JEE Main 2015 Paper 1 (B.E./B.Tech) Apr 4 Shift 1 (morning) Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2015, Annual (offline), conducted by the Central Board of Secondary Education (CBSE) in pen and paper mode on 4 April 2015, Single sitting. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. If the function.

$$g(x) = \begin{cases} k\sqrt{x+1}, & 0 \le x \le 3 \\ mx + 2, & 3 < x \le 5 \end{cases}$$

is differentiable, then the value of $k + m$ is : [4 marks]

(A) $\frac{10}{3}$

(B) 4

(C) 2

(D) $\frac{16}{5}$

Correct Answer: (C) 2

Solution:

Concept:

- Differentiability at an interior joining point requires both continuity and equality of the left and right derivatives.
- For the two branches, the derivatives are $k/(2\sqrt{x+1})$ and m, respectively. The only interior junction is $x = 3$.

Step 1: Impose continuity at the junction At $x = 3$, the first branch gives $g(3) = k\sqrt{4} = 2k$. The right-hand limit of the linear branch is $3m + 2$. Thus continuity requires

$$2k = 3m + 2.$$

Equality of derivatives alone would not ensure differentiability if the function jumped at the junction.

Step 2: Equate the one-sided derivatives The left derivative at the junction is $k/(2\sqrt{4}) = k/4$, while the right derivative is m. Therefore

$$m = \frac{k}{4}.$$

Substitute this relation into continuity:

$$2k = \frac{3k}{4} + 2, \qquad \frac{5k}{4} = 2.$$

Hence $k = 8/5$, $m = 2/5$. Both conditions are satisfied; the branches are otherwise differentiable on their respective interiors.

Step 3: Conclusion Therefore $k + m = 8/5 + 2/5 = 2$. The other three proposed sums do not satisfy both junction conditions. Hence, the correct option is (C).

Quick Tip to Solve: At a piecewise junction, write the value equation and the slope equation separately before solving for the constants.

2. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is : [4 marks]

(A) 15.8

(B) 14.0

(C) 16.8

(D) 16.0

Correct Answer: (B) 14.0

Solution:

Concept:

- The arithmetic mean is the total of all observations divided by their number: $\bar{x} = S/n$.
- When observations are deleted or added, update the sum and the count separately; the mean itself is not additive.

Step 1: Recover the original sum The original data have $n = 16$ observations and mean 16. Consequently their total is

$$S = 16 \times 16 = 256.$$

Deleting one observation whose value is 16 leaves a sum of $256 - 16 = 240$ and a count of $16 - 1 = 15$. Since the deleted value equalled the original mean, the mean of these 15 observations is still 16.

Step 2: Update the sum and the sample size The three new observations contribute $3 + 4 + 5 = 12$ to the sum and increase the count by three. Thus

$$S_{\text{new}} = 240 + 12 = 252, \qquad n_{\text{new}} = 15 + 3 = 18.$$

It follows that

$$\bar{x}_{\text{new}} = \frac{252}{18} = 14.0.$$

Using the old count of 16 would be incorrect because there are now two more observations overall.

Step 3: Conclusion The resultant mean is 14.0. It must decrease because the three added values are below the former mean; 16.8 is therefore impossible, and 15.8 and 16.0 do not equal the updated sum divided by 18. Hence, the correct option is (B).

Quick Tip to Solve: For a changed data set, keep a two-column tally of its sum and its count; divide only after both have been updated.

3. $\lim\limits_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ **is equal to** [4 marks]

(A) 2

(B) $\frac{1}{2}$

(C) 4

(D) 3

Correct Answer: (A) 2

Solution:

Concept:

- Use the identity $1-\cos 2x = 2\sin^2 x$ to remove the vanishing cosine difference.
- For angles in radians, $\sin x/x \to 1$, $\tan 4x/(4x) \to 1$, and $\cos x \to 1$ as $x \to 0$.

Step 1: Separate the standard factors For nonzero x sufficiently close to zero, rewrite the expression as

$$\frac{2\sin^2 x(3+\cos x)}{x\tan 4x} = \frac{1}{2}\left(\frac{\sin x}{x}\right)^2 (3+\cos x)\frac{4x}{\tan 4x}.$$

The factor $1/2$ includes the factor 4 introduced by the argument of the tangent. This scaling must not be discarded.

Step 2: Take the limit of each factor All factors in the rewritten expression have finite limits. Therefore

$$L = \frac{1}{2}\times 1^2 \times (3+1)\times 1 = 2.$$

Equivalently, the leading terms are $1-\cos 2x \sim 2x^2$ and $\tan 4x \sim 4x$, which give $2x^2\cdot 4/(4x^2) = 2$. This provides an independent order-of-vanishing check.

Step 3: Conclusion The limit equals 2. The values $1/2$, 4, and 3 fail to include all of the limiting factors with their correct scales. Hence, the correct option is (A).

Quick Tip to Solve: Convert a cosine difference into a sine square, and preserve the argument multiplier when using a standard tangent limit.

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4. Let α and β be the roots of equation $x^2-6x-2=0$. If $a_n=\alpha^n-\beta^n$, for $n\geq 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is equal to : [4 marks]

(A) 3

(B) -3

(C) 6

(D) -6

Correct Answer: (A) 3

Solution:

Concept:

- Every root r of $r^2-6r-2=0$ satisfies $r^2=6r+2$.

- Multiplying the root identity by a power and subtracting the identities for the two roots gives a recurrence for $a_n = \alpha^n - \beta^n$.

Step 1: Derive the recurrence Multiply the root identity by r^{n-2}, for $n \geq 2$, to obtain

$$r^n = 6r^{n-1} + 2r^{n-2}.$$

Apply this once to α and once to β, then subtract:

$$a_n = 6a_{n-1} + 2a_{n-2}.$$

This avoids computing very large tenth powers explicitly.

Step 2: Use the recurrence at the required index For $n = 10$,

$$a_{10} = 6a_9 + 2a_8, \qquad a_{10} - 2a_8 = 6a_9.$$

Division is legitimate: the roots are $3 + \sqrt{11}$ and $3 - \sqrt{11}$, which are distinct and have distinct ninth powers because the ninth-power function is strictly increasing on the real line. Thus $a_9 \neq 0$. Hence

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{6a_9}{2a_9} = 3.$$

Step 3: Conclusion The required ratio is 3, independent of the order in which the roots are named. The negative choices have the wrong sign, and 6 misses the factor 2 in the denominator. Hence, the correct option is (A).

Quick Tip to Solve: For expressions involving powers of quadratic roots, convert the quadratic equation into a recurrence before expanding any powers.

5. If 12 different balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is : [4 marks]

(A) $220\left(\frac{1}{3}\right)^{12}$

(B) $22\left(\frac{1}{3}\right)^{11}$

(C) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$

(D) $55\left(\frac{2}{3}\right)^{10}$

Correct Answer: (C) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$

Solution:

Concept:

- Under independent uniform placement of each distinct ball, a specified box receives each ball with probability $1/3$; it does not receive it with probability $2/3$.
- The count in a specified box is binomial: $P(X = r) = \binom{n}{r}p^r(1-p)^{n-r}$. The phrase one of the boxes needs a specified-box interpretation to match the supplied key.

Step 1: Calculate the probability for a specified box Choose which three of the 12 balls enter that box. The remaining nine must go to either of the other two boxes. Thus

$$P(X=3)=\binom{12}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^9=\frac{220\cdot 2^9}{3^{12}}.$$

Since $220\cdot 2^9=55\cdot 2^{11}$, this equals

$$\frac{55}{3}\left(\frac{2}{3}\right)^{11}.$$

This is the intended keyed option C; the other expressions do not equal the binomial result.

Step 2: Distinguish the literal alternative event If the wording means that at least one of the three boxes has exactly three balls, it is a different event. Inclusion-exclusion gives

$$P=\frac{3\binom{12}{3}2^9-3\binom{12}{3}\binom{9}{3}}{3^{12}}=\frac{282480}{531441}.$$

Two boxes can each contain three balls, so simply multiplying the specified-box answer by three overcounts these cases. Three such boxes are impossible because there are twelve balls. None of the listed options equals this at-least-one probability.

Step 3: Conclusion The key selects option (C), $\frac{55}{3}(\frac{2}{3})^{11}$, for one specified box. Literally reading the phrase one of the boxes as at least one gives a different, unlisted probability, so the wording is ambiguous.

Quick Tip to Solve: Distinguish a specified box from an existence event; for the latter, check whether two boxes can satisfy the requirement simultaneously.

6. The integral $\int_2^4 \frac{\log x^2}{\log x^2+\log(36-12x+x^2)}dx$ **is equal to :** [4 marks]

(A) 1

(B) 6

(C) 2

(D) 4

Correct Answer: (A) 1

Solution:

Concept:

- For an integrable function on $[a,b]$, reflection preserves the integral: $\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx$.
- Here $36-12x+x^2=(6-x)^2$, and reflection about the midpoint 3 interchanges x and $6-x$.

Step 1: Reflect the integrand Write

$$f(x)=\frac{\log x^2}{\log x^2+\log(6-x)^2},\qquad I=\int_2^4 f(x)\,dx.$$

On this interval both x and $6-x$ lie between 2 and 4. Their squared arguments exceed 1, so the logarithms are defined and the denominator does not vanish. Under the substitution $u = 6 - x$, the reversed limits and the negative differential cancel.

Step 2: Add the two equivalent integrals The reflected integrand is

$$f(6-x) = \frac{\log(6-x)^2}{\log(6-x)^2 + \log x^2}.$$

Consequently $f(x) + f(6-x) = 1$ everywhere on the interval. Therefore

$$2I = \int_2^4 \{f(x) + f(6-x)\}\, dx = \int_2^4 1\, dx = 2,$$

which yields $I = 1$. No antiderivative of the individual logarithmic fraction is needed.

Step 3: Conclusion The integral equals 1. Its interval length is 2, and the reflected pair averages to $1/2$, excluding the alternatives 6, 2, and 4. Hence, the correct option is (A).

Quick Tip to Solve: When a logarithmic denominator contains two terms exchanged by $x \mapsto a + b - x$, add the reflected integrand before integrating.

7. Let $y(x)$ be the solution of the differential equation $(x \log x)\frac{dy}{dx} + y = 2x \log x, (x \geq 1)$. Then $y(e)$ is equal to : [4 marks]

(A) 2

(B) $2e$

(C) e

(D) 0

Correct Answer: (A) 2

Solution:

Concept:

- For $x > 1$, dividing the equation by $x \log x$ gives a first-order linear differential equation with integrating factor $\log x$.
- The stated domain includes $x = 1$. A differentiable solution there is finite and continuous; this endpoint condition fixes the integration constant.

Step 1: Integrate after multiplying by the integrating factor For $x > 1$,

$$y' + \frac{y}{x \log x} = 2.$$

Since $\exp(\int dx/(x \log x)) = \log x$, multiplication gives

$$\frac{d}{dx}(y \log x) = 2 \log x.$$

Integrating,

$$y \log x = 2x \log x - 2x + C.$$

The original equation at $x = 1$ also requires $y(1) = 0$.

Step 2: Enforce finiteness at the endpoint As $x \to 1^+$, continuity and finiteness imply $y(x) \log x \to 0$. The right-hand side tends to $-2 + C$, so $C = 2$. Hence

$$y(x) = \frac{2x \log x - 2x + 2}{\log x} \quad (x > 1).$$

Its limit at 1 is zero, consistent with the equation. Evaluating at $x = e$, where $\log e = 1$, gives

$$y(e) = 2e - 2e + 2 = 2.$$

Retaining an arbitrary constant would ignore the included endpoint.

Step 3: Conclusion The required value is 2, rather than $2e$, e, or 0. Hence, the correct option is (A).

Quick Tip to Solve: When the coefficient of y' vanishes at an included endpoint, use continuity there to determine the integration constant.

8. The area (in sq. units) of the region described by

$\{(x, y) : y^2 \leq 2x$ **and** $y \geq 4x - 1\}$ **is :** [4 marks]

(A) $\frac{15}{64}$

(B) $\frac{9}{32}$

(C) $\frac{7}{32}$

(D) $\frac{5}{64}$

Correct Answer: (B) $\frac{9}{32}$

Solution:

Concept:

- For horizontal strips, the area enclosed between two curves is $\int (x_{\text{right}} - x_{\text{left}})\, dy$. Expressing both boundaries in terms of y avoids splitting the region into separate upper and lower branches of the parabola.
- The inequalities determine which side of each boundary is included: $y^2 \leq 2x$ means $x \geq y^2/2$, while $y \geq 4x - 1$ means $x \leq (y + 1)/4$.

Step 1: Find the intersection ordinates The parabola is the left boundary and the line is the right boundary wherever the interval between them is nonempty. At their intersections,

$$\frac{y^2}{2} = \frac{y + 1}{4} \quad \Longrightarrow \quad 2y^2 - y - 1 = 0.$$

Factoring gives $(2y + 1)(y - 1) = 0$, so the limiting ordinates are $y = -1/2$ and $y = 1$. The strip width is nonnegative between these roots and vanishes at both endpoints.

Step 2: Integrate the horizontal width The required area is therefore

$$\begin{aligned}\mathcal{A} &= \int_{-1/2}^{1}\left(\frac{y+1}{4}-\frac{y^2}{2}\right)dy \\ &= \left[\frac{y^2}{8}+\frac{y}{4}-\frac{y^3}{6}\right]_{-1/2}^{1} \\ &= \frac{5}{24}-\left(-\frac{7}{96}\right)=\frac{27}{96}=\frac{9}{32}.\end{aligned}$$

Both boundaries have been used once, so no symmetry multiplier is needed. The other three listed fractions differ from the exact definite integral and hence do not represent the enclosed area.

Step 3: Conclusion The area is $\boxed{9/32}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: When a sideways parabola meets an oblique line, integrate with respect to y: one horizontal-strip integral often replaces two vertical-strip integrals.

9. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals : [4 marks]

(A) $-\left(x^4+1\right)^{\frac{1}{4}}+c$

(B) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

(C) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

(D) $\left(x^4+1\right)^{\frac{1}{4}}+c$

Correct Answer: (B) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

Solution:

Concept:

- A useful substitution is $u=(x^4+1)^{1/4}/x$, for $x\neq 0$. Differentiating this quotient produces the given integrand directly, without expanding the fourth-root expression.
- An indefinite integral is determined on an interval in its domain. A real fourth root is nonnegative, so $(x^4)^{1/4}=|x|$; the sign of x matters when comparing an antiderivative with the printed options.

Step 1: Differentiate the suggested quotient Put $g(x)=(x^4+1)^{1/4}$, so that $g'(x)=x^3(x^4+1)^{-3/4}$. The quotient rule then gives

$$\begin{aligned}\frac{d}{dx}\left(\frac{g(x)}{x}\right) &= x^2(x^4+1)^{-3/4}-\frac{(x^4+1)^{1/4}}{x^2} \\ &= \frac{x^4-(x^4+1)}{x^2(x^4+1)^{3/4}} \\ &= -\frac{1}{x^2(x^4+1)^{3/4}}.\end{aligned}$$

Thus the negative of this quotient has exactly the derivative required in the question.

Step 2: Integrate and compare the root expressions Consequently, on either interval avoiding zero,

$$\int \frac{dx}{x^2(x^4+1)^{3/4}} = -\frac{(x^4+1)^{1/4}}{x} + C.$$

For $x > 0$, this is $-((x^4+1)/x^4)^{1/4} + C$, namely option (B). Option (C) has the opposite derivative there. Options (A) and (D) differentiate to expressions proportional to x^3, so they lack the required factor x^{-2}. If $x < 0$, the same quotient antiderivative is instead the positive fourth root in option (C). This distinction follows from the absolute value and cannot be absorbed into an integration constant.

Step 3: Conclusion The antiderivative valid for every $x \neq 0$, interval by interval, is $\boxed{-(x^4+1)^{1/4}/x + C}$. With the positive-$x$ convention used by the listed key, it is option (B); the sign adjustment on negative intervals is as derived above. Hence, the correct option is (B) on $x > 0$.

Quick Tip to Solve: Check a radical antiderivative by differentiation, and retain $(x^4)^{1/4} = |x|$ until the sign interval has been specified.

10. The set of all values of λ for which the system of linear equations:

$$\begin{aligned} 2x_1 - 2x_2 + x_3 &= \lambda x_1 \\ 2x_1 - 3x_2 + 2x_3 &= \lambda x_2 \\ -x_1 + 2x_2 &= \lambda x_3 \end{aligned}$$

has a non-trivial solution [4 marks]

(A) contains two elements

(B) contains more than two elements

(C) in an empty set

(D) is a singleton

Correct Answer: (A) contains two elements

Solution:

Concept:

- A homogeneous system $(A - \lambda I)X = 0$ has a nonzero solution precisely when its coefficient matrix is singular: $\det(A - \lambda I) = 0$. An invertible coefficient matrix would force all three unknowns to vanish.
- The question asks for the number of distinct parameter values, rather than the multiplicities of roots of the determinant polynomial. A repeated root supplies only one element of the required set.

Step 1: Form the coefficient determinant Moving the right-hand sides to the left gives

$$\begin{pmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

Expanding the determinant along its first row, its value is

$$D = (2-\lambda)(\lambda^2+3\lambda-4) + 2(2-2\lambda) + (1-\lambda).$$

The three cofactors include the signs of the entries of the original system; in particular the third row has first entry -1, not $+1$.

Step 2: Factor the parameter equation Collecting powers of λ yields

$$\begin{aligned} D &= -\lambda^3 - \lambda^2 + 5\lambda - 3 \\ &= -(\lambda-1)^2(\lambda+3). \end{aligned}$$

It vanishes exactly for $\lambda = 1$ or $\lambda = -3$. Each makes the homogeneous matrix singular and hence permits a nontrivial solution. For example, $(1,1,1)$ works at 1, while $(1,2,-1)$ works at -3. The double occurrence of 1 in the factorization does not produce a third distinct value. Thus the set is neither empty nor a singleton, and does not contain more than two elements.

Step 3: Conclusion The required set is $\boxed{\{-3,1\}}$, which contains two elements. Hence, the correct option is (A).

Quick Tip to Solve: After factoring a homogeneous-system determinant, count distinct roots; algebraic multiplicity does not count repeated parameter values as new elements.

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11. Let $f(x)$ be a polynomial of degree four having extreme values at $x = 1$ and $x = 2$. If $\lim\limits_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$, then f$(2)$ is equal to : [4 marks]

(A) 0

(B) 4

(C) -8

(D) -4

Correct Answer: (A) 0

Solution:

Concept:

- For a polynomial, the finite limit of $f(x)/x^2$ at zero forces the constant and linear coefficients to vanish. Its limit is then the coefficient of x^2.
- At an interior extremum of a differentiable function, the derivative is zero. The given extreme points at 1 and 2 therefore provide two equations for the remaining quartic coefficients.

Step 1: Use the limit to determine the low-degree terms Write $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, with $a \neq 0$. The stated limit gives $\lim_{x\to 0} f(x)/x^2 = 2$. But

$$\frac{f(x)}{x^2} = ax^2 + bx + c + \frac{d}{x} + \frac{e}{x^2}.$$

A finite two-sided limit requires $e = 0$ and $d = 0$; otherwise the last terms diverge. The remaining

limit is c, so $c = 2$. Consequently $f(x) = ax^4 + bx^3 + 2x^2$.

Step 2: Apply both stationary-point conditions The derivative is $f\prime(x) = 4ax^3 + 3bx^2 + 4x$. Substituting the two extreme points gives

$$4a + 3b + 4 = 0, \qquad 32a + 12b + 8 = 0.$$

Subtracting four times the first equation from the second gives $16a - 8 = 0$, hence $a = 1/2$ and then $b = -2$. Thus

$$f(x) = \frac{1}{2}x^4 - 2x^3 + 2x^2 = \frac{1}{2}x^2(x-2)^2.$$

As a consistency check, $f\prime(x) = 2x(x-1)(x-2)$ changes sign at both stated points, so they really are extrema.

Step 3: Evaluate the requested value At $x = 2$, the factor $(x-2)^2$ vanishes, giving $f(2) = 0$. Direct substitution likewise gives $8 - 16 + 8 = 0$, ruling out 4, -8 and -4. The extra stationary point at zero is allowed: the question does not say that the two listed points are the only extrema.

Step 4: Conclusion $\boxed{f(2) = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: A finite polynomial quotient limit first fixes the lowest coefficients; apply the stationary conditions only after eliminating those terms.

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12. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9\mathbf{I}$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to : [4 marks]

(A) $(2, 1)$

(B) $(-2, -1)$

(C) $(2, -1)$

(D) $(-2, 1)$

Correct Answer: (B) $(-2, -1)$

Solution:

Concept:

- The entry (i, j) of AA^T is the dot product of row i and row j of A. Therefore $AA^T = 9I$ says the rows are mutually perpendicular and each has squared length 9.
- Off-diagonal entries give linear equations in the unknown entries a and b. After solving them, check the diagonal norm condition as well, since orthogonality alone does not specify the scale.

Step 1: Set the off-diagonal products equal to zero The rows are $r_1 = (1, 2, 2)$, $r_2 = (2, 1, -2)$ and $r_3 = (a, 2, b)$. The first two already satisfy $r_1 \cdot r_2 = 2 + 2 - 4 = 0$. Their dot products with the third

row must also vanish:

$$r_1 \cdot r_3 = a + 4 + 2b = 0,$$

$$r_2 \cdot r_3 = 2a + 2 - 2b = 0.$$

The second equation simplifies to $a - b = -1$, or $a = b - 1$.

Step 2: Solve the equations and verify all row lengths Substitute $a = b - 1$ into the first equation to get $b - 1 + 4 + 2b = 0$, hence $3b + 3 = 0$. Therefore $b = -1$ and $a = -2$. The three row norms now give

$$\|r_1\|^2 = 1 + 4 + 4 = 9, \qquad \|r_2\|^2 = 4 + 1 + 4 = 9,$$

$$\|r_3\|^2 = 4 + 4 + 1 = 9.$$

Together with all three zero off-diagonal products, these confirm the full matrix identity. The ordered pair is (a, b), so reversing the signs or swapping their roles would fail the displayed linear equations. None of the other listed pairs satisfies both equations.

Step 3: Conclusion The ordered pair is $\boxed{(a, b) = (-2, -1)}$. Hence, the correct option is (B).

Quick Tip to Solve: For $AA^T = kI$, use row dot products rather than full matrix multiplication; finish by checking the row norms against k.

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13. Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$,
where $|x| < \frac{1}{\sqrt{3}}$. **Then a value of** y **is :** [4 marks]

(A) $\frac{3x-x^3}{1+3x^2}$
(B) $\frac{3x+x^3}{1+3x^2}$
(C) $\frac{3x-x^3}{1-3x^2}$
(D) $\frac{3x+x^3}{1-3x^2}$

Correct Answer: (C) $\frac{3x-x^3}{1-3x^2}$

Solution:

Concept:

- The principal inverse tangent lies in $(-\pi/2, \pi/2)$. Its addition formulas must be checked against these branch limits before identifying inverse tangent expressions.
- If $\theta = \tan^{-1} x$, then $\tan 2\theta = 2x/(1-x^2)$ and $\tan 3\theta = (3x - x^3)/(1 - 3x^2)$, whenever the displayed denominators are nonzero.

Step 1: Use the given bound to control the angles Put $\theta = \tan^{-1} x$. Since $|x| < 1/\sqrt{3}$, monotonicity of inverse tangent gives $|\theta| < \pi/6$. Thus 2θ lies strictly between $-\pi/3$ and $\pi/3$, inside the principal inverse tangent range. Also $1 - x^2 > 0$. Consequently,

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\theta.$$

The right-hand side of the original equation is therefore 3θ, which lies in $(-\pi/2, \pi/2)$. No correction by an integer multiple of π is needed.

Step 2: Take the tangent and simplify Taking the tangent of $\tan^{-1} y = 3\theta$, we get

$$\begin{aligned} y &= \tan 3\theta \\ &= \frac{x + \dfrac{2x}{1-x^2}}{1 - x\dfrac{2x}{1-x^2}} \\ &= \frac{3x - x^3}{1 - 3x^2}. \end{aligned}$$

The assumed bound ensures $1 - 3x^2 > 0$, so this result is defined on the entire given interval. Option (A) has the wrong sign in its denominator; options (B) and (D) also have the wrong sign of the cubic numerator term. The signs follow directly from the tangent addition denominator $1 - uv$, rather than $1 + uv$.

Step 3: Conclusion $\boxed{y = (3x - x^3)/(1 - 3x^2)}$. Hence, the correct option is (C).

Quick Tip to Solve: When inverse tangent expressions resemble double and triple angles, check the principal ranges first; the bound on x usually supplies that branch check.

14. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio $1 : 3$, then locus of P is : [4 marks]

(A) $y^2 = 2x$

(B) $x^2 = 2y$

(C) $x^2 = y$

(D) $y^2 = x$

Correct Answer: (B) $x^2 = 2y$

Solution:

Concept:

- The vertex of $x^2 = 8y$ is $O = (0, 0)$. If P divides OQ in the ratio $OP : PQ = 1 : 3$, then P is one quarter of the way from O to Q.
- The section formula gives $P = (X/4, Y/4)$ for $Q = (X, Y)$. A locus equation is obtained by expressing X, Y through the coordinates of P and substituting into the equation satisfied by Q.

Step 1: Relate the coordinates of the two points Let the moving point be $P = (x, y)$ and the original point on the parabola be $Q = (X, Y)$. The internal section formula yields

$$x = \frac{3 \cdot 0 + 1 \cdot X}{1 + 3} = \frac{X}{4}, \qquad y = \frac{3 \cdot 0 + 1 \cdot Y}{1 + 3} = \frac{Y}{4}.$$

Therefore $X = 4x$ and $Y = 4y$. The factor is $1/4$, not $1/3$, because the whole segment has four equal ratio parts and OP contains one of them.

