JEE Main 2014 Paper 1 (B.E./B.Tech) Apr 9 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2014, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 9 April 2014, Single slot. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. If $\frac{1}{\sqrt{\alpha}}, \frac{1}{\sqrt{\beta}}$ are the roots of $ax^2 + bx + 1 = 0$, where $a \neq 0$ and $a, b \in \mathbb{R}$, then the equation $x(x + b^3) + (a^3 - 3abx) = 0$ has roots: [4 marks]

(1) $\sqrt{\alpha\beta}$ and $\alpha\beta$

(2) $\alpha^{-3/2}$ and $\beta^{-3/2}$

(3) $\alpha\beta^{1/2}$ and $\alpha^{1/2}\beta$

(4) $\alpha^{3/2}$ and $\beta^{3/2}$

Correct Answer: (4) $\alpha^{3/2}$ and $\beta^{3/2}$

Solution:

Concept:

- For a quadratic $Ax^2+Bx+C = 0$, the sum and product of its roots are $-B/A$ and C/A, respectively.
- The identity $s^3 + t^3 = (s + t)^3 - 3st(s + t)$ converts the original root relations into relations for the cubes.

Step 1: Express the coefficients through the original roots Put $s = \sqrt{\alpha}$ and $t = \sqrt{\beta}$; the stated roots are $1/s$ and $1/t$, so neither is zero. Their product and sum give

$$\frac{1}{st} = \frac{1}{a}, \qquad \frac{s + t}{st} = -\frac{b}{a}.$$

Consequently $a = st$ and $b = -(s + t)$. The constant term 1 is essential: it fixes the coefficient scale instead of giving only a ratio.

Step 2: Rewrite the new quadratic Collect its terms before identifying its roots:

$$x^2 + (b^3 - 3ab)x + a^3 = 0.$$

Now substitute $a = st$, $b = -(s + t)$:

$$b^3 - 3ab = -(s + t)^3 + 3st(s + t) = -(s^3 + t^3), \qquad a^3 = s^3t^3.$$

The equation therefore factors as

$$x^2 - (s^3 + t^3)x + s^3t^3 = (x - s^3)(x - t^3) = 0.$$

Both the sum and the product have been checked; checking only one of them could select a distractor.

Step 3: Compare the listed pairs The roots are the cubes of s and t, not the cubes of the original reciprocal roots. Option (2) therefore applies the reciprocal transformation in the wrong direction. The pairs in (1) and (3) generally have neither the required sum nor the required product. Option (4) matches both relations exactly.

Step 4: Conclusion The roots are $\alpha^{3/2}$ and $\beta^{3/2}$. Hence, the correct option is (4).

Quick Tip to Solve: When reciprocal square roots are given, rename their denominators first; the cubic identity then exposes the new quadratic immediately.

2. If equations $ax^2 + bx + c = 0$, where $a, b, c \in \mathbb{R}$, $a \neq 0$, and $2x^2 + 3x + 4 = 0$ have a common root, then $a : b : c$ equals: [4 marks]

(1) 2 : 3 : 4

(2) 4 : 3 : 2

(3) 1 : 2 : 3

(4) 3 : 2 : 1

Correct Answer: (1) 2 : 3 : 4

Solution:

Concept:

- Nonreal roots of a polynomial with real coefficients occur in complex conjugate pairs.
- Two genuine quadratic polynomials with the same two roots differ only by a nonzero constant factor; their coefficient triples are proportional.

Step 1: Check whether the common root is real For the fixed quadratic, the discriminant is

$$\Delta = 3^2 - 4(2)(4) = 9 - 32 = -23 < 0.$$

Thus both its roots are nonreal, and the quadratic formula gives

$$r = \frac{-3 + i\sqrt{23}}{4}, \qquad \overline{r} = \frac{-3 - i\sqrt{23}}{4}.$$

Whichever of these is shared, it cannot be treated as a lone real root. This is the crucial information supplied by the particular coefficients 2, 3 and 4.

Step 2: Use the conjugate-root condition The coefficients a, b, c are real and $a \neq 0$, so if r is a root of $ax^2 + bx + c$, its conjugate $\overline{r}$ must be another root. A quadratic has exactly two roots counted with multiplicity. Hence its two roots are precisely those of $2x^2 + 3x + 4$, and for a nonzero real constant λ,

$$ax^2 + bx + c = \lambda(2x^2 + 3x + 4).$$

Comparing coefficients yields $a = 2\lambda$, $b = 3\lambda$, $c = 4\lambda$.

Step 3: Check the proposed ratios It follows that $a : b : c = 2 : 3 : 4$. The scale λ may be positive or negative but cancels from the ratio. Reversing the outer coefficients, as in (2), changes the polynomial. Neither (3) nor (4) is proportional to $(2, 3, 4)$, so neither can have the required nonreal common root with real coefficients.

Step 4: Conclusion $a : b : c = 2 : 3 : 4$. Hence, the correct option is (1).

Quick Tip to Solve: For two real quadratics, first test the discriminant: sharing a nonreal root forces sharing its conjugate and hence proportional coefficients.

3. Let w, with $\operatorname{Im} w \neq 0$, be a complex number. Then the set of all complex numbers z satisfying $w - \overline{w}z = k(1 - z)$, for some real number k, is: [4 marks]

(1) $\{z : z \neq 1\}$

(2) $\{z : |z| = 1,\ z \neq 1\}$

(3) $\{z : z = \overline{z}\}$

(4) $\{z : |z| = 1\}$

Correct Answer: (2) $\{z : |z| = 1,\ z \neq 1\}$

Solution:

Concept:

- A complex number and its conjugate have equal modulus: $|u| = |\overline{u}|$. For real k, $\overline{k - w} = k - \overline{w}$.
- A locus must be checked in both directions: every solution lies on it, and every proposed point is obtained for an allowed parameter.

Step 1: Solve for the parameterised point Rearrange the equation without taking moduli prematurely:

$$w - k = (\overline{w} - k)z, \qquad z = \frac{k - w}{k - \overline{w}}.$$

The denominator is nonzero because k is real but $\operatorname{Im} w \neq 0$. The numerator and denominator are conjugates, so

$$|z| = \frac{|k - w|}{|k - \overline{w}|} = 1.$$

However $z = 1$ in the original equation would imply $w = \overline{w}$, contradicting the hypothesis. Thus every solution lies on the unit circle with 1 removed.

Step 2: Prove that all remaining circle points occur Conversely, choose $|z| = 1$, $z \neq 1$, and define

$$k = \frac{w - \overline{w}z}{1 - z}.$$

Since $\overline{z} = 1/z$, conjugating and multiplying numerator and denominator by z gives

$$\overline{k} = \frac{\overline{w} - w/z}{1 - 1/z} = \frac{\overline{w}z - w}{z - 1} = k.$$

Therefore this parameter is real and satisfies the original equation. The sufficiency check prevents a mere necessary modulus condition from being mistaken for the full locus.

Step 3: Eliminate the other sets Option (1) includes points off the unit circle, whereas all solutions have modulus 1. Option (3) restricts points to the real axis and misses nonreal solutions. Option (4) incorrectly retains 1, where no real parameter can make the equation hold.

Step 4: Conclusion The locus is $\{z : |z| = 1,\ z \neq 1\}$. Hence, the correct option is (2).

Quick Tip to Solve: After a conjugate-ratio calculation gives a unit circle, test excluded denominator points directly in the original equation.

4. The sum of the digits in the unit's place of all the 4 - digit numbers formed by using the numbers 3, 4, 5 and 6, without repetition is [4 marks]

(1) 18

(2) 36

(3) 108

(4) 432

Correct Answer: (3) 108

Solution:

Concept:

- With four distinct digits used without repetition, the number of complete arrangements is $4! = 24$.
- Fixing a digit in the units position leaves the other three distinct digits to occupy the remaining positions in $3! = 6$ ways.

Step 1: Count appearances in the units place There is no zero digit, so every permutation of 3, 4, 5 and 6 is a valid four-digit number. If 3 is fixed at the units position, the thousands, hundreds and tens positions may be filled by 4, 5 and 6 in $3! = 6$ arrangements. Exactly the same reasoning works for 4, 5 or 6 in the units position. Thus each allowed digit occurs six times as the final digit. This counts $4 \times 6 = 24$ final digits, one for each number, so none is missed or counted twice.

Step 2: Add the contributions of all four digits The question asks for the sum of the digits occupying the units places, rather than the sum of the complete four-digit numbers. Hence place-value factors for the other columns do not enter:

$$S = 6(3 + 4 + 5 + 6) = 6 \times 18 = 108.$$

Equivalently, the average units digit is $(3 + 4 + 5 + 6)/4 = 4.5$; multiplying this by the 24 numbers again gives $24 \times 4.5 = 108$. This independent symmetry check confirms the counting.

Step 3: Distinguish the distractors Option (1), 18, adds each available digit only once and ignores its repeated occurrence across different numbers. Option (2), 36, uses too small a repetition count. Option (4), 432, is four times the required sum and counts all four positional digit totals instead of only the units column. Only (3) includes the correct six appearances of each digit.

Step 4: Conclusion The required sum is 108. Hence, the correct option is (3).

Quick Tip to Solve: For all permutations of distinct digits, each digit occurs equally often in any fixed position; fix that position and permute the remaining digits.

5. Given an A.P. whose terms are all positive integers. The sum of its first nine terms is greater than 200 and less than 220. If the second term in it is 12, then its 4th term is [4 marks]

(1) 8

(2) 24

(3) 20

(4) 16

Correct Answer: (3) 20

Solution:

Concept:

- An arithmetic progression with first term a and common difference d has $T_n = a + (n-1)d$.
- The sum of its first nine terms is nine times the middle term: $S_9 = 9(a+4d)$. Integer terms make d an integer.

Step 1: Express the sum using the given second term The condition on the second term is $a + d = 12$, hence $a = 12 - d$. Substituting this into the sum formula gives

$$S_9 = \frac{9}{2}(2a + 8d) = 9(a + 4d) = 9(12 + 3d) = 108 + 27d.$$

The bounds on the sum are strict; neither 200 nor 220 is included. Using these exact inequalities will isolate the common difference rather than merely estimate it.

Step 2: Use integrality to determine the difference The printed condition yields

$$200 < 108 + 27d < 220, \qquad 92 < 27d < 112.$$

Therefore

$$\frac{92}{27} < d < \frac{112}{27}, \quad \text{that is,} \quad 3.407\ldots < d < 4.148\ldots.$$

Because consecutive terms are integers, their difference is an integer. The only integer in this interval is $d = 4$, giving $a = 8$. These values also respect positivity, and $S_9 = 216$ lies strictly between the stated bounds.

Step 3: Find the fourth term and test the options Now

$$T_4 = a + 3d = 8 + 12 = 20.$$

Alternatively, move two common differences beyond the given second term: $T_4 = 12 + 2(4) = 20$. Option (1), 8, is the first term; (4), 16, is the third term; (2), 24, is the fifth term. This identifies the positional errors behind the other choices.

Step 4: Conclusion The fourth term is 20. Hence, the correct option is (3).

Quick Tip to Solve: For an odd number of AP terms, replace the sum by the number of terms times the middle term before applying integer bounds.

6. If the sum $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\cdots$ up to 20 terms equals $\frac{k}{21}$, then k equals: [4 marks]

(1) 240

(2) 120

(3) 60

(4) 180

Correct Answer: (2) 120

Solution:

Concept:

- The sum of the first n squares is $1^2+2^2+\cdots+n^2=n(n+1)(2n+1)/6$.
- The identity $1/[n(n+1)]=1/n-1/(n+1)$ causes consecutive terms to cancel in a telescoping sum.

Step 1: Write and simplify the general summand The numerators 3, 5, 7, and so on correspond to $2n+1$ for $n=1,2,3,\ldots$, while the denominator of the nth term is the sum of the first n squares. Consequently

$$t_n=\frac{2n+1}{1^2+2^2+\cdots+n^2}=\frac{2n+1}{n(n+1)(2n+1)/6}=\frac{6}{n(n+1)}.$$

For example, $t_1=3$, agreeing with the printed first term. Cancelling the factor $2n+1$ before summing is the essential simplification.

Step 2: Telescope the twenty terms Decompose each simplified term and add from $n=1$ through $n=20$:

$$S_{20}=6\sum_{n=1}^{20}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$
$$=6\left(1-\frac{1}{21}\right)=\frac{120}{21}.$$

All intermediate fractions cancel pairwise. The surviving negative fraction is $1/21$, not $1/20$, because the last denominator is $20+1$. The number 20 specifies the count of summands, not the value of the last square.

Step 3: Compare with the required form The given form is $k/21$, so equality with $120/21$ forces $k=120$. The alternatives 240, 60 and 180 are respectively twice, half and one-and-a-half times this value and do not match the exact telescoping boundary terms. There is no approximation in the calculation, so the numerator is fixed uniquely.

Step 4: Conclusion $k=120$. Hence, the correct option is (2).

Quick Tip to Solve: When an odd numerator sits above a sum of consecutive squares, substitute the square-sum formula before attempting any addition.

7. The number of terms in the expansion of $(1+x)^{101}(1-x+x^2)^{100}$ in powers of x is: [4 marks]

(1) 301

(2) 302

(3) 101

(4) 202

Correct Answer: (4) 202

Solution:

Concept:

- The polynomial identity $(1+x)(1-x+x^2) = 1+x^3$ combines the two factors before expansion.
- The binomial theorem gives nonzero coefficients $\binom{100}{r}$ for $r = 0, 1, \ldots, 100$; distinct powers count as separate terms.

Step 1: Combine the factors strategically Factor out one copy of $1+x$ from the factor with exponent 101 and pair the remaining 100 copies with the second factor:

$$\begin{aligned}(1+x)^{101}(1-x+x^2)^{100} &= (1+x)\left[(1+x)(1-x+x^2)\right]^{100} \\ &= (1+x)(1+x^3)^{100}.\end{aligned}$$

The cancellation inside the identity is exact. The degree is 301, but knowing the degree alone does not mean that every intermediate power has a nonzero coefficient.

Step 2: List the two disjoint families of powers By the binomial theorem,

$$(1+x)(1+x^3)^{100} = \sum_{r=0}^{100}\binom{100}{r}x^{3r} + \sum_{r=0}^{100}\binom{100}{r}x^{3r+1}.$$

The first sum supplies 101 exponents congruent to 0 modulo 3, from 0 to 300. The second supplies 101 exponents congruent to 1 modulo 3, from 1 to 301. No exponent can belong to both families, and every listed binomial coefficient is positive, so no terms cancel. Powers congruent to 2 modulo 3 are absent.

Step 3: Count the nonzero terms The total number is therefore $101 + 101 = 202$. Option (2), 302, counts all powers from 0 through 301 even though many are missing. Option (3), 101, omits the second family created by multiplication by x. Option (1), 301, does not correspond to the actual nonzero exponents. Counting the two residue classes gives the precise answer.

Step 4: Conclusion The expansion contains 202 nonzero terms. Hence, the correct option is (4).

Quick Tip to Solve: Before multiplying large binomial powers, look for a low-degree product identity; then count the residue classes of the surviving exponents.

8. If $\csc\theta = \dfrac{p+q}{p-q}$, $p \neq q$, $p \neq 0$, **then** $\left|\cot\left(\dfrac{\pi}{4}+\dfrac{\theta}{2}\right)\right|$ **is equal to:** [4 marks]

(1) pq

(2) $\sqrt{\dfrac{p}{q}}$

(3) $\sqrt{\dfrac{q}{p}}$

(4) $\sqrt{pq}$

Correct Answer: (3) $\sqrt{\dfrac{q}{p}}$

Solution:

Concept:

- The reciprocal identity gives $\sin\theta = (p-q)/(p+q)$, whenever the given cosecant exists.
- The half-angle identity $\cot^2(\pi/4+\theta/2) = (1-\sin\theta)/(1+\sin\theta)$ removes the sign of the cotangent. The absolute value is its nonnegative square root.

Step 1: Express the required square using sine Put $u = \pi/4 + \theta/2$. Since $2u = \pi/2 + \theta$, we have $\cos 2u = -\sin\theta$. Applying $\cot^2 u = (1+\cos 2u)/(1-\cos 2u)$ gives

$$\cot^2 u = \frac{1-\sin\theta}{1+\sin\theta}.$$

This step avoids choosing a sign for $\cos\theta$, which would be unsafe when no quadrant is specified.

Step 2: Substitute and compare the options Insert the sine found from the stated reciprocal relation:

$$\cot^2 u = \frac{1-\frac{p-q}{p+q}}{1+\frac{p-q}{p+q}} = \frac{2q/(p+q)}{2p/(p+q)} = \frac{q}{p}.$$

The premise excludes a zero denominator in the given cosecant; $p \neq 0$ also makes the last ratio defined. For real admissible data this ratio is nonnegative. Taking the principal square root therefore gives the absolute value. Option (2) reverses numerator and denominator and would correspond to the reciprocal trigonometric quantity. Options (1) and (4) depend on the product, which cannot remain invariant when both p and q are scaled together, whereas the premise and required answer do.

Step 3: Conclusion

$$\left|\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right| = \sqrt{\frac{q}{p}}.$$

Hence, the correct option is (3).

Quick Tip to Solve: For an absolute half-angle expression, first square it and use sine; this avoids an unjustified quadrant sign.

9. The number of values of $\alpha \in [0, 2\pi]$ for which $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$ is: [4 marks]

(1) 3

(2) 1

(3) 6

(4) 4

Correct Answer: (1) 3

Solution:

Concept:

- Substitute $s = \sin\alpha$ to reduce the equation to a polynomial, retaining the restriction $-1 \le s \le 1$.
- Each admissible sine value must then be counted on the specified closed interval. A value of 1 occurs once, while a value strictly between 0 and 1 occurs twice.

Step 1: Factor the polynomial Move all terms to the left and use $s = \sin\alpha$:

$$2s^3 - 7s^2 + 7s - 2 = 0.$$

Substitution of $s = 1$ gives zero, so division by $s - 1$ leaves $2s^2 - 5s + 2$. Hence

$$(s - 1)(2s - 1)(s - 2) = 0.$$

The algebraic candidates are $s = 1, \frac{1}{2}, 2$. The last value must be rejected because no real angle has sine greater than one. Counting all three polynomial roots as angles would ignore the range of sine.

Step 2: Count the distinct angles For $\sin\alpha = 1$, the only angle in the stated interval is $\alpha = \pi/2$. For $\sin\alpha = 1/2$, the angles are $\alpha = \pi/6$ and $5\pi/6$. Neither endpoint 0 nor 2π satisfies the equation: its left side is then zero rather than two. Thus there are exactly three distinct solutions, all in the upper half of the unit circle. Option (2) counts only the sine value one; option (3) would overcount by assigning two angles to every algebraic root; option (4) also fails to remove the inadmissible or repeated cases.

Step 3: Conclusion The solutions are $\pi/6, \pi/2, 5\pi/6$, so the number is 3. Hence, the correct option is (1).

Quick Tip to Solve: After factoring in sine, reject roots outside [-1,1] before counting angles; the maximum sine value gives only one angle per full turn.

10. Given three points P, Q, R with $P(5, 3)$ and R on the x-axis. If the equation of RQ is $x - 2y = 2$ and PQ is parallel to the x-axis, then the centroid of $\triangle PQR$ lies on the line: [4 marks]

(1) $x - 2y + 1 = 0$

(2) $2x + y - 9 = 0$

(3) $2x - 5y = 0$

(4) $5x - 2y = 0$

Correct Answer: (3) $2x - 5y = 0$

Solution:

Concept:

- A point on the x-axis has zero ordinate. Points on a horizontal line have equal ordinates.
- The centroid of vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is $((x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3)$.

Step 1: Locate the unknown vertices Since R belongs to the x-axis, substitute $y = 0$ into its line $x - 2y = 2$. This gives $x = 2$, so $R = (2, 0)$. Next, PQ is parallel to the x-axis and passes through $P = (5, 3)$, so every point of PQ has ordinate 3. The vertex Q is also on RQ; substituting $y = 3$ gives $x - 6 = 2$, or $x = 8$. Thus $Q = (8, 3)$. Both coordinates have been fixed by intersections, rather than by assuming a shape for the triangle.

Step 2: Compute the centroid and test each line The arithmetic means of the coordinates give

$$G = \left(\frac{5 + 8 + 2}{3}, \frac{3 + 3 + 0}{3}\right) = (5, 2).$$

Substitute this pair into the four candidate equations. In (1), $5 - 2(2) + 1 = 2 \neq 0$. In (2), $2(5) + 2 - 9 = 3 \neq 0$. In (3), $2(5) - 5(2) = 0$, so it is satisfied. In (4), $5(5) - 2(2) = 21 \neq 0$. This direct test distinguishes the two equations with similar-looking coefficients and confirms that the coordinate averages, rather than a midpoint of one side, are required.

Step 3: Conclusion The centroid is $(5, 2)$, which lies on $2x - 5y = 0$. Hence, the correct option is (3).

Quick Tip to Solve: Find each unknown vertex by its two line conditions, average all three coordinates, and test the candidate equations directly.

11. Let a and b be any two numbers satisfying $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4}$. Then, the foot of perpendicular from the origin on the variable line $\frac{x}{a}+\frac{y}{b}=1$ lies on: [4 marks]

(1) A circle of radius 2

(2) A hyperbola with each semi-axis $\sqrt{2}$

(3) A hyperbola with each semi-axis 2

(4) A circle of radius $\sqrt{2}$

Correct Answer: (1) A circle of radius 2

Solution:

Concept:

- For a line $Ax+By=1$, its perpendicular distance from the origin is $1/\sqrt{A^2+B^2}$.
- The foot of the perpendicular is the closest point on that line. If its distance from the origin is constant, it lies on a circle centred at the origin.

Step 1: Put the line into normal form Take $A=1/a$ and $B=1/b$; both are finite because the given reciprocal squares exist. The constraint is

$$A^2+B^2=\frac{1}{4}.$$

Thus the line $Ax+By=1$ has a normal vector of constant length $1/2$. Its orientation may change as a and b change, but its distance from the origin is fixed. Dividing the line by $\sqrt{A^2+B^2}$ would give unit-normal coefficients and a constant right-hand side of two.

Step 2: Find the radius and identify the locus If $H=(h,k)$ is the perpendicular foot, it lies along the normal vector, so $H=\lambda(A,B)$. Substitution into the line gives $\lambda(A^2+B^2)=1$, hence $\lambda=4$. Therefore

$$h^2+k^2=16(A^2+B^2)=4.$$

This is the equation of a circle of radius 2. With finite nonzero intercepts, some axis points may be excluded from the parametrisation, but every allowed foot lies on that same circle, exactly as asked. The hyperbola options require a difference of squares rather than this positive sum. The circle of radius $\sqrt{2}$ would require squared distance two, inconsistent with the computed distance four.

Step 3: Conclusion The foot lies on $x^2+y^2=4$, a circle of radius 2. Hence, the correct option is (1).

Quick Tip to Solve: When reciprocals of intercepts have a fixed sum of squares, use the normal distance of the intercept-form line immediately.

12. If the point $(1, 4)$ lies inside the circle $x^2 + y^2 - 6x + 10y + p = 0$ and the circle does not touch or intersect the coordinate axes, then the set of all possible values of p is the interval: [4 marks]

(1) $(25, 39)$

(2) $(25, 29)$

(3) $(0, 25)$

(4) $(9, 25)$

Correct Answer: No printed option is correct: the conditions give an empty set of values of p.

Solution:

Concept:

- Completing the squares puts the circle in centre-radius form: $(x-3)^2 + (y+5)^2 = 34 - p$. Its centre is $(3, -5)$.
- A real circle avoiding both coordinate axes has radius strictly smaller than each distance of its centre from those axes. A point strictly inside has distance from the centre strictly smaller than the radius.

Step 1: Apply the axis-avoidance conditions The distance from the centre to the y-axis is 3, and the distance to the x-axis is 5. Consequently the stronger restriction is $0 < r < 3$. As $r^2 = 34 - p$, this requires

$$0 < 34 - p < 9, \qquad 25 < p < 34.$$

Equality at radius three would give tangency, which is explicitly excluded. A value $p \geq 34$ gives a degenerate point or no real circle, so such values cannot be admitted either.

Step 2: Apply the printed point condition For the printed point $(1, 4)$, the squared distance from $(3, -5)$ is

$$(1-3)^2 + (4+5)^2 = 4 + 81 = 85.$$

Strict interior membership therefore requires $85 < r^2 = 34 - p$, or $p < -51$. This contradicts $p > 25$ from the axes condition. Equivalently, a circle centred below the x-axis that never intersects it cannot contain a point above that axis. None of the four intervals can satisfy both demands. In particular, the often tempting interval $(25, 29)$ would give $5 < r^2 < 9$, far smaller than the required squared distance eighty-five. The point and signs must be retained as printed; replacing a coordinate would solve a different question.

Step 3: Conclusion For the question as printed, the admissible set is empty. There is no correct option among (1)–(4); the printed conditions are inconsistent.

Quick Tip to Solve: Before choosing an interval, compare the point and centre sides of each axis; a circle avoiding an axis stays entirely on one side.

