JEE Main 2014 Paper 1 (B.E./B.Tech) Apr 11 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2014, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 11 April 2014, Single slot. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. If α and β are roots of the equation, $x^2 - 4\sqrt{2}kx + 2e^{4\ln k} - 1 = 0$ for some k, and $\alpha^2 + \beta^2 = 66$, then $\alpha^3 + \beta^3$ is equal to: [4 marks]

(1) $248\sqrt{2}$

(2) $280\sqrt{2}$

(3) $-32\sqrt{2}$

(4) $-280\sqrt{2}$

Correct Answer: (2) $280\sqrt{2}$

Solution:

Concept:

- For a monic quadratic $x^2 - Sx + P = 0$, the sum and product of its roots are $S = \alpha + \beta$ and $P = \alpha\beta$. These relations avoid solving separately for the two roots.
- The power-sum identities are $\alpha^2 + \beta^2 = S^2 - 2P$ and $\alpha^3 + \beta^3 = S^3 - 3PS$. The real logarithm $\ln k$ requires $k > 0$.

Step 1: Express the root sum and product in terms of k Comparing coefficients and simplifying the exponential gives

$$S = 4\sqrt{2}k, \qquad P = 2e^{4\ln k} - 1 = 2k^4 - 1.$$

The equality $e^{4\ln k} = k^4$ is valid because $k > 0$. Retaining this domain condition will determine which square root of k^2 is admissible.

Step 2: Use the given sum of squares to determine k Substitute the expressions for S and P in the first power-sum identity:

$$66 = 32k^2 - 2(2k^4 - 1) = 32k^2 - 4k^4 + 2.$$

Rearranging and dividing by four yields

$$k^4 - 8k^2 + 16 = 0 \quad \Longrightarrow \quad (k^2 - 4)^2 = 0.$$

Thus $k^2 = 4$, and the logarithm domain excludes $k = -2$; hence $k = 2$. Consequently $S = 8\sqrt{2}$ and $P = 31$.

Step 3: Evaluate the required cubic power sum Apply the cubic identity without finding the individual roots:

$$\begin{aligned} \alpha^3 + \beta^3 &= (8\sqrt{2})^3 - 3(31)(8\sqrt{2}) \\ &= 1024\sqrt{2} - 744\sqrt{2} \\ &= 280\sqrt{2}. \end{aligned}$$

Options (3) and (4) have the wrong sign. Option (1), $248\sqrt{2}$, does not equal the verified difference $1024\sqrt{2} - 744\sqrt{2}$.

Step 4: Conclusion $\boxed{\alpha^3 + \beta^3 = 280\sqrt{2}}$. The supplied key gives the negative value $-280\sqrt{2}$, which conflicts with the logarithm domain $k > 0$ and the exact calculation $1024\sqrt{2} - 744\sqrt{2}$ above. Hence,

the correct option is (2).

Quick Tip to Solve: Introduce $S = \alpha + \beta$ and $P = \alpha\beta$ before expanding powers; use the logarithm domain to reject the spurious negative value of k.

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2. If z_1, z_2 and z_3, z_4 are 2 pairs of complex conjugate numbers, then $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$ equals [4 marks]

(A) 0
(B) $\frac{\pi}{2}$
(C) $\frac{3\pi}{2}$
(D) π

Correct Answer: (A) 0

Solution:

Concept:

- For conjugate complex numbers, $z_2 = \overline{z_1}$ and $z_4 = \overline{z_3}$. Conjugation changes the sign of an argument.
- For nonzero complex numbers, the argument of a quotient equals the difference of the arguments, modulo 2π. A nonzero number times its conjugate is positive real.

Step 1: Express the two quotients as conjugates Put $w = z_1/z_4$. Since the denominator is nonzero, conjugating the quotient gives

$$\overline{w} = \frac{\overline{z_1}}{\overline{z_4}} = \frac{z_2}{z_3}.$$

Thus the two numbers whose arguments are added have equal moduli and opposite phases. This uses both conjugate pairs in their stated order; conjugating only the numerator would not establish the result.

Step 2: Add the arguments and check the alternatives Write $w = \rho e^{i\phi}$, with $\rho > 0$. Its conjugate is $\rho e^{-i\phi}$, so the two phase angles cancel:

$$\arg w + \arg \overline{w} \equiv \phi - \phi \equiv 0 \pmod{2\pi}.$$

Equivalently,

$$\frac{z_1}{z_4}\frac{z_2}{z_3} = \frac{|z_1|^2}{|z_3|^2} > 0.$$

A positive real product has phase zero. The proposed values $\pi/2$, $3\pi/2$ and π would give a non-real or negative product, and hence are inconsistent. Argument addition here is understood modulo a complete revolution, as in the quotient rule.

Step 3: Conclusion The argument sum is 0 modulo 2π, corresponding to the listed value 0. Hence, the correct option is (A).

Quick Tip to Solve: Before expanding real and imaginary parts, test whether the two quotients are conjugates; their product is then positive real and their phases cancel.

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3. An eight digit number divisible by 9 is to be formed using digits from 0 to 9 without repeating the digits. The number of ways in which this can be done is [4 marks]

(A) 72(7!)

(B) 18(7!)

(C) 40(7!)

(D) 36(7!)

Correct Answer: (D) 36(7!)

Solution:

Concept:

- A decimal integer is divisible by 9 exactly when the sum of its digits is divisible by 9. The sum of all ten distinct digits is 45.
- An eight-digit selection omits two digits. Its admissible arrangements are $8!$ if zero is absent, and $8! - 7!$ if zero is present, because a leading zero is forbidden.

Step 1: Find the possible omitted digit pairs If the omitted distinct digits are u, v, the retained sum is $45 - u - v$. Divisibility by 9 therefore requires $u + v$ to be a multiple of 9. For two distinct digits between zero and nine the sum is between 1 and 17, so it must be 9. The unordered omitted pairs are

$$(0, 9),\ (1, 8),\ (2, 7),\ (3, 6),\ (4, 5).$$

This enumeration is exhaustive: exchanging the two omitted digits does not create a new selection.

Step 2: Arrange the retained digits and exclude leading zero Omitting $(0, 9)$ leaves eight nonzero digits, giving $8!$ numbers. Each of the other four pairs leaves zero among the eight retained digits. Of their $8!$ permutations, exactly $7!$ start with zero and have only seven digits as integers. The selections are disjoint, so the required total is

$$N = 8! + 4(8! - 7!) = (8 + 4 \cdot 7)7! = 36(7!).$$

Counting all five selections as $8!$ would give $40(7!)$, which explains the tempting wrong option. The values $72(7!)$ and $18(7!)$ do not account correctly for the unordered omitted pairs and the leading-zero restriction.

Step 3: Conclusion The number of permissible eight-digit integers is $\boxed{36(7!)}$. Hence, the correct option is (D).

Quick Tip to Solve: For a large selection of distinct digits, count the omitted digits first; handle zero separately only when arranging each retained selection.

4. In a geometric progression, if the ratio of the sum of first 5 terms to the sum of their reciprocals is 49 , and the sum of the first and the third term is 35. Then the first term of this geometric progression is [4 marks]

(A) 7

(B) 21

(C) 28

(D) 42

Correct Answer: (C) 28

Solution:

Concept:

- For a real geometric progression with nonzero first term a and common ratio r, the first five terms are a, ar, ar^2, ar^3, ar^4.
- Reversing the five reciprocal terms shows that their sum is the sum of the original five terms divided by a^2r^4. This avoids any restriction such as $r \neq 1$ in a geometric-sum formula.

Step 1: Relate the two sums to the middle term Let $S = a + ar + ar^2 + ar^3 + ar^4$, and let R be the sum of the reciprocals. Reversing their order gives

$$R = \frac{1}{ar^4}(1 + r + r^2 + r^3 + r^4) = \frac{S}{a^2r^4}.$$

Since the stated quotient $S/R = 49$ exists, the sums involved are nonzero. Consequently

$$a^2r^4 = 49, \qquad (ar^2)^2 = 49,$$

so the third term ar^2 is either 7 or -7. Retaining both signs at this stage prevents an unjustified square-root choice.

Step 2: Use the sum of the first and third terms The other condition is $a + ar^2 = 35$. If $ar^2 = 7$, then $a = 28$, giving $r^2 = 7/28 = 1/4$, a valid real ratio. If $ar^2 = -7$, then $a = 42$, which would force $r^2 = -7/42 = -1/6$, impossible for a real progression. Thus the apparent candidate 42 must be rejected. The options 7 and 21 also fail: subtracting them from 35 gives third terms 28 and 14, whose squares are not 49. The valid ratio can be either $1/2$ or $-1/2$; both give the same first term.

Step 3: Conclusion The first term is $\boxed{28}$. Hence, the correct option is (C).

Quick Tip to Solve: For an odd number of GP terms, compare the sum with the reversed reciprocal sum: their quotient is the square of the middle term.

5. The sum of the first 20 terms common between the series 3 + 7 + 11 + 15+ and 1 + 6 + 11+ 16 + ... is [4 marks]

(A) 4000

(B) 4020

(C) 4200

(D) 4220

Correct Answer: (B) 4020

Solution:

Concept:

- The terms of the first arithmetic progression have the form $3 + 4m$, and those of the second have the form $1 + 5n$, with m, n nonnegative integers.
- Common terms recur at intervals equal to $\text{lcm}(4, 5) = 20$. The sum of N terms of an arithmetic progression is $N[2a + (N - 1)d]/2$.

Step 1: Locate the first common term and the common interval The first progression begins $3, 7, 11, 15, 19, \ldots$, while the second begins $1, 6, 11, 16, 21, \ldots$. Their first common term is therefore 11. Every further common term differs from it by a multiple of both 4 and 5, hence by a multiple of 20. Conversely $11 + 20k$ belongs to both progressions for every integer $k \geq 0$, because

$$11 + 20k = 3 + 4(2 + 5k) = 1 + 5(2 + 4k).$$

Thus no common term is missed: the required common sequence is $11, 31, 51, \ldots$.

Step 2: Sum exactly twenty common terms The twentieth term is obtained with index $k = 19$, not 20:

$$a_{20} = 11 + 19(20) = 391.$$

Pairing the first and last terms, the second and penultimate terms, and so on gives ten equal pairs. Therefore

$$S_{20} = \frac{20}{2}(11 + 391) = 10(402) = 4020.$$

The value 4000 would correspond to an incorrect starting offset, and 4220 or 4200 cannot equal the sum determined by the endpoints 11 and 391. The direct AP formula gives the same check: $10[2(11) + 19(20)] = 4020$.

Step 3: Conclusion The first twenty common terms sum to $\boxed{4020}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the first common term explicitly, use the LCM of the original differences, and remember that twenty terms contain nineteen intervals.

6. The coefficient of x^{50} in the binomial expansion of

$$(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+\cdots+x^{1000}$$

is: [4 marks]

(1) $\dfrac{(1000)!}{(50)(!95\varphi!}$

(2) $\dfrac{(1000)!}{(49)(!95)!}$

(3) $\dfrac{(1001)!}{(51)(!95\phi!}$

(4) $\dfrac{(1001)!}{(50)(!95)!}$

Correct Answer: The coefficient is $\dfrac{1001!}{50!\,951!}$; none of the printed options states this value unambiguously.

Solution:

Concept:

- A finite geometric sum with successive terms $a^{n-j}b^j$ satisfies $(a-b)\sum_{j=0}^{n} a^{n-j}b^j = a^{n+1}-b^{n+1}$. It can simplify a sum of many binomial expansions before any coefficient is extracted.
- The binomial theorem gives $[x^r](1+x)^N = \binom{N}{r} = N!/[r!(N-r)!]$, where $[x^r]$ denotes the coefficient of x^r.

Step 1: Collapse the finite sum before expanding Write the given polynomial as

$$S(x)=\sum_{j=0}^{1000} x^j(1+x)^{1000-j}.$$

Here the successive factors are $a=1+x$, $b=x$, and $a-b=1$. The finite-sum identity therefore gives the exact polynomial equality

$$S(x)=(1+x)^{1001}-x^{1001}.$$

It can also be checked by multiplying the sum by $(1+x)-x$: the intermediate terms cancel in pairs.

Step 2: Extract the coefficient of the requested power The subtracted monomial has degree 1001, so it contributes nothing to the coefficient of x^{50}. Thus

$$[x^{50}]S(x)=\binom{1001}{50}=\frac{1001!}{50!\,951!}.$$

The second factorial in the denominator is $(1001-50)!=951!$; it is not $95!$, $950!$, or a factorial involving ϕ. This subtraction of indices is an independent check of the denominator.

Step 3: Compare with the literal printed choices The printed choices contain damaged factorial notation. Choice (1) has numerator $1000!$ and a malformed denominator; choice (2) also has numerator $1000!$ and shows 49 and 95 in its denominator. Choice (3) has numerator $1001!$, but its denominator begins with 51 and includes malformed symbols. Choice (4) has numerator $1001!$, but the denominator visibly ends with $(95)!$, rather than the required $951!$, and does not correctly state $50!\,951!$. None is an unambiguous representation of the calculated coefficient; changing a printed digit or inserting missing factorial symbols would change a choice.

Step 4: Conclusion $\boxed{[x^{50}]S(x) = \binom{1001}{50} = \frac{1001!}{50!\,951!}}$. The supplied key selects option (4), but its literal denominator does not state this value. Therefore the printed options do not permit a defensible unique selection.

Quick Tip to Solve: For sums of $x^j(1+x)^{n-j}$, multiply by $(1+x)-x=1$ to telescope, then verify both factorial indices before matching an option.

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7. If $2\cos\theta + \sin\theta = 1$ $\left(\theta \neq \frac{\pi}{2}\right)$, then $7\cos\theta + 6\sin\theta$ is equal to [4 marks]

(A) $\frac{1}{2}$
(B) 2
(C) $\frac{11}{2}$
(D) $\frac{46}{5}$

Correct Answer: (B) 2

Solution:

Concept:

- The identity $\sin^2\theta + \cos^2\theta = 1$ lets us combine a linear relation between sine and cosine with the unit-circle condition.
- When squaring after isolating a trigonometric function, substitute the resulting candidates back into the original relation and apply the stated exclusion.

Step 1: Solve the given relation for sine and cosine Put $c = \cos\theta$ and $s = \sin\theta$. From $2c + s = 1$, obtain $s = 1 - 2c$. Substituting into the identity yields

$$c^2 + (1-2c)^2 = 1,$$

and hence

$$5c^2 - 4c = 0, \qquad c(5c-4) = 0.$$

The candidates are $c = 0$ and $c = 4/5$. The first gives $s = 1$, the right-angle branch. The printed exclusion removes $\theta = \pi/2$; taking the intended non-right-angle branch gives the remaining candidate below. For the remaining branch,

$$c = \frac{4}{5}, \qquad s = 1 - \frac{8}{5} = -\frac{3}{5}.$$

They satisfy both original equations.

Step 2: Evaluate the requested linear combination It is useful to write $7c + 6s = 6(2c+s) - 5c$, since the parenthesis is already known to be one. Thus

$$7c + 6s = 6 - 5\left(\frac{4}{5}\right) = 6 - 4 = 2.$$

The negative sign of sine is essential. None of $1/2$, $11/2$ or $46/5$ equals the verified result.

Step 3: Conclusion The listed value is $\boxed{2}$; the supplied key value $46/5$ incorrectly takes sine positive. Literally, $\theta = \pi/2 + 2k\pi$, $k \neq 0$, also satisfies the printed exclusion and gives the unlisted value 6; the listed answer uses the intended exclusion of that right-angle branch. Hence, the correct option is (B).

Quick Tip to Solve: After finding cosine, rewrite the target as a multiple of the given linear relation plus one trigonometric term to reduce arithmetic.

8. The base of an equilateral triangle is along the line given by 3x + 4y = 9. If a vertex of the triangle is (1, 2), then the length of a side of the triangle is [4 marks]

(1) $\frac{2\sqrt{3}}{15}$

(2) $\frac{4\sqrt{3}}{15}$

(3) $\frac{4\sqrt{3}}{5}$

(4) $\frac{2\sqrt{3}}{5}$

Correct Answer: (2) $\frac{4\sqrt{3}}{15}$

Solution:

Concept:

- The perpendicular distance of (x_0, y_0) from $ax + by + c = 0$ is $h = |ax_0 + by_0 + c|/\sqrt{a^2 + b^2}$. This distance is the altitude when the opposite side lies on that line.
- An equilateral triangle of side s has altitude $h = \sqrt{3}\, s/2$. The altitude bisects the base, producing two right triangles with hypotenuse s.

Step 1: Identify the vertex opposite the base The supplied vertex is $V = (1, 2)$. Substitution into the base equation gives $3(1) + 4(2) = 11$, not 9, so this vertex does not lie on the base. It is therefore the opposite vertex, and its perpendicular distance from $3x + 4y - 9 = 0$ gives the required altitude. This avoids confusing a coordinate difference with the actual perpendicular distance.

Step 2: Compute the altitude Use the normal coefficients 3 and 4 of the base line. Their squared sum is 25, so the normal has length 5. Hence

$$h = \frac{|3(1) + 4(2) - 9|}{\sqrt{3^2 + 4^2}} = \frac{|2|}{5} = \frac{2}{5}.$$

The absolute value ensures that the geometrical length is positive, regardless of which side of the line contains the vertex.

Step 3: Convert the altitude into the side For an equilateral triangle, $s = 2h/\sqrt{3}$. Substituting the altitude and rationalising gives

$$s = \frac{2(2/5)}{\sqrt{3}} = \frac{4}{5\sqrt{3}} = \frac{4\sqrt{3}}{15}.$$

The first option is half this value. The third option is three times this value, and the fourth option is $3/2$ times it. Thus they cannot produce the computed altitude when multiplied by $\sqrt{3}/2$.

Step 4: Conclusion The length of a side is $\boxed{\frac{4\sqrt{3}}{15}}$. Hence, the correct option is (2).

Quick Tip to Solve: Find the perpendicular height from the opposite vertex first; then use side $= 2\times$ height $/\sqrt{3}$ for an equilateral triangle.

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9. The set of all real values of λ for which exactly two common tangents can be drawn to the circles $x^2 + y^2 - 4x - 4y + 6 = 0$ and $x^2 + y^2 - 10x - 10y + \lambda = 0$ is the interval: [4 marks]

(1) (12, 32)

(2) (18, 42)

(3) (12, 24)

(4) (18, 48)

Correct Answer: (2) (18, 42)

Solution:

Concept:

- For two circles with radii r, R and distance d between their centres, exactly two common tangents exist when they intersect at two points: $|R - r| < d < R + r$.
- Complete the squares to read each centre and radius. Equality in either tangency condition gives a different number of common tangents, so interval endpoints must be excluded.

Step 1: Find the centres and radii Complete the squares separately:

$$(x - 2)^2 + (y - 2)^2 = 2, \qquad (x - 5)^2 + (y - 5)^2 = 50 - \lambda.$$

Thus the centres are $C_1 = (2, 2)$ and $C_2 = (5, 5)$, with $r = \sqrt{2}$ and $R = \sqrt{50 - \lambda}$. Their separation is

$$d = \sqrt{(5 - 2)^2 + (5 - 2)^2} = 3\sqrt{2}.$$

We will impose a positive real radius through the geometric inequalities below.

Step 2: Impose the two-intersection condition Exactly two common tangents are the two direct tangents for intersecting circles. The inequalities are

$$|R - \sqrt{2}| < 3\sqrt{2} < R + \sqrt{2}.$$

The right-hand inequality gives $R > 2\sqrt{2}$. The left-hand one gives $-2\sqrt{2} < R < 4\sqrt{2}$. Combining these restrictions, remembering that a radius is nonnegative, gives

$$2\sqrt{2} < R < 4\sqrt{2}.$$

Internal or external tangency is excluded because the inequalities are strict.

Step 3: Translate the radius bounds to the parameter Both bounds are positive, so squaring preserves their order:

$$8 < 50 - \lambda < 32.$$

Solving the left and right inequalities gives $\lambda < 42$ and $\lambda > 18$, respectively. Consequently $18 < \lambda < 42$. Option (1) wrongly includes parameters below 18; option (3) both includes those parameters and omits valid values above 24. Option (4) includes parameters above 42, where the two-intersection condition fails.

Step 4: Conclusion The required parameter interval is $\boxed{(18, 42)}$. Hence, the correct option is (2).

Quick Tip to Solve: For exactly two common tangents, start with intersecting circles: the centre distance lies strictly between the difference and sum of the radii.

10. Let L_1 be the length of the common chord of the curves $x^2 + y^2 = 9$ and $y^2 = 8x$, and L_2 be the length of the latus rectum of $y^2 = 8x$, then: [4 marks]

(1) $L_1 > L_2$

(2) $L_1 = L_2$

(3) $L_1 < L_2$

(4) $\dfrac{L_1}{L_2} = \sqrt{2}$

Correct Answer: (3) $L_1 < L_2$

Solution:

Concept:

- The common chord joins the two intersection points of the curves. Symmetry about the x-axis makes its length twice the positive intersection ordinate.
- A parabola $y^2 = 4ax$ has latus rectum length $4a$. A negative candidate for x is inadmissible in $y^2 = 8x$ for real points.

Step 1: Eliminate the ordinate At an intersection, both $x^2 + y^2 = 9$ and $y^2 = 8x$ must hold. Substitute the second relation into the first:

$$x^2 + 8x = 9 \quad \Longrightarrow \quad x^2 + 8x - 9 = (x - 1)(x + 9) = 0.$$

The algebraic candidates are therefore $x = 1$ and $x = -9$. Before calculating a distance, we must check which candidate corresponds to real points on the parabola.

Step 2: Find the real common chord Because $y^2 \geq 0$, the equation $y^2 = 8x$ requires $x \geq 0$. Hence $x = -9$ must be rejected. For $x = 1$, $y^2 = 8$, so the two intersection points are $(1, 2\sqrt{2})$ and $(1, -2\sqrt{2})$. Their common chord is vertical and has length

$$L_1 = |2\sqrt{2} - (-2\sqrt{2})| = 4\sqrt{2}.$$

Both points also satisfy the circle, since $1 + 8 = 9$.

Step 3: Compare with the latus rectum Comparing $y^2 = 8x$ with $y^2 = 4ax$ gives $a = 2$. Thus $L_2 = 4a = 8$, and

$$\frac{L_1}{L_2} = \frac{4\sqrt{2}}{8} = \frac{1}{\sqrt{2}} < 1.$$

Therefore neither $L_1 > L_2$ nor $L_1 = L_2$ is possible. The proposed ratio $\sqrt{2}$ in option (4) is the reciprocal of the actual ratio, so that option also fails.

Step 4: Conclusion We obtain $L_1 = 4\sqrt{2} < 8 = L_2$. Hence, the correct option is (3).

Quick Tip to Solve: After eliminating y^2, reject negative x for this right-opening parabola before finding the chord; its symmetric ordinates then give the length directly.

11. A stair-case of length l rests against a vertical wall and a floor of a room. Let P be a point on the stair-case, nearer to its end on the wall, that divides its length in the ratio $1 : 2$. If the staircase begins to slide on the floor, then the locus of P is: [4 marks]

(1) an ellipse of eccentricity $\frac{1}{2}$

(2) an ellipse of eccentricity $\frac{\sqrt{3}}{2}$

(3) a circle of radius $\frac{l}{2}$

(4) a circle of radius $\frac{\sqrt{3}}{2}l$

Correct Answer: (2) an ellipse of eccentricity $\frac{\sqrt{3}}{2}$

Solution:

Concept:

- If the ends of a fixed-length ladder are $A = (0, a)$ on the wall and $B = (b, 0)$ on the floor, Pythagoras gives $a^2 + b^2 = l^2$.
- Internal division in a fixed ratio makes the point coordinates fixed multiples of the endpoint distances. An ellipse with semiaxes $u > v$ has eccentricity $e = \sqrt{1 - v^2/u^2}$.

Step 1: Use the specified division ratio Take the wall as the y-axis and the floor as the x-axis, meeting at the origin. Let the ladder ends be $A = (0, a)$ and $B = (b, 0)$, where $a, b \geq 0$. Since P is nearer the wall end, $AP : PB = 1 : 2$. By the section formula,

$$P = (x, y) = \left(\frac{b}{3}, \frac{2a}{3}\right).$$

Thus $b = 3x$ and $a = 3y/2$. The ratio is assigned using the stated nearer endpoint, rather than arbitrarily choosing one orientation.

Step 2: Eliminate the changing endpoint positions The length of the staircase remains l as the ends slide, so

$$a^2 + b^2 = l^2.$$

Substitute $a = 3y/2$ and $b = 3x$ to obtain

$$9x^2 + \frac{9}{4}y^2 = l^2 \quad \Longrightarrow \quad \frac{x^2}{l^2/9} + \frac{y^2}{4l^2/9} = 1.$$

This is an ellipse with semiminor axis $v = l/3$ and semimajor axis $u = 2l/3$. Physically the point traces the first-quadrant arc while both ends stay on the given wall and floor.

Step 3: Calculate the eccentricity and reject the circles The unequal semiaxes rule out both circular options, since a circle would require equal coefficients of x^2 and y^2 after normalisation. The eccentricity is

$$e = \sqrt{1 - \frac{(l/3)^2}{(2l/3)^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}.$$

The ratio $v/u = 1/2$ is not the eccentricity; using that ratio directly would incorrectly select option (1).

Step 4: Conclusion The locus lies on an ellipse of eccentricity $\boxed{\sqrt{3}/2}$. Hence, the correct option is (2).

Quick Tip to Solve: For a point dividing a sliding ladder, write its coordinates from the section formula and substitute into the fixed-length equation; compute eccentricity only after identifying both semiaxes.

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12. Let $P(3\sec\theta, 2\tan\theta)$ and $Q(3\sec\phi, 2\tan\phi)$ where $\theta + \phi = \frac{\pi}{2}$, be two distinct points on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$. Then the ordinate of the point of intersection of the normals at P and Q is [4 marks]

(1) $\frac{11}{3}$

(2) $-\frac{11}{3}$

(3) $\frac{13}{2}$

(4) $-\frac{13}{2}$

Correct Answer: (4) $-\frac{13}{2}$

Solution:

Concept:

- For $x^2/a^2 - y^2/b^2 = 1$, implicit differentiation gives tangent slope $b^2x/(a^2y)$. The normal slope is its negative reciprocal when the slopes are finite.
- At $(a\sec t, b\tan t)$, the normal can be written $ax\sin t + by = (a^2 + b^2)\tan t$. Complementary parameters satisfy $\sin\phi = \cos\theta$ and $\tan\phi = \cot\theta$.

Step 1: Write the normal at a parameter point Here $a = 3$, $b = 2$, so the tangent slope at $(3\sec t, 2\tan t)$ is

$$\frac{4(3\sec t)}{9(2\tan t)} = \frac{2\sec t}{3\tan t}.$$

Its normal slope is $-3\sin t/2$. Using the point-slope form and multiplying by 2,

$$2y - 4\tan t = -3x\sin t + 9\tan t,$$

which simplifies to $3x\sin t + 2y = 13\tan t$. The coefficient 13 comes from $a^2 + b^2$, not their difference.

Step 2: Use the complementary parameters Put $s = \sin\theta$, $c = \cos\theta$. Since $\phi = \pi/2 - \theta$, the two normals give

$$3sx + 2y = 13\frac{s}{c}, \qquad 3cx + 2y = 13\frac{c}{s}.$$

Both s and c are nonzero because the secants and tangents defining both points must be finite. Also $s \neq c$; equality, together with complementary parameters, would make $P = Q$, contrary to the condition that they are distinct.

Step 3: Eliminate the abscissa Subtract the equations and factor $s^2 - c^2$:

$$3(s-c)x = 13\frac{s^2 - c^2}{sc} \quad \Longrightarrow \quad x = \frac{13(s+c)}{3sc}.$$

Substituting in the first normal yields

$$2y = \frac{13s}{c} - \frac{13(s+c)}{c} = -13, \qquad y = -\frac{13}{2}.$$

This fixed ordinate is independent of the choice of the distinct points. The positive value in option (3) has the wrong sign, while the values involving $11/3$ cannot satisfy the derived normal equations.

