JEE Main 2013 Paper 1 (B.E./B.Tech) Apr 7 Shift 1 (morning) Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2013, Annual (offline), conducted by the Central Board of Secondary Education (CBSE) in pen and paper mode on 7 April 2013, Single sitting. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of A $\times$ B having 3 or more elements is : [4 marks]

(A) 219

(B) 211

(C) 256

(D) 220

Correct Answer: (A) 219

Solution:

Concept:

- The Cartesian product contains all ordered pairs, so $|A \times B| = |A||B|$. Distinct ordered pairs are the elements whose subsets are counted.
- A set with n elements has 2^n subsets. Exactly $\binom{n}{r}$ of them contain r elements; the empty set counts as one subset.

Step 1: Count the available elements Each of the two elements of A can be paired with each of the four elements of B. Hence the product has $2 \times 4 = 8$ elements. We count subsets of this eight-element set, rather than subsets of either factor separately.

Step 2: Subtract the excluded subset sizes The complement of having at least three elements is having zero, one or two elements. Therefore

$$N = 2^8 - \left[\binom{8}{0} + \binom{8}{1} + \binom{8}{2}\right] = 256 - (1 + 8 + 28) = 219.$$

There is no ordering of the selected pairs within a subset, so permutations must not be used. The value 256 includes every excluded subset; 220 forgets the empty set. The proposed count 211 also does not equal the required complement.

Step 3: Conclusion The number of subsets containing three or more elements is $\boxed{219}$. Hence, the correct option is (A).

Quick Tip to Solve: For an at-least-three subset count, subtract sizes zero, one and two from the power-set size.

2. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given? [4 marks]

(A) median

(B) mode

(C) variance

(D) mean

Correct Answer: (C) variance

Solution:

Concept:

- Adding a constant shifts every observation and its mean by the same amount. Deviations from the mean therefore remain unchanged.
- For observations $x_1, \ldots, x_n$, variance is $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$; it measures spread rather than location.

Step 1: Translate the observations and their mean Write the original scores as x_i, and the revised scores as $y_i = x_i + 10$. Their mean is

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n}(x_i + 10) = \bar{x} + 10.$$

Thus the mean changes. The revised score differs from the revised mean by $y_i - \bar{y} = x_i - \bar{x}$.

Step 2: Compare spread and position measures Using the unchanged deviations gives

$$\mathrm{Var}(Y) = \frac{1}{n}\sum_{i=1}^{n}[(x_i + 10) - (\bar{x} + 10)]^2 = \mathrm{Var}(X).$$

The ordering and frequencies of scores do not change, but every middle score increases by 10, so the median increases by 10. The most frequent score also increases by 10, so the mode shifts as well. Consequently none of the three position measures remains fixed; only variance does.

Step 3: Conclusion The variance remains unchanged after the uniform addition. Hence, the correct option is (C).

Quick Tip to Solve: Adding a constant changes location measures; multiplying by a constant changes variance by its square.

3. $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ **is equal to** [4 marks]

(A) $-\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 1

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- Use $1-\cos 2x = 2\sin^2 x$ to expose the quadratic zero in the numerator.
- The standard limits, with angles in radians, are $\lim_{x\to 0} \sin x/x = 1$ and $\lim_{u\to 0} \tan u/u = 1$.

Step 1: Rewrite into standard-limit factors Direct substitution produces an indeterminate ratio, so cancelling by an identity is necessary. For nonzero x near zero,

$$\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x} = \frac{1}{2}\left(\frac{\sin x}{x}\right)^2 (3+\cos x)\frac{4x}{\tan 4x}.$$

The factor $1/2$ comes from the factor 2 in the cosine identity divided by the factor 4 in the tangent argument.

Step 2: Take each finite limit All factors on the right have finite limits: $\sin x/x \to 1$, $3+\cos x \to 4$, and $4x/\tan 4x \to 1$. Thus

$$L = \frac{1}{2}\cdot 1^2 \cdot 4 \cdot 1 = 2.$$

The expression is positive sufficiently close to zero, excluding the negative option. Values $1/2$ and 1 omit one of the limiting constants; neither retains both the cosine factor 4 and the tangent scale 4.

Step 3: Conclusion The limit equals $\boxed{2}$. Hence, the correct option is (D).

Quick Tip to Solve: For trigonometric zero-over-zero limits, track the argument multipliers before taking limits.

4. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dp}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is [4 marks]

(A) 2500

(B) 3000

(C) 3500

(D) 4500

Correct Answer: (C) 3500

Solution:

Concept:

- The integral of dP/dx over an interval is the change in production, not the final production.

- The variable x counts additional workers; hence the initial condition is $P(0) = 2000$. The power rule gives $\int x^{1/2}dx = \frac{2}{3}x^{3/2}$.

Step 1: Integrate over the additional workers The change when the firm hires 25 workers is

$$P(25) - P(0) = \int_0^{25} (100 - 12\sqrt{x})\, dx.$$

Using a definite integral automatically applies the starting level at zero additional workers and avoids confusing the rate at the endpoint with the accumulated increase.

Step 2: Evaluate and add the initial output Integration gives

$$\begin{aligned} P(25) - 2000 &= [100x - 8x^{3/2}]_0^{25} \\ &= 2500 - 8(125) = 1500. \end{aligned}$$

Therefore $P(25) = 2000 + 1500 = 3500$ items. The rate decreases from 100 to 40 over this interval but stays positive, so an increase of 1500 is consistent with the bounds on the rate. The alternatives 2500, 3000 and 4500 do not equal the initial output plus the integrated change; 4500 would ignore the decreasing-rate term.

Step 3: Conclusion The new production level is $\boxed{3500 \text{ items}}$. Hence, the correct option is (C).

Quick Tip to Solve: Integrate a marginal production rate to get the increase, then add the initial production.

5. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is : [4 marks]

(A) 9

(B) 36

(C) 18

(D) $\frac{27}{4}$

Correct Answer: (A) 9

Solution:

Concept:

- Horizontal strips are convenient because $y = \sqrt{x}$ becomes $x = y^2$, while the line becomes $x = 2y + 3$.
- The first-quadrant condition gives $y \geq 0$. The strip width is right boundary minus left boundary, and area is its integral over the appropriate heights.

Step 1: Find the upper intersection height At the intersection, $y^2 = 2y + 3$, so $(y - 3)(y + 1) = 0$. Only $y = 3$ lies in the first quadrant; the corresponding point is $(9, 3)$. The lower boundary is the x-axis, $y = 0$. For $0 \leq y \leq 3$, the parabola $x = y^2$ is on the left and the line $x = 2y + 3$ is on the right.

Step 2: Integrate the horizontal widths The entire enclosed region is covered once by the strips,

giving

$$A = \int_0^3 (2y + 3 - y^2)\, dy$$
$$= \left[y^2 + 3y - \frac{y^3}{3} \right]_0^3$$
$$= 9 + 9 - 9 = 9.$$

The width vanishes at the upper intersection and is 3 at the axis, as expected. No factor of two is needed: the problem asks only for the region in the first quadrant. The other proposed areas do not equal this boundary integral.

Step 3: Conclusion The enclosed area is $\boxed{9}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: Use horizontal strips when a square-root curve becomes a simple polynomial in the height.

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6. Statement-1 : The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ **is equal to** $\pi/6$

Statement-2 : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx.$ [4 marks]

(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(C) Statement- 1 is true; Statement-2 is False.

(D) Statement-1 is false; Statement-2 is true.

Correct Answer: (D) Statement-1 is false; Statement-2 is true.

Solution:

Concept:

- For an integrable function, reflection of the variable gives $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$.
- Here the bounds sum to $\pi/2$, and $\tan(\pi/2 - x) = 1/\tan x$. The tangent is positive throughout the interval, so the square roots are real and reciprocal.

Step 1: Evaluate Statement-2 independently Substitute $u = a + b - x$. Then $dx = -du$, while the endpoints $x = a, b$ become $u = b, a$. Reversing the endpoints removes the minus sign and proves the displayed identity. Thus Statement-2 is true under the ordinary integrability assumption.

Step 2: Pair the reflected integrands Let the stated integral be I, and put $t = \sqrt{\tan x} > 0$. Reflection yields

$$I = \int_{\pi/6}^{\pi/3} \frac{t}{1+t}\, dx.$$

Adding this to the original expression gives

$$2I = \int_{\pi/6}^{\pi/3} \left(\frac{1}{1+t} + \frac{t}{1+t} \right) dx = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}.$$

Therefore $I = \pi/12$, not $\pi/6$; Statement-1 is false. Options asserting that it is true are all excluded, regardless of their explanation claims.

Step 3: Conclusion Statement-1 is false and Statement-2 is true. Hence, the correct option is (D).

Quick Tip to Solve: When reflected integrands add to one, the integral is half the interval length, not the whole length.

7. If $\int f(x)\,dx = \psi(x)$, then $\int x^5 f\left(x^3\right) dx$ is equal to [4 marks]

(A) $\frac{1}{3}\left[x^3\psi\left(x^3\right) - \int x^2\psi\left(x^3\right) dx\right] + C$

(B) $\frac{1}{3}x^3\psi\left(x^3\right) - 3\int x^3\psi\left(x^3\right) dx + C$

(C) $\frac{1}{3}x^3\psi\left(x^3\right) - \int x^2\psi\left(x^3\right) dx + C$

(D) $\frac{1}{3}\left[x^3\psi\left(x^3\right) - \int x^3\psi\left(x^3\right) dx\right] + C$

Correct Answer: (C) $\frac{1}{3}x^3\psi\left(x^3\right) - \int x^2\psi\left(x^3\right) dx + C$

Solution:

Concept:

- The given antiderivative implies $\psi'(x) = f(x)$. Hence $d[\psi(x^3)] = 3x^2 f(x^3)\,dx$.
- Use integration by parts, $\int u\,dv = uv - \int v\,du$, separating $x^5 = x^3 x^2$ so the second factor accompanies $f(x^3)$.

Step 1: Choose the factors for integration by parts Take $u = x^3$ and $dv = x^2 f(x^3)\,dx$. The chain rule gives $du = 3x^2 dx$ and $v = \frac{1}{3}\psi(x^3)$. Thus

$$\int x^5 f(x^3)\,dx = \frac{x^3}{3}\psi(x^3) - \int x^2\psi(x^3)\,dx + C.$$

The coefficient in the remaining integral is one because the factors 3 and $1/3$ cancel.

Step 2: Verify the result by differentiation Differentiate the obtained expression. The derivative of its first term is $x^2\psi(x^3) + x^5 f(x^3)$; the derivative of the subtracted antiderivative is $x^2\psi(x^3)$. These terms cancel, leaving precisely $x^5 f(x^3)$. Option A keeps an incorrect factor $1/3$ on the remaining integral. Options B and D use x^3 rather than x^2 in that integral, so their derivatives do not cancel the extra product-rule term.

Step 3: Conclusion The antiderivative is $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)\,dx + C$. Hence, the correct option is (C).

Quick Tip to Solve: When the inner function is a cube, reserve an outer factor of x^2 for the chain-rule differential.

8. If x, y, z are in A.P. and $\tan^{-1}x, \tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then : [4 marks]

(A) $x = y = z$

(B) $2x = 3y = 6z$

(C) $6x = 3y = 2z$

(D) $6x = 4y = 3z$

Correct Answer: The valid triple (1,0,-1) satisfies both progressions but none of the options, so no option follows universally.

Solution:

Concept:

- For three numbers in arithmetic progression, $x + z = 2y$. For their principal inverse tangents, write $\alpha = \tan^{-1} x$, $\beta = \tan^{-1} y$, and $\gamma = \tan^{-1} z$, so $\alpha + \gamma = 2\beta$.
- The sine and cosine addition identities remain valid even when a tangent addition denominator is zero; they avoid losing exceptional solutions.

Step 1: Combine the two progression conditions Using positive square-root denominators for principal inverse tangents gives $\sin(\alpha + \gamma) = (x + z)/\sqrt{(1 + x^2)(1 + z^2)}$ and $\cos(\alpha + \gamma) = (1 - xz)/\sqrt{(1 + x^2)(1 + z^2)}$. Equating the cross-products with $\sin 2\beta = 2y/(1 + y^2)$ and $\cos 2\beta = (1 - y^2)/(1 + y^2)$, and using $x + z = 2y$, yields

$$2y(xz - y^2) = 0.$$

Step 2: Check the exceptional case If $y \neq 0$, then $xz = y^2$. Together with $x + z = 2y$, this implies $(x - z)^2 = 0$, hence $x = y = z$. But if $y = 0$, every triple $(t, 0, -t)$ satisfies both progressions because inverse tangent is odd. For example $(1, 0, -1)$ satisfies the hypotheses and violates every listed relation, including equality. The other three proposed proportionality conditions also fail this explicit counterexample.

Step 3: Conclusion As printed, no listed option is necessarily true. The complete possibilities are $x = y = z$ or $y = 0,\ z = -x$. The usual keyed equality requires an extra restriction excluding the nonzero symmetric triples; it cannot be concluded from the stated hypotheses.

Quick Tip to Solve: Before dividing by the middle term of a progression, test whether it is zero; odd functions preserve symmetric triples.

9. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having centre at $(0, 3)$ is : [4 marks]

(A) $x^2 + y^2 - 6y - 7 = 0$

(B) $x^2 + y^2 - 6y + 7 = 0$

(C) $x^2 + y^2 - 6y - 5 = 0$

(D) $x^2 + y^2 - 6y + 5 = 0$

Correct Answer: (A) $x^2 + y^2 - 6y - 7 = 0$

Solution:

Concept:

- For an ellipse $x^2/a^2 + y^2/b^2 = 1$, with $a > b$, the foci are $(\pm c, 0)$, where $c^2 = a^2 - b^2$.
- A circle with centre (h, k) and radius r has equation $(x - h)^2 + (y - k)^2 = r^2$.

Step 1: Locate the two foci Here $a^2 = 16$ and $b^2 = 9$, so $c^2 = 16 - 9 = 7$. Thus the two points through which the circle must pass are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$. These are symmetric about the vertical axis, consistent with the specified centre $(0, 3)$.

Step 2: Calculate the radius and expand The squared distance from the centre to either focus is

$$r^2 = (\sqrt{7} - 0)^2 + (0 - 3)^2 = 7 + 9 = 16.$$

Therefore

$$x^2 + (y - 3)^2 = 16 \quad \Longrightarrow \quad x^2 + y^2 - 6y - 7 = 0.$$

To check the alternatives directly, put $y = 0$ and $x^2 = 7$. Only the constant -7 makes the left side vanish; the constants $+7, -5, +5$ do not. The centre is also confirmed by completing the square.

Step 3: Conclusion The required circle is $x^2 + y^2 - 6y - 7 = 0$. Hence, the correct option is (A).

Quick Tip to Solve: Use the squared focal distance directly: the circle radius squared equals $a^2 - b^2 + k^2$ when its centre is $(0, k)$.

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10. If $y = \sec\left(\tan^{-1} x\right)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to : [4 marks]

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{1}{2}$

(C) 1

(D) $\sqrt{2}$

Correct Answer: (A) $\frac{1}{\sqrt{2}}$

Solution:

Concept:

- The principal angle $\theta = \tan^{-1} x$ lies in $(-\pi/2, \pi/2)$, where cosine and secant are positive.
- The identity $\sec^2 \theta = 1 + \tan^2 \theta$ gives $\sec(\tan^{-1} x) = \sqrt{1 + x^2}$. The positive square root is essential.

Step 1: Simplify the composite function Put $\theta = \tan^{-1} x$. Since $\tan\theta = x$, the Pythagorean identity yields $y^2 = 1 + x^2$. Positivity of secant on the principal interval fixes the sign, giving

$$y = (1 + x^2)^{1/2}.$$

This simplification is valid for every real x, so differentiation near $x = 1$ is legitimate.

Step 2: Differentiate and substitute Apply the power rule and chain rule together:

$$\frac{dy}{dx} = \frac{1}{2}(1 + x^2)^{-1/2}(2x) = \frac{x}{\sqrt{1 + x^2}}.$$

At $x = 1$, this is $1/\sqrt{2}$. Independently, the unsimplified chain rule gives $\sec\theta\tan\theta/(1 + x^2)$, which at $\theta = \pi/4$ is $\sqrt{2}/2$, the same value. The value $\sqrt{2}$ is the function value, not its derivative; $1/2$ omits a factor, and 1 is not obtained by the correct chain rule.

Step 3: Conclusion At $x = 1$, $dy/dx = 1/\sqrt{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Simplify a secant of an inverse tangent to a positive square root before applying the chain rule.

11. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have : [4 marks]

(A) any value

(B) exactly one value

(C) exactly two values

(D) exactly three values

Correct Answer: (C) exactly two values

Solution:

Concept:

- Two lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$, with directions $\mathbf{d}_1, \mathbf{d}_2$, are coplanar precisely when $(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{d}_1 \times \mathbf{d}_2) = 0$.
- The denominators in symmetric line equations are direction ratios. A zero direction ratio means the corresponding coordinate is constant, and is interpreted parametrically.

Step 1: Identify points and direction vectors Take $\mathbf{a}_1 = (2, 3, 4)$, $\mathbf{d}_1 = (1, 1, -k)$, $\mathbf{a}_2 = (1, 4, 5)$, and $\mathbf{d}_2 = (k, 2, 1)$. Then $\mathbf{a}_2 - \mathbf{a}_1 = (-1, 1, 1)$. Their cross product is

$$\mathbf{d}_1 \times \mathbf{d}_2 = (1 + 2k, -k^2 - 1, 2 - k).$$

Neither direction vector is zero for any real k.

Step 2: Solve the coplanarity condition The scalar triple product gives

$$-(1 + 2k) + (-k^2 - 1) + (2 - k) = 0,$$

so

$$-k^2 - 3k = 0 \quad \Longrightarrow \quad k(k+3) = 0.$$

Thus $k = 0$ or $k = -3$, two distinct real values. At $k = 0$, the first line has constant $z = 4$ and the second has constant $x = 1$; they intersect at $(1, 2, 4)$, so this root is valid under the parametric interpretation. The condition is not an identity, and its quadratic factorisation has two roots, excluding the other three counts.

Step 3: Conclusion There are exactly two possible values, $k = 0$ and $k = -3$. Hence, the correct option is (C).

Quick Tip to Solve: Form the scalar triple product before expanding; factor the resulting polynomial and check zero direction ratios parametrically.

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12. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching X-axis, the equation of the reflected ray is : [4 marks]

(A) $y = x + \sqrt{3}$
(B) $\sqrt{3}y = x - \sqrt{3}$
(C) $y = \sqrt{3}x - \sqrt{3}$
(D) $\sqrt{3}y = x - 1$

Correct Answer: (B) $\sqrt{3}y = x - \sqrt{3}$

Solution:

Concept:

- Reflection in the X-axis preserves the horizontal component of a direction vector and reverses its vertical component.
- The incident and reflected rays pass through the same point on the reflecting axis. Their supporting lines therefore have opposite slopes.

Step 1: Find the point of incidence The incident supporting line is $x + \sqrt{3}y = \sqrt{3}$, or

$$y = -\frac{x}{\sqrt{3}} + 1.$$

Setting $y = 0$ gives the point $P = (\sqrt{3}, 0)$. This point must also lie on the reflected ray, regardless of which direction along the incident line the light travels toward the axis.

Step 2: Reverse the slope and form the line The incident slope is $-1/\sqrt{3}$. Reflection in the horizontal axis changes this to $+1/\sqrt{3}$, so the reflected line through P is

$$y - 0 = \frac{1}{\sqrt{3}}(x - \sqrt{3}),$$

giving $\sqrt{3}y = x - \sqrt{3}$. The first and third alternatives have slopes 1 and $\sqrt{3}$, inconsistent with reflection. The fourth has the correct slope but crosses the axis at $x = 1$, rather than $x = \sqrt{3}$, so it cannot pass through the reflection point. The required option satisfies both the slope and incidence

tests.

Step 3: Conclusion The reflected ray lies along $\sqrt{3}y = x - \sqrt{3}$. Hence, the correct option is (B).

Quick Tip to Solve: Check reflection questions using two independent facts: the slope changes sign and the point on the mirror stays fixed.

13. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as $(0,1)(1,1)$ and $(1,0)$ is : [4 marks]

(A) $2+\sqrt{2}$

(B) $2-\sqrt{2}$

(C) $1+\sqrt{2}$

(D) $1-\sqrt{2}$

Correct Answer: (B) $2-\sqrt{2}$

Solution:

Concept:

- If D, E, F are the three side midpoints, the original vertices are $E+F-D$, $F+D-E$, and $D+E-F$.
- For a right triangle with perpendicular legs a, b and hypotenuse c, the inradius is $r = (a+b-c)/2$. The incentre is distance r from each leg.

Step 1: Reconstruct the original triangle Use $D = (0,1)$, $E = (1,1)$, $F = (1,0)$. The recovered vertices are

$$A = E + F - D = (2,0), \quad B = F + D - E = (0,0), \quad C = D + E - F = (0,2).$$

Their pairwise midpoints reproduce all three given points, confirming that we have reconstructed the original triangle rather than used the smaller medial triangle.

Step 2: Find the inradius and the coordinate This is an isosceles right triangle with legs along the positive coordinate axes, each of length 2, and hypotenuse $2\sqrt{2}$. The angle bisector at the origin is $y = x$, so its incentre is (r, r). Hence

$$r = \frac{2+2-2\sqrt{2}}{2} = 2-\sqrt{2}.$$

Thus the required horizontal coordinate is $2-\sqrt{2}$, which is positive and less than 1. The alternatives $2+\sqrt{2}$ and $1+\sqrt{2}$ lie beyond the triangle, and $1-\sqrt{2}$ is negative. These location checks independently exclude them.

Step 3: Conclusion The incentre is $(2-\sqrt{2}, 2-\sqrt{2})$, so its x-coordinate is $2-\sqrt{2}$. Hence, the correct option is (B).

Quick Tip to Solve: When side midpoints are given, reconstruct the vertices first; the incentre of the medial triangle generally differs from the required incentre.

14. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is [4 marks]

(A) 4

(B) 120

(C) 210

(D) 310

Correct Answer: (C) 210

Solution:

Concept:

- Use $a^3+1=(a+1)(a^2-a+1)$ and $b^2-1=(b-1)(b+1)$ to simplify the expression before expanding.
- In $(u+v)^{10}$, the general term is $\binom{10}{r}u^{10-r}v^r$, where $r=0,1,\ldots,10$. A constant term has total exponent of x equal to zero.

Step 1: Factor the two fractions With $a=x^{1/3}$, the first fraction simplifies to

$$\frac{x+1}{x^{2/3}-x^{1/3}+1}=x^{1/3}+1.$$

With $b=x^{1/2}$, the second fraction becomes

$$\frac{x-1}{x-x^{1/2}}=\frac{(b-1)(b+1)}{b(b-1)}=1+x^{-1/2}.$$

The original real expression requires $x>0$ and $x\neq 1$. On that domain the difference of the fractions is $x^{1/3}-x^{-1/2}$.

Step 2: Select the zero exponent The general term of the resulting tenth power is

$$T_{r+1}=\binom{10}{r}(-1)^r x^{(10-r)/3-r/2}.$$

Its exponent vanishes when

$$\frac{10-r}{3}-\frac{r}{2}=0 \quad\Longrightarrow\quad 20-5r=0 \quad\Longrightarrow\quad r=4.$$

Since this exponent is linear in r, no other term is constant. The coefficient is $\binom{10}{4}(-1)^4=210$. The negative sign contributes positively because r is even. The other numerical options do not equal this coefficient; in particular the root $r=4$ is a term index, not the constant term itself.

Step 3: Conclusion The term independent of x is 210. Hence, the correct option is (C).

Quick Tip to Solve: Simplify nested fractional powers before expanding, then set the general term exponent to zero and retain the binomial coefficient.

15. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n$ = 10, then the value of n is : [4 marks]

(A) 7

(B) 5

(C) 10

(D) 8

Correct Answer: (B) 5

Solution:

Concept:

- Three distinct vertices determine one triangle when no three selected vertices are collinear. A regular polygon is cyclic, and a straight line intersects its circumcircle in at most two points.
- The number of unordered selections of three vertices from n vertices is $T_n = \binom{n}{3}$. Pascal's identity gives $\binom{n+1}{3} - \binom{n}{3} = \binom{n}{2}$.

Step 1: Count the newly formed triangles Add the extra vertex to the collection of vertices. Triangles not using it are already counted by T_n. Every new triangle uses the new vertex together with two of the other n vertices. Hence the increase is the number of unordered pairs of old vertices:

$$T_{n+1} - T_n = \binom{n}{2} = \frac{n(n-1)}{2}.$$

There is no factor of six in this last expression, since only two old vertices are being chosen.

Step 2: Solve for the admissible number of sides The given difference therefore gives

$$\frac{n(n-1)}{2} = 10 \quad \Longrightarrow \quad n^2 - n - 20 = 0.$$

Factoring,

$$(n-5)(n+4) = 0.$$

Thus $n = 5$ or $n = -4$. A polygon must have a positive integral number of sides, at least three, so the negative root is inadmissible. For the other listed numbers $7, 10, 8$, the increases would be $21, 45, 28$, respectively, rather than ten. As a check, $\binom{6}{3} - \binom{5}{3} = 20 - 10 = 10$.

Step 3: Conclusion The polygon has $n = 5$ sides. Hence, the correct option is (B).

Quick Tip to Solve: For the difference between consecutive triangle counts, count triangles using the newly added vertex; this immediately reduces a three-vertex count to a two-vertex count.

16. The number of values of k, for which the system of equations

$$(k+1)x + 8y = 4k,$$
$$kx + (k+3)y = 3k - 1$$

has no solution, is [4 marks]

(A) infinite

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Concept:

- A pair of linear equations has a unique solution when the coefficient determinant is nonzero. Only values making that determinant zero can produce no solution.
- When the coefficients are proportional, compare the constants too: proportional constants give coincident lines, whereas nonproportional constants give parallel distinct lines.

Step 1: Find all singular values of the parameter The coefficient determinant is

$$D = (k+1)(k+3) - 8k = k^2 - 4k + 3 = (k-1)(k-3).$$

Therefore, for every real k except 1 and 3, the system has exactly one solution. It remains necessary to check these two exceptional values separately. A vanishing determinant alone does not distinguish infinitely many solutions from no solution.

Step 2: Test the two exceptional systems For $k = 1$, the equations become

$$2x + 8y = 4, \qquad x + 4y = 2.$$

The first is twice the second, including its right-hand side, so they describe the same line and have infinitely many solutions. For $k = 3$, they become

$$4x + 8y = 12, \qquad 3x + 6y = 8.$$

Dividing by their first coefficients yields

$$x + 2y = 3, \qquad x + 2y = \frac{8}{3}.$$

These demand two unequal values for the same expression and hence are inconsistent. Thus only $k = 3$ has no solution. There cannot be two, three or infinitely many such values, since the determinant already restricts the candidates to two and one of those is consistent.

Step 3: Conclusion Exactly one value, $k = 3$, makes the system inconsistent. Hence, the correct option is (B).

Quick Tip to Solve: After setting the determinant to zero, always test the constants. A singular system may represent coincident lines rather than an inconsistent pair.

17. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$, have a common root, then $a : b : c$ is [4 marks]

(A) $1 : 2 : 3$

(B) $3 : 2 : 1$

(C) $1 : 3 : 2$

(D) $3 : 1 : 2$

Correct Answer: (A) $1 : 2 : 3$

Solution:

Concept:

- Nonreal roots of a polynomial with real coefficients occur in conjugate pairs. If α is a root, its conjugate $\overline{\alpha}$ is also a root.
- Two genuine quadratic polynomials sharing the same two roots are scalar multiples, so corresponding coefficients have equal ratios.

Step 1: Establish that the shared root is nonreal The discriminant of the first quadratic is

$$\Delta = 2^2 - 4(1)(3) = -8 < 0.$$

Its roots are

$$-1 + i\sqrt{2} \quad \text{and} \quad -1 - i\sqrt{2}.$$

Therefore any common root is nonreal. Since a, b, c are real, the second quadratic must also have the conjugate of the shared root. It thus shares both distinct roots with the first, rather than merely one independent real root.

Step 2: Compare the coefficients of the two quadratics Consequently the second polynomial is a scalar multiple of the first:

$$ax^2 + bx + c = a(x^2 + 2x + 3).$$

Comparing the coefficients gives

$$b = 2a, \qquad c = 3a,$$

and hence

$$a : b : c = 1 : 2 : 3.$$

Equivalently, substitute a root satisfying $x^2 = -2x - 3$ into the second equation to get $(b - 2a)x + (c - 3a) = 0$. Its imaginary part forces $b = 2a$, and its real part then forces $c = 3a$. The other proposed ratios fail at least one of these two required equalities. Here the equations are understood as quadratic equations, so the leading coefficient is nonzero.

Step 3: Conclusion The coefficient ratio is $a : b : c = 1 : 2 : 3$. Hence, the correct option is (A).

Quick Tip to Solve: For two real quadratics, one shared nonreal root automatically supplies the second shared root through conjugation; compare coefficients instead of solving both equations.

18. If z is a complex number of unit modulus and argument θ, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals : [4 marks]

(A) $-\theta$

(B) $\frac{\pi}{2} - \theta$

(C) θ

(D) $\pi - \theta$

Correct Answer: (C) θ

Solution:

Concept:

- For a complex number of unit modulus, $z\bar{z} = 1$. Thus $\bar{z} = 1/z$, with $z \neq 0$.
- The argument of a complex number is unchanged when the number itself is unchanged. Arguments may differ by integer multiples of 2π; use the same argument convention as for θ.

Step 1: Simplify the quotient using unit modulus Use $\bar{z} = 1/z$ in the denominator. Wherever the printed quotient is defined,

$$\frac{1+z}{1+\bar{z}} = \frac{1+z}{1+1/z} = \frac{1+z}{(z+1)/z} = z.$$

Cancellation is permitted when $z + 1 \neq 0$. At $z = -1$, both numerator and denominator vanish, so the original quotient has no argument; this point is necessarily excluded by the question asking for its argument.

Step 2: Read the resulting argument and check the alternatives Since the quotient equals z, its argument is θ, modulo 2π. For a direct check, write $z = e^{i\theta}$, so

$$1 + z = 2\cos(\theta/2)e^{i\theta/2}, \quad 1 + \bar{z} = 2\cos(\theta/2)e^{-i\theta/2}.$$

Their quotient is $e^{i\theta}$ whenever the common cosine factor is nonzero. The alternatives $-\theta$, $\pi/2 - \theta$ and $\pi - \theta$ do not give this identity for general θ. For example $\theta = \pi/3$ yields $\pi/3$, excluding all three of those alternatives at once.

Step 3: Conclusion Where the quotient is defined, its argument is θ with the same argument convention as z. Hence, the correct option is (C).

Quick Tip to Solve: For unit-modulus complex numbers, replace the conjugate by the reciprocal before applying argument rules. This often cancels the entire quotient.

19. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in $[0, 1]$

[4 marks]

(A) lies between 1 and 2

(B) lies between 2 and 3

(C) lies between -1 and 0

(D) does not exist.

Correct Answer: (D) does not exist.

Solution:

Concept:

- A differentiable function whose derivative is strictly positive throughout an interval is strictly increasing there, and cannot take the same value twice.
- Adding a constant k shifts a graph vertically without changing its derivative or its monotonicity.

Step 1: Differentiate the polynomial For an arbitrary real constant k, define

$$f(x) = 2x^3 + 3x + k.$$

Its derivative is

$$f\prime(x) = 6x^2 + 3.$$

Since $x^2 \geq 0$, this derivative is at least three and is therefore strictly positive for every real x. In particular, f is strictly increasing throughout the closed interval $[0, 1]$, regardless of the chosen value of k.

Step 2: Rule out two distinct roots If two distinct numbers $r, s \in [0, 1]$, with $r < s$, were both roots, strict increase would imply $f(r) < f(s)$. But both values would equal zero, a contradiction. Equivalently, Rolle's theorem would require a point between the roots where $f\prime = 0$, impossible here. Endpoint roots do not avoid this argument: $f(0) = k$ and $f(1) = 5 + k$ cannot both vanish. A root occurs in the interval precisely for $-5 \leq k \leq 0$, by continuity and endpoint signs, but even then it is unique. Hence neither the first two positive intervals nor the negative interval in option (C) supplies two distinct roots.

Step 3: Conclusion No real value of k gives two distinct roots in $[0, 1]$. Hence, the correct option is (D).

Quick Tip to Solve: Before solving a cubic with a parameter, check its derivative. A derivative that never changes sign can settle the number of real roots immediately.

20. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as: [4 marks]

(A) $\sin A \cos A + 1$

(B) $\sec A \cos ecA + 1$

(C) $\tan A + \cot A$

(D) $\sec A + \cos ecA$

Correct Answer: (B) $\sec A \cos ecA + 1$

Solution:

Concept:

- Use $\cot A = 1/\tan A$ wherever both are defined. Replacing the trigonometric functions by one variable turns the expression into a rational function.
- The identity $\tan A + \cot A = 1/(\sin A \cos A) = \sec A \csc A$ follows from $\sin^2 A + \cos^2 A = 1$.

Step 1: Combine the rational expressions Put $t = \tan A$, so $\cot A = 1/t$. The original expression requires both trigonometric functions to exist and both denominators to be nonzero; in particular $t \neq 0, 1$. Then

$$\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = \frac{t^2}{t-1} - \frac{1}{t(t-1)} = \frac{t^3-1}{t(t-1)}.$$

The minus sign in the second term comes from $1 - t = -(t-1)$.

Step 2: Factor and return to trigonometric form Factor the difference of cubes and cancel the nonzero factor:

$$\frac{(t-1)(t^2+t+1)}{t(t-1)} = t + 1 + \frac{1}{t}.$$

Therefore the expression is

$$\tan A + \cot A + 1 = \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} + 1 = \sec A \csc A + 1.$$

Option (C) omits the additional one. Option (A) replaces the reciprocal product by the product itself. Option (D) uses a sum of reciprocals instead of their product plus one. For instance, at $A = \pi/6$, these alternatives do not reproduce $1 + 4/\sqrt{3}$, the value of the simplified expression.

Step 3: Conclusion The expression equals $\sec A \csc A + 1$ on its original domain. Hence, the correct option is (B).

Quick Tip to Solve: When tangent and cotangent occur together, set $t = \tan A$. A difference-of-cubes factor often appears after taking a common denominator.

21. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is : [4 marks]

(A) $\sqrt{18}$

(B) $\sqrt{72}$

(C) $\sqrt{33}$

(D) $\sqrt{45}$

Correct Answer: (C) $\sqrt{33}$

Solution:

Concept:

- The median from a vertex joins that vertex to the midpoint of the opposite side. The midpoint position vector is the arithmetic mean of the endpoint vectors.
- For perpendicular unit vectors, the magnitude of $u\hat{i} + v\hat{j} + w\hat{k}$ is $\sqrt{u^2 + v^2 + w^2}$.

Step 1: Form the vector from the vertex to the midpoint Let M be the midpoint of BC. Take A as origin so that the position vectors of B and C are the two given side vectors. By the midpoint formula,

$$\overrightarrow{AM} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}.$$

Adding corresponding components gives

$$\overrightarrow{AB} + \overrightarrow{AC} = (3+5)\hat{i} + (0-2)\hat{j} + (4+4)\hat{k} = 8\hat{i} - 2\hat{j} + 8\hat{k}.$$

Hence $\overrightarrow{AM} = 4\hat{i} - \hat{j} + 4\hat{k}$.

Step 2: Calculate the median length The components refer to orthogonal coordinate axes, so

$$AM = \sqrt{4^2 + (-1)^2 + 4^2} = \sqrt{16 + 1 + 16} = \sqrt{33}.$$

The negative sign in the middle component contributes a positive square. The vector difference $\overrightarrow{AC} - \overrightarrow{AB}$ would instead give the side BC, not the requested median. Likewise, omitting the factor of one half would give twice the median and the wrong magnitude. The squared length is exactly 33, so the listed values $\sqrt{18}, \sqrt{72}, \sqrt{45}$ are all excluded. A vector midpoint calculation does not require a separate geometric drawing.

Step 3: Conclusion The length of the median through A is $\sqrt{33}$. Hence, the correct option is (C).

Quick Tip to Solve: For a median from the common tail of two side vectors, add the vectors and halve the sum before taking its magnitude; subtraction gives the opposite side.

22. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is : [4 marks]

(A) $\frac{3}{2}$

(B) $\frac{5}{2}$

(C) $\frac{7}{2}$

(D) $\frac{9}{2}$

Correct Answer: (C) $\frac{7}{2}$

Solution:

Concept:

- For parallel planes with the same normal coefficients, their perpendicular separation is the absolute difference of their constant terms divided by the length of the common normal.
- For a normal vector (a, b, c), its length is $\sqrt{a^2 + b^2 + c^2}$. Multiplying a plane equation by a nonzero number does not change the plane, but its coefficients must be normalized before comparing constants.

Step 1: Normalize the second plane The first equation is $2x + y + 2z - 8 = 0$. Divide every term of the second equation by 2, including its constant term, to obtain

$$2x + y + 2z + \frac{5}{2} = 0.$$

Both equations now have precisely the same coefficients of x, y, z, so their common normal is $(2, 1, 2)$. Its length is $\sqrt{4 + 1 + 4} = 3$.

Step 2: Calculate the perpendicular separation The constants in the normalized equations are -8 and $5/2$. Their difference in magnitude is $21/2$, not $11/2$, because the constants have opposite signs. Hence

$$d = \frac{|\frac{5}{2} - (-8)|}{3} = \frac{21/2}{3} = \frac{7}{2}.$$

The other listed distances $3/2, 5/2, 9/2$ do not satisfy this normalized distance formula. Comparing the unnormalized constants would give an incorrect result.

Step 3: Conclusion The distance between the planes is $\boxed{7/2}$ units. Hence, the correct option is (C).

Quick Tip to Solve: First make the normal coefficients identical; only then subtract the constants and divide by the normal length.

23. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is : [4 marks]

(A) $\frac{17}{3^5}$

(B) $\frac{13}{3^5}$

(C) $\frac{11}{3^5}$

(D) $\frac{10}{3^5}$

Correct Answer: (C) $\frac{11}{3^5}$

Solution:

Concept:

- Independent random guesses give a correct answer with probability $p = 1/3$ and an incorrect answer with probability $1 - p = 2/3$.
- The number X of correct answers has binomial distribution: $P(X = k) = \binom{5}{k}(1/3)^k(2/3)^{5-k}$. The phrase four or more means exactly four or exactly five.

Step 1: Count the outcomes with exactly four correct answers There are $\binom{5}{4} = 5$ ways to choose the one incorrectly answered question. For each such choice, the four correct guesses contribute $(1/3)^4$, while the incorrect guess contributes $2/3$. Therefore

$$P(X = 4) = 5\left(\frac{1}{3}\right)^4 \frac{2}{3} = \frac{10}{3^5}.$$

The factor 2 records the two wrong alternatives; omitting it would count only one wrong answer per question.

Step 2: Include the outcome with all five answers correct All five questions are correct with probability

$$P(X = 5) = \left(\frac{1}{3}\right)^5 = \frac{1}{3^5}.$$

The two events are disjoint, so their probabilities add:

$$P(X \geq 4) = \frac{10 + 1}{3^5} = \frac{11}{3^5}.$$

Option (D) counts exactly four correct answers and omits the all-correct outcome. The numerators in (A) and (B) do not count the favourable guessing outcomes. There are 3^5 equally likely complete answer strings.

Step 3: Conclusion The required probability is $\boxed{11/3^5}$. Hence, the correct option is (C).

Quick Tip to Solve: For at least four successes in five trials, add the exactly-four and exactly-five terms; keep both wrong alternatives.

24. The circle passing through $(1, -2)$ and touching the axis of x at $(3, 0)$ also passes through the point : [4 marks]

(A) $(-5, 2)$

(B) $(2, -5)$

(C) $(5, -2)$

(D) $(-2, 5)$

Correct Answer: (C) $(5, -2)$

Solution:

Concept:

- The radius at a point of tangency is perpendicular to the tangent. Since the circle touches the x-axis at $(3, 0)$, its centre lies on the vertical line $x = 3$.
- If the centre is $(3, k)$, the radius is $|k|$, and the circle equation is $(x-3)^2 + (y-k)^2 = k^2$. A point belongs to the circle precisely when it satisfies this equation.

Step 1: Determine the centre from the given point Substitute the known point $(1, -2)$ in the tangent-circle equation:

$$(1-3)^2 + (-2-k)^2 = k^2.$$

Expanding gives $4 + 4 + 4k + k^2 = k^2$, so $k = -2$. Thus the centre is $(3, -2)$, the radius is 2, and the equation becomes

$$(x-3)^2 + (y+2)^2 = 4.$$

The negative centre ordinate is consistent with the given point below the axis.

Step 2: Test the listed points For option (C), $(5, -2)$, the left side is $(5-3)^2 + 0^2 = 4$, so it lies on the circle. For (A), $(-5, 2)$, the left side is $64 + 16 = 80$. For (B), $(2, -5)$, it is $1 + 9 = 10$. For (D), $(-2, 5)$, it is $25 + 49 = 74$. These three values differ from the required 4, ruling out those options. Geometrically, $(1, -2)$ and $(5, -2)$ are opposite endpoints of a horizontal diameter.

Step 3: Conclusion The circle also passes through $\boxed{(5, -2)}$. Hence, the correct option is (C).

Quick Tip to Solve: A circle tangent to an axis has its centre directly above or below the contact point; use the known point to find the signed ordinate.

25. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}x$.

Statement-1 : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement-2 : If the line, $y = mx + \frac{\sqrt{5}}{m}$ $(m \neq 0)$ is their common tangent, then m satiesfies $m^4 - 3m^2 + 2 = 0$. [4 marks]

(A) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(C) Statement-1 is true; Statement-2 is false.

(D) Statement-1 is false Statement-2 is true.

Correct Answer: (B) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

Solution:

Concept:

- The circle is $x^2 + y^2 = 5/2$, with centre at the origin and radius $\sqrt{5/2}$. A line is tangent when its distance from the centre equals the radius.
- For the parabola $y^2 = 4ax$, a nonvertical tangent of slope $m \neq 0$ is $y = mx + a/m$. Here $a = \sqrt{5}$.

Step 1: Check Statement-1 directly The line $y = x + \sqrt{5}$ is the parabola tangent with $m = 1$, since its intercept equals $a/m = \sqrt{5}$. Its distance from the circle centre is

$$\frac{|\sqrt{5}|}{\sqrt{1^2 + (-1)^2}} = \sqrt{\frac{5}{2}},$$

which is exactly the circle radius. It is therefore tangent to both curves, establishing that Statement-1 is true without relying on Statement-2.

Step 2: Derive the actual common-tangent condition For $y = mx + \sqrt{5}/m$, tangency to the circle requires

$$\frac{5/m^2}{m^2 + 1} = \frac{5}{2}, \qquad m^4 + m^2 - 2 = 0.$$

This factors as $(m^2 - 1)(m^2 + 2) = 0$, so every real common tangent has $m = \pm 1$. Both values also satisfy the printed equation $m^4 - 3m^2 + 2 = (m^2 - 1)(m^2 - 2) = 0$. Thus Statement-2, which asserts only a necessary condition using the word "if", is true. It does not say every root gives a common tangent.

Step 3: Test whether the necessary condition explains the tangent The printed polynomial additionally admits $m^2 = 2$. At either of these slopes, the squared distance of the line from the origin is $5/[m^2(m^2 + 1)] = 5/6$, whereas the circle has squared radius $5/2$. Such a line is not tangent to the circle. Therefore Statement-2 is insufficient to establish a common tangent: it cannot explain Statement-1. Options (C) and (D) incorrectly call a statement false, while (A) incorrectly treats the necessary condition as an explanation.

Step 4: Conclusion Both statements are true, but Statement-2 is not a correct explanation of Statement-1. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish an "if necessary condition from an "if and only if characterization; extraneous roots can prevent a condition from explaining an assertion.

26. $ABCD$ **is a trapezium such that** AB **and** CD **are parallel and** $BC \perp CD$. **If** $\angle ADB = \theta$, $BC = p$ **and** $CD = q$, **then AB is equal to:** [4 marks]

(A) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$

(B) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$

(C) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$

(D) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

Correct Answer: (A) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$

Solution:

Concept:

- Because $BC \perp CD$ and $AB \parallel CD$, triangle BCD is right-angled and $BD = \sqrt{p^2+q^2}$.
- The sine rule in triangle ABD relates AB, opposite θ, to BD, opposite $\angle DAB$. The angle-addition formula resolves the resulting denominator.

Step 1: Express the diagonal angle using the right triangle Put $\beta = \angle ABD$. The parallel sides show that this is the acute angle between BD and a horizontal side. In right triangle BCD, the opposite leg is $BC = p$ and the adjacent leg is $CD = q$. With $L = BD$,

$$L = \sqrt{p^2+q^2}, \qquad \sin\beta = \frac{p}{L}, \qquad \cos\beta = \frac{q}{L}.$$

In triangle ABD, the remaining angle is $\pi - (\theta+\beta)$, whose sine equals $\sin(\theta+\beta)$.

Step 2: Apply the sine rule and simplify The sine rule gives

$$\frac{AB}{\sin\theta} = \frac{L}{\sin(\theta+\beta)}.$$

Now $\sin(\theta+\beta) = (q\sin\theta + p\cos\theta)/L$. Therefore

$$AB = \frac{L^2\sin\theta}{p\cos\theta+q\sin\theta} = \frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}.$$

This is option (A). Option (B) has a different angle dependence in its numerator. Options (C) and (D) are dimensionless, whereas AB must have the dimension of length, so neither can be correct.

Step 3: Conclusion $AB = \boxed{\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the right triangle to express the diagonal angle, then apply the sine rule in the triangle containing the requested side.

27. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t|\,dt, x \in R$, which are parallel to the line $y = 2x$, are equal to : [4 marks]

(A) ± 1

(B) ± 2

(C) ± 3

(D) ± 4

Correct Answer: (A) ± 1

Solution:

Concept:

- The fundamental theorem of calculus gives $y\prime = |x|$, because the integrand $|t|$ is continuous. A tangent parallel to $y = 2x$ must have slope 2.
- The ordinate is $y = x^2/2$ for $x \geq 0$ and $y = -x^2/2$ for $x < 0$. The negative branch accounts for the reversed integration limits when x is negative.

Step 1: Find both points where the tangent slope is two Solve $|x| = 2$ to obtain $x = 2$ and $x = -2$. For $x = 2$,

$$y = \int_0^2 t\,dt = 2.$$

For $x = -2$, the integrand on the interval is $-t$, so

$$y = \int_0^{-2} (-t)\,dt = -2.$$

Hence the two points of contact are $(2, 2)$ and $(-2, -2)$. Their abscissae are not the requested intercepts; those must be obtained from the tangent equations.

Step 2: Set the tangent ordinate equal to zero At $(2, 2)$, the tangent is $y - 2 = 2(x - 2)$, or $y = 2x - 2$. Setting $y = 0$ gives $x = 1$. At $(-2, -2)$, the tangent is $y + 2 = 2(x + 2)$, or $y = 2x + 2$, whose intercept is $x = -1$. Thus the intercepts are ± 1. Option (B) confuses the contact abscissae with the intercepts; (C) and (D) satisfy neither tangent equation at $y = 0$.

Step 3: Conclusion The two intercepts on the x-axis are $\boxed{\pm 1}$. Hence, the correct option is (A).