13. If OB is the semi-minor axis of an ellipse, F_1 and F_2 are its foci and the angle between F_1B and F_2B is a right angle, then the square of the eccentricity of the ellipse is: [4 marks]

(1) $\frac{1}{4}$

(2) $\frac{1}{\sqrt{2}}$

(3) $\frac{1}{2}$

(4) $\frac{1}{2\sqrt{2}}$

Correct Answer: (3) $\frac{1}{2}$

Solution:

Concept:

- For an ellipse with semi-major axis a, semi-minor axis b, and focal distance c, $c^2 = a^2 - b^2$ and $e = c/a$.
- Two nonzero vectors are perpendicular exactly when their scalar product is zero. Place the centre at the origin and the major axis along the x-axis.

Step 1: Translate the right angle into a relation Choose $O = (0, 0)$, $B = (0, b)$, $F_1 = (-c, 0)$, and $F_2 = (c, 0)$. This coordinate placement uses only the symmetries of an ellipse and does not restrict its size. The vectors from B to the foci are $\overrightarrow{BF_1} = (-c, -b)$ and $\overrightarrow{BF_2} = (c, -b)$. Their scalar product is

$$-c^2 + b^2.$$

Since the included angle is a right angle, this product is zero; hence $b^2 = c^2$. The same result follows from the isosceles right triangle whose hypotenuse is the distance $F_1F_2 = 2c$.

Step 2: Use the ellipse identity Substitute $c^2 = b^2$ into the defining identity:

$$a^2 = b^2 + c^2 = 2c^2.$$

Therefore

$$e^2 = \frac{c^2}{a^2} = \frac{1}{2}.$$

The eccentricity itself would be $1/\sqrt{2}$, which appears as option (2); that option answers a different demand. Option (1) would require $a = 2c$, whereas here $a = \sqrt{2}c$. Option (4) introduces an additional square-root factor with no support in either the perpendicularity relation or the ellipse identity. The requested square is a dimensionless ratio, so the scale of the ellipse cancels entirely.

Step 3: Conclusion The square of the eccentricity is $e^2 = 1/2$. Hence, the correct option is (3).

Quick Tip to Solve: At a minor-axis endpoint, perpendicular focal joins give b=c; then distinguish eccentricity from its square when comparing options.

14. If $f(x)$ is continuous and $f\left(\frac{9}{2}\right) = \frac{2}{9}$, then $\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right)$ equals: [4 marks]

(1) $\frac{8}{9}$
(2) 0
(3) $\frac{2}{9}$
(4) $\frac{9}{2}$

Correct Answer: (3) $\frac{2}{9}$

Solution:

Concept:

- The identity $1-\cos u = 2\sin^2(u/2)$, together with $\lim_{t\to 0} \sin t/t = 1$, evaluates the inner limit.
- Continuity at a point L allows $\lim_{x\to 0} f(g(x)) = f(L)$ whenever $g(x) \to L$. The function value and its input must be kept distinct.

Step 1: Evaluate the argument limit Direct substitution in the quotient gives the indeterminate form $0/0$, so rewrite its numerator before taking the limit:

$$\frac{1-\cos 3x}{x^2} = \frac{2\sin^2(3x/2)}{x^2} = \frac{9}{2}\left(\frac{\sin(3x/2)}{3x/2}\right)^2.$$

As $x \to 0$, the variable $t = 3x/2$ also tends to zero from either side. The standard sine limit is one, so the inner quotient tends to $9/2$. The square makes this conclusion valid for both positive and negative small x.

Step 2: Use continuity at the limiting input The result of the first step is the input approached by f, not yet the value of the required limit. Since f is continuous, it is continuous in particular at $9/2$, and hence

$$\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right) = f\left(\frac{9}{2}\right) = \frac{2}{9}.$$

Option (4) stops after finding the argument limit and forgets the outer function. Option (2) would follow from treating a zero numerator as enough to make a quotient zero, despite its denominator also tending to zero. Option (1) is neither the inner limit nor the specified continuous function value. No formula for f away from the limiting point is needed.

Step 3: Conclusion The required limit is $f(9/2) = 2/9$. Hence, the correct option is (3).

Quick Tip to Solve: For a continuous outer function, first find the inner limit, then insert that limiting input into the given function value.

15. The contrapositive of the statement "I go to school if it does not rain" is [4 marks]

(1) If it rains, I go to school

(2) If it rains, I do not go to school

(3) If I go to school, it rains

(4) If I do not go to school, it rains

Correct Answer: (4) If I do not go to school, it rains

Solution:

Concept:

- An implication $p \Rightarrow q$ has contrapositive $\neg q \Rightarrow \neg p$. The two implications are logically equivalent.
- In the wording "q if p", the statement following "if" is the hypothesis, so the original implication runs from p to q.

Step 1: Identify the hypothesis and conclusion Let r mean "it rains" and let s mean "I go to school". The printed sentence says that absence of rain guarantees going to school. Therefore its implication is $\neg r \Rightarrow s$. It does not say that going to school guarantees absence of rain: reversing the direction would give the converse, which need not have the same truth value.

Step 2: Reverse and negate both statements The contrapositive begins with the negation of the original conclusion. Negating s gives "I do not go to school". Its conclusion is the negation of the original hypothesis, namely $\neg(\neg r) = r$, or "it rains". Thus the required implication is $\neg s \Rightarrow r$. Option (1) changes only the hypothesis; option (2) gives $r \Rightarrow \neg s$, the inverse of the original implication; and option (3) has $s \Rightarrow r$. None has the required premise and conclusion. Although the supplied key indicates (2), that statement is the inverse; the literal printed implication proves (4).

Step 3: Conclusion The printed implication is $\neg r \Rightarrow s$, whose contrapositive is $\neg s \Rightarrow r$. This differs from the supplied key by applying the required logical operation. The contrapositive is "If I do not go to school, it rains". Hence, the correct option is (4).

Quick Tip to Solve: First rewrite a sentence containing "if" as an arrow; then reverse the arrow and negate both ends. Do not merely negate the English sentence.

16. In a set of $2n$ distinct observations, each of the observations below the median of all the observations is increased by 5 and each of the remaining observations is decreased by 3. Then, the mean of the new set of observations: [4 marks]

(1) Increases by 2

(2) Increases by 1

(3) Decreases by 2

(4) Decreases by 1

Correct Answer: (2) Increases by 1

Solution:

Concept:

- The arithmetic mean equals the sum of all observations divided by their number: $\bar{x} = S/(2n)$.
- For $2n$ distinct ordered observations, the median lies strictly between the nth and $(n+1)$th observations. Exactly n observations lie below it.

Step 1: Count the two groups Write the original ordered data as $x_1 < x_2 < \cdots < x_{2n}$. Their median is $(x_n + x_{n+1})/2$, so the first n observations form the group increased by 5. The remaining n observations form the group decreased by 3. Distinctness ensures that none equals the median, so there is no observation requiring a separate treatment.

Step 2: Compute the net change in the sum Let the original sum be S. Increasing each member of the first group adds $5n$ to the sum, while decreasing each member of the second group subtracts $3n$. Consequently,

$$S_{\text{new}} = S + 5n - 3n = S + 2n.$$

The number of observations is unchanged. Hence

$$\bar{x}_{\text{new}} = \frac{S + 2n}{2n} = \bar{x} + 1.$$

The values might change their order afterwards, but reordering cannot change their sum or mean. The increases of 2 and the proposed decreases are incompatible with this positive net change of $2n$.

Step 3: Conclusion The new mean exceeds the original mean by 1. Hence, the correct option is (2).

Quick Tip to Solve: When equally sized groups receive different increments, average those increments: here $(5 + (-3))/2 = 1$. The median identifies the group sizes.

17. Let P be the relation defined on the set of all real numbers such that $P = \{(a, b) : \sec^2 a - \tan^2 b = 1\}$. Then, P is [4 marks]

(1) reflexive and symmetric but not transitive

(2) symmetric and transitive but not reflexive

(3) reflexive and transitive but not symmetric

(4) an equivalence relation

Correct Answer: (2) symmetric and transitive but not reflexive

Solution:

Concept:

- A relation on $\mathbb{R}$ is reflexive only if (a, a) belongs to it for every real number a. Symmetry reverses related pairs; transitivity chains two pairs.
- The identity $\sec^2 t = 1 + \tan^2 t$ holds where the trigonometric functions are defined. Their common real domain is $D = \mathbb{R} \setminus \{\pi/2 + k\pi : k \in \mathbb{Z}\}$.

Step 1: Rewrite the condition on its actual domain For $a, b \in D$, the defining condition is equivalent to

$$1 + \tan^2 a - \tan^2 b = 1 \quad \Longleftrightarrow \quad \tan^2 a = \tan^2 b.$$

This is equality of the values of the function $t \mapsto \tan^2 t$. Thus if $(a, b) \in P$, reversing the equality proves $(b, a) \in P$. If $(a, b) \in P$ and $(b, c) \in P$, then $\tan^2 a = \tan^2 b = \tan^2 c$, so $(a, c) \in P$. These arguments establish symmetry and transitivity.

Step 2: Check reflexivity on the printed underlying set The question specifies all real numbers, not just D. At $a = \pi/2$, both $\sec a$ and $\tan a$ are undefined. Therefore $(\pi/2, \pi/2)$ is not a pair satisfying the defining equality, and reflexivity on $\mathbb{R}$ fails. On the restricted set D, every diagonal pair would satisfy the identity, giving an equivalence relation there. That restricted-domain result cannot establish reflexivity on the larger printed set. The supplied key selects (4), which is valid only after restricting the underlying set to the common domain D.

Step 3: Conclusion The diagonal pair $(\pi/2, \pi/2)$ is absent, while equality of tangent-squares is symmetric and transitive. The literal all-real universe therefore gives a different answer from the supplied key. On the printed set $\mathbb{R}$, P is symmetric and transitive but not reflexive. Hence, the correct option is (2).

Quick Tip to Solve: Before using a trigonometric identity to prove reflexivity, check that the functions exist at every member of the stated underlying set.

18. If B is a 3×3 matrix such that $B^2 = 0$, then $\det\left[(I+B)^{50} - 50B\right]$ is equal to: [4 marks]

(1) 1

(2) 2

(3) 3

(4) 50

Correct Answer: (1) 1

Solution:

Concept:

- The identity matrix I commutes with every square matrix. Consequently the binomial expansion applies to $(I+B)^{50}$.
- If $B^2 = 0$, every higher power B^k with $k \geq 2$ is also the zero matrix, regardless of whether B itself is zero.

Step 1: Expand the matrix power Because the two matrix terms commute, write

$$(I+B)^{50} = I + \binom{50}{1}B + \binom{50}{2}B^2 + \cdots + B^{50}.$$

The first term is $I^{50} = I$, and the linear term is $50I^{49}B = 50B$. For each $k \geq 2$, the factor $B^k = B^{k-2}B^2$ vanishes by the given condition. Thus no higher binomial term survives; there is no need to determine the individual nine entries of B.

Step 2: Subtract before taking the determinant The expression inside the determinant simplifies exactly to

$$(I+B)^{50} - 50B = (I + 50B) - 50B = I.$$

Since B has order 3×3, the identity here is I_3. Its determinant is the product of its three diagonal entries,

$$\det I_3 = 1 \cdot 1 \cdot 1 = 1.$$

The order 3 determines the size of the identity, rather than the numerical value of its determinant. In particular 3 is its trace, not its determinant. Neither the exponent 50 nor the coefficient 50 remains after cancellation, ruling out the other listed values.

Step 3: Conclusion The required determinant equals 1. Hence, the correct option is (1).

Quick Tip to Solve: For $B^2 = 0$, immediately replace $(I+B)^m$ by $I + mB$ for any positive integer m, then simplify the matrix before evaluating its determinant.

19. If a, b, c are non-zero real numbers and if the system of equations

$$(a-1)x = y + z,$$
$$(b-1)y = x + z,$$
$$(c-1)z = x + y$$

has a non-trivial solution, then $ab + bc + ca$ equals: [4 marks]

(1) -1

(2) a + b + c

(3) abc

(4) 1

Correct Answer: (3) abc

Solution:

Concept:

- A non-trivial solution of a homogeneous linear system is a solution vector different from the zero vector. Its existence forces the coefficient matrix to be singular.
- The common sum $s = x + y + z$ converts the printed equations into $ax = s$, $by = s$, and $cz = s$. The non-zero conditions on a, b, c permit division.

Step 1: Introduce the common sum Add x to both sides of the first equation, y to the second, and z to the third. The resulting equations are

$$ax = x + y + z = s, \quad by = s, \quad cz = s.$$

If $s = 0$, non-zero a, b, c would force $x = y = z = 0$, contradicting the stated non-triviality. Hence $s \neq 0$, and it is legitimate to divide the next relation by s. This check prevents accidentally retaining only the trivial solution.

Step 2: Sum the variables and cancel the non-zero sum Express each variable in terms of s:

$$x = \frac{s}{a}, \qquad y = \frac{s}{b}, \qquad z = \frac{s}{c}.$$

Their sum must equal s, giving

$$s = s\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right), \qquad 1 = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

Multiplication by the non-zero product abc yields $abc = bc + ca + ab$. This also agrees with setting the determinant of the original coefficient matrix to zero. The alternatives involving -1, $a + b + c$, or 1 do not express the condition imposed by the system.

Step 3: Conclusion The relation required by a non-trivial solution is $ab + bc + ca = abc$. Hence, the correct option is (3).

Quick Tip to Solve: Whenever three equations can be rewritten as $ax = by = cz = x + y + z$, introduce the common sum and first prove it is non-zero before cancelling it.

20. If $y = e^{nx}$, then $\frac{d^2y}{dx^2} \cdot \frac{d^2x}{dy^2}$ is equal to: [4 marks]

(1) ne^{-nx}

(2) $-ne^{-nx}$

(3) ne^{nx}

(4) 1

Correct Answer: (2) $-ne^{-nx}$

Solution:

Concept:

- For $y = e^{nx}$, differentiation with respect to x gives $dy/dx = ny$. The inverse exists locally when the constant $n \neq 0$.
- To obtain d^2x/dy^2, differentiate the inverse relation with respect to y. Second derivatives of inverse functions are not reciprocals of one another.

Step 1: Differentiate the exponential twice Treat n as constant. Applying the chain rule to the exponential and then differentiating once more gives

$$\frac{dy}{dx} = ne^{nx}, \qquad \frac{d^2y}{dx^2} = n^2e^{nx}.$$

The value $y = e^{nx}$ is positive, so the logarithm used for the inverse is defined for every value of y reached by this function. The use of d^2x/dy^2 presumes an inverse and thus excludes $n = 0$, when y would be the constant 1.

Step 2: Differentiate the inverse and multiply Take logarithms to obtain $x = (\ln y)/n$. Therefore

$$\frac{dx}{dy} = \frac{1}{ny}, \qquad \frac{d^2x}{dy^2} = -\frac{1}{ny^2}.$$

Multiplying the two second derivatives and using $y = e^{nx}$ gives

$$\frac{d^2y}{dx^2}\frac{d^2x}{dy^2} = (n^2y)\left(-\frac{1}{ny^2}\right) = -\frac{n}{y} = -ne^{-nx}.$$

The negative sign arises from differentiating y^{-1}. A positive exponential or a positive sign therefore cannot be correct. The product 1 would follow from a false reciprocal rule for second derivatives.

Step 3: Conclusion The required product is $-ne^{-nx}$, for the invertible case $n \neq 0$. Hence, the correct option is (2).

Quick Tip to Solve: For an inverse function, $d^2x/dy^2 = -y''/(y')^3$. The reciprocal rule applies directly only to first derivatives.

21. If $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$, $x \in \mathbb{R}$, then the equation $f(x) = 0$ has: [4 marks]

(1) No solution

(2) More than two solutions

(3) One solution

(4) Two solutions

Correct Answer: (3) One solution

Solution:

Concept:

- For a base a with $0 < a < 1$, the exponential a^x is continuous and strictly decreasing on all real numbers.
- A continuous function changing sign has a zero between the sign-check points. Strict monotonicity ensures that this zero is unique.

Step 1: Show that there cannot be two roots Both bases $3/5$ and $4/5$ lie strictly between 0 and 1. Differentiating gives

$$f'(x) = \left(\frac{3}{5}\right)^x \ln\frac{3}{5} + \left(\frac{4}{5}\right)^x \ln\frac{4}{5}.$$

Each exponential factor is positive and each logarithm is negative. Hence $f'(x) < 0$ for every real x. Thus f is strictly decreasing: if $x_1 < x_2$, then $f(x_1) > f(x_2)$. Two distinct zeros would contradict this strict inequality.

Step 2: Find a root and combine the two facts The familiar Pythagorean triple $3, 4, 5$ suggests testing $x = 2$. Direct substitution gives

$$f(2) = \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 - 1 = \frac{9}{25} + \frac{16}{25} - 1 = 0.$$

This supplies a real root without a numerical approximation. Strict decrease already proved that there is at most one, so $x = 2$ is the unique solution. Merely spotting this root would not rule out further roots; the monotonicity argument is necessary for choosing between the one-root and multiple-root alternatives.

Step 3: Conclusion The equation has exactly one solution, namely $x = 2$. Hence, the correct option is (3).

Quick Tip to Solve: When the bases are $3/5$ and $4/5$, try exponent 2 using $3^2 + 4^2 = 5^2$; then use monotonicity to establish uniqueness.

22. If Rolle's theorem holds for the function $f(x) = 2x^3 + ax^2 + bx$ in the interval $[-1, 1]$ for the point $c = \frac{1}{2}$, then the value of $2a + b$ is: [4 marks]

(1) -1

(2) 2

(3) 1

(4) -2

Correct Answer: (1) -1

Solution:

Concept:

- Rolle's theorem requires continuity on the closed interval, differentiability on the open interval, and equal values at the endpoints. A polynomial already satisfies the first two requirements.
- The stated Rolle point must satisfy $f\prime(c) = 0$. The endpoint condition determines one coefficient, and the derivative condition determines the other.

Step 1: Apply the endpoint condition The endpoints are -1 and 1, not zero and one. Their function values are

$$f(-1) = -2 + a - b, \qquad f(1) = 2 + a + b.$$

Equating them gives

$$-2 + a - b = 2 + a + b \implies 2b = -4 \implies b = -2.$$

Notice that the even-power term cancels in this comparison, so this equation alone does not determine a.

Step 2: Use the given stationary point Differentiate the polynomial and substitute the specified point $c = \frac{1}{2}$:

$$f\prime(x) = 6x^2 + 2ax + b, \qquad 0 = f\prime(1/2) = \frac{3}{2} + a - 2.$$

Thus $a = \frac{1}{2}$. It follows that

$$2a + b = 2\left(\frac{1}{2}\right) - 2 = -1.$$

As a check, the derivative becomes $6x^2 + x - 2 = (3x + 2)(2x - 1)$, so the required point is indeed a zero of the derivative lying inside the interval. The other stationary point does not change the coefficients or the requested combination. The alternatives $2, 1, -2$ do not equal this combination; -2 is only the value of b.

Step 3: Conclusion The required value is $2a + b = -1$. Hence, the correct option is (1).

Quick Tip to Solve: For a polynomial on a symmetric interval, use the equality of endpoint values before substituting the given Rolle point into the derivative.

23. $\int \dfrac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}\, dx$ **is equal to:** [4 marks]

(1) $-\frac{1}{2}\sin 2x + C$

(2) $-\sin^2 x + C$

(3) $-\frac{1}{2}\sin x + C$

(4) $\frac{1}{2}\sin 2x + C$

Correct Answer: (1) $-\frac{1}{2}\sin 2x + C$

Solution:

Concept:

- Use the difference-of-squares identity repeatedly to reduce the numerator before attempting a substitution.
- The identities $\sin^2 x + \cos^2 x = 1$ and $\cos 2x = \cos^2 x - \sin^2 x$ turn the simplified integrand into a standard trigonometric function.

Step 1: Factor and cancel the common factor Write $u = \sin^2 x$ and $v = \cos^2 x$, so that $u + v = 1$. Then

$$\sin^8 x - \cos^8 x = (u^2 - v^2)(u^2 + v^2) = (u - v)(u + v)(u^2 + v^2).$$

Also,

$$u^2 + v^2 = (u + v)^2 - 2uv = 1 - 2\sin^2 x \cos^2 x.$$

Therefore the quotient in the question is exactly $u - v = \sin^2 x - \cos^2 x = -\cos 2x$. Cancellation is legitimate for every real x: since $0 \le uv \le 1/4$, its denominator is at least $1/2$ and never vanishes.

Step 2: Integrate and check the options The integral is now elementary:

$$I = -\int \cos 2x\, dx = -\frac{1}{2}\sin 2x + C.$$

The factor $1/2$ compensates for the derivative of the inner angle $2x$. Differentiating gives

$$\frac{d}{dx}\left(-\frac{1}{2}\sin 2x\right) = -\cos 2x,$$

exactly the reduced integrand. Option (2) differentiates to $-\sin 2x$, option (3) to $-\frac{1}{2}\cos x$, and option (4) to $\cos 2x$. None of these has the necessary derivative, so neither a sign change nor a missing factor can be absorbed into the arbitrary constant.

Step 3: Conclusion The integral is $-\frac{1}{2}\sin 2x + C$. Hence, the correct option is (1).

Quick Tip to Solve: When eighth powers occur with $1 - 2\sin^2 x \cos^2 x$, factor the numerator until the factor $\sin^4 x + \cos^4 x$ appears.

24. The integral $\int_0^{1/2} \frac{\ln(1+2x)}{1+4x^2}\,dx$ equals: [4 marks]

(1) $\frac{\pi}{4}\ln 2$

(2) $\frac{\pi}{16}\ln 2$

(3) $\frac{\pi}{8}\ln 2$

(4) $\frac{\pi}{32}\ln 2$

Correct Answer: (2) $\frac{\pi}{16}\ln 2$

Solution:

Concept:

- For a denominator $1+4x^2$, use $2x=\tan t$; the differential becomes $dx/(1+4x^2)=dt/2$.
- For a definite integral over $[0,a]$, replacing t by $a-t$ leaves its value unchanged and can simplify a logarithmic sum.

Step 1: Transform the limits with the substitution Let $2x=\tan t$. At $x=0$, $t=0$; at $x=1/2$, $t=\pi/4$. Since $dx=\frac{1}{2}\sec^2 t\,dt$,

$$I=\frac{1}{2}\int_0^{\pi/4}\ln(1+\tan t)\,dt.$$

Denote the integral on the right without its factor $1/2$ by J. The factor from the differential must be retained throughout; losing it would produce twice the correct answer.

Step 2: Pair the reflected logarithms Under $t\mapsto\pi/4-t$,

$$J=\int_0^{\pi/4}\ln\left(1+\frac{1-\tan t}{1+\tan t}\right)dt=\int_0^{\pi/4}\ln\left(\frac{2}{1+\tan t}\right)dt.$$

All logarithmic arguments are positive on this interval. Adding this expression to the original gives

$$2J=\int_0^{\pi/4}\ln 2\,dt=\frac{\pi}{4}\ln 2.$$

Consequently

$$J=\frac{\pi}{8}\ln 2,\qquad I=\frac{J}{2}=\frac{\pi}{16}\ln 2.$$

The coefficient $\pi/8$ corresponds to J, not the requested integral. The remaining coefficients $\pi/4$ and $\pi/32$ likewise fail to retain both factors of two from the substitution and symmetry.

Step 3: Conclusion The value is $\frac{\pi}{16}\ln 2$. Hence, the correct option is (2).

Quick Tip to Solve: After $2x=\tan t$, pair the integrand at t and $\pi/4-t$; their logarithms add to the constant $\ln 2$.

25. Let $A = \{(x, y) : y^2 \le 4x,\ y - 2x \ge -4\}$. The area of the region A, in square units, is: [4 marks]

(1) 10

(2) 8

(3) 9

(4) 11

Correct Answer: (3) 9

Solution:

Concept:

- The inequalities determine horizontal strips: $y^2 \le 4x$ gives $x \ge y^2/4$, while $y - 2x \ge -4$ gives $x \le (y + 4)/2$.
- Area is the integral of right boundary minus left boundary over the full range of the strip variable. Intersection points supply that range.

Step 1: Find the vertical limits of the bounded region The boundaries meet when

$$\frac{y^2}{4} = \frac{y + 4}{2} \quad \Longrightarrow \quad y^2 - 2y - 8 = 0 \quad \Longrightarrow \quad (y - 4)(y + 2) = 0.$$

Thus the limits are $y = -2$ and $y = 4$; the corresponding points are $(1, -2)$ and $(4, 4)$. Between these values, the line is to the right of the parabola, as can be checked at $y = 0$, where the two x-coordinates are 2 and 0. Outside this range the required right endpoint is smaller than the left endpoint, so there are no further points satisfying both inequalities.

Step 2: Integrate the width of each strip The area therefore equals

$$S = \int_{-2}^{4} \left(\frac{y + 4}{2} - \frac{y^2}{4} \right) dy$$
$$= \left[\frac{y^2}{4} + 2y - \frac{y^3}{12} \right]_{-2}^{4}.$$

The upper endpoint gives $4 + 8 - 16/3 = 20/3$, while the lower endpoint gives $1 - 4 + 2/3 = -7/3$. Hence

$$S = \frac{20}{3} - \left(-\frac{7}{3} \right) = 9.$$

This accounts for both the negative and positive portions of the y-range. Values $10, 8, 11$ do not match the complete integral; using only positive y would omit a nonzero part of the region.

