JEE Main 2015 Paper 1 (B.E./B.Tech) Apr 10 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2015, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 10 April 2015, Single slot. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. The largest value of r, for which the region represented by the set $\{\omega \in \mathbb{C} : |\omega - 4 - i| \leq r\}$ is contained in the region represented by the set $\{z \in \mathbb{C} : |z - 1| \leq |z + i|\}$, is equal to [4 marks]

(1) $2\sqrt{2}$

(2) $\frac{3}{2}\sqrt{2}$

(3) $\sqrt{17}$

(4) $\frac{5}{2}\sqrt{2}$

Correct Answer: (4) $\frac{5}{2}\sqrt{2}$

Solution:

Concept:

- For $z = x + iy$, its modulus measures distance in the Argand plane. Squaring an inequality between nonnegative moduli preserves its direction.
- The set $|\omega - (4 + i)| \leq r$ is the closed disc of centre $(4, 1)$ and radius r. Its largest radius inside a closed half-plane equals the perpendicular distance from its centre to the boundary.

Step 1: Convert the modulus inequality to a half-plane Write $z = x + iy$. The distances from $(1, 0)$ and $(0, -1)$ satisfy

$$(x - 1)^2 + y^2 \leq x^2 + (y + 1)^2.$$

Cancelling the square terms gives $-2x \leq 2y$, or $x + y \geq 0$. The boundary is the straight line $x + y = 0$, and the disc centre satisfies $4 + 1 > 0$, so it lies in the required half-plane.

Step 2: Find the limiting radius and check containment The perpendicular distance of $(4, 1)$ from that line is

$$d = \frac{|4 + 1|}{\sqrt{1^2 + 1^2}} = \frac{5}{\sqrt{2}} = \frac{5}{2}\sqrt{2}.$$

For any point of a disc of radius r, its signed distance toward the other half-plane can decrease by at most r. Thus every point remains in $x + y \geq 0$ precisely when $r \leq d$. At equality the disc is tangent to the line; for a larger radius it crosses it. The values in (1) and (2) are smaller and are not maximal, while $\sqrt{17} > 5/\sqrt{2}$ crosses the boundary.

Step 3: Conclusion The largest radius is $\boxed{\frac{5}{2}\sqrt{2}}$. Hence, the correct option is (4).

Quick Tip to Solve: Convert equal-distance modulus inequalities into a line first; use the centre-to-line distance rather than distance to either fixed point.

2. If $2+3i$ is one of the roots of the equation $2x^3-9x^2+kx-13=0$, $k\in\mathbb{R}$, then the real root of this equation (where $i^2=-1$): [4 marks]

(1) Exists and is equal to $\frac{1}{2}$

(2) Does not exist

(3) Exists and is equal to 1

(4) Exists and is equal to $-\frac{1}{2}$

Correct Answer: (1) Exists and is equal to $\frac{1}{2}$

Solution:

Concept:

- A polynomial with real coefficients has its non-real roots in conjugate pairs. Therefore $2-3i$ accompanies the root $2+3i$.
- For $ax^3+bx^2+cx+d=0$, the sum of roots is $-b/a$ and the product of roots is $-d/a$. These relations determine the remaining root without finding k.

Step 1: Use conjugacy and the sum of roots Let the third root be t. Because k and all other coefficients are real, the first two roots are $2+3i$ and $2-3i$. Their sum is 4. Applying the sum-of-roots relation gives

$$(2+3i)+(2-3i)+t=\frac{9}{2}, \qquad t=\frac{9}{2}-4=\frac{1}{2}.$$

This explicitly establishes that a real root exists, so the assertion in option (2) is false.

Step 2: Verify using the product and factorisation The conjugate product is $(2+3i)(2-3i)=4+9=13$. The product of all three roots must be $13/2$, so $13t=13/2$, again giving $t=1/2$, with positive sign. In fact

$$(x^2-4x+13)(2x-1)=2x^3-9x^2+30x-13.$$

Thus $k=30$ is real and the factorisation confirms existence with every coefficient consistent. The values 1 and $-1/2$ fail the sum and product tests; the latter also gives the wrong sign of the constant term.

Step 3: Conclusion The real root exists and is $\boxed{\frac{1}{2}}$. Hence, the correct option is (1).

Quick Tip to Solve: For a real cubic with one complex root, insert its conjugate immediately and use the sum of roots to find the real root.

3. The number of ways of selecting 15 teams from 15 men and 15 women, such that each team consists of a man and a woman is [4 marks]

(1) 1960

(2) $15!$

(3) $(15!)^2$

(4) $14!$

Correct Answer: (2) $15!$

Solution:

Concept:

- A team is a pair containing one man and one woman. Selecting all 15 such teams uses every person exactly once.
- Fixing the order of the distinct men gives a unique label for each pair; the women can then be matched bijectively to those 15 men in $15!$ ways.

Step 1: Fix the men and choose their partners List the men in an arbitrary fixed order $M_1, M_2, \ldots, M_{15}$. The first man can be paired with any of 15 women. After that choice, the second has 14 available partners, then the third has 13, and so on until the last man has one. The multiplication principle gives

$$15 \times 14 \times \cdots \times 2 \times 1 = 15!.$$

Every choice sequence gives a collection of disjoint mixed pairs, so no person is repeated.

Step 2: Check that teams have not been overcounted The fixed list of men is a counting device, not an extra ordering of the selected teams. Any completed collection identifies exactly one partner of M_1, one of M_2, and so forth. Hence it occurs once in the calculation. If one instead permutes both the men and women, there are $(15!)^2$ ordered lists of pairs, but each collection occurs $15!$ times from reordering the teams. Dividing gives the same $15!$. The option $14!$ would incorrectly fix one partnership in advance; 1960 cannot equal the complete product.

Step 3: Conclusion The required number of ways is $\boxed{15!}$. Hence, the correct option is (2).

Quick Tip to Solve: For complete mixed pairings, keep one group fixed and permute only the members of the other group to avoid counting team order.

4. The value of $\sum_{r=16}^{30}(r+2)(r-3)$ is equal to: [4 marks]

(1) 7775

(2) 7785

(3) 7780

(4) 7770

Correct Answer: (3) 7780

Solution:

Concept:

- Expand the summand before applying linearity: $(r+2)(r-3) = r^2 - r - 6$. The bounds 16 through 30 include 15 integers.
- The standard identities are $\sum_{r=1}^{n} r = n(n+1)/2$ and $\sum_{r=1}^{n} r^2 = n(n+1)(2n+1)/6$. Subtract sums through 15 to retain the requested bounds.

Step 1: Evaluate the separate sums over the correct interval The sum of the integers from 16 to 30 is

$$\sum_{r=16}^{30} r = \frac{30 \cdot 31}{2} - \frac{15 \cdot 16}{2} = 465 - 120 = 345.$$

Their squares sum to

$$\sum_{r=16}^{30} r^2 = \frac{30 \cdot 31 \cdot 61}{6} - \frac{15 \cdot 16 \cdot 31}{6} = 9455 - 1240 = 8215.$$

There are $30 - 16 + 1 = 15$ constant contributions. Subtracting through 16 instead of through 15 would wrongly remove the first required term.

Step 2: Combine the contributions and check the arithmetic By linearity, the required value is

$$\sum_{r=16}^{30}(r+2)(r-3) = 8215 - 345 - 6 \cdot 15 = 8215 - 345 - 90 = 7780.$$

As a separate check, put $r = 23 + s$, where $s = -7, \ldots, 7$. Then the summand is $500 + 45s + s^2$, so the total is $15 \cdot 500 + 2(1^2 + \cdots + 7^2) = 7500 + 280 = 7780$. The odd terms cancel. This check excludes the nearby alternatives differing by 5 or 10.

Step 3: Conclusion The sum is $\boxed{7780}$. Hence, the correct option is (3).

Quick Tip to Solve: For an interval sum, subtract the standard prefix sum through one less than the lower bound; count constant terms separately.

5. Let the sum of the first three terms of an A.P. be 39 and the sum of its last four terms be 178. If the first term of this A.P. is 10, then the median of the A.P. is [4 marks]

(1) 26.5

(2) 29.5

(3) 28

(4) 31

Correct Answer: (2) 29.5

Solution:

Concept:

- An arithmetic progression with first term a and common difference d has nth term $a + (n-1)d$. Here $a = 10$.
- The median of a finite increasing arithmetic progression is the average of its first and last terms. This holds for both odd and even numbers of terms because the terms are equally spaced.

Step 1: Find the common difference The first three terms are $10, 10+d, 10+2d$. Their given sum yields

$$30 + 3d = 39, \qquad d = 3.$$

Thus the progression is $10, 13, 16, \ldots$, which is increasing. Let its final term be l. Working backwards, its last four terms are $l-9, l-6, l-3, l$, since each adjacent difference is 3.

Step 2: Find the final term and locate the median The given sum of the last four terms is

$$(l-9) + (l-6) + (l-3) + l = 178, \qquad 4l - 18 = 178, \qquad l = 49.$$

The number of terms satisfies $49 = 10 + (n-1)3$, giving $n = 14$. The two middle terms, in positions 7 and 8, are 28 and 31. Therefore the median is

$$\frac{28+31}{2} = 29.5 = \frac{10+49}{2}.$$

The values 28 and 31 are the individual central terms, not the median of this even-sized list. The value 26.5 does not lie halfway between its endpoints and corresponds to an incorrect final term.

Step 3: Conclusion The median is $\boxed{29.5}$. Hence, the correct option is (2).

Quick Tip to Solve: Once the first and last terms of an arithmetic progression are known, their average gives its median without listing all terms.

6. If the coefficients of three successive terms in the binomial expansion of $(1+x)^n$ are in the ratio $1:7:42$, then the first of these terms in the expansion is [4 marks]

(1) 9th

(2) 6th

(3) 8th

(4) 7th

Correct Answer: (4) 7th

Solution:

Concept:

- The term with exponent r in $(1+x)^n$ is $\binom{n}{r}x^r$, and it is the $(r+1)$th term because indexing starts with exponent zero.
- Successive coefficient ratios obey $\binom{n}{r+1}/\binom{n}{r} = (n-r)/(r+1)$. The two ratios in $1:7:42$ are 7 and 6, rather than 7 and 42.

Step 1: Translate the coefficient ratios into equations Let the three coefficients be $\binom{n}{r}, \binom{n}{r+1}, \binom{n}{r+2}$. The first ratio gives

$$\frac{n-r}{r+1} = 7, \qquad n = 8r+7.$$

The ratio of the third to the second is $42/7 = 6$, giving

$$\frac{n-r-1}{r+2} = 6, \qquad n = 7r+13.$$

These equations use adjacent coefficients, so each numerator and denominator shifts by one for the second comparison.

Step 2: Solve and convert exponent to term number Equating the two expressions for n gives $8r+7 = 7r+13$, hence $r = 6$, and $n = 55$. Direct substitution checks both ratios:

$$\frac{55-6}{6+1} = 7, \qquad \frac{55-6-1}{6+2} = 6.$$

Thus the first coefficient multiplies x^6, which belongs to the seventh term, not the sixth. Options 8th and 9th correspond to different exponents and fail the simultaneous equations. Distinguishing the exponent from the ordinal term avoids the tempting sixth-term error.

Step 3: Conclusion The first of the three successive terms is the $\boxed{7\text{th}}$ term. Hence, the correct option is (4).

Quick Tip to Solve: First form adjacent ratios, then add one to the first exponent to obtain its ordinal term number in a binomial expansion.

7. In a ΔABC, a/b = 2 + $\sqrt{3}$, and $\angle$C = 60°. Then the ordered pair ($\angle$A, $\angle$B) is equal to [4 marks]

(1) (105°, 15°)

(2) (15°, 105°)

(3) (45°, 75°)

(4) (75°, 45°)

Correct Answer: (1) (105°, 15°)

Solution:

Concept:

- The sine rule relates opposite sides and angles: $a/b = \sin A/\sin B$. The angle sum gives $A + B = 120°$ because $C = 60°$.
- The identity $\sin(A + B) = \sin A\cos B + \cos A\sin B$, or direct expansion of $\sin(120° - B)$, converts the ratio into an equation for B.

Step 1: Express the side ratio using one unknown angle Put $A = 120° - B$. Since both are triangle angles, $0 < B < 120°$, and $\sin B > 0$. By the sine rule,

$$2 + \sqrt{3} = \frac{\sin(120° - B)}{\sin B} = \frac{\sqrt{3}}{2}\cot B + \frac{1}{2}.$$

The expansion uses $\cos 120° = -1/2$, which produces a positive $\frac{1}{2}\sin B$ term after subtraction.

Step 2: Determine the angles and reject reversed ordering Solving gives

$$\cot B = \frac{2}{\sqrt{3}}\left(\frac{3}{2} + \sqrt{3}\right) = 2 + \sqrt{3}, \qquad \tan B = 2 - \sqrt{3} = \tan 15°.$$

Tangent is negative for angles between 90° and 120°, so the positive value forces B acute and uniquely gives $B = 15°$. Then $A = 105°$. Also $a/b > 1$ requires $A > B$, ruling out the reversed pair in (2). The other pairs have sine ratios at most $\sin 75°/\sin 45°$, which is much smaller than $2 + \sqrt{3}$.

Step 3: Conclusion The ordered pair is $\boxed{(105°, 15°)}$. Hence, the correct option is (1).

Quick Tip to Solve: Check which side is larger before solving for the angles; the larger opposite angle fixes the ordering of any candidate pair.

8. Let L be the line passing through the point $P(1,2)$ such that its intercepted segment between the co-ordinate axes is bisected at P. If L_1 is the line perpendicular to L and passing through the point $(-2,1)$, then the point of intersection of L and L_1 is [4 marks]

(1) $\left(\frac{3}{5}, \frac{23}{10}\right)$
(2) $\left(\frac{4}{5}, \frac{12}{5}\right)$
(3) $\left(\frac{11}{20}, \frac{29}{10}\right)$
(4) $\left(\frac{3}{10}, \frac{17}{5}\right)$

Correct Answer: (2) $\left(\frac{4}{5}, \frac{12}{5}\right)$

Solution:

Concept:

- The midpoint of intercepts $(a,0)$ and $(0,b)$ is $(a/2, b/2)$. The given midpoint therefore determines both intercepts, rather than just the slope.
- For two nonvertical perpendicular lines their slopes have product -1. Their intersection must satisfy both equations simultaneously.

Step 1: Recover the intercepted line The intercepted segment is bisected at $P(1,2)$. Thus $a/2 = 1$ and $b/2 = 2$, giving $a = 2$, $b = 4$. Use the intercept equation to obtain

$$\frac{x}{2} + \frac{y}{4} = 1, \qquad 2x + y = 4.$$

Hence the slope of L is -2; the coordinates of P indeed satisfy this equation.

Step 2: Construct the perpendicular and intersect The slope of L_1 is $1/2$. Since it passes through $(-2,1)$, its equation is

$$y - 1 = \frac{1}{2}(x + 2), \qquad y = \frac{x}{2} + 2.$$

Substitution into $2x + y = 4$ yields $5x/2 = 2$, so $x = 4/5$ and $y = 12/5$. This ordered pair satisfies both equations. Each of the other listed pairs fails at least one equation, so merely lying on one of the lines is insufficient.

Step 3: Conclusion The intersection is $\left(\frac{4}{5}, \frac{12}{5}\right)$. Hence, the correct option is (2).

Quick Tip to Solve: When a point bisects the axis-intercept segment, double its coordinates to get the two intercepts immediately.

9. The points $\left(0, \frac{8}{3}\right)$, $(1, 3)$ and $(82, 30)$ [4 marks]

(1) form an obtuse angled triangle

(2) form an acute angled triangle

(3) lie on a straight line

(4) form a right angled triangle

Correct Answer: (3) lie on a straight line

Solution:

Concept:

- Three distinct points are collinear when the slopes of two segments sharing a point are equal. This is equivalent to the determinant or triangle-area test giving zero.
- The slope between (x_1, y_1) and (x_2, y_2) is $(y_2 - y_1)/(x_2 - x_1)$ when the denominator is nonzero. Keep the fraction $8/3$ exact.

Step 1: Compare the segment slopes Name the points $A = (0, 8/3)$, $B = (1, 3)$, and $C = (82, 30)$. Their horizontal separations are nonzero, so the slope test applies without a vertical-line exception. Calculate

$$m_{AB} = \frac{3 - 8/3}{1 - 0} = \frac{1}{3}, \qquad m_{BC} = \frac{30 - 3}{82 - 1} = \frac{27}{81} = \frac{1}{3}.$$

The identical slopes show that the two segments have the same direction.

Step 2: Verify the common line and exclude triangle types The line through A with this slope is $y = x/3 + 8/3$, or $3y = x + 8$. For B, both sides equal 9; for C, both sides equal 90. Thus all three points lie on this single line. Their enclosed area is zero. Consequently they do not form a nondegenerate triangle, so none of the acute, obtuse or right-angled triangle descriptions applies.

Step 3: Conclusion The three points lie on a straight line. Hence, the correct option is (3).

Quick Tip to Solve: Before classifying a triangle by its angles, check whether the three given points are collinear.

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10. If $y + 3x = 0$ is the equation of a chord of the circle $x^2 + y^2 - 30x = 0$, then the equation of the circle with this chord as diameter is [4 marks]

(1) $x^2 + y^2 + 3x - 9y = 0$

(2) $x^2 + y^2 - 3x + 9y = 0$

(3) $x^2 + y^2 + 3x + 9y = 0$

(4) $x^2 + y^2 - 3x - 9y = 0$

Correct Answer: (2) $x^2 + y^2 - 3x + 9y = 0$

Solution:

Concept:

- To find the endpoints of a chord, solve the line equation and circle equation together. A line through the origin can make one intersection particularly easy to identify.
- For diameter endpoints (x_1, y_1), (x_2, y_2), the circle is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$, because

the angle subtended by a diameter is a right angle.

Step 1: Locate both ends of the chord On the given chord $y = -3x$. Substitute this into the original circle rather than using its centre as a diameter endpoint:

$$x^2 + 9x^2 - 30x = 0, \qquad 10x(x - 3) = 0.$$

Therefore the two intersections are $A = (0, 0)$ and $B = (3, -9)$. Their midpoint, which will be the new centre, is $(3/2, -9/2)$.

Step 2: Write and check the new circle Using the diameter-endpoint equation gives

$$x(x - 3) + y(y + 9) = 0,$$

$$x^2 + y^2 - 3x + 9y = 0.$$

Its centre agrees with the midpoint just found. Its squared radius is $(3^2+9^2)/4 = 45/2$, a positive value. The other equations have different centre signs and do not contain both required chord endpoints; substituting $(3, -9)$ is a quick way to reject them.

Step 3: Conclusion The required equation is $x^2 + y^2 - 3x + 9y = 0$. Hence, the correct option is (2).

Quick Tip to Solve: If one diameter endpoint is the origin, the circle equation is simply $x^2 + y^2 - x_2x - y_2y = 0$.

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11. Let the tangents drawn to the circle $x^2 + y^2 = 16$ from the point $P(0, h)$ meet the x-axis at points A and B. If the area of $\triangle APB$ is minimum, then the positive value of h is [4 marks]

(1) $4\sqrt{2}$
(2) $3\sqrt{2}$
(3) $4\sqrt{3}$
(4) $3\sqrt{3}$

Correct Answer: (1) $4\sqrt{2}$

Solution:

Concept:

- A line is tangent to a circle when its perpendicular distance from the centre equals the radius. Here the radius is 4, and two real tangents from $(0, h)$ require $h > 4$.
- The triangle area is one half of base times perpendicular height. Express the base in terms of h before minimizing, and check the boundary behaviour to identify a global minimum.

Step 1: Express the tangent intercepts and area A tangent through P has equation $y = mx + h$. Its distance from the origin is $h/\sqrt{m^2 + 1} = 4$. Hence $m^2 = (h^2 - 16)/16$. The two slopes have opposite signs, so their x-intercepts are opposite and the base length is

$$AB = \frac{2h}{|m|} = \frac{8h}{\sqrt{h^2 - 16}}.$$

The height from P to the x-axis is h. Thus the area is $S(h) = 4h^2/\sqrt{h^2-16}$.

Step 2: Minimize on the admissible domain Differentiate the area for $h > 4$:

$$S'(h) = \frac{4h(h^2-32)}{(h^2-16)^{3/2}}.$$

The denominator and $4h$ are positive, so the derivative changes from negative to positive at $h^2 = 32$. Also $S(h)$ tends to infinity as h approaches 4 from above and as h tends to infinity. This critical point is therefore the global minimum. The other positive options do not satisfy the stationary condition, even when they lie in the admissible domain.

Step 3: Conclusion The positive minimizing value is $h = 4\sqrt{2}$. Hence, the correct option is (1).

Quick Tip to Solve: For this symmetric tangent triangle, first obtain the intercepts from the centre-to-line distance; then minimize a single-variable area.

12. If the tangent to the conic $y - 6 = x^2$ at $(2, 10)$ touches the circle $x^2 + y^2 + 8x - 2y = k$ (for some fixed k) at a point (α, β), then (α, β) is [4 marks]

(1) $\left(-\frac{7}{17}, \frac{6}{17}\right)$
(2) $\left(-\frac{8}{17}, \frac{2}{17}\right)$
(3) $\left(-\frac{6}{17}, \frac{10}{17}\right)$
(4) $\left(-\frac{4}{17}, \frac{1}{17}\right)$

Correct Answer: (2) $\left(-\frac{8}{17}, \frac{2}{17}\right)$

Solution:

Concept:

- The derivative of the parabola gives the tangent slope. A tangent to a circle is perpendicular to the radius at its point of contact.
- For a circle $x^2 + y^2 + 2gx + 2fy + c = 0$, the centre is $(-g, -f)$. Its point of contact with a given tangent is the perpendicular projection of this centre onto the line.

Step 1: Find the tangent and the circle centre Write the conic as $y = x^2 + 6$. Its derivative is $dy/dx = 2x$, so the slope at $(2, 10)$ is 4. The tangent is therefore

$$y - 10 = 4(x - 2), \qquad 4x - y + 2 = 0.$$

Completing squares in the circle gives $(x+4)^2 + (y-1)^2 = k + 17$, so its centre is $C = (-4, 1)$. The unknown radius does not affect the direction of the perpendicular radius.

Step 2: Project the centre onto the tangent For the line $ax + by + c = 0$, the foot from (x_0, y_0) is $(x_0 - ad, y_0 - bd)$, with $d = (ax_0 + by_0 + c)/(a^2 + b^2)$. Here

$$d = \frac{4(-4) - 1 + 2}{4^2 + (-1)^2} = -\frac{15}{17}.$$

Consequently

$$\alpha = -4 + \frac{60}{17} = -\frac{8}{17}, \qquad \beta = 1 - \frac{15}{17} = \frac{2}{17}.$$

This point lies on the tangent and its radius has slope $-1/4$. The other listed pairs fail these simultaneous contact conditions.

Step 3: Conclusion The contact point is $\left(-\frac{8}{17}, \frac{2}{17}\right)$. Hence, the correct option is (2).

Quick Tip to Solve: When a circle tangent is known, its contact point is the foot from the centre; solving for the radius is unnecessary.

13. An ellipse passes through the foci of the hyperbola $9x^2 - 4y^2 = 36$, and its major and minor axes lie along the transverse and conjugate axes of the hyperbola respectively. If the product of eccentricities of the two conics is $\frac{1}{2}$, then which of the following points does not lie on the ellipse? [4 marks]

(1) $\left(\frac{\sqrt{39}}{2}, \sqrt{3}\right)$

(2) $\left(\frac{\sqrt{13}}{2}, \frac{\sqrt{3}}{2}\right)$

(3) $\left(\sqrt{\frac{13}{2}}, \sqrt{6}\right)$

(4) $(\sqrt{13}, 0)$

Correct Answer: (2) $\left(\frac{\sqrt{13}}{2}, \frac{\sqrt{3}}{2}\right)$

Solution:

Concept:

- For $x^2/a_h^2 - y^2/b_h^2 = 1$, the hyperbola has focal distance $c_h^2 = a_h^2 + b_h^2$ and eccentricity $e_h = c_h/a_h$.
- An ellipse centred at the origin with horizontal major axis has equation $x^2/a^2 + y^2/b^2 = 1$, where $b^2 = a^2(1 - e^2)$. A point lies on it only when substitution gives exactly one.

Step 1: Determine the ellipse from the conics The hyperbola is $x^2/4 - y^2/9 = 1$, so its foci are $(\pm\sqrt{13}, 0)$ and its eccentricity is $\sqrt{13}/2$. The ellipse has the same coordinate axes as its principal axes and passes through these points on its major axis; hence $a^2 = 13$. The given eccentricity product yields

$$e\frac{\sqrt{13}}{2} = \frac{1}{2}, \qquad e = \frac{1}{\sqrt{13}}, \qquad b^2 = 13\left(1 - \frac{1}{13}\right) = 12.$$

Thus the ellipse is $x^2/13 + y^2/12 = 1$.

Step 2: Test every printed point For option (1), substitution gives $(39/4)/13 + 3/12 = 3/4 + 1/4 = 1$. For option (2), it gives

$$\frac{13/4}{13} + \frac{3/4}{12} = \frac{1}{4} + \frac{1}{16} = \frac{5}{16} \neq 1.$$

Option (3) gives $(13/2)/13 + 6/12 = 1/2 + 1/2 = 1$, and option (4) gives $13/13 + 0 = 1$. Therefore only the second point fails the ellipse equation; it is inside the ellipse rather than on its boundary.

Step 3: Conclusion The point $\left(\frac{\sqrt{13}}{2}, \frac{\sqrt{3}}{2}\right)$ does not lie on the ellipse. Hence, the correct option is (2).

Quick Tip to Solve: After finding the ellipse equation, square each printed coordinate carefully: a denominator outside a radical also gets squared.