Step 4: Conclusion The ordinate of the intersection is $\boxed{-\frac{13}{2}}$. Hence, the correct option is (4).

Quick Tip to Solve: For normals at complementary parameters, write the two linear equations using sine and cosine, subtract them, and substitute back; the ordinate simplifies without finding either parameter.

13. If $\lim\limits_{x\to 2} \dfrac{\tan(x - 2\{x^2 + k + 2x - 2k\}}{x^2 - 4x + 4} = 5$**, then** k **is equal to:** [4 marks]

(1) 0

(2) 1

(3) 2

(4) 3

Correct Answer: The numerator has an unmatched tangent parenthesis, so the printed expression does not define an unambiguous limit or a unique value of k.

Solution:

Concept:

- A limit must begin with a well-defined function in a punctured neighbourhood of the approach point. Parentheses specify the argument of a trigonometric function; an absent closing parenthesis can change the expression being evaluated.

- The denominator is $x^2 - 4x + 4 = (x-2)^2$. A finite nonzero quotient limit requires a numerator vanishing to second order, rather than a nonzero value or a simple zero.

Step 1: Check the exact function notation The numerator is printed as $\tan(x - 2\{x^2 + k + 2x - 2k\}$, with an opening parenthesis after $\tan$ and no matching closing parenthesis. Therefore it does not specify where the tangent argument ends. In particular, one cannot silently turn it into a product involving $\tan(x-2)$, or supply a different exponent in the polynomial. Such changes would define a different question.

Step 2: Test whether placing the missing parenthesis at the end resolves it This is a conditional consistency check, not a change to the printed question. If the tangent were applied to the entire displayed expression, its argument would be

$$g(x) = x - 2(x^2 + k + 2x - 2k) = -2x^2 - 3x + 2k.$$

At $x = 2$, $g(2) = 2k - 14$. For the four choices $k = 0, 1, 2, 3$, these values are $-14, -12, -10, -8$, respectively. None is a multiple of π: a nonzero integer cannot equal a nonzero integer multiple of the irrational number π. Thus the numerator would not tend to zero, while the denominator tends to zero, so no displayed choice would give the finite limit 5.

Step 3: Check that a special k cannot cure this interpretation Even if an unrestricted real k made $g(2) = m\pi$ for some integer m, the derivative is $g\prime(2) = -11$. Writing $h = x - 2$, the tangent would then have expansion $\tan(g(2 + h)) = -11h + O(h^2)$, and division by h^2 would diverge. A tangent pole likewise cannot give the stated finite limit. Hence adding that one closing parenthesis would not produce the required second-order vanishing; a unique answer would require further unspecified changes.

Step 4: Conclusion The printed numerator does not define an unambiguous function, and the four options cannot be justified from it. The supplied key selects option (4), $k = 3$, but substituting that value in the conditional full-argument reading leaves $\tan(-8) \neq 0$ over a vanishing denominator. No unique value of k follows from the question as printed.

Quick Tip to Solve: Factor a vanishing denominator first and check the required order of the numerator zero; never choose a limit answer by silently supplying missing function punctuation.

14. The proposition $\sim(p \vee \sim q) \vee \sim(p \vee q)$ is logically equivalent to [4 marks]

(1) p

(2) q

(3) $\sim$p

(4) $\sim$q

Correct Answer: (3) $\sim$p

Solution:

Concept:

- De Morgan laws give $\neg(A \vee B) = (\neg A) \wedge (\neg B)$, while double negation gives $\neg(\neg A) = A$. Negation must be applied to the whole parenthesised statement.
- Distributivity gives $(A \wedge B) \vee (A \wedge C) = A \wedge (B \vee C)$. The excluded-middle statement $q \vee \neg q$ is always true.

Step 1: Apply De Morgan laws to each term Write the printed proposition using $\neg$ for the negation symbol:

$$E = \neg(p \vee \neg q) \vee \neg(p \vee q).$$

For the first term, negating both members changes the connective to AND, and the double negation of q cancels. The second term is handled in the same way. Therefore

$$\neg(p \vee \neg q) = \neg p \wedge q, \qquad \neg(p \vee q) = \neg p \wedge \neg q.$$

This step keeps the scope of each negation explicit.

Step 2: Factor the common statement Both resulting terms contain $\neg p$. Apply distributivity:

$$E = (\neg p \wedge q) \vee (\neg p \wedge \neg q) = \neg p \wedge (q \vee \neg q).$$

The bracket is true for either possible value of q, so

$$E = \neg p \wedge \text{True} = \neg p.$$

Thus the truth value of the original expression depends only on p, and is opposite to that of p.

Step 3: Check that the other options are not equivalent When p is true, both negated disjunctions in the original expression are false, so the result is false, whereas option (1), p, is true. Holding p false makes the expression true for both choices of q: one of the two AND terms must then hold. Hence neither q nor $\neg q$, options (2) and (4), can agree for all truth assignments. Only $\neg p$ has the required truth values.

Step 4: Conclusion The given proposition is logically equivalent to $\boxed{\neg p}$. Hence, the correct option is (3).

Quick Tip to Solve: After using De Morgan laws, look for a common literal; a remaining bracket of the form $q \vee \neg q$ collapses to true.

15. Two ships A and B are sailing straight away from a fixed point O along routes such that $\angle AOB$ is always 120°. At a certain instance, $OA = 8\,\text{km}$, $OB = 6\,\text{km}$, and ship A is sailing at the rate of $20\,\text{km/hr}$ while ship B is sailing at the rate of $30\,\text{km/hr}$. Then the distance between A and B is changing at the rate (in km/hr): [4 marks]

(A) $\frac{260}{\sqrt{37}}$

(B) $\frac{260}{37}$

(C) $\frac{80}{\sqrt{37}}$

(D) $\frac{80}{37}$

Correct Answer: (A) $\frac{260}{\sqrt{37}}$

Solution:

Concept:

- For a triangle with sides a, b, c and included angle θ, the cosine rule gives $c^2 = a^2 + b^2 - 2ab\cos\theta$. Here $a = OA$, $b = OB$, and $c = AB$.
- When the included angle is constant, differentiate the distance relation with respect to time. The instantaneous speeds $\dot{a}, \dot{b}$ are positive because both ships move away from O.

Step 1: Write the relation between the distances The angle remains 120°, so $\cos 120^\circ = -1/2$. Therefore the separation obeys

$$c^2 = a^2 + b^2 + ab.$$

At the specified instant,

$$c^2 = 8^2 + 6^2 + 8 \cdot 6 = 148, \qquad c = 2\sqrt{37}\,\text{km}.$$

The positive square root is required because c is a physical distance.

Step 2: Differentiate before substituting the speeds Apply the product rule to ab, since both distances vary:

$$2c\dot{c} = 2a\dot{a} + 2b\dot{b} + a\dot{b} + b\dot{a}.$$

Substituting $a = 8$, $b = 6$, $\dot{a} = 20$, $\dot{b} = 30$ gives

$$2c\dot{c} = 320 + 360 + 240 + 120 = 1040.$$

Consequently

$$\dot{c} = \frac{1040}{4\sqrt{37}} = \frac{260}{\sqrt{37}}\,\text{km/hr}.$$

This positive value agrees with the increasing separation.

Step 3: Check the alternatives The denominator contains $\sqrt{37}$, since the differentiated equation divides by the distance c, not by c^2. This excludes (B) and (D). Option (C) has the correct type of denominator but its numerator omits contributions from the moving sides; inserting it into the differentiated equation gives 320 rather than 1040.

Step 4: Conclusion The ships separate at $\boxed{260/\sqrt{37}\,\text{km/hr}}$. Hence, the correct option is (A).

Quick Tip to Solve: Differentiate the cosine-rule equation while both sides are still variables; substitute their lengths and speeds only afterwards.

16. The angle of elevation of the top of a vertical tower from a point P on the horizontal ground was observed to be α. After moving a distance 2 metres from P towards the foot of the tower, the angle of elevation changes to β. Then the height (in metres) of the tower is: [4 marks]

(A) $\dfrac{2\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$

(B) $\dfrac{\sin\alpha\sin\beta}{\cos(\beta-\alpha)}$

(C) $\dfrac{2\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$

(D) $\dfrac{\cos(\beta-\alpha)}{\sin\alpha\sin\beta}$

Correct Answer: (A) $\dfrac{2\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$

Solution:

Concept:

- For a vertical tower of height h and horizontal distance d from its foot, the angle of elevation θ satisfies $\tan\theta = h/d$, or $d = h\cot\theta$.
- The identity $\sin(\beta-\alpha) = \sin\beta\cos\alpha - \cos\beta\sin\alpha$ converts a difference of cotangents to a single quotient.

Step 1: Express the two horizontal distances Let the original distance from P to the foot of the tower be d metres and let its height be h metres. The initial observation gives

$$d = h\cot\alpha.$$

Moving 2 metres towards the tower reduces the distance to $d-2$, so

$$d - 2 = h\cot\beta.$$

The tower height is unchanged, and the closer observation has the larger angle, $\beta > \alpha$.

Step 2: Subtract to find the height Subtract the second distance equation from the first:

$$2 = h(\cot\alpha - \cot\beta).$$

Thus

$$h = \frac{2}{\cot\alpha - \cot\beta}.$$

Now

$$\cot\alpha - \cot\beta = \frac{\cos\alpha\sin\beta - \cos\beta\sin\alpha}{\sin\alpha\sin\beta} = \frac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}.$$

Substitution gives the required positive height.

Step 3: Compare the four expressions It follows that

$$h = \frac{2\sin\alpha\sin\beta}{\sin(\beta-\alpha)}\,\text{m}.$$

Option (B) uses a cosine of the angle difference and also loses the displacement factor 2. Option (C) reverses the trigonometric ratio; (D) has both the wrong ratio and the wrong difference function. As β approaches α, a fixed 2-metre shift corresponds to a very tall tower, consistent with the small sine in the denominator.

Step 4: Conclusion The height is $\boxed{\frac{2\sin\alpha\sin\beta}{\sin(\beta-\alpha)}}$ metres. Hence, the correct option is (A).

Quick Tip to Solve: Write each horizontal distance as height times cotangent. Their difference equals the measured movement, which eliminates the unknown original distance.

17. Let $A(2,3,5)$, $B(-1,3,2)$ and $C(\lambda,5,\mu)$ be the vertices of a triangle ABC. If the median through A is equally inclined to the coordinate axes, then: [4 marks]

(A) $5\lambda - 8\mu = 0$

(B) $8\lambda - 5\mu = 0$

(C) $10\lambda - 7\mu = 0$

(D) $7\lambda - 10\mu = 0$

Correct Answer: (C) $10\lambda - 7\mu = 0$

Solution:

Concept:

- A median from A passes through the midpoint M of BC. The midpoint coordinates are the componentwise averages of the endpoints.
- If a directed line is equally inclined to the positive coordinate axes, its three direction cosines are equal. Its direction ratios are therefore equal as well.

Step 1: Find the midpoint and direction ratios The midpoint of $B(-1,3,2)$ and $C(\lambda,5,\mu)$ is

$$M = \left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right).$$

Subtract the coordinates of $A(2,3,5)$ to obtain

$$\overrightarrow{AM} = \left(\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}\right).$$

Multiplying all three components by 2 gives the equivalent direction ratios $\lambda - 5, 2, \mu - 8$.

Step 2: Use equal inclination to determine the coordinates Equal angles with the coordinate axes give equal direction cosines, rather than merely equal magnitudes. Since normalization divides each direction ratio by the same positive length,

$$\lambda - 5 = 2 = \mu - 8.$$

Hence

$$\lambda = 7, \qquad \mu = 10.$$

As a check, the original direction vector becomes $(1, 1, 1)$, whose three direction cosines all equal $1/\sqrt{3}$.

Step 3: Test the printed relations For $(\lambda, \mu) = (7, 10)$,

$$10\lambda - 7\mu = 70 - 70 = 0.$$

The remaining left-hand sides are

$$5\lambda - 8\mu = -45, \quad 8\lambda - 5\mu = 6, \quad 7\lambda - 10\mu = -51.$$

None vanishes, so only (C) meets the condition. Using the vector AB or AC in place of AM would describe a side of the triangle rather than the required median.

Step 4: Conclusion We obtain $\lambda = 7$ and $\mu = 10$, so $\boxed{10\lambda - 7\mu = 0}$. Hence, the correct option is (C).

Quick Tip to Solve: For a median, form the midpoint first. Then equalize the components of the vector from the vertex to that midpoint.

18. Let A be a 3×3 matrix such that

$$A \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Then A^{-1} is: [4 marks]

(A) $\begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 1 & 3 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & 3 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

Solution:

Concept:

- If $AM = N$, with M and N invertible, then $A = NM^{-1}$ and $A^{-1} = MN^{-1}$. Matrix multiplication order must be preserved.
- A permutation matrix is orthogonal: $N^{-1} = N^T$. Right multiplication by it rearranges the columns of a matrix, which is faster than evaluating a full inverse.

Step 1: Identify the two known matrices Write

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}, \qquad N = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

The given equation is $AM = N$. Here $\det M = 2 - 3 = -1$ and N is a permutation matrix, so both matrices are invertible and the required inverse exists.

Step 2: Rearrange the equation without reversing factors Right multiplication by M^{-1} gives $A = NM^{-1}$. Invert this product in reverse order:

$$A^{-1} = MN^{-1}, \qquad N^{-1} = N^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

Each column of the product is M times the corresponding column of N^{-1}, so the columns of M appear in the order third, first, second.

Step 3: Compute and check the result Thus

$$A^{-1} = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}.$$

A quick independent check is to call this result X and multiply it by N:

$$XN = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} = M,$$

exactly as $X = MN^{-1}$ requires. Option (B) interchanges its second and third columns; (C) has a different first column; (D) retains the original first row instead of the required cyclic permutation. Each therefore fails the column test.

Step 4: Conclusion $\boxed{A^{-1} = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}}$. Hence, the correct option is (A).

Quick Tip to Solve: When one side is a permutation matrix, find its transpose and permute columns instead of computing the unknown matrix first.

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19. Let, for $i = 1, 2, 3$, $p_i(x)$ be a polynomial of degree 2 in x, and let $p_i'(x)$ and $p_i''(x)$ be its first and second derivatives respectively. Let

$$A(x) = \begin{bmatrix} p_1(x) & p_1'(x) & p_1''(x) \\ p_2(x) & p_2'(x) & p_2''(x) \\ p_3(x) & p_3'(x) & p_3''(x) \end{bmatrix}, \qquad B(x) = [A(x)]^T A(x).$$

Then the determinant of $B(x)$: [4 marks]

(A) is a polynomial of degree 6 in x.

(B) is a polynomial of degree 3 in x.

(C) is a polynomial of degree 2 in x.

(D) does not depend on x.

Correct Answer: (D) does not depend on x.

Solution:

Concept:

- For square matrices, $\det(A^T A) = \det(A^T) \det A = (\det A)^2$. It is enough to determine whether $\det A(x)$ varies with x.
- Write each quadratic as $p_i(x) = a_i x^2 + b_i x + c_i$. Differentiation gives $p_i'(x) = 2a_i x + b_i$ and $p_i''(x) = 2a_i$.

Step 1: Separate the constant coefficients from the variable Form the constant coefficient matrix

$$C = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}.$$

The three columns of A are the values of the polynomials and their first two derivatives. They can be expressed simultaneously as

$$A(x) = C \begin{bmatrix} x^2 & 2x & 2 \\ x & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = CT(x).$$

Multiplying any row (a_i, b_i, c_i) by T gives precisely (p_i, p_i', p_i'').

Step 2: Evaluate the determinant of the variable factor Expand $\det T(x)$ using its last row:

$$\det T(x) = \det \begin{bmatrix} 2x & 2 \\ 1 & 0 \end{bmatrix} = -2.$$

The cofactor sign for position $(3,1)$ is positive. Hence

$$\det A(x) = \det C \det T(x) = -2 \det C,$$

a constant, even though the entries of A contain x.

Step 3: Apply the product rule for determinants It follows that

$$\det B(x) = (\det A(x))^2 = 4(\det C)^2.$$

The coefficients a_i, b_i, c_i are fixed, so this expression is independent of x. If C is singular the determinant is identically zero, which still does not depend on x. Options (A), (B), and (C) incorrectly infer a positive degree by counting entry degrees before the cancellations forced by differentiation.

Step 4: Conclusion $\boxed{\det B(x) = 4(\det C)^2}$ is independent of x. The supplied key indicates the first option, but the exact factorization proves that its claimed degree 6 is incorrect. Hence, the correct option is (D).

Quick Tip to Solve: For a matrix built from polynomials and their derivatives, factor out the coefficient matrix before estimating the determinant degree.

20. Let f be an odd function defined on the set of real numbers such that, for $x \geq 0$, $f(x) = 3\sin x + 4\cos x$. Then $f(x)$ at $x = -\frac{11\pi}{6}$ is equal to: [4 marks]

(A) $\frac{3}{2} + 2\sqrt{3}$

(B) $-\frac{3}{2} + 2\sqrt{3}$

(C) $\frac{3}{2} - 2\sqrt{3}$

(D) $-\frac{3}{2} - 2\sqrt{3}$

Correct Answer: No such odd function exists: oddness requires f(0)=0, whereas the printed rule gives f(0)=4. The intended nonzero-input calculation yields the third option.

Solution:

Concept:

- For an odd function, $f(-t) = -f(t)$. A value at a negative nonzero input is therefore obtained by evaluating the given rule at its positive opposite and reversing the sign.
- At $11\pi/6 = 2\pi - \pi/6$, sine is negative and cosine is positive: $\sin(11\pi/6) = -1/2$, $\cos(11\pi/6) = \sqrt{3}/2$.

Step 1: Evaluate the expression at the positive input The requested argument is $-11\pi/6$, so first take $t = 11\pi/6 > 0$, where the stated trigonometric expression applies. The reference angle is $\pi/6$, in the fourth quadrant. Thus

$$f(t) = 3\left(-\frac{1}{2}\right) + 4\left(\frac{\sqrt{3}}{2}\right) = -\frac{3}{2} + 2\sqrt{3}.$$

The separate signs of sine and cosine matter; using first-quadrant signs would give a different option.

Step 2: Use oddness at the requested nonzero point The stipulated oddness gives

$$f\left(-\frac{11\pi}{6}\right) = -f\left(\frac{11\pi}{6}\right) = \frac{3}{2} - 2\sqrt{3}.$$

This is negative because $2\sqrt{3} > 3/2$. Option (B) is the positive-input value before negation. Options (A) and (D) use the wrong sign for the sine contribution at the positive input, so neither equals this conditional value.

Step 3: Check the consistency of the printed assumptions There is a literal endpoint inconsistency in the question. An odd function on all real numbers must satisfy $f(0) = -f(0)$, hence $f(0) = 0$. But the displayed rule, stated for $x \geq 0$, gives $f(0) = 4$. Therefore no function satisfies all the printed assumptions simultaneously. The calculation above is the intended nonzero-input evaluation if the displayed rule is understood for $x > 0$; it does not remove the inconsistency at zero.

Step 4: Conclusion The stipulated nonzero-input calculation gives $\boxed{\frac{3}{2} - 2\sqrt{3}}$, which is option (C). Strictly, the printed inclusion of zero makes the assumptions inconsistent, so no such odd function exists on all real numbers.

Quick Tip to Solve: For negative inputs, evaluate the positive opposite before applying oddness. Also check f(0)=0 whenever an odd function is specified on all real numbers.

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21. Let $f(x) = x|x|$, $g(x) = \sin x$ and $h(x) = (g \circ f)(x)$. Then: [4 marks]

(A) $h(x)$ is not differentiable at $x = 0$.

(B) $h(x)$ is differentiable at $x = 0$, but $h'(x)$ is not continuous at $x = 0$.

(C) $h'(x)$ is continuous at $x = 0$, but it is not differentiable at $x = 0$.

(D) $h'(x)$ is differentiable at $x = 0$.

Correct Answer: (C) $h'(x)$ is continuous at $x = 0$, but it is not differentiable at $x = 0$.

Solution:

Concept:

- The composition is $h(x) = \sin(x|x|)$. Since $x|x| = x^2$ for $x \geq 0$ and $-x^2$ for $x < 0$, it must be differentiated piecewise.
- Differentiability at a junction is checked from a difference quotient. Continuity of the first derivative does not by itself imply the existence of a second derivative.

Step 1: Write the composition and differentiate away from zero Using that sine is odd,

$$h(x) = \begin{cases} \sin(x^2), & x \geq 0, \\ -\sin(x^2), & x < 0. \end{cases}$$

Therefore, for nonzero x,

$$h'(x) = \begin{cases} 2x\cos(x^2), & x > 0, \\ -2x\cos(x^2), & x < 0, \end{cases} = 2|x|\cos(x^2).$$

The chain rule introduces the factor $2x$, with an extra minus sign on the negative branch.

Step 2: Check the derivative and its continuity at zero We have $h(0) = 0$. Since $|\sin u| \leq |u|$,

$$\left|\frac{h(x) - h(0)}{x}\right| \leq \frac{|x|^2}{|x|} = |x| \longrightarrow 0.$$

Hence $h'(0) = 0$. Also

$$\lim_{x \to 0} 2|x|\cos(x^2) = 0 = h'(0),$$

so the first derivative is continuous there. These facts exclude (A) and (B).

Step 3: Test whether the first derivative is differentiable The derivative of h' at zero would require

$$\lim_{x \to 0} \frac{h'(x) - h'(0)}{x} = \lim_{x \to 0} 2\frac{|x|}{x}\cos(x^2).$$

Its right-hand limit is 2, whereas its left-hand limit is -2. Since the one-sided limits differ, h' is not differentiable at zero, excluding (D). Its continuity and failure of differentiability are exactly the statement in (C).

Step 4: Conclusion h' is continuous at zero but is not differentiable there. Hence, the correct option is (C).

Quick Tip to Solve: After finding a piecewise first derivative, use its difference quotient at the junction; matching derivative values alone does not prove a second derivative exists.

22. For the curve $y = 3\sin\theta\cos\theta$, $x = e^{\theta}\sin\theta$, $0 \le \theta \le \pi$, the tangent is parallel to the x-axis when θ is: [4 marks]

(1) $\frac{3\pi}{4}$

(2) $\frac{\pi}{2}$

(3) $\frac{\pi}{4}$

(4) $\frac{\pi}{6}$

Correct Answer: (3) $\frac{\pi}{4}$

Solution:

Concept:

- For a parametrically defined curve, the tangent slope is $dy/dx = (dy/d\theta)/(dx/d\theta)$ whenever $dx/d\theta \neq 0$. A horizontal tangent requires slope zero.
- If both parametric derivatives vanish, the quotient cannot be evaluated by merely setting the numerator to zero. Its limit must be checked at that parameter.

Step 1: Differentiate both coordinates Write $y = \frac{3}{2}\sin 2\theta$. Differentiating with respect to the same parameter gives

$$\frac{dy}{d\theta} = 3\cos 2\theta, \qquad \frac{dx}{d\theta} = e^{\theta}(\sin\theta + \cos\theta).$$

Thus the possible horizontal-tangent parameters satisfy $\cos 2\theta = 0$. In $[0, \pi]$, these are $\pi/4$ and $3\pi/4$.

Step 2: Check the exceptional candidate At $\theta = \pi/4$, the denominator is $\sqrt{2}e^{\pi/4} \neq 0$, so the slope is zero. At $\theta = 3\pi/4$, both first derivatives vanish. Differentiating their numerator and denominator once to compute the limiting slope yields

$$\lim_{\theta \to 3\pi/4} \frac{3\cos 2\theta}{e^{\theta}(\sin\theta + \cos\theta)} = \frac{-6\sin(3\pi/2)}{2e^{3\pi/4}\cos(3\pi/4)} = -3\sqrt{2}e^{-3\pi/4} \neq 0.$$

Hence option (1) is not a horizontal tangent. At $\pi/2$, the slope is $-3e^{-\pi/2}$, while at $\pi/6$ its numerator is $3/2$, so options (2) and (4) also fail.

Step 3: Conclusion The tangent is parallel to the x-axis at $\theta = \pi/4$. Hence, the correct option is (3).

Quick Tip to Solve: In parametric tangent questions, check the denominator at every zero of $dy/d\theta$; a simultaneous zero can conceal a nonzero limiting slope.

23. The volume of the largest possible right circular cylinder that can be inscribed in a sphere of radius $\sqrt{3}$ is: [4 marks]

(1) $\frac{4\sqrt{3}\pi}{3}$

(2) $\frac{8\sqrt{3}\pi}{3}$

(3) 4π

(4) 2π

Correct Answer: (3) 4π

Solution:

Concept:

- An axial cross-section of a cylinder inscribed in a sphere is a rectangle. If its radius is r, its full height is h, and the sphere radius is R, then $r^2 + (h/2)^2 = R^2$.
- The cylinder volume is $V = \pi r^2 h$. Use the geometric constraint to express this in one variable and maximize it over the physically possible interval.

Step 1: Express the volume using its height The sphere has $R = \sqrt{3}$, so the radius and height satisfy

$$r^2 = 3 - \frac{h^2}{4}, \qquad 0 \le h \le 2\sqrt{3}.$$

The use of $h/2$, rather than h, is essential because the centre of the sphere lies halfway between the bases of an inscribed cylinder. Substituting in the volume gives

$$V(h) = \pi\left(3h - \frac{h^3}{4}\right).$$

Step 2: Locate and verify the maximum Differentiate and set the derivative to zero:

$$V'(h) = \pi\left(3 - \frac{3h^2}{4}\right) = 0 \quad \Longrightarrow \quad h^2 = 4.$$

The positive stationary height is $h = 2$, and

$$V''(2) = -3\pi < 0.$$

Moreover, the volume is zero at both endpoints of the feasible interval, and this is its only interior stationary point, so it is the global maximum. At this height $r^2 = 2$, giving

$$V_{\max} = \pi(2)(2) = 4\pi.$$

Options (1), (2), and (4) give different values and therefore cannot be the maximum. In particular, maximizing height alone would collapse the cylinder radius and yield zero volume.

Step 3: Conclusion The maximum volume is 4π cubic units. Hence, the correct option is (3).

Quick Tip to Solve: Use the full height in $V = \pi r^2 h$, but use half the height in the sphere constraint; this prevents the common factor-of-two error.

24. The integral $\int x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$, $x>0$, is equal to: [4 marks]

(1) $-x+(1+x^2)\tan^{-1}x+c$

(2) $x-(1+x^2)\cot^{-1}x+c$

(3) $-x+(1+x^2)\cot^{-1}x+c$

(4) $x-(1+x^2)\tan^{-1}x+c$

Correct Answer: (1) $-x+(1+x^2)\tan^{-1}x+c$

Solution:

Concept:

- The identity $\cos 2t=(1-\tan^2 t)/(1+\tan^2 t)$ converts the inverse cosine to an inverse tangent, provided the principal-value range is checked.
- For $x>0$, $t=\tan^{-1}x$ lies in $(0,\pi/2)$, so $2t\in(0,\pi)$ and $\cos^{-1}(\cos 2t)=2t$.
- Integration by parts uses $\int u\,dv=uv-\int v\,du$, and $x^2/(1+x^2)=1-1/(1+x^2)$.

Step 1: Simplify the inverse cosine Set $t=\tan^{-1}x$. Then

$$\frac{1-x^2}{1+x^2}=\cos(2t),\qquad \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)=2\tan^{-1}x.$$

The domain condition $x>0$ makes this identity valid on the principal branch; no absolute value or additional multiple of π is needed. Thus the required integral is $2\int x\tan^{-1}x\,dx$.

Step 2: Integrate by parts and check the result Take $u=\tan^{-1}x$ and $dv=2x\,dx$, so that $du=dx/(1+x^2)$ and $v=x^2$. Therefore,

$$\begin{aligned}I&=x^2\tan^{-1}x-\int\frac{x^2}{1+x^2}\,dx\\&=x^2\tan^{-1}x-x+\tan^{-1}x+c\\&=(1+x^2)\tan^{-1}x-x+c.\end{aligned}$$

Differentiation gives $2x\tan^{-1}x+1-1=2x\tan^{-1}x$, exactly the transformed integrand. Option (4) gives its negative. Options (2) and (3) contain $\cot^{-1}x$; using its derivative $-1/(1+x^2)$ shows that neither differentiates to the required expression. For example, option (2) differs from the correct primitive by the nonconstant term $-\pi(1+x^2)/2+2x$.