Quick Tip to Solve: After solving the slope equation, form each tangent and set its ordinate to zero; contact coordinates are not intercepts.

28. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,.....,is [4 marks]

(A) $\frac{7}{81}\left(179-10^{-20}\right)$

(B) $\frac{7}{9}\left(99-10^{-20}\right)$

(C) $\frac{7}{81}\left(179+10^{-20}\right)$

(D) $\frac{7}{9}\left(99+10^{-20}\right)$

Correct Answer: (C) $\frac{7}{81}\left(179+10^{-20}\right)$

Solution:

Concept:

- The term with n digits equal to seven is $a_n = 7(10^{-1}+10^{-2}+\cdots+10^{-n}) = \frac{7}{9}(1-10^{-n})$.
- The finite geometric sum is $\sum_{n=1}^{N} r^n = r(1-r^N)/(1-r)$, for $r \neq 1$. Apply it to $r = 1/10$ after separating the constant part of each term.

Step 1: Write the twenty-term sum in a usable form The decimal terms are not themselves a geometric progression, since their successive ratios change. Their small remainders from $7/9$, however, form a geometric progression. Summing the expression for a_n gives

$$S_{20} = \frac{7}{9}\left(20 - \sum_{n=1}^{20} 10^{-n}\right).$$

The finite sum starts with 10^{-1}, has common ratio 10^{-1}, and contains exactly twenty terms.

Step 2: Evaluate the geometric correction The correction is

$$\sum_{n=1}^{20} 10^{-n} = \frac{(1/10)(1-10^{-20})}{1-1/10} = \frac{1}{9}(1-10^{-20}).$$

Consequently,

$$S_{20} = \frac{7}{9}\left(20 - \frac{1}{9} + \frac{10^{-20}}{9}\right) = \frac{7}{81}(179+10^{-20}).$$

The plus sign on the tiny last term follows from subtracting $1-10^{-20}$. Option (A) reverses that sign. Options (B) and (D) are near 77, impossible since each of twenty terms is below 1. The answer is just below $20 \times 7/9$, as expected.

Step 3: Conclusion The sum is $\boxed{\frac{7}{81}(179+10^{-20})}$. Hence, the correct option is (C).

Quick Tip to Solve: Express each repeated-digit decimal as its infinite recurring value minus a geometric remainder; then sum the remainders.

29. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to : [4 marks]

(A) 4

(B) 11

(C) 5

(D) 0

Correct Answer: (B) 11

Solution:

Concept:

- For a square matrix of order n, $A \operatorname{adj} A = |A| I$. Taking determinants gives $|\operatorname{adj} A| = |A|^{n-1}$ when A is invertible.
- The given matrix P is the adjoint, not the original matrix. Its determinant must therefore equal the square of the given determinant because the order is three.

Step 1: Find the determinant required of P The given value $|A| = 4$ is nonzero, so the determinant identity applies directly. With $n = 3$,

$$|P| = |\operatorname{adj} A| = |A|^2 = 4^2 = 16.$$

It would be incorrect to set $|P| = 4$; that would confuse a matrix with its adjoint.

Step 2: Expand the displayed determinant Expand along the first row, keeping the alternating cofactor signs:

$$\begin{aligned} |P| &= 1(3 \cdot 4 - 3 \cdot 4) - \alpha(1 \cdot 4 - 3 \cdot 2) + 3(1 \cdot 4 - 3 \cdot 2) \\ &= 0 - \alpha(-2) + 3(-2) = 2\alpha - 6. \end{aligned}$$

The two equal entries in the first-column minors make this expansion short. The middle term is positive because both the cofactor sign and its minor are negative.

Step 3: Solve and check the parameter Equate the two expressions for the same determinant:

$$2\alpha - 6 = 16 \quad \Longrightarrow \quad 2\alpha = 22 \quad \Longrightarrow \quad \alpha = 11.$$

Substitution gives $2(11) - 6 = 16$, as required. The alternative values $4, 5, 0$ give determinants $2, 4, -6$, respectively, so none satisfies the adjoint identity.

Step 4: Conclusion The required value is $\alpha = 11$. Hence, the correct option is (B).

Quick Tip to Solve: For a three-by-three adjoint problem, write $|\operatorname{adj} A| = |A|^2$ before expanding the determinant; this avoids equating the wrong determinants.

30. Consider :

Statement – I : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ **is a fallacy.**

Statement – II : $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ **is a tautology.** [4 marks]

(A) Statement - I is True; Statement -II is true; Statement-II is not a correct explanation for Statement-I

(B) Statement -I is True; Statement -II is False.

(C) Statement -I is False; Statement -II is True

(D) Statement -I is True; Statement -II is True; Statement-II is a correct explanation for Statement-I

Correct Answer: (A) Statement - I is True; Statement -II is true; Statement-II is not a correct explanation for Statement-I

Solution:

Concept:

- A fallacy is a compound proposition that is false for every truth assignment; a tautology is true for every truth assignment.
- An implication and its contrapositive are logically equivalent: $p \rightarrow q \equiv \neg q \rightarrow \neg p$. Whether this explains a separate statement must be checked independently.

Step 1: Test the conjunction in Statement I Associativity and commutativity allow the factors to be regrouped:

$$(p \wedge \neg q) \wedge (\neg p \wedge q) \equiv (p \wedge \neg p) \wedge (q \wedge \neg q).$$

A proposition and its negation cannot both be true. Therefore each contradictory pair is false, and the full conjunction is false for all four possible assignments to p, q. Thus Statement I is true: the displayed proposition is a fallacy.

Step 2: Test the biconditional in Statement II Use the standard implication identity $u \rightarrow v \equiv \neg u \vee v$:

$$p \rightarrow q \equiv \neg p \vee q, \qquad \neg q \rightarrow \neg p \equiv q \vee \neg p.$$

The two right-hand sides are identical by commutativity. A biconditional between equivalent propositions is always true, even when both propositions happen to be false. Statement II is consequently true.

Step 3: Check the explanatory relationship Statement I follows from the contradictory factors $p \wedge \neg p$ and $q \wedge \neg q$. Statement II concerns equivalence of an implication with its contrapositive and does not establish that contradiction. It therefore supplies no explanation of Statement I. Options (B) and (C) incorrectly deny one true statement; option (D) incorrectly asserts an explanatory connection.

Step 4: Conclusion Both statements are true, but Statement II is not the correct explanation of Statement I. Hence, the correct option is (A).

Quick Tip to Solve: In statement-and-explanation questions, decide the truth of each statement first, then test the claimed explanation as a separate logical step.

Physics (120 marks)

31. A circular loop of radius $0.3\ cm$ lies parallel to a much bigger circular loop of radius $20\ cm$. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is $15\ cm$. If a current of $2.0\ A$ flows through the smaller loop, then the flux linked with bigger loop is [4 marks]

(A) 9.1×10^{-11} weber

(B) 6×10^{-11} weber

(C) 3.3×10^{-11} weber

(D) 6.6×10^{-9} weber

Correct Answer: (A) 9.1×10^{-11} weber

Solution:

Concept:

- Mutual inductance is reciprocal: if M is the mutual inductance of two fixed loops, the flux through either loop due to current I in the other is MI. This lets us calculate the easier of the two magnetic fields.
- A circular loop of radius R carrying current I produces the axial field $B = \mu_0 I R^2/[2(R^2+z^2)^{3/2}]$ at axial distance z. Across a much smaller coaxial loop, this field is approximately uniform and normal to its area.

Step 1: Reverse the source and receiving loops Direct integration of the small loop's nonuniform field over the large loop is unnecessary. Imagine the same current $I = 2.0\,\mathrm{A}$ flowing in the large loop instead. The small loop has radius $a = 0.3\,\mathrm{cm} = 0.003\,\mathrm{m}$, the large loop has radius $R = 0.20\,\mathrm{m}$, and their separation is $z = 0.15\,\mathrm{m}$. Since a is much smaller than both relevant distances, use the field at the small loop's centre over its area. The planes are parallel, so no extra cosine factor is needed.

Step 2: Calculate the axial field and the flux First simplify the geometric denominator:

$$R^2 + z^2 = 0.20^2 + 0.15^2 = 0.0625\,\mathrm{m}^2, \qquad (R^2+z^2)^{3/2} = 0.015625\,\mathrm{m}^3.$$

Thus the field produced by the imagined current in the larger loop is

$$B = \frac{(4\pi \times 10^{-7})(2.0)(0.20)^2}{2(0.015625)} = 3.217 \times 10^{-6}\,\mathrm{T}.$$

Multiply by the smaller loop's area:

$$\Phi = B\pi a^2 = (3.217 \times 10^{-6})\pi(0.003)^2 = 9.10 \times 10^{-11}\,\mathrm{Wb}.$$

Step 3: Apply reciprocity to the stated current This calculated flux equals MI. Mutual-inductance reciprocity gives exactly the same flux linkage in the larger loop when the current of the same magnitude flows in the smaller loop, as in the question. The value is closest to (A); (B) and (C) have incorrect numerical factors, while (D) is far too large. In particular, using the field at the large loop's centre times its full area would wrongly treat a strongly varying field as uniform.

Step 4: Conclusion The flux linked with the bigger loop is approximately $9.1 \times 10^{-11}\,\mathrm{Wb}$. Hence, the

correct option is (A).

Quick Tip to Solve: For two very unequal coaxial loops, reverse the current by mutual-inductance reciprocity and calculate the large loop's nearly uniform field over the small loop.

32. In a hydrogen like atom electron make transition from an energy level with quantum number n to another with quantum number $(n-1)$. If $n >> 1$, the frequency of radiation emitted is proportional to : [4 marks]

(A) $\frac{1}{n}$
(B) $\frac{1}{n^2}$
(C) $\frac{1}{n^{\frac{3}{2}}}$
(D) $\frac{1}{n^3}$

Correct Answer: (D) $\frac{1}{n^3}$

Solution:

Concept:

- The Bohr energies of a hydrogen-like atom are $E_n = -K/n^2$, where $K > 0$ is constant for the particular atom.
- The photon frequency satisfies $h\nu = E_n - E_{n-1}$. At large n, the spacing between neighbouring levels is determined by a difference of inverse squares, not by the energy of one level alone.

Step 1: Write the positive emitted energy The final level $n-1$ is lower in energy than the initial level n, so the photon carries

$$h\nu = K\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right].$$

The bracket is positive. Reversing its order would give a negative photon energy, which is impossible for emission. The constants K and h do not change as the initial quantum number varies.

Step 2: Simplify before taking the large-n limit Combine the two fractions exactly:

$$\begin{aligned}\nu &= \frac{K}{h}\frac{n^2-(n-1)^2}{n^2(n-1)^2} \\ &= \frac{K}{h}\frac{2n-1}{n^2(n-1)^2}.\end{aligned}$$

For $n \gg 1$, the numerator behaves as $2n$, while the denominator behaves as n^4. Thus the leading dependence is

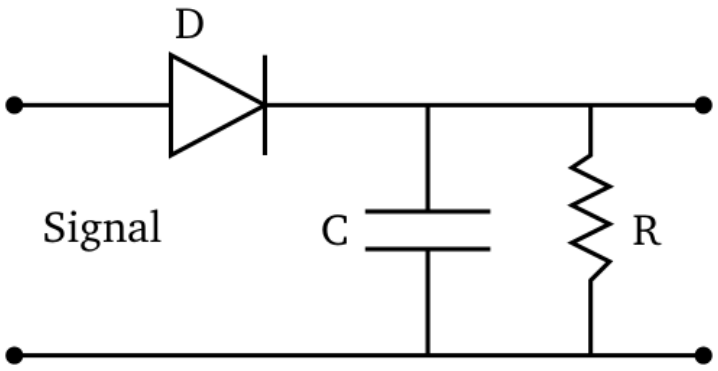
$$\nu \simeq \frac{2K}{h}\frac{1}{n^3}.$$

Step 3: Distinguish the competing powers Option (B), n^{-2}, describes the magnitude of a single Bohr energy rather than the separation of successive high levels. Taking that difference introduces one further inverse power of n. Neither n^{-1} nor $n^{-3/2}$ follows from the exact rational expression. As an independent check, differentiating $-K/n^2$ gives $2K/n^3$, the same leading spacing for a unit change in n.

Step 4: Conclusion The frequency is proportional to $1/n^3$ for $n \gg 1$. Hence, the correct option is (D).

Quick Tip to Solve: For transitions between neighbouring large quantum numbers, differentiate the level-energy formula to identify the leading power, then confirm it with the exact difference.

33. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 picofarad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it. [4 marks]



(A) $10.62\ MHz$

(B) $10.62\ kHz$

(C) $5.31\ MHz$

(D) $5.31\ kHz$

Correct Answer: (B) $10.62\ kHz$

Solution:

Concept:

- In the usual approximate envelope-detector criterion, the discharge rate of the capacitor must follow the modulation: $m\omega_m RC \lesssim 1$, where m is modulation index and $\omega_m = 2\pi f_m$.
- The capacitor and load resistance set the discharge time $\tau = RC$. The limiting frequency estimate is $f_{m,\max} \simeq 1/(2\pi mRC)$, distinct from the carrier frequency.

Step 1: Convert the data and find the time constant The modulation depth is 60%, hence $m = 0.60$. Express resistance and capacitance in SI units:

$$R = 100 \times 10^3\,\Omega, \qquad C = 250 \times 10^{-12}\,\text{F}.$$

Their product is

$$RC = (100 \times 10^3)(250 \times 10^{-12}) = 2.50 \times 10^{-5}\,\text{s}.$$

This is the capacitor discharge time through the load; it is neither the modulation period nor the carrier period.

Step 2: Apply the limiting modulation-frequency estimate At the approximate upper limit, put

$m\omega_m RC = 1$. Since $\omega_m = 2\pi f_m$,

$$\begin{aligned} f_{m,\max} &\simeq \frac{1}{2\pi(0.60)(2.50 \times 10^{-5})} \\ &= 1.061 \times 10^4\,\text{Hz} \\ &\simeq 10.62\,\text{kHz}. \end{aligned}$$

The factor 2π converts angular frequency into cycles per second. The modulation percentage enters as its fractional value, rather than the number 60.

Step 3: Compare the proposed frequencies The estimate is of order 10^4 Hz, so the MHz values in (A) and (C) are too large by roughly three orders of magnitude. Option (D) has an extra factor of two in the denominator relative to this criterion. The value in (B) agrees with the computed modulation-frequency limit. At higher modulation frequencies the capacitor cannot discharge rapidly enough to track the falling envelope, causing distortion.

Step 4: Conclusion The standard approximate detector estimate gives $f_{m,\max} \simeq 10.62\,\text{kHz}$. Hence, the correct option is (B).

Quick Tip to Solve: Compute RC in seconds and convert modulation percentage to a fraction before using $1/(2\pi mRC)$; the resulting frequency is in hertz.

34. The magnetic field in a travelling electromagnetic wave has a peak value of $20\ nT$. The peak value of electric field strength is : [4 marks]

(A) $3V/m$

(B) $6V/m$

(C) $9V/m$

(D) $12V/m$

Correct Answer: (B) $6V/m$

Solution:

Concept:

- For a travelling electromagnetic wave in vacuum, the electric and magnetic peak amplitudes obey $E_0 = cB_0$, where $c = 3.0 \times 10^8\,\text{m}\,\text{s}^{-1}$.
- The prefix nano denotes 10^{-9}. Peak amplitudes must be compared with peak amplitudes; no root-mean-square conversion is required here.

Step 1: Convert the magnetic amplitude to SI units The stated peak magnetic field is

$$B_0 = 20\,\text{nT} = 20 \times 10^{-9}\,\text{T} = 2.0 \times 10^{-8}\,\text{T}.$$

Keeping the power of ten explicit prevents a factor-of-a-thousand prefix error. The word peak means this value is already the amplitude of the magnetic oscillation, rather than its average or root-mean-square value.

Step 2: Apply the electromagnetic-wave amplitude relation For a plane wave, Faraday's law and the electromagnetic wave equations relate the two transverse fields by their propagation speed. Therefore

$$\begin{aligned} E_0 &= cB_0 \\ &= (3.0 \times 10^8)(20 \times 10^{-9}) \\ &= 60 \times 10^{-1} = 6.0\,\mathrm{V\,m^{-1}}. \end{aligned}$$

The unit is correct because $\mathrm{T\,m\,s^{-1}} = \mathrm{V\,m^{-1}}$.

Step 3: Check the option values The ratio E_0/B_0 must equal $3.0 \times 10^8\,\mathrm{m\,s^{-1}}$. Using $6\,\mathrm{V\,m^{-1}}$ reproduces this value. The proposed amplitudes $3, 9, 12\,\mathrm{V\,m^{-1}}$ would instead give speeds $1.5, 4.5, 6.0$ times $10^8\,\mathrm{m\,s^{-1}}$, respectively, so they are inconsistent. Introducing a factor of $\sqrt{2}$ would also be inappropriate because both amplitudes are specified as peak values.

Step 4: Conclusion The peak electric field is $6\,\mathrm{V\,m^{-1}}$. Hence, the correct option is (B).

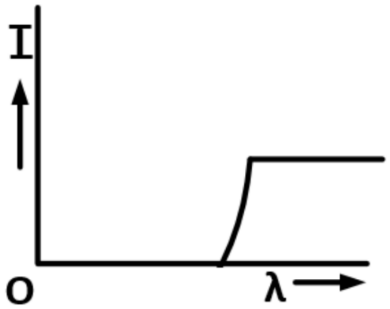
Quick Tip to Solve: Convert nanotesla to tesla first, then multiply by 3×10^8; use the same peak or rms convention for both fields.

35. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows : [4 marks]

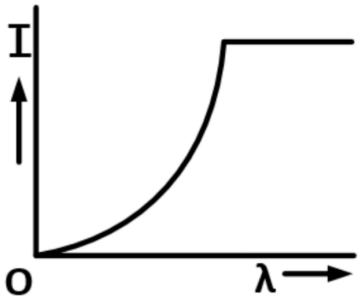
(A)



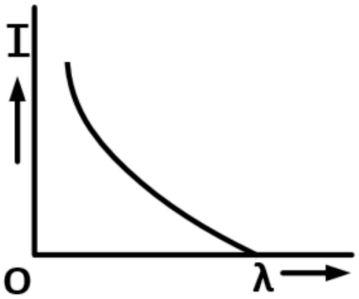
(B)



(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- The photoelectric threshold is $\lambda_0 = hc/W$, where W is the work function. Light with $\lambda > \lambda_0$ cannot liberate photoelectrons, so the photocurrent vanishes beyond the threshold.
- The measured current of a practical photocell is its spectral response at the fixed collecting voltage. The photon arrival rate and the efficiency of emission together determine that response.

Step 1: Identify the long-wavelength cutoff Einstein's photoelectric equation is

$$K_{\max} = \frac{hc}{\lambda} - W.$$

As wavelength increases, photon energy decreases. At $\lambda = \lambda_0$, the available energy just equals the work function; beyond this wavelength photoemission ceases. The current must therefore become zero on the long-wavelength side. Options (B) and (C), which retain a positive plateau as wavelength

increases, fail this necessary threshold test.

Step 2: Read the spectral response before the cutoff For the usual photocell response represented by these graphs, current first rises from a small short-wavelength response, reaches a maximum in the sensitive spectral region, and then falls to zero at the threshold. This is the peaked response shown in (A). A practical cathode does not have a wavelength-independent emission efficiency: the number of incident photons and the fraction that actually yield collected electrons both matter. Fixed anode voltage means the collection condition is unchanged while the light wavelength is scanned.

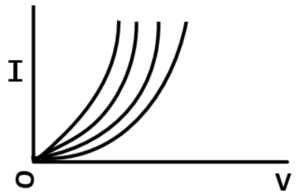
Step 3: Distinguish the decreasing curve Graph (D) represents a current that starts largest at the shortest wavelengths and then decreases monotonically. It omits the initial rise and maximum of the usual spectral-response curve. Thus a decrease in maximum electron kinetic energy with increasing wavelength must not itself be mistaken for a complete current-versus-wavelength law. The threshold fixes the vanishing current at long wavelengths; the practical spectral response supplies the peaked shape before that cutoff.

Step 4: Conclusion The usual spectral-response curve of the photocell is the peaked curve ending at the threshold wavelength. Hence, the correct option is (A).

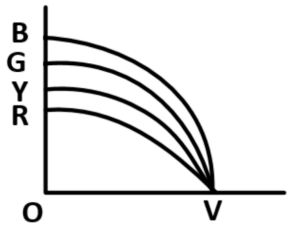
Quick Tip to Solve: For a photocell wavelength scan, separate the threshold condition from the current response: electron kinetic energy and collected photocurrent are different quantities.

36. The I-V characteristic of an LED is [4 marks]

(A)



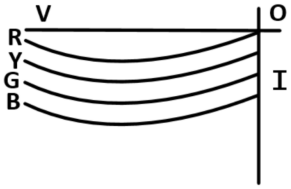
(B)



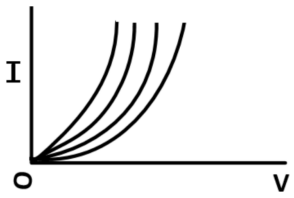
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- An LED is a forward-biased semiconductor p–n junction. Electron–hole recombination releases light; appreciable current starts near the junction threshold voltage.
- The forward current grows rapidly and nonlinearly with forward voltage. Its characteristic is diode-like, rather than a straight line obeying a constant resistance.

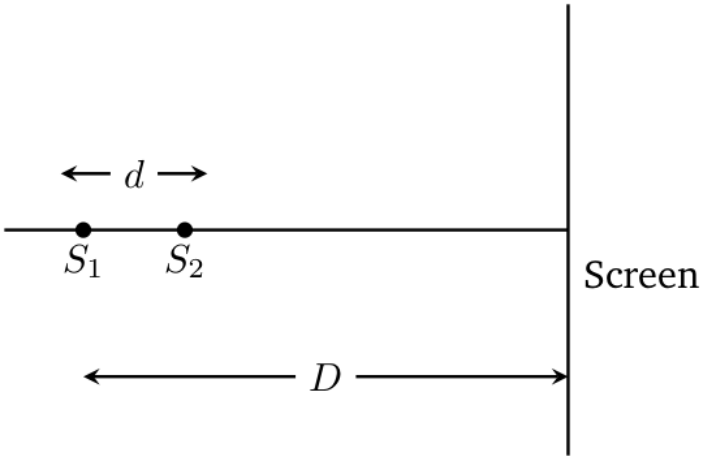
Step 1: Identify the operating quadrant An LED emits light when forward biased. Positive forward voltage produces positive current, so the relevant characteristic lies in the first quadrant of the current–voltage axes. Below the turn-on region the current is small, whereas beyond that region it rises steeply. Different semiconductor band gaps shift the turn-on voltage; the family of rising curves in option (A) is consistent with such forward characteristics.

Step 2: Compare the four graphical shapes Option (A) shows a positive current that rises increasingly rapidly as voltage increases, the required diode behaviour. Option (B) instead has a nonzero current at zero voltage and decreases to zero as voltage increases; this resembles a light-generating source characteristic rather than the forward input characteristic of an LED. Option (C) includes a falling-current segment, implying negative differential resistance, which is not the normal LED characteristic. Option (D) uses negative-coordinate curves and does not represent the forward light-emitting operating region.

Step 3: Conclusion The increasing nonlinear forward current–voltage characteristic is the graph in (A). Hence, the correct option is (A).

Quick Tip to Solve: For an LED graph, first locate the forward-bias quadrant, then look for a knee followed by a rapid current increase.

37. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be [4 marks]



(A) points

(B) straight lines

(C) semi-circles

(D) concentric circles

Correct Answer: (D) concentric circles

Solution:

Concept:

- For two coherent sources, interference intensity depends on their optical path difference. Bright fringes satisfy $\Delta = m\lambda$, and dark fringes satisfy $\Delta = (m + \frac{1}{2})\lambda$.
- When the line joining the two sources is normal to the screen, rotational symmetry makes the path difference depend only on radial distance from the axis.

