JEE Main 2015 Paper 1 (B.E./B.Tech) Apr 11 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2015, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 11 April 2015, Single slot. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Physics, Chemistry and Mathematics, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Physics (120 marks)

1. If electronic charge e, electron mass m, speed of light in vacuum c and Planck's constant h are taken as fundamental quantities, the permeability of vacuum μ_0 can be expressed in units of [4 marks]

(A) $\left(\frac{hc}{me^2}\right)$
(B) $\left(\frac{h}{me^2}\right)$
(C) $\left(\frac{h}{ce^2}\right)$
(D) $\left(\frac{mc^2}{he^2}\right)$

Correct Answer: (C) $\left(\frac{h}{ce^2}\right)$

Solution:

Concept:

- The dimensions of magnetic permeability follow from the force between parallel currents: $F/\ell = \mu_0 I_1 I_2/(2\pi d)$, so $[\mu_0] = MLT^{-2}I^{-2}$.
- Electric charge has dimensions $Q = IT$; hence $[\mu_0] = MLQ^{-2}$. Planck's constant has dimensions ML^2T^{-1}, and speed has dimensions LT^{-1}.

Step 1: Set up a dimensional product Write $\mu_0 \propto h^a c^b m^d e^f$. The right side has dimensions $M^{a+d}L^{2a+b}T^{-a-b}Q^f$. Comparing this with MLQ^{-2} gives four independent equations:

$$a + d = 1, \qquad 2a + b = 1, \qquad -a - b = 0, \qquad f = -2.$$

Numerical constants are irrelevant because the question asks for units, not the numerical value of permeability.

Step 2: Solve the exponent equations and test the options The time equation gives $b = -a$. Substitution into the length equation gives $a = 1$, so $b = -1$, $d = 0$, and $f = -2$. Therefore

$$\mu_0 \propto \frac{h}{ce^2}.$$

The electron mass cancels entirely. Option (A) has dimensions $L^3T^{-2}Q^{-2}$, and option (B) has dimensions $L^2T^{-1}Q^{-2}$; neither contains the required mass dimension. Option (D) has dimensions $T^{-1}Q^{-2}$, also different from permeability. Only (C) gives MLQ^{-2}.

Step 3: Conclusion The required unit combination is $h/(ce^2)$. Hence, the correct option is (C).

Quick Tip to Solve: Express current as charge divided by time before matching powers; then the absence of electron mass follows immediately.

2. A vector $\vec{A}$ is rotated by a small angle $\Delta\theta$ radians ($\Delta\theta \ll 1$) to get a new vector $\vec{B}$. In that case $\left|\vec{B}-\vec{A}\right|$ is [4 marks]

(A) 0

(B) $\left|\vec{A}\right|\left(1-\frac{\Delta\theta^2}{2}\right)$

(C) $\left|\vec{A}\right|\Delta\theta$

(D) $\left|\vec{B}\right|\Delta\theta-\left|\vec{A}\right|$

Correct Answer: (C) $\left|\vec{A}\right|\Delta\theta$

Solution:

Concept:

- A rotation preserves magnitude: $|\vec{A}|=|\vec{B}|=A$. The angle between the original and rotated vectors is $\Delta\theta$.
- The magnitude of a difference is found from $|\vec{B}-\vec{A}|^2=A^2+B^2-2AB\cos\Delta\theta$. For a small positive rotation angle, $\sin(\Delta\theta/2)\simeq\Delta\theta/2$.

Step 1: Find the exact magnitude of the change The two vectors have a common tail. Their difference joins their endpoints, so its length is a chord rather than the change of their magnitudes. Applying the scalar-product identity,

$$|\vec{B}-\vec{A}|^2=2A^2(1-\cos\Delta\theta)=4A^2\sin^2(\Delta\theta/2).$$

Thus the exact result is $|\vec{B}-\vec{A}|=2A|\sin(\Delta\theta/2)|$. A zero change of magnitude does not imply a zero change of vector because the direction changes.

Step 2: Use the small-angle limit and compare choices Keeping the leading term for the stated small rotation gives

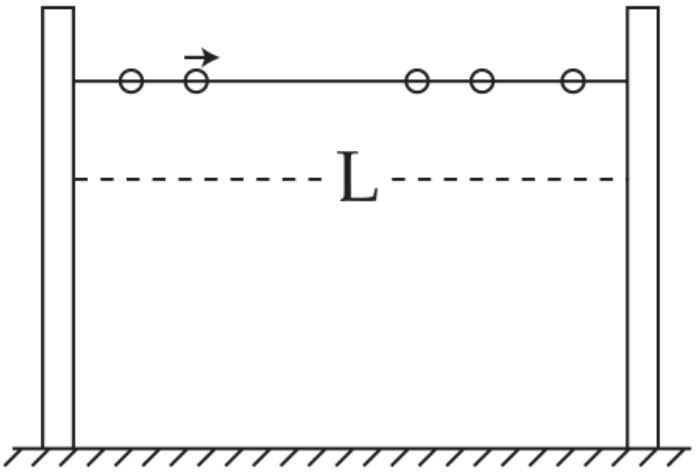
$$|\vec{B}-\vec{A}|\simeq A\Delta\theta.$$

The angle is in radians, which is essential for the approximation. Option (A) would require no directional change. Option (B) approaches A, not zero, as the rotation angle tends to zero; it resembles a projection rather than a vector difference. Since $B=A$, option (D) approaches $-A$ for a very small angle and cannot be a magnitude. Option (C) alone has the correct positive linear dependence and the correct zero-angle limit.

Step 3: Conclusion To first order in the small rotation, $|\vec{B}-\vec{A}|=|\vec{A}|\Delta\theta$. Hence, the correct option is (C).

Quick Tip to Solve: A small rotation changes a vector by the arc length at its tip: magnitude times the angle in radians.

3. A large number (n) of identical beads, each of mass m and radius r are strung on a thin smooth rigid horizontal rod of length L (L $\gg$ r) and are at rest at random positions. The rod is mounted between two rigid supports (see figure). If one of the beads is now given a speed v, the average force experienced by each support after a long time is (assume all collisions are elastic): [4 marks]



(A) (mv2)/(L−nr)

(B) (mv2)/(L−2nr)

(C) (mv2)/(2(L−nr))

(D) Zero

Correct Answer: (B) (mv2)/(L−2nr)

Solution:

Concept:

- In a one-dimensional elastic collision between equal masses, the velocities are exchanged. A moving bead therefore transfers its speed to the next bead.
- The mean force on a support is its accumulated impulse divided by elapsed time. Reflection of a bead reverses its momentum and transfers impulse $2mv$ to that support.

Step 1: Replace velocity transfer by a moving pulse Only one bead initially carries speed v. When it strikes a stationary identical bead, it stops and the struck bead moves with the same speed. Thus the kinetic energy and momentum pulse travels through the line of beads. Random initial bead positions affect individual collision times but not the long-time mean traversal distance. The finite radii must still be included: adjoining bead centres collide when separated by $2r$, so those contact intervals are crossed by velocity exchange without any travel time.

Step 2: Find the effective traversal time The bead centres have a total available span $L - 2r$ between their extreme positions at the supports. There are $n - 1$ internal contacts, each bypassing a centre separation $2r$. The effective distance travelled by the pulse from one support to the other is consequently

$$\ell_{\text{eff}} = L - 2r - 2r(n-1) = L - 2nr.$$

The time between successive impacts on the same support is the round-trip time $T = 2(L - 2nr)/v$.

Step 3: Divide support impulse by the return period Each return delivers impulse $2mv$. Hence

$$F_{\text{av}} = \frac{2mv}{2(L-2nr)/v} = \frac{mv^2}{L-2nr}.$$

Both supports experience this same average magnitude. Options (A) and (C) omit part of the total excluded bead diameter, while (C) also introduces an unwarranted factor of two. Option (D) confuses zero net impulse on the whole stationary enclosure with the separate nonzero forces on its supports.

Step 4: Conclusion The average force on each support is $mv^2/(L-2nr)$. Hence, the correct option is (B).

Quick Tip to Solve: For identical elastic beads, follow the velocity pulse and subtract every bead diameter from the rod length.

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4. A particle is moving in a circle of radius r under the action of a force $F = \alpha r^2$ which is directed towards centre of the circle. Total mechanical energy (kinetic energy + potential energy) of the particle is (take potential energy = 0 for $r = 0$) [4 marks]

(A) αr^3

(B) $\frac{1}{2}\alpha r^3$

(C) $\frac{4}{3}\alpha r^3$

(D) $\frac{5}{6}\alpha r^3$

Correct Answer: (D) $\frac{5}{6}\alpha r^3$

Solution:

Concept:

- For uniform circular motion, the inward force supplies centripetal acceleration: $mv^2/r = F(r)$. Thus kinetic energy is $K = F(r)r/2$.
- A radial conservative force satisfies $F_r = -dU/dr$, where outward radial direction is positive. The stated inward force therefore has signed component $F_r = -\alpha r^2$.

Step 1: Find the kinetic energy of the circular orbit The magnitude of the force is αr^2, so the circular-motion equation gives

$$\frac{mv^2}{r} = \alpha r^2, \qquad mv^2 = \alpha r^3.$$

It follows that

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\alpha r^3.$$

This is only the kinetic contribution: although the force does no work during motion around a fixed circle, its potential energy at that radius need not vanish.

Step 2: Integrate the signed force to obtain the potential Using the force-potential relation with the correct radial sign,

$$\frac{dU}{dr} = \alpha r^2.$$

With the specified reference $U(0) = 0$,

$$U(r) = \int_0^r \alpha s^2 \, ds = \frac{1}{3}\alpha r^3.$$

The potential increases outwards because an external agent must do positive work against the attractive force. Setting potential energy to zero everywhere would contradict the nonzero radial force.

Step 3: Add the energies and eliminate distractors The mechanical energy is therefore

$$E = K + U = \left(\frac{1}{2} + \frac{1}{3}\right)\alpha r^3 = \frac{5}{6}\alpha r^3.$$

Option (B) retains only kinetic energy. Options (A) and (C) give incorrect sums of the independently determined kinetic and potential terms. No negative potential sign is required here: its zero was explicitly placed at the centre, rather than at infinity.

Step 4: Conclusion The total mechanical energy is $E = 5\alpha r^3/6$. Hence, the correct option is (D).

Quick Tip to Solve: Write the inward force with a minus sign before integrating; the chosen zero of potential determines the additive constant.

5. A uniform thin rod AB of length L has linear mass density μ(x) = a + (bx)/(L), where x is measured from A. If the CM of the rod lies at a distance of ((7)/(12)L) from A, then a and b are related as: [4 marks]

(A) 2a = b

(B) a = 2b

(C) a = b

(D) 3a = 2b

Correct Answer: (A) 2a = b

Solution:

Concept:

- For a thin rod, an element of length dx has mass $dm = \mu(x)\,dx$. Its centre of mass is $x_{\text{CM}} = \int x\,dm / \int dm$.
- The supplied density is $\mu(x) = a + bx/L$, with $x = 0$ at end A and $x = L$ at B. Both a and b have dimensions of linear mass density.

Step 1: Calculate total mass and first moment The total mass follows by integrating the density over the rod:

$$M = \int_0^L \left(a + \frac{bx}{L}\right) dx = L\left(a + \frac{b}{2}\right).$$

The corresponding first moment about A is

$$\int_0^L x\,dm = \int_0^L \left(ax + \frac{bx^2}{L}\right) dx = L^2\left(\frac{a}{2} + \frac{b}{3}\right).$$

Weighting by mass is necessary; the geometrical midpoint $L/2$ applies only when the density is constant.

Step 2: Apply the given centre-of-mass position Dividing moment by mass gives

$$\frac{x_{\text{CM}}}{L} = \frac{a/2 + b/3}{a + b/2} = \frac{7}{12}.$$

Cross multiplication yields

$$6a + 4b = 7a + \frac{7}{2}b, \qquad \frac{b}{2} = a.$$

Therefore $b = 2a$. This relation makes the density larger towards B, consistently shifting the centre of mass beyond $L/2$ towards B.

Step 3: Check all proposed relations Option (A), $2a = b$, reproduces the required ratio $7/12$. In contrast, $a = 2b$ gives $8/15$, $a = b$ gives $5/9$, and $3a = 2b$ gives $4/7$. These are different from $7/12$; checking them also distinguishes the correct relation from its inverted version. The overall magnitude of the density cancels, as a centre-of-mass position should depend on relative distribution alone.

Step 4: Conclusion The relation is $b = 2a$, or $2a = b$. Hence, the correct option is (A).

Quick Tip to Solve: Integrate density once for mass and multiply by x before integrating for the first moment; never average the endpoint densities for the moment.

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6. A particle of mass 2 kg is on a smooth horizontal table and moves in a circular path of radius 0.6 m. The height of the table from the ground is 0.8 m. If the angular speed of the particle is 12 rad s-1 , the magnitude of its angular momentum about a point on the ground right under the center of the circle is [4 marks]

(A) 14.4 kg m2 s-1

(B) 11.52 kg m2 s-1

(C) 20.16 kg m2 s-1

(D) 8.64 kg m2 s-1

Correct Answer: (A) 14.4 kg m2 s-1

Solution:

Concept:

- Angular momentum about a chosen point is $\vec{L} = \vec{r} \times m\vec{v}$, so its magnitude is mv times the perpendicular distance from that point to the instantaneous line of motion.
- The speed in circular motion is $v = \omega R$. Here $R = 0.6\,\text{m}$, height $h = 0.8\,\text{m}$, and angular speed $\omega = 12\,\text{rad}\,\text{s}^{-1}$.

Step 1: Locate the position and velocity directions Let the origin O be the ground point directly below the circle centre. The position of the particle has one horizontal radial component R and one vertical component h. Its velocity is horizontal and tangential. Consequently it is perpendicular to both components of the position vector, and therefore to the complete position vector. Thus

$$r_O = \sqrt{R^2 + h^2} = \sqrt{0.6^2 + 0.8^2} = 1.0\,\mathrm{m}.$$

The angle in the cross product is ninety degrees.

Step 2: Calculate the full angular-momentum magnitude The particle speed is

$$v = \omega R = 12(0.6) = 7.2\,\mathrm{m\,s^{-1}}.$$

Hence the required magnitude is

$$L_O = mr_O v = 2(1.0)(7.2) = 14.4\,\mathrm{kg\,m^2\,s^{-1}}.$$

The radius in the speed formula is still the horizontal circle radius, not the longer distance from O; these lengths play different roles.

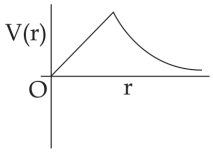
Step 3: Compare the result with alternative values Using $mR^2\omega$ alone gives $8.64\,\mathrm{kg\,m^2\,s^{-1}}$, option (D), which is the vertical component rather than the full magnitude about O. The horizontal component is $mvh = 11.52\,\mathrm{kg\,m^2\,s^{-1}}$, option (B). The two components are perpendicular, so their magnitudes combine quadratically, not arithmetically: their sum 20.16, option (C), is incorrect.

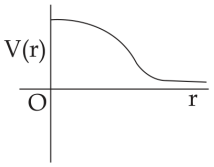
Step 4: Conclusion The angular-momentum magnitude about the specified ground point is $14.4\,\mathrm{kg\,m^2\,s^{-1}}$. Hence, the correct option is (A).

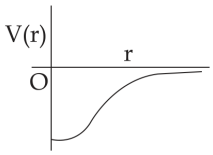
Quick Tip to Solve: When the reference point is displaced vertically, calculate the full cross product rather than only its vertical component.

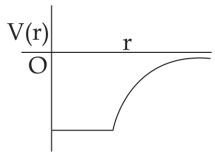
7. Which of the following most closely depicts the correct variation of the gravitation potential V(r) due to a large planet of radius R and uniform mass density ρ ? (figures are not drawn to scale)

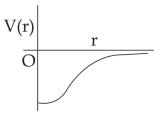
[4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- With gravitational potential zero at infinity, the exterior potential of a spherical planet is $V(r) = -GM/r$ for $r \geq R$. It is negative and approaches zero from below.
- Inside a uniform sphere, the enclosed mass is $M(r) = Mr^3/R^3$. The gravitational field is inward with magnitude GMr/R^3, and $dV/dr = GMr/R^3$.

Step 1: Find the interior potential and its initial slope At the surface, continuity fixes $V(R) = -GM/R$. Integrating the interior derivative from the surface to a radius r gives

$$V(r) = -\frac{GM}{R} + \frac{GM}{2R^3}(r^2 - R^2) = -\frac{GM}{2R^3}(3R^2 - r^2).$$

At the centre, $V(0) = -3GM/(2R)$, which is finite and most negative. Its slope is zero at the centre and then increases linearly outwards. Therefore the initial graph must be a rising concave-up curve below the horizontal axis.

Step 2: Join to the exterior inverse-radius curve For $r > R$, $V = -GM/r$ increases towards zero with derivative $GM/r^2 > 0$. Its second derivative is negative, so this exterior portion is concave down and gradually flattens. At $r = R$, both potential and first derivative agree with the interior expressions. The correct graph therefore has a smooth join and changes concavity near the surface without crossing the zero-potential axis.

Step 3: Select the graph and reject the others Graph (C) has the required negative central value, horizontal tangent at the centre, rising curved interior and asymptotic exterior tail. Graphs (A) and (B) show positive potentials with the wrong radial behaviour. Graph (D) has a constant interior potential, which would describe a thin spherical shell, not a solid body with uniform mass density; the nonzero interior field of this planet rules it out.

Step 4: Conclusion The potential has the radial variation shown in graph (C). Hence, the correct option is (C).

Quick Tip to Solve: Check three features first: negative potential, zero central slope, and a nonconstant interior potential for a solid uniform sphere.

8. A cylindrical block of wood (density = 650 kg m−3), of base area 30 cm2 and height 54 cm, floats in a liquid of density 900 kg m−3. The block is depressed slightly and then released. The time period of the resulting oscillations of the block would be equal to that of a simple pendulum of length (nearly) [4 marks]

(A) 52 cm

(B) 26 cm

(C) 39 cm

(D) 65 cm

Correct Answer: (C) 39 cm

Solution:

Concept:

- A floating body at equilibrium displaces liquid whose weight equals its own weight. A small additional immersion produces an additional upward buoyant force.
- For vertical displacement x, the restoring force is $F = -\rho_l A g x$. Comparing with $F = -kx$ gives an effective spring constant $k = \rho_l A g$.

Step 1: Find the mass and restoring force Let the block density be $\rho_b = 650\,\mathrm{kg\,m^{-3}}$, its height be $H = 0.54\,\mathrm{m}$, and its base area be A. Its mass is $m = \rho_b A H$. At equilibrium the upward buoyancy balances mg. If the block is pushed down through a small distance x, its immersed volume increases by Ax, so the extra buoyancy is $\rho_l g A x$, directed upward. The equilibrium forces therefore cancel and only this restoring increment remains. The motion is simple harmonic while the block remains partly immersed.

Step 2: Compare the two time periods The block has angular frequency $\omega^2 = \rho_l A g/m = \rho_l g/(\rho_b H)$, and therefore

$$T = 2\pi\sqrt{\frac{\rho_b H}{\rho_l g}}.$$

A simple pendulum of length ℓ has $T_p = 2\pi\sqrt{\ell/g}$. Equating these expressions gives

$$\ell = \frac{\rho_b H}{\rho_l} = \frac{650 \times 54}{900}\,\mathrm{cm} = 39\,\mathrm{cm}.$$

The base area cancels because both the inertia and the change in buoyancy are proportional to it. The choices 52 cm, 26 cm and 65 cm do not equal this density-weighted height.

Step 3: Conclusion The equivalent pendulum length is $39\,\mathrm{cm}$. Hence, the correct option is (C).

Quick Tip to Solve: For a floating uniform prism, cancel its base area immediately: the equivalent pendulum length is the equilibrium immersed depth, $\rho_b H/\rho_l$.

9. A beaker contains a fluid of density ρ kg/m^3, specific heat S J/kg°C and viscosity η. The beaker is filled up to height h. To estimate the rate of heat transfer per unit area ($\dot{Q}$/A) by convection when beaker is put on a hot plate, a student proposes that it should depend on η, (S$\Delta\theta$/h) and (1/ρg) when $\Delta\theta$ (in °C) is the difference in the temperature between the bottom and top of the fluid. In that situation the correct option for ($\dot{Q}$/A) is [4 marks]

(A) η(S$\Delta\theta$/h)

(B) η(S$\Delta\theta$/h)(1/ρg)

(C) (S$\Delta\theta$/ηh)(1/ρg)

(D) S$\Delta\theta$/(ηh)

Correct Answer: (A) η(S$\Delta\theta$/h)

Solution:

Concept:

- Heat-transfer rate per unit area has dimensions of power divided by area: $[\dot{Q}/A] = MT^{-3}$.
- Dynamic viscosity has dimensions $[\eta] = ML^{-1}T^{-1}$, while specific heat has $[S] = L^2T^{-2}\Theta^{-1}$, where Θ denotes temperature.

Step 1: Find the dimensions of the proposed factors The temperature difference $\Delta\theta$ has dimension Θ, and height has dimension L. Consequently,

$$\left[\frac{S\Delta\theta}{h}\right] = \frac{L^2T^{-2}\Theta^{-1}\Theta}{L} = LT^{-2}.$$

Multiplying this by viscosity gives

$$\left[\eta\frac{S\Delta\theta}{h}\right] = (ML^{-1}T^{-1})(LT^{-2}) = MT^{-3}.$$

This agrees exactly with heat flow per unit area, since $[\dot{Q}/A] = (ML^2T^{-3})/L^2$. No additional length or mass dimension is needed.

Step 2: Reject the other dimensional combinations The factor involving density and gravity has dimensions

$$\left[\frac{1}{\rho g}\right] = \frac{1}{(ML^{-3})(LT^{-2})} = M^{-1}L^2T^2.$$

Option (B) therefore has dimensions L^2T^{-1}, rather than MT^{-3}. Also,

$$\left[\frac{S\Delta\theta}{\eta h}\right] = M^{-1}L^2T^{-1},$$

so option (D) fails. Multiplying that expression by $1/(\rho g)$, as in option (C), gives $M^{-2}L^4T$, which fails as well. Dimensional matching determines which offered expression is admissible; it does not claim to calculate a dimensionless convection coefficient.

Step 3: Conclusion The dimensionally consistent expression is $\dot{Q}/A = \eta(S\Delta\theta/h)$, up to a dimensionless factor. Hence, the correct option is (A).

Quick Tip to Solve: Reduce heat flux to MT^{-3} first; temperature cancels between specific heat and the temperature difference.

10. An experiment takes 10 min to raise the temperature of water in a container from 0oC to 100oC and another 55 min to convert it totally into steam by a heater supplying heat at a uniform rate. Neglecting the specific heat of the container and taking specific heat of the water to be 1 cal (g◦C)-1 , the heat of vaporization according to this experiment will come out to be [4 marks]

(A) 530 cal g-1

(B) 550 cal g-1

(C) 540 cal g-1

(D) 560 cal g-1

Correct Answer: (B) 550 cal g-1

Solution:

Concept:

- Sensible heating requires $Q_1 = mc\Delta T$, where m is the water mass and c its specific heat.
- At constant heater power P, supplied heat is proportional to heating time: $Q = Pt$. Vaporization at the boiling point requires $Q_2 = mL_v$.

Step 1: Relate the heater power to the temperature rise The first stage heats liquid water from $0^\circ\mathrm{C}$ to $100^\circ\mathrm{C}$, so $\Delta T = 100^\circ\mathrm{C}$. With $c = 1\,\mathrm{cal\,g^{-1}\,^\circ C^{-1}}$, a mass of m grams needs

$$Q_1 = m \times 1 \times 100 = 100m\,\mathrm{cal}.$$

The time for this stage is 10 min. Since the container heat capacity is neglected, the heater power in calories per minute is $P = 100m/10 = 10m$. The unknown water mass does not need to be supplied because it will cancel in the ratio.

Step 2: Use the vaporization interval The second stage lasts another 55 min. This interval is used entirely to change water at its boiling point into steam, rather than to raise its temperature. Thus

$$Q_2 = P(55) = 550m\,\mathrm{cal} = mL_v.$$

Dividing by the water mass gives

$$L_v = 550\,\mathrm{cal\,g^{-1}}.$$

Equivalently, $L_v = c\Delta T(t_2/t_1) = 1 \times 100 \times 55/10$. Options 530, 540 and 560 do not follow from the measured time ratio. Although a tabulated latent heat may be near one of those values, the question asks for the value obtained in this experiment under its stated assumptions.

Step 3: Conclusion The experimental heat of vaporization is $550\,\mathrm{cal\,g^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: With the same heater, compare the two intervals: $L_v/(c\Delta T) = t_{\mathrm{boil}}/t_{\mathrm{warm}}$, so mass and power both cancel.

11. Using equipartition of energy, the specific heat (in $J\,kg^{-1}K^{-1}$) of aluminium at room temperature can be estimated to be (atomic weight of aluminium = 27) [4 marks]

(A) 25

(B) 1850

(C) 410

(D) 925

Correct Answer: (D) 925

Solution:

Concept:

- An atom in a classical solid oscillates in three independent spatial directions. Each direction contributes a kinetic and a potential quadratic energy term.
- Equipartition gives $k_BT/2$ for each quadratic term, so the internal energy is $3k_BT$ per atom and $3RT$ per mole.

