JEE Main 2016 Paper 1 (B.E./B.Tech) Apr 3 Shift 1 (morning) Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2016, Annual (offline), conducted by the Central Board of Secondary Education (CBSE) in pen and paper mode on 3 April 2016, Single sitting. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. For $x \in R,\ f(x) = |\log 2 - \sin x|$ **and** $g(x) = f(f(x))$, **then :** [4 marks]

(A) g is not differentiable at $x = 0$

(B) $g'(0) = \cos(\log 2)$

(C) $g'(0) = -\cos(\log 2)$

(D) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

Correct Answer: (B) $g'(0) = \cos(\log 2)$

Solution:

Concept:

- For a composition $g = f \circ f$, the chain rule gives $g\prime(x) = f\prime(f(x))f\prime(x)$ wherever both relevant derivatives exist.
- The modulus $|u(x)|$ equals $u(x)$ on a neighbourhood where $u(x) > 0$. Thus check the signs at both the inner and outer evaluation points before differentiating.

Step 1: Remove the inner modulus near zero Put $a = \log 2$, so $0 < a < 1$. At $x = 0$, the quantity inside the first modulus is $a - \sin 0 = a > 0$. By continuity it remains positive for all sufficiently small x. Consequently, near zero,

$$f(x) = a - \sin x, \qquad f(0) = a, \qquad f\prime(0) = -1.$$

The absolute-value function therefore causes no corner at this evaluation point.

Step 2: Check the outer modulus and apply the chain rule Since $0 < a < 1$ and $\sin a < a$, we have $a - \sin a > 0$. Continuity again gives a neighbourhood of a on which $f(t) = a - \sin t$. Hence $f\prime(a) = -\cos a$, and

$$g\prime(0) = f\prime(f(0))f\prime(0) = (-\cos a)(-1) = \cos a.$$

Option (A) incorrectly treats any modulus as a corner. Option (C) misses one negative factor, and option (D) differentiates the sine incorrectly; the derivative of $-\sin t$ is $-\cos t$, not $-\sin t$.

Step 3: Conclusion The composition is differentiable at zero and $g\prime(0) = \cos(\log 2)$. Hence, the correct option is (B).

Quick Tip to Solve: For nested absolute values, test the sign at both $x = 0$ and $f(0)$; only then use the chain rule.

2. Let $p = \lim_{x \to 0^+} \left(1 + \tan^2 \sqrt{x}\right)^{1/(2x)}$. Then $\log p$ is equal to: [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) 2

(D) 1

Correct Answer: (A) $\frac{1}{2}$

Solution:

Concept:

- The logarithm of a power $a(x)^{b(x)}$, with $a(x) > 0$, is $b(x) \log a(x)$. Taking logarithms resolves an indeterminate form of type 1^∞.
- The standard limits are $\log(1+u)/u \to 1$ as $u \to 0$, and $\tan t/t \to 1$ as $t \to 0$, with angles measured in radians.

Step 1: Take logarithms and substitute the square root For $x > 0$ near zero, the base $1 + \tan^2 \sqrt{x}$ is positive. Let $t = \sqrt{x}$, so $x = t^2$ and $t \to 0^+$. The logarithm of the expression whose limit defines p becomes

$$\frac{\log(1 + \tan^2 \sqrt{x})}{2x} = \frac{\log(1 + \tan^2 t)}{2t^2}.$$

It suffices to find this finite logarithmic limit, after which continuity of the exponential will establish existence and positivity of p.

Step 2: Factor into standard limits Multiply and divide by $\tan^2 t$:

$$\frac{\log(1 + \tan^2 t)}{2t^2} = \frac{1}{2} \frac{\log(1 + \tan^2 t)}{\tan^2 t} \left(\frac{\tan t}{t}\right)^2.$$

As $t \to 0^+$, both variable factors tend to 1, giving $\log p = 1/2$, and consequently $p = e^{1/2}$. The question asks for $\log p$, not p. Option (B) incorrectly introduces an additional factor of two; option (C) reverses the constant factor; option (D) omits the 2 in the original exponent. None agrees with the explicit product of standard limits.

Step 3: Conclusion For the printed exponent $1/(2x)$, the logarithmic factorisation gives $\log p = (1/2) \cdot 1 \cdot 1 = 1/2$, rather than 1. Hence, the correct option is (A).

Quick Tip to Solve: With $\sqrt{x}$ inside a trigonometric limit, set $t = \sqrt{x}$; then pair $\tan^2 t$ with t^2 before taking the limit.

3. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true? [4 marks]

(A) $3a^2$ - $26a$ + 55 = 0

(B) $3a^2$ - $32a$ + 84 = 0

(C) $3a^2$ - $34a$ + 91 = 0

(D) $3a^2$ - $23a$ + 44 = 0

Correct Answer: (B) $3a^2$ - $32a$ + 84 = 0

Solution:

Concept:

- For N observations, the population variance is $\sigma^2 = \frac{1}{N}\sum x_i^2 - \left(\frac{1}{N}\sum x_i\right)^2$.
- The standard deviation is the non-negative square root of the variance. Thus $\sigma = 3.5$ means $\sigma^2 = 49/4$, rather than 3.5.

Step 1: Compute the two sums There are four observations: $2, 3, a, 11$. Their sum and sum of squares are

$$\sum x_i = a + 16, \qquad \sum x_i^2 = a^2 + 134.$$

The mean is therefore $(a + 16)/4$. Use denominator 4 because these four given numbers form the complete data set; the sample-variance denominator 3 is not the convention used for this question.

Step 2: Impose the given standard deviation Substitute into the variance identity:

$$\frac{a^2 + 134}{4} - \frac{(a + 16)^2}{16} = \frac{49}{4}.$$

Multiplying by 16 gives

$$4a^2 + 536 - (a^2 + 32a + 256) = 196,$$

and hence

$$3a^2 - 32a + 84 = 0.$$

As a check, its roots are $a = 6$ and $a = 14/3$; either gives variance $49/4$. The remaining options have different linear coefficients or constant terms and do not express this variance condition. For example, at the valid value $a = 6$, their left sides are respectively 7, -5 and 14, whereas option (B) is zero.

Step 3: Conclusion The required relation is $3a^2 - 32a + 84 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: For a small data set with one unknown, use mean of squares minus square of mean; it avoids expanding four squared deviations.

4. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\ \sin\theta}$ is purely imaginary, is : [4 marks]

(A) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(B) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

Correct Answer: (B) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Solution:

Concept:

- A complex number is purely imaginary when its real part is zero; multiply a quotient by the conjugate of its denominator to expose that real part.
- For real s, the denominator $1+4s^2$ is strictly positive, so a zero real part is determined solely by the real numerator.

Step 1: Rationalise the denominator Write $s=\sin\theta$. Multiplication by $1+2is$, the conjugate of $1-2is$, gives

$$\frac{2+3is}{1-2is}=\frac{(2+3is)(1+2is)}{1+4s^2}=\frac{2-6s^2+7is}{1+4s^2}.$$

Here the real term $-6s^2$ comes from $6i^2s^2$; using $i^2=-1$ is essential. The original denominator cannot vanish for real θ.

Step 2: Set the real part to zero and test the options The required condition is

$$2-6s^2=0 \quad\Longrightarrow\quad \sin^2\theta=\frac{1}{3}.$$

The proposed angle $\theta=\sin^{-1}(1/\sqrt{3})$ satisfies it, with nonzero imaginary part because $s\neq 0$. For option (A), $s^2=3/16$, so the real numerator is $7/8$, not zero. For (C), $s^2=3/4$, giving $-5/2$; for (D), $s^2=1/4$, giving $1/2$. Thus each of these alternatives retains a nonzero real part.

Step 3: Conclusion A suitable value is $\theta=\sin^{-1}(1/\sqrt{3})$. Hence, the correct option is (B).

Quick Tip to Solve: Rationalise once and set only the real numerator to zero; there is no need to calculate the final imaginary value.

5. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true? [4 marks]

(A) E_1 and E_2 are independent.

(B) E_2 and E_3 are independent.

(C) E_1 and E_3 are independent.

(D) E_1, E_2 and E_3 are independent.

Correct Answer: (D) E_1, E_2 and E_3 are independent.

Solution:

Concept:

- Two events E, F are independent when $P(E \cap F) = P(E)P(F)$.
- Mutual independence of three events requires all pairwise conditions and the triple-intersection condition. Pairwise independence alone is insufficient.

Step 1: Check every pair The 36 ordered dice outcomes are equally likely. We have

$$P(E_1) = P(E_2) = \frac{1}{6}, \qquad P(E_3) = \frac{1}{2}.$$

There is one outcome in $E_1 \cap E_2$, namely $(4, 2)$, so its probability is $1/36$, the product of the marginals. With die $B = 2$, the sum is odd exactly when die A is 1, 3 or 5. Therefore $P(E_2 \cap E_3) = 3/36 = 1/12$. Likewise, with die $A = 4$, die B must be odd, and $P(E_1 \cap E_3) = 1/12$. Both equal $(1/6)(1/2)$, confirming that (A), (B) and (C) are true.

Step 2: Test the triple intersection When E_1 and E_2 both occur, the dice show $(4, 2)$, whose sum is 6 and hence even. The triple intersection is empty:

$$P(E_1 \cap E_2 \cap E_3) = 0.$$

However,

$$P(E_1)P(E_2)P(E_3) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{72} \neq 0.$$

Consequently the three events are not mutually independent. This does not contradict the pairwise checks: conditioning on both specified even faces destroys the possibility of an odd sum, although conditioning on either face alone does not change its probability.

Step 3: Conclusion The statement that E_1, E_2, E_3 are independent is NOT true. Hence, the correct option is (D).

Quick Tip to Solve: After verifying pairwise independence, always test the intersection of all three events; this example fails exactly at that final condition.

6. The area (in sq. units) of the region $\{(x, y) : y^2 \geq 2x \ \ and \ \ x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is : [4 marks]

(A) $\pi - \frac{4\sqrt{2}}{3}$

(B) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$

(C) $\pi - \frac{4}{3}$

(D) $\pi - \frac{8}{3}$

Correct Answer: (D) $\pi - \frac{8}{3}$

Solution:

Concept:

- In the first quadrant, $y^2 \geq 2x$ means $y \geq \sqrt{2x}$, while $x^2 + y^2 \leq 4x$ means $y \leq \sqrt{4x - x^2}$.
- The area between an upper and a lower curve is the integral of their difference over the interval where both bounds are compatible.

Step 1: Find the common interval The circle is $(x - 2)^2 + y^2 = 4$, with centre $(2, 0)$ and radius 2. Its intersection with the parabola follows by substituting $y^2 = 2x$:

$$x^2 + 2x = 4x \quad \Longrightarrow \quad x(x - 2) = 0.$$

The first-quadrant boundary points are $(0, 0)$ and $(2, 2)$. The upper circle boundary lies above $\sqrt{2x}$ precisely when $4x - x^2 \geq 2x$, which gives $0 \leq x \leq 2$. Thus no part with $x > 2$ belongs to the required region.

Step 2: Subtract the two bounded areas The desired area is

$$A = \int_0^2 \sqrt{4x - x^2}\, dx - \int_0^2 \sqrt{2x}\, dx.$$

The first integral is the upper-left quarter of the radius-2 circle, so it equals π. For the second,

$$\int_0^2 \sqrt{2x}\, dx = \sqrt{2}\left[\frac{2}{3}x^{3/2}\right]_0^2 = \frac{8}{3}.$$

Hence $A = \pi - 8/3$. Options (A) and (B) leave spurious $\sqrt{2}$ factors after evaluating $2^{3/2}$; option (C) halves the parabola contribution. Option (B) also incorrectly halves the circle contribution. The inequalities, rather than the full circle alone, determine the integration interval.

Step 3: Conclusion The area is $\pi - 8/3$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Translate both inequalities into upper and lower bounds before integrating; the compatible interval is only $0 \leq x \leq 2$.

7. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)dx = x\,dy$, then $f\left(-\frac{1}{2}\right)$ is equal to : [4 marks]

(A) $\frac{2}{5}$

(B) $\frac{4}{5}$

(C) $-\frac{2}{5}$

(D) $-\frac{4}{5}$

Correct Answer: (B) $\frac{4}{5}$

Solution:

Concept:

- The equation $y' = y/x + y^2$ becomes linear after the reciprocal substitution $v = 1/y$, using $v' = -y'/y^2$.
- A first-order linear equation $v' + v/x = -1$ has integrating factor x on an interval not containing zero; its integrated algebraic relation can then be simplified.

Step 1: Use the reciprocal substitution For the solution through $(1, -1)$, $y \neq 0$ locally. Dividing the differential equation by $x\,dx$, initially near $x = 1$, gives

$$y' = \frac{y}{x} + y^2.$$

Set $v = 1/y$. Then

$$v' = -\frac{y'}{y^2} = -\frac{v}{x} - 1, \qquad v' + \frac{v}{x} = -1.$$

Multiplying by x converts the left side into a product derivative: $(xv)' = -x$.

Step 2: Apply the initial condition and evaluate Integration gives

$$xv = -\frac{x^2}{2} + C, \qquad \frac{x}{y} = -\frac{x^2}{2} + C.$$

At $(1, -1)$, $-1 = -1/2 + C$, so $C = -1/2$. Solving for y yields

$$y = -\frac{2x}{x^2 + 1}.$$

This rational expression extends smoothly through zero and directly satisfies the original differential equation there as well. Substitution at $x = -1/2$ gives

$$f(-1/2) = \frac{1}{1 + 1/4} = \frac{4}{5}.$$

For a direct check, $y' = 2(x^2-1)/(x^2+1)^2$, and both sides of $y(1+xy) = xy'$ equal $2x(x^2-1)/(x^2+1)^2$. The negative options have the wrong sign; option (A) loses a factor of two in the reciprocal integration.

Step 3: Conclusion $f(-1/2) = 4/5$. Hence, the correct option is (B).

Quick Tip to Solve: When a differential equation contains $y/x + y^2$, try $1/y$; solve for the constant before substituting the requested input.

8. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$ is equal to : [4 marks]

(A) $\frac{x^5}{2(x^5+x^3+1)^2}+C$

(B) $\frac{-x^{10}}{2(x^5+x^3+1)^2}+C$

(C) $\frac{-x^5}{(x^5+x^3+1)^2}+C$

(D) $\frac{x^{10}}{2(x^5+x^3+1)^2}+C$

Correct Answer: (D) $\frac{x^{10}}{2(x^5+x^3+1)^2}+C$

Solution:

Concept:

- An antiderivative can be identified by differentiating the proposed answer. The chain rule must include the derivative of the complete denominator.
- Write $f(x)=x^5+x^3+1$. Then $f'(x)=5x^4+3x^2$, and use the product rule on $x^{10}f(x)^{-2}/2$.

Step 1: Differentiate the candidate Let $F(x)=x^{10}/(2f(x)^2)$. On every interval on which $f(x)\neq 0$, differentiation gives

$$F'(x)=\frac{5x^9}{f(x)^2}-\frac{x^{10}(5x^4+3x^2)}{f(x)^3}.$$

The second term has a minus sign because differentiating the power -2 produces that sign. Bring both terms to the common denominator before simplifying.

Step 2: Simplify and compare every option The numerator becomes

$$\begin{aligned}5x^9(x^5+x^3+1)-x^{10}(5x^4+3x^2) &= 5x^{14}+5x^{12}+5x^9-5x^{14}-3x^{12}\\ &= 2x^{12}+5x^9.\end{aligned}$$

This is exactly the given integrand. Option B is the negative of this answer, so its derivative has the opposite sign. Options A and C contain x^5 instead of x^{10}; their derivatives have numerator proportional to $x^4(-5x^5-x^3+5)$, which does not match. The arbitrary constant differentiates to zero and cannot correct any of those mismatches.

Step 3: Conclusion

$$\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}\,dx=\frac{x^{10}}{2(x^5+x^3+1)^2}+C.$$

Hence, the correct option is (D).

Quick Tip to Solve: When the options share a squared denominator, differentiate the most plausible numerator first; the quotient rule often produces the cubic denominator immediately.

9. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side $= x$ units and a circle of radius $= r$ units. If the sum of the areas of the square and the circle so formed is minimum, then: [4 marks]

(A) $x = 2r$

(B) $2x = r$

(C) $2x = (\pi + 4)\, r$

(D) $(4 - \pi)\, x = \pi\ r$

Correct Answer: (A) $x = 2r$

Solution:

Concept:

- The total wire length is fixed: the perimeter of the square plus the circumference of the circle is $4x + 2\pi r = 2$.
- The combined area is $A = x^2 + \pi r^2$. Eliminate one variable and use the second derivative to distinguish a minimum from a maximum.

Step 1: Express the area using one variable From the length constraint,

$$r = \frac{1 - 2x}{\pi}, \qquad 0 \le x \le \frac{1}{2}.$$

Therefore the area is

$$A(x) = x^2 + \frac{(1 - 2x)^2}{\pi}.$$

Both terms are areas and remain nonnegative. Using the perimeter constraint is essential: independently varying the two dimensions would ignore the fact that both shapes use the same wire.

Step 2: Find and verify the stationary point Differentiate and set the derivative to zero:

$$A'(x) = 2x - \frac{4(1 - 2x)}{\pi} = 0.$$

Thus $\pi x = 2(1 - 2x)$, or $x = 2(1 - 2x)/\pi = 2r$. Moreover,

$$A''(x) = 2 + \frac{8}{\pi} > 0.$$

The function is strictly convex, so the interior stationary point is its unique global minimum. Explicitly, $r = 1/(4 + \pi)$ and $x = 2/(4 + \pi)$, both positive. Option B reverses the ratio. Substitution of $x = 2r$ in C would require $\pi = 0$, and in D would require $\pi = 8/3$; neither is true. Hence those conditions do not describe the minimizing cut.

Step 3: Conclusion The minimum combined area occurs when the square side equals the circle diameter, $x = 2r$. Hence, the correct option is (A).

Quick Tip to Solve: For a wire split between shapes, write the perimeter constraint before differentiating the sum of their areas.

10. The system of linear equations

$$x + \lambda y - z = 0$$
$$\lambda x - y - z = 0$$
$$x + y - \lambda z = 0$$

has a non-trivial solution for : [4 marks]

(A) infinitely many values of λ.

(B) exactly one value of λ.

(C) exactly two values of λ.

(D) exactly three values of λ.

Correct Answer: (D) exactly three values of λ.

Solution:

Concept:

- A homogeneous square system always has the zero solution. A non-trivial solution exists exactly when its coefficient matrix is singular.
- Thus compute the determinant of the coefficient matrix and count its distinct real zeros, rather than attempting to divide by expressions involving λ.

Step 1: Form the determinant condition The matrix of coefficients, in the variable order (x, y, z), is

$$M = \begin{pmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{pmatrix}.$$

The condition is $\det M = 0$. If the determinant were nonzero, multiplying the homogeneous equations by the inverse would force $x = y = z = 0$, contrary to the requested non-trivial solution.

Step 2: Expand and count the parameter values Expanding along the first row, with alternating cofactor signs, gives

$$\begin{aligned} \det M &= (\lambda + 1) - \lambda(1 - \lambda^2) - (\lambda + 1) \\ &= \lambda^3 - \lambda \\ &= \lambda(\lambda - 1)(\lambda + 1). \end{aligned}$$

Consequently $\lambda = -1, 0, 1$. These are three different real values. As a direct check, nonzero solution vectors at these values are respectively $(0, 1, -1)$, $(1, -1, 1)$, and $(1, 0, 1)$. Each satisfies all three equations at its stated parameter. The determinant polynomial is not identically zero, ruling out infinitely many values. The three distinct roots also rule out exactly one or exactly two values; multiplicity is not being confused with the number of admissible parameters.

Step 3: Conclusion The three admissible values are $\lambda = -1, 0, 1$. Hence, the correct option is (D).

Quick Tip to Solve: For a homogeneous system, test singularity first; a determinant factorization usually counts the permitted parameter values directly.

11. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj $A = A\,A^T$, then $5a + b$ is equal to : [4 marks]

(A) 4

(B) 13

(C) -1

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- For every square matrix, $A \operatorname{adj} A = (\det A)I$. This identity holds even when A is singular.
- The entries of AA^T are dot products of the rows of A. Equal matrices have equal corresponding diagonal and off-diagonal entries.

Step 1: Compute both sides without assuming invertibility Here $\det A = 10a+3b$, while multiplication by the transpose gives

$$AA^T = \begin{pmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{pmatrix}.$$

Since this equals $(10a + 3b)I$, the off-diagonal entry must vanish and the lower diagonal entry must equal the determinant. Therefore

$$15a - 2b = 0, \qquad 10a + 3b = 13.$$

In particular the determinant is 13, so invertibility follows from the condition; it need not be presumed at the beginning.

Step 2: Solve and verify the remaining entry The first relation gives $b = 15a/2$. Substitution into the second gives

$$10a + \frac{45a}{2} = 13, \qquad \frac{65a}{2} = 13.$$

Thus $a = 2/5$ and $b = 3$. The upper diagonal entry is then $25(2/5)^2 + 3^2 = 4 + 9 = 13$, so it too agrees with the required scalar matrix. This check matters because using only two entries must not leave a conflicting third condition. Finally, $5a + b = 2 + 3 = 5$. The other listed numbers, 4, 13 and -1, are not this sum: 13 is the determinant and is therefore a plausible but incorrect distractor.

Step 3: Conclusion The required value is $5a + b = 5$. Hence, the correct option is (D).

Quick Tip to Solve: Use the fixed row of a matrix to find the scalar diagonal entry of AA^T before solving for the unknown row entries.

12. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : [4 marks]

(A) $\frac{2}{\sqrt{3}}$

(B) $\sqrt{3}$

(C) $\frac{4}{3}$

(D) $\frac{4}{\sqrt{3}}$

Correct Answer: (A) $\frac{2}{\sqrt{3}}$

Solution:

Concept:

- For a hyperbola with transverse semiaxis a, conjugate semiaxis b, and focal distance c, $c^2 = a^2 + b^2$ and $e = c/a$.
- The full conjugate axis has length $2b$, the distance between the foci is $2c$, and the full latus rectum has length $2b^2/a$.

Step 1: Translate the length comparison carefully The question compares full lengths, not semiaxes. It says

$$2b = \frac{1}{2}(2c) = c.$$

Hence $c = 2b$. Inserting this in the hyperbola identity yields

$$4b^2 = a^2 + b^2, \qquad a^2 = 3b^2.$$

Since semiaxis lengths are positive, $a = \sqrt{3}\,b$. This ratio already fixes the eccentricity, independently of the overall size of the conic.

Step 2: Compute the eccentricity and check the scale Therefore

$$e = \frac{c}{a} = \frac{2b}{\sqrt{3}\,b} = \frac{2}{\sqrt{3}}.$$

The latus rectum condition is consistent: $2b^2/a = 8$ becomes $2b/\sqrt{3} = 8$, so $b = 4\sqrt{3}$, $a = 12$, and $c = 8\sqrt{3}$. These values give $c^2 = 192 = 144 + 48 = a^2 + b^2$, and the conjugate axis $8\sqrt{3}$ is half the interfocal distance $16\sqrt{3}$. Squaring the proposed eccentricities is a quick distractor check: the necessary value is $e^2 = 4/3$, whereas the other choices give 3, $16/9$, and $16/3$. Thus none satisfies the length comparison.

Step 3: Conclusion The eccentricity is $e = 2/\sqrt{3}$. Hence, the correct option is (A).

Quick Tip to Solve: Write full axis and interfocal lengths explicitly; confusing $2b$ with b changes the eccentricity even when the algebra is correct.

13. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at $(-3, 2)$, then the radius of S is : [4 marks]

(A) 5

(B) 10

(C) $5\sqrt{2}$

(D) $5\sqrt{3}$

Correct Answer: (D) $5\sqrt{3}$

Solution:

Concept:

- Completing the square in a circle equation gives its centre and radius. A diameter has its midpoint at that centre.
- The line joining a circle centre to the midpoint of a chord is perpendicular to the chord. The radius, midpoint distance and half-chord form a right triangle.

Step 1: Find the chord midpoint and half-length Complete the squares in the given equation:

$$(x-2)^2 + (y+3)^2 = 25.$$

Thus the first circle has centre $C = (2, -3)$ and radius 5. Any of its diameters has midpoint C and length 10. When that diameter is regarded as a chord of circle S, its half-length remains 5. The fact that it is a diameter of the first circle does not make it a diameter of S.

