JEE Main 2014 Paper 1 (B.E./B.Tech) Apr 19 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2014, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 19 April 2014, Single slot. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Physics, Chemistry and Mathematics, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Physics (120 marks)

1. A bullet loses ((1)/(n))th of its velocity passing through one plank. Considering uniform retardation, the number of such planks that are required to stop the bullet can be [4 marks]

(A) Infinite

(B) n

(C) $(n^2)/((2n-1))$

(D) $(2n^2)/((n-1))$

Correct Answer: (C) $(n^2)/((2n-1))$

Solution:

Concept:

- For constant retardation of magnitude a, the speed after travelling distance s satisfies $v^2 = u^2 - 2as$. Here u is the incident speed and each plank has the same thickness d.
- Equal thicknesses under the same retarding force remove equal amounts of kinetic energy, not equal amounts of speed. Thus the total stopping distance divided by one plank thickness gives the equivalent number of planks.

Step 1: Find the speed after the first plank The bullet loses the fraction $1/n$ of its initial velocity, so its emerging speed is

$$v_1 = u - \frac{u}{n} = u\frac{n-1}{n}.$$

Applying the constant-acceleration equation to this plank gives

$$2ad = u^2 - v_1^2 = u^2\left[1 - \frac{(n-1)^2}{n^2}\right] = u^2\frac{2n-1}{n^2}.$$

The square of speed, rather than speed itself, is the quantity whose decrement is proportional to thickness.

Step 2: Compare with the full stopping distance If the stopping distance is Nd, set the final speed equal to zero:

$$0 = u^2 - 2aNd, \qquad N = \frac{u^2}{2ad} = \frac{n^2}{2n-1}.$$

This is finite for the physical range $n \geq 1$. Option (B) incorrectly assumes the same speed loss in every plank; option (A) incorrectly assumes an indefinitely decreasing speed; option (D) has the wrong energy decrement. If whole physical planks must be counted, the bullet stops in the plank containing this equivalent thickness.

Step 3: Conclusion The equivalent number of plank thicknesses is $\boxed{n^2/(2n-1)}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the fraction of kinetic energy lost in the first plank: the reciprocal of that fraction is the total stopping thickness in plank units.

2. A heavy box is to be dragged along a rough horizontal floor. To do so, person A pushes it at an angle 30° from the horizontal and requires a minimum force FA, while person B pulls the box at an angle 60° from the horizontal and needs minimum force FB. If the coefficient of friction between the box and the floor is (√3)/(5), the ratio (FA)/(FB) is [4 marks]

(A) √(3/2)

(B) 2/√3

(C) √3

(D) 5/√3

Correct Answer: (B) 2/√3

Solution:

Concept:

- At impending horizontal motion, limiting friction is $f = \mu N$, where N is the normal reaction. A downward push increases N, whereas an upward pull reduces it.
- Resolve the applied force horizontally and vertically before comparing the two cases. The box has no vertical acceleration, so vertical force balance determines the normal reaction separately for each person.

Step 1: Resolve the downward push Let the box weight be W. For person A, $N_A = W + F_A \sin 30^\circ$. At the threshold of motion,

$$F_A \cos 30^\circ = \mu(W + F_A \sin 30^\circ).$$

Therefore

$$F_A = \frac{\mu W}{\cos 30^\circ - \mu \sin 30^\circ} = \frac{\mu W}{2\sqrt{3}/5},$$

using $\mu = \sqrt{3}/5$. The minus sign in the denominator represents the extra friction created by pushing downward.

Step 2: Resolve the upward pull and form the ratio For person B, $N_B = W - F_B \sin 60^\circ$, and hence

$$F_B = \frac{\mu W}{\cos 60^\circ + \mu \sin 60^\circ} = \frac{\mu W}{4/5}.$$

The weight and coefficient cancel in the ratio:

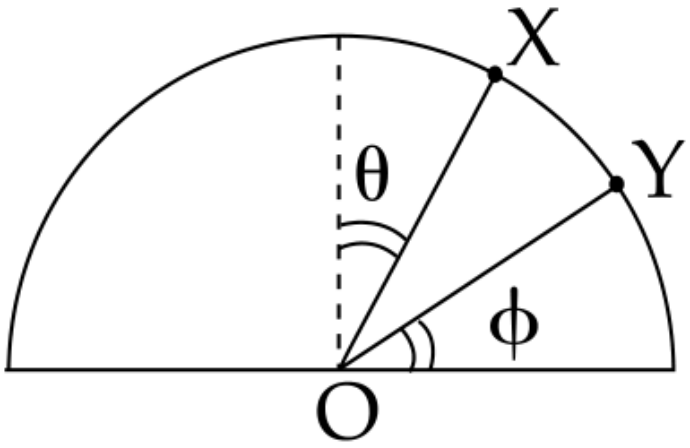
$$\frac{F_A}{F_B} = \frac{4/5}{2\sqrt{3}/5} = \frac{2}{\sqrt{3}}.$$

The upward component reduces the normal reaction in the pulling case. The remaining listed values do not follow these two distinct normal reactions: using the same normal force for both cases would omit the essential vertical components.

Step 3: Conclusion The ratio is $\boxed{F_A/F_B = 2/\sqrt{3}}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the two threshold-force denominators first: pushing gives cos minus mu sin, while pulling gives cos plus mu sin.

3. A particle is released on a vertical smooth semicircular track from point X so that OX makes angle θ from the vertical (see figure). The normal reaction of the track on the particle vanishes at the point Y where OY makes an angle ϕ with the horizontal. Then: [4 marks]



(A) $\sin\phi = (2)/(3)\cos\theta$

(B) $\sin\phi = (3)/(4)\cos\theta$

(C) $\sin\phi = (1)/(2)\cos\theta$

(D) $\sin\phi = \cos\theta$

Correct Answer: (A) $\sin\phi = (2)/(3)\cos\theta$

Solution:

Concept:

- On a smooth track, mechanical energy is conserved. With the centre O as the height reference and track radius R, the heights of X and Y are $R\cos\theta$ and $R\sin\phi$.
- For motion on the outside of the circular track, the inward component of gravity is $mg\sin\phi$. At loss of contact the normal reaction vanishes, so gravity alone supplies the centripetal force.

Step 1: Use conservation of energy from release to separation The particle is released from rest at X. The track does no work, because its normal force is perpendicular to the displacement. The loss in gravitational potential energy becomes kinetic energy:

$$\frac{1}{2}mv^2 = mgR(\cos\theta - \sin\phi).$$

Thus

$$v^2 = 2gR(\cos\theta - \sin\phi).$$

This relates the speed at Y to its vertical height and uses the angular references shown in the figure, rather than measuring both angles from the same axis.

Step 2: Set the normal reaction equal to zero Taking the inward radial direction toward O, the radial equation before separation is

$$mg\sin\phi - N = \frac{mv^2}{R}.$$

At Y, $N = 0$, so $v^2 = gR\sin\phi$. Combining this with the energy result gives

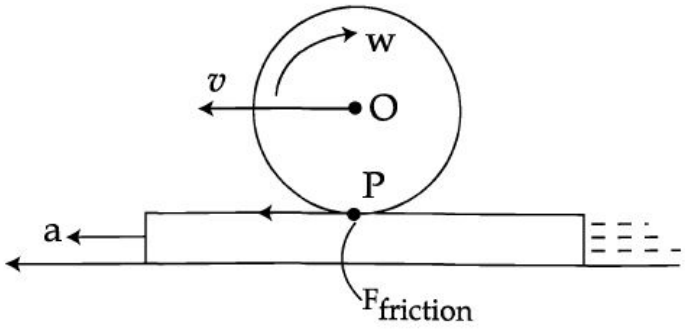
$$2(\cos\theta - \sin\phi) = \sin\phi, \qquad 3\sin\phi = 2\cos\theta.$$

The coefficients $3/4$, $1/2$, and 1 in the other options do not satisfy both energy conservation and the zero-reaction radial equation.

Step 3: Conclusion $\boxed{\sin\phi = \frac{2}{3}\cos\theta}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert each angle into a height first; at detachment replace the radial equation by gravity alone providing the centripetal acceleration.

4. Consider a cylinder of mass M resting on a rough horizontal rug that is pulled out from under it with acceleration 'a' perpendicular to the axis of the cylinder. What is $F_{friction}$ at point P? It is assumed that the cylinder does not slip. [4 marks]



(A) Ma

(B) Mg

(C) Ma/3

(D) Ma/2

Correct Answer: (C) Ma/3

Solution:

Concept:

- For a uniform solid cylinder of radius R, the moment of inertia about its axis is $I = MR^2/2$. Friction provides both the horizontal force on its centre and the torque about that centre.
- No slipping is relative to the accelerating rug. The acceleration of the cylinder contact point along the rug must match the acceleration a of the rug; it is not zero as it would be for a fixed ground surface.

Step 1: Write translation and rotation equations Take leftward as positive, in the direction of the rug acceleration shown. If the friction magnitude is f, the cylinder centre has acceleration $A = f/M$.

Friction at the lower point also produces clockwise angular acceleration of magnitude

$$\alpha = \frac{fR}{I} = \frac{2f}{MR}.$$

Both translation and the tangential acceleration due to rotation contribute leftward to the contact point acceleration. The normal reaction and weight balance vertically and exert no relevant torque about the centre.

Step 2: Apply rolling relative to the moving rug At the lower contact point, the horizontal no-slip acceleration condition is

$$a = A + R\alpha.$$

The centripetal term is radial and therefore has no horizontal component at this point. Substituting the force and torque equations yields

$$a = \frac{f}{M} + \frac{2f}{M} = \frac{3f}{M}, \qquad f = \frac{Ma}{3}.$$

Option (A) ignores rotation; option (B) identifies friction with the weight instead of using the dynamics. Option (D) also fails the solid-cylinder moment of inertia and the moving-contact constraint.

Step 3: Conclusion The friction magnitude at P is $\boxed{Ma/3}$, directed with the rug acceleration on the cylinder. Hence, the correct option is (C).

Quick Tip to Solve: For rolling on an accelerating belt, match the contact point acceleration to the belt: combine centre acceleration with rotational tangential acceleration.

5. A ball of mass 160 g is thrown up at an angle of 60° to the horizontal at a speed of 10 m s^{-1}. The angular momentum of the ball at the highest point of the trajectory with respect to the point from which the ball is thrown is nearly (g = 10 m s^{-2}) [4 marks]

(A) 1.73 kg m^2 s^{-1}

(B) 3.46 kg m^2 s^{-1}

(C) 3.0 kg m^2 s^{-1}

(D) 6.0 kg m^2 s^{-1}

Correct Answer: (C) 3.0 kg m^2 s^{-1}

Solution:

Concept:

- Angular momentum about the point of projection is $\vec{L} = \vec{r} \times m\vec{v}$. Its magnitude is momentum multiplied by the perpendicular distance from the reference point to the line of motion.
- At the highest point of projectile motion, the vertical velocity is zero, but the horizontal velocity remains $u\cos 60°$. The perpendicular lever arm for this horizontal momentum is the maximum height.

Step 1: Find the horizontal speed and maximum height Convert the mass to SI units: $m = 0.160\,\text{kg}$.

The horizontal speed is

$$v_x = 10\cos 60^\circ = 5\,\mathrm{m\,s^{-1}}.$$

From the vertical equation of motion,

$$H = \frac{u^2\sin^2 60^\circ}{2g} = \frac{100(3/4)}{20} = 3.75\,\mathrm{m}.$$

The upward component of the launch velocity has become zero at this point; using the initial speed as the speed at the top would therefore be incorrect.

Step 2: Evaluate the cross product With horizontal and vertical axes in the plane of motion, $\vec{r} = x\hat{i} + H\hat{j}$ and $\vec{p} = mv_x\hat{i}$. Thus

$$\vec{L} = -mHv_x\hat{k}, \qquad |\vec{L}| = 0.160 \times 3.75 \times 5 = 3.0\,\mathrm{kg\,m^2\,s^{-1}}.$$

The horizontal coordinate x drops out because its contribution is parallel to momentum. The direction is perpendicular to the trajectory plane, clockwise for a projectile moving right. Options (A), (B), and (D) give incorrect products of the mass, top speed, and perpendicular lever arm.

Step 3: Conclusion The angular momentum magnitude is $\boxed{3.0\,\mathrm{kg\,m^2\,s^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: At a projectile apex, angular momentum about launch equals mass times horizontal speed times maximum height; the horizontal range to the apex is irrelevant.

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6. Match List - I (Event) with List - II (Order of the time interval for the happening of the event) and select the correct option from the options given below the lists.

List - I: (a) The rotation period of earth, (b) Revolution period of earth, (c) Period of a light wave, (d) Period of a sound wave

List - II: (i) 10^5 s, (ii) 10^7 s, (iii) 10^{-15} s, (iv) 10^{-3} s [4 marks]

(A) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

(B) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

(C) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

(D) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Correct Answer: (A) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

Solution:

Concept:

- The order of a time interval is its characteristic power-of-ten scale in seconds. A period is the reciprocal of frequency, $T = 1/f$.
- Terrestrial rotation and revolution are macroscopic astronomical periods. Light oscillations occur far faster than audible sound oscillations, so their periods are correspondingly much smaller.

Step 1: Estimate the two astronomical periods One rotation of the Earth takes approximately one

day:

$$T_{\text{rotation}} = 24 \times 60 \times 60 = 86400\,\text{s} \approx 10^5\,\text{s}.$$

One revolution takes approximately one year:

$$T_{\text{revolution}} = 365 \times 86400\,\text{s} \approx 3.15 \times 10^7\,\text{s}.$$

Thus event (a) matches (i) and event (b) matches (ii). Interchanging them, as in options (B) and (D), would make the Earth orbit the Sun faster than it rotates on its axis.

Step 2: Estimate optical and acoustic periods Visible light has frequencies of order 10^{14} to 10^{15} Hz, so its period has the supplied scale 10^{-15} s. A typical audible tone of frequency 10^3 Hz has period

$$T_{\text{sound}} = \frac{1}{10^3} = 10^{-3}\,\text{s}.$$

Hence (c) matches (iii) and (d) matches (iv). Option (C) reverses these two wave timescales; option (D) reverses both pairs. Only option (A) preserves the astronomical as well as wave-period comparisons.

Step 3: Conclusion Rotation, revolution, light and sound match the time scales in the order listed in option (A). Hence, the correct option is (A).

Quick Tip to Solve: Memorise the useful scales: a day is about 10^5 seconds, a year about 10^7 seconds, and optical periods are vastly shorter than sound periods.

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7. The gravitational field in a region is given by $\vec{g} = 5\,\text{N/kg}\,\hat{i} + 12\,\text{N/kg}\,\hat{j}$. The change in the gravitational potential energy of a particle of mass 2 kg when it is taken from the origin to a point (7 m, −3 m) is : [4 marks]

(A) 71J

(B) $13\sqrt{58}$ J

(C) -71 J

(D) 1J

Correct Answer: The printed data give $\boxed{\Delta U = +2\,\text{J}}$. None of the listed options is correct; the value 1 J in option (D) omits the given mass factor.

Solution:

Concept:

- For a uniform gravitational field $\vec{g}$, the force on mass m is $\vec{F} = m\vec{g}$. The field work during displacement $\Delta\vec{r}$ is the dot product $m\vec{g} \cdot \Delta\vec{r}$.
- The change in gravitational potential energy is the negative of work done by the gravitational force: $\Delta U = -m\vec{g} \cdot \Delta\vec{r}$. A dot product must retain component signs; multiplying field and displacement magnitudes is valid only when they are parallel.

Step 1: Evaluate the signed field work The displacement from the origin is

$$\Delta\vec{r} = (7\hat{i} - 3\hat{j})\,\text{m}.$$

With the printed field $\vec{g} = (5\hat{i} + 12\hat{j})\,\text{N kg}^{-1}$,

$$\vec{g}\cdot\Delta\vec{r} = 5(7) + 12(-3) = 35 - 36 = -1\,\text{J kg}^{-1}.$$

Multiplying by the given mass, $2\,\text{kg}$, gives gravitational work $W_g = -2\,\text{J}$. The negative vertical contribution slightly exceeds the positive horizontal contribution.

Step 2: Reverse the sign to find the potential-energy change The required change is

$$\Delta U = -W_g = +2\,\text{J}.$$

Options (A) and (C), with magnitude $71\,\text{J}$, do not use the signed dot product. Option (B) corresponds to a magnitude-based expression and neglects the angle between the field and displacement. Option (D), $1\,\text{J}$, equals the increase in potential per unit mass, rather than the potential energy increase of the stated $2\,\text{kg}$ particle. Consequently none of the four printed options equals the result of the given data.

Step 3: Conclusion The printed data give $\boxed{\Delta U = +2\,\text{J}}$. None of the listed options is correct; the value 1 J in option (D) omits the given mass factor.

Quick Tip to Solve: Compute the signed dot product first, multiply by mass, and then change its sign; distinguish potential in J/kg from potential energy in J.

8. The velocity of water in a river is 18 km h^{-1} near the surface. If the river is 5 m deep, find the shearing stress between the horizontal layers of water. The coefficient of viscosity of water= 10^{-2} poise [4 marks]

(A) 10^{-4} N m^{-2}

(B) 10^{-3} N m^{-2}

(C) 10^{-2} N m^{-2}

(D) 10^{-1} N m^{-2}

Correct Answer: (B) 10^{-3} N m^{-2}

Solution:

Concept:

- For adjacent fluid layers, Newton's law of viscosity gives the shear stress $\tau = \eta\, dv/dy$, where η is dynamic viscosity and dv/dy is the velocity gradient normal to the layers.
- The river bed supplies the stationary boundary, while the stated surface speed is the upper boundary speed. Use the mean gradient across the given depth, $dv/dy = \Delta v/h$. The SI conversion is $1\,\text{poise} = 0.1\,\text{Pa s}$.

Step 1: Convert the speed and viscosity The surface speed must be expressed in metres per second

before combining it with the depth. Thus

$$v = 18 \times \frac{1000}{3600} = 5\,\mathrm{m\,s^{-1}}.$$

The viscosity given in poise is

$$\eta = 10^{-2} \times 0.1 = 10^{-3}\,\mathrm{Pa\,s}.$$

In particular, the numerical value in poise cannot be substituted unchanged into an otherwise SI calculation.

Step 2: Evaluate the average velocity gradient Water immediately next to the stationary bed has zero velocity. Across the $5\,\mathrm{m}$ depth, the mean velocity change is therefore $5 - 0\,\mathrm{m\,s^{-1}}$:

$$\frac{\Delta v}{h} = \frac{5}{5} = 1\,\mathrm{s^{-1}}.$$

The stress concerns transfer of tangential momentum between horizontal layers, rather than the hydrostatic pressure due to the water depth.

Step 3: Apply the viscous-stress law Substitute the viscosity and gradient:

$$\tau = \eta\frac{\Delta v}{h} = 10^{-3} \times 1 = 10^{-3}\,\mathrm{N\,m^{-2}}.$$

The units are $\mathrm{Pa\,s} \times \mathrm{s^{-1}} = \mathrm{Pa}$. Option (C) is ten times too large because it effectively leaves viscosity in poise; (A) is ten times too small and (D) is one hundred times too large. Neither the depth alone nor the speed alone has the units of a shear stress.

Step 4: Conclusion The shearing stress is $10^{-3}\,\mathrm{N\,m^{-2}}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert poise to SI first, then divide the surface speed by the depth; the velocity gradient here happens to be exactly $1\,\mathrm{s^{-1}}$.

9. A large number of liquid drops each of radius r coalesce to form a single drop of radius R. The energy released in the process is converted into kinetic energy of the big drop so formed. The speed of the big drop is (given surface tension of liquid T, density ρ) [4 marks]

(A) $\sqrt{\frac{2T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$

(B) $\sqrt{\frac{6T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$

(C) $\sqrt{\frac{4T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$

(D) $\sqrt{\frac{T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$

Correct Answer: (B) $\sqrt{\frac{6T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$

Solution:

Concept:

- A liquid drop has one free surface, so its surface energy is $T \times 4\pi r^2$, where T is surface tension. Coalescence conserves volume but reduces total free-surface area.
- The given conversion of released surface energy into kinetic energy means $T(A_i - A_f) = \frac{1}{2}mv^2$. The final mass is $m = \frac{4}{3}\pi R^3\rho$, with ρ the liquid density.

Step 1: Use conservation of liquid volume Let the number of identical small drops be N. No liquid is lost in forming the large drop, hence

$$N\frac{4\pi r^3}{3} = \frac{4\pi R^3}{3}, \qquad N = \frac{R^3}{r^3}.$$

The total original surface area is

$$A_i = N4\pi r^2 = \frac{4\pi R^3}{r},$$

whereas the final surface area is $A_f = 4\pi R^2$. One must count all the original drops, rather than subtracting the areas of just one small drop and one large drop.

Step 2: Find the surface energy released The decrease in area is

$$A_i - A_f = 4\pi R^3\left(\frac{1}{r}-\frac{1}{R}\right).$$

Multiplying by surface tension gives

$$E = 4\pi T R^3\left(\frac{1}{r}-\frac{1}{R}\right).$$

Since coalescence makes $R > r$, the bracket is positive and energy is released. There is no factor of two for a second surface: these are liquid drops, not thin soap bubbles.

Step 3: Equate the release to kinetic energy With the final mass $m = \frac{4}{3}\pi R^3\rho$, the prescribed energy balance becomes

$$4\pi T R^3\left(\frac{1}{r}-\frac{1}{R}\right) = \frac{1}{2}\frac{4}{3}\pi R^3\rho v^2.$$

Cancelling the common volume factor yields

$$v^2 = \frac{6T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right).$$

The coefficients 2, 4 and 1 in (A), (C) and (D) respectively do not follow this balance. The radicand has dimensions $\mathrm{m}^2\,\mathrm{s}^{-2}$, as required.

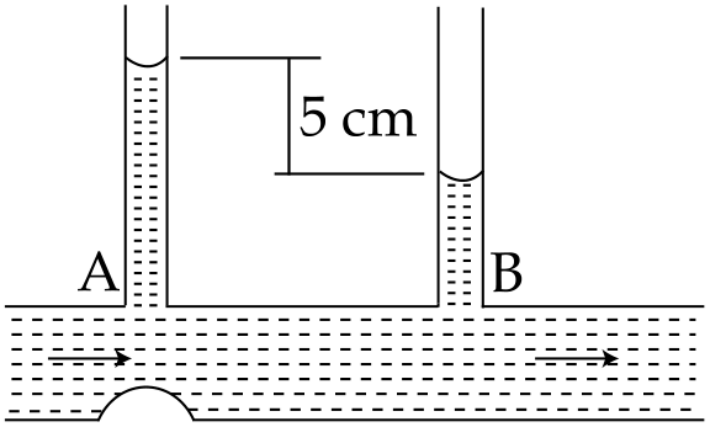
Step 4: Conclusion The speed is

$$v = \sqrt{\frac{6T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Eliminate the number of drops with volume conservation before computing area; the common $4\pi R^3$ then cancels from the energy balance.

10. In the diagram shown, the difference in the two tubes of the manometer is 5 cm, the cross-section of the tube at A and B is 6 mm^2 and 10 mm^2 respectively. The rate at which water flows through the tube is (g = 10 m s^{-2}) [4 marks]



(A) 7.5 cc s^{-1}

(B) 12.5 cc s^{-1}

(C) 8.0 cc s^{-1}

(D) 10.0 cc s^{-1}

Correct Answer: (A) 7.5 cc s^{-1}

Solution:

Concept:

- For steady incompressible flow, the volume rate is conserved: $Q = A_A v_A = A_B v_B$, where A is area and v is speed. Use SI units throughout the substitution.
- For two sections at the same height, Bernoulli gives the pressure-difference magnitude $|p_A - p_B| = \frac{1}{2}\rho|v_A^2 - v_B^2|$. A water-column height difference h measures a pressure difference of magnitude $\rho g h$.

Step 1: Convert the areas and relate the speeds The stated cross-sections are $A_A = 6 \times 10^{-6}\,\mathrm{m}^2$ and $A_B = 10 \times 10^{-6}\,\mathrm{m}^2$. Continuity therefore gives $v_B = (3/5)v_A$. The smaller stated area has the larger speed. The difference in column levels is $h = 5\,\mathrm{cm} = 0.05\,\mathrm{m}$; this is a pressure-head difference, not a flow speed.

Step 2: Combine the pressure head with Bernoulli The sketch places the water column at A higher than at B, whereas the stated smaller area at A requires a lower pressure there. Thus the schematic height direction is inconsistent with the stated areas. The given 5 cm separation still supplies the magnitude of the pressure head. Using that magnitude,

$$v_A^2 - v_B^2 = 2gh = 2(10)(0.05) = 1.0\,\mathrm{m}^2\,\mathrm{s}^{-2}.$$

Hence

$$\left(1 - \frac{9}{25}\right) v_A^2 = 1, \qquad v_A = \frac{5}{4}\,\mathrm{m\,s}^{-1}.$$

The volume rate is

$$Q = A_A v_A = 6 \times 10^{-6} \times 1.25 = 7.5 \times 10^{-6}\,\mathrm{m}^3\,\mathrm{s}^{-1} = 7.5\,\mathrm{cm}^3\,\mathrm{s}^{-1}.$$

The factor 10^6 converts cubic metres to cubic centimetres. Options (B), (C), and (D) give different flow rates and do not satisfy the combined continuity and head equations with the stated areas.

Step 3: Conclusion The flow rate is $7.5\,\mathrm{cc\,s}^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the column difference as pressure head, combine it with continuity, and convert the final cubic metres per second into cc per second.