Step 1: Count the energy contributions correctly For each atom, three translational displacement coordinates describe the vibrations around its equilibrium site. Each coordinate has both kinetic energy and elastic potential energy. The six quadratic terms therefore contribute

$$u = 6\left(\frac{1}{2}k_BT\right) = 3k_BT.$$

For one mole, $U_m = 3N_Ak_BT = 3RT$. Differentiating with respect to temperature gives the molar heat capacity $C_m = 3R$. Counting only kinetic terms would incorrectly halve the result; conversely, counting each quadratic term as k_BT would incorrectly double it.

Step 2: Convert molar heat capacity to specific heat The atomic weight 27 corresponds to a molar mass of $27\,\mathrm{g\,mol^{-1}} = 0.027\,\mathrm{kg\,mol^{-1}}$. Hence the specific heat per unit mass is

$$c = \frac{3R}{0.027} = \frac{3 \times 8.314}{0.027}\,\mathrm{J\,kg^{-1}K^{-1}} \simeq 924\,\mathrm{J\,kg^{-1}K^{-1}}.$$

The nearest offered value is 925. The value 25 is approximately the molar heat capacity in $\mathrm{J\,mol^{-1}K^{-1}}$, not the requested mass-specific value. The value 1850 double-counts the energy, while 410 is not obtained from the classical three-dimensional solid model. This is the Dulong–Petit estimate at room temperature.

Step 3: Conclusion The estimated specific heat is about $925\,\mathrm{J\,kg^{-1}K^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a classical solid start with $3R$ per mole, then divide by molar mass expressed in kilograms per mole.

12. A pendulum with the time period of 1 s is losing energy due to damping. At a certain time, its energy is 45 J. If after completing 15 oscillations its energy has become 15 J, then its damping constant (in s^{-1}) will be [4 marks]

(A) 1/2

(B) (1/15) ln 3

(C) (1/30) ln 3

(D) 2

Correct Answer: (B) (1/15) ln 3

Solution:

Concept:

- For weak linear damping the oscillation energy decreases exponentially. Write $E(t) = E(0)e^{-\gamma t}$, where γ is the energy-decay constant.
- The logarithmic energy decrement gives $\gamma = \ln[E(0)/E(t)]/t$. The amplitude decays at half this rate because energy is proportional to amplitude squared.

Step 1: Determine the elapsed time and energy ratio The pendulum completes one oscillation in 1 s, so 15 complete oscillations take

$$t = 15 \times 1 = 15\,\text{s}.$$

During that interval the mechanical energy decreases from 45 J to 15 J. Thus the remaining fraction is $15/45 = 1/3$. The same equilibrium reference must be used for both energies. There is no need to know the pendulum mass, length or initial amplitude because only a ratio of its energies enters the exponential law.

Step 2: Extract the damping constant Using the energy-decay convention for the damping constant,

$$\frac{15}{45} = e^{-15\gamma}.$$

Taking natural logarithms yields

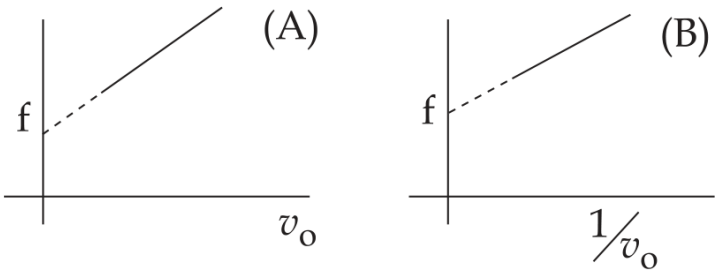
$$-\ln 3 = -15\gamma, \qquad \gamma = \frac{\ln 3}{15}\,\text{s}^{-1}.$$

The value $(\ln 3)/30$ would be the amplitude-decay constant, since $A(t)/A(0) = e^{-\gamma t/2}$; it is half the energy-decay constant used here. The constants $1/2$ and 2 would predict energy ratios $e^{-7.5}$ and e^{-30}, respectively, rather than the observed $1/3$. Keeping the decay convention explicit avoids a factor-of-two error when translating between amplitude and energy.

Step 3: Conclusion With $E(t) = E(0)e^{-\gamma t}$, the damping constant is $\gamma = (\ln 3)/15\,\text{s}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Always state whether the exponent describes amplitude or energy: the energy-decay rate is twice the amplitude-decay rate.

13. A source of sound emits sound waves at frequency f0. It is moving towards an observer with fixed speed vs (vs $<$ v, where v is the speed of sound in air). If the observer were to move towards the source with speed v0, one of the following two graphs (A and B) will give the correct variation of the frequency f heard by the observer as v0 is changed. The variation of f with v0 is given correctly by [4 marks]



(A) Graph A with slope =(f0)/((v−vs))

(B) Graph A with slope =(f0)/((v+vs))

(C) Graph B with slope =(f0)/((v−vs))

(D) Graph B with slope =(f0)/((v+vs))

Correct Answer: (A) Graph A with slope =(f0)/((v−vs))

Solution:

Concept:

- For a source approaching an observer through still air, the wavelength ahead of the source is $\lambda = (v - v_s)/f_0$.
- An observer moving toward the source meets wavefronts at relative speed $v + v_0$, so the received frequency is $f = (v + v_0)/\lambda$.

Step 1: Combine source and observer motions Over one source period $1/f_0$, a wavefront moves a distance v/f_0, while the source advances v_s/f_0 toward the observer. Their difference is the separation of successive forward wavefronts:

$$\lambda = \frac{v - v_s}{f_0}.$$

The approaching observer travels into these wavefronts, so their rate of arrival is

$$f = \frac{v + v_0}{\lambda} = f_0 \frac{v + v_0}{v - v_s}.$$

The sound speed and source speed are fixed in the question. Only the observer speed v_0 is varied, so the denominator remains constant.

Step 2: Read the slope and graph axes Write the frequency explicitly as a linear function:

$$f = \frac{f_0 v}{v - v_s} + \frac{f_0}{v - v_s} v_0.$$

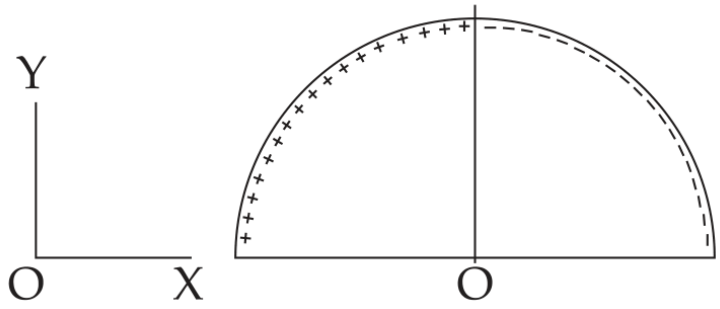
This is a straight line with positive intercept and positive slope $f_0/(v - v_s)$ when plotted against v_0, which is graph A. Graph B uses $1/v_0$ as its horizontal coordinate, so the same equation would contain the reciprocal of that coordinate and would not be the straight line drawn. The alternative

denominator $v + v_s$ applies to a receding source and is inconsistent with the stated approaching source. Both the axis label and the sign of the source correction are therefore decisive.

Step 3: Conclusion Graph A is correct, and its slope is $f_0/(v - v_s)$. Hence, the correct option is (A).

Quick Tip to Solve: First obtain f as a function of the variable on the horizontal axis; a graph versus $1/v_0$ is different from one versus v_0.

14. A wire, of length L (= 20 cm), is bent into a semi-circular arc. If the two equal halves of the arc, were each to be uniformly charged with charges ±Q, [|Q| = 103 ε0 Coulomb where ε0 is the permittivity (in SI units) of free space] the net electric field at the centre O of the semi-circular arc would be: [4 marks]



(A) (25 × 103) ^ i N C−1

(B) (50 × 103) ^ i N C−1

(C) (25 × 103) ^ j N C−1

(D) (50 × 103) ^ j N C−1

Correct Answer: (A) (25 × 103) ^ i N C−1

Solution:

Concept:

- At the centre of a circular arc, the field from a positive charge element points away from that element, while the field from a negative element points toward it.
- Use $dE = k\,dq/R^2$, where $k = 1/(4\pi\varepsilon_0)$, and resolve the elemental fields into horizontal and vertical components.

Step 1: Use symmetry and the charge density The semicircle has radius $R = L/\pi$. Its left quarter carries $+Q$, and its right quarter carries $-Q$, as shown in the figure. Each quarter has length $\pi R/2$, so the magnitude of its linear charge density is

$$\lambda = \frac{2|Q|}{\pi R}.$$

For mirror-image elements, the vertical fields cancel: the positive element produces a downward component, while the negative one produces an equal upward component. The horizontal components both point toward positive x. Thus the net field is along $\hat{\mathbf{i}}$, ruling out the two options along $\hat{\mathbf{j}}$.

Step 2: Integrate the horizontal components Parameterize the positive left quarter by polar angle θ, from $\pi/2$ to π. Since $dq = \lambda R\, d\theta$, its horizontal field is

$$E_{x,+} = -\frac{k\lambda}{R}\int_{\pi/2}^{\pi} \cos\theta\, d\theta = \frac{k\lambda}{R}.$$

The negative right quarter contributes the same amount. Therefore

$$E_x = \frac{2k\lambda}{R} = \frac{|Q|}{\pi^2\varepsilon_0 R^2} = \frac{|Q|}{\varepsilon_0 L^2}.$$

Substituting $L = 0.20\,\mathrm{m}$ gives a numerical multiplier $1/L^2 = 25$. Accordingly the field is 25 times the printed factor $|Q|/\varepsilon_0$, directed along $\hat{\mathbf{i}}$. The factor 50 would double-count the two quarters after their contributions have already been added.

Step 3: Conclusion The net field is $\mathbf{E} = [25(|Q|/\varepsilon_0)]\hat{\mathbf{i}}\,\mathrm{N\,C^{-1}}$, namely the expression in option (A). Hence, the correct option is (A).

Quick Tip to Solve: For equal opposite charges on mirror-image quarter arcs, the vertical components cancel and the horizontal components reinforce; then replace πR by the total arc length.

15. An electric field E = (25 $\hat{\imath}$ + 30 $\hat{\jmath}$) N C−1 exists in a region of space. If the potential at the origin is taken to be zero then the potential at x = 2 m, y = 2 m is [4 marks]

(A) −110 J C−1

(B) −140 J C−1

(C) −130 J C−1

(D) −120 J C−1

Correct Answer: (A) −110 J C−1

Solution:

Concept:

- The potential difference is the negative line integral of the electric field: $V(P) - V(O) = -\int_O^P \mathbf{E}\cdot d\mathbf{r}$. The electric field points toward decreasing potential.
- For a spatially constant field, this integral is path independent and reduces to $-\mathbf{E}\cdot\Delta\mathbf{r}$. Each Cartesian component multiplies the displacement along its own axis.

Step 1: Identify the displacement The reference point is the origin, where the potential is zero. The destination has coordinates $x = 2\,\mathrm{m}$ and $y = 2\,\mathrm{m}$, so $\Delta\mathbf{r} = 2\hat{\imath} + 2\hat{\jmath}$ metres. No third component contributes because the given field has only the two indicated Cartesian components.

Step 2: Evaluate the scalar product The field is $\mathbf{E} = 25\hat{\imath} + 30\hat{\jmath}$ in $\mathrm{N\,C^{-1}}$. Since the unit vectors along different coordinate axes are perpendicular, the cross terms vanish. Hence

$$\mathbf{E}\cdot\Delta\mathbf{r} = 25(2) + 30(2) = 110\,\mathrm{J\,C^{-1}}.$$

Both coordinate displacements are positive, so both contributions have the same sign.

Step 3: Apply the sign of potential change Therefore

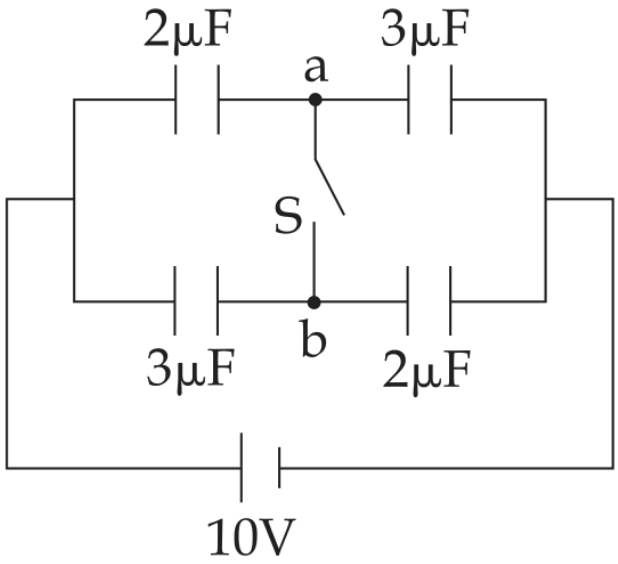
$$V(2,2) - 0 = -110\,\mathrm{J\,C^{-1}}.$$

The negative sign is essential: the displacement has a component along the field, so the potential decreases. The other three options have the correct sign but their magnitudes do not equal the sum of the two component contributions; none is obtained by the required dot product.

Step 4: Conclusion The potential is $-110\,\mathrm{J\,C^{-1}}$, equivalently $-110\,\mathrm{V}$. Hence, the correct option is (A).

Quick Tip to Solve: For a uniform field, calculate $-E_x x - E_y y$ directly after fixing the reference potential; do not replace the dot product by a product of magnitudes.

16. In the figure is shown a system of four capacitors connected across a 10 V battery. The charge that will flow from switch S when it is closed is: [4 marks]



(A) Zero

(B) 20 μC from a to b

(C) 5 μC from b to a

(D) 5 μC from a to b

Correct Answer: (C) 5 μC from b to a

Solution:

Concept:

- A capacitor plate connected to node a carries signed charge $C(V_a - V_{\text{other}})$. The algebraic sum of the charges on all plates at an isolated junction is conserved.
- When an ideal switch joins two junctions, they reach the same potential. The charge crossing the switch equals the change in net charge on either original junction, with its direction fixed by the

sign.

Step 1: Find the initial midpoint potentials Take the right battery terminal as zero and its left, positive terminal as $10\,\text{V}$. Before closing the switch, the two capacitors in each branch are in series. The top branch has equivalent capacitance $2 \times 3/(2+3) = 1.2\,\mu\text{F}$, so each capacitor carries magnitude $12\,\mu\text{C}$. The drop across its left $2\,\mu\text{F}$ capacitor is $6\,\text{V}$, giving $V_a = 4\,\text{V}$. The lower left $3\,\mu\text{F}$ capacitor has drop $4\,\text{V}$, giving $V_b = 6\,\text{V}$.

Step 2: Find the common final potential After closure, the left capacitors are parallel with combined capacitance $5\,\mu\text{F}$; the right capacitors likewise have $5\,\mu\text{F}$. These equal groups are in series across $10\,\text{V}$. Their common midpoint is consequently at $5\,\text{V}$. Initially the net charge at each separate floating midpoint was zero.

Step 3: Calculate charge transferred to node a The final signed charge on the plates originally attached to a is

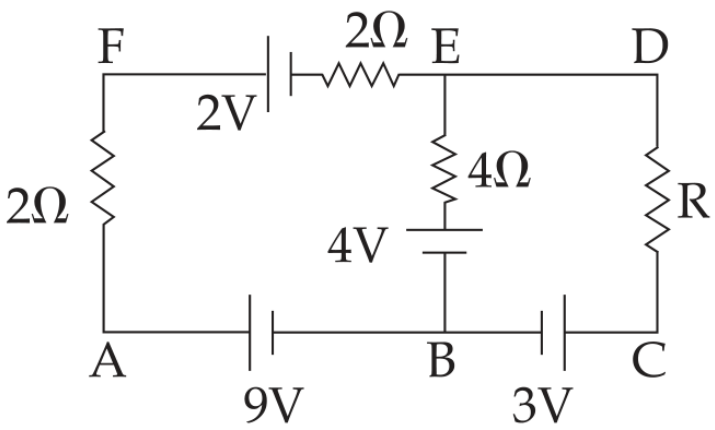
$$Q_a = 2(5-10) + 3(5-0) = 5\,\mu\text{C}.$$

Its increase from zero means positive charge has entered a through the switch from b. The corresponding charge change at b is $-5\,\mu\text{C}$, confirming conservation. Thus zero and $20\,\mu\text{C}$ are incorrect magnitudes, while the opposite direction reverses the physical charge change.

Step 4: Conclusion The charge through S is $5\,\mu\text{C}$ from b to a. Hence, the correct option is (C).

Quick Tip to Solve: Find signed plate charges at a midpoint before and after closure; adding capacitor charge magnitudes can give the wrong switch charge.

17. In the electric network shown, when no current flows through the 4 Ω resistor in the arm EB, the potential difference between the points A and D will be: [4 marks]



(A) 5 V

(B) 3 V

(C) 4 V

(D) 6 V

Correct Answer: (A) 5 V

Solution:

Concept:

- An ideal cell fixes the potential difference between its terminals: the long plate is positive. A resistor has zero voltage drop when the current through it is zero.
- Points connected by an ideal conducting wire have equal potentials. Thus a zero-current branch can determine a node potential without solving all other branch currents.

Step 1: Choose the reference potential Set $V_B = 0$. The $9\,\mathrm{V}$ battery on AB has its long plate on the A side. It therefore makes A positive relative to B, giving

$$V_A - V_B = 9\,\mathrm{V}, \qquad V_A = 9\,\mathrm{V}.$$

This statement follows from the cell polarity and does not require the currents in either outer arm.

Step 2: Use the zero-current condition in EB The vertical branch EB contains the $4\,\Omega$ resistor and the $4\,\mathrm{V}$ cell. The long plate of that cell is its upper plate, toward E. Since no current flows through the resistor, its voltage drop IR is zero. Consequently

$$V_E - V_B = 4\,\mathrm{V}, \qquad V_E = 4\,\mathrm{V}.$$

The $4\,\Omega$ value must not be multiplied by a current from a different branch.

Step 3: Compare A and D E and D are joined directly by an ideal wire, so $V_D = V_E = 4\,\mathrm{V}$. Hence

$$V_A - V_D = 9 - 4 = 5\,\mathrm{V}.$$

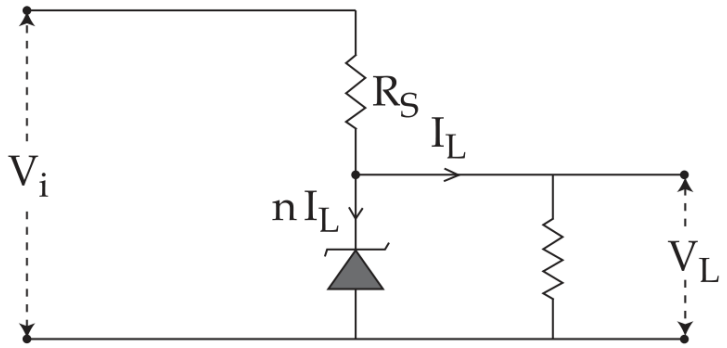
The unknown resistance R and the other cells affect currents needed to maintain the stated balance, but cannot change these node constraints. The alternatives 3, 4, and $6\,\mathrm{V}$ do not equal the difference between the established potentials of A and D.

Step 4: Conclusion A is $5\,\mathrm{V}$ above D, so the required potential difference is $5\,\mathrm{V}$. Hence, the correct option is (A).

Quick Tip to Solve: At balance, first replace the zero-current resistor by a zero voltage drop, then follow battery polarities between the requested nodes.

18. The value of the resistor, RS, needed in the DC voltage regulator circuit shown here, equals:

[4 marks]



(A) ((Vi+VL))/((n+1)IL)

(B) ((Vi−VL))/(nIL)

(C) ((Vi+VL))/(nIL)

(D) ((Vi−VL))/((n+1)IL)

Correct Answer: (D) ((Vi−VL))/((n+1)IL)

Solution:

Concept:

- In a shunt voltage regulator the load and Zener diode are connected across the same output voltage V_L. The series resistor drops the difference between input and output voltages.
- Kirchhoff's current law at the output junction gives $I_S = I_L + I_Z$. Ohm's law then gives $R_S = (V_i - V_L)/I_S$, with all currents in the directions shown.

Step 1: Read the branch currents from the circuit The figure labels the load current as I_L, directed toward the load, and the downward Zener current as nI_L. These are separate currents leaving the junction at the lower end of the series resistor. Thus the current entering that junction through R_S is

$$I_S = I_L + nI_L = (n+1)I_L.$$

Using only the Zener current would omit the current which supplies the load.

Step 2: Determine the resistor voltage The input upper rail is at V_i relative to the common lower rail. The output upper rail, at the lower end of R_S, is at V_L relative to the same rail. Accordingly the voltage drop in the direction of series current is

$$V_{R_S} = V_i - V_L.$$

The two labelled voltages share a reference, so their difference, rather than their sum, appears across the resistor.

Step 3: Apply Ohm's law and compare options Combining the voltage and current expressions yields

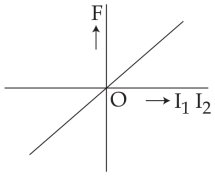
$$R_S = \frac{V_{R_S}}{I_S} = \frac{V_i - V_L}{(n+1)I_L}.$$

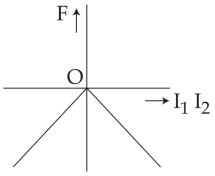
The dimension is volt per ampere, hence ohm. Options A and C incorrectly add input and output voltages. Option B has the correct voltage difference but neglects the load branch in its denominator. Only the remaining expression accounts for both current paths and the correct voltage drop.

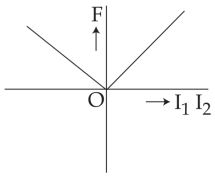
Step 4: Conclusion The required series resistance is $R_S = \dfrac{V_i - V_L}{(n+1)I_L}$. Hence, the correct option is (D).

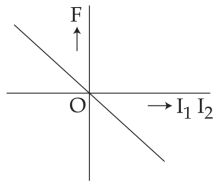
Quick Tip to Solve: For a shunt regulator, add the load and Zener currents before dividing the input-to-output voltage drop by the series current.

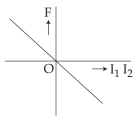
19. Two long straight parallel wires, carrying (adjustable) currents I1 and I2, are kept at a distance d apart. If the force F between the two wires is taken as 'positive' when the wires repel each other and 'negative' when the wires attract each other, the graph showing the dependence of F, on the product I1I2, would be [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- The force magnitude on length ℓ of either long parallel wire is $\mu_0 \ell |I_1 I_2|/(2\pi d)$. It vanishes if either current is zero.
- Currents in the same direction attract; currents in opposite directions repel. Treating the two currents as signed along a common reference direction makes their product distinguish these cases.

Step 1: Fix the sign for positive current product Let $x = I_1 I_2$, with the signed currents measured along the same reference direction. For $x > 0$, the currents are parallel and flow in the same direction. Their interaction is attractive. Because the question defines attractive force as negative,

$$F = -\frac{\mu_0 \ell}{2\pi d}\, x \qquad (x > 0).$$

Thus the graph must lie below the horizontal axis on its positive-product side.

Step 2: Check the negative-product side For $x < 0$, the currents have opposite signs and therefore opposite directions. The interaction is repulsive, which the question defines as positive. The same

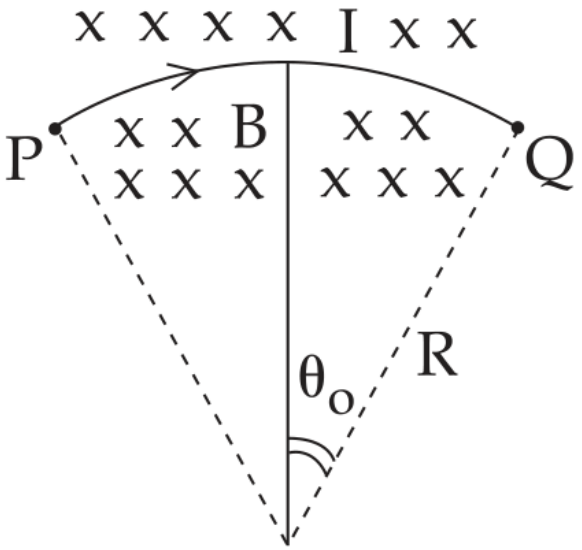
linear relation gives a positive result because its coefficient is negative and x is negative. At $x = 0$, at least one current is zero and the magnetic interaction is zero. The graph must pass continuously through the origin.

Step 3: Select the graph with the correct slope The factor $\mu_0\ell/(2\pi d)$ is a positive constant for the fixed geometry. Therefore the entire relation is one straight line with negative slope, extending from the second quadrant to the fourth quadrant. A positive-slope line reverses attraction and repulsion. Either V-shaped graph assigns the same force sign to both product signs and is also incompatible with the physical distinction. The fourth supplied graph has the required negative slope.

Step 4: Conclusion The force obeys $F = -\dfrac{\mu_0\ell}{2\pi d} I_1 I_2$, so the fourth graph is correct. Hence, the correct option is (D).

Quick Tip to Solve: With a signed force convention, test the same-direction case first; a positive current product must represent attraction and therefore negative force here.

20. A wire carrying current I is tied between points P and Q and is in the shape of a circular arc of radius R due to a uniform magnetic field B (perpendicular to the plane of the paper, as shown in the figure) in the vicinity of the wire. If the wire subtends an angle 2θ0 at the centre of the circle (of which it forms an arch) then the tension in the wire is: [4 marks]



(A) IBR

(B) (IBR)/(sinθ0)

(C) (IBR)/(2sinθ0)

(D) (IBRθ0)/(sinθ0)

Correct Answer: (A) IBR

Solution:

Concept:

- A current element in a magnetic field experiences force $d\mathbf{F} = I\, d\boldsymbol{\ell} \times \mathbf{B}$. In this figure the magnetic field is perpendicular to the plane of the curved wire.
- For a small arc of a circle of radius R, the tangent turns through $d\phi$, its length is $R\, d\phi$, and the resultant of its endpoint tensions is $T\, d\phi$ to first order.