Step 2: Substitute into the original parabola Since Q lies on the given curve, $X^2 = 8Y$. Substitution gives

$$(4x)^2 = 8(4y) \quad \Longrightarrow \quad 16x^2 = 32y \quad \Longrightarrow \quad x^2 = 2y.$$

Conversely, every point (x, y) satisfying this equation corresponds to $Q = (4x, 4y)$ on $X^2 = 8Y$, so the equation describes the whole locus. The axis remains the positive y-axis under this scaling, ruling out options (A) and (D). Option (C) has the wrong scale factor: it would not satisfy the substitution into the original curve.

Step 3: Conclusion The locus of P is $\boxed{x^2 = 2y}$. Hence, the correct option is (B).

Quick Tip to Solve: For a section point measured from the origin, use its fraction of the whole segment, then scale both coordinates before substituting into the curve.

15. Locus of the image of the point $(2, 3)$ in the line $(2x - 3y + 4) + k\,(x - 2y + 3) = 0$, $k \in R$, is a : [4 marks]

(A) circle of radius $\sqrt{2}$.

(B) circle of radius $\sqrt{3}$.

(C) straight line parallel to x-axis

(D) straight line parallel to y-axis

Correct Answer: (A) circle of radius $\sqrt{2}$.

Solution:

Concept:

- A line of the form $L_1 + kL_2 = 0$ passes through the intersection of $L_1 = 0$ and $L_2 = 0$, because both expressions vanish there.
- Reflection in a line preserves distance from every point on the mirror line. Thus the image of a fixed point under mirrors through a fixed centre lies on a circle about that centre.

Step 1: Find the fixed point of the family Solve the two equations that form the pencil of lines. From $x - 2y + 3 = 0$, we have $x = 2y - 3$. Substitution into the other equation gives

$$2(2y - 3) - 3y + 4 = 0 \quad \Longrightarrow \quad y = 2, \quad x = 1.$$

Therefore every reflecting line passes through the fixed point $C = (1, 2)$. The given point is $P = (2, 3)$, which differs from this centre.

Step 2: Use the distance preserved by reflection Let $P' = (X, Y)$ be the image of P in any line of the family. Since C belongs to that mirror line, reflection fixes C and gives $CP' = CP$. Consequently

$$CP^2 = (2 - 1)^2 + (3 - 2)^2 = 2,$$

$$(X - 1)^2 + (Y - 2)^2 = 2.$$

This is the equation of the circle with centre $(1, 2)$ and radius $\sqrt{2}$. In particular the distance from C

remains constant, rather than either coordinate remaining constant. The radius is not $\sqrt{3}$, since the sum of the squared coordinate differences is exactly 2. Hence neither of the proposed horizontal or vertical straight lines describes the locus.

Step 3: Conclusion The locus is a circle of radius $\sqrt{2}$, centred at $(1, 2)$. Hence, the correct option is (A).

Quick Tip to Solve: For reflection in a pencil of lines, find the common point first: its distance from the original point is the radius of the image locus.

16. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices $(0, 0)$ $(0, 41)$ and $(41, 0)$ is : [4 marks]

(A) 820

(B) 780

(C) 901

(D) 861

Correct Answer: (B) 780

Solution:

Concept:

- An interior point lies strictly on the inner side of every edge. Here the edges are $x = 0$, $y = 0$, and $x + y = 41$.
- The sum of the first N positive integers is $N(N + 1)/2$; strict inequalities ensure that boundary lattice points are excluded.

Step 1: Translate the interior condition into inequalities The two coordinate axes form the perpendicular edges of the triangle, and the hypotenuse joins $(0, 41)$ to $(41, 0)$. Therefore the required integer coordinates must satisfy

$$x > 0, \qquad y > 0, \qquad x + y < 41.$$

Since $x + y$ is an integer, the last condition is equivalently $x + y \leq 40$. Neither zero coordinate is allowed, and points on $x + y = 41$ must also be left out.

Step 2: Count by fixing the first coordinate For $x = 1$, the second coordinate can be any of $1, 2, \ldots, 39$, giving 39 points. For $x = 2$, it can be $1, 2, \ldots, 38$, giving 38 points. Continuing, $x = 39$ permits just $y = 1$; $x = 40$ permits none. Thus

$$N = 39 + 38 + \cdots + 1 = \frac{39 \cdot 40}{2} = 780.$$

Every interior point has exactly one first coordinate, so these rows are disjoint and complete. The value 820 would result from admitting the hypotenuse points with both coordinates positive, which are boundary points. The larger values 861 and 901 likewise cannot count the strict interior. This direct count avoids treating an edge point as an interior point.

Step 3: Conclusion There are 780 integer-coordinate points strictly inside the triangle. Hence, the

correct option is (B).

Quick Tip to Solve: For the triangle bounded by the axes and $x + y = n$, count positive integer pairs with sum at most $n - 1$: the result is $(n-1)(n-2)/2$.

17. If m is the A.M. of two distinct real numbers l and n $(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals: [4 marks]

(A) $4\,lmn^2$

(B) $4\,l^2m^2n^2$

(C) $4\,l^2m\,n$

(D) $4\,l\,m^2n$

Correct Answer: (D) $4\,l\,m^2n$

Solution:

Concept:

- If three geometric means are inserted between positive numbers l and n, the five terms l, G_1, G_2, G_3, n form a geometric progression.
- For common ratio r, these terms are l, lr, lr^2, lr^3, lr^4, while their endpoints have arithmetic mean $m = (l + n)/2$.

Step 1: Express the fourth powers through the endpoints Write $G_1 = lr$, $G_2 = lr^2$, $G_3 = lr^3$ and $n = lr^4$. The endpoint relation gives $r^4 = n/l$. Therefore

$$G_1^4 = l^4r^4 = l^3n,$$

$$G_2^4 = l^4r^8 = l^2n^2, \qquad G_3^4 = l^4r^{12} = ln^3.$$

The fourth powers make the calculation especially simple: we do not need to extract a fourth root to find r explicitly. Positivity of l and n ensures that the usual real geometric means exist.

Step 2: Factor the required sum and use the arithmetic mean Substitute the three expressions, then take out the common factor ln:

$$\begin{aligned} G_1^4 + 2G_2^4 + G_3^4 &= l^3n + 2l^2n^2 + ln^3 \\ &= ln(l^2 + 2ln + n^2) \\ &= ln(l + n)^2. \end{aligned}$$

Since $m = (l + n)/2$, we have $l + n = 2m$. Consequently the sum becomes

$$ln(2m)^2 = 4lm^2n.$$

The alternatives containing only one power of m cannot replace the squared sum $(l + n)^2$, and the expression with $l^2m^2n^2$ introduces an extra factor ln absent from the factorisation.

Step 3: Conclusion The required expression equals $4lm^2n$. Hence, the correct option is (D).

Quick Tip to Solve: With three inserted geometric means, take fourth powers first: each resulting term becomes an integer-power product of the endpoints.

18. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is: [4 marks]

(A) 120

(B) 72

(C) 216

(D) 192

Correct Answer: (D) 192

Solution:

Concept:

- The multiplication principle counts successive digit choices; without repetition, each chosen digit reduces the number available for later positions.
- Numbers formed from five available digits can have at most five digits. Separate the four-digit and five-digit cases because their lower-bound restrictions differ.

Step 1: Count the eligible four-digit numbers No one-, two-, or three-digit number can exceed 6000. For a four-digit number, the thousands digit must be 6, 7, or 8, giving three choices. If it is 6, the remaining digits are positive, so the number is still strictly greater than 6000. After the first digit is chosen, fill the other three positions with distinct digits selected from the four remaining ones:

$$N_4 = 3 \times 4 \times 3 \times 2 = 72.$$

Leading digits 3 and 5 would give numbers smaller than 6000, so they are correctly excluded in this case.

Step 2: Include every five-digit permutation Every five-digit number formed from these digits is greater than 6000, even if its leading digit is 3 or 5. With all five digits distinct and nonzero, every permutation is valid:

$$N_5 = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120.$$

The cases have different lengths, so no number is counted twice. Their total is

$$N = N_4 + N_5 = 72 + 120 = 192.$$

The answers 72 and 120 each omit one entire permissible length. The value 216 exceeds the exhaustive total of 192, so it cannot be correct. Admitting all five leading digits in the four-digit case would incorrectly include numbers beginning with 3 or 5. Exhausting the possible lengths proves that no further integers have been missed.

Step 3: Conclusion The total number of integers is 192. Hence, the correct option is (D).

Quick Tip to Solve: When a digit-counting question gives only a lower bound, check all possible lengths; do not stop after counting numbers with the same length as the bound.

19. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\overline{z_2}}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a : [4 marks]

(A) circle of radius 2.

(B) circle of radius $\sqrt{2}$.

(C) straight line parallel to x-axis

(D) straight line parallel to y-axis.

Correct Answer: (A) circle of radius 2.

Solution:

Concept:

- A complex quotient is unimodular when its modulus is one, so the numerator and denominator have equal moduli, provided the denominator is nonzero.
- For any complex number w, $|w|^2 = w\overline{w}$. Expanding both squared moduli lets conjugate cross terms cancel.

Step 1: Convert the unimodular condition into an equation The given quotient is defined and has modulus one. Hence

$$|z_1 - 2z_2|^2 = |2 - z_1\overline{z_2}|^2.$$

Expanding the left side gives

$$|z_1|^2 + 4|z_2|^2 - 2z_1\overline{z_2} - 2\overline{z_1}z_2.$$

Expanding the right side gives

$$4 + |z_1|^2|z_2|^2 - 2z_1\overline{z_2} - 2\overline{z_1}z_2.$$

The two cross terms are identical on both sides, including their signs, so they cancel exactly.

Step 2: Use the restriction on the second complex number Collecting the remaining terms and factorising yields

$$|z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2|z_2|^2 = 0,$$

$$(|z_1|^2 - 4)(1 - |z_2|^2) = 0.$$

The hypothesis says z_2 is not unimodular, so $|z_2| \neq 1$; therefore the second factor is nonzero. It follows that $|z_1|^2 = 4$, or $|z_1| = 2$. Writing $z_1 = x + iy$, this becomes

$$x^2 + y^2 = 4.$$

This is the circle centred at the origin with radius 2, not radius $\sqrt{2}$. Neither coordinate is fixed, so the two proposed axis-parallel lines are excluded. The non-unimodular restriction is essential: without it the factored equation would not force the first factor to vanish.

Step 3: Conclusion The point representing z_1 lies on the circle $|z_1| = 2$. Hence, the correct option is (A).

Quick Tip to Solve: When two complex moduli are equated, square and expand before substituting real and imaginary parts; the mixed conjugate terms often cancel.

20. Let A and B be two sets containing four and two elements respectively. Then, the number of subsets of the set A $\times$ B , each having atleast three elements are [4 marks]

(A) 219

(B) 256

(C) 275

(D) 510

Correct Answer: (A) 219

Solution:

Concept:

- The Cartesian product has $|A \times B| = |A|\,|B|$ elements, since each element of A pairs with each element of B.
- A set with n elements has 2^n subsets, and exactly $\binom{n}{r}$ of these have r elements. Complementary counting is efficient when only a few small sizes are excluded.

Step 1: Find the size of the product set There are four possible first entries and two possible second entries in an ordered pair belonging to $A \times B$. Hence

$$|A \times B| = 4 \times 2 = 8.$$

Ordered pairs remain distinct when either coordinate differs. This result does not require A and B to be disjoint: the product counts pairs, rather than members of their union. The total number of unrestricted subsets is therefore $2^8 = 256$.

Step 2: Subtract subsets with fewer than three elements The required subsets have sizes 3 through 8. Instead of adding six binomial coefficients, remove the subsets with 0, 1, and 2 elements:

$$N = 2^8 - \binom{8}{0} - \binom{8}{1} - \binom{8}{2}.$$

There is one empty subset, eight singleton subsets, and $8 \cdot 7/2 = 28$ two-element subsets. Therefore

$$N = 256 - 1 - 8 - 28 = 219.$$

The three removed categories are disjoint, and every remaining subset has at least three elements, as requested. Option 256 includes all the forbidden small subsets. The values 275 and 510 exceed the total number of subsets of an eight-element set, so neither can be correct even before the detailed subtraction.

Step 3: Conclusion The number of subsets having at least three elements is 219. Hence, the correct option is (A).

Quick Tip to Solve: To count subsets with at least a small number of elements, subtract the few smaller sizes from 2^n; include the empty subset among the exclusions.

21. The number of common tangents to the circles $x^2+y^2-4x-6x-12=0$ and $x^2+y^2+6x+18y+26=0$, is : [4 marks]

(A) 3

(B) 4

(C) 1

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- For $x^2+y^2+2gx+2fy+c=0$, the centre is $(-g,-f)$ and the radius is $\sqrt{g^2+f^2-c}$. Like terms must be combined exactly as written.
- Two circles intersect at two points when $|r_1-r_2|<d<r_1+r_2$, where d is the distance between their centres. In that case they have two common tangents.

Step 1: Find the centres and radii from the given equations The first equation contains both $-4x$ and $-6x$. Combining these terms gives $x^2+y^2-10x-12=0$. Completing squares for the two circles yields

$$(x-5)^2+y^2=37,$$

$$(x+3)^2+(y+9)^2=64.$$

Thus $C_1=(5,0)$, $r_1=\sqrt{37}$, $C_2=(-3,-9)$, and $r_2=8$. Their centre distance is

$$d=\sqrt{(5+3)^2+(0+9)^2}=\sqrt{145}.$$

The repeated x term is retained as given; replacing it with a y term would describe a different circle.

Step 2: Compare the centre distance with the radius sum and difference All distances are positive. To check the upper inequality, compare squares:

$$(r_1+r_2)^2=101+16\sqrt{37}>145=d^2,$$

since $16\sqrt{37}>44$. Also

$$(r_2-r_1)^2=101-16\sqrt{37}<145=d^2.$$

Therefore $|r_1-r_2|<d<r_1+r_2$, so the circles cross at two points. There are two direct common tangents and no transverse common tangents. Four tangents would require externally separate circles, three external contact, and one internal contact; none of those distance equalities or inequalities holds here.

Step 3: Conclusion The supplied answer key gives three tangents, but the two repeated x terms in the given first equation lead to intersecting circles. For the equations given, the circles intersect and have 2 common tangents. Hence, the correct option is (D).

Quick Tip to Solve: Combine repeated coordinate terms before extracting a circle centre, then compare the centre distance with both the radius sum and the radius difference.

22. The distance of the point $(1, 0, 2)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is : [4 marks]

(A) $3\sqrt{21}$

(B) 13

(C) $2\sqrt{14}$

(D) 8

Correct Answer: (B) 13

Solution:

Concept:

- A point on a line in symmetric form is obtained by setting its three ratios equal to a parameter t. The intersection with a plane must satisfy the plane equation as well.
- The distance between (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Step 1: Parameterise the given line Set each ratio equal to t. The line is then described by

$$x = 2 + 3t, \qquad y = -1 + 4t, \qquad z = 2 + 12t.$$

In particular, the negative sign in $y + 1$ means that the fixed coordinate is -1, not 1. We must first locate the intersection; the distance from the given point to the entire line is a different quantity.

Step 2: Find its intersection with the plane Substitute the parameterised coordinates into $x - y + z = 16$:

$$(2 + 3t) - (-1 + 4t) + (2 + 12t) = 16.$$

Thus $5 + 11t = 16$, giving $t = 1$. The intersection point is therefore $Q = (5, 3, 14)$. As a check, $5 - 3 + 14 = 16$, and all three line ratios are equal to one.

Step 3: Calculate the required point-to-point distance For the specified point $P = (1, 0, 2)$, the displacement to Q has components $(4, 3, 12)$. Hence

$$PQ = \sqrt{(5 - 1)^2 + (3 - 0)^2 + (14 - 2)^2} = \sqrt{16 + 9 + 144} = 13.$$

The three alternative values do not have square equal to 169: their squares are $189, 56, 64$, respectively. This also distinguishes the correct option without approximating radicals.

Step 4: Conclusion The required distance is $\boxed{13}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the common line parameter to find the actual intersection first; then apply the ordinary three-dimensional distance formula.

23. Let $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ be three non-zero vectors such that no two of them are collinear and $\left(\overrightarrow{a} \times \overrightarrow{b}\right) \times \overrightarrow{c} = \frac{1}{3}\left|\overrightarrow{b}\right||\overrightarrow{c}|\overrightarrow{a}$. If θ is the angle between vectors $\overrightarrow{b}$ and $\overrightarrow{c}$, then a value of $\sin\theta$ is : [4 marks]

(A) $\frac{2}{3}$

(B) $\frac{-2\sqrt{3}}{3}$

(C) $\frac{2\sqrt{2}}{3}$

(D) $\frac{-\sqrt{2}}{3}$

Correct Answer: (C) $\frac{2\sqrt{2}}{3}$

Solution:

Concept:

- The vector triple-product identity is $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{a}(\vec{b} \cdot \vec{c})$. The order matters because the vector product is not associative.
- For two non-collinear non-zero vectors, $u\vec{a} + v\vec{b} = \vec{0}$ implies $u = v = 0$. Also $\vec{b} \cdot \vec{c} = |\vec{b}||\vec{c}|\cos\theta$, with $0 \le \theta \le \pi$.

Step 1: Expand the left-hand side Apply the triple-product identity to the stated equation, keeping its original grouping:

$$\vec{b}(\vec{a} \cdot \vec{c}) - \vec{a}(\vec{b} \cdot \vec{c}) = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}.$$

Move all terms to the left and collect coefficients of $\vec{a}$ and $\vec{b}$:

$$(\vec{a} \cdot \vec{c})\vec{b} - \left(\vec{b} \cdot \vec{c} + \frac{1}{3}|\vec{b}||\vec{c}|\right)\vec{a} = \vec{0}.$$

Step 2: Use the non-collinearity condition Since $\vec{a}$ and $\vec{b}$ are non-collinear, both scalar coefficients must vanish. Consequently $\vec{a} \cdot \vec{c} = 0$ and

$$\vec{b} \cdot \vec{c} = -\frac{1}{3}|\vec{b}||\vec{c}|.$$

The vectors are non-zero, so division by their magnitudes is valid. It gives $\cos\theta = -1/3$, placing the angle in the second quadrant.

Step 3: Find the permissible sine The angle between two vectors lies between zero and π, so its sine is non-negative. Therefore

$$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}.$$

Option (A) has square $4/9$, whereas the required sine squared is $8/9$. Options (B) and (D) are negative, and (B) additionally has magnitude exceeding one. Thus only option (C) meets both the equation and the angle convention.

Step 4: Conclusion $\boxed{\sin\theta = 2\sqrt{2}/3}$. Hence, the correct option is (C).

Quick Tip to Solve: Expand the grouped vector product before taking magnitudes; the independent coefficients determine the cosine directly.

24. The sum of coefficients of integral power of x in the binomial expansion $(1-2\sqrt{x})^{50}$ is : [4 marks]

(A) $\frac{1}{2}\left(3^{50}-1\right)$

(B) $\frac{1}{2}\left(2^{50}+1\right)$

(C) $\frac{1}{2}\left(3^{50}+1\right)$

(D) $\frac{1}{2}\left(3^{50}\right)$

Correct Answer: (C) $\frac{1}{2}\left(3^{50}+1\right)$

Solution:

Concept:

- The general term of $(1-2\sqrt{x})^{50}$, indexed from $r=0$, is $\binom{50}{r}(-2)^r x^{r/2}$. Its power of x is integral precisely when r is even.
- For a polynomial $F(t)$, the sum of its even-power terms is $[F(t)+F(-t)]/2$. Adding cancels every odd-power term without changing the even terms.

Step 1: Identify exactly which coefficients are required Put $r=2k$, where $k=0,1,\ldots,25$. The corresponding term is

$$\binom{50}{2k}2^{2k}x^k.$$

Thus every retained coefficient is positive, and the required sum is $S=\sum_{k=0}^{25}\binom{50}{2k}2^{2k}$. The constant term is included because zero is an integral power. Substituting $x=1$ in the original expression alone would mix the wanted terms with the odd-index terms, so it would not give this sum.

Step 2: Extract the even-index terms Introduce $F(t)=(1-2t)^{50}$, treating t as an auxiliary variable. In $F(-t)=(1+2t)^{50}$, each odd-index term has the opposite sign and each even-index term has the same sign. Therefore

$$\frac{(1-2t)^{50}+(1+2t)^{50}}{2}=\sum_{k=0}^{25}\binom{50}{2k}2^{2k}t^{2k}.$$

Now set $t=1$; the right-hand side becomes the sum of the required coefficients.

Step 3: Evaluate and compare the options The exponent 50 is even, so $(-1)^{50}=1$. Consequently

$$S=\frac{(1-2)^{50}+(1+2)^{50}}{2}=\frac{1+3^{50}}{2}.$$

The minus-one expression in (A) selects the odd-index sum instead. Option (B) incorrectly uses 2^{50} rather than 3^{50}, while (D) omits the $F(1)=1$ contribution before halving.

Step 4: Conclusion The sum is $\boxed{(3^{50}+1)/2}$. Hence, the correct option is (C).

Quick Tip to Solve: For integral powers in an expansion involving a square root, retain even binomial indices by averaging the plus and minus expansions.

25. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is : [4 marks]

(A) $\frac{27}{2}$

(B) 27

(C) $\frac{27}{4}$

(D) 18

Correct Answer: (B) 27

Solution:

Concept:

- For $x^2/a^2 + y^2/b^2 = 1$, with $a > b$, the foci are $(\pm c, 0)$, where $c^2 = a^2 - b^2$. The latus rectum through each focus is perpendicular to the major axis.
- The tangent at a point (x_1, y_1) on the ellipse is $xx_1/a^2 + yy_1/b^2 = 1$. A quadrilateral with perpendicular diagonals of lengths d_1, d_2 has area $d_1 d_2/2$.

Step 1: Find the four endpoints Here $a^2 = 9$, $b^2 = 5$, and hence $c = \sqrt{9-5} = 2$. The two latera recta lie on $x = 2$ and $x = -2$. Substitution of either value into the ellipse gives

$$\frac{4}{9} + \frac{y^2}{5} = 1 \quad \Rightarrow \quad y = \pm\frac{5}{3}.$$

Their four endpoints are therefore $(2, 5/3), (2, -5/3), (-2, 5/3), (-2, -5/3)$. These are points of tangency, not the vertices of the requested quadrilateral.

Step 2: Write the tangents and obtain their intersections Using the tangent formula at each endpoint yields the four lines

$$2x + 3y = 9, \quad 2x - 3y = 9, \quad -2x + 3y = 9, \quad -2x - 3y = 9.$$

The first two intersect at $(9/2, 0)$, and the last two at $(-9/2, 0)$. Intersecting the two upper tangents gives $(0, 3)$; the corresponding lower tangents give $(0, -3)$. These four intersections form a rhombus centred at the origin.

Step 3: Calculate its area The horizontal diagonal joins $(-9/2, 0)$ to $(9/2, 0)$, so its length is 9. The vertical diagonal joins $(0, -3)$ to $(0, 3)$, so its length is 6. They are perpendicular, giving

$$A = \frac{1}{2} \times 9 \times 6 = 27.$$

Using only half of one diagonal would incorrectly produce $27/2$; halving both would give $27/4$. The value 18 also fails the actual diagonal calculation.

Step 4: Conclusion The quadrilateral has area $\boxed{27 \text{ square units}}$. Hence, the correct option is (B).

Quick Tip to Solve: Find intersections of the tangent lines before computing area; the latus-rectum endpoints themselves are not the required vertices.

26. The sum of first 9 terms of the series.

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+.....$$

[4 marks]

(A) 142

(B) 192

(C) 71

(D) 96

Correct Answer: (D) 96

Solution:

Concept:

- The sum of the first n cubes is $1^3+2^3+\cdots+n^3=[n(n+1)/2]^2$, while the sum of the first n odd natural numbers is n^2.
- The square-sum identity $\sum_{k=1}^{m}k^2=m(m+1)(2m+1)/6$ converts the remaining series into a numerical calculation.

Step 1: Simplify the general term The nth numerator is the sum of cubes from 1 through n. The corresponding denominator contains exactly the first n odd numbers, ending at $2n-1$. Thus

$$T_n=\frac{1^3+2^3+\cdots+n^3}{1+3+\cdots+(2n-1)}=\frac{n^2(n+1)^2/4}{n^2}=\frac{(n+1)^2}{4}.$$

The cancellation is valid for every index used here because $n\geq 1$. As a direct check, $T_1=1$ and $T_2=9/4$, agreeing with the first two printed fractions.

Step 2: Shift the summation limits carefully There are nine terms, with $n=1$ through $n=9$. Therefore $n+1$ runs from 2 through 10, not from 1 through 9:

$$S_9=\frac{1}{4}\sum_{n=1}^{9}(n+1)^2=\frac{1}{4}\left(\sum_{k=1}^{10}k^2-1\right).$$

Subtracting 1^2 removes the single unwanted term introduced by starting the standard square sum at one. No final term is omitted.

Step 3: Evaluate the square sum Use the formula with $m=10$:

$$\sum_{k=1}^{10}k^2=\frac{10\cdot 11\cdot 21}{6}=385.$$

It follows that

$$S_9=\frac{385-1}{4}=96.$$

Option (B), 192, would result from using one half instead of one quarter after simplifying the cubes. Options (A) and (C) do not equal the exact shifted square sum, so neither can be the required total.

Step 4: Conclusion The sum of the first nine terms is $\boxed{96}$. Hence, the correct option is (D).

Quick Tip to Solve: Simplify a ratio of cumulative sums into its nth term first, then write the shifted first and last indices explicitly.

27. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at $(1, 1)$ [4 marks]

(A) meets the curve again in the third quadrant.

(B) meets the curve again in the fourth quadrant.

(C) does not meet the curve again.

(D) meets the curve again in the second quadrant.

Correct Answer: (B) meets the curve again in the fourth quadrant.

Solution:

Concept:

- The polynomial $x^2 + 2xy - 3y^2$ factors as $(x - y)(x + 3y)$. Thus the given curve is the union of two straight lines, $y = x$ and $y = -x/3$.
- At a regular point with tangent slope $m \neq 0$, the normal has slope $-1/m$. To find whether it meets the full curve again, intersect it with every component of that curve.

