JEE Main 2016 Paper 1 (B.E./B.Tech) Apr 9 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2016, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 9 April 2016, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. Let x, y, z be positive real numbers such that x + y + z = 12 and $x^3y^4z^5 = (0.1)(600)^3$. Then $x^3 + y^3 + z^3$ is equal to : [4 marks]

(A) 270

(B) 258

(C) 342

(D) 216

Correct Answer: (D) 216

Solution:

Concept:

- For positive numbers the arithmetic mean is at least the geometric mean, with equality only when all the numbers being averaged are equal. Apply this to twelve entries: three copies of $x/3$, four of $y/4$, and five of $z/5$. Their sum is $x + y + z = 12$, so their arithmetic mean is exactly 1.
- Check the computed result against the printed options after completing the calculation.

Step 1: Bound the given product The arithmetic-geometric mean inequality gives

$$\left(\frac{x}{3}\right)^3 \left(\frac{y}{4}\right)^4 \left(\frac{z}{5}\right)^5 \leq 1.$$

Consequently $x^3y^4z^5 \leq 3^3 4^4 5^5 = 21,600,000$. The specified value is $0.1(600)^3 = 21,600,000$, so this upper bound is attained exactly. The equality condition is therefore compulsory, rather than an arbitrary guess of convenient values.

Step 2: Use equality and evaluate the cubes Equality requires $x/3 = y/4 = z/5 = 1$, giving $x = 3, y = 4, z = 5$. These values satisfy the sum and the original product. Hence

$$x^3 + y^3 + z^3 = 27 + 64 + 125 = 216.$$

The other offered values do not equal this sum, and no different positive triple can attain the product maximum under the same sum constraint.

Step 3: Conclusion The required sum is 216. Hence, the correct option is (D).

Quick Tip to Solve: When the powers add to the fixed sum, divide each variable by its exponent and apply weighted AM-GM; equality determines the variables.

2. If the mean deviation of the numbers 1, 1 + d, ..., 1 +100d from their mean is 255, then a value of d is : [4 marks]

(A) 10.1

(B) 20.2

(C) 10

(D) 5.05

Correct Answer: (A) 10.1

Solution:

Concept:

- Mean deviation about the mean is the average of the absolute differences from that mean. For an arithmetic progression, terms equidistant from the middle have equal absolute deviations. Since the question asks for a value of the common difference, its magnitude is sufficient.
- Check the computed result against the printed options after completing the calculation.

Step 1: Locate the mean and pair the deviations There are 101 terms, indexed by $k = 0, 1, \ldots, 100$, with value $1 + kd$. Their mean is the middle term $1 + 50d$. Subtracting this from the kth term gives $(k - 50)d$, whose absolute value is $|k - 50||d|$. The middle deviation is zero and every integer distance from 1 through 50 appears twice.

Step 2: Evaluate the mean deviation and solve Thus the sum of the absolute deviations is

$$2|d|(1 + 2 + \cdots + 50) = 2|d|\frac{50 \cdot 51}{2} = 2550|d|.$$

Dividing by the number of observations, not by 100 intervals, gives

$$255 = \frac{2550|d|}{101}, \qquad |d| = 10.1.$$

Both signs give the same mean deviation, and the positive value 10.1 is listed. Substitution of 20.2 would double the required deviation, while 10 and 5.05 give different results.

Step 3: Conclusion A possible value is $d = 10.1$. Hence, the correct option is (A).

Quick Tip to Solve: For an odd number of equally spaced data, centre the indices at zero; paired absolute deviations turn the calculation into an arithmetic sum.

3. If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = (\tan x)^A + C(\tan x)^B + k$,
where k is a constant of integration, then A + B +C equals : [4 marks]

(A) $\frac{21}{5}$
(B) $\frac{16}{5}$
(C) $\frac{7}{10}$
(D) $\frac{27}{10}$

Correct Answer: (B) $\frac{16}{5}$

Solution:

Concept:

- Use $t = \tan x$, so $dx = \cos^2 x\, dt$ and $\sin 2x = 2\tan x \cos^2 x$. On an interval with $\cos x > 0$ and $\tan x > 0$, the square root becomes $2\sqrt{t}\cos x$. The resulting antiderivative identifies the powers and coefficient in the question.
- Check the computed result against the printed options after completing the calculation.

Step 1: Transform the integrand With the stated substitution, combine the factors carefully:

$$\frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = \frac{\cos^2 x\, dt}{2\cos^4 x\sqrt{t}} = \frac{1+t^2}{2\sqrt{t}}\, dt.$$

The extra factor $1 + t^2$ comes from $\sec^2 x$; omitting it would discard one of the two powers. The original integrand is real where $\sin 2x > 0$, away from zeros of its denominator.

Step 2: Integrate the powers and identify constants Integrating term by term gives

$$\frac{1}{2}\int (t^{-1/2} + t^{3/2})\, dt = t^{1/2} + \frac{1}{5}t^{5/2} + k.$$

Therefore $A = 1/2, B = 5/2, C = 1/5$. Differentiating this expression gives $(1+t^2)/(2\sqrt{t})$, confirming the transformed integral. These are the constants in the displayed form; on intervals with negative cosine the corresponding antiderivative has the opposite overall sign.

Step 3: Conclusion Thus $A + B + C = \frac{1}{2} + \frac{5}{2} + \frac{1}{5} = \frac{16}{5}$. Hence, the correct option is (B).

Quick Tip to Solve: Before integrating after a tangent substitution, simplify the entire differential; the square root contributes an additional cosine factor.

4. If the function

$$\mathbf{f(x)} = \begin{cases} -x & x < 1 \\ a + \cos^{-1}(x+b), & 1 \le x \le 2 \end{cases}$$

is differentiable at x = 1, then $\frac{a}{b}$ is equal to : [4 marks]

(A) $\frac{\pi-2}{2}$

(B) $\frac{-\pi-2}{2}$

(C) $\frac{\pi+2}{2}$

(D) $-1-\cos^{-1}(2)$

Correct Answer: (C) $\frac{\pi+2}{2}$

Solution:

Concept:

- Differentiability at a joining point requires both continuity and equality of the one-sided derivatives. The derivative of $\arccos u$ is $-u'/\sqrt{1-u^2}$ when $|u| < 1$. First determine the shift from the slopes, then determine the additive constant from continuity.
- Check the computed result against the printed options after completing the calculation.

Step 1: Match the one-sided derivatives The branch $-x$ has left derivative -1 at 1. The right branch has derivative $-1/\sqrt{1-(x+b)^2}$. Matching these at the joining point gives

$$-\frac{1}{\sqrt{1-(1+b)^2}} = -1, \qquad (1+b)^2 = 0, \qquad b = -1.$$

The argument at the join is therefore zero, inside the inverse-cosine domain. The shifted argument ranges from zero to one on the given right-hand interval.

Step 2: Enforce continuity at the join The left limit is -1. With $b = -1$, the value from the second branch at 1 is $a + \arccos 0 = a + \pi/2$. Thus

$$a + \frac{\pi}{2} = -1, \qquad a = -1 - \frac{\pi}{2}.$$

Both slope and value conditions are now satisfied. Dividing by $b = -1$ reverses the sign; retaining the negative value of a as the ratio would be an error.

Step 3: Conclusion The ratio is $a/b = 1 + \pi/2 = (\pi+2)/2$. Hence, the correct option is (C).

Quick Tip to Solve: For a piecewise inverse-trigonometric branch, match the derivative first to find the horizontal shift, then use continuity for the vertical shift.

5. If f(x) is a differentiable function in the interval (0, ∞) such that f (1) = 1 and $\lim_{t\to x} \frac{t^2 f(x) - x^2 f(t)}{t-x} = 1$, for each x > 0, then $f\left(\frac{3}{2}\right)$ equal to : [4 marks]

(A) $\frac{13}{6}$

(B) $\frac{23}{18}$

(C) $\frac{25}{9}$

(D) $\frac{31}{18}$

Correct Answer: (D) $\frac{31}{18}$

Solution:

Concept:

- Rewrite the numerator so the definition of the derivative applies. The resulting equation is linear in f. Multiplication by an integrating factor converts it into an exact derivative; the condition at x=1 determines its constant uniquely on the positive interval.
- Check the computed result against the printed options after completing the calculation.

Step 1: Convert the limit into a differential equation Add and subtract $x^2 f(x)$ in the numerator. Then the quotient becomes

$$f(x)\frac{t^2 - x^2}{t - x} - x^2 \frac{f(t) - f(x)}{t - x}.$$

As $t \to x$, differentiability gives $2xf(x) - x^2 f'(x) = 1$. Since $x > 0$, division by $-x^2$ is permitted and yields $f' - 2f/x = -1/x^2$. The minus sign before the derivative is essential.

Step 2: Integrate and apply the initial condition The integrating factor is x^{-2}. Hence

$$\left(\frac{f(x)}{x^2}\right)' = -x^{-4}, \qquad \frac{f(x)}{x^2} = \frac{1}{3x^3} + K.$$

The condition $f(1) = 1$ implies $K = 2/3$, so $f(x) = 1/(3x) + 2x^2/3$. Substitution back gives $2xf - x^2 f' = 1$, confirming the equation. Evaluating at $3/2$ gives $2/9 + 3/2 = 31/18$, excluding all other listed fractions.

Step 3: Conclusion Therefore $f(3/2) = 31/18$. Hence, the correct option is (D).

Quick Tip to Solve: Split a mixed difference quotient into a polynomial difference and the derivative quotient before taking its limit.

6. Let a and b respectively be the semitransverse and semi-conjugate axes of a hyperbola whose eccentricity satisfies the equation $9e^2 - 18e + 5 = 0$. If S(5, 0) is a focus and $5x = 9$ is the corresponding directrix of this hyperbola, then $a^2 - b^2$ is equal to : [4 marks]

(A) 7

(B) -7

(C) 5

(D) -5

Correct Answer: (B) -7

Solution:

Concept:

- For a hyperbola, eccentricity exceeds one and satisfies $b^2 = a^2(e^2 - 1)$. Its centre-to-focus distance is ae, while the corresponding directrix lies a distance a/e from the centre on the same side. The focus-directrix separation is therefore $a(e - 1/e)$.
- Check the computed result against the printed options after completing the calculation.

Step 1: Select the hyperbolic eccentricity Factor the given equation:

$$9e^2 - 18e + 5 = (3e - 1)(3e - 5) = 0.$$

The roots are $1/3$ and $5/3$. The former is inadmissible for a hyperbola, so $e = 5/3$. The focus is at x=5 and its directrix is at x=9/5, giving separation $5 - 9/5 = 16/5$. This positive separation is independent of the unknown centre.

Step 2: Find the two squared semiaxes Using that separation,

$$a\left(\frac{5}{3} - \frac{3}{5}\right) = \frac{16}{5}, \qquad a = 3.$$

Thus $a^2 = 9$, and

$$b^2 = 9\left(\frac{25}{9} - 1\right) = 16.$$

Indeed the centre is at x=0, the focus distance is $ae = 5$, and $a/e = 9/5$ reproduces the directrix. The requested difference is negative because the conjugate semiaxis is longer here; a hyperbola does not require a to exceed b.

Step 3: Conclusion Hence $a^2 - b^2 = 9 - 16 = -7$. Hence, the correct option is (B).

Quick Tip to Solve: Choose e greater than one before using hyperbola formulae; the focus-directrix separation also avoids assuming the centre in advance.

7. For $x \in R$, $x \neq -1$,

if $(1+x)^{2016} + x(1+x)^{2015} + x^2(1+x)^{2014} + \ldots + x^{2016} = \sum_{i=0}^{2016} a_i x^i$, then a_{17} is equal to : [4 marks]

(A) $\frac{2017!}{17!\ 2000!}$

(B) $\frac{2016!}{17!\ 1999!}$

(C) $\frac{2017!}{2000!}$

(D) $\frac{2016!}{16!}$

Correct Answer: (A) $\frac{2017!}{17!\ 2000!}$

Solution:

Concept:

- The expression is a finite geometric sum in two bases, $1+x$ and x. The identity $\sum_{k=0}^{n} a^{n-k}b^k = (a^{n+1} - b^{n+1})/(a-b)$ follows by multiplying the sum by a-b and cancelling neighbouring terms. After this reduction use the binomial theorem.
- Check the computed result against the printed options after completing the calculation.

Step 1: Collapse the finite sum Set $a = 1+x, b = x, n = 2016$. Their difference is exactly 1, so the polynomial in the question becomes

$$\sum_{k=0}^{2016} (1+x)^{2016-k} x^k = (1+x)^{2017} - x^{2017}.$$

The subtraction cancels the highest-degree term and leaves degree 2016, consistent with the required expansion. This derivation uses the polynomial identity directly, so it does not introduce a problematic ratio or require division by x.

Step 2: Extract the coefficient of the required power By the binomial theorem the coefficient of x^{17} in $(1+x)^{2017}$ is $\binom{2017}{17}$. The subtracted monomial x^{2017} has no contribution to that coefficient. Consequently

$$a_{17} = \binom{2017}{17} = \frac{2017!}{17!\,2000!}.$$

The coefficient from a power 2016 would ignore the contributions of the remaining summands. The expressions without both factorial factors are permutations rather than this binomial coefficient.

Step 3: Conclusion The coefficient is $2017!/(17!\,2000!)$. Hence, the correct option is (A).

Quick Tip to Solve: Recognise descending powers of one base paired with ascending powers of another; telescope first and extract the binomial coefficient afterward.

8. If A and B are any two events such that P(A) = $\frac{2}{5}$ and P (A $\cap$ B) = $\frac{3}{20}$, hen the conditional probability, P(A | (A' $\cup$ B')), where A' denotes the complement of A, is equal to : [4 marks]

(A) $\frac{1}{4}$

(B) $\frac{5}{17}$

(C) $\frac{8}{17}$

(D) $\frac{11}{20}$

Correct Answer: (B) $\frac{5}{17}$

Solution:

Concept:

- Conditional probability is $P(A \mid C) = P(A \cap C)/P(C)$, provided $P(C) > 0$. The conditioning event changes the sample space.
- De Morgan's law gives $A' \cup B' = (A \cap B)'$. Thus the excluded outcomes are precisely those belonging to both events.

Step 1: Identify the conditioning event Set $C = A' \cup B'$. Its complement is $A \cap B$, whose probability is given. Hence

$$P(C) = 1 - P(A \cap B) = 1 - \frac{3}{20} = \frac{17}{20}.$$

This is positive, so the required conditional probability is defined. No separate value of $P(B)$ is needed.

Step 2: Find the surviving part of A Within A, the event C removes the overlap with B. Therefore $A \cap C = A \cap B'$, and the disjoint decomposition of A gives

$$P(A \cap C) = P(A) - P(A \cap B) = \frac{2}{5} - \frac{3}{20} = \frac{1}{4}.$$

The probability $1/4$ is the numerator, rather than the final conditional probability.

Step 3: Divide by the new sample-space probability Using the definition,

$$P(A \mid C) = \frac{1/4}{17/20} = \frac{5}{17}.$$

Option (A) omits the conditioning denominator. Option (C) would retain all of A rather than remove its overlap with B. Option (D) does not equal the required ratio. These checks also show why treating the two original events as independent would be unjustified.

Step 4: Conclusion The conditional probability is $\boxed{5/17}$. Hence, the correct option is (B).

Quick Tip to Solve: For conditioning on a union of complements, use De Morgan's law first; then subtract the excluded overlap from the numerator as well.

9. A circle passes through (−2, 4) and touches the y-axis at (0, 2). Which one of the following equations can represent a diameter of this circle? [4 marks]

(A) 4x + 5y − 6 = 0

(B) 2x − 3y + 10 = 0

(C) 3x + 4y − 3 = 0

(D) 5x + 2y + 4 = 0

Correct Answer: (B) 2x − 3y + 10 = 0

Solution:

Concept:

- A tangent to a circle is perpendicular to the radius at the point of contact. A circle touching the y-axis at $(0, 2)$ therefore has centre $(h, 2)$.
- Every diameter lies on a line through the centre. Once the centre is known, substitution in each candidate line is enough; finding diameter endpoints is unnecessary.

Step 1: Use the given point to locate the centre The radius is the distance of $(h, 2)$ from the tangent $x = 0$, so $r = |h|$. Since $(-2, 4)$ lies on the circle,

$$(-2 - h)^2 + (4 - 2)^2 = h^2.$$

Expanding gives $h^2 + 4h + 4 + 4 = h^2$, so $4h + 8 = 0$ and $h = -2$. Thus the centre is $C = (-2, 2)$, and the radius is 2. This also checks the stated point, which lies directly above the centre by one radius.

Step 2: Test which line passes through the centre Substitute $x = -2, y = 2$ into the four left-hand sides. For (A), $4(-2) + 5(2) - 6 = -4$. For (B), $2(-2) - 3(2) + 10 = 0$. For (C), $3(-2) + 4(2) - 3 = -1$. For (D), $5(-2) + 2(2) + 4 = -2$. Thus only the line in option (B) passes through the centre. Since its normal vector is nonzero, it is an ordinary line, and its intersection with the circle is a diameter. Any other candidate misses the centre and can at most define a non-diametral chord.

Step 3: Conclusion The required diameter is $\boxed{2x - 3y + 10 = 0}$. Hence, the correct option is (B).

Quick Tip to Solve: First locate the centre from tangency, then test the centre in each line. A diameter line must pass through it.

10. The area (in sq. units) of the region described by

A= {(x, y) |y$\geq$ x^2 $-$ 5x + 4, x + y $\geq$ 1, y $\leq$ 0} is : [4 marks]

(A) $\frac{7}{2}$

(B) $\frac{19}{6}$

(C) $\frac{13}{6}$

(D) $\frac{17}{6}$

Correct Answer: (B) $\frac{19}{6}$

Solution:

Concept:

- The inequalities impose upper boundary $y = 0$ and lower boundary equal to the larger of x^2-5x+4 and $1 - x$. Area is the integral of upper minus lower boundary.
- The lower boundary can change where the line and parabola meet. Their intersections must be found before selecting the integration intervals.

Step 1: Find the feasible interval and crossings Write $p(x) = x^2 - 5x + 4 = (x - 1)(x - 4)$ and $\ell(x) = 1 - x$. To admit a point below $y = 0$, both lower bounds must be nonpositive. Thus $1 \leq x \leq 4$. The two boundaries meet when

$$p(x) - \ell(x) = x^2 - 4x + 3 = (x - 1)(x - 3) = 0.$$

Their crossing abscissae are 1 and 3. Between 1 and 3, this difference is negative, so the line is higher. Between 3 and 4, the parabola is higher.

Step 2: Integrate the two vertical strip heights The region therefore has area

$$\mathcal{A} = \int_1^3 (x - 1)\, dx + \int_3^4 (-x^2 + 5x - 4)\, dx.$$

The first integral is $[(x - 1)^2/2]_1^3 = 2$. For the second,

$$\left[-\frac{x^3}{3} + \frac{5x^2}{2} - 4x\right]_3^4 = \frac{8}{3} - \frac{3}{2} = \frac{7}{6}.$$

Consequently $\mathcal{A} = 2 + 7/6 = 19/6$. Both strip heights are nonnegative on their respective intervals, as an area calculation requires. Integrating just one lower curve over the full interval would include points that violate the other inequality. None of the remaining three numerical choices equals this sum.

Step 3: Conclusion The area is $\boxed{19/6}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: When two inequalities give lower bounds on y, integrate above their maximum; switch boundaries at their intersections.

11. If the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from -1, then $|b|$ is equal to : [4 marks]

(A) $\sqrt{2}$

(B) 2

(C) 3

(D) $\sqrt{3}$

Correct Answer: (D) $\sqrt{3}$

Solution:

Concept:

- A common root must satisfy both equations simultaneously. Eliminating the parameter using one equation gives a necessary algebraic condition on that root.
- Complex coefficients and roots are permitted by the statement. For $b = u + iv$, its modulus is $|b| = \sqrt{u^2 + v^2}$.

Step 1: Eliminate the parameter using the common root Let the common root be $\alpha \neq -1$. From the second quadratic, $b = -\alpha^2 - \alpha$. Substitute this into the first:

$$\alpha^2 + (-\alpha^2 - \alpha)\alpha - 1 = 0,$$

which simplifies to $\alpha^3 = -1$. Factor to obtain

$$(\alpha + 1)(\alpha^2 - \alpha + 1) = 0.$$

Since the common root is explicitly different from -1, it must satisfy $\alpha^2 - \alpha + 1 = 0$. Thus $\alpha = (1 \pm i\sqrt{3})/2$. The excluded root would instead lead to $b = 0$, so retaining it would violate the condition.

Step 2: Recover b and calculate its modulus The remaining quadratic identity gives $\alpha^2 = \alpha - 1$. Hence

$$b = -\alpha^2 - \alpha = 1 - 2\alpha = \mp i\sqrt{3}.$$

For either conjugate possibility,

$$|b| = \sqrt{0^2 + (\sqrt{3})^2} = \sqrt{3}.$$

Substitution of $b = -\alpha^2 - \alpha$ makes the second equation zero by construction, and $\alpha^3 = -1$ makes the first zero, so these values really do give a common root. The other listed moduli $\sqrt{2}, 2, 3$ cannot arise from either allowed value of b.

Step 3: Conclusion The parameter has modulus $\boxed{\sqrt{3}}$. Hence, the correct option is (D).

Quick Tip to Solve: For common-root problems, eliminate the parameter first and enforce any excluded-root condition before evaluating the requested quantity.

12. If m and M are the minimum and the maximum values of $4 + \frac{1}{2}\sin^2 2x - 2\cos^4 x$, $x \in \mathbb{R}$, then $M - m$ is equal to : [4 marks]

(A) $\frac{15}{4}$

(B) $\frac{9}{4}$

(C) $\frac{7}{4}$

(D) $\frac{1}{4}$

Correct Answer: (B) $\frac{9}{4}$

Solution:

Concept:

- Use $\sin^2 2x = 4\sin^2 x \cos^2 x$ and $\sin^2 x = 1 - \cos^2 x$ to express the function in one bounded variable.
- If $t = \cos^2 x$, then every value in $[0, 1]$ is attained for real x. The range is therefore obtained by optimizing a quadratic on this closed interval.

Step 1: Reduce to a quadratic and complete the square Put $t = \cos^2 x$. The expression becomes

$$f(t) = 4 + 2t(1 - t) - 2t^2 = 4 + 2t - 4t^2.$$

Completing the square gives

$$f(t) = \frac{17}{4} - 4\left(t - \frac{1}{4}\right)^2.$$

The coefficient of the squared term is negative, so the vertex is a maximum. Its location $t = 1/4$ lies inside the permitted interval, and is attainable, for example when $\cos x = 1/2$. Thus $M = 17/4$.

Step 2: Find the minimum and subtract For a concave quadratic on a closed interval, the minimum occurs at an endpoint. Evaluate $f(0) = 4$ and $f(1) = 2$, hence $m = 2$. These endpoint values correspond to real angles with $\cos x = 0$ and $\cos x = \pm 1$, respectively. Therefore

$$M - m = \frac{17}{4} - 2 = \frac{9}{4}.$$

The quadratic range is $[2, 17/4]$, which confirms both extrema and their difference. Maximizing the sine term and minimizing the cosine term separately would be invalid because their values are linked by the same angle. The other choices do not equal the difference of the actual attained extrema.

Step 3: Conclusion The required difference is $\boxed{9/4}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert even powers of sine and cosine to t = cos squared x, retain 0 to 1 as its domain, and check both endpoints as well as the vertex.

13. The value of $\sum_{r=1}^{15} r^2 \left(\frac{^{15}C_r}{^{15}C_{r-1}}\right)$ is equal to : [4 marks]

(A) 560

(B) 680

(C) 1240

(D) 1085

Correct Answer: (B) 680

Solution:

Concept:

- For adjacent binomial coefficients, factorial cancellation gives $\binom{n}{r}/\binom{n}{r-1} = (n-r+1)/r$. Here the denominator is nonzero for every $r = 1, \ldots, 15$.
- The standard sums are $\sum_{r=1}^{n} r = n(n+1)/2$ and $\sum_{r=1}^{n} r^2 = n(n+1)(2n+1)/6$.

Step 1: Cancel the factorials before summing For an individual summand,

$$\frac{\binom{15}{r}}{\binom{15}{r-1}} = \frac{15!}{r!(15-r)!}\frac{(r-1)!(16-r)!}{15!} = \frac{16-r}{r}.$$

Multiplying by r^2 leaves $r(16-r) = 16r - r^2$. This cancellation is valid over the whole stated range, including $r = 1$ and $r = 15$. It removes the binomial coefficients entirely; a binomial-theorem expansion is unnecessary.

Step 2: Evaluate the two elementary sums Therefore

$$S = 16\sum_{r=1}^{15} r - \sum_{r=1}^{15} r^2.$$

Now $\sum r = 15 \cdot 16/2 = 120$, while $\sum r^2 = 15 \cdot 16 \cdot 31/6 = 1240$. Hence

$$S = 16(120) - 1240 = 1920 - 1240 = 680.$$

As an additional sign check, every summand $r(16-r)$ is positive for the permitted indices; the resulting total is positive as expected. Option (C), 1240, is only the square-sum appearing in the subtraction, not the requested total. Options (A) and (D) also fail the evaluated identity. Keeping the ratio orientation correct is essential, since reversing it would not produce this quadratic summand.

