JEE Main 2014 Paper 1 (B.E./B.Tech) Apr 6 Shift 1 (morning) Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2014, Annual (offline), conducted by the Central Board of Secondary Education (CBSE) in pen and paper mode on 6 April 2014, Single sitting. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. The variance of first 50 even natural numbers is [4 marks]

(A) 833

(B) 437

(C) $\frac{437}{4}$

(D) $\frac{833}{4}$

Correct Answer: (A) 833

Solution:

Concept:

- For equally likely observations, variance is the mean of the squares minus the square of the mean: $\text{Var}(X) = E(X^2) - [E(X)]^2$.
- Multiplying every observation by a constant a multiplies the variance by a^2; adding a constant does not change it.

Step 1: Represent the even numbers The first 50 even natural numbers are $2, 4, 6, \ldots, 100$. Write each observation as $X = 2K$, where K runs uniformly through $1, 2, \ldots, 50$. Thus there are 50 observations, and the last term is 100. We must use population variance because the question asks for the variance of this entire specified collection, rather than an estimate from a sample.

Step 2: Calculate the two means Using the standard sums of integers and their squares,

$$E(K) = \frac{50+1}{2} = \frac{51}{2}, \qquad E(K^2) = \frac{(50+1)(2\cdot 50+1)}{6} = \frac{5151}{6}.$$

Therefore

$$\text{Var}(K) = \frac{5151}{6} - \frac{2601}{4} = \frac{2499}{12} = \frac{833}{4}.$$

The common factor of 2 in the observations has not yet been incorporated in this intermediate result.

Step 3: Scale the variance and compare Since $X = 2K$,

$$\text{Var}(X) = 2^2 \,\text{Var}(K) = 4\left(\frac{833}{4}\right) = 833.$$

Option (D) is the variance of the first 50 natural numbers before scaling, so it misses the factor of four. Options (B) and (C) do not equal the result of the correct sum-of-squares calculation. Variance is nonnegative and grows quadratically with the common difference, as this result confirms.

Step 4: Conclusion The variance of the first 50 even natural numbers is $\boxed{833}$. Hence, the correct option is (A).

Quick Tip to Solve: For the first n terms of an arithmetic progression with common difference d, use $d^2(n^2-1)/12$; its starting value does not affect variance.

2. $\lim_{x\to 0} \frac{\sin\left(\pi\cos^2 x\right)}{x^2}$ **is equal to :** [4 marks]

(A) $-\pi$

(B) π

(C) $\frac{\pi}{2}$

(D) 1

Correct Answer: (B) π

Solution:

Concept:

- The standard small-angle limit is $\lim_{u\to 0} \sin u/u = 1$, with angles measured in radians.
- Use $\cos^2 x = 1 - \sin^2 x$ and $\sin(\pi - u) = \sin u$ before applying a limit.

Step 1: Rewrite the angle near zero The numerator tends to $\sin \pi = 0$, while the denominator tends to zero. Direct substitution therefore gives the indeterminate form $0/0$, not the value zero. Re-express the numerator using an exact identity:

$$\sin(\pi \cos^2 x) = \sin\left(\pi - \pi \sin^2 x\right) = \sin(\pi \sin^2 x).$$

This changes an angle approaching π into an angle approaching zero without changing the sign of the sine.

Step 2: Separate the standard limits For nonzero x sufficiently close to zero, write

$$\frac{\sin(\pi \sin^2 x)}{x^2} = \frac{\sin(\pi \sin^2 x)}{\pi \sin^2 x}\, \pi \left(\frac{\sin x}{x}\right)^2.$$

Here $u = \pi \sin^2 x \to 0$, so the first factor tends to one by the sine limit. The squared factor also tends to one. The identity holds on both sides of zero; no one-sided sign adjustment is needed.

Step 3: Evaluate and eliminate the alternatives Taking the limit of the product gives

$$L = 1 \cdot \pi \cdot 1 = \pi.$$

In particular, the numerator is nonnegative close to zero, so the proposed value $-\pi$ cannot be right. There is no extra factor of $1/2$, since the departure of $\cos^2 x$ from 1 is exactly $\sin^2 x$, whose ratio to x^2 tends to one. Option (D) omits the factor π from the argument of the outer sine.

Step 4: Conclusion The required limit is $\boxed{\pi}$. Hence, the correct option is (B).

Quick Tip to Solve: When the sine argument approaches π, first replace it by the complementary small angle; retain every constant in that angle.

3. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log|x| + \beta x^2 + x$ then [4 marks]

(A) $\alpha = 2, \beta = -\frac{1}{2}$

(B) $\alpha = 2, \beta = \frac{1}{2}$

(C) $\alpha = -6, \beta = \frac{1}{2}$

(D) $\alpha = -6, \beta = -\frac{1}{2}$

Correct Answer: (A) $\alpha = 2, \beta = -\frac{1}{2}$

Solution:

Concept:

- At an interior differentiable local extremum, the derivative is zero. Both stated points lie in the domain $x \neq 0$.
- For either positive or negative nonzero x, $d(\log|x|)/dx = 1/x$.

Step 1: Differentiate on the domain Treat α and β as constants. The function is differentiable on each of the intervals $(-\infty, 0)$ and $(0, \infty)$, so

$$f\prime(x) = \frac{\alpha}{x} + 2\beta x + 1.$$

The absolute value in the logarithm permits negative arguments of x, but it does not change the derivative to $1/|x|$. Using $1/x$ at -1 is essential for the correct signs.

Step 2: Use the two stationary conditions At $x = -1$, the zero-derivative condition gives

$$-\alpha - 2\beta + 1 = 0, \qquad \alpha + 2\beta = 1.$$

At $x = 2$, it gives

$$\frac{\alpha}{2} + 4\beta + 1 = 0, \qquad \alpha + 8\beta = -2.$$

Subtract the first equation from the second: $6\beta = -3$, hence $\beta = -1/2$. Substituting back yields $\alpha = 1 - 2\beta = 2$.

Step 3: Confirm that the stationary points are extrema With these values,

$$f\prime\prime(x) = -\frac{2}{x^2} - 1.$$

It is negative for every nonzero x, and specifically $f\prime\prime(-1) = -3$ and $f\prime\prime(2) = -3/2$. Thus both points are indeed local maxima on their respective domain intervals, as allowed by the question. Option (B) has the wrong sign of β; options (C) and (D) have the wrong α, and fail the two simultaneous derivative equations.

Step 4: Conclusion The parameters are $\boxed{\alpha = 2,\ \beta = -\frac{1}{2}}$. Hence, the correct option is (A).

Quick Tip to Solve: For parameter problems involving $\log|x|$, use $1/x$ at negative stationary points and then verify the second derivative.

4. The area of the region described by

$A = \{(x, y) : x^2 + y^2 \leq 1$ **and** $y^2 \leq 1 - x\}$ **is :** [4 marks]

(A) $\frac{\pi}{2} - \frac{2}{3}$

(B) $\frac{\pi}{2} + \frac{2}{3}$

(C) $\frac{\pi}{2} + \frac{4}{3}$

(D) $\frac{\pi}{2} - \frac{4}{3}$

Correct Answer: (C) $\frac{\pi}{2} + \frac{4}{3}$

Solution:

Concept:

- The required region is the intersection of the unit disk $x^2 + y^2 \leq 1$ and the region $x \leq 1 - y^2$.
- For horizontal strips, area is $\int (x_{\text{right}} - x_{\text{left}})\, dy$, with boundaries chosen from both inequalities.

Step 1: Determine the horizontal boundaries The disk requires $-1 \leq y \leq 1$, and for a fixed such y, its horizontal interval is

$$-\sqrt{1 - y^2} \leq x \leq \sqrt{1 - y^2}.$$

The second inequality adds the upper bound $x \leq 1 - y^2$. Put $u = 1 - y^2$. Since $0 \leq u \leq 1$, we have $u \leq \sqrt{u}$. Therefore the parabola, rather than the right semicircle, is the active right boundary throughout this interval.

Step 2: Integrate the strip width The left boundary remains the negative semicircle because $1 - y^2 \geq 0$. Consequently

$$\begin{aligned} \mathcal{A} &= \int_{-1}^{1} \left(1 - y^2 + \sqrt{1 - y^2}\right) dy \\ &= \left[y - \frac{y^3}{3}\right]_{-1}^{1} + \int_{-1}^{1} \sqrt{1 - y^2}\, dy. \end{aligned}$$

The polynomial integral equals $4/3$. The remaining integral is the area under the upper half of the unit circle, namely $\pi/2$.

Step 3: Add the contributions and check the options Thus

$$\mathcal{A} = \frac{\pi}{2} + \frac{4}{3}.$$

The region contains the entire left half-disk and a positive part on its right, so options (A) and (D), which subtract from $\pi/2$, cannot represent it. Option (B) counts only half the polynomial-strip contribution: the right-hand portion extends over both positive and negative y. The result is also less than the full disk area π, as required.

Step 4: Conclusion The area is $\boxed{\frac{\pi}{2} + \frac{4}{3}}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: When a parabola is written as $x = 1 - y^2$, horizontal strips often avoid splitting the intersection integral into separate ranges.

5. Let the population of rabbits surviving at time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then $p(t)$ equals: [4 marks]

(A) $600 - 500\,e^{t/2}$

(B) $400 - 300\,e^{-t/2}$

(C) $400 - 300\,e^{t/2}$

(D) $300 - 200\,e^{-t/2}$

Correct Answer: (C) $400 - 300\,e^{t/2}$

Solution:

Concept:

- A constant particular solution can be found by setting $dp/dt = 0$; here the equilibrium population is 400.
- Shifting by the equilibrium reduces the linear equation to the separable form $dq/dt = q/2$.

Step 1: Shift the dependent variable The given equation is

$$p\prime(t) = \frac{1}{2}p(t) - 200 = \frac{1}{2}\big(p(t) - 400\big).$$

Define $q(t) = p(t) - 400$. Since the derivative of a constant is zero, this gives $q\prime(t) = q(t)/2$. This change separates the constant forcing term from the exponential part of the solution; the equilibrium 400 is fixed by the equation itself, not by the initial condition.

Step 2: Solve and impose the initial population The solution of the shifted equation is

$$q(t) = Ce^{t/2}, \qquad p(t) = 400 + Ce^{t/2}.$$

At $t = 0$, the exponential is one. Therefore

$$100 = 400 + C, \qquad C = -300,$$

and hence $p(t) = 400 - 300e^{t/2}$. The exponent has a positive sign because the coefficient of q in its differential equation is positive.

Step 3: Verify the differential equation and alternatives Differentiate the result to obtain $p\prime(t) = -150e^{t/2}$. Substitution on the right gives

$$\frac{1}{2}(400 - 300e^{t/2}) - 200 = -150e^{t/2},$$

so both sides agree. Option (A) starts at 100 but has the wrong equilibrium and derivative. Option (B) starts correctly but predicts an initially increasing population, whereas the equation gives $p\prime(0) = -150$. Option (D) also starts at 100 but fails the required derivative. As a population model, the formula is physically relevant until the population first reaches zero.

Step 4: Conclusion The solution satisfying the initial condition is $\boxed{p(t) = 400 - 300e^{t/2}}$. Hence, the correct option is (C).

Quick Tip to Solve: For $p\prime = ap - b$, first shift by the equilibrium b/a; then use the initial value to determine the exponential coefficient.

6. The integral $\int_0^{\pi} \sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ equals: [4 marks]

(A) $4\sqrt{3}-4$

(B) $4\sqrt{3}-4-\frac{\pi}{3}$

(C) $\pi-4$

(D) $\frac{2\pi}{3}-4-4\sqrt{3}$

Correct Answer: (B) $4\sqrt{3}-4-\frac{\pi}{3}$

Solution:

Concept:

- The principal square root of a square is an absolute value: $\sqrt{v^2}=|v|$, not always v.
- A definite integral involving an absolute value must be split wherever the expression inside changes sign.

Step 1: Recognise the perfect square Writing $s=\sin(x/2)$, the radicand is

$$1+4s^2-4s=(2s-1)^2.$$

Therefore the integral is $I=\int_0^{\pi}|2\sin(x/2)-1|\,dx$. On this interval, $x/2$ runs from 0 to $\pi/2$, so its sine increases from 0 to 1. The expression inside the absolute value vanishes once, when $\sin(x/2)=1/2$, giving $x=\pi/3$.

Step 2: Split at the sign change Before $\pi/3$, the expression is negative; after it, the expression is positive. Thus

$$I=\int_0^{\pi/3}\left(1-2\sin\frac{x}{2}\right)dx+\int_{\pi/3}^{\pi}\left(2\sin\frac{x}{2}-1\right)dx.$$

Antiderivatives of the two integrands are respectively $x+4\cos(x/2)$ and $-4\cos(x/2)-x$. The factor 4 comes from the chain rule for the half-angle.

Step 3: Evaluate both pieces and compare The first part equals

$$\frac{\pi}{3}+4\cos\frac{\pi}{6}-4=\frac{\pi}{3}+2\sqrt{3}-4.$$

The second part equals

$$-\pi-\left(-2\sqrt{3}-\frac{\pi}{3}\right)=2\sqrt{3}-\frac{2\pi}{3}.$$

Adding gives $I=4\sqrt{3}-4-\pi/3$. Option (A) loses the net angular term. Options (C) and (D) are negative, whereas this continuous nonnegative integrand has a strictly positive integral. This sign check independently rejects those alternatives.

Step 4: Conclusion The integral is $\boxed{4\sqrt{3}-4-\frac{\pi}{3}}$. Hence, the correct option is (B).

Quick Tip to Solve: After spotting a squared expression under a radical, locate its zeros before removing the absolute value; a missed sign change changes the integral.

7. If A is a 3×3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals: [4 marks]

(A) B^{-1}

(B) $\left(B^{-1}\right)'$

(C) $I + B$

(D) I

Correct Answer: (D) I

Solution:

Concept:

- Transposition reverses a matrix product: $(XY)\prime = Y\prime X\prime$. For an invertible matrix, $(A^{-1})\prime = (A\prime)^{-1}$.
- The given equality $AA\prime = A\prime A$ states that A commutes with its transpose. No symmetry assumption $A = A\prime$ is needed.

Step 1: Transpose the definition of B Start from $B = A^{-1}A\prime$. Both A and $A\prime$ are invertible because A is nonsingular. Applying the transpose rules in the correct order gives

$$B\prime = (A^{-1}A\prime)\prime = A(A^{-1})\prime = A(A\prime)^{-1}.$$

It would be incorrect to preserve the order of the factors while taking this transpose.

Step 2: Use the commutation condition Multiplying the expressions for B and $B\prime$,

$$BB\prime = A^{-1}A\prime A(A\prime)^{-1}.$$

Now use $A\prime A = AA\prime$ on the middle pair, without assuming arbitrary matrices commute:

$$BB\prime = A^{-1}(AA\prime)(A\prime)^{-1} = (A^{-1}A)\left(A\prime(A\prime)^{-1}\right) = I.$$

All cancellations are between adjacent inverse factors after the stated commutation has been used.

Step 3: Interpret the result and reject the alternatives The identity $BB\prime = I$ means that B is orthogonal, so $B\prime = B^{-1}$. It does not mean that $BB\prime$ is generally B^{-1} or its transpose; those matrices need not equal I. For instance a planar rotation embedded in three dimensions supplies admissible nonsymmetric matrices and yields a nonidentity B. Option (C), $I + B$, cannot equal I because B is invertible and hence is not the zero matrix. The identity matrix in the answer has order three.

Step 4: Conclusion For the given nonsingular matrix, $\boxed{BB\prime = I}$. Hence, the correct option is (D).

Quick Tip to Solve: In products involving inverses and transposes, reverse the order first; use the stated commutation only on the adjacent middle factors.

8. If $\alpha, \beta \neq 0$, $f(n) = \alpha^n + \beta^n$, and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2,$$

then K is equal to: [4 marks]

(A) 1

(B) -1

(C) $\alpha\beta$

(D) $\frac{1}{\alpha\beta}$

Correct Answer: (A) 1

Solution:

Concept:

- A matrix of power sums can be factored as a matrix times its transpose. The determinant identity $\det(VV^T) = (\det V)^2$ then avoids expanding nine entries.
- For three numbers u, v, w, the Vandermonde determinant with columns $(1, u, u^2)^T$, $(1, v, v^2)^T$, $(1, w, w^2)^T$ is $(v-u)(w-u)(w-v)$.

Step 1: Express every entry as a power sum Put $s_j = 1 + \alpha^j + \beta^j$. In particular, $s_0 = 3$, so the determinant in the question is the determinant of the matrix whose entry in row r, column s is s_{r+s-2}. Thus the top-left entry also fits the same rule; it is not an extra coefficient to be taken outside the determinant. Define

$$V = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{pmatrix}.$$

The dot product of row r of V with row s of V is precisely $1 + \alpha^{r+s-2} + \beta^{r+s-2}$. Hence the given matrix is VV^T.

Step 2: Evaluate the determinant through its factors By the product and transpose rules,

$$\det(VV^T) = \det V \det V^T = (\det V)^2.$$

Using the three column parameters $1, \alpha, \beta$,

$$\det V = (\alpha - 1)(\beta - 1)(\beta - \alpha).$$

Consequently the required determinant equals

$$(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2.$$

Comparing the polynomial identity gives the constant coefficient 1. There is no remaining factor of $\alpha\beta$ or its reciprocal, so (C) and (D) do not match. Squaring the determinant also rules out the negative coefficient in (B). If two parameters coincide, both sides vanish; the same identity still holds without dividing by a zero factor.

Step 3: Conclusion The determinant has exactly the displayed squared factors with coefficient $K = 1$. Hence, the correct option is (A).

Quick Tip to Solve: When a determinant contains consecutive power sums, try a Vandermonde matrix times its transpose before expanding.

9. The integral $\int \left(1 + x - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx$ is equal to [4 marks]

(A) $(x+1)e^{x+\frac{1}{x}} + c$

(B) $-xe^{x+\frac{1}{x}} + c$

(C) $(x-1)e^{x+\frac{1}{x}} + c$

(D) $xe^{x+\frac{1}{x}} + c$

Correct Answer: (D) $xe^{x+\frac{1}{x}} + c$

Solution:

Concept:

- The product rule gives $\frac{d}{dx}[u(x)v(x)] = u\,v\prime + u\prime v$. Reverse this rule to recognise an antiderivative without expanding an exponential.
- For $h(x) = x + 1/x$, the chain rule gives $h\prime(x) = 1 - 1/x^2$ on either interval $x > 0$ or $x < 0$. The integrand is undefined at zero.

Step 1: Differentiate the common exponential Write $E = e^{x+1/x}$. All four candidates contain this factor, so first calculate its derivative once:

$$E\prime = e^{x+1/x}\left(1 - \frac{1}{x^2}\right).$$

The coefficient required in the question is $1 + x - 1/x$. Multiplying the chain-rule factor by x gives $x - 1/x$, leaving exactly the additional constant 1 supplied by differentiating the prefactor x.

Step 2: Check each proposed prefactor For option (D), apply the product rule explicitly:

$$\frac{d}{dx}(xE) = E + xE\left(1 - \frac{1}{x^2}\right) = \left(1 + x - \frac{1}{x}\right)E.$$

This is precisely the given integrand. Option (B) has the opposite derivative. Option (A), with prefactor $x + 1$, adds the unwanted term $E(1 - 1/x^2)$; option (C), with $x - 1$, subtracts that term. Neither agrees identically on an interval. The arbitrary additive constant differentiates to zero.

Step 3: Conclusion Thus

$$\int \left(1 + x - \frac{1}{x}\right) e^{x+1/x}\, dx = xe^{x+1/x} + C, \qquad x \neq 0.$$

The constant may be chosen separately on the two intervals separated by zero. Hence, the correct

option is (D).

Quick Tip to Solve: When every option contains the same exponential, differentiate the prefactors first; the chain-rule term often identifies the answer immediately.

10. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then $g'(x)$ is equal to: [4 marks]

(A) $\frac{1}{1+\{g(x)\}^5}$

(B) $1 + \{g(x)\}^5$

(C) $1 + x^5$

(D) $5x^4$

Correct Answer: (B) $1 + \{g(x)\}^5$

Solution:

Concept:

- If $g = f^{-1}$, then $f(g(x)) = x$ on the domain of the inverse. Differentiating this identity uses the chain rule.
- The inverse derivative is $g\prime(x) = 1/f\prime(g(x))$, wherever the derivative exists and the denominator is nonzero. The input of $f\prime$ is $g(x)$, not x.

Step 1: Differentiate the inverse identity Begin with $f(g(x)) = x$. The inside function in this composite is $g(x)$, so differentiating both sides with respect to x gives

$$f\prime(g(x))\, g\prime(x) = 1.$$

Solving for the unknown derivative yields

$$g\prime(x) = \frac{1}{f\prime(g(x))}.$$

This relation is local to an interval on which the stated inverse is defined. It does not require finding an explicit antiderivative for f, nor an explicit formula for its inverse.

Step 2: Evaluate the derivative at the correct argument The supplied derivative is $f\prime(t) = 1/(1 + t^5)$. Replacing its argument t by $g(x)$ gives

$$f\prime(g(x)) = \frac{1}{1 + [g(x)]^5}.$$

Taking the reciprocal as required by the inverse identity,

$$g\prime(x) = 1 + [g(x)]^5.$$

All formulae are understood at points in the inverse's domain where the given derivative is defined. Option (A) is $f\prime(g(x))$ itself and forgets the reciprocal. Option (C) takes the reciprocal at x instead of at $g(x)$; these are different arguments. Option (D) differentiates x^5, which is neither the inverse

identity nor the given function. Only (B) performs both the substitution and reciprocal correctly.

Step 3: Conclusion Where the inverse is differentiable, $g\prime(x) = 1 + \{g(x)\}^5$. Hence, the correct option is (B).

Quick Tip to Solve: Write the identity $f(g(x)) = x$ first: it tells you both where to evaluate the derivative and why to take its reciprocal.

11. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is : [4 marks]

(A) $\left(x^2 + y^2\right)^2 = 6x^2 + 2y^2$

(B) $\left(x^2 + y^2\right)^2 = 6x^2 - 2y^2$

(C) $\left(x^2 - y^2\right)^2 = 6x^2 + 2y^2$

(D) $\left(x^2 - y^2\right)^2 = 6x^2 - 2y^2$

Correct Answer: (A) $\left(x^2 + y^2\right)^2 = 6x^2 + 2y^2$

Solution:

Concept:

- The tangent to $x^2/a^2 + y^2/b^2 = 1$ at $(a\cos t, b\sin t)$ is $x\cos t/a + y\sin t/b = 1$.
- For a line $Ax + By = 1$, its perpendicular foot from the origin is $(A/(A^2 + B^2), B/(A^2 + B^2))$. This follows because the origin-to-foot vector is normal to the line.

Step 1: Parameterise the tangent and its perpendicular foot The ellipse is

$$\frac{x^2}{6} + \frac{y^2}{2} = 1,$$

so its centre is the origin and its semi-axes are $a = \sqrt{6}$, $b = \sqrt{2}$. Write a variable tangent as $Ax + By = 1$, where

$$A = \frac{\cos t}{\sqrt{6}}, \qquad B = \frac{\sin t}{\sqrt{2}}.$$

These coefficients satisfy $6A^2 + 2B^2 = 1$. Let the perpendicular foot have coordinates (h, k). Since the perpendicular follows the normal vector (A, B), substitution into the line gives

$$h = \frac{A}{A^2 + B^2}, \qquad k = \frac{B}{A^2 + B^2}.$$

Step 2: Eliminate the tangent parameter Squaring and adding the foot coordinates gives

$$h^2 + k^2 = \frac{1}{A^2 + B^2}.$$

It follows that $A = h/(h^2 + k^2)$ and $B = k/(h^2 + k^2)$. Substitute these expressions into the tangent condition:

$$6\frac{h^2}{(h^2 + k^2)^2} + 2\frac{k^2}{(h^2 + k^2)^2} = 1.$$

Therefore

$$(h^2 + k^2)^2 = 6h^2 + 2k^2.$$

Relabel h, k as x, y to obtain option (A). The negative y^2 terms in (B) and (D) contradict the positive semi-axis squares of the ellipse. Option (C) incorrectly replaces the squared distance $h^2 + k^2$ by a difference. The origin is an extraneous isolated point of the implicit equation; an actual perpendicular foot is nonzero.

Step 3: Conclusion The requested implicit locus is $(x^2 + y^2)^2 = 6x^2 + 2y^2$, with the origin excluded as an actual foot. Hence, the correct option is (A).

Quick Tip to Solve: Use the normal form of the tangent: the foot lies along its normal, so the distance squared naturally produces $x^2 + y^2$.

12. Let A and B be two events such that $P\left(\overline{A \cup B}\right) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P\left(\overline{A}\right) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are : [4 marks]

(A) independent but not equally likely.

(B) independent and equally likely.

(C) mutually exclusive and independent.

(D) equally likely but not independent.

Correct Answer: (A) independent but not equally likely.

Solution:

Concept:

- The complement rule gives $P(E) = 1 - P(\overline{E})$. The addition rule is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- Events are independent exactly when $P(A \cap B) = P(A)P(B)$. They are equally likely when their probabilities are equal, and mutually exclusive when their intersection has probability zero.

Step 1: Recover the individual event probabilities The supplied probability of the complement of the union gives

$$P(A \cup B) = 1 - \frac{1}{6} = \frac{5}{6}.$$

Similarly,

$$P(A) = 1 - P(\overline{A}) = 1 - \frac{1}{4} = \frac{3}{4}.$$

Now use the addition rule to solve for the other probability, rather than assuming the two events are equally likely:

$$\begin{aligned} P(B) &= P(A \cup B) - P(A) + P(A \cap B) \\ &= \frac{5}{6} - \frac{3}{4} + \frac{1}{4} = \frac{1}{3}. \end{aligned}$$

The probabilities $3/4$ and $1/3$ are different, already excluding both options that claim equal likelihood.