Step 3: Conclusion The integral is $-x+(1+x^2)\tan^{-1}x+c$. Hence, the correct option is (1).

Quick Tip to Solve: Check the inverse-function range before replacing $\cos^{-1}(\cos 2t)$ by $2t$, then verify the proposed primitive by differentiation.

25. If for $n \geq 1$, $P_n = \int_1^e (\log x^n)\, dx$, then $P_{10} - 90P_8$ is equal to: [4 marks]

(1) -9

(2) $10e$

(3) $-9e$

(4) 10

Correct Answer: The literal integrand $\log(x^n)$ gives -710, absent from the options; $(\log x)^n$ would give option (3).

Solution:

Concept:

- Under the usual logarithm convention, $\log x^n = \log(x^n) = n \log x$ for $x > 0$. This is different from $(\log x)^n$. The displayed exponent is attached to x.
- Integration by parts gives $\int \log x\, dx = x \log x - x + c$, using natural logarithms and $\log e = 1$.
- An objective question has no defensible unique option when its literal expression gives a result absent from every listed choice. An alternative placement of an exponent must be identified explicitly rather than silently substituted.

Step 1: Evaluate the integral as printed For positive x, apply the power rule inside the logarithm:

$$P_n = \int_1^e \log(x^n)\, dx = n \int_1^e \log x\, dx.$$

The remaining definite integral is

$$[x \log x - x]_1^e = (e - e) - (0 - 1) = 1.$$

Consequently $P_n = n$, and the expression asked for is

$$P_{10} - 90P_8 = 10 - 90(8) = -710.$$

None of $-9, 10e, -9e, 10$ equals -710, so none of the four printed options answers this literal question.

Step 2: Distinguish the alternative exponent placement If the integrand were instead $(\log x)^n$, denote its integral by I_n. Integration by parts would give $I_n = e - nI_{n-1}$ for $n \geq 1$, since the boundary contribution at $x = 1$ vanishes. In that different problem,

$$I_{10} = e - 10I_9 = e - 10(e - 9I_8) = -9e + 90I_8,$$

so $I_{10} - 90I_8 = -9e$, option (3). This calculation explains the listed choice but depends on moving the exponent from x to the whole logarithm. It does not change the evaluation of $\log(x^n)$ actually displayed.

Step 3: Conclusion As printed, $P_{10} - 90P_8 = -710$, so no listed option is correct. Option (3), $-9e$, applies only to the different integrand $(\log x)^n$; the notation therefore prevents an unqualified unique option.

Quick Tip to Solve: Before applying the logarithmic-power recurrence, distinguish $\log(x^n)$ from $(\log x)^n$; only the second has the recurrence involving nI_{n-1}.

26. If the general solution of the differential equation $y' = \frac{y}{x} + \Phi\left(\frac{x}{y}\right)$, for some function Φ, is given by $y \ln|cx| = x$, where c is an arbitrary constant, then $\Phi(2)$ is equal to: [4 marks]

(1) 4
(2) $\frac{1}{4}$
(3) -4
(4) $-\frac{1}{4}$

Correct Answer: (4) $-\frac{1}{4}$

Solution:

Concept:

- An implicit family $y \ln|cx| = x$ can be differentiated with c held constant to recover its differential equation.
- For $cx \neq 0$, $d \ln|cx|/dx = 1/x$. The original family lets us replace $\ln|cx|$ by x/y wherever the stated equation is defined.

Step 1: Differentiate the implicit solution Apply the product rule to the supplied family, without treating the arbitrary constant as a variable:

$$y' \ln|cx| + \frac{y}{x} = 1.$$

The relation $y \ln|cx| = x$ gives $\ln|cx| = x/y$. Substitute this into the differentiated equation:

$$y' \frac{x}{y} + \frac{y}{x} = 1.$$

Multiplying by y/x, which is legitimate on a solution segment where the original equation is defined, gives

$$y' + \frac{y^2}{x^2} = \frac{y}{x}.$$

Step 2: Identify the function and substitute its argument Rearranging yields

$$y' = \frac{y}{x} - \frac{y^2}{x^2}.$$

Compare this with the specified form

$$y' = \frac{y}{x} + \Phi\left(\frac{x}{y}\right).$$

Writing $t = x/y$, the extra term is $-1/t^2$, rather than $-t^2$. Thus $\Phi(t) = -t^{-2}$, and

$$\Phi(2) = -\frac{1}{2^2} = -\frac{1}{4}.$$

Option (2) has the wrong sign. Option (3) incorrectly uses the reciprocal ratio y/x as the function argument, while option (1) has both that reciprocal error and the wrong sign. A direct substitution

into the recovered equation confirms the negative reciprocal-square dependence.

Step 3: Conclusion The requested value of the function is $-1/4$. Hence, the correct option is (4).

Quick Tip to Solve: After differentiating an implicit family, replace the logarithm using the original relation and name the exact argument $t = x/y$ before identifying $\Phi(t)$.

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27. If $|\vec{c}|^2 = 60$ and $\vec{c} \times (\hat{i} + 2\hat{j} + 5\hat{k}) = \vec{0}$, then a value of $\vec{c} \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$ is: [4 marks]

(1) $4\sqrt{2}$

(2) 12

(3) 24

(4) $12\sqrt{2}$

Correct Answer: (4) $12\sqrt{2}$

Solution:

Concept:

- The cross product of two nonzero vectors vanishes exactly when they are parallel. Therefore $\vec{c} \times (\hat{i} + 2\hat{j} + 5\hat{k}) = \vec{0}$ means $\vec{c} = \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$.
- The squared magnitude is the sum of the squares of Cartesian components, while the scalar product is the sum of componentwise products. A magnitude condition fixes $|\lambda|$, but can leave either sign possible.

Step 1: Use the magnitude to find the proportionality factor Let $\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k}$. Its squared magnitude is

$$|\vec{a}|^2 = 1^2 + 2^2 + 5^2 = 30.$$

Since $\vec{c} = \lambda\vec{a}$, the given condition gives

$$60 = |\vec{c}|^2 = \lambda^2|\vec{a}|^2 = 30\lambda^2, \qquad \lambda = \pm\sqrt{2}.$$

Both directions along the same line satisfy the cross-product condition, so neither sign should be discarded merely because the magnitude is positive.

Step 2: Evaluate the required scalar product Using $\vec{b} = -7\hat{i} + 2\hat{j} + 3\hat{k}$, compute

$$\vec{a} \cdot \vec{b} = (1)(-7) + (2)(2) + (5)(3) = -7 + 4 + 15 = 12.$$

It follows that

$$\vec{c} \cdot \vec{b} = \lambda(\vec{a} \cdot \vec{b}) = \pm 12\sqrt{2}.$$

The wording asks for a value, so the positive possibility is enough when selecting from the options. Only option (4) gives one of these two values. Option (1), $4\sqrt{2}$, has the correct irrational factor but the wrong coefficient; options (2) and (3) omit the necessary factor arising from the prescribed magnitude. The negative value is possible too, but it is not among the printed choices.

Step 3: Conclusion A possible scalar product is $12\sqrt{2}$, with the other possible value $-12\sqrt{2}$. Hence,

the correct option is (4).

Quick Tip to Solve: A zero cross product gives proportional components; use the squared norm to determine the factor, keeping both signs until the requested quantity or options resolve them.

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28. The plane containing the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and parallel to the line $\frac{x}{1}=\frac{y}{1}=\frac{z}{4}$ passes through the point: [4 marks]

(1) $(1,-2,5)$

(2) $(1,0,5)$

(3) $(0,3,-5)$

(4) $(-1,-3,0)$

Correct Answer: (2) $(1,0,5)$

Solution:

Concept:

- A plane containing a line must contain its direction vector and any point on it. If the plane is also parallel to another line, that second direction vector lies parallel to the plane.
- The cross product of two nonparallel vectors in a plane gives a normal vector. A plane through $\vec{r}_0$ with normal $\vec{n}$ satisfies $\vec{n}\cdot(\vec{r}-\vec{r}_0)=0$.

Step 1: Find the two directions and a normal The contained line supplies the point $(1,2,3)$ and direction $\vec{a}=(1,2,3)$. The second line has direction $\vec{b}=(1,1,4)$. These directions are not proportional, so they determine a unique normal direction:

$$\vec{n}=\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&3\\1&1&4\end{vmatrix}=5\hat{i}-\hat{j}-\hat{k}.$$

The location of the second line is irrelevant; only its direction is needed for the parallelism condition.

Step 2: Form the plane and test the points The equation through $(1,2,3)$ is

$$5(x-1)-(y-2)-(z-3)=0,\qquad 5x-y-z=0.$$

Its normal is orthogonal to both line directions, since $5-2-3=0$ and $5-1-4=0$, which checks both geometric requirements. Now substitute the four candidate points. The values of $5x-y-z$, in printed order, are

$$\begin{aligned}(1,-2,5):&\ 5+2-5=2,\\(1,0,5):&\ 5-0-5=0,\\(0,3,-5):&\ 0-3+5=2,\\(-1,-3,0):&\ -5+3-0=-2.\end{aligned}$$

Only the second point satisfies the equation; the nonzero residuals explicitly rule out options (1), (3), and (4).

Step 3: Conclusion The required plane is $5x - y - z = 0$, and it passes through $(1, 0, 5)$. Hence, the correct option is (2).

Quick Tip to Solve: Use a point from the contained line and the cross product of the two direction vectors; check options directly in the resulting plane equation.

29. A set S contains 7 elements. A non-empty subset A of S and an element x of S are chosen at random. Then the probability that $x \in A$ is: [4 marks]

(1) $\frac{1}{2}$

(2) $\frac{64}{127}$

(3) $\frac{63}{128}$

(4) $\frac{31}{128}$

Correct Answer: (2) $\frac{64}{127}$

Solution:

Concept:

- A set of seven elements has 2^7 subsets, including the empty set. Uniform choice of a non-empty subset therefore gives $2^7 - 1$ equally likely possibilities.
- For any fixed element x, a subset containing x is formed by including x and freely choosing any subset of the remaining six elements. Independence lets us condition on the selected element.

Step 1: Count all possible choices There are seven possible elements and $2^7 - 1 = 127$ non-empty subsets. Thus the ordered pairs (A, x) number 127×7. Each has the same probability, because the subset and the element are selected uniformly. The empty subset must be excluded from the denominator: retaining it would answer a different experiment.

Step 2: Count pairs for which the element belongs to the subset Fix one particular element x. Include it in A, and decide independently whether to include each of the other six elements. This gives $2^6 = 64$ subsets containing that element. All these subsets are automatically non-empty. Repeating for the seven possible values of x, the favourable pairs number 7×64. Hence

$$P(x \in A) = \frac{7 \times 64}{7 \times 127} = \frac{64}{127}.$$

Step 3: Check the alternatives The result is slightly larger than $1/2$: excluding the empty subset removes only outcomes in which membership fails. Thus option (1) would hold if all subsets, including the empty one, were allowed, while the smaller values in (3) and (4) cannot be correct. This also verifies that no extra factor of seven belongs in the final probability.

Step 4: Conclusion The required probability is $\boxed{64/127}$. Hence, the correct option is (2).

Quick Tip to Solve: For a fixed element of an n-element set, exactly 2^{n-1} subsets contain it; use $2^n - 1$ in the denominator when the empty subset is excluded.

30. If X has a binomial distribution, $B(n,p)$ with parameters n and p such that $P(X=2)=P(X=3)$, then $E(X)$, the mean of variable X, is [4 marks]

(1) $2-p$

(2) $3-p$

(3) $\frac{p}{2}$

(4) $\frac{p}{3}$

Correct Answer: (2) $3-p$

Solution:

Concept:

- For a binomial variable, $P(X=r)=\binom{n}{r}p^r(1-p)^{n-r}$. Write $q=1-p$ and compare consecutive probabilities by a ratio.
- The expectation is $E(X)=np$. For the non-degenerate binomial distribution described by equal positive probabilities at 2 and 3, $0<p<1$ and $n\geq 3$.

Step 1: Form the ratio of the given probabilities Using the binomial mass function,

$$\binom{n}{2}p^2q^{n-2}=\binom{n}{3}p^3q^{n-3}.$$

Cancel the common nonzero factor p^2q^{n-3}, or equivalently divide the probability at 3 by that at 2. The ratio of the binomial coefficients is $\binom{n}{3}/\binom{n}{2}=(n-2)/3$, so

$$1=\frac{n-2}{3}\frac{p}{q}.$$

Step 2: Express the expectation in terms of p Multiply through to obtain $(n-2)p=3q$. Substituting $q=1-p$ gives

$$np-2p=3-3p, \qquad np=3-p.$$

Therefore the mean is $E(X)=3-p$. The subtraction comes from retaining the $-2p$ term on the left; it is not legitimate to replace $n-2$ by n in the probability ratio.

Step 3: Check the options and admissible size As a check, for $n=5$ the equality requires $p=1/2$, and the mean is $5/2=3-1/2$. Option (1) would give $3/2$, while (3) and (4) would give $1/4$ and $1/6$, respectively. These values fail the probability-ratio relation. In general $0<p<1$ puts the required mean between 2 and 3, also eliminating the two small fractions.

Step 4: Conclusion $\boxed{E(X)=3-p}$. Hence, the correct option is (2).

Quick Tip to Solve: For equal consecutive binomial probabilities, use $P(r+1)/P(r)=(n-r)p/((r+1)(1-p))$ before expanding any factorials.

Physics (120 marks)

31. In terms of resistance R and time T, the dimensions of ratio $\frac{\mu}{\varepsilon}$ of the permeability μ and permittivity ε is: [4 marks]

(1) $[RT^{-2}]$

(2) $[R^2T^{-1}]$

(3) $[R^2]$

(4) $[R^2T^2]$

Correct Answer: (3) $[R^2]$

Solution:

Concept:

- Permeability μ enters the magnetic force per unit length between parallel currents, so $[\mu] = [F]/[I]^2$. Permittivity ε enters Coulomb's law, $F = q^2/(4\pi\varepsilon r^2)$.
- Electrical resistance has dimension $[R] = [V]/[I] = ML^2T^{-3}I^{-2}$. Here I denotes the electric-current dimension, while T denotes time.

Step 1: Find the two electromagnetic dimensions From $F/\ell = \mu I^2/(2\pi r)$, the distance factors cancel when solving for permeability, giving

$$[\mu] = MLT^{-2}I^{-2}.$$

Coulomb's law gives $[\varepsilon] = [q]^2/([F]L^2)$. Since $[q] = IT$,

$$[\varepsilon] = M^{-1}L^{-3}T^4I^2.$$

Pure numbers such as 2π and 4π do not affect dimensions.

Step 2: Divide and compare with resistance Taking the ratio gives

$$\left[\frac{\mu}{\varepsilon}\right] = M^2L^4T^{-6}I^{-4}.$$

The potential difference is energy per charge, so $[V] = ML^2T^{-3}I^{-1}$, and therefore $[R^2] = M^2L^4T^{-6}I^{-4}$. This is exactly the dimension found for the ratio; there is no additional time factor.

Step 3: Eliminate the remaining dimension combinations Option (1) contains only one power of resistance and cannot reproduce the squared mass and current exponents. Option (2) has the correct resistance power but adds T^{-1}, changing the time exponent to -7. Option (4) adds T^2, changing it to -4. Only option (3) keeps the required exponent -6. The result is also consistent with the fact that $\sqrt{\mu/\varepsilon}$ is an electromagnetic wave impedance.

Step 4: Conclusion The dimensions of μ/ε are $\boxed{[R^2]}$. Hence, the correct option is (3).

Quick Tip to Solve: Remember that $\sqrt{\mu/\varepsilon}$ is a wave impedance, with the dimensions of resistance; square it for this ratio.

32. The initial speed of a bullet fired from a rifle is 630 m/s. The rifle is fired at the centre of a target 700 m away at the same level as the target. How far above the centre of the target ? [4 marks]

(1) $1.0\,\mathrm{m}$

(2) $4.2\,\mathrm{m}$

(3) $6.1\,\mathrm{m}$

(4) $9.8\,\mathrm{m}$

Correct Answer: (3) $6.1\,\mathrm{m}$

Solution:

Concept:

- The horizontal velocity remains constant in ideal projectile motion, while the downward displacement is $y = gt^2/2$ for a bullet fired horizontally.
- For horizontal range D and speed u, the flight time is $t = D/u$. The vertical aiming correction compensates the gravitational drop; air resistance is neglected.

Step 1: Calculate the time to reach the target The rifle and the target centre start at the same level, with horizontal separation $D = 700\,\mathrm{m}$. Using the given initial speed $u = 630\,\mathrm{m/s}$, the horizontal-flight estimate gives

$$t = \frac{D}{u} = \frac{700}{630} = \frac{10}{9}\,\mathrm{s}.$$

The time depends on the horizontal distance and horizontal speed, not on D/g.

Step 2: Find the gravitational drop The initial vertical component is zero when the rifle is pointed directly at the target centre. Taking downward displacement as positive and using $g = 9.8\,\mathrm{m/s^2}$,

$$y = \frac{1}{2}gt^2 = \frac{9.8}{2}\left(\frac{10}{9}\right)^2 = \frac{490}{81}\,\mathrm{m} \approx 6.05\,\mathrm{m}.$$

Thus the uncompensated shot falls about $6.1\,\mathrm{m}$ below the centre. To compensate, the rifle must be aimed the corresponding distance above the centre. The required elevation is small, so the horizontal-speed approximation is consistent.

Step 3: Compare the listed distances Rounding the displacement to the precision of the options gives $6.1\,\mathrm{m}$. The values $1.0\,\mathrm{m}$, $4.2\,\mathrm{m}$, and $9.8\,\mathrm{m}$ fail $y = gD^2/(2u^2)$. For example, $9.8\,\mathrm{m}$ would require a flight time $\sqrt{2}\,\mathrm{s}$, rather than $10/9\,\mathrm{s}$. The drop is a distance, not an acceleration.

Step 4: Conclusion The aiming correction is approximately $\boxed{6.1\,\mathrm{m}}$ above the target centre. Hence, the correct option is (3).

Quick Tip to Solve: First calculate D/u, then use $g(D/u)^2/2$. The bullet drops below a horizontal line of aim, so compensate by aiming above it.

33. A body of mass 5 kg under the action of constant force $\vec{F} = F_x\hat{i} + F_y\hat{j}$ has velocity at $t = 0\,\text{s}$ as $\vec{v} = (6\hat{i} - 2\hat{j})\,\text{m/s}$ and at $t = 10\,\text{s}$ as $\vec{v} = +6\hat{j}\,\text{m/s}$.

The force $\vec{F}$ is: [4 marks]

(1) $(-3\hat{i} + 4\hat{j})\,\text{N}$

(2) $\left(-\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right)\,\text{N}$

(3) $(3\hat{i} - 4\hat{j})\,\text{N}$

(4) $\left(\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}\right)\,\text{N}$

Correct Answer: (1) $(-3\hat{i} + 4\hat{j})\,\text{N}$

Solution:

Concept:

- A constant force acting on a fixed mass produces constant acceleration: $\vec{F} = m\vec{a}$, with $\vec{a} = (\vec{v}_f - \vec{v}_i)/\Delta t$.
- Subtract velocity components along the same axes. A missing component of a vector is zero, and the signs determine the direction of force.

Step 1: Subtract the initial velocity from the final velocity The initial velocity is $\vec{v}_i = (6\hat{i} - 2\hat{j})\,\text{m/s}$, while the final velocity is $\vec{v}_f = 6\hat{j}\,\text{m/s}$. In particular, its final horizontal component is zero. Therefore

$$\Delta\vec{v} = (0 - 6)\hat{i} + [6 - (-2)]\hat{j} = (-6\hat{i} + 8\hat{j})\,\text{m/s}.$$

The vertical change is $+8$, since the initial vertical velocity was negative.

Step 2: Calculate acceleration and force The change takes place over $10\,\text{s}$, so

$$\vec{a} = \frac{-6\hat{i} + 8\hat{j}}{10} = \left(-\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right)\,\text{m/s}^2.$$

Multiplying by the mass $m = 5\,\text{kg}$ gives

$$\vec{F} = 5\vec{a} = (-3\hat{i} + 4\hat{j})\,\text{N}.$$

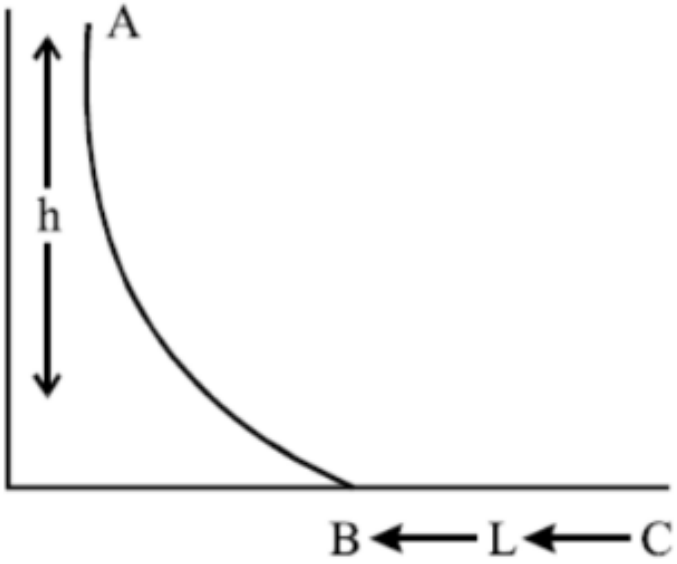
The negative horizontal force removes the initial positive horizontal velocity; the positive vertical force reverses the initially downward vertical component.

Step 3: Check the proposed vectors Option (2) lists the numerical components of acceleration rather than force and omits the multiplication by mass. Options (3) and (4) reverse both component signs; they correspond to subtracting the final velocity from the initial velocity. As a direct check, $\vec{v}_i + \vec{F}\Delta t/m = (6\hat{i} - 2\hat{j}) + (-6\hat{i} + 8\hat{j}) = 6\hat{j}\,\text{m/s}$, precisely the specified final velocity.

Step 4: Conclusion $\boxed{\vec{F} = (-3\hat{i} + 4\hat{j})\,\text{N}}$. Hence, the correct option is (1).

Quick Tip to Solve: Write each velocity as an ordered pair first, including zero components, and calculate final minus initial before multiplying by $m/\Delta t$.

34. A small ball of mass m starts at a point A with speed v_0 and moves along a frictionless track AB as shown. The track BC has coefficient of friction μ. The ball comes to stop at C after travelling a distance L which is: [4 marks]



(1) $\frac{2h}{\mu} + \frac{v_0^2}{2\mu g}$

(2) $\frac{h}{\mu} + \frac{v_0^2}{2\mu g}$

(3) $\frac{h}{2\mu} + \frac{v_0^2}{\mu g}$

(4) $\frac{h}{2\mu} + \frac{v_0^2}{2\mu g}$

Correct Answer: (2) $\frac{h}{\mu} + \frac{v_0^2}{2\mu g}$

Solution:

Concept:

- On the frictionless descending track, gravitational potential energy lost becomes kinetic energy. The normal reaction does no work along the track.
- On the horizontal rough track, the normal reaction is mg, so sliding friction has magnitude μmg. Its negative work over length L is $-\mu mgL$.

Step 1: Calculate the kinetic energy at B Take the horizontal level of BC as the zero of gravitational potential energy. At A, the ball is at height h and already has speed v_0. Conservation of energy along frictionless AB gives

$$K_B = \frac{1}{2}mv_0^2 + mgh.$$

Equivalently $v_B^2 = v_0^2 + 2gh$. The curved shape of AB does not change the result: only its vertical drop matters, because gravity is conservative and no energy is dissipated on this segment.

Step 2: Apply the stopping-work condition on BC The ball stops at C, so its final kinetic energy is zero. On horizontal BC, gravity and the normal force have zero work, leaving only friction to remove

K_B. Hence

$$\mu mgL = \frac{1}{2}mv_0^2 + mgh.$$

Cancel the mass and divide by μg:

$$L = \frac{v_0^2}{2\mu g} + \frac{h}{\mu}.$$

The two terms are the stopping distances associated with the initial motion and with the gained gravitational energy, respectively.

Step 3: Check coefficients and limiting cases If $v_0 = 0$, the formula reduces to $L = h/\mu$; option (1) gives twice this distance and options (3) and (4) give half of it. If $h = 0$, the usual horizontal stopping distance $v_0^2/(2\mu g)$ is recovered; option (3) incorrectly doubles that contribution. Both terms have units of length, and increasing friction decreases the stopping distance, as expected.

Step 4: Conclusion $\boxed{L = h/\mu + v_0^2/(2\mu g)}$. Hence, the correct option is (2).

Quick Tip to Solve: Equate the energy available at the foot of the smooth track to μmgL; the ball already has kinetic energy at the starting point.

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35. A thin bar of length L has a mass per unit length λ, that increases linearly with distance from one end. If its total mass is M and its mass per unit length at the lighter end is λ_0, then the distance of the centre of mass from the lighter end is: [4 marks]

(1) $\frac{L}{2} - \frac{\lambda_0 L^2}{4M}$

(2) $\frac{L}{3} + \frac{\lambda_0 L^2}{8M}$

(3) $\frac{L}{3} + \frac{\lambda_0 L^2}{4M}$

(4) $\frac{2L}{3} - \frac{\lambda_0 L^2}{6M}$

Correct Answer: (4) $\frac{2L}{3} - \frac{\lambda_0 L^2}{6M}$

Solution:

Concept:

- For a thin bar with linear density $\lambda(x)$, an element has mass $dm = \lambda(x)\,dx$, and $x_{\text{cm}} = M^{-1}\int_0^L x\lambda(x)\,dx$.
- A density increasing linearly from the lighter end is written $\lambda(x) = \lambda_0 + kx$, where $x = 0$ is the lighter end and $k > 0$. Use the given total mass to eliminate k.

Step 1: Relate the density gradient to the total mass Integrate the mass elements along the entire length:

$$M = \int_0^L (\lambda_0 + kx)\,dx = \lambda_0 L + \frac{kL^2}{2}.$$

Consequently $k = 2(M - \lambda_0 L)/L^2$. A positive gradient implies $M > \lambda_0 L$.

Step 2: Calculate the first moment about the lighter end The first moment is

$$\int_0^L x\,dm = \int_0^L (\lambda_0 x + kx^2)\,dx = \frac{\lambda_0 L^2}{2} + \frac{kL^3}{3}.$$

Substitute the expression for k:

$$Mx_{\text{cm}} = \frac{\lambda_0 L^2}{2} + \frac{2L}{3}(M - \lambda_0 L) = \frac{2ML}{3} - \frac{\lambda_0 L^2}{6}.$$

Dividing by M produces $x_{\text{cm}} = 2L/3 - \lambda_0 L^2/(6M)$.

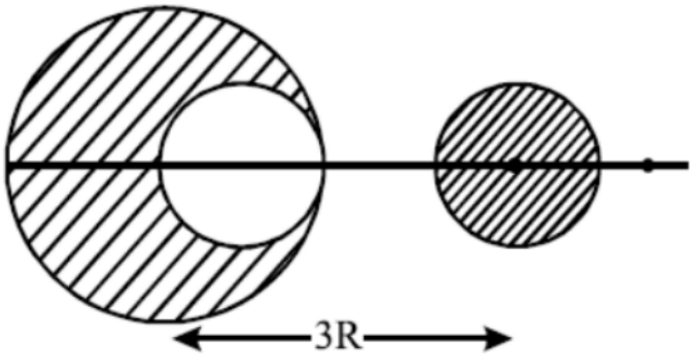
Step 3: Check the distribution limits and other options For uniform density $M = \lambda_0 L$, the answer is $L/2$. For $\lambda_0 = 0$, it is $2L/3$, whereas option (1) gives $L/2$ and options (2) and (3) give $L/3$. Only (4) passes both limits; the centre lies towards the heavier end.

