JEE Main 2014 Paper 1 (B.E./B.Tech) Apr 12 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2014, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 12 April 2014, Single slot. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. The sum of the roots of the equation, $x^2 + |2x - 3| - 4 = 0$, is [4 marks]

(A) 2

(B) -2

(C) $\sqrt{2}$

(D) $-\sqrt{2}$

Correct Answer: (C) $\sqrt{2}$

Solution:

Concept:

- The expression $|2x - 3|$ equals $2x - 3$ when $x \geq 3/2$, and $3 - 2x$ when $x < 3/2$. Each algebraic root must satisfy the condition of the branch that produced it.
- A quadratic equation $x^2 + bx + c = 0$ has roots $(-b \pm \sqrt{b^2 - 4c})/2$. The roots of two different branches cannot be added before rejecting inadmissible values.

Step 1: Solve the upper branch For $x \geq 3/2$, replace the absolute value by $2x - 3$. Then

$$x^2 + 2x - 7 = 0, \qquad x = -1 \pm 2\sqrt{2}.$$

The positive value $-1 + 2\sqrt{2} \approx 1.828$ satisfies this branch. The other value is negative and must be rejected from this branch.

Step 2: Solve the lower branch For $x < 3/2$, the equation becomes

$$x^2 - 2x - 1 = 0, \qquad x = 1 \pm \sqrt{2}.$$

Here $1 - \sqrt{2} \approx -0.414$ is admissible, whereas $1 + \sqrt{2} \approx 2.414$ violates the branch condition. Thus exactly two real roots survive. Substitution into their respective absolute-value branches verifies both.

Step 3: Add only admissible roots The required sum is

$$(-1 + 2\sqrt{2}) + (1 - \sqrt{2}) = \sqrt{2}.$$

The values 2 and -2 would be possible Vieta sums for individual quadratic branches, but neither is the sum of the admissible roots of the original equation. The remaining negative radical has the wrong sign.

Step 4: Conclusion The sum of the roots is $\sqrt{2}$. Hence, the correct option is (C).

Quick Tip to Solve: Split an absolute-value equation at its zero, then check every quadratic root against the interval used to derive it.

2. Let z $\neq -$i be any complex number such that (z$-$i)/(z+i) is a purely imaginary number. Then z +(1)/(z) is [4 marks]

(A) 0

(B) any non-zero real number other than 1

(C) any non-zero real number

(D) a purely imaginary number

Correct Answer: (C) any non-zero real number

Solution:

Concept:

- Write $z = x + iy$, where x, y are real. Rationalising a complex quotient separates its real and imaginary parts.
- A purely imaginary number has zero real part. On the unit circle $z\overline{z} = 1$, so $1/z = \overline{z}$.

Step 1: Find the locus Because $z \neq -i$, the denominator is nonzero. Multiply numerator and denominator by the conjugate of $x + i(y + 1)$:

$$\frac{z - i}{z + i} = \frac{x^2 + y^2 - 1 - 2ix}{x^2 + (y + 1)^2}.$$

Its real part vanishes precisely when $x^2 + y^2 = 1$. Thus z lies on the unit circle and in particular is not zero.

Step 2: Evaluate the requested expression Using $1/z = \overline{z} = x - iy$, obtain

$$z + \frac{1}{z} = (x + iy) + (x - iy) = 2x.$$

This is real. With the usual convention here that a purely imaginary quotient is nonzero, its imaginary part $-2x/[x^2 + (y + 1)^2]$ is nonzero, so $x \neq 0$. The requested expression is therefore a nonzero real number.

Step 3: Check the alternatives The value 1 is possible: take $z = \frac{1}{2} + i\frac{\sqrt{3}}{2}$, which is on the unit circle and gives a nonzero purely imaginary quotient. This rules out the exclusion in (B). The result is real, so (D) fails, and the nonzero condition rules out (A). Here the phrase in (C) identifies the number as a nonzero real quantity; its actual range also satisfies $-2 \leq 2x \leq 2$.

Step 4: Conclusion Thus $z + 1/z$ is a nonzero real number, under the nonzero purely imaginary convention of the question. Hence, the correct option is (C).

Quick Tip to Solve: For a quotient constrained to be purely imaginary, set its rationalised real numerator to zero before simplifying the target expression.

3. 8-digit numbers are formed using the digits 1, 1, 2, 2, 2, 3, 4, 4. The number of such numbers in which the odd digits do no occupy odd places, is [4 marks]

(A) 160

(B) 120

(C) 60

(D) 48

Correct Answer: (B) 120

Solution:

Concept:

- An eight-digit arrangement has four odd positions $1, 3, 5, 7$ and four even positions $2, 4, 6, 8$. Odd digits here are the two copies of 1 and the single 3.
- Identical digits must not be counted as distinct. A multiset of n symbols with multiplicities $n_1, n_2, \ldots$ has $n!/(n_1!n_2!\cdots)$ arrangements.

Step 1: Place all odd digits in even positions The restriction forces the three odd digits $1, 1, 3$ into three of the four even positions. First choose those positions in $\binom{4}{3} = 4$ ways. In the selected positions, the digits can be arranged in $3!/2! = 3$ ways, since interchanging the two copies of 1 does not change the number. This gives $4 \times 3 = 12$ partial arrangements.

Step 2: Fill every remaining position After the odd digits have been placed, five positions remain: all four odd positions and the unused even position. Fill them with $2, 2, 2, 4, 4$. These are unrestricted even digits, so the number of fillings is

$$\frac{5!}{3!\,2!} = 10.$$

No leading-zero adjustment is necessary because zero is not among the supplied digits.

Step 3: Multiply and check the counting For each of the 12 odd-digit placements there are exactly 10 fillings of the remaining positions. The choices are independent, giving

$$N = \binom{4}{3}\frac{3!}{2!}\frac{5!}{3!\,2!} = 120.$$

Each finished number determines its odd-digit positions uniquely, so there is no overcount. The other proposed totals 160, 60 and 48 do not equal this complete product; omitting repeated-digit factors or a choice of even position would miscount.

Step 4: Conclusion There are 120 such eight-digit numbers. Hence, the correct option is (B).

Quick Tip to Solve: Place the restricted digits first, divide by factorials for identical copies, then fill all unrestricted positions.

4. Let G be the geometric mean of two positive numbers a and b, and M be the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$. If $\frac{1}{M} : G$ is $4 : 5$, then $a : b$ can be [4 marks]

(A) 1 : 4

(B) 1 : 2

(C) 2 : 3

(D) 3 : 4

Correct Answer: (A) 1 : 4

Solution:

Concept:

- For positive a, b, their geometric mean is $G = \sqrt{ab}$. The arithmetic mean of their reciprocals is $M = (1/a + 1/b)/2 = (a + b)/(2ab)$.
- Ratios of homogeneous expressions depend only on $a : b$. A positive scale factor common to a and b cancels from $(1/M)/G$.

Step 1: Translate the ratio into an equation The reciprocal of the arithmetic mean is $1/M = 2ab/(a+b)$. Therefore

$$\frac{1/M}{G} = \frac{2\sqrt{ab}}{a+b} = \frac{4}{5}.$$

Cross-multiplying gives $10\sqrt{ab} = 4(a + b)$, or equivalently $5\sqrt{ab} = 2(a + b)$. All quantities are positive, so these operations preserve the equality.

Step 2: Solve for the ratio Let $t = \sqrt{a/b} > 0$. Dividing by b yields

$$5t = 2(t^2 + 1), \qquad 2t^2 - 5t + 2 = 0.$$

Factorising, $(2t - 1)(t - 2) = 0$, so $t = 1/2$ or 2. Squaring gives $a/b = 1/4$ or 4. The reciprocal possibilities reflect the symmetry in a and b.

Step 3: Check the listed choices Only $1 : 4$ occurs among these two ratios in the list. For a direct check set $a = 1, b = 4$: then $G = 2$, $M = 5/8$, and $(1/M) : G = (8/5) : 2 = 4 : 5$. The ratios $1 : 2$, $2 : 3$ and $3 : 4$ do not solve the factored equation, so none can satisfy the stated mean ratio.

Step 4: Conclusion The ratio $a : b$ can be $1 : 4$. Hence, the correct option is (A).

Quick Tip to Solve: When a ratio of means involves positive numbers, put $t = \sqrt{a/b}$ to turn radicals into a quadratic.

5. The least positive integer n such that $1 - \frac{2}{3} - \frac{2}{3^2} - \cdots - \frac{2}{3^{n-1}} < \frac{1}{100}$, is [4 marks]

(A) 4

(B) 5

(C) 6

(D) 7

Correct Answer: (C) 6

Solution:

Concept:

- The subtracted terms form a geometric progression with first term $2/3$, common ratio $1/3$, and $n-1$ terms.
- The finite sum of a geometric progression is $a(1-r^k)/(1-r)$. Subtracting this sum from its limiting total leaves an exact geometric remainder.

Step 1: Evaluate the finite sum For $n \geq 2$, apply the geometric-sum formula:

$$\sum_{k=1}^{n-1} \frac{2}{3^k} = \frac{(2/3)(1-(1/3)^{n-1})}{1-1/3} = 1 - \frac{1}{3^{n-1}}.$$

Consequently the entire left-hand side of the inequality is simply $3^{-(n-1)}$. This remainder formula also holds for $n = 1$, when the subtracted sum is empty and the expression is 1.

Step 2: Convert the inequality to powers The requirement becomes

$$\frac{1}{3^{n-1}} < \frac{1}{100} \quad \Longleftrightarrow \quad 3^{n-1} > 100.$$

The powers immediately around 100 are $3^4 = 81$ and $3^5 = 243$. Since powers of 3 increase strictly as the integer exponent increases, the least allowed exponent is $n - 1 = 5$.

Step 3: Verify minimality and reject the alternatives Thus $n = 6$. At $n = 5$, the remainder is $1/81 > 1/100$, so 5 fails, and the still smaller choice 4 fails as well. At $n = 6$, the remainder is $1/243 < 1/100$, so 6 works. The choice 7 also satisfies the inequality, but it is not the least positive integer. The strict inequality has therefore been checked on both sides of the boundary.

Step 4: Conclusion The least positive integer is $n = 6$. Hence, the correct option is (C). This differs from the supplied key selecting the second option: at $n = 5$ the literal remainder is $1/81 > 1/100$, so that key cannot satisfy the printed inequality.

Quick Tip to Solve: For a truncated geometric sum close to its limiting value, simplify the remainder first and compare neighbouring integer powers.

6. If $1 + x^4 + x^5 = \sum_{i=0}^{5} a_i(1 + x^i)$, for all $x \in \mathbb{R}$, then a_2 is: [4 marks]

(A) -4

(B) 6

(C) -8

(D) 10

Correct Answer: The identity as printed gives $a_2 = 0$, which is absent from the listed options.

Solution:

Concept:

- Two polynomials equal for every real x have equal coefficients of each power of x. This allows us to determine an individual coefficient without finding every unknown first.
- In the printed expression $a_i(1 + x^i)$, the exponent belongs to x, so its expansion is simply $a_i + a_i x^i$. It is not the binomial power $a_i(1 + x)^i$.

Step 1: Expand the sum exactly as printed Separate the term with $i = 0$, for which $x^0 = 1$. The right-hand side becomes

$$2a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5.$$

Thus only the $i = 2$ summand contributes an x^2 term. The constant contributions from the other summands do not affect this coefficient.

Step 2: Compare coefficients and test consistency The left-hand polynomial $1 + x^4 + x^5$ has zero coefficient of x^2. Equality for every real x therefore requires

$$a_2 = 0.$$

More fully, comparison of the other positive powers gives $a_1 = a_3 = 0$ and $a_4 = a_5 = 1$. The constant coefficient then requires $2a_0 + 2 = 1$, hence $a_0 = -1/2$. These coefficients reproduce the entire left-hand side, confirming that the literal identity is consistent. Each listed value, namely $-4, 6, -8, 10$, would instead introduce a nonzero x^2 term and violates the identity.

Step 3: Conclusion For the identity as printed, $\boxed{a_2 = 0}$. None of the four printed options is correct. Every listed value contradicts the zero coefficient of x^2, so no unique listed option can be selected.

Quick Tip to Solve: Check the position of an exponent before applying the binomial theorem: $1+x^i$ contributes one nonconstant power, whereas $(1 + x)^i$ contributes several.

7. If $\left(2+\frac{x}{3}\right)^{55}$ is expanded in the ascending powers of x and the coefficients of powers of x in two consecutive terms of the expansion are equal, then these terms are [4 marks]

(A) 7th and 8th

(B) 8th and 9th

(C) 28th and 29th

(D) 27th and 28th

Correct Answer: (B) 8th and 9th

Solution:

Concept:

- In the expansion of $(a+bx)^n$, the coefficient of x^r is $\binom{n}{r}a^{n-r}b^r$. The term containing x^r is the $(r+1)$th term.
- The ratio of consecutive binomial coefficients is $\binom{n}{r+1}/\binom{n}{r} = (n-r)/(r+1)$. Include the powers of the constants as well as the combinatorial factor.

Step 1: Write the actual coefficients For the given expansion, denote the coefficient of x^r by c_r. Then

$$c_r = \binom{55}{r}\frac{2^{55-r}}{3^r}, \qquad 0 \le r \le 55.$$

The next coefficient is $c_{r+1} = \binom{55}{r+1}2^{54-r}/3^{r+1}$. All these coefficients are positive, so equality may be tested by taking their ratio.

Step 2: Set the ratio equal to one Cancel common factors carefully:

$$\frac{c_{r+1}}{c_r} = \frac{55-r}{r+1}\cdot\frac{1}{2}\cdot\frac{1}{3} = \frac{55-r}{6(r+1)}.$$

Equality means $55-r = 6r+6$, hence $7r = 49$ and $r = 7$. This is an admissible index, so the equal coefficients multiply x^7 and x^8.

Step 3: Convert exponents to term numbers The constant term is term 1. Therefore x^7 occurs in term 8 and x^8 in term 9. The pair 7th and 8th is the common off-by-one error of identifying an exponent with a term number. The central pairs 27th/28th and 28th/29th would ignore the unequal constants 2 and $1/3$; their coefficient ratios are not one. The linear equation for r has a unique solution.

Step 4: Conclusion The equal coefficients occur in the 8th and 9th terms. Hence, the correct option is (B).

Quick Tip to Solve: Compare consecutive coefficients by their ratio, then add one to each exponent to obtain its term number.

8. If a line intercepted between the coordinate axes is trisected at a point $A(4,3)$, which is nearer to x-axis, then its equation is [4 marks]

(A) 4x $-$3y = 7

(B) 3x + 2y = 18

(C) 3x + 8y = 36

(D) x + 3y = 13

Correct Answer: (B) 3x + 2y = 18

Solution:

Concept:

- A line cutting the axes at $P(a,0)$ and $Q(0,b)$ has intercept form $x/a + y/b = 1$.
- The point of trisection nearer the x-axis is one third of the way from P to Q, so its coordinates are $(2a/3, b/3)$.

Step 1: Use the specified trisection point The distance of a point from the x-axis is the absolute value of its ordinate. Of the two internal trisection points of the intercepted segment, the one nearer this axis has ordinate $b/3$, rather than $2b/3$. Therefore the given point satisfies

$$(4,3) = \left(\frac{2a}{3}, \frac{b}{3}\right).$$

Comparing coordinates gives $a = 6$ and $b = 9$. These are the axis intercepts of the required line, not the coordinates of a second trisection point.

Step 2: Write and check the line equation Substitute the intercepts into intercept form:

$$\frac{x}{6} + \frac{y}{9} = 1 \quad \Longrightarrow \quad 3x + 2y = 18.$$

The point $(4,3)$ satisfies this equation since $12 + 6 = 18$. The other trisection point is $(2,6)$, whose distance from the x-axis is 6, whereas the given point has distance 3, confirming the required choice. Merely substituting A into the options does not settle the question: all four displayed lines pass through A. Options (A), (C) and (D) fail the specified intercept-trisection relation. In particular, (C) uses the other possible trisection ratio for the given point.

Step 3: Conclusion The required equation is $\boxed{3x + 2y = 18}$. Hence, the correct option is (B).

Quick Tip to Solve: For a trisection point nearer the x-axis, use one third of the y-intercept and two thirds of the x-intercept; passing through the given point alone is insufficient.

9. If the three distinct lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4ay + a = 0$ are concurrent, then the point (a, b) lies on a: [4 marks]

(A) circle

(B) hyperbola

(C) straight line

(D) parabola

Correct Answer: (C) straight line

Solution:

Concept:

- Concurrent lines pass through a common point. Subtracting their equations eliminates common terms and often locates that point.
- The word distinct excludes parameter choices that make two displayed line equations identical.

Step 1: Find the intersection of the first and third lines The first and third equations are

$$x + 2ay + a = 0, \qquad x + 4ay + a = 0.$$

Subtracting gives $2ay = 0$. We cannot take $a = 0$, since both equations would then be $x = 0$, contradicting the condition that the three lines are distinct. Consequently $y = 0$. Substitution in the first equation gives $x = -a$. Thus the common point, if it exists, must be $(-a, 0)$. This is the concurrency point in the coordinate plane of the given lines; it is different from the parameter point (a, b) whose locus is requested.

Step 2: Impose the second equation and identify the parameter locus The second line also passes through $(-a, 0)$, so

$$-a + 3b(0) + b = 0 \quad \Longrightarrow \quad b = a.$$

For every nonzero value of a with $b = a$, the three equations have different coefficients of y, namely $2a, 3a, 4a$, and pass through $(-a, 0)$. Thus the distinctness condition is respected. The parameter points lie on $b - a = 0$, with the origin excluded. This linear relation describes a straight line. A circle, hyperbola or parabola would require a nondegenerate quadratic relation; none is produced by the concurrency condition. Excluding the origin restricts the allowed points but does not change the line on which they lie.

Step 3: Conclusion The point (a, b) lies on the straight line $b = a$, with $a \neq 0$. Hence, the correct option is (C).

Quick Tip to Solve: When two line equations differ only in a parameter coefficient, subtract them first, but exclude parameter values that violate the distinct-lines condition.

10. For the two circles $x^2 + y^2 = 16$ and $x^2 + y^2 - 2y = 0$, there is/are: [4 marks]

(A) one pair of common tangents

(B) two pair of common tangents

(C) three pair of common tangents

(D) no common tangent

Correct Answer: (D) no common tangent

Solution:

Concept:

- For circles of radii $r_1 > r_2$ and centre distance d, strict containment occurs when $d < r_1 - r_2$.
- A line tangent to the outer circle cannot reach a strictly enclosed inner circle, so strictly nested circles have no common tangent.

Step 1: Find both centres and radii The first circle is $x^2 + y^2 = 16$, with centre $O_1 = (0, 0)$ and radius $r_1 = 4$. Complete the square for the second circle:

$$x^2 + y^2 - 2y = 0 \quad \Longrightarrow \quad x^2 + (y - 1)^2 = 1.$$

It has centre $O_2 = (0, 1)$ and radius $r_2 = 1$. The distance between their centres is therefore $d = 1$. It is essential to compare this distance with the difference of the radii, rather than only their sum, because one circle may lie completely inside the other.

Step 2: Classify the position and reject common tangency Here

$$d = 1 < 4 - 1 = 3 = r_1 - r_2.$$

Thus the smaller circle lies strictly inside the larger circle without touching it. Equivalently, every point of the smaller circle is at distance at most $d + r_2 = 2$ from the origin, which is strictly less than the larger radius 4. Every tangent to the larger circle is at perpendicular distance 4 from the origin and therefore cannot touch any point of the smaller circle. This directly rules out even a single common tangent, and hence also rules out the pairs claimed in (A), (B) and (C). The situation differs from internal tangency, where $d = r_1 - r_2$ and one common tangent would exist; the present inequality is strict.

Step 3: Conclusion The circles have $\boxed{\text{no common tangent}}$. Hence, the correct option is (D).

Quick Tip to Solve: Complete squares to read the radii, then compare the centre distance with both the sum and the difference of the radii before counting common tangents.

11. Two tangents are drawn from a point $(-2,-1)$ to the curve, $y^2 = 4x$. If α is the angle between them, then $|\tan\alpha|$ is equal to [4 marks]

(A) $\frac{1}{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\sqrt{3}$

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- A tangent of finite slope $m \neq 0$ to $y^2 = 4ax$ is $y = mx + a/m$; here $a = 1$.
- For slopes m_1, m_2, the angle between the lines satisfies $|\tan\alpha| = |(m_1 - m_2)/(1 + m_1 m_2)|$, unless the denominator vanishes.

Step 1: Find the two tangent slopes A tangent to $y^2 = 4x$ has equation $y = mx + 1/m$. Requiring passage through $(-2,-1)$ gives

$$-1 = -2m + \frac{1}{m}.$$

Multiplication by m, followed by rearrangement, yields

$$2m^2 - m - 1 = 0 = (2m+1)(m-1).$$

Thus the slopes are $m_1 = 1$ and $m_2 = -1/2$. Both are real and distinct, so there are indeed two tangents of the stated kind. The vertical tangent $x = 0$ does not pass through the given point and supplies no additional possibility.

Step 2: Evaluate the angle formula and compare the options Substitute the slopes:

$$|\tan\alpha| = \left|\frac{1-(-1/2)}{1+(1)(-1/2)}\right| = \frac{3/2}{1/2} = 3.$$

The denominator is nonzero, so the lines are not perpendicular. Direct substitution also verifies the two tangent equations $y = x + 1$ and $y = -x/2 - 2$, both of which contain $(-2,-1)$. Options (B) and (C) are familiar special-angle values but do not follow from these slopes. Option (A), $1/3$, is the reciprocal of the required tangent and would arise by interchanging numerator and denominator; it is not the value of the stated expression. Taking the absolute value makes the choice of the acute or supplementary angle immaterial.

Step 3: Conclusion $\boxed{|\tan\alpha| = 3}$. Hence, the correct option is (D).

Quick Tip to Solve: First solve the quadratic for the tangent slopes; then use their difference over one plus their product, keeping the denominator in that order.

12. The minimum area of a triangle formed by any tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{81}=1$ and the co-ordinate axes is: [4 marks]

(A) 12

(B) 18

(C) 26

(D) 36

Correct Answer: (D) 36

Solution:

Concept:

- At (x_1, y_1) on $x^2/a^2 + y^2/b^2 = 1$, the tangent is $xx_1/a^2 + yy_1/b^2 = 1$.
- A triangle with axis intercepts u, v has area $|uv|/2$. For nonnegative s, t with $s+t = 1$, their product is at most $1/4$.

Step 1: Express the tangent triangle area Here $a = 4$ and $b = 9$. Let (x_1, y_1) be a point of tangency with both coordinates nonzero. The tangent is

$$\frac{xx_1}{16}+\frac{yy_1}{81}=1.$$

Its intercepts are $u = 16/x_1$ and $v = 81/y_1$. Consequently the positive geometric area is

$$A=\frac{1}{2}\left|\frac{16}{x_1}\frac{81}{y_1}\right|=\frac{648}{|x_1y_1|}.$$

Tangents at an axis endpoint are parallel to an axis and do not form a finite triangle with both axes, so they do not furnish a smaller admissible area.

Step 2: Maximize the denominator and attain the bound Set $s = x_1^2/16$ and $t = y_1^2/81$. The point lies on the ellipse, so $s + t = 1$, and

$$|x_1y_1| = 36\sqrt{st} \le 36 \cdot \frac{1}{2} = 18.$$

It follows that $A \ge 648/18 = 36$. Equality is possible because $s = t = 1/2$ gives actual points of the ellipse, for example $x_1 = 4/\sqrt{2}$, $y_1 = 9/\sqrt{2}$. Their tangent intercepts are $4\sqrt{2}$ and $9\sqrt{2}$, giving area $(4\sqrt{2})(9\sqrt{2})/2 = 36$. Thus the bound is attained rather than merely approached. Each of 12, 18 and 26 is below this proved lower bound, so none of options (A), (B) and (C) is possible.

Step 3: Conclusion The minimum area is $\boxed{36}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: For an ellipse with semiaxes a and b, the least tangent-intercept triangle area is ab; equality occurs when the squared normalized coordinates are equal.

13. Let p, q, r denote arbitrary statements. Then the logically equivalent of the statement p $\Rightarrow$(q $\vee$r) is [4 marks]

(A) (p $\vee$q) $\Rightarrow$r

(B) (p $\Rightarrow$q) $\vee$(p $\Rightarrow$r)

(C) (p $\Rightarrow\sim$q) $\wedge$(p $\Rightarrow$r)

(D) (p $\Rightarrow$q) $\wedge$(p $\Rightarrow\sim$r)

Correct Answer: (B) (p $\Rightarrow$q) $\vee$(p $\Rightarrow$r)

Solution:

Concept:

- An implication $u \Rightarrow v$ is equivalent to $\neg u \vee v$; it is false only when u is true and v is false.
- Disjunction is associative and idempotent, so repeated copies of a statement can be removed from an OR expression.