Step 2: Use the right triangle in the second circle Let $O = (-3, 2)$ be the centre of S and let P be an endpoint of the chord. The midpoint theorem for a chord gives $OC \perp CP$. Also,

$$OC^2 = (2+3)^2 + (-3-2)^2 = 25 + 25 = 50.$$

By Pythagoras the radius $R = OP$ satisfies

$$R^2 = OC^2 + CP^2 = 50 + 25 = 75.$$

Hence $R = 5\sqrt{3}$, taking the positive square root because a radius is a length. The value 5 is only the half-chord, $5\sqrt{2}$ is only the distance between the two centres, and 10 is the full chord length. These correspond to the other options and none is the hypotenuse required here. No orientation of the chosen diameter changes this right-triangle relation.

Step 3: Conclusion The radius of S is $5\sqrt{3}$ units. Hence, the correct option is (D).

Quick Tip to Solve: When one circle's diameter is another circle's chord, use its centre as the chord midpoint and apply $R^2 = d^2 + (L/2)^2$.

14. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus? [4 marks]

(A) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

(B) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$

(C) $(-3, -9)$

(D) $(-3, -8)$

Correct Answer: (A) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

Solution:

Concept:

- The given side lines meet at one vertex. The diagonals of a rhombus bisect each other at right angles.
- Knowing one vertex and the intersection of the diagonals determines the first diagonal; the perpendicular line through that intersection is the second diagonal.

Step 1: Find the known vertex and diagonal The first side is $y = x + 1$ and the second is $y = 7x - 5$. Equating them gives $6x = 6$, so their intersection is $A = (1, 2)$. Let $O = (-1, -2)$ be the intersection of the diagonals. The slope of AO is

$$\frac{2 - (-2)}{1 - (-1)} = 2.$$

Consequently the other diagonal has slope $-1/2$ and passes through O. Its equation is

$$y + 2 = -\frac{1}{2}(x + 1), \qquad x + 2y + 5 = 0.$$

Step 2: Intersect the other diagonal with the sides On the side $y = 7x - 5$, substitution gives

$$x + 2(7x - 5) + 5 = 0, \qquad 15x = 5.$$

Thus one adjacent vertex is $B = (1/3, -8/3)$. The opposite vertex to A is $2O - A = (-3, -6)$, and that opposite to B is $2O - B = (-7/3, -4/3)$. The latter lies on $y = x + 1$, verifying the remaining side direction. The four vertices are therefore fixed. Only option A occurs in this list: $(-10/3, -7/3)$, $(-3, -9)$, and $(-3, -8)$ are not vertices. Using the complete vertex list also prevents mistaking a point on an extended side for a corner of the rhombus.

Step 3: Conclusion One vertex is $\left(\frac{1}{3}, -\frac{8}{3}\right)$. Hence, the correct option is (A).

Quick Tip to Solve: For a rhombus, join the known vertex to the diagonal midpoint, then intersect its perpendicular through the midpoint with a side.

15. If the $2^{nd}, 5^{th}$ and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is : [4 marks]

(A) 1

(B) $\frac{7}{4}$

(C) $\frac{8}{5}$

(D) $\frac{4}{3}$

Correct Answer: (D) $\frac{4}{3}$

Solution:

Concept:

- In an arithmetic progression with first term a and common difference d, the term in position n is $a + (n-1)d$. The non-constant condition means $d \neq 0$.
- Three consecutive terms u, v, w of a geometric progression satisfy $v^2 = uw$; their common ratio is v/u when $u \neq 0$.

Step 1: Express the three selected terms The positions in the question are not consecutive positions of the A.P. Their terms are therefore $a + d$, $a + 4d$ and $a + 8d$, in that order. Applying the G.P. condition gives

$$(a + 4d)^2 = (a + d)(a + 8d).$$

The spacing of the indices matters: replacing these by consecutive A.P. terms would describe a different problem.

Step 2: Use the non-constant condition Expand both sides and cancel the common terms:

$$a^2 + 8ad + 16d^2 = a^2 + 9ad + 8d^2.$$

Thus

$$d(8d - a) = 0.$$

Since the A.P. is non-constant, $d = 0$ is excluded. It follows that $a = 8d$. The three selected terms consequently become $9d, 12d, 16d$.

Step 3: Calculate and check the ratio As $d \neq 0$, both adjacent ratios exist and agree:

$$r = \frac{12d}{9d} = \frac{4}{3}, \qquad \frac{16d}{12d} = \frac{4}{3}.$$

The ratio 1 would require equal selected terms and hence a constant A.P. The proposed ratios $7/4$ and $8/5$ do not equal either of these verified ratios, so they are excluded.

Step 4: Conclusion The common ratio of the G.P. is $\boxed{4/3}$. Hence, the correct option is (D).

Quick Tip to Solve: For terms selected from an A.P., write their actual indices first, then impose the middle-square condition for the G.P.

16. The sum of all real values of x satisfying the equation $\left(x^2-5x+5\right)^{x^2+4x-60}=1$ is : [4 marks]

(A) 6

(B) 5

(C) 3

(D) -4

Correct Answer: (C) 3

Solution:

Concept:

- For a real power to equal 1, check a base of 1, a zero exponent with nonzero base, and a base of -1 when the exponent is an even integer.
- A positive base other than 1 gives 1 only at exponent zero. A negative base, whenever its real power is defined, can have magnitude 1 only if the base itself has magnitude 1.

Step 1: Check the base equal to 1 Put $x^2-5x+5=1$. Then

$$x^2-5x+4=(x-1)(x-4)=0,$$

so $x=1,4$. Both are admissible because 1 raised to any real exponent remains 1. These values must be retained even though their exponents need not vanish.

Step 2: Check the exponent equal to zero Put

$$x^2+4x-60=(x+10)(x-6)=0.$$

This gives $x=-10,6$. Their bases are respectively 155 and 11, both nonzero, so both powers are defined and equal 1. A zero base cannot supply any additional solution: positive powers of zero give zero, while zero or negative exponents are not admissible here.

Step 3: Check the negative unit base For $x^2-5x+5=-1$, factor

$$x^2-5x+6=(x-2)(x-3)=0.$$

At $x=2$ the exponent is -48, an even integer, giving $(-1)^{-48}=1$. At $x=3$ it is -39, an odd integer, giving -1, so 3 is excluded. No other negative base can have a defined real power of magnitude 1 with nonzero exponent.

Step 4: Add the distinct admissible values The complete solution set is $\{-10,1,2,4,6\}$. Its sum is

$$-10+1+2+4+6=3.$$

Thus 6, 5 and -4, the other proposed sums, do not represent all the admissible values.

Step 5: Conclusion The sum of all real solutions is $\boxed{3}$. Hence, the correct option is (C).

Quick Tip to Solve: In equations of the form $A(x)^{B(x)}=1$, remember to test $A=-1$ separately and check the parity of its exponent.

17. If all the words (with or without meaning) having five letters,formed using the letters of the word SMALL and arranged as in a dictionary, then the position of the word SMALL is : [4 marks]

(A) 46^{th}

(B) 59^{th}

(C) 52^{nd}

(D) 58^{th}

Correct Answer: (D) 58^{th}

Solution:

Concept:

- Dictionary order compares words at the first letter where they differ. The letters here have the alphabetical order $A < L < M < S$.
- The two L letters are identical. Arrangements of four letters containing two L letters number $4!/2!$, rather than $4!$. The rank is one more than the number of preceding words.

Step 1: Count words whose first letter precedes S Words beginning with A leave L,L,M,S, giving $4!/2! = 12$ arrangements. Words beginning with L leave A,L,M,S, all distinct, giving $4! = 24$. Words beginning with M leave A,L,L,S, giving another $4!/2! = 12$. Therefore

$$12 + 24 + 12 = 48$$

words precede every word beginning with S. The repeated L must not be treated as two separately labelled choices.

Step 2: Fix S and count second letters preceding M The target begins SM. After fixing S, the available letters are A,L,L,M. A second letter A gives the prefix SA, with L,L,M remaining, so it contributes $3!/2! = 3$. A second letter L gives SL, with A,L,M remaining, so it contributes $3! = 6$. Both prefixes precede SM, adding

$$3 + 6 = 9$$

words.

Step 3: Complete the target prefix and obtain its rank After SM is fixed, the remaining letters are A,L,L. The target takes A next, which is the smallest available letter. The final LL is forced. Thus there are no further preceding words in the SM block before SMALL itself. The number before the target is $48 + 9 = 57$, and its one-based position is

$$57 + 1 = 58.$$

Hence neither 46, 59 nor 52 is the dictionary rank; in particular, adding two instead of one after counting preceding words would incorrectly give 59.

Step 4: Conclusion The position of SMALL is $\boxed{58^{\text{th}}}$. Hence, the correct option is (D).

Quick Tip to Solve: Count only prefixes smaller than the target, divide by factorials for repeated remaining letters, and add one at the very end.

18. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$**, and** $S = \{x \in \mathbf{R} : f(x) = f(-x)\}$**; then** S : [4 marks]

(A) is an empty set.

(B) contains exactly one element.

(C) contains exactly two elements.

(D) contains more than two elements.

Correct Answer: (C) contains exactly two elements.

Solution:

Concept:

- The identity holds for every nonzero input, so replacing x by $1/x$ supplies a second equation in $f(x)$ and $f(1/x)$.
- The equality $f(x) = f(-x)$ is imposed only at the points belonging to S; it does not assert that the whole function is even. The input zero is excluded throughout.

Step 1: Obtain a pair of linear equations The supplied relation and the relation at reciprocal input are

$$f(x) + 2f(1/x) = 3x,$$

$$2f(x) + f(1/x) = \frac{3}{x}.$$

Both are legitimate for $x \neq 0$, since the reciprocal is also nonzero. These are simultaneous linear equations for the two function values, so no assumption about continuity or a polynomial form is necessary.

Step 2: Eliminate the reciprocal function value Multiply the second equation by 2 and subtract the first. This yields

$$3f(x) = \frac{6}{x} - 3x, \qquad f(x) = \frac{2}{x} - x.$$

A direct check gives $f(1/x) = 2x - 1/x$, and substitution recovers $3x$, confirming the derived formula rather than merely guessing it.

Step 3: Solve the defining condition for S At the opposite input,

$$f(-x) = -\frac{2}{x} + x = -f(x).$$

Consequently equality of the two values requires $f(x) = 0$. Since zero is excluded from the domain, we may multiply by x:

$$\frac{2}{x} - x = 0 \quad \Longrightarrow \quad x^2 = 2.$$

Both $\sqrt{2}$ and $-\sqrt{2}$ are real nonzero inputs and satisfy the equation. There are no further roots because the resulting quadratic has exactly these two roots. Thus S is neither empty nor a singleton, and it cannot contain more than two elements.

Step 4: Conclusion $S = \{-\sqrt{2}, \sqrt{2}\}$ contains exactly two elements. Hence, the correct option is (C).

Quick Tip to Solve: For a reciprocal functional equation, write the identity at $1/x$ and eliminate one function value before imposing any extra condition.

19. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is: [4 marks]

(A) 7

(B) 9

(C) 3

(D) 5

Correct Answer: (A) 7

Solution:

Concept:

- The sum-to-product identity is $\cos u + \cos v = 2\cos((u+v)/2)\cos((u-v)/2)$. Pairing outer and inner angles gives a common factor.
- A product vanishes when at least one factor vanishes. Count the union of the root sets, so an angle satisfying two factors is counted only once.

Step 1: Factor the sum without dividing by a variable factor Pair $\cos x$ with $\cos 4x$, and $\cos 2x$ with $\cos 3x$. Then

$$\cos x + \cos 4x = 2\cos\frac{5x}{2}\cos\frac{3x}{2},$$

$$\cos 2x + \cos 3x = 2\cos\frac{5x}{2}\cos\frac{x}{2}.$$

Applying the same identity to the two remaining cosines transforms the complete sum into

$$4\cos\frac{5x}{2}\cos x\cos\frac{x}{2} = 0.$$

No zero factor has been cancelled, so every solution is retained.

Step 2: List the roots of each factor in the given interval For $0 \le x < 2\pi$, the first factor gives

$$x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}.$$

The factor $\cos x$ gives $x = \pi/2, 3\pi/2$. The factor $\cos(x/2)$ gives only $x = \pi$. Neither endpoint supplies an additional root: at zero the original sum is 4, and 2π is excluded in any case.

Step 3: Remove overlaps and count The root π from the last factor already occurs in the first list. Neither $\pi/2$ nor $3\pi/2$ belongs to that list. Thus the union contains

$$5 + 2 + 1 - 1 = 7$$

distinct angles. Counting a repeated root twice, or overlooking the roots from $\cos x$, changes the count incorrectly. The other proposed counts 9, 3 and 5 omit or add angles not present in this union.

Step 4: Conclusion The number of distinct real values is $\boxed{7}$. Hence, the correct option is (A).

Quick Tip to Solve: Factor a trigonometric sum before solving, and write the root sets explicitly to remove duplicates such as $x = \pi$.

20. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\sqrt{3}}{2}\left(\vec{b} + \vec{c}\right)$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is: [4 marks]

(A) $\frac{2\pi}{3}$

(B) $\frac{5\pi}{6}$

(C) $\frac{3\pi}{4}$

(D) $\frac{\pi}{2}$

Correct Answer: (B) $\frac{5\pi}{6}$

Solution:

Concept:

- The vector triple-product identity is $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$. Its order determines the minus sign.
- Nonparallel vectors $\vec{b}, \vec{c}$ are linearly independent, so their coefficients in equal linear combinations agree. For unit vectors, their scalar product is the cosine of the included angle.

Step 1: Expand the given vector identity Using the stated order of the cross products gives

$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{\sqrt{3}}{2}\vec{b} + \frac{\sqrt{3}}{2}\vec{c}.$$

Move all terms to one side and collect the independent directions:

$$\left(\vec{a} \cdot \vec{c} - \frac{\sqrt{3}}{2}\right)\vec{b} + \left(-\vec{a} \cdot \vec{b} - \frac{\sqrt{3}}{2}\right)\vec{c} = \vec{0}.$$

Comparing individual Cartesian components is unnecessary.

Step 2: Compare coefficients in the independent directions Since $\vec{b}$ is not parallel to $\vec{c}$, neither direction can cancel a nonzero multiple of the other. Each coefficient must therefore be zero. Hence

$$\vec{a} \cdot \vec{c} = \frac{\sqrt{3}}{2}, \qquad \vec{a} \cdot \vec{b} = -\frac{\sqrt{3}}{2}.$$

The negative sign in the second equation is essential; dropping it would produce an acute angle instead of the required obtuse one.

Step 3: Convert the required dot product into an angle Let θ be the angle between $\vec{a}$ and $\vec{b}$, with $0 \le \theta \le \pi$. Both vectors are unit vectors, so

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = -\frac{\sqrt{3}}{2}.$$

Thus $\theta = 5\pi/6$. The cosines of the other options $2\pi/3, 3\pi/4, \pi/2$ are respectively $-1/2, -1/\sqrt{2}, 0$, none equal to this dot product.

Step 4: Conclusion The angle between $\vec{a}$ and $\vec{b}$ is $\boxed{5\pi/6}$. Hence, the correct option is (B).

Quick Tip to Solve: Expand a vector triple product before comparing coefficients, and keep the minus sign on the coefficient of its last vector.

21. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + ...is\ \frac{16}{5}m$, then m is equal to : [4 marks]

(A) 100

(B) 99

(C) 102

(D) 101

Correct Answer: (D) 101

Solution:

Concept:

- The unsquared mixed numbers are $8/5, 12/5, 16/5, 20/5, 24/5, \ldots$, an A.P. of common difference $4/5$.
- The sum of consecutive squares is $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$. Apply it after factoring out the common square $(4/5)^2$.

Step 1: Identify the general term and its endpoints The first mixed number is $1 + 3/5 = 8/5$, and the next is $2 + 2/5 = 12/5$. Thus the unsquared n-th number is

$$a_n = \frac{8}{5} + (n-1)\frac{4}{5} = \frac{4}{5}(n+1).$$

For ten terms, $n+1$ runs from 2 through 11, not from 1 through 10. This endpoint shift matters in the square sum even though there are still exactly ten terms.

Step 2: Sum the squares using the shifted index Let S denote the requested sum. Factoring out the common square gives

$$S = \frac{16}{25}\sum_{k=2}^{11} k^2 = \frac{16}{25}\left(\sum_{k=1}^{11} k^2 - 1\right).$$

Evaluate the standard formula:

$$\sum_{k=1}^{11} k^2 = \frac{11 \cdot 12 \cdot 23}{6} = 506.$$

Subtracting the unwanted first square leaves 505, so

$$S = \frac{16}{25} \cdot 505.$$

This sums the squares of the terms, rather than squaring their total.

Step 3: Compare with the specified multiple The question defines the same sum by $S = (16/5)m$. Consequently

$$m = \frac{5}{16}S = \frac{5}{16} \cdot \frac{16}{25} \cdot 505 = \frac{505}{5} = 101.$$

As a numerical check, S is 323.2, which equals $(16/5) \cdot 101$. The options 100, 99 and 102 give different multiples and do not agree with the exact square sum.

Step 4: Conclusion The required value is $\boxed{m = 101}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert mixed numbers to improper fractions first; after taking out the common factor, check the first and last indices of the square sum.

22. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is : [4 marks]

(A) 243

(B) 729

(C) 64

(D) 2187

Correct Answer: (B) 729

Solution:

Concept:

- For three summands, the uncollected multinomial expansion has one term for each triple of nonnegative integers (a, b, c) with $a+b+c=n$. Their number is $\binom{n+2}{2}$.
- The sum of coefficients of a polynomial in $1/x$ is its value at $x=1$, because every power of $1/x$ then becomes one.

Step 1: Count the multinomial terms The stated term count uses the multinomial convention, before terms with the same power of x are collected. Hence

$$\frac{(n+1)(n+2)}{2}=28, \qquad n^2+3n-54=0.$$

Factoring gives $(n-6)(n+9)=0$. The exponent is a nonnegative integer, so $n=6$; the root -9 cannot be an expansion exponent of this kind. Combining equal powers first would give a different term-count convention, which is not the one used in this question.

Step 2: Evaluate the sum of coefficients Set $x=1$ in the original expression, retaining the negative sign in its middle summand:

$$S=(1-2+4)^6=3^6=729.$$

This substitution includes every coefficient, including negative ones, without computing any individual multinomial coefficient. Option (A), $243=3^5$, uses the wrong exponent; option (D), $2187=3^7$, likewise uses a different exponent. Option (C), $64=2^6$, does not use the actual sum of the three base coefficients, which is 3 rather than 2.

Step 3: Conclusion The exponent is $n=6$ and the coefficient sum is 729. Hence, the correct option is (B).

Quick Tip to Solve: For a multinomial coefficient sum, find the exponent from the term count first, then put the variable equal to 1; do not replace negative coefficients by their magnitudes.

23. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)\cdots(3n)}{n^{2n}}\right)^{1/n}$ **is equal to:** [4 marks]

(A) $9/e^2$

(B) $3\log 3 - 2$

(C) $18/e^4$

(D) $27/e^2$

Correct Answer: (D) $27/e^2$

Solution:

Concept:

- A positive product raised to $1/n$ is simplified by taking its natural logarithm: the logarithm converts products into sums.
- The limit $\frac{1}{n}\sum_{k=1}^{2n} f(1+k/n)$ is a Riemann sum for $\int_1^3 f(t)\,dt$; here the mesh is $1/n$.

Step 1: Take logarithms of the product Let the requested limit be L. Every factor is positive, so the logarithm is defined. There are exactly $2n$ factors from $n+1$ through $3n$. Dividing each by n accounts for the denominator n^{2n}, giving

$$\log L = \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{2n}\log\left(1+\frac{k}{n}\right).$$

The sample points run from $1+1/n$ to 3, so the integration interval has length 2, not 1.

Step 2: Evaluate the integral and exponentiate The function $\log t$ is continuous on $[1,3]$, so its Riemann sums converge. Therefore

$$\log L = \int_1^3 \log t\,dt = [t\log t - t]_1^3 = 3\log 3 - 2.$$

The question asks for L, rather than its logarithm. Exponentiating yields

$$L = \exp(3\log 3 - 2) = \frac{27}{e^2}.$$

Option (B) is the intermediate logarithm and is not the product limit. Option (A) would require a logarithmic numerator $2\log 3$ instead of $3\log 3$. Option (C) has both a different multiplicative constant and exponential power; neither results from the evaluated integral.

Step 3: Conclusion The supplied key gives a different answer, but the printed product gives $\log L = 3\log 3 - 2$, so its value is $27/e^2$, not $9/e^2$. The required product limit is $\frac{27}{e^2}$. Hence, the correct option is (D).

Quick Tip to Solve: Count the factors before taking logarithms: here 2n factors create an integral from 1 to 3, and the integral gives the logarithm of the requested limit.

24. The Boolean expression

$(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ **is equivalent to :** [4 marks]

(A) $\sim p \wedge q$

(B) $p \wedge q$

(C) $p \vee q$

(D) $p \vee \sim q$

Correct Answer: (C) $p \vee q$

Solution:

Concept:

- Disjunction and conjunction obey absorption: $q \vee (r \wedge q) = q$. A term already implied by another disjunct can be removed.
- The distributive law gives $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$, and $q \vee \sim q$ is always true.

Step 1: Absorb the redundant conjunction Write the expression as

$$E = (p \wedge \sim q) \vee \big[q \vee (\sim p \wedge q)\big].$$

Whenever $\sim p \wedge q$ is true, q is already true. Thus the bracket is simply q, giving $E = (p \wedge \sim q) \vee q$. This step uses absorption and does not assume particular truth values for either proposition.

Step 2: Distribute and identify the equivalent expression Apply the distributive law to obtain

$$E = (p \vee q) \wedge (\sim q \vee q) = (p \vee q) \wedge \text{True} = p \vee q.$$

The resulting statement is false only when both propositions are false. To check the alternatives, let p be true and q false: the original expression is true, but both $\sim p \wedge q$ and $p \wedge q$ are false, eliminating (A) and (B). Let both propositions be false: the original is false, whereas $p \vee \sim q$ is true, eliminating (D). Option (C) agrees for all four possible truth-value assignments, as the identities establish.

Step 3: Conclusion The expression is equivalent to $p \vee q$. Hence, the correct option is (C).

Quick Tip to Solve: In a long OR expression, first absorb every conjunction containing a disjunct already present; then distribute only the terms that remain.

25. If the line, $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the planes, $lx + my - z = 9$, then $l^2 + m^2$ is equal to : [4 marks]

(A) 5

(B) 2

(C) 26

(D) 18

Correct Answer: (B) 2

Solution:

Concept:

- A line lies entirely in a plane when one point on the line belongs to the plane and its direction vector is perpendicular to the plane normal.
- Equivalently, substituting a parametric form of the line into the plane equation must give an identity in the parameter: both its constant and variable coefficients must match.

Step 1: Parameterize the line and substitute Set the common ratio equal to t. The coordinates of every point on the line are

$$x = 3 + 2t, \qquad y = -2 - t, \qquad z = -4 + 3t.$$

Substitution into $lx + my - z = 9$ yields

$$l(3 + 2t) + m(-2 - t) - (-4 + 3t) = 9,$$

$$(3l - 2m + 4) + (2l - m - 3)t = 9.$$

Since the whole line is contained in the plane, this equality must hold for every real t, not just for one intersection point.

Step 2: Solve the two containment conditions Equating the coefficient of t to zero and the constant term to 9 gives

$$2l - m = 3, \qquad 3l - 2m = 5.$$

From the first equation, $m = 2l - 3$. Substituting into the second gives $3l - 4l + 6 = 5$, hence $l = 1$ and $m = -1$. The line point $(3, -2, -4)$ then gives $3 + 2 + 4 = 9$, and its direction $(2, -1, 3)$ has zero dot product with $(1, -1, -1)$, confirming containment. Finally, $l^2 + m^2 = 1 + 1 = 2$. The other listed values, 5, 26 and 18, do not equal the uniquely determined sum.

Step 3: Conclusion $l = 1$, $m = -1$, so $l^2 + m^2 = 2$. Hence, the correct option is (B).

Quick Tip to Solve: For a line contained in a plane, impose both point membership and zero normal-direction dot product; using just one condition leaves the plane undetermined.

26. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is : [4 marks]

(A) 6

(B) 10

(C) 20

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- For a vertical pillar of height h, viewed from horizontal distance d, the angle of elevation satisfies $\tan\theta = h/d$.
- At uniform speed, travel times are proportional to distances. The pillar height and walking speed can therefore be eliminated by comparing the two observation positions.