Step 2: Test independence and mutual exclusivity separately Multiply the two individual probabilities:

$$P(A)P(B) = \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}.$$

This is precisely the given $P(A \cap B)$, so the events are independent. Their intersection has positive probability, however, so they cannot be mutually exclusive. Thus option (C) fails its mutual-exclusivity claim. Option (B) combines independence with equal likelihood, but its second claim fails because $3/4 \neq 1/3$. Option (D) has both an incorrect equal-likelihood claim and an incorrect denial of independence. Option (A) is the only description consistent with all three given probabilities; independence places a product condition on the intersection, not an equality condition on the separate events.

Step 3: Conclusion The individual event probabilities are $3/4$ and $1/3$, and their product equals the intersection probability $1/4$. They are independent but not equally likely. Hence, the correct option is (A).

Quick Tip to Solve: Compute both marginal probabilities before naming the relationship: equal likelihood and independence are separate tests.

---

13. Let PS be the median of the triangle with vertices $P(2,2)$, $Q(6,-1)$ and $R(7,3)$. The equation of the line passing through $(1,-1)$ band parallel to PS is : [4 marks]

(A) $4x + 7y + 3 = 0$

(B) $2x - 9y - 11 = 0$

(C) $4x - 7y - 11 = 0$

(D) $2x + 9y + 7 = 0$

Correct Answer: (D) $2x + 9y + 7 = 0$

Solution:

Concept:

- A median joins a triangle vertex to the midpoint of its opposite side. The midpoint of (x_1, y_1) and (x_2, y_2) is $((x_1 + x_2)/2, (y_1 + y_2)/2)$.
- Parallel lines have the same direction vector or slope. A direction (u, v) gives a line through (x_0, y_0) as $v(x - x_0) - u(y - y_0) = 0$.

Step 1: Find the direction of the median Since PS is the median from P, its endpoint S is the midpoint of QR, not of either side containing P. Using $Q(6,-1)$ and $R(7,3)$,

$$S = \left(\frac{6+7}{2}, \frac{-1+3}{2}\right) = \left(\frac{13}{2}, 1\right).$$

Subtract the coordinates of $P(2,2)$:

$$\overrightarrow{PS} = \left(\frac{13}{2} - 2, 1 - 2\right) = \left(\frac{9}{2}, -1\right).$$

An equivalent direction vector is $(9, -2)$, obtained by multiplying both components by two. Hence the median's slope is $-2/9$.

Step 2: Construct the parallel line through the specified point The required line passes through $(1, -1)$ and has the same slope, so

$$y + 1 = -\frac{2}{9}(x - 1).$$

Multiplying by nine and collecting all terms gives

$$9y + 9 = -2x + 2,$$

$$2x + 9y + 7 = 0.$$

As a check, substituting $(1, -1)$ gives $2 - 9 + 7 = 0$, and its normal $(2, 9)$ is perpendicular to the median direction because $2(9) + 9(-2) = 0$. Option (A) has slope $-4/7$, while (B) and (C) have positive slopes $2/9$ and $4/7$. None of those slopes is $-2/9$; (D) satisfies both the direction and point conditions.

Step 3: Conclusion The required parallel line is $2x + 9y + 7 = 0$. Hence, the correct option is (D).

Quick Tip to Solve: Find the midpoint of the side opposite the named vertex, then preserve the median direction when shifting the line to a new point.

---

14. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then : [4 marks]

(A) $3bc - 2ad = 0$

(B) $3bc + 2ad = 0$

(C) $2bc - 3ad = 0$

(D) $2bc + 3ad = 0$

Correct Answer: (A) $3bc - 2ad = 0$

Solution:

Concept:

- A point equidistant from the coordinate axes satisfies $|x| = |y|$. In the fourth quadrant $x > 0$ and $y < 0$, so its equation is $y = -x$.
- An intersection point satisfies both line equations. Substituting its geometric constraint into both equations lets us eliminate its coordinate and find a relation among the coefficients.

Step 1: Use the quadrant to select the correct bisector The distance of a point (x, y) from the x-axis is $|y|$, and its distance from the y-axis is $|x|$. Equality of these distances gives two possible bisectors, $y = x$ and $y = -x$. The fourth quadrant has opposite coordinate signs, so only $y = -x$ is applicable. Write the intersection as $(t, -t)$ with $t > 0$. The strict inequality also confirms that the point is not the origin; no zero-coordinate solution is relevant here.

Step 2: Substitute and eliminate the coordinate Putting $x = t$, $y = -t$ in the first line yields

$$4at - 2at + c = 0 \quad \Rightarrow \quad t = -\frac{c}{2a}.$$

The second line similarly gives

$$5bt - 2bt + d = 0 \quad \Rightarrow \quad t = -\frac{d}{3b}.$$

The coefficients a, b are nonzero, so equating these expressions is valid:

$$\frac{c}{2a} = \frac{d}{3b} \quad \Rightarrow \quad 3bc = 2ad.$$

Thus $3bc - 2ad = 0$. The quadrant premise additionally makes both coordinate expressions positive, but no extra sign choice is needed in the requested coefficient equality. Option (B) has the wrong sign; (C) and (D) interchange the factors two and three, which came specifically from $4-2$ and $5-2$. As all four coefficients are nonzero, these incorrect alternatives cannot coincide with the derived relation.

Step 3: Conclusion The intersection conditions require $3bc - 2ad = 0$. Hence, the correct option is (A).

Quick Tip to Solve: For a point equally distant from the axes, choose $y = x$ or $y = -x$ using its quadrant before substituting into the lines.

15. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. then the common ratio of the G.P. is : [4 marks]

(A) $2 - \sqrt{3}$

(B) $2 + \sqrt{3}$

(C) $\sqrt{2} + \sqrt{3}$

(D) $3 + \sqrt{2}$

Correct Answer: (B) $2 + \sqrt{3}$

Solution:

Concept:

- Write three successive terms of a geometric progression as a, ar, ar^2, where $a > 0$ and r is the common ratio.
- Three numbers u, v, w are in arithmetic progression exactly when $2v = u + w$. Since the given geometric progression is increasing and positive, $r > 1$.

Step 1: Express the changed middle term The original terms are a, ar, ar^2. Doubling only the middle term gives $a, 2ar, ar^2$. The endpoints stay unchanged; in particular, the new middle term is not ar. Applying the arithmetic-progression condition gives

$$2(2ar) = a + ar^2.$$

Division by the positive number a is valid, and yields $4r = 1 + r^2$.

Step 2: Solve and select the increasing progression Rearrange the equation and complete the square:

$$r^2 - 4r + 1 = 0, \qquad (r-2)^2 = 3.$$

Thus $r = 2 \pm \sqrt{3}$. Both roots are positive, but $0 < 2 - \sqrt{3} < 1$, which would produce a decreasing progression. Only $2 + \sqrt{3} > 1$ obeys the stated increasing order. Option (A) is therefore rejected by monotonicity. Options (C) and (D) are not roots of the required quadratic, so they do not make the changed terms an arithmetic progression.

Step 3: Conclusion The common ratio is $\boxed{2 + \sqrt{3}}$. Hence, the correct option is (B).

Quick Tip to Solve: After doubling the middle term, apply $2 \times$ new middle $=$ first $+$ last; use increasing order to choose between reciprocal roots.

16. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$: [4 marks]

(A) is strictly greater that $\frac{5}{2}$

(B) is strictly greater that $\frac{3}{2}$ but less than $\frac{5}{2}$

(C) is equal to $\frac{5}{2}$

(D) lie in the interval (1, 2)

Correct Answer: (D) lie in the interval (1, 2)

Solution:

Concept:

- The reverse triangle inequality gives $|u + v| \geq \big||u| - |v|\big|$. It provides a lower bound on the distance from a fixed point.
- A lower bound is the minimum only if an allowed complex number attains it. Here $|z| \geq 2$ describes the exterior and boundary of a circle centred at the origin.

Step 1: Find a universal lower bound Apply the reverse triangle inequality with $u = z$ and $v = 1/2$:

$$\left|z + \frac{1}{2}\right| \geq \left||z| - \frac{1}{2}\right|.$$

Since $|z| \geq 2$, the expression inside the outer absolute value is positive. Therefore

$$\left|z + \frac{1}{2}\right| \geq |z| - \frac{1}{2} \geq 2 - \frac{1}{2} = \frac{3}{2}.$$

This argument holds for every allowed real or non-real value of z.

Step 2: Check equality and compare the options Take the negative real number $z = -2$. It satisfies $|z| = 2$, and

$$\left|-2 + \frac{1}{2}\right| = \left|-\frac{3}{2}\right| = \frac{3}{2}.$$

The bound is attained, so the minimum is exactly $3/2$. This number lies strictly between 1 and 2, as option (D) states. Option (B) is false because it requires strict inequality above $3/2$; equality actually occurs. Options (A) and (C), which refer to a value above or equal to $5/2$, are ruled out by the admissible example $z = -2$.

Step 3: Conclusion The minimum equals $\boxed{3/2}$, which belongs to $(1, 2)$. The admissible value $z = -2$ attains $3/2$, so the key claim of a strictly greater value is excluded. Hence, the correct option is (D).

Quick Tip to Solve: Use the reverse triangle inequality first, then test the point on the negative real axis nearest to $-1/2$ to confirm equality.

---

17. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is : [4 marks]

(A) $\frac{\sqrt{34}}{9}$

(B) $\frac{2\sqrt{13}}{9}$

(C) $\frac{\sqrt{61}}{9}$

(D) $\frac{2\sqrt{17}}{9}$

Correct Answer: (B) $\frac{2\sqrt{13}}{9}$

Solution:

Concept:

- For $px^2 + qx + r = 0$, Vieta's relations give $\alpha + \beta = -q/p$ and $\alpha\beta = r/p$. The reciprocal condition requires both roots to be nonzero.
- The arithmetic-progression condition on the coefficients is $2q = p + r$. The squared difference is $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$.

Step 1: Determine the coefficient ratios Using the sum of reciprocals,

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-q/p}{r/p} = -\frac{q}{r} = 4.$$

Thus $q = -4r$, where $r \neq 0$. Substitute this into $2q = p + r$:

$$-8r = p + r, \qquad p = -9r.$$

Consequently the root sum is $-q/p = -4/9$, and the root product is $r/p = -1/9$. Keeping these signs is essential; the product is negative.

Step 2: Compute the separation of the roots Now calculate

$$(\alpha - \beta)^2 = \left(-\frac{4}{9}\right)^2 - 4\left(-\frac{1}{9}\right)$$
$$= \frac{16}{81} + \frac{36}{81} = \frac{52}{81}.$$

The discriminant is positive, so the roots are real and their absolute difference is the positive square root: $|\alpha - \beta| = \sqrt{52}/9 = 2\sqrt{13}/9$. The other choices have squared values $34/81$, $61/81$ and $68/81$, respectively; none agrees with the derived value $52/81$.

Step 3: Conclusion $\boxed{|\alpha - \beta| = 2\sqrt{13}/9}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the sum and product of the roots from coefficient ratios; the identity for the squared difference avoids finding each root separately.

18. If $a \in R$ and the equation $-3(x-[x])^2+2(x-[x])+a^2=0$ (where [x] denotes the greater integer $\leq x$) has no integral solution, then all possible values of a lie in the interval : [4 marks]

(A) $(-2,-1)$

(B) $(-\infty,-2)\cup(2,\infty)$

(C) $(-1,0)\cup(0,1)$

(D) $(1,2)$

Correct Answer: For the question as printed, $\boxed{a \in \mathbb{R}\setminus\{0\}}$. No offered option gives all possible values. In particular, option (C) omits valid values such as $a=3$; the literal integer test proves why that selection cannot be made uniquely.

Solution:

Concept:

- The fractional part $t=x-[x]$ belongs to $[0,1)$. For every integral value of x, it is exactly zero.
- The printed demand is that there be no integral solution. This tests the equation at integers, rather than excluding all real solutions or requiring a non-integral solution to exist.

Step 1: Test every integer directly If $x=n\in\mathbb{Z}$, then $[n]=n$, so $x-[x]=0$. The equation reduces to

$$-3(0)^2+2(0)+a^2=0, \qquad a^2=0.$$

For real a, this holds exactly when $a=0$. Thus when $a=0$, every integer is a solution, and when $a\neq 0$, no integer is a solution. This is both necessary and sufficient: the expression at an integer contains no other dependence on n.

Step 2: Compare the complete parameter set with the choices The full answer to the literal condition is therefore $\mathbb{R}\setminus\{0\}$. None of the four proposed sets equals this set. For instance, $a=3$ gives no integral solution but lies in neither (A), (C), nor (D); $a=1/2$ also gives no integral solution but is excluded by (B). Each listed set contains some acceptable values, but none contains all of them. One must not replace the printed words with a requirement that a non-integral real solution exist: that additional requirement would change the question and would lead to a narrower set.

Step 3: Conclusion For the question as printed, $\boxed{a \in \mathbb{R}\setminus\{0\}}$. No offered option gives all possible values. In particular, option (C) omits valid values such as $a=3$; the literal integer test proves why that selection cannot be made uniquely.

Quick Tip to Solve: When a question says no integral solution and includes a fractional part, substitute an arbitrary integer before solving the fractional-part quadratic.

19. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ where $x \in R$ and $k \geq 1$.

Then $f_4(x) - f_6(x)$ equals : [4 marks]

(A) $\frac{1}{4}$

(B) $\frac{1}{12}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

Correct Answer: (B) $\frac{1}{12}$

Solution:

Concept:

- Use $\sin^2 x + \cos^2 x = 1$, and set $u = \sin^2 x$, $v = \cos^2 x$, so $u + v = 1$.
- The power identities $u^2 + v^2 = (u+v)^2 - 2uv$ and $u^3 + v^3 = (u+v)^3 - 3uv(u+v)$ reduce fourth and sixth trigonometric powers to the same product.

Step 1: Express the two function values Let $P = uv = \sin^2 x \cos^2 x$. From $u + v = 1$,

$$\sin^4 x + \cos^4 x = 1 - 2P,$$

and

$$\sin^6 x + \cos^6 x = 1 - 3P.$$

The definition of f_k includes its own factor $1/k$, so $f_4 = (1-2P)/4$ whereas $f_6 = (1-3P)/6$. These two denominators must be retained separately.

Step 2: Subtract and check the alternatives Take the common denominator 12:

$$\begin{aligned} f_4(x) - f_6(x) &= \frac{1-2P}{4} - \frac{1-3P}{6} \\ &= \frac{3-6P-2+6P}{12} = \frac{1}{12}. \end{aligned}$$

The variable terms cancel identically, so the result is independent of x, not merely true at a special angle. As a quick check, at $x = 0$ the two values are $1/4$ and $1/6$, whose difference is $1/12$. This check also rejects (A), (C) and (D): none equals the difference even at $x = 0$. The algebra establishes it for all real x.

Step 3: Conclusion $\boxed{f_4(x) - f_6(x) = 1/12}$ for every real x. Hence, the correct option is (B).

Quick Tip to Solve: For even trigonometric powers, set $u = \sin^2 x$, $v = \cos^2 x$; use $u + v = 1$ before expanding large powers.

20. The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$ and $l^2 = m^2 + n^2$ is : [4 marks]

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{4}$

Correct Answer: (C) $\frac{\pi}{3}$

Solution:

Concept:

- Direction cosines (l, m, n) give the direction of a line and satisfy $l^2 + m^2 + n^2 = 1$. A nonzero proportional vector gives the same line direction.
- For two line directions $\mathbf{a}, \mathbf{b}$, the smaller angle satisfies $\cos\theta = |\mathbf{a} \cdot \mathbf{b}|/(|\mathbf{a}||\mathbf{b}|)$. Opposite direction vectors describe the same line.

Step 1: Find the two possible line directions The first condition gives $l = -(m + n)$. Substitute it into $l^2 = m^2 + n^2$:

$$(m + n)^2 = m^2 + n^2, \qquad 2mn = 0.$$

Hence either $m = 0$ or $n = 0$. If $m = 0$, then $l = -n$, giving a direction proportional to $(1, 0, -1)$. If $n = 0$, then $l = -m$, giving a direction proportional to $(1, -1, 0)$. Neither vector is zero, and both satisfy the stated equations. Simultaneously setting $m = n = 0$ would force $l = 0$, which is not an allowed direction.

Step 2: Evaluate the dot product Take $\mathbf{a} = (1, 0, -1)$ and $\mathbf{b} = (1, -1, 0)$. Their dot product and lengths are

$$\mathbf{a} \cdot \mathbf{b} = 1, \qquad |\mathbf{a}| = |\mathbf{b}| = \sqrt{2}.$$

Therefore $\cos\theta = 1/2$, and the smaller angle is $\theta = \pi/3$. The cosines corresponding to (A), (B) and (D) are $\sqrt{3}/2$, 0 and $1/\sqrt{2}$, respectively, rather than $1/2$. Changing the sign of a direction vector does not change the smaller angle between the lines.

Step 3: Conclusion The angle between the lines is $\boxed{\pi/3}$. Hence, the correct option is (C).

Quick Tip to Solve: Eliminate one direction cosine first; a product such as $mn = 0$ splits the problem into the two line directions immediately.

21. If $(10)^9 + 2(11)^1(10^8) + 3(11)^2(10)^7 + \ldots\ldots\ldots + 10(11)^9 = k(10)^9$**,, then k is equal to :** [4 marks]

(A) 100

(B) 110

(C) $\frac{121}{10}$

(D) $\frac{441}{100}$

Correct Answer: (A) 100

Solution:

Concept:

- After factoring out 10^9, the sum is $1 + 2r + 3r^2 + \cdots + 10r^9$, with $r = 11/10$. This is an arithmetic-geometric series.
- Subtracting a shifted copy of a finite series reduces its coefficients to a geometric progression. The final boundary term must be included.

Step 1: Factor out the common power of ten The term with coefficient $j+1$ is $(j+1)11^j 10^{9-j}$, where $j = 0, 1, \ldots, 9$. Divide the stated equality by 10^9, obtaining

$$k = S = 1 + 2r + 3r^2 + \cdots + 10r^9, \qquad r = \frac{11}{10}.$$

There are ten terms, ending at power 9. This rewriting keeps the coefficient 10 of the last term; it must not be treated as an ordinary geometric series.

Step 2: Subtract the shifted series and evaluate Multiplying by r and subtracting gives

$$(1-r)S = 1 + r + r^2 + \cdots + r^9 - 10r^{10}.$$

Use the finite geometric-sum formula:

$$(1-r)^2 S = 1 - r^{10} - 10r^{10}(1-r) = 1 - 11r^{10} + 10r^{11}.$$

At $r = 11/10$, $10r^{11} = 11r^{10}$, so the last two terms cancel. Also $(1-r)^2 = (-1/10)^2 = 1/100$. Thus $S = 100$. Options (B), (C) and (D) do not satisfy this exact finite-sum identity; their different values cannot arise after the boundary-term cancellation.

Step 3: Conclusion $\boxed{k = 100}$. Hence, the correct option is (A).

Quick Tip to Solve: For $1 + 2r + \cdots + nr^{n-1}$, subtract rS from S. Here $10r = 11$ makes the endpoint powers cancel exactly.

22. If f and g are differentiable functions in $[0,1]$ satisfying

$f(0)=2=g(1), g(0)=0$ **and** $f(1)=6$, **then for some** $c \in]0,1[$ [4 marks]

(A) $f'(c)=g'(c)$

(B) $f'(c)=2g'(c)$

(C) $2f'(c)=g'(c)$

(D) $2f'(c)=3g'(c)$

Correct Answer: (B) $f'(c)=2g'(c)$

Solution:

Concept:

- Rolle's theorem states that a function continuous on $[a,b]$, differentiable on (a,b), and equal at its endpoints has zero derivative at some interior point.
- A linear combination of differentiable functions is differentiable. Choose its coefficients so the known endpoint changes cancel; the same coefficients then relate the derivatives.

Step 1: Calculate the endpoint changes The hypotheses give $f(1)-f(0)=6-2=4$ and $g(1)-g(0)=2-0=2$. Thus the change in f is twice that in g. It is the difference of these changes that we must make zero; applying the ordinary mean value theorem to each function separately would not guarantee the same intermediate point.

Step 2: Construct a function with equal endpoint values Define $h(x)=f(x)-2g(x)$. Its endpoint values are

$$h(0)=2-2(0)=2, \qquad h(1)=6-2(2)=2.$$

Differentiability of f and g gives the continuity and differentiability needed for Rolle's theorem. Hence there is some $c \in (0,1)$ for which

$$h'(c)=f'(c)-2g'(c)=0.$$

Step 3: Check the alternatives The required relation is therefore $f'(c)=2g'(c)$. The other ratios are not guaranteed: for example, $f(x)=2+4x$ and $g(x)=2x$ satisfy every endpoint condition but have constant derivatives 4 and 2. They satisfy option (B), while (A), (C) and (D) all fail at every point.

Step 4: Conclusion At a common interior point, the derivative of f is twice the derivative of g. Hence, the correct option is (B).

Quick Tip to Solve: Use the ratio of endpoint increments to construct $f-kg$; equal endpoint values let Rolle's theorem supply one common point.

23. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is [4 marks]

(A) $\frac{1}{8}$
(B) $\frac{2}{3}$
(C) $\frac{1}{2}$
(D) $\frac{3}{2}$

Correct Answer: (C) $\frac{1}{2}$

Solution:

Concept:

- For $y^2 = 4ax$, a tangent of nonzero slope m has equation $y = mx + a/m$.
- A line is tangent to a parabola when substitution produces a quadratic with a repeated root, so its discriminant is zero.

Step 1: Write the tangent to the first parabola Here $a = 1$, so a tangent to $y^2 = 4x$ with slope $m \neq 0$ is

$$y = mx + \frac{1}{m}.$$

We seek a line of this form that also touches $x^2 = -32y$. A horizontal line cannot touch the first parabola, and a vertical tangent of the first parabola is not a tangent of the second, so no omitted exceptional direction is a common tangent.

Step 2: Enforce tangency to the second parabola Substitute the line into the second equation to obtain

$$x^2 = -32\left(mx + \frac{1}{m}\right), \qquad x^2 + 32mx + \frac{32}{m} = 0.$$

A point of tangency gives equal roots in this equation for x. Consequently,

$$(32m)^2 - 4\frac{32}{m} = 0.$$

Multiplying by the nonzero m yields

$$1024m^3 - 128 = 0, \qquad m^3 = \frac{1}{8}, \qquad m = \frac{1}{2}.$$

Step 3: Verify the common line and exclude the other slopes At $m = 1/2$, the line is $y = x/2 + 2$. Substitution in the first parabola gives $(x-4)^2 = 0$, while substitution in the second gives $(x+8)^2 = 0$. Thus it touches both, at different points as expected. The other listed slopes $1/8$, $2/3$ and $3/2$ do not satisfy $m^3 = 1/8$ and therefore cannot produce a common tangent.

Step 4: Conclusion The slope of the common tangent is $\boxed{1/2}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the slope form of the tangent for one parabola, then impose a zero discriminant after substitution into the other.

24. Let C be the circle with centre at $(1,1)$ and radius $= 1$. If T is the circle centred at $(0,y)$, passing through origin and touching the circle C externally, then the radius of T is equal to : [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{\sqrt{3}}{\sqrt{2}}$

(D) $\frac{\sqrt{3}}{2}$

Correct Answer: (B) $\frac{1}{4}$

Solution:

Concept:

- For two circles touching externally, the distance between their centres equals the sum of their radii.
- A circle with centre $(0,y)$ passing through the origin has radius $r = |y|$. Keeping this absolute value prevents a premature assumption about the centre's position.

Step 1: Express the radius and centre separation Let the radius of T be r. Since the origin lies on T, its distance from the centre $(0,y)$ gives $r = |y|$. The centre of C is $(1,1)$, and its radius is 1. The separation of the two centres is therefore

$$d = \sqrt{(1-0)^2 + (1-y)^2} = \sqrt{1 + (1-y)^2}.$$

Step 2: Apply external tangency and determine the sign The external-contact condition is

$$\sqrt{1 + (1-y)^2} = 1 + |y|.$$

Both sides are nonnegative, so squaring preserves the equation. Expanding gives

$$2 - 2y + y^2 = 1 + 2|y| + y^2, \qquad 1 = 2y + 2|y|.$$

If $y \le 0$, the right-hand side is zero, which is impossible. Hence $y > 0$, so $|y| = y$, and

$$1 = 4y, \qquad y = \frac{1}{4}.$$

Step 3: Verify the radius and reject the distractors The radius is therefore $r = 1/4$. Directly, the distance between centres is

$$\sqrt{1 + (1 - 1/4)^2} = \sqrt{\frac{25}{16}} = \frac{5}{4} = 1 + \frac{1}{4},$$

which confirms external contact rather than internal contact. For positive $y = r$, the squared tangency equation requires $4r = 1$; none of $1/2$, $\sqrt{3}/\sqrt{2}$, or $\sqrt{3}/2$ meets that requirement.

Step 4: Conclusion The radius of T is $\boxed{1/4}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the unknown radius as the distance to a known point on the circle, and use the sum of radii only for external contact.

25. If the coefficints of x^3 and x^4 in the expansion of $\left(1+ax+bx^2\right)(1-2x)^{18}$ in powers of x are both zero, then $(a,\ b)$ is equal to: [4 marks]

(A) $\left(14, \frac{272}{3}\right)$
(B) $\left(16, \frac{272}{3}\right)$
(C) $\left(16, \frac{251}{3}\right)$
(D) $\left(14, \frac{251}{3}\right)$

Correct Answer: (B) $\left(16, \frac{272}{3}\right)$

Solution:

Concept:

- In $(1-2x)^{18}$, the coefficient of x^k is $\binom{18}{k}(-2)^k$.
- Multiplication by $1+ax+bx^2$ shifts these coefficients by zero, one and two powers. Each requested zero coefficient gives a linear equation in a and b.

Step 1: List the coefficients needed for the product Only the terms up to degree four can affect the two specified coefficients. The binomial theorem gives

$$(1-2x)^{18} = 1 - 36x + 612x^2 - 6528x^3 + 48960x^4 + \cdots .$$

Here $\binom{18}{2}4 = 612$, $\binom{18}{3}(-8) = -6528$, and $\binom{18}{4}16 = 48960$. The alternating signs come from the negative term $-2x$.