Step 3: Conclusion The area is 9 square units. Hence, the correct option is (3).

Quick Tip to Solve: Use horizontal strips here: each boundary is a single expression for x in terms of y, so no split integral is needed.

26. If the differential equation representing the family of all circles touching the x-axis at the origin is $(x^2 - y^2)\frac{dy}{dx} = g(x)y$, then $g(x)$ equals: [4 marks]

(1) $\frac{1}{2}x^2$

(2) $2x$

(3) $\frac{1}{2}x$

(4) $2x^2$

Correct Answer: (2) $2x$

Solution:

Concept:

- A circle tangent to the x-axis at the origin has its centre on the y-axis, say $(0, a)$, and radius $|a|$.
- To obtain the differential equation of a one-parameter family, differentiate once and eliminate the parameter using the original equation.

Step 1: Write and differentiate the circle equation The geometric condition gives

$$x^2 + (y - a)^2 = a^2 \quad \Longrightarrow \quad x^2 + y^2 = 2ay.$$

This allows centres above or below the axis; the same family equation covers both signs of a. Differentiation with respect to x, treating a as constant for each individual circle, gives

$$2x + 2y\frac{dy}{dx} = 2a\frac{dy}{dx}.$$

It is essential to differentiate y^2 with the chain rule, since y depends on x.

Step 2: Eliminate the centre coordinate Multiply the differentiated equation by y and use $2ay = x^2 + y^2$:

$$2xy + 2y^2\frac{dy}{dx} = (x^2 + y^2)\frac{dy}{dx}.$$

Rearranging,

$$(x^2 - y^2)\frac{dy}{dx} = 2xy.$$

This form eliminates the parameter without dividing by y, so it retains the algebraic relation at the origin as well. Comparing directly with $(x^2 - y^2)y' = g(x)y$ gives $g(x) = 2x$. The choices containing x^2 would give a different power of x on the right side and cannot represent this family. The choice $x/2$ has the correct power but an incorrect coefficient, arising from a lost differentiation factor.

Step 3: Conclusion The required function is $g(x) = 2x$. Hence, the correct option is (2).

Quick Tip to Solve: For tangency at the origin, place the centre on the normal to the tangent axis, then eliminate its single coordinate after differentiating.

27. If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $|2\vec{a} - \vec{b}| = 5$, then $|2\vec{a} + \vec{b}|$ equals: [4 marks]

(1) 5

(2) 7

(3) 17

(4) 1

Correct Answer: (1) 5

Solution:

Concept:

- The identity $|\vec{u} \pm \vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 \pm 2\vec{u} \cdot \vec{v}$ relates vector magnitudes to a scalar product.
- Adding the plus and minus identities eliminates the scalar product: $|\vec{u}+\vec{v}|^2+|\vec{u}-\vec{v}|^2 = 2(|\vec{u}|^2+|\vec{v}|^2)$.

Step 1: Apply the parallelogram identity to the correct vectors Take $\vec{u} = 2\vec{a}$ and $\vec{v} = \vec{b}$. The scaling factor must be included before squaring:

$$|\vec{u}| = 2|\vec{a}| = 4, \qquad |\vec{v}| = 3, \qquad |\vec{u} - \vec{v}| = 5.$$

Thus

$$|\vec{u} + \vec{v}|^2 + 25 = 2(4^2 + 3^2) = 50,$$

and therefore $|\vec{u} + \vec{v}|^2 = 25$. This calculation uses magnitudes rather than assuming the vectors are collinear.

Step 2: Take the magnitude and verify the geometry A magnitude is nonnegative, so

$$|2\vec{a} + \vec{b}| = \sqrt{25} = 5.$$

Independently, expanding the given difference shows

$$25 = 4|\vec{a}|^2 + |\vec{b}|^2 - 4\vec{a} \cdot \vec{b} = 16 + 9 - 4\vec{a} \cdot \vec{b},$$

hence $\vec{a} \cdot \vec{b} = 0$. The vectors are perpendicular, so the sum and difference indeed have the same magnitude. The alternative 7 would require parallel vectors in the same direction; 1 would require opposite directions. Neither is consistent with the specified difference of magnitude 5. The value 17 even exceeds the maximum $4 + 3 = 7$ allowed by the triangle inequality.

Step 3: Conclusion The magnitude $|2\vec{a} + \vec{b}|$ is 5. Hence, the correct option is (1).

Quick Tip to Solve: When the magnitude of a vector difference is given, add the squared sum and difference identities to eliminate the unknown angle immediately.

28. Equation of the plane which passes through the point of intersection of the lines $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$, and has the largest distance from the origin, is: [4 marks]

(1) 4x + 3y + 5z = 50

(2) 3x + 4y + 5z = 49

(3) 5x + 4y + 3z = 57

(4) 7x + 2y + 4z = 54

Correct Answer: (1) 4x + 3y + 5z = 50

Solution:

Concept:

- For every plane passing through a fixed point P, the distance of the plane from the origin is at most $|OP|$. Equality holds when OP is normal to the plane.
- Parametric forms of the two lines determine their intersection. The coordinates of that intersection then provide the normal vector of the required plane.

Step 1: Find the common point of the two lines For the first line write

$$x = 1 + 3t, \quad y = 2 + t, \quad z = 3 + 2t.$$

For the second write

$$x = 3 + s, \quad y = 1 + 2s, \quad z = 2 + 3s.$$

Equating the first two coordinates gives $3t - s = 2$ and $t - 2s = -1$. Solving yields $t = s = 1$. The third coordinate also agrees, since both give $z = 5$. Thus their intersection is $P = (4, 3, 5)$, and $|OP| = \sqrt{4^2 + 3^2 + 5^2} = \sqrt{50}$.

Step 2: Choose the orientation that maximises the distance A plane through P with nonzero normal $\vec{n}$ has equation $\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{OP}$. Its distance from the origin is

$$d = \frac{|\vec{n} \cdot \vec{OP}|}{|\vec{n}|} \leq |OP|$$

by the scalar-product inequality. Equality is attained when $\vec{n}$ is parallel to $(4, 3, 5)$. Therefore the plane is

$$4(x - 4) + 3(y - 3) + 5(z - 5) = 0,$$

or $4x+3y+5z = 50$. Every listed plane passes through P, but the other normals $(3, 4, 5), (5, 4, 3), (7, 2, 4)$ are not parallel to OP. Their distances are strictly smaller; incidence alone cannot select the required option.

Step 3: Conclusion The plane is $4x + 3y + 5z = 50$. Hence, the correct option is (1).

Quick Tip to Solve: The farthest plane through a fixed point is perpendicular to the line joining that point to the origin; use the point itself as the plane normal.

29. A line in the 3-dimensional space makes an angle θ, $0 < \theta \leq \pi/2$, with both the X- and Y-axes. Then, the set of all values of θ is in the interval: [4 marks]

(1) $(\pi/3, \pi/2]$

(2) $(0, \pi/4]$

(3) $[\pi/4, \pi/2]$

(4) $[\pi/6, \pi/3]$

Correct Answer: (3) $[\pi/4, \pi/2]$

Solution:

Concept:

- The direction cosines l, m, n of any line satisfy $l^2 + m^2 + n^2 = 1$. They are the cosines of its angles with the positive coordinate axes.
- A real direction cosine has a nonnegative square. Conversely, any real triple whose squares sum to one determines a direction in space.

Step 1: Use the two equal angles The angle with each of the first two axes is θ, so $l = m = \cos\theta$. The angle with the third axis is unrestricted and must simply give a real value of n. Substitution into the direction-cosine identity gives

$$2\cos^2\theta + n^2 = 1, \qquad n^2 = 1 - 2\cos^2\theta.$$

Step 2: Impose reality and check the endpoints Therefore $2\cos^2\theta \leq 1$. On the stated interval $0 < \theta \leq \pi/2$, cosine is nonnegative and decreases monotonically; hence $\cos\theta \leq 1/\sqrt{2}$ means $\theta \geq \pi/4$. At $\theta = \pi/4$, choose $n = 0$; at $\theta = \pi/2$, choose $n = \pm 1$. Both endpoints are possible. For every intermediate angle, $n = \pm\sqrt{1 - 2\cos^2\theta}$ is real, so there are no further restrictions. Option 1 omits valid angles between $\pi/4$ and $\pi/3$; 2 includes impossible smaller angles; 4 both includes impossible angles and excludes possible ones.

Step 3: Conclusion The complete range is $\boxed{\pi/4 \leq \theta \leq \pi/2}$. Hence, the correct option is (3).

Quick Tip to Solve: When two direction angles are equal, replace their two squared direction cosines by $2\cos^2\theta$ and require the remaining square to be nonnegative.

30. If Aand B are two events such that P(A ∪B) = P(A ∩B), then the incorrect statement amongst the following statements is [4 marks]

(1) P(A) + P(B) = 1

(2) P(A ∩B′) = 0

(3) A & B are equally likely

(4) P(A′∩B) = 0

Correct Answer: (1) P(A) + P(B) = 1

Solution:

Concept:

- For two events, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. The disjoint pieces $A \cap B$, $A \cap B'$, and $A' \cap B$ partition their union.
- Probabilities are nonnegative. A sum of two such numbers is zero only when each number is zero; events of zero probability need not be empty events.

Step 1: Separate the common and exclusive pieces Write $p = P(A \cap B)$, $x = P(A \cap B')$, and $y = P(A' \cap B)$. The stated equality becomes $p + x + y = p$. Thus $x + y = 0$, and nonnegativity gives $x = y = 0$. This proves statements 2 and 4, without making any unsupported assumption that the events themselves must be identical.

Step 2: Compare the individual event probabilities Using the same disjoint decompositions, $P(A) = p + x = p$ and $P(B) = p + y = p$. Consequently A and B are equally likely, as stated in statement 3. However, their sum is $2p$, and the hypothesis imposes no condition $p = 1/2$. For a concrete counterexample, take A and B to be the same event of probability $1/4$. The union and intersection then both have probability $1/4$, whereas $P(A) + P(B) = 1/2$, not 1. The word incorrect asks for the statement that is not forced by the given condition; Statement 1 could hold for a special choice but is not generally true.

Step 3: Conclusion The sum of the individual event probabilities need not equal one. Hence, the correct option is (1).

Quick Tip to Solve: For an equality between union and intersection probabilities, split the union into its common part and its two exclusive parts; the exclusive probabilities must vanish.

Physics (120 marks)

31. An experiment is performed to obtain the value of acceleration due to gravity g by using a simple pendulum of length L. In this experiment time for 100 oscillations is measured by using a watch of 1 second least count and the value is 90.0 seconds. The length L is measured by using a meter scale of least count 1 mm and the value is 20.0 cm. The error in the determination of g would be [4 marks]

(1) 4.4%

(2) 2.27%

(3) 1.7%

(4) 2.7%

Correct Answer: (4) 2.7%

Solution:

Concept:

- A simple pendulum obeys $T = 2\pi\sqrt{L/g}$, so $g = 4\pi^2 L/T^2$. For N oscillations timed together, $T = t/N$.
- For maximum fractional errors in a product or quotient, add absolute fractional contributions multiplied by their powers: $\Delta g/g = \Delta L/L + 2\Delta t/t$, taking the counted number N as exact.

Step 1: Find each relative instrumental error The length is $L = 20.0\,\text{cm} = 200\,\text{mm}$ and the meter scale least count is $\Delta L = 1\,\text{mm}$. Thus $\Delta L/L = 1/200 = 0.005$. The measured total time is $t = 90.0\,\text{s}$, with $\Delta t = 1\,\text{s}$, giving $\Delta t/t = 1/90$. Timing 100 oscillations improves the relative error compared with timing one; it does not allow the least count to be divided by 100 while retaining the total time in the denominator.

Step 2: Add the contributions and compare the options Because time appears squared in the expression for g, its fractional error is doubled. Therefore

$$\frac{\Delta g}{g} = \frac{1}{200} + \frac{2}{90} = 0.02722\ldots.$$

The percentage error is approximately 2.722%, which rounds to 2.7% among the choices. The value 2.27% does not follow from the stated least counts; 1.7% would underestimate the squared time contribution, and 4.4% is too large. This is the usual limiting-error calculation using the least count as the instrument error, rather than a statistical quadrature estimate.

Step 3: Conclusion The percentage error in g is $\boxed{2.7\%}$. Hence, the correct option is (4).

Quick Tip to Solve: Use the fractional error of the total measured time; dividing both time and its uncertainty by the oscillation count leaves that fraction unchanged.

32. The position of a projectile launched from the origin at $t = 0$ is given by $\vec{r} = (40\hat{i} + 50\hat{j})\,\text{m}$ at $t = 2\,\text{s}$. If the projectile was launched at an angle θ from the horizontal, then θ is (take $g = 10\,\text{m s}^{-2}$). [4 marks]

(1) $\tan^{-1}(3/2)$

(2) $\tan^{-1}(2/3)$

(3) $\tan^{-1}(7/4)$

(4) $\tan^{-1}(4/5)$

Correct Answer: (3) $\tan^{-1}(7/4)$

Solution:

Concept:

- With horizontal initial speed u_x and vertical initial speed u_y, projectile coordinates are $x = u_x t$ and $y = u_y t - \frac{1}{2}gt^2$, neglecting air resistance.
- The launch angle satisfies $\tan\theta = u_y/u_x$. The position-vector slope y/x describes the line joining the origin to the current point, not the initial velocity direction.

Step 1: Recover the horizontal component At $t = 2\,\text{s}$, the vector gives $x = 40\,\text{m}$ and $y = 50\,\text{m}$. There is no horizontal acceleration, so

$$u_x = \frac{x}{t} = \frac{40}{2} = 20\,\text{m s}^{-1}.$$

The axes in the position vector distinguish horizontal and vertical components; the two numbers must not be interchanged.

Step 2: Undo the gravitational displacement vertically In two seconds, gravity subtracts $\frac{1}{2}gt^2 = \frac{1}{2}(10)(2)^2 = 20\,\text{m}$ from the vertical displacement. Thus the initial vertical motion would have covered $50 + 20 = 70\,\text{m}$, giving

$$u_y = \frac{50 + 20}{2} = 35\,\text{m s}^{-1}, \qquad \tan\theta = \frac{35}{20} = \frac{7}{4}.$$

Both components are positive, so the angle is the acute inverse tangent of this ratio. Ratios $3/2$ and $2/3$ do not recover the observed vertical position. The ratio $4/5$ reverses the instantaneous position-coordinate ratio and also ignores gravity. Substitution of the recovered components gives $y = 35(2) - 20 = 50\,\text{m}$, confirming the result.

Step 3: Conclusion The launch angle is $\boxed{\theta = \tan^{-1}(7/4)}$. Hence, the correct option is (3).

Quick Tip to Solve: Before dividing vertical by horizontal displacement to get a launch angle, add $gt^2/2$ back to the vertical displacement.

33. Water is flowing at a speed of $1.5\,\mathrm{m\,s^{-1}}$ through a horizontal tube of cross-sectional area $10^{-2}\,\mathrm{m^2}$ and you are trying to stop the flow by your palm. Assuming that the water stops immediately after hitting the palm, the minimum force that you must exert should be (density of water $= 10^3\,\mathrm{kg\,m^{-3}}$):

[4 marks]

(1) $33.7\,\mathrm{N}$

(2) $45\,\mathrm{N}$

(3) $15\,\mathrm{N}$

(4) $22.5\,\mathrm{N}$

Correct Answer: (4) $22.5\,\mathrm{N}$

Solution:

Concept:

- Newton's second law for a continuous stream is force equal to the rate of change of momentum. The mass crossing tube area A per unit time is $\dot{m} = \rho A v$.
- When a stream stops instead of rebounding, each kilogram loses momentum v in the incident direction. The force magnitude is therefore $F = \dot{m}v = \rho A v^2$.

Step 1: Calculate the mass arriving each second The water has density $\rho = 10^3\,\mathrm{kg\,m^{-3}}$, area $A = 10^{-2}\,\mathrm{m^2}$, and speed $v = 1.5\,\mathrm{m\,s^{-1}}$. In a small time Δt, a length $v\Delta t$ reaches the palm and its mass is $\rho A v \Delta t$. Hence

$$\dot{m} = (10^3)(10^{-2})(1.5) = 15\,\mathrm{kg\,s^{-1}}.$$

The number 15 is the mass flow rate, not yet the force.

Step 2: Use the complete loss of forward momentum After contact, the stated forward velocity is zero, so the force exerted by the palm on the water opposes the flow and has magnitude

$$F = \dot{m}(v - 0) = 15(1.5) = 22.5\,\mathrm{N}.$$

By Newton's third law, the water pushes the palm with an equal force in the forward direction. The required holding force must balance it. A value $45\,\mathrm{N}$ would correspond to reversal at the same speed, giving twice the momentum change; the question specifies stopping. Neither $15\,\mathrm{N}$ nor $33.7\,\mathrm{N}$ equals the momentum flux for the given speed and area. The units also check: $(\mathrm{kg\,s^{-1}})(\mathrm{m\,s^{-1}}) = \mathrm{N}$.

Step 3: Conclusion The minimum force magnitude is $\boxed{22.5\,\mathrm{N}}$, opposite to the incoming flow. Hence, the correct option is (4).

Quick Tip to Solve: For a jet striking a fixed surface, first calculate mass per second and then multiply by the change in velocity; stopping and reversing give different forces.

34. A block A of mass 4 kg is placed on another block B of mass 5 kg, and the block B rests on a smooth horizontal table. If the minimum force that can be applied on A so that both the blocks move together is 12 N, the maximum force that can be applied on B for the blocks to move together will be [4 marks]

(1) 0 N

(2) 25 N

(3) 48 N

(4) 27 N

Correct Answer: The stated nonzero minimum force is incompatible with static friction; the maximum force on B cannot be determined from this premise.

Solution:

Concept:

- Static friction supplies whatever tangential force is required up to $f_{\max} = \mu_s m_A g$; it is not necessarily at its maximum during common motion.
- On a smooth horizontal table the two-block system has acceleration $a = F/(m_A + m_B) = F/9$. Common motion is possible for arbitrarily small positive horizontal forces.

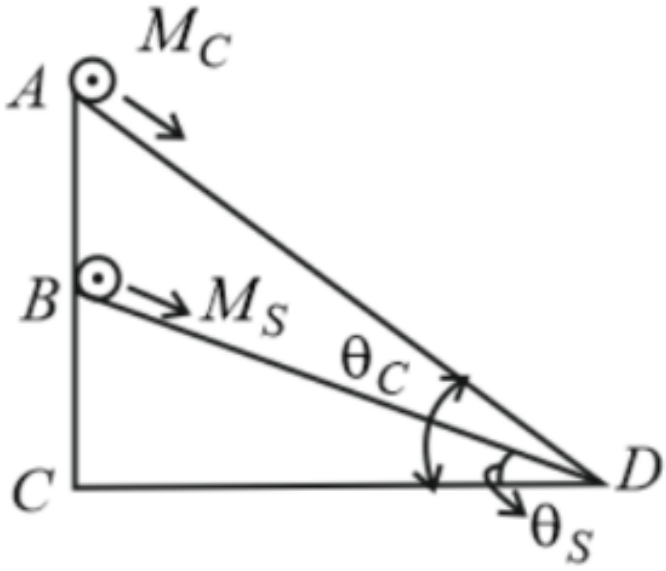
Step 1: Check the stated minimum against the force equations When a horizontal force F is applied on the upper block A, friction alone accelerates the lower block B. Hence $f = m_B a = 5F/9$. The no-slip condition is $5F/9 \le f_{\max}$, or $F \le 9f_{\max}/5$. This gives an upper threshold, not a positive lower threshold. For every sufficiently small positive force both blocks move together; zero force leaves them together at rest. Thus a claimed minimum of $12\,\mathrm{N}$ cannot specify the friction coefficient in this model.

Step 2: Test whether a maximum on B can nevertheless be inferred If force is applied to B, friction accelerates A, giving $f = 4F/9$. Thus the maximum force on B would be $F_{B,\max} = 9f_{\max}/4$, which requires knowing $f_{\max}$. The stated minimum does not provide that information. Even replacing the word minimum by maximum without authority would give $f_{\max} = 5(12)/9$ and $F_{B,\max} = 15\,\mathrm{N}$, absent from the options. Option 1, zero, describes the infimum of the initially applied force, not the requested maximum on B. Values 25, 48 and 27 N likewise are not determined by the literal premise.

Step 3: Conclusion The question as printed has no uniquely defensible option: its nonzero minimum-force premise is inconsistent with common motion governed by static friction, and the requested maximum force on B is undetermined.

Quick Tip to Solve: For static-friction common-motion problems, write the no-slip inequality first; it usually establishes a maximum applied force, so do not turn a stated minimum into a threshold without checking.

35. A cylinder of mass M_c and sphere of mass M_s are placed at points A and B of two inclines, respectively (see figure). If they roll on the incline without slipping such that their accelerations are the same, then the ratio $\sin\theta_c/\sin\theta_s$ is: [4 marks]



(1) $8/7$

(2) $\sqrt{8/7}$

(3) $\sqrt{15/14}$

(4) $15/14$

Correct Answer: (4) $15/14$

Solution:

Concept:

- For a body rolling without slipping down a fixed incline, $a = g\sin\theta/(1 + I/(MR^2))$. This follows from translational motion and the torque of static friction together.
- For a uniform solid cylinder, $I_c = \frac{1}{2}M_cR_c^2$; for a uniform solid sphere, $I_s = \frac{2}{5}M_sR_s^2$. Their masses and radii cancel from their respective acceleration formulae.

Step 1: Obtain the acceleration on each incline For either rolling body, write $Mg\sin\theta - f = Ma$, $fR = I\alpha$, and the no-slip relation $a = \alpha R$. Eliminating friction gives $Mg\sin\theta = (M + I/R^2)a$. Substituting the solid-cylinder and solid-sphere moments gives

$$a_c = \frac{2}{3}g\sin\theta_c, \qquad a_s = \frac{5}{7}g\sin\theta_s.$$

The angles are measured from the same horizontal base, as indicated in the figure; using the angle between the two ramps instead would be incorrect.

Step 2: Equate the accelerations and simplify The stated condition is equality of translational accelerations down the respective inclines, so

$$\frac{2}{3}g\sin\theta_c = \frac{5}{7}g\sin\theta_s, \qquad \frac{\sin\theta_c}{\sin\theta_s} = \frac{5}{7}\frac{3}{2} = \frac{15}{14}.$$

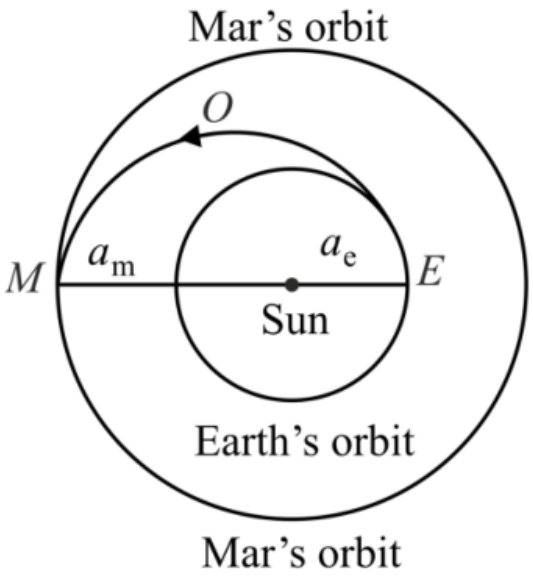
The cylinder has the larger ratio of rotational inertia to MR^2, so at an equal slope it accelerates less than the sphere. It must therefore have the steeper incline, consistent with a sine ratio slightly greater than one. The square-root options would arise from confusing acceleration with a speed or energy comparison; no square root occurs when equal accelerations are imposed. The value $8/7$ does not use the cylinder's correct inertia factor.

Step 3: Conclusion The required ratio is $\boxed{\sin\theta_c/\sin\theta_s = 15/14}$. Hence, the correct option is (4).

Quick Tip to Solve: For equal rolling accelerations, the ratio of slope sines equals the ratio of $1 + I/(MR^2)$; masses cancel before any numerical substitution.

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36. India's Mangalyan was sent to the Mars by launching it into a transfer orbit EOM around the sun. It leaves the earth at E and meets Mars at M. If the semi-major axis of Earth's orbit is $a_e = 1.5 \times 10^{11}$ m, that of Mar's orbit $a_m = 2.28 \times 10^{11}$ m, taking Kepler's laws, give the estimate of time for Mangalyan to reach Mars from Earth. [4 marks]



(1) 220 days

(2) 500 days

(3) 260 days

(4) 320 days

Correct Answer: (3) 260 days

Solution:

Concept:

- For orbits around the same Sun, Kepler's third law is $T^2/a^3 =$ constant, where T is the full orbital period and a the semi-major axis.
- The transfer ellipse touches the Earth orbit at perihelion and the Mars orbit at aphelion. Travel from one end to the other takes half its orbital period.

Step 1: Find the transfer semi-major axis The two apsidal distances add to the major axis of the ellipse. Hence

$$a_t = \frac{a_e + a_m}{2} = \frac{1.50 + 2.28}{2} \times 10^{11} = 1.89 \times 10^{11}\ \text{m}.$$

The spacecraft travels along half of this ellipse, rather than around a circular orbit of radius equal to the Earth–Mars separation. Using the difference of the radii would therefore give the wrong orbital scale.