Step 1: Express the path difference on the screen Let the normal through the sources meet the screen at O, and consider a point P at radial distance $\rho = OP$. With the source positions shown, their distances from the screen along the common axis are D and $D - d$. The two distances to P are therefore

$$r_1 = \sqrt{D^2 + \rho^2}, \qquad r_2 = \sqrt{(D-d)^2 + \rho^2}.$$

Their difference $\Delta = r_1 - r_2$ contains ρ, but no azimuthal angle around the axis.

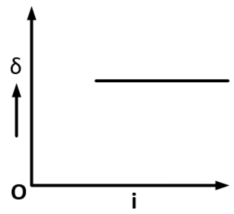
Step 2: Identify the constant-phase loci Every point on a circle centred at O has the same ρ, so it has the same path difference and interference intensity. Each allowed bright or dark condition consequently gives a circular ring, and successive conditions give rings sharing the centre O. The pattern is not a collection of isolated points because rotation about the source axis changes neither path length. Straight fringes correspond to the familiar transverse source separation, whereas here the source separation is along the screen normal. There is also no restriction to half of the screen that could select semicircles.

Step 3: Conclusion The fringes are concentric circles about the point where the source axis meets the screen. Hence, the correct option is (D).

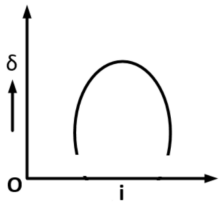
Quick Tip to Solve: Check the source-separation direction: sources placed along the screen normal give circular symmetry, unlike the usual transverse double slit.

38. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by [4 marks]

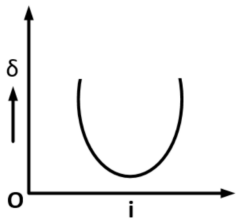
(A)



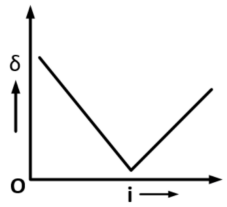
(B)



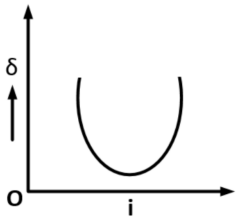
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- For a prism of angle A, the deviation is $\delta = i + e - A$, where i and e are the angles of incidence and emergence.
- The deviation reaches a minimum at the symmetric passage of the ray: $i = e$ and $r_1 = r_2 = A/2$. On either side of this condition the deviation is larger.

Step 1: Follow the deviation as incidence changes For emergent rays, increasing the angle of incidence initially reduces the angle of deviation. This reduction continues until the ray passes symmetrically through the prism. At that point

$$\delta_{\min} = 2i_{\min} - A.$$

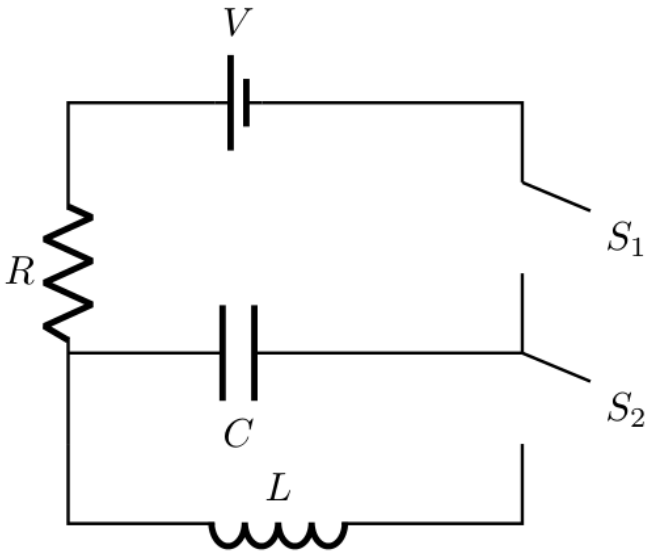
Beyond the symmetric condition, increasing incidence increases deviation again. The graph therefore has a smooth minimum at a nonzero deviation, and rises on either side. This behaviour follows from refraction at both faces; the emergence angle is itself changing as incidence changes.

Step 2: Match the smooth minimum to the options Option (C) is the smooth U-shaped curve with a lowest value, matching the decrease followed by increase. Option (A) would make deviation independent of incidence, contrary to the two-face refraction relation. Option (B) has a maximum rather than a minimum and reverses the physical trend. Option (D) contains two straight segments meeting at a sharp corner. Refraction changes continuously and smoothly through the symmetric configuration, so that angular corner is not the actual deviation curve. The U shape is qualitative; it need not be an exact parabola.

Step 3: Conclusion The deviation–incidence graph is a smooth curve with a minimum, represented by (C). Hence, the correct option is (C).

Quick Tip to Solve: Remember the prism sequence: deviation decreases, reaches minimum at symmetric passage, then increases; select a smooth valley.

39. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is Capacitance time constant). Which of the following statement is correct ? [4 marks]



(A) Work done by the battery is half of the energy dissipated in the resistor

(B) $t = \tau$, $q = CV/2$

(C) At $t = 2\tau$, $q = CV\left(1 - e^{-2}\right)$

(D) At $t = 2\tau$, $q = CV\left(1 - e^{-1}\right)$

Correct Answer: (C) At $t = 2\tau$, $q = CV\left(1 - e^{-2}\right)$

Solution:

Concept:

- With S_1 closed and S_2 open, the inductor branch is disconnected. The active circuit is a battery charging an initially uncharged capacitor through R.
- Kirchhoff's loop law gives $V = R\,dq/dt + q/C$. The charging time constant is $\tau = RC$.

Step 1: Solve the charging equation The initial condition is $q(0) = 0$. Separating the first-order loop equation, or using its standard exponential solution, gives

$$q(t) = CV\left(1 - e^{-t/(RC)}\right) = CV\left(1 - e^{-t/\tau}\right).$$

The limiting charge is CV, approached asymptotically rather than reached after one finite time constant. The open switch S_2 prevents current in the inductor, so an LC oscillation formula is inapplicable to this switching condition.

Step 2: Test the stated times and energy relation At $t = 2\tau$, substitution gives

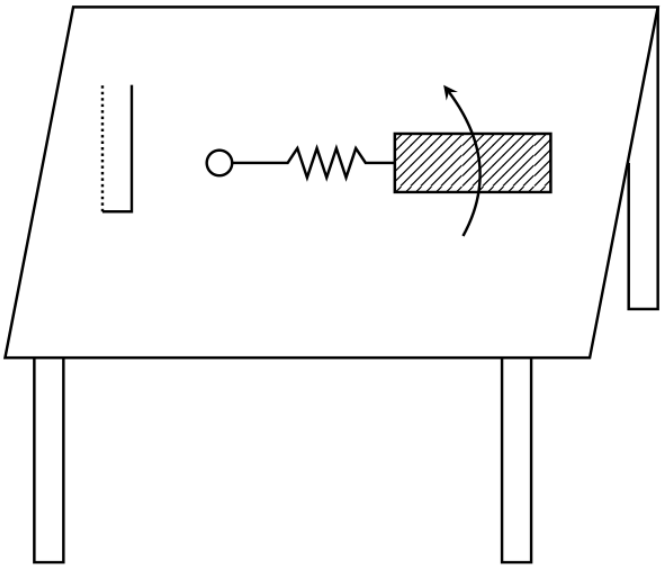
$$q(2\tau) = CV(1 - e^{-2}),$$

exactly as in (C). At $t = \tau$, the charge is $CV(1-e^{-1}) \simeq 0.632CV$, so (B) is false. Option (D) incorrectly uses the one-time-constant exponent for the two-time-constant instant. During complete charging the battery supplies $W = V(CV) = CV^2$; the capacitor stores $CV^2/2$ and the resistor dissipates $CV^2/2$. Battery work is therefore twice, rather than half, the dissipated energy, rejecting (A).

Step 3: Conclusion At two time constants the charge is $CV(1 - e^{-2})$. Hence, the correct option is (C).

Quick Tip to Solve: Insert the dimensionless ratio t/τ directly in the charging exponent; disconnect any branch whose switch remains open.

40. A metallic rod of length $'\ell'$ is tied to a string of length 2ℓ and made to rotate with angular speed w on a horizontal table with one end of the string fixed. If there is a vertical magnetic field $'B'$ in the region, the $e.m.f$ induced across the ends of the rod is [4 marks]



(A) $\frac{2B\omega\ell}{2}$

(B) $\frac{3B\omega\ell}{2}$

(C) $\frac{4B\omega\ell^2}{2}$

(D) $\frac{5B\omega\ell^2}{2}$

Correct Answer: (D) $\frac{5B\omega\ell^2}{2}$

Solution:

Concept:

- A conductor moving in a magnetic field experiences magnetic force on its free charges. The motional emf magnitude is $\mathcal{E} = \int(\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$.
- For radial rotation at angular speed ω in a perpendicular field, the local speed is $v = \omega r$, so an element contributes $d\mathcal{E} = B\omega r\, dr$.

Step 1: Locate the ends of the rotating rod The string of length 2ℓ joins the fixed centre to the near end of the rod. In the rotating radial arrangement shown, the near end is therefore at $r_1 = 2\ell$, and the far end is at $r_2 = 3\ell$. The whole rod has angular speed ω, but its linear speed increases from $2\omega\ell$ to $3\omega\ell$. It is consequently incorrect to assign the same endpoint speed to all elements.

Step 2: Integrate the magnetic contribution along the rod The rod lies horizontally and the magnetic field is vertical, so the velocity is perpendicular to the field and its magnetic contribution acts along the radial rod. Integrating gives

$$\mathcal{E} = B\omega \int_{2\ell}^{3\ell} r\, dr = \frac{B\omega}{2}\left[(3\ell)^2 - (2\ell)^2\right] = \frac{5B\omega\ell^2}{2}.$$

Equivalently, the rod's mean speed is $\omega(2\ell + 3\ell)/2$, and multiplying by $B\ell$ gives the same result. Options (A) and (B), as printed, lack a second length factor and have incorrect dimensions for emf. Option (C) has the right dimensions but the wrong radial-square difference.

Step 3: Conclusion The emf magnitude between the rod ends is $5B\omega\ell^2/2$. Hence, the correct option is (D).

Quick Tip to Solve: For a rotating radial rod use $B\omega(r_2^2 - r_1^2)/2$; measure both endpoint radii from the fixed rotation centre.

41. Diameter of a plano-convex lens is $6\ cm$ and thickness at the center is $3mm$. If speed of light in material of lens is $2 \times 10^8\ m/s$, the focal length of the lens is [4 marks]

(A) $15\ cm$

(B) $20\ cm$

(C) $30\ cm$

(D) $10\ cm$

Correct Answer: (C) $30\ cm$

Solution:

Concept:

- The refractive index of the lens material is $n = c/v$. For a thin plano-convex lens in air, the lens maker formula reduces to $1/f = (n-1)/R$.
- The spherical surface sagitta h, aperture radius a, and curvature radius R obey $a^2 + (R-h)^2 = R^2$. For a shallow cap, $R \simeq a^2/(2h)$.

Step 1: Calculate the refractive index and curvature With $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$ and the given material speed,

$$n = \frac{3 \times 10^8}{2 \times 10^8} = 1.5.$$

The diameter is $6\,\mathrm{cm}$, so the aperture radius is $a = 3\,\mathrm{cm}$, not $6\,\mathrm{cm}$. The central thickness is $h = 3\,\mathrm{mm} = 0.3\,\mathrm{cm}$. The shallow-cap relation gives

$$R \simeq \frac{3^2}{2(0.3)} = 15\,\mathrm{cm}.$$

Retaining the small sagitta-squared correction gives $R = (a^2 + h^2)/(2h) = 15.15\,\mathrm{cm}$, consistent with the same rounded option.

Step 2: Apply the plano-convex lens formula The plane face has infinite radius of curvature and contributes zero to the lens maker expression. Therefore

$$f = \frac{R}{n-1} \simeq \frac{15}{0.5} = 30\,\mathrm{cm}.$$

Keeping the geometrical correction would give about $30.3\,\mathrm{cm}$; the listed answer is the usual thin-lens, shallow-cap value $30\,\mathrm{cm}$. Option (A) confuses curvature radius with focal length. The alternatives $20\,\mathrm{cm}$ and $10\,\mathrm{cm}$ do not satisfy the measured curvature together with $n - 1 = 0.5$.

Step 3: Conclusion The focal length is approximately $30\,\mathrm{cm}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert thickness and diameter to one unit, halve the diameter, find the spherical radius, then divide it by $n - 1$.

42. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is [4 marks]

(A) I_0

(B) $\frac{I_0}{2}$

(C) $\frac{I_0}{4}$

(D) $\frac{I_0}{8}$

Correct Answer: (C) $\frac{I_0}{4}$

Solution:

Concept:

- An ideal polaroid transmits half the intensity of incident unpolarised light, because the average squared projection of all equally probable electric-field directions is $1/2$.
- For linearly polarised incident light, Malus' law is $I = I_{\text{in}} \cos^2\theta$, where θ is the angle between the transmission axes.

Step 1: Find the intensity transmitted by the first polaroid The incident beam is unpolarised with intensity I_0. Polaroid A selects the electric-field component along its transmission axis. Averaging over the random incident directions gives

$$I_A = \frac{I_0}{2}.$$

The beam after A is now linearly polarised, so it is this reduced intensity, rather than the original I_0, that must be used as the input to the second polaroid. This first loss occurs even when no second polaroid is present.

Step 2: Apply Malus law at the second polaroid The transmission axis of B makes 45° with that of A. Thus

$$I_B = I_A \cos^2 45^\circ = \frac{I_0}{2} \times \frac{1}{2} = \frac{I_0}{4}.$$

Option (A) ignores both reductions. Option (B) accounts for only one reduction, as would be appropriate for the first polaroid alone or for a second axis parallel to the first. Option (D) introduces an extra factor of one-half that is not present: two polaroids give precisely the first unpolarised-light average and then the single Malus-law projection. The intensity factor involves the square of cosine, because intensity is proportional to the square of electric-field amplitude.

Step 3: Conclusion The emergent intensity is $I_0/4$. Hence, the correct option is (C).

Quick Tip to Solve: For unpolarised light followed by two ideal polaroids, use $I = (I_0/2)\cos^2\theta$, with θ their relative angle.

43. Two short bar magnets of length $1\ cm$ each have magnetic moments $1.20\ Am^2$ and $1.00\ Am^2$ respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of $20.0\ cm$. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of earth's magnetic induction is $3.6 \times 10.5 Wb/m^2$) [4 marks]

(A) $3.6 \times 10.5\ Wb/m^2$

(B) $2.56 \times 10.4\ Wb/m^2$

(C) $3.50 \times 10.4\ Wb/m^2$

(D) $5.80 \times 10.4\, Wb/m^2$

Correct Answer: (A) $3.6 \times 10.5\ Wb/m^2$

Solution:

Concept:

- The equatorial field of a short magnetic dipole has magnitude $B_{\text{eq}} = \mu_0 m/(4\pi r^3)$ and points opposite to its magnetic moment.
- Magnetic moment points from south pole to north pole. With both north poles pointing geographical south, the two equatorial fields point north and add to the northward horizontal Earth field.

Step 1: Identify the midpoint geometry The magnets share a magnetic equator and their centres are separated by $20.0\,\text{cm}$. Their common midpoint is therefore $r = 10.0\,\text{cm} = 0.100\,\text{m}$ from each centre, on the equatorial line of each magnet. Since their lengths are only $1\,\text{cm}$, this distance is large compared with each magnet length, so the short-dipole approximation applies.

Step 2: Add the two dipole fields Both moments are parallel and point south, so their equatorial fields at the midpoint point north. Their combined magnitude is

$$B_m = 10^{-7}\frac{1.20 + 1.00}{(0.100)^3}\,\text{T} = 2.20 \times 10^{-4}\,\text{T}.$$

The fields do not cancel: the midpoint is equatorial rather than an axial point between opposite ends. The moment sum, rather than the difference, must therefore be used.

Step 3: Use the literal numerical factors and compare The given horizontal Earth-field expression evaluates literally to

$$B_H = 3.6 \times 10.5\,\text{T} = 37.8\,\text{T}.$$

Adding the northward contributions gives

$$B_{\text{net}} = 37.8 + 0.000220 = 37.80022\,\text{T}.$$

Option (A) is $37.8\,\text{T}$, differing by only $0.00022\,\text{T}$. The other printed values are $26.624\,\text{T}$, $36.4\,\text{T}$, and $60.32\,\text{T}$, respectively. Thus (A) is unambiguously closest to the resultant using the numbers actually given; replacing the decimal factors by powers would be a different calculation.

Step 4: Conclusion The literal resultant is $37.80022\,\text{T}$, closest to $3.6 \times 10.5\,\text{Wb}\,\text{m}^{-2}$. Hence, the correct option is (A). This differs from the question-sheet key: with the literal decimal factors given, the keyed second entry is $26.624\,\text{T}$, whereas the calculated result is $37.80022\,\text{T}$.

Quick Tip to Solve: At a dipole equator the field opposes the magnetic moment. Establish that direction before adding the Earth field and evaluate the printed numerical factors literally.

44. The supply voltage to room is $120V$. The resistance of the lead wires is 6Ω. A $60\ W$ bulb is already switched on. What is the decrease of voltage across the bulb, when a $240\ W$ heater is switched on in parallel to the bulb? [4 marks]

(A) zero

(B) 2.9 Volt

(C) 13.3 Volt

(D) 10.04 Volt

Correct Answer: None of the printed options equals the calculated decrease of 10.41 V.

Solution:

Concept:

- A device rated at power P on voltage V has resistance $R = V^2/P$, using the fixed-resistance model for this circuit problem.
- The lead resistance is in series with the room appliances. Switching on a parallel appliance lowers their equivalent resistance and increases the lead-wire voltage drop.
- The common appliance voltage follows the divider relation $V_{\text{load}} = V_s R_{\text{load}}/(r + R_{\text{load}})$, where r is the total lead resistance.

Step 1: Calculate the appliance resistances Use the common rated supply of $120\,\text{V}$. The bulb and heater resistances are respectively

$$R_b = \frac{120^2}{60} = 240\,\Omega, \qquad R_h = \frac{120^2}{240} = 60\,\Omega.$$

Their rated powers specify resistances; they are not constant powers once the lead wires lower the actual terminal voltage.

Step 2: Find the bulb voltage before switching With only the bulb connected, the lead wires and bulb form a series circuit. Thus

$$I_1 = \frac{120}{240 + 6} = \frac{20}{41}\,\text{A}, \qquad V_1 = I_1 R_b = \frac{4800}{41}\,\text{V} \approx 117.073\,\text{V}.$$

The bulb therefore does not initially receive the full source voltage.

Step 3: Find the new parallel-load voltage The heater is across the bulb, so

$$R_p = \frac{240 \times 60}{240 + 60} = 48\,\Omega.$$

The new common voltage is

$$V_2 = 120\frac{48}{48 + 6} = 106.667\,\text{V}.$$

Consequently

$$\Delta V = 117.073 - 106.667 = 10.406\,\text{V}.$$

A zero drop would require zero lead resistance. The values 2.9 V and 13.3 V are not the decrease between the two operating states; 13.3 V is approximately the final lead drop alone. The printed 10.04 V is the only nearby choice, but differs from the exact fixed-resistance result.

Step 4: Conclusion The decrease calculated from the stated values is $10.41\,\mathrm{V}$. None of the printed options is correct; (D), $10.04\,\mathrm{V}$, appears intended but is not the result of the given circuit values.

Quick Tip to Solve: Compute the terminal voltage twice; subtracting the final lead drop from the supply alone does not give the change in bulb voltage.

45. This questions has Statement - I and Statement - II. Of the four choices given after the Statements, choose the one that best describes into two Statements.

Statement - I : Higher the range, greater is the resistance of ammeter.

Statement - II : To increase the range of ammeter, additional shunt needs to be used across it.

[4 marks]

(A) Statement - I is true, Statement - II is true, Statement - II is the correct explanation of statement - I.

(B) Statement - I is true, Statement - II is true, Statement - II is not the correct explanation of statement - I.

(C) Statement - I is true, Statement - II is false

(D) Statement - I is false, Statement - II is true

Correct Answer: (D) Statement - I is false, Statement - II is true

Solution:

Concept:

- An ammeter is inserted in series and should have low effective resistance, so that its insertion changes the circuit current as little as possible.
- A galvanometer of resistance G and full-scale current I_g becomes an ammeter of range I when a shunt $S = I_gG/(I - I_g)$ carries the excess current.

Step 1: Evaluate Statement I At full scale the voltage across the galvanometer, and therefore across the complete ammeter, is $V_g = I_gG$. Its effective resistance is

$$R_A = \frac{V_g}{I} = \frac{I_gG}{I}.$$

Keeping the movement fixed, increasing the full-scale current I decreases R_A. Equivalently, $R_A = GS/(G + S)$ falls when a smaller parallel shunt is connected. Thus the claim that a higher-range ammeter has greater resistance is false.

Step 2: Evaluate Statement II The movement may safely carry only its designed full-scale current. To measure a greater total current, an additional low-resistance bypass is connected across it, diverting more current around the sensitive movement. At the desired new range, equal parallel-branch voltages require

$$I_gG = (I - I_g)S.$$

A larger I requires smaller effective S; an additional parallel shunt achieves this. Statement II is true.

Step 3: Select the statement combination The options must be checked as combinations, rather than choosing a statement because shunts are associated with ammeters. Options (A) and (B) both assert Statement I to be true and are excluded. Option (C) also accepts Statement I and rejects the

valid shunt principle, so it fails both tests. Option (D) alone states that I is false and II is true. No explanation relationship can make a false Statement I correct.

Step 4: Conclusion Statement I is false and Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: For a fixed galvanometer movement, remember $IR_A = I_g G$: a larger current range means a smaller effective ammeter resistance.

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46. Two charges, each equals to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement $(y << a)$ along the y-axis, the net force acting on the particle is proportional to [4 marks]

(A) y

(B) $-y$

(C) $\frac{1}{y}$

(D) $-\frac{1}{y}$

Correct Answer: (A) y

Solution:

Concept:

- Coulomb force acts along the joining line, with magnitude $k|qq_0|/r^2$, where $k = 1/(4\pi\varepsilon_0)$.
- Symmetry cancels the horizontal components at $(0, y)$, while the vertical components add. Here $qq_0 = q^2/2 > 0$, so each force is repulsive.

Step 1: Resolve the two electric forces For a displacement to $(0, y)$, the distance from either fixed charge is $r = \sqrt{a^2 + y^2}$. A single force has vertical component

$$F_{1y} = \frac{kqq_0}{r^2}\frac{y}{r} = \frac{kqq_0 y}{(a^2 + y^2)^{3/2}}.$$

The second charge has the same vertical component and an opposite horizontal component. This remains true for either sign of q, because the test charge always has the same sign as both source charges.

Step 2: Use the small-displacement condition The net vertical force is therefore

$$F_y = \frac{2kqq_0 y}{(a^2 + y^2)^{3/2}} = \frac{kq^2 y}{(a^2 + y^2)^{3/2}}.$$

Since $|y| \ll a$, the denominator is approximately a^3, giving

$$F_y \simeq \frac{kq^2}{a^3}\, y.$$

The proportionality coefficient is positive, independent of the sign chosen for the two equal charges.

Step 3: Check the sign and eliminate alternatives For positive y, both repulsions have an upward component; for negative y, both point downward. The force thus acts away from the origin along the

displacement, rather than restoring the particle to it. This excludes $-y$. The force is zero at $y = 0$ and varies smoothly nearby, whereas $1/y$ and $-1/y$ diverge there. The mass affects any subsequent acceleration but does not enter the electrostatic force itself.