Step 1: Consider a small element of the circular wire Take a small arc subtending $d\phi$ at the centre. Its length is $d\ell = R\, d\phi$. Since the magnetic field is perpendicular to the wire element, the magnetic force magnitude is

$$dF_B = IB\, d\ell = IBR\, d\phi.$$

The current direction and inward field shown make this magnetic force radially outward for the pictured upper arc.

Step 2: Balance the radial force with tension The tensions on the two ends of the element have equal magnitude T and act along the local tangents. Their tangential components cancel. Their resultant points toward the centre and has magnitude

$$2T \sin(d\phi/2).$$

In the limit of a small element this equals $T\, d\phi$. Static equilibrium therefore requires $T\, d\phi = IBR\, d\phi$.

Step 3: Cancel the angle and check the alternatives Cancelling the common infinitesimal angle gives $T = IBR$. The result is local: it depends on the curvature radius, current and field, rather than the total angular span of the wire. As a whole-wire check, the magnetic resultant is $IB(2R\sin\theta_0)$, while the two endpoint tensions provide $2T\sin\theta_0$; balancing them gives the same result. The other options retain an extra angular factor or a factor of two which cancels in the correct balance.

Step 4: Conclusion The tension in the wire is $T = IBR$. Hence, the correct option is (A).

Quick Tip to Solve: Balance forces on a small circular element: magnetic force per unit length is IB and the inward force per unit length due to tension is T/R.

21. A short bar magnet is placed in the magnetic meridian of the earth with North Pole pointing north. Neutral points are found at a distance of 30 cm from the magnet on the East-West line, drawn through the middle point of the magnet. The magnetic moment of the magnet in Am2 is close to: (Given (μ0)/(4π)= 10−7 in SI units and BH= Horizontal component of earth's magnetic field = 3.6 × 10−5 Tesla.) [4 marks]

(A) 4.9

(B) 14.6

(C) 19.4

(D) 9.7

Correct Answer: (D) 9.7

Solution:

Concept:

- For a short bar magnet of moment M, the equatorial magnetic field has magnitude $B_{\text{eq}} = \mu_0 M/(4\pi r^3)$, directed opposite to the magnetic moment.
- A neutral point occurs where the magnet's field cancels the Earth's horizontal field. The East-West line through the centre is the magnet's equatorial line when its north pole points north.

Step 1: Choose the appropriate dipole field The magnet lies in the magnetic meridian with its north pole pointing north. Its magnetic moment is northward, in the direction of the Earth's horizontal field. At an equatorial point on the East-West line, however, the magnet's field is southward. Cancellation is therefore possible there. The equatorial formula applies; using the axial formula would incorrectly introduce an extra factor of two.

Step 2: Set the field magnitudes equal At either stated neutral point,

$$\frac{\mu_0}{4\pi}\frac{M}{r^3} = B_H.$$

First convert the distance before substitution: $r = 30\,\text{cm} = 0.30\,\text{m}$. With $\mu_0/(4\pi) = 10^{-7}$ and $B_H = 3.6 \times 10^{-5}\,\text{T}$, the required moment is

$$M = \frac{(3.6 \times 10^{-5})(0.30)^3}{10^{-7}}.$$

Step 3: Evaluate and compare the results Since $(0.30)^3 = 0.027$,

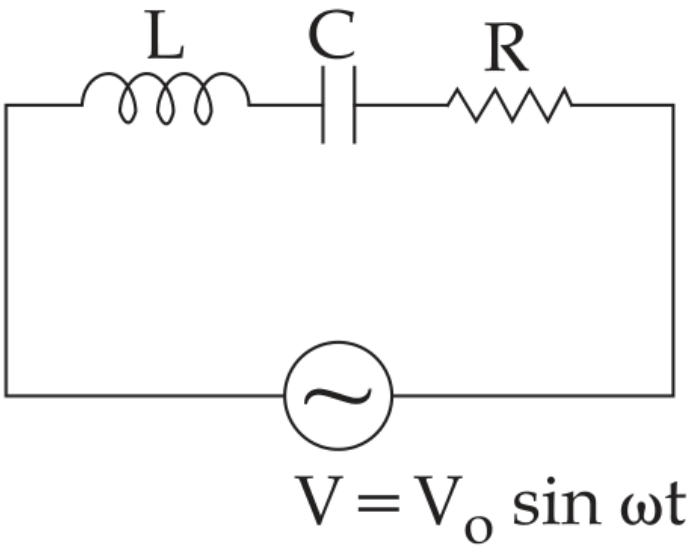
$$M = \frac{9.72 \times 10^{-7}}{10^{-7}} = 9.72\,\text{A m}^2.$$

This is closest to $9.7\,\text{A m}^2$. A value near 4.9 arises from mistakenly using the axial factor of two. The values 14.6 and 19.4 do not satisfy the equatorial cancellation equation; the latter is approximately twice the required moment. The SI unit is ampere square metre, as required for a magnetic dipole moment.

Step 4: Conclusion The magnetic moment is approximately $9.7\,\text{A m}^2$. Hence, the correct option is (D).

Quick Tip to Solve: Identify whether the neutral point is axial or equatorial before inserting the dipole-field formula, and cube the distance only after converting it to metres.

22. For the LCR circuit, shown here, the current is observed to lead the applied voltage. An additional capacitor C′ , when joined with the capacitor C present in the circuit, makes the power factor of the circuit unity. The capacitor C′ , must have been connected in: [4 marks]



(A) Parallel with C and has a magnitude $\frac{1-\omega^2 LC}{\omega^2 L}$

(B) Series with C and has a magnitude $\frac{1-\omega^2 LC}{\omega^2 L}$

(C) Series with C and has a magnitude $\frac{C}{\omega^2 LC-1}$

(D) Parallel with C and has a magnitude $\frac{C}{\omega^2 LC-1}$

Correct Answer: (A) Parallel with C and has a magnitude $\frac{1-\omega^2 LC}{\omega^2 L}$

Solution:

Concept:

- For a series LCR circuit, the impedance is $Z = R + \mathrm{i}(\omega L - 1/(\omega C))$. The sign of the net reactance decides whether the current leads or lags the voltage.
- Unity power factor means zero net reactance: $\omega L = 1/(\omega C_{\text{eq}})$. Capacitors in parallel add their capacitances, whereas a series combination has a capacitance smaller than either constituent.

Step 1: Use the observed phase relation The current leads the applied voltage, so the circuit is capacitive. Consequently, the magnitude of capacitive reactance exceeds the inductive reactance:

$$\frac{1}{\omega C} > \omega L \quad \Longrightarrow \quad \omega^2 LC < 1.$$

To remove this excess capacitive reactance, the effective capacitance must increase, since capacitive reactance varies inversely with capacitance. Adding a capacitor in series would decrease the capacitance and increase the unwanted reactance.

Step 2: Calculate the parallel capacitance With the extra capacitor connected in parallel, $C_{\text{eq}} = C + C'$.

The resonance condition gives

$$\omega L = \frac{1}{\omega(C + C')}, \qquad C + C' = \frac{1}{\omega^2 L}.$$

Therefore

$$C' = \frac{1}{\omega^2 L} - C = \frac{1 - \omega^2 LC}{\omega^2 L}.$$

The numerator is positive by the observed leading current, as required for an ordinary capacitor. Option (B) gives the correct algebraic magnitude but the wrong connection. Options (C) and (D) contain a negative denominator for this circuit, and hence cannot describe the required positive capacitance.

Step 3: Conclusion The added capacitor is in parallel with C, with $C' = (1 - \omega^2 LC)/(\omega^2 L)$. Hence, the correct option is (A).

Quick Tip to Solve: Leading current means excess capacitive reactance; increase the capacitance by adding a parallel capacitor to reach resonance.

23. For plane electromagnetic waves propagating in the z direction, which one of the following combination gives the correct possible direction for $\vec{E}$ and $\vec{B}$ field respectively ? [4 marks]

(A) $\left(\hat{i} + 2\hat{j}\right)$ and $\left(2\hat{i} - \hat{j}\right)$

(B) $\left(-2\hat{i} - 3\hat{j}\right)$ and $\left(3\hat{i} - 2\hat{j}\right)$

(C) $\left(2\hat{i} + 3\hat{j}\right)$ and $\left(\hat{i} + 2\hat{j}\right)$

(D) $\left(3\hat{i} + 4\hat{j}\right)$ and $\left(4\hat{i} - 3\hat{j}\right)$

Correct Answer: (B) $\left(-2\hat{i} - 3\hat{j}\right)$ and $\left(3\hat{i} - 2\hat{j}\right)$

Solution:

Concept:

- In a plane electromagnetic wave, **E** and **B** are mutually perpendicular and each is perpendicular to the propagation direction.
- The propagation direction is that of the Poynting vector, $\mathbf{S} \propto \mathbf{E} \times \mathbf{B}$. For transverse vectors, the sign of $E_x B_y - E_y B_x$ distinguishes positive and negative z.

Step 1: Check the perpendicularity condition Both vectors in every option lie in the xy-plane, so they are transverse to z. We must still test their mutual perpendicularity. In option (B), the electric direction is $-2\hat{\mathrm{i}} - 3\hat{\mathrm{j}}$ and the magnetic direction is $3\hat{\mathrm{i}} - 2\hat{\mathrm{j}}$. Their scalar product is

$$(-2)(3) + (-3)(-2) = -6 + 6 = 0.$$

Thus they satisfy the necessary perpendicularity condition. The magnitudes of the direction vectors need not equal the actual electric and magnetic field magnitudes; their role here is to specify orientation.

Step 2: Use the cross product to choose the positive direction For option (B),

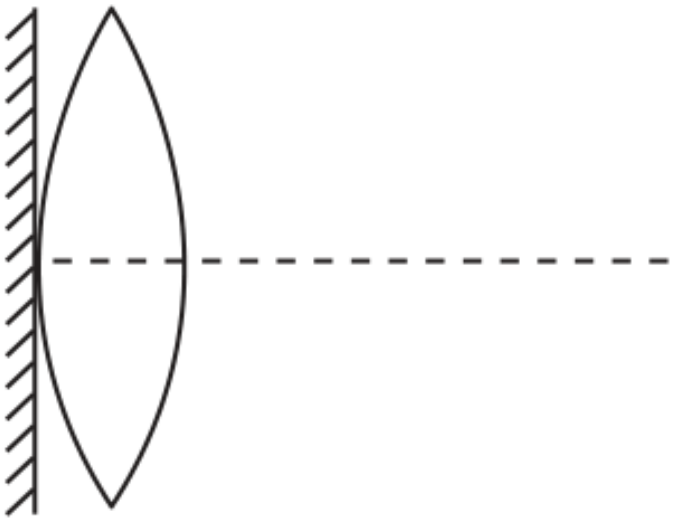
$$\mathbf{E} \times \mathbf{B} \propto [(-2)(-2) - (-3)(3)]\hat{\mathrm{k}} = 13\hat{\mathrm{k}},$$

which points along positive z. In (A), the scalar product is zero, but the cross-product coefficient is $1(-1) - 2(2) = -5$, giving negative z. In (D), it is likewise $3(-3) - 4(4) = -25$, again negative z. Option (C) fails even the perpendicularity test because its scalar product is $2(1) + 3(2) = 8 \neq 0$. Hence checking perpendicularity alone would leave incorrect alternatives; the right-hand rule is essential.

Step 3: Conclusion The pair in option (B) is perpendicular and has $\mathbf{E} \times \mathbf{B}$ along positive z. Hence, the correct option is (B).

Quick Tip to Solve: For transverse vectors, calculate both the dot product and the signed two-dimensional determinant; zero dot product alone does not fix propagation sense.

24. A thin convex lens of focal length 'f' is put on a plane mirror as shown in the figure. When an object is kept at a distance 'a' from the lens - mirror combination, its image is formed at a distance (a)/(3) in front of the combination. The value of 'a' is : [4 marks]



(A) f

(B) 2f

(C) 3f

(D) (3)/(2)f

Correct Answer: (B) 2f

Solution:

Concept:

- A thin convex lens placed immediately against a plane mirror is traversed twice. Its round-trip converging power is twice the lens power, so the combination behaves as a concave mirror of focal length $F = f/2$.
- For a real object and a real reflected image in front of the combination, positive distance magnitudes obey $1/u + 1/v = 1/F$. The stated image distance is $v = a/3$, while $u = a$.

Step 1: Account for the double passage through the lens On its way to the plane mirror, a paraxial ray is refracted by the convex lens. After reflection it passes through the same lens again and acquires a second equal converging deviation. The plane mirror contributes no optical power, and the lens and mirror are shown in contact, so no finite propagation separation needs to be included. The equivalent reflecting system therefore has

$$\frac{1}{F} = \frac{2}{f}, \qquad F = \frac{f}{2}.$$

This effective focal length must not be confused with the focal length f of the lens alone; using f directly would miss the second refraction.

Step 2: Apply the conjugate-distance relation The object is at distance a and its real image is at distance $a/3$ in front of the combination. Substitution in the equivalent mirror relation gives

$$\frac{1}{a} + \frac{1}{a/3} = \frac{2}{f}.$$

Thus

$$\frac{4}{a} = \frac{2}{f} \quad \Longrightarrow \quad a = 2f.$$

Check this explicitly: when $u = 2f$, the equation gives $1/v = 2/f - 1/(2f) = 3/(2f)$, hence $v = 2f/3 = u/3$, precisely the required image location. The alternatives f, $3f$ and $3f/2$ do not satisfy this relation; their predicted image-to-object ratios differ from one-third.

Step 3: Conclusion The required object distance is $a = 2f$. Hence, the correct option is (B).

Quick Tip to Solve: For a lens touching a plane mirror, count two refractions: use the effective reflecting focal length $f/2$, then apply the object-image relation.

25. In a Young's double slit experiment with light of wavelength λ, the separation of slits is d and distance of screen is D such that D $\gg$ d $\gg \lambda$. If the Fringe width is β, the distance from point of maximum intensity to the point where intensity falls to half of the maximum intensity on either side is [4 marks]

(A) $(\beta)/(4)$

(B) $(\beta)/(3)$

(C) $(\beta)/(6)$

(D) $(\beta)/(2)$

Correct Answer: (A) $(\beta)/(4)$

Solution:

Concept:

- For equal coherent slit contributions, the resultant intensity is $I = I_{\max}\cos^2(\delta/2)$, where δ is their phase difference.
- The fringe width is $\beta = \lambda D/d$. A displacement x from a bright fringe changes the phase by $2\pi x/\beta$, so the local intensity is $I_{\max}\cos^2(\pi x/\beta)$.

Step 1: Express the intensity relative to a maximum At any bright fringe the two contributions arrive in phase, so the phase difference is an integer multiple of 2π. Measuring x from that maximum absorbs this integer phase and gives

$$\frac{I(x)}{I_{\max}} = \cos^2\left(\frac{\pi x}{\beta}\right).$$

The approximations $D \gg d \gg \lambda$ justify the small-angle path-difference expression dx/D and therefore the constant fringe spacing used here. The requested distance is to the nearest point on either side where this ratio is one-half, rather than to the next minimum.

Step 2: Solve for the nearest half-intensity points Set

$$\cos^2\left(\frac{\pi x}{\beta}\right) = \frac{1}{2}.$$

The smallest nonzero magnitude of the argument is $\pi/4$, giving

$$\frac{\pi|x|}{\beta} = \frac{\pi}{4} \quad \Longrightarrow \quad |x| = \frac{\beta}{4}.$$

The two points are symmetrically situated at $x = \pm\beta/4$. As a check on the remaining options, $x = \beta/3$ gives $I/I_{\max} = 1/4$, $x = \beta/6$ gives $3/4$, and $x = \beta/2$ gives zero intensity. None is the requested half-maximum point. The separation between the two half-maximum points would be $\beta/2$, but the question asks the distance from the maximum to either point.

Step 3: Conclusion The distance on each side of a maximum is $\beta/4$. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish the distance from the bright centre to one half-maximum point, $\beta/4$, from the full half-maximum width, $\beta/2$.

26. Unpolarized light of intensity I0 is incident on surface of a block of glass at Brewster's angle. In that case, which one of the following statements is true? [4 marks]

(A) transmitted light is partially polarized with intensity (I0)/(2)

(B) transmitted light is completely polarized with intensity less than (I0)/(2)

(C) reflected light is completely polarized with intensity less than (I0)/(2)

(D) reflected light is partially polarized with intensity (I0)/(2)

Correct Answer: (C) reflected light is completely polarized with intensity less than (I0)/(2)

Solution:

Concept:

- At Brewster incidence, the reflection coefficient for the electric component parallel to the plane of incidence is zero. The reflected beam is therefore completely plane-polarized perpendicular to that plane.
- Unpolarized incident light has equal mean intensities $I_0/2$ in the two perpendicular polarization components. Only a fraction of the perpendicular component is reflected at an ordinary air-glass interface.

Step 1: Identify the polarization of the reflected beam Resolve the incident unpolarized light into electric components perpendicular and parallel to the plane of incidence. Each contributes an average intensity $I_0/2$. At Brewster angle, $\tan i_B = n_{\text{glass}}/n_{\text{air}}$, and the parallel component is transmitted without reflection. Consequently the reflected light contains only the perpendicular electric component and is completely polarized, not partially polarized. This rules out option (D), and also shows why the transmitted beam generally retains both polarization components rather than becoming completely polarized.

Step 2: Compare its intensity with the incident components Let R_s be the intensity reflection coefficient for the perpendicular component. Then

$$I_{\text{reflected}} = R_s \frac{I_0}{2}.$$

At an ordinary glass interface, $0 < R_s < 1$, since part of this component also passes into the glass. Thus

$$I_{\text{reflected}} < \frac{I_0}{2}.$$

The transmitted beam receives the full parallel component together with the transmitted part of the perpendicular component; it is partially polarized, with intensity greater than $I_0/2$ for a transparent block. Therefore option (A) has the wrong intensity, option (B) has both the wrong polarization and intensity, and option (D) has the wrong reflected polarization. Option (C) gives both required properties.

Step 3: Conclusion The reflected light is completely polarized and has intensity less than $I_0/2$. Hence, the correct option is (C).

Quick Tip to Solve: At Brewster angle the parallel component does not reflect; the reflected intensity is only a fraction of the incident perpendicular half.

27. The de-Broglie wavelength associated with the electron in the n = 4 level is [4 marks]

(A) Half of the de-Broglie wavelength of the electron in the ground state

(B) Four times the de-Broglie wavelength of the electron in the ground state

(C) (1)/(4)th of the de-Broglie wavelength of the electron in the ground state

(D) Two times the de-Broglie wavelength of the electron in the ground state

Correct Answer: (B) Four times the de-Broglie wavelength of the electron in the ground state

Solution:

Concept:

- The de Broglie wavelength of an electron is $\lambda = h/p = h/(m_e v)$. A larger orbital speed therefore corresponds to a smaller wavelength.
- In the Bohr model for the same atom, $r_n \propto n^2$ and $v_n \propto 1/n$. Equivalently, the standing-wave condition is $2\pi r_n = n\lambda_n$.

Step 1: Obtain the dependence on the orbit number The Coulomb force supplies the centripetal force and the angular momentum is quantized:

$$\frac{m_e v_n^2}{r_n} = \frac{Ze^2}{4\pi\varepsilon_0 r_n^2}, \qquad m_e v_n r_n = n\hbar.$$

Eliminating r_n gives

$$v_n = \frac{Ze^2}{4\pi\varepsilon_0\hbar}\frac{1}{n}.$$

For the same atom, all factors except n are fixed, so $v_4 = v_1/4$. Although the fourth orbit is larger, the wavelength is controlled by the momentum, not by the radius directly.

Step 2: Compare the electron wavelengths Use the inverse momentum dependence:

$$\frac{\lambda_4}{\lambda_1} = \frac{h/(m_e v_4)}{h/(m_e v_1)} = \frac{v_1}{v_4} = 4.$$

A second check follows from the standing-wave condition: $r_4 = 16r_1$, so $\lambda_4 = 2\pi(16r_1)/4 = 4(2\pi r_1) = 4\lambda_1$. Option (A) predicts a shorter wavelength despite the reduced momentum, and option (C) reverses the correct ratio. Option (D) gives a factor two, which would arise from the wrong square-root dependence on n; the Bohr speed varies as $1/n$, not $1/\sqrt{n}$.

Step 3: Conclusion The wavelength in the $n = 4$ orbit is four times the ground-state wavelength: $\lambda_4 = 4\lambda_1$. Hence, the correct option is (B).

Quick Tip to Solve: For a fixed Bohr atom, remember $r_n \propto n^2$, $v_n \propto n^{-1}$, and hence $\lambda_n \propto n$.

28. Let N_β be the number of β particles emitted by 1 gram of Na^{24} radioactive nuclei (half life = 15 hrs) in 7.5 hours, N_β is close to (Avogadro number = 6.023×10^{23} /g. mole): [4 marks]

(A) 1.75×10^{22}

(B) 6.2×10^{21}

(C) 7.5×10^{21}

(D) 1.25×10^{22}

Correct Answer: (C) 7.5×10^{21}

Solution:

Concept:

- Radioactive nuclei obey $N(t) = N_0 e^{-\lambda t} = N_0\, 2^{-t/T_{1/2}}$, with decay constant $\lambda = \ln 2/T_{1/2}$.
- The number of emitted beta particles equals the number of parent nuclei that decay when one beta particle is released per decay: $N_\beta = N_0 - N(t)$. The initial number of nuclei in mass m is $(m/M)N_A$.

Step 1: Count the initial sodium nuclei For $1\,\mathrm{g}$ of sodium-24, take its molar mass as $24\,\mathrm{g\,mol^{-1}}$. The number of parent nuclei is

$$N_0 = \frac{1}{24}(6.023 \times 10^{23}) = 2.5096 \times 10^{22}.$$

The interval 7.5 hours is half of the stated half-life of 15 hours. Therefore the surviving fraction is

$$\frac{N(t)}{N_0} = 2^{-7.5/15} = 2^{-1/2} = \frac{1}{\sqrt{2}} \simeq 0.7071.$$

Half a half-life does not mean that one quarter of the nuclei decay: radioactive decay is exponential rather than a constant-rate loss.

Step 2: Convert the decayed fraction into beta particles The fraction decayed is

$$1 - \frac{1}{\sqrt{2}} \simeq 0.2929.$$

Hence

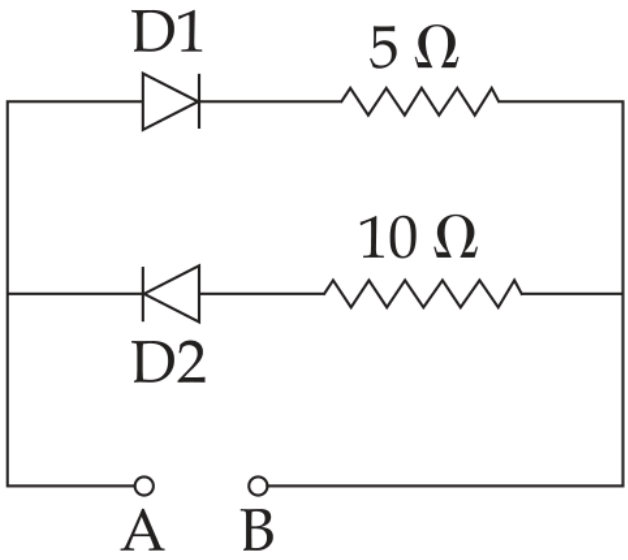
$$N_\beta = (2.5096 \times 10^{22})(0.2929) \simeq 7.35 \times 10^{21}.$$

This is closest to 7.5×10^{21}, the value in option (C). Option (A), 1.75×10^{22}, is close to the number of nuclei remaining rather than decayed. Option (D), 1.25×10^{22}, corresponds approximately to decay over a full half-life. Option (B), 6.2×10^{21}, is close to the erroneous linear estimate of one quarter of the initial nuclei. Counting emissions requires the decrease in parent nuclei, not the surviving population.

Step 3: Conclusion Approximately 7.35×10^{21} beta particles are emitted, closest to 7.5×10^{21}. Hence, the correct option is (C).

Quick Tip to Solve: Calculate the surviving fraction as $2^{-t/T_{1/2}}$, then subtract it from one before multiplying by the initial nuclei count.

29. A 2 V battery is connected across AB as shown in the figure. The value of the current supplied by the battery when in one case battery's positive terminal is connected to A and in other case when positive terminal of battery is connected to B will respectively be: [4 marks]



(A) 0.2 A and 0.1 A

(B) 0.1 A and 0.2 A

(C) 0.2 A and 0.4 A

(D) 0.4 A and 0.2 A

Correct Answer: (D) 0.4 A and 0.2 A

Solution:

Concept:

- An ideal forward-biased diode conducts with negligible resistance, whereas a reverse-biased diode blocks its branch. Conventional current leaves the positive battery terminal.
- For a conducting resistor branch, Ohm's law gives $I = V/R$. The two branches are parallel, so whichever branch conducts receives the full battery voltage of $2\,\mathrm{V}$.

Step 1: Connect the positive terminal to A A is the left-hand node and B is the right-hand node. With A positive, current would travel from left to right through the upper branch. Diode D1 is forward biased in that direction, while D2 is reverse biased and blocks the lower branch. Consequently only the upper resistor is included in the conducting path, giving

$$I_A = \frac{2}{5} = 0.4\,\mathrm{A}.$$

The lower resistor does not draw an additional current because its series diode is off.