Step 3: Conclusion The sum is $\boxed{680}$. Hence, the correct option is (B).

Quick Tip to Solve: For a ratio of neighbouring binomial coefficients, cancel factorials first; the remaining summand often reduces to a low-degree polynomial.

14. If P = $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Q = PAP^T, then P^T Q^{2015} P is : [4 marks]

(A) $\begin{bmatrix} 0 & 2015 \\ 0 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 2015 & 1 \\ 0 & 2015 \end{bmatrix}$

(C) $\begin{bmatrix} 2015 & 0 \\ 1 & 2015 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

Correct Answer: (D) $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

Solution:

Concept:

- The given real matrix P is orthogonal: $P^TP = PP^T = I$. This follows because each column has squared length $3/4 + 1/4 = 1$ and their scalar product is zero.
- If $Q = PAP^{-1}$, then $Q^n = PA^nP^{-1}$. Also, if $N^2 = 0$, the binomial expansion yields $(I + N)^n = I + nN$.

Step 1: Cancel the orthogonal matrices in the power Here $P^{-1} = P^T$, so $Q = PAP^T$. For example, $Q^2 = PA(P^TP)AP^T = PA^2P^T$, and repeated cancellation gives

$$Q^{2015} = PA^{2015}P^T.$$

Multiplying by the stated outside matrices produces

$$P^TQ^{2015}P = (P^TP)A^{2015}(P^TP) = A^{2015}.$$

Thus the rotation entries in P disappear from the required answer. Directly multiplying Q 2015 times would obscure this simple similarity relation.

Step 2: Use the nilpotent part of A Write

$$A = I + N, \qquad N = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

Direct multiplication gives $N^2 = 0$, so all binomial terms of degree two or higher vanish. Hence

$$A^{2015} = I + 2015N = \begin{pmatrix} 1 & 2015 \\ 0 & 1 \end{pmatrix}.$$

The diagonal entries remain one for every positive integer power, ruling out the first three displayed

alternatives. The upper-right entry increases additively with each multiplication, while the lower-left entry remains zero. This independently checks the orientation of the off-diagonal term.

Step 3: Conclusion The matrix is $\boxed{\begin{pmatrix} 1 & 2015 \\ 0 & 1 \end{pmatrix}}$. Hence, the correct option is (D).

Quick Tip to Solve: When an orthogonal similarity is raised to a power, cancel adjacent transpose factors first; then exploit nilpotence for triangular matrices.

---

15. The point (2, 1) is translated parallel to the line L : x− y = 4 by $2\sqrt{3}$ units. If the newpoint Q lies in the third quadrant, then the equation of the line passing through Q and perpendicular to L is : [4 marks]

(A) x + y = $2 - \sqrt{6}$

(B) x + y = $3 - 3\sqrt{6}$

(C) x + y = $3 - 2\sqrt{6}$

(D) 2x + 2y = $1 - \sqrt{6}$

Correct Answer: (C) x + y = $3 - 2\sqrt{6}$

Solution:

Concept:

- Translation parallel to a line changes position along its direction vector. The line $x - y = 4$ has direction $(1, 1)$, whose length is $\sqrt{2}$.
- A perpendicular line has slope -1 and hence equation $x + y = c$. Determine the constant from the translated point, not from the original point.

Step 1: Find the possible displacement vectors The required displacement has length $2\sqrt{3}$ along either direction parallel to the given line. Normalising its direction vector gives

$$\mathbf{d} = \pm 2\sqrt{3}\frac{(1,1)}{\sqrt{2}} = \pm(\sqrt{6}, \sqrt{6}).$$

Both coordinates therefore change by equal amounts; using the full displacement as each coordinate change would give the wrong distance.

Step 2: Use the third-quadrant condition Translating $P = (2, 1)$ gives the candidates $(2 + \sqrt{6}, 1 + \sqrt{6})$ and $(2 - \sqrt{6}, 1 - \sqrt{6})$. The first has both coordinates positive. Since $\sqrt{6} > 2$, both coordinates of the second are negative, so

$$Q = (2 - \sqrt{6}, 1 - \sqrt{6}).$$

This checks the quadrant requirement as well as the displacement length.

Step 3: Write the perpendicular through Q The original line has slope 1, so the required perpendicular has equation $x + y = c$. Substituting Q yields

$$c = (2 - \sqrt{6}) + (1 - \sqrt{6}) = 3 - 2\sqrt{6}.$$

Options (A) and (B) have different intercept constants. Dividing option (D) by two also gives a different constant, so none passes through Q.

Step 4: Conclusion The required line is $\boxed{x + y = 3 - 2\sqrt{6}}$. Hence, the correct option is (C).

Quick Tip to Solve: Normalise the parallel direction first, then use the stated quadrant to choose the displacement sign.

16. If a variable line drawn through the intersection of the lines $\frac{x}{3} + \frac{y}{4} = 1$ and $\frac{x}{4} + \frac{y}{3} = 1$, meets the coordinate axes at A and B, (A $\neq$ B), then the locus of the midpoint of AB is : [4 marks]

(A) 6xy = 7(x + y)

(B) $4(x + y)^2 - 28(x + y) + 49 = 0$

(C) 7xy = 6(x + y)

(D) $14(x + y)^2 - 97(x + y) + 168 = 0$

Correct Answer: (C) 7xy = 6(x + y)

Solution:

Concept:

- A line with nonzero intercepts a, b on the axes is $X/a + Y/b = 1$. Its intercept midpoint is $(a/2, b/2)$.
- The fixed intersection of the two given lines belongs to every member of the variable-line family. Substitute it into the intercept equation to obtain the midpoint locus.

Step 1: Locate the fixed point Multiplying the given equations by twelve gives

$$4X + 3Y = 12, \qquad 3X + 4Y = 12.$$

Subtraction gives $X = Y$, and either equation then gives $7X = 12$. Thus every variable line passes through

$$P = \left(\frac{12}{7}, \frac{12}{7}\right).$$

This point lies on neither axis, so a line through P cannot coincide with an axis.

Step 2: Express the intercepts using the midpoint Let M have coordinates (x, y). If $A = (a, 0)$ and $B = (0, b)$, then the midpoint formula gives $a = 2x$, $b = 2y$. The intercept line therefore has equation

$$\frac{X}{2x} + \frac{Y}{2y} = 1.$$

The finite distinct intercept condition excludes the line through the origin and lines parallel to an axis; in particular the intercept denominators here are nonzero.

Step 3: Eliminate the variable line Because P lies on the line,

$$\frac{12/7}{2x} + \frac{12/7}{2y} = 1.$$

Multiplication by $7xy$ gives

$$6y + 6x = 7xy.$$

Thus the algebraic locus is the hyperbola in option (C), with the inadmissible degenerate intercept case excluded. Option (A) reverses the coefficients. Options (B) and (D) depend only on $x + y$ and do not express this intercept relation.

Step 4: Conclusion The midpoint locus has equation $\boxed{7xy = 6(x + y)}$, with nondegenerate intercepts understood. Hence, the correct option is (C).

Quick Tip to Solve: For an intercept-midpoint locus, replace the intercepts by twice the midpoint coordinates before imposing the fixed-point condition.

17. The shortest distance between the lines $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$ lies in the interval : [4 marks]

(A) [0, 1)

(B) [1, 2)

(C) (2, 3]

(D) (3, 4]

Correct Answer: (C) (2, 3]

Solution:

Concept:

- For lines through points $\mathbf{a}_1, \mathbf{a}_2$ with nonparallel directions $\mathbf{b}_1, \mathbf{b}_2$, the shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|/|\mathbf{b}_1 \times \mathbf{b}_2|$.
- The cross product supplies a common normal to both lines. The absolute scalar projection of their connecting vector onto this unit normal is the required distance.

Step 1: Read points and directions carefully From the first symmetric equation take $\mathbf{a}_1 = (0, 0, 0)$ and $\mathbf{b}_1 = (2, 2, 1)$. From the second take $\mathbf{a}_2 = (-2, 4, 5)$ and $\mathbf{b}_2 = (-1, 8, 4)$. In particular, the numerator $x + 2$ corresponds to the coordinate -2, not $+2$. The connecting vector is therefore $(-2, 4, 5)$.

Step 2: Compute the common normal and projection The cross product is

$$\mathbf{b}_1 \times \mathbf{b}_2 = (2 \cdot 4 - 1 \cdot 8,\ 1(-1) - 2 \cdot 4,\ 2 \cdot 8 - 2(-1)) = (0, -9, 18).$$

Its magnitude is $\sqrt{81 + 324} = 9\sqrt{5}$. The scalar triple product is

$$(-2, 4, 5) \cdot (0, -9, 18) = -36 + 90 = 54.$$

The nonzero result also shows that the two lines do not intersect.

Step 3: Identify the correct interval The distance is

$$d = \frac{54}{9\sqrt{5}} = \frac{6}{\sqrt{5}}.$$

Since $d > 0$, compare its square with the interval endpoints: $d^2 = 36/5 = 7.2$, so $4 < d^2 < 9$, giving $2 < d < 3$. Thus it lies in $(2, 3]$, not in either interval below two or the interval above three.

Step 4: Conclusion The shortest distance is $\boxed{6/\sqrt{5}}$, approximately 2.683, in $(2, 3]$. Hence, the correct option is (C).

Quick Tip to Solve: Read the base point signs directly from the numerators, and compare squared distances to avoid unnecessary decimal approximations.

---

18. In a triangle ABC, right angled at the vertex A, if the position vectors of A, B and C are respectively $3\hat{i} + \hat{j} - \hat{k}$, $-\hat{i} + 3\hat{j} + p\hat{k}$ and $5\hat{i} + q\hat{j} - 4\hat{k}$, then the point (p, q) lies on a line : [4 marks]

(A) parallel to x-axis.

(B) parallel to y-axis.

(C) making an acute angle with the positive direction of x-axis.

(D) making an obtuse angle with the positive direction of x-axis.

Correct Answer: (C) making an acute angle with the positive direction of x-axis.

Solution:

Concept:

- A right angle at A means $\overrightarrow{AB} \cdot \overrightarrow{AC} = 0$. Use vectors beginning at the right-angle vertex.
- For a line $y = mx + c$, its inclination θ satisfies $m = \tan\theta$, with $0 \le \theta < \pi$. A finite positive slope gives an acute inclination.

Step 1: Construct the two sides at A The supplied position vectors give $A = (3, 1, -1)$, $B = (-1, 3, p)$, and $C = (5, q, -4)$. Subtract the coordinates of A from those of each other vertex:

$$\overrightarrow{AB} = (-4, 2, p + 1), \qquad \overrightarrow{AC} = (2, q - 1, -3).$$

The third components are $p + 1$ and -3; their signs matter in the perpendicularity condition.

Step 2: Apply the right-angle condition The dot product must vanish, hence

$$(-4)(2) + 2(q - 1) + (p + 1)(-3) = 0.$$

Expanding and collecting terms gives

$$-8 + 2q - 2 - 3p - 3 = 0, \qquad 2q - 3p - 13 = 0.$$

Therefore the point whose coordinates are (p, q) satisfies

$$q = \frac{3}{2}p + \frac{13}{2}.$$

This is the required straight line in the p-q coordinate plane.

Step 3: Classify the inclination With p as the horizontal coordinate and q as the vertical coordinate, the slope is $3/2$. It is positive and finite, so the inclination is $\theta = \tan^{-1}(3/2)$, strictly between zero

and $\pi/2$. A horizontal line would have zero slope; a vertical line would have undefined slope. An obtuse inclination would require a negative slope, which the derived relation does not have.

Step 4: Conclusion The point (p, q) lies on $2q - 3p - 13 = 0$, whose inclination is acute. Hence, the correct option is (C).

Quick Tip to Solve: For a right angle at a named vertex, subtract that vertex first; then read the slope with the first listed parameter as the horizontal coordinate.

---

19. If $\lim_{x\to\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x}=e^3$, **then 'a' is equal to :** [4 marks]

(A) 2

(B) $\frac{3}{2}$

(C) $\frac{2}{3}$

(D) $\frac{1}{2}$

Correct Answer: (B) $\frac{3}{2}$

Solution:

Concept:

- For a positive base approaching one, take natural logarithms to evaluate an indeterminate power of the form 1^∞.
- The standard small-increment limit is $\log(1+u)/u \to 1$ as $u \to 0$. Terms of order x^{-2} vanish after multiplication by x.

Step 1: Introduce the small increment Put $u_x = a/x - 4/x^2$. For each fixed real a, $u_x \to 0$ as $x \to \infty$, and $1 + u_x$ is positive for all sufficiently large x. Thus taking the natural logarithm of the limiting expression is legitimate. If its limit is L, then

$$\log L = \lim_{x\to\infty} 2x \log(1+u_x).$$

The exponent is $2x$, so its factor two must be retained.

Step 2: Evaluate the logarithmic limit Using the standard logarithmic limit, write

$$2x\log(1+u_x) = 2xu_x\frac{\log(1+u_x)}{u_x}.$$

The ratio tends to one, with its continuous limiting interpretation when the increment is zero. Meanwhile

$$2xu_x = 2a - \frac{8}{x} \longrightarrow 2a.$$

Therefore $\log L = 2a$, or equivalently $L = e^{2a}$. The term involving four contributes no finite constant to this limit.

Step 3: Match the specified value The question gives $L = e^3$. Since the real exponential function is

one-to-one,

$$e^{2a} = e^3 \quad \Rightarrow \quad 2a = 3 \quad \Rightarrow \quad a = \frac{3}{2}.$$

As a check, the other proposed values $2, 2/3, 1/2$ would give limits $e^4, e^{4/3}, e$, respectively; none equals e^3.

Step 4: Conclusion $\boxed{a = 3/2}$. Hence, the correct option is (B).

Quick Tip to Solve: For a base $1 + a/x + O(x^{-2})$ raised to kx, the limit is e^{ka}; retain the multiplier in the exponent.

---

20. The point represented by 2 + *i* in the Argand plane moves 1 unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south-westwardsdirection. Then its new position in the Argand plane is at the point represented by : [4 marks]

(A) 2 + 2i

(B) 1 + i

(C) -1 − i

(D) -2 −2i

Correct Answer: (B) 1 + i

Solution:

Concept:

- A complex number $z = x + iy$ represents the point (x, y). East increases its real coordinate and north increases its imaginary coordinate.
- A south-west displacement has equal negative horizontal and vertical components. Each component is the displacement length divided by $\sqrt{2}$.

Step 1: Apply the east and north movements The initial point is $z_0 = 2 + i$, or $(2, 1)$. Moving one unit east gives $(3, 1)$. Moving two units north from there gives $(3, 3)$, represented by

$$z_2 = 3 + 3i.$$

These are successive translations, so each movement starts from the endpoint of the preceding one rather than from the original position.

Step 2: Resolve the final diagonal movement South-west is equally directed toward negative real and negative imaginary axes. Its unit direction vector is $(-1/\sqrt{2}, -1/\sqrt{2})$. Multiplying by the stated length $2\sqrt{2}$ gives

$$\Delta(x, y) = 2\sqrt{2}\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = (-2, -2).$$

Its magnitude is indeed $\sqrt{(-2)^2 + (-2)^2} = 2\sqrt{2}$; assigning the full length to each component would incorrectly enlarge the movement.

Step 3: Add the displacement and check the options The final point is

$$(3,3)+(-2,-2)=(1,1),$$

so its complex representation is $1+i$. Option (A) would correspond to a diagonal movement of only $\sqrt{2}$ from $(3,3)$. Options (C) and (D) place the endpoint too far south-west and require larger movements than specified. Only option (B) has exactly the correct displacement.

Step 4: Conclusion The new position is represented by $\boxed{1+i}$. Hence, the correct option is (B).

Quick Tip to Solve: For a diagonal compass direction, resolve the distance into components before adding it to the complex position.

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21. **If** $2\int_0^1 \tan^{-1}x\,dx = \int_0^1 \cot^{-1}\left(1-x+x^2\right)dx,$

then $\int_0^1 \tan^{-1}\left(1-x+x^2\right)dx$ **is equalto :** [4 marks]

(A) log4

(B) $\frac{\pi}{2}$ + log2

(C) log2

(D) $\frac{\pi}{2}$ − log4

Correct Answer: (C) log2

Solution:

Concept:

- For a positive real t, principal inverse trigonometric values satisfy $\tan^{-1}t+\cot^{-1}t=\pi/2$.
- Integration by parts with $u=\tan^{-1}x$, $dv=dx$ evaluates $\int_0^1 \tan^{-1}x\,dx$. The logarithm here is natural.

Step 1: Use the complementary inverse-function identity Let

$$I=\int_0^1 \tan^{-1}(1-x+x^2)\,dx, \qquad J=\int_0^1 \tan^{-1}x\,dx.$$

On the interval, $1-x+x^2=(x-1/2)^2+3/4>0$, so the principal-value complementary identity applies without any branch adjustment. Integrating that identity over an interval of length one yields

$$I+\int_0^1 \cot^{-1}(1-x+x^2)\,dx=\frac{\pi}{2}.$$

Step 2: Substitute the given integral relation The relation stated in the question makes the second integral equal to $2J$. Consequently

$$I=\frac{\pi}{2}-2J.$$

This step uses the supplied equality directly and eliminates the quadratic argument. There is no need

to integrate the inverse tangent of a quadratic explicitly; only the simpler integral J remains.

Step 3: Evaluate the remaining integral Integration by parts gives

$$J = \left[x \tan^{-1} x\right]_0^1 - \int_0^1 \frac{x}{1+x^2}\, dx = \frac{\pi}{4} - \frac{1}{2}\log 2.$$

Therefore

$$I = \frac{\pi}{2} - 2\left(\frac{\pi}{4} - \frac{1}{2}\log 2\right) = \log 2.$$

The $\pi/2$ terms cancel exactly. Option (A) is twice this logarithm; options (B) and (D) retain extra pi terms and do not follow from the cancellation.

Step 4: Conclusion The requested value is $\boxed{\log 2}$. Hence, the correct option is (C).

Quick Tip to Solve: When inverse tangent and inverse cotangent share a positive argument, add their integrals first and use their complementary-angle identity.

22. For $x \in R$, $x \neq 0$, Let $f_0(x) = \frac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x))$, $n = 0, 1, 2, \ldots$.
Then the value of $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$ is equal to : [4 marks]

(A) $\frac{8}{3}$
(B) $\frac{5}{3}$
(C) $\frac{4}{3}$
(D) $\frac{1}{3}$

Correct Answer: (B) $\frac{5}{3}$

Solution:

Concept:

- Composition means applying the inner function first. Write $T(x) = 1/(1-x)$; the definition makes $f_n = T^{n+1}$, rather than T^n.
- An iteration may repeat periodically. Compute successive compositions algebraically before reducing a large index; every intermediate denominator must be nonzero.

Step 1: Find the cycle For the inputs used here, neither 0 nor 1 occurs in the cycle, so every required composition exists. Direct substitution gives

$$T^2(x) = \frac{1}{1 - \frac{1}{1-x}} = \frac{x-1}{x}, \qquad T^3(x) = \frac{1}{1 - \frac{x-1}{x}} = x.$$

Thus three applications return the input. Consequently $f_0 = T$, $f_1 = T^2$, $f_2 = T^3$, and this sequence repeats every three indices. The apparent exclusion of only zero in the introductory text does not affect these particular inputs.

Step 2: Reduce the index and add Since $101 = 3(33) + 2$, the function f_{100} is T^2. Therefore

$$f_{100}(3) = \frac{3-1}{3} = \frac{2}{3}, \qquad f_1\left(\frac{2}{3}\right) = \frac{\frac{2}{3}-1}{\frac{2}{3}} = -\frac{1}{2}, \qquad f_2\left(\frac{3}{2}\right) = \frac{3}{2}.$$

Adding with common denominator 6,

$$\frac{2}{3} - \frac{1}{2} + \frac{3}{2} = \frac{4-3+9}{6} = \frac{5}{3}.$$

The other three listed fractions do not equal this sum; using the index itself as the number of compositions would shift the cycle and give an incorrect result.

Step 3: Conclusion The required value is $5/3$. Hence, the correct option is (B).

Quick Tip to Solve: Count compositions as $n+1$ for f_n, then reduce that number modulo the period of the iteration.

23. The minimum distance of a point on the curve $y = x^2 - 4$ from the origin is : [4 marks]

(A) $\frac{\sqrt{19}}{2}$

(B) $\sqrt{\frac{15}{2}}$

(C) $\frac{\sqrt{15}}{2}$

(D) $\sqrt{\frac{19}{2}}$

Correct Answer: (C) $\frac{\sqrt{15}}{2}$

Solution:

Concept:

- The distance from (x, y) to the origin is $d = \sqrt{x^2 + y^2}$. Since the square root is increasing on nonnegative numbers, minimizing d^2 gives the same point.
- Use the curve equation to express the squared distance in one variable. Completing a square establishes a global minimum without relying only on a stationary-point test.

Step 1: Express the squared distance A general point of the parabola is $(x, x^2 - 4)$, with x any real number. Hence

$$d^2 = x^2 + (x^2 - 4)^2 = x^4 - 7x^2 + 16.$$

The terms are squared coordinate differences from the origin, so no perpendicular-distance-to-a-line formula applies. Set $u = x^2$, which imposes the important restriction $u \geq 0$. Then the problem becomes minimizing the quadratic $u^2 - 7u + 16$ on that interval.

Step 2: Complete the square and check attainment Rewrite the quadratic exactly as

$$d^2 = \left(u - \frac{7}{2}\right)^2 + \frac{15}{4}.$$

The squared term is nonnegative and becomes zero at $u = 7/2$, an admissible value. This corresponds

to the two points with $x = \pm\sqrt{7/2}$ and $y = -1/2$. Each gives $x^2 + y^2 = 7/2 + 1/4 = 15/4$. Thus this lower bound is attained and is the global minimum, not merely a local stationary value. Taking the positive square root yields $d_{\min} = \sqrt{15}/2$. Options containing 19 have the wrong minimum squared distance, while $\sqrt{15/2}$ has squared value $15/2$, twice the required one.

Step 3: Conclusion The minimum distance is $\sqrt{15}/2$. Hence, the correct option is (C).

Quick Tip to Solve: Minimize squared distance first; substituting $u = x^2$ often converts a symmetric parabola problem into a simple quadratic.

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24. If the four letter words (need not be meaningful) are to be formed using the letters from the word "MEDITERRANEAN" such that the first letter is R and the fourth letter is E, then the total number of all such words is : [4 marks]

(A) $\frac{11!}{(2!)^3}$

(B) 110

(C) 56

(D) 59

Correct Answer: (D) 59

Solution:

Concept:

- Words formed from a given word use each letter no more times than it occurs. Identical copies of a letter do not produce different words.
- Once the first and fourth positions are fixed, count ordered choices for the two middle positions. Separate unequal letters from equal-letter pairs.

Step 1: List the letters still available The word MEDITERRANEAN has the eight distinct letters $\{M, E, D, I, T, R, A, N\}$. Its multiplicities are three E's, two R's, two A's, two N's, and one each of M, D, I and T. Fix one R in the first position and one E in the fourth position. The remaining supply therefore has two E's, two A's, two N's, one R, and one each of M, D, I and T. All eight letter types remain available for the second and third positions. A word has the form $R__E$; exchanging unequal middle letters produces a different word.

Step 2: Count unequal and equal middle pairs If the two middle letters differ, there are eight choices for the second position and seven for the third:

$$N_{\text{unequal}} = 8 \times 7 = 56.$$

If they are equal, that letter must have at least two copies left after fixing the ends. Exactly E, A and N satisfy this requirement. The additional pairs are EE, AA and NN, giving three further words. The two cases are disjoint and exhaustive, so

$$N = 56 + 3 = 59.$$

The answer 56 omits the valid equal-letter pairs. The value 110 treats available physical copies as distinguishable in an unsuitable count; the factorial expression counts an arrangement of far more

positions than the four required here.

Step 3: Conclusion The total number of distinct four-letter words is 59. Hence, the correct option is (D).

Quick Tip to Solve: Fix required end letters first, update multiplicities, and add repeated middle pairs separately to the ordered unequal-letter count.

25. The number of distinct real roots of the equation,

$$\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$ **in the interval** $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ **is :** [4 marks]

(A) 4

(B) 3

(C) 2

(D) 1

Correct Answer: (C) 2

Solution:

Concept:

- A matrix with diagonal entries a and all off-diagonal entries b has determinant $(a-b)^2(a+2b)$ in order three.
- When counting distinct roots, multiplicity of a factor does not create extra roots. Endpoints of a closed interval are included.

Step 1: Factor the determinant Put $c = \cos x$ and $s = \sin x$. Expanding the determinant, or using the equal-entry matrix identity, gives

$$D = c^3 - 3cs^2 + 2s^3 = (c-s)^2(c+2s).$$

For example, the vector $(1,1,1)$ has eigenvalue $c+2s$, and the two independent directions whose coordinates sum to zero have eigenvalue $c-s$, confirming the factorization. Thus the determinant vanishes precisely when $\cos x - \sin x = 0$ or $\cos x + 2\sin x = 0$. A repeated eigenvalue affects algebraic multiplicity, not the count of distinct real values of x.