14. $\lim\limits_{x\to 0}\dfrac{e^{x^2}-\cos x}{\sin^2 x}$ **is equal to** [4 marks]

(1) 2

(2) $\frac{3}{2}$

(3) $\frac{5}{4}$

(4) 3

Correct Answer: (2) $\frac{3}{2}$

Solution:

Concept:

- For small x, both numerator and denominator tend to zero. The first nonzero terms, of order x^2, determine the ratio.
- The standard limits are $(e^t-1)/t \to 1$, $(1-\cos x)/x^2 \to 1/2$, and $\sin x/x \to 1$. They allow an exact limit calculation without an indeterminate substitution.

Step 1: Separate the two vanishing contributions Rewrite the numerator as $(e^{x^2}-1)+(1-\cos x)$, which separates the exponential and trigonometric contributions. For nonzero x near zero, divide numerator and denominator by x^2:

$$\frac{e^{x^2}-\cos x}{\sin^2 x}=\frac{\dfrac{e^{x^2}-1}{x^2}+\dfrac{1-\cos x}{x^2}}{\left(\dfrac{\sin x}{x}\right)^2}.$$

Each ratio on the right has a standard finite limit, and the denominator tends to one, so division is valid.

Step 2: Apply the limits and compare options Set $t=x^2$ in the exponential standard limit. It gives $(e^{x^2}-1)/x^2 \to 1$. Also $1-\cos x = 2\sin^2(x/2)$, so the second numerator ratio tends to $1/2$. Therefore

$$\lim_{x\to 0}\frac{e^{x^2}-\cos x}{\sin^2 x}=\frac{1+1/2}{1}=\frac{3}{2}.$$

As a check, the expansions $e^{x^2}=1+x^2+O(x^4)$ and $\cos x = 1-x^2/2+O(x^4)$ give the same leading coefficient. Neither 2, $5/4$, nor 3 has that coefficient.

Step 3: Conclusion The limit equals $\frac{3}{2}$. Hence, the correct option is (2).

Quick Tip to Solve: When an exponential and cosine cancel at zero, split the numerator at one so their standard quadratic limits can be added.

15. The contrapositive of the statement "If it is raining, then I will not come", is [4 marks]

(1) if I will come, then it is not raining

(2) if I will come, then it is raining

(3) if I will not come, then it is raining

(4) if I will not come, then it is not raining

Correct Answer: (1) if I will come, then it is not raining

Solution:

Concept:

- For propositions p and q, the contrapositive of $p \Rightarrow q$ is $\neg q \Rightarrow \neg p$. It is logically equivalent to the original implication.
- The converse reverses the implication without negating either proposition, while the inverse negates both without reversing them. These need not be equivalent to the original.

Step 1: Identify the propositions Let p mean "it is raining" and let q mean "I will not come". The original sentence has precisely the form $p \Rightarrow q$. The negation of q is "I will come", since the word "not" is already part of q. The negation of p is "it is not raining".

Step 2: Reverse and negate both sides Place $\neg q$ first and $\neg p$ second. The result is: "If I will come, then it is not raining." Merely reversing the original statement would give "If I will not come, then it is raining", which is its converse. Negating without reversing would give "If it is not raining, then I will come". The other offered sentences therefore do not have the required contrapositive form.

Step 3: Conclusion The contrapositive is "If I will come, then it is not raining." Hence, the correct option is (1).

Quick Tip to Solve: Write $p \Rightarrow q$ first; then reverse the arrow and negate both complete propositions, paying attention to an existing "not".

16. A factory is operating in two shifts, day and night, with 70 and 30 workers, respectively. If per day mean wage of the day shift workers is ₹ 54 and per day mean wage of all the workers is ₹ 60, then per day mean wage of the night shift workers (in ₹) is: [4 marks]

(1) 75

(2) 74

(3) 69

(4) 66

Correct Answer: (2) 74

Solution:

Concept:

- A combined mean is weighted by the number of observations in each group:

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}.$$

- For wages measured per worker per day, the number of workers times the group mean gives the total daily wage bill of that group.

Step 1: Compute the total and day-shift wage bills There are $70 + 30 = 100$ workers altogether. Their total daily wages are $100 \times 60 = 6000$ rupees. The day-shift workers receive $70 \times 54 = 3780$ rupees per day. These totals must be compared before taking another average; the two shifts have unequal populations.

Step 2: Divide the remaining wage bill by the night workforce The night-shift wage bill is $6000 - 3780 = 2220$ rupees. Consequently its mean wage is

$$\bar{x}_{\text{night}} = \frac{2220}{30} = 74 \text{ rupees per day.}$$

As a check, $0.7(54) + 0.3(74) = 37.8 + 22.2 = 60$. The other proposed means, 75, 69 and 66, would give combined means 60.3, 58.5 and 57.6 respectively, so they do not satisfy the stated overall mean.

Step 3: Conclusion The mean daily wage of a night-shift worker is 74 rupees. Hence, the correct option is (2).

Quick Tip to Solve: For two groups of unequal sizes, recover their total amounts first; never average the two group means without weights.

17. In a certain town, 25% of the families own a phone and 15% own a car; 65% families own neither a phone nor a car and 2000 families own both a car and a phone. Consider the following three statements: (i) 5% families own both a car and a phone. (ii) 35% families own either a car or a phone. (iii) 40000 families live in the town. Then, [4 marks]

(1) Only (ii) and (iii) are correct

(2) Only (i) and (ii) are correct

(3) All (i), (ii) and (iii) are correct

(4) Only (i) and (iii) are correct

Correct Answer: (3) All (i), (ii) and (iii) are correct

Solution:

Concept:

- For events P and C, $P(P \cup C) = P(P) + P(C) - P(P \cap C)$. The complement of the union represents owning neither item.
- An observed number belonging to a known fraction of the population determines the total population by division.

Step 1: Find the union and overlap Let P denote owning a phone and C owning a car. Since 65 percent own neither,

$$P(P \cup C) = 1 - 0.65 = 0.35.$$

Inclusion-exclusion gives

$$P(P \cap C) = 0.25 + 0.15 - 0.35 = 0.05.$$

Thus 5 percent own both, establishing statement (i), and 35 percent own a car or a phone in the inclusive union sense, establishing statement (ii). The overlap must be subtracted once because it was counted in both individual percentages.

Step 2: Use the overlap to count all families If N is the number of families, the given count of families owning both implies $0.05N = 2000$. Hence

$$N = \frac{2000}{0.05} = 40000.$$

This establishes statement (iii). All three statements follow from the same overlap calculation, so each option that omits one of these true statements is incomplete.

Step 3: Conclusion All statements (i), (ii) and (iii) are correct. Hence, the correct option is (3).

Quick Tip to Solve: Start with the percentage owning neither: its complement gives the union, which then determines the overlap by inclusion-exclusion.

18. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then which one of the following statements is not correct? [4 marks]

(1) $A^3 + I = A(A^3 - I)$

(2) $A^4 - I = A^2 + I$

(3) $A^2 + I = A(A^2 - I)$

(4) $A^3 - I = A(A - I)$

Correct Answer: (3) $A^2 + I = A(A^2 - I)$

Solution:

Concept:

- Matrix powers are formed by matrix multiplication. For the given square matrix, computing A^2 first reduces every higher power to a multiple of I or A, where I is the identity matrix of order two.
- A matrix identity is true only when its two sides are equal as matrices. Since the question asks for the statement that is not correct, each displayed identity must be checked, rather than merely finding one true identity.

Step 1: Compute the short cycle of powers Multiply the given matrix by itself, preserving the minus sign in its upper-right entry:

$$A^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I.$$

It follows that $A^3 = -A$ and $A^4 = I$. These relations let us test all four alternatives without repeatedly multiplying full matrices.

Step 2: Test the first two identities For (1), the left side is $A^3 + I = I - A$, while its right side is

$$A(A^3 - I) = A(-A - I) = -A^2 - A = I - A.$$

Thus (1) is correct. For (2), $A^4 - I = I - I = 0$ and $A^2 + I = -I + I = 0$, where 0 denotes the zero matrix. Therefore (2) is also correct.

Step 3: Compare the remaining two identities For (3), the left side is $A^2 + I = 0$. Its right side is

$$A(A^2 - I) = A(-I - I) = -2A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix},$$

which is not the zero matrix. Therefore (3) is false. Finally, for (4), $A^3 - I = -A - I$, whereas

$$A(A - I) = A^2 - A = -I - A.$$

These expressions coincide, so (4) is correct. This last check ensures that the negative wording singles out exactly one alternative.

Step 4: Conclusion The identity $A^2 + I = A(A^2 - I)$ is not correct, because its left side is the zero matrix and its right side is the nonzero matrix $-2A$. Hence, the correct option is (3).

Quick Tip to Solve: When a small matrix satisfies $A^2 = -I$, reduce all higher powers using the four-term cycle $I, A, -I, -A$ before testing identities.

19. The least value of the product xyz (such that x, y and z are positive real numbers) for which the determinant

$$\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix}$$

is non-negative is [4 marks]

(1) -1

(2) $-16\sqrt{2}$

(3) -8

(4) $-2\sqrt{2}$

Correct Answer: No listed option is possible; the admissible products have infimum zero and no attained minimum.

Solution:

Concept:

- Expansion of the printed determinant gives

$$D = \begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix} = xyz - x - y - z + 2.$$

- The stipulated positivity $x, y, z > 0$ forces $xyz > 0$. A negative listed value cannot be the product of these positive numbers.

Step 1: Test whether a positive minimum is attained Take $x = y = z = t$, where $0 < t \leq 1$. The determinant becomes

$$D = t^3 - 3t + 2 = (t-1)^2(t+2) \geq 0.$$

Thus every such choice is allowed by the printed non-negativity condition, and its product is t^3. In particular, taking t closer to zero gives admissible products as small and positive as desired.

Step 2: Compare the literal result with the options Every allowed product is strictly positive, but for any proposed positive lower bound m, one can choose $0 < t < \min(1, m^{1/3})$ and obtain $xyz = t^3 < m$. Therefore the infimum is zero and no least product is attained under strict positivity. The four options are -1, $-16\sqrt{2}$, -8 and $-2\sqrt{2}$, all negative. None can answer the question as printed. This is not resolved by selecting the smallest of those numbers, since each violates the explicit domain.

Step 3: Conclusion For the stated positive domain, $\inf xyz = 0$, with no attained least value. None of the printed options is correct.

Quick Tip to Solve: Check the sign and domain before optimizing: a product of strictly positive variables cannot equal any negative option.

20. If f(x) = 2 $\tan^{-1}x$ + $\sin^{-1}(2x/(1+x^2))$, x > 1, then f(5) is equal to [4 marks]

(1) $\pi/2$

(2) $\tan^{-1}(65/156)$

(3) π

(4) 4 $\tan^{-1}5$

Correct Answer: (3) π

Solution:

Concept:

- The principal range of $\sin^{-1}$ is $[-\pi/2, \pi/2]$, so $\sin^{-1}(\sin u) = u$ only when u lies in that interval.
- If $\theta = \tan^{-1} x$, the double-angle identity gives $\sin 2\theta = 2x/(1 + x^2)$. The restriction $x > 1$ fixes the correct principal-value branch.

Step 1: Locate the double angle For $x > 1$, $\theta = \tan^{-1} x$ lies in $(\pi/4, \pi/2)$. Thus $2\theta \in (\pi/2, \pi)$, outside the principal range of inverse sine. Its sine is positive, and the first-quadrant angle having that sine is $\pi - 2\theta$. Consequently

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \pi - 2\tan^{-1} x.$$

Step 2: Substitute into the given function For every $x > 1$,

$$f(x) = 2\tan^{-1} x + \pi - 2\tan^{-1} x = \pi.$$

In particular, 5 satisfies the domain condition and $f(5) = \pi$. Taking the inverse sine as $2\tan^{-1} 5$ would incorrectly produce $4\tan^{-1} 5$; that double angle is greater than $\pi/2$. The value $\pi/2$ and the small principal arctangent of $65/156$ also fail this branch calculation.

Step 3: Conclusion $f(5) = \pi$. Hence, the correct option is (3).

Quick Tip to Solve: For $x > 1$, use $\sin^{-1}(2x/(1 + x^2)) = \pi - 2\tan^{-1} x$, checking the principal range before cancellation.

21. If Rolle's theorem holds for the function $f(x) = 2x^3 + bx^2 + cx$, $x \in [-1, 1]$, at the point $x = \frac{1}{2}$, then $2b + c$ is equal to [4 marks]

(1) 2

(2) 1

(3) -1

(4) -3

Correct Answer: (3) -1

Solution:

Concept:

- Rolle's theorem applies to a function continuous on a closed interval and differentiable on its interior when its endpoint values are equal. It then guarantees a point in the interior where the derivative vanishes.
- A polynomial is continuous and differentiable everywhere, so the required conditions here reduce to $f(-1) = f(1)$ and the specified stationary-point condition $f'(1/2) = 0$. These are two independent equations for b and c.

Step 1: Use equality of the endpoint values Substitute the two endpoints into the given cubic:

$$f(-1) = -2 + b - c, \qquad f(1) = 2 + b + c.$$

The endpoint condition in Rolle's theorem therefore gives

$$-2 + b - c = 2 + b + c \quad \Longrightarrow \quad -2c = 4 \quad \Longrightarrow \quad c = -2.$$

The coefficient b cancels because the quadratic term has the same value at both endpoints. The cubic and linear terms change sign.

Step 2: Apply the derivative condition at the given point Differentiate the polynomial and then put $x = 1/2$, which is strictly inside $[-1, 1]$:

$$f'(x) = 6x^2 + 2bx + c, \qquad f'(1/2) = \frac{3}{2} + b + c = 0.$$

Using $c = -2$ yields $3/2 + b - 2 = 0$, hence $b = 1/2$. The endpoint condition alone cannot determine both coefficients.

Step 3: Evaluate the requested combination and check the conditions Now calculate

$$2b + c = 2\left(\frac{1}{2}\right) - 2 = -1.$$

As a direct check, $f(-1) = f(1) = 1/2$ for these coefficients, and $f'(1/2) = 3/2 + 1/2 - 2 = 0$. Thus all the theorem's conditions hold. The other offered values 2, 1 and -3 are excluded by this calculation.

Step 4: Conclusion Rolle's theorem gives $c = -2$ and $b = 1/2$, so $2b + c = -1$. Hence, the correct option is (3).

Quick Tip to Solve: On a symmetric interval, first equate endpoint values to eliminate the even-power terms; then use the specified Rolle point in the derivative.

22. The distance from the origin, of the normal to the curve, $x = 2\cos t + 2t\sin t$, $y = 2\sin t - 2t\cos t$ at $t = \frac{\pi}{4}$, is [4 marks]

(1) $\sqrt{2}$

(2) $2\sqrt{2}$

(3) 4

(4) 2

Correct Answer: (4) 2

Solution:

Concept:

- For a parametric curve, the tangent slope is $dy/dx = (dy/dt)/(dx/dt)$ when $dx/dt \neq 0$. A normal is perpendicular to the tangent, so its slope is the negative reciprocal.
- The perpendicular distance of $(0, 0)$ from $Ax + By + C = 0$ is $|C|/\sqrt{A^2 + B^2}$. The curve parameter must be substituted into both coordinates and the slope.

Step 1: Differentiate both coordinates Differentiating the products in the two parametric coordinates gives

$$x'(t) = -2\sin t + 2\sin t + 2t\cos t = 2t\cos t,$$

$$y'(t) = 2\cos t - 2\cos t + 2t\sin t = 2t\sin t.$$

Thus the tangent slope is $\tan t$. At $t = \pi/4$ it is 1, and the normal slope is -1. Both derivatives are nonzero here, so this reciprocal-slope argument is valid.

Step 2: Find the normal and its distance At the specified parameter the point is

$$x_0 = \sqrt{2}\left(1 + \frac{\pi}{4}\right), \qquad y_0 = \sqrt{2}\left(1 - \frac{\pi}{4}\right).$$

Therefore $x_0 + y_0 = 2\sqrt{2}$. The normal through this point has equation

$$y - y_0 = -(x - x_0), \qquad x + y - 2\sqrt{2} = 0.$$

Its distance from the origin is

$$d = \frac{2\sqrt{2}}{\sqrt{1^2 + 1^2}} = 2.$$

The value $2\sqrt{2}$ is the axis intercept, rather than the perpendicular distance; division by $\sqrt{2}$ is essential. The other proposed distances do not satisfy the point-to-line formula.

Step 3: Conclusion The required perpendicular distance is 2 units. Hence, the correct option is (4).

Quick Tip to Solve: For a normal of slope -1, compute the sum of the contact coordinates and divide its absolute value by $\sqrt{2}$.

23. The integral $\int \frac{dx}{(x+1)^{3/4}(x-2)^{5/4}}$, is equal to [4 marks]

(1) $4\left(\frac{x-2}{x+1}\right)^{1/4}+c$

(2) $-\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{1/4}+c$

(3) $4\left(\frac{x+1}{x-2}\right)^{1/4}+c$

(4) $-\frac{4}{3}\left(\frac{x-2}{x+1}\right)^{1/4}+c$

Correct Answer: (2) $-\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{1/4}+c$

Solution:

Concept:

- A fractional-power integrand involving two linear factors often simplifies by using their ratio as a substitution. Differentiate the ratio before choosing its orientation.
- For the real fourth roots in the printed denominator, take $x > 2$. An antiderivative can be verified by differentiation on this interval, where all fractional powers have their usual positive values.

Step 1: Introduce the ratio substitution Put $u = (x+1)/(x-2)$. The quotient rule gives

$$\frac{du}{dx} = \frac{(x-2)-(x+1)}{(x-2)^2} = -\frac{3}{(x-2)^2}.$$

Also,

$$\frac{1}{(x+1)^{3/4}(x-2)^{5/4}} = \frac{u^{-3/4}}{(x-2)^2}.$$

The negative sign comes from differentiating the ratio; overlooking it would reverse the sign of the antiderivative.

Step 2: Integrate and check the result The integral becomes

$$I = -\frac{1}{3}\int u^{-3/4}\,du = -\frac{4}{3}u^{1/4} + C.$$

Returning to x,

$$I = -\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{1/4} + C.$$

For an exact check, differentiating this expression gives

$$-\frac{4}{3}\cdot\frac{1}{4}u^{-3/4}\cdot\frac{-3}{(x-2)^2} = \frac{1}{(x+1)^{3/4}(x-2)^{5/4}},$$

which is precisely the printed integrand. Option (3) has the same ratio but a coefficient giving minus three times the required derivative. Options (1) and (4) reverse the ratio, changing the powers of the two linear factors, so their derivatives cannot equal the integrand.

Step 3: Conclusion The integral is $-\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{1/4}+C$. Hence, the correct option is (2).

Quick Tip to Solve: Check both the ratio orientation and its derivative: the factor -3 fixes the coefficient -4/3 immediately.

24. For $x > 0$, let $f(x) = \int_1^x \frac{\log t}{1+t}\,dt$. Then $f(x) + f\left(\frac{1}{x}\right)$ is equal to [4 marks]

(1) $\frac{1}{2}(\log x)^2$

(2) $\log x$

(3) $\frac{1}{4}\log x^2$

(4) $\frac{1}{4}(\log x)^2$

Correct Answer: (1) $\frac{1}{2}(\log x)^2$

Solution:

Concept:

- The fundamental theorem of calculus states that $d[\int_1^x g(t)\,dt]/dx = g(x)$ for continuous g. Composing the upper limit with $1/x$ also requires the chain rule.
- Here $x > 0$, so logarithms and both integrals are defined. A constant of integration is fixed by evaluating at $x = 1$, where each integral has equal limits.

Step 1: Differentiate the required sum Write $F(x) = f(x) + f(1/x)$. Then

$$f'(x) = \frac{\log x}{1+x}.$$

For the second term,

$$\frac{d}{dx}f(1/x) = \frac{\log(1/x)}{1+1/x}\left(-\frac{1}{x^2}\right) = \frac{\log x}{x(x+1)}.$$

The two negatives cancel because $\log(1/x) = -\log x$. Hence

$$F'(x) = \frac{\log x}{x+1} + \frac{\log x}{x(x+1)} = \frac{\log x}{x}.$$

Step 2: Integrate and determine the constant Since the derivative of $\frac{1}{2}(\log x)^2$ is $(\log x)/x$,

$$F(x) = \frac{1}{2}(\log x)^2 + C.$$

At $x = 1$, $f(1) = 0$, so $F(1) = 0$ and $C = 0$. Thus the required identity holds throughout the connected domain $x > 0$. Options (2) and (3), with $\log x^2 = 2\log x$, are linear in $\log x$, and their derivatives lack the necessary logarithmic factor. Option (4) has the squared logarithm but only half the required coefficient. The derivative and the value at 1 together determine the answer uniquely.

Step 3: Conclusion For every $x > 0$, $f(x) + f(1/x) = \frac{1}{2}(\log x)^2$. Hence, the correct option is (1).

Quick Tip to Solve: For a sum involving $f(x)$ and $f(1/x)$, differentiate first; the chain rule often removes the need to evaluate either integral.

25. The area (in square units) of the region bounded by the curves $y + 2x^2 = 0$ and $y + 3x^2 = 1$, is equal to [4 marks]

(1) $\frac{3}{4}$ sq. units

(2) $\frac{1}{3}$ sq. units

(3) $\frac{3}{5}$ sq. units

(4) $\frac{4}{3}$ sq. units

Correct Answer: (4) $\frac{4}{3}$ sq. units

Solution:

Concept:

- The area enclosed between two graphs is the integral of the upper ordinate minus the lower ordinate over their intersection interval. Determine the intersections before choosing the bounds.
- For the curves here, the ordinates are $y_1 = -2x^2$ and $y_2 = 1 - 3x^2$. Their difference $1 - x^2$ is even, allowing a symmetric integral to be evaluated using twice the integral from 0 to 1.

Step 1: Find intersections and the upper curve Set the two ordinates equal:

$$-2x^2 = 1 - 3x^2 \quad \Longrightarrow \quad x^2 = 1.$$

The boundaries therefore meet at $(-1, -2)$ and $(1, -2)$. Between these values,

$$y_2 - y_1 = (1 - 3x^2) - (-2x^2) = 1 - x^2 \geq 0.$$

Thus y_2 is the upper graph throughout the bounded region. The curve with the more negative quadratic coefficient is still above the other near the origin because its vertical intercept is 1.

Step 2: Integrate the vertical separation The required area is

$$A = \int_{-1}^{1} (1 - x^2)\, dx = 2\int_{0}^{1} (1 - x^2)\, dx.$$

Using the elementary power rule,

$$A = 2\left[x - \frac{x^3}{3}\right]_0^1 = 2\left(1 - \frac{1}{3}\right) = \frac{4}{3}.$$

This is positive, as an area must be. A useful scale check is that the region lies inside a rectangle of width 2 and height 1, so its area must be less than 2. The value $1/3$ omits part of the integral, while $3/4$ and $3/5$ are not obtained by integrating the actual separation over the full intersection interval. Both halves of the symmetric region must be included.

Step 3: Conclusion The enclosed area is $\frac{4}{3}$ square units. Hence, the correct option is (4).

Quick Tip to Solve: Subtract the two parabola equations first; their complicated individual shapes reduce here to the simple even height $1 - x^2$.

26. If $y(x)$ is the solution of the differential equation $(x+2)\frac{dy}{dx} = x^2 + 4x - 9$, $x \neq -2$ and $y(0) = 0$, then $y(-4)$ is equal to [4 marks]

(1) -1

(2) 1

(3) 0

(4) 2

Correct Answer: The condition at 0 fixes only $x > -2$; $y(-4)$ is arbitrary on $x < -2$. The conventional same-constant answer is 0 (3).

Solution:

Concept:

- An initial condition determines the integration constant on the connected interval containing the initial point. It does not determine a constant on a separate interval across a singularity of the differential equation.
- Polynomial division gives $y' = x + 2 - 13/(x+2)$. The coefficient is undefined at $x = -2$, separating the domains $x > -2$ and $x < -2$. The initial point 0 and requested point -4 lie in different components.

Step 1: Integrate on each admissible interval For $x \neq -2$, divide by $x + 2$ and integrate:

$$y' = \frac{(x+2)^2 - 13}{x+2} = x + 2 - \frac{13}{x+2},$$

$$y(x) = \frac{x^2}{2} + 2x - 13\log|x+2| + C.$$

On $x > -2$, the condition $y(0) = 0$ gives $C_+ = 13\log 2$. Hence the branch containing the initial point is uniquely specified. It cannot be continued through -2 as a finite solution, since the logarithmic term diverges there.

Step 2: Check the requested point and the conventional value At $x = -4$, the separate branch has its own arbitrary constant C_-, giving

$$y(-4) = 8 - 8 - 13\log 2 + C_- = C_- - 13\log 2.$$

No stated condition fixes C_-. Indeed, choosing $C_- = a + 13\log 2$ gives $y(-4) = a$ for any real number a, while preserving the same branch through $(0, 0)$. Thus every listed numerical value can be realized by a solution on the disconnected domain. If one additionally chooses the same integration constant on both components, substitution gives $y(-4) = 0$, the conventional option (3). That equality of constants is an extra assumption, however, and does not follow from the printed initial condition.

Step 3: Conclusion The printed initial-value problem does not uniquely determine $y(-4)$, since -4 and 0 are separated by the excluded point -2. The conventional same-constant calculation gives 0, option (3), but it requires an additional assumption.

Quick Tip to Solve: Before using an initial condition at a distant point, check whether the differential equation has a singularity between that point and the initial point.

27. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a}+\vec{b}|=\sqrt{3}$. If $\vec{c}=\vec{a}+2\vec{b}+(\vec{a}\times\vec{b})$, then $2|\vec{c}|$ is equal to: [4 marks]

(1) $\sqrt{51}$
(2) $\sqrt{37}$
(3) $\sqrt{43}$
(4) $\sqrt{55}$

Correct Answer: The literal printed vectors give $\sqrt{31}$, which is absent from all four options.

Solution:

Concept:

- For unit vectors, $|\vec{a}+\vec{b}|^2=2+2\vec{a}\cdot\vec{b}$. This relation finds the angle information without choosing coordinates.
- The cross product $\vec{a}\times\vec{b}$ is perpendicular to both vectors, so it is also perpendicular to $\vec{a}+2\vec{b}$. Its squared magnitude is $|\vec{a}|^2|\vec{b}|^2-(\vec{a}\cdot\vec{b})^2$.