11. A black coloured solid sphere of radius R and mass M is inside a cavity with a vacuum inside. The walls of the cavity are maintained at temperature T_0. The initial temperature of the sphere is $3T_0$. If the specific heat of the material of the sphere varies as αT^3 per unit mass with the temperature T of the sphere, where α is a constant, then the time taken for the sphere to cool down to temperature $2T_0$ will be (σ is Stefan Boltzmann constant) [4 marks]

(A) $\frac{M\alpha}{16\pi R^2\sigma}\ln\frac{3}{2}$

(B) $\frac{M\alpha}{16\pi R^2\sigma}\ln\frac{16}{3}$

(C) $\frac{M\alpha}{4\pi R^2\sigma}\ln\frac{3}{2}$

(D) $\frac{M\alpha}{4\pi R^2\sigma}\ln\frac{16}{3}$

Correct Answer: (B) $\frac{M\alpha}{16\pi R^2\sigma}\ln\frac{16}{3}$

Solution:

Concept:

- A black sphere of area $A = 4\pi R^2$ in a cavity at T_0 has net radiative heat loss $P = A\sigma(T^4 - T_0^4)$. Radiation from the cavity must be included even though the surrounding space is a vacuum.
- The specific heat per unit mass is $c(T) = \alpha T^3$, so a small temperature change changes the internal energy by $dU = M\alpha T^3\,dT$. Equate the energy loss rate to the net outward power.

Step 1: Write the cooling equation The sphere emits radiation and also absorbs radiation from the cavity walls. Therefore

$$-M\alpha T^3\frac{dT}{dt} = 4\pi R^2\sigma(T^4 - T_0^4).$$

Both sides are positive during cooling because $dT/dt < 0$. Vacuum suppresses conduction and convection; it does not remove the radiation arriving from the walls.

Step 2: Separate variables with the cooling limits The sphere cools from $3T_0$ to $2T_0$. Reverse the temperature limits to obtain a positive elapsed time:

$$t = \frac{M\alpha}{4\pi R^2\sigma}\int_{2T_0}^{3T_0}\frac{T^3\,dT}{T^4 - T_0^4}.$$

Let $u = T^4 - T_0^4$; then $du = 4T^3dT$. This substitution is especially simple because the given heat capacity supplies exactly the cubic factor needed to differentiate the radiative denominator.

Step 3: Evaluate the logarithm and compare Integration gives

$$t = \frac{M\alpha}{16\pi R^2\sigma}\ln\left(\frac{(3T_0)^4 - T_0^4}{(2T_0)^4 - T_0^4}\right).$$

The logarithm reduces to

$$\ln\frac{81-1}{16-1} = \ln\frac{80}{15} = \ln\frac{16}{3}.$$

Options (A) and (C) use $\ln(3/2)$, which ignores the cavity absorption term. Option (D) omits the factor $1/4$ from the integral. The ratio inside the logarithm is dimensionless and greater than one.

Step 4: Conclusion The cooling time is

$$t = \frac{M\alpha}{16\pi R^2\sigma} \ln \frac{16}{3}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Keep the background term T_0^4: with heat capacity proportional to T^3, substitute $u = T^4 - T_0^4$ directly.

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12. A monoatomic gas is compressed from a volume of 2 m^3 to a volume of 1 m^3 at a constant pressure of 100 N m^{-2}. Then it is heated at constant volume by supplying 150 J of energy. As a result, the internal energy of the gas [4 marks]

(A) Decreases by 50 J

(B) Increases by 250 J

(C) Decreases by 250 J

(D) 0 J

Correct Answer: (D) 0 J

Solution:

Concept:

- For a fixed quantity of ideal monoatomic gas, $U = \frac{3}{2}nRT = \frac{3}{2}PV$. Internal energy is a state function, so its changes in successive stages add algebraically.
- The first law in the convention of work done by the gas is $\Delta U = Q - W$. At constant volume, $W = 0$, so all supplied heat increases internal energy.

Step 1: Find the change during constant-pressure compression At the stated constant pressure $P = 100\,\mathrm{N\,m^{-2}}$, volume decreases from 2 to $1\,\mathrm{m^3}$. Because $PV = nRT$, this stage also lowers the temperature. Its internal-energy change is

$$\Delta U_1 = \frac{3}{2}P(V_2 - V_1) = \frac{3}{2}(100)(1-2) = -150\,\mathrm{J}.$$

Compression alone does not specify an adiabatic process. It is incorrect to identify the work done on the gas with its internal-energy change while ignoring the heat exchange required to keep pressure constant.

Step 2: Find the change during constant-volume heating The gas next receives $150\,\mathrm{J}$ while its volume remains fixed. Since the boundary does not move,

$$W_2 = \int P\,dV = 0.$$

The first law consequently gives

$$\Delta U_2 = Q_2 - W_2 = 150 - 0 = 150\,\mathrm{J}.$$

Although pressure rises in this stage, there is no expansion work because $dV = 0$.

Step 3: Add the state-function changes The final change relative to the original state is

$$\Delta U = \Delta U_1 + \Delta U_2 = -150 + 150 = 0\,\text{J}.$$

The heating exactly restores the energy lost during the isobaric compression. Thus (A) and (C), which give net decreases, and (B), which gives an increase of $250\,\text{J}$, are excluded. In particular, adding the compression work $100\,\text{J}$ to the heat $150\,\text{J}$ would falsely treat the first stage as adiabatic.

Step 4: Conclusion The listed key gives an increase of $250\,\text{J}$, but that would incorrectly count the compression as adiabatic. The stated constant-pressure compression gives a change of $-150\,\text{J}$, cancelled by the supplied $150\,\text{J}$. The internal energy therefore has no net change: $\Delta U = 0\,\text{J}$. Hence, the correct option is (D).

Quick Tip to Solve: Use $\Delta U = \frac{3}{2}\Delta(PV)$ for the isobaric stage and $\Delta U = Q$ for the isochoric stage; add their signed changes.

13. A gas molecule of mass M at the surface of the earth has kinetic energy equivalent to 0 °C. If it were to go up straight without colliding with any other molecules, how high it would rise? Assume that the height attained is much less than the radius of the earth. (kB is Boltzmann constant) [4 marks]

(A) (273kB)/(2Mg)

(B) (819kB)/(2Mg)

(C) 0

(D) (546kB)/(3Mg)

Correct Answer: (B) (819kB)/(2Mg)

Solution:

Concept:

- The mean translational kinetic energy associated with absolute temperature T is $K = \frac{3}{2}k_BT$, where k_B is Boltzmann's constant. Celsius temperature must first be converted to kelvin.
- For a collision-free molecule moving vertically and rising a height small compared with Earth's radius, gravity is effectively constant. Conservation of mechanical energy gives $Mgh = K$ at the highest point.

Step 1: Interpret the stated thermal kinetic energy The temperature $0\,^\circ\text{C}$ corresponds to $273\,\text{K}$ at the precision of the answer choices. The kinetic energy equivalent to this temperature is

$$K = \frac{3}{2}k_B(273) = \frac{819}{2}k_B.$$

The three translational degrees of freedom determine this energy-temperature relation. The subsequent statement that the molecule travels straight upwards specifies the direction in which this given kinetic energy acts; it does not reduce it to one third.

Step 2: Convert kinetic energy into gravitational energy With no intervening molecular collision,

no energy is transferred to other molecules. The initial kinetic energy is exhausted at the top of the vertical flight, so

$$Mgh = \frac{819}{2}k_B.$$

As h is much smaller than Earth's radius, replacing the change of gravitational potential by Mgh is consistent with the question. Thus

$$h = \frac{819k_B}{2Mg}.$$

The mass in this equation is that of the individual molecule, not the molar mass.

Step 3: Check the other expressions Option (A) uses only $\frac{1}{2}k_BT$, which is the mean energy of a single Cartesian component rather than the total translational energy specified here. Option (C) confuses zero Celsius temperature with zero absolute temperature; molecular motion remains nonzero at 273 K. Option (D) has the wrong numerical coefficient. Finally, $k_BT/(Mg)$ has units of energy divided by force, hence metres, as a height must.

Step 4: Conclusion The molecule rises to

$$h = \frac{819k_B}{2Mg}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Zero Celsius is 273 K, not zero kinetic energy. Equate the stated molecular energy $\frac{3}{2}k_BT$ to Mgh.

14. A body is in simple harmonic motion with time period T= 0.5 s and amplitude A = 1 cm. Find the average velocity in the interval in which it moves from equilibrium position to half of its amplitude

[4 marks]

(A) 16 cm/s

(B) 6 cm/s

(C) 4 cm/s

(D) 12 cm/s

Correct Answer: (D) 12 cm/s

Solution:

Concept:

- For an oscillator starting at equilibrium and moving towards its positive extreme, $x = A\sin(\omega t)$, where $\omega = 2\pi/T$. The first arrival at a displacement fixes the corresponding phase.
- Average velocity over a specified interval is displacement divided by elapsed time, $\bar{v} = \Delta x/\Delta t$. It differs from instantaneous velocity and from the average speed over a full oscillation.

Step 1: Find the phase at half amplitude Take the equilibrium crossing as $t = 0$, with motion initially

in the positive direction. At the first arrival at $x = A/2$,

$$\frac{A}{2} = A\sin(\omega\Delta t), \qquad \sin(\omega\Delta t) = \frac{1}{2}.$$

In this first quarter-cycle the phase lies between 0 and $\pi/2$, so

$$\omega\Delta t = \frac{\pi}{6}.$$

The other sine solutions occur after additional motion and do not represent the stated first interval from equilibrium to half amplitude.

Step 2: Calculate the time and displacement Using $\omega = 2\pi/T$, the elapsed time is

$$\Delta t = \frac{\pi/6}{2\pi/T} = \frac{T}{12} = \frac{0.5}{12} = \frac{1}{24}\,\text{s}.$$

The net displacement is

$$\Delta x = \frac{A}{2} = 0.5\,\text{cm}.$$

Neither $T/4$ nor $T/8$ is the correct interval: displacement in SHM is sinusoidal, not proportional to time.

Step 3: Divide displacement by elapsed time The average velocity in the direction of this motion is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0.5}{1/24} = 12\,\text{cm}\,\text{s}^{-1}.$$

Equivalently, $\bar{v} = 6A/T$. The values 16, 6 and 4 in options (A), (B) and (C) do not use the phase interval $\pi/6$. As a check, this average is below the maximum speed $\omega A = 4\pi\,\text{cm}\,\text{s}^{-1}$, consistent with the oscillator slowing as it leaves equilibrium.

Step 4: Conclusion The average velocity is $12\,\text{cm}\,\text{s}^{-1}$ along the direction of travel. Hence, the correct option is (D).

Quick Tip to Solve: Use phase to get the time first: the first journey from $x = 0$ to $x = A/2$ lasts $T/12$, so its average velocity is $6A/T$.

15. The total length of a sonometer wire fixed between two bridges is 110 cm. Now, two more bridges are placed to divide the length of the wire in the ratio 6 : 3 : 2. If the tension in the wire is 400 N and the mass per unit length of the wire is 0.01 kg m^{-1}, then the minimum common frequency with which all the three parts can vibrate, is [4 marks]

(A) 1000 Hz

(B) 1100 Hz

(C) 100 Hz

(D) 110 Hz

Correct Answer: (A) 1000 Hz

Solution:

Concept:

- The speed of a transverse wave is $v = \sqrt{T/\mu}$, where T is tension and μ is mass per unit length.
- A string fixed at both ends has allowed frequencies $f_n = nv/(2L)$, for positive integers n. A common frequency must be an allowed harmonic for every segment.

Step 1: Find the segment lengths The ratio has $6 + 3 + 2 = 11$ parts, so each part is $110/11 = 10\,\text{cm}$. The three vibrating lengths are therefore $L_1 = 0.60\,\text{m}$, $L_2 = 0.30\,\text{m}$, and $L_3 = 0.20\,\text{m}$. Bridges impose nodes, so each segment behaves as an independently fixed string; the same tension and linear density give the same wave speed in all three.

Step 2: Calculate the fundamental frequencies Using SI units gives

$$v = \sqrt{\frac{400}{0.01}} = 200\,\text{m}\,\text{s}^{-1}.$$

Hence

$$f_1 = \frac{200}{1.20} = \frac{500}{3}\,\text{Hz}, \quad f_2 = \frac{200}{0.60} = \frac{1000}{3}\,\text{Hz}, \quad f_3 = \frac{200}{0.40} = 500\,\text{Hz}.$$

Do not take an average of these frequencies: every segment must simultaneously have an integral harmonic number.

Step 3: Find the lowest shared harmonic For a shared frequency, $n_1/0.60 = n_2/0.30 = n_3/0.20$, so $n_1 : n_2 : n_3 = 6 : 3 : 2$. This is already the smallest positive integral ratio. The common frequency is

$$f = 6\left(\frac{500}{3}\right) = 3\left(\frac{1000}{3}\right) = 2(500) = 1000\,\text{Hz}.$$

Frequencies 100 Hz and 110 Hz are below even the first segment fundamental, while 1100 Hz gives nonintegral harmonic numbers and cannot work.

Step 4: Conclusion The minimum common frequency is $1000\,\text{Hz}$. Hence, the correct option is (A).

Quick Tip to Solve: When wave speed is common, harmonic numbers at a common frequency are proportional to segment lengths; reduce that length ratio to its smallest integers.

16. The electric field in a region of space is given by, $E = E_0\,\hat{i} + 2E_0\,\hat{j}$ where $E_0 = 100\ N\ C^{-1}$. The flux of this field through a circular surface of radius 0.02 m parallel to the Y-Z plane is nearly [4 marks]

(A) 0.02 N m^2 C^{-1}

(B) 0.005 N m^2 C^{-1}

(C) 0.125 N m^2 C^{-1}

(D) 3.14 N m^2 C^{-1}

Correct Answer: (C) 0.125 N m^2 C^{-1}

Solution:

Concept:

- Electric flux through a flat surface is $\Phi = \vec{E} \cdot \vec{A}$; the area vector is perpendicular to the surface.
- For a circular surface of radius r, $A = \pi r^2$. Only the field component normal to the surface contributes.

Step 1: Identify the normal component The surface is parallel to the Y–Z plane, so its unit normal lies along the x axis. Choosing the positive x normal, $\vec{A} = A\hat{i}$. The given field is $\vec{E} = E_0\hat{i} + 2E_0\hat{j}$. Thus

$$\Phi = (E_0\hat{i} + 2E_0\hat{j}) \cdot (A\hat{i}) = E_0 A.$$

The y component lies along the plane and gives zero dot product. It must not be added to the x component or used as the total field magnitude.

Step 2: Evaluate the area and flux The radius is already in metres. Therefore

$$A = \pi(0.02)^2 = 4\pi \times 10^{-4}\,\mathrm{m}^2.$$

With $E_0 = 100\,\mathrm{N\,C^{-1}}$,

$$\Phi = 100(4\pi \times 10^{-4}) = 0.04\pi \simeq 0.1257\,\mathrm{N\,m^2\,C^{-1}}.$$

The options quote positive magnitudes; reversing the chosen normal would reverse the sign but not this magnitude. A factor of two from the y component would be erroneous. Neither 0.02 nor 0.005 matches the circular area calculation, while 3.14 is too large by a factor of about 25.

Step 3: Conclusion The flux magnitude is nearly $0.125\,\mathrm{N\,m^2\,C^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: For flux through a coordinate plane, select the perpendicular field component first, then multiply by the area; the full field magnitude usually is unnecessary.

17. The gap between the plates of a parallel plate capacitor of area A and distance between plates d, is filled with a dielectric whose relative permittivity varies linearly from ε_1 at one plate to ε_2 at the other. The capacitance of the capacitor is [4 marks]

(A) $\dfrac{\epsilon_0(\epsilon_2-\epsilon_1)A}{d\,\ln(\epsilon_2/\epsilon_1)}$

(B) $\dfrac{\epsilon_0(\epsilon_2+\epsilon_1)A}{2d}$

(C) $\dfrac{\epsilon_0(\epsilon_1+\epsilon_2)A}{d}$

(D) $\dfrac{\epsilon_0 A}{d\,\ln(\epsilon_2/\epsilon_1)}$

Correct Answer: (A) $\dfrac{\epsilon_0(\epsilon_2-\epsilon_1)A}{d\,\ln(\epsilon_2/\epsilon_1)}$

Solution:

Concept:

- Layers perpendicular to the electric field act as capacitors in series: their inverse capacitances add.
- For a thin layer of thickness dx, area A, and relative permittivity $\epsilon_r(x)$, $d(1/C) = dx/[\epsilon_0 A\epsilon_r(x)]$.

Step 1: Describe the linear permittivity Let $x = 0$ be the plate with relative permittivity ϵ_1, and $x = d$ the plate with ϵ_2. The linear profile fixed by these endpoint conditions is

$$\epsilon_r(x) = \epsilon_1 + \frac{\epsilon_2-\epsilon_1}{d}x.$$

Each thin slab occupies the full plate area. Because the variation is along the separation, the potential drops across these slabs add; they are not parallel capacitors. The free charge and electric displacement normal to the layers are common.

Step 2: Integrate the inverse capacitance Adding the series contributions yields

$$\frac{1}{C} = \frac{1}{\epsilon_0 A}\int_0^d \frac{dx}{\epsilon_1 + (\epsilon_2-\epsilon_1)x/d}.$$

For distinct positive endpoint permittivities, integration gives

$$\frac{1}{C} = \frac{d}{\epsilon_0 A(\epsilon_2-\epsilon_1)}\ln\left(\frac{\epsilon_2}{\epsilon_1}\right).$$

Equivalently, integrate $E(x) = Q/[\epsilon_0 A\epsilon_r(x)]$ to obtain the voltage, then use $C = Q/V$. Both methods account for the stronger field in the lower-permittivity region.

Step 3: Invert and check the result Taking the reciprocal gives

$$C = \frac{\epsilon_0 A(\epsilon_2-\epsilon_1)}{d\ln(\epsilon_2/\epsilon_1)}.$$

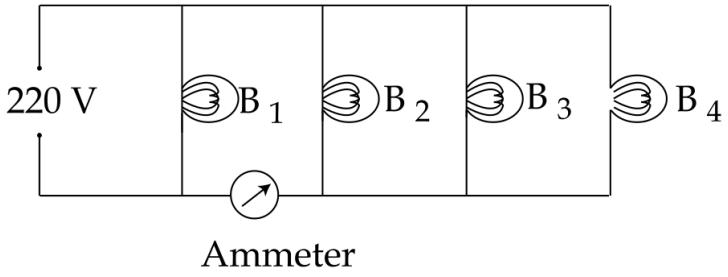
If the endpoints approach the same value ϵ, the ratio $(\epsilon_2-\epsilon_1)/\ln(\epsilon_2/\epsilon_1)$ tends to ϵ, recovering $C = \epsilon_0\epsilon A/d$. An arithmetic average, as in (B), corresponds to a different arrangement or approximation, not this exact series integral. Option (C) doubles the uniform-dielectric limit, and (D) lacks the

endpoint difference needed for that finite limit.

Step 4: Conclusion The capacitance is $\epsilon_0 A(\epsilon_2 - \epsilon_1)/[d\ln(\epsilon_2/\epsilon_1)]$. Hence, the correct option is (A).

Quick Tip to Solve: If a dielectric varies along plate separation, integrate its reciprocal permittivity; averaging permittivity directly is appropriate for side-by-side parallel regions.

18. Four bulbs B_1, B_2, B_3 and B_4 of 100 W each are connected to 220 V main as shown in the figure. The reading in an ideal ammeter will be [4 marks]



(A) 0.90 A

(B) 1.35 A

(C) 0.45 A

(D) 1.80 A

Correct Answer: (B) 1.35 A

Solution:

Concept:

- For a bulb operating at its rated voltage, electrical power is $P = VI$, so its current is $I = P/V$.
- Kirchhoff's junction rule makes the current in a connecting wire equal to the sum of currents from the branches whose return paths pass through that wire. An ideal ammeter has zero resistance.

Step 1: Identify which branches use the ammeter The diagram connects all four bulbs between the upper and lower supply rails. Each is therefore across the 220 V mains. The ammeter lies in the lower rail between the return connection of B_1 and those of B_2, B_3, B_4. The return current of B_1 reaches the supply without crossing the meter. In contrast, the return currents of the three bulbs to its right combine and pass through it. The meter therefore measures three branch currents, rather than the entire four-bulb supply current.

Step 2: Calculate one bulb current Each bulb consumes 100 W at the applied 220 V, giving

$$I_b = \frac{P}{V} = \frac{100}{220} = \frac{5}{11}\,\text{A} \simeq 0.4545\,\text{A}.$$

This current is the same for every bulb because their ratings and applied voltages are the same. Their effective operating resistance would be $V^2/P = 484\,\Omega$, leading to the same current. There is no series voltage division between these separate parallel branches.

Step 3: Add only the measured branch currents By the junction rule at the return rail,

$$I_A = I_2 + I_3 + I_4 = 3I_b = \frac{300}{220}\,\text{A} \simeq 1.36\,\text{A}.$$

The listed 1.35 A is the nearest value, using approximately 0.45 A for each bulb. Option (C) counts just one bulb, (A) counts two, and (D) counts all four. The last would apply to an ammeter in the supply lead before the first branch, which is not its shown position.

Step 4: Conclusion The ammeter reads approximately $1.35\,\text{A}$ among the listed choices. Hence, the correct option is (B).

Quick Tip to Solve: Trace the return wire on each side of an ammeter before adding currents; a meter drawn on a supply rail need not measure every parallel branch.

19. An example of a perfect diamagnet is a superconductor. This implies that when a superconductor is put in a magnetic field of intensity B, the magnetic field Bs inside the superconductor will be such that [4 marks]

(A) Bs= B

(B) Bs= 0

(C) Bs< Bbut Bs≠0

(D) Bs= –B

Correct Answer: (B) Bs= 0

Solution:

Concept:

- A superconductor in its superconducting Meissner state expels magnetic flux from its bulk: its interior magnetic induction is zero.
- In SI notation, $\vec{B}_{\text{inside}} = \mu_0(\vec{H} + \vec{M})$; perfect diamagnetism corresponds to $\vec{M} = -\vec{H}$ and susceptibility $\chi = -1$.

Step 1: Distinguish applied and internal fields The question asks about the resulting field inside a perfect diamagnet, denoted B_s, rather than only the magnetic field created by its induced currents. Applying an external field produces screening currents at the surface. Their magnetic effect opposes the applied field so that the sum vanishes in the superconducting bulk. This is the Meissner effect in the superconducting state, under the conditions implicit in calling the material a perfect diamagnet.

Step 2: Apply perfect screening If the applied induction is $\vec{B}$, the screening contribution in the bulk is $-\vec{B}$. Consequently,

$$\vec{B}_s = \vec{B} + (-\vec{B}) = \vec{0}.$$

Equivalently, $\vec{M} = -\vec{H}$ gives $\mu_0(\vec{H} + \vec{M}) = 0$. The minus sign belongs to the induced contribution, not to the net interior induction. Surface penetration over a microscopic length does not change the ideal bulk statement being tested.

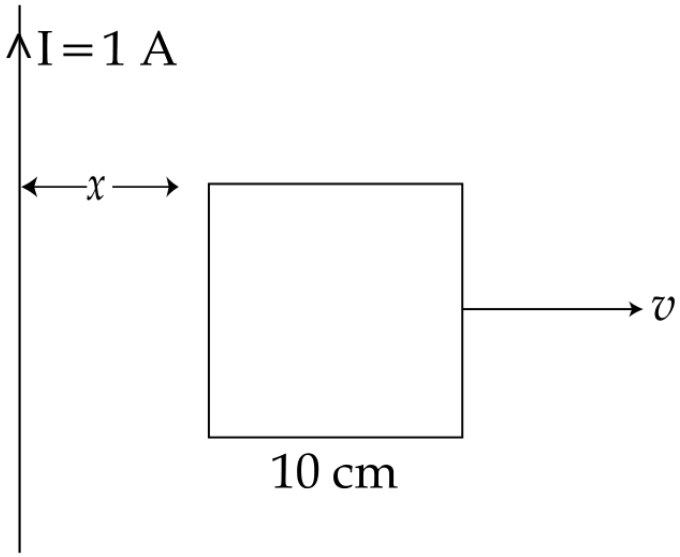
Step 3: Compare the alternatives Option (A), $B_s = B$, would indicate no expulsion. Option (C), a

nonzero field smaller than the applied field, describes incomplete screening and does not express perfect diamagnetism. Option (D), $B_s = -B$, mistakes the opposing induced field for the resulting internal field. Only a zero net field states complete flux exclusion. A material with merely high conductivity is not by that fact a perfect diamagnet; the superconducting Meissner property is the reason for the zero induction here.

Step 4: Conclusion The magnetic field inside is $B_s = 0$. Hence, the correct option is (B).

Quick Tip to Solve: For perfect diamagnetism, add applied and screening fields: the induced field opposes the applied field, whereas their net field inside the bulk is zero.

20. A square frame of side 10 cm and a long straight wire carrying current 1 A are in the plane of the paper. Starting from close to the wire, the frame moves towards the right with a constant speed of 10 m s^{-1} (see figure). The e.m.f induced at the time the left arm of the frame is at x = 10 cm from the wire is [4 marks]



(A) 0.75 μV

(B) 1 μV

(C) 2 μV

(D) 0.5 μV

Correct Answer: (B) 1 μV

Solution:

Concept:

- A long straight wire produces magnetic field $B(r) = \mu_0 I/(2\pi r)$, where r is the perpendicular distance from the wire.
- For a rectangular loop moving away from the wire, the two transverse sides give opposing motional emfs. The net magnitude is $\mathcal{E} = lv[B(x) - B(x + l)]$.

Step 1: Identify the unequal magnetic fields The wire current is upward, so the field at the square to its right is directed into the page. Its magnitude decreases with distance. The loop has side $l = 0.10\,\mathrm{m}$, its near vertical side is at $x = 0.10\,\mathrm{m}$, and its far side is at $x + l = 0.20\,\mathrm{m}$. Thus the two vertical sides do not have equal field magnitudes. Their induced emfs oppose around the circuit; adding their magnitudes would give the wrong result.

