JEE Main 2016 Paper 1 (B.E./B.Tech) Apr 10 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2016, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 10 April 2016, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. $\lim\limits_{x\to 0} \frac{(1-\cos 2x)^2}{2x\tan x - x\tan 2x}$ **is :** [4 marks]

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

Correct Answer: (A) -2

Solution:

Concept:

- Use the exact identities $1-\cos 2x = 2\sin^2 x$ and $\tan 2x = 2\tan x/(1-\tan^2 x)$. These expose the cancellation in the denominator before a limit is taken.
- The standard limits are $\sin x/x \to 1$, $\cos x \to 1$ and $\cos 2x \to 1$ as $x \to 0$, with angles measured in radians.

Step 1: Simplify the denominator exactly The two terms in the denominator have the same leading term, so replacing each tangent by its argument immediately would lose the first nonzero contribution. Put $t = \tan x$ and combine them first:

$$2x\tan x - x\tan 2x = x\left(2t - \frac{2t}{1-t^2}\right) = -\frac{2xt^3}{1-t^2}.$$

For sufficiently small nonzero x, this expression is nonzero, so the following cancellation is valid on a punctured neighbourhood of zero.

Step 2: Convert to standard limits The numerator is $4\sin^4 x$. Consequently the quotient becomes

$$-\frac{2\sin^4 x(1-\tan^2 x)}{x\tan^3 x} = -2\frac{\sin x}{x}\cos x\cos 2x.$$

Here we used $\sin^4 x/\tan^3 x = \sin x\cos^3 x$ and $1-\tan^2 x = \cos 2x/\cos^2 x$. Each remaining factor has an ordinary finite limit. The minus sign comes from the subtraction in the original denominator and must be retained.

Step 3: Evaluate and compare the options Taking the limit factor by factor gives

$$-2\times 1\times 1\times 1 = -2.$$

Thus the positive choices have the wrong sign. The value $-1/2$ also has the wrong magnitude: the exact coefficient after cancellation is -2, not its reciprocal.

Step 4: Conclusion The required limit is $\boxed{-2}$. Hence, the correct option is (A).

Quick Tip to Solve: When the leading tangent terms cancel, combine them with the double-angle identity before using standard limits.

2. If $\frac{^{n+2}C_6}{^{n-2}P_2}$ = 11, then n satisfies the equation : [4 marks]

(A) $n^2 + 3n - 108 = 0$

(B) $n^2 + 5n - 84 = 0$

(C) $n^2 + 2n - 80 = 0$

(D) $n^2 + n - 110 = 0$

Correct Answer: (A) $n^2 + 3n - 108 = 0$

Solution:

Concept:

- For integers $N \geq r \geq 0$, $^{N}C_r = N!/[r!(N-r)!]$, whereas $^{N}P_r = N!/(N-r)!$. The distinction between selection and ordering supplies the factor $6!$.
- Both symbols in the question are defined for integers $n \geq 4$. On this domain, a product of positive factors that each increase with n is strictly increasing.

Step 1: Expand and cancel the factorials The numerator and denominator of the given ratio are

$$^{n+2}C_6 = \frac{(n+2)!}{720(n-4)!}, \qquad ^{n-2}P_2 = \frac{(n-2)!}{(n-4)!}.$$

Dividing cancels the common factorial $(n-4)!$. Hence

$$\frac{^{n+2}C_6}{^{n-2}P_2} = \frac{(n+2)(n+1)n(n-1)}{720} = 11.$$

The desired product of four consecutive integers therefore equals 7920.

Step 2: Determine the admissible integer The four consecutive factors $8, 9, 10, 11$ have product

$$8 \times 9 \times 10 \times 11 = 72 \times 110 = 7920.$$

They correspond to $n - 1 = 8$, or $n = 9$. This is the only admissible value: for $n \geq 4$, all four factors are positive and each strictly increases when n increases. As a direct check, $^{11}C_6 = 462$ and $^{7}P_2 = 42$, giving the required ratio $462/42 = 11$.

Step 3: Test the proposed equations Substitute $n = 9$ into each left-hand side:

$$\begin{aligned} n^2 + 3n - 108 &= 81 + 27 - 108 = 0, \\ n^2 + 5n - 84 &= 81 + 45 - 84 = 42, \\ n^2 + 2n - 80 &= 81 + 18 - 80 = 19, \\ n^2 + n - 110 &= 81 + 9 - 110 = -20. \end{aligned}$$

Only the first equation vanishes; the other equations do not describe the integer determined by the factorial relation.

Step 4: Conclusion We obtain $n = 9$, which satisfies $\boxed{n^2 + 3n - 108 = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: Cancel factorials before multiplying: here six descending factors reduce to four because the permutation contributes two.

3. The sum $\sum_{r=1}^{10} (r^2+1) \times (r!)$ is equal to : [4 marks]

(A) (11)!

(B) 10 × (11!)

(C) 101 × (10!)

(D) 11 × (11!)

Correct Answer: (B) 10 × (11!)

Solution:

Concept:

- A sum telescopes when its summand is written as a difference $F(r) - F(r-1)$; all intermediate terms then cancel.
- The factorial identity $(r+1)! = (r+1)r!$ allows a polynomial times $r!$ to be compared with consecutive factorial expressions.

Step 1: Find a consecutive difference We want to express $(r^2+1)r!$ using factorials at adjacent indices. Consider $F(r) = r(r+1)!$. Its preceding value is $F(r-1) = (r-1)r!$, and

$$\begin{aligned} F(r) - F(r-1) &= r(r+1)! - (r-1)r! \\ &= \big[r(r+1) - (r-1)\big]r! \\ &= (r^2+1)r!. \end{aligned}$$

This is an exact identity for every positive integer r; no approximation to the factorial is needed.

Step 2: Cancel the internal terms Summing the identity from $r = 1$ to $r = 10$ yields

$$\sum_{r=1}^{10}(r^2+1)r! = \sum_{r=1}^{10}\big[F(r) - F(r-1)\big] = F(10) - F(0).$$

For example, the first two differences are $[1(2!) - 0(1!)]$ and $[2(3!) - 1(2!)]$; the term $1(2!)$ cancels. The same cancellation continues through the sum, leaving only its upper and lower endpoint terms.

Step 3: Evaluate the endpoints and check the choices Here $F(10) = 10(11!)$, while $F(0) = 0(1!) = 0$. Therefore

$$\sum_{r=1}^{10}(r^2+1)r! = 10(11!).$$

In units of $10!$, this is $110(10!)$. The four options respectively equal $11(10!)$, $110(10!)$, $101(10!)$, and $121(10!)$. This comparison rules out the tempting last-summand value $101(10!)$: the other nine positive summands must also be included.

Step 4: Conclusion The sum is $\boxed{10(11!)}$. Hence, the correct option is (B).

Quick Tip to Solve: For a quadratic times $r!$, try differences involving $r(r+1)!$; the lower endpoint can be zero even though the sum begins at 1.

4. For x $\in$ R, x $\neq$ 0, if y(x) is a differentiable function such that x $\int_1^x y$ (t) dt = (x + 1) $\int_1^x ty$ (t) dt, then y (x) equals : (where C is a constant.) [4 marks]

(A) $\frac{C}{x}e^{-\frac{1}{x}}$

(B) $\frac{C}{x^2}e^{-\frac{1}{x}}$

(C) $\frac{C}{x^3}e^{-\frac{1}{x}}$

(D) $Cx^3\,\frac{1}{e^x}$

Correct Answer: The intended differential-equation family is option (C), but the printed lower limit 1 forces the zero solution on x>0, which every option includes for C=0.

Solution:

Concept:

- The fundamental theorem of calculus gives $A'(x) = y(x)$ and $B'(x) = xy(x)$ for $A(x) = \int_1^x y(t)\,dt$ and $B(x) = \int_1^x ty(t)\,dt$.
- Differentiating an integral identity gives a necessary differential equation; conditions lost through differentiation must subsequently be checked in the original identity.

Step 1: Differentiate the given identity twice The identity is $xA = (x+1)B$. Differentiating once gives

$$A + xy = B + (x+1)xy, \qquad A - B = x^2y.$$

Differentiate this last relation. Since $A' - B' = (1-x)y$, we obtain

$$(1-x)y = 2xy + x^2y', \qquad x^2y' + (3x-1)y = 0.$$

The term $2xy$ comes from differentiating x^2y; omitting it would change the power of x in the answer.

Step 2: Solve the necessary differential equation For $x \neq 0$, the linear equation is

$$y' + \left(\frac{3}{x} - \frac{1}{x^2}\right)y = 0.$$

On an interval not containing zero its integrating factor can be taken proportional to $x^3e^{1/x}$. Thus

$$\frac{d}{dx}\left(x^3e^{1/x}y\right) = 0, \qquad y = \frac{C}{x^3}e^{-1/x}.$$

This is the functional form in (C). Its logarithmic derivative is $-3/x + 1/x^2$; the powers x^{-1}, x^{-2}, and the form x^3e^{-x} in the other choices do not satisfy this equation for nonzero C.

Step 3: Check the lower-limit constraint The printed integral identity imposes a further restriction. At $x = 1$, both A and B are zero. The first differentiated relation therefore gives $y(1) = 0$, which forces $C = 0$ on the interval $x > 0$. Nonzero values of C in the family above do not satisfy the full original identity. Consequently the literal problem admits the zero solution there; since each displayed family includes zero when $C = 0$, the original condition does not distinguish a unique option without the customary interpretation of asking for the differential-equation family.

Step 4: Conclusion The differential-equation family is $\boxed{y = Cx^{-3}e^{-1/x}}$, corresponding to option (C). The printed definite-integral condition further forces $C = 0$ on $x > 0$; under that literal condition all four displayed families contain the same zero solution, so the MCQ is not uniquely distinguishing.

Quick Tip to Solve: After differentiating a definite-integral equation, substitute its lower limit: the resulting initial condition can eliminate nonzero integration constants.

5. Let a, b $\in$ R, (a $\neq$ 0). If the function *f* defined as

$$f(x) = \begin{cases} \frac{2x^2}{a}, & 0 \le x < 1 \\ a, & 1 \le x < \sqrt{2} \\ \frac{2b^2-4b}{x^3}, & \sqrt{2} \le x < \infty \end{cases}$$

is continuous in the interval [0, ∞), then an ordered pair (a, b) is : [4 marks]

(A) $(\sqrt{2}, 1-\sqrt{3})$
(B) $(-\sqrt{2}, 1+\sqrt{3})$
(C) $(\sqrt{2}, -1+\sqrt{3})$
(D) $(-\sqrt{2}, 1-\sqrt{3})$
Correct Answer: (A) $(\sqrt{2}, 1-\sqrt{3})$

Solution:

Concept:

- Each formula is continuous in the interior of its own interval. Continuity on the full domain therefore requires matching one-sided values only at $x = 1$ and $x = \sqrt{2}$.
- The parameters a, b must both be real, with $a \neq 0$. A formally possible sign of a must be rejected if it requires a nonreal value of b.

Step 1: Match the values at the first junction At $x = 1$, the middle branch supplies $f(1) = a$, whereas the left-hand limit equals $2/a$. Hence

$$\frac{2}{a} = a, \qquad a^2 = 2, \qquad a = \pm\sqrt{2}.$$

The right-hand limit at 1 also equals a, since the middle branch is constant. The endpoint $x = 0$ presents no extra restriction: the first formula has value zero and is right-continuous there.

Step 2: Apply continuity at the second junction At $x = \sqrt{2}$, the left-hand limit is a, while the third branch gives

$$f(\sqrt{2}) = \frac{2b^2 - 4b}{(\sqrt{2})^3} = \frac{b^2 - 2b}{\sqrt{2}}.$$

Therefore

$$b^2 - 2b = a\sqrt{2}, \qquad (b-1)^2 = 1 + a\sqrt{2}.$$

This condition uses the cube in the denominator; replacing it by x^2 would produce an incorrect parameter equation.

Step 3: Enforce reality and compare the ordered pairs For $a = \sqrt{2}$, the equation is $(b-1)^2 = 3$, giving $b = 1 \pm \sqrt{3}$. For $a = -\sqrt{2}$, it becomes $(b-1)^2 = -1$, which is impossible for real b. Thus both choices with negative first coordinate fail. Among the remaining displayed pairs, (A) uses the valid value $1 - \sqrt{3}$. The second coordinate of (C), $-1 + \sqrt{3}$, is not $1 + \sqrt{3}$ or $1 - \sqrt{3}$, so it does not meet the second-junction condition. Both valid pairs have $a \neq 0$, as required.

Step 4: Conclusion One valid ordered pair is $\boxed{(a, b) = (\sqrt{2}, 1 - \sqrt{3})}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the sign possibilities from the first junction, then use a squared expression at the second junction to reject nonreal parameter choices.

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6. A ray of light is incident along a line which meets another line, 7x − y + 1 = 0, at the point (0, 1). The ray is then reflected from this point along the line, y + 2x = 1. Then the equation of the line of incidence of the ray of light is : [4 marks]

(A) 41x − 38y + 38 = 0

(B) 41x + 25y − 25 = 0

(C) 41x + 38y − 38 = 0

(D) 41x − 25y + 25 = 0

Correct Answer: (A) 41x − 38y + 38 = 0

Solution:

Concept:

- Reflection in a line preserves the component of a direction vector parallel to the mirror and reverses its perpendicular component. For a direction v and mirror vector u, the reflected direction is $2\,\mathrm{proj}_u\, v - v$.
- Reflection is reversible. Thus reflecting the outgoing line in the mirror gives the incident line; reversing a direction vector does not change its underlying straight line.

Step 1: Choose directions for the mirror and reflected ray The reflecting line is $7x - y + 1 = 0$, or $y = 7x + 1$, so a direction vector is $u = (1, 7)$. The outgoing line is $y + 2x = 1$, so take $v = (1, -2)$. Both lines contain the reflection point $P = (0, 1)$. It is enough to reflect the outgoing direction in the mirror and then use this same point to recover the incident line.

Step 2: Reflect the vector The relevant dot products are

$$v \cdot u = 1 - 14 = -13, \qquad u \cdot u = 1 + 49 = 50.$$

Hence the reflected direction is

$$\begin{aligned} w &= 2\frac{v \cdot u}{u \cdot u} u - v \\ &= -\frac{13}{25}(1, 7) - (1, -2) \\ &= \left(-\frac{38}{25}, -\frac{41}{25}\right). \end{aligned}$$

Multiplying a direction vector by a nonzero scalar does not change its line. We may therefore use

$(38, 41)$, with slope $41/38$.

Step 3: Write and verify the line equation The line through $(0, 1)$ with this slope is

$$y - 1 = \frac{41}{38}x, \qquad 41x - 38y + 38 = 0.$$

Its angle with the mirror has tangent

$$\left|\frac{7 - 41/38}{1 + 7(41/38)}\right| = \frac{9}{13}.$$

The outgoing line gives $|(7 - (-2))/(1 + 7(-2))| = 9/13$, confirming equal angles. The other choices have slopes $-41/25$, $-41/38$, and $41/25$, none of which is the reflected direction just calculated.

Step 4: Conclusion The incident line is $\boxed{41x - 38y + 38 = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: Reflect a direction vector in the mirror using a projection; then place the resulting line through the stated point of incidence.

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7. Let ABC be a triangle whose circumcentre is at P. If the position vectors of A, B, C and P are $\vec{a}, \vec{b}, \vec{c}$ and $\frac{\vec{a}+\vec{b}+\vec{c}}{4}$ respectively, then the position vector of the orthocentre of this triangle, is : [4 marks]

(A) $\vec{a} + \vec{b} + \vec{c}$

(B) $-\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

(C) $\vec{0}$

(D) $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

Correct Answer: (D) $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

Solution:

Concept:

- If p is the circumcentre vector and a, b, c are the vertex vectors, the orthocentre vector is $h = a + b + c - 2p$. This identity is valid relative to any chosen origin.
- To verify a proposed orthocentre, show that its displacement from each vertex is perpendicular to the opposite side, using a zero dot product.

Step 1: Shift the origin to the circumcentre Write the vertex vectors relative to P as $u = a - p$, $v = b - p$, and $w = c - p$. Since P is the circumcentre,

$$|u| = |v| = |w| = R,$$

where R is the circumradius. Consider the point whose vector relative to P is $u + v + w$. Its displacement from vertex A is $v + w$, and the direction of side BC is $v - w$.

Step 2: Check the altitudes Their dot product is

$$(v+w)\cdot(v-w) = |v|^2 - |w|^2 = R^2 - R^2 = 0.$$

Thus the proposed point lies on the altitude from A. By cyclically interchanging u, v, w, the same calculation proves that it lies on the altitudes from B and C. Since the vertices form a triangle, these altitudes determine its orthocentre. Translating back to the original origin gives

$$h = p + (u+v+w) = a+b+c-2p.$$

Step 3: Substitute the given circumcentre Let $s = a+b+c$. The question supplies $p = s/4$, so

$$h = s - 2\left(\frac{s}{4}\right) = \frac{s}{2}.$$

The subtraction of twice the circumcentre vector is essential; simply adding the three vertex vectors would work only when the origin itself is the circumcentre. Option (A) omits this translation, option (B) has the wrong sign, and option (C) would require the additional condition $s = 0$, which the question does not impose.

Step 4: Conclusion The orthocentre has position vector $\boxed{(\vec{a}+\vec{b}+\vec{c})/2}$. Hence, the correct option is (D).

Quick Tip to Solve: Remember the origin-independent identity H = A + B + C - 2P; the simpler sum of the vertex vectors assumes P is the origin.

8. The solution of the differential equation

$$\frac{dy}{dx} + \frac{y}{2}\sec x = \frac{\tan x}{2y},$$

where $0 \le x < \frac{\pi}{2}$, and y (0) = 1, is given by : [4 marks]

(A) $y = 1 - \frac{x}{\sec x + \tan x}$

(B) $y^2 = 1 + \frac{x}{\sec x + \tan x}$

(C) $y^2 = 1 - \frac{x}{\sec x + \tan x}$

(D) $y = 1 + \frac{x}{\sec x + \tan x}$

Correct Answer: (C) $y^2 = 1 - \frac{x}{\sec x + \tan x}$

Solution:

Concept:

- Multiply by $2y$ and put $z = y^2$, so that $z\prime = 2y\,dy/dx$. This converts the equation into a first-order linear equation.
- For $z\prime + P(x)z = Q(x)$, multiply by the integrating factor $e^{\int P(x)\,dx}$. Here $\sec x + \tan x$ is positive throughout the stated interval.

Step 1: Reduce the equation to linear form Multiplication by $2y$ gives

$$2y\frac{dy}{dx} + y^2 \sec x = \tan x.$$

Thus, with $z = y^2$,

$$\frac{dz}{dx} + z \sec x = \tan x.$$

The initial value is $z(0) = 1$. It is the square of the unknown, rather than y itself, which satisfies this linear equation.

Step 2: Apply the integrating factor and initial value Since $\int \sec x\, dx = \log(\sec x + \tan x)$, take $I = \sec x + \tan x$. Then

$$\frac{d}{dx}(Iz) = I \tan x.$$

Use $I\prime = I \sec x$ and $(\sec x - \tan x)I = 1$ to write

$$I \tan x = I \sec x - 1 = I\prime - 1.$$

Integration therefore gives $Iz = I - x + C$. At $x = 0$, both I and z equal 1, hence $C = 0$. Consequently,

$$y^2 = 1 - \frac{x}{\sec x + \tan x}.$$

The initial condition selects the positive square root for y. The expression under that root is positive: $I(0) = 1$ and $I\prime \geq 1$, so $I \geq 1 + x > x$. Option (B) has the wrong sign, while (A) and (D) omit the square required by the substitution.

Step 3: Conclusion The required relation is $y^2 = 1 - \frac{x}{\sec x + \tan x}$, with the positive root determined by $y(0) = 1$. Hence, the correct option is (C).

Quick Tip to Solve: When the equation contains both y and $1/y$, multiply by $2y$ and test the substitution $z = y^2$ before separating variables.

9. The mean of 5 observations is 5 and their variance is 124. If three of the observations are 1, 2 and 6 ; then the mean deviation from the mean of the data is : [4 marks]

(A) 2.4

(B) 2.8

(C) 2.5

(D) 2.6

Correct Answer: None of the listed options: the printed data give $(8 + 24\sqrt{2})/5$.

Solution:

Concept:

- For five observations with mean $\bar{x}$, the population variance is $\sigma^2 = \frac{1}{5}\sum x_i^2 - \bar{x}^2$. The mean deviation about the mean is $\frac{1}{5}\sum |x_i - \bar{x}|$.

- The last two observations must satisfy both their sum, fixed by the mean, and their sum of squares, fixed by the variance. Absolute deviations must be evaluated after determining their positions relative to the mean.

Step 1: Find the remaining two observations Let the unknown observations be u, v. From the stated mean,

$$u + v = 5(5) - (1 + 2 + 6) = 16.$$

Using the printed variance 124 gives

$$1^2 + 2^2 + 6^2 + u^2 + v^2 = 5(124 + 5^2) = 745,$$

so $u^2 + v^2 = 704$. Hence

$$uv = \frac{(u+v)^2 - (u^2+v^2)}{2} = \frac{256 - 704}{2} = -224.$$

The roots of $t^2 - 16t - 224 = 0$ are therefore

$$u = 8 + 12\sqrt{2}, \qquad v = 8 - 12\sqrt{2}.$$

They sum to 16 and their squared sum is 704, so they satisfy both statistical conditions exactly.

Step 2: Calculate the absolute deviations and compare the choices The first three absolute deviations from 5 are $4, 3, 1$. The other two are $|3 + 12\sqrt{2}|$ and $|3 - 12\sqrt{2}|$. Since $12\sqrt{2} > 3$, these equal $3 + 12\sqrt{2}$ and $12\sqrt{2} - 3$, respectively. Thus

$$\begin{aligned} \text{MD} &= \frac{4 + 3 + 1 + (3 + 12\sqrt{2}) + (12\sqrt{2} - 3)}{5} \\ &= \frac{8 + 24\sqrt{2}}{5} \approx 8.3882. \end{aligned}$$

None of 2.4, 2.8, 2.5 or 2.6 equals this result. Choosing one of these would require changing a stated numerical datum; it cannot follow from the mean and variance as printed. In particular, replacing absolute deviations with signed deviations would incorrectly make their sum zero.

Step 3: Conclusion With variance 124 as printed, the mean deviation is $\frac{8 + 24\sqrt{2}}{5} \approx 8.3882$. None of options (A)–(D) is correct for these data.

Quick Tip to Solve: Use the mean to get the missing sum and variance to get the missing squared sum; evaluate each absolute deviation only after locating the unknown observations.

10. A hyperbola whose transverse axis is along the major axis of the conic, $\frac{x^2}{3}+\frac{y^2}{4}=4$ and has vertices at the foci of this conic. If the eccentricity of the hyperbola is $\frac{3}{2}$, then which of the following points does NOT lie on it? [4 marks]

(A) (0, 2)

(B) $\left(\sqrt{5}, 2\sqrt{2}\right)$

(C) $\left(\sqrt{10}, 2\sqrt{3}\right)$

(D) $\left(5, 2\sqrt{3}\right)$

Correct Answer: (D) $\left(5, 2\sqrt{3}\right)$

Solution:

Concept:

- For an ellipse $x^2/b_e^2 + y^2/a_e^2 = 1$, with $a_e > b_e$, the foci lie on the y-axis at $(0, \pm\sqrt{a_e^2 - b_e^2})$.
- A hyperbola with transverse axis along the y-axis has equation $y^2/a_h^2 - x^2/b_h^2 = 1$, and eccentricity satisfies $e^2 = 1 + b_h^2/a_h^2$. The ellipse and hyperbola have different semi-axis parameters.

Step 1: Locate the vertices of the hyperbola First divide the given conic equation by 4:

$$\frac{x^2}{12}+\frac{y^2}{16}=1.$$

It is an ellipse with major axis on the y-axis. Its focal distance satisfies $c_e^2 = 16 - 12 = 4$, hence its foci are $(0, \pm 2)$. These are the specified vertices of the hyperbola. Therefore the hyperbola has centre at the origin, transverse axis on the y-axis, and $a_h = 2$. Its eccentricity is $3/2$, so

$$\frac{9}{4} = 1 + \frac{b_h^2}{4} \quad \Longrightarrow \quad b_h^2 = 5.$$

The required hyperbola is consequently

$$\frac{y^2}{4} - \frac{x^2}{5} = 1.$$

Step 2: Test each listed point in the equation A point lies on the hyperbola precisely when the left-hand side equals 1. Substituting the four options gives

$$\begin{aligned}
\text{(A)}: &\quad \frac{2^2}{4} - 0 = 1, \\
\text{(B)}: &\quad \frac{(2\sqrt{2})^2}{4} - \frac{(\sqrt{5})^2}{5} = 2 - 1 = 1, \\
\text{(C)}: &\quad \frac{(2\sqrt{3})^2}{4} - \frac{(\sqrt{10})^2}{5} = 3 - 2 = 1, \\
\text{(D)}: &\quad \frac{(2\sqrt{3})^2}{4} - \frac{5^2}{5} = 3 - 5 = -2.
\end{aligned}$$

Only (D) fails the equation; (A), (B) and (C) all satisfy it.

Step 3: Conclusion $\left(5, 2\sqrt{3}\right)$ is not on the hyperbola. Hence, the correct option is (D).

Quick Tip to Solve: Normalize the original conic before finding its foci, and label the ellipse and hyperbola parameters separately to avoid mixing their semi-axes.

11. If A > 0, B > 0 and A + B = $\frac{\pi}{6}$,
then the minimum value of tanA + tanB is : [4 marks]

(A) $\sqrt{3}-\sqrt{2}$

(B) $2-\sqrt{3}$

(C) $4-2\sqrt{3}$

(D) $\frac{2}{\sqrt{3}}$

Correct Answer: (C) $4-2\sqrt{3}$

Solution:

Concept:

- For angles A, B with positive cosines, $\tan A+\tan B=\sin(A+B)/(\cos A\cos B)$. Fixing the sum of the angles fixes the numerator.
- The product-to-sum identity $2\cos A\cos B=\cos(A+B)+\cos(A-B)$, together with $\cos(A-B)\le 1$, gives the largest possible denominator. Equality is possible when $A=B$.

Step 1: Express the sum using the fixed angle Because $A, B>0$ and $A+B=\pi/6$, both angles are strictly between 0 and $\pi/6$. Their cosines are therefore positive. Combining the two tangents over a common denominator,

$$\tan A+\tan B=\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B}=\frac{1}{2\cos A\cos B}.$$

The numerator is fixed, so minimizing this positive fraction means maximizing $\cos A\cos B$, rather than separately minimizing either tangent.

Step 2: Bound the denominator and check attainment Using the product-to-sum identity,

$$2\cos A\cos B=\frac{\sqrt{3}}{2}+\cos(A-B)\le\frac{\sqrt{3}}{2}+1.$$

Since all denominators are positive, inversion reverses this bound:

$$\tan A+\tan B\ge\frac{1}{1+\sqrt{3}/2}=\frac{2}{2+\sqrt{3}}=4-2\sqrt{3}.$$

Equality requires $\cos(A-B)=1$. In the permitted angle range this means $A=B=\pi/12$, which satisfies both strict positivity conditions. Directly, $2\tan(\pi/12)=2(2-\sqrt{3})$ gives the same value. Option (B) gives only one tangent at equality; the other listed expressions do not equal the attained lower bound. Numerically the minimum is about 0.536, providing a quick check on its scale.

Step 3: Conclusion The minimum is $4-2\sqrt{3}$, attained at $A=B=\pi/12$. Hence, the correct option is (C).

Quick Tip to Solve: For a fixed positive sum of two acute angles, rewrite the tangent sum as a fixed sine divided by a cosine product; equality occurs at equal angles.

