamin D

(4) Vitamin C

Correct Answer: (3) Vitamin D

Solution:

Concept:

- Vitamin D supports calcium and phosphate absorption and their use in mineralising bone. Adequate mineralisation gives bones their normal hardness.
- Vitamin D deficiency causes rickets in children; the corresponding defective mineralisation in adults is termed osteomalacia.

Step 1: Connect the disease with mineralisation Rickets is a disorder of growing bone in which the bone matrix and growth regions fail to mineralise normally. The resulting bones become soft and may bend under body weight. A supply of calcium in the diet alone is not sufficient if its absorption and regulation are impaired; vitamin D is needed for normal calcium and phosphate homeostasis.

Step 2: Identify the deficient vitamin Vitamin D, through its active metabolites, promotes intestinal absorption of calcium and phosphate and supports mineral deposition in bone. When it is deficient, mineral availability for the developing skeleton falls, producing the characteristic bone defects of rickets. Therefore vitamin D is the vitamin linked directly with the disease named in the question.

Step 3: Rule out the alternatives Vitamin A deficiency is classically associated with night blindness and epithelial problems, rather than rickets. Deficiencies of B-group vitamins cause disorders such as beriberi or other metabolic and neurological conditions depending on the specific vitamin; they do not identify rickets. Vitamin C deficiency causes scurvy through defective collagen formation, with bleeding gums and poor wound healing. These disease associations distinguish option (3) from the other choices.

Step 4: Conclusion Rickets is caused by deficiency of vitamin D. Hence, the correct option is (3).

Quick Tip to Solve: For vitamin-deficiency MCQs, pair D with bone mineralisation and rickets, A with night blindness, and C with scurvy.