Step 4: Conclusion The net force is proportional to $+y$ for a small transverse displacement. Hence, the correct option is (A).

Quick Tip to Solve: Cancel the symmetric components before expanding: here the remaining transverse repulsive components add, so the force has the same sign as the displacement.

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47. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released,. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frquency [4 marks]

(A) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$

(B) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$

(C) $\frac{1}{2\pi}\sqrt{\frac{A\gamma P_0}{V_0 M}}$

(D) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

Correct Answer: None of the options equals $f = A\sqrt{\gamma P_0/(MV_0)}/(2\pi)$.

Solution:

Concept:

- For an isolated ideal gas undergoing a small reversible displacement, PV^γ is constant; $\gamma = C_P/C_V$ is the heat-capacity ratio.
- A small upward piston displacement x changes the gas volume by $\delta V = Ax$. Differentiating the adiabatic law gives $\delta P = -\gamma P_0 \delta V/V_0$.
- The equilibrium external pressure and piston weight cancel the equilibrium gas force. Only the change in gas force provides the restoring force for the displacement.

Step 1: Find the gas-pressure change At equilibrium the piston experiences zero net force. For a small upward displacement, the expanded gas exerts a smaller pressure, while the external forces are unchanged. Linearising the adiabatic equation gives

$$\delta P = -\frac{\gamma P_0}{V_0}\delta V = -\frac{\gamma P_0 A}{V_0}x.$$

A downward displacement reverses both signs, producing an increased upward gas force.

Step 2: Obtain the restoring-force constant The gas pressure acts on piston area A, so the change in upward force is

$$F = A\delta P = -\frac{\gamma P_0 A^2}{V_0}x.$$

Newton's second law is therefore

$$M\ddot{x} = -\frac{\gamma P_0 A^2}{V_0} x.$$

This is simple harmonic motion with

$$\omega^2 = \frac{\gamma P_0 A^2}{M V_0}, \qquad f = \frac{A}{2\pi}\sqrt{\frac{\gamma P_0}{M V_0}}.$$

The two factors of area arise separately from converting displacement into volume and pressure into force.

Step 3: Check every printed option Neither (A) nor (B) has the square-root dependence required by $\omega = \sqrt{k/M}$. Option (C) resembles the frequency but contains only A, rather than A^2, under its radical. Its dimensions are $\mathrm{m^{-1}s^{-1}}$, not frequency. Option (D) inverts the dynamical ratio and has dimensions $\mathrm{m\,s}$. Thus none is the frequency for the stated area definition, and selecting (C) would discard a necessary area factor.

Step 4: Conclusion The frequency is $f = \frac{A}{2\pi}\sqrt{\frac{\gamma P_0}{M V_0}}$. None of the printed options is correct.

Quick Tip to Solve: Two area factors are needed: one in $\delta V = Ax$ and another in $\delta F = A\delta P$. A dimensional check detects a missing area factor in the frequency.

48. A sonometer wire of length $1.5\ m$ is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are $7.7 \times 10^3\ kg/m^3$ and $2.2 \times 10^{11}\ N/m^2$ respectively ? [4 marks]

(A) $188.5\ Hz$

(B) $178.2\ Hz$

(C) $200.5\ Hz$

(D) $770\ Hz$

Correct Answer: (B) $178.2\ Hz$

Solution:

Concept:

- For a wire fixed at both ends, the fundamental standing wave has wavelength $2L$, so $f_1 = (1/2L)\sqrt{T/\mu}$.
- Young's modulus is stress divided by strain: $Y = (T/A)/\epsilon$. The mass per unit length is $\mu = \rho A$; hence the cross-sectional area cancels.

Step 1: Express the wave speed using the strain The applied tension produces strain $\epsilon = 1\% = 0.01$, not unity. Writing the tension as $T = YA\epsilon$, and the linear mass density as ρA, gives

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{YA\epsilon}{\rho A}} = \sqrt{\frac{Y\epsilon}{\rho}}.$$

The radius or diameter of the wire is therefore unnecessary: a thicker wire requires proportionally more tension to produce the same strain.

Step 2: Substitute the given material constants Using the SI values in the question,

$$v = \sqrt{\frac{(2.2 \times 10^{11})(0.01)}{7.7 \times 10^{3}}} = \sqrt{2.85714 \times 10^{5}}\,\mathrm{m\,s^{-1}} \approx 534.52\,\mathrm{m\,s^{-1}}.$$

This is the transverse wave speed on the tensioned wire; the longitudinal sound speed $\sqrt{Y/\rho}$ would omit the strain factor and would not describe the sonometer mode.

Step 3: Apply the fundamental-mode condition The lowest mode has nodes at both bridges and one antinode between them, so its half-wavelength equals the stated wire length. Thus

$$f_1 = \frac{534.52}{2(1.5)}\,\mathrm{Hz} = 178.17\,\mathrm{Hz} \approx 178.2\,\mathrm{Hz}.$$

This agrees with (B). The frequencies in (A) and (C) do not follow from the supplied constants and length; 770 Hz is much too large and is not the fundamental obtained from $v/(2L)$.

Step 4: Conclusion The fundamental frequency is approximately $178.2\,\mathrm{Hz}$. Hence, the correct option is (B).

Quick Tip to Solve: When strain is supplied instead of tension and diameter, use $v = \sqrt{Y\epsilon/\rho}$ directly, converting the percentage strain to a fraction.

49. Two capacitors C_1 and C_2 are charged to $120\ V$ and $200\ V$ respectively. It is found that connecting them together the potential on each one can be made zero. Then [4 marks]

(A) $5C_1 = 3C_2$

(B) $3C_1 = 5C_2$

(C) $3C_1 + 5C_2 = 0$

(D) $9C_1 = 4C_2$

Correct Answer: (B) $3C_1 = 5C_2$

Solution:

Concept:

- The charge magnitude on a capacitor is $Q = CV$. A capacitor charged to zero final voltage has zero final plate charge in the ideal capacitance model.
- When charged capacitors are connected in parallel, the total charge on each isolated connected plate node is conserved. Reversing one capacitor allows opposite initial node charges to cancel.

Step 1: Choose the polarity needed for cancellation If the positive plates are joined together, their charges add and the common final voltage is positive, so it cannot vanish. To make the voltage zero, connect the positive plate of the first capacitor to the negative plate of the second, and vice versa. On one resulting node the initial charges are $+Q_1$ and $-Q_2$. The other node carries their equal and opposite total.

Step 2: Conserve the connected-node charge Choose the final voltage direction to agree with the initial polarity of C_1. The connected pair has effective capacitance $C_1 + C_2$, and hence

$$(C_1 + C_2)V_f = 120C_1 - 200C_2.$$

The condition $V_f = 0$ requires

$$120C_1 = 200C_2 \quad \Rightarrow \quad 3C_1 = 5C_2.$$

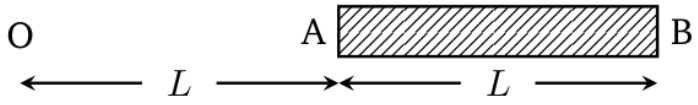
It is charge, rather than initial voltage or stored energy, that must cancel at the nodes.

Step 3: Check the alternatives and physical meaning The required ratio is $C_1/C_2 = 5/3$: the capacitor charged to the lower voltage must have the larger capacitance to carry the same initial charge magnitude. Option (A) gives the reciprocal ratio and fails this check. Option (C) would require a sum of positive capacitance terms to vanish, impossible for ordinary capacitors. Option (D) gives $C_1/C_2 = 4/9$, also inconsistent with equal charges. Energy need not be conserved as capacitor energy alone: transient currents dissipate it during connection.

Step 4: Conclusion The necessary condition is $3C_1 = 5C_2$. Hence, the correct option is (B).

Quick Tip to Solve: Zero final voltage requires equal opposite initial plate charges: equate C_1V_1 and C_2V_2, rather than the two voltages.

50. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance L from the end A is [4 marks]



(A) $\frac{Q}{8\pi\varepsilon_0 L}$
(B) $\frac{3Q}{4\pi\varepsilon_0 L}$
(C) $\frac{Q}{4\pi\varepsilon_0 L \ln 2}$
(D) $\frac{Q \ln 2}{4\pi\varepsilon_0 L^s}$

Correct Answer: No literal option matches: $V = Q \ln 2/(4\pi\varepsilon_0 L)$, while (D) has an undefined exponent s on L.

Solution:

Concept:

- The electric potential of a small charge element is $dV = dq/(4\pi\varepsilon_0 x)$, where x is its distance from the observation point. Potential is a scalar, so the element contributions add directly.
- For uniform total charge Q on length L, linear charge density is $\lambda = Q/L$. The figure places the rod along the same line as O, with its near and far ends at distances L and $2L$.

Step 1: Express the charge element and integration limits Take a coordinate x measured from

O towards the rod. A small rod segment of length dx carries charge $dq = \lambda dx = (Q/L)dx$. Its contribution is

$$dV = \frac{Q}{4\pi\varepsilon_0 L}\frac{dx}{x}.$$

The integration runs from $x = L$ at A to $x = 2L$ at B. The position-dependent distance cannot be replaced by one endpoint distance for the entire rod.

Step 2: Integrate the potential With potential zero at infinity,

$$V(O) = \frac{Q}{4\pi\varepsilon_0 L}\int_L^{2L}\frac{dx}{x} = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{2L}{L}\right).$$

$$\boxed{V(O) = \frac{Q\ln 2}{4\pi\varepsilon_0 L}}.$$

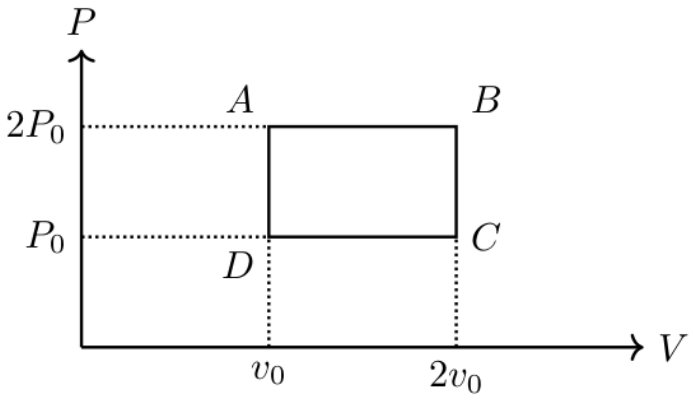
The logarithm acts on a ratio of distances, so its argument is dimensionless. The sign of the potential follows the sign of Q.

Step 3: Compare the literal printed options Options A and B give coefficients $1/2$ and 3, respectively, instead of $\ln 2$. Option C puts $\ln 2$ in the denominator, whereas integration multiplies by it. Option D prints L^s in the denominator with an undefined superscript s. For arbitrary L, this is not the derived expression, and the question does not define $s = 1$. Thus no literal option is uniquely valid as written. Omitting the superscript would give the derived result, but that would change the printed option rather than solve it.

Step 4: Conclusion The potential is $\boxed{Q\ln 2/(4\pi\varepsilon_0 L)}$. No literal printed option matches: (D) contains the undefined exponent s. The answer-key selection (D) agrees only after removing that exponent, so the printed multiple-choice item is ambiguous.

Quick Tip to Solve: For a collinear uniformly charged rod, integrate dq divided by distance; the answer contains the logarithm of far-end distance divided by near-end distance.

51. The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monatomic gas. The amount of heat, extracted from the source in a single cycle is [4 marks]



(A) p_0v_0

(B) $\left(\frac{13}{2}\right)p_0v_0$

(C) $\left(\frac{11}{2}\right)p_0v_0$

(D) $4p_0v_0$

Correct Answer: (B) $\left(\frac{13}{2}\right)p_0v_0$

Solution:

Concept:

- For an ideal monatomic gas, internal energy is $U = \frac{3}{2}nRT = \frac{3}{2}pv$. Thus $\Delta U = \frac{3}{2}\Delta(pv)$.
- With work done by the gas positive, the first law is $Q = \Delta U + W$, where $W = \int p\,dv$. Heat extracted from the source means total positive heat input, rather than net heat over the cycle.

Step 1: Identify the heat-absorbing paths The engine traverses the rectangle clockwise: $D \to A \to B \to C \to D$. On $D \to A$, volume remains v_0 while pressure rises from p_0 to $2p_0$. On $A \to B$, pressure remains $2p_0$ and volume rises from v_0 to $2v_0$. Both paths increase temperature and require positive heat. The other two paths reject heat.

Step 2: Calculate heat during constant-volume heating No work is done on the vertical path, since $dv = 0$. Therefore,

$$Q_{DA} = \frac{3}{2}(2p_0v_0 - p_0v_0) = \frac{3}{2}p_0v_0.$$

This is the first contribution to heat supplied by the source.

Step 3: Calculate heat during expansion and add On the upper horizontal path,

$$\Delta U_{AB} = \frac{3}{2}(4p_0v_0 - 2p_0v_0) = 3p_0v_0.$$

$$W_{AB} = 2p_0(2v_0 - v_0) = 2p_0v_0.$$

$$Q_{AB} = 5p_0v_0, \qquad Q_{\text{in}} = Q_{DA} + Q_{AB} = \frac{13}{2}p_0v_0.$$

Option A gives the net cycle work, the rectangular area, rather than the heat input. Options C and D

omit part of the required heating.

Step 4: Conclusion The heat extracted from the source per cycle is $\boxed{\frac{13}{2}p_0v_0}$. Hence, the correct option is (B).

Quick Tip to Solve: For a rectangular engine cycle, add heat on the upward and rightward paths; the enclosed area gives work, not heat input.

52. Assume that a drop of liquid evaporates by decreases in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible ? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization. [4 marks]

(A) $\rho L/T$

(B) $\sqrt{T/\rho L}$

(C) $T/\rho L$

(D) $2T/\rho L$

Correct Answer: No positive minimum radius exists in this model; evaporation is possible for $0 < r \leq 2T/(\rho L)$. Option (D) gives the critical maximum radius.

Solution:

Concept:

- A spherical liquid drop of radius r has surface energy $E_s = 4\pi r^2 T$, where T is surface tension. A small decrease in radius releases surface energy.
- The mass of the drop is $m = \frac{4}{3}\pi r^3 \rho$. Evaporating mass dm requires latent heat $L\,dm$, without changing its temperature.

Step 1: Find the surface energy released Let the radius decrease by a small positive amount δr. To first order, the released surface energy is

$$\delta E_s = 8\pi r T\,\delta r.$$

Use this differential change, rather than equating the entire surface energy to the entire mass latent heat: evaporation proceeds through successive thin shells.

Step 2: Find the latent heat required The thin spherical layer has volume $4\pi r^2 \delta r$ and mass $4\pi r^2 \rho\,\delta r$. Its vaporization requires

$$\delta Q = 4\pi r^2 \rho L\,\delta r.$$

At the limiting radius, surface-energy release exactly supplies this requirement, so

$$8\pi r T\,\delta r = 4\pi r^2 \rho L\,\delta r \quad \Rightarrow \quad r_c = \frac{2T}{\rho L}.$$

Step 3: Interpret the limiting value and compare options The ratio of available energy to required

heat is

$$\frac{\delta E_s}{\delta Q} = \frac{2T}{\rho L r}.$$

Hence sufficient surface energy is available when $r \le r_c$. Strictly, the wording "minimum radius" is inaccurate: the calculation gives the largest admissible radius, or critical radius, and smaller drops also satisfy the energy condition. Option D is this limiting scale. Option A has dimensions of inverse length, option B is not a length, and option C misses the factor two arising from the area and volume derivatives.

Step 4: Conclusion The condition is $0 < r \le \boxed{2T/(\rho L)}$, so there is no positive minimum radius and none of the options answers the literal request. The answer key selects (D), which gives the critical maximum radius rather than a minimum.

Quick Tip to Solve: Differentiate surface area and volume before balancing evaporation energy; their derivatives produce the factor two and reveal which side of the threshold is possible.

53. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$? [4 marks]

(A) $\frac{5GmM}{6R}$
(B) $\frac{2GmM}{3R}$
(C) $\frac{GmM}{2R}$
(D) $\frac{GmM}{3R}$

Correct Answer: (A) $\frac{5GmM}{6R}$

Solution:

Concept:

- At distance r from a spherical planet, gravitational potential energy is $U = -GMm/r$, with zero at infinity.
- For a circular orbit, gravity supplies centripetal force, giving $v^2 = GM/r$ and total orbital energy $E = -GMm/(2r)$. Minimum launch energy is the increase in total mechanical energy, neglecting atmospheric losses and planetary rotation.

Step 1: Set the initial and final radii The satellite begins at the surface, at radial distance R, with no initial kinetic energy in the minimum-energy model. Its initial energy is

$$E_i = -\frac{GMm}{R}.$$

Altitude is measured above the surface, so the orbit at altitude $2R$ has radius $r = R + 2R = 3R$, not $2R$.

Step 2: Calculate the energy in the circular orbit At $r = 3R$, the circular speed satisfies

$$\frac{mv^2}{3R} = \frac{GMm}{(3R)^2} \quad \Rightarrow \quad v^2 = \frac{GM}{3R}.$$

Therefore,

$$K_f = \frac{GMm}{6R}, \qquad U_f = -\frac{GMm}{3R},$$

$$E_f = K_f + U_f = -\frac{GMm}{6R}.$$

The final kinetic energy must be included: merely lifting the satellite does not establish an orbit.

Step 3: Take the total-energy difference The launch must supply

$$\Delta E = E_f - E_i = -\frac{GMm}{6R} + \frac{GMm}{R} = \frac{5GMm}{6R}.$$

Option B is only the potential-energy increase and omits orbital kinetic energy. Option C corresponds to a different final energy; option D is not the energy difference for the specified initial and final states. The positive result is less than the escape energy GMm/R, as expected for a bound orbit.

Step 4: Conclusion The minimum required energy is $\boxed{5GMm/(6R)}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert altitude to distance from the centre first, then subtract total orbital energies rather than gravitational potentials alone.

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54. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement - I: A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$, then $f = \left(\frac{m}{M+m}\right)$.

Statement - II: Maximum energy loss occurs when the particles get stuck together as a result of the collision. [4 marks]

(A) Statement - I is true, Statement - II is true; Statement - II is the correct explanation of Statement - I.

(B) Statement - I is true, Statement - II is true; Statement - II is not the correct explanation of Statement - I.

(C) Statement - I is true, Statement - II is false

(D) Statement - I is false, Statement - II true.

Correct Answer: (D) Statement - I is false, Statement - II true.

Solution:

Concept:

- In an isolated collision, total momentum is conserved. The initial momentum is mv and the initial kinetic energy is $K_i = \frac{1}{2}mv^2$.
- For a fixed total momentum, the final kinetic energy is smallest when both particles move together, so their relative kinetic energy vanishes. This is a perfectly inelastic collision.

Step 1: Test Statement II using the energy minimum Write the total final kinetic energy as the kinetic energy of the centre of mass plus kinetic energy of relative motion. The centre-of-mass term is fixed by the conserved momentum; the relative term is nonnegative. Maximum energy loss therefore occurs when relative speed is zero, meaning that the particles stick together. Statement II is true.

Step 2: Calculate the common speed If the particles stick, conservation of momentum gives

$$mv = (m+M)V \quad \Rightarrow \quad V = \frac{mv}{m+M}.$$

Their final kinetic energy is

$$K_f = \frac{1}{2}(m+M)V^2 = \frac{m^2v^2}{2(m+M)}.$$

No assumption of kinetic-energy conservation has been made; the lost mechanical energy can become deformation, heat or sound.

Step 3: Evaluate the claimed loss fraction The maximum loss is

$$\Delta K_{\max} = \frac{1}{2}mv^2 - \frac{m^2v^2}{2(m+M)} = \frac{1}{2}mv^2\frac{M}{m+M}.$$

Thus the fraction lost is $f = M/(m+M)$, whereas Statement I claims $m/(m+M)$. Its expression is actually the fraction of initial kinetic energy remaining. The equality when $m = M$ is a special case and does not make a general assertion true. Options A and B wrongly declare Statement I true, and C also wrongly rejects Statement II.

Step 4: Conclusion Statement I is false, while Statement II is true: $\boxed{f = M/(M+m)}$. Hence, the correct option is (D).

Quick Tip to Solve: In a sticking collision with a stationary target, the moving mass divided by total mass gives the energy fraction remaining; the target mass divided by total mass gives the fraction lost.

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55. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a mass-less spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is: [4 marks]

(A) $\frac{Mg}{k}$
(B) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$
(C) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$
(D) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

Correct Answer: (C) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$

Solution:

Concept:

- Archimedes' principle gives buoyant force $F_b = \sigma g V_{\text{sub}}$, where σ is the liquid density and V_{sub} is displaced liquid volume.
- A spring extended by x_0 exerts upward force kx_0. Static equilibrium requires the sum of upward forces to equal the cylinder's weight Mg.

Step 1: Calculate the submerged volume The cylinder has uniform cross-sectional area A, and exactly

half its length L lies under the liquid surface. The displaced volume is therefore

$$V_{\text{sub}} = A\frac{L}{2}.$$

Consequently the upward buoyant force is

$$F_b = \frac{LA\sigma g}{2}.$$

The whole cylinder volume LA must not be used because the upper half displaces no liquid.

Step 2: Balance the vertical forces The downward force is Mg; spring force and buoyancy act upward. The equilibrium condition is

$$kx_0 + \frac{LA\sigma g}{2} = Mg.$$

Solving for the extension gives

$$x_0 = \frac{Mg}{k} - \frac{LA\sigma g}{2k} = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right).$$

The stated extension requires a nonnegative bracket.

Step 3: Check the physical sign and reject alternatives Buoyancy supports part of the weight, so the extension is smaller than Mg/k. Option A ignores buoyancy, while B incorrectly uses full immersion. Option D adds a downward buoyant contribution and also uses the full volume; both its sign and magnitude contradict the force balance. As $\sigma \to 0$, the result reduces to the familiar extension Mg/k, which checks the expression.

Step 4: Conclusion The equilibrium extension is $\boxed{x_0 = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)}$. Hence, the correct option is (C).

Quick Tip to Solve: Treat buoyancy as a reduction of the load supported by the spring, using only the immersed volume.

56. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the center of the hoop is zero. What will be the velocity of the center of the hoop when it cases to slip? [4 marks]

(A) $\frac{r\omega_0}{4}$

(B) $\frac{r\omega_0}{3}$

(C) $\frac{r\omega_0}{2}$

(D) $r\omega_0$

Correct Answer: (C) $\frac{r\omega_0}{2}$

Solution:

Concept:

- For a thin hoop, the moment of inertia about its centre is $I = mr^2$. During slipping, friction changes both translational and rotational motion.
- If friction supplies impulse J, the centre gains speed $v = J/m$, while the magnitude of angular speed decreases by Jr/I. Slipping ends at the rolling condition $v = r\omega$.

Step 1: Relate linear speed to frictional impulse Initially the centre is at rest and the hoop spins with angular speed ω_0. Friction opposes the contact point's sliding motion; it accelerates the centre in the direction in which the hoop will roll. Integrating the translational equation over the slipping interval gives

$$mv = J.$$

This impulse method avoids needing the coefficient of friction or the duration of slipping.

Step 2: Calculate the decrease in angular speed The same friction produces a torque of magnitude Fr opposite to the original spin. Angular impulse about the centre gives

$$I(\omega_0 - \omega) = Jr.$$

With $I = mr^2$ and $J = mv$,

$$\omega = \omega_0 - \frac{v}{r}.$$

Friction increases translational kinetic energy but decreases rotational kinetic energy; total kinetic energy is not conserved because sliding friction dissipates energy.

Step 3: Impose rolling without slipping At the instant slipping ceases,

$$v = r\omega = r\left(\omega_0 - \frac{v}{r}\right) = r\omega_0 - v.$$

Hence

$$2v = r\omega_0 \quad \Rightarrow \quad v = \frac{r\omega_0}{2}.$$

The angular speed is then $\omega_0/2$, consistent with the rolling condition. Options A and B underestimate the speed for the hoop's inertia; D would require the initial angular speed to remain unchanged despite the frictional torque. None satisfies both impulse equations.