12. ABC is a triangle in a plane with vertices

A(2, 3, 5), B(-1, 3, 2) and C(λ, 5, μ).

If the median through A is equally inclined to the coordinate axes, then the value of ($\lambda^3 + \mu^3 + 5$) is : [4 marks]

(A) 1130

(B) 1348

(C) 676

(D) 1077

Correct Answer: (B) 1348

Solution:

Concept:

- A median joins a vertex to the midpoint of the opposite side. If M is the midpoint of BC, then $\overrightarrow{AM}$ provides direction ratios for the median through A.
- For a directed line equally inclined to the positive coordinate axes, its three direction cosines are equal. They have the same sign, so the corresponding direction ratios are equal, not merely equal in magnitude.

Step 1: Calculate a direction vector of the median The midpoint of $B(-1, 3, 2)$ and $C(\lambda, 5, \mu)$ is

$$M = \left(\frac{\lambda - 1}{2}, 4, \frac{\mu + 2}{2}\right).$$

Subtracting the coordinates of $A(2, 3, 5)$ gives

$$\overrightarrow{AM} = \left(\frac{\lambda - 5}{2}, 1, \frac{\mu - 8}{2}\right).$$

The middle component is 1, so this vector is nonzero and is oriented with a positive y-component. Equal inclination to the coordinate axes thus requires its x- and z-components to have this same positive value.

Step 2: Enforce equal direction cosines and evaluate Dividing each direction ratio by the same positive vector length gives the direction cosines. Their equality is therefore equivalent to

$$\frac{\lambda - 5}{2} = 1 = \frac{\mu - 8}{2}.$$

Consequently $\lambda = 7$ and $\mu = 10$. As a check, these values give $M = (3, 4, 6)$, so $\overrightarrow{AM} = (1, 1, 1)$. Its direction cosines are all $1/\sqrt{3}$, exactly as required. Now compute the requested expression:

$$\lambda^3 + \mu^3 + 5 = 7^3 + 10^3 + 5 = 343 + 1000 + 5 = 1348.$$

Only option (B) equals the computed value.

Step 3: Conclusion $\lambda = 7$, $\mu = 10$, and $\lambda^3 + \mu^3 + 5 = 1348$. Hence, the correct option is (B).

Quick Tip to Solve: Find the opposite side midpoint first, then subtract the given vertex; equal coordinate direction angles mean equal signed direction cosines.

13. If the coefficients of x^{-2} and x^{-4} in the expansion of $\left(x^{\frac{1}{3}}+\frac{1}{2x^{\frac{1}{3}}}\right)^{18}, (x>0)$, are m and n respectively, then $\frac{m}{n}$ is equal to : [4 marks]

(A) 182

(B) $\frac{4}{5}$

(C) $\frac{5}{4}$

(D) 27

Correct Answer: (A) 182

Solution:

Concept:

- In $(a+b)^{18}$, the term obtained by choosing b exactly r times is $T_{r+1}=\binom{18}{r}a^{18-r}b^r$. The coefficient and the power of x must be tracked separately.
- Here $a=x^{1/3}$, $b=\frac{1}{2}x^{-1/3}$. The general term has coefficient $\binom{18}{r}2^{-r}$ and exponent $(18-2r)/3$.

Step 1: Locate the two required terms Writing out the general term explicitly gives

$$\begin{aligned} T_{r+1} &= \binom{18}{r}(x^{1/3})^{18-r}\left(\frac{1}{2x^{1/3}}\right)^r \\ &= \binom{18}{r}\frac{1}{2^r}x^{6-2r/3}. \end{aligned}$$

To obtain x^{-2}, solve $6-2r/3=-2$, giving $r=12$. To obtain x^{-4}, solve $6-2r/3=-4$, giving $r=15$. Both indices lie between 0 and 18, so both requested terms occur. Their coefficients are

$$m=\frac{\binom{18}{12}}{2^{12}}, \qquad n=\frac{\binom{18}{15}}{2^{15}}.$$

Step 2: Simplify the ratio without expanding the polynomial On division, the powers of 2 contribute $2^{15-12}=8$, not its reciprocal. Using symmetry of binomial coefficients,

$$\begin{aligned} \frac{m}{n} &= 8\frac{\binom{18}{6}}{\binom{18}{3}} \\ &= 8\frac{3!\,15!}{6!\,12!} \\ &= 8\frac{15\cdot 14\cdot 13}{6\cdot 5\cdot 4} = 8\cdot\frac{91}{4} = 182. \end{aligned}$$

The values $4/5$ and $5/4$ do not include the actual binomial-coefficient ratio. The value 27 also fails the exact factorial calculation. The condition $x>0$ ensures the fractional powers are unambiguous but does not change these constant coefficients.

Step 3: Conclusion The coefficient ratio is $m/n=182$. Hence, the correct option is (A).

Quick Tip to Solve: Equate the exponent in the general term to each requested power first; only then form the coefficient ratio, retaining the factor 2^{15-12}.

14. Let f(x) = $\sin^4 x + \cos^4 x$. Then f is an increasing function in the interval : [4 marks]

(A) $]0, \frac{\pi}{4}[$

(B) $]\frac{\pi}{4}, \frac{\pi}{2}[$

(C) $]\frac{\pi}{2}, \frac{5\pi}{8}[$

(D) $]\frac{5\pi}{8}, \frac{3\pi}{4}[$

Correct Answer: (B) $]\frac{\pi}{4}, \frac{\pi}{2}[$

Solution:

Concept:

- A differentiable function is strictly increasing on an interval when its derivative is positive there. The sign must hold throughout the proposed interval, not just at one endpoint.
- The identities $\sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$ and $\sin^2(2x) = (1 - \cos 4x)/2$ reduce the fourth powers to a single multiple-angle cosine.

Step 1: Rewrite and differentiate the function Use $\sin^2 x + \cos^2 x = 1$ to obtain

$$\begin{aligned} f(x) &= 1 - 2\sin^2 x \cos^2 x \\ &= 1 - \frac{1}{2}\sin^2(2x) = \frac{3}{4} + \frac{1}{4}\cos 4x. \end{aligned}$$

Differentiating with the chain rule gives

$$f\prime(x) = -\sin 4x.$$

The factor 4 from the inner angle cancels the coefficient $1/4$. Thus an increasing interval must have $\sin 4x < 0$.

Step 2: Test the sign on all four intervals For option (A), $0 < x < \pi/4$ gives $0 < 4x < \pi$. Sine is positive there, so $f\prime < 0$: the function decreases.

For option (B), $\pi/4 < x < \pi/2$ gives $\pi < 4x < 2\pi$. Sine is negative throughout, hence $f\prime > 0$ throughout this interval.

For option (C), the transformed interval is $2\pi < 4x < 5\pi/2$, where sine is positive. For option (D), it is $5\pi/2 < 4x < 3\pi$, where sine is also positive. Thus both (C) and (D) are decreasing intervals. At the endpoints of (B) the derivative vanishes, but those endpoints are excluded and do not affect strict increase inside. The endpoint values also agree with the sign test: $f(\pi/4) = 1/2$ and $f(\pi/2) = 1$.

Step 3: Conclusion The function is increasing on $(\pi/4, \pi/2)$. Hence, the correct option is (B).

Quick Tip to Solve: Reduce fourth powers to $3/4 + (1/4)\cos 4x$; multiply each candidate interval by 4 and read the sign of sine.

15. The integral $\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$ is equal to :

(where C is a constant of integration.) [4 marks]

(A) $-2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

(B) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

(C) $-\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

(D) $2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

Correct Answer: (B) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

Solution:

Concept:

- The radical satisfies $\sqrt{x-x^2}=\sqrt{x}\sqrt{1-x}$ on the real domain $0<x<1$. A substitution must transform both the radical and dx.
- After setting $u=\sqrt{x}$, a half-angle substitution $u=\cos\theta$ gives the standard integral involving $1+\cos\theta$. The constant C accounts for any fixed additive term.

Step 1: Remove the square root of x Put $u=\sqrt{x}$, so $x=u^2$, $dx=2u\,du$, and $0<u<1$. Since u is positive, the denominator radical becomes $u\sqrt{1-u^2}$. Therefore

$$I=2\int\frac{du}{(1+u)\sqrt{1-u^2}}.$$

The factor u cancels; retaining it would give an incorrect antiderivative.

Step 2: Apply a trigonometric substitution Take $u=\cos\theta$ with $0<\theta<\pi/2$. Then $du=-\sin\theta\,d\theta$, and $\sqrt{1-u^2}=\sin\theta$. Using $1+\cos\theta=2\cos^2(\theta/2)$, obtain

$$I=-2\int\frac{d\theta}{1+\cos\theta}=-\int\sec^2(\theta/2)\,d\theta=-2\tan(\theta/2)+C.$$

The negative sign comes from differentiating cosine; the factor 2 comes from the half-angle argument.

Step 3: Return to x and check the sign The positive half-angle identity on this interval gives

$$\tan(\theta/2)=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}.$$

Differentiating the resulting expression gives exactly $1/[(1+\sqrt{x})\sqrt{x-x^2}]$. Option (C) is half the required antiderivative. Options (A) and (D) use the reciprocal radical and yield a denominator containing $1-\sqrt{x}$, so neither has the required derivative.

Step 4: Conclusion $I=-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$, on $0<x<1$. Hence, the correct option is (B).

Quick Tip to Solve: First remove $\sqrt{x}$ by setting $x=u^2$; then $u=\cos\theta$ turns the remaining denominator into a half-angle identity.

16. Let $a_1, a_2, a_3, \ldots\ldots\ldots, a_n, \ldots\ldots$ be in A.P.
If $a_3 + a_7 + a_{11} + a_{15} = 72$,
then the sum of its first 17 terms is equal to : [4 marks]

(A) 306
(B) 153
(C) 612
(D) 204

Correct Answer: (A) 306

Solution:

Concept:

- An arithmetic progression has general term $a_n = a + (n-1)d$, where a is its first term and d its common difference.
- For an odd number of consecutive AP terms, the sum equals the number of terms times the middle term. Thus $S_{17} = 17a_9$, which can be found without determining a and d separately.

Step 1: Express the four given terms The indices 3, 7, 11 and 15 are symmetric about 9. Substituting their AP expressions into the given condition gives

$$(a + 2d) + (a + 6d) + (a + 10d) + (a + 14d) = 72.$$

Collect the first-term contributions and the common-difference contributions:

$$4a + 32d = 72 \quad \Longrightarrow \quad a + 8d = 18.$$

The left-hand side is exactly a_9. The information fixes the middle term even though it does not uniquely fix the entire progression.

Step 2: Sum the first seventeen terms Use the AP sum formula with $n = 17$:

$$S_{17} = \frac{17}{2}\big(2a + 16d\big) = 17(a + 8d) = 17 \times 18 = 306.$$

Equivalently, the pairs (a_1, a_{17}), (a_2, a_{16}), and so on each sum to $2a_9 = 36$. There are eight such pairs and the unpaired middle term 18, giving $8 \times 36 + 18 = 306$. This checks both the term count and the arithmetic.

Step 3: Compare the options Option (B), 153, is half the required sum and would result from keeping the factor $1/2$ after already replacing $2a + 16d$ by twice the middle term. Option (C), 612, doubles the required value. Option (D), 204, also fails the exact AP sum relation $S_{17} = 17a_9$. Only 306 is consistent with the middle term 18.

Step 4: Conclusion The sum of the first 17 terms is $\boxed{306}$. Hence, the correct option is (A).

Quick Tip to Solve: Average the symmetric given terms to identify a_9; an AP with 17 terms has sum $17a_9$, so finding a and d individually is unnecessary.

17. If x is a solution of the equation, $\sqrt{2x+1}-\sqrt{2x-1}=1,\ \left(x\geq\frac{1}{2}\right)$, then $\sqrt{4x^2-1}$ is equal to :

[4 marks]

(A) $\frac{3}{4}$

(B) $\frac{1}{2}$

(C) 2

(D) $2\sqrt{2}$

Correct Answer: (A) $\frac{3}{4}$

Solution:

Concept:

- For nonnegative radicands, put $u=\sqrt{2x+1}$ and $v=\sqrt{2x-1}$. The given domain $x\geq 1/2$ ensures these are real, with $u>v\geq 0$.
- The identity $(u-v)(u+v)=u^2-v^2$ recovers the sum from the difference. Also $uv=\sqrt{(2x+1)(2x-1)}=\sqrt{4x^2-1}$, the expression required.

Step 1: Find the sum of the two radicals The equation states that $u-v=1$. Their squared values differ by a constant:

$$u^2-v^2=(2x+1)-(2x-1)=2.$$

Factoring this difference gives

$$(u-v)(u+v)=2 \quad\Longrightarrow\quad u+v=2.$$

Division by $u-v$ is valid because the given difference equals 1. This method avoids repeated squaring and its possible extraneous roots.

Step 2: Evaluate their product Add and subtract $u+v=2$ and $u-v=1$ to get

$$u=\frac{3}{2},\qquad v=\frac{1}{2}.$$

Consequently,

$$\sqrt{4x^2-1}=uv=\frac{3}{2}\times\frac{1}{2}=\frac{3}{4}.$$

As a direct check, $v^2=2x-1=1/4$ gives $x=5/8$, which belongs to the specified domain. Substitution in the original equation gives $3/2-1/2=1$, confirming that no invalid root has been introduced.

Step 3: Check the requested expression and options At $x=5/8$, the quantity under the final radical is

$$4x^2-1=\frac{25}{16}-1=\frac{9}{16}.$$

Its principal square root is $3/4$, not $1/2$, which is only the smaller original radical. The values 2 and $2\sqrt{2}$ cannot equal the product of the two radical values obtained above. Thus only option (A) satisfies the demand.

Step 4: Conclusion $\sqrt{4x^2-1}=\boxed{\frac{3}{4}}$. Hence, the correct option is (A).

Quick Tip to Solve: When a difference of two radicals is given, multiply by their conjugate to find the sum; the required larger radical here is their product.

18. Let P = {θ : sinθ − cosθ = $\sqrt{2}\cos\theta$}
and Q = {θ : sinθ + cosθ = $\sqrt{2}\sin\theta$} be two sets. Then [4 marks]

(A) P $\subset$ Q and Q − P $\neq \phi$

(B) Q $\not\subset$ P

(C) P $\not\subset$ Q

(D) P = Q

Correct Answer: (D) P = Q

Solution:

Concept:

- Two sets are equal when their defining conditions are equivalent in both directions. Rearranging equations is safer here than dividing immediately by a trigonometric function that could be zero.
- The conjugate identity $(\sqrt{2}-1)(\sqrt{2}+1)=1$ relates the two coefficients. The common nonzero tangent value is $1+\sqrt{2}=\tan(3\pi/8)$.

Step 1: Rewrite the condition defining P For membership in P, the printed equation is

$$\sin\theta-\cos\theta=\sqrt{2}\cos\theta.$$

Move the cosine term to the right:

$$\sin\theta=(1+\sqrt{2})\cos\theta.$$

If $\cos\theta=0$, this equation would require $\sin\theta=0$, impossible for a real angle. Thus all members have nonzero cosine, and the condition could also be written as $\tan\theta=1+\sqrt{2}$.

Step 2: Rewrite the condition defining Q For membership in Q, rearrange its equation as

$$\sin\theta+\cos\theta=\sqrt{2}\sin\theta \quad\Longleftrightarrow\quad \cos\theta=(\sqrt{2}-1)\sin\theta.$$

Multiply both sides by $\sqrt{2}+1$. Since the product of the conjugate coefficients is 1, this becomes

$$(\sqrt{2}+1)\cos\theta=\sin\theta.$$

This is precisely the condition obtained for P. Every step is reversible, since the multiplier is a nonzero real constant; hence neither equation admits additional angles.

Step 3: Compare the solution sets Both sets therefore consist of the angles

$$\theta=\frac{3\pi}{8}+k\pi, \qquad k\in\mathbb{Z}.$$

Every member of P belongs to Q, and every member of Q belongs to P. In particular, $Q-P$ is empty, contradicting option (A). The non-inclusion claims in options (B) and (C) are also false. The equality in option (D) is the only valid relation.

Step 4: Conclusion The two defining equations have exactly the same solutions, so $\boxed{P=Q}$. Hence, the correct option is (D).

Quick Tip to Solve: Multiply the equation containing $\sqrt{2}-1$ by its conjugate $\sqrt{2}+1$; this exposes equivalence without losing angles through division.

19. A straight line through origin O meets the lines 3y = 10 − 4x and 8x + 6y + 5 = 0 at points A and B respectively. Then O divides the segment AB in the ratio : [4 marks]

(A) 2 : 3

(B) 1 : 2

(C) 4 : 1

(D) 3 : 4

Correct Answer: (C) 4 : 1

Solution:

Concept:

- Points on a line through the origin can be written as scalar multiples of one nonzero direction vector: $\mathbf{r} = t\mathbf{v}$. Ratios of distances from the origin are ratios of the absolute scalar parameters.
- The two given lines are parallel because they have proportional normal vectors. Writing both with the same linear form $L(x, y) = 4x + 3y$ makes their intersection parameters directly comparable.

Step 1: Put the parallel lines into a common form The first line $3y = 10 - 4x$ becomes $4x + 3y = 10$. Divide the second equation by 2 to obtain $4x + 3y = -5/2$. Thus

$$L(A) = 10, \qquad L(B) = -\frac{5}{2}, \qquad L(O) = 0.$$

The constants have opposite signs. On any line through O that meets both parallel lines, the points A and B are therefore on opposite sides of the origin. The requested division is internal.

Step 2: Find the distances from the origin Write a general point on the intersecting line as $(x, y) = t(p, q)$, where $(p, q) \neq (0, 0)$. If $k = 4p + 3q$, the premise that intersections exist ensures $k \neq 0$. At the two intersections,

$$t_A k = 10, \qquad t_B k = -\frac{5}{2}.$$

Hence $t_A/t_B = -4$. As distances are nonnegative,

$$\frac{OA}{OB} = \frac{|t_A|}{|t_B|} = 4.$$

The direction-vector length cancels, so the answer does not depend on the slope of the line through the origin.

Step 3: Verify the order of the ratio The statement that O divides AB asks for $AO : OB$, not its reciprocal. The point on the level $L = 10$ is four times as far from the origin as the point on the level $L = -5/2$, giving $4 : 1$. Indeed, the internal section point $(A+4B)/5$ has L-value $(10+4(-5/2))/5 = 0$, as required at O. The ratios in the other options do not meet this condition.

Step 4: Conclusion O divides AB internally in the ratio $\boxed{AO : OB = 4 : 1}$. Hence, the correct option is (C).

Quick Tip to Solve: For parallel lines written as $L(x, y) = c_1$ and $L(x, y) = c_2$, an intersecting line through the origin gives $OA : OB = |c_1| : |c_2|$.

20. The value of the integral

$$\int_4^{10} \frac{[x^2]\,dx}{[x^2-28x+196]+[x^2]},$$

where [x] denotes the greatest integer less than or equal to x, is : [4 marks]

(A) 6

(B) 3

(C) 7

(D) $\frac{1}{3}$

Correct Answer: (B) 3

Solution:

Concept:

- The greatest-integer function $[t] = \lfloor t \rfloor$ is the integer not exceeding t. It must be retained exactly; generally $[a] + [b] \neq [a + b]$.
- The substitution $x \mapsto a + b - x$ preserves a definite integral on $[a, b]$. Here $a + b = 14$, and this reflection swaps x^2 with $(14 - x)^2$, making the two numerators complementary.

Step 1: Identify the symmetric pair in the denominator Complete the square inside the first floor function:

$$x^2 - 28x + 196 = (14 - x)^2.$$

Define

$$f(x) = \frac{\lfloor x^2 \rfloor}{\lfloor (14-x)^2 \rfloor + \lfloor x^2 \rfloor}.$$

For $4 \leq x \leq 10$, both x and $14 - x$ are at least 4. Each floor in the denominator is therefore at least 16, so the denominator is positive everywhere. The finitely many step discontinuities pose no difficulty for the definite integral.

Step 2: Reflect the interval and add the two forms Let $I = \int_4^{10} f(x)\,dx$. Under $u = 14 - x$, the endpoints 4 and 10 interchange and the differential reverses sign. Thus

$$I = \int_4^{10} \frac{\lfloor (14-x)^2 \rfloor}{\lfloor x^2 \rfloor + \lfloor (14-x)^2 \rfloor}\,dx.$$

Adding this to the original representation gives a pointwise numerator equal to the denominator:

$$2I = \int_4^{10} 1\,dx = 10 - 4 = 6.$$

No approximation of either floor function has been used.

Step 3: Extract the integral value Dividing by 2 yields $I = 3$. Option (A), 6, is the value of the sum of the original and reflected integrals, not of either integral separately. Option (C), 7, is the midpoint of the interval and is unrelated to its integrated width. Option (D), $1/3$, is the reciprocal of the correct value. Only option (B) follows from the exact reflection identity.

Step 4: Conclusion The given integral has value $\boxed{3}$. Hence, the correct option is (B).

Quick Tip to Solve: When a denominator is $g(x) + g(a + b - x)$, add the integral to its reflected form; each numerator then supplies exactly half the interval length.

21. Let A be a 3 × 3 matrix such that $A^2 - 5A + 7I = 0$

Statement - I :

$A^{-1} = \frac{1}{7}(5I - A)$.

Statement - II :

The polynomial $A^3 - 2A^2 - 3A + I$ can be reduced to $5(A - 4I)$.

Then : [4 marks]

(A) Statement-I is true, but Statement-II is false.

(B) Statement-I is false, but Statement-II is true.

(C) Both the statements are true.

(D) Both the statements are false

Correct Answer: (C) Both the statements are true.

Solution:

Concept:

- An identity such as $A^2 - 5A + 7I = 0$ allows every higher power of A to be reduced to a linear combination of A and the identity matrix I. Scalar constants in a matrix polynomial multiply I.
- A matrix B is the inverse of A when $AB = BA = I$. A polynomial in A commutes with A, so the given relation can directly construct its inverse without assuming invertibility first.

Step 1: Check Statement I by constructing the inverse Rearrange the supplied matrix equation:

$$5A - A^2 = 7I \quad \Longrightarrow \quad A(5I - A) = 7I.$$

Since $5I - A$ commutes with A, multiplication in the reverse order also gives $7I$. Dividing by the nonzero scalar 7 gives

$$A\left(\frac{5I - A}{7}\right) = \left(\frac{5I - A}{7}\right)A = I.$$

Thus A is invertible and $A^{-1} = (5I - A)/7$. Statement I is true; the existence of this inverse is proved rather than presumed.

Step 2: Reduce the polynomial in Statement II From the original identity, $A^2 = 5A - 7I$. Multiplying it by A yields $A^3 = 5A^2 - 7A$. Substitute successively into the required polynomial:

$$\begin{aligned} A^3 - 2A^2 - 3A + I &= (5A^2 - 7A) - 2A^2 - 3A + I \\ &= 3A^2 - 10A + I \\ &= 3(5A - 7I) - 10A + I \\ &= 5A - 20I = 5(A - 4I). \end{aligned}$$

All operations preserve matrix order, and only powers of the same matrix are involved. Statement II is therefore also true.

Step 3: Select the truth-value combination Statement I follows by the verified inverse product, while Statement II follows by direct polynomial reduction. Option (A) rejects a valid second identity, option (B) rejects the proved inverse identity, and option (D) rejects both. Since both computations hold for every matrix satisfying the given equation, the correct combination is option (C).

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (C).

Quick Tip to Solve: Use $A^2 = 5A - 7I$ repeatedly to reduce higher powers, and rearrange the same relation as $A(5I - A) = 7I$ to read off the inverse.

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22. If A = $\begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$,

then the determinant of the matrix ($A^{2016} - 2A^{2015} - A^{2014}$) is : [4 marks]

(A) 2014

(B) -175

(C) 2016

(D) -25

Correct Answer: (D) -25

Solution:

Concept:

- The Cayley–Hamilton theorem states that a square matrix satisfies its own characteristic equation. For a two-by-two matrix it is $A^2 - (\operatorname{tr} A)A + (\det A)I = 0$, where I is the identity matrix.
- Determinants obey $\det(BC) = \det B \det C$ and $\det(cB) = c^2 \det B$ for a two-by-two matrix. These rules evaluate a determinant without computing enormous matrix powers.

Step 1: Find the characteristic equation From the entries of the given matrix, its trace and determinant are

$$\operatorname{tr} A = -4 + 1 = -3, \qquad \det A = (-4)(1) - (-1)(3) = -1.$$

Equivalently, direct expansion gives $\det(\mu I - A) = \mu^2 + 3\mu - 1$. Therefore Cayley–Hamilton yields

$$A^2 + 3A - I = 0, \qquad A^2 = I - 3A.$$

All these equalities are matrix equalities; the constant term multiplies the identity.

Step 2: Reduce the matrix polynomial before taking its determinant Factor the common power of A, then use the quadratic identity:

$$\begin{aligned} A^{2016} - 2A^{2015} - A^{2014} &= A^{2014}(A^2 - 2A - I) \\ &= A^{2014}\big((I - 3A) - 2A - I\big) \\ &= -5A^{2015}. \end{aligned}$$

This reduction also avoids confusing the determinant of a difference with the difference of determinants, which is not a valid rule.

Step 3: Apply the determinant rules and compare the options Because the matrix order is two, the scalar factor contributes $(-5)^2$, while the odd power preserves the negative sign of $\det A$:

$$\det(-5A^{2015}) = 25(\det A)^{2015} = 25(-1)^{2015} = -25.$$

The positive numbers 2014 and 2016 cannot match this negative determinant. The other negative option, -175, does not follow from the scalar factor and determinant power.

Step 4: Conclusion The required determinant is $\boxed{-25}$. Hence, the correct option is (D).

Quick Tip to Solve: For a high matrix power inside a determinant, first reduce its polynomial using Cayley–Hamilton; remember that a scalar multiplies a two-by-two determinant by its square.

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23. Let z = 1 + ai be a complex number, a > 0, such that z^3 is a real number. Then the sum 1 + z + z^2 + ...+ z^{11} is equal to : [4 marks]

(A) $-1250\sqrt{3}\,i$

(B) $1250\sqrt{3}\,i$

(C) $1365\sqrt{3}i$

(D) $-1365\sqrt{3}i$

Correct Answer: (D) $-1365\sqrt{3}i$

Solution:

Concept:

- A complex number is real exactly when its imaginary part vanishes. Expand $(1+ai)^3$ using $i^2 = -1$, and impose this condition before evaluating the sum.
- For a geometric progression with ratio $z \neq 1$, the first twelve terms have sum $S = (z^{12} - 1)/(z - 1)$. Computing z^3 first simplifies the large power.

Step 1: Determine the positive value of a Separate the real and imaginary terms in the cube:

$$(1 + ai)^3 = (1 - 3a^2) + i(3a - a^3).$$

Since the cube is real, its imaginary coefficient must be zero:

$$3a - a^3 = a(3 - a^2) = 0.$$

The condition $a > 0$ excludes both $a = 0$ and $a = -\sqrt{3}$. Thus $a = \sqrt{3}$, giving $z = 1+i\sqrt{3}$. Substitution into the real part gives

$$z^3 = 1 - 3(3) = -8.$$

Step 2: Evaluate the twelve-term geometric sum The sum includes the powers from zero to eleven,

so it has twelve terms. Since $z-1=i\sqrt{3}\neq 0$, apply the geometric-sum formula:

$$\begin{aligned} z^{12} &= (z^3)^4 = (-8)^4 = 4096, \\ S &= 1+z+z^2+\cdots+z^{11} \\ &= \frac{4096-1}{i\sqrt{3}} = \frac{4095}{i\sqrt{3}} \\ &= -\frac{4095i}{\sqrt{3}} = -1365\sqrt{3}\,i. \end{aligned}$$

The minus sign follows from $1/i=-i$, not from the numerator, which is positive.