Step 2: Apply the period ratio Take the Earth period as $T_e = 365$ days. The solar mass and gravitational constant cancel when the two Kepler relations are divided:

$$\frac{T_t}{T_e} = \left(\frac{a_t}{a_e}\right)^{3/2} = \left(\frac{1.89}{1.50}\right)^{3/2} = 1.26^{3/2} \simeq 1.414.$$

Thus the full transfer period is approximately 516 days, and the required flight time is

$$t = \frac{T_t}{2} \simeq 258\,\text{days}.$$

The 500-day option is close to the full period and misses the half-orbit condition. Neither 220 nor 320 days follows from the given semi-major axes.

Step 3: Conclusion The estimated Earth-to-Mars transfer time is about 260 days. Hence, the correct option is (3).

Quick Tip to Solve: For a tangent Earth–Mars transfer, average the two orbital radii first, apply $T \propto a^{3/2}$, and finally halve the period.

37. In materials like aluminium and copper, the correct order of magnitude of various elastic modulii is [4 marks]

(1) Young's moduli $<$ shear moduli $<$ bulk moduli

(2) Bulk moduli $<$ shear moduli $<$ Young's moduli

(3) Shear moduli $<$ Young's moduli $<$ bulk moduli

(4) Bulk moduli $<$ Young's moduli $<$ shear moduli

Correct Answer: (3) Shear moduli $<$ Young's moduli $<$ bulk moduli

Solution:

Concept:

- For an isotropic elastic solid, Young modulus Y, shear modulus G and bulk modulus K satisfy $Y = 2G(1+\nu) = 3K(1-2\nu)$.
- The Poisson ratio ν of aluminium and copper is about one third; these metals have $\nu > 1/3$. This material information determines the comparison of bulk and Young moduli.

Step 1: Compare shear and Young moduli Poisson ratio is positive for ordinary aluminium and copper,

so $2(1+\nu) > 2$. Therefore

$$\frac{Y}{G} = 2(1+\nu) > 2, \qquad G < Y.$$

For example, at $\nu = 1/3$, the ratio is $Y/G = 8/3$. A material resists volume change and longitudinal extension differently from a pure change of shape, so one cannot simply equate the three elastic constants. Options placing Young modulus below shear modulus are inconsistent with this relation.

Step 2: Compare bulk and Young moduli Use the second isotropic-elasticity identity:

$$\frac{Y}{K} = 3(1-2\nu).$$

For these metals, $\nu > 1/3$, so $3(1-2\nu) < 1$; more specifically their Poisson ratios are slightly greater than $1/3$, giving Y/K close to 1 and slightly below it. Aluminium has approximately $G = 26$, $Y = 70$, $K = 76$ GPa; copper similarly has approximately $G = 45$, $Y = 120$, $K = 140$ GPa. Thus both named metals satisfy $G < Y < K$. The alternatives that put bulk modulus first reverse this observed ordering. The result is a property of the named metals, rather than a universal ordering for every solid.

Step 3: Conclusion Shear modulus is smaller than Young modulus, which is smaller than bulk modulus. Hence, the correct option is (3).

Quick Tip to Solve: For aluminium and copper remember $G < Y < K$; the relations with Poisson ratio explain why the ordering is material dependent.

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38. A capillary tube is immersed vertically in water and the height of the water column is x. When this arrangement is taken into a mine of depth d, the height of the water column is y. If R is the radius of earth, the ratio x/y is: [4 marks]

(1) $\dfrac{R-d}{R+d}$

(2) $\dfrac{R+d}{R-d}$

(3) $1-\dfrac{2d}{R}$

(4) $1-\dfrac{d}{R}$

Correct Answer: (4) $1-\dfrac{d}{R}$

Solution:

Concept:

- Capillary rise is $h = 2\gamma\cos\theta/(\rho r g)$, where γ is surface tension, θ contact angle and r the capillary radius.
- In the usual uniform-density Earth approximation, gravity at depth d is $g_d = g(1-d/R)$. The liquid and capillary properties remain the same.

Step 1: Relate rise to gravity The upward force due to surface tension is $2\pi r\gamma\cos\theta$, while the water-column weight is $\rho\pi r^2 hg$. Balancing them gives the capillary-rise expression. At the surface

and in the mine respectively,

$$x = \frac{2\gamma\cos\theta}{\rho r g}, \qquad y = \frac{2\gamma\cos\theta}{\rho r g_d}.$$

The same liquid and tube are used, so all factors other than gravitational acceleration cancel in their ratio. In particular, the ratio is not obtained by taking the ratio of distances from the centre directly without considering gravity.

Step 2: Use gravity at the mine For a spherical Earth with the standard uniform-density approximation, the mass inside radius $R - d$ scales as $(R - d)^3$. The inverse-square field of that enclosed mass consequently scales as $R - d$:

$$\frac{g_d}{g} = \frac{R - d}{R}.$$

Divide the two rise expressions carefully in the requested order:

$$\frac{x}{y} = \frac{g_d}{g} = 1 - \frac{d}{R}.$$

The result is less than one because weaker gravity produces a larger capillary rise underground. The $2d/R$ expression belongs to the first-order change of gravity with height above Earth, and the ratios involving $R + d$ do not describe the interior linear law.

Step 3: Conclusion $x/y = 1 - d/R$. Hence, the correct option is (4).

Quick Tip to Solve: Capillary height varies inversely with gravity. At depth gravity decreases linearly, so divide surface height by mine height as g_d/g.

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39. Water of volume $2\,\mathrm{L}$ in a closed container is heated with a coil of $1\,\mathrm{kW}$. While water is heated, the container loses energy at a rate of $160\,\mathrm{J/s}$. In how much time will the temperature of water rise from $27^\circ\mathrm{C}$ to $77^\circ\mathrm{C}$? (Specific heat of water is $4.2\,\mathrm{kJ/kg}$ and that of the container is negligible.) [4 marks]

(1) 8 min 20 s

(2) 7 min

(3) 14 min

(4) 6 min 2 s

Correct Answer: (1) 8 min 20 s

Solution:

Concept:

- The heat required for a temperature rise is $Q = mc\Delta T$, with m the water mass and c its specific heat capacity.
- The useful heating power is input power minus heat-loss rate. With constant rates, the heating time is $t = Q/P_{\text{net}}$.

Step 1: Calculate the required heat Two litres of water have mass approximately $2\,\mathrm{kg}$, using water

density $1\,\mathrm{kg/L}$. The temperature change is

$$\Delta T = 77 - 27 = 50\,\mathrm{K}.$$

Temperature differences have the same numerical value in kelvin and degrees Celsius. Use the quoted specific heat as $4.2 \times 10^3\,\mathrm{J\,kg^{-1}\,K^{-1}}$, so

$$Q = 2 \times 4.2 \times 10^3 \times 50 = 4.20 \times 10^5\,\mathrm{J}.$$

The container contribution is negligible as stated; the problem describes heating liquid water and gives no phase change, so no latent-heat term is included.

Step 2: Subtract losses and obtain the time The electrical coil supplies $1000\,\mathrm{J/s}$, but $160\,\mathrm{J/s}$ escapes. The energy balance over a time t is

$$(1000 - 160)t = Q, \qquad P_{\text{net}} = 840\,\mathrm{W}.$$

Hence

$$t = \frac{420000}{840} = 500\,\mathrm{s} = 8\,\mathrm{min}\ 20\,\mathrm{s}.$$

Ignoring heat loss would give $420\,\mathrm{s} = 7$ minutes, explaining option (2). Adding the loss to the input would incorrectly make the water heat faster. The other suggested durations do not deliver the required heat at the available net power.

Step 3: Conclusion The water requires $500\,\mathrm{s}$, or $8\,\mathrm{min}\ 20\,\mathrm{s}$. Hence, the correct option is (1).

Quick Tip to Solve: Convert litres to kilograms, find $mc\Delta T$, then divide by useful power after subtracting the continuous heat loss.

40. The equation of state for a gas is given by $PV = nRT + \alpha V$, where n is the number of moles and α is a positive constant. The initial temperature and pressure of one mole of the gas contained in a cylinder are T_0 and P_0 respectively. The work done by the gas when its temperature doubles isobarically will be [4 marks]

(1) $P_0T_0R\ln 2$
(2) $\dfrac{P_0T_0R}{P_0+\alpha}$
(3) $\dfrac{P_0T_0R}{P_0-\alpha}$
(4) P_0T_0R

Correct Answer: (3) $\dfrac{P_0T_0R}{P_0-\alpha}$

Solution:

Concept:

- The supplied gas law is $(P-\alpha)V = nRT$; it must be used instead of the ideal-gas law $PV = nRT$.
- For an isobaric process, the gas work is $W = \int P\,dV = P_0(V_f - V_i)$. The volume is positive for the

stated positive temperature only when $P_0 > \alpha$.

Step 1: Find the initial and final volumes Here $n = 1$ mole and the pressure remains P_0. Rearranging the equation of state at the initial temperature gives

$$V_i = \frac{RT_0}{P_0 - \alpha}.$$

When temperature doubles to $2T_0$, the pressure and α are unchanged, hence

$$V_f = \frac{2RT_0}{P_0 - \alpha} = 2V_i.$$

This volume doubling follows because the effective coefficient $P_0 - \alpha$ is constant along this particular path. It does not permit replacing the actual mechanical pressure by that coefficient in the work integral.

Step 2: Integrate work at the actual pressure The volume change is

$$\Delta V = V_f - V_i = \frac{RT_0}{P_0 - \alpha}.$$

The gas expands against the constant pressure P_0, so its work is positive:

$$W = P_0 \Delta V = \frac{P_0 RT_0}{P_0 - \alpha}.$$

The minus sign in the denominator comes directly from moving αV to the left of the supplied equation. A logarithmic expression would correspond to an isothermal pressure-varying path, not the present isobaric one. In the limit $\alpha \to 0$, this result becomes RT_0, the expected ideal-gas isobaric work for doubling the temperature. The expression $P_0 RT_0$ also has an extra pressure factor and cannot represent work by itself.

Step 3: Conclusion $W = \frac{P_0 T_0 R}{P_0 - \alpha}$. Hence, the correct option is (3).

Quick Tip to Solve: Rearrange the unusual gas law first, but use the actual constant pressure P_0, rather than $P_0 - \alpha$, when calculating work.

41. Modern vacuum pumps can evacuate a vessel down to a pressure of $4.0 \times 10^{-15}\,\text{atm}$ at room temperature ($300\,\text{K}$). Taking $R = 8.3\,\text{J K}^{-1}\,\text{mol}^{-1}$, $1\,\text{atm} = 10^5\,\text{Pa}$ and $N_{\text{Avogadro}} = 6 \times 10^{23}\,\text{mol}^{-1}$, the mean distance between molecules of gas in an evacuated vessel will be of the order of [4 marks]

(1) $0.2\,\mu\text{m}$

(2) $0.2\,\text{mm}$

(3) $0.2\,\text{cm}$

(4) $0.2\,\text{nm}$

Correct Answer: (2) $0.2\,\text{mm}$

Solution:

Concept:

- For a dilute gas, $PV = nRT$ and $N = nN_A$, giving molecular number density $N/V = PN_A/(RT)$.
- The characteristic intermolecular separation is the cube root of volume available per molecule: $\ell \sim (V/N)^{1/3}$. It is distinct from the mean free path.

Step 1: Calculate number density in SI units Convert the residual pressure before substituting:

$$P = 4.0 \times 10^{-15} \times 10^5 = 4.0 \times 10^{-10}\,\text{Pa}.$$

The gas is extremely dilute, so the ideal-gas relation gives

$$\frac{N}{V} = \frac{(4.0 \times 10^{-10})(6 \times 10^{23})}{8.3 \times 300} \simeq 9.64 \times 10^{10}\,\text{m}^{-3}.$$

Avogadro constant converts the molar ideal-gas relation into a molecular count. Omitting this conversion would give a volume per mole instead of a volume per molecule, and therefore an entirely different length scale.

Step 2: Take the cube root of volume per molecule The typical volume assigned to one molecule is

$$\frac{V}{N} \simeq 1.04 \times 10^{-11}\,\text{m}^3.$$

Treating this as a cube of side ℓ estimates the mean separation:

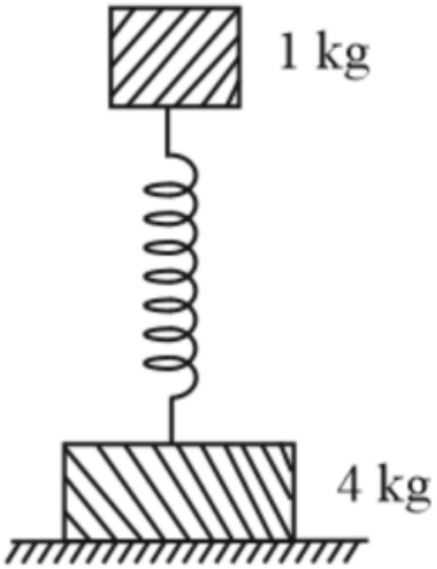
$$\ell \sim \left(1.04 \times 10^{-11}\right)^{1/3} \simeq 2.18 \times 10^{-4}\,\text{m} = 0.218\,\text{mm}.$$

This is an order-of-magnitude estimate, so no microscopic packing arrangement is required. The micrometre and nanometre choices are much too small for this extremely low number density; the centimetre choice is ten times larger than the calculated separation. A mean-free-path calculation would require a collision cross section, which is neither given nor needed here.

Step 3: Conclusion The separation is of the order of $0.2\,\text{mm}$. Hence, the correct option is (2).

Quick Tip to Solve: For intermolecular spacing use $(RT/PN_A)^{1/3}$. Convert atm to pascals before taking the cube root.

42. Two bodies of masses $1\,\text{kg}$ and $4\,\text{kg}$ are connected to a vertical spring, as shown in the figure. The smaller mass executes simple harmonic motion of angular frequency $25\,\text{rad}\,\text{s}^{-1}$, and amplitude $1.6\,\text{cm}$ while the bigger mass remains stationary on the ground. The maximum force exerted by the system on the floor is (take $g = 10\,\text{m}\,\text{s}^{-2}$). [4 marks]



(1) 20 N

(2) 60 N

(3) 40 N

(4) 10 N

Correct Answer: (2) 60 N

Solution:

Concept:

- For the upper mass on the spring, $\omega^2 = k/m$; the spring-force change over amplitude A is $kA = m\omega^2 A$.
- The lower block remains at rest. Its upward floor reaction balances its weight plus the downward force exerted by the compressed spring.

Step 1: Find the greatest spring compression force The small mass is $m = 1\,\text{kg}$ and its angular frequency is $25\,\text{rad}\,\text{s}^{-1}$. Therefore

$$k = m\omega^2 = 1 \times 25^2 = 625\,\text{N/m}.$$

At equilibrium the spring pushes the upper body upward with force $mg = 10\,\text{N}$. During the oscillation its force varies by

$$kA = 625 \times 0.016 = 10\,\text{N}.$$

The amplitude has been converted from centimetres to metres. At the lowest position the spring is most compressed, and its upward force on the upper body is $mg + kA = 20\,\text{N}$. By Newton third law it then pushes the lower body downward with the same magnitude.

Step 2: Balance forces on the stationary lower block The lower block has mass $M = 4\,\text{kg}$, so its weight is $Mg = 40\,\text{N}$. If N is the upward floor reaction, its instantaneous force balance is

$$N = Mg + F_{\text{spring}}.$$

Consequently the largest reaction is

$$N_{\max} = 40 + 20 = 60\,\text{N}.$$

The force exerted by the system on the floor has this same magnitude and acts downward. The $40\,\text{N}$ choice omits the spring force, while $20\,\text{N}$ counts only the maximum spring load. At the highest point the spring force falls to zero and the lower body still has a positive reaction, consistent with its remaining on the floor.

Step 3: Conclusion The maximum downward force on the floor is $60\,\text{N}$. Hence, the correct option is (2).

Quick Tip to Solve: The greatest floor load equals total equilibrium weight plus $m\omega^2 A$, because only the upper mass oscillates.

---

43. The amplitude of a simple pendulum, oscillating in air with a small spherical bob, decreases from 10 cm to 8 cm in 40 seconds. Assuming that Stokes law is valid, and ratio of the coefficient of viscosity of air to that of carbon dioxide is 1.3, the time in which amplitude of this pendulum will reduce from 10 cm to 5 cm in carbondioxide will be close to (ln 5 = 1.601, ln 2 = 0.693) [4 marks]

(1) 231 s

(2) 142 s

(3) 208 s

(4) 161 s

Correct Answer: (4) 161 s

Solution:

Concept:

- Under Stokes drag, the resistive force is proportional to velocity, and a weakly damped oscillator has amplitude $A(t) = A_0 e^{-\beta t}$. The decay constant β is proportional to the viscosity of the surrounding fluid.
- For the same bob, the ratio of decay constants equals the viscosity ratio: $\beta_{\text{air}}/\beta_{\text{CO}_2} = 1.3$. Amplitude ratios, rather than absolute amplitudes, determine the elapsed time.

Step 1: Find the decay constant in air The amplitude decreases to $8/10$ of its initial value in 40 seconds. Hence

$$e^{-40\beta_{\text{air}}} = \frac{4}{5}, \qquad 40\beta_{\text{air}} = \ln(5/4).$$

The logarithm is positive, as required for a positive decay constant. The amplitude itself decays exponentially; its square would instead describe the oscillator energy.

Step 2: Use the viscosity ratio for carbon dioxide The smaller viscosity of carbon dioxide gives $\beta_{CO_2} = \beta_{air}/1.3$. Halving the amplitude requires $\beta_{CO_2} t = \ln 2$, so

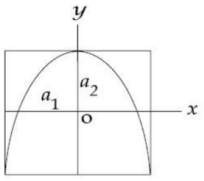
$$t = \frac{1.3(40)\ln 2}{\ln(5/4)}.$$

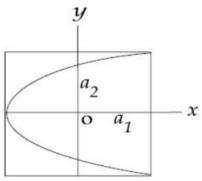
Using $\ln(5/4) \simeq 0.2231$ gives $t \simeq 161.5\,\text{s}$. The printed value $\ln 5 = 1.601$, used literally, gives $\ln(5/4) = 1.601 - 2(0.693) = 0.215$ and $t \simeq 167.6\,\text{s}$; the closest listed time is still 161 s. Multiplying the decay rate by 1.3 instead would incorrectly make the less viscous gas damp faster. The other listed times do not follow the viscosity-scaled logarithmic decay law.

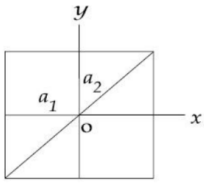
Step 3: Conclusion The closest listed time is $161\,\text{s}$. Hence, the correct option is (4).

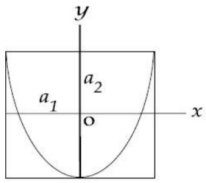
Quick Tip to Solve: Use $t \propto \ln(A_0/A)/\eta$ for a fixed bob; a less viscous gas needs longer to produce the same fractional decrease.

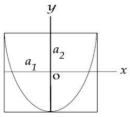
44. A particle which is simultaneously subjected to two perpendicular simple harmonic motions represented by $x = a_1 \cos\omega t$ and $y = a_2 \cos 2\omega t$ traces a curve given by: [4 marks]

(1) Curve (1) 

(2) Curve (2) 

(3) Curve (3) 

(4) Curve (4) 

Correct Answer: (4) Curve (4) 

Solution:

Concept:

- The path of a particle is found by eliminating time between its two displacement equations. The identity $\cos 2\theta = 2\cos^2\theta - 1$ directly relates the two coordinates here.
- The amplitudes restrict the accessible coordinates to $-a_1 \le x \le a_1$ and $-a_2 \le y \le a_2$; only that segment of the resulting algebraic curve is traced.

Step 1: Eliminate the time parameter From the horizontal motion, $\cos\omega t = x/a_1$. Substituting this in the vertical motion gives

$$y = a_2(2\cos^2\omega t - 1) = \frac{2a_2}{a_1^2}x^2 - a_2.$$

This equation is quadratic in x and linear in y. Its axis of symmetry is the vertical axis and its quadratic coefficient is positive, so it opens upwards. No sign ambiguity occurs in the elimination because the

double-angle identity contains the square of the cosine.

Step 2: Identify the vertex and end points At $x = 0$, the vertical displacement is $y = -a_2$; this is the lowest point of the path. At either extremity $x = \pm a_1$, the vertical displacement is $y = a_2$. Therefore the particle traces the upward-opening parabolic arc with vertex at the bottom of the illustrated rectangle. Curve (1) has the wrong opening and vertex; curve (2) opens sideways; curve (3) is a straight line and would correspond to a different frequency relation. Curve (4) alone has the required symmetry, vertex and two upper end points. As time evolves the same bounded arc is retraced, rather than an unbounded parabola being covered.

Step 3: Conclusion The locus is $y = 2a_2x^2/a_1^2 - a_2$, represented by curve (4). Hence, the correct option is (4).

Quick Tip to Solve: For cosine motions with frequency ratio 1:2, use the double-angle identity and check the point at $x = 0$ to decide the parabola orientation.

---

45. A transverse wave is represented by $y = \frac{10}{\pi}\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$. For what value of the wavelength is the wave velocity twice the maximum particle velocity? [4 marks]

(1) 40 cm

(2) 60 cm

(3) 10 cm

(4) 20 cm

Correct Answer: (1) 40 cm

Solution:

Concept:

- For a travelling sinusoidal wave $y = A\sin(\omega t - kx)$, the propagation speed is $v = \omega/k = \lambda/T$. It describes the motion of a fixed phase along the medium.
- The transverse velocity of a material particle is $\partial y/\partial t = A\omega\cos(\omega t - kx)$, so its maximum magnitude is $u_{\max} = A\omega = 2\pi A/T$.

Step 1: Read the amplitude and angular frequency Comparison with the given expression gives $A = 10/\pi$ in the length unit used by the answer choices and $\omega = 2\pi/T$. Thus

$$u_{\max} = \frac{10}{\pi}\frac{2\pi}{T} = \frac{20}{T}.$$

The amplitude is the coefficient outside the sine; it is not the displacement at every point. The particle velocity must be obtained by differentiation with respect to time at fixed position.

Step 2: Apply the required speed ratio The condition states that the wave speed is twice the greatest particle speed, not the converse. Therefore

$$\frac{\lambda}{T} = 2\frac{20}{T}, \qquad \lambda = 40\,\text{cm}.$$

Equivalently, $v/u_{\max} = \lambda/(2\pi A)$, and setting this ratio to 2 gives $\lambda = 4\pi A$. The period cancels because

both velocities have the same inverse dependence on it. For the proposed wavelengths 60 cm, 10 cm and 20 cm, the ratios would be 3, one-half and 1 respectively, so they fail the specified factor of two. The result depends on amplitude and wavelength, rather than the instantaneous phase of the wave.

Step 3: Conclusion The required wavelength is $40\,\mathrm{cm}$. Hence, the correct option is (1).

Quick Tip to Solve: Cancel the period early: the ratio of wave speed to maximum particle speed is $\lambda/(2\pi A)$.

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46. The magnitude of the average electric field normally present in the atmosphere just above the surface of the Earth is about $150\,\mathrm{N/C}$, directed inward towards the center of the Earth. This gives the total net surface charge carried by the Earth to be: [Given: $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/(N\,m^2)}$, $R_E = 6.37 \times 10^6\,\mathrm{m}$]. [4 marks]

(1) -680 kC

(2) + 670 kC

(3) + 680 kC

(4) $-$ 670 kC

Correct Answer: (1) -680 kC

Solution:

Concept:

- Gauss's law relates electric flux through a closed surface to its enclosed charge: $\oint \mathbf{E} \cdot d\mathbf{A} = Q/\varepsilon_0$. For the stated average radial field at the Earth surface, $Q = 4\pi\varepsilon_0 R_E^2 E_r$.
- The outward radial direction is positive. An inward electric field has negative flux and therefore implies negative net enclosed charge.

Step 1: Determine the sign and surface area The Gaussian surface is a sphere just outside the Earth, with area $4\pi R_E^2$. The supplied field points towards the centre, opposite to the outward area vector, so $E_r = -150\,\mathrm{N/C}$. Hence

$$Q = -4\pi\varepsilon_0 R_E^2 (150).$$

Taking the field magnitude as positive without accounting for its direction would give the wrong sign. The near-surface average field is treated as the uniform radial average specified in the question.

Step 2: Substitute the constants Using the radius in metres and the permittivity in SI units,

$$\begin{aligned} |Q| &= 4\pi(8.85 \times 10^{-12})(6.37 \times 10^6)^2(150) \\ &\simeq 6.77 \times 10^5\,\mathrm{C} \\ &\simeq 677\,\mathrm{kC}. \end{aligned}$$

The nearest listed rounded magnitude is 680 kC. Both positively signed alternatives contradict the observed inward field. The negative 670 kC value is less close to the numerical result than negative 680 kC: the respective deviations are about 7 kC and 3 kC. Squaring the radius is essential because Gauss's law uses the whole spherical area rather than a circumference or a length.

Step 3: Conclusion The total net surface charge is approximately $-680\,\mathrm{kC}$. Hence, the correct option

is (1).