Step 2: Derive the net emf Integrating the magnetic flux through the square gives

$$\Phi = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{x+l}{x}\right).$$

With $dx/dt = v$, Faraday's law yields the magnitude

$$\mathcal{E} = \frac{\mu_0 I l v}{2\pi}\left(\frac{1}{x} - \frac{1}{x+l}\right).$$

This also follows immediately from the difference of the motional emfs Blv on the two vertical sides. The horizontal sides have no contribution along their length.

Step 3: Substitute and compare Using $\mu_0/(2\pi) = 2 \times 10^{-7}$ in SI units,

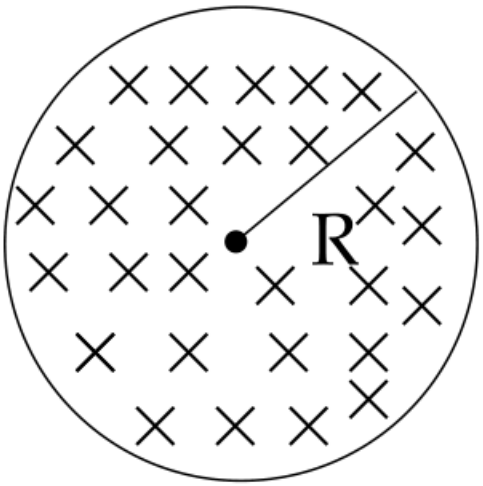
$$\mathcal{E} = (2 \times 10^{-7})(1)(0.10)(10)\,(10 - 5) = 10^{-6}\,\mathrm{V} = 1\,\mu\mathrm{V}.$$

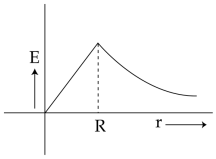
As the loop moves away, into-page flux decreases, so the induced current would be clockwise to reinforce it, consistent with Lenz's law. Option (C) is the near-side emf alone; option (D) is the far-side emf alone. Neither represents their difference. The value 0.75 microvolt likewise does not follow from the two specified distances.

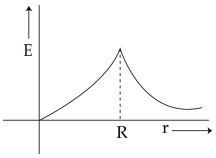
Step 4: Conclusion The induced emf magnitude is $1\,\mu\mathrm{V}$. Hence, the correct option is (B).

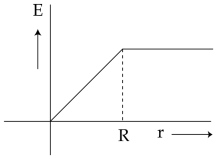
Quick Tip to Solve: For a loop receding from a straight wire, subtract the far-side motional emf from the near-side value; retain the wire-distance dependence before substituting.

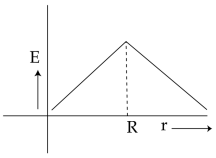
21. The figure shows a circular area of the radius R where a uniform magnetic field B is going into the plane of the paper and increasing in magnitude at a constant rate. In that case, which of the following graphs, drawn schematically, correctly shows the variation of the induced electric field E(r)? [4 marks]

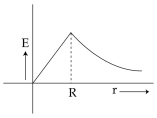


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- Faraday's law is $\oint \vec{E} \cdot d\vec{l} = -d\Phi_B/dt$. Circular symmetry makes the induced electric field tangential and constant in magnitude on a circle centred on the magnetic region.
- The enclosed magnetic-flux area is πr^2 inside the region but saturates at πR^2 outside it.

Step 1: Find the field within the magnetic region Write $k = dB/dt > 0$, a constant. For a concentric circular path with $r \leq R$, all its enclosed area has the changing field. Taking magnitudes in Faraday's law gives

$$E(r)\,2\pi r = \pi r^2 k, \qquad E(r) = \frac{kr}{2}.$$

Thus the field vanishes at the centre and rises linearly with distance. The direction is counterclockwise for an increasing into-page magnetic field: the circulation opposes the increase in into-page flux.

Step 2: Find the field outside the magnetic region For $r \geq R$, increasing the path radius adds no changing magnetic flux outside the given circular area. Therefore

$$E(r)\,2\pi r = \pi R^2 k, \qquad E(r) = \frac{kR^2}{2r}.$$

The electric field is not zero outside the field region. A changing enclosed flux induces circulation even where the local magnetic field is absent. Since the same circulation is spread over a circumference proportional to r, its magnitude decreases as $1/r$.

Step 3: Match the two regions to the graphs At the boundary both formulas give $E(R) = kR/2$, so the field is continuous and has its maximum there. The required graph is a straight rising line up to R, followed by a decreasing inverse-radius curve. This is graph (A). Graph (B) incorrectly curves upward inside. Graph (C) keeps the exterior field constant, ignoring the growing circumference. Graph (D) makes it decrease linearly to zero at a finite radius, whereas $1/r$ remains positive at any finite distance and tends to zero only asymptotically.

Step 4: Conclusion The dependence is $E = kr/2$ for $r \leq R$, and $E = kR^2/(2r)$ for $r \geq R$. Hence, the correct option is (A).

Quick Tip to Solve: For a circular Faraday-law path, first decide whether the changing-flux area grows with the path or has reached its fixed maximum; then divide by the circumference.

22. If microwaves, X-rays, infrared, gamma rays, ultraviolet, radio waves and visible parts of the electromagnetic spectrum are denoted respectively by M, X, I, G, U, R and V the following is the arrangement in ascending order of the wavelength [4 marks]

(A) I, M, R, U, V, X and G

(B) R, M, I, V, U, X and G

(C) M, R, V, X, U, G and I

(D) G, X, U, V, I, M and R

Correct Answer: (D) G, X, U, V, I, M and R

Solution:

Concept:

- All electromagnetic waves travel with the same vacuum speed c, and $c = \nu\lambda$. Thus increasing wavelength means decreasing frequency.
- The conventional spectrum runs from gamma rays through X-rays, ultraviolet, visible, infrared and microwaves to radio waves. Adjacent bands can overlap, but this is their standard order.

Step 1: Start at the shortest wavelengths Gamma rays have the greatest characteristic frequencies and hence the smallest wavelengths in this list. X-rays follow gamma rays; ultraviolet has a longer wavelength than X-rays but a shorter wavelength than visible light. Thus the beginning of the required ascending sequence must be G, X, U, V.

Step 2: Continue beyond visible light Infrared lies beyond the red end of the visible spectrum, so its wavelength exceeds that of visible light. Microwaves have longer wavelengths than infrared, and the

usual radio band lies beyond microwaves. Combining both parts gives

$$\lambda_G < \lambda_X < \lambda_U < \lambda_V < \lambda_I < \lambda_M < \lambda_R.$$

This comparison concerns wavelength, rather than photon energy; the energy ordering is reversed because $E = hc/\lambda$.

Step 3: Check the offered sequences Option A starts with infrared and puts high-energy gamma rays last, so it is not increasing wavelength. Option B is the reverse of the requested order. Option C places microwaves before radio but then incorrectly puts visible and X-rays after them. Only D preserves every adjacent comparison.

Step 4: Conclusion The ascending order of wavelength is G, X, U, V, I, M and R. Hence, the correct option is (D).

Quick Tip to Solve: First place gamma rays at the short-wavelength end and radio waves at the long-wavelength end; then fill in X, U, V, I and M between them.

23. The diameter of the objective lens of microscope makes an angle β at the focus of the microscope. Further, the medium between the object and the lens is an oil of refractive index n. Then the resolving power of the microscope [4 marks]

(A) Increases with decreasing value of β

(B) Increases with increasing value of $n \sin 2\beta$

(C) Increases with increasing value of $\frac{1}{n \sin 2\beta}$

(D) Increases with decreasing value of n

Correct Answer: (B) Increases with increasing value of $n \sin 2\beta$

Solution:

Concept:

- A microscope resolves two nearby points when their diffraction images satisfy the resolution criterion. For fixed vacuum wavelength λ, the minimum separation is $d_{\min} = 0.61\lambda/(n \sin \alpha)$.
- The numerical aperture is $\mathrm{NA} = n \sin \alpha$, where α is the semi-angle subtended by the objective at the object. Resolving power is proportional to $1/d_{\min}$.

Step 1: Identify the aperture angle The complete objective diameter subtends angle β, so its radius subtends the semi-angle $\alpha = \beta/2$. The numerical aperture of the oil-immersion objective is therefore

$$\mathrm{NA} = n \sin(\beta/2).$$

It is the semi-angle, not the full diameter angle, which enters the diffraction formula.

Step 2: Relate aperture to resolving power Taking the reciprocal of the limiting separation gives

$$\mathrm{RP} = \frac{n \sin(\beta/2)}{0.61\lambda}.$$

At fixed wavelength, increasing the refractive index or increasing the accepted cone angle increases the resolving power. Oil immersion improves resolution because its larger refractive index increases numerical aperture.

Step 3: Apply the keyed interpretation The answer key reads the printed quantity $n \sin 2\beta$ as the numerical aperture $n \sin(\beta/2)$ of the half-angle. Under that interpretation B states the required increasing dependence. A decreases the cone angle; C uses the reciprocal dependence; D decreases the refractive index. Each of those changes would reduce resolving power rather than increase it.

Step 4: Conclusion The resolving power increases with numerical aperture $n \sin(\beta/2)$; with the stated interpretation of the printed expression, the keyed selection is (B). Hence, the correct option is (B).

Quick Tip to Solve: When a complete lens diameter subtends an angle, halve that angle before using numerical aperture; resolving power increases with numerical aperture.

24. A ray of light is incident from a denser to a rarer medium. The critical angle for total internal reflection is θiC and Brewster's angle of incidence is θiB, such that sinθiC / sinθiB = η = 1.28. The relative refractive index of the two media is [4 marks]

(A) 0.4

(B) 0.2

(C) 0.9

(D) 0.8

Correct Answer: (D) 0.8

Solution:

Concept:

- For incidence from a denser medium of index n_d into a rarer medium of index n_r, define the relative index $\mu = n_r/n_d < 1$. The critical-angle relation is $\sin\theta_{iC} = \mu$.
- Brewster's law for this same direction of incidence is $\tan\theta_{iB} = \mu$. Both relations must use the same relative index.

Step 1: Express both sines with one index From $\tan\theta_{iB} = \mu$, use a right triangle whose opposite and adjacent sides are μ and 1. Its hypotenuse is $\sqrt{1+\mu^2}$, so

$$\sin\theta_{iB} = \frac{\mu}{\sqrt{1+\mu^2}}.$$

The critical-angle sine is simply μ; there is no need to determine either angle separately.

Step 2: Use the given ratio Substitute into the ratio supplied in the question:

$$\eta = \frac{\sin\theta_{iC}}{\sin\theta_{iB}} = \frac{\mu}{\mu/\sqrt{1+\mu^2}} = \sqrt{1+\mu^2}.$$

Therefore

$$\mu = \sqrt{\eta^2 - 1} = \sqrt{1.28^2 - 1} = \sqrt{0.6384} \simeq 0.799.$$

The positive root is required because refractive indices are positive.

Step 3: Compare the numerical options The result rounds to 0.8, corresponding to D. The alternatives 0.4, 0.2 and 0.9 would give ratios $\sqrt{1+\mu^2}$ of approximately 1.077, 1.020 and 1.345, respectively, rather than 1.28. The reciprocal index is greater than one, but the relative index for the stated denser-to-rarer direction is less than one.

Step 4: Conclusion The refractive index of the rarer medium relative to the denser medium is approximately 0.8. Hence, the correct option is (D).

Quick Tip to Solve: Use the same direction for both laws: $\sin C = \mu$ and $\tan B = \mu$; their sine ratio immediately gives $\sqrt{1+\mu^2}$.

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25. In Young's double-slit experiment, the distance between the two identical slits is 6.1 times larger than the slit width. Then the number of intensity maxima observed within the central maximum of the single-slit diffraction pattern is [4 marks]

(A) 3

(B) 6

(C) 24

(D) 12

Correct Answer: For the stated ratio $d/a = 6.1$, the standard count is 13, including the central bright fringe. No printed option is correct. This differs from the sheet key (D: 12), which omits the central maximum.

Solution:

Concept:

- For slit width a, the central single-slit diffraction maximum lies between the first minima $a\sin\theta = \pm\lambda$.
- The usual interference maxima for slit separation d satisfy $d\sin\theta = m\lambda$, with integer order m. Count orders on both sides and include the central order $m = 0$.

Step 1: Find the allowed interference orders Within the central diffraction maximum, $|\sin\theta| < \lambda/a$. Substitute the interference condition to obtain

$$\left|\frac{m\lambda}{d}\right| < \frac{\lambda}{a} \quad \Longrightarrow \quad |m| < \frac{d}{a} = 6.1.$$

Hence the permitted integer orders are $m = -6, -5, \ldots, -1, 0, 1, \ldots, 5, 6$. Neither limiting diffraction minimum is counted as a bright fringe.

Step 2: Count the fringes explicitly There are six negative orders, six positive orders and one central order. Thus

$$N = 6 + 6 + 1 = 13.$$

The outer orders $m = \pm 6$ are still inside the diffraction boundary because 6 is smaller than 6.1. They can be weak, but the problem specifies no detection threshold by which they could be discarded. The central maximum is itself an intensity maximum and cannot be left out of the count.

Step 3: Assess the available choices The answer key selects D, namely 12, but this counts the six noncentral orders on either side while omitting the central bright fringe. The printed ratio 6.1 instead gives 13 in the standard interference-order count. Values 3 and 6 discard further orders; 24 doubles an already two-sided count. None of A–D gives the complete count for the stated data. If one interprets "6.1 times larger" as 7.1 times as large, the count becomes 15, which still does not validate any offered value.

Step 4: Conclusion For the stated ratio $d/a = 6.1$, the standard count is 13, including the central bright fringe. No printed option is correct. This differs from the sheet key (D: 12), which omits the central maximum.

Quick Tip to Solve: Write $|m| < d/a$, list the allowed integer orders, and count $m = 0$ once. Avoid rounding a continuous envelope-width ratio to obtain a fringe count.

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26. Match List - I (Experiment performed) with List - II (Phenomena discovered/associated) and select the correct option from the options given below the lists

List - I: (a) Davisson and Germer experiment, (b) Millikan's oil drop experiment, (c) Rutherford experiment, (d) Franck-Hertz experiment

List - II: (i) Wave nature of electrons, (ii) Charge of an electron, (iii) Quantisation of energy levels, (iv) Existence of the nucleus [4 marks]

(A) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(B) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

(C) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

(D) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

Correct Answer: (B) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

Solution:

Concept:

- Match each experiment to the specific physical result it established, rather than to a broad subject such as atomic physics.
- Electron diffraction establishes wave behaviour; oil-drop balance measures charge; large-angle scattering reveals a compact nucleus; discrete collision-energy losses establish quantised atomic excitation.

Step 1: Match the electron experiments Davisson and Germer observed diffraction of electrons by a nickel crystal. A diffraction pattern requires wave behaviour, and its wavelength agrees with the de Broglie relation $\lambda = h/p$. Thus (a) matches (i). Millikan balanced the electrical and gravitational forces on charged oil drops and obtained charges in integer multiples of the elementary charge. Thus (b) matches (ii). These experiments did not interchange their roles: diffraction measures a wave property, whereas the oil-drop method determines charge.

Step 2: Match scattering and atomic excitation In Rutherford scattering, most alpha particles passed through the foil, while a small fraction underwent large deflections. This required a very small, massive, positively charged nucleus, so (c) matches (iv). Franck and Hertz found discrete energy

losses in electron collisions with atoms. The fixed excitation energies demonstrated quantisation of atomic energy levels, so (d) matches (iii).

Step 3: Select the complete mapping Combining the four results gives (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii), which is B. A and C already assign the first two experiments incorrectly. D gets the electron experiments right but swaps Rutherford and Franck-Hertz: existence of the nucleus is the scattering result, while quantised energy levels are the collision-excitation result.

Step 4: Conclusion The experiments establish electron waves, elementary charge, the nucleus and discrete atomic energy levels, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Associate each apparatus with its signature observation: crystal diffraction, balanced oil drops, rare backward alpha scattering, and discrete electron-energy losses.

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27. A piece of wood from a recently cut tree shows 20 decays per minute. A wooden piece of same size placed in a museum (obtained from a tree cut many years back) shows 2 decays per minute. If half life of C^{14} is 5730 years, then age of the wooden piece placed in the museum is approximately [4 marks]

(A) 10439 years

(B) 39049 years

(C) 19039 years

(D) 13094 years

Correct Answer: (C) 19039 years

Solution:

Concept:

- Activity is proportional to the number of surviving radioactive nuclei: $A = \lambda N$. For a fixed decay constant, the activity ratio equals the surviving fraction.
- Radioactive decay gives $A = A_0 e^{-\lambda t}$, with $\lambda = \ln 2/T_{1/2}$. Equal-sized wood samples allow comparison of their activities as specified.

Step 1: Form the surviving fraction The recently cut wood provides initial activity $A_0 = 20$ decays per minute for the corresponding sample size. The museum wood has activity $A = 2$ decays per minute. Therefore

$$\frac{A}{A_0} = \frac{2}{20} = \frac{1}{10}.$$

Once cut, the wood no longer replenishes its carbon-14 by exchange with the living environment, so its activity decreases with time.

Step 2: Solve the exponential decay law Taking natural logarithms of the decay equation gives

$$t = \frac{1}{\lambda} \ln \frac{A_0}{A} = \frac{5730}{\ln 2} \ln 10.$$

Substituting $\ln 10 \simeq 2.3026$ and $\ln 2 \simeq 0.6931$,

$$t \simeq 5730(3.322) \simeq 1.90 \times 10^4 \text{ years}.$$

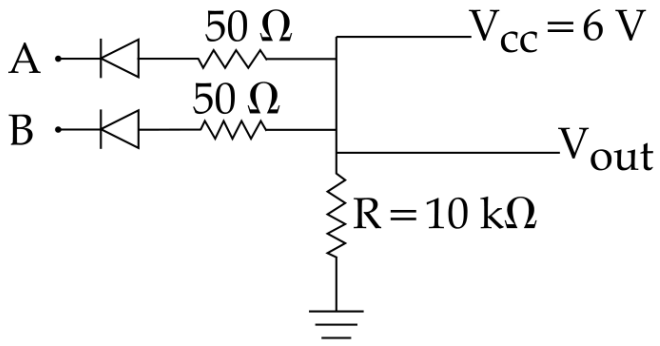
The number of half-lives is about 3.322, which is consistent with a reduction by a factor of ten.

Step 3: Choose the closest listed age The value 19039 years is the closest option, C. As a quick independent check, three half-lives reduce the activity from 20 to 2.5 decays per minute; a little more than three half-lives is needed to reach 2. Ages 10439 and 13094 years are too short for that decrease, while 39049 years corresponds to nearly seven half-lives and would leave much less activity.

Step 4: Conclusion The museum wood is approximately 1.90×10^4 years old, nearest to 19039 years. Hence, the correct option is (C).

Quick Tip to Solve: Use the activity ratio directly: a factor-of-ten decrease is about 3.322 half-lives. There is no need to calculate the number of carbon-14 atoms.

28. Given, A and B are input terminals. Logic 1 is $>$ 5 V, Logic 0 is $<$ 1 V. Which logic gate operation, the following circuit does ? [4 marks]



Given : A and B are input terminals.

(A) AND Gate

(B) OR Gate

(C) XOR Gate

(D) NOR Gate

Correct Answer: (A) AND Gate

Solution:

Concept:

- For the intended positive-logic operation, a low input supplies a conducting diode path that holds the output low; a high output is obtained only when both input paths permit it.
- Positive logic assigns 1 to a voltage above 5 V and 0 to a voltage below 1 V. An AND operation has Boolean form $Y = A \cdot B$.

Step 1: Test the cases with a low input Use the intended diode switching action to compare the input combinations. If A is low, its diode branch provides the low-level clamping action; the output is low regardless of whether B is low or high. Similarly, if B alone is low, its diode branch clamps the output. Thus the combinations (0,0), (0,1) and (1,0) all produce output 0 in the intended logic interpretation.

Step 2: Test the case with both inputs high When both A and B are high, neither input branch imposes a low-level clamp. The intended output is then high. The two-input truth table is therefore conveniently written as

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

The decisive property is that both conditions must hold together; a single high input is insufficient.

Step 3: Compare with the proposed gates The table is the AND table, matching A. An OR gate would already give output 1 for (0,1) or (1,0), contrary to the intended low-input clamping action. An XOR gate would give 1 for unequal inputs but 0 for (1,1), which reverses the relevant cases. A NOR gate would give 1 only for (0,0). Neither XOR nor NOR has the high-high output required here. Thus the answer follows from the complete table rather than from the presence of two diodes alone.

Step 4: Conclusion In the intended positive-logic interpretation, $Y = A \cdot B$: the output is high only when both inputs are high. Hence, the correct option is (A).

Quick Tip to Solve: Check the unequal-input cases first: if a single low input forces a low output, test high-high next to identify the AND truth table.

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29. Long-range radio transmission is possible when the radio waves are reflected from the ionosphere. For this to happen the frequency of the radio waves must be in the range [4 marks]

(A) 150 $-$500 kHz

(B) 80 $-$150 MHz

(C) 1 $-$3 MHz

(D) 8 $-$25 MHz

Correct Answer: (D) 8 $-$25 MHz

Solution:

Concept:

- In sky-wave propagation, the ionised layers of the upper atmosphere refract suitable radio waves back towards Earth. This makes communication possible beyond the direct line of sight.
- The useful frequency depends on the ionospheric electron density and the angle of incidence. Frequencies in the short-wave band are used for long-distance sky-wave broadcasting.

Step 1: Identify the propagation mechanism The question explicitly requires the ionosphere to return the transmitted wave. We therefore select the sky-wave band, rather than a ground-wave band or a space-wave band. The ionosphere does not act as an equally effective reflector for all radio frequencies; waves with sufficiently high frequency pass through it instead of returning to the receiver.

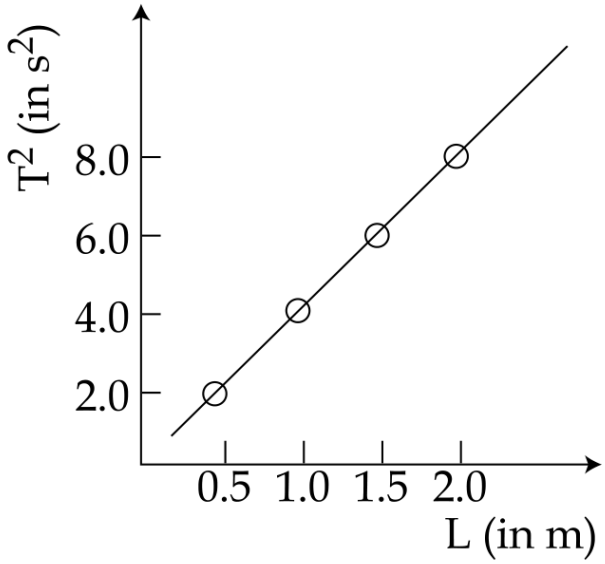
Step 2: Compare the frequency intervals The interval 8–25 MHz lies in the short-wave region commonly used for long-range ionospheric transmission. The interval 150–500 kHz is a low-frequency

ground-wave range. The interval 80–$150\,\text{MHz}$ is too high for ordinary sky-wave return and is associated with line-of-sight communication. The lower interval 1–$3\,\text{MHz}$ is not the intended short-wave long-range transmission band among these choices; ionospheric absorption is appreciably stronger at lower frequencies.

Step 3: Conclusion The suitable listed band is 8–$25\,\text{MHz}$. Hence, the correct option is (D).

Quick Tip to Solve: Identify whether a question describes ground wave, sky wave or line-of-sight propagation before comparing numerical frequency ranges.

30. In an experiment to determine the gravitational acceleration g of a place with the help of a simple pendulum, the measured time period squared is plotted against the string length of the pendulum in the figure. What is the value of g at the place? [4 marks]



(A) 10.0 m s^{-2}

(B) 9.87 m s^{-2}

(C) 9.91 m s^{-2}

(D) 9.81 m s^{-2}

Correct Answer: (B) 9.87 m s^{-2}

Solution:

Concept:

- For small-amplitude oscillations of a simple pendulum, $T = 2\pi\sqrt{L_{\text{eff}}/g}$. Squaring gives a linear relation between period squared and effective length.
- The slope of a graph of T^2 against length is $m = 4\pi^2/g$. A constant offset between string length and effective pendulum length changes the intercept, but not this slope.

Step 1: Read the slope from the plotted line The graph has T^2 in s^2 on the vertical axis and string length L in metres on the horizontal axis. Moving between the plotted levels near $T^2 = 4$ and $T^2 = 8$,

the horizontal change is $1.0\,\mathrm{m}$. Thus

$$m = \frac{\Delta T^2}{\Delta L} = \frac{8-4}{2.0-1.0} = 4.0\,\mathrm{s^2\,m^{-1}}.$$

Use the separation of two points on the straight line, rather than measuring the angle of the line on the page: the axes have different physical units.

Step 2: Calculate gravitational acceleration Equating the measured slope to the pendulum result,

$$g = \frac{4\pi^2}{m} = \frac{4\pi^2}{4.0} = \pi^2\,\mathrm{m\,s^{-2}} \simeq 9.87\,\mathrm{m\,s^{-2}}.$$

The units follow because the reciprocal slope has units $\mathrm{m\,s^{-2}}$. This agrees with (B). Option (A) rounds too coarsely to the stated alternatives, while (C) and (D) do not follow from the plotted slope. One must infer the local acceleration from the experimental graph instead of automatically substituting a memorised value of $9.81\,\mathrm{m\,s^{-2}}$.

Step 3: Conclusion The measured gravitational acceleration is $9.87\,\mathrm{m\,s^{-2}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a period-squared versus length graph, determine its slope first and use g = 4 pi squared divided by slope; do not use the line angle.