Step 1: Express both distances using the height Let O be the foot of the pillar, and let its height be $h > 0$. Since the man walks towards it along the same straight path, the order is A, B, O. From the two right triangles,

$$AO = \frac{h}{\tan 30^\circ} = \sqrt{3}h, \qquad BO = \frac{h}{\tan 60^\circ} = \frac{h}{\sqrt{3}}.$$

Consequently $AO = 3BO$. The larger elevation angle corresponds to the smaller horizontal distance, consistent with the direction of motion.

Step 2: Compare the covered and remaining distances The distance already covered in 10 minutes is

$$AB = AO - BO = 3BO - BO = 2BO.$$

If the uniform speed is v, then $10v = 2BO$, so the remaining time is

$$t = \frac{BO}{v} = \frac{10}{2} = 5 \text{ minutes}.$$

The 10-minute interval covers twice the remaining distance; it does not cover the remaining distance itself. Option (B) would incorrectly equate AB with BO, and option (C) reverses their ratio. Option (A), 6 minutes, also contradicts the exact tangent ratio. No numerical value for the height is needed because it cancels from the distance comparison.

Step 3: Conclusion The man takes 5 minutes to reach the pillar from B. Hence, the correct option is (D).

Quick Tip to Solve: Convert elevation angles into horizontal-distance ratios before using speed: the tangent ratio gives AO:BO = 3:1, so AB:BO = 2:1.

27. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on : [4 marks]

(A) a circle

(B) an ellipse which is not a circle

(C) a hyperbola

(D) a parabola

Correct Answer: (D) a parabola

Solution:

Concept:

- Completing squares gives the centre and radius of a circle. External tangency means the centre distance equals the sum of the two radii.
- A circle tangent to the x-axis with centre (h, k) has radius $|k|$. Eliminating that radius yields a quadratic relation for the centre coordinates.

Step 1: Find the fixed circle and impose tangency The fixed circle can be written as

$$(x - 4)^2 + (y - 4)^2 = 36.$$

Its centre is $(4, 4)$ and radius is 6. Let a variable circle have centre (h, k) and radius $r > 0$. Tangency to the x-axis gives $r = |k|$. External tangency to the fixed circle requires

$$\sqrt{(h - 4)^2 + (k - 4)^2} = 6 + |k|.$$

Both sides are positive, so squaring preserves this distance condition.

Step 2: Eliminate the radius and classify the locus For centres above the axis, $k > 0$, squaring and cancelling k^2 gives

$$(h - 4)^2 + (k - 4)^2 = (k + 6)^2,$$

$$(h - 4)^2 = 20(k + 1).$$

This is a parabola with vertical axis. Centres below the axis satisfy $k < 0$, and the same calculation with $|k| = -k$ gives $(h - 4)^2 = 4(5 - k)$, again a parabola, with the appropriate half-plane restriction. Thus either admissible family has parabolic form. There is only one squared coordinate after cancellation, whereas a circle or ellipse would require two squared coordinates of the same sign, and a hyperbola would require two of opposite signs. Options (A), (B) and (C) therefore do not give the conic type of these families.

Step 3: Conclusion The admissible centres follow parabolic loci. Hence, the correct option is (D).

Quick Tip to Solve: Replace a circle tangent to an axis by its centre and radius r=|k|; combine this with centre distance = sum of radii before classifying the resulting quadratic.

28. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y+6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is: [4 marks]

(A) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

(B) $x^2 + y^2 - 4x + 9y + 18 = 0$

(C) $x^2 + y^2 - 4x + 8y + 12 = 0$

(D) $x^2 + y^2 - x + 4y - 12 = 0$

Correct Answer: (C) $x^2 + y^2 - 4x + 8y + 12 = 0$

Solution:

Concept:

- The centre of $x^2 + (y+6)^2 = 1$ is $C = (0, -6)$. Minimizing distance is equivalent to minimizing its square, avoiding a square root.
- A point on $y^2 = 8x$ has coordinates $(y^2/8, y)$. A circle with centre (a, b) passing through C has radius equal to the distance from (a, b) to C.

Step 1: Minimize the squared distance Write a general parabola point as $P = (u^2/8, u)$, where u is real. Its squared distance from C is

$$D(u) = \frac{u^4}{64} + (u+6)^2.$$

Differentiation gives

$$D'(u) = \frac{u^3}{16} + 2u + 12.$$

Setting this equal to zero gives $u^3 + 32u + 192 = 0$, which factors as $(u+4)(u^2 - 4u + 48) = 0$. The quadratic has negative discriminant, so the only real critical point is $u = -4$. Also $D''(u) = 3u^2/16 + 2 > 0$, so this is the unique global minimum.

Step 2: Form the required circle At $u = -4$, the nearest point is $P = (2, -4)$. The required circle has this centre and passes through $C = (0, -6)$, so

$$R^2 = (2-0)^2 + (-4+6)^2 = 8.$$

Its equation is therefore

$$(x-2)^2 + (y+4)^2 = 8,$$

$$x^2 + y^2 - 4x + 8y + 12 = 0.$$

The radius 1 of the original circle does not enter this last construction: the question requires passage through its centre, not tangency to that circle. Options (A), (B) and (D) have different centres, as their linear coefficients show; only (C) has centre $(2, -4)$ and squared radius 8.

Step 3: Conclusion The required circle is $x^2 + y^2 - 4x + 8y + 12 = 0$. Hence, the correct option is (C).

Quick Tip to Solve: Parameterize the parabola by y and minimize the squared distance; after locating P, compute PC rather than using the radius of the given circle.

29. The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along the line $x = y = z$ is : [4 marks]

(A) $\frac{10}{\sqrt{3}}$

(B) $\frac{20}{3}$

(C) $3\sqrt{10}$

(D) $10\sqrt{3}$

Correct Answer: (D) $10\sqrt{3}$

Solution:

Concept:

- A distance measured in a specified direction is found along the line through the point parallel to that direction, rather than along the plane normal.
- The direction ratios of $x = y = z$ are $(1, 1, 1)$. A displacement $t(1, 1, 1)$ has length $\sqrt{3}|t|$.

Step 1: Write the line through the given point Let $P = (1, -5, 9)$. A point on the line through P, parallel to the specified line, is

$$Q = (1 + t, -5 + t, 9 + t).$$

The same parameter is added to all three coordinates because the direction ratios are equal. The line $x = y = z$ itself need not contain P; it specifies the direction in which the separation is measured.

Step 2: Find the intersection with the plane Substitute the coordinates of Q into $x - y + z = 5$:

$$(1 + t) - (-5 + t) + (9 + t) = 5.$$

Thus $15 + t = 5$, giving $t = -10$ and $Q = (-9, -15, -1)$. Direct substitution gives $-9 + 15 - 1 = 5$, confirming that the endpoint lies in the plane.

Step 3: Calculate the requested distance The displacement is $Q - P = (-10, -10, -10)$, so

$$PQ = \sqrt{(-10)^2 + (-10)^2 + (-10)^2} = 10\sqrt{3}.$$

The value $10/\sqrt{3}$ in (A) is the perpendicular distance and therefore answers a different measurement. Neither $20/3$ nor $3\sqrt{10}$ equals the length of this displacement, excluding (B) and (C).

Step 4: Conclusion The distance in the specified direction is $\boxed{10\sqrt{3}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a directional point-to-plane distance, parametrize the line through the point first; the usual perpendicular-distance formula applies only along the plane normal.

30. Consider :

$\mathbf{f}(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$.

A normal to $y = \mathbf{f}(x)$ at $x = \frac{\pi}{6}$ also passes through the point: [4 marks]

(A) $\left(\frac{\pi}{6}, 0\right)$

(B) $\left(\frac{\pi}{4}, 0\right)$

(C) $(0, 0)$

(D) $\left(0, \frac{2\pi}{3}\right)$

Correct Answer: (D) $\left(0, \frac{2\pi}{3}\right)$

Solution:

Concept:

- On the given interval, the half-angle identity gives $\sqrt{(1+\sin x)/(1-\sin x)} = \tan(\pi/4 + x/2)$.
- The slope of the normal is the negative reciprocal of the nonzero tangent slope: $m_n = -1/f\prime(x)$.

Step 1: Simplify the inverse tangent using its range Because $0 < x < \pi/2$, we have $\cos x > 0$ and

$$\sqrt{\frac{1+\sin x}{1-\sin x}} = \frac{1+\sin x}{\cos x} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right).$$

The angle $\pi/4 + x/2$ lies between $\pi/4$ and $\pi/2$, inside the principal range of inverse tangent. Therefore $f(x) = \pi/4 + x/2$, with no branch adjustment.

Step 2: Form the normal at the specified point The derivative is $f\prime(x) = 1/2$, so the normal slope is -2. At $x = \pi/6$, the ordinate is

$$f\left(\frac{\pi}{6}\right) = \frac{\pi}{4} + \frac{\pi}{12} = \frac{\pi}{3}.$$

The point-slope equation of the normal is consequently

$$y - \frac{\pi}{3} = -2\left(x - \frac{\pi}{6}\right), \qquad y = -2x + \frac{2\pi}{3}.$$

Step 3: Check the listed points Setting $x = 0$ gives $y = 2\pi/3$, exactly the point in (D). At $x = \pi/6$, the normal has ordinate $\pi/3$, not zero, so (A) fails. At $x = \pi/4$, its ordinate is $\pi/6$, excluding (B). Its nonzero vertical intercept also excludes the origin in (C).

Step 4: Conclusion The normal passes through $\boxed{(0, 2\pi/3)}$. Hence, the correct option is (D).

Quick Tip to Solve: Before differentiating an inverse trigonometric expression, simplify it with half-angle identities and check that the resulting angle lies in the principal range.

Physics (120 marks)

31. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be: [4 marks]

(A) $= v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

(B) $= v\left(\frac{3}{4}\right)^{\frac{1}{2}}$

(C) $> v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

(D) $< v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

Correct Answer: (C) $> v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where $\phi > 0$ is the work function.
- The maximum kinetic energy is $K_{\max} = mv^2/2$; photon energy scales as the reciprocal of wavelength, but the work function is unchanged.

Step 1: Write the initial and final energy equations For the original radiation, the fastest electron satisfies

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi.$$

After the wavelength becomes $3\lambda/4$, the photon energy increases by a factor $4/3$. If u is the new maximum speed, then

$$\frac{1}{2}mu^2 = \frac{4hc}{3\lambda} - \phi.$$

The same metal has the same work function in both equations; this energy cost must not be multiplied by $4/3$.

Step 2: Eliminate the incident photon energy From the first equation, $hc/\lambda = mv^2/2 + \phi$. Inserting this into the second gives

$$\frac{1}{2}mu^2 = \frac{4}{3}\left(\frac{1}{2}mv^2 + \phi\right) - \phi = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{\phi}{3}.$$

Consequently

$$u^2 = \frac{4}{3}v^2 + \frac{2\phi}{3m} > \frac{4}{3}v^2.$$

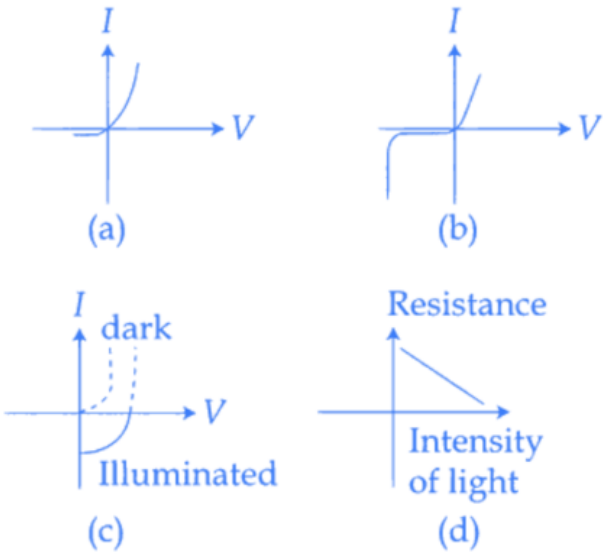
The inequality is strict because an ordinary photocathode has positive work function.

Step 3: Compare with the options Taking positive square roots yields $u > v\sqrt{4/3}$. Equality in (A) would require zero work function. Option (B) predicts a decrease of speed when photon energy increases and contradicts the energy balance. Option (D) has the opposite inequality to the positive extra term derived above.

Step 4: Conclusion The new maximum speed is $\boxed{u > v\sqrt{4/3}}$. Hence, the correct option is (C).

Quick Tip to Solve: Never scale photoelectron speed directly with photon energy: subtract the unchanged work function before comparing kinetic energies.

32. Identify the semiconductor devices whose characteristics are given below, in the order $(a), (b), (c), (d)$: [4 marks]



(A) Simple diode, Zener diode, Solar cell, Light dependent resistance

(B) Zener diode, Simple diode, Light dependent resistance, Solar cell

(C) Solar cell, Light dependent resistance, Zener diode, simple diode

(D) Zener diode, Solar cell, simple diode, Light dependent resistance

Correct Answer: (A) Simple diode, Zener diode, Solar cell, Light dependent resistance

Solution:

Concept:

- A forward-biased ordinary diode has a rapidly increasing current; a Zener diode also has a sharp reverse-breakdown region.
- A solar cell supplies photocurrent under illumination. In a light dependent resistor, increased illumination creates carriers and reduces resistance.

Step 1: Identify the two diode characteristics Graph (a) shows very small reverse current and a rising forward current. There is no marked reverse-breakdown branch in the displayed range, so it is the characteristic of a simple semiconductor diode. Graph (b) has a similar forward branch but also a steep reverse-current rise at a particular negative voltage. This controlled reverse-breakdown feature identifies a Zener diode, used for voltage regulation.

Step 2: Identify the light-sensitive devices In graph (c), the illuminated current-voltage curve is displaced below the dark curve. A nonzero current exists at zero applied voltage, and the curve crosses the positive voltage axis: the device can deliver electrical power under illumination. This is a solar cell. In graph (d), the plotted quantity is resistance, which falls as light intensity rises. That is the characteristic signature of a light dependent resistance, rather than the current-voltage graph of a diode.

Step 3: Match the complete sequence The required order is therefore simple diode, Zener diode, solar cell and light dependent resistance. Option (B) interchanges the first two and also the final two

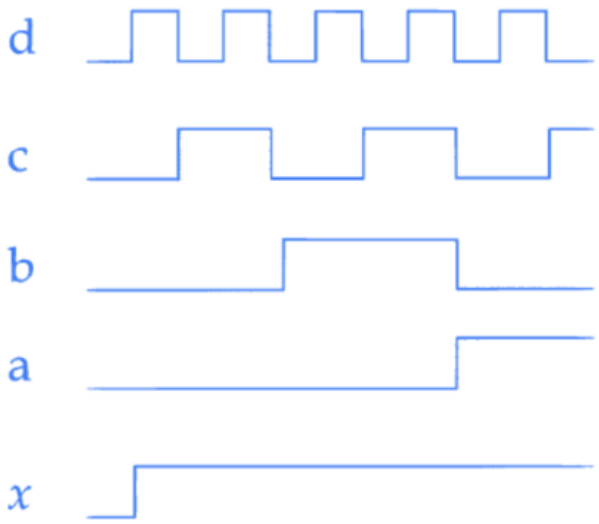
devices. Option (C) incorrectly starts with a solar cell and assigns light dependent resistance to a diode curve. Option (D) incorrectly calls (a) a Zener diode and (b) a solar cell. Only (A) assigns all four characteristic shapes correctly.

Step 4: Conclusion The sequence is simple diode, Zener diode, solar cell, light dependent resistance. Hence, the correct option is (A).

Quick Tip to Solve: Read both axes before identifying a device: reverse breakdown singles out a Zener diode, whereas falling resistance with illumination singles out an LDR.

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33. If a, b, c, d are inputs to a gate and x is its output, then, as per the following time graph, the gate is : [4 marks]



(A) OR

(B) $NAND$

(C) NOT

(D) AND

Correct Answer: (A) OR

Solution:

Concept:

- A multi-input OR gate produces output 1 whenever at least one input is 1: $x = a + b + c + d$, where addition denotes Boolean OR.
- An AND gate needs every input high; NAND is its complement. A NOT gate inverts a single input.

Step 1: Read the first output transition Initially all four inputs a, b, c, d are low and the output x is low. At the first rising edge of d, the other three inputs remain low, but the output becomes high. A single high input is therefore sufficient to produce a high output. This directly rules out AND, which would still produce zero for the input state $(a, b, c, d) = (0, 0, 0, 1)$.

Step 2: Check the subsequent time intervals When the first pulse of d ends, c becomes high, so at least one input remains high. In the central part of the graph, b remains high while c and d pulse. Later a becomes high before those earlier high intervals disappear. Thus the output remains continuously high after the first transition, just as the OR of the four input waveforms requires. The high intervals overlap; the gate has no memory or latching action.

Step 3: Exclude the remaining alternatives For NAND, all-zero inputs would give a high output, contrary to the initial low level shown. A NOT gate has one input and would invert that input; it cannot implement the observed four-input dependence. The waveform agrees with $x = a \vee b \vee c \vee d$ at both the initial all-low state and every subsequent interval, so OR is the only consistent listed gate.

Step 4: Conclusion The gate is a four-input OR gate, with $\boxed{x = a \vee b \vee c \vee d}$. Hence, the correct option is (A).

Quick Tip to Solve: Use an interval where all inputs are zero and another where exactly one is high; these two tests quickly distinguish OR, AND and NAND.

34. In an experiment for determination of refractive index of glass of a prism by $i-\delta$, plot it was found thata ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index? [4 marks]

(A) 1.7

(B) 1.8

(C) 1.5

(D) 1.6

Correct Answer: (C) 1.5

Solution:

Concept:

- For a prism, the deviation is $\delta = i + e - A$, where A is the prism angle and i, e are measured from the normals.
- At minimum deviation, $\mu = \sin[(A+\delta_{\min})/2]/\sin(A/2)$. Any measured deviation provides an upper bound on $\delta_{\min}$.

Step 1: Determine the prism angle The measured incident angle, emergent angle and deviation give

$$A = i + e - \delta = 35^\circ + 79^\circ - 40^\circ = 74^\circ.$$

The value 40° is a measured deviation at the stated incidence, not automatically the minimum deviation. Indeed, the unequal values of i and e show that this ray is not the symmetric minimum-deviation ray.

Step 2: Use the minimum-deviation inequality Since the minimum deviation cannot exceed an observed deviation, $\delta_{\min} \le 40^\circ$. On the relevant angular interval the sine in the prism formula

increases with $\delta_{\min}$, and hence

$$\mu \le \frac{\sin[(74^\circ + 40^\circ)/2]}{\sin(74^\circ/2)} = \frac{\sin 57^\circ}{\sin 37^\circ}.$$

For the nonsymmetric measured ray the bound is strict, but using it as an upper limit gives the requested maximum possible estimate.

Step 3: Evaluate and compare the choices Using $\sin 57^\circ \simeq 0.839$ and $\sin 37^\circ \simeq 0.602$, the upper limiting estimate is

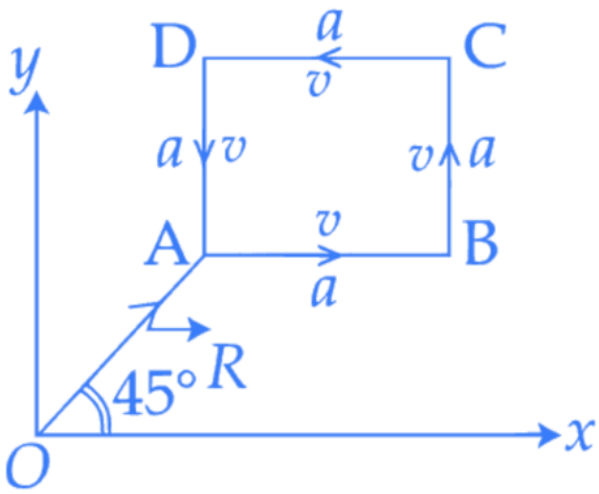
$$\mu_{\text{upper}} \simeq 1.394.$$

Among the listed numbers, 1.5 is closest to this bound: its difference is approximately 0.106, compared with $0.206, 0.306, 0.406$ for $1.6, 1.7, 1.8$. The question asks for the closest choice, not an exact equality to the index of the particular measured ray.

Step 4: Conclusion The upper bound is about 1.394, whose closest listed value is $\boxed{1.5}$. Hence, the correct option is (C).

Quick Tip to Solve: An arbitrary point on the deviation-versus-incidence graph gives $\delta_{\min} \le \delta$; use this inequality before inserting the measured deviation into the minimum-deviation formula.

35. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed v in the x-y plane as shown in the figure :
Which of the following statements is false for the angular momentum $\vec{L}$ about the origin ? [4 marks]



(A)

$$\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$$

when the particle is moving from B to C.

(B)

$$\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$$

when the particle is moving from D to A.

(C)

$$\overrightarrow{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$$

when the particle is moving from A to B

(D)

$$\overrightarrow{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$$

when the particle is moving from C to D.

Correct Answer: (B)

$$\overrightarrow{L} = \frac{mv}{\sqrt{2}}R\hat{k}$$

when the particle is moving from D to A.

Solution:

Concept:

- For planar motion, $\boldsymbol{L} = \boldsymbol{r} \times m\boldsymbol{v} = m(xv_y - yv_x)\hat{\boldsymbol{k}}$; the sign follows the cross product.
- The figure places A at distance R from the origin along $45°$, so both coordinates of A equal $h = R/\sqrt{2}$.

Step 1: Assign the coordinates from the figure Let $h = R/\sqrt{2}$. The vertices are

$$A = (h, h), \quad B = (h + a, h), \quad C = (h + a, h + a), \quad D = (h, h + a).$$

The velocity is rightward on AB, upward on BC, leftward on CD and downward on DA. On each straight side the perpendicular distance from the origin to the line of motion is constant, so the angular momentum is constant along that side.

Step 2: Evaluate the four cross products For $A \to B$, $v_x = v, v_y = 0$, giving $\boldsymbol{L} = -mvh\hat{\boldsymbol{k}}$. For $B \to C$, $v_x = 0, v_y = v$, giving $\boldsymbol{L} = mv(h + a)\hat{\boldsymbol{k}}$. For the other two sides,

$$C \to D: \quad L_z = 0 - (h + a)m(-v) = mv(h + a),$$
$$D \to A: \quad L_z = hm(-v) - 0 = -mvh.$$

The negative sign on DA is essential: downward momentum at positive x gives angular momentum into the plane.

Step 3: Identify the false statements Option (A) correctly gives $mv(h + a)\hat{\boldsymbol{k}}$ on BC, and (C) correctly gives $-mvh\hat{\boldsymbol{k}}$ on AB. Option (B) has the wrong sign for DA. Option (D) gives $mv(h - a)\hat{\boldsymbol{k}}$ on CD, whereas the correct lever arm is $h + a$, so it too is false. Thus the printed question has two false statements rather than one.

Step 4: Conclusion Both $\boxed{\text{(B) and (D)}}$ are false. The official answer key accepts both (B) and (D); choosing (B) is valid. Hence, the correct option is (B).

Quick Tip to Solve: Compute $L_z = xp_y - yp_x$ separately on each side; using only the magnitude of the perpendicular distance loses the crucial sign.

36. Arrange the following electromagnetic radiations per quantum in the order of increasing energy :

A : Blue light B : Yellow light

C : X-ray D : Radiowave. [4 marks]

(A) C, A, B, D

(B) B, A, D, C

(C) D, B, A, C

(D) A, B, D, C

Correct Answer: (C) D, B, A, C

Solution:

Concept:

- A photon is one quantum of electromagnetic radiation, with energy $E = h\nu = hc/\lambda$. Here h is Planck's constant, ν is frequency and λ is wavelength in vacuum.
- All four radiations travel at the same vacuum speed; the energy per quantum is controlled by frequency, not by the intensity of a beam.

Step 1: Order the wavelengths Radio waves have much longer wavelengths than visible light. Within visible light, yellow has a longer wavelength than blue. X-rays have wavelengths much shorter than either visible colour. Thus the wavelength order, from longest to shortest, is

$$\lambda_D > \lambda_B > \lambda_A > \lambda_C.$$

The letters here identify the radiations in the question: D is radio, B is yellow, A is blue and C is X-ray.

Step 2: Reverse the order for photon energy Since hc is a positive constant, dividing it by a longer wavelength gives a smaller photon energy. Consequently

$$E_D < E_B < E_A < E_C.$$

Option (A) starts with the highest-energy radiation and runs in the reverse direction. Options (B) and (D) place radio waves after visible light, although a radio photon has less energy than either a yellow or a blue photon. Only option (C) preserves all three comparisons.