Step 2: Form the two zero-coefficient equations The x^3 coefficient receives contributions from 1, ax, and bx^2, respectively. Therefore

$$-6528 + 612a - 36b = 0, \qquad 17a - b = \frac{544}{3}.$$

Likewise the x^4 coefficient is

$$48960 - 6528a + 612b = 0, \qquad 544a - 51b = 4080.$$

Terms of degree five or higher cannot contribute because the multiplying polynomial has no negative powers.

Step 3: Solve simultaneously and compare the pairs The first equation gives $b = 17a - 544/3$. Substitution into the second yields

$$544a - 51\left(17a - \frac{544}{3}\right) = 4080,$$

$$-323a + 9248 = 4080, \qquad a = 16.$$

Hence

$$b = 272 - \frac{544}{3} = \frac{272}{3}.$$

Options (A) and (D) use $a = 14$, which cannot satisfy the simultaneous equations; option (C) has the right a but an incorrect b.

Step 4: Conclusion The required pair is $\boxed{(a, b) = (16, 272/3)}$. Hence, the correct option is (B).

Quick Tip to Solve: For a quadratic multiplier, record just three adjacent binomial coefficients for each requested power, then solve the resulting linear equations.

26. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It files off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is : [4 marks]

(A) $20\sqrt{2}$

(B) $20\left(\sqrt{3}-1\right)$

(C) $40\left(\sqrt{2}-1\right)$

(D) $40\left(\sqrt{3}-\sqrt{2}\right)$

Correct Answer: (B) $20\left(\sqrt{3}-1\right)$

Solution:

Concept:

- For an angle of elevation θ, $\tan\theta$ is the vertical height divided by the horizontal distance from the observer.
- Horizontal flight leaves the height unchanged. Flight straight away from the observer increases the horizontal distance along the same ground direction.

Step 1: Find the initial horizontal distance Let A be the foot of the pole and P the bird's initial position. The height is $AP = 20\,\text{m}$. If $OA = x_1$, the initial right triangle gives

$$\tan 45^\circ = \frac{20}{x_1}, \qquad x_1 = 20\,\text{m}.$$

This is the horizontal distance, not the initial slant distance OP. The latter must not be subtracted from a horizontal distance.

Step 2: Find the horizontal distance after one second Let the bird's new position be Q and its vertical projection on the ground be B. Its altitude remains $BQ = 20\,\text{m}$. The final elevation is 30°, so

$$\tan 30^\circ = \frac{20}{OB}, \qquad OB = \frac{20}{1/\sqrt{3}} = 20\sqrt{3}\,\text{m}.$$

Since the bird moves horizontally straight away from O, the projections O, A, B are collinear in that order.

Step 3: Calculate the displacement per second The bird's horizontal flight distance is the increase in these ground distances:

$$PQ = AB = OB - OA = 20\sqrt{3} - 20 = 20(\sqrt{3}-1)\,\text{m}.$$

Over the stated one-second interval, its speed is

$$v = \frac{20(\sqrt{3}-1)}{1} = 20(\sqrt{3}-1)\,\mathrm{m\,s^{-1}}.$$

Numerically this is about $14.64\,\mathrm{m\,s^{-1}}$. The other options give about 28.28, 16.57, and 12.71, respectively, and do not equal the required horizontal distance per second.

Step 4: Conclusion The bird's speed is $\boxed{20(\sqrt{3}-1)\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use height times the difference of cotangents: horizontal travel is $h(\cot\theta_2 - \cot\theta_1)$ when the height stays fixed.

27. The statement $\sim (p \leftrightarrow \sim q)$ is : [4 marks]

(A) equivalent to $\sim p \leftrightarrow q$

(B) a tautology

(C) a fallacy

(D) equivalent to $p \leftrightarrow q$

Correct Answer: (D) equivalent to $p \leftrightarrow q$

Solution:

Concept:

- The biconditional $p \leftrightarrow q$ is true precisely when the two propositions have the same truth value.
- Negating a biconditional reverses its truth value. Replacing one input by its negation also reverses whether the inputs agree.

Step 1: Interpret the inner biconditional The proposition $p \leftrightarrow \sim q$ is true when p and $\sim q$ agree, which is exactly when p and q disagree. Its negation $\sim (p \leftrightarrow \sim q)$ must consequently be true exactly when p and q agree. That is the defining condition for $p \leftrightarrow q$.

Step 2: Verify every truth assignment A truth table removes any ambiguity about the placement of negation:

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim (p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

The final two columns coincide for all four possible assignments, which proves logical equivalence rather than agreement in just one example.

Step 3: Reject the remaining descriptions The output column contains both true and false entries. Thus the statement is neither a tautology, which would be true in every row, nor a fallacy, which would be false in every row. Moreover $\sim p \leftrightarrow q$, the statement in option (A), is true when p and q differ, so it has the opposite truth column. For example, at $p = q = T$, the given statement is true but option (A)'s biconditional is false.

Step 4: Conclusion The statement is equivalent to $\boxed{p \leftrightarrow q}$. Hence, the correct option is (D).

Quick Tip to Solve: Negating one side of a biconditional flips its truth value; an outside negation flips it back. Confirm with equal and unequal inputs.

28. If $\left[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}\right] = \lambda\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2$ **then** λ **is equal to :** [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Concept:

- The scalar triple product is $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$. Its value is unchanged by cyclic permutations.
- The vector identity $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \mathbf{v}(\mathbf{u} \cdot \mathbf{w}) - \mathbf{w}(\mathbf{u} \cdot \mathbf{v})$ expands a nested cross product without changing its order.

Step 1: Introduce the original scalar triple product Write $D = [\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$. The left-hand side of the question is the scalar triple product of three cross products, namely

$$L = (\mathbf{a} \times \mathbf{b}) \cdot \big((\mathbf{b} \times \mathbf{c}) \times (\mathbf{c} \times \mathbf{a})\big).$$

It is important to keep this order: reversing two entries of a scalar triple product would change its sign.

Step 2: Expand the inner cross product Apply the vector triple-product identity with $\mathbf{u} = \mathbf{b} \times \mathbf{c}$, $\mathbf{v} = \mathbf{c}$, and $\mathbf{w} = \mathbf{a}$. This gives

$$\begin{aligned}(\mathbf{b} \times \mathbf{c}) \times (\mathbf{c} \times \mathbf{a}) &= \mathbf{c}\big((\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a}\big) \\ &\quad - \mathbf{a}\big((\mathbf{b} \times \mathbf{c}) \cdot \mathbf{c}\big) \\ &= D\mathbf{c}.\end{aligned}$$

The second dot product vanishes because a cross product is perpendicular to each of its factors.

Step 3: Take the remaining dot product and compare It follows that

$$L = D(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = D[\mathbf{c}\,\mathbf{a}\,\mathbf{b}] = D^2.$$

The last equality uses cyclic invariance. Hence the coefficient in this general identity is 1. A quick independent check is $\mathbf{a} = \mathbf{i}$, $\mathbf{b} = \mathbf{j}$, $\mathbf{c} = \mathbf{k}$: the three cross products are $\mathbf{k}, \mathbf{i}, \mathbf{j}$, whose scalar triple product is 1. This excludes coefficients 0, 2, and 3.

Step 4: Conclusion The identity has coefficient $\boxed{\lambda = 1}$. Hence, the correct option is (B).

Quick Tip to Solve: Expand the cross product of the last two entries first; perpendicular dot products vanish, leaving the square of the original scalar triple product.

29. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line : [4 marks]

(A) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$

(B) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

(C) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

(D) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

Correct Answer: (C) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

Solution:

Concept:

- A line can be written as $\mathbf{r} = \mathbf{p} + t\mathbf{d}$, where $\mathbf{p}$ is a point on it and $\mathbf{d}$ is its direction vector.
- Reflection in $\mathbf{n} \cdot \mathbf{r} + c = 0$ maps a point to $\mathbf{p}\prime = \mathbf{p} - 2(\mathbf{n} \cdot \mathbf{p} + c)\mathbf{n}/|\mathbf{n}|^2$. A direction parallel to the plane is unchanged.

Step 1: Check the direction relative to the plane The given point is $P = (1, 3, 4)$, its direction is $\mathbf{d} = (3, 1, -5)$, and the normal to the mirror plane is $\mathbf{n} = (2, -1, 1)$. Their scalar product is

$$\mathbf{d} \cdot \mathbf{n} = 3(2) + 1(-1) + (-5)(1) = 0.$$

Thus the line is parallel to the plane. Reflection translates it to a parallel line; it does not rotate its direction. Reversing all direction ratios also describes the same direction.

Step 2: Reflect one point of the line For $P = (1, 3, 4)$, the plane expression is $2 - 3 + 4 + 3 = 6$, while $|\mathbf{n}|^2 = 4 + 1 + 1 = 6$. Hence

$$P\prime = (1, 3, 4) - \frac{2(6)}{6}(2, -1, 1) = (-3, 5, 2).$$

The midpoint $(-1, 4, 3)$ lies on the plane because $-2 - 4 + 3 + 3 = 0$, verifying that the perpendicular displacement has the correct magnitude and sign.

Step 3: Write and compare the reflected line Using $P\prime$ and the unchanged direction gives

$$\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}.$$

Options (A) and (B) represent the same line through $(3, -5, 2)$, which does not contain $P\prime$. Option (D) passes through $(-3, 5, -2)$; at $x = -3$, the reflected line has $z = 2$, so (D) also fails.

Step 4: Conclusion The image is $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$. Hence, the correct option is (C).

Quick Tip to Solve: First dot the line direction with the mirror normal. When the result is zero, reflect just one point and retain the direction ratios.

30. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to : [4 marks]

(A) X

(B) Y

(C) N

(D) $Y - X$

Correct Answer: (B) Y

Solution:

Concept:

- If every element of X belongs to Y, then $X \subseteq Y$ and consequently $X \cup Y = Y$.
- The binomial theorem applied to $4^n = (1+3)^n$ separates the first two terms from terms divisible by 9. Natural numbers here start at 1.

Step 1: Prove divisibility for every natural index Let $x_n = 4^n - 3n - 1$. For $n \geq 2$, expand the power:

$$4^n = 1 + 3n + \sum_{k=2}^{n} \binom{n}{k} 3^k.$$

Subtracting the first two terms gives

$$x_n = 9 \sum_{k=2}^{n} \binom{n}{k} 3^{k-2}.$$

The sum on the right is an integer and is nonnegative. The separate case $n = 1$ gives $x_1 = 4 - 3 - 1 = 0$. Therefore every element of X is a nonnegative multiple of 9, including its first element zero.

Step 2: Identify the containing set The elements of Y are obtained by substituting successive natural numbers:

$$Y = \{0, 9, 18, 27, 36, \ldots\}.$$

For any $x_n = 9k$ found above, k is a nonnegative integer. Choosing the natural index $m = k + 1$ gives $9(m-1) = x_n$. This explicitly proves $X \subseteq Y$, rather than relying on a few numerical examples.

Step 3: Take the union and exclude the other choices Adding a subset to its containing set leaves the latter unchanged, so $X \cup Y = Y$. It is not X: the first values of X are $0, 9, 54$, and $x_{n+1} - x_n = 3(4^n - 1) > 0$, so $18 \in Y$ is absent from X. It is not N, since 1 is absent. It is not $Y - X$, since zero belongs to the union but is removed from that difference.

Step 4: Conclusion Every element of X is in Y, and therefore $X \cup Y = Y$. Hence, the correct option is (B).

Quick Tip to Solve: For expressions such as $4^n - 3n - 1$, write $4 = 1 + 3$. Removing the constant and linear binomial terms reveals divisibility by 3^2.

Physics (120 marks)

31. Hydrogen $({}_1H^1)$, Deuterium $({}_1H^2)$, singly ionised Helium $({}_2He^4)^+$ and doubly ionised lithium $({}_3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wavelengths of emitted radiation are $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively then approximately which one of the following is correct? [4 marks]

(A) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(B) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(C) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

(D) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

Correct Answer: (C) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

Solution:

Concept:

- For a one-electron ion, the Bohr energy is approximately $E_n = -13.6Z^2/n^2\,\text{eV}$, where Z is the atomic number.
- The emitted photon satisfies $hc/\lambda = E_{\text{initial}} - E_{\text{final}}$. For the same pair of levels, $\lambda \propto 1/Z^2$; isotope reduced-mass corrections are small.

Step 1: Find the energy for the common transition All four species contain one electron, so the hydrogen-like energy formula applies. For the transition from $n = 2$ to $n = 1$, the positive energy released is

$$\Delta E = 13.6Z^2\left(1 - \frac{1}{4}\right) = 10.2Z^2\,\text{eV}.$$

The factor from the energy levels is identical for every species; only the nuclear charge changes the leading energy scale.

Step 2: Insert the nuclear charges Hydrogen and deuterium are isotopes with the same nuclear charge $Z = 1$, although their nuclear masses differ. Singly ionised helium has $Z = 2$, and doubly ionised lithium has $Z = 3$. Thus, neglecting the small reduced-mass differences as required by the word approximately,

$$\lambda_1 : \lambda_2 : \lambda_3 : \lambda_4 = 1 : 1 : \frac{1}{4} : \frac{1}{9}.$$

Adding neutrons to hydrogen does not double its nuclear charge, so deuterium does not acquire a factor-of-two change of wavelength.

Step 3: Compare the proposed equalities Multiplying the helium wavelength by four and the lithium wavelength by nine puts both on the hydrogen scale:

$$\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4.$$

Options (A), (B) and (D) impose large differences between hydrogen and deuterium and also fail the inverse-square dependence on nuclear charge. Only (C) agrees with both the isotope comparison and the ion charge scaling.

Step 4: Conclusion Approximately, $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$. Hence, the correct option is (C).

Quick Tip to Solve: For identical transitions of hydrogen-like species, compare atomic numbers, not mass numbers: photon energy scales as Z^2 and wavelength as Z^{-2}.

32. The current voltage relation of diode is given by $\mathrm{I} = \left(e^{100V/T} - 1\right) mA$, where the applied voltage V is in volts and the temperature T is in degree kelvin. If a student makes an error measuring $\pm 0.01\,V$ while measuring the current of $5\ mA$ at $300\ K$, what will be the error in the value of current on mA? [4 marks]

(A) $0.2\ mA$

(B) $0.02\ mA$

(C) $0.5\ mA$

(D) $0.05\ mA$

Correct Answer: (B) $0.02\ mA$

Solution:

Concept:

- For a small voltage uncertainty at fixed temperature, the absolute current uncertainty is $|\delta I| \simeq |dI/dV|\,|\delta V|$.
- In $I = (e^{100V/T} - 1)\,\mathrm{mA}$, the numerical current in milliamperes obeys $e^{100V/T} = I + 1$. The additive constant must be retained when differentiating.

Step 1: Evaluate the exponential at the operating point The measured current is $5\,\mathrm{mA}$. Using milliamperes as the numerical current unit in the given relation,

$$5 = e^{100V/T} - 1 \quad \Rightarrow \quad e^{100V/T} = 6.$$

The temperature is fixed at $T = 300\,\mathrm{K}$; the stated uncertainty concerns voltage alone. There is no need to calculate the operating voltage, because the exponential is already determined by the current.

Step 2: Differentiate the relation at fixed temperature The sensitivity of current to voltage is

$$\frac{dI}{dV} = \frac{100}{T} e^{100V/T}\,\mathrm{mA\,V^{-1}}.$$

At the specified operating point this becomes

$$\frac{dI}{dV} = \frac{100}{300} \times 6 = 2\,\mathrm{mA\,V^{-1}}.$$

Multiplying by the absolute voltage error $0.01\,\mathrm{V}$ gives

$$|\delta I| \simeq 2 \times 0.01 = 0.02\,\mathrm{mA}.$$

Step 3: Check the approximation and the choices The exponent changes by only $(100/300)(0.01) = 1/300$, much less than one, so first-order error propagation is appropriate. The exact upward current change is $6(e^{1/300} - 1)\,\mathrm{mA}$, which is about $0.02003\,\mathrm{mA}$, consistent with the differential result. The values 0.2, 0.5 and $0.05\,\mathrm{mA}$ do not equal the local slope times the given uncertainty.

Step 4: Conclusion The uncertainty in current is approximately $\pm 0.02\,\mathrm{mA}$, with magnitude $0.02\,\mathrm{mA}$. Although the supplied key gives the value $0.2\,\mathrm{mA}$, the printed relation gives $\delta I = (100/300)(5 +$

$1)(0.01) = 0.02\,\text{mA}$, which proves (B) rather than $0.2\,\text{mA}$. Hence, the correct option is (B).

Quick Tip to Solve: In an exponential diode law, replace the exponential by $I + 1$ before inserting numbers into the derivative; this avoids an unnecessary logarithm.

33. During the propagation of electromagnetic waves in a medium : [4 marks]

(A) Electric energy density is double of the magnetic energy density.

(B) Electric energy density is half of the magnetic energy density.

(C) Electric energy density is equal to the magnetic energy density.

(D) Both electric and magnetic energy of densities are zero.

Correct Answer: (C) Electric energy density is equal to the magnetic energy density.

Solution:

Concept:

- In a uniform ideal medium, the electric and magnetic energy densities of an electromagnetic wave are $u_E = \frac{1}{2}\varepsilon E^2$ and $u_B = B^2/(2\mu)$.
- The wave speed is $v = 1/\sqrt{\mu\varepsilon}$, and its fields satisfy $E = vB$. Here ε and μ are the medium permittivity and permeability.

Step 1: Relate the electric and magnetic field amplitudes The electric and magnetic fields in a travelling electromagnetic wave are coupled parts of the same wave. Their magnitudes satisfy

$$E = vB, \qquad v^2 = \frac{1}{\mu\varepsilon}.$$

In a medium one must use its wave speed rather than automatically replacing v by the vacuum value c. The relation holds both for instantaneous magnitudes at a common phase and for the corresponding peak amplitudes.

Step 2: Compare the two energy densities Substitute the wave field relation into the electric energy expression:

$$u_E = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}\varepsilon v^2 B^2.$$

Now use the speed formula to simplify:

$$u_E = \frac{1}{2}\varepsilon \frac{B^2}{\mu\varepsilon} = \frac{B^2}{2\mu} = u_B.$$

Thus the electric and magnetic contributions are equal. The same equality holds for their time averages, since averaging both identical expressions preserves equality.

Step 3: Interpret the options physically The wave carries nonzero energy when its fields are present, with total density $u = u_E + u_B = 2u_E = 2u_B$. Options (A) and (B) introduce an extra factor of two between the individual contributions; that factor belongs between the total and either contribution instead. Option (D) would describe an absence of electromagnetic energy and cannot represent a propagating nonzero wave. The equality stated in (C) is the standard electromagnetic-wave result.

Step 4: Conclusion The electric energy density equals the magnetic energy density during propagation. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish the total energy density from either field contribution: the two contributions are equal, and each is half of their sum.

34. The radiation corresponding to $3 \to 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field $3 \times 10^{-4}\,T$. If the radius of the larger circular path followed by these electrons is $10.0\ mm$, the work function of the metal is close to: [4 marks]

(A) $1.8\ eV$

(B) $1.1\ eV$

(C) $0.8\ eV$

(D) $1.6\ eV$

Correct Answer: (B) $1.1\ eV$

Solution:

Concept:

- For hydrogen, the photon energy from n_i to n_f is $13.6(1/n_f^2 - 1/n_i^2)\,\text{eV}$. Einstein's equation is $K_{\max} = E_\gamma - \phi$.
- For an electron moving perpendicular to a magnetic field, $p = eBr$ and $K = p^2/(2m_e)$. The largest circular radius corresponds to the largest electron kinetic energy.

Step 1: Calculate the incident photon energy The transition is from $n = 3$ to $n = 2$, so the emitted energy is positive:

$$E_\gamma = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = 13.6\frac{5}{36} = 1.889\,\text{eV}.$$

This is the energy available to release an electron and supply its kinetic energy; it is not itself the work function. The work function must be smaller than this photon energy for emission to occur.

Step 2: Find the largest photoelectron kinetic energy Convert the largest radius to SI units: $r = 10.0\,\text{mm} = 1.00 \times 10^{-2}\,\text{m}$. With $B = 3.0 \times 10^{-4}\,\text{T}$, magnetic circular motion gives

$$p = eBr = (1.60 \times 10^{-19})(3.0 \times 10^{-4})(1.00 \times 10^{-2}) = 4.80 \times 10^{-25}\,\text{kg m s}^{-1}.$$

Hence, with $m_e = 9.11 \times 10^{-31}\,\text{kg}$,

$$K_{\max} = \frac{p^2}{2m_e} = 1.26 \times 10^{-19}\,\text{J} = 0.79\,\text{eV}.$$

Step 3: Subtract the kinetic energy from the photon energy Einstein's photoelectric equation now yields

$$\phi = E_\gamma - K_{\max} = 1.889 - 0.79 \simeq 1.10\,\text{eV}.$$

The magnetic field does no work; it measures the momentum by bending the path, so its effect must

not be added to the photon energy. Option (C), near $0.8\,\text{eV}$, is the kinetic energy rather than the work function. Options (A) and (D) leave kinetic energies far smaller than the radius implies.

Step 4: Conclusion The work function is close to $1.1\,\text{eV}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the maximum orbit radius to obtain $K_{\max} = e^2B^2r^2/(2m_e)$, convert it to electronvolts, and only then subtract it from the photon energy.

35. Match List - 1 (Electromagnetic wave type) with List - 2 (Its association/application) and select the correct option from the choices given below the lists: [4 marks]

List 1	List 2
1. Infrared waves	(i) To treat muscular strain
2. Radio waves	(ii) For broadcasting
3. X-rays	(iii) To detect fracture of bones
4. Ultraviolet rays	(iv) Absorbed by the ozone layer of the atmosphere

(A) $1-iv,\ 2-iii,\ 3-ii,\ 4-i$

(B) $1-i,\ 2-ii,\ 3-iv,\ 4-iii$

(C) $1-iii,\ 2-ii,\ 3-i,\ 4-iv$

(D) $1-i,\ 2-ii,\ 3-iii,\ 4-iv$

Correct Answer: (D) $1-i,\ 2-ii,\ 3-iii,\ 4-iv$

Solution:

Concept:

- Electromagnetic waves have the same nature but different frequencies and wavelengths; their interaction with matter determines their practical uses.
- Infrared radiation produces heating, radio waves carry broadcast signals, X-rays penetrate soft tissue, and atmospheric ozone absorbs much harmful ultraviolet radiation.

Step 1: Match the first two wave types Infrared waves are used in heat treatment because absorption of their energy warms tissue. Thus infrared radiation corresponds to treatment of muscular strain, giving $1 \to i$. Radio waves can be generated and received electronically and travel over useful communication distances. They are used for broadcasting, giving $2 \to ii$. This already rules out (A), which puts infrared with ozone absorption, and (C), which puts infrared with bone-fracture detection.

Step 2: Complete and verify the remaining matches X-rays penetrate soft tissue more readily than bone, so their differential absorption produces an image that can reveal a fracture: $3 \to iii$. Ultraviolet radiation is strongly absorbed by ozone in the atmosphere: $4 \to iv$. Option (B) interchanges these last two associations and is therefore incorrect even though its first two matches are right. The complete ordered matching is

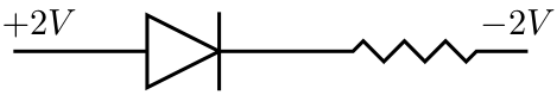
$$1-i, \qquad 2-ii, \qquad 3-iii, \qquad 4-iv.$$

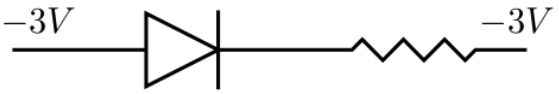
All four associations, rather than only one familiar application, must agree with the selected code.

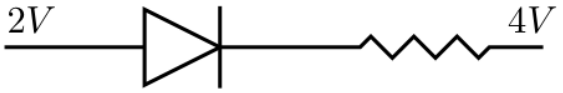
Step 3: Conclusion Infrared corresponds to muscular-strain treatment, radio waves to broadcasting, X-rays to fracture detection and ultraviolet rays to absorption by atmospheric ozone. The ordered matching is $1-i, 2-ii, 3-iii, 4-iv$. Hence, the correct option is (D).

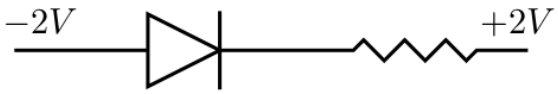
Quick Tip to Solve: For spectrum matching, attach one physical effect to each band: infrared heats, radio communicates, X-rays image bones, and ultraviolet is screened by ozone.

36. The forward biased diode connection is: [4 marks]

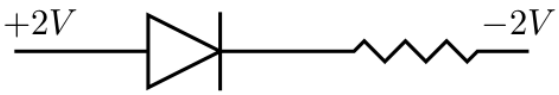
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- A diode is forward biased when its anode (p-side) is at a higher electric potential than its cathode (n-side). The bar in the diode symbol marks the cathode.
- The relevant quantity is the signed voltage difference $V_A - V_K$, not whether an individual terminal is positive or negative relative to an arbitrary zero.

Step 1: Identify the terminals in the diagrams In every displayed connection the anode is on the left of the diode and the cathode bar is on the right. Compare the left and right applied potentials. A series resistor limits the resulting current; it does not reverse the polarity imposed by the two terminals. Before conduction starts there is no resistor drop, so the applied potential difference directly tells us which bias is being attempted.

