JEE Main 2016 Paper 2 (B.Arch/B.Plan) Apr 3 Single sitting (Offline) Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 390**

General Instructions

(i) This paper is Paper 2 (B.Arch/B.Plan.) of JEE (Main) 2016, Annual (offline), conducted by the Central Board of Secondary Education (CBSE) in pen and paper mode on 3 April 2016, Single sitting. The total duration is 3 hours.

(ii) The paper has 82 questions: Aptitude Test (50), Mathematics (30), Drawing Test (2), in that order. Maximum marks: 390.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) The Drawing Test questions are answered on the drawing sheet; marks for each are indicated against it.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Aptitude Test (200 marks)

1. Which is best used as a sound absorbing material in partition walls ? [4 marks]

(A) Glass-wool

(B) Glass pieces

(C) Stone chips

(D) Steel

Correct Answer: (A) Glass-wool

Solution:

Concept:

- Sound absorption reduces reflected sound energy and reverberation inside a room. A useful absorber is porous and fibrous rather than a hard, smooth reflecting surface.
- In a fibrous material, air moves through small interconnected spaces. Viscous friction and thermal losses convert part of the acoustic energy into heat.

Step 1: Identify the required property The question asks for sound absorption within a partition, so we need a material that dissipates sound energy in its pores. Glass-wool consists of fine glass fibres with air spaces between them. It can be placed in the cavity of a partition to absorb sound and reduce cavity resonance.

Step 2: Eliminate the alternatives Glass pieces and stone chips are relatively hard, dense fragments; they do not provide the fine, distributed fibrous pore structure of glass-wool. Steel is also hard and reflects much incident sound. A heavy steel layer may contribute to sound insulation, but insulation by mass and absorption within a cavity are different functions.

Step 3: Conclusion Glass-wool is the suitable sound absorbing material for partition walls. Hence, the correct option is (A).

Quick Tip to Solve: Separate absorption from insulation: porous fibres absorb sound, while dense barriers mainly resist its transmission.

2. Interior of any room will appear larger when painted with which colour ? [4 marks]

(A) Blue colour

(B) Black colour

(C) White colour

(D) Grey colour

Correct Answer: (C) White colour

Solution:

Concept:

- The apparent size of an interior depends on its brightness, surface colours and the visibility of its boundaries, not only on its physical dimensions.

- Light colours generally reflect more incident light than dark colours. A bright, neutral wall finish helps an enclosed room feel open.

Step 1: Choose the most reflective neutral finish White reflects a broad range of visible light and gives a high overall brightness for the same lighting conditions. Using white on the enclosing surfaces reduces the visual heaviness of the walls, so the room generally appears larger and more spacious.

Step 2: Compare the listed colours Black absorbs much of the incident light and tends to make the enclosure feel darker and more confined. Grey is less bright than white. Blue can have a visually receding effect, particularly in a pale shade, but no shade is specified here. Among the stated general choices, white is the standard choice for maximising the impression of interior space.

Step 3: Conclusion Painting the interior white makes the room appear larger. Hence, the correct option is (C).

Quick Tip to Solve: For a general room-size question, select a light, highly reflective neutral colour; do not confuse apparent space with actual floor area.

3. Which one of the following is not an architect ? [4 marks]

(A) B.V. Doshi

(B) Zakir Hussain

(C) Hafiz Contractor

(D) Raj Rewal

Correct Answer: (B) Zakir Hussain

Solution:

Concept:

- An architect designs buildings and built environments. The question is a classification question asking for the person outside that professional group.
- B. V. Doshi, Hafiz Contractor and Raj Rewal are associated with architecture, whereas Zakir Hussain is associated with a different field.

Step 1: Classify the architects B. V. Doshi was an Indian architect known for work on housing, institutions and urban design. Hafiz Contractor is an Indian architect with a substantial building-design practice. Raj Rewal is also an Indian architect, associated with major institutional and public buildings. These three names therefore belong to the required professional group.

Step 2: Find the exception Zakir Hussain is the name associated with the renowned tabla musician, rather than an architectural practice. Thus he is the exception in this list. The wording asks who is not an architect, so selecting one of the three building designers would reverse the demand of the question.

Step 3: Conclusion Zakir Hussain is not an architect. Hence, the correct option is (B).

Quick Tip to Solve: Underline the word "not" before checking a list of professions; identify the three members of the group and select the exception.

4. Howrah Bridge is : [4 marks]

(A) Resting on concrete pillars

(B) Cable hung structure

(C) Resting on brick arches

(D) A steel structure

Correct Answer: (D) A steel structure

Solution:

Concept:

- Bridges may be classified by their material and by their structural system. Steel is a material, while suspension, arch and cantilever describe structural forms.
- Howrah Bridge across the Hooghly River is a steel cantilever bridge. Its main river span is carried by the cantilever structural system rather than by brick arches or suspension cables.

Step 1: Identify the bridge system The characteristic main structure of Howrah Bridge is a large steel cantilever truss. The interconnected steel members transfer the loads through the cantilever arms to the principal supports near the banks. Its classification therefore directly supports the description "a steel structure".

Step 2: Reject the mismatched descriptions A cable-hung structure would carry its deck primarily through suspension cables, which is not the structural type of Howrah Bridge. Brick arches are also not its load-carrying system. The description of resting on concrete pillars does not identify its famous steel cantilever form and suggests the wrong arrangement of intermediate river supports. Material used in foundations should not be confused with the bridge superstructure being asked about.

Step 3: Conclusion Howrah Bridge is a steel structure, specifically a steel cantilever bridge. Hence, the correct option is (D).

Quick Tip to Solve: Associate Howrah Bridge with a steel cantilever truss; distinguish the main superstructure from its foundations.

5. The famous work of Leonardo Da Vinci is : [4 marks]

(A) Elizabeth

(B) Mona Lisa

(C) The King

(D) Cleopatra

Correct Answer: (B) Mona Lisa

Solution:

Concept:

- Questions about artistic awareness test the association between a recognised artist and a named work. Leonardo da Vinci was an Italian Renaissance artist.
- The Mona Lisa is the celebrated portrait painted by Leonardo da Vinci. It is recognised for its subtle

modelling, atmospheric effects and the sitter's expression.

Step 1: Recall the established attribution The name Mona Lisa is directly associated with Leonardo da Vinci. It is a portrait rather than a general title for a ruler or a historical subject. Recognising this artist-work pairing is sufficient to choose from the four options; no inference from the appearance of the sitter is needed.

Step 2: Check the other titles Elizabeth, The King and Cleopatra are not the recognised Leonardo work named by this standard art-awareness question. They name a person or a broad subject and do not match the established attribution of the Mona Lisa. The decision rests on authorship, not merely on whether a title could describe a portrait.

Step 3: Conclusion The famous work of Leonardo da Vinci in the given list is Mona Lisa. Hence, the correct option is (B).

Quick Tip to Solve: Learn artist-work pairs as a unit: Leonardo da Vinci and Mona Lisa form a direct association in art-awareness questions.

6. Which person is famous for the extensive brickwork in Kerala ? [4 marks]

(A) Hafeez Contractor

(B) Charles Correa

(C) Achyut Kanvinde

(D) Laurie Baker

Correct Answer: (D) Laurie Baker

Solution:

Concept:

- Architectural awareness includes identifying designers through their characteristic materials, construction methods and regional practice.
- Laurie Baker is closely associated with economical, climate-responsive architecture in Kerala and with extensive, expressive use of exposed brickwork.

Step 1: Match the material and location The combination of Kerala and extensive brickwork points to Laurie Baker. His buildings use brick as both a construction material and a visible architectural surface. Brick screens, careful bonding and economical wall construction are characteristic associations, alongside attention to ventilation and local conditions.

Step 2: Distinguish the other architects Hafeez Contractor, Charles Correa and Achyut Kanvinde are important architects, but the specific reputation described here is Laurie Baker's. Contractor is associated with a large contemporary architectural practice; Correa with climate-sensitive modern design and major public projects; Kanvinde with modern institutional architecture. Those broader associations do not replace Baker's particularly strong link to brick construction in Kerala.

Step 3: Conclusion Laurie Baker is famous for the extensive use of brickwork in Kerala. Hence, the correct option is (D).

Quick Tip to Solve: Use both clues together: the regional clue Kerala and the material clue brickwork identify Laurie Baker.

7. Nalanda is : [4 marks]

(A) A Temple

(B) Ancient center of higher learning

(C) A Fort in Bihar

(D) An ancient town in Sri Lanka

Correct Answer: (B) Ancient center of higher learning

Solution:

Concept:

- Nalanda is identified by its historic function as a major centre of higher learning in ancient India.
- A monument or settlement may contain religious buildings, but its usual historical classification follows the principal institution for which it is known.

Step 1: Identify the institution Nalanda, in present-day Bihar, was a renowned residential monastic university. Students and teachers pursued advanced study there, so the description ancient centre of higher learning directly matches its historical identity.

Step 2: Eliminate the other descriptions A temple is a place of worship, whereas Nalanda is famous for its organised teaching and scholarship. A fort is a defensive structure and does not describe this institution. Although Nalanda is in Bihar, that geographical clue does not make it a fort. It is also an Indian site, so the description of an ancient town in Sri Lanka is incorrect.

Step 3: Conclusion Nalanda is an ancient centre of higher learning. Hence, the correct option is (B).

Quick Tip to Solve: Associate Nalanda with ancient education in Bihar; distinguish the institution's function from its location.

8. Which one of the following is a sound reflecting material ? [4 marks]

(A) Wood

(B) Mirror

(C) Cotton Cloth

(D) Woolen cloth

Correct Answer: (B) Mirror

Solution:

Concept:

- Hard, smooth and nonporous surfaces generally reflect sound effectively, while soft porous materials absorb a greater proportion of incident sound.
- Acoustic reflection depends on the surface and material; it is distinct from the optical reflection that produces an image.

Step 1: Compare the reflecting surfaces A mirror has a hard, smooth glass surface. It offers little porous structure for the air motion of a sound wave to penetrate, and is therefore the best sound-reflecting material among the listed choices. Its familiar reflection of light is not itself the reason for

this acoustic behaviour.

Step 2: Rule out the absorbers Cotton cloth and woollen cloth are soft fibrous materials. Their structure dissipates some sound energy and makes them useful for reducing reflections. Wood can also reflect sound, especially when it is a solid panel, but its behaviour depends on the panel and mounting. The smooth rigid mirror is the intended strongest reflector in this comparison.

Step 3: Conclusion The hard, smooth surface of a mirror reflects sound effectively. Hence, the correct option is (B).

Quick Tip to Solve: For acoustic-material questions, compare hard smooth surfaces with soft porous fabrics before selecting the reflector.

9. Eiffel Tower is located in : [4 marks]

(A) Australia

(B) Paris

(C) Beijing

(D) London

Correct Answer: (B) Paris

Solution:

Concept:

- The Eiffel Tower is a landmark of Paris, the capital of France.
- Questions about landmark locations require matching the named structure to its city, rather than identifying its appearance or construction material.

Step 1: Identify the city of the tower The Eiffel Tower stands in Paris on the Champ de Mars, close to the River Seine. Its name is associated with the engineer Gustave Eiffel, but the question asks only where the tower is located. Paris is therefore the direct geographical match.

Step 2: Check every alternative Australia is a country and is not the location of the original Eiffel Tower. Beijing is the capital of China and has a different group of major historic landmarks. London is the capital of the United Kingdom and likewise does not contain this Parisian landmark. None of these alternatives matches the city of the named original structure.

Step 3: Conclusion The Eiffel Tower is located in Paris, France. Hence, the correct option is (B).

Quick Tip to Solve: Keep the pair Eiffel Tower–Paris together; do not confuse a landmark's original location with replicas elsewhere.

10. Shahjahanabad is a part of which one of the following cities ? [4 marks]

(A) Delhi

(B) Aurangabad

(C) Allahabad

(D) Lucknow

Correct Answer: (A) Delhi

Solution:

Concept:

- Shahjahanabad is the historic walled city founded under the Mughal emperor Shah Jahan in Delhi.
- Historic urban names often refer to an older district within a present-day city rather than to a separate modern city.

Step 1: Connect the old and present names Shahjahanabad forms the core of what is commonly called Old Delhi. Its major landmarks include the Red Fort and Jama Masjid. These features identify the historic settlement as part of Delhi, rather than as a district of another city.

Step 2: Eliminate the competing cities Aurangabad is a different historic city in Maharashtra and is not the modern city containing Shahjahanabad. Allahabad, now called Prayagraj, is in Uttar Pradesh and is associated with the confluence of the Ganga and Yamuna. Lucknow is another city in Uttar Pradesh, associated with the Awadh region. Neither of those two cities is the location of Shahjahanabad.

Step 3: Conclusion Shahjahanabad is part of Delhi, specifically its historic Old Delhi area. Hence, the correct option is (A).

Quick Tip to Solve: Remember the urban-history link Shah Jahan–Shahjahanabad–Old Delhi, and distinguish it from other Mughal-era cities.

11. The temple of Angkorvat is in : [4 marks]

(A) Vietnam

(B) Myanmar

(C) Cambodia

(D) Laos

Correct Answer: (C) Cambodia

Solution:

Concept:

- Angkor Wat is a major temple complex in Cambodia, near Siem Reap.
- The name Angkorvat in the question refers to Angkor Wat; the landmark must be matched to its country.

Step 1: Locate the temple complex Angkor Wat belongs to the historic Khmer architectural tradition. Its monumental layout, galleries and towers are associated with the Angkor region of Cambodia. This country identification directly answers the question; the spelling used in the question does not

indicate a different monument.

Step 2: Compare the neighbouring countries Vietnam, Myanmar and Laos are all countries of mainland Southeast Asia, which makes them plausible geographical distractors. However, Angkor Wat is not located in any of them. Myanmar is especially associated with the temple landscape of Bagan, while the Angkor complex belongs to Cambodia. The regional proximity of the alternatives does not change the national location of the named temple.

Step 3: Conclusion The temple of Angkor Wat is in Cambodia. Hence, the correct option is (C).

Quick Tip to Solve: Use the association Angkor Wat–Khmer architecture–Cambodia to separate it from other Southeast Asian temple sites.

12. Aswan dam is situated on which river : [4 marks]

(A) Nile River

(B) Rhine River

(C) Irrawaddy River

(D) Amazon River

Correct Answer: (A) Nile River

Solution:

Concept:

- The Aswan Dam is built across the Nile in Egypt.
- A dam-location question is answered by identifying the river crossed by the structure, rather than the reservoir or the nearest city alone.

Step 1: Identify the river at Aswan Aswan is a city in southern Egypt on the Nile. The Aswan dams control the flow of this river; the High Dam also forms the large reservoir known as Lake Nasser. Both the city and the dam are therefore directly associated with the Nile river system.

Step 2: Exclude the other river systems The Rhine is a European river and cannot be the river crossed by this Egyptian dam. The Irrawaddy flows through Myanmar in Southeast Asia, while the Amazon belongs to South America. These rivers are in different geographical regions from Aswan. The Nile is the only listed river consistent with the dam's location in Egypt.

Step 3: Conclusion The Aswan Dam is situated on the Nile River. Hence, the correct option is (A).

Quick Tip to Solve: Link Aswan with Egypt and the Nile; use the continents of the other rivers to eliminate them quickly.

13. Buland Darwaza is located in : [4 marks]

(A) Red Fort

(B) Agra Fort

(C) Golconda

(D) Fatehpur Sikri

Correct Answer: (D) Fatehpur Sikri

Solution:

Concept:

- Buland Darwaza is the monumental gateway at Fatehpur Sikri, the Mughal city associated with Emperor Akbar. It is an architectural landmark of that complex.
- An identification question is decided by the building and its actual site. A similarity of period, building material or regional location does not establish that two monuments belong to the same complex.