Step 1: Identify the component through the point The point $(1, 1)$ satisfies $x-y = 0$, but $x+3y = 4 \neq 0$. Therefore the relevant local branch is the line $y = x$, whose tangent slope is 1. The point is not the crossing of the two branches, so the tangent and normal there are uniquely defined. Equivalently, implicit differentiation gives

$$2x + 2y + (2x - 6y)\frac{dy}{dx} = 0,$$

which at $(1, 1)$ gives $dy/dx = 1$.

Step 2: Write the normal and find the other intersection The normal slope is -1, and the equation through $(1, 1)$ is

$$y - 1 = -(x - 1) \quad \Rightarrow \quad y = 2 - x.$$

Its intersection with $y = x$ is just the original point. For the other component, impose $x + 3y = 0$:

$$x + 3(2 - x) = 0 \quad \Rightarrow \quad x = 3, \qquad y = -1.$$

Thus there is indeed a second intersection with the curve; treating the curve as only its branch through the original point would miss it.

Step 3: Determine the quadrant The second point is $(3, -1)$, with positive x-coordinate and negative y-coordinate, so it lies in the fourth quadrant. It also checks directly in the original curve: $9-6-3 = 0$. The third-quadrant alternative would require both coordinates negative, and the second-quadrant alternative would require x negative and y positive. The no-intersection alternative is ruled out by the explicit second point.

Step 4: Conclusion The normal meets the curve again at $\boxed{(3, -1)}$, in the fourth quadrant. Hence, the correct option is (B).

Quick Tip to Solve: Factor a homogeneous quadratic curve before differentiating: a normal to one component can intersect the other component again.

28. The equation of the plane containing the line $2x - 5y + z = 3; x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is : [4 marks]

(A) $x + 3y + 6z = 7$

(B) $2x + 6y + 12z = -13$

(C) $2x + 6y + 12z = 13$

(D) $x + 3y + 6z = -7$

Correct Answer: (A) $x + 3y + 6z = 7$

Solution:

Concept:

- Every plane through the intersection of two distinct planes $P_1 = 0$, $P_2 = 0$ can be written as $P_1 + \lambda P_2 = 0$, including the separate limiting member $P_2 = 0$.
- Two planes are parallel when their normal vectors are proportional. All coefficients, including the constant, must be scaled together when comparing equivalent plane equations.

Step 1: Form the family of planes through the line The given line is the intersection of $2x-5y+z-3 = 0$ and $x + y + 4z - 5 = 0$. A plane containing it therefore has equation

$$(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0.$$

Collecting coefficients gives

$$(2 + \lambda)x + (-5 + \lambda)y + (1 + 4\lambda)z = 3 + 5\lambda.$$

The separate member $x + y + 4z = 5$ is not parallel to the specified plane, so a finite parameter will capture the required member.

Step 2: Make its normal parallel to the required normal The plane $x + 3y + 6z = 1$ has normal $(1, 3, 6)$. Proportionality of the first two coefficients requires

$$-5 + \lambda = 3(2 + \lambda),$$

so $\lambda = -11/2$. At this value the family normal is $(-7/2, -21/2, -21) = (-7/2)(1, 3, 6)$. The third coefficient therefore has the required ratio as well; this check is necessary since matching only two coefficients does not by itself prove parallelism.

Step 3: Simplify the equation and check the alternatives The constant is $3 + 5(-11/2) = -49/2$, giving

$$-\frac{7}{2}x - \frac{21}{2}y - 21z = -\frac{49}{2}.$$

Dividing every term by $-7/2$ gives $x + 3y + 6z = 7$. In doubled form the constant would be 14, not the positive or negative 13 in options (C) and (B). Option (D) has the wrong constant sign. The family construction guarantees that the entire specified line lies in the resulting plane.

Step 4: Conclusion The required plane is $\boxed{x + 3y + 6z = 7}$. Hence, the correct option is (A).

Quick Tip to Solve: Build a linear combination of the two planes defining the line, then match its normal ratios and scale the constant with the coefficients.

29. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, $AB : BC$, is : [4 marks]

(A) $1 : \sqrt{3}$

(B) $2 : 3$

(C) $\sqrt{3} : 1$

(D) $\sqrt{3} : \sqrt{2}$

Correct Answer: (C) $\sqrt{3} : 1$

Solution:

Concept:

- For an observation point on level ground, $\tan\alpha = h/x$, where h is the tower height and x is the horizontal distance from its foot.
- As the angle of elevation increases, the distance from the tower decreases. Distances between successive observation points are differences of their distances from the tower.

Step 1: Express the distances from the tower Let O be the foot and $h > 0$ the height of the tower. The observation points lie along the line approaching its foot, so their order is A, B, C, O. Applying the tangent ratio to each right triangle gives

$$OA = h\cot 30^\circ = \sqrt{3}h, \quad OB = h\cot 45^\circ = h,$$

$$OC = h\cot 60^\circ = \frac{h}{\sqrt{3}}.$$

This ordering is consistent with the three increasingly steep lines of sight.

Step 2: Subtract successive distances and simplify The segment AB is not the distance from A to the foot: subtract OB from OA. Similarly subtract OC from OB for BC:

$$AB = (\sqrt{3} - 1)h, \qquad BC = \left(1 - \frac{1}{\sqrt{3}}\right)h.$$

Consequently,

$$\frac{AB}{BC} = \frac{\sqrt{3} - 1}{1 - 1/\sqrt{3}} = \sqrt{3}.$$

The unknown tower height cancels. Option (A) is the reciprocal and would reverse the order of the two segments. Neither the rational ratio in (B) nor the extra $\sqrt{2}$ in (D) follows from these three cotangents.

Step 3: Conclusion Therefore $AB : BC = \sqrt{3} : 1$. Hence, the correct option is (C).

Quick Tip to Solve: Write all three ground distances as $h\cot\alpha$ before subtracting; taking ratios of the tangents directly gives the wrong segments.

30. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to : [4 marks]

(A) $s \vee (r \vee \sim s)$

(B) $s \wedge r$

(C) $s \wedge \sim r$

(D) $s \wedge (r \wedge \sim s)$

Correct Answer: (B) $s \wedge r$

Solution:

Concept:

- De Morgan's laws give $\neg(P \vee Q) = (\neg P) \wedge (\neg Q)$ and $\neg(P \wedge Q) = (\neg P) \vee (\neg Q)$.
- The distributive law and complement law give $s \wedge (r \vee \neg s) = (s \wedge r) \vee (s \wedge \neg s) = s \wedge r$.

Step 1: Negate the outer disjunction Denote the given statement by $T = (\neg s) \vee ((\neg r) \wedge s)$. We need the negation of the complete statement, including both terms of this disjunction. Hence

$$\neg T = \neg(\neg s) \wedge \neg((\neg r) \wedge s).$$

The first term becomes s. Negating the second conjunction changes it into a disjunction and negates each constituent, so

$$\neg T = s \wedge (r \vee \neg s).$$

Step 2: Eliminate the contradictory term Distribute s over the parentheses:

$$\neg T = (s \wedge r) \vee (s \wedge \neg s) = s \wedge r.$$

The second term is always false, because a statement and its negation cannot both hold. As a direct check, the original disjunction is false only if s is true and r is also true; those are exactly the cases in which its negation must be true. Option (A) is a tautology because it contains $s \vee \neg s$. Option (C) instead requires r false, and option (D) contains the contradiction $s \wedge \neg s$, so neither is equivalent to the required negation.

Step 3: Conclusion The required equivalent statement is $s \wedge r$. Hence, the correct option is (B).

Quick Tip to Solve: Negate from the outermost connective inward: exchange AND and OR, negate each term, and then simplify complementary terms.

Physics (120 marks)

31. As an electron makes a transition from an excited state to the ground state of a hydrogen - like atom/ion : [4 marks]

(A) kinetic energy decreases, potential energy increases but total energy remains same

(B) kinetic energy and total energy decrease but potential energy increases

(C) its kinetic energy increases but potential energy and total energy decrease

(D) kinetic energy, potential energy and total energy decrease

Correct Answer: (C) its kinetic energy increases but potential energy and total energy decrease

Solution:

Concept:

- For a hydrogen-like ion, the bound-state total energy is $E_n = -13.6Z^2/n^2$ eV, measured relative to a free electron at infinity.
- For a Coulomb orbit, $U_n = -2K_n$ and $E_n = -K_n$. Thus kinetic energy is positive while potential and total energies are negative.

Step 1: Compare the level energies before and after transition Let the excited state have principal quantum number $n > 1$; the ground state has $n = 1$. The total energy changes from $-13.6Z^2/n^2$ to $-13.6Z^2$ in electronvolts. The final value is more negative, and therefore the electron's total energy decreases. The lost energy is emitted as a photon; it does not remain in the electron's mechanical energy. This rules out any statement that the total electron energy stays unchanged.

Step 2: Use the Coulomb energy relations Writing $a = 13.6Z^2\,\text{eV} > 0$, the initial and final kinetic energies are

$$K_i = \frac{a}{n^2}, \qquad K_f = a,$$

so the kinetic energy increases. Meanwhile,

$$U_i = -\frac{2a}{n^2}, \qquad U_f = -2a,$$

so the potential energy decreases. Equivalently, the electron enters an orbit closer to the nucleus, where attraction is stronger and the potential energy is more negative. Options (A) and (B) incorrectly predict an increase in potential energy. Option (D) correctly lowers potential and total energies but incorrectly lowers kinetic energy. The distinction is between algebraic energy and its magnitude: the magnitude of the negative potential energy grows while its actual value falls.

Step 3: Conclusion The kinetic energy increases, while both potential energy and total energy decrease. Hence, the correct option is (C).

Quick Tip to Solve: For a bound Coulomb orbit remember $K = -E$ and $U = 2E$; a downward transition makes E more negative, immediately fixing all three changes.

32. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of $1\ m$ from the diode is : [4 marks]

(A) $5.48\ V/m$

(B) $7.75\ V/m$

(C) $1.73\ V/m$

(D) $2.45\ V/m$

Correct Answer: (D) $2.45\ V/m$

Solution:

Concept:

- For radiation emitted uniformly in all directions, conservation of power gives the mean intensity $I = P/(4\pi r^2)$.
- A sinusoidal electromagnetic wave with peak electric field E_0 has mean intensity $I = \frac{1}{2}\varepsilon_0 c E_0^2$. The factor $1/2$ is required when the requested field is the amplitude.

Step 1: Spread the radiated power over a sphere The LED supplies $P = 0.1\,\mathrm{W}$, and the observation point is $r = 1\,\mathrm{m}$ away. The area of the surrounding sphere is $4\pi r^2$, not πr^2. Thus

$$I = \frac{0.1}{4\pi(1)^2} = 7.96 \times 10^{-3}\,\mathrm{W\,m^{-2}}.$$

The quoted wattage is the total power, while intensity is power per unit area at the specified distance.

Step 2: Recover the peak electric field Using $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$ and $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$, solve the mean-intensity relation:

$$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2(7.96 \times 10^{-3})}{(8.85 \times 10^{-12})(3.00 \times 10^8)}}.$$

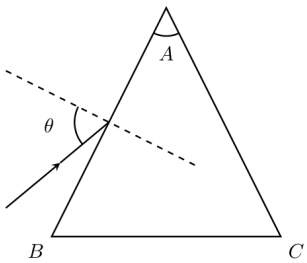
$$E_0 \approx \sqrt{6.00} = 2.45\,\mathrm{V\,m^{-1}}.$$

The value $1.73\,\mathrm{V\,m^{-1}}$ in (C) is the rms field $E_0/\sqrt{2}$, not the amplitude. Values (A) and (B) produce larger intensities than the given isotropic power permits. The red colour fixes the frequency range but does not enter this intensity-to-field conversion.

Step 3: Conclusion The electric-field amplitude is $2.45\,\mathrm{V\,m^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: First divide isotropic power by $4\pi r^2$; then use $2I$ under the square root for peak field, or I for rms field.

33. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ. on the face AB would get transmitted through the face AC of the prism provided : [4 marks]



(A) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(B) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(C) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(D) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

Correct Answer: (C) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

Solution:

Concept:

- For an air-to-glass interface, Snell's law is $\sin\theta = \mu \sin r_1$, with all incidence and refraction angles measured from their respective normals.
- Inside a prism of angle A, $r_1 + r_2 = A$. A ray emerges from glass into air when its incidence r_2 is below the critical angle $i_c = \sin^{-1}(1/\mu)$.

Step 1: Apply the emergence condition at the second face Let r_1 be the refracted angle at face AB, and r_2 the internal incidence angle at face AC. At the critical value $r_2 = i_c$, the refracted ray just grazes the second face. Transmission across that face therefore requires

$$r_2 < i_c = \sin^{-1}\left(\frac{1}{\mu}\right).$$

Using the prism geometry gives

$$A - r_1 < i_c \quad \Rightarrow \quad r_1 > A - i_c.$$

A larger entrance angle increases r_1, which decreases the incidence at the exit face and makes emergence easier.

Step 2: Convert to a condition on the entrance angle Snell's law at the first face gives $\sin\theta = \mu \sin r_1$. In the ordinary prism geometry, the relevant angles lie on the increasing branch of sine. The limiting entrance angle satisfies

$$\sin\theta_c = \mu \sin\left(A - \sin^{-1}\frac{1}{\mu}\right).$$

Thus the required inequality is

$$\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)\right].$$

The minus sign follows directly from $r_1 = A - r_2$; the outer inverse sine follows from Snell's law. The cosine and plus-sign expressions in (A) and (B) do not express this limiting condition, while (D) reverses the necessary inequality. The threshold formula is understood for a real, physically accessible limiting angle.

Step 3: Conclusion Transmission through face AC requires the stated inverse-sine lower bound on θ. Hence, the correct option is (C).

Quick Tip to Solve: Work backward from the exit face: set its internal incidence to the critical angle, use $r_1 + r_2 = A$, then apply Snell's law at entrance.

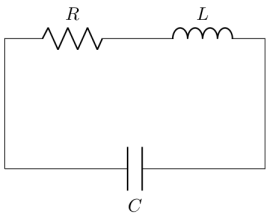
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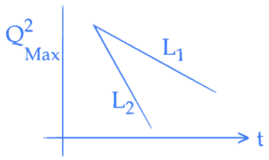
34. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below :

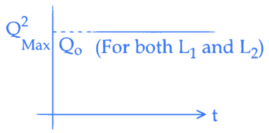
If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time(t) for two different values L_1 and L_2 $(L_1 > L_2)$ of L then which of the following represents this graph correctly ?

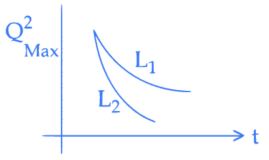
(plots are schematic and niot drawn to scale)

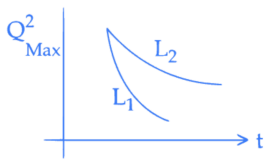
[4 marks]



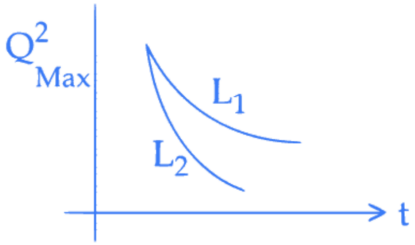
(A) 

(B) 

(C) 

(D) 

Correct Answer: (C)



Solution:

Concept:

- In an underdamped series LCR circuit, charge obeys $L\ddot{q} + R\dot{q} + q/C = 0$. The charge envelope decays as $Q_{\max} = Q_0 e^{-Rt/(2L)}$.
- Squaring the envelope gives $Q^2_{\max} = Q_0^2 e^{-Rt/L}$. Its decay time is L/R, so larger inductance means slower damping when the resistance is unchanged.

Step 1: Obtain the envelope of the squared maximum charge The capacitor initially stores energy, which then oscillates between the electric field of the capacitor and magnetic field of the inductor. The resistor dissipates energy, so successive charge maxima decrease. For the oscillatory solution of the series LCR equation, the amplitude factor is

$$Q_{\max}(t) = Q_0 \exp\left(-\frac{Rt}{2L}\right).$$

The question asks for its square, rather than the instantaneous charge squared. Therefore the plotted smooth envelope is

$$Q^2_{\max}(t) = Q_0^2 \exp\left(-\frac{Rt}{L}\right).$$

Step 2: Compare the two inductances Both traces start with the same ordinate Q_0^2. For $L_1 > L_2$ and any $t > 0$, the exponent $-Rt/L_1$ is less negative than $-Rt/L_2$. Consequently,

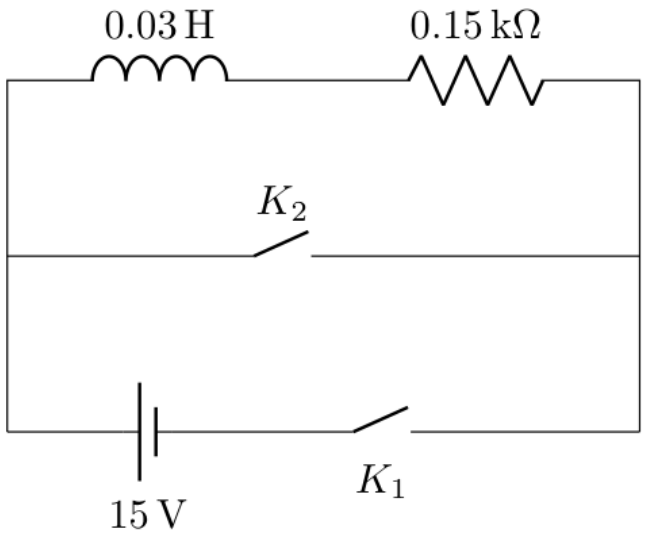
$$Q^2_{\max,1}(t) > Q^2_{\max,2}(t).$$

Both curves decrease exponentially toward zero, with L_1 above L_2. This is the arrangement shown in (C). The straight lines in (A) incorrectly imply a constant rate of loss of squared amplitude. The horizontal trace in (B) would mean no damping despite the resistor. The curves in (D) reverse the dependence on inductance and therefore assign the faster decay to the larger L.

Step 3: Conclusion The correct graph shows two exponential decays with the L_1 curve above the L_2 curve. Hence, the correct option is (C).

Quick Tip to Solve: Squaring an exponential envelope doubles its decay exponent, but the larger L/R still gives the slower decay.

35. An inductor $(L = 0.03\ H)$ and a resistor $(R = 0.15\,k\Omega)$ are connected in series to a battery of $15V$ EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1\ ms$, the current in the circuit will be : $(e^5 \cong 150)$ [4 marks]



(A) $6.7\ mA$

(B) $0.67\ mA$

(C) $100\ mA$

(D) $67\ mA$

Correct Answer: (B) $0.67\ mA$

Solution:

Concept:

- After a series LR circuit has reached its dc steady state, an ideal inductor has zero induced emf and the current is $I_0 = V/R$.
- When the source is removed and a closed LR loop remains, the current is continuous initially and decays as $I(t) = I_0 e^{-Rt/L}$, with time constant $\tau = L/R$.

Step 1: Find the current immediately before switching With K_1 closed for a long time, the inductor is in steady state. Convert the resistance before substituting:

$$R = 0.15\,\text{k}\Omega = 150\,\Omega, \qquad I_0 = \frac{15}{150} = 0.10\,\text{A}.$$

Closing K_2 as K_1 opens provides the discharge path through the inductor and resistor. The battery no longer supplies that loop. The current cannot jump to zero because an instantaneous change would require an infinite inductive emf.

Step 2: Calculate the decay over one millisecond The decay time is

$$\tau = \frac{L}{R} = \frac{0.03}{150} = 2.0 \times 10^{-4}\,\text{s} = 0.20\,\text{ms}.$$

At the required time $t = 1\,\text{ms} = 5\tau$, use the supplied approximation $e^5 \simeq 150$:

$$I = 0.10e^{-5}\,\text{A} = \frac{0.10}{150}\,\text{A} = 6.67 \times 10^{-4}\,\text{A} \simeq 0.67\,\text{mA}.$$

Option (C), $100\,\text{mA}$, is the initial steady current rather than its value after decay. Options (A) and (D) are too large; at five time constants the remaining current is about one part in 150, so neither can satisfy the exponential law. Switching the source off requires the decay formula, not the current-growth expression.

Step 3: Conclusion The current at $1\,\text{ms}$ is approximately $0.67\,\text{mA}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert $\text{k}\Omega$ and ms first, then compute t/τ; five time constants here means divide the initial current by 150.

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36. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam : [4 marks]

(A) bends down wards

(B) bends upwards

(C) becomes narrower

(D) goes horizontally without any deflection

Correct Answer: (B) bends upwards

Solution:

Concept:

- The speed of light in a medium of refractive index n is $v = c/n$. A larger refractive index therefore means a smaller propagation speed.
- According to Huygens' principle, each point on a wavefront sends out a secondary wavelet. Different speeds at different heights rotate the wavefront and bend the ray, which is normal to that wavefront.

Step 1: Compare the speeds above and below the beam The question explicitly states that refractive index is smallest near the ground and increases with height. Thus the lower part of an initially vertical wavefront advances farther horizontally in the same short time than its upper part. This comparison follows from the stated index gradient; the word night does not change that given condition.

Step 2: Use the rotation of the wavefront Because the bottom advances faster, the new wavefront has its bottom farther forward than its top. Its forward normal consequently tilts upwards. Equivalently, a ray in a smoothly varying medium bends towards the region of higher refractive index, which here is the air at greater height. The direction of bending is therefore upwards rather than downwards.

Step 3: Check the alternatives Downward bending would require refractive index to increase downwards, contrary to the question. No deflection would require equal advance of the upper and lower parts, which is impossible with this gradient. Becoming narrower is not the directional consequence predicted by the Huygens construction for this horizontally launched beam.

Step 4: Conclusion The beam bends towards the higher-index air above it, so it bends upwards. Hence, the correct option is (B).

Quick Tip to Solve: First write $v = c/n$: the faster, lower-index side advances farther, and the ray turns towards the slower, higher-index side.

37. Two coaxial solenoids of different radius carry current I in the same direction. $\overrightarrow{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\overrightarrow{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then : [4 marks]

(A) $\overrightarrow{F_1}$ is radially in wards and $\overrightarrow{F_2} = 0$

(B) $\overrightarrow{F_1}$ is radially outwards and $\overrightarrow{F_2} = 0$

(C) $\overrightarrow{F_1} = \overrightarrow{F_2} = 0$

(D) $\overrightarrow{F_1}$ is radially inwards and $\overrightarrow{F_2}$ is radially outards

Correct Answer: (C) $\overrightarrow{F_1} = \overrightarrow{F_2} = 0$

Solution:

Concept:

- The force on a complete solenoid is the vector sum of the magnetic forces on all its current elements: $\boldsymbol{F} = \int I\,\mathrm{d}\boldsymbol{\ell} \times \boldsymbol{B}$. Local forces that stretch its turns must be distinguished from this resultant.
- In the standard coaxial long-solenoid model, the field is axial inside and negligible outside. Rotational symmetry makes equal radial force elements on opposite sides cancel as vectors.

Step 1: Examine the inner winding element by element The inner winding lies in the axial magnetic field of the outer solenoid. Its azimuthal current elements each experience a local outward radial force, since $\mathrm{d}\boldsymbol{F} = I\,\mathrm{d}\boldsymbol{\ell} \times \boldsymbol{B}$. However, the outward directions differ around a complete turn. Each element has a diametrically opposite partner with the same force magnitude and the opposite vector direction. Thus the winding may experience radial stress even though its total translational force is zero.

Step 2: Sum the forces on each complete solenoid For a complete inner turn in the uniform axial field of the outer solenoid,

$$\boldsymbol{F}_1 = I\left(\oint \mathrm{d}\boldsymbol{\ell}\right) \times \boldsymbol{B}_{\text{outer}} = \boldsymbol{0}.$$

The closed-loop displacement integral vanishes. Summing complete turns preserves this result. The outer winding lies outside the inner ideal long solenoid, where its field is negligible, so $\boldsymbol{F}_2 = \boldsymbol{0}$ as well. These are the resultant forces on the solenoids, which are the vectors asked for in the question.

Step 3: Distinguish radial stress from the requested force Options (A), (B) and (D) assign a single radial direction to a force on the entire inner solenoid. Such a direction cannot be the resultant on a rotationally symmetric winding: opposite radial contributions cancel. Option (B) describes the local outward loading on elements of the inner winding, but confuses it with the total vector force. Option (C) correctly states that both complete-solenoid resultant forces vanish.

Step 4: Conclusion The resultant magnetic forces are $\boldsymbol{F}_1 = \boldsymbol{F}_2 = \boldsymbol{0}$. Local radial stress on a winding

does not constitute a nonzero net force. Hence, the correct option is (C).

Quick Tip to Solve: When a question gives force vectors for an entire circular winding, sum opposite current-element forces before interpreting a local radial push as a resultant.

38. Assuming human pupil to have a radius of $0.25\ cm$ and a comfortable viewing distance of $25\ cm$, the minimum separation between two objects that human eye can resolve at $500\ nm$ wavelength is : [4 marks]

(A) $100\,\mu m$

(B) $300\,\mu m$

(C) $1\,\mu m$

(D) $30\,\mu m$

Correct Answer: (D) $30\,\mu m$

Solution:

Concept:

- The pupil acts as a circular aperture. Its diffraction-limited angular resolution is $\theta_{\min} = 1.22\lambda/D$, where D is the pupil diameter and λ is the wavelength.
- For two nearby objects at distance s, the small-angle relation is $d_{\min} = s\theta_{\min}$. All lengths must be expressed in consistent units before substitution.

Step 1: Convert the given aperture size The stated $0.25\,\text{cm}$ is the radius, not the diameter. Therefore the diameter used in the circular-aperture formula is

$$D = 2(0.25\,\text{cm}) = 0.50\,\text{cm} = 5.0 \times 10^{-3}\,\text{m}.$$

The viewing distance and wavelength become $s = 0.25\,\text{m}$ and $\lambda = 500 \times 10^{-9}\,\text{m}$. The viewing distance converts an angular limit into a separation in the object plane.

Step 2: Calculate the smallest resolvable angle Apply the Rayleigh criterion for a circular pupil:

$$\theta_{\min} = \frac{1.22(500 \times 10^{-9})}{5.0 \times 10^{-3}} = 1.22 \times 10^{-4}\,\text{rad}.$$

This angle is much smaller than a radian, so replacing its tangent by the angle is consistent with the small-angle separation formula.