Step 2: Reverse the battery polarity With B positive, current travels from the right-hand node towards the left-hand node. D2 now conducts in its forward direction through the lower branch, while D1

blocks the upper branch. The battery therefore supplies

$$I_B = \frac{2}{10} = 0.2\,\text{A}.$$

The requested order is A positive first and B positive second. Options A and B incorrectly halve the branch currents, while option C reverses the two correct values.

Step 3: Conclusion The battery currents are $0.4\,\text{A}$ and $0.2\,\text{A}$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Mark the positive node first, then keep only the branch whose diode permits current from that node to the negative node.

30. The AC voltage across a resistance can be measured using a [4 marks]

(A) Moving magnet galvanometer

(B) Moving coil galvanometer

(C) Hot wire voltmeter

(D) Potentiometer

Correct Answer: (C) Hot wire voltmeter

Solution:

Concept:

- An alternating voltage reverses polarity periodically. A measuring instrument must respond to an effect that remains nonzero after averaging over a complete cycle.
- Joule heating is proportional to I^2R, so it is independent of current direction. A hot-wire instrument uses this heating effect and can be calibrated to display an RMS voltage.

Step 1: Identify the effect that survives a polarity reversal Across a resistance, an AC voltage produces an alternating current. Although the mean current of a symmetric AC waveform is zero, its mean square is positive. Thus

$$P_{\text{av}} = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}.$$

A hot-wire voltmeter uses the expansion of a wire caused by this heating. The wire is heated during both half-cycles, allowing a steady reading related to the effective voltage.

Step 2: Eliminate instruments that cannot directly measure this AC voltage A conventional moving-coil galvanometer has torque proportional to the signed instantaneous current. Under rapidly reversing AC its average torque vanishes, so it does not directly give an AC-voltage reading. The ordinary moving-magnet galvanometer likewise responds to current direction and is not the required AC voltmeter. A standard potentiometer compares a steady potential difference by a DC null balance; without additional AC-specific arrangements it does not directly measure the given alternating voltage. The hot-wire instrument is therefore the suitable choice among the four listed devices.

Step 3: Conclusion A hot-wire voltmeter measures the AC voltage through its heating effect. Hence, the correct option is (C).

Quick Tip to Solve: For AC instruments, distinguish effects proportional to current from effects proportional to current squared; only the latter survive reversal automatically.

Chemistry (120 marks)

31. $\mathrm{A} + 2\,\mathrm{B} + 3\,\mathrm{C} \rightleftharpoons \mathrm{AB_2C_3}$ **Reaction of 6.0 g of A,** 6.0×10^{23} **atoms of B, and 0.036 mol of C yields 4.8 g of compound** $\mathrm{AB_2C_3}$**. If the atomic mass of A and C are 60 and 80 amu, respectively, the atomic mass of B is (Avogadro no. =** 6×10^{23}**):** [4 marks]

(A) 40 amu

(B) 60 amu

(C) 70 amu

(D) 50 amu

Correct Answer: (D) 50 amu

Solution:

Concept:

- The reaction consumes A, B and C in the mole ratio $1 : 2 : 3$. The limiting reagent has the smallest value of available moles divided by its stoichiometric coefficient.
- If the extent of reaction is ξ, the amount of product is ξ mole and its molar mass is $M_A + 2M_B + 3M_C$. Product mass divided by product moles determines that molar mass.

Step 1: Determine the limiting reagent The initial amount of A is

$$n_A = \frac{6.0}{60} = 0.100\,\mathrm{mol}.$$

Dividing the given number of B atoms by the given Avogadro number gives $n_B = 1.0\,\mathrm{mol}$. For C, $n_C = 0.036\,\mathrm{mol}$. The possible reaction extents are therefore

$$\frac{n_A}{1} = 0.100, \qquad \frac{n_B}{2} = 0.500, \qquad \frac{n_C}{3} = 0.012.$$

C is exhausted first, so the reaction produces $0.012\,\mathrm{mol}$ of the compound; A and B are both in excess.

Step 2: Use the product mass to find the unknown atomic mass The product molar mass obtained from its yield is

$$M = \frac{4.8}{0.012} = 400\,\mathrm{g\,mol^{-1}}.$$

Its composition requires

$$60 + 2M_B + 3(80) = 400.$$

Hence $2M_B = 100$, giving $M_B = 50\,\mathrm{g\,mol^{-1}}$. The corresponding atomic mass is 50 amu. As a check, the limiting amount of C contributes 2.88 g and the consumed A contributes 0.72 g, leaving 1.20 g of B in the 4.8 g product. Dividing that by the consumed 0.024 mol also gives 50. Values 40, 60 and 70 would instead predict product molar masses 380, 420 and 440, inconsistent with the yield.

Step 3: Conclusion The atomic mass of B is $50\,\mathrm{amu}$. Hence, the correct option is (D).

Quick Tip to Solve: Before using a product yield, compare moles divided by coefficients; the smallest ratio fixes the product amount.

32. When does a gas deviate the most from its ideal behavior? [4 marks]

(A) At low pressure and high temperature

(B) At high pressure and high temperature

(C) At low pressure and low temperature

(D) At high pressure and low temperature

Correct Answer: (D) At high pressure and low temperature

Solution:

Concept:

- The ideal-gas model neglects both intermolecular attractions and the finite volume occupied by gas molecules. Real gases depart from this model when either neglected effect becomes significant.
- In the van der Waals model, $\left(P + an^2/V^2\right)(V - nb) = nRT$. The attraction parameter a and excluded-volume parameter b describe the corrections.

Step 1: Examine the pressure dependence At high pressure, the molecules are packed into a smaller volume. Their own excluded volume is then appreciable compared with the container volume, so replacing the available volume by the entire container volume becomes inaccurate. Molecular separations are also reduced, making interactions more important. Low pressure does the opposite: molecules are far apart and their volume occupies only a small fraction of the gas volume, approaching the ideal limit.

Step 2: Examine the temperature dependence and combine the effects At low temperature, molecules have less average translational kinetic energy. Intermolecular attractions consequently have a greater influence relative to their thermal motion, increasing departure from ideal behaviour and potentially promoting condensation. High temperature reduces this relative influence. The combination of high pressure and low temperature therefore makes both neglected physical effects important. Option A, low pressure and high temperature, describes the closest approach to ideal behaviour. Options B and C each combine one condition that enhances nonideality with another that suppresses it, whereas option D combines both enhancing conditions.

Step 3: Conclusion A gas deviates most from ideal behaviour at high pressure and low temperature. Hence, the correct option is (D).

Quick Tip to Solve: Remember the ideal limit as hot and dilute; reverse both conditions to identify the strongest real-gas effects.

33. At temperature T, the average kinetic energy of any particle is (3)/(2)kT. The de Broglie wavelength follows the order: [4 marks]

(A) Visible photon > Thermal electron > Thermal neutron

(B) Thermal proton > Visible photon > Thermal electron

(C) Thermal proton > Thermal electron > Visible photon

(D) Visible photon > Thermal neutron > Thermal electron

Correct Answer: (A) Visible photon > Thermal electron > Thermal neutron

Solution:

Concept:

- For a nonrelativistic thermal particle, $K = p^2/(2m) = 3kT/2$. Its de Broglie wavelength is therefore $\lambda = h/\sqrt{3mkT}$.
- At the same temperature, the thermal wavelength decreases as the square root of mass increases. An electron is far lighter than a proton or neutron, while a visible photon has an optical wavelength.

Step 1: Compare the massive thermal particles For electrons and neutrons at the same temperature,

$$\frac{\lambda_e}{\lambda_n} = \sqrt{\frac{m_n}{m_e}} \approx \sqrt{1839} \approx 43.$$

Thus the thermal electron has a substantially longer wavelength than the thermal neutron. Protons and neutrons have nearly equal masses, so a thermal proton also has a much shorter wavelength than a thermal electron. This immediately rules out options B and C, which place the proton above the electron, and option D, which places the neutron above the electron.

Step 2: Compare with the optical scale Visible photons have wavelengths of roughly 400 to $700\,\text{nm}$. For thermal particles at ordinary temperatures, inserting $T \approx 300\,\text{K}$ gives

$$\lambda_e \approx 6.3\,\text{nm}, \qquad \lambda_n \approx 0.15\,\text{nm}.$$

The optical wavelength is therefore larger than either thermal wavelength on this usual thermal scale. Combining the optical comparison with the exact inverse-square-root mass dependence gives visible photon, thermal electron, thermal neutron in descending order. The thermal relation refers to massive particles; a photon itself satisfies $p = h/\lambda$ and has no rest mass to substitute into the nonrelativistic kinetic-energy expression.

Step 3: Conclusion The order is visible photon > thermal electron > thermal neutron. Hence, the correct option is (A).

Quick Tip to Solve: At a common thermal temperature, compare massive-particle wavelengths using inverse square root of mass, then compare with the optical wavelength scale.

34. Molecule AB has a bond length of 1.617 Å and a dipole moment of 0.38 D. The fractional charge on each atom(absolute magnitude) is: ($e_0 = 4.802 \times 10^{-10}$ esu) [4 marks]

(A) 0.5

(B) 0.05

(C) 0

(D) 1.0

Correct Answer: (B) 0.05

Solution:

Concept:

- For two equal and opposite charges of magnitude q separated by bond length r, the dipole moment is $\mu = qr$. The fractional charge in units of electronic charge is $f = q/e_0$.
- In electrostatic cgs units, $1\,\text{D} = 10^{-18}\,\text{esu cm}$ and $1\,\text{Å} = 10^{-8}\,\text{cm}$. Consistent units are essential before dividing the dipole moment by the bond length.

Step 1: Calculate the dipole moment for a full electronic charge The bond length is

$$r = 1.617 \times 10^{-8}\,\text{cm}.$$

If one full electronic charge were separated by this distance, the dipole moment would be

$$\mu_{\text{full}} = (4.802 \times 10^{-10})(1.617 \times 10^{-8}) = 7.764834 \times 10^{-18}\,\text{esu cm}.$$

Thus $\mu_{\text{full}} \approx 7.765\,\text{D}$. This is the reference value for a fractional charge of one; the observed dipole is much smaller.

Step 2: Divide the observed dipole by the full-charge dipole The fraction of the electronic charge is

$$f = \frac{\mu}{e_0 r} = \frac{0.38}{7.764834} \approx 0.04894 \approx 0.05.$$

Equivalently, the charge magnitude itself is approximately 2.35×10^{-11} esu, about five per cent of e_0. Option A, 0.5, would predict a dipole near 3.88 D; option D, 1.0, would predict about 7.76 D. Neither agrees with 0.38 D. Option C would give zero dipole, also inconsistent with the observation. The magnitude is requested, so there is no sign to attach to the fractional charge.

Step 3: Conclusion The fractional charge on each atom is approximately 0.05. Hence, the correct option is (B).

Quick Tip to Solve: Compute the full-charge dipole in Debye first; the measured dipole divided by that value directly gives the fractional ionic charge.

35. For the equilibrium, A(g) $\rightleftharpoons$ B(g), ΔH is -40 kJ/mol. If the ratio of the activation energies of the forward (Ef) and reverse (Eb) reactions is (2)/(3) then: [4 marks]

(A) Ef= 30 kJ / mol, Eb= 70 kJ / mol

(B) Ef= 70 kJ / mol, Eb= 30 kJ / mol

(C) Ef= 80 kJ / mol, Eb= 120 kJ / mol

(D) Ef = 60 kJ/mol ; Eb = 100 kJ/mol

Correct Answer: (C) Ef= 80 kJ / mol, Eb= 120 kJ / mol

Solution:

Concept:

- The forward and reverse reactions pass through the same transition state. Thus $E_f = E_{\text{TS}} - E_A$ and $E_b = E_{\text{TS}} - E_B$.
- Subtracting gives $E_f - E_b = E_B - E_A = \Delta H$. For an exothermic forward reaction, the product energy is lower, so the reverse activation barrier is larger.

Step 1: Translate the energy ratio into one variable The given ratio is $E_f/E_b = 2/3$. Write

$$E_f = 2x, \qquad E_b = 3x.$$

Since the forward reaction releases $40\,\text{kJ}\,\text{mol}^{-1}$, its products lie that much below its reactants. The difference between the barriers must therefore be

$$E_f - E_b = -40\,\text{kJ}\,\text{mol}^{-1}.$$

Substituting the ratio gives $2x - 3x = -40$, so $x = 40\,\text{kJ}\,\text{mol}^{-1}$.

Step 2: Calculate both barriers and test the alternatives The individual activation energies are consequently

$$E_f = 80\,\text{kJ}\,\text{mol}^{-1}, \qquad E_b = 120\,\text{kJ}\,\text{mol}^{-1}.$$

Their ratio is $80/120 = 2/3$, and their difference is $-40\,\text{kJ}\,\text{mol}^{-1}$, satisfying both independent conditions. Option A has the correct difference but ratio $3/7$, so it fails the ratio condition. Option B has a positive difference, corresponding to an endothermic forward reaction, and also the wrong ratio. Option D has the correct difference but ratio $3/5$. Only option C satisfies the sign, magnitude and ratio simultaneously.

Step 3: Conclusion $E_f = 80\,\text{kJ}\,\text{mol}^{-1}$ and $E_b = 120\,\text{kJ}\,\text{mol}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: Use both checks: the barrier difference equals the reaction enthalpy, while the barrier ratio fixes their relative sizes.

36. Determination of the molar mass of acetic acid in benzene using freezing point depression is affected by [4 marks]

(A) Complex formation

(B) Association

(C) Partial ionization

(D) Dissociation

Correct Answer: (B) Association

Solution:

Concept:

- Freezing-point depression is a colligative property: $\Delta T_f = iK_f m$, where m is the analytical molality, K_f the solvent constant and i the van't Hoff factor.
- Association joins solute molecules and reduces the number of independently moving solute particles; it therefore gives $i < 1$.

Step 1: Identify the molecular interaction Acetic acid contains both a hydrogen-bond donor, the carboxylic O–H group, and a hydrogen-bond acceptor, the carbonyl oxygen. In nonpolar benzene, two acetic-acid molecules associate through two intermolecular hydrogen bonds to form a cyclic dimer. Benzene does not favour separation into charged ions, so dissociation or partial ionization is not the relevant process.

Step 2: Relate dimerization to the measured molar mass Write the association as

$$2\,\mathrm{CH_3COOH} \rightleftharpoons (\mathrm{CH_3COOH})_2.$$

If a fraction α of the original molecules dimerizes, one initial mole gives $1-\alpha$ moles of monomers and $\alpha/2$ moles of dimers. Hence

$$i = 1 - \frac{\alpha}{2} < 1.$$

The depression is smaller than for unassociated molecules at the same analytical molality. Applying the formula with $i = 1$ consequently gives an apparent molar mass $M_{\text{app}} = M/i$, larger than the actual molecular molar mass. Complete dimerization would give twice the normal value. Complex formation with a different species is unnecessary; the acid associates with itself.

Step 3: Conclusion Acetic acid undergoes association through hydrogen-bonded dimer formation in benzene. Hence, the correct option is (B).

Quick Tip to Solve: For carboxylic acids in a nonpolar solvent, check hydrogen-bonded dimerization first: fewer particles mean a larger apparent molar mass.

37. The increase of pressure on ice $\rightleftharpoons$water system at constant temperature will lead to [4 marks]

(A) A decrease in the entropy of the system

(B) An increase in the Gibbs energy of the system

(C) A shift of the equilibrium in the forward direction

(D) No effect on the equilibrium

Correct Answer: (C) A shift of the equilibrium in the forward direction

Solution:

Concept:

- Le Chatelier's principle states that increasing pressure favours the equilibrium direction that reduces total volume.
- Near the ordinary melting point, liquid water is denser than ice, so the same mass occupies less volume as liquid than as solid.

Step 1: Compare the volumes of the phases For the same mass, $V = m/\rho$. Since $\rho_{\text{water}} > \rho_{\text{ice}}$, melting changes the volume by

$$\Delta V = V_{\text{water}} - V_{\text{ice}} < 0.$$

The open hydrogen-bonded lattice of ice explains its unusually large volume. Thus the pressure effect is opposite to that of substances whose solid is denser than the liquid.

Step 2: Apply the pressure change to melting For a phase at constant temperature, $d\mu = V_m\, dP$, where μ is chemical potential and V_m molar volume. Ice has the larger V_m, so raising pressure raises its chemical potential more than that of water. Relative to ice, liquid water becomes more favourable. Accordingly the system responds by melting some ice, moving ice $\rightleftharpoons$ water towards the right. Option (A) is not the direction of the phase change: melting increases entropy. Option (B) describes neither the relative phase stability nor the equilibrium response and is not the appropriate equilibrium conclusion. Option (D) would require no volume difference, contrary to the densities of these phases.

Step 3: Conclusion Increasing pressure favours conversion of ice into the smaller-volume liquid phase; the equilibrium shifts forward. Hence, the correct option is (C).

Quick Tip to Solve: For pressure changes in a solid–liquid equilibrium, compare densities rather than counting particles: pressure favours the denser phase.

38. At 298 K, the standard reduction potentials are 1.51 V for MnO4−|Mn2+, 1.36 V for Cl2|Cl−, 1.07 V for Br2|Br−, and 0.54 V for I2|I−. At pH = 3, permanganate is expected to oxidize: (RT/F = 0.059 V) [4 marks]

(A) Cl− and Br−

(B) Br− and I−

(C) I− only

(D) Cl− , Br− and I−

Correct Answer: The supplied value RT/F = 0.059 V conflicts with 298 K: taking it literally gives (C), whereas the temperature-consistent coefficient gives (B).

Solution:

Concept:

- An oxidant can oxidize a halide when its reduction potential is greater than the reduction potential of the corresponding halogen/halide couple.
- For acidic permanganate, $\mathrm{MnO_4^-} + 8\,\mathrm{H^+} + 5\,\mathrm{e^-} \longrightarrow \mathrm{Mn^{2+}} + 4\,\mathrm{H_2O}$. The hydrogen-ion concentration therefore strongly affects its reduction potential.

Step 1: Calculate the permanganate potential at the stated temperature With unit activities of the manganese species, the Nernst equation gives

$$E = 1.51 - \frac{2.303RT}{5F}\log_{10}\left(\frac{1}{[\mathrm{H^+}]^8}\right).$$

At 298 K, the common-logarithm coefficient $2.303RT/F$ is approximately $0.059\,\mathrm{V}$, and $[\mathrm{H^+}] = 10^{-3}$. Consequently

$$E = 1.51 - \frac{0.059}{5}(8)(3) = 1.2268\,\mathrm{V}.$$

The parenthetical value in the question labels $0.059\,\mathrm{V}$ as RT/F; that is inconsistent with 298 K. It is the usual common-logarithm coefficient $2.303RT/F$, while RT/F itself is about $0.0257\,\mathrm{V}$. The temperature-consistent calculation above makes that distinction explicit.

Step 2: Apply the supplied constant literally If the explicitly supplied $RT/F = 0.059\,\mathrm{V}$ is used literally, it belongs in the natural-logarithm form, not directly before a common logarithm:

$$E = 1.51 - \frac{0.059}{5}\ln(10^{24}) \simeq 0.858\,\mathrm{V}.$$

This exceeds $0.54\,\mathrm{V}$ for iodine/iodide, but is less than $1.07\,\mathrm{V}$ for bromine/bromide and $1.36\,\mathrm{V}$ for chlorine/chloride. Thus the literal supplied constant leads to iodide only, option (C). It is not consistent to use this value as both RT/F and $2.303RT/F$.

Step 3: Compare each halogen couple For oxidation of a halide, $E_{\mathrm{cell}} = E_{\mathrm{permanganate}} - E_{\mathrm{halogen/halide}}$. The three differences are

$$\begin{aligned}\mathrm{Cl^-}: &\quad 1.2268 - 1.36 = -0.1332\,\mathrm{V},\\ \mathrm{Br^-}: &\quad 1.2268 - 1.07 = +0.1568\,\mathrm{V},\\ \mathrm{I^-}: &\quad 1.2268 - 0.54 = +0.6868\,\mathrm{V}.\end{aligned}$$

Positive cell potentials favour bromide and iodide oxidation; chloride oxidation is not favoured. Thus (A) and (D) incorrectly include chloride, and (C) wrongly excludes bromide at 298 K.

Step 4: Conclusion The data are inconsistent: the literal supplied RT/F gives iodide only, option (C), whereas the physical value at 298 K gives bromide and iodide, option (B), which matches the supplied answer key. The printed conditions therefore do not establish a unique choice without resolving the conflicting constant.

Quick Tip to Solve: Write the balanced permanganate half-reaction before applying pH: its eight hydrogen ions and five electrons give the factor $8/5$.

39. A + 2B →C , the rate equation for the reaction is given as Rate = k [A] [B] If the concentration of A is kept the same but that of B is doubled what will happen to the rate itself? [4 marks]

(A) Doubled

(B) Halved

(C) The same

(D) Quadrupled

Correct Answer: (A) Doubled

Solution:

Concept:

- The experimental rate law, rather than the stoichiometric equation alone, determines how changing a concentration changes the reaction rate.
- Here $r = k[A][B]$: the reaction is first order in A and first order in B, with overall order two.

Step 1: Write the rates before and after the change Let the initial concentrations be $[A] = a$ and $[B] = b$. Then the initial rate is

$$r_1 = kab.$$

The concentration of A is held fixed, and B is doubled to $2b$. At unchanged temperature, the rate constant k does not change. The new rate is therefore

$$r_2 = ka(2b) = 2kab.$$

There is no additional factor from A because its concentration has not changed.

Step 2: Form a ratio and check the alternatives Dividing cancels both the unchanged concentration and the rate constant:

$$\frac{r_2}{r_1} = \frac{k[A](2[B])}{k[A][B]} = 2.$$

Thus the rate doubles. The coefficient 2 before B in the balanced equation specifies the amount of B consumed relative to A; it does not replace the exponent of B in the explicitly supplied rate law. Option (D), quadrupled, would follow from a second-order dependence $[B]^2$, which is absent here. Option (B), halved, would require an inverse dependence, and option (C), unchanged, would require zero-order dependence on B. Neither is consistent with the stated law.

Step 3: Conclusion The new rate is twice the initial rate: $r_2 = 2r_1$. Hence, the correct option is (A).

Quick Tip to Solve: Take the ratio of the two rate laws. Only the changed concentrations survive, each raised to its experimental rate-law exponent.

40. Under ambient conditions, which among the following surfactants will form micelles in aqueous solution at lowest molar concentration? [4 marks]

(A) CH3(CH2)15N⊕(CH3)3Br⊖

(B) CH3−(CH2)13−OSO3−Na+

(C) CH3−(CH2)8−COO⊖Na+

(D) CH3(CH2)11N⊕(CH3)3Br−

Correct Answer: (A) CH3(CH2)15N⊕(CH3)3Br⊖

Solution:

Concept:

- Micelles form when surfactant concentration reaches its critical micelle concentration (CMC); the surfactant has a hydrophilic ionic head and a hydrophobic hydrocarbon tail.
- For comparable ionic surfactants under the same conditions, increasing the hydrophobic chain length strongly lowers the CMC by making association of the tails more favourable.

Step 1: Count the hydrocarbon tails Option (A) has $CH_3(CH_2)_{15}{}^-$, a sixteen-carbon alkyl tail attached to a trimethylammonium head. Option (D) has the same type of head but the shorter twelve-carbon tail $CH_3(CH_2)_{11}{}^-$. Option (B) has a fourteen-carbon alkyl chain attached through oxygen to sulfate. Option (C) has a still shorter hydrocarbon chain, $CH_3(CH_2)_8{}^-$, attached to carboxylate. Keeping the head and tail roles separate avoids incorrectly counting the methyl groups attached to nitrogen as one continuous hydrophobic chain.

Step 2: Relate chain length to aggregation In water, exposed hydrocarbon tails are unfavourable. In a micelle the tails cluster inward while the ionic head groups remain in contact with water. The longer the tail, the greater the hydrophobic driving force per surfactant molecule, so micelles become favourable at a smaller dissolved concentration. The sixteen-carbon surfactant in (A) therefore has the lowest CMC among the given choices. The direct A–D comparison is especially clear because their ionic heads are the same: shortening the tail from sixteen to twelve carbons raises the CMC. The shorter chains in (B) and (C) do not give the lowest threshold in this set under ambient aqueous conditions.

Step 3: Conclusion $CH_3(CH_2)_{15}N^+(CH_3)_3Br^-$ forms micelles at the lowest molar concentration among the listed surfactants. Hence, the correct option is (A).

Quick Tip to Solve: When comparing CMC values for ordinary ionic surfactants, first identify the continuous hydrophobic tail: a longer tail usually means a lower CMC.

41. Choose the incorrect formula out of the four compounds for an element X below : [4 marks]

(A) X2O3

(B) X2(SO4)3

(C) XPO4

(D) X2Cl3

Correct Answer: (D) X2Cl3

Solution:

Concept:

- An electrically neutral ionic compound has total positive charge equal to total negative charge.
- Oxide, sulfate, phosphate and chloride have charges O^{2-}, $SO_4{}^{2-}$, $PO_4{}^{3-}$ and Cl^-, respectively.

Step 1: Determine the common oxidation state of X In X_2O_3, the three oxide ions contribute -6. The two X atoms must therefore contribute $+6$, giving $+3$ for each X. In $X_2(SO_4)_3$, three sulfate groups again contribute -6, so X is also trivalent. In XPO_4, one phosphate ion has charge -3, requiring one X^{3+}. Thus the first three options consistently describe compounds of trivalent X.