Step 2: Count the roots in the stated interval Throughout $[-\pi/4, \pi/4]$, cosine is positive, so division by it loses no solution. The first equation becomes $\tan x = 1$, whose only solution here is the included right endpoint $x = \pi/4$. The second becomes $\tan x = -1/2$, whose only solution is $x = -\tan^{-1}(1/2)$, strictly between $-\pi/4$ and 0. Tangent is strictly increasing on this whole interval, so neither equation has another solution there. These two roots differ. The squared first factor does not count the endpoint twice; hence the counts 3 and 4 overcount, while 1 misses one of the factors.

Step 3: Conclusion There are 2 distinct real roots in the given closed interval. Hence, the correct option is (C).

Quick Tip to Solve: Factor equal-diagonal, equal-off-diagonal determinants before doing trigonometry, and count distinct roots rather than their multiplicities.

26. The distance of the point (1, − 2, 4) from the plane passing through the point (1, 2, 2) and perpendicular to the planes x − y + 2z = 3 and 2x − 2y + z + 12 = 0, is : [4 marks]

(A) $2\sqrt{2}$

(B) 2

(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{2}}$

Correct Answer: (A) $2\sqrt{2}$

Solution:

Concept:

- Normals to perpendicular planes have zero dot product. A plane perpendicular to two given planes has a normal perpendicular to both given normals.
- For $ax+by+cz+d=0$, the perpendicular distance of (x_0, y_0, z_0) is $|ax_0+by_0+cz_0+d|/\sqrt{a^2+b^2+c^2}$.

Step 1: Determine the required plane The normals of the given planes are $\mathbf{n}_1 = (1, -1, 2)$ and $\mathbf{n}_2 = (2, -2, 1)$. A suitable normal to the required plane is their cross product:

$$\mathbf{n}_1 \times \mathbf{n}_2 = (3, 3, 0).$$

We may divide by 3 and use $(1, 1, 0)$. Its dot product with each of the original normals is zero, verifying both perpendicularity requirements. The plane passes through $(1, 2, 2)$, so its equation is

$$(x - 1) + (y - 2) = 0, \qquad x + y - 3 = 0.$$

There is no z term because the normal's third component is zero; the given point's third coordinate consequently does not enter this equation.

Step 2: Substitute the point into the distance formula For $P = (1, -2, 4)$, substitution into the plane expression gives $1 - 2 - 3 = -4$. The normal has length $\sqrt{1^2 + 1^2} = \sqrt{2}$, so

$$d = \frac{|-4|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

The absolute value is essential: distance cannot be negative. Using the unscaled normal $(3, 3, 0)$ would multiply both numerator and denominator by 3 and produce the same result. The other listed values do not have squared distance 8; simply using the numerator without normalizing would not be a perpendicular distance.

Step 3: Conclusion The required distance is $2\sqrt{2}$ units. Hence, the correct option is (A).

Quick Tip to Solve: Find a plane perpendicular to two planes by crossing their normals, then use its point-normal equation before computing distance.

27. If the tangent at a point on the ellipse $\frac{x^2}{27} + \frac{y^2}{3} = 1$ meets the coordinate axes at A and B, and O is the origin, then the minimum area (in sq. units) of the triangle OAB is : [4 marks]

(A) $\frac{9}{2}$

(B) $3\sqrt{3}$

(C) $9\sqrt{3}$

(D) 9

Correct Answer: (D) 9

Solution:

Concept:

- The tangent to $x^2/a^2 + y^2/b^2 = 1$ at $(a\cos\theta, b\sin\theta)$ is $x\cos\theta/a + y\sin\theta/b = 1$.
- A triangle formed by the coordinate axes and a line with finite intercepts u, v has area $|uv|/2$. Also $|\sin 2\theta| \le 1$.

Step 1: Write the tangent and its intercepts Here $a = \sqrt{27} = 3\sqrt{3}$ and $b = \sqrt{3}$, so $ab = 9$. Parametrize the point of contact as $(a\cos\theta, b\sin\theta)$. The tangent meets the x-axis at $A = (a/\cos\theta, 0)$ and the y-axis at $B = (0, b/\sin\theta)$. The condition that both intercepts exist requires $\sin\theta\cos\theta \neq 0$; the endpoint tangents parallel to an axis do not form the stated finite triangle. Negative intercepts cause no difficulty because geometric area uses the absolute product.

Step 2: Minimize the area Using the intercepts,

$$\mathcal{A} = \frac{1}{2}\left|\frac{a}{\cos\theta}\frac{b}{\sin\theta}\right| = \frac{ab}{|2\sin\theta\cos\theta|} = \frac{9}{|\sin 2\theta|}.$$

For every admissible tangent, $0 < |\sin 2\theta| \le 1$. Therefore $\mathcal{A} \ge 9$, and equality is attained, for example, at $\theta = \pi/4$. At that point both intercepts are finite and nonzero, so the bound corresponds to an actual triangle. The smaller values $9/2$ and $3\sqrt{3}$ violate the lower bound, whereas $9\sqrt{3}$ is larger than the attainable minimum. The factor of one-half in triangle area has already been absorbed by the double-angle identity.

Step 3: Conclusion The minimum area of $\triangle OAB$ is 9 square units. Hence, the correct option is (D).

Quick Tip to Solve: For an ellipse with semiaxes a, b, a tangent intercept triangle has area $ab/|\sin 2\theta|$, so its minimum is ab.

28. If the tangent at a point P, with parameter t, on the curve $x = 4t^2 + 3$, $y = 8t^3 - 1$, $t \in R$, meets the curve again at a point Q, then the coordinates of Q are : [4 marks]

(A) $(t^2 + 3, -t^3 - 1)$

(B) $(4t^2 + 3, -8t^3 - 1)$

(C) $(t^2 + 3, t^3 - 1)$

(D) $(16t^2 + 3, -64t^3 - 1)$

Correct Answer: (A) $(t^2 + 3, -t^3 - 1)$

Solution:

Concept:

- For a parametrized curve, the tangent slope is $(dy/dt)/(dx/dt)$ whenever the denominator is nonzero; a limiting slope can handle a singular parameter.
- To find another intersection, use a new parameter s for that point. Tangency produces a repeated root at $s = t$, which must be separated from the other root.

Step 1: Form the tangent equation At $P = (4t^2 + 3, 8t^3 - 1)$, the derivatives are $dx/dt = 8t$ and $dy/dt = 24t^2$. For $t \neq 0$, the tangent slope is $3t$. Thus the tangent line is

$$y - (8t^3 - 1) = 3t\big(x - (4t^2 + 3)\big).$$

Introduce a separate parameter s for a general intersection on the curve, with coordinates $(4s^2 + 3, 8s^3 - 1)$. Keeping these parameters distinct prevents substituting the contact point back in and missing the second intersection.

Step 2: Factor the intersection equation Substitute the coordinates with parameter s into the tangent equation:

$$8(s^3 - t^3) = 12t(s^2 - t^2), \qquad 2s^3 - 3ts^2 + t^3 = 0.$$

The factorization is

$$(s - t)^2(2s + t) = 0.$$

The repeated root $s = t$ represents contact at P. The remaining root is $s = -t/2$, giving

$$Q = \left(4\frac{t^2}{4} + 3,\ 8\left(-\frac{t^3}{8}\right) - 1\right) = (t^2 + 3, -t^3 - 1).$$

At $t = 0$, the limiting tangent is $y = -1$; the intersections coalesce at $(3, -1)$, consistent with the same coordinate formula. For nonzero t, Q is the distinct second point intended by the question. Options B, C and D use incorrect parameter values or the wrong sign for the cubic term.

Step 3: Conclusion The coordinates are $Q = (t^2 + 3, -t^3 - 1)$. Hence, the correct option is (A).

Quick Tip to Solve: Use a second parameter for the other intersection and factor out the double root corresponding to tangency before finding its coordinates.

29. The number of x $\in$ [0, 2π] for which

$\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$ **is :** [4 marks]

(A) 2

(B) 4

(C) 6

(D) 8

Correct Answer: (D) 8

Solution:

Concept:

- Use $\sin^2 x = (1 - \cos 2x)/2$ and $\cos^2 x = (1 + \cos 2x)/2$ to express both radicands in one variable.
- For nonnegative square roots A, B, the equation $|A - B| = 1$ gives $A^2 + B^2 - 2AB = 1$; any subsequent squaring must be checked.

Step 1: Introduce a single trigonometric variable Put $t = \cos 2x$, so $-1 \le t \le 1$. Denote the two square roots in the question by A and B, respectively. Their radicands become

$$A^2 = \frac{t^2 + 16t + 19}{2}, \qquad B^2 = \frac{t^2 - 16t + 19}{2}.$$

Thus $A^2 + B^2 = t^2 + 19$, while $A^2 - B^2 = 16t$. This substitution keeps the absolute value intact and does not impose a sign on $A - B$.

Step 2: Solve the algebraic condition Squaring the given absolute-value equation yields $2AB = t^2 + 18$. Both sides are nonnegative, so squaring this equation is reversible. Hence

$$(t^2 + 19)^2 - 256t^2 = (t^2 + 18)^2 \quad \Longrightarrow \quad 254t^2 = 37.$$

Consequently $t = \pm\sqrt{37/254}$. Both values lie strictly between -1 and 1, and satisfy $2AB = t^2 + 18$, giving $(A - B)^2 = 1$ as required.

Step 3: Count the angles in the specified interval For each fixed value c strictly between -1 and 1, $\cos 2x = c$ has four solutions when $0 \le x \le 2\pi$, because $2x$ covers two complete periods. The two distinct values of c give disjoint sets of four solutions. Neither endpoint is a solution, since there $t = 1$.

Step 4: Conclusion The total number is $4 + 4 = 8$. Hence, the correct option is (D).

Quick Tip to Solve: Count the solutions only after checking the allowed cosine values; here the doubled angle covers two full periods.

30. Consider the following two statements :

P : If 7 is an odd number, then 7 is divisible by 2.

Q : If 7 is a prime number, then 7 is an odd number

If V_1 is the truth value of the contrapositive of P and V_2 is the truth value of contrapositive of Q, then the ordered pair (V_1 , V_2) equals : [4 marks]

(A) (T, T)

(B) (T, F)

(C) (F, T)

(D) (F, F)

Correct Answer: (C) (F, T)

Solution:

Concept:

- An implication $p \Rightarrow q$ is false exactly when its antecedent is true and its consequent is false.
- The contrapositive $\neg q \Rightarrow \neg p$ always has the same truth value as the original implication.

Step 1: Write the contrapositive of P For P the antecedent is "7 is an odd number", which is true, and the consequent is "7 is divisible by 2", which is false. Its contrapositive is: "If 7 is not divisible by 2, then 7 is not an odd number." Here the new antecedent is true, but the new consequent is false. Therefore this implication is false and $V_1 = \mathrm{F}$. Notice that reversing the original statement without negating would produce its converse, not its contrapositive.

Step 2: Evaluate the contrapositive of Q For Q, both "7 is a prime number" and "7 is an odd number" are true, so Q is true. Its contrapositive is: "If 7 is not an odd number, then 7 is not a prime number." The antecedent of this statement is false. In propositional logic an implication with a false antecedent is true, irrespective of the consequent. Thus $V_2 = \mathrm{T}$, in agreement with the equivalence between an implication and its contrapositive.

Step 3: Compare the ordered truth values Keep the order specified in the question: the value from P comes first and that from Q comes second. Options (A) and (B) incorrectly make the first value true, while option (D) incorrectly makes the second value false. Only the pair with a false first entry and a true second entry satisfies both evaluations. No assumption about all odd numbers or all prime numbers is necessary: the statements concern the particular number 7.

Step 4: Conclusion The ordered pair is $(V_1, V_2) = (\mathrm{F}, \mathrm{T})$. Hence, the correct option is (C).

Quick Tip to Solve: Use equivalence with the original implication to evaluate a contrapositive quickly; do not confuse it with the converse.

Physics (120 marks)

31. An experiment is performed to determine the *I* - V characteristics of a Zener diode, which has a protective resistance of R = 100 Ω, and a maximum power of dissipation rating of 1 W. The minimum voltage range of the DC source in the circuit is : [4 marks]

(A) 0 – 5 V

(B) 0 – 8 V

(C) 0 – 12 V

(D) 0 – 24 V

Correct Answer: (D) 0 – 24 V

Solution:

Concept:

- For a Zener diode in breakdown, the series protective resistor R carries current $I = (V - V_Z)/R$, where V is the supply voltage and V_Z is the diode voltage.
- The diode dissipation is $P_Z = V_Z I = V_Z(V - V_Z)/R$. For a fixed supply, this quadratic has maximum $V^2/(4R)$.

Step 1: Write the diode power in terms of supply voltage The 1 W rating refers to the Zener diode whose characteristic is to be investigated. The protective resistance drops the part of the supply voltage not appearing across the diode. Thus

$$V = V_Z + IR, \qquad P_Z = V_Z I = \frac{V_Z(V - V_Z)}{R}.$$

Using V^2/R directly would calculate a resistor power for the full source voltage across the resistor; it would not calculate the Zener's dissipation. Both the diode voltage and the resistor drop must be included in the experiment.

Step 2: Find the least supply voltage capable of the rated power Complete the square in the diode-power expression:

$$P_Z = \frac{1}{R}\left[\frac{V^2}{4} - \left(V_Z - \frac{V}{2}\right)^2\right] \le \frac{V^2}{4R}.$$

Therefore attaining the stated maximum diode dissipation of 1 W requires at least

$$V \ge 2\sqrt{P_Z R} = 2\sqrt{1 \times 100} = 20\,\text{V}.$$

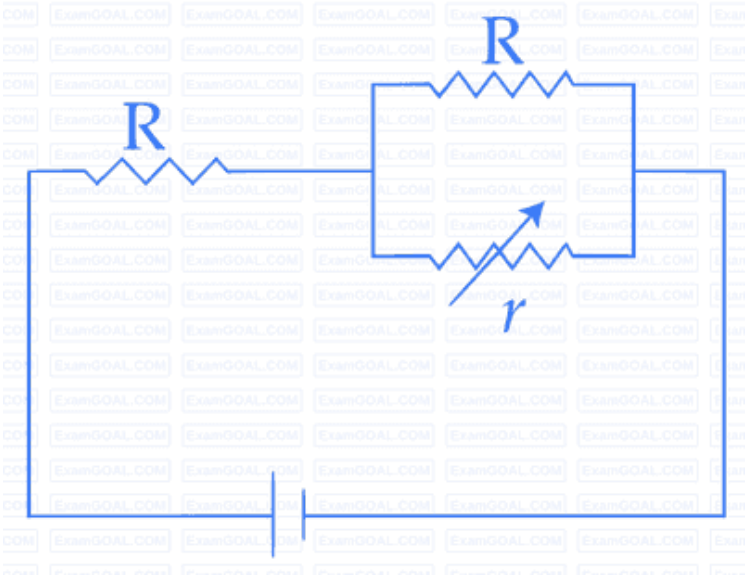
The lower bound corresponds to equal voltage drops across the diode and the protective resistor. Equivalently, at a specified diode power, $V = V_Z + P_Z R/V_Z$ is minimized at $V_Z = \sqrt{P_Z R}$.

Step 3: Compare the available DC source ranges The upper limits 5 V, 8 V and 12 V are all below the minimum 20 V scale. In particular, even the largest possible diode dissipation with a 12 V source and this resistance is $12^2/(4 \times 100) = 0.36\,\text{W}$, below 1 W. Of the stated choices, only a 0–24 V supply reaches beyond 20 V, and it is therefore the required range. During measurement the voltage is swept gradually and the diode rating must not be exceeded.

Step 4: Conclusion The minimum suitable listed DC source range is 0–$24\,\mathrm{V}$. Hence, the correct option is (D).

Quick Tip to Solve: For a diode fed through a series resistor, use its own power $V_Z(V - V_Z)/R$; the source must supply both voltage drops.

32. In the circuit shown, the resistance r is a variable resistance. If for r = *fR*, the heat generation in r is maximum then the value of *f* is : [4 marks]



(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) $\frac{3}{4}$
(D) 1

Correct Answer: (B) $\frac{1}{2}$

Solution:

Concept:

- A resistive network viewed from a load can be replaced by its Thevenin voltage V_{th} and series resistance R_{th}.
- For a variable load r, its power is $P_r = V_{\text{th}}^2 r/(R_{\text{th}} + r)^2$, maximized at $r = R_{\text{th}}$.

Step 1: Find the resistance seen by the variable load Remove r and suppress the ideal voltage source, replacing it by a short circuit. In the given circuit, the left-hand resistor R then joins the same two terminal nodes as the upper resistor R. These two resistances are in parallel as viewed from the terminals of r. Therefore

$$R_{\text{th}} = R \parallel R = \frac{R \cdot R}{R + R} = \frac{R}{2}.$$

The fact that the two fixed resistors appear in different branches in the original drawing does not make them series resistors after the source is suppressed.

Step 2: Apply the maximum-power condition The Thevenin source drives the load through R_{th}, so its current is $I = V_{\text{th}}/(R_{\text{th}} + r)$. The rate of heat generation in the load is

$$P_r = \frac{V_{\text{th}}^2 r}{(R_{\text{th}} + r)^2}, \qquad \frac{dP_r}{dr} = \frac{V_{\text{th}}^2 (R_{\text{th}} - r)}{(R_{\text{th}} + r)^3}.$$

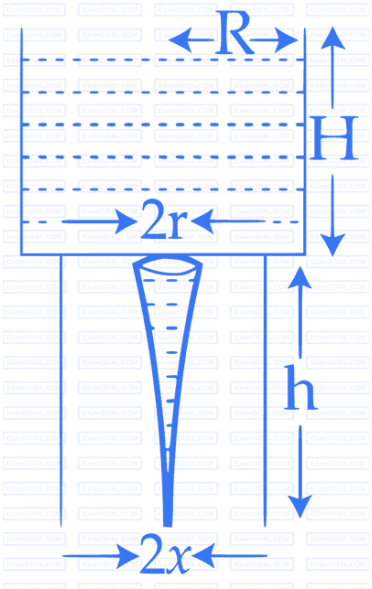
The derivative changes from positive to negative at $r = R_{\text{th}}$; hence this stationary point is a maximum, rather than a minimum.

Step 3: Express the optimum as a fraction of R At maximum heat generation, $r = R/2 = fR$, which gives $f = 1/2$. Values $1/4$ and $3/4$ lie on opposite sides of the optimum, and neither makes the derivative vanish. The value 1 would incorrectly use one fixed resistor instead of the parallel equivalent. As an additional check, the load power tends to zero both for $r \to 0$ and $r \to \infty$, so an interior maximum is physically necessary.

Step 4: Conclusion The required value is $f = \frac{1}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Suppress the ideal source before finding the resistance seen by a variable load; redraw the resulting parallel connections mentally.

33. Consider a water jar of radius R that has water filled up to height H and is kept on astand of height h (see figure). Through a hole of radius r (r << R) at its bottom, the water leaks out and the stream of water coming down towards the ground has a shape like a funnel as shown in the figure. If the radius of the cross-section of water stream when it hits the ground is x. Then : [4 marks]



(A) $x = r\left(\frac{H}{H+h}\right)$

(B) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{2}}$

(C) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{4}}$

(D) $x = r\left(\frac{H}{H+h}\right)^{2}$

Correct Answer: (C) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{4}}$

Solution:

Concept:

- For a small outlet in a large jar, Torricelli's law gives the outlet speed $u = \sqrt{2gH}$, neglecting the free-surface speed.
- An incompressible steady stream obeys $Av =$ constant, and a downward fall through height h increases squared speed by $2gh$.

Step 1: Calculate the speeds at the outlet and ground Because $r \ll R$, the water surface moves much more slowly than the water emerging from the hole. Both the free surface and the stream are exposed to atmospheric pressure. Bernoulli's equation between the surface and the outlet therefore gives

$$u^2 = 2gH.$$

After the water falls through the additional distance h, conservation of mechanical energy gives

$$v^2 = u^2 + 2gh = 2g(H + h).$$

Thus the stream accelerates as it falls. Its narrowing is a consequence of this increase in speed, rather than a decrease in the volume flow rate.

Step 2: Apply continuity to the two cross-sections At the outlet the cross-sectional area is πr^2, while at the ground it is πx^2. The same volume of incompressible water crosses each section per unit time, so

$$\pi r^2 u = \pi x^2 v, \qquad \frac{x^2}{r^2} = \frac{u}{v} = \sqrt{\frac{H}{H+h}}.$$

Taking the positive square root, since radii are positive, yields

$$x = r\left(\frac{H}{H+h}\right)^{1/4}.$$

Step 3: Check the exponent and limiting cases The speed ratio contributes one square root and converting the area ratio to a radius ratio contributes another, producing the fourth-root dependence. Option (B) would confuse the area ratio with the radius ratio; (A) and (D) likewise give incorrect exponents. If $h = 0$, the expression gives $x = r$; for $h > 0$, it gives $x < r$, consistent with the funnel-shaped narrowing shown.

Step 4: Conclusion The radius is $x = r\big(H/(H+h)\big)^{1/4}$. Hence, the correct option is (C).

Quick Tip to Solve: In a falling jet, use energy for the speed and continuity for the area; take one more square root when converting area to radius.

34. To know the resistance G of a galvanometer by half deflection method, a battery of emf V_E and resistance R is used to deflect the galvanometer by angle θ. If a shunt of resistance S is needed to get half deflection then G, R and S are related by the equation : [4 marks]

(A) 2S (R + G) = RG

(B) S (R + G) = RG

(C) 2S = G

(D) 2G = S

Correct Answer: (B) S (R + G) = RG

Solution:

Concept:

- For a galvanometer in its linear range, deflection is proportional to its current. Half deflection therefore means half the initial galvanometer current.
- Adding a shunt changes the total current drawn from a source with series resistance R; current division alone must include this change.

Step 1: Write the original and shunted galvanometer currents Before the shunt is connected, the current through the galvanometer is

$$I_g = \frac{V_E}{R+G}.$$

After connecting S in parallel with G, their equivalent resistance is $GS/(G+S)$. The total current entering the parallel combination is $V_E/[R+GS/(G+S)]$. The fraction entering the galvanometer branch is $S/(G+S)$, by the current-divider rule. Consequently

$$I_g' = \frac{V_E S}{R(G+S)+GS}.$$

Step 2: Impose the half-deflection condition Set $I_g' = I_g/2$, with the same source emf in both measurements:

$$\frac{V_E S}{RG+RS+GS} = \frac{V_E}{2(R+G)}.$$

Cancel V_E and cross-multiply to obtain

$$2S(R+G) = RG+RS+GS \quad \Longrightarrow \quad S(R+G) = RG.$$

This is the exact relation for the shown series-source resistance; it does not require approximating R as infinitely large.

Step 3: Distinguish the exact relation from shortcuts Solving for the galvanometer resistance gives $G = RS/(R-S)$, which requires $R > S$, as expected for a positive resistance. If $R \gg G$, the exact relation approaches $S \simeq G$, the familiar approximate half-deflection result. Options (C) and (D) assert fixed factors of two that do not follow from current division. Option (A) retains an extra factor of two because it neglects the change in current supplied when the shunt is connected.

Step 4: Conclusion The relation is $S(R+G) = RG$. Hence, the correct option is (B).

Quick Tip to Solve: Do not assume that the source current remains constant when a shunt is added; write the circuit current again before using half deflection.

35. A car of weight W is on an inclined road that rises by 100 m over a distance of 1 km and applies a constant frictional force $\frac{W}{20}$ on the car. While moving uphill on the road at a speed of 10 ms^{-1}, the car needs power P. If it needs power $\frac{p}{2}$ while moving downhill at speed v then value of v is : [4 marks]

(A) 20 ms^{-1}

(B) 15 ms^{-1}

(C) 10 ms^{-1}

(D) 5 ms^{-1}

Correct Answer: (B) 15 ms^{-1}

Solution:

Concept:

- For steady motion along an incline, the driving or braking force balances the component of weight and the frictional resistance.
- Mechanical power magnitude is $P = Fv$. Friction always opposes motion, whereas the component of weight always points downhill.

Step 1: Calculate the uphill power The rise is 100 m along a road length of 1000 m, so $\sin\theta = 100/1000 = 1/10$. The component of weight down the road is $W\sin\theta = W/10$. While the car moves uphill, gravity and the given friction $W/20$ both oppose its motion. The required driving force is therefore

$$F_{\text{up}} = \frac{W}{10} + \frac{W}{20} = \frac{3W}{20}.$$

At $10\,\text{m}\,\text{s}^{-1}$, the power is $P = (3W/20)(10) = 3W/2$, with W here denoting the car's weight, not the watt symbol.

Step 2: Find the downhill force and power magnitude During downhill motion, gravity acts along the velocity, while friction acts uphill. The remaining downhill force is

$$F_{\text{net}} = \frac{W}{10} - \frac{W}{20} = \frac{W}{20}.$$

For a constant downhill speed, the car must absorb this tendency through a braking force of magnitude $W/20$. Thus the power required in magnitude is $(W/20)v$. Its signed value as engine work would be negative: the relevant comparison in the question is the magnitude of the downhill power.