Chemistry (120 marks)

31. The ionization energy of gaseous Na atoms is 495.5 kJ mol^{-1}. The lowest possible frequency of light that ionizes a sodium atom is (h = 6.626 $\times$ 10^{-34} Js, NA = 6.022 $\times$ 10^{23} mol^{-1}) [4 marks]

(A) 1.24 $\times$ 10^{15} s^{-1}

(B) 7.50 $\times$ 10^{4} s^{-1}

(C) 4.76 $\times$ 10^{14} s^{-1}

(D) 3.15 $\times$ 10^{15} s^{-1}

Correct Answer: (A) 1.24 $\times$ 10^{15} s^{-1}

Solution:

Concept:

- At the ionisation threshold, one photon supplies exactly the energy needed to remove an electron: $h\nu_0 = E_{\mathrm{atom}}$. Frequencies below this threshold cannot ionise an isolated ground-state atom.
- A molar ionisation energy must be divided by Avogadro's constant before it is equated to the energy of one photon. Convert kilojoules into joules first.

Step 1: Convert the molar energy to energy per atom The given $495.5\,\mathrm{kJ\,mol^{-1}}$ refers to one mole of gaseous sodium atoms, rather than a single atom. Hence the required energy per atom is

$$E_{\mathrm{atom}} = \frac{495.5 \times 10^3}{6.022 \times 10^{23}} = 8.228 \times 10^{-19}\,\mathrm{J}.$$

The large Avogadro factor is essential: using the molar energy directly in the photon equation would produce an unphysical frequency.

Step 2: Calculate the threshold frequency and compare Using the supplied value of Planck's constant,

$$\nu_0 = \frac{E_{\text{atom}}}{h} = \frac{8.228 \times 10^{-19}}{6.626 \times 10^{-34}} = 1.242 \times 10^{15}\,\text{s}^{-1}.$$

The exponent follows from $10^{-19}/10^{-34} = 10^{15}$. Option (A) agrees after rounding. The much smaller frequencies in (B) and (C) supply less than the ionisation energy, so they cannot ionise sodium at threshold. Option (D) supplies more energy than necessary and therefore cannot be the lowest possible frequency.

Step 3: Conclusion The lowest ionising frequency is $1.24 \times 10^{15}\,\text{s}^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: For a molar ionisation energy, use threshold frequency = energy in J per mole divided by the product of Avogadro and Planck constants.

32. Which one of the following has largest ionic radius ? [4 marks]

(A) F^-

(B) B^{3+}

(C) O^{2-}

(D) Li^+

Correct Answer: (C) O^{2-}

Solution:

Concept:

- Ionic size depends on the number of occupied electron shells and on the nuclear attraction experienced by the electrons. Anions expand relative to their atoms, whereas cations contract.
- For an isoelectronic series, the ion with smaller nuclear charge has the larger radius: the same number of electrons is held less tightly.

Step 1: Count the electrons and occupied shells For O^{2-}, the electron count is $8 + 2 = 10$; for F^-, it is $9 + 1 = 10$. Both have a neon-like configuration with their outer electrons in the second shell. By contrast, Li^+ has $3 - 1 = 2$ electrons, while B^{3+} has $5 - 3 = 2$. These cations have only the compact first shell occupied, making them smaller than the two second-shell anions.

Step 2: Compare the isoelectronic anions Oxygen and fluorine anions contain the same ten electrons, but their nuclear charges differ: oxygen has eight protons and fluorine has nine. The larger nuclear charge of fluorine draws the same electron cloud inward more strongly. Thus

$$r(O^{2-}) > r(F^-).$$

This comparison eliminates (A), while the smaller shell count of the helium-like cations eliminates (B) and (D). Choosing merely the most positively charged ion would reverse the physical effect of electron removal and increased effective attraction.

Step 3: Conclusion Among the listed ions, O^{2-} has the largest ionic radius. Hence, the correct option is (C).

Quick Tip to Solve: First group ions by their electron configurations; within an isoelectronic group, arrange radii in the reverse order of nuclear charge.

33. Which one of the following molecules is paramagnetic? [4 marks]

(A) N_2

(B) O_3

(C) CO

(D) NO

Correct Answer: (D) NO

Solution:

Concept:

- Paramagnetism requires at least one unpaired electron. A species in which all electrons are paired is diamagnetic, even if it contains lone pairs.
- Molecular orbitals are filled according to the Aufbau principle, Pauli exclusion principle and Hund's rule. An odd total electron count necessarily leaves an unpaired electron in an ordinary neutral molecule.

Step 1: Identify the odd-electron molecule For nitric oxide, nitrogen contributes seven electrons and oxygen contributes eight. Thus neutral NO has $7 + 8 = 15$ total electrons, an odd number. Equivalently, it has eleven valence electrons. Its molecular-orbital filling leaves one electron unpaired in an antibonding π^* orbital. This unpaired electron gives a magnetic moment and makes the molecule paramagnetic.

Step 2: Check each competing molecule N_2 has fourteen total electrons and a closed-shell molecular-orbital configuration; its bonding and nonbonding electrons are paired. CO is isoelectronic with N_2, also has fourteen total electrons, and is diamagnetic. Ground-state ozone, O_3, is also a closed-shell diamagnetic molecule; it must not be confused with dioxygen, O_2, which has two unpaired electrons. Therefore (A), (B) and (C) do not meet the required condition. Counting lone pairs as unpaired electrons would incorrectly classify closed-shell molecules.

Step 3: Conclusion NO contains an unpaired electron and is paramagnetic. Hence, the correct option is (D).

Quick Tip to Solve: An odd electron count quickly identifies NO as paramagnetic here; distinguish ozone from the familiar paramagnetic oxygen molecule.

34. Sulphur dioxide and oxygen were allowed to diffuse through a porous partition. 20 dm^3 of SO_2 diffuses through the porous partition in 60 seconds. The volume of O_2 in dm^3 which diffuses under the similar condition in 30 seconds will be (atomic mass of sulphur = 32 u); [4 marks]

(A) 28.2

(B) 10.0

(C) 14.1

(D) 7.09

Correct Answer: (C) 14.1

Solution:

Concept:

- Graham's law states that, at the same temperature and pressure, diffusion rate is inversely proportional to the square root of molar mass: $r_1/r_2 = \sqrt{M_2/M_1}$.
- For volumes measured under the same conditions, the rate is volume divided by elapsed time, $r = V/t$. Different diffusion times must be included explicitly.

Step 1: Find the molar masses and the rate ratio Using sulfur mass 32 and oxygen mass 16,

$$M(\mathrm{SO_2}) = 32 + 2(16) = 64, \qquad M(\mathrm{O_2}) = 32\,\mathrm{g\,mol^{-1}}.$$

The lighter oxygen molecules diffuse faster. Thus

$$\frac{r_{\mathrm{O_2}}}{r_{\mathrm{SO_2}}} = \sqrt{\frac{64}{32}} = \sqrt{2}.$$

The sulfur dioxide diffusion rate is $20/60 = 1/3\,\mathrm{dm^3\,s^{-1}}$.

Step 2: Apply the shorter time for oxygen In thirty seconds, the oxygen volume is

$$V_{\mathrm{O_2}} = r_{\mathrm{O_2}} t = \sqrt{2}\left(\frac{20}{60}\right)(30) = 10\sqrt{2} = 14.14\,\mathrm{dm^3}.$$

This rounds to 14.1, matching (C). Option (A) corresponds approximately to allowing oxygen to diffuse for the full sixty seconds. Option (B) simply halves the given volume without accounting for the faster oxygen rate. Option (D) reverses the molar-mass ratio and incorrectly makes the lighter gas diffuse more slowly.

Step 3: Conclusion The oxygen volume is $14.1\,\mathrm{dm^3}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the rate as V/t before applying Graham's law; a correct mass ratio alone is insufficient when the stated times differ.

35. For the decomposition of the compound, represented as $NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$ the $K_p = 2.9 \times 10^{-5}$ atm^3. If the reaction is started with 1 mole of the compound, the total pressure at equilibrium would be [4 marks]

(A) 7.66×10^{-2} atm

(B) 38.8×10^{-2} atm

(C) 5.82×10^{-2} atm

(D) 1.94×10^{-2} atm

Correct Answer: (C) 5.82×10^{-2} atm

Solution:

Concept:

- A pure solid has unit activity and does not appear in the pressure equilibrium expression. For this decomposition, $K_p = p_{NH_3}^2 p_{CO_2}$.
- When the gases are formed only by this reaction, their mole ratio and partial-pressure ratio are $2:1$. Dalton's law gives total pressure as the sum of partial pressures.

Step 1: Express all pressures with one variable The balanced decomposition is

$$NH_2COONH_4(s) \rightleftharpoons 2\,NH_3(g) + CO_2(g).$$

Let $p_{CO_2} = x\,\text{atm}$. Because twice as much ammonia is formed, $p_{NH_3} = 2x\,\text{atm}$. Therefore the total gas pressure is $P = 3x$. The initial one mole is solid; it is not an additional gaseous mole contributing to the pressure.

Step 2: Use the equilibrium constant Substitute the stoichiometric partial pressures into the equilibrium expression:

$$K_p = (2x)^2 x = 4x^3.$$

$$x = \left(\frac{2.9 \times 10^{-5}}{4}\right)^{1/3} = 1.935 \times 10^{-2}\,\text{atm}.$$

Consequently,

$$P = 3x = 5.805 \times 10^{-2}\,\text{atm} \simeq 5.82 \times 10^{-2}\,\text{atm}.$$

The small rounding difference is consistent with the given constant. Option (D) is approximately the carbon dioxide partial pressure alone. Options (A) and (B) do not satisfy $K_p = 4P^3/27$, so neither is the equilibrium total pressure.

Step 3: Conclusion The total equilibrium pressure is approximately $5.82 \times 10^{-2}\,\text{atm}$. Hence, the correct option is (C).

Quick Tip to Solve: For a 2:1 gaseous product mixture, set partial pressures to 2x and x, calculate x from Kp, and multiply by three only at the end.

36. Zirconium phosphate [$Zr_3(PO_4)_4$] dissociates into three zirconium cations of charge +4 and four phosphate anions of charge −3. If molar solubility of zirconium phosphate is denoted by S and its solubility product by K_{sp} then which of the following relationship between S and Ksp is correct ? [4 marks]

(A) $S = \{K_{sp}/6912\}^7$

(B) $S = \{K_{sp}/144\}^{1/7}$

(C) $S = \{K_{sp}/(6912)^{1/7}\}$

(D) $S = (K_{sp}/6912)^{1/7}$

Correct Answer: (D) $S = (K_{sp}/6912)^{1/7}$

Solution:

Concept:

- Molar solubility S is the amount of salt dissolved per litre of saturated solution. Its stoichiometric coefficients determine the individual equilibrium ion concentrations.
- The solubility product is the product of equilibrium ionic concentrations, each raised to its coefficient in the dissociation equation. The pure undissolved solid is omitted.

Step 1: Write the dissolution equilibrium One formula unit gives three zirconium ions and four phosphate ions:

$$Zr_3(PO_4)_4(s) \rightleftharpoons 3\,Zr^{4+}(aq) + 4\,{PO_4}^{3-}(aq).$$

Consequently, dissolving S moles per litre gives $[Zr^{4+}] = 3S$ and $[{PO_4}^{3-}] = 4S$. The charges confirm electrical neutrality: the total positive and negative charges are both twelve.

Step 2: Construct and simplify the product Use the ion concentrations, not merely the solubility itself:

$$K_{sp} = (3S)^3(4S)^4 = 27 \times 256\, S^{3+4} = 6912S^7.$$

The seventh power arises because seven ions are produced per formula unit. Thus the seventh root must apply to the entire quotient, including its numerical denominator.

Step 3: Compare the offered expressions Rearrangement gives

$$S = \left(\frac{K_{sp}}{6912}\right)^{1/7}.$$

Option (A) raises the quotient to seven instead of taking its seventh root. Option (B) uses 144, which is not $3^3 4^4$. Option (C) roots only the denominator and leaves K_{sp} to the first power; substituting it into the product would not recover the given equilibrium constant.

Step 4: Conclusion The correct relationship is $S = (K_{sp}/6912)^{1/7}$. Hence, the correct option is (D).

Quick Tip to Solve: For a salt giving a cations and b anions, first write $K_{sp} = a^a b^b S^{a+b}$; then take the root of the entire quotient.

37. How many electrons are involved in the following redox reaction ? $Cr_2O_7^{2-} + Fe^{2+} + C_2O_4^{2-} \rightarrow Cr^{3+} + Fe^{3+} + CO_2$ (Unbalanced) [4 marks]

(A) 6

(B) 5

(C) 3

(D) 4

Correct Answer: (A) 6

Solution:

Concept:

- Oxidation releases electrons and reduction consumes electrons. In a balanced redox equation the total numbers released and consumed are equal.
- In dichromate, oxygen has oxidation state -2, so chromium is $+6$. Reduction to Cr^{3+} requires three electrons per chromium atom.

Step 1: Count the reduction requirement For $Cr_2O_7{}^{2-}$, $2x + 7(-2) = -2$, giving $x = +6$. There are two chromium atoms, each changing from $+6$ to $+3$. Therefore the electron gain per dichromate ion is

$$2 \times (6 - 3) = 6.$$

The reduction half-reaction in acidic medium is

$$Cr_2O_7{}^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O.$$

Both atoms and net charge balance.

Step 2: Check the oxidised species Ferrous ions lose one electron each: $Fe^{2+} \longrightarrow Fe^{3+} + e^-$. In oxalate the carbon oxidation state is $+3$, while carbon in carbon dioxide is $+4$, so one oxalate ion releases two electrons:

$$C_2O_4{}^{2-} \longrightarrow 2\,CO_2 + 2\,e^-.$$

The electrons supplied by these two reducing species must together meet the six-electron requirement of a dichromate ion.

Step 3: Verify a balanced combination For example, two ferrous ions and two oxalate ions supply $2 + 4 = 6$ electrons. Their sum with the reduction half-reaction is

$$Cr_2O_7{}^{2-} + 14\,H^+ + 2\,Fe^{2+} + 2\,C_2O_4{}^{2-} \longrightarrow 2\,Cr^{3+} + 7\,H_2O + 2\,Fe^{3+} + 4\,CO_2.$$

Options 5, 3 and 4 do not equal the total reduction requirement for the two chromium atoms in the dichromate unit. Three counts only one chromium atom.

Step 4: Conclusion Six electrons are transferred per dichromate ion in the balanced reaction. Hence, the correct option is (A).

Quick Tip to Solve: Count all atoms of the element being reduced: dichromate has two chromium atoms, so multiply the three-electron change by two.

38. Consider the reaction : $H_2SO_3(aq) + Sn^{4+}(aq) + H_2O(l) \rightarrow Sn^{2+}(aq) + HSO_4^-(aq) + 3H^+(aq)$ Which of the following statements is correct ? [4 marks]

(A) Sn^{4+} is the oxidizing agent because it undergoes oxidation

(B) Sn^{4+} is the reducing agent because it undergoes oxidation

(C) H_2SO_3 is the reducing agent because it undergoes reduction

(D) H_2SO_3 is the reducing agent because it undergoes oxidation

Correct Answer: (D) H_2SO_3 is the reducing agent because it undergoes oxidation

Solution:

Concept:

- A reducing agent supplies electrons to another reactant and is itself oxidised. An oxidising agent accepts electrons and is itself reduced.
- An increase in oxidation state indicates oxidation; a decrease indicates reduction. Agent names describe the effect on the other reactant, not the change undergone by the agent.

Step 1: Determine the sulphur oxidation states In sulphurous acid, let sulphur have oxidation state x. From $2(+1) + x + 3(-2) = 0$, obtain $x = +4$. In hydrogen sulphate, $(+1) + y + 4(-2) = -1$, giving $y = +6$. Therefore sulphur changes from $+4$ to $+6$ and loses two electrons. The corresponding oxidation half-reaction is

$$H_2SO_3 + H_2O \longrightarrow HSO_4^- + 3\,H^+ + 2\,e^-.$$

Step 2: Identify the tin reduction Tin changes from $+4$ in Sn^{4+} to $+2$ in Sn^{2+}. It gains the same two electrons:

$$Sn^{4+} + 2\,e^- \longrightarrow Sn^{2+}.$$

Tin(IV) therefore acts as the oxidising agent. Adding these two half-reactions cancels the electrons and reproduces the printed overall equation, including the three hydrogen ions.

Step 3: Test each statement Option (A) correctly calls tin(IV) an oxidising agent but incorrectly says that it undergoes oxidation; it is reduced. Option (B) assigns both the wrong agent and the wrong process to tin(IV). Option (C) correctly identifies sulphurous acid as the reducing agent but incorrectly associates it with reduction. Only option (D) pairs the correct reducing agent with the oxidation it actually undergoes. The water and hydrogen ions balance the acidic medium and do not change this classification.

Step 4: Conclusion H_2SO_3 is the reducing agent because sulphur is oxidised from $+4$ to $+6$. Hence, the correct option is (D).

Quick Tip to Solve: Write the oxidation-state change first, then assign the agent: the species oxidised is always the reducing agent.

39. Amongst LiCl, RbCl, $BeCl_2$ and $MgCl_2$ the compounds with the greatest and the least ionic character, respectively are [4 marks]

(A) $MgCl_2$ and $BeCl_2$

(B) RbCl and $MgCl_2$

(C) LiCl and RbCl

(D) RbCl and $BeCl_2$

Correct Answer: (D) RbCl and $BeCl_2$

Solution:

Concept:

- Fajans' rules relate covalent character to distortion of an anion's electron cloud by a cation. A smaller cation and a larger cationic charge give greater polarising power.
- For chlorides, the anion is the same in every comparison. The key differences are therefore the cation size and charge; stronger polarisation means less ionic character.

Step 1: Find the least polarising cation Rubidium forms the large, singly charged ion Rb^+. Its positive charge is spread over a much larger ion than in Li^+, so it distorts the chloride electron cloud less strongly. Consequently $RbCl$ has the greatest ionic character among the four salts. The two divalent cations have a higher charge and stronger attraction for the chloride electron cloud.

Step 2: Find the most polarising cation Beryllium forms the exceptionally small ion Be^{2+}, which has a high charge density. It polarises chloride strongly and promotes sharing of electron density, producing appreciable covalent bonding. Magnesium also forms a divalent ion, but Mg^{2+} is larger than Be^{2+}. Thus magnesium chloride is more ionic than beryllium chloride. Within these salts, $BeCl_2$ is the least ionic.

Step 3: Check the order in the options The question requests greatest ionic character first and least ionic character second. Option (A) correctly places beryllium chloride second but misses rubidium chloride as the greatest. Option (B) gets the first member right but wrongly replaces beryllium chloride by magnesium chloride. Option (C) reverses the trend between lithium and rubidium chlorides: the larger alkali-metal ion gives the more ionic chloride. Option (D) alone includes both correct extremes in the requested order.

Step 4: Conclusion The greatest and least ionic characters belong to $RbCl$ and $BeCl_2$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: When all anions are chloride, compare cations: large and singly charged favours ionic bonding; very small and doubly charged favours covalent bonding.

40. Example of a three-dimensional silicate is [4 marks]

(A) Ultramarines

(B) Feldspars

(C) Zeolites

(D) Beryls

Correct Answer: Ultramarines, feldspars and zeolites are three-dimensional framework aluminosilicates; beryl is a ring silicate. There is no unique correct option. The official key dropped this question and awarded bonus marks to all.

Solution:

Concept:

- Silicates are classified by the way SiO_4 tetrahedra share their oxygen atoms. Sharing through an extended three-dimensional network produces a framework silicate.
- Aluminosilicate frameworks permit some silicon sites to be replaced by aluminium. Additional cations balance the resulting negative framework charge; this does not turn the framework into isolated rings.

Step 1: Identify framework structures Feldspars are framework aluminosilicates. Their tetrahedra form an extended three-dimensional network, with alkali-metal or alkaline-earth cations compensating for aluminium substitution. Zeolites likewise contain a three-dimensional aluminosilicate framework, commonly with channels or cavities holding exchangeable cations and water. Thus both (B) and (C) satisfy the question; a unique answer is already impossible.

Step 2: Classify the remaining alternatives Ultramarines are members of the sodalite-related aluminosilicate family. Their cage-containing framework is also three-dimensional, with trapped sulphur species responsible for characteristic colour. They therefore also provide a framework example, rather than an isolated-chain or sheet structure. Beryl, on the other hand, is classified as a ring silicate: its characteristic anionic units are six-membered $Si_6O_{18}{}^{12-}$ rings. It is not the intended framework category merely because its crystal occupies three-dimensional space.

Step 3: Apply the multiple-choice requirement The wording asks for one example but several printed alternatives describe framework silicates. In particular, feldspars and zeolites independently establish the defect without relying on any subtle classification of ultramarines. Selecting only one would incorrectly imply that the others are not frameworks. The official answer key dropped this question, awarding bonus marks to all, which is consistent with its lack of a single defensible selection.

Step 4: Conclusion Ultramarines, feldspars and zeolites are three-dimensional framework aluminosilicates; beryl is a ring silicate. There is no unique correct option. The official key dropped this question and awarded bonus marks to all.

Quick Tip to Solve: Classify by tetrahedral connectivity, not by the fact that every crystal is three-dimensional: feldspars and zeolites are frameworks, while beryl contains silicate rings.

41. Which of these statements is not true ? [4 marks]

(A) $LiAlH_4$ is versatile reducing agent in organic synthesis

(B) NO^+ is not isoelectronic with O_2

(C) Boron is always covalent in its compounds

(D) In aqueous solution, the Tl^+ ion is much more stable than Tl (III)

Correct Answer: No listed statement is false in the standard treatment, so there is no correct option to the negative question. The official key dropped this question and awarded bonus marks to all.

Solution:

Concept:

- To answer a negative multiple-choice question, test every statement rather than choosing one merely because its wording is unfamiliar.
- Isoelectronic species have equal electron counts. The inert-pair effect stabilises the lower oxidation state of heavy p-block elements such as thallium.

Step 1: Check the reducing agent and electron count Lithium aluminium hydride, $LiAlH_4$, is a powerful and versatile reducing agent used in organic synthesis, for example for reducing carbonyl derivatives. Statement (A) is true. For nitrosonium, NO^+, the electron count is $7 + 8 - 1 = 14$. An oxygen molecule has $2 \times 8 = 16$ electrons. The two species are therefore not isoelectronic, exactly as statement (B) says; confusing NO^+ with neutral nitric oxide would give a wrong assessment.

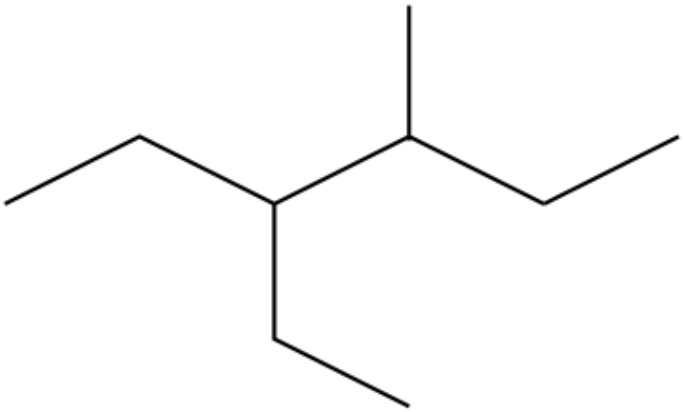
Step 2: Check boron and thallium Boron has a small atomic size and high ionisation energies, so formation of a simple B^{3+} ion is energetically unfavourable. Its compounds are described in the standard p-block treatment as covalent; even in salts containing borate ions, the bonds involving boron in the anion are covalent. Statement (C) expresses that textbook behaviour. For thallium the $6s^2$ electron pair is relatively reluctant to participate in bonding. The inert-pair effect stabilises $+1$ relative to $+3$, making Tl^+ much more stable in aqueous solution, so statement (D) is also true.

Step 3: Check whether a negative answer exists All four printed statements are true under the usual chemistry treatment: (A) describes a standard reagent, (B) passes direct electron counting, (C) describes boron bonding and (D) follows the inert-pair effect. Consequently none answers the request for a statement that is not true. The official answer key dropped the question and awarded bonus marks to all; a false selection should not be manufactured.

Step 4: Conclusion No listed statement is false in the standard treatment, so there is no correct option to the negative question. The official key dropped this question and awarded bonus marks to all.

Quick Tip to Solve: For NO^+, subtract one electron: its total is 14, whereas O_2 has 16. Always notice both the ion charge and the word "not".

42. The correct IUPAC name of the following compound is [4 marks]



(A) 4 - methyl - 3 - ethylhexane

(B) 3 - ethyl - 4 - methylhexane

(C) 4 - ethyl - 3 - methylhexane

(D) 3, 4 - ethylmethylhexane

Correct Answer: (B) 3 - ethyl - 4 - methylhexane

Solution:

Concept:

- The parent name of an acyclic saturated hydrocarbon comes from its longest continuous carbon chain. A line end or an unlabelled corner in a skeletal formula represents a carbon atom.
- Number the parent chain to obtain the lowest substituent locants. When the locant sets tie, give the lower number to the substituent cited first alphabetically; list different substituents alphabetically.

Step 1: Choose the six-carbon parent chain Trace the continuous chain from the left terminal carbon through the central branching region to the right terminal carbon. It contains six carbons, so the parent is hexane. The two-carbon branch at one central carbon is ethyl, while the one-carbon branch at the adjacent central carbon is methyl. An alternative longest path through the lower branch has the same length and yields the same substituent pattern; the drawing does not provide a seven-carbon continuous path.

Step 2: Number and arrange the substituents Numbering the selected six-carbon chain from either end gives the locant set $\{3, 4\}$. This is a tie, so alphabetical order decides which substituent receives 3: ethyl precedes methyl. The substituents are therefore 3-ethyl and 4-methyl, and the full name is 3-ethyl-4-methylhexane. Option (A) lists methyl before ethyl and is not in alphabetical order. Option (C) assigns the lower locant to methyl despite the tie. Option (D) combines the two substituents incorrectly instead of giving each its own locant and name. These checks use both numbering and name order; the right carbon count alone is not sufficient.