Step 1: Identify the monument and its setting Buland Darwaza, often described as the lofty or great gateway, forms the imposing entrance associated with the Jama Masjid complex at Fatehpur Sikri. Its scale, monumental stairway and Mughal architectural treatment make it a distinctive landmark. Therefore, the site to match with its name is Fatehpur Sikri.

Step 2: Eliminate the other sites The Red Fort is the famous Mughal fort complex in Delhi; it is not the site of Buland Darwaza. Agra Fort is another major Mughal complex, but it is separate from Fatehpur Sikri, even though both are associated with the Agra region. Golconda is the historic fort near Hyderabad and belongs to a different architectural setting. Thus neither a general association with Mughal architecture nor proximity to Agra makes options (A) or (B) correct, and (C) is unrelated to the named gateway.

Step 3: Conclusion Buland Darwaza is located at Fatehpur Sikri. Hence, the correct option is (D).

Quick Tip to Solve: Learn a monument together with its specific complex: Buland Darwaza–Fatehpur Sikri. Do not replace the site with the nearest famous fort or city.

14. There are maximum forests in which State of India : [4 marks]

(A) Karnataka

(B) Madhya Pradesh

(C) Himachal Pradesh

(D) Uttar Pradesh

Correct Answer: (B) Madhya Pradesh

Solution:

Concept:

- For this geographical comparison, maximum forests means the greatest absolute area of forest, rather than the greatest percentage of a state covered by forest.

- Absolute forest area and percentage forest cover are different measures: a large state can have more forest in square kilometres even when a smaller state has a higher forest percentage.

Step 1: Interpret the basis of comparison The question asks which of the listed states has the maximum forests. In the standard geographical usage of this question, the comparison is by total forest area. It does not ask for the densest forest, the largest proportion of forest land, or the state with the greatest altitude. Keeping this distinction prevents the mountainous landscape of Himachal Pradesh from misleading the comparison.

Step 2: Compare the four listed states Madhya Pradesh has extensive forest tracts spread across central India and is the correct state for the largest absolute forest area in the comparison. Karnataka also has substantial forests, particularly in the Western Ghats, but its total is smaller. Himachal Pradesh has important Himalayan forests, yet its smaller geographical area means that forest percentage and forest area must not be confused. Uttar Pradesh has forests in its northern and other belts but does not exceed Madhya Pradesh in total forest area. Thus (A), (C) and (D) do not satisfy the required maximum.

Step 3: Conclusion The state with the maximum absolute forest area in this comparison is Madhya Pradesh. Hence, the correct option is (B).

Quick Tip to Solve: Before choosing a forest-cover answer, check whether the question compares square kilometres or percentage of state area; the two rankings need not agree.

15. Which one of the following is an Earthquake resistant structure ? [4 marks]

(A) RCC framed

(B) Load bearing brick walled

(C) Random stone masonary

(D) Mud walls

Correct Answer: (A) RCC framed

Solution:

Concept:

- Earthquakes impose horizontal inertia forces on a building. A suitable structural system must transfer these forces through connected members to the foundation without sudden brittle collapse.
- A properly designed and detailed reinforced cement concrete (RCC) frame combines concrete in compression with steel reinforcement that provides tensile resistance and ductility. Continuity and ductile detailing are essential to its seismic performance.

Step 1: Identify the frame action In an RCC framed building, slabs, beams and columns form a connected load-resisting system. Under earthquake shaking, the frame can resist lateral loads through bending and the action of its connected members. Proper reinforcement and seismic detailing provide deformation capacity, allowing the structure to absorb energy instead of failing abruptly. This makes an appropriately engineered RCC frame the earthquake-resistant choice among the systems listed.

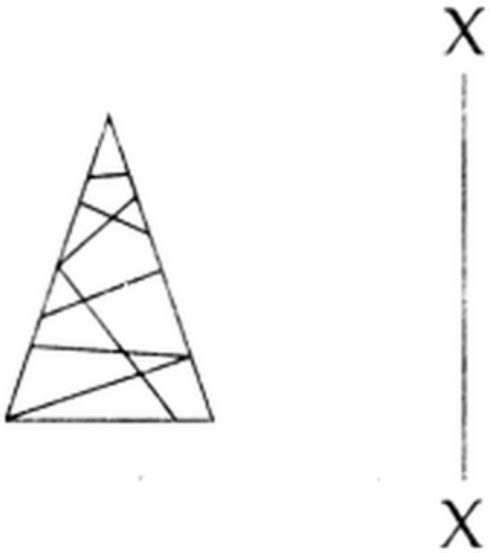
Step 2: Compare the masonry alternatives Unreinforced load-bearing brick walls are relatively

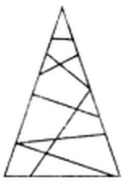
brittle and are vulnerable to lateral shaking and out-of-plane failure unless special seismic measures are provided. Random stone masonry may have weak bonding between irregular stones and poor tensile resistance. Ordinary mud walls also lack the continuity and tensile capacity needed to resist strong horizontal shaking. Such buildings can be improved with seismic bands and other engineering measures, but those measures are not specified in the options. The comparison therefore favours the RCC framed system, rather than treating any material as automatically earthquake-proof.

Step 3: Conclusion A properly designed RCC framed structure is the earthquake-resistant system among the listed choices. Hence, the correct option is (A).

Quick Tip to Solve: For earthquake-resistance comparisons, look for a continuous load path, adequate connections and ductility; do not judge safety from compressive strength alone.

16. Directions : (For Q. 16 to 22). Which one of the answer figures is the correct mirror image of the problem figure with respect to X–X ? [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- Reflection in the vertical line X–X reverses left and right while preserving top and bottom. Each point lies at the same perpendicular distance from the mirror as its image.
- Every line segment and intersection must be reflected together. A correct answer cannot merely resemble the outline: the internal diagonals, their endpoints and their crossings must also match.

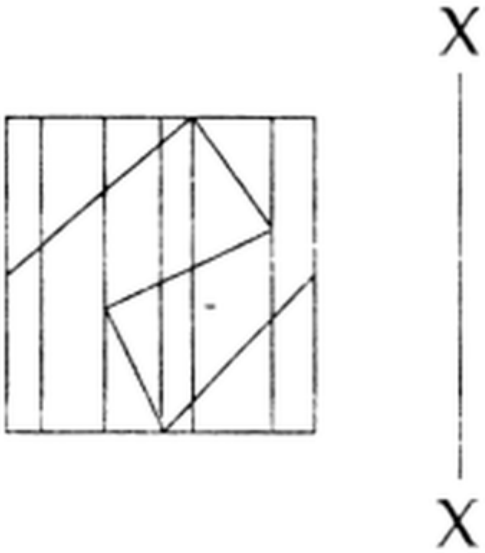
Step 1: Reflect the main landmarks The problem figure is a pointed triangle containing several transverse and diagonal segments. Its apex remains at the top and its base remains horizontal after reflection. The long diagonal that starts at the lower left corner must therefore start at the lower right corner of the image. The rising and falling directions of the oblique internal lines are reversed, whereas their relative heights are retained.

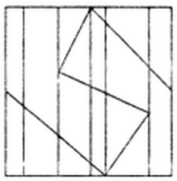
Step 2: Check all the internal segments Option (A) preserves the triangular outline and gives the reflected arrangement of the short cross-lines near the apex, the crossing diagonals in the upper part, and the longer diagonal segments in the lower part. In particular, the lower-left origin of the prominent source diagonal becomes a lower-right origin. Option (B) introduces an additional diagonal reaching the lower-right vertex. Option (C) adds a different network of diagonals from the lower-left region. Option (D) includes a near-vertical internal segment that is not the reflection of a corresponding source segment. These extra or altered connections rule out (B), (C) and (D).

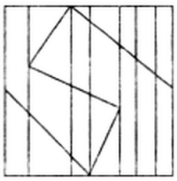
Step 3: Conclusion Only option (A) reproduces the complete left-right reflection of the problem figure in X–X. Hence, the correct option is (A).

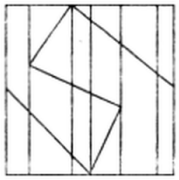
Quick Tip to Solve: Use a distinctive corner-to-side diagonal as an anchor, then count and trace every remaining segment; matching the triangular outline alone is insufficient.

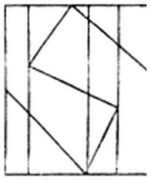
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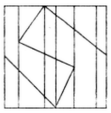


(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- A vertical mirror reverses horizontal order but preserves vertical position. A diagonal rising to the right in the source rises to the left in the reflected image.
- Parallel stripes remain parallel, but their unequal horizontal spacings appear in reverse order. Both the outer boundary and every internal polygon edge must be retained.

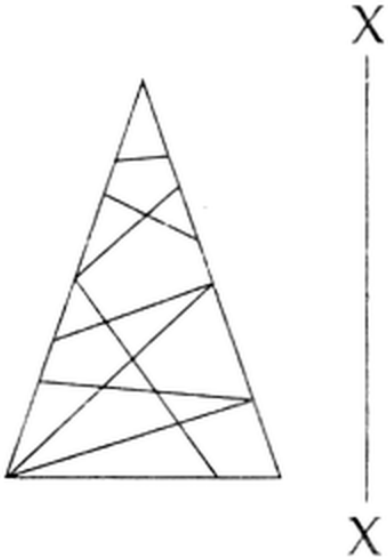
Step 1: Reverse the polygon landmarks The source has a rectangular boundary with vertical stripes and an oblique polygonal arrangement. Its top vertex lies slightly to the right of the centre and must move to the corresponding position left of centre. The lower vertex moves from slightly left of centre to slightly right of centre. Likewise, the upper-right side junction becomes an upper-left junction, and the lower-left side junction becomes a lower-right junction.

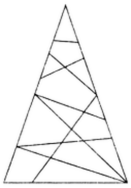
Step 2: Compare the stripes and the boundary Option (C) has the required reversal of the polygon vertices and preserves the correct number and reversed spacing of the vertical stripes. This stripe check distinguishes close-looking alternatives. Option (A) leaves the stripe arrangement in the wrong horizontal order, so its internal pattern is not the complete mirror image. Option (B) includes an extra vertical stripe near the right side and therefore does not preserve the source line pattern. Option (D) has an incomplete right-hand boundary and missing portions of the pattern. The correct reflection must retain a closed rectangle and reproduce all existing lines without adding any new ones.

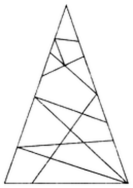
Step 3: Conclusion The complete mirror image, including the reversed stripe spacing and polygon, is shown in option (C). Hence, the correct option is (C).

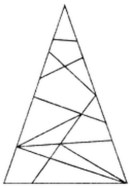
Quick Tip to Solve: When several reflected shapes look alike, compare the order of the narrow and wide stripe gaps as well as the polygon; reflection reverses the entire pattern.

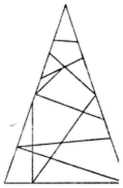
18. Directions : (For Q. 16 to 22). Which one of the answer figures is the correct mirror image of the problem figure with respect to X–X ? [4 marks]

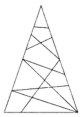


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- For reflection in the vertical mirror X–X, left and right exchange places but all heights remain unchanged. Distances and intersections within the figure are preserved.
- A triangular outline alone does not identify a reflected answer. The distinctive diagonals through the interior must connect the corresponding reflected points on the opposite sides.

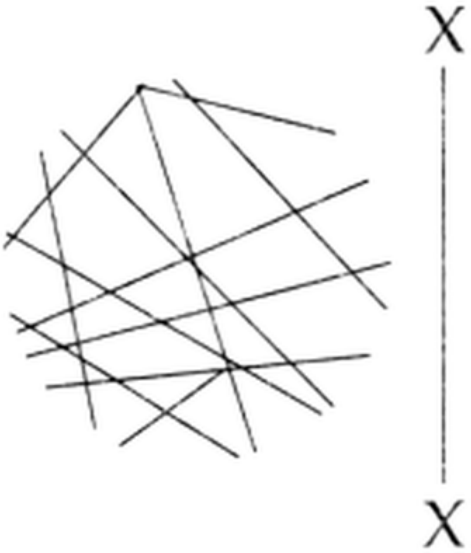
Step 1: Trace the major reflected diagonals In the source, a long diagonal joins the bottom-left vertex to a point on the right sloping edge above the lower cross-line. After reflection, this diagonal must run from the bottom-right vertex to the corresponding point on the left sloping edge. The downward diagonal ending on the base near the right side must instead end near the left side. The short upper cross-line stays near the apex, while the two upper oblique segments exchange their left-right orientations.

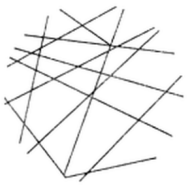
Step 2: Eliminate altered internal patterns Option (A) carries all these reflected features: the correct corner-to-side diagonal, the reversed base endpoint and the unchanged vertical ordering of the transverse lines. Option (B) introduces an additional short diagonal in the upper region that the source does not contain. Option (C) changes the lower interior network by adding diagonals from a left-side junction rather than retaining the required complete reflection. Option (D) adds a near-vertical internal line and extra connections. Since reflection cannot introduce a new segment or change which lines meet, these three options are excluded.

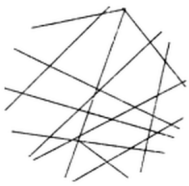
Step 3: Conclusion The full reflected triangular pattern is option (A). Hence, the correct option is (A).

Quick Tip to Solve: Trace the longest corner-to-side diagonal first, then verify the short lines near the apex. This catches alternatives that add one small but incorrect segment.

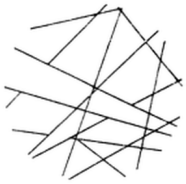
19. Directions : (For Q. 16 to 22). Which one of the answer figures is the correct mirror image of the problem figure with respect to X–X ? [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- A reflection in a vertical mirror reverses left and right while preserving the vertical position of every point. A point above another point remains above it.
- The number of strokes, their endpoints and their intersections are preserved. A sloping line reverses its horizontal sense; reflection neither rotates the whole design nor adds a connecting segment.

Step 1: Locate the distinctive upper junction Begin at the highest junction of the problem figure. It is left of centre and joins several sloping strokes. Its reflected location must be right of centre at the same height. The long shallow stroke extending rightward from this junction must now extend leftward. This immediately distinguishes a left-right reversal from a rotation: the junction must remain near the top, rather than moving to the bottom.

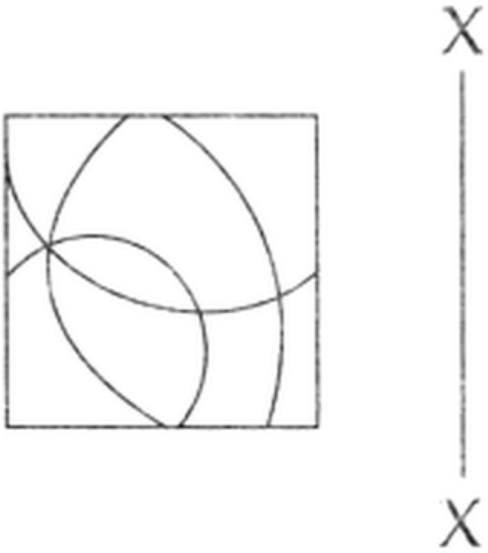
Step 2: Track the long lines through the middle Trace the long steep stroke from the upper junction down towards the lower-right side in the problem. It must descend towards the lower-left side in the answer. Next follow the two prominent oblique lines crossing the centre. Their crossing survives, but each line has the opposite horizontal slope. Option (B) retains these straight continuous strokes and reproduces the lower crossing network without introducing a new triangular connection.