Step 3: Use the specified power ratio Equate the downhill power magnitude to half the uphill power:

$$\frac{W}{20}v = \frac{P}{2} = \frac{3W}{4} \quad \Longrightarrow \quad v = 15\,\text{m}\,\text{s}^{-1}.$$

The weight cancels, so the result is independent of the mass of the car. The uphill force is three times the downhill braking-force magnitude. Therefore half the power can be absorbed at one and a half times the uphill speed. The other offered speeds fail this force-times-speed ratio.

Step 4: Conclusion The downhill speed is $15\,\text{m}\,\text{s}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Reverse friction when reversing the direction of travel, but keep gravity pointing downhill; compare power magnitudes for braking.

36. 200 g water is heated from 40°C to 60°C. Ignoring the slight expansion of water, the change in its internal energy is close to (Given specific heat of water = 4184 J/kg/K) : [4 marks]

(A) 8.4 kJ

(B) 4.2 kJ

(C) 16.7 kJ

(D) 167.4 kJ

Correct Answer: (C) 16.7 kJ

Solution:

Concept:

- The heat needed for a temperature change without a change of state is $Q = mc\Delta T$, where m is mass and c is specific heat capacity.
- The first law gives $\Delta U = Q - W$. Boundary work is $W = p\Delta V$; neglecting expansion makes this work negligible.

Step 1: Convert the mass and temperature interval The mass must be in kilograms because the specific heat is given per kilogram. The temperature difference has the same numerical value in kelvin and degrees Celsius. Thus

$$m = 200\,\text{g} = 0.200\,\text{kg}, \qquad \Delta T = 60 - 40 = 20\,\text{K}.$$

The final temperature itself must not be used as the temperature rise. There is no melting or boiling in this interval, so no latent-heat term is needed.

Step 2: Calculate the heat and internal-energy change Substitute the given values into the calorimetric expression:

$$Q = 0.200 \times 4184 \times 20 = 16736\,\text{J} = 16.736\,\text{kJ}.$$

Because expansion is neglected, the supplied heat is stored as internal energy rather than doing appreciable external work. Hence $\Delta U \simeq Q$. The increase is positive because the water is heated. Option (A) is about half this result, (B) is about one quarter, and (D) is ten times too large; these values do not use the given mass and temperature rise correctly.

Step 3: Conclusion The change in internal energy is approximately $+16.7\,\text{kJ}$. Hence, the correct option is (C).

Quick Tip to Solve: First calculate the temperature rise, then use mass in kilograms; when expansion is negligible, the heat supplied equals the internal-energy increase.

37. A uniformly tapering conical wire is made from a material of Young's modulus Y and has a normal, unextended length L. The radii, at the upper and lower ends of this conical wire, have values R and 3 R, respectively. The upper end of the wire is fixed to a rigid support and a mass M is suspended from its lower end. The equilibrium extended length, of this wire, would equal : [4 marks]

(A) L $\left(1+\frac{2}{9}\frac{Mg}{\pi YR^2}\right)$

(B) L $\left(1+\frac{1}{3}\frac{Mg}{\pi YR^2}\right)$

(C) L $\left(1+\frac{1}{9}\frac{Mg}{\pi YR^2}\right)$

(D) L $\left(1+\frac{2}{3}\frac{Mg}{\pi YR^2}\right)$

Correct Answer: (B) L $\left(1+\frac{1}{3}\frac{Mg}{\pi YR^2}\right)$

Solution:

Concept:

- For a short wire element of length dx, cross-sectional area $A(x)$, and tensile force F, Hooke's law gives $d\ell = F\,dx/[YA(x)]$.
- The radius changes linearly along a uniformly tapering cone. The total extension is the sum of the extensions of all elements, not the extension computed with one end area.

Step 1: Write the local radius and extension Measure x downward from the fixed upper end. The radius grows from R to $3R$ over length L, so

$$r(x) = R\left(1+\frac{2x}{L}\right), \quad A(x) = \pi R^2\left(1+\frac{2x}{L}\right)^2.$$

The suspended mass supplies the tensile load $F = Mg$, constant through the wire when its own weight is neglected, as in the usual loaded-wire model. Therefore

$$d\ell = \frac{Mg}{\pi YR^2}\frac{dx}{(1+2x/L)^2}.$$

The narrow part is more compliant than the broad part, so averaging the end areas is not justified.

Step 2: Integrate and add the original length Integrating along the unextended wire gives

$$\Delta L = \frac{Mg}{\pi YR^2}\int_0^L \frac{dx}{(1+2x/L)^2}.$$

With $u = 1+2x/L$, the limits become 1 and 3:

$$\Delta L = \frac{MgL}{2\pi YR^2}\left[-\frac{1}{u}\right]_1^3 = \frac{MgL}{3\pi YR^2}.$$

Hence the extended length is $L+\Delta L$. The factor multiplying $Mg/(\pi YR^2)$ is $1/3$, whereas the other options use $2/9$, $1/9$, or $2/3$, none of which equals the integral.

Step 3: Conclusion The equilibrium length is $L\left(1+\frac{Mg}{3\pi YR^2}\right)$. Hence, the correct option is (B).

Quick Tip to Solve: For a linearly tapered circular wire with end radii a and b, integration gives extension FL/(pi Yab); here ab equals 3R squared.

38. A magnetic dipole is acted upon by two magnetic fields which are inclined to each other at an angle of 75°. One of the fields has a magnitude of 15 mT. The dipole attains stable equilibrium at an angle of 30° with this field. The magnitude of the other field (in mT) is close to [4 marks]

(A) 11

(B) 36

(C) 1

(D) 1060

Correct Answer: (A) 11

Solution:

Concept:

- A magnetic dipole of moment m experiences torque $mB\sin\theta$ in a uniform magnetic field.
- Stable equilibrium places the dipole parallel to the resultant field, so the opposing torque components about that direction must cancel.

Step 1: Find the angles with both fields Let $B_1 = 15\,\mathrm{mT}$ and let B_2 denote the unknown field. The fields are separated by 75°. Their resultant lies between them because both field magnitudes are positive. In stable equilibrium the dipole is along this resultant and makes 30° with B_1. Its angle with B_2 is consequently

$$\theta_2 = 75^\circ - 30^\circ = 45^\circ.$$

Using 75° as the dipole angle with the second field would confuse the angle between the fields with the angle entering the torque formula.

Step 2: Balance the torques and evaluate the field The two torques are opposite in sense, and the common dipole moment cancels:

$$mB_1\sin 30^\circ = mB_2\sin 45^\circ.$$

Thus

$$B_2 = 15\frac{1/2}{1/\sqrt{2}} = \frac{15}{\sqrt{2}}\,\mathrm{mT} \simeq 10.61\,\mathrm{mT}.$$

Both fields have positive components along the chosen dipole direction, so the resultant points along the dipole rather than opposite to it; this confirms stable rather than unstable equilibrium. The closest listed magnitude is 11 mT. Values 36, 1, and 1060 mT cannot satisfy the torque balance with these angles and the given 15 mT field.

Step 3: Conclusion The second field has magnitude about $10.6\,\mathrm{mT}$, close to $11\,\mathrm{mT}$. Hence, the correct option is (A).

Quick Tip to Solve: Draw the resultant direction mentally: its two angles add to the angle between the fields, and the transverse components must cancel.

39. When photons of wavelength λ_1 are incident on an isolated sphere, the corresponding stopping potential is found to be V. When photons of wavelength λ_2 are used, the corresponding stopping potential was thrice that of the above value. If light of wavelength λ_3 is used then find the stopping potential for this case : [4 marks]

(A) $\frac{hc}{e}\left[\frac{1}{\lambda_3}-\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$

(B) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$

(C) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right]$

(D) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{1}{\lambda_1}\right]$

Correct Answer: (C) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right]$

Solution:

Concept:

- Einstein's photoelectric equation is $eV_s = hc/\lambda - \phi$, where ϕ is the fixed work function of the sphere.
- Two known stopping potentials allow the work function to be eliminated. Keep photon energy and stopping potential distinct by retaining the factor e.

Step 1: Use the two given wavelengths to eliminate V For the first and second illuminations, respectively,

$$eV = \frac{hc}{\lambda_1} - \phi, \qquad 3eV = \frac{hc}{\lambda_2} - \phi.$$

Multiply the first equation by 3 and compare it with the second:

$$\frac{3hc}{\lambda_1} - 3\phi = \frac{hc}{\lambda_2} - \phi.$$

Rearranging gives

$$\phi = \frac{hc}{2}\left(\frac{3}{\lambda_1} - \frac{1}{\lambda_2}\right).$$

The work function must be the same for both measurements, since the emitting material has not changed.

Step 2: Substitute the work function for the third illumination Let the required stopping potential be V_3. Substitution gives

$$eV_3 = \frac{hc}{\lambda_3} - \frac{hc}{2}\left(\frac{3}{\lambda_1} - \frac{1}{\lambda_2}\right),$$

$$V_3 = \frac{hc}{e}\left(\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1}\right).$$

As an algebraic check, setting $\lambda_3 = \lambda_1$ recovers V, and setting $\lambda_3 = \lambda_2$ recovers $3V$. The other expressions fail these two constraints. This expression describes a stopping potential when the third photon energy is above the emission threshold; below threshold there is no photoelectric emission to stop.

Step 3: Conclusion The stopping potential is $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right]$. Hence, the correct option is (C).

Quick Tip to Solve: Eliminate the common work function with the first two measurements, then test the final formula by restoring either known wavelength.

40. The potential (in volts) of a charge distribution is given by.

V(z) = 30 − 5x^2 for $|z| \leq 1$ m.

V(z) = 35 − 10 $|z|$ for $|z| \geq 1$ m.

V(z) does not depend on x and y. If this potential is generated by a constant charge per unit volume ρ_0 (in units of ε_0) which is spread over a certain region, then choose the correct statement. [4 marks]

(A) $\rho_0 = 10\ \varepsilon_0$ for $|z| \leq 1$ m and $\rho_0 = 0$ elsewhere

(B) $\rho_0 = 20\ \varepsilon_0$ in the entire region

(C) $\rho_0 = 40\ \varepsilon_0$ in the entire region

(D) $\rho_0 = 20\ \varepsilon_0$ for $|z| \leq 1$ m and $\rho_0 = 0$ elsewhere

Correct Answer: The inner potential contains x squared despite being stated independent of x; option (A) follows only if this is read as z squared.

Solution:

Concept:

- Electrostatics relates charge density to potential through Poisson's equation, $\rho = -\varepsilon_0 \nabla^2 V$.
- If potential depends only on z, this reduces to $\rho = -\varepsilon_0\, d^2V/dz^2$. A proposed potential must also satisfy its stated coordinate dependence and boundary continuity.

Step 1: Check the literal coordinate dependence The inner expression is written as $V(z) = 30 - 5x^2$, while the statement explicitly says that $V(z)$ does not depend on x or y. These conditions conflict: differentiating the inner expression with respect to x gives $-10x$, generally nonzero. Taken literally, it is therefore not a potential depending only on z. It also does not join the outer value uniformly on the planes $z = \pm 1$, because the outer value there is 25 V whereas the inner value would vary with x. Thus a consistent charge distribution cannot satisfy all the stated expressions simultaneously.

Step 2: Identify the conditional result for the consistent quadratic form If the quadratic term is interpreted as $-5z^2$, the intended one-dimensional calculation is

$$\frac{d^2V}{dz^2} = -10 \quad (|z| < 1), \qquad \frac{d^2V}{dz^2} = 0 \quad (|z| > 1).$$

Poisson's equation then gives $\rho = 10\varepsilon_0$ inside and zero outside. The potential is 25 V at both boundaries, and its first derivative matches the outer derivative there, so no extra surface charge is required. This conditional result corresponds to (A); (B) and (C) wrongly charge the exterior and (D) doubles the interior value. However, this requires changing the printed coordinate, so it is not a literal resolution of the inconsistency.

Step 3: Conclusion As written, the coordinate dependence is inconsistent and no unique physically consistent option follows. If the inner term is $-5z^2$, the result is $\rho = 10\varepsilon_0$ for $|z| \leq 1$ m, zero elsewhere,

corresponding to option (A).

Quick Tip to Solve: Before differentiating a piecewise potential, check its coordinate dependence and match its value and first derivative at the boundaries.

41. In Young's double slit experiment, the distance between slits and the screen is 1.0 m and monochromatic light of 600 nm is being used. A person standing near the slits is looking at the fringe pattern. When the separation between the slits is varied, the interference pattern disappears for a particular distance d_0 between the slits. If the angular resolution of the eye is $(\frac{1}{60})^o$, the value of d_0 is close to : [4 marks]

(A) 1 mm

(B) 2 mm

(C) 4 mm

(D) 3 mm

Correct Answer: (B) 2 mm

Solution:

Concept:

- The spacing of adjacent bright fringes in Young's experiment is $\beta = \lambda D/d$, where D is the slit-to-screen distance and d the slit separation.
- An observer near the slits sees adjacent fringes at angular separation $\beta/D \simeq \lambda/d$. They cease to be resolved when this equals the eye's angular resolution.

Step 1: Convert the limiting angular resolution to radians The given angular resolution is one minute of arc, not one degree. Since $180^\circ = \pi$ radians,

$$\theta_{\min} = \frac{1}{60}\frac{\pi}{180} = \frac{\pi}{10800}\,\text{rad} \simeq 2.909 \times 10^{-4}\,\text{rad}.$$

The wavelength is $600\,\text{nm} = 6.00 \times 10^{-7}\,\text{m}$. The small-angle approximation is excellent at this angular scale, so a separation on the screen subtends its linear spacing divided by the viewing distance.

Step 2: Equate the fringe angle to the resolution limit The person is near the slit plane, about D from the screen, so

$$\frac{\beta}{D} = \frac{\lambda D/d_0}{D} = \frac{\lambda}{d_0} = \theta_{\min}.$$

Consequently,

$$d_0 = \frac{6.00 \times 10^{-7}}{2.909 \times 10^{-4}}\,\text{m} \simeq 2.06 \times 10^{-3}\,\text{m} = 2.06\,\text{mm}.$$

The screen distance cancels because it affects the fringe spacing and viewing distance equally. At larger slit separations the fringes become even closer angularly and cannot be distinguished. Of the listed values, 2 mm is closest; 1, 3, and 4 mm are farther from the calculated threshold.

Step 3: Conclusion The limiting slit separation is about $2.06\,\text{mm}$, close to $2\,\text{mm}$. Hence, the correct option is (B).

Quick Tip to Solve: For an observer near the slit plane, use the angular fringe spacing lambda/d and convert minutes of arc to radians before substituting.

42. A hydrogen atom makes a transition from n = 2 to n = 1 and emits a photon. This photon strikes a doubly ionized lithium atom (z = 3) in excited state and completely removes the orbiting electron. The least quantum number for the excited state of the ion for the process is : [4 marks]

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (C) 4

Solution:

Concept:

- The energy of level n of a hydrogen-like one-electron ion is $E_n = -13.6Z^2/n^2\,\text{eV}$. Its ionization energy from that level is the positive magnitude $13.6Z^2/n^2\,\text{eV}$.
- A photon can remove the electron when its energy is at least the binding energy. Any excess appears as kinetic energy of the ejected electron.

Step 1: Calculate the hydrogen photon energy The hydrogen atom has $Z = 1$. In the transition from $n = 2$ to $n = 1$, its energy decreases and the photon carries away the positive energy difference:

$$E_\gamma = 13.6\left(1 - \frac{1}{4}\right)\text{eV} = 10.2\,\text{eV}.$$

The sign is positive because this is emitted energy, although the bound-state energies themselves are negative. This photon energy is fixed by hydrogen and does not acquire an additional factor of Z^2 when it subsequently strikes another ion.

Step 2: Find the lowest lithium level that can be ionized Doubly ionized lithium is Li^{2+}, which has one electron and nuclear charge $Z = 3$. Its binding energy in level n is

$$I_n = \frac{13.6 \times 9}{n^2}\,\text{eV} = \frac{122.4}{n^2}\,\text{eV}.$$

Ionization requires

$$10.2 \geq \frac{122.4}{n^2} \quad \Rightarrow \quad n^2 \geq 12.$$

The least integer satisfying this is $n = 4$. Specifically, $I_2 = 30.6$ eV and $I_3 = 13.6$ eV exceed the photon energy, while $I_4 = 7.65$ eV is smaller. Level 5 can also be ionized but is not the least possible level; for level 4 the remaining 2.55 eV becomes electron kinetic energy.

Step 3: Conclusion The least initial quantum number of the excited lithium ion is $n = 4$. Hence, the correct option is (C).

Quick Tip to Solve: Compare photon energy with the positive binding energy 13.6 Z squared/n squared; solve the inequality, then choose the next allowed integer.

43. Two particles are performing simple harmonic motion in a straight line about the same equilibrium point. The amplitude and time period for both particles are same and equal to A and *I*, respectively. At time t = 0 one particle has displacement A while the other one has displacement $\frac{-A}{2}$ and they are moving towards each other. If they cross each other at time t, then t is : [4 marks]

(A) $\frac{T}{6}$
(B) $\frac{5T}{6}$
(C) $\frac{T}{3}$
(D) $\frac{T}{4}$

Correct Answer: (A) $\frac{T}{6}$

Solution:

Concept:

- For SHM of amplitude A and angular frequency ω, displacement can be written $x = A\cos(\omega t + \phi)$, where $\omega = 2\pi/T$.
- The initial displacement fixes the cosine of the phase; the initial direction of velocity fixes its sign. The first crossing occurs when the displacements are equal.

Step 1: Choose phases consistent with the initial motion Take positive displacement towards the initial position of the first particle. It starts at its positive extreme, so $x_1 = A\cos\omega t$; just after release it moves towards the other particle. The second starts at $-A/2$ and moves in the positive direction. Therefore its phase is $-2\pi/3$, since its cosine is $-1/2$ and its velocity $-A\omega\sin(-2\pi/3)$ is positive. Thus $x_2 = A\cos(\omega t - 2\pi/3)$.

Step 2: Find the earliest equality of displacement Writing $u = \omega t$, the crossing condition is $\cos u = \cos(u - 2\pi/3)$. The branch with equal arguments modulo 2π is impossible, because the fixed phase difference is not a full cycle. The other branch gives

$$u = -(u - 2\pi/3) + 2n\pi, \qquad u = \pi/3 + n\pi.$$

The earliest positive value is $u = \pi/3$, so $t = (\pi/3)/(2\pi/T) = T/6$. Both particles are then at $A/2$, confirming a crossing. The later value $T/3$ is not an equality, while $T/4$ places the first at zero; $5T/6$ is not the first crossing.

Step 3: Conclusion The particles first cross at $t = T/6$. Hence, the correct option is (A).

Quick Tip to Solve: Use the initial velocity direction to choose between the two phases having the same cosine, then equate the two displacements.

44. A simple pendulum made of a bob of mass *m* and a metallic wire of negligible mass has time period 2 s at T=0°C. If the temperature of the wire is increased and the corresponding change in its time period is plotted against its temperature, the resulting graph is a line of slope S. If the coefficient of linear expansion of metal is α then the value of S is : [4 marks]

(A) α

(B) $\frac{\alpha}{2}$

(C) 2α

(D) $\frac{1}{\alpha}$

Correct Answer: (A) α

Solution:

Concept:

- A simple pendulum has period $P = 2\pi\sqrt{\ell/g}$. Its bob mass does not enter the period when the wire mass is negligible.
- For a small temperature rise $\Delta\vartheta$, linear expansion gives $\ell = \ell_0(1 + \alpha\Delta\vartheta)$. To first order, the fractional period change is half the fractional length change.

Step 1: Express the period in terms of temperature Let $P_0 = 2\,\text{s}$ be the period at 0°C, and let ϑ denote the temperature rise in degrees Celsius. Substituting the expanded wire length gives

$$P = 2\pi\sqrt{\frac{\ell_0(1+\alpha\vartheta)}{g}} = P_0\sqrt{1+\alpha\vartheta}.$$

The increase in length makes the period increase, so the slope must be positive for a positive expansion coefficient.

Step 2: Linearise and identify the slope The straight-line thermal-expansion relation uses the first-order approximation $(1+z)^{1/2} \simeq 1 + z/2$, valid for $|\alpha\vartheta| \ll 1$. Consequently

$$\Delta P = P - P_0 \simeq \frac{P_0\alpha}{2}\vartheta, \qquad S = \frac{\Delta P}{\vartheta} = \frac{P_0\alpha}{2}.$$

With $P_0 = 2\,\text{s}$, the numerical coefficient is $S = \alpha$ in seconds per degree when α is quoted per degree. Option (B) omits the initial period, (C) uses the full length response with an extra factor, and (D) incorrectly makes the slope inversely proportional to expansion. An exactly square-root dependence has curvature; the stated line is the standard first-order approximation.

Step 3: Conclusion For the stated linear thermal-expansion approximation, $S = P_0\alpha/2 = \alpha$. Hence, the correct option is (A).

Quick Tip to Solve: First write $\Delta P/P_0 = \frac{1}{2}\alpha\Delta\vartheta$; substitute the given initial period only after finding the slope.

45. A 50 Ω resistance is connected to a battery of 5 V. A galvanometer of resistance 100 Ω is to be used as an ammeter to measure current through the resistance, for this a resistance r_s is connected to the galvanometer. Which of the following connections should be employed if the measured current is within 1% of thecurrent without the ammeter in the circuit ? [4 marks]

(A) $r_s = 0.5\ \Omega$ in parallel with the galvanometer

(B) $r_s = 0.5\ \Omega$ in series with the galvanometer

(C) $r_s = 1\ \Omega$ in series with galvanometer

(D) $r_s = 1\ \Omega$ in parallel with galvanometer

Correct Answer: (A) $r_s = 0.5\ \Omega$ in parallel with the galvanometer

Solution:

Concept:

- An ammeter is inserted in series with the load. Its effective resistance must be small, otherwise it reduces the current it is intended to measure.
- A low shunt resistance r_s is placed in parallel with galvanometer resistance G, giving $R_A = Gr_s/(G + r_s)$.

Step 1: Find the permitted ammeter resistance Without the meter, the load current is $I_0 = 5/50 = 0.100\,\text{A}$. On inserting an ammeter of resistance R_A, it becomes $I = 5/(50 + R_A)$. The fractional reduction must not exceed 0.01, so

$$\frac{I_0 - I}{I_0} = \frac{R_A}{50 + R_A} \le 0.01, \qquad R_A \le \frac{0.5}{0.99} = 0.50505\ldots\ \Omega.$$

This condition concerns the total resistance of the parallel galvanometer-shunt combination.

Step 2: Test the proposed connections For a $0.5\,\Omega$ parallel shunt,

$$R_A = \frac{100(0.5)}{100 + 0.5} = \frac{100}{201}\,\Omega \approx 0.4975\,\Omega.$$

Its current reduction is $0.4975/(50.4975) \approx 0.00985$, or 0.985%, which meets the condition. A $1\,\Omega$ parallel shunt gives $R_A = 100/101 \approx 0.9901\,\Omega$, reducing the current by about 1.94%, so (D) fails. The series connections in (B) and (C) give meter resistances $100.5\,\Omega$ and $101\,\Omega$, respectively; these are far above the allowed value and greatly disturb the current. The parallel shunt provides the necessary low-resistance current path.

Step 3: Conclusion Connect $r_s = 0.5\,\Omega$ in parallel with the galvanometer. The resulting loading error is approximately 0.985%. Hence, the correct option is (A).

Quick Tip to Solve: Apply the error limit to the effective ammeter resistance, not directly to the shunt resistance; then compare the parallel combinations.

46. The ratio of work done by an ideal monoatomic gas to the heat supplied to it in an isobaric process is : [4 marks]

(A) $\frac{3}{5}$
(B) $\frac{2}{3}$
(C) $\frac{3}{2}$
(D) $\frac{2}{5}$

Correct Answer: (D) $\frac{2}{5}$

Solution:

Concept:

- At constant pressure, an ideal gas does work $W = p\Delta V = nR\Delta T$, using $pV = nRT$.
- A monatomic ideal gas has $C_V = 3R/2$ and $C_P = C_V + R = 5R/2$. The supplied heat in an isobaric process is $Q = nC_P\Delta T$.

Step 1: Express the work for the temperature change Let the amount of gas be n moles, and suppose its temperature rises by ΔT. Because the pressure remains constant, the ideal-gas equation implies $p(V_f - V_i) = nR(T_f - T_i)$. Thus

$$W = p\Delta V = nR\Delta T.$$

This is the energy transferred mechanically as the gas expands against the external pressure. It is not the whole heat input because the internal energy also increases.

Step 2: Calculate the heat input and take the ratio The internal-energy increase for a monatomic gas is $\Delta U = (3/2)nR\Delta T$. With work taken positive when done by the gas, the first law gives

$$Q = \Delta U + W = \frac{3}{2}nR\Delta T + nR\Delta T = \frac{5}{2}nR\Delta T.$$

Therefore

$$\frac{W}{Q} = \frac{nR\Delta T}{(5/2)nR\Delta T} = \frac{2}{5}.$$

The amount of gas and temperature change cancel. The fraction $3/5$ in (A) is instead $\Delta U/Q$. The fraction $2/3$ in (B) is $W/\Delta U$, and $3/2$ in (C) is its reciprocal. These distinguish mechanical work from the heat that also changes internal energy.