Step 3: Conclusion The correct IUPAC name is 3-ethyl-4-methylhexane. Hence, the correct option is (B).

Quick Tip to Solve: After finding the longest chain, compare locant sets in both directions; if they tie, ethyl receives the lower number before methyl.

43. The total number of octahedral void (s) per atom present in a cubic close packed structure is [4 marks]

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (A) 1

Solution:

Concept:

- In a close-packed structure, an octahedral void is surrounded by six atoms. The number of such voids must be counted with unit-cell sharing, rather than by counting their surrounding atoms.
- A cubic close-packed arrangement is face-centred cubic (fcc). An fcc unit cell contains four atoms after corner and face sharing are included.

Step 1: Count the atoms in the unit cell The eight corner atoms each contribute one eighth, and the six face-centred atoms each contribute one half. Thus the effective number of atoms is

$$N = 8\left(\frac{1}{8}\right) + 6\left(\frac{1}{2}\right) = 1 + 3 = 4.$$

Counting all fourteen visible positions as complete atoms would double-count atoms shared with neighbouring cells.

Step 2: Count and normalise the octahedral sites One octahedral site lies at the body centre and belongs wholly to the cell. Twelve more lie at edge centres; each edge is shared by four cells. Therefore

$$N_{\text{oct}} = 1 + 12\left(\frac{1}{4}\right) = 4.$$

The number per atom is consequently

$$\frac{N_{\text{oct}}}{N} = \frac{4}{4} = 1.$$

The value 2 belongs to the number of tetrahedral voids per atom, not octahedral voids. The choices 3 and 4 also do not give the required ratio; 4 is the number of octahedral sites per conventional fcc cell, before dividing by four atoms.

Step 3: Conclusion There is one octahedral void per atom in a cubic close-packed structure. Hence, the correct option is (A).

Quick Tip to Solve: In any close packing of N atoms, remember N octahedral sites and 2N tetrahedral sites; distinguish a per-atom count from a per-cell count.

44. For an ideal solution of two components A & B, which of the following is true? [4 marks]

(A) ΔHmixing$>$ 0 (zero)

(B) A $-$B interaction is stronger than A $-$A & B $-$B. interactions

(C) A$-$A, B$-$B & A$-$B interactions are identical

(D) ΔHmixing $<$ 0 (zero)

Correct Answer: (C) A$-$A, B$-$B & A$-$B interactions are identical

Solution:

Concept:

- An ideal solution obeys Raoult's law throughout the composition range: each component contributes a partial pressure proportional to its liquid mole fraction.
- Ideal mixing has $\Delta H_{\text{mix}} = 0$ and $\Delta V_{\text{mix}} = 0$. The energetic reason is that unlike and like molecular interactions are effectively equivalent.

Step 1: Compare the interactions before and after mixing Before mixing, molecules encounter mainly A–A and B–B attractions. Mixing replaces some of these by A–B contacts. If all three types have the same effective strength, breaking the original contacts and forming the new ones produces no net heat change. This is precisely the ideal-solution condition represented by option (C).

Step 2: Use the heat of mixing to eliminate the alternatives The ideal condition is

$$\Delta H_{\text{mix}} = 0.$$

Option (A), which requires a positive heat of mixing, and option (D), which requires a negative heat of mixing, both contradict this condition. Option (B) makes A–B attraction stronger than the original attractions. Such preferential unlike attraction ordinarily releases energy and gives negative deviation from Raoult's law, rather than ideal behaviour. Thus the zero enthalpy condition and the equivalence of intermolecular attractions identify the same choice. Entropy can still increase on mixing: zero heat of mixing does not mean that mixing has no thermodynamic driving force.

Step 3: Conclusion The A–A, B–B and A–B interactions are identical in the ideal-solution model. Hence, the correct option is (C).

Quick Tip to Solve: For an ideal solution, link equal like/unlike attractions with zero enthalpy and volume changes; do not confuse zero enthalpy with zero entropy change.

45. The observed osmotic pressure for a 0.10 M solution of $Fe(NH_4)_2(SO_4)_2$ at 25°C is 10.8 atm. The expected and experimental (observed) values of Van't Hoff factor 'i' will be respectively; (R = 0.082 L atm k^{-1} mol^{-1}) [4 marks]

(A) 4 and 4.00

(B) 3 and 5.42

(C) 5 and 4.42

(D) 5 and 3.42

Correct Answer: (C) 5 and 4.42

Solution:

Concept:

- Osmotic pressure is $\pi = iCRT$, where i is the van't Hoff factor, C is formula-unit molarity and T is absolute temperature.
- The expected factor for complete dissociation equals the total number of independent ions generated by one formula unit. The observed factor follows from the measured colligative property.

Step 1: Count the ions on complete dissociation Mohr's salt dissociates as

$$\mathrm{Fe(NH_4)_2(SO_4)_2} \longrightarrow \mathrm{Fe^{2+} + 2\,NH_4^{+} + 2\,SO_4^{2-}}.$$

There is one iron ion, two ammonium ions and two sulphate ions, giving

$$i_{\text{expected}} = 1 + 2 + 2 = 5.$$

This counts ions, not distinct ion species: counting three species would incorrectly give 3.

Step 2: Calculate the factor from the osmotic pressure Convert the temperature to kelvin: $T = 25 + 273 = 298\,\mathrm{K}$. Substitution gives

$$i_{\text{observed}} = \frac{10.8}{(0.10)(0.082)(298)} = \frac{10.8}{2.4436} \approx 4.42.$$

The concentration in mol per litre and the stated gas constant in litre-atmosphere units are consistent, so no volume conversion is necessary. Option (C) gives both the five-ion theoretical value and the calculated observed value. Options (A) and (B) have the wrong theoretical count, while option (D) has the correct count but an observed value that does not reproduce 10.8 atm. A measured factor below five reflects nonideal ionic behaviour or association rather than an arithmetic change in ion stoichiometry.

Step 3: Conclusion The expected and observed van't Hoff factors are 5 and approximately 4.42, respectively. Hence, the correct option is (C).

Quick Tip to Solve: First count every ion with its coefficient, then calculate the experimental factor independently from pi/(CRT), always using kelvin.

46. Choose the correct statement with respect to the vapour pressure of a liquid among the following

[4 marks]

(A) Increases non-linearly with increasing temperature

(B) Decreases non-linearly with increasing temperature

(C) Decreases linearly with increasing temperature

(D) Increases linearly with increasing temperature

Correct Answer: (A) Increases non-linearly with increasing temperature

Solution:

Concept:

- Equilibrium vapour pressure is the pressure of vapour in dynamic equilibrium with its liquid at a specified temperature. It rises as more molecules can escape the liquid surface.
- The Clausius–Clapeyron relation is $d\ln p/dT = \Delta H_{\text{vap}}/(RT^2)$, with positive enthalpy of vaporisation for an ordinary liquid.

Step 1: Determine the direction of the change Vaporisation requires energy, so $\Delta H_{\text{vap}} > 0$. The Clausius–Clapeyron equation therefore gives a positive temperature derivative of $\ln p$, and hence a positive derivative of p. Increasing temperature shifts the liquid–vapour equilibrium to a higher vapour pressure. The statements in options (B) and (C), both of which predict decreasing pressure, are inconsistent with this result.

Step 2: Determine whether the increase is linear For an approximately temperature-independent enthalpy of vaporisation, integration gives

$$\ln p = -\frac{\Delta H_{\text{vap}}}{RT} + K, \qquad p = p_0 \exp\left(-\frac{\Delta H_{\text{vap}}}{RT}\right).$$

Pressure itself is therefore not a linear function of temperature. A straight-line relation is obtained approximately for $\ln p$ against $1/T$, not for p against T. Option (D) mistakes these different variables. Although a sufficiently small segment of any smooth curve can appear approximately straight, the general dependence asked for here is nonlinear. This leaves the increasing, nonlinear statement in option (A).

Step 3: Conclusion The vapour pressure of a liquid increases nonlinearly with increasing temperature. Hence, the correct option is (A).

Quick Tip to Solve: Keep the plot variables distinct: vapour pressure versus temperature is curved, whereas ln p versus inverse absolute temperature is approximately straight.

47. For the reaction, 3A + 2B →C + D, the differential rate law can be written as [4 marks]

(A) $+(1)/(3)(d[A])/(dt) = -(d[C])/(dt) = k[A]^n[B]^m$

(B) $(1)/(3)(d[A])/(dt) = (d[C])/(dt) = k[A]^n[B]^m$

(C) $-(1)/(3)(d[A])/(dt) = (d[C])/(dt) = k[A]^n[B]^m$

(D) $-(d[A])/(dt) = (d[C])/(dt) = k[A]^n[B]^m$

Correct Answer: (C) $-(1)/(3)(d[A])/(dt) = (d[C])/(dt) = k[A]^n[B]^m$

Solution:

Concept:

- The reaction rate uses the negative concentration derivative for a reactant and the positive derivative for a product, each divided by its stoichiometric coefficient.
- The experimentally determined orders n and m appear in $r = k[A]^n[B]^m$; they need not equal the stoichiometric coefficients 3 and 2.

Step 1: Relate the concentration changes to reaction progress For $3\,\mathrm{A} + 2\,\mathrm{B} \longrightarrow \mathrm{C} + \mathrm{D}$, each unit of reaction progress consumes three units of A and two of B while producing one unit each of C and D. Thus $d[A]/dt$ is negative and $d[C]/dt$ is positive. Dividing by the coefficients gives a common, positive reaction rate:

$$r = -\frac{1}{3}\frac{d[A]}{dt} = -\frac{1}{2}\frac{d[B]}{dt} = \frac{d[C]}{dt} = \frac{d[D]}{dt}.$$

Step 2: Match the rate expression to the options Combining this stoichiometric definition with the stated concentration form yields

$$-\frac{1}{3}\frac{d[A]}{dt} = \frac{d[C]}{dt} = k[A]^n[B]^m.$$

This is option (C). In option (A), the reactant derivative is positive and the product derivative negative, making both expressions negative rather than equal to the positive forward rate. Option (B) places the decreasing reactant concentration and increasing product concentration on the same sign footing, so they cannot be equal during forward reaction. Option (D) includes the correct reactant sign but omits the divisor 3: A disappears three times as rapidly as C forms. None of these eliminations requires knowing the experimental orders.

Step 3: Conclusion The correct rate law is $-\frac{1}{3}d[A]/dt = d[C]/dt = k[A]^n[B]^m$. Hence, the correct option is (C).

Quick Tip to Solve: Attach a minus sign to reactant disappearance and divide every concentration derivative by its own coefficient before comparing reaction rates.

48. Which one of the following ores is known as Malachite [4 marks]

(A) Cu_2S

(B) Cu_2O

(C) $Cu(OH)_2{\cdot}CuCO_3$

(D) $CuFeS_2$

Correct Answer: (C) $Cu(OH)_2{\cdot}CuCO_3$

Solution:

Concept:

- An ore name identifies a particular mineral composition. Malachite is a basic copper carbonate, containing both carbonate and hydroxide in the same mineral.
- The formula $CuCO_3 \cdot Cu(OH)_2$ is equivalently written $Cu_2CO_3(OH)_2$; these forms express the same atom counts.

Step 1: Identify the carbonate–hydroxide composition The term basic carbonate indicates carbonate together with hydroxide rather than a simple carbonate alone. Malachite has composition

$$CuCO_3 \cdot Cu(OH)_2.$$

Reversing the order of the two formula fragments gives the option

$$Cu(OH)_2 \cdot CuCO_3.$$

The dot does not change the composition or introduce water; the complete formula contains two copper atoms, one carbonate group and two hydroxide groups.

Step 2: Distinguish the other copper ores Option (A), Cu_2S, is copper glance (chalcocite), a sulphide mineral rather than a basic carbonate. Option (B), Cu_2O, is cuprite, an oxide mineral. Option (D), $CuFeS_2$, is copper pyrites (chalcopyrite), which contains iron and sulphur. None contains the combination of carbonate and hydroxide required for malachite. Only option (C) contains both groups in the correct ratio. As a charge check, two copper(II) ions give total charge $+4$, balanced by one carbonate ion (-2) and two hydroxide ions (-2). This verifies that the combined expression is electrically neutral and consistent with a copper(II) basic carbonate.

Step 3: Conclusion Malachite is $Cu(OH)_2 \cdot CuCO_3$, the basic carbonate of copper. Hence, the correct option is (C).

Quick Tip to Solve: Associate malachite with both carbonate and hydroxide; use the anion type to distinguish it quickly from copper sulphide and oxide ores.

49. An octahedral complex with molecular composition $M.5NH_3.Cl.SO_4$ has two isomers, A and B. The solution of A gives a white precipitate with $AgNO_3$ solution and the solution of B gives white precipitate with $BaCl_2$ solution .The type of isomerism exhibited by the complex is [4 marks]

(A) Coordinate isomerism

(B) Geometrical isomerism

(C) Ionisation isomerism

(D) Linkage isomerism

Correct Answer: (C) Ionisation isomerism

Solution:

Concept:

- Ionisation isomers differ by exchange of a coordinated anion with an anion outside the coordination sphere. Dissolution therefore produces different free ions.
- A free chloride ion forms white $AgCl$ with silver nitrate, while a free sulphate ion forms white $BaSO_4$ with barium chloride.

Step 1: Locate the ions detected in each solution The white precipitate from solution A with silver nitrate identifies chloride outside the coordination sphere:

$$Ag^+ + Cl^- \longrightarrow AgCl(s).$$

The white precipitate from solution B with barium chloride identifies sulphate outside the coordination sphere:

$$Ba^{2+} + {SO_4}^{2-} \longrightarrow BaSO_4(s).$$

The tests distinguish free counter-ions rather than merely the total elemental composition.

Step 2: Write the exchanged coordination arrangements Five ammonia ligands occupy five of the six octahedral positions. The remaining position can be occupied by sulphate or chloride, giving the arrangements

$$A: \ [M(NH_3)_5(SO_4)]Cl, \qquad B: \ [M(NH_3)_5Cl]SO_4.$$

They have the same overall composition but release different counter-ions in water. This is ionisation isomerism. Coordination isomerism involves redistribution of ligands between complex cations and complex anions, which are not both present here. Geometrical isomerism changes relative ligand positions without exchanging a free counter-ion. Linkage isomerism changes the donor atom of an ambidentate ligand; it does not explain the chloride/sulphate counter-ion exchange observed in these tests.

Step 3: Conclusion The complexes exchange chloride and sulphate between the coordination sphere and the counter-ion position, showing ionisation isomerism. Hence, the correct option is (C).

Quick Tip to Solve: Use precipitate tests to identify the free counter-ion first; different ions released by otherwise compositionally identical complexes signal ionisation isomerism.

50. Nickel (Z=28) combines with a uninegative monodentate ligand to form a diamagnetic complex $[NiL_4]^{2-}$. The hybridisation involved and the number of unpaired electrons present in the complex are respectively [4 marks]

(A) dsp^2, zero

(B) sp^3, zero

(C) dsp^2, one

(D) sp^3, two

Correct Answer: (A) dsp^2, zero

Solution:

Concept:

- In a complex, the metal oxidation state follows from the overall charge and the charges on the ligands. A uninegative ligand has charge -1.
- Nickel has atomic number 28 and ground-state configuration $[\mathrm{Ar}]3d^8 4s^2$. Diamagnetism means that every electron in the complex is paired.

Step 1: Find the metal ion and its configuration Let the oxidation state of nickel be x. Four ligands contribute a total charge of -4, while the complex has charge -2. Therefore

$$x - 4 = -2 \quad \Rightarrow \quad x = +2.$$

The ion is thus Ni^{2+}, with configuration $[\mathrm{Ar}]3d^8$; the two electrons are removed from the $4s$ orbital first.

Step 2: Use the stated magnetic behaviour A tetrahedral sp^3 arrangement of a d^8 nickel(II) ion retains two unpaired electrons. It would therefore be paramagnetic, contrary to the condition in the question. For the stated diamagnetic four-coordinate complex, the $3d$ electrons pair in four orbitals, leaving one $3d$ orbital available. This orbital mixes with one $4s$ and two $4p$ orbitals to form four dsp^2 hybrid orbitals in a square plane.

Step 3: Compare the alternatives The square-planar arrangement has no unpaired electrons. Option (B) incorrectly combines tetrahedral hybridisation with zero unpaired electrons. Option (C) contradicts diamagnetism directly. Option (D) describes the paramagnetic tetrahedral possibility, rather than the diamagnetic complex specified here.

Step 4: Conclusion The hybridisation is dsp^2, and the number of unpaired electrons is zero. Hence, the correct option is (A).

Quick Tip to Solve: For a four-coordinate nickel(II) complex, the word diamagnetic distinguishes square-planar dsp^2 from tetrahedral sp^3.

51. Amongst the following, identify the species with an atom in +6 oxidation state [4 marks]

(A) CrO_2Cl_2

(B) $[Cr(CN)_6]^{3-}$

(C) Cr_2O_3

(D) $[MnO_4]^-$

Correct Answer: (A) CrO_2Cl_2

Solution:

Concept:

- The sum of oxidation states equals the total charge of a molecule or ion. Oxygen normally has oxidation state -2, and chlorine in the given oxychloride has oxidation state -1.
- A cyanide ligand, CN^-, has total charge -1. Use the ligand charge without separately assigning carbon and nitrogen oxidation states.

Step 1: Calculate the chromium states In neutral chromyl chloride, CrO_2Cl_2, let the chromium oxidation state be x. Then

$$x + 2(-2) + 2(-1) = 0 \quad \Rightarrow \quad x = +6.$$

For $[Cr(CN)_6]^{3-}$, the six cyanide ligands give -6, so $x-6 = -3$, giving $x = +3$. In Cr_2O_3, $2x+3(-2) = 0$, again giving chromium in the $+3$ state.

Step 2: Check the manganese alternative For permanganate, $MnO_4{}^-$, charge balance gives

$$x + 4(-2) = -1 \quad \Rightarrow \quad x = +7.$$

The charge on the entire ion is not the oxidation state of manganese: the four oxygens must be included. Thus this alternative contains manganese in the $+7$ state and oxygen in the -2 state, with no atom in the requested $+6$ state.

Step 3: Select the matching species The central-metal oxidation states in options (A), (B), (C) and (D), respectively, are $+6, +3, +3, +7$. Only the first species satisfies the condition. This comparison also shows why merely recognizing a strongly oxidizing transition-metal compound is insufficient; the exact charge balance decides the answer.

Step 4: Conclusion Chromium is in oxidation state $+6$ in CrO_2Cl_2. Hence, the correct option is (A).

Quick Tip to Solve: Write charge balance once for each formula: neutral oxychloride gives zero, whereas a complex ion must equal its displayed charge.

52. The major product formed when 1, 1, 1- trichloropropane is treated with aqueous potassium hydroxide is [4 marks]

(A) 2 - Propanol

(B) Propyne

(C) Propionic acid

(D) 1 - Propanol

Correct Answer: (C) Propionic acid

Solution:

Concept:

- Aqueous potassium hydroxide supplies hydroxide ions for hydrolysis of carbon–halogen bonds. Three halogens attached to the same terminal carbon lead to a carboxyl group.
- The substrate is $CH_3CH_2CCl_3$. Its three-carbon skeleton is retained during hydrolysis; the terminal trichloromethyl carbon becomes the carboxyl carbon.

Step 1: Identify the site of hydrolysis In 1,1,1-trichloropropane, all three chlorine atoms are on carbon 1. Replacement by hydroxyl groups gives the formal intermediate $CH_3CH_2C(OH)_3$. This geminal triol is unstable and loses water to produce the carboxyl group. A useful overall representation of the hydrolysis is

$$CH_3CH_2CCl_3 + 3\,KOH \longrightarrow CH_3CH_2COOH + 3\,KCl + H_2O.$$

Step 2: Identify the organic product The acid corresponding to CH_3CH_2COOH is propionic acid, also named propanoic acid. In an excess of alkaline reagent it is present as potassium propionate, since the acid is neutralized by hydroxide. The carboxylic-acid product class represented in the options is therefore propionic acid; acidification gives the free acid. The hydrolysis neither removes nor adds a carbon atom.

Step 3: Eliminate the other product classes Both 1-propanol and 2-propanol contain only one hydroxyl oxygen and are not the product of hydrolysing a terminal carbon bearing three chlorines. Propyne is an unsaturated hydrocarbon; its formation would require an elimination pathway, whereas the reagent here is aqueous alkali and the terminal trihalide hydrolyses to a carboxyl derivative. Thus none of these three alternatives represents the stated transformation.

Step 4: Conclusion The hydrolysis gives the propionic-acid product, CH_3CH_2COOH (propionate in alkaline medium). Hence, the correct option is (C).

Quick Tip to Solve: Count halogens on the same carbon: a terminal $-CCl_3$ group hydrolyses to $-COOH$, while keeping the carbon chain intact.

53. Which of the following compounds will not be soluble in sodium bicarbonate? [4 marks]

(A) 2, 4, 6 - Trinitrophenol

(B) Benzene sulphonic acid

(C) o - Nitrophenol

(D) Benzoic acid

Correct Answer: (C) o - Nitrophenol

Solution:

Concept:

- Sodium bicarbonate converts acids stronger than carbonic acid into water-soluble sodium salts, with evolution of carbon dioxide. Weak phenols normally do not undergo this reaction.
- Electron-withdrawing nitro groups stabilize a phenoxide ion and increase phenol acidity. The number of nitro groups matters: picric acid is much stronger than a mononitrophenol.

Step 1: Apply the bicarbonate test The acid-base reaction can be expressed as

$$\mathrm{HA} + \mathrm{NaHCO_3} \longrightarrow \mathrm{NaA} + \mathrm{CO_2} + \mathrm{H_2O}.$$

It proceeds for the sufficiently strong acids in this list and creates ionic salts that dissolve in the aqueous medium. Benzoic acid is a carboxylic acid and reacts to form sodium benzoate. Benzene sulphonic acid is a strong organic acid and similarly forms its sodium sulphonate.

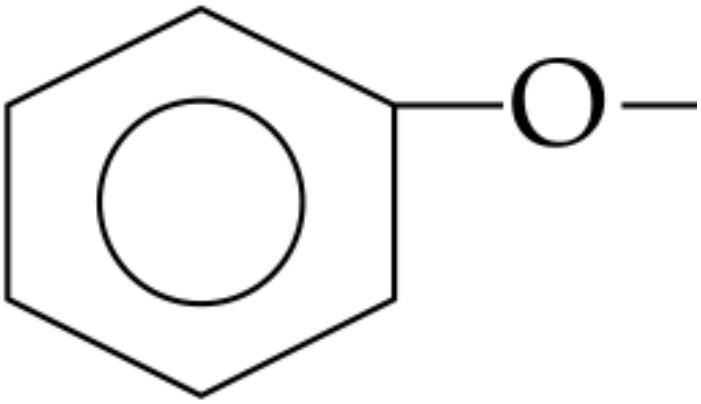
Step 2: Compare the two phenols 2,4,6-Trinitrophenol, or picric acid, has three strongly electron-withdrawing nitro groups. They stabilize its conjugate base by inductive and resonance effects, making it sufficiently acidic to react with bicarbonate. In contrast, *o*-nitrophenol has just one nitro group. Although more acidic than phenol, it is still weaker than carbonic acid in the bicarbonate test and is not appreciably converted to its sodium salt by this reagent.

Step 3: Choose the compound that remains undissolved Options (A), (B) and (D) acquire ionic character through salt formation and are soluble in sodium bicarbonate solution. Option (C) does not form the salt under these conditions and therefore is the compound described as insoluble in this qualitative test. Do not confuse bicarbonate with sodium hydroxide: the stronger hydroxide base can dissolve ordinary phenols as phenoxides.

Step 4: Conclusion *o*-Nitrophenol does not dissolve by reaction with sodium bicarbonate. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish the bases: bicarbonate tests sufficiently strong acids, while sodium hydroxide also forms salts of much weaker phenols.

54. Which one of the following substituents at para-position is most effective in stabilizing the phenoxide ion ? [4 marks]



(A) -CH_3

(B) -OCH_3

(C) -$COCH_3$

(D) -CH_2OH

Correct Answer: (C) -$COCH_3$

Solution:

Concept:

- The negative charge of a phenoxide ion is delocalised from oxygen into the aromatic ring. Its resonance contributors place extra electron density at the ortho and para positions.
- A substituent at the para position stabilises phenoxide most effectively when it withdraws electron density by resonance. Direct conjugation matters: an intervening saturated carbon interrupts the resonance pathway.

Step 1: Identify the conjugated electron-withdrawing group The acetyl substituent, $-COCH_3$, attaches to the ring through its carbonyl carbon. The carbonyl is an electron acceptor by the resonance effect. At the para position it can accept the additional ring electron density of phenoxide, allowing charge delocalisation towards the carbonyl oxygen. This reduces the concentration of negative charge and stabilises the anion. It is the only strongly resonance-withdrawing substituent among the listed choices.

Step 2: Compare the other three substituents The methyl group, $-CH_3$, donates electron density through its inductive effect and hyperconjugation, so it does not provide the required electron sink. Methoxy, $-OCH_3$, has an inductive withdrawing effect but donates strongly to the aromatic ring through oxygen lone-pair resonance at para; this does not stabilise the phenoxide charge as acetyl does. In $-CH_2OH$, the saturated methylene carbon separates the oxygen from the ring. Consequently, its oxygen cannot withdraw the ring charge by direct conjugation. Comparing the actual ring attachment and resonance effects therefore selects acetyl over all three alternatives.

Step 3: Conclusion The para substituent $-COCH_3$ gives the strongest stabilisation of phenoxide among these options. Hence, the correct option is (C).

Quick Tip to Solve: For para-substituted phenoxide, look for a directly conjugated electron acceptor; a carbonyl attached through its carbonyl carbon can receive the extra charge.