Step 2: Test the chloride formula The chloride of the same trivalent X must balance one X^{3+} with three Cl^- ions, giving XCl_3. If the formula is doubled without changing composition, it becomes X_2Cl_6, not X_2Cl_3. Indeed, the printed option X_2Cl_3 would require an average oxidation state $+3/2$ for X, inconsistent with the trivalent formulas established by (A), (B) and (C). Under the ordinary ionic valency interpretation tested here, it is the incorrect formula. The sulfate parentheses in (B) are essential: the subscript three applies to each complete sulfate ion, and that option is charge balanced.

Step 3: Conclusion The incorrect formula is X_2Cl_3; the chloride corresponding to trivalent X is XCl_3. Hence, the correct option is (D).

Quick Tip to Solve: Find the valency of X from the oxide first, then balance each complete polyatomic ion as a single charged unit.

42. Calamine is an ore of [4 marks]

(A) Iron

(B) Copper

(C) Aluminium

(D) Zinc

Correct Answer: (D) Zinc

Solution:

Concept:

- An ore is a naturally occurring mineral deposit from which a metal can be extracted economically; the mineral name identifies its characteristic metal compound.
- In the standard mineral nomenclature used here, calamine is zinc carbonate, $ZnCO_3$, and therefore supplies zinc.

Step 1: Identify the composition of calamine Calamine is the carbonate ore of zinc, conventionally represented as $ZnCO_3$. The carbonate ion carries charge -2, balanced by Zn^{2+}. The metal in the mineral is thus zinc. This distinguishes it from zinc blende, ZnS, another important zinc ore with a different anion. The name of the ore, rather than the carbonate group alone, is what determines the answer.

Step 2: Connect the mineral to extraction and eliminate the other metals On calcination, zinc carbonate decomposes into zinc oxide and carbon dioxide:

$$ZnCO_3 \xrightarrow{\Delta} ZnO + CO_2.$$

The oxide can subsequently be reduced to obtain the metal. This extraction connection confirms why calamine is classed as an ore of zinc. The alternative metals have different familiar ores: iron occurs in hematite, Fe_2O_3, and magnetite, Fe_3O_4; copper in copper pyrites, $CuFeS_2$; and aluminium principally in bauxite, a hydrated aluminium oxide material. None of those identities makes calamine an ore of the corresponding metal. Thus iron, copper and aluminium in options (A), (B) and (C) are excluded.

Step 3: Conclusion Calamine is an ore of zinc. Hence, the correct option is (D).

Quick Tip to Solve: Associate the zinc ores with their anions: calamine is carbonate $ZnCO_3$, while zinc blende is sulfide ZnS.

43. Which physical property of di-hydrogen is wrong? [4 marks]

(A) Tasteless gas

(B) Odourless gas

(C) Colourless gas

(D) Non-inflammable gas

Correct Answer: (D) Non-inflammable gas

Solution:

Concept:

- Dihydrogen, H_2, is a colourless, odourless and tasteless molecular gas at ordinary temperature and pressure.
- Inflammability means the ability to burn in a suitable oxidising atmosphere; it is different from supporting combustion of another substance.

Step 1: Check the sensory properties Pure dihydrogen has no visible colour, characteristic smell or taste. Thus tasteless in (A), odourless in (B), and colourless in (C) are all correct descriptions. These properties refer to the pure gas; an impurity can introduce an odour without making odour a property of hydrogen itself. No chemical reaction is required to observe a colour, whereas inflammability concerns its response to an ignition source.

Step 2: Test the statement about burning Hydrogen burns in oxygen or air when ignited, producing

water and releasing considerable energy. The balanced reaction is

$$2\,H_2 + O_2 \longrightarrow 2\,H_2O.$$

Consequently hydrogen is an inflammable gas. The absence of oxygen in a vessel of pure hydrogen does not make hydrogen non-inflammable: combustion requires both fuel and oxidant. Likewise, a gas need not support combustion to be a fuel. Option (D) denies the ability of hydrogen to burn and is therefore the wrong statement asked for.

Step 3: Conclusion Dihydrogen is inflammable, so describing it as a non-inflammable gas is wrong. Hence, the correct option is (D).

Quick Tip to Solve: For a question asking for the wrong property, first separate fuel behaviour from the ability to support combustion; hydrogen is a fuel.

44. Which of the alkaline earth metal halides given below is essentially covalent in nature? [4 marks]

(A) BeCl2

(B) MgCl2

(C) SrCl2

(D) CaCl2

Correct Answer: (A) BeCl2

Solution:

Concept:

- Fajans' rules connect covalent character with polarisation: a small, highly charged cation distorts the electron cloud of an anion.
- All four chlorides contain a Group 2 metal and chloride; comparing the metal-ion sizes therefore reveals the strongest polarising cation.

Step 1: Compare the cation sizes The ions in these compounds are formally Be^{2+}, Mg^{2+}, Sr^{2+} and Ca^{2+}. Their charges are equal, but beryllium is at the top of the group and has the smallest ion. It therefore has the highest charge density and the greatest ability to attract and distort chloride electron density. Down the group, increasing ionic radius reduces this polarising power.

Step 2: Identify the chloride with the strongest sharing The strong distortion by beryllium produces substantial sharing of electron density between Be and Cl, so $BeCl_2$ is essentially covalent. Its bonding is described by electron-deficient covalent units rather than simply isolated ions. The larger magnesium cation polarises chloride less strongly, and calcium and strontium are still larger; their chlorides are predominantly ionic. Thus the common anion and common cation charge do not imply identical bonding. Options (B), (C) and (D) all have weaker polarisation than option (A).

Step 3: Conclusion $BeCl_2$ has the strongest covalent character among the listed alkaline earth metal halides. Hence, the correct option is (A).

Quick Tip to Solve: With the same anion and cation charge, choose the smallest cation for greatest covalent character.

45. Which of the following compounds has a P −P Bond? [4 marks]

(A) (HPO3)3

(B) H4P2O6

(C) H4P2O7

(D) H4P2O5

Correct Answer: (B) H4P2O6

Solution:

Concept:

- A direct phosphorus–phosphorus bond must be distinguished from a phosphorus–oxygen–phosphorus bridge.
- Hypophosphoric acid, $H_4P_2O_6$, consists of two phosphorus centres joined directly, each carrying one oxo oxygen and two hydroxyl groups.

Step 1: Write the connectivity of hypophosphoric acid The relevant condensed structural formula is

$$(HO)_2P(=O)-P(=O)(OH)_2.$$

Counting atoms gives four hydroxyl hydrogens, two phosphorus atoms and six oxygen atoms, exactly $H_4P_2O_6$. The central linkage is P–P, with no intervening oxygen atom. Each phosphorus has one oxo group, two hydroxyl groups and the bond to the other phosphorus; this directly establishes the bond asked for.

Step 2: Distinguish the other oxoacids Trimetaphosphoric acid, $(HPO_3)_3$, has a cyclic framework containing P–O–P bridges rather than direct P–P bonds. Pyrophosphoric acid, $H_4P_2O_7$, has connectivity $(HO)_2P(=O)-O-P(=O)(OH)_2$, so its central atom is oxygen. Pyrophosphorous acid, $H_4P_2O_5$, similarly contains a P–O–P bridge, with P–H bonds at the phosphorus centres. Its lower oxygen count does not by itself establish a direct P–P bond. These structural differences rule out (A), (C) and (D).

Step 3: Conclusion Hypophosphoric acid, $H_4P_2O_6$, contains a direct P–P bond. Hence, the correct option is (B).

Quick Tip to Solve: Remember hypophosphoric acid as the P–P-linked acid; pyro acids generally join phosphorus centres through oxygen.

46. Chlorine water on standing loses its color and forms [4 marks]

(A) HOCl and HOCl2

(B) HCl only

(C) HCl and HClO2

(D) HCl and HOCl

Correct Answer: (B) HCl only

Solution:

Concept:

- Chlorine dissolves in water and establishes the disproportionation equilibrium $Cl_2 + H_2O \rightleftharpoons HCl + HOCl$.
- Hypochlorous acid is unstable on standing, especially in light, and decomposes to hydrochloric acid and oxygen.

Step 1: Distinguish the initial solution from the aged solution Fresh chlorine water contains dissolved molecular chlorine, which accounts for its characteristic colour. Some chlorine reacts with water to give hydrochloric acid and hypochlorous acid. Thus HCl and $HOCl$, listed in (D), describe the initial hydrolysis products. The question specifically asks what happens on standing, so stopping at this first equilibrium would miss the subsequent chemical change.

Step 2: Combine the reactions on standing Hypochlorous acid decomposes according to

$$2\,HOCl \longrightarrow 2\,HCl + O_2.$$

Removal of this species allows more dissolved chlorine to react. Combining this decomposition with twice the hydrolysis reaction gives

$$2\,Cl_2 + 2\,H_2O \longrightarrow 4\,HCl + O_2.$$

The coloured dissolved chlorine is gradually consumed, oxygen escapes as a gas, and hydrochloric acid remains in the solution. Neither $HOCl_2$ in (A) nor $HClO_2$ in (C) is the relevant stable solution product. In the answer choices, "HCl only" refers to the acid left in solution; it does not deny the evolution of oxygen.

Step 3: Conclusion On standing, chlorine water loses colour and leaves hydrochloric acid in solution, with oxygen evolved. Hence, the correct option is (B).

Quick Tip to Solve: Fresh chlorine water gives HCl and HOCl; on standing, remember to decompose HOCl before choosing the final product.

47. Which of the following statements is false ? [4 marks]

(A) CrO42− is tetrahedral in shape

(B) Cr2O72− has a Cr−O−Cr bond

(C) Na2Cr2O7 is a primary standard in volumetry

(D) Na2Cr2O7 is less soluble than K2Cr2O7

Correct Answer: Both (C) and (D) are false, so the printed options do not give a unique answer.

Solution:

Concept:

- The chromate ion, CrO_4^{2-}, has tetrahedral coordination around chromium; dichromate consists of two tetrahedral units sharing an oxygen.
- A primary standard must have reliably known composition and be suitable for accurate direct weighing. Hygroscopic sodium dichromate is unsuitable, whereas potassium dichromate is commonly used.

Step 1: Check the structural statements In CrO_4^{2-}, four oxygen atoms surround the chromium centre tetrahedrally, so statement (A) is true. In $Cr_2O_7^{2-}$, two chromium centres are connected through a shared oxygen, producing a Cr–O–Cr linkage. The remaining oxygen atoms are terminal. Thus statement (B) is also true; the bridging oxygen does not require a direct Cr–Cr bond.

Step 2: Check both statements about the sodium salt Sodium dichromate is hygroscopic, so its mass can change through uptake of atmospheric moisture. It is therefore not the usual primary standard for volumetric work; potassium dichromate is suitable because of its stable composition and convenient purity. Consequently statement (C), which names sodium dichromate as a primary standard, is false. Sodium dichromate is also more soluble in water than potassium dichromate, not less soluble. Hence statement (D) is false as well. Both errors are independent of the correct structural statements. The wording asks for one false statement, but the options as printed supply two, so selecting just one would conceal a defect in the question.

Step 3: Conclusion Both (C) and (D) are false as printed: sodium dichromate is not a suitable primary standard and is more soluble than potassium dichromate. The printed question therefore has no unique single-option answer.

Quick Tip to Solve: Use potassium dichromate for the primary-standard comparison; sodium dichromate is hygroscopic and more water-soluble.

48. When concentrated HCl is added to aqueous solution of CoCl2, its colour changes from reddish pink to deep blue. Which complex ion gives blue colour in reaction? [4 marks]

(A) [CoCl6]4−

(B) [CoCl6]3−

(C) [Co(H2O)6]2+

(D) [CoCl4]2−

Correct Answer: (D) [CoCl4]2−

Solution:

Concept:

- Aqueous cobalt(II) is principally the reddish-pink hexaaqua complex, $[Co(H_2O)_6]^{2+}$.
- High chloride concentration favours replacement of water ligands by chloride, forming the blue tetrahedral tetrachlorocobaltate(II) ion.

Step 1: Write the ligand-substitution equilibrium Concentrated hydrochloric acid supplies a large concentration of chloride ions. The equilibrium is

$$[Co(H_2O)_6]^{2+} + 4\,Cl^- \rightleftharpoons [CoCl_4]^{2-} + 6\,H_2O.$$

Increasing chloride concentration drives this equilibrium toward the right. This is a change in coordination environment, from a six-coordinate water complex to a four-coordinate chloride complex, rather than oxidation of the cobalt centre.

Step 2: Check the charge and colour of each candidate Cobalt remains in oxidation state +2. Four chloride ligands each have charge -1, so the complex charge is $+2-4=-2$, consistent with $[CoCl_4]^{2-}$. This tetrahedral complex is deep blue. Option (C) is the original pink hexaaqua ion and therefore cannot account for the new colour. Options (A) and (B) both contain six chlorides rather than the four ligands of the blue species; the latter additionally implies cobalt(III), because $+3-6=-3$. The addition of hydrochloric acid in this reaction does not oxidise cobalt(II) to cobalt(III). These checks identify (D) unambiguously.

Step 3: Conclusion The deep-blue ion is $[CoCl_4]^{2-}$, tetrachlorocobaltate(II). Hence, the correct option is (D).

Quick Tip to Solve: For cobalt chloride colour changes, associate pink with six water ligands and blue with four chloride ligands.

49. Which of the following complex ions has electrons that are symmetrically filled in both t2g and eg orbitals? [4 marks]

(A) [Co(NH3)6]2+

(B) [Mn(CN)6]4−

(C) [CoF6]3−

(D) [FeF6]3−

Correct Answer: (D) [FeF6]3−

Solution:

Concept:

- In an octahedral field the five d orbitals split into three t_{2g} orbitals and two e_g orbitals.
- Symmetric filling within both sets occurs for high-spin d^5: each of the five d orbitals contains one electron, giving $t_{2g}^3 e_g^2$.

Step 1: Determine the metal configurations In (A), neutral ammonia makes cobalt +2, hence d^7, with octahedral occupation $t_{2g}^5 e_g^2$; the lower set is not symmetrically singly occupied. In (B), six cyanide ligands and total charge -4 make manganese +2, hence d^5. Cyanide is a strong-field ligand and gives low-spin $t_{2g}^5 e_g^0$, which does not meet the stated condition. In (C), cobalt is +3, hence d^6; fluoride gives high-spin $t_{2g}^4 e_g^2$, with unequal occupancies in the three lower orbitals.

Step 2: Identify the high-spin half-filled ion For $[FeF_6]^{3-}$, charge balance gives $x - 6 = -3$, so iron is +3. Neutral iron is $[Ar]3d^6 4s^2$; removing two 4s electrons and one 3d electron leaves d^5. Fluoride is weak field, so pairing is avoided until all five d orbitals have received an electron. The resulting configuration is

$$t_{2g}^3 e_g^2.$$

Each of the three lower orbitals and each of the two upper orbitals is singly occupied. Thus both sets are symmetrically filled, unlike the other listed configurations.

Step 3: Conclusion High-spin $[FeF_6]^{3-}$ has the symmetric configuration $t_{2g}^3 e_g^2$. Hence, the correct option is (D).

Quick Tip to Solve: First find oxidation state and d count, then check ligand strength: only high-spin d5 places one electron in every d orbital.

50. Addition of phosphate fertilizers to water bodies causes [4 marks]

(A) Deposition of calcium phosphate

(B) Enhanced growth of algae

(C) Increase in fish population

(D) Increase in amount of dissolved oxygen in water

Correct Answer: (B) Enhanced growth of algae

Solution:

Concept:

- Phosphate is a nutrient for aquatic plants and algae. A large external supply can remove the nutrient limitation on their growth.
- Eutrophication is nutrient enrichment of a water body, followed by excessive growth of algae and increased oxygen demand during decomposition.

Step 1: Identify the direct effect of phosphate addition Phosphate fertilizers contain phosphorus in forms that can enter ponds and lakes with agricultural runoff. Phosphorus is needed in nucleic acids, ATP and other cellular constituents. When its availability limits growth, an extra supply allows algae to multiply rapidly. The immediate characteristic response is therefore an algal bloom, which means enhanced growth of algae. This identifies option (B).

Step 2: Distinguish the bloom from its later consequences A bloom does not imply a sustained increase in the oxygen available to fish. Algae respire, and after they die, microorganisms decompose the organic matter and consume dissolved oxygen. Thus the biological oxygen demand rises and the dissolved oxygen concentration can fall, especially at night or during decay. Fish may die under these oxygen-poor conditions, so an increase in fish population is not the characteristic consequence. Although insoluble phosphates can form under particular chemical conditions, deposition of calcium phosphate is not the general environmental effect being asked about. Option (D) also confuses photosynthetic oxygen production with the net ecological outcome of excessive nutrient loading.

Step 3: Conclusion Addition of phosphate fertilizers causes enhanced growth of algae, the initial visible feature of eutrophication. Hence, the correct option is (B).

Quick Tip to Solve: For fertilizer runoff questions, connect phosphate enrichment to an algal bloom, then connect decomposition of that bloom to oxygen depletion.

51. Match the organic compounds in column - I with the Lassaigne's test results in column - II appropriately:

Column - I: (A) Aniline, (B) Benzene sulfonic acid, (C) Thiourea

Column - II: (i) Red color with FeCl3, (ii) Violet color with sodium nitroprusside, (iii) Blue color with hot and acidic solution of FeSO4 [4 marks]

(A) A - ii ; B - iii ; C - i

(B) A - iii ; B - ii ; C - i

(C) A - ii ; B - i ; C - iii

(D) A - iii ; B - i ; C - ii

Correct Answer: (B) A - iii ; B - ii ; C - i

Solution:

Concept:

- Sodium fusion converts covalently bound nitrogen and sulphur into water-soluble ionic species, which are detected in Lassaigne's extract.
- Nitrogen alone gives cyanide; sulphur alone gives sulphide. When nitrogen and sulphur occur together, thiocyanate can give a blood-red ferric complex.

Step 1: Match the compound containing nitrogen alone Aniline, $C_6H_5NH_2$, contains nitrogen but no sulphur. Sodium fusion produces $NaCN$. With ferrous ions, cyanide forms ferrocyanide; subsequent acidic treatment and oxidation provide ferric ions, producing Prussian blue ferric ferrocyanide. Consequently aniline corresponds to test (iii), not the sulphide or thiocyanate colour tests. This already excludes options (A) and (C).

Step 2: Match sulphur alone and nitrogen with sulphur Benzene sulphonic acid contains sulphur but no nitrogen. Its sodium-fusion extract contains S^{2-}, which gives the violet nitroprusside test:

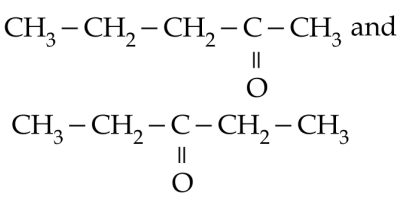
$$[Fe(CN)_5NO]^{2-} + S^{2-} \longrightarrow [Fe(CN)_5NOS]^{4-}.$$

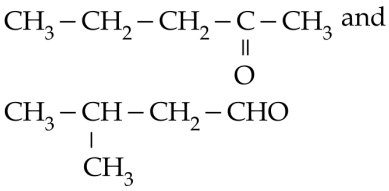
Thus B corresponds to (ii). Thiourea, NH_2CSNH_2, contains both nitrogen and sulphur. Fusion can form SCN^-, which gives a blood-red complex with Fe^{3+} from $FeCl_3$. Thus C corresponds to (i). The assignment B–(i) in option (D) is wrong: a sulphur-only compound cannot supply the nitrogen required for thiocyanate. The complete matching is fixed by the elemental composition, rather than by memorising the compounds independently.

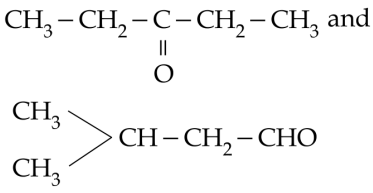
Step 3: Conclusion A–(iii), B–(ii), C–(i). Hence, the correct option is (B).

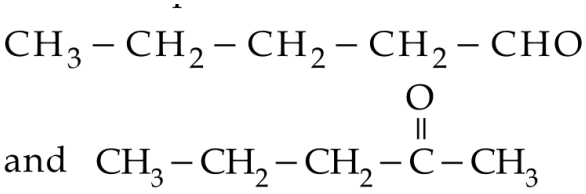
Quick Tip to Solve: First list N and S in each compound: N alone gives Prussian blue, S alone gives violet with nitroprusside, and N plus S can give blood red with ferric ions.

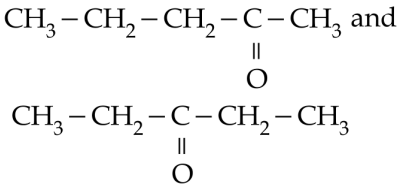
52. Which of the following pairs of compounds are positional isomers? [4 marks]

(A) 

(B)  and

(C)  and

(D)  and

Correct Answer: (A)  and

Solution:

Concept:

- Positional isomers have the same molecular formula, carbon skeleton and functional group, but differ in the position of that functional group or substituent.
- Functional isomers instead contain different functional groups. A ketone and an aldehyde with the same formula are functional isomers, not positional isomers.

Step 1: Identify the structures in option (A) The first structure is $CH_3CH_2CH_2COCH_3$, pentan-2-one, numbered from the end nearest the carbonyl group. The second is $CH_3CH_2COCH_2CH_3$, pentan-3-one. Each contains five carbon atoms in an unbranched chain and one internal carbonyl group. Both therefore have molecular formula $C_5H_{10}O$ and the ketone functional group. The carbonyl carbon occupies position 2 in one compound and position 3 in the other. This is precisely a change of position without changing the skeleton or functional class.

Step 2: Eliminate pairs that change the functional group In option (B), pentan-2-one is paired with the branched aldehyde $CH_3CH(CH_3)CH_2CHO$. In option (C), pentan-3-one is paired with the same branched aldehyde. Both pairs change a ketone into an aldehyde, and also differ in chain arrangement, so neither meets the requirement for positional isomerism. In option (D), pentanal, $CH_3CH_2CH_2CH_2CHO$, is paired with pentan-2-one. Although both have an unbranched five-carbon skeleton, the terminal $-CHO$ group belongs to the aldehyde class, whereas an internal $C=O$ group is a ketone. This pair is classified as functional isomers.

Step 3: Conclusion Pentan-2-one and pentan-3-one in option (A) are positional isomers. Hence, the correct option is (A).

Quick Tip to Solve: Before comparing functional-group locants, check that both structures belong to the same functional class; an aldehyde–ketone pair is functional isomerism.

53. The number of structural isomers for C6H14 is [4 marks]

(A) 4

(B) 5

(C) 6

(D) 3

Correct Answer: (B) 5

Solution:

Concept:

- Structural isomers have the same molecular formula but different atom connectivity. Rotations around single bonds do not create new structural isomers.
- For an acyclic alkane, enumerate distinct carbon skeletons systematically by decreasing the length of the longest continuous chain.

Step 1: Enumerate the six-carbon and five-carbon chains The formula C_6H_{14} is that of an acyclic saturated hydrocarbon. A straight chain using all six carbon atoms gives hexane, $CH_3CH_2CH_2CH_2CH_2CH_3$. With a longest chain of five carbon atoms, one carbon remains as a methyl substituent. The distinct possibilities are 2-methylpentane, $CH_3CH(CH_3)CH_2CH_2CH_3$, and 3-methylpentane, $CH_3CH_2CH(CH_3)CH_2CH_3$. Positions 4 and 2 are equivalent on reversing the numbering direction. A methyl group at position 1 would merely extend the chain and reproduce hexane.

Step 2: Enumerate the four-carbon chains and count once With a longest chain of four carbon atoms, the remaining two carbons can be methyl branches. The distinct structures are 2,2-dimethylbutane, $CH_3C(CH_3)_2CH_2CH_3$, and 2,3-dimethylbutane, $CH_3CH(CH_3)CH(CH_3)CH_3$. Putting a branch at an end extends the selected longest chain and reproduces an already counted skeleton. An apparent ethyl-substituted butane also contains a longer chain and is already included in the five-carbon-chain cases. No additional skeleton with a longest chain of only three carbons can accommodate six carbons within carbon valency four. The count is therefore

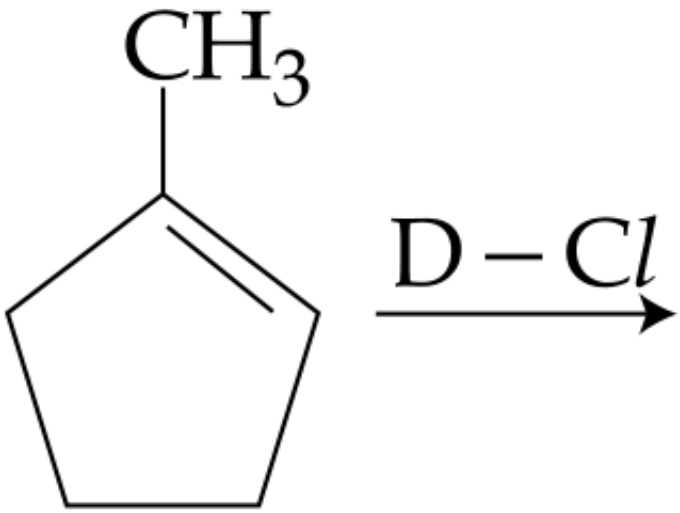
$$1 + 2 + 2 = 5.$$

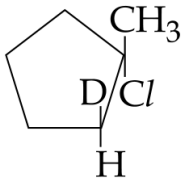
Thus 4 and 3 omit valid skeletons, while 6 double-counts a structure or conformation.