Step 3: Conclusion The ratio of work done by the gas to heat supplied is $W/Q = 2/5$. Hence, the correct option is (D).

Quick Tip to Solve: For constant-pressure heating, divide R by C_P; a monatomic gas has $C_P = 5R/2$.

47. In the following 'I' refers to current and other symbols have their usual meaning. Choose the option that corresponds to the dimensions of electrical conductivity : [4 marks]

(A) $ML^{-3}\ T^{-3}\ I2$

(B) $M^{-1}\ L^{3}\ T^{3}\ I$

(C) $M^{-1}\ L^{-3}\ T^{3}\ I^{2}$

(D) $M^{-1}\ L^{-3}\ T^{3}\ I$

Correct Answer: (C) $M^{-1}\ L^{-3}\ T^{3}\ I^{2}$

Solution:

Concept:

- Electrical conductivity σ is the reciprocal of resistivity ρ. For a uniform wire, $R = \rho\ell/A$, where ℓ is length and A is cross-sectional area.
- The potential difference has dimensions $[V] = ML^2T^{-3}I^{-1}$, since voltage is power divided by current. Resistance is voltage divided by current.

Step 1: Derive the dimensions of resistance Power is energy per unit time, so its dimensions are ML^2T^{-3}. Dividing by current gives voltage, and applying Ohm's law $V = IR$ gives

$$[R] = \frac{[V]}{[I]} = ML^2T^{-3}I^{-2}.$$

Here I denotes the base dimension of electric current, not a numerical value. Keeping both current factors is essential: voltage already contains one inverse power of current.

Step 2: Convert resistance to resistivity and invert From $\rho = RA/\ell$, the area-to-length factor contributes one power of length. Consequently

$$[\rho] = ML^2T^{-3}I^{-2}\frac{L^2}{L} = ML^3T^{-3}I^{-2}.$$

Taking the reciprocal yields

$$[\sigma] = [\rho]^{-1} = M^{-1}L^{-3}T^3I^2.$$

This matches (C). Option (A) retains the wrong signs for mass and time; option (B) has the wrong length exponent and current power; option (D) has only one power of current. The same result follows from $\mathbf{J} = \sigma\mathbf{E}$: current density has dimensions IL^{-2}, and electric field has dimensions $MLT^{-3}I^{-1}$. Their ratio again gives the stated conductivity dimensions.

Step 3: Conclusion The dimensions of electrical conductivity are $M^{-1}L^{-3}T^3I^2$. Hence, the correct option is (C).

Quick Tip to Solve: Use $\sigma = \ell/(RA)$: conductivity has dimensions of inverse resistance divided by length, which prevents confusing it with conductance.

48. A rocket is fired vertically from the earth with an acceleration of 2g, where g is the gravitational acceleration. On an inclined plane inside the rocket, making an angle θ with the horizontal, a point object of mass *m* is kept. The minimum coefficient of friction μ_{min} between the mass and the inclined surface such that the mass does not move is : [4 marks]

(A) tanθ

(B) 2tanθ

(C) 3tanθ

(D) tan2θ

Correct Answer: (A) tanθ

Solution:

Concept:

- In a frame accelerating upward with acceleration a, a body experiences a downward pseudo-force ma, in addition to its real weight mg.
- Static friction adjusts up to $f_{\max} = \mu N$. On an incline at angle θ, resolve the effective downward force into parallel and perpendicular components.

Step 1: Find the effective gravity inside the rocket The rocket accelerates vertically upward at $a = 2g$. In its frame the pseudo-force is vertically downward with magnitude $2mg$, so the total effective downward force is

$$mg + ma = m(g + 2g) = 3mg.$$

The effective gravitational acceleration is therefore $g_{\text{eff}} = 3g$. Gravity and the pseudo-force point in the same direction; the latter must not be treated as an upward force in the rocket frame.

Step 2: Balance the components along and normal to the plane The component tending to slide the mass down the plane is $3mg\sin\theta$, while the normal reaction is $N = 3mg\cos\theta$. To keep the body at rest relative to the plane, static friction acts up the plane and must satisfy

$$3mg\sin\theta \le \mu(3mg\cos\theta).$$

At the limiting condition,

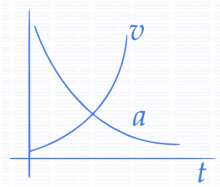
$$\mu_{\min} = \frac{3mg\sin\theta}{3mg\cos\theta} = \tan\theta.$$

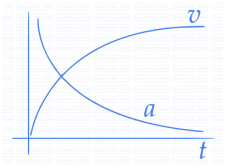
The enhanced effective gravity multiplies both the sliding tendency and the normal force by the same factor. Therefore factors 2 and 3 do not remain in the ratio, ruling out (B) and (C). No double-angle relation arises, so (D) also fails. For a horizontal surface the expression correctly gives zero required friction.

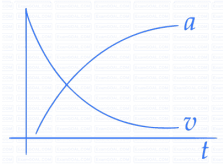
Step 3: Conclusion The minimum coefficient of friction is $\mu_{\min} = \tan\theta$. Hence, the correct option is (A).

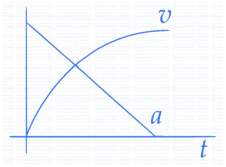
Quick Tip to Solve: When the effective gravity remains vertical, its magnitude cancels from the ratio of downslope force to normal force.

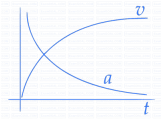
49. Which of the following option correctly describes the variation of the speed *v* and acceleration *'a'* of a point mass falling vertically in a viscous medium that applies a force F = − *kv,* where 'k' is a constant, on the body ? (Graphs are schematic and not drawn to scale) [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- For a body falling downward through a medium with drag $F = -kv$, Newton's law along the downward direction is $m\,dv/dt = mg - kv$. Drag opposes the velocity.
- Terminal speed is $v_T = mg/k$. A body released from rest approaches this speed asymptotically, while its downward acceleration decreases from g to zero.

Step 1: Solve the velocity equation for release from rest Choose downward as positive. The equation can be written $dv/dt + (k/m)v = g$, with $v(0) = 0$. Its solution is

$$v(t) = \frac{mg}{k}\left(1 - e^{-kt/m}\right).$$

Thus the speed starts at zero, rises rapidly initially, and then increases more slowly towards the positive constant mg/k. Since $dv/dt > 0$ but $d^2v/dt^2 < 0$, its graph is increasing and concave downward, with a horizontal terminal-speed asymptote.

Step 2: Identify the acceleration curve and compare the options Differentiating the velocity gives

$$a(t) = \frac{dv}{dt} = g\,e^{-kt/m}.$$

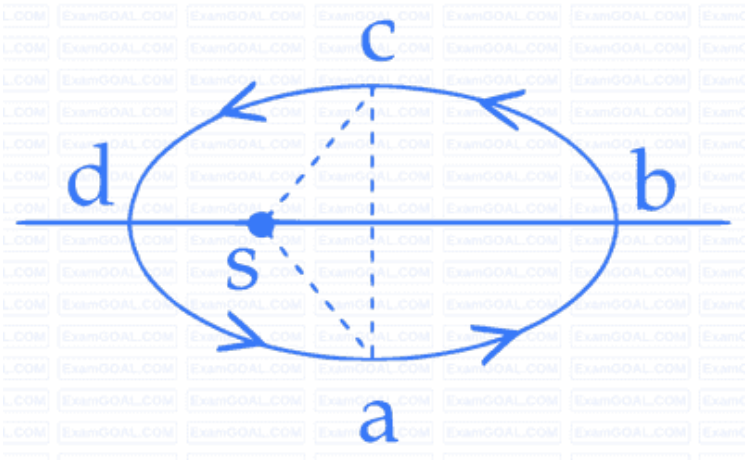
The acceleration starts at g, remains positive at every finite time, and decreases exponentially towards zero. Its graph is decreasing and concave upward, rather than a straight line. Option (B) shows precisely this pair: speed rising with decreasing slope towards a plateau, and acceleration falling asymptotically to zero. In (A), the speed curve bends upward and keeps steepening, contrary to decreasing net force. In (C), the rising curve is labelled acceleration and the falling curve speed, reversing the physical behaviour. In (D), the acceleration decreases linearly to zero at a finite time, incompatible with the exponential solution. The schematic curves need not have a meaningful intersection because speed and acceleration have different units.

Step 3: Conclusion Option (B) correctly shows $v \to mg/k$ from below and $a \to 0$ exponentially.

Hence, the correct option is (B).

Quick Tip to Solve: Check the endpoints first: release from rest means $v(0) = 0$, $a(0) = g$; linear drag makes the final speed constant and acceleration vanish.

50. Figure shows elliptical path abcd of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse. (See figure) With db as the semimajor axis, and ca as the semiminor axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then : [4 marks]



(A) $t_1 = t_2$

(B) $t_1 = 2t_2$

(C) $t_1 = 3t_2$

(D) $t_1 = 4t_2$

Correct Answer: (C) $t_1 = 3t_2$

Solution:

Concept:

- Kepler's second law states that the line joining the planet to the Sun sweeps equal areas in equal times. Consequently the travel time along an arc is proportional to the area swept about the focus S.
- The minor axis ca divides an ellipse into two equal halves. The triangle cSa must be added to one half and subtracted from the other when computing areas swept about the off-centre focus.

Step 1: Identify the two swept regions Let the total area of the ellipse be A. The area of triangle cSa is given as $A/4$. The arc abc lies on the side of the minor axis away from S, so its swept region contains the right half of the ellipse together with triangle cSa. The other arc cda sweeps the left half with this triangle removed. The two regions partition the full ellipse; their areas must therefore add to A.

Step 2: Calculate the times from the areas Using the stated triangle area gives

$$A_1 = \frac{A}{2} + \frac{A}{4} = \frac{3A}{4}, \qquad A_2 = \frac{A}{2} - \frac{A}{4} = \frac{A}{4}.$$

The same constant areal speed applies to both portions of this orbit, so

$$\frac{t_1}{t_2} = \frac{A_1}{A_2} = 3.$$

Equal times would require equal swept areas, which these are not. Ratios 2 and 4 likewise do not match the given one-quarter triangle area. The planet moves faster near S, explaining why the shorter-time left-side passage can cover its arc more quickly.

Step 3: Conclusion Thus $t_1 = 3t_2$. Hence, the correct option is (C).

Quick Tip to Solve: Draw the two radii from the focus to the minor-axis endpoints; add the triangle on the far side and subtract it on the near side.

51. The truth table given in fig. represents :

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

[4 marks]

(A) AND - Gate

(B) OR - Gate

(C) NAND - Gate

(D) NOR - Gate

Correct Answer: (B) OR - Gate

Solution:

Concept:

- A logic gate is identified by its complete truth table, not merely by one row. Inputs and output take binary values 0 and 1.
- For an OR gate, $Y = A + B$, where the plus sign denotes Boolean OR: the output is 1 whenever at least one input is 1.

Step 1: Read the distinguishing rows The supplied output sequence for inputs $(A, B) = (0,0), (0,1), (1,0), (1,1)$ is respectively $0, 1, 1, 1$. In particular, either single high input is sufficient to make the output high. It is low only when both inputs are low. This is precisely the inclusive OR condition; both inputs being high also gives a high output.

Step 2: Compare every candidate gate For an AND gate the output sequence would be $0, 0, 0, 1$, since both inputs must be high. It fails the middle two rows. A NAND gate has the complementary AND sequence $1, 1, 1, 0$; it fails both the first and last rows. A NOR gate has the complementary OR sequence $1, 0, 0, 0$, which is opposite to the given output in all four rows. Only the OR gate reproduces every entry:

$$0 + 0 = 0, \quad 0 + 1 = 1, \quad 1 + 0 = 1, \quad 1 + 1 = 1.$$

The last equality is Boolean addition, not ordinary arithmetic addition, so its output remains a single binary 1.

Step 3: Conclusion The given truth table represents an OR gate. Hence, the correct option is (B).

Quick Tip to Solve: An OR table has just one zero output, at input 00; AND instead has just one high output, at 11.

52. Microwave oven acts on the principle of : [4 marks]

(A) transferring electrons from lower to higher energy levels in water molecule

(B) giving rotational energy to water molecules

(C) giving vibrational energy to water molecules

(D) giving translational energy to water molecules

Correct Answer: (B) giving rotational energy to water molecules

Solution:

Concept:

- Water is a polar molecule with a permanent electric dipole moment. An alternating electric field exerts a changing torque on that dipole.
- Microwave heating couples primarily to molecular rotational or orientational motion. Intermolecular interactions dissipate this motion and increase the thermal energy of the food.

Step 1: Identify the direct interaction The microwave electric field repeatedly reverses direction. The dipole of a water molecule tends to align with this field, so the changing field drives repeated reorientation. A dipole in an electric field experiences the torque $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$. Thus the initial molecular response is rotational motion, rather than an electronic transition. In liquid water this is a lossy dielectric response, rather than free rotation of isolated molecules.

Step 2: Distinguish the other energy transfers Electronic excitation requires much higher photon energies, typically in the visible or ultraviolet range. Molecular vibrational excitations are primarily associated with infrared radiation, so vibration is not the direct mechanism selected by microwaves in this question. Translation and other molecular motions do gain energy after collisions and relaxation; that is the resulting heating, not the initial dipolar coupling. The distinction is between how the electric field first does work on polar molecules and how that energy later becomes shared as heat. The option specifying rotational energy therefore describes the microwave mechanism; the vibrational-energy option does not.

Step 3: Conclusion Microwaves give rotational energy to water molecules, which is dissipated as heat. Hence, the correct option is (B). The supplied key selects the vibrational-energy choice, but that mechanism does not describe microwave heating.

Quick Tip to Solve: Associate microwave absorption with rotational or dipolar orientation, infrared with vibration, and visible or ultraviolet radiation with electronic excitation.

53. To find the focal length of a convex mirror, a student records the following data :

Object Pin	Convex Lens	Convex Mirror	Image Pin
22.2 cm	32.2 cm	45.8 cm	71.2 cm

The focal length of the convex lens is f_1 and that of mirror is f_2. Then taking index correction to be negligibly small, f_1 and f_2 are close to : [4 marks]

(A) $f_1 = 12.7$ cm $f_2 = 7.8$ cm

(B) $f_1 = 7.8$ cm $f_2 = 12.7$ cm

(C) $f_1 = 7.8$ cm $f_2 = 25.4$ cm

(D) $f_1 = 15.6$ cm $f_2 = 25.4$ cm

Correct Answer: (B) $f_1 = 7.8$ cm $f_2 = 12.7$ cm

Solution:

Concept:

- For a thin convex lens producing a real image, $1/f_1 = 1/u + 1/v$, with u, v used as positive distance magnitudes.
- In the convex-mirror experiment the pin image coincides with the centre of curvature of the mirror when reflected rays retrace their paths. Hence the mirror focal length is half the mirror-to-image-pin distance.

Step 1: Use the optical-bench readings for the lens The object pin, convex lens and image pin have readings 22.2 cm, 32.2 cm and 71.2 cm. Distances are differences of readings; a bench coordinate is not itself an object distance. Therefore

$$u = 32.2 - 22.2 = 10.0\,\text{cm}, \qquad v = 71.2 - 32.2 = 39.0\,\text{cm}.$$

The positive focal-length magnitude follows as

$$f_1 = \frac{uv}{u+v} = \frac{10.0 \times 39.0}{49.0}\,\text{cm} \simeq 7.96\,\text{cm}.$$

This is close to the listed value 7.8 cm rather than 12.7 cm or 15.6 cm.

Step 2: Find the mirror radius and focal length The convex mirror is at 45.8 cm, while the lens alone would bring the incident rays to the image pin at 71.2 cm. In the no-parallax adjustment that point is the mirror's centre of curvature, so the converging rays strike normally and retrace their paths. Thus

$$R = 71.2 - 45.8 = 25.4\,\text{cm}, \qquad f_2 = \frac{R}{2} = 12.7\,\text{cm}.$$

The 25.4 cm appearing in other options is the radius, not the mirror focal length. Combining both independently computed values selects only one pair.

Step 3: Conclusion The closest listed pair is $f_1 = 7.8\,\text{cm}$, $f_2 = 12.7\,\text{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: Subtract optical-bench readings first, and remember that the convex mirror gives its radius in this retracing experiment; divide it by two.

54. Three capacitors each of 4 μF are to be connected in such a way that the effective capacitance is 6 μF. This can be done by connecting them : [4 marks]

(A) all in series

(B) two in series and one in parallel

(C) all in parallel

(D) two in parallel and one in series

Correct Answer: (B) two in series and one in parallel

Solution:

Concept:

- For capacitors in parallel, capacitances add: $C_{\rm p} = C_1 + C_2$. The connected capacitors share the same potential difference.
- For capacitors in series, reciprocal capacitances add: $1/C_{\rm s} = 1/C_1 + 1/C_2$. Two equal capacitors of capacitance C consequently have equivalent capacitance $C/2$.

Step 1: Construct the required equivalent capacitance Each available capacitor has capacitance $4\,\mu$F. Put two of them in series first. Their equivalent capacitance is

$$C_{12} = \frac{4 \times 4}{4 + 4}\,\mu\text{F} = 2\,\mu\text{F}.$$

Connect that entire two-capacitor branch in parallel with the third capacitor. Both branches then experience the same applied voltage and their charges add. Therefore

$$C_{\rm eq} = C_{12} + 4\,\mu\text{F} = 6\,\mu\text{F}.$$

This explicitly uses all three capacitors and produces the required value.

Step 2: Check the alternative connections Putting all three in series gives $C_{\rm eq} = 4/3\,\mu$F, which is too small. Putting all three in parallel gives $12\,\mu$F, which is too large. For two in parallel followed by the third in series, the parallel pair is $8\,\mu$F, and the complete value becomes

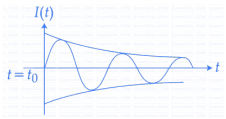
$$C_{\rm eq} = \frac{8 \times 4}{8 + 4}\,\mu\text{F} = \frac{8}{3}\,\mu\text{F},$$

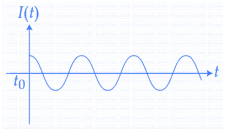
again different from the target. Thus the order and grouping of series and parallel connections matter; the words describe whole branches, not an unspecified mixture.

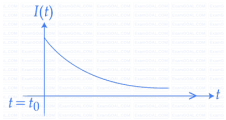
Step 3: Conclusion Two capacitors in series, with the third in parallel with that pair, give $6\,\mu$F. Hence, the correct option is (B).

Quick Tip to Solve: For three equal capacitors, series-pair plus parallel single gives $C/2 + C = 3C/2$, so substitute C only at the end.

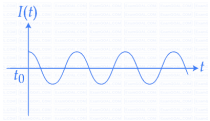
55. A series *LR* circuit is connected to a voltage source with $V(t) = V_0 \sin\Omega t$. After very large time, current *I(t)* behaves as ($t_0 >> \frac{L}{R}$) : [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- The series LR equation is $L\,dI/dt + RI = V_0 \sin(\Omega t)$. Its complementary response decays with time constant $\tau = L/R$.
- In steady state the current is sinusoidal at the driving frequency. Its amplitude is $I_0 = V_0/\sqrt{R^2 + (\Omega L)^2}$, and it lags the voltage by $\phi = \tan^{-1}(\Omega L/R)$.

Step 1: Separate transient and forced responses The complete solution is the sum of a transient term and a forced sinusoid:

$$I(t) = Ke^{-Rt/L} + \frac{V_0}{\sqrt{R^2 + (\Omega L)^2}} \sin(\Omega t - \phi).$$

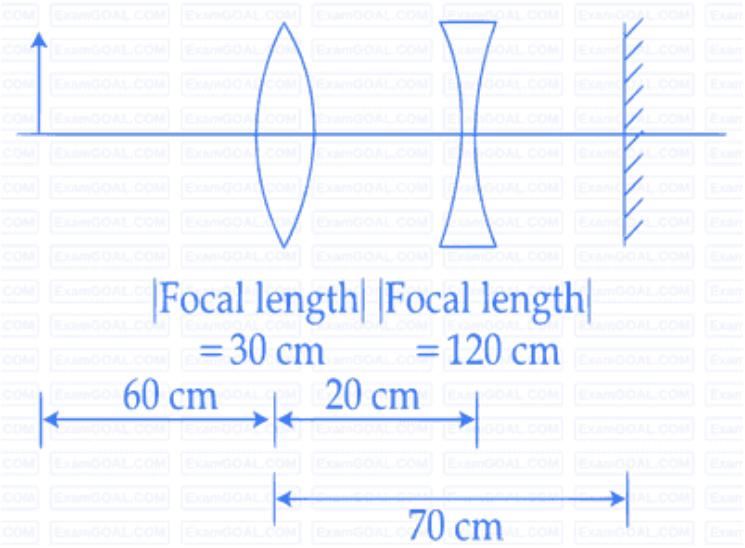
Here K depends on the initial condition. The problem specifically chooses $t_0 \gg L/R$, so the exponential has already become negligible. One must therefore examine the persistent forced response rather than a switching transient.

Step 2: Select the correct long-time graph After t_0, the current continues to alternate about zero with constant amplitude and constant period $2\pi/\Omega$. Graph (B) has this behaviour. Graph (A) shows an oscillation whose envelope decays; such an envelope is not the maintained steady response to a sinusoidal source. Graph (C) shows a purely decaying current, and graph (D) approaches a constant nonzero current, as could occur for a different, direct-current driving condition. Neither describes an LR circuit driven indefinitely by the given sinusoidal voltage. The inductor changes amplitude and phase; it does not remove the driving frequency or make the sustained current decay.

Step 3: Conclusion The long-time current is a constant-amplitude sinusoid, lagging the applied voltage. Hence, the correct option is (B).

Quick Tip to Solve: For a driven linear circuit at times much larger than its time constant, discard the exponential transient and keep the sinusoidal particular solution.

56. A convex lens, of focal length 30 cm, a concave lens of focal length 120 cm, and aplane mirror are arranged as shown. For an object kept at a distance of 60 cm from the convex lens, the final image, formed by the combination, is a real image, at a distance of : [4 marks]



(A) 60 cm from the convex lens

(B) 60 cm from the concave lens

(C) 70 cm from the convex lens

(D) 70 cm from the concave lens

Correct Answer: For the diagram's 20 cm lens separation and 70 cm convex-lens-to-mirror distance, the final real image is 75 cm from the convex lens. No listed option is correct. The supplied key selects (A), but 60 cm does not follow from the stated focal lengths and displayed separations.

Solution:

Concept:

- Apply $1/v - 1/u = 1/f$ at each lens, measuring signed distances along the current propagation direction. Use $f_1 = +30\,\text{cm}$ and $f_2 = -120\,\text{cm}$.
- A plane mirror at coordinate m maps an incident geometrical image at x to a reflected image at $2m - x$. The diagram places the convex lens at x=0, the concave lens at x=20 cm and the mirror at x=70 cm.

Step 1: Find the forward images Take rightward coordinates in centimetres. The object is at x=-60. At the convex lens, $u = -60$, giving

$$\frac{1}{v_1} = \frac{1}{30} - \frac{1}{60} = \frac{1}{60}, \qquad x_1 = 60.$$

The rays then meet the concave lens at x=20 while converging toward x=60, so its object is virtual with $u = +40$. Hence

$$\frac{1}{v_2} = -\frac{1}{120} + \frac{1}{40} = \frac{1}{60}, \qquad x_2 = 20 + 60 = 80.$$

The mirror at x=70 reflects this converging bundle to an image at $x_3 = 2(70) - 80 = 60$.

Step 2: Trace the return through both lenses After reflection the rays cross at x=60 and then travel leftward toward the concave lens. Its real object is 40 cm to its right; with the positive direction now leftward, $u = -40$. Thus

$$\frac{1}{v_3} = -\frac{1}{120} - \frac{1}{40} = -\frac{1}{30}.$$

This virtual image is 30 cm to the right of the concave lens, at x=50. The returning light reaches the convex lens diverging as if from x=50, so the real object distance for that pass is $u = -50$. Consequently

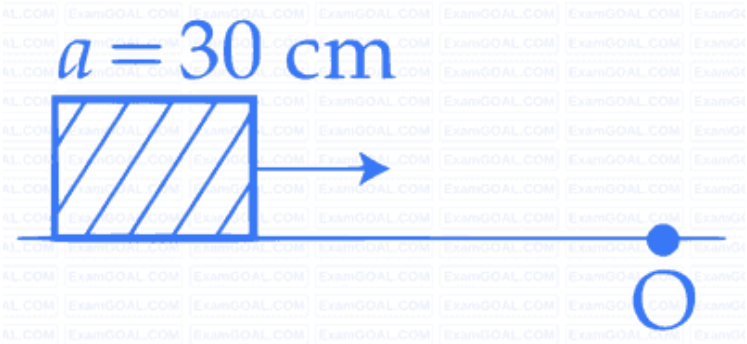
$$\frac{1}{v_4} = \frac{1}{30} - \frac{1}{50} = \frac{1}{75}, \qquad v_4 = 75\,\text{cm}.$$

The final image is therefore real and lies at x=-75, on the original object side. It is 75 cm from the convex lens and 95 cm from the concave lens. None of the four listed distances equals either result.