55. Williamson synthesis of ether is an example of [4 marks]

(A) Nucleophilic substitution

(B) Electrophilic addition

(C) Nucleophilic addition

(D) Electrophilic substitution

Correct Answer: (A) Nucleophilic substitution

Solution:

Concept:

- Williamson ether synthesis forms an ether by reacting an alkoxide ion with a suitable alkyl halide. The alkoxide is an electron-pair donor and therefore a nucleophile.
- The usual mechanism is bimolecular nucleophilic substitution, S_N2: carbon–oxygen bond formation accompanies displacement of a leaving group.

Step 1: Identify the reacting nucleophile An alkoxide, written RO^-, has a negatively charged oxygen with lone pairs. It attacks the electrophilic carbon bonded to the halogen in $R'X$. The general conversion is

$$RO^- + R'X \longrightarrow ROR' + X^-.$$

For example, sodium ethoxide and methyl bromide form methoxyethane and sodium bromide. The oxygen-containing ion supplies the new group attached to carbon.

Step 2: Classify the bond changes The attacked carbon loses its bond to the leaving halide and gains a bond to oxygen. The halide leaving group is replaced by alkoxide oxygen, so this is substitution rather than addition. The attacking species is the electron-rich alkoxide rather than an electron-deficient electrophile; hence the substitution is nucleophilic. With a methyl or primary alkyl halide the backside attack occurs in a single S_N2 step, without a carbocation intermediate.

Step 3: Exclude the other descriptions Electrophilic addition involves an electrophile adding across a multiple bond, which is not the reaction shown. Nucleophilic addition adds a nucleophile without displacement, commonly at a carbonyl carbon, and is also inapplicable. Electrophilic substitution replaces a group through attack by an electrophile, as in aromatic substitution. Here the leaving halide and the alkoxide attacker identify nucleophilic substitution unambiguously.

Step 4: Conclusion Williamson ether synthesis is a nucleophilic substitution reaction. Hence, the correct option is (A).

Quick Tip to Solve: Track the leaving group: if alkoxide replaces halide at an alkyl carbon, classify the ether formation as nucleophilic substitution.

56. The final product formed when methylamine is treated with $NaNO_2$ and HCl followed by hydrolysis is [4 marks]

(A) Nitromethane

(B) Methylcyanide

(C) Methyl alcohol

(D) Diazomethane

Correct Answer: (C) Methyl alcohol

Solution:

Concept:

- Sodium nitrite and hydrochloric acid generate nitrous acid in situ: $NaNO_2 + HCl \longrightarrow HNO_2 + NaCl$. Primary aliphatic amines react with nitrous acid with evolution of nitrogen.
- An aliphatic diazonium intermediate is unstable in water. Its decomposition replaces the amino group by a hydroxyl group, producing the corresponding alcohol.

Step 1: Identify the amine and reagent Methylamine, CH_3NH_2, is a primary aliphatic amine. The combination of $NaNO_2$ and HCl provides HNO_2, so the relevant reaction is deamination by nitrous acid. The overall transformation in water is

$$CH_3NH_2 + HNO_2 \longrightarrow CH_3OH + N_2 + H_2O.$$

This equation conserves one carbon atom and both nitrogen atoms and makes the gaseous nitrogen product explicit.

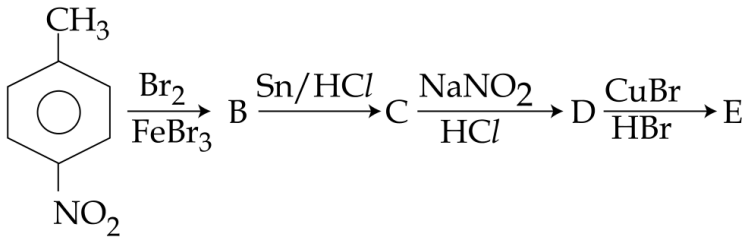
Step 2: Follow the unstable intermediate Initial nitrosation and diazotization produce a short-lived methyl diazonium species. Unlike an aromatic diazonium salt, a primary aliphatic diazonium species does not remain as a stable isolable salt under these conditions. Nitrogen is expelled, and reaction with water gives methanol. The question asks for the final hydrolysis product, so the intermediate is not the required answer.

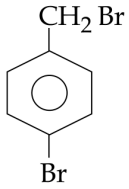
Step 3: Check the alternatives against the carbon skeleton Methyl alcohol is methanol, CH_3OH, and matches replacement of the amino group by hydroxyl. Nitromethane would require conversion to a nitro group rather than loss of nitrogen. Methylcyanide contains an additional carbon atom not supplied by these reagents. Diazomethane is not the final product of this aqueous nitrous-acid reaction and would not represent the stated hydrolysis. These distinctions select the alcohol uniquely.

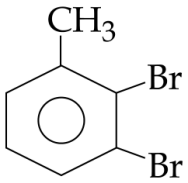
Step 4: Conclusion The final product is methyl alcohol (methanol), CH_3OH. Hence, the correct option is (C).

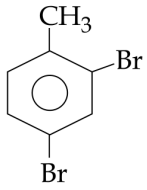
Quick Tip to Solve: For a primary aliphatic amine with nitrous acid, replace $-NH_2$ by $-OH$ and account for the evolved N_2.

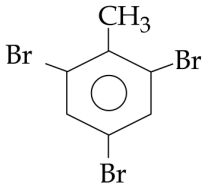
57. In a set of reactions p - nitrotoluene yielded a product E The product E would be [4 marks]

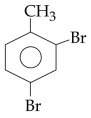


(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- In electrophilic aromatic bromination, methyl is an ortho/para director, while nitro is a meta director. On para-nitrotoluene these directives both favour positions 2 and 6 relative to methyl.
- Sn/HCl reduces an aromatic nitro group to an amino group. Diazotisation with $NaNO_2/HCl$ forms an arenediazonium salt; $CuBr/HBr$ then replaces the diazonium group by bromine in the Sandmeyer reaction.

Step 1: Locate the first bromine substituent Number the starting ring with methyl at carbon 1 and nitro at carbon 4. Positions 2 and 6 are ortho to methyl and meta to nitro; they are equivalent by symmetry. Bromination with $Br_2/FeBr_3$ therefore gives 2-bromo-4-nitrotoluene as B. The catalyst indicates ring bromination, not a free-radical substitution on the methyl side chain. The methyl group remains CH_3.

Step 2: Follow reduction, diazotisation and replacement Reduction converts B into 2-bromo-4-aminotoluene, C, without changing the carbon skeleton or the existing ring bromine. Treatment with nitrous acid generated from $NaNO_2/HCl$ changes the amino group at carbon 4 into the diazonium group, D. Copper(I) bromide in hydrobromic acid replaces this group by Br, releasing nitrogen. Thus E has methyl at 1 and bromines at 2 and 4: it is 2,4-dibromotoluene. Option (A), with a brominated side chain, is inconsistent with the initial electrophilic conditions. Option (B) puts the second bromine at the wrong position. Option (D) adds a third bromine although the sequence only installs two; none of the later reagents adds an extra ring bromine elsewhere.

Step 3: Conclusion Product E is 2,4-dibromotoluene, shown in option (C). Hence, the correct option

is (C).

Quick Tip to Solve: Keep the original ring numbering throughout a multistep sequence: reduction and diazotisation change the group at carbon 4, and Sandmeyer replaces that group in place.

58. Which one of the following is an example of thermosetting polymer? [4 marks]

(A) Nylon -6,6

(B) Bakelite

(C) Buna - N

(D) Teflon

Correct Answer: (B) Bakelite

Solution:

Concept:

- Thermosetting polymers form extensive covalent cross-links during curing. Their three-dimensional network cannot be melted and reshaped repeatedly by heating.
- The distinction is based on molecular architecture and behaviour on heating, rather than merely on whether a polymer was formed by addition or condensation.

Step 1: Identify the cross-linked resin Bakelite is a phenol-formaldehyde resin. Phenol reacts with formaldehyde initially to form products containing links between aromatic rings; further heating and curing produce an extensively cross-linked network. The covalent bridges restrict movement of the chains, making the final material hard and rigid. Heating does not allow the chains to slide past one another as they would in a linear thermoplastic. Consequently Bakelite retains its moulded shape and cannot be repeatedly softened and remoulded.

Step 2: Compare every other candidate Nylon-6,6 is a polyamide whose chains are predominantly linear and are held together by intermolecular forces, including hydrogen bonds; it is classified as a thermoplastic. Teflon is polytetrafluoroethylene, another thermoplastic, with the repeat unit $-CF_2-CF_2-$. Buna-N is a nitrile rubber made from butadiene and acrylonitrile and is classified as an elastomer. Although rubbers can be cross-linked during processing, Buna-N is not the thermosetting resin sought in this standard classification. Thus none of these three replaces Bakelite as the listed thermosetting polymer.

Step 3: Conclusion Bakelite is the thermosetting polymer because curing gives a permanent cross-linked network. Hence, the correct option is (B).

Quick Tip to Solve: For this list, identify the phenol-formaldehyde network: Bakelite is thermosetting, nylon and Teflon are thermoplastics, and Buna-N is an elastomer.

59. Among the following organic acids, the acid present in rancid butter is [4 marks]

(A) Pyruvic acid

(B) Acetic acid

(C) Butyric acid

(D) Lactic acid

Correct Answer: (C) Butyric acid

Solution:

Concept:

- Rancidity is deterioration of fats that creates substances with unpleasant odours. Hydrolysis of butter fat releases short-chain carboxylic acids.
- Butyric acid is the common name of butanoic acid, a four-carbon monocarboxylic acid with formula $CH_3CH_2CH_2COOH$.

Step 1: Identify the characteristic acid Butter contains triglycerides, which are esters of glycerol with fatty acids. When the ester links undergo hydrolysis, free fatty acids are released. Butyric acid is characteristic of butter fat and has a strongly unpleasant smell when liberated, so it is associated with rancid butter. Its structure is $CH_3CH_2CH_2COOH$: the carboxyl carbon is included in the four-carbon chain. The common name butyric acid and the systematic name butanoic acid refer to the same compound, so either name identifies the required acid.

Step 2: Distinguish the alternatives Acetic acid, CH_3COOH, is the two-carbon acid familiar as the main acid in vinegar; that association does not make it the characteristic acid of rancid butter. Lactic acid, $CH_3CH(OH)COOH$, is a hydroxy acid associated with lactic fermentation and sour milk. Pyruvic acid, $CH_3COCOOH$, is a three-carbon keto acid involved in metabolism. Their functional groups and customary associations differ from those of butyric acid. Rancidity can involve a mixture of degradation products, but the question asks for the characteristic organic acid among these four names, which singles out butyric acid.

Step 3: Conclusion The acid characteristic of rancid butter is butyric acid (butanoic acid), $CH_3CH_2CH_2COOH$. Hence, the correct option is (C).

Quick Tip to Solve: Link rancid butter with butyric acid; keep vinegar with acetic acid and sour milk with lactic acid to avoid mixing the common names.

60. The reason for double helical structure of DNA is the operation of [4 marks]

(A) Van der Waals forces

(B) Electrostatic attractions

(C) Hydrogen bonding

(D) Dipole - Dipole interactions

Correct Answer: (C) Hydrogen bonding

Solution:

Concept:

- DNA contains two complementary polynucleotide strands. Hydrogen bonds form between the nitrogenous bases on opposite strands and maintain specific base pairing.
- Adenine pairs with thymine through two hydrogen bonds, while guanine pairs with cytosine through three; the sugar-phosphate backbones lie toward the outside.

Step 1: Locate the interstrand interaction The bases carry suitable hydrogen-bond donor and acceptor groups. Their complementary arrangement allows adenine to pair with thymine and guanine to pair with cytosine. These hydrogen bonds join the two strands without requiring covalent bonds between each pair of bases. Repetition of the complementary pairs along the strands helps maintain their ordered double-helical arrangement. Within each strand the nucleotides are connected by covalent phosphodiester bonds, but those backbone bonds alone do not identify the interaction asked for between the paired strands.

Step 2: Evaluate the listed forces Van der Waals forces and base-stacking interactions do contribute to the overall stability of DNA; it would be inaccurate to say that hydrogen bonds are its only stabilising interactions. Nevertheless, van der Waals forces do not by themselves express the specific complementary pairing of the two strands tested here. General electrostatic attraction is also unsuitable: both phosphate backbones carry negative charges and require screening rather than simple mutual attraction. Ordinary dipole-dipole interactions are too general a description of the directional donor-acceptor contacts between complementary bases. Of the listed alternatives, hydrogen bonding is the standard specific explanation.

Step 3: Conclusion Complementary bases on the two DNA strands are linked by hydrogen bonds, supporting the double-helical structure. Hence, the correct option is (C).

Quick Tip to Solve: Remember A–T has two hydrogen bonds and G–C has three. Phosphodiester bonds join nucleotides along a strand rather than pairing the strands. Use the paired bases to identify the interstrand link.

Mathematics (120 marks)

61. The equation $\sqrt{3x^2+x+5} = x-3$, where x is real, has [4 marks]

(A) no solution

(B) exactly four solutions

(C) exactly one solution

(D) exactly two solutions

Correct Answer: (A) no solution

Solution:

Concept:

- The principal square root is nonnegative. Thus $\sqrt{3x^2+x+5} = x-3$ can hold only if $x \geq 3$.
- Squaring a radical equation produces a necessary algebraic condition, but any roots obtained must still satisfy the original equation and its sign restrictions.

Step 1: Use the nonnegative square root The radicand is positive for every real x, since

$$3x^2+x+5 = 3\left(x+\frac{1}{6}\right)^2 + \frac{59}{12} > 0.$$

The left-hand side is therefore strictly positive, which requires $x-3>0$, or $x>3$. In particular, any putative solution must lie on the positive side beyond 3; values below this bound cannot satisfy the equation even before any calculation is performed.

Step 2: Square and test the resulting quadratic Squaring both sides gives

$$3x^2+x+5 = (x-3)^2 = x^2-6x+9.$$

Hence

$$2x^2+7x-4=0, \qquad (2x-1)(x+4)=0.$$

The only candidates are $x=\frac{1}{2}$ and $x=-4$. Neither satisfies $x>3$; their right-hand sides are negative, while the square root is positive. In fact substitution gives a positive left side and its negative on the right in each case. Thus both are extraneous roots introduced by squaring. The algebra supplies no other candidates, so the original equation has no real solution. In particular, the possibilities of exactly one, exactly two, or exactly four solutions are all ruled out.

Step 3: Conclusion There is no real solution to the given equation. Hence, the correct option is (A).

Quick Tip to Solve: Before squaring, write the sign condition imposed by the principal square root; here it rejects both quadratic roots immediately.

62. For all complex numbers z of the form 1 + iα, $\alpha \in$ R, if $z^2 = x + iy$, then [4 marks]

(A) $y^2 - 4x + 4 = 0$

(B) $y^2 + 4x - 4 = 0$

(C) $y^2 - 4x + 2 = 0$

(D) $y^2 + 4x + 2 = 0$

Correct Answer: (B) $y^2 + 4x - 4 = 0$

Solution:

Concept:

- For $z = 1 + i\alpha$, with real α, expand z^2 using $i^2 = -1$, then equate real and imaginary parts.
- The locus in the (x, y)-plane follows by eliminating the real parameter α. Check the constant term and the sign of x carefully.

Step 1: Separate real and imaginary components Expand the square without replacing i^2 by $+1$:

$$z^2 = (1 + i\alpha)^2 = 1 + 2i\alpha - \alpha^2.$$

Since $z^2 = x + iy$, equality of complex numbers yields

$$x = 1 - \alpha^2, \qquad y = 2\alpha.$$

Both x and y are real because α is real. Notice that $x \leq 1$, and $\alpha = 0$ gives the point $(1, 0)$. These facts provide independent checks on the sign and constant in the eventual equation.

Step 2: Eliminate the parameter and compare options From $y = 2\alpha$, obtain $\alpha = y/2$. Substituting into the expression for x,

$$x = 1 - \frac{y^2}{4} \quad \Longrightarrow \quad y^2 + 4x - 4 = 0.$$

Conversely, any real point satisfying this equation arises from the real value $\alpha = y/2$, so the elimination does not add spurious points. Option (A) has the wrong sign of x: taking $\alpha = 1$ gives $(x, y) = (0, 2)$, which fails that expression. Options (C) and (D) have incorrect constants, already exposed by $(1, 0)$. Option (B) alone has the correct sign and intercept and holds for every real parameter value.

Step 3: Conclusion The required relation is $y^2 + 4x - 4 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: After eliminating the parameter, test $\alpha = 0$, which gives $(1, 0)$, and $\alpha = 1$, which gives $(0, 2)$, to catch sign errors. The vertex check is especially quick for the constant term.

63. Two women and some men participated in a chess tournament in which every participant played two games with each of the other participants. If the number of games that the men played between themselves exceeds the number of games that the men played with the women by 66, then the number of men who participated in the tournament lies in the interval [4 marks]

(A) (11, 13]

(B) (14, 17)

(C) [10, 12)

(D) [8, 9]

Correct Answer: (C) [10, 12)

Solution:

Concept:

- If each pair of distinct participants plays two games, count unordered pairs first and multiply by two; this avoids counting the same game twice.
- With m men and two women, there are $\binom{m}{2}$ male-male pairs and $2m$ male-female pairs. The question compares the numbers of games, not merely pairs.

Step 1: Count the two types of games The number of pairs consisting of two men is $\binom{m}{2} = m(m-1)/2$. As each pair plays twice, male-male games number

$$G_{MM} = 2\binom{m}{2} = m(m-1).$$

Each of the m men plays each of the two women twice, so male-female games number

$$G_{MW} = m \cdot 2 \cdot 2 = 4m.$$

The two women also play each other, but those games belong to neither category being compared and must not enter the stated difference.

Step 2: Solve the counting equation and locate the interval The male-male total exceeds the male-female total by 66:

$$m(m-1) - 4m = 66.$$

Rearranging and factoring,

$$m^2 - 5m - 66 = 0, \qquad (m-11)(m+6) = 0.$$

Thus $m = 11$ or $m = -6$. A number of participants cannot be negative, leaving $m = 11$. Checking gives $G_{MM} = 110$ and $G_{MW} = 44$, whose difference is indeed 66. The interval in option (A), $(11, 13]$, excludes its lower endpoint 11. Option (B), $(14, 17)$, and option (D), $[8, 9]$, do not contain 11 either. The half-open interval $[10, 12)$ includes all real values at least 10 and less than 12, so it contains 11. The integer count therefore belongs to option (C).

Step 3: Conclusion There are 11 men, and $11 \in [10, 12)$. Hence, the correct option is (C).

Quick Tip to Solve: Count each male-male unordered pair twice, but count each man-woman pair only once before applying the two-games factor; then check open interval endpoints.

64. Let $f(n) = \left[\frac{1}{3} + \frac{3n}{100}\right] n$, where [n] denotes the greatest integer less than or equal to n. Then $\sum_{n=1}^{56} f(n)$ is equal to [4 marks]

(A) 56

(B) 1287

(C) 1399

(D) 689

Correct Answer: (C) 1399

Solution:

Concept:

- The greatest integer function $\lfloor x \rfloor$ is constant between successive integers. First locate the values of n at which $1/3 + 3n/100$ crosses an integer.
- For consecutive integers, $\sum_{n=p}^{q} n = (q - p + 1)(p + q)/2$. The factor outside the square brackets is n, so it must remain in every summand.

Step 1: Find the constant-value intervals For $1 \leq n \leq 56$, the bracketed argument lies between $109/300$ and $151/75$. Its first crossing is determined by $1/3 + 3n/100 \geq 1$, giving $n \geq 200/9$; therefore the first integer is 23. Its second crossing satisfies $n \geq 500/9$, so it occurs at 56. Thus the floor is zero for 1 through 22, one for 23 through 55, and two at 56. In particular, the endpoint 56 must not be placed in the middle interval.

Step 2: Sum the three contributions The first interval contributes nothing. In the second interval $f(n) = n$, whereas $f(56) = 112$. Consequently,

$$\sum_{n=1}^{56} f(n) = \sum_{n=23}^{55} n + 112 = \frac{33(23 + 55)}{2} + 112 = 1287 + 112 = 1399.$$

Option (B) is only the middle contribution and misses the last term. Option (A) does not account for the weighted sum, and option (D) also disagrees with the interval calculation.

Step 3: Conclusion The required sum is 1399. Hence, the correct option is (C).

Quick Tip to Solve: Convert each floor threshold into an inequality for the integer index; check the last index separately when it sits just beyond a threshold.

65. The number of terms in an A.P. is even; the sum of the odd terms in it is 24 and that the even terms is 30. If the last term exceeds the first term by $10\frac{1}{2}$, then the number of terms in the A.P. is [4 marks]

(A) 4

(B) 8

(C) 16

(D) 12

Correct Answer: (B) 8

Solution:

Concept:

- An arithmetic progression has terms $a, a+d, \ldots$. With $2m$ terms, there are exactly m terms in odd positions and m in even positions.
- Each even-positioned term exceeds the preceding odd-positioned term by d. The difference of the two positional sums is therefore md, while the last-minus-first difference is $(2m-1)d$.

Step 1: Use the positional sums Write the pairs as $(a, a+d), (a+2d, a+3d), \ldots$. Subtracting the odd-position sum from the even-position sum gives

$$md = 30 - 24 = 6, \qquad d = \frac{6}{m}.$$

This uses the difference within each pair and does not require us to find the first term. Since the difference of the sums is positive and $m > 0$, the common difference is positive.

Step 2: Apply the endpoint condition The given mixed number $10\frac{1}{2}$ equals $21/2$. There are $2m-1$ steps from the first term to the last, so

$$(2m-1)\frac{6}{m} = \frac{21}{2}.$$

Multiplication by $2m$ yields $24m - 12 = 21m$, hence $m = 4$ and the full number of terms is $2m = 8$. As a check, $d = 3/2$ and seven steps give $7(3/2) = 21/2$, exactly the required excess. For the other proposed totals 4, 16 and 12, m would be 2, 8 and 6; the formula $12 - 6/m$ gives 9, $45/4$ and 11, respectively, rather than $21/2$.

Step 3: Conclusion The A.P. contains 8 terms. Hence, the correct option is (B).

Quick Tip to Solve: For an even-length A.P., subtract the odd-position sum from the even-position sum to obtain half the number of terms times the common difference.

66. The coefficient of x^{1012} in the expansion of $(1+x^n+x^{253})^{10}$, (where n $\leq$ 22 is any positive integer), is [4 marks]

(A) $^{253}C_4$

(B) $^{10}C_4$

(C) 4n

(D) 1

Correct Answer: (B) $^{10}C_4$

Solution:

Concept:

- Choose one term from each of the ten factors. If k factors supply x^{253}, l supply x^n and the rest supply 1, the resulting power is $253k+nl$.
- The selection numbers are nonnegative integers with $k+l \leq 10$. These integer constraints, together with $1 \leq n \leq 22$, can rule out all but one kind of selection.

Step 1: Determine the possible selections To obtain the requested power we need

$$253k + nl = 1012 = 4 \cdot 253.$$

If $k \geq 5$, the power already exceeds 1012 before any x^n is chosen. If $k \leq 3$, the deficit is at least 253, but even all ten factors of the small power contribute at most $10n \leq 220$. Thus the small-power selections cannot fill that deficit. These two bounds force $k = 4$. The exponent equation then gives $nl = 0$, so $l = 0$ because n is positive.

Step 2: Count and check the coefficient Exactly four factors must supply x^{253}, and the remaining six must supply 1. We can choose those four positions in

$$\binom{10}{4} = \frac{10!}{4!6!} = 210$$

ways. Each selection has coefficient one, so their total coefficient is 210. The exponent 253 is not the number of factors, so $\binom{253}{4}$ does not count the choices. The expression $4n$ is unrelated to their number, and 1 would count only one arrangement rather than all four-position choices. The coefficient is independent of the permitted value of n.

Step 3: Conclusion The coefficient is $\binom{10}{4} = 210$. Hence, the correct option is (B).

Quick Tip to Solve: Before expanding a trinomial, bound the exponent contributed by its smaller-power term; a large gap may force a unique selection pattern.

67. If a line L is perpendicular to the line 5x −y = 1, and the area of the triangle formed by the line L and the coordinate axes is 5 sq units, then the distance of the line L from the line x + 5y = 0 is [4 marks]

(A) $7/\sqrt{5}$ units

(B) $7/\sqrt{13}$ units

(C) $5/\sqrt{13}$ units

(D) $5/\sqrt{7}$ units

Correct Answer: (C) $5/\sqrt{13}$ units

Solution:

Concept:

- Perpendicular nonvertical lines have slopes whose product is -1. A line perpendicular to $5x - y = 1$ therefore has slope $-1/5$.
- A line with axis intercepts p, q forms a triangle of area $|pq|/2$. Parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ are separated by $|C_1 - C_2|/\sqrt{A^2 + B^2}$.

Step 1: Find the constant in the line equation The given line is $y = 5x - 1$. Hence write the required line in the form $x + 5y = c$, with $c \neq 0$ because its intercept triangle has positive area. Putting $y = 0$ gives its x-intercept c, and putting $x = 0$ gives its y-intercept $c/5$. The intercepts have the same sign, but the area is positive in either case. Thus

$$\frac{1}{2}\left|c \cdot \frac{c}{5}\right| = 5, \qquad c^2 = 50.$$

Either sign of c gives the same distance from the parallel line through the origin.

Step 2: Calculate the separation The required comparison line is $x + 5y = 0$, which has the same normal vector as L. Their perpendicular separation is therefore

$$D = \frac{|c|}{\sqrt{1^2 + 5^2}} = \frac{\sqrt{50}}{\sqrt{26}} = \sqrt{\frac{25}{13}} = \frac{5}{\sqrt{13}} \text{ units.}$$

This normalization by the normal-vector length is essential: the intercept magnitude by itself is not the distance. Of the listed expressions, only option (C) has both the numerator 5 and denominator $\sqrt{13}$; the other three do not equal the derived value.