Step 1: Find the scalar product Squaring the given magnitude gives

$$3=(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})=1+1+2\vec{a}\cdot\vec{b}.$$

Consequently $\vec{a}\cdot\vec{b}=1/2$, corresponding to an angle of 60°. The planar combination has squared magnitude

$$|\vec{a}+2\vec{b}|^2=1+4+4(1/2)=7.$$

The mixed scalar-product coefficient is 4, because the coefficient 2 on $\vec{b}$ occurs in both cross terms when squaring.

Step 2: Combine the perpendicular components The squared cross-product magnitude is

$$|\vec{a}\times\vec{b}|^2=1-(1/2)^2=\frac{3}{4}.$$

As the cross product is perpendicular to $\vec{a}+2\vec{b}$, the mixed term vanishes in the square of $\vec{c}$. Therefore

$$|\vec{c}|^2=7+\frac{3}{4}=\frac{31}{4},\qquad 2|\vec{c}|=\sqrt{31}.$$

None of $\sqrt{51},\sqrt{37},\sqrt{43},\sqrt{55}$ equals $\sqrt{31}$. The literal printed coefficient of $\vec{b}$ is 2, and replacing it by another coefficient to match an option would change the question. The calculation can also be checked with $\vec{a}=(1,0,0)$, $\vec{b}=(1/2,\sqrt{3}/2,0)$, which yields $\vec{c}=(2,\sqrt{3},\sqrt{3}/2)$ and the same squared norm.

Step 3: Conclusion The printed data give $2|\vec{c}|=\sqrt{31}$. None of the listed options is correct.

Quick Tip to Solve: Use perpendicularity before expanding a norm: the cross product contributes its squared magnitude but no mixed dot-product term.

28. If the points $(1, 1, \lambda)$ and $(-3, 0, 1)$ are equidistant from the plane $3x + 4y - 12z + 13 = 0$, then λ satisfies the equation: [4 marks]

(1) $3x^2 + 10x + 7 = 0$

(2) $3x^2 + 10x - 13 = 0$

(3) $3x^2 - 10x + 7 = 0$

(4) $3x^2 - 10x + 21 = 0$

Correct Answer: (3) $3x^2 - 10x + 7 = 0$

Solution:

Concept:

- The perpendicular distance of (x_0, y_0, z_0) from $Ax + By + Cz + D = 0$ is $|Ax_0 + By_0 + Cz_0 + D|/\sqrt{A^2 + B^2 + C^2}$.
- For two points and the same plane, the denominator is common. Equate the absolute values of the signed numerators; squaring retains both the same-side and opposite-side possibilities.

Step 1: Evaluate the two distance numerators The normal length of the plane is

$$\sqrt{3^2 + 4^2 + (-12)^2} = 13.$$

At $(1, 1, \lambda)$, the signed numerator is $3+4-12\lambda+13 = 20-12\lambda$. At $(-3, 0, 1)$, it is $-9+0-12+13 = -8$. Therefore equal distances require

$$\frac{|20 - 12\lambda|}{13} = \frac{8}{13}, \qquad |20 - 12\lambda| = 8.$$

The absolute value is essential because equal-distance points may lie on opposite sides of the plane.

Step 2: Obtain the quadratic and compare options Squaring and simplifying gives

$$(20 - 12\lambda)^2 = 64,$$

$$144\lambda^2 - 480\lambda + 336 = 0.$$

Dividing by the common factor 48 produces

$$3\lambda^2 - 10\lambda + 7 = 0.$$

For a check, the two roots are $\lambda = 1$ and $\lambda = 7/3$, corresponding respectively to signed numerators 8 and -8. Both have distance $8/13$ from the plane. The options write the polynomial using the dummy variable x; its coefficients, rather than that dummy letter, determine the choice. The positive-linear-term options have the wrong root sum, while the last option has the wrong constant term and cannot have both verified roots.

Step 3: Conclusion The required equation is $3\lambda^2 - 10\lambda + 7 = 0$, written as $3x^2 - 10x + 7 = 0$ in the options. Hence, the correct option is (3).

Quick Tip to Solve: When equating distances to one plane, cancel the normal-length denominator first, then retain both signs of the numerator.

29. If the shortest distance between the line $\frac{x-1}{\alpha} = \frac{y+1}{-1} = \frac{z}{1}$, $\alpha \neq -1$, and $x+y+z+1 = 0 = 2x-y+z+3$ is $\frac{1}{\sqrt{3}}$, then the value of α is: [4 marks]

(1) $-\frac{19}{16}$

(2) $\frac{32}{19}$

(3) $-\frac{16}{19}$

(4) $\frac{19}{32}$

Correct Answer: (2) $\frac{32}{19}$

Solution:

Concept:

- For two nonparallel lines through position vectors $\mathbf{p}, \mathbf{q}$, with directions $\mathbf{u}, \mathbf{v}$, the distance is $d = |(\mathbf{q}-\mathbf{p}) \cdot (\mathbf{u} \times \mathbf{v})|/|\mathbf{u} \times \mathbf{v}|$.
- The direction of the intersection of two planes is the cross product of their normal vectors.

Step 1: Find the directions and reference points The first line has $\mathbf{p} = (1, -1, 0)$ and $\mathbf{u} = (\alpha, -1, 1)$. For the second line, crossing the plane normals gives $\mathbf{v} = (1, 1, 1) \times (2, -1, 1) = (2, 1, -3)$. Setting $z = 0$ in its plane equations gives $\mathbf{q} = (-4/3, 1/3, 0)$.

Step 2: Apply the distance condition Now $\mathbf{u} \times \mathbf{v} = (2, 3\alpha + 2, \alpha + 2)$, and its scalar product with $\mathbf{q} - \mathbf{p} = (-7/3, 4/3, 0)$ is $4\alpha - 2$. Thus

$$\frac{|4\alpha - 2|}{\sqrt{10\alpha^2 + 16\alpha + 12}} = \frac{1}{\sqrt{3}}.$$

Squaring gives $38\alpha^2 - 64\alpha = 0$, so $\alpha = 0$ or $32/19$. The latter is the only listed value. Substitution, rather than sign alone, rules out each other listed fraction.

Step 3: Conclusion Among the supplied choices, $\alpha = \frac{32}{19}$. Hence, the correct option is (2). Algebra also admits $\alpha = 0$ under a parametric interpretation of the first line; it is not among the choices.

Quick Tip to Solve: Obtain an intersection-line direction from the plane normals before applying the scalar triple-product formula.

30. Let X be a set containing 10 elements and $P(X)$ be its power set. If A and B are picked up at random from $P(X)$, with replacement, then the probability that A and B have equal numbers of elements is: [4 marks]

(1) $\frac{2^{10}-1}{2^{20}}$

(2) $\frac{\binom{20}{10}}{2^{20}}$

(3) $\frac{\binom{20}{10}}{2^{10}}$

(4) $\frac{2^{10}-1}{2^{10}}$

Correct Answer: (2) $\frac{\binom{20}{10}}{2^{20}}$

Solution:

Concept:

- A set of 10 elements has 2^{10} subsets. Independent sampling with replacement makes all 2^{20} ordered pairs (A, B) equally likely.
- There are $\binom{10}{k}$ subsets of size k; equal size does not require the subsets themselves to be equal.

Step 1: Count equal-size ordered pairs For a fixed size k, choose A in $\binom{10}{k}$ ways and independently choose B in the same number of ways. Include both the empty subset and the full subset. Thus the favourable count is

$$\sum_{k=0}^{10} \binom{10}{k}^2.$$

The total count is the square of the number of subsets, since replacement preserves every second choice.

Step 2: Use the binomial convolution By symmetry, $\binom{10}{k} = \binom{10}{10-k}$. The sum is therefore the coefficient of t^{10} in $(1+t)^{10}(1+t)^{10} = (1+t)^{20}$, namely $\binom{20}{10}$. Dividing by 2^{20} gives the required probability. The options using $2^{10}-1$ do not count equal-size pairs; a denominator of 2^{10} ignores the independent second subset.

Step 3: Conclusion The probability is $\frac{\binom{20}{10}}{2^{20}}$. Hence, the correct option is (2).

Quick Tip to Solve: For equal subset sizes, sum squared binomial coefficients and apply Vandermonde's identity.

Physics (120 marks)

31. If the capacitance of a nanocapacitor is measured in terms of a unit u, made by combining the electronic charge e, Bohr radius a_0, Planck's constant h and speed of light c, then [4 marks]

(1) $u = \dfrac{e^2 a_0}{hc}$

(2) $u = \dfrac{hc}{e^2 a_0}$

(3) $u = \dfrac{e^2 c}{h a_0}$

(4) $u = \dfrac{e^2 h}{c a_0}$

Correct Answer: (1) $u = \dfrac{e^2 a_0}{hc}$

Solution:

Concept:

- Capacitance satisfies $C = Q/V$, while electrical energy is QV. Consequently capacitance has dimensions charge squared divided by energy.
- Planck's constant has dimensions energy times time, and c/a_0 has dimensions inverse time. Their product hc/a_0 therefore has the dimensions of energy.

Step 1: Construct an energy scale The available length and speed determine the time scale a_0/c. Dividing h by that time gives

$$E_0 = \frac{hc}{a_0}.$$

This has dimensions ML^2T^{-2}, regardless of the numerical values of the constants. Dimensional analysis determines a possible unit, not the detailed capacitance of a particular geometry.

Step 2: Form and check the capacitance unit Use charge squared divided by this energy:

$$u = \frac{e^2}{E_0} = \frac{e^2 a_0}{hc}.$$

It has dimensions $Q^2M^{-1}L^{-2}T^2$, as required. The reciprocal expression has inverse capacitance dimensions. Replacing a_0/c by c/a_0 gives the wrong powers of length and time, while multiplying by h instead of dividing gives the wrong mass dimension.

Step 3: Conclusion The correct unit is $u = \dfrac{e^2 a_0}{hc}$. Hence, the correct option is (1).

Quick Tip to Solve: Rewrite capacitance as charge squared over energy, and recognise hc/a_0 as an energy.

32. A block of mass $m = 10\,\text{kg}$ rests on a horizontal table. The coefficient of friction between the block and the table is 0.05. When hit by a bullet of mass 50 g moving with speed v, that gets embedded in it, the block moves and comes to stop after moving a distance of 2 m on the table. If a freely falling object were to acquire speed $v/10$ after being dropped from height H, then neglecting energy losses and taking $g = 10\,\text{m s}^{-2}$, the value of H is close to [4 marks]

(1) $0.2\,\text{km}$

(2) $0.5\,\text{km}$

(3) $0.4\,\text{km}$

(4) None of these

Correct Answer: (4) None of these

Solution:

Concept:

- The embedded bullet and block share a common speed immediately after collision. Momentum is conserved during the short impact, but kinetic energy is not.
- After impact, work done by sliding friction removes the combined kinetic energy: $V^2 = 2\mu g s$. A freely falling body satisfies $u^2 = 2gH$.

Step 1: Recover the bullet speed from the stopping distance Let V be the common post-impact speed. Friction produces retardation $\mu g = 0.05(10) = 0.5\,\text{m s}^{-2}$, so

$$V^2 = 2(0.5)(2) = 2, \qquad V = \sqrt{2}\,\text{m s}^{-1}.$$

With bullet mass $0.050\,\text{kg}$, momentum conservation gives $0.050v = 10.050V$, hence $v = 201\sqrt{2}\,\text{m s}^{-1}$. The block mass alone must not replace the combined moving mass.

Step 2: Calculate the free-fall height The required falling speed is $v/10$, rather than the post-impact speed. Starting from rest,

$$H = \frac{(v/10)^2}{2g} = \frac{201^2(2)}{100(20)} = 40.401\,\text{m} = 0.040401\,\text{km}.$$

This is far smaller than all three numerical choices: 0.2, 0.5 and 0.4 km. In particular, confusing metres with kilometres would produce an incorrect match.

Step 3: Conclusion $H \simeq 40.4\,\text{m}$, so none of the stated numerical distances is correct. Hence, the correct option is (4).

Quick Tip to Solve: Work backwards: stopping distance gives the common speed; momentum then gives the original bullet speed.

33. A block of mass $m = 0.1\,\mathrm{kg}$ is connected to a spring of unknown spring constant k. It is compressed to a distance x from its equilibrium position and released from rest. After approaching half the distance $(x/2)$ from the equilibrium position, it hits another block and comes to rest momentarily, while the other block moves with velocity $3\,\mathrm{m\,s^{-1}}$. The total initial energy of the spring is: [4 marks]

(1) $0.6\,\mathrm{J}$

(2) $0.8\,\mathrm{J}$

(3) $1.5\,\mathrm{J}$

(4) $0.3\,\mathrm{J}$

Correct Answer: Insufficient collision data; 0.6 J follows only with an additional elastic-collision assumption.

Solution:

Concept:

- For a spring, potential energy is $U = \frac{1}{2}ky^2$. At displacement $x/2$, it retains one quarter of its initial energy $E = \frac{1}{2}kx^2$.
- Under the usual lossless elastic-collision interpretation, a moving block that stops while a stationary block moves away transfers its speed to an equal-mass block. The printed statement does not explicitly specify that collision assumption or the other mass.

Step 1: Evaluate the moving block's energy at impact Before impact, conservation of spring-plus-block energy gives

$$\frac{1}{2}mu^2 = E - \frac{1}{4}E = \frac{3}{4}E.$$

For an elastic impact, momentum and kinetic-energy conservation with final speed $w = 3\,\mathrm{m\,s^{-1}}$ and the first block at rest imply $mu = Mw$ and $mu^2 = Mw^2$. Dividing yields $u = w = 3\,\mathrm{m\,s^{-1}}$, and then $M = m$.

Step 2: State the result and its assumption The kinetic energy just before that elastic impact is $\frac{1}{2}(0.1)(3)^2 = 0.45\,\mathrm{J}$. Therefore

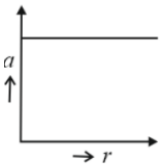
$$\frac{3}{4}E = 0.45, \qquad E = 0.60\,\mathrm{J}.$$

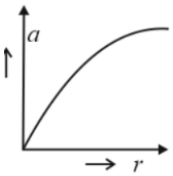
The other numerical options do not satisfy this energy balance. Literally, if an inelastic collision and an unspecified second mass are permitted, momentum alone gives $u = 30M$ in SI units, leaving E undetermined. The numerical answer consequently uses the standard elastic interpretation.

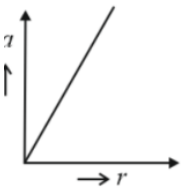
Step 3: Conclusion The literal statement does not determine a unique initial energy. If the collision is additionally assumed elastic, the result would be $0.6\,\mathrm{J}$, corresponding to option (1), but that assumption is not specified.

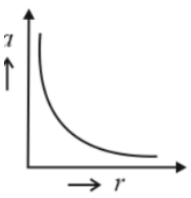
Quick Tip to Solve: At half the original compression the spring retains one quarter of its energy; identify the collision assumption before using the outgoing speed.

34. If a body moving in a circular path maintains constant speed of $10\,\mathrm{m\,s^{-1}}$, then which of the following correctly describes the relation between acceleration and radius? [4 marks]

(1) 

(2) 

(3) 

(4) 

Correct Answer: (4) 

Solution:

Concept:

- Uniform speed does not imply zero acceleration: the velocity direction changes continuously on a circular path.
- The magnitude of centripetal acceleration is $a = v^2/r$, where v is tangential speed and $r > 0$ is the radius.

Step 1: Insert the specified constant speed The speed is fixed at $10\,\mathrm{m\,s^{-1}}$, so its square is fixed at $100\,\mathrm{m^2\,s^{-2}}$. Consequently

$$a = \frac{100}{r}.$$

The product ar is constant, not the acceleration itself. For example, changing the radius from 1 m to 2 m reduces acceleration from 100 to $50\,\mathrm{m\,s^{-2}}$.

Step 2: Identify the graph and exclude alternatives For positive radius, acceleration is positive and decreases as radius grows. Its slope is $-100/r^2$ and its curvature is $200/r^3 > 0$: the curve is decreasing and convex, approaching both coordinate axes asymptotically. This is a rectangular hyperbola in the first quadrant. A horizontal line would mean acceleration is independent of radius. Either increasing graph reverses the inverse dependence and therefore cannot represent constant-speed circular motion.

Step 3: Conclusion The decreasing hyperbolic graph represents $a = 100/r$. Hence, the correct option is (4).

Quick Tip to Solve: Hold the stated speed fixed before comparing graphs: $ar = v^2$ remains constant.

35. Consider a thin uniform square sheet made of a rigid material. If its side is a, mass m and moment of inertia I about one of its diagonals, then: [4 marks]

(1) $I = \dfrac{ma^2}{24}$

(2) $\dfrac{ma^2}{24} < I < \dfrac{ma^2}{12}$

(3) $I > \dfrac{ma^2}{12}$

(4) $I = \dfrac{ma^2}{12}$

Correct Answer: (4) $I = \dfrac{ma^2}{12}$

Solution:

Concept:

- The perpendicular-axis theorem for a plane lamina states $I_z = I_{d_1} + I_{d_2}$, where the two in-plane axes are perpendicular and intersect at the same point as the normal axis.
- The diagonals of a square are mutually perpendicular and equivalent by symmetry. Thus their moments of inertia are equal.

Step 1: Calculate the central normal moment Take central in-plane axes parallel to the sides. A uniform square has $I_x = ma^2/12$ and $I_y = ma^2/12$. Equivalently, these follow by integrating squared distances over a uniform strip distribution from $-a/2$ to $a/2$. Adding them by the perpendicular-axis theorem gives

$$I_z = I_x + I_y = \frac{ma^2}{6}.$$

All axes here pass through the sheet's centre; no parallel-axis shift is needed.

Step 2: Resolve the normal moment into the diagonal moments Let each diagonal moment be I. The square's symmetry makes the two diagonal moments identical, while their perpendicularity permits a second application of the theorem:

$$2I = I_z = \frac{ma^2}{6}, \qquad I = \frac{ma^2}{12}.$$

The smaller value $ma^2/24$ would make their sum too small. The strict intermediate and greater-than choices also contradict the exact symmetry result.

Step 3: Conclusion The moment of inertia about either diagonal is $I = ma^2/12$. Hence, the correct option is (4).

Quick Tip to Solve: For a square, any two perpendicular central in-plane axes share the normal moment equally.

36. A uniform solid cylindrical roller of mass m is being pulled on a horizontal surface with force F parallel to the surface and applied at its centre. If the acceleration of the cylinder is a and it is rolling without slipping then the value of F is [4 marks]

(1) $\frac{3}{2}ma$

(2) $2ma$

(3) $\frac{5}{3}ma$

(4) ma

Correct Answer: (1) $\frac{3}{2}ma$

Solution:

Concept:

- Rolling without slipping couples translation and rotation through $a = \alpha R$, where R is the cylinder radius and α its angular acceleration.
- A uniform solid cylinder has moment of inertia $I = \frac{1}{2}mR^2$ about its axis. Static friction provides the torque because the applied force acts at the centre.

Step 1: Identify the force that produces rotation The applied pull acts through the centre, so it has zero torque about the axis. The contact friction f acts opposite the pull and supplies the clockwise torque needed for rolling. Taking forward translation and the corresponding rotation as positive gives $F - f = ma$ and $fR = I\alpha$. Friction here is static: its role is to prevent relative motion at the contact, rather than to represent sliding resistance.

Step 2: Combine translation and rotation Insert the solid-cylinder inertia and the rolling constraint into the torque equation:

$$fR = \frac{1}{2}mR^2\frac{a}{R}, \qquad f = \frac{1}{2}ma.$$

Hence the translational equation gives

$$F = ma + f = \frac{3}{2}ma.$$

The value ma neglects rotational acceleration. The value $2ma$ would apply to a thin hoop with $I = mR^2$, not a solid cylinder. The factor $5/3$ likewise does not correspond to this inertia.

Step 3: Conclusion The required force is $F = \frac{3}{2}ma$, so the correct option is (1).

Quick Tip to Solve: For a central pull and pure rolling, use $F = ma[1 + I/(mR^2)]$; insert the inertia of the actual body.

37. A very long (length L) cylindrical galaxy is made of uniformly distributed mass and has radius R ($R \ll L$). A star outside the galaxy is orbiting the galaxy in a plane perpendicular to the galaxy and passing through its centre. If the time period of the star is T and its distance from the galaxy's axis is r, then [4 marks]

(1) $T \propto \sqrt{r}$

(2) $T \propto r$

(3) $T \propto r^2$

(4) $T^2 \propto r^3$

Correct Answer: (2) $T \propto r$

Solution:

Concept:

- Far from the ends of a very long uniform cylinder, cylindrical symmetry makes its gravitational field radial. Outside it, the enclosed mass per unit length λ is constant.
- Gravitational Gauss law gives $g(r) = 2G\lambda/r$. Circular motion requires $v^2/r = g(r)$, and the period is $T = 2\pi r/v$.

Step 1: Find the radial dependence of the gravitational field Take a coaxial Gaussian cylinder of radius $r > R$ and length ℓ. The field is normal to its curved surface, while its flux through the ends vanishes in the long-cylinder approximation. The enclosed mass is $\lambda\ell$, so

$$g(2\pi r\ell) = 4\pi G\lambda\ell \quad \Rightarrow \quad g = \frac{2G\lambda}{r}.$$

Here $\lambda = \rho\pi R^2$ depends on the galaxy, not on the orbital radius.

Step 2: Use the field as the centripetal acceleration For the star in its circular orbit,

$$\frac{v^2}{r} = \frac{2G\lambda}{r} \quad \Rightarrow \quad v = \sqrt{2G\lambda}.$$

The speed is therefore independent of r, giving

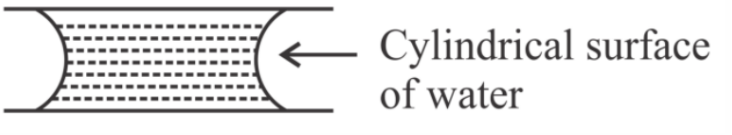
$$T = \frac{2\pi}{\sqrt{2G\lambda}}r.$$

A square-root period, a quadratic period and the point-mass law $T^2 \propto r^3$ all have different radial dependences. The last applies to a spherical or pointlike mass, not to the cylindrical field used here.

Step 3: Conclusion The orbital period varies directly with distance from the axis: $T \propto r$. Hence, the correct option is (2).

Quick Tip to Solve: First identify source symmetry: an effectively infinite line gives $g \propto 1/r$ and constant circular speed.

38. If two glass plates have water between them and are separated by very small distance (see figure), it is very difficult to pull them apart. It is because the water in between forms cylindrical surface on the side that gives rise to lower pressure in the water in comparison to atmosphere. If the radius of the cylindrical surface is R and surface tension of water is T then the pressure in water between the plates is lower by: [4 marks]



(1) $\frac{T}{R}$
(2) $\frac{T}{4R}$
(3) $\frac{4T}{R}$
(4) $\frac{2T}{R}$

Correct Answer: (1) $\frac{T}{R}$

Solution:

Concept:

- Across a single liquid-air interface, the pressure difference is $\Delta p = T(1/R_1 + 1/R_2)$, where T is the surface tension and R_1, R_2 are the principal radii of curvature.
- A cylindrical surface curves in one principal direction and is straight in the other: its radii are R and infinity. The concave water meniscus makes the water pressure lower than atmospheric pressure.

Step 1: Identify the curvature of the meniscus The water surface between the closely spaced plates is explicitly cylindrical. In a cross-section perpendicular to the cylinder axis it is curved with radius R, whereas along the cylinder axis it has no curvature. Thus $1/R_1 = 1/R$ and $1/R_2 = 0$. The two visible sides in the drawing do not mean that pressure jumps must be added: each side separately separates the same interior water from the atmosphere.

Step 2: Apply the pressure jump and compare the factors The atmospheric pressure pushes inward across the concave meniscus, and the water pressure is lower by

$$p_{\text{atm}} - p_{\text{water}} = T\left(\frac{1}{R} + 0\right) = \frac{T}{R}.$$

A factor $2T/R$ would be the pressure jump for one spherical liquid surface, with two finite equal principal radii. A soap bubble has two spherical interfaces and gives $4T/R$. Neither model describes this cylindrical water surface; $T/(4R)$ has no corresponding curvature here.

Step 3: Conclusion The pressure reduction is T/R. Hence, the correct option is (1). The supplied answer key instead gives $2T/R$, but the explicitly cylindrical single interface gives T/R by the Laplace-pressure relation.

Quick Tip to Solve: Count principal curvatures, not the number of menisci visible in a cross-section: cylindrical means one nonzero curvature.

39. If it takes 5 minutes to fill a 15 litre bucket from a water tap of diameter $\frac{2}{\sqrt{\pi}}$ cm, then the Reynolds number for the flow is (density of water $= 10^3\,\mathrm{kg/m^3}$ and viscosity of water $= 10^{-3}\,\mathrm{Pa\,s}$) close to:

[4 marks]

(1) 5500

(2) 550

(3) 1100

(4) 11,000

Correct Answer: (1) 5500

Solution:

Concept:

- The Reynolds number for a pipe of diameter D is $\mathrm{Re} = \rho v D/\eta$, where ρ is density, v is mean speed and η is dynamic viscosity.
- The volume flow rate is $Q = \mathcal{V}/t = Av$, with circular area $A = \pi D^2/4$. Use SI units consistently.