Quick Tip to Solve: Choose the sign from the field direction before calculating: inward electric flux means negative enclosed charge.

47. Three capacitances, each of 3 μF, are provided. These cannot be combined to provide the resultant capacitance of [4 marks]

(1) 2 μF

(2) 6 μF

(3) 1.5 μF

(4) 1 μF

Correct Answer: Both (2) and (3) are impossible when all three capacitors must be used; no listed value is impossible if subsets are allowed.

Solution:

Concept:

- Capacitors in parallel add directly, $C_{\text{p}} = C_1 + C_2$, while capacitors in series satisfy $1/C_{\text{s}} = 1/C_1 + 1/C_2$.
- Whether every provided capacitor must be used matters here. The printed wording does not produce a unique impossible value under either natural interpretation.

Step 1: Enumerate combinations using all three capacitors Let each capacitance be $C = 3\,\mu\text{F}$. All three in parallel give $3C = 9\,\mu\text{F}$, and all three in series give $C/3 = 1\,\mu\text{F}$. Putting two in parallel and that pair in series with the third gives

$$C_{\text{eq}} = \frac{(2C)C}{2C + C} = \frac{2C}{3} = 2\,\mu\text{F}.$$

Putting two in series and that pair in parallel with the third gives

$$C_{\text{eq}} = \frac{C}{2} + C = \frac{3C}{2} = 4.5\,\mu\text{F}.$$

These exhaust the distinct connected series-parallel arrangements of three identical two-terminal capacitors. A connection that leaves one inactive reduces to a subset case, rather than an additional all-three arrangement.

Step 2: Compare both readings with the listed options When all three must participate, the achievable set is $\{1, 2, 4.5, 9\}\,\mu\text{F}$. Consequently both 6 and 1.5 microfarads, options (2) and (3), are impossible, while 2 and 1 microfarads are possible. If it is permissible to use only two of the provided capacitors, the two in parallel give 6 microfarads and the two in series give 1.5 microfarads. Along with the all-three constructions above, every listed option then becomes possible. Thus interpreting the unused capacitor differently does not rescue a single correct choice. Selecting either (2) or (3) alone would require an extra restriction absent from the printed question.

Step 3: Conclusion The printed question has no unique defensible option: using all three makes both

(2) and (3) impossible, while allowing subsets makes every listed value achievable.

Quick Tip to Solve: List all-series, all-parallel and the two mixed arrangements before deciding that a three-capacitor value is impossible.

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48. A d.c. main supply of e.m.f. $220\,\mathrm{V}$ is connected across a storage battery of e.m.f. $200\,\mathrm{V}$ through a resistance of $1\,\Omega$. The battery terminals are connected to external resistance R. The minimum value of R, so that a current passes through the battery to charge it is: [4 marks]

(1) 9 Ω

(2) 7 Ω

(3) 11 Ω

(4) 0

Correct Answer: (3) 11 Ω

Solution:

Concept:

- At the boundary between charging and discharging, the battery current is zero and its terminal voltage equals its emf, $200\,\mathrm{V}$. A voltage exceeding the emf drives current into the positive terminal and charges the battery.
- The supply current through the 1-ohm series resistor splits into load current through R and charging current through the battery. Both branches have the same terminal voltage.

Step 1: Find the supply and load currents at the threshold At zero battery current, the series resistor has a voltage drop of $220 - 200 = 20\,\mathrm{V}$, and therefore the current from the mains is

$$I_{\text{supply}} = \frac{220 - 200}{1} = 20\,\mathrm{A}.$$

The load current at this same terminal voltage is $I_R = 200/R$. A nonzero charging current is possible only if the mains provides more current than the load consumes; otherwise the load takes all the mains current or the battery must supply the deficit.

Step 2: Impose the charging condition Writing the battery current as positive into its positive terminal gives

$$I_{\text{charge}} = 20 - \frac{200}{R}.$$

For charging this expression must be positive, so

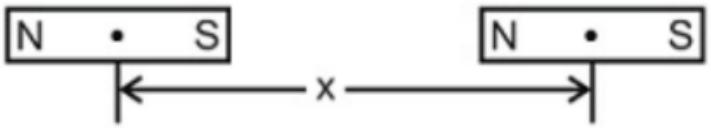
$$20 > \frac{200}{R}, \qquad R > 10\,\Omega.$$

At exactly 10 ohms the current into the battery is zero, so this is a strict inequality. Of the listed values 9, 7, 11 and 0 ohms, only 11 ohms lies above the threshold. The 9-ohm and 7-ohm loads demand more than 20 A at 200 V and cannot leave charging current; zero ohms would short the battery terminals. There is no smallest real number greater than 10, but the requested minimum among the offered discrete choices is unambiguous.

Step 3: Conclusion The charging condition is $R > 10\,\Omega$, so the suitable listed minimum is $11\,\Omega$. Hence, the correct option is (3).

Quick Tip to Solve: At the charging threshold set the battery current to zero, then compare the supply current with the parallel load current.

49. The mid points of two small magnetic dipoles of length d in end-on positions are separated by a distance x, where $x \gg d$. The magnitude of force between them is proportional to x^{-n}, where n is: [4 marks]



(1) 3

(2) 4

(3) 2

(4) 1

Correct Answer: (2) 4

Solution:

Concept:

- At a distance much larger than a bar magnet's length, its field is the dipole field. Along its axis, $B = \mu_0(2m)/(4\pi x^3)$, where m is magnetic dipole moment.
- A small magnetic dipole in a nonuniform axial field experiences force $F = m\,dB/dx$ for a fixed alignment. Thus the force depends on the field gradient, rather than on the field alone.

Step 1: Write the axial field of the first dipole Because $x \gg d$, the second magnet samples the far field of the first,

$$B_1(x) = \frac{\mu_0}{4\pi}\frac{2m_1}{x^3}.$$

The finite length corrections are smaller by powers of d/x and do not determine the leading distance dependence. A bar magnet has no isolated magnetic monopole, so the inverse-square field of a single pole cannot be used as the net far field of the entire magnet.

Step 2: Differentiate to find the force scaling For the second dipole aligned along the common axis,

$$F_x = m_2\frac{dB_1}{dx} = -\frac{\mu_0}{4\pi}\frac{6m_1m_2}{x^4}.$$

The sign depends on the orientation convention; the magnitude always varies as x^{-4} for these fixed end-on dipoles. Comparing this with the given proportionality $|F| \propto x^{-n}$ gives $n = 4$. The exponent 3 belongs to the magnetic field itself, not to the force. The exponent 2 would correspond to an isolated-pole interaction rather than two complete short dipoles, and an inverse-first-power force does not follow from either dipole law. The pictured parallel orientation affects attraction versus repulsion but cannot alter the exponent.

Step 3: Conclusion The magnitude of the interaction force is proportional to x^{-4}, so $n = 4$. Hence, the correct option is (2).

Quick Tip to Solve: For a dipole in another dipole's field, differentiate the x^{-3} field once: the force varies as x^{-4}.

50. The magnetic field of earth at the equator is approximately $4 \times 10^{-5}\,\mathrm{T}$. The radius of earth is $6.4 \times 10^{6}\,\mathrm{m}$. Then the dipole moment of the earth will be nearly of the order of: [4 marks]

(1) $10^{20}\,\mathrm{A\,m^2}$

(2) $10^{16}\,\mathrm{A\,m^2}$

(3) $10^{10}\,\mathrm{A\,m^2}$

(4) $10^{23}\,\mathrm{A\,m^2}$

Correct Answer: (4) $10^{23}\,\mathrm{A\,m^2}$

Solution:

Concept:

- Earth can be approximated by a magnetic dipole. At a point on its equatorial line, the field magnitude is $B_{\text{eq}} = \mu_0 m/(4\pi R^3)$, where m is magnetic dipole moment and R is distance from the centre.
- Use SI units throughout: $\mu_0/(4\pi) = 10^{-7}\,\mathrm{T\,m\,A^{-1}}$. The magnetic moment therefore has unit $\mathrm{A\,m^2}$.

Step 1: Select the equatorial field relation The location is the equator, so the field is the equatorial rather than the axial dipole field. Rearranging the equatorial expression isolates the unknown magnetic moment:

$$m = \frac{B_{\text{eq}} R^3}{10^{-7}}.$$

The axial expression would contain an extra factor of two and is not the appropriate starting point here.

Step 2: Substitute the field and radius Insert the given surface field and Earth radius without dropping the negative exponent in the field:

$$\begin{aligned} m &= \frac{(4 \times 10^{-5})(6.4 \times 10^{6})^3}{10^{-7}} \\ &= 4 \times 6.4^3 \times 10^{20} \\ &= 1048.576 \times 10^{20} \\ &= 1.048576 \times 10^{23}\,\mathrm{A\,m^2}. \end{aligned}$$

Cubing the radius produces 10^{18}, which is why a very large terrestrial magnetic moment is compatible with the small field measured at its surface.

Step 3: Compare the proposed orders The coefficient of the scientific-notation result is close to unity. Thus its order is 10^{23}, rather than 10^{20}. The latter would arise by overlooking the factor 6.4^3 and the coefficient of the field. Values of 10^{16} and 10^{10} are many powers of ten too small to reproduce the

stated equatorial field.

Step 4: Conclusion The dipole moment is approximately $1.05 \times 10^{23}\,\mathrm{A\,m^2}$, of order $10^{23}\,\mathrm{A\,m^2}$. Hence, the correct option is (4).

Quick Tip to Solve: For a planetary dipole, first identify equatorial or axial field and retain the full cubed radius before judging the order of magnitude.

51. When the rms voltages V_L, V_C and V_R are measured respectively across the inductor L, the capacitor C and the resistor R in a series LCR circuit connected to an AC source, it is found that the ratio $V_L : V_C : V_R = 1 : 2 : 3$. If the rms voltage of the AC source is $100\,\mathrm{V}$, then V_R is close to: [4 marks]

(1) 50 V

(2) 70 V

(3) 100 V

(4) 90 V

Correct Answer: (4) 90 V

Solution:

Concept:

- In a series LCR circuit the same rms current flows through all components. The resistor voltage is in phase with current, while the inductor and capacitor voltages are opposite along the quadrature direction.
- The source rms voltage is the magnitude of a phasor sum: $V^2 = V_R^2 + (V_L - V_C)^2$. Component rms voltage magnitudes cannot generally be added arithmetically.

Step 1: Express the measured voltages with one scale Let the common voltage scale be $k\,\mathrm{V}$. The measured ratio gives

$$V_L = k, \qquad V_C = 2k, \qquad V_R = 3k.$$

The net reactive voltage is therefore $V_L - V_C = -k$. Its negative sign indicates a capacitive circuit; its magnitude is still k in the voltage triangle.

Step 2: Apply the source voltage condition The resistor voltage is perpendicular to the net reactive voltage, so use Pythagoras in the phasor triangle:

$$\begin{aligned} 100^2 &= (3k)^2 + (k - 2k)^2 \\ &= 9k^2 + k^2 = 10k^2. \end{aligned}$$

Taking the positive root because k is a voltage magnitude gives

$$k = \frac{100}{\sqrt{10}}\,\mathrm{V}, \qquad V_R = \frac{300}{\sqrt{10}}\,\mathrm{V} \approx 94.87\,\mathrm{V}.$$

Step 3: Choose the closest listed value The value $90\,\mathrm{V}$ differs from the calculated resistor voltage by about $4.87\,\mathrm{V}$, whereas $100\,\mathrm{V}$ differs by about $5.13\,\mathrm{V}$. Thus 90 V is slightly closer. The alternatives 50

V and 70 V are much further away. The near equality of 90 V and 100 V as possible estimates makes retaining a few decimal places useful here. A simple arithmetic division of the source among the three components would ignore their phase differences.

Step 4: Conclusion $V_R \approx 94.87\,\text{V}$, whose nearest listed value is $90\,\text{V}$. Hence, the correct option is (4).

Quick Tip to Solve: Use the voltage triangle, then compare actual distances to the nearby options; rounding 94.87 too early can select the wrong one.

52. Match List I (Wavelength range of electromagnetic spectrum) with List II (Method of production of these waves) and select the correct option from the options given below the lists.

List I	List II
(a) $700\,\text{nm}$ **to** $1\,\text{mm}$	**(i) Vibration of atoms and molecules.**
(b) $1\,\text{nm}$ **to** $400\,\text{nm}$	**(ii) Inner shell electrons in atoms moving from one energy level to a lower level.**
(c) $< 10^{-3}\,\text{nm}$	**(iii) Radioactive decay of the nucleus.**
(d) $1\,\text{mm}$ **to** $0.1\,\text{m}$	**(iv) Magnetron valve.**

[4 marks]

(1) (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)

(2) (a) - (i), (b) - (ii), (c) - (iii), (d) - (iv)

(3) (a) - (iv), (b) - (iii), (c) - (ii), (d) - (i)

(4) (a) - (ii), (b) - (iii), (c) - (iv), (d) - (i)

Correct Answer: (2) (a) - (i), (b) - (ii), (c) - (iii), (d) - (iv)

Solution:

Concept:

- Electromagnetic radiation can be identified both by wavelength and by the physical process that produces it. Infrared emission is associated with atomic and molecular vibrations, while nuclear transitions produce gamma radiation.
- Microwaves can be generated by a magnetron. Electronic transitions involving inner atomic shells produce short-wavelength electronic radiation, conventionally associated with X rays in this matching list.

Step 1: Match the infrared and microwave intervals The interval $700\,\text{nm}$ to $1\,\text{mm}$ is the infrared band. It corresponds to vibrations of atoms and molecules, so (a) matches (i). The interval $1\,\text{mm}$ to $0.1\,\text{m}$ is the microwave band. A magnetron valve is a microwave generator, so (d) matches (iv). These two direct matches already select the second proposed matching.

Step 2: Match the shortest wavelengths Radiation with wavelength less than $10^{-3}\,\text{nm}$ lies at the very high energy end of the spectrum and is gamma radiation. Radioactive decay of a nucleus can emit gamma photons, so (c) matches (iii). The remaining supplied electronic production mechanism, inner-shell electrons falling from one atomic level to a lower level, is matched with (b), giving (b)–(ii). The printed interval for (b) is broad, including ultraviolet as well as part of the X-ray range; the intended correspondence is nevertheless fixed uniquely by the other three unambiguous matches.

Step 3: Check the complete code The resulting order is (a)–(i), (b)–(ii), (c)–(iii), (d)–(iv). The first code incorrectly assigns nuclear decay to infrared and vibrations to the shortest wavelengths. The third assigns a magnetron to infrared and vibrations to microwaves. The fourth reverses the physical associations of both the long- and short-wavelength bands. Only the second code preserves the wavelength-production associations required by the lists.

Step 4: Conclusion The matching is (a)–(i), (b)–(ii), (c)–(iii), (d)–(iv). Hence, the correct option is (2).

Quick Tip to Solve: Anchor spectrum matching with microwaves–magnetron and gamma rays–nucleus before assigning the remaining electronic and vibrational processes.

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53. A diver looking up through the water sees the outside world contained in a circular horizon. The refractive index of water is $4/3$, and the diver's eyes are $15\,\text{cm}$ below the surface of the water. Then the radius of the circle is: [4 marks]

(1) $\dfrac{15\sqrt{7}}{3}\,\text{cm}$

(2) $\dfrac{15\times 3}{\sqrt{7}}\,\text{cm}$

(3) $15\times 3\sqrt{7}\,\text{cm}$

(4) $15\times 3\sqrt{5}\,\text{cm}$

Correct Answer: (2) $\dfrac{15\times 3}{\sqrt{7}}\,\text{cm}$

Solution:

Concept:

- Light from the outside world enters water through refraction. The limiting ray that grazes the surface in air refracts at the critical angle c in water, where $\sin c = 1/\mu$.
- The circular window is the intersection of the limiting cone of rays with the water surface. If the eye is depth h below the surface, the radius is $r = h\tan c$, measured horizontally from the point directly above the eye.

Step 1: Find the limiting angle in water Apply Snell's law to the limiting ray with angle 90° to the normal in air:

$$1\sin 90^\circ = \frac{4}{3}\sin c.$$

Hence $\sin c = 3/4$. The angle is measured from the normal, not from the horizontal water surface. Using its complement would interchange the relevant tangent and cotangent.

Step 2: Obtain the cone radius The limiting ray, the vertical depth, and the surface radius form a right triangle. First calculate the cosine and then the tangent:

$$\cos c = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}, \qquad \tan c = \frac{3}{\sqrt{7}}.$$

Substituting the given depth yields

$$r = 15 \tan c = \frac{45}{\sqrt{7}}\,\text{cm} \approx 17.0\,\text{cm}.$$

This geometrical relation uses a tangent because the radius is opposite the critical angle while the depth is adjacent to it.

Step 3: Compare the option expressions The second expression is exactly $15 \times 3/\sqrt{7}\,\text{cm}$. The first uses the reciprocal ratio and would correspond to an incorrect use of the complement angle. The third and fourth multiply by square roots and produce much larger circles, inconsistent with $\tan c = 3/\sqrt{7}$. In particular, the given refractive index fixes the limiting cone completely; no additional height or diameter factor is involved.

Step 4: Conclusion The circular horizon has radius $\frac{45}{\sqrt{7}}\,\text{cm}$. Hence, the correct option is (2).

Quick Tip to Solve: For an underwater viewing window use $r = h \tan c$, with the critical angle measured from the vertical normal, not from the surface.

---

54. The focal lengths of objective lens and eye lens of a Galilean Telescope are respectively 30 cm and 3.0 cm. Telescope produces virtual, erect image of an object situated far away from it at least distance of distinct vision from the eye lens. In this condition the Magnifying Power of the Galilean Telescope should be [4 marks]

(1) +8. 8

(2) −11. 2

(3) +11. 2

(4) −8. 8

Correct Answer: (1) +8. 8

Solution:

Concept:

- A Galilean telescope uses a converging objective and a diverging eyepiece. A distant object forms an intermediate image at the objective focal plane; the eyepiece intercepts the rays before they converge.
- The final image is virtual and erect. For a final image at the least distance of distinct vision $D = 25\,\text{cm}$, the angular magnification is positive and differs from the relaxed-eye value $f_o/|f_e|$.

Step 1: Find the eyepiece object distance Use the Cartesian lens convention for the concave eyepiece: $f_e = -3\,\text{cm}$ and final image distance $v_e = -25\,\text{cm}$. Its object is virtual, on the right of the eyepiece, because the objective rays are still converging. The lens equation gives

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}, \qquad -\frac{1}{25} - \frac{1}{u_e} = -\frac{1}{3}.$$

Thus $1/u_e = 1/3 - 1/25 = 22/75$, so $u_e = 75/22\,\text{cm}$.

Step 2: Calculate the angular magnification If the distant object subtends a small angle α, the objective image height is approximately $h = f_o\alpha$. The ray through the eyepiece centre has angle h/u_e, giving the magnitude

$$M = \frac{h/u_e}{h/f_o} = \frac{f_o}{u_e} = \frac{30 \times 22}{75} = 8.8.$$

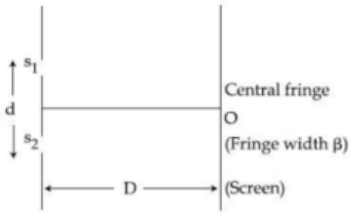
Equivalently, $M = (f_o/|f_e|)(1 - |f_e|/D) = 10(1 - 3/25)$. This form specifically incorporates the virtual final image at the near point.

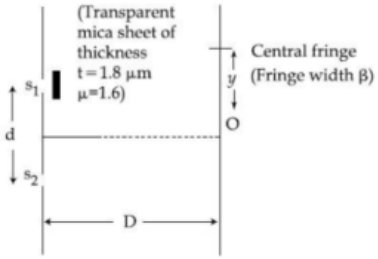
Step 3: Assign the image orientation and sign The Galilean arrangement gives an erect image, as explicitly stated in the question, so the magnifying power is positive. This eliminates both negative choices. The value $+11.2$ would follow from using a plus sign in the near-point factor appropriate to a different eyepiece arrangement; the diverging eyepiece here requires $1 - |f_e|/D$. The relaxed-eye value would be $+10$, but the problem specifies a finite final image distance.

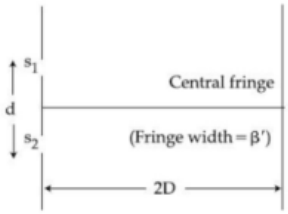
Step 4: Conclusion The angular magnifying power is $+8.8$. This differs from the supplied key's $+11.2$; the diverging eyepiece lens equation gives the minus sign in the near-point factor. Hence, the correct option is (1).

Quick Tip to Solve: For a Galilean telescope use a negative eyepiece focal length in the lens equation; the near-point factor is $1 - |f_e|/D$.

55. Using monochromatic light of wavelength λ, an experimentalist sets up the Young's double slit experiment in three ways as shown. If she observes that $y = \beta'$, the wavelength of light used is: [4 marks]







(1) 520 nm

(2) 580 nm

(3) 540 nm

(4) 560 nm

Correct Answer: (3) 540 nm

Solution:

Concept:

- In Young's double-slit experiment the fringe width at screen distance D is $\beta = \lambda D/d$, where d is slit separation. Doubling the screen distance doubles the fringe width.
- A transparent sheet of thickness t and refractive index μ placed before one slit adds optical path $(\mu - 1)t$. It shifts the central fringe by $y = D(\mu - 1)t/d$ without changing the fringe width.

Step 1: Read the three arrangements The first arrangement has slit separation d, screen distance D, and fringe width β. In the second, the transparent mica sheet is placed before slit s_1, with $t = 1.8\,\mu\text{m}$ and $\mu = 1.6$, while the screen remains at distance D. The central fringe shifts toward that slit by y. In the third arrangement the screen is at $2D$ without the sheet, so

$$\beta' = \frac{2\lambda D}{d}.$$

Step 2: Equate the observed shift and new width The additional optical path in the mica sheet is the material path minus the air path it replaces, so it is $(\mu - 1)t$, rather than μt. The geometrical path difference at the shifted central fringe cancels this excess, giving

$$y = \frac{D}{d}(\mu - 1)t.$$

The observation $y = \beta'$ therefore implies

$$\frac{D}{d}(\mu - 1)t = \frac{2\lambda D}{d}, \qquad \lambda = \frac{(\mu - 1)t}{2}.$$

Both the unknown slit spacing and screen distance cancel.

Step 3: Evaluate the wavelength and check the choices Substitute the mica data:

$$\lambda = \frac{(1.6 - 1)(1.8\,\mu\text{m})}{2} = 0.54\,\mu\text{m} = 540\,\text{nm}.$$

The proposed values 520 nm, 580 nm, and 560 nm do not satisfy the equality of shift and doubled-distance fringe width. Omitting the factor two from the third arrangement would instead yield 1080 nm, showing why the change of screen position must be included explicitly.

Step 4: Conclusion The wavelength of the monochromatic light is $540\,\text{nm}$. Hence, the correct option is (3).

Quick Tip to Solve: Write the sheet-induced shift and the fringe width at the new screen distance separately; cancel D/d only after equating them.

56. For which of the following particles will it be most difficult to experimentally verify the de-Broglie relationship? [4 marks]

(1) An electron

(2) An α-particle

(3) A dust particle

(4) A proton

Correct Answer: (3) A dust particle

Solution:

Concept:

- The de Broglie wavelength of a moving particle is $\lambda = h/p$, where h is Planck's constant and p is its momentum. For a nonrelativistic particle this becomes $\lambda = h/(mv)$.
- Diffraction and interference are easiest to detect when the wavelength is comparable with an available slit width or lattice spacing. A wavelength enormously smaller than practical structures makes wave effects extremely difficult to resolve.

Step 1: Compare microscopic and macroscopic masses Electrons, protons, and alpha particles are microscopic particles. Their masses are very small: the proton and alpha particle are heavier than the electron, but all remain on the atomic or subatomic mass scale. A dust particle contains an enormous number of atoms, so its mass and typical momentum are vastly greater than those of individual microscopic particles. The inverse relationship between wavelength and momentum is the key comparison, rather than electric charge.

Step 2: Relate wavelength to experimental resolution For a given nonrelativistic speed, increasing mass decreases λ in exactly the same ratio. As an illustrative scale, even a dust particle of mass $10^{-12}\,\mathrm{kg}$ moving at only $1\,\mathrm{m\,s^{-1}}$ has

$$\lambda = \frac{6.63 \times 10^{-34}}{10^{-12} \times 1}\,\mathrm{m} = 6.63 \times 10^{-22}\,\mathrm{m}.$$

This is far below atomic spacings and cannot give readily resolvable diffraction in ordinary apparatus. This illustrative estimate explains the physical difficulty; it is not an extra condition needed to answer the qualitative comparison.

Step 3: Choose the least accessible wave behaviour Electron diffraction is experimentally observable, and microscopic ions such as protons and alpha particles also have matter wavelengths accessible in suitable conditions. The dust particle's macroscopic inertia makes its wavelength by far the smallest under ordinary comparable conditions. Environmental interactions also make maintaining coherent macroscopic motion much harder. None of this means that the de Broglie relation ceases to apply; it means its wave signature is the hardest to demonstrate.

Step 4: Conclusion It is most difficult to verify the de Broglie relation experimentally for a dust particle. Hence, the correct option is (3).