Step 4: Conclusion The distance from the lighter end is $\boxed{x_{\text{cm}} = \frac{2L}{3} - \frac{\lambda_0 L^2}{6M}}$. This differs from the supplied key's third option: when $\lambda_0 = 0$, the defining integral gives $2L/3$, whereas the third option gives $L/3$. Hence, the correct option is (4).

Quick Tip to Solve: Write $\lambda = \lambda_0 + kx$, use total mass to remove k, then integrate $x\,dm$; test the uniform and zero-end-density limits.

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36. From a sphere of mass M and radius R, a smaller sphere of radius $\frac{R}{2}$ is carved out such that the cavity made in the original sphere is between its centre and the periphery (See figure). For the configuration in the figure where the distance between the centre of the original sphere and the removed sphere is $3R$, the gravitational force between the two spheres is: [4 marks]



(1) $\frac{41GM^2}{3600R^2}$

(2) $\frac{41GM^2}{450R^2}$

(3) $\frac{59GM^2}{450R^2}$

(4) $\frac{GM^2}{225R^2}$

Correct Answer: (1) $\frac{41GM^2}{3600R^2}$

Solution:

Concept:

- For a uniform sphere, mass is proportional to the cube of its radius: a sphere of radius $R/2$ has mass $m = M/8$.
- For two non-overlapping spherically symmetric bodies, the gravitational force is the same as if their masses were concentrated at their centres.
- A cavity can be represented by subtracting the gravitational field of the removed mass from that of the complete sphere. Forces superpose vectorially.

Step 1: Determine the masses and centre separations The detached small sphere has mass $m = M(R/2)^3/R^3 = M/8$. The figure places the centre of the spherical cavity a distance $R/2$ from the original centre, on the side nearer the detached sphere. The detached centre is $3R$ from the original centre. Hence its distance from the cavity centre is $3R - R/2 = 5R/2$. Both relevant pairs of spheres are separated, so the external-sphere formula applies.

Step 2: Subtract the attraction of the absent sphere The full sphere would attract the detached sphere with force

$$F_{\text{full}} = \frac{GM(M/8)}{(3R)^2} = \frac{GM^2}{72R^2}.$$

The portion now absent would have contributed attraction in the same direction, of magnitude

$$F_{\text{absent}} = \frac{G(M/8)(M/8)}{(5R/2)^2} = \frac{GM^2}{400R^2}.$$

Subtract this contribution; do not treat the remaining irregular body as a point mass at its original centre.

Step 3: Simplify and compare the options The required force magnitude is

$$F = \frac{GM^2}{R^2}\left(\frac{1}{72} - \frac{1}{400}\right) = \frac{GM^2}{R^2}\frac{50-9}{3600} = \frac{41GM^2}{3600R^2}.$$

Options (2) and (3) have much larger coefficients and option (4) is too small; none includes the correct difference of the two centre-to-centre attractions.

Step 4: Conclusion The attraction is $\frac{41GM^2}{3600R^2}$, directed along the common line of centres. Hence, the correct option is (1).

Quick Tip to Solve: For a spherical cavity, use a complete positive-mass sphere minus the missing sphere; keep the two distinct centre distances in the force terms.

37. The bulk moduli of ethanol, mercury and water are given as 0.9, 25 and 2.2 respectively in units of $10^9\,\mathrm{N\,m^{-2}}$. For a given value of pressure, the fractional compression in volume is $\frac{\Delta V}{V}$. Which of the following statements about $\frac{\Delta V}{V}$ for these three liquids is correct? [4 marks]

(1) Ethanol > Water > Mercury

(2) Water > Ethanol > Mercury

(3) Mercury > Ethanol > Water

(4) Ethanol > Mercury > Water

Correct Answer: (1) Ethanol > Water > Mercury

Solution:

Concept:

- The bulk modulus is $B = -\Delta P/(\Delta V/V)$. The minus sign records that positive pressure decreases volume.
- The positive fractional compression is $c = |\Delta V|/V = \Delta P/B$. For equal pressure changes it is inversely proportional to bulk modulus.

Step 1: Arrange the given bulk moduli The three moduli are

$$B_{\text{ethanol}} = 0.9 \times 10^9\,\text{Pa}, \quad B_{\text{water}} = 2.2 \times 10^9\,\text{Pa}, \quad B_{\text{mercury}} = 25 \times 10^9\,\text{Pa}.$$

Thus ethanol has the smallest bulk modulus, water lies in between, and mercury has the largest. A larger modulus means more resistance to a given fractional change in volume, so it must correspond to less compression for the same applied pressure.

Step 2: Invert the moduli to compare fractional compressions For the common pressure increment p,

$$c_{\text{ethanol}} : c_{\text{water}} : c_{\text{mercury}} = \frac{1}{0.9} : \frac{1}{2.2} : \frac{1}{25}.$$

Numerically the relative factors are about $1.111 : 0.455 : 0.040$. All share the same positive factor $p/10^9$, which does not affect their ordering. Therefore

$$c_{\text{ethanol}} > c_{\text{water}} > c_{\text{mercury}}.$$

The question uses fractional compression as a positive magnitude, rather than the signed volume change.

Step 3: Eliminate the other orderings Option (2) incorrectly places water above ethanol even though its modulus is greater. Option (3) places mercury first, reversing the consequence of its very large bulk modulus. Option (4) correctly starts with ethanol but incorrectly puts mercury ahead of water. Only option (1) respects the inverse dependence for both successive comparisons.

Step 4: Conclusion The fractional compression decreases in the order ethanol, water, mercury. Hence, the correct option is (1).

Quick Tip to Solve: For equal pressure, rank fractional compression by reversing the order of the bulk moduli; do not rank by density or by the moduli themselves.

38. The average mass of rain drops is $3.0 \times 10^{-5}\,\text{kg}$ and their average terminal velocity is $9\,\text{m/s}$. Calculate the energy transferred by rain to each square metre of the surface at a place which receives $100\,\text{cm}$ of rain in a year. [4 marks]

(1) $3.5 \times 10^5\,\text{J}$

(2) $4.05 \times 10^4\,\text{J}$

(3) $3.0 \times 10^5\,\text{J}$

(4) $9.0 \times 10^4\,\text{J}$

Correct Answer: (2) $4.05 \times 10^4\,\text{J}$

Solution:

Concept:

- Each drop arrives with kinetic energy $K = \frac{1}{2}mv^2$, where m is its mass and v its terminal speed.
- Rainfall depth measures accumulated volume per unit area. A depth h over area A gives water mass $M = \rho Ah$, with $\rho = 10^3\,\text{kg m}^{-3}$.

Step 1: Convert the rainfall depth into total mass The annual rainfall depth is $100\,\text{cm} = 1\,\text{m}$. For one square metre of surface, the collected volume is

$$V = Ah = (1\,\text{m}^2)(1\,\text{m}) = 1\,\text{m}^3.$$

Therefore the mass delivered during the year is

$$M = \rho V = (10^3\,\text{kg m}^{-3})(1\,\text{m}^3) = 1000\,\text{kg}.$$

This is the mass of all the rain over the specified area, not the mass of just one drop.

Step 2: Sum the impact energy of all drops The number of drops is $N = M/m = 1000/(3.0 \times 10^{-5})$. Since every drop reaches the stated average terminal speed, summing their kinetic energies gives

$$E = N\left(\frac{1}{2}mv^2\right) = \frac{1}{2}Mv^2.$$

Substituting the total mass and speed,

$$E = \frac{1}{2}(1000)(9)^2 = 500 \times 81 = 40500\,\text{J} = 4.05 \times 10^4\,\text{J}.$$

The individual drop mass cancels; its presence in the question does not change the total.

Step 3: Check the interpretation and options The energy transferred on striking the surface is the arriving kinetic energy. At terminal speed, air resistance has already dissipated energy during the fall, so an unspecified cloud height must not be inserted into Mgh. Options (1), (3) and (4) do not equal $40500\,\text{J}$ and would require a different rainfall mass or impact speed.

Step 4: Conclusion The annual impact energy on each square metre is $4.05 \times 10^4\,\text{J}$. Hence, the correct option is (2).

Quick Tip to Solve: Convert rain depth to water mass first, then use one-half times total mass times terminal speed squared; the mass of a single drop cancels.

39. A tank with a small hole at the bottom has been filled with water and kerosene (specific gravity 0.8). The height of water is $3\,\mathrm{m}$ and that of kerosene $2\,\mathrm{m}$. When the hole is opened the velocity of fluid coming out from it is nearly: (take $g = 10\,\mathrm{m\,s^{-2}}$ and density of water $= 10^3\,\mathrm{kg\,m^{-3}}$) [4 marks]

(1) $10.7\,\mathrm{m\,s^{-1}}$

(2) $9.6\,\mathrm{m\,s^{-1}}$

(3) $8.5\,\mathrm{m\,s^{-1}}$

(4) $7.6\,\mathrm{m\,s^{-1}}$

Correct Answer: (2) $9.6\,\mathrm{m\,s^{-1}}$

Solution:

Concept:

- At rest, a layer of density ρ and height h contributes hydrostatic pressure ρgh. Contributions from stacked layers add.
- At a small bottom outlet, the pressure excess drives the lower liquid with $\frac{1}{2}\rho_w v^2 = \Delta P$. The large tank surface speed is negligible.

Step 1: Identify the liquid leaving the bottom Kerosene has density $\rho_k = 0.8\rho_w = 800\,\mathrm{kg\,m^{-3}}$, less than water, so it forms the upper layer. Water occupies the lower $3\,\mathrm{m}$ and emerges first from the bottom hole. The kerosene contributes pressure but its density must not be used for the kinetic energy of the emerging water.

Step 2: Find the pressure excess above the outside atmosphere Both the upper free surface and the outgoing stream are exposed to atmospheric pressure. Thus the atmospheric contribution cancels, leaving

$$\Delta P = \rho_w g(3) + \rho_k g(2).$$

Substituting the data,

$$\Delta P = 1000(10)(3) + 800(10)(2) = 46000\,\mathrm{Pa}.$$

Equivalently, the water has an effective pressure head $3 + 0.8(2) = 4.6\,\mathrm{m}$, rather than the full geometric height of $5\,\mathrm{m}$.

Step 3: Convert the pressure into efflux speed Apply Bernoulli's pressure-to-speed relation to the outflowing water:

$$\frac{1}{2}\rho_w v^2 = 46000, \qquad v^2 = \frac{2(46000)}{1000} = 92\,\mathrm{m^2\,s^{-2}}.$$

Therefore $v = \sqrt{92} = 9.59\,\mathrm{m\,s^{-1}}$, nearly $9.6\,\mathrm{m\,s^{-1}}$. The other listed speeds do not satisfy $v^2 = 92$; treating both liquid layers as water would also ignore their different densities.

Step 4: Conclusion The initial efflux speed is approximately $9.6\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (2).

Quick Tip to Solve: For layers above a bottom water outlet, replace each upper-layer height by its specific gravity times its height before applying Torricelli's formula.

40. An air bubble of radius $0.1\,\text{cm}$ is in a liquid having surface tension $0.06\,\text{N/m}$ and density $10^3\,\text{kg/m}^3$. The pressure inside the bubble is $1100\,\text{N m}^{-2}$ greater than the atmospheric pressure. At what depth is the bubble below the surface of the liquid? ($g = 9.8\,\text{m s}^{-2}$) [4 marks]

(1) $0.1\,\text{m}$

(2) $0.15\,\text{m}$

(3) $0.20\,\text{m}$

(4) $0.25\,\text{m}$

Correct Answer: (1) $0.1\,\text{m}$

Solution:

Concept:

- An air bubble within a liquid has one interface. Its inside pressure exceeds the surrounding liquid pressure by $2T/r$, with surface tension T and bubble radius r.
- At depth h, surrounding liquid pressure is $P_{\text{atm}} + \rho gh$. Thus $P_{\text{inside}} - P_{\text{atm}} = \rho gh + 2T/r$.

Step 1: Convert the bubble radius and find the capillary contribution The radius is $r = 0.1\,\text{cm} = 1.0 \times 10^{-3}\,\text{m}$. The pressure jump across its liquid-air interface is therefore

$$\Delta P_{\text{surface}} = \frac{2T}{r} = \frac{2(0.06)}{1.0 \times 10^{-3}} = 120\,\text{Pa}.$$

This is an air bubble in a liquid, not a soap bubble with two interfaces; using $4T/r$ would count a second surface that is absent.

Step 2: Separate hydrostatic pressure from the stated pressure excess The stated $1100\,\text{Pa}$ is the total excess over atmospheric pressure. It includes both the depth contribution and the surface-tension contribution:

$$1100 = \rho gh + 120.$$

Thus the hydrostatic part alone is

$$\rho gh = 1100 - 120 = 980\,\text{Pa}.$$

Adding the capillary term again would overestimate depth, while identifying all $1100\,\text{Pa}$ with ρgh would neglect the curved interface.

Step 3: Evaluate the depth and test the choices Using $\rho = 10^3\,\text{kg m}^{-3}$ and $g = 9.8\,\text{m s}^{-2}$,

$$h = \frac{980}{1000(9.8)} = 0.100\,\text{m}.$$

At the alternative depths 0.15, 0.20 and $0.25\,\text{m}$, hydrostatic pressure alone would exceed the allowed $980\,\text{Pa}$, so each would give a total internal pressure excess greater than the stated value.

Step 4: Conclusion The bubble is $0.1\,\text{m}$ below the surface. Hence, the correct option is (1).

Quick Tip to Solve: For a submerged air bubble, subtract the single-interface excess pressure from the given excess over atmosphere before dividing by density times gravity.

41. A hot body, obeying Newton's law of cooling is cooling down from its peak value 80°C to an ambient temperature of 30°C. It takes 5 minutes in cooling down from 80°C to 40°C. How much time will it take to cool down from 62°C to 32°C? (Given $\ln 2 = 0.693$, $\ln 5 = 1.609$) [4 marks]

(1) 3.75 minutes

(2) 8.6 minutes

(3) 9.6 minutes

(4) 6.5 minutes

Correct Answer: (2) 8.6 minutes

Solution:

Concept:

- Newton's law gives $d\theta/dt = -k\theta$, where $\theta = T - T_a$ is temperature excess above the ambient temperature T_a.
- Integration gives $\theta_f = \theta_i e^{-kt}$, so $t = k^{-1}\ln(\theta_i/\theta_f)$. The same body in the same surroundings has the same constant k.

Step 1: Determine the cooling constant from the first interval The ambient temperature is 30°C. Cooling from 80°C to 40°C reduces the temperature excess from 50°C to 10°C, a factor of five. Therefore

$$5 = \frac{1}{k}\ln\left(\frac{50}{10}\right) = \frac{\ln 5}{k}, \qquad k = \frac{1.609}{5} = 0.3218\,\text{min}^{-1}.$$

Ratios must use excess temperatures; directly using $80/40$ would contradict the stated ambient temperature.

Step 2: Apply exponential cooling to the second interval The required initial and final excesses are $62 - 30 = 32^\circ\text{C}$ and $32 - 30 = 2^\circ\text{C}$. Hence

$$t = \frac{1}{k}\ln\left(\frac{32}{2}\right) = \frac{5\ln 16}{\ln 5} = \frac{5(4\ln 2)}{\ln 5}.$$

The excess falls by a factor of sixteen; the body cools more slowly as it approaches the surroundings, so a constant degrees-per-minute rule cannot be used.

Step 3: Substitute the logarithms and compare Using the given values,

$$t = \frac{5(4)(0.693)}{1.609} = \frac{13.86}{1.609} = 8.61\,\text{min}.$$

This rounds to $8.6\,\text{min}$. Options (1), (3) and (4) fail the required exponential ratio: for the calibrated constant the elapsed time must satisfy $kt = \ln 16 = 2.772$.

Step 4: Conclusion The required cooling time is approximately 8.6 minutes. Hence, the correct option is (2).

Quick Tip to Solve: Subtract the ambient temperature at both endpoints, then compare the logarithms of the two excess-temperature ratios to avoid solving for the constant separately.

42. During an adiabatic compression, $830\,\mathrm{J}$ of work is done on 2 moles of a diatomic ideal gas to reduce its volume by 50%. The change in its temperature is nearly: ($R = 8.3\,\mathrm{J\,K^{-1}\,mol^{-1}}$) [4 marks]

(1) $40\,\mathrm{K}$

(2) $33\,\mathrm{K}$

(3) $20\,\mathrm{K}$

(4) $14\,\mathrm{K}$

Correct Answer: (3) $20\,\mathrm{K}$

Solution:

Concept:

- The first law in terms of work done on a gas is $\Delta U = Q + W_{\text{on}}$. An adiabatic process has $Q = 0$.
- For an ordinary diatomic ideal gas with rotational modes active and vibrations neglected, $C_V = 5R/2$ per mole and $\Delta U = nC_V\Delta T$.

Step 1: Use the sign and heat condition correctly The surroundings do $830\,\mathrm{J}$ of work on the gas during compression. Because the process is adiabatic, no heat is exchanged:

$$Q = 0, \qquad \Delta U = +830\,\mathrm{J}.$$

The positive sign means the internal energy and temperature increase. Writing negative work without specifying the work-by-gas convention would incorrectly predict cooling during this compression.

Step 2: Calculate the total heat capacity at constant volume There are $n = 2$ moles. For the diatomic gas,

$$nC_V = 2\left(\frac{5}{2}R\right) = 5R = 5(8.3) = 41.5\,\mathrm{J\,K^{-1}}.$$

The constant-volume heat capacity is used here because it relates temperature to internal energy, even though the actual volume changes. The constant-pressure heat capacity would include expansion work and does not give ΔU.

Step 3: Find the temperature rise and reject alternatives Now use the internal-energy change directly:

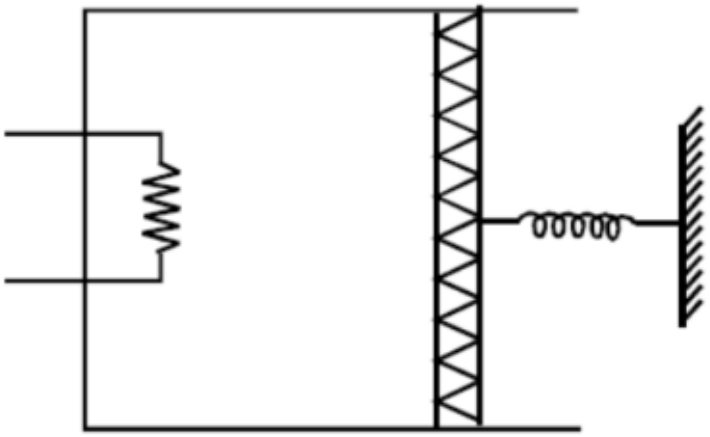
$$\Delta T = \frac{\Delta U}{nC_V} = \frac{830}{41.5} = 20\,\mathrm{K}.$$

The stated halving of volume is compatible with an adiabatic compression, but the known work already determines the temperature rise; initial temperature is not needed. The values 40, 33 and $14\,\mathrm{K}$ do not give $830\,\mathrm{J}$ when multiplied by the correct total heat capacity $41.5\,\mathrm{J\,K^{-1}}$.

Step 4: Conclusion The gas temperature increases by $20\,\mathrm{K}$. Hence, the correct option is (3).

Quick Tip to Solve: When adiabatic work is given, find the temperature change directly from work divided by total constant-volume heat capacity; the volume ratio may be redundant.

43. An ideal monoatomic gas is confined in a cylinder by a spring loaded piston of cross section $8.0 \times 10^{-3}\,\mathrm{m}^2$. Initially the gas is at $300\,\mathrm{K}$ and occupies a volume of $2.4 \times 10^{-3}\,\mathrm{m}^3$ and the spring is in its relaxed state as shown in figure. The gas is heated by a small heater until the piston moves out slowly by $0.1\,\mathrm{m}$. The force constant of the spring is $8000\,\mathrm{N/m}$ and the atmospheric pressure is $1.0 \times 10^5\,\mathrm{N/m}^2$. The cylinder and the piston are thermally insulated. The piston and the spring are massless and there is no friction between the piston and the cylinder. The final temperature of the gas will be: (Neglect the heat loss through the lead wires of the heater. The heat capacity of the heater coil is also negligible). [4 marks]



(1) 300 K

(2) 800 K

(3) 500 K

(4) 1000 K

Correct Answer: (2) 800 K

Solution:

Concept:

- For a fixed amount of ideal gas, $pV = nRT$, so the ratio of final to initial temperature is $T_f/T_i = p_f V_f/(p_i V_i)$. Pressures must be absolute.
- With a massless, frictionless piston moving slowly, the outward force of the gas balances atmospheric pressure and the spring force. At displacement x, $p_f A = p_0 A + kx$, where A is piston area and k is the spring constant.

Step 1: Find the final volume The initially relaxed spring exerts no force, hence $p_i = p_0 = 1.0 \times 10^5\,\mathrm{Pa}$. The piston increases the volume by its area times its displacement:

$$\Delta V = Ax = (8.0 \times 10^{-3})(0.1) = 8.0 \times 10^{-4}\,\mathrm{m}^3.$$

$$V_f = 2.4 \times 10^{-3} + 0.8 \times 10^{-3} = 3.2 \times 10^{-3}\,\mathrm{m}^3.$$

The positive displacement is an expansion, so the final volume must exceed the initial volume.

Step 2: Balance piston forces The compressed spring opposes the outward motion with force

$kx = 8000(0.1) = 800\,\text{N}$. The pressure increase required to overcome it is

$$p_f - p_0 = \frac{kx}{A} = \frac{800}{8.0 \times 10^{-3}} = 1.0 \times 10^5\,\text{Pa}.$$

Therefore $p_f = 2.0 \times 10^5\,\text{Pa}$. Using only atmospheric pressure here would ignore the loading of the spring.

Step 3: Use the ideal-gas temperature ratio The gas amount is unchanged, and therefore

$$T_f = 300\frac{(2.0 \times 10^5)(3.2 \times 10^{-3})}{(1.0 \times 10^5)(2.4 \times 10^{-3})} = 300 \times 2 \times \frac{4}{3} = 800\,\text{K}.$$

Monoatomicity would be needed to calculate the supplied heat, but the two equilibrium states already determine the temperature. Thus 300 K, 500 K and 1000 K do not satisfy the final pressure-volume product.

Step 4: Conclusion The final gas temperature is $800\,\text{K}$. Since the absolute pressure doubles and the volume grows by a factor $4/3$, the temperature must grow by $8/3$; the alternative $500\,\text{K}$ is incompatible with these equilibrium states. Hence, the correct option is (2).

Quick Tip to Solve: For a spring-loaded piston, calculate both $p_f = p_0 + kx/A$ and $V_f = V_i + Ax$ before taking the ratio pV/T.

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44. The angular frequency of the damped oscillator is given by, $\omega = \sqrt{\frac{k}{m} - \frac{r^2}{4m^2}}$ where k is the spring constant, m is the mass of the oscillator and r is the damping constant. If the ratio $\frac{r^2}{mk}$ is 8%, the change in time period compared to the undamped oscillator is approximately as follows: [4 marks]

(1) increases by 1%

(2) increases by 8%

(3) decreases by 1%

(4) decreases by 8%

Correct Answer: (1) increases by 1%

Solution:

Concept:

- The undamped angular frequency is $\omega_0 = \sqrt{k/m}$, and the given damped frequency is $\omega = \omega_0\sqrt{1 - r^2/(4mk)}$.
- The time period is inversely proportional to angular frequency: $T = 2\pi/\omega$. For small positive z, the binomial approximation gives $(1 - z)^{-1/2} \simeq 1 + z/2$.

Step 1: Identify the small damping correction The dimensionless ratio provided is $r^2/(mk) = 8\% =$

0.08. The factor in the frequency formula contains an additional divisor of four:

$$z = \frac{r^2}{4mk} = \frac{0.08}{4} = 0.02.$$

Thus $\omega = \omega_0\sqrt{0.98}$. Damping reduces the angular frequency, so it must increase the time taken for one oscillation; the options predicting a decrease already have the wrong sign.

Step 2: Convert the frequency correction to a period correction Keeping the same spring and oscillator mass, the exact ratio of periods is

$$\frac{T}{T_0} = \frac{\omega_0}{\omega} = \frac{1}{\sqrt{1-z}}.$$

Expanding to first order gives

$$\frac{T}{T_0} \simeq 1 + \frac{z}{2} = 1 + \frac{0.02}{2} = 1.01.$$

Hence $(T - T_0)/T_0 \simeq 0.01$, which is an increase of approximately 1%.

Step 3: Check the size and reject the other changes The exact numerical ratio $1/\sqrt{0.98} \simeq 1.01015$ confirms that the small-correction estimate is accurate. The quoted 8% is a ratio involving the square of damping, rather than the percentage change in the period; using it directly would miss both the factor four and the square root. A decrease by either 1% or 8% contradicts $\omega < \omega_0$.

Step 4: Conclusion The time period increases by approximately 1%. Hence, the correct option is (1).

Quick Tip to Solve: Factor out k/m first: the fractional period increase is approximately $r^2/(8mk)$, with a positive sign.

45. Two factories are sounding their sirens at 800 Hz. A man goes from one factory to other at a speed of 2 m/s. The velocity of sound is 320 m/s. The number of beats heard by the person in one second will be [4 marks]

(1) 2

(2) 4

(3) 8

(4) 10

Correct Answer: (4) 10

Solution:

Concept:

- For a stationary source of frequency f, an observer moving towards it at speed u hears $f_+ = f(v+u)/v$; when moving away the observer hears $f_- = f(v-u)/v$. Here v is sound speed in stationary air.
- Two simultaneous tones produce a beat frequency equal to the absolute difference between their observed frequencies: $f_b = |f_+ - f_-|$.

Step 1: Assign approaching and receding sources The man moves from one factory to the other. He

therefore recedes from the factory behind him while approaching the factory ahead. Both sirens have the same emitted frequency $800\,\text{Hz}$, but their received frequencies are different. Since the factories are fixed, observer-motion formulae apply; replacing the denominator by $v \pm u$ would instead describe source motion.

Step 2: Calculate each received frequency For the factory ahead, the observer crosses wavefronts faster:

$$f_+ = 800\frac{320+2}{320} = 805\,\text{Hz}.$$

For the factory behind, he crosses them more slowly:

$$f_- = 800\frac{320-2}{320} = 795\,\text{Hz}.$$

Both shifts have magnitude $5\,\text{Hz}$, one upwards and the other downwards.

Step 3: Take the difference to obtain the beats The superposed sounds alternately reinforce and weaken at the difference frequency:

$$f_b = 805 - 795 = 10\,\text{Hz}.$$

Equivalently, $f_b = 2fu/v = 2(800)(2)/320 = 10\,\text{Hz}$. This means ten beats in one second. The speed 2 m/s is not itself a frequency, and the other proposed counts 2, 4 and 8 do not equal the difference of the two Doppler-shifted tones. The equal original siren frequencies would give no beats only for a stationary listener.

Step 4: Conclusion The man hears 10 beats in one second. Hence, the correct option is (4).

Quick Tip to Solve: When moving between identical stationary sirens, the two Doppler shifts have opposite signs, so the beat frequency is twice one shift: $2fu/v$.

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46. A cone of base radius R and height h is located in a uniform electric field $\vec{E}$ parallel to its base. The electric flux entering the cone is: [4 marks]

(1) $\frac{1}{2}EhR$

(2) EhR

(3) $2EhR$

(4) $4EhR$

Correct Answer: (2) EhR

Solution:

Concept:

- Electric flux through a surface is $\Phi = \int \vec{E}\cdot d\vec{A}$. In a uniform field its magnitude equals field strength times the area projected on a plane perpendicular to the field.
- The entering and exiting parts of a closed uncharged surface have equal flux magnitudes. The signed net flux is zero, but each individual entering or exiting magnitude can be nonzero.

Step 1: Choose the surface carrying the entering flux The field is parallel to the plane of the circular base. Its scalar product with the base normal is therefore zero, so no flux crosses the base. The curved conical surface has an upstream half where field lines enter and a downstream half where they leave. The question asks for the entering magnitude, rather than the signed flux through the whole closed cone.

Step 2: Find the perpendicular projection Look along the direction of the horizontal electric field. The silhouette of the cone is a triangle with base equal to the base diameter $2R$ and height equal to the cone height h. Every field line crossing this triangular region first enters the upstream curved surface, so the entering half projects once onto this triangle. Its area is

$$A_{\perp} = \frac{1}{2}(2R)h = Rh.$$

The slant height is irrelevant because the cosine in the flux integral removes the obliquity of each surface element.