Step 3: Calculate the linear separation and compare Multiply by the object distance:

$$d_{\min} = 0.25(1.22 \times 10^{-4}) = 3.05 \times 10^{-5}\,\text{m} = 30.5\,\mu\text{m}.$$

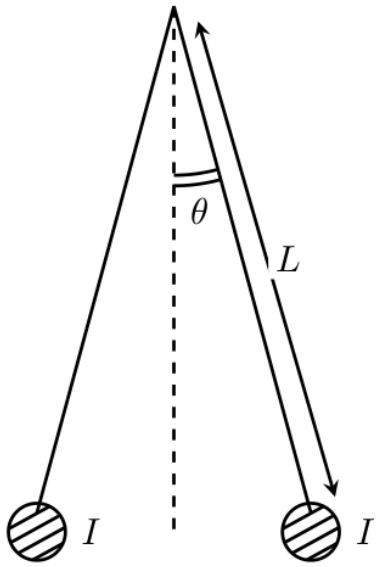
Thus the closest offered value is $30\,\mu\text{m}$. The $1\,\mu\text{m}$ choice is below the diffraction limit, while 100 and $300\,\mu\text{m}$ are larger separations and do not represent the minimum calculated from the given aperture.

Step 4: Conclusion The minimum separation is approximately $30.5\,\mu\text{m}$, or $30\,\mu\text{m}$ among the choices. Hence, the correct option is (D).

Quick Tip to Solve: For eye-resolution questions, double a given pupil radius before using $1.22\lambda/D$, then multiply the resulting angle by the viewing distance.

39. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle $'\theta'$ with the vertical. If wires have mass λ per unit-length then the value of I is :

($g = gravitational\ acceleration$) [4 marks]



(A) $2\sqrt{\frac{\pi gL}{\mu_0}\tan\theta}$

(B) $\sqrt{\frac{\pi\lambda gL}{\mu_0}\tan\theta}$

(C) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$

(D) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$

Correct Answer: (D) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$

Solution:

Concept:

- Two long parallel wires separated by distance d, each carrying current of magnitude I, exert magnetic force per unit length $f = \mu_0 I^2/(2\pi d)$. The outward deflection shown corresponds to repulsion.
- A wire of mass per unit length λ has weight per unit length λg. Balancing the horizontal and vertical components of thread tension gives $f/(\lambda g) = \tan\theta$.

Step 1: Find the distance between the wires Each thread has length L and makes angle θ with the vertical. Each wire is displaced horizontally by $L\sin\theta$ from the central vertical line, so the separation of the two wires is

$$d = 2L\sin\theta.$$

Both displacements must be included. Using just one displacement would lose a factor of two in the separation and give an incorrect current.

Step 2: Eliminate the tension from equilibrium Consider equal lengths of the two wires so that magnetic force and weight can both be written per unit length. If τ is the supporting tension per unit

wire length, then

$$\tau \sin\theta = f, \qquad \tau \cos\theta = \lambda g.$$

Dividing yields $f = \lambda g \tan\theta$. The thread force supplies the inward component that balances the outward magnetic repulsion.

Step 3: Substitute the magnetic force and solve for current Using the separation found above,

$$\frac{\mu_0 I^2}{4\pi L \sin\theta} = \lambda g \tan\theta.$$

Hence

$$I^2 = \frac{4\pi\lambda g L \sin^2\theta}{\mu_0 \cos\theta}, \qquad I = 2\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0 \cos\theta}}.$$

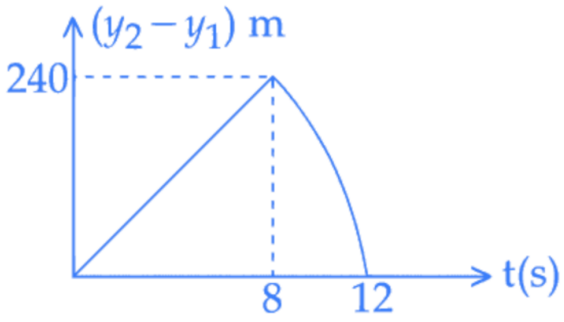
Option (A) lacks the mass density and is dimensionally wrong; (B) has the wrong angular dependence, and (C) is smaller by a factor of two.

Step 4: Conclusion The current magnitude is $I = 2\sin\theta\sqrt{\pi\lambda g L/(\mu_0\cos\theta)}$. Hence, the correct option is (D).

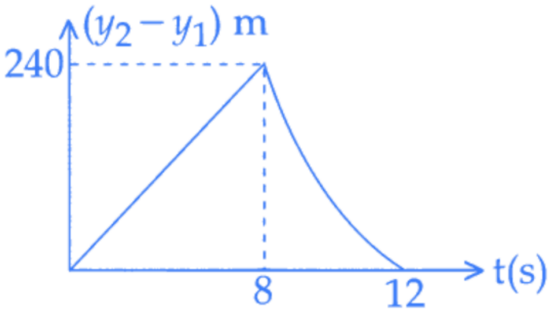
Quick Tip to Solve: Use force per unit length throughout, and calculate the full wire separation $2L\sin\theta$ before eliminating the tension.

40. Two stones are thrown up simultaneously from the edge of a cliff $240\ m$ high with initial speed of $10\ m/s$ and $40\ m/s$ respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?
(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10 m/s^2$)
(The figures are schematic and not drawn to scale) [4 marks]

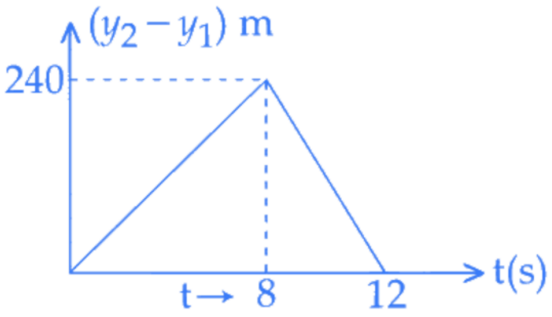
(A)



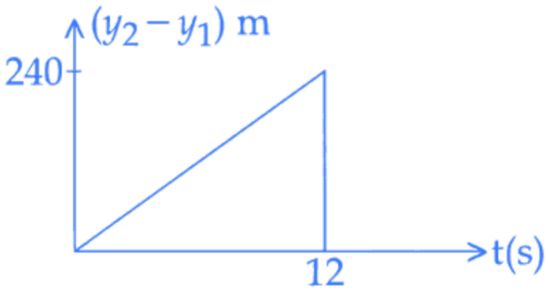
(B)



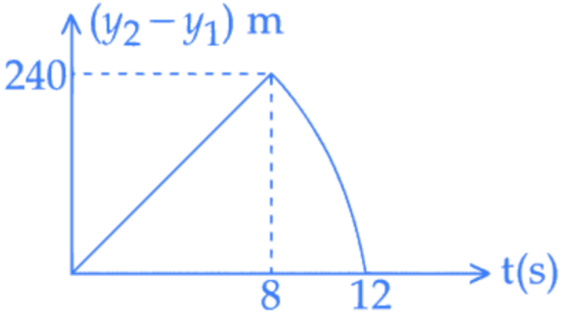
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- While both stones are freely moving, they have the same gravitational acceleration, so their relative acceleration is zero. Their relative displacement therefore grows at the constant relative velocity $40 - 10 = 30\,\mathrm{m\,s^{-1}}$.
- After a stone reaches the ground it remains there. Its position must then be held constant; continuing its free-fall parabola below ground would give the wrong relative-position graph.

Step 1: Find when each stone reaches the ground Take upward as positive and the ground as height zero. While airborne,

$$h_1 = 240 + 10t - 5t^2, \qquad h_2 = 240 + 40t - 5t^2.$$

Solving $h_1 = 0$ gives the positive root $t = 8\,\mathrm{s}$, while solving $h_2 = 0$ gives $t = 12\,\mathrm{s}$. The first stone therefore comes to rest four seconds before the second.

Step 2: Obtain the relative position in each interval Before the first impact the common height and gravitational terms cancel. After it, $h_1 = 0$, so the relative position equals the height of the second

stone:

$$h_2 - h_1 = \begin{cases} 30t, & 0 \le t \le 8, \\ 240 + 40t - 5t^2, & 8 \le t \le 12, \\ 0, & t \ge 12. \end{cases}$$

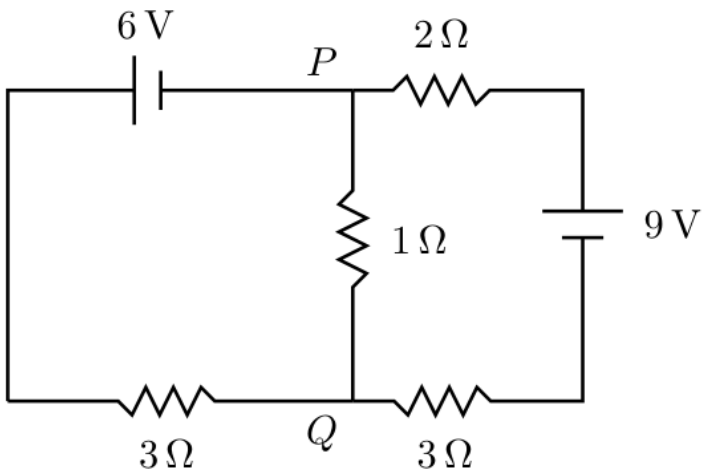
It rises linearly to $240\,\mathrm{m}$ at $t = 8\,\mathrm{s}$, then falls continuously to zero at $t = 12\,\mathrm{s}$.

Step 3: Identify the curvature of the descending segment During the second interval, the slope is $40 - 10t$, changing from -40 to $-80\,\mathrm{m\,s^{-1}}$. It becomes more negative, so the curve is concave down and steepens as it approaches the ground. Option (A) shows exactly that behavior. Option (B) flattens instead, (C) gives a straight fall, and (D) incorrectly delays the end of the linear rise and then inserts a discontinuous drop.

Step 4: Conclusion The relative position rises linearly until $8\,\mathrm{s}$, then decreases along a concave-down parabola to zero at $12\,\mathrm{s}$. Hence, the correct option is (A).

Quick Tip to Solve: Equal accelerations cancel only while both objects are airborne. At the first impact, switch to the remaining stone's height above the ground.

41. In the circuit shown, the current in the 1Ω resistor is : [4 marks]



(A) $0.13\ A$, from Q to P

(B) $0.13\ A$, from P to Q

(C) $1.3A$ from P to Q

(D) $0A$

Correct Answer: (A) $0.13\ A$, from Q to P

Solution:

Concept:

- Kirchhoff's current law states that the algebraic sum of currents leaving a junction is zero. Take $V_Q = 0$ and write $V = V_P - V_Q$.

- A battery gives a rise from its short plate to its long plate. In the diagram the 6 V cell is positive at the left, while the 9 V cell is positive at the top. These polarities must be retained when writing branch currents.

Step 1: Write the currents in the three branches Choose every branch current as positive from P to Q. The middle branch contains only $1\,\Omega$, giving $i_m = V/1$. In the left branch, moving from P crosses the 6 V cell from negative to positive before reaching its $3\,\Omega$ resistor, so $i_L = (V+6)/3$. The right branch contains $2+3=5\,\Omega$ and a 9 V drop from top to bottom, giving $i_R = (V-9)/5$.

Step 2: Apply the node equation No external current enters the P junction, so

$$\frac{V+6}{3} + V + \frac{V-9}{5} = 0.$$

Multiplication by 15 gives

$$5V + 30 + 15V + 3V - 27 = 0, \qquad 23V = -3.$$

Therefore $V = -3/23\,\text{V}$. This negative sign means that P is below Q in potential; it is not an error in the initially chosen current direction.

Step 3: Interpret the current and check its magnitude The middle-branch current in the chosen P-to-Q direction is

$$i_m = -\frac{3}{23}\,\text{A} \simeq -0.1304\,\text{A}.$$

Its physical direction is therefore Q to P. For a check, the left and right branch currents are $45/23$ and $-42/23\,\text{A}$, whose sum with $-3/23\,\text{A}$ is zero. Option (B) reverses the direction, (C) also uses a tenfold magnitude, and (D) incorrectly assumes a balanced zero-current bridge.

Step 4: Conclusion The current in the $1\,\Omega$ resistor is approximately $0.13\,\text{A}$, flowing from Q to P. Hence, the correct option is (A).

Quick Tip to Solve: Use a single unknown node voltage across P and Q; a negative current then gives the actual direction without rewriting the circuit equations.

42. When $5V$ potential difference is applied across a wire of length $0.1\ m$, the drift speed of electrons is $2.5 \times 10^{-4}\ ms^{-1}$. If the electron density in the wire is $8 \times 10^{28}\ m^{-3}$, the resistivity of the material is close to : [4 marks]

(A) $1.6 \times 10^{-6} \Omega m$

(B) $1.6 \times 10^{-5} \Omega m$

(C) $1.6 \times 10^{-8} \Omega m$

(D) $1.6 \times 10^{-7} \Omega m$

Correct Answer: (B) $1.6 \times 10^{-5} \Omega m$

Solution:

Concept:

- The magnitude of current density is $J = nev_d$, where n is the free-electron number density, e is the magnitude of the electron charge and v_d is the drift speed.
- The electric field in a uniform wire is $E = V/L$. The local form of Ohm's law, $E = \rho J$, therefore gives $\rho = V/(Lnev_d)$, independently of the wire cross-sectional area.

Step 1: Calculate the electric field The applied potential difference is $5\,\mathrm{V}$ and the wire length is $0.1\,\mathrm{m}$. Hence

$$E = \frac{V}{L} = \frac{5}{0.1} = 50\,\mathrm{V\,m^{-1}}.$$

Using the full voltage as the electric field would omit the division by length and produce an incorrect power of ten in the resistivity.

Step 2: Calculate the current density from drift motion Use $e = 1.6 \times 10^{-19}\,\mathrm{C}$ and the given density and drift speed:

$$\begin{aligned} J &= (8 \times 10^{28})(1.6 \times 10^{-19})(2.5 \times 10^{-4}) \\ &= 32 \times 10^{5} = 3.2 \times 10^{6}\,\mathrm{A\,m^{-2}}. \end{aligned}$$

The electron drift direction is opposite to conventional current, but only their magnitudes are needed to find the positive material resistivity.

Step 3: Divide field by current density From the local Ohm law,

$$\rho = \frac{50}{3.2 \times 10^{6}} = 1.5625 \times 10^{-5}\,\Omega\,\mathrm{m}.$$

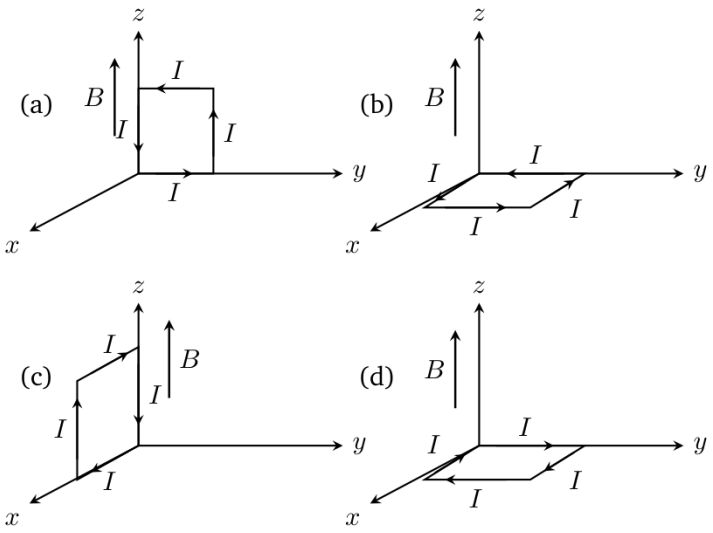
Rounding to two significant figures yields $1.6 \times 10^{-5}\,\Omega\,\mathrm{m}$. The other choices differ by powers of ten: (A) is ten times smaller, (C) is a thousand times smaller, and (D) is a hundred times smaller. No area is needed because current density already accounts for charge flow per unit area.

Step 4: Conclusion The resistivity is approximately $1.6 \times 10^{-5}\,\Omega\,\mathrm{m}$. Hence, the correct option is (B).

Quick Tip to Solve: Combine $E = V/L$ and $J = nev_d$ directly as $\rho = E/J$; keep the powers of ten separate from the numerical product.

43. A rectangular loop of sides $10\ cm$ and $5\ cm$ carrying a current 1 of $12A$ is placed in different orientations as shown in the figures below :

If there is a uniform magnetic field of $0.3\ T$ in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium ? [4 marks]



(A) (B) and (D), respectively

(B) (B) and (C), respectively

(C) (A) and (B), respectively

(D) (A) and (C), respectively

Correct Answer: (A) (B) and (D), respectively

Solution:

Concept:

- A plane current loop behaves as a magnetic dipole with moment $\boldsymbol{m} = IA\hat{n}$, where the right-hand curl in the current direction fixes the unit normal $\hat{n}$.
- The torque is $\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B}$, and the potential energy is $U = -\boldsymbol{m} \cdot \boldsymbol{B} = -mB\cos\theta$. Parallel alignment is a minimum of energy, while antiparallel alignment is a maximum.

Step 1: Identify the orientations with zero torque The field points along positive z. In drawings (a) and (c), the rectangular loop lies in a vertical plane, so its normal and its magnetic moment are perpendicular to z. Each therefore experiences a nonzero torque and cannot be either equilibrium orientation. In (b) and (d), the loop lies in the xy-plane, making its magnetic moment parallel or antiparallel to the field. The magnitude is

$$m = IA = 12(0.10)(0.05) = 0.060\ \mathrm{A\,m^2}.$$

This magnitude sets the energy and torque scales but does not decide which zero-torque orientation is stable.

Step 2: Apply the current-direction rule and test stability Curl the fingers of the right hand along the current arrows. In (b), the arrows give the normal along positive z, hence $\boldsymbol{m}$ is parallel to $\boldsymbol{B}$. Its

energy is $U = -mB = -0.018\,\text{J}$, the minimum, so a small tilt produces a restoring torque. In (d), the current reverses around the horizontal loop and the normal points along negative z. Its energy is $U = +mB = +0.018\,\text{J}$, the maximum, so a small tilt grows rather than being restored. Thus the pair is (B) followed by (D), using the capital orientation labels in the answer choices. The other choices include vertical-plane orientations that have nonzero torque.

Step 3: Conclusion The second drawing gives stable equilibrium, and the fourth drawing gives unstable equilibrium. Hence, the correct option is (A).

Quick Tip to Solve: First require the loop normal to be along the magnetic field, then use the current arrows: parallel moment is stable and antiparallel moment is unstable.

---

44. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to :

($g = gravitational\ acceleration$) [4 marks]

(A) $\left[1 - \left(\frac{T_M}{T}\right)^2\right] \frac{A}{Mg}$

(B) $\left[1 - \left(\frac{T}{T_M}\right)^2\right] \frac{A}{Mg}$

(C) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{A}{Mg}$

(D) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{Mg}{A}$

Correct Answer: (C) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{A}{Mg}$

Solution:

Concept:

- For small oscillations, a simple pendulum of effective length L has period $T = 2\pi\sqrt{L/g}$. The period changes here because the wire stretches, rather than because the period directly depends on bob mass.
- Young's modulus is stress divided by longitudinal strain: $Y = (F/A)/(\Delta L/L)$. For the additional load, $F = Mg$; the elastic strain is small and the same length is used to the accuracy of this relation.

Step 1: Relate the extra load to the extension Let the wire length before adding the mass be L, and let the added mass produce an extension ΔL. The additional tensile stress is Mg/A. In the usual small-strain approximation for the wire, Hooke's law gives

$$\frac{\Delta L}{L} = \frac{Mg}{AY}.$$

The extension is positive, so the effective pendulum length and its time period must both increase. This gives an immediate sign check on any proposed answer.

Step 2: Eliminate the wire length using the periods Before and after loading, use the simple-pendulum formula with the two lengths:

$$T^2 = \frac{4\pi^2 L}{g}, \qquad T_M^2 = \frac{4\pi^2 (L + \Delta L)}{g}.$$

Taking their ratio cancels both g and the numerical factor, leaving

$$\left(\frac{T_M}{T}\right)^2 - 1 = \frac{\Delta L}{L} = \frac{Mg}{AY}.$$

Solving this last equality for the reciprocal modulus therefore gives the expression to compare with the options.

Step 3: Check the sign and dimensions Because $T_M > T$, the factor $(T_M/T)^2 - 1$ is positive. Option (A) has the opposite sign. Option (B) uses the inverse period ratio, which is not the fractional increase of length obtained above. Finally, $A/(Mg)$ has dimensions of reciprocal pressure, as $1/Y$ must; the factor Mg/A in (D) instead has dimensions of pressure.

Step 4: Conclusion

$$\boxed{\frac{1}{Y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}}.$$

Hence, the correct option is (C).

Quick Tip to Solve: For an elastic pendulum, square the period ratio first: it directly gives the length ratio. Then subtract 1 to obtain the strain.

45. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}$, $\frac{5V_0}{4}$, $\frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then [4 marks]

(A) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

(B) $2R < R_4$

(C) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

(D) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

Correct Answer: (A) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

Solution:

Concept:

- A uniformly charged solid sphere has an internal field proportional to distance from its centre. Its potential inside is $V(r) = \frac{V_0}{2}(3 - r^2/R^2)$, where $V_0 = kQ/R$.
- Outside a spherically symmetric charge distribution, $V(r) = kQ/r = V_0 R/r$. The internal formula must be used for radii below R, and the external formula for radii above R.

Step 1: Find the two internal equipotential radii The largest potential is at the centre, where the

internal formula gives $V(0) = 3V_0/2$. Thus the first specified equipotential degenerates to the central point:

$$R_1 = 0.$$

For the second potential, substitute $r = R_2$ into the internal formula and divide by V_0:

$$\frac{5}{4} = \frac{1}{2}\left(3 - \frac{R_2^2}{R^2}\right).$$

Consequently $R_2^2/R^2 = 1/2$, and the nonnegative radial coordinate is $R_2 = R/\sqrt{2}$. This lies inside the charged sphere, as required.

Step 2: Find the two external equipotential radii The remaining potential magnitudes are smaller than the surface value, so they occur outside. Apply $V(r) = V_0 R/r$ separately:

$$\frac{3}{4} = \frac{R}{R_3} \quad \Longrightarrow \quad R_3 = \frac{4R}{3}, \qquad \frac{1}{4} = \frac{R}{R_4} \quad \Longrightarrow \quad R_4 = 4R.$$

Both radii exceed R, confirming the choice of the external expression. Their separation is

$$R_4 - R_3 = 4R - \frac{4R}{3} = \frac{8R}{3}.$$

Step 3: Test every printed statement Since $R/\sqrt{2} < 8R/3$, statement (A) is true, whereas (C) reverses that inequality. Statement (D) is already false because it requires $R_1 \neq 0$; its separation inequality also fails. Independently, (B) is true because $R_4 = 4R > 2R$. Thus this printed question contains two true listed statements, rather than a unique true option.

Step 4: Conclusion

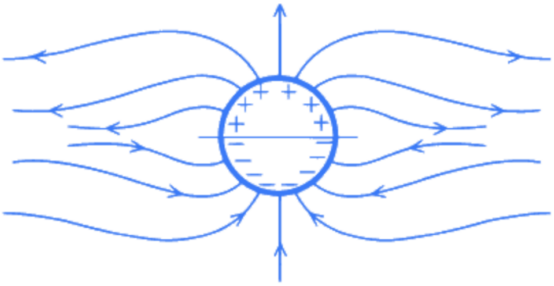
$$R_1 = 0, \qquad R_2 = \frac{R}{\sqrt{2}}, \qquad R_3 = \frac{4R}{3}, \qquad R_4 = 4R.$$

Hence, the correct option is (A). Option (B) is also true; the official answer key accepts both (A) and (B).

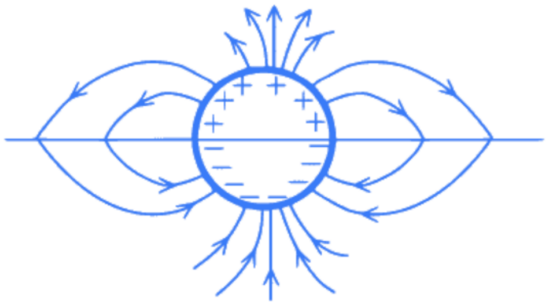
Quick Tip to Solve: Check which equipotentials lie inside the sphere before using the point-charge formula; inside, potential is quadratic in radius, not inversely proportional to it.

46. A long cylindrical shell carries positives surfaces change σ in the upper half and negative surface charge - σ in the lower half. The electric field lines around the cylinder will look like figure given in : (*figures are schematic and not drawn to scale*) [4 marks]

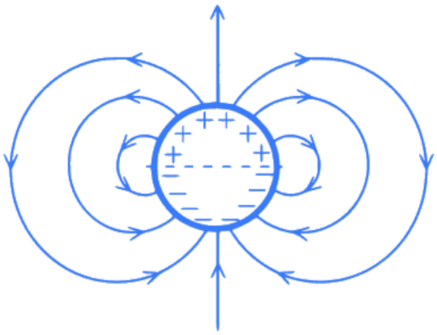
(A)



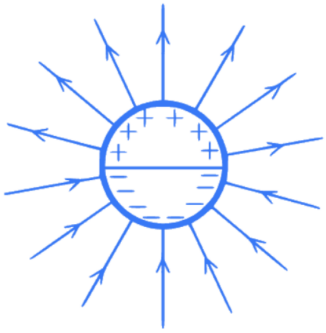
(B)



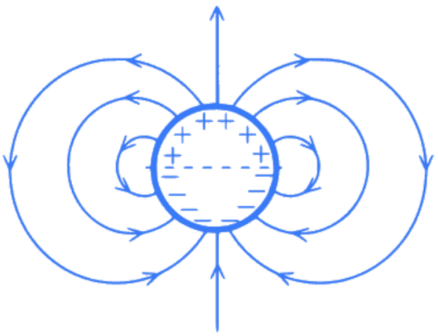
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- Electric field lines originate on positive charge and terminate on negative charge. At every regular point, their tangent gives the unique direction of the electric field; two field lines cannot cross.