Quick Tip to Solve: When comparing observability of matter waves, rank momenta: the macroscopic object has an extremely short wavelength and negligible visible diffraction.

57. If the binding energy of the electron in a hydrogen atom is $13.6\,\text{eV}$, the energy required to remove the electron from the first excited state of Li^{2+} is: [4 marks]

(1) $13.6\,\text{eV}$

(2) $3.4\,\text{eV}$

(3) $122.4\,\text{eV}$

(4) $30.6\,\text{eV}$

Correct Answer: (4) $30.6\,\text{eV}$

Solution:

Concept:

- A hydrogen-like ion contains one electron. Its Bohr energy is $E_n = -13.6Z^2/n^2\,\text{eV}$, where Z is the nuclear charge and n the principal quantum number.
- Ionisation takes the electron to infinity, where its energy is zero. The required energy is therefore the positive binding energy $-E_n$.

Step 1: Identify the ion and the occupied level Lithium has atomic number $Z = 3$. Removing two electrons leaves Li^{2+}, a one-electron ion, so the hydrogen-like expression applies. The ground state is $n = 1$, and the first excited state is $n = 2$. The charge on the ion does not mean that $Z = 2$; the nucleus still contains three protons.

Step 2: Calculate the energy needed for ionisation Using the given hydrogen binding energy as the energy scale,

$$E_2 = -13.6\frac{3^2}{2^2} = -30.6\,\text{eV}.$$

The final energy is zero, so

$$\Delta E = 0 - (-30.6) = 30.6\,\text{eV}.$$

Thus the supplied energy must be positive. The value $13.6\,\text{eV}$ belongs to ground-state hydrogen, $3.4\,\text{eV}$ to first-excited hydrogen, and $122.4\,\text{eV}$ to ground-state lithium ion. None describes the specified ion and level.

Step 3: Conclusion The energy required is $30.6\,\text{eV}$. Hence, the correct option is (4).

Quick Tip to Solve: For ionisation of a one-electron ion, scale the hydrogen binding energy by Z^2/n^2; first excited always means $n = 2$.

58. An n-p-n transistor has three leads A, B and C. Connecting B and C by moist fingers, A to the positive lead of an ammeter, and C to the negative lead of the ammeter, one finds large deflection. Then, A, B and C refer respectively to: [4 marks]

(1) Base, emitter and collector

(2) Base, collector and emitter

(3) Emitter, base and collector

(4) Collector, emitter and base

Correct Answer: (2) Base, collector and emitter

Solution:

Concept:

- An n-p-n transistor has a p-type base between n-type emitter and collector regions. Forward bias of the base-emitter junction requires the base to be positive relative to the emitter.
- The emitter is heavily doped and acts as the principal carrier-injecting region. A forward-connected base-emitter junction gives substantial current; reversing that junction gives only a small leakage current.

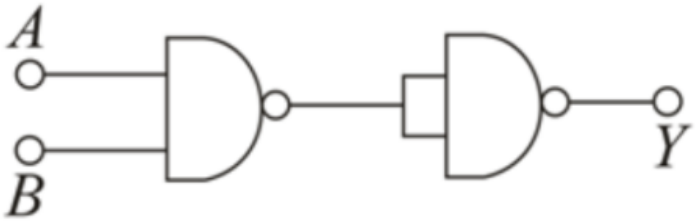
Step 1: Use the polarity of the current test The positive lead is attached to A and the negative lead to C. For the strong forward-conduction condition of an n-p-n transistor, the p-type base must be on the positive side and the n-type emitter on the negative side. Thus A is identified as the base and C as the emitter. The remaining lead B is the collector. The moist fingers provide a conducting link between B and C during this identification test. The observation is understood as a current test with the meter setup supplying the test bias. Moist fingers form a high-resistance path, rather than an ideal short circuit. With the emitter directly at the negative terminal, its strongly conducting base-emitter path is not limited by that finger resistance.

Step 2: Compare all proposed assignments The assignment is therefore A = base, B = collector and C = emitter. Option (1) places the collector directly at the negative terminal, so the heavily doped emitter is reached through the high-resistance finger path; it does not give the same strong direct base-emitter conduction. Option (3) puts an n-type emitter on the positive side and the collector on the negative side, leaving the base at B; it does not provide the required base-positive, emitter-negative connection. Option (4) places the base at the negative lead C, again inconsistent with forward bias of the base-emitter junction in an n-p-n transistor. These polarity checks distinguish the intended terminal assignment.

Step 3: Conclusion A, B and C are respectively base, collector and emitter. Hence, the correct option is (2).

Quick Tip to Solve: First identify the p-type base from the positive polarity required for an n-p-n base-emitter junction; then check the emitter at the negative lead.

59. Identify the gate and match A, B, Y in the bracket to check. [4 marks]



(1) OR (A = 1, B = 1, Y= 0)

(2) XOR (A = 0, B = 0, Y= 0)

(3) NOT (A = 1, B = 1, Y= 1)

(4) AND (A = 1, B = 1, Y= 1)

Correct Answer: (4) AND (A = 1, B = 1, Y= 1)

Solution:

Concept:

- A NAND gate produces the complement of the AND operation: $X = \overline{AB}$. An output bubble denotes inversion.
- When both inputs of a NAND gate are connected to the same signal X, its output is $\overline{XX} = \overline{X}$. Thus a NAND gate with tied inputs acts as a NOT gate.

Step 1: Read the two stages in the circuit The left-hand gate has the AND shape with a bubble at its output, so it is a NAND gate. Its output is $X = \overline{AB}$. This signal is fed to both inputs of the second NAND gate. The second stage therefore inverts X, rather than forming a second independent two-input operation. Reading both the gate body and the output bubble is necessary; ignoring either bubble would change the overall function.

Step 2: Simplify and test the stated input combination The complete circuit has output

$$Y = \overline{XX} = \overline{X} = \overline{\overline{AB}} = AB.$$

It is consequently an AND gate. For $A = B = 1$, the first NAND output is 0, and the second NAND output is 1, so $Y = 1$. Option (1) has both the wrong gate and wrong output for these inputs. Option (2) gives an input-output triple that also occurs for an AND gate, but labels the whole circuit XOR, which fails at $A = B = 1$. Option (3) calls the circuit NOT despite its two independent inputs and is therefore incorrect.

Step 3: Conclusion The combination is AND, with $A = 1$, $B = 1$ giving $Y = 1$. Hence, the correct option is (4).

Quick Tip to Solve: A NAND gate with its inputs tied is an inverter. Cancel the two successive inversions to recognise the net AND operation.

60. A transmitting antenna at the top of a tower has a height $32\,\mathrm{m}$ and the height of the receiving antenna is $50\,\mathrm{m}$. What is the maximum distance between them for satisfactory communication in line of sight (LOS) mode? [4 marks]

(1) $45.5\,\mathrm{km}$

(2) $54.5\,\mathrm{km}$

(3) $55.4\,\mathrm{km}$

(4) $455\,\mathrm{km}$

Correct Answer: (1) $45.5\,\mathrm{km}$

Solution:

Concept:

- For an antenna at height $h \ll R$ above a spherical Earth, the distance to the horizon is approximately $d = \sqrt{2Rh}$, where R is Earth's radius.
- The maximum line-of-sight range between two raised antennas is the sum of their individual horizon distances: $d_{\max} = \sqrt{2Rh_t} + \sqrt{2Rh_r}$.

Step 1: Apply the horizon-distance relation to both towers A straight ray can reach from one antenna to the other until it just grazes the Earth between them. Both antenna heights contribute to this limit. With $h_t = 32\,\mathrm{m}$, $h_r = 50\,\mathrm{m}$, and the standard terrestrial radius $R = 6.4 \times 10^6\,\mathrm{m}$, use

$$d_{\max} = \sqrt{2(6.4 \times 10^6)(32)} + \sqrt{2(6.4 \times 10^6)(50)}.$$

All lengths are in metres before taking square roots.

Step 2: Evaluate and select the matching range The two terms are

$$d_t = \sqrt{4.096 \times 10^8} \simeq 20239\,\mathrm{m}, \qquad d_r = \sqrt{6.4 \times 10^8} \simeq 25298\,\mathrm{m}.$$

Thus

$$d_{\max} \simeq 45537\,\mathrm{m} = 45.5\,\mathrm{km}.$$

The values $54.5\,\mathrm{km}$ and $55.4\,\mathrm{km}$ do not equal the sum of the two horizon distances. The value $455\,\mathrm{km}$ is too large by a factor of ten. Adding the heights inside one square root would also be incorrect, because the two horizon distances must be added separately.

Step 3: Conclusion The maximum LOS separation is approximately $45.5\,\mathrm{km}$. Hence, the correct option is (1).

Quick Tip to Solve: Compute one horizon distance for each antenna, then add; do not replace the two square roots by a square root of the sum of the heights.

Chemistry (120 marks)

61. Dissolving $120\,\mathrm{g}$ of a compound of (mol. wt. 60) in $1000\,\mathrm{g}$ of water gave a solution of density $1.12\,\mathrm{g/mL}$. The molarity of the solution is: [4 marks]

(1) 2.50 M

(2) 1.00 M

(3) 2.00 M

(4) 4.00 M

Correct Answer: (3) 2.00 M

Solution:

Concept:

- Molarity is the number of moles of solute per litre of solution: $M = n/V_{\text{solution}}$. It uses solution volume, rather than the mass of solvent.
- The solution mass is the sum of solute and solvent masses. Its volume follows from $V = m_{\text{solution}}/\rho$, with consistent mass and volume units.

Step 1: Find solute moles and total solution mass The solute has mass $120\,\mathrm{g}$ and molar mass $60\,\mathrm{g\,mol^{-1}}$. Therefore

$$n = \frac{120}{60} = 2.00\,\mathrm{mol}.$$

The water contributes $1000\,\mathrm{g}$, so the total mass after dissolution is

$$m_{\text{solution}} = 120 + 1000 = 1120\,\mathrm{g}.$$

Using only the water mass with the given solution density would omit the solute mass and produce an incorrect solution volume.

Step 2: Convert solution mass to volume and calculate molarity Using the given density,

$$V_{\text{solution}} = \frac{1120\,\mathrm{g}}{1.12\,\mathrm{g\,mL^{-1}}} = 1000\,\mathrm{mL} = 1.000\,\mathrm{L}.$$

Consequently

$$M = \frac{2.00\,\mathrm{mol}}{1.000\,\mathrm{L}} = 2.00\,\mathrm{mol\,L^{-1}}.$$

Option (2) would correspond to only one mole in this volume. Options (1) and (4) would require smaller solution volumes than the density permits. Although the molality happens also to be 2 here, that numerical coincidence is not a reason to use solvent mass in the molarity definition.

Step 3: Conclusion The solution has molarity $2.00\,\mathrm{M}$. Hence, the correct option is (3).

Quick Tip to Solve: When density is given, add solute and solvent masses first; divide their total by density to obtain the solution volume needed for molarity.

62. The amount of oxygen in 3.6 moles of water is: [4 marks]

(1) $57.6\,\text{g}$

(2) $115.2\,\text{g}$

(3) $18.4\,\text{g}$

(4) $28.8\,\text{g}$

Correct Answer: (1) $57.6\,\text{g}$

Solution:

Concept:

- Each molecule of water, H_2O, contains one oxygen atom. Hence one mole of water contains one mole of oxygen atoms.
- The mass of oxygen present is $m_O = n_O M_O$, with atomic molar mass $M_O = 16\,\text{g mol}^{-1}$.

Step 1: Convert moles of water to moles of oxygen atoms The formula H_2O has one oxygen atom and two hydrogen atoms per molecule. The sample of $3.6\,\text{mol}$ of water therefore contains $3.6\,\text{mol}$ of oxygen atoms. There is no extra factor of two for oxygen: the subscript 2 in the formula belongs to hydrogen. The question asks for oxygen present in water, not for the total mass of the water sample.

Step 2: Calculate and cross-check the oxygen mass The oxygen mass is

$$m_O = 3.6 \times 16 = 57.6\,\text{g}.$$

An equivalent mass-fraction check gives total water mass $3.6 \times 18 = 64.8\,\text{g}$, of which oxygen is the fraction $16/18$:

$$m_O = 64.8 \times \frac{16}{18} = 57.6\,\text{g}.$$

Using $32\,\text{g mol}^{-1}$ directly with 3.6 mol would produce $115.2\,\text{g}$, but that is the molar mass of O_2, not oxygen atoms in water. If oxygen were collected as O_2, it would be 1.8 mol and still weigh 57.6 g. The remaining listed masses do not satisfy this elemental balance.

Step 3: Conclusion The amount of oxygen is $57.6\,\text{g}$. Hence, the correct option is (1).

Quick Tip to Solve: Read the formula subscripts before multiplying: one mole of water contains one mole of O atoms, so use 16 g per mole of water for its oxygen content.

63. The energy of an electron in first Bohr's orbit of H atom is $-13.6\,\text{eV}$. The energy value of electron in the first excited state of Li^{2+} is: [4 marks]

(1) $27.2\,\text{eV}$

(2) $-30.6\,\text{eV}$

(3) $30.6\,\text{eV}$

(4) $-27.2\,\text{eV}$

Correct Answer: (2) $-30.6\,\text{eV}$

Solution:

Concept:

- The energy of a one-electron ion is $E_n = -13.6Z^2/n^2\,\text{eV}$. The negative sign means the electron is bound relative to a free electron at infinity.
- For Li^{2+}, the nuclear charge is $Z = 3$, while the first excited state has principal quantum number $n = 2$.

Step 1: Determine the charge and state factors Lithium has three protons in its nucleus. Losing two electrons leaves only one electron, so the same Bohr expression as for hydrogen applies, with an increased attraction represented by $Z^2 = 9$. The first excited state is the level immediately above $n = 1$, namely $n = 2$, so its energy is divided by $2^2 = 4$. The ion charge $+2$ counts missing electrons and must not be substituted for the nuclear charge.

Step 2: Calculate the signed electron energy Relative to the given ground-state hydrogen value,

$$E_2(Li^{2+}) = (-13.6)\frac{9}{4} = -30.6\,\text{eV}.$$

This is the energy of the electron in the bound state. It differs in sign from the energy required to remove that electron, which would be $+30.6\,\text{eV}$. Option (3) uses that positive removal energy instead of the requested level energy. Options (1) and (4), whose magnitude is $27.2\,\text{eV}$, do not follow the required $Z^2/n^2 = 9/4$ scaling. The sign and magnitude together uniquely select the negative 30.6 eV option.

Step 3: Conclusion The electron energy is $-30.6\,\text{eV}$. Hence, the correct option is (2).

Quick Tip to Solve: Distinguish level energy from ionisation energy: a bound Bohr level has negative energy, while the energy supplied to reach infinity is positive.

64. In the following sets of reactants which two sets best exhibit the amphoteric character of $Al_2O_3 \cdot xH_2O$?

Set I: $Al_2O_3 \cdot xH_2O\,(s) + OH^-(aq)$

Set II: $Al_2O_3 \cdot xH_2O\,(s) + H_2O\,(l)$

Set III: $Al_2O_3 \cdot xH_2O\,(s) + H^+(aq)$

Set IV: $Al_2O_3 \cdot xH_2O\,(s) + NH_3(aq)$

[4 marks]

(1) 2 and 4

(2) 1 and 2

(3) 1 and 3

(4) 3 and 4

Correct Answer: (3) 1 and 3

Solution:

Concept:

- Amphoteric oxides react both with acids and with strong bases. The same oxide therefore displays basic behaviour toward an acid and acidic behaviour toward a base.
- Hydrated aluminium oxide retains the amphoteric character of alumina; water of hydration does not change the two tests needed to demonstrate that character.

Step 1: Identify the acid and strong base Set I supplies hydroxide ions, OH^-, and tests reaction with a strong base. Set III supplies hydrogen ions, H^+, and tests reaction with an acid. A positive reaction in both these sets is precisely the evidence required: an amphoteric substance must respond to both opposite acid-base conditions.

Step 2: Write the two characteristic reactions Ignoring water of hydration when writing the net oxide reactions, dissolution in acid can be represented as

$$Al_2O_3 + 6\,H^+ \longrightarrow 2\,Al^{3+} + 3\,H_2O.$$

In aqueous alkali, soluble hydroxoaluminate forms:

$$Al_2O_3 + 2\,OH^- + 3\,H_2O \longrightarrow 2\,[Al(OH)_4]^-.$$

The first reaction shows basic behaviour, while the second shows acidic behaviour. Both reactions refer to the same aluminium oxide framework.

Step 3: Exclude the other combinations Set II contains only water, which does not establish either of the two required contrasting reactions. Aqueous ammonia in Set IV is a weak base and is not the standard reagent for demonstrating dissolution of alumina in strong alkali. Moreover, any pair containing Set IV but omitting either Set I or Set III fails to provide the direct strong-base and acid comparison. Thus the proposed pairs 2 and 4, 1 and 2, and 3 and 4 are not the best tests.

Step 4: Conclusion The appropriate sets are I and III, namely 1 and 3. Hence, the correct option is (3).

Quick Tip to Solve: To test amphoterism, look for one reaction with H^+ and another with OH^-; reaction with water alone is not the test.

65. The number and type of bonds in C_2^{2-} ion in CaC_2 are: [4 marks]

(1) Two σ-bonds and two π-bonds

(2) One σ-bond and two π-bonds

(3) One σ-bond and one π-bond

(4) Two σ-bonds and one π-bond

Correct Answer: (2) One σ-bond and two π-bonds

Solution:

Concept:

- Calcium carbide is described ionically as $Ca^{2+}C_2^{2-}$. The question asks about bonding within the two-carbon acetylide ion, rather than counting ionic attraction to calcium.
- A carbon-carbon triple bond consists of one axial σ bond and two mutually perpendicular lateral π bonds.

Step 1: Count the valence electrons Each carbon supplies four valence electrons and the dianionic charge supplies two more:

$$N_{\text{valence}} = 2(4) + 2 = 10.$$

The Lewis structure has a carbon-carbon triple bond and one lone pair on each carbon. Six electrons occupy the three shared pairs and four electrons form the two lone pairs. The count therefore uses all ten electrons without placing extra bonds between the same two nuclei.

Step 2: Identify the orbital overlaps Each carbon is described as sp-hybridised. One hybrid orbital on each carbon overlaps along the internuclear axis to give the single σ bond. Each carbon also has two unhybridised p orbitals. Sideways overlap of the corresponding p orbitals gives two separate π bonds in perpendicular planes. The remaining hybrid orbital on each carbon contains its lone pair. Lone pairs are nonbonding and must not be included in the bond count.

Step 3: Compare all the choices The triple bond therefore has three bonding components, arranged as one sigma and two pi. Two sigma bonds between the same pair of carbon atoms are not possible in the ordinary axial-overlap description, so the choices containing two sigma bonds fail. One sigma and one pi would describe a double bond, which is inconsistent with the acetylide triple bond and the electron count. Calcium does not add another covalent sigma component to the carbon-carbon bond being counted.

Step 4: Conclusion The acetylide ion contains one σ-bond and two π-bonds. Hence, the correct option is (2).

Quick Tip to Solve: For a bond of order three, count one axial sigma bond first; the other two bonding components are pi bonds, not additional sigma bonds.

66. Which of the following has unpaired electron(s)? [4 marks]

(1) O_2^-

(2) N_2^{2+}

(3) O_2^{2-}

(4) N_2

Correct Answer: (1) O_2^-

Solution:

Concept:

- Electrons fill molecular orbitals according to increasing energy, the Pauli exclusion principle and Hund's rule. Degenerate orbitals receive single electrons before pairing.
- An unpaired molecular-orbital electron makes a species paramagnetic. Electron count and the occupancy of the highest occupied orbitals decide the answer.

Step 1: Examine the oxygen species Neutral O_2 has two electrons in its pair of degenerate antibonding π^* orbitals, one in each. Adding one electron gives superoxide, O_2^-. The resulting three electrons in these two orbitals have occupancies two and one, leaving exactly one electron unpaired. Adding two electrons instead gives peroxide, O_2^{2-}. Its four π^* electrons occupy the two orbitals as two complete pairs. Thus superoxide is paramagnetic and peroxide is diamagnetic.

Step 2: Examine both nitrogen species The ten valence electrons of N_2 occupy

$$(\sigma_{2s})^2(\sigma^*_{2s})^2(\pi_{2p_x})^2(\pi_{2p_y})^2(\sigma_{2p_z})^2.$$

Every occupied orbital is doubly occupied. For N_2^{2+}, remove the two electrons from the highest occupied σ_{2p_z} orbital. That orbital becomes empty, while all the remaining occupied orbitals stay doubly occupied. Consequently neither nitrogen species in the options has an unpaired electron.

Step 3: Select the uniquely paramagnetic ion The superoxide option has one unpaired electron. The nitrogen dication, peroxide and neutral nitrogen have none. It would be incorrect to infer paramagnetism merely from a positive or negative ionic charge: removing or adding an even number of electrons can leave a closed-shell configuration, as the other two ionic options demonstrate.

Step 4: Conclusion Only O_2^- has an unpaired electron. Hence, the correct option is (1).

Quick Tip to Solve: Use the last degenerate orbitals: three electrons in two π^* orbitals leave one unpaired, whereas four fill both as pairs.

67. The temperature at which oxygen molecules have the same root mean square speed as helium atoms have at 300 K is : (Atomic masses : He = 4 u, O = 16 u) [4 marks]

(1) 600 K

(2) 2400 K

(3) 1200 K

(4) 300 K

Correct Answer: (2) 2400 K

Solution:

Concept:

- The root mean square speed of a gas is $v_{\text{rms}} = \sqrt{3RT/M}$, where T is absolute temperature and M is molar mass in consistent units.
- For two gases to have the same root mean square speed, their ratios T/M must be equal. Oxygen is molecular O_2, while helium is monatomic.

Step 1: Use the molecular masses The atomic mass of oxygen is 16 u, but each oxygen molecule contains two oxygen atoms. Hence the molar masses are

$$M_{O_2} = 32\,\text{g mol}^{-1}, \qquad M_{\text{He}} = 4\,\text{g mol}^{-1}.$$

The mass ratio is eight. Using 16 for oxygen here would mistakenly substitute the atomic mass for the molecular molar mass and would produce the tempting 1200 K choice.

Step 2: Equate the squared speeds Let the required oxygen temperature be T. Squaring the equality of speeds avoids unnecessary square-root calculations:

$$\frac{3RT}{M_{O_2}} = \frac{3R(300)}{M_{\text{He}}}.$$

The factor $3R$ cancels on both sides. Therefore

$$T = 300\frac{M_{O_2}}{M_{\text{He}}} = 300\frac{32}{4} = 2400\,\text{K}.$$

Only the mass ratio is needed, so grams per mole may be used consistently in the ratio without a conversion to kilograms per mole.

Step 3: Check the physical trend and other options At the same temperature, the much heavier oxygen molecules move more slowly than helium atoms. Oxygen must therefore be heated above 300 K to match the helium speed. The required temperature is eight times as large, rather than twice or four times as large. Thus 600 K and 1200 K do not compensate sufficiently for the mass difference, and 300 K would leave the speed ratio equal to the inverse square root of eight.

Step 4: Conclusion The required temperature is $2400\,\text{K}$. Hence, the correct option is (2).

Quick Tip to Solve: For equal rms speeds write $T_1/M_1 = T_2/M_2$, and first check whether the gas is atomic or molecular.

68. Van der Waal's equation for a gas is stated as

$$P = \frac{nRT}{V - nb} - a\left(\frac{n}{V}\right)^2.$$

This equation reduces to the perfect gas equation, $P = nRT/V$, when: [4 marks]

(1) temperature is sufficiently high and pressure is low.

(2) both temperature and pressure are very low.

(3) both temperature and pressure are very high.

(4) temperature is sufficiently low and pressure is high.

Correct Answer: (1) temperature is sufficiently high and pressure is low.

Solution:

Concept:

- The van der Waals constant a describes the effect of intermolecular attraction on pressure. The constant b accounts for the finite excluded volume of gas molecules.
- The ideal-gas limit requires both corrections to be negligible: $nb \ll V$, and $a(n/V)^2$ small compared with the kinetic pressure nRT/V.

Step 1: Reduce the volume correction at low pressure At a given sufficiently high temperature, low pressure corresponds to a large volume per mole, V/n, and therefore low number density. Then the excluded volume is a very small fraction of the container volume:

$$\frac{nb}{V} \ll 1, \qquad V - nb \simeq V.$$

Consequently the first term in the equation approaches nRT/V. High pressure does the opposite: it compresses the gas and makes the finite molecular volume important.

Step 2: Reduce the attraction correction at high temperature Compare the attraction term with the ideal pressure term:

$$\frac{a(n/V)^2}{nRT/V} = \frac{an}{RTV}.$$

This ratio becomes small when the temperature is high and the molar volume is large. Physically, high molecular kinetic energy makes attractions less effective relative to thermal motion, while low density separates the molecules. Under both conditions,

$$P \simeq \frac{nRT}{V} - 0 = \frac{nRT}{V}.$$

Step 3: Eliminate the remaining conditions Low temperature increases the relative importance of attractions and may bring a gas towards condensation. High pressure increases density and excluded-volume effects even at high temperature. Low temperature with high pressure enhances both deviations. Thus all three remaining choices retain at least one significant correction. High temperature together with low pressure makes both corrections small.