Step 1: Rewrite the given statement without implication The given statement becomes

$$p \Rightarrow (q \vee r) \equiv \neg p \vee q \vee r.$$

This says that either p is false, or at least one of q, r is true. Now rewrite option (B):

$$(p \Rightarrow q) \vee (p \Rightarrow r) \equiv (\neg p \vee q) \vee (\neg p \vee r).$$

Combining the disjunctions and removing the repeated $\neg p$ yields precisely $\neg p \vee q \vee r$. Thus (B) is equivalent for all assignments of truth values, not merely for a particular example.

Step 2: Check that the other options fail For option (A), choose p false, q true and r false. The original implication is true because its antecedent is false, whereas $(p \vee q) \Rightarrow r$ is false. For option (C), choose p true, q true and r false. The original is true but the factor $p \Rightarrow \neg q$ is false, making (C) false. For option (D), choose p true, q false and r true. The original is true but $p \Rightarrow q$ is false. These counterexamples establish that no other listed option is universally equivalent. A conjunction of implications imposes two simultaneous conditions; it cannot replace the disjunctive condition in the given consequent.

Step 3: Conclusion The equivalent statement is $\boxed{(p \Rightarrow q) \vee (p \Rightarrow r)}$. Hence, the correct option is (B).

Quick Tip to Solve: Replace every implication by not-antecedent OR consequent, simplify repeated terms, and use a single truth assignment to disprove each remaining candidate.

14. Let $\overline{X}$ and M.D. be the mean and the mean deviation about $\overline{X}$ of n observations $x_i,\ i = 1, 2, \ldots, n$. If each of the observations is increased by 5, then the new mean and the mean deviation about the new mean, respectively, are: [4 marks]

(A) $\overline{X},\ \text{M.D.}$

(B) $\overline{X} + 5,\ \text{M.D.}$

(C) $\overline{X},\ \text{M.D.} + 5$

(D) $\overline{X} + 5,\ \text{M.D.} + 5$

Correct Answer: (B) $\overline{X} + 5,\ \text{M.D.}$

Solution:

Concept:

- The arithmetic mean is $\overline{X} = (1/n)\sum_{i=1}^{n} x_i$, and mean deviation about it is $\text{M.D.} = (1/n)\sum_{i=1}^{n} |x_i - \overline{X}|$.
- Adding the same constant to every observation shifts the centre by that constant but leaves all deviations from the new centre unchanged.

Step 1: Compute the mean after the increase Let the new observations be $y_i = x_i + 5$. Their mean is

$$\overline{Y} = \frac{1}{n}\sum_{i=1}^{n}(x_i + 5) = \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{5n}{n} = \overline{X} + 5.$$

The addition is performed once for each observation, so the sum rises by $5n$ and the mean rises by 5. It does not remain fixed, as options (A) and (C) would require. This reasoning is valid for any number of observations and for arbitrary values, positive or negative.

Step 2: Compute the new absolute deviations For each individual observation,

$$y_i - \overline{Y} = (x_i + 5) - (\overline{X} + 5) = x_i - \overline{X}.$$

Taking absolute values preserves equality. Averaging gives

$$\frac{1}{n}\sum_{i=1}^{n} |y_i - \overline{Y}| = \frac{1}{n}\sum_{i=1}^{n} |x_i - \overline{X}| = \text{M.D.}$$

Thus the mean deviation about the new mean is exactly the original mean deviation. In particular, it does not increase by 5, so option (D) fails even though its new mean is correct. The common translation changes location but not spread. One must measure the new deviations about the new mean; measuring about the old mean would answer a different question.

Step 3: Conclusion The new mean and mean deviation are $\boxed{\overline{X} + 5,\ \text{M.D.}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a common additive shift, shift the mean equally and cancel the added constant inside each deviation before computing any absolute values.

15. A relation on the set $A = \{x : |x| < 3,\ x \in \mathbb{Z}\}$, where $\mathbb{Z}$ is the set of integers, is defined by $R = \{(x, y) : y = |x|,\ x \neq -1\}$. Then the number of elements in the power set of R is: [4 marks]

(A) 32

(B) 16

(C) 8

(D) 64

Correct Answer: (B) 16

Solution:

Concept:

- A relation on a set is a set of ordered pairs satisfying its defining condition. Count distinct pairs, rather than distinct second coordinates.
- For a finite set with n elements, every element can either belong or not belong to a subset. Its power set therefore contains 2^n subsets.

Step 1: List the elements of the underlying set The strict inequality $|x| < 3$, together with the integer condition, gives

$$A = \{-2, -1, 0, 1, 2\}.$$

The endpoints -3 and 3 are excluded. Since the relation is on A, both coordinates must belong to this set; each absolute value needed here does belong to it.

Step 2: Apply both conditions defining the relation The condition $x \neq -1$ removes just the input -1. Substituting each remaining input into $y = |x|$ gives

$$R = \{(-2, 2), (0, 0), (1, 1), (2, 2)\}.$$

The pairs $(-2, 2)$ and $(2, 2)$ are different because their first coordinates differ, even though their second coordinates coincide. Thus $|R| = 4$, not three.

Step 3: Count all subsets and reject the other counts There are two independent choices for including each of these four pairs, so

$$|\mathcal{P}(R)| = 2^4 = 16.$$

The count includes both the empty subset and R itself. The option 32 would count all five possible inputs before the exclusion; 8 would wrongly count only three pairs; 64 would require six elements in the relation, which are not present.

Step 4: Conclusion The relation contains four distinct ordered pairs and its power set contains 16 elements. Hence, the correct option is (B).

Quick Tip to Solve: Enumerate the ordered pairs before applying 2^n; repeated output values do not make two ordered pairs identical.

16. If $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ **be such that** $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$**, then:** [4 marks]

(A) y = 2x

(B) y = −2x

(C) y = x

(D) y = −x

Correct Answer: (A) y = 2x

Solution:

Concept:

- The product of a row and a column is the sum of the products of corresponding entries. A 2×3 matrix multiplied by a 3×1 matrix gives a 2×1 column.
- Equality of column matrices means equality of corresponding entries, producing two simultaneous linear equations for x and y.

Step 1: Multiply the matrices row by row The first row contributes $1 \cdot y + 2 \cdot x + x \cdot 1 = y + 3x$. The second row contributes $3y - x + 2$. Therefore

$$AB = \begin{bmatrix} y + 3x \\ 3y - x + 2 \end{bmatrix}.$$

Equating this with the specified column gives

$$y + 3x = 6, \qquad 3y - x = 6.$$

The constant 2 in the second row must be moved to the other side of the equation $3y - x + 2 = 8$.

Step 2: Solve the simultaneous equations From the first equation, $y = 6 - 3x$. Substitute this in the second:

$$3(6 - 3x) - x = 6 \quad \Longrightarrow \quad 10x = 12.$$

Thus

$$x = \frac{6}{5}, \qquad y = 6 - \frac{18}{5} = \frac{12}{5}.$$

Substitution verifies $y + 3x = 30/5 = 6$ and $3y - x + 2 = 36/5 - 6/5 + 2 = 8$, so both entries of the target column are satisfied.

Step 3: Compare the stated relationships The computed values obey $y = 2x$. They cannot satisfy $y = -2x$ or $y = -x$, since both are positive and nonzero. They do not satisfy $y = x$, because $12/5 \neq 6/5$. This checks all the alternatives directly rather than choosing a relation from only one of the matrix equations.

Step 4: Conclusion The values $x = 6/5$ and $y = 12/5$ give $y = 2x$. Hence, the correct option is (A).

Quick Tip to Solve: Write one scalar equation per output entry before solving; the last column contributes constants as well as variable terms.

17. If $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix} = k\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$, $\lambda \neq 0$, **then** k **is equal to :** [4 marks]

(A) $4\lambda abc$

(B) $-4\lambda abc$

(C) $4\lambda^2$

(D) $-4\lambda^2$

Correct Answer: (C) $4\lambda^2$

Solution:

Concept:

- Subtracting a multiple of one row from another leaves a determinant unchanged. A scalar common to a row can be taken outside the determinant.
- The determinant is linear in each row separately. A term with two identical rows is zero, so only independent row components contribute.

Step 1: Remove the common quadratic row Write the determinant on the left as D, and denote the row vectors (a^2, b^2, c^2), (a, b, c), and $(1, 1, 1)$ by U, V, W, respectively. Expanding the squares makes the three rows U, $U + 2\lambda V + \lambda^2 W$, and $U - 2\lambda V + \lambda^2 W$. Apply $R_2 \leftarrow R_2 - R_1$ and $R_3 \leftarrow R_3 - R_1$. These operations do not introduce a sign change, and give

$$D = \det \begin{bmatrix} U \\ 2\lambda V + \lambda^2 W \\ -2\lambda V + \lambda^2 W \end{bmatrix}.$$

Step 2: Combine the lower rows and extract factors Now use $R_3 \leftarrow R_3 + R_2$, retaining the already transformed second row. The third row becomes $2\lambda^2 W$. Consequently

$$D = 2\lambda^2 \det \begin{bmatrix} U \\ 2\lambda V + \lambda^2 W \\ W \end{bmatrix}$$

$$= 4\lambda^3 \det \begin{bmatrix} U \\ V \\ W \end{bmatrix}.$$

In the second equality the contribution involving W in the middle row vanishes, because it would repeat the last row. No row interchange has been used, so the coefficient is positive.

Step 3: Compare the multiplier with the given form The question writes the same determinant as $k\lambda \det[U; V; W]$. Comparing the determinant identity gives $k\lambda = 4\lambda^3$, hence $k = 4\lambda^2$ since $\lambda \neq 0$. This coefficient is independent of a, b, c: those quantities remain inside the common determinant, so the two alternatives containing abc are not the multiplier in the identity. The negative alternative has the wrong sign. Even if particular values make the common determinant zero, the row-operation identity determines the stated general multiplier.

Step 4: Conclusion The multiplier is $k = 4\lambda^2$. Hence, the correct option is (C).

Quick Tip to Solve: Expand shifted squares as row vectors, subtract the original row, then add the two new rows to isolate the constant row.

18. If $f(\theta) = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\sin\theta & 1 & -\cos\theta \\ -1 & \sin\theta & 1 \end{vmatrix}$ **and** A **and** B **are respectively the maximum and the minimum values of** $f(\theta)$**, then** (A, B) **is equal to:** [4 marks]

(A) $(3, -1)$

(B) $(4, 2-\sqrt{2})$

(C) $(2+\sqrt{2}, 2-\sqrt{2})$

(D) $(2+\sqrt{2}, -1)$

Correct Answer: (C) $(2+\sqrt{2}, 2-\sqrt{2})$

Solution:

Concept:

- Expand a third-order determinant along a convenient row, retaining the alternating cofactor signs $+, -, +$.
- The identities $2\sin\theta\cos\theta = \sin 2\theta$ and $\cos^2\theta - \sin^2\theta = \cos 2\theta$ reduce the result to a single sinusoid.

Step 1: Expand the determinant Put $s = \sin\theta$ and $c = \cos\theta$. Expansion along the first row gives

$$\begin{aligned} f(\theta) &= (1 + cs) - c(-s - c) + (1 - s^2) \\ &= 2 + 2sc + c^2 - s^2. \end{aligned}$$

In particular, the middle minor is $-s - c$, and its cofactor is subtracted. Retaining that sign is essential for obtaining both cross terms with the same sign.

Step 2: Express the variable terms as one sine Using the double-angle identities yields

$$f(\theta) = 2 + \sin 2\theta + \cos 2\theta = 2 + \sqrt{2}\sin\left(2\theta + \frac{\pi}{4}\right).$$

As θ varies over the real numbers, the argument of this sine takes all real values. Its extreme values -1 and 1 are both attained, so the range obtained from the amplitude is exact, rather than only a bound.

Step 3: Read the maximum and minimum in order The maximum is $A = 2 + \sqrt{2}$ and the minimum is $B = 2 - \sqrt{2}$. The extremes occur when the displayed sine equals 1 and -1, respectively. Options with minimum -1 fail because $2 - \sqrt{2} > 0$; the option with maximum 4 fails because the amplitude is $\sqrt{2}$, not 2. Thus only the third pair has both entries correct.

Step 4: Conclusion $(A, B) = (2+\sqrt{2}, 2-\sqrt{2})$. Hence, the correct option is (C).

Quick Tip to Solve: After determinant expansion, combine $\sin 2\theta + \cos 2\theta$ into an amplitude-$\sqrt{2}$ sinusoid to read both extrema immediately.

19. Statement I: The equation $(\sin^{-1} x)^3 + (\cos^{-1} x)^3 - a\pi^3 = 0$ has a solution for all $a \geq \frac{1}{32}$. Statement II: For any $x \in \mathbb{R}$, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ and $0 \leq (\sin^{-1} x - \frac{\pi}{4})^2 \leq \frac{9\pi^2}{16}$. [4 marks]

(A) Both statements I and II are true.

(B) Both statements I and II are false.

(C) Statement I is true and statement II is false.

(D) Statement I is false and statement II is true.

Correct Answer: (B) Both statements I and II are false.

Solution:

Concept:

- The real principal inverse sine and inverse cosine are defined only for $-1 \leq x \leq 1$. On that domain their sum is $\pi/2$.
- If $u + v = S$, then $u^3 + v^3 = S^3 - 3uvS$. Rewriting uv in terms of $(u - S/2)^2$ determines the exact range.

Step 1: Determine the range relevant to Statement I Let $u = \sin^{-1} x$ and $v = \cos^{-1} x = \pi/2 - u$. Since $u \in [-\pi/2, \pi/2]$,

$$u^3 + v^3 = \frac{\pi^3}{32} + \frac{3\pi}{2}\left(u - \frac{\pi}{4}\right)^2.$$

Here $u - \pi/4 \in [-3\pi/4, \pi/4]$. Its square ranges continuously from zero to $9\pi^2/16$, with the minimum at $x = 1/\sqrt{2}$ and the maximum at $x = -1$.

Step 2: Check whether every stated parameter is possible The preceding formula shows that the equation has a real solution precisely when

$$\frac{1}{32} \leq a \leq \frac{7}{8}.$$

It does not have a solution for every $a \geq 1/32$. For example, $a = 1$ exceeds the maximum $7/8$. Consequently Statement I is false: its missing upper restriction matters, even though the lower endpoint is correct.

Step 3: Check the domain in Statement II The sum identity and the square bounds are valid when $x \in [-1, 1]$. Statement II, however, asserts them for any real x. At $x = 2$, neither real inverse sine nor real inverse cosine is defined. Thus the universal assertion over $\mathbb{R}$ is false. The both-true option and either mixed option each require at least one true statement, so all three are ruled out.

Step 4: Conclusion Both statements I and II are false: I omits the upper bound on a, and II uses an invalid real domain. Hence, the correct option is (B).

Quick Tip to Solve: Check the principal inverse-function domain first, then test a parameter statement at a value beyond the exact maximum.

20. If $f(x) = x^2 - x + 5$, $x > \frac{1}{2}$, and $g(x)$ is its inverse function, then $g'(7)$ equals [4 marks]

(A) $-\frac{1}{3}$

(B) $\frac{1}{13}$

(C) $\frac{1}{3}$

(D) $-\frac{1}{13}$

Correct Answer: (C) $\frac{1}{3}$

Solution:

Concept:

- For an invertible differentiable function, differentiating $f(g(y)) = y$ gives $g\prime(y) = 1/f\prime(g(y))$, provided the denominator is nonzero.
- The prescribed restriction $x > 1/2$ selects the increasing branch of the quadratic and determines which preimage must be used.

Step 1: Find the preimage of the requested argument The number $g(7)$ is the allowed value of x for which $f(x) = 7$. Solve

$$x^2 - x + 5 = 7 \quad \Longrightarrow \quad x^2 - x - 2 = (x - 2)(x + 1) = 0.$$

The algebraic roots are 2 and -1. Only 2 lies in the given domain $x > 1/2$, so $g(7) = 2$. Using -1 would correspond to a different inverse branch and violate the question.

Step 2: Apply the reciprocal derivative formula The derivative of the original quadratic is $f\prime(x) = 2x - 1$. It is positive throughout the restricted domain, so the inverse exists there and the derivative formula is applicable. At the identified preimage,

$$f\prime(2) = 3, \qquad g\prime(7) = \frac{1}{f\prime(2)} = \frac{1}{3}.$$

The derivative is evaluated at $g(7)$, not at 7 itself.

Step 3: Check the alternative values The value $1/13$ comes from wrongly calculating $1/f\prime(7)$. The negative choice $-1/3$ would arise from the excluded root -1, while $-1/13$ has both the wrong sign and evaluation point. Since the chosen branch is increasing, its inverse must have positive derivative, which also rejects both negative answers.

Step 4: Conclusion $g\prime(7) = 1/3$. Hence, the correct option is (C).

Quick Tip to Solve: Solve $f(x) = y$ and select the allowed root before taking the reciprocal of $f\prime(x)$; do not substitute y directly into $f\prime$.

21. Let $f, g : \mathbb{R} \to \mathbb{R}$ be two functions defined by $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ and $g(x) = xf(x)$.

Statement I : f is a continuous function at $x = 0$.

Statement II : g is a differentiable function at $x = 0$. [4 marks]

(A) Both statement I and II are false.

(B) Both statement I and II are true.

(C) Statement I is true, statement II is false.

(D) Statement I is false, statement II is true.

Correct Answer: (B) Both statement I and II are true.

Solution:

Concept:

- The squeeze theorem controls an oscillating sine by using $|\sin t| \leq 1$, without requiring the sine itself to have a limit.
- Continuity at zero requires the function limit to equal its value there. Differentiability at zero is checked from the difference quotient rather than a formula valid only away from zero.

Step 1: Test continuity of f directly For every nonzero x, the bound on sine gives

$$|f(x)| = \left| x \sin \frac{1}{x} \right| \leq |x|.$$

The right side tends to zero as $x \to 0$, so the squeeze theorem yields $\lim_{x \to 0} f(x) = 0$. Since the stated definition has $f(0) = 0$, the limit equals the function value. Thus Statement I is true, despite the increasingly rapid oscillations of the sine factor.

Step 2: Form the difference quotient for g By the definition $g(x) = xf(x)$, one has $g(0) = 0$ and $g(x) = x^2 \sin(1/x)$ for nonzero x. The derivative at zero is therefore

$$g\prime(0) = \lim_{h \to 0} \frac{g(h) - g(0)}{h} = \lim_{h \to 0} h \sin \frac{1}{h}.$$

Its absolute value is at most $|h|$, so the limit exists and equals zero. Statement II is consequently true.

Step 3: Distinguish oscillation from failure of a limit The sine factor alone has no limit at zero, but multiplication by a sufficiently small power of the variable forces the required expression to zero. Continuity of f and differentiability of g are exactly the two properties just proved; no assertion about differentiability of f is needed. Every other listed alternative declares at least one of these established statements false and is therefore excluded.

Step 4: Conclusion Statement I and Statement II are both true, with $g\prime(0) = 0$. Hence, the correct option is (B).

Quick Tip to Solve: For oscillating expressions near zero, bound the absolute value first and apply the derivative definition at the special point.

22. Let f and g be two differentiable functions on $\mathbb{R}$ such that $f'(x) > 0$ and $g'(x) < 0$ for all $x \in \mathbb{R}$. Then for all x [4 marks]

(A) f(g(x)) $>$ f(g(x -1))

(B) f(g(x)) $>$ f(g(x + 1))

(C) g(f(x)) $>$ g(f(x -1))

(D) g(f(x)) $<$ g(f(x + 1))

Correct Answer: (B) f(g(x)) $>$ f(g(x + 1))

Solution:

Concept:

- A differentiable function with positive derivative everywhere is strictly increasing; one with negative derivative everywhere is strictly decreasing.
- An increasing outer function preserves inequalities, whereas a decreasing outer function reverses them. Both compositions here have negative derivative by the chain rule.

Step 1: Compare the inner function values Since $x - 1 < x < x + 1$ and g is strictly decreasing,

$$g(x-1) > g(x) > g(x+1).$$

Applying the strictly increasing function f preserves both signs, giving

$$f(g(x-1)) > f(g(x)) > f(g(x+1)).$$

Thus option (A) reverses the first inequality, while option (B) states the second one correctly.

Step 2: Check the other composition The function f is strictly increasing, so

$$f(x-1) < f(x) < f(x+1).$$

Applying the decreasing function g reverses these inequalities:

$$g(f(x-1)) > g(f(x)) > g(f(x+1)).$$

Hence (C) and (D) both have the wrong direction. Equivalently, $(g \circ f)\prime(x) = g\prime(f(x))f\prime(x) < 0$, while $(f \circ g)\prime(x) = f\prime(g(x))g\prime(x) < 0$. These comparisons hold for every real x, because the derivative assumptions are global.

Step 3: Conclusion For all real x, $f(g(x)) > f(g(x+1))$. Hence, the correct option is (B).

Quick Tip to Solve: When composing monotone functions, count the decreasing functions: exactly one decreasing function makes the composition decreasing.

23. The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2}\,dx$ is equal to: [4 marks]

(A) $\frac{1}{1+\cot^3 x} + c$

(B) $-\frac{1}{3(1+\tan^3 x)} + c$

(C) $\frac{\sin^3 x}{1+\cos^3 x} + c$

(D) $-\frac{\cos^3 x}{3(1+\sin^3 x)} + c$

Correct Answer: (B) $-\frac{1}{3(1+\tan^3 x)} + c$

Solution:

Concept:

- For $t = \tan x$, $dt = \sec^2 x\,dx$. Factoring powers of $\cos x$ converts this integral into a rational function of t.
- The derivative of $(1+t^3)^{-1}$ is $-3t^2(1+t^3)^{-2}$, which gives the required antiderivative directly.

Step 1: Convert to a rational integral Work on an interval where the original integrand is defined and $\cos x \neq 0$. Since $\sin^3 x + \cos^3 x = \cos^3 x(1+\tan^3 x)$,

$$I = \int \frac{\tan^2 x \sec^2 x}{(1+\tan^3 x)^2}\,dx.$$

Put $t = \tan x$, obtaining

$$I = \int \frac{t^2}{(1+t^3)^2}\,dt.$$

Now set $u = 1 + t^3$, so $du = 3t^2 dt$. The square in the original denominator is essential.

Step 2: Integrate and check the options It follows that

$$I = \frac{1}{3}\int u^{-2}\,du = -\frac{1}{3u} + c = -\frac{1}{3(1+\tan^3 x)} + c.$$

Differentiating this expression gives $\tan^2 x \sec^2 x/(1+\tan^3 x)^2$, exactly the original integrand. Option (A) equals $\tan^3 x/(1+\tan^3 x) + c$; its derivative is three times the required integrand. Options (C) and (D) have denominators $1+\cos^3 x$ and $1+\sin^3 x$, respectively, rather than the sum of cubes needed here, and their derivatives do not match.

Step 3: Conclusion $I = -\frac{1}{3(1+\tan^3 x)} + c$, on each interval of definition. Hence, the correct option is (B).

Quick Tip to Solve: For a denominator involving equal powers of sine and cosine, divide by the appropriate power of cosine and try the tangent substitution.

24. If $[\,]$ denotes the greatest integer function, then the integral $\int_0^{\pi} [\cos x]\,dx$ is equal to: [4 marks]

(A) $\frac{\pi}{2}$
(B) 0
(C) -1
(D) $-\frac{\pi}{2}$

Correct Answer: (D) $-\frac{\pi}{2}$

Solution:

Concept:

- The greatest integer $[u]$ is the largest integer not exceeding u; in particular a negative number between -1 and zero has greatest integer -1.
- Changing the integrand at finitely many points does not change a definite integral. Split the interval where the sign of cosine changes.

Step 1: Find the constant values on each interval For $0 < x < \pi/2$, we have $0 < \cos x < 1$, hence $[\cos x] = 0$. For $\pi/2 < x < \pi$, we have $-1 < \cos x < 0$, hence $[\cos x] = -1$. At the isolated endpoints, $[\cos 0] = 1$, $[\cos(\pi/2)] = 0$, and $[\cos \pi] = -1$. These single-point values do not contribute any area and therefore do not affect the calculation.

Step 2: Add the signed areas The integral is therefore

$$I = \int_0^{\pi/2} 0\,dx + \int_{\pi/2}^{\pi} (-1)\,dx$$
$$= 0 - \left(\pi - \frac{\pi}{2}\right) = -\frac{\pi}{2}.$$

The floor function destroys the cancellation that occurs for the ordinary cosine integral. In particular, the negative half contributes a rectangle of height -1, while the positive half contributes zero. Option (A) has the wrong sign, (B) incorrectly treats the integrand as an odd cancellation about the midpoint, and (C) confuses the function value on the second interval with its integral over an interval of length $\pi/2$.