- The equal and opposite charges on the upper and lower halves give zero net charge per unit length. At distances large compared with the shell radius, the leading field pattern is therefore dipolar, rather than the radial field of a uniformly charged cylinder.

Step 1: Determine the required direction and symmetry Consider a cross-section perpendicular to the long cylinder. The upper semicircle carries positive surface charge and the lower semicircle negative surface charge. Lines must leave the positive upper region and bend towards the negative lower region on both sides. Reflection in the vertical plane through the axis gives left-right symmetry. The far-away pattern has no monopole component, because the two semicircles have equal area and opposite charge density. Thus an everywhere radial pattern cannot describe this shell.

Step 2: Inspect the candidate curves In option (C), the outside curves emerge from the upper positive region, turn downwards around either side, and enter the lower negative region. The arrows have the required sense, and the curves form a smooth dipole-like pattern. The upper axial arrow points away from the positive part; the lower axial arrow points towards the negative part. These axial directions are also consistent with the signs of the charges.

Step 3: Reject the inconsistent alternatives Option (A) sends the side lines away approximately horizontally rather than producing the proper far-field return pattern; the displayed side arrows also give incompatible flow through the equatorial region. Option (B) has sharp meetings/crossings of the depicted curves at the equatorial side points: a regular electrostatic field cannot have two different tangent directions there. Option (D) depicts straight radial rays, like a monopole field, rather than the curved field joining the two oppositely charged halves. The problem states that the figures are schematic, so these qualitative direction and topology tests, rather than exact distances between curves, decide the selection.

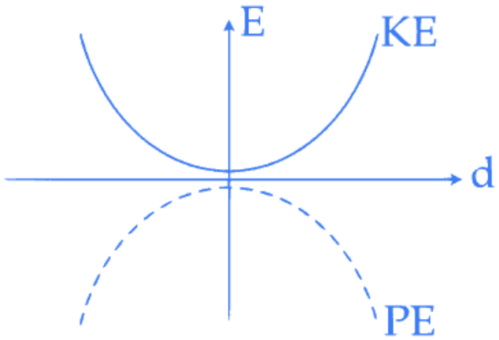
Step 4: Conclusion The smooth dipole-like pattern in which lines leave the positive upper half and enter the negative lower half is the required one. Hence, the correct option is (C).

Quick Tip to Solve: For a field-line MCQ, first check arrow direction and forbidden crossings; then check whether the net charge predicts a radial or dipole-like far field.

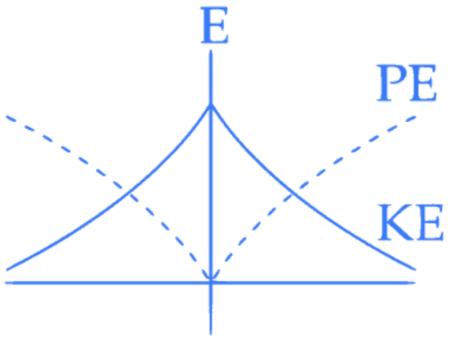
47. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly?

$(graphs\ are\ schematic\ and\ not\ drawn\ to\ scale)$ [4 marks]

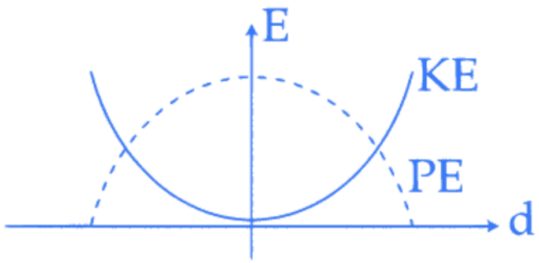
(A)



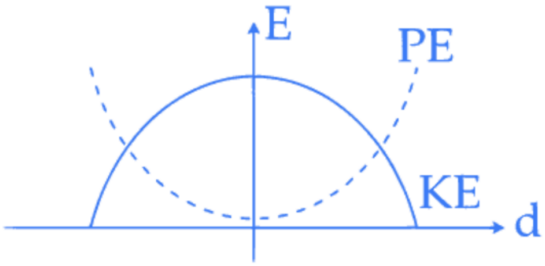
(B)



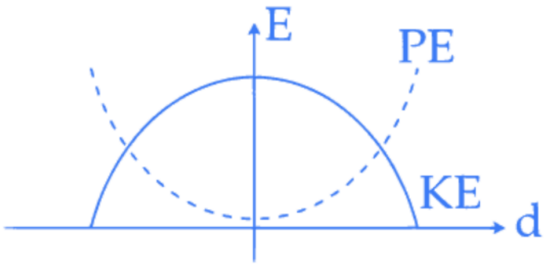
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- For a simple pendulum in the small-oscillation regime, displacement d from equilibrium obeys simple harmonic motion. With effective restoring constant $k = mg/L$, its potential energy relative to equilibrium is $PE = \frac{1}{2}kd^2$.
- The conserved total mechanical energy for amplitude a is $E = \frac{1}{2}ka^2$. Hence $KE = E - PE = \frac{1}{2}k(a^2 - d^2)$ for $-a \le d \le a$.

Step 1: Identify the potential-energy curve The restoring force is opposite to displacement and is

proportional to it for small angular oscillations. Integrating this force gives a quadratic potential energy:

$$PE(d) = \frac{1}{2}\frac{mg}{L}d^2.$$

It has its minimum at $d = 0$, is symmetric under $d \mapsto -d$, and increases towards either turning point. With the usual zero at equilibrium, its graph is an upward-opening parabola through the origin. A cusp at equilibrium would imply a discontinuous restoring-force direction with nonzero limiting magnitude, which is not the pendulum restoring force.

Step 2: Identify the kinetic-energy curve Conservation of energy gives

$$KE(d) = \frac{1}{2}ka^2 - \frac{1}{2}kd^2.$$

Therefore kinetic energy is greatest at the equilibrium position and falls symmetrically to zero at $d = \pm a$. Its graph is a downward-opening parabola over the physically allowed displacement range. Beyond the turning points the formal quadratic would be negative, so that portion does not describe accessible pendulum motion. At any allowed displacement, the two plotted energies must add to the same constant.

Step 3: Compare the labels and shapes Option (D) labels the upward-opening curve as PE and the downward-opening curve as KE, with maximum KE at the centre and vanishing KE at the extremes. Option (A) and option (C) assign the energy labels in the wrong sense. Option (B) has a pointed, nonquadratic PE minimum and the wrong smooth curvature for the SHM energy graphs. These differences remain meaningful even though the drawings are schematic and not to scale.

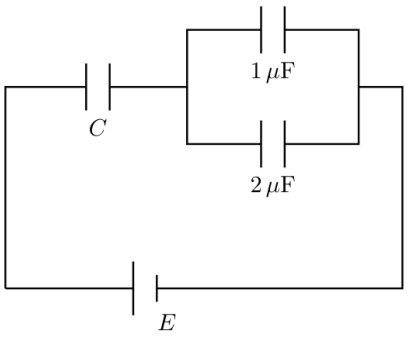
Step 4: Conclusion Potential energy rises quadratically from the equilibrium position, while kinetic energy decreases quadratically from its central maximum to zero at the turning points. Hence, the correct option is (D).

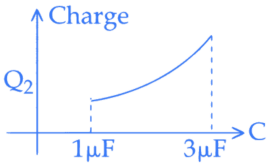
Quick Tip to Solve: At equilibrium a pendulum is fastest and lowest: KE must be maximum and PE minimum there. This immediately checks the energy labels before examining curvature.

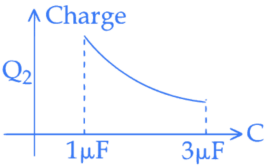
48. In the given circuit, charges Q_2 on the $2\mu F$ capacitor changes as C is varied from $1\,\mu F$ to $3\mu F$. Q_2 as a function of 'C' is given properly by:

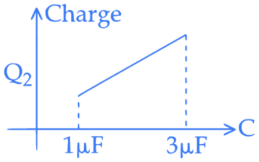
(figures are drawn schematically and are not to scale)

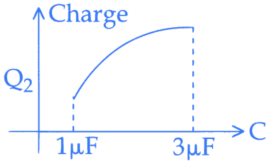
[4 marks]



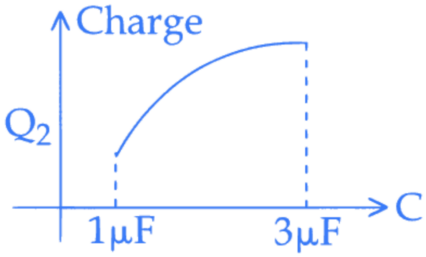
(A) 

(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- Parallel capacitors have the same potential difference and add directly. Capacitors in series carry equal total charge, and their equivalent capacitance is the product divided by the sum.
- The charge on a capacitor is $Q = CV$. The shape of a charge-versus-capacitance graph is determined by both its slope and how that slope changes, not merely by whether charge rises.

Step 1: Reduce the circuit and find the parallel-branch voltage The $1\,\mu$F and $2\,\mu$F capacitors are parallel, so their equivalent capacitance is $3\,\mu$F. That parallel combination is in series with the variable capacitor. Let $c = C/(1\,\mu\text{F})$ be its numerical capacitance in microfarads, so the stated variation is

$1 \le c \le 3$. The equivalent series capacitance and common series charge are

$$C_{\text{eq}} = \frac{3c}{c+3}\,\mu\text{F}, \qquad Q = \frac{3cE}{c+3}\,\mu\text{C},$$

when E is measured in volts. The voltage across the parallel pair is therefore

$$V_p = \frac{Q}{3\,\mu\text{F}} = \frac{cE}{c+3}.$$

Step 2: Derive the charge and determine the curve shape The required charge belongs only to the $2\,\mu$F branch, not to the whole parallel combination. Hence

$$Q_2 = (2\,\mu\text{F})V_p = \frac{2cE}{c+3}\,\mu\text{C}.$$

For a positive battery-voltage magnitude,

$$\frac{dQ_2}{dc} = \frac{6E}{(c+3)^2} > 0, \qquad \frac{d^2Q_2}{dc^2} = -\frac{12E}{(c+3)^3} < 0.$$

Thus charge increases with a progressively smaller slope. At the two limits, $Q_2(1) = E/2\,\mu$C and $Q_2(3) = E\,\mu$C. The graph must be increasing and concave downward over the displayed interval. The increasing concave-up curve, decreasing curve, and straight line fail these tests; the fourth curve has the required shape.

Step 3: Conclusion The charge varies as $Q_2 = 2cE/(c+3)\,\mu$C, rising with decreasing slope as the variable capacitance increases. Hence, the correct option is (D).

Quick Tip to Solve: Combine the two parallel capacitors before finding the voltage; for a series variable capacitor, the resulting ratio c/(c+constant) rises but flattens.

49. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is: [4 marks]

(A) $T \propto \frac{1}{R}$
(B) $T \propto \frac{1}{R^3}$
(C) $T \propto e^{-R}$
(D) $T \propto e^{-3R}$

Correct Answer: (A) $T \propto \frac{1}{R}$

Solution:

Concept:

- For the radiation gas, write the given energy density as $u = aT^4$, with constant a. The total energy is $U = aVT^4$, and the given radiation pressure is $p = aT^4/3$.

- An adiabatic expansion transfers no heat, so the first law is $dU = -p\,dV$. The spherical cavity volume is $V = 4\pi R^3/3$; its differential obeys $dV/V = 3\,dR/R$.

Step 1: Differentiate the total radiation energy The energy density is proportional to the fourth power of temperature, but the total energy also depends on the changing cavity volume. Differentiate both factors in $U = aVT^4$:

$$dU = aT^4\,dV + 4aVT^3\,dT.$$

Retaining the first term is essential: using only the temperature derivative would incorrectly treat the expanding cavity as having constant volume. The work term has the sign convention that positive outward expansion lowers the internal energy.

Step 2: Apply the adiabatic first law Substitute the pressure into $dU = -p\,dV$ and cancel the common factor aT^4:

$$aT^4\,dV + 4aVT^3\,dT = -\frac{aT^4}{3}\,dV,$$

$$4V\frac{dT}{T} = -\frac{4}{3}\,dV.$$

Separate the variables and integrate along the adiabatic path:

$$\frac{dT}{T} = -\frac{1}{3}\frac{dV}{V} \quad \Longrightarrow \quad \ln T = -\frac{1}{3}\ln V + \text{constant}.$$

Thus $TV^{1/3}$ remains constant during the expansion.

Step 3: Replace the volume by the sphere radius Since V is proportional to R^3, its cube root is proportional to R. The relation therefore reduces to

$$TR = \text{constant}, \qquad T \propto R^{-1}.$$

The temperature decreases on expansion, consistent with the radiation doing work. Option (B) would require $T \propto V^{-1}$, which does not satisfy the differential first law derived above. Options (C) and (D) are exponential functions of radius rather than the required power law; neither keeps $TV^{1/3}$ constant. No ordinary monatomic-gas heat-capacity ratio has been assumed.

Step 4: Conclusion

$$\boxed{T \propto \frac{1}{R}}.$$

Hence, the correct option is (A).

Quick Tip to Solve: For a photon gas, start with the given $U = aVT^4$ and pressure. Do not insert the adiabatic exponent of a material ideal gas.

50. A solid body of constant heat capacity $1\ J/^{\circ}C$ is being heated by keeping it in contact with reservoirs in two ways:

(i) **Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.**

(ii) **Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.**

In both the cases body is brought from initial temperature $100^{\circ}C$ to final temperature $200^{\circ}C$. Entropy change of the body in the two cases respectively is : [4 marks]

(A) $ln2, 2ln2$

(B) $2ln2, 8ln2$

(C) $ln2, 4ln2$

(D) $ln2, ln2$

Correct Answer: No printed option has the correct magnitude for 100 degrees Celsius to 200 degrees Celsius; both changes are ln(473.15/373.15) J/K.

Solution:

Concept:

- Entropy is a state function: for a body of constant heat capacity C, $\Delta S = \int_{T_i}^{T_f} C\,dT/T$. Temperatures in this formula must be absolute temperatures.
- The actual heat transfer can be irreversible; the entropy change of the body is evaluated along any reversible path between the same equilibrium states.

Step 1: Convert the endpoint temperatures The printed temperatures are in degrees Celsius, so $T_i = 373.15\,\mathrm{K}$ and $T_f = 473.15\,\mathrm{K}$. A heat capacity of $1\,\mathrm{J/^{\circ}C}$ is also $1\,\mathrm{J\,K^{-1}}$, because a Celsius interval equals a kelvin interval.

Step 2: Integrate the entropy change In either procedure the body has the same initial and final temperatures. Therefore

$$\Delta S = C \ln \frac{T_f}{T_i} = \ln \frac{473.15}{373.15}\,\mathrm{J\,K^{-1}} \simeq 0.2374\,\mathrm{J\,K^{-1}}.$$

Splitting the heating among two or eight reservoirs cannot change this state-function difference. It can change the entropy generated in the combined body-reservoir system.

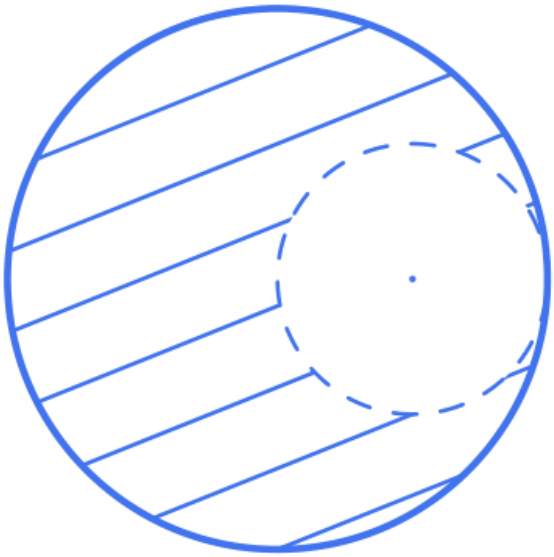
Step 3: Check the offered pairs Only the fourth pair expresses equal changes, explaining the intended selection in the supplied key. Its magnitude $\ln 2$ nevertheless results from using $200/100$, which is invalid for Celsius temperatures. The other pairs additionally violate path independence. None of the numerical pairs equals the change for the stated endpoints.

Step 4: Conclusion Both entropy changes are $\ln(473.15/373.15)\,\mathrm{J\,K^{-1}}$. No option is numerically correct as printed. The supplied key selects (D), which captures equality but has an incorrect magnitude.

Quick Tip to Solve: For every entropy logarithm, convert both endpoint temperatures to kelvin before taking their ratio.

51. From a solid sphere of mass M and radius R, a spherical portion of radius $R/2$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the center of the cavity thus formed is:

($G = gravitational\ constant$) [4 marks]



(A) $\frac{-2GM}{3R}$

(B) $\frac{-2GM}{R}$

(C) $\frac{-GM}{2R}$

(D) $\frac{-GM}{R}$

Correct Answer: (D) $\frac{-GM}{R}$

Solution:

Concept:

- Gravitational potential superposes algebraically. Removing a spherical portion is equivalent to subtracting the potential of the material that occupied that portion.
- Inside a uniform sphere of mass M and radius R, $V(r) = -GM(3R^2 - r^2)/(2R^3)$, with zero potential at infinity.

Step 1: Locate the cavity centre and removed mass The cavity has radius $R/2$ and is internally tangent to the original surface, as shown. Its centre is therefore at distance $r = R/2$ from the original centre. Equal density gives the removed mass

$$m = M\left(\frac{R/2}{R}\right)^3 = \frac{M}{8}.$$

The mass M denotes the original sphere, not the remaining material.

Step 2: Subtract the two sphere potentials At the cavity centre the full sphere would give

$$V_{\text{full}} = -\frac{GM}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) = -\frac{11GM}{8R}.$$

The removed sphere, evaluated at its own centre, would give

$$V_{\text{removed}} = -\frac{3G(M/8)}{2(R/2)} = -\frac{3GM}{8R}.$$

Thus $V = V_{\text{full}} - V_{\text{removed}} = -GM/R$. The subtraction makes the potential less negative, as required after removing attractive mass. None of the other coefficients equals this difference.

Step 3: Conclusion The potential at the cavity centre is $-GM/R$. Hence, the correct option is (D).

Quick Tip to Solve: Use potential superposition with a negative-density sphere for the cavity; evaluate each sphere at the correct distance from its own centre.

52. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is: $\left(\gamma = \frac{C_p}{C_v}\right)$ [4 marks]

(A) $\frac{\gamma+1}{2}$
(B) $\frac{\gamma-1}{2}$
(C) $\frac{3\gamma+5}{6}$
(D) $\frac{3\gamma-5}{6}$

Correct Answer: (A) $\frac{\gamma+1}{2}$

Solution:

Concept:

- The mean time between molecular collisions is $\tau = \lambda/\bar{v}$, where λ is mean free path and $\bar{v}$ is a characteristic molecular speed.
- For fixed molecule number and collision diameter, $\lambda \propto 1/n \propto V$. A quasistatic adiabatic change obeys $TV^{\gamma-1} =$ constant.

Step 1: Find the speed dependence For an ideal gas of fixed molecular mass, $\bar{v} \propto \sqrt{T}$. Using the usual adiabatic relation intended here gives

$$T \propto V^{1-\gamma}, \qquad \bar{v} \propto V^{(1-\gamma)/2}.$$

Expansion reduces the number density and, along this adiabatic path, reduces the molecular speed.

Step 2: Combine mean free path and speed For molecular diameter d, $\lambda = (\sqrt{2}\pi d^2 n)^{-1}$. Consequently

$$\tau \propto \frac{V}{V^{(1-\gamma)/2}} = V^{(\gamma+1)/2}.$$

The exponent must include both the increase in free path and the increase in collision time caused by cooling. The exponent $(\gamma - 1)/2$ includes the latter alone. The other two offered expressions do not follow from either density or speed scaling.

Step 3: Conclusion $q = (\gamma + 1)/2$. Hence, the correct option is (A).

Quick Tip to Solve: Write collision time as free path divided by speed first; density and adiabatic cooling each contribute a power of volume.

53. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to: [4 marks]

(A) 56%

(B) 62%

(C) 44%

(D) 50%

Correct Answer: (A) 56%

Solution:

Concept:

- Total linear momentum is conserved in each perpendicular direction during the collision. A perfectly inelastic collision leaves the particles moving together.
- Kinetic energy is not conserved. For the combined mass $3m$ and momentum magnitude p, the final kinetic energy is $p^2/(6m)$.

Step 1: Calculate initial energy and momentum The two incoming momenta are perpendicular, not parallel:

$$\mathbf{p} = 2mv\,\hat{\mathbf{i}} + 2mv\,\hat{\mathbf{j}}, \qquad p^2 = 8m^2v^2.$$

The initial kinetic energy is

$$K_i = \tfrac{1}{2}m(2v)^2 + \tfrac{1}{2}(2m)v^2 = 3mv^2.$$

Adding the momentum magnitudes as scalars would incorrectly omit the right angle between the velocities.

Step 2: Find the final energy and loss The common velocity is $\mathbf{u} = (2v/3)(\hat{\mathbf{i}} + \hat{\mathbf{j}})$. Thus

$$K_f = \frac{p^2}{2(3m)} = \frac{4}{3}mv^2.$$

The loss as a fraction of the initial energy is

$$\frac{K_i - K_f}{K_i} = \frac{3 - 4/3}{3} = \frac{5}{9}, \qquad \text{percentage loss} = 55.56\%.$$

The complementary 44.44% is the energy retained, so it cannot be the requested loss; neither 62 percent nor 50 percent matches the conserved momentum.

Step 3: Conclusion The energy loss is approximately 56%. Hence, the correct option is (A).

Quick Tip to Solve: For a sticking collision with perpendicular incoming velocities, add momentum vectors before using $K_f = p^2/(2M_{\text{total}})$.

54. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its face is: [4 marks]

(A) $\frac{4MR^2}{9\sqrt{3\pi}}$

(B) $\frac{4MR^2}{3\sqrt{3\pi}}$

(C) $\frac{MR^2}{32\sqrt{2\pi}}$

(D) $\frac{MR^2}{16\sqrt{2\pi}}$

Correct Answer: No option equals $4MR^2/(9\pi\sqrt{3})$: the printed radicals include π inside their bounds.

Solution:

Concept:

- The largest cube inside a sphere has its body diagonal equal to the diameter: $a\sqrt{3} = 2R$, where a is its side.
- Uniform density is $\rho = M/(4\pi R^3/3)$. For a cube of mass m_c, the moment of inertia about a central axis normal to a face is $I = m_c a^2/6$.

Step 1: Determine the mass of the cube The sphere mass is not the mass of the cut-out cube. From $a = 2R/\sqrt{3}$,

$$m_c = \rho a^3 = \frac{3M}{4\pi R^3}\frac{8R^3}{3\sqrt{3}} = \frac{2M}{\pi\sqrt{3}}.$$

The mass ratio is simply the volume ratio because the sphere is solid and uniform.

Step 2: Calculate the moment of inertia The axis is parallel to one cube edge through its centre, so the two transverse side lengths are both a. Hence

$$I = \frac{m_c}{12}(a^2 + a^2) = \frac{m_c a^2}{6} = \frac{4MR^2}{9\pi\sqrt{3}}.$$

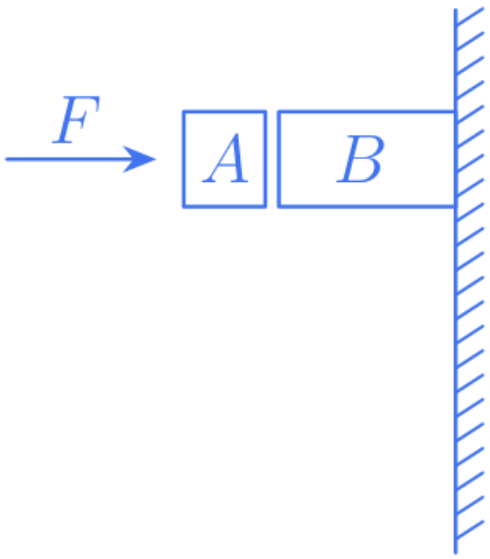
This has the required dimensions of mass times length squared. The denominator contains π outside the square root, since it arose from the sphere volume.

Step 3: Compare with the printed radical bounds The first offered denominator is $9\sqrt{3\pi}$, rather than $9\pi\sqrt{3}$; the second has $3\sqrt{3\pi}$, and the other two contain $\sqrt{2\pi}$. None equals the derived coefficient. The supplied answer key identifies the first choice, which is evidently the intended location, but it does not equal the formula with the radical bound as printed.

Step 4: Conclusion The moment of inertia is $4MR^2/(9\pi\sqrt{3})$. No option is exact as printed; the supplied key selects (A), whose square root incorrectly includes π.

Quick Tip to Solve: Find the cube mass from the common density before using its moment-of-inertia formula; the given M belongs to the sphere.

55. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is : [4 marks]



(A) $120\ N$

(B) $150\ N$

(C) $100\ N$

(D) $80\ N$

Correct Answer: (A) $120\ N$

Solution:

Concept:

- In static equilibrium, actual friction balances the required tangential forces; μN is only its upper limit.
- For the combined pair of blocks, the friction between A and B is an internal force and cancels. The wall supplies the external upward support.

Step 1: Balance the smaller block vertically Block A has weight $20\,\mathrm{N}$. The only upward force on it is friction from B, so this contact friction must be $20\,\mathrm{N}$. By Newton's third law, A exerts an equal downward friction force of $20\,\mathrm{N}$ on B. The horizontal applied force presses both interfaces together; it is not itself a vertical support.

Step 2: Balance block B and check static limits Block B must support its own $100\,\mathrm{N}$ weight plus the downward $20\,\mathrm{N}$ contact force. Therefore

$$f_{\text{wall}} = 100 + 20 = 120\,\mathrm{N},$$

directed upward. Equivalently, treat both blocks as one system of total weight $120\,\mathrm{N}$. The coefficients determine whether equilibrium is possible: $0.1F \geq 20$ and $0.15F \geq 120$, so sufficient pressing requires $F \geq 800\,\mathrm{N}$. They do not set the actual friction equal to its maximum at every applied force. Choosing

100 N omits A, while 80 N reverses the contact force on B; 150 N is not the required support.

Step 3: Conclusion The wall exerts an upward frictional force of $120\,\mathrm{N}$ on B in equilibrium. Hence, the correct option is (A).