Step 3: Conclusion The increasing energy order per quantum is radio wave, yellow light, blue light, X-ray: D, B, A, C. Hence, the correct option is (C).

Quick Tip to Solve: First write the spectrum in wavelength order, then reverse it whenever the question asks for energy per photon.

37. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to : [4 marks]

(A) 0.044 H

(B) 0.065 H

(C) 80 H

(D) 0.08 H

Correct Answer: (B) 0.065 H

Solution:

Concept:

- Treat the operating lamp as a resistance $R = V/I$. An ideal series inductor provides reactance $X_L = \omega L = 2\pi f L$ without dissipating average power.
- For a series resistance and inductor, the resistive and inductive voltage drops are perpendicular phasors, so $V_{\text{rms}}^2 = (IR)^2 + (IX_L)^2$.

Step 1: Find the operating resistance and impedance The lamp operating point gives

$$R = \frac{80}{10} = 8\,\Omega.$$

To maintain the specified current of $10\,\text{A}$ with a $220\,\text{V}$ rms source, the required total impedance magnitude is

$$Z = \frac{220}{10} = 22\,\Omega.$$

Use rms current with rms supply voltage. The lamp voltage and the inductor voltage cannot simply be subtracted as ordinary in-phase quantities.

Step 2: Calculate the required inductance The impedance relation gives

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{22^2 - 8^2} = \sqrt{420}\,\Omega.$$

Therefore

$$L = \frac{\sqrt{420}}{2\pi(50)}\,\text{H} \simeq 0.0652\,\text{H}.$$

For comparison, $0.044\,\text{H}$ gives a reactance of about $13.8\,\Omega$, too small to limit the current as required; $0.08\,\text{H}$ gives about $25.1\,\Omega$, too large. An inductance of $80\,\text{H}$ would virtually choke off the current.

Step 3: Conclusion The required series inductance is close to $0.065\,\text{H}$. Hence, the correct option is (B).

Quick Tip to Solve: For a series choke, obtain the inductive drop by a phasor square root; subtracting 80 V directly from 220 V gives the wrong reactance.

38. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say $b_{\min}$) when : [4 marks]

(A) $a = \sqrt{\lambda L}$ and $b_{\min} = \sqrt{4\lambda L}$

(B) $a = \frac{\lambda^2}{L}$ and $b_{\min} = \sqrt{4\lambda L}$

(C) $a = \frac{\lambda^2}{L}$ and $b_{\min} = \left(\frac{2\lambda^2}{L}\right)$

(D) $a = \sqrt{\lambda L}$ and $b_{\min} = \left(\frac{2\lambda^2}{L}\right)$

Correct Answer: (A) $a = \sqrt{\lambda L}$ and $b_{\min} = \sqrt{4\lambda L}$

Solution:

Concept:

- The stated model adds a geometrical spread, increasing with aperture radius a, and a diffraction spread, decreasing as $1/a$. In the scale approximation used here, $b(a) = a + \lambda L/a$.
- A positive sum $x + K/x$ has its minimum when its two terms are equal. Equivalently, set its derivative to zero and check positive curvature.

Step 1: Balance the two contributions Write the spread in the specified geometrical-plus-diffraction approximation as

$$b(a) = a + \frac{\lambda L}{a}, \qquad a > 0.$$

A very small hole reduces the geometrical contribution but greatly increases diffraction. A very large hole has little diffraction but a large geometrical spot. Differentiating yields

$$\frac{db}{da} = 1 - \frac{\lambda L}{a^2} = 0 \quad \Rightarrow \quad a = \sqrt{\lambda L}.$$

Also $d^2b/da^2 = 2\lambda L/a^3 > 0$, confirming a minimum.

Step 2: Evaluate the minimum and compare the choices At this radius the two spread contributions are equal, each being $\sqrt{\lambda L}$. Hence

$$b_{\min} = \sqrt{\lambda L} + \frac{\lambda L}{\sqrt{\lambda L}} = 2\sqrt{\lambda L} = \sqrt{4\lambda L}.$$

Only (A) gives both required expressions. Choices (B) and (C) have $a = \lambda^2/L$, which does not solve the minimisation condition. Choice (D) gives the right radius but its minimum spread has the wrong wavelength and camera-length dependence.

Step 3: Conclusion The minimum occurs at $a = \sqrt{\lambda L}$, with $b_{\min} = \sqrt{4\lambda L}$. Hence, the correct option is (A).

Quick Tip to Solve: When one blur contribution grows as aperture size and the other shrinks inversely, equate the two to locate the optimum.

39. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be : [4 marks]

(A) 92 ± 1.8 s

(B) 92 ± 3 s

(C) 92 ± 2 s

(D) 92 ± 5.0 s

Correct Answer: (C) 92 ± 2 s

Solution:

Concept:

- The mean of repeated observations is $\bar{t} = \sum t_i/n$. Their mean absolute deviation is $\Delta t = \sum |t_i - \bar{t}|/n$.
- Report an experimental uncertainty with suitable precision and keep the measured value to the same decimal place. The clock resolves only whole seconds.

Step 1: Calculate the mean measured duration The four recorded durations each refer to the same set of 100 oscillations. Their arithmetic mean is

$$\bar{t} = \frac{90 + 91 + 95 + 92}{4} = \frac{368}{4} = 92\,\text{s}.$$

The options ask for this measured duration, not the period of one oscillation; dividing the answer by 100 would answer a different reporting question.

Step 2: Find and report the uncertainty The absolute deviations from the mean are

$$|90 - 92| = 2, \quad |91 - 92| = 1, \quad |95 - 92| = 3, \quad |92 - 92| = 0$$

in seconds. Thus

$$\Delta t = \frac{2 + 1 + 3 + 0}{4} = 1.5\,\text{s}.$$

Reporting the uncertainty to one significant digit at the clock's whole-second resolution gives $2\,\text{s}$. Keep the central value at the same whole-second place. The value $1.8\,\text{s}$ is not the mean absolute deviation and suggests unjustified precision. The $3\,\text{s}$ choice uses the largest deviation alone, while $5.0\,\text{s}$ is the full range rather than the uncertainty of the mean.

Step 3: Conclusion The reported mean measured duration is $(92 \pm 2)\,\text{s}$. Hence, the correct option is (C).

Quick Tip to Solve: Average the absolute deviations from the mean, then round the uncertainty appropriately before deciding the final number of decimal places.

40. An observer looks at a distant tree of height $10\ m$ with a telescope of magnifying power of 20. To the observer the tree appears: [4 marks]

(A) 20 times taller

(B) 20 times nearer

(C) 10 times taller

(D) 10 times nearer

Correct Answer: (B) 20 times nearer

Solution:

Concept:

- The magnifying power of a telescope is angular magnification: $M = \theta_{\text{image}}/\theta_{\text{object}}$. It changes apparent angular size, not the physical height of a distant object.
- For a distant object of height h at distance d, the small-angle relation is $\theta \simeq h/d$. An increased angle can be interpreted as viewing that same object from a smaller distance.

Step 1: Express the unaided and aided angles Let the tree be at distance d. Its unaided angular height is approximately

$$\theta = \frac{10}{d}.$$

With telescope magnifying power 20, the angular height becomes

$$\theta_{\text{image}} = 20\theta = \frac{20 \times 10}{d}.$$

The stated height merely helps interpret the angle: the telescope does not make the actual tree taller.

Step 2: Find the equivalent apparent distance If the same tree were viewed directly from an apparent distance d_{app}, its angular height would be $10/d_{\text{app}}$. Equating this with the telescopic angular height gives

$$\frac{10}{d_{\text{app}}} = \frac{20 \times 10}{d} \quad \Rightarrow \quad d_{\text{app}} = \frac{d}{20}.$$

Thus its angular appearance corresponds to a tree twenty times nearer. Options saying ten times taller or nearer use a magnification of ten rather than twenty. Calling the tree twenty times taller confuses an increase in viewing angle with a change in the tree's linear height; a telescope magnifies angles.

Step 3: Conclusion The tree appears 20 times nearer to the observer. Hence, the correct option is (B).

Quick Tip to Solve: For a telescope, translate angular magnification M into the apparent distance d/M of the same object; do not multiply its physical height.

41. Two identical wires A and B, each of length 'l', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic fields at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is: [4 marks]

(A) $\frac{\pi^2}{16}$
(B) $\frac{\pi^2}{8\sqrt{2}}$
(C) $\frac{\pi^2}{8}$
(D) $\frac{\pi^2}{16\sqrt{2}}$

Correct Answer: (B) $\frac{\pi^2}{8\sqrt{2}}$

Solution:

Concept:

- At the centre of a circular loop, $B_A = \mu_0 I/(2R)$. Equal wire lengths imply $2\pi R = 4a = l$.
- For a finite straight current segment, $B = \mu_0 I(\sin\theta_1 + \sin\theta_2)/(4\pi r)$, where r is the perpendicular distance to the segment.

Step 1: Find the centre field of the square The perpendicular distance from the square centre to any side is $a/2$. Each endpoint subtends 45° with that perpendicular. One side therefore contributes

$$B_{\text{side}} = \frac{\mu_0 I}{4\pi(a/2)}(\sin 45^\circ + \sin 45^\circ) = \frac{\mu_0 I\sqrt{2}}{2\pi a}.$$

The four side fields have the same direction perpendicular to the square, so

$$B_B = 4B_{\text{side}} = \frac{2\sqrt{2}\mu_0 I}{\pi a}.$$

Step 2: Use the common wire length and form the ratio From $2\pi R = 4a$, we have $a/R = \pi/2$. Consequently

$$\frac{B_A}{B_B} = \frac{\mu_0 I}{2R}\frac{\pi a}{2\sqrt{2}\mu_0 I} = \frac{\pi a}{4\sqrt{2}R} = \frac{\pi^2}{8\sqrt{2}}.$$

The current and permeability cancel, as required for identical-current loops. Option (C) omits the endpoint-angle factor $\sqrt{2}$; (A) and (D) have incorrect numerical factors. Numerically the ratio is about 0.872, consistent with a slightly larger field at the square centre for equal perimeter.

Step 3: Conclusion The ratio is $B_A/B_B = \pi^2/(8\sqrt{2})$. Hence, the correct option is (B).

Quick Tip to Solve: For equal-length loops, eliminate the lengths only after summing all four finite-wire fields at the square centre.

42. A galvanometer having a coil resistance of $100\,\Omega$ gives a full scale deflection, when a currect of $1\ mA$ is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of $10\ A$, is : [4 marks]

(A) $0.1\,\Omega$

(B) $3\,\Omega$

(C) $0.01\,\Omega$

(D) $2\,\Omega$

Correct Answer: (C) $0.01\,\Omega$

Solution:

Concept:

- Convert a galvanometer into an ammeter by placing a low shunt resistance S in parallel with its coil of resistance G. Most of the measured current then bypasses the coil.
- Equal voltages across the parallel paths give $I_gG = (I - I_g)S$, where I_g is the full-scale galvanometer current and I is the desired ammeter range.

Step 1: Identify the currents in the two paths The galvanometer must still carry only its full-scale current

$$I_g = 1\,\text{mA} = 10^{-3}\,\text{A}.$$

At a total ammeter current of $10\,\text{A}$, the parallel shunt carries

$$I_s = 10 - 0.001 = 9.999\,\text{A}.$$

The voltage across the galvanometer at full scale is

$$V_g = I_gG = (10^{-3})(100) = 0.1\,\text{V}.$$

A series resistance would not protect the sensitive coil by diverting current; a parallel path is essential.

Step 2: Calculate and compare the shunt resistance The shunt must have the same potential difference as the coil, so

$$S = \frac{V_g}{I_s} = \frac{0.1}{9.999}\,\Omega \simeq 0.010001\,\Omega.$$

This is approximately $0.01\,\Omega$, which is much smaller than $100\,\Omega$, as expected for the branch carrying almost all the current. A $0.1\,\Omega$ shunt would carry only about $1\,\text{A}$ at this coil voltage; $2\,\Omega$ and $3\,\Omega$ would carry still less. None would provide the required 10 A full-scale range.

Step 3: Conclusion A shunt resistance of approximately $0.01\,\Omega$ must be connected in parallel. Hence, the correct option is (C).

Quick Tip to Solve: First calculate the tiny full-scale coil voltage, then divide it by the bypass current to obtain the shunt resistance.

43. The temperature dependence of resistance of Cu and undoped Si in the temperature range $300 - 400\ K$, is best described by : [4 marks]

(A) Linear increases for Cu, exponential decrease of Si.

(B) Linear decrease for Cu, linear decrease for Si

(C) Linear increase for Cu, linear increase for Si.

(D) Linear increase for Cu, exponential increase for Si

Correct Answer: (A) Linear increases for Cu, exponential decrease of Si.

Solution:

Concept:

- A metal has nearly constant carrier concentration; its resistance rises when lattice vibrations increase the scattering of electrons.
- In an intrinsic semiconductor, thermally generated electron-hole pairs increase rapidly with temperature; this carrier generation dominates the resistance trend.

Step 1: Analyse copper Copper is a metal. Over the modest interval from 300 to $400\,\mathrm{K}$, its positive temperature coefficient can be treated as approximately constant. Writing R_{300} for the resistance at $300\,\mathrm{K}$,

$$R(T) \simeq R_{300}[1 + \alpha(T - 300\,\mathrm{K})], \qquad \alpha > 0.$$

Thus the resistance increases approximately linearly. A linear decrease would require a negative coefficient and does not describe copper in this interval.

Step 2: Analyse undoped silicon Undoped silicon is an intrinsic semiconductor. Its intrinsic carrier concentration varies approximately as

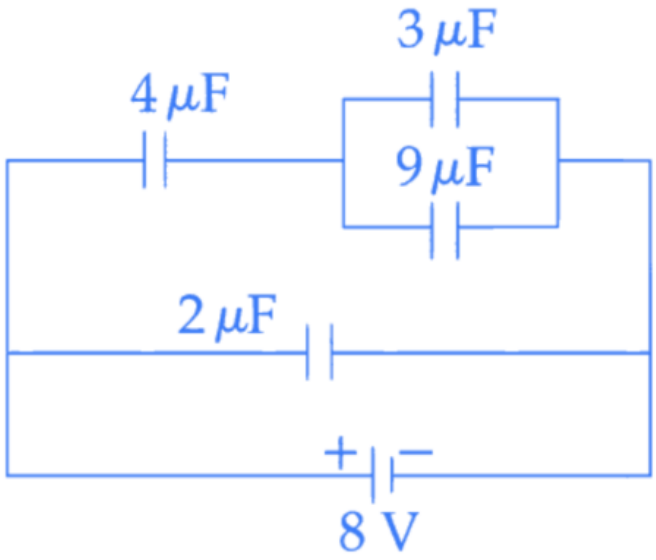
$$n_i \propto T^{3/2} \exp\left(-\frac{E_g}{2k_B T}\right).$$

Here E_g is the band gap and k_B is Boltzmann's constant. As temperature increases, the exponential suppression becomes weaker, generating many additional mobile carriers. Since conductivity is $\sigma = en_i(\mu_e + \mu_h)$, this carrier increase dominates the reduction of mobility. Resistance therefore decreases strongly, with the characteristic exponential dependence. A linear increase and an exponential increase both give the wrong sign; a linear decrease misses the characteristic intrinsic-carrier dependence.

Step 3: Conclusion Copper shows an approximately linear increase of resistance, whereas undoped silicon shows an exponential decrease. Hence, the correct option is (A).

Quick Tip to Solve: For a metal, focus on increased scattering; for an intrinsic semiconductor, focus on the rapid thermal generation of carriers.

44. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the $4\ \mu F$ and $9\ \mu F$ capacitors), at a point distance $30\ m$ from it, would equal : [4 marks]



(A) $420 N/C$

(B) $480 N/C$

(C) $240 N/C$

(D) $360 N/C$

Correct Answer: (A) $420 N/C$

Solution:

Concept:

- Capacitors in parallel have a common voltage and capacitances add. Series groups carry equal charge magnitudes and their reciprocal capacitances add.
- The field of an isolated point charge at distance r is $E = kQ/r^2$, where $k = 9 \times 10^9\ \mathrm{N\,m^2/C^2}$.

Step 1: Reduce the upper branch The $3\,\mu\mathrm{F}$ and $9\,\mu\mathrm{F}$ capacitors are in parallel, giving $C_p = 12\,\mu\mathrm{F}$. This group is in series with $4\,\mu\mathrm{F}$. Thus

$$C_s = \frac{4 \times 12}{4 + 12} = 3\,\mu\mathrm{F}.$$

The entire upper branch is across the ideal $8\,\mathrm{V}$ battery. The $2\,\mu\mathrm{F}$ branch is separately in parallel with the battery and does not change this voltage. The series charge is

$$q_s = C_s V = 3 \times 8 = 24\,\mu\mathrm{C}.$$

Consequently the charge magnitude on the $4\,\mu\mathrm{F}$ capacitor is $24\,\mu\mathrm{C}$.

Step 2: Find the specified charge and its field The voltage across the parallel pair is

$$V_p = \frac{24\,\mu\text{C}}{12\,\mu\text{F}} = 2\,\text{V}.$$

The $9\,\mu$F capacitor therefore carries $q_9 = 9 \times 2 = 18\,\mu$C. The point charge specified in the question has magnitude equal to the sum of these two capacitor charges:

$$Q = 24 + 18 = 42\,\mu\text{C}.$$

At $r = 30\,\text{m}$,

$$E = \frac{9 \times 10^9 \times 42 \times 10^{-6}}{30^2} = 420\,\text{N/C}.$$

The values 480, 240 and $360\,\text{N/C}$ would correspond to different charge sums, and none follows from the individual capacitor charges calculated above.

Step 3: Conclusion The electric-field magnitude is $420\,\text{N/C}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the voltage across the parallel pair before calculating an individual capacitor charge; the series charge belongs to the whole parallel group.

45. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is: [4 marks]

(A) $A\sqrt{3}$

(B) $\frac{7A}{3}$

(C) $\frac{A}{3}\sqrt{41}$

(D) $3A$

Correct Answer: (B) $\frac{7A}{3}$

Solution:

Concept:

- For simple harmonic motion with angular frequency ω, speed and displacement satisfy $v^2 = \omega^2(A^2 - x^2)$.
- An instantaneous change of speed leaves the displacement unchanged. With the same restoring force and mass, ω is unchanged but the total energy and amplitude change.

Step 1: Calculate the original speed at the stated position The particle is at $x = 2A/3$, where A denotes its original amplitude. Substituting this displacement into the SHM relation gives

$$v^2 = \omega^2\left(A^2 - \frac{4A^2}{9}\right) = \frac{5}{9}\omega^2 A^2.$$

The particle is neither at equilibrium nor at a turning point, so its energy is shared between kinetic and potential energy. This is why multiplying its speed by three does not simply multiply its amplitude by three.

Step 2: Apply the new energy at the same displacement Let A_{new} denote the amplitude immediately after the speed becomes $v_{\text{new}} = 3v$. The speed changes at a fixed position, so the potential energy remains unchanged. Using the same SHM relation for the new motion,

$$\begin{aligned} A_{\text{new}}^2 &= x^2 + \frac{v_{\text{new}}^2}{\omega^2} \\ &= \frac{4A^2}{9} + 9\left(\frac{5A^2}{9}\right) \\ &= \frac{49A^2}{9}. \end{aligned}$$

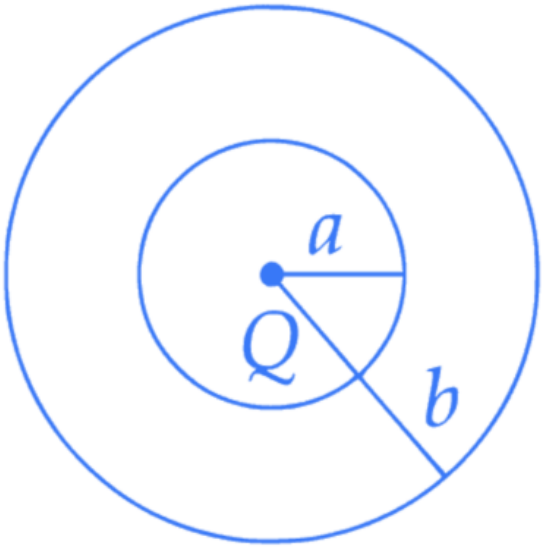
Taking the positive square root yields $A_{\text{new}} = 7A/3$. The value $3A$ would follow only if the speed were trebled at equilibrium. The other proposed values, $A\sqrt{3}$ and $A\sqrt{41}/3$, do not satisfy the new energy equation with the unchanged displacement.

Step 3: Conclusion The new amplitude is $7A/3$. Hence, the correct option is (B).

Quick Tip to Solve: After a sudden speed change in SHM, use $A_{\text{new}}^2 = x^2 + v_{\text{new}}^2/\omega^2$; keep the instantaneous displacement fixed.

---

46. The region between two concentric spheres of radii $'a'$ and $'b'$, respectively (see figure), have volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the center. A such that the electric field in the region between the spheres will be constant, is : [4 marks]



(A) $\frac{2Q}{\pi(a^2-b^2)}$

(B) $\frac{2Q}{\pi\, a^2}$

(C) $\frac{Q}{2\pi\, a^2}$

(D) $\frac{Q}{2\pi\,(b^2-a^2)}$

Correct Answer: (C) $\frac{Q}{2\pi\, a^2}$

Solution:

Concept:

- Spherical symmetry gives $E(r)\,4\pi r^2 = Q_{\text{enc}}/\varepsilon_0$ on a spherical Gaussian surface.
- The figure places charge Q at the centre. The surrounding charged region contributes $dq = \rho\,4\pi r^2 dr$, with $\rho = A/r$.

Step 1: Integrate the charge enclosed in the annular region Choose a Gaussian sphere of radius r with $a < r < b$. It encloses the central charge Q and the part of the volume distribution from a to r. Using s as the integration variable avoids confusing the variable radius with the fixed outer limit.

$$\begin{aligned} Q_{\text{enc}}(r) &= Q + \int_a^r \frac{A}{s}\,4\pi s^2\,ds \\ &= Q + 2\pi A(r^2 - a^2). \end{aligned}$$

Charges lying between r and b are outside the Gaussian sphere and do not contribute to the enclosed charge; by spherical symmetry their net field inside is zero.

Step 2: Remove the radial dependence of the field Gauss's law now yields

$$\begin{aligned} E(r) &= \frac{Q + 2\pi A(r^2 - a^2)}{4\pi\varepsilon_0 r^2} \\ &= \frac{A}{2\varepsilon_0} + \frac{Q - 2\pi A a^2}{4\pi\varepsilon_0 r^2}. \end{aligned}$$

For the field to be constant throughout this region, the coefficient of $1/r^2$ must vanish. Therefore

$$Q - 2\pi A a^2 = 0, \qquad A = \frac{Q}{2\pi a^2}.$$

This result depends on the inner radius and central charge, not on the outer boundary b. Both options involving $b^2 - a^2$ fail that condition. The value $2Q/(\pi a^2)$ has the right dimensions but is four times too large and leaves a nonzero inverse-square term.

Step 3: Conclusion The required constant is $A = Q/(2\pi a^2)$, for which $E = Q/(4\pi\varepsilon_0 a^2)$ in the region. Hence, the correct option is (C).

Quick Tip to Solve: For a radius-independent spherical field, make the enclosed charge proportional to r^2; cancel the constant part of Q_{enc}.

47. A uniform string of length $20\ m$ is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the supports is :

(take $g = 10ms^{-2}$) [4 marks]

(A) $2\sqrt{2}\,s$

(B) $2\pi\sqrt{2}\,s$

(C) $2\pi\sqrt{2}\,s$

(D) $\frac{2}{s}$

Correct Answer: (A) $2\sqrt{2}\,s$

Solution:

Concept:

- The speed of a small transverse pulse on a string is $v = \sqrt{T/\mu}$, where T is the local tension and μ is mass per unit length.
- In a uniform hanging string, the tension at height x above the free end equals the weight $\mu x g$ of the string below that point.

Step 1: Express the varying pulse speed Measure x upward from the lower free end. Since each section supports the length of string beneath it, the local tension is $T(x) = \mu g x$. Substitution into the wave-speed formula gives

$$v(x) = \sqrt{\frac{\mu g x}{\mu}} = \sqrt{gx}.$$

The linear density cancels: the result is independent of how heavy the uniform string is. The pulse accelerates as it moves upward, because the tension is greater nearer the support. Its speed therefore cannot be treated as a constant equal to the speed at the upper end.