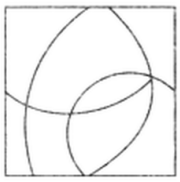
Step 3: Eliminate the altered networks Option (A) places the distinctive multi-line junction at the bottom, so it changes the vertical ordering. Option (C) changes the upper connections and introduces short internal links forming additional small regions. Option (D) interrupts or redirects strokes around the central and upper-left parts of the reflected network. These are changes of connectivity, which a plane mirror cannot produce. Only (B) preserves the complete intersection pattern after reversing left and right.

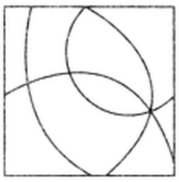
Step 4: Conclusion The upper junction, steep diagonal and lower crossings all match the horizontal reversal in option (B). Hence, the correct option is (B).

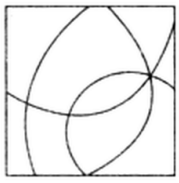
Quick Tip to Solve: For a dense line network, trace one unusual endpoint and one multi-line junction before comparing the remaining intersections.

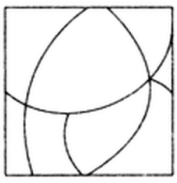
20. Directions : (For Q. 16 to 22). Which one of the answer figures is the correct mirror image of the problem figure with respect to X–X ? [4 marks]

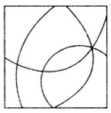


(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- A vertical mirror reverses horizontal position but keeps height fixed. A curve meeting the left border must therefore meet the right border at the same level.
- Reflection preserves continuity and intersections of curves. The outer square retains its shape, so boundary contacts are useful landmarks for distinguishing similar options.

Step 1: Reflect the boundary contacts The problem has a curve descending from the upper part of the left border towards an interior crossing on the left. Its reflected counterpart must descend from the upper part of the right border to a crossing on the right. The curve that meets the right border near mid-height must instead meet the left border at that same height. The upper and lower contacts on the horizontal sides keep their respective heights, though their horizontal positions reverse.

Step 2: Follow the closed lens and long arc The lens-like region in the lower-left portion of the

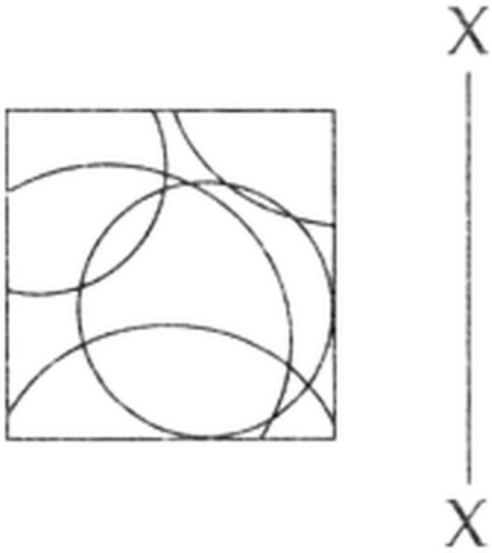
problem must occupy the lower-right portion of the answer. Its two bounding arcs must still meet, rather than terminate partway through the figure. At the same time, the large arc bowing towards the right side of the original must bow towards the left in the reflection. Option (C) shows these features together, including the curve continuing from the right-side crossing upwards to the right boundary.

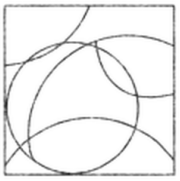
Step 3: Compare the competing drawings Option (A) resembles much of the reflected pattern but omits the upper-right continuation of the curve that originates at the boundary in the problem. Option (B) moves the lens to the upper-right region, changing the heights of its intersections. Option (D) breaks the lower curved connection into a different junction and consequently changes the lens outline. Thus none of these three preserves every curve and boundary contact under a left-right reversal.

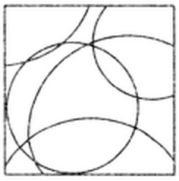
Step 4: Conclusion Option (C) preserves the curve connections and all boundary contacts after reflection in the vertical line X–X. Hence, the correct option is (C).

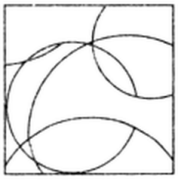
Quick Tip to Solve: Check where each curve enters the square: matching border contacts quickly reveals a missing or wrongly placed arc.

21. Directions : (For Q. 16 to 22). Which one of the answer figures is the correct mirror image of the problem figure with respect to X–X ? [4 marks]

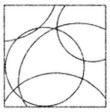


(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- A mirror parallel to the vertical sides of the square reverses left and right, without interchanging top and bottom.
- Every visible arc and crossing must remain present in a reflection. Similarity of the central oval alone is insufficient: the smaller boundary arcs must also agree.

Step 1: Map the major oval The prominent rounded loop lies mainly in the lower and central parts of the problem. Reflect its outline across a vertical line: the left and right bulges exchange places, while its lowest contact with the bottom edge remains at the bottom. The broad arc crossing the lower part of the square must continue to curve upwards in the middle; it does not become a top arch through a rotation.

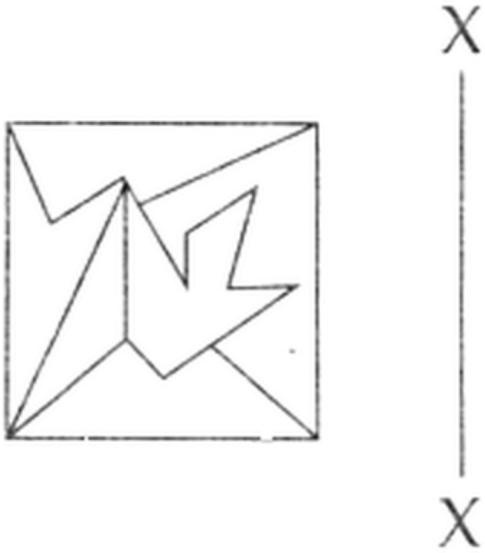
Step 2: Check the two top contacts and the right lens Two separate curved strokes meet the top border near the middle of the original. After reflection, both top contacts must still be present, in reversed horizontal order. The small lens formed near the upper-left side of the original becomes an upper-right lens. Its boundary must connect to the right side of the square. Option (B) preserves both the close top contacts and this complete right-hand lens, together with the central oval and lower arch.

Step 3: Reject missing arcs Option (A) lacks one of the two curves entering from the top edge, so its upper network cannot be the full reflection. Option (C) omits the continuing lower-right portion of the oval where it should cross the lower arch. Option (D) loses the inner upper-right lens boundary. Each deletion changes the number or continuity of the original strokes. Option (B) alone retains all these local checks simultaneously.

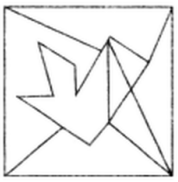
Step 4: Conclusion The complete left-right reversal, including both top contacts and the upper-right lens, is option (B). Hence, the correct option is (B).

Quick Tip to Solve: When curved options look alike, count contacts on each side and inspect the smallest enclosed lens rather than judging the overall outline.

22. Directions : (For Q. 16 to 22). Which one of the answer figures is the correct mirror image of the problem figure with respect to X–X ? [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- For a vertical mirror, left and right exchange while all vertical levels are unchanged. The square boundary is unchanged by this reflection.
- The reflected angular outline must retain every corner and segment connection. Extra diagonals or missing notches are not permissible even if the broad silhouette looks similar.

Step 1: Reflect the distinctive zigzag In the problem, the short folded line near the upper-left corner must move to the upper-right corner. The pointed irregular outline projecting towards the right must become a left-pointing outline. Trace that outline corner by corner: the shallow horizontal projection, narrow upper notch and lower inward corner must all appear in reversed horizontal order. Option (D) retains this full sequence.

Step 2: Trace the main triangular subdivision The long line joining the upper interior vertex to

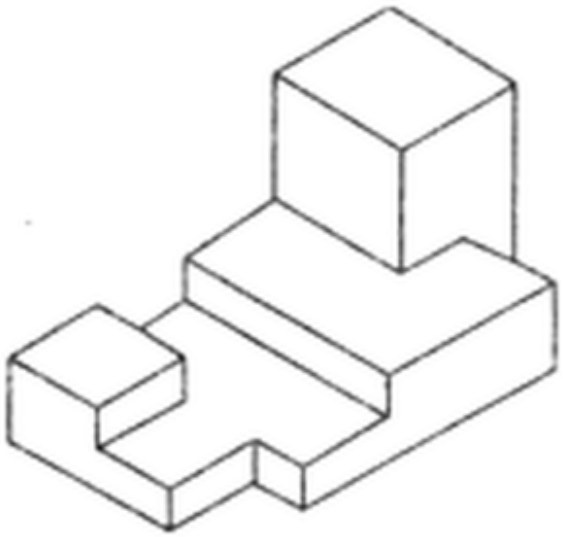
the lower-left corner in the problem must join the reflected upper vertex to the lower-right corner. The nearly vertical segment dropping from the upper interior vertex remains vertical, with its lower junction at the same height. The original diagonal from the lower interior region towards the lower-right corner must emerge towards the lower-left corner in the reflected square. The selected figure shows precisely this arrangement.

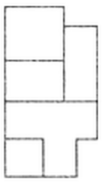
Step 3: Use local errors to reject the others Option (A) changes the central zigzag by inserting a horizontal junction across the notch, instead of preserving the original continuous pointed outline. Option (B) adds a crossing diagonal through the right-hand triangular region and alters the small upper-right fold. Option (C) has an extra vertical segment from the lower-left sloping boundary down to the bottom border. No such extra segment is created by reflection. Option (D) avoids all three alterations and matches every required edge.

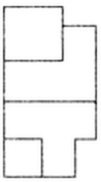
Step 4: Conclusion The correct reflected angular outline and internal segments occur in option (D). Hence, the correct option is (D).

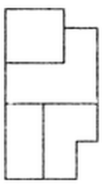
Quick Tip to Solve: Trace an angular mirror image as a sequence of connected corners; then check for an extra line in each distractor.

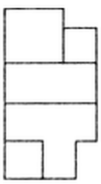
23. Directions : (For Q. 23 to 26). The 3–D problem figure shows the view of an object. Identify the correct top view from amongst the answer figures. [4 marks]

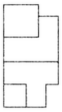


(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- A top view is an orthographic projection along the vertical direction. Height is suppressed, but changes of level leave visible edges in the plan.
- A vertical face projects to a line rather than a strip. Only edges that bound a visible horizontal surface, or form a visible change of level, should remain in the top view.

Step 1: Establish the footprint Look vertically down on the object. Its base has an offset outer boundary with a rectangular notch near the front-right side. This outline survives in the plan. The raised rear portion occupies the rear part of the footprint, while the small block at the front-left occupies a separate corner. Keeping these two blocks at their correct plan positions fixes the orientation of the answer before internal lines are compared.

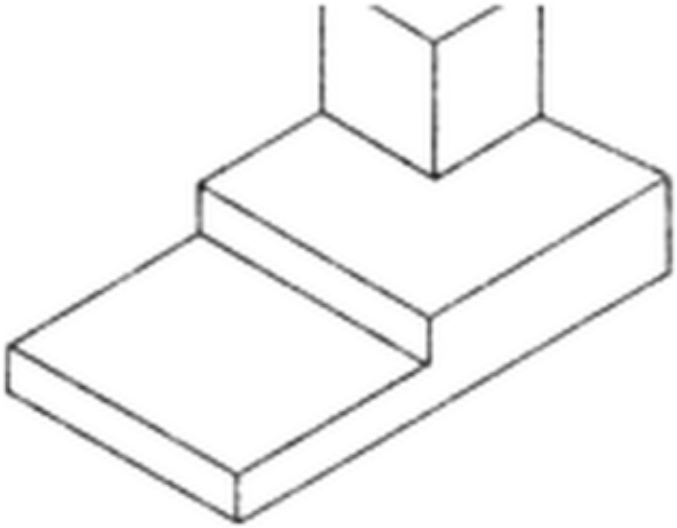
Step 2: Locate the visible level boundaries The tall rear block has a rectangular top footprint at the rear-left of the plan. Its two visible vertical faces do not become additional strips in a top view. The main step across the body creates a transverse line separating the higher rear platform from the lower front platform. The small front-left block adds its own rectangular boundary. Option (B) shows these actual level changes and leaves the remaining horizontal platform areas undivided.

Step 3: Exclude lines belonging to elevations Option (A) introduces an extra transverse line beneath the rear block, treating the apparent thickness of a vertical face as a horizontal band. Option (D) similarly inserts an additional band through the raised rear platform. Option (C) instead extends an internal line through the lower platform where the object has one continuous horizontal surface; this does not correspond to a visible change of level. Option (B) keeps the notch and both block footprints without these false subdivisions.

Step 4: Conclusion The correct top view is the plan with the rear block footprint, the transverse step and the front-left block, shown in option (B). Hence, the correct option is (B).

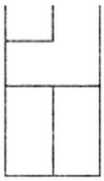
Quick Tip to Solve: In a top view, collapse each vertical face to its boundary line; do not preserve the apparent thickness seen in the perspective drawing.

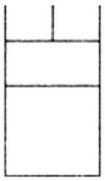
24. Directions : (For Q. 23 to 26). The 3–D problem figure shows the view of an object. Identify the correct top view from amongst the answer figures. [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- An orthographic top view records horizontal position and footprint, not the apparent height or thickness of a vertical riser.
- A continuous horizontal surface remains one region in the plan. An internal line is justified only by a visible top edge, a footprint boundary or a change of level.

Step 1: Identify the two platform levels The object has a lower rectangular platform in front and a higher platform behind it. Viewed from directly above, the riser between them reduces to one transverse boundary. It does not occupy a visible horizontal strip. Therefore the plan has one line across the width separating the front lower region from the rear upper region, with no extra parallel line added to show the riser thickness.

Step 2: Place the upright block on the rear platform The upright portion stands on part of the

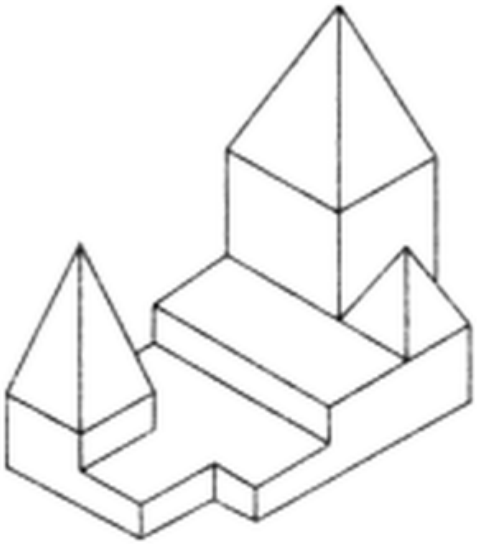
rear platform. Its footprint appears as a rectangular boundary at the rear-left of the top view in the orientation used by the options. From above, the vertical height of this portion is irrelevant: its projected boundary does not continue forward through the exposed platform. Option (B) leaves the rest of the upper platform as a single region and the lower platform as a single region.

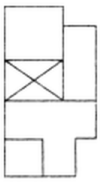
Step 3: Reject artificial subdivisions Option (A) extends a longitudinal line across the rear platform and adds another transverse segment, representing perspective face edges as extra plan divisions. Option (C) inserts a longitudinal line through the lower front platform, though this visible surface is uninterrupted in the solid. Option (D) adds a second transverse boundary and carries a dividing line along the topmost area. These additional divisions are absent from the horizontal surface arrangement. Only option (B) preserves the block footprint and the one genuine step.

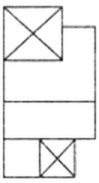
Step 4: Conclusion The projection contains the rear block footprint and one transverse step boundary, with no subdivision of the lower platform. Hence, the correct option is (B).