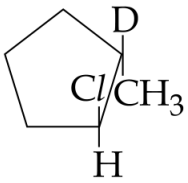
Step 3: Conclusion There are five structural isomers of hexane. Hence, the correct option is (B).

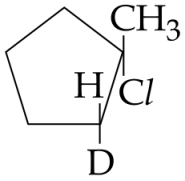
Quick Tip to Solve: Count hexane skeletons by longest chain: one with six carbons, two with five, and two with four; reverse-chain numbering must not be counted twice.

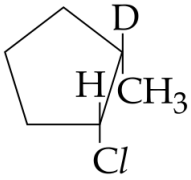
54. What is the major product expected from the following reaction? Where D is an isotope of hydrogen [4 marks]

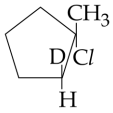


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- Deuterium, D, is an isotope of hydrogen and behaves as the electrophilic hydrogen in addition of DCl to an alkene.
- Markovnikov addition proceeds by the more stable carbocation: D attaches to the less substituted alkene carbon, leaving positive charge on the more substituted carbon for chloride attack.

Step 1: Choose the favoured carbocation The supplied alkene is 1-methylcyclopentene. Of the two double-bond carbons, one already carries a methyl group and has no hydrogen; the other carries a hydrogen. Addition of D^+ to the carbon already bearing hydrogen leaves a tertiary carbocation at the methyl-bearing carbon. The positively charged carbon is then bonded to the methyl group and two ring carbons. The alternative direction would leave a secondary carbocation, which is less stable. The favoured pathway therefore places D on the initially less substituted double-bond carbon.

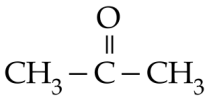
Step 2: Attach chloride and compare the products Chloride attacks the tertiary carbocation at the carbon bearing the methyl substituent. Thus Cl and CH_3 must be on the same ring carbon, while D and the original H must be on the adjacent ring carbon. The ring itself is saturated after addition; no carbon skeleton rearrangement is needed. In the supplied drawings this connectivity is shown by option (A). Option (C) has the same regioselectivity but shows D and Cl on the same side of the ring; option (A) shows the anti arrangement, favoured by chloride attack from the face opposite the newly added D in the initially formed ion pair. The carbocation mechanism is not absolutely stereospecific, so this comparison concerns the major product rather than excluding all syn product formation. Options (B) and (D) place D at the methyl-bearing addition centre and correspond to the unfavoured direction or an incorrect arrangement of groups. The choice is made by tracing the two original alkene carbons rather than by treating D as a new carbon-containing substituent.

Step 3: Conclusion D adds to the less substituted alkene carbon, and Cl adds to the methyl-bearing carbon, as shown in option (A). Hence, the correct option is (A).

Quick Tip to Solve: Treat DCl exactly like HCl for regioselectivity, then track D separately from the H already present so that the isotope label is retained correctly.

55. In the reaction sequence 2CH3CHO →(OH−) A →(Δ) B; the product B is: [4 marks]

(A) CH3 −CH = CH −CHO

(B) 

(C) CH3 −CH2 −CH2 −CH3

(D) CH3 −CH2 −CH2 −CH2 −OH

Correct Answer: (A) CH3 −CH = CH −CHO

Solution:

Concept:

- An aldehyde containing an alpha hydrogen undergoes aldol addition in dilute base. The base generates an enolate, which attacks the carbonyl carbon of a second aldehyde molecule to form a new carbon-carbon bond.
- The aldol product is a beta-hydroxy aldehyde. On heating, elimination of water gives an alpha,beta-unsaturated aldehyde, with the original aldehyde group retained and a double bond conjugated with it.

Step 1: Form the aldol from two ethanal molecules Ethanal, CH_3CHO, contains alpha hydrogens on its methyl group. Hydroxide removes an alpha proton, producing the resonance-stabilised enolate. Its carbon nucleophile attacks the carbonyl carbon of another ethanal molecule. Protonation of the resulting alkoxide by water regenerates hydroxide and gives the four-carbon addition product:

$$2\,CH_3CHO \xrightarrow{OH^-} CH_3CH(OH)CH_2CHO.$$

Thus A is 3-hydroxybutanal. Numbering from the aldehyde carbon places the hydroxyl group at the beta carbon. Both carbon atoms of each starting molecule are incorporated; no carbon atom is lost in this addition step. The aldehyde group on the enolate donor remains an aldehyde in A.

Step 2: Dehydrate the beta-hydroxy aldehyde on heating Heating promotes removal of the beta hydroxyl group together with an alpha hydrogen as water. The new double bond lies between those two carbons:

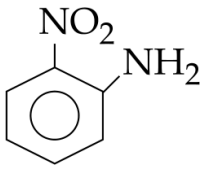
$$CH_3CH(OH)CH_2CHO \xrightarrow{\Delta} CH_3CH{=}CHCHO + H_2O.$$

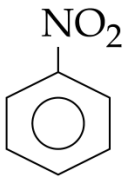
Therefore B is but-2-enal, also called crotonaldehyde. Conjugation between the carbon-carbon double bond and the carbonyl group favours this condensation product. Option (A) is exactly this four-carbon unsaturated aldehyde. The supplied structure in (B) is acetone, CH_3COCH_3, a three-carbon ketone; it cannot be obtained by this carbon-preserving ethanal aldol sequence. Option (C) is butane and would require removal of oxygen and hydrogenation, whereas option (D) is butan-1-ol and would require reduction. Neither reducing reagents nor hydrogenation conditions occur in the sequence.

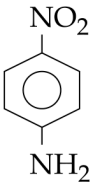
Step 3: Conclusion Product B is crotonaldehyde, $CH_3CH{=}CHCHO$, formed by dehydration of 3-hydroxybutanal. Hence, the correct option is (A).

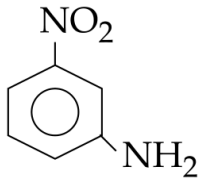
Quick Tip to Solve: For ethanal plus dilute base followed by heat, first write the four-carbon beta-hydroxy aldehyde, then remove water to give the conjugated aldehyde.

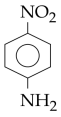
56. Which compounds exhibits maximum dipole moment among the following? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- Molecular dipole moment is the vector sum of bond and electronic-polarisation contributions; it depends on both group polarity and relative orientation.
- The amino group donates electron density into a benzene ring by resonance, while the nitro group withdraws it. Para placement permits strong donor-to-acceptor conjugation.

Step 1: Identify and compare the four substitution patterns The supplied structures are ortho-nitroaniline in (A), nitrobenzene in (B), para-nitroaniline in (C), and meta-nitroaniline in (D). In nitroaniline the amino group is an electron donor through its lone pair, whereas NO_2 is an electron acceptor. The donor and acceptor generate a polarised conjugated system. In the para isomer these

groups lie opposite one another and can be connected through the ring by charge-separated resonance structures, with the amino end relatively positive and the nitro end relatively negative.

Step 2: Use vector addition and resonance to select the maximum The para arrangement gives a large net separation of charge along the donor–acceptor axis. The electronic effects reinforce the molecular polarisation rather than cancelling merely because the groups are opposite: the groups are different, and their donor/acceptor roles are opposite. Nitrobenzene lacks the additional amino donor, so its polarisation is smaller. In the meta isomer the amino donor and nitro acceptor are not coupled by the same direct resonance pathway; the group contributions also do not reinforce along one common para axis. In the ortho isomer their directions are not arranged for the same large net resultant, and local interactions can reduce the simple donor–acceptor contribution. Therefore the greatest dipole moment among the given compounds is that of para-nitroaniline, not nitrobenzene or the ortho/meta isomers.

Step 3: Conclusion Para-nitroaniline, shown in option (C), has the maximum dipole moment among the compounds given. Hence, the correct option is (C).

Quick Tip to Solve: For nitroaniline isomers, combine donor–acceptor resonance with vector orientation; unlike identical para substituents, an amino–nitro pair does not cancel by symmetry.

57. Which one of the following structures represents the neoprene polymer? [4 marks]

(A) $-\left(CH_2-\underset{CN}{CH}\right)_n-$

(B) $-\left(CH_2-\underset{Cl}{CH}\right)_n-$

(C) $-\left(CH_2-\underset{Cl}{C}=CH-CH_2\right)_n-$

(D) $-\left(\underset{C_6H_5}{CH}-CH_2\right)_n-$

Correct Answer: (C) $-\left(CH_2-\underset{Cl}{C}=CH-CH_2\right)_n-$

Solution:

Concept:

- Neoprene is a synthetic elastomer obtained by addition polymerisation of chloroprene, whose systematic name is 2-chlorobuta-1,3-diene.
- In 1,4-addition of a conjugated diene, the terminal carbon atoms form the new chain links and the double bond remains between the two central carbon atoms.

Step 1: Identify the monomer and its repeat unit Chloroprene has the structure $CH_2=C(Cl)-CH=CH_2$. Number its four carbon atoms from the end nearest chlorine. On polymerisation, carbon atoms 1 and 4 become attached to neighbouring monomer units, while a double bond is retained between carbon atoms 2 and 3. Thus the chain contains a four-carbon repeat unit with chlorine on one of the unsaturated carbon atoms:

$$[-CH_2-C(Cl)=CH-CH_2-]_n .$$

The chlorine is a substituent on the carbon backbone, not a separate link in the chain.

Step 2: Compare each displayed structure Option (C) shows exactly this four-carbon unit, with its internal double bond and chlorine attached to the appropriate central carbon. Option (A) has a two-carbon saturated backbone with a nitrile group and represents polyacrylonitrile. Option (B) has a two-carbon saturated backbone carrying chlorine and represents polyvinyl chloride. Option (D) carries a phenyl substituent on a two-carbon saturated backbone and represents polystyrene. Consequently, merely finding chlorine in a structure does not identify neoprene: the residual double bond and four-carbon repeat are also essential.

Step 3: Conclusion Neoprene is the polymer with repeat unit $[-CH_2-C(Cl)=CH-CH_2-]_n$, shown in option (C). Hence, the correct option is (C).

Quick Tip to Solve: For neoprene, look for both chlorine and a residual carbon-carbon double bond within a four-carbon repeat unit; chlorine alone could indicate PVC.

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58. Accumulation of which of the following molecules in the muscles occurs as a result of vigorous exercise? [4 marks]

(A) Glycogen

(B) Pyruvic acid

(C) Glucose

(D) L-lactic acid

Correct Answer: (D) L-lactic acid

Solution:

Concept:

- Glycolysis converts glucose to pyruvate and provides ATP. During vigorous exercise, oxygen supply may be insufficient for the rate at which muscle cells require energy.
- Reduction of pyruvate to lactate regenerates oxidised nicotinamide adenine dinucleotide, enabling glycolysis to continue under these conditions.

Step 1: Trace the fate of pyruvate during vigorous exercise Glucose is broken down through glycolysis to pyruvate. When the energy demand is high and oxygen delivery cannot keep pace, the muscle increases the conversion of pyruvate into lactate through lactate dehydrogenase. The reduced coenzyme is oxidised in this step, so glycolysis can continue supplying ATP. In the terminology of the question, this product is L-lactic acid. At physiological pH it is predominantly present as its conjugate base, L-lactate, but this acid-base distinction does not alter the selection among the given molecules.

Step 2: Eliminate the other molecules Glycogen is a stored carbohydrate that is mobilised to supply fuel during exercise; it is not the characteristic product that accumulates from anaerobic glucose breakdown. Glucose is the starting fuel, rather than the named end product of this pathway. Pyruvic acid is an intermediate formed by glycolysis, but under the oxygen-limited conditions in this question it is reduced further instead of being the characteristic accumulating product. L-lactic acid is therefore the only listed molecule corresponding to the anaerobic metabolic response of exercising muscle.

Step 3: Conclusion Vigorous exercise causes accumulation of L-lactic acid, conventionally describing the increased muscle lactate produced from pyruvate. Hence, the correct option is (D).

Quick Tip to Solve: Follow the sequence glycogen or glucose to pyruvate to lactate; the last conversion regenerates the coenzyme needed to keep glycolysis running.

59. Which artificial sweetener contains chlorine? [4 marks]

(A) Sucralose

(B) Alitame

(C) Saccharin

(D) Aspartame

Correct Answer: (A) Sucralose

Solution:

Concept:

- Artificial sweeteners are distinguished by their chemical structures rather than by their common use as sugar substitutes.
- Sucralose is a chlorinated derivative of sucrose: three hydroxyl groups of the parent sugar are replaced by chlorine atoms.

Step 1: Identify the chlorinated sugar derivative Sucrose is a disaccharide containing carbon, hydrogen and oxygen. In sucralose, selective substitution of three hydroxyl groups by chlorine gives a sweet compound that still resembles the sugar skeleton. Its molecular formula is $C_{12}H_{19}Cl_3O_8$, which directly shows three chlorine atoms. Thus the structural modification itself supplies the test asked for: among the listed sweeteners, sucralose is the chlorine-containing one. The presence of chlorine is a compositional feature, independent of how much of the sweetener is used.

Step 2: Check the remaining names Alitame is a peptide-based high-intensity sweetener derived from amino-acid components and does not contain chlorine. Aspartame is the methyl ester of an aspartic acid and phenylalanine dipeptide, so its characteristic atoms are carbon, hydrogen, nitrogen and oxygen, without chlorine. Saccharin contains a sulfonimide group and therefore contains sulfur and nitrogen, but it too lacks chlorine. These alternatives illustrate why the shared property of sweetness cannot decide the question; their functional groups and elemental compositions must be distinguished.

Step 3: Conclusion Sucralose contains chlorine, with three chlorine atoms in each molecule. Hence, the correct option is (A).

Quick Tip to Solve: Associate the name sucralose with chlorinated sucrose; saccharin contains sulfur, while aspartame and alitame are peptide-based sweeteners.

60. A pink coloured salt turns blue on heating. The presence of which cation is most likely ? [4 marks]

(A) Cu^{2+}

(B) Fe^{2+}

(C) Zn^{2+}

(D) Co^{2+}

Correct Answer: (D) Co^{2+}

Solution:

Concept:

- Transition-metal ions can show different colours when their coordination environment changes, because their ligand-field energy levels change.
- Hydrated cobalt(II) chloride is pink, whereas the anhydrous salt is blue; heating removes its water of crystallisation.

Step 1: Relate heating to loss of hydration water A familiar example of the stated change is cobalt(II) chloride hexahydrate. On heating it loses water of crystallisation, and the pink hydrated material becomes blue anhydrous cobalt(II) chloride. This can be represented by

$$CoCl_2 \cdot 6\,H_2O \xrightarrow{\text{heat}} CoCl_2 + 6\,H_2O.$$

The cobalt oxidation state remains +2 in both salts. The colour change is therefore evidence of a change in hydration and coordination, not evidence that heating necessarily oxidises or reduces the metal ion.

Step 2: Compare the candidate cations Copper(II) sulfate pentahydrate is blue and becomes white on losing water, so it has the opposite initial colour and the wrong final colour for this observation. Iron(II) salts are commonly pale green rather than pink, and the standard pink-to-blue dehydration test does not identify iron(II). Zinc(II) has a filled d subshell and its ordinary salts are colourless or white, so it is not the characteristic coloured ion here. Cobalt(II) gives the familiar reversible hydration colour change: adding water to blue anhydrous cobalt chloride restores the pink hydrated form.

Step 3: Conclusion The most likely cation is Co^{2+}, identified by the pink hydrated salt turning blue on dehydration. Hence, the correct option is (D).

Quick Tip to Solve: Remember the cobalt chloride hydration pair as pink when hydrated and blue when dry; do not confuse it with blue hydrated copper sulfate.

Mathematics (120 marks)

61. Let A = {x1, x2, ... , x7} and B = {y1, y2, y3} be two sets containing seven and three distinct elements respectively. Then the total number of functions f: A → B that are onto, if there exist exactly three elements x in A such that f(x) = y2, is equal to: [4 marks]

(A) 12 ·7C2

(B) 16 ·7C3

(C) 14 · 7C3

(D) 14 ·7C2

Correct Answer: (C) 14 · 7C3

Solution:

Concept:

- An onto function must have every element of the codomain as an image of at least one element of its domain.
- Choose the elements assigned to the prescribed image first, then count assignments of the remaining elements using inclusion-exclusion.

Step 1: Choose the preimages of the prescribed value Exactly three of the seven distinct domain elements must map to y_2. Their identities can be chosen in $\binom{7}{3}$ ways. Once they are chosen, their function values are fixed, and none of the remaining four domain elements may map to y_2. Each of those four must therefore map to either y_1 or y_3. The function is onto only if both of these values actually occur.

Step 2: Count onto assignments of the remaining four elements There are $2^4 = 16$ assignments to two labelled values before imposing the onto condition. Exactly two assignments are inadmissible: all four elements could map to y_1, or all four could map to y_3. These cases are distinct, and excluding them leaves $16 - 2 = 14$ assignments. Consequently,

$$N = \binom{7}{3}(2^4 - 2) = 14\binom{7}{3} = 490.$$

The coefficient 16 in option (B) fails to remove the two constant assignments. Options (A) and (D) use a choice of two domain elements, whereas the specified fibre contains three; option (A) also has the wrong assignment factor. The codomain values are labelled, so no division by a permutation factor is appropriate.

Step 3: Conclusion The required number is $14\binom{7}{3} = 490$. Hence, the correct option is (C).

Quick Tip to Solve: Fix the fibre of the specially constrained codomain value first; for the two remaining values, subtract the two constant assignments to enforce onto behaviour.

62. If z is a non-real complex number, then the minimum value of (Im z^5)/((Im z)^5) is: [4 marks]

(A) -2

(B) -4

(C) -5

(D) -1

Correct Answer: (B) -4

Solution:

Concept:

- Write a non-real complex number as $z = x + iy$, with $x, y \in \mathbb{R}$ and $y \neq 0$. The denominator is then nonzero.
- The binomial expansion separates the imaginary part of a power; a quartic in x/y can be minimised by completing a square.

Step 1: Express the ratio in terms of a real variable In the expansion of $(x + iy)^5$, only the terms with odd powers of i contribute to the imaginary part. Since $i^3 = -i$ and $i^5 = i$,

$$\mathrm{Im}(z^5) = 5x^4y - 10x^2y^3 + y^5.$$

Dividing by y^5, and putting $t = x/y$, gives

$$\frac{\mathrm{Im}(z^5)}{(\mathrm{Im}\, z)^5} = 5t^4 - 10t^2 + 1.$$

The variable t can be any real number because $y \neq 0$ and there is no further restriction on x.

Step 2: Complete the square and check attainability Rearrange the quartic as

$$5t^4 - 10t^2 + 1 = 5(t^2 - 1)^2 - 4 \geq -4.$$

Equality holds when $t^2 = 1$, or $x = \pm y$. For example, $z = 1 + i$ is non-real and satisfies the equality condition, proving that the lower bound is attained rather than merely approached. The value -5 in option (C) is below the rigorous lower bound and is impossible. Values -2 and -1 are above an attained value of -4, so neither can be the minimum. Keeping the denominator as y^5, rather than replacing it by a modulus, is essential to obtaining this exact expression.

Step 3: Conclusion The minimum value is -4, attained whenever $\mathrm{Re}\, z = \pm \mathrm{Im}\, z \neq 0$. Hence, the correct option is (B).

Quick Tip to Solve: For imaginary-part ratios, expand with a real and imaginary part, divide by the appropriate power of the imaginary part, and complete a square in their ratio.

63. If the two roots of the equation, $(a-1)(x^4+x^2+1)+(a+1)(x^2+x+1)^2=0$ are real and distinct, then the set of all values of a is equal to [4 marks]

(A) $(-\infty, -2) \cup (2, \infty)$

(B) $(-1/2, 0) \cup (0, 1/2)$

(C) $(0, 1/2)$

(D) $(-1/2, 0)$

Correct Answer: (B) $(-1/2, 0) \cup (0, 1/2)$

Solution:

Concept:

- The factorisation $x^4+x^2+1=(x^2+x+1)(x^2-x+1)$ exposes a common quadratic factor.
- A real quadratic has two distinct real roots when its leading coefficient is nonzero and its discriminant is strictly positive.

Step 1: Factor the given polynomial without dividing by a parameter Taking out the common factor gives

$$\begin{aligned}&(a-1)(x^4+x^2+1)+(a+1)(x^2+x+1)^2\\&=(x^2+x+1)\big[(a-1)(x^2-x+1)+(a+1)(x^2+x+1)\big]\\&=2(x^2+x+1)(ax^2+x+a).\end{aligned}$$

For real x, the first factor is $(x+\frac{1}{2})^2+\frac{3}{4}>0$, so it contributes no real roots. Every real root must therefore come from $ax^2+x+a=0$.

Step 2: Impose two distinct real roots and exclude degeneration When $a\neq 0$, the remaining factor is quadratic with discriminant $\Delta=1-4a^2$. Two distinct real roots require $1-4a^2>0$, giving $-\frac{1}{2}<a<\frac{1}{2}$. The value $a=0$ must be excluded separately: the factor becomes x, so only one real root remains. The endpoints also fail because they give a repeated root. Thus

$$a\in\left(-\frac{1}{2},0\right)\cup\left(0,\frac{1}{2}\right).$$

Option (A) lies outside the positive-discriminant range. Options (C) and (D) retain only one sign of the parameter, although the discriminant condition permits either sign. Option (B) is precisely the required union.

Step 3: Conclusion The parameter set is $\left(-\frac{1}{2},0\right)\cup\left(0,\frac{1}{2}\right)$. Hence, the correct option is (B).

Quick Tip to Solve: Factor out the quadratic with no real zeros first, then test both the discriminant and the parameter value that would make the remaining quadratic linear.

64. If A is a 3 × 3 matrix such that |5 adjA| = 5, then |A| is equal to: [4 marks]

(A) ±(1)/(5)

(B) ±5

(C) ±1

(D) ±(1)/(25)

Correct Answer: (A) ±(1)/(5)

Solution:

Concept:

- For an order-n matrix, multiplying the entire matrix by a scalar c multiplies its determinant by c^n, because each of its n rows acquires that factor.
- The adjugate identity is $\det(\operatorname{adj} A) = (\det A)^{n-1}$. Here the order is three, so the power on $\det A$ is two.

Step 1: Separate the scalar from the determinant The bars denote a determinant, not the magnitude of each entry. Since $5 \operatorname{adj} A$ is also a matrix of order three, all three rows contain the scalar factor five. Therefore

$$\det(5 \operatorname{adj} A) = 5^3 \det(\operatorname{adj} A) = 5.$$

Dividing by 125 gives $\det(\operatorname{adj} A) = 1/25$. Using only one factor of five would ignore two of the scaled rows.

Step 2: Apply the adjugate identity Put $d = \det A$. The order-three identity now gives

$$d^2 = \frac{1}{25}, \qquad d = \pm\frac{1}{5}.$$

The data constrain the square of the determinant, so neither sign may be discarded. Both signs satisfy $125d^2 = 5$. Values ± 5, ± 1 and $\pm 1/25$ instead give respectively 3125, 125 and $1/5$ for the determinant in the question, so they cannot satisfy it.

Step 3: Conclusion Thus $|A| = \pm\frac{1}{5}$. Hence, the correct option is (A).

Quick Tip to Solve: For $\det(c \operatorname{adj} A)$, account for the matrix order twice: the scalar contributes c^n, while the adjugate contributes $(\det A)^{n-1}$.

65. If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = ax - 12$**, then 'a' is equal to:** [4 marks]

(A) -24

(B) 24

(C) -12

(D) 12

Correct Answer: (B) 24

Solution:

Concept:

- A determinant identity stated in the variable x holds at any convenient substituted value. If the unknown coefficient appears linearly on the right, choose a value that leaves it present.
- Expansion along a row or column with a zero reduces the arithmetic. Cofactor signs across the first row are $+, -, +$.

Step 1: Substitute a convenient value At $x = 0$, the right side is -12, which contains no a, so that substitution cannot determine the requested coefficient. Choose $x = 1$ instead. The identity becomes

$$\begin{vmatrix} 2 & 2 & -1 \\ 4 & 3 & 0 \\ 6 & 1 & 1 \end{vmatrix} = a - 12.$$

Each entry follows directly from its polynomial: in particular the lower-left entry is $1 + 2 + 3 = 6$, and the upper-right entry is $1 - 2 = -1$. Keeping these signs is essential to the expansion.

Step 2: Expand and isolate the coefficient Expanding the first row, with the cofactor signs included, gives

$$\begin{aligned} D(1) &= 2(3 \cdot 1 - 0 \cdot 1) - 2(4 \cdot 1 - 0 \cdot 6) \\ &\quad + (-1)(4 \cdot 1 - 3 \cdot 6) \\ &= 6 - 8 + 14 = 12. \end{aligned}$$

Thus $a - 12 = 12$, so $a = 24$. As a check on the alternatives, $a = -24, -12, 12$ would make $a - 12$ equal $-36, -24, 0$, respectively; none equals the evaluated determinant. This substitution suffices because the question already specifies the form of the identity.

Step 3: Conclusion The required coefficient is $a = 24$. Hence, the correct option is (B).

Quick Tip to Solve: When a determinant is already stated to equal $ax + b$, evaluate it at one simple nonzero value of x rather than fully expanding all polynomial entries.

66. If in a regular polygon the number of diagonals is 54, then the number of sides of this polygon is [4 marks]

(A) 12

(B) 10

(C) 6

(D) 9

Correct Answer: (A) 12

Solution:

Concept:

- An n-sided polygon has $\binom{n}{2}$ pairs of vertices. These pairs include both its sides and its diagonals.
- From any vertex there are $n-3$ diagonals: exclude the vertex itself and its two adjacent vertices. Counting from all vertices counts every diagonal twice.