Step 3: Check the sign and magnitude against the choices A useful independent grouping is $(1+z+z^2)(1+z^3+z^6+z^9)$. Here $1+z+z^2=3\sqrt{3}\,i$, while the second factor is $1-8+64-512=-455$. Their product is again $-1365\sqrt{3}\,i$. Thus the positive-imaginary choices have the wrong sign, and the negative choice with coefficient 1250 has the wrong magnitude.

Step 4: Conclusion The sum is $\boxed{-1365\sqrt{3}\,i}$. Hence, the correct option is (D).

Quick Tip to Solve: When a power of the common ratio is simple, compute that power first; track the sign in $1/i=-i$ when simplifying the geometric sum.

24. P and Q are two distinct points on the parabola, $y^2 = 4x$, with parameters t and t_1 respectively. If the normal at P passes through Q, then the minimum value of t_1^2 is : [4 marks]

(A) 2

(B) 4

(C) 6

(D) 8

Correct Answer: (D) 8

Solution:

Concept:

- On $y^2=4x$, a point with parameter t is $(t^2,2t)$. Its normal has equation $y=-tx+2t+t^3$, obtained from the tangent slope $1/t$ when $t\neq 0$.
- Substituting a second parametrised point into the normal gives a polynomial relation. Factor out the known intersection corresponding to the original point before using the distinctness condition.

Step 1: Substitute Q into the normal at P Write $P=(t^2,2t)$ and $Q=(t_1^2,2t_1)$. Differentiating the parabola gives $2y\,dy/dx=4$, so the normal slope at P is $-t$. Its equation is therefore

$$y-2t=-t(x-t^2).$$

If $t=0$, the normal is $y=0$, which meets the parabola only at $(0,0)$; it cannot contain a distinct point Q. Hence $t\neq 0$ for the required configuration.

Step 2: Use the distinct intersection to relate the parameters Because Q lies on the normal, substitution

and rearrangement yield

$$\begin{aligned} 2t_1 &= -tt_1^2 + 2t + t^3, \\ 0 &= tt_1^2 + 2t_1 - 2t - t^3 \\ &= (t_1 - t)(tt_1 + t^2 + 2). \end{aligned}$$

Distinct points have $t_1 \neq t$, since their y-coordinates are twice their parameters. Thus the second factor must vanish:

$$t_1 = -t - \frac{2}{t}.$$

Step 3: Minimise the square and verify attainability Squaring the relation produces

$$t_1^2 = \left(t + \frac{2}{t}\right)^2 = t^2 + 4 + \frac{4}{t^2}.$$

For $u = t^2 > 0$, $u + 4/u \geq 4$, because $(u-2)^2/u \geq 0$. Hence $t_1^2 \geq 8$, with equality when $t^2 = 2$. At equality, $t_1 = -2t$, which is distinct from t since $t \neq 0$; the lower bound is therefore actually reached. The values 2, 4 and 6 are below this bound and cannot occur as the minimum.

Step 4: Conclusion The minimum value is $\boxed{t_1^2 = 8}$. Hence, the correct option is (D).

Quick Tip to Solve: In a normal–parabola intersection, remove the factor for the original point first; then minimise the expression using $t^2 > 0$.

25. The number of distinct real values of λ for which the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{\lambda^2}$ and $\frac{x-3}{1} = \frac{y-2}{\lambda^2} = \frac{z-1}{2}$ are coplanar is : [4 marks]

(A) 4

(B) 1

(C) 2

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- Two lines through position vectors $\mathbf{a}$ and $\mathbf{b}$, with direction vectors $\mathbf{d}_1$ and $\mathbf{d}_2$, are coplanar when their scalar triple product $(\mathbf{b} - \mathbf{a}) \cdot (\mathbf{d}_1 \times \mathbf{d}_2)$ vanishes.
- A parameter occurring as λ^2 is conveniently replaced by $u = \lambda^2 \geq 0$. After solving for u, recover and count the distinct real values of λ, including zero only once.

Step 1: Read the points and direction vectors Set $u = \lambda^2$. The first line passes through $\mathbf{a} = (1, 2, -3)$ with direction $(1, 2, u)$; the second passes through $\mathbf{b} = (3, 2, 1)$ with direction $(1, u, 2)$. Thus

$$\mathbf{b} - \mathbf{a} = (2, 0, 4).$$

Use the parametric interpretation of symmetric line equations: a zero direction component means that

the corresponding coordinate remains fixed. In particular $u = 0$ is meaningful and must be checked, rather than discarded because of a displayed zero denominator.

Step 2: Apply the scalar triple-product condition Compute the cross product and then dot it with the joining vector:

$$\mathbf{d}_1 \times \mathbf{d}_2 = (4 - u^2,\ u - 2,\ u - 2),$$
$$(2, 0, 4) \cdot (4 - u^2, u - 2, u - 2) = 2(4 - u^2) + 4(u - 2)$$
$$= -2u^2 + 4u = 2u(2 - u).$$

Coplanarity therefore requires $u = 0$ or $u = 2$. At $u = 2$ the direction vectors coincide, so the lines are parallel and hence coplanar; the zero cross product correctly includes this case.

Step 3: Recover and count the real parameters The two equations for the original parameter give

$$\lambda^2 = 0 \Rightarrow \lambda = 0, \qquad \lambda^2 = 2 \Rightarrow \lambda = \pm\sqrt{2}.$$

For $u = 0$, the lines can also be checked directly: their parametric forms meet at $(1, 2, -3)$. Thus all three values are admissible. Counting the two values of u would give the wrong count 2; counting zero twice would give 4. A count of 1 would omit valid possibilities.

Step 4: Conclusion The distinct real values are $0, \sqrt{2}, -\sqrt{2}$, so their number is $\boxed{3}$. Hence, the correct option is (D).

Quick Tip to Solve: Solve the coplanarity determinant in $u = \lambda^2$, then convert back to λ; a zero direction component still defines a valid parametric line.

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26. Equation of the tangent to the circle, at the point (1, −1), whose centre is the point of intersection of the straight lines x − y = 1 and 2x + y = 3 is : [4 marks]

(A) 4x + y − 3 = 0

(B) x + 4y + 3 = 0

(C) 3x − y − 4 = 0

(D) x − 3y − 4 = 0

Correct Answer: (B) x + 4y + 3 = 0

Solution:

Concept:

- The centre is found by solving the two given straight-line equations simultaneously. The tangent at a point of a circle is perpendicular to the radius through that point.
- If $C = (h, k)$ is the centre and $P = (x_1, y_1)$ is the contact point, the radius vector $(x_1 - h, y_1 - k)$ is a normal vector to the tangent. Hence the tangent equation is $(x_1 - h)(x - x_1) + (y_1 - k)(y - y_1) = 0$.

Step 1: Locate the centre from the intersecting lines The centre $C = (h, k)$ satisfies $h - k = 1$ and $2h + k = 3$. Adding these equations eliminates k:

$$3h = 4, \qquad h = \frac{4}{3}, \qquad k = h - 1 = \frac{1}{3}.$$

Thus the centre is $C = (4/3, 1/3)$. The point of contact is the given point $P = (1, -1)$, rather than the intersection just calculated.

Step 2: Form the radius vector and the tangent equation The vector from the centre to the contact point is

$$\overrightarrow{CP} = \left(1 - \frac{4}{3}, -1 - \frac{1}{3}\right) = \left(-\frac{1}{3}, -\frac{4}{3}\right).$$

A point (x, y) lies on the tangent when its displacement from P is perpendicular to this vector. Therefore

$$-\frac{1}{3}(x-1) - \frac{4}{3}(y+1) = 0.$$

Multiplying by -3 and collecting the constant terms gives

$$(x-1) + 4(y+1) = 0, \qquad x + 4y + 3 = 0.$$

Step 3: Verify the contact point and perpendicularity Substituting $(1, -1)$ gives $1 - 4 + 3 = 0$, so the line passes through the required point. The radius slope is $(-4/3)/(-1/3) = 4$, and the tangent slope is $-1/4$; their product is -1. The other three options also pass through P, but their slopes are -4, 3 and $1/3$, respectively. None is perpendicular to the radius, so passing through the point alone is insufficient.

Step 4: Conclusion The tangent is $\boxed{x + 4y + 3 = 0}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the radius vector directly as the tangent's normal vector; this avoids a slope calculation and works even for horizontal or vertical radii.

27. The angle of elevation of the top of a vertical tower from a point A, due east of it is 45°. The angle of elevation of the top of the same tower from a point B, due south of A is 30°. If the distance between A and B is $54\sqrt{2}\,m$, then the height of the tower (in metres), is : [4 marks]

(A) $36\sqrt{3}$

(B) 54

(C) $54\sqrt{3}$

(D) 108

Correct Answer: (B) 54

Solution:

Concept:

- For a vertical tower of height h, the tangent of an angle of elevation equals h divided by the horizontal distance to the tower's foot.
- East–west and north–south ground directions are perpendicular. A point due east of the tower and another due south of that point form a right ground triangle with the tower's foot.

Step 1: Express both ground distances in terms of the height Let O be the tower's foot and T its top,

with $OT = h > 0$. The angle from A is 45°, so

$$\tan 45^\circ = \frac{h}{OA} = 1, \qquad OA = h.$$

The angle from B is 30°, which gives

$$\tan 30^\circ = \frac{h}{OB} = \frac{1}{\sqrt{3}}, \qquad OB = \sqrt{3}\,h.$$

These are two different horizontal distances: OB is measured from the tower's foot, not from A.

Step 2: Apply Pythagoras to the ground triangle Since A is due east of O, OA runs east–west. Since B is due south of A, AB runs north–south. Therefore $OA \perp AB$ and the right angle in OAB is at A, with hypotenuse OB. Substitute $AB = 54\sqrt{2}\,\text{m}$:

$$\begin{aligned} OB^2 &= OA^2 + AB^2, \\ (\sqrt{3}h)^2 &= h^2 + (54\sqrt{2})^2, \\ 3h^2 &= h^2 + 5832, \\ h^2 &= 2916 = 54^2. \end{aligned}$$

A tower has positive height, so take $h = 54\,\text{m}$.

Step 3: Check the result against both elevation angles With this height, $OA = 54\,\text{m}$ and $OB = 54\sqrt{3}\,\text{m}$. These give $h/OA = 1$ and $h/OB = 1/\sqrt{3}$, as required. The option $54\sqrt{3}$ is the distance OB, not the height. The proposed heights $36\sqrt{3}$ and 108 do not satisfy $2h^2 = (54\sqrt{2})^2$.

Step 4: Conclusion The height of the tower is $\boxed{54\,\text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: First map the compass directions onto the ground plane; identify the right angle before combining the two elevation-angle equations.

28. An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is : [4 marks]

(A) $\frac{240}{729}$
(B) $\frac{192}{729}$
(C) $\frac{256}{729}$
(D) $\frac{496}{729}$

Correct Answer: (C) $\frac{256}{729}$

Solution:

Concept:

- If success is twice as likely as failure, their probabilities obey $p = 2q$ and $p + q = 1$.
- In six independent repetitions with constant success probability p, the number of successes X is binomial: $P(X = r) = \binom{6}{r} p^r (1-p)^{6-r}$. At least five successes includes both five and six successes.

Step 1: Find the success and failure probabilities Write $p = P(\text{success})$ and $q = P(\text{failure})$. Since the outcomes are exhaustive and success occurs twice as often,

$$p = 2q, \qquad 2q + q = 1, \qquad q = \frac{1}{3}, \qquad p = \frac{2}{3}.$$

For independent trials, $X \sim \text{Bin}(6, 2/3)$.

Step 2: Calculate the probability of exactly five successes Five successes can occupy any five of the six positions; equivalently, the single failure can be in any one of six positions. All these arrangements are disjoint and have probability $p^5 q$, so

$$\begin{aligned} P(X = 5) &= \binom{6}{5} \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) \\ &= 6 \cdot \frac{32}{243} \cdot \frac{1}{3} = \frac{192}{729}. \end{aligned}$$

Step 3: Include all six successes and compare the choices Six successes have probability

$$P(X = 6) = \binom{6}{6} \left(\frac{2}{3}\right)^6 = \frac{64}{729}.$$

Add the mutually exclusive cases:

$$P(X \geq 5) = P(X = 5) + P(X = 6) = \frac{192 + 64}{729} = \frac{256}{729}.$$

The option $192/729$ omits the all-success case. The other two numerators, 240 and 496, do not equal $6 \cdot 2^5 + 2^6 = 256$.

Step 4: Conclusion The probability of at least five successes is $\boxed{256/729}$. Hence, the correct option is (C).

Quick Tip to Solve: For at least five successes in six trials, add only the two top binomial terms; retain the factor $\binom{6}{5} = 6$ for the possible failure positions.

29. Let C be a curve given by y(x) = 1 + $\sqrt{4x-3}, x > \frac{3}{4}$. If P is a point on C, such that the tangent at P has slope $\frac{2}{3}$, then a point through which the normal at P passes, is : [4 marks]

(A) (2, 3)

(B) (4, −3)

(C) (1, 7)

(D) (3, − 4),

Correct Answer: (C) (1, 7)

Solution:

Concept:

- For a differentiable curve, the derivative dy/dx is the slope of its tangent. Here the square root requires $x > 3/4$, so the derivative is finite and positive.
- The tangent and normal at the same point are perpendicular. If the tangent slope is $m \neq 0$, the normal slope is $-1/m$; its line through (x_1, y_1) is $y - y_1 = m_n(x - x_1)$.

Step 1: Locate the point with the specified tangent slope Differentiate the curve using the chain rule, keeping the inner derivative of $4x - 3$:

$$\frac{dy}{dx} = \frac{4}{2\sqrt{4x-3}} = \frac{2}{\sqrt{4x-3}}.$$

Set this equal to the given value:

$$\frac{2}{\sqrt{4x-3}} = \frac{2}{3} \quad \Longrightarrow \quad \sqrt{4x-3} = 3 \quad \Longrightarrow \quad x = 3.$$

The corresponding ordinate is $y = 1 + 3 = 4$, so the point of tangency is $P = (3, 4)$. It lies inside the stated domain.

Step 2: Form the normal and test the listed points The negative reciprocal of $2/3$ is $-3/2$. Therefore the normal is

$$y - 4 = -\frac{3}{2}(x - 3), \qquad 3x + 2y = 17.$$

For options (A), (B), (C) and (D), respectively, $3x + 2y$ equals $12, 6, 17, 1$. Only $(1, 7)$ satisfies the required equation. In particular, the point $(3, -4)$ has the correct abscissa of tangency but the wrong ordinate and is not on the normal.

Step 3: Conclusion The normal passes through $(1, 7)$. Hence, the correct option is (C).

Quick Tip to Solve: Find the point of tangency before forming the normal; then substitute each option in the single linear equation $3x + 2y = 17$.

30. The contrapositive of the following statement,

"If the side of a square doubles, then its area increases four times", is : [4 marks]

(A) If the side of a square is not doubled, then its area does not increase four times.

(B) If the area of a square increases four times, then its side is doubled.

(C) If the area of a square increases four times, then its side is not doubled.

(D) If the area of a square does not increase four times, then its side is not doubled.

Correct Answer: (D) If the area of a square does not increase four times, then its side is not doubled.

Solution:

Concept:

- For propositions p and q, an implication has the form $p \Rightarrow q$. Its contrapositive is $\neg q \Rightarrow \neg p$, obtained by reversing the order and negating both propositions.
- The contrapositive is logically equivalent to the original implication. The converse $q \Rightarrow p$ and the inverse $\neg p \Rightarrow \neg q$ are different logical constructions.

Step 1: Identify the hypothesis and the conclusion Let p denote "the side of a square doubles" and let q denote "its area increases four times". The given sentence is precisely $p \Rightarrow q$. Its hypothesis concerns the side, while its conclusion concerns the area. The question asks for a logical transformation of this sentence, so we must follow the implication rule rather than simply write another geometrically related assertion.

Step 2: Reverse the implication and negate both statements The negation of q is "the area of a square does not increase four times", and the negation of p is "its side is not doubled". Placing $\neg q$ first gives:

$$\neg q \Rightarrow \neg p.$$

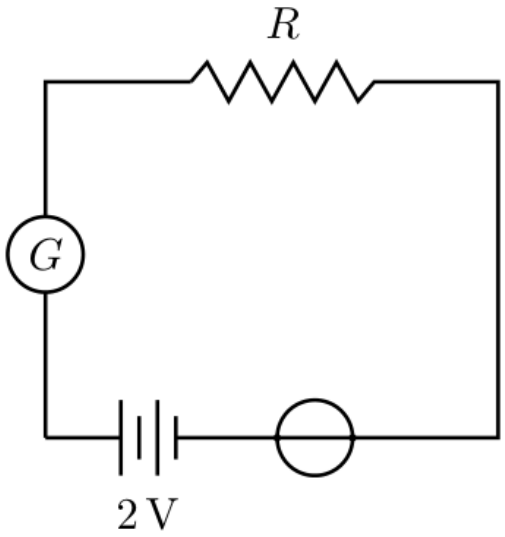
Thus the required statement reads: "If the area of a square does not increase four times, then its side is not doubled." Option (A) is the inverse, since it negates without reversing. Option (B) is the converse, since it reverses without negating. Option (C) reverses but negates only the side statement; that is not a contrapositive.

Step 3: Conclusion If the area of a square does not increase four times, then its side is not doubled. Hence, the correct option is (D).

Quick Tip to Solve: For a contrapositive, perform both operations: exchange the hypothesis and conclusion, then negate each. Reversing alone gives the converse.

Physics (120 marks)

31. A galvanometer has a 50 division scale. Battery has no internal resistance. It is found that there is deflection of 40 divisions when R = 2400 Ω. Deflection becomes 20 divisions when resistance taken from resistance box is 4900 Ω. Then we can conclude : [4 marks]



(A) Resistance of galvanometer is 200 Ω

(B) Full scale deflection current is 2 mA.

(C) Current sensitivity of galvanometer is 20 μA/division.

(D) Resistance required on R.B. for a deflection of 10 divisions is 9800 Ω.

Correct Answer: (C) Current sensitivity of galvanometer is 20 μA/division.

Solution:

Concept:

- For a moving coil galvanometer, the deflection n is proportional to the current I. With an ideal battery of emf E, a series resistance R, and galvanometer resistance G, the current is $I = E/(R+G)$.
- The current corresponding to one division is $k = I/n$. Once k and G are known, any desired deflection requires $R = E/(nk) - G$. The circuit supplies $E = 2\,\text{V}$.

Step 1: Determine the galvanometer resistance from the ratio of deflections The two deflections obey

$$\frac{40}{20} = \frac{I_1}{I_2} = \frac{4900 + G}{2400 + G}.$$

Therefore $4800 + 2G = 4900 + G$, giving $G = 100\,\Omega$. In particular, option (A), which states $200\,\Omega$, is incorrect. The ratio uses total series resistance, including G, rather than just the resistance box value.

Step 2: Calculate the current per division and check the other claims With $R = 2400\,\Omega$, the current is

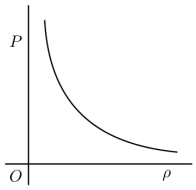
$$I_1 = \frac{2}{2400 + 100} = 0.8\,\text{mA}, \qquad k = \frac{0.8\,\text{mA}}{40} = 20\,\mu\text{A/division}.$$

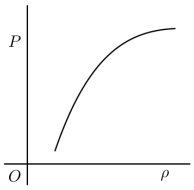
For the full 50-division scale, $I_{\text{fs}} = 50k = 1.0\,\text{mA}$, so option (B) is false. For 10 divisions, the current must be $0.20\,\text{mA}$, requiring a total resistance $2/(0.20 \times 10^{-3}) = 10000\,\Omega$. The resistance box must therefore supply $10000 - 100 = 9900\,\Omega$, not $9800\,\Omega$. Strictly, $20\,\mu\text{A/division}$ is the current figure of merit; deflection sensitivity is its reciprocal, $0.05\,\text{division}/\mu\text{A}$. The wording of (C) uses current sensitivity for that current-per-division value.

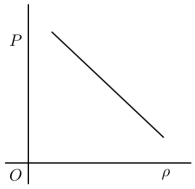
Step 3: Conclusion The current per scale division is $20\,\mu\text{A/division}$, which is the quantity stated in option (C). Hence, the correct option is (C).

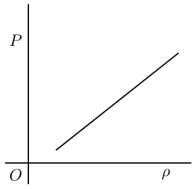
Quick Tip to Solve: Use the deflection ratio to find G before using the battery voltage; when setting a new deflection, subtract G from the total required resistance.

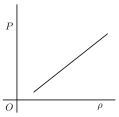
32. Which of the following shows the correct relationship between the pressure 'P' and density ρ of an ideal gas at constant temperature ? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- The ideal gas equation is $PV = nRT$. For a gas of molar mass M, the mass is $m = nM$ and the mass density is $\rho = m/V$.
- At constant temperature and fixed gas composition, R, T, and M are constants. Thus the dependence of pressure on density is determined by the positive constant RT/M.

Step 1: Express the gas equation in terms of density Divide the ideal gas equation by volume and use $n = m/M$:

$$P = \frac{nRT}{V} = \frac{mRT}{MV} = \frac{RT}{M}\rho.$$

This is a direct proportionality between pressure and mass density. Equivalently, kinetic theory gives $P = \rho v_{\text{rms}}^2/3$, and v_{rms}^2 is constant at fixed temperature for a given gas. Both descriptions lead to the same dependence, so pressure increases when density increases.

Step 2: Identify the graph with the correct slope and intercept Writing $P = k\rho$ with $k = RT/M > 0$ shows that the graph is a straight line of positive slope. Its extrapolation passes through the origin:

reducing the amount of gas per unit volume towards zero reduces the pressure towards zero. Option (D) is the rising straight line through the origin. Option (A) decreases nonlinearly, whereas option (B) rises with changing slope. Option (C) decreases linearly. None of these three has the required direct proportionality. In particular, the inverse pressure-volume relation at fixed gas amount must not be mistaken for an inverse pressure-density relation; increasing density corresponds to decreasing volume.

Step 3: Conclusion At fixed temperature, $P = (RT/M)\rho$, so the correct graph is the straight line through the origin with positive slope. Hence, the correct option is (D).

Quick Tip to Solve: Replace n/V by ρ/M in $PV = nRT$; this immediately shows both the positive slope and the zero intercept of the pressure-density graph.

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33. Within a spherical charge distribution of charge density ρ(r), N equipotential surfaces of potential V_0, $V_0 + \Delta V$, $V_0 + 2\Delta V$, ... $V_0 + N\Delta V$ ($\Delta V > 0$), are drawn and have increasing radii r_0, r_1, r_2,.....r_N, respectively. If the difference in the radii of the surfaces is constant for all values of V_0 and ΔV then : [4 marks]

(A) ρ (r) α r

(B) ρ (r) = constant

(C) ρ (r) $\alpha \frac{1}{r}$

(D) ρ (r) $\alpha \frac{1}{r^2}$

Correct Answer: (C) ρ (r) $\alpha \frac{1}{r}$

Solution:

Concept:

- For spherical symmetry, the electric field is radial and $E_r = -dV/dr$. Equal potential increments occurring at equal radial increments imply a constant potential gradient over the region considered.
- Gauss's law in differential form is $\rho(r) = \varepsilon_0 r^{-2}\, d(r^2 E_r)/dr$ for a radial field. This links the local volume charge density to the radial variation of the field.

Step 1: Use the equal spacing of equipotential surfaces The statement holds for all choices of V_0 and ΔV, so this is not merely a special finite sequence. Equal increments of potential correspond to equal increments of radius throughout the region. Consequently the potential is affine in radius:

$$V(r) = ar + b, \qquad E_r = -\frac{dV}{dr} = -a.$$

Since the potentials increase with the radii, $a > 0$ and the field points inward. The magnitude of the radial field is constant; the direction changes from point to point because it is radial.

Step 2: Recover the charge density with spherical Gauss law Insert the constant radial field into the spherical divergence formula:

$$\rho(r) = \frac{\varepsilon_0}{r^2}\frac{d}{dr}(-ar^2) = -\frac{2\varepsilon_0 a}{r}.$$

Thus the required radial dependence is $1/r$, with a negative coefficient for the stated outward

increase in potential. A constant charge density would instead give an interior field proportional to r, inconsistent with equal potential spacing. Densities proportional to r or $1/r^2$ likewise do not produce this constant radial field over an interval.

Step 3: Conclusion The charge density has the dependence $\rho(r) \propto 1/r$, for $r > 0$ in the region described. Hence, the correct option is (C).

Quick Tip to Solve: Equal radial spacing for every equal potential increment means constant dV/dr; then use the spherical factor r^2 in Gauss's law rather than a Cartesian derivative.

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34. Consider an electromagnetic wave propagating in vacuum. Choose the correct statement :

[4 marks]

(A) For an electromagnetic wave propagating in +x direction the electric field is $\vec{E} = \frac{1}{\sqrt{2}} E_{yz}\,(x,t)\,(\hat{y} - \hat{z})$ and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}} B_{yz}\,(x,t)\,(\hat{y} + \hat{z})$

(B) For an electromagnetic wave propagating in +x direction the electric field is $\vec{E} = \frac{1}{\sqrt{2}} E_{yz}\,(y,z,t)\,(\hat{y} + \hat{z})$ and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}} B_{yz}\,(y,z,t)\,(\hat{y} + \hat{z})$

(C) For an electromagnetic wave propagating in + y direction the electric field is $\overrightarrow{E} = \frac{1}{\sqrt{2}} E_{yz}\,(x,t)\,\widehat{y}$ and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}} B_{yz}\,(x,t)\,\widehat{z}$

(D) For an electromagnetic wave propagating in + y direction the electric field is $\overrightarrow{E} = \frac{1}{\sqrt{2}} E_{yz}\,(x,t)\,\widehat{z}$ and the magnetic field is $\overrightarrow{B} = \frac{1}{\sqrt{2}} B_{z}\,(x,t)\,\widehat{y}$

Correct Answer: (A) For an electromagnetic wave propagating in +x direction the electric field is $\vec{E} = \frac{1}{\sqrt{2}} E_{yz}\,(x,t)\,(\hat{y} - \hat{z})$ and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}} B_{yz}\,(x,t)\,(\hat{y} + \hat{z})$

Solution:

Concept:

- A plane electromagnetic wave in vacuum is transverse: $\vec{E}$, $\vec{B}$, and the propagation direction are mutually perpendicular. The fields are in phase and their magnitudes obey $E = cB$.
- The energy flow and propagation direction are along $\vec{E} \times \vec{B}$. A wave travelling along the x-axis has its phase dependence on x and t, with both fields in the transverse yz-plane.

Step 1: Check the geometry and direction in option A In option (A), the electric and magnetic directions are $(\hat{y} - \hat{z})/\sqrt{2}$ and $(\hat{y} + \hat{z})/\sqrt{2}$, respectively. Their dot product is zero, and both are perpendicular to $\hat{x}$. Their vector product is

$$\frac{1}{2}(\hat{y} - \hat{z}) \times (\hat{y} + \hat{z}) = \frac{1}{2}(\hat{x} + \hat{x}) = \hat{x}.$$

Thus the stated directions give energy flow along $+x$. The dependence on (x,t) is also appropriate. The scalar field amplitudes are related by $E_{yz} = cB_{yz}$, as required for a vacuum wave.

Step 2: Rule out the other field arrangements In option (B), both fields have the same direction

$\hat{y} + \hat{z}$, so they are parallel and their cross product vanishes. Its stated spatial dependence is also inconsistent with a plane wave travelling along x. In option (C), the electric field is along the claimed $+y$ propagation direction, violating transversality; moreover $\hat{y} \times \hat{z} = \hat{x}$, not $\hat{y}$. In option (D), the magnetic field is longitudinal for the claimed direction and $\hat{z} \times \hat{y} = -\hat{x}$. Hence neither of those can describe the stated wave.