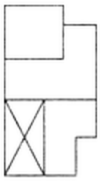
Quick Tip to Solve: Flatten the solid mentally from above, retaining changes of level but discarding the visible vertical thickness of each step.

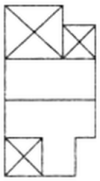
25. Directions : (For Q. 23 to 26). The 3–D problem figure shows the view of an object. Identify the correct top view from amongst the answer figures. [4 marks]

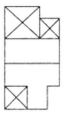


(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- A top view is obtained by looking vertically downward: height disappears, while the horizontal footprint and visible edges of the upper surfaces remain.
- A square pyramidal roof projects as a square with diagonals meeting at the projected apex. Vertical walls underneath it do not remove its roof edges from the plan.

Step 1: Locate the three roofs The object has a large pyramidal roof at the rear, a smaller pyramidal roof beside it on the right, and another pyramidal roof on the projecting front-left block. Each roof has an apex and sloping edges down to the corners of its base. Looking from above, each apex lies within its base square, so the roof lines must appear inside that square. The three roof locations must all be retained; the fact that the rear roof is taller does not turn it into an unmarked rectangle.

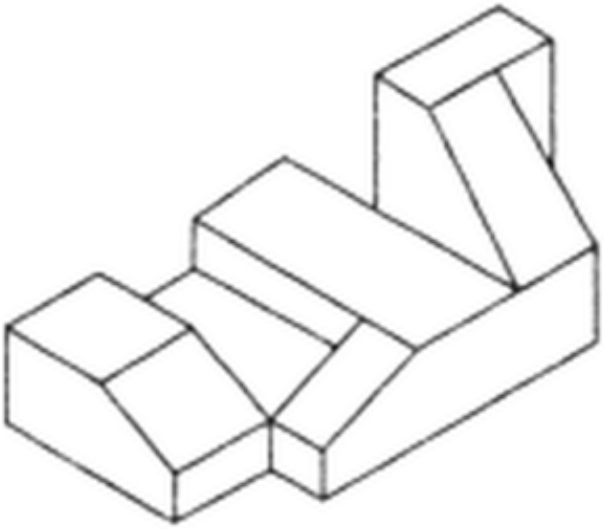
Step 2: Project the base and intermediate platform Ignore the vertical wall heights and trace the outline of the base. It is stepped, with a projection at the front and a recess beside that projection. The long intermediate raised platform crosses the width between the roof groups. In plan, its parallel front and rear edges give the horizontal strip between the rear roofs and the forward roof. These features establish the orientation before comparing the small details of the answer figures.

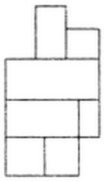
Step 3: Eliminate plans which lose or move a roof Option (A) puts a crossed region in the middle and leaves the rear and forward roof locations without their necessary diagonal roof edges. Option (B) retains the large rear roof and a forward roof but omits the small roof beside the rear one. Option (C) again loses the rear roofs and places a crossed region in the wrong part of the footprint. Option (D) alone retains all three roof plans in the correct locations, as well as the intermediate strip and the stepped outer boundary.

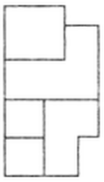
Step 4: Conclusion The correct top view is (D). Although the supplied answer key selects the first choice, that figure omits the projected edges of visible pyramidal roofs and relocates a crossed roof region; the three roof footprints in the given object match (D). Hence, the correct option is (D).

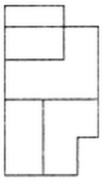
Quick Tip to Solve: For a top-view problem with several pyramidal roofs, first count the roof apexes and mark their base locations; then compare the connecting platforms and outline.

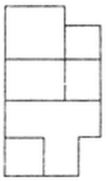
26. Directions : (For Q. 23 to 26). The 3–D problem figure shows the view of an object. Identify the correct top view from amongst the answer figures. [4 marks]

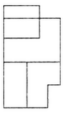


(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- In an orthographic top view, an inclined surface retains its horizontal projection; an upright wall is represented by its upper boundary rather than by its height.
- Trace the complete footprint first, then insert only the visible boundaries between the upper flat surfaces, ramps and raised end blocks.

Step 1: Follow the stepped outer footprint The object has the same general stepped footprint as its platforms: a broad central region, a rear projection and a forward projection with a notch beside it. A top view must preserve these positions. The slope of an edge in the pictorial view results partly from perspective, so it cannot be copied directly as an oblique line in the plan. Instead, mentally drop each corner vertically onto the horizontal base.

Step 2: Mark the two ramp transitions At the back is a raised end block with a narrow horizontal top.

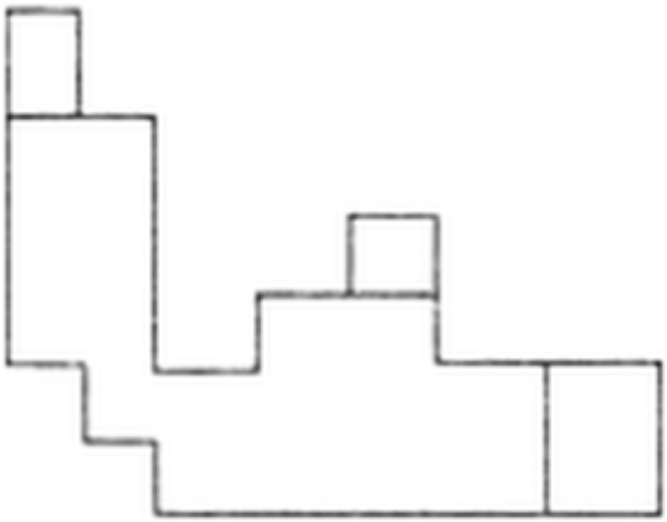
Its cap therefore appears as a narrow strip at the far end of the rear rectangular region. Below the cap, the inclined face continues toward the central platform. At the front-left is another raised flat top, followed by an inclined surface extending toward the front projection. Their junctions must remain in their corresponding rectangles. The broad central platform supplies a pair of parallel transverse boundaries.

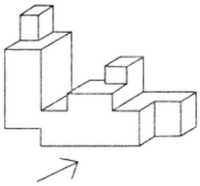
Step 3: Compare the visible subdivisions Option (C) preserves the stepped outline, the narrow rear cap strip, the transverse central platform and the front-left surface division. Option (A) changes the arrangement of the rear projection and its adjoining platform; its long central strips do not describe the same top-surface divisions. Option (B) omits the narrow rear cap boundary and inserts an incorrect division in the forward area. Option (D) places its main subdivisions across the wrong regions and also loses the rear cap strip. Thus the distinguishing feature is the cap and ramp arrangement, not simply the stepped silhouette.

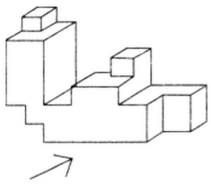
Step 4: Conclusion The top projection agrees with (C). The supplied answer key gives the first choice, but the narrow rear cap, its adjoining inclined face and the forward ramp subdivisions are represented together in (C), whereas the keyed choice changes their arrangement. Hence, the correct option is (C).

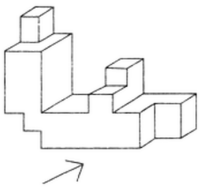
Quick Tip to Solve: For ramps, trace the outer footprint and the line where each slope meets a flat top; do not replace a sloping surface by an arbitrary diagonal in plan.

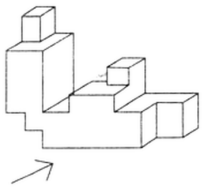
27. Directions : (For Q. 27 and 28). Identify the correct 3-D figure from amongst the answer figures, which has the same elevation, as given in the problem figure on the left, looking in the direction of the arrow. [4 marks]

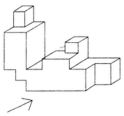


(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- An elevation is an orthographic projection in the viewing direction: depth is collapsed, but height and lateral position are preserved.
- Two solids with similar overall mass can have different elevations because a recess, lower step or vertical joint projects in a different position.

Step 1: Read the elevation as a sequence of levels Starting at the left, the elevation shows a tall block with a narrow rectangular projection above it. The bottom boundary then descends through two successive offsets to the long lowest base. Across the upper boundary, the tall left block drops into a low recess, rises to the central platform and then to a small raised block before dropping again toward the right-hand end. These changes must be reproduced together, rather than matching only the tall tower.

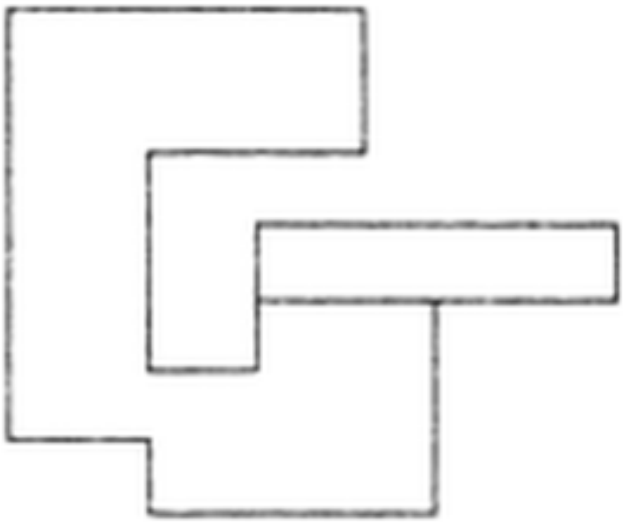
Step 2: Project the fourth solid along its arrow In option (D), looking along the indicated arrow collapses the receding edges of the solid. The tall left prism and its small top block produce the two left-hand height levels. The recess beside it remains below the middle platform, while the small block on that platform gives the short upper projection. The right-hand extension reaches the same low level as in the given elevation, and its visible end division is retained. The lower offsets also fall in the required lateral sequence.

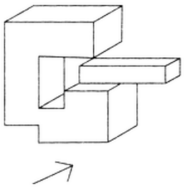
Step 3: Reject the close alternatives Option (A) does not reproduce the two-stage lower offset beneath the tall left portion: its lower silhouette makes the wrong transition to the base. Option (B) has a different terminal arrangement at the right, so its projected vertical division and end block do not match the given low right-hand rectangle. Option (C) changes the central ledge beside the left recess, placing an extra visible division where the target has a continuous level. Option (D) is the one which satisfies the upper profile, lower stepped profile and right-end detail simultaneously.

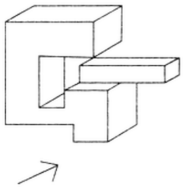
Step 4: Conclusion The solid in (D) gives the stated elevation when viewed along its arrow. Hence, the correct option is (D).

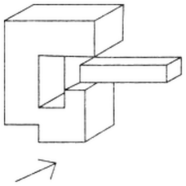
Quick Tip to Solve: When several solids look nearly identical, compare the lower outline and small internal vertical edges after checking the main upper silhouette.

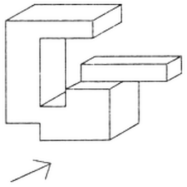
28. Directions : (For Q. 27 and 28). Identify the correct 3-D figure from amongst the answer figures, which has the same elevation, as given in the problem figure on the left, looking in the direction of the arrow. [4 marks]

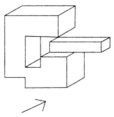


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- For an elevation, project every visible edge parallel to the arrow and retain the resulting heights and horizontal positions.
- A correct match must reproduce the outer silhouette and the edges of the opening and projecting arm; a similar letter-like outline is insufficient.

Step 1: Identify the essential features of the target The elevation has a tall left member, a broad upper overhang and a rectangular opening below that overhang. A lower block projects to the right of the opening. Near its top, a long narrow horizontal member continues to the right. At the bottom, the left boundary changes level by a short offset before meeting the lowest horizontal edge. The central opening, the short lower offset and the long right-hand arm are therefore useful independent checks.

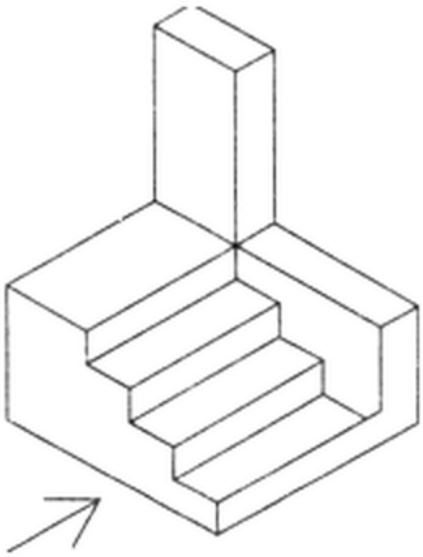
Step 2: Examine the projection of option A Looking in the direction of the arrow in (A) removes the apparent depth of its receding edges. The upper thick member projects as the broad top overhang. The inner left faces bound the rectangular opening. The lower block provides the correct bottom step and the long lowest edge, while the projecting right-hand bar gives the narrow horizontal strip. The front edge of that strip also supplies the internal horizontal line shown in the target. No extra gap is introduced between the bar and the central mass.

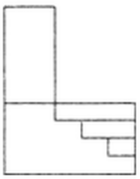
Step 3: Use the lower block and upper overhang to eliminate the others In (B), the bottom offset is moved too far toward the right, shortening the lower block in the projection. In (C), the central lower block and its junction with the horizontal arm are shifted, so the inner edge does not lie at the required position. In (D), the upper overhang is too shallow vertically and the separation around the rightward arm differs from the target. Matching only the open rectangular space would overlook these differences; matching the full set selects (A).

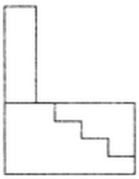
Step 4: Conclusion Option (A) has the same elevation as the problem figure in the indicated direction. Hence, the correct option is (A).

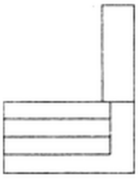
Quick Tip to Solve: In an elevation with an opening, compare both the opening and the bottom step; matching the opening alone often leaves more than one tempting choice.

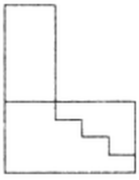
29. Directions : (For Q. 29 to 31). The 3-D figure shows the view of an object. Identify the correct front view from amongst the answer figures, in the direction of the arrow. [4 marks]

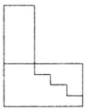


(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- An orthographic front view is taken along the arrow, so parallel depth edges collapse and the horizontal and vertical dimensions facing the viewer remain.
- A staircase viewed from its side gives a descending stepped profile; viewed along its run, its tread edges would instead appear as parallel horizontal lines.

Step 1: Choose the viewing face before tracing edges The arrow points toward the stair object from the lower-left of the pictorial view. This direction exposes the stair profile rather than looking straight down the length of every tread. Consequently, the change in level from the high platform to the lowest tread must appear as a descending stair-shaped line. The tall rear post projects on the left of this elevation. Its width must remain consistent with the platform edge below it.

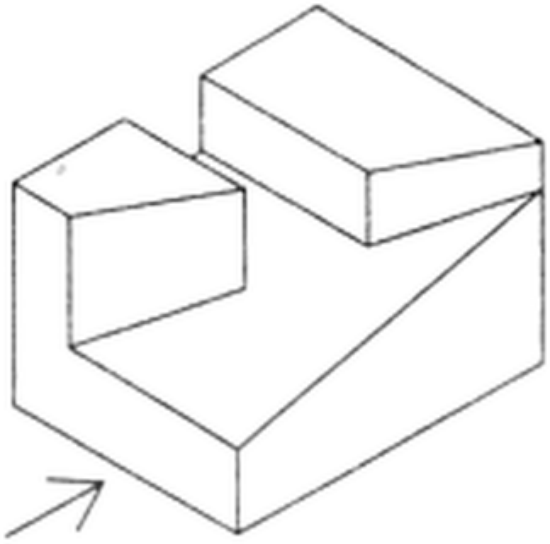
Step 2: Trace the front projection from left to right The left-hand platform gives a horizontal edge at the level from which the stairs descend. The tall post rises above the left part of that edge. To its right, the profile descends successively through the stair levels until it reaches the low right-hand side. The supporting base forms the long horizontal bottom edge, with an uninterrupted outside boundary. This combination gives a tall rectangular part at the left and a descending sequence within the lower rectangular part.