Step 4: Conclusion The equation approaches the ideal-gas form at sufficiently high temperature and low pressure. Hence, the correct option is (1).

Quick Tip to Solve: For near-ideal behaviour remember both corrections: high temperature weakens attraction effects, and low pressure makes molecular volume negligible.

69. The standard enthalpy of formation of NH_3 is $-46.0\,kJ\,mol^{-1}$. If bond enthalpy of H_2 is $-436\,kJ\,mol^{-1}$ and that of N_2 is $-712\,kJ\,mol^{-1}$, the average bond enthalpy of the $N-H$ bond in NH_3 is: [4 marks]

(1) $-964\,kJ\,mol^{-1}$

(2) $-1102\,kJ\,mol^{-1}$

(3) $+1056\,kJ\,mol^{-1}$

(4) $+352\,kJ\,mol^{-1}$

Correct Answer: (4) $+352\,kJ\,mol^{-1}$

Solution:

Concept:

- Hess's law gives the reaction enthalpy as energy absorbed in breaking reactant bonds minus energy released in forming product bonds.
- A bond dissociation enthalpy is positive. The negative values supplied for the elemental bonds correspond to bond formation; their positive magnitudes are used for breaking those bonds.

Step 1: Write the formation reaction for one mole The standard formation equation for one mole of ammonia is

$$\tfrac{1}{2}N_2(g) + \tfrac{3}{2}H_2(g) \longrightarrow NH_3(g), \qquad \Delta H = -46.0\,kJ\,mol^{-1}.$$

Thus the reactants contain half a mole of nitrogen-nitrogen bonds and one and a half moles of hydrogen-hydrogen bonds per mole of ammonia produced. The ammonia molecule has three nitrogen-hydrogen bonds, so three average bond enthalpies must be used for the product.

Step 2: Apply the supplied bond-energy magnitudes Let D be the average positive dissociation enthalpy of one mole of nitrogen-hydrogen bonds. Breaking the reactant bonds costs

$$\frac{712}{2} + \frac{3(436)}{2} = 356 + 654 = 1010\,kJ\,mol^{-1}.$$

Forming three nitrogen-hydrogen bonds releases $3D$. The signed energy balance is therefore

$$-46 = 1010 - 3D.$$

Solving gives

$$3D = 1056, \qquad D = 352\,kJ\,mol^{-1}.$$

The negative signs printed for the elemental bond energies are not used as negative bond-breaking costs: bond breaking requires energy.

Step 3: Check the choice and the sign The positive 1056 value is the total dissociation energy for all three bonds in one mole of ammonia molecules, not the average for one mole of bonds. Dividing by three is essential. The negative numerical choices cannot represent a positive dissociation enthalpy. Substituting 352 back gives 1010 minus 1056, or minus 46, which reproduces the stated exothermic formation enthalpy.

Step 4: Conclusion Using the supplied elemental bond energies as bond-formation values, the average

bond dissociation enthalpy is $+352\,\text{kJ mol}^{-1}$. Hence, the correct option is (4).

Quick Tip to Solve: Balance the reaction for one mole of product, use positive bond-breaking magnitudes, and divide the total product bond energy by the number of bonds.

70. At a certain temperature, only 50% HI is dissociated into H_2 and I_2 at equilibrium. The equilibrium constant is: [4 marks]

(1) 0.5

(2) 4.0

(3) 1.0

(4) 0.25

Correct Answer: (4) 0.25

Solution:

Concept:

- For $2\,\text{HI}(g) \rightleftharpoons H_2(g) + I_2(g)$, the equilibrium constant is $K_c = [H_2][I_2]/[HI]^2$. Stoichiometric coefficients become powers in the expression.
- If the initial concentration of hydrogen iodide is c and its degree of dissociation is α, the equilibrium concentrations are $c(1-\alpha)$, $c\alpha/2$ and $c\alpha/2$.

Step 1: Construct the equilibrium composition Take initially pure hydrogen iodide, with concentration c, and no hydrogen or iodine. The statement that half dissociates means $\alpha = 0.50$. Accordingly,

$$[HI]_{eq} = c(1-0.50) = \frac{c}{2}.$$

Two moles of hydrogen iodide consumed produce one mole each of hydrogen and iodine. Each product therefore has half the concentration of hydrogen iodide consumed:

$$[H_2]_{eq} = [I_2]_{eq} = \frac{c(0.50)}{2} = \frac{c}{4}.$$

Assigning $c/2$ to each product would ignore the reaction stoichiometry.

Step 2: Substitute into the mass-action expression Insert these concentrations into the dissociation equilibrium constant:

$$K_c = \frac{(c/4)(c/4)}{(c/2)^2} = \frac{c^2/16}{c^2/4} = \frac{1}{4} = 0.25.$$

The initial concentration cancels because the total stoichiometric coefficient is the same on the two sides. Equivalently,

$$K_c = \frac{\alpha^2}{4(1-\alpha)^2}.$$

Putting $\alpha = 1/2$ gives the same result. Since the change in gaseous mole number is zero, $K_p = K_c$ for this reaction as well.

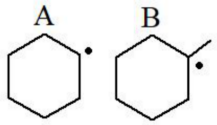
Step 3: Check the alternative numbers The value 0.5 is the fraction dissociated, not the equilibrium

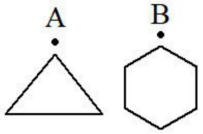
constant. The value 1 would follow from mistakenly giving each product the full concentration of hydrogen iodide that disappeared. The value 4 is the reciprocal constant for the reverse reaction forming hydrogen iodide. The question asks for the dissociation direction, so its constant must be 0.25.

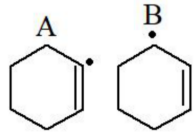
Step 4: Conclusion For $2\,HI \rightleftharpoons H_2 + I_2$, the equilibrium constant is 0.25. Hence, the correct option is (4).

Quick Tip to Solve: For dissociation of HI, halve the consumed amount for each product before substituting; 50 percent dissociation does not mean $K = 0.5$.

71. In which of the following pairs A is more stable than B? [4 marks]

(1) 

(2) 

(3) 

(4) A: $Ph_3C^{\bullet}$, B: $(CH_3)_3C^{\bullet}$

Correct Answer: (4) A: $Ph_3C^{\bullet}$, B: $(CH_3)_3C^{\bullet}$

Solution:

Concept:

- A free radical contains an unpaired electron. Delocalisation of this electron over a conjugated system lowers its energy and stabilises the radical.
- For comparable alkyl radicals, hyperconjugation favours a tertiary centre over a secondary one. A radical directly on an alkene carbon is vinylic; a radical beside the double bond is allylic and can be resonance-stabilised.

Step 1: Compare the first two pairs In pair (1), the radical centre of A is secondary, whereas the methyl-substituted radical centre in B is tertiary. The additional alkyl substitution stabilises B, so A is not more stable. In pair (2), A is a cyclopropyl radical, with the radical centre on the strained three-membered ring itself. It is not a cyclopropylmethyl radical. Compared with the ordinary cyclohexyl radical B, its constrained ring geometry does not provide the proposed extra stability; this pair does not give A greater stability.

Step 2: Identify the resonance-stabilised member In pair (3), the radical in A is directly on a carbon of the double bond: it is vinylic. The radical in B is adjacent to the double bond: it is allylic and resonance-stabilised, so B is more stable. In pair (4), A is the triphenylmethyl radical, $Ph_3C^{\bullet}$. Its unpaired electron can delocalise through the attached phenyl rings. B is the tert-butyl radical, $(CH_3)_3C^{\bullet}$, whose stabilisation is mainly hyperconjugative. The extensive resonance stabilisation of A exceeds that of B. Thus this is the pair with the required inequality.

Step 3: Conclusion Only pair (4), triphenylmethyl radical A and tert-butyl radical B, has A more

stable than B. Hence, the correct option is (4).

Quick Tip to Solve: Locate the radical dot before comparing stability: vinylic and allylic centres are different, and a cyclopropyl radical is not cyclopropylmethyl.

72. In a face centered cubic lattice atoms A are at the corner points and atoms B at the face centered points. If atom B is missing from one of the face centered points, the formula of the ionic compound is [4 marks]

(1) A_2B_3

(2) AB_2

(3) A_5B_2

(4) A_2B_5

Correct Answer: (4) A_2B_5

Solution:

Concept:

- An atom at a corner is shared by eight unit cells, so its contribution to one cell is $1/8$. A face-centred atom is shared by two cells, so its contribution is $1/2$.
- The formula of the compound is the simplest whole-number ratio of the effective numbers of A and B atoms in the specified unit cell. Vacancies must be included before simplifying the ratio.

Step 1: Count the A atoms at the corners There are eight corner positions and all are occupied by A. Their effective contribution is therefore

$$n_A = 8\left(\frac{1}{8}\right) = 1.$$

Counting eight complete A atoms would be incorrect, because each corner belongs to eight neighbouring unit cells. The corner contribution remains unchanged by a vacancy at a face centre.

Step 2: Count the occupied B positions and reduce the ratio An ordinary face-centred cubic cell has six face-centred positions. One B atom is absent from one such position, leaving five occupied face centres. Each contributes one half, giving

$$n_B = (6-1)\left(\frac{1}{2}\right) = \frac{5}{2}.$$

Hence the required ratio is

$$A : B = 1 : \frac{5}{2} = 2 : 5.$$

The usual complete-cell ratio would be $1 : 3$, but that does not apply after the vacancy. AB_2 would require only four occupied faces, while A_5B_2 reverses the correct atom ratio. A_2B_3 likewise does not follow the shared contributions.

Step 3: Conclusion The simplest formula is A_2B_5. Hence, the correct option is (4).

Quick Tip to Solve: Count occupied sites first, apply their sharing fractions next, and only then multiply the atom ratio to clear fractions.

73. A current of $10.0\,\mathrm{A}$ flows for $2.00\,\mathrm{h}$ through an electrolytic cell containing a molten salt of metal X. This results in the decomposition of $0.250\,\mathrm{mol}$ of metal X at the cathode. The oxidation state of X in the molten salt is: ($F = 96,500\,\mathrm{C}$) [4 marks]

(1) $1+$

(2) $4+$

(3) $3+$

(4) $2+$

Correct Answer: (3) $3+$

Solution:

Concept:

- The charge delivered by a constant current is $Q = It$, where I is in amperes and t is in seconds. One mole of electrons carries the Faraday charge F.
- If a metal has oxidation state $+z$, its deposition is $\mathrm{X^{z+}} + \mathrm{ze^-} \longrightarrow \mathrm{X}$. Thus z moles of electrons deposit one mole of metal and $Q = zFn_X$.

Step 1: Calculate the charge and electron amount Convert the stated two-hour interval to seconds before using the current:

$$t = 2.00 \times 3600 = 7200\,\mathrm{s}.$$

The total charge is

$$Q = 10.0 \times 7200 = 72,000\,\mathrm{C}.$$

Consequently the amount of electrons supplied to the cathode is

$$n_e = \frac{Q}{F} = \frac{72,000}{96,500} \simeq 0.746\,\mathrm{mol}.$$

The cathodic metal ion gains electrons, so its initial oxidation state must be positive.

Step 2: Find electrons required per metal atom The metal deposited is $n_X = 0.250\,\mathrm{mol}$. Therefore

$$z = \frac{n_e}{n_X} = \frac{72,000}{96,500 \times 0.250} \simeq 2.98 \simeq 3.$$

The data are rounded, and the chemically meaningful electron count is the integer three. As a check, deposition of $0.250\,\mathrm{mol}$ of a trivalent ion requires $0.750F = 72,375\,\mathrm{C}$, very close to the supplied charge. The other proposed states require $24,125$, $48,250$, or $96,500\,\mathrm{C}$, respectively; none matches the observation nearly as well.

Step 3: Conclusion The ion accepts three electrons per atom, so X has oxidation state $+3$. Hence, the correct option is (3).

Quick Tip to Solve: Use charge divided by Faraday to obtain moles of electrons, then divide by moles of deposited metal to obtain its ionic charge.

74. The standard electrode potentials $\left(E^{\circ}_{M^+/M}\right)$ of four metals A, B, C and D are $-1.2\,V$, $0.6\,V$, $0.85\,V$ and $-0.76\,V$, respectively. The sequence of deposition of metals on applying potential is [4 marks]

(1) B > D > C > A

(2) A > C > B > D

(3) C > B > D > A

(4) D > A > B > C

Correct Answer: (3) C > B > D > A

Solution:

Concept:

- A standard electrode potential written as $E^{\circ}_{M^+/M}$ is a reduction potential for $M^+ + e^- \longrightarrow M$. A more positive value means a greater tendency for the ion to be reduced.
- In the electrode-potential comparison asked here, deposition at the cathode proceeds from the most readily reduced metal ion toward the least readily reduced one. Arrange the signed reduction potentials in descending order.

Step 1: Associate each potential with its metal The word respectively fixes the correspondence:

$$E^{\circ}_A = -1.20\,V, \quad E^{\circ}_B = +0.60\,V,$$
$$E^{\circ}_C = +0.85\,V, \quad E^{\circ}_D = -0.76\,V.$$

The largest reduction potential belongs to C, followed by B. Both positive values exceed the two negative values. Among the negative values, -0.76 is greater than -1.20.

Step 2: Arrange the order of reduction and deposition The complete numerical ordering is

$$+0.85 > +0.60 > -0.76 > -1.20.$$

It follows that ions of C are reduced most easily, followed by those of B, D and A. The requested deposition sequence is therefore

$$C > B > D > A.$$

Ordering by absolute magnitude would incorrectly place A near the beginning; the signs must be retained. Option (1) fails by putting D before C, option (2) begins with the least readily reduced ion A, and option (4) also puts negative-potential ions ahead of C and B. These orders do not follow the stated reduction potentials.

Step 3: Conclusion The sequence is $C > B > D > A$. Hence, the correct option is (3).

Quick Tip to Solve: For cathodic deposition, rank the signed reduction potentials from most positive to most negative; do not rank their absolute values.

75. The half-life period of a first-order reaction is 15 minutes. The amount of substance left after one hour will be [4 marks]

(1) $\frac{1}{16}$ of the original amount

(2) $\frac{1}{8}$ of the original amount

(3) $\frac{1}{32}$ of the original amount

(4) $\frac{1}{4}$ of the original amount

Correct Answer: (1) $\frac{1}{16}$ of the original amount

Solution:

Concept:

- A first-order reaction has a constant half-life: in each interval $t_{1/2}$, half the amount present at the start of that interval remains.
- If N_0 is the initial amount, the fraction left after time t is $N/N_0 = (1/2)^{t/t_{1/2}}$. This formula counts successive halvings, rather than subtracting a fixed amount.

Step 1: Count the elapsed half-lives The half-life is $15\,\text{min}$, while the elapsed time is one hour. Use matching time units:

$$t = 60\,\text{min}, \qquad n = \frac{t}{t_{1/2}} = \frac{60}{15} = 4.$$

Thus the reaction proceeds through four complete half-life intervals. Their end times are 15, 30, 45 and 60 minutes. The first-order designation is essential, because it makes each of these intervals the same length.

Step 2: Apply four successive halvings The amount remaining changes according to

$$N_0 \longrightarrow \frac{N_0}{2} \longrightarrow \frac{N_0}{4} \longrightarrow \frac{N_0}{8} \longrightarrow \frac{N_0}{16}.$$

Equivalently,

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}.$$

This is the amount left, not the amount consumed: the latter would be $15N_0/16$. The fraction $1/8$ corresponds to three half-lives or 45 minutes; $1/32$ corresponds to five or 75 minutes; $1/4$ corresponds to two or 30 minutes. All three describe different elapsed times from the one asked.

Step 3: Conclusion The substance left after one hour is $N_0/16$, or $1/16$ of the original amount. Hence, the correct option is (1).

Quick Tip to Solve: Convert the total time to the half-life unit, count the intervals, and raise one half to that count.

76. The form of iron obtained from blast furnace is [4 marks]

(1) Cast Iron

(2) Wrought Iron

(3) Steel

(4) Pig Iron

Correct Answer: (4) Pig Iron

Solution:

Concept:

- A blast furnace reduces iron oxides from the ore using carbon monoxide generated from coke. The iron melts and collects below the slag at the bottom of the furnace.
- The immediate furnace product is carbon-rich and still contains impurities such as silicon, phosphorus, manganese and sulphur. This crude product is called pig iron; further processing gives other commercial forms.

Step 1: Identify the immediate furnace product Inside the blast furnace, carbon monoxide removes oxygen from iron oxide, represented by

$$\mathrm{Fe_2O_3 + 3\,CO \longrightarrow 2\,Fe + 3\,CO_2}.$$

At the high operating temperature the reduced iron melts. It dissolves carbon from the coke and carries other impurities reduced from the ore and flux mixture. The metal tapped from the furnace is therefore not pure iron. This crude, carbon-rich material is the product conventionally termed pig iron.

Step 2: Distinguish the refined forms in the alternatives Cast iron is obtained by processing and remelting pig iron, commonly with scrap iron; it is a later commercial product and should not replace the name of the initial furnace output. Wrought iron is a much purer, very low-carbon form obtained after extensive removal of carbon and impurities. Steel is produced by controlled refining, with its carbon content and alloy composition adjusted for useful mechanical properties. Those forms correspond to subsequent treatment rather than the direct reduction and melting stage described here. Therefore the names cast iron, wrought iron and steel do not answer the specific question about what comes out of the blast furnace.

Step 3: Conclusion The iron obtained directly from the blast furnace is pig iron. Hence, the correct option is (4).

Quick Tip to Solve: Keep the production stages distinct: blast-furnace tapping gives pig iron, while refining or remelting produces the other commercial forms.

77. The gas evolved on heating CaF_2 and SiO_2 with concentrated H_2SO_4, on hydrolysis gives a white gelatinous precipitate. The precipitate is [4 marks]

(1) Silicic acid

(2) Calcium fluorosilicate

(3) Hydrofluorosilicic acid

(4) Silica gel

Correct Answer: (1) Silicic acid

Solution:

Concept:

- Concentrated sulphuric acid releases hydrogen fluoride from calcium fluoride. Hydrogen fluoride reacts with silica to form volatile silicon tetrafluoride, SiF_4.
- Hydrolysis of SiF_4 gives a gelatinous precipitate of silicic acid, conventionally represented as H_2SiO_3, together with hydrofluorosilicic acid in solution. The precipitate and soluble acid must be distinguished.

Step 1: Identify the gas produced on heating The first reaction supplies hydrogen fluoride:

$$CaF_2 + H_2SO_4 \longrightarrow CaSO_4 + 2\,HF.$$

Silica then reacts with this reagent:

$$SiO_2 + 4\,HF \longrightarrow SiF_4 + 2\,H_2O.$$

Thus the gas whose hydrolysis is being considered is silicon tetrafluoride, rather than calcium fluorosilicate or sulphur dioxide. The silicon in SiF_4 remains in oxidation state $+4$; the subsequent process is hydrolysis, not a redox change.

Step 2: Identify the gelatinous hydrolysis product The overall hydrolysis may be written in its conventional textbook form as

$$3\,SiF_4 + 3\,H_2O \longrightarrow H_2SiO_3\downarrow + 2\,H_2SiF_6.$$

This equation conserves three silicon atoms, twelve fluorine atoms, six hydrogen atoms and three oxygen atoms. The white gelatinous precipitate is silicic acid, H_2SiO_3. Hydrofluorosilicic acid, H_2SiF_6, is the soluble coproduct, so option (3) names the wrong phase. Calcium fluorosilicate is not produced by this hydrolysis. Silica gel is related to hydrated silica obtained by ageing or drying silica-containing gels; the named freshly precipitated hydrolysis product in this reaction is silicic acid.

Step 3: Conclusion The white gelatinous precipitate is silicic acid, H_2SiO_3; hydrofluorosilicic acid is the soluble coproduct, so the printed reaction supports option (1) despite the supplied key naming hydrofluorosilicic acid. Hence, the correct option is (1).

Quick Tip to Solve: For the fluoride test with silica, follow the sequence HF, then SiF4, then gelatinous silicic acid on hydrolysis; the fluorosilicic acid remains in solution.

78. Which of the following is not formed when H_2S reacts with acidic $K_2Cr_2O_7$ solution? [4 marks]

(1) K_2SO_4

(2) $Cr_2(SO_4)_3$

(3) S

(4) $CrSO_4$

Correct Answer: (4) $CrSO_4$

Solution:

Concept:

- Acidified dichromate is an oxidising agent: chromium is reduced from oxidation state $+6$ to $+3$.
- Hydrogen sulphide acts as the reducing agent. Sulphur changes from oxidation state -2 in H_2S to zero in elemental sulphur.

Step 1: Write the reduction and oxidation changes The acidic dichromate half-reaction is

$$Cr_2O_7^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O.$$

Each molecule of hydrogen sulphide loses two electrons:

$$H_2S \longrightarrow S + 2\,H^+ + 2\,e^-.$$

Multiplying the second equation by three matches the six electrons accepted by one dichromate ion. Thus the chromium product contains trivalent chromium, not divalent chromium.

Step 2: Combine the changes and identify the salts Adding the half-reactions and cancelling electrons and excess hydrogen ions gives

$$Cr_2O_7^{2-} + 8\,H^+ + 3\,H_2S \longrightarrow 2\,Cr^{3+} + 7\,H_2O + 3\,S.$$

In sulphuric acid, potassium and chromium form their sulphate salts. The complete equation is

$$K_2Cr_2O_7 + 4\,H_2SO_4 + 3\,H_2S \longrightarrow K_2SO_4 + Cr_2(SO_4)_3 + 7\,H_2O + 3\,S.$$

This equation is balanced for potassium, chromium, sulphur, hydrogen and oxygen. Options (1), (2) and (3) are therefore actual products. The salt in (4) would require chromium in the $+2$ state, which is not the dichromate reduction product under these conditions.

Step 3: Conclusion $CrSO_4$ is not formed; the chromium salt is $Cr_2(SO_4)_3$. Hence, the correct option is (4).

Quick Tip to Solve: First identify the chromium oxidation state in each proposed salt: acidic dichromate normally gives Cr^{3+}, so a chromium(II) product is immediately suspect.

79. The correct statement about the magnetic properties of $[Fe(CN)_6]^{3-}$ and $[FeF_6]^{3-}$ is: $(Z = 26)$.

[4 marks]

(1) Both are paramagnetic.

(2) Both are diamagnetic.

(3) $[Fe(CN)_6]^{3-}$ is paramagnetic, $[FeF_6]^{3-}$ is diamagnetic.

(4) $[Fe(CN)_6]^{3-}$ is diamagnetic, $[FeF_6]^{3-}$ is paramagnetic.

Correct Answer: (1) Both are paramagnetic.

Solution:

Concept:

- An unpaired electron makes a complex paramagnetic; diamagnetism requires every electron to be paired.
- An octahedral field splits the five d orbitals into lower t_{2g} and upper e_g levels. Strong-field ligands favour pairing; weak-field ligands favour high spin.

Step 1: Determine the metal electron count Each cyanide or fluoride ligand has charge -1. If the oxidation state of iron is x, then

$$x + 6(-1) = -3 \quad \Rightarrow \quad x = +3.$$

Neutral iron has configuration $[\mathrm{Ar}]3d^6 4s^2$. Removing the two $4s$ electrons and one $3d$ electron gives Fe^{3+} with $3d^5$. Thus both complexes contain five d electrons; changing the ligand does not change the metal oxidation state.

Step 2: Distribute the five d electrons in each field Cyanide is a strong-field ligand. The low-spin arrangement is

$$[\mathrm{Fe(CN)_6}]^{3-} : \quad t_{2g}^5 e_g^0.$$

Three lower orbitals containing five electrons have two pairs and one unpaired electron. Fluoride is weak field, so the high-spin arrangement is

$$[\mathrm{FeF_6}]^{3-} : \quad t_{2g}^3 e_g^2.$$

All five electrons are unpaired. Both complexes are consequently paramagnetic, although their magnetic moments differ. The spin-only estimates are $\sqrt{3}$ and $\sqrt{35}$ BM respectively. Options (2), (3) and (4) each wrongly call at least one of these odd-electron complexes diamagnetic.

Step 3: Conclusion Both complexes have unpaired electrons and are paramagnetic. Hence, the correct option is (1).

Quick Tip to Solve: Do not equate a strong-field ligand with diamagnetism. A low-spin d^5 ion still has one unpaired electron.

80. A compound of vanadium chloride has spin only magnetic moment of $1.73\,\mathrm{BM}$. Its formula is [4 marks]

(1) VCl_2

(2) VCl_5

(3) VCl_4

(4) VCl_3

Correct Answer: (3) VCl_4

Solution:

Concept:

- The spin-only moment is $\mu = \sqrt{n(n+2)}\,\mathrm{BM}$, where n is the number of unpaired electrons.
- Vanadium has atomic number 23 and ground-state configuration $[\mathrm{Ar}]3d^3 4s^2$. On ionisation the $4s$ electrons are removed before the $3d$ electrons.

Step 1: Infer the unpaired-electron count The given value is essentially $\sqrt{3}\,\mathrm{BM}$. Squaring the formula gives

$$n(n+2) = (1.73)^2 \simeq 3.$$

The nonnegative integer solution is $n = 1$, since $1(1+2) = 3$. The small difference between 1.73 and $\sqrt{3}$ is just rounding. We therefore need the vanadium oxidation state whose d configuration contains one unpaired electron, rather than one with two or three unpaired electrons.