Step 1: Translate the diagonal count into an equation Using the second counting argument, the number of diagonals is $n(n-3)/2$. Set it equal to the given value:

$$\frac{n(n-3)}{2} = 54 \quad \Longrightarrow \quad n^2 - 3n - 108 = 0.$$

The same equation follows from $\binom{n}{2} - n = 54$: subtract the n sides from all vertex pairs. Regularity does not change this count; it guarantees a familiar polygon but the counting depends only on its number of vertices.

Step 2: Factor and retain the admissible root Two numbers with product -108 and sum -3 are -12 and 9. Hence

$$(n-12)(n+9) = 0, \qquad n = 12 \text{ or } n = -9.$$

A number of sides is a positive integer at least three, so discard -9. Direct substitution checks the result: $12 \cdot 9/2 = 54$. The other offered positive values give $10 \cdot 7/2 = 35$, $6 \cdot 3/2 = 9$ and $9 \cdot 6/2 = 27$ diagonals, respectively, rather than the specified count.

Step 3: Conclusion The polygon has 12 sides. Hence, the correct option is (A).

Quick Tip to Solve: Use $n(n-3)/2$ for diagonals, then reject any negative quadratic root because a vertex count must be a positive integer.

67. The term independent of x in the binomial expansion of $\left(1-\frac{1}{x}+3x^5\right)\left(2x^2-\frac{1}{x}\right)^8$ is: [4 marks]

(A) -496

(B) -400

(C) 496

(D) 400

Correct Answer: (D) 400

Solution:

Concept:

- The general term of $(u+v)^8$ is $\binom{8}{r}u^{8-r}v^r$, where r is an integer from zero to eight.
- A constant term in a product comes from powers whose exponents add to zero. Each of the three terms in the first factor must therefore be checked separately.

Step 1: Write the power and coefficient of a general term Take $u=2x^2$ and $v=-1/x$. The term indexed by r is

$$T_{r+1}=\binom{8}{r}2^{8-r}(-1)^r x^{16-3r}.$$

Multiplication by the three terms $1, -x^{-1}, 3x^5$ changes its exponent to $16-3r$, $15-3r$, and $21-3r$, respectively. The first vanishes only at $r=16/3$, which is not an integer, so the factor 1 contributes no constant term.

Step 2: Calculate the two admissible contributions For multiplication by $-x^{-1}$, take $r=5$. Its constant contribution is

$$-\binom{8}{5}2^3(-1)^5=56\cdot 8=448.$$

For multiplication by $3x^5$, take $r=7$. Its contribution is

$$3\binom{8}{7}2^1(-1)^7=-48.$$

Adding all admissible contributions gives $448-48=400$. The value 496 results from adding magnitudes and ignoring the negative second contribution. The two negative choices also fail because the larger contribution is positive. Both indices lie within the allowed range, so neither contribution may be omitted.

Step 3: Conclusion The term independent of x is 400. Hence, the correct option is (D).

Quick Tip to Solve: For a product with several monomials outside a binomial power, match exponents separately for each monomial before adding signed coefficients.

68. The sum of the 3rd and the 4th terms of a G. P. is 60 and the product of its first three terms is 1000. If the first term of this G. P. is positive, then its 7th term is [4 marks]

(A) 320

(B) 640

(C) 2430

(D) 7290

Correct Answer: (A) 320

Solution:

Concept:

- If the first term is a and the common ratio is r, then the nth term is ar^{n-1}.
- The product of the first three terms is $a \cdot ar \cdot ar^2 = (ar)^3$. For real terms its cube root gives the second term directly.

Step 1: Use the product to obtain the second term From the stated product,

$$(ar)^3 = 1000 \quad \Longrightarrow \quad ar = 10.$$

In particular neither a nor r is zero. The sum of the third and fourth terms is

$$ar^2 + ar^3 = ar\,r(1 + r) = 10r(1 + r) = 60.$$

After dividing by ten, $r^2 + r - 6 = 0$. Notice that the product determines ar, not a itself; identifying the first term with ten would give the wrong progression.

Step 2: Select the ratio using positivity and find the seventh term Factor the quadratic:

$$(r - 2)(r + 3) = 0, \qquad r = 2 \text{ or } -3.$$

Since $a > 0$ and $ar = 10 > 0$, the ratio must be positive. Thus $r = 2$, and $a = 10/2 = 5$. Consequently

$$T_7 = ar^6 = 5 \cdot 2^6 = 320.$$

As a direct check, the first four terms are $5, 10, 20, 40$; their required product and sum are 1000 and 60. The negative root would require $a = -10/3$, contradicting the hypothesis. None of the other offered seventh terms follows from the unique admissible a, r.

Step 3: Conclusion The seventh term is 320. Hence, the correct option is (A).

Quick Tip to Solve: The product of three consecutive GP terms is the cube of the middle term; combine this with any sign condition before choosing a ratio root.

69. If $\sum_{n=1}^{5} 1/(n(n+1)(n+2)(n+3)) = k/3$, then k is equal to : [4 marks]

(A) 17/105

(B) 55/336

(C) 19/112

(D) 1/6

Correct Answer: (B) 55/336

Solution:

Concept:

- Adjacent reciprocal products can be subtracted to shorten a finite sum. Here the numerators of two adjacent three-factor fractions differ by three.
- The parameter k is three times the displayed sum, so keep the outside factor separate until the telescoping cancellation is complete.

Step 1: Decompose the summand For every positive integer n,

$$\frac{1}{n(n+1)(n+2)} - \frac{1}{(n+1)(n+2)(n+3)} = \frac{(n+3)-n}{n(n+1)(n+2)(n+3)} = \frac{3}{n(n+1)(n+2)(n+3)}.$$

Thus each summand in the question is one third of this difference. All denominators are nonzero for $n = 1, \ldots, 5$, so the decomposition is valid throughout the given range.

Step 2: Cancel adjacent terms and identify the parameter Let the given sum be S. On summing the differences, every interior triple product cancels with the next term. Only the first and last boundary fractions survive:

$$S = \frac{1}{3}\left(\frac{1}{1\cdot 2\cdot 3} - \frac{1}{6\cdot 7\cdot 8}\right).$$

Since $S = k/3$, multiply by three to obtain

$$k = \frac{1}{6} - \frac{1}{336} = \frac{56-1}{336} = \frac{55}{336}.$$

The option $1/6$ omits the final boundary term. Neither $17/105$ nor $19/112$ equals the exact difference above, so those alternatives fail. The end denominator uses six, seven and eight because the upper summation index is five.

Step 3: Conclusion Therefore $k = \frac{55}{336}$. Hence, the correct option is (B).

Quick Tip to Solve: For four consecutive factors in the denominator, subtract adjacent three-factor reciprocals; write the final uncancelled boundary term explicitly.

70. Let k be a non-zero real number. If $f(x) = \begin{cases} \dfrac{(e^x-1)^2}{\sin\left(\frac{x}{k}\right)\log\left(1+\frac{x}{4}\right)}, & x \neq 0 \\ 12, & x = 0 \end{cases}$ **is a continuous function, then the value of k is:** [4 marks]

(A) 2

(B) 4

(C) 3

(D) 1

Correct Answer: (C) 3

Solution:

Concept:

- Continuity at zero requires the limit as $x \to 0$ to equal the defined value $f(0) = 12$.
- The standard limits are $(e^x - 1)/x \to 1$, $\sin t/t \to 1$, and $\log(1+t)/t \to 1$. The fixed parameter k is nonzero.

Step 1: Separate the standard limit factors For sufficiently small nonzero x, factor the expression as

$$f(x) = \left(\frac{e^x - 1}{x}\right)^2 \frac{x}{\sin(x/k)} \frac{x}{\log(1 + x/4)}.$$

The logarithm is defined near zero because $1 + x/4 > 0$ there. Since $k \neq 0$, its sine argument tends to zero along with x, allowing use of the standard sine limit. This factorisation preserves both powers of x in the squared numerator.

Step 2: Evaluate the limit and enforce continuity The individual limits are

$$\frac{e^x - 1}{x} \to 1, \qquad \frac{x}{\sin(x/k)} \to k, \qquad \frac{x}{\log(1 + x/4)} \to 4.$$

Their product is therefore $4k$. Equating it to the function value gives

$$4k = 12 \quad \Longrightarrow \quad k = 3.$$

This is nonzero as required. Substituting the other offered values $k = 2, 4, 1$ gives limits $8, 16, 4$, respectively, which do not match twelve. At every sufficiently close nonzero point the displayed quotient is a quotient of continuous functions with a nonzero denominator, so this limit condition resolves the only possible discontinuity at the origin.

Step 3: Conclusion Continuity requires $k = 3$. Hence, the correct option is (C).

Quick Tip to Solve: Normalise every small argument separately: $\sin(x/k) \sim x/k$ and $\log(1 + x/4) \sim x/4$, so their product contributes $x^2/(4k)$.

71. The equation of a normal to the curve, $\sin y = x \sin\left(\frac{\pi}{3} + y\right)$ at x = 0, is: [4 marks]

(A) 2x $-\sqrt{3}$ y = 0

(B) 2y $-\sqrt{3}$ x = 0

(C) 2y + $\sqrt{3}$ x = 0

(D) 2x + $\sqrt{3}$ y = 0

Correct Answer: (D) 2x + $\sqrt{3}$ y = 0

Solution:

Concept:

- For an implicit curve, differentiate both sides with respect to x, remembering that y is a function of x. Its derivative dy/dx gives the tangent slope at a regular point.
- The tangent and normal are perpendicular. If the finite nonzero tangent slope is m, the normal slope is $-1/m$, and a line through (x_0, y_0) has equation $y - y_0 = m(x - x_0)$.

Step 1: Locate a point and differentiate At $x = 0$, the equation gives $\sin y = 0$. Choose $y = 0$, so the origin is a point of the curve and supplies the normal requested by the wording "a normal". Implicit differentiation gives

$$\cos y\, y\prime = \sin(\pi/3 + y) + x\cos(\pi/3 + y)y\prime.$$

Therefore, at the origin, $y\prime = \sin(\pi/3) = \sqrt{3}/2$. The coefficient of the derivative is nonzero there, so this is a regular tangent slope. One must differentiate the factor involving y as well as the explicit factor x, even though its contribution subsequently vanishes at the origin.

Step 2: Write the perpendicular line The normal slope is $-2/\sqrt{3}$. Since this normal passes through the origin,

$$y = -\frac{2}{\sqrt{3}}x \quad \Longleftrightarrow \quad 2x + \sqrt{3}y = 0.$$

Option (A) has positive slope $2/\sqrt{3}$; option (B) has the tangent slope $\sqrt{3}/2$; and option (C) has slope $-\sqrt{3}/2$, whose product with the tangent slope is $-3/4$, not -1. Only option (D) gives the negative reciprocal. The curve also meets the axis at other multiples of π, but the line displayed in (D) is indeed one normal, as required.

Step 3: Conclusion The normal through the origin is $2x + \sqrt{3}y = 0$. Hence, the correct option is (D).

Quick Tip to Solve: Find the point before differentiating, then take the negative reciprocal of the tangent slope to obtain the normal slope.

72. Let k and K be the minimum and the maximum values of the function $f(x) = \dfrac{(1+x)^{0.6}}{1+x^{0.6}}$ in $[0,1]$ respectively, then the ordered pair (k, K) is equal to : [4 marks]

(A) $(2^{-0.4}, 1)$

(B) $(2^{-0.6}, 1)$

(C) $(2^{-0.4}, 2^{0.6})$

(D) $(1, 2^{0.6})$

Correct Answer: (A) $(2^{-0.4}, 1)$

Solution:

Concept:

- For a positive function, logarithmic differentiation simplifies products, quotients and fractional powers. On the open interval, the sign of the logarithmic derivative is the sign of the ordinary derivative.
- A continuous function on a closed interval attains its maximum and minimum. Compare endpoint values after establishing monotonicity; an endpoint can be an extremum even when the derivative there is undefined.

Step 1: Differentiate the logarithm The numerator and denominator are positive throughout the interval, so take the natural logarithm without changing the domain. For $0 < x < 1$,

$$\frac{f'(x)}{f(x)} = \frac{0.6}{1+x} - \frac{0.6x^{-0.4}}{1+x^{0.6}} = \frac{0.6(1-x^{-0.4})}{(1+x)(1+x^{0.6})}.$$

The cancellation uses $x\,x^{-0.4} = x^{0.6}$. Since $x^{-0.4} > 1$, the numerator is negative and the denominator is positive. Hence the function strictly decreases on the open interval.

Step 2: Compare the endpoint values Continuity extends this ordering to both endpoints. Direct substitution gives

$$f(0) = 1, \qquad f(1) = \frac{2^{0.6}}{2} = 2^{-0.4}.$$

Thus the smaller endpoint value occurs at one and the larger at zero. Option (B) uses an incorrect exponent: dividing by two subtracts one from the exponent, not 1.2. Options (C) and (D) assign a maximum greater than one, which is impossible for a function decreasing from its value one at zero. The derivative vanishes at the right endpoint only; that does not create a larger interior value.

Step 3: Conclusion The ordered pair is $(k, K) = (2^{-0.4}, 1)$. Hence, the correct option is (A).

Quick Tip to Solve: For fractional-power quotients, logarithmically differentiate and combine the terms before testing the sign; the mixed powers often cancel.

73. From the top of a 64 metres high tower, a stone is thrown upwards vertically with the velocity of 48 m/s. The greatest height (in metres) attained by the stone, assuming the value of the gravitational acceleration g = 32 m/s^2, is: [4 marks]

(A) 112

(B) 88

(C) 128

(D) 100

Correct Answer: (D) 100

Solution:

Concept:

- With constant downward gravitational acceleration of magnitude g, the upward velocity after time t is $v = u - gt$. At the highest point, the instantaneous vertical velocity is zero.
- The additional rise above the launch point follows from $v^2 = u^2 - 2gh$, giving $h = u^2/(2g)$. Height above the ground includes both this rise and the initial tower height.

Step 1: Find the time and rise to the highest point Choose upward as positive and use the given numerical value of gravitational acceleration, $g = 32$. The stone starts with upward speed $u = 48\,\text{m/s}$. Its turning time is

$$t = \frac{u}{g} = \frac{48}{32} = \frac{3}{2}\,\text{s}.$$

The extra rise is consequently

$$h = ut - \frac{1}{2}gt^2 = 48\left(\frac{3}{2}\right) - 16\left(\frac{3}{2}\right)^2 = 72 - 36 = 36\,\text{m}.$$

Equivalently, setting the final speed to zero gives $h = 48^2/(2 \times 32) = 36\,\text{m}$. This second calculation checks the sign and the numerical result.

Step 2: Measure the height from the ground The launch point is already 64 metres above the ground. Therefore add the rise, rather than replacing the initial height:

$$H = 64 + 36 = 100\,\text{m}.$$

Options (A), (B) and (C), namely 112, 88 and 128, do not satisfy this height calculation. In particular, the initial speed is not a distance and cannot simply be added to the tower height. The result also exceeds 64 metres, as it must because the initial velocity is upward, while remaining finite because gravity continually reduces that velocity until the turning point.

Step 3: Conclusion The greatest height above the ground is $100\,\text{m}$. Hence, the correct option is (D).

Quick Tip to Solve: Compute the extra rise from the launch point first, then add the tower height; setting the velocity to zero identifies the turning point.

74. If $\int \frac{\log\left(t+\sqrt{1+t^2}\right)}{\sqrt{1+t^2}}\,dt = \frac{1}{2}\left(g(t)\right)^2 + C$, where C is a constant, then g(2) is equal to: [4 marks]

(A) $\frac{1}{\sqrt{5}}\log\left(2+\sqrt{5}\right)$

(B) $2\log\left(2+\sqrt{5}\right)$

(C) $\log\left(2+\sqrt{5}\right)$

(D) $\frac{1}{2}\log\left(2+\sqrt{5}\right)$

Correct Answer: (C) $\log\left(2+\sqrt{5}\right)$

Solution:

Concept:

- A substitution is useful when the integrand contains a function multiplied by its derivative. Here the logarithm $u = \log(t+\sqrt{1+t^2})$ has derivative $1/\sqrt{1+t^2}$.
- The elementary integral $\int u\,du = u^2/2 + C$ supplies the required square. Since $\sqrt{1+t^2} > |t|$, the logarithm has a positive argument for every real t.

Step 1: Differentiate the logarithmic expression Let $u = \log(t+\sqrt{1+t^2})$. Applying the chain rule explicitly,

$$\frac{du}{dt} = \frac{1+t/\sqrt{1+t^2}}{t+\sqrt{1+t^2}} = \frac{1}{\sqrt{1+t^2}}.$$

The cancellation is valid because the denominator is strictly positive. Thus the factor multiplying the logarithm in the given integrand is exactly the differential needed for this substitution. There is no additional factor of t or $\sqrt{1+t^2}$ left after the substitution.

Step 2: Integrate and compare the offered values The integral becomes

$$\int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\{\log(t+\sqrt{1+t^2})\}^2 + C.$$

Hence take $g(t) = \log(t+\sqrt{1+t^2})$, giving $g(2) = \log(2+\sqrt{5})$. Differentiating this final squared expression recovers the given integrand exactly. The factors $1/\sqrt{5}$, 2, and $1/2$ in the other options are not produced by evaluating g; the first confuses its derivative with its value. A negative choice of g gives the same square, but among the displayed positive values the direct antiderivative identification is precisely option (C).

Step 3: Conclusion $g(2) = \log(2+\sqrt{5})$. Hence, the correct option is (C).

Quick Tip to Solve: Differentiate the entire logarithm first; its derivative cancels the square root denominator and leaves the integral of u.

75. Let f: R $\to$ R be a function such that f(2 $-$ x) = f(2 + x) and f(4 $-$ x) = f(4 + x), for all x $\in$ R and $\int_0^2 f(x)\,dx = 5$. Then the value of $\int_{10}^{50} f(x)\,dx$ is: [4 marks]

(A) 100

(B) 125

(C) 80

(D) 200

Correct Answer: (A) 100

Solution:

Concept:

- The relations $f(2-x) = f(2+x)$ and $f(4-x) = f(4+x)$ express reflection symmetry about $x = 2$ and $x = 4$. Composing two reflections gives a translation.
- For an integrable periodic function of period T, the integral over any interval of length T is the same. An interval containing an integer number of periods has that many times the single-period integral.

Step 1: Derive the period and one-period integral Writing the reflection identities for an arbitrary argument t gives $f(t) = f(4-t)$ and $f(t) = f(8-t)$. Consequently,

$$f(t+4) = f(8-(t+4)) = f(4-t) = f(t).$$

Thus four is a period, whether or not it is the smallest one. Reflection about two also gives

$$\int_2^4 f(x)\,dx = \int_0^2 f(4-u)\,du = \int_0^2 f(u)\,du = 5.$$

Therefore $\int_0^4 f(x)\,dx = 10$. This uses the given integral as the half-period area; it is not the whole-period area.

Step 2: Count complete periods in the requested interval The requested interval has length $50 - 10 = 40 = 10 \times 4$. Partition it into ten adjacent intervals of length four. Periodicity makes the integral over each such interval equal to ten, even though the first interval begins at ten rather than at a multiple of four. Hence

$$\int_{10}^{50} f(x)\,dx = 10\int_0^4 f(x)\,dx = 10 \times 10 = 100.$$

The alternative values 125, 80 and 200 do not use both the derived period and its correct area. In particular, 200 doubles the result by counting twice as many full-period areas. No explicit formula for the function is needed: its reflection symmetries and the one supplied integral determine the answer.

Step 3: Conclusion $\int_{10}^{50} f(x)\,dx = 100$. Hence, the correct option is (A).

Quick Tip to Solve: Compose the two reflections to find the period, then find the area of one full period before counting periods.

76. Let f: $(-1, 1) \to \mathbf{R}$ be a continuous function. If $\int_0^{\sin x} f(t)\,dt = \frac{\sqrt{3}}{2}x$, then $f\left(\frac{\sqrt{3}}{2}\right)$ is equal to : [4 marks]

(A) $\frac{\sqrt{3}}{2}$

(B) $\sqrt{3}$

(C) $\sqrt{\frac{3}{2}}$

(D) $\frac{1}{2}$

Correct Answer: (B) $\sqrt{3}$

Solution:

Concept:

- For continuous f, differentiation of an integral with upper limit $h(x)$ gives $\frac{d}{dx}\int_0^{h(x)} f(t)\,dt = f(h(x))h\prime(x)$. Both the fundamental theorem of calculus and the chain rule are required.
- To evaluate f at a particular argument, first choose an angle whose sine is that argument. The convenient angle for $\sqrt{3}/2$ is $\pi/3$, with cosine $1/2$.

Step 1: Differentiate the given identity The variable upper limit is $\sin x$, rather than x. Differentiate the identity with respect to x on the branch containing zero with $-\pi/2 < x < \pi/2$:

$$f(\sin x)\cos x = \frac{\sqrt{3}}{2}.$$

The continuity assumption permits the fundamental theorem to be applied. The extra factor $\cos x$ comes from differentiating the upper limit and cannot be omitted. On this branch it is positive, and the sine takes every value in $(-1, 1)$, the stated domain of the function.

Step 2: Substitute the angle giving the target argument Choose $x = \pi/3$, for which $\sin x = \sqrt{3}/2$. Then

$$f\left(\frac{\sqrt{3}}{2}\right)\frac{1}{2} = \frac{\sqrt{3}}{2}, \qquad f\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}.$$

Equivalently, on the same branch $f(t) = \sqrt{3}/(2\sqrt{1-t^2})$, whose integral from zero to $\sin x$ is $(\sqrt{3}/2)x$, checking the calculation. Option (A) omits division by the cosine; option (C) takes an unjustified square root of the result; and option (D) gives the cosine rather than the integrand value. The chosen angle is strictly inside the relevant interval, so no endpoint or zero denominator is involved.

Step 3: Conclusion $f(\sqrt{3}/2) = \sqrt{3}$. Hence, the correct option is (B).

Quick Tip to Solve: Differentiate with the chain rule before substituting; a sine upper limit contributes a cosine factor.

77. The solution of the differential equation $y\,dx - (x + 2y^2)\,dy = 0$ is $x = f(y)$. If $f(-1) = 1$, then $f(1)$ is equal to : [4 marks]

(A) 2

(B) 3

(C) 4

(D) 1

Correct Answer: (B) 3

Solution:

Concept:

- Treat x as a function of y. A first-order linear equation $dx/dy + P(y)x = Q(y)$ is solved by multiplying by an integrating factor $e^{\int P(y)\,dy}$.
- Where division by the independent variable is used, first solve on intervals excluding zero. Check the resulting polynomial directly in the original differential equation to confirm its continuation through zero.

Step 1: Solve with y as the independent variable Rearrange the equation on an interval where $y \neq 0$:

$$\frac{dx}{dy} - \frac{x}{y} = 2y.$$

An integrating factor can be taken as $1/y$, since a nonzero constant multiplier does not change the integrating-factor method on either such interval. Multiplication then gives

$$\frac{d}{dy}\left(\frac{x}{y}\right) = 2.$$

Integrating yields

$$\frac{x}{y} = 2y + C, \qquad x = 2y^2 + Cy.$$

Substitution checks it in the original form: $y(4y + C) - (2y^2 + Cy + 2y^2) = 0$, identically, including at zero.

Step 2: Apply the condition and evaluate Use the prescribed value at minus one:

$$1 = 2(-1)^2 + C(-1) = 2 - C, \qquad C = 1.$$

Thus the solution is $f(y) = 2y^2 + y$, and

$$f(1) = 2 + 1 = 3.$$

This selects option (B), rather than 2, 4 or 1. The value two drops the linear term, four has an incorrect constant, and one confuses the given value at minus one with the value at plus one. The polynomial also gives a smooth continuation through zero. For a differentiable solution across zero, its derivative there fixes the same coefficient on the two sides, so the initial condition determines the requested continuation.

Step 3: Conclusion The solution is $x = 2y^2 + y$, giving $f(1) = 3$. Hence, the correct option is (B).

Quick Tip to Solve: When the differential form contains y dx and an expression times dy, test whether treating x as the dependent variable makes the equation linear.

78. A straight line L through the point (3, −2) is inclined at an angle of 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is: [4 marks]

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$

(B) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$

(C) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

(D) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

Correct Answer: (D) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

Solution:

Concept:

- The inclination of a line is measured anticlockwise from the positive x-axis; a nonvertical line with inclination θ has slope $\tan\theta$.
- For two intersecting lines the smaller angle between their directions is used. A line parallel to the x-axis does not intersect that axis unless it coincides with it.

Step 1: Find both possible line directions The given line is $\sqrt{3}x + y = 1$, so its slope is $-\sqrt{3}$ and its inclination is $120°$. A line making an acute angle $60°$ with it can have inclination $60°$ or $180°$, the latter representing the horizontal direction. Thus the two possible slopes are $\sqrt{3}$ and zero. Equivalently the slope relation is $\left|(m + \sqrt{3})/(1 - \sqrt{3}m)\right| = \sqrt{3}$; solving its two signs gives exactly these slopes. Both directions must be considered before applying the extra condition.

Step 2: Apply the intersection condition and the given point The horizontal candidate through $(3, -2)$ is $y = -2$. It is parallel to the x-axis and never meets it, so the condition in the question excludes it. Therefore the required slope is $\sqrt{3}$. By the point-slope equation,

$$y + 2 = \sqrt{3}(x - 3).$$

Rearranging gives

$$y - \sqrt{3}x + 2 + 3\sqrt{3} = 0.$$

Its x-intercept is $3 + 2/\sqrt{3}$, which exists, and substituting $(3, -2)$ gives zero. Option (A) has slope $-\sqrt{3}$, making it parallel to the reference line. Options (B) and (C) have slopes $1/\sqrt{3}$ and $-1/\sqrt{3}$, respectively, giving acute angles $90°$ and $30°$ with the reference direction rather than the required $60°$.