Step 3: Conclusion Option (A) has mutually perpendicular transverse fields and $\vec{E} \times \vec{B}$ along the specified propagation direction. Hence, the correct option is (A).

Quick Tip to Solve: Check transversality first, then take the ordered cross product $\vec{E} \times \vec{B}$. Reversing that order reverses the propagation direction.

35. Two stars are 10 light years away from the earth. They are seen through a telescope of objective diameter 30 cm. The wavelength of light is 600 nm. To see the stars just resolved by the telescope, the minimum distance between them should be (1 light year = 9.46 $\times$ 10^{15} m) of the order of : [4 marks]

(A) 10^6 km

(B) 10^8 km

(C) 10^{11} km

(D) 10^{10} km

Correct Answer: (B) 10^8 km

Solution:

Concept:

- For a circular telescope objective, the Rayleigh criterion gives the minimum resolvable angular separation $\theta_{\min} = 1.22\lambda/D$, where λ is the wavelength and D is the objective diameter.
- For two distant stars at approximately the same distance L, their small transverse separation is $s = L\theta$, with θ expressed in radians. Convert every length to compatible units before evaluating.

Step 1: Calculate the diffraction limited angular separation The given wavelength and aperture diameter are

$$\lambda = 600 \times 10^{-9}\,\text{m} = 6.00 \times 10^{-7}\,\text{m}, \qquad D = 30\,\text{cm} = 0.30\,\text{m}.$$

Therefore the limiting angular separation is

$$\theta_{\min} = \frac{1.22(6.00 \times 10^{-7})}{0.30} = 2.44 \times 10^{-6}\,\text{rad}.$$

This angle is very small, so the linear small-angle relation is appropriate.

Step 2: Convert the angular limit to the separation of the stars The distance from Earth is $L = 10(9.46 \times 10^{15}) = 9.46 \times 10^{16}\,\text{m}$. Hence the minimum transverse separation is

$$s_{\min} = L\theta_{\min} = (9.46 \times 10^{16})(2.44 \times 10^{-6}) = 2.31 \times 10^{11}\,\text{m} = 2.31 \times 10^{8}\,\text{km}.$$

Thus the listed order is $10^8\,\text{km}$. The 10^{11} option corresponds approximately to the numerical scale in metres, not kilometres; the 10^6 and 10^{10} kilometre choices are each two powers of ten away from the

required scale.

Step 3: Conclusion The minimum separation is approximately $2.3 \times 10^8\,\text{km}$, of the order of $10^8\,\text{km}$. Hence, the correct option is (B).

Quick Tip to Solve: Use $s = 1.22\lambda L/D$ directly, but postpone conversion from metres to kilometres until the final step to avoid a factor of 10^3 error.

36. A photoelectric surface is illuminated successively by monochromatic light of wavelengths λ and $\frac{\lambda}{2}$. If the maximum kinetic energy of the emitted photoelectrons in the second case is 3 times that in the first case, the work function of the surface is : [4 marks]

(A) $\frac{hc}{3\lambda}$
(B) $\frac{hc}{2\lambda}$
(C) $\frac{hc}{\lambda}$
(D) $\frac{3\,hc}{\lambda}$

Correct Answer: (B) $\frac{hc}{2\lambda}$

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where ϕ is the work function of the surface and hc/λ is the incident photon energy.
- The work function is a property of the surface, so it stays unchanged when the illuminating wavelength changes. Halving the wavelength doubles photon energy; it does not directly double the electron's kinetic energy.

Step 1: Express both maximum kinetic energies Put $E = hc/\lambda$. For the first illumination, the photon gives energy E, of which ϕ releases the electron and the remainder becomes its maximum kinetic energy. In the second illumination the photon energy is $hc/(\lambda/2) = 2E$. Therefore,

$$K_1 = E - \phi, \qquad K_2 = 2E - \phi.$$

The same energy cost ϕ is subtracted in each case. Comparing photon energies alone would miss this fixed cost.

Step 2: Apply the given kinetic-energy ratio The stated condition is $K_2 = 3K_1$, so

$$2E - \phi = 3(E - \phi)$$

and hence

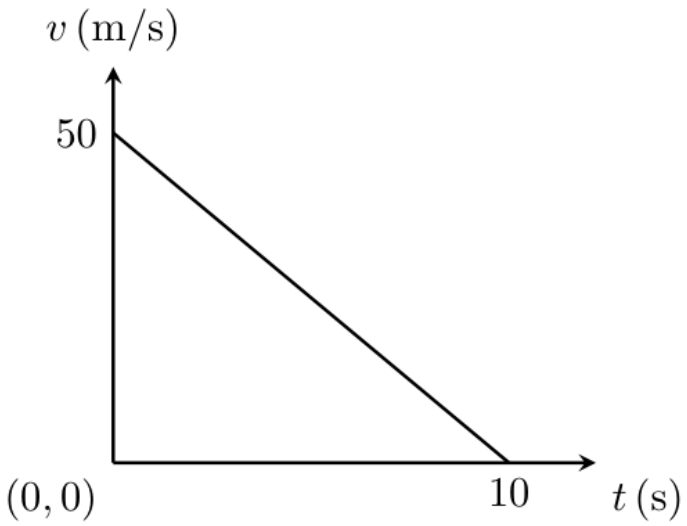
$$2\phi = E, \qquad \phi = \frac{E}{2} = \frac{hc}{2\lambda}.$$

As a check, this gives $K_1 = E/2$ and $K_2 = 3E/2$, both positive and in the required ratio. Option (A) would give a ratio $5/2$, while option (C) makes the first kinetic energy zero. Option (D) would prevent photoemission for both wavelengths, since its work function exceeds both photon energies.

Step 3: Conclusion The work function is $\boxed{\phi = hc/(2\lambda)}$. Hence, the correct option is (B).

Quick Tip to Solve: First write the photon energies as E and 2E, subtract the same work function from each, and only then apply the kinetic-energy ratio.

37. Velocity-time graph for a body of mass 10 kg is shown in figure. Work-done on the body in first two seconds of the motion is : [4 marks]



(A) 12000 J

(B) − 12000 J

(C) − 4500 J

(D) − 9300 J

Correct Answer: (C) − 4500 J

Solution:

Concept:

- The net work done on a body equals its change in kinetic energy: $W_{\text{net}} = \frac{1}{2}m(v_f^2 - v_i^2)$. Only the speeds at the beginning and end of the interval are needed.
- The slope of a velocity-time graph gives acceleration. The area under that graph gives displacement, not work, so the graph must first be used to read the required velocities.

Step 1: Read the velocities from the graph The straight line joins $(t, v) = (0, 50\,\text{m s}^{-1})$ and $(10\,\text{s}, 0)$. Its constant slope is

$$a = \frac{0 - 50}{10 - 0} = -5\,\text{m s}^{-2}.$$

Thus, during the first two seconds,

$$v_i = 50\,\text{m s}^{-1}, \qquad v_f = 50 - 5(2) = 40\,\text{m s}^{-1}.$$

The velocity is still positive, but its magnitude decreases, so the kinetic energy and the net work must decrease and be negative, respectively.

Step 2: Calculate the change in kinetic energy Substituting the given mass $m = 10\,\text{kg}$ and the two velocities gives

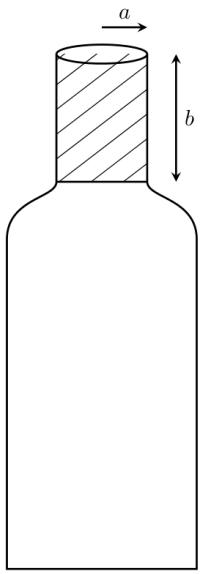
$$W_{\text{net}} = \frac{1}{2}(10)(40^2 - 50^2) = 5(1600 - 2500) = -4500\,\text{J}.$$

The initial and final energies are 12500 J and 8000 J, so the difference is indeed negative. As a second check, the net force is $ma = -50\,\text{N}$, while the displacement in this interval is $\frac{1}{2}(50 + 40)2 = 90\,\text{m}$; their product is again $-4500\,\text{J}$. The positive alternative has the wrong sign, and the other negative alternatives do not equal this energy change.

Step 3: Conclusion The work done on the body in its first two seconds is $\boxed{-4500\,\text{J}}$. Hence, the correct option is (C).

Quick Tip to Solve: For work from a velocity-time graph, find the endpoint speeds and subtract their squared values; the graph's area alone is displacement.

38. A bottle has an opening of radius a and length b. A cork of length b and radius (a + Δa) where (Δa $<<$ a) is compressed to fit into the opening completely (See figure). If the bulk modulus of cork is *B* and frictional coefficient between the bottle and cork is μ then the force needed to push the cork into the bottle is : [4 marks]



(A) (π μ B b) Δa

(B) (2π μ B b) Δa

(C) (π μ B b) a

(D) (4π μ B b) Δa

Correct Answer: (D) (4π μ B b) Δa

Solution:

Concept:

- The bulk modulus relates compressive pressure to the fractional decrease in volume: $p = B\,|\Delta V|/V$. Here the small radial compression gives the pressure exerted between the cork and the neck.
- Limiting friction is μN. For contact over a curved cylindrical surface, the total normal load is pressure multiplied by the contact area $2\pi ab$; it is not the vector resultant of the radial normal forces.

Step 1: Find the pressure caused by compression The cork radius changes from $a + \Delta a$ to a, with its stated length b. Its initial and compressed volumes are

$$V_i = \pi(a + \Delta a)^2 b, \qquad V_f = \pi a^2 b.$$

Since $\Delta a \ll a$, retain only first-order terms:

$$|\Delta V| \simeq 2\pi ab\,\Delta a, \qquad \frac{|\Delta V|}{V_i} \simeq \frac{2\Delta a}{a}.$$

Hence the contact pressure in the given bulk-modulus model is $p \simeq 2B\Delta a/a$. The factor of two arises because cross-sectional area is proportional to radius squared.

Step 2: Sum the friction over the neck surface When the cork occupies the complete neck length, its curved contact area is $S = 2\pi ab$. Each surface element contributes friction $\mu p\,dS$ opposing the axial push, so all those frictional contributions add:

$$F = \mu pS = \mu\left(\frac{2B\Delta a}{a}\right)(2\pi ab) = 4\pi\mu Bb\,\Delta a.$$

Although opposite radial normal forces cancel as vectors, their friction forces act in the same axial direction and do not cancel. Option (B) misses a factor of two in volume strain, and option (A) also loses a factor in contact area. Option (C) remains nonzero when Δa tends to zero, which is inconsistent with vanishing compression and friction.

Step 3: Conclusion The required force, to first order in $\Delta a/a$, is $\boxed{F = 4\pi\mu Bb\,\Delta a}$. Hence, the correct option is (D).

Quick Tip to Solve: Use twice the fractional radius decrease for cylindrical volume strain, then multiply the resulting pressure by the curved contact area and friction coefficient.

39. A particle of mass m is acted upon by a force F given by the empirical law F $= \frac{R}{t^2} v\,(t)$. If this law is to be tested experimentally by observing the motion starting from rest, the best way is to plot : [4 marks]

(A) v(t) against t^2

(B) log v(t) against $\frac{1}{t^2}$

(C) log v(t) against t

(D) log v(t) against $\frac{1}{t}$

Correct Answer: (D) log v(t) against $\frac{1}{t}$

Solution:

Concept:

- Newton's second law relates the net force to velocity by $F = m\,dv/dt$. A force proportional to velocity gives an equation that can be separated into a velocity factor and a time factor.
- An empirical law is conveniently tested by transforming it into a straight-line relation. A plot is linear when the dependent variable equals a constant plus a constant multiple of the horizontal variable.

Step 1: Separate and integrate the equation of motion Substitution of the given force gives, for a nonzero measured velocity at $t > 0$,

$$m\frac{dv}{dt} = \frac{R}{t^2}v, \qquad \frac{dv}{v} = \frac{R}{m}\frac{dt}{t^2}.$$

Integrating yields

$$\ln v = C - \frac{R}{mt},$$

or, in a dimensionally explicit measured form using a reference observation (t_0, v_0),

$$\ln\left(\frac{v(t)}{v_0}\right) = -\frac{R}{m}\left(\frac{1}{t} - \frac{1}{t_0}\right).$$

Thus the logarithm is linear in reciprocal time, not time or reciprocal time squared.

Step 2: Identify the appropriate straight-line plot Plotting $\ln v$ against $1/t$, using a fixed velocity unit for the numerical logarithm, gives slope $-R/m$ and constant intercept. Changing the logarithm base changes the slope by a constant factor and preserves linearity. By contrast, $v = v_* \exp[-R/(mt)]$ is not linear in t^2; neither $\log v$ against t nor against $1/t^2$ has the required constant slope. The force law is singular at $t = 0$, so the measurement and logarithm must use positive times and nonzero speeds. For positive R, the nontrivial solution has $v \to 0$ as $t \to 0^+$, consistent with the stated start from rest as a limiting condition. One must not substitute $t = 0$ directly into the empirical law or take the logarithm of zero.

Step 3: Conclusion The straight-line test is a plot of $\boxed{\log v(t) \text{ against } 1/t}$. Hence, the correct option is (D).

Quick Tip to Solve: Separate dv/v from the time factor before choosing axes: integrating t to the power -2 produces -1/t, while integrating dv/v produces a logarithm.

40. A neutron moving with a speed 'v' makes a head on collision with a stationary hydrogen atom in ground state. The minimum kinetic energy of the neutron for which inelastic collision will take place is : [4 marks]

(A) 10.2 eV

(B) 16.8 eV

(C) 12.1 eV

(D) 20.4 eV

Correct Answer: (D) 20.4 eV

Solution:

Concept:

- The first excitation of ground-state hydrogen is from $n = 1$ to $n = 2$. With $E_n = -13.6/n^2\,\text{eV}$, its minimum internal-energy increase is $\Delta E = 10.2\,\text{eV}$.
- Momentum conservation prevents the complete incident kinetic energy from becoming excitation energy. At threshold the outgoing particles have zero relative velocity, so the final kinetic energy is the minimum consistent with their total momentum.

Step 1: Calculate the smallest excitation energy The smallest internal-energy change is the first excitation:

$$\Delta E = E_2 - E_1 = -\frac{13.6}{4} - (-13.6) = 10.2\,\text{eV}.$$

Higher excitations and ionisation require more energy. Treat the neutron and hydrogen atom as equal masses m, appropriate to the options.

Step 2: Account for the unavoidable recoil energy Let the incident neutron speed be v. The initial momentum is mv, and the initial kinetic energy is $K = \frac{1}{2}mv^2$. At threshold the two outgoing particles share a common velocity u, giving

$$mv = 2mu, \qquad u = v/2.$$

Their combined translational kinetic energy is therefore

$$K_f = 2\left(\frac{1}{2}mu^2\right) = \frac{mv^2}{4} = \frac{K}{2}.$$

Only the remaining $K/2$ is available for excitation. Energy conservation consequently requires

$$\frac{K}{2} = 10.2\,\text{eV}, \qquad K = 20.4\,\text{eV}.$$

Option (A) ignores recoil. For options (B) and (C), the available energy $K/2$ is below 10.2 eV, so excitation cannot occur.

Step 3: Conclusion The minimum incident kinetic energy is $\boxed{20.4\,\text{eV}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a projectile striking an equal stationary mass, only half its laboratory kinetic energy is available for internal excitation at threshold.

41. A, B, C and D are four different physical quantities having different dimensions. None of them is dimensionless. But we know that the equation AD = C ln (BD) holds true. Then which of the combination is not a meaningful quantity ? [4 marks]

(A) $A^2 - B^2C^2$

(B) $\frac{(A-C)}{D}$

(C) $\frac{A}{B} - C$

(D) $\frac{C}{BD} - \frac{AD^2}{C}$

Correct Answer: (B) $\frac{(A-C)}{D}$

Solution:

Concept:

- The argument of a logarithm must be dimensionless. Thus BD has no dimensions, and the equation $AD = C\ln(BD)$ requires $[A][D] = [C]$.
- Physical quantities can be added or subtracted only if their dimensions match. Products and quotients may involve different dimensions, but every subtraction inside an expression must itself be meaningful.

Step 1: Derive the dimensional relations Write $[B] = X$ and $[C] = Y$. Since $[B][D] = 1$, the logarithm condition gives $[D] = X^{-1}$. The equality of the dimensions of the two sides then gives

$$[A] = \frac{[C]}{[D]} = XY.$$

Hence the four dimensions are XY, X, Y, X^{-1}, respectively. The problem explicitly says they are different and none is dimensionless. These facts must be used when deciding whether two terms can be subtracted.

Step 2: Test the two meaningful combinations In option (A),

$$[A^2] = X^2Y^2 = [B^2C^2],$$

so that subtraction is dimensionally allowed. In option (C), $[A/B] = (XY)/X = Y = [C]$, making $A/B - C$ meaningful too. Different symbols need not prevent a subtraction if the complete terms have identical dimensions.

Step 3: Check the two invalid subtractions Option (B) contains $A - C$. The given dimensions of A and C are different, so this numerator is already meaningless; dividing it by D cannot repair it. For option (D),

$$\left[\frac{C}{BD}\right] = Y = [C], \qquad \left[\frac{AD^2}{C}\right] = \frac{(XY)X^{-2}}{Y} = X^{-1} = [D].$$

Since C and D have different dimensions, this subtraction is also invalid. Therefore the printed question admits two choices, in agreement with its multiple-accepted-option key, rather than a unique invalid expression.

Step 4: Conclusion Both (B) and (D) are not meaningful quantities. Taking (B) as the selected answer, the correct option is (B); the official answer key accepts both (B) and (D).

Quick Tip to Solve: Make the logarithm's argument dimensionless first, then compare dimensions separately on each side of every addition or subtraction.

42. A particle of mass M is moving in a circle of fixed radius R in such a way that its centripetal acceleration at time t is given by n^2 R t^2 where n is a constant. The power delivered to the particle by the force acting on it, is : [4 marks]

(A) M n^2 R^2 t

(B) M n R^2 t

(C) M n R^2 t^2

(D) $\frac{1}{2}$ M n^2 R^2 t^2

Correct Answer: (A) M n^2 R^2 t

Solution:

Concept:

- In circular motion the centripetal acceleration is $a_c = v^2/R$. It determines the speed even when the motion is nonuniform and a separate tangential acceleration is present.
- The net instantaneous power is $P = \mathbf{F} \cdot \mathbf{v} = dK/dt$, where $K = \frac{1}{2}Mv^2$. A radial force is perpendicular to the tangential velocity and does no work; the tangential component changes the speed and kinetic energy.

Step 1: Obtain the speed and kinetic energy Equate the usual centripetal acceleration to the expression given in the question:

$$\frac{v^2}{R} = n^2Rt^2, \qquad v^2 = n^2R^2t^2.$$

The radius is fixed, so its time derivative is zero. The constant n also remains unchanged. Therefore the kinetic energy at time t is

$$K(t) = \frac{1}{2}Mn^2R^2t^2.$$

This expression already includes the variation in speed; it does not assume that the total acceleration is only radial.

Step 2: Differentiate the kinetic energy The power supplied by the force acting on the particle is the time rate of change of this kinetic energy:

$$P = \frac{dK}{dt} = \frac{1}{2}Mn^2R^2(2t) = Mn^2R^2t.$$

For positive n and time, the speed is $v = nRt$, giving tangential acceleration $a_t = nR$ and the independent check $P = (Ma_t)v = Mn^2R^2t$. The centripetal force contributes zero to this dot product. Option (B) omits one factor of n, while option (C) has both an incorrect power of n and an extra time factor. Option (D) is the kinetic energy itself, not its derivative, and therefore has energy dimensions rather than power dimensions. At the starting instant the power is zero because the speed is zero even though tangential acceleration need not vanish.

Step 3: Conclusion The instantaneous power is $\boxed{P = Mn^2R^2t}$. Hence, the correct option is (A).

Quick Tip to Solve: Use centripetal acceleration to find v squared, then differentiate one-half M v squared; multiplying the radial force by speed would give the wrong power.

43. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute(rpm) to ensure proper mixing is close to :

(Take the radius of the drum to be 1.25 m and its axle to be horizontal) : [4 marks]

(A) 0.4

(B) 1.3

(C) 8.0

(D) 27.0

Correct Answer: (D) 27.0

Solution:

Concept:

- A particle on the inside of a rotating drum needs centripetal acceleration $\omega^2 r$, directed towards the horizontal axle. Here ω is angular speed and r is drum radius.
- At the highest point, gravity and the inward normal force both act towards the centre. Loss of contact begins when the required normal force becomes zero: $\omega_c^2 r = g$.

Step 1: Identify the limiting condition If the drum turns sufficiently fast, each ingredient can remain in contact with its inner wall throughout a full revolution. At the top, the radial equation for a small mass m is

$$mg + N = m\omega^2 r.$$

Thus $N = m(\omega^2 r - g)$. The drum wall can push inward, but cannot pull the ingredient outward to maintain contact. For material to fall away from the top and tumble, the speed should be below the limiting value at which $N = 0$. This falling and tumbling is what enables proper mixing.

Step 2: Calculate the limiting angular speed Using $r = 1.25\,\mathrm{m}$ and $g \simeq 9.8\,\mathrm{m\,s^{-2}}$,

$$\omega_c = \sqrt{\frac{g}{r}} = \sqrt{\frac{9.8}{1.25}} \simeq 2.80\,\mathrm{rad\,s^{-1}}.$$

The mass cancels, so cement, sand and stones have the same ideal threshold in this model.

Step 3: Convert angular speed into rpm One revolution corresponds to 2π radians, and one minute contains 60 seconds. Therefore

$$n_c = \frac{60\omega_c}{2\pi} = \frac{60(2.80)}{2\pi} \simeq 26.7\,\mathrm{rpm}.$$

The alternatives 0.4, 1.3 and 8.0 rpm are all below this threshold; none is the maximum speed asked for. In particular, the frequency $\omega_c/(2\pi) \simeq 0.45\,\mathrm{s^{-1}}$ must not be mistaken for rpm.

Step 4: Conclusion The limiting rotational speed is approximately $27\,\mathrm{rpm}$; operate just below it to allow tumbling. Hence, the correct option is (D).

Quick Tip to Solve: At the top of an inside circular path, set the normal force to zero to find the sticking threshold, then multiply revolutions per second by 60.

44. An astronaut of mass m is working on a satellite orbiting the earth at a distance h from the earth's surface. The radius of the earth is R, while its mass is M. The gravitational pull F_G on the astronaut is : [4 marks]

(A) Zero since astronaut feels weightless

(B) $0 < F_G < \frac{GMm}{R^2}$

(C) $\frac{GMm}{(R+h)^2} < F_G < \frac{GMm}{R^2}$

(D) $F_G = \frac{GMm}{(R+h)^2}$

Correct Answer: (D) $F_G = \frac{GMm}{(R+h)^2}$

Solution:

Concept:

- Newton's gravitational law gives $F_G = GMm/r^2$ for an astronaut of mass m a distance r from the centre of a spherical Earth of mass M.
- Apparent weight is a support force, whereas gravitational pull is a real force. An orbiting astronaut can have zero apparent weight while gravity provides centripetal acceleration.

Step 1: Measure distance from the centre of Earth The satellite is at height h above Earth's surface. Since Earth's radius is R, the centre-to-astronaut distance is

$$r = R + h.$$

The height alone is not the distance to use in the inverse-square law. Treat the astronaut as at the satellite's orbital radius, as assumed in the question.

Step 2: Apply the gravitational law Substitute this distance directly:

$$F_G = \frac{GMm}{(R+h)^2}.$$

This inward pull makes the astronaut accelerate with the satellite. Both are continually falling around Earth. The absence of a support force on the astronaut explains the sensation of weightlessness; it does not remove Earth's attraction. Thus option (A) incorrectly identifies weightlessness with zero gravity.

Step 3: Check all four options For the stated orbit above the surface, $h > 0$, so $(R+h)^2 > R^2$. Consequently

$$0 < \frac{GMm}{(R+h)^2} < \frac{GMm}{R^2}.$$

Option (B) is therefore also a true inequality, although it does not specify the force exactly. Option (C) is false because it requires the actual pull to be strictly greater than the very expression to which it is equal. Option (D) gives the exact value, and is the most direct response. This also explains why more than one option can be accepted without any contradiction in the physics.

Step 4: Conclusion The exact gravitational pull is $F_G = GMm/(R+h)^2$. Hence, the correct option is (D). The official answer key accepts both B and D, since the inequality in B is also a valid bound for $h > 0$.

Quick Tip to Solve: For an orbiting object distinguish zero apparent weight from zero gravity, and always insert the distance from Earth's centre into the inverse-square law.

45. The resistance of an electrical toaster has a temperature dependence given by R(T) = R_0 [1 + α(T − T_0)] in its range of operation. At T_0 = 300 K, R = 100 Ω and at T = 500 K, R = 120 Ω. The toaster is connected to a voltage source at 200 V and its temperature is raised at a constant rate from 300 to 500 K in 30 s. The total work done in raising the temperature is : [4 marks]

(A) 400 $\ln \frac{1.5}{1.3}$ J

(B) 200 $\ln \frac{2}{3}$ J

(C) 60000 $\ln \frac{6}{5}$ J

(D) 300 J

Correct Answer: (C) 60000 $\ln \frac{6}{5}$ J

Solution:

Concept:

- At fixed applied voltage V, instantaneous electrical power is $P(t) = V^2/R(t)$. The supplied work is $W = \int P(t)\,dt$, not generally final power multiplied by time.
- If temperature rises uniformly with time and resistance is linear in temperature, resistance is also linear in time. Its two endpoint values determine this function completely.

Step 1: Determine the temperature coefficient At $T_0 = 300\,\mathrm{K}$, $R_0 = 100\,\Omega$. At $500\,\mathrm{K}$, the stated law gives

$$120 = 100[1 + \alpha(500 - 300)], \qquad \alpha = 1.0 \times 10^{-3}\,\mathrm{K}^{-1}.$$

The temperature rises by $200\,\mathrm{K}$ in $30\,\mathrm{s}$, hence

$$T(t) - 300 = \frac{200}{30}t.$$

Here t is in seconds after switching on the prescribed heating interval.

Step 2: Express resistance and integrate power Substituting the temperature law,

$$R(t) = 100\left(1 + \frac{t}{150}\right)\,\Omega.$$

Thus the instantaneous power is

$$P(t) = \frac{200^2}{100(1 + t/150)} = \frac{400}{1 + t/150}\,\mathrm{W}.$$

The total electrical work over the interval is

$$W = 400\int_0^{30} \frac{dt}{1 + t/150} = 60000\,[\ln(1 + t/150)]_0^{30} = 60000\ln\frac{6}{5}\,\mathrm{J}.$$

The logarithm contains a dimensionless ratio, as required.

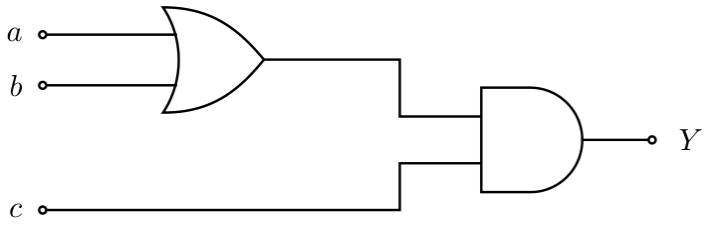
Step 3: Check the magnitude and the alternatives Numerically the work is about 1.094×10^4 J. The initial and final powers are $400\,\mathrm{W}$ and $333.3\,\mathrm{W}$; hence the work must lie between 10000 and $12000\,\mathrm{J}$. Option (A) is much too small, (B) is negative despite a passive resistor absorbing energy, and (D) is also much too small. The stated temperature history is used directly; no separate heat capacity is

needed to compute electrical work.

Step 4: Conclusion The total work done is $60000\ln(6/5)$ J, approximately 10.94 kJ. Hence, the correct option is (C).