Step 3: Conclusion $\int_0^{\pi} [\cos x]\,dx = -\frac{\pi}{2}$. Hence, the correct option is (D).

Quick Tip to Solve: For floor-function integrals, first identify intervals on which the floor is constant; remember that the floor of a small negative number is minus one.

25. If for a continuous function $f(x)$, $\int_{-\pi}^{t}(f(x)+x)\,dx = \pi^2 - t^2$, for all $t \geq -\pi$, then $f\left(-\frac{\pi}{3}\right)$ is equal to: [4 marks]

(A) π
(B) $\frac{\pi}{2}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{6}$

Correct Answer: (A) π

Solution:

Concept:

- If the integrand is continuous, differentiating an integral with variable upper limit returns its integrand evaluated at that limit.
- Differentiate both sides with respect to the upper limit t. The fixed lower limit contributes no derivative.

Step 1: Differentiate the given identity The supplied identity holds for every $t \geq -\pi$, and f is continuous. The fundamental theorem of calculus therefore gives, for interior values of this interval,

$$\frac{d}{dt}\int_{-\pi}^{t}(f(x)+x)\,dx = f(t)+t.$$

The derivative of the right-hand side is $-2t$. Consequently,

$$f(t)+t = -2t, \qquad f(t) = -3t.$$

The integration variable x is a dummy variable and is replaced by the upper limit only after differentiation.

Step 2: Evaluate the required function value The point $-\pi/3$ belongs to the interval in which the identity holds. Substitute it into the expression for f:

$$f\left(-\frac{\pi}{3}\right) = -3\left(-\frac{\pi}{3}\right) = \pi.$$

As a consistency check, substituting $f(x) = -3x$ back gives

$$\int_{-\pi}^{t}(-2x)\,dx = [-x^2]_{-\pi}^{t} = \pi^2 - t^2,$$

which reproduces the original equation including the lower-limit constant. Options (B), (C), and (D) give $\pi/2, \pi/3, \pi/6$, none of which equals the uniquely determined value π.

Step 3: Conclusion $f(-\pi/3) = \pi$. Hence, the correct option is (A).

Quick Tip to Solve: For an identity containing a variable upper limit, differentiate first; the original integral then provides a useful check on any constant terms.

26. The general solution of the differential equation $\sin 2x\left(\frac{dy}{dx}-\sqrt{\tan x}\right)-y=0$ is: [4 marks]

(A) $y\sqrt{\tan x}=x+c$

(B) $y\sqrt{\cot x}=\tan x+c$

(C) $y\sqrt{\tan x}=\cot x+c$

(D) $y\sqrt{\cot x}=x+c$

Correct Answer: (D) $y\sqrt{\cot x}=x+c$

Solution:

Concept:

- A first-order linear equation $y'+P(x)y=Q(x)$ has integrating factor $\exp(\int P(x)\,dx)$. Multiplying by it converts the left side into a product derivative.
- Here $P(x)=-1/\sin 2x$. Since $d(\ln|\tan x|)/dx=2/\sin 2x$, the integrating factor is $1/\sqrt{\tan x}$ on an interval with $\tan x>0$.

Step 1: Write the linear equation and integrating factor On an interval where the real differential equation is defined and $\sin 2x\neq 0$, divide through to obtain

$$y'-\frac{y}{\sin 2x}=\sqrt{\tan x}.$$

The integrating factor is

$$\exp\left(-\int\frac{dx}{\sin 2x}\right)=\exp\left(-\frac{1}{2}\ln|\tan x|\right)=\sqrt{\cot x}.$$

We use the positive square root on intervals where $\tan x>0$, as required for this real expression.

Step 2: Integrate the product derivative and verify Multiply the linear equation by $\sqrt{\cot x}$. The right side becomes one and the left becomes a product derivative:

$$\frac{d}{dx}\left(y\sqrt{\cot x}\right)=1.$$

Thus

$$y\sqrt{\cot x}=x+c.$$

To check, write $y=(x+c)\sqrt{\tan x}$; then $y'=\sqrt{\tan x}+y/\sin 2x$, reproducing the original equation. Options (A) and (C) use the reciprocal integrating factor, so their product derivatives have the wrong sign of the coefficient. Option (B) has derivative $\sec^2 x$ instead of one on the right.

Step 3: Conclusion The general solution is $y\sqrt{\cot x}=x+c$, on each appropriate interval of definition. Hence, the correct option is (D).

Quick Tip to Solve: Use the identity $\int\csc 2x\,dx=\frac{1}{2}\ln|\tan x|$ to find the integrating factor quickly, then check the product derivative.

27. If $\hat{x}, \hat{y}, \hat{z}$ are three unit vectors in three-dimensional space, then the minimum value of $|\hat{x}+\hat{y}|^2 + |\hat{y}+\hat{z}|^2 + |\hat{z}+\hat{x}|^2$ is: [4 marks]

(A) $\frac{3}{2}$

(B) 3

(C) $3\sqrt{3}$

(D) 6

Correct Answer: (B) 3

Solution:

Concept:

- For vectors $\mathbf{a}, \mathbf{b}$, $|\mathbf{a}+\mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a}\cdot\mathbf{b}$. A squared norm is always nonnegative.
- The three unit vectors have total squared length three. Relating the desired sum to the norm of their total gives a sharp lower bound.

Step 1: Rewrite the sum using the total vector Let $S = |\hat{x}+\hat{y}|^2 + |\hat{y}+\hat{z}|^2 + |\hat{z}+\hat{x}|^2$, and let $T = \hat{x}\cdot\hat{y} + \hat{y}\cdot\hat{z} + \hat{z}\cdot\hat{x}$. Expanding and using the unit lengths gives

$$S = 6 + 2T.$$

On the other hand,

$$|\hat{x}+\hat{y}+\hat{z}|^2 = 3 + 2T.$$

Eliminating T yields

$$S = 3 + |\hat{x}+\hat{y}+\hat{z}|^2 \geq 3.$$

This bound accounts for the restrictions among the three pairwise dot products; they cannot independently all equal minus one.

Step 2: Show that the bound is attained Choose three coplanar unit vectors separated by 120°:

$$\hat{x} = (1,0,0), \quad \hat{y} = (-1/2, \sqrt{3}/2, 0), \quad \hat{z} = (-1/2, -\sqrt{3}/2, 0).$$

Their vector sum is zero, so the squared norm in the bound vanishes and $S = 3$. Each pairwise sum has unit length, giving the same result directly. Thus (A) lies below the rigorous bound, whereas (C) and (D) exceed an attainable value and cannot be minima.

Step 3: Conclusion The minimum value is 3, attained when $\hat{x}+\hat{y}+\hat{z} = 0$. Hence, the correct option is (B).

Quick Tip to Solve: To minimize sums of squared pairwise vector sums, express them in terms of the squared norm of the total vector and then check equality.

28. A symmetrical form of the line of intersection of the planes $x = ay + b$ and $z = cy + d$ is: [4 marks]

(A) $\frac{x-b}{a} = \frac{y-1}{1} = \frac{z-d}{c}$

(B) $\frac{x-b-a}{a} = \frac{y-1}{1} = \frac{z-d-c}{c}$

(C) $\frac{x-a}{b} = \frac{y-0}{1} = \frac{z-c}{d}$

(D) $\frac{x-b-a}{b} = \frac{y-1}{0} = \frac{z-d-c}{d}$

Correct Answer: (B) $\frac{x-b-a}{a} = \frac{y-1}{1} = \frac{z-d-c}{c}$

Solution:

Concept:

- The line satisfying $x = ay+b$ and $z = cy+d$ can be parametrized by taking y itself as the parameter.
- A line through (x_0, y_0, z_0) with direction ratios (l, m, n) has symmetric form $(x - x_0)/l = (y - y_0)/m = (z - z_0)/n$, with zero-denominator cases interpreted through its parametric equations.

Step 1: Find a point and direction on the intersection Put $y = t$. The two plane equations then give

$$x = at + b, \qquad y = t, \qquad z = ct + d.$$

Hence the direction ratios are $(a, 1, c)$. Taking $t = 1$ gives the point $(a + b, 1, c + d)$, which satisfies both planes. Writing $t = 1 + s$ displays the same line as

$$x = a + b + as, \quad y = 1 + s, \quad z = c + d + cs.$$

The coefficient of y ensures a valid line parameter even if either a or c is zero.

Step 2: Convert to symmetric form and eliminate other choices Eliminating s gives

$$\frac{x-b-a}{a} = \frac{y-1}{1} = \frac{z-d-c}{c}.$$

Conversely, setting all three ratios equal to s recovers $x = a(y - 1) + a + b = ay + b$ and $z = c(y - 1) + c + d = cy + d$, so the equation satisfies both planes identically. Option (A) combines the point corresponding to $y = 0$ with $y - 1$, which shifts the line incorrectly. Option (C) has the wrong point and direction in general. Option (D) gives zero change in y and direction ratios $(b, 0, d)$, inconsistent with the parametrization.

Step 3: Conclusion The required form is $\frac{x-b-a}{a} = \frac{y-1}{1} = \frac{z-d-c}{c}$. Hence, the correct option is (B).

Quick Tip to Solve: Take one coordinate as the parameter, then choose the reference point matching the coordinate shift printed in the options.

29. If the distance between planes, $4x-2y-4z+1=0$ and $4x-2y-4z+d=0$ is 7, then d is: [4 marks]

(A) 41 or -42

(B) 42 or -43

(C) -41 or 43

(D) -42 or 44

Correct Answer: (C) -41 or 43

Solution:

Concept:

- Two planes with proportional normal vectors are parallel. Their distance is the absolute difference of their constant terms after their normal coefficients have been made identical.
- For planes $ax+by+cz+d_1=0$ and $ax+by+cz+d_2=0$, the perpendicular distance is $D=|d_2-d_1|/\sqrt{a^2+b^2+c^2}$. The absolute value retains both possible relative positions.

Step 1: Identify the common normal The coefficients of the coordinates are identical in the two equations, so their common normal is $\mathbf{n}=4\hat{\mathbf{i}}-2\hat{\mathbf{j}}-4\hat{\mathbf{k}}$. Its magnitude is

$$|\mathbf{n}|=\sqrt{4^2+(-2)^2+(-4)^2}=\sqrt{36}=6.$$

We must divide by this magnitude: the constant difference alone is not the geometric separation.

Step 2: Use the prescribed separation The first constant is 1 and the second is d. Inserting the distance 7 gives

$$7=\frac{|d-1|}{6}, \qquad |d-1|=42.$$

Either the second plane lies on one side of the first, or it lies on the other side. Therefore both signs of the constant difference are necessary.

Step 3: Solve both signs and compare The positive case gives $d-1=42$, hence $d=43$. The negative case gives $d-1=-42$, hence $d=-41$. Each value gives an absolute constant difference of 42 and therefore distance 7. Option (A) uses 41 or -42, option (B) uses 42 or -43, and option (D) uses -42 or 44; none reproduces the required pair.

Step 4: Conclusion

$$\boxed{d=-41 \text{ or } 43}.$$

Hence, the correct option is (C).

Quick Tip to Solve: Keep the absolute value until the last step; a fixed distance between parallel planes normally gives two possible constants.

30. A number x is chosen at random from the set $\{1, 2, 3, 4, \ldots, 100\}$. Define the event: $A =$ the chosen number x satisfies $\dfrac{(x-10)(x-50)}{x-30} \geq 0$. Then $P(A)$ is: [4 marks]

(A) 0.71

(B) 0.70

(C) 0.51

(D) 0.20

Correct Answer: (A) 0.71

Solution:

Concept:

- For a uniformly selected integer, probability equals the number of favourable integers divided by the total number of integers. All 100 integers here are equally likely.
- A rational expression changes sign only at numerator zeros and denominator zeros. A denominator zero is excluded even when a neighbouring interval satisfies the inequality.

Step 1: Find the critical integers Write the expression as $f(x) = (x-10)(x-50)/(x-30)$. Its numerator vanishes at 10 and 50, while it is undefined at 30. Divide the real line into the four intervals separated by these three values. The zeros 10 and 50 may be included because the inequality is non-strict.

Step 2: Determine the accepted intervals For $x < 10$, the two numerator factors are negative and the denominator is negative, so $f(x) < 0$. For $10 < x < 30$, the numerator and denominator are both negative, giving $f(x) > 0$. For $30 < x < 50$, the numerator is negative but the denominator is positive, so $f(x) < 0$. For $x > 50$, every factor is positive. Thus the admissible real values are $[10, 30) \cup [50, \infty)$.

Step 3: Count integers in the sample space The first accepted group is $10, 11, \ldots, 29$, containing $29 - 10 + 1 = 20$ integers. The second is $50, 51, \ldots, 100$, containing $100 - 50 + 1 = 51$ integers. The groups do not overlap, so there are 71 favourable outcomes. Including 30 would be an error because the denominator there is zero; excluding 50 would incorrectly discard a permitted zero.

Step 4: Compare the probabilities Therefore $P(A) = (20 + 51)/100 = 0.71$. The value 0.70 misses an allowed endpoint, 0.51 counts only the upper group, and 0.20 counts only the lower group. Both accepted intervals must contribute to the probability.

Step 5: Conclusion

$$\boxed{P(A) = 0.71}.$$

Hence, the correct option is (A).

Quick Tip to Solve: Mark numerator zeros with closed endpoints and denominator zeros with open endpoints before counting integers.

Physics (120 marks)

31. From the following combinations of physical constants (expressed through their usual symbols) the only combination, that would have the same value in different systems of units, is [4 marks]

(A) $\frac{ch}{2\pi\varepsilon_0^2}$

(B) $\frac{e^2}{2\pi\varepsilon_0 G m_e^2}$

(C) $\frac{\mu_0\varepsilon_0}{c^2}\frac{G}{he^2}$

(D) $\frac{2\pi\sqrt{\mu_0\varepsilon_0}}{ce^2}\frac{h}{G}$

Correct Answer: (B) $\frac{e^2}{2\pi\varepsilon_0 G m_e^2}$

Solution:

Concept:

- A combination has the same numerical value in all systems of units only when it is dimensionless. A dimensionful quantity changes its numerical measure when its base units change.
- Write dimensions using mass M, length L, time T and charge Q. Coulomb's law gives $[\varepsilon_0] = Q^2T^2M^{-1}L^{-3}$, while $[G] = M^{-1}L^3T^{-2}$.
- The remaining dimensions are $[h] = ML^2T^{-1}$, $[c] = LT^{-1}$, $[m_e] = M$, and $[e] = Q$. Numerical factors such as 2π do not affect dimensions.

Step 1: Test the electrostatic and gravitational combination For option (B), the denominator apart from its numerical factor has dimensions

$$[\varepsilon_0 G m_e^2] = (Q^2T^2M^{-1}L^{-3})(M^{-1}L^3T^{-2})M^2 = Q^2.$$

The numerator e^2 also has dimensions Q^2. Hence the quotient has no dimensions. It is proportional to the ratio of the electrostatic and gravitational forces between two electrons at the same separation; the common inverse-square distance dependence cancels.

Step 2: Exclude the other combinations For option (A), direct substitution gives

$$[ch/\varepsilon_0^2] = M^3L^9T^{-6}Q^{-4},$$

so it is not dimensionless. For the remaining options use $\mu_0\varepsilon_0 = 1/c^2$, whose dimensions are T^2L^{-2}. Option (C) consequently has dimensions

$$[\mu_0\varepsilon_0 G/(c^2he^2)] = M^{-2}L^{-3}T^3Q^{-2},$$

whereas option (D) has dimensions

$$[\sqrt{\mu_0\varepsilon_0}h/(ce^2G)] = M^2L^{-3}T^3Q^{-2}.$$

Both retain nonzero powers of the base dimensions. Thus none of (A), (C) or (D) can have a

unit-independent numerical value.

Step 3: Conclusion Only $\boxed{e^2/(2\pi\varepsilon_0 G m_e^2)}$ is dimensionless and therefore retains its numerical value on changing the system of units. Hence, the correct option is (B).

Quick Tip to Solve: Recognise $e^2/(\varepsilon_0 G m_e^2)$ as an electric-to-gravitational force ratio; this identifies the dimensionless expression before doing a full dimensional table.

32. A person climbs up a stalled escalator in 60 s. If standing on the same but escalator running with constant velocity he takes 40 s. How much time is taken by the person to walk up the moving escalator? [4 marks]

(A) 37 s

(B) 27 s

(C) 24 s

(D) 45 s

Correct Answer: (C) 24 s

Solution:

Concept:

- The walking velocity relative to the steps and the velocity of the steps relative to the ground add when both motions are in the same direction.
- For a fixed path length L, speed is L/t. Working with fractions of the escalator length per second eliminates the unknown physical length.

Step 1: Find the walking speed Let the length of the escalator along its inclined path be L. When the escalator is stalled, all displacement is produced by walking. The time is 60 s, so the walking speed relative to the steps is

$$v_w = \frac{L}{60}.$$

This is the speed the person maintains relative to the steps while walking on the moving escalator as well.

Step 2: Find the escalator speed When the person stands still on the moving steps, the escalator alone carries the person through distance L in 40 s. Thus its speed relative to the ground is

$$v_e = \frac{L}{40}.$$

The standing time measures the escalator speed; it must not be treated as an additional waiting time to be added to the walking time.

Step 3: Add the two speeds When the person walks upward on the upward-moving escalator, the velocities point in the same direction. The ground speed is consequently

$$v = v_w + v_e = L\left(\frac{1}{60} + \frac{1}{40}\right) = \frac{5L}{120} = \frac{L}{24}.$$

The length cancels in the time calculation, giving $t = L/v = 24\,\text{s}$.

Step 4: Check the result The combined motion must be faster than either walking alone or riding alone, so its time must be below both 60 s and 40 s. The exact reciprocal-time calculation selects 24 s. The values 37 s, 27 s and 45 s do not satisfy $1/t = 1/60 + 1/40$; 45 s even exceeds the time for riding without walking.

Step 5: Conclusion

$$\boxed{t = 24\,\text{s}}.$$

Hence, the correct option is (C).

Quick Tip to Solve: For the same escalator length and motion in the same direction, add reciprocal times: $1/t = 1/t_w + 1/t_e$.

33. A bullet of mass 4 g is fired horizontally with a speed of 300 m/s into 0.8 kg block of wood at rest on a table. If the coefficient of friction between the block and the table is 0.3, how far will the block slide approximately? [4 marks]

(A) 0.19 m

(B) 0.379 m

(C) 0.569 m

(D) 0.758 m

Correct Answer: (B) 0.379 m

Solution:

Concept:

- When a bullet embeds in a wooden block, their brief impact is completely inelastic. Horizontal momentum is conserved during the impact to the usual impulsive approximation.
- After impact, kinetic friction converts the kinetic energy of the combined mass into heat. On a horizontal table its deceleration has magnitude μg, independent of mass.

Step 1: Convert the mass and find momentum The bullet mass is $m = 4\,\text{g} = 0.004\,\text{kg}$, and the block mass is $M = 0.8\,\text{kg}$. The block initially has zero momentum. Therefore the initial total horizontal momentum is

$$p = mu = 0.004(300) = 1.2\,\text{kg m s}^{-1}.$$

Converting grams before substitution is essential.

Step 2: Find the common speed after embedding The bullet and block move together immediately after the collision, with total mass $M + m = 0.804\,\text{kg}$. Conservation of momentum gives

$$(M + m)V = mu, \qquad V = \frac{1.2}{0.804} \simeq 1.493\,\text{m s}^{-1}.$$

Kinetic energy is not conserved during embedding; using the original bullet kinetic energy for the sliding stage would substantially overestimate the distance.

Step 3: Calculate the stopping distance The retarding acceleration is $a = \mu g = 0.3(9.8) = 2.94\,\mathrm{m\,s^{-2}}$. With final speed zero, $0 = V^2 - 2as$, so

$$s = \frac{V^2}{2\mu g} = \frac{(1.493)^2}{5.88} \simeq 0.379\,\mathrm{m}.$$

Using $g = 10\,\mathrm{m\,s^{-2}}$ gives about 0.37 m, consistent with the same approximate choice.

Step 4: Check the options The calculated distance matches 0.379 m. The value 0.19 m is approximately half the required distance, while 0.569 m and 0.758 m are too large. The two stages must remain separate: momentum sets the post-impact speed, and friction then sets the stopping distance.

Step 5: Conclusion

$$\boxed{s \simeq 0.379\,\mathrm{m}}.$$

Hence, the correct option is (B).

Quick Tip to Solve: For an embedded bullet, first use momentum, then use $s = V^2/(2\mu g)$; never conserve kinetic energy across the impact.

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34. A spring of unstretched length l has a mass m with one end fixed to a rigid support. Assuming spring to be made of a uniform wire, the kinetic energy possessed by it if its free end is pulled with uniform velocity v is: [4 marks]

(A) $\frac{1}{2}mv^2$

(B) mv^2

(C) $\frac{1}{3}mv^2$

(D) $\frac{1}{6}mv^2$

Correct Answer: (D) $\frac{1}{6}mv^2$

Solution:

Concept:

- A uniform spring has equal mass per unit unstretched length. The fixed end has zero velocity, while the free end has velocity v.
- In the standard uniform-extension model, a point at fractional distance x/L from the fixed end moves with speed vx/L. Kinetic energy is the sum of $\frac{1}{2}\,dm\,v_x^2$ over the spring.

Step 1: Set up a mass element Let the unstretched length be L, and measure x from the fixed end in the unstretched spring. Uniform mass distribution gives linear density m/L, so a small segment of reference length dx has mass

$$dm = \frac{m}{L}\,dx.$$

This uses a material coordinate: the segment retains its mass while the spring extends.

Step 2: Express the local velocity Uniform extension makes displacement proportional to distance

from the fixed end. The same proportionality applies to velocity, so

$$v_x = \frac{x}{L}v.$$

This expression satisfies both endpoint conditions: it is zero at $x = 0$ and equals v at $x = L$. Treating the whole mass as moving at v would overestimate the kinetic energy.

Step 3: Integrate the kinetic energy The differential kinetic energy is

$$dK = \frac{1}{2}\frac{m}{L}\left(\frac{xv}{L}\right)^2 dx.$$

Integrating from the fixed end to the free end gives

$$K = \frac{mv^2}{2L^3}\int_0^L x^2\,dx = \frac{mv^2}{2L^3}\frac{L^3}{3} = \frac{1}{6}mv^2.$$

The length cancels, as expected for an expression involving only mass and endpoint speed.

Step 4: Distinguish the alternatives The mean square speed is $v^2/3$, not the square of the mean speed and not the endpoint speed squared. Hence $\frac{1}{2}mv^2$ treats all segments as moving at the free-end speed, while mv^2 and $\frac{1}{3}mv^2$ are also too large. The kinetic-energy factor $\frac{1}{2}$ must multiply the mean square speed.

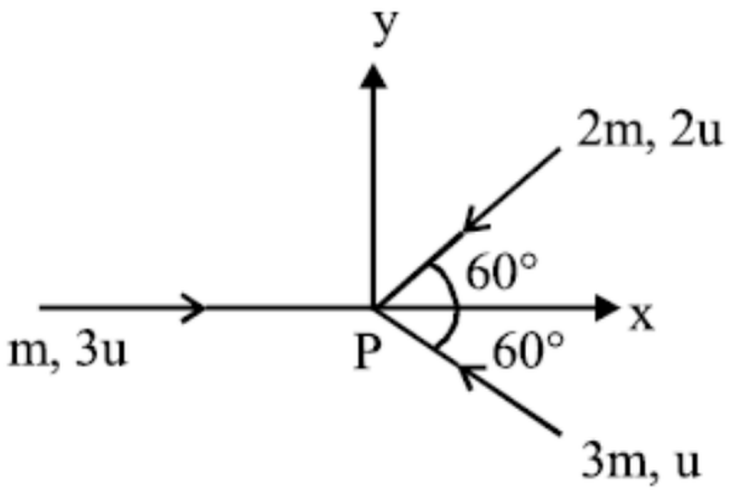
Step 5: Conclusion

$$\boxed{K = \frac{1}{6}mv^2}.$$

Hence, the correct option is (D).

Quick Tip to Solve: A uniform spring with one end fixed has effective kinetic mass $m/3$ when its free-end speed is used.