Step 2: Compare all four voltage differences In option (A) the left terminal is $+2\,\mathrm{V}$ and the right is

$-2\,\mathrm{V}$. Thus the anode is driven above the cathode by

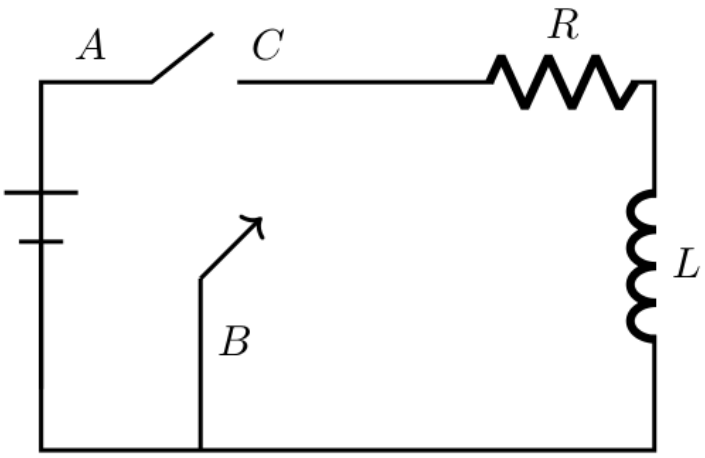
$$+2-(-2)=+4\,\mathrm{V},$$

giving forward bias. In (B), both terminals are at $-3\,\mathrm{V}$, so the applied bias is zero. In (C), the difference is $2-4=-2\,\mathrm{V}$, which is reverse bias. In (D), it is $-2-(+2)=-4\,\mathrm{V}$, also reverse bias. Only (A) has the required higher anode potential. The resistor shares the applied voltage after current begins, but the diode remains forward biased.

Step 3: Conclusion Option (A) alone connects the anode toward the higher potential and the cathode toward the lower potential. Its applied difference is $+4\,\mathrm{V}$; the other connections give zero or reverse bias. Hence, the correct option is (A).

Quick Tip to Solve: Locate the cathode bar, then subtract cathode potential from anode potential. A positive result indicates forward bias even if both potentials are negative.

37. In the circuit shown here, the point $'C'$ is kept connected to point $'A'$ till the current flowing through the circuit becomes constant. Afterward, suddenly, point $'C'$ is disconnected from point $'A'$ and connected to point $'B'$ at time $t=0$. Ratio of the voltage across resistance and the inductor at $t=L/R$ will be equal to : [4 marks]



(A) $\frac{e}{1-e}$

(B) 1

(C) -1

(D) $\frac{1-e}{e}$

Correct Answer: (C) -1

Solution:

Concept:

- After the source is removed, an ideal series resistance R and inductance L obey $L\,di/dt+Ri=0$. The time constant is $\tau=L/R$.

- Using the same passive polarity convention around the discharge loop, $V_R = Ri$ and $V_L = L\,di/dt$. These signed voltages add to zero because no source remains.

Step 1: Write the discharge current after switching When C is connected to A for a long time, the inductor carries its steady current I_0. The inductor current cannot change instantaneously. Switching C to B removes the battery and leaves the R-L discharge loop. Solving its loop equation gives

$$i(t) = I_0 e^{-Rt/L}, \qquad \frac{di}{dt} = -\frac{R}{L} I_0 e^{-Rt/L}.$$

At the specified time $t = L/R$, the current is therefore I_0/e. The negative derivative expresses decay, not reversal of the current.

Step 2: Calculate the signed voltage ratio The resistor voltage at this instant is

$$V_R = \frac{RI_0}{e}.$$

The inductor voltage, with the same loop sign convention, is

$$V_L = L\frac{di}{dt} = -\frac{RI_0}{e}.$$

Hence

$$\frac{V_R}{V_L} = -1.$$

The exponential factor cancels: this ratio holds throughout the source-free decay. Option (B) gives the ratio of magnitudes only and misses the opposite polarity; (A) and (D) introduce factors appropriate to a different switching or growth calculation, not this discharge loop.

Step 3: Conclusion At $t = L/R$, the resistance and inductor have equal voltage magnitudes and opposite signed voltages. Thus $V_R/V_L = -1$. Hence, the correct option is (C).

Quick Tip to Solve: For a source-free series RL loop, apply Kirchhoff immediately: $V_R + V_L = 0$. The signed ratio is -1 without needing the initial battery voltage.

38. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is : [4 marks]

(A) $f_1 = f_2 < f$

(B) $f_1 > f$ and f_2 becomes negative

(C) $f_2 > f$ and f_1 becomes negative

(D) f_1 and f_2 both become negative

Correct Answer: (B) $f_1 > f$ and f_2 becomes negative

Solution:

Concept:

- For a thin lens immersed in a medium of refractive index μ_m, $1/f_m = (\mu_l/\mu_m - 1)K$, where $K = 1/R_1 - 1/R_2$ depends only on its two surface curvatures.
- The lens material has index $\mu_l = 3/2$. A convex lens has $K > 0$, so its power is positive when its index exceeds that of the surrounding medium and negative when it is smaller.

Step 1: Find the curvature factor from the lens in air In air the surrounding index is approximately one. Thus the given positive focal length satisfies

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right) K = \frac{K}{2}, \qquad K = \frac{2}{f}.$$

Immersion changes the refractive-index contrast rather than the physical curvature, so this same factor must be used for both liquids.

Step 2: Substitute the two liquid indices For the first liquid,

$$\frac{1}{f_1} = \left(\frac{3/2}{4/3} - 1\right) \frac{2}{f} = \left(\frac{9}{8} - 1\right) \frac{2}{f} = \frac{1}{4f}, \qquad f_1 = 4f.$$

For the second liquid,

$$\frac{1}{f_2} = \left(\frac{3/2}{5/3} - 1\right) \frac{2}{f} = \left(\frac{9}{10} - 1\right) \frac{2}{f} = -\frac{1}{5f}, \qquad f_2 = -5f.$$

The first lens remains converging but weaker; the second becomes diverging. Option A wrongly equates the two focal lengths, option C reverses which is negative, and option D wrongly makes the first focal length negative.

Step 3: Conclusion The results are $f_1 = 4f > f$ and $f_2 = -5f < 0$. Hence, the correct option is (B).

Quick Tip to Solve: Compare the lens index with the liquid index first: a convex glass lens becomes diverging when immersed in a liquid optically denser than the glass.

39. Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then $\frac{\mathrm{I}_A}{\mathrm{I}_B}$ equals: [4 marks]

(A) 3

(B) $\frac{3}{2}$

(C) 1

(D) $\frac{1}{3}$

Correct Answer: (D) $\frac{1}{3}$

Solution:

Concept:

- Malus law gives the transmitted intensity $I = I_0 \cos^2 \phi$, where ϕ is the angle between the incident polarization direction and the transmission axis of the analyser.
- The beams are polarized in perpendicular directions. Consequently, when the analyser makes angle θ with beam A, its angle with beam B is $90^\circ - \theta$, and the corresponding transmission factors are $\cos^2 \theta$ and $\sin^2 \theta$.

Step 1: Identify the two angles after rotation Initially the analyser axis is parallel to the polarization of A, giving its maximum transmission, and perpendicular to B, giving zero transmission. Rotating the analyser through 30° therefore makes angles 30° and 60° with A and B respectively. The initial beam intensities I_A, I_B refer to the incident beams, before these analyser factors are applied.

Step 2: Equate the final transmitted intensities The two beams appear equally bright after the rotation, so

$$I_A \cos^2 30^\circ = I_B \cos^2 60^\circ.$$

Using $\cos^2 30^\circ = 3/4$ and $\cos^2 60^\circ = 1/4$,

$$\frac{3}{4} I_A = \frac{1}{4} I_B, \qquad 3I_A = I_B, \qquad \frac{I_A}{I_B} = \frac{1}{3}.$$

A has the larger transmission factor and must therefore have the smaller incident intensity for the outputs to match. This excludes ratios 3 and 3/2 in options A and B. Equal incident intensities, option C, would give transmitted intensities in the ratio 3:1, not equal brightness.

Step 3: Conclusion The incident intensity ratio is $I_A/I_B = 1/3$. Hence, the correct option is (D).

Quick Tip to Solve: For perpendicular incident polarizations, write the two transmitted intensities as $I_A \cos^2 \theta$ and $I_B \sin^2 \theta$; equal brightness then gives $I_A/I_B = \tan^2 \theta$.

40. A green light is incident from the water to the air - water interface at the critical angle (θ). Select the correct statement. [4 marks]

(A) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal.

(B) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

(C) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.

(D) The entire spectrum of visible light will come out of the water at various angles to the normal.

Correct Answer: (B) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

Solution:

Concept:

- For light passing from water into air, the critical angle obeys $\sin\theta_c = n_a/n_w$, where n_a and n_w are the refractive indices at that light frequency.
- Under normal dispersion in visible light, water has a larger refractive index at higher frequency. The critical angle is therefore smaller for higher-frequency light and larger for lower-frequency light.

Step 1: Use the green-light limiting condition The common incidence angle is fixed at the critical angle for green light. Writing the relative water-to-air index as μ_g,

$$\sin\theta = \frac{1}{\mu_g}.$$

Snell's law for any other visible component at this same incidence gives

$$\sin r = \mu \sin\theta = \frac{\mu}{\mu_g}.$$

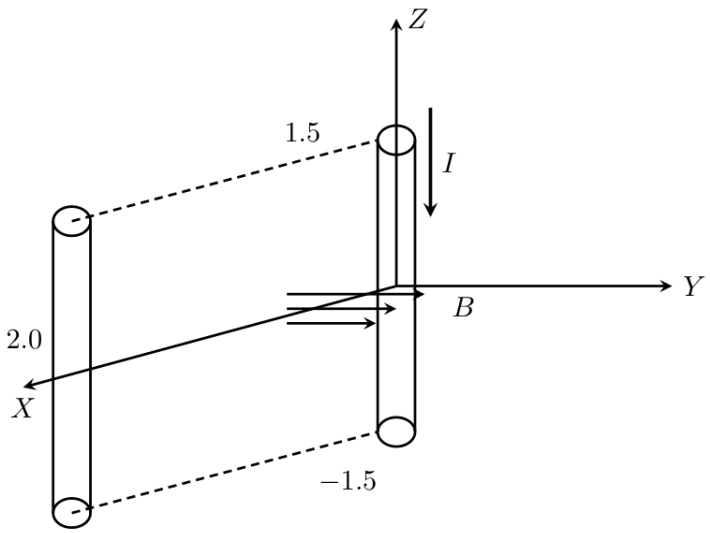
Green itself has $\sin r = 1$, so its limiting refracted ray lies along the interface.

Step 2: Compare the other frequencies with green For lower-frequency light, such as red, normal dispersion gives $\mu < \mu_g$. Hence $\sin r < 1$, a real refracted ray exists, and this component emerges into air at an angle less than 90°. For higher-frequency light, such as violet, $\mu > \mu_g$, and the formal value $\sin r > 1$ is impossible: this component undergoes total internal reflection. Option A incorrectly gives every wavelength the green critical condition. Option C selects precisely the frequency side that is totally internally reflected. Option D wrongly allows the entire spectrum to escape despite the critical-angle threshold.

Step 3: Conclusion Visible components of frequency less than that of green emerge into air; higher-frequency components are totally internally reflected. Hence, the correct option is (B).

Quick Tip to Solve: In normal dispersion, the red end has the larger critical angle. At the green critical angle, red can escape while the violet end is already beyond its critical angle.

41. A conductor lies along the z-axis at $-1.5 \le z < 1.5\,m$ and carries a fixed current of $10.0\ A$ in $-\hat{a}_z$ direction (see figure). For a field $\overrightarrow{B} = 3.0 \times 10^{-4}\, e^{-0.2x}\ \hat{a}_y\ T$, find the power required to move the conductor at constant speed to $x = 2.0\ m$, $y = 0\ m$ in $5 \times 10^{-3} s$. Assume parallel motion along the x-axis. [4 marks]



(A) $1.57W$

(B) $2.97W$

(C) $14.85\ W$

(D) $29.7W$

Correct Answer: (B) $2.97W$

Solution:

Concept:

- The force on a straight current-carrying wire is $\mathbf{F} = I\boldsymbol{\ell} \times \mathbf{B}$. Its magnitude here is $F = I\ell B(x)$, since the wire and field are perpendicular.
- For a position-dependent force, calculate transferred work by $W = \int F(x)\, dx$; the average power magnitude for the whole trip is $P = |W|/\Delta t$.

Step 1: Find the force as a function of position The conductor extends from $z = -1.5\,\text{m}$ to $z = 1.5\,\text{m}$, so its length is $\ell = 3.0\,\text{m}$. With $I = 10.0\,\text{A}$,

$$F(x) = 10(3)(3.0 \times 10^{-4})e^{-0.2x} = 9.0 \times 10^{-3} e^{-0.2x}\,\text{N}.$$

The direction is $(-\hat{\mathbf{a}}_z) \times \hat{\mathbf{a}}_y = +\hat{\mathbf{a}}_x$. Thus the magnetic force assists positive-x motion, and a controller maintaining constant speed must oppose it.

Step 2: Integrate the work and divide by travel time The magnitude of magnetic work in moving

from $x = 0$ to $x = 2.0\,\mathrm{m}$ is

$$\begin{aligned} |W| &= 9.0 \times 10^{-3} \int_0^2 e^{-0.2x}\, dx \\ &= \frac{9.0 \times 10^{-3}}{0.2}(1 - e^{-0.4}) \\ &= 0.045(1 - 0.67032) = 0.0148356\,\mathrm{J}. \end{aligned}$$

Therefore the average power magnitude is

$$P = \frac{0.0148356}{5.0 \times 10^{-3}} = 2.96712\,\mathrm{W} \simeq 2.97\,\mathrm{W}.$$

The force changes during the trip, so substituting only the endpoint field into Fv would not give this average. The other three numerical values fail this work-time calculation.

Step 3: Conclusion The average mechanical power magnitude is $2.97\,\mathrm{W}$. With the stated current and field directions, the external controller actually absorbs this power (its signed work is negative); the positive option lists its magnitude. Hence, the correct option is (B).

Quick Tip to Solve: When the magnetic field depends on position, integrate $I\ell B(x)$ over displacement before dividing by the total time; do not use the endpoint force for average power.

42. The coercivity of a small magnet where the ferromagnet gets demagnetized is $3 \times 10^3\ Am^{-1}$. The current required to be passed in a solenoid of length $10\ cm$ and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is : [4 marks]

(A) $30\ mA$

(B) $60\ mA$

(C) $3\ A$

(D) $6A$

Correct Answer: (C) $3\ A$

Solution:

Concept:

- Coercivity H_c is the magnitude of the reverse magnetic-field intensity needed to reduce the magnetization of a magnet to zero; its unit is $\mathrm{A\,m^{-1}}$.
- For an ideal solenoid, magnetic-field intensity is $H = nI = NI/\ell$, with N turns over length ℓ and current I. The field must be directed opposite to the original magnetization.

Step 1: Convert the length and set the required field The specified coercive field is

$$H_c = 3 \times 10^3\,\mathrm{A\,m^{-1}}.$$

The solenoid length is $10\,\text{cm} = 0.10\,\text{m}$, and it contains $N = 100$ turns. Thus its turn density is

$$n = \frac{100}{0.10} = 1000\,\text{m}^{-1}.$$

To demagnetize the small magnet inside the solenoid, choose the current polarity so the resulting field opposes its magnetization and set the field magnitude equal to H_c.

Step 2: Calculate the current and check the alternatives Using $nI = H_c$,

$$I = \frac{H_c}{n} = \frac{3 \times 10^3}{1000} = 3\,\text{A}.$$

Equivalently, $I = H_c\ell/N = (3 \times 10^3)(0.10)/100$. This uses magnetic-field intensity directly: inserting an additional μ_0 would confuse H with flux density B. Option A gives only $30\,\text{A m}^{-1}$, option B gives $60\,\text{A m}^{-1}$, and option D gives $6000\,\text{A m}^{-1}$, twice the specified coercivity. Only option C supplies the stated demagnetizing field.

Step 3: Conclusion The current magnitude is $3\,\text{A}$, with polarity chosen to produce a field opposite to the magnetization. Hence, the correct option is (C).

Quick Tip to Solve: Coercivity is an H-field value, so use $I = H_c\ell/N$ directly and convert the solenoid length to metres before substituting.

---

43. In a large building, three are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of electric mains is 220 V. The minimum capacity of the main fuse of the building will be: [4 marks]

(A) 8 A

(B) 10 A

(C) 12 A

(D) 14 A

Correct Answer: (C) 12 A

Solution:

Concept:

- The rated powers of appliances used simultaneously add because each appliance receives the mains potential difference in a parallel domestic supply.
- For total real power P at voltage V, the supply current is $I = P/V$. A fuse must carry this normal current; among the available ratings choose the smallest rating greater than it.

Step 1: Add the simultaneous power demand The fifteen smaller bulbs together consume $15 \times 40 = 600\,\text{W}$. The five larger bulbs consume $5 \times 100 = 500\,\text{W}$, while the fans consume $5 \times 80 = 400\,\text{W}$. Convert the heater rating before adding: $1\,\text{kW} = 1000\,\text{W}$. Thus

$$P_{\text{total}} = 600 + 500 + 400 + 1000 = 2500\,\text{W}.$$

We use all appliances in finding the maximum normal load on the main fuse, rather than the current in any one branch.

Step 2: Calculate the current and compare the ratings At the specified mains voltage, the full-load current is

$$I_{\text{total}} = \frac{2500}{220} = 11.36\,\text{A} \quad \text{(approximately)}.$$

Both 8 A and 10 A are below this current and would not support the stated load. A 12 A fuse exceeds 11.36 A and can carry it. Although a 14 A fuse would also carry the load, it is a larger rating than necessary and therefore is not the minimum among the listed choices. The fuse is selected from current, not by directly comparing its rating with power.

Step 3: Conclusion The building draws approximately $11.36\,\text{A}$; the minimum listed main-fuse rating is $12\,\text{A}$. Hence, the correct option is (C).

Quick Tip to Solve: Add all simultaneous wattages, convert kW to W, divide by the mains voltage, then choose the next higher available fuse rating.

---

44. A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is: [4 marks]

(A) $\frac{1}{6}m$
(B) $\frac{2}{3}m$
(C) $\frac{1}{3}m$
(D) $\frac{1}{2}m$

Correct Answer: (A) $\frac{1}{6}m$

Solution:

Concept:

- At any point of a smooth curve, the local surface is its tangent. If this tangent makes angle θ with the horizontal, $\tan\theta = dy/dx$.
- For a block at rest, gravity has tangential component $mg\sin\theta$ and normal component $mg\cos\theta$. Static friction can balance the tangential component only when $|\tan\theta| \le \mu$.

Step 1: Find the limiting local slope With no other applied force, the normal reaction is $N = mg\cos\theta$. Friction acts up the local slope and its limiting value is μN. At the last position before slipping,

$$mg\sin\theta = \mu mg\cos\theta, \qquad |\tan\theta| = \mu = 0.5.$$

The mass and gravitational acceleration cancel. A more massive block therefore has the same limiting position on this specified surface.

Step 2: Use the curve to obtain the height Differentiate the given cross section, taking the stated coordinates in metres:

$$y = \frac{x^3}{6}, \qquad \frac{dy}{dx} = \frac{x^2}{2}.$$

The no-slip condition becomes $x^2/2 \le 1/2$, so $|x| \le 1$. On the part above ground, the largest allowed height occurs at $x = 1\,\mathrm{m}$, giving

$$y_{\max} = \frac{1^3}{6} = \frac{1}{6}\,\mathrm{m}.$$

Every other listed positive height is greater than this: for the increasing positive branch it requires $x > 1$ and hence a tangent slope greater than 0.5. The friction available could not then maintain equilibrium.

Step 3: Conclusion The maximum height at which static friction can hold the block is $\frac{1}{6}\,\mathrm{m}$. Hence, the correct option is (A).

Quick Tip to Solve: For equilibrium on a curved track, compare the local derivative with the friction coefficient before substituting into the height equation.

---

45. A parallel plate capacitor is made of two circular plates separated by a distance $5\ mm$ and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is $3 \times 10^4\ V/m$ the charge density of the positive plate will be close to: [4 marks]

(A) $6 \times 10^{-7}\ C/m^2$

(B) $3 \times 10^{-7}\ C/m^2$

(C) $3 \times 10^{4}\ C/m^2$

(D) $6 \times 10^{4}\ C/m^2$

Correct Answer: (A) $6 \times 10^{-7}\ C/m^2$

Solution:

Concept:

- For a parallel-plate capacitor completely filled with a linear dielectric, $E = \sigma_{\text{free}}/(K\varepsilon_0)$, where K is its dielectric constant.
- The charge density on a conducting plate is the free surface charge per unit area. Thus $\sigma_{\text{free}} = K\varepsilon_0 E$, with $\varepsilon_0 \simeq 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$.

Step 1: Relate the field to the plate charge The field between oppositely charged large plates in the dielectric is smaller than the vacuum field for the same free plate charge by the factor K. In terms of electric displacement,

$$D = \varepsilon E = K\varepsilon_0 E = \sigma_{\text{free}}.$$

We require the free charge density of the positive conducting plate, not the bound charge density on the dielectric. Since the field itself is supplied, neither the circular plate area nor the plate separation is needed. The separation would enter a calculation of potential difference, $V = Ed$.

Step 2: Substitute the values and identify the scale Using the given relative permittivity and field,

$$\sigma_{\text{free}} = 2.2(8.85 \times 10^{-12})(3 \times 10^4) = 5.841 \times 10^{-7}\,\mathrm{C\,m^{-2}}.$$

This is close to $6 \times 10^{-7}\,\mathrm{C\,m^{-2}}$. The value 3×10^{-7} is approximately the result of omitting the dielectric factor, which is required here. The two options of order $10^4\,\mathrm{C\,m^{-2}}$ mistake the electric-field scale for

a charge-density scale and are many orders of magnitude too large. The units also agree because permittivity multiplied by field gives charge per area.

Step 3: Conclusion The positive plate carries approximately $5.84 \times 10^{-7}\,\mathrm{C\,m^{-2}}$, closest to $6 \times 10^{-7}\,\mathrm{C\,m^{-2}}$. Hence, the correct option is (A).

Quick Tip to Solve: When the field inside the dielectric is already given, obtain free plate charge density directly from $\sigma = K\varepsilon_0 E$.

---

46. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2\ m$ is : [4 marks]

(A) $120\ J/C$

(B) $-120\ J/C$

(C) $-80\ J/C$

(D) $80\ J/C$

Correct Answer: (C) $-80\ J/C$

Solution:

Concept:

- The electrostatic potential change is the negative line integral of the field: $V_A - V_O = -\int_O^A \vec{E} \cdot d\vec{r}$.
- Here the field is along the positive x-direction and depends only on x, so along the x-axis the integrand is $E_x\,dx = 30x^2\,dx$.

Step 1: Set up the potential change with its sign Take a path from the origin to $x = 2\,\mathrm{m}$ along the positive x-axis. On this path, $d\vec{r} = dx\,\hat{i}$, and therefore

$$V_A - V_O = -\int_0^2 30x^2\,dx.$$

The field is positive over this interval, so potential must decrease as we move from O to A. This sign check immediately rules out both positive options. Since the field varies quadratically with position, replacing it by a constant endpoint field would be incorrect.

Step 2: Evaluate the definite integral Integrate first and then substitute the limits:

$$V_A - V_O = -30\left[\frac{x^3}{3}\right]_0^2 = -10(2^3 - 0^3) = -80\,\mathrm{V}.$$

A volt is a joule per coulomb, matching the units used in the answer choices. The magnitude 120 is not the value of this integral: the cubic endpoint factor must be accompanied by the factor $1/3$ from integration. We have calculated the specified order $V_A - V_O$; reversing the two points would reverse the sign and produce $+80\,\mathrm{J/C}$, but that is not the quantity asked. The result is independent of an arbitrary common zero chosen for potential.

Step 3: Conclusion The potential difference is $V_A - V_O = -80\,\mathrm{J/C}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the potential difference in the requested order before integrating, and check that potential falls in the direction of the electric field.

47. A pipe of length $85\ cm$ is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below $1250\ Hz$. The velocity of sound in air is $340\ m/s$. [4 marks]

(A) 12

(B) 8

(C) 6

(D) 4

Correct Answer: (C) 6

Solution:

Concept:

- A pipe closed at one end has a displacement node at the closed end and an antinode at the open end. Its allowed frequencies are $f_n = (2n-1)v/(4L)$, with $n = 1, 2, \ldots$.
- Only odd multiples of the fundamental $f_1 = v/(4L)$ occur; counting all integer multiples would incorrectly include modes incompatible with the end conditions.

Step 1: Calculate the fundamental frequency Convert the pipe length to SI units, $L = 85\,\text{cm} = 0.85\,\text{m}$. For the ideal pipe with the stated length,

$$f_1 = \frac{340}{4(0.85)} = 100\,\text{Hz}.$$

The wavelength of this lowest mode is $4L = 3.4\,\text{m}$. Higher modes add half wavelengths while retaining the closed-end node and open-end antinode, so the frequencies are $100, 300, 500, \ldots$ Hz, not $100, 200, 300, \ldots$.

Step 2: Count the allowed modes below the bound The frequency restriction is strict:

$$(2n-1)100 < 1250, \qquad 2n - 1 < 12.5.$$

The allowed positive odd multipliers are $1, 3, 5, 7, 9, 11$. Thus the six frequencies are

$$100,\ 300,\ 500,\ 700,\ 900,\ 1100\,\text{Hz}.$$

The next allowed mode is $1300\,\text{Hz}$, which is already above 1250 Hz. Counting twelve would count the absent even harmonics; eight and four respectively overcount and undercount the actual sequence. The requested number is the number of distinct natural modes in this frequency interval, not the harmonic index of its highest member.

Step 3: Conclusion There are six allowed natural oscillation frequencies below $1250\,\text{Hz}$. Hence, the correct option is (C).

Quick Tip to Solve: For a pipe with one closed end, list only odd multiples of $v/(4L)$, then count those satisfying the frequency inequality.

48. A particle moves with simple harmonic motion in a straight line. In first τs, after starting from rest it travels a distance a, and in next τs it travels $2a$, in same direction, then: [4 marks]

(A) amplitude of motion is $3a$

(B) time period of oscillations is 8τ

(C) amplitude of motion is $4a$

(D) time period of oscillations is 6τ

Correct Answer: (D) time period of oscillations is 6τ

Solution:

Concept:

- A particle starting from rest in nontrivial SHM starts at an extreme position. Choosing that extreme as $x = A$, its coordinate is $x = A\cos\omega t$.
- Before the first reversal, distance from the starting extreme is $s(t) = A(1-\cos\omega t)$. The time period is $T = 2\pi/\omega$.