Step 3: Conclusion For the diagram's 20 cm lens separation and 70 cm convex-lens-to-mirror distance, the final real image is 75 cm from the convex lens. No listed option is correct. The supplied key selects (A), but 60 cm does not follow from the stated focal lengths and displayed separations.

Quick Tip to Solve: Reverse the distance-sign direction on the return journey, and carry each image coordinate forward as the next optical element's object.

57. A cubical block of side 30 cm is moving with velocity 2 ms^{-1} on a smooth horizontal surface. The surface has a bump at a point O as shown in figure. The angular velocity (in rad/s) of the block immediately after it hits the bump, is : [4 marks]



(A) 5.0

(B) 6.7

(C) 9.4

(D) 13.3

Correct Answer: (A) 5.0

Solution:

Concept:

- An impact with a fixed bump exerts an external impulse, so linear momentum and kinetic energy need not be conserved. Its moment about the bump O is zero during the short collision, allowing conservation of angular momentum about O.

- For a cube of side a rotating about an edge parallel to one of its edges, the centre-of-mass moment of inertia is $I_C = ma^2/6$. The distance from the centre to the corner axis satisfies $d^2 = a^2/2$.

Step 1: Find the initial angular momentum The centre moves horizontally with speed v and lies a/2 above O. Its perpendicular lever arm is therefore a/2, not the diagonal distance to O. Initially there is no spin, so

$$L_O = mv\frac{a}{2}.$$

Gravity and the normal force have negligible angular impulse during the very brief impact.

Step 2: Use the moment of inertia about the bump The parallel-axis theorem gives

$$I_O = \frac{ma^2}{6} + m\frac{a^2}{2} = \frac{2}{3}ma^2.$$

Immediately after the lower leading edge is arrested by O, the block rotates about that edge. Hence

$$\omega = \frac{mva/2}{(2/3)ma^2} = \frac{3v}{4a} = \frac{3(2)}{4(0.30)} = 5.0\ \text{rad}\,\text{s}^{-1}.$$

Using energy conservation would overestimate the speed because this impact is inelastic. The other three numerical options fail the angular-momentum equation.

Step 3: Conclusion The angular speed is $5.0\ \text{rad}\,\text{s}^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: For a cube caught at its leading lower edge, combine the lever arm a/2 with I about the edge equal to $2ma^2/3$.

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58. An audio signal consists of two distinct sounds : one a human speech signal in the frequency band of 200 Hz to 2700 Hz, while the other is a high frequency music signal in the frequency band of 10200 Hz to 15200 Hz. The ratio of the AM signal bandwidth required to send both the signals together to the AM signal bandwidth required to send just the human speech is : [4 marks]

(A) 3

(B) 5

(C) 6

(D) 2

Correct Answer: No exact listed option: the conventional AM channel-span ratio is $152/27 \simeq 5.63$; (C) is the nearest rounded choice.

Solution:

Concept:

- For ordinary double-sideband AM, a modulating frequency f produces sidebands at $f_c - f$ and $f_c + f$. The required channel span is twice the highest modulating frequency, even when some frequencies inside that span are absent.
- For several simultaneous audio components, use the largest upper frequency. Do not add the widths of the occupied audio bands when calculating the full AM channel bandwidth.

Step 1: Calculate the two channel spans For speech alone, the highest audio frequency is 2700 Hz, giving

$$B_s = 2(2700) = 5400 \text{ Hz}.$$

The lower cutoff 200 Hz locates the inner edge of each sideband but does not change the full lower-to-upper sideband span. When music is included, the highest frequency becomes 15200 Hz, so

$$B_{s+m} = 2(15200) = 30400 \text{ Hz}.$$

The unused interval between speech and music does not shorten that enclosing frequency interval.

Step 2: Compare the ratio with the choices The numerical ratio is

$$\frac{B_{s+m}}{B_s} = \frac{15200}{2700} \simeq 5.63.$$

Among the listed integer choices, 6 is the nearest and matches the supplied key when a rounded answer is intended. A ratio of 3 follows from adding only occupied band widths; it does not represent the complete AM channel span. Ratios 5 and 2 likewise do not follow from the given upper frequencies. The precise ratio from the printed data is 5.63 rather than exactly 6.

Step 3: Conclusion The complete AM channel-span ratio is $152/27 \simeq 5.63$. None of the listed choices is exact, and no rounding rule is stated. The supplied key (C), 6, is the nearest integer choice; counting only occupied sideband widths instead gives (A), 3. The question therefore does not have an unqualified exact listed answer.

Quick Tip to Solve: AM channel bandwidth uses the highest audio frequency, not the width of a separated audio band; retain the exact ratio before choosing a rounded option.

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59. Two engines pass each other moving in opposite directions with uniform speed of 30 m/s. One of them is blowing a whistle of frequency 540 Hz. Calculate the frequency heard by driver of second engine before they pass each other. Speed of sound is 330 m/sec : [4 marks]

(A) 450 Hz

(B) 540 Hz

(C) 648 Hz

(D) 270 Hz

Correct Answer: (C) 648 Hz

Solution:

Concept:

- The Doppler effect depends on source and observer speeds relative to the sound medium. For an observer and a source moving toward one another, the numerator increases while the denominator decreases.
- Writing u for sound speed, v_o for observer speed, v_s for source speed, and f for emitted frequency gives $f_{obs} = f(u + v_o)/(u - v_s)$ during approach.

Step 1: Choose the approach signs Before the engines pass, the driver moves toward the oncoming wavefronts at 30 m/s, and the whistling engine moves toward the driver at 30 m/s. The observer meets wavefronts at a speed $330+30 = 360\ \mathrm{m/s}$. Meanwhile, successive emitted crests are compressed to a wavelength

$$\lambda = \frac{330-30}{540} = \frac{300}{540}\ \mathrm{m}.$$

Both effects increase the observed pitch; the relative speed 60 m/s should not simply replace one of the two speeds in the Doppler formula.

Step 2: Calculate the received frequency Divide the wavefront encounter speed by the compressed wavelength:

$$f_{obs} = \frac{360}{300/540} = 540\frac{360}{300} = 648\ \mathrm{Hz}.$$

This exceeds 540 Hz as required for approach. The 450 Hz option corresponds to recession, with both signs reversed. The unchanged value 540 Hz would require no Doppler shift, while 270 Hz is inconsistent with the modest speeds given. The sound speed is used in the medium frame throughout.

Step 3: Conclusion The second driver hears $648\ \mathrm{Hz}$ before the engines cross. Hence, the correct option is (C).

Quick Tip to Solve: For approach, write plus for the observer in the numerator and minus for the source in the denominator; the result must exceed the emitted frequency.

60. An unknown transistor needs to be identified as a *npn* or *pnp* type. A multimeter, with + ve and − ve terminals, is used to measure resistance between different terminals of transistor. If terminal 2 is the base of the transistor then which of the following is correct for a pnp transistor ? [4 marks]

(A) + ve termial 1, −ve terminal 2, resistance high

(B) + ve termial 2, −ve terminal 1, resistance high

(C) + ve termial 3, −ve terminal 2, resistance high

(D) + ve termial 2, −ve terminal 3, resistance low

Correct Answer: (B) + ve termial 2, −ve terminal 1, resistance high

Solution:

Concept:

- A pnp transistor has two p-type outer regions and an n-type base. Each outer-region-to-base junction therefore behaves as a diode with its p side at terminal 1 or 3 and its n side at terminal 2.
- A p-n junction has low resistance under forward bias, with the p region positive and n region negative. Reversing those connections gives high resistance.

Step 1: Test the polarity of each junction Terminal 2 is the n-type base. Connecting the positive multimeter terminal to 1 and the negative terminal to 2 forward biases the emitter-base or collector-base junction. The resistance should consequently be low, so (A), which says high, is false. The same argument applies to positive at 3 and negative at 2: (C) is also false. This reasoning does not require deciding which outer terminal is the emitter.

Step 2: Reverse the base connection With the positive lead at base terminal 2 and the negative lead at outer terminal 1, the n region is positive relative to the p region. This reverse biases that junction and gives high resistance, exactly as stated in (B). With positive at 2 and negative at 3, the other junction is also reverse biased, giving high rather than low resistance, so (D) is false. Thus only (B) agrees with both the stated pnp structure and the stated lead polarities. The supplied key selects (A), but that would require low resistance rather than its printed high resistance.

Step 3: Conclusion Positive at terminal 2 and negative at terminal 1 gives high resistance. Hence, the correct option is (B). The supplied key contradicts the printed resistance for a pnp junction.

Quick Tip to Solve: Mark the p and n regions first: positive on p and negative on n means low resistance; reversing the leads means high resistance.

Chemistry (120 marks)

61. The group of molecules having identical shape is : [4 marks]

(A) SF_4 , XeF_4 , CCl_4

(B) ClF_3 , $XeOF_2$, XeF_3^+

(C) BF_3 , PCl_3 , XeO_3

(D) PCl_5 , IF_5 , XeO_2F_2

Correct Answer: (B) ClF_3 , $XeOF_2$, XeF_3^+

Solution:

Concept:

- VSEPR counts sigma-bond domains and lone pairs around a central atom. A multiple bond counts as one domain when assigning the electron-pair arrangement, although its repulsion is stronger.
- Three bonded atoms and two equatorial lone pairs in a trigonal-bipyramidal domain arrangement give an AX_3E_2, T-shaped molecule.

Step 1: Check the T-shaped set In ClF_3, the chlorine has three bonds and two lone pairs. In $XeOF_2$, xenon has one oxygen bond domain, two fluorine bond domains and two lone pairs. In $XeF_3{}^+$, xenon likewise has three Xe–F bonds and two lone pairs after accounting for the positive charge. Each is therefore AX_3E_2. The two lone pairs occupy equatorial sites to minimise 90-degree repulsions, leaving a T-shaped arrangement of the three bonded atoms.

Step 2: Eliminate the other sets Option (A) mixes seesaw SF_4, square-planar XeF_4, and tetrahedral CCl_4. Option (C) mixes trigonal-planar BF_3 with trigonal-pyramidal PCl_3 and XeO_3. Option (D) contains trigonal-bipyramidal PCl_5, square-pyramidal IF_5, and seesaw XeO_2F_2. In the last species the two oxygen double bonds still count as two domains, and xenon has one lone pair. Thus none of these groups has one common molecular shape.

Step 3: Conclusion ClF_3, $XeOF_2$, and $XeF_3{}^+$ are all T-shaped. Hence, the correct option is (B).

Quick Tip to Solve: Count each double bond as one VSEPR domain and include the ionic charge before assigning the central atom lone pairs.

62. The reaction of ozone with oxygen atoms in the presence of chlorine atoms can occur by a two step process shown below :

$O_3(g) + Cl^{\bullet}(g) \rightarrow O_2(g) + ClO^{\bullet}(g)$ . . . . . .(i)

$k_i = 5.2 \times 10^9$ L mol^{-1} s^{-1}

$ClO^{\bullet}(g) + O^{\bullet}(g) \rightarrow O_2(g) + Cl^{\bullet}(g)$ . . . . . (ii)

$k_{ii} = 2.6 \times 10^{10}$ L mol^{-1} s^{-1}

The closest rate constant for the overall reaction $O_3(g) + O^{\bullet}(g) \rightarrow 2\,O_2(g)$ is : [4 marks]

(A) 5.2×10^9 L mol^{-1} s^{-1}

(B) 2.6×10^{10} L mol^{-1} s^{-1}

(C) 3.1×10^{10} L mol^{-1} s^{-1}

(D) 1.4×10^{20} L mol^{-1} s^{-1}

Correct Answer: No unique overall rate constant follows without concentration conditions; (A) is only a conditional slow-step estimate, while the supplied key (D) is dimensionally inconsistent.

Solution:

Concept:

- Adding elementary steps cancels intermediates and regenerates a catalyst. That addition gives the net stoichiometric equation; it does not mean that elementary rate constants are added or multiplied.
- For a simplified slow-step estimate with comparable reactant concentrations, the smaller elementary rate constant identifies the slower step. A rigorous overall rate law also depends on concentrations and the treatment of the intermediate.

Step 1: Identify the slow step and catalyst Adding the two given equations cancels ClO and Cl, leaving $\mathrm{O_3 + O \longrightarrow 2\,O_2}$. Chlorine atoms are regenerated and hence act as catalyst; chlorine monoxide is the intermediate. The given constants obey

$$k_{ii} = 5k_i.$$

Thus the first step is the slower elementary step on the usual comparable-concentration approximation. Its rate is $r_i = k_i[\mathrm{O_3}][\mathrm{Cl}]$, so ozone consumption is controlled by that step in this approximation.

Step 2: Check the available numerical estimates The slow-step estimate uses

$$k_i = 5.2 \times 10^9\ \mathrm{L\,mol^{-1}\,s^{-1}},$$

which is option (A). Adding the constants to get option (C) has no mechanistic justification, and option (B) uses the faster step. Multiplying them gives the numerical value in (D), but its actual dimensions would be $\mathrm{L^2\,mol^{-2}\,s^{-2}}$, not the units printed for a bimolecular rate constant. Consequently the supplied key (D) cannot be justified by multiplying the two elementary constants. Without concentration conditions there is no unique exact overall bimolecular constant; (A) is the defensible slow-step estimate among the choices.

Step 3: Conclusion The printed data do not determine a unique overall rate constant. Under a comparable-concentration, slow-step approximation, (A), $5.2 \times 10^9\ \mathrm{L\,mol^{-1}\,s^{-1}}$, is an estimate. The

supplied key (D) cannot be obtained consistently: multiplying the elementary constants produces different dimensions. Thus no unique unconditional option can be selected.

Quick Tip to Solve: Add elementary equations to find stoichiometry, but never multiply their rate constants to obtain an overall rate constant; always check dimensions.

63. A reaction at 1 bar is non-spontaneous at low temperature but becomes spontaneous at high temperature. Identify the correct statement about the reaction among the following : [4 marks]

(A) Both ΔH and ΔS are negative.

(B) Both ΔH and ΔS are positive.

(C) ΔH is positive while ΔS is negative.

(D) ΔH is negative while ΔS is positive.

Correct Answer: (B) Both ΔH and ΔS are positive.

Solution:

Concept:

- At fixed pressure, spontaneity is determined by Gibbs energy: $\Delta G = \Delta H - T\Delta S$. A reaction is spontaneous when $\Delta G < 0$ and non-spontaneous when $\Delta G > 0$.
- Over the temperature range used in this qualitative test, take the signs of enthalpy and entropy changes as fixed. Increasing T makes the entropy contribution more important.

Step 1: Infer the entropy sign from the temperature effect The process changes from non-spontaneous to spontaneous as temperature rises. Therefore ΔG must decrease with temperature. Since

$$\left(\frac{\partial \Delta G}{\partial T}\right)_p = -\Delta S,$$

this requires positive ΔS. If entropy decreased, the term $-T\Delta S$ would become increasingly positive, giving the opposite trend. This already excludes options (A) and (C), both of which assign negative entropy change.

Step 2: Infer the enthalpy sign and crossover At low temperature, the positive entropy term is too small to make the process spontaneous. The enthalpy contribution must therefore be positive: $\Delta H > 0$. At higher temperature, the positive quantity $T\Delta S$ can exceed ΔH, giving

$$\Delta G < 0 \quad \text{when} \quad T > \frac{\Delta H}{\Delta S}.$$

If enthalpy were negative while entropy were positive, as in (D), both contributions to Gibbs energy would favour spontaneity at every positive temperature; that would contradict the low-temperature observation. Thus both changes must be positive.

Step 3: Conclusion Both ΔH and ΔS are positive. Hence, the correct option is (B).

Quick Tip to Solve: A reaction that becomes spontaneous only on heating needs positive entropy change to overcome an initially unfavourable positive enthalpy change.

64. Which intermolecular force is most responsible in allowing xenon gas to liquefy? [4 marks]

(A) Dipole - dipole

(B) Ion - dipole

(C) Instantaneous dipole - induced dipole

(D) Ionic

Correct Answer: (C) Instantaneous dipole - induced dipole

Solution:

Concept:

- London dispersion forces arise when a momentary fluctuation of an electron cloud creates an instantaneous dipole, which induces a dipole in a neighbouring atom.
- A noble-gas atom has no permanent molecular dipole. Its attractive interactions nevertheless grow stronger as its electron cloud becomes larger and more polarizable.

Step 1: Identify the particles and their polarity Xenon consists of electrically neutral, monatomic particles, Xe. A spherically averaged xenon atom has no permanent separation of positive and negative charge. Therefore permanent dipole–dipole attraction cannot provide the explanation for liquefaction. The gas also contains neither cations nor anions, so ionic attraction and ion–dipole attraction are absent.

Step 2: Identify the attraction that remains At any instant, electrons need not be distributed symmetrically about the xenon nucleus. A temporary dipole can distort the electron cloud of a neighbouring atom, creating an induced dipole and an attractive interaction. Xenon has many electrons and a relatively diffuse electron cloud, so these London dispersion interactions are appreciable. On cooling, the reduction in thermal motion allows these attractions to hold atoms together in the liquid. Option (C) names precisely this instantaneous-dipole–induced-dipole mechanism; the remaining three options require permanent dipoles or ions that xenon gas does not possess.

Step 3: Conclusion Xenon liquefies primarily because of instantaneous dipole–induced dipole (London dispersion) attractions. Hence, the correct option is (C).

Quick Tip to Solve: For monatomic noble gases, look for London dispersion forces; a larger, more polarizable electron cloud strengthens their attraction.

65. Which one of the following complexes will consume more equivalents of aqueous solution of $Ag(NO_3)$? [4 marks]

(A) $Na_3[CrCl_6]$

(B) $[Cr(H_2O)_5Cl]Cl_2$

(C) $[Cr(H_2O)_6]Cl_3$

(D) $Na_2[CrCl_5(H_2O)]$

Correct Answer: (C) $[Cr(H_2O)_6]Cl_3$

Solution:

Concept:

- An aqueous solution of $AgNO_3$ precipitates freely ionizable chloride as $AgCl$: $Ag^+ + Cl^- \longrightarrow AgCl\,(s)$.
- Chloride ligands inside square brackets belong to the coordination sphere; chloride counterions outside the brackets dissociate in water and give the immediate precipitation test.

Step 1: Count chloride counterions outside each complex For $Na_3[CrCl_6]$, all six chlorides are coordinated inside the complex anion, so there is no free chloride counterion. In $[Cr(H_2O)_5Cl]Cl_2$, one chloride is coordinated and two are outside the brackets. Thus one mole of this salt supplies two moles of free Cl^-. In $[Cr(H_2O)_6]Cl_3$, all three chloride ions are outside the coordination sphere. Finally, $Na_2[CrCl_5(H_2O)]$ has sodium counterions and no chloride outside its brackets.

Step 2: Compare the silver nitrate consumption The stoichiometric requirement is one mole of silver ion for every mole of free chloride ion. The four salts therefore consume, respectively, $0, 2, 3, 0$ equivalents of silver nitrate per mole in this test. The largest value belongs to the hexaaquachromium(III) chloride salt, not to the salt with the greatest total number of chlorine atoms. Its precipitation may be represented by

$$[Cr(H_2O)_6]Cl_3 + 3\,AgNO_3 \longrightarrow [Cr(H_2O)_6](NO_3)_3 + 3\,AgCl\,(s).$$

This distinguishes the three outer-sphere chlorides from chlorido ligands that remain bonded to chromium during the usual aqueous test.

Step 3: Conclusion $[Cr(H_2O)_6]Cl_3$ consumes the most silver nitrate: three equivalents per mole of complex salt. Hence, the correct option is (C).

Quick Tip to Solve: For the silver nitrate test, count chloride ions outside square brackets, rather than all chlorine atoms in the formula.

66. Bouveault-Blanc reduction reaction involves : [4 marks]

(A) Reduction of an acyl halide with H_2/Pd.

(B) Reduction of an ester with Na/C_2H_5OH.

(C) Reduction of a carbonyl compound with Na/Hg and HCl.

(D) Reduction of an anhydride with $LiAlH_4$.

Correct Answer: (B) Reduction of an ester with Na/C_2H_5OH.

Solution:

Concept:

- The Bouveault–Blanc reaction reduces an ester to alcohols using sodium metal in an alcohol solvent, commonly ethanol.
- The ester acyl carbon becomes a primary-alcohol carbon. The alkoxy fragment is also released as its corresponding alcohol after protonation.

Step 1: Recall the substrate and the reducing system The identifying combination is an ester together with sodium metal and ethanol, $\mathrm{Na}/\mathrm{C_2H_5OH}$. Sodium supplies electrons, while the alcohol medium supplies protons in the reduction sequence. For a general ester the overall organic transformation is

$$\mathrm{RCOOR' + 4[H] \longrightarrow RCH_2OH + R'OH.}$$

Thus the carbonyl group of the ester is converted into a primary alcohol without requiring catalytic hydrogen gas.

Step 2: Check every proposed description Option (B) correctly pairs the ester substrate with sodium in ethanol. Option (A), reduction of an acyl halide with hydrogen over palladium, describes a different reduction family: under the controlled, poisoned-catalyst conditions it is the Rosenmund route to an aldehyde. Option (C) instead names a carbonyl compound with sodium amalgam and acid; it is not the defining ester/sodium/alcohol combination of the Bouveault–Blanc reaction. Option (D) uses lithium aluminium hydride, a hydride reducing agent which can indeed reduce anhydrides, but it is a different reagent system. The ability of several reagents to produce alcohols does not make their named reductions interchangeable; both substrate and reagent must match.

Step 3: Conclusion Bouveault–Blanc reduction is the reduction of an ester with $\mathrm{Na}/\mathrm{C_2H_5OH}$. Hence, the correct option is (B).

Quick Tip to Solve: Remember the pair for Bouveault–Blanc: ester substrate plus sodium in alcohol; do not identify a named reaction by its product alone.

67. The total number of orbitals associated with the principal quantum number 5 is : [4 marks]

(A) 5

(B) 10

(C) 20

(D) 25

Correct Answer: (D) 25

Solution:

Concept:

- For a principal quantum number n, the allowed orbital angular-momentum quantum numbers are $l = 0, 1, \ldots, n-1$.
- Each subshell with quantum number l has $2l+1$ orbitals, corresponding to $m_l = -l, \ldots, +l$. The number of orbitals in a shell is n^2.

Step 1: List the subshells in the fifth shell With $n = 5$, the allowed values of l are $0, 1, 2, 3, 4$. These correspond to the $5s$, $5p$, $5d$, $5f$ and $5g$ subshells. Their orbital counts follow directly from the number of magnetic quantum-number values:

$$l = 0 : 1, \quad l = 1 : 3, \quad l = 2 : 5,$$
$$l = 3 : 7, \quad l = 4 : 9.$$

These are allowed quantum states of the shell whether or not they are occupied in the ground state of a particular atom.

Step 2: Sum orbital counts and distinguish electrons from orbitals The required total is

$$N = 1 + 3 + 5 + 7 + 9 = 25 = 5^2.$$

Each orbital can accommodate two electrons of opposite spin, so the maximum electron capacity of this shell would be $2 \times 25 = 50$; the question asks for orbitals, not electron capacity. Five, option (A), counts subshells rather than orbitals. Ten, option (B), is neither the complete orbital count nor the shell capacity; twenty, option (C), misses allowed states. Only option (D) includes all five permitted subshells and their magnetic states. In general, the sum of the first n odd numbers is n^2, giving the shell formula directly.

Step 3: Conclusion The shell with principal quantum number 5 contains 25 orbitals. Hence, the correct option is (D).

Quick Tip to Solve: Use n^2 for orbitals and $2n^2$ for maximum electrons; the number of subshells is only n.

68. The test to distinguish primary, secondary and tertiary amines is : [4 marks]

(A) Carbylamine reaction

(B) $C_6H_5SO_2Cl$

(C) Sandmeyer's reaction

(D) Mustard oil test

Correct Answer: (B) $C_6H_5SO_2Cl$

Solution:

Concept:

- The Hinsberg reagent is benzenesulphonyl chloride, $C_6H_5SO_2Cl$, used with aqueous alkali to distinguish primary, secondary and tertiary amines.
- The distinction depends on sulphonamide formation and on whether the product retains an acidic nitrogen–hydrogen bond, which controls its solubility in alkali.

Step 1: Compare the three amine classes with Hinsberg reagent A primary amine, RNH_2, forms $C_6H_5SO_2NHR$. This sulphonamide retains an $N-H$ bond and is acidic enough to dissolve in aqueous alkali as a salt; acidification precipitates the neutral sulphonamide again. A secondary amine, R_2NH, forms $C_6H_5SO_2NR_2$, which has no $N-H$ bond and remains insoluble in alkali. A tertiary amine, R_3N, lacks an $N-H$ bond in the starting amine and does not form this sulphonamide; the unchanged amine dissolves in dilute acid through ammonium-salt formation.

Step 2: Select the test that gives three distinct outcomes These different reactions and solubility behaviours separate all three classes, so the reagent in option (B) is the answer. The carbylamine reaction, option (A), is given by primary amines only, so a negative result does not distinguish secondary from tertiary amines. Sandmeyer chemistry, option (C), concerns replacement of an aromatic diazonium group and is not a general classification test for all three amine classes. The mustard oil test, option (D), likewise identifies primary amines rather than providing the characteristic three-way distinction offered by the Hinsberg test.