Step 4: Conclusion The centre moves with speed $\boxed{r\omega_0/2}$ when slipping ceases. Hence, the correct option is (C).

Quick Tip to Solve: Use the same frictional impulse in translation and rotation, then impose rolling; conserving kinetic energy during slipping gives a wrong result.

57. A projectile is given an initial velocity of $\left(\hat{i}+2\hat{j}\right)$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is: [4 marks]

(A) y = x - 5x^2

(B) y = 2x - 5x^2

(C) 4y = 2x - 5x^2

(D) 4y = 2x - 25x^2

Correct Answer: (B) y = 2x - 5x^2

Solution:

Concept:

- In projectile motion without air resistance, horizontal acceleration is zero while vertical acceleration is $-g$. The two components therefore obey independent equations of motion.
- With the launch point as the origin, $x = u_x t$ and $y = u_y t - \frac{1}{2}gt^2$. Eliminating the same time t gives the Cartesian trajectory.

Step 1: Identify the velocity components The coefficient of $\hat{i}$ is the horizontal speed and the coefficient of $\hat{j}$ is the initial upward speed. Thus $u_x = 1\,\mathrm{m\,s^{-1}}$, $u_y = 2\,\mathrm{m\,s^{-1}}$, and the downward acceleration has magnitude $10\,\mathrm{m\,s^{-2}}$. These are components, so the resultant speed $\sqrt{5}\,\mathrm{m\,s^{-1}}$ must not replace either one.

Step 2: Eliminate the flight time The horizontal equation gives $x = t$ when numerical distances are in metres and time in seconds. The vertical equation is

$$y = 2t - \frac{1}{2}(10)t^2 = 2t - 5t^2.$$

Substituting $t = x$ gives

$$y = 2x - 5x^2.$$

Equivalently, the general formula $y = (u_y/u_x)x - gx^2/(2u_x^2)$ gives the same result.

Step 3: Check the coefficients against the choices At launch, the slope must be $u_y/u_x = 2$. Option (A) instead has slope 1; options (C) and (D), after division by 4, both have slope $1/2$. They cannot describe this launch. The negative quadratic coefficient also correctly describes a downward-curving path under gravity.

Step 4: Conclusion The trajectory is $\boxed{y = 2x - 5x^2}$, with x, y measured in metres. Hence, the correct option is (B).

Quick Tip to Solve: Check the linear coefficient first: the trajectory starts with slope u_y/u_x, which eliminates three choices here.

58. Let [ε_0] denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then: [4 marks]

(A) $\varepsilon_0 = \left[M^{-1}L^{-3}T^2A\right]$

(B) $\varepsilon_0 = \left[M^{-1}L^{-3}T^4A^2\right]$

(C) $\varepsilon_0 = \left[M^1L^2T^1A^2\right]$

(D) $\varepsilon_0 = \left[M^1L^2T^1A\right]$

Correct Answer: (B) $\varepsilon_0 = \left[M^{-1}L^{-3}T^4A^2\right]$

Solution:

Concept:

- Coulomb's law in vacuum is $F = q_1q_2/(4\pi\varepsilon_0 r^2)$. Numerical constants such as 4π have no dimensions.
- Electric charge is current multiplied by time, so $[q] = AT$. Force has dimensions MLT^{-2}; current A is an independent base dimension.

Step 1: Isolate the dimensional expression Rearranging Coulomb's law gives $\varepsilon_0 = q_1q_2/(4\pi Fr^2)$. Both charges have the same dimensions, and the separation contributes a factor L^2. Therefore

$$[\varepsilon_0] = \frac{[q]^2}{[F][r]^2}.$$

This step avoids guessing from the symbol or treating electric charge itself as a base current.

Step 2: Substitute and simplify the powers Insert $[q] = AT$, $[F] = MLT^{-2}$ and $[r] = L$:

$$[\varepsilon_0] = \frac{A^2T^2}{(MLT^{-2})L^2} = M^{-1}L^{-3}T^4A^2.$$

The two powers of time from the two charges combine with the extra T^2 produced by dividing by force. Likewise the length power is $-1 - 2 = -3$.

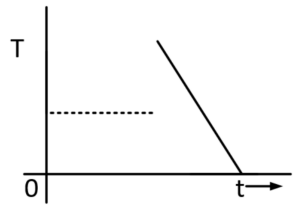
Step 3: Reject inconsistent expressions Option (A) contains only one power of current and only T^2, whereas two charges require A^2T^2 before division by force. Options (C) and (D) have positive mass and length powers, contradicting the inverse force and inverse squared separation in the rearranged equation. Only (B) contains every required exponent.

Step 4: Conclusion $\boxed{[\varepsilon_0] = M^{-1}L^{-3}T^4A^2}$. Hence, the correct option is (B).

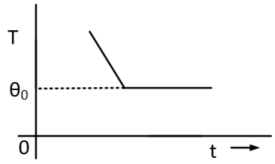
Quick Tip to Solve: Derive electrical dimensions from a defining law, and replace charge by AT before collecting powers.

59. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closest to [4 marks]

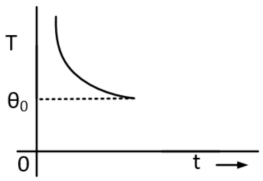
(A)



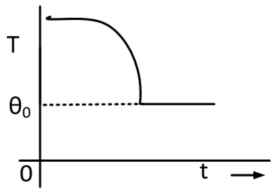
(B)



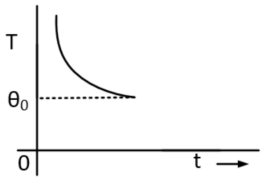
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- Newton's law of cooling states that the rate of decrease of temperature is proportional to the excess temperature above the surroundings: $dT/dt = -k(T - \theta_0)$, where $k > 0$.
- For a fixed room temperature and an initially hotter metal, the solution is $T(t) = \theta_0 + (\theta - \theta_0)e^{-kt}$. The temperature excess decays exponentially.

Step 1: Determine the shape and limiting temperature At $t = 0$, the metal is at $T = \theta > \theta_0$, so its initial slope is negative. As it cools, $T - \theta_0$ becomes smaller, so the magnitude of that slope decreases. In the long-time limit, e^{-kt} tends to zero and T approaches θ_0 from above. It does not continue below the room temperature during passive cooling.

Step 2: Check the curvature Differentiating the cooling law again gives

$$\frac{d^2T}{dt^2} = k^2(\theta - \theta_0)e^{-kt} > 0.$$

Thus the descending temperature curve is concave upward: steep initially and progressively flatter later. It approaches the horizontal line $T = \theta_0$ smoothly and asymptotically, rather than meeting that

line at a sharp corner in finite time.

Step 3: Compare all four graphical choices Graph (C) has exactly this falling, concave-up shape and approaches the marked room-temperature line. Graph (A) wrongly continues toward zero with a constant slope. Graph (B) is a straight-line fall followed by an abrupt horizontal segment. Graph (D) initially becomes steeper and then meets the room-temperature line abruptly, contrary to the decreasing cooling rate.

Step 4: Conclusion The temperature decreases exponentially toward the room temperature θ_0, as shown by graph (C). Hence, the correct option is (C).

Quick Tip to Solve: For Newton cooling, inspect both the final asymptote and the slope: the curve approaches room temperature with progressively smaller slope magnitude.

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60. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in $5s$. In another $10s$ it will decrease to α times its original magnitude, where α equals [4 marks]

(A) 0.7

(B) 0.81

(C) 0.729

(D) 0.6

Correct Answer: (C) 0.729

Solution:

Concept:

- For linear damping, an oscillator's amplitude has an exponential envelope $A(t) = A_0 e^{-\beta t}$, where A_0 is the original amplitude and $\beta > 0$ is the damping constant.
- Equal time intervals multiply an exponential amplitude by the same factor. The amplitude envelope must be distinguished from the oscillating displacement and from the energy, which is proportional to amplitude squared.

Step 1: Use the measured five-second reduction The supplied observation is

$$\frac{A(5)}{A_0} = e^{-5\beta} = 0.9.$$

Thus each successive interval of $5\,\mathrm{s}$ reduces the amplitude to 90 percent of its value at the start of that interval. There is no need to calculate a logarithm or find β separately.

Step 2: Interpret the additional time correctly The phrase another 10 s refers to ten more seconds after the first five seconds. The total elapsed time is therefore $15\,\mathrm{s}$, which is three five-second intervals. Consequently

$$\alpha = \frac{A(15)}{A_0} = e^{-15\beta} = (e^{-5\beta})^3 = (0.9)^3 = 0.729.$$

The same result follows by multiplying the remaining fractions $0.9 \times 0.9 \times 0.9$.

Step 3: Check the alternative values Option (B), 0.81, is the fraction after only 10 s from the original

instant; it omits the first five-second interval. A linear subtraction of 0.1 per interval would give 0.7, option (A), but damping is exponential rather than linear. Option (D), 0.6, follows neither the measured equal-interval ratio nor the amplitude envelope.

Step 4: Conclusion After a total of $15\,\text{s}$, $\boxed{\alpha = 0.729}$. Hence, the correct option is (C).

Quick Tip to Solve: Count the equal time intervals from the original instant, then raise the surviving amplitude fraction to that count.

Chemistry (120 marks)

61. A piston filled with 0.04 mol of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of 37.0°C. As it does so, it absorbs 208J of heat. The values of q and w for the process will be :

(R = 8.314 J/mol K) (l n 7.5 = 2.01) [4 marks]

(A) q = – 208 J, w = – 208 J

(B) q = – 208 J, w = + 208 J

(C) q = + 208 J, w = + 208 J

(D) q = + 208 J, w = – 208 J

Correct Answer: (D) q = + 208 J, w = – 208 J

Solution:

Concept:

- Using the chemistry sign convention, heat absorbed by the gas is positive and work done on the gas is positive. During expansion the gas does work on the surroundings, so its work term is negative.
- For an ideal gas, internal energy depends only on temperature. An isothermal process has $\Delta U = 0$, and the first law $\Delta U = q + w$ therefore gives $w = -q$.

Step 1: Assign the sign of the absorbed heat The system is the ideal gas in the piston. It absorbs $208\,\text{J}$ from its surroundings, so

$$q = +208\,\text{J}.$$

The temperature remains constant even though heat enters because the energy supplied is transferred out as expansion work, instead of increasing the gas's internal energy.

Step 2: Apply the first law and verify the work At constant temperature for an ideal gas,

$$0 = q + w, \qquad w = -208\,\text{J}.$$

For a reversible isothermal expansion one may also calculate

$$w = -nRT\ln\left(\frac{V_f}{V_i}\right).$$

Here $T = 310.15\,\text{K}$, $V_f/V_i = 375/50 = 7.5$, and the given logarithm yields

$$w = -(0.04)(8.314)(310.15)(2.01)\,\text{J} \approx -207.3\,\text{J}.$$

This agrees with about $-208\,\text{J}$ to the accuracy of the supplied rounded data.

Step 3: Compare the sign pairs Both (A) and (B) assign negative heat, contradicting absorption. Option (C) assigns positive work to an expanding gas and would make $q + w$ positive rather than zero. The pair in (D) alone has heat entering, expansion work leaving, and zero internal-energy change.

Step 4: Conclusion $\boxed{q = +208\,\text{J}, \quad w = -208\,\text{J}}$. Hence, the correct option is (D).

Quick Tip to Solve: For an isothermal ideal-gas expansion in chemistry notation, immediately use $\Delta U = 0$ and $w = -q$, with absorbed heat positive.

62. How many litres of water must be added to 1 litre of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2? [4 marks]

(A) 0.1 L

(B) 0.9 L

(C) 2.0 L

(D) 9.0 L

Correct Answer: (D) 9.0 L

Solution:

Concept:

- For a dilute strong acid such as HCl, dissociation is essentially complete and $[\text{H}^+]$ equals the analytical acid concentration. The relation is $\text{pH} = -\log_{10}[\text{H}^+]$.
- Dilution conserves the number of moles of acid: $C_i V_i = C_f V_f$. The volume of water added is the final solution volume minus the original solution volume, taking volumes as additive.

Step 1: Convert each pH to concentration The initial pH is 1 and the desired final pH is 2. Therefore

$$C_i = 10^{-1} = 0.1\,\text{mol}\,\text{L}^{-1}, \qquad C_f = 10^{-2} = 0.01\,\text{mol}\,\text{L}^{-1}.$$

Increasing pH by one unit means reducing the hydrogen-ion concentration by a factor of ten; it does not mean merely doubling the solution volume. At these concentrations, hydrogen ions from water are negligible.

Step 2: Find the required final volume The initial litre contains

$$n = C_i V_i = (0.1)(1) = 0.1\,\text{mol}.$$

No acid is removed when water is added. To place these same moles at the final concentration requires

$$V_f = \frac{n}{C_f} = \frac{0.1}{0.01} = 10\,\text{L}.$$

This is the total volume of the diluted solution, not the volume of added water.

Step 3: Subtract the original solution volume The added water is

$$V_{\text{water}} = V_f - V_i = 10 - 1 = 9.0\,\text{L}.$$

Adding 0.1 L, 0.9 L or 2.0 L would give final volumes of only 1.1 L, 1.9 L or 3.0 L respectively. None produces the tenfold dilution needed for the stated change of pH.

Step 4: Conclusion $\boxed{9.0\,\text{L}}$ of water must be added. Hence, the correct option is (D).

Quick Tip to Solve: A one-unit pH rise for a strong acid requires ten times the final volume; subtract the original volume to find the water added.

63. An unknown alcohol is treated with the "Lucas reagent" to determine whether the alcohol is primary, secondary or tertiary. Which alcohol reacts fastest and by what mechanism: [4 marks]

(A) tertiary alcohol by S_N1

(B) secondary alcohol by S_N2

(C) tertiary alcohol by S_N2

(D) secondary alcohol by S_N1

Correct Answer: (A) tertiary alcohol by S_N1

Solution:

Concept:

- Lucas reagent is concentrated HCl with anhydrous $ZnCl_2$. It converts alcohols to alkyl chlorides; insoluble chloride produces the turbidity used in the test.
- In an S_N1 substitution, the slow ionization step forms a carbocation. Its stability is generally tertiary greater than secondary greater than primary, because alkyl groups stabilize positive charge by hyperconjugation and their electron-releasing inductive effect.

Step 1: Explain how the reagent activates the alcohol The hydroxyl group alone is a poor leaving group. The acidic medium and Lewis-acid action of $ZnCl_2$ facilitate its departure. For a suitable alcohol this allows formation of a carbocation, followed by rapid attack by Cl^-, producing the corresponding alkyl chloride. The rate depends strongly on how readily the carbocation forms.

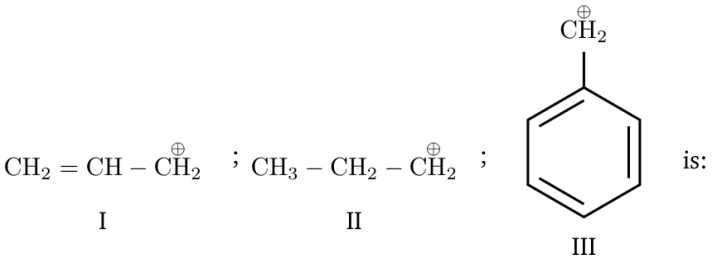
Step 2: Compare the alcohol classes A tertiary alcohol forms the most stabilized carbocation of these ordinary alcohol classes and therefore reacts rapidly at room temperature, producing immediate turbidity. Secondary alcohols react more slowly, while ordinary primary alcohols do not readily generate primary carbocations and usually require heating for the Lucas test. Thus the fastest class is tertiary, and its favoured mechanism is S_N1.

Step 3: Reject the other mechanism and class combinations Option (C) pairs a tertiary alcohol with S_N2, but bulky alkyl groups around a tertiary carbon obstruct the backside attack required for that mechanism. Options (B) and (D) both name a secondary alcohol, which is slower in this test than a tertiary alcohol. Although secondary substrates can undergo S_N1, that does not make them the fastest class.

Step 4: Conclusion Tertiary alcohol reacts fastest with Lucas reagent by the S_N1 mechanism. Hence, the correct option is (A).

Quick Tip to Solve: For the Lucas test, connect immediate turbidity to easy tertiary-carbocation formation and hence to S_N1.

64. The order of stability of the following carbocations : [4 marks]



(A) III > II > I

(B) II > III > I

(C) I > II > III

(D) III > I > II

Correct Answer: (D) III > I > II

Solution:

Concept:

- A carbocation is stabilised when its positive charge is delocalised through an adjacent conjugated pi system.
- Resonance stabilisation generally outweighs the inductive and hyperconjugative stabilisation of an ordinary primary alkyl carbocation.

Step 1: Identify the three carbocations I is the allyl cation, $CH_2{=}CH{-}CH_2^+$, in which the empty p orbital overlaps with the neighbouring double bond. II is the primary propyl cation, $CH_3CH_2CH_2^+$, which lacks a conjugated pi system. III is the benzyl cation, $C_6H_5CH_2^+$, whose empty p orbital is conjugated with the benzene ring.

Step 2: Compare the available delocalisation In the allyl cation, resonance distributes the positive charge between the two terminal carbons. In the benzyl cation, the charge can be distributed over the benzylic carbon and the ortho and para ring positions. This more extensive conjugation makes III more stable than I. The primary propyl cation has only limited sigma-bond donation and is the least stable. Thus any order placing II above either resonance-stabilised cation is wrong, eliminating the first three alternatives.

Step 3: Conclusion The stability order is III > I > II. Hence, the correct option is (D).

Quick Tip to Solve: First separate resonance-stabilised carbocations from ordinary alkyl carbocations, then compare the extent of charge delocalisation.

65. An organic compound A upon reacting with NH_3 gives B. On heating B gives C. C in presence of KOH reacts with Br_2 to given $CH_3CH_2NH_2$2. A is : [4 marks]

(A) CH_3COOH

(B) $CH_3CH_2CH_2COOH$

(C) $CH_3CH(CH_3)COOH$

(D) CH_3CH_2COOH

Correct Answer: (D) CH_3CH_2COOH

Solution:

Concept:

- Hofmann bromamide degradation converts a primary amide $RCONH_2$ to RNH_2, losing the carbonyl carbon.
- A carboxylic acid first forms its ammonium salt with ammonia; heating the salt eliminates water to give the corresponding amide.

Step 1: Count the carbons backwards The product ethylamine is $CH_3CH_2NH_2$, containing two carbon atoms. Its amide precursor must therefore contain three carbon atoms: $CH_3CH_2CONH_2$. The alkyl group migrates intact during Hofmann degradation; bromine and alkali do not add a carbon to the chain. The removed carbonyl carbon is recovered as carbonate in the strongly alkaline reaction mixture.

Step 2: Identify the acid and check the sequence The acid must be propanoic acid. The reactions are

$$CH_3CH_2COOH + NH_3 \longrightarrow CH_3CH_2COONH_4$$

$$CH_3CH_2COONH_4 \xrightarrow{\Delta} CH_3CH_2CONH_2 + H_2O$$

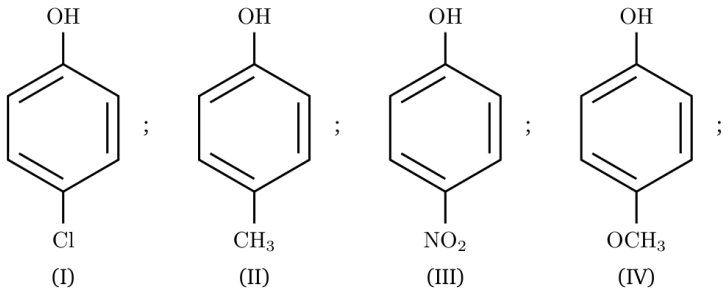
$$CH_3CH_2CONH_2 + Br_2 + 4\,KOH \longrightarrow CH_3CH_2NH_2 + 2\,KBr + K_2CO_3 + 2\,H_2O$$

Ethanoic acid would give methylamine, whereas butanoic acid and 2-methylpropanoic acid would give three-carbon amines, so the other options fail.

Step 3: Conclusion Compound A is propanoic acid, CH_3CH_2COOH. Hence, the correct option is (D).

Quick Tip to Solve: For a Hofmann degradation, work backwards by adding the carbonyl carbon to the carbon skeleton of the final amine.

66. Arrange the following compounds in order of decreasing acidity : [4 marks]



(A) II > IV > I > III

(B) I > II > III > IV

(C) III > I > II > IV

(D) IV > III > I > II

Correct Answer: (C) III > I > II > IV

Solution:

Concept:

- A phenol is more acidic when its conjugate phenoxide ion is stabilised. Electron-withdrawing substituents favour deprotonation, while electron-donating groups oppose it.
- At the para position, resonance effects can operate through the aromatic ring and must be considered along with inductive effects.

Step 1: Compare the electron-withdrawing groups III is para-nitrophenol. Its nitro group withdraws electrons by both induction and resonance, strongly stabilising the phenoxide ion, so III is the most acidic. I is para-chlorophenol. Chlorine has a withdrawing inductive effect and a competing donating resonance effect; the net effect still makes I more acidic than the phenols bearing the electron-donating groups shown.

Step 2: Order the electron-donating groups II is para-cresol, bearing a methyl group, and IV is para-methoxyphenol. Methyl donates electron density by induction and hyperconjugation. Methoxy is a stronger resonance donor at the para position; its donation destabilises the electron-rich phenoxide ion more strongly in this comparison. IV is therefore less acidic than II. Combining the comparisons gives III, then I, then II, then IV. The other listed orders either put nitrophenol too low or put methoxyphenol first, contrary to conjugate-base stabilisation.

Step 3: Conclusion The decreasing acidity is III > I > II > IV. Hence, the correct option is (C).

Quick Tip to Solve: Rank phenol acidity by stabilisation of the phenoxide ion; a para nitro group withdraws strongly, while para methoxy donates by resonance.

67. Compound (A), C_8H_9Br, gives a white precipitate when warmed with alcoholic $AgNO_3$. Oxidation of (A) gives an acid (B), $C_8H_6O_4$. (B) easily forms anhydride on heating. Identify the compound (A). [4 marks]

(A) m-$CH_3C_6H_4CH_2Br$

(B) o-$BrC_6H_4CH_2CH_3$

(C) p-$CH_3C_6H_4CH_2Br$

(D) o-$CH_3C_6H_4CH_2Br$

Correct Answer: (D) o-$CH_3C_6H_4CH_2Br$

Solution:

Concept:

- Benzylic bromides readily release bromide in the alcoholic silver nitrate test, producing a precipitate of $AgBr$; aryl bromides resist this substitution.
- Oxidation of alkylbenzene side chains bearing benzylic hydrogen gives carboxyl groups on the ring. Adjacent carboxyl groups can form a five-membered cyclic anhydride.

Step 1: Use the oxidation product The product $C_8H_6O_4$ is a benzenedicarboxylic acid. Easy anhydride formation requires the two carboxyl groups in adjacent positions: the product is phthalic acid, benzene-1,2-dicarboxylic acid. Thus A must have two oxidisable side chains in the ortho positions, not the meta or para arrangement.

Step 2: Apply the halide test and reject alternatives The structure o-$CH_3C_6H_4CH_2Br$ has formula C_8H_9Br. Its benzylic bromide group passes the silver nitrate test, while oxidation of both the methyl and bromomethyl groups gives adjacent carboxyl groups. The meta and para methylbenzyl bromides yield isophthalic and terephthalic acids, which do not readily close to this cyclic anhydride. The ortho bromoethylbenzene has bromine directly on the ring and only one alkyl side chain, so it fails both deductions.

Step 3: Conclusion A is ortho-methylbenzyl bromide, o-$CH_3C_6H_4CH_2Br$. Hence, the correct option is (D).