Step 2: Test the proposed chloride formulas Each chloride has charge -1, so the metal oxidation state equals the number of chlorides in each neutral formula. The possibilities are

$$\begin{aligned} VCl_2: &\quad V^{2+},\ d^3,\ n = 3,\\ VCl_5: &\quad V^{5+},\ d^0,\ n = 0,\\ VCl_4: &\quad V^{4+},\ d^1,\ n = 1,\\ VCl_3: &\quad V^{3+},\ d^2,\ n = 2. \end{aligned}$$

A single d electron must be unpaired regardless of the ordinary ligand-field splitting. Thus the predicted moments for (1), (2) and (4) are $\sqrt{15}$, zero and $\sqrt{8}$ BM, respectively, whereas (3) gives $\sqrt{3}$ BM. This simultaneously rules out every distractor.

Step 3: Conclusion The required formula is VCl_4, with vanadium(IV) in the d^1 configuration. Hence, the correct option is (3).

Quick Tip to Solve: Memorise the diagnostic moments: one, two and three unpaired electrons give about 1.73, 2.83 and 3.87 BM, respectively; then check the oxidation state.

81. An octahedral complex of Co^{3+} is diamagnetic. The hybridisation involved in the formation of the complex is: [4 marks]

(1) dsp^2

(2) d^2sp^3

(3) dsp^3d

(4) sp^3d^2

Correct Answer: (2) d^2sp^3

Solution:

Concept:

- In valence-bond theory, an octahedral complex uses six hybrid orbitals. Inner-orbital octahedral hybridisation is d^2sp^3; outer-orbital octahedral hybridisation is sp^3d^2.
- Diamagnetic means that no unpaired electrons remain. For a d^6 metal ion this requires a low-spin arrangement.

Step 1: Find the electronic configuration of cobalt(III) Neutral cobalt has atomic number 27 and electron configuration $[\mathrm{Ar}]3d^74s^2$. Formation of Co^{3+} removes the two $4s$ electrons and one $3d$ electron, leaving

$$\mathrm{Co}^{3+}: \quad [\mathrm{Ar}]3d^6.$$

In a weak octahedral field, six electrons would occupy the split levels as $t_{2g}^4e_g^2$, leaving four unpaired electrons. That arrangement is inconsistent with the stated diamagnetism.

Step 2: Pair the electrons and identify the six hybrids In the required low-spin state the six electrons pair in the three lower d orbitals:

$$t_{2g}^6e_g^0.$$

In the valence-bond description this pairing leaves two inner $3d$ orbitals available for bonding. Those two orbitals combine with one $4s$ and three $4p$ orbitals to make six equivalent octahedral hybrids:

$$2(3d) + 1(4s) + 3(4p) \longrightarrow d^2sp^3.$$

Option (1) uses four hybrids and is associated with square-planar coordination. Option (3), printed as dsp^3d, is not the conventional inner-orbital notation: it separates the two d contributions rather than specifying the two inner $3d$ orbitals together. The standard inner-orbital hybridisation is explicitly d^2sp^3, as in (2). Option (4) uses outer d orbitals and is the outer-orbital description, rather than the paired inner-orbital complex demanded here.

Step 3: Conclusion The diamagnetic octahedral Co^{3+} complex has inner-orbital d^2sp^3 hybridisation. Hence, the correct option is (2).

Quick Tip to Solve: For octahedral d^6, diamagnetism means t_{2g}^6. In the valence-bond notation that is the inner-orbital d^2sp^3 case.

82. Allyl phenyl ether can be prepared by heating: [4 marks]

(1) $CH_2=CH-Br + C_6H_5-CH_2-ONa$

(2) $C_6H_5-CH=CH-Br + CH_3-ONa$

(3) $CH_2=CH-CH_2-Br + C_6H_5ONa$

(4) $C_6H_5Br + CH_2=CH-CH_2-ONa$

Correct Answer: (3) $CH_2=CH-CH_2-Br + C_6H_5ONa$

Solution:

Concept:

- Williamson ether synthesis couples an alkoxide or phenoxide ion with a suitable alkyl halide by an S_N2 displacement.
- Allyl phenyl ether has connectivity $C_6H_5-O-CH_2-CH=CH_2$: oxygen must connect directly to the phenyl ring and to the saturated allylic carbon.

Step 1: Choose the correct bond to form Disconnect the ether at the oxygen–allyl bond. This gives sodium phenoxide, C_6H_5ONa, as the nucleophile and allyl bromide, $CH_2=CH-CH_2-Br$, as the electrophile. The carbon carrying bromine is primary and allylic, which is well suited to backside nucleophilic substitution. The reaction is

$$C_6H_5ONa + CH_2{=}CH{-}CH_2{-}Br \longrightarrow C_6H_5{-}O{-}CH_2{-}CH{=}CH_2 + NaBr.$$

The phenoxide oxygen supplies the electron pair, while bromide leaves the saturated carbon.

Step 2: Check why the other reagent pairs fail In (1), bromine is bonded directly to a vinyl carbon, $CH_2=CH-Br$; such a carbon does not undergo ordinary Williamson S_N2 substitution. The benzyl alkoxide in that option also gives the wrong oxygen-bound aromatic connectivity. Option (2) likewise contains a vinylic bromide and methoxide, so it is neither a suitable substrate nor the required phenoxide–allyl pairing. Option (4) places bromine on the phenyl ring. An unactivated aryl bromide does not undergo the ordinary S_N2 process with allyloxide because attack at its sp^2 carbon is unsuitable. Thus reversing the two fragments is not a valid equivalent preparation; the location of the leaving group matters as much as the identities of the fragments.

Step 3: Conclusion Heating allyl bromide with sodium phenoxide gives allyl phenyl ether. Hence, the correct option is (3).

Quick Tip to Solve: For an aryl–alkyl ether, put the aryl group on the phenoxide nucleophile and the leaving group on the alkyl fragment; avoid aryl or vinyl halides in Williamson synthesis.

83. For the compounds CH_3Cl, CH_3Br, CH_3I and CH_3F, the correct order of increasing C-halogen bond length is: [4 marks]

(1) $CH_3F < CH_3Br < CH_3Cl < CH_3I$

(2) $CH_3F < CH_3I < CH_3Br < CH_3Cl$

(3) $CH_3F < CH_3Cl < CH_3Br < CH_3I$

(4) $CH_3Cl < CH_3Br < CH_3F < CH_3I$

Correct Answer: (3) $CH_3F < CH_3Cl < CH_3Br < CH_3I$

Solution:

Concept:

- For a comparable series of single covalent bonds, bond length follows the sizes of the two bonded atoms; approximately $d(\mathrm{C-X}) \simeq r_{\mathrm{C}} + r_{\mathrm{X}}$.
- Halogen covalent radius increases down group 17: $r_{\mathrm{F}} < r_{\mathrm{Cl}} < r_{\mathrm{Br}} < r_{\mathrm{I}}$, because successive elements have additional occupied electron shells.

Step 1: Hold the carbon environment fixed All four compounds are methyl halides. The carbon is sp^3-hybridised and bonded to three hydrogen atoms and one halogen. Each C–halogen bond is a single bond, so neither a change of bond order nor a change of carbon hybridisation is involved. This makes halogen size the direct comparison variable. A larger halogen has its bonding electron cloud extending farther from the nucleus and hence a larger equilibrium separation from carbon.

Step 2: Apply the periodic size trend and compare options Fluorine is the smallest of the four atoms, followed by chlorine, bromine and iodine. Consequently,

$$d(\mathrm{C-F}) < d(\mathrm{C-Cl}) < d(\mathrm{C-Br}) < d(\mathrm{C-I}).$$

In molecular notation the increasing order is

$$CH_3F < CH_3Cl < CH_3Br < CH_3I.$$

Option (1) incorrectly places C–Br below C–Cl in length. Option (2) puts the largest halogen, iodine, before bromine and chlorine. Option (4) places the short C–F bond after both C–Cl and C–Br. Option (3) alone respects the increasing atomic size at every adjacent comparison. Electronegativity alone should not be used as a direct numerical measure of bond length; here the common carbon environment makes the radius trend decisive.

Step 3: Conclusion The increasing C–halogen bond-length order is $CH_3F < CH_3Cl < CH_3Br < CH_3I$. Hence, the correct option is (3).

Quick Tip to Solve: When the carbon skeleton and bond order are unchanged, rank C–halogen lengths using halogen radii: F, Cl, Br, I from shortest to longest.

84. In a nucleophilic substitution reaction:

$$\mathrm{R{-}Br} + \mathrm{Cl^-} \xrightarrow{\mathrm{DMF}} \mathrm{R{-}Cl} + \mathrm{Br^-},$$

which one of the following undergoes complete inversion of configuration? [4 marks]

(1) $C_6H_5CH(C_6H_5)Br$

(2) $C_6H_5C(CH_3)(C_6H_5)Br$

(3) $C_6H_5CH_2Br$

(4) $C_6H_5CH_2CH_2Br$

Correct Answer: No unique literal stereochemical answer; (4) is the intended elementary mechanism choice.

Solution:

Concept:

- An S_N2 displacement involves backside attack and inverts the tetrahedral arrangement at the attacked carbon. A change between distinguishable stereoisomers is observable only when that carbon is stereogenic.
- DMF is a polar aprotic solvent, so it favours anionic nucleophiles and S_N2 substitution. Benzylic substrates can also react rapidly by this mechanism.

Step 1: Identify the intended elementary mechanism comparison Option (4) is the primary non-benzylic bromide $C_6H_5CH_2CH_2Br$. The carbon bearing bromine is one carbon removed from the aromatic ring, so direct departure of bromide would not produce a resonance-stabilised benzylic carbocation. Chloride attack in DMF is therefore expected to use the concerted S_N2 pathway, with backside displacement. The recorded answer key selects (4). (1) and (2) can form resonance-stabilised diaryl carbocations, and (2) is additionally too hindered for ordinary S_N2 attack.

Step 2: Check the literal stereochemical wording The printed options do not contain a stereogenic carbon bearing bromine. In (1) that carbon carries two identical phenyl groups; (2) also has two identical phenyl groups. In (3) and (4) the brominated carbon carries two identical hydrogen atoms. Thus none has two distinguishable configurations at that carbon whose complete inversion could be established. Moreover, (3) is benzyl bromide, $C_6H_5CH_2Br$, which is also an excellent S_N2 substrate with chloride in a polar aprotic solvent. Resonance stabilisation of a possible carbocation does not prove that (3) must undergo S_N1 under these conditions. Therefore a strict claim that only (4) undergoes backside displacement is also unsupported by the supplied conditions.

Step 3: Conclusion The intended key is (4), the primary non-benzylic substrate favouring S_N2. Literally, however, all printed substrates are achiral at the reaction centre and benzyl bromide can also undergo S_N2; the question does not establish a unique example of observable complete inversion.

Quick Tip to Solve: Distinguish a backside-attack mechanism from observable stereochemical inversion: first check for four different groups at the carbon carrying the leaving group.

85. In the hydroboration - oxidation reaction of propene with diborane, H_2O_2 and $NaOH$, the organic compound formed is: [4 marks]

(1) $CH_3CH_2CH_2OH$

(2) CH_3CH_2OH

(3) $CH_3CHOHCH_3$

(4) $(CH_3)_3COH$

Correct Answer: (1) $CH_3CH_2CH_2OH$

Solution:

Concept:

- Hydroboration followed by alkaline hydrogen peroxide gives anti-Markovnikov hydration of an alkene: the hydroxyl group occupies the less substituted carbon.
- Boron and hydrogen add across the double bond in a concerted syn addition. Oxidation replaces the carbon-boron bond by a carbon-oxygen bond without rearranging the carbon skeleton.

Step 1: Locate the less substituted carbon Propene is $CH_3-CH=CH_2$. The terminal carbon of the double bond bears two hydrogen atoms, while the internal carbon bears one hydrogen and a methyl group. During hydroboration, boron preferentially attaches to the terminal carbon; the hydrogen from the borane attaches to the internal carbon. This regioselectivity is the opposite of acid-catalysed Markovnikov hydration.

Step 2: Follow hydroboration and oxidation Diborane supplies borane, which reacts with the alkene to form organoborane intermediates. Alkaline H_2O_2 then oxidizes the carbon-boron bonds to alcohol groups. For each propene unit, the net transformation is

$$CH_3CH=CH_2 \xrightarrow[2.\ H_2O_2,\,OH^-]{1.\ BH_3} CH_3CH_2CH_2OH.$$

No carbon atom is added or removed, and no carbocation rearrangement is involved.

Step 3: Check the other products Propan-2-ol would put the hydroxyl group on the more substituted carbon, corresponding to Markovnikov hydration. Ethanol has only two carbon atoms, so it cannot result from this hydration of a three-carbon alkene. Tert-butanol contains four carbon atoms and a branched skeleton, neither of which this reaction produces.

Step 4: Conclusion The product is propan-1-ol, $CH_3CH_2CH_2OH$. Hence, the correct option is (1).

Quick Tip to Solve: For hydroboration-oxidation, first preserve the alkene carbon skeleton, then place OH on its less substituted double-bond carbon.

86. Which is the major product formed when acetone is heated with iodine and potassium hydroxide?

[4 marks]

(1) Acetophenone

(2) Iodoform

(3) Iodoacetone

(4) Acetic acid

Correct Answer: (2) Iodoform

Solution:

Concept:

- A methyl ketone, $RCOCH_3$, reacts with iodine in alkali by the haloform reaction to yield CHI_3 and a carboxylate salt.
- Acetone, CH_3COCH_3, contains the required methyl-ketone group. The three alpha hydrogens on one methyl group are replaced by iodine before cleavage.

Step 1: Identify the reactive group Acetone has a carbonyl carbon bonded to two methyl groups. It therefore satisfies the structural test for the iodoform reaction. In alkaline solution, removal of an alpha hydrogen produces an enolate, which reacts with iodine. Repeated iodination converts one methyl group next to the carbonyl into a triiodomethyl group.

Step 2: Apply cleavage by hydroxide Hydroxide attacks the carbonyl group of the triiodinated ketone. Cleavage releases the stabilized triiodomethyl species, which gains a proton to give yellow iodoform. The other fragment remains as acetate because the reaction mixture is strongly alkaline. The balanced overall equation is

$$CH_3COCH_3 + 3\,I_2 + 4\,KOH \longrightarrow CHI_3 + CH_3COOK + 3\,KI + 3\,H_2O.$$

Both carbon atoms and iodine atoms balance in this equation.

Step 3: Distinguish the listed products Iodoacetone can be an intermediate stage of iodination, but continued reaction with iodine and alkali leads to haloform cleavage. Acetophenone would require formation of a phenyl-containing skeleton, which acetone and these reagents cannot supply. Acetic acid is not the neutral organic product isolated under alkaline conditions: its corresponding acetate salt is formed along with iodoform. Thus the diagnostic product among the listed compounds is iodoform.

Step 4: Conclusion Acetone undergoes the iodoform reaction to form CHI_3. Hence, the correct option is (2).

Quick Tip to Solve: Look for the CH_3CO^- group when iodine and alkali are given; its characteristic product is yellow CHI_3.

87. Which one of the following reactions will not result in the formation of carbon-carbon bond ?

[4 marks]

(1) Friedel Craft's acylation

(2) Reimer-Tieman reaction

(3) Cannizaro reaction

(4) Wurtz reaction

Correct Answer: (3) Cannizaro reaction

Solution:

Concept:

- A carbon-carbon bond-forming reaction joins a carbon atom of one reacting fragment to a carbon atom of another fragment.
- The Cannizzaro reaction is base-induced disproportionation of a non-enolizable aldehyde: one molecule is oxidized to carboxylate and another is reduced to alcohol, with no coupling of their carbon skeletons.

Step 1: Check Friedel-Crafts and Reimer-Tiemann reactions In Friedel-Crafts acylation, an acyl electrophile replaces an aromatic hydrogen, forming the bond $\mathrm{Ar{-}C(O)R}$. The carbonyl carbon is newly bonded to the aromatic ring. In the Reimer-Tiemann reaction, phenol reacts with chloroform and alkali and, after hydrolysis, acquires a formyl group, chiefly at the ortho position. The new bond joins the ring carbon to the carbon of the introduced $-\mathrm{CHO}$ group. Both reactions therefore form a carbon-carbon bond.

Step 2: Examine the Cannizzaro transformation For an aldehyde with no alpha hydrogen, concentrated alkali transfers hydride between two aldehyde molecules. For example,

$$2\,\mathrm{C_6H_5CHO} + \mathrm{NaOH} \longrightarrow \mathrm{C_6H_5COONa} + \mathrm{C_6H_5CH_2OH}.$$

The carbon framework of each benzaldehyde molecule remains separate. One aldehyde carbon changes oxidation state upward and the other downward. No new bond connects the carbon atoms of these separate molecules.

Step 3: Check Wurtz coupling The Wurtz reaction couples alkyl halides with sodium in dry ether:

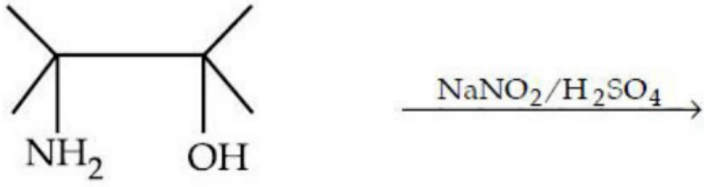
$$2\,\mathrm{R{-}X} + 2\,\mathrm{Na} \longrightarrow \mathrm{R{-}R} + 2\,\mathrm{NaX}.$$

Its defining change is the formation of a bond between the two alkyl carbon fragments. Thus it cannot be the requested exception. Among the four named transformations, only Cannizzaro is a disproportionation rather than a carbon-carbon coupling or carbon introduction reaction.

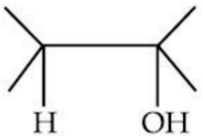
Step 4: Conclusion The Cannizzaro reaction does not create a carbon-carbon bond. Hence, the correct option is (3).

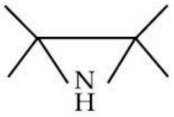
Quick Tip to Solve: Draw the before-and-after carbon skeleton: Cannizzaro changes oxidation states, whereas Wurtz and aromatic carbon substitutions extend a skeleton.

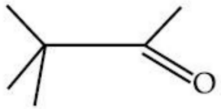
88. The major product of the reaction is [4 marks]

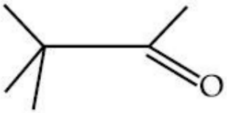


(1) 

(2) 

(3) 

(4) 

Correct Answer: (4) 

Solution:

Concept:

- Sodium nitrite with sulfuric acid generates nitrous acid. A primary aliphatic amine is converted into an unstable aliphatic diazonium group, which loses N_2.
- In a vicinal amino alcohol, loss of nitrogen can be accompanied by a 1,2-alkyl shift and formation of a carbonyl group. This semipinacol rearrangement stabilizes the electron-deficient centre.

Step 1: Identify the adjacent amino alcohol centres The substrate has adjacent carbons bearing NH_2 and OH; each of these carbons also carries two methyl groups. Denote the amino-bearing carbon by A and the hydroxyl-bearing carbon by B. Nitrous acid transforms the amino group into a leaving diazonium group at A. Loss of nitrogen generates strong electron deficiency at A, next to the alcohol centre B.

Step 2: Follow methyl migration and carbonyl formation A methyl group on B migrates with its bonding electron pair from B to A as nitrogen is expelled. The oxygen lone pair forms the carbonyl bond at B; deprotonation then gives the neutral ketone. Since the two methyl groups on B are equivalent, either migration gives the same product. Carbon A now bears three methyl groups, while carbon B is the carbonyl carbon bonded to its remaining methyl group. The product is therefore $(CH_3)_3C-C(=O)-CH_3$, pinacolone, or 3,3-dimethylbutan-2-one.

Step 3: Compare the four displayed structures The fourth structure is precisely this rearranged ketone: a tert-butyl group lies on one side of the carbonyl and a methyl group on the other. The first is an alkene and lacks the retained oxygen. The second is the unrearranged deaminated alcohol, which does not reflect the neighbouring alcohol-assisted carbonyl-forming migration. The third retains nitrogen in a three-membered ring, whereas diazotization of this primary aliphatic amine expels nitrogen gas.

Step 4: Conclusion The major product is the rearranged ketone $(CH_3)_3CCOCH_3$, shown in structure (4). Hence, the correct option is (4).

Quick Tip to Solve: For a vicinal amino alcohol treated with nitrous acid, track loss of N_2, migration from the OH-bearing carbon, and formation of $C=O$.

89. Structure of some important polymers are given. Which one represents Buna-S ? [4 marks]

(1) $\left[-CH_2-CH=CH-CH_2-CH(CN)-CH_2-\right]_n$

(2) $\left[-CH_2-CH=CH-CH_2-CH(C_6H_5)-CH_2-\right]_n$

(3) $\left[-CH_2-C(CH_3)=CH-CH_2-\right]_n$

(4) $\left[-CH_2-C(Cl)=CH-CH_2-\right]_n$

Correct Answer: (2) $\left[-CH_2-CH=CH-CH_2-CH(C_6H_5)-CH_2-\right]_n$

Solution:

Concept:

- Buna-S is the copolymer of 1,3-butadiene and styrene. Styrene contributes a pendant phenyl group, C_6H_5, on the saturated part of the polymer chain.
- A 1,4-addition unit of butadiene retains a carbon-carbon double bond inside the chain. The identity of the substituent distinguishes several common synthetic rubbers.

Step 1: Identify the monomer signatures Butadiene has the structure $CH_2{=}CH{-}CH{=}CH_2$, and styrene is $CH_2{=}CH{-}C_6H_5$. Addition polymerization incorporates both double-bond monomers into one chain. The butadiene-derived portion contains $-CH_2-CH{=}CH-CH_2-$, while the styrene-derived portion contains $-CH_2-CH(C_6H_5)-$. Consequently the decisive identifying feature of Buna-S is a pendant phenyl group together with the butadiene segment.

Step 2: Select the phenyl-containing chain Option (2) displays the butadiene segment and a chain carbon carrying C_6H_5. This matches the two monomers of Buna-S. The schematic grouping in the option identifies its constituent units; an actual copolymer need not have a perfectly alternating monomer sequence for its chemical identity to be Buna-S. It is the presence of styrene units, rather than a particular drawing orientation, that settles the choice.

Step 3: Identify the other polymer substituents The pendant CN group in option (1) identifies acrylonitrile units and therefore the butadiene-acrylonitrile rubber Buna-N. The methyl-substituted diene repeat in option (3) is the polyisoprene-type unit associated with natural rubber. The chlorine-substituted diene repeat in option (4) is polychloroprene, or neoprene. None contains the styrene phenyl group required for Buna-S.

Step 4: Conclusion The phenyl-containing butadiene-styrene copolymer is structure (2). Hence, the correct option is (2).

Quick Tip to Solve: Use the pendant group as a fingerprint: phenyl means Buna-S, cyano means Buna-N, and chlorine means neoprene.

90. Which one of the following class of compounds is obtained by polymerization of acetylene ?

[4 marks]

(1) Poly-ene

(2) Poly-amide

(3) Poly-yne

(4) Poly-ester

Correct Answer: (1) Poly-ene

Solution:

Concept:

- Acetylene, $HC{\equiv}CH$, undergoes addition polymerization to form polyacetylene, whose chain contains alternating single and double carbon-carbon bonds.
- An ene denotes a carbon-carbon double bond. A polyene has multiple such double bonds, whereas amide and ester polymers require nitrogen- or oxygen-containing functional groups.

Step 1: Write the polymer repeat unit In chain polymerization of acetylene, one component of the triple bond is used to connect successive monomer units, while unsaturation remains within the chain. The resulting transformation is represented as

$$n\,HC{\equiv}CH \longrightarrow \left[-CH = CH-\right]_n.$$

The carbon atoms retain one hydrogen each, so the empirical composition remains CH. Polymerization adds the monomers together without elimination of a small molecule.

Step 2: Classify the bonds in the chain Adjacent repeat units are joined by carbon-carbon single bonds, with a carbon-carbon double bond within each $-CH{=}CH-$ unit. Thus the extended chain has alternating single and double bonds and is a conjugated polyene. Although the monomer contains a triple bond, the name of the polymer class follows the bonds present in the resulting chain, not simply the suffix of the monomer name.

Step 3: Eliminate the other classes A polyamide contains repeated $-CONH-$ linkages and therefore nitrogen and oxygen. A polyester contains repeated $-COO-$ linkages and therefore oxygen. Neither can be obtained from acetylene alone by this addition process. A polyyne would retain repeated triple bonds in its backbone; that is not the $-CH{=}CH-$ chain produced by polymerization of acetylene. The preserved double bonds instead establish the polyene classification.

Step 4: Conclusion Polymerization of acetylene produces polyacetylene, a polyene. The supplied answer key selects poly-yne, but the literal polymerization gives the double-bond repeat unit rather than a triple-bond chain. Hence, the correct option is (1).

Quick Tip to Solve: Write the repeat unit before naming the polymer: acetylene gives $-CH{=}CH-$, so the polymer retains double bonds rather than triple bonds.