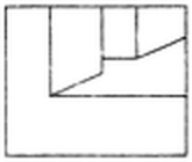
Step 3: Compare all four drawings Option (D) has the left post at the proper width and position and shows the descending stair line meeting the right-hand boundary at the lower level. Option (C) places the tall post on the right and represents the steps mainly as full horizontal bands, so it is the wrong viewing orientation. Option (B) makes the tall post too narrow relative to the left platform. Option (A) adds horizontal divisions extending across the right-hand portion rather than retaining just the visible stair profile. Those extra lines describe a different projection, leaving (D) as the complete match.

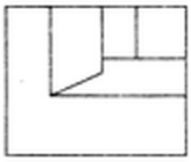
Step 4: Conclusion The required front view is (D): the post is at the left and the staircase descends toward the right. Hence, the correct option is (D).

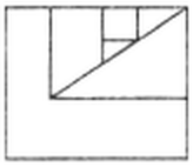
Quick Tip to Solve: Check whether the arrow looks across a staircase or along its run; that determines whether the treads appear as a zigzag profile or as horizontal bands.

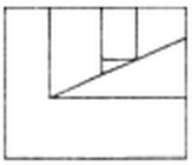
30. Directions : (For Q. 29 to 31). The 3-D figure shows the view of an object. Identify the correct front view from amongst the answer figures, in the direction of the arrow. [4 marks]

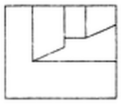


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- A front view removes depth in the arrow direction while retaining visible boundaries between surfaces of different height and inclination.
- A sloping edge can be partly hidden by a raised block. The visible projection must show the interruption at that block rather than continuing through it.

Step 1: Establish the frame of the elevation The arrow directs the view onto the recessed face of the object. The outer mass projects as a rectangle, with a thick lower and left border surrounding the recessed part. The raised blocks at the upper left and upper right contribute vertical boundaries and horizontal underside edges. The ramp inside the recess rises toward the far upper region; its visible edge must be considered together with the block that stands in front of part of it.

Step 2: Follow the visible inclined edge Starting at the lower-left corner of the recess, the inclined

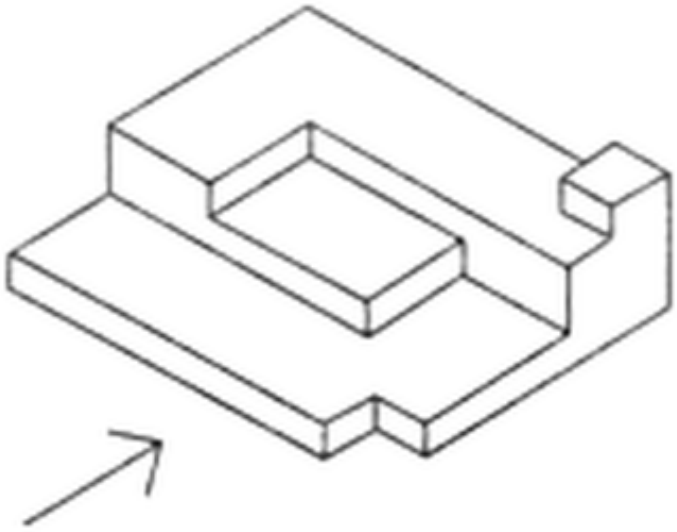
boundary rises toward the middle vertical division. Its continuation is interrupted by the projecting upper block, so the visible line does not simply run in one unbroken diagonal to the outer upper-right corner. Beyond the block, another inclined segment is visible toward the right-hand boundary. The middle horizontal ledge and its short vertical junction remain as separate edges. This is precisely the interrupted arrangement shown by (A).

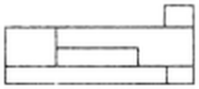
Step 3: Rule out incorrect line continuations Option (B) replaces the right-hand inclined portion with a horizontal boundary, losing the rise of the ramp. Option (C) uses an uninterrupted straight diagonal across the recess to the upper-right corner, ignoring the obstruction produced by the raised block. Option (D) continues the lower diagonal through the central junction and gives the wrong relationship between the ramp and the projecting block. Option (A) alone keeps the lower diagonal segment, the short vertical interruption, the ledge and the upper inclined segment in their correct order.

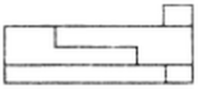
Step 4: Conclusion The front view with the correctly interrupted inclined boundary is (A). Hence, the correct option is (A).

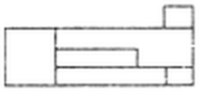
Quick Tip to Solve: In a recessed solid with a ramp, follow a sloping edge only while it is visible; a raised block may hide a middle portion and create a stepped projection.

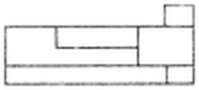
31. Directions : (For Q. 29 to 31). The 3-D figure shows the view of an object. Identify the correct front view from amongst the answer figures, in the direction of the arrow. [4 marks]

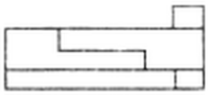


(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- A front elevation is an orthographic projection in the direction of the arrow. Depth along that direction disappears, while height and horizontal position remain.
- A visible edge is drawn only where a surface actually ends or changes level in the projected view. A nearer solid surface conceals edges behind it.

Step 1: Establish the outline and the viewing direction Look along the arrow from the lower left towards the object. The thin base projects as a narrow horizontal band. The raised rear portion gives the long upper horizontal outline, and the small raised block at the right gives a short rectangular projection above it. All four answers resemble this outer outline, so the outer boundary alone cannot decide the answer. The internal visible edges must also be checked; do not retain the diagonal lines merely because they appear in the pictorial view.

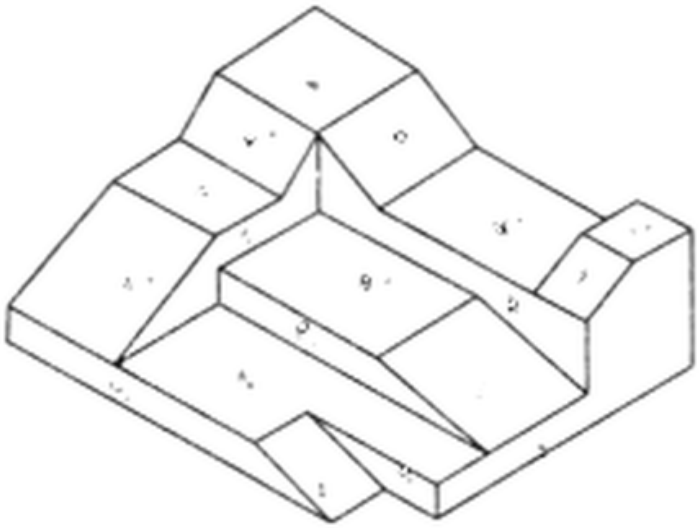
Step 2: Trace the changes of level inside the outline Start at the left end of the inner raised portion. Its vertical boundary descends to the intermediate level and then turns horizontally towards the right. The next vertical edge drops to the base-level band. Thus the internal boundary is a descending stepped line. The nearer surfaces hide the continuation that would otherwise seem to run vertically through the whole height at the left of this step. The small notch at the right remains visible in the lower band. This combination is shown in (B).

Step 3: Eliminate the misleading internal lines Option (A) introduces an extra full vertical subdivision at the left of the intermediate feature. Option (C) also continues that subdivision down to the base and therefore shows an edge through an area where the projection is continuous. Option (D) closes off the intermediate feature on the right in a way that does not reproduce the descending visible step. These are changes to visible boundaries, rather than alternative ways of drawing the same elevation.

Step 4: Conclusion The descending internal step, the narrow base band and the raised block at the right agree with the object only in (B). Hence, the correct option is (B).

Quick Tip to Solve: When several elevations have the same outline, trace one internal edge from its starting point and apply visibility at each change of level.

32. Directions : (For Q. 32 to 35). Find the total number of surfaces of the object given below in the problem figure. [4 marks]



(A) 19

(B) 22

(C) 21

(D) 24

Correct Answer: (C) 21

Solution:

Concept:

- Count each distinct flat boundary surface of the solid once, including inclined faces and faces concealed by the pictorial viewpoint.
- A single plane may have an irregular outline. An edge separating two different planes creates two faces, but an arbitrary subdivision of one continuous plane creates no extra surface.

Step 1: Count the horizontal and inclined surfaces Sweep across the upper part from the left sloping end, over the raised rear portion, and towards the small cap at the right. The horizontal surfaces are the high rear cap, the left intermediate shelf, the long right shelf, the small right cap, the central platform and the exposed low floor. This gives six horizontal surfaces. The inclined surfaces are the lower left ramp, the slope above the left shelf, the slope descending from the rear cap on the right, the slope rising to the small right cap, the slope descending from the central platform, and the small slope at the front notch. There are six inclined surfaces.

Step 2: Account for the visible upright boundaries There are six visible upright surfaces: the two inner walls beside the recessed floor, the front riser of the central platform, the long front outer boundary, the right outer boundary, and the upright face beside the small front slope. Follow the edges to distinguish these from the ramps. In particular, the central inclined face and the little front inclined face are not upright walls and were already counted in the preceding step. The visible tally is therefore

$$6 + 6 + 6 = 18.$$

Step 3: Include the faces hidden from this viewpoint The two exterior boundary surfaces at the rear and the left are concealed by the upper features. The underside of the whole object is also a surface even though it cannot be seen from above. These add three to the visible count:

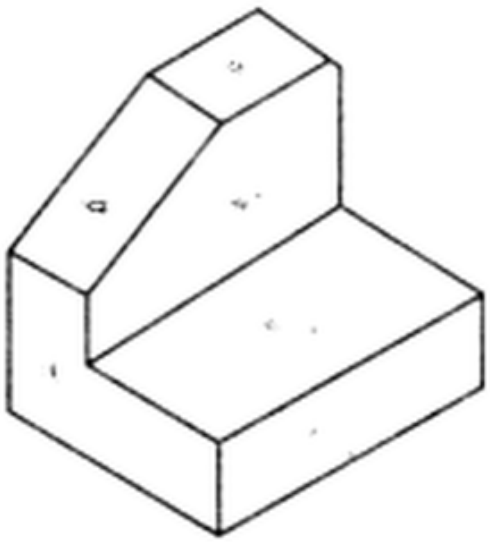
$$N = 18 + 3 = 21.$$

The value 19 misses two boundary surfaces, 22 counts one extra subdivision, and 24 overcounts three surfaces. None of those choices matches a complete sweep of the exterior.

Step 4: Conclusion The object has 21 surfaces in total, including the concealed exterior faces and its underside. Hence, the correct option is (C).

Quick Tip to Solve: Make separate tallies for horizontal, inclined and upright faces, then explicitly add the concealed rear faces and the underside.

33. Directions : (For Q. 32 to 35). Find the total number of surfaces of the object given below in the problem figure. [4 marks]



(A) 7

(B) 9

(C) 11

(D) 5

Correct Answer: (B) 9

Solution:

Concept:

- Each distinct plane forming the outside of a solid is one surface. Both exposed and hidden boundary faces belong in the total.
- For a prism with an unchanged polygonal profile, each edge of that profile sweeps out one lateral

face, and there are two end faces. Thus an n-sided profile gives $n + 2$ surfaces.

Step 1: Identify the stepped profile The solid has a low projecting platform and a higher portion whose top changes from a slope to a small flat cap. Follow its profile around the boundary rather than treating the shape as separate overlapping cuboids. Starting along the bottom and proceeding round the outside, the profile includes the low outer upright edge, the low horizontal shelf, the tall upright rise, the short high horizontal top, the inclined edge, and the remaining outer upright edge, in addition to the bottom edge. This is a seven-sided profile.

Step 2: Count the lateral and end surfaces Extending each of those seven profile edges through the width produces seven distinct surfaces. They include the underside, the low shelf, the high top and the sloping top, together with the upright boundary surfaces. The two matching end boundaries close the solid and each contributes one more surface. Even though an end boundary has an irregular outline, it is a single flat face; its outline does not split it into several surfaces. Hence

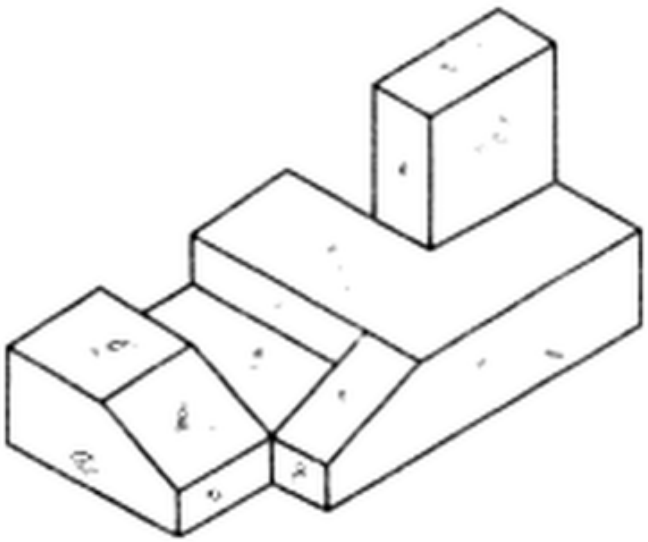
$$N = 7 + 2 = 9.$$

Step 3: Check the count against the viewpoint and options Several rear or lower surfaces are concealed in the given view, but concealment does not remove them from the solid. Counting only the more prominent planes can lead to 5 or 7, which omits boundary faces. On the other hand, splitting an irregular flat end face into rectangles produces an excessive count such as 11. The complete boundary count is 9, so only the second choice is consistent with the prism construction.

Step 4: Conclusion The seven lateral surfaces and the two end surfaces give a total of 9 surfaces. Hence, the correct option is (B).

Quick Tip to Solve: For a solid with a constant cross-section, count the edges of the section and add two; do not subdivide its flat end faces.

34. Directions : (For Q. 32 to 35). Find the total number of surfaces of the object given below in the problem figure. [4 marks]



(A) 19

(B) 16

(C) 18

(D) 17

Correct Answer: (C) 18

Solution:

Concept:

- A surface is a distinct planar part of the boundary of the solid. Inclined faces, recess walls, concealed outer walls and the underside all count.
- Joined coplanar regions are counted as one continuous surface. Contact faces inside a composite object do not belong to its exposed boundary.

Step 1: Sweep the visible upper surfaces Divide the task by orientation instead of trying to remember which parts have already been counted. There are six visible upward-facing surfaces: the small top of the upright rear block, the large raised shelf, the flat cap at the left end, the slope descending from that cap, the low central floor, and the slope along the right of that floor. The two inclined surfaces belong in this group even though they are not horizontal. Each is separated from its neighbour by a genuine change of plane.