35. Three masses m, $2m$ and $3m$ are moving in $x-y$ plane with speed $3u$, $2u$ and u respectively as shown in figure. The three masses collide at the same point at P and stick together. The velocity of resulting mass will be: [4 marks]



(A) $\frac{u}{12}(\hat{\imath}+\sqrt{3}\hat{\jmath})$

(B) $\frac{u}{12}(\hat{\imath}-\sqrt{3}\hat{\jmath})$

(C) $\frac{u}{12}(-\hat{\imath}+\sqrt{3}\hat{\jmath})$

(D) $\frac{u}{12}(-\hat{\imath}-\sqrt{3}\hat{\jmath})$

Correct Answer: (D) $\frac{u}{12}(-\hat{\imath}-\sqrt{3}\hat{\jmath})$

Solution:

Concept:

- During a brief collision with negligible external impulse, total momentum is conserved independently along the two coordinate axes.
- When all three masses stick, their common velocity equals the vector sum of initial momenta divided by total mass $m+2m+3m=6m$. Momentum directions follow the arrows toward P.

Step 1: Resolve the first two momenta The mass m travels along positive x with speed $3u$, so $\mathbf{p}_1 = 3mu\hat{\mathbf{i}}$. The mass $2m$ approaches P along the upper ray; its direction is opposite to the outward ray at $60°$. Thus

$$\mathbf{p}_2 = 4mu(-\cos 60°\hat{\mathbf{i}} - \sin 60°\hat{\mathbf{j}}) = -2mu\hat{\mathbf{i}} - 2\sqrt{3}mu\hat{\mathbf{j}}.$$

The arrow, rather than the location of the mass, determines each sign.

Step 2: Resolve the third momentum The mass $3m$ approaches P from the lower right at speed u. Its x component is negative and its y component is positive. Therefore

$$\mathbf{p}_3 = 3mu(-\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}) = -\frac{3}{2}mu\hat{\mathbf{i}} + \frac{3\sqrt{3}}{2}mu\hat{\mathbf{j}}.$$

The upper and lower vertical components do not cancel because their momentum magnitudes are different.

Step 3: Add momenta and divide by mass Adding all three momenta yields

$$p_x = mu(3 - 2 - 3/2) = -mu/2,$$
$$p_y = mu(-2\sqrt{3} + 3\sqrt{3}/2) = -\sqrt{3}mu/2.$$

The total mass after sticking is $6m$. Hence

$$\mathbf{V} = \frac{\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3}{6m} = \frac{u}{12}(-\hat{\mathbf{i}} - \sqrt{3}\hat{\mathbf{j}}).$$

Step 4: Check the velocity direction Both net components are negative, so the combined mass moves left and downward. Options (A), (B) and (C) each have at least one positive component and therefore contradict the resolved momentum sum. Option (D) has both the correct signs and the correct coefficient.

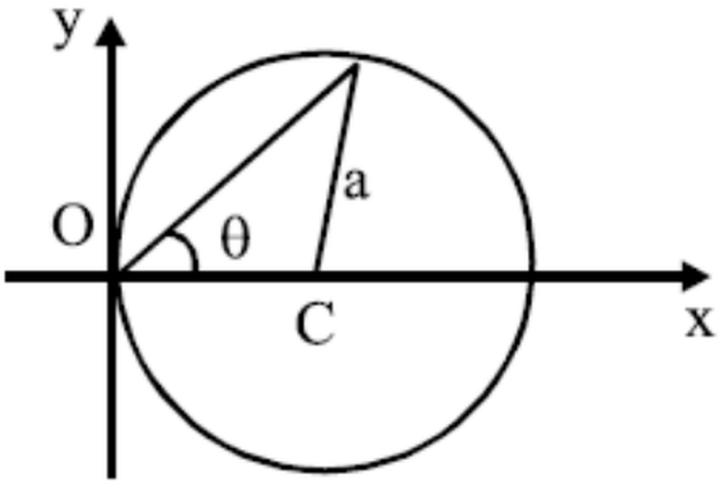
Step 5: Conclusion

$$\boxed{\mathbf{V} = \frac{u}{12}(-\hat{\mathbf{i}} - \sqrt{3}\hat{\mathbf{j}})}.$$

Hence, the correct option is (D).

Quick Tip to Solve: For particles approaching a point along inclined rays, reverse the outward radial direction before resolving their momenta.

36. A particle is moving in a circular path of radius a, with a constant velocity v as shown in the figure. The centre of circle is marked by 'C'. The angular momentum from the origin O can be written as: [4 marks]



(A) va (1 + cos 2θ)

(B) va (1 + cos θ)

(C) va cos 2θ

(D) va

Correct Answer: (A) va (1 + cos 2θ)

Solution:

Concept:

- Angular momentum about O is $\boldsymbol{L} = \boldsymbol{r} \times m\boldsymbol{v}$. Its magnitude equals momentum times the perpendicular distance from O to the tangent along the velocity.
- The options have dimensions of angular momentum per unit mass: av. Thus compare the specific angular momentum L/m; for a particle of mass m, multiply the resulting expression by m.

Step 1: Relate the chord to the marked angle Let the particle be at P, so $OP = r$, $CP = a$, and $OC = a$. The angle $POC = \theta$ is the marked angle. From the circle equation with centre $(a, 0)$, $x^2 + y^2 = 2ax$. Writing $x = r\cos\theta$, $y = r\sin\theta$ gives

$$r^2 = 2ar\cos\theta, \qquad r = 2a\cos\theta.$$

This uses the actual origin on the circumference rather than measuring position from the centre.

Step 2: Find the perpendicular lever arm The radius CP is normal to the tangent. In the isosceles triangle OCP, the angle between OP and CP has magnitude θ. Hence the sine of the angle between OP and the tangential velocity has magnitude $\cos\theta$. Therefore

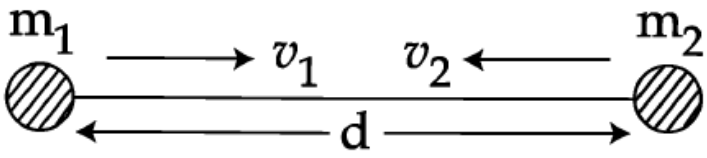
$$\frac{L}{m} = rv\cos\theta = 2av\cos^2\theta = av(1 + \cos 2\theta).$$

Option B uses the wrong angular dependence. Option C omits the constant term and can become negative as a magnitude. Option D is the constant specific angular momentum about C, not about the stated origin O.

Step 3: Conclusion The expression represented by the options is $L/m = av(1 + \cos 2\theta)$; the angular momentum itself is $mav(1 + \cos 2\theta)$. Hence, the correct option is (A).

Quick Tip to Solve: For angular momentum about a point on the circle, use the distance to the tangent, not the circle radius; keep the mass factor separate when the options give specific angular momentum.

37. Two hypothetical planets of masses m_1 and m_2 are at rest when they are at infinite distance apart. Because of the gravitational force they move towards each other along the line joining their centres. What is their speed when their separation is 'd'? (Speed of m_1 is v_1 and that of m_2 is v_2) [4 marks]



(A) $v_1 = v_2$

(B) $v_1 = m_2\sqrt{\dfrac{2G}{d(m_1+m_2)}},\ v_2 = m_1\sqrt{\dfrac{2G}{d(m_1+m_2)}}$

(C) $v_1 = m_1\sqrt{\dfrac{2G}{d(m_1+m_2)}},\ v_2 = m_2\sqrt{\dfrac{2G}{d(m_1+m_2)}}$

(D) $v_1 = m_2\sqrt{\dfrac{2G}{m_1}},\quad v_2 = m_2\sqrt{\dfrac{2G}{m_2}}$

Correct Answer: (B) $v_1 = m_2\sqrt{\dfrac{2G}{d(m_1+m_2)}},\ v_2 = m_1\sqrt{\dfrac{2G}{d(m_1+m_2)}}$

Solution:

Concept:

- With no external force, the total linear momentum stays zero. Speeds toward one another obey $m_1v_1 = m_2v_2$, with opposite velocity directions.
- Taking gravitational potential energy zero at infinity, energy conservation gives $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = Gm_1m_2/d$. Both bodies move, so both kinetic energies are required.

Step 1: Use the stationary centre of mass Initially both planets are at rest, so their total momentum is zero. Their gravitational forces are internal and equal and opposite. At separation d, write

$$m_1v_1 - m_2v_2 = 0, \qquad v_2 = \frac{m_1}{m_2}v_1.$$

The lighter body moves faster; equal speeds are possible only for equal masses. This already rules out option A as a general result and the mass dependence in option C.

Step 2: Combine momentum and energy The initial total mechanical energy is zero, whereas the final gravitational energy is $-Gm_1m_2/d$. Substitution into the kinetic-energy equation gives

$$\frac{1}{2}m_1v_1^2\left(1+\frac{m_1}{m_2}\right) = \frac{Gm_1m_2}{d}.$$

Thus

$$v_1^2 = \frac{2Gm_2^2}{d(m_1+m_2)}, \qquad v_2^2 = \frac{2Gm_1^2}{d(m_1+m_2)}.$$

Taking positive square roots gives the speeds, while the directions remain toward each other. Option D lacks the separation d, is dimensionally inconsistent as a speed, and cannot approach zero as d tends to infinity.

Step 3: Conclusion $v_1 = m_2\sqrt{\dfrac{2G}{d(m_1+m_2)}}$ and $v_2 = m_1\sqrt{\dfrac{2G}{d(m_1+m_2)}}$. Hence, the correct option is (B).

Quick Tip to Solve: For gravitational infall of two freely moving bodies, impose zero total momentum before using energy; treating one body as fixed gives the wrong mass factors.

38. Steel ruptures when a shear of $3.5 \times 10^8\,\mathrm{N\,m^{-2}}$ is applied. The force needed to punch a $1\,\mathrm{cm}$ diameter hole in a steel sheet $0.3\,\mathrm{cm}$ thick is nearly: [4 marks]

(A) $1.4 \times 10^4\,\mathrm{N}$

(B) $2.7 \times 10^4\,\mathrm{N}$

(C) $3.3 \times 10^4\,\mathrm{N}$

(D) $1.1 \times 10^4\,\mathrm{N}$

Correct Answer: (C) $3.3 \times 10^4\,\mathrm{N}$

Solution:

Concept:

- Shearing stress is tangential force divided by the area over which the material is sheared: $\tau = F/A_s$. Rupture occurs when the stated limiting shear stress is reached.
- A circular punch cuts the cylindrical wall around the hole. Therefore the shearing area is circumference times thickness, $A_s = \pi D t$, rather than the circular face area $\pi D^2/4$.

Step 1: Identify and convert the cutting dimensions The hole has diameter $D = 1\,\mathrm{cm} = 1.0 \times 10^{-2}\,\mathrm{m}$, while the sheet thickness is $t = 0.3\,\mathrm{cm} = 3.0 \times 10^{-3}\,\mathrm{m}$. Every point on the perimeter must shear through this thickness. The total area resisting the punch is therefore

$$A_s = \pi(1.0 \times 10^{-2})(3.0 \times 10^{-3}) = 3.0\pi \times 10^{-5}\,\mathrm{m^2}.$$

The diameter must not be substituted as a radius; doing that doubles the perimeter and the predicted force.

Step 2: Multiply the area by the rupture stress Using the limiting stress $\tau = 3.5 \times 10^8\,\mathrm{N\,m^{-2}}$,

$$F = \tau A_s = (3.5 \times 10^8)(3.0\pi \times 10^{-5})\,\mathrm{N} = 1.05\pi \times 10^4\,\mathrm{N}.$$

With $\pi \simeq 3.14$, this is $3.30 \times 10^4\,\mathrm{N}$. The dimensions check: stress times area is force. Options A, B and D do not agree with the perimeter-times-thickness cutting area and the given rupture stress; the nearest listed value is option C.

Step 3: Conclusion The necessary punching force is approximately $3.3 \times 10^4\,\mathrm{N}$. Hence, the correct option is (C).

Quick Tip to Solve: In punching or shearing a hole, use the area of the cylindrical cut surface, $\pi D t$; the area of the removed circular disc is not the shear area.

39. A cylindrical vessel of cross-section A contains water to a height h. There is a hole in the bottom of radius 'a'. The time in which it will be emptied is: [4 marks]

(A) $\frac{2A}{\pi a^2}\sqrt{\frac{h}{g}}$

(B) $\frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$

(C) $\frac{2\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$

(D) $\frac{A}{\sqrt{2}\pi a^2}\sqrt{\frac{h}{g}}$

Correct Answer: (B) $\frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$

Solution:

Concept:

- Torricelli's law gives the efflux speed $u = \sqrt{2gy}$ when the instantaneous water height above the hole is y. The hole area is $b = \pi a^2$.
- Continuity equates the rate of loss of volume to the discharge: $-A\,dy/dt = b\sqrt{2gy}$. The speed decreases as the level falls, so a single initial-speed estimate cannot give the total time.

Step 1: Write the differential volume balance At height y, the vessel holds volume Ay. During a short interval dt, the volume leaving the bottom is $b\sqrt{2gy}\,dt$, and the surface falls by $-dy$. Consequently

$$-A\,dy = b\sqrt{2gy}\,dt, \qquad dt = -\frac{A}{b\sqrt{2g}}y^{-1/2}\,dy.$$

Here the vessel is treated as broad relative to its small outlet, as required for the usual Torricelli approximation. The positive elapsed time corresponds to a decreasing height.

Step 2: Integrate over the falling water level Integrate from the initial level h down to zero:

$$t = \frac{A}{b\sqrt{2g}}\int_0^h y^{-1/2}\,dy = \frac{2A\sqrt{h}}{b\sqrt{2g}}.$$

Since $b = \pi a^2$, this simplifies to

$$t = \frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}.$$

The factor $A/(\pi a^2)$ is dimensionless and $\sqrt{h/g}$ has units of time. Options A and C have respectively incorrect factors $\sqrt{2}$ and 2 relative to this result. Option D is half the integrated time, corresponding to using the initial discharge throughout.

Step 3: Conclusion The emptying time is $\frac{\sqrt{2}A}{\pi a^2}\sqrt{h/g}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the instantaneous level as a variable and integrate $dy/\sqrt{y}$; using the initial efflux speed for the entire drainage underestimates the time by a factor of two.

40. Two soap bubbles coalesce to form a single bubble. If V is the subsequent change in volume of contained air and S change in total surface area, T is the surface tension and P atmospheric pressure, then which of the following relation is correct? [4 marks]

(A) 4PV + 3ST = 0

(B) 3PV + 4ST = 0

(C) 2PV + 3ST = 0

(D) 3PV + 2ST = 0

Correct Answer: (B) 3PV + 4ST = 0

Solution:

Concept:

- The excess pressure inside a soap bubble of radius r is $4T/r$, because the soap film has two air-liquid surfaces. Thus $p = P + 4T/r$.
- For the same trapped air at the same temperature, the ideal-gas law conserves the sum of $p\mathcal{V}$ over the bubbles. Here $\mathcal{V}$ denotes an individual volume, while the question uses V for its change.

Step 1: Express the pressure-volume sum initially Let the initial radii be r_1, r_2, and the final radius be R. For each spherical bubble, $\mathcal{V} = 4\pi r^3/3$, so

$$p\mathcal{V} = P\frac{4\pi r^3}{3} + \frac{16\pi T r^2}{3}.$$

Conservation of the contained air at the same temperature equates the final expression to the sum for the two initial bubbles. This calculation uses the volume of gas and the spherical area $4\pi r^2$; the two interfaces have already supplied the factor 4 in the excess pressure.

Step 2: Collect the volume and area changes Define the signed changes as

$$V = \frac{4\pi}{3}(R^3 - r_1^3 - r_2^3), \qquad S = 4\pi(R^2 - r_1^2 - r_2^2).$$

Subtracting the initial pressure-volume sum from the final one gives

$$PV + \frac{4}{3}TS = 0, \qquad 3PV + 4TS = 0.$$

Surface area decreases on coalescence, so $S < 0$, and the equation predicts $V > 0$, consistent with lower excess pressure. Option A reverses the coefficient ratio; options C and D use a different ratio and cannot follow from the two-surface bubble pressure.

Step 3: Conclusion The changes satisfy $3PV + 4ST = 0$. Hence, the correct option is (B).

Quick Tip to Solve: Use absolute pressure $P + 4T/r$ in the gas law for soap bubbles; neglecting atmospheric pressure or counting the two interfaces twice changes the coefficient ratio.

41. Hot water cools from 60°C to 50°C in the first 10 minutes and to 42°C in the next 10 minutes. The temperature of the surroundings is [4 marks]

(A) 25°C

(B) 10°C

(C) 15°C

(D) 20°C

Correct Answer: (B) 10°C

Solution:

Concept:

- Newton's cooling law gives $T(t) - T_s = (T_0 - T_s)e^{-kt}$, where T_s is the constant surrounding temperature and $k > 0$ is the cooling constant.
- For equal time intervals, the excess temperatures form a geometric sequence. Thus the same multiplicative factor relates $60 - T_s$, $50 - T_s$, and $42 - T_s$.

Step 1: Compare the two equal cooling intervals Both intervals last ten minutes. Define $r = e^{-10k}$, so

$$50 - T_s = r(60 - T_s), \qquad 42 - T_s = r(50 - T_s).$$

Eliminating r gives

$$(50 - T_s)^2 = (60 - T_s)(42 - T_s).$$

This compares excess temperatures above the surroundings, not absolute Celsius readings. Using $50/60$ and $42/50$ directly would implicitly set the surroundings to zero and violate the cooling law.

Step 2: Solve and verify the surrounding temperature Expand both sides:

$$2500 - 100T_s + T_s^2 = 2520 - 102T_s + T_s^2.$$

The quadratic terms cancel, leaving $2T_s = 20$, so $T_s = 10°\text{C}$. Check: the excess temperatures are $50, 40, 32$ degrees, and both ratios are $40/50 = 32/40 = 0.8$. For options A, C and D the two excess-temperature ratios are unequal, so a single cooling constant cannot fit both intervals. The successive drops, 10 and 8 degrees, also have the ratio 0.8, providing a separate consistency check.

Step 3: Conclusion The temperature of the surroundings is $10°\text{C}$. Hence, the correct option is (B).

Quick Tip to Solve: At equally spaced times, square the middle excess temperature and equate it to the product of the neighbouring excess temperatures; the unknown ambient temperature must be subtracted first.

42. A Carnot engine absorbs 1000 J of heat energy from a reservoir at 127°C and rejects 600 J of heat energy during each cycle. The efficiency of engine and temperature of sink will be [4 marks]

(A) 20% and $-43°$C

(B) 40% and $-33°$C

(C) 50% and $-20°$C

(D) 70% and $-10°$C

Correct Answer: (B) 40% and $-33°$C

Solution:

Concept:

- A cyclic heat engine produces net work $W = Q_H - Q_C$, and its efficiency is $\eta = W/Q_H = 1 - Q_C/Q_H$. Here Q_H and Q_C are positive heat magnitudes.
- For a reversible Carnot engine, $Q_C/Q_H = T_C/T_H$, where both reservoir temperatures are measured on the absolute Kelvin scale. A Celsius ratio cannot be used in this relation.

Step 1: Calculate the work and thermal efficiency The engine absorbs $Q_H = 1000\,\text{J}$ and rejects $Q_C = 600\,\text{J}$ during a complete cycle. Since its internal energy returns to the initial value, energy conservation gives

$$W = 1000 - 600 = 400\,\text{J}.$$

Therefore

$$\eta = \frac{400}{1000} = 0.40 = 40\%.$$

This immediately rules out options A, C and D, whose efficiency values do not agree with the given heat exchanges. The rejected heat is not the useful work: using $600/1000$ as the efficiency would give 60 percent incorrectly.

Step 2: Use the Carnot temperature ratio The hot reservoir has absolute temperature

$$T_H = 127 + 273 = 400\,\text{K}.$$

The Carnot condition then gives

$$T_C = T_H \frac{Q_C}{Q_H} = 400 \frac{600}{1000} = 240\,\text{K}.$$

Returning to Celsius,

$$t_C = 240 - 273 = -33°\text{C}.$$

The sink temperature is below the hot reservoir, as required for heat rejection. Its negative Celsius value is physically allowed; its absolute temperature remains positive. Substitution checks $1 - 240/400 = 0.40$, agreeing with the independent energy calculation.

Step 3: Conclusion The efficiency is 40%, and the sink temperature is $-33°\text{C}$. Hence, the correct option is (B).

Quick Tip to Solve: Find efficiency directly from the two heat quantities, then convert the hot temperature to kelvin before applying the Carnot ratio; convert back to Celsius only at the end.

43. At room temperature a diatomic gas is found to have an r.m.s. speed of $1930\,\mathrm{m\,s^{-1}}$. The gas is [4 marks]

(A) H_2

(B) Cl_2

(C) O_2

(D) F_2

Correct Answer: (A) H_2

Solution:

Concept:

- The root-mean-square speed is $v_{\mathrm{rms}} = \sqrt{3RT/M}$, where R is the gas constant, T is absolute temperature and M is molar mass in kilograms per mole.
- At the same temperature, molecular speed varies inversely as the square root of molar mass; lighter molecules move faster.

Step 1: Estimate the molar mass Room temperature is approximately $300\,\mathrm{K}$. Rearrange the rms-speed relation before substituting the given speed:

$$M = \frac{3RT}{v_{\mathrm{rms}}^2} = \frac{3(8.314)(300)}{1930^2}\,\mathrm{kg\,mol^{-1}} \approx 2.01 \times 10^{-3}\,\mathrm{kg\,mol^{-1}}.$$

Thus the gas has a molar mass close to $2\,\mathrm{g\,mol^{-1}}$. The stated speed is an rms speed, so the factor 3 is used rather than the factors appropriate to average or most-probable speed.

Step 2: Compare the four diatomic gases H_2 has molar mass approximately $2\,\mathrm{g\,mol^{-1}}$, whereas Cl_2, O_2 and F_2 have molar masses approximately 71, 32 and 38 respectively. For hydrogen, $\sqrt{3(8.314)(300)/0.002} \approx 1934\,\mathrm{m\,s^{-1}}$, matching the observation. The other three gases are much heavier, giving speeds only about 325, 484 and 444 $\mathrm{m\,s^{-1}}$; none can give the stated value at room temperature.

Step 3: Conclusion The gas is hydrogen, H_2. Hence, the correct option is (A).

Quick Tip to Solve: Convert the inferred molar mass from kilograms per mole to grams per mole before identifying the gas; a room-temperature speed near 2 km/s indicates hydrogen.

44. Which of the following expressions corresponds to simple harmonic motion along a straight line, where x is the displacement and a, b, c are positive constants? [4 marks]

(A) $a + bx - cx^2$

(B) bx^2

(C) $a - bx + cx^2$

(D) $-bx$

Correct Answer: (D) $-bx$

Solution:

Concept:

- For straight-line simple harmonic motion, the restoring acceleration obeys $\ddot{x} = -\omega^2 x$, with displacement measured from equilibrium.
- The restoring force is linear in displacement and points towards equilibrium: $F = -kx$, with $k > 0$.

Step 1: Apply the linear restoring condition The characteristic displacement dependence must contain a first power of x with a negative coefficient. If the expression represents acceleration, $-bx$ gives $\ddot{x} + bx = 0$, so $\omega = \sqrt{b}$. If it represents force, Newton's law gives $m\ddot{x} = -bx$, so $\omega = \sqrt{b/m}$. In either usual interpretation the sign makes the acceleration oppose displacement, and its magnitude grows directly in proportion to the distance from equilibrium.

Step 2: Reject the nonlinear alternatives The expressions $a + bx - cx^2$ and $a - bx + cx^2$ contain a quadratic term. They therefore do not have an exactly linear restoring dependence, even though a small-oscillation approximation about a suitable equilibrium can sometimes be harmonic. The expression bx^2 is also nonlinear and remains positive for both positive and negative x, so it cannot restore towards the origin on both sides. The question asks for the expression corresponding to SHM itself, not merely an approximate motion after linearising a nonlinear force.

Step 3: Conclusion The required expression is $-bx$, the linear restoring dependence. Hence, the correct option is (D).

Quick Tip to Solve: Identify SHM by checking both requirements together: linear displacement dependence and an acceleration directed back towards equilibrium.

45. A source of sound A emitting waves of frequency 1800 Hz is falling towards ground with a terminal speed v. The observer B on the ground directly beneath the source receives waves of frequency 2150 Hz. The source A receives waves, reflected from ground of frequency nearly: (Speed of sound = 343 m/s) [4 marks]

(A) 2150 Hz

(B) 2500 Hz

(C) 1800 Hz

(D) 2400 Hz

Correct Answer: (B) 2500 Hz

Solution:

Concept:

- For a source approaching a stationary observer, $f_g = f_0 c/(c-v)$, where c is sound speed and v is source speed.
- Reflection at a stationary ground surface preserves frequency in the ground frame. A receiver moving towards the reflected wave then hears $f_r = f_g(c+v)/c$.