Quick Tip to Solve: When voltage is fixed and resistance changes linearly with time, integrate its reciprocal; use endpoint powers to check that the energy has a sensible magnitude.

46. To get an output of 1 from the circuit shown in figure the input must be : [4 marks]



(A) a = 0, b = 1, c = 0

(B) a = 1, b = 0, c = 0

(C) a = 1, b = 0, c = 1

(D) a = 0, b = 0, c = 1

Correct Answer: (C) a = 1, b = 0, c = 1

Solution:

Concept:

- An OR gate produces 1 when at least one of its inputs is 1: $x = a \vee b$. An AND gate produces 1 only when both of its inputs are 1.
- In the shown circuit the OR output and the separate input c feed the AND gate. Thus $Y = (a \vee b) \wedge c$. The open input/output terminals do not denote logical inversion.

Step 1: Read the order of the two gates The first gate, connected to a and b, has the curved OR-gate outline. Denote its output by x. The second gate has a flat input face and rounded output face, identifying it as an AND gate. Its upper input receives x, while its lower input receives c directly. Therefore

$$x = a \vee b, \qquad Y = x \wedge c.$$

No NOT operation occurs on either path; terminal circles outside the gates are simply connection points.

Step 2: State the condition for a high final output To obtain $Y = 1$, the AND gate requires $c = 1$ and $x = 1$ simultaneously. The latter condition means that a and b cannot both be zero. A high c alone is insufficient. Conversely, a high OR output alone is insufficient when $c = 0$. This separates the roles of the two stages without constructing an unnecessary eight-row truth table.

Step 3: Test each listed input combination For (A), $(a, b, c) = (0, 1, 0)$, so $x = 1$ but $Y = 1 \wedge 0 = 0$. For (B), $(1, 0, 0)$ again gives $x = 1$, $Y = 0$. For (C), $(1, 0, 1)$ gives $x = 1$, $Y = 1 \wedge 1 = 1$. For (D), $(0, 0, 1)$

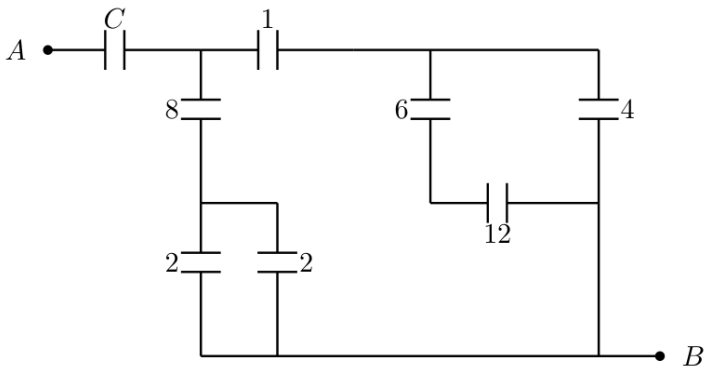
gives $x = 0$ and hence $Y = 0 \wedge 1 = 0$. Only the third listed combination satisfies both requirements at the final gate.

Step 4: Conclusion The required inputs are $a = 1,\ b = 0,\ c = 1$. Hence, the correct option is (C).

Quick Tip to Solve: Work backwards from the final AND gate: require both its inputs high, then apply the OR condition to the earlier pair of inputs.

47. Figure shows a network of capacitors where the numbers indicates capacitances in micro Farad. The value of capacitance C if the equivalent capacitance between point A and B is to be 1 μF is :

[4 marks]



(A) $\frac{31}{23}\,\mu F$

(B) $\frac{32}{23}\,\mu F$

(C) $\frac{33}{23}\,\mu F$

(D) $\frac{34}{23}\,\mu F$

Correct Answer: (B) $\frac{32}{23}\,\mu F$

Solution:

Concept:

- Capacitances in parallel add: $C_p = C_1 + C_2$. For two in series, $C_s = C_1C_2/(C_1 + C_2)$. Apply these rules only after identifying the actual common junctions.
- All given capacitances are in μF. The unknown capacitor next to terminal A is in series with the entire network to its right.

Step 1: Reduce the left lower branch Call the junction immediately after the unknown capacitor X. From X, the downward branch contains $8\,\mu$F, followed by two $2\,\mu$F capacitors in parallel to the bottom wire connected to B. The parallel pair gives

$$C_{2,2} = 2 + 2 = 4\,\mu\text{F}.$$

Its series combination with 8 gives

$$C_L = \frac{8 \times 4}{8 + 4} = \frac{8}{3}\,\mu\text{F}.$$

The two 2-valued capacitors share both junctions, so they are not in series.

Step 2: Reduce the right-hand branch The $6\,\mu\text{F}$ and $12\,\mu\text{F}$ capacitors are in series between the upper right junction and B, giving

$$C_{6,12} = \frac{6 \times 12}{6 + 12} = 4\,\mu\text{F}.$$

This is parallel with the rightmost $4\,\mu\text{F}$ capacitor, giving $8\,\mu\text{F}$. The result is in series with the $1\,\mu\text{F}$ capacitor on the upper wire:

$$C_R = \frac{1 \times 8}{1 + 8} = \frac{8}{9}\,\mu\text{F}.$$

Both reduced branches connect X to B, so

$$C_{XB} = C_L + C_R = \frac{8}{3} + \frac{8}{9} = \frac{32}{9}\,\mu\text{F}.$$

Step 3: Use the required terminal capacitance Let c denote the numerical value of the unknown capacitance in microfarads. Its series combination with $32/9$ must equal 1:

$$1 = \frac{c(32/9)}{c + 32/9}.$$

Multiplication gives $c + 32/9 = 32c/9$, hence

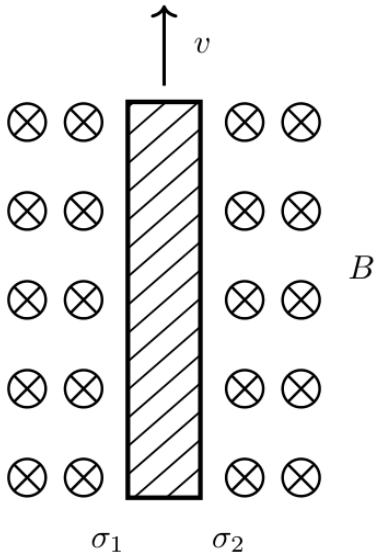
$$c = \frac{32}{23}.$$

The three other listed numerators would not give the specified terminal capacitance when substituted into this equation. The resulting equivalent is smaller than each of the two series capacitances, which is a useful consistency check.

Step 4: Conclusion The unknown capacitance is $C = (32/23)\,\mu\text{F}$. Hence, the correct option is (B).

Quick Tip to Solve: Collapse the two lower parallel capacitors and the right-hand 6–12 series pair first; only then combine the two branches after the unknown capacitor.

48. Consider a thin metallic sheet perpendicular to the plane of the paper moving with speed *'v'* in a uniform magnetic field B going into the plane of the paper (See figure). If charge densities σ_1 and σ_2 are induced on the left and right surfaces, respectively, of the sheet then (ignore fringe effects) : [4 marks]



(A) $\sigma_1 = \in_0 \upsilon$ B, $\sigma_2 = -\in_0 \upsilon$ B

(B) $\sigma_1 = \frac{\in_0 \upsilon B}{2}$, $\sigma_2 = \frac{-\in_0 \upsilon B}{2}$

(C) $\sigma_1 = \sigma_2 = \in_0 \upsilon B$

(D) $\sigma_1 = \frac{-\in_0 \upsilon B}{2}$, $\sigma_2 = \frac{\in_0 \upsilon B}{2}$,

Correct Answer: (A) $\sigma_1 = \in_0 \upsilon$ B, $\sigma_2 = -\in_0 \upsilon$ B

Solution:

Concept:

- A charge moving with the metal feels magnetic force $q\mathbf{v} \times \mathbf{B}$. Charges redistribute until the electric and magnetic forces cancel: $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$.
- Two large oppositely charged faces with densities $+\sigma$ and $-\sigma$ produce an electric field of magnitude σ/ε_0 between them, ignoring edge effects.

Step 1: Determine which face becomes positive Choose rightward as positive x, upward as positive y, and out of the paper as positive z. The figure has $\mathbf{v} = v\hat{\mathbf{y}}$ and $\mathbf{B} = -B\hat{\mathbf{z}}$. Therefore

$$\mathbf{v} \times \mathbf{B} = -vB\hat{\mathbf{x}}.$$

Positive charge is driven left; mobile electrons are driven right. The left face acquires positive surface charge, $\sigma_1 > 0$, and the right face negative surface charge, $\sigma_2 < 0$. The electric field created inside the sheet consequently points right.

Step 2: Balance the forces after separation In the steady state the transverse charge separation stops growing when the net Lorentz force vanishes. Thus

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} = vB\hat{\mathbf{x}}, \qquad E = vB.$$

The initially neutral sheet merely redistributes its charge, so the two parallel faces have equal and opposite densities: $\sigma_2 = -\sigma_1$. No external source adds a net charge to the sheet.

Step 3: Relate field to the face densities Each individual large charged face contributes field magnitude $\sigma/(2\varepsilon_0)$. Between the positive left face and negative right face these fields point in the same direction and add, giving

$$E = \frac{\sigma_1}{\varepsilon_0}.$$

Combining with $E = vB$,

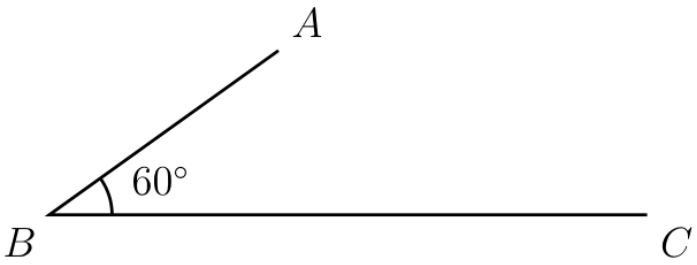
$$\sigma_1 = \varepsilon_0 vB, \qquad \sigma_2 = -\varepsilon_0 vB.$$

The half-sized densities in (B) and (D) would not generate the required balancing field. Option (D) also reverses the charge signs. The equal positive densities in (C) violate charge conservation and the required electric-field direction.

Step 4: Conclusion The left and right surface densities are $\sigma_1 = \varepsilon_0 vB$ and $\sigma_2 = -\varepsilon_0 vB$, respectively. Hence, the correct option is (A).

Quick Tip to Solve: Use the cross product first to fix the signs, then remember that the fields of the two opposite faces add inside the metal, eliminating the factor of two.

49. In the figure shown ABC is a uniform wire. If centre of mass of wire lies vertically below point A, then $\frac{BC}{AB}$ is close to : [4 marks]



(A) 1.85

(B) 1.37

(C) 1.5

(D) 3

Correct Answer: (B) 1.37

Solution:

Concept:

- A straight uniform wire segment has its centre of mass at its midpoint. Its mass is its length multiplied by the common linear mass density λ.
- The horizontal coordinate of the combined centre of mass is $x_{\text{cm}} = \sum m_i x_i / \sum m_i$. A point vertically below A has the same horizontal coordinate as A.

Step 1: Assign lengths and midpoint coordinates The figure has a wire segment AB inclined at 60°

to the horizontal segment BC, both starting at B. Let $AB = a$, $BC = b$, and place B at the origin with BC along positive x. Then

$$x_A = a\cos 60^\circ = \frac{a}{2}.$$

The midpoint of AB has $x_1 = a/4$, and that of BC has $x_2 = b/2$. Their respective masses are λa and λb; the longer piece must not be assigned the same mass as the shorter piece.

Step 2: Impose the vertical alignment condition The horizontal coordinate of the total centre of mass is

$$x_{\text{cm}} = \frac{\lambda a(a/4) + \lambda b(b/2)}{\lambda(a+b)} = \frac{a^2/4 + b^2/2}{a+b}.$$

The centre of mass lies vertically below A, so set this equal to $a/2$:

$$\frac{a^2}{4} + \frac{b^2}{2} = \frac{a(a+b)}{2}, \qquad 2b^2 - 2ab - a^2 = 0.$$

The common density has cancelled, so its numerical value is unnecessary.

Step 3: Solve for the positive length ratio Write $r = b/a$. Dividing by $a^2 > 0$ yields

$$2r^2 - 2r - 1 = 0, \qquad r = \frac{1 \pm \sqrt{3}}{2}.$$

Only the positive root represents a ratio of lengths:

$$r = \frac{1+\sqrt{3}}{2} \simeq 1.366.$$

Also the vertical centre coordinate is $\lambda a(\sqrt{3}a/4)/[\lambda(a+b)] = \sqrt{3}a^2/[4(a+b)]$, which is positive and smaller than $y_A = \sqrt{3}a/2$, confirming that the point is below A, rather than above it. The other listed positive ratios do not satisfy the horizontal moment equation.

Step 4: Conclusion The required ratio is $BC/AB = (1+\sqrt{3})/2 \simeq 1.37$. Hence, the correct option is (B).

Quick Tip to Solve: For a bent uniform wire replace each straight segment by its midpoint mass, weighting that mass by segment length before taking moments.

50. A thin 1 m long rod has a radius of 5 mm. A force of 50 πkN is applied at one end to determine its Young's modulus. Assume that the force is exactly known. If the least count in the measurement of all lengths is 0.01 mm, which of the following statements is false ? [4 marks]

(A) $\frac{\Delta\gamma}{\gamma}$ gets minimum contribution from the uncertainty in the length.

(B) The figure of merit is the largest for the length of the rod.

(C) The maximum value of γ that can be determined is 2×10^{14} N/m^2

(D) $\frac{\Delta\gamma}{\gamma}$ gets its maximum contribution from the uncertainty in strain

Correct Answer: The key selects C using a two-reading uncertainty convention; a direct extension least count gives the value in C, so the measurement protocol is needed.

Solution:

Concept:

- For a rod of original length L, radius r and extension x, Young's modulus is $\gamma = FL/(\pi r^2 x)$. The force is exactly known, so only length measurements contribute uncertainty.
- The limiting fractional uncertainty is $\Delta\gamma/\gamma = \Delta L/L + 2\Delta r/r + \Delta x/x$. An extension obtained by subtracting two independent length readings has the sum of their limiting absolute uncertainties.

Step 1: Compare the initial length and radius measurements The common least count is $d = 0.01\,\text{mm}$. Since $L = 1000\,\text{mm}$ and $r = 5\,\text{mm}$,

$$\frac{d}{L} = 10^{-5}, \qquad \frac{2d}{r} = 0.004.$$

The initial length therefore makes the smallest contribution. Its ratio of measured length to least count is the largest, so it has the best figure of merit. Thus (A) and (B) describe the relatively precise original-length measurement correctly.

Step 2: Include both readings used to find the extension The elongation is the difference of the final and initial lengths. Each reading has limiting uncertainty d, so $\Delta x = 2d = 0.02\,\text{mm}$. If one uses this combined uncertainty as the smallest reliably measurable extension,

$$\gamma_{\max} = \frac{(50\pi \times 10^3)(1)}{\pi(5 \times 10^{-3})^2(2 \times 10^{-5})} = 1.0 \times 10^{14}\,\text{N m}^{-2}.$$

Using only one reading's least count instead would give $2 \times 10^{14}\,\text{N m}^{-2}$, the value in (C), which is also a possible interpretation if extension is read directly with that least count. Measurement resolution and limiting uncertainty are distinct, and the question does not specify this convention. Near the resolution limit, the uncertainty in the small extension, and hence in strain, dominates; this is the situation described by (D).

Step 3: Conclusion The key selects (C), consistently with treating the two-reading uncertainty as the limiting extension. With a directly resolved extension of one least count, however, the stated ceiling $2 \times 10^{14}\,\text{N m}^{-2}$ is obtained exactly, so (C) is not demonstrably false without the measurement convention. Hence, the correct option is (C) only under the two-reading uncertainty convention; the literal wording is ambiguous.

Quick Tip to Solve: When extension is measured as a difference, include the uncertainties of both length readings before estimating the largest measurable modulus.

51. A conducting metal circular-wire-loop of radius r is placed perpendicular to a magnetic field which varies with time as

$B = B_0 e^{\frac{-t}{r}}$, where B_0 and τ are constants, at time t = 0. If the resistance of the loop is R then the heat generated in the loop after a long time ($t \to \infty$) is : [4 marks]

(A) $\frac{\pi^2 r^4 B_0^4}{2\tau R}$

(B) $\frac{\pi^2 r^4 B_0^2}{2\tau R}$

(C) $\frac{\pi^2 r^4 B_0^2 R}{\tau}$

(D) $\frac{\pi^2 r^4 B_0^2}{\tau R}$

Correct Answer: The exponent prints the radius r in place of a time scale; (B) follows when the declared constant tau is the decay time.

Solution:

Concept:

- Faraday's law gives the induced emf $\mathcal{E} = -d\Phi/dt$, where $\Phi = \pi r^2 B$ for a uniform field perpendicular to the circular loop. Joule power is $P = \mathcal{E}^2/R$.
- An exponential decay must have a dimensionless exponent. Denote its decay time by T, so $B = B_0 e^{-t/T}$. The given statement names τ as a constant and all relevant choices use τ, although its displayed exponent contains r.

Step 1: Differentiate the decaying magnetic flux Keeping the time constant explicit avoids confusing the radius with a time. The flux and emf magnitude are

$$\Phi(t) = \pi r^2 B_0 e^{-t/T}, \qquad |\mathcal{E}(t)| = \frac{\pi r^2 B_0}{T} e^{-t/T}.$$

The current is $I = \mathcal{E}/R$. Reversing the field direction reverses the current but cannot change the heat because the power contains its square.

Step 2: Integrate the power over the entire decay Starting at $t = 0$, the total heat is

$$Q = \int_0^\infty \frac{\mathcal{E}^2}{R}\, dt = \frac{\pi^2 r^4 B_0^2}{RT^2} \int_0^\infty e^{-2t/T}\, dt.$$

For $T > 0$, the integral is $T/2$, and hence

$$Q = \frac{\pi^2 r^4 B_0^2}{2RT}.$$

The square of the exponential is essential: omitting it loses the factor $1/2$. The field enters quadratically, excluding (A); the resistance belongs in the denominator, excluding (C); (D) misses the factor of two.

Step 3: Address the symbol inconsistency explicitly For the time constant $T = \tau$ identified by the sentence and the options, the result is precisely (B). If the displayed r in the exponent is read literally as the loop radius, t/r is dimensional and no physical decay law is specified. The literal printed equation therefore does not justify any option; (B) is the result for its dimensionally consistent time-constant reading.

Step 4: Conclusion The time-constant reading gives $Q = \pi^2 r^4 B_0^2/(2\tau R)$. Hence, the correct option is

(B) under that reading. The displayed use of radius r instead of a decay time makes a strictly literal reading inconsistent.

Quick Tip to Solve: Square the induced emf before integrating; exponential Joule heating decays twice as fast as the magnetic field.

52. In an engine the piston undergoes vertical simple harmonic motion with amplitude 7 cm. A washer rests on top of the piston and moves with it. The motor speed is slowly increased. The frequency of the piston at which the washer no longer stays in contact with the piston, is close to : [4 marks]

(A) 0.1 Hz

(B) 1.2 Hz

(C) 0.7 Hz

(D) 1.9 Hz

Correct Answer: (D) 1.9 Hz

Solution:

Concept:

- A body executing vertical SHM has acceleration $a_y = -\omega^2 y$ when upward displacement y is measured from equilibrium. The maximum downward acceleration is $\omega^2 A$, at the upper extreme.
- A washer can be pushed upward by a piston but cannot be pulled downward by it. Contact requires a nonnegative normal reaction N, and separation begins when the required N first becomes zero.

Step 1: Find the reaction required during the motion Let the washer have mass m. With upward taken positive, Newton's second law while contact lasts is

$$N - mg = ma_y = -m\omega^2 y.$$

Consequently $N = m(g - \omega^2 y)$. The smallest reaction occurs at $y = A$, not at the mean position or the lowest point. As motor speed increases, the acceleration needed at the top grows until the piston would have to pull the washer downward to keep it moving with it.

Step 2: Impose the limiting condition and calculate the frequency At first separation,

$$N_{\min} = 0 \quad \Rightarrow \quad \omega^2 A = g.$$

The amplitude is $A = 7\,\text{cm} = 0.070\,\text{m}$. Therefore

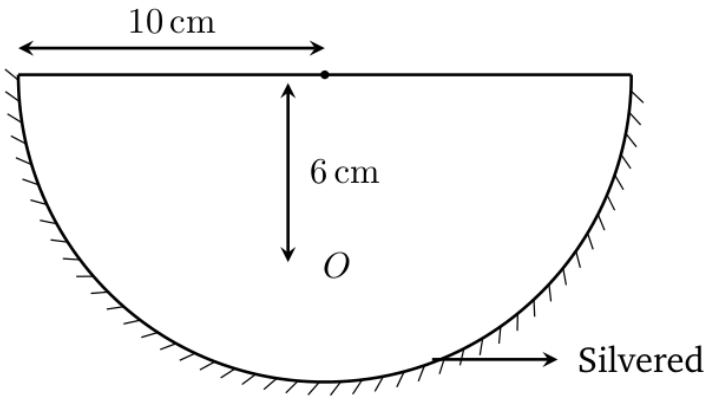
$$\omega = \sqrt{\frac{9.8}{0.070}} = \sqrt{140} = 11.83\,\text{rad}\,\text{s}^{-1}, \qquad f = \frac{\omega}{2\pi} = 1.88\,\text{Hz}.$$

The angular frequency must be divided by 2π to obtain hertz. The lower suggested frequencies all have $(2\pi f)^2 A < g$, so gravity is sufficient to provide the downward acceleration while the reaction stays positive.

Step 3: Conclusion The limiting frequency is approximately $1.9\,\text{Hz}$. Above this threshold the washer loses contact near the upper extreme. Hence, the correct option is (D).

Quick Tip to Solve: For an object riding on a vertical SHM platform, test the topmost point and set the largest downward acceleration equal to g.

53. A hemispherical glass body of radius 10 cm and refractive index 1.5 is silvered on its curved surface. A small air bubble is 6 cm below the flat surface inside it along the axis. The position of the image of the air bubble made by the mirror is seen : [4 marks]



(A) 14 cm below flat surface

(B) 30 cm below flat surface

(C) 20 cm below flat surface

(D) 16 cm below flat surface

Correct Answer: (C) 20 cm below flat surface

Solution:

Concept:

- The silvered curved surface acts as a concave spherical mirror for light travelling inside the glass. Its focal length is $f = R/2$; the mirror law uses geometrical distances in the same medium on both sides of reflection.
- An image viewed from air through a plane glass surface has apparent depth $d_{\text{app}} = d/\mu$, where d is its depth in glass and $\mu = 1.5$. Apply this only after finding the mirror image.

Step 1: Locate the bubble relative to the mirror pole The centre of the hemispherical surface lies in the flat face and the mirror pole is $10\,\text{cm}$ below that face. The bubble is $6\,\text{cm}$ below the face, so its distance in front of the mirror pole is $10-6=4\,\text{cm}$. The mirror focal length is $5\,\text{cm}$. Thus the bubble is inside the focal distance and the mirror produces an enlarged virtual image behind its curved surface.

Step 2: Calculate the mirror image before refraction Using positive object distance in front of a concave mirror and positive real-image distance,

$$\frac{1}{5}=\frac{1}{4}+\frac{1}{v} \quad \Rightarrow \quad \frac{1}{v}=-\frac{1}{20}, \qquad v=-20\,\text{cm}.$$

The minus sign means the image is behind the mirror, $20\,\text{cm}$ deeper than its pole. Its depth measured from the flat face is therefore $10 + 20 = 30\,\text{cm}$, rather than $20\,\text{cm}$.

Step 3: Account for observation through the flat surface The reflected rays travel upward through glass and refract into air at the flat face. Their apparent origin has depth

$$d_{\text{app}} = \frac{30}{1.5} = 20\,\text{cm}.$$

The value $30\,\text{cm}$ in (B) is the intermediate mirror-image depth before this refraction. Using the bubble's apparent depth before applying the mirror formula would combine distances from different media incorrectly and cannot produce the correct final image position.

Step 4: Conclusion The image is seen $20\,\text{cm}$ below the flat surface. Hence, the correct option is (C).

Quick Tip to Solve: Find the mirror image using distances inside glass, convert its position to depth below the plane face, and then divide by the refractive index.

54. To determine refractive index of glass slab using a travelling microscope, minimum number of readings required are : [4 marks]

(A) Two

(B) Three

(C) Four

(D) Five

Correct Answer: (B) Three

Solution:

Concept:

- For a plane parallel glass slab viewed nearly normally from air, its refractive index is $\mu = t/t'$, the ratio of real thickness to apparent thickness. The microscope provides vertical positions, so both thicknesses must be obtained from differences of readings.
- One common reference is a mark underneath the slab. Three focus settings determine the uncovered mark, the mark viewed through the slab, and the upper surface of the slab.

Step 1: Identify the three distinct focus positions First focus the microscope on the mark without the slab and record its vertical position z_1. Place the slab over the mark, focus on the mark's raised image and record z_2. Finally focus on a small speck or fine powder at the upper surface and record z_3. These are three independent readings. The speck identifies the surface sharply; the microscope is not required to focus on two transparent surfaces simultaneously.

Step 2: Form both thicknesses from those readings Choose a vertical coordinate increasing upward. The real position of the mark is z_1, its apparent raised position is z_2, and the top is z_3. Consequently,

$$t = z_3 - z_1, \qquad t' = z_3 - z_2, \qquad \mu = \frac{z_3 - z_1}{z_3 - z_2}.$$

The top-surface reading is shared by the two differences. Two readings provide only one independent

difference and are insufficient to determine both real and apparent thickness by this method. A fourth or fifth reading may be useful for repetition or checking, but is not part of the minimum set. The ratio is dimensionless and exceeds unity because the apparent depth is smaller than the real depth.

Step 3: Conclusion Three focus readings suffice to determine the refractive index of the glass slab. Hence, the correct option is (B).

Quick Tip to Solve: Count distinct focus positions rather than the number of differences: real and apparent thickness share the same top-surface reading.

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55. A fighter plane of length 20 m, wing span (distance from tip of one wing to the tip of the other wing) of 15 m and height 5 m is flying towards east over Delhi. Its speed is 240 ms^{-1}. The earth's magnetic field over Delhi is 5 $\times 10^{-5}$ T with the declination angle ~ 0° and dip of θ such that sin θ = 2/3 . If the voltage developed is V_B between the lower and upper side of the plane and V_W between the tips of the wings then V_B and V_W are close to : [4 marks]

(A) V_B = 45 m_V; V_W = 120 m_V with right side of pilot at higher voltage

(B) V_B = 45 m_V; V_W = 120 m_V with left side of pilot at higher voltage

(C) V_B = 40 m_V; V_W = 135 m_V with right side of pilot at high voltage

(D) V_B = 40 m_V; V_W = 135 m_V with left side of pilot at higher voltage

Correct Answer: (B) V_B = 45 m_V; V_W = 120 m_V with left side of pilot at higher voltage

Solution:

Concept:

- Charges in an aircraft moving with velocity $\mathbf{v}$ experience $q\mathbf{v} \times \mathbf{B}$. In electrostatic equilibrium the voltage increases in the direction of $\mathbf{v} \times \mathbf{B}$, and its magnitude across a transverse dimension ℓ is $vB_\perp \ell$.
- With zero declination in Delhi, the horizontal magnetic field points north and its vertical component points down. The given dip implies $B_V = B\sin\theta = 2B/3$ and $B_H = B\cos\theta = \sqrt{5}B/3$.

Step 1: Resolve the field and obtain the vertical voltage The speed is $240\,\mathrm{m\,s^{-1}}$ and $B = 5 \times 10^{-5}\,\mathrm{T}$. The northward component is

$$B_H = 5 \times 10^{-5}\frac{\sqrt{5}}{3} = 3.73 \times 10^{-5}\,\mathrm{T}.$$

For eastward motion, east crossed with north points upward. The voltage magnitude between the bottom and top, separated by $5\,\mathrm{m}$, is therefore

$$V_B = vB_H(5) = 0.0447\,\mathrm{V} \simeq 45\,\mathrm{mV}.$$

The upper side is at higher potential.