Step 3: Multiply by the field and distinguish the options The entering flux magnitude is therefore

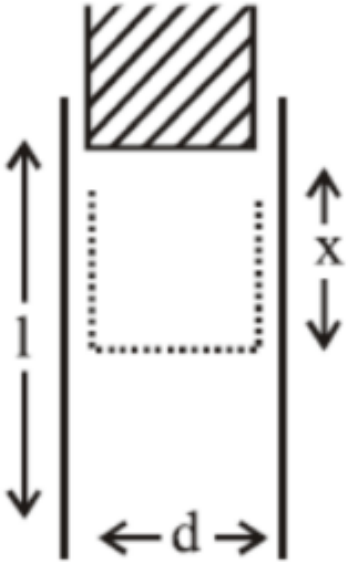
$$\Phi_{\text{in}} = EA_{\perp} = ERh.$$

The value $EhR/2$ incorrectly uses the radius as the triangle base. The value $2EhR$ adds entering and exiting magnitudes even though only entering flux was asked for; $4EhR$ has no corresponding projected area. With outward normals the entering contribution has a negative sign, but its magnitude is the positive quantity listed in the options.

Step 4: Conclusion The magnitude of electric flux entering the cone is EhR. Hence, the correct option is (2).

Quick Tip to Solve: For flux entering a convex surface in a uniform field, use its silhouette perpendicular to the field; the cone gives a triangle of base $2R$.

47. A parallel plate capacitor is made of two plates of length l, width w and separated by distance d. A dielectric slab (dielectric constant K) that fits exactly between the plates is held near the edge of the plates. It is pulled into the capacitor by a force $F = -\frac{\partial U}{\partial x}$ where U is the energy of the capacitor when dielectric is inside the capacitor up to distance x (See figure). If the charge on the capacitor is Q then the force on the dielectric when it is near the edge is: [4 marks]



(1) $\frac{Q^2 d}{2wl^2\varepsilon_0}K$

(2) $\frac{Q^2 w}{2dl^2\varepsilon_0}(K-1)$

(3) $\frac{Q^2 d}{2wl^2\varepsilon_0}(K-1)$

(4) $\frac{Q^2 w}{2dl^2\varepsilon_0}K$

Correct Answer: (3) $\frac{Q^2 d}{2wl^2\varepsilon_0}(K-1)$

Solution:

Concept:

- The filled and empty portions of the capacitor share the same potential difference, so their capacitances add in parallel. For slab insertion x, $C(x) = \varepsilon_0 w[l + (K-1)x]/d$.
- With isolated charge Q, the electrostatic energy is $U = Q^2/(2C)$. The mechanical force in the insertion direction is $F = -dU/dx$, with Q held fixed.

Step 1: Write capacitance as a function of insertion The dielectric-filled area is wx, leaving empty area $w(l-x)$. Their individual capacitances are $K\varepsilon_0 wx/d$ and $\varepsilon_0 w(l-x)/d$. Thus

$$C(x) = \frac{\varepsilon_0 w}{d}\big[l + (K-1)x\big], \qquad \frac{dC}{dx} = \frac{\varepsilon_0 w(K-1)}{d}.$$

Increasing x increases the capacitance for a dielectric with $K > 1$.

Step 2: Differentiate the energy at fixed charge Use the specified energy derivative with the charge kept constant:

$$F = -\frac{d}{dx}\left(\frac{Q^2}{2C}\right) = \frac{Q^2}{2C^2}\frac{dC}{dx}.$$

The force is positive in the direction of increasing insertion. At the edge, $x \simeq 0$, so the capacitance approaches $C_0 = \varepsilon_0 wl/d$. Consequently

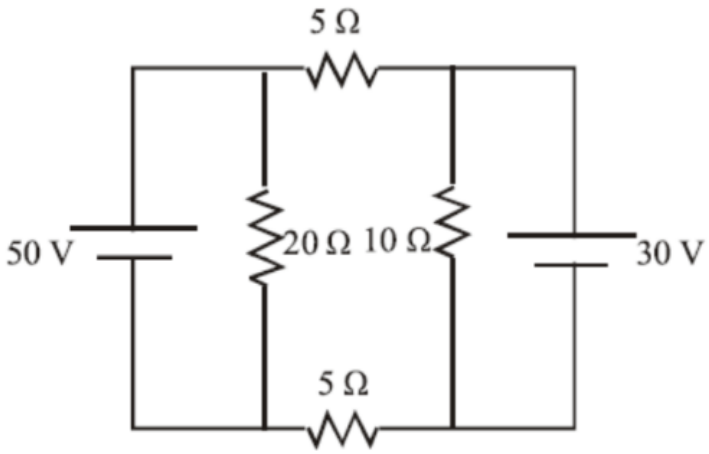
$$F_{\text{edge}} = \frac{Q^2}{2(\varepsilon_0 wl/d)^2}\frac{\varepsilon_0 w(K-1)}{d} = \frac{Q^2 d(K-1)}{2\varepsilon_0 wl^2}.$$

Step 3: Check the limiting behaviour If $K = 1$, insertion replaces vacuum by an identical medium and cannot change the energy; the force must vanish. This eliminates options proportional to K alone. Of the two options proportional to $K-1$, only option (3) has the factor $d/(wl^2)$ obtained by differentiating the actual fixed-charge energy. Using a fixed-voltage energy expression here would apply the wrong electrical constraint.

Step 4: Conclusion The inward force near the edge is $\frac{Q^2 d}{2wl^2\varepsilon_0}(K-1)$. Hence, the correct option is (3).

Quick Tip to Solve: First identify whether charge or voltage is fixed. For fixed charge, differentiating $Q^2/(2C(x))$ avoids sign and power errors.

48. In the circuit shown, current (in A) through 50 V and 30 V batteries are, respectively: [4 marks]



(1) 2.5 and 3
(2) 3.5 and 2
(3) 4.5 and 1
(4) 3 and 2.5

Correct Answer: (3) 4.5 and 1

Solution:

Concept:

- An ideal battery fixes the potential difference between its terminals. Ohm's law then gives the current in a resistor directly across it: $I = V/R$.
- Kirchhoff's voltage law determines a loop current from the net emf and total series resistance; Kirchhoff's current law gives the total battery current by adding currents leaving its positive terminal.

Step 1: Calculate the two local resistor currents Both batteries have their positive terminals at the upper nodes, as shown by the longer battery plates. The $20\,\Omega$ resistor is directly across the $50\,\text{V}$ battery, so its downward current is

$$I_{20} = \frac{50}{20} = 2.5\,\text{A}.$$

Similarly the $10\,\Omega$ resistor lies across $30\,\text{V}$, giving

$$I_{10} = \frac{30}{10} = 3.0\,\text{A}.$$

These branch currents are not yet the full battery currents.

Step 2: Find the current between the battery branches The outer loop passes through both horizontal $5\,\Omega$ resistors and both opposing batteries. The stronger source drives a current towards the weaker source along the upper link:

$$I_{\text{link}} = \frac{50 - 30}{5 + 5} = 2.0\,\text{A}.$$

The lower link carries the same current back towards the $50\,\text{V}$ negative terminal. Adding the two emfs instead would ignore their opposing polarity around this loop.

Step 3: Apply current balance at the upper nodes The $50\,\text{V}$ battery supplies both its local resistor and the upper link, so

$$I_{50} = 2.5 + 2.0 = 4.5\,\text{A}.$$

At the other upper node, the $10\,\Omega$ resistor takes $3.0\,\text{A}$, of which $2.0\,\text{A}$ arrives from the link. The $30\,\text{V}$ battery therefore supplies the remaining

$$I_{30} = 3.0 - 2.0 = 1.0\,\text{A}.$$

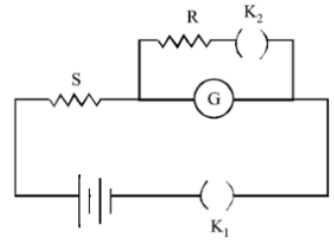
Only the third pair satisfies these two node balances; using 2.5 A and 3 A merely reports the resistor currents.

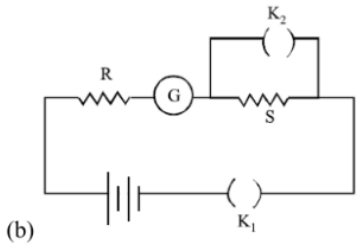
Step 4: Conclusion The battery currents are $4.5\,\text{A}$ and $1.0\,\text{A}$, respectively. The alternative $2.5\,\text{A}$ and $3.0\,\text{A}$ gives only the resistor currents and omits the $2.0\,\text{A}$ link current. Hence, the correct option is (3).

Quick Tip to Solve: Find the local currents across each ideal battery, then add or subtract the link current at each battery terminal.

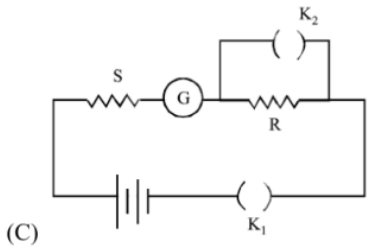
49. In the circuit diagrams (A, B, C and D) shown below, R is a high resistance and S is a resistance of the order of galvanometer resistance G. The correct circuit, corresponding to the half deflection method for finding the resistance and figure of merit of the galvanometer, is the circuit labelled as:

(a) [4 marks]

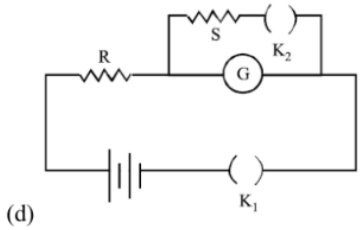




(b)



(C)



(d)

(1) Circuit A with $G = \dfrac{RS}{R-S}$

(2) Circuit B with $G = S$

(3) Circuit C with $G = S$

(4) Circuit D with $G = \dfrac{RS}{R-S}$

Correct Answer: (4) Circuit D with $G = \dfrac{RS}{R-S}$

Solution:

Concept:

- Galvanometer deflection θ is proportional to its current. In the half-deflection method, a shunt S is connected directly across the galvanometer while a high series resistance R limits and nearly stabilises the supply current.
- For an ideal supply of emf E, the initial galvanometer current is $I_g = E/(R+G)$. After inserting

the shunt, exact circuit analysis gives $I_g' = ES/[R(G+S)+GS]$.

Step 1: Identify the required switching arrangement The initial measurement requires the galvanometer to be in series with the high resistance R, with the shunt branch disconnected. Closing the second key must add the lower resistance S in parallel with the galvanometer, reducing its current and deflection. Circuit D has exactly this arrangement. In A, the high resistance is put in the shunt branch instead; in B and C the second key bypasses a separate series resistor rather than connecting S across the galvanometer.

Step 2: Calculate the galvanometer current with the shunt In circuit D, the resistance of the galvanometer-shunt pair is $GS/(G+S)$. The total current is $E/[R+GS/(G+S)]$, of which the fraction $S/(G+S)$ flows through the galvanometer. Hence

$$I_g' = \frac{ES}{R(G+S)+GS}.$$

This accounts for the slight change in total supply current when the second key is closed.

Step 3: Impose half deflection and solve for resistance Half deflection requires $I_g' = I_g/2$. Cancelling the nonzero emf gives

$$2S(R+G) = R(G+S)+GS.$$

Rearrangement yields

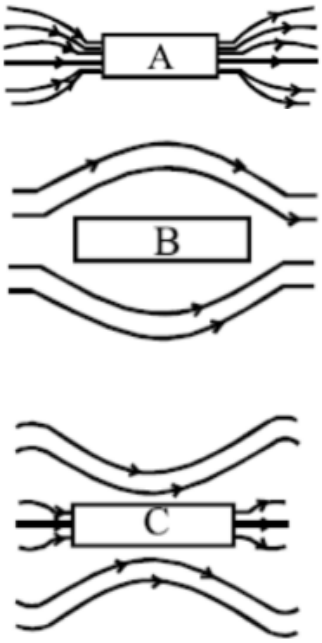
$$RS+SG = RG, \qquad G(R-S) = RS, \qquad G = \frac{RS}{R-S}.$$

For $R \gg S$, this approaches $G \simeq S$, but the exact expression in option (4) retains the finite series-resistance correction. Once G is known, the figure of merit is $I_g/\theta = E/[(R+G)\theta]$.

Step 4: Conclusion The correct arrangement is circuit D, with $G = \frac{RS}{R-S}$. Hence, the correct option is (4).

Quick Tip to Solve: In half deflection, look for high resistance in series and a switched low resistance directly across the galvanometer; keep the small current-change correction for exact G.

50. Three identical bars A, B and C are made of different magnetic materials. When kept in a uniform magnetic field, the field lines around them look as follows. Make the correspondence of these bars with their material being diamagnetic (D), ferromagnetic (F) and paramagnetic (P): [4 marks]



(1) $A \leftrightarrow D,\ B \leftrightarrow P,\ C \leftrightarrow F$

(2) $A \leftrightarrow F,\ B \leftrightarrow D,\ C \leftrightarrow P$

(3) $A \leftrightarrow P,\ B \leftrightarrow F,\ C \leftrightarrow D$

(4) $A \leftrightarrow F,\ B \leftrightarrow P,\ C \leftrightarrow D$

Correct Answer: (2) $A \leftrightarrow F,\ B \leftrightarrow D,\ C \leftrightarrow P$

Solution:

Concept:

- The density of magnetic field lines indicates the magnetic induction. A material that draws many field lines into itself has a greater magnetic permeability than its surroundings.
- Diamagnetic substances have a small negative susceptibility, $\chi_m < 0$; paramagnetic substances have a small positive susceptibility; ferromagnetic substances have a very large positive susceptibility in their unsaturated response.

Step 1: Identify the strongest concentration of field lines In diagram A, many external lines bend sharply towards the bar and crowd into its ends. This is the strongest attraction of magnetic flux among the three diagrams. Such a pronounced increase of induction is characteristic of a ferromagnetic material, whose domains align strongly with the applied field. Therefore $A \leftrightarrow F$. Merely having lines through the bar is not enough to identify it: the substantial crowding distinguishes A from the weaker response of C.

Step 2: Compare exclusion with weak attraction In B, the lines bow outwards around the specimen and avoid its interior. The induced magnetisation opposes the applied field, reducing the induction inside the material. This identifies B as diamagnetic: $B \leftrightarrow D$. In C, some lines bend into the bar, but

the concentration is much less pronounced than in A. Its magnetisation is along the applied field and produces a weak enhancement, so $C \leftrightarrow P$, corresponding to a paramagnetic material.

Step 3: Check the complete correspondence The three independent observations give

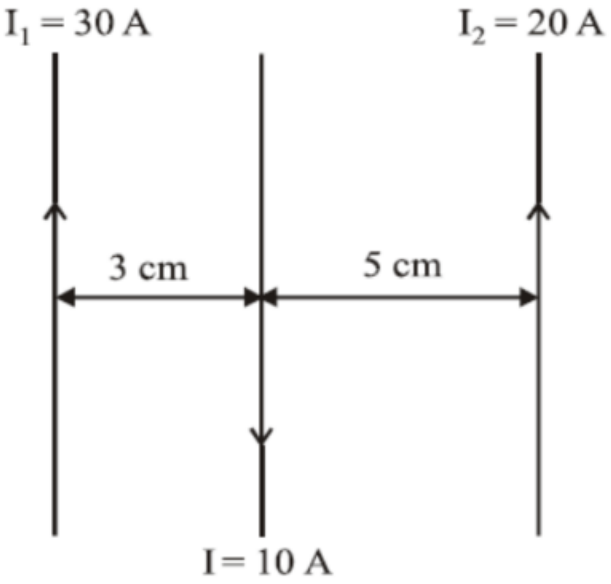
$$A \leftrightarrow F, \qquad B \leftrightarrow D, \qquad C \leftrightarrow P.$$

Option (1) makes A diamagnetic even though its field lines crowd strongly into the bar. Option (3) makes B ferromagnetic although its lines are expelled. Option (4) interchanges the weakly attracted and repelled specimens: B cannot be paramagnetic and C cannot be diamagnetic. Only option (2) respects all three patterns.

Step 4: Conclusion A is ferromagnetic, B is diamagnetic and C is paramagnetic. Hence, the correct option is (2).

Quick Tip to Solve: First locate the bar that pushes field lines away: it is diamagnetic. Of the two remaining bars, the one with much stronger line crowding is ferromagnetic; the weaker attraction identifies the paramagnetic bar.

51. Three straight parallel current carrying conductors are shown in the figure. The force experienced by the middle conductor of length $25\,\text{cm}$ is: [4 marks]



(1) $3 \times 10^{-4}\,\text{N}$ toward right

(2) $6 \times 10^{-4}\,\text{N}$ toward right

(3) $9 \times 10^{-4}\,\text{N}$ toward right

(4) Zero

Correct Answer: (1) $3 \times 10^{-4}\,\text{N}$ toward right

Solution:

Concept:

- Two long parallel wires carrying currents I_a and I_b, separated by distance r, exert a force $F = \mu_0 I_a I_b L/(2\pi r)$ on a length L. Currents in the same direction attract; currents in opposite directions repel.
- The resultant force on the middle conductor is the vector sum of the separate forces exerted by the left and right conductors. Use SI lengths and $\mu_0/(2\pi) = 2 \times 10^{-7}\,\mathrm{N\,A^{-2}}$.

Step 1: Assign the force directions from the currents The middle wire carries $10\,\mathrm{A}$ downward, while the left and right wires carry $30\,\mathrm{A}$ and $20\,\mathrm{A}$ upward. Both outer currents are opposite to the middle current, so both forces are repulsive. The left wire pushes the middle wire towards the right. The right wire pushes it towards the left. These forces must therefore be subtracted, not added. The relevant middle-wire length is $L = 25\,\mathrm{cm} = 0.25\,\mathrm{m}$.

Step 2: Calculate the two individual magnetic forces For the left wire, the separation is $r_L = 3\,\mathrm{cm} = 0.03\,\mathrm{m}$. Thus

$$F_L = 2 \times 10^{-7}\frac{30 \times 10 \times 0.25}{0.03} = 5.0 \times 10^{-4}\,\mathrm{N}$$

towards the right. For the right wire, the separation from the middle wire is $r_R = 5\,\mathrm{cm} = 0.05\,\mathrm{m}$, giving

$$F_R = 2 \times 10^{-7}\frac{20 \times 10 \times 0.25}{0.05} = 2.0 \times 10^{-4}\,\mathrm{N}$$

towards the left. The $8\,\mathrm{cm}$ distance between the outer wires is not a separation involving the middle wire and does not enter either calculation.

Step 3: Subtract the opposing forces and select the option Taking rightward as positive,

$$F_{\text{net}} = F_L - F_R = (5.0 - 2.0) \times 10^{-4} = 3.0 \times 10^{-4}\,\mathrm{N}.$$

The positive sign establishes the rightward direction. Options (2) and (3) give magnitudes inconsistent with these pairwise forces. Option (4) would require equal values of I/r for the outer wires, whereas $30/0.03 = 1000$ and $20/0.05 = 400$ are unequal.

Step 4: Conclusion The middle conductor experiences a net force of $3 \times 10^{-4}\,\mathrm{N}$ towards the right. Hence, the correct option is (1).

Quick Tip to Solve: Fix attraction or repulsion before calculating. For a common middle-wire current and length, compare each outer wire's I/r: the larger value determines the direction when the two forces oppose.

52. A coil of circular cross-section having 1000 turns and $4\,\mathrm{cm}^2$ face area is placed with its axis parallel to a magnetic field which decreases by $10^{-2}\,\mathrm{Wb\,m}^{-2}$ in $0.01\,\mathrm{s}$. The e.m.f. induced in the coil is: [4 marks]

(1) $400\,\mathrm{mV}$

(2) $200\,\mathrm{mV}$

(3) $4\,\mathrm{mV}$

(4) $0.4\,\mathrm{mV}$

Correct Answer: (1) $400\,\mathrm{mV}$

Solution:

Concept:

- Faraday's law gives the magnitude of the induced emf as $|\mathcal{E}| = N|\Delta\Phi|/\Delta t$, where N is the number of turns and Φ is flux through one turn.
- Magnetic flux through a flat turn is $\Phi = BA\cos\theta$, with θ measured between the magnetic field and the normal to the coil. The coil axis is its normal.
- Convert area to square metres before substitution: $1\,\mathrm{cm}^2 = 10^{-4}\,\mathrm{m}^2$. The direction of induced emf is fixed by Lenz's law; only its magnitude is requested here.

Step 1: Identify the flux change The axis is parallel to the field, so $\theta = 0$ and the flux per turn is BA, not zero. The stated decrease has magnitude $\Delta B = 10^{-2}\,\mathrm{Wb\,m}^{-2}$. The face area is

$$A = 4\,\mathrm{cm}^2 = 4 \times 10^{-4}\,\mathrm{m}^2.$$

Hence the magnitude of the flux change through one turn is

$$|\Delta\Phi| = A|\Delta B| = 4 \times 10^{-6}\,\mathrm{Wb}.$$

Step 2: Apply Faraday's law to all turns The flux linkages of the 1000 turns add. For the stated time interval,

$$|\mathcal{E}| = \frac{1000(4 \times 10^{-6})}{0.01} = 0.4\,\mathrm{V} = 400\,\mathrm{mV}.$$

The time is already in seconds. Using the flux change of just one turn would miss the factor 1000. Options 200 mV, 4 mV and 0.4 mV do not equal the emf from the stated area, turn count and rate of field change. In particular, no extra factor of one-half is used in Faraday's law.

Step 3: Conclusion The induced emf has magnitude $400\,\mathrm{mV}$. Hence, the correct option is (1).

Quick Tip to Solve: For a coil whose axis is along the field, use $NA\Delta B/\Delta t$ directly; convert cm^2 to m^2 before multiplying.

53. An electromagnetic wave of frequency 1×10^{14} hertz is propagating along z-axis. The amplitude of electric field is $4\,\mathrm{V/m}$. If $\varepsilon_0 = 8.8 \times 10^{-12}\,\mathrm{C^2/(N\,m^2)}$, then average energy density of electric field will be: [4 marks]

(1) $35.2 \times 10^{-10}\,\mathrm{J/m^3}$

(2) $35.2 \times 10^{-11}\,\mathrm{J/m^3}$

(3) $35.2 \times 10^{-12}\,\mathrm{J/m^3}$

(4) $35.2 \times 10^{-13}\,\mathrm{J/m^3}$

Correct Answer: (3) $35.2 \times 10^{-12}\,\mathrm{J/m^3}$

Solution:

Concept:

- The instantaneous electric-field energy density in vacuum is $u_E = \frac{1}{2}\varepsilon_0 E^2$. This is the electric contribution alone, not the sum of electric and magnetic energies.
- For a sinusoidal field of amplitude E_0, the time average satisfies $\langle E^2 \rangle = E_0^2/2$, so $\langle u_E \rangle = \varepsilon_0 E_0^2/4$.
- The propagation direction and frequency do not change the cycle-averaged energy density at a specified electric amplitude. Frequency only fixes how rapidly the field oscillates.

Step 1: Average the squared electric field Write the travelling electric field as $E = E_0 \cos(kz - \omega t)$, with $E_0 = 4\,\mathrm{V\,m^{-1}}$. Squaring and averaging over a complete cycle gives

$$\langle E^2 \rangle = 16 \langle \cos^2(kz - \omega t) \rangle = 8\,\mathrm{V^2\,m^{-2}}.$$

The amplitude is not the rms field: the rms field is $4/\sqrt{2}\,\mathrm{V\,m^{-1}}$. This distinction supplies the additional factor of one-half.

Step 2: Calculate the electric contribution Using the supplied value $\varepsilon_0 = 8.8 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$,

$$\langle u_E \rangle = \frac{1}{2}(8.8 \times 10^{-12})(8) = 35.2 \times 10^{-12}\,\mathrm{J\,m^{-3}}.$$

The total average electromagnetic energy would be twice this, since the magnetic contribution equals the electric contribution. That total is not what is asked. Options (1), (2) and (4) differ by powers of ten and do not follow from the supplied permittivity and the squared amplitude.

Step 3: Conclusion The average energy density of the electric field is $35.2 \times 10^{-12}\,\mathrm{J\,m^{-3}}$. Hence, the correct option is (3).

Quick Tip to Solve: When a question gives the field amplitude and asks for average electric energy, remember both halves: one in $\varepsilon_0 E^2/2$, and one from averaging the squared sinusoid.

54. An object is located in a fixed position in front of a screen. Sharp image is obtained on the screen for two positions of a thin lens separated by 10 cm. The size of the images in two situations are in the ratio 3 : 3. What is the distance between the screen and the object? [4 marks]

(1) 124.5 cm

(2) 144.5 cm

(3) 65.0 cm

(4) 99.0 cm

Correct Answer: The printed image-size ratio 3:3 forces zero lens displacement, contradicting the stated 10 cm separation; none of the options satisfies both conditions.

Solution:

Concept:

- For a real image of a fixed object on a fixed screen, let the positive object and image distances be u and v. Their sum is the object-screen distance $D = u + v$.
- The thin-lens equation is $1/f = 1/u + 1/v$, and the magnitude of the linear magnification is $m = v/u$. The two conjugate lens positions exchange u and v.
- Therefore the two magnifications have product one, and the separation of the two lens positions is $d = |v - u|$. Equal image sizes force equal conjugate distances.

Step 1: Use the printed image-size ratio The question states that the image sizes in the two situations are in the ratio $3 : 3$, which is unity. Since the object size is unchanged, the magnification magnitudes obey

$$\frac{m_1}{m_2} = 1.$$

In the two conjugate positions, $m_1 = v/u$ and $m_2 = u/v$. Thus

$$\frac{v^2}{u^2} = 1.$$

As both distances are positive for a real object and a real image, this implies $u = v$, and both magnifications have magnitude one.

Step 2: Test the stated lens separation Equal conjugate distances imply

$$d = |v - u| = 0.$$

The two conjugate lens positions then coincide at the midpoint between object and screen. This contradicts the separately stated separation $d = 10\,\text{cm}$. The contradiction is independent of the screen distance or the focal length. Consequently none of 124.5 cm, 144.5 cm, 65.0 cm or 99.0 cm can satisfy all the printed data. A nonunit image-size ratio could describe two distinct positions, but changing the ratio would change the question and cannot be used to select an answer here.

Step 3: Conclusion As printed, the ratio $3 : 3$ requires coincident lens positions, whereas their separation is stated to be $10\,\text{cm}$. The data are inconsistent, so no listed option is defensible.

Quick Tip to Solve: Before applying the displacement formula, check that two distinct lens positions give reciprocal magnifications; equal image sizes mean the two positions have merged.

55. In a compound microscope the focal length of objective lens is 1.2 cm and focal length of eye piece is 3.0 cm. When object is kept at 1.25 cm in front of objective, final image is formed at infinity. Magnifying power of the compound microscope should be [4 marks]

(1) 200

(2) 100

(3) 400

(4) 150

Correct Answer: (1) 200

Solution:

Concept:

- The objective of a compound microscope first produces a real enlarged intermediate image. With Cartesian signs its lens equation is $1/f_o = 1/v_o - 1/u_o$.
- For a final image at infinity, the intermediate image lies in the focal plane of the eyepiece. The eyepiece angular magnification is then $M_e = D/f_e$, where $D = 25\,\text{cm}$ is the conventional near-point distance.
- The total magnifying power in magnitude is the objective linear magnification multiplied by the eyepiece angular magnification: $|M| = |v_o/u_o| D/f_e$.

Step 1: Find the objective image distance The object is in front of the objective, so $u_o = -1.25\,\text{cm}$, while $f_o = +1.2\,\text{cm}$. Hence

$$\frac{1}{v_o} = \frac{1}{1.2} - \frac{1}{1.25} = \frac{5}{6} - \frac{4}{5} = \frac{1}{30}\,\text{cm}^{-1},$$

giving $v_o = 30\,\text{cm}$. The intermediate image is real and inverted. Its linear magnification has magnitude

$$|m_o| = \frac{30}{1.25} = 24.$$

Use the actual supplied object position here rather than replacing the objective magnification by a rough tube-length approximation.