Step 1: Determine the source speed from the ground frequency The falling source approaches the ground observer, so

$$2150 = 1800\frac{343}{343-v}.$$

Dividing and rearranging gives

$$\frac{v}{c} = 1 - \frac{1800}{2150} = \frac{7}{43}, \qquad v = 343\frac{7}{43} \approx 55.8\,\mathrm{m\,s^{-1}}.$$

Terminal speed means this value is constant; no acceleration correction is needed while the relevant waves travel.

Step 2: Apply the second Doppler shift to the reflected wave The stationary ground reflects sound of frequency $2150\,\mathrm{Hz}$ upwards. The source now acts as a moving receiver travelling downwards towards that returning wave. Hence

$$f_r = 2150\left(1+\frac{7}{43}\right) = 2150\frac{50}{43} = 2500\,\mathrm{Hz}.$$

Equivalently, the combined factor is $(c+v)/(c-v)$. The value 2150 Hz accounts only for reception at the ground, while 1800 Hz neglects both shifts. The value 2400 Hz does not satisfy the two-stage calculation.

Step 3: Conclusion The source receives the reflected waves at approximately $2500\,\mathrm{Hz}$. Hence, the correct option is (B).

Quick Tip to Solve: For an echo received by a moving source, calculate the frequency at the reflector first, then treat the original source as a moving observer.

46. A spherically symmetric charge distribution is characterised by a charge density having the following variations:

$$\rho(r) = \rho_0\left(1 - \frac{r}{R}\right) \text{ for } r < R, \qquad \rho(r) = 0 \text{ for } r \geq R.$$

Where r is the distance from the centre of the charge distribution, ρ_0 is a constant. The electric field at an internal point $(r < R)$ is [4 marks]

(A) $\frac{\rho_0}{4\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

(B) $\frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

(C) $\frac{\rho_0}{3\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

(D) $\frac{\rho_0}{12\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

Correct Answer: (B) $\frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

Solution:

Concept:

- Spherical symmetry makes the electric field radial and constant in magnitude on a concentric Gaussian sphere: $E\,4\pi r^2 = Q_{\text{enc}}/\varepsilon_0$.
- A thin spherical shell of radius s and thickness ds has charge $dq = \rho(s)4\pi s^2 ds$. Only charge inside the Gaussian sphere contributes to its flux.

Step 1: Integrate the enclosed charge Use a separate integration variable s, since r is the radius of the observation point. For $0 < r < R$,

$$Q_{\text{enc}} = 4\pi\rho_0 \int_0^r \left(1 - \frac{s}{R}\right) s^2\, ds = 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^4}{4R}\right).$$

The density is not constant, so multiplying its endpoint value by the sphere volume would be incorrect. Outer shells do not change the result inside a spherically symmetric distribution.

Step 2: Divide by the Gaussian surface area Substitute the enclosed charge in Gauss's law:

$$E(r) = \frac{Q_{\text{enc}}}{4\pi\varepsilon_0 r^2} = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right).$$

The factors 4π cancel entirely. Options (A), (C) and (D) introduce extra divisors 4, 3 and 12 respectively and therefore do not reproduce the charge integral. The field tends to zero as $r \to 0$, as required by symmetry, and has units of electric field because $\rho_0 r/\varepsilon_0$ does.

Step 3: Conclusion $E(r) = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$, directed radially outward for positive ρ_0. Hence, the correct option is (B).

Quick Tip to Solve: For a varying spherical density, integrate $4\pi s^2\rho(s)$ first; use the observation radius only after finding the enclosed charge.

47. The space between the plates of a parallel plate capacitor is filled with a 'dielectric' whose 'dielectric constant' varies with distance as per the relation:

$$K(x) = K_0 + \lambda x \qquad (\lambda = \textbf{a constant}).$$

The capacitance C, of the capacitor, would be related to its vacuum capacitance C_0 for the relation

[4 marks]

(A) $C = \dfrac{\lambda d}{\ln(1 + K_o \lambda d)} C_o$

(B) $C = \dfrac{\lambda}{d \cdot \ln(1 + K_o \lambda d)} C_o$

(C) $C = \dfrac{\lambda d}{\ln\left(1 + \dfrac{\lambda d}{K_o}\right)} C_o$

(D) $C = \dfrac{\lambda}{d \cdot \ln\left(1 + \dfrac{K_o}{\lambda d}\right)} C_o$

Correct Answer: (C) $C = \dfrac{\lambda d}{\ln\left(1 + \dfrac{\lambda d}{K_o}\right)} C_o$

Solution:

Concept:

- Dielectric layers perpendicular to the electric field act as capacitors in series. A slice of thickness dx has inverse capacitance $d(1/C) = dx/[\varepsilon_0 A K(x)]$.
- The vacuum capacitance is $C_0 = \varepsilon_0 A/d$, where A is plate area and d is their separation.

Step 1: Add the inverse capacitances of the layers Take x from one plate towards the other, so $K(x) = K_0 + \lambda x$ and $0 \le x \le d$. Every thin layer carries the same free charge, while their voltage drops add. Thus

$$\frac{1}{C} = \frac{1}{\varepsilon_0 A} \int_0^d \frac{dx}{K_0 + \lambda x} = \frac{1}{\varepsilon_0 A \lambda} \ln\left(\frac{K_0 + \lambda d}{K_0}\right).$$

This expression assumes the dielectric constant remains positive throughout the gap. An arithmetic average of K would correspond to a different arrangement of dielectric regions.

Step 2: Express the result using the vacuum capacitance Invert the integrated expression and substitute $\varepsilon_0 A = C_0 d$:

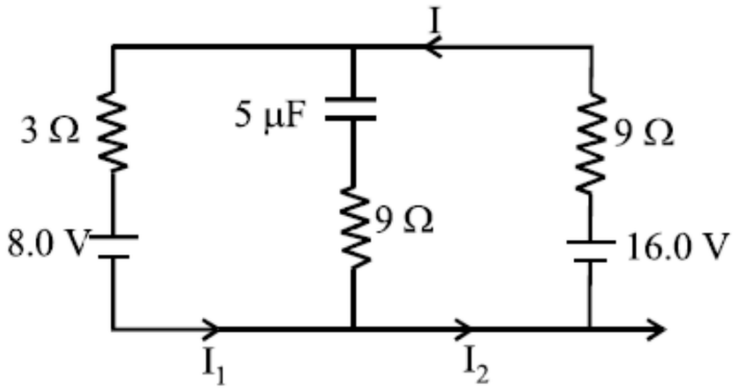
$$C = \frac{\varepsilon_0 A \lambda}{\ln(1 + \lambda d/K_0)} = \frac{\lambda d}{\ln(1 + \lambda d/K_0)} C_0.$$

The factor multiplying C_0 must be dimensionless because λ has dimensions of inverse length. Options (B) and (D) fail this test. Option (A) has the wrong logarithm argument; in the uniform limit $\lambda \to 0$, the correct formula tends to $K_0 C_0$, while (A) tends to C_0/K_0.

Step 3: Conclusion $C = \dfrac{\lambda d}{\ln(1 + \lambda d/K_0)} C_0$. Hence, the correct option is (C).

Quick Tip to Solve: For a dielectric varying through the plate separation, integrate $dx/K(x)$; verify the result tends to $K_0 C_0$ when the gradient vanishes.

48. The circuit shown here has two batteries of $8.0\,\mathrm{V}$ and $16.0\,\mathrm{V}$ and three resistors $3\,\Omega$, $9\,\Omega$ and $9\,\Omega$ and a capacitor of $5.0\,\mu\mathrm{F}$. How much is the current I in the circuit in steady state? [4 marks]



(A) 1.6 A

(B) 0.67 A

(C) 2.5 A

(D) 0.25 A

Correct Answer: (B) 0.67 A

Solution:

Concept:

- In a direct-current circuit after a long time, an ideal capacitor is fully charged and its branch carries zero steady conduction current.
- Kirchhoff's loop rule equates the net emf around the conducting loop to the sum of its resistive voltage drops.

Step 1: Identify the conducting loop The middle branch contains the capacitor in series with a $9\,\Omega$ resistor. At steady state the capacitor acts as an open circuit, so that entire branch carries no current. Consequently this middle resistor does not contribute to the resistance of the remaining loop. The conducting path goes through the left $3\,\Omega$ resistor, the right $9\,\Omega$ resistor and both batteries. Its total resistance is

$$R_{\text{loop}} = 3 + 9 = 12\,\Omega.$$

The capacitor value determines stored charge and transients, but not this final loop current.

Step 2: Find the net emf and current direction Both batteries have their positive terminals at the top, so their emfs oppose each other around the outer loop. The $16.0\,\mathrm{V}$ battery drives current up the right branch and leftwards along the top wire, the direction marked I. Traversing the loop in that direction gives

$$16.0 - 8.0 - I(9 + 3) = 0, \qquad I = \frac{8}{12} = 0.667\,\mathrm{A}.$$

Adding the battery voltages would incorrectly treat them as aiding. Including the middle resistor would incorrectly allow steady current through the capacitor. None of 1.6, 2.5 or 0.25 A satisfies this loop equation.

Step 3: Conclusion The steady current is $I \approx 0.67\,\mathrm{A}$, leftwards along the upper wire as shown. Hence,

the correct option is (B).

Quick Tip to Solve: First open every fully charged capacitor branch, then use the polarities of the remaining batteries to decide whether their emfs add or subtract.

49. In the experiment of calibration of voltmeter, a standard cell of e.m.f. 1.1 volt is balanced against 440 cm of potential wire. The potential difference across the ends of resistance is found to balance against 220 cm of the wire. The corresponding reading of voltmeter is 0.5 volt. The error in the reading of volmeter will be [4 marks]

(A) -0.15 volt

(B) 0.15 volt

(C) 0.5 volt

(D) -0.05 volt

Correct Answer: (D) -0.05 volt

Solution:

Concept:

- A uniform potentiometer wire has potential difference proportional to balancing length: $V = k\ell$, where k is its potential gradient.
- The error of a measuring instrument is its indicated reading minus the true value. The correction to be added has the opposite sign.

Step 1: Calibrate the potential gradient The standard cell provides a known emf of $1.1\,\mathrm{V}$ balanced by $440\,\mathrm{cm}$. Therefore

$$k = \frac{1.1}{440} = 0.0025\,\mathrm{V\,cm^{-1}}.$$

At balance the potentiometer draws no current from the tested branch, so its balance gives the actual potential difference. Keep the lengths in the same units when forming their ratio; converting both to metres gives the same gradient in volts per metre.

Step 2: Compare the true and indicated voltages The tested resistance balances against $220\,\mathrm{cm}$, half the standard-cell length. Its true potential difference is consequently

$$V_{\text{true}} = k(220) = 0.55\,\mathrm{V}.$$

The voltmeter indicates $0.50\,\mathrm{V}$, so

$$\text{error} = V_{\text{indicated}} - V_{\text{true}} = 0.50 - 0.55 = -0.05\,\mathrm{V}.$$

The negative sign means the instrument reads low. A positive $0.05\,\mathrm{V}$ would instead be the correction to add. The alternatives $\pm 0.15\,\mathrm{V}$ do not follow from the balance lengths, and 0.5 V is the reading itself, not its error.

Step 3: Conclusion The error in the voltmeter reading is $-0.05\,\mathrm{V}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the balance-length ratio to obtain the true voltage, then calculate reading minus true value; do not confuse instrument error with its correction.

50. A positive charge q of mass m is moving along the $+x$ axis. We wish to apply a uniform magnetic field B for time Δt so that the charge reverses its direction crossing the y axis at a distance d. Then: [4 marks]

(A) $B = \dfrac{mv}{qd}$ and $\Delta t = \dfrac{\pi d}{v}$

(B) $B = \dfrac{mv}{2qd}$ and $\Delta t = \dfrac{\pi d}{2v}$

(C) $B = \dfrac{2mv}{qd}$ and $\Delta t = \dfrac{\pi d}{2v}$

(D) $B = \dfrac{2mv}{qd}$ and $\Delta t = \dfrac{\pi d}{v}$

Correct Answer: (C) $B = \dfrac{2mv}{qd}$ and $\Delta t = \dfrac{\pi d}{2v}$

Solution:

Concept:

- A magnetic force perpendicular to velocity changes its direction without changing speed. The trajectory in a uniform perpendicular field is circular.
- The radius is $r = mv/(qB)$, and the time to describe an angle θ is $r\theta/v$.

Step 1: Identify the required circular arc Take the initial position as the origin, with initial velocity along the positive x axis. To reverse that velocity the particle must turn through an angle of π, describing a semicircle. Its final point on the y axis is the opposite end of a diameter. The stated displacement d is therefore the diameter, not the radius. Thus

$$2r = d, \qquad r = \frac{d}{2}.$$

The field must be perpendicular to the plane of the motion, with the sense chosen to bend toward the specified crossing.

Step 2: Find the field and the duration Equate the magnetic force to the centripetal force and then use the semicircular path length:

$$qvB = \frac{mv^2}{r}, \qquad B = \frac{mv}{qr} = \frac{2mv}{qd}.$$

$$\Delta t = \frac{\pi r}{v} = \frac{\pi d}{2v}.$$

Options (A) and (B) give incorrect radii because their field magnitudes do not yield $r = d/2$. Option (D) has the right field but twice the required time; that time would complete a full circle and restore the original velocity rather than reverse it.

Step 3: Conclusion The required magnitudes are $B = 2mv/(qd)$ and $\Delta t = \pi d/(2v)$. Hence, the correct option is (C).

Quick Tip to Solve: For reversal in a uniform magnetic field, first draw the semicircle mentally: the final transverse displacement is its diameter.

51. Consider two thin identical conducting wires covered with very thin insulating material. One of the wires is bent into a loop and produces magnetic field B_1 at its centre when a current I passes through it. The second wire is bent into a coil with three identical loops adjacent to each other and produces magnetic field B_2 at the centre of the loops when current $\frac{I}{3}$ passes through it. The ratio $B_1 : B_2$ is : [4 marks]

(A) $1 : 1$

(B) $1 : 3$

(C) $1 : 9$

(D) $9 : 1$

Correct Answer: (B) $1 : 3$

Solution:

Concept:

- The magnetic field at the centre of a circular coil of N closely adjacent turns, each of radius r, carrying current i, is $B = \mu_0 Ni/(2r)$. The fields of the turns add because their directions at the common centre agree.
- Identical wires have equal lengths. Bending the same length into more turns decreases the radius of each turn; both the number of turns and the changed radius must be included in the comparison.

Step 1: Relate the radii using the equal wire lengths Let the first wire form a single circular loop of radius R. Its length is $2\pi R$. The second wire has exactly the same length but forms three identical closely adjacent loops. If each of these has radius r, conservation of wire length gives

$$3(2\pi r) = 2\pi R, \qquad r = \frac{R}{3}.$$

The very thin insulation permits the turns to lie close together without electrically shorting across them. We therefore add their central magnetic fields as those of a three-turn coil. It would be incorrect to give the second coil the same radius as the first loop: that would require three times the available wire length.

Step 2: Compare the fields with the specified currents The first loop carries current I, so its central field is

$$B_1 = \frac{\mu_0 I}{2R}.$$

Each of the three turns of the second coil carries the specified current $I/3$. Using its radius $R/3$, its total central field is

$$B_2 = \frac{\mu_0\, 3(I/3)}{2(R/3)} = \frac{3\mu_0 I}{2R} = 3B_1.$$

The factor of three from the turn count cancels the factor of one third from the current, but the smaller radius supplies another factor of three. Thus equal fields, as in (A), would overlook this radius change. Option (C) would omit the reduced current and incorrectly give a ninefold field; option (D) also reverses the physically correct ordering, since the second coil has the stronger field.

Step 3: Conclusion The three-turn coil produces three times the central field of the single loop. Hence $B_1 : B_2 = 1 : 3$, and the correct option is (B).

Quick Tip to Solve: For a fixed wire length, r is inversely proportional to the number of turns, so the central field scales as $N^2 i$. Here $3^2(I/3)/I = 3$.

52. A sinusoidal voltage V(t) = 100 sin(500t) is applied across a pure inductance of L = 0.02H. The current through the coil is [4 marks]

(A) 10 cos(500t)

(B) -10 cos(500t)

(C) 10 sin(500t)

(D) -10 sin(500t)

Correct Answer: (B) -10 cos(500t)

Solution:

Concept:

- For a pure inductance, the applied voltage satisfies $V = L\,di/dt$. Its sinusoidal steady-state current lags voltage by $\pi/2$.
- The inductive reactance is $X_L = \omega L$, and the current amplitude is $I_0 = V_0/X_L$.

Step 1: Calculate the inductive reactance Compare the given voltage with $V(t) = V_0 \sin(\omega t)$. Here $V_0 = 100\,\text{V}$, $\omega = 500\,\text{rad}\,\text{s}^{-1}$, and $L = 0.02\,\text{H}$. Consequently

$$X_L = \omega L = 500(0.02) = 10\,\Omega, \qquad I_0 = \frac{100}{10} = 10\,\text{A}.$$

The amplitude alone cannot distinguish the options, since all four have magnitude ten. The phase is essential.

Step 2: Apply the phase lag and check the sign The steady-state current is

$$i(t) = I_0 \sin(\omega t - \pi/2) = -10\cos(500t)\,\text{A}.$$

Differentiate to check directly:

$$L\frac{di}{dt} = 0.02\,[5000\sin(500t)] = 100\sin(500t) = V(t).$$

The positive cosine in (A) produces the opposite voltage. The sine currents in (C) and (D) have the wrong phase: their derivatives are cosines. The sinusoidal steady-state condition excludes an arbitrary constant direct-current offset that integration alone might introduce.

Step 3: Conclusion The current is $i(t) = -10\cos(500t)$ A. Hence, the correct option is (B).

Quick Tip to Solve: For an inductor driven by a sine voltage, write the current as a negative cosine and then calculate its amplitude from omega L.

53. A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electrical power to electromagnetic waves and consumes 100 W of power. The amplitude of the electric field associated with the electromagnetic radiation at a distance of 5 m from the lamp will be nearly [4 marks]

(A) 1.34 V/m

(B) 2.68 V/m

(C) 4.02 V/m

(D) 5.36 V/m

Correct Answer: (B) 2.68 V/m

Solution:

Concept:

- Only the stated conversion efficiency contributes to radiated electromagnetic power. Isotropic power spreads over a sphere of area $4\pi r^2$.
- The time-averaged intensity is $I = \frac{1}{2}\varepsilon_0 c E_0^2$, where E_0 is the peak electric-field amplitude.

Step 1: Determine the intensity at the observation point The electrical consumption is $100\,\mathrm{W}$, but only three percent is emitted as the light being considered. Therefore

$$P_{\text{rad}} = \frac{3}{100}(100) = 3\,\mathrm{W}.$$

At radius $r = 5\,\mathrm{m}$, the same power crosses a sphere of area $4\pi(5)^2 = 100\pi\,\mathrm{m}^2$. Uniform emission gives

$$I = \frac{3}{100\pi} \simeq 9.55 \times 10^{-3}\,\mathrm{W\,m^{-2}}.$$

Using the entire electrical power here would overestimate the field.

Step 2: Convert average intensity to peak field Solve the electromagnetic intensity relation for its peak field:

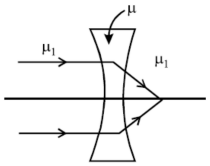
$$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2(9.55 \times 10^{-3})}{(8.85 \times 10^{-12})(3.00 \times 10^{8})}}.$$

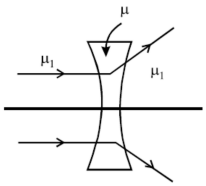
The quantity under the square root is about $7.19\,\mathrm{V^2\,m^{-2}}$, so $E_0 \simeq 2.68\,\mathrm{V\,m^{-1}}$. Option (A) is half this amplitude, while (C) and (D) are larger factors unsupported by either the radiated power or the spherical spreading. The rms field would be $E_0/\sqrt{2}$, which is a different quantity.

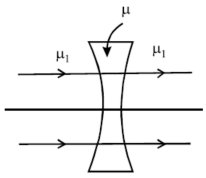
Step 3: Conclusion The electric-field amplitude is nearly $2.68\,\mathrm{V\,m^{-1}}$. Hence, the correct option is (B).

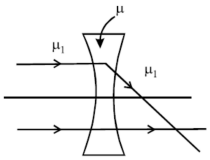
Quick Tip to Solve: Apply the efficiency before dividing by spherical area, and retain the factor one-half when converting average intensity into peak field.

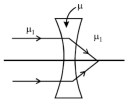
54. The refractive index of the material of a concave lens is μ. It is immersed in a medium of refractive index μ_1. A parallel beam of light is incident on the lens. The path of the emergent rays when $\mu_1 > \mu$ is: [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- A lens bends light according to its refractive index relative to the surrounding medium: $\mu_{\text{rel}} = \mu/\mu_1$.
- The lens-maker relation is $1/f = (\mu_{\text{rel}} - 1)(1/R_1 - 1/R_2)$; changing the sign of the index factor reverses the optical behaviour.

Step 1: Determine the sign of the relative index factor The surrounding medium has index $\mu_1 > \mu$, so

$$\frac{\mu}{\mu_1} < 1, \qquad \frac{\mu}{\mu_1} - 1 < 0.$$

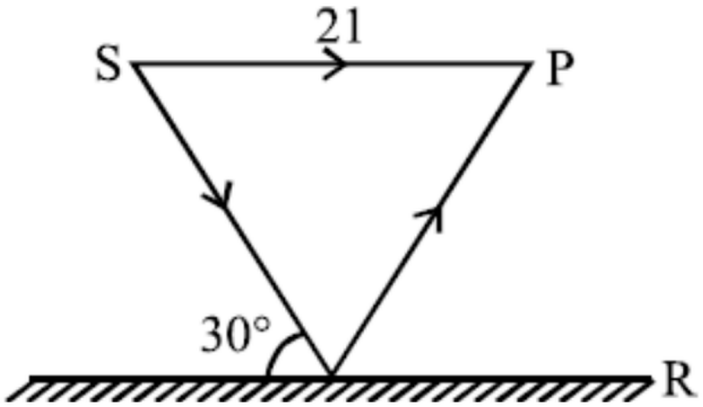
For a biconcave lens with light entering from the left, the Cartesian radius signs are $R_1 < 0$ and $R_2 > 0$. Hence $1/R_1 - 1/R_2 < 0$. Both factors in the lens-maker expression are negative, giving $1/f > 0$: this concave lens acts as a converging lens in the stated medium.

Step 2: Compare the illustrated emergent rays A parallel beam must consequently emerge toward the principal axis and meet at a real focus on the far side. The first figure depicts precisely this converging behaviour. The second figure shows divergence, appropriate to the opposite relative-index condition. The third leaves every ray undeviated, which would require index matching $\mu = \mu_1$, contrary to the strict inequality. The fourth bends the upper ray while leaving an equally displaced lower ray parallel, inconsistent with the symmetric lens and symmetric incident beam. The central axial ray is undeviated in the valid diagram.

Step 3: Conclusion The lens becomes converging because its material is optically rarer than the surrounding medium; the emergent rays follow the first diagram. Hence, the correct option is (A).

Quick Tip to Solve: Use the relative index of lens to medium. A concave lens changes from diverging to converging when that ratio falls below one.

55. Interference pattern is observed at P due to superimposition of two rays coming out from a source S as shown in the figure. The value of l for which maxima is obtained at P is: (R is perfect reflecting surface) [4 marks]



(A) $l = \dfrac{2n\lambda}{\sqrt{3}-1}$

(B) $l = \dfrac{(2n-1)\lambda}{2(\sqrt{3}-1)}$

(C) $l = \dfrac{(2n-1)\lambda\sqrt{3}}{4(2-\sqrt{3})}$

(D) $l = \dfrac{(2n-1)\lambda}{\sqrt{3}-1}$

Correct Answer: (C) $l = \dfrac{(2n-1)\lambda\sqrt{3}}{4(2-\sqrt{3})}$

Solution:

Concept:

- The two rays interfere according to their optical path difference. A reflection at the perfectly reflecting surface contributes a phase reversal of π.
- With one phase reversal, constructive interference requires a geometrical path difference $(2n-1)\lambda/2$, for positive integers n.