Step 3: Conclusion Benzenesulphonyl chloride, $C_6H_5SO_2Cl$, is used in the Hinsberg test. Hence, the correct option is (B).

Quick Tip to Solve: In the Hinsberg test, primary gives an alkali-soluble sulphonamide, secondary an insoluble one, and tertiary forms no sulphonamide.

69. The gas evolved on heating CH_3MgBr in methanol is : [4 marks]

(A) HBr

(B) Methane

(C) Ethane

(D) Propane

Correct Answer: (B) Methane

Solution:

Concept:

- A Grignard reagent RMgX is strongly basic: its carbon behaves as a carbanion equivalent and removes a proton from an alcohol.
- Protonation converts the organic group R into the hydrocarbon RH; it does not join two organic groups together.

Step 1: Identify the proton donor and reactive carbon Methanol, $\mathrm{CH_3OH}$, contains an active hydrogen in its hydroxyl group. In methylmagnesium bromide, $\mathrm{CH_3MgBr}$, the carbon–magnesium bond is highly polar, with electron density concentrated toward carbon. The methyl group therefore acts as a strong base and abstracts the hydroxyl proton. The carbon group carried by the Grignard reagent has one carbon atom; adding a proton changes it into $\mathrm{CH_4}$, not into a two-carbon hydrocarbon.

Step 2: Write the reaction and eliminate the alternatives The acid–base reaction is

$$\mathrm{CH_3MgBr + CH_3OH \longrightarrow CH_4\uparrow + CH_3OMgBr.}$$

Methane is the gaseous product and magnesium methoxide bromide is the other product. Hydrogen bromide, option (A), is not released: bromide remains associated with the magnesium-containing product. Ethane, option (C), would require coupling two methyl groups, but the alcohol proton quenches the Grignard reagent instead. Propane, option (D), would additionally require creating a three-carbon skeleton, which this proton-transfer reaction cannot do. Heating does not change the fundamental active-hydrogen reaction; this is why Grignard preparations require dry apparatus and solvents free of hydroxyl protons.

Step 3: Conclusion Methylmagnesium bromide is protonated by methanol and evolves methane, $\mathrm{CH_4}$. Hence, the correct option is (B).

Quick Tip to Solve: To find the gas from a Grignard reagent and an alcohol, replace the $-\mathrm{MgX}$ group by hydrogen: $\mathrm{R-MgX \longrightarrow R-H}$.

70. An organic compound contains C, H and S. The minimum molecular weight of the compound containing 8% sulphur is :

(atomic weight of S = 32 amu) [4 marks]

(A) 200 g mol^{-1}

(B) 400 g mol^{-1}

(C) 600 g mol^{-1}

(D) 300 g mol^{-1}

Correct Answer: (B) 400 g mol^{-1}

Solution:

Concept:

- Mass percentage of sulphur is the mass of sulphur in one mole of compound divided by the compound molar mass, multiplied by 100.
- A molecule containing sulphur has an integral positive number of sulphur atoms. The smallest possible molar mass for a fixed sulphur percentage corresponds to one sulphur atom per molecule.

Step 1: Express the sulphur fraction in terms of molar mass Let the compound contain x sulphur atoms per molecule and have molar mass M in $\mathrm{g\,mol^{-1}}$. Since sulphur has atomic mass 32, one mole of compound contains $32x$ grams of sulphur. The given mass percentage implies

$$\frac{32x}{M} \times 100 = 8.$$

Rearranging gives

$$M = \frac{32x \times 100}{8} = 400x \ \mathrm{g\,mol^{-1}}.$$

The presence of carbon and hydrogen does not change this sulphur mass-balance relation; their masses make up the remaining fraction.

Step 2: Choose the smallest permitted integer count The number x cannot be a fraction because each molecule contains whole atoms. Since the compound contains sulphur, $x \geq 1$, and the minimum occurs at $x = 1$. Thus $M_{\text{min}} = 400 \ \mathrm{g\,mol^{-1}}$. Checking the options gives inferred sulphur-atom counts $M/400$ of $0.5, 1, 1.5, 0.75$, respectively. Only option (B) gives a positive integer. For any larger allowed sulphur count, the molar mass would be $800, 1200, \ldots$, so it could not lower the minimum. No complete empirical formula is needed: the elemental percentage and the smallest atomic count already set the bound.

Step 3: Conclusion The minimum molar mass is $400 \ \mathrm{g\,mol^{-1}}$, corresponding to one sulphur atom per molecule. Hence, the correct option is (B).

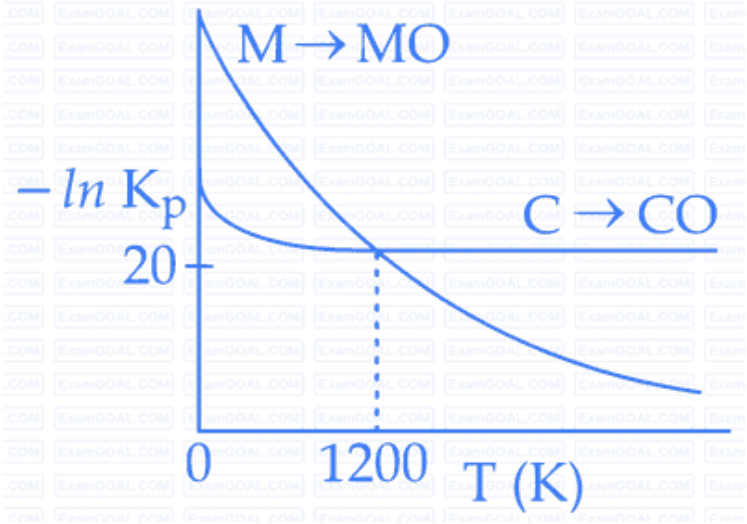
Quick Tip to Solve: For an element of atomic mass A present at $p\%$, the minimum molar mass is $100A/p$, taking one atom per molecule.

71. The plot shows the variation of $-ln$ K_p versus temperature for the two reactions.

M(s) + $\frac{1}{2}$ O_2(g) $\rightarrow$ MO(s) and

C(s) + $\frac{1}{2}$ O_2(g) $\rightarrow$ CO(s)

Identify the correct statement : [4 marks]



(A) At T > 1200 K, carbon will reduce MO(s) to M(s).

(B) At T < 1200 K, the reaction

MO(s) + C(s) $\rightarrow$ M(s) + CO(g) is spontaneous.

(C) At T < 1200 K, oxidation of carbon is unfavourable

(D) Oxidation of carbon is favourable at all temperatures.

Correct Answer: (B) At T < 1200 K, the reaction

MO(s) + C(s) $\rightarrow$ M(s) + CO(g) is spontaneous.

Solution:

Concept:

- For a reaction at temperature T, $\Delta G^\circ = -RT \ln K_p$. Thus the ordinate $-\ln K_p$ is $\Delta G^\circ/(RT)$, and lower ordinate means a more favourable oxidation at the same temperature.
- Carbon reduces a metal oxide when carbon oxidation has a lower Gibbs energy than metal oxidation. Subtracting metal oxidation from carbon oxidation gives the oxide-reduction reaction.

Step 1: Compare the two curves below their intersection The curves intersect at $1200\,\mathrm{K}$. Below this temperature the carbon-oxidation curve is below the metal-oxidation curve. Therefore $\Delta G^\circ_{\mathrm{C}} < \Delta G^\circ_{\mathrm{M}}$. Compare their vertical positions at one temperature; comparing slopes alone would give the wrong criterion for reduction.

Step 2: Form the reduction reaction and check the statements Add carbon oxidation to the reverse of metal oxidation. The resulting reaction is

$$\mathrm{MO\,(s) + C\,(s) \longrightarrow M\,(s) + CO\,(g).}$$

Its standard Gibbs energy is

$$\Delta G^\circ_{\text{red}} = \Delta G^\circ_{\text{C}} - \Delta G^\circ_{\text{M}} < 0 \qquad (T < 1200\,\text{K}).$$

Thus statement (B) expresses the favourable reduction. Above the intersection the metal-oxidation curve is lower, reversing the sign of this difference, so (A) is false. Carbon oxidation being less favourable than metal oxidation is also different from being unfavourable in absolute terms. Its plotted ordinate is positive, so the graph does not support oxidation being favourable at every temperature; (D) is false. However, the carbon curve also lies above zero below the crossing, so its standard Gibbs energy is positive there. Consequently statement (C), read literally as a standard-state claim, is true as well: a positive carbon-oxidation Gibbs energy can still give a negative reduction Gibbs energy after subtracting the larger metal-oxidation value.

Step 3: Conclusion Below $1200\,\text{K}$, carbon can reduce the oxide, giving the keyed option (B). The graph also makes (C) true in standard-state terms because the carbon ordinate is positive; therefore both (B) and (C) are supported as printed, and there is no unique literal answer.

Quick Tip to Solve: For oxide reduction, subtract the metal-oxidation ordinate from the carbon-oxidation ordinate at the same temperature; a negative difference favours reduction.

72. The hydrocarbon with seven carbon atoms containing a neopentyl and a vinyl group is : [4 marks]

(A) 2, 2-dimethyl-4-pentene

(B) Isopropyl-2-butene

(C) 4, 4-dimethylpentene

(D) 2, 2-dimethyl-3-pentene

Correct Answer: (C) 4, 4-dimethylpentene

Solution:

Concept:

- The neopentyl group is $(CH_3)_3C{-}CH_2{}^-$, containing five carbon atoms, and the vinyl group is $CH_2{=}CH^-$, containing two.
- Join groups through their free valencies. In naming an alkene, select a chain containing the double bond and number it to give the double bond the lowest possible locant.

Step 1: Join the groups and count the carbon atoms Connecting the free valencies gives

$$(CH_3)_3C{-}CH_2{-}CH{=}CH_2.$$

This contains the required seven carbon atoms. Its longest chain containing the double bond has five carbon atoms, and the remaining two methyl groups are on the same carbon. The connectivity, rather than the overall carbon count alone, identifies the requested hydrocarbon.

Step 2: Number from the alkene end and examine the names Writing the chain from the double-bond end gives

$$CH_2{=}CH{-}CH_2{-}C(CH_3)_2{-}CH_3.$$

The preferred systematic name is 4,4-dimethylpent-1-ene, corresponding to the abbreviated name in (C). The methyl groups occupy carbon 4 when the double bond starts at carbon 1. Option (B) does not specify the required neopentyl skeleton. In (D), the double bond would occupy the internal position and the vinyl group is lost. However, the literal name 2,2-dimethyl-4-pentene in (A), though numbered contrary to the lowest-double-bond-locant rule, describes $CH_3-C(CH_3)_2-CH_2-CH=CH_2$, exactly the same connectivity. Reversing that chain converts its locants into those of (C). Therefore the preferred nomenclature answer is (C), but a literal structural reading also admits (A).

Step 3: Conclusion The required structure is $CH_2=CH-CH_2-C(CH_3)_2-CH_3$, properly named 4,4-dimethylpent-1-ene. The keyed nomenclature choice is (C); as printed, (A) also encodes the same molecule with incorrect numbering, so the structural alternatives are ambiguous.

Quick Tip to Solve: First connect the free bonds of the two named groups, then number the resulting alkene from the end nearer its double bond.

73. The amount of arsenic pentasulphide that can be obtained when 35.5 g arsenic acid istreated with excess H_2S in the presence of conc. HCl (assuming 100% conversion) is : [4 marks]

(A) 0.50 mol

(B) 0.25 mol

(C) 0.125 mol

(D) 0.333 mol

Correct Answer: (C) 0.125 mol

Solution:

Concept:

- Arsenic acid has formula H_3AsO_4, with molar mass $3(1) + 75 + 4(16) = 142\,g\,mol^{-1}$.
- Each formula unit of arsenic pentasulphide, As_2S_5, contains two arsenic atoms. Therefore two moles of arsenic acid supply one mole of pentasulphide when conversion is complete.

Step 1: Calculate the amount of arsenic acid The given mass is $35.5\,g$, so

$$n(H_3AsO_4) = \frac{35.5\,g}{142\,g\,mol^{-1}} = 0.250\,mol.$$

The hydrogen sulphide is in excess; it cannot limit the product. Concentrated hydrochloric acid supplies the reaction medium and does not supply arsenic, so the arsenic balance fixes the product amount.

Step 2: Apply the balanced mole ratio The overall formation of the requested pentasulphide can be represented as

$$2\,H_3AsO_4 + 5\,H_2S \longrightarrow As_2S_5 + 8\,H_2O.$$

There are two arsenic atoms on each side, five sulphur atoms on each side, and sixteen hydrogen

atoms and eight oxygen atoms on each side. Thus

$$n(\mathrm{As_2S_5}) = \frac{1}{2}n(\mathrm{H_3AsO_4}) = \frac{0.250}{2} = 0.125\,\mathrm{mol}.$$

Option (B) is the amount of starting acid, not the product. Option (A) incorrectly doubles the acid amount, while (D) has no basis in the two-to-one arsenic balance. The assumption of complete conversion means no yield correction is required.

Step 3: Conclusion The amount of arsenic pentasulphide obtained is $\boxed{0.125\,\mathrm{mol}}$. Hence, the correct option is (C).

Quick Tip to Solve: Count the conserved arsenic atoms first: one arsenic per acid molecule but two per pentasulphide formula unit halves the product mole number.

74. What will occur if a block of copper metal is dropped into a beaker containing a solution of 1M $ZnSO_4$? [4 marks]

(A) The copper metal will dissolve and zinc metal will be deposited.

(B) The copper metal will dissolve with evolution of hydrogen gas.

(C) The copper metal will dissolve with evolution of oxygen gas.

(D) No reaction will occur.

Correct Answer: (D) No reaction will occur.

Solution:

Concept:

- A metal displaces another metal from its salt only when the corresponding electron-transfer reaction is spontaneous. Copper is less readily oxidised than zinc.
- The standard reduction potentials are approximately $E^\circ(\mathrm{Zn^{2+}/Zn}) = -0.76\,\mathrm{V}$ and $E^\circ(\mathrm{Cu^{2+}/Cu}) = +0.34\,\mathrm{V}$. A proposed cell reaction is spontaneous when its cell potential is positive.

Step 1: Test displacement of zinc by copper The proposed reaction behind option (A) would be

$$\mathrm{Cu\,(s) + Zn^{2+}(aq) \longrightarrow Cu^{2+}(aq) + Zn\,(s)}.$$

Zinc ions would have to accept electrons, while copper would have to lose them. Using reduction potentials,

$$E^\circ_{\mathrm{cell}} = -0.76 - (+0.34) = -1.10\,\mathrm{V}.$$

The negative result corresponds to a positive standard Gibbs energy, because $\Delta G^\circ = -nFE^\circ_{\mathrm{cell}}$. Hence this is not a spontaneous displacement under the stated conditions.

Step 2: Check the suggested gas-producing reactions The beaker contains aqueous zinc sulphate, not an oxidising acid. Sulphate acts as a spectator ion here, and the solution offers no suitable oxidant that would dissolve a block of copper with hydrogen evolution. Copper also cannot release oxygen merely by being immersed in this salt solution; oxygen evolution is an oxidation process normally requiring a driving source such as an applied potential. Thus neither gas-evolution alternative supplies a viable

spontaneous electron-transfer reaction. Neglecting incidental surface changes due to dissolved air, the expected bulk observation is unchanged copper and no deposited zinc. The reverse combination, zinc metal in copper sulphate, would be spontaneous, which provides a useful check on the sign.

Step 3: Conclusion Copper does not displace zinc from $1\,\mathrm{M}$ zinc sulphate, and no stated gas-evolution reaction occurs. Hence, the correct option is (D).

Quick Tip to Solve: A metal above another in the activity series displaces it from solution; zinc displaces copper, so copper cannot perform the reverse displacement.

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75. The solubility of N_2 in water at 300 K and 500 torr partial pressure is 0.01 g L^{-1}. The solubility (in g L^{-1}) at 750 torr partial pressure is : [4 marks]

(A) 0.0075

(B) 0.015

(C) 0.02

(D) 0.005

Correct Answer: (B) 0.015

Solution:

Concept:

- At a fixed temperature, the solubility of a dilute, nonreacting gas in a liquid is directly proportional to its partial pressure above the solution: $S = kP$.
- For the same gas, solvent and temperature, the proportionality constant cancels, giving $S_2/S_1 = P_2/P_1$. The concentration units need not be converted when used consistently.

Step 1: Identify what changes between the two conditions The gas is nitrogen and the solvent is water in both cases. The temperature remains $300\,\mathrm{K}$, so the Henry-law constant is unchanged. Only nitrogen partial pressure increases, from 500 to $750\,\mathrm{torr}$. Their ratio is

$$\frac{P_2}{P_1} = \frac{750}{500} = 1.5.$$

Thus the solubility must increase by the same factor. Using an inverse ratio would incorrectly predict less dissolved gas at higher pressure.

Step 2: Calculate and compare the solubilities Using $S_1 = 0.010\,\mathrm{g\,L^{-1}}$,

$$S_2 = S_1\frac{P_2}{P_1} = 0.010 \times \frac{750}{500} = 0.015\,\mathrm{g\,L^{-1}}.$$

The pressure units cancel because both pressures are in torr; conversion to atmospheres is unnecessary. Option (A) would correspond to a decrease, while option (D) would halve the initial value. Both contradict the direction required by Henry's law. Option (C) doubles the solubility and would require a doubling of pressure to $1000\,\mathrm{torr}$, rather than the stated increase to $750\,\mathrm{torr}$. The value in (B) has the required factor of 1.5 and the requested mass-concentration unit.

Step 3: Conclusion The nitrogen solubility is $\boxed{0.015\,\text{g}\,\text{L}^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: At constant temperature, scale gas solubility by the pressure ratio; pressures expressed in the same unit cancel directly.

76. 5 L of an alkane requires 25 L of oxygen for its complete combustion. If all volumes are measured at constant temperature and pressure, the alkane is : [4 marks]

(A) Ethane

(B) Propane

(C) Butane

(D) Isobutane

Correct Answer: (B) Propane

Solution:

Concept:

- An alkane has molecular formula $\text{C}_n\text{H}_{2n+2}$. Complete combustion produces carbon dioxide and water.
- Gas volumes measured at identical temperature and pressure are proportional to amounts in moles. Therefore the oxygen-to-alkane volume ratio equals the stoichiometric coefficient ratio.

Step 1: Obtain the oxygen coefficient for a general alkane Balance carbon and hydrogen first: one mole of $\text{C}_n\text{H}_{2n+2}$ produces n moles of carbon dioxide and $n+1$ moles of water. The products contain $2n+(n+1)=3n+1$ moles of oxygen atoms. Therefore the required oxygen coefficient is

$$\nu(\text{O}_2)=\frac{3n+1}{2}.$$

This coefficient counts oxygen molecules, so forgetting the division by two would give an incorrect carbon count.

Step 2: Use the measured volume ratio and select the alkane The given oxygen-to-fuel ratio is

$$\frac{V(\text{O}_2)}{V(\text{alkane})}=\frac{25}{5}=5.$$

Hence

$$\frac{3n+1}{2}=5 \quad\Longrightarrow\quad 3n+1=10 \quad\Longrightarrow\quad n=3.$$

The alkane is therefore C_3H_8, propane. Its equation

$$\text{C}_3\text{H}_8+5\,\text{O}_2\longrightarrow 3\,\text{CO}_2+4\,\text{H}_2\text{O}$$

confirms that $5\,\text{L}$ requires $25\,\text{L}$ oxygen. Ethane requires 3.5 volumes of oxygen per volume of fuel, while butane and isobutane both require 6.5 volumes. Branching does not change combustion oxygen demand when the molecular formula is unchanged, so neither of the four-carbon alternatives fits.

Step 3: Conclusion The alkane is $\boxed{\text{propane, } C_3H_8}$. Hence, the correct option is (B).

Quick Tip to Solve: For an alkane, equate its oxygen-to-fuel gas-volume ratio to $(3n+1)/2$; combustion distinguishes molecular formulas, not chain isomers.

77. Which one of the following species is stable in aqueous solution? [4 marks]

(A) Cr^{2+}

(B) Cu^{+}

(C) MnO_4^{3-}

(D) MnO_4^{2-}

Correct Answer: (D) MnO_4^{2-}

Solution:

Concept:

- Aqueous stability depends on resistance to oxidation, reduction and disproportionation, rather than on the existence of a formal oxidation state alone.
- Chromium(II) readily oxidises to chromium(III); copper(I) disproportionates in ordinary aqueous solution. Manganate(VI), $MnO_4{}^{2-}$, is the familiar aqueous oxoanion stabilised particularly in alkaline medium.

Step 1: Eliminate the readily transformed ions Chromium(II) has a d^4 configuration and is a strong reducing agent. It readily loses an electron to form the more stable d^3 chromium(III) ion; consequently Cr^{2+} is not the stable aqueous choice. Copper(I), despite its d^{10} configuration, is not stable as the simple hydrated ion because copper(II) is strongly stabilised by hydration. It undergoes

$$2\,Cu^{+} \longrightarrow Cu^{2+} + Cu\,(s).$$

Thus a filled subshell alone does not establish aqueous stability.

Step 2: Compare the two manganese oxoanions In $MnO_4{}^{3-}$, oxygen contributes -8 and the ion charge is -3, giving manganese oxidation state $+5$. This hypomanganate species is not the ordinary stable aqueous oxoanion; it readily disproportionates unless special strongly alkaline conditions are maintained. In $MnO_4{}^{2-}$, manganese is in oxidation state $+6$. The green manganate(VI) ion is the stable choice among those listed in the usual alkaline aqueous context. Its stability is medium-dependent: in neutral or acidic solution it can disproportionate, for example

$$3\,MnO_4{}^{2-} + 4\,H^{+} \longrightarrow 2\,MnO_4{}^{-} + MnO_2 + 2\,H_2O.$$

Thus choosing manganate does not mean it persists unchanged at every pH. The other alternatives are more readily transformed under ordinary aqueous comparison conditions.

Step 3: Conclusion The intended stable aqueous species is manganate(VI), $MnO_4{}^{2-}$, especially in alkaline solution. Hence, the correct option is (D).

Quick Tip to Solve: For aqueous transition-metal stability, check hydration and disproportionation as well as electronic configuration; manganate requires an alkaline medium.

78. Identify the correct trend given below : (Atomic No.=Ti : 22, Cr : 24 and Mo : 42) [4 marks]

(A) Δ^o of $[Cr(H_2O)_6]^{2+} >$ $[Mo(H_2O)_6]^{2+}$ and Δ^o of $[Ti(H_2O)_6]^{3+} > [Ti(H_2O)_6]^{2+}$

(B) Δ^o of $[Cr(H_2O)_6]^{2+} >$ $[Mo(H_2O)_6]^{2+}$ and Δ^o of $[Ti(H_2O)_6]^{3+} < [Ti(H_2O)_6]^{2+}$

(C) Δ^o of $[Cr(H_2O)_6]^{2+}$ $< [Mo(H_2O)_6]^{2+}$ and Δ^o of $[Ti(H_2O)_6]^{3+} > [Ti(H_2O)_6]^{2+}$

(D) Δ^o of $[Cr(H_2O)_6]^{2+}$ $< [Mo(H_2O)_6]^{2+}$ and Δ^o of $[Ti(H_2O)_6]^{3+} < [Ti(H_2O)_6]^{2+}$

Correct Answer: (C) Δ^o of $[Cr(H_2O)_6]^{2+}$ $< [Mo(H_2O)_6]^{2+}$ and Δ^o of $[Ti(H_2O)_6]^{3+} > [Ti(H_2O)_6]^{2+}$

Solution:

Concept:

- In an octahedral complex, the five metal d orbitals split into lower-energy t_{2g} and higher-energy e_g sets. Their separation is Δ_o.
- For comparable complexes with the same ligand and oxidation state, the splitting generally increases from the 3d to the 4d series. For the same metal and ligand, a higher oxidation state increases the splitting.

Step 1: Compare chromium with molybdenum Both ions have oxidation state +2, both are coordinated by six water ligands, and both complexes are octahedral. Chromium is a 3d metal whereas molybdenum is a 4d metal. The more spatially extended 4d orbitals interact more strongly with the ligand field. Consequently,

$$\Delta_o\big([Cr(H_2O)_6]^{2+}\big) < \Delta_o\big([Mo(H_2O)_6]^{2+}\big).$$

The atomic numbers identify the series; it would be incorrect to infer the opposite order simply from the smaller size of chromium.

Step 2: Compare the two titanium oxidation states The ligand and coordination number are unchanged, so the relevant difference is the metal charge. The +3 ion attracts the ligand electron density more strongly than the +2 ion and gives a larger separation between the two d-orbital sets:

$$\Delta_o\big([Ti(H_2O)_6]^{3+}\big) > \Delta_o\big([Ti(H_2O)_6]^{2+}\big).$$

Options (A) and (B) reverse the chromium–molybdenum comparison. Option (D) has the first comparison right but reverses the titanium comparison. Only (C) combines both trends correctly.

Step 3: Conclusion The splitting is larger for the molybdenum(II) complex than for the chromium(II) complex, and larger for titanium(III) than for titanium(II). Hence, the correct option is (C).