Step 2: Integrate the travel time An element of travel time is $dt = dx/v(x)$. Integrating over the length $L = 20\,\mathrm{m}$,

$$\begin{aligned} t &= \int_0^L \frac{dx}{\sqrt{gx}} \\ &= \frac{1}{\sqrt{g}}\left[2\sqrt{x}\right]_0^L \\ &= 2\sqrt{\frac{L}{g}} = 2\sqrt{\frac{20}{10}} = 2\sqrt{2}\,\mathrm{s}. \end{aligned}$$

Although the ideal expression for speed is zero at the very bottom, the integral converges there; its lower-limit contribution is zero. No oscillation period is being calculated, so a factor of π is inappropriate. Thus both printed options containing $2\pi\sqrt{2}$ are excluded, while $2/s$ has dimensions of inverse time rather than time and cannot be a travel time.

Step 3: Conclusion The pulse reaches the support after $2\sqrt{2}\,\mathrm{s}$. Hence, the correct option is (A).

Quick Tip to Solve: For a hanging string, measure distance from the free end: $T = \mu g x$ immediately gives $v = \sqrt{gx}$, then integrate dx/v.

48. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now : [4 marks]

(A) $2f$

(B) f

(C) $\frac{f}{2}$

(D) $\frac{3f}{4}$

Correct Answer: (B) f

Solution:

Concept:

- An air column open at both ends has displacement antinodes at both ends and fundamental frequency $v/(2L)$.
- An air column closed at one end has a displacement node at the closed end, an antinode at the open end, and fundamental frequency $v/(4\ell)$.

Step 1: Write the original open-pipe frequency Let the full length of the pipe be L, and let the sound speed in air be v. For the fundamental mode of a pipe open at both ends, half of one wavelength fits inside the pipe. Hence

$$\lambda_1 = 2L, \qquad f = \frac{v}{\lambda_1} = \frac{v}{2L}.$$

The important boundary condition is that both ends initially permit air motion. The given frequency f already includes the effect of the original length, so it is convenient to compare the new frequency directly with this expression.

Step 2: Apply the new length and boundary condition together When half the pipe is immersed vertically in water, the water rises inside to the external water level. The remaining air column has length $\ell = L/2$. Its lower end is bounded by water, which acts as a closed end for the air column, while the upper end remains open. Therefore the fundamental now satisfies

$$\lambda_{\text{new}} = 4\ell = 4\left(\frac{L}{2}\right) = 2L.$$

It follows that

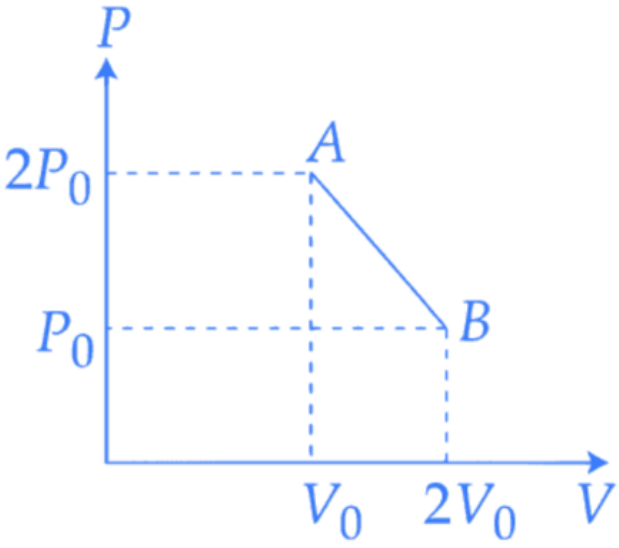
$$f_{\text{new}} = \frac{v}{4\ell} = \frac{v}{2L} = f.$$

Using only the halving of length would incorrectly suggest $2f$; that ignores the change to an open-closed pipe. Conversely, applying only the closed-end factor would suggest $f/2$, ignoring the halved air length. The fraction $3f/4$ satisfies neither fundamental standing-wave condition.

Step 3: Conclusion The fundamental frequency of the remaining air column is unchanged and equals f. Hence, the correct option is (B).

Quick Tip to Solve: When water enters an open pipe, update both the air-column length and its end conditions before comparing frequencies.

49. **'n' moles of an ideal gas undergoes a process $A \to B$ as shown in the figure. The maximum temperature of the gas during the process will be :** [4 marks]



(A) $\frac{9P_0V_0}{2nR}$
(B) $\frac{9P_0V_0}{nR}$
(C) $\frac{9P_0V_0}{4nR}$
(D) $\frac{3P_0V_0}{2nR}$

Correct Answer: (C) $\frac{9P_0V_0}{4nR}$

Solution:

Concept:

- For n moles of ideal gas, $T = PV/(nR)$, so maximizing temperature means maximizing the product PV.
- A straight line on a pressure-volume diagram gives pressure as a linear function of volume; the product PV is then a quadratic function.

Step 1: Write the equation of the path The graph gives $A = (V_0, 2P_0)$ and $B = (2V_0, P_0)$. Its slope is

$$\frac{P_0 - 2P_0}{2V_0 - V_0} = -\frac{P_0}{V_0}.$$

Using point A, the pressure along the path is

$$P - 2P_0 = -\frac{P_0}{V_0}(V - V_0), \qquad P = 3P_0 - \frac{P_0 V}{V_0}.$$

This equation reproduces both endpoint pressures, so it describes the given straight process rather than an isothermal or adiabatic curve.

Step 2: Maximize the pressure-volume product Substitute the path equation into the ideal-gas law:

$$T(V) = \frac{P_0}{nR}\left(3V - \frac{V^2}{V_0}\right).$$

Differentiating and setting the derivative to zero gives

$$\frac{dT}{dV} = \frac{P_0}{nR}\left(3 - \frac{2V}{V_0}\right) = 0, \qquad V = \frac{3V_0}{2}.$$

The second derivative is negative, and this volume lies between V_0 and $2V_0$, so it is the maximum on the actual path. The corresponding pressure is $3P_0/2$. Consequently

$$T_{\max} = \frac{(3P_0/2)(3V_0/2)}{nR} = \frac{9P_0V_0}{4nR}.$$

Both endpoints have $T = 2P_0V_0/(nR)$, less than this maximum. Options (A) and (B) are too large, while (D) is below even the endpoint temperature; none can be the maximum on the path.

Step 3: Conclusion The maximum temperature is $9P_0V_0/(4nR)$. Hence, the correct option is (C).

Quick Tip to Solve: Along a straight PV segment, write $P = mV + c$ first and maximize PV; a temperature maximum need not be at an endpoint.

50. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by $PV^n =$ constant, then n is given by (Here C_p and C_v are molar specific heat at constant pressure and constant volume, respectively: [4 marks]

(A) $n = \frac{C_p - C}{C - C_v}$

(B) $n = \frac{C - C_v}{C - C_p}$

(C) $n = \frac{C_p}{C_v}$

(D) $n = \frac{C - C_p}{C - C_v}$

Correct Answer: (D) $n = \frac{C - C_p}{C - C_v}$

Solution:

Concept:

- For one mole in a reversible process, the first law is $C\,dT = C_v\,dT + P\,dV$. Here C refers to this particular process, whereas C_v is a property of the gas.
- The ideal-gas equation is $PV = RT$, and Mayer's relation is $C_p - C_v = R$. A polytropic process obeys $PV^n =$ constant.

Step 1: Differentiate the prescribed process Taking the logarithmic differential gives $dP/P + n\,dV/V = 0$. Differentiating the ideal-gas equation, and then eliminating dP, yields

$$R\,dT = P\,dV + V\,dP = (1 - n)P\,dV.$$

The factor is $1-n$, not $n-1$; this sign is essential when using the first law.

Step 2: Eliminate the infinitesimal work The first law also gives $P\,dV = (C - C_v)dT$. Substituting this in the preceding equation and cancelling the temperature differential gives

$$R = (1-n)(C - C_v).$$

Hence

$$n = 1 - \frac{R}{C - C_v} = \frac{C - C_v - (C_p - C_v)}{C - C_v} = \frac{C - C_p}{C - C_v}.$$

For the isochoric limiting case $C = C_v$, the finite-exponent formula is singular, as expected for constant volume.

Step 3: Check the alternatives and limits Option (A) reverses the numerator sign and therefore gives the wrong sign for the exponent. Option (B) is the reciprocal of the required expression. Option (C) applies only to the adiabatic case $C = 0$, rather than a general constant heat capacity. Indeed, our expression gives $n = C_p/C_v$ when $C = 0$, and $n = 0$ for the isobaric case $C = C_p$.

Step 4: Conclusion

$$\boxed{n = \frac{C - C_p}{C - C_v}}$$

Hence, the correct option is (D).

Quick Tip to Solve: Differentiate both $PV^n = \text{constant}$ and $PV = RT$; checking the adiabatic limit immediately detects a reversed sign.

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51. A pendulum clock loses $12\ s$ a day if the temperature is $40°C$ and gains $4\ s$ a day if the temperature is $20°C$. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively : [4 marks]

(A) $30°C;\ \alpha = 1.85 \times 10^{-3}/°C$

(B) $55°C;\ \alpha = 1.85 \times 10^{-2}/°C$

(C) $25°C;\ \alpha = 1.85 \times 10^{-5}/°C$

(D) $60°C;\ \alpha = 1.85 \times 10^{-4}/°C$

Correct Answer: (C) $25°C;\ \alpha = 1.85 \times 10^{-5}/°C$

Solution:

Concept:

- A simple pendulum has period $T = 2\pi\sqrt{L/g}$. For a small temperature change, $\Delta T/T = \alpha\Delta\theta/2$, because the rod length changes by $\Delta L/L = \alpha\Delta\theta$.
- A longer period makes a clock lose time. Take the daily loss as positive and the daily gain as negative; a day contains $86400\,\text{s}$.

Step 1: Express the two signed errors Let θ_0 be the temperature at which the clock is correct. To first

order in thermal expansion, the daily loss is

$$e = \frac{86400\alpha}{2}(\theta - \theta_0).$$

At 40°C, $e = 12\,\text{s}$; at 20°C, $e = -4\,\text{s}$. Thus the increase in loss over 20°C is $12 - (-4) = 16\,\text{s}$, not eight seconds.

Step 2: Find the expansion coefficient Subtract the two error equations:

$$16 = 43200\alpha(40 - 20).$$

Therefore

$$\alpha = \frac{16}{864000} = 1.85185 \times 10^{-5}\,^\circ\text{C}^{-1}.$$

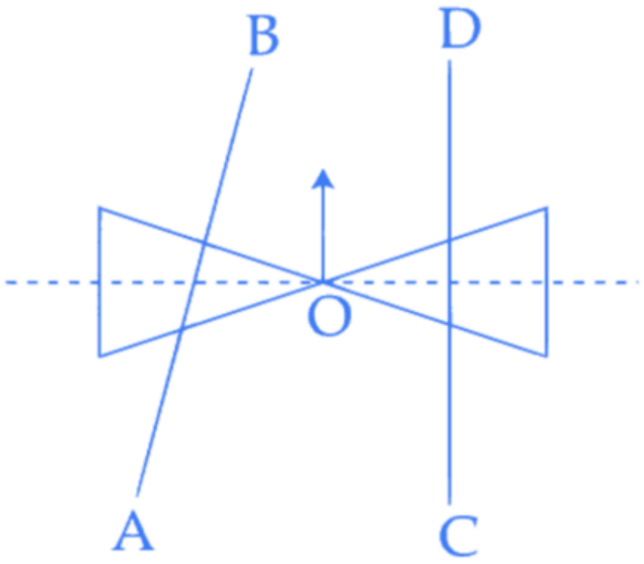
The small value justifies the linear approximation used for the pendulum period; its fractional change over twenty degrees is only about 1.85×10^{-4}.

Step 3: Locate the zero-error temperature The daily loss increases by $16/20 = 0.8\,\text{s}$ per degree. Starting from a gain of four seconds at twenty degrees, a rise of $4/0.8 = 5^\circ\text{C}$ removes the error. Consequently $\theta_0 = 25^\circ\text{C}$. Options (A), (B) and (D) give both incorrect zero-error temperatures and coefficients inconsistent with the measured error slope. The correct temperature must lie between the two temperatures where the error has opposite signs.

Step 4: Conclusion The correct-time temperature is 25°C, and $\alpha \simeq 1.85 \times 10^{-5}\,^\circ\text{C}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: Use signed daily errors: a gain of four seconds is a loss of minus four seconds. Interpolate the zero before calculating α.

52. A roller is made by joining together two cones at their vertices 0. It is kept on two rails AB and CD, which are placed asymmetrically (see figure), with its axis perpendicular to CD and its center O at the center of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its center O moving parallel to CD in the direction shown. As it moves, the roller will tend to : [4 marks]



(A) go straight

(B) turn left and right alternately

(C) turn left

(D) turn right

Correct Answer: (C) turn left

Solution:

Concept:

- For rolling without slipping, a contact with local radius r advances by $r\,d\phi$ when the roller rotates through $d\phi$. Both cones share the same angular rotation.
- Each cone widens away from the common vertex O. Therefore a contact nearer O has a smaller rolling radius than one farther from it.

Step 1: Read the rail geometry in the direction of motion The arrow shows motion upwards, parallel to the right rail CD. The left rail AB slopes towards CD in this direction. Initially the axis is perpendicular to CD, and O is midway between the two contact positions, so the two local cone radii initially agree. As the roller begins to advance upwards, the left contact moves inwards towards the narrow vertex while the right rail remains parallel to the initial motion.

Step 2: Compare the forward distances at the contacts The left cone consequently rolls on a smaller local radius than the right cone. Denote these radii by r_L and r_R. Immediately after the initial advance,

$$r_L < r_R, \qquad ds_L = r_L\,d\phi < r_R\,d\phi = ds_R.$$

The common axle cannot keep moving straight while its right end travels farther than its left end. It must turn towards the side covering the smaller distance, just as a vehicle turns towards its slower wheel.

Step 3: Identify the sense of turning Viewed in the upward direction indicated, the right-hand contact advances more, so the axle turns anticlockwise on the page and the centre bends towards the left rail. Going straight would require equal effective rolling radii throughout. Turning right would require the opposite inequality. Alternating left and right is not the initial tendency selected by the one-sided convergence of the rails in this arrangement.

Step 4: Conclusion The roller tends to turn left. Hence, the correct option is (C).

Quick Tip to Solve: For a joined-cone roller, compare the local radii at the rail contacts: the larger-radius side advances farther and the roller turns towards the smaller-radius side.

53. A person trying to lose weight by burning fat lifts a mass of $10\ kg$ upto a height of $1\ m$ 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies $3.8 \times 10^7 J$ of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\, ms^{-2}$: [4 marks]

(A) $9.89 \times 10^{-3}\ kg$

(B) $12.89 \times 10^{-3}\, kg$

(C) $2.45 \times 10^{-3}\ kg$

(D) $6.45 \times 10^{-3}\ kg$

Correct Answer: (B) $12.89 \times 10^{-3}\, kg$

Solution:

Concept:

- The useful work in raising a mass through height h is mgh. For N identical lifts, the total upward mechanical work is $W = Nmgh$.
- Efficiency is useful mechanical output divided by chemical energy input: $\eta = W/E_{\text{in}}$. Fat mass used is $m_f = E_{\text{in}}/q$, where q is energy supplied per kilogram of fat.

Step 1: Calculate the positive lifting work Each lift raises a $10\,\text{kg}$ mass through $1\,\text{m}$, so the useful work per lift is

$$W_1 = 10 \times 9.8 \times 1 = 98\,\text{J}.$$

There are one thousand lifts, giving

$$W = 1000 \times 98 = 9.8 \times 10^4\,\text{J}.$$

The energy released during lowering is dissipated and cannot be reused for the next lift. We count the specified upward work only; we do not subtract the downward gravitational work.

Step 2: Convert mechanical work into chemical energy The efficiency is $20\% = 0.20$, so only one fifth of the consumed fat energy becomes the calculated lifting work:

$$E_{\text{in}} = \frac{9.8 \times 10^4}{0.20} = 4.9 \times 10^5\,\text{J}.$$

Multiplying by the efficiency instead would incorrectly make the required chemical input smaller than the useful output.

Step 3: Find the fat mass and compare the options Using the stated energy supply $q = 3.8 \times 10^7\,\text{J kg}^{-1}$,

$$m_f = \frac{4.9 \times 10^5}{3.8 \times 10^7} = 1.28947 \times 10^{-2}\,\text{kg}.$$

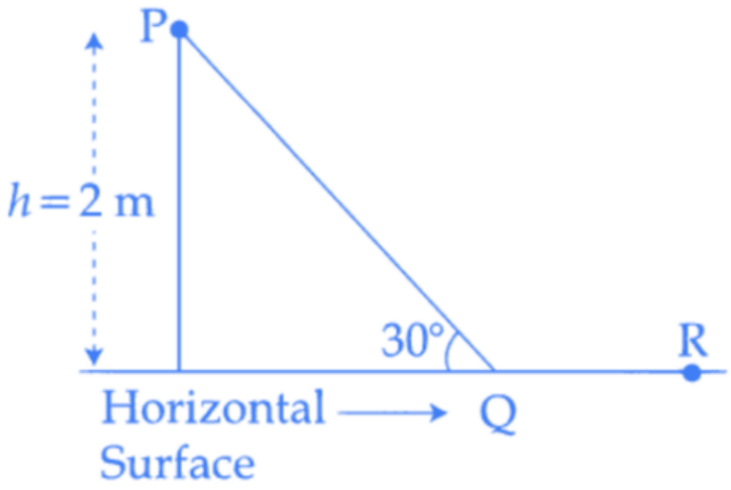
This is approximately $12.89\,\text{g}$. The other three offered masses do not supply the required $9.8 \times 10^4\,\text{J}$ of mechanical work when multiplied by $0.20q$; only option (B) satisfies that energy balance.

Step 4: Conclusion The fat consumed is $12.89 \times 10^{-3}\,\text{kg}$. Hence, the correct option is (B).

Quick Tip to Solve: Combine the calculation as $m_f = Nmgh/(\eta q)$. Divide by efficiency because the chemical energy input exceeds the useful work.

54. A point particle of mass m, moves long the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest from the point P and it comes to rest at point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other , and no energy is lost when particle changes direction from PQ to QR.

The value of the coefficient of friction μ and the distance $x\ (= QR)$, are, respectively close to: [4 marks]



(A) 0.29 and $3.5\ m$

(B) 0.29 and $6.5\ m$

(C) 0.2 and $6.5\ m$

(D) 0.2 and $3.5\ m$

Correct Answer: (A) 0.29 and $3.5\ m$

Solution:

Concept:

- The particle begins and ends at rest, so its initial gravitational potential energy mgh is entirely dissipated by friction. Equal losses on the two segments mean each loss is $mgh/2$.
- On an incline of angle θ, friction has magnitude $\mu mg\cos\theta$. On a horizontal track, its magnitude is μmg. Frictional energy loss equals force times path length.

Step 1: Determine the inclined path length The figure gives vertical height $h = 2\,\text{m}$ and inclination $\theta = 30°$. The inclined length is therefore

$$PQ = \frac{h}{\sin 30°} = 4\,\text{m}.$$

The gravitational energy available is $2mg$, and the stipulated equal dissipation assigns mg to each segment. No additional energy is removed at the junction Q.

Step 2: Use the inclined loss to calculate friction The loss along PQ is

$$\mu mg\cos 30°\,(4\,\text{m}) = \frac{mg(2\,\text{m})}{2}.$$

Cancelling mg gives

$$\mu = \frac{1}{4\cos 30^\circ} = \frac{1}{2\sqrt{3}} \simeq 0.2887.$$

Thus the value near 0.29, rather than 0.2, is required. The normal reaction on the slope is $mg\cos 30^\circ$, not simply mg.

Step 3: Use the horizontal loss to find the distance On QR, the loss is μmgx. Equating this to the same $mg(1\,\text{m})$ gives

$$x = \frac{1\,\text{m}}{\mu} = 2\sqrt{3}\,\text{m} \simeq 3.46\,\text{m}.$$

Equivalently, equality of the two frictional losses gives $x = PQ\cos 30^\circ$, the horizontal projection of the incline. Hence option (B) has the wrong stopping distance, and options (C) and (D) also have the wrong coefficient.

Step 4: Conclusion $\mu \simeq 0.29$ and $x \simeq 3.5\,\text{m}$. Hence, the correct option is (A).

Quick Tip to Solve: Equal frictional losses imply $x = PQ\cos\theta$. Combine this with total loss mgh to find μ without calculating the speed at Q.

---

55. A satellite is revolving in a circular orbit at a height $'h'$ from the earth's surface (radius of earth $R; h << R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.) [4 marks]

(A) $\sqrt{2gR}$

(B) $\sqrt{gR}$

(C) $\sqrt{gR/2}$

(D) $\sqrt{gR}(\sqrt{2}-1)$

Correct Answer: (D) $\sqrt{gR}(\sqrt{2}-1)$

Solution:

Concept:

- At distance r from the Earth's centre, a circular orbit has speed $v_c = \sqrt{GM/r}$. Escape from the same position requires at least $v_e = \sqrt{2GM/r}$.
- The smallest velocity increment is applied in the existing direction of motion. It raises the speed from v_c to v_e, so $\Delta v = v_e - v_c$.

Step 1: Calculate the two speeds at the orbital radius The satellite is already moving at radius $r = R + h$. Balancing gravitational force with centripetal force gives

$$\frac{GMm}{r^2} = \frac{mv_c^2}{r}, \qquad v_c = \sqrt{\frac{GM}{R+h}}.$$

For marginal escape the final total mechanical energy is zero. Therefore $mv_e^2/2 - GMm/r = 0$, which gives $v_e = \sqrt{2}\,v_c$.

Step 2: Subtract the initial speed Because the required quantity is the increase rather than the final

speed, the minimum boost is

$$\Delta v = (\sqrt{2} - 1)\sqrt{\frac{GM}{R+h}}.$$

A boost parallel to the initial velocity maximizes the final speed for any fixed boost magnitude. Any other direction would require a larger increment to reach the escape-speed threshold.

Step 3: Use the low-altitude approximation The condition $h \ll R$ allows $R + h \simeq R$. With surface gravity $g = GM/R^2$,

$$\sqrt{\frac{GM}{R+h}} \simeq \sqrt{\frac{GM}{R}} = \sqrt{gR}.$$

Option (A) is approximately the total escape speed, not the increase. Option (B) is approximately the original orbital speed. Option (C) is neither the difference nor the escape threshold. Only option (D) retains the required factor $\sqrt{2} - 1$.

Step 4: Conclusion The minimum increase is $\boxed{\Delta v \simeq \sqrt{gR}(\sqrt{2} - 1)}$. Hence, the correct option is (D).

Quick Tip to Solve: For a circular orbit, escape speed is $\sqrt{2}$ times orbital speed. Subtract the speed already present before making the low-altitude approximation.

56. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line? [4 marks]

(A) 0.70 mm

(B) 0.50 mm

(C) 0.75 mm

(D) 0.80 mm

Correct Answer: (D) 0.80 mm

Solution:

Concept:

- The least count of a screw gauge is its pitch divided by the number of circular-scale divisions. The observed reading is the main-scale reading plus the circular-scale contribution.
- Zero correction is the negative of zero error. When the reference coincides with division n just before zero on an N-division scale, the negative zero error is $-(N - n)$ least counts.

Step 1: Calculate the least count and uncorrected reading The screw advances $0.5\,\text{mm}$ in one turn, divided into fifty equal circular divisions. Thus

$$\text{LC} = \frac{0.5}{50} = 0.01\,\text{mm}.$$

With the sheet inserted, the indicated thickness is

$$t_{\text{obs}} = 0.5 + 25(0.01) = 0.75\,\text{mm}.$$

This is a raw reading; it must still be corrected for the nonzero indication when the jaws touch.

Step 2: Interpret the contact reading with its sign At jaw contact, the forty-fifth circular division aligns and the main-scale zero is barely visible. The circular scale is five divisions short of completing fifty, corresponding to a negative zero error rather than a positive $0.45\,\text{mm}$ error:

$$e_0 = -(50 - 45)(0.01) = -0.05\,\text{mm}.$$

The visibility of the main-scale zero distinguishes this negative-error situation. Therefore the required correction is $+0.05\,\text{mm}$.

Step 3: Apply the correction and reject the distractors The true thickness equals observed reading minus zero error:

$$t = t_{\text{obs}} - e_0 = 0.75 - (-0.05) = 0.80\,\text{mm}.$$

Option (C) ignores the zero correction. Option (A) applies the correction in the wrong direction. Option (B) omits the circular reading and also fails to implement the measured contact offset.