Step 1: Express the distances in the two equal intervals Put $\theta = \omega\tau$. The first interval gives

$$A(1-\cos\theta) = a.$$

The particle continues in the same direction through the next interval, so its total distance by time 2τ is $a + 2a = 3a$. Consequently,

$$A(1-\cos 2\theta) = 3a.$$

This use of distance as a directed change is justified by the explicit same-direction condition; it would not remain valid if the particle had crossed a turning point during these intervals.

Step 2: Solve for the phase and compare every statement Eliminate a/A between the equations and use $\cos 2\theta = 2\cos^2\theta - 1$:

$$1-\cos 2\theta = 3(1-\cos\theta),$$

$$2\cos^2\theta - 3\cos\theta + 1 = 0.$$

Writing $c = \cos\theta$, this is $(2c-1)(c-1) = 0$. The root $c = 1$ would make $a = 0$ and is excluded by the travelled distance. Thus $c = 1/2$. The uninterrupted initial motion requires $0 < 2\theta \le \pi$, selecting $\theta = \pi/3$, rather than later phases. Therefore

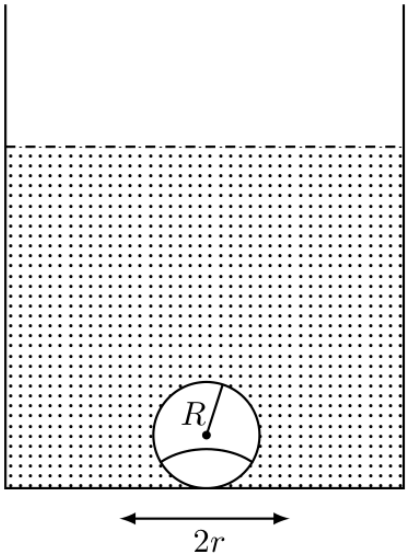
$$\omega\tau = \frac{\pi}{3}, \qquad T = 6\tau, \qquad A = \frac{a}{1-1/2} = 2a.$$

Both amplitude claims, $3a$ and $4a$, fail; the claimed period 8τ also fails. Only the stated period 6τ agrees with both distance conditions.

Step 3: Conclusion The amplitude is $2a$ and the time period is 6τ. Hence, the correct option is (D).

Quick Tip to Solve: When SHM starts from rest, measure displacement from an extreme as $A(1-\cos\omega t)$, and use the cumulative distance at the second time.

49. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom R and making a circular contact of radius r with the bottom of the vessel. If $r << R$ and the surface tension of water is T, value of r just before bubbles detach is: (density of water is ρ_w) [4 marks]



(A) $R^2\sqrt{\frac{\rho_w g}{3T}}$

(B) $R^2\sqrt{\frac{\rho_w g}{6T}}$

(C) $R^2\sqrt{\frac{\rho_w g}{T}}$

(D) $R^2\sqrt{\frac{2\rho_w g}{3T}}$

Correct Answer: (D) $R^2\sqrt{\frac{2\rho_w g}{3T}}$

Solution:

Concept:

- The upward buoyant force is the weight of displaced water: $F_B = \rho_w g V$. For a nearly spherical bubble, $V \simeq 4\pi R^3/3$.
- Surface tension T acts tangentially along the circular line of contact. Its vertical component holds the bubble against the bottom; detachment occurs when buoyancy just balances this retaining force.

Step 1: Resolve the surface-tension force Let θ be the small angle that the tangent to the bubble at its contact circle makes with the horizontal. The radius to that circle and the spherical geometry give $\sin\theta = r/R$. The contact circumference is $2\pi r$, so its total tangential force has magnitude $2\pi r T$. Horizontal components around the circle cancel, while the downward vertical components add:

$$F_T = 2\pi r T \sin\theta = \frac{2\pi T r^2}{R}.$$

Using the full circumference force as a vertical force would incorrectly omit the geometrical projection.

Step 2: Balance the forces at detachment The bubble contains vapour whose weight is negligible compared with the displaced water. Since $r \ll R$, the small contact patch changes the spherical volume only negligibly. At the threshold of detachment,

$$\frac{4\pi R^3}{3}\rho_w g = \frac{2\pi T r^2}{R}.$$

Cancelling 2π and multiplying by R/T gives

$$r^2 = \frac{2\rho_w g R^4}{3T}, \qquad r = R^2\sqrt{\frac{2\rho_w g}{3T}}.$$

The positive root is required because r is a radius. Options (A), (B) and (C) have incorrect numerical factors; only (D) retains both the sphere-volume factor and the contact-line projection.

Step 3: Conclusion The contact radius immediately before detachment is

$$\boxed{r = R^2\sqrt{\frac{2\rho_w g}{3T}}}.$$

Hence, the correct option is (D).

Quick Tip to Solve: For a bubble attached over a small circle, first project the contact-line tension using $\sin\theta = r/R$; then balance it against the buoyancy of the whole bubble.

50. The pressure that has to be applied to the ends of a steel wire of length $10\ cm$ to keep its length constant when its temperature is raised by $100°C$ is: (For steel Young's modulus is $2 \times 10^{11}\ Nm^{-2}$ and coefficient of thermal expansion is $1.1 \times 10^{-5}\ K^{-1}$) [4 marks]

(A) $2.2 \times 10^8\ Pa$

(B) $2.2 \times 10^9\ Pa$

(C) $2.2 \times 10^7\ Pa$

(D) $2.2 \times 10^6\ Pa$

Correct Answer: (A) $2.2 \times 10^8\ Pa$

Solution:

Concept:

- Thermal strain in a freely heated wire is $\alpha\Delta T$, where α is its linear expansion coefficient and ΔT is the temperature rise.
- Young's modulus relates longitudinal stress to elastic strain: $Y = \sigma/\epsilon$. Holding the length fixed requires compression that cancels thermal expansion.

Step 1: Find the expansion that must be prevented The temperature interval is $100\,\mathrm{K}$, numerically the same as a rise of $100°\mathrm{C}$. Thus the free thermal strain would be

$$\epsilon_{\text{th}} = \alpha\Delta T = (1.1 \times 10^{-5})(100) = 1.1 \times 10^{-3}.$$

For the stated length $L = 0.10\,\mathrm{m}$, the corresponding free increase would be $\Delta L = L\epsilon_{\mathrm{th}} = 1.1 \times 10^{-4}\,\mathrm{m}$. The wire must be shortened elastically by this same amount relative to its heated, stress-free length.

Step 2: Apply the compensating compressive stress Equal and opposite thermal and elastic strains give

$$\frac{\sigma}{Y} = \alpha\Delta T, \qquad \sigma = Y\alpha\Delta T.$$

Substituting the supplied constants,

$$\sigma = (2 \times 10^{11})(1.1 \times 10^{-5})(100) = 2.2 \times 10^{8}\,\mathrm{Pa}.$$

The pressure is the magnitude of this compressive stress. Its unit is $\mathrm{N\,m^{-2}}$, equivalent to pascals. The length cancels because both the thermal increase and elastic decrease are proportional to it. Options (B), (C) and (D) differ by incorrect powers of ten and cannot cancel the computed thermal strain.

Step 3: Conclusion The required compressive pressure is $2.2 \times 10^{8}\,\mathrm{Pa}$. Hence, the correct option is (A).

Quick Tip to Solve: For completely restrained thermal expansion, calculate $Y\alpha\Delta T$ directly; neither wire length nor cross-sectional area enters the stress.

---

51. Three rods of Copper, Brass and Steel are welded together to form a Y shaped structure. Area of cross - section of each rod $= 4cm^2$. End of copper rod is maintained at $100°C$ where as ends of brass and steel are kept at $0°C$. Lengths of the copper, brass and steel rods are $46, 13$ and $12\ cms$ respectively. The rods are thermally insulated from surroundings excepts at ends. Thermal conductivities of copper, brass and steel are $0.92, 0.26$ and $0.12\ CGS$ units respectively. Rate of heat flow through copper rod is: [4 marks]

(A) $1.2\ cal/s$

(B) $2.4\ cal/s$

(C) $4.8\ cal/s$

(D) $6.0\ cal/s$

Correct Answer: (C) $4.8\ cal/s$

Solution:

Concept:

- For a uniform insulated rod, the steady heat current is $H = (kA/L)\Delta T$; kA/L is its thermal conductance.
- At a junction in steady state, heat entering equals heat leaving. The brass and steel rods provide two parallel paths from the common junction to the cold ends.

Step 1: Calculate the three thermal conductances Keep all lengths in centimetres and use the supplied CGS conductivities, so the heat currents will be in calories per second. With $A = 4\,\mathrm{cm^2}$,

$$G_{\mathrm{Cu}} = \frac{0.92(4)}{46} = 0.08, \quad G_{\mathrm{Br}} = \frac{0.26(4)}{13} = 0.08, \quad G_{\mathrm{St}} = \frac{0.12(4)}{12} = 0.04.$$

Each conductance has units $\mathrm{cal\,s^{-1}\,K^{-1}}$. The cold-side conductance is therefore $0.08 + 0.04 = 0.12$, since both cold ends are at the same temperature.

Step 2: Determine the junction temperature and copper current Let the junction temperature be T in degrees Celsius. Heat conservation gives

$$0.08(100 - T) = 0.08T + 0.04T.$$

Hence $8 = 0.20T$, so $T = 40^{\circ}\mathrm{C}$. The required copper heat current is

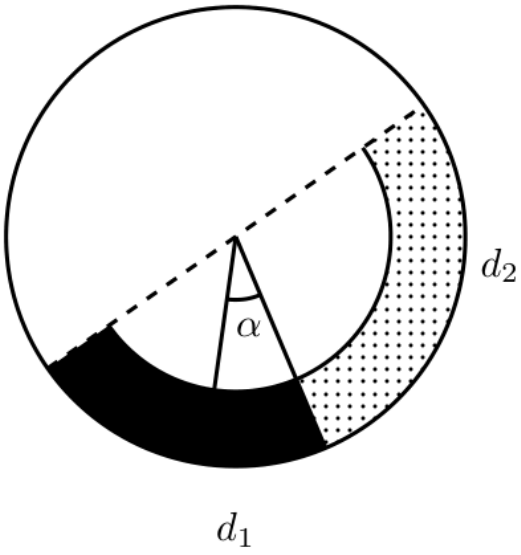
$$H_{\mathrm{Cu}} = 0.08(100 - 40) = 4.8\,\mathrm{cal\,s^{-1}}.$$

As a check, brass carries $0.08(40) = 3.2$ and steel carries $0.04(40) = 1.6\,\mathrm{cal\,s^{-1}}$, adding to 4.8. Thus options (A), (B) and (D) fail the junction heat balance. In particular, the copper temperature drop is 60, not 100, so treating its cold end as being at zero is incorrect.

Step 3: Conclusion The rate of heat flow through copper is $4.8\,\mathrm{cal\,s^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the common junction temperature before calculating a branch current; add kA/L for rods connected to the same cold reservoir.

52. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at center. Radius joining their interface makes an angle α with vertical. Radio $\frac{d_1}{d_2}$ is : [4 marks]



(A) $\frac{1+\sin\alpha}{1-\sin\alpha}$
(B) $\frac{1+\cos\alpha}{1-\cos\alpha}$
(C) $\frac{1+\tan\alpha}{1-\tan\alpha}$
(D) $\frac{1+\sin\alpha}{1-\cos\alpha}$

Correct Answer: (C) $\frac{1+\tan\alpha}{1-\tan\alpha}$

Solution:

Concept:

- In a liquid at rest, pressure increases by ρgh on descending through a vertical height h; arc length is irrelevant to this pressure change.
- The two free surfaces are exposed to the same pressure. At their common interface the liquid pressures must be equal, giving $d_1 h_1 = d_2 h_2$.

Step 1: Read the vertical depths from the circular geometry Let the tube radius be R, and take the centre as vertical coordinate zero, positive upwards. The interface radius makes angle α with the downward vertical, so its height is $-R\cos\alpha$. Each liquid occupies a quarter circle. The left free surface of liquid d_1 is therefore at height $-R\sin\alpha$, whereas the right free surface of liquid d_2 is at $+R\sin\alpha$. The vertical depths of the same interface below these surfaces are

$$h_1 = R(\cos\alpha - \sin\alpha), \qquad h_2 = R(\cos\alpha + \sin\alpha).$$

These are vertical projections, not the equal quarter-circle arc lengths.

Step 2: Equate the pressures at the interface Writing the common free-surface pressure as p_0, continuity of pressure gives

$$p_0 + d_1 g h_1 = p_0 + d_2 g h_2.$$

Cancel p_0, g and R, and divide numerator and denominator by $\cos\alpha$:

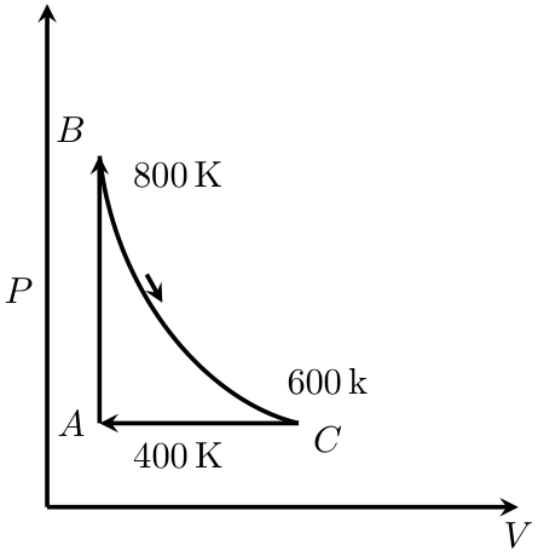
$$\frac{d_1}{d_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1 + \tan\alpha}{1 - \tan\alpha}.$$

The geometry shown has a positive denominator. The expressions involving $1 \pm \sin\alpha$ or $1 \pm \cos\alpha$ use an incorrect surface height, while option (D) also combines incompatible projections. Only option (C) includes both required height differences.

Step 3: Conclusion The density ratio is $d_1/d_2 = (1 + \tan\alpha)/(1 - \tan\alpha)$. Hence, the correct option is (C).

Quick Tip to Solve: For liquids in a curved tube, locate the free surfaces and interface vertically, then use ρh; equal arc lengths do not mean equal pressure heads.

53. One mole of a diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are $400\ K$, $800\ K$ and $600\ K$ respectively. Choose the correct statement : [4 marks]



(A) The change in internal energy in whole cyclic process is $250\ R$.

(B) The change in internal energy in the process CA is $700\ R$.

(C) The change in internal energy in the process AB is - $350\ R$.

(D) The change in internal energy in the process BC is - $500\ R$.

Correct Answer: (D) The change in internal energy in the process BC is - $500\ R$.

Solution:

Concept:

- For one mole of a diatomic ideal gas with rotational modes active and vibrational modes neglected, $C_V = 5R/2$ and $U = C_V T$ up to a reference constant.
- Internal energy is a state function: $\Delta U = nC_V(T_f - T_i)$, independent of the path, and its net change over a complete cycle is zero.

Step 1: Calculate the internal-energy change along BC For the stated one mole, the temperature decreases from $800\,\mathrm{K}$ at B to $600\,\mathrm{K}$ at C. Hence

$$\Delta U_{BC} = \frac{5R}{2}(600 - 800) = -500R.$$

Here the numerical coefficient represents a temperature interval in kelvin; with R in $\mathrm{J\,mol^{-1}\,K^{-1}}$, the resulting energy is in joules. The negative sign means the gas loses internal energy. Although BC is adiabatic, this does not make ΔU zero: zero heat transfer allows work to be supplied from internal energy.

Step 2: Check every remaining statement For the other two processes, apply the same endpoint formula:

$$\Delta U_{AB} = \frac{5R}{2}(800 - 400) = 1000R,$$

$$\Delta U_{CA} = \frac{5R}{2}(400 - 600) = -500R.$$

Thus option (C), which gives $-350R$ for AB, has both an incorrect sign and magnitude, and option (B), which gives $700R$ for CA, is incorrect. Summing the three changes gives

$$1000R - 500R - 500R = 0.$$

This also follows immediately because A is both the initial and final state of the full cycle, excluding option (A). No area calculation on the pressure-volume graph is required for an internal-energy change.

Step 3: Conclusion In process BC, $\Delta U = -500R$, an energy decrease. Hence, the correct option is (D).

Quick Tip to Solve: Use temperature endpoints for ideal-gas internal energy. A closed cycle has zero net ΔU, even when its work and heat are nonzero.

54. When a rubber-band is stretched by a distance x, it exerts restoring force of magnitude $F = ax+bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is :

[4 marks]

(A) $aL^2 + bL^3$

(B) $\frac{1}{2}\left(aL^2 + bL^3\right)$

(C) $\frac{aL^2}{2} + \frac{bL^3}{3}$

(D) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

Correct Answer: (C) $\frac{aL^2}{2} + \frac{bL^3}{3}$

Solution:

Concept:

- For a small extension dx, the work supplied by a slowly applied stretching force is $dW = F(x)\,dx$. Total work is the area under the force-extension graph.
- The restoring force opposes extension. To calculate the positive work done in stretching, integrate its stated magnitude $ax + bx^2$ from zero to the final extension.

Step 1: Set up the work integral At extension x, the external force balancing the rubber band is $F_{\text{ext}} = ax + bx^2$. Starting from the unstretched state means that the lower integration limit is zero. Thus

$$W = \int_0^L (ax + bx^2)\,dx = a\int_0^L x\,dx + b\int_0^L x^2\,dx.$$

The force changes throughout the motion, so multiplying the final force by L would overestimate the area. Similarly, using one-half of the final force times the extension is valid for a purely linear spring, but not for an additional quadratic term.

Step 2: Integrate each power and compare the options Using $\int x\,dx = x^2/2$ and $\int x^2\,dx = x^3/3$,

$$W = \left[\frac{ax^2}{2} + \frac{bx^3}{3}\right]_0^L = \frac{aL^2}{2} + \frac{bL^3}{3}.$$

Both terms have dimensions of energy: a has dimensions of force per length, and b of force per length squared. Option (A) omits both integration denominators. Option (B) incorrectly assigns the quadratic force term the same factor of one-half as the linear term. Option (D) introduces an extra factor of one-half after the integration. Setting $b = 0$ recovers the familiar elastic energy $aL^2/2$, providing a useful consistency check.

Step 3: Conclusion The work done in stretching is $W = aL^2/2 + bL^3/3$. Hence, the correct option is (C).

Quick Tip to Solve: Integrate each force term separately: a force proportional to x^n contributes a factor $1/(n+1)$ to the work.

---

55. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed $\omega\,rad/s$ about the vertical. About the point of suspension: [4 marks]

(A) angular momentum is conserved

(B) angular momentum changes in magnitude but not in direction.

(C) angular momentum changes in direction but not in magnitude.

(D) angular momentum changes both in direction and magnitude.

Correct Answer: (C) angular momentum changes in direction but not in magnitude.

Solution:

Concept:

- Angular momentum about the suspension point is $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$, with $\mathbf{r}$ measured from that point.
- In steady conical motion, the string length and its angle to the vertical are constant, while the horizontal radial and tangential directions rotate with the bob.

Step 1: Evaluate the angular-momentum vector Let the string make angle θ with the downward vertical. In cylindrical unit vectors,

$$\mathbf{r} = l\sin\theta\,\hat{\mathbf{e}}_r - l\cos\theta\,\hat{\mathbf{e}}_z, \qquad \mathbf{v} = \omega l\sin\theta\,\hat{\mathbf{e}}_\phi.$$

Using $\hat{\mathbf{e}}_r \times \hat{\mathbf{e}}_\phi = \hat{\mathbf{e}}_z$ and $\hat{\mathbf{e}}_z \times \hat{\mathbf{e}}_\phi = -\hat{\mathbf{e}}_r$,

$$\mathbf{L} = m\omega l^2\left(\sin^2\theta\,\hat{\mathbf{e}}_z + \sin\theta\cos\theta\,\hat{\mathbf{e}}_r\right).$$

Its vertical component is fixed, but its horizontal component follows the rotating radial direction.

Step 2: Compare its magnitude and direction The two displayed components are perpendicular,

giving

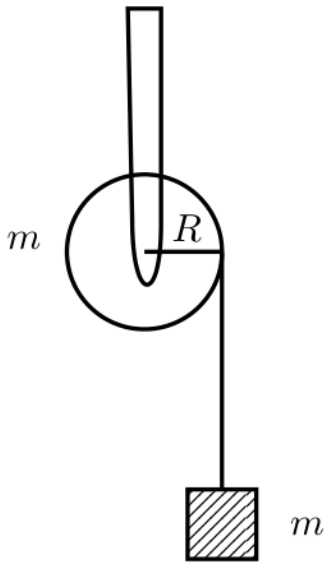
$$|\mathbf{L}| = m\omega l^2 \sin\theta \sqrt{\sin^2\theta + \cos^2\theta} = m\omega l^2 \sin\theta.$$

All factors are constant in the steady motion, so the magnitude remains constant. The horizontal component rotates, however, so the full vector changes direction. Tension acts through the suspension point and has zero torque about it, but the weight has a nonzero horizontal torque, consistent with this changing vector. Thus option (A) confuses conservation of the vertical component with conservation of the whole vector. Options (B) and (D) incorrectly claim a changing magnitude.

Step 3: Conclusion About the suspension point, angular momentum changes in direction but not in magnitude. Hence, the correct option is (C).

Quick Tip to Solve: Conservation of one component does not conserve a vector. For a conical pendulum, inspect the rotating horizontal component of $\mathbf{L}$.

56. A mass $'m'$ is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall or release? [4 marks]



(A) $\frac{2g}{3}$

(B) $\frac{g}{2}$

(C) $\frac{5g}{6}$

(D) g

Correct Answer: (B) $\frac{g}{2}$

Solution:

Concept:

- For the falling mass, Newton's second law is $mg - T = ma$, taking downward as positive and denoting the string tension by T.

- For a uniform thin hollow cylinder about its fixed symmetry axis, $I = mR^2$. The no-slip condition connects the string acceleration and angular acceleration: $a = R\beta$.

Step 1: Relate tension to the cylinder acceleration The cylinder rotates about its supported central axle. Its weight and the axle reaction exert no torque about this axis. The tangential string tension supplies torque TR, so

$$TR = I\beta.$$

With the stated cylinder mass equal to the hanging mass and $I = mR^2$, and using $\beta = a/R$,

$$TR = mR^2\frac{a}{R} = mRa, \qquad T = ma.$$

The radius cancels because a larger torque arm is accompanied by a larger rotational inertia. The cylinder is hollow, so the solid-cylinder value $mR^2/2$ must not be used.

Step 2: Solve the falling-mass equation Substitute the tension into the translational equation:

$$mg - T = ma \quad \Longrightarrow \quad mg - ma = ma.$$

Therefore

$$2ma = mg, \qquad a = \frac{g}{2}.$$

This is downward and smaller than free-fall acceleration because some gravitational energy drives the cylinder's rotation. Option (A), $2g/3$, would correspond to a solid cylinder of equal mass, not the hollow cylinder specified. Option (D), g, would require zero tension and no rotational inertia. Option (C), $5g/6$, does not satisfy either the torque equation or the resulting force balance. As an energy check, the translational and rotational kinetic energies are equal because $I = mR^2$ and the rim speed equals the falling speed.

Step 3: Conclusion The hanging mass falls with acceleration $a = g/2$ downward. Hence, the correct option is (B).

Quick Tip to Solve: Use $a = g/(1 + I/(mR^2))$ for a mass unwinding a fixed-axis rotor; substitute the hollow-cylinder inertia before simplifying.

57. An open glass tube is immersed in mercury in such a way that a length of $8\ cm$ extends above the mercury level. The open end of the tube is then closed and scaled and the tube is raised vertically up by additional $46\ cm$. What will be length of the air column above mercury in the tube now? (Atmospheric pressure $= 76\ cm$ of Hg) [4 marks]

(A) $16\ cm$

(B) $22\ cm$

(C) $38\ cm$

(D) $6\ cm$

Correct Answer: (A) $16\ cm$

Solution:

Concept:

- The pressure of trapped air is found by hydrostatic equilibrium with the external atmosphere. A mercury level higher inside the tube means a lower air pressure inside.
- For a fixed amount of gas at constant temperature, Boyle's law gives $P_1V_1 = P_2V_2$. In a uniform tube of cross-sectional area A, the air volume is $V = Ax$.

Step 1: Identify the initial trapped air Before sealing, the air in the open tube is at atmospheric pressure, and the mercury inside is at the external mercury level. Sealing traps an air column of length $8\,\text{cm}$, so $P_1 = 76\,\text{cm of Hg}$ and $V_1 = 8A$, with lengths measured in centimetres.

Step 2: Express the final pressure in terms of the air length Let the final air-column length be x centimetres. After the tube rises by $46\,\text{cm}$, its sealed top is $8 + 46 = 54\,\text{cm}$ above the external mercury surface. Thus the mercury surface inside is $54 - x$ centimetres above the outside surface. Hydrostatic balance gives

$$P_2 = 76 - (54 - x) = 22 + x \quad \text{cm of Hg.}$$

The rise of the tube is not itself the mercury-level difference; the air expands and moves the internal meniscus.

Step 3: Apply Boyle's law and select the physical root Cancelling the constant area from Boyle's law,

$$76 \times 8 = (22 + x)x.$$

$$x^2 + 22x - 608 = 0 = (x - 16)(x + 38).$$

A length must be positive, so $x = 16\,\text{cm}$, while $-38\,\text{cm}$ is rejected. At this length the internal mercury rise is $38\,\text{cm}$, making the final pressure $38\,\text{cm of Hg}$; the pressure halves and the air length doubles. Options (B), (C) and (D) fail this pressure-volume relation.

Step 4: Conclusion The air column is $16\,\text{cm}$ long. Hence, the correct option is (A).

Quick Tip to Solve: Write the top height as air length plus the mercury-level rise before applying Boyle's law; tube displacement and mercury rise are different.

58. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is : [4 marks]

(A) $\sqrt{\frac{GM}{R}}$

(B) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

(C) $\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

(D) $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

Correct Answer: (D) $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

Solution:

Concept:

- The equally spaced particles occupy the four vertices of a square inscribed in the circle. The distance to an adjacent particle is $\sqrt{2}R$, and the distance to the opposite particle is $2R$.
- Each pair attracts with force GM^2/r^2. Symmetry cancels tangential components, leaving a net inward force Mv^2/R.