Step 3: Conclusion The distance is $5/\sqrt{13}$ units. Hence, the correct option is (C).

Quick Tip to Solve: Write a perpendicular line with the same normal as the parallel comparison line; use the intercept area to determine only the magnitude of its constant.

68. The circumcentre of a triangle lies at the origin and its centroid is the midpoint of the line segment joining the points $(a^2 + 1, a^2 + 1)$ and $(2a, -2a)$, $a \neq 0$. Then for any a, the orthocentre of this triangle lies on the line [4 marks]

(A) $y - (a^2 + 1)x = 0$

(B) $y - 2ax = 0$

(C) $y + x = 0$

(D) $(a - 1)^2x - (a + 1)^2y = 0$

Correct Answer: (D) $(a - 1)^2x - (a + 1)^2y = 0$

Solution:

Concept:

- The centroid is the coordinate-wise average of the vertices. In a triangle with circumcentre O, centroid G and orthocentre H, the vector relation is $\overrightarrow{OH} = 3\overrightarrow{OG}$.
- This relation follows by taking the circumcentre as origin: equal vertex-vector lengths imply that the sum of the three vertex vectors lies on every altitude. Since $O = (0, 0)$, the coordinates of H are three times those of G.

Step 1: Calculate the centroid coordinates The midpoint of the specified endpoints is

$$G = \left(\frac{a^2 + 1 + 2a}{2}, \frac{a^2 + 1 - 2a}{2}\right) = \left(\frac{(a + 1)^2}{2}, \frac{(a - 1)^2}{2}\right).$$

Here it is important to take the midpoint, rather than treat either endpoint as the centroid. The opposite signs of the $2a$ terms produce two different squared factors.

Step 2: Locate the orthocentre and test the line With the circumcentre at the origin, the Euler relation gives

$$H = \left(\frac{3}{2}(a + 1)^2, \frac{3}{2}(a - 1)^2\right).$$

Substitute these coordinates into the candidate equation:

$$(a - 1)^2x - (a + 1)^2y = \frac{3}{2}(a - 1)^2(a + 1)^2 - \frac{3}{2}(a + 1)^2(a - 1)^2 = 0.$$

Thus option (D) holds for every admissible a, including $a = 1$ and $a = -1$; no division by a potentially zero squared factor was used. To exclude a universal claim for the other options, take $a = 1$, giving $H = (6, 0)$. The left sides of (A), (B), and (C) are then $-12, -12, 6$, respectively, so none is the required line for every a.

Step 3: Conclusion The orthocentre lies on $(a - 1)^2x - (a + 1)^2y = 0$. Hence, the correct option is (D).

Quick Tip to Solve: When the circumcentre is the origin, multiply the centroid coordinates by three; test the line by substitution instead of dividing by coordinates that may vanish.

69. The equation of the circle described on the chord $3x + y + 5 = 0$ of the circle $x^2 + y^2 = 16$ as the diameter is [4 marks]

(A) $x^2 + y^2 + 3x + y + 1 = 0$

(B) $x^2 + y^2 + 3x + y - 22 = 0$

(C) $x^2 + y^2 + 3x + y - 11 = 0$

(D) $x^2 + y^2 + 3x + y - 2 = 0$

Correct Answer: (C) $x^2 + y^2 + 3x + y - 11 = 0$

Solution:

Concept:

- The perpendicular from the centre of a circle to a chord bisects the chord. Thus the midpoint of the given chord is also the centre of the circle built on that chord as diameter.
- If the original radius is R and the centre-to-chord distance is d, the squared half-chord length is $R^2 - d^2$. This half-chord length is the radius of the new circle.

Step 1: Find the centre of the new circle The original circle has centre $(0, 0)$ and radius 4. For the chord line $3x + y + 5 = 0$, the foot of the perpendicular from the origin is

$$M = \left(-\frac{3 \cdot 5}{3^2 + 1^2}, -\frac{1 \cdot 5}{3^2 + 1^2}\right) = \left(-\frac{3}{2}, -\frac{1}{2}\right).$$

The point lies on the chord because $3(-3/2) - 1/2 + 5 = 0$. It is the midpoint of the chord, hence the required centre. Its squared distance from the original centre is $d^2 = 9/4 + 1/4 = 5/2$.

Step 2: Find the radius and expand the equation The squared half-chord length is $16 - 5/2 = 27/2$, so the new circle is

$$\left(x + \frac{3}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{27}{2}.$$

Expanding and collecting terms gives

$$x^2 + y^2 + 3x + y + \frac{5}{2} - \frac{27}{2} = 0, \qquad x^2 + y^2 + 3x + y - 11 = 0.$$

All four choices have the same linear terms and hence the same centre, so the radius decides between them. Constants $1, -22, -2$ would give squared radii $3/2, 49/2, 9/2$, respectively, rather than $27/2$. Only the constant -11 gives the correct half-chord radius.

Step 3: Conclusion The circle is $x^2 + y^2 + 3x + y - 11 = 0$. Hence, the correct option is (C).

Quick Tip to Solve: For a circle on a chord as diameter, locate the chord midpoint by dropping the perpendicular from the original centre, then use the right triangle for its radius.

70. A chord is drawn through the focus of the parabola $y^2 = 6x$ such that its distance from the vertex of this parabola is $(\sqrt{5})/(2)$, then its slope can be [4 marks]

(A) $(\sqrt{5})/(2)$

(B) $2/\sqrt{3}$

(C) $(\sqrt{3})/(2)$

(D) $2/\sqrt{5}$

Correct Answer: (A) $(\sqrt{5})/(2)$

Solution:

Concept:

- For a parabola $y^2 = 4ax$, the vertex is $(0, 0)$ and the focus is $(a, 0)$. Here $4a = 6$, so the focus is $(3/2, 0)$.
- The distance from (x_0, y_0) to $Ax + By + C = 0$ is $|Ax_0 + By_0 + C|/\sqrt{A^2 + B^2}$. A focal chord line must pass through the focus.

Step 1: Express the chord line using its slope Let its finite slope be m. Because it passes through $(3/2, 0)$, the line is

$$y = m\left(x - \frac{3}{2}\right), \quad \text{or} \quad mx - y - \frac{3m}{2} = 0.$$

Its perpendicular distance from the vertex is therefore

$$D = \frac{3|m|}{2\sqrt{m^2 + 1}}.$$

The vertical focal line would have distance $3/2$, not the specified $\sqrt{5}/2$, so the finite-slope description does not omit a possible listed answer.

Step 2: Use the distance condition and check the choices Substitute the specified positive distance and square both sides:

$$\frac{3|m|}{2\sqrt{m^2 + 1}} = \frac{\sqrt{5}}{2}, \qquad 9m^2 = 5(m^2 + 1).$$

Thus $4m^2 = 5$ and $m = \pm\sqrt{5}/2$. Squaring introduces no unwanted magnitude because both original sides are nonnegative. The question asks which slope can occur, and the positive value $\sqrt{5}/2$ is listed. The squared slopes in the remaining options are $4/3, 3/4, 4/5$, none equal to $5/4$, so they fail the distance condition. A line through the interior focus intersects this parabola at two points, giving the required focal chord.

Step 3: Conclusion The slope can be $\sqrt{5}/2$. Hence, the correct option is (A).

Quick Tip to Solve: Use the focal point to write the line immediately, then apply the vertex-to-line distance formula; retain both slope signs until comparing with the choices.

71. The tangent at an extremity (in the first quadrant) of the latus rectum of the hyperbola $\frac{x^2}{4}-\frac{y^2}{5}=1$ meets the x-axis and y-axis at A and B, respectively. Then OA^2-OB^2, where O is the origin, equals

[4 marks]

(A) $-\frac{20}{9}$

(B) $\frac{16}{9}$

(C) 4

(D) $-\frac{4}{3}$

Correct Answer: (A) $-\frac{20}{9}$

Solution:

Concept:

- For the hyperbola $x^2/a^2-y^2/b^2=1$, the focal distance is $c=\sqrt{a^2+b^2}$. A latus rectum is perpendicular to the transverse axis through a focus.
- The tangent at a point (x_1,y_1) on this hyperbola is $xx_1/a^2-yy_1/b^2=1$. Axis intercepts are obtained by setting the other coordinate equal to zero.

Step 1: Find the first-quadrant endpoint Here $a^2=4$ and $b^2=5$, so $c=\sqrt{4+5}=3$. The right-hand latus rectum has equation $x=3$. Substitution into the hyperbola gives

$$\frac{9}{4}-\frac{y^2}{5}=1 \quad\Longrightarrow\quad y^2=\frac{25}{4}.$$

The required endpoint lies in the first quadrant, so we take the positive ordinate. Thus $(x_1,y_1)=(3,5/2)$; the negative ordinate would select the lower endpoint instead.

Step 2: Calculate both tangent intercepts The tangent at this point is

$$\frac{3x}{4}-\frac{(5/2)y}{5}=1 \quad\Longrightarrow\quad \frac{3x}{4}-\frac{y}{2}=1.$$

At the x-axis, $y=0$, giving $A=(4/3,0)$. At the y-axis, $x=0$, giving $B=(0,-2)$. Notice that the tangent meets the negative y-axis even though its point of contact is in the first quadrant. Distances from the origin are consequently $OA=4/3$ and $OB=2$.

Step 3: Compare the squared distances Use distances rather than signed coordinates when interpreting OA and OB. Their squares give

$$OA^2-OB^2=\left(\frac{4}{3}\right)^2-2^2=\frac{16}{9}-\frac{36}{9}=-\frac{20}{9}.$$

The value $16/9$ omits the y-intercept contribution; 4 is only OB^2, and $-4/3$ is not the difference of the squared intercept distances.

Step 4: Conclusion Therefore $OA^2-OB^2=-20/9$. Hence, the correct option is (A).

Quick Tip to Solve: Find the focus first, substitute its abscissa into the hyperbola, and use the point-form tangent; a negative intercept still contributes a positive squared distance.

72. The contrapositive of the statement "if I am not feeling well, then I will go to the doctor" is

[4 marks]

(A) if I will go to the doctor, then I am not feeling well.

(B) if I am feeling well, then I will not go to the doctor.

(C) if I will not go to the doctor, then I am feeling well.

(D) if I will go to the doctor, then I am feeling well.

Correct Answer: (C) if I will not go to the doctor, then I am feeling well.

Solution:

Concept:

- For an implication $p \Rightarrow q$, the contrapositive is $\neg q \Rightarrow \neg p$. It reverses the order and negates both statements.
- The contrapositive has exactly the same truth value as the original implication. The converse reverses the order alone; the inverse negates both statements without reversing them.

Step 1: Name the two component statements Let p mean I am not feeling well and let q mean I will go to the doctor. The given sentence has the form $p \Rightarrow q$, with the health condition as its hypothesis and the visit as its conclusion. The negative wording inside p must be retained while identifying the logical form; it does not change the rule for taking a contrapositive.

Step 2: Reverse and negate both components The negation $\neg q$ is I will not go to the doctor. The negation $\neg p$ is I am feeling well, because it negates the complete statement I am not feeling well. Therefore

$$p \Rightarrow q \quad \text{has contrapositive} \quad \neg q \Rightarrow \neg p.$$

In words this says: if I will not go to the doctor, then I am feeling well. This is precisely the sentence in option (C).

Step 3: Distinguish the other forms Option (A) is $q \Rightarrow p$, the converse, and option (B) is $\neg p \Rightarrow \neg q$, the inverse. Neither is generally equivalent to the original implication. Option (D) is $q \Rightarrow \neg p$, which reverses the order while negating only the original hypothesis. For example, the original statement allows someone who is feeling well to visit a doctor; it does not assert either the converse or the inverse. The two required operations are both essential.

Step 4: Conclusion The contrapositive is if I will not go to the doctor, then I am feeling well. Hence, the correct option is (C).

Quick Tip to Solve: Write the hypothesis and conclusion separately before using not conclusion implies not hypothesis; negate the complete phrase, especially when it already contains not.

73. Let $\overline{x}$, M and σ^2 be respectively the mean, mode and variance of n observations $x_1, x_2, \ldots, x_n$ and $d_i = -x_i - a$, $i = 1, 2, \ldots, n$, where a is any number.

Statement I : Variance of $d_1, d_2, \ldots, d_n$ is σ^2.

Statement II : Mean and mode of $d_1, d_2, \ldots, d_n$ are $-\overline{x} - a$ and $-M - a$, respectively [4 marks]

(A) Statement I and Statement II are both false

(B) Statement I and Statement II are both true

(C) Statement I is true and Statement II is false

(D) Statement I is false and Statement II is true

Correct Answer: (B) Statement I and Statement II are both true

Solution:

Concept:

- If every observation is changed by $d_i = bx_i + c$, then its mean changes to $b\overline{x} + c$, while its variance becomes $b^2\sigma^2$.
- A one-to-one transformation preserves the frequency attached to each observed value. Thus a modal value transforms by the same rule as the observations.

Step 1: Compute the transformed mean The transformation is $d_i = -x_i - a$, with multiplier -1 and constant shift $-a$. By summing all observations,

$$\overline{d} = \frac{1}{n}\sum_{i=1}^{n}(-x_i - a) = -\frac{1}{n}\sum_{i=1}^{n} x_i - a = -\overline{x} - a.$$

The shift is applied to every observation, so the sum contains $-na$, which becomes $-a$ after division by n. This proves the mean part of Statement II directly.

Step 2: Check the variance about the new mean For each observation, its deviation from the transformed mean is

$$d_i - \overline{d} = (-x_i - a) - (-\overline{x} - a) = -(x_i - \overline{x}).$$

Squaring removes the minus sign. Hence

$$\text{Var}(d) = \frac{1}{n}\sum_{i=1}^{n}(d_i - \overline{d})^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2 = \sigma^2.$$

The variance is unchanged by a translation and by reflection of all observations. Thus Statement I is true.

Step 3: Track the modal value and compare choices The map $x \mapsto -x - a$ is one-to-one: distinct original values remain distinct after transformation. Every occurrence of the modal value M becomes an occurrence of $-M - a$, without any change in frequency. Therefore the transformed mode is $-M - a$, proving the remaining part of Statement II. Both statements are true, so (A), (C), and (D), each of which labels at least one statement false, do not describe this transformation.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (B).

Quick Tip to Solve: A shift changes location but not variance; multiplying observations by -1 reverses deviations, whose squares remain unchanged.

74. Let A and B be any two 3 × 3 matrices. If A is symmetric and B is skew symmetric, then the matrix AB − BA is [4 marks]

(A) skew symmetric

(B) I or − I, where I is an identity matrix

(C) symmetric

(D) neither symmetric nor skew symmetric

Correct Answer: (C) symmetric

Solution:

Concept:

- A symmetric matrix satisfies $A^T = A$, and a skew symmetric matrix satisfies $B^T = -B$. These identities hold entry by entry.
- The transpose reverses the order of a product: $(AB)^T = B^T A^T$. For matrices, reversing product order is essential because multiplication need not commute.

Step 1: Transpose the entire expression Put $C = AB - BA$. Transpose both terms and reverse the factors in each product:

$$C^T = (AB - BA)^T = B^T A^T - A^T B^T.$$

It would be incorrect to replace $(AB)^T$ by $A^T B^T$. The two products have the same dimensions here, but equality of their dimensions does not allow their factors to be interchanged.

Step 2: Use the given symmetry properties Substitute $A^T = A$ and $B^T = -B$:

$$\begin{aligned} C^T &= (-B)A - A(-B) \\ &= -BA + AB \\ &= AB - BA = C. \end{aligned}$$

Thus the transpose equals the original matrix for every pair satisfying the hypotheses. This is exactly the defining condition for symmetry, independent of the particular numerical entries or of whether either product is zero.

Step 3: Check the alternatives Option (A) would require $C^T = -C$ in general, whereas the computation gives $C^T = C$. For instance, choosing $A = \mathrm{diag}(1, 2, 0)$ and B with $b_{12} = 1$, $b_{21} = -1$, all other entries zero, produces nonzero equal off-diagonal entries in C. Hence C need not be skew symmetric or I or $-I$. Option (D) is ruled out by the general transpose identity. A special zero result can belong to both symmetry classes, but the property guaranteed for every pair is symmetry.

Step 4: Conclusion The matrix $AB - BA$ is symmetric. Hence, the correct option is (C).

Quick Tip to Solve: For a commutator such as $AB - BA$, transpose first, reverse each product, and only then substitute the symmetry signs.

75. If $\Delta_r = \begin{vmatrix} r & 2r-1 & 3r-2 \\ \frac{n}{2} & n-1 & a \\ \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n+4) \end{vmatrix}$, **then the value of** $\sum_{r=1}^{n-1} \Delta_r$ [4 marks]

(A) Is independent of both a and n

(B) Depends only on a

(C) Depends only on n

(D) Depends both on a and n

Correct Answer: (A) Is independent of both a and n

Solution:

Concept:

- A determinant is linear in any one row when the other rows remain fixed. A sum of such determinants can therefore be evaluated by summing that row.
- Adding a multiple of one row to another does not change a determinant. If two columns of a minor are proportional, that minor is zero.

Step 1: Sum the varying first row Only the first row depends on r; the second and third rows are fixed throughout the sum. Let $S = \sum_{r=1}^{n-1} \Delta_r$. The three first-row sums are

$$\sum r = \frac{n(n-1)}{2}, \qquad \sum(2r-1) = (n-1)^2,$$

$$\sum(3r-2) = \frac{3n(n-1)}{2} - 2(n-1) = \frac{(n-1)(3n-4)}{2}.$$

Thus S is a single determinant whose first row consists of these three sums, and whose other rows are those printed in the question.

Step 2: Subtract the third row from the first The first two entries of this summed first row equal the corresponding entries of the third row. The last entries differ by

$$\frac{(n-1)(3n-4)}{2} - \frac{(n-1)(3n+4)}{2} = -4(n-1).$$

Apply $R_1 \leftarrow R_1 - R_3$. The new first row is $(0, 0, -4(n-1))$. Expanding along this row gives

$$S = -4(n-1)\begin{vmatrix} n/2 & n-1 \\ n(n-1)/2 & (n-1)^2 \end{vmatrix}.$$

The cofactor sign in position $(1,3)$ is positive.

Step 3: Evaluate the remaining minor The lower row of this minor is $(n-1)$ times the upper row, so

$$\frac{n}{2}(n-1)^2 - (n-1)\frac{n(n-1)}{2} = 0.$$

Consequently $S = 0$ for all allowed n, whatever the value of a. The entry a drops out on expansion because the only surviving cofactor uses the first two columns. Therefore neither dependence on a nor dependence on n remains, ruling out (B), (C), and (D).

Step 4: Conclusion $\sum_{r=1}^{n-1} \Delta_r = 0$, independent of both a and n. Hence, the correct option is (A).

Quick Tip to Solve: Before expanding many determinants, sum the row that varies; here the summed row nearly matches an existing row and leaves a zero minor.

76. The principal value of $\tan^{-1}\left(\cot\frac{43\pi}{4}\right)$ is [4 marks]

(A) $\frac{3\pi}{4}$

(B) $-\frac{\pi}{4}$

(C) $-\frac{3\pi}{4}$

(D) $\frac{\pi}{4}$

Correct Answer: (B) $-\frac{\pi}{4}$

Solution:

Concept:

- The principal-value range of $\tan^{-1} u$ is $(-\pi/2, \pi/2)$. Any final angle must lie in this open interval.
- Cotangent has period π, so its argument may be reduced by any integer multiple of π before applying the inverse tangent.

Step 1: Reduce the angle inside cotangent Write the given argument as

$$\frac{43\pi}{4} = 10\pi + \frac{3\pi}{4}.$$

Since 10π is an integer multiple of the cotangent period,

$$\cot\frac{43\pi}{4} = \cot\frac{3\pi}{4}.$$

The reduced angle lies in the second quadrant, where sine is positive and cosine is negative; the cotangent must therefore be negative.

Step 2: Evaluate and select the principal angle At $3\pi/4$, the sine and cosine have equal magnitudes, giving

$$\cot\frac{3\pi}{4} = \frac{-1/\sqrt{2}}{1/\sqrt{2}} = -1.$$

We need the unique angle $\theta \in (-\pi/2, \pi/2)$ with $\tan\theta = -1$. That angle is $\theta = -\pi/4$, so

$$\tan^{-1}\left(\cot\frac{43\pi}{4}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}.$$

Although adding an integer multiple of π gives other angles with the same tangent, the principal-value condition excludes them.

Step 3: Rule out the other listed angles Option (A), $3\pi/4$, has tangent -1 but lies outside the inverse-tangent principal range. Option (C), $-3\pi/4$, also lies outside that range and has positive tangent. Option (D), $\pi/4$, lies in the range but has tangent $+1$, inconsistent with the negative cotangent found above. Thus the range check and the sign check together leave exactly one choice.

Step 4: Conclusion The principal value is $-\pi/4$. Hence, the correct option is (B).

Quick Tip to Solve: Reduce the inner trigonometric angle first, then enforce the inverse function range; inverse tangent does not return every angle having the required tangent.

77. The function $f(x) = |\sin 4x| + |\cos 2x|$, is a periodic function with a fundamental period [4 marks]

(A) π

(B) 2π

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

Correct Answer: (D) $\frac{\pi}{2}$

Solution:

Concept:

- The fundamental period of $|\sin kx|$ or $|\cos kx|$ is $\pi/|k|$. A common period of both summands is a period of their sum.
- To establish the fundamental period, one must also exclude every smaller positive period. The zeros of a nonnegative function provide a useful exact test.

Step 1: Find a common positive period The summand $|\sin 4x|$ has period $\pi/4$, while $|\cos 2x|$ has period $\pi/2$. A shift by $\pi/2$ changes their arguments by 2π and π, respectively. Therefore

$$|\sin(4x + 2\pi)| = |\sin 4x|, \qquad |\cos(2x + \pi)| = |\cos 2x|.$$

This shows directly that $f(x + \pi/2) = f(x)$ for all real x. It establishes a period, but minimality still needs proof.

Step 2: Locate all zeros to prove minimality Both terms are nonnegative, so their sum is zero exactly when each term vanishes. The equation $\cos 2x = 0$ gives

$$x = \frac{\pi}{4} + k\frac{\pi}{2}, \qquad k \in \mathbb{Z}.$$

At each of these points, $4x = \pi + 2k\pi$, and hence $\sin 4x = 0$ as well. Thus these and only these points are zeros of f. Their consecutive spacing is $\pi/2$. Any positive period T must carry the zero $\pi/4$ to another zero, forcing T to be a positive integer multiple of $\pi/2$.

Step 3: Compare all four choices The smallest possible positive period is therefore $\pi/2$, and Step 1 has already shown it works. The proposed smaller value $\pi/4$ fails directly: $f(0) = 1$, whereas $f(\pi/4) = 0$. The values π and 2π are periods because they are multiples of $\pi/2$, but they are not

fundamental. This distinguishes the least period from merely a valid repeating shift.

Step 4: Conclusion The fundamental period is $\pi/2$. Hence, the correct option is (D).

Quick Tip to Solve: A common period need not be minimal: for this nonnegative sum, find the complete zero set and use the spacing between consecutive zeros to prove minimality.

78. Let f: R $\to$ R be defined by f(x) = (|x|−1)/(|x|+1), then f is [4 marks]

(A) one-one but not onto

(B) neither one-one nor onto

(C) both one-one and onto

(D) onto but not one-one

Correct Answer: (B) neither one-one nor onto

Solution:

Concept:

- A function is one-one when equal outputs force equal inputs. To disprove this property, it is enough to exhibit two distinct real inputs with the same image.
- A function from $\mathbb{R}$ to $\mathbb{R}$ is onto only if every real number belongs to its range. Here the absolute value makes the output depend on $t = |x| \geq 0$.

Step 1: Test the one-one condition The denominator $|x| + 1$ is positive for every real input, so the stated function is defined everywhere on its domain. However, replacing x by $-x$ leaves its value unchanged. In particular,

$$f(1) = \frac{1-1}{1+1} = 0, \qquad f(-1) = \frac{1-1}{1+1} = 0.$$

These inputs are distinct; hence the function is not one-one. This immediately rules out both options that describe it as one-one.

Step 2: Determine the range Write $t = |x|$, so that

$$f(x) = \frac{t-1}{t+1} = 1 - \frac{2}{t+1}, \qquad t \geq 0.$$

Since $t + 1 \geq 1$, its values satisfy $-1 \leq f(x) < 1$. The lower endpoint is attained at $x = 0$; the upper endpoint is approached as $|x|$ increases but is never attained. In fact every value between these bounds occurs: solving $y = (t-1)/(t+1)$ gives $t = (1+y)/(1-y) \geq 0$ for $-1 \leq y < 1$. Thus the range is exactly $[-1, 1)$. In particular, the codomain value 2 has no preimage, so the function is not onto. The option saying onto but not one-one therefore also fails.

Step 3: Conclusion The function is neither one-one nor onto as a map from $\mathbb{R}$ to $\mathbb{R}$. Hence, the correct option is (B).

Quick Tip to Solve: For a function involving only $|x|$, test x and $-x$ first; then rewrite the expression to expose its range.

79. If the function $f(x) = \begin{cases} \dfrac{\sqrt{2+\cos x}-1}{(\pi - x)^2}, & x \neq \pi \\ k, & x = \pi \end{cases}$ **is continuous at** $x = \pi$**, then** k **equals** [4 marks]

(A) $\frac{1}{4}$
(B) 0
(C) 2
(D) $\frac{1}{2}$

Correct Answer: (A) $\frac{1}{4}$

Solution:

Concept:

- Continuity at the specified point requires $k = \lim_{x\to\pi} f(x)$. Direct substitution gives the indeterminate form $0/0$, so the numerator must first be simplified.
- Rationalising $\sqrt{2+\cos x}-1$ produces $1+\cos x$. With $h = x-\pi$, the identity $1-\cos h = 2\sin^2(h/2)$ permits the standard limit $\sin u/u \to 1$.