Step 1: Calculate the outlet speed from the filling time The collected volume is $15\,\mathrm{L} = 0.015\,\mathrm{m^3}$ and the filling time is $300\,\mathrm{s}$. Thus

$$Q = \frac{0.015}{300} = 5.0 \times 10^{-5}\,\mathrm{m^3/s}.$$

The printed diameter is $D = 2 \times 10^{-2}/\sqrt{\pi}\,\mathrm{m}$, not simply 2 cm. Therefore

$$A = \frac{\pi}{4}\frac{4 \times 10^{-4}}{\pi} = 10^{-4}\,\mathrm{m^2}, \qquad v = \frac{Q}{A} = 0.50\,\mathrm{m/s}.$$

Step 2: Evaluate the dimensionless flow parameter Substitute the density and viscosity stated in the question:

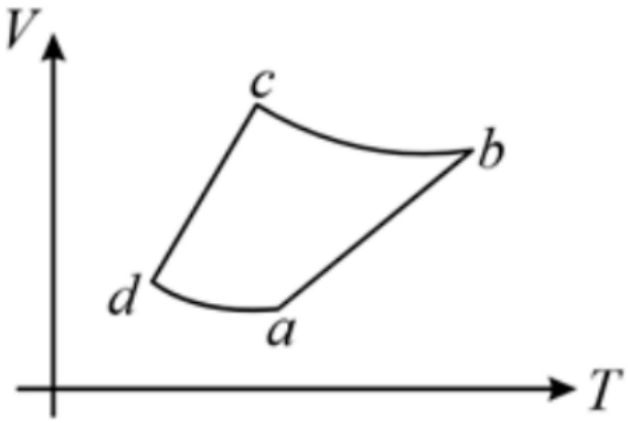
$$\mathrm{Re} = \frac{10^3(0.50)(2 \times 10^{-2}/\sqrt{\pi})}{10^{-3}} = \frac{10^4}{\sqrt{\pi}} \simeq 5642.$$

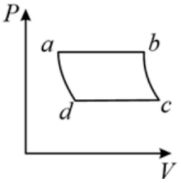
The closest listed value is 5500. The value 550 is too small by about a factor of ten, and 1100 is too small by about five. Using the radius in the diameter-based formula, or a spurious factor of two, changes the result incorrectly; 11,000 is approximately twice the required value.

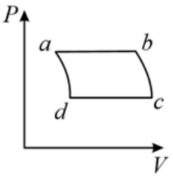
Step 3: Conclusion The Reynolds number is approximately 5.64×10^3, closest to 5500. Hence, the correct option is (1).

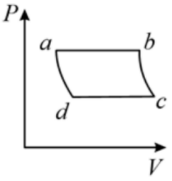
Quick Tip to Solve: For a fixed flow rate, eliminate speed directly: $\mathrm{Re} = 4\rho Q/(\pi\eta D)$, with D in metres.

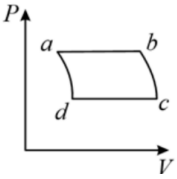
40. An ideal gas goes through a reversible cycle $a \to b \to c \to d$ and has the V-T diagram shown below. Processes $d \to a$ and $b \to c$ are adiabatic. The corresponding P-V diagram for the process is (all figures are schematic and not drawn to scale): [4 marks]



(1) Diagram (1) 

(2) Diagram (2) 

(3) Diagram (3) 

(4) Diagram (4) 

Correct Answer: The printed first and third diagrams show the same physically admissible cycle; the supplied key selects diagram (1).

Solution:

Concept:

- For a fixed amount of ideal gas, $PV = nRT$, so constant-pressure branches have $V \propto T$ and are horizontal in a P-V plot.
- A reversible adiabatic branch satisfies $PV^{\gamma} = \text{constant}$, with $\gamma > 1$. Its pressure decreases as volume increases, and its graph is convex upward.

Step 1: Translate the straight branches and state ordering The straight rising branches $a \to b$ and $c \to d$ in the schematic V-T plot represent the two constant-pressure branches. The lower-slope branch ab has the higher pressure because $V/T = nR/P$. It therefore becomes the upper horizontal branch in the P-V plot, while cd becomes the lower one. Expansion is from a to b, and compression is from c to d.

Step 2: Check both adiabatic branches rather than just their endpoints The transition $b \to c$ is

adiabatic expansion and $d \to a$ is adiabatic compression. Along either branch,

$$P = KV^{-\gamma}, \quad \frac{dP}{dV} = -\frac{\gamma P}{V} < 0, \quad \frac{d^2P}{dV^2} = \frac{\gamma(\gamma+1)P}{V^2} > 0.$$

Thus each adiabat must fall to the right with upward convexity; the smaller-volume branch is steeper at the same pressure. The first and third printed diagrams both show this orientation and curvature, with the same corner labels and pressure levels. Their slight stylistic differences do not supply a physical reason to reject one. The second and fourth diagrams bend the adiabats in the opposite direction, violating the positive second derivative. The source therefore does not distinguish a unique diagram between (1) and (3).

Step 3: Conclusion The physically required cycle is shown by diagrams (1) and (3), which are indistinguishable in the printed choices. The supplied answer key selects (1); the printed figures also admit (3), so a unique selection is ambiguous.

Quick Tip to Solve: On a P-V diagram, test an adiabat with both $dP/dV < 0$ and $d^2P/dV^2 > 0$; do not rely on corner labels alone.

41. In an ideal gas at temperature T, the average force that a molecule applies on the walls of a closed container depends on T as T^q. A good estimate for q is: [4 marks]

(1) 2

(2) $\frac{1}{2}$

(3) 1

(4) $\frac{1}{4}$

Correct Answer: (3) 1

Solution:

Concept:

- For an ideal gas molecule, typical kinetic energy scales with absolute temperature: $\frac{1}{2}m\langle v^2\rangle \propto T$, hence a typical molecular speed scales as $\sqrt{T}$.
- The average force on a wall equals momentum transferred per collision multiplied by collision frequency. The dimensions of the closed container are fixed.

Step 1: Find the temperature dependence of the collision impulse For an elastic collision with a wall perpendicular to the x-axis, the normal velocity reverses, and the magnitude of momentum delivered is $\Delta p_x = 2m|v_x|$. Since a typical $|v_x|$ scales as $T^{1/2}$, the typical impulse per impact has this same temperature dependence. Considering only the impulse would incorrectly suggest $q = 1/2$, because force is impulse per unit time, not impulse alone.

Step 2: Include how frequently the molecule reaches the wall For walls separated by L, successive returns to the same wall occur at intervals $2L/|v_x|$. The average force due to the molecule is therefore

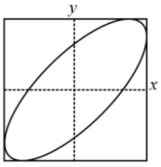
$$F_{\text{avg}} = \frac{2m|v_x|}{2L/|v_x|} = \frac{mv_x^2}{L}.$$

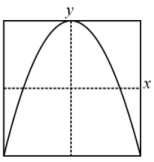
Averaging over velocities gives $\langle F\rangle = m\langle v_x^2\rangle/L = k_{\mathrm{B}}T/L$. Thus force scales linearly with temperature. The exponents 2 and $1/4$ do not follow from either the impulse or the collision-frequency dependence.

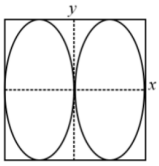
Step 3: Conclusion The average molecular force varies as T^1, so $q = 1$. Hence, the correct option is (3).

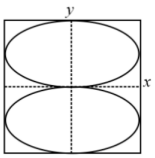
Quick Tip to Solve: For molecular force, count both impulse and collision rate: each contributes one factor of speed, so force scales as v^2.

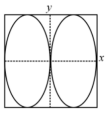
42. The x and y displacements of a particle are given as $x(t) = a\sin\omega t$ and $y(t) = a\sin 2\omega t$. Its trajectory will look like: [4 marks]

(1) Diagram (1) 

(2) Diagram (2) 

(3) Diagram (3) 

(4) Diagram (4) 

Correct Answer: (3) Diagram (3) 

Solution:

Concept:

- Two perpendicular oscillations with frequency ratio $1:2$ can be combined using $\sin 2\theta = 2\sin\theta\cos\theta$. The path is determined by eliminating the phase $\theta = \omega t$.
- The signs of both sine and cosine matter. Squaring removes the phase sign, so the physical trajectory must include both upper and lower branches allowed by time evolution.

Step 1: Eliminate time to obtain the locus Put $u = x/a = \sin\omega t$. Then $y = 2a\sin\omega t\cos\omega t = 2x\cos\omega t$, and $\cos^2\omega t = 1 - x^2/a^2$. Hence

$$y^2 = 4x^2\left(1 - \frac{x^2}{a^2}\right), \qquad -a \le x \le a.$$

This equation is even in both x and y, so the path is symmetric about both coordinate axes. At $x = 0$, it requires $y = 0$; the curve crosses at the origin.

Step 2: Identify the loop direction and reject the alternatives During the first half-period, $x \ge 0$, while y first becomes positive and then negative before returning to zero. This traces the right-hand loop. During the second half-period, $x \le 0$, giving the left-hand loop. The outer horizontal points are $(a, 0)$ and $(-a, 0)$, and maximum vertical displacement occurs at $x = \pm a/\sqrt{2}$. Thus the trajectory has two side-by-side loops, as in the third figure. It is neither a tilted ellipse nor a parabola; the vertical

figure-eight reverses the frequency assignment.

Step 3: Conclusion The trajectory is a horizontal figure-eight, with two lobes meeting at the origin. Hence, the correct option is (3).

Quick Tip to Solve: The coordinate with the lower frequency sets the direction of the two lobes: here x has frequency ω, so the lobes lie left and right.

43. A simple harmonic oscillator of angular frequency $2\,\mathrm{rad\,s^{-1}}$ is acted upon by an external force $F = \sin t\,\mathrm{N}$. If the oscillator is at rest in its equilibrium position at $t = 0$, its position at later times is proportional to: [4 marks]

(1) $\sin t + \frac{1}{2}\cos 2t$

(2) $\cos t - \frac{1}{2}\sin 2t$

(3) $\sin t - \frac{1}{2}\sin 2t$

(4) $\sin t + \frac{1}{2}\sin 2t$

Correct Answer: (3) $\sin t - \frac{1}{2}\sin 2t$

Solution:

Concept:

- For a mass m with natural angular frequency ω_0, Newton's law gives $\ddot{x} + \omega_0^2 x = F(t)/m$. Here $\omega_0 = 2\,\mathrm{rad\,s^{-1}}$, so the coefficient of displacement is 4.
- The complete motion is the sum of a particular forced response and the free response. Both constants of the free response must be fixed by the initial displacement and velocity, not just by the force.

Step 1: Find a particular response The driving force has angular frequency $1\,\mathrm{rad\,s^{-1}}$, different from the natural frequency. Put $x_p = K\sin t$ in the equation of motion. Then $-K\sin t + 4K\sin t = \sin t/m$, giving $K = 1/(3m)$. Thus the forced term is $\sin t/(3m)$; there is no resonant term proportional to time.

Step 2: Apply both initial conditions The general solution is

$$x = A\cos 2t + B\sin 2t + \frac{\sin t}{3m}.$$

At equilibrium initially, $x(0) = 0$, so $A = 0$. Differentiating gives $\dot{x} = 2B\cos 2t + \cos t/(3m)$. The oscillator is initially at rest, so $2B + 1/(3m) = 0$, or $B = -1/(6m)$. Therefore

$$x = \frac{1}{3m}\left(\sin t - \frac{1}{2}\sin 2t\right).$$

Step 3: Check the proposed alternatives Option 1 has a nonzero initial displacement due to its cosine term. Option 2 also has nonzero initial displacement. Option 4 starts at the equilibrium position but has nonzero initial velocity because the two derivative contributions add. Only the minus sign with the half-amplitude free sine term satisfies both initial conditions.

Step 4: Conclusion The displacement is proportional to $\sin t - \frac{1}{2}\sin 2t$. Hence, the correct option is (3).

Quick Tip to Solve: For forced motion starting from rest, check both the expression and its first derivative at zero; the free oscillation cancels the initial forced velocity.

44. A bat moving at $10\,\mathrm{m\,s^{-1}}$ towards a wall sends a sound signal of $8000\,\mathrm{Hz}$ towards it. On reflection, it hears a sound of frequency f. The value of f in Hz is close to (speed of sound $= 320\,\mathrm{m\,s^{-1}}$): [4 marks]

(1) 8258

(2) 8424

(3) 8000

(4) 8516

Correct Answer: (4) 8516

Solution:

Concept:

- For sound travelling through stationary air, a source moving towards a stationary receiver increases the received frequency by $v/(v - u_s)$. The denominator changes because the moving source compresses wavefront spacing.
- A receiver moving towards a stationary source receives frequency multiplied by $(v + u_o)/v$. A stationary wall reflects the frequency it receives; reflection therefore requires two successive Doppler shifts.

Step 1: Find the frequency reaching the wall The bat acts first as a moving source and the wall is a stationary receiver. Use $f_0 = 8000\,\mathrm{Hz}$, sound speed $v = 320\,\mathrm{m\,s^{-1}}$, and bat speed $u = 10\,\mathrm{m\,s^{-1}}$. The incident frequency at the wall is

$$f_w = f_0\frac{v}{v-u} = 8000\frac{320}{310}.$$

It exceeds 8000 Hz because successive wavefronts leave the approaching source closer together.

Step 2: Apply the return Doppler shift The wall is stationary, so its reflected wave has frequency f_w in the air frame. The bat now acts as a receiver moving towards this reflected wave. Therefore

$$f = f_w\frac{v+u}{v} = f_0\frac{v+u}{v-u} = 8000\frac{330}{310} = 8516.13\,\mathrm{Hz}.$$

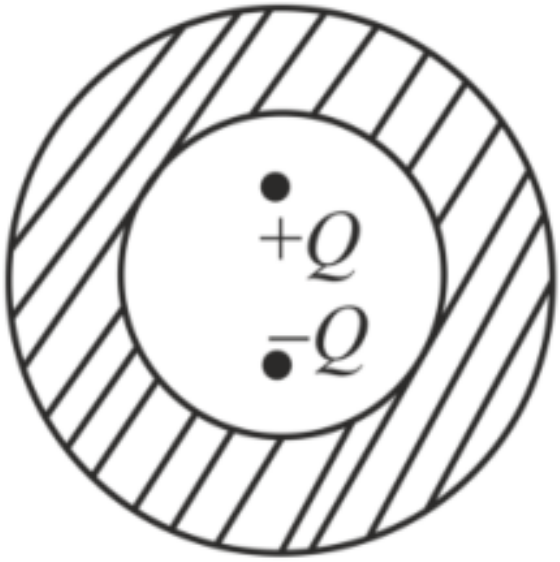
Both shifts increase the frequency: the outward source effect and the returning observer effect must both be included.

Step 3: Compare the numerical choices The nearest listed value is 8516 Hz. The value 8258 Hz is approximately the one-way source shift, so it misses the second encounter. The original frequency 8000 Hz would require no relative approach, while 8424 Hz does not result from the correct product of the two Doppler factors. The reflected wave is heard at a higher pitch than the emitted signal.

Step 4: Conclusion The reflected frequency heard by the bat is approximately $8516\,\mathrm{Hz}$. Hence, the correct option is (4).

Quick Tip to Solve: For an echo from a fixed wall heard by the approaching source, combine the shifts directly as $f_{\text{echo}} = f_0(v+u)/(v-u)$.

45. Shown in the figure are two point charges $+Q$ and $-Q$ inside the cavity of a spherical shell. The charges are kept near the surface of the cavity on opposite sides of the centre of the shell. If σ_1 is the surface charge on the inner surface and Q_1 net charge on it and σ_2 the surface charge on the outer surface and Q_2 net charge on it then: [4 marks]



(1) $\sigma_1 = 0,\ Q_1 = 0,\ \sigma_2 = 0,\ Q_2 = 0$

(2) $\sigma_1 \neq 0,\ Q_1 = 0,\ \sigma_2 \neq 0,\ Q_2 = 0$

(3) $\sigma_1 \neq 0,\ Q_1 \neq 0,\ \sigma_2 \neq 0,\ Q_2 \neq 0$

(4) $\sigma_1 \neq 0,\ Q_1 = 0,\ \sigma_2 = 0,\ Q_2 = 0$

Correct Answer: (4) $\sigma_1 \neq 0,\ Q_1 = 0,\ \sigma_2 = 0,\ Q_2 = 0$

Solution:

Concept:

- In electrostatic equilibrium the electric field inside the conducting material is zero. A Gaussian surface drawn entirely in that material has zero flux and must enclose zero net charge.
- Net induced charge and local surface charge density are different. An inner surface can have positive and negative induced regions whose integral vanishes even though the local density is nonzero.

Step 1: Find the net charge on the inner surface Take a closed Gaussian surface within the shell material surrounding the cavity. Since the field is zero everywhere on it, Gauss's law gives

$$Q_1 + (+Q) + (-Q) = 0.$$

Hence $Q_1 = 0$. This statement determines the integral of the inner surface charge, not its value at each point. The positive and negative internal charges create a nonzero field in the cavity and rearrange the free charges on the wall.

Step 2: Distinguish the local density from its total The inner cavity wall acquires negative induced

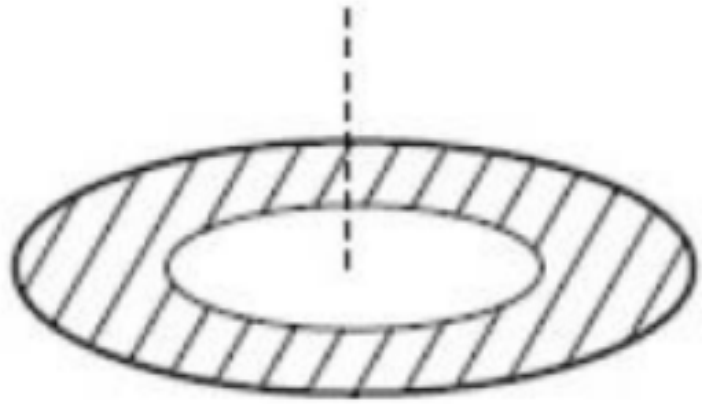
charge near $+Q$ and positive induced charge near $-Q$, ensuring that the conductor stays equipotential. These induced contributions sum to zero because the total cavity charge is zero. Thus $\sigma_1 \neq 0$ in the sense that the inner surface density is not identically zero, while $Q_1 = 0$. It need not be nonzero at every individual surface point.

Step 3: Determine the outer surface charge For the initially uncharged shell in the stated isolated configuration, charge conservation gives $Q_2 = -Q_1 = 0$. Electrostatic shielding prevents the internal dipole from producing an external field. The spherical outer surface is an equipotential; with no external charges and zero total outer charge, its charge density vanishes: $\sigma_2 = 0$. Thus option 1 incorrectly removes the inner polarization, option 2 assigns an external density despite shielding, and option 3 incorrectly gives nonzero net induced charges.

Step 4: Conclusion The inner wall has nonuniform induced charge with zero total: $\sigma_1 \neq 0,\ Q_1 = 0$. The outer surface has $\sigma_2 = 0,\ Q_2 = 0$. Hence, the correct option is (4).

Quick Tip to Solve: Apply Gauss's law for the net inner charge first; then separately ask whether nearby cavity charges require a nonzero local induced density.

46. A thin disc of radius $b = 2a$ has a concentric hole of radius a in it (see figure). It carries uniform surface charge σ on it. If the electric field on its axis at a height h, where $h \ll a$, from its centre is given as Ch, then the value of C is: [4 marks]



(1) $\frac{\sigma}{4a\varepsilon_0}$
(2) $\frac{\sigma}{a\varepsilon_0}$
(3) $\frac{\sigma}{5a\varepsilon_0}$
(4) $\frac{\sigma}{2a\varepsilon_0}$

Correct Answer: (1) $\frac{\sigma}{4a\varepsilon_0}$

Solution:

Concept:

- A uniformly charged annulus can be treated by superposition as a full disc of outer radius b minus a disc of hole radius a, both with the same surface charge density σ.
- The axial field of a disc of radius R at positive height h is $E_R = \sigma[1 - h/\sqrt{h^2 + R^2}]/(2\varepsilon_0)$. For

$h \ll a$, retain the leading term linear in height.

Step 1: Subtract the field of the missing disc The electric fields from the two fictitious discs point along the same positive axis for positive surface charge; subtract the smaller disc because its charge is absent. Thus

$$E = \frac{\sigma h}{2\varepsilon_0}\left(\frac{1}{\sqrt{h^2+a^2}} - \frac{1}{\sqrt{h^2+b^2}}\right).$$

The constant terms in the two disc fields cancel, as expected: the annulus has no charge at its centre and its axial field vanishes in the plane.

Step 2: Use the small-height approximation Because $h \ll a$ and $b = 2a$, one also has $h \ll b$. In the parentheses, replace the square roots by their leading values a and b. Corrections to these reciprocals are of order h^2, and after multiplying by h they contribute only cubic terms. Consequently

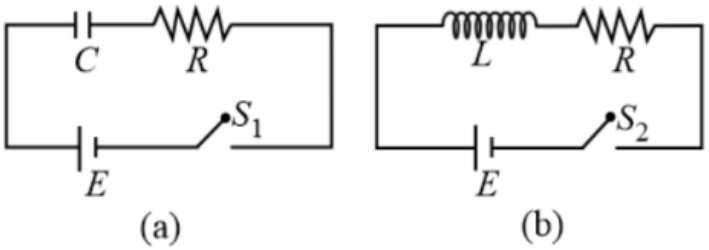
$$E \simeq \frac{\sigma h}{2\varepsilon_0}\left(\frac{1}{a} - \frac{1}{2a}\right) = \frac{\sigma h}{4a\varepsilon_0}.$$

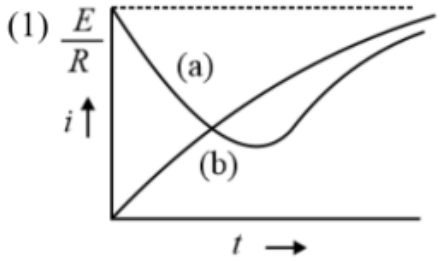
Step 3: Identify the coefficient and check it Comparison with $E = Ch$ gives $C = \sigma/(4a\varepsilon_0)$. The factor one half comes from the disc field formula and another one half from the difference $1/a - 1/(2a)$. The other denominators would miss this combination or introduce an unsupported numerical factor. Dimensionally, σ/ε_0 is an electric field, so division by a correctly makes C a field per unit length.

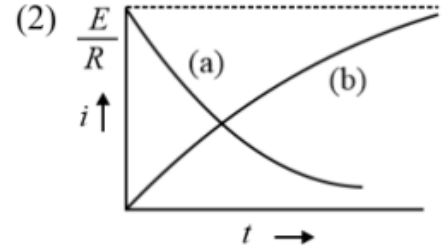
Step 4: Conclusion To leading order in h/a, $C = \dfrac{\sigma}{4a\varepsilon_0}$. Hence, the correct option is (1).

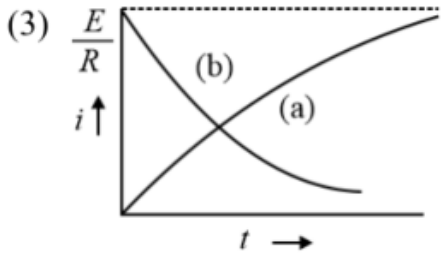
Quick Tip to Solve: For a disc with a concentric hole, subtract full-disc fields before approximating; their constant terms cancel exactly.

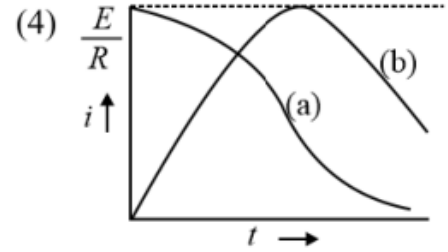
47. In the given circuits (a) and (b), switches S_1 and S_2 are closed at $t = 0$ and kept close for a long time. The variation of currents in the two circuits for $t \geq 0$ are shown in the options. (Figures are schematic and not drawn to scale.) [4 marks]



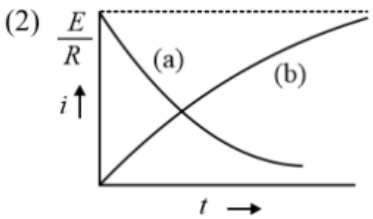
(1)


(2)


(3)


(4)


Correct Answer: (2)

(2)


Solution:

Concept:

- In a series RC charging circuit, an initially uncharged capacitor behaves like a short circuit immediately after closing the switch and like an open circuit after a long time. The current decays exponentially.
- An inductor prevents an instantaneous change of current. In a series RL circuit connected to a constant emf E, current starts at zero and grows towards E/R, with time constant L/R.

Step 1: Find the current in circuit (a) Circuit (a) contains a capacitor C, resistor R, battery emf E, and switch S_1. Immediately after closure, the initially uncharged capacitor has zero potential difference, so the resistor initially gets the entire battery voltage. Hence $i_a(0^+) = E/R$. As charge accumulates, the capacitor voltage increases and reduces the voltage across the resistor. Solving $Ri_a + q/C = E$, with $i_a = dq/dt$, gives

$$i_a(t) = \frac{E}{R} e^{-t/(RC)}.$$

This curve decreases monotonically towards zero.

Step 2: Find the current in circuit (b) Circuit (b) has an inductor L in series with the resistor and battery. Its back emf initially opposes the entire battery emf, so $i_b(0^+) = 0$. The equation $L\, di_b/dt + Ri_b = E$ gives

$$i_b(t) = \frac{E}{R}\left(1 - e^{-Rt/L}\right).$$

This curve rises monotonically and approaches E/R from below. Neither current overshoots or oscillates because each circuit has only one energy-storage component and positive resistance.

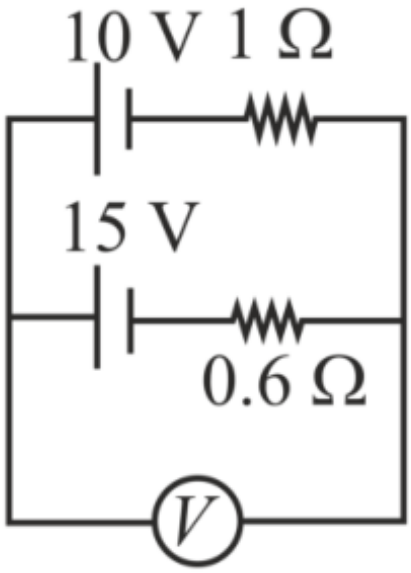
Step 3: Choose the graph with the correct endpoints The correct graph must label the falling curve as (a), from E/R to zero, and the rising curve as (b), from zero to E/R. Option 2 has precisely this behavior. Option 1 makes the decreasing current turn upwards, option 3 interchanges the circuit labels, and option 4 makes the inductive current rise and then fall rather than approach a steady nonzero value.