Step 2: Count the visible upright faces The rear upright block shows two vertical faces. The raised shelf has its inner vertical riser. The left end has its broad outer side and its small low front face. On the right there is the short upright face at the near end of the ramp and the long exterior side of the base. These give seven visible upright surfaces. Count the long continuous exterior side once, rather than dividing it according to the features above it. The visible total is therefore

$$N_{\text{visible}} = 6 + 7 = 13.$$

Step 3: Complete the concealed boundary Following the boundary round the back adds four concealed upright surfaces: the remaining side of the upright rear block and the three remaining exterior wall regions around the stepped base and raised portion. The flat underside adds one further surface. Faces where the rear block joins the shelf are internal contact areas and must not be included. Thus

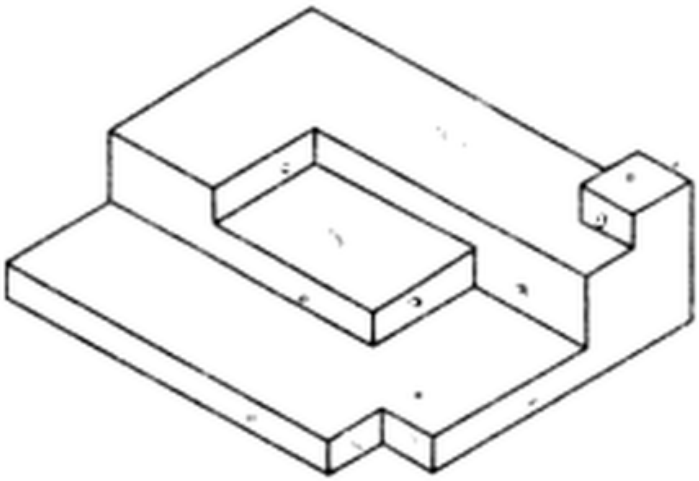
$$N = 13 + 4 + 1 = 18.$$

The alternatives 16 and 17 omit boundary surfaces, while 19 introduces an extra face by subdividing or counting a contact region.

Step 4: Conclusion Including all exposed and concealed boundary planes, the object has 18 surfaces. Hence, the correct option is (C).

Quick Tip to Solve: For a composite block, tally upper faces and upright walls separately, then inspect the concealed boundary and add the underside once.

35. Directions : (For Q. 32 to 35). Find the total number of surfaces of the object given below in the problem figure. [4 marks]



(A) 19

(B) 16

(C) 18

(D) 17

Correct Answer: (B) 16

Solution:

Concept:

- Every continuous planar boundary region counts as one surface, regardless of its outline or whether the viewpoint hides part of it.
- Recesses introduce inner walls as well as horizontal shelves. Coplanar adjoining wall regions are counted together; internal contact areas are excluded.

Step 1: Count the horizontal levels There are four distinct horizontal surfaces: the exposed low base, the rectangular intermediate platform, the higher L-shaped ledge, and the small cap at the far right. The higher ledge is one connected flat plane even though its outline turns a corner. Similarly, the low exposed floor goes round the platform and is counted once. Counting an L-shaped plane as two rectangles would incorrectly add a face. This gives a horizontal tally of four.

Step 2: Trace the visible upright surfaces Trace the walls in a continuous sweep, starting at the near long edge of the base. The lower perimeter gives four visible upright faces: the long near face, the two short faces of the inward notch, and the long outer right face. The walls surrounding the intermediate platform and ledge give four more distinct visible planes. At the left, the end of the raised ledge and the near riser of the intermediate platform belong to one continuous plane, so they must not be counted twice. Finally, the little cap adds its short inward-facing riser. The visible upright tally is nine.

Step 3: Finish with the concealed surfaces The concealed left and rear exterior walls add two upright surfaces. The underside adds one horizontal surface. Do not add the lower face of the intermediate platform or of the small cap: these lie inside the solid and are not exterior boundaries. Thus the total

is

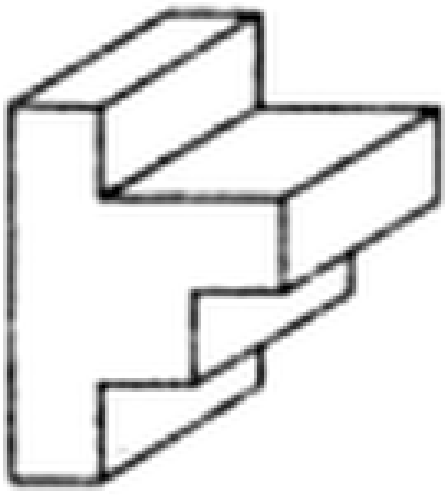
$$N = 4 + 9 + 2 + 1 = 16.$$

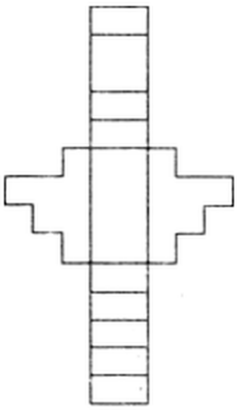
Option 17 typically results from splitting the continuous stepped wall into two planes. The larger values 18 and 19 also overcount planes or internal joins, so none matches the boundary tally.

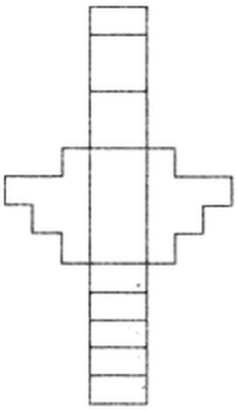
Step 4: Conclusion The solid has 16 distinct surfaces in all. Hence, the correct option is (B).

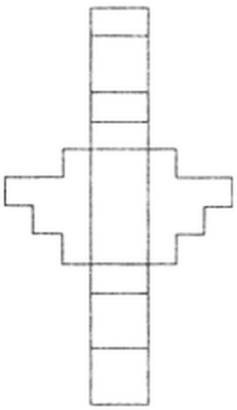
Quick Tip to Solve: Before counting a stepped wall twice, extend its two parts mentally: if they are connected and coplanar, they form one surface.

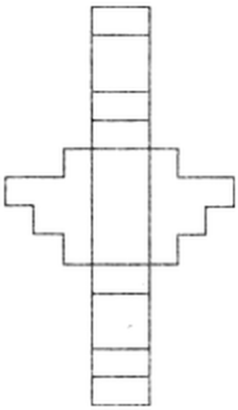
36. Directions : (For Q. 36 to 40). Which one of the answer figures shows the correct view of the 3–D problem figure after the problem figure is opened up ? [4 marks]



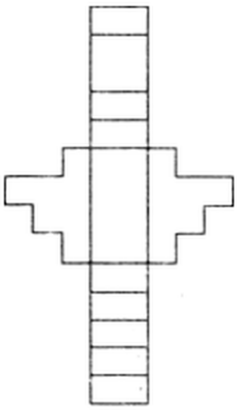
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- The development of a prism has two congruent end profiles and a strip of rectangular lateral panels. Each edge of the end profile corresponds to one panel in the strip.
- The widths of consecutive panels must follow the edge lengths around the profile in their cyclic order. Matching the two end silhouettes alone does not establish a correct net.

Step 1: Read the boundary of the stepped end face The object has a long straight upright edge and a series of shorter steps on its opposite boundary. Its end face is the stepped profile seen in the alternatives. Taking a short step as one unit, the boundary includes one long edge of four units, one

horizontal edge of two units, and eight edges of one unit. There are consequently ten boundary edges. The corresponding lateral development needs ten rectangular panels, not merely enough rectangles to approximate the outline.

Step 2: Trace the strip in the first alternative In (A), the long central rectangle corresponds to the four-unit edge, and the two congruent stepped end faces attach to its opposite sides. The rectangles continuing above it have relative widths 1, 2, 1 and 1 when traced from the free end towards the central panel. The five rectangles below it each have unit width. Including the central panel, this accounts for every one of the ten edges:

$$4 + 1 + 2 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 14$$

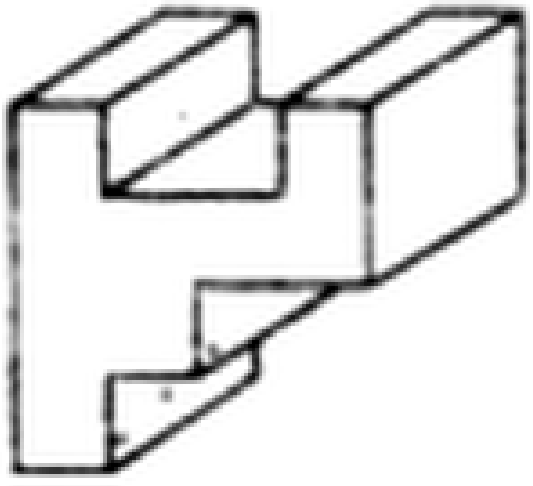
units of boundary length. Folding this strip makes each short rectangle meet its matching step, while the two-unit panel closes the longer ledge.

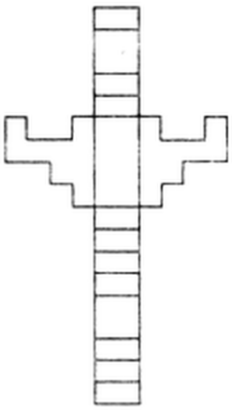
Step 3: Reject nets with missing or merged panels In (B), the strip above the central rectangle lacks a required short panel. It cannot cover all ten profile edges. Option (C) combines short boundary lengths into long panels in its lower strip; these long faces cannot turn at the intermediate steps of the object. Option (D) likewise has an incorrectly merged lower panel and therefore lacks a required fold. Only (A) preserves both the panel count and the order of short and long faces around the profile.

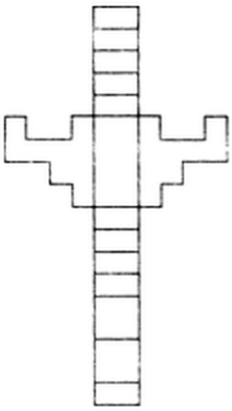
Step 4: Conclusion The strip and the two stepped end faces in (A) fold to give the stated three-dimensional object. Hence, the correct option is (A).

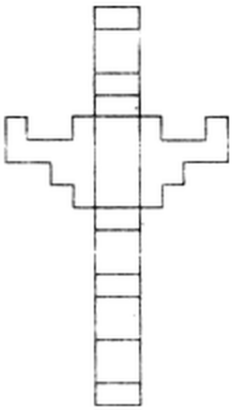
Quick Tip to Solve: Trace the perimeter of one end face and write its edge-length sequence; compare that sequence with the widths of the net's rectangular panels.

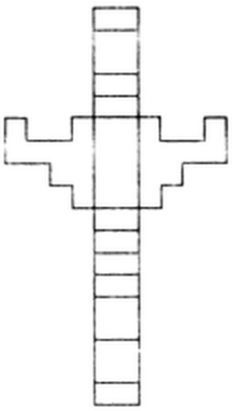
37. Directions : (For Q. 36 to 40). Which one of the answer figures shows the correct view of the 3–D problem figure after the problem figure is opened up ? [4 marks]



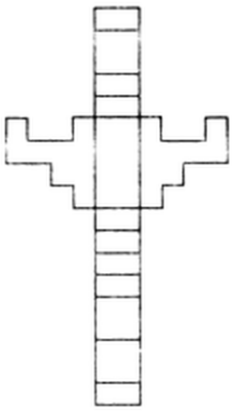
(A) 

(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- A uniform solid with a stepped cross-section opens into two identical end profiles and a strip of rectangular lateral faces. Each rectangle corresponds to one straight edge of the end profile.
- The order of the fold lines matters as much as the total strip length. Walking around the boundary of the profile gives the order of the narrow and wide rectangular panels.

Step 1: Identify the matching end profiles The solid has an upper recess and a stepped lower boundary. The two large side pieces in the developments reproduce this same outline. They must fold into parallel, congruent ends; the rectangular strip between them supplies the depth of the

solid. Consequently, comparing only these two silhouettes cannot decide the answer, because the alternatives use essentially the same silhouettes.

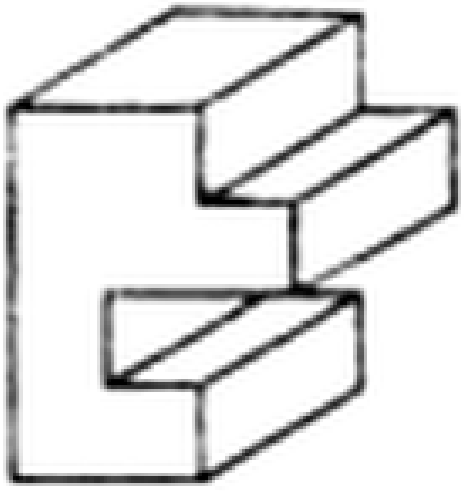
Step 2: Trace the rectangular strip around the outline Begin at the upper end of the rectangle joining the two profiles and follow the stepped edge around one profile. Every corner requires a fold, and a longer straight edge requires a wider panel. In (D), the sequence above the joining rectangle has the narrow, narrow, wide, narrow intervals when read outward from that rectangle. Continuing around the remaining edge gives the lower sequence of narrow panels, the wider panel, and the final narrow panels. The panels can therefore wrap round the recesses and outer steps without a fold falling halfway along a straight edge.

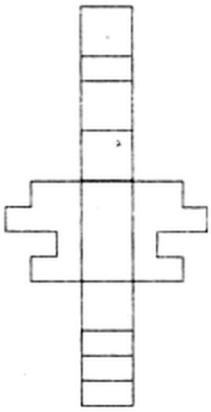
Step 3: Reject the other panel arrangements In (A), the placement of a wider interval in the lower run does not agree with the order of the lower steps. In (B), the upper run subdivides the long straight portion at the wrong places. In (C), the upper and lower runs do not retain the complete required ordering of folds. A net cannot repair such a mismatch by rotating a single panel: all the connected panels must fold together while preserving their adjacency.

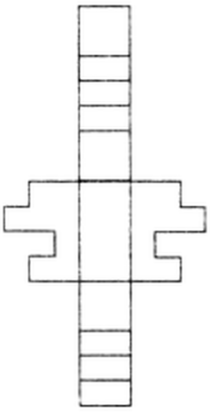
Step 4: Conclusion Only development (D) preserves the stepped end profile and the required cyclic order of its lateral faces. Hence, the correct option is (D).

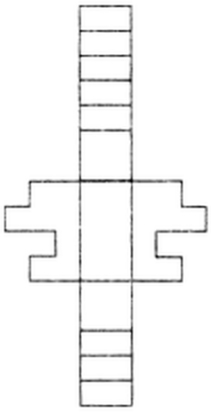
Quick Tip to Solve: For a stepped prism, trace the perimeter from the attached rectangle and compare successive panel widths; the overall cross-shaped silhouette alone is insufficient.

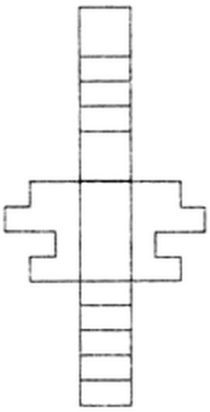
38. Directions : (For Q. 36 to 40). Which one of the answer figures shows the correct view of the 3–D problem figure after the problem figure is opened up ? [4 marks]



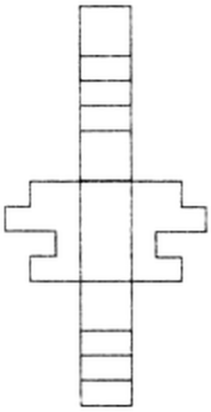
(A) 

(B) 

(C) 

(D) 

Correct Answer: (B)



Solution:

Concept:

- For a prism, unfolding the lateral surface produces a strip whose consecutive panel widths equal consecutive edge lengths of the cross-section.
- A re-entrant notch contributes its own edges. These must remain separate panels even when the neighbouring parts of the strip are all rectangular.

Step 1: Read the cross-section of the solid The end of the solid is a C-shaped profile with upper and lower projecting arms and a recess between them. There are two congruent copies of that profile in the net. The central strip joins them along the long straight back edge. The remaining rectangles must

turn round both projecting arms and into the recess, returning to the starting edge without stretching or overlapping.