Quick Tip to Solve: Compare one factor at a time: 4d versus 3d at fixed charge, then oxidation state for the same metal.

79. The non-metal that does not exhibit positive oxidation state is : [4 marks]

(A) Oxygen

(B) Iodine

(C) Chlorine

(D) Fluorine

Correct Answer: (D) Fluorine

Solution:

Concept:

- Oxidation state is the formal charge assigned by giving bonding electrons to the more electronegative atom. A positive value requires an atom to be bonded to a more electronegative partner.
- Fluorine is the most electronegative element. In its compounds it has oxidation state -1, while elemental F_2 has oxidation state zero.

Step 1: Apply the electronegativity rule to fluorine No other element is more electronegative than fluorine. Therefore, when fluorine bonds to another element, the bonding electrons are assigned to fluorine, not away from it. Fluorine consequently does not acquire a positive oxidation state in its compounds. Its elemental form is an exception only in the sense that the oxidation state is zero, not positive. This eliminates the possibility of treating zero in elemental fluorine as a positive value.

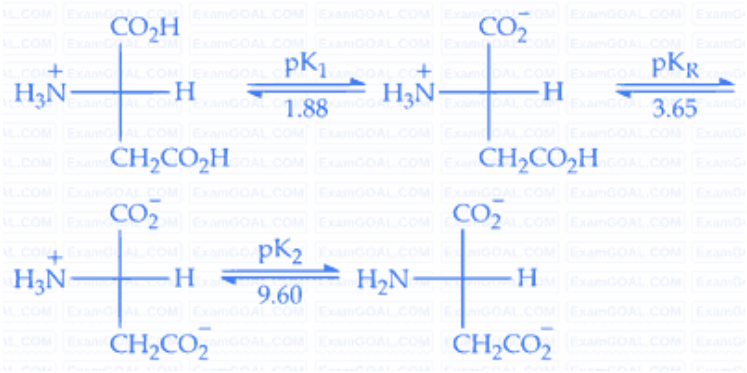
Step 2: Check why the other choices are possible Oxygen can have positive oxidation states when bonded to fluorine: in OF_2, two fluorine atoms contribute a total of -2, so oxygen is $+2$. Chlorine can be positive in ClF, where chlorine is $+1$, and iodine can be positive in IF_5, where iodine is $+5$. These examples are sufficient to reject oxygen, iodine and chlorine. Although these elements commonly occur with negative oxidation states, that does not prevent them from also showing positive states with more electronegative atoms.

Step 3: Conclusion Fluorine alone among the listed non-metals does not exhibit a positive oxidation state. Hence, the correct option is (D).

Quick Tip to Solve: To test whether a non-metal can be positive, look for its compound with fluorine; fluorine itself has no more electronegative partner.

80. Consider the following sequence for aspartic acid :

The *pI* (isoelectric point) of aspartic acid is : [4 marks]



(A) 1.88

(B) 3.65

(C) 5.74

(D) 2.77

Correct Answer: (D) 2.77

Solution:

Concept:

- The isoelectric point, pI, is the pH at which an amino acid has zero average net charge. It is the average of the two pK_a values bordering its electrically neutral zwitterion.
- Aspartic acid contains two carboxyl groups and one amino group. For this acidic amino acid, the neutral form lies between the first and second carboxyl deprotonations, not between carboxyl and ammonium deprotonations.

Step 1: Locate the neutral species in the sequence At sufficiently low pH, both carboxyl groups are protonated and the amino group is $-NH_3^+$, so the molecule has net charge $+1$. Removal of the first carboxyl proton at $pK_1 = 1.88$ produces one carboxylate while the amino group remains protonated: the net charge is then zero. Removal of the second carboxyl proton at $pK_2 = 3.65$ produces a species of net charge -1. The neutral zwitterion is therefore bracketed by 1.88 and 3.65. The value 9.60 belongs to loss of the ammonium proton and is not one of the boundaries of the neutral form.

Step 2: Average the two bracketing values For the $+1$, 0 and -1 sequence, zero average charge occurs when the concentrations of the $+1$ and -1 forms are equal. Combining the two successive acid dissociation equilibria then gives $[H^+]^2 = K_1K_2$. Taking negative logarithms yields

$$pI = \frac{pK_1 + pK_2}{2} = \frac{1.88 + 3.65}{2} = 2.765 \simeq 2.77.$$

The options 1.88 and 3.65 are individual dissociation constants expressed as pK values, not their mean. The value 5.74 would come from averaging 1.88 and 9.60, incorrectly skipping the acidic side-chain ionization.

Step 3: Conclusion The isoelectric point of aspartic acid is $pI = 2.77$. Hence, the correct option is (D).

Quick Tip to Solve: Write the net charges along the ionization sequence, then average only the two pKa values immediately around the zero-charge form.

81. For the reaction,

A(g) + B(g) $\rightarrow$ C(g) + D(g), ΔH^o and ΔS^o are, respectively, $-$ 29.8 kJ mol^{-1} and $-$0.100 kJ K^{-1} mol^{-1} at 298 K. The equilibrium constant for the reaction at 298 K is : [4 marks]

(A) 1.0×10^{-10}

(B) 1.0×10^{10}

(C) 10

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- The standard Gibbs energy change at temperature T is $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$. Enthalpy and the entropy term must use the same energy unit.
- The thermodynamic equilibrium constant satisfies $\Delta G^\circ = -RT\ln K$, where R is the gas constant. If the standard Gibbs energy change is zero, the equilibrium constant is unity.

Step 1: Calculate the entropy contribution The temperature is $298\,\mathrm{K}$ and the stated standard entropy change is negative. Using the given kilojoule units throughout,

$$T\Delta S^\circ = (298\,\mathrm{K})(-0.100\,\mathrm{kJ\,K^{-1}\,mol^{-1}}) = -29.8\,\mathrm{kJ\,mol^{-1}}.$$

The reaction is exothermic, which favours product formation, but its entropy decrease opposes that tendency at positive temperature. At the particular temperature supplied, these two contributions have equal magnitude.

Step 2: Find the standard Gibbs energy and K Substitute the signed quantities, taking care that the negative entropy term is subtracted:

$$\Delta G^\circ = -29.8 - (-29.8) = 0\,\mathrm{kJ\,mol^{-1}}.$$

Therefore,

$$0 = -RT\ln K \quad \Rightarrow \quad \ln K = 0 \quad \Rightarrow \quad K = e^0 = 1.$$

The constants 10^{-10}, 10^{10} and 10 would all require a nonzero standard Gibbs energy change and hence do not satisfy the supplied data. A zero standard Gibbs energy change does not mean that the equilibrium concentrations are all identical; it specifies the equilibrium activity ratio.

Step 3: Conclusion The equilibrium constant at $298\,\mathrm{K}$ is $K = 1$. Hence, the correct option is (D).

Quick Tip to Solve: Check whether enthalpy and T times entropy cancel before calculating a logarithm; exact cancellation gives K equal to one.

82. The most appropriate method of making egg-albumin sol is : [4 marks]

(A) Break an egg carefully and transfer the transparent part of the content to 100 mL of 5% w/V saline solution and stir well

(B) Break an egg carefully and transfer only the yellow part of the content to 100 mL of 5% w/V saline solution and stir well.

(C) Keep the egg in boiling water for 10 minutes. After removing the shell, transfer the white part of the content to 100 mL of 5% w/V saline solution and homogenize with a mechanical shaker.

(D) Keep the egg in boiling water for 10 minutes. After removing the shell, transfer the yellow part of the content to 100 mL of 5% w/V saline solutionand homogenize with a mechanical shaker.

Correct Answer: (A) Break an egg carefully and transfer the transparent part of the content to 100 mL of 5% w/V saline solution and stir well

Solution:

Concept:

- Egg albumin is a protein present in the transparent egg white. Proteins can form lyophilic colloidal dispersions in a suitable aqueous medium.
- Heating an egg strongly denatures and coagulates its proteins. A coagulated protein cannot be restored to its original native sol merely by shaking it in saline solution.

Step 1: Select the correct part of the raw egg The transparent material surrounding the yolk is egg white, the source of albumin required for an egg-albumin sol. Transferring this portion of a raw egg into the stated saline solution and stirring distributes the protein through the liquid. The protein molecules are solvated and form a colloidal dispersion. The yellow yolk instead contains a different mixture rich in lipids and lipoproteins; it is not the appropriate starting material for the egg-albumin preparation specified here. Thus, choosing the yellow portion does not achieve the stated aim.

Step 2: Avoid coagulation during preparation Boiling for ten minutes visibly changes the transparent egg white into a firm white solid. This is denaturation followed by aggregation or coagulation of its proteins. Mechanical shaking can break the solid into smaller pieces, but it does not reverse the denaturation and regenerate the original albumin sol. Hence option (C) is unsuitable even though it uses the white portion, while option (D) both boils the egg and selects the yolk. Option (B) avoids boiling but still selects the wrong portion. Only option (A) uses raw egg white with gentle mixing in the given dispersion medium.

Step 3: Conclusion Transfer the transparent part of the raw egg to the saline solution and stir well. Hence, the correct option is (A).

Quick Tip to Solve: For an albumin sol, choose raw egg white; boiling produces coagulated protein rather than a reversible preparation step.

83. The correct order of the solubility of alkaline-earth metal sulphates in water is : [4 marks]

(A) Mg $<$ Ca $<$ Sr $<$ Ba

(B) Mg $<$ Sr $<$ Ca $<$ Ba

(C) Mg $>$ Sr $>$ Ca $>$ Ba

(D) Mg $>$ Ca $>$ Sr $>$ Ba

Correct Answer: (D) Mg $>$ Ca $>$ Sr $>$ Ba

Solution:

Concept:

- Dissolving an ionic solid requires disruption of its lattice and hydration of its ions. The balance of lattice enthalpy, hydration enthalpy and entropy determines the solubility.
- Down Group 2, the metal-ion radius increases. For the large sulphate ion, the fall in hydration strength of the cation dominates the comparatively smaller change in lattice stabilization.

Step 1: Follow the hydration trend down the group The cations occur in the size order $Mg^{2+} < Ca^{2+} < Sr^{2+} < Ba^{2+}$. Smaller ions have greater charge density and attract water molecules more strongly. Magnesium therefore has the most favourable, most negative hydration enthalpy of these cations, while barium has the least favourable hydration. The sulphate ion is the same in every salt, so its hydration contribution does not generate the changing order.

Step 2: Balance hydration against the lattice effect The sulphate anion is relatively large. Increasing the cation radius changes the total cation–anion separation by a smaller proportion than it changes the effective hydration of the cation. Thus the lattice enthalpy decreases down the group, but not sufficiently to compensate for the loss of favourable hydration. Dissolution becomes less favourable and the familiar sulphate solubility trend is

$$MgSO_4 > CaSO_4 > SrSO_4 > BaSO_4.$$

Magnesium sulphate is readily soluble, calcium sulphate is sparingly soluble, and barium sulphate is extremely sparingly soluble. Options (A) and (B) wrongly put barium at the most soluble end; option (C) incorrectly interchanges calcium and strontium.

Step 3: Conclusion The solubility order is Mg sulphate $>$ Ca sulphate $>$ Sr sulphate $>$ Ba sulphate. Hence, the correct option is (D).

Quick Tip to Solve: Remember the Group 2 sulphates become less soluble down the group; the strong hydration of magnesium favours its dissolution.

84. A particular adsorption process has the following characteristics : (i) It arises due to van der Waals forces and (ii) it is reversible. Identify the correct statement that describes the above adsorption process : [4 marks]

(A) Enthalpy of adsorption is greater than 100 kJ mol^{-1}.

(B) Energy of activation is low.

(C) Adsorption is monolayer.

(D) Adsorption increases with increase in temperature.

Correct Answer: (B) Energy of activation is low.

Solution:

Concept:

- Adsorption caused by weak van der Waals attractions is physisorption. It is commonly reversible because it does not require formation of strong chemical bonds between adsorbate and surface.
- Physisorption usually has little or no activation barrier and a relatively small, negative enthalpy of adsorption. It can form multiple layers and is favoured by lower temperature.

Step 1: Identify the type of adsorption The question supplies two matching characteristics: van der Waals forces and reversibility. Together these identify physical adsorption. Molecules can be retained on the surface without the bond-making process characteristic of chemisorption. Consequently, the energy barrier to adsorption is generally very small, and adsorption can take place readily at low temperature. This is exactly the low activation energy described in option (B). The statement concerns the barrier to reaching the adsorbed state, which should be distinguished from the heat released after adsorption occurs.

Step 2: Eliminate the other descriptions An adsorption enthalpy magnitude greater than $100\,\mathrm{kJ\,mol^{-1}}$ is characteristic of much stronger interactions, commonly chemical adsorption, rather than ordinary physisorption, so (A) is unsuitable. Physical adsorption is not restricted to a monolayer: an adsorbed layer can attract additional molecules by van der Waals forces, so (C) is not its defining description. Since adsorption is exothermic, increasing temperature favours desorption and generally reduces physical adsorption at a fixed pressure. This rejects (D), which states the opposite temperature trend. These checks all support the low-barrier choice.

Step 3: Conclusion The process is physisorption, for which the energy of activation is low. Hence, the correct option is (B).

Quick Tip to Solve: Van der Waals attraction plus reversibility points to physisorption: low barrier, possible multilayers, and greater adsorption on cooling.

85. The artificial sweetener that has the highest sweetness value in comparison to cane sugar is :

[4 marks]

(A) Aspartane

(B) Saccharin

(C) Sucralose

(D) Alitame

Correct Answer: (D) Alitame

Solution:

Concept:

- Sweetness is compared with sucrose (cane sugar) assigned a relative sweetness of 1. A greater relative value means a smaller amount provides comparable sweetness.
- The common approximate textbook values are aspartame 100–200, saccharin about 550, sucralose about 600 and alitame about 2000 times the sweetness of sucrose.

Step 1: Compare the listed sweeteners Aspartame, written as Aspartane in the option, is a peptide-based sweetener and is substantially sweeter than sucrose, but its sweetness is well below that of alitame. Saccharin also gives strong sweetness at low concentration; its value is commonly quoted near 550 times that of cane sugar. Neither is the greatest among these choices.

Step 2: Select the largest relative value Sucralose is a chlorinated sucrose derivative and is about 600 times as sweet as sucrose. Alitame is a dipeptide sweetener with relative sweetness around 2000. Thus the relevant increasing order is aspartame, saccharin, sucralose, alitame. The question asks for sweetness intensity, not heat stability, calorific content or suitability for cooking. Comparing that single property eliminates A, B and C directly.

Step 3: Conclusion Alitame has the highest sweetness value among the listed artificial sweeteners, approximately 2000 times that of cane sugar. Hence, the correct option is (D).

Quick Tip to Solve: Remember the approximate sequence 200, 550, 600, 2000 for aspartame, saccharin, sucralose and alitame; the last is the sweetest here.

86. Match the items in Column I with its main use listed in Column II :

Column I	Column II
(A) Silica gel	(i) Transistor
(B) Silicon	(ii) Ion-exchanger
(C) Silicone	(iii) Drying agent
(D) Silicate	(iv) Sealant

[4 marks]

(A) -(iii), (B)-(i), (C)-(iv), (D)-(ii)

(B) -(iv), (B)-(i), (C)-(ii), (D)-(iii)

(C) -(ii), (B)-(iv), (C)-(i), (D)-(iii)

(D) -(ii), (B)-(i), (C)-(iv), (D)-(iii)

Correct Answer: (A) -(iii), (B)-(i), (C)-(iv), (D)-(ii)

Solution:

Concept:

- Silica gel is porous hydrated silica. Its large surface area enables it to adsorb water vapour, making it useful as a drying agent.
- Elemental silicon is a semiconductor; silicones are organosilicon polymers; certain silicates, such as zeolites, exchange cations.

Step 1: Match silica gel and silicon Silica gel removes moisture from its surroundings by adsorption, so item (A) matches drying agent (iii). It should not be confused with silicone, whose name differs by the final e. Silicon, item (B), is the elemental semiconductor used in electronic devices such as transistors, so it matches (i). These two matches already sharply restrict the possible answer.

Step 2: Match silicone and silicate Silicones have a silicon–oxygen backbone with organic groups attached to silicon. Their flexibility, resistance to water and chemical stability make them useful as sealants, giving (C)-(iv). Silicates containing exchangeable cations can act as ion-exchangers; zeolites are the standard example in water softening. Hence (D)-(ii). The complete combination is therefore (A)-(iii), (B)-(i), (C)-(iv), (D)-(ii). The other listed combinations misassign silica gel or confuse the distinct silicon materials.

Step 3: Conclusion Silica gel is the drying agent; silicon is used in transistors; silicone is the sealant; silicate is the ion-exchanger. Hence, the correct option is (A).

Quick Tip to Solve: Associate gel with moisture adsorption, silicon with semiconductors, silicone with sealing, and zeolitic silicate with ion exchange.

87. **Assertion :**

Rayon is a semisynthetic polymer whose properties are better than natural cotton.

Reason :

Mechanical and aesthetic properties of cellulose can be improved by acetylation. [4 marks]

(A) Both assertion and reason are correct, and the reason is the correct explanation for the assertion.

(B) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion.

(C) Assertion is incorrect statement, but the reason is correct.

(D) Both assertion and reason are incorrect.

Correct Answer: (B) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion.

Solution:

Concept:

- Rayon is a semisynthetic fibre obtained by processing natural cellulose. Chemical processing allows useful fibre properties and appearance to be controlled.
- Acetylation replaces hydroxyl-group hydrogens in cellulose with acetyl groups, producing cellulose acetate. An assertion–reason answer also requires checking the causal link.

Step 1: Evaluate the assertion The assertion classifies rayon as semisynthetic, which is correct in the textbook classification: it originates from the natural polymer cellulose but is processed into a useful man-made fibre. Rayon is valued for its silk-like appearance, absorbency and suitability for textiles. The broad comparison with natural cotton is intended to refer to such improved or controllable textile properties, rather than claiming superiority in every possible property.

Step 2: Evaluate the reason Acetylation changes cellulose into cellulose acetate and can improve mechanical and aesthetic characteristics for useful applications. Thus the reason states a valid chemical modification of cellulose. However, ordinary viscose rayon is regenerated cellulose: cellulose is processed through a soluble derivative and then regenerated. It is not produced simply by acetylating cotton.

Step 3: Test whether the reason explains the assertion Both statements can be accepted as textbook statements about semisynthetic materials, but they concern different processing routes. Acetylation explains the properties of cellulose acetate, not the specific origin and properties of regenerated rayon. Therefore the reason is not the correct explanation of the given assertion. This distinction rules out A; the truth of both statements rules out C and D.

Step 4: Conclusion Both the assertion and reason are correct in the intended textbook classification, but the reason is not the correct explanation of the assertion. Hence, the correct option is (B).

Quick Tip to Solve: In assertion–reason questions, check the processing route: viscose rayon is regenerated cellulose, whereas acetylation produces cellulose acetate.

88. Identify the incorrect statement regarding heavy water : [4 marks]

(A) It reacts with Al_4C_3 to produce CD_4 and $Al(OD)_3$.

(B) It is used as a coolant in nuclear reactors.

(C) It reacts with CaC_2 to produce C_2D_2 and $Ca(OD)_2$.

(D) It reacts with SO_3 to form deuterated sulphuric acid (D_2SO_4).

Correct Answer: All four statements are scientifically correct; the supplied key selects B by contrasting moderator and coolant.

Solution:

Concept:

- Heavy water is deuterium oxide, D_2O. Its reactions resemble those of ordinary water, with deuterium replacing hydrogen in the products.
- A moderator slows neutrons, whereas a coolant removes heat. These are different functions, but a material can perform both in a suitable reactor design.

Step 1: Verify the three chemical reactions Aluminium carbide gives deuterated methane and aluminium deuteroxide:

$$Al_4C_3 + 12\,D_2O \longrightarrow 3\,CD_4 + 4\,Al(OD)_3.$$

Calcium carbide gives deuterated acetylene:

$$CaC_2 + 2\,D_2O \longrightarrow C_2D_2 + Ca(OD)_2.$$

Sulfur trioxide combines with heavy water to form deuterated sulfuric acid:

$$SO_3 + D_2O \longrightarrow D_2SO_4.$$

Each equation conserves every element and supports A, C and D respectively.

Step 2: Examine the reactor statement carefully Heavy water is well known as a neutron moderator because it slows neutrons with relatively little neutron absorption. It is also used as the primary coolant in pressurised heavy-water reactor designs. Therefore saying that it is used as a coolant in nuclear reactors is not, as an unrestricted scientific statement, false. Moderator and coolant describe different jobs; the distinction does not prohibit the same substance from serving in both systems.

Step 3: Compare the literal result with the supplied key The supplied answer key selects B, evidently expecting the familiar textbook association of heavy water with moderation. That association is true but does not establish that cooling is an impossible use. Since A, C and D are chemically correct and B also describes an actual application, the printed question has no uniquely incorrect statement. A scientifically complete answer must retain this qualification rather than deny a real coolant application.

Step 4: Conclusion All four statements are scientifically correct as written, so there is no uniquely incorrect option. The supplied key selects (B), apparently intending the usual moderator association; that intended selection overlooks heavy water's use as a reactor coolant.

Quick Tip to Solve: Distinguish reactor functions without treating them as mutually exclusive: heavy water can moderate neutrons and can also remove reactor heat.

89. BOD stands for : [4 marks]

(A) Biological Oxygen Demand

(B) Bacterial Oxidation Demand

(C) Biochemical Oxygen Demand

(D) Biochemical Oxidation Demand

Correct Answer: (C) Biochemical Oxygen Demand

Solution:

Concept:

- BOD means Biochemical Oxygen Demand: the dissolved oxygen required by microorganisms to oxidise biodegradable organic matter in a water sample under specified conditions.
- BOD is a measure of the oxygen demand associated with biodegradable pollution. It is commonly expressed as mass of oxygen per unit volume, such as milligrams per litre.

Step 1: Identify the meaning of each word The B in the standard expansion is biochemical, indicating that microbial activity drives the chemical oxidation of organic material. The O is oxygen, the dissolved gas consumed during this process. The D is demand, referring to the amount required rather than an instantaneous oxygen concentration. Thus the standard technical expression is Biochemical Oxygen Demand.

Step 2: Eliminate the distractors Bacterial Oxidation Demand in B is not the standard expansion; microbial oxygen consumption is not restricted to a term defined simply as bacterial oxidation. Biochemical Oxidation Demand in D incorrectly replaces oxygen with oxidation. Biological Oxygen Demand in A is sometimes encountered as an informal synonymous expression, but this question supplies the precise standard term Biochemical Oxygen Demand in C. That is the expected scientific expansion.

Step 3: Relate the abbreviation to water quality A high BOD indicates that microorganisms need considerable oxygen to decompose the biodegradable material present. Their consumption can deplete the dissolved oxygen available to aquatic organisms. BOD therefore differs from dissolved oxygen itself: high oxygen demand usually signals a larger organic pollution load, rather than oxygen-rich clean water.

Step 4: Conclusion BOD stands for Biochemical Oxygen Demand. Hence, the correct option is (C).

Quick Tip to Solve: Read BOD as an amount of oxygen demanded by biodegradation; do not replace the word oxygen with oxidation.

90. At very high pressures, the compressibility factor of one mole of a gas is given by : [4 marks]

(A) $\frac{pb}{RT}$

(B) $1 + \frac{pb}{RT}$

(C) $1 - \frac{pb}{RT}$

(D) $1 - \frac{b}{VRT}$

Correct Answer: (B) $1 + \frac{pb}{RT}$

Solution:

Concept:

- For one mole, the compressibility factor is $Z = pV/(RT)$. It equals 1 for an ideal gas at the same pressure, molar volume and temperature.
- The van der Waals equation for one mole is $(p + a/V^2)(V - b) = RT$, where a accounts for attractions and b for excluded molecular volume.

Step 1: Apply the high-pressure approximation At very high pressure the repulsive or excluded-volume effect dominates the correction due to intermolecular attraction. Neglecting the attraction correction in comparison with the applied pressure gives

$$p(V - b) = RT.$$

The volume accessible for translational motion is $V - b$, not the full molar volume. This is why the finite-size correction remains important in this limit.

Step 2: Rearrange before forming the ratio Expanding the preceding equation gives

$$pV - pb = RT, \qquad pV = RT + pb.$$

Divide throughout by RT, using the definition of Z:

$$Z = \frac{pV}{RT} = 1 + \frac{pb}{RT}.$$

The correction is positive because finite molecular size produces a positive deviation from ideal behaviour under these conditions.

Step 3: Check the options Option A retains only the correction and drops the ideal contribution 1. It may resemble the leading divergent term in an even more extreme asymptotic comparison, but the standard high-pressure van der Waals expression retains both terms. C has the wrong sign for the excluded-volume correction. D has neither the correct dependence nor a dimensionless correction: b/(VRT) is not dimensionless. Thus B is the consistent expression.

Step 4: Conclusion At very high pressures, the standard van der Waals approximation gives $Z = 1 + pb/(RT)$. Hence, the correct option is (B).

Quick Tip to Solve: At high pressure neglect the attraction term first, then expand p(V-b)=RT; dividing by RT preserves the ideal contribution 1.