Step 4: Conclusion The thickness of the aluminium sheet is $0.80\,\text{mm}$. Hence, the correct option is (D).

Quick Tip to Solve: For negative zero error, count the divisions remaining to complete the circular scale; then add their least-count value to the observed measurement.

57. Choose the correct statement : [4 marks]

(A) In amplitude modulation the amplitude of the high frequency carrier wave is make to vary in proportion to the amplitude of the audio signal.

(B) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(C) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(D) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.

Correct Answer: (A) In amplitude modulation the amplitude of the high frequency carrier wave is make to vary in proportion to the amplitude of the audio signal.

Solution:

Concept:

- Modulation transfers information from a low-frequency message signal onto a high-frequency carrier by varying a carrier parameter.
- In amplitude modulation (AM), the carrier amplitude changes in proportion to the instantaneous

message amplitude; its frequency remains fixed.

- In frequency modulation (FM), the instantaneous carrier frequency changes with message amplitude while the carrier amplitude remains constant.

Step 1: Write the amplitude-modulated signal Let the audio signal be $m(t)$, the unmodulated carrier amplitude be A_c, and its angular frequency be ω_c. An AM signal has the form

$$s(t) = [A_c + k_a m(t)] \cos(\omega_c t).$$

Thus the envelope is $A_c + k_a m(t)$. A positive or negative change in the audio amplitude changes the envelope proportionally, without changing the fixed carrier frequency $\omega_c/(2\pi)$. This is exactly the statement in option (A).

Step 2: Compare the remaining statements Option (B) changes frequency and therefore describes the varying parameter used in FM rather than AM. Both (C) and (D) claim that carrier amplitude is varied in FM, although an ideal FM wave has constant amplitude. Moreover, the frequency deviation in FM is proportional to the instantaneous message amplitude, not the message frequency. Hence none of these three statements gives the correct definition of its named modulation scheme.

Step 3: Conclusion In AM the high-frequency carrier amplitude varies in proportion to the audio-signal amplitude. Hence, the correct option is (A).

Quick Tip to Solve: Identify which carrier parameter changes: AM changes amplitude, whereas FM changes frequency; both changes track the message amplitude.

---

58. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is :

[4 marks]

(A) $\alpha = \frac{\beta}{1+\beta}$

(B) $\alpha = \frac{\beta^2}{1+\beta^2}$

(C) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$

(D) $\alpha = \frac{\beta}{1-\beta^2}$

Correct Answer: (B) $\alpha = \frac{\beta^2}{1+\beta^2}$

Solution:

Concept:

- The common-base and common-emitter current gains are $\alpha = I_C/I_E$ and $\beta = I_C/I_B$, respectively.
- Transistor currents obey $I_E = I_C + I_B$. Eliminating the currents gives $\alpha = \beta/(1+\beta)$.
- A proposed relationship must hold generally for physical current gains; agreement at one special value is not sufficient.

Step 1: Derive the gain relationship Divide the emitter-current identity by I_C:

$$\frac{I_E}{I_C} = 1 + \frac{I_B}{I_C} \quad \Longrightarrow \quad \frac{1}{\alpha} = 1 + \frac{1}{\beta}.$$

Inverting gives $\alpha = \beta/(1+\beta)$. Therefore options (A) and (C) are equivalent correct identities and cannot answer a question asking for an incorrect relationship.

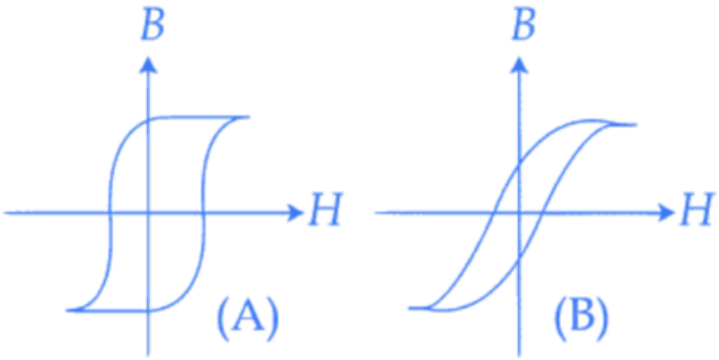
Step 2: Test both other expressions Option (B) replaces β by β^2, which is not permitted by the current identity. For example, at $\beta = 2$, the actual value is $\alpha = 2/3$, whereas (B) gives $4/5$. Option (D) is also incorrect: at the same value it gives $2/(1-4) = -2/3$, although a forward-active transistor has positive $\alpha < 1$. In fact for the usual $\beta > 1$, (D) predicts a negative gain. Thus there are two incorrect relationships, rather than a unique incorrect option.

Step 3: Conclusion Both (B) and (D) are incorrect identities. Choosing (B) is valid; the official answer key accepts options (B) and (D). Hence, the correct option is (B).

Quick Tip to Solve: Derive the gain formulas from current conservation first; a test value such as $\beta = 2$ quickly exposes invalid alternatives.

59. Hysteresis loops for two magnetic materials A and B are given below :

These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use : [4 marks]



(A) A for transformers and B for electric generators.

(B) B for electromagnets and transformers.

(C) A for electric generators and transformers.

(D) A for electromagnets and B for electric generators.

Correct Answer: (B) B for electromagnets and transformers.

Solution:

Concept:

- The area enclosed by a hysteresis loop measures the energy dissipated per unit volume during one complete magnetisation cycle.
- A transformer core should be magnetically soft, with a narrow loop and small coercive field, so

repeated reversal produces little heating.

- An electromagnet should magnetise and demagnetise readily. Permanent magnets instead need substantial coercivity and retentivity.

Step 1: Read the two loops Material A has the broader, more nearly rectangular loop, with a larger separation between the two branches and a larger enclosed area. Material B has the narrower loop. The field required to reduce its induction to zero is smaller, so B is the softer magnetic material. It can follow changes in the applied magnetising field with relatively little energy loss.

Step 2: Match material properties to applications A transformer is driven by alternating current and its core undergoes a magnetisation cycle repeatedly; selecting B reduces hysteresis heating. An electromagnet also benefits from easy magnetisation and demagnetisation, so B suits its core. Material A is more suitable where magnetisation must be retained, as in a permanent generator magnet. Option (A) incorrectly assigns the broad loop to a transformer, and (C) makes the same error. Option (D) incorrectly assigns A to an electromagnet and B to a permanent generator magnet.

Step 3: Conclusion Material B is appropriate for both electromagnet cores and transformer cores. Hence, the correct option is (B).

Quick Tip to Solve: For alternating-field cores choose the narrow hysteresis loop; for permanent magnets look for high coercivity and retentivity.

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60. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed number of A and B nuclei will be: [4 marks]

(A) $1:4$

(B) $5:4$

(C) $1:16$

(D) $4:1$

Correct Answer: (B) $5:4$

Solution:

Concept:

- After time t, the surviving nuclei are $N = N_0(1/2)^{t/T_{1/2}}$, where $T_{1/2}$ is the half-life.
- The number that have decayed is $D = N_0 - N$, not the surviving number N.
- Equal initial populations allow the common factor N_0 to cancel in the requested ratio.

Step 1: Find the surviving populations after eighty minutes For A, the elapsed time contains $80/20 = 4$ half-lives, so

$$N_A = N_0\left(\frac{1}{2}\right)^4 = \frac{N_0}{16}.$$

For B, it contains $80/40 = 2$ half-lives, giving

$$N_B = N_0 \left(\frac{1}{2}\right)^2 = \frac{N_0}{4}.$$

The ratio of surviving nuclei would therefore be $1 : 4$, but the question specifically asks how many have decayed.

Step 2: Subtract survivors and form the ratio The decayed populations are

$$D_A = N_0 - \frac{N_0}{16} = \frac{15N_0}{16}, \qquad D_B = N_0 - \frac{N_0}{4} = \frac{3N_0}{4}.$$

Consequently,

$$\frac{D_A}{D_B} = \frac{15/16}{3/4} = \frac{5}{4}.$$

Option (A) describes survivors instead of decays. Options (C) and (D) also do not result from subtracting the surviving fractions. Since A decays faster, its decayed population should be larger, which is consistent with $5 : 4$.

Step 3: Conclusion The ratio of the numbers of decayed nuclei is $D_A : D_B = 5 : 4$. Hence, the correct option is (B).

Quick Tip to Solve: Underline whether a decay question asks for remaining or decayed nuclei; for decayed nuclei use $N_0 - N$ before taking a ratio.

Chemistry (120 marks)

61. The reaction of zinc with dilute and concentrated nitric acid, respectively, produces : [4 marks]

(A) NO_2 and NO

(B) NO and N_2O

(C) NO_2 and N_2O

(D) N_2O and NO_2

Correct Answer: (D) N_2O and NO_2

Solution:

Concept:

- Nitric acid is an oxidising acid: zinc is oxidised to Zn^{2+}, while nitrate nitrogen is reduced to a nitrogen oxide.
- The reduction product depends on acid concentration and the reducing metal. Zinc with dilute nitric acid gives N_2O; concentrated acid gives NO_2.
- Nitrogen has oxidation state $+5$ in nitrate, an average of $+1$ in N_2O, and $+4$ in NO_2.

Step 1: Write the reaction with dilute acid Zinc is a sufficiently active metal to reduce dilute nitric

acid to nitrous oxide. The balanced reaction is

$$4\,Zn + 10\,HNO_3 \longrightarrow 4\,Zn(NO_3)_2 + N_2O + 5\,H_2O.$$

The four zinc atoms lose eight electrons in total. The two nitrogen atoms that form N_2O together gain eight electrons, consistent with the average nitrogen oxidation state changing from $+5$ to $+1$. Thus the first gas in the required pair is N_2O, not NO or NO_2.

Step 2: Write the concentrated-acid reaction and choose the ordered pair With concentrated nitric acid, the familiar brown nitrogen dioxide gas is obtained:

$$Zn + 4\,HNO_3 \longrightarrow Zn(NO_3)_2 + 2\,NO_2 + 2\,H_2O.$$

Here one zinc atom supplies two electrons and two nitrate nitrogen atoms each accept one. The order in the question is dilute acid followed by concentrated acid, so the gases must be N_2O followed by NO_2. Options (A), (B) and (C) all give the wrong first product and also fail this ordered comparison.

Step 3: Conclusion The products are N_2O with dilute nitric acid and NO_2 with concentrated nitric acid, respectively. Hence, the correct option is (D).

Quick Tip to Solve: For nitric-acid reactions remember both the metal and the concentration; the dilute-acid product for zinc differs from the familiar copper reaction.

62. Which of the following compounds is metallic and ferromagnetic ? [4 marks]

(A) CrO_2

(B) VO_2

(C) MnO_2

(D) TiO_2

Correct Answer: (A) CrO_2

Solution:

Concept:

- Metallic conductivity requires mobile electronic carriers; the presence of a metal atom in an oxide does not by itself make the oxide metallic.
- Ferromagnetism is the cooperative parallel alignment of magnetic moments, producing a net magnetisation even without an applied field.
- Chromium dioxide, CrO_2, is a standard example of a transition-metal oxide combining metallic electrical behaviour with ferromagnetism.

Step 1: Identify the oxide with both required properties Chromium dioxide has chromium formally in oxidation state $+4$, corresponding to a $3d^2$ configuration. Its electronic structure supports conducting electrons as well as ferromagnetic alignment. The question asks for both properties simultaneously, so an oxide that merely has unpaired electrons or displays conduction under some conditions is insufficient. CrO_2 meets the combined criterion.

Step 2: Exclude the other oxides VO_2 is well known for a temperature-dependent metal-insulator transition, but is not the standard metallic ferromagnet in this list. MnO_2 does not show the required combination of metallic conduction and ferromagnetic order under ordinary conditions; unpaired manganese electrons alone do not prove ferromagnetism. In TiO_2, titanium is Ti^{4+}, with a d^0 configuration, so it does not supply the partially occupied magnetic d states needed for this answer. These comparisons leave chromium dioxide as the only matching compound.

Step 3: Conclusion CrO_2 is metallic and ferromagnetic. Hence, the correct option is (A).

Quick Tip to Solve: Keep CrO_2 as the standard metallic ferromagnetic oxide; do not infer ferromagnetism from unpaired electrons alone.

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63. Decomposition of H_2O_2 follows a first order reaction. In fifty minutes the concentration of H_2O_2 decreases from 0.5 to 0.125 M in one such decomposition. When the concentration of H_2O_2 reaches 0.05 M, the rate of formation of O_2 will be : [4 marks]

(A) 6.93×10^{-4} mol min^{-1}

(B) 6.96×10^{-2} mol min^{-1}

(C) 1.34×10^{-2} mol min^{-1}

(D) 2.66 L min^{-1} at STP

Correct Answer: (A) 6.93×10^{-4} mol min^{-1}

Solution:

Concept:

- For first-order decomposition, $k = t^{-1}\ln(C_0/C_t)$, and the disappearance rate of the reactant is kC.
- The balanced decomposition is $2\,H_2O_2 \longrightarrow 2\,H_2O + O_2$; oxygen forms at half the molar concentration rate at which peroxide disappears.
- Concentration data determine rate per unit volume. An amount rate in $\mathrm{mol\,min^{-1}}$ additionally requires the solution volume.

Step 1: Calculate the first-order rate constant The concentration decreases by a factor of four in fifty minutes. Therefore

$$k = \frac{1}{50}\ln\frac{0.5}{0.125} = \frac{\ln 4}{50} = 0.027726\,\mathrm{min^{-1}}.$$

Equivalently this is two half-lives, so $t_{1/2} = 25\,\mathrm{min}$ and $k = 0.693/25$. The rate constant is fixed for the reaction conditions and does not change when the peroxide concentration reaches its later value.

Step 2: Use the later concentration and oxygen stoichiometry At $[H_2O_2] = 0.05\,\mathrm{mol\,L^{-1}}$,

$$-\frac{d[H_2O_2]}{dt} = k[H_2O_2] = 1.3863 \times 10^{-3}\,\mathrm{mol\,L^{-1}\,min^{-1}}.$$

Thus

$$\frac{d[O_2]}{dt} = \frac{1}{2}k[H_2O_2] = 6.93 \times 10^{-4}\,\mathrm{mol\,L^{-1}\,min^{-1}}.$$

This coefficient matches (A). Options (B) and (C) have incorrect numerical rates, and (D) is neither the corresponding per-litre gas-volume rate nor inferable without a sample volume. The printed molar

amount units omit the per-litre factor; an absolute amount rate would require the solution volume.

Step 3: Conclusion The oxygen formation rate per litre is $6.93 \times 10^{-4}\,\mathrm{mol\,L^{-1}\,min^{-1}}$. The numerical value corresponds to (A), whose printed units omit the per-litre factor. Hence, the correct option is (A).

Quick Tip to Solve: After finding kC, use the balanced equation: two peroxide molecules produce one oxygen molecule, so halve the peroxide disappearance rate.

64. Galvanization is applying a coating of : [4 marks]

(A) Cr

(B) Cu

(C) Zn

(D) Pb

Correct Answer: (C) Zn

Solution:

Concept:

- Galvanization protects iron or steel by applying a coating of zinc, Zn. The coating separates the iron surface from oxygen and moisture.
- Zinc is more readily oxidized than iron. It therefore provides sacrificial protection as well as a physical barrier against corrosion.

Step 1: Identify the protective coating Galvanization is the established name for zinc-coating iron or steel, commonly by dipping the cleaned metal in molten zinc. The zinc surface develops a protective layer of corrosion products. This prevents direct contact of the underlying iron with moist air and slows the electrochemical processes responsible for rust formation.

Step 2: Check the electrochemical protection Even if the coating is scratched, zinc is preferentially oxidized:

$$\mathrm{Zn} \longrightarrow \mathrm{Zn^{2+}} + 2\,\mathrm{e^-}.$$

The electrons help keep the exposed iron cathodic, so zinc is consumed instead of iron. Chromium coating is called chrome plating and is used for a hard, bright surface. Copper and lead coatings are different metal coatings; neither defines galvanization. Thus selecting any metal merely because it can form a surface coating would miss the specific meaning of the process.

Step 3: Conclusion Galvanization applies a zinc, Zn, coating to iron or steel. Hence, the correct option is (C).

Quick Tip to Solve: Associate galvanization with zinc and sacrificial protection; distinguish it from decorative chromium plating.

65. 18 g glucose ($C_6H_{12}O_6$) is added to 178.2 g water. The vapor pressure of water (in torr) for this aqueous solution is : [4 marks]

(A) 76.0

(B) 752.4

(C) 759.0

(D) 7.6

Correct Answer: (B) 752.4

Solution:

Concept:

- For a nonvolatile solute, Raoult's law gives $p = x_{\text{water}} p^0$, where p^0 is the vapour pressure of pure water at the same temperature.
- Glucose does not dissociate into ions in water. Its molar mass is $180\,\text{g mol}^{-1}$, while that of water is $18\,\text{g mol}^{-1}$.

Step 1: Calculate the solvent mole fraction The amounts of glucose and water are

$$n_{\text{glucose}} = \frac{18}{180} = 0.100\,\text{mol}, \qquad n_{\text{water}} = \frac{178.2}{18} = 9.900\,\text{mol}.$$

Therefore

$$x_{\text{water}} = \frac{9.900}{9.900 + 0.100} = 0.990.$$

It is the mole fraction, rather than the mass fraction, that enters Raoult's law.

Step 2: Evaluate the pressure and identify the needed datum The resulting pressure is

$$p = 0.990 p^0.$$

Taking the intended pure-water vapour pressure as $760\,\text{torr}$, this gives

$$p = 0.990(760) = 752.4\,\text{torr}.$$

The decrease is $7.6\,\text{torr}$, so option (D) represents the lowering rather than the remaining vapour pressure. The values 76.0 and $759.0\,\text{torr}$ do not follow from this mole fraction. Strictly, the printed question supplies neither temperature nor p^0, so it determines the ratio $p/p^0 = 0.990$, not an unconditional numerical pressure.

Step 3: Conclusion With the intended $p^0 = 760\,\text{torr}$, the solution vapour pressure is $752.4\,\text{torr}$. Hence, the correct option is (B). The numerical value requires this unstated pure-water pressure.

Quick Tip to Solve: First calculate the solvent mole fraction; keep the vapour pressure distinct from its lowering, and check that the pure-solvent pressure is known.

66. The reaction of propene with HOCl (Cl_2 + H_2O) proceeds through the intermediate: [4 marks]

(A) $CH_3 - CH^+ - CH_2 - OH$

(B) $CH_3 - CHCl - CH_2^+$

(C) CH_3 - CH^+ - CH_2 - OH

(D) $CH_3 - CH^+ - CH_2 - Cl$

Correct Answer: (D) $CH_3 - CH^+ - CH_2 - Cl$

Solution:

Concept:

- In chlorohydrin formation, the alkene reacts with electrophilic chlorine, followed by attack by water and loss of a proton.
- The polarized intermediate has greater positive character at the more substituted carbon. For propene this is the middle, secondary carbon.

Step 1: Place chlorine and positive charge Propene is $CH_3{-}CH{=}CH_2$. Adding electrophilic chlorine to the terminal carbon gives the open-chain representation

$$CH_3 - CH^+ - CH_2 - Cl.$$

This places positive charge at a secondary carbon. Placing chlorine on the middle carbon instead would give $CH_3 - CHCl - CH_2^+$, the less favourable primary-cation representation shown in option (B).

Step 2: Relate the intermediate to the product Water attacks the carbon with positive character, and deprotonation gives $CH_3{-}CH(OH){-}CH_2Cl$. Thus the hydroxyl group enters at the more substituted carbon. More precisely, halogen addition is described using a bridged chloronium ion rather than a freely isolated carbocation; option (D) expresses the relevant charge distribution in the open-chain notation used by the choices. Options (A) and (C) put an OH group on the terminal carbon and a positive charge on the middle carbon; they do not represent electrophilic chlorine addition leading to this chlorohydrin. The two similarly written OH-containing choices are therefore both excluded.

Step 3: Conclusion The suitable listed intermediate is $CH_3 - CH^+ - CH_2 - Cl$. Hence, the correct option is (D).

Quick Tip to Solve: For chlorohydrin formation, put OH on the more substituted alkene carbon and chlorine on the other carbon; use this to check the intermediate.

67. At 300 K and 1 atm, 15 mL of a gaseous hydrocarbon requires 375 mL air containing 20% O_2 by volume for complete combustion. After combustion the gases occupy 345 mL. Assuming that the water formed is in liquid form and the volumes were measured at the same temperature and pressure, the formula of the hydrocarbon is: [4 marks]

(A) C_3H_8

(B) C_4H_8

(C) C_4H_{10}

(D) C_3H_6

Correct Answer: (A) C_3H_8

Solution:

Concept:

- At the same temperature and pressure, gas volumes are proportional to their amounts in moles. Air supplies oxygen and an unreactive residual gas fraction.
- Complete combustion obeys $C_xH_y + (x^+y/_4)O_2 \longrightarrow xCO_2 + (y/_2)H_2O$. Liquid water does not contribute to the final gas volume.

Step 1: Separate oxygen and residual air The air contains

$$V_{O_2} = 0.20(375) = 75\,\text{mL}, \qquad V_{\text{inert}} = 375 - 75 = 300\,\text{mL}.$$

Since the stated air volume is required for complete combustion, its oxygen is consumed. The final gas consists of the unchanged inert component and carbon dioxide; condensed water is excluded. Hence

$$V_{CO_2} = 345 - 300 = 45\,\text{mL}.$$

Step 2: Determine carbon and hydrogen counts Each volume of hydrocarbon gives x volumes of carbon dioxide, so

$$x = \frac{45}{15} = 3.$$

The oxygen-to-hydrocarbon volume ratio is

$$x + \frac{y}{4} = \frac{75}{15} = 5.$$

Substituting $x = 3$ gives $y = 8$. The balanced check is

$$C_3H_8 + 5\,O_2 \longrightarrow 3\,CO_2 + 4\,H_2O.$$

Thus 15 mL gives 45 mL carbon dioxide, leaving 345 mL gas including inert air. Both four-carbon options would yield 60 mL carbon dioxide. Propene has three carbons but needs only 4.5 volumes of oxygen per volume of fuel and therefore fails the stated oxygen requirement.

Step 3: Conclusion The hydrocarbon is propane, C_3H_8. Hence, the correct option is (A).

Quick Tip to Solve: Subtract the unchanged non-oxygen part of air before using the carbon dioxide volume to count carbon atoms.

68. The distillation technique most suited for separating glycerol from spent-lye in the soap industry is : [4 marks]

(A) Simple distillation

(B) Fractional distillation

(C) Steam distillation

(D) Distillation under reduced pressure

Correct Answer: (D) Distillation under reduced pressure

Solution:

Concept:

- A liquid boils when its vapour pressure equals the external pressure. Lowering the external pressure lowers its boiling temperature.
- Reduced-pressure distillation is suitable for high-boiling liquids that may decompose on heating to their normal boiling point.

Step 1: Identify the difficulty with glycerol Spent-lye from soap manufacture contains glycerol together with water and dissolved inorganic material. Glycerol is a high-boiling substance, with a normal boiling point near $290\,^{\circ}\mathrm{C}$, and strong heating can cause decomposition. The useful separation must therefore allow glycerol to vaporize at a lower temperature while the nonvolatile salts remain in the residue.

Step 2: Choose the method and exclude alternatives Applying a vacuum lowers the pressure above the liquid. Glycerol then boils when its vapour pressure reaches this reduced value, before reaching its normal boiling temperature. Its vapour can be condensed and collected. Ordinary simple distillation at atmospheric pressure does not remove the overheating problem. Fractional distillation mainly improves separation of volatile liquids with relatively close boiling points; a fractionating column alone does not lower the pressure. Steam distillation is normally used for substances sufficiently volatile with steam and essentially immiscible with water. Glycerol is miscible with water and is not appropriately recovered by that method.

Step 3: Conclusion Glycerol is separated from spent-lye by distillation under reduced pressure. Hence, the correct option is (D).

Quick Tip to Solve: For a high-boiling liquid that can decompose when strongly heated, first consider lowering the pressure to lower its boiling temperature.

69. The equilibrium constant at 298 K for a reaction A + B ⇋ C + D is 100. If the initial concentration of all the four species were 1M each, then equilibrium concentration of D (in mol L^{-1}) will be:

[4 marks]

(A) 0.818

(B) 1.818

(C) 1.182

(D) 0.182

Correct Answer: (B) 1.818

Solution:

Concept:

- For $\mathrm{A} + \mathrm{B} \rightleftharpoons \mathrm{C} + \mathrm{D}$, $K_c = [C][D]/([A][B])$. All concentrations in this expression are equilibrium values.
- A reaction extent x changes each concentration by the same magnitude because every stoichiometric coefficient is one.