Step 2: Combine the objective and eyepiece factors For relaxed viewing, the final rays emerging from the eyepiece are parallel. Thus

$$M_e = \frac{25}{3}, \qquad |M| = 24\left(\frac{25}{3}\right) = 200.$$

The final image is inverted relative to the original object; the options give the positive magnifying power. The finite-near-point expression $1 + D/f_e$ is not applicable because the final image is at infinity. Values 100, 400 and 150 do not equal the product of the objective and eyepiece magnifications calculated from these data.

Step 3: Conclusion The magnitude of the microscope's magnifying power is 200. Hence, the correct option is (1).

Quick Tip to Solve: With the final image at infinity, use D/f_e for the eyepiece and calculate the objective magnification from the actual object distance.

56. Two monochromatic light beams of intensity 16 and 9 units are interfering. The ratio of intensities of bright and dark parts of the resultant pattern is [4 marks]

(1) $\frac{16}{9}$

(2) $\frac{4}{3}$

(3) $\frac{7}{1}$

(4) $\frac{49}{1}$

Correct Answer: (4) $\frac{49}{1}$

Solution:

Concept:

- For two coherent beams, the resultant intensity at phase difference δ is $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$.
- Constructive interference gives $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$, while destructive interference gives $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$. Intensities are proportional to the squared amplitudes.
- The dark-fringe intensity is zero only when the two component amplitudes are equal. Unequal input intensities give a nonzero minimum even at phase difference π.

Step 1: Convert the intensities to relative amplitudes The two input intensities are 16 and 9 in the same units. Their field amplitudes are therefore proportional to

$$a_1 = \sqrt{16} = 4, \qquad a_2 = \sqrt{9} = 3.$$

We add or subtract these amplitudes before squaring. Adding the intensities directly would omit the interference term. The phase difference changes across the pattern, giving maxima when the fields reinforce and minima when they oppose.

Step 2: Evaluate the bright and dark intensities At a bright fringe the amplitudes add, so

$$I_{\max} = (4 + 3)^2 = 49.$$

At a dark fringe they subtract, so

$$I_{\min} = (4 - 3)^2 = 1.$$

Both values use the original intensity unit, which cancels in their ratio:

$$\frac{I_{\max}}{I_{\min}} = \frac{49}{1}.$$

Option (1) is the ratio of the individual beam intensities, not the resultant extremes. Option (2) is the input amplitude ratio. Option (3) is the ratio of maximum and minimum resultant amplitudes and must be squared to obtain the intensity ratio.

Step 3: Conclusion The bright-to-dark intensity ratio is $49 : 1$. Hence, the correct option is (4).

Quick Tip to Solve: Take square roots of the input intensities first; square the sum for the bright fringe and the difference for the dark fringe.

57. A photon of wavelength λ is scattered from an electron, which was at rest. The wavelength shift $\Delta\lambda$ is three times of λ and the angle of scattering θ is 60°. The angle at which the electron recoiled is ϕ. The value of $\tan\phi$ is: (electron speed is much smaller than the speed of light) [4 marks]

(1) 0.16

(2) 0.22

(3) 0.25

(4) 0.28

Correct Answer: (3) 0.25

Solution:

Concept:

- For a photon, momentum is $p = h/\lambda$. The electron is initially at rest, so the vector sum of the scattered photon momentum and recoil electron momentum equals the incident photon momentum.
- Choose the incident photon direction as the positive horizontal axis. Resolve momentum parallel and perpendicular to this axis; the recoil angle follows from the ratio of the electron momentum components.

Step 1: Find the scattered photon wavelength The shift is the change in wavelength, not the final wavelength. Thus

$$\Delta\lambda = 3\lambda, \qquad \lambda' = \lambda + \Delta\lambda = 4\lambda.$$

Consequently, if the incident photon momentum is $p = h/\lambda$, the scattered photon momentum is $p' = h/\lambda' = p/4$. Using 3λ as the final wavelength would give an incorrect recoil direction.

Step 2: Resolve the conserved momentum Let P be the electron recoil momentum. The electron recoils on the opposite side of the incident direction from the scattered photon. The component equations are

$$P\cos\phi = p - p'\cos\theta, \qquad P\sin\phi = p'\sin\theta.$$

Dividing the magnitudes gives

$$\tan\phi = \frac{p'\sin\theta}{p - p'\cos\theta} = \frac{\sin 60^\circ}{4 - \cos 60^\circ} = \frac{\sqrt{3}/2}{4 - 1/2} = \frac{\sqrt{3}}{7} \approx 0.2474.$$

The recoil momentum magnitude cancels, so neither the electron mass nor its speed is needed. The values 0.16, 0.22 and 0.28 do not agree with the momentum-component ratio; 0.25 is its stated precision.

Step 3: Conclusion $\tan\phi = \sqrt{3}/7 \approx 0.25$. The keyed value 0.22 fails the momentum-component ratio for the stated $\lambda\prime = 4\lambda$. Hence, the correct option is (3).

Quick Tip to Solve: First convert the wavelength shift to the final wavelength. Then divide transverse photon momentum by the horizontal momentum transferred to the electron.

58. Match the List-I (Phenomenon associated with electromagnetic radiation) with List-II (Part of electromagnetic spectrum) and select the correct code from the choices given below these lists:

	List I		List II
I	Doublet of sodium	(A)	Visible radiation
II	Wavelength corresponding to temperature associated with the isotropic radiation filling all space	(B)	Microwave
III	Wavelength emitted by atomic hydrogen in interstellar space	(C)	Short radio wave
IV	Wavelength of radiation arising from two close energy levels in hydrogen	(D)	X-rays

[4 marks]

(1) (I)-(A), (II)-(B), (III)-(B), (IV)-(C)

(2) (I)-(A), (II)-(B), (III)-(C), (IV)-(C)

(3) (I)-(D), (II)-(C), (III)-(A), (IV)-(B)

(4) (I)-(B), (II)-(A), (III)-(D), (IV)-(A)

Correct Answer: (1) (I)-(A), (II)-(B), (III)-(B), (IV)-(C)

Solution:

Concept:

- Electromagnetic radiation is classified by wavelength and frequency, related by $c = \nu\lambda$. A familiar source or transition can identify its spectral region.
- The sodium doublet is visible yellow light. Cosmic background radiation and the interstellar hydrogen hyperfine line are microwaves, while very close hydrogen energy levels can produce radio emission.

Step 1: Identify the first two phenomena The sodium doublet consists of the familiar yellow lines near $589\,\text{nm}$. These are visible wavelengths, so I matches (A), not X-rays or microwaves. The isotropic radiation filling space is the cosmic microwave background. Its temperature is about $2.7\,\text{K}$, corresponding to millimetre-scale radiation. Thus II matches (B), microwave. Already these two assignments rule out the third and fourth codes, which assign sodium to X-rays and microwaves respectively.

Step 2: Match hydrogen radiation and select the code Neutral atomic hydrogen in interstellar space emits its characteristic $21\,\text{cm}$ hyperfine line, of frequency approximately $1.42\,\text{GHz}$. It lies in the microwave part of the radio spectrum, so III matches (B) in the supplied classification. This distinguishes the first code from the second, which places III in (C). For two very close energy levels, the photon energy difference is small. Since $\lambda = hc/\Delta E$, a small separation gives a long wavelength, represented here by short radio waves, so IV matches (C). The complete sequence is therefore I–A, II–B, III–B, IV–C. These assignments use the phenomena listed, rather than assuming that each spectral category must be used once.

Step 3: Conclusion The correct code matches sodium to visible radiation, the cosmic background and interstellar hydrogen line to microwaves, and the close-level transition to short radio waves. The keyed fourth code instead assigns sodium to microwaves, contradicting its visible yellow doublet.

Hence, the correct option is (1).

Quick Tip to Solve: Use sodium yellow light and the cosmic microwave background as anchors; the 21 cm hydrogen line then distinguishes the two remaining codes.

59. A radioactive nuclei with decay constant 0.5/s is being produced at a constant rate of 100 nuclei/s. If at t = 0 there were no nuclei, the time when there are 50 nuclei is [4 marks]

(1) $1\,\text{s}$

(2) $2\ln\left(\frac{4}{3}\right)\,\text{s}$

(3) $\ln 2\,\text{s}$

(4) $\ln\left(\frac{4}{3}\right)\,\text{s}$

Correct Answer: (2) $2\ln\left(\frac{4}{3}\right)\,\text{s}$

Solution:

Concept:

- When radioactive nuclei are supplied at a constant rate R and decay with constant λ, their number obeys $dN/dt = R - \lambda N$. Production increases the population while decay removes existing nuclei.
- With $N(0) = 0$, the solution is $N(t) = \frac{R}{\lambda}(1 - e^{-\lambda t})$. The limiting population R/λ is reached when production balances decay.

Step 1: Set up and solve the population equation Here $R = 100\,\text{s}^{-1}$ and $\lambda = 0.5\,\text{s}^{-1}$. The decay rate depends on the number already present, so using simply $N = Rt$ would ignore the loss of newly created nuclei. Separating the linear equation, or multiplying it by the integrating factor $e^{\lambda t}$, gives

$$\frac{d}{dt}(Ne^{\lambda t}) = Re^{\lambda t}.$$

Integrating from zero to t, with no nuclei initially, yields

$$N = \frac{R}{\lambda}(1 - e^{-\lambda t}) = 200(1 - e^{-t/2}).$$

Step 2: Apply the required population and compare choices Set the population equal to 50:

$$50 = 200(1 - e^{-t/2}), \qquad e^{-t/2} = \frac{3}{4}.$$

Taking logarithms gives

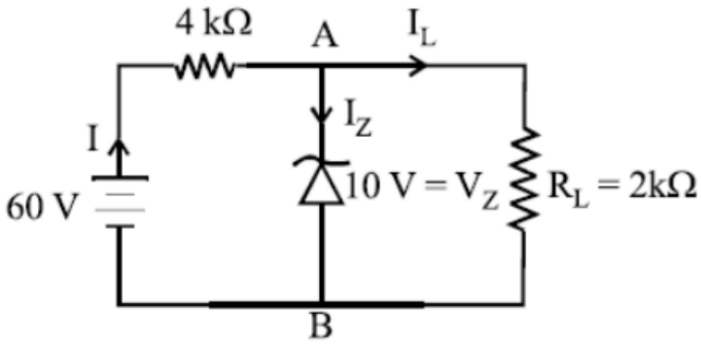
$$-\frac{t}{2} = \ln\frac{3}{4}, \qquad t = 2\ln\frac{4}{3}\,\text{s} \approx 0.5754\,\text{s}.$$

The result exceeds $0.5\,\text{s}$, the time production alone would need, as expected because some nuclei decay during production. Neither $1\,\text{s}$ nor $\ln 2\,\text{s}$ gives the required population. The last choice omits the factor $1/\lambda = 2\,\text{s}$, so it is also incorrect.

Step 3: Conclusion The time is $2\ln(4/3)\,\text{s}$. Hence, the correct option is (2).

Quick Tip to Solve: Compute the equilibrium population R divided by the decay constant first; the target fraction immediately gives the exponential decay factor.

60. A Zener diode is connected to a battery and a load as show below: The currents, I, IZ and IL are respectively [4 marks]



(1) 15 mA, 5 mA, 10 mA

(2) 15 mA, 7.5 mA, 7.5 mA

(3) 12.5 mA, 5 mA, 7.5 mA

(4) 12.5 mA, 7.5 mA, 5 mA

Correct Answer: (4) 12.5 mA, 7.5 mA, 5 mA

Solution:

Concept:

- A reverse-biased Zener diode in breakdown holds the parallel load voltage at its Zener voltage V_Z, provided a positive diode current remains available.
- Ohm's law gives the current in each resistor. At junction A, Kirchhoff's current law gives $I = I_Z + I_L$.

Step 1: Calculate the supply and load currents The $10\,\text{V}$ Zener diode is connected in parallel with the $2\,\text{k}\Omega$ load between A and B. Taking the regulated output as $V_{AB} = 10\,\text{V}$, the $4\,\text{k}\Omega$ series resistor has the remaining battery voltage across it:

$$V_R = 60 - 10 = 50\,\text{V}.$$

Therefore the current supplied to junction A is

$$I = \frac{50}{4000}\,\text{A} = 12.5\,\text{mA}.$$

The load is across the Zener voltage, not the full battery, so

$$I_L = \frac{10}{2000}\,\text{A} = 5\,\text{mA}.$$

Step 2: Find and check the Zener current At A the series current divides between the downward Zener branch and the load branch. Thus

$$I_Z = I - I_L = 12.5 - 5 = 7.5\,\text{mA}.$$

This positive value confirms that the series resistor can supply both the load and a reverse-breakdown current, consistent with the assumed regulation. The ordered triple requested is (I, I_Z, I_L), so the last two entries must not be interchanged. The first two options use $15\,\text{mA}$, which neglects the Zener voltage drop when evaluating the series current. The third option has the correct total but an incorrect split and load current.

Step 3: Conclusion $I = 12.5\,\text{mA}$, $I_Z = 7.5\,\text{mA}$, and $I_L = 5\,\text{mA}$. Hence, the correct option is (4).

Quick Tip to Solve: Apply the Zener voltage to the load and the remaining battery voltage to the series resistor; subtract the load current only after calculating both.

Chemistry (120 marks)

61. A gaseous compound of nitrogen and hydrogen contains 12.5% (by mass) of hydrogen. The density of the compound relative to hydrogen is 16. The molecular formula of the compound is [4 marks]

(1) NH_2

(2) N_3H

(3) NH_3

(4) N_2H_4

Correct Answer: (4) N_2H_4

Solution:

Concept:

- The empirical formula follows from the simplest whole-number mole ratio. Convert each mass percentage to moles by dividing by the corresponding atomic mass.
- For a gas, density relative to hydrogen gas equals its molar mass divided by $2\,\text{g mol}^{-1}$. The molecular formula is an integer multiple of the empirical formula.

Step 1: Determine the empirical composition Take $100\,\text{g}$ of the compound. It contains $12.5\,\text{g}$ hydrogen and $87.5\,\text{g}$ nitrogen. Using atomic masses H = 1 and N = 14, the mole amounts are

$$n(\text{H}) = \frac{12.5}{1} = 12.5, \qquad n(\text{N}) = \frac{87.5}{14} = 6.25.$$

Divide both quantities by 6.25:

$$\text{N} : \text{H} = 1 : 2.$$

Thus the empirical formula is NH_2, whose formula mass is $14 + 2 = 16$. This ratio alone gives the simplest composition, not yet the molecular formula.

Step 2: Use the gas density and test the alternatives The stated relative density refers to molecular

hydrogen, H_2. Hence

$$M = 16 \times 2 = 32\,\text{g mol}^{-1}.$$

The molecular-to-empirical mass ratio is $32/16 = 2$. Multiplying every subscript in NH_2 by two gives N_2H_4. Its hydrogen percentage is $4/32 \times 100 = 12.5\%$, confirming the composition. The choice NH_2 has the correct empirical ratio but only half the required molar mass. Neither N_3H nor NH_3 has the required nitrogen-to-hydrogen ratio or molar mass.

Step 3: Conclusion The molecular formula is N_2H_4. Hence, the correct option is (4).

Quick Tip to Solve: Relative density to hydrogen uses H2, so double the density to obtain molar mass; then compare it with the empirical formula mass.

62. If λ_0 and λ be threshold wavelength and wavelength of incident light, the velocity of photoelectron ejected from the metal surface is: [4 marks]

(1) $\sqrt{\frac{2h}{m}(\lambda_0 - \lambda)}$

(2) $\sqrt{\frac{2hc}{m}(\lambda_0 - \lambda)}$

(3) $\sqrt{\frac{2hc}{m}\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)}$

(4) $\sqrt{\frac{2h}{m}\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right)}$

Correct Answer: (3) $\sqrt{\frac{2hc}{m}\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)}$

Solution:

Concept:

- A photon of wavelength λ has energy hc/λ. At the threshold wavelength λ_0, its energy just equals the metal work function: $\Phi = hc/\lambda_0$.
- The maximum kinetic energy of an emitted electron is the incident photon energy minus the work function: $\frac{1}{2}mv^2 = hc/\lambda - hc/\lambda_0$, where m is the electron mass.

Step 1: Subtract the threshold energy Photoemission requires $\lambda \leq \lambda_0$, so that the incident photon carries at least the energy needed for escape. Einstein's energy balance gives

$$\frac{1}{2}mv^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right).$$

The reciprocal wavelengths must be subtracted in this order: for light shorter than threshold, the kinetic energy must be positive. Putting the difference over a common denominator gives

$$\frac{1}{\lambda} - \frac{1}{\lambda_0} = \frac{\lambda_0 - \lambda}{\lambda\lambda_0}.$$

Step 2: Solve for the speed and check the expression Multiplying by $2/m$ and taking the positive square root yields

$$v = \sqrt{\frac{2hc}{m}\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)}.$$

At $\lambda = \lambda_0$, this becomes zero, as required at threshold. The complete factor under the root has units $\mathrm{m^2\,s^{-2}}$. Options (1) and (2) use a direct wavelength difference without the reciprocal-wavelength denominator, so they do not reproduce photon energy and are dimensionally incorrect. Option (4) lacks c and reverses the reciprocal difference; it would give negative kinetic energy for light capable of emission.

Step 3: Conclusion $v = \sqrt{\frac{2hc}{m}\frac{\lambda_0 - \lambda}{\lambda\lambda_0}}$. Hence, the correct option is (3).

Quick Tip to Solve: Write photon energy minus threshold photon energy before taking the square root; the reciprocal difference prevents both sign and dimension errors.

63. Based on the equation:

$$\Delta E = -2.0 \times 10^{-18}\ \mathrm{J}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

the wavelength of the light that must be absorbed to excite hydrogen electron from level $n = 1$ to level $n = 2$ will be: ($h = 6.625 \times 10^{-34}\ \mathrm{J\,s}$, $C = 3 \times 10^8\ \mathrm{m\,s^{-1}}$) [4 marks]

(1) $1.325 \times 10^{-7}\ \mathrm{m}$

(2) $1.325 \times 10^{-10}\ \mathrm{m}$

(3) $2.650 \times 10^{-7}\ \mathrm{m}$

(4) $5.300 \times 10^{-10}\ \mathrm{m}$

Correct Answer: (1) $1.325 \times 10^{-7}\ \mathrm{m}$

Solution:

Concept:

- An absorbed photon raises the hydrogen electron from the initial level to the final level. The required photon energy is the positive level difference ΔE.
- Photon energy and wavelength satisfy $E = hc/\lambda$. Use the energy coefficient supplied in the question to calculate the requested wavelength consistently.

Step 1: Evaluate the stated energy difference For excitation from $n_1 = 1$ to $n_2 = 2$, substitute directly into the supplied equation:

$$\Delta E = -2.0 \times 10^{-18}\left(\frac{1}{2^2} - \frac{1}{1^2}\right)\ \mathrm{J}.$$

The bracket is $1/4 - 1 = -3/4$, so

$$\Delta E = (-2.0 \times 10^{-18})(-3/4) = 1.5 \times 10^{-18}\ \mathrm{J}.$$

The positive sign confirms absorption. Retaining the coefficient provided, rather than substituting a

different rounded Rydberg energy, is necessary for agreement with the choices.

Step 2: Convert the absorbed energy to wavelength Use $h = 6.625 \times 10^{-34}\,\mathrm{J\,s}$ and $c = 3 \times 10^{8}\,\mathrm{m\,s^{-1}}$:

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.625 \times 10^{-34})(3 \times 10^{8})}{1.5 \times 10^{-18}}\,\mathrm{m} = 1.325 \times 10^{-7}\,\mathrm{m}.$$

The units reduce to metres because hc has units joule-metres. This is $132.5\,\mathrm{nm}$, in the ultraviolet, consistent with excitation from the hydrogen ground state. Option (2) is smaller by a factor of one thousand; option (3) is twice the required wavelength and would carry half the needed energy; option (4) also has an incorrect power of ten.

Step 3: Conclusion The wavelength is $1.325 \times 10^{-7}\,\mathrm{m}$. Hence, the correct option is (1).

Quick Tip to Solve: Calculate the positive energy gap before applying hc divided by energy, and keep the energy constant given in the question throughout.

64. Which of the following series correctly represents relations between the elements from X to Y?
$X \to Y$ [4 marks]

(1) ${}_{3}\mathrm{Li} \to {}_{19}\mathrm{K}$: Ionization enthalpy increases

(2) ${}_{9}\mathrm{F} \to {}_{35}\mathrm{Br}$: Electron gain enthalpy (negative sign) increases

(3) ${}_{6}\mathrm{C} \to {}_{32}\mathrm{Ge}$: Atomic radii increases

(4) ${}_{18}\mathrm{Ar} \to {}_{54}\mathrm{Xe}$: Noble character increases

Correct Answer: (3) ${}_{6}\mathrm{C} \to {}_{32}\mathrm{Ge}$: Atomic radii increases

Solution:

Concept:

- On moving down a group, an extra occupied electron shell generally increases atomic size and shielding.
- Ionization enthalpy measures removal of an electron, whereas electron gain enthalpy describes addition of an electron; their signs and trends must be distinguished.

Step 1: Compare size and ionization trends Carbon and germanium are in group 14, with germanium below carbon. Germanium has more occupied shells, so its atomic radius is larger. Thus the stated increase from carbon to germanium is correct. In contrast, potassium has a larger radius and greater shielding than lithium, so its first ionization enthalpy is lower, not higher.

Step 2: Check the other two statements The negative electron gain enthalpy does not become larger in magnitude from fluorine to bromine: the compact halogens have their own well-known fluorine/chlorine exception, and bromine is less exothermic than fluorine. Argon to xenon gives larger, more polarizable atoms with lower ionization energies. Xenon forms compounds more readily, so noble or inert character decreases rather than increases.

Step 3: Conclusion Atomic radii increase along $\mathrm{C} \to \mathrm{Ge}$. Hence, the correct option is (3).

Quick Tip to Solve: For down-group questions, first count shells; then check whether the statement concerns size, electron removal, or chemical inertness.

65. The correct order of bond dissociation energy among $N_2, O_2, {O_2}^-$ is shown in which of the following arrangements? [4 marks]

(1) $N_2 > {O_2}^- > O_2$

(2) ${O_2}^- > O_2 > N_2$

(3) $N_2 > O_2 > {O_2}^-$

(4) $O_2 > {O_2}^- > N_2$

Correct Answer: (3) $N_2 > O_2 > {O_2}^-$

Solution:

Concept:

- For these related diatomic molecules, a larger bond order corresponds to a stronger bond and a larger bond dissociation energy.
- Bond order is $\frac{1}{2}(N_b - N_a)$, where N_b and N_a count electrons in bonding and antibonding molecular orbitals.

Step 1: Calculate the neutral-molecule bond orders In nitrogen, the valence molecular orbitals contain eight bonding and two antibonding electrons, giving $\mathrm{BO}(N_2) = (8-2)/2 = 3$. In oxygen there are eight bonding and four antibonding valence electrons, giving $\mathrm{BO}(O_2) = (8-4)/2 = 2$. Nitrogen therefore has the strongest bond of the three species.

Step 2: Account for the added electron Forming superoxide adds one electron to an antibonding π^* orbital of oxygen. Therefore

$$\mathrm{BO}({O_2}^-) = \frac{8-5}{2} = \frac{3}{2}.$$

This weakens the oxygen-oxygen bond compared with neutral oxygen. The options placing superoxide above oxygen reverse the effect of an antibonding electron; the option placing oxygen above nitrogen also contradicts their bond orders.

Step 3: Conclusion The order is $N_2 > O_2 > {O_2}^-$. Hence, the correct option is (3).

Quick Tip to Solve: When an electron is added to an antibonding orbital, reduce the bond order by one-half before comparing bond strengths.

66. Which one of the following does not have a pyramidal shape? [4 marks]

(1) $(CH_3)_3N$

(2) $(SiH_3)_3N$

(3) $P(CH_3)_3$

(4) $P(SiH_3)_3$

Correct Answer: (2) $(SiH_3)_3N$

Solution:

Concept:

- A localized lone pair on a nitrogen or phosphorus atom bonded to three groups generally produces a trigonal pyramidal molecular shape.
- In trisilylamine, donation of the nitrogen lone pair toward the silicon substituents favors a planar nitrogen environment and partial multiple-bond character in the nitrogen-silicon bonds.

Step 1: Identify the exception at nitrogen In $(SiH_3)_3N$, the nitrogen lone pair is delocalized into the silicon-containing bonding framework. The usual textbook description is nitrogen-to-silicon back donation. Nitrogen adopts an approximately trigonal planar arrangement, with its three bonds lying in a plane, rather than the pyramidal arrangement expected from an entirely localized lone pair.

Step 2: Compare the remaining molecules Trimethylamine $(CH_3)_3N$ retains a localized nitrogen lone pair and is pyramidal. The phosphorus analogues $P(CH_3)_3$ and $P(SiH_3)_3$ also retain pyramidal phosphorus centers; the planarization characteristic of nitrogen bonded to silicon does not apply in the same effective way. Thus merely counting three bonds and one formal lone pair would miss the specific delocalization exception in trisilylamine.

Step 3: Conclusion $(SiH_3)_3N$ does not have a pyramidal shape. Hence, the correct option is (2).

Quick Tip to Solve: Before applying the three-bonds-plus-lone-pair rule mechanically, check whether the lone pair is delocalized into neighboring bonds.

67. In allene (C_3H_4), the type(s) of hybridization of the carbon atoms is (are): [4 marks]

(1) sp and sp^3

(2) sp^2 and sp

(3) only sp^2

(4) sp^2 and sp^3

Correct Answer: (2) sp^2 and sp

Solution:

Concept:

- Allene has the connectivity $H_2C{=}C{=}CH_2$: its middle carbon participates in two double bonds.
- Hybridization follows the number of sigma-bond directions: two directions give sp, while three directions give sp^2. A double bond counts as one sigma direction.

Step 1: Examine the central carbon The middle carbon makes one sigma bond to each terminal

carbon. Its two sigma-bond directions are linear, so it is sp-hybridized. The two remaining, mutually perpendicular unhybridized p orbitals form the two pi bonds. Counting each double bond twice as an electron domain would give the wrong geometry.

Step 2: Examine both terminal carbons Each terminal carbon makes two sigma bonds to hydrogen and one sigma bond to the middle carbon. Its three sigma directions form a trigonal planar set, so it is sp^2-hybridized. Neither terminal carbon has four sigma bonds, so there is no sp^3 carbon. An answer with only sp^2 misses the linear central atom, while either option containing sp^3 introduces a hybridization absent from allene.

Step 3: Conclusion Allene contains sp^2 and sp carbon atoms. Hence, the correct option is (2).

Quick Tip to Solve: For a cumulene, count sigma-bond directions at each carbon separately; do not count a pi bond as an extra direction.

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68. The initial volume of a gas cylinder is $750.0\,\text{mL}$. If the pressure of gas inside the cylinder changes from $840.0\,\text{mmHg}$ to $360.0\,\text{mmHg}$, the final volume the gas will be: [4 marks]

(1) $1.750\,\text{L}$

(2) $3.60\,\text{L}$

(3) $4.032\,\text{L}$

(4) $7.50\,\text{L}$

Correct Answer: (1) $1.750\,\text{L}$

Solution:

Concept:

- At fixed temperature and fixed amount of gas, Boyle's law gives $P_1V_1 = P_2V_2$.
- Use the same pressure units on both sides, so the pressure ratio is dimensionless; convert the calculated volume to the unit used in the options.

Step 1: Set up the pressure-volume relation The question describes a pressure change of the same gas, with the standard constant-temperature interpretation of Boyle's law. The initial and final pressures are both given in mmHg. Thus

$$V_2 = \frac{P_1}{P_2}V_1 = \frac{840.0}{360.0} \times 750.0\,\text{mL}.$$

Since the pressure falls, the volume must increase; this provides a useful direction check.