Quick Tip to Solve: Easy cyclic anhydride formation identifies ortho carboxyl groups; then use the silver nitrate test to distinguish benzylic from aryl bromine.

68. The gas leaked from a storage tank of the Union Carbide plant in Bhopal gas tragedy was: [4 marks]

(A) Methylamine

(B) Ammonia

(C) Phosgene

(D) Methyl isocyanate

Correct Answer: (D) Methyl isocyanate

Solution:

Concept:

- The Bhopal disaster involved leakage of methyl isocyanate, abbreviated MIC, from a pesticide-manufacturing plant.
- Methyl isocyanate has the connectivity $CH_3-N=C=O$. Its functional group differs from those of methylamine, ammonia and phosgene.

Step 1: Identify the stored toxic intermediate The substance named in the historical Bhopal gas tragedy is methyl isocyanate. MIC was stored as an intermediate in pesticide manufacture at the Union Carbide plant. A runaway reaction in a storage tank caused a large release of this volatile, highly toxic substance; the question asks for the leaked chemical, rather than every chemical used in the plant.

Step 2: Distinguish the listed gases Methylamine, CH_3NH_2, is an amine and is chemically distinct from MIC. Ammonia is NH_3, with no carbon atom. Phosgene is carbonyl chloride, $COCl_2$, and is also distinct from the methyl isocyanate stored in the tank. Although these are hazardous chemicals, their toxicity alone does not identify the gas responsible for this particular disaster.

Step 3: Conclusion The leaked gas was methyl isocyanate, CH_3NCO. Hence, the correct option is (D).

Quick Tip to Solve: Remember MIC as methyl isocyanate: the name and the group N=C=O distinguish it from methylamine and phosgene.

69. Which of the following complex species is not expected to exhibit optical isomerism? [4 marks]

(A) $[Co\ (en)_2\ Cl_2]^-$

(B) $[Co\ (NH_3)_3\ Cl_3]$

(C) $[Co\ (en)\ (NH_3)_2\ Cl_2]^+$

(D) $[Co\ (en)_3]^{3+}$

Correct Answer: (B) $[Co\ (NH_3)_3\ Cl_3]$

Solution:

Concept:

- Optical isomers are non-superimposable mirror-image coordination structures. A plane of symmetry makes a structure achiral.

- The bidentate ligand en occupies two adjacent octahedral sites. Several chelate arrangements are chiral, whereas the fac and mer forms of $[Ma_3b_3]$ have symmetry planes.

Step 1: Examine the three-ammine complex In $[Co(NH_3)_3Cl_3]$, all ligands are monodentate and the octahedral ligand pattern is Ma_3b_3. Its facial form has three identical ligands on one face, and its meridional form has them along a meridian. Each form has a mirror plane, so neither gives a pair of optical isomers. Geometrical isomerism here does not imply optical isomerism.

Step 2: Compare the chelate alternatives The cis arrangement of the bis(ethylenediamine) dichloride complex can be chiral. The mixed complex containing en, two ammines and two chlorides also admits a chiral cis arrangement. Finally, three en chelate rings in $[Co(en)_3]^{3+}$ wind around the metal in right- and left-handed arrangements, denoted Δ and Λ. Thus these species can exhibit optical isomerism even though some of their geometrical forms may be achiral. The question asks which species is not expected to do so in any such form.

Step 3: Conclusion $[Co(NH_3)_3Cl_3]$ is not expected to exhibit optical isomerism. Hence, the correct option is (B).

Quick Tip to Solve: For octahedral complexes, check symmetry of every geometrical form; an achiral trans form does not rule out a chiral cis form.

70. Synthesis of each molecule of glucose in photosynthesis involves: [4 marks]

(A) 10 molecules of ATP

(B) 8 molecules of ATP

(C) 6 molecules of ATP

(D) 18 molecules of ATP

Correct Answer: (D) 18 molecules of ATP

Solution:

Concept:

- The Calvin cycle fixes carbon dioxide into carbohydrate using ATP for energy and NADPH as reducing power.
- The net formation of one three-carbon triose phosphate requires fixation of three carbon dioxide molecules, consuming nine ATP and six NADPH.

Step 1: Scale the carbon requirement to glucose Glucose contains six carbon atoms. Since each carbon dioxide supplies one carbon atom, formation of one glucose requires fixation of six carbon dioxide molecules. This corresponds to two net triose-phosphate equivalents; the energy requirement must therefore be twice that for a three-carbon product, rather than equating ATP molecules with the number of carbon atoms.

Step 2: Count ATP in reduction and regeneration For six carbon dioxide molecules, reduction of the twelve molecules of 3-phosphoglycerate uses twelve ATP. Regeneration of six molecules of the carbon

dioxide acceptor, ribulose-1,5-bisphosphate, uses another six ATP. Therefore

$$N_{\text{ATP}} = 12 + 6 = 18.$$

The accompanying reducing-power requirement is twelve NADPH, but NADPH is not ATP and is not added to the requested count. The values 10, 8 and 6 omit essential energy-consuming steps and do not equal the complete Calvin-cycle ATP requirement.

Step 3: Conclusion Synthesis of one glucose molecule involves 18 ATP molecules. Hence, the correct option is (D).

Quick Tip to Solve: Use the Calvin-cycle ratio: each carbon dioxide fixed costs three ATP and two NADPH; glucose requires six carbon dioxide molecules.

71. A compound with molecular mass 180 is acylated with CH_3COCl to get a compound with molecular mass 390. The number of amino groups present per molecule of the former compound is: [4 marks]

(A) 2

(B) 5

(C) 4

(D) 6

Correct Answer: (B) 5

Solution:

Concept:

- Acylation replaces one hydrogen on an amino group by an acetyl group, CH_3CO, forming an amide.
- The increase in molecular mass per amino group is the mass of CH_3CO minus the mass of the displaced hydrogen; the hydrogen leaves as HCl.

Step 1: Calculate the mass added per amino group The reaction of one primary amino group is

$$RNH_2 + CH_3COCl \longrightarrow RNHCOCH_3 + HCl.$$

The acetyl group has relative mass $2(12) + 3(1) + 16 = 43$. Because it replaces a hydrogen already present on nitrogen, the net increase in molecular mass is

$$43 - 1 = 42.$$

Adding the whole mass of acetyl chloride would incorrectly count chlorine, which leaves in hydrogen chloride.

Step 2: Use the total molecular-mass increase The molecular mass rises from 180 to 390, so the total increase is

$$390 - 180 = 210.$$

If the compound has n acylatable amino groups, complete acylation therefore gives

$$42n = 210, \qquad n = \frac{210}{42} = 5.$$

As a check, five replacements predict $180 + 5(42) = 390$, exactly the stated product mass. The other proposed counts 2, 4 and 6 predict molecular masses 264, 348 and 432 respectively, so none agrees with the observed mass.

Step 3: Conclusion The former compound contains $\boxed{5}$ amino groups per molecule. Hence, the correct option is (B).

Quick Tip to Solve: For acetylation, count the net mass increment as 42 per amino group: an acetyl group of mass 43 replaces hydrogen of mass 1.

72. The rate of a reaction doubles when its temperature changes from 300K to 310K. Activation energy of such a reaction will be:(R = 8.314 JK^{-1} mol^{-1} and log 2 = 0.301) [4 marks]

(A) 48.6 kJ mol^{-1}

(B) 58.5 kJ mol^{-1}

(C) 60.5 kJ mol^{-1}

(D) 53.6 kJ mol^{-1}

Correct Answer: (D) 53.6 kJ mol^{-1}

Solution:

Concept:

- For an unchanged reaction mechanism, the Arrhenius equation is $k = Ae^{-E_a/(RT)}$, where E_a is activation energy and R is the gas constant.
- At the same reactant concentrations, a doubling of reaction rate means $k_2/k_1 = 2$. The two-temperature form eliminates the pre-exponential factor.

Step 1: Write the two-temperature relation Take $T_1 = 300\,\mathrm{K}$ and $T_2 = 310\,\mathrm{K}$. Using common logarithms gives

$$\log \frac{k_2}{k_1} = \frac{E_a}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right).$$

The reciprocal-temperature difference is positive because the higher temperature gives the larger rate constant. Thus the calculated activation energy must also be positive.

Step 2: Substitute and convert the energy unit With the given $\log 2 = 0.301$, rearrangement gives

$$E_a = \frac{2.303(8.314)(0.301)(300)(310)}{310 - 300}.$$

The temperatures are absolute temperatures; Celsius values would invalidate the reciprocal difference. Evaluating,

$$E_a \simeq 5.36 \times 10^4\,\mathrm{J\,mol^{-1}} = 53.6\,\mathrm{kJ\,mol^{-1}}.$$

The factor 2.303 is necessary with common logarithms. The alternatives 48.6, 58.5 and 60.5 do not satisfy the supplied rate ratio and temperature interval.

Step 3: Conclusion The activation energy is $\boxed{53.6\,\text{kJ mol}^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use either natural logarithms with R or common logarithms with 2.303R; mixing these conventions changes the activation energy.

73. Which of the following arrangements does not represent the correct order of the property stated against it? [4 marks]

(A) $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$: ionic size

(B) $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$: stability in aqueous solution

(C) Sc < Ti < Cr < Mn : number of oxidation states

(D) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: paramagnetic behaviour

Correct Answer: (D) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: paramagnetic behaviour

Solution:

Concept:

- For free high-spin first-row divalent transition-metal ions, paramagnetism is compared using the number n of unpaired d electrons.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, which increases with n. Remove the 4s electrons before counting the d electrons.

Step 1: Test the proposed magnetic order The relevant configurations and unpaired-electron counts are

$$\begin{aligned} V^{2+} &: 3d^3, \quad n = 3, \\ Cr^{2+} &: 3d^4, \quad n = 4, \\ Mn^{2+} &: 3d^5, \quad n = 5, \\ Fe^{2+} &: 3d^6, \quad n = 4. \end{aligned}$$

The sixth d electron in the iron ion pairs with one already present, so iron(II) does not continue the increase from manganese(II). The moments are respectively $\sqrt{15}, \sqrt{24}, \sqrt{35}, \sqrt{24}$ in Bohr magnetons.

Step 2: Check why the remaining arrangements are accepted Across the series the increasing effective nuclear charge contracts comparable divalent ions, giving the stated size order $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$. For the trivalent ions, aqueous stability increases from readily reduced cobalt(III) through iron(III) and chromium(III) to scandium(III), the stable oxidation state of scandium. The range of accessible oxidation states generally grows from scandium towards manganese, so the sequence in (C) is accepted. The magnetic order instead places iron(II) above manganese(II), contrary to the unpaired-electron calculation.

Step 3: Conclusion The incorrect arrangement is (D). The spin-only trend is $V^{2+} < Cr^{2+} = Fe^{2+} < Mn^{2+}$. Hence, the correct option is (D).

Quick Tip to Solve: For magnetic-order questions, count unpaired electrons rather than total d electrons: the count starts falling after the half-filled d5 state.

74. Four successive members of the first row transition elements are listed below with atomic numbers. Which one of them is expected to have the highest $E^0_{M^{3+}/M^{2+}}$ value? [4 marks]

(A) Mn (Z = 25)

(B) Fe (Z = 26)

(C) Co (Z = 27)

(D) Cr (Z = 24)

Correct Answer: (C) Co (Z = 27)

Solution:

Concept:

- The potential $E^\circ_{\mathrm{M^{3+}/M^{2+}}}$ describes the reduction $\mathrm{M^{3+} + e^- \longrightarrow M^{2+}}$ in aqueous solution.
- A more positive reduction potential means a greater thermodynamic tendency of the trivalent ion to gain an electron. Electronic configuration and hydration both affect this tendency.

Step 1: Compare the relevant electronic configurations Chromium changes from $\mathrm{Cr^{3+}}$, $3d^3$, to $\mathrm{Cr^{2+}}$, $3d^4$; the trivalent state is especially stable, so this reduction is not strongly favoured. Manganese changes from $3d^4$ to the stable half-filled $3d^5$ state, giving a large positive reduction potential. Iron changes from the stable half-filled $3d^5$ trivalent ion to $3d^6$, so its reduction potential is lower than that of manganese.

Step 2: Identify the greatest aqueous reduction tendency Cobalt changes from $\mathrm{Co^{3+}}$, $3d^6$, to $\mathrm{Co^{2+}}$, $3d^7$. The high third ionisation energy makes aqueous cobalt(III) strongly oxidising: it readily gains an electron to form cobalt(II). For comparison, the characteristic aqueous potentials are approximately

$$\begin{aligned}\mathrm{Cr^{3+}/Cr^{2+}} &: -0.41\,\mathrm{V},\\ \mathrm{Fe^{3+}/Fe^{2+}} &: +0.77\,\mathrm{V},\\ \mathrm{Mn^{3+}/Mn^{2+}} &: +1.51\,\mathrm{V},\\ \mathrm{Co^{3+}/Co^{2+}} &: +1.97\,\mathrm{V}.\end{aligned}$$

Thus neither chromium, iron nor even manganese exceeds cobalt. Considering only the half-filled configuration would miss the complete aqueous-energy balance.

Step 3: Conclusion Cobalt has the highest $E^\circ_{\mathrm{M^{3+}/M^{2+}}}$ among these elements. Hence, the correct option is (C).

Quick Tip to Solve: For aqueous cobalt, remember that Co3+ is a strong oxidising agent; half-filled-shell arguments alone do not rank all reduction potentials.

75. Given

$E^o_{Cr^{2+}/Cr}$ = -0.74 V; $E^o_{MnO_4^-/Mn^{2+}}$ = 1.51 V

$E^o_{Cr_2O_7^{2-}/Cr^{3+}}$ = 1.33 V; E^o_{Cl/Cl^-} = 1.36 V

Based on the data given above, strongest oxidising agent will be : [4 marks]

(A) Cr^{3+}

(B) Mn^{2+}

(C) MnO_4^-

(D) Cl^-

Correct Answer: (C) MnO_4^-

Solution:

Concept:

- An oxidising agent accepts electrons and is itself reduced. A larger standard reduction potential indicates a stronger tendency to accept electrons.
- Compare potentials as written for reduction. Do not multiply an electrode potential by the number of electrons when balancing a half-reaction.

Step 1: Compare the supplied positive potentials The permanganate couple has $E^\circ = 1.51\,\text{V}$, greater than the chlorine couple at $1.36\,\text{V}$ and the dichromate couple at $1.33\,\text{V}$. The chromium-metal couple is still lower at $-0.74\,\text{V}$. Therefore the oxidised member of the permanganate couple has the greatest tendency to gain electrons under the standard conditions represented by the data.

Step 2: Identify the oxidised member rather than its product In acidic medium, the relevant reduction is

$$\text{MnO}_4{}^- + 8\,\text{H}^+ + 5\,\text{e}^- \longrightarrow \text{Mn}^{2+} + 4\,\text{H}_2\text{O}.$$

Here $\text{MnO}_4{}^-$ receives electrons, whereas Mn^{2+} is the reduced product; selecting manganese(II) would reverse the role of the species. Similarly, chloride is the reduced member of the chlorine/chloride couple and is not the oxidant associated with its positive potential. Chromium(III) is not the species undergoing the highest-potential reduction in the supplied list. The comparison must be made between electron-accepting species, not simply between any ions containing those elements.

Step 3: Conclusion The strongest oxidising agent listed is $\boxed{\text{MnO}_4{}^-}$. Hence, the correct option is (C).

Quick Tip to Solve: Circle the electron acceptor in each reduction half-equation before comparing potentials; the product of reduction is not that couple's oxidising agent.

76. The molarity of a solution obtained by mixing 750 mL of 0.5 (M) HCl with 250 mL of 2(M) HCl will be: [4 marks]

(A) 1.00 M

(B) 1. 75 M

(C) 0.975 M

(D) 0.875 M

Correct Answer: (D) 0.875 M

Solution:

Concept:

- Molarity is the amount of solute in moles divided by the final solution volume in litres: $M = n/V$.
- When two solutions of the same solute are mixed without reaction, their solute moles add. For the stated dilution calculation, use the sum of the supplied solution volumes.

Step 1: Calculate the moles in each portion Convert the given volumes to litres before multiplying by molarity. The first solution contains

$$n_1 = (0.5\,\text{mol}\,\text{L}^{-1})(0.750\,\text{L}) = 0.375\,\text{mol}.$$

The second solution contains

$$n_2 = (2\,\text{mol}\,\text{L}^{-1})(0.250\,\text{L}) = 0.500\,\text{mol}.$$

Although its volume is smaller, the more concentrated second portion contributes more hydrogen chloride. Mixing the same acid with itself causes no neutralisation that would consume either amount.

Step 2: Divide the total moles by the total volume The final amount of HCl is

$$n = 0.375 + 0.500 = 0.875\,\text{mol}.$$

The total solution volume is

$$V = 0.750 + 0.250 = 1.000\,\text{L}.$$

Therefore

$$M = \frac{0.875}{1.000} = 0.875\,\text{mol}\,\text{L}^{-1}.$$

A simple arithmetic mean of 0.5 and 2 would be inappropriate because the volumes are unequal. The result must lie between the initial concentrations and closer to 0.5 M because three quarters of the mixture comes from that solution. The other listed concentrations do not follow from the conserved total moles.

Step 3: Conclusion The mixed solution has molarity $\boxed{0.875\,\text{M}}$. Hence, the correct option is (D).

Quick Tip to Solve: For the same solute, use the volume-weighted mean: final molarity equals (M1V1 + M2V2)/(V1 + V2), with consistent volume units.

77. A solution of (–*l*) – chloro –1 – phenylethane in toluene racemises slowly in the presence of a small amount of $SbCl_5$, due to the formation of : [4 marks]

(A) carbene

(B) carbocation

(C) free radical

(D) carbanion

Correct Answer: (B) carbocation

Solution:

Concept:

- A chiral benzylic halide can racemise if its carbon–halogen bond ionises to produce a planar carbocation.
- $SbCl_5$ is a Lewis acid. It promotes chloride removal by accepting a chloride ion, facilitating heterolytic cleavage of the carbon–chlorine bond.

Step 1: Identify the intermediate formed on ionisation The stereogenic carbon of 1-chloro-1-phenylethane is attached to phenyl, methyl, hydrogen and chlorine. Lewis-acid-assisted loss of chloride generates the benzylic ion

$$C_6H_5-CH(Cl)-CH_3 \longrightarrow C_6H_5-CH^+-CH_3 + Cl^-.$$

The positive charge is stabilised by resonance with the adjacent phenyl ring. The carbon carrying the positive charge is approximately sp^2-hybridised, with a vacant p orbital and three substituents arranged in a plane.

Step 2: Connect planarity with loss of optical purity Because this intermediate is planar, chloride can return from either face. Attack on the two faces gives the two opposite configurations at the benzylic carbon, progressively reducing the optical purity of the initially levorotatory sample. Repeated ionisation and recombination therefore produce racemisation. A carbene would be a neutral divalent carbon species and is not produced by simple loss of chloride here. A free radical would require homolytic cleavage rather than the Lewis-acid-assisted heterolysis. A carbanion would carry negative charge at carbon, the opposite of the species formed when chloride leaves with its bonding electron pair.

Step 3: Conclusion Racemisation occurs through a planar benzylic $\boxed{\text{carbocation}}$. Hence, the correct option is (B).

Quick Tip to Solve: When a Lewis acid promotes ionisation of a chiral benzylic halide, test for a planar carbocation and attack from both faces.

78. Stability of the species Li_2, Li_2^- and Li_2^+ increases in the order of: [4 marks]

(A) $Li_2 < Li_2^+ < Li_2^-$

(B) $Li_2^+ < Li_2^- < Li_2$

(C) $Li_2^- < Li_2^+ < Li_2$

(D) $Li_2^- < Li_2 < Li_2^+$

Correct Answer: (C) $Li_2^- < Li_2^+ < Li_2$

Solution:

Concept:

- Bond order measures the net bonding contribution: $\text{BO} = (N_b - N_a)/2$, where N_b and N_a count bonding and antibonding electrons.
- The filled inner-shell orbitals of lithium cancel in the bond-order calculation. The valence orbitals are the bonding σ_{2s} and antibonding σ_{2s}^* orbitals.

Step 1: Count the valence electrons Each lithium atom supplies one valence electron. Thus Li_2 has two, Li_2^+ has one and Li_2^- has three valence electrons. Fill the lower-energy bonding orbital before the antibonding orbital. The configurations are $Li_2 : (\sigma_{2s})^2$, $Li_2^+ : (\sigma_{2s})^1$, and $Li_2^- : (\sigma_{2s})^2(\sigma_{2s}^*)^1$.

Step 2: Compare bond order and antibonding occupation The neutral molecule has bond order $(2-0)/2 = 1$, whereas each ion has bond order $1/2$. Hence the neutral molecule is the most stable of the three. To distinguish the ions in this conventional molecular-orbital comparison, note that the anion contains an electron in the destabilising antibonding orbital; the cation has no valence antibonding electron. Accordingly the anion is placed below the cation in the stated stability order. Bond order alone establishes the neutral species above both ions, but does not by itself distinguish their equal half-unit bond orders. Options (A) and (D) incorrectly make the neutral molecule less stable than an ion; option (B) reverses the ionic order.

Step 3: Conclusion The increasing stability order is $Li_2^- < Li_2^+ < Li_2$. Hence, the correct option is (C).

Quick Tip to Solve: Write the valence MO occupations first; when comparing these two ions, do not overlook the antibonding electron in the anion.

79. In which of the following pairs of molecules/ions, both the species are not likely to exist? [4 marks]

(A) H_2^+, He_2^{2-}

(B) H_2, He_2^{2-}

(C) H_2^{2+}, He_2

(D) H_2, He_2^{2+}

Correct Answer: (C) H_2^{2+}, He_2

Solution:

Concept:

- A stable ordinary diatomic covalent bond requires a positive bond order, $(N_b - N_a)/2$. A zero bond order means that bonding and antibonding contributions cancel.
- For hydrogen and helium species, fill σ_{1s} before σ^*_{1s}. Count electrons after applying the ionic charge, rather than counting only nuclei.

Step 1: Check the two species in option (C) Two hydrogen nuclei supply two electrons before ionisation. The dication H_2^{2+} has lost both, so there are no electrons to bind its two protons: its bond order is zero. Neutral He_2 has four electrons and configuration $(\sigma_{1s})^2(\sigma^*_{1s})^2$. Therefore its bond order is $(2-2)/2 = 0$. Neither member forms an ordinary stable covalent molecule in this elementary MO treatment. Very weak van der Waals association is not the covalent bond meant by this question.

Step 2: Eliminate the remaining pairs The hydrogen molecular cation H_2^+ in option (A) has one bonding electron and bond order $1/2$, so that pair cannot have both members non-existent. Neutral H_2, appearing in options (B) and (D), has two electrons, both in the bonding orbital, and bond order one. Thus those options do not give two zero-bond-order species. Independently, He_2^{2+} in (D) has two electrons in the bonding orbital and bond order one. These electron counts are sufficient to reject (A), (B) and (D), without making assumptions about the extra-electron helium species. The answer depends on both members of the pair: finding one positive bond order is enough to eliminate an option.

Step 3: Conclusion Both H_2^{2+} and He_2 have zero bond order. Hence, the correct option is (C).

Quick Tip to Solve: For a question asking whether both species fail to form a bond, reject a pair as soon as one member has positive elementary MO bond order.

80. Which one of the following molecules is expected to exhibit diamagnetic behaviour? [4 marks]

(A) N_2

(B) O_2

(C) S_2

(D) C_2

Correct Answer: (A) N_2

Solution:

Concept:

- A diamagnetic molecule has all electrons paired. A paramagnetic molecule has at least one unpaired electron; total electron count alone is not sufficient.
- Hund's rule requires singly occupying degenerate orbitals before pairing. The ordering of the $2p$ molecular orbitals must be appropriate to the molecule.