Step 1: Resolve the two adjacent attractions Consider one vertex of the rotating square. Each adjacent mass exerts a force

$$F_a = \frac{GM^2}{(\sqrt{2}R)^2} = \frac{GM^2}{2R^2}.$$

Each of these forces makes an angle of 45° with the inward radius. Their tangential components are equal and opposite. Their combined radial component is

$$2F_a \cos 45^\circ = \frac{GM^2}{\sqrt{2}R^2}.$$

It would be incorrect to add both adjacent forces as if both acted directly along the radius.

Step 2: Add the opposite attraction The diagonally opposite mass lies on the same diameter and pulls directly inward. Its contribution is

$$F_o = \frac{GM^2}{(2R)^2} = \frac{GM^2}{4R^2}.$$

Therefore the total centripetal force is

$$F = \frac{GM^2}{4R^2}(1+2\sqrt{2}).$$

All four particles have the same force magnitude and hence the same speed by symmetry.

Step 3: Equate the force to the centripetal requirement Using the orbit radius R, rather than the square side, gives

$$\frac{Mv^2}{R} = \frac{GM^2}{4R^2}(1+2\sqrt{2}),$$

$$v^2 = \frac{GM}{4R}(1+2\sqrt{2}), \qquad v = \frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}.$$

Option (A) misses the geometric factor. Option (B) does not include the correct opposite-particle term,

while option (C) is twice the required speed and would demand four times the available force.

Step 4: Conclusion The speed is $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$. Hence, the correct option is (D).

Quick Tip to Solve: For a regular polygon of gravitating masses, resolve every pair force along the inward radius before using Mv^2/R.

---

59. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is: [4 marks]

(A) $2gH = n^2u^2$

(B) $gH = (n - 2)^2u^2$

(C) $2gH = nu^2(n - 2)$

(D) $gH = (n - 2)u^2$

Correct Answer: (C) $2gH = nu^2(n - 2)$

Solution:

Concept:

- With upward displacement positive, uniform gravitational acceleration is $-g$, so $y = ut - \frac{1}{2}gt^2$. The ground is at displacement $-H$ from the launch point.
- The highest point is reached when vertical velocity vanishes: $0 = u - gt_{\text{up}}$. The specified total flight time is n times this ascent time.

Step 1: Calculate the ascent time and the total flight time At the highest point, the initial upward velocity has been reduced to zero by gravity. Thus

$$t_{\text{up}} = \frac{u}{g}, \qquad T = nt_{\text{up}} = \frac{nu}{g}.$$

The quantity T includes both the upward part and the entire downward journey to the ground. It is not only the descent time from the highest point. Since the launch point is above the ground, T must exceed the return time $2u/g$, so a positive tower height requires $n > 2$.

Step 2: Apply the displacement equation to the ground Take the origin at the top of the tower. The displacement at the instant of impact is negative, even though the height H itself is positive. Therefore

$$-H = uT - \frac{1}{2}gT^2.$$

Substitute the total time:

$$-H = \frac{nu^2}{g} - \frac{n^2u^2}{2g}.$$

Multiplying by $2g$ and rearranging gives

$$2gH = n^2u^2 - 2nu^2 = nu^2(n - 2).$$

Both sides have dimensions of speed squared, as required.

Step 3: Check the options against a limiting case When the tower height tends to zero, the projectile returns to its launch level in twice the ascent time: $n = 2$. The derived expression correctly vanishes then. Option (A) does not vanish at $n = 2$. Options (B) and (D) omit the factor from substituting the full flight time into the quadratic displacement equation. Only option (C) reproduces the full result, not merely the zero-height limit.

Step 4: Conclusion The required relation is $\boxed{2gH = nu^2(n-2)}$. Hence, the correct option is (C).

Quick Tip to Solve: Place the origin at the launch point and use ground displacement $-H$; this keeps the upward launch speed and gravitational sign consistent.

---

60. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it? [4 marks]

(A) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.

(B) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.

(C) A meter scale.

(D) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.

Correct Answer: (D) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.

Solution:

Concept:

- The last quoted decimal place indicates the measurement resolution intended by the reported value. Writing $3.50\,\text{cm}$ retains a hundredth of a centimetre, including its significant final zero.
- A screw gauge has least count equal to pitch divided by circular-scale divisions. A direct vernier has least count $1\,\text{MSD} - 1\,\text{VSD}$.

Step 1: Determine the intended resolution The length is quoted to $0.01\,\text{cm}$, which equals $0.1\,\text{mm}$. Under the usual reporting convention, the instrument should have this least count. The zero after the 5 is significant: $3.50\,\text{cm}$ communicates finer resolution than $3.5\,\text{cm}$. A more precise reading could be rounded, but the question identifies the instrument from the precision with which the unrounded measurement is conventionally recorded.

Step 2: Compute the vernier least count The main scale has ten divisions per centimetre, so

$$1\,\text{MSD} = 0.1\,\text{cm}.$$

Since ten vernier divisions equal nine main-scale divisions,

$$1\,\text{VSD} = \frac{9}{10}(0.1) = 0.09\,\text{cm}.$$

Hence

$$\text{LC} = 0.1 - 0.09 = 0.01\,\text{cm}.$$

This matches the last retained decimal place of the recorded length.

Step 3: Compare the other instruments For option (A), the screw-gauge least count is $1/100 = 0.01\,\text{mm} = 0.001\,\text{cm}$; its reading is normally recorded to three decimal places in centimetres. For option (B), it is $1/50 = 0.02\,\text{mm} = 0.002\,\text{cm}$, again finer than the stated resolution. A conventional metre scale in option (C) resolves $1\,\text{mm} = 0.1\,\text{cm}$, which is coarser. The vernier in (D) is therefore the instrument matching the quoted precision.

Step 4: Conclusion The intended instrument is the vernier calliper with least count $0.01\,\text{cm}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert every instrument's least count into the same unit as the reading, and preserve trailing zeros when interpreting precision.

Chemistry (120 marks)

61. Resistance of 0.2 M solution of an electrolyte is 50 Ω. The specific conductance of the solution is 1.4 S m^{-1}. The resistance of 0.5 M solution of the same electrolyte is 280 Ω. The molar conductivity of 0.5 M solution of the electrolyte in S m^2 mol^{-1} is : [4 marks]

(A) 5×10^3

(B) 5×10^2

(C) 5×10^{-4}

(D) 5×10^{-3}

Correct Answer: (C) 5×10^{-4}

Solution:

Concept:

- Conductivity κ, resistance R, and cell constant l/A are related by $\kappa = (l/A)/R$. Using the same cell keeps l/A constant.
- Molar conductivity in SI units is $\Lambda_m = \kappa/c$, with concentration c expressed in $\text{mol}\,\text{m}^{-3}$, not $\text{mol}\,\text{L}^{-1}$.

Step 1: Find the cell constant from the first measurement The first solution has resistance $R_1 = 50\,\Omega$ and conductivity $\kappa_1 = 1.4\,\text{S}\,\text{m}^{-1}$. Therefore

$$\frac{l}{A} = \kappa_1 R_1 = 1.4 \times 50 = 70\,\text{m}^{-1}.$$

The resistance alone is not a conductivity; it must be combined with the geometry of the conductivity cell. The stated data let us calibrate that geometry without knowing the electrode separation or area individually.

Step 2: Obtain the conductivity of the second solution For the second concentration, the same cell

has $R_2 = 280\,\Omega$. Consequently

$$\kappa_2 = \frac{70}{280} = 0.25\,\mathrm{S\,m^{-1}}.$$

Equivalently, $\kappa_2/\kappa_1 = R_1/R_2$. Conductance is inversely proportional to resistance, so multiplying by the resistance ratio in the opposite direction would give the wrong result.

Step 3: Convert concentration and calculate molar conductivity The second concentration is $0.5\,\mathrm{mol\,L^{-1}}$. Since $1\,\mathrm{m^3} = 1000\,\mathrm{L}$,

$$c_2 = 0.5 \times 1000 = 500\,\mathrm{mol\,m^{-3}}.$$

Thus

$$\Lambda_m = \frac{0.25}{500} = 5 \times 10^{-4}\,\mathrm{S\,m^2\,mol^{-1}}.$$

The unit follows from $(\mathrm{S\,m^{-1}})/(\mathrm{mol\,m^{-3}})$. Options (A) and (B) have large positive powers inconsistent with this division; option (D) is ten times the calculated value. Only (C) uses the measured cell constant and the SI concentration correctly.

Step 4: Conclusion The molar conductivity is $5 \times 10^{-4}\,\mathrm{S\,m^2\,mol^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use κR to transfer the cell calibration, then convert molarity to $\mathrm{mol\,m^{-3}}$ before computing SI molar conductivity.

62. Consider separate solutions of 0.500 M C_2H_5OH(aq), 0.100 M $Mg_3(PO_4)_2$(aq), 0.250 M KBr(aq) and 0.125 M Na_3PO_4(aq) at 25°C. Which statement is true about these solutions, assuming all salts to be strong electrolytes? [4 marks]

(A) 0.125 M Na_3PO_4(aq) has the highest osmotic pressure.

(B) 0.500 M C_2H_5OH(aq) has the highest osmotic pressure

(C) They all have the same osmotic pressure

(D) 0.100 M $Mg_3(PO_4)_2$(aq) has the highest osmotic pressure.

Correct Answer: (C) They all have the same osmotic pressure

Solution:

Concept:

- For an ideal dilute solution, osmotic pressure is $\pi = iCRT$, where i is the van't Hoff factor and C is the formula-unit molar concentration.
- At the same temperature, compare iC. A nonelectrolyte has $i = 1$, while each completely dissociated salt contributes its total number of ions per formula unit.

Step 1: Count the dissolved particles from each solute Ethanol, C_2H_5OH, is a molecular nonelectrolyte and remains as one solute particle per molecule, so its factor is 1. Complete dissociation of the salts gives

$$\mathrm{Mg_3(PO_4)_2 \longrightarrow 3\,Mg^{2+} + 2\,{PO_4}^{3-}},$$

$$\mathrm{KBr \longrightarrow K^+ + Br^-}, \qquad \mathrm{Na_3PO_4 \longrightarrow 3\,Na^+ + {PO_4}^{3-}}.$$

Their factors are 5, 2 and 4, respectively. Ionic charge does not multiply the count again: a phosphate ion is one particle even though its charge is -3.

Step 2: Calculate the effective particle concentrations The products controlling osmotic pressure are

$$\begin{aligned} \text{Ethanol:}\ & iC = 1(0.500) = 0.500, \\ \mathrm{Mg_3(PO_4)_2}:\ & iC = 5(0.100) = 0.500, \\ \mathrm{KBr}:\ & iC = 2(0.250) = 0.500, \\ \mathrm{Na_3PO_4}:\ & iC = 4(0.125) = 0.500. \end{aligned}$$

Every value is in $\mathrm{mol\,L^{-1}}$ of independently moving solute particles. The common temperature means the remaining factors R and T are also identical.

Step 3: Compare the osmotic pressures Since all four values of iC are equal, multiplying them by the common RT gives the same osmotic pressure. Comparing just the listed molarities would incorrectly choose ethanol, because it ignores dissociation of the salts. Choosing sodium phosphate or magnesium phosphate merely because it produces several ions also fails: its lower formula-unit concentration exactly compensates for the larger number of particles. Thus none of the three solutions singled out in (A), (B) and (D) exceeds the others.

Step 4: Conclusion All four solutions have the same osmotic pressure under the stated complete-dissociation assumption. Hence, the correct option is (C).

Quick Tip to Solve: For equal-temperature osmotic-pressure comparisons, make a quick list of iC, counting ions rather than charges.

63. For the estimation of nitrogen, 1.4 g of organic compound was digested by Kjeldahl method and the evolved ammonia was absorbed in 60 mL of M/10 sulphuric acid. The unreacted acid required 20 ml of M/10 sodium hydroxide for complete neutralization. The percentage of nitrogen in the compound is: [4 marks]

(A) 3%

(B) 5%

(C) 6%

(D) 10%

Correct Answer: (D) 10%

Solution:

Concept:

- Kjeldahl digestion converts nitrogen into ammonia. Each mole of $\mathrm{NH_3}$ contains one mole of nitrogen atoms and consumes one mole of acid equivalents.
- Sulphuric acid is diprotic: $\mathrm{H_2SO_4 + 2\,NaOH \longrightarrow Na_2SO_4 + 2\,H_2O}$. Back titration measures the acid left over, which must be subtracted from the initial acid equivalents.

Step 1: Calculate the initial acid capacity The concentration $M/10$ means $0.1\,\mathrm{mol\,L^{-1}}$. The initial

amount of sulphuric acid is

$$n(\mathrm{H_2SO_4}) = 0.060 \times 0.1 = 0.0060\,\mathrm{mol}.$$

Each mole can neutralize two moles of ammonia:

$$\mathrm{H_2SO_4} + 2\,\mathrm{NH_3} \longrightarrow (\mathrm{NH_4})_2\mathrm{SO_4}.$$

Hence the initial acid capacity is $2 \times 0.0060 = 0.0120\,\mathrm{mol}$ of ammonia, or equivalently the same amount of hydrogen-ion equivalents.

Step 2: Subtract the unreacted acid equivalents The sodium hydroxide used in the back titration is

$$n(\mathrm{NaOH}) = 0.020 \times 0.1 = 0.0020\,\mathrm{mol}.$$

As hydroxide is monovalent, this represents $0.0020\,\mathrm{mol}$ of acid equivalents remaining after ammonia absorption. Therefore the acid equivalents consumed by ammonia are

$$0.0120 - 0.0020 = 0.0100\,\mathrm{mol}.$$

Thus the evolved ammonia, and hence nitrogen atoms in the sample, amount to $0.0100\,\mathrm{mol}$. One must subtract equivalents, rather than directly subtracting moles of two acids or bases of different basicity.

Step 3: Convert the nitrogen amount into percentage Using the molar mass of nitrogen atoms,

$$m(\mathrm{N}) = 0.0100 \times 14 = 0.140\,\mathrm{g}.$$

The original organic sample weighed $1.4\,\mathrm{g}$, so

$$\%\mathrm{N} = \frac{0.140}{1.4} \times 100 = 10\%.$$

Options (A), (B) and (C) would imply nitrogen masses of 0.042, 0.070 and $0.084\,\mathrm{g}$, respectively, inconsistent with the ammonia amount found by back titration. In particular, using sulphuric acid as monoprotic loses its factor of two and does not represent complete neutralization.

Step 4: Conclusion The compound contains 10% nitrogen by mass. Hence, the correct option is (D).

Quick Tip to Solve: In Kjeldahl back titrations, convert the initial acid and leftover base to equivalents first; their difference equals the moles of ammonia.

64. In which of the following reactions H_2O_2 acts as a reducing agent?

1. $H_2O_2 + 2H^+ + 2e^- \rightarrow 2H_2O$

2. $H_2O_2 - 2e^- \rightarrow O_2 + 2H^+$

3. $H_2O_2 + 2e^- \rightarrow 2OH^-$

4. $H_2O_2 + 2OH^- - 2e^- \rightarrow O_2 + 2H_2O$ [4 marks]

(A) 1, 2

(B) 3, 4

(C) 1, 3

(D) 2, 4

Correct Answer: (D) 2, 4

Solution:

Concept:

- A reducing agent donates electrons and is itself oxidised; its oxidation number increases during the reaction.
- In hydrogen peroxide, H_2O_2, oxygen has oxidation number -1. It becomes 0 in O_2 and -2 in water or hydroxide.

Step 1: Check the two reactions forming water or hydroxide In reaction 1, peroxide accepts two electrons and produces water. Oxygen changes from -1 to -2, so peroxide undergoes reduction and acts as an oxidising agent. In reaction 3 it again accepts two electrons, now producing hydroxide ions. The oxidation number of oxygen in OH^- is also -2, so reaction 3 likewise shows oxidising behaviour. Neither of these can belong to the required pair.

Step 2: Check the reactions forming oxygen In reaction 2, the written subtraction of electrons represents electron loss. Its equivalent oxidation half-reaction is

$$H_2O_2 \longrightarrow O_2 + 2\,H^+ + 2\,e^-.$$

Oxygen rises from -1 to 0. In alkaline reaction 4 the corresponding form is

$$H_2O_2 + 2\,OH^- \longrightarrow O_2 + 2\,H_2O + 2\,e^-.$$

Here too peroxide loses electrons and is oxidised. Thus both 2 and 4 describe peroxide reducing some other species.

Step 3: Compare the offered pairs Option (A) includes reaction 1, which is reduction of peroxide. Option (B) includes reaction 3, also reduction of peroxide. Option (C) contains only those two oxidising-agent examples. Only option (D) groups the two electron-loss half-reactions.

Step 4: Conclusion H_2O_2 acts as a reducing agent in reactions 2 and 4. Hence, the correct option is (D).

Quick Tip to Solve: For peroxide, inspect the oxygen product: formation of O_2 means peroxide is oxidised and therefore acts as the reducing agent.

65. For the reaction SO_2 (g) + $\frac{1}{2}O_2(g) \leftrightharpoons$ SO_3(g).
if $K_P = K_C(RT)^x$ where the symbols have usual meaning then the value of x is: (assuming ideality)

[4 marks]

(A) -1
(B) -1/2
(C) 1/2
(D) 1

Correct Answer: (B) -1/2

Solution:

Concept:

- For ideal gases, $K_p = K_c(RT)^{\Delta n_g}$, where Δn_g is the sum of gaseous product coefficients minus the sum of gaseous reactant coefficients.
- Stoichiometric coefficients, including fractional coefficients, must be used exactly as the equilibrium equation is written.

Step 1: Count gaseous coefficients on each side The equilibrium is

$$\mathrm{SO_2(g)} + \frac{1}{2}\,\mathrm{O_2(g)} \rightleftharpoons \mathrm{SO_3(g)}.$$

There is one mole of gaseous product per reaction as written. The reactant side contains one mole of sulfur dioxide and half a mole of oxygen. Thus the total gaseous reactant coefficient is $1 + \frac{1}{2} = \frac{3}{2}$. No solid or liquid needs to be excluded in this particular reaction because every species is gaseous.

Step 2: Substitute the change in gaseous amount Calculate the signed difference:

$$\Delta n_g = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}.$$

Therefore the pressure and concentration constants obey

$$K_p = K_c(RT)^{-1/2}.$$

Comparing its exponent with the given expression identifies $x = -\frac{1}{2}$. The negative sign follows because the product side has fewer gaseous moles.

Step 3: Eliminate the alternative exponents The value -1 would follow from doubling the entire chemical equation, which would also square both equilibrium constants and describe a different normalisation. The positive exponents in (C) and (D) use the wrong sign for the product-minus-reactant difference. Counting only the number of distinct gas species would also be incorrect; coefficients measure the amounts that enter the equilibrium law.

Step 4: Conclusion The required exponent is $x = -\frac{1}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Compute gaseous product coefficients minus reactant coefficients before comparing K_p and K_c; retain the fraction in the equation.

66. The equivalent conductance of NaCl at concentration C and at infinite dilution are λ_C and λ_∞, respectively. The correct relationship between λ_C and λ_∞ is given as: (where the constant B is positive) [4 marks]

(A) $\lambda_C = \lambda_\infty + (B)C$

(B) $\lambda_C = \lambda_\infty - (B)C$

(C) $\lambda_C = \lambda_\infty - (B)\sqrt{C}$

(D) $\lambda_C = \lambda_\infty + (B)\sqrt{C}$

Correct Answer: (C) $\lambda_C = \lambda_\infty - (B)\sqrt{C}$

Solution:

Concept:

- For a strong electrolyte at low concentration, the square-root law is $\lambda_C = \lambda_\infty - B\sqrt{C}$, with positive constant B.
- Equivalent conductance approaches its limiting value from below on dilution, because interactions between oppositely charged ions weaken.

Step 1: Identify the electrolyte and applicable dependence Sodium chloride is a strong electrolyte: it is substantially dissociated into Na^+ and Cl^- in dilute aqueous solution. Its concentration dependence is therefore governed by the strong-electrolyte square-root relation, rather than by a linear term in concentration. The ionic atmosphere and associated electrophoretic and relaxation effects reduce the mobility of ions at a finite concentration.

Step 2: Apply the limiting-conductance relation Writing the dilute-solution law in the notation of the question gives

$$\lambda_C = \lambda_\infty - B\sqrt{C}.$$

Since $B > 0$ and $C > 0$, the correction is negative and $\lambda_C < \lambda_\infty$. As $C \to 0$, the square-root term vanishes and the expression correctly gives $\lambda_C \to \lambda_\infty$. This agrees with the physical increase of conductance per equivalent upon dilution.

Step 3: Check the form and sign of every option Options (A) and (D) have positive corrections. They would make conductance exceed the infinite-dilution limit at finite concentration, contrary to strong-electrolyte behaviour. Option (B) has the correct negative sign but uses C, whereas the leading dilute-solution dependence is on $\sqrt{C}$. Option (C) alone has both the negative sign and the required square-root concentration dependence. The relation is understood in its dilute-solution range, rather than as an exact law at arbitrary concentration.

Step 4: Conclusion $\lambda_C = \lambda_\infty - B\sqrt{C}$. Hence, the correct option is (C).

Quick Tip to Solve: For a strong electrolyte, remember both checks: the finite-concentration value lies below the limit and its leading correction varies as $\sqrt{C}$.

67. For complete combustion of ethanol, $C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$ the amount of heat produced as measured in bomb calorimeter, is 1364.47 kJ mol^{-1} at 25°C. Assuming ideality the Enthalpy of combustion, $\Delta_C H$, for the reaction will be : (R = 8.314 kJ mol^{-1}) [4 marks]

(A) –1460.50 kJ mol^{-1}

(B) – 1350.50 kJ mol^{-1}

(C) – 1366.95 kJ mol^{-1}

(D) – 1361.95 kJ mol^{-1}

Correct Answer: The supplied gas-constant value has incompatible units; the question was dropped in the official answer key.

Solution:

Concept:

- A bomb calorimeter measures constant-volume heat: $q_v = \Delta U$. Heat produced by combustion corresponds to negative ΔU for the reacting system.
- For an ideal-gas reaction, $\Delta H = \Delta U + \Delta n_g RT$, with R expressed in energy per mole per kelvin and T in kelvin.

Step 1: Obtain the internal-energy change and gaseous mole change The measured heat release gives

$$\Delta U = -1364.47\,\text{kJ mol}^{-1}.$$

For the written reaction, liquid ethanol and liquid water do not contribute to the gaseous mole count. There are two moles of gaseous carbon dioxide as products and three moles of gaseous oxygen as reactants, so

$$\Delta n_g = 2 - 3 = -1, \qquad T = 25 + 273.15 = 298.15\,\text{K}.$$

Thus enthalpy must be more negative than internal energy for a physically consistent gas constant.

Step 2: Check the stated gas-constant datum The question states $R = 8.314\,\text{kJ mol}^{-1}$, which lacks the required inverse-kelvin dimension and has the wrong energy scale for the universal gas constant. It cannot be inserted as a dimensionally valid gas constant. Even if a missing K^{-1} were provisionally supplied while retaining its stated kilojoule magnitude, the correction would be $-8.314(298.15) = -2478.8191\,\text{kJ mol}^{-1}$, giving $-3843.2891\,\text{kJ mol}^{-1}$, which is none of the options.

Step 3: Distinguish the conventional numerical result With the standard physical constant $R = 0.008314\,\text{kJ mol}^{-1}\,\text{K}^{-1}$, the calculation would instead be

$$\Delta H = -1364.47 - 0.008314(298.15) = -1366.9488\,\text{kJ mol}^{-1}.$$

This rounds to the numerical value in (C). Option (D) reverses the correction sign, and (A) and (B) give neither correction. However, this conventional result requires repairing the supplied datum and must not be presented as an unconditional answer to its literal wording.

Step 4: Conclusion The stated value and units of R do not define a physically consistent calculation, so no option is unconditionally correct as printed. The official answer key dropped this question and awarded bonus credit to all. With the standard gas constant, the conventional result is $-1366.95\,\text{kJ mol}^{-1}$, the value shown in option (C).

Quick Tip to Solve: For bomb-calorimeter problems, first attach a negative sign to released heat, then count gaseous moles and check the energy and temperature units of R.

68. The ratio of masses of oxygen and nitrogen in a particular gaseous mixture is 1 : 4. The ratio of number of their molecule is: [4 marks]

(A) 1: 8

(B) 3 : 16

(C) 1 : 4

(D) 7 : 32

Correct Answer: (D) 7 : 32

Solution:

Concept:

- For a substance of mass m and molar mass M, the number of molecules is $N = (m/M)N_A$. Hence molecule ratios equal mole ratios.
- Oxygen and nitrogen gases are diatomic: O_2 has molar mass $32\,g\,mol^{-1}$ and N_2 has molar mass $28\,g\,mol^{-1}$.

Step 1: Choose masses in the stated ratio Let the oxygen mass be m and the nitrogen mass be $4m$, measured in the same mass unit. The absolute size of the sample is irrelevant because the common mass factor cancels. The number of molecules is proportional to mass divided by molar mass, not to the mass alone: equal masses of different gases generally contain different numbers of molecules.

Step 2: Convert the mass ratio to a molecule ratio Using the molecular molar masses gives

$$\frac{N_{O_2}}{N_{N_2}} = \frac{(m/32)N_A}{(4m/28)N_A} = \frac{28}{128} = \frac{7}{32}.$$

Equivalently, take $1\,g$ of oxygen and $4\,g$ of nitrogen. Their amounts are $1/32$ and $4/28 = 1/7$ mole, respectively. Multiplying their ratio by the common denominator yields $7 : 32$, in oxygen-to-nitrogen order.

Step 3: Check the other choices Option (C), $1 : 4$, simply repeats the mass ratio and ignores the unequal molar masses. Options (A) and (B) would require different molecular masses or an incorrect division by the given masses. Because oxygen molecules are heavier than nitrogen molecules, the oxygen fraction by molecule count must be slightly smaller than its mass-ratio value $1/4$. The obtained fraction $7/32$ passes this check, whereas a direct use of the mass ratio does not.