Step 1: Calculate the direct and reflected distances The direct distance SP in the diagram is $2l$. The reflection point lies midway horizontally between S and P, since incidence and reflection angles are equal and the two endpoints are at the same height. Each sloping segment has horizontal projection l. The given angle between the incident segment and the horizontal reflector is $30°$, so each segment has length $l/\cos 30° = 2l/\sqrt{3}$. Thus

$$L_{\text{ref}} = \frac{4l}{\sqrt{3}}, \qquad L_{\text{dir}} = 2l.$$

Step 2: Impose constructive interference including reflection The extra geometric distance is

$$\delta = \frac{4l}{\sqrt{3}} - 2l = \frac{2l(2-\sqrt{3})}{\sqrt{3}}.$$

Because only the lower path undergoes a reflection, maxima require

$$\frac{2l(2-\sqrt{3})}{\sqrt{3}} = \frac{(2n-1)\lambda}{2}.$$

Solving gives

$$l = \frac{(2n-1)\lambda\sqrt{3}}{4(2-\sqrt{3})}.$$

The first option has even-order dependence and misses the reflection reversal. The second and fourth use a different geometric factor $\sqrt{3}-1$, incompatible with the marked thirty-degree angle and the direct separation $2l$.

Step 3: Conclusion Maxima occur for $l = (2n-1)\lambda\sqrt{3}/[4(2-\sqrt{3})]$. Hence, the correct option is (C).

Quick Tip to Solve: Read the angle to the reflecting surface carefully, compute both lengths, and then include the extra half-wavelength from one reflection.

56. In an experiment of single slit diffraction pattern, first minimum for red light coincides with first maximum of some other wavelength. If wavelength of red light is 6600Å, then wavelength of first maximum will be [4 marks]

(A) 3300Å

(B) 4400Å

(C) 5500Å

(D) 6600Å

Correct Answer: (B) 4400Å

Solution:

Concept:

- The first single-slit minimum satisfies $a\sin\theta = \lambda$, where a is slit width.
- The usual secondary-maximum approximation places the first side maximum at $a\sin\theta \simeq 3\lambda/2$; the exact extrema satisfy $\tan\beta = \beta$.

Step 1: Use the common angular position The first minimum for red light and the first secondary maximum for the other light occur at the same point of the diffraction pattern. They therefore have the same θ and the same value of $a\sin\theta$. For red light,

$$a\sin\theta = \lambda_r = 6600\,\text{Å}.$$

The central maximum is not being counted as a secondary maximum: it is at zero angle and cannot coincide with a nonzero-angle first minimum.

Step 2: Find the wavelength using the secondary-maximum condition For the first side maximum, the standard approximate condition is

$$a\sin\theta \simeq \frac{3\lambda}{2}.$$

Equate this to the red-light minimum condition:

$$6600 \simeq \frac{3\lambda}{2}, \qquad \lambda \simeq 4400\,\text{Å}.$$

The choices 3300, 5500, and $6600\,$Å do not satisfy this first-side-maximum relation. More exactly, $\beta = \pi a \sin\theta/\lambda$ obeys $\tan\beta = \beta$, whose first side root is about 4.493; that refinement gives about $4615\,$Å. The listed answer uses the conventional $3\pi/2$ approximation, so it is the intended approximation rather than an exact extremum.

Step 3: Conclusion Using the standard first-secondary-maximum approximation, the wavelength is $4400\,$Å. Hence, the correct option is (B).

Quick Tip to Solve: At a shared screen position, equate a sin theta for the two wavelengths; use lambda for the first minimum and three lambda over two for the first side maximum.

57. A beam of light has two wavelengths of $4972\,$Å and $6216\,$Å with a total intensity of $3.6\times10^{-3}\,\text{W m}^{-2}$ equally distributed among the two wavelengths. The beam falls normally on an area of $1\,\text{cm}^2$ of a clean metallic surface of work function $2.3\,\text{eV}$. Assume that there is no loss of light by reflection and that each capable photon ejects one electron. The number of photoelectrons liberated in $2\,\text{s}$ is approximately: [4 marks]

(A) 6×10^{11}

(B) 9×10^{11}

(C) 11×10^{11}

(D) 15×10^{11}

Correct Answer: (B) 9×10^{11}

Solution:

Concept:

- A photon can eject an electron only if its energy $E = hc/\lambda$ exceeds the work function ϕ. Intensity controls photon number, whereas wavelength determines whether emission is possible.
- For a monochromatic component, the number of incident photons is $N = IAt/(hc/\lambda)$, where I is its intensity, A the illuminated area and t the exposure time.

Step 1: Check both photon energies Use $hc \simeq 12400\,\text{eV Å}$. The two photon energies are

$$E_1 = \frac{12400}{4972} \simeq 2.49\,\text{eV}, \qquad E_2 = \frac{12400}{6216} \simeq 2.00\,\text{eV}.$$

Only the shorter wavelength exceeds $2.3\,\text{eV}$. Photons of the longer wavelength do not contribute any photoelectrons, even though they carry half the total incident power.

Step 2: Count the capable photons The useful intensity is half the stated total, $I_1 = 1.8\times10^{-3}\,\text{W m}^{-2}$, and $A = 10^{-4}\,\text{m}^2$. Hence the energy received in two seconds in this component is

$$U_1 = I_1 At = 1.8\times10^{-3}\times10^{-4}\times2 = 3.6\times10^{-7}\,\text{J}.$$

Each capable photon carries $hc/\lambda_1 \simeq 4.0 \times 10^{-19}$ J. Therefore

$$N = \frac{3.6 \times 10^{-7}}{4.0 \times 10^{-19}} \simeq 9 \times 10^{11}.$$

With one electron per capable photon and no reflection, this equals the electron count. Options A, C and D disagree with this photon balance; including the incapable component would overcount.

Step 3: Conclusion The number of photoelectrons liberated is approximately 9×10^{11}. Hence, the correct option is (B).

Quick Tip to Solve: Apply the work-function threshold to each wavelength before converting the corresponding intensity into a photon count.

58. For LED's to emit light in visible region of electromagnetic light, it should have energy band gap in the range of [4 marks]

(A) 0.1eV to 0.4eV

(B) 0.5eV to 0.8eV

(C) 0.9eV to 1.6eV

(D) 1.7eV to 3.0eV

Correct Answer: (D) 1.7eV to 3.0eV

Solution:

Concept:

- In a light-emitting diode, radiative recombination across the semiconductor band gap releases photons whose energy is approximately $E_g = hc/\lambda$.
- Visible photons have energies of order a few electronvolts. The useful conversion is $E(\text{eV}) \simeq 1240/\lambda(\text{nm})$.

Step 1: Convert the visible wavelength scale Take the visible wavelength range as approximately 400 to $750\,\text{nm}$. Its photon energies run from

$$\frac{1240}{750} \simeq 1.65\,\text{eV} \quad \text{to} \quad \frac{1240}{400} \simeq 3.10\,\text{eV}.$$

Thus a semiconductor used to emit visible light must have a gap in approximately the 1.7 to $3.0\,\text{eV}$ range. The limits describe the colour range approximately; shorter wavelengths require larger gaps.

Step 2: Compare the given gap ranges Option D lies in this visible-energy window. Option A, 0.1 to $0.4\,\text{eV}$, would give wavelengths from about 12400 to $3100\,\text{nm}$, far into the infrared. Option B, 0.5 to $0.8\,\text{eV}$, corresponds to about 2480 to $1550\,\text{nm}$, also infrared. Option C, 0.9 to $1.6\,\text{eV}$, corresponds to about 1378 to $775\,\text{nm}$, beyond the red visible boundary. These lower gaps cannot supply the energy of ordinary visible photons. The comparison concerns photon energy, rather than an increase of intensity at the same wavelength.

Step 3: Conclusion The required band-gap range is approximately $1.7\,\text{eV}$ to $3.0\,\text{eV}$. Hence, the correct option is (D).

Quick Tip to Solve: Remember that a red photon has about 2 eV energy and a violet photon about 3 eV; an LED gap must match its emitted photon energy.

59. A piece of bone of an animal from a ruin is found to have ^{14}C activity of 12 disintegrations per minute per gm of its carbon content. The ^{14}C activity of a living animal is 16 disintegrations per minute per gm. How long ago nearly did the animal die? (Given half life of ^{14}C is $t_{1/2} = 5760$ years) [4 marks]

(A) 1672 years

(B) 2391 years

(C) 3291 years

(D) 4453 years

Correct Answer: (B) 2391 years

Solution:

Concept:

- After death, replenishment of carbon-14 stops, and its abundance decreases according to $N = N_0 e^{-\lambda t}$. Activity $A = \lambda N$ obeys the same exponential law.
- The decay constant and half-life satisfy $\lambda = \ln 2/t_{1/2}$. Compare activities per gram of carbon so that different sample masses do not affect the ratio.

Step 1: Find the surviving fraction The living activity is the initial activity per gram of carbon. The bone now has activity 12 compared with the original 16 in the same units. Therefore

$$\frac{A}{A_0} = \frac{12}{16} = \frac{3}{4}.$$

The loss is one quarter of the original activity, not one quarter of a half-life: radioactive decay is exponential. Since more than half remains, the age must be less than the given half-life of 5760 years.

Step 2: Take logarithms of the decay law From $A/A_0 = e^{-\lambda t}$,

$$t = \frac{1}{\lambda} \ln \frac{A_0}{A} = \frac{5760}{\ln 2} \ln \frac{4}{3}.$$

Using $\ln(4/3) \simeq 0.28768$ and $\ln 2 \simeq 0.69315$,

$$t \simeq 5760 \frac{0.28768}{0.69315} \simeq 2391 \text{ years}.$$

The disintegration-per-minute units cancel in the ratio; no conversion between minutes and years is needed because the decay constant was expressed through a half-life in years. Options A, C and D give different surviving fractions under the exponential law and do not match the observed ratio of three quarters.

Step 3: Conclusion The animal died approximately 2391 years ago. Hence, the correct option is (B).

Quick Tip to Solve: For dating from activity, use the logarithm of initial activity divided by present activity; never treat the percentage decrease as linear in time.

60. For sky wave propagation, the radio waves must have a frequency range in between [4 marks]

(A) 1MHz to 2MHz

(B) 5MHz to 25MHz

(C) 35MHz to 40MHz

(D) 45MHz to 50MHz

Correct Answer: (B) 5MHz to 25MHz

Solution:

Concept:

- Sky waves reach distant receivers after refraction back toward Earth by ionised layers of the upper atmosphere. Their useful frequencies must be high enough to avoid excessive absorption and low enough to return from the ionosphere.
- The standard sky-wave band is in the high-frequency region, roughly a few megahertz up to about $30\,\mathrm{MHz}$, with actual usable limits depending on ionisation and path.

Step 1: Identify the propagation mechanism At suitably chosen radio frequencies the ionosphere refracts a wave progressively and can return it to Earth, allowing communication beyond the direct line of sight. Frequencies that are too high penetrate the ionosphere instead of returning. The exact critical frequency depends on electron density, while the maximum usable frequency also depends on incidence angle; the question asks for the conventional operating range among the choices.

Step 2: Select the appropriate frequency interval The interval 5 to $25\,\mathrm{MHz}$ lies comfortably in the usual high-frequency sky-wave band. The intervals 35 to $40\,\mathrm{MHz}$ and 45 to $50\,\mathrm{MHz}$ exceed the usual upper limit of about $30\,\mathrm{MHz}$, so options C and D are unsuitable for ordinary ionospheric communication. The 1 to $2\,\mathrm{MHz}$ interval is the lower-frequency region associated primarily with ground-wave propagation and increased ionospheric absorption, rather than the standard sky-wave band being tested. Night-time or special-path exceptions do not make it the intended general band. Thus option B is the listed interval consistent with the usual sky-wave operating window.

Step 3: Conclusion The appropriate frequency interval is $5\,\mathrm{MHz}$ to $25\,\mathrm{MHz}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the rough 3 to 30 MHz high-frequency window for ordinary sky waves; substantially higher frequencies usually pass through the ionosphere.

Chemistry (120 marks)

61. The amount of BaSO4 formed upon mixing 100 mL of 20.8%BaCl2 solution with 50 mL of 9.8% H2SO4 solution will be: (Ba = 137, Cl = 35.5, S = 32, H = 1 and O = 16) [4 marks]

(A) 23.3 g

(B) 11.65 g

(C) 30.6 g

(D) 33.2 g

Correct Answer: (B) 11.65 g

Solution:

Concept:

- Barium chloride and sulphuric acid form insoluble barium sulphate in a one-to-one mole ratio: $\mathrm{BaCl_2 + H_2SO_4 \longrightarrow BaSO_4\downarrow + 2\,HCl}$.
- For the stated percentage solutions, use grams of solute per 100 mL of solution. The reagent supplying fewer stoichiometric moles limits the mass of precipitate.

Step 1: Convert both solutes to moles The $100\,\mathrm{mL}$ portion contains $20.8\,\mathrm{g}$ of $\mathrm{BaCl_2}$. Its molar mass is

$$M(\mathrm{BaCl_2}) = 137 + 2(35.5) = 208\,\mathrm{g\,mol^{-1}}.$$

Thus $n(\mathrm{BaCl_2}) = 20.8/208 = 0.100\,\mathrm{mol}$. The $50\,\mathrm{mL}$ acid portion contains $9.8 \times 50/100 = 4.9\,\mathrm{g}$ of $\mathrm{H_2SO_4}$. Since its molar mass is $2 + 32 + 64 = 98\,\mathrm{g\,mol^{-1}}$, it supplies $4.9/98 = 0.050\,\mathrm{mol}$.

Step 2: Use the limiting reagent to find precipitate mass The balanced equation consumes one mole of each reactant per mole of precipitate. Only $0.050\,\mathrm{mol}$ of acid is present, so the acid is exhausted first and forms $0.050\,\mathrm{mol}$ of $\mathrm{BaSO_4}$. Its molar mass is

$$M(\mathrm{BaSO_4}) = 137 + 32 + 4(16) = 233\,\mathrm{g\,mol^{-1}}.$$

Consequently,

$$m(\mathrm{BaSO_4}) = 0.050 \times 233 = 11.65\,\mathrm{g}.$$

Option A, 23.3 g, would incorrectly consume all 0.100 mol of barium salt despite insufficient acid. Options C and D exceed even that barium-salt upper bound. Excess barium chloride stays in solution; it cannot create additional sulphate ions.

Step 3: Conclusion The mass of $\mathrm{BaSO_4}$ formed is $11.65\,\mathrm{g}$. Hence, the correct option is (B). The supplied key gives 23.3 g, but the printed acid amount supplies only $0.050\,\mathrm{mol}$ of sulphate; therefore the stoichiometric result is (B).

Quick Tip to Solve: Convert both percentage solutions to moles before using their reaction ratio; the smaller solution volume often changes which reagent is limiting.

62. If m and e are the mass and charge of the revolving electron in the orbit of radius r for hydrogen atom, the total energy of the revolving electron will be: [4 marks]

(A) $\frac{1}{2}\frac{e^2}{r}$
(B) $-\frac{e^2}{r}$
(C) $\frac{me^2}{r}$
(D) $-\frac{1}{2}\frac{e^2}{r}$

Correct Answer: (D) $-\frac{1}{2}\frac{e^2}{r}$

Solution:

Concept:

- For an electron bound by Coulomb attraction in a circular orbit, centripetal balance gives $mv^2/r = ke^2/r^2$, where k is the electrostatic constant.
- Taking zero potential energy at infinite separation, $U = -ke^2/r$ and $K = mv^2/2$. The printed options use the electrostatic-unit convention $k = 1$.

Step 1: Find kinetic energy from orbital balance Multiply the force balance by $r/2$:

$$\frac{1}{2}mv^2 = \frac{ke^2}{2r}.$$

Thus the kinetic energy is positive and equals half the magnitude of the potential energy. The electron mass affects the allowed orbit and speed, but it cancels when the energy is expressed in terms of the given radius. Therefore an extra multiplicative mass factor, as in option C, does not belong in this expression.

Step 2: Add kinetic and potential energies The attraction gives negative potential energy relative to infinity. Therefore

$$E = K + U = \frac{ke^2}{2r} - \frac{ke^2}{r} = -\frac{ke^2}{2r}.$$

In the convention used by the options this is $-e^2/(2r)$. In SI units the same physical result is $-e^2/(8\pi\varepsilon_0 r)$; the factor k must then be retained. Option A has the correct magnitude but the positive sign of kinetic energy alone. Option B gives the potential energy without including kinetic energy. Option C has an extraneous mass and the wrong sign. A negative total energy is essential because the electron is bound, and energy must be supplied to separate it from the nucleus to infinity.

Step 3: Conclusion The total orbital energy in the convention of the options is $E = -e^2/(2r)$. Hence, the correct option is (D).

Quick Tip to Solve: For a circular Coulomb orbit remember K = -U/2 and E = U/2; total energy is negative and half the potential-energy magnitude.

63. The de-Broglie wavelength of a particle of mass $6.63\,\mathrm{g}$ moving with a velocity of $100\,\mathrm{m\,s^{-1}}$ is:

[4 marks]

(A) $10^{-33}\,\mathrm{m}$

(B) $10^{-35}\,\mathrm{m}$

(C) $10^{-31}\,\mathrm{m}$

(D) $10^{-25}\,\mathrm{m}$

Correct Answer: (A) $10^{-33}\,\mathrm{m}$

Solution:

Concept:

- The de Broglie wavelength of a nonrelativistic particle is $\lambda = h/p = h/(mv)$, where p is its momentum and h is Planck's constant.
- Use SI quantities consistently: mass in kilograms and speed in metres per second, so that $h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$ gives the wavelength in metres.

Step 1: Convert the mass and find momentum The given mass is in grams, not kilograms. Therefore

$$m = 6.63\,\mathrm{g} = 6.63 \times 10^{-3}\,\mathrm{kg}.$$

At $v = 100\,\mathrm{m\,s^{-1}} = 10^{2}\,\mathrm{m\,s^{-1}}$, the momentum is

$$p = mv = 6.63 \times 10^{-3} \times 10^{2} = 6.63 \times 10^{-1}\,\mathrm{kg\,m\,s^{-1}}.$$

The speed is very small compared with the speed of light, so the ordinary expression $p = mv$ is adequate; relativistic corrections are unnecessary.

Step 2: Divide Planck constant by momentum Substitution gives

$$\lambda = \frac{6.63 \times 10^{-34}}{6.63 \times 10^{-1}} = 10^{-33}\,\mathrm{m}.$$

The numerical coefficients cancel, and the exponent is $-34 - (-1) = -33$. The units reduce correctly because $\mathrm{J\,s}/(\mathrm{kg\,m\,s^{-1}}) = \mathrm{m}$. The extremely short wavelength is expected for a macroscopic mass; wave effects are not apparent for an object of this size at ordinary speeds. Options B, C and D have powers of ten inconsistent with the correctly converted momentum. In particular, treating the gram value as a kilogram value or misplacing the speed exponent changes the result by spurious orders of magnitude.

Step 3: Conclusion The de Broglie wavelength is $10^{-33}\,\mathrm{m}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert grams to kilograms before calculating momentum, then subtract denominator exponents carefully when dividing h by mv.

64. Excited hydrogen atom emits light in the ultraviolet region at 2.47×10^{15} Hz. With this frequency, the energy of a single photon is: $h = 6.63 \times 10^{-34}$ J s. [4 marks]

(A) 8.041×10^{-40} J

(B) 2.680×10^{-19} J

(C) 1.640×10^{-18} J

(D) 6.111×10^{-17} J

Correct Answer: (C) 1.640×10^{-18} J

Solution:

Concept:

- A photon carries energy $E = h\nu$, where h is Planck's constant and ν is its frequency.
- The energy is proportional to frequency; the ultraviolet classification does not introduce any additional factor. The product of joule seconds and reciprocal seconds is joules.

Step 1: Identify the supplied quantities The emitted light has frequency $\nu = 2.47 \times 10^{15}$ Hz, and the supplied constant is $h = 6.63 \times 10^{-34}$ J s. The question asks for one photon, so neither Avogadro's constant nor the number of atoms enters the calculation. Frequency has units s^{-1}.

Step 2: Multiply the coefficient and the power of ten Apply the photon-energy relation directly:

$$E = (6.63 \times 10^{-34})(2.47 \times 10^{15}) = 16.3761 \times 10^{-19}\ \mathrm{J}.$$

Writing the coefficient between one and ten gives

$$E = 1.63761 \times 10^{-18}\ \mathrm{J} \approx 1.640 \times 10^{-18}\ \mathrm{J}.$$

The three significant figures of the input justify reporting about 1.64×10^{-18} J.

Step 3: Compare the choices Option (A) has an energy many powers of ten too small and cannot result from adding the exponents $-34 + 15 = -19$. Option (B) has the wrong coefficient after normalization. Option (D) is too large. Only (C) agrees with both the coefficient and the exponent of $h\nu$.

Step 4: Conclusion The energy of a single emitted photon is 1.640×10^{-18} J. Hence, the correct option is (C).

Quick Tip to Solve: Add the powers of ten before normalizing the coefficient: $16.38 \times 10^{-19} = 1.638 \times 10^{-18}$.

65. Similarity in chemical properties of the atoms of elements in a group of the Periodic table is most closely related to [4 marks]

(A) atomic numbers

(B) atomic masses

(C) number of principal energy levels

(D) number of valence electrons

Correct Answer: (D) number of valence electrons

Solution:

Concept:

- The outermost occupied shell contains the valence electrons that participate most directly in chemical bonding and reactions.
- Elements in a group generally have corresponding valence-shell configurations. This recurring outer electronic structure explains their family resemblance.

Step 1: Identify the electronic cause of chemical behavior Chemical reactions rearrange electrons, especially those in the outer shell. Elements having the same number of valence electrons tend to show similar valencies, form analogous compounds and undergo related reactions. For example, the alkali metals possess one outer s electron, which they readily lose to form singly charged cations.

Step 2: Distinguish group similarity from period similarity Descending a group adds occupied principal shells. Therefore, the number of principal energy levels is not constant within the group; it changes from one member to the next. A common number of occupied shells instead characterizes a period. Atomic number and atomic mass also increase down a group, so their numerical equality cannot explain a group's chemical resemblance.

Step 3: Evaluate the alternatives Option (A), atomic number, determines the complete electronic configuration but is different for every element. Option (B), atomic mass, mainly reflects the nucleus and is not the direct basis of ordinary chemical bonding. Option (C) concerns the number of shells, which differs within a group. Option (D) identifies the shared valence-electron count and is therefore the closest connection asked for. The wording concerns the general periodic trend rather than requiring identical quantitative reactivities.

Step 4: Conclusion Atomic numbers differ down a group, while the recurring valence-electron count governs similar bonding; thus the question-sheet key is inconsistent with the printed question. Similarity within a group is most closely related to the number of valence electrons. Hence, the correct option is (D).

Quick Tip to Solve: Associate a group with a recurring outer-shell configuration, and a period with the number of occupied principal shells.

66. Which of the following arrangements represents the increasing order (smallest to largest) of ionic radii of the given species $O^{2-}, S^{2-}, N^{3-}, P^{3-}$? [4 marks]

(A) $O^{2-} < N^{3-} < S^{2-} < P^{3-}$

(B) $O^{2-} < P^{3-} < N^{3-} < S^{2-}$

(C) $N^{3} < O^{2-} < P^{3-} < S^{2-}$

(D) $N^{3-} < S^{2-} < O^{2-} < P^{3-}$

Correct Answer: (A) $O^{2-} < N^{3-} < S^{2-} < P^{3-}$

Solution:

Concept:

- For isoelectronic ions, a larger nuclear charge attracts the same electron population more strongly and gives a smaller radius.
- Adding an occupied principal shell increases ionic size substantially. Compare shell number first, then nuclear charge within an isoelectronic pair.

Step 1: Group the ions by electron count For O^{2-}, the electron count is $8 + 2 = 10$, while N^{3-} has $7 + 3 = 10$. These ions have the neon configuration with their outer electrons in the second shell. Similarly, S^{2-} and P^{3-} each have eighteen electrons, the argon configuration, and an occupied third shell.

Step 2: Order each isoelectronic pair Oxygen has eight protons whereas nitrogen has seven, so oxygen binds the ten-electron cloud more strongly:

$$r(O^{2-}) < r(N^{3-}).$$

Sulfur has sixteen protons whereas phosphorus has fifteen, giving

$$r(S^{2-}) < r(P^{3-}).$$

The third-shell anions are larger than the second-shell anions, so these two comparisons combine without reversing the shell-size trend.

Step 3: Combine the comparisons and reject reversals The increasing sequence is

$$O^{2-} < N^{3-} < S^{2-} < P^{3-}.$$

Option (B) incorrectly puts phosphorus before nitrogen and sulfur. Option (C) reverses both isoelectronic-pair trends. Option (D) places the larger third-shell sulfur ion before second-shell oxygen. Option (A) alone respects both principles.

Step 4: Conclusion The increasing ionic-radius order is $O^{2-} < N^{3-} < S^{2-} < P^{3-}$. Hence, the correct option is (A).

Quick Tip to Solve: Count electrons before comparing ionic sizes: within an isoelectronic pair, the ion with more protons is smaller.