Step 3: Conclusion The required line is $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$. Hence, the correct option is (D).

Quick Tip to Solve: When an angle condition produces two directions, use the remaining geometric condition to choose between them; here it excludes the horizontal line.

79. If the incentre of an equilateral triangle is (1, 1) and the equation of its one side is 3x + 4y + 3 = 0, then the equation of the circumcircle of this triangle is [4 marks]

(A) $x^2 + y^2 - 2x - 2y - 2 = 0$

(B) $x^2 + y^2 - 2x - 2y + 2 = 0$

(C) $x^2 + y^2 - 2x - 2y - 7 = 0$

(D) $x^2 + y^2 - 2x - 2y - 14 = 0$

Correct Answer: (D) $x^2 + y^2 - 2x - 2y - 14 = 0$

Solution:

Concept:

- The incentre and circumcentre of an equilateral triangle coincide, since its angle bisectors, medians and perpendicular bisectors are the same lines.
- The perpendicular distance from the common centre to a side is the inradius r, and the circumradius is $R = 2r$.

Step 1: Calculate the inradius The given centre is $I = (1, 1)$. For a line $ax + by + c = 0$, its distance from (h, k) is $|ah + bk + c|/\sqrt{a^2 + b^2}$. Consequently the distance to the given side is

$$r = \frac{|3(1) + 4(1) + 3|}{\sqrt{3^2 + 4^2}} = \frac{10}{5} = 2.$$

The absolute value ensures that distance is positive and independent of which side of the line contains the centre.

Step 2: Obtain the circumradius and expand the equation In an equilateral triangle the centroid divides each altitude in the ratio $2 : 1$. The distance from the common centre to a vertex is therefore twice its distance from the opposite side. Thus $R = 2r = 4$, while the centre remains $(1, 1)$. The circumcircle is

$$(x - 1)^2 + (y - 1)^2 = 16.$$

Expanding both squares gives $x^2 + y^2 - 2x - 2y + 2 = 16$, so its constant term is -14. The constants in the other alternatives give squared radii 4, 0, or 9; none equals the required 16.

Step 3: Conclusion The circumcircle is $x^2 + y^2 - 2x - 2y - 14 = 0$. Hence, the correct option is (D).

Quick Tip to Solve: For an equilateral triangle, first find the centre-to-side distance, then double it to obtain the circumradius.

80. If a circle passing through the point $(-1, 0)$ touches y-axis at (0, 2), then the length of the chord of the circle along the x-axis is: [4 marks]

(A) (5)/(2)

(B) 5

(C) (3)/(2)

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- A circle tangent to the y-axis at $(0, 2)$ has its centre on the horizontal normal $y = 2$.
- If the centre is $(h, 2)$, its radius is $|h|$; the circle has equation $(x - h)^2 + (y - 2)^2 = h^2$.

Step 1: Use the given point to determine the centre The point of tangency is $(0, 2)$, so the centre-to-tangency radius is perpendicular to the vertical axis. Write the centre as $(h, 2)$. Substituting the given point $(-1, 0)$ into the circle equation gives

$$(-1 - h)^2 + (0 - 2)^2 = h^2.$$

After expanding, $h^2 + 2h + 1 + 4 = h^2$, whence $2h + 5 = 0$ and $h = -5/2$. Therefore the radius is $5/2$, and the centre lies to the left of the vertical axis. Its sign is consistent with the circle passing through a point with negative abscissa.

Step 2: Find the length cut off on the horizontal axis The intercept of a circle on the x-axis is the length of the chord cut by $y = 0$, rather than the radius or an individual abscissa. Put $y = 0$ to get

$$(x + 5/2)^2 + 4 = 25/4, \qquad (x + 5/2)^2 = 9/4.$$

Thus the two intersection abscissae are -1 and -4, and their separation is $|-1 - (-4)| = 3$. The value $5/2$ in option (A) is the radius, and option (B), 5, is the diameter, not this chord. Option (C), $3/2$, is half the chord length. Only the full length 3 answers the intercept question.

Step 3: Conclusion The circle cuts an intercept of length 3 on the x-axis. Hence, the correct option is (D).

Quick Tip to Solve: For a circle, an axis intercept means the distance between its two intersections with that axis, so subtract the two roots.

81. If the distance between the foci of an ellipse is half the length of its latus rectum, then the eccentricity of the ellipse is [4 marks]

(A) 1/2

(B) $\sqrt{2}-1$

(C) $(\sqrt{2}-1)/2$

(D) $(2\sqrt{2}-1)/2$

Correct Answer: (B) $\sqrt{2}-1$

Solution:

Concept:

- For an ellipse with semi-major axis a, semi-minor axis b, and eccentricity e, the focal separation is $2ae$.
- The full length of the latus rectum is $2b^2/a$, and $b^2 = a^2(1-e^2)$, with $0 \le e < 1$.

Step 1: Translate the length condition The question compares the separation of the two foci with half the full latus rectum, rather than with the full latus rectum. Therefore

$$2ae = \frac{1}{2}\left(\frac{2b^2}{a}\right) = \frac{b^2}{a}.$$

Multiplication by a gives $2a^2e = b^2$. Since an ellipse has $a > 0$, division by a^2 is valid. Substitute $b^2/a^2 = 1-e^2$ to obtain $2e = 1-e^2$.

Step 2: Solve and retain the admissible root Rearranging the last equation produces

$$e^2 + 2e - 1 = 0, \qquad e = -1 \pm \sqrt{2}.$$

The root $-1-\sqrt{2}$ is negative, so it cannot represent an eccentricity. The other root $\sqrt{2}-1$ lies strictly between zero and one and thus describes an ellipse. As a direct check, $1-(\sqrt{2}-1)^2 = 2(\sqrt{2}-1)$, recovering the required length relation. Option (A), $e = 1/2$, gives $1-e^2 = 3/4$ but $2e = 1$. Option (C) is half the required root and fails the quadratic. Option (D), $e = \sqrt{2} - 1/2$, is approximately 0.914; then $2e \approx 1.828$, whereas $1-e^2 \approx 0.164$, so it fails the required relation.

Step 3: Conclusion The eccentricity is $\sqrt{2}-1$. Hence, the correct option is (B).

Quick Tip to Solve: Write both lengths in terms of the semi-major axis before cancelling it; keep the factor of one-half in the condition.

82. Let PQ be a double ordinate of the parabola, $y^2 = -4x$, where P lies in the second quadrant. If R divides PQ in the ratio 2 : 1, then the locus of R is : [4 marks]

(A) $3y^2 = -2x$

(B) $9y^2 = 4x$

(C) $9y^2 = -4x$

(D) $3y^2 = 2x$

Correct Answer: (C) $9y^2 = -4x$

Solution:

Concept:

- A double ordinate of $y^2 = -4x$ joins points with the same x-coordinate and opposite y-coordinates.
- For internal division $PR : RQ = m : n$, the section formula is $R = (nP + mQ)/(m + n)$.

Step 1: Parametrize the two endpoints Write $P = (-t^2, 2t)$ with $t > 0$, so P lies in the second quadrant as stated. The other endpoint on the parabola with the same abscissa is $Q = (-t^2, -2t)$. Both satisfy $y^2 = 4t^2 = -4(-t^2)$. The segment is vertical and crosses the axis of the parabola at $(-t^2, 0)$. This parametrization captures every nondegenerate double ordinate with its upper endpoint designated P.

Step 2: Apply the section formula and eliminate the parameter Because $PR : RQ = 2 : 1$, the point nearer Q is

$$R = \frac{P + 2Q}{3} = \left(-t^2, -\frac{2t}{3}\right).$$

Thus, if $R = (x, y)$, then $x = -t^2$ and $y^2 = 4t^2/9$. Elimination of t gives

$$9y^2 = 4t^2 = -4x.$$

The negative sign is essential: all the points have negative abscissa, so the positive-sign alternatives (B) and (D) cannot hold. Option (A) uses the wrong scale factor; for example $t = 1$ gives $3y^2 = 4/3$ whereas $-2x = 2$. The stated internal division selects the lower branch, but its Cartesian conic equation is the listed one.

Step 3: Conclusion The locus satisfies $9y^2 = -4x$. Hence, the correct option is (C).

Quick Tip to Solve: For a double ordinate use endpoint ordinates v and $-v$; a 2:1 division makes the new ordinate one-third of an endpoint ordinate.

83. The shortest distance between the z-axis and the line x + y + 2z − 3 = 0 = 2x + 3y + 4z − 4, is: [4 marks]

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Concept:

- The given space line is the intersection of its two defining planes; its direction vector is perpendicular to both plane normals.
- The shortest distance between lines through A, B, with direction vectors u, v, is $|(B - A) \cdot (u \times v)|/|u \times v|$ when the directions are not parallel.

Step 1: Find a point and direction for the given line The two planes are $x + y + 2z = 3$ and $2x + 3y + 4z = 4$. Subtract twice the first from the second to obtain $y = -2$. The first then gives $x + 2z = 5$. Hence the line is

$$(x, y, z) = (5, -2, 0) + t(-2, 0, 1).$$

The other line, the z-axis, is $(0, 0, s)$. In particular, every point of the given line has ordinate -2, while every point of the axis has ordinate zero.

Step 2: Minimize the separation Take a point $(5 - 2t, -2, t)$ on the given line and $(0, 0, s)$ on the axis. Their squared distance is

$$D^2 = (5 - 2t)^2 + 4 + (t - s)^2.$$

Both variable squared terms can vanish simultaneously: choose $t = 5/2$, $s = 5/2$. Therefore $D^2_{\text{min}} = 4$, and the shortest distance is 2. The joining segment is parallel to the y-axis and perpendicular to both line directions, confirming minimality geometrically. A distance 1 is impossible because the ordinate difference alone is 2. Distances 3 and 4 are larger than the exhibited perpendicular segment, so cannot be shortest.

Step 3: Conclusion The shortest distance is 2. Hence, the correct option is (B).

Quick Tip to Solve: Eliminate one coordinate from the two plane equations first; a fixed coordinate may expose the common perpendicular immediately.

84. A plane containing the point (3, 2, 0) and the line (x−1)/1 = (y−2)/5 = (z−3)/4 also contains the point [4 marks]

(A) (0, 7, −10)

(B) (0, 7, 10)

(C) (0, 3, 1)

(D) (0, −3, 1)

Correct Answer: (B) (0, 7, 10)

Solution:

Concept:

- A plane containing a line has a normal perpendicular to the direction vector of that line.
- A second vector in the plane joins any point on the line to the additional given point; their cross product supplies a normal.

Step 1: Construct two vectors in the required plane Using the symmetric equation, put all three ratios equal to t. The line becomes

$$(x, y, z) = (1, 2, 3) + t(1, 5, 4).$$

So a point on the line is $A = (1, 2, 3)$, and its direction is $u = (1, 5, 4)$. The extra point is $P = (3, 2, 0)$, giving $AP = (2, 0, -3)$. These vectors are not parallel, so together they determine a unique plane. A normal is their cross product

$$u \times AP = (-15, 11, -10).$$

Step 2: Form the equation and test the alternatives Using the normal and the point A, the plane equation is

$$-15(x - 1) + 11(y - 2) - 10(z - 3) = 0,$$

which simplifies to $-15x + 11y - 10z + 23 = 0$. The given extra point satisfies it: $-45 + 22 + 23 = 0$, as required. Evaluating its left-hand side at the proposed points gives 200 for (A), 0 for (B), 46 for (C), and -20 for (D). Thus precisely $(0, 7, 10)$ lies in the plane. In particular, changing the sign of the last coordinate from 10 to -10 changes the plane expression by 200, so the visually similar first alternative fails.

Step 3: Conclusion The plane contains $(0, 7, 10)$. Hence, the correct option is (B).

Quick Tip to Solve: After obtaining the normal, test every candidate in one linear equation; also substitute the supplied point to catch a sign error.

85. In a parallelogram ABCD, $\left|\overrightarrow{AB}\right| = a$, $\left|\overrightarrow{AD}\right| = b$ and $\left|\overrightarrow{AC}\right| = c$, then $\overrightarrow{DB} \cdot \overrightarrow{AB}$ has the value : [4 marks]

(A) $\frac{1}{2}\left(a^2 - b^2 + c^2\right)$

(B) $\frac{1}{4}\left(a^2 + b^2 - c^2\right)$

(C) $\frac{1}{3}\left(b^2 + c^2 - a^2\right)$

(D) $\frac{1}{2}\left(a^2 + b^2 + c^2\right)$

Correct Answer: None of the listed expressions equals the requested scalar product in general.

Solution:

Concept:

- Write the adjacent side vectors as $\mathbf{u} = \overrightarrow{AB}$ and $\mathbf{v} = \overrightarrow{AD}$, so $|\mathbf{u}| = a$ and $|\mathbf{v}| = b$.
- The diagonals satisfy $\overrightarrow{AC} = \mathbf{u} + \mathbf{v}$ and $\overrightarrow{DB} = \mathbf{u} - \mathbf{v}$. The squared norm of a sum is obtained by taking its scalar product with itself.

Step 1: Express the side scalar product using the given diagonal Taking A as the vector origin gives B at $\mathbf{u}$, D at $\mathbf{v}$, and C at $\mathbf{u} + \mathbf{v}$. Consequently the specified diagonal length gives

$$c^2 = (\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} + \mathbf{v}) = a^2 + b^2 + 2\mathbf{u} \cdot \mathbf{v}.$$

Therefore $\mathbf{u} \cdot \mathbf{v} = (c^2 - a^2 - b^2)/2$. The direction of the other diagonal matters: the displacement from D to B is the position of B minus the position of D, rather than their sum.

Step 2: Evaluate the requested product and test the alternatives Use that directed displacement in the requested scalar product:

$$\overrightarrow{DB} \cdot \overrightarrow{AB} = (\mathbf{u} - \mathbf{v}) \cdot \mathbf{u} = a^2 - \frac{c^2 - a^2 - b^2}{2} = \frac{3a^2 + b^2 - c^2}{2}.$$

None of the four printed alternatives has these coefficients. For an explicit check, take perpendicular sides with $a = 2$, $b = 1$, so $c^2 = 5$. The requested product is then 4, whereas (A), (B), (C), and (D) give $4, 0, 2/3, 5$, respectively. Although (A) happens to agree for perpendicular sides, it is not an identity: for $a = b = 1$ and the included angle 60°, we have $c^2 = 3$, the product is $1/2$, and (A) gives $3/2$. This also rules out its apparent agreement in the first check.

Step 3: Conclusion As written, the scalar product is $\boxed{(3a^2 + b^2 - c^2)/2}$. No listed option is correct in general; in particular, the expression in (A) does not equal this result for an arbitrary parallelogram.

Quick Tip to Solve: Write each directed diagonal as endpoint position minus starting position before taking any scalar product.

86. If the lengths of the sides of a triangle are decided by the three throws of a single fair die, then the probability that the triangle is of maximum area given that it is an isosceles triangle, is [4 marks]

(A) 1/27

(B) 1/26

(C) 1/21

(D) 1/15

Correct Answer: (C) 1/21

Solution:

Concept:

- Treat an isosceles triangle here as having exactly two equal sides, so the equilateral outcomes are excluded. Write its side lengths as (r, r, s), where $r \neq s$ and $s < 2r$.
- Ordered outcomes of three fair-die throws are equally likely. Conditional probability is the number of favourable ordered outcomes divided by the number of ordered outcomes satisfying the condition.
- An isosceles triangle with equal sides r and base s has area $K = \frac{s}{4}\sqrt{4r^2 - s^2}$.

Step 1: Count the valid isosceles outcomes The equal side r can be any integer from 1 to 6. For each value, count unequal base values $s \in \{1, 2, 3, 4, 5, 6\}$ satisfying the strict triangle inequality $s < 2r$. For $r = 1, 2, 3, 4, 5, 6$, the respective numbers of permissible bases are $0, 2, 4, 5, 5, 5$. Their total is 21. Every such pair (r, s) has exactly three ordered die outcomes, according to the position of its unequal side. The conditioning event therefore contains $3 \times 21 = 63$ equally likely outcomes; degenerate triples with $s = 2r$ are not triangles.

Step 2: Identify the triangle of greatest area For fixed base s, the area increases with the equal side r. When $s \leq 5$, its largest possible value is consequently obtained with $r = 6$. For this equal side, $K^2 = s^2(144 - s^2)/16$ increases over $1 \leq s \leq 5$, so the largest such area occurs at $(6, 6, 5)$, giving $K^2 = 2975/16$. If $s = 6$, the largest allowed unequal repeated side is $r = 5$, which gives $K^2 = 144$, smaller than $2975/16$. Thus the favourable outcomes are precisely the three permutations of $(6, 6, 5)$.

Step 3: Form the conditional probability Dividing the three favourable ordered outcomes by all 63 ordered outcomes in the conditioning event gives

$$P(\text{maximum area} \mid \text{isosceles}) = \frac{3}{63} = \frac{1}{21}.$$

Options (A), (B), and (D) have denominators that do not follow from this count of valid triangles. Counting the 21 unordered repeated-side/base choices also gives the same ratio, because each has the same three permutations. The exclusion of equilateral triangles is relevant to the interpretation: if they were included, the unique largest triangle would instead be $(6, 6, 6)$, and there would be 69 conditioned ordered outcomes.

Step 4: Conclusion Using the exactly-two-equal-sides interpretation of isosceles triangle, the required probability is $\boxed{1/21}$. Hence, the correct option is (C). The answer key gives a different value, but $1/27$ contradicts the explicit count of three favourable outcomes among 63 valid ordered outcomes; the stated conditions give (C).

Quick Tip to Solve: Count repeated-side/base pairs first, enforce the strict triangle inequality, and only then count their three permutations.

87. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than or equal to one is [4 marks]

(A) 1/16

(B) 9/16

(C) 3/4

(D) 15/16

Correct Answer: (D) 15/16

Solution:

Concept:

- For a binomial variable with parameters n, p, the mean is np and the variance is $np(1-p)$. Here n is the number of independent trials and p the success probability.
- The event $X \geq 1$ is the complement of no successes. Thus $P(X \geq 1) = 1 - P(X = 0)$, with $P(X = 0) = (1-p)^n$.

Step 1: Determine the success probability The two given moments describe the same distribution, so divide the variance by the mean to eliminate the unknown trial count. This is valid because the mean is nonzero. We obtain

$$1 - p = \frac{np(1-p)}{np} = \frac{1}{2}, \qquad p = \frac{1}{2}.$$

Both success and failure therefore have probability one half; the variance is smaller than the mean as required for a nondegenerate binomial distribution.

Step 2: Find the number of trials and use the complement Substitute the success probability into the mean:

$$n\left(\frac{1}{2}\right) = 2 \quad \Rightarrow \quad n = 4.$$

For zero successes, all four trials must fail, and independence allows multiplication of their failure probabilities:

$$P(X = 0) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}.$$

Consequently

$$P(X \geq 1) = 1 - \frac{1}{16} = \frac{15}{16}.$$

Option (A) is the complementary event, not the requested event. Options (B) and (C) do not equal the complement for these four trials.

Step 3: Conclusion The required probability is $\boxed{15/16}$. Hence, the correct option is (D).

Quick Tip to Solve: Divide variance by mean to obtain the failure probability, then use the complement for at least one success.

88. If cos α + cos β = (3)/(2) and sin α + sin β = (1)/(2) and θ is the arithmetic mean of α and β, then sin 2θ + cos 2θ is equal to: [4 marks]

(A) (3)/(5)

(B) (7)/(5)

(C) (4)/(5)

(D) (8)/(5)

Correct Answer: (B) (7)/(5)

Solution:

Concept:

- Put $\theta = (\alpha+\beta)/2$ and $\delta = (\alpha-\beta)/2$. Then the cosine and sine sums are $2\cos\theta\cos\delta$ and $2\sin\theta\cos\delta$.
- The double-angle identities give $\sin 2\theta = 2\tan\theta/(1+\tan^2\theta)$ and $\cos 2\theta = (1-\tan^2\theta)/(1+\tan^2\theta)$ whenever the tangent is defined.

Step 1: Take the ratio of the given sums The cosine sum is $3/2$, which is nonzero. Thus both $\cos\theta$ and $\cos\delta$ are nonzero, and division does not discard a possible solution. Using the two sum-to-product identities,

$$\frac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta} = \frac{2\sin\theta\cos\delta}{2\cos\theta\cos\delta} = \tan\theta = \frac{1/2}{3/2} = \frac{1}{3}.$$

There is no need to determine either original angle separately. A possible addition of an integer multiple of π to θ does not change its double-angle sine or cosine.

Step 2: Evaluate both double-angle terms Substitute $\tan\theta = 1/3$ into the rational forms of the double-angle identities:

$$\sin 2\theta = \frac{2/3}{1+1/9} = \frac{3}{5}, \qquad \cos 2\theta = \frac{1-1/9}{1+1/9} = \frac{4}{5}.$$

Therefore

$$\sin 2\theta + \cos 2\theta = \frac{3}{5} + \frac{4}{5} = \frac{7}{5}.$$

Options (A) and (C) each give only one of the two terms. Option (D) gives neither their sum nor either individual value.

Step 3: Conclusion The requested sum is $\boxed{7/5}$. Hence, the correct option is (B).

Quick Tip to Solve: Divide the sine sum by the cosine sum to get the tangent of the mean angle directly.

89. Let 10 vertical poles standing at equal distances on a straight line, subtend the same angle of elevation α at a point O on this line and all the poles are on the same side of O. If the height of the longest pole is h and the distance of the foot of the smallest pole from O is a; then the distance between two consecutive poles, is [4 marks]

(A) (h sinα + a cosα)/(9 sinα)

(B) (h cosα − a sinα)/(9 sinα)

(C) (h cosα − a sinα)/(9 cosα)

(D) (h sinα + a cosα)/(9 cosα)

Correct Answer: (B) (h cosα − a sinα)/(9 sinα)

Solution:

Concept:

- For a vertical pole at horizontal distance x from the observation point, its height is $x\tan\alpha$ when its angle of elevation is α.
- Ten equally spaced pole feet have nine intervening gaps. If the common spacing is d, the farthest foot is at distance $a+9d$.

Step 1: Locate the longest pole All poles are on the same side of the observation point, so their positive distances increase in their order along the line. Since the angle of elevation is the same for every pole, height is proportional to distance. Hence the longest pole is the farthest one, and the shortest is the nearest. The ten foot distances form the arithmetic progression

$$a,\ a+d,\ a+2d,\ldots,\ a+9d.$$

The last distance has nine spacings added to the first, rather than ten; the poles themselves are ten in number.

Step 2: Use the right triangle for the farthest pole The right triangle with vertical side h and horizontal side $a+9d$ gives

$$\tan\alpha=\frac{h}{a+9d}.$$

Rearranging in successive steps,

$$a+9d=h\cot\alpha,\qquad d=\frac{h\cot\alpha-a}{9}=\frac{h\cos\alpha-a\sin\alpha}{9\sin\alpha}.$$

The subtraction is essential because the nearest pole is already a distance a from the observer. Options (A) and (D) incorrectly add that initial distance. Option (C) has a cosine denominator and therefore does not reproduce the tangent relation.

Step 3: Conclusion The spacing is $\boxed{\frac{h\cos\alpha-a\sin\alpha}{9\sin\alpha}}$. Hence, the correct option is (B).

Quick Tip to Solve: Count the gaps between the poles first: ten poles give nine equal spacings, then subtract the nearest-foot distance.

90. Consider the following statements: P: Suman is brilliant Q: Suman is rich R: Suman is honest The negation of the statement, "Suman is brilliant and dishonest if and only if Suman is rich" can be equivalently expressed as [4 marks]

(A) ~Q ↔~P∨R

(B) ~Q ↔P∨~R

(C) ~Q ↔P∧~R

(D) ~Q ↔~P∧R

Correct Answer: (C) ~Q ↔P∧~R

Solution:

Concept:

- The statement that Suman is brilliant and dishonest is $P \wedge \neg R$, since R denotes honesty.
- A biconditional $A \leftrightarrow Q$ is true when its two truth values agree. Its negation is true when they differ, equivalently $A \leftrightarrow \neg Q$.

Step 1: Translate the complete statement Keep the conjunction within the first side of the biconditional. The original statement is

$$(P \wedge \neg R) \leftrightarrow Q.$$

Its negation applies to this entire expression; it does not mean separately negating every constituent word. Introduce $A = P \wedge \neg R$ to make the logical structure transparent. The requested statement is then $\neg(A \leftrightarrow Q)$.

Step 2: Negate the equivalence and compare the options If A and Q are both true or both false, the original equivalence is true and its negation false. If exactly one is true, the negation is true. Replacing Q by $\neg Q$ in the equivalence gives precisely that truth pattern:

$$\neg(A \leftrightarrow Q) \equiv A \leftrightarrow \neg Q \equiv \neg Q \leftrightarrow (P \wedge \neg R).$$

This is option (C). Options (A) and (B) replace the required conjunction by a disjunction, altering which combinations make the first side true. Option (D) instead describes being non-brilliant and honest, which is a different conjunction from being brilliant and dishonest.

Step 3: Conclusion The equivalent negation is $\boxed{\neg Q \leftrightarrow (P \wedge \neg R)}$. Hence, the correct option is (C).

Quick Tip to Solve: Negate a biconditional by complementing exactly one of its two sides; retain the internal conjunction unchanged.