Step 2: Calculate and compare the options The pressure ratio is $7/3$, giving

$$V_2 = 1750\,\text{mL} = 1.750\,\text{L}.$$

The product check is $840.0 \times 0.7500 = 360.0 \times 1.750$ in mmHg L. The other three listed volumes do not satisfy this equality for the stated pressure change. Dividing by the pressure ratio instead of multiplying would incorrectly predict contraction on lowering the pressure.

Step 3: Conclusion The final volume is $1.750\,\text{L}$. Hence, the correct option is (1).

Quick Tip to Solve: Keep both pressures in mmHg, calculate their ratio, and convert mL to L only once at the end.

69. The molar heat capacity (C_p) of CD_2O is 10 cals at $1000\,K$. The change in entropy associated with cooling of $32\,g$ of CD_2O vapour from $1000\,K$ to $100\,K$ at constant pressure will be: (D = deuterium, atomic mass = $2\,u$) [4 marks]

(1) $23.03\,cal\,deg^{-1}$

(2) $-23.03\,cal\,deg^{-1}$

(3) $2.303\,cal\,deg^{-1}$

(4) $-2.303\,cal\,deg^{-1}$

Correct Answer: (2) $-23.03\,cal\,deg^{-1}$

Solution:

Concept:

- At constant pressure, with constant molar heat capacity over the stated interval, $\Delta S = nC_p \ln(T_2/T_1)$. Temperatures must be in kelvin.
- Deuterium has mass number 2, so $M(CD_2O) = 12 + 2(2) + 16 = 32\,g\,mol^{-1}$.

Step 1: Find the amount and temperature ratio The sample contains

$$n = \frac{32\,g}{32\,g\,mol^{-1}} = 1\,mol.$$

The temperature ratio for cooling is $T_2/T_1 = 100/1000 = 0.1$. Taking the supplied molar heat capacity as $10\,cal\,mol^{-1}\,K^{-1}$, as intended by the numerical choices, the coefficient nC_p is $10\,cal\,K^{-1}$.

Step 2: Evaluate the logarithm and sign Integrating $dS = nC_p\,dT/T$ gives

$$\Delta S = 10\ln(0.1) = -10\ln 10 = -23.03\,cal\,K^{-1}.$$

Cooling lowers the entropy of the vapour, so the positive alternatives have the wrong sign. The alternative -2.303 omits the factor of 10 from the heat capacity. A temperature interval of one kelvin equals one Celsius degree, so the printed unit cal deg^{-1} is equivalent for this entropy difference.

Step 3: Conclusion $\Delta S = -23.03\,cal\,deg^{-1}$. Hence, the correct option is (2).

Quick Tip to Solve: Use the final-to-initial temperature ratio inside the logarithm; cooling must produce a negative entropy change for the sample.

70. Consider the following equilibrium

$$AgCl\downarrow + 2\,NH_3 \rightleftharpoons [Ag(NH_3)_2]^+ + Cl^-$$

White precipitate of $AgCl$ appears on adding which of the following? [4 marks]

(1) NH_3

(2) aqueous $NaCl$

(3) aqueous HNO_3

(4) aqueous NH_4Cl

Correct Answer: (3) aqueous HNO_3

Solution:

Concept:

- Ammonia dissolves silver chloride by forming the diamminesilver(I) complex, $[Ag(NH_3)_2]^+$.
- Acid consumes free ammonia through $NH_3 + H^+ \longrightarrow NH_4{}^+$, reducing the ligand concentration and favoring decomposition of the complex.

Step 1: Apply acid-base chemistry to the ligand Adding aqueous nitric acid supplies hydrogen ions, which protonate ammonia. As free NH_3 is removed, the stated equilibrium shifts toward solid $AgCl$. The net reaction can be written

$$[Ag(NH_3)_2]^+ + Cl^- + 2\,H^+ \longrightarrow AgCl\downarrow + 2\,NH_4{}^+.$$

Nitrate does not provide a competing precipitate with silver under these conditions. The reappearing silver chloride is white.

Step 2: Compare the additions carefully Extra ammonia stabilizes the soluble complex and favors dissolution, so it does not produce the requested precipitate. Nitric acid is the standard qualitative-analysis reagent for recovering silver chloride from its ammoniacal solution. Chloride salts cannot, however, be dismissed as inert: adding chloride raises the product concentration and can also favor precipitation if the ion product exceeds the solubility product. The outcome for sodium chloride or ammonium chloride depends on their concentrations and the excess ammonia, which are not supplied. Thus the conventional single-answer interpretation selects nitric acid, while the literal unrestricted wording does not uniquely exclude the chloride additions.

Step 3: Conclusion The conventional keyed answer is aqueous HNO_3, option (3), through protonation of ammonia. The concentration-free wording also permits chloride-induced precipitation under suitable conditions, so uniqueness requires that conventional qualitative-analysis interpretation.

Quick Tip to Solve: For an ammine complex, check both ligand protonation and common-ion effects before claiming that only one added reagent can precipitate the salt.

71. Assuming that the degree of hydrolysis is small, the pH of $0.1\,\mathrm{M}$ solution of sodium acetate ($K_a = 1.0 \times 10^{-5}$) will be: [4 marks]

(1) 5.0

(2) 6.0

(3) 8.0

(4) 9.0

Correct Answer: (4) 9.0

Solution:

Concept:

- Sodium acetate is a salt of a strong base and a weak acid. Its acetate ion accepts a proton from water, generating hydroxide ions and making the solution basic.
- For the conjugate base of an acid, $K_b = K_w/K_a$. At the usual aqueous reference temperature of $298\,\mathrm{K}$, $K_w = 10^{-14}$.
- If hydrolysis is small and the salt concentration is C, the equilibrium hydroxide concentration is $[\mathrm{OH^-}] \simeq \sqrt{K_b C}$.

Step 1: Write the hydrolysis equilibrium The sodium ion is a spectator ion. Acetate undergoes the equilibrium

$$\mathrm{CH_3COO^- + H_2O \rightleftharpoons CH_3COOH + OH^-}.$$

Initially the acetate concentration is $C = 0.1\,\mathrm{mol\,L^{-1}}$. If x is hydrolysed, the two product concentrations are each x, and $K_b = x^2/(C - x)$. The stated small-hydrolysis approximation permits $C - x \simeq C$.

Step 2: Calculate the hydroxide concentration Use the given acid constant to obtain

$$K_b = \frac{10^{-14}}{10^{-5}} = 10^{-9}.$$

Hence

$$x = \sqrt{10^{-9} \times 0.1} = \sqrt{10^{-10}} = 10^{-5}\,\mathrm{mol\,L^{-1}}.$$

The hydrolysed fraction is $x/C = 10^{-4}$, confirming that neglecting the change in acetate concentration is justified.

Step 3: Convert pOH into pH and check the choices The hydroxide concentration gives

$$\mathrm{pOH} = -\log_{10}(10^{-5}) = 5.$$

Therefore

$$\mathrm{pH} = 14 - 5 = 9.$$

The values 5 and 6 are acidic, contradicting acetate hydrolysis. Although 8 is basic, it would require a hydroxide concentration of only $10^{-6}\,\mathrm{M}$, inconsistent with the calculated equilibrium constant.

Step 4: Conclusion The solution has $\boxed{\mathrm{pH} = 9.0}$. Hence, the correct option is (4).

Quick Tip to Solve: For a weak-acid salt, use $\mathrm{pH} = \frac{1}{2}(pK_w + pK_a + \log C)$; here it is $(14 + 5 - 1)/2 = 9$.

72. Which of the following statements about Na_2O_2 is not correct? [4 marks]

(1) It is diamagnetic in nature

(2) It is derivative of H_2O_2

(3) Na_2O_2 oxidises Cr^{3+} to $CrO_4{}^{2-}$ in acid medium.

(4) It is the super oxide of sodium

Correct Answer: Both (3) and (4) are incorrect as printed: chromate formation requires alkaline conditions, and Na_2O_2 is peroxide, not superoxide.

Solution:

Concept:

- The peroxide ion is $O_2{}^{2-}$, whereas the superoxide ion is $O_2{}^{-}$. Sodium peroxide is Na_2O_2, not sodium superoxide.
- Peroxide has all electrons paired and is diamagnetic. Its oxygen oxidation state is -1, as in hydrogen peroxide.
- Oxidation of chromium(III) to chromate by peroxide is the familiar alkaline-medium reaction; acidic peroxide chemistry does not justify the printed claim.

Step 1: Check the two correct statements In Na_2O_2, the two sodium cations balance one $O_2{}^{2-}$ anion. Its molecular-orbital electrons are paired, so statement (1), diamagnetism, is correct. Replacing the acidic hydrogen atoms of H_2O_2 by sodium gives the corresponding peroxide, so statement (2), describing it as a hydrogen-peroxide derivative, is also correct.

Step 2: Check the oxidation medium in statement (3) The printed option explicitly says acid medium. Chromate formation by peroxide oxidation of chromium(III) is associated with alkaline conditions. One balanced equation is

$$2\,Cr(OH)_3 + 3\,H_2O_2 + 4\,OH^- \longrightarrow 2\,CrO_4{}^{2-} + 8\,H_2O.$$

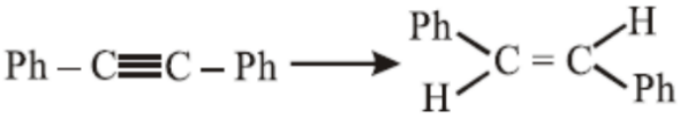
The hydroxide ions are essential to this alkaline formulation. Moreover, acid favours dichromate rather than chromate, and acidic hydrogen peroxide can reduce chromium(VI) to chromium(III). Thus the acid-medium assertion in (3) is not a correct statement as printed.

Step 3: Distinguish peroxide from superoxide in statement (4) Statement (4) calls Na_2O_2 the superoxide of sodium. This is false: it contains peroxide, $O_2{}^{2-}$. A superoxide formula with a monovalent sodium ion would be NaO_2, containing $O_2{}^{-}$. Consequently (4) is false independently of the acid-medium error in (3); the wording cannot select exactly one incorrect statement.

Step 4: Conclusion As printed, both (3) and (4) are incorrect, so this question has no unique answer. The supplied key selects (4), which is certainly false, but (3) is also false because it specifies acid medium instead of the alkaline conditions for chromate formation.

Quick Tip to Solve: Check both the oxygen ion and the reaction medium: peroxide and superoxide differ in charge, while chromium oxidation to chromate depends on alkaline conditions.

73. The reagent needed for converting [4 marks]



(1) Cat. Hydrogenation

(2) H_2/Lindlar Cat.

(3) Li/NH_3

(4) $LiAlH_4$

Correct Answer: (3) Li/NH_3

Solution:

Concept:

- The starting material is diphenylacetylene, $Ph{-}C{\equiv}C{-}Ph$, and the displayed product is trans-stilbene: the two phenyl groups are on opposite sides of its double bond.
- Dissolving-metal reduction of an alkyne with lithium in liquid ammonia adds two hydrogen atoms overall by an anti pathway and gives a trans alkene.
- Lindlar-catalysed hydrogenation gives the cis alkene, whereas ordinary catalytic hydrogenation generally continues to the alkane.

Step 1: Identify the required stereochemistry The reactant contains a carbon–carbon triple bond between the two phenyl groups. In the product diagram, one phenyl group is above the double bond on the left carbon and the other is below it on the right carbon. The required change is therefore partial reduction of the triple bond to the E, or trans, alkene. Simply adding hydrogen without controlling the extent and geometry would not specify this product.

Step 2: Choose the anti-reduction reagent Lithium in liquid ammonia transfers an electron to the alkyne, followed by protonation, a second electron transfer and another protonation. The stepwise dissolving-metal pathway favours the anti arrangement of the new hydrogen atoms, giving trans-stilbene:

$$Ph{-}C{\equiv}C{-}Ph \xrightarrow{Li/NH_3} \text{trans}{-}PhCH{=}CHPh.$$

The phenyl groups are consequently opposite, exactly as drawn in the question.

Step 3: Reject reagents with the wrong product Ordinary catalytic hydrogenation is not the selective dissolving-metal process: with sufficient hydrogen it gives the saturated carbon chain. Hydrogen with Lindlar catalyst stops at the alkene but delivers hydrogen to the same face, giving cis-stilbene, contrary to the displayed trans geometry. Lithium aluminium hydride is used for polar functional-group reductions and is not the standard reagent for this isolated alkyne-to-trans-alkene conversion.

Step 4: Conclusion Lithium in liquid ammonia, Li/NH_3, gives the displayed trans-stilbene. Hence, the correct option is (3).

Quick Tip to Solve: First read the relative positions of the substituents: Lindlar gives cis, while an alkali metal in liquid ammonia gives trans.

74. The gas liberated by the electrolysis of Dipotassium succinate solution is [4 marks]

(1) Ethane

(2) Ethyne

(3) Ethene

(4) Propene

Correct Answer: (3) Ethene

Solution:

Concept:

- Kolbe electrolysis oxidises a carboxylate ion at the anode, removing carbon dioxide and generating a carbon-centred radical.
- Succinate is $^{-}OOC-CH_2-CH_2-COO^{-}$; loss of its two carboxylate groups leaves adjacent carbon centres, which form an alkene.
- The carbon skeleton retained after double decarboxylation contains two carbon atoms, so the characteristic organic gas must be a two-carbon product.

Step 1: Write the ion present in solution Dipotassium succinate dissociates to two potassium ions and the succinate dianion. The carbon chain is

$$^{-}OOC-CH_2-CH_2-COO^{-}.$$

Its terminal carboxylate carbons are not retained in the hydrocarbon; each leaves as carbon dioxide during anodic oxidation. Only the central pair of methylene carbons remains.

Step 2: Carry out anodic double decarboxylation Each carboxylate group loses one electron and then releases CO_2. The two adjacent radical centres left on the central carbons combine by forming a second bond between those carbons. The overall anodic change is

$$^{-}OOC-CH_2-CH_2-COO^{-} \longrightarrow CH_2{=}CH_2 + 2\,CO_2 + 2\,e^{-}.$$

This equation balances carbon, hydrogen and charge. The hydrocarbon produced is ethene, C_2H_4.

Step 3: Compare the listed gases Ethane would require a saturated C_2H_6 product and does not follow from joining the adjacent centres left by this double decarboxylation. Ethyne would require removal of two further hydrogen atoms beyond the displayed process. Propene contains three retained carbon atoms, whereas succinate leaves only two after both carboxylate carbons are removed. Carbon dioxide at the anode and hydrogen at the cathode may also be evolved, but the listed organic gas sought here is ethene.

Step 4: Conclusion The characteristic hydrocarbon gas is $\boxed{CH_2{=}CH_2}$, ethene. Hence, the correct option is (3).

Quick Tip to Solve: For an alkane dicarboxylate, delete both carboxylate carbons first and inspect how the remaining radical centres can bond.

75. Which of the following statements about the depletion of ozone layer is correct? [4 marks]

(1) The problem of ozone depletion is less serious at poles because NO_2 solidifies and is not available for consuming $ClO^{\bullet}$ radicals.

(2) The problem of ozone depletion is more serious at poles because ice crystals in the clouds over poles act as catalyst for photochemical reactions involving the decomposition of ozone of $Cl^{\bullet}$ and $ClO^{\bullet}$ radicals.

(3) Freons, chlorofluorocarbons, are inert. Chemically, they do not react with ozone in stratosphere.

(4) Oxides of nitrogen also do not react with ozone in stratosphere.

Correct Answer: (3) Freons, chlorofluorocarbons, are inert. Chemically, they do not react with ozone in stratosphere.

Solution:

Concept:

- Freons are chlorofluorocarbons with high chemical stability. The intact molecules do not directly react with ozone in the stratosphere.
- Chemical inertness does not exclude photochemical bond cleavage. Ultraviolet light can release chlorine radicals from a freon; these radicals, rather than the intact freon, destroy ozone.
- Polar stratospheric clouds activate chlorine through heterogeneous surface chemistry. This is distinct from ice directly catalysing photochemical ozone decomposition.

Step 1: Distinguish the carrier from the active species Option (3) refers to freon molecules themselves. Their ordinary chemical inertness makes them persistent enough to reach the stratosphere, and they do not directly consume ozone there. This statement does not imply that freons are harmless: the decisive step is photochemical cleavage. For example,

$$CF_2Cl_2 \xrightarrow{h\nu} CF_2Cl^{\bullet} + Cl^{\bullet}.$$

The chlorine radical produced is a new reactive species, distinct from the original stable chlorofluorocarbon.

Step 2: Show the indirect ozone-destruction pathway The liberated chlorine enters a catalytic cycle, represented in simplified form by

$$Cl^{\bullet} + O_3 \longrightarrow ClO^{\bullet} + O_2$$

$$ClO^{\bullet} + O \longrightarrow Cl^{\bullet} + O_2.$$

Chlorine is regenerated and the net change is $O_3 + O \longrightarrow 2\,O_2$. Thus ozone loss caused indirectly by freon-derived radicals is entirely consistent with the claim that intact freon molecules do not chemically react with ozone.

Step 3: Check the other statements precisely Option (1) is incorrect because ozone depletion is especially severe over the poles, not less serious. Option (2) correctly points toward polar clouds but gives the wrong mechanistic role: ice surfaces enable heterogeneous reactions involving chlorine reservoir species, thereby activating chlorine. The subsequent sunlight-driven radical chemistry destroys ozone; ice crystals do not directly act as catalysts for photochemical ozone decomposition by chlorine and chlorine monoxide radicals as stated. Option (4) is false because nitrogen oxides do

react in ozone chemistry; for example, $NO + O_3 \longrightarrow NO_2 + O_2$.

Step 4: Conclusion The intact freon molecules are chemically inert toward ozone; their photolysis products cause the ozone destruction. Hence, the correct option is (3).

Quick Tip to Solve: Keep the molecular species distinct: a stable freon carries chlorine upward, ultraviolet light releases $Cl^{\bullet}$, and that radical reacts with ozone.

76. The appearance of colour in solid alkali metal halides is generally due to [4 marks]

(1) Schottky defect

(2) Frenkel defect

(3) Interstitial position

(4) F-centres

Correct Answer: (4) F-centres

Solution:

Concept:

- An F-centre is an anion vacancy occupied by an electron. The trapped electron maintains local charge compensation in a metal-excess ionic crystal.
- The electron can absorb visible light and be excited to a higher energy state. Selective absorption gives the crystal its observed colour.
- Schottky and Frenkel defects describe missing ions or displaced ions; their definitions alone do not provide the electron-filled vacancy responsible for this characteristic coloration.

Step 1: Identify the relevant defect For example, sodium chloride heated in sodium vapour can acquire excess sodium. The sodium atoms supply sodium ions and electrons. Electrons occupy vacant chloride-ion sites in the crystal rather than ordinary interstitial positions. Each such electron-containing anion vacancy is an F-centre, where the letter F refers to the colour centre.

Step 2: Connect the defect with visible absorption The electron is confined in the electrostatic environment of the vacancy and has discrete energy levels. Absorption of a photon promotes it between these levels when

$$\Delta E = h\nu.$$

If the absorbed photons lie in the visible region, some wavelengths are removed from white light. The remaining transmitted or reflected light makes the otherwise colourless crystal appear coloured. The exact colour depends on the host crystal and the vacancy environment, not merely on having an empty lattice site.

Step 3: Rule out the other defect names A Schottky defect consists of compensating cation and anion vacancies and commonly reduces density; it is not specifically an electron trapped at an anion vacancy. A Frenkel defect moves an ion from its normal lattice site to an interstitial site, leaving a vacancy–interstitial pair. The phrase interstitial position alone does not specify the colour-producing electronic centre. Of the listed alternatives, only F-centres directly name the accepted mechanism for colour in alkali-metal halides.

Step 4: Conclusion The colour is generally caused by electrons trapped in anion vacancies, called F-centres. Hence, the correct option is (4).

Quick Tip to Solve: Associate colour with electronic absorption: an empty anion site plus a trapped electron is an F-centre, rather than an ordinary vacancy defect.

77. In some solutions, the concentration of H_3O^+ remains constant even when small amounts of strong acid or strong base are added to them. These solutions are known as: [4 marks]

(1) Ideal solutions

(2) Colloidal solutions

(3) True solutions

(4) Buffer solutions

Correct Answer: (4) Buffer solutions

Solution:

Concept:

- A buffer solution resists a change in pH when a small amount of strong acid or strong base is added.
- Since $\mathrm{pH} = -\log_{10}[H_3O^+]$, approximately constant hydronium concentration means approximately constant pH.
- A typical acidic buffer contains a weak acid and its conjugate base; the pair consumes added base and added acid respectively.

Step 1: Translate the stated property into pH language The question says that the hydronium-ion concentration remains constant on adding small quantities of strong acid or base. Because pH depends logarithmically on that concentration, the solution resists an appreciable change of pH. This is the defining property of a buffer. In practice the change is very small rather than mathematically zero, and the ability to resist change is limited by the amounts of the buffer components.

Step 2: Explain how the two components respond Consider a mixture containing a weak acid HA and its conjugate base A^-. Added acid is consumed by

$$A^- + H_3O^+ \longrightarrow HA + H_2O,$$

while added hydroxide is consumed by

$$HA + OH^- \longrightarrow A^- + H_2O.$$

Thus neither small addition remains free in solution to produce the large pH change it would cause in pure water. Both a proton donor and its conjugate proton acceptor are needed for this two-sided action.

Step 3: Distinguish the other solution categories An ideal solution is defined by its thermodynamic mixing behaviour, commonly obedience to Raoult's law, rather than by acid–base resistance. Colloidal solutions are classified by dispersed-particle size. True solutions contain molecular or ionic solutes

dispersed homogeneously, but an ordinary true solution need not contain the conjugate acid–base pair required for buffering. These classifications therefore do not identify the special property in the question.

Step 4: Conclusion These solutions are buffer solutions. Hence, the correct option is (4).

Quick Tip to Solve: Whenever small acid and base additions leave pH nearly unchanged, look for a conjugate acid–base pair and identify the solution as a buffer.

78. **Given**

$$\begin{aligned} \mathrm{Fe^{3+}(aq) + e^- \longrightarrow Fe^{2+}(aq);} \quad & E^0 = +0.77\,\mathrm{V} \\ \mathrm{Al^{3+}(aq) + 3\,e^- \longrightarrow Al(s);} \quad & E^0 = -1.66\,\mathrm{V} \\ \mathrm{Br_2(aq) + 2\,e^- \longrightarrow 2\,Br^-;} \quad & E^0 = +1.09\,\mathrm{V} \end{aligned}$$

Considering the electrode potentials, which of the following represents the correct order of reducing power? [4 marks]

(1) $\mathrm{Fe^{2+} < Al < Br^-}$

(2) $\mathrm{Br^- < Fe^{2+} < Al}$

(3) $\mathrm{Al < Br^- < Fe^{2+}}$

(4) $\mathrm{Al < Fe^{2+} < Br^-}$

Correct Answer: (2) $\mathrm{Br^- < Fe^{2+} < Al}$

Solution:

Concept:

- A reducing agent donates electrons and is itself oxidised. Compare the oxidation tendency of the reduced forms $\mathrm{Fe^{2+}}$, Al and $\mathrm{Br^-}$, rather than the oxidising tendency of the species on the left of the printed half-reactions.
- Reversing a standard reduction half-reaction changes the sign of its potential: $E^0_{\mathrm{ox}} = -E^0_{\mathrm{red}}$. A more positive oxidation potential corresponds to a stronger reducing agent in this comparison.

Step 1: Reverse each relevant half-reaction For iron the reducing species is $\mathrm{Fe^{2+}}$, because it can lose one electron to become $\mathrm{Fe^{3+}}$. Its oxidation potential is $-0.77\,\mathrm{V}$. Similarly, aluminium metal gives $\mathrm{Al \longrightarrow Al^{3+} + 3\,e^-}$, with $E^0_{\mathrm{ox}} = +1.66\,\mathrm{V}$. Bromide gives $\mathrm{2\,Br^- \longrightarrow Br_2 + 2\,e^-}$, with $E^0_{\mathrm{ox}} = -1.09\,\mathrm{V}$. Do not multiply an electrode potential by the number of electrons: potential is an intensive quantity.

Step 2: Arrange the electron-donating tendencies The oxidation potentials increase as

$$-1.09 < -0.77 < +1.66.$$

Thus bromide is the weakest electron donor among the three, ferrous ion is intermediate and aluminium is the strongest. The corresponding increasing order of reducing power is

$$\mathrm{Br^- < Fe^{2+} < Al.}$$

Option (1) wrongly places bromide strongest; options (3) and (4) wrongly place aluminium weakest.

Both errors result from using the reduction potentials without reversing their physical interpretation.

Step 3: Conclusion The printed electrode potentials prove $Br^- < Fe^{2+} < Al$. The supplied key places aluminium weakest, but that reverses the aluminium comparison and is inconsistent with the given potentials. Hence, the correct option is (2).

Quick Tip to Solve: For reducing power, rank the species on the right of reduction half-reactions in the reverse order of their reduction potentials.

79. In the reaction of formation of sulphur trioxide by contact process

$$2\,SO_2 + O_2 \rightleftharpoons 2\,SO_3$$

the rate of reaction was measured as $\frac{d[O_2]}{dt} = -2.5 \times 10^{-4}\,\text{mol}\,L^{-1}\,s^{-1}$**. The rate of reaction in terms of** $[SO_2]$**, in** $\text{mol}\,L^{-1}\,s^{-1}$**, will be:** [4 marks]

(1) -1.25×10^{-4}

(2) -2.50×10^{-4}

(3) -3.75×10^{-4}

(4) -5.00×10^{-4}

Correct Answer: (4) -5.00×10^{-4}

Solution:

Concept:

- In $2\,SO_2 + O_2 \longrightarrow 2\,SO_3$, every mole of oxygen consumed accompanies consumption of two moles of sulphur dioxide. The concentration derivatives therefore have a fixed stoichiometric ratio.
- The positive, stoichiometrically normalised reaction rate is $r = -\frac{1}{2}d[SO_2]/dt = -d[O_2]/dt$. A reactant concentration derivative itself is negative.

Step 1: Relate the two reactant concentration derivatives Let the reaction advance by an amount corresponding to oxygen consumption $\Delta[O_2]$. Since the coefficient of sulphur dioxide is twice that of oxygen, its concentration decreases twice as quickly:

$$\frac{d[SO_2]}{dt} = 2\frac{d[O_2]}{dt}.$$

Both derivatives have the same negative sign, because both species are consumed. This equation compares concentration changes, not the stoichiometrically divided positive rate.

Step 2: Substitute the measured oxygen consumption Using the given value,

$$\frac{d[SO_2]}{dt} = 2(-2.5 \times 10^{-4}) = -5.00 \times 10^{-4}\,\text{mol}\,L^{-1}\,s^{-1}.$$

The magnitude of sulphur dioxide disappearance is 5.00×10^{-4}, whereas the normalised reaction rate is 2.5×10^{-4}. Since all the choices are negative, the question is asking for the sulphur dioxide concentration derivative. Option (1) incorrectly halves the oxygen derivative, option (2) ignores the

coefficient ratio and option (3) uses an unsupported factor of $3/2$.

Step 3: Conclusion The concentration of SO_2 changes at $-5.00 \times 10^{-4}\,\text{mol}\,\text{L}^{-1}\,\text{s}^{-1}$; its disappearance rate is the corresponding positive magnitude. Hence, the correct option is (4).

Quick Tip to Solve: Divide each concentration derivative by its own stoichiometric coefficient before equating reaction rates; restore the negative sign for reactant derivatives.

80. For the reaction, $2\,N_2O_5 \longrightarrow 4\,NO_2 + O_2$, the rate equation can be expressed in two ways

$$-\frac{d[N_2O_5]}{dt} = k[N_2O_5]$$

and

$$+\frac{d[NO_2]}{dt} = k'[N_2O_5].$$

k and k' are related as: [4 marks]

(1) $k = k'$

(2) $2k = k'$

(3) $k = 2k'$

(4) $k = 4k'$

Correct Answer: (2) $2k = k'$

Solution:

Concept:

- For $2\,N_2O_5 \longrightarrow 4\,NO_2 + O_2$, four molecules of nitrogen dioxide form for every two molecules of dinitrogen pentoxide consumed. The rate of product formation is therefore twice the rate of reactant disappearance.
- The normalised reaction rate is $r = -\frac{1}{2}d[N_2O_5]/dt = \frac{1}{4}d[NO_2]/dt$. Constants multiplying the same concentration must respect this stoichiometric relation.