Step 2: Find the wing voltage and its polarity The downward component is $B_V = 3.33 \times 10^{-5}\,\mathrm{T}$. East crossed with down points north. Thus positive charges move toward the north wing, which is the

pilot's left when facing east. Across the $15\,\mathrm{m}$ wing span,

$$V_W = vB_V(15) = 240\left(\frac{2}{3}\right)(5\times10^{-5})(15) = 0.120\,\mathrm{V} = 120\,\mathrm{mV}.$$

The $20\,\mathrm{m}$ fuselage length lies along the velocity, so it contributes neither of these transverse voltage differences. Choices (C) and (D) have incorrect magnitudes, while (A) reverses the wing polarity.

Step 3: Conclusion The magnitudes are $V_B \simeq 45\,\mathrm{mV}$ and $V_W \simeq 120\,\mathrm{mV}$, with the pilot's left wing at higher potential. Hence, the correct option is (B).

Quick Tip to Solve: Use the horizontal field for the vertical aircraft dimension and the vertical field for the wing span; check polarity with the cross product.

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56. A Carnot freezer takes heat from water at 0°C inside it and rejects it to the room at a temperature of 27°C. The latent heat of ice is 336×10^3 J kg^{-1}. If 5 kg of water at 0°C is converted into ice at 0°C by the freezer, then the energy consumed by the freezer is close to : [4 marks]

(A) 1.67×10^5 J

(B) 1.68×10^6 J

(C) 1.51×10^5 J

(D) 1.71×10^7 J

Correct Answer: (A) 1.67×10^5 J

Solution:

Concept:

- A Carnot refrigerator operating between cold temperature T_C and hot temperature T_H has coefficient of performance $\mathrm{COP} = Q_C/W = T_C/(T_H - T_C)$. Both temperatures must be expressed in kelvin.
- Freezing water already at $0^\circ\mathrm{C}$ removes latent heat $Q_C = mL$, without a sensible-heat term. This heat removed is larger than the external work consumed by an efficient refrigerator.

Step 1: Calculate the heat removed while the water freezes The mass is $5\,\mathrm{kg}$ and the specific latent heat is $336\times10^3\,\mathrm{J\,kg^{-1}}$. Thus

$$Q_C = mL = 5(336\times10^3) = 1.680\times10^6\,\mathrm{J}.$$

The water and ice are both at the freezing temperature, so the calculation requires no specific heat capacity. This heat is not itself the energy consumed electrically: it is transferred from the water to the room, with additional work supplied to the refrigerator.

Step 2: Use the Carnot coefficient of performance to obtain work The cold and hot temperatures are $T_C = 273\,\mathrm{K}$ and $T_H = 300\,\mathrm{K}$. Therefore

$$\mathrm{COP} = \frac{273}{300-273} = \frac{273}{27} \simeq 10.11,$$

$$W = \frac{Q_C}{\text{COP}} = 1.680 \times 10^6 \frac{27}{273} \simeq 1.66 \times 10^5\,\text{J}.$$

Using $273.15\,\text{K}$ changes this only slightly. Option (B) gives the latent heat instead of the input work; (D) would imply a coefficient of performance smaller than one despite this small temperature lift. Option (C) is outside the result of the stated temperatures and latent heat. Conservation of energy gives $Q_H = Q_C + W$, consistently larger than the extracted heat.

Step 3: Conclusion The energy consumed is about $1.66 \times 10^5\,\text{J}$, closest to $1.67 \times 10^5\,\text{J}$. Hence, the correct option is (A).

Quick Tip to Solve: For a freezer, first compute latent heat removed, then multiply by the temperature lift divided by the cold absolute temperature.

57. A modulated signal $C_m(t)$ has the form $C_m(t) = 30 \sin 300\,\pi t + 10 (\cos 200\,\pi t - \cos 400\,\pi t)$. The carrier frequency c, the modulating frequency (message frequency) f_ω, and the modulation index μ are respectively given by : [4 marks]

(A) $f_c = 200$ Hz; $f_\omega = 50$ Hz; $\mu = \frac{1}{2}$
(B) $f_c = 150$ Hz; $f_\omega = 50$ Hz; $\mu = \frac{2}{3}$
(C) $f_c = 150$ Hz; $f_\omega = 30$ Hz; $\mu = \frac{1}{3}$
(D) $f_c = 200$ Hz; $f_\omega = 30$ Hz; $\mu = \frac{1}{2}$

Correct Answer: (B) $f_c = 150$ Hz; $f_\omega = 50$ Hz; $\mu = \frac{2}{3}$

Solution:

Concept:

- An amplitude-modulated sinusoidal carrier can be written as $C_m(t) = A_c[1 + \mu \sin(\omega_m t)] \sin(\omega_c t)$, where A_c is carrier amplitude and μ is modulation index.
- The identity $2 \sin a \sin b = \cos(a - b) - \cos(a + b)$ identifies the sidebands at angular frequencies $\omega_c - \omega_m$ and $\omega_c + \omega_m$. Frequency is angular frequency divided by 2π.

Step 1: Identify the carrier and sideband frequencies The standalone carrier term is $30 \sin(300\pi t)$, so $\omega_c = 300\pi\,\text{rad}\,\text{s}^{-1}$. The two sidebands have angular frequencies 200π and $400\pi\,\text{rad}\,\text{s}^{-1}$, equally spaced about the carrier. Thus

$$f_c = \frac{300\pi}{2\pi} = 150\,\text{Hz}, \qquad f_m = \frac{400\pi - 300\pi}{2\pi} = 50\,\text{Hz}.$$

The difference of the two sideband frequencies is twice the message frequency, not the message frequency itself.

Step 2: Factor the signal and read the modulation index Applying the product identity gives

$$10[\cos(200\pi t) - \cos(400\pi t)] = 20 \sin(300\pi t) \sin(100\pi t).$$

Consequently,

$$C_m(t) = 30\left[1 + \frac{2}{3}\sin(100\pi t)\right]\sin(300\pi t).$$

Comparison with the standard AM form yields $\mu = 20/30 = 2/3$. Each sideband amplitude is $\mu A_c/2 = 10$, providing an independent check. Options (A) and (D) give the wrong carrier; (C) gives both the wrong message frequency and modulation index.

Step 3: Conclusion The carrier frequency is $150\,\text{Hz}$, message frequency is $50\,\text{Hz}$, and modulation index is $2/3$. Hence, the correct option is (B).

Quick Tip to Solve: In an AM spectrum, average the sideband frequencies to find the carrier; take half their separation for the message frequency, and use twice a sideband amplitude divided by carrier amplitude for the index.

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58. A toy-car, blowing its horn, is moving with a steady speed of 5 m/s, away from a wall. An observer, towards whom the toy car is moving, is able to hear 5 beats per second. If the velocity of sound in air is 340 m/s, the frequency of the horn of the toy car is close to : [4 marks]

(A) 680 Hz

(B) 510 Hz

(C) 340 Hz

(D) 170 Hz

Correct Answer: (D) 170 Hz

Solution:

Concept:

- For a stationary listener and a moving source, the received frequency is $f_0 v/(v-u)$ for approach and $f_0 v/(v+u)$ for recession, where v is sound speed and u is source speed.
- A stationary wall reflects sound without changing its frequency in the laboratory frame. Beats arise from the difference between the direct and reflected frequencies reaching the stationary observer.

Step 1: Calculate the direct and reflected frequencies The car moves towards the observer but away from the wall. The directly received frequency is therefore increased:

$$f_d = f_0\frac{340}{340-5} = f_0\frac{340}{335}.$$

The frequency incident on the wall is reduced because the source recedes from it. Since the wall and observer are stationary, no additional Doppler shift occurs on reflection:

$$f_r = f_0\frac{340}{340+5} = f_0\frac{340}{345}.$$

Using an extra moving-reflector factor would incorrectly treat the fixed wall as moving.

Step 2: Use the beat frequency to recover the horn frequency The faster oscillation is the direct wave,

so

$$5 = f_d - f_r = f_0\, 340 \left(\frac{1}{335} - \frac{1}{345} \right).$$

Solving gives

$$f_0 = \frac{5(335)(345)}{340(10)} = 169.963\ldots \text{ Hz} \approx 170 \text{ Hz}.$$

As a check, since $u \ll v$, the frequency difference is approximately $2f_0u/v$, which gives $f_0 \approx 5 \times 340/(2 \times 5) = 170$ Hz. The other frequencies, 340, 510 and 680 Hz, would produce approximately 10, 15 and 20 beats per second respectively.

Step 3: Conclusion The frequency of the horn is close to 170 Hz. Hence, the correct option is (D).

Quick Tip to Solve: For a source moving away from a fixed wall towards a stationary listener, compare one approach-shifted direct wave with one recession-shifted reflected wave; the wall adds no second shift.

59. The ratio (R) of output resistance r_0, and the input resistance ri in measurements of input and output characteristics of a transistor is typically in the range: [4 marks]

(A) R $\sim 10^2 - 10^3$

(B) R $\sim 1 - 10$

(C) R $\sim 0.1 - 0.01$

(D) R $\sim 0.1 - 1.0$

Correct Answer: (A) R $\sim 10^2 - 10^3$

Solution:

Concept:

- Transistor input and output resistances are small-signal slopes of characteristic curves, not simply the applied voltage divided by total current. In common-emitter measurements, $r_i = (\Delta V_{BE}/\Delta I_B)_{V_{CE}}$ and $r_o = (\Delta V_{CE}/\Delta I_C)_{I_B}$.
- The forward-biased input junction gives relatively small input resistance, whereas the nearly flat active-region output characteristic gives much larger output resistance. Thus the ratio $R = r_o/r_i$ is substantially greater than unity.

Step 1: Compare the slopes of the two characteristic curves For a small change of base-emitter voltage, the base current changes appreciably because the input junction is forward biased. Its resistance is typically of the order of a few hundred ohms. On the output side, at fixed base current, a substantial collector-emitter voltage change gives only a small collector-current change in the active region. The output resistance is consequently of the order of tens of kilohms or larger. These are differential resistances at the operating point, and their precise values depend on the transistor and bias.

Step 2: Form the resistance ratio and assess the choices Using representative characteristic slopes,

$$r_i \sim 10^2 \text{ to } 10^3\ \Omega, \qquad r_o \sim 10^5\ \Omega,$$

gives the textbook order-of-magnitude range

$$R = \frac{r_o}{r_i} \sim 10^2 \text{ to } 10^3.$$

For example, $r_i = 500\,\Omega$ and $r_o = 100\,\text{k}\Omega$ give $R = 200$. Options (C) and (D) reverse the usual large-output/small-input relationship. Option (B) allows only comparable or modestly different resistances and is inconsistent with this standard characteristic comparison.

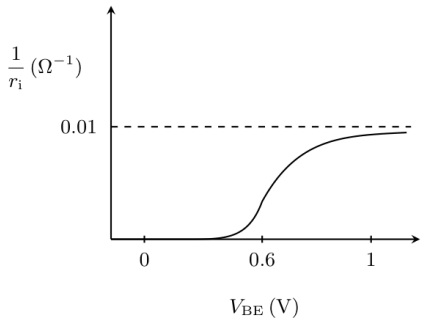
Step 3: Conclusion The standard transistor-characteristic estimate is $R \sim 10^2$ to 10^3. Hence, the correct option is (A). The supplied answer key lists the range 1 to 10; that conflicts with the usual input/output differential-resistance comparison above.

Quick Tip to Solve: Read resistance from voltage change divided by current change at the stated fixed parameter; a nearly horizontal output-current curve corresponds to a large output resistance, not a small one.

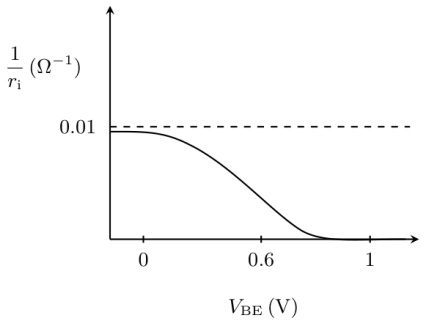
60. A realistic graph depicting the variation of the reciprocal of input resistance in an input characteristics measurement in a commonemitter transistor configuration is :

[4 marks]

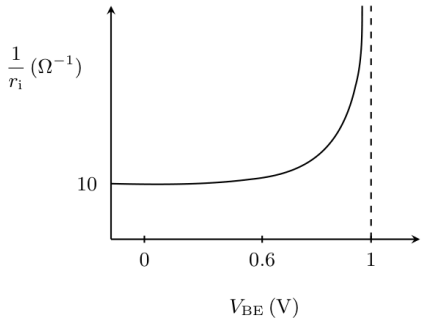
(A)



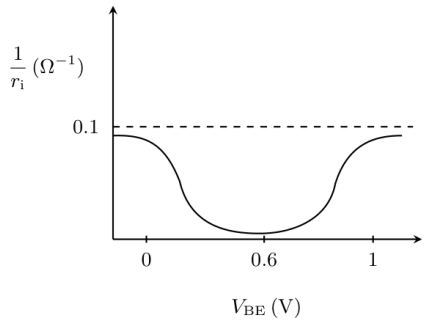
(B)



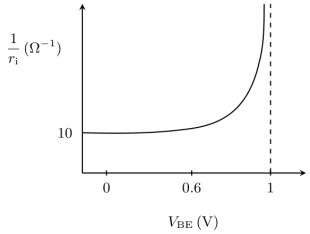
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- In common-emitter input characteristics, base current I_B is measured against base-emitter voltage V_{BE} at fixed collector-emitter voltage. The dynamic input resistance is $r_i = (\partial V_{BE}/\partial I_B)_{V_{CE}}$.
- The reciprocal input resistance is the differential conductance $1/r_i = (\partial I_B/\partial V_{BE})_{V_{CE}}$. A forward-biased base-emitter junction has an approximately exponential current-voltage characteristic.

Step 1: Relate reciprocal resistance to the slope of input current For the forward-biased base-emitter junction, a useful approximation is

$$I_B = I_{B0}\left[\exp\left(\frac{V_{BE}}{\eta V_T}\right) - 1\right],$$

where I_{B0} is a current scale, $V_T = kT/q$ is thermal voltage, and η is the ideality factor. At a fixed operating point, reciprocal dynamic resistance is the local slope of this curve. Differentiating with respect to the input voltage gives

$$\frac{1}{r_i} = \frac{I_{B0}}{\eta V_T}\exp\left(\frac{V_{BE}}{\eta V_T}\right).$$

Thus the conductance is small at low forward voltage and rises increasingly rapidly as the forward voltage increases.

Step 2: Choose the curve with increasing slope The required graph is therefore positive, increasing and upward-curving in the normal forward-input operating range. It has an initially shallow part followed by a sharp rise, which is shown by option (C). Option (A) approaches a plateau; that would require the differential conductance to saturate, contrary to the forward-junction exponential dependence in the stated measurement. Option (B) decreases with increasing forward bias, whereas conduction should become stronger. Option (D) first decreases and then rises, producing a minimum that is not characteristic of the forward-biased input junction. The plotted quantity is conductance, so confusing it with resistance would reverse the physical trend.

Step 3: Conclusion The reciprocal input resistance increases approximately exponentially with base-emitter voltage. Hence, the correct option is (C).

Quick Tip to Solve: For a reciprocal-resistance graph, first rewrite the vertical quantity as the slope $\mathrm{d}I_B/\mathrm{d}V_{BE}$. The slope of a forward diode characteristic becomes steeper as its voltage rises.

Chemistry (120 marks)

61. The correct statement about the synthesis of erythritol ($C(CH_2OH)4$) used in the preparation of PETN is : [4 marks]

(A) The synthesis requires four aldol condensations between methanol and ethanol.

(B) The synthesis requires two aldol condensations and two Cannizzaro reactions.

(C) The synthesis requires three aldol condensations and one Cannizzaro reaction.

(D) Alpha hydrogens of ethanol and methanol are involved in this reaction.

Correct Answer: (C) The synthesis requires three aldol condensations and one Cannizzaro reaction.

Solution:

Concept:

- An aldehyde containing alpha hydrogens can form an enolate and add to formaldehyde in an aldol-type hydroxymethylation. Each such addition replaces one alpha hydrogen by a CH_2OH group.
- An aldehyde with no alpha hydrogen can undergo a Cannizzaro reaction. In crossed Cannizzaro reduction with formaldehyde, the other aldehyde is reduced to an alcohol while formaldehyde is oxidized to formate.

Step 1: Identify the carbon skeleton and count the aldol additions The formula given, $C(CH_2OH)_4$, identifies pentaerythritol, the alcohol used to make PETN; ordinary erythritol has a different formula. The relevant starting aldehydes are ethanal, CH_3CHO, and methanal, $HCHO$. Ethanal has three alpha hydrogens on its methyl carbon. Successive hydroxymethylation by three molecules of methanal replaces all three:

$$CH_3CHO + 3\,HCHO \longrightarrow (HOCH_2)_3CCHO.$$

There are therefore three aldol addition steps, often termed aldol condensations in this question. No dehydration is needed for the retained hydroxymethyl groups.

Step 2: Reduce the remaining aldehyde by Cannizzaro reaction The intermediate $(HOCH_2)_3CCHO$ now has a quaternary alpha carbon and no alpha hydrogen. A further molecule of methanal enables crossed Cannizzaro reaction in base:

$$(HOCH_2)_3CCHO + HCHO + OH^- \longrightarrow C(CH_2OH)_4 + HCOO^-.$$

This one redox step converts the remaining $-CHO$ group into the fourth $-CH_2OH$ group. Two aldol steps would leave an alpha hydrogen and only two introduced hydroxymethyl groups; four aldol additions are impossible because ethanal has only three alpha hydrogens. The alcohol reactants named in (A) and (D) are also inappropriate for this aldehyde reaction sequence.

Step 3: Conclusion The synthesis of the compound specified by $C(CH_2OH)_4$ uses three aldol additions followed by one crossed Cannizzaro reaction. Hence, the correct option is (C).

Quick Tip to Solve: Count ethanal's three alpha hydrogens: each installs one hydroxymethyl group, and reduction of its original aldehyde supplies the fourth alcohol group.

62. The following statements concern elements in the periodic table. Which of the following is true ? [4 marks]

(A) All the elements in Group 17 are gases.

(B) The Group 13 elements are all metals.

(C) Elements of Group 16 have lower ionization enthalpy values compared to those of Group 15 in the corresponding periods.

(D) For Group 15 elements, the stability of + 5 oxidation state increases down the group.

Correct Answer: (C) Elements of Group 16 have lower ionization enthalpy values compared to those of Group 15 in the corresponding periods.

Solution:

Concept:

- First ionization enthalpy is the energy needed to remove an electron from an isolated gaseous atom. A half-filled np^3 subshell has extra exchange stabilization, while np^4 contains one paired orbital with additional electron repulsion.
- Periodic comparisons must consider electronic configuration as well as increasing nuclear charge. Down the heavier p-block groups, the inert-pair effect stabilizes lower oxidation states.

Step 1: Examine the Group 15 and Group 16 comparison Group 15 atoms have outer configuration ns^2np^3; Group 16 atoms have ns^2np^4. Removing one paired electron from the latter relieves repulsion and yields a half-filled np^3 subshell. This explains the familiar exceptions

$$I_1(\mathrm{O}) < I_1(\mathrm{N}), \qquad I_1(\mathrm{S}) < I_1(\mathrm{P}).$$

Option (C) expresses this standard comparison, which is the one selected by the answer key. It should not be read as an exceptionless rule for every heavier period: nuclear-charge and other effects compete, and the ordering reverses for pairs such as antimony and tellurium.

Step 2: Eliminate the remaining statements Option (A) is false because the halogens are not all gases: bromine is liquid and iodine is solid at ordinary room temperature. Option (B) is false because boron, the first Group 13 element, is a metalloid rather than a metal. Option (D) reverses the inert-pair trend: down Group 15, the ns^2 pair becomes less available for bonding, so the $+3$ state gains stability relative to $+5$. For example, bismuth strongly favors $+3$, whereas the $+5$ state is much less stable. Thus none of these three alternatives describes the accepted trend.

Step 3: Conclusion The keyed answer is (C), referring to the half-filled-subshell ionization-enthalpy exception, especially the N/O and P/S pairs. Its wording is overgeneralized for heavier periods; it is not a universal inequality for all Group 15/16 pairs. Hence, the correct option is (C).

Quick Tip to Solve: When comparing N with O or P with S, write the p-orbital occupancies first: the paired electron in the Group 16 atom is easier to remove than a singly occupied electron from the half-filled Group 15 atom.

63. Identify the correct statement : [4 marks]

(A) Iron corrodes in oxygen-free water.

(B) Iron corrodes more rapidly in salt water because its electrochemical potential is higher.

(C) Corrosion of iron can be minimized by forming a contact with another metal with a higher reduction potential.

(D) Corrosion of iron can be minimized by forming an impermeable barrier at its surface.

Correct Answer: (D) Corrosion of iron can be minimized by forming an impermeable barrier at its surface.

Solution:

Concept:

- Rusting is an electrochemical process: iron dissolves anodically as $\mathrm{Fe} \longrightarrow \mathrm{Fe}^{2+} + 2\,\mathrm{e}^-$, while dissolved oxygen is reduced at cathodic regions in moist, aerated conditions.
- A continuous impermeable coating blocks water and oxygen from reaching iron. Sacrificial protection instead couples iron to a metal that oxidizes more readily, such as zinc or magnesium.

Step 1: Test the statements about water and salt Ordinary rusting requires both moisture and oxygen. Its cathodic reaction in neutral aerated water is

$$\mathrm{O_2} + 2\,\mathrm{H_2O} + 4\,\mathrm{e}^- \longrightarrow 4\,\mathrm{OH}^-.$$

Consequently, oxygen-free water does not sustain the usual oxygen-driven rusting process described here, so (A) is incorrect. Salt water does accelerate corrosion, but the principal reason is greater electrolyte conductivity and easier ionic charge transport; chloride can also disrupt protective surface films. It is not correct to explain this merely by saying the electrochemical potential of iron is higher. Thus (B) contains an incorrect reason even though its observation about faster corrosion is familiar.

Step 2: Compare galvanic contact with a protective barrier A metal with a higher reduction potential is more noble than iron. In an electrically conducting couple exposed to an electrolyte, iron tends to become the anode and dissolve, while the nobler metal becomes the cathode. Such contact can intensify localized corrosion, so (C) reverses the sacrificial-anode principle. A less noble metal such as zinc protects iron by oxidizing preferentially. An intact impermeable coating, in contrast, isolates iron from the corrosive environment and interrupts access of oxygen and water. Suitable paint, grease or a continuous protective layer therefore reduces corrosion without requiring iron to serve as an anode.

Step 3: Conclusion Corrosion of iron can be minimized by forming an impermeable barrier on its surface, provided the layer remains intact. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish barrier protection from sacrificial protection: an intact coating excludes oxygen and water, whereas a sacrificial metal must have a lower reduction potential than iron.

64. Oxidation of succinate ion produces ethylene and carbon dioxide gases. On passing 0.2 Faraday electricity through an aqueous solution of potassium succinate, the total volume of gases (at both cathode and anode) at STP (1 atm and 273 K) is : [4 marks]

(A) 2.24 L

(B) 4.48 L

(C) 6.72 L

(D) 8.96 L

Correct Answer: (D) 8.96 L

Solution:

Concept:

- Faraday's law links charge to electrons: one faraday transfers one mole of electrons. Both electrodes pass the same charge.
- Anodic oxidation of one succinate ion releases two electrons and produces one molecule of ethylene plus two molecules of carbon dioxide. Water reduction at the cathode produces hydrogen.

Step 1: Write the electrode reactions The balanced anodic process is

$$^{-}\mathrm{OOC}-\mathrm{CH_2}-\mathrm{CH_2}-\mathrm{COO}^{-} \longrightarrow \mathrm{CH_2}=\mathrm{CH_2} + 2\,\mathrm{CO_2} + 2\,\mathrm{e}^{-}.$$

At the cathode, potassium ions are not deposited from water; instead,

$$2\,\mathrm{H_2O} + 2\,\mathrm{e}^{-} \longrightarrow \mathrm{H_2} + 2\,\mathrm{OH}^{-}.$$

Thus a common charge of two faradays produces three moles of gas at the anode and one mole at the cathode.

Step 2: Calculate the total gas volume For 0.2 faraday, the succinate oxidised is $0.2/2 = 0.1$ mol. Hence the gases comprise 0.1 mol ethylene, 0.2 mol carbon dioxide and 0.1 mol hydrogen:

$$n_{\text{total}} = 0.1 + 0.2 + 0.1 = 0.4 \text{ mol}.$$

At the specified STP, use the molar volume $22.4\ \mathrm{L\,mol^{-1}}$:

$$V_{\text{total}} = 0.4(22.4) = 8.96 \text{ L}.$$

The smaller options omit part of the anodic gas or the cathodic hydrogen; the question explicitly asks for gases at both electrodes.

Step 3: Conclusion The total volume is 8.96 L. Hence, the correct option is (D).

Quick Tip to Solve: Write both electrode reactions before converting faradays to gas moles; the same charge passes through both electrodes.

65. The transition metal ions responsible for color in ruby and emerald are, respectively : [4 marks]

(A) Cr^{3+} and Co^{3+}

(B) Co^{3+} and Cr^{3+}

(C) Co^{3+} and Co^{3+}

(D) Cr^{3+} and Cr^{3+}

Correct Answer: (D) Cr^{3+} and Cr^{3+}

Solution:

Concept:

- Transition-metal ions in a crystal are surrounded by ligands. The crystal field splits their d orbitals, allowing absorption of selected visible wavelengths.
- The colour depends on both the metal ion and its coordination environment; different colours do not necessarily mean different metal ions.

Step 1: Identify the colouring ion in ruby Ruby is corundum, Al_2O_3, containing a small proportion of chromium(III) ions in place of aluminium(III). The ion responsible for its red colour is therefore Cr^{3+}, not Co^{3+}. Chromium has atomic number 24, and chromium(III) has the $3d^3$ configuration. Its interaction with the surrounding oxide ions determines which wavelengths are absorbed.

Step 2: Identify the colouring ion in emerald Emerald is a variety of beryl, $Be_3Al_2Si_6O_{18}$, whose characteristic green colour is also associated with chromium(III) substituting for aluminium. The environment around Cr^{3+} differs from that in ruby, changing the crystal-field splitting and absorption spectrum. Thus the two hosts can give red and green gemstones even though the colouring ion is the same. Options involving cobalt(III) for either of these standard examples therefore fail.

Step 3: Conclusion Both ruby and emerald owe their characteristic colour to Cr^{3+}. Hence, the correct option is (D).

Quick Tip to Solve: For ruby and emerald, remember the same chromium(III) ion in different crystal hosts; the host changes the colour.

66. A solid XY kept in an evacuated sealed container undergoes decomposition to form a mixture of gases X and Y at temperature T. The equilibrium pressure is 10 bar in this vessel. K_p for this reaction is : [4 marks]

(A) 5

(B) 10

(C) 25

(D) 100

Correct Answer: (C) 25

Solution:

Concept:

- For a pure solid in a heterogeneous equilibrium, its activity is taken as unity and is omitted from the equilibrium-constant expression.
- Dalton's law gives the total pressure as the sum of the partial pressures. Equal stoichiometric amounts of two ideal gases have equal partial pressures in the same vessel.

Step 1: Use the decomposition stoichiometry The decomposition described is

$$\mathrm{XY\,(s)} \rightleftharpoons \mathrm{X\,(g)} + \mathrm{Y\,(g)}.$$

The container is initially evacuated, so neither product gas is present at the start. Every mole of solid decomposed gives one mole of X and one mole of Y. Therefore their equilibrium mole numbers, mole fractions and partial pressures are equal. The solid does not itself contribute to the gas pressure.