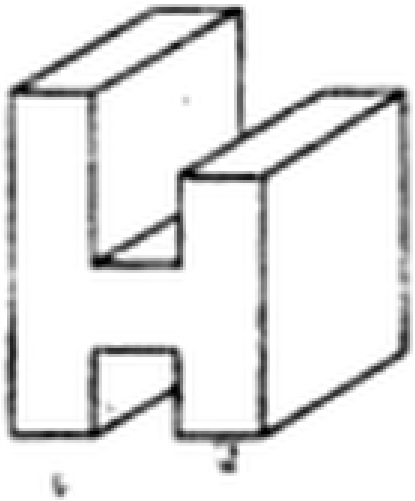
Step 2: Match the ordered widths in the strip Use the shortest step in the profile as one unit. In development (B), the upper run, read from the joining rectangle outwards, has widths in the proportion 2, 1, 1, 1, 2. The lower run starts with a two-unit panel and then the successive one-unit panels. This sequence supplies the long outer surfaces and the short surfaces at the two corners of the recess in their required order. The two profiles then close the ends of the folded strip.

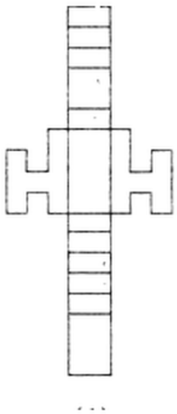
Step 3: Compare each competing net Option (A) places the long interval in the upper run at the wrong position and creates the wrong sequence of short faces in the lower run. Option (C) divides the upper run into too many equal short intervals, losing a required wider face. Option (D) gives an incorrect succession of equal panels along the lower run. Equal overall strip lengths would not make these alternatives correct: each fold must coincide with a corner of the actual C-shaped boundary.

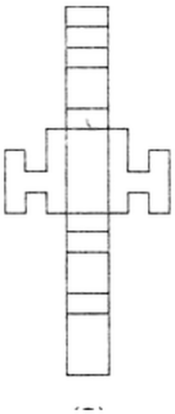
Step 4: Conclusion Development (B) has the appropriate two end profiles and the correct order of long and short lateral panels. Hence, the correct option is (B).

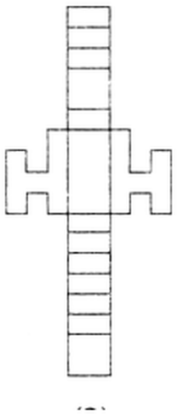
Quick Tip to Solve: Mark the long back edge first, then walk round the C-shaped boundary, counting the edges inside the notch as well as the outside edges.

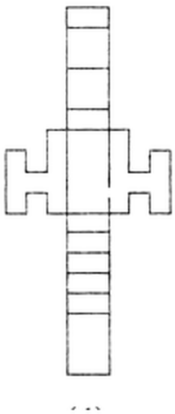
39. Directions : (For Q. 36 to 40). Which one of the answer figures shows the correct view of the 3–D problem figure after the problem figure is opened up ? [4 marks]



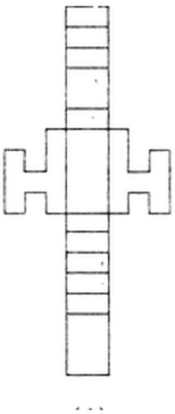
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- The lateral faces of a prism correspond one-to-one with the straight boundary segments of its end profile.
- A valid development must preserve both the number of corners and the cyclic order of the panel widths; a missing crease merges two distinct faces.

Step 1: Identify the H-shaped end The solid has two tall upright portions joined by a middle bridge, with recesses above and below the bridge. Its development must therefore have two matching H-shaped end pieces. The long central rectangle between them represents one of the straight edges of

this profile; all the other edges must be represented by the narrow and wide rectangles extending from it.

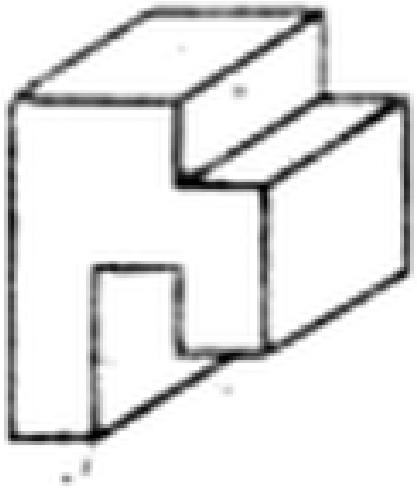
Step 2: Follow the perimeter through the folds In (A), start at the central rectangle and trace the upper strip towards its free end. The narrow intervals, including the wider interval required by the profile, occur in the same order as the boundary segments reached while walking round the H-shaped end. Continuing from the bottom of the central rectangle, the short segments are retained individually and the long unbroken outer edge is represented by the long panel at the far end of the lower strip. Each change of direction in the H outline therefore has a corresponding crease.

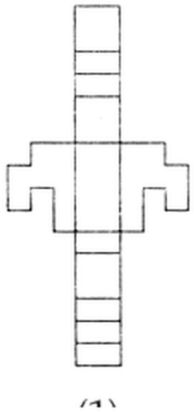
Step 3: Eliminate the alternatives Option (B) merges or relocates creases in the upper run, so it does not reproduce the same sequence of short faces. Option (C) puts the long unbroken interval on the wrong part of the strip and changes the lower sequence. Option (D) likewise shifts the wider upper interval and the lower crease pattern. These changes cannot be corrected simply by turning the drawing on the page: the two attached H profiles fix the positions from which the strip must wrap.

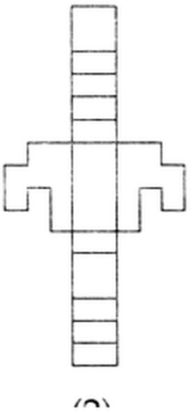
Step 4: Conclusion The crease pattern in (A) is the one that closes around the given H-shaped solid. Hence, the correct option is (A).

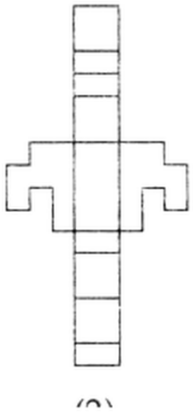
Quick Tip to Solve: Check the longest uninterrupted edge and its panel first, then count the short crease intervals around both recesses to distinguish similar H-shaped nets.

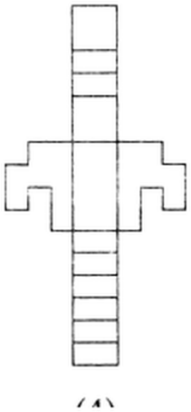
40. Directions : (For Q. 36 to 40). Which one of the answer figures shows the correct view of the 3–D problem figure after the problem figure is opened up ? [4 marks]



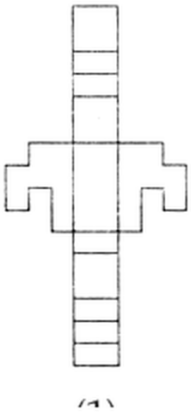
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- A development keeps the lengths and adjacency of all faces, while allowing them to rotate about their common edges.
- For the illustrated stepped prism, the two end faces must be congruent and the connecting rectangular strip must follow the complete notched perimeter.

Step 1: Locate the distinctive corners The solid has a raised portion on the top, a lower ledge beside it, and a rectangular opening in the lower part of the end profile. These features create both long outer edges and short edges around the opening. A correct net must include separate lateral rectangles for

all these edges; the opening cannot be represented by one large rectangle that ignores its two inner corners.

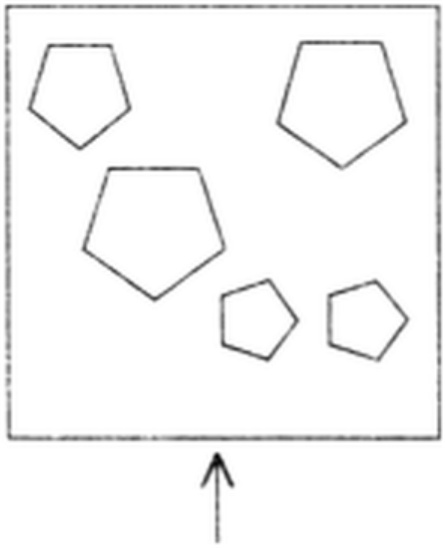
Step 2: Test the strip in option (A) Read the upper run in (A) outward from the rectangle between the two profiles. Its widths follow the proportion 2, 1, 1, 2, matching the two longer edges with the two short edges between them. From the lower end of the joining rectangle, the strip starts with a short panel, then the longer panel, followed by the remaining short panels. This arrangement places the folds at the top step and at each corner of the lower recess, and the second end profile closes the solid.

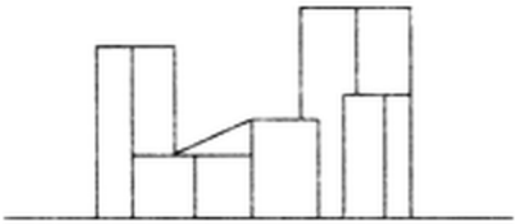
Step 3: Distinguish the incorrect fold patterns In (B), the upper run is divided into equal short panels where the end profile requires longer uninterrupted faces. In (C), the order of the first long and short intervals near the free end of the upper run is exchanged. In (D), the lower run puts a long interval immediately against the central rectangle instead of after the first short panel. Thus the folds would meet the wrong edges of the end profile in each of these three cases.

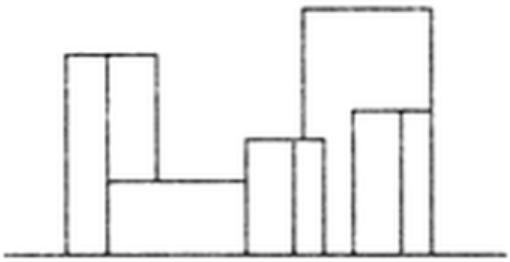
Step 4: Conclusion Option (A) preserves the required order of long and short panels around the stepped, notched outline. Hence, the correct option is (A).

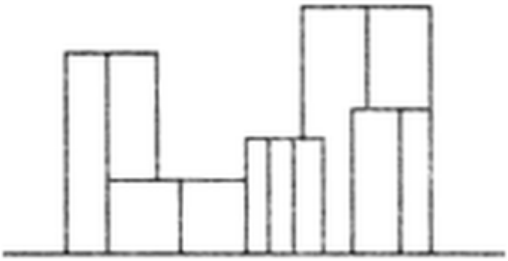
Quick Tip to Solve: Anchor the net at its central rectangle and compare panel widths in order on both sides; do not select a net merely because the end faces look right.

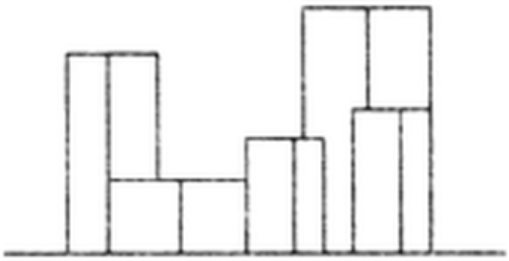
41. Directions : (For Q. 41 to 43). The problem figure shows the top view of objects. Looking in the direction of the arrow, identify the correct elevation, from amongst the answer figures. [4 marks]



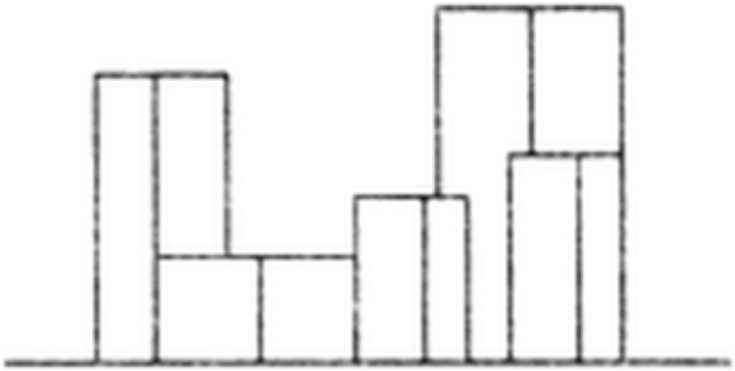
(A) 

(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- An elevation is an orthographic projection: positions along the viewing direction collapse, while left-to-right order across the view is preserved.
- Objects nearer the viewing arrow hide overlapping portions of objects farther away. Only genuinely visible vertical edges appear; horizontal roof slopes must not be invented from a plan.

Step 1: Fix the direction of projection The arrow points upwards from the bottom of the plan, so the observer is below the square and looks towards its top. Preserve the horizontal positions of the five pentagonal objects. The isolated object on the left must appear as a separate left-hand group, while the larger rear object on the right lies behind the two smaller objects nearer the observer. The central object is also partly concealed wherever a nearer object overlaps it in projection.

Step 2: Apply occlusion and compare the edges Project the corners of each pentagon vertically into the elevation and remove any edge hidden by a nearer face. The resulting view has the separated left-hand upright, the lower middle portion and the group of overlapping uprights on the right. The visible central vertical edges in the left upright, the low middle portion and the exposed top of the tall rear-right upright agree with (D). In particular, the front objects interrupt the lower portions of the

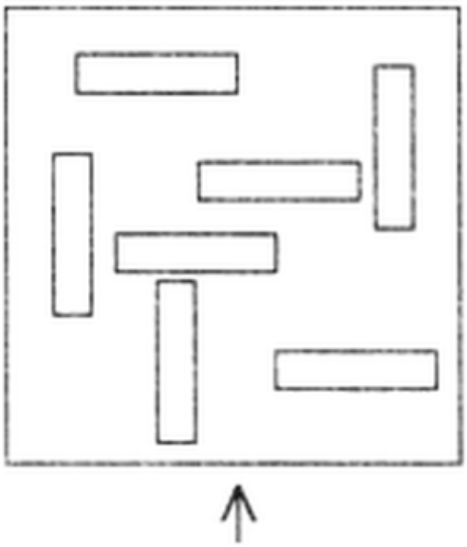
rear objects rather than making all their edges visible through them.

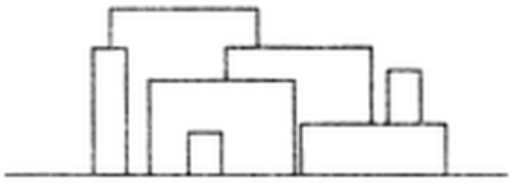
Step 3: Reject the incompatible views Option (A) introduces a sloping line across the middle, which is not a visible edge obtained by this elevation of the upright objects. Option (B) omits the visible middle edge in the low central portion and the visible edge in the exposed top of the rear-right object. Option (C) introduces an additional exposed division in the front upright of the right-hand group. Option (D) is the one that maintains the correct projected edges and their interruption by the foreground objects.

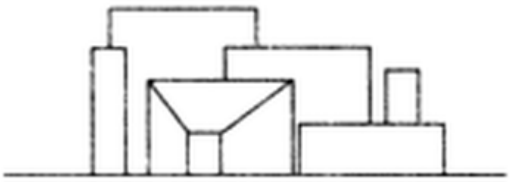
Step 4: Conclusion The correct elevation, viewed in the direction of the arrow, is (D). Hence, the correct option is (D).

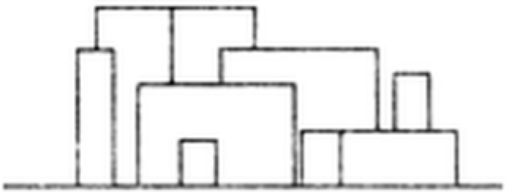
Quick Tip to Solve: Look from the arrow and resolve overlaps from nearest to farthest. Erase hidden edges before comparing otherwise similar elevation options.