Quick Tip to Solve: For a stack pressed horizontally against a wall, sum both weights to get wall friction; use coefficients only to test whether slipping can be prevented.

56. Distance of the center of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to : [4 marks]

(A) $\frac{5h}{8}$
(B) $\frac{3h^2}{8R}$
(C) $\frac{h^2}{4R}$
(D) $\frac{3h}{4}$

Correct Answer: (D) $\frac{3h}{4}$

Solution:

Concept:

- For a uniform body, $z_0 = \int z\,dm / \int dm$. Measure z along the cone axis from the vertex, so the base lies at $z = h$.
- Similar triangles give the radius of a thin circular slice as $r(z) = Rz/h$; its mass is proportional to $z^2\,dz$.

Step 1: Express the slice mass For constant volume density ρ, a slice of thickness dz has

$$dm = \rho\pi r(z)^2\,dz = \frac{\rho\pi R^2}{h^2}z^2\,dz.$$

The slices widen toward the base, so equal axial intervals do not contain equal mass. A midpoint result would therefore be inappropriate.

Step 2: Integrate the first moment The common constant cancels between numerator and denominator:

$$z_0 = \frac{\int_0^h z^3\,dz}{\int_0^h z^2\,dz} = \frac{h^4/4}{h^3/3} = \frac{3h}{4}.$$

Thus the centre is closer to the broad base than to the vertex; it is $h/4$ above the base. The terms containing h^2/R cannot describe the axial centre of mass for cones of every base radius, since the radius scale cancels in the ratio. The remaining value $5h/8$ disagrees with the exact first-moment integral.

Step 3: Conclusion $z_0 = 3h/4$, measured from the vertex. Hence, the correct option is (D).

Quick Tip to Solve: Remember the reference point: a solid cone has its centre of mass $h/4$ from the base, or $3h/4$ from the vertex.

57. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using wrist watch of 1 s resolution. The accuracy in the determination of g is: [4 marks]

(A) 1 %

(B) 5 %

(C) 2 %

(D) 3 %

Correct Answer: (D) 3 %

Solution:

Concept:

- The pendulum relation gives $g = 4\pi^2 L/T^2$, where L is its length and T is the period of one oscillation.
- For a product or quotient with powers, the maximum fractional uncertainties add with the magnitudes of the powers: $\Delta g/g = \Delta L/L + 2\Delta T/T$. Exact constants do not contribute uncertainty.

Step 1: Convert the length uncertainty The measured length is $L = 20.0\,\text{cm} = 200\,\text{mm}$, while its stated accuracy is $\Delta L = 1\,\text{mm}$. Hence the fractional uncertainty contributed by the length is

$$\frac{\Delta L}{L} = \frac{1}{200} = 0.005 = 0.5\%.$$

Keep both quantities in the same unit; using 1 mm directly with a length in centimetres would overestimate this contribution.

Step 2: Use the uncertainty of the total measured time The watch measures the combined time $t = 90\,\text{s}$ for exactly 100 oscillations, with uncertainty $\Delta t = 1\,\text{s}$. Thus $T = t/100 = 0.90\,\text{s}$ and $\Delta T = \Delta t/100 = 0.01\,\text{s}$. Dividing both by the same exact count leaves the fractional error unchanged:

$$\frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{1}{90}.$$

The 1 s resolution belongs to the entire timed interval, not to each oscillation.

Step 3: Combine the independent contributions conservatively Using the maximum-error rule for the inferred value of gravity gives

$$\frac{\Delta g}{g} = \frac{1}{200} + \frac{2}{90} = 0.02722\ldots.$$

Therefore the percentage uncertainty is 2.72%, which rounds to approximately 3%. Options 1%, 2% and 5% do not represent this combined uncertainty; omitting the square of the period would incorrectly reduce the timing contribution.

Step 4: Conclusion The accuracy in determining g is approximately 3%. Hence, the correct option is (D).

Quick Tip to Solve: First write the measured quantity as a product of powers. When many oscillations are timed together, use the fractional uncertainty of the total time, then multiply it by the period exponent.

58. A train is moving on a straight track with speed $20\,ms^{-1}$. It is blowing its whistle at the frequency of $1000\ Hz$. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound $= 320\,ms^{-1}$) close to : [4 marks]

(A) 18%

(B) 24%

(C) 6%

(D) 12%

Correct Answer: (D) 12%

Solution:

Concept:

- For a stationary listener and a source moving along the line of hearing at speed u, the approaching and receding frequencies are $f_a = fv/(v-u)$ and $f_r = fv/(v+u)$. Here v is the speed of sound.
- Passing the listener changes the source motion from approaching to receding, so the observed frequency drops from f_a to f_r. Both sides of the passage must be included.

Step 1: Calculate the approaching frequency The train emits $f = 1000\,\text{Hz}$, with $u = 20\,\text{m s}^{-1}$ and $v = 320\,\text{m s}^{-1}$. Ahead of the approaching train the wavefronts are compressed, so

$$f_a = 1000\frac{320}{320-20} = 1066.67\,\text{Hz}.$$

The listener is at rest, so the sound speed remains in the numerator; the source motion changes the denominator.

Step 2: Calculate the receding frequency After the train passes, the source moves away and successive wavefronts reaching the listener are farther apart. Therefore

$$f_r = 1000\frac{320}{320+20} = 941.18\,\text{Hz}.$$

The decrease between the two observed frequencies is

$$\Delta f = f_a - f_r = 125.49\,\text{Hz}.$$

A calculation using only the approaching shift would miss the additional fall below the emitted frequency after passage.

Step 3: Express the drop as a percentage Relative to the frequency just before passing, the fractional decrease is

$$\frac{f_a - f_r}{f_a} = 1 - \frac{v-u}{v+u} = \frac{2u}{v+u}.$$

Hence the percentage decrease is $100(40/340) = 11.76\%$, close to 12%. The usual small-speed estimate $200u/v = 12.5\%$ gives the same closest choice. Six percent describes roughly a one-sided shift; 18% and 24% are substantially too large for this speed ratio.

Step 4: Conclusion The frequency decreases by approximately 12% as the train passes. Hence, the correct option is (D).

Quick Tip to Solve: For a passing source, calculate both approach and recession frequencies. At small source speeds the total change is roughly twice a one-sided Doppler shift.

59. Match List - I (Fundamental Experiment) with List - II (its conclusion) and select the correct option from the choices given below the list: [4 marks]

List - I		List - II	
A.	Franck-Hertz	(i)	Particle nature of light
B.	Photo-electric	(ii)	Discrete energy levels of atom
C.	Davison-Germer experiment	(iii)	Wave nature of electron
		(iv)	Structure of atom

(A) $A-ii;\ B-i,\ C-iii$

(B) $A-iv;\ B-iii,\ C-ii$

(C) $A-i;\ B-iv,\ C-iii$

(D) $A-ii;\ B-iv,\ C-iii$

Correct Answer: (A) $A-ii;\ B-i,\ C-iii$

Solution:

Concept:

- Each experiment must be matched to the physical conclusion directly supported by its observations, rather than to a broad subject such as atomic physics.
- Quantised excitation energies, photon energy transfer and matter-wave diffraction establish three different features of microscopic physics.

Step 1: Identify the Franck–Hertz result In the Franck–Hertz experiment, electrons accelerated through a gas lose energy in inelastic collisions only when their kinetic energy reaches particular excitation thresholds. The repeated features in the current versus accelerating-voltage curve show that atoms absorb definite amounts of energy. This is evidence for discrete atomic energy levels, so List I entry A matches List II entry (ii), not the general structure-of-atom statement (iv).

Step 2: Identify the photoelectric result The photoelectric effect has a threshold frequency and gives the maximum photoelectron kinetic energy through $K_{\max} = h\nu - \phi$. A single photon transfers its energy $h\nu$ to an electron. This supports the particle nature of light, so entry B matches (i). It is not an experiment demonstrating the wave nature of electrons; that distinction rules out the B–(iii) assignment in option (B).

Step 3: Identify the Davisson–Germer result and compare the codes Davisson–Germer electron scattering from a crystal produced diffraction maxima. The crystal serves as a diffraction grating, and the inferred wavelength agrees with de Broglie's $\lambda = h/p$. This demonstrates the wave nature of electrons, so C matches (iii). Combining the three independent results gives A–(ii), B–(i), C–(iii). Option (C) assigns the particle nature of light to Franck–Hertz; option (D) assigns atomic structure to the photoelectric effect. Both therefore fail, even though they retain the correct C match.

Step 4: Conclusion The correct matching is A–(ii), B–(i), C–(iii). Hence, the correct option is (A).

Quick Tip to Solve: Remember one decisive observation per experiment: excitation thresholds imply discrete levels, photoelectron energies imply photons, and electron diffraction implies matter waves.

60. A signal of $5\ kHz$ frequency is amplitude modulated on a carrier wave of frequency $2\ MHz$. The frequencies of the resultant signal is/are : [4 marks]

(A) $2005\ kHz, 2000\ kHz$ and $1995\ kHz$

(B) $2000\ kHz$ and $1995\ kHz$

(C) $2\ MHz$ only

(D) $2005\ kHz$ and $1995\ kHz$

Correct Answer: (A) $2005\ kHz, 2000\ kHz$ and $1995\ kHz$

Solution:

Concept:

- In ordinary amplitude modulation, a sinusoidal message of frequency f_m varies the amplitude of a carrier of frequency f_c. The carrier itself remains present.
- The product identity for cosines generates two sidebands at $f_c + f_m$ and $f_c - f_m$, in addition to the original carrier at f_c.

Step 1: Write the modulated signal and expand its product Let the modulation index be μ. A standard AM wave can be expressed as

$$s(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t).$$

Using $\cos x \cos y = [\cos(x + y) + \cos(x - y)]/2$, its time dependence consists of the carrier cosine and two further cosines, each with amplitude $\mu A_c/2$. These terms have distinct frequencies, so the complete signal cannot be described by either sideband alone.

Step 2: Put both given frequencies in the same unit The carrier frequency is $f_c = 2\,\text{MHz} = 2000\,\text{kHz}$, and the message frequency is $f_m = 5\,\text{kHz}$. Therefore the upper and lower sidebands are

$$f_c + f_m = 2005\,\text{kHz}, \qquad f_c - f_m = 1995\,\text{kHz}.$$

The unshifted carrier remains at $2000\,\text{kHz}$. The message frequency is a separation from the carrier, not a replacement for it.

Step 3: Check the listed combinations The complete AM spectrum therefore contains 1995, 2000 and 2005 kHz. Option (B) omits the upper sideband, and option (C) omits both sidebands. Option (D) contains the two sidebands but leaves out the carrier; that would describe the spectral locations of a suppressed-carrier double-sideband signal, whereas this question asks for ordinary amplitude modulation.

Step 4: Conclusion The resultant signal contains $2005\,\text{kHz}$, $2000\,\text{kHz}$ and $1995\,\text{kHz}$. Hence, the correct option is (A).

Quick Tip to Solve: For standard AM, write three frequencies in order: lower sideband, carrier and upper sideband. Convert MHz to kHz before adding or subtracting the message frequency.

Chemistry (120 marks)

61. The vapour pressure of acetone at 20°C is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100 g of acetone at 20°C, its vapour pressure was 183 torr. The molar mass (g mol^{-1}) of the substance is: [4 marks]

(A) 64

(B) 128

(C) 488

(D) 32

Correct Answer: (A) 64

Solution:

Concept:

- For an ideal solution containing a nonvolatile solute, Raoult's law gives $p = x_1 p^0$, where p^0 is pure-solvent vapour pressure and x_1 is solvent mole fraction.
- The relative lowering $(p^0 - p)/p^0 = x_2$ equals the solute mole fraction. Acetone, C_3H_6O, has molar mass $58\,\text{g mol}^{-1}$.

Step 1: Calculate the relative vapour-pressure lowering At the same temperature the pressure falls from $185\,\text{torr}$ to $183\,\text{torr}$, so $\Delta p = 2\,\text{torr}$. Consequently

$$x_2 = \frac{p^0 - p}{p^0} = \frac{2}{185}.$$

The temperature is held fixed, so this lowering is caused by dilution of the solvent rather than a temperature change. The pressure units cancel in the ratio.

Step 2: Relate the mole ratio to the solute mass The amount of acetone is $n_1 = 100/58\,\text{mol}$. If the unknown molar mass is M, the solute amount is $n_2 = 1.2/M\,\text{mol}$. The exact mole-fraction relation gives

$$\frac{n_2}{n_1} = \frac{x_2}{1 - x_2} = \frac{2}{183}.$$

Substituting the two amounts,

$$\frac{1.2/M}{100/58} = \frac{2}{183}.$$

This form retains the solute contribution in the total number of moles.

Step 3: Solve and compare with the choices Rearranging gives

$$M = \frac{1.2 \times 58 \times 183}{100 \times 2} = 63.684\,\text{g mol}^{-1}.$$

Thus the closest listed value is 64. The dilute-solution approximation, replacing solute mole fraction by n_2/n_1, instead gives $1.2 \times 58 \times 185/(100 \times 2) = 64.38$, leading to the same choice. Values 128 and 32 differ by a factor near two, while 488 would produce a much smaller lowering than observed.

Step 4: Conclusion The molar mass of the nonvolatile solute is approximately $64\,\text{g mol}^{-1}$. Hence, the

correct option is (A).

Quick Tip to Solve: The exact ratio is $n_2/n_1 = (p^0 - p)/p$. For a very dilute solution, using p^0 instead of p gives a close estimate, but check the pressure difference carefully.

62. Which of the following compounds will exhibit geometrical isomerism? [4 marks]

(A) 3-Phenyl-1-butene

(B) 2-Phenyl-1-butene

(C) 1,1-Diphenyl-1-propane

(D) 1-Phenyl-2-butene

Correct Answer: (D) 1-Phenyl-2-butene

Solution:

Concept:

- Rotation about a carbon–carbon double bond is restricted. Distinct geometrical isomers are possible only when each double-bonded carbon carries two different substituents.
- A terminal $CH_2=$ carbon has two identical hydrogen substituents and prevents geometrical isomerism, regardless of substitution elsewhere in the molecule.

Step 1: Reject the two terminal alkenes For 3-phenyl-1-butene, the condensed structure is

$$CH_2{=}CH{-}CH(C_6H_5){-}CH_3.$$

Its first alkene carbon is CH_2, so interchanging its two hydrogens creates no new configuration. For 2-phenyl-1-butene,

$$CH_2{=}C(C_6H_5){-}CH_2{-}CH_3,$$

the same terminal CH_2 feature rules out geometrical isomerism. A phenyl substituent does not remove the need for different groups at both ends of the double bond.

Step 2: Check the saturated compound The name 1,1-diphenyl-1-propane represents a saturated propane framework with two phenyl substituents on carbon 1. It has no side-chain carbon–carbon double bond and no ring arrangement providing the required cis/trans restriction. Ordinary single-bond rotation does not generate isolable geometrical isomers. The aromatic bonds inside a phenyl ring are not treated as independently configurable alkene bonds.

Step 3: Test the internal alkene For 1-phenyl-2-butene the relevant structure is

$$C_6H_5{-}CH_2{-}CH{=}CH{-}CH_3.$$

One double-bonded carbon carries H and $CH_2C_6H_5$, while the other carries H and CH_3. Each end therefore has two different groups. Because the double bond prevents free rotation, the two hydrogens may lie on the same side or on opposite sides, producing distinct cis/trans, equivalently Z/E, configurations.

Step 4: Conclusion Only 1-phenyl-2-butene satisfies the substituent requirement for geometrical

isomerism. Hence, the correct option is (D).

Quick Tip to Solve: Inspect the two alkene carbons separately. Finding even one $CH_2=$ end immediately rules out geometrical isomerism, whatever groups occur farther along the chain.

63. In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is: (at. Mass Ag = 108; Br = 80) [4 marks]

(A) 48

(B) 60

(C) 36

(D) 24

Correct Answer: (D) 24

Solution:

Concept:

- In Carius halogen estimation, the halogen in a known mass of organic compound is converted quantitatively to a silver halide precipitate. For bromine the precipitate is AgBr.
- Each mole of AgBr contains one mole of bromine atoms, so the bromine mass fraction in the precipitate is $M(\mathrm{Br})/M(\mathrm{AgBr})$. The desired percentage uses the original sample mass as denominator.

Step 1: Determine the bromine fraction in silver bromide Using the supplied atomic masses, the molar mass of silver bromide is

$$M(\mathrm{AgBr}) = 108 + 80 = 188\,\mathrm{g\,mol^{-1}}.$$

Of every 188 g of this precipitate, only 80 g is bromine; the remaining 108 g is silver introduced by the reagent. Thus the measured precipitate mass must first be converted to the mass of the halogen itself.

Step 2: Calculate the mass of bromine in the sample The precipitate obtained weighs $141\,\mathrm{mg}$. Conservation of bromine atoms gives

$$m(\mathrm{Br}) = 141 \times \frac{80}{188} = 60\,\mathrm{mg}.$$

The mass units can stay in milligrams throughout because all subsequent expressions use ratios of masses. Taking the entire 141 mg as bromine would incorrectly count the silver mass too.

Step 3: Calculate the percentage in the original organic compound The organic sample weighed $250\,\mathrm{mg}$, so its bromine percentage is

$$\%\mathrm{Br} = \frac{60}{250} \times 100 = 24\%.$$

This equals $141 \times 80 \times 100/(188 \times 250)$. The 60 in the calculation is the mass of bromine in milligrams, not a percentage; hence option (B) is incorrect. Neither 48% nor 36% follows from the one-to-one mole relation between silver bromide and bromine atoms.

Step 4: Conclusion The organic compound contains 24% bromine by mass. Hence, the correct option is (D).

Quick Tip to Solve: Convert silver halide mass to halogen mass first, then divide by the original sample mass. Keep both masses in the same unit to avoid unnecessary conversion errors.

64. The standard Gibbs energy change at 300 K for the reaction 2A $\rightleftharpoons$ B + C is 2494.2 J. At a given time, the composition of the reaction mixture is [A] = 1/2, [B] = 2 and [C] = 1/2. The reaction proceeds in the: [R = 8.314 J/K/mol, e = 2.718] [4 marks]

(A) reverse direction because Q $>$ K_c

(B) forward direction because Q $<$ K_c

(C) reverse direction because Q $<$ K_c

(D) forward direction because Q $>$ K_c

Correct Answer: (A) reverse direction because Q $>$ K_c

Solution:

Concept:

- The standard Gibbs energy change and equilibrium constant are related by $\Delta_r G^\circ = -RT \ln K_c$, where R is the gas constant and T is absolute temperature.
- The reaction quotient uses the current concentrations with stoichiometric powers: for $2\,\mathrm{A} \rightleftharpoons \mathrm{B} + \mathrm{C}$, $Q_c = [\mathrm{B}][\mathrm{C}]/[\mathrm{A}]^2$. The actual driving force is $\Delta_r G = RT \ln(Q_c/K_c)$.

Step 1: Calculate the equilibrium constant Use the given standard energy, not the instantaneous concentrations, to calculate the equilibrium constant at 300 K. Since $8.314 \times 300 = 2494.2$,

$$\ln K_c = -\frac{2494.2}{8.314 \times 300} = -1.$$

Thus $K_c = e^{-1} = 1/2.718 \simeq 0.368$. A positive standard Gibbs energy does not by itself decide the direction for every possible mixture; the current quotient must also be considered.

Step 2: Evaluate the current reaction quotient Insert the concentrations exactly as given and square the concentration of A because its coefficient is 2:

$$Q_c = \frac{(2)(1/2)}{(1/2)^2} = \frac{1}{1/4} = 4.$$

The stoichiometric change in gas moles is zero, so the usual $(RT)^{\Delta n}$ conversion does not introduce a different numerical constant here.

Step 3: Compare the quotient with equilibrium Here $Q_c/K_c = 4e > 1$, hence $\Delta_r G = RT \ln(4e) > 0$ for forward reaction. The spontaneous change is therefore in the reverse direction: B and C are consumed and A is formed until the quotient falls to its equilibrium value. Option B uses the wrong inequality, C pairs reverse reaction with the wrong inequality, and D assigns forward reaction to an excess of products.

Step 4: Conclusion $Q_c = 4 > K_c \simeq 0.368$, so the reaction proceeds in the reverse direction. Hence,

the correct option is (A).

Quick Tip to Solve: First calculate Q, then compare it with K: excess products, $Q > K$, drive the reaction backwards.

65. The following reaction is performed at 298 K

$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$

The standard free energy of formation of NO(g) is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of NO_2(g) at 298 K? (KP = 1.6×10^{12}) [4 marks]

(A) 86600 + R(298) ln(1.6×10^{12})

(B) 86600 - $\frac{ln(1.6\times10^{12})}{R(298)}$

(C) 0.5[2×86,600 – R(298) ln(1.6×10^{12})]

(D) R(298) ln(1.6×10^{12}) – 86600

Correct Answer: (C) 0.5[2×86,600 – R(298) ln(1.6×10^{12})]

Solution:

Concept:

- For the reaction as balanced, $\Delta_r G^\circ = -RT \ln K_p$. The stoichiometric coefficients belong to this complete reaction, not to a single mole of product.
- The reaction Gibbs energy is the sum of product formation energies minus the sum of reactant formation energies. An element in its standard state, such as $O_2(g)$, has zero standard formation energy.

Step 1: Write the formation-energy balance Let $x = \Delta_f G^\circ(NO_2(g))$ in $J\,mol^{-1}$. Convert the given value for NO before combining it with R in SI units: $86.6\ kJ\,mol^{-1} = 86600\ J\,mol^{-1}$. For $2\,NO\,(g) + O_2(g) \rightleftharpoons 2\,NO_2(g)$,

$$\Delta_r G^\circ = 2x - [2(86600) + 0] = 2x - 173200.$$

Step 2: Relate the balanced reaction to its equilibrium constant At 298 K the large equilibrium constant gives a negative reaction Gibbs energy:

$$2x - 173200 = -R(298)\ln(1.6 \times 10^{12}).$$

Rearrange without dropping the coefficient of the product:

$$x = \frac{2(86600) - R(298)\ln(1.6 \times 10^{12})}{2}.$$

This is the formation energy of one mole of nitrogen dioxide, whereas the original reaction forms two moles.

Step 3: Check the expression against the alternatives The expression equals option C. As a sign and scale check, $\ln(1.6 \times 10^{12}) \simeq 28.10$, so the correction to 86600 is about $34800\ J\,mol^{-1}$, giving $x \simeq 51.8\ kJ\,mol^{-1}$. Option A has the wrong sign and coefficient; B divides by RT rather than multiplying, so its dimensions fail; D reverses the required balance.

Step 4: Conclusion $\Delta_f G^\circ(NO_2) = 0.5[2(86600) - R(298)\ln(1.6 \times 10^{12})]\ \text{J mol}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: Write products minus reactants first, then use $-RT\ln K$. Divide by the coefficient of the species whose formation energy is required.

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66. The molecular formula of a commercial resin used for exchanging ions in water softening is $C_8H_7SO_3Na$ (Mol. Wt. 206). What would be the maximum uptake of Ca^{2+} ions by the resin when expressed in mole per gram resin? [4 marks]

(A) 2/309

(B) 1/412

(C) 1/103

(D) 1/206

Correct Answer: (B) 1/412

Solution:

Concept:

- Each formula unit of $C_8H_7SO_3Na$ supplies one monovalent sodium ion and one fixed negatively charged sulfonate site.
- Calcium is divalent: one Ca^{2+} must replace two Na^+ ions to preserve charge balance. The maximum capacity is therefore one mole of calcium for every two moles of resin formula units.

Step 1: Count the available exchange sites Take a basis of 1 g of resin. With the stated molar mass $206\ \text{g mol}^{-1}$, its amount of formula units is

$$n_{\text{resin}} = \frac{1\ \text{g}}{206\ \text{g mol}^{-1}} = \frac{1}{206}\ \text{mol}.$$

There is only one sodium atom in each formula unit, so this is also the amount of sodium ions that can be released upon complete exchange.

Step 2: Apply charge balance in the exchange Write R for the organic residue attached to the sulfonate group. The exchange is

$$2\,\text{R}{-}\text{SO}_3\text{Na} + \text{Ca}^{2+} \longrightarrow (\text{R}{-}\text{SO}_3)_2\text{Ca} + 2\,\text{Na}^+.$$

Two singly charged fixed sites hold one calcium ion. Consequently,

$$n_{Ca^{2+}} = \frac{1}{2}n_{\text{resin}} = \frac{1}{412}\ \text{mol}$$

for the chosen 1 g sample. This is a maximum because it assumes all exchange sites are accessible and used.

Step 3: Compare capacities rather than molar masses alone The requested quantity is moles of calcium per gram of resin, not moles of sodium per gram. Option D, $1/206$, counts sodium sites

without dividing by the calcium charge. Option C, $1/103 = 2/206$, incorrectly multiplies by two; option A also exceeds the maximum allowed by the available negative charge. Only option B obeys both mass and charge balance.

Step 4: Conclusion The maximum calcium uptake is $\frac{1}{412}\ \text{mol g}^{-1}$ of resin. Hence, the correct option is (B).

Quick Tip to Solve: Convert grams to moles of exchange sites, then divide by the incoming ion charge when each site is singly charged.

67. Two Faraday of electricity is passed through a solution of $CuSO_4$. The mass of copper deposited at the cathode is: (at. mass of Cu = 63.5 amu) [4 marks]

(A) 63.5 g

(B) 2 g

(C) 127 g

(D) 0 g

Correct Answer: (A) 63.5 g

Solution:

Concept:

- One Faraday is the charge carried by one mole of electrons. The amount of metal deposited depends on its ionic charge, not only on its atomic mass.
- Copper sulfate supplies Cu^{2+}, whose cathodic reduction is $Cu^{2+} + 2\,e^- \longrightarrow Cu\,(s)$. Thus one mole of copper requires two Faradays.