67. Which of the following molecules has two sigma (σ) and two pi (π) bonds? [4 marks]

(A) C_2H_4

(B) N_2F_2

(C) $C_2H_2Cl_2$

(D) HCN

Correct Answer: (D) HCN

Solution:

Concept:

- A single bond is one σ bond. A double bond contains one σ and one π bond, while a triple bond contains one σ and two π bonds.
- Count every bond between a pair of atoms, including bonds to hydrogen or a halogen; counting only the multiple bond misses additional sigma bonds.

Step 1: Count the bonds in hydrogen cyanide The connectivity of hydrogen cyanide is $H-C\equiv N$. Its $H-C$ single bond provides one σ bond. The $C\equiv N$ triple bond provides another σ bond and two π bonds. Thus the complete molecule contains

$$N_\sigma = 1 + 1 = 2, \qquad N_\pi = 0 + 2 = 2.$$

Step 2: Check the carbon-containing alternatives Ethene, C_2H_4, contains four $C-H$ sigma bonds and one sigma component of $C=C$, for five sigma and one pi bond. Dichloroethene, $C_2H_2Cl_2$, similarly contains one carbon-carbon sigma bond, two carbon-hydrogen and two carbon-chlorine sigma bonds, plus one pi bond: again five sigma and one pi.

Step 3: Check the nitrogen fluoride alternative In N_2F_2, the usual molecular connectivity is $F-N=N-F$. The two nitrogen-fluorine single bonds and the sigma component of the nitrogen-nitrogen double bond give three sigma bonds; the double bond supplies only one pi bond. Hence this alternative also fails the required count. The triple bond in HCN supplies exactly the required two pi bonds without adding too many single bonds.

Step 4: Conclusion HCN contains two sigma bonds and two pi bonds. Hence, the correct option is (D).

Quick Tip to Solve: Convert each triple bond into $1\sigma + 2\pi$, then add the sigma bonds joining the remaining atoms.

68. The S° of the following substances are: $CH_4(g)$, $186.2\,J\,K^{-1}\,mol^{-1}$; $O_2(g)$, $205.2\,J\,K^{-1}\,mol^{-1}$; $CO_2(g)$, $213.6\,J\,K^{-1}\,mol^{-1}$; $H_2O\,(g)$, $69.9\,J\,K^{-1}\,mol^{-1}$. The entropy change ΔS° for the reaction $CH_4(g) + 2\,O_2(g) \longrightarrow CO_2(g) + 2\,H_2O\,(l)$ is: [4 marks]

(A) $-312.5\,J\,K^{-1}\,mol^{-1}$

(B) $-242.8\,J\,K^{-1}\,mol^{-1}$

(C) $-108.1\,J\,K^{-1}\,mol^{-1}$

(D) $-37.6\,J\,K^{-1}\,mol^{-1}$

Correct Answer: The supplied entropies give $-243.2\,J\,K^{-1}\,mol^{-1}$, absent from the options; the water phase is inconsistent.

Solution:

Concept:

- The standard entropy change is the stoichiometric sum of product entropies minus that of reactant entropies: $\Delta S^\circ = \sum \nu S^\circ_{\text{p}} - \sum \nu S^\circ_{\text{r}}$.
- Use the supplied numerical entropies and the coefficients of the stated reaction. Entropy change has units $J\,K^{-1}\,mol^{-1}$ for the reaction as written.

Step 1: Write the stoichiometric subtraction The reaction is $CH_4(g) + 2\,O_2(g) \longrightarrow CO_2(g) + 2\,H_2O\,(l)$. Each water term and each oxygen term must be multiplied by two. Substituting the tabulated numbers supplied in the question gives

$$\Delta S^\circ = [213.6 + 2(69.9)] - [186.2 + 2(205.2)].$$

No bond energies or enthalpies are needed, because the given quantities are absolute standard molar entropies.

Step 2: Evaluate products and reactants separately The product sum is

$$213.6 + 139.8 = 353.4\,J\,K^{-1}\,mol^{-1},$$

and the reactant sum is

$$186.2 + 410.4 = 596.6\,J\,K^{-1}\,mol^{-1}.$$

Therefore

$$\Delta S^\circ = 353.4 - 596.6 = -243.2\,J\,K^{-1}\,mol^{-1}.$$

The supplied numbers do not give any listed option exactly.

Step 3: Assess the listed values The negative sign is consistent with the stated conversion of three gaseous reactant molecules to one gaseous product molecule together with liquid water. Among the listed values, -242.8 is very close to the directly calculated -243.2; the other alternatives differ substantially. Thus (B) is only the nearest listed numerical value, not an exact result: the supplied entropies produce a difference of $0.4\,J\,K^{-1}\,mol^{-1}$.

Step 4: Conclusion The given values give $\Delta S^\circ = -243.2\,J\,K^{-1}\,mol^{-1}$; the nearest listed value is $-242.8\,J\,K^{-1}\,mol^{-1}$. No listed option equals the result from the supplied data, and water is labelled gas in the entropy list but liquid in the reaction. Therefore the literal question has no exact correct option; (B) is merely the nearest value.

Quick Tip to Solve: Calculate the two stoichiometric sums separately before subtracting; both oxygen and water carry coefficients of two.

69. The standard enthalpy of formation $\Delta_f H^\circ_{298}$ for methane, CH_4, is $-74.9\,kJ\,mol^{-1}$. In order to calculate the average energy given out in the formation of a $C-H$ bond from this it is necessary to know which one of the following? [4 marks]

(A) The dissociation energy of the hydrogen molecule, H_2.

(B) The first four ionisation energies of carbon.

(C) The dissociation energy of H_2 and enthalpy and sublimation of carbon (graphite).

(D) The first four ionisation energies of carbon and electron affinity of hydrogen.

Correct Answer: (C) The dissociation energy of H_2 and enthalpy and sublimation of carbon (graphite).

Solution:

Concept:

- Standard formation enthalpy refers to producing one mole of a compound from elements in their standard states.
- A bond enthalpy relates gaseous atoms and gaseous molecules. Hess's law connects the standard-state formation process to atomization followed by bond formation.

Step 1: Write the formation process Methane forms from $C(\text{graphite}) + 2\,H_2(g) \longrightarrow CH_4(g)$, with $\Delta_f H^\circ = -74.9\,kJ\,mol^{-1}$. This energy is not simply four times the energy released by making one carbon-hydrogen bond: the starting materials are graphite and hydrogen molecules, not separate gaseous carbon and hydrogen atoms.

Step 2: Introduce the gaseous-atom intermediate First convert $C(\text{graphite})$ to $C\,(g)$, requiring the sublimation or atomization enthalpy of graphite. Next dissociate two moles of $H_2(g)$ into four moles of $H\,(g)$, requiring twice the hydrogen-hydrogen dissociation energy. Formation of the four carbon-hydrogen bonds from these atoms then releases the corresponding bonding energy.

Step 3: Apply Hess's law and compare the data sets If D_{C-H} denotes the average bond dissociation enthalpy, the energy balance is

$$\Delta_f H^\circ = \Delta_{sub} H^\circ(C) + 2D(H{-}H) - 4D_{C-H}.$$

Both the graphite sublimation enthalpy and hydrogen dissociation energy are needed. Option (A) supplies only the latter. Options (B) and (D) concern ionic processes, whereas neutral gaseous atoms are the required intermediate. Option (C) supplies both necessary atomization terms.

Step 4: Conclusion Because carbon starts as graphite, hydrogen dissociation alone cannot complete the Hess cycle, so the question-sheet key omits an essential atomization term. The necessary extra data are the dissociation energy of H_2 and sublimation enthalpy of graphite. Hence, the correct option is (C).

Quick Tip to Solve: Before extracting a bond energy from a formation enthalpy, convert every standard-state element into its neutral gaseous atoms.

70. What happens when an inert gas is added to an equilibrium keeping volume unchanged? [4 marks]

(A) More product will form

(B) Less product will form

(C) More reactant will form

(D) Equilibrium will remain unchanged

Correct Answer: (D) Equilibrium will remain unchanged

Solution:

Concept:

- At a fixed temperature, a chemical equilibrium is determined by the activities or partial pressures of the reacting species.
- For an ideal gas, the partial pressure of species i is $p_i = n_i RT/V$. At constant volume and temperature, adding an inert gas does not change the reacting species' partial pressures.

Step 1: Separate total pressure from partial pressure Adding an inert gas increases the total number of gas particles and therefore raises total pressure in the fixed vessel. However, it neither reacts with the equilibrium mixture nor changes the number of moles of any reactant or product. With the same temperature and vessel volume, each reacting component retains its original value of $n_i RT/V$.

Step 2: Compare the reaction quotient with its equilibrium value For a general reaction $aA + bB \rightleftharpoons cC + dD$, the pressure quotient is

$$Q_p = \frac{p_C^c p_D^d}{p_A^a p_B^b}.$$

Every partial pressure in this expression remains unchanged, so Q_p stays equal to K_p. The inert gas is absent from the expression. There is consequently no driving force favoring either the forward or the reverse reaction.

Step 3: Evaluate all alternatives Options (A), (B) and (C) each predict a shift and therefore require a change in the reacting partial pressures, which has not occurred. Option (D) states the correct result. A pressure increase caused by compression would be different because compression changes the volume and hence the partial pressures of the reacting gases. Here the stated constant-volume condition is decisive.

Step 4: Conclusion At unchanged volume, adding an inert gas leaves the equilibrium composition unchanged. Hence, the correct option is (D).

Quick Tip to Solve: For an inert-gas addition, ask whether volume or pressure is fixed; at fixed volume the reacting partial pressures stay unchanged.

71. The conjugate base of hydrazoic acid is [4 marks]

(A) N^{-3}

(B) N_3^-

(C) N_2^-

(D) HN_3^-

Correct Answer: (B) N_3^-

Solution:

Concept:

- A conjugate base is formed when an acid donates one proton, H^+. Removing a proton changes the charge by minus one without removing any other atom.
- Hydrazoic acid has the molecular formula HN_3. Its nitrogen skeleton contains three nitrogen atoms, and that skeleton remains intact during acid-base ionization.

Step 1: Write the proton-loss equation Treat hydrazoic acid as a proton donor. Separate its hydrogen as a proton rather than as a neutral atom or a hydride ion. The acid-base change is

$$HN_3 \longrightarrow H^+ + N_3^-.$$

The product N_3^- is the azide ion. The total charge is conserved: the neutral acid produces a singly positive proton and a singly negative anion. All three nitrogen atoms of the original acid appear in that anion.

Step 2: Compare the atoms and charge in every option The required species must simultaneously contain three nitrogen atoms, contain no remaining acidic hydrogen, and carry one negative charge. The second option has all three properties. The first option represents a single nitrogen atom carrying charge minus three, so it is nitride rather than azide. The third option has only two nitrogen atoms and cannot arise by proton loss from this acid. The fourth option still contains hydrogen and therefore does not represent the conjugate base obtained by removing the acid's proton. The distinction between a subscript, which counts atoms, and a superscript, which records charge, is decisive here.

Step 3: Conclusion The conjugate base of HN_3 is the azide ion, N_3^-. Hence, the correct option is (B).

Quick Tip to Solve: Form a conjugate base by removing exactly one H^+: preserve the remaining atom counts and reduce the total charge by one.

72. Hydrogen peroxide acts both as an oxidising and as a reducing agent depending upon the nature of the reacting species. In which of the following cases H_2O_2 acts as a reducing agent in acid medium?

[4 marks]

(A) MnO_4^-

(B) $Cr_2O_7^{2-}$

(C) SO_3^{2-}

(D) KI

Correct Answer: Both permanganate and dichromate oxidize hydrogen peroxide in acid; options (A) and (B) are correct.

Solution:

Concept:

- In H_2O_2, oxygen has oxidation number -1. Peroxide is a reducing agent when it loses electrons and its oxygen is oxidized to O_2, in which oxygen has oxidation number zero.
- Acidified permanganate and acidified dichromate are both oxidizing agents. Each can accept the electrons released by peroxide.

Step 1: Identify the peroxide oxidation half-reaction The relevant half-reaction in acidic solution is

$$H_2O_2 \longrightarrow O_2 + 2\,H^+ + 2\,e^-.$$

Here hydrogen peroxide donates electrons, so it reduces the other reactant. Acidic conditions do not limit this behaviour to a single oxidant: any sufficiently strong oxidizing species can accept the electrons.

Step 2: Test both oxidizing oxyanions Permanganate is reduced to manganese(II), giving

$$2\,MnO_4^- + 5\,H_2O_2 + 6\,H^+ \longrightarrow 2\,Mn^{2+} + 5\,O_2 + 8\,H_2O.$$

Dichromate is reduced to chromium(III), giving

$$Cr_2O_7^{2-} + 3\,H_2O_2 + 8\,H^+ \longrightarrow 2\,Cr^{3+} + 3\,O_2 + 7\,H_2O.$$

Both balanced equations show peroxide becoming oxygen and acting as the reducing agent. Thus both of the first two options satisfy the stated condition; selecting only one would omit another chemically correct case.

Step 3: Exclude sulfite and iodide Sulfite is oxidized to sulfate by peroxide: $SO_3^{2-} + H_2O_2 \longrightarrow SO_4^{2-} + H_2O$. Likewise iodide is oxidized to iodine: $H_2O_2 + 2\,I^- + 2\,H^+ \longrightarrow I_2 + 2\,H_2O$. In these reactions peroxide gains electrons and acts as an oxidizing agent. They therefore do not answer the question.

Step 4: Conclusion Both MnO_4^- and $Cr_2O_7^{2-}$ oxidize H_2O_2 to O_2 in acid. Options (A) and (B) are correct; the printed alternatives do not give a unique answer.

Quick Tip to Solve: Track the peroxide oxygen: a change from -1 to zero means peroxide is the reducing agent; a change to -2 means it is the oxidizing agent.

73. Which one of the following acids does not exhibit optical isomerism? [4 marks]

(A) Lactic acid

(B) Tartaric acid

(C) Maleic acid

(D) α-amino acids

Correct Answer: (C) Maleic acid

Solution:

Concept:

- Optical isomerism requires chirality: a molecule and its mirror image must be nonsuperimposable. A tetrahedral carbon attached to four different groups is a common source of chirality.
- Maleic acid is cis-butenedioic acid, $HOOC-CH=CH-COOH$. Its double-bond carbons are trigonal planar rather than tetrahedral stereogenic centres.

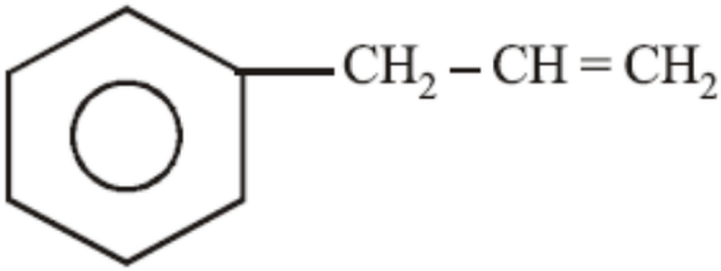
Step 1: Examine the structure of maleic acid Both central carbon atoms of maleic acid belong to a carbon-carbon double bond. The substituents around these atoms lie in a planar arrangement, and the molecule has no tetrahedral carbon bearing four different substituents. The cis arrangement distinguishes maleic acid from the trans geometrical isomer, fumaric acid, but cis-trans isomerism does not by itself produce optical activity. Maleic acid is superimposable on its mirror image and hence does not exhibit optical isomerism.

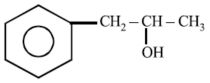
Step 2: Compare the remaining classes Lactic acid, $CH_3-CH(OH)-COOH$, has a carbon attached to hydrogen, hydroxyl, methyl and carboxyl groups, so it has two enantiomeric forms. Tartaric acid contains stereogenic centres and includes an optically active enantiomeric pair, even though its meso form is inactive. In general an α-amino acid, $R-CH(NH_2)-COOH$, has a chiral alpha carbon when the side chain differs from hydrogen. Glycine is the familiar achiral exception, but the class does include optical isomers. Thus these alternatives cannot replace maleic acid as the named acid that lacks optical isomerism.

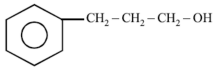
Step 3: Conclusion Maleic acid has geometrical isomerism but no optical isomerism. Hence, the correct option is (C).

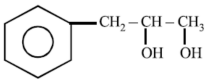
Quick Tip to Solve: Distinguish geometrical isomerism about a double bond from chirality; a cis label alone is not evidence of optical activity.

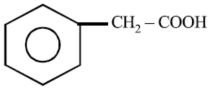
74. The following compound on mercuration-demercuration produces the major product: [4 marks]

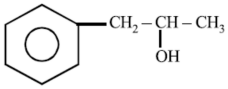


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- Oxymercuration followed by demercuration hydrates an alkene in Markovnikov orientation: hydrogen goes to the less substituted alkene carbon and hydroxyl to the more substituted one.
- The bridged mercurinium intermediate avoids a freely rearranging carbocation. The carbon skeleton is retained while the double bond becomes a carbon-carbon single bond.

Step 1: Identify the two alkene carbons The substrate is allylbenzene, $C_6H_5-CH_2-CH=CH_2$. Its terminal alkene carbon is the CH_2 group, whereas the internal alkene carbon is attached to the benzyl group and to hydrogen. During hydration the internal carbon receives hydroxyl and the terminal carbon receives hydrogen. The benzene ring and its adjacent methylene group remain in their original positions.

Step 2: Write the hydration product The net conversion is

$$C_6H_5-CH_2-CH=CH_2 + H_2O \longrightarrow C_6H_5-CH_2-CH(OH)-CH_3.$$

The first structure depicts precisely this secondary alcohol. The second structure is a primary alcohol with hydroxyl at the terminal carbon and corresponds to the opposite orientation. The third contains two hydroxyl groups, which would require dihydroxylation rather than the single hydration being performed. The fourth is a carboxylic acid and would require oxidative chemistry that is not part of mercuration-demercuration. No shift of the benzyl group or other skeletal rearrangement is needed.

Step 3: Conclusion The major product is $C_6H_5-CH_2-CH(OH)-CH_3$, the secondary alcohol shown first. Hence, the correct option is (A).

Quick Tip to Solve: For oxymercuration-demercuration, add one H and one OH across the double bond in Markovnikov orientation without rearranging the carbon skeleton.

75. In the presence of peroxide, HCl and HI do not give anti-Markownikoff's addition of alkenes because [4 marks]

(A) One of the steps is endothermic in HCl and HI

(B) Both HCl and HI are strong acids

(C) HCl is oxidizing and the HI is reducing

(D) All the steps are exothermic is HCl and HI

Correct Answer: (A) One of the steps is endothermic in HCl and HI

Solution:

Concept:

- The peroxide effect depends on a sustainable radical chain, rather than simply on the acid strength of a hydrogen halide.
- The propagation steps are halogen-radical addition to the alkene followed by hydrogen abstraction from HX. Both must be energetically favourable for an efficient chain.

Step 1: Write the radical propagation sequence A halogen radical adds to the carbon-carbon double bond, producing a carbon radical. That carbon radical then abstracts hydrogen from a hydrogen halide molecule, producing the addition product and regenerating the halogen radical. Anti-Markovnikov orientation follows from formation of the more stable carbon-radical intermediate. Peroxide initiates radicals, but initiation alone cannot make an unfavourable propagation reaction proceed efficiently.

Step 2: Identify the unfavourable step for each reagent For hydrogen chloride, the strong hydrogen-chlorine bond makes hydrogen abstraction by the carbon radical endothermic. For hydrogen iodide, addition of an iodine radical to the double bond is endothermic because the carbon-iodine bond formed is comparatively weak. Consequently the chain does not continue effectively in either case. Hydrogen bromide is the usual reagent for which both propagation steps are favourable. The first option correctly identifies an endothermic step in each unsuccessful reagent. Their classification as strong acids is irrelevant to this radical mechanism. Ordinary oxidizing or reducing descriptions do not explain the specific chain failure, and the claim that all propagation steps are exothermic is the opposite of the required energy balance.

Step 3: Conclusion An endothermic propagation step prevents the peroxide-induced chain with both HCl and HI. Hence, the correct option is (A).

Quick Tip to Solve: Remember the failing steps separately: HCl resists hydrogen abstraction, whereas iodine-radical addition to the alkene is unfavourable for HI.

76. Global warming is due to increase of [4 marks]

(A) methane and nitrous oxide in atmosphere

(B) methane and CO2 in atmosphere

(C) methane and O3 in atmosphere

(D) methane and CO in atmosphere

Correct Answer: The usual intended answer is (B), but (A) and (C) also name greenhouse gases, so the wording does not establish a unique option.

Solution:

Concept:

- Greenhouse gases absorb outgoing infrared radiation and reduce the rate at which the Earth's surface loses energy to space. An increase in their atmospheric abundance can increase the greenhouse effect.
- Carbon dioxide and methane are important greenhouse gases. Nitrous oxide and tropospheric ozone also contribute to warming, so a list need not contain carbon dioxide to name gases capable of producing the stated effect.

Step 1: Identify the conventional pair Option (B) gives methane and carbon dioxide, the pair conventionally emphasized in introductory descriptions of global warming. Both gases absorb terrestrial infrared radiation. Increased carbon dioxide from fossil-fuel combustion and increased methane from biological and industrial sources strengthen the greenhouse effect. Thus (B) is a defensible and usual intended answer to this question.

Step 2: Check whether the other pairs are excluded Option (A) gives methane and nitrous oxide, which are also greenhouse gases. Nitrous oxide is not an inert atmospheric gas: its infrared absorption contributes to warming. Option (C) gives methane and ozone; tropospheric ozone is likewise a greenhouse gas. The wording does not restrict attention to the two most familiar gases or specify the atmospheric layer for ozone. Consequently neither (A) nor (C) can be categorically rejected as a pair whose increase may cause warming. Carbon monoxide in (D) is not normally listed as a principal direct greenhouse gas, although it can have indirect atmospheric effects. That distinction does not remove the multiple valid pairs in the first three alternatives.

Step 3: Conclusion The conventional intended answer is methane and carbon dioxide, option (B). However, options (A) and (C) also contain greenhouse gases; as worded, the question does not supply a unique scientifically defensible choice.

Quick Tip to Solve: Recognize methane, carbon dioxide, nitrous oxide and tropospheric ozone as greenhouse gases; test every listed pair before assuming only the familiar pair can be correct.

77. In a monoclinic unit cell, the relation of sides and angles are respectively [4 marks]

(A) $a = b \neq c$ and $\alpha = \beta = \gamma = 90^\circ$

(B) $a \neq b \neq c$ and $\alpha = \beta = \gamma = 90^\circ$

(C) $a \neq b \neq c$ and $\beta = \gamma = 90^\circ \neq \alpha$

(D) $a \neq b \neq c$ and $\alpha \neq \beta \neq \gamma \neq 90^\circ$

Correct Answer: (C) $a \neq b \neq c$ and $\beta = \gamma = 90^\circ \neq \alpha$

Solution:

Concept:

- A crystal unit cell is described by three edge lengths a, b, c and three interaxial angles α, β, γ. A crystal system specifies restrictions on these six parameters.
- In a monoclinic unit cell the three lengths are unequal and two angles are right angles; the remaining angle is oblique. Axis names can be permuted without changing the crystal system.

Step 1: Apply the monoclinic restrictions The term monoclinic describes a cell with a single oblique interaxial angle. Thus the edges obey $a \neq b \neq c$, and exactly two of the three angles are 90°. Although a common convention takes $\alpha = \gamma = 90^\circ$ and $\beta \neq 90^\circ$, the printed alternatives use a different axis convention: $\beta = \gamma = 90^\circ$ and $\alpha \neq 90^\circ$. Relabelling axes changes the name of the non-right angle, not the geometry or the crystal system.

Step 2: Classify all alternatives The first alternative has two equal edge lengths and three right angles, which is tetragonal rather than monoclinic. The second has three unequal edge lengths but all three angles right, which is orthorhombic. The third has three unequal edges, two right angles and one non-right angle, satisfying the monoclinic definition. The last gives three unequal edges and three non-right angles, which describes the general triclinic case. Comparing the angle restrictions is therefore essential: unequal edges alone cannot distinguish orthorhombic, monoclinic and triclinic cells.

Step 3: Conclusion In the printed axis convention the monoclinic relations are $a \neq b \neq c$ and $\beta = \gamma = 90^\circ \neq \alpha$. Hence, the correct option is (C).

Quick Tip to Solve: Count the oblique angles before checking edge equality: monoclinic has one, orthorhombic has none, and a general triclinic cell has three.