Step 4: Conclusion The capacitor circuit has exponentially decreasing current, while the inductor circuit has exponentially increasing current approaching E/R. Hence, the correct option is (2).

Quick Tip to Solve: For switching graphs, mark the initial and final currents before considering the detailed curve: capacitor current falls and inductor current rises.

48. A $10\,\mathrm{V}$ battery with internal resistance $1\,\Omega$ and a $15\,\mathrm{V}$ battery with internal resistance $0.6\,\Omega$ are connected in parallel to a voltmeter (see figure). The reading in the voltmeter will be close to:

[4 marks]



(1) $11.9\,\mathrm{V}$

(2) $13.1\,\mathrm{V}$

(3) $12.5\,\mathrm{V}$

(4) $24.5\,\mathrm{V}$

Correct Answer: (2) $13.1\,\mathrm{V}$

Solution:

Concept:

- Cells connected with like terminals together share a common terminal voltage. When their emfs differ, an internal circulating current flows even if an ideal voltmeter draws negligible current.
- The common open-circuit voltage of two parallel cells is the conductance-weighted average $V = (E_1/r_1 + E_2/r_2)/(1/r_1 + 1/r_2)$; it is not the arithmetic sum of their emfs.

Step 1: Find the circulating current The diagram joins the positive terminals together and the negative terminals together. The 15 V cell therefore drives current through the 10 V cell. Around this closed internal loop the emfs oppose, giving net emf $15 - 10 = 5\,\mathrm{V}$. The internal resistances are in series around the loop, so

$$I = \frac{15 - 10}{1 + 0.6} = 3.125\,\mathrm{A}.$$

This current charges the weaker battery and is supplied by the stronger battery.

Step 2: Calculate the common terminal voltage The stronger cell is discharging; its terminal voltage is its emf minus its internal voltage drop:

$$V = 15 - I(0.6) = 15 - 1.875 = 13.125\,\mathrm{V}.$$

The weaker cell is charging, so its terminal voltage is its emf plus its internal drop:

$$V = 10 + I(1) = 10 + 3.125 = 13.125\,\text{V}.$$

The equality is an independent consistency check, since the voltmeter is connected across those same two terminals.

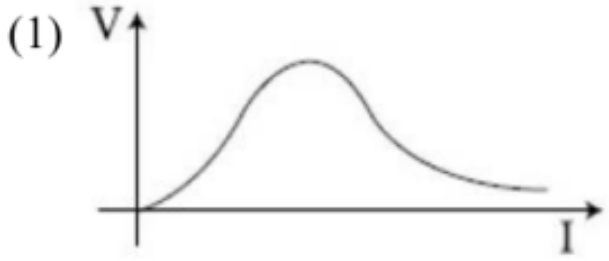
Step 3: Compare with the listed readings An ideal voltmeter reads the common potential difference, approximately 13.1 V. The terminal voltage lies between the emfs and is closer to 15 V because that battery has the smaller internal resistance and hence greater conductance. The arithmetic mean 12.5 V would apply only to equal internal resistances. A value near the sum of emfs would describe a series-aiding connection, absent here; 11.9 V does not satisfy either battery's internal-drop equation.

Step 4: Conclusion The common terminal potential difference is $13.125\,\text{V}$, close to $13.1\,\text{V}$. Hence, the correct option is (2).

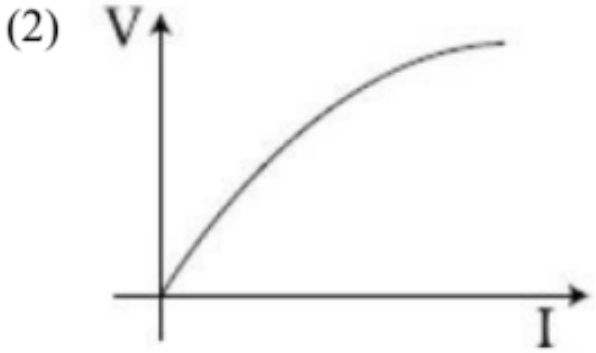
Quick Tip to Solve: For unequal parallel cells, calculate the circulating current from the emf difference and check the terminal voltage using both cells.

49. Suppose the drift velocity v_d in a material varied with the applied electric field E as $v_d \propto \sqrt{E}$. Then the $V - I$ graph for a wire made of such a material is best given by: [4 marks]

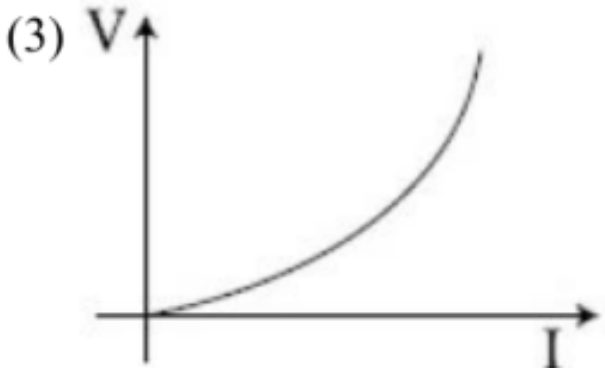
(1)



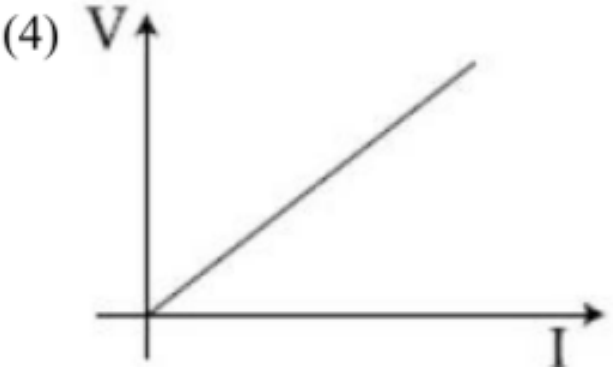
(2)



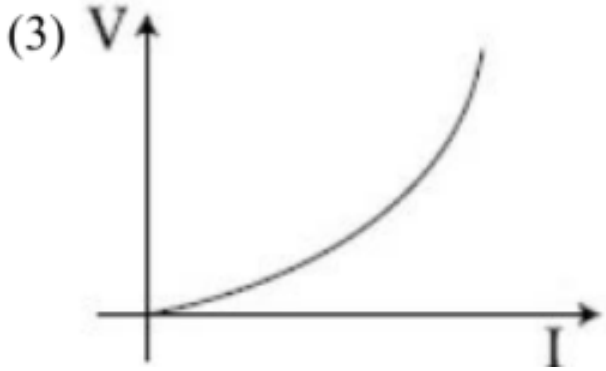
(3)



(4)



Correct Answer: (3)



Solution:

Concept:

- For a wire with carrier number density n, cross-sectional area A, carrier charge magnitude e, and drift speed v_d, the current magnitude is $I = neAv_d$. These geometric and material factors remain constant.
- For a uniform wire of length ℓ, the applied electric field magnitude is $E = V/\ell$. The given drift law therefore determines how current depends on the potential difference.

Step 1: Convert the drift law into a current law Write the stated dependence as $v_d = k\sqrt{E}$, with positive constant k. Substituting into the current formula yields

$$I = neAk\sqrt{\frac{V}{\ell}} = K\sqrt{V}, \qquad K = \frac{neAk}{\sqrt{\ell}} > 0.$$

Thus the current increases with voltage but is not directly proportional to it. Ohm's law with a constant

resistance cannot be imposed on this material because that would contradict the specified drift-speed dependence.

Step 2: Express the vertical quantity in terms of the horizontal quantity The graph has voltage on its vertical axis and current on its horizontal axis. Solve the preceding relation for voltage rather than plotting current as a function of voltage:

$$V = \frac{I^2}{K^2}.$$

This is a parabola through the origin in the first quadrant. Its gradient is $dV/dI = 2I/K^2$, which starts at zero and grows as current increases. Its second derivative $2/K^2$ is positive, so the curve bends upwards.

Step 3: Compare the shapes in the options Option 3 shows a rising curve through the origin with an increasing slope, exactly as $V \propto I^2$ requires. Option 2 is concave down and would fit a square-root dependence of voltage on current, reversing the relation. Option 4 would require a constant drift mobility and a straight ohmic characteristic. Option 1 first rises and then falls, whereas the derived voltage increases monotonically for positive current.

Step 4: Conclusion The required graph is the upward-curving parabola $V \propto I^2$. Hence, the correct option is (3).

Quick Tip to Solve: Always read the graph axes: $I \propto \sqrt{V}$ means $V \propto I^2$, so a voltage-versus-current plot curves upwards.

50. A 25 cm long solenoid has the radius 2 cm and 500 turns. It carries a current of 15 A. If it is equivalent to a magnet of the same size and magnetization $\vec{M}$ (Magnetic moment/volume), then $|\vec{M}|$ is: [4 marks]

(1) 3π A m^{-1}

(2) 30000 A m^{-1}

(3) 30000π A m^{-1}

(4) 300 A m^{-1}

Correct Answer: (2) 30000 A m^{-1}

Solution:

Concept:

- The magnetic dipole moment of a coil of N turns, current I and cross-sectional area A is $\mu = NIA$. Each turn contributes the same directed area, so the moments add.
- Magnetization is magnetic moment per unit volume: $M = \mu/\mathcal{V}$. A cylindrical magnet with the same dimensions has volume $\mathcal{V} = A\ell$, where ℓ is the solenoid length.

Step 1: Express the moment and volume The radius is $r = 2$ cm $= 0.02$ m, so the cross-sectional area is $A = \pi r^2$. The solenoid has $N = 500$, carries $I = 15$ A, and has length $\ell = 25$ cm $= 0.25$ m. Thus its magnetic moment is $500 \times 15 \times \pi(0.02)^2$ A m^2. The equivalent magnet occupies the cylinder of volume $\pi(0.02)^2 \times 0.25$ m^3.

Step 2: Divide the moment by the volume The cross-sectional area cancels because the magnet and solenoid have the same radius. Therefore,

$$M = \frac{NIA}{A\ell} = \frac{NI}{\ell} = \frac{500 \times 15}{0.25} = 30000 \text{ A m}^{-1}.$$

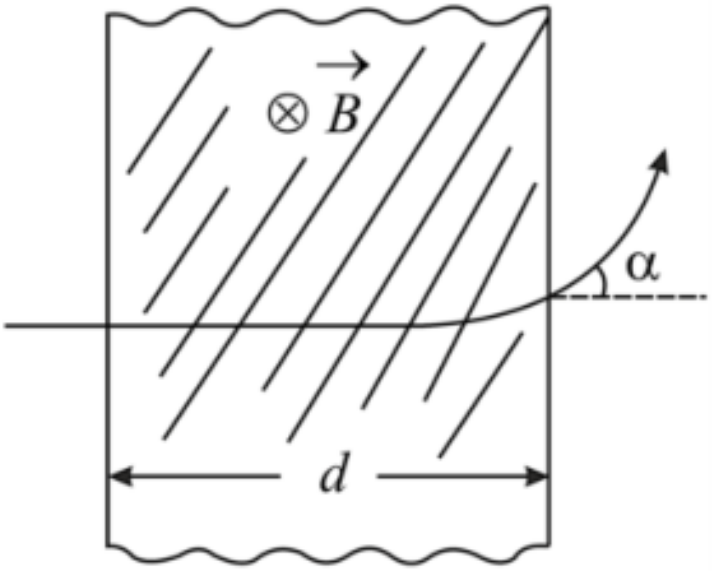
Magnetization is not the total magnetic moment: dividing by the volume changes both its numerical value and its unit.

Step 3: Check the option values The result contains no π, since both magnetic moment and volume contain the same factor πr^2. This excludes (1) and (3). Option (4) is smaller by a factor of one hundred, as would arise from failing to convert the length from centimetres to metres. Only (2) has the required magnitude and unit.

Step 4: Conclusion The magnitude of magnetization is $M = 3.0 \times 10^4$ A m^{-1}. Hence, the correct option is (2).

Quick Tip to Solve: For a solenoid replaced by a magnet of identical dimensions, use $M = NI/\ell$; the radius cancels before substitution.

51. A proton (mass m) accelerated by a potential difference V flies through a uniform transverse magnetic field B. The field occupies a region of space by width d. If α be the angle of deviation of proton from the initial direction of motion (see figure), the value of $\sin\alpha$ will be: [4 marks]



(1) $\frac{B}{2}\sqrt{\frac{qd}{mV}}$

(2) $Bd\sqrt{\frac{q}{2mV}}$

(3) $\frac{B}{d}\sqrt{\frac{q}{2mV}}$

(4) $qV\sqrt{\frac{Bd}{2m}}$

Correct Answer: (2) $Bd\sqrt{\frac{q}{2mV}}$

Solution:

Concept:

- Acceleration from rest through potential difference V gives kinetic energy qV, so $\frac{1}{2}mv^2 = qV$. Here q is the positive proton charge and v its speed at entry.
- A uniform magnetic field perpendicular to velocity supplies centripetal force without changing speed: $qvB = mv^2/r$, giving $r = mv/(qB)$. The trajectory inside the field is a circular arc.

Step 1: Find the proton speed and orbit radius Equating electric work to kinetic energy gives

$$v = \sqrt{\frac{2qV}{m}}.$$

The magnetic force is perpendicular to the instantaneous velocity, so the energy remains constant while the direction changes. Its radius is consequently

$$r = \frac{mv}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}.$$

Step 2: Relate the arc to the field width At entry the tangent is horizontal, and at exit the tangent has rotated through angle α. The corresponding radii also enclose angle α. The horizontal displacement of the circular arc is $r\sin\alpha$, which equals the width d of the magnetic region. Thus

$$\sin\alpha = \frac{d}{r} = Bd\sqrt{\frac{q}{2mV}}.$$

This expression applies when the particle reaches the far boundary, as shown in the figure; it requires $d \le r$.

Step 3: Compare physical dependences The deviation increases with field strength and with the width traversed, but decreases when the accelerating voltage increases. Option (3), proportional to $1/d$, therefore fails immediately. Options (1) and (4) have incorrect dependences on width or voltage and do not equal the derived dimensionless ratio d/r.

Step 4: Conclusion $\sin\alpha = Bd\sqrt{q/(2mV)}$. Hence, the correct option is (2).

Quick Tip to Solve: For a transverse magnetic strip, first write $\sin\alpha = d/r$, then substitute $r = \sqrt{2mV/q}/B$.

52. When the current in a coil changes from 5 A to 2 A in 0.1 s, an average voltage of 50 V is produced. The self- inductance of the coil is [4 marks]

(1) 1.67 H

(2) 6 H

(3) 3 H

(4) 0.67 H

Correct Answer: (1) 1.67 H

Solution:

Concept:

- Self-induction produces an emf opposing the change of current in a coil. For a constant self-inductance L, its average value is $\overline{\mathcal{E}} = -L\,\Delta I/\Delta t$.
- When only the magnitude of voltage is specified, use $|\overline{\mathcal{E}}| = L|\Delta I|/\Delta t$. The SI unit of inductance is the henry, equal to one volt-second per ampere.

Step 1: Find the current change and its rate The initial current is $I_i = 5$ A, the final current is $I_f = 2$ A, and the time interval is $\Delta t = 0.1$ s. Therefore

$$\Delta I = I_f - I_i = 2 - 5 = -3 \text{ A}.$$

The negative sign indicates that the current decreases; its rate of change in magnitude is

$$\frac{|\Delta I|}{\Delta t} = \frac{3}{0.1} = 30 \text{ A s}^{-1}.$$

Step 2: Use the induced voltage to obtain inductance The stated average induced voltage has magnitude 50 V. Rearranging the self-induction relation gives

$$L = \frac{|\overline{\mathcal{E}}|\Delta t}{|\Delta I|} = \frac{50 \times 0.1}{3} = \frac{5}{3} \text{ H} \simeq 1.67 \text{ H}.$$

The sign of the emf is fixed by Lenz's law, but inductance itself is positive. It is not appropriate to use either the initial or final current in place of the change.

Step 3: Check the answer against the data An inductance of 1.67 H multiplied by the rate 30 A s^{-1} produces approximately 50 V, as required. The alternatives 6, 3 and 0.67 H would instead give 180, 90 and approximately 20 V, respectively. They cannot reproduce the stated average voltage.

Step 4: Conclusion The self-inductance is $L \simeq 1.67$ H. Hence, the correct option is (1).

Quick Tip to Solve: For an average induced voltage, divide by the average rate of change of current; use $5 - 2$, not 5 or 2 alone.

53. An electromagnetic wave travelling in the x-direction has frequency of 2×10^{14} Hz and electric field amplitude of $27\ \mathrm{V\,m^{-1}}$ oscillates in Y-direction. From the options given below, which one describes the magnetic field for this wave? [4 marks]

(1) $\vec{B}(x,t) = (9 \times 10^{-8}\ \mathrm{T})\hat{j}\sin[1.5 \times 10^{-6}x - 2 \times 10^{14}t]$

(2) $\vec{B}(x,t) = (9 \times 10^{-8}\ \mathrm{T})\hat{\imath}\sin[2\pi(1.5 \times 10^{-8}x - 2 \times 10^{14}t)]$

(3) $\vec{B}(x,t) = (3 \times 10^{-8}\ \mathrm{T})\hat{j}\sin 2\pi\left[\dfrac{x}{1.5 \times 10^{-8}} - 2 \times 10^{14}t\right]$

(4) $\vec{B}(x,t) = (9 \times 10^{-8}\ \mathrm{T})\hat{k}\sin 2\pi\left[\dfrac{x}{1.5 \times 10^{-6}} - 2 \times 10^{14}t\right]$

Correct Answer: (4) $\vec{B}(x,t) = (9 \times 10^{-8}\ \mathrm{T})\hat{k}\sin 2\pi\left[\dfrac{x}{1.5 \times 10^{-6}} - 2 \times 10^{14}t\right]$

Solution:

Concept:

- For a plane electromagnetic wave in vacuum, the field amplitudes obey $E_0 = cB_0$ and the wavelength is $\lambda = c/\nu$, where $c = 3 \times 10^8\ \mathrm{m\,s^{-1}}$.
- The electric field, magnetic field and direction of travel are mutually perpendicular. Their orientation satisfies $\vec{E} \times \vec{B}$ along the direction of propagation. A wave moving along positive x has phase $2\pi(x/\lambda - \nu t)$.

Step 1: Calculate amplitude and wavelength The electric amplitude is $27\ \mathrm{V\,m^{-1}}$, so

$$B_0 = \frac{E_0}{c} = \frac{27}{3 \times 10^8} = 9 \times 10^{-8}\ \mathrm{T}.$$

From the frequency $\nu = 2 \times 10^{14}$ Hz,

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{2 \times 10^{14}} = 1.5 \times 10^{-6}\ \mathrm{m}.$$

These are independent checks on the numerical amplitude and spatial period of the proposed expressions.

Step 2: Determine direction and phase The electric field oscillates along y, and the wave travels along $+x$. The magnetic field must therefore point along z, since $\hat{j} \times \hat{k} = \hat{\imath}$. Electric and magnetic fields oscillate in phase. Combining these facts gives

$$\vec{B}(x,t) = (9 \times 10^{-8}\ \mathrm{T})\hat{k}\,\sin 2\pi\left[\frac{x}{1.5 \times 10^{-6}\ \mathrm{m}} - (2 \times 10^{14}\ \mathrm{s^{-1}})t\right].$$

The phase is dimensionless, as required.

Step 3: Eliminate the remaining expressions Options (1) and (3) put the magnetic field along y, parallel to the electric field; that is inconsistent with a transverse electromagnetic wave. Option (2) puts it along the propagation direction x, which is also impossible. In addition, the wavelength and amplitude in (3) do not have the calculated values. Option (4) satisfies direction, amplitude, wavelength and frequency simultaneously.

Step 4: Conclusion The required magnetic field is the z-directed wave with amplitude 9×10^{-8} T

and wavelength 1.5×10^{-6} m. Hence, the correct option is (4).

Quick Tip to Solve: Check an electromagnetic-wave option in three passes: $B_0 = E_0/c$, $\lambda = c/\nu$, and the direction of $\vec{E} \times \vec{B}$.

54. A telescope has an objective lens of focal length 150 cm and an eyepiece of focal length 5 cm. If a 50 m tall tower at a distance of 1 km is observed through this telescope in a normal setting, the angle formed by the image of the tower is θ, then θ is close to [4 marks]

(1) 30°

(2) 15°

(3) 1°

(4) 86°

Correct Answer: (4) 86°

Solution:

Concept:

- For an astronomical telescope in normal adjustment, the final image is at infinity and the magnitude of angular magnification is $|M| = f_o/f_e$, where f_o and f_e are the objective and eyepiece focal lengths.
- An object of height h at distance D subtends a small angle $\alpha \simeq h/D$ radians. The standard paraxial telescope relation then gives the image angle $|\theta| \simeq |M|\alpha$.

Step 1: Calculate the angle subtended by the tower The tower height is $h = 50$ m, and its distance is $D = 1$ km $= 1000$ m. Since $h \ll D$, the tower subtends the small angle

$$\alpha \simeq \frac{h}{D} = \frac{50}{1000} = 0.05 \text{ rad}.$$

The objective forms an inverted image, but only the magnitude of the angular size is requested here.

Step 2: Apply the angular magnification The telescope has objective focal length $f_o = 150$ cm and eyepiece focal length $f_e = 5$ cm. Their units cancel in the ratio:

$$|M| = \frac{150}{5} = 30.$$

Using the usual normal-adjustment telescope formula gives

$$|\theta| \simeq 30 \times 0.05 = 1.5 \text{ rad}.$$

Both angles in the magnification relation must be expressed in the same units.

Step 3: Convert to degrees and identify the choice Convert the predicted image angle to degrees:

$$|\theta| \simeq 1.5 \times \frac{180^\circ}{\pi} \simeq 85.9^\circ.$$

Thus the listed value closest to the standard telescope estimate is 86°. The alternatives 30°, 15° and

1° do not equal the angular magnification multiplied by the original angular size. This is the paraxial formula intended by the question; the very large resulting angle lies beyond a precise wide-field optical treatment.

Step 4: Conclusion The standard normal-adjustment estimate gives $\theta \simeq 86^\circ$. Hence, the correct option is (4).

Quick Tip to Solve: Convert the distant tower to a small angle h/D, multiply by f_o/f_e, and only then convert radians to degrees.

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55. You are asked to design a shaving mirror assuming that a person keeps it at 10 cm from his face and views the magnified image of the face at the closest comfortable distance of 25 cm. The radius of curvature of the mirror would then be [4 marks]

(1) 24 cm

(2) 30 cm

(3) 60 cm

(4) -24 cm

Correct Answer: (3) 60 cm

Solution:

Concept:

- A concave shaving mirror produces an erect, enlarged virtual image when the face is between the pole and the focus. The spherical-mirror formula is $1/f = 1/u + 1/v$, with the Cartesian sign convention.
- The distance at which the person views the image is the distance from the eye to the virtual image, not the distance from the mirror to that image. The magnitude of the radius of curvature is $R = 2|f|$.

Step 1: Locate the virtual image relative to the mirror The mirror is $10\ \text{cm}$ in front of the face, and the comfortable eye-to-image distance is $25\ \text{cm}$. Since the image is virtual and behind the mirror, that total distance is the eye-to-mirror distance plus the image distance behind the mirror. Therefore

$$v = 25 - 10 = +15\ \text{cm}.$$

The face is in front of the mirror, so its Cartesian object distance is $u = -10\ \text{cm}$.

Step 2: Apply the mirror formula Substitute the signed object and image distances:

$$\frac{1}{f} = \frac{1}{-10} + \frac{1}{15} = \frac{-3+2}{30} = -\frac{1}{30}\ \text{cm}^{-1}.$$

Thus $f = -30\ \text{cm}$, the negative sign confirming that the mirror is concave. Its curvature radius has magnitude

$$R = 2|f| = 2 \times 30 = 60\ \text{cm}.$$

The corresponding signed Cartesian radius would be $-60\ \text{cm}$; the offered positive value specifies the

physical size.

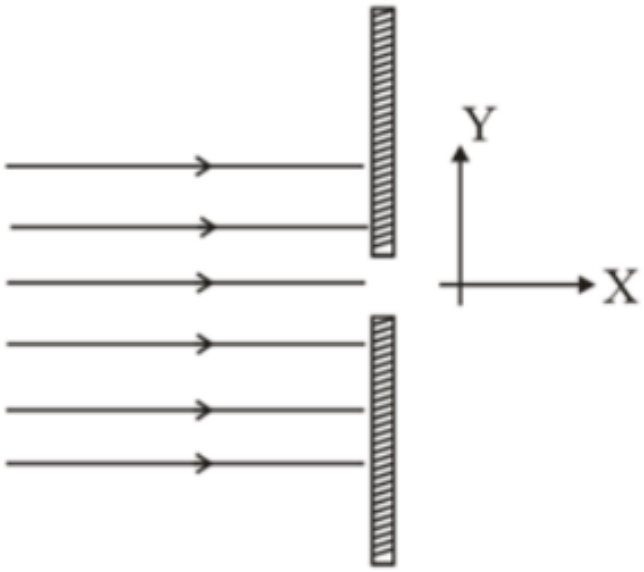
Step 3: Check magnification and reject the distractors The linear magnification is $m = -v/u = 1.5$, so the image is erect and larger than the face, exactly as a shaving mirror requires. Treating 25 cm as the mirror-to-image distance would yield an incorrect geometry. The radii 24, 30 and -24 cm do not satisfy the mirror equation with the required eye-to-image distance. Only the magnitude 60 cm fits all the conditions.

Step 4: Conclusion A concave mirror of radius of curvature 60 cm is required. Hence, the correct option is (3).

Quick Tip to Solve: In a shaving-mirror question, subtract the eye-to-mirror distance from the near-point distance before using the mirror formula.