Step 1: Determine the direction and write concentrations Initially every concentration is $1\,\mathrm{mol\,L^{-1}}$, giving

$$Q_c = \frac{1 \times 1}{1 \times 1} = 1.$$

Since $Q_c < K_c = 100$, the reaction moves forward. If $x\,\mathrm{mol\,L^{-1}}$ reacts, then

$$[A] = [B] = 1 - x, \qquad [C] = [D] = 1 + x.$$

Physical concentrations require $0 \le x < 1$.

Step 2: Solve the equilibrium equation and check choices Substitution gives

$$100 = \frac{(1+x)^2}{(1-x)^2}.$$

Both numerator and denominator are positive in the physical interval, so

$$10 = \frac{1+x}{1-x}, \qquad 10 - 10x = 1 + x.$$

Therefore

$$x = \frac{9}{11} = 0.81818\ldots, \qquad [D] = 1 + x = \frac{20}{11} = 1.81818\ldots\ \mathrm{mol\,L^{-1}}.$$

Option (A) is the change in concentration rather than the final concentration. Option (D) is the remaining reactant concentration, $1 - x$. Option (C) gives a product/reactant ratio inconsistent with $K_c = 100$. Indeed $(20/11)^2/(2/11)^2 = 100$, verifying the result.

Step 3: Conclusion The equilibrium concentration of D is $1.818\,\mathrm{mol\,L^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: When products are present initially, add the equilibrium change to their initial concentration; do not report the change alone.

70. The heats of combustion of carbon and carbon monoxide are –393.5 and –283.5 kJ mol^{-1}, respectively. The heat of formation (in kJ) of carbon monoxide per mole is : [4 marks]

(A) 676.5

(B) -676.5

(C) –110

(D) 110.5

Correct Answer: (C) –110

Solution:

Concept:

- Hess's law states that the enthalpy change depends only on initial and final states; reaction equations and their enthalpy changes can be added.
- The formation reaction for one mole of carbon monoxide is $\mathrm{C(s)} + \frac{1}{2}\,\mathrm{O_2(g)} \longrightarrow \mathrm{CO\,(g)}$, starting with carbon in its standard state.

Step 1: Write the given combustion equations The two supplied combustion reactions are

$$\mathrm{C(s) + O_2(g) \longrightarrow CO_2(g)}, \quad \Delta H_1 = -393.5\,\mathrm{kJ\,mol^{-1}},$$

and

$$\mathrm{CO\,(g) + \frac{1}{2}\,O_2(g) \longrightarrow CO_2(g)}, \quad \Delta H_2 = -283.5\,\mathrm{kJ\,mol^{-1}}.$$

Both end at carbon dioxide, so subtracting the second reaction from the first eliminates this common product.

Step 2: Reverse the carbon monoxide combustion and add Reversing the second equation changes its enthalpy sign:

$$\mathrm{CO_2(g) \longrightarrow CO\,(g) + \frac{1}{2}\,O_2(g)}, \quad \Delta H = +283.5\,\mathrm{kJ\,mol^{-1}}.$$

Adding and cancelling carbon dioxide and half a mole of oxygen gives

$$\mathrm{C(s) + \frac{1}{2}\,O_2(g) \longrightarrow CO\,(g)}.$$

Thus

$$\Delta H_f = -393.5 + 283.5 = -110.0\,\mathrm{kJ\,mol^{-1}}.$$

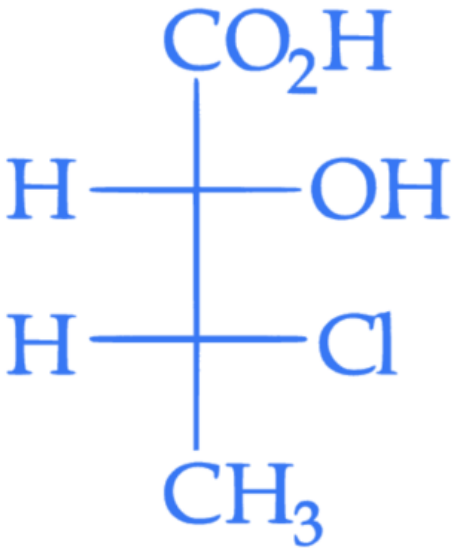
The sum of the two negative combustion enthalpies would describe no valid formation reaction, excluding the large-magnitude options (A) and (B). A positive value such as (D) has the wrong sign; formation in the stated direction releases heat.

Step 3: Conclusion The heat of formation of one mole of carbon monoxide is $-110\,\mathrm{kJ\,mol^{-1}}$, using the supplied data. Hence, the correct option is (C).

Quick Tip to Solve: For formation of CO, subtract its combustion enthalpy from that of carbon; reverse the entire equation and its enthalpy sign together.

71. The absolute configuration of

is : [4 marks]



(A) $(2S, 3S)$

(B) $(2R, 3R)$

(C) $(2R, 3S)$

(D) $(2S, 3R)$

Correct Answer: (D) $(2S, 3R)$

Solution:

Concept:

- Assign priorities at each stereocentre using atomic number; compare tied carbon ligands at their first point of difference.
- In a Fischer projection, horizontal bonds point towards the observer and vertical bonds point away. Reverse the apparent sense when priority 4 is horizontal.

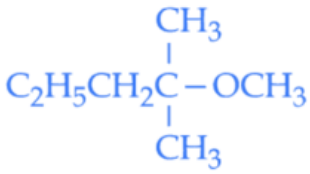
Step 1: Rank the groups at carbon 2 The carboxyl carbon is C1, so the upper crossing is C2. Oxygen of the hydroxyl group has priority 1 and hydrogen has priority 4. Compare the two carbon ligands: C3 is attached to $\mathrm{Cl}, \mathrm{C}, \mathrm{H}$, whereas the carboxyl carbon is treated as attached to $\mathrm{O}, \mathrm{O}, \mathrm{O}$, counting its double bond twice. Since chlorine has higher atomic number than oxygen, the downward C3 ligand has priority 2 and the upward carboxyl ligand has priority 3.

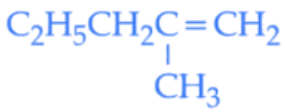
Step 2: Determine both configurations At C2, the sequence right $\rightarrow$ down $\rightarrow$ up is clockwise. Hydrogen lies on the horizontal left bond, pointing towards us, so reverse the sense to obtain $2S$. At C3, chlorine on the right has priority 1, the upward C2 ligand has priority 2, the downward methyl group has priority 3, and hydrogen on the left has priority 4. The upper ligand outranks methyl because its next atoms include oxygen. The sequence right $\rightarrow$ up $\rightarrow$ down is anticlockwise; reversing because hydrogen is horizontal gives $3R$. Thus A has C3 wrong, B has C2 wrong, and C has both wrong.

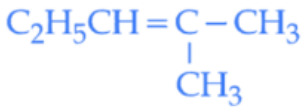
Step 3: Conclusion The absolute configuration is $(2S, 3R)$. Hence, the correct option is (D).

Quick Tip to Solve: At C2, compare chlorine with oxygen before deciding the order of the two carbon ligands; then reverse for horizontal hydrogen.

72. 2-chloro-2-methylpentane on reaction with sodium methoxide in methanol yields: [4 marks]

(a) 

(b) 

(c) 

(A) All of these

(B) (a) and (c)

(C) (c) only

(D) (a) and (b)

Correct Answer: (C) (c) only

Solution:

Concept:

- A tertiary alkyl chloride reacts preferentially by elimination with a strong alkoxide base; steric crowding disfavors direct substitution at its tertiary carbon.
- With a small base such as methoxide, the more substituted alkene is the major elimination product according to the Zaitsev rule.

Step 1: Identify the competing beta positions The substrate is $(CH_3)_2C(Cl)CH_2CH_2CH_3$. Chloride is bonded to a tertiary carbon. Methoxide can abstract a beta hydrogen as the carbon-chlorine bond breaks in an E2 elimination. The two methyl groups attached to that carbon are equivalent; removing hydrogen from either gives the terminal alkene $CH_2{=}C(CH_3)CH_2CH_2CH_3$, structure (b). Removing hydrogen from the adjacent propyl-chain methylene gives the internal alkene $(CH_3)_2C{=}CHCH_2CH_3$, structure (c).

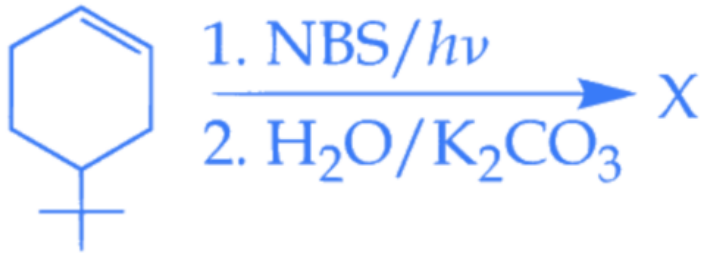
Step 2: Select the major product and the matching option The internal double bond of (c) has three alkyl substituents, whereas the terminal double bond of (b) has only two. The greater alkyl substitution stabilises (c) through hyperconjugation and the electron-releasing effect of alkyl groups. Methoxide is a small, unhindered base, so the usual Zaitsev orientation favours 2-methylpent-2-ene, structure (c). The tertiary substrate also prevents the direct backside substitution that would produce ether (a). Consequently, in the standard major-product interpretation of this question, options A and B include an inappropriate substitution product, while D includes that ether and omits the favoured

alkene. Although a minor terminal alkene may accompany elimination, it is not the principal product selected in this MCQ.

Step 3: Conclusion The major product is 2-methylpent-2-ene, structure (c). Hence, the correct option is (C).

Quick Tip to Solve: With a tertiary chloride and a small strong base, apply elimination first and compare alkene substitution to locate the major product.

73. The product of the reaction given below is : [4 marks]



(A)  5-tert-butylcyclohex-2-en-1-one

(B) 5-tert-butylcyclohex-2-ene-1-carboxylic acid 

(C) 4-tert-butylcyclohex-1-ene 

(D) 5-tert-butylcyclohex-2-en-1-ol 

Correct Answer: (D) 5-tert-butylcyclohex-2-en-1-ol 

Solution:

Concept:

- N-bromosuccinimide (NBS) under light provides radical allylic bromination rather than ordinary addition of bromine across the double bond.
- Hydrolysis of an allylic bromide with water and potassium carbonate replaces the bromine by hydroxyl, retaining an alkene.

Step 1: Apply the first reagent The starting ring is a tert-butyl-substituted cyclohexene. Under NBS and $h\nu$, an allylic hydrogen is abstracted and the resulting resonance-stabilised allylic radical is trapped by bromine. The important change is formation of an allylic carbon-bromine bond; the ring and the tert-butyl substituent are retained. Because the allylic radical is delocalised, the alkene position must be followed together with the newly substituted carbon when comparing the drawings. This is not a reagent combination that inserts a new carbon atom.

Step 2: Apply hydrolysis and compare the products Treatment with H_2O/K_2CO_3 converts the allylic bromide to the corresponding allylic alcohol. The product shown in option D has hydroxyl on an allylic ring carbon, next to the retained carbon-carbon double bond, and the tert-butyl group remains unchanged. Numbering from the alcohol carbon gives 5-tert-butylcyclohex-2-en-1-ol. Option A is an enone and would require oxidation of the alcohol; these reagents do not perform that oxidation.

Option B has an extra carboxyl carbon, which neither stage can introduce. Option C is merely an unsubstituted-at-the-allylic-position alkene and omits the hydroxyl introduced by hydrolysis.

Step 3: Conclusion The product is the allylic alcohol depicted in option D, 5-tert-butylcyclohex-2-en-1-ol. Hence, the correct option is (D).

Quick Tip to Solve: Read this reagent pair as allylic H to Br, then allylic Br to OH; do not convert the alcohol to a ketone without an oxidant.

74. In the Hofmann bromamide degradation reaction, the number of moles of NaOH and Br_2 used per mole of amine produced are: [4 marks]

(A) Four moles of NaOH and two moles of Br_2.

(B) Two moles of NaOH and two moles of Br_2

(C) Four moles of NaOH and one mole of Br_2

(D) One mole of NaOH and one mole of Br_2

Correct Answer: (C) Four moles of NaOH and one mole of Br_2

Solution:

Concept:

- Hofmann bromamide degradation converts a primary amide to a primary amine with one fewer carbon atom.
- The required reagent amounts follow from the balanced overall equation, including both bromide and carbonate formed in alkaline solution.

Step 1: Write the balanced reaction For a primary amide $RCONH_2$, the overall reaction is

$$RCONH_2 + Br_2 + 4\,NaOH \longrightarrow RNH_2 + Na_2CO_3 + 2\,NaBr + 2\,H_2O.$$

The carbonyl carbon is removed from the organic product and appears in carbonate; the group R becomes attached to nitrogen during rearrangement. Thus one mole of primary amide gives one mole of amine, so the coefficients of bromine and hydroxide in this equation can be read directly per mole of amine formed.

Step 2: Check the reagent coefficients The single Br_2 molecule supplies two bromine atoms, which finish as two $NaBr$. Four sodium atoms are needed in total: two for Na_2CO_3 and two for the bromide salts, requiring four $NaOH$. Oxygen also balances: the amide oxygen plus four hydroxide oxygens equal the three carbonate oxygens plus two water oxygens. This confirms that the stoichiometric requirement is four moles of hydroxide and one mole of bromine. Option A incorrectly doubles bromine. Option B both doubles bromine and supplies insufficient hydroxide. Option D supplies only one-quarter of the hydroxide needed by the balanced reaction.

Step 3: Conclusion Per mole of amine produced, four moles of $NaOH$ and one mole of Br_2 are required. Hence, the correct option is (C).

Quick Tip to Solve: Balance the carbonate and the two sodium bromide molecules to recover the four-hydroxide coefficient without memorising intermediate steps.

75. Which one of the following complexes shows optical isomerism?
(en = ethylenediamine) [4 marks]

(A) cis$[Co(en)_2Cl_2]Cl$

(B) trans$[Co(en)_2Cl_2]Cl$

(C) $[Co(NH_3)_4Cl_2]Cl$

(D) $[Co(NH_3)_3Cl_3]$

Correct Answer: (A) cis$[Co(en)_2Cl_2]Cl$

Solution:

Concept:

- Optical isomers are non-superimposable mirror images. An internal mirror plane or centre of symmetry makes a coordination entity achiral.
- Ethylenediamine, en, is a bidentate ligand. Two such ligands form two chelate rings around an octahedral metal centre.

Step 1: Examine the two chelated complexes In cis-$[Co(en)_2Cl_2]^+$, the chlorido ligands occupy adjacent octahedral positions. The two en chelate rings can wrap around the metal in two opposite arrangements, conventionally described as Δ and Λ. These are mirror images that cannot be superimposed by rotation. The cis arrangement lacks the internal symmetry needed to make these two forms identical. The chloride outside the square brackets is a counter-ion and does not change the stereochemistry of the coordination sphere.

Step 2: Reject the achiral alternatives For trans-$[Co(en)_2Cl_2]^+$, the chlorido ligands occupy opposite positions and the arrangement has symmetry making it optically inactive. In $[Co(NH_3)_4Cl_2]^+$, an octahedral MA_4B_2 entity can have cis and trans geometrical forms, but each is achiral: monodentate ligands do not create the chelate arrangement present in A. The $[Co(NH_3)_3Cl_3]$ entity has facial and meridional geometrical isomers, both possessing a mirror plane. Therefore B, C and D do not show the optical isomerism asked for. Geometrical isomerism alone is not evidence of optical activity.

Step 3: Conclusion The complex cis-$[Co(en)_2Cl_2]Cl$ shows optical isomerism. Hence, the correct option is (A).

Quick Tip to Solve: For octahedral complexes with two en rings and two identical monodentate ligands, test the cis form first for a mirror-image pair.

76. Thiol group is present in : [4 marks]

(A) Cytosine

(B) Cystine

(C) Cysteine

(D) Methionine

Correct Answer: (C) Cysteine

Solution:

Concept:

- A thiol group is $-SH$: sulfur must be bonded to hydrogen. Merely containing sulfur does not make a compound a thiol.
- Cysteine contains a sulfhydryl side chain. Oxidation joins two cysteine residues through a disulfide linkage to form cystine.

Step 1: Identify the sulfhydryl side chain Cysteine has the structure $HS-CH_2-CH(NH_2)-COOH$. Its side chain is $-CH_2SH$, so it contains the required sulfur-hydrogen bond. This free thiol can participate in oxidation-reduction chemistry and in forming disulfide bonds between peptide chains or different parts of one chain. The amino and carboxyl groups do not determine the answer here; the question specifically asks for the functional group in the side chain.

Step 2: Compare all the alternatives Cystine is the oxidation product of two cysteine molecules: their sulfur atoms are joined as $-CH_2-S-S-CH_2-$. Thus cystine contains a disulfide linkage rather than a free thiol. Methionine has a thioether side chain, $-CH_2CH_2-S-CH_3$, in which sulfur is bonded to two carbon groups and has no hydrogen attached. Cytosine is a nitrogenous pyrimidine base containing amino and carbonyl functionality and no sulfur atom. These distinctions rule out B, D and A, respectively. In particular, the similar names cysteine and cystine must not be confused: only the former retains the free sulfhydryl group.

Step 3: Conclusion The thiol group is present in cysteine. Hence, the correct option is (C).

Quick Tip to Solve: Look specifically for an S-H bond: cysteine has SH, cystine has S-S, and methionine has C-S-C.

77. The pair having the same magnetic moment is :

[At. No.: Cr = 24, Mn = 25, Fe = 26, Co = 27] [4 marks]

(A) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

(B) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$

(C) $[CoCl_4]^{2-}$ and $[Fe(H_2O)_6]^{2+}$

(D) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$

Correct Answer: (A) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

Solution:

Concept:

- For these transition-metal complexes, compare the spin-only moment $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons.
- Water and chloride are weak-field ligands here. The aqua complexes are high-spin octahedral, while $[CoCl_4]^{2-}$ is high-spin tetrahedral.

Step 1: Determine the metal-ion configurations All four metal ions are in oxidation state +2: water is neutral, and four chlorides contribute -4 to the cobalt complex of charge -2. The resulting d counts are $Cr^{2+} : d^4$, $Mn^{2+} : d^5$, $Fe^{2+} : d^6$, and $Co^{2+} : d^7$. In high-spin octahedral fields, d^4 has four unpaired electrons, d^5 has five, and d^6 has four. In a tetrahedral field, high-spin d^7 has configuration $e^4t_2^3$, with three unpaired electrons.

Step 2: Compare the four pairs The chromium and iron aqua complexes therefore both have

$$\mu = \sqrt{4(4+2)}\,\mu_B = \sqrt{24}\,\mu_B \approx 4.90\,\mu_B.$$

The manganese aqua complex has $\sqrt{35}\,\mu_B$, approximately $5.92\,\mu_B$, while the cobalt chloride complex has $\sqrt{15}\,\mu_B$, approximately $3.87\,\mu_B$. Thus option B compares five unpaired electrons with four, C compares three with four, and D compares four with three. Only A compares two species having the same number of unpaired electrons and consequently the same spin-only magnetic moment.

Step 3: Conclusion The pair is $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$, each with four unpaired electrons. Hence, the correct option is (A).

Quick Tip to Solve: Compute the d count after oxidation first; high-spin octahedral d4 and d6 both have four unpaired electrons.

78. A stream of electrons from a heated filament was passed between two charged plates kept at a potential difference V esu. If e and m are charge and mass of an electron, respectively, then the value of h / λ (where λ is wavelength associated with electron wave) is given by : [4 marks]

(A) 2meV

(B) $\sqrt{meV}$

(C) $\sqrt{2meV}$

(D) meV

Correct Answer: (C) $\sqrt{2meV}$

Solution:

Concept:

- De Broglie associates wavelength λ with momentum p through $\lambda = h/p$, where h is Planck's constant.
- An electron accelerated through a potential difference of magnitude V gains kinetic energy eV, where e denotes the magnitude of its charge. In the nonrelativistic treatment, $K = p^2/(2m)$.

Step 1: Identify the quantity required The ratio in the question is a momentum, not an energy or a wavelength. Rearranging the matter-wave relation gives

$$\frac{h}{\lambda} = p = mv.$$

The mass is the electron mass and the speed is its speed after acceleration. The heated filament supplies the electrons; the electric field between the plates supplies their directed kinetic energy. We use the usual accelerating-voltage approximation, neglecting the initial thermal kinetic energy compared with the energy gained.

Step 2: Use the accelerating potential The magnitude of the electric work is eV. Equate this to the kinetic energy and solve for the positive speed:

$$\frac{1}{2}mv^2 = eV, \qquad v = \sqrt{\frac{2eV}{m}}.$$

Multiplication by m now yields

$$p = m\sqrt{\frac{2eV}{m}} = \sqrt{2meV}.$$

The positive square root is appropriate because a wavelength determines the magnitude of momentum. The stated electrostatic units do not alter the algebra when the charge and potential are expressed in a consistent system.

Step 3: Reject the remaining expressions Option (B), $\sqrt{meV}$, misses the factor of two coming from kinetic energy. Options (A) and (D) have the dimensions of momentum squared rather than momentum: meV is mass times energy. Thus neither can equal h/λ.

Step 4: Conclusion

$$\boxed{\frac{h}{\lambda} = \sqrt{2meV}}$$

Hence, the correct option is (C).

Quick Tip to Solve: Identify h/λ as momentum first, then use $p^2 = 2mK$ with $K = eV$.

79. The species in which the N atom is in a state of sp hybridization is : [4 marks]

(A) NO_2^+

(B) NO_2^-

(C) NO_3^-

(D) NO_3^-

Correct Answer: (A) NO_2^+

Solution:

Concept:

- Hybridisation is assigned from the number of sigma-bonding domains and lone pairs around the central atom. A multiple bond contributes one domain, not one domain for each bond line.
- Two electron domains give linear geometry and sp hybridisation; three give trigonal-planar electron geometry and sp^2 hybridisation.

Step 1: Count domains in the nitronium ion For NO_2^+, the total number of valence electrons is $5 + 2(6) - 1 = 16$. The Lewis structure is $O{=}N^+{=}O$, with the positive charge on nitrogen in the usual formal-charge representation. Nitrogen has two sigma bonds, one to each oxygen, and no lone pair. Its steric number is therefore

$$2 + 0 = 2.$$

The two sigma-bond directions are linear, with an $O{-}N{-}O$ angle of 180°, so the central nitrogen uses sp hybrid orbitals. The two remaining unhybridised p orbitals can participate in pi bonding.

Step 2: Check the other printed species In NO_2^-, there are $5+12+1 = 18$ valence electrons. Nitrogen has two sigma bonds and one lone pair, so its steric number is three. It is assigned sp^2 hybridisation and the molecular shape is bent. In NO_3^-, nitrogen has three sigma bonds and no lone pair: it again has steric number three and sp^2 hybridisation. Resonance distributes the pi bonding over the nitrate ion but does not turn its three sigma-bond directions into two.

Step 3: Select the uniquely linear species Options (C) and (D) both print nitrate, and both are rejected by the same three-domain count. The duplicate printed species does not affect the answer: neither nitrate entry nor the bent nitrite ion has an sp-hybridised nitrogen. Only the nitronium ion satisfies the two-domain requirement.

Step 4: Conclusion The nitrogen atom in NO_2^+ has two electron domains and is sp-hybridised. Hence, the correct option is (A).

Quick Tip to Solve: Count sigma bonds plus lone pairs at nitrogen; do not count each pi bond as an additional hybridisation domain.

80. Which of the following atoms has the highest first ionization energy? [4 marks]

(A) Na

(B) K

(C) Sc

(D) Rb

Correct Answer: (C) Sc

Solution:

Concept:

- First ionisation energy is the energy needed for $\mathrm{M(g)} \longrightarrow \mathrm{M^+(g) + e^-}$. It depends on the nuclear attraction experienced by the outermost electron.
- Increasing atomic size and shielding generally lowers ionisation energy down a group; increasing effective nuclear charge generally raises it across a period.

Step 1: Compare the three alkali metals Sodium, potassium and rubidium are Group 1 atoms with one outermost s electron. Their valence configurations are $\mathrm{Na} : [Ne]3s^1$, $\mathrm{K} : [Ar]4s^1$ and $\mathrm{Rb} : [Kr]5s^1$. Moving down this group increases the principal shell and shielding, so removal of the valence electron becomes easier. Consequently,

$$I_1(\mathrm{Na}) > I_1(\mathrm{K}) > I_1(\mathrm{Rb}).$$

This already rules out potassium and rubidium as the largest of the four choices. Sodium is the strongest competitor among the alkali-metal options.