Step 2: Evaluate the pressure product If each product has partial pressure p, the measured total gives

$$p_X + p_Y = 2p = 10 \text{ bar}, \qquad p_X = p_Y = 5 \text{ bar}.$$

For the reaction as written,

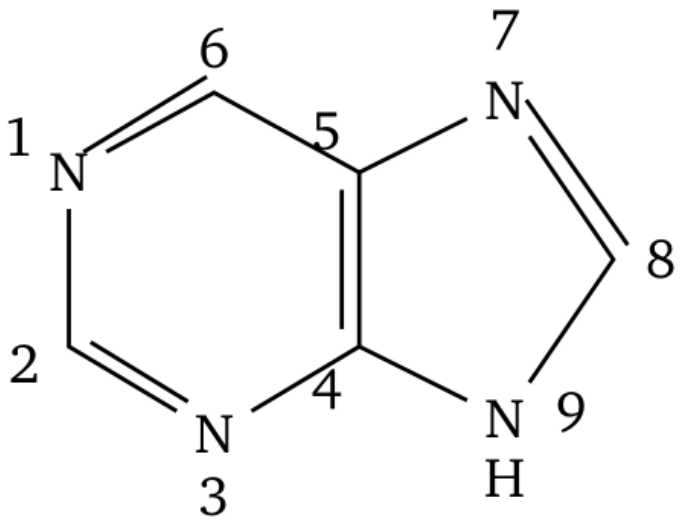
$$K_p = p_X p_Y = (5)(5) = 25.$$

This is the usual numerical K_p when pressures are expressed in bar; with a standard-state formulation, each pressure is divided by 1 bar, giving the same numerical value. The value 5 is only one partial pressure, 10 is the total pressure, and 100 incorrectly treats each partial pressure as 10.

Step 3: Conclusion The numerical equilibrium constant is $K_p = 25$. Hence, the correct option is (C).

Quick Tip to Solve: In an initially evacuated vessel, use the gaseous product ratio to split the total pressure before writing the equilibrium product.

67. The "N" which does not contribute to the basicity for the compound is : [4 marks]



(A) N 7

(B) N 9

(C) N 1

(D) N 3

Correct Answer: (B) N 9

Solution:

Concept:

- A nitrogen lone pair can accept a proton readily when it is not needed to maintain the aromatic electron system. A pyridine-type ring nitrogen has such an available lone pair.
- In a pyrrole-type nitrogen, the lone pair occupies a p orbital and contributes two electrons to aromatic delocalisation. Protonating that lone pair sacrifices this stabilisation.

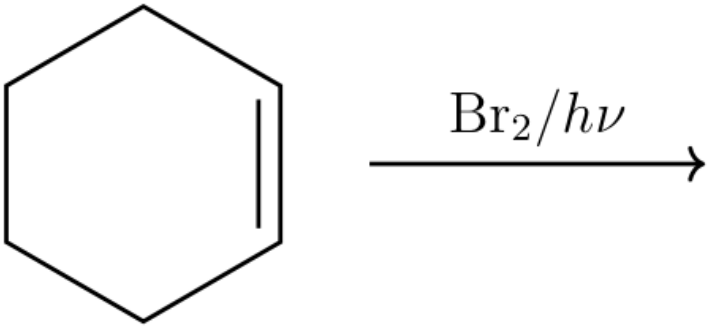
Step 1: Locate the pyrrole-type nitrogen In the displayed fused purine framework, N_9 bears hydrogen and is bonded to two neighbouring ring atoms by single bonds. It is the pyrrole-type nitrogen of the five-membered ring. Its lone pair participates in the conjugated aromatic system, rather than remaining as a freely available pair for proton binding. This is the nitrogen sought by the question.

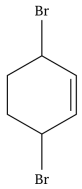
Step 2: Compare the other labelled nitrogen atoms The nitrogens at positions 1, 3 and 7 are pyridine-type nitrogens. Each participates in the ring pi bonding using a p electron, while its lone pair lies in a different, approximately in-plane orbital. Those lone pairs can contribute to proton acceptance without being the aromatic electron pair supplied by N_9. Their precise basic strengths can differ because of the fused-ring environment, but the qualitative distinction is the availability of the lone pair. Thus the labels 7, 1 and 3 do not answer the question.

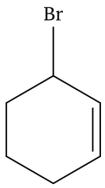
Step 3: Conclusion The nitrogen whose lone pair does not contribute to basicity is N_9. Hence, the correct option is (B).

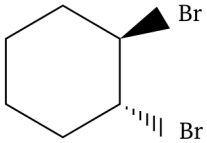
Quick Tip to Solve: Look for the ring nitrogen bearing H with two single bonds: its lone pair is often the pyrrole-type pair sustaining aromaticity.

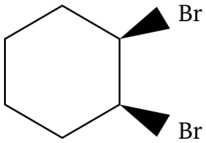
68. Bromination of cyclohexene under conditions given below yields : [4 marks]

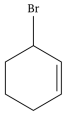


(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- Light promotes homolytic cleavage of Br_2, initiating a radical chain. Allylic carbon–hydrogen bonds are susceptible to substitution because an allylic radical is resonance-stabilised.
- The conditions distinguish radical allylic substitution from ordinary electrophilic addition of bromine across a double bond.

Step 1: Identify the radical substitution site The reagent over the arrow is Br_2 under $h\nu$. A bromine radical abstracts an allylic hydrogen from cyclohexene, producing an allylic radical; reaction with another bromine molecule then forms the allylic bromide and regenerates a bromine radical. In this chain sequence a hydrogen adjacent to the double bond is replaced, while a carbon–carbon double bond remains in the ring.

Step 2: Select the matching structure The mono-brominated cyclohexene in option (B) places bromine on an allylic carbon and retains the double bond. In cyclohexene the symmetry-related allylic positions lead to the same constitutional product, commonly named 3-bromocyclohex-1-ene. Option (A) shows two bromine substituents while retaining the alkene, which is not the single allylic substitution product. Options (C) and (D) show dibromides in which the alkene has been consumed; they represent addition structures rather than the photochemical allylic substitution asked here. Thus the light condition is the decisive discriminator.

Step 3: Conclusion The product is the allylic monobromide, 3-bromocyclohex-1-ene. Hence, the

correct option is (B).

Quick Tip to Solve: Read the condition over the arrow first: light signals a radical chain, so inspect allylic positions before choosing a vicinal dibromide.

69. If 100 mole of H_2O_2 decompose at 1 bar and 300 K, the work done (kJ) by one mole of $O_2(g)$ as it expands against 1 bar pressure is :

$2H_2O_2(l) \rightleftharpoons 2H_2O(l) + O_2(g)$

(R = 8.3 J K $^{-1}$ mol^{-1}) [4 marks]

(A) 62.25

(B) 124.50

(C) 249.00

(D) 498.00

Correct Answer: No listed option equals the literal one-mole work, 2.49 kJ; the total reaction gives 124.50 kJ, while the supplied key selects 249.00 kJ.

Solution:

Concept:

- The magnitude of expansion work done by a gas against a constant external pressure is $W_{\text{by}} = P_{\text{ext}}\Delta V$. Chemistry instead writes work on the system as $w = -P_{\text{ext}}\Delta V$.
- For ideal gas formed at the stated pressure and temperature, neglecting the liquid-volume change, $P\Delta V = nRT$. The number of moles must match the amount whose work is requested.

Step 1: Calculate the work for one mole of oxygen The wording explicitly asks for the work by one mole of O_2, at 300 K, expanding against 1 bar. With the gas at this same pressure, $\Delta V = RT/P$ for one mole, so the pressure cancels:

$$W_{\text{by,1 mol}} = RT = (8.3)(300) = 2490 \text{ J} = 2.49 \text{ kJ}.$$

This is positive work done by the gas. In the work-on-system convention it is -2.49 kJ. Neither sign convention yields a listed magnitude.

Step 2: Check the amount from the complete decomposition The equation $2\,H_2O_2(l) \longrightarrow 2\,H_2O\,(l) + O_2(g)$ shows that 100 mol peroxide forms 50 mol oxygen. The expansion work for all that oxygen is

$$W_{\text{by,total}} = 50RT = 124500 \text{ J} = 124.50 \text{ kJ}.$$

Thus the listed value 124.50 belongs to the entire evolved gas, not one mole. The supplied key selects 249.00, but that value would require 100 mol oxygen at this temperature, contrary to the stated stoichiometry. The other magnitudes likewise do not equal the literal one-mole result. No given option is defensible as written.

Step 3: Conclusion For the stated one mole of oxygen the work done is 2.49 kJ; none of the options matches. The full decomposition gives 124.50 kJ, whereas the supplied key value 249.00 kJ is

inconsistent with both the one-mole wording and the reaction stoichiometry.

Quick Tip to Solve: Before using nRT, distinguish moles of reactant, total moles of gas produced, and the one-mole amount explicitly requested.

70. Aqueous solution of which salt will not contain ions with the electronic configuration $1s^2 2s^2 2p^6 3s^2 3p^6$? [4 marks]

(A) NaF

(B) NaCl

(C) KBr

(D) CaI_2

Correct Answer: (A) NaF

Solution:

Concept:

- The stated configuration contains $2 + 2 + 6 + 2 + 6 = 18$ electrons, so it is the closed-shell configuration of argon, written $[\mathrm{Ar}]$.
- In aqueous solution a soluble salt separates into its constituent ions. Count electrons using atomic number minus positive charge or plus negative charge.

Step 1: Count electrons in the sodium salts For NaF, the ions are Na^+ and F^-. Sodium has atomic number 11, so Na^+ has 10 electrons. Fluorine has atomic number 9, so F^- also has 10 electrons. Both have the neon configuration, $1s^2 2s^2 2p^6$, rather than the specified 18-electron configuration. In contrast, NaCl supplies Cl^-, which has $17 + 1 = 18$ electrons and therefore does meet the stated configuration.

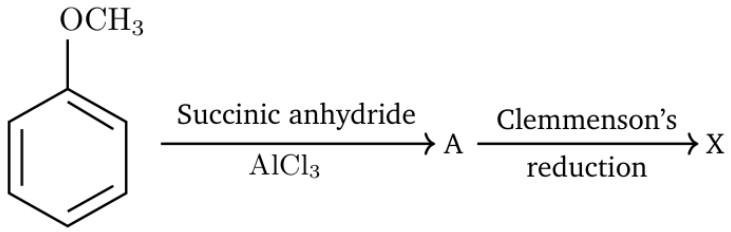
Step 2: Check potassium and calcium salts Potassium has atomic number 19, so K^+ from KBr has $19 - 1 = 18$ electrons. Calcium has atomic number 20, so Ca^{2+} from $\mathrm{CaI_2}$ has $20 - 2 = 18$ electrons. Hence each of these solutions contains an argon-configuration cation, regardless of the configuration of its accompanying anion. The condition is absence of any ion with the specified configuration, not absence of both types of ion. Only sodium fluoride has neither such cation nor such anion.

Step 3: Conclusion Aqueous NaF has Na^+ and F^-, both with 10 electrons, so it lacks an 18-electron constituent ion. Hence, the correct option is (A).

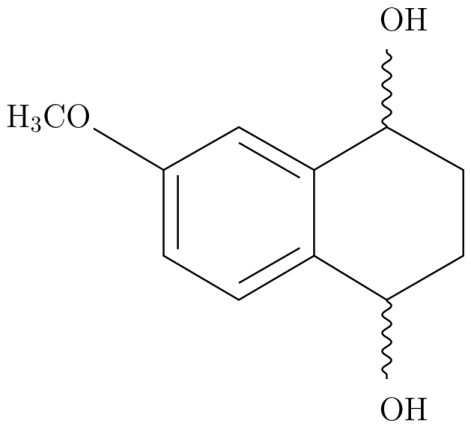
Quick Tip to Solve: Add the superscripts to identify the electron count, then check both ions: one matching ion is enough to eliminate a salt.

71. Consider the reaction sequence below :

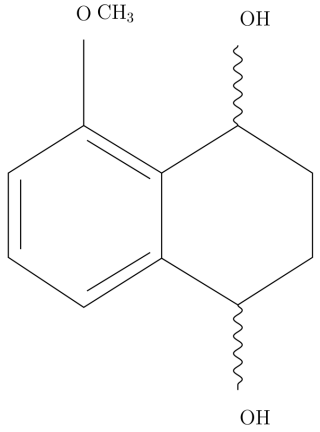
X is: [4 marks]



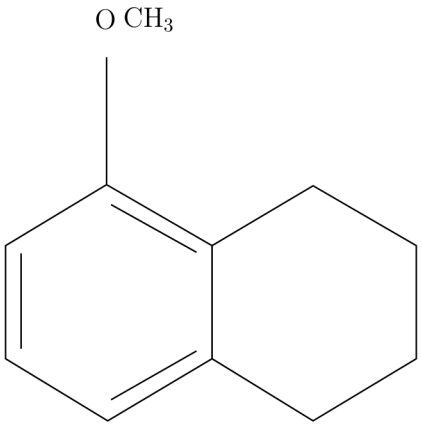
(A)



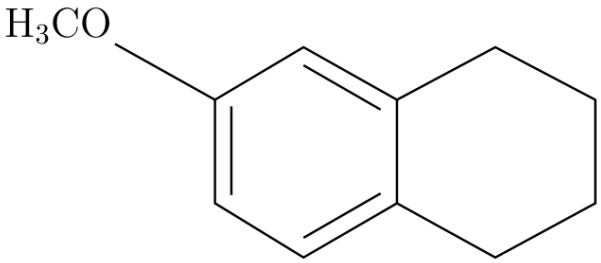
(B)



(C)



(D)



Correct Answer: No displayed option follows from the two stated steps: the product is principally 4-(4-methoxyphenyl)butanoic acid. The supplied answer key selects (D), but that fused-ring product requires additional transformations.

Solution:

Concept:

- Succinic anhydride undergoes Friedel–Crafts acylation with an aromatic ring in the presence of $AlCl_3$, introducing the chain $-COCH_2CH_2COOH$.
- Clemmensen reduction replaces the ketone carbonyl group by CH_2. It does not, by itself, remove the carboxyl group and install a second fused ring.

Step 1: Form the acylation product The starting aromatic compound is anisole. Its methoxy group directs electrophilic substitution to the ortho and para positions; the less hindered para acylation product is the principal product. Opening the succinic-anhydride ring introduces a four-carbon side

chain, with one ketone carbonyl and one terminal carboxyl group. Thus the principal intermediate A is

$$\text{p-}CH_3OC_6H_4COCH_2CH_2COOH.$$

This step is acylation, not attachment of an already formed saturated six-membered ring.

Step 2: Apply the specified reduction Clemmensen reduction converts the aryl ketone carbonyl into a methylene group while retaining the carboxylic acid. The resulting principal product is

$$\text{p-}CH_3OC_6H_4CH_2CH_2CH_2COOH,$$

namely 4-(4-methoxyphenyl)butanoic acid. An ortho acylation product would similarly give the corresponding ortho-substituted open-chain acid. Neither positional alternative is a fused-ring tetralin at this stage.

Step 3: Compare the actual product with every option Options (A) and (B) show fused bicyclic frameworks bearing hydroxyl groups, rather than the retained terminal carboxyl group. Options (C) and (D) show methoxy-substituted tetralin frameworks and also lack that carboxyl group. Reaching a tetralin by the usual Haworth sequence requires cyclization of the phenylbutanoic acid to a tetralone and a further reduction of that cyclic ketone. Those are additional transformations beyond the two arrows given here. Selecting a methoxytetralin solely from its substitution pattern would silently insert those missing steps.

Step 4: Conclusion The two stated steps give principally $\text{p-}CH_3OC_6H_4CH_2CH_2CH_2COOH$, so none of the displayed options is the literal product. The supplied answer key selects (D); that selection requires additional cyclization and reduction steps absent from the stated sequence.

Quick Tip to Solve: Track both carbonyl groups of succinic anhydride: acylation leaves a terminal carboxyl group, and Clemmensen reduction removes only the ketone oxygen.

72. The rate law for the reaction below is given by the expression k [A] [B]

A + B $\rightarrow$ Product

If the concentration of B is increased from 0.1 to 0.3 mole, keeping the value of A at 0.1 mole, the rate constant will be : [4 marks]

(A) k

(B) k/3

(C) 3k

(D) 9k

Correct Answer: (A) k

Solution:

Concept:

- The rate law is $r = k[A][B]$, where r is the reaction rate and k is the rate constant at the specified temperature.

- Changing reactant concentration changes the rate through the concentration factors. For the same reaction at a fixed temperature and unchanged conditions, it does not change k.

Step 1: Identify the quantity being asked The question asks for the rate constant after increasing the amount or concentration of B, rather than for the new reaction rate. The given rate expression is first order in A and first order in B. Its overall order is two. The coefficient k is a property of this reaction under the specified conditions, whereas $[A]$ and $[B]$ describe its current composition.

Step 2: Compare the initial and final rates Keeping A fixed and increasing B by a factor of three gives

$$\frac{r_2}{r_1} = \frac{k[A](0.3)}{k[A](0.1)} = 3.$$

Thus the reaction runs three times as fast. If both concentrations are expressed consistently, the initial concentration product is $0.1 \times 0.1 = 0.01$ and the final product is $0.1 \times 0.3 = 0.03$. Dividing either rate by its corresponding product recovers the same coefficient k.

Step 3: Distinguish concentration effects from changes in k The Arrhenius relation $k = A\exp(-E_a/RT)$ explains why temperature can change the rate constant. No temperature, catalyst or reaction-medium change is specified here. The alternatives $k/3$, $3k$ and $9k$ incorrectly transfer concentration factors into the rate constant; in particular, $3k$ confuses the tripling of rate with a tripling of k.

Step 4: Conclusion The rate becomes three times its original value, but the rate constant remains k. Hence, the correct option is (A).

Quick Tip to Solve: Before applying a concentration ratio, check whether the question asks for the rate or the rate constant: only the rate acquires that ratio at fixed temperature.

73. The bond angle H - X - H is the greatest in the compound : [4 marks]

(A) CH_4

(B) NH_3

(C) H_2O

(D) PH_3

Correct Answer: (A) CH_4

Solution:

Concept:

- VSEPR theory compares repulsions between electron pairs around a central atom. A tetrahedral arrangement of four equivalent bonding pairs has bond angles of $109.5°$.
- At a comparable central atom, lone pairs repel bonding pairs more strongly and compress the angle between bonds. Heavier group-15 hydrides also have angles nearer $90°$.

Step 1: Determine the geometry of methane In CH_4, carbon forms four equivalent C–H bonds and has no lone pair. Its four bonding electron pairs occupy the vertices of a regular tetrahedron. Therefore

every H–C–H bond angle is approximately

$$\theta_{CH_4} = 109.5^\circ.$$

There is no lone-pair compression of this angle.

Step 2: Compare ammonia and water Nitrogen in NH_3 has three bonds and one lone pair. The molecular shape is trigonal pyramidal and the H–N–H angle is about 107°. Oxygen in H_2O has two bonds and two lone pairs, producing a bent molecule with an H–O–H angle of about 104.5°. These are both smaller than the methane angle because of the stronger lone-pair repulsions around their central atoms.

Step 3: Check phosphine and rank the alternatives The H–P–H angle in PH_3 is about 93.5°. Its bonding has much less s–p mixing than a simple ideal tetrahedral picture would suggest, so the angle lies close to the 90° angle between p orbitals. A larger central atom does not imply a larger bond angle. The relevant order is

$$CH_4 > NH_3 > H_2O > PH_3$$

when the symbols are read as the respective H–X–H bond angles. Thus none of the other three molecules exceeds methane.

Step 4: Conclusion The greatest H–X–H bond angle is 109.5° in CH_4. Hence, the correct option is (A).

Quick Tip to Solve: Use methane as the ideal tetrahedral reference, then check lone pairs and the near-90-degree bonding of heavier hydrides before comparing angles.

74. Observation of "Rhumann's purple" is a confirmatory test for the presence of : [4 marks]

(A) Reducing sugar

(B) Cupric ion

(C) Protein

(D) Starch

Correct Answer: (C) Protein

Solution:

Concept:

- Ninhydrin reacts with free primary amino groups to give the characteristic blue-violet colour called Ruhemann's purple.
- Amino acids are the building units of proteins. Their amino groups, and accessible amino groups in proteins, underpin the ninhydrin test.

Step 1: Recognise the named coloured product The distinctive name in the question identifies the ninhydrin reaction. A typical α-amino acid reacts with ninhydrin on heating, undergoing oxidative deamination and decarboxylation. Carbon dioxide and an aldehyde containing one fewer carbon atom are formed. The released nitrogen participates in formation of the conjugated blue-violet product

known as Ruhemann's purple.

Step 2: Connect the response with the biomolecule Proteins are made from amino-acid residues. Their terminal amino groups, and free amino groups on suitable side chains, can give the characteristic ninhydrin response. The most direct use of this reagent is detection of amino acids and primary amino groups. Since the alternatives do not list amino acids, the protein alternative is the matching biomolecular category. The test should not be interpreted as a universal purple response from every nitrogen compound: secondary amino acids such as proline show a different colour.

Step 3: Eliminate the other tests Reducing sugars are commonly detected by reduction of the copper(II) reagent in Fehling's or Benedict's test, giving a brick-red Cu_2O precipitate. Starch gives a blue-black complex with iodine. A cupric ion may produce characteristic complexes or precipitates with suitable reagents, but it does not account for the named ninhydrin purple product. These alternatives confuse different analytical colours rather than identifying the amino-group reaction.

Step 4: Conclusion Among the listed choices, the ninhydrin purple response corresponds to protein and its accessible amino groups. Hence, the correct option is (C).

Quick Tip to Solve: Associate ninhydrin purple with amino groups; distinguish it from iodine blue-black for starch and copper(I) oxide red for reducing sugars.

75. Fluorination of an aromatic ring is easily accomplished by treating a diazonium salt with HBF_4. Which of the following conditions is correct about this reaction ? [4 marks]

(A) Only heat

(B) $NaNO_2$/Cu

(C) Cu_2O/H_2O

(D) NaF/Cu

Correct Answer: (A) Only heat

Solution:

Concept:

- The Balz–Schiemann reaction converts an aromatic diazonium tetrafluoroborate into an aryl fluoride by thermal decomposition.
- Tetrafluoroboric acid supplies BF_4^-, which forms the diazonium tetrafluoroborate salt. Heating expels nitrogen and produces the C–F bond.

Step 1: Identify the intermediate salt Treating an aromatic diazonium salt with HBF_4 gives the corresponding aromatic diazonium tetrafluoroborate. For the benzene example this intermediate is $C_6H_5N_2^+\,BF_4^-$. The diazonium group is a useful leaving group because it can depart as stable nitrogen gas. The tetrafluoroborate counterion distinguishes this fluorination from other diazonium substitutions.

Step 2: Apply the required condition On heating the isolated diazonium tetrafluoroborate, it decomposes according to

$$C_6H_5N_2^+\,BF_4^- \xrightarrow{\Delta} C_6H_5F + BF_3 + N_2.$$

Thus the additional condition needed after treatment with HBF_4 is heat. No copper salt or separate sodium fluoride reagent is required for this characteristic transformation. The aryl framework is retained while its diazonium group is replaced by fluorine.

Step 3: Distinguish the other conditions Copper-mediated diazonium substitutions are associated with other replacements, such as the Sandmeyer preparation of aryl chlorides, bromides and nitriles. Nitrite with copper does not specify the Balz–Schiemann fluorination, nor does cuprous oxide in water. Sodium fluoride with copper is likewise not the necessary condition for decomposition of the tetrafluoroborate already formed here. Selecting a copper reagent merely because a diazonium salt is present misses the special role of BF_4^-.

Step 4: Conclusion The diazonium tetrafluoroborate needs only heating to form the aryl fluoride. Hence, the correct option is (A).

Quick Tip to Solve: For an aryl diazonium salt treated with tetrafluoroboric acid, think Balz–Schiemann: form the tetrafluoroborate, then heat it.

76. Sodium extract is heated with concentrated HNO_3 before testing for halogens because : [4 marks]

(A) Silver halides are totally insoluble in nitric acid.

(B) Ag_2S and AgCN are soluble in acidic medium.

(C) S_2^- and CN^-, if present, are decomposed by conc. HNO_3 and hence do not interfere in the test.

(D) Ag reacts faster with halides in acidic medium.

Correct Answer: (C) S_2^- and CN^-, if present, are decomposed by conc. HNO_3 and hence do not interfere in the test.

Solution:

Concept:

- Sodium fusion converts covalently bound halogens into water-soluble halide ions. Addition of silver nitrate then gives the corresponding silver halide precipitate.
- If nitrogen or sulphur is also present, the sodium extract may contain cyanide or sulphide. These ions also precipitate with silver ions and must be removed first.

Step 1: Identify the intended halide test The halogen test uses $AgNO_3$ after acidifying the sodium extract. A halide ion X^- reacts as

$$Ag^+ + X^- \longrightarrow AgX\,(s).$$

The colour and subsequent behaviour of this precipitate help identify the halogen. The interpretation is valid only if other ions that precipitate silver have first been removed from the extract.

Step 2: Explain the interference An extract containing nitrogen or sulphur may contain CN^- or S^{2-}, respectively. Without pretreatment these give

$$Ag^+ + CN^- \longrightarrow AgCN\,(s),$$

$$2\,Ag^{+} + S^{2-} \longrightarrow Ag_2S\,(s).$$

These unwanted silver compounds can obscure or imitate the response being used to detect halide. Heating with concentrated nitric acid decomposes or removes the interfering cyanide and sulphide species before silver nitrate is added.

Step 3: Compare the explanations in the options The crucial purpose of the nitric-acid treatment is removal of the interfering ions, exactly as stated in option (C). The resistance of silver halides to nitric acid helps the procedure, but option (A) does not explain why heating the extract is needed. Option (B) incorrectly offers solubility of both interfering silver salts as the general explanation. Option (D) refers to faster reaction of silver rather than the elimination of false precipitates; the analytical reagent supplies silver ions, not metallic silver.

Step 4: Conclusion Concentrated HNO_3 removes interfering S^{2-} and CN^{-} before the halide precipitation test. Hence, the correct option is (C).

Quick Tip to Solve: Before interpreting a silver-nitrate precipitate as halide, remove cyanide and sulphide from the sodium extract by the prescribed nitric-acid treatment.

77. Which of the following is an example of homoleptic complex? [4 marks]

(A) $[Co(NH_3)_6]Cl_3$

(B) $[Pt(NH_3)_2Cl_2]$

(C) $[Co(NH_3)_4Cl_2]$

(D) $[Co(NH_3)_5Cl]Cl_2$

Correct Answer: (A) $[Co(NH_3)_6]Cl_3$

Solution:

Concept:

- A homoleptic complex contains only one kind of ligand bound to the central metal ion. A heteroleptic complex contains two or more kinds of coordinated ligand.
- Ligands are counted inside the coordination sphere, shown by square brackets. Counterions outside the brackets balance charge and are not coordinated ligands.

Step 1: Inspect the first coordination sphere In $[Co(NH_3)_6]Cl_3$, the coordination entity is $[Co(NH_3)_6]^{3+}$. All six donor ligands attached to cobalt are ammonia molecules. The three chloride ions lie outside the brackets, so they balance the charge without introducing a second ligand type. Thus this complex has six ligands but only one kind of ligand and is homoleptic.

Step 2: Check the platinum alternative In $[Pt(NH_3)_2Cl_2]$, both ammonia molecules and chloride ions are inside the coordination sphere. Platinum is therefore bonded to two chemically distinct kinds of ligand: neutral ammine and anionic chlorido. Irrespective of whether a cis or trans arrangement is considered, the presence of these two ligand types makes the complex heteroleptic.