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56. A parallel beam of electrons travelling in x-direction falls on a slit of width d (see figure). If after passing the slit, an electron acquires momentum p_y in the y-direction then for a majority of electrons passing through the slit (h is Planck's constant): [4 marks]



(1) $|p_y|d < h$

(2) $|p_y|d > h$

(3) $|p_y|d \simeq h$

(4) $|p_y|d \gg h$

Correct Answer: (1) $|p_y|d < h$

Solution:

Concept:

- Electrons have de Broglie wavelength $\lambda = h/p$, where p is their momentum magnitude. Passage through a narrow slit produces diffraction, so the outgoing momentum is spread over different directions.

- For a slit of width d, the first diffraction minima satisfy $d\sin\theta = \lambda$. A large majority of the transmitted particles lies in the central diffraction maximum between these first minima.

Step 1: Relate transverse momentum to the diffraction angle The incident electrons move along x, so initially their transverse momentum is zero. After diffraction an electron travelling at angle θ to the original direction has transverse momentum

$$p_y = p\sin\theta.$$

The slit restricts position in the y-direction and thereby spreads the possible p_y values. The beam is not constrained to emerge with one definite transverse momentum.

Step 2: Evaluate momentum at the first minimum At the boundary of the central maximum, the single-slit condition gives

$$d\sin\theta_1 = \lambda = \frac{h}{p}.$$

Multiplication by p yields

$$p\,d\sin\theta_1 = h.$$

Hence $|p_y|d = h$ at either first-minimum boundary. For particles inside the central maximum, $|\sin\theta| < \sin\theta_1$, and therefore

$$|p_y|d < h.$$

The equality is a boundary condition, rather than a value shared by all outgoing electrons.

Step 3: Interpret the majority condition Most transmitted electrons arrive in the central maximum, where the intensity is concentrated. Thus the question asks for the inequality describing that main population, not the very weak large-angle side lobes. Options (2) and (4) describe particles outside the central-maximum bound, while (3) is only an order-of-magnitude uncertainty estimate. The resolved central diffraction maximum gives the more specific choice (1).

Step 4: Conclusion For a majority of the transmitted electrons, $|p_y|d < h$. Hence, the correct option is (1).

Quick Tip to Solve: Use the first diffraction minimum as the central-maximum boundary: $d\sin\theta = \lambda$ becomes $|p_y|d = h$.

57. De-Broglie wavelength of an electron accelerated by a voltage of $50\,\mathrm{V}$ is close to ($|e| = 1.6 \times 10^{-19}\,\mathrm{C},\ m_e = 9.1 \times 10^{-31}\,\mathrm{kg},\ h = 6.6 \times 10^{-34}\,\mathrm{J\,s}$) [4 marks]

(1) $0.5\,\text{Å}$

(2) $1.2\,\text{Å}$

(3) $1.7\,\text{Å}$

(4) $2.4\,\text{Å}$

Correct Answer: (3) $1.7\,\text{Å}$

Solution:

Concept:

- An electron accelerated from rest through a potential difference V acquires kinetic energy $K = |e|V$. The positive magnitude of the electronic charge is used in this energy expression.
- The de Broglie relation is $\lambda = h/p$. For nonrelativistic motion, $p = \sqrt{2m_eK}$, so $\lambda = h/\sqrt{2m_e|e|V}$. One angstrom is $10^{-10}\,\mathrm{m}$.

Step 1: Find the acquired kinetic energy The field does work equal to charge magnitude times voltage. With the supplied values,

$$K = (1.6 \times 10^{-19})(50) = 8.0 \times 10^{-18}\,\mathrm{J}.$$

This is only $50\,\mathrm{eV}$, far below the electron rest energy, so the ordinary kinetic-energy formula is adequate; a relativistic correction is unnecessary.

Step 2: Calculate the momentum and wavelength Substitute the energy into the momentum relation, retaining the powers of ten:

$$p = \sqrt{2(9.1 \times 10^{-31})(8.0 \times 10^{-18})} = \sqrt{1.456 \times 10^{-47}} \simeq 3.82 \times 10^{-24}\,\mathrm{kg\,m\,s^{-1}}.$$

Therefore

$$\lambda = \frac{6.6 \times 10^{-34}}{3.82 \times 10^{-24}} \simeq 1.73 \times 10^{-10}\,\mathrm{m} = 1.73\,\text{Å}.$$

The nearest listed value is 1.7 angstrom. Values 0.5, 1.2 and 2.4 angstrom do not agree with this momentum; the wavelength scales as the inverse square root of voltage, rather than its inverse.

Step 3: Conclusion The de Broglie wavelength is approximately $1.7\,\text{Å}$. Hence, the correct option is (3).

Quick Tip to Solve: For an electron at modest voltage, use $\lambda(\text{Å}) \simeq 12.3/\sqrt{V(\mathrm{V})}$; at 50 V this immediately gives about 1.74 angstrom.

58. If one were to apply the Bohr model to a particle of mass 'm' and charge 'q' moving in a plane under the influence of a magnetic field 'B', the energy of the charged particle in the nth level will be: [4 marks]

(1) $n\left(\frac{hqB}{\pi m}\right)$

(2) $n\left(\frac{hqB}{4\pi m}\right)$

(3) $n\left(\frac{hqB}{2\pi m}\right)$

(4) $n\left(\frac{hqB}{8\pi m}\right)$

Correct Answer: (2) $n\left(\frac{hqB}{4\pi m}\right)$

Solution:

Concept:

- A magnetic force perpendicular to velocity provides the centripetal force for circular motion: $qvB = mv^2/r$, with q understood as charge magnitude.
- The stipulated Bohr prescription quantizes orbital angular momentum as $mvr = n\hbar = nh/(2\pi)$. The magnetic field does no work, so the mechanical energy here is kinetic energy $K = mv^2/2$.

Step 1: Relate the orbit to the magnetic field For motion in the plane perpendicular to the field, balancing radial forces gives

$$qvB = \frac{mv^2}{r} \quad \Longrightarrow \quad r = \frac{mv}{qB}.$$

The circular angular frequency is consequently $v/r = qB/m$. The sign of charge changes the direction of circulation but does not change the positive kinetic-energy magnitude.

Step 2: Apply angular-momentum quantization Insert the radius into the Bohr angular-momentum rule:

$$mvr = mv\left(\frac{mv}{qB}\right) = \frac{m^2v^2}{qB} = \frac{nh}{2\pi}.$$

Multiplying by $qB/(2m)$ gives

$$E_n = \frac{1}{2}mv^2 = \frac{nhqB}{4\pi m}.$$

Thus the energy is proportional to both n and the field strength. The denominator $2\pi m$ would miss the factor of one half in kinetic energy; denominators πm and $8\pi m$ likewise give the wrong factor. This calculation uses precisely the Bohr model requested, not the full quantum Landau-level spectrum.

Step 3: Conclusion Within the stated Bohr prescription, $E_n = n\,(hqB/(4\pi m))$. Hence, the correct option is (2).

Quick Tip to Solve: Combine $E = \frac{1}{2}L\omega$, $L = n\hbar$ and $\omega = qB/m$ to obtain the energy without first solving for the orbit radius.

59. In an unbiased p–n junction electrons diffuse from n-region to p-region because: [4 marks]

(1) electrons travel across the junction due to potential difference

(2) only electrons move from n to p region and not the vice-versa

(3) electron concentration in n-region is more as compared to that in p-region

(4) holes in p-region attract them

Correct Answer: (3) electron concentration in n-region is more as compared to that in p-region

Solution:

Concept:

- Diffusion is the net thermal transport of particles from higher concentration to lower concentration. In a doped semiconductor, electrons are majority carriers on the n-side and minority carriers on the p-side.
- An unbiased junction can have a built-in electric field even though there is no externally applied voltage. At thermal equilibrium, drift and diffusion currents balance, leaving zero net current.

Step 1: Identify the electron-concentration gradient Donor doping makes the electron concentration in the n-region much greater than that in the p-region. Random thermal motion occurs in both directions across the junction, but more electrons initially cross from the high-concentration n-side into the low-concentration p-side. That imbalance of the two microscopic flows is the electron diffusion current. It exists because of unequal concentrations, not because motion in the reverse direction is impossible.

Step 2: Distinguish diffusion from field-driven drift As electrons and holes diffuse and recombine near the junction, fixed ionized donors and acceptors form the depletion region. Its built-in electric field drives a drift motion opposing further majority-carrier diffusion. Therefore potential difference is associated with drift and the barrier; it is not the cause named for diffusion here. Holes do not simply pull electrons across as an isolated Coulomb attraction mechanism. At equilibrium both drift and diffusion continue and cancel in net current. Hence option (1) confuses the two transport processes, (2) denies reverse thermal motion, and (4) misses the concentration-gradient cause.

Step 3: Conclusion Electron concentration is greater in the n-region than in the p-region, producing net electron diffusion from n to p. Hence, the correct option is (3).

Quick Tip to Solve: When a junction question says diffusion, compare carrier concentrations; when it says drift, look for the electric field and the carrier charge.

60. Diameter of a steel ball is measured using a Vernier calipers which has divisions of $0.1\,\text{cm}$ on its main scale (MS) and 10 divisions of its Vernier scale (VS) match 9 divisions on the main scale. Three such measurements for a ball are given as:

S.No.	MS (cm)	VS divisions
1.	0.5	8
2.	0.5	4
3.	0.5	6

If the zero error is $-0.03\,\text{cm}$, then mean corrected diameter is: [4 marks]

(1) $0.53\,\text{cm}$

(2) $0.56\,\text{cm}$

(3) $0.59\,\text{cm}$

(4) $0.52\,\text{cm}$

Correct Answer: (3) $0.59\,\text{cm}$

Solution:

Concept:

- For a direct vernier, least count is one main-scale division minus one vernier-scale division: $\text{LC} = 1\,\text{MSD} - 1\,\text{VSD}$.
- The observed reading is $\text{MSR} + k\,\text{LC}$, where k is the coincident vernier division. Correct reading equals observed reading minus zero error, so a negative zero error requires a positive correction.

Step 1: Determine the least count The main-scale division is $0.1\,\text{cm}$, and ten vernier divisions span nine main-scale divisions. Thus

$$1\,\text{VSD} = \frac{9}{10}(0.1) = 0.09\,\text{cm}, \qquad \text{LC} = 0.1 - 0.09 = 0.01\,\text{cm}.$$

The vernier contribution must therefore be in hundredths of a centimetre, rather than the tenths shown directly on the main scale.

Step 2: Average the readings and correct the zero error The three observed diameters are

$$d_1 = 0.5 + 8(0.01) = 0.58\,\text{cm}, \quad d_2 = 0.5 + 4(0.01) = 0.54\,\text{cm}, \quad d_3 = 0.5 + 6(0.01) = 0.56\,\text{cm}.$$

Their mean is

$$\overline{d}_{\text{obs}} = \frac{0.58 + 0.54 + 0.56}{3} = 0.56\,\text{cm}.$$

Subtracting the given negative zero error gives

$$\overline{d}_{\text{true}} = 0.56 - (-0.03) = 0.59\,\text{cm}.$$

Option (2) leaves the zero error uncorrected, while option (1) applies the correction with the wrong sign. The value 0.52 cm agrees with neither the mean nor the specified correction.

Step 3: Conclusion The mean corrected diameter of the ball is $0.59\,\text{cm}$. Hence, the correct option is (3).

Quick Tip to Solve: Average the coincident divisions first: $(8+4+6)/3 = 6$. Then calculate $0.5+6(0.01)+0.03$ in one line.

Chemistry (120 marks)

61. A sample of a hydrate of barium chloride weighing $61\,\mathrm{g}$ was heated until all the water of hydration is removed. The dried sample weighed $52\,\mathrm{g}$. The formula of the hydrated salt is: (atomic mass, Ba = 137 amu, Cl = 35.5 amu) [4 marks]

(1) $\mathrm{BaCl_2 \cdot H_2O}$

(2) $\mathrm{BaCl_2 \cdot 3\,H_2O}$

(3) $\mathrm{BaCl_2 \cdot 4\,H_2O}$

(4) $\mathrm{BaCl_2 \cdot 2\,H_2O}$

Correct Answer: (4) $\mathrm{BaCl_2 \cdot 2\,H_2O}$

Solution:

Concept:

- A hydrate contains a definite number of water molecules per formula unit of salt. Its formula can be written $\mathrm{BaCl_2}.x\mathrm{H_2O}$, where x is the ratio of water moles to anhydrous-salt moles.
- Heating removes the water of crystallization while leaving anhydrous $\mathrm{BaCl_2}$. Thus the mass lost is the mass of water, and moles are calculated using $n = m/M$.

Step 1: Separate the water and salt masses The initial hydrate has mass 61 g and the fully dried residue has mass 52 g. Hence

$$m(\mathrm{H_2O}) = 61 - 52 = 9\,\mathrm{g}.$$

The anhydrous salt mass is 52 g, not the original hydrate mass. From the given atomic masses,

$$M(\mathrm{BaCl_2}) = 137 + 2(35.5) = 208\,\mathrm{g\,mol^{-1}}.$$

Using $M(\mathrm{H_2O}) = 18\,\mathrm{g\,mol^{-1}}$, both components can now be converted independently to moles.

Step 2: Obtain the simplest mole ratio The salt and water amounts are

$$n(\mathrm{BaCl_2}) = \frac{52}{208} = 0.25\,\mathrm{mol}, \qquad n(\mathrm{H_2O}) = \frac{9}{18} = 0.50\,\mathrm{mol}.$$

Consequently

$$x = \frac{0.50}{0.25} = 2.$$

The hydrate therefore contains two waters per salt formula unit. As a mass check, its molar mass is $208 + 2(18) = 244\,\mathrm{g\,mol^{-1}}$; one quarter mole weighs 61 g and leaves 52 g of salt. Monohydrate, trihydrate and tetrahydrate would contain respectively 4.5, 13.5 and 18 g of water with this residue, rather than the observed 9 g.

Step 3: Conclusion The formula is $\mathrm{BaCl_2 \cdot 2\,H_2O}$, barium chloride dihydrate. Hence, the correct option is (4).

Quick Tip to Solve: For a hydrate, divide the moles of mass lost as water by the moles of dried salt; never divide the two masses directly.

62. If the principal quantum number $n = 6$, the correct sequence of filling of electrons will be:

[4 marks]

(1) $ns \to (n-1)d \to (n-2)f \to np$

(2) $ns \to np \to (n-1)d \to (n-2)f$

(3) $ns \to (n-2)f \to np \to (n-1)d$

(4) $ns \to (n-2)f \to (n-1)d \to np$

Correct Answer: (4) $ns \to (n-2)f \to (n-1)d \to np$

Solution:

Concept:

- The Aufbau ordering uses the $n+l$ rule: an orbital with smaller $n+l$ fills first. When two subshells have equal $n+l$, the subshell with smaller principal quantum number fills first.
- The azimuthal quantum numbers are $l = 0, 1, 2, 3$ for s, p, d and f respectively. For the indicated sixth-shell sequence, the subshells are $6s, 4f, 5d, 6p$.

Step 1: Evaluate the ordering numbers Substitute $n = 6$ into each subshell and calculate the sum used in the Aufbau rule:

Subshell	$6s$	$4f$	$5d$	$6p$
$n+l$	$6+0$	$4+3$	$5+2$	$6+1$
	6	7	7	7

The 6s subshell has the uniquely smaller sum of 6, so it is filled before the other three subshells. The identical sums of 7 for the remaining subshells require the second part of the rule.

Step 2: Resolve the equal-sum subshells Among 4f, 5d and 6p, the principal quantum numbers increase in the order 4, 5 and 6. Therefore the standard filling order is

$$6s \to 4f \to 5d \to 6p.$$

Rewriting this in the notation of the question gives

$$ns \to (n-2)f \to (n-1)d \to np.$$

The first option reverses 4f and 5d. The second puts 6p before the smaller-principal-number equal-sum subshells, and the third wrongly places 6p before 5d. Detailed exceptions in individual ground-state electronic configurations do not replace the ordering rule being tested here.

Step 3: Conclusion The correct sequence is $ns \to (n-2)f \to (n-1)d \to np$. Hence, the correct option is (4).

Quick Tip to Solve: For equal $n+l$, compare the actual principal quantum numbers of the subshells; 4f precedes 5d, which precedes 6p.

63. In the long form of the periodic table, the valence shell electronic configuration of $5s^2 5p^4$ corresponds to the element present in: [4 marks]

(1) Group 16 and period 5

(2) Group 17 and period 5

(3) Group 16 and period 6

(4) Group 17 and period 6

Correct Answer: (1) Group 16 and period 5

Solution:

Concept:

- For a main-group element, the period number is the highest occupied principal quantum number in its ground-state electronic configuration.
- In the p-block, a valence configuration ns^2np^x has group number $12 + x$, equivalently 10 plus the total number of valence s and p electrons. Configurations ns^2np^4 belong to group 16.

Step 1: Read the period from the outer shell Both the outer s and p subshells are specified with principal quantum number 5: $5s^2$ and $5p^4$. The valence shell is therefore the fifth shell. Consequently the element lies in period 5. The superscripts 2 and 4 count electrons in the subshells; their sum does not specify a period. In particular, adding them to obtain 6 is not a valid way to identify the period.

Step 2: Count the valence electrons and identify the group The valence-shell electron total is

$$N_{\text{valence}} = 2 + 4 = 6.$$

Since this is a p-block configuration, the modern group number is

$$G = 10 + N_{\text{valence}} = 10 + 6 = 16.$$

Equivalently, using the p-electron count directly gives $12 + 4 = 16$. The configuration belongs to the oxygen-family or chalcogen group, with tellurium as the period-5 member. Group 17 requires ns^2np^5, which has one more p electron than the configuration given. The period-6 options would require an outer $6s, 6p$ shell rather than $5s, 5p$. These checks eliminate each alternative without requiring the full inner-shell configuration.

Step 3: Conclusion The element is in group 16 and period 5. Hence, the correct option is (1).

Quick Tip to Solve: Read period from the leading shell number, then count valence s and p electrons: 5 gives period 5 and $2 + 4 + 10 = 16$ gives the group.

64. The shape of $XeOF_4$ by VSEPR theory is: [4 marks]

(1) Trigonal bipyramidal

(2) Square pyramidal

(3) Pentagonal planar

(4) Octahedral

Correct Answer: (2) Square pyramidal

Solution:

Concept:

- VSEPR theory counts regions of electron density around the central atom. A multiple bond counts as one region when determining the arrangement of electron domains.
- For xenon, the valence-shell electron count is eight. Bonding and lone-pair domains together determine electron-pair geometry, whereas the molecular shape describes only the positions of the surrounding atoms.

Step 1: Count the bonding and lone-pair domains In $XeOF_4$, xenon is bonded to four fluorine atoms and one oxygen atom. The oxygen double bond supplies one bonding domain, not two. The four single bonds supply four more domains. Xenon retains one lone pair, giving five bonding domains and one lone-pair domain: the VSEPR type is AX_5E.

Step 2: Distinguish electron geometry from molecular shape Six domains adopt an octahedral electron-pair arrangement. One octahedral vertex is occupied by the lone pair; the other five vertices contain atoms. Removing the lone-pair position from the shape description leaves a square base and one apical atom, hence a square pyramid. Octahedral describes all six electron domains, not the five-atom shape; trigonal bipyramidal requires only five domains without a lone pair, and pentagonal planar does not follow from an octahedral arrangement.

Step 3: Conclusion The molecular shape of $XeOF_4$ is square pyramidal. Hence, the correct option is (2).

Quick Tip to Solve: Count a double bond as one VSEPR domain, then remove lone-pair positions when naming molecular shape.

65. After understanding the assertion and reason, choose the correct option. Assertion: In the bonding molecular orbital (MO) of H_2, electron density is increased between the nuclei. Reason: The bonding MO is $\psi_A + \psi_B$, which shows destructive interference of the combining electron waves. [4 marks]

(1) Assertion and Reason are correct, but Reason is not the correct explanation for the Assertion.

(2) Assertion and Reason are correct and Reason is the correct explanation for the Assertion.

(3) Assertion is incorrect, Reason is correct.

(4) Assertion is correct, Reason is incorrect.

Correct Answer: (4) Assertion is correct, Reason is incorrect.

Solution:

Concept:

- A bonding molecular orbital results from constructive overlap of atomic wavefunctions with matching phases; it concentrates electron density in the internuclear region.
- Electron density is proportional to the square of the wavefunction. The interference term in $(\psi_A + \psi_B)^2$ is positive in the bonding region.

Step 1: Evaluate the assertion For the hydrogen molecule, the bonding orbital is proportional to $\psi_A + \psi_B$, with the appropriate normalization factor understood. Electron density between the two nuclei is greater than for the separated atomic orbitals. Both nuclei attract this shared density, lowering the orbital energy. The assertion therefore correctly states a characteristic property of a bonding molecular orbital.

Step 2: Evaluate the reason and its explanation The sum of in-phase wavefunctions gives constructive, not destructive, interference. Algebraically,

$$(\psi_A + \psi_B)^2 = \psi_A^2 + \psi_B^2 + 2\psi_A\psi_B.$$

The positive cross term increases internuclear density. Destructive overlap corresponds to the antibonding combination, with a node between the nuclei. Although the reason writes the bonding combination, it incorrectly calls its interference destructive. A statement with that incorrect claim is false and cannot explain the true assertion. Thus both options asserting a true reason fail, as does the option calling the assertion incorrect.

Step 3: Conclusion The assertion is correct, but the reason is incorrect. Hence, the correct option is (4).

Quick Tip to Solve: In an assertion-reason item, check every clause of the reason: a correct orbital formula does not rescue an incorrect description of interference.

66. Which of the following is not an assumption of the kinetic theory of gases? [4 marks]

(1) Collisions of gas particles are perfectly elastic.

(2) A gas consists of many identical particles which are in continual motion.

(3) At high pressure, gas particles are difficult to compress.

(4) Gas particles have negligible volume.

Correct Answer: (3) At high pressure, gas particles are difficult to compress.

Solution:

Concept:

- The ideal-gas kinetic model treats molecules as particles of negligible volume undergoing continuous random motion and perfectly elastic collisions.
- An assumption of a model must be distinguished from a high-pressure property of real gases, where finite molecular size can no longer be neglected.

Step 1: Identify the actual ideal-gas assumptions A gas contains a very large number of particles in continual motion. For a pure gas these particles are identical. Collisions between particles and with the container walls are perfectly elastic, so translational kinetic energy is conserved in each collision even though velocities change. The particles themselves occupy negligible volume compared with the volume available to the gas. These are the standard statements represented by options (1), (2), and (4).

Step 2: Examine the high-pressure statement The claim that particles are difficult to compress at high pressure is not an assumption used to derive the ideal-gas kinetic equation. At high pressure, real molecules are closely packed and their finite size becomes important. This concerns real-gas behavior and the failure of the negligible-volume idealization, rather than a postulate of the elementary kinetic model. In the ideal model, pressure arises from momentum transfer to the walls; no separate assumption about difficulty of compressing particles at high pressure is required. Therefore the other three choices are genuine assumptions and this statement is the exception.

Step 3: Conclusion At high pressure, gas particles are difficult to compress is not an assumption of the kinetic theory of gases. Hence, the correct option is (3).

Quick Tip to Solve: For a question asking which is not an assumption, separate ideal-model postulates from real-gas observations at extreme pressure.

67. The heat of atomization of methane and ethane are $360\,\mathrm{kJ\,mol^{-1}}$ and $620\,\mathrm{kJ\,mol^{-1}}$, respectively. The longest wavelength of light capable of breaking the $\mathrm{C{-}C}$ bond is (Avogadro's number $= 6.023 \times 10^{23}$, $h = 6.62 \times 10^{-34}\,\mathrm{J\,s}$) [4 marks]

(1) $2.48 \times 10^{4}\,\mathrm{nm}$

(2) $1.49 \times 10^{4}\,\mathrm{nm}$

(3) $2.48 \times 10^{3}\,\mathrm{nm}$

(4) $1.49 \times 10^{3}\,\mathrm{nm}$

Correct Answer: (4) $1.49 \times 10^{3}\,\mathrm{nm}$

Solution:

Concept:

- Atomization enthalpy is the energy needed to convert one mole of gaseous molecules into separated gaseous atoms; it equals the sum of bond dissociation enthalpies in the bond-energy approximation.
- A photon can just break a bond when hc/λ equals the energy per bond. The longest capable wavelength corresponds to the minimum required photon energy.

Step 1: Extract the carbon-carbon bond energy Methane contains four carbon-hydrogen bonds. Taking their common bond energy as $D_{\mathrm{C-H}}$,

$$4D_{\mathrm{C-H}} = 360 \quad \Rightarrow \quad D_{\mathrm{C-H}} = 90\,\mathrm{kJ\,mol^{-1}}.$$

Ethane has six such bonds and one carbon-carbon bond. Therefore

$$D_{\mathrm{C-C}} = 620 - 6(90) = 80\,\mathrm{kJ\,mol^{-1}}.$$

Using the whole atomization enthalpy of ethane as a single-bond energy would overestimate the threshold.

Step 2: Convert the molar threshold to wavelength The energy for one carbon-carbon bond is

$$E = \frac{80 \times 10^{3}}{6.023 \times 10^{23}}\,\mathrm{J}.$$

Hence, using $c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$,

$$\lambda_{\max} = \frac{hcN_A}{80 \times 10^{3}} = \frac{(6.62 \times 10^{-34})(3.00 \times 10^{8})(6.023 \times 10^{23})}{80 \times 10^{3}}\,\mathrm{m}.$$

This gives $1.495 \times 10^{-6}\,\mathrm{m} = 1.495 \times 10^{3}\,\mathrm{nm}$. All the other listed wavelengths are larger than this threshold wavelength, so their photons have insufficient energy to break the bond in this calculation.

Step 3: Conclusion The longest wavelength is approximately $1.49 \times 10^{3}\,\mathrm{nm}$. Hence, the correct option is (4).

Quick Tip to Solve: Subtract the six carbon-hydrogen bond energies from ethane atomization first, then divide molar energy by Avogadro's number before applying photon energy.