Step 1: Examine nitrogen and carbon molecules For these light diatomics, write the filled 2s bonding and antibonding orbitals together as a paired core. Beyond that core, nitrogen has

$$(\pi_{2p_x})^2(\pi_{2p_y})^2(\sigma_{2p_z})^2.$$

Every occupied orbital is doubly occupied, so nitrogen is diamagnetic. Carbon has two fewer electrons and its remaining 2p configuration is

$$(\pi_{2p_x})^2(\pi_{2p_y})^2.$$

The two bonding pi orbitals lie below the sigma 2p orbital for carbon. Thus its electrons are all paired too. Carbon in option (D) is also diamagnetic; the wording does not yield a unique chemical answer.

Step 2: Check oxygen and sulphur and interpret the key In O_2, the last two electrons occupy the separate degenerate π^* orbitals singly, producing two unpaired electrons. It is consequently paramagnetic. The corresponding valence occupation of S_2 similarly has two unpaired electrons in antibonding π^* orbitals, so it is paramagnetic as well. Options (B) and (C) must therefore be rejected. The decisive issue is pairing within the degenerate orbitals, rather than the presence of an even number of electrons. Both (A) and (D) satisfy the requested diamagnetic behaviour, and the official answer key accepts both options.

Step 3: Conclusion N_2 is diamagnetic. Hence, the correct option is (A). C_2 is also diamagnetic, and the official answer key accepts options (A) and (D).

Quick Tip to Solve: For C_2, use the light-diatomic MO ordering: the two bonding pi orbitals fill completely before the sigma 2p orbital.

81. Which of the following represents the correct order of increasing first ionization enthalpy for Ca, Ba, S, Se and Ar? [4 marks]

(A) S < Se < Ca < Ba < Ar

(B) Ba < Ca < Se < S < Ar

(C) Ca < Ba < S < Se < Ar

(D) Ca < S < Ba < Se < Ar

Correct Answer: (B) Ba < Ca < Se < S < Ar

Solution:

Concept:

- First ionisation enthalpy is the energy required to remove the outermost electron from an isolated gaseous atom: $\mathrm{M(g)} \longrightarrow \mathrm{M^+(g) + e^-}$.
- Ionisation enthalpy generally decreases down a group because the valence shell becomes larger and more shielded, and increases across a period as effective nuclear attraction grows.

Step 1: Order the members within their groups Calcium and barium are both group 2 metals with two outer s electrons. Barium lies below calcium, so its outer electron is farther from the nucleus and more effectively shielded by filled inner shells. Consequently $I_1(\mathrm{Ba}) < I_1(\mathrm{Ca})$. Sulphur and selenium are group 16 elements; selenium lies below sulphur, and the same radius-and-shielding argument gives $I_1(\mathrm{Se}) < I_1(\mathrm{S})$. These comparisons must not be reversed by considering nuclear charge alone, because shell distance and shielding also change.

Step 2: Join the comparisons and place argon The group 2 metals lose their outer s electron more readily than sulphur or selenium, whose valence electrons experience greater effective nuclear attraction. Thus calcium belongs below selenium in this list. Argon has a closed valence shell, $3s^2 3p^6$, and is particularly resistant to removal of an electron; it has the greatest first ionisation enthalpy among these elements. Combining the comparisons gives $\mathrm{Ba} < \mathrm{Ca} < \mathrm{Se} < \mathrm{S} < \mathrm{Ar}$. Option (A) incorrectly places the chalcogens below the metals and reverses both within-group trends. Options (C) and (D) incorrectly place calcium below barium, so they fail immediately.

Step 3: Conclusion The increasing first ionisation enthalpy order is $\mathrm{Ba} < \mathrm{Ca} < \mathrm{Se} < \mathrm{S} < \mathrm{Ar}$. Hence, the correct option is (B).

Quick Tip to Solve: First compare elements in the same group, then connect the groups; keep the closed-shell noble gas at the high-energy end here.

82. The first ionization potential of Na is 5.1 eV. The value of electron gain enthalpy of Na+ will be: [4 marks]

(A) – 5.1 eV

(B) –10.2 eV

(C) + 2.55 eV

(D) – 2.55 eV

Correct Answer: (A) – 5.1 eV

Solution:

Concept:

- Ionisation and electron attachment are reverse processes when they involve the same gaseous atom and its singly charged cation.
- Reversing a reaction reverses the sign of its energy change without altering its magnitude. Energy released on electron attachment has a negative sign.

Step 1: Write the stated ionisation process The given first ionisation potential refers to removing one electron from neutral sodium. In energy-per-particle form the process is

$$\mathrm{Na(g)} \longrightarrow \mathrm{Na^{+}(g)} + \mathrm{e^{-}}, \qquad \Delta E = +5.1\,\mathrm{eV}.$$

The positive sign expresses energy absorption. It is essential to identify the reactant as the neutral atom here: the question subsequently asks about adding an electron to $\mathrm{Na^{+}}$, not adding an extra electron to neutral sodium.

Step 2: Reverse the process and compare options The requested attachment is

$$\mathrm{Na^{+}(g)} + \mathrm{e^{-}} \longrightarrow \mathrm{Na(g)}, \qquad \Delta E = -5.1\,\mathrm{eV}.$$

It is exactly the reverse reaction, so the energy released equals the energy originally supplied. There is no factor of two, since one electron is transferred in each direction and the initial and final species are the same pair. Option (B), with magnitude $10.2\,\mathrm{eV}$, incorrectly doubles the energy. Options (C) and (D), with magnitude $2.55\,\mathrm{eV}$, incorrectly halve it; (C) also uses the wrong sign for the energy-releasing reverse process. No conversion to energy per mole is necessary because all choices use electron volts.

Step 3: Conclusion The electron gain enthalpy of $\mathrm{Na^{+}}$ is $-5.1\,\mathrm{eV}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the two electron-transfer equations explicitly: when one is exactly the reverse of the other, change only the sign of the given energy.

83. Energy of an electron is given by $E = -2.178 \times 10^{-18} J \left(\frac{Z^2}{n^2}\right)$. Wavelength of light required to excite an electron in an hydrogen atom from level n = 1 to n = 2 will be
(h = 6.62 $\times$ 10^{-34} Js and c = 3.0 $\times$ 10^{8} ms^{-1}) [4 marks]

(A) 2.816×10^{-7} m
(B) 6.500×10^{-7} m
(C) 8.500×10^{-7} m
(D) 1.214×10^{-7} m

Correct Answer: (D) 1.214×10^{-7} m

Solution:

Concept:

- The hydrogen energy levels are $E_n = -2.178 \times 10^{-18}/n^2$ J, because hydrogen has nuclear charge $Z = 1$.
- An absorbed photon supplies the positive difference $\Delta E = E_f - E_i$. Its wavelength satisfies $\lambda = hc/\Delta E$, where h is Planck's constant and c is the speed of light.

Step 1: Calculate the excitation energy The initial state has $n_i = 1$ and the final state has $n_f = 2$. Substituting each principal quantum number separately gives

$$E_1 = -2.178 \times 10^{-18}\,\text{J}, \qquad E_2 = -5.445 \times 10^{-19}\,\text{J}.$$

The final energy is less negative and therefore higher. Their difference is

$$\Delta E = 2.178 \times 10^{-18}\left(1 - \frac{1}{4}\right) = 1.6335 \times 10^{-18}\,\text{J}.$$

This positive energy must be absorbed; using either individual level energy would give a different transition.

Step 2: Convert photon energy into wavelength Using the constants stated in the question,

$$\lambda = \frac{(6.62 \times 10^{-34})(3.0 \times 10^{8})}{1.6335 \times 10^{-18}}\,\text{m} = 1.2158 \times 10^{-7}\,\text{m}.$$

The closest listed value is $1.214 \times 10^{-7}\,\text{m}$; the small difference reflects rounding of the listed constants and answer. This is ultraviolet radiation for the first Lyman excitation. Options (B) and (C) lie in the visible/near-infrared region and carry far too little photon energy. Option (A) also has a substantially longer wavelength than required and therefore insufficient energy. Only option (D) matches the computed energy gap.

Step 3: Conclusion The required wavelength is approximately $1.216 \times 10^{-7}\,\text{m}$, corresponding to the listed $1.214 \times 10^{-7}\,\text{m}$. Hence, the correct option is (D).

Quick Tip to Solve: For excitation from 1 to 2 in hydrogen, multiply the ground-state energy magnitude by three quarters before applying the photon formula.

84. Consider the following reaction:

$xMnO_4^- + yC_2O_4^{2-}$ **+ zH⁺** $\rightarrow$ **xMn²⁺ + 2yCO₂ +** $\frac{z}{2}H_2O$

The value's of x, y and z in the reaction are, respectively : [4 marks]

(A) 5, 2 and 16

(B) 2, 5 and 8

(C) 2, 5 and 16

(D) 5, 2 and 8

Correct Answer: (C) 2, 5 and 16

Solution:

Concept:

- In acidic medium, balance redox reactions using electron conservation, then hydrogen and oxygen with H^+ and H_2O.
- Manganese changes from oxidation state +7 in permanganate to +2, gaining five electrons per Mn. Carbon changes from +3 in oxalate to +4 in carbon dioxide, losing one electron per carbon.

Step 1: Balance the oxidation and reduction half-reactions The permanganate reduction half-reaction is

$$\mathrm{MnO_4}^- + 8\,\mathrm{H}^+ + 5\,\mathrm{e}^- \longrightarrow \mathrm{Mn}^{2+} + 4\,\mathrm{H_2O}.$$

Oxygen is balanced with four waters, hydrogen with eight protons, and charge with five electrons. Oxalate has two carbon atoms and loses two electrons in total:

$$\mathrm{C_2O_4}^{2-} \longrightarrow 2\,\mathrm{CO_2} + 2\,\mathrm{e}^-.$$

The least common electron count is ten. Multiply the first equation by two and the second by five so that electrons cancel exactly when adding the equations.

Step 2: Add the equations and verify the coefficients Adding the scaled half-reactions gives

$$2\,\mathrm{MnO_4}^- + 5\,\mathrm{C_2O_4}^{2-} + 16\,\mathrm{H}^+ \longrightarrow 2\,\mathrm{Mn}^{2+} + 10\,\mathrm{CO_2} + 8\,\mathrm{H_2O}.$$

Therefore $x = 2$, $y = 5$, and $z = 16$. The printed water coefficient $z/2$ consequently equals eight. As a check, the left side has charge $-2 - 10 + 16 = +4$, equal to the right-side manganese charge $+4$. There are ten carbon atoms and twenty-eight oxygen atoms on each side, with sixteen hydrogens in the eight waters. Options (A) and (D) reverse the electron-balanced Mn-to-oxalate ratio; option (B) has too few protons and fails hydrogen, oxygen and charge balance.

Step 3: Conclusion The coefficients are $x = 2$, $y = 5$, and $z = 16$. Hence, the correct option is (C).

Quick Tip to Solve: Find the Mn-to-oxalate ratio from electron transfer first, then use the acidic half-reaction to determine the proton coefficient.

85. Experimentally it was found that a metal oxide has formula $M_{0.98}O$. Metal M, present as M^{2+} and M^{3+} in its oxide. Fraction of the metal which exists as M^{3+} would be [4 marks]

(A) 7.01%

(B) 4.08%

(C) 6.05%

(D) 5.08%

Correct Answer: (B) 4.08%

Solution:

Concept:

- A bulk oxide must be electrically neutral: the sum of the positive ionic charges equals the magnitude of the negative ionic charges.
- For one formula unit of $M_{0.98}O$, there are 0.98 metal ions and one O^{2-} ion. Each M^{3+} supplies one more positive charge than M^{2+}.

Step 1: Choose a basis and count the ions Use one mole of oxide formula units as the basis. Let x mole of the metal ions be M^{3+}. The amount of M^{2+} must then be $(0.98 - x)$ mole, because the total amount of metal is fixed at 0.98 mole. The oxygen contributes 2 mole of negative charge equivalents.

Step 2: Apply electrical neutrality The positive charge equivalents must equal the negative charge equivalents. Thus

$$2(0.98 - x) + 3x = 2.$$

Simplifying gives

$$1.96 + x = 2, \qquad x = 0.04.$$

The value 0.04 is the number of trivalent metal ions per oxide formula unit; it is not yet the fraction of the metal ions.

Step 3: Express the result as a percentage of metal Divide the amount of trivalent metal by the total amount of metal, rather than by the amount of oxygen:

$$f = \frac{0.04}{0.98} = 0.040816\ldots, \qquad 100f = 4.0816\ldots\%.$$

The alternatives 7.01%, 6.05% and 5.08% would supply too much positive charge for one oxide ion and therefore fail neutrality.

Step 4: Conclusion The fraction of metal present as M^{3+} is 0.0408, or approximately 4.08%. Hence, the correct option is (B).

Quick Tip to Solve: In $M_{1-\delta}O$ containing divalent and trivalent metal, the trivalent fraction is $2\delta/(1-\delta)$; divide by the actual metal occupancy.

86. A gaseous hydrocarbon gives upon combustion 0.72 g of water and 3.08 g of CO_2. The empirical formula of the hydrocarbon is [4 marks]

(A) C_2H_4

(B) C_3H_4

(C) C_6H_5

(D) C_7H_8

Correct Answer: (D) C_7H_8

Solution:

Concept:

- Complete combustion converts each carbon atom in a hydrocarbon into one CO_2 molecule and each pair of hydrogen atoms into one H_2O molecule.
- The empirical formula gives the smallest whole-number ratio of the numbers of carbon and hydrogen atoms. Product masses must first be converted to mole amounts.

Step 1: Determine the amount of carbon Use the molar mass $M(CO_2) = 44\,\text{g mol}^{-1}$. Every mole of carbon dioxide contains one mole of carbon atoms, so

$$n(\text{C}) = n(CO_2) = \frac{3.08}{44} = 0.070\,\text{mol}.$$

The oxygen in carbon dioxide comes from the oxidant and is not part of the hydrocarbon formula.

Step 2: Determine the amount of hydrogen Using $M(H_2O) = 18\,\text{g mol}^{-1}$, the amount of water formed is

$$n(H_2O) = \frac{0.72}{18} = 0.040\,\text{mol}.$$

Each water molecule contains two hydrogen atoms. Therefore

$$n(\text{H}) = 2(0.040) = 0.080\,\text{mol}.$$

Forgetting this factor of two would give an incorrect carbon-to-hydrogen ratio.

Step 3: Reduce to the simplest integer ratio The atomic ratio is

$$\text{C} : \text{H} = 0.070 : 0.080 = 7 : 8.$$

The integers seven and eight have no common divisor, so this is already the empirical ratio. Option (A) has ratio $1 : 2$, option (B) has ratio $3 : 4$, and option (C) has ratio $6 : 5$; none equals $7 : 8$. A molecular mass is unnecessary because only the empirical formula is requested.

Step 4: Conclusion The empirical formula is C_7H_8. Hence, the correct option is (D).

Quick Tip to Solve: For hydrocarbon combustion, compare $m(CO_2)/44$ with $2m(H_2O)/18$, then reduce that atomic mole ratio.

87. For gaseous state, if most probable speed is denoted by C*, average speed by $\overline{C}$ and mean square speed by C, then for a large number of molecules the ratios of these speeds are: [4 marks]

(A) C*: $\overline{C}$: C = 1.128 : 1.225 : 1

(B) C*: $\overline{C}$: C = 1.225 : 1.128 : 1

(C) C*: $\overline{C}$: C = 1 : 1.225 : 1.128

(D) C*: $\overline{C}$: C = 1 : 1.128 : 1.225

Correct Answer: No option is valid for the literal mean square speed; option (D) applies only to root-mean-square speed.

Solution:

Concept:

- The most probable speed is $C^* = \sqrt{2RT/M}$, and the average speed is $\overline{C} = \sqrt{8RT/(\pi M)}$, where T is absolute temperature, R is the gas constant and M is molar mass.
- Mean square speed is $\langle v^2 \rangle = 3RT/M$, whereas root-mean-square speed is $\sqrt{\langle v^2 \rangle} = \sqrt{3RT/M}$. The former has units of speed squared; these are different quantities.

Step 1: Preserve the quantity stated in the question The printed third quantity is mean square speed, denoted by C. Taking that wording literally gives $C = \langle v^2 \rangle = 3RT/M$, not its square root. The first two quantities have dimension LT^{-1}, but this third quantity has dimension L^2T^{-2}. Therefore the three cannot have a universal dimensionless numerical ratio of the form offered in the options.

Step 2: Check the literal ratio Let $u = \sqrt{RT/M}$, which has units of speed. Then

$$C^* : \overline{C} : C = \sqrt{2}\,u : \sqrt{8/\pi}\,u : 3u^2.$$

Dividing by the most probable speed gives

$$1 : \sqrt{4/\pi} : \frac{3}{\sqrt{2}}u.$$

Its third entry still depends on the gas temperature, molar mass and unit of speed. Hence none of the four fixed dimensionless triples answers the question literally.

Step 3: Distinguish the familiar speed formula If the third quantity were root-mean-square speed instead, all three entries would contain the common speed factor u. Their ratio would then be

$$1 : \sqrt{4/\pi} : \sqrt{3/2} = 1 : 1.128\ldots : 1.225\ldots.$$

This is option (D), while (A) and (B) put the largest characteristic speed last with the smallest factor, and (C) interchanges the average and root-mean-square factors. This explains the familiar answer (D), but obtaining it requires replacing the stated mean square quantity with a different quantity.

Step 4: Conclusion As printed, the question has no valid option: mean square speed has different dimensions from the other two quantities. Option (D) is the ratio for root-mean-square speed, and does not answer the literal wording.

Quick Tip to Solve: Check units before comparing molecular-speed ratios: mean square speed is in $\mathrm{m^2\,s^{-2}}$, while root-mean-square speed is in $\mathrm{m\,s^{-1}}$.

88. Which of the following exists as covalent crystals in the solid state? [4 marks]

(A) Silicon

(B) Sulphur

(C) Phosphorous

(D) Iodine

Correct Answer: (A) Silicon

Solution:

Concept:

- A covalent or network crystal consists of atoms connected throughout the solid by an extended network of strong covalent bonds.
- A molecular crystal consists of discrete molecules. Covalent bonds inside each molecule do not make the whole solid a covalent network; the forces between its molecules determine its crystal classification.

Step 1: Examine the bonding in silicon In crystalline silicon, each silicon atom forms four covalent bonds to neighbouring silicon atoms in an approximately tetrahedral arrangement. These bonds continue throughout the three-dimensional lattice, giving a giant covalent structure similar in connectivity to diamond. There are no separate small silicon molecules acting as the lattice units. Silicon therefore satisfies the definition of a covalent crystal.

Step 2: Distinguish the molecular alternatives Ordinary crystalline sulphur contains discrete S_8 rings, while the familiar molecular form of phosphorus contains P_4 units. Crystalline iodine contains I_2 molecules. The atoms within each unit are covalently bonded, but neighbouring units are held together predominantly by intermolecular dispersion forces. In the standard classification intended by these options, these are molecular solids.

Step 3: Select the extended network The deciding feature is whether covalent bonds join atoms across the crystal rather than merely inside a molecule. Silicon has such a continuous network and is option (A). Sulphur, phosphorus in its molecular form, and iodine represent the molecular alternatives, so options (B), (C) and (D) do not give the intended covalent network crystal. The presence of covalent bonds alone is insufficient to classify a solid as a network solid.

Step 4: Conclusion Silicon exists as an extended covalent crystal in the solid state. Hence, the correct option is (A).

Quick Tip to Solve: Ask what occupies the lattice: separate molecules indicate a molecular solid, while a continuous framework of covalently bonded atoms indicates a network crystal.

89. The coagulating power of electrolytes having ions Na^+, Al^{3+} and Ba^{2+} for arsenic sulphide sol increases in the order: [4 marks]

(A) $Na^+ < Ba^{2+} < Al^{3+}$

(B) $Al^{3+} < Ba^{2+} < Na^+$

(C) $Ba^{2+} < Na^+ < Al^{3+}$

(D) $Al^{3+} < Na^+ < Ba^{2+}$

Correct Answer: (A) $Na^+ < Ba^{2+} < Al^{3+}$

Solution:

Concept:

- An arsenic sulphide sol, As_2S_3, is negatively charged under the usual preparation conditions; positive counter-ions are therefore responsible for its coagulation.
- The Hardy–Schulze rule states that the coagulating power of a counter-ion increases strongly with its valency. Greater coagulating power means a smaller concentration is required to destabilise the sol.

Step 1: Identify the ions that neutralise the sol Colloidal particles resist aggregation because like surface charges repel one another. Adding an electrolyte supplies ions; the ions of charge opposite to the particles are preferentially effective at reducing their electrical stabilisation. For negatively charged arsenic sulphide, the relevant species are the cations listed in the question: Na^+, Ba^{2+}, and Al^{3+}.

Step 2: Rank the counter-ion valencies The counter-ion charges have magnitudes

$$z(Na^+) = 1, \qquad z(Ba^{2+}) = 2, \qquad z(Al^{3+}) = 3.$$

Applying the Hardy–Schulze rule gives increasing coagulating power in the same order as these charge magnitudes. Thus the monovalent sodium ion is least effective and the trivalent aluminium ion is most effective.

Step 3: Compare the printed orders The required order is

$$Na^+ < Ba^{2+} < Al^{3+}.$$

Option (B) reverses the effectiveness completely. Option (C) incorrectly places divalent barium below monovalent sodium. Option (D) incorrectly puts trivalent aluminium below both other cations. These comparisons concern coagulating power; the order of the minimum concentrations required for coagulation would be reversed.

Step 4: Conclusion The coagulating power increases as $Na^+ < Ba^{2+} < Al^{3+}$. Hence, the correct option is (A).

Quick Tip to Solve: First find the sign of the sol, then rank only the oppositely charged ions by valency; do not confuse coagulating power with coagulation concentration.

90. Which of the following is the wrong statement? [4 marks]

(A) O_3 molecule is bent

(B) Ozone is violet-black in solid state

(C) Ozone is diamagnetic gas

(D) ONCl and ONO^- are isoelectronic

Correct Answer: (D) ONCl and ONO^- are isoelectronic

Solution:

Concept:

- Ozone, O_3, has a bent molecular structure and all its electrons are paired, so it is diamagnetic.
- Isoelectronic species contain the same total number of electrons. For a molecular ion, add electrons for a negative charge and subtract them for a positive charge; matching valence-electron counts alone does not establish equal total electron counts.

Step 1: Check the three statements about ozone The central oxygen atom in ozone has two bonding regions and a lone pair. Its electron-pair arrangement produces a bent molecular shape, so statement (A) is correct. Ozone is violet-black in its solid state, making statement (B) correct. Unlike dioxygen, ozone has no unpaired electrons in its ground state, so its diamagnetic character makes statement (C) correct as well.

Step 2: Count electrons in nitrosyl chloride For neutral $ONCl$, sum the atomic numbers of its atoms. Oxygen contributes eight electrons, nitrogen seven, and chlorine seventeen. Thus

$$N_e(ONCl) = 8 + 7 + 17 = 32.$$

There is no charge correction because this species is neutral.

Step 3: Count electrons in nitrite and test the claim The formula ONO^- represents the nitrite ion, with two oxygen atoms and one nitrogen atom. Its negative charge adds one electron:

$$N_e(ONO^-) = 8 + 7 + 8 + 1 = 24.$$

Since $32 \neq 24$, the two species are not isoelectronic. They each have eighteen valence electrons, which can make their Lewis-structure counts look similar, but their total electron counts differ. Therefore statement (D), rather than any of the true ozone statements, is the wrong statement requested.

Step 4: Conclusion $ONCl$ has 32 electrons whereas ONO^- has 24, so the claim that they are isoelectronic is wrong. Hence, the correct option is (D).

Quick Tip to Solve: When testing whether species are isoelectronic, sum atomic numbers and apply the ionic charge; do not count only the outer-shell electrons.