Step 1: Rationalise the expression For $x \neq \pi$ near the point, multiply the numerator and denominator by the conjugate:

$$\frac{\sqrt{2+\cos x}-1}{(\pi-x)^2} = \frac{1+\cos x}{(\pi-x)^2(\sqrt{2+\cos x}+1)}.$$

The conjugate tends to 2, not 1; keeping this factor is essential. It is positive near π, so this transformation introduces no difficulty with division.

Step 2: Evaluate the two-sided limit Put $h = x-\pi$. Then $\cos x = -\cos h$ and $(\pi-x)^2 = h^2$, giving

$$k = \lim_{h\to 0} \frac{1-\cos h}{h^2(\sqrt{2-\cos h}+1)}.$$

Now

$$\frac{1-\cos h}{h^2} = \frac{1}{2}\left(\frac{\sin(h/2)}{h/2}\right)^2 \longrightarrow \frac{1}{2}.$$

The remaining denominator tends to $\sqrt{1}+1 = 2$. Consequently the complete limit is $(1/2)/2 = 1/4$. The squared sine ratio gives the same limit from either side. The option 0 ignores the simultaneous vanishing of the denominator; $1/2$ omits the conjugate factor, while 2 inverts or misuses the small-angle limit.

Step 3: Conclusion Continuity forces $k = \frac{1}{4}$. Hence, the correct option is (A).

Quick Tip to Solve: For a square-root difference tending to zero, rationalise before using a trigonometric small-angle limit.

80. Let f: R $\to$R be a function such that $|f(x)| \leq x^2$, for all x $\in$R. Then, at x = 0, f is [4 marks]

(A) differentiable but not continuous

(B) neither continuous nor differentiable

(C) continuous as well as differentiable

(D) continuous but not differentiable

Correct Answer: (C) continuous as well as differentiable

Solution:

Concept:

- The bound $|f(x)| \leq x^2$ controls the function without requiring a formula for it. The squeeze theorem applies because $x^2 \to 0$ as $x \to 0$.
- Differentiability at zero must be checked from $f\prime(0) = \lim_{h\to 0}[f(h) - f(0)]/h$. A quadratic bound on the numerator becomes a bound by $|h|$ on this difference quotient.

Step 1: Establish the value and continuity at zero Substitute $x = 0$ into the given inequality:

$$0 \leq |f(0)| \leq 0, \qquad f(0) = 0.$$

For all real x, the same hypothesis yields

$$-x^2 \leq f(x) \leq x^2.$$

Both bounding functions tend to zero as $x \to 0$. Hence the squeeze theorem gives $\lim_{x\to 0} f(x) = 0 = f(0)$. This proves continuity at zero rather than merely asserting it from the small size of the function.

Step 2: Apply the definition of derivative For $h \neq 0$, use $f(0) = 0$ and divide the absolute-value inequality by $|h|$:

$$\left|\frac{f(h) - f(0)}{h}\right| = \frac{|f(h)|}{|h|} \leq \frac{h^2}{|h|} = |h|.$$

Since the right-hand side tends to zero from both sides, the difference quotient tends to zero. Thus $f\prime(0) = 0$, so the function is differentiable at the specified point. No claim about differentiability away from zero is needed or implied. The choices denying continuity are contradicted by the first limit; the choice asserting continuity without differentiability is contradicted by this direct derivative calculation.

Step 3: Conclusion At $x = 0$, the function is continuous as well as differentiable, with $f(0) = f\prime(0) = 0$. Hence, the correct option is (C).

Quick Tip to Solve: If a function is bounded in magnitude by x^2 near zero, divide by $|x|$ to squeeze its difference quotient.

81. If the volume of a spherical ball is increasing at the rate of 4π cc/sec then the rate of increase of its radius (in cm/sec), when the volume is 288π cc is [4 marks]

(A) $\frac{1}{24}$

(B) $\frac{1}{6}$

(C) $\frac{1}{9}$

(D) $\frac{1}{36}$

Correct Answer: (D) $\frac{1}{36}$

Solution:

Concept:

- The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. Both volume and radius depend on time, so differentiating requires the chain rule.
- The related-rate formula is $dV/dt = 4\pi r^2\, dr/dt$. Use the instantaneous radius corresponding to the stated volume before substituting into this formula.

Step 1: Find the radius at the given instant The volume is $288\pi\,\text{cm}^3$. Substitute this into the spherical-volume formula:

$$\frac{4}{3}\pi r^3 = 288\pi \quad \Longrightarrow \quad r^3 = 216.$$

The physical radius is nonnegative, so $r = 6\,\text{cm}$. This is the radius at the instant in question; it is not itself the rate of increase. In particular, neither the diameter nor the cube of the radius should be substituted for r in the differentiated relation.

Step 2: Differentiate and substitute the rate Differentiating with respect to time gives

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}.$$

Since the volume increases at $4\pi\,\text{cm}^3\text{s}^{-1}$,

$$4\pi = 4\pi(6)^2\frac{dr}{dt}, \qquad \frac{dr}{dt} = \frac{1}{36}\,\text{cm}\,\text{s}^{-1}.$$

The sign is positive, as expected for an expanding ball. The surface area $4\pi r^2$ is the factor relating the two rates; using r, r^3, or an incorrect derivative coefficient instead would produce the distractor values. The other listed fractions do not satisfy the rate equation with $r = 6$.

Step 3: Conclusion The radius increases at $\frac{1}{36}\,\text{cm}\,\text{s}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: In a spherical related-rate problem, first recover the instantaneous radius from the volume, then divide the volume rate by $4\pi r^2$.

82. If non-zero real numbers b and c are such that min f(x) $>$ max g(x), where $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$, $(x \in R)$; then $|c/b|$ lies in the interval [4 marks]

(A) $(\sqrt{2}, \infty)$

(B) $[(1)/(\sqrt{2}), \sqrt{2}]$

(C) $(0, (1)/(2))$

(D) $[(1)/(2), (1)/(\sqrt{2}))$

Correct Answer: (A) $(\sqrt{2}, \infty)$

Solution:

Concept:

- Completing the square identifies global extrema of quadratics on $\mathbb{R}$. A positive coefficient of x^2 gives a minimum; a negative coefficient gives a maximum.
- The strict comparison concerns the two extremal values, even though they may occur at different inputs. Since $b \neq 0$, division by $b^2 > 0$ preserves an inequality.

Step 1: Compute both extremal values Complete the square in each polynomial:

$$f(x) = (x + b)^2 + 2c^2 - b^2, \qquad g(x) = -(x + c)^2 + c^2 + b^2.$$

The squared term in f is nonnegative, so $\min f = 2c^2 - b^2$, attained at $x = -b$. The negative squared term in g is nonpositive, so $\max g = c^2 + b^2$, attained at $x = -c$. These extrema exist for all real b, c; we must compare their values rather than compare $f(x)$ and $g(x)$ at an arbitrary common input.

Step 2: Translate the strict comparison into a ratio The hypothesis becomes

$$2c^2 - b^2 > c^2 + b^2 \quad \Longrightarrow \quad c^2 > 2b^2.$$

Divide by the positive quantity b^2:

$$\left(\frac{c}{b}\right)^2 > 2 \quad \Longrightarrow \quad \left|\frac{c}{b}\right| > \sqrt{2}.$$

There is no upper bound on this absolute ratio. Equality at $\sqrt{2}$ is excluded because the original inequality is strict; at equality the two extrema would coincide. The bounded interval in (B) is incompatible with being strictly larger than $\sqrt{2}$, and the intervals in (C) and (D) lie below this threshold. Thus none of those three choices meets the hypothesis.

Step 3: Conclusion The required interval is $(\sqrt{2}, \infty)$. Hence, the correct option is (A).

Quick Tip to Solve: Complete both squares first; compare the constant remainders, and preserve the strict endpoint when taking the absolute ratio.

83. If m is a non-zero number and $\int \frac{x^{5m-1}+2x^{4m-1}}{(x^{2m}+x^m+1)^3}\,dx = f(x)+c$, then f(x) is equal to [4 marks]

(A) $\frac{x^{5m}-x^{4m}}{2m\,(x^{2m}+x^m+1)^2}$

(B) $\frac{x^{4m}}{2m\,(x^{2m}+x^m+1)^2}$

(C) $\frac{x^{5m}}{2m\,(x^{2m}+x^m+1)^2}$

(D) $\frac{2m\,(x^{5m}+x^{4m})}{(x^{2m}+x^m+1)^2}$

Correct Answer: (B) $\frac{x^{4m}}{2m\,(x^{2m}+x^m+1)^2}$

Solution:

Concept:

- When powers occur in multiples of m, use $u = x^m$, so $du = mx^{m-1}dx$. The formula is considered on any interval where the real powers and their derivatives are defined.
- A proposed indefinite integral can be verified by differentiation. For $D = u^2+u+1$, differentiate u^4D^{-2} using the product and chain rules.

Step 1: Reduce the integral to a rational expression Factor x^{m-1} out of the numerator and substitute $u = x^m$:

$$I = \frac{1}{m}\int \frac{u^4+2u^3}{(u^2+u+1)^3}\,du.$$

The factor $1/m$ is legitimate because $m \neq 0$. Since the denominator is cubed, an antiderivative with denominator $(u^2+u+1)^2$ is a natural candidate. The numerator can be found by testing a fourth power rather than expanding a lengthy partial-fraction decomposition.

Step 2: Differentiate the candidate and compare options Let $D = u^2+u+1$. Then

$$\frac{d}{du}\left(\frac{u^4}{D^2}\right) = \frac{4u^3D - 2u^4(2u+1)}{D^3} = \frac{2u^4+4u^3}{D^3}.$$

Therefore

$$I = \frac{u^4}{2mD^2} + C = \frac{x^{4m}}{2m(x^{2m}+x^m+1)^2} + C.$$

This calculation verifies the entire numerator, not just its highest power. To distinguish the remaining choices, differentiating a numerator u^5 produces a term u^6 after bringing to denominator D^3; that unwanted term persists in (C) and in the u^5-u^4 numerator of (A). The u^5+u^4 numerator in (D) also produces this extra term, and its coefficient $2m$ is inconsistent with the substitution factor. Only (B) differentiates to the stated integrand.

Step 3: Conclusion An antiderivative is $f(x) = \frac{x^{4m}}{2m(x^{2m}+x^m+1)^2}$, with the additive constant already represented by c. Hence, the correct option is (B).

Quick Tip to Solve: For an MCQ integral with a cubed quadratic in x^m, substitute $u = x^m$ and differentiate the proposed squared-denominator answers.

84. Let the function F be defined as $F(x) = \int_1^x \frac{e^t}{t}\,dt$, x > 0, then the value of the integral $\int_1^x \frac{e^t}{t+a}\,dt$, where a > 0, is [4 marks]

(A) $e^a[F(x) - F(1 + a)]$

(B) $e^{-a}[F(x + a) - F(a)]$

(C) $e^a[F(x + a) - F(1 + a)]$

(D) $e^{-a}[F(x + a) - F(1 + a)]$

Correct Answer: (D) $e^{-a}[F(x + a) - F(1 + a)]$

Solution:

Concept:

- A change of variable in a definite integral changes both the integrand and its endpoints. With $u = t + a$, the exponential becomes $e^t = e^{u-a} = e^{-a}e^u$.
- The definition $F(s) = \int_1^s e^u/u\,du$ gives $\int_p^q e^u/u\,du = F(q) - F(p)$ for positive endpoints. Here $a > 0$ and $x > 0$ ensure the shifted endpoints are positive.

Step 1: Shift the integration variable and limits Denote the required integral by J. Put $u = t + a$, so $dt = du$, $t = u - a$, and $t + a = u$. The lower limit $t = 1$ becomes $u = 1 + a$, while the upper limit $t = x$ becomes $u = x + a$. Consequently

$$J = \int_{1+a}^{x+a} \frac{e^{u-a}}{u}\,du = e^{-a}\int_{1+a}^{x+a} \frac{e^u}{u}\,du.$$

The multiplier is e^{-a}, not e^a, because the original exponent is $u - a$.

Step 2: Express the result through the defined function By additivity of definite integrals,

$$F(x + a) - F(1 + a) = \int_{1+a}^{x+a} \frac{e^u}{u}\,du.$$

It follows that

$$J = e^{-a}\big[F(x + a) - F(1 + a)\big].$$

This remains valid when $0 < x < 1$, since reversing endpoints reverses the sign automatically. At $x = 1$ the expression vanishes, agreeing with an integral whose limits are equal. Option (A) fails to shift the upper argument and uses the wrong exponential factor; option (B) uses a in place of the shifted lower limit $1 + a$; option (C) has the correct limits but the wrong multiplier. Option (D) satisfies both the transformed integrand and both endpoints.

Step 3: Conclusion The value is $e^{-a}[F(x + a) - F(1 + a)]$. Hence, the correct option is (D).

Quick Tip to Solve: When shifting a denominator by a, shift both integration limits and rewrite the exponential before identifying the given integral function.

85. The area of the region in square units above the x-axis bounded by the curve y = tan x, $0 \leq x \leq \pi/2$ and the tangent to the curve at $x = \pi/4$ is [4 marks]

(A) (1)/(2)(log 2 − (1)/(2))

(B) (1)/(2)(1 + log 2)

(C) (1)/(2)(1 − log 2)

(D) (1)/(2)(log 2 + (1)/(2))

Correct Answer: (A) (1)/(2)(log 2 − (1)/(2))

Solution:

Concept:

- The tangent at a point $x = a$ has equation $y = f(a) + f\prime(a)(x - a)$. For $f(x) = \tan x$, the slope is $\sec^2 x$.
- The required finite region lies above the x-axis and below the curve, with the tangent replacing the lower boundary only after its positive x-intercept. Areas are computed by definite integration.

Step 1: Find the tangent and its intercept At $x = \pi/4$, the point of contact is $(\pi/4, 1)$ and the slope is 2. Hence the tangent is

$$y - 1 = 2(x - \pi/4), \qquad y = 2x + 1 - \pi/2.$$

Its intercept on the x-axis is $a = \pi/4 - 1/2$. The curve $\tan x$ is convex on this interval, so its tangent lies below it. The finite enclosed region ends at their contact point; integrating up to the vertical asymptote would give an unrelated unbounded area.

Step 2: Subtract the triangular portion The area under the curve from 0 to the contact point is

$$\int_0^{\pi/4} \tan x \, dx = [-\log(\cos x)]_0^{\pi/4} = \tfrac{1}{2}\log 2.$$

The tangent and the x-axis form a triangle under this area, with base $\pi/4 - a = 1/2$ and height 1. Its area is $\frac{1}{2}(1/2)(1) = 1/4$. Therefore

$$A = \tfrac{1}{2}\log 2 - \tfrac{1}{4} = \tfrac{1}{2}(\log 2 - \tfrac{1}{2}).$$

Options B and D add positive contributions instead of subtracting this triangle. Option C reverses the logarithmic and constant terms and does not equal the integral difference.

Step 3: Conclusion The area is $\frac{1}{2}(\log 2 - \frac{1}{2})$ square units. Hence, the correct option is (A).

Quick Tip to Solve: Find the tangent intercept first: the required area is the area under $\tan x$ up to contact minus a simple triangle, avoiding a piecewise integral.

86. If (dy)/(dx)+ ytan x=sin 2x and y(0) = 1, then y(π) is equal to [4 marks]

(A) -1

(B) 5

(C) 1

(D) -5

Correct Answer: (D) -5

Solution:

Concept:

- A first-order linear equation $y' + P(x)y = Q(x)$ has integrating factor $e^{\int P(x)dx}$, on intervals where its coefficients are defined.
- Here $P(x) = \tan x$, so an integrating factor is $\sec x$. The identity $\sin 2x = 2\sin x \cos x$ simplifies the transformed right side.

Step 1: Multiply by the integrating factor On an interval not containing a zero of $\cos x$, multiply the equation by $\sec x$. The product rule gives

$$\frac{d}{dx}(y \sec x) = \sin 2x \sec x = 2\sin x.$$

Integrating,

$$y \sec x = -2\cos x + C, \qquad y = C\cos x - 2\cos^2 x.$$

The sign in the antiderivative is negative because the derivative of cosine is minus sine. No extra factor of two is introduced when integrating $2\sin x$.

Step 2: Apply the condition and evaluate The initial condition gives

$$1 = C - 2, \qquad C = 3.$$

Thus the continued solution expression is $y = 3\cos x - 2\cos^2 x$. At $x = \pi$, $\cos \pi = -1$, and therefore

$$y(\pi) = 3(-1) - 2(1) = -5.$$

The equation itself has a singular coefficient at $\pi/2$; the intended evaluation uses this smooth solution expression continued across that point. Its derivative at $\pi/2$ is -3, so smooth continuation fixes the same constant on the next interval. Options A, B and C fail direct substitution in the expression fixed by $y(0) = 1$. In particular, replacing $\cos^2 \pi$ by -1 would be an incorrect sign change.

Step 3: Conclusion Using the smooth continuation of the solution determined by the initial condition, $y(\pi) = -5$. Hence, the correct option is (D).

Quick Tip to Solve: After finding the integrating factor, simplify $\sin 2x \sec x$ before integrating; at the final endpoint keep $\cos^2 \pi = 1$.

87. If $\mathbf{x} = 3\hat{\imath} - 6\hat{\jmath} - \hat{k}$, $\mathbf{y} = \hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ and $\mathbf{z} = 3\hat{\imath} - 4\hat{\jmath} - 12\hat{k}$, then the magnitude of the projection of $\mathbf{x} \times \mathbf{y}$ on $\mathbf{z}$ is [4 marks]

(A) 14

(B) 12

(C) 15

(D) 10

Correct Answer: (A) 14

Solution:

Concept:

- The scalar projection magnitude of a vector $\mathbf{a}$ onto nonzero $\mathbf{z}$ is $|\mathbf{a} \cdot \mathbf{z}|/|\mathbf{z}|$. The absolute value removes the orientation sign.
- The vector cross product is computed using the determinant with unit vectors in its first row. The second component carries the alternating minus sign.

Step 1: Compute the cross product Using the components exactly as given,

$$\mathbf{x} \times \mathbf{y} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -6 & -1 \\ 1 & 4 & -3 \end{vmatrix}.$$

The first component is $(-6)(-3) - (-1)(4) = 22$. The second is $(-1)(1) - 3(-3) = 8$, while the third is $3(4) - (-6)(1) = 18$. Consequently,

$$\mathbf{x} \times \mathbf{y} = 22\hat{\imath} + 8\hat{\jmath} + 18\hat{k}.$$

As a useful check, this vector has zero dot product with each of the original two vectors.

Step 2: Project onto the specified direction For $\mathbf{z} = (3, -4, -12)$,

$$|\mathbf{z}| = \sqrt{9 + 16 + 144} = 13.$$

Its dot product with the cross product is

$$22(3) + 8(-4) + 18(-12) = 66 - 32 - 216 = -182.$$

The requested magnitude, rather than the signed scalar component, is therefore

$$\frac{|-182|}{13} = 14.$$

The negative sign only indicates that the cross product points partly opposite to $\mathbf{z}$. Dividing by $|\mathbf{z}|^2$ would instead give the coefficient used in a vector projection and would not answer this magnitude question. The alternatives 12, 15 and 10 do not match this exact scalar product divided by 13.

Step 3: Conclusion The magnitude of the projection is 14. Hence, the correct option is (A).

Quick Tip to Solve: For a projection magnitude, take the absolute scalar triple product and divide by the length of the vector defining the projection direction.

88. If the angle between the line $2(x+1) = y = z+4$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is $\frac{\pi}{6}$, then the value of λ is [4 marks]

(A) $\frac{135}{7}$

(B) $\frac{45}{11}$

(C) $\frac{45}{7}$

(D) $\frac{135}{11}$

Correct Answer: (C) $\frac{45}{7}$

Solution:

Concept:

- If a line has direction vector $\mathbf{d}$ and a plane has normal $\mathbf{n}$, their acute angle θ satisfies $\sin\theta = |\mathbf{d}\cdot\mathbf{n}|/(|\mathbf{d}||\mathbf{n}|)$.
- The common parameter in a symmetric line equation gives its direction ratios. The coefficients of x, y and z in a plane equation give its normal vector.

Step 1: Identify the two relevant vectors Put $2(x+1) = y = z+4 = t$. Then $x = t/2 - 1$, $y = t$, and $z = t - 4$. A direction vector is therefore $(1/2, 1, 1)$, or equivalently $\mathbf{d} = (1, 2, 2)$, of length 3. The normal to the plane is $\mathbf{n} = (2, -1, \sqrt{\lambda})$, with length $\sqrt{5+\lambda}$. The constant 4 fixes the position of the plane but has no effect on this angle. The scalar product simplifies to

$$\mathbf{d}\cdot\mathbf{n} = 2 - 2 + 2\sqrt{\lambda} = 2\sqrt{\lambda}.$$

Step 2: Use the line-plane angle formula Because $\sin(\pi/6) = 1/2$ and $\lambda \geq 0$,

$$\frac{1}{2} = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}.$$

Squaring both nonnegative sides gives

$$9(5+\lambda) = 16\lambda, \qquad 45 = 7\lambda.$$

Hence $\lambda = 45/7$. Substitution in the unsquared ratio yields $1/2$, so no extraneous value was introduced. The other proposed fractions fail this equality. In particular, using cosine instead of sine would compute the angle with the normal rather than with the plane, and would lead to a different equation.

Step 3: Conclusion The required value is $\lambda = \frac{45}{7}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert a line-plane angle to a sine formula with the plane normal; scale direction ratios to integers before taking the dot product.

89. Equation of the line of the shortest distance between the lines (x)/(1) = (y)/(−1) = (z)/(1) and (x−1)/(0) = (y+1)/(−2) = (z)/(1) is [4 marks]

(A) (x)/(−2)=(y)/(1)=(z)/(2)

(B) (x)/(1)=(y)/(−1)=(z)/(−2)

(C) (x−1)/(1)=(y+1)/(−1)=(z)/(−2)

(D) (x−1)/(1)=(y+1)/(−1)=(z)/(1)

Correct Answer: (C) (x−1)/(1)=(y+1)/(−1)=(z)/(−2)

Solution:

Concept:

- The line of shortest distance is perpendicular to both skew lines, so its direction is the cross product of their direction vectors.
- A valid common perpendicular must also meet both given lines. Its direction alone does not fix its position, so an intersection check is necessary.

Step 1: Find the perpendicular direction The first line is $L_1 : (x, y, z) = (t, -t, t)$, with direction $\mathbf{a} = (1, -1, 1)$. The zero denominator in the second symmetric equation means $x = 1$; its parametric form is $L_2 : (x, y, z) = (1, -1 - 2s, s)$, with direction $\mathbf{b} = (0, -2, 1)$. Thus

$$\mathbf{a} \times \mathbf{b} = (1, -1, -2).$$

This vector is perpendicular to both since its dot products with $\mathbf{a}$ and $\mathbf{b}$ are each zero. Options A and D do not have this direction. Options B and C have the right direction, so their positions must be checked.

Step 2: Locate the common perpendicular Option C passes through $(1, -1, 0)$, which is on L_2 at $s = 0$. Write its points as

$$(x, y, z) = (1 + u, -1 - u, -2u).$$

At $u = -1/3$ it gives $(2/3, -2/3, 2/3)$, lying on L_1 with $t = 2/3$. It therefore meets both lines and is perpendicular to each. Option B passes through the origin on L_1, but to meet L_2 its x-coordinate would require its parameter to equal 1; it would then give $(1, -1, -2)$, whereas the second line at y=-1 has z=0. Hence option B does not meet the second line and cannot be the common perpendicular.

Step 3: Conclusion The line is $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{-2}$. Hence, the correct option is (C).

Quick Tip to Solve: Use a cross product to eliminate wrong directions, then check that the remaining line actually intersects both skew lines; a perpendicular direction alone is insufficient.

90. Let A and E be any two events with positive probabilities: Statement - I : P(E/A) $\geq$ P(A/E)P(E). Statement - II : P(A/E) $\geq$ P(A$\cap$E) [4 marks]

(A) Both the statements are false

(B) Both the statements are true

(C) Statement - I is false, Statement - II is true

(D) Statement - I is true, Statement - II is false

Correct Answer: (B) Both the statements are true

Solution:

Concept:

- For events with positive probabilities, $P(E \mid A) = P(A \cap E)/P(A)$ and $P(A \mid E) = P(A \cap E)/P(E)$.
- Every probability is at most 1 and nonnegative. Dividing a nonnegative number by a positive number at most 1 cannot make it smaller.

Step 1: Check Statement I directly Let $p = P(A \cap E)$, so $p \geq 0$. The right side of Statement I simplifies without any independence assumption:

$$P(A \mid E)P(E) = \frac{p}{P(E)}P(E) = p.$$

Its left side is $p/P(A)$. Since $0 < P(A) \leq 1$,

$$\frac{p}{P(A)} \geq p.$$

Therefore Statement I is true. Equality is allowed, for example when the intersection has probability zero or when $P(A) = 1$. The condition that the individual events have positive probabilities ensures that the conditional probabilities are defined.

Step 2: Check Statement II and compare the choices Likewise, Statement II asks whether

$$\frac{p}{P(E)} \geq p.$$

This follows from $0 < P(E) \leq 1$. More explicitly, their difference is

$$\frac{p}{P(E)} - p = \frac{p[1 - P(E)]}{P(E)} \geq 0,$$

as every factor has the required sign. Thus Statement II is also true, including the boundary case $p = 0$. Option A incorrectly declares both inequalities false; C incorrectly rejects the first, and D incorrectly rejects the second. Neither statement requires A and E to be independent or mutually exclusive: each is simply a consequence of the conditional-probability definition and the upper bound 1 on probabilities.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (B).

Quick Tip to Solve: Replace conditional probabilities by intersection probabilities first. The remaining comparisons reduce to dividing a nonnegative quantity by a probability no greater than 1.