Step 4: Conclusion The ratio of the number of oxygen molecules to nitrogen molecules is $7 : 32$. Hence, the correct option is (D).

Quick Tip to Solve: Turn each gas mass into moles first. In a molecule ratio, Avogadro's constant cancels immediately.

69. The equation which is balanced and represents the correct product(s) is : [4 marks]

(A) $[Mg\ (H_2O)_6\ ]^{2+} + (EDTA)^{4-} \xrightarrow{excess\ NaOH} [Mg\ (EDTA)\ ]^{2+} + 6H_2O$

(B) $CuSO_4 + KCN \rightarrow K_2\ [Cu\ (CN)_4] + K_2SO_4$

(C) $Li_2O + 2KCl \rightarrow 2LiCl + K_2O$

(D) $[CoCl(NH_3)_5]^+ + 5H^+ \rightarrow Co^{2+} + 5NH_4^+ + Cl^-$

Correct Answer: (D) $[CoCl(NH_3)_5]^+ + 5H^+ \rightarrow Co^{2+} + 5NH_4^+ + Cl^-$

Solution:

Concept:

- A valid reaction must conserve every element and total electric charge, and must describe chemically plausible products under its stated conditions.
- Coordinated ammonia is protonated to NH_4^+ in acid when the complex dissociates; ligand charge determines the oxidation state of the central metal.

Step 1: Check the magnesium and cyanide equations In (A), combining Mg^{2+} with $EDTA^{4-}$ must give $[Mg(EDTA)]^{2-}$, not the printed positively charged complex. The left-hand charge is -2 whereas the printed product charge is $+2$, so (A) fails charge conservation. In (B), the single KCN supplies only one potassium and one cyanide, while the products require four potassium atoms and four cyanide ligands. Copper(II) also undergoes redox chemistry with cyanide. The printed equation is neither balanced nor a correct description.

Step 2: Check the oxide exchange and the cobalt complex Option (C) conserves atoms formally, but the proposed salt exchange does not represent the correct reaction. Stable lithium oxide does not simply exchange oxygen for chloride with potassium chloride to produce the more reactive potassium oxide. A balanced arrangement of formulae alone is insufficient. For (D), the complex $[CoCl(NH_3)_5]^+$ has neutral ammonia and one Cl^- ligand, so cobalt is in the $+2$ oxidation state.

Step 3: Verify the valid equation completely Acid dissociation and protonation are represented by

$$[CoCl(NH_3)_5]^+ + 5\,H^+ \longrightarrow Co^{2+} + 5\,NH_4^+ + Cl^-.$$

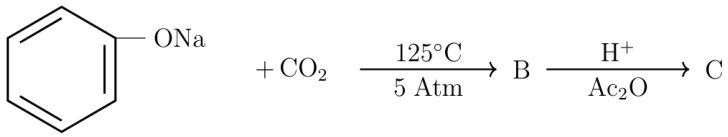
Cobalt and chloride each appear once on each side; five nitrogen atoms and twenty hydrogen atoms also balance. Total charge is $+1 + 5 = +6$ on the left and $+2 + 5 - 1 = +6$ on the right. Cobalt remains in oxidation state $+2$, so there is no missing electron-transfer reagent. All five ammonia ligands become ammonium ions in the acidic medium.

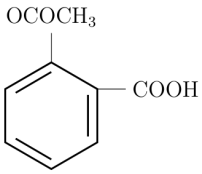
Step 4: Conclusion The cobalt-complex dissociation in option (D) is balanced and gives the correct products. Hence, the correct option is (D).

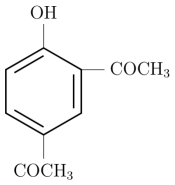
Quick Tip to Solve: Check atoms, net charge and chemical feasibility separately. Formal atom balance cannot by itself validate a proposed salt-exchange reaction.

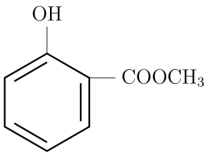
70. Sodium phenoxide when heated with CO_2 under pressure at $125°C$ yields a product which on acetylation products C.

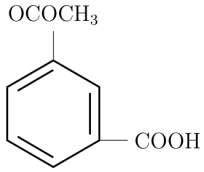
The major product C would be [4 marks]

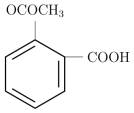


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- In the Kolbe–Schmitt reaction, sodium phenoxide reacts with CO_2 under pressure to introduce a carboxyl group mainly ortho to the phenoxide oxygen. Acidification gives salicylic acid.
- Acetic anhydride, $(CH_3CO)_2O$, acetylates a phenolic hydroxyl group to $-OCOCH_3$. This esterification retains the ring position and the carboxylic acid group.

Step 1: Identify the product of carboxylation The starting material is sodium phenoxide, C_6H_5ONa. Its oxygen donates electron density to the aromatic ring, making the ortho and para positions reactive toward carboxylation. Under the stated conditions, the major Kolbe product has the carboxyl group next to the oxygen-bearing carbon. After the acidic treatment shown over the second arrow, its acid form is salicylic acid, namely 2-hydroxybenzoic acid. Thus the two substituents that must be followed are an ortho pair: $-OH$ and $-COOH$. Neither a ring ketone nor a methyl ester is formed in this first transformation.

Step 2: Acetylate the phenolic oxygen and compare the structures Acetic anhydride supplies an acetyl group to the phenolic oxygen. The transformation is

$$HO-C_6H_4-COOH + (CH_3CO)_2O \longrightarrow CH_3COO-C_6H_4-COOH + CH_3COOH.$$

The product is 2-acetoxybenzoic acid, commonly called acetylsalicylic acid or aspirin. The $-COOH$ group remains intact, while the neighboring $-OH$ becomes $-OCOCH_3$. Option (A) shows precisely this ortho arrangement. Option (B) retains the phenolic hydroxyl and introduces two carbon-attached acetyl groups, which is not oxygen acetylation. Option (C) contains $-COOCH_3$, a methyl ester

that requires a methylating or methanol-based esterification step absent here. Option (D) places the carboxyl and acetoxy substituents meta rather than ortho, contrary to the major carboxylation product.

Step 3: Conclusion Product C is acetylsalicylic acid (aspirin), with adjacent $-OCOCH_3$ and $-COOH$ groups. Hence, the correct option is (A).

Quick Tip to Solve: Follow the oxygen substituent separately: Kolbe carboxylation adds ortho $-COOH$, while acetic anhydride converts $-OH$ into $-OCOCH_3$ without moving either ring substituent.

71. On heating an aliphatic primary amine with chloroform and ethanolic potassium hydroxide, the organic compound formed is: [4 marks]

(A) an alkyl cyanide

(B) an alkyl isocyanide

(C) an alkanol

(D) an alkanediol

Correct Answer: (B) an alkyl isocyanide

Solution:

Concept:

- The carbylamine reaction is the characteristic conversion of a primary amine into an isocyanide on heating with chloroform and alcoholic potassium hydroxide.
- An isocyanide has the connectivity $R-N\equiv C$, whereas a nitrile (alkyl cyanide) has the different connectivity $R-C\equiv N$. The atom joined directly to the alkyl group decides the name.

Step 1: Identify the reaction from its reagents The starting compound is an aliphatic primary amine, RNH_2. Chloroform and ethanolic KOH, with heating, form the familiar carbylamine reagent combination. The nitrogen already attached to the alkyl group remains attached to it; the carbon supplied by chloroform becomes the terminal carbon of the isocyanide group. This test applies to primary amines, including aliphatic primary amines.

Step 2: Write the balanced transformation The overall reaction is

$$RNH_2 + CHCl_3 + 3\,KOH \longrightarrow RNC + 3\,KCl + 3\,H_2O.$$

Thus the organic product is RNC, an alkyl isocyanide, also called an alkyl carbylamine. The three chlorine atoms leave as chloride and are accounted for in three formula units of potassium chloride.

Step 3: Eliminate the other proposed products Option (A), an alkyl cyanide, would require the alkyl group to be bonded to carbon of the cyanide group, which is not the connectivity produced here. Options (C) and (D), an alkanol and an alkanediol, would require replacement by one or two hydroxyl groups on an alkyl skeleton. Alcoholic alkali is a reagent here, not evidence that an alcohol or diol must form.

Step 4: Conclusion The primary amine gives an alkyl isocyanide, RNC, in the carbylamine reaction. Hence, the correct option is (B).

Quick Tip to Solve: For primary amine + chloroform + alcoholic alkali, recall carbylamine: the product is RNC, not the nitrile RCN.

72. In S_N2 reactions, the correct order of reactivity for the following compounds: CH_3Cl, CH_3CH_2Cl, $(CH_3)_2CHCl$ and $(CH_3)_3CCl$ is: [4 marks]

(A) $CH_3CH_2Cl > CH_3Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

(B) $(CH_3)_2CHCl > CH_3CH_2Cl > CH_3Cl > (CH_3)_3CCl$

(C) $CH_3Cl > (CH_3)_2CHCl > CH_3CH_2Cl > (CH_3)_3CCl$

(D) CH3Cl $> CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

Correct Answer: (D) CH3Cl $> CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

Solution:

Concept:

- An S_N2 substitution is a single-step process in which the nucleophile attacks the carbon bearing the leaving group from the side opposite that group.
- With the same leaving group and comparable conditions, crowding around that carbon controls accessibility: methyl halide reacts faster than primary, secondary and tertiary alkyl halides.

Step 1: Classify each carbon bearing chlorine In CH_3Cl, the reacting carbon has no attached alkyl groups; this is a methyl halide. In CH_3CH_2Cl, it has one alkyl group and is primary. The reacting carbon of $(CH_3)_2CHCl$ has two alkyl groups and is secondary. In $(CH_3)_3CCl$, it has three alkyl groups and is tertiary. All four contain chlorine, so a change in leaving-group identity does not determine the comparison.

Step 2: Compare the ease of backside attack The nucleophile must approach the carbon closely while the carbon–chlorine bond breaks. Increasing the number and size of alkyl groups obstructs that approach and raises the activation barrier. Methyl chloride has the least steric hindrance, ethyl chloride is next, and the secondary chloride is slower. The tertiary chloride is extremely hindered and does not normally undergo substitution by the S_N2 pathway.

Step 3: Match the decreasing order to the options Therefore the decreasing reactivity is

$$CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl.$$

Option (A) incorrectly makes ethyl chloride faster than methyl chloride. Option (B) incorrectly puts the secondary chloride first. Option (C) incorrectly puts the secondary compound above the primary compound. Only option (D) follows the progressively increasing steric obstruction.

Step 4: Conclusion The order is methyl chloride > ethyl chloride > isopropyl chloride > tert-butyl chloride. Hence, the correct option is (D).

Quick Tip to Solve: For an SN2 comparison with the same leaving group, count alkyl groups on the reacting carbon: fewer groups mean easier backside attack.

73. Which one of the following bases is not present in DNA? [4 marks]

(A) Cytosine

(B) Thymine

(C) Quinoline

(D) Adenine

Correct Answer: (C) Quinoline

Solution:

Concept:

- Deoxyribonucleic acid (DNA) contains four standard nitrogenous bases: adenine, guanine, cytosine and thymine.
- Adenine and guanine are purines; cytosine and thymine are pyrimidines. These bases form the complementary pairs that stabilise the DNA double helix.

Step 1: Recall the complete DNA base set A DNA nucleotide contains a deoxyribose sugar, a phosphate group and a nitrogenous base. For the normal bases relevant to this question, the complete list is adenine, guanine, cytosine and thymine. Adenine pairs with thymine through two hydrogen bonds, while guanine pairs with cytosine through three hydrogen bonds. This pairing provides a direct check that adenine, thymine and cytosine belong in DNA.

Step 2: Check the listed compounds individually Option (A), cytosine, is one of the two DNA pyrimidine bases, so it cannot be the answer to the negative question. Option (B), thymine, is the other DNA pyrimidine and is also present. Option (D), adenine, is a purine base found in DNA. The remaining compound, quinoline in option (C), is a fused heteroaromatic compound but is not one of the standard bases incorporated into DNA nucleotides.

Step 3: Distinguish the biochemical use of base A nitrogen-containing organic compound may be basic in the chemical acid–base sense without being a nucleic-acid base. Quinoline contains a ring nitrogen, but that fact does not put it into the DNA base set. Also, uracil is the familiar RNA base that replaces thymine; it is not among the printed options. The answer must be selected from the actual list rather than replacing quinoline by a more familiar distractor.

Step 4: Conclusion Quinoline is not present as a standard nitrogenous base in DNA; cytosine, thymine and adenine are present. Hence, the correct option is (C).

Quick Tip to Solve: For a negative DNA-base question, first write A, G, C and T, then compare every printed option with that complete set.

74. In the reaction,

the product C is: [4 marks]

$$CH_3COOH \xrightarrow{LiAlH_4} A \xrightarrow{PCl_5} B \xrightarrow[BaSO_4]{Alc.KOH} C,$$

(A) Ethylene

(B) Acetyl chloride

(C) Acetaldehyde

(D) Acetylene

Correct Answer: (A) Ethylene

Solution:

Concept:

- Lithium aluminium hydride, $LiAlH_4$, reduces a carboxylic acid to the corresponding primary alcohol without changing its carbon skeleton.
- Phosphorus pentachloride, PCl_5, replaces an alcohol hydroxyl group by chlorine. Alcoholic KOH then favors elimination of hydrogen chloride from a haloalkane to form an alkene.

Step 1: Determine intermediates A and B in order The starting compound is acetic acid, CH_3COOH. Complete reduction by $LiAlH_4$ converts its carboxyl carbon to the alcohol carbon, giving ethanol as A:

$$CH_3COOH + 4\,[H] \longrightarrow CH_3CH_2OH + H_2O.$$

Both carbons are retained. The next reagent acts on this alcohol, rather than on the original acid. Consequently PCl_5 gives chloroethane as B:

$$CH_3CH_2OH + PCl_5 \longrightarrow CH_3CH_2Cl + POCl_3 + HCl.$$

Acetyl chloride would be obtained from acetic acid treated directly with PCl_5, but that is not the sequence shown: the acid has already been reduced before this step.

Step 2: Apply alcoholic alkali to the haloalkane Alcoholic KOH removes a beta hydrogen from the methyl carbon while chloride leaves the adjacent carbon of chloroethane. A carbon–carbon double bond forms:

$$CH_3CH_2Cl + KOH \longrightarrow CH_2{=}CH_2 + KCl + H_2O.$$

Thus C is ethylene (ethene). The $BaSO_4$ printed below the last arrow does not change the carbon skeleton or supply an oxidizing reagent; it does not convert this elimination into oxidation or a Rosenmund reduction. The active organic transformation specified above that arrow is dehydrohalogenation by alcoholic KOH. Option (B) ignores the initial reduction. Option (C), acetaldehyde, would require oxidation or dehydrogenation of ethanol, not the chlorination followed by elimination shown here. Option (D), acetylene, would need elimination from a suitable dihalide; the intermediate here contains only one chlorine atom, so this sequence gives one double bond, not a triple bond.

Step 3: Conclusion A is ethanol, B is chloroethane, and C is ethylene, $CH_2{=}CH_2$. Hence, the correct option is (A).

Quick Tip to Solve: Identify each intermediate before reading the next reagent. Here reduction removes the acid group first, so the following PCl_5 step makes an alkyl chloride rather than an acyl chloride.

75. Considering the basic strength of amines in aqueous solution, which one has the smallest pK_b value? [4 marks]

(A) $(CH_3)_3N$

(B) $C_6H_5NH_2$

(C) $(CH_3)_2NH$

(D) CH_3NH_2

Correct Answer: (C) $(CH_3)_2NH$

Solution:

Concept:

- For an amine base B in water, $B + H_2O \rightleftharpoons BH^+ + OH^-$. A larger K_b means a stronger base, and $pK_b = -\log_{10} K_b$, so the strongest base has the smallest pK_b.
- Basicity in water reflects both the electron-releasing effect of alkyl groups and hydration of the protonated amine. Resonance reduces the availability of the nitrogen lone pair in aniline.

Step 1: Translate smallest pKb into strongest base The question asks for the smallest pK_b, not the smallest K_b. Because of the negative logarithm, decreasing pK_b corresponds to increasing basic strength. We must therefore identify the amine whose nitrogen most favourably accepts a proton in aqueous solution, considering stabilisation of both the free base and its conjugate acid.

Step 2: Compare the methylamines in water Methyl groups donate electron density to nitrogen by their inductive effect. However, simply counting methyl groups would give the wrong aqueous trend, because the conjugate acid must also be solvated. Dimethylamine, $(CH_3)_2NH$, combines strong electron donation by two methyl groups with effective hydration of its conjugate acid $(CH_3)_2NH_2{}^+$. Trimethylamine is more sterically crowded and its conjugate acid is less effectively hydrated. For these methylamines in water, the basic-strength order is secondary > primary > tertiary.

Step 3: Exclude aniline and identify the minimum In aniline, $C_6H_5NH_2$, the nitrogen lone pair is delocalised into the benzene ring, making it less available for protonation. It is consequently much weaker than the methylamines. Thus option (B) cannot have the smallest pK_b; options (A) and (D) are weaker bases than dimethylamine under the stated aqueous conditions. Option (C) is therefore the unique strongest base in this list.

Step 4: Conclusion Dimethylamine, $(CH_3)_2NH$, has the greatest basic strength in aqueous solution and hence the smallest pK_b. Hence, the correct option is (C).

Quick Tip to Solve: Convert a pKb question into a basic-strength question first; in water, include conjugate-acid hydration instead of using alkyl donation alone.

76. The major organic compound formed by the reaction of 1, 1, 1– trichloroethane with silver powder is: [4 marks]

(A) 2- Butyne

(B) 2- Butene

(C) Acetylene

(D) Ethene

Correct Answer: (A) 2- Butyne

Solution:

Concept:

- Silver powder removes halogen from suitable polyhalogenated organic compounds, forming silver halide. Two molecules of a geminal trihalide can couple while losing their halogens.
- For a compound of the form $\mathrm{R{-}CCl_3}$, complete silver-mediated dehalogenative coupling gives an alkyne $\mathrm{R{-}C{\equiv}C{-}R}$. Carbon atoms and substituents must be conserved in selecting the product.

Step 1: Write the correct starting structure 1,1,1-Trichloroethane is $\mathrm{CH_3{-}CCl_3}$: all three chlorine atoms are attached to the same terminal carbon. It is not 1,2-dichloroethane, and it is not chloroform. Two molecules provide two carbon centres bearing chlorine and two methyl groups that remain attached during the net coupling transformation.

Step 2: Form the alkyne and balance halogen removal Silver removes the six chlorine atoms from two substrate molecules as six units of silver chloride. The two originally trichlorinated carbon atoms become joined by a triple bond. The net reaction is

$$2\,\mathrm{CH_3CCl_3} + 6\,\mathrm{Ag} \longrightarrow \mathrm{CH_3C{\equiv}CCH_3} + 6\,\mathrm{AgCl}.$$

Carbon balance gives four carbon atoms on each side, hydrogen balance gives six hydrogen atoms, and all six chlorine atoms are accounted for in AgCl.

Step 3: Name the product and reject the alternatives The product $\mathrm{CH_3C{\equiv}CCH_3}$ has a four-carbon chain with its triple bond between carbons 2 and 3, so its name is but-2-yne (2-butyne). Option (B), 2-butene, would contain a double bond and two additional hydrogen atoms, which the stated net reaction does not supply. Option (C), acetylene, and option (D), ethene, each have only two carbon atoms and omit the retained methyl groups from coupling of the two trichloroethane molecules.

Step 4: Conclusion The major organic product is $\mathrm{CH_3C{\equiv}CCH_3}$, 2-butyne. Hence, the correct option is (A).

Quick Tip to Solve: For silver powder acting on R-CCl3, couple two halogen-bearing carbons and retain both R groups: the product skeleton is $\mathrm{R{-}C{\equiv}C{-}R}$.

77. The most suitable reagent for the conversion of R - CH_2 - OH $\rightarrow$ R - CHO is: [4 marks]

(A) CrO_3

(B) PCC (Pyridinium Chlorochromate)

(C) $KMnO_4$

(D) $K_2Cr_2O_7$

Correct Answer: (B) PCC (Pyridinium Chlorochromate)

Solution:

Concept:

- A primary alcohol RCH_2OH can be oxidised first to an aldehyde $RCHO$ and then, under stronger aqueous oxidation conditions, to a carboxylic acid $RCOOH$.
- Pyridinium chlorochromate (PCC) is a selective oxidant used under anhydrous conditions to stop the oxidation of a primary alcohol at the aldehyde stage.

Step 1: Identify the required oxidation level The starting group is a primary alcohol, $-CH_2OH$, while the target is an aldehyde group, $-CHO$. The carbon skeleton R is unchanged. The desired conversion removes two hydrogen atoms overall and creates the carbonyl group; it does not require cleavage of the carbon chain or oxidation all the way to a carboxylic acid.

Step 2: Select the reagent that stops at the aldehyde PCC, pyridinium chlorochromate, performs this controlled oxidation in a dry organic medium. The net transformation may be written

$$RCH_2OH + [O] \xrightarrow{PCC} RCHO + H_2O.$$

Under these conditions the aldehyde is obtained without the ready further oxidation associated with many strongly oxidising aqueous reagents. The selectivity, rather than merely the ability to oxidise an alcohol, is what makes PCC the most suitable choice.

Step 3: Compare the other chromium and permanganate reagents Potassium permanganate, option (C), and potassium dichromate under the usual acidic aqueous oxidation conditions, option (D), generally oxidise primary alcohols beyond the aldehyde to carboxylic acids. Chromium trioxide, option (A), can be used in different reagent systems, and carefully specified anhydrous conditions can affect its behaviour, but bare CrO_3 does not specify the selective PCC conditions sought in this standard conversion. Among the listed reagents, PCC is the clearest and most suitable choice for isolating the aldehyde.

Step 4: Conclusion Use PCC to convert RCH_2OH selectively into $RCHO$. Hence, the correct option is (B).

Quick Tip to Solve: When a primary alcohol must stop at an aldehyde, look for PCC; strong aqueous oxidants usually continue to the carboxylic acid.

78. The octahedral complex of a metal ion M^{3+} with four monodentate ligands L_1, L_2, L_3 and L_4 absorb wavelengths in the region of red, green, yellow and blue, respectively. The increasing order of ligand strength of the four ligands is : [4 marks]

(A) $L_3 < L_2 < L_4 < L_1$

(B) $L_1 < L_2 < L_4 < L_3$

(C) $L_4 < L_3 < L_2 < L_1$

(D) $L_1 < L_3 < L_2 < L_4$

Correct Answer: (D) $L_1 < L_3 < L_2 < L_4$

Solution:

Concept:

- In an octahedral complex, the metal d orbitals split into lower-energy t_{2g} and higher-energy e_g sets. Their separation is the crystal field splitting energy Δ_o.
- Absorption promotes an electron between the split levels, so the absorbed photon obeys $\Delta_o = h\nu = hc/\lambda$. A stronger ligand produces a larger splitting for the same metal ion and geometry.

Step 1: Arrange the absorbed colours by wavelength The given colours are the colours absorbed, rather than the complementary colours observed. Within the visible spectrum, red has the longest wavelength, followed by yellow, then green, while blue has the shortest wavelength among these four. Thus

$$\lambda_{\text{red}} > \lambda_{\text{yellow}} > \lambda_{\text{green}} > \lambda_{\text{blue}}.$$

Using the assignments in the question, this becomes $\lambda_1 > \lambda_3 > \lambda_2 > \lambda_4$.

Step 2: Convert wavelength order to ligand strength Because h and c are positive constants, taking the reciprocal reverses the wavelength order. Therefore

$$\Delta_o(L_1) < \Delta_o(L_3) < \Delta_o(L_2) < \Delta_o(L_4).$$

Ligand strength increases in exactly this order. Option (A) incorrectly makes the red-absorbing ligand strongest. Option (B) interchanges the yellow and green positions and also misplaces blue. Option (C) reverses the endpoint strengths, so none of these alternatives satisfies the inverse-wavelength relation.

Step 3: Conclusion The increasing ligand-strength order is $L_1 < L_3 < L_2 < L_4$. Hence, the correct option is (D).

Quick Tip to Solve: For absorption questions, order the photon energies using $E = hc/\lambda$; do not use the complementary colour seen by the observer.

79. The correct statement for the molecule, CsI_3 is : [4 marks]

(A) it contains Cs^{3+} and I^- ions.

(B) it contains Cs^+, I^- and lattice I_2 molecule.

(C) it is a covalent molecule

(D) it contains Cs^+ and I_3^- ions

Correct Answer: (D) it contains Cs^+ and I_3^- ions

Solution:

Concept:

- Caesium is a Group 1 metal and normally forms the singly charged cation Cs^+ by loss of its outermost electron.
- Iodide combines with an iodine molecule to form the polyhalide ion triiodide: $I^- + I_2 \rightleftharpoons I_3^-$. Charge balance determines the ions present in its salt.

Step 1: Identify the cation and the required anion In CsI_3, the caesium contribution is Cs^+. The iodine-containing unit must consequently carry one negative charge for the formula unit to be electrically neutral. The three iodine atoms are associated as the triiodide ion I_3^-, giving the ionic description

$$CsI_3 \longrightarrow Cs^+ + I_3^-.$$

This representation states the identities of the constituent ions; it does not mean that every bond within the polyatomic anion is ionic.

Step 2: Distinguish triiodide from the other descriptions The anion may be understood as the association of I^- and I_2, but the product contains a triiodide unit rather than a separate lattice iodine molecule alongside independent iodide ions. Thus option (B) misses the polyhalide ion. Option (A) would require the highly unfavourable Cs^{3+} state instead of the characteristic alkali-metal cation. Option (C) incorrectly describes the entire compound as a covalent molecule: the interaction between Cs^+ and I_3^- is ionic, even though bonding within the anion has covalent character.

Step 3: Conclusion Caesium triiodide contains Cs^+ and I_3^- ions. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish bonding inside a polyatomic ion from bonding between ions: a salt can contain a covalently bonded polyhalide anion.