Step 1: Identify the reacting ion and electron requirement In aqueous $CuSO_4$, copper is in the $+2$ oxidation state. The cathode gains electrons and the copper ion is reduced to metallic copper. The balanced half-reaction requires two moles of electrons for every mole of copper, rather than the one electron needed for a singly charged metal ion. The sulfate ion does not alter this electron count.

Step 2: Use the charge passed to find copper moles The charge passed is $Q = 2F$, so the number of moles of electrons is

$$n_e = \frac{Q}{F} = 2\ \text{mol}.$$

Using the reduction stoichiometry,

$$n_{\text{Cu}} = \frac{n_e}{2} = 1\ \text{mol}.$$

With the supplied atomic mass, the molar mass is $63.5\ \text{g mol}^{-1}$, hence

$$m = n_{\text{Cu}} M = 1 \times 63.5 = 63.5\ \text{g}.$$

Step 3: Check using the equivalent-mass formula The same result follows directly from Faraday's law, $m = (Q/F)(M/z)$, with $z = 2$:

$$m = 2\left(\frac{63.5}{2}\right) = 63.5\ \text{g}.$$

Option C, 127 g, would incorrectly assign only one electron to each copper ion. Option B confuses the number of electron moles with a mass in grams. Option D would imply no deposition despite a reducible copper ion and the stated passage of charge.

Step 4: Conclusion Two Faradays deposit one mole of copper, with mass $63.5\ \mathrm{g}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $m = (Q/F)(M/z)$; for Cu^{2+}, every two Faradays give one mole of copper.

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68. The intermolecular interaction that is dependent on the inverse cube of distance between the molecule is: [4 marks]

(A) ion-dipole interaction

(B) London force

(C) hydrogen bond

(D) ion-ion interaction

Correct Answer: (C) hydrogen bond

Solution:

Concept:

- Intermolecular interaction here is compared through its interaction energy. For fixed dipole orientations, the electrostatic dipole–dipole energy varies as the inverse cube of separation.
- A hydrogen bond is a directional attraction involving a hydrogen atom bonded to an electronegative atom and an electron-rich electronegative atom. In the idealised electrostatic classification used here it is treated as a strong, oriented dipole–dipole interaction.

Step 1: Identify the inverse-cube energy dependence For two permanent dipoles held at fixed relative orientations, the electrostatic interaction energy can be written as

$$U_{dd} = \frac{\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2 - 3(\boldsymbol{\mu}_1 \cdot \hat{\mathbf{r}})(\boldsymbol{\mu}_2 \cdot \hat{\mathbf{r}})}{4\pi\varepsilon_0 r^3}.$$

The orientation factor controls the sign and magnitude, while the explicit distance dependence is r^{-3}. The directional, strong dipolar attraction represented by a hydrogen bond is the listed interaction corresponding to this classification.

Step 2: Eliminate the other energy laws For an ion and a permanent dipole at a fixed orientation, the interaction energy is proportional to $q\mu\cos\theta/r^2$, so option A has inverse-square energy, not inverse-cube. The attractive London dispersion energy is proportional to $-r^{-6}$, excluding B. For two ions the Coulomb potential energy is proportional to q_1q_2/r, giving inverse-first-power dependence and excluding D.

Step 3: Distinguish the stated energy classification from force If an interaction energy varies as r^{-n}, its radial force varies as $r^{-(n+1)}$. Thus an ion–dipole force does have an inverse-cube dependence, but that is not the interaction-energy comparison intended in this question. Likewise, freely rotating dipoles have a different thermally averaged law. A real hydrogen bond is not exactly described

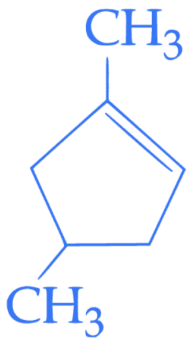
by a universal inverse-cube potential at every distance; the answer follows the fixed-orientation electrostatic model among these choices.

Step 4: Conclusion In the intended interaction-energy classification, the inverse-cube dependence corresponds to hydrogen bonding, treated as an oriented dipolar interaction. Hence, the correct option is (C).

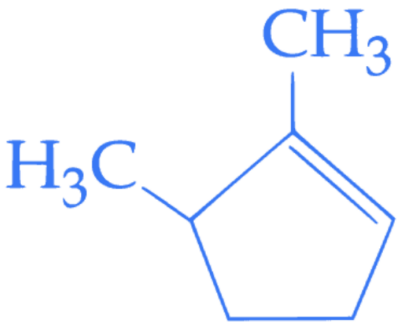
Quick Tip to Solve: For interaction energies, remember the fixed-orientation powers: ion–ion r^{-1}, ion–dipole r^{-2}, dipole–dipole r^{-3}, and London attraction r^{-6}.

69. Which compound would give 5 - keto - 2 - methylhexanal upon ozonolysis? [4 marks]

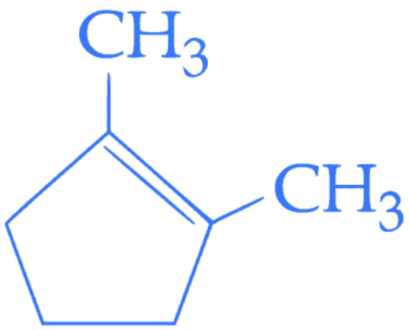
(A)



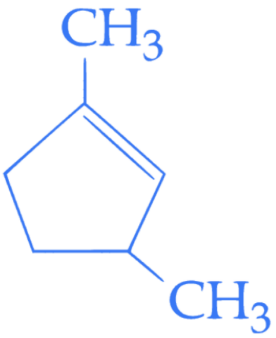
(B)



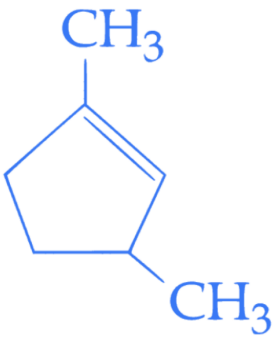
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- Ozonolysis cleaves the carbon–carbon double bond and converts each double-bond carbon into a carbonyl carbon. A double-bond carbon bearing hydrogen gives an aldehyde; one bearing two carbon groups gives a ketone.
- Cleaving one double bond in a cycloalkene opens the ring. Reconnecting the two carbonyl carbons of the product reconstructs the original double bond and ring size.

Step 1: Write the required open-chain product Number the aldehyde carbon as carbon 1. The named product, 5-keto-2-methylhexanal, has the condensed structure

$$\mathrm{OHC{-}CH(CH_3){-}CH_2{-}CH_2{-}CO{-}CH_3}.$$

Its aldehyde carbon and ketone carbon are the two carbon atoms that originally formed the alkene double bond. The terminal methyl attached to the ketone was therefore already attached to one alkene carbon.

Step 2: Reconstruct the ring and methyl positions Reconnect product carbons 1 and 5 with a double bond and remove their carbonyl oxygens. The ring consists of carbons 1 through 5, so it is a cyclopentene. One methyl is attached to the ketone-derived double-bond carbon, while the second methyl is attached to the saturated carbon immediately next to the aldehyde-derived double-bond carbon. This is 1,3-dimethylcyclopent-1-ene, shown in option D.

Step 3: Verify every graphical alternative In option D, opening the double bond and numbering from the aldehyde end gives exactly $\mathrm{OHC{-}CH(CH_3){-}CH_2{-}CH_2{-}CO{-}CH_3}$. Option A places the

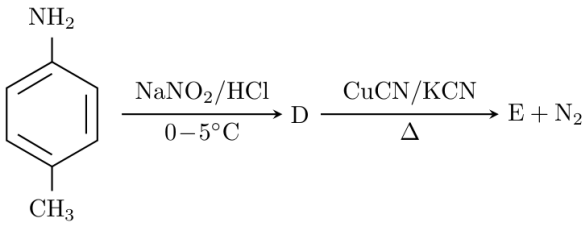
saturated-ring methyl one carbon farther from the aldehyde and gives the 3-methyl isomer. Option B puts it next to the ketone end and gives the 4-methyl isomer. In option C both double-bond carbons carry methyl groups, so both become ketones and no aldehyde is obtained. The aldehyde and ketone identities therefore check both substitution and connectivity.

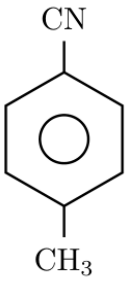
Step 4: Conclusion 1,3-Dimethylcyclopent-1-ene gives the required aldehyde–ketone on ozonolysis. Hence, the correct option is (D).

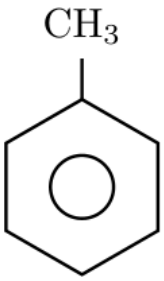
Quick Tip to Solve: To reverse ozonolysis, join the two carbonyl carbons by a double bond, then check the substituents from the aldehyde end.

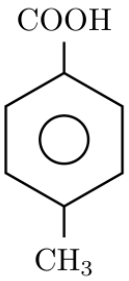
70. In the reaction

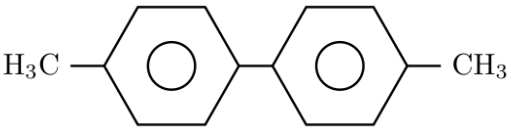
The product E is : [4 marks]



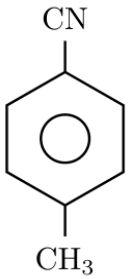
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- A primary aromatic amine reacts with nitrous acid, generated from $NaNO_2/HCl$ at 0–5 °C, to form an aryl diazonium salt.
- In the Sandmeyer reaction, copper(I) cyanide replaces the diazonium group by $-CN$ with nitrogen evolution. Substituents already on the aromatic ring retain their positions.

Step 1: Identify the intermediate D The starting compound is para-methylaniline: the amino and methyl groups occupy opposite positions on the same benzene ring. Cold acidified sodium nitrite converts the amino group into a diazonium group without moving the methyl group:

$$\text{p-}CH_3C_6H_4NH_2 + NaNO_2 + 2\,HCl$$

$$\longrightarrow \text{p-}CH_3C_6H_4N_2{}^+Cl^- + NaCl + 2\,H_2O.$$

Thus D is para-methylbenzenediazonium chloride. The low temperature preserves the diazonium intermediate before its subsequent substitution.

Step 2: Apply copper(I)-assisted cyanide substitution Treatment of this aryl diazonium salt with the displayed $CuCN/KCN$ on warming replaces $-N_2{}^+$ by $-CN$. The two nitrogen atoms of the diazonium group leave as N_2:

$$\text{p-}CH_3C_6H_4N_2{}^+ \xrightarrow{CuCN/KCN} \text{p-}CH_3C_6H_4CN + N_2.$$

The new group attaches through its carbon atom, giving an aromatic nitrile, not an isocyanide.

Step 3: Compare the product structures The methyl group remains para to the newly introduced nitrile, so E is 4-methylbenzonitrile, also called para-tolunitrile. Option A shows precisely this arrangement. Option B is toluene, which would require replacing the diazonium group by hydrogen. Option C is the carboxylic acid that would require later nitrile hydrolysis, absent here. Option D couples two aromatic rings, whereas the displayed reagents perform cyanation rather than biaryl formation.

Step 4: Conclusion E is para-methylbenzonitrile, $\text{p-}CH_3C_6H_4CN$. Hence, the correct option is (A).

Quick Tip to Solve: In a Sandmeyer sequence, first replace aromatic $-NH_2$ by $-N_2{}^+$, then replace that group with the reagent-supplied substituent; retain the ring positions.

71. Match the catalysts to the correct processes :

	Catalyst		Process
(A)	$TiCl_3$	(i)	Wacker process
(B)	$PdCl_2$	(ii)	Ziegler - Natta polymerization
(C)	$CuCl_2$	(iii)	Contact process
(D)	V_2O_5	(iv)	Deacon's process

[4 marks]

(A) $(A)-(ii)$, $(B)-(iii)$, $(C)-(iv)$, $(D)-(i)$
(B) $(A)-(iii)$, $(B)-(i)$, $(C)-(ii)$, $(D)-(iv)$
(C) $(A)-(iii)$, $(B)-(ii)$, $(C)-(iv)$, $(D)-(i)$
(D) $(A)-(ii)$, $(B)-(i)$, $(C)-(iv)$, $(D)-(iii)$
Correct Answer: (D) $(A)-(ii)$, $(B)-(i)$, $(C)-(iv)$, $(D)-(iii)$

Solution:

Concept:

- A catalyst is identified by the particular industrial transformation it promotes, not merely by the presence of a transition metal. Match each named process to its characteristic catalyst independently.
- The four relevant transformations are alkene polymerisation, oxidation of ethene to ethanal, oxidation of hydrogen chloride to chlorine, and oxidation of sulfur dioxide to sulfur trioxide.

Step 1: Match titanium and palladium catalysts Ziegler–Natta catalysts combine a titanium halide, such as $TiCl_3$, with an organoaluminium compound. They promote coordination polymerisation of alkenes. Thus catalyst (A) belongs to process (ii). Wacker oxidation converts ethene into ethanal using $PdCl_2$ with a copper salt as co-catalyst. Hence the characteristic palladium catalyst (B) matches Wacker process (i). These two matches already exclude options (B) and (C).

Step 2: Match the copper and vanadium catalysts In the Deacon process, hydrogen chloride is oxidised by oxygen in the presence of $CuCl_2$:

$$4\,HCl + O_2 \longrightarrow 2\,Cl_2 + 2\,H_2O.$$

Therefore (C) matches (iv). In the Contact process, V_2O_5 catalyses the oxidation

$$2\,SO_2 + O_2 \rightleftharpoons 2\,SO_3.$$

Therefore (D) matches (iii). This is distinct from Wacker oxidation even though both industrial processes involve an oxidation step.

Step 3: Compare the complete sequence The combined assignment is

$$(A)-(ii), \qquad (B)-(i), \qquad (C)-(iv), \qquad (D)-(iii).$$

Option (A) incorrectly assigns palladium chloride to the Contact process and vanadium pentoxide to the Wacker process. Options (B) and (C) incorrectly place titanium chloride in the Contact process.

Only the fourth sequence preserves all four standard catalyst–process pairs.

Step 4: Conclusion Titanium chloride matches polymerisation, palladium chloride matches Wacker oxidation, copper chloride matches the Deacon process, and vanadium pentoxide matches the Contact process. Hence, the correct option is (D).

Quick Tip to Solve: Recall the distinctive anchors first: titanium halide for Ziegler–Natta polymerisation and vanadium pentoxide for the Contact process; then check the two remaining matches.

72. Which of the vitamins given below is water soluble? [4 marks]

(A) Vitamin D

(B) Vitamin E

(C) Vitamin K

(D) Vitamin C

Correct Answer: (D) Vitamin C

Solution:

Concept:

- Vitamins are classified as water soluble or fat soluble according to their behaviour in water and lipid media. The standard fat-soluble group is A, D, E and K.
- The water-soluble group comprises the B-complex vitamins and vitamin C. Vitamin C is also called ascorbic acid and contains several oxygen-containing groups that interact strongly with water.

Step 1: Identify the water-soluble candidate Vitamin C is ascorbic acid. Its multiple hydroxyl groups can form hydrogen bonds with water, and its polar oxygen-containing structure favours dissolution in water. It therefore belongs to the water-soluble class. This classification directly answers the question; neither vitamin function nor deficiency symptoms need to be inferred to select the option.

Step 2: Eliminate the three fat-soluble vitamins Vitamin D belongs to the fat-soluble group and is associated with calcium and phosphate regulation. Vitamin E is a fat-soluble antioxidant, while vitamin K is a fat-soluble vitamin needed in normal blood coagulation. Their substantial nonpolar portions favour lipid environments. Thus the first three listed vitamins, D, E and K, all belong to the same fat-soluble class and cannot be the requested water-soluble vitamin.

Step 3: Check the option order The printed choices list vitamin D first, vitamin E second, vitamin K third, and vitamin C fourth. Since only vitamin C is in the water-soluble group, its position is the fourth choice. Do not confuse the option letter D with vitamin D: the letter labels the choice, whereas the vitamin name is the substance being classified. The distinction is especially helpful here because the first option itself names vitamin D.

Step 4: Conclusion Vitamin C is water soluble; vitamins D, E and K are fat soluble. Hence, the correct option is (D).

Quick Tip to Solve: Use the group ADEK for fat-soluble vitamins. Any vitamin C or B-complex choice belongs to the water-soluble group.

73. The synthesis of alkyl fluorides is best accomplished by: [4 marks]

(A) Sandmeyer's reaction

(B) Finkelstein reaction

(C) Swarts reaction

(D) Free radical fluorination

Correct Answer: (C) Swarts reaction

Solution:

Concept:

- The Swarts reaction prepares alkyl fluorides by replacing chlorine or bromine in an alkyl halide with fluorine using a metallic fluoride.
- A named reaction must be matched both to the desired halogen and to the substrate class. Reactions of aryl diazonium salts do not serve as the general preparation of alkyl fluorides.

Step 1: Choose the controlled halogen-exchange method In the Swarts reaction an alkyl chloride or alkyl bromide is heated with a suitable metallic fluoride, such as AgF, Hg_2F_2, or SbF_3. Fluorine replaces the original halogen without relying on uncontrolled direct fluorination. A representative substitution is

$$\mathrm{RCl} + \mathrm{AgF} \longrightarrow \mathrm{RF} + \mathrm{AgCl}.$$

Here R is an alkyl group and the product RF is the required alkyl fluoride.

Step 2: Distinguish the other named reactions The Finkelstein reaction is a halogen exchange commonly used to prepare alkyl iodides using NaI in dry acetone. The precipitation of the chloride or bromide salt helps drive that exchange; it is not the standard route to alkyl fluorides. Sandmeyer reactions involve replacement of the diazonium group of an aromatic diazonium salt, commonly by chlorine, bromine or cyanide using copper salts. Their substrate is aromatic rather than the alkyl halide required here.

Step 3: Reject direct radical fluorination Direct reaction of hydrocarbons with fluorine is highly exothermic and difficult to control. It can cause extensive substitution and unwanted products rather than a clean, selective preparation of one alkyl fluoride. Therefore free radical fluorination is not the best general synthetic method among these choices. Swarts halogen exchange is the specifically named preparation that delivers the target carbon–fluorine bond under controlled conditions.

Step 4: Conclusion Alkyl fluorides are best prepared by the Swarts reaction. Hence, the correct option is (C).

Quick Tip to Solve: Link the names to the product halogen: Finkelstein usually gives an alkyl iodide, whereas Swarts gives an alkyl fluoride.

74. The colour of $KMnO_4$ is due to : [4 marks]

(A) d – d transition

(B) L $\rightarrow$ M charge transfer transition

(C) σ – σ^* transition

(D) M $\rightarrow$ L charge transfer transition

Correct Answer: (B) L $\rightarrow$ M charge transfer transition

Solution:

Concept:

- In $KMnO_4$, potassium is present as K^+ and the coloured species is the permanganate ion, MnO_4^-. Manganese has oxidation state $+7$.
- A metal-centred d–d transition requires electrons in the metal d orbitals. A ligand-to-metal charge-transfer transition instead promotes an electron from an occupied ligand-based orbital to an empty metal-based orbital.

Step 1: Find the manganese electron count For MnO_4^-, let the oxidation number of manganese be x. Oxygen contributes four times -2 and the total ion charge is -1, so

$$x + 4(-2) = -1, \qquad x = +7.$$

Neutral manganese has configuration $[\mathrm{Ar}]\,3d^5 4s^2$. Removal of seven electrons in the formal oxidation-state description leaves manganese(VII) with a d^0 configuration. It therefore has no d electron available for a conventional d–d excitation.

Step 2: Identify the allowed charge-transfer direction The oxygen ligands have occupied orbitals, and the highly oxidised manganese centre has empty orbitals capable of accepting electron density on excitation. Absorption of visible light can therefore transfer an electron from a ligand-based orbital to a metal-based orbital. This is ligand-to-metal charge transfer, abbreviated LMCT or L to M. Such transitions can be intense and account for the strong purple colour of permanganate.

Step 3: Compare the alternative explanations Option (A) is ruled out by the d^0 count. Option (C), a sigma-to-sigma-star excitation, does not describe the characteristic visible absorption responsible for the permanganate colour. Option (D) reverses the required transfer direction: the high-oxidation-state metal is the electron acceptor, not the donor. Thus only the L to M charge-transfer option gives the correct origin of the colour.

Step 4: Conclusion The colour of $KMnO_4$ is caused by a ligand-to-metal charge-transfer transition. Hence, the correct option is (B).

Quick Tip to Solve: For an intensely coloured ion with a formally empty d subshell, check charge transfer before assigning the colour to a d–d transition.

75. In the following sequence of reactions:

The product C is [4 marks]

$$\text{Toluene} \xrightarrow{KMnO_4} A \xrightarrow{SOCl_2} B \xrightarrow[BaSO_4]{H_2/Pd} C$$

(A) $C_6H_5CH_3$

(B) $C_6H_5CH_2OH$

(C) C_6H_5CHO

(D) C_6H_5COOH

Correct Answer: (C) C_6H_5CHO

Solution:

Concept:

- Permanganate oxidises an alkyl side chain on benzene, provided a benzylic hydrogen is present, to the carboxylic acid. Toluene therefore gives benzoic acid.
- Thionyl chloride converts a carboxylic acid into its acid chloride. Hydrogen with palladium supported on barium sulfate reduces an acid chloride to an aldehyde in the Rosenmund reaction.

Step 1: Identify product A after side-chain oxidation The starting compound is toluene, $C_6H_5CH_3$. Its methyl group contains benzylic hydrogens, so oxidation with $KMnO_4$ converts this side chain into the carboxyl group. The aromatic ring is retained. Thus

$$C_6H_5CH_3 \xrightarrow{KMnO_4} C_6H_5COOH.$$

Product A is benzoic acid, rather than benzyl alcohol or benzaldehyde, because permanganate is a strong oxidising reagent for this transformation.

Step 2: Identify the acid chloride B The next reagent is $SOCl_2$. A carboxylic acid reacts with thionyl chloride by replacement of its hydroxyl group with chlorine:

$$C_6H_5COOH + SOCl_2 \longrightarrow C_6H_5COCl + SO_2 + HCl.$$

Consequently B is benzoyl chloride. This step changes the acid functional group into an acyl chloride; it does not remove the carbonyl group or change the aromatic carbon skeleton.

Step 3: Apply the selective final reduction The final reagent is hydrogen over palladium on $BaSO_4$, characteristic of Rosenmund reduction. Benzoyl chloride is reduced selectively to benzaldehyde:

$$C_6H_5COCl + H_2 \xrightarrow{Pd/BaSO_4} C_6H_5CHO + HCl.$$

The supported catalyst limits further reduction, so the required product is not benzyl alcohol. Toluene is the original starting compound and benzoic acid is intermediate A; neither is the final product C.

Step 4: Conclusion Product C is benzaldehyde, C_6H_5CHO. Hence, the correct option is (C).

Quick Tip to Solve: Read the final catalyst carefully: hydrogen with Pd on barium sulfate converts an acid chloride into an aldehyde, rather than reducing it all the way to an alcohol.

76. The number of geometric isomers that can exist for square planar [Pt (Cl) (py) (NH_3) (NH_2OH)]+ is (py = pyridine) : [4 marks]

(A) 3

(B) 4

(C) 6

(D) 2

Correct Answer: (A) 3

Solution:

Concept:

- A square-planar complex has four ligand positions in one plane. Geometrical isomers differ in which ligand pairs occupy opposite, or trans, positions.
- For four different monodentate ligands, fix any one ligand and choose its trans partner. The remaining two ligands occupy the other opposite pair; exchanging them gives an equivalent arrangement by rotation.

Step 1: Identify the four distinguishable ligands The complex contains chloride, pyridine (py), ammonia, and hydroxylamine. These are four different ligands, so represent the coordination entity as square-planar $MABCD$. The fact that two ligands contain nitrogen does not make them identical: NH_3, pyridine and NH_2OH have different chemical identities. Only their positions around the metal are being counted; the question does not ask about possible alternative donor-atom linkages.

Step 2: Enumerate the distinct trans relationships Fix chloride at one vertex of the square. There are exactly three choices for the ligand opposite chloride: pyridine, ammonia, or hydroxylamine. When chloride is trans to pyridine, ammonia is trans to hydroxylamine. When chloride is trans to ammonia, pyridine is trans to hydroxylamine. When chloride is trans to hydroxylamine, pyridine is trans to ammonia. These three sets of opposite pairs are distinct and cannot be converted into one another by rotation of the square.

Step 3: Avoid counting equivalent arrangements twice For each chosen trans partner, interchanging the two ligands in the remaining opposite positions does not create a new geometrical isomer: a suitable rotation of the planar arrangement superposes them. Equivalently, placing four different ligands around a square gives $(4-1)!/2 = 3$ arrangements after rotational and reversal equivalences are removed. Counts of six or four retain redundant placements, while two is the familiar cis/trans count for MA_2B_2, not for four different ligands.

Step 4: Conclusion There are three geometrical isomers of the given square-planar complex. Hence, the correct option is (A).

Quick Tip to Solve: For square-planar complexes with four different ligands, fix one ligand and count its three possible trans partners.

77. Which of the following compounds is not colored yellow? [4 marks]

(A) $K_3[Co(NO_2)_6]$

(B) $(NH_4)_3[As(Mo_3O_{10})_4]$

(C) $BaCrO_4$

(D) $Zn_2[Fe(CN)_6]$

Correct Answer: (D) $Zn_2[Fe(CN)_6]$

Solution:

Concept:

- The requested compound is the exception to the yellow colour shared by the other three listed salts. Use the characteristic colours of the named inorganic compounds.
- Zinc(II) is formally d^{10}, while the iron(II) centre in ferrocyanide is low-spin d^6. The colour of an entire salt must be identified from that actual compound, not inferred from the mere presence of a transition element.

Step 1: Identify the three yellow compounds Potassium hexanitritocobaltate(III), $K_3[Co(NO_2)_6]$, is a yellow compound associated with the cobaltinitrite test. The ammonium arsenomolybdate compound $(NH_4)_3[As(Mo_3O_{10})_4]$ is also yellow. Barium chromate, $BaCrO_4$, forms a characteristic yellow precipitate. Therefore the first, second and third choices all satisfy the yellow-colour description and cannot be the exception requested.