78. How many electrons would be required to deposit 6.35 g of copper at the cathode during the electrolysis of an aqueous solution of copper sulphate? (Atomic mass of copper = 63.5u, NA= Avogadro's constant) [4 marks]

(A) $N_A/20$

(B) $N_A/10$

(C) $N_A/5$

(D) $N_A/2$

Correct Answer: (C) $N_A/5$

Solution:

Concept:

- The cathodic reduction is $\mathrm{Cu}^{2+} + 2\,\mathrm{e}^- \longrightarrow \mathrm{Cu}$: each copper ion needs two electrons.
- The amount of copper is its mass divided by molar mass, and one mole of electrons contains N_A electrons.

Step 1: Calculate the amount of copper The stated atomic mass gives the molar mass $M_{\mathrm{Cu}} = 63.5\,\mathrm{g\,mol^{-1}}$. The mass deposited is a mass of copper metal, so the relevant amount is not the number of moles of hydrated copper sulphate.

$$n(\mathrm{Cu}) = \frac{6.35\,\mathrm{g}}{63.5\,\mathrm{g\,mol^{-1}}} = 0.100\,\mathrm{mol}.$$

Thus the cathode gains one tenth of a mole of copper atoms.

Step 2: Use the electron stoichiometry In aqueous copper sulphate the copper is present as Cu^{2+}. Reducing it to neutral copper removes a charge of two positive units per ion, requiring two electrons per deposited atom.

$$n(e^-) = 2n(\mathrm{Cu}) = 2(0.100) = 0.200\,\mathrm{mol}.$$

$$N(e^-) = 0.200N_A = \frac{N_A}{5}.$$

The option $N_A/10$ counts copper atoms but omits the factor two. The options $N_A/20$ and $N_A/2$ correspond to incorrect electron amounts of 0.05 and 0.5 mole, respectively. No electrolysis time or applied current is needed because the deposited mass already determines the charge transferred.

Step 3: Conclusion The required number is $N_A/5$ electrons. Hence, the correct option is (C).

Quick Tip to Solve: First write the metal-ion reduction half-equation; multiply moles of deposited metal by the number of electrons in that equation.

79. The rate coefficient k for a particular reactions is $1.3\times10^{-4}\,\mathrm{M^{-1}\,s^{-1}}$ at $100^\circ\mathrm{C}$, and $1.3\times10^{-3}\,\mathrm{M^{-1}\,s^{-1}}$ at $150^\circ\mathrm{C}$. What is the energy of activation E_A (in kJ) for this reaction? (R = molar gas constant $= 8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$) [4 marks]

(A) 16

(B) 60

(C) 99

(D) 132

Correct Answer: (B) 60

Solution:

Concept:

- The Arrhenius law is $k = A\exp[-E_A/(RT)]$, where E_A is the molar activation energy.
- For two temperatures, $\ln(k_2/k_1) = (E_A/R)(1/T_1 - 1/T_2)$; temperatures must be in kelvin.

Step 1: Form the rate ratio and temperatures The coefficient rises from 1.3×10^{-4} to 1.3×10^{-3} in identical units. Therefore their ratio is dimensionless and the common numerical factor cancels.

$$\frac{k_2}{k_1} = 10, \qquad T_1 = 373\,\mathrm{K}, \qquad T_2 = 423\,\mathrm{K}.$$

Using Celsius directly in reciprocal temperatures would be invalid because the exponential law is defined on the absolute temperature scale.

Step 2: Calculate the activation energy Rearrange the two-temperature equation before substituting:

$$E_A = \frac{R\ln(k_2/k_1)T_1T_2}{T_2 - T_1}.$$

$$E_A = \frac{8.314 \times 2.3026 \times 373 \times 423}{50}\,\mathrm{J\,mol^{-1}} \approx 6.04 \times 10^4\,\mathrm{J\,mol^{-1}}.$$

Dividing by 1000 gives approximately $60.4\,\mathrm{kJ\,mol^{-1}}$, which matches 60 among the stated choices. The positive sign is consistent with a larger rate coefficient at higher temperature. The other values, 16, 99 and 132, do not reproduce the observed tenfold increase over this interval; the common second-order units of the coefficients cancel and do not alter this calculation.

Step 3: Conclusion The activation energy is approximately $60\,\mathrm{kJ\,mol^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: For two Arrhenius data points, use the ratio of rate constants and kelvin temperatures; convert joules to kilojoules only at the end.

80. Which of the following xenon-oxo compounds may not be obtained by hydrolysis of xenon fluorides? [4 marks]

(A) XeO_2F_2

(B) $XeOF_4$

(C) XeO_3

(D) XeO_4

Correct Answer: (D) XeO_4

Solution:

Concept:

- Hydrolysis replaces Xe–F bonding by Xe–O bonding with formation of HF.
- Xenon in XeF_6 and its usual partial-hydrolysis products has oxidation state +6; XeO_4 contains xenon in +8.

Step 1: Identify the obtainable hydrolysis products Controlled hydrolysis of xenon hexafluoride produces oxyfluorides, and complete hydrolysis gives xenon trioxide. The successive balanced equations illustrate the three accessible products:

$$XeF_6 + H_2O \longrightarrow XeOF_4 + 2\,HF$$

$$XeOF_4 + H_2O \longrightarrow XeO_2F_2 + 2\,HF$$

$$XeO_2F_2 + H_2O \longrightarrow XeO_3 + 2\,HF.$$

Each step replaces two fluorine atoms by one oxygen atom, conserving the oxidation state of xenon. Thus the compounds listed in options A, B and C are consistent with the ordinary hydrolysis sequence.

Step 2: Compare xenon tetroxide In XeO_4, four oxygen atoms at oxidation state -2 require xenon to be +8. Merely replacing fluoride by oxide during hydrolysis of XeF_6 cannot raise Xe from +6 to +8. Xenon tetroxide is instead associated with the chemistry of xenon(VIII), such as preparation from perxenates; it is not a direct product of hydrolysis of xenon fluorides. This distinguishes a change in oxidation state from a substitution of ligands by water. The presence of oxygen alone does not imply that every xenon oxide can be made by the same hydrolysis reaction.

Step 3: Conclusion XeO_4 is not obtained by the stated hydrolysis route. Hence, the correct option is (D).

Quick Tip to Solve: Check the xenon oxidation state: ordinary hydrolysis of XeF6 gives Xe(VI) oxyfluorides and XeO3, whereas XeO4 requires Xe(VIII).

81. Which one of the following exhibits the large number of oxidation states? [4 marks]

(A) Ti(22)

(B) V(23)

(C) Cr(24)

(D) Mn(25)

Correct Answer: (D) Mn(25)

Solution:

Concept:

- Both the 4s and 3d electrons can participate in bonding in first-row transition metals.
- The range of accessible oxidation states generally grows towards manganese, which has five 3d and two 4s valence electrons.

Step 1: Write the valence configurations The neutral atoms have the configurations $\mathrm{Ti} : [\mathrm{Ar}]3d^2 4s^2$, $\mathrm{V} : [\mathrm{Ar}]3d^3 4s^2$, $\mathrm{Cr} : [\mathrm{Ar}]3d^5 4s^1$, and $\mathrm{Mn} : [\mathrm{Ar}]3d^5 4s^2$. The atomic numbers supplied identify these consecutive first-row metals. Counting only the two 4s electrons would miss the essential reason why transition metals display variable oxidation states.

Step 2: Compare their common oxidation-state ranges Titanium commonly shows +2, +3 and +4; vanadium extends through +5; chromium has an important +6 state in chromate and dichromate as well as lower states. Manganese shows the widest familiar range among these choices, from +2 through +7. Examples include Mn^{2+}, manganese(III) compounds, $\mathrm{MnO_2}$ with +4, manganate with +6 and permanganate with +7; +5 also occurs. The seven valence electrons in the $3d^5 4s^2$ arrangement explain its high maximum. Thus Ti, V and Cr each provide a narrower usual set of states than Mn in the comparison requested. This is a comparison of available chemical oxidation states, not a claim that all electrons are removed as isolated gas-phase ions.

Step 3: Conclusion Manganese, atomic number 25, exhibits the largest number of oxidation states here. Hence, the correct option is (D).

Quick Tip to Solve: Across the early 3d series, compare both d and s valence electrons; manganese marks the broadest familiar oxidation-state range.

82. Copper becomes green when exposed to moist air for a long period. This is due to [4 marks]

(A) the formation of a layer of cupric oxide on the surface of copper.

(B) the formation of a layer of basic carbonate of copper on the surface of copper.

(C) the formation of a layer of cupric hydroxide on the surface of copper.

(D) the formation of basic copper sulphate layer on the surface of the metal.

Correct Answer: (B) the formation of a layer of basic carbonate of copper on the surface of copper.

Solution:

Concept:

- Moist air provides water, oxygen and carbon dioxide, allowing copper to corrode slowly.

- The characteristic green patina is basic copper carbonate, often represented as $CuCO_3 \cdot Cu(OH)_2$.

Step 1: Identify the green surface product Copper exposed for a long time to the atmosphere develops a green coating rather than retaining its original reddish metallic surface. Oxygen oxidises the metal, and moisture together with atmospheric carbon dioxide produces the basic carbonate. The overall composition may be represented by

$$2\,Cu + O_2 + CO_2 + H_2O \longrightarrow CuCO_3 \cdot Cu(OH)_2.$$

The product contains both carbonate and hydroxide and is therefore called a basic carbonate, rather than simply copper carbonate.

Step 2: Eliminate the competing layers Cupric oxide CuO is black, so a layer of it does not account for the characteristic green patina in option A. Copper(II) hydroxide is blue and lacks the carbonate constituent acquired from atmospheric carbon dioxide, excluding C. Basic copper sulphates can occur in particular polluted environments, but D is not the conventional product described by this general moist-air observation. Under the stated conditions the expected board-level identification is basic copper carbonate. The long exposure matters because the adherent corrosion film develops progressively through reactions at the damp metal surface; it is not merely a temporary wet colour or a physical deposit of water.

Step 3: Conclusion The green coating is a layer of basic copper carbonate on copper, formed by reaction with oxygen, water and carbon dioxide as shown above. Hence, the correct option is (B). The supplied key lists basic copper sulphate, but no sulphur-containing reactant is given in this moist-air question; basic copper sulphate therefore does not describe the standard green patina asked for here.

Quick Tip to Solve: Associate the familiar green patina of copper in moist air with basic copper carbonate; black copper oxide describes a different product.

83. Among the following species the one which causes the highest CFSE, $\Delta 0$ as a ligand is [4 marks]

(A) CN^-

(B) NH_3

(C) F^-

(D) CO

Correct Answer: (D) CO

Solution:

Concept:

- Ligands split the metal d orbitals into sets of different energies; Δ_o denotes octahedral splitting.
- The spectrochemical series ranks ligand field strength; among these choices $F^- < NH_3 < CN^- < CO$.

Step 1: Compare ligand field strengths The question uses CFSE and Δ_o to ask which ligand produces the greatest ligand-field splitting. With the same metal centre, oxidation state and geometry, the ligand further towards the strong-field end of the spectrochemical series produces the larger splitting.

Fluoride is a weak-field ligand, ammonia is intermediate to strong, and cyanide and carbon monoxide are strong-field ligands. Thus fluoride and ammonia cannot be the maximum in the supplied list.

Step 2: Distinguish carbon monoxide from cyanide Both cyanide and carbon monoxide can accept electron density from filled metal d orbitals into suitable ligand antibonding orbitals. Such pi acceptance contributes to their strong ligand fields. In the standard spectrochemical ordering carbon monoxide lies beyond cyanide, so it gives the largest Δ_o of the four. Strictly, the numerical crystal-field stabilisation energy also depends on the metal d-electron count and occupancy, which the question does not specify. The stated Δ_o and the comparison of ligands make the intended quantity ligand-field splitting. Option A is strong but is below CO; B and C are weaker still. No metal electron configuration needs to be assumed to answer this ranking question.

Step 3: Conclusion Carbon monoxide, CO, produces the highest ligand-field splitting among the listed ligands. Hence, the correct option is (D).

Quick Tip to Solve: For a ligand-only comparison, rank the ligands in the spectrochemical series; CO lies at the strong-field end beyond cyanide.

84. Which one of the following complexes will most likely absorb visible light? (At nos. Sc = 21, Ti = 22, V = 23, Zn = 30) [4 marks]

(A) $[Sc(H_2O)_6]^{3+}$

(B) $[Ti(NH_3)_6]^{4+}$

(C) $[V(NH_3)_6]^{3+}$

(D) $[Zn(NH_3)_6]^{2+}$

Correct Answer: (C) $[V(NH_3)_6]^{3+}$

Solution:

Concept:

- Visible absorption in many transition-metal complexes arises from d–d transitions between crystal-field-split d orbitals.
- A partially filled d subshell can support such transitions; d^0 and d^{10} ions have no ordinary d–d absorption.

Step 1: Determine each metal-ion configuration Water and ammonia are neutral ligands, so the overall complex charge is the oxidation state of the metal. The four ions are therefore Sc^{3+}, Ti^{4+}, V^{3+} and Zn^{2+}. Remove 4s electrons before 3d electrons from their neutral configurations. Scandium(III) and titanium(IV) both have the argon core and no d electrons, giving d^0. Vanadium(III) has $3d^2$, whereas zinc(II) retains a completely filled $3d^{10}$ subshell.

Step 2: Select the complex with accessible d–d absorption In the octahedral vanadium(III) ammine complex, the two d electrons occupy the lower-energy d orbitals and can be promoted to the higher-energy set by absorbing radiation. Such a partially filled transition-metal d shell makes visible absorption and colour likely. In the scandium and titanium choices there is no d electron to promote, while in the zinc choice both split sets are full, preventing an ordinary d–d excitation. Other mechanisms such as charge-transfer absorption can exist in chemistry, but these simple neutral-ligand

complexes are being compared by their metal configurations. Thus the d2 vanadium species is the clear selection, while A, B and D lack the requisite partially filled d shell.

Step 3: Conclusion $[V(NH_3)_6]^{3+}$ is most likely to absorb visible light. Hence, the correct option is (C).

Quick Tip to Solve: Find the oxidation state before counting d electrons: neutral ammonia leaves the metal charge equal to the charge written on the complex.

85. Conversion of benzene diazonium chloride to chlorobenzene is an example of which of the following reactions? [4 marks]

(A) Claisen

(B) Friedel-craft

(C) Sandmeyer

(D) Wurtz

Correct Answer: (C) Sandmeyer

Solution:

Concept:

- Aryl diazonium salts undergo substitution in which the diazonium group is replaced by another group with loss of nitrogen gas.
- The Sandmeyer reaction replaces the diazonium group by chlorine, bromine or cyanide using the corresponding copper(I) salt.

Step 1: Identify the group being replaced Benzene diazonium chloride contains the aryl diazonium group, $C_6H_5{-}N_2{}^+$, paired with chloride. Chlorobenzene has chlorine directly attached to the aromatic ring. Thus the transformation replaces the diazonium group by chlorine; it neither joins two alkyl groups nor introduces an acyl or alkyl group onto benzene.

Step 2: Match the transformation with its named reaction Treatment with cuprous chloride in hydrochloric acid gives the substitution:

$$C_6H_5N_2Cl \xrightarrow{CuCl/HCl} C_6H_5Cl + N_2.$$

This is the characteristic Sandmeyer conversion. Escape of nitrogen gas accompanies removal of the diazonium group.

Step 3: Eliminate the remaining names Claisen reactions involve carbonyl compounds or rearrangement of allyl aryl ethers, rather than this diazonium substitution. Friedel–Crafts reactions introduce alkyl or acyl groups on an aromatic ring using a Lewis acid. Wurtz coupling joins alkyl halide fragments using sodium in dry ether. None describes the given replacement.

Step 4: Conclusion The benzene diazonium chloride to chlorobenzene conversion is the Sandmeyer reaction. Hence, the correct option is (C).

Quick Tip to Solve: For an aryl diazonium salt, replacement by Cl, Br or CN with a copper(I) salt immediately suggests Sandmeyer substitution.

86. The major product obtained in the photo catalysed bromination of 2-methylbutane is [4 marks]

(A) 1-bromo-2-methylbutane

(B) 1-bromo-3-methylbutane

(C) 2-bromo-3-methylbutane

(D) 2-bromo-2-methylbutane

Correct Answer: (D) 2-bromo-2-methylbutane

Solution:

Concept:

- Photochemical bromination of an alkane proceeds by a free radical chain mechanism: light initiates homolysis of Br_2.
- Bromination is selective: abstraction of a tertiary hydrogen is favoured because it produces the most stable alkyl radical.

Step 1: Locate the different hydrogen environments Write 2-methylbutane in condensed form:

$$CH_3-CH(CH_3)-CH_2-CH_3$$

Carbon 2 is tertiary and carries one hydrogen; carbon 3 is secondary and carries two hydrogens. The methyl groups carry primary hydrogens. Substitution can therefore form several monobromo isomers, but they are not formed in equal amounts.

Step 2: Choose the favoured radical pathway A bromine radical preferentially abstracts the tertiary hydrogen from carbon 2, giving $CH_3-C^{\bullet}(CH_3)-CH_2-CH_3$. The tertiary radical is stabilised by electron donation and hyperconjugation from three alkyl groups. Its reaction with Br_2 gives $CH_3-C(Br)(CH_3)-CH_2-CH_3$ and regenerates a bromine radical.

Step 3: Name the product and compare the choices The product has bromine and a methyl substituent both at carbon 2, so its name is 2-bromo-2-methylbutane. The two 1-bromo choices result from substitution at primary carbon positions. The 2-bromo-3-methylbutane choice results from the secondary radical pathway. These pathways are less favoured than tertiary substitution in selective bromination.

Step 4: Conclusion The major product is 2-bromo-2-methylbutane. Hence, the correct option is (D).

Quick Tip to Solve: In photochemical bromination, first find the hydrogen whose removal gives the most stable radical; selectivity usually outweighs the number of primary hydrogens.

87. In the Victor-Meyer's test, the colour given by 1°, 2° and 3° alcohols are respectively [4 marks]

(A) Red, colourless, blue

(B) Red, blue, colourless

(C) Colourless, red, blue

(D) Red, blue, violet

Correct Answer: (B) Red, blue, colourless

Solution:

Concept:

- Victor Meyer's test distinguishes primary, secondary and tertiary alcohols through their conversion to iodides and then nitrogen-containing derivatives.
- After treatment with nitrous acid followed by alkali, primary alcohol derivatives give red colour, secondary derivatives give blue colour, and tertiary alcohols give no characteristic colour.

Step 1: Follow the primary alcohol sequence A primary alcohol is first converted into the corresponding alkyl iodide, which on treatment with silver nitrite gives a primary nitroalkane. It has the required hydrogen atoms on the carbon bearing the nitro group. Reaction with nitrous acid produces a nitrolic acid; its alkaline salt is red. Thus the first colour in the ordered sequence must be red.

Step 2: Compare the secondary and tertiary cases A secondary alcohol gives a secondary nitroalkane through the analogous preliminary steps. Reaction with nitrous acid produces a pseudonitrole, responsible for the characteristic blue colour. A tertiary alcohol does not undergo the corresponding colour-forming sequence, since the necessary hydrogen on the relevant carbon is absent. Its test is recorded as colourless.

Step 3: Match the ordered set The question asks for primary, secondary and tertiary alcohols in that order, so the sequence is red, blue, colourless. The first choice interchanges the secondary and tertiary results. The third incorrectly makes the primary result colourless. The fourth incorrectly assigns violet to a tertiary alcohol.

Step 4: Conclusion The respective colours are red, blue and colourless. Hence, the correct option is (B).

Quick Tip to Solve: Remember Victor Meyer's sequence as primary-red, secondary-blue, tertiary-no colour; preserve that order when checking the options.

88. Phthalic acid reacts with resorcinol in the presence of concentrated H_2SO_4 to give [4 marks]

(A) Phenolphthalein

(B) Alizarin

(C) Coumarin

(D) Fluorescein

Correct Answer: (D) Fluorescein

Solution:

Concept:

- Phthalic acid can form phthalic anhydride under dehydrating conditions; its condensation with phenols gives characteristic dyes.
- Resorcinol is benzene-1,3-diol. Its acid-catalysed condensation with the phthalic component gives the xanthene dye fluorescein.

Step 1: Identify the phenolic starting material Resorcinol contains two hydroxyl groups in the meta positions of a benzene ring. These groups activate the ring toward electrophilic substitution. This distinction matters because the dye formed depends on the phenol used; an answer associated with ordinary phenol cannot automatically be assigned to resorcinol.

Step 2: Apply the condensation reaction Concentrated H_2SO_4 supplies strongly acidic and dehydrating conditions. Phthalic acid provides the phthalic anhydride component, which condenses with resorcinol to build the fluorescein framework. The familiar laboratory preparation uses phthalic anhydride and two equivalents of resorcinol; starting with phthalic acid under these conditions leads to the same dye.

Step 3: Distinguish the other products Phenolphthalein is associated with the condensation of the phthalic component with phenol, not resorcinol. Alizarin is a dihydroxyanthraquinone dye, with an anthraquinone framework. Coumarin is a benzopyrone and is not the product of this condensation. Fluorescein is therefore the only listed dye matching both reactants and the acidic conditions.

Step 4: Conclusion Phthalic acid reacts with resorcinol in concentrated sulphuric acid to form fluorescein. Hence, the correct option is (D).

Quick Tip to Solve: Keep the phenolic partner distinct: phenol gives phenolphthalein, whereas resorcinol gives fluorescein with the phthalic component.

89. Aminoglycosides are usually used as [4 marks]

(A) antibiotic

(B) analgesic

(C) hypnotic

(D) antifertility

Correct Answer: (A) antibiotic

Solution:

Concept:

- Aminoglycosides are a class of antibacterial antibiotics containing amino sugars linked by glycosidic bonds.
- An antibiotic inhibits growth of susceptible microorganisms or kills them; this therapeutic action differs from pain relief, sleep induction and contraception.

Step 1: Recognise the drug family Examples of aminoglycosides include streptomycin, gentamicin and neomycin. They are used against susceptible bacterial infections. The common suffix or a chemical functional group alone does not determine a drug application, so the relevant fact is the recognised pharmacological class of the entire family.

Step 2: Connect the family with its action Aminoglycosides act on bacterial protein synthesis through interaction with the small ribosomal subunit. Their disruption of normal translation gives antibacterial activity. Consequently their usual therapeutic classification is antibiotic. This identification concerns their general use, rather than a particular formulation or an unrelated side effect.

Step 3: Reject the other therapeutic categories Analgesics are primarily used to relieve pain; that is not the characteristic role of aminoglycosides. Hypnotics are medicines used to induce sleep, a different action on the nervous system. Antifertility medicines are used to prevent conception and commonly act through reproductive hormonal pathways. None of these alternatives describes the antibacterial role of aminoglycosides.

Step 4: Conclusion Aminoglycosides are usually used as antibiotics. Hence, the correct option is (A).

Quick Tip to Solve: Link streptomycin or gentamicin with the aminoglycoside family; their treatment of bacterial infections identifies the antibiotic option.

90. Which of the following will not show mutarotation? [4 marks]

(A) Maltose

(B) Lactose

(C) Glucose

(D) Sucrose

Correct Answer: (D) Sucrose

Solution:

Concept:

- Mutarotation is the change in optical rotation as the alpha and beta anomers of a sugar interconvert in solution through an open-chain form.
- A free anomeric hemiacetal or hemiketal group permits ring opening. If all anomeric centres are fixed in glycosidic acetal or ketal linkages, normal mutarotation does not occur.

Step 1: Check glucose and the reducing disaccharides Glucose has a free anomeric hydroxyl group, so its cyclic anomers interconvert through the open-chain aldehyde form. Maltose has one anomeric carbon involved in its glycosidic bond but retains a free anomeric carbon on its other glucose unit. Lactose similarly retains a free anomeric centre. Each can therefore undergo mutarotation in aqueous solution.

Step 2: Inspect the linkage in sucrose Sucrose joins the anomeric carbon C1 of glucose to the anomeric carbon C2 of fructose. Both anomeric hydroxyl groups are consumed in the glycosidic linkage. There is no free hemiacetal or hemiketal centre available for reversible ring opening without breaking that linkage. Sucrose is consequently a non-reducing sugar and does not show mutarotation.

Step 3: Select the exception The question asks which carbohydrate will not show the phenomenon. Maltose, lactose and glucose are excluded because their free anomeric centres allow interconversion. Sucrose alone has both anomeric centres locked. Hydrolysis would create different sugars; it is not the mutarotation of intact sucrose asked about here.

Step 4: Conclusion Sucrose does not show mutarotation because both anomeric centres participate in its glycosidic bond. Hence, the correct option is (D).

Quick Tip to Solve: For mutarotation, look for a free anomeric hydroxyl group; sucrose locks both anomeric carbons and therefore fails this test.