68. Gaseous N_2O_4 dissociates into gaseous NO_2 according to the reaction $N_2O_4(g) \rightleftharpoons 2\,NO_2(g)$. At $300\,K$ and $1\,atm$ pressure, the degree of dissociation of N_2O_4 is 0.2. If one mole of N_2O_4 gas is contained in a vessel, then the density of the equilibrium mixture is: [4 marks]

(1) $3.11\,g\,L^{-1}$

(2) $1.56\,g\,L^{-1}$

(3) $4.56\,g\,L^{-1}$

(4) $6.22\,g\,L^{-1}$

Correct Answer: (1) $3.11\,g\,L^{-1}$

Solution:

Concept:

- For $N_2O_4(g) \rightleftharpoons 2\,NO_2(g)$, dissociating a fraction α of an initial mole produces $1+\alpha$ moles in total.
- Total mass is conserved during dissociation. For an ideal equilibrium gas mixture, $PV = n_{total}RT$, and density is total mass divided by the mixture volume.

Step 1: Determine equilibrium moles and total mass Start with one mole of N_2O_4. With $\alpha = 0.2$, the undissociated amount is 0.8 mole and the amount of NO_2 formed is $2(0.2) = 0.4$ mole. Thus

$$n_{total} = 0.8 + 0.4 = 1.2\,mol.$$

The total mass remains $92\,g$, the molar mass of the initial N_2O_4. Dissociation changes particle number but neither creates nor removes mass.

Step 2: Find the mixture volume and density At $300\,K$ and total pressure $1\,atm$, take $R = 0.0821\,L\,atm\,mol^{-1}\,K^{-1}$. Then

$$V = \frac{nRT}{P} = \frac{(1.2)(0.0821)(300)}{1} = 29.556\,L.$$

Consequently,

$$\rho = \frac{92}{29.556} = 3.11\,g\,L^{-1}.$$

Equivalently, the average molar mass is $92/1.2$, giving the same answer from $\rho = P\overline{M}/(RT)$. Treating the gas as wholly undissociated misses the extra moles; treating it as fully dissociated overestimates them. The other numerical options do not satisfy both mass conservation and the stated dissociation fraction.

Step 3: Conclusion The density of the equilibrium mixture is $3.11\,g\,L^{-1}$. Hence, the correct option is (1).

Quick Tip to Solve: For a dissociating gas, conserve the starting mass but use the final total number of moles when finding the volume.

69. Permanent hardness in water cannot be cured by [4 marks]

(1) Treatment with washing soda

(2) Ion exchange method

(3) Calgon's method

(4) Boiling

Correct Answer: (4) Boiling

Solution:

Concept:

- Permanent hardness is caused mainly by dissolved calcium and magnesium chlorides and sulphates. These salts are not removed simply by heating water to boiling.
- Temporary hardness, caused by bicarbonates, is different: boiling decomposes the bicarbonates and can produce insoluble deposits.

Step 1: Check the three chemical treatment methods Washing soda supplies carbonate ions, which precipitate hardness-producing calcium and magnesium ions as insoluble carbonates. For example,

$$\mathrm{CaCl_2 + Na_2CO_3 \longrightarrow CaCO_3\downarrow + 2\,NaCl.}$$

Ion-exchange resins remove the dissolved hardness ions and replace them with suitable other ions. Calgon, sodium hexametaphosphate, binds calcium ions in soluble complexes, preventing their usual hardness effects. Thus the treatments in (1), (2), and (3) are effective methods for dealing with permanent hardness.

Step 2: Identify why boiling is ineffective Boiling removes temporary bicarbonate hardness because bicarbonates decompose on heating. For example,

$$\mathrm{Ca(HCO_3)_2 \longrightarrow CaCO_3\downarrow + CO_2\uparrow + H_2O.}$$

The chlorides and sulphates responsible for permanent hardness do not undergo the analogous removal merely by boiling. Evaporation may even concentrate these dissolved salts rather than eliminate them. The question asks for the treatment that cannot cure permanent hardness, so it is essential not to apply the familiar temporary-hardness result to all hard water. Boiling is the exceptional method among the four choices.

Step 3: Conclusion Permanent hardness cannot be cured by boiling. Hence, the correct option is (4).

Quick Tip to Solve: First identify the dissolved salt: bicarbonate suggests boiling, while permanent chloride or sulphate hardness needs chemical or ion-exchange treatment.

70. The correct order of thermal stability of hydroxides is [4 marks]

(1) $Mg(OH)_2 < Sr(OH)_2 < Ca(OH)_2 < Ba(OH)_2$

(2) $Mg(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Ba(OH)_2$

(3) $Ba(OH)_2 < Sr(OH)_2 < Ca(OH)_2 < Mg(OH)_2$

(4) $Ba(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Mg(OH)_2$

Correct Answer: (2) $Mg(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Ba(OH)_2$

Solution:

Concept:

- The thermal stability of alkaline-earth metal hydroxides generally increases down group 2 as the cation becomes larger and its polarizing power decreases.
- On heating, these hydroxides can decompose into the corresponding oxide and water: $M(OH)_2 \longrightarrow MO + H_2O$. Greater resistance to this decomposition means greater thermal stability.

Step 1: Relate cation size to polarization All four cations have charge $+2$, but ionic radius increases from magnesium through calcium and strontium to barium. Consequently, charge density and polarizing power decrease down the group. The smaller magnesium ion distorts the hydroxide electron cloud more strongly and favors its breakdown on heating. The larger ions exert less distortion, so their hydroxides are more resistant to thermal decomposition. The trend must therefore be read as increasing, rather than decreasing, stability down the group.

Step 2: Place the given hydroxides in order The periodic order is magnesium, calcium, strontium, barium. Applying the stability trend gives

$$Mg(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Ba(OH)_2.$$

Option (1) incorrectly interchanges calcium and strontium. Options (3) and (4) put barium before the less stable hydroxides and hence reverse the essential down-group trend; (4) also misorders calcium and strontium. Only the sequence in (2) keeps both the correct direction of the trend and the correct periodic positions of all four elements.

Step 3: Conclusion Thermal stability increases as $Mg(OH)_2 < Ca(OH)_2 < Sr(OH)_2 < Ba(OH)_2$. Hence, the correct option is (2).

Quick Tip to Solve: For group 2 hydroxide decomposition, move down the group toward larger, less polarizing cations to identify increasing thermal stability.

71. 1.4 g of an organic compound was digested according to Kjeldahl's method and the ammonia evolved was absorbed in 60 mL of M/10 H_2SO_4 solution. The excess sulphuric acid required 20 mL of M/10 NaOH solution for neutralization. The percentage of nitrogen in the compound is: [4 marks]

(1) 24

(2) 3

(3) 5

(4) 10

Correct Answer: (4) 10

Solution:

Concept:

- Kjeldahl digestion converts the compound's nitrogen into ammonia; each mole of ammonia contains one mole of nitrogen.
- Acid equivalents consumed by ammonia equal the initial acid equivalents minus the acid equivalents remaining after absorption. Sulphuric acid supplies two acid equivalents per mole.

Step 1: Calculate the initial acid capacity The acid molarity is $M/10 = 0.10\,\text{mol}\,\text{L}^{-1}$. Both acidic protons of sulphuric acid are neutralized in this titration, so its normality is $0.20\,\text{eq}\,\text{L}^{-1}$. Thus the initial acid capacity is

$$n_{\text{eq,initial}} = 0.060 \times 0.10 \times 2 = 0.012\,\text{eq}.$$

It is essential to distinguish molarity from normality here: treating the diprotic acid as monoprotic would underestimate the ammonia absorbed.

Step 2: Subtract the excess acid and find nitrogen The excess acid consumes $20\,\text{mL}$ of $0.10\,\text{M}$ sodium hydroxide. Since each mole of NaOH supplies one hydroxide equivalent,

$$n_{\text{eq,excess}} = 0.020 \times 0.10 = 0.002\,\text{eq}.$$

Hence ammonia consumed $0.012 - 0.002 = 0.010$ acid equivalents. From $\text{NH}_3 + \text{H}^+ \longrightarrow \text{NH}_4{}^+$, this corresponds to $0.010\,\text{mol}$ of ammonia and therefore of nitrogen atoms.

Step 3: Convert nitrogen mass to a percentage Taking the molar mass of nitrogen atoms as $14\,\text{g}\,\text{mol}^{-1}$,

$$m_N = 0.010 \times 14 = 0.140\,\text{g}, \qquad \%N = \frac{0.140}{1.4} \times 100 = 10.$$

The other percentages do not satisfy the measured acid consumption; in particular, using the total acid without subtracting its excess would give a different, incorrect result.

Step 4: Conclusion The compound contains 10% nitrogen by mass. Hence, the correct option is (4).

Quick Tip to Solve: Convert both titrants to equivalents first; for Kjeldahl analysis one acid equivalent consumed corresponds to one mole of nitrogen atoms.

72. The optically inactive compound from the following is [4 marks]

(1) 2-chloropropanal

(2) 2-chloro-2-methylbutane

(3) 2-chlorobutane

(4) 2-chloropentane

Correct Answer: (2) 2-chloro-2-methylbutane

Solution:

Concept:

- A tetrahedral carbon is a stereogenic centre only when its four substituents are different. Two identical substituents prevent chirality at that carbon.
- Optical inactivity of an individual constitution can follow from achirality; a racemic mixture is a separate issue and is not specified here.

Step 1: Inspect the carbon bearing chlorine In 2-chloro-2-methylbutane the substituted carbon is bonded to chlorine, an ethyl group, and two methyl groups:

$$CH_3-C(Cl)(CH_3)-CH_2-CH_3.$$

Because the two CH_3 groups are identical, this carbon does not have four different substituents. No other carbon in this molecule is a stereogenic centre. Its mirror image is therefore superimposable, so the molecule is achiral and optically inactive.

Step 2: Compare the remaining structures In 2-chloropropanal, $CH_3-CH(Cl)-CHO$, the carbon bearing chlorine has CH_3, CHO, Cl and H as four different groups. In 2-chlorobutane, the corresponding groups are methyl, ethyl, chlorine and hydrogen. In 2-chloropentane they are methyl, propyl, chlorine and hydrogen. Each of these three constitutions permits a pair of enantiomers, whereas 2-chloro-2-methylbutane does not.

Step 3: Apply the molecular criterion The question distinguishes the listed compounds by their molecular structures. It does not state that any of the chiral compounds is supplied as a racemic mixture. Although equal amounts of two enantiomers can cancel their rotations, that sample-dependent cancellation cannot identify one unique constitution here. The structural distinction is the repeated methyl group at carbon 2 in the second compound.

Step 4: Conclusion 2-Chloro-2-methylbutane is achiral and optically inactive. Hence, the correct option is (2).

Quick Tip to Solve: At a suspected stereocentre, list the four complete substituents; two identical methyl groups immediately rule out that carbon as a chiral centre.

73. A compound A with molecular formula $C_{10}H_{13}Cl$, gives a white precipitate on adding silver nitrate solution. A on reacting with alcoholic KOH gives compound B as the main product. B on ozonolysis gives C and D. C gives Cannizzaro reaction, but not aldol condensation. D gives aldol condensation, but not Cannizzaro reaction. A is [4 marks]

(1) $C_6H_5-CH_2-CH_2-CHCl-CH_3$

(2) $C_6H_5-CH_2-C(Cl)(CH_3)_2$

(3) $C_6H_5-CH_2-CH_2-CH_2-CH_2-Cl$

(4) o-$C_6H_4(CH_2CH_2CH_3)(CH_2Cl)$

Correct Answer: (2) $C_6H_5-CH_2-C(Cl)(CH_3)_2$

Solution:

Concept:

- Ozonolysis cleaves a carbon-carbon double bond into carbonyl compounds; each alkene carbon retains its original substituents.
- Benzaldehyde has no alpha hydrogen and undergoes Cannizzaro reaction; acetone has alpha hydrogens and undergoes aldol condensation.

Step 1: Infer the alkene from the carbonyl reactions The carbonyl product C must have no alpha hydrogen. A suitable aromatic aldehyde is benzaldehyde, C_6H_5CHO. Product D must have alpha hydrogens, as acetone does. Joining the carbonyl carbons back together gives the conjugated alkene

$$C_6H_5-CH=C(CH_3)_2.$$

Its ozonolysis yields C_6H_5CHO and $(CH_3)_2CO$. The former satisfies the Cannizzaro condition and the latter the aldol condition.

Step 2: Identify the chloride giving that alkene The second printed structure is

$$C_6H_5-CH_2-C(Cl)(CH_3)_2.$$

Alcoholic KOH removes a beta hydrogen from the benzylic CH_2 and chloride from the adjacent carbon, producing $C_6H_5-CH=C(CH_3)_2$. This product is both more substituted and conjugated with the phenyl ring, favouring it over the alternative terminal alkene. The tertiary chloride also releases chloride readily in the silver nitrate test, giving white AgCl.

Step 3: Check the competing options The first structure gives carbonyl products both containing alpha hydrogens, so it fails the Cannizzaro condition. The third, nonbenzylic primary chloride does give formaldehyde (Cannizzaro) and a phenyl-substituted aldehyde (aldol) after elimination and ozonolysis. However, it does not readily ionize in the usual silver-nitrate test upon adding the reagent, unlike the tertiary chloride in option (2). Thus the silver-nitrate observation, not ozonolysis alone, distinguishes these candidates. The fourth, benzylic chloride can release chloride but cannot undergo the specified ordinary beta elimination to the required alkene.

Step 4: Conclusion A is $C_6H_5-CH_2-C(Cl)(CH_3)_2$. Hence, the correct option is (2).

Quick Tip to Solve: Reconstruct an alkene by joining the two carbonyl carbons from ozonolysis, then locate a chloride that eliminates to that alkene.

74. Photochemical smog consists of excessive amount of X, in addition to aldehydes, ketones, peroxy acetyl nitrile (PAN). X is: [4 marks]

(1) CH_4

(2) CO_2

(3) O_3

(4) CO

Correct Answer: (3) O_3

Solution:

Concept:

- Photochemical smog forms when nitrogen oxides and organic pollutants react in sunlight, producing secondary oxidants.
- Ozone in the lower atmosphere is a major oxidizing component of photochemical smog, unlike protective stratospheric ozone.

Step 1: Identify the secondary pollutant In sunlight nitrogen dioxide undergoes photolysis, releasing an oxygen atom. That atom combines with molecular oxygen to form ozone:

$$NO_2 \xrightarrow{\text{sunlight}} NO + O, \qquad O + O_2 \longrightarrow O_3.$$

The second process requires a collision partner to remove excess energy, although the simplified equation captures the chemical change. Ozone accumulates when reactions of organic radicals prevent its rapid removal by nitric oxide.

Step 2: Relate ozone to the smog mixture The mixture also includes aldehydes, ketones and PAN, as the question states. These are formed by atmospheric oxidation of hydrocarbons and their interactions with nitrogen oxides. Their presence identifies oxidizing, photochemical smog. Ozone is responsible for strong oxidative effects and is therefore the missing species X. The formula O_3, rather than a single oxygen atom or ordinary O_2, names the persistent secondary pollutant in the listed options.

Step 3: Exclude the other gases Methane, CH_4, can participate in atmospheric chemistry but is not the characteristic excess oxidant requested. Carbon dioxide, CO_2, is chiefly associated with the greenhouse effect and does not account for the smog mixture described. Carbon monoxide, CO, is a toxic primary combustion pollutant, not the principal secondary oxidant identified by the sunlight-driven reactions. Thus none of these three alternatives replaces ozone in the stated composition.

Step 4: Conclusion X is ozone, O_3. Hence, the correct option is (3).

Quick Tip to Solve: For photochemical smog, connect sunlight and nitrogen oxides to secondary ozone; distinguish this from carbon dioxide and its greenhouse role.

75. A solution at $20^{\circ}C$ is composed of 1.5 mol of benzene and 3.5 mol of toluene. If the vapour pressure of pure benzene and pure toluene at this temperature are 74.7 torr and 22.3 torr respectively, then the total vapour pressure of the solution and the benzene mole fraction in equilibrium with it will be, respectively: [4 marks]

(1) 30.5 torr and 0.389

(2) 35.0 torr and 0.480

(3) 38.0 torr and 0.589

(4) 35.8 torr and 0.280

Correct Answer: (3) 38.0 torr and 0.589

Solution:

Concept:

- For an ideal liquid solution, Raoult law gives each partial pressure as $p_i = x_i P_i^{\circ}$, where x_i is its liquid mole fraction.
- The vapour mole fraction is $y_i = p_i / P_{\text{total}}$. A more volatile component is enriched in the equilibrium vapour.

Step 1: Find liquid mole fractions and partial pressures The total number of liquid moles is $1.5+3.5 = 5.0$. Hence

$$x_{\text{benzene}} = \frac{1.5}{5.0} = 0.30, \qquad x_{\text{toluene}} = 0.70.$$

Using the pure-component pressures at the stated temperature,

$$p_{\text{benzene}} = 0.30(74.7) = 22.41\,\text{torr}, \qquad p_{\text{toluene}} = 0.70(22.3) = 15.61\,\text{torr}.$$

The given pressures already refer to the same temperature, so no temperature conversion or gas-law correction is needed.

Step 2: Add pressures and determine the vapour fraction Dalton law gives the total pressure above the liquid:

$$P_{\text{total}} = 22.41 + 15.61 = 38.02\,\text{torr} \simeq 38.0\,\text{torr}.$$

The benzene mole fraction in the equilibrium vapour is consequently

$$y_{\text{benzene}} = \frac{22.41}{38.02} = 0.5894 \simeq 0.589.$$

The phrase in equilibrium with the solution refers to the vapour composition. Its mole fraction is distinct from the liquid value of 0.30.

Step 3: Check the physical consistency Benzene has the larger pure-component vapour pressure, 74.7 torr compared with 22.3 torr for toluene. The calculated vapour fraction 0.589 is therefore correctly larger than its liquid fraction 0.30. The total pressure lies between the two pure-liquid pressures as an ideal mixture requires. Only the third listed pair agrees with both the weighted pressure and the partial-pressure ratio; the remaining pairs fail the numerical calculation.

Step 4: Conclusion The total vapour pressure is $38.0\,\text{torr}$, and the benzene vapour mole fraction is 0.589. Hence, the correct option is (3).

Quick Tip to Solve: First use liquid mole fractions in Raoult law, then divide a partial pressure by the total to obtain the vapour mole fraction.

76. A variable, the opposite external potential E_{ext} is applied to the cell $Zn|Zn^{2+}(1\,M) \parallel Cu^{2+}(1\,M)|Cu$, of potential 1.1 V. When $E_{ext} < 1.1\,V$ and $E_{ext} > 1.1\,V$, respectively electrons flow from [4 marks]

(1) anode to cathode in both the cases

(2) anode to cathode and cathode to anode

(3) cathode to anode and anode to cathode

(4) cathode to anode in both cases

Correct Answer: (2) anode to cathode and cathode to anode

Solution:

Concept:

- The spontaneous Daniell-cell reaction oxidizes zinc and reduces copper ions; electrons travel through the external circuit from zinc to copper.
- An opposing applied potential smaller than the cell emf cannot reverse the reaction. A larger opposing potential drives the reverse electrolysis.

Step 1: Consider an opposing potential below the emf The cell has an emf of $1.1\,V$. In its galvanic operation, zinc is the anode and copper the cathode:

$$Zn \longrightarrow Zn^{2+} + 2\,e^-, \qquad Cu^{2+} + 2\,e^- \longrightarrow Cu.$$

When $E_{ext} < 1.1\,V$, the net driving emf is still in the spontaneous direction. Electrons therefore leave zinc and travel towards copper through the external circuit, that is, from the original anode to the original cathode.

Step 2: Increase the opposing potential beyond the emf When $E_{ext} > 1.1\,V$, the applied source overcomes the cell emf and reverses the current. Copper is oxidized and zinc ions are reduced:

$$Cu \longrightarrow Cu^{2+} + 2\,e^-, \qquad Zn^{2+} + 2\,e^- \longrightarrow Zn.$$

Relative to the electrode names of the original galvanic cell, the electron flow is now from cathode to anode. At exact equality the ideal net current would vanish, but equality is not either case asked.

Step 3: Interpret the electrode terminology consistently Anode and cathode are defined physically by oxidation and reduction. In the driven reverse process their actual reaction roles interchange. The answer options use the labels of the original zinc-copper cell to compare directions, so one must retain those reference labels when reading the choices. The first and fourth options incorrectly keep the same direction for both inequalities; the third reverses the order of the two cases. Only the second describes the smaller and larger opposing potentials in the stated order.

Step 4: Conclusion The directions are anode to cathode, then cathode to anode, relative to the original cell labels. Hence, the correct option is (2).

Quick Tip to Solve: Compare the opposing external voltage with the emf first; below it the cell discharges, above it the original reaction runs backwards.

77. The reaction $2\,N_2O_5(g) \longrightarrow 4\,NO_2(g) + O_2(g)$ follows first order kinetics. The pressure of a vessel containing only N_2O_5 was found to increase from 50 mm Hg to 87.5 mm Hg in 30 min. The pressure exerted by the gases after 60 min will be (assume temperature remains constant): [4 marks]

(1) 106.25 mm Hg

(2) 125 mm Hg

(3) 116.25 mm Hg

(4) 150 mm Hg

Correct Answer: (1) 106.25 mm Hg

Solution:

Concept:

- At constant temperature and vessel volume, gas pressure is proportional to the total number of moles. Reaction stoichiometry determines the increase as decomposition proceeds.
- For a first-order reaction the fraction of reactant remaining is multiplied by the same factor in each equal time interval.

Step 1: Express total pressure using the decomposed fraction Let α be the fraction of initial N_2O_5 decomposed. Per mole of initial reactant, the reaction produces 2α moles of NO_2 and $\alpha/2$ moles of O_2, while $1-\alpha$ mole remains. Thus

$$n_{\text{total}} = n_0\left(1+\frac{3}{2}\alpha\right), \qquad P_t = 50\left(1+\frac{3}{2}\alpha\right).$$

Complete decomposition would give $P_\infty = 125\,\text{mm Hg}$. This is the limiting pressure, not automatically the pressure after 60 minutes.

Step 2: Use the pressure after thirty minutes Insert the observed pressure:

$$87.5 = 50 + 75\alpha_{30} \quad \Longrightarrow \quad \alpha_{30} = \frac{37.5}{75} = \frac{1}{2}.$$

Exactly half the reactant remains after 30 minutes, so the first-order half-life is 30 minutes. After another equal interval, half of that remaining amount survives. The remaining fraction after 60 minutes is $(1/2)^2 = 1/4$, and therefore $\alpha_{60} = 3/4$.

Step 3: Calculate and compare the final pressure The total pressure at 60 minutes is

$$P_{60} = 50 + 75\left(\frac{3}{4}\right) = 106.25\,\text{mm Hg}.$$

It lies between 87.5 and the complete-decomposition limit of 125 mm Hg. The rise in pressure in the second interval is smaller than that in the first, as expected for first-order decay. A linear extrapolation of the first pressure increase is invalid. The 150 mm Hg option even exceeds the stoichiometric limiting pressure, while 116.25 mm Hg does not correspond to two half-lives.

Step 4: Conclusion After 60 minutes the total pressure is $106.25\,\text{mm Hg}$. Hence, the correct option is (1).

Quick Tip to Solve: Find the complete-decomposition pressure from stoichiometry; the fraction of the total possible pressure rise equals the fraction decomposed.

78. The following statements relate to the adsorption of gases on a solid surface. Identify the incorrect statement among them [4 marks]

(1) Entropy of adsorption is negative

(2) Enthalpy of adsorption is negative

(3) On adsorption decrease in surface energy appears as heat

(4) On adsorption, the residual forces on the surface are increased

Correct Answer: (4) On adsorption, the residual forces on the surface are increased

Solution:

Concept:

- Adsorption is accumulation of gas molecules at the surface of a solid. Attractive residual forces at surface sites bind the adsorbate and lower the surface free energy.
- Gas molecules lose translational freedom on adsorption, so the entropy change is normally negative: $\Delta S_{\text{ads}} < 0$. Ordinary gas adsorption is exothermic, with $\Delta H_{\text{ads}} < 0$.

Step 1: Check the entropy and enthalpy statements Before adsorption the gas molecules move freely throughout the available volume. After adsorption they are restricted to surface sites, so their accessible configurations decrease. Thus statement (1) correctly describes the negative entropy change. Forming attractive interactions between the gas and the solid releases energy; consequently statement (2), which says that the enthalpy of adsorption is negative, is also correct.

Step 2: Identify what happens to the surface forces Surface atoms possess unbalanced attractive forces because their coordination differs from that of atoms inside the solid. Adsorbed molecules occupy sites and satisfy these residual forces. The surface energy consequently decreases and the released energy appears as heat, as statement (3) says. The residual forces are reduced by adsorption; they are not increased. Therefore statement (4) reverses the physical effect and is the requested incorrect statement. The question asks for the incorrect statement, so the three true statements must not be selected.

Step 3: Conclusion Adsorption satisfies residual surface forces rather than increasing them. Hence, the correct option is (4).

Quick Tip to Solve: Underline "incorrect" first. On adsorption, remember the linked changes: less molecular freedom, lower surface energy and satisfaction of residual forces.

79. In the isolation of metals, calcination process usually results in [4 marks]

(1) Metal oxide

(2) Metal carbonate

(3) Metal sulphide

(4) Metal hydroxide

Correct Answer: (1) Metal oxide

Solution:

Concept:

- Calcination heats an ore strongly in the absence or limited supply of air. Carbonate and hydrated ores are commonly converted into metal oxides by this treatment.
- Thermal decomposition removes volatile components such as CO_2 and water. Roasting, in contrast, uses excess air and is commonly applied to sulphide ores.

Step 1: Apply calcination to a carbonate ore A carbonate loses carbon dioxide on heating, leaving the corresponding oxide. For example, zinc carbonate gives zinc oxide:

$$\mathrm{ZnCO_3} \xrightarrow{\Delta} \mathrm{ZnO} + \mathrm{CO_2}.$$

The metal remains in the solid product while the gaseous carbon dioxide escapes. Therefore a metal carbonate is generally a starting material for calcination rather than the usual final product. This directly supports the oxide option.