Step 3: Check both remaining cobalt alternatives The bracketed entity $[Co(NH_3)_4Cl_2]$ likewise contains both ammine and chlorido ligands. In $[Co(NH_3)_5Cl]Cl_2$, five ammonia ligands and one chloride ligand are attached to cobalt; the other two chloride ions are counterions. Both are heteroleptic

because their inner coordination spheres contain two ligand types. The distinction depends on ligand identity, not simply on the coordination number, the metal, the total number of atoms or the presence of chlorine anywhere in the formula.

Step 4: Conclusion Only $[Co(NH_3)_6]Cl_3$ has a coordination sphere containing one ligand type exclusively. Hence, the correct option is (A).

Quick Tip to Solve: Look inside the square brackets first: identical ligands there mean homoleptic, even if different ions occur outside the brackets.

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78. An aqueous solution of a salt MX_2 at certain temperature has a van't Hoff factor of 2. The degree of dissociation for this solution of the salt is : [4 marks]

(A) 0.33

(B) 0.50

(C) 0.67

(D) 0.80

Correct Answer: (B) 0.50

Solution:

Concept:

- The van't Hoff factor i is the actual number of dissolved solute particles divided by the number of formula units introduced. Dissociation increases the particle count, so a partially dissociated electrolyte has a factor between the undissociated and fully dissociated limits.
- If one formula unit gives ν ions and its degree of dissociation is α, the total particle amount per initial mole is $1 + (\nu - 1)\alpha$. Here α is a fraction, not a percentage.

Step 1: Count the species produced The salt has the stoichiometry MX_2, so its dissociation is

$$MX_2 \longrightarrow M^{2+} + 2\,X^{-}.$$

One formula unit therefore gives three ions when completely dissociated. Starting with one mole, the undissociated salt contributes $1 - \alpha$ mole of particles, the cation contributes α mole, and the anions contribute 2α mole. These contributions must all be counted, including the salt that remains undissociated.

Step 2: Use the measured factor The total amount of particles is

$$n_{\text{particles}} = (1 - \alpha) + \alpha + 2\alpha = 1 + 2\alpha.$$

As the initial amount is one mole,

$$i = 1 + 2\alpha.$$

Substituting the given value yields

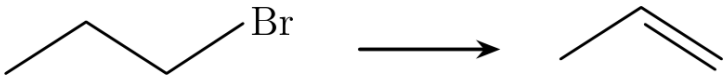
$$2 = 1 + 2\alpha \quad \Rightarrow \quad \alpha = \frac{1}{2} = 0.50.$$

This corresponds to 50% dissociation. As a check, $\alpha = 0.33, 0.67, 0.80$ would give factors approximately $1.66, 2.34, 2.60$, respectively, rather than the specified value 2. Complete dissociation would give $i = 3$, so the result is physically within the correct bounds.

Step 3: Conclusion The degree of dissociation is $\boxed{\alpha = 0.50}$. Hence, the correct option is (B).

Quick Tip to Solve: Count the total ions from one formula unit before using $\alpha = (i-1)/(\nu-1)$; for MX_2, the denominator is 2, not 3.

79. Which one of the following reagents is not suitable for the elimination reaction ? [4 marks]



(A) $NaOH/H_2O$

(B) NaOEt/EtOH

(C) $NaOH/H_2O$-EtOH

(D) NaI

Correct Answer: (D) NaI

Solution:

Concept:

- A beta-elimination removes a hydrogen from a carbon adjacent to the carbon bearing the leaving group, while the leaving group departs. A carbon-carbon double bond forms between those two carbons, so a reagent capable of removing the beta-hydrogen is required.
- The tendency to cause elimination depends on basicity as well as reaction conditions. A good nucleophile need not be a good base: iodide is nucleophilic but is a very weak proton acceptor, whereas hydroxide and ethoxide can act as bases.

Step 1: Determine the reagent function The depicted conversion is the loss of hydrogen halide from a halo compound to give an alkene. In base-promoted elimination, the base removes the beta-hydrogen as the carbon-halogen bond breaks and the alkene bond develops. Thus we should compare the ability of the supplied anions to abstract a proton, rather than compare only how readily they attack an electrophilic carbon. Changing a halide by nucleophilic substitution would not produce the alkene shown.

Step 2: Compare the listed anions Options (A) and (C) supply hydroxide, OH^-, and option (B) supplies ethoxide, $C_2H_5O^-$. These anions can act as bases and promote dehydrohalogenation under suitable conditions; the alcoholic ethoxide reagent is a familiar elimination reagent. Aqueous hydroxide can favour substitution, but it is still a basic reagent and elimination can compete depending on substrate and conditions. Sodium iodide instead supplies I^-. This ion is the conjugate base of the strong acid HI, making it much too weak a base for the required beta-proton abstraction. Its useful nucleophilicity is associated with substitution or halide exchange, not the base-induced elimination shown. Thus the aqueous solvent in (A) should not distract from the much more decisive absence of a suitable base in (D).

Step 3: Conclusion $\boxed{\text{NaI}}$ is not suitable for the required elimination reaction. Hence, the correct option is (D).

Quick Tip to Solve: For dehydrohalogenation, look for a proton-abstracting base. Iodide can be a good nucleophile while still being a very weak base.

80. The volume of 0.1N dibasic acid sufficient to neutralize 1 g of a base that furnishes 0.04 mole of OH^- in aqueous solution is : [4 marks]

(A) 200 mL

(B) 400 mL

(C) 600 mL

(D) 800 mL

Correct Answer: (B) 400 mL

Solution:

Concept:

- Neutralization consumes one mole of H^+ for every mole of OH^-: $H^+ + OH^- \longrightarrow H_2O$. Thus the total hydroxide amount fixes the acid equivalents needed, independently of the base's formula.
- Normality is the number of equivalents per litre. For acid-base neutralization, the number of acid equivalents is NV, with V in litres. A dibasic acid can supply two hydrogen ions per molecule, but that factor is already included in its stated normality.

Step 1: Find the hydroxide requirement The question states that the given 1 g sample of the base furnishes 0.04 mole of OH^-. Since each hydroxide ion consumes one hydrogen ion, this sample requires

$$n(H^+) = n(OH^-) = 0.04 \text{ mol}.$$

The corresponding neutralization requirement is 0.04 equivalent. The sample mass has already been accounted for in the stated hydroxide yield, so no molar mass or extra multiplication by 1 g is needed.

Step 2: Convert acid equivalents to volume For the $0.1\,N$ acid,

$$V = \frac{\text{equivalents required}}{N} = \frac{0.04}{0.1} = 0.40 \text{ L}.$$

Therefore,

$$V = 0.40 \times 1000 = 400 \text{ mL}.$$

A molecular calculation gives the same check: a fully neutralized dibasic acid of normality $0.1\,N$ has molarity $0.05\,\text{mol}\,L^{-1}$; the required 0.04 mole of hydrogen ions needs 0.02 mole of this acid, and $0.02/0.05 = 0.40$ litre. The listed 200, 600 and 800 mL quantities would provide $0.02, 0.06, 0.08$ equivalent, respectively, so none gives exact neutralization.

Step 3: Conclusion The sufficient volume is $\boxed{400 \text{ mL}}$. Hence, the correct option is (B).

Quick Tip to Solve: When normality is supplied, use equivalents directly. Multiplying by the acid basicity again would count the same hydrogen ions twice.

81. Initially, the *root mean square (rms)* velocity of N_2 molecules at certain temperature is u. If this temperature is doubled and all the nitrogen molecules dissociate into nitrogen atoms, then the new *rms* velocity will be : [4 marks]

(A) u/2

(B) 2u

(C) 4u

(D) 14u

Correct Answer: (B) 2u

Solution:

Concept:

- For ideal-gas particles of molar mass M at absolute temperature T, the root mean square speed is $v_{\text{rms}} = \sqrt{3RT/M}$. Equivalently it is $\sqrt{3k_{\text{B}}T/m}$ for a particle of mass m.
- The speed depends on both temperature and particle mass. Dissociating N_2 into nitrogen atoms halves the mass per freely moving particle; doubling the absolute temperature produces a separate increase in the mean-square speed.

Step 1: Write the initial and final conditions Let M be the molar mass of nitrogen molecules and T their initial absolute temperature. The initial speed satisfies

$$u = \sqrt{\frac{3RT}{M}}.$$

After the stated complete dissociation, the particles are nitrogen atoms rather than N_2 molecules. Each atom has molar mass $M/2$, while the final temperature is explicitly given as $2T$. These are the two changes to insert into the same kinetic-theory expression.

Step 2: Take the ratio of rms speeds The final speed is

$$u_{\text{f}} = \sqrt{\frac{3R(2T)}{M/2}} = \sqrt{4\frac{3RT}{M}} = 2u.$$

Alternatively,

$$\frac{u_{\text{f}}}{u} = \sqrt{\frac{T_{\text{f}}}{T}\frac{M}{M_{\text{f}}}} = \sqrt{2 \times 2} = 2.$$

Counting twice as many particles does not introduce an additional multiplier: its effect on speed is already captured through the smaller mass of an atom. The value $u/2$ has the wrong direction of change, and $4u$ confuses the factor in the mean-square speed with that in the speed itself. The remaining option $14u$ does not follow from either of the given changes. The specified final temperature is used directly, so no separate energy-balance calculation is required.

Step 3: Conclusion The new rms velocity is $\boxed{2u}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the ratio $v_{\text{f}}/v_{\text{i}} = \sqrt{(T_{\text{f}}/T_{\text{i}})(M_{\text{i}}/M_{\text{f}})}$, so temperature and dissociation are both included under one square root.

82. The commercial name for calcium oxide is : [4 marks]

(A) Milk of lime

(B) Slaked lime

(C) Limestone

(D) Quick lime

Correct Answer: (D) Quick lime

Solution:

Concept:

- Calcium oxide has the formula CaO and is commercially called quick lime. The common names of calcium compounds distinguish the oxide, hydroxide and carbonate even though all contain the same metal.
- Quick lime reacts with water to form calcium hydroxide, called slaked lime. Limestone is calcium carbonate, and milk of lime is a suspension of calcium hydroxide in water. The physical form and chemical formula must both be matched to the name.

Step 1: Identify the oxide in the lime cycle Calcium oxide is obtained by strongly heating calcium carbonate:

$$CaCO_3 \xrightarrow{\Delta} CaO + CO_2.$$

The carbonate starting material is limestone; the oxide product is quick lime. This reaction removes carbon dioxide from the carbonate, so limestone and calcium oxide cannot be used as two names for the same chemical substance. The question specifically asks for the commercial name of CaO, which is the oxide at this stage.

Step 2: Distinguish the other listed names Adding water to calcium oxide gives the hydroxide in an exothermic reaction:

$$CaO + H_2O \longrightarrow Ca(OH)_2.$$

The product $Ca(OH)_2$ is slaked lime, so option (B) refers to the hydrated reaction product rather than the original oxide. Milk of lime, option (A), is a suspension of this hydroxide in water; it is therefore also distinct from CaO. Limestone, option (C), is $CaCO_3$, the carbonate used to manufacture the oxide. The only option that names calcium oxide itself is quick lime, option (D). This identification uses composition rather than the general word "lime, which appears in several different common names.

Step 3: Conclusion The commercial name of CaO is $\boxed{\text{quick lime}}$. Hence, the correct option is (D).

Quick Tip to Solve: Remember the sequence limestone $CaCO_3$, quick lime CaO, and slaked lime $Ca(OH)_2$; heating and then hydration connect the three.

83. Which of the following polymers is synthesized using a free radical polymerization technique ?

[4 marks]

(A) Teflon

(B) Terylene

(C) Melamine polymer

(D) Nylon 6, 6

Correct Answer: (A) Teflon

Solution:

Concept:

- Free-radical addition polymerization joins unsaturated monomers by opening their carbon-carbon double bonds. A radical adds to a monomer, leaves a new radical at the chain end, and permits repeated chain growth without eliminating a small molecule in each addition.
- Teflon is polytetrafluoroethene (PTFE), formed from tetrafluoroethene, $CF_2{=}CF_2$. Terylene, nylon 6,6 and melamine-formaldehyde polymer are instead identified by condensation reactions of monomers with suitable functional groups.

Step 1: Identify the addition monomer and repeat unit Tetrafluoroethene contains the double bond needed for chain addition. Its polymerization is represented by

$$n\,CF_2{=}CF_2 \longrightarrow [-CF_2{-}CF_2{-}]_n.$$

Under suitable radical-initiating conditions, an initiator-derived radical attacks the double bond, and successive monomer molecules add to the growing radical chain. The carbon-fluorine bonds are retained, while the double bond becomes the carbon-carbon single-bonded polymer backbone. This is the free-radical polymerization route associated with Teflon.

Step 2: Exclude the condensation polymers Terylene is a polyester produced from ethylene glycol and terephthalic acid, forming ester links by condensation. Nylon 6,6 is a polyamide produced from hexamethylenediamine and adipic acid, forming amide links by condensation. Melamine polymer in this list is the melamine-formaldehyde resin, a network condensation polymer formed by reaction of the amino groups of melamine with formaldehyde and subsequent cross-linking. These routes rely on functional-group reactions rather than the repeated radical opening of an alkene double bond. Thus options (B), (C) and (D) belong to a different polymerization class from the process asked for. Teflon alone supplies the required free-radical addition example among the four choices.

Step 3: Conclusion $\boxed{\text{Teflon}}$ is synthesized by free-radical polymerization of tetrafluoroethene. Hence, the correct option is (A).

Quick Tip to Solve: Look for an alkene monomer and a carbon-chain repeat unit for radical addition; polyester, polyamide and amino-resin links point to condensation.

84. Gold numbers of some colloids are : Gelatin : 0.005 - 0.01, Gum Arabic : 0.15 - 0.25; Oleate : 0.04 - 1.0; Starch : 15 - 25. Which among these is a better protective colloid ? [4 marks]

(A) Gelatin

(B) Gum Arabic

(C) Oleate

(D) Starch

Correct Answer: (A) Gelatin

Solution:

Concept:

- The gold number is the minimum mass, in milligrams, of a protective colloid needed to prevent coagulation of 10 mL of a standard gold sol when 1 mL of a 10% sodium chloride solution is added. It compares the efficiency of protective action under the same test conditions.
- A smaller gold number means greater protective power: less protective colloid is needed to stabilize the same amount of gold sol against the specified electrolyte addition. The numerical ordering is therefore the reverse of the ordering of protection efficiency.

Step 1: Interpret the comparison correctly A protective lyophilic colloid helps stabilize the particles of a lyophobic sol by associating with their surfaces and maintaining a solvated protective layer. The gold-number test measures the amount required for this protection. Consequently, a large gold number does not indicate better protection; it means a larger dose is needed to prevent the same coagulation. We must select the smallest value or range, rather than the largest listed number.

Step 2: Compare the four supplied ranges The supplied ranges are gelatin 0.005–0.01, gum arabic 0.15–0.25, oleate 0.04–1.0, and starch 15–25. The whole range for gelatin lies below the ranges of every other listed substance: even its upper value 0.01 is below the lowest value 0.04 quoted for oleate. Thus the comparison is decisive despite the overlapping ranges of gum arabic and oleate. Gum arabic and oleate require more material than gelatin, while starch has by far the largest gold number and is the weakest protector among these choices. No averaging of a range, conversion to a percentage, or direct proportionality between gold number and protective power is needed to identify the best colloid.

Step 3: Conclusion $\boxed{\text{Gelatin}}$ has the lowest gold number and hence the greatest protective power. Hence, the correct option is (A).

Quick Tip to Solve: Gold number measures the dose needed for protection: the smaller the required dose, the better the protective colloid.

85. Which one of the following substances used in dry cleaning is a better strategy to control environmental pollution? [4 marks]

(A) Tetrachloroethylene

(B) Carbon dioxide

(C) Sulphur dioxide

(D) Nitrogen dioxide

Correct Answer: (B) Carbon dioxide

Solution:

Concept:

- Green chemistry reduces hazardous substances at their source by choosing cleaner processes and solvents, rather than only treating the waste after it forms.
- Liquid carbon dioxide, CO_2, can act as a dry-cleaning medium under suitable pressure. A detergent may be used with it to help remove dirt from fabrics.

Step 1: Identify the pollution associated with the conventional solvent Tetrachloroethylene is a chlorinated organic solvent traditionally used in dry cleaning. Its toxicity and the hazards associated with its release make reducing its use desirable. Therefore, option (A) describes the solvent that the greener process seeks to replace; it is not the improved choice among these alternatives.

Step 2: Select the cleaner dry-cleaning medium Liquid CO_2 provides an alternative to the hazardous chlorinated solvent. After cleaning, reducing the pressure allows carbon dioxide to leave the fabric as a gas, and a suitably designed closed process can recover and reuse it. The important comparison is the reduction of hazardous dry-cleaning solvent use. Carbon dioxide still requires responsible handling and recovery; calling this a greener process does not mean that unlimited atmospheric release is harmless.

Step 3: Rule out the other gases Sulphur dioxide, SO_2, and nitrogen dioxide, NO_2, are harmful air pollutants associated with respiratory irritation and acid-forming pollution. Neither is the cleaner dry-cleaning replacement intended here. Thus options (C) and (D) do not meet the stated environmental objective.

Step 4: Conclusion Carbon dioxide is the better dry-cleaning alternative among the listed substances. Hence, the correct option is (B).

Quick Tip to Solve: For green dry cleaning, recall liquid CO_2 as the replacement for the conventional chlorinated solvent.

86. Assertion : Among the carbon allotropes, diamond is an insulator, whereas, graphite is a good conductor of electricity.

Reason : Hybridization of carbon in diamond and graphite are sp^3 and sp^2, respectively. [4 marks]

(A) Both assertion and reason are correct, and the reason is the correct explanation for the assertion.

(B) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion

(C) Assertion is incorrect statement, but the reason is correct.

(D) Both assertion and reason are incorrect.

Correct Answer: (A) Both assertion and reason are correct, and the reason is the correct explanation for the assertion.

Solution:

Concept:

- Electrical conduction requires charge carriers that can move through the solid. Electrons tightly localized in covalent bonds do not provide the same mobility as delocalized electrons.
- Carbon has four valence electrons. Diamond has an sp^3 tetrahedral network, whereas graphite has sp^2 carbon atoms arranged in planar layers.

Step 1: Evaluate the assertion for diamond and graphite In diamond, each carbon atom forms four strong covalent sigma bonds with four neighbouring carbon atoms. Its valence electrons are used in this three-dimensional bonding network, leaving no readily mobile electron system under ordinary conditions. Diamond is therefore an electrical insulator. Graphite, in contrast, has a delocalized electronic system within its layers and conducts electricity, especially along those layers. The assertion correctly states this contrast.

Step 2: Evaluate the stated hybridizations The four equivalent bonding directions of diamond correspond to sp^3 hybridization. In graphite, three sp^2 hybrid orbitals per carbon form three in-plane sigma bonds. One unhybridized p orbital remains perpendicular to the plane and participates in a delocalized pi-electron system. Thus the reason correctly identifies both hybridizations.

Step 3: Connect the reason with the assertion The hybridization difference accounts for the bonding and electron availability: all four valence electrons are tied into localized bonds in diamond, while graphite retains the delocalized pi system needed for conduction. This is precisely the structural explanation of the electrical behaviour. Option (B) misses that connection; (C) and (D) incorrectly deny statements that are true.

Step 4: Conclusion Both assertion and reason are correct. The supplied answer-key entry is B, but the sp^3 network localizes all four valence electrons in sigma bonds, whereas sp^2 graphite retains a delocalized pi-electron system; this establishes the explanatory link. Thus the structural reasoning supports code A rather than the supplied key. Hence, the correct option is (A).

Quick Tip to Solve: Count sigma bonds around carbon: four in diamond leave no pi electron system; three in graphite leave a delocalized pi system.

87. Extraction of copper by smelting uses silica as an additive to remove : [4 marks]

(A) Cu_2S

(B) FeO

(C) FeS

(D) Cu_2O

Correct Answer: (B) FeO

Solution:

Concept:

- A flux is added during smelting to react with an impurity and form a fusible slag that can be separated from the metal-bearing material.
- Silica, SiO_2, is an acidic flux. It combines with the basic oxide FeO to form iron(II) silicate, $FeSiO_3$.

Step 1: Identify the iron-containing impurity at the slag-forming stage The concentrated copper ore commonly contains iron sulphide along with copper compounds. During the oxidizing stages of extraction, the iron sulphide is converted into iron(II) oxide. The relevant oxidation may be represented as

$$2\,FeS + 3\,O_2 \longrightarrow 2\,FeO + 2\,SO_2.$$

Therefore, although iron may initially occur as sulphide, the substance directly removed by silica is its oxide, not the sulphide.

Step 2: Write the reaction with silica The acidic flux reacts with the basic oxide in the furnace:

$$FeO + SiO_2 \longrightarrow FeSiO_3.$$

The product is the slag, which can be separated from the copper-containing matte. This separates the iron impurity while retaining the copper compound for the subsequent steps of extraction. The equation has one iron, one silicon and three oxygen atoms on each side.

Step 3: Distinguish the remaining options Cu_2S is a copper-bearing component of the matte and must be retained for obtaining copper; it is not the impurity removed by this flux. FeS is the precursor that is first oxidized to FeO. Cu_2O participates in copper extraction and is also not the basic iron oxide caught by silica. Hence only option (B) gives the direct reactant in slag formation.

Step 4: Conclusion Silica removes FeO as iron silicate slag, $FeSiO_3$. Hence, the correct option is (B).

Quick Tip to Solve: Pair the acidic flux SiO_2 with the basic impurity FeO; their product is $FeSiO_3$ slag.

88. Identify the reaction which does not liberate hydrogen : [4 marks]

(A) Reaction of zinc with aqueous alkali.

(B) Electrolysis of acidified water using Pt electrodes.

(C) Allowing a solution of sodium in liquid ammonia to stand.

(D) Reaction of lithium hydride with B_2H_6.

Correct Answer: (D) Reaction of lithium hydride with B_2H_6.

Solution:

Concept:

- Hydrogen gas can form when an active metal reacts with a suitable hydrogen-containing medium, or when hydride hydrogen reacts with a proton source.
- Lithium hydride supplies hydride ions. Diborane accepts hydride to form tetrahydroborate ions, BH_4^-, without requiring hydrogen gas as a product.

Step 1: Check zinc with aqueous alkali and water electrolysis Zinc is amphoteric and reacts with aqueous alkali to liberate hydrogen. An ionic form is

$$Zn + 2\,OH^- + 2\,H_2O \longrightarrow [Zn(OH)_4]^{2-} + H_2.$$

During electrolysis of acidified water using inert platinum electrodes, hydrogen is liberated at the cathode:

$$2\,H^+ + 2\,e^- \longrightarrow H_2.$$

Oxygen forms at the anode. Thus both (A) and (B) involve hydrogen liberation.

Step 2: Check sodium in liquid ammonia on standing The initial blue solution contains solvated electrons. On standing, sodium can react with ammonia to form sodium amide and hydrogen:

$$2\,Na + 2\,NH_3 \longrightarrow 2\,NaNH_2 + H_2.$$

Consequently, option (C) is also a hydrogen-liberating process. It is important to distinguish formation of the initial solution from the subsequent reaction specified by allowing it to stand.

Step 3: Check the reaction of lithium hydride with diborane Diborane reacts with lithium hydride to form lithium borohydride:

$$B_2H_6 + 2\,LiH \longrightarrow 2\,LiBH_4.$$

The six hydrogen atoms from diborane and two from lithium hydride are all incorporated into the two tetrahydroborate ions. This balanced addition reaction has no H_2 product. Therefore, it is the exception asked for in the question.

Step 4: Conclusion The reaction of lithium hydride with B_2H_6 forms $LiBH_4$ without liberating hydrogen. Hence, the correct option is (D).

Quick Tip to Solve: For $B_2H_6 + 2\,LiH \longrightarrow 2\,LiBH_4$, count all eight hydrogen atoms in the product; none remains to form H_2.

89. Which of the following is a bactericidal antibiotic ? [4 marks]

(A) Erythromycin

(B) Tetracycline

(C) Chloramphenicol

(D) Ofloxacin

Correct Answer: (D) Ofloxacin

Solution:

Concept:

- Bactericidal antibiotics kill susceptible bacteria, whereas bacteriostatic antibiotics primarily inhibit bacterial growth and multiplication.
- In the standard classification used here, ofloxacin is bactericidal; erythromycin, tetracycline and chloramphenicol are bacteriostatic antibiotics.

Step 1: Identify the property the question asks for The word bactericidal refers to killing bacteria, rather than merely stopping their growth. Both types may be therapeutically useful, but their classifications are different. Therefore, it is insufficient to identify a drug as an antibiotic: its usual antibacterial action must also match the word used in the question.

Step 2: Classify the listed antibiotics Ofloxacin is a fluoroquinolone with bactericidal activity against susceptible bacteria. It interferes with bacterial enzymes involved in DNA replication, including DNA gyrase and topoisomerase IV, disrupting the maintenance and copying of bacterial DNA. Erythromycin, tetracycline and chloramphenicol instead belong to the standard bacteriostatic examples in this classification. Their inhibition of bacterial protein synthesis primarily arrests growth.

Step 3: Select the killing agent and eliminate the distractors Option (D) is the only listed drug classified as bactericidal. Erythromycin in (A), tetracycline in (B) and chloramphenicol in (C) are all antibiotics, but that shared label does not make them bactericidal. Their placement as growth-inhibiting examples eliminates them for the requested category. This is a classification question; no dosage, treatment choice or clinical inference is needed to select the answer.

Step 4: Conclusion Ofloxacin is the bactericidal antibiotic among these choices. Hence, the correct option is (D).

Quick Tip to Solve: In this textbook list, group erythromycin, tetracycline and chloramphenicol as bacteriostatic; identify ofloxacin as bactericidal.

90. Identify the incorrect statement : [4 marks]

(A) S_2 is paramagnetic like oxygen.

(B) Rhombic and monoclinic sulphur have S_8 molecules.

(C) S_8 ring has a crown shape.

(D) The S-S-S bond angles in the S_8 and S_6 rings are the same.

Correct Answer: (D) The S-S-S bond angles in the S_8 and S_6 rings are the same.

Solution:

Concept:

- Sulphur forms molecular allotropes with different ring sizes. Both rhombic and monoclinic sulphur contain puckered S_8 molecules.
- The number of unpaired electrons controls molecular paramagnetism, while the ring size and conformation determine the bond angles within a sulphur ring.

Step 1: Check the statements about diatomic and crystalline sulphur In the molecular-orbital description, S_2 has two unpaired electrons, analogous to O_2, and is paramagnetic. Statement (A) is therefore correct. Rhombic and monoclinic sulphur differ in their crystal arrangement and stability ranges, but each contains S_8 molecules. Hence statement (B) is also correct; a difference in crystal form does not require a different molecular formula.

Step 2: Check the shape of the eight-membered ring The eight-membered sulphur ring is nonplanar and puckered, with the familiar crown conformation. This avoids treating the molecule as a flat regular octagon and is the structure described in statement (C). Thus (C) is correct as well. The ring geometry depends on the spatial arrangement of sulphur atoms and the bonding and nonbonding electron pairs.

Step 3: Compare the angles of the two ring sizes The S_8 crown has an S–S–S bond angle of about 108°, whereas the six-membered S_6 ring has a chair conformation with a smaller angle, about 102°. Their bond angles are therefore not equal. The claim in statement (D) overlooks the effect of ring size and conformation, making it the incorrect statement requested. Since (A), (B) and (C) are valid, they must not be selected in this negatively worded question.

Step 4: Conclusion The S–S–S bond angles in S_8 and S_6 rings are different, so statement (D) is incorrect. Hence, the correct option is (D).

Quick Tip to Solve: Underline the word incorrect, then compare ring geometries: S_8 is a crown, while S_6 is a chair.