80. Which one of the following properties is not shown by NO? [4 marks]

(A) It combines with oxygen to form nitrogen dioxide

(B) It's bond order is 2.5

(C) It is diamagnetic in gaseous state

(D) It is a neutral oxide

Correct Answer: (C) It is diamagnetic in gaseous state

Solution:

Concept:

- The magnetic character of a molecule is determined by its electron pairing. An unpaired electron makes the molecule paramagnetic, while a species with all electrons paired is diamagnetic.
- In molecular orbital theory, bond order equals half the difference between bonding and antibonding electron populations. Nitric oxide, NO, has an odd total number of electrons and bond order 2.5.

Step 1: Determine the magnetic behavior of gaseous NO Nitrogen contributes five valence electrons and oxygen contributes six, so one NO molecule has eleven valence electrons. The odd number requires an unpaired electron in its ground state. In its molecular orbital description, the singly occupied orbital is an antibonding π^* orbital. Therefore gaseous nitric oxide is paramagnetic: it is attracted by a magnetic field. It cannot be described as diamagnetic in its ordinary gaseous molecular state. Counting the valence populations gives eight bonding and three antibonding electrons, so

$$\text{Bond order} = \frac{8-3}{2} = 2.5.$$

The magnetic and bond-order statements must be tested separately; a nonintegral bond order does not by itself mean that every electron is paired.

Step 2: Verify the other three properties Nitric oxide reacts readily with oxygen to give nitrogen dioxide:

$$2\,\mathrm{NO} + \mathrm{O_2} \longrightarrow 2\,\mathrm{NO_2}.$$

Thus the oxygen-combination property in option (A) is shown by NO. The molecular orbital calculation confirms the bond order in option (B). Nitric oxide is also classified as a neutral oxide: it is neither an acidic oxide that forms an acid simply on reacting with water nor a basic oxide that forms a base with water. This confirms option (D). The term neutral oxide concerns acid–base behavior and does not imply absence of all chemical reactivity, as its oxidation to NO_2 demonstrates. Only option (C), the claim of diamagnetism in the gaseous state, contradicts the unpaired-electron structure.

Step 3: Conclusion Gaseous NO is paramagnetic, so the property it does not show is diamagnetism. Hence, the correct option is (C).

Quick Tip to Solve: For an isolated odd-electron molecule such as NO, check electron parity first: an odd population cannot have all electrons paired, immediately excluding diamagnetism.

81. Which series of reactions correctly represents chemical relations related to iron and its compound?

[4 marks]

(A) $Fe \xrightarrow{dil.\ H_2SO_4} FeSO_4 \xrightarrow{H_2SO_4, O_2} Fe_2(SO_4)_3 \xrightarrow{heat} Fe$

(B) $Fe \xrightarrow{O_2, heat} FeO \xrightarrow{dil.\ H_2SO_4} FeSO_4 \xrightarrow{heat} Fe$

(C) $Fe \xrightarrow{Cl_2, heat} FeCl_3 \xrightarrow{heat, air} FeCl_2 \xrightarrow{Zn} Fe$

(D) $Fe \xrightarrow{O_2, heat} Fe_3O_4 \xrightarrow{CO, 600^oC} FeO \xrightarrow{CO, 700^oC} Fe$

Correct Answer: (D) $Fe \xrightarrow{O_2, heat} Fe_3O_4 \xrightarrow{CO, 600^oC} FeO \xrightarrow{CO, 700^oC} Fe$

Solution:

Concept:

- Hot iron reacts with oxygen to form magnetite, Fe_3O_4. Carbon monoxide acts as a reducing agent in the extraction of iron from its oxides.
- Reduction removes oxygen from an oxide and transfers it to CO, producing CO_2. Successive oxide-reduction steps must conserve both iron and oxygen atoms.

Step 1: Verify the oxide sequence in option D The first conversion is the oxidation of iron to magnetite:

$$3\,Fe + 2\,O_2 \longrightarrow Fe_3O_4.$$

The next conversion reduces magnetite to iron(II) oxide at the stated temperature:

$$Fe_3O_4 + CO \xrightarrow{600\ ^\circ C} 3\,FeO + CO_2.$$

Finally, carbon monoxide reduces the iron(II) oxide to metal:

$$FeO + CO \xrightarrow{700\ ^\circ C} Fe + CO_2.$$

Each reaction conserves atoms, and the oxidation state of iron falls during the two carbon monoxide reductions.

Step 2: Check why the remaining reaction chains fail Option (A) correctly begins with iron reacting with dilute sulphuric acid, but heating ferric sulphate alone does not reduce it to metallic iron; sulphates decompose to oxide products. Option (B) similarly gives iron metal merely by heating ferrous sulphate, which is incorrect: heating instead yields ferric oxide and sulphur oxides. Also, burning iron in oxygen normally forms magnetite. Option (C) correctly starts with ferric chloride formation, but heating ferric chloride in air is not a reduction to ferrous chloride. A reducing agent would be needed for that change, so the complete chain cannot be accepted.

Step 3: Conclusion The sequence $Fe \longrightarrow Fe_3O_4 \longrightarrow FeO \longrightarrow Fe$, with the reagents and temperatures in the question, is correct. Hence, the correct option is (D).

Quick Tip to Solve: Check every arrow in a reaction sequence: one impossible conversion invalidates the entire option even if its first step is familiar.

82. For the non – stoichiometre reaction 2A + B $\rightarrow$ C + D, the following kinetic data were obtained in three separate experiments, all at 298 K.

Initial Concentration (A)	Initial Concentration (B)	Initial rate of formation of C ($\text{mol L}^{-1}\,\text{s}^{-1}$)
0.1 M	**0.1 M**	1.2×10^{-3}
0.1 M	**0.2 M**	1.2×10^{-3}
0.2 M	**0.1 M**	2.4×10^{-3}

The rate law for the formation of C is: [4 marks]

(A) $\frac{dc}{dt} = k[A][B]^2$

(B) $\frac{dc}{dt} = k[A]$

(C) $\frac{dc}{dt} = k[A][B]$

(D) $\frac{dc}{dt} = k[A^2][B]$

Correct Answer: (B) $\frac{dc}{dt} = k[A]$

Solution:

Concept:

- The experimental rate law is $r = k[A]^m[B]^n$, where m and n are determined from measured concentration effects. They are not generally the stoichiometric coefficients of an overall reaction.
- Comparing experiments in which only one concentration changes isolates its reaction order. A concentration doubling that leaves rate unchanged gives order zero; one that doubles the rate gives order one.

Step 1: Find the order with respect to B Compare the first and second experiments at the common temperature of $298\,\text{K}$. The concentration of A stays at $0.1\,\text{M}$, while B doubles from 0.1 to $0.2\,\text{M}$. The rate remains $1.2 \times 10^{-3}\,\text{mol L}^{-1}\,\text{s}^{-1}$. Dividing the two rate laws cancels k and the A factor:

$$\frac{r_2}{r_1} = 1 = \left(\frac{0.2}{0.1}\right)^n = 2^n.$$

Hence $n = 0$. The rate has no dependence on B over the tested conditions, so every option containing a positive power of B is already inconsistent with this comparison.

Step 2: Find the order with respect to A and check the law Compare the first and third experiments. B remains at $0.1\,\text{M}$, A doubles from 0.1 to $0.2\,\text{M}$, and the rate doubles from 1.2×10^{-3} to 2.4×10^{-3}. Thus

$$\frac{r_3}{r_1} = 2 = \left(\frac{0.2}{0.1}\right)^m = 2^m, \qquad m = 1.$$

Therefore

$$\frac{d[C]}{dt} = k[A]^1[B]^0 = k[A].$$

As a consistency check,

$$k = \frac{1.2 \times 10^{-3}}{0.1} = 1.2 \times 10^{-2}\,\text{s}^{-1},$$

and the third experiment gives the same value. Option (A) would make doubling B quadruple the rate; options (C) and (D) would make it double. Neither response occurs. Option (D) also incorrectly

makes the rate quadruple when A doubles. Option (B) alone reproduces all three measured rates.

Step 3: Conclusion The formation rate of C is $d[C]/dt = k[A]$: first order in A and zero order in B. Hence, the correct option is (B).

Quick Tip to Solve: Choose pairs with one concentration held fixed. Do not copy the coefficient 2 of A into the rate law; these measurements directly show an exponent of 1.

83. Given below are the half-cell reactions:

$Mn^{2+} + 2e^- \rightarrow Mn$; $E^o = -1.18$ V

$2(Mn^{3+} + e^- \rightarrow Mn^{2+})$; $E^o = +1.51$ V

The E^o for $3Mn^{2+} \rightarrow Mn + 2Mn^{3+}$ will be : [4 marks]

(A) – 0.33 V; the reaction will not occur

(B) – 0.33 V; the reaction will occur

(C) – 2.69 V; the reaction will not occur

(D) – 2.69 V; the reaction will occur

Correct Answer: (C) – 2.69 V; the reaction will not occur

Solution:

Concept:

- Standard cell potential is $E^\circ_{\text{cell}} = E^\circ_{\text{reduction,cathode}} - E^\circ_{\text{reduction,anode}}$. Use the tabulated potentials as reduction potentials.
- Potential is an intensive quantity: multiplying a half-reaction does not multiply its potential. The standard free-energy change is $\Delta G^\circ = -nFE^\circ_{\text{cell}}$, where n is the transferred electron count.

Step 1: Combine the reduction and oxidation half-reactions For the requested disproportionation, one manganese(II) ion is reduced to manganese metal:

$$\mathrm{Mn^{2+} + 2\,e^- \longrightarrow Mn}, \qquad E^\circ_{\text{red}} = -1.18\,\mathrm{V}.$$

Two other manganese(II) ions are oxidised:

$$\mathrm{2\,Mn^{2+} \longrightarrow 2\,Mn^{3+} + 2\,e^-}.$$

The associated reduction potential is $+1.51\,\mathrm{V}$, so the potential of this reversed oxidation half-reaction is $-1.51\,\mathrm{V}$. Electron cancellation gives precisely $\mathrm{3\,Mn^{2+} \longrightarrow Mn + 2\,Mn^{3+}}$.

Step 2: Calculate the potential and assess feasibility Adding the reduction and oxidation potentials gives

$$E^\circ_{\text{cell}} = -1.18 - 1.51 = -2.69\,\mathrm{V}.$$

Since two electrons are transferred, $\Delta G^\circ = -2F(-2.69) > 0$. The reaction as written is therefore not spontaneous under standard conditions. Options (A) and (B) use an incorrect potential, arising from combining the numbers without reversing the appropriate half-reaction or weighting potentials incorrectly. Option (D) has the correct potential but the wrong feasibility statement; a negative

standard cell potential cannot imply spontaneous forward reaction in standard conditions.

Step 3: Conclusion $E^\circ = -2.69\,\text{V}$; the disproportionation will not occur spontaneously under standard conditions. Hence, the correct option is (C).

Quick Tip to Solve: Balance electrons by multiplying half-reactions, but never multiply their electrode potentials; reverse the sign only when reversing a half-reaction.

84. The correct set of four quantum numbers for the valence elections of rubidium atom (Z= 37) is: [4 marks]

(A) 5, 1, 1, + 1/2

(B) 5, 1, 0, + 1/2

(C) 5, 0, 0, + 1/2

(D) 5, 0, 1, + 1/2

Correct Answer: (C) 5, 0, 0, + 1/2

Solution:

Concept:

- The four quantum numbers are (n, l, m_l, m_s). For a shell n, $l = 0, 1, \ldots, n-1$, while m_l takes integer values from $-l$ to $+l$.
- An s electron has $l = 0$ and therefore necessarily $m_l = 0$. Its spin quantum number may be either $+\frac{1}{2}$ or $-\frac{1}{2}$.

Step 1: Locate the valence electron of rubidium Rubidium has atomic number 37. Filling the ground-state orbitals gives

$$\text{Rb}: \ [\text{Kr}]\, 5s^1.$$

The krypton core contains 36 electrons, leaving the single outer electron in the 5s orbital. Thus its principal quantum number is $n = 5$, not 4, even though occupied inner d orbitals are also present in the electronic configuration. The question asks for the valence electron, so it is the outermost occupied orbital that determines the quantum numbers.

Step 2: Apply the orbital and spin restrictions For the 5s orbital, the azimuthal quantum number is $l = 0$. The only allowed magnetic quantum number is then $m_l = 0$, because its allowed range runs from $-l$ to $+l$. Taking the offered spin orientation gives

$$(n, l, m_l, m_s) = \left(5, 0, 0, +\tfrac{1}{2}\right).$$

Options (A) and (B) both have $l = 1$, so they describe 5p rather than 5s electrons. Option (D) combines $l = 0$ with $m_l = 1$, which violates the allowed magnetic-quantum-number range. The choice $+\frac{1}{2}$ is allowed for the unpaired electron; no second electron is present to impose pairing.

Step 3: Conclusion The valid set is $5, 0, 0, +\frac{1}{2}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the valence subshell first; its letter fixes l, after which the allowed range of m_l immediately rules out impossible sets.

85. Among the following oxoacids, the correct decreasing order of acid strength is : [4 marks]

(A) $HClO_4 > HClO_3 > HClO_2 > HOCl$

(B) $HClO_2 > HClO_4 > HClO_3 > HOCl$

(C) $HOCl > HClO_2 > HClO_3 > HClO_4$

(D) $HClO_4 > HOCl > HClO_3 > HClO_2$

Correct Answer: (A) $HClO_4 > HClO_3 > HClO_2 > HOCl$

Solution:

Concept:

- For oxoacids of the same central atom, acidity depends on how readily the O–H bond loses a proton and how stable the resulting conjugate base is.
- Additional oxygen atoms withdraw electron density through the inductive effect and allow the negative charge of the conjugate base to be distributed over more oxygen atoms.

Step 1: Identify the comparable acid family All four acids contain chlorine as the central atom and one ionisable hydrogen attached to oxygen. They differ in the number of oxygen atoms, so the identity of the central atom is held fixed in this comparison. Their deprotonation produces ClO_4^-, ClO_3^-, ClO_2^- and ClO^-, respectively. A more stable conjugate base has a smaller tendency to recapture a proton and therefore corresponds to a stronger acid.

Step 2: Compare electron withdrawal and conjugate-base stability The extra terminal oxygen atoms exert an electron-withdrawing effect, increasing polarisation of the O–H bond. They also stabilise the negative charge after proton loss. These effects become progressively stronger as the oxygen count rises from one to four. Thus perchloric acid is strongest, followed by chloric acid and chlorous acid, while hypochlorous acid is weakest:

$$HClO_4 > HClO_3 > HClO_2 > HOCl.$$

Option (B) wrongly places chlorous acid above perchloric acid. Option (C) reverses the trend. Option (D) wrongly puts hypochlorous acid above both chloric and chlorous acids; it has the least stabilised conjugate base in this family.

Step 3: Conclusion The decreasing acid strength is $HClO_4 > HClO_3 > HClO_2 > HOCl$. Hence, the correct option is (A).

Quick Tip to Solve: For oxoacids with the same central atom and one acidic O–H group, compare the number of additional oxygen atoms: more oxygen generally means a stronger acid.

86. The metal that cannot be obtained by electrolysis of an aqueous solution of its salts is : [4 marks]

(A) Cu

(B) Cr

(C) Ag

(D) Ca

Correct Answer: (D) Ca

Solution:

Concept:

- At the cathode of an aqueous electrolytic cell, metal-ion reduction competes with reduction of water or hydrogen ions.
- Very electropositive metals such as calcium require molten salts because hydrogen is liberated preferentially from their aqueous salt solutions.

Step 1: Identify the competing cathode reactions To deposit a metal from its salt, its cation must accept electrons at the cathode. For calcium the required reaction would be

$$\mathrm{Ca}^{2+} + 2\,\mathrm{e}^- \longrightarrow \mathrm{Ca}.$$

In water there is also a competing reduction:

$$2\,\mathrm{H_2O} + 2\,\mathrm{e}^- \longrightarrow \mathrm{H_2} + 2\,\mathrm{OH}^-.$$

Calcium is strongly electropositive; its reduction potential is very negative. Consequently, water is reduced more readily than calcium ions in the aqueous electrolyte. Passing current therefore produces hydrogen rather than a recoverable calcium deposit.

Step 2: Distinguish calcium from the other metals Copper and silver are much less electropositive and are readily deposited from suitable aqueous solutions of their salts. Chromium can also be deposited electrolytically from suitably controlled aqueous chromium-containing baths; its deposition involves kinetic and bath-condition considerations, so a simplistic rule excluding every metal above hydrogen is not valid. Calcium is the clear exclusion among the listed choices. Even calcium momentarily formed in contact with water would react with it, further preventing its collection as metal. Its industrial electrolytic preparation uses an anhydrous molten salt, eliminating water from the cathode competition. Thus (A), (B) and (C) do not answer the question asking which metal cannot be obtained by the specified aqueous route.

Step 3: Conclusion Calcium cannot be obtained by electrolysis of an aqueous solution of its salts; hydrogen is evolved instead. Hence, the correct option is (D).

Quick Tip to Solve: For alkali and alkaline-earth metals, immediately check whether the electrolyte is aqueous or molten: water prevents their isolation at the cathode.

87. Which one is classified as a condensation polymer? [4 marks]

(A) Teflon

(B) Acrylonitrile

(C) Dacron

(D) Neoprene

Correct Answer: (C) Dacron

Solution:

Concept:

- Condensation polymerisation joins bifunctional monomers through repeated reactions that eliminate small molecules such as water.
- Addition polymerisation links unsaturated monomers without the repeated elimination of a small molecule. Dacron is the polyester polyethylene terephthalate.

Step 1: Examine the formation of Dacron Dacron is formed from ethylene glycol, $HOCH_2CH_2OH$, and terephthalic acid, $HOOC-C_6H_4-COOH$. Each monomer has two reactive functional groups, allowing chain growth in both directions. The hydroxyl groups and carboxyl groups react to form ester linkages, with water eliminated during esterification. Its characteristic repeat unit can be written as

$$[-OCH_2CH_2OCOC_6H_4CO-]_n .$$

The repeated formation of ester bonds accompanied by loss of water identifies Dacron as a condensation polymer, rather than merely an addition product of carbon–carbon double bonds.

Step 2: Eliminate the remaining choices by their monomers Teflon is polytetrafluoroethylene, obtained by addition polymerisation of tetrafluoroethene, $CF_2=CF_2$. Acrylonitrile, $CH_2=CHCN$, is itself an unsaturated monomer; its polymer polyacrylonitrile is an addition polymer, so either reading of that option does not supply a condensation polymer. Neoprene is made by addition polymerisation of chloroprene, $CH_2=CCl-CH=CH_2$. In these addition processes, the unsaturated carbon framework provides chain growth without the repeated elimination of water. Thus only Dacron has the bifunctional acid–alcohol condensation route required by the question.

Step 3: Conclusion Dacron is classified as a condensation polymer because its polyester chain is formed by repeated esterification with elimination of water. Hence, the correct option is (C).

Quick Tip to Solve: Recognise Dacron as a polyester: the acid plus diol route and ester linkages distinguish it from vinyl addition polymers.

88. If Z is a compressibility factor, van der Waals equation at low pressure can be written as: [4 marks]

(A) Z = 1 + $\frac{RT}{Pb}$

(B) Z = 1 - $\frac{a}{VRT}$

(C) Z = 1 - $\frac{Pb}{RT}$

(D) Z = 1 + $\frac{Pb}{RT}$

Correct Answer: (B) Z = 1 - $\frac{a}{VRT}$

Solution:

Concept:

- The compressibility factor for one mole is $Z = PV/(RT)$, where V is molar volume, P is pressure, T is temperature and R is the gas constant.
- The van der Waals equation for one mole is $\left(P + a/V^2\right)(V - b) = RT$. The attraction parameter a lowers the measured pressure; b represents excluded volume.

Step 1: Use the conventional low-pressure approximation At low pressure the molar volume is large compared with the excluded volume, so in the usual attraction-dominated low-pressure form we take $V - b \simeq V$. Retaining the attractive-pressure correction gives

$$\left(P + \frac{a}{V^2}\right) V \simeq RT.$$

Expanding this expression and isolating the measurable pressure–volume product,

$$PV + \frac{a}{V} \simeq RT, \qquad PV \simeq RT - \frac{a}{V}.$$

This negative correction is physically consistent: intermolecular attractions reduce the pressure compared with an ideal gas at the same molar volume and temperature.

Step 2: Convert the result into the compressibility factor Divide through by RT:

$$Z = \frac{PV}{RT} \simeq 1 - \frac{a}{VRT}.$$

This is option (B). Option (A) contains a reciprocal-pressure correction and does not approach unity as pressure tends to zero. Options (C) and (D) contain the excluded-volume parameter rather than the retained attraction term; (D) is associated with a repulsion-dominated approximation. More generally, keeping both first-order low-density corrections gives $Z \simeq 1 + (b - a/(RT))/V$. The offered expression is the conventional low-pressure approximation obtained by neglecting b, rather than a claim that every real gas must have $Z < 1$ at every temperature.

Step 3: Conclusion With the conventional low-pressure approximation $V - b \simeq V$, the equation becomes $Z = 1 - a/(VRT)$. Hence, the correct option is (B).

Quick Tip to Solve: At low pressure, start from the attraction correction and divide the resulting PV equation by RT; check that Z approaches 1 as volume grows.

89. CsCl crystallises in body centred cubic lattice. If 'a' is its edge length then which of the following expressions is correct? [4 marks]

(A) $r_{Cs^+} + r_{Cl^-} = \frac{\sqrt{3}}{2}a$

(B) $r_{Cs^+} + r_{Cl^-} = \frac{3}{2}a$

(C) $r_{Cs^+} + r_{Cl^-} = \sqrt{3}a$

(D) $r_{Cs^+} + r_{Cl^-} = 3a$

Correct Answer: (A) $r_{Cs^+} + r_{Cl^-} = \frac{\sqrt{3}}{2}a$

Solution:

Concept:

- In the CsCl arrangement, one ion occupies the body centre of a cube and the oppositely charged ions occupy its eight corners; nearest neighbours touch along a body diagonal.
- A cube of edge length a has body-diagonal length $\sqrt{3}a$. The centre-to-corner separation is half this length.

Step 1: Find the nearest unlike-ion separation Choose any corner of the cubic unit cell and the ion at its body centre. Their coordinate displacement has components $a/2$, $a/2$ and $a/2$. The distance between their centres is therefore

$$d = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{3}}{2}a.$$

The same result follows by bisecting the body diagonal. Every corner is equally far from the body centre, giving the familiar eightfold coordination in this structure.

Step 2: Apply the contact condition and compare the options Since adjacent unlike ions touch along this line, the distance between their centres is the sum of their radii:

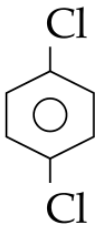
$$r_{Cs^+} + r_{Cl^-} = d = \frac{\sqrt{3}}{2}a.$$

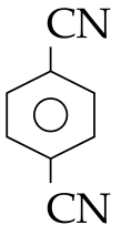
Equivalently, the full body diagonal contains two such unlike-ion contact distances, so $2(r_{Cs^+} + r_{Cl^-}) = \sqrt{3}a$. Option (C) incorrectly assigns the whole body diagonal to just one radius sum. Options (B) and (D) replace the three-dimensional diagonal factor $\sqrt{3}$ by 3 and therefore do not satisfy the cube geometry. Although the arrangement is often described as body-centred cubic in elementary questions, the two occupied positions contain different ions; the contact relation, rather than a monatomic bcc packing formula, is the relevant result.

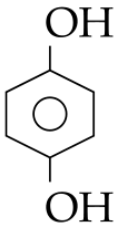
Step 3: Conclusion The correct relation is $r_{Cs^+} + r_{Cl^-} = (\sqrt{3}/2)a$. Hence, the correct option is (A).

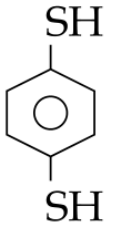
Quick Tip to Solve: Locate the touching unlike ions first: in CsCl the contact distance is half the body diagonal, whereas an edge-contact formula belongs to a different arrangement.

90. For which of the following molecule significant $\mu \neq 0$? [4 marks]

(a) 

(b) 

(c) 

(d) 

(A) Only (c)

(B) (c) and (d)

(C) Only (a)

(D) (a) and (b)

Correct Answer: (B) (c) and (d)

Solution:

Concept:

- The permanent dipole moment μ is a vector sum of bond and group dipoles. Identical groups at opposite positions cancel only when their dipoles are equal and oppositely directed.
- The shapes of substituents matter: a chlorine atom and a linear cyano group contribute along their attachment directions, whereas hydroxyl and thiol groups contain bent $C-O-H$ and $C-S-H$ arrangements.

Step 1: Check the two groups directed along the para axis In structure (a), the two identical $C-Cl$ bonds occupy opposite para positions on the benzene ring. Their equal bond-dipole vectors point in opposite directions, so they cancel. Structure (b) similarly has two identical linear $-C\equiv N$ groups. Their group dipoles lie along opposite ring-to-group directions and cancel. Thus neither (a) nor (b) has the significant resultant asked for. The presence of polar bonds alone does not establish a polar molecule: their directions must also be included.

Step 2: Account for the bent hydroxyl and thiol groups In (c), each $-OH$ group includes both a $C-O$ bond and an $O-H$ bond. Because the oxygen centre is bent, the $O-H$ contribution is not collinear with the para substitution axis. Cancellation of the opposed ring-to-oxygen contributions therefore does not enforce cancellation of all the group contributions. The hydroxyl compound has a significant resultant dipole in the molecular treatment used here. The corresponding argument applies to the bent $C-S-H$ groups in (d): the $S-H$ contributions are not forced to cancel by para substitution alone. Thus (c) and (d) are the polar structures. Option (A) leaves out (d), while (C) and (D) incorrectly include the symmetric chlorine compound; (D) also includes the symmetric dicyano compound.

Step 3: Conclusion The two molecules containing hydroxyl and thiol substituents have the significant nonzero resultant dipole asked for. Hence, the correct option is (B).

Quick Tip to Solve: For identical para substituents, inspect the whole substituent geometry before cancelling dipoles. A bent $-\mathrm{OH}$ or $-\mathrm{SH}$ group cannot be replaced by a single ring-axis bond dipole.