Step 2: Check hydrated ores and eliminate the alternatives Hydrated oxides and hydroxides lose water on heating. A representative decomposition is

$$2\,\mathrm{Al(OH)_3} \xrightarrow{\Delta} \mathrm{Al_2O_3} + 3\,\mathrm{H_2O}.$$

Again, the nonvolatile product is a metal oxide. Thus both major ore types treated by calcination point to the same answer. A metal hydroxide is generally dehydrated instead of being produced. A metal sulphide is not the characteristic product of this process; sulphide ores are usually roasted in air to form oxides and sulphur dioxide. The word "usually" refers to this standard metallurgical preparation of an oxide before reduction, rather than to every possible thermal treatment of every metal compound.

Step 3: Conclusion Calcination usually converts the concentrated ore into a metal oxide. Hence, the correct option is (1).

Quick Tip to Solve: Distinguish the atmosphere: calcination uses little or no air, while roasting uses excess air. Carbonate or hydrated ore on heating usually leaves an oxide.

80. The least number of oxyacids are formed by [4 marks]

(1) Chlorine

(2) Fluorine

(3) Sulphur

(4) Nitrogen

Correct Answer: (2) Fluorine

Solution:

Concept:

- An oxyacid contains hydrogen, oxygen and another element. The range of oxidation states available to that element helps determine the variety of oxyacids it forms.
- Fluorine is the most electronegative element. Unlike the other halogens, it does not show the usual positive oxidation states needed for a family of conventional halogen oxyacids.

Step 1: Compare fluorine with chlorine Chlorine forms a familiar series of oxyacids: $HClO$, $HClO_2$, $HClO_3$ and $HClO_4$. These correspond to several positive oxidation states of chlorine. Fluorine does not form an analogous series because it is more electronegative than oxygen and normally has oxidation state -1 in its compounds. Hypofluorous acid, HOF, is the exceptional oxyacid associated with fluorine; it does not establish a series comparable with the chlorine oxyacids.

Step 2: Check sulphur and nitrogen Sulphur forms numerous oxyacids, including sulphurous acid H_2SO_3, sulphuric acid H_2SO_4, pyrosulphuric acid $H_2S_2O_7$ and peroxoacids. Nitrogen also has several oxyacids, notably nitrous acid HNO_2, nitric acid HNO_3 and hyponitrous acid $H_2N_2O_2$. Thus neither sulphur nor nitrogen has the exceptional restriction seen for fluorine. Chlorine is ruled out by its four standard oxyacids alone. Counting the common examples is sufficient to establish the comparison: fluorine forms the least number among the four given elements. It is the unusual electronegativity and oxidation-state behaviour of fluorine that decides the answer.

Step 3: Conclusion Fluorine forms the least number of oxyacids among the listed elements. Hence, the correct option is (2).

Quick Tip to Solve: For a halogen oxyacid comparison, separate fluorine first: its extreme electronegativity prevents the usual positive-oxidation-state oxyacid series.

81. The cation that will not be precipitated by H_2S in the presence of dilute HCl is: [4 marks]

(1) Co^{2+}

(2) As^{3+}

(3) Pb^{2+}

(4) Cu^{2+}

Correct Answer: (1) Co^{2+}

Solution:

Concept:

- Hydrogen sulphide is a weak acid. Added HCl suppresses its ionisation through the common-ion effect, making the concentration of S^{2-} very small.
- A metal sulphide precipitates only when its ionic product exceeds its solubility product. Very insoluble group-II sulphides precipitate even from acidic solution; cobalt sulphide requires a higher sulphide-ion concentration.

Step 1: Determine the sulphide concentration in acid The overall dissociation can be represented as

$$H_2S \rightleftharpoons 2\,H^+ + S^{2-}.$$

Increasing H^+ shifts this equilibrium toward undissociated hydrogen sulphide. In terms of the two acid dissociation constants,

$$[S^{2-}] = \frac{K_{a1}K_{a2}[H_2S]}{[H^+]^2}.$$

Thus dilute hydrochloric acid keeps the sulphide concentration low. The presence of hydrogen sulphide alone does not mean every metal ion will precipitate.

Step 2: Compare the four cations The sulphides of arsenic, lead and copper are sufficiently insoluble to precipitate under these acidic conditions: As_2S_3, PbS and CuS are the relevant solids. Cobalt(II) belongs to the later sulphide group: the ionic product for CoS does not reach its precipitation condition at the low sulphide concentration imposed by dilute acid. It is precipitated using hydrogen sulphide in an ammoniacal or alkaline medium, where more sulphide ion is available. Consequently As^{3+}, Pb^{2+} and Cu^{2+} are eliminated, leaving Co^{2+} as the cation that remains unprecipitated in the stated test.

Step 3: Conclusion Co^{2+} is not precipitated by H_2S in dilute HCl. Hence, the correct option is (1).

Quick Tip to Solve: Use the medium as the clue: acidic hydrogen sulphide precipitates very insoluble group-II sulphides; cobalt needs the higher sulphide concentration of an alkaline medium.

82. An aqueous solution of a salt X turns blood red on treatment with SCN^- and blue on treatment with $K_4[Fe(CN)_6]$. X also gives a positive chromyl chloride test. The salt X is: [4 marks]

(1) $FeCl_3$

(2) $Fe(NO_3)_3$

(3) $CuCl_2$

(4) $Cu(NO_3)_2$

Correct Answer: (1) $FeCl_3$

Solution:

Concept:

- Ferric ions give a blood-red thiocyanato complex with SCN^-. They also give a deep blue ferric ferrocyanide precipitate with potassium ferrocyanide.
- The chromyl chloride test identifies chloride ions. On heating a chloride with dichromate and concentrated sulphuric acid, red vapours of CrO_2Cl_2 are formed.

Step 1: Identify the cation from the two colour tests The blood-red colour on adding thiocyanate is characteristic of ferric ion, represented by

$$Fe^{3+} + SCN^- \rightleftharpoons [FeSCN]^{2+}.$$

The blue product with $K_4[Fe(CN)_6]$ provides a second confirmation: ferric ions form Prussian blue, commonly represented as $Fe_4[Fe(CN)_6]_3$. Both tests point to Fe^{3+}, rather than Cu^{2+}. Therefore both copper salts, options (3) and (4), are ruled out before using the anion test.

Step 2: Identify the anion and combine the results A positive chromyl chloride test requires chloride. A representative ionic equation is

$$Cr_2O_7^{2-} + 4\,Cl^- + 6\,H^+ \longrightarrow 2\,CrO_2Cl_2 + 3\,H_2O.$$

Nitrate does not provide the chloride needed to form chromyl chloride. Between the two ferric salts, $Fe(NO_3)_3$ can account for the iron tests but not the positive chloride test, so option (2) fails. $FeCl_3$ supplies both Fe^{3+} and Cl^- and satisfies all three observations. The salt must fit the cation tests and the anion test simultaneously; matching only the blood-red colour would leave an incomplete identification.

Step 3: Conclusion The cation is Fe^{3+} and the anion is Cl^-, so X is $FeCl_3$. Hence, the correct option is (1).

Quick Tip to Solve: Use colour tests to select the metal ion, then use chromyl chloride to select the anion. Ferric tests alone cannot distinguish ferric chloride from ferric nitrate.

83. Which molecule/ion among the following cannot act as a ligand in complex compounds? [4 marks]

(1) CN^-

(2) CH_4

(3) CO

(4) Br^-

Correct Answer: (2) CH_4

Solution:

Concept:

- A conventional ligand is a Lewis base that donates an available electron pair to a metal centre to form a coordinate bond. A donor atom must possess an accessible pair or suitable donor electron density.
- Neutrality does not prevent ligand behaviour: both neutral molecules and anions may be ligands. The deciding property is availability of electrons for coordination, rather than charge alone.

Step 1: Examine cyanide, carbon monoxide and bromide Cyanide, CN^-, is an established ligand with donor electron density, commonly coordinating through carbon. Carbon monoxide, CO, is a neutral ligand and donates through its carbon lone pair; its bonding can also involve metal-to-ligand back donation. Bromide, Br^-, has available lone pairs and acts as a halide ligand. Hence options (1), (3) and (4) all contain species capable of conventional coordination. In particular, rejecting carbon monoxide merely because it is neutral would be incorrect.

Step 2: Check methane against the donor requirement In methane, CH_4, carbon forms four saturated covalent sigma bonds to hydrogen. There is no free lone pair on carbon or hydrogen available for the ordinary electron-pair donation described in coordination chemistry. Methane therefore cannot act as a normal ligand in the sense used in this question. The contrast with CO is useful: both are neutral carbon compounds, but carbon monoxide has a donor lone pair whereas methane does not. The conclusion follows from the electronic structure of each species and not from its physical state or whether its overall charge is zero.

Step 3: Conclusion CH_4 lacks an available donor lone pair and is the required species. Hence, the correct option is (2).

Quick Tip to Solve: Locate a donor lone pair before classifying a ligand. A neutral molecule such as carbon monoxide can coordinate, but saturated methane has no conventional donor lone pair.

84. The correct statement on the isomerism associated with the following complex ions, (a) $[Ni(H_2O)_5NH_3]^{2+}$, (b) $[Ni(H_2O)_4(NH_3)_2]^{2+}$ and (c) $[Ni(H_2O)_3(NH_3)_3]^{2+}$, is: [4 marks]

(1) (a) and (b) show only geometrical isomerism

(2) (a) and (b) show geometrical and optical isomerism

(3) (b) and (c) show geometrical and optical isomerism

(4) (b) and (c) show only geometrical isomerism

Correct Answer: (4) (b) and (c) show only geometrical isomerism

Solution:

Concept:

- The six monodentate ligands around these nickel ions give octahedral arrangements. Geometrical isomers differ in relative positions; optical isomers are nonsuperimposable mirror images.
- For an octahedral complex, MA_5B has one arrangement, MA_4B_2 has cis and trans forms, and MA_3B_3 has facial and meridional forms. Here A is water and B is ammonia.

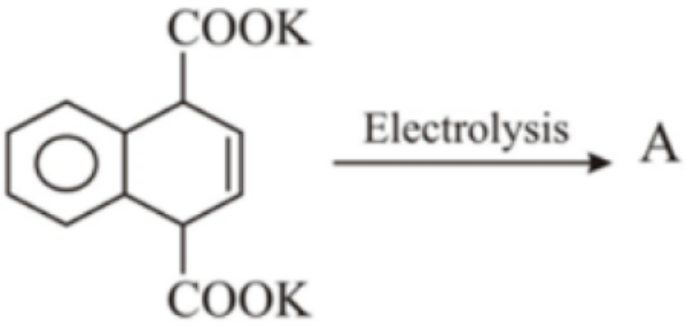
Step 1: Classify complexes (a) and (b) Complex (a), $[Ni(H_2O)_5NH_3]^{2+}$, is of type MA_5B. All six octahedral positions are equivalent for the single ammonia ligand, so changing its position does not produce a distinct isomer. It therefore shows no geometrical isomerism. Complex (b), $[Ni(H_2O)_4(NH_3)_2]^{2+}$, is of type MA_4B_2: the two ammonia ligands can be adjacent at 90° (cis) or opposite at 180° (trans). These are geometrical isomers, and both have symmetry that prevents optical activity.

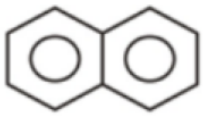
Step 2: Classify complex (c) and select the statement Complex (c), $[Ni(H_2O)_3(NH_3)_3]^{2+}$, is of type MA_3B_3. In the facial form the three ammonia ligands occupy one triangular face; in the meridional form they lie in a plane through the metal and include an opposite pair. The fac and mer arrangements are geometrically distinct but each is achiral for these identical monodentate ligand sets. Thus (b) and (c) show only geometrical isomerism. Options (1) and (2) wrongly include (a) as geometrically isomeric. Option (3) wrongly assigns optical isomerism to (b) and (c). No drawing is needed to establish the octahedral composition and symmetry classification.

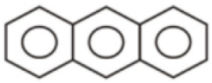
Step 3: Conclusion The second complex has cis/trans forms and the third complex has fac/mer forms; neither has optical isomerism. Hence, the correct option is (4).

Quick Tip to Solve: For octahedral complexes with two kinds of monodentate ligands, remember the sequence: 5:1 gives one form, 4:2 gives cis/trans, and 3:3 gives fac/mer.

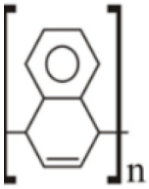
85. In the electrolysis shown below, A is: [4 marks]

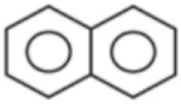


(1) Naphthalene. 

(2) Anthracene. 

(3) The bridged product shown. 

(4) The polymer shown. 

Correct Answer: (1) Naphthalene. 

Solution:

Concept:

- Kolbe electrolysis oxidises a carboxylate at the anode and removes its carboxyl carbon as carbon dioxide: $\mathrm{RCOO^- \longrightarrow R^\bullet + CO_2 + e^-}$. The radical remains on the carbon originally bonded to the carboxyl group.
- An appropriate cyclic diradical can redistribute its radical electrons and an existing π-electron pair into additional conjugated double bonds. Formation of an aromatic ring is strongly stabilising.

Step 1: Locate the two radicals after decarboxylation The two COOK groups are bonded to the top and bottom carbons of the right-hand ring. Each carboxylate loses an electron and then $\mathrm{CO_2}$, giving radical centres at these two ring carbons. Potassium ions do not become part of the organic product. The six-carbon ring skeleton remains intact. Between the two radical-bearing carbons along the outer side of this ring is the original carbon–carbon double bond, so the reactive portion has the form $\mathrm{^\bullet C-C=C-C^\bullet}$.

Step 2: Redistribute the electrons to obtain the aromatic product The two radical electrons and the existing double-bond electron pair supply four electrons. Redistribute these into two new π bonds: the top radical carbon forms a double bond to its adjacent outer carbon, and the bottom radical carbon forms a double bond to the other outer carbon. The former central double bond becomes single. The resulting fused conjugated system is naphthalene, with aromatic stabilisation. This electron redistribution does not add a new carbon–carbon sigma bridge across the ring. Thus the strained bridged alternative (3) is not selected. Anthracene, alternative (2), requires an additional ring and extra carbon atoms absent from the retained skeleton. Alternative (4) is a polymer, whereas this

substrate can form the stable fused aromatic molecule directly.

Step 3: Conclusion The product A is naphthalene, structure (1), formed after two decarboxylations and aromatising electron redistribution. Hence, the correct option is (1).

Quick Tip to Solve: After removing both carboxylates, inspect any $^{\bullet}C-C=C-C^{\bullet}$ segment: rearranging its four electrons into two π bonds can restore aromaticity instead of making a strained cross-ring bond.

86. In the presence of a small amount of phosphorous, aliphatic carboxylic acid reacts with chlorine or bromine to yield a reaction in which, $\alpha-$hydrogen is been replaced by halogen. This reaction is known as [4 marks]

(1) Etard reaction

(2) Wolff-Kischner reaction

(3) Rosenmund reaction

(4) Hell-volhard-zelinsky reaction

Correct Answer: (4) Hell-volhard-zelinsky reaction

Solution:

Concept:

- The Hell–Volhard–Zelinsky (HVZ) reaction replaces an α-hydrogen of a carboxylic acid by chlorine or bromine in the presence of phosphorus.
- The α-carbon is the carbon directly adjacent to the $-COOH$ group. A suitable acid must possess at least one hydrogen on that carbon.

Step 1: Identify the site and reagents The question specifies an aliphatic carboxylic acid, a small amount of phosphorus, and either chlorine or bromine. It also states that the hydrogen replaced is at the α-position. These are the characteristic reagent and position clues for HVZ halogenation. For a typical acid with an α-methylene group, the overall change may be represented as

$$RCH_2COOH \xrightarrow{X_2, P} RCHXCOOH, \qquad X = Cl \text{ or } Br.$$

The carboxyl group is retained: the change concerns a neighbouring carbon–hydrogen bond.

Step 2: Distinguish the other named reactions Phosphorus helps generate an acyl halide, whose enol can undergo halogenation; hydrolysis ultimately restores the acid group. The Etard reaction oxidises a benzylic methyl group to an aldehyde using chromyl chloride, so it is not this substitution. Wolff–Kishner reduction converts an aldehyde or ketone carbonyl group to a methylene group using hydrazine and strong base. Rosenmund reduction converts an acyl chloride to an aldehyde using a poisoned palladium catalyst. None of those alternatives describes halogen replacement of an acid's α-hydrogen.

Step 3: Conclusion The stated α-halogenation of an aliphatic carboxylic acid is the Hell–Volhard–Zelinsky reaction. Hence, the correct option is (4).

Quick Tip to Solve: Carboxylic acid + Cl_2 or Br_2 + phosphorus, with substitution beside $-COOH$, is the diagnostic HVZ combination.

87. The correct order of basicity is [4 marks]

(1) Aniline < p-methoxyaniline < p-nitroaniline < methylamine.

(2) p-Methoxyaniline < p-nitroaniline < aniline < methylamine.

(3) p-Nitroaniline < aniline < p-methoxyaniline < methylamine.

(4) Methylamine < aniline < p-methoxyaniline < p-nitroaniline.

Correct Answer: (3) p-Nitroaniline < aniline < p-methoxyaniline < methylamine.

Solution:

Concept:

- A base accepts a proton through an available lone pair. Greater availability of the nitrogen lone pair generally increases the basicity of an amine.
- In aniline, the nitrogen lone pair is delocalised into the benzene ring. Para electron-withdrawing substituents reduce basicity, whereas a para electron-donating substituent can increase it.

Step 1: Compare the three aromatic amines The $-NO_2$ group withdraws electron density strongly through both inductive and resonance effects. At the para position it makes the amino nitrogen's lone pair less available, so p-nitroaniline is less basic than aniline. A para $-OCH_3$ group has an electron-donating resonance effect that outweighs its inductive withdrawal in this comparison. It enriches the aromatic system and increases the basicity relative to aniline. Thus the aromatic members follow

$$\text{p-nitroaniline} < \text{aniline} < \text{p-methoxyaniline}.$$

Step 2: Place methylamine and eliminate the other orders In methylamine, CH_3NH_2, the nitrogen lone pair is not delocalised into an aromatic ring. The methyl group's positive inductive effect also supports electron density on nitrogen. Consequently methylamine is more basic than all three aromatic amines here. Option (1) incorrectly puts p-nitroaniline above p-methoxyaniline. Option (2) starts with p-methoxyaniline as the weakest aromatic base. Option (4) wrongly places methylamine below the aromatic bases. Only the third order simultaneously places the nitro compound lowest and methylamine highest.

Step 3: Conclusion The increasing basicity order is p-nitroaniline < aniline < p-methoxyaniline < methylamine, which is printed structure order (3). Hence, the correct option is (3).

Quick Tip to Solve: First separate aliphatic from aromatic amines; then rank the aromatic compounds by whether the para substituent withdraws or donates electron density.

88. Match the polymers in column-A with their main uses in column-B and choose the correct answer:

Column-A	Column-B
A. Polystyrene	i. Paints and lacquers
B. Glyptal	ii. Rain coats
C. Polyvinyl chloride	iii. Manufacture of toys
D. Bakelite	iv. Computer discs

[4 marks]

(1) A - iii , B - i , C - ii , D - iv

(2) A - ii , B - i , C - iii , D - iv

(3) A - ii , B - iv , C - iii , D - i

(4) A - iii , B - iv , C - ii , D - i

Correct Answer: (1) A - iii , B - i , C - ii , D - iv

Solution:

Concept:

- Polymer uses depend on properties such as flexibility, ease of moulding, resistance to chemicals, and electrical insulation. Match each named polymer to its standard application before reading the combined codes.
- Polystyrene and polyvinyl chloride are addition polymers; glyptal and Bakelite are condensation polymers with characteristic commercial applications.

Step 1: Match polystyrene and glyptal Polystyrene is readily moulded into rigid objects and is used in manufacturing toys. It therefore matches A with iii. Glyptal is an alkyd polyester prepared from ethylene glycol and phthalic acid or anhydride. It is used in paints and lacquers, so B matches i. These first two matches already distinguish the correct code from alternatives that associate polystyrene with rain coats or glyptal with computer discs. Do not confuse a material's broad possible applications with the specific standard matching set supplied here.

Step 2: Match PVC and Bakelite and check every code Polyvinyl chloride, or PVC, is used in flexible waterproof products such as rain coats, giving C with ii. Bakelite is a hard, mouldable phenol–formaldehyde thermosetting resin; within the listed application set it is paired with computer discs, giving D with iv. The complete matching is therefore

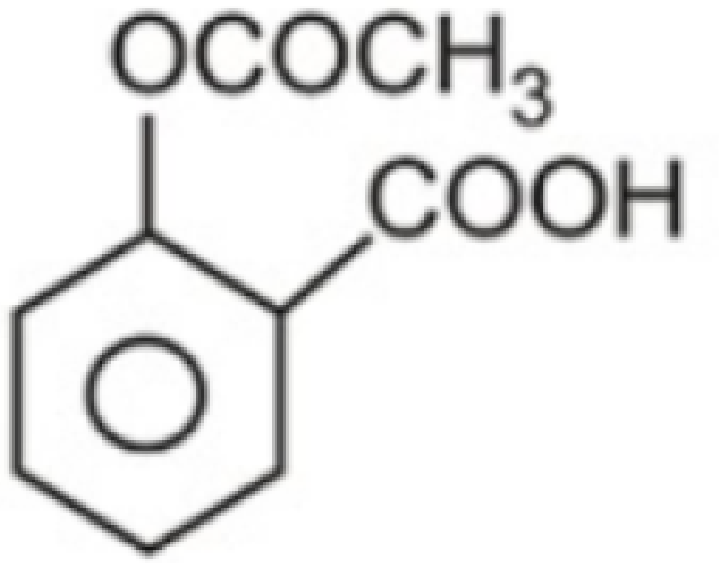
$$A : iii, \qquad B : i, \qquad C : ii, \qquad D : iv.$$

Option (2) interchanges the applications assigned to polystyrene and PVC. Options (3) and (4) pair glyptal with computer discs and Bakelite with paints and lacquers, which do not match the prescribed commercial uses. Only option (1) contains all four required pairs.

Step 3: Conclusion The matching is A–iii, B–i, C–ii, D–iv. Hence, the correct option is (1).

Quick Tip to Solve: Anchor glyptal to paints and lacquers and PVC to rain coats; checking these two distinctive pairs quickly narrows the matching codes.

89. The compound shown below is used as: [4 marks]



(1) Antacid

(2) Insecticide

(3) Antihistamine

(4) Analgesic

Correct Answer: (4) Analgesic

Solution:

Concept:

- The displayed compound has a benzene ring bearing adjacent $-\mathrm{COOH}$ and $-\mathrm{OCOCH_3}$ groups. This is acetylsalicylic acid, commonly called aspirin.
- An analgesic relieves pain. Antacids neutralise excess stomach acid, antihistamines oppose histamine action, and insecticides control insects.

Step 1: Identify the functional groups and compound Read the top substituent carefully: the ring is bonded through oxygen to $\mathrm{COCH_3}$, so it is an acetoxy group, not a free phenolic $-\mathrm{OH}$. The adjacent substituent is a carboxylic acid group. The molecule is therefore the acetylated derivative of salicylic acid, namely 2-acetoxybenzoic acid or acetylsalicylic acid. Recognising that ester linkage is essential: salicylic acid and its acetylated derivative are different substances, even though both retain the carboxyl group.

Step 2: Classify its therapeutic action and reject other choices Acetylsalicylic acid is aspirin, a non-narcotic analgesic that is used to reduce pain; it also has antipyretic and anti-inflammatory actions. Its pain-relieving action is associated with inhibition of prostaglandin synthesis. The presence of an acidic group does not make it an antacid: an antacid neutralises excess gastric acid, whereas aspirin can irritate the stomach. The structure is not one of the insecticidal compounds suggested by option (2), and aspirin is not classified as an antihistamine. Thus only the analgesic category fits the displayed medicine.

Step 3: Conclusion The compound is aspirin (acetylsalicylic acid), which is used as an analgesic.

Hence, the correct option is (4).

Quick Tip to Solve: An ortho $-COOH$ group beside $-OCOCH_3$ on benzene identifies aspirin; distinguish its acetoxy group from the hydroxyl group of salicylic acid.

90. Complete hydrolysis of starch gives [4 marks]

(1) Galactose and fructose in equimolar amounts

(2) Glucose and galactose in equimolar amounts

(3) Glucose and fructose in equimolar amounts

(4) Glucose only

Correct Answer: (4) Glucose only

Solution:

Concept:

- Starch is a polysaccharide made entirely from α-D-glucose units. Its two components are amylose and amylopectin.
- Complete hydrolysis cleaves the glycosidic bonds of a polysaccharide to release its constituent monosaccharides. Partial hydrolysis may stop at smaller carbohydrates, but complete hydrolysis does not.

Step 1: Determine the monomer in both components of starch Amylose contains glucose residues joined primarily by $\alpha(1\rightarrow4)$ glycosidic linkages. Amylopectin contains the same glucose residues, with $\alpha(1\rightarrow4)$ chains and $\alpha(1\rightarrow6)$ branch points. Branching changes the architecture of the polymer, not the identity of its monosaccharide building blocks. Neither starch component contains alternating fructose or galactose units.

Step 2: Apply complete rather than partial hydrolysis Water cleaves each glycosidic linkage under suitable acidic or enzymatic conditions. When hydrolysis is complete, no residual starch, dextrin or maltose is the final carbohydrate product; the residues are released as glucose. The usual repeat-unit representation is

$$(C_6H_{10}O_5)_n + nH_2O \longrightarrow nC_6H_{12}O_6.$$

The molecular formula on the right must be interpreted using the known monomer: glucose, not an arbitrary hexose. Option (1) gives galactose and fructose, neither of which forms starch. Option (2) incorrectly includes galactose, while option (3) introduces fructose as if the starting carbohydrate were sucrose. Option (4) alone lists only the actual monomer.

Step 3: Conclusion Complete hydrolysis of starch gives glucose only. Hence, the correct option is (4).

Quick Tip to Solve: Identify a polysaccharide by its monomer before hydrolysing it: starch and cellulose give glucose, whereas sucrose gives glucose together with fructose.