Step 2: Compare scandium with the alkali metals Scandium has configuration $[\mathrm{Ar}]3d^1 4s^2$. Relative to potassium in the same period, scandium has a greater nuclear charge and its outer electron is more strongly bound overall. Its first ionisation energy is approximately $633\,\mathrm{kJ\,mol^{-1}}$, greater than sodium's approximately $496\,\mathrm{kJ\,mol^{-1}}$. These values illustrate the stronger binding; memorising the exact numbers is unnecessary. The relevant comparison includes effective nuclear attraction as well as shell number, so simply selecting the element with the smallest period number would give the wrong result.

Step 3: Apply the comparison to the options The resulting order for the listed atoms is

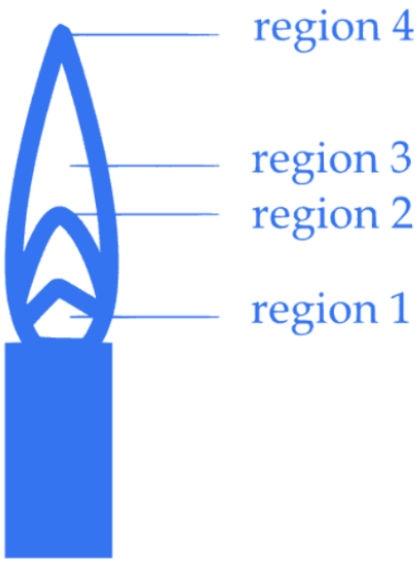
$$I_1(\mathrm{Sc}) > I_1(\mathrm{Na}) > I_1(\mathrm{K}) > I_1(\mathrm{Rb}).$$

Thus options (A), (B) and (D) all correspond to less energy required to remove the first electron than scandium does. We compare neutral gaseous atoms throughout, rather than their ions or bulk metallic samples.

Step 4: Conclusion Scandium has the highest first ionisation energy among the listed atoms. Hence, the correct option is (C).

Quick Tip to Solve: First eliminate K and Rb using the down-group trend, then compare Na with the more strongly binding transition element Sc.

81. The hottest region of Bunsen flame shown in the figure below is : [4 marks]



(A) region 1

(B) region 2

(C) region 3

(D) region 4

Correct Answer: (B) region 2

Solution:

Concept:

- A Bunsen burner mixes fuel gas with air. The different flame zones correspond to unburnt gas, partial combustion and efficient combustion with the surrounding oxygen.
- The most useful heating zone is just above the tip of the inner cone, where combustion is efficient and the temperature is highest. Its position must be read from the labelled figure.

Step 1: Locate the heating zone in the drawing The figure labels four positions along the flame. Region 1 is close to the burner mouth in the inner low-temperature zone. Region 2 points to the neighbourhood of the tip of the inner cone. Regions 3 and 4 are higher in the extended outer flame, with region 4 at the top. The comparison concerns these indicated locations, rather than the height of the flame alone. In a properly adjusted Bunsen flame, the standard point of maximum temperature for heating is the region immediately above the inner-cone tip, indicated here by region 2.

Step 2: Explain why this region is hottest The gas-air mixture is still undergoing combustion in the inner part of the flame; unburnt gas near the base has not yet released all of its chemical energy. Around the inner-cone tip, hot combustible intermediates meet an adequate supply of oxygen and combustion proceeds most effectively. This produces the hottest local heating region. An object to be heated strongly is therefore placed just above the inner cone rather than in the unburnt-gas zone or simply at the highest visible point. The distribution of temperature depends on mixing and reaction, not on a rule that temperature always increases with height.

Step 3: Eliminate the remaining labels Region 1 is rejected because it is near the relatively cool inner

gas zone. Regions 3 and 4 are rejected because the labelled points lie beyond the local maximum associated with the inner-cone tip; hot products can transfer heat to the surroundings as they rise. Thus the marked region 2, rather than the extreme outer tip, is the appropriate answer for this diagram.

Step 4: Conclusion The hottest indicated region is region 2, just above the inner-cone tip. Hence, the correct option is (B).

Quick Tip to Solve: For strong heating, use the zone immediately above the inner blue cone; do not automatically select the highest flame tip.

82. The pair in which phosphorous atoms have a formal oxidation state of +3 is : [4 marks]

(A) Orthophosphorous and pyrophosphorous acids

(B) Pyrophosphorous and hypophosphoric acids

(C) Orthophosphorus and hypophosphoric acids

(D) Pyrophosphorous and pyrophosphoric acids

Correct Answer: (A) Orthophosphorous and pyrophosphorous acids

Solution:

Concept:

- For a neutral oxoacid, the algebraic sum of oxidation states is zero. Use $+1$ for hydrogen and -2 for oxygen in these compounds.
- An acid name must first be matched to its correct molecular formula. Oxidation state and basicity are different quantities: a P–H bond does not determine the oxidation state by counting ionisable hydrogens.

Step 1: Evaluate orthophosphorous acid Orthophosphorous acid has formula H_3PO_3. If the oxidation state of phosphorus is x, electrical neutrality gives

$$3(+1) + x + 3(-2) = 0, \qquad x = +3.$$

Thus this acid is a suitable member of the required pair. Although its structure contains a P–H bond and only two ionisable O–H hydrogens, all three hydrogens enter the oxidation-state sum; the number of acidic protons must not replace the molecular formula.

Step 2: Evaluate pyrophosphorous acid Pyrophosphorous acid is $H_4P_2O_5$, obtained formally by loss of water from two molecules of H_3PO_3:

$$2\,H_3PO_3 \longrightarrow H_4P_2O_5 + H_2O.$$

Its two phosphorus atoms are equivalent for this oxidation-state assignment. Therefore,

$$4(+1) + 2x + 5(-2) = 0, \qquad 2x = 6, \qquad x = +3.$$

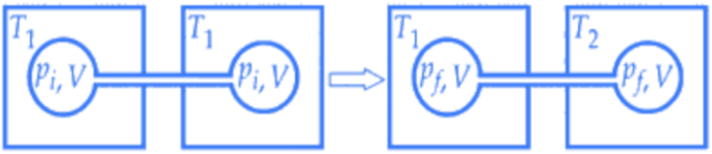
Both members in option (A) now meet the stated requirement.

Step 3: Reject the acids in the other pairs Hypophosphoric acid has formula $H_4P_2O_6$, giving $4 + 2x - 12 = 0$, so each phosphorus is $+4$. Pyrophosphoric acid is $H_4P_2O_7$, giving $4 + 2x - 14 = 0$, so each phosphorus is $+5$. Options (B) and (C) include hypophosphoric acid and hence fail; option (D) includes pyrophosphoric acid and fails. Similar names do not imply identical oxidation states.

Step 4: Conclusion Phosphorus is in oxidation state $+3$ in both orthophosphorous acid, H_3PO_3, and pyrophosphorous acid, $H_4P_2O_5$. Hence, the correct option is (A).

Quick Tip to Solve: Write each oxoacid formula before calculating; distinguish hypophosphoric $H_4P_2O_6$ from pyrophosphoric $H_4P_2O_7$.

83. Two closed bulbs of equal volume (V) containing an ideal gas initially at pressure p_i and temperature T_1 are connected through a narrow tube of negligible volume as shown in the figure below. The temperature of one of the bulbs is then raised to T_2. The final pressure p_f is : [4 marks]



(A) $2p_i\left(\frac{T_2}{T_1+T_2}\right)$

(B) $2p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$

(C) $p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$

(D) $2p_i\left(\frac{T_1}{T_1+T_2}\right)$

Correct Answer: (A) $2p_i\left(\frac{T_2}{T_1+T_2}\right)$

Solution:

Concept:

- For an ideal gas, $n = pV/(RT)$, with temperature measured on the absolute scale. Connected bulbs reach a common mechanical pressure even when their temperatures differ.
- The system is closed, so the total number of moles is conserved. The separate bulbs retain equal fixed volumes V, while their final temperatures are T_1 and T_2.

Step 1: Count the initial amount of gas Initially each bulb is at pressure p_i, temperature T_1 and volume V. The narrow tube has negligible volume, so it adds no appreciable gas capacity. Each bulb contains $p_iV/(RT_1)$ moles, and the total is

$$n_{\text{total}} = \frac{2p_iV}{RT_1}.$$

Heating one bulb can move gas through the tube, so the amount in an individual bulb need not remain fixed. Only the amount in the two-bulb system is constant.

Step 2: Write the final mole balance At equilibrium the connecting tube ensures a common pressure

p_f. One bulb remains at T_1 and the other is at T_2. Thus conservation gives

$$\frac{p_f V}{RT_1} + \frac{p_f V}{RT_2} = \frac{2p_i V}{RT_1}.$$

Cancel V/R and rearrange:

$$p_f \frac{T_1 + T_2}{T_1 T_2} = \frac{2p_i}{T_1}, \qquad p_f = \frac{2p_i T_2}{T_1 + T_2}.$$

No single final temperature should be assigned to the whole gas, since the two bulbs are maintained at different temperatures.

Step 3: Check dimensions and limiting behaviour Options (B) and (C) multiply a pressure by a quantity with temperature dimensions, so neither is a pressure. If $T_2 > T_1$, heating must raise the pressure above p_i; option (D) instead predicts a decrease. The derived expression gives $p_f = p_i$ when $T_2 = T_1$, and for heating gives $p_i < p_f < 2p_i$, consistent with gas redistribution between fixed-volume bulbs.

Step 4: Conclusion

$$\boxed{p_f = 2p_i \left(\frac{T_2}{T_1 + T_2} \right)}$$

Hence, the correct option is (A).

Quick Tip to Solve: Use one common final pressure but two temperatures; conserve total moles instead of the moles in either bulb separately.

84. Which of the following is an anionic detergent ? [4 marks]

(A) Sodium stearate

(B) Sodium lauryl sulphate

(C) Cetyltrimethyl ammonium bromide

(D) Glyceryl oleate

Correct Answer: (B) Sodium lauryl sulphate

Solution:

Concept:

- An anionic synthetic detergent has a long hydrophobic hydrocarbon chain and a negatively charged hydrophilic head group, commonly a sulphate or sulphonate.
- Soaps are sodium or potassium salts of long-chain fatty acids. They are distinguished from synthetic detergents even though their cleansing ions are also negatively charged.

Step 1: Identify the sulphate detergent Sodium lauryl sulphate is a sodium alkyl sulphate with the representative formula $C_{12}H_{25}OSO_3Na$. In water it provides sodium ions and the surface-active lauryl sulphate anion:

$$C_{12}H_{25}OSO_3Na \longrightarrow Na^+ + C_{12}H_{25}OSO_3{}^-.$$

The long alkyl chain associates with greasy material while the charged sulphate-containing head interacts with water. Because the surface-active portion carries a negative charge, this compound belongs to the anionic synthetic-detergent class.

Step 2: Distinguish soap from synthetic detergent Sodium stearate is $C_{17}H_{35}COONa$, the sodium salt of a long-chain carboxylic acid. It is a soap, not the anionic synthetic detergent sought in this classification. Its stearate ion can form insoluble calcium and magnesium salts in hard water, which is a familiar distinction between soaps and many synthetic detergents. Thus merely observing a sodium counterion or a negative cleansing ion is insufficient to select option (A). The chemical nature of the head group matters.

Step 3: Exclude the cationic and uncharged choices Cetyltrimethyl ammonium bromide contains a positively charged quaternary ammonium surface-active ion, $C_{16}H_{33}N(CH_3)_3{}^+$, so it is a cationic detergent and option (C) fails. Glyceryl oleate is an ester derived from glycerol and oleic acid; it does not provide a sulphate or sulphonate anionic head group and is not the required anionic detergent. Therefore option (D) also fails, leaving sodium lauryl sulphate as the correct classification.

Step 4: Conclusion Sodium lauryl sulphate is an anionic synthetic detergent. Hence, the correct option is (B).

Quick Tip to Solve: Classify the surface-active ion: sulphate/sulphonate heads are anionic, quaternary ammonium heads are cationic, and fatty-acid salts are soaps.

85. Which one of the following ores is best concentrated by froth floatation method? [4 marks]

(A) Siderite

(B) Galena

(C) Malachite

(D) Magnetite

Correct Answer: (B) Galena

Solution:

Concept:

- Froth flotation separates an ore from gangue by the difference in their wettability. Sulphide ore particles are preferentially wetted by oil, whereas the gangue is wetted by water.
- Collectors enhance the water-repellent character of sulphide particles; frothers stabilise the bubbles that carry these particles to the surface.

Step 1: Identify the chemical nature of each ore Galena is lead sulphide, PbS. Siderite is iron(II) carbonate, $FeCO_3$; malachite is basic copper carbonate, $CuCO_3 \cdot Cu(OH)_2$; and magnetite is the oxide Fe_3O_4. Thus, among the four listed ores, galena is the sulphide ore. This chemical classification determines which ore is suited to the flotation process.

Step 2: Apply the flotation principle In the flotation cell, the finely powdered ore is agitated with water, suitable reagents and air. The oil-wetted galena particles attach to air bubbles and rise as a froth, which is skimmed off. The water-wetted gangue remains in the aqueous mixture. The separation

therefore enriches the sulphide without requiring the ore itself to dissolve.

Step 3: Eliminate the remaining choices Siderite and malachite are carbonate ores, so they are not the standard sulphide-ore application asked here. Magnetite is an oxide with magnetic properties and is ordinarily associated with magnetic separation when the gangue permits it. None of these three is the best choice for the stated froth flotation method.

Step 4: Conclusion Galena, PbS, is best concentrated by froth flotation among the given ores. Hence, the correct option is (B).

Quick Tip to Solve: First classify each ore as sulphide, oxide or carbonate: the usual froth flotation choice is the sulphide.

86. The concentration of fluoride, lead, nitrate and iron in a water sample from an underground lake was found to be 1000 ppb, 40 ppb, 100 ppm and 0.2 ppm, respectively. This water is unsuitable for drinking due to high concentration of : [4 marks]

(A) Lead

(B) Nitrate

(C) Iron

(D) Fluoride

Correct Answer: (B) Nitrate

Solution:

Concept:

- Compare contaminant concentrations only after expressing them in the same units. For these dilute aqueous concentrations, $1000\,\text{ppb} = 1\,\text{ppm}$.
- The textbook comparison used here permits lead up to about $0.05\,\text{ppm}$, iron up to about $0.2\,\text{ppm}$, and nitrate up to $50\,\text{ppm}$. Fluoride near $1\,\text{ppm}$ is not an excessive fluoride concentration.

Step 1: Convert the values given in ppb The sample contains fluoride at $1000\,\text{ppb}$ and lead at $40\,\text{ppb}$. Converting them gives

$$c_{\text{fluoride}} = 1.0\,\text{ppm}, \qquad c_{\text{lead}} = 0.040\,\text{ppm}.$$

The nitrate and iron values are already in ppm: $100\,\text{ppm}$ and $0.2\,\text{ppm}$, respectively. Comparing the unconverted numbers 1000 and 100 would be misleading.

Step 2: Compare nitrate with its stated textbook limit The nitrate concentration is twice the usual textbook upper limit:

$$\frac{100\,\text{ppm}}{50\,\text{ppm}} = 2.$$

Excess nitrate can be reduced to nitrite, which interferes with the oxygen-carrying function of haemoglobin. This explains why the nitrate level, rather than merely the largest numerical value in the original list, is the relevant concern in this question.

Step 3: Check the other three alternatives Lead at $0.040\,\text{ppm}$ is below the textbook limit of $0.05\,\text{ppm}$. Iron at $0.2\,\text{ppm}$ does not exceed the quoted iron level. Fluoride at $1.0\,\text{ppm}$ is the customary ben-

eficial concentration, rather than an excess. Therefore none of options (A), (C) or (D) is the high-concentration impurity identified by these data.

Step 4: Conclusion The excessive nitrate concentration, $100\,\text{ppm}$, makes the sample unsuitable under the textbook criteria used in the question. Hence, the correct option is (B).

Quick Tip to Solve: Convert every ppb value to ppm before comparing it with a ppm limit; then check which limit is actually exceeded.

87. Which of the following statements about low density polythene is FALSE ? [4 marks]

(A) Its synthesis requires high pressure.

(B) It is a poor conductor of electricity.

(C) Its synthesis requires dioxygen or a peroxide initiator as a catalyst.

(D) It is used in the manufacture of buckets, dust-bins etc.

Correct Answer: (D) It is used in the manufacture of buckets, dust-bins etc.

Solution:

Concept:

- Low-density polythene (LDPE) is formed by free-radical addition polymerisation of ethene under high pressure and elevated temperature, initiated by traces of dioxygen or a peroxide.
- Branching prevents close packing of LDPE chains, making it relatively soft and flexible. More linear high-density polythene (HDPE) packs more closely and is harder and more rigid.

Step 1: Check the preparation statements In the standard preparation of LDPE, ethene is polymerised at high pressure, typically about 1000–$2000\,\text{atm}$, and elevated temperature. A trace of dioxygen or a peroxide supplies radicals to initiate the chain reaction. Thus statement (A) is true, and statement (C), using the conventional initiator terminology of the question, also describes LDPE preparation.

Step 2: Check the electrical property Polythene has a covalently bonded hydrocarbon backbone and does not contain mobile ions or delocalised electrons that could transport charge readily. LDPE is consequently a poor conductor of electricity and is useful as an electrical insulating material. Statement (B) is true; being low in density does not imply that it conducts electricity.

Step 3: Distinguish the uses of the two forms LDPE is soft, flexible and suitable for films, squeeze bottles, toys and insulation. Buckets and dustbins require comparatively rigid, tough moulded plastic and are standard uses of HDPE. Its straighter chains pack closely, giving the required density and rigidity. Therefore statement (D) assigns a characteristic HDPE application to LDPE and is the false statement sought here.

Step 4: Conclusion The statement that low-density polythene is used in the manufacture of buckets and dustbins is false in the standard LDPE/HDPE comparison. Hence, the correct option is (D).

Quick Tip to Solve: Link branched LDPE with flexible films and insulation; link closely packed HDPE with rigid buckets and dustbins.

88. The main oxides formed on combustion of Li, Na and K in excess of air are, respectively : [4 marks]

(A) LiO_2, Na_2O_2 and K_2O

(B) Li_2O_2, Na_2O_2 and KO_2

(C) Li_2O, Na_2O_2 and KO_2

(D) Li_2O, Na_2O and KO_2

Correct Answer: (C) Li_2O, Na_2O_2 and KO_2

Solution:

Concept:

- Alkali metals form different oxygen compounds on combustion in excess air: lithium mainly forms an oxide, sodium a peroxide, and potassium a superoxide.
- The relevant oxygen ions are oxide O^{2-}, peroxide ${O_2}^{2-}$, and superoxide ${O_2}^{-}$. Their charges determine the formula when combined with a monovalent alkali-metal cation.

Step 1: Write the product for lithium Lithium forms the normal oxide as its main combustion product. Two Li^+ ions balance one oxide ion, giving Li_2O:

$$4\,Li + O_2 \longrightarrow 2\,Li_2O.$$

The small lithium cation stabilises this oxide lattice strongly. Therefore a choice beginning with LiO_2 or Li_2O_2 does not give the main product asked for.

Step 2: Write the products for sodium and potassium Sodium mainly forms peroxide in excess air, whereas potassium mainly forms superoxide:

$$2\,Na + O_2 \longrightarrow Na_2O_2, \qquad K + O_2 \longrightarrow KO_2.$$

The peroxide ion carries a total charge of -2, so it requires two sodium ions. The superoxide ion carries -1, so it requires only one potassium ion. Larger alkali cations increasingly stabilise the larger oxygen-containing anions.

Step 3: Match the sequence to the choices The required order is lithium, sodium and potassium, so the sequence is Li_2O, Na_2O_2, KO_2. Option (A) has the wrong lithium and potassium products. Option (B) incorrectly gives lithium peroxide as the main lithium product. Option (D) gives sodium oxide instead of sodium peroxide. Only option (C) has all three products in the correct order.

Step 4: Conclusion The main oxides are Li_2O, Na_2O_2 and KO_2, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Remember the down-group progression for excess air: Li gives oxide, Na peroxide, and K superoxide; verify each formula using ion charges.

89. For a linear plot of log (x/m) versus log p in a Freundlich adsorption isotherm, which of the following statements is correct ? (k and n are constants) [4 marks]

(A) Both k and 1/n appear in the slope term.

(B) 1/n appears as the intercept.

(C) Only 1/n appears as the slope

(D) log (1/n) appears as the intercept.

Correct Answer: (C) Only 1/n appears as the slope

Solution:

Concept:

- The Freundlich adsorption isotherm at a fixed temperature is $x/m = kp^{1/n}$, where x is the mass adsorbed, m the adsorbent mass, p the gas pressure, and k, n empirical constants.
- A straight-line graph in the form $Y = a + bX$ has slope b and vertical intercept a. Taking logarithms converts the power law into this form.

Step 1: Take logarithms of the isotherm Start with the pressure-dependent relation and use the logarithm rules for a product and a power:

$$\log\left(\frac{x}{m}\right) = \log\left(kp^{1/n}\right) = \log k + \frac{1}{n}\log p.$$

The same logarithm base must be used on both axes. The constant k becomes an additive term after the logarithm is taken; it does not multiply the pressure-dependent term.

Step 2: Read off the slope and intercept For the specified axes, let $Y = \log(x/m)$ and $X = \log p$. The equation becomes

$$Y = \log k + \frac{1}{n}X.$$

Consequently the rise per unit change in X is $1/n$, while the intercept at $X = 0$ is $\log k$. Changing k shifts the line vertically without changing this slope when n is held fixed.

Step 3: Eliminate the incorrect statements Option (A) is incorrect because k appears in the intercept rather than the slope. Option (B) confuses the slope with the intercept. Option (D) is also incorrect: the intercept is $\log k$, not $\log(1/n)$. The coefficient of the horizontal-axis variable contains only $1/n$, exactly as stated in option (C).

Step 4: Conclusion Only $1/n$ appears as the slope of the plot of $\log(x/m)$ against $\log p$. Hence, the correct option is (C).

Quick Tip to Solve: Write the logarithmic equation as vertical variable = intercept + slope times horizontal variable before identifying either graph parameter.

90. Which one of the following statements about water is FALSE? [4 marks]

(A) Water is oxidized to oxygen during photosynthesis

(B) Water can act both as an acid and as a base.

(C) There is extensive intramolecular hydrogen bonding in the condensed phase.

(D) Ice formed by heavy water sinks in normal water

Correct Answer: (C) There is extensive intramolecular hydrogen bonding in the condensed phase.

Solution:

Concept:

- Hydrogen bonding in liquid water and ice occurs between different water molecules: a hydrogen attached to oxygen in one molecule interacts with oxygen in another molecule.
- Water is amphoteric, can be oxidised to oxygen in photosynthesis, and has a heavy-water analogue D_2O whose ice is denser than ordinary liquid water.

Step 1: Locate the false description of hydrogen bonding A water molecule has two hydrogen atoms covalently bonded to one oxygen atom. In its condensed phases, the extensive hydrogen-bond network links neighbouring molecules. These are therefore *intermolecular* hydrogen bonds. Calling this network intramolecular, as statement (C) does, incorrectly places the bonds within a single water molecule.

Step 2: Verify the oxidation and acid-base statements During oxygenic photosynthesis, water is oxidised and supplies the oxygen released. The oxidation half-reaction can be written as

$$2\,H_2O \longrightarrow O_2 + 4\,H^+ + 4\,e^-.$$

Thus (A) is true. Water accepts a proton from an acid to form H_3O^+, and donates a proton to a base to form OH^-. Its ability to do both makes (B) true.

Step 3: Check the heavy-water statement Ice made from heavy water contains deuterium in place of ordinary hydrogen. Its density is approximately $1.02\,g\,cm^{-3}$, greater than that of ordinary liquid water, approximately $1.00\,g\,cm^{-3}$. It therefore sinks in normal water, so statement (D) is true. Ordinary ice floating on ordinary water must not be confused with this comparison involving heavy-water ice.

Step 4: Conclusion The false statement is the claim of extensive intramolecular hydrogen bonding: the water network is intermolecular. Hence, the correct option is (C).

Quick Tip to Solve: Check the prefix carefully: inter means between different molecules; intra means within one molecule. Water has the former hydrogen-bond network.