Step 1: Eliminate the common reaction rate Equating the two expressions for the normalised rate gives

$$-\frac{1}{2}\frac{d[N_2O_5]}{dt} = \frac{1}{4}\frac{d[NO_2]}{dt}.$$

Multiplying through by four,

$$\frac{d[NO_2]}{dt} = 2\left(-\frac{d[N_2O_5]}{dt}\right).$$

The minus sign converts the negative concentration derivative of the reactant into a positive disappearance rate. Both sides of this last equation are positive during the forward decomposition.

Step 2: Compare the two printed rate laws The question defines the reactant disappearance rate as $k[N_2O_5]$ and the nitrogen dioxide formation rate as $k'[N_2O_5]$. Substituting these definitions,

$$k'[N_2O_5] = 2k[N_2O_5],$$

so, at any nonzero reactant concentration,

$$k' = 2k.$$

The constants have the same dimensions, but not the same numerical value because the concentration derivatives being described differ. Option (1) neglects the coefficient ratio; option (3) reverses it; option (4) gives the product coefficient alone instead of comparing four with two.

Step 3: Conclusion The nitrogen dioxide formation constant is twice the dinitrogen pentoxide disappearance constant: $2k = k'$. Hence, the correct option is (2).

Quick Tip to Solve: When two rate laws use the same concentration factor, first compare the actual species derivatives through the balanced equation; then compare their constants.

81. Shapes of certain interhalogen compounds are stated below. Which one of them is not correctly stated? [4 marks]

(1) IF_7: pentagonal bipyramid

(2) BrF_5: trigonal bipyramid

(3) BrF_3: planar T-shaped

(4) ICl_3: planar dimeric

Correct Answer: (2) BrF_5: trigonal bipyramid

Solution:

Concept:

- VSEPR distinguishes electron-pair geometry from molecular shape: all bonding and lone pairs determine the former, while the positions of bonded atoms determine the latter.
- A halogen central atom has seven valence electrons. Five single bonds in BrF_5 leave one lone pair, giving six electron domains and an octahedral electron-pair arrangement.

Step 1: Determine the shape of bromine pentafluoride Bromine forms five Br – F bonds and retains one lone pair. Thus BrF_5 is an AX_5E species. Six electron domains occupy an approximately octahedral arrangement; removing the lone-pair vertex from the molecular outline leaves a square pyramid. A trigonal bipyramid requires five domains, as in an AX_5 species without a lone pair, and is therefore not the shape of bromine pentafluoride.

Step 2: Check the remaining stated structures In IF_7, iodine has seven bonding pairs and no lone pair, so the seven fluorine atoms occupy a pentagonal bipyramid: five equatorial positions and two axial positions. In BrF_3, three bonds and two lone pairs give AX_3E_2; the lone pairs prefer equatorial sites in the trigonal-bipyramidal domain arrangement, leaving a planar T-shaped molecule. The stated planar dimeric description of ICl_3 refers to its chlorine-bridged dimeric form, I_2Cl_6. These distinguishable structures leave the pentafluoride statement as the incorrect one. In particular, confusing five bonded atoms with five total electron domains would miss the bromine lone pair.

Step 3: Conclusion BrF_5 is square pyramidal, not trigonal bipyramidal. Hence, the correct option is (2).

Quick Tip to Solve: Count lone pairs before naming a shape: five bonds plus one lone pair give a square pyramid, whereas five bonds with no lone pair give a trigonal bipyramid.

82. Which of the following name formula combinations is not correct? [4 marks]

(1) $K_2[Pt(CN)_4]$ – Potassium tetracyanoplatinate (II)

(2) $[Mn(CN)_5]^{2-}$ – Pentacyanomagnate (II) ion

(3) $K[Cr(NH_3)_2Cl_4]$ – Potassium diammine tetrachlorochromate (III)

(4) $[Co(NH_3)_4(H_2O)I]SO_4$ – Magnetron value.

Correct Answer: Both printed combinations (2) and (4) are incorrect: Mn is (III), while Magnetron value is not the name of the cobalt salt.

Solution:

Concept:

- The oxidation state of the metal is found by adding ligand charges to the metal charge and equating the result to the complex charge. Cyanide and chloride each have charge -1; ammonia and water are neutral.
- A coordination name must specify the ligand identities, metal and metal oxidation state. A nonchemical phrase cannot be a valid coordination-compound name; all printed alternatives must be checked before choosing a unique incorrect pair.

Step 1: Check the chemically named alternatives For $K_2[Pt(CN)_4]$, the complex charge is -2. Writing $x - 4 = -2$ gives platinum(II), so potassium tetracyanoplatinate(II) is consistent. For $[Mn(CN)_5]^{2-}$, charge balance instead gives

$$x + 5(-1) = -2 \quad \Rightarrow \quad x = +3.$$

The printed name in option (2), "Pentacyanomagnate (II) ion", has the wrong oxidation numeral as well as an incorrect metal-name spelling. With the stated formula, the oxidation numeral must be (III). In option (3), the complex is -1, two ammonia ligands are neutral and four chloride ligands contribute -4. Hence $x - 4 = -1$, giving chromium(III), consistent with the printed name.

Step 2: Examine the fourth printed combination Option (4) prints $[Co(NH_3)_4(H_2O)I]SO_4$ followed by the words "Magnetron value." Sulfate has charge -2, so the coordination cation has charge $+2$. Its four ammonia and one aqua ligand are neutral, while iodide contributes -1, giving $x - 1 = +2$ and cobalt(III). A valid name is tetraammineaquaiodidocobalt(III) sulfate. "Magnetron value" names neither the ligands nor this salt and is also an incorrect formula–name combination. Consequently both (2) and (4), as actually printed, satisfy "not correct"; it would be chemically unjustified to declare the fourth combination correct merely to force a single answer.

Step 3: Conclusion Option (2) is chemically incorrect because its manganese is (III), not (II), which agrees with the supplied key. However, option (4) is also incorrect as printed: "Magnetron value is not the name of the cobalt salt. Therefore the literal question has two incorrect combinations, (2) and (4), and no unique correct option.

Quick Tip to Solve: Check charge balance and the entire printed name independently. A keyed oxidation-state error does not make another nonsensical printed name valid.

83. Consider the coordination compound, $[Co(NH_3)_6]Cl_3$. In the formation of this complex, the species which acts as the Lewis acid is: [4 marks]

(1) $[Co(NH_3)_6]^{3+}$

(2) Cl^-

(3) Co^{3+}

(4) NH_3

Correct Answer: (3) Co^{3+}

Solution:

Concept:

- A Lewis acid accepts an electron pair and a Lewis base donates one. Coordinate bonds form when a ligand donates a lone pair to a suitable acceptor orbital on the central metal ion.
- In $[Co(NH_3)_6]Cl_3$, the complex ion is $[Co(NH_3)_6]^{3+}$ and the three chloride ions are counterions. Since ammonia is neutral, the metal in the complex is Co^{3+}.

Step 1: Identify the donor and acceptor during bond formation Each ammonia molecule has a lone pair on nitrogen. It donates that pair to the cobalt ion, so the six ammonia molecules act as Lewis bases. The central Co^{3+} ion accepts these donated pairs to form six coordinate bonds:

$$Co^{3+} + 6\,NH_3 \longrightarrow [Co(NH_3)_6]^{3+}.$$

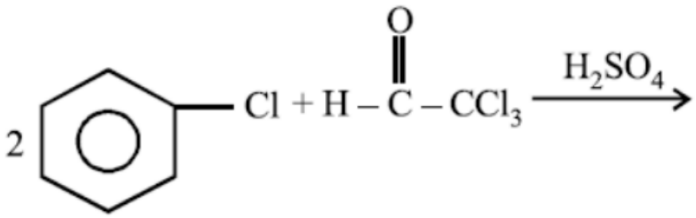
The electron pair remains shared in each coordinate bond; identifying its original donor distinguishes the base from the acid. A positive charge alone is not the definition of an acid, but here the cation is also specifically the electron-pair acceptor.

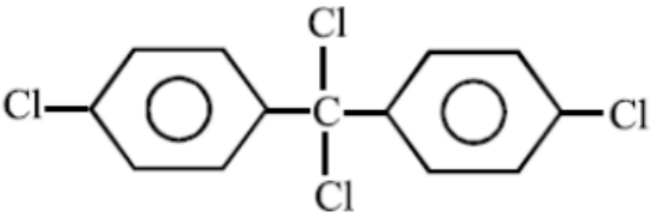
Step 2: Distinguish the complex from the species forming it The three Cl^- ions outside the brackets balance the complex ion's charge and are not the electron-pair acceptors in forming the six metal–ammonia bonds. Ammonia is the donor, excluding option (4). The ion $[Co(NH_3)_6]^{3+}$ in option (1) is the coordination product already formed; it is not the central acceptor species from which that same complex is formed. Option (2), chloride, is an anion with available lone pairs rather than the stated metal-centre acceptor. Thus the process named in the question singles out the free cobalt(III) ion in option (3).

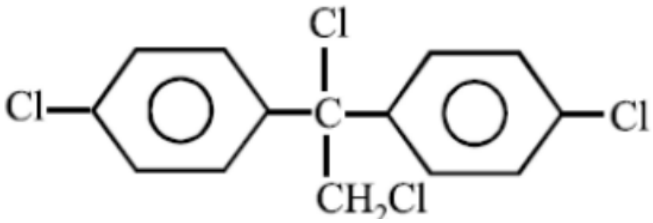
Step 3: Conclusion Co^{3+} accepts the nitrogen lone pairs and is the Lewis acid. The supplied key points to the completed complex, but the formation equation identifies Co^{3+} as the acceptor. Hence, the correct option is (3).

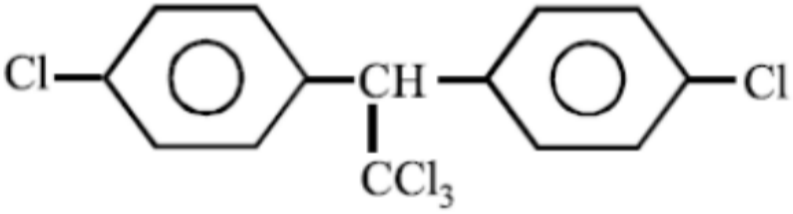
Quick Tip to Solve: In a complex-formation question, trace the ligand lone pair to its acceptor: the ligand is the Lewis base and the central metal ion is the Lewis acid.

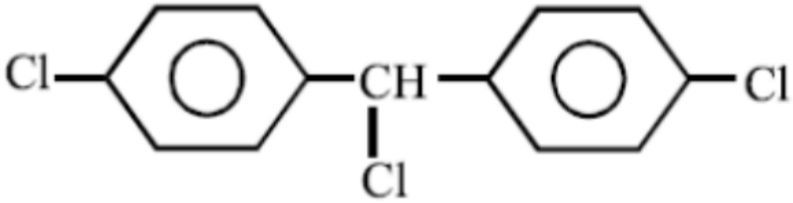
84. Chlorobenzene reacts with trichloro acetaldehyde in the presence of H_2SO_4. The major product formed is: [4 marks]



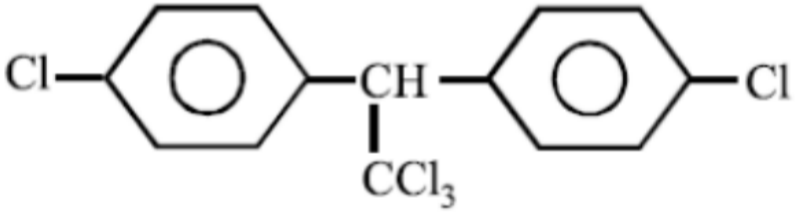
(1) 

(2) 

(3) 

(4) 

Correct Answer: (3)



Solution:

Concept:

- Chloral is trichloroacetaldehyde, CCl_3CHO. Condensation of chloral with two molecules of chlorobenzene in concentrated sulfuric acid forms DDT.
- Chlorine deactivates a benzene ring toward electrophilic substitution but directs substitution mainly to the ortho and para positions. Steric effects favour the para-substituted major product in this condensation.

Step 1: Track the two carbons of chloral The reagent CCl_3CHO contains a trichloromethyl carbon and an aldehyde carbon. The trichloromethyl group is retained intact. The aldehyde carbon retains its hydrogen and becomes bonded to two chlorophenyl groups as the oxygen is removed in the condensation. Thus the product skeleton must contain $Ar_2CH-CCl_3$, where Ar denotes a chlorophenyl group. It must not contain a central carbon bearing two chlorine atoms or a chloromethyl group replacing CCl_3.

Step 2: Select the major ring-substitution pattern Sulfuric acid activates the aldehyde toward electrophilic attack. The chlorine substituent directs attachment on each chlorobenzene ring predominantly to its para position, giving two para-chlorophenyl rings joined to the same CH centre. The net condensation is

$$2\,C_6H_5Cl + CCl_3CHO \longrightarrow (ClC_6H_4)_2CHCCl_3 + H_2O.$$

This is 1,1,1-trichloro-2,2-bis(4-chlorophenyl)ethane, commonly called DDT. Structure (3) shows both para chlorines, the central CH and its intact CCl_3 substituent. Structure (1) incorrectly makes the bridge CCl_2; structure (2) puts Cl and CH_2Cl at that centre; structure (4) omits the second carbon and the trichloromethyl group. These alternatives fail the carbon connectivity and substituent tracking.

Step 3: Conclusion The major product is DDT, $(p\text{-}ClC_6H_4)_2CHCCl_3$, represented by structure (3). Hence, the correct option is (3).

Quick Tip to Solve: Track chloral as CHO attached to CCl3: the aldehyde carbon becomes the diaryl CH bridge, while the CCl3 group remains attached unchanged.

85. Tischenko reaction is a modification of [4 marks]

(1) Aldol condensation

(2) Claisen condensation

(3) Cannizzaro reaction

(4) Pinacol-pinacolon reaction

Correct Answer: (3) Cannizzaro reaction

Solution:

Concept:

- The Cannizzaro reaction is disproportionation of an aldehyde: one aldehyde molecule is oxidised while another is reduced through hydride transfer.
- The Tischenko reaction couples two aldehyde molecules into an ester, usually with an aluminium

alkoxide catalyst. The ester combines the oxidised and reduced portions of the aldehyde.

Step 1: Identify the characteristic transformation The Tischenko process converts an aldehyde into an ester without a separate external oxidant or reductant. Its overall transformation can be represented as

$$2\,\mathrm{RCHO} \xrightarrow{\mathrm{Al(OR\prime)_3}} \mathrm{RCOOCH_2R}.$$

For example, acetaldehyde produces ethyl acetate. One carbonyl carbon ends up at the carboxylic-acid oxidation level and the other at the alcohol oxidation level.

Step 2: Relate the reaction to Cannizzaro disproportionation In the Cannizzaro reaction, an aldehyde lacking an alpha hydrogen reacts with concentrated alkali to give a carboxylate salt and an alcohol. The Tischenko modification retains the same oxidation–reduction relationship between two aldehyde molecules, but the products are combined as an ester. The crucial similarity is intermolecular hydride transfer, not carbon–carbon bond formation.

Step 3: Eliminate the other named reactions Aldol condensation joins carbonyl compounds through an enolate and produces a beta-hydroxy carbonyl compound, often followed by dehydration. Claisen condensation forms a carbon–carbon bond between esters to give a beta-keto ester. Pinacol–pinacolone rearrangement converts a vicinal diol into a carbonyl compound through migration. None of these describes aldehyde disproportionation into an ester.

Step 4: Conclusion Tischenko reaction is a modification of the Cannizzaro reaction. Hence, the correct option is (3).

Quick Tip to Solve: Recognise Tischenko by aldehyde-to-ester conversion; compare the two aldehyde carbon atoms to see the Cannizzaro-type oxidation and reduction.

86. Which one of the following statements is not correct? [4 marks]

(1) Alcohols are weaker acids than water

(2) Acid strength of alcohols decreases in the following order: $\mathrm{RCH_2OH} > \mathrm{R_2CHOH} > \mathrm{R_3COH}$.

(3) Carbon-oxygen bond length in methanol, $\mathrm{CH_3OH}$, is shorter than that of C–O bond length in phenol.

(4) The bond angle $\angle\mathrm{COH}$ in methanol is 108.9°.

Correct Answer: (3) Carbon-oxygen bond length in methanol, $\mathrm{CH_3OH}$, is shorter than that of C–O bond length in phenol.

Solution:

Concept:

- Phenol is more acidic than an aliphatic alcohol because the phenoxide ion is stabilised by resonance, whereas an alkoxide ion does not have equivalent aromatic resonance stabilisation.
- Conjugation of an oxygen lone pair with the benzene ring gives the phenolic C–O bond partial double-bond character. This affects its length as well as its strength.

Step 1: Compare the carbon–oxygen bond lengths In methanol the carbon atom bonded to oxygen is sp^3-hybridised and its C–O bond is essentially a single bond. In phenol the bonded ring carbon is sp^2-hybridised and oxygen participates in conjugation with the aromatic ring. The phenolic C–O bond

consequently has partial double-bond character. Typical bond lengths are approximately $142\,\text{pm}$ for methanol and $136\,\text{pm}$ for phenol. Therefore the methanol bond is longer, not shorter.

Step 2: Check the acidity statements The usual textbook comparison treats ordinary alkyl alcohols as weaker acids than water because alkyl groups donate electron density and destabilise the negative charge on the conjugate-base oxygen. Increasing alkyl substitution also hinders solvation of the alkoxide ion. These considerations account for the stated decrease

$$\text{RCH}_2\text{OH} > \text{R}_2\text{CHOH} > \text{R}_3\text{COH}$$

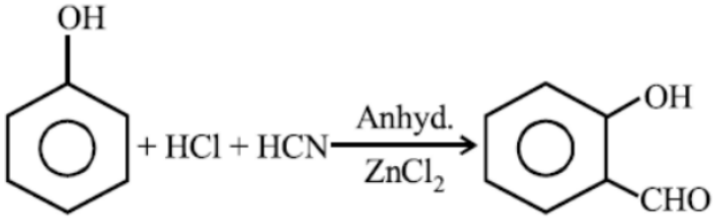
in acidic strength. Thus neither of the first two statements is the intended incorrect bond-property statement.

Step 3: Check the methanol angle and select the false statement The oxygen of methanol has two bonds and two lone pairs, so the C–O–H arrangement is bent rather than linear. The stated angle 108.9° is consistent with the molecular structure. The third statement alone reverses the established methanol-versus-phenol C–O bond-length comparison: resonance shortens the phenolic bond.

Step 4: Conclusion The statement that the C–O bond in methanol is shorter than that in phenol is incorrect. Hence, the correct option is (3).

Quick Tip to Solve: For phenol, use lone-pair conjugation to remember both effects: a more stable conjugate base and a shorter C–O bond.

87. The following reaction is known as: [4 marks]



(1) Perkin reaction

(2) Gatterman-Koch Formylation

(3) Kolbe's reaction

(4) Gattermann reaction

Correct Answer: (4) Gattermann reaction

Solution:

Concept:

- Formylation introduces the aldehyde group $-\text{CHO}$ onto an aromatic ring. The reagents used distinguish closely named aromatic formylation reactions.
- The Gattermann reaction uses hydrogen cyanide and hydrogen chloride in the presence of a Lewis acid; hydrolysis of the intermediate gives an aromatic aldehyde.

Step 1: Read the substrate, reagents and product The starting aromatic compound is phenol. The

reaction arrow shows HCN, HCl and anhydrous $ZnCl_2$. The product retains the phenolic $-OH$ group and gains a neighbouring $-CHO$ group, giving the ortho-hydroxy aromatic aldehyde shown. Thus the chemical change is aromatic formylation, not addition of an acyl side chain or a carboxyl group.

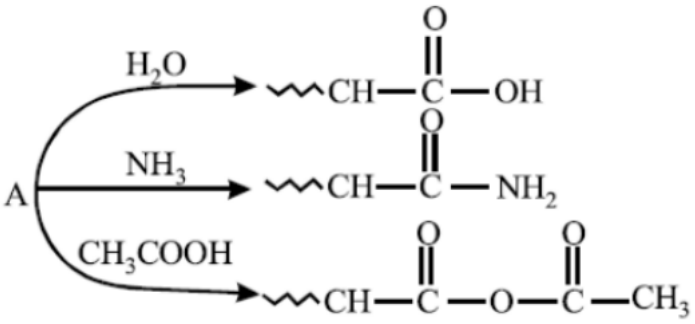
Step 2: Identify the HCN-based formylation Hydrogen cyanide and hydrogen chloride generate a formylating system under Lewis-acid conditions. The aromatic ring undergoes electrophilic substitution, and the imine-derived intermediate is hydrolysed to the aldehyde. This sequence is the Gattermann reaction. The electron-donating phenolic group activates the aromatic ring and directs substitution to its ortho and para positions; the source specifically depicts the ortho product.

Step 3: Distinguish the competing reaction names Gattermann–Koch formylation instead uses carbon monoxide and hydrogen chloride, typically with $AlCl_3/CuCl$, so the explicit HCN reagent rules it out. Perkin reaction is condensation of an aromatic aldehyde with an acid anhydride to form an unsaturated acid. Kolbe's reaction of phenoxide with carbon dioxide gives a hydroxybenzoic acid after acidification. Neither gives the depicted HCN-based introduction of $-CHO$.

Step 4: Conclusion The depicted reaction is the Gattermann reaction. Hence, the correct option is (4).

Quick Tip to Solve: Distinguish the formylation names by the one-carbon reagent: HCN identifies Gattermann, whereas CO identifies Gattermann–Koch.

88. An organic compound A, C_5H_8O, reacts with H_2O, NH_3 and CH_3COOH as described below. A is: [4 marks]



(1) $CH_3CH{=}C(CH_3)CHO$

(2) $CH_2{=}CHCH(CH_3)CHO$

(3) $CH_3CH_2C(CH_3){=}C{=}O$

(4) $CH_3CH_2C(=CH_2)CHO$

Correct Answer: (3) $CH_3CH_2C(CH_3){=}C{=}O$

Solution:

Concept:

- A ketene contains the cumulated double-bond unit $C{=}C{=}O$. A substituted ketene $R_2C{=}C{=}O$ reacts rapidly with compounds containing an O–H or N–H bond.
- Addition of water, ammonia and a carboxylic acid to a ketene gives, respectively, a carboxylic acid,

an amide and an acid anhydride.

Step 1: Identify the products common to the scheme The three branches show a carboxylic acid $R_2CHCOOH$, an amide $R_2CHCONH_2$, and a mixed anhydride $R_2CHCO-O-COCH_3$. The carbon skeleton is retained while the carbonyl-containing carbon acquires $-OH$, $-NH_2$, or $-OCOCH_3$. At the adjacent carbon a hydrogen is added. This pattern points to addition across a ketene unit.

Step 2: Apply each reagent to the ketene option For $CH_3CH_2C(CH_3)=C=O$, take the two substituents on the first ketene carbon to be ethyl and methyl. Water adds H and OH to give $CH_3CH_2CH(CH_3)COOH$. Ammonia adds H and NH_2 to give $CH_3CH_2CH(CH_3)CONH_2$. Acetic acid adds H and acetate to give the mixed anhydride $CH_3CH_2CH(CH_3)CO-O-COCH_3$. All three transformations match the displayed functional groups.

Step 3: Verify the formula and exclude the aldehydes The ketene option has five carbon atoms, eight hydrogen atoms and one oxygen atom, agreeing with C_5H_8O. The first, second and fourth options are unsaturated aldehydes containing $-CHO$, rather than the ketene $C=C=O$ unit. Ordinary addition of water to an aldehyde gives a hydrate, and its reaction with ammonia does not directly generate the carboxamide shown. Those options cannot account for the whole three-reagent pattern.

Step 4: Conclusion A is $CH_3CH_2C(CH_3)=C=O$, the substituted ketene. Hence, the correct option is (3).

Quick Tip to Solve: A single compound giving acid with water, amide with ammonia and anhydride with a carboxylic acid is a strong signature of a ketene.

89. Complete reduction of benzene-diazonium chloride with Zn/HCl gives [4 marks]

(1) Aniline

(2) Phenylhydrazine

(3) Azobenzene

(4) Hydrazobenzene

Correct Answer: (1) Aniline

Solution:

Concept:

- A diazonium group $-N_2^+$ contains two nitrogen atoms. Its reduction products depend on how far the nitrogen-containing group is reduced.
- Complete reduction of an aryl diazonium salt with zinc and hydrochloric acid gives the corresponding primary aromatic amine; partial reduction can stop at an arylhydrazine.

Step 1: Identify the group undergoing complete reduction Benzene-diazonium chloride has the formula $C_6H_5N_2^+Cl^-$. The question specifies complete reduction with Zn/HCl, so reduction must be followed beyond the hydrazine stage. The nitrogen atom directly attached to the benzene ring remains as the amino-group nitrogen, while the second nitrogen is removed as an ammonia-derived species in the acidic medium.

Step 2: Write the overall conversion Using [H] to denote reducing hydrogen equivalents, the overall

change may be written as

$$C_6H_5N_2Cl + 6\,[H] \longrightarrow C_6H_5NH_2 + NH_3 + HCl.$$

The equation conserves both nitrogen atoms and gives $C_6H_5NH_2$, aniline, as the aromatic organic product. In the actual acidic reaction mixture, the basic nitrogen products are protonated; the named organic product corresponds to aniline after normal work-up.

Step 3: Reject the less reduced and coupled products Phenylhydrazine, $C_6H_5NHNH_2$, still retains an N–N bond and represents incomplete reduction relative to the stated complete process. Azobenzene, $C_6H_5N{=}NC_6H_5$, and hydrazobenzene, $C_6H_5NHNHC_6H_5$, each contain two phenyl groups. They are coupled products rather than the primary aromatic amine produced by complete reduction of this diazonium salt. Hence those names do not fit the specified transformation.

Step 4: Conclusion Complete reduction gives aniline, $C_6H_5NH_2$. Hence, the correct option is (1).

Quick Tip to Solve: For diazonium reduction, check whether the question says partial or complete: retention of an N–N bond indicates a hydrazine stage, not the final aniline product.

90. Which one of the following is used as Antihistamine? [4 marks]

(1) Omeprazole

(2) Chloramphenicol

(3) Diphenhydramine

(4) Norethindrone

Correct Answer: (3) Diphenhydramine

Solution:

Concept:

- Histamine is a chemical mediator involved in allergic responses. Classical antihistamine drugs reduce such symptoms by blocking histamine H1 receptors.
- Drug classification depends on the biological target and therapeutic action. An antibiotic, a proton-pump inhibitor and a hormonal contraceptive belong to different drug classes.

Step 1: Identify the antihistamine among the names Diphenhydramine is a first-generation H1 antihistamine. By inhibiting the action of histamine at H1 receptors, it helps control allergic manifestations such as sneezing, itching and a runny nose. Its ability to enter the central nervous system also explains the sedative effect associated with this class. The relevant identification here is its antihistamine action.

Step 2: Distinguish the two non-hormonal distractors Omeprazole suppresses gastric-acid secretion by inhibiting the proton pump in gastric parietal cells. It is an acid-suppressing drug, but that mechanism does not make it a classical antihistamine. Chloramphenicol is an antibiotic that inhibits bacterial protein synthesis by acting on the bacterial ribosome. Its role is treatment of susceptible bacterial infections, not blockade of the histamine-mediated allergic response.

Step 3: Classify the remaining option and choose Norethindrone is a synthetic progestin used in

hormonal preparations, including oral contraceptives. Its effects arise from progestational hormonal activity rather than H1-receptor blockade. Therefore three options can be separated by their characteristic actions: omeprazole affects gastric proton pumps, chloramphenicol affects bacteria, and norethindrone acts as a hormone. Diphenhydramine alone matches the antihistamine class asked for.

Step 4: Conclusion Diphenhydramine is used as an antihistamine. Hence, the correct option is (3).

Quick Tip to Solve: Associate each name with its target: diphenhydramine–H1 receptor; omeprazole–proton pump; chloramphenicol–bacterial ribosome; norethindrone–progestin.