Step 2: Examine the zinc ferrocyanide salt The fourth compound is zinc ferrocyanide, $Zn_2[Fe(CN)_6]$. Its ferrocyanide ion is $[Fe(CN)_6]^{4-}$, and two Zn^{2+} ions balance this charge. Zinc ferrocyanide is a white precipitate, so it does not share the yellow appearance of the other three compounds. Do not confuse it with intensely blue iron-containing ferrocyanide pigments, which involve a different metal-ion combination and electronic interactions.

Step 3: Use the negative wording to select the option The wording asks which compound is *not* coloured yellow. After verifying that the cobaltinitrite, arsenomolybdate and barium chromate compounds are yellow, the white zinc ferrocyanide is the sole remaining choice. Its position in the printed list is fourth. Selecting barium chromate would reverse a well-known precipitation-test colour; selecting either of the first two would likewise contradict their standard yellow appearance.

Step 4: Conclusion $Zn_2[Fe(CN)_6]$ is white rather than yellow. Hence, the correct option is (D).

Quick Tip to Solve: When a colour question asks for the exception, mark the familiar yellow precipitates first; zinc ferrocyanide is the white member of this list.

78. Higher order (>3) reactions are rare due to [4 marks]

(A) increase in entropy and activation energy as more molecules are involved

(B) shifting of equilibrium towards reactants due to elastic collisions

(C) loss of active species on collision

(D) low probability of simultaneous collision of all the reacting species

Correct Answer: (D) low probability of simultaneous collision of all the reacting species

Solution:

Concept:

- For an elementary reaction, molecularity is the number of reacting particles participating together in one elementary event. Effective collisions require suitable energy and orientation.
- The order of a reaction is obtained from its experimental rate law. Order and molecularity coincide for a simple elementary step, but need not coincide for a multistep reaction.

Step 1: Identify what simultaneous participation requires A bimolecular event brings two reactant particles into contact. A termolecular event requires three particles to meet within the same very small region and collision time. Bringing four or more particles together is still less probable, and all must also have an arrangement capable of producing products. Thus increasing the number of simultaneous participants sharply reduces the frequency of effective events.

Step 2: Connect the collision probability to the reaction order The usual elementary explanation for the rarity of reactions of order greater than three is this low probability of a simultaneous collision involving all reactants. Most reactions proceed instead through sequences of simpler steps. A high overall stoichiometric coefficient alone does not establish a high order: the measured rate law determines the order. This distinction prevents us from treating every equation with many reactants as a many-body elementary collision.

Step 3: Eliminate the other explanations Option A incorrectly makes an increase of entropy and activation energy a universal consequence of involving more molecules. Neither statement supplies the collision-probability explanation. Elastic collisions do not in themselves shift chemical equilibrium towards reactants, so B is unsuitable. Loss of active species, C, is not the general cause of the rarity of simultaneous many-particle events.

Step 4: Conclusion Simultaneous effective collision of more than three reacting species has a very low probability. Hence, the correct option is (D).

Quick Tip to Solve: For this question, focus on the probability of a many-particle elementary encounter; do not infer reaction order from the balanced equation.

79. Assertion : Nitrogen and Oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen.

Reason : The reaction between nitrogen and oxygen requires high temperature. [4 marks]

(A) Both assertion and reason are correct, and the reason is the correct explanation for the assertion

(B) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion

(C) The assertion is incorrect, but the reason is correct

(D) Both the assertion and reason are incorrect

Correct Answer: (A) Both assertion and reason are correct, and the reason is the correct explanation for the assertion

Solution:

Concept:

- Molecular nitrogen, N_2, contains a very strong triple bond. Its large activation-energy barrier makes direct reaction with O_2 negligible at ordinary atmospheric temperature.
- At sufficiently high temperature, nitrogen and oxygen can combine to form nitric oxide: $N_2(g) + O_2(g) \rightleftharpoons 2\,NO\,(g)$. Heating increases the fraction of collisions energetic enough to react.

Step 1: Evaluate the assertion in its atmospheric context Nitrogen and oxygen are the principal components of ordinary air. Although they coexist and continually collide, they do not appreciably combine to form nitrogen oxides under normal atmospheric conditions. The assertion describes this ordinary-temperature behaviour. It is not a claim that nitrogen oxides can never form: electrical discharges such as lightning can supply the necessary energy.

Step 2: Evaluate the reason The direct reaction between N_2 and O_2 needs high temperature because appreciable reaction requires overcoming the large energy barrier associated with the strong nitrogen bond. At ordinary temperature, the proportion of suitably energetic collisions is extremely small. At elevated temperature, that proportion increases, and NO formation becomes possible. Thus the reason is correct.

Step 3: Check whether the reason explains the assertion The need for high temperature explains why mere coexistence of nitrogen and oxygen in normal air does not cause appreciable reaction. Both statements are therefore correct and causally related. Option B denies that explanatory link, C rejects the atmospheric-context assertion, and D rejects the correct high-temperature requirement; each is inconsistent with the kinetic explanation.

Step 4: Conclusion Under ordinary atmospheric conditions both statements are correct, and the high-temperature requirement explains the lack of appreciable reaction. Hence, the correct option is (A).

Quick Tip to Solve: In assertion-reason questions about inertness, distinguish a kinetic barrier at ordinary temperature from an absolute impossibility of reaction.

80. Which one has the highest boiling point? [4 marks]

(A) Ne

(B) Kr

(C) Xe

(D) He

Correct Answer: (C) Xe

Solution:

Concept:

- Noble gases are monatomic and have no permanent molecular dipole. Their interatomic attractions arise predominantly from London dispersion forces.
- A larger, more easily distorted electron cloud has greater polarizability. Stronger dispersion attractions require a higher temperature to separate atoms into the gas phase.

Step 1: Compare the electron clouds The listed elements belong to Group 18. Moving down the group from helium through neon and krypton to xenon increases the number of electrons and the spatial extent of the electron cloud. The outer electrons are held less tightly relative to the size of the atom, so the cloud is more easily distorted. Xenon is therefore the most polarizable of these four gases.

Step 2: Relate polarizability to boiling point Temporary fluctuations in the electron distribution of one atom induce a dipole in a neighbouring atom. More polarizable atoms develop stronger induced dipoles, producing stronger London attractions in the condensed phase. Boiling requires overcoming these interatomic attractions, so the boiling point increases in the order

$$\mathrm{He} < \mathrm{Ne} < \mathrm{Kr} < \mathrm{Xe}.$$

This is an intermolecular-force comparison, not an argument about breaking covalent bonds inside a molecule; noble-gas particles are individual atoms.

Step 3: Check the alternatives Helium has the smallest and least polarizable cloud, so D gives the lowest rather than the highest boiling point. Neon is more polarizable than helium but much less than xenon, excluding A. Krypton is below neon but above xenon in the group; its dispersion attractions are weaker than those of xenon, excluding B. Only C identifies the largest listed noble-gas atom.

Step 4: Conclusion Xenon has the strongest London dispersion attractions and the highest boiling point among the gases listed. Hence, the correct option is (C).

Quick Tip to Solve: For noble gases, rank boiling points by polarizability: the larger atom lower in Group 18 usually boils at the higher temperature.

81. The ionic radii (in Å) of N^{3-}, O^{2-} and F^{-} are respectively: [4 marks]

(A) 1.36, 1.71 and 1.40

(B) 1.71, 1.40 and 1.36

(C) 1.71, 1.36 and 1.40

(D) 1.36, 1.40 and 1.71

Correct Answer: (B) 1.71, 1.40 and 1.36

Solution:

Concept:

- Isoelectronic ions have the same number of electrons. When their electron configurations are the same, increasing nuclear charge pulls the electron cloud inward and decreases ionic radius.
- For N^{3-}, O^{2-} and F^{-}, the electron count is ten in every case, but the nuclear charges are respectively 7, 8 and 9.

Step 1: Verify the common electron configuration A nitrogen atom has seven electrons and gains three to form N^{3-}; oxygen has eight and gains two to form O^{2-}; fluorine has nine and gains one to form F^{-}. Hence each ion has ten electrons with configuration $1s^2 2s^2 2p^6$. Their occupied shells and electron counts are identical, making comparison by nuclear attraction appropriate.

Step 2: Rank the radii Along this sequence the number of protons increases from seven to nine while the number of electrons remains ten. The stronger nuclear attraction contracts the cloud. Therefore

$$r(N^{3-}) > r(O^{2-}) > r(F^{-}).$$

Do not rank these ions merely by the neutral-atom periodic trend: they are ions with different charges, and the isoelectronic argument explains the trend directly.

Step 3: Match the descending numerical sequence The three supplied radii are 1.71, 1.40 and 1.36 Å. Assign the largest to nitride, the intermediate value to oxide and the smallest to fluoride. This gives $1.71, 1.40, 1.36$ in the question's stated order. Option A and D make nitride too small; C puts fluoride above oxide and reverses their required relative sizes.

Step 4: Conclusion The respective ionic radii are 1.71 Å, 1.40 Å and 1.36 Å. Hence, the correct option is (B).

Quick Tip to Solve: First count electrons. In an isoelectronic series, the ion with the most protons has the smallest radius.

82. Which of the following is the energy of a possible excited state of hydrogen? [4 marks]

(A) –6.8 eV

(B) –3.4 eV

(C) +6.8 eV

(D) +13.6 eV

Correct Answer: (B) –3.4 eV

Solution:

Concept:

- With the free electron at rest at infinity assigned zero energy, the bound-state energies of hydrogen are $E_n = -13.6/n^2\,\text{eV}$, where $n = 1, 2, 3, \ldots$.
- The ground state has $n = 1$; an excited bound state has an integer $n > 1$. Bound energies remain negative, whereas positive energies belong to the ionized continuum.

Step 1: Test the negative energy values For a proposed bound energy E, compute $n^2 = 13.6/|E|$. For option A,

$$n^2 = \frac{13.6}{6.8} = 2, \qquad n = \sqrt{2}.$$

The principal quantum number cannot be $\sqrt{2}$, so $-6.8\,\text{eV}$ is not an allowed stationary-state energy of hydrogen. An energy lying between two allowed levels is not an additional level.

Step 2: Identify the allowed excited level For option B,

$$n^2 = \frac{13.6}{3.4} = 4, \qquad n = 2.$$

This is an allowed integer greater than one, and direct substitution gives

$$E_2 = -\frac{13.6}{2^2} = -3.4\,\text{eV}.$$

It is the first excited state. The energy needed to excite the ground-state atom to this level would be $10.2\,\text{eV}$, but that transition energy is different from the absolute level energy asked here.

Step 3: Reject the positive energies Options C and D give $+6.8\,\text{eV}$ and $+13.6\,\text{eV}$. Relative to the usual zero at infinity, these are positive continuum energies of an unbound electron, not discrete excited bound levels. Thus neither satisfies the question about an excited state of the hydrogen atom.

Step 4: Conclusion The possible excited-state energy is $-3.4\,\text{eV}$, corresponding to $n = 2$. Hence, the correct option is (B).

Quick Tip to Solve: Check both conditions: a hydrogen bound-state energy must be negative and $\sqrt{13.6/|E|}$ must be a positive integer.

83. 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is: [4 marks]

(A) 36 mg

(B) 42 mg

(C) 54 mg

(D) 18 mg

Correct Answer: (D) 18 mg

Solution:

Concept:

- The amount of solute adsorbed equals its initial amount in solution minus the amount left in the filtrate, taking the solution volume as unchanged.
- Normality is equivalents per litre. Acetic acid, CH_3COOH, is monoprotic, so its equivalent mass is its molar mass, $60\,\mathrm{g\,equiv^{-1}}$.

Step 1: Calculate the decrease in concentration The initial and final acid strengths are $0.060\,\mathrm{N}$ and $0.042\,\mathrm{N}$. The decrease represents acid removed from the liquid by adsorption on charcoal:

$$\Delta N = 0.060 - 0.042 = 0.018\,\mathrm{equiv\,L^{-1}}.$$

Convert the solution volume before multiplying: $50\,\mathrm{mL} = 0.050\,\mathrm{L}$. The time of contact describes the experimental procedure and does not enter this mass balance.

Step 2: Find the total adsorbed mass The equivalents of acid adsorbed are

$$n_{\mathrm{eq}} = 0.018 \times 0.050 = 9.0 \times 10^{-4}\,\mathrm{equiv}.$$

Multiplication by equivalent mass gives

$$x = 9.0 \times 10^{-4} \times 60 = 0.054\,\mathrm{g} = 54\,\mathrm{mg}.$$

As a check, the initial acid mass is $180\,\mathrm{mg}$ and the remaining mass is $126\,\mathrm{mg}$; their difference is again $54\,\mathrm{mg}$.

Step 3: Express the result per gram of charcoal The mass $54\,\mathrm{mg}$ is adsorbed by the entire $3\,\mathrm{g}$ sample. Therefore

$$\frac{x}{m} = \frac{54}{3} = 18\,\mathrm{mg\,g^{-1}}.$$

Option C is the total adsorbed mass, not the requested mass per gram. The values in A and B do not follow from the concentration decrease and charcoal mass. The normalization by charcoal mass is essential.

Step 4: Conclusion The amount adsorbed per gram of charcoal is $18\,\mathrm{mg}$, or $18\,\mathrm{mg\,g^{-1}}$ of charcoal. Hence, the correct option is (D).

Quick Tip to Solve: Use concentration lost, not final concentration: $x/m = (N_i - N_f)VE_{\mathrm{acid}}/m$, with volume in litres.

84. In the context of the Hall-Heroult process for the extraction of Al, which of the following statements is false? [4 marks]

(A) Al_2O_3 is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity

(B) Al^{3+} is reduced at the cathode to form Al

(C) Na_3AlF_6 serves as the electrolyte

(D) CO and CO_2 are produced in this process

Correct Answer: (C) Na_3AlF_6 serves as the electrolyte

Solution:

Concept:

- The Hall-Heroult process electrolytically reduces alumina, Al_2O_3, dissolved in a molten fluoride bath containing cryolite, Na_3AlF_6, and additives such as CaF_2.
- The alumina supplies the aluminium ions that are reduced. Cryolite is principally the bath solvent and flux; it lowers the operating melting temperature and helps the bath conduct electricity.

Step 1: Identify the electrolyte being decomposed Alumina is the substance electrolysed to obtain aluminium. It is dissolved in molten cryolite rather than electrolysed as pure high-melting alumina. In the distinction tested by these options, cryolite is the solvent or supporting bath medium, not the aluminium-source electrolyte undergoing the intended decomposition. Thus calling Na_3AlF_6 itself the electrolyte in place of alumina is the false statement in C. This does not mean the molten cryolite bath is electrically nonconducting.

Step 2: Check the cathodic and bath statements At the cathode, aluminium species accept electrons and yield liquid metal, represented in the usual simplified form by

$$Al^{3+} + 3\,e^- \longrightarrow Al.$$

Thus B is correct. The fluoride additives help lower the melting temperature of the working mixture and provide a conducting molten bath, so the function described in A is consistent with the process.

Step 3: Check what happens at the carbon anode Oxide-derived oxygen reacts with the hot carbon anodes. Both carbon dioxide and carbon monoxide can be produced, represented by the simplified anode reactions

$$C + 2\,O^{2-} \longrightarrow CO_2 + 4\,e^-,$$

$$C + O^{2-} \longrightarrow CO + 2\,e^-.$$

The carbon anodes are therefore consumed and require replacement. This verifies D rather than making it the false statement. Only C confuses the bath solvent with the material being electrolysed.

Step 4: Conclusion The false statement is that Na_3AlF_6 serves as the electrolyte; in this classification it acts as the solvent/flux for the alumina being electrolysed. Hence, the correct option is (C).

Quick Tip to Solve: For Hall-Heroult questions separate three roles: alumina supplies aluminium, cryolite forms the molten solvent bath, and carbon anodes are consumed.

85. Which of the following compounds is not an antacid? [4 marks]

(A) Cimetidine

(B) Ranitidine

(C) Phenelzine

(D) Aluminium hydroxide

Correct Answer: (C) Phenelzine

Solution:

Concept:

- An antacid treatment reduces gastric acidity, either by neutralising acid already present or by suppressing its secretion in the stomach.
- Histamine H2-receptor antagonists reduce acid secretion, whereas monoamine oxidase inhibitors belong to the antidepressant class.

Step 1: Classify the acid-lowering medicines Cimetidine and ranitidine block histamine H2 receptors in the stomach and therefore decrease the secretion of hydrochloric acid. In the classification used in this question, both are medicines for treating hyperacidity. Thus options (A) and (B) do not satisfy the word "not. Their action is suppression of acid production, rather than direct neutralisation.

Step 2: Check the remaining compounds Aluminium hydroxide is a weak base that neutralises excess gastric acid. Its reaction can be written as

$$\mathrm{Al(OH)_3 + 3\,HCl \longrightarrow AlCl_3 + 3\,H_2O.}$$

Phenelzine instead inhibits monoamine oxidase and is used as an antidepressant. It neither belongs to the H2-blocker group nor serves as the acid-neutralising base among these choices. Hence option (D) is also an acid-control agent, while option (C) is the exception.

Step 3: Conclusion Phenelzine is an antidepressant, not an antacid. Hence, the correct option is (C).

Quick Tip to Solve: For drug-classification exceptions, identify the therapeutic target: cimetidine and ranitidine concern gastric acid, while phenelzine concerns monoamine metabolism.

86. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy? [4 marks]

(A) $BeSO_4$

(B) $BaSO_4$

(C) $SrSO_4$

(D) $CaSO_4$

Correct Answer: (A) $BeSO_4$

Solution:

Concept:

- Dissolution of an ionic solid requires separation of its lattice and hydration of the resulting ions: $\Delta H_{\text{sol}} = \Delta H_{\text{latt,diss}} + \Delta H_{\text{hyd}}$.
- Lattice dissociation enthalpy is positive and hydration enthalpy is negative. Comparing their strengths here means comparing magnitudes, so hydration outweighs lattice separation when $|\Delta H_{\text{hyd}}| > \Delta H_{\text{latt,diss}}$.

Step 1: Compare hydration down the group The sulphate ion is the same in all four salts. The important change is the size of the divalent metal ion: Be^{2+} is much smaller than Ca^{2+}, Sr^{2+} and Ba^{2+}. A small cation has a high charge density and interacts strongly with water molecules. Consequently beryllium has the greatest magnitude of cation hydration enthalpy among the listed choices.

Step 2: Balance hydration against lattice separation For these alkaline earth sulphates the large sulphate anion makes the lattice-energy variation less pronounced than the fall in hydration strength down the group. The hydration energy of $BeSO_4$ is sufficient to outweigh its lattice dissociation energy. The larger-cation sulphates do not give the required comparison: calcium sulphate is sparingly soluble, and strontium and barium sulphates are still less soluble. Thus (B), (C) and (D) are rejected in favour of the smallest-cation salt.

Step 3: Conclusion The required sulphate is $BeSO_4$; hydration enthalpy exceeds lattice enthalpy in magnitude. Hence, the correct option is (A).

Quick Tip to Solve: With a common large anion such as sulphate, compare the cation hydration strengths first: the smallest divalent cation is hydrated most strongly.

87. From the following statements regarding H_2O_2, choose the incorrect statement : [4 marks]

(A) It decomposes on exposure to light

(B) It has to be stored in plastic or wax lined glass bottles in dark

(C) It has to be kept away from dust

(D) It can act only as an oxidizing agent

Correct Answer: (D) It can act only as an oxidizing agent

Solution:

Concept:

- In H_2O_2, oxygen has oxidation state -1, intermediate between its states in water (-2) and molecular oxygen (0).
- Hydrogen peroxide can therefore be reduced to water or oxidised to oxygen, allowing it to act as either an oxidising or a reducing agent.

Step 1: Verify the storage statements Hydrogen peroxide decomposes according to

$$2\,H_2O_2 \longrightarrow 2\,H_2O + O_2.$$

Light accelerates this decomposition, so statement (A) is correct. Storage in the dark in suitable plastic or wax-lined glass bottles reduces exposure to light and contact with catalytic impurities; thus statement (B) is correct. Dust can introduce substances that catalyse decomposition, making the precaution in statement (C) correct as well.

Step 2: Test the claim that it is only an oxidant When hydrogen peroxide oxidises iodide, its oxygen is reduced from -1 to -2:

$$H_2O_2 + 2\,I^- + 2\,H^+ \longrightarrow I_2 + 2\,H_2O.$$

In contrast, it reduces acidified permanganate while being oxidised to oxygen:

$$2\,MnO_4^- + 5\,H_2O_2 + 6\,H^+ \longrightarrow 2\,Mn^{2+} + 5\,O_2 + 8\,H_2O.$$

The second reaction directly disproves the restriction “only. The question asks for the incorrect statement, so the three valid storage facts must not be selected.

Step 3: Conclusion Statement (D) is incorrect: H_2O_2 acts as both an oxidising agent and a reducing agent. Hence, the correct option is (D).

Quick Tip to Solve: Use the intermediate oxygen oxidation state -1 to remember both roles of peroxide; then pay attention to the word “incorrect.

88. Which among the following is the most reactive? [4 marks]

(A) Br_2

(B) I_2

(C) ICl

(D) Cl_2

Correct Answer: (C) ICl

Solution:

Concept:

- Interhalogen compounds contain two different halogens. Their unlike-atom bond is polar because the two atoms have different electronegativities.
- The standard qualitative comparison treats interhalogens as more reactive than their constituent halogens, apart from fluorine, because their bonds are generally weaker and readily attacked.

Step 1: Identify the exceptional type of compound Among Br_2, I_2, ICl and Cl_2, only ICl is an interhalogen. Chlorine is more electronegative than iodine, so the iodine end has partial positive character and the chlorine end has partial negative character. This charge separation makes the unlike-atom molecule susceptible to attack by suitable reactants, unlike a nonpolar bond joining two identical halogen atoms.

Step 2: Compare the offered choices Within the ordinary halogen series, chlorine is more reactive than bromine and iodine, so Br_2 and I_2 cannot be preferred simply because they contain heavier atoms. However, the question also includes the interhalogen ICl. Applying the usual interhalogen-reactivity rule places it ahead of the constituent elemental halogens in this qualitative comparison. Thus (D) is the tempting elemental-halogen answer, but (C) accounts for the presence of the more reactive interhalogen. No fluorine molecule, the familiar exception, is listed.

Step 3: Conclusion The most reactive species in the stated comparison is iodine monochloride, ICl. Hence, the correct option is (C).

Quick Tip to Solve: Before ranking halogens down a group, check whether a choice is an interhalogen; its unlike-atom bond changes the comparison.

89. Which polymer is used in the manufacture of paints and lacquers? [4 marks]

(A) Glyptal

(B) Polypropene

(C) Poly vinyl chloride

(D) Bakelite

Correct Answer: (A) Glyptal

Solution:

Concept:

- Glyptal is a polyester prepared by condensation of ethylene glycol with phthalic acid or its anhydride; its resin-forming character makes it useful in coatings.
- Polymer applications depend on functional groups, structure and processing: a coating resin is selected for its ability to form an adherent protective film.

Step 1: Identify the coating resin Glyptal contains ester linkages produced by reaction of the hydroxyl groups of ethylene glycol with the carboxyl groups of phthalic acid. In the condensation route using the acid, water is eliminated as these linkages form. The resulting polyester resin can be used in the manufacture of paints and lacquers, where a continuous film is required on the coated surface. This is the standard application associated with glyptal among the listed polymers.

Step 2: Eliminate polymers with other standard uses Polypropene is an addition polymer commonly used for fibres, ropes and moulded articles; it is not the named polyester coating resin. Polyvinyl chloride is associated with pipes, cable insulation and related plastic products. Bakelite, a phenol–formaldehyde thermosetting resin, is familiar in electrical fittings and handles because of its rigidity and insulating character. These general uses distinguish (B), (C) and (D) from the particular paints-and-lacquers application asked here. Identifying an ester-forming condensation polymer therefore selects (A).

Step 3: Conclusion Glyptal is the polymer used in the manufacture of paints and lacquers. Hence, the correct option is (A).

Quick Tip to Solve: Associate glyptal with polyester coating resins, PVC with pipes and insulation, and Bakelite with rigid electrical fittings.

90. Sodium metal crystallizes in a body centred cubic lattice with a unit cell edge of 4.29 Å . The radius of sodium atom is approximately : [4 marks]

(A) 5.72 Å

(B) 0.93 Å

(C) 1.86 Å

(D) 3.22 Å

Correct Answer: (C) 1.86 Å

Solution:

Concept:

- In a body-centred cubic (bcc) unit cell, atoms touch along the body diagonal, not along a cube edge or a face diagonal.
- For cube edge a, the body diagonal is $\sqrt{3}a$. The touching sequence corner–centre–corner spans four atomic radii, so $4r = \sqrt{3}a$.

Step 1: Write the correct contact relation A bcc cell has an atom at each corner and one at its body centre. Along a body diagonal, the distance from the first corner centre to the central atom is $2r$, and the distance from that atom to the opposite corner centre is another $2r$. Hence

$$\sqrt{3}a = 2r + 2r = 4r, \qquad r = \frac{\sqrt{3}}{4}a.$$

Using $a = 2r$ would instead describe edge contact in a simple cubic lattice and is inappropriate here.

Step 2: Insert the edge length and compare With the stated edge length $a = 4.29\,\text{Å}$,

$$r = \frac{1.732 \times 4.29}{4}\,\text{Å} = 1.8576\,\text{Å} \approx 1.86\,\text{Å}.$$

The unit remains angstrom because the multiplier is dimensionless. The corresponding diameter is about $3.72\,\text{Å}$, so a value close to twice the calculated radius must not be used as the radius. Options (A), (B) and (D) do not satisfy the bcc relation $4r = \sqrt{3}a$ with the given cell edge.

Step 3: Conclusion The sodium atomic radius is approximately $1.86\,\text{Å}$. Hence, the correct option is (C).

Quick Tip to Solve: Remember the contact line first: bcc gives $4r = \sqrt{3}a$, whereas fcc gives $4r = \sqrt{2}a$; neither uses edge contact.