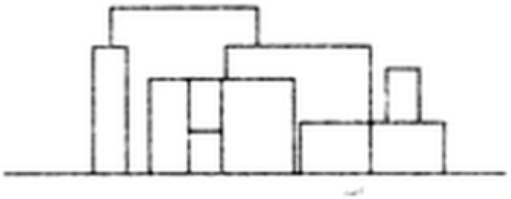
42. Directions : (For Q. 41 to 43). The problem figure shows the top view of objects. Looking in the direction of the arrow, identify the correct elevation, from amongst the answer figures. [4 marks]



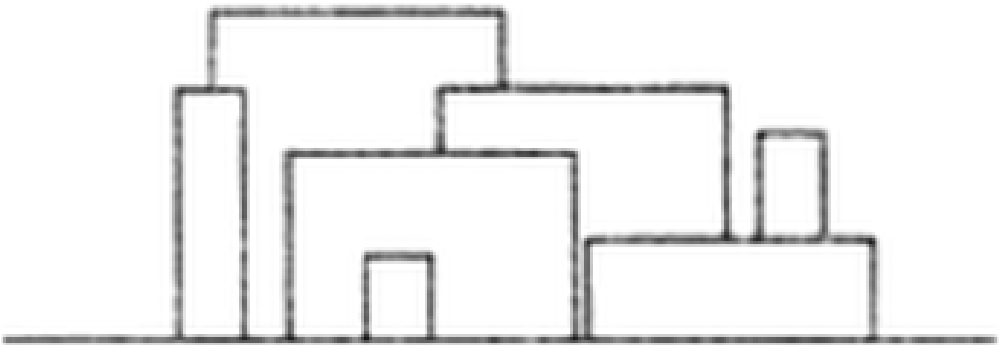
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- In orthographic elevation, the left-to-right extent of each plan shape is preserved, but its depth along the line of sight is suppressed.
- A rectangular upright has flat vertical faces. Its elevation has straight vertical boundaries; diagonal or extra central edges cannot be introduced unless the plan supplies them.

Step 1: Establish which objects are in front The observer is at the bottom of the plan and looks upwards along the arrow. The short horizontal rectangle near the lower right is therefore in front of the right-hand objects. The narrow vertical rectangle near the lower centre is also near the observer. The long horizontal shapes and the rear-left shape lie farther away. Their visible parts must be found after the foreground rectangles have covered any overlapping portions.

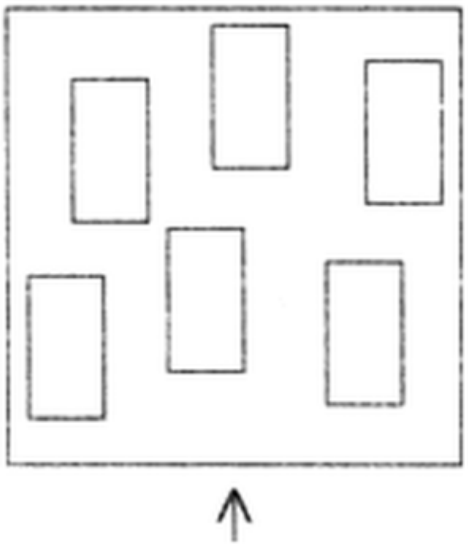
Step 2: Build the projected outline Project the horizontal span of each rectangle onto a common baseline. The left narrow object remains an upright near the left side. The lower central narrow object appears as a small rectangular upright in front of the larger central block. The front horizontal object makes the low wide block on the right and hides the lower parts behind it. The rear objects rise above these foreground portions without acquiring interior edges down the middle of their flat faces. This is the arrangement drawn in (A).

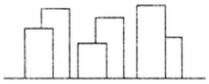
Step 3: Check all alternatives Option (B) adds diagonal lines meeting in the central foreground area, implying sloping or triangular faces absent from the rectangular plan. Option (C) adds a vertical division in the tall rear block, although that flat face has no corresponding edge. Option (D) similarly adds exposed vertical divisions and an incorrect central edge pattern. Option (A) alone retains the small front rectangle, the right low block and the uninterrupted rear faces while respecting which object hides which.

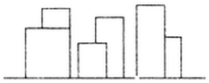
Step 4: Conclusion The elevation consistent with the rectangular plan and the viewing arrow is (A). Hence, the correct option is (A).

Quick Tip to Solve: Draw mental vertical projection lines through the left and right ends of each plan rectangle, then let the nearest rectangle hide the ones behind it.

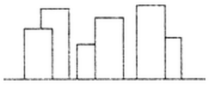
43. Directions : (For Q. 41 to 43). The problem figure shows the top view of objects. Looking in the direction of the arrow, identify the correct elevation, from amongst the answer figures. [4 marks]

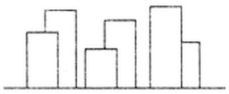


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- The arrow gives the viewing direction. Depth is suppressed in an elevation, while each object retains its left-to-right position and projected width.
- A nearer object hides the overlapping part of an object behind it. Its front outline must therefore interrupt any rear edge covered by it.

Step 1: Project the footprints Look from the bottom of the plan towards the top. There are three pairs of rectangles. In each pair the front rectangle lies to the left of the rectangle behind it, with only part of their horizontal spans overlapping. The three pairs remain separated by gaps; they cannot merge into a single continuous block.

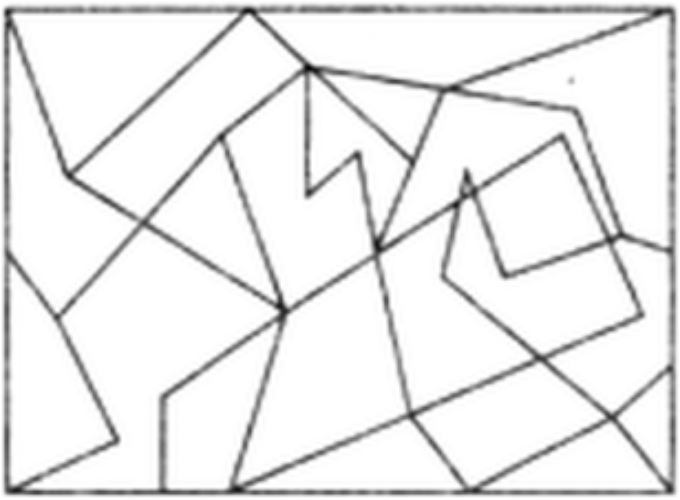
Step 2: Check the overlap and widths In (A), each front rectangle retains its full width and the exposed rear rectangle continues beyond its right edge. The hidden portion of each rear vertical edge is omitted. In (B), the left front outline extends too far across the rear object. In (C), an inclined edge is introduced in the middle pair although the plan contains rectangular objects. In (D), the middle front rectangle loses part of its projected width at the overlap. These are changes to the projected outlines, rather than permissible changes in viewing depth.

Step 3: Conclusion The three overlapping pairs are represented consistently by (A). Hence, the

correct option is (A).

Quick Tip to Solve: Project the left and right edges of each footprint before comparing heights; the widths and overlaps often eliminate the distractors immediately.

44. Directions : (For Q. 44 to 46). One of the following answer figures is hidden in the problem figure in the same size and direction. Select the correct one. [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: More than one supplied outline satisfies the printed embedded-figure condition.

Solution:

Concept:

- An embedded figure may be traced along selected portions of existing lines. Extra lines crossing its interior do not invalidate a match.
- The instruction permits translation only: the size, orientation, corner order and inward notch must remain unchanged.

Step 1: Trace the keyed outline The six-sided concave outline in (D) can be traced in the central part of the problem diagram. Start at the upper central junction, follow the vertical line downward to the bottom of the notch, go diagonally up and right, then down the short sloping side. Continue

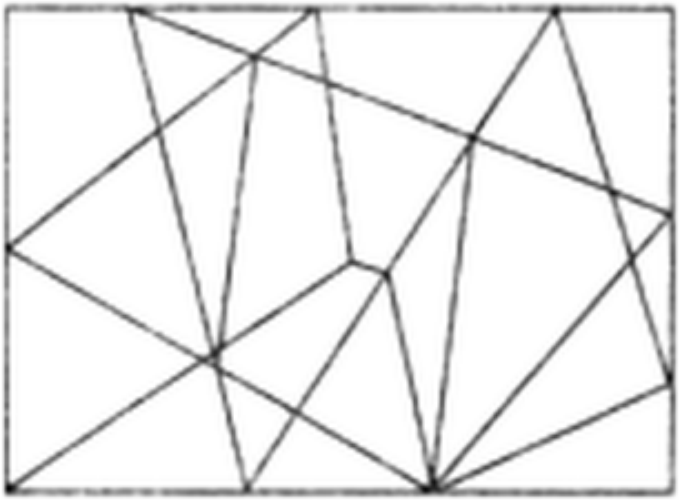
down-left to the lower central junction and return up-left along the long sloping side. This preserves the inward step and the direction of every segment.

Step 2: Check whether the match is unique There is also a complete trace of (A) immediately to the right of the same upper central junction. Follow the short upper edge rightwards to the next junction, descend diagonally leftwards to the lower tip, return up-left to the small inward corner, go down-left to the end of the short vertical segment, and close vertically upwards. Each segment is present in the problem figure. In contrast, the convex outline (B) and the larger inward-pointing outline (C) do not have the required complete, same-size boundary. Since both (A) and (D) are traceable, rejecting (A) would require an extra rule absent from the question.

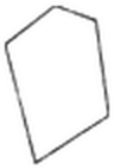
Step 3: Conclusion The supplied key selects (D), which is indeed embedded. However, (A) is also embedded in the same size and direction, so the printed figure does not support a unique answer: both (A) and (D) satisfy the stated condition.

Quick Tip to Solve: For an embedded-figure test, trace every edge and closure; also check uniqueness instead of accepting the first familiar outline.

45. Directions : (For Q. 44 to 46). One of the following answer figures is hidden in the problem figure in the same size and direction. Select the correct one. [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: More than one supplied outline satisfies the printed embedded-figure condition.

Solution:

Concept:

- A hidden figure is formed by existing line segments in their printed direction and size; the surrounding lines are distractions.
- An open outline must remain open. A matching path need not enclose an isolated blank region, and intersections may lie along its edges.

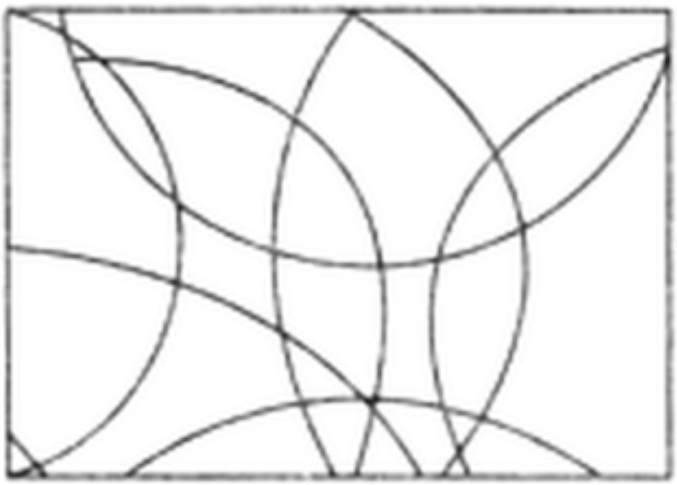
Step 1: Trace the keyed open path Option (D) is an open angular path. In the upper central-right part of the problem figure, start at the upper intersection on the long descending diagonal. Follow the steep segment downwards to the central meeting point, turn down-right for the short lower segment, then follow the rising diagonal towards the upper right. Its separate upper arm lies on the long descending diagonal. The gap between the two right-hand ends remains open, just as in (D).

Step 2: Test the remaining complete boundaries Option (B) also has a same-size trace on the left of the central region. Its upper short sides follow the rising diagonal to the upper junction and then the descending diagonal to the next intersection. Its right side runs down to the central junction; its lower side runs down-left to the multi-line junction; its left side returns up-left to the starting point. Thus the five-sided outline is present despite the crossing lines. The triangle (A) does not have the required complete three sides at its printed size, and the quadrilateral (C) lacks its matching lower and side connections. The presence of both (B) and (D) prevents a unique selection under the stated rule.

Step 3: Conclusion The supplied key selects (D), and its open path is present. Nevertheless, (B) is also present at the same size and orientation. The literal figure therefore admits both (B) and (D), rather than a unique answer.

Quick Tip to Solve: Treat open ends as features: do not close a gap merely because another line in the background passes nearby.

46. Directions : (For Q. 44 to 46). One of the following answer figures is hidden in the problem figure in the same size and direction. Select the correct one. [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- For a curved embedded figure, match each arc, its curvature direction and the sequence of corners, rather than the overall silhouette alone.
- Crossing curves can continue through an intersection. Follow the original arc through that crossing instead of switching to an unrelated curve.

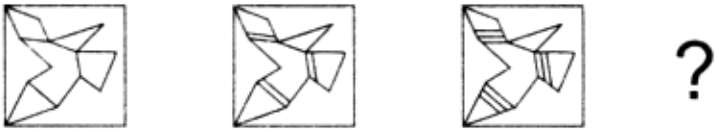
Step 1: Locate the broad curved upper boundary In the upper central part of the problem figure, locate the pointed junction on the top border. The broad arc descending towards the right from this junction supplies the curved upper-right boundary of (B). The left boundary descends from the same top point towards the central-left portion of the design. Following these arcs preserves the wider top and narrower lower portion of the answer figure.

Step 2: Follow the remaining arcs and compare From the right-hand end of the broad upper arc, follow the inward-curving boundary towards the short lower edge; the opposite inward arc returns towards the left boundary. The complete outline has the same two lower corners and the same curved sides as (B). Option (A) has a tall narrow profile and a long oblique lower edge, rather than this broad curved head. Option (C) is an elongated curved strip, with a different sequence of end curves. Option (D) has a straighter right end and a different lower boundary. Neither the proportions nor the connections of those distractors reproduce the traced upper-central outline.

Step 3: Conclusion The complete curved outline matches (B) without rotation or change of size. Hence, the correct option is (B).

Quick Tip to Solve: Begin with the most distinctive arc and its two endpoints; then trace the boundary continuously through the distracting intersections.

47. Directions : (For Q. 47 to 50). Which one of the answer figure will complete the sequence of the three problem figures ? [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- The outer square and the angular inner outline remain fixed. The changing feature is the number of short parallel strokes in three small peripheral regions.
- A sequence rule must apply to all changing regions simultaneously; a match in just one region is insufficient.

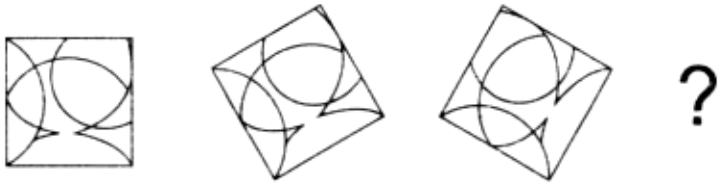
Step 1: Separate the fixed and changing details The upper-left, lower-left and right-hand peripheral regions occupy the same positions in all three problem figures. Their boundary lines do not move. The first figure has no added internal stripe in these regions; the next has one, and the third has two. Thus each successive figure adds one short internal line to each of the three regions. The central white angular region is unchanged.

Step 2: Require one more stroke in every region The missing figure must therefore contain three internal parallel strokes in each of the upper-left, lower-left and right-hand regions, with the original outline unchanged. Option (D) has all three required groups. Option (A) leaves the lower-left group one stage behind. Option (B) leaves the upper-left group one stage behind. Option (C) does not complete the required set in the right-hand group. These differences matter even though the square and the central outline look almost identical in all four choices.

Step 3: Conclusion Only (D) advances every stripe group by one stage. Hence, the correct option is (D).

Quick Tip to Solve: Count internal strokes in each changing region separately; never count the fixed boundary as an added stripe.

48. Directions : (For Q. 47 to 50). Which one of the answer figure will complete the sequence of the three problem figures ? [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- A rigid rotation moves every line, intersection and gap together; it changes orientation without changing the internal connectivity of the design.
- The successive square frames show equal angular turns. Compare a distinctive gap as well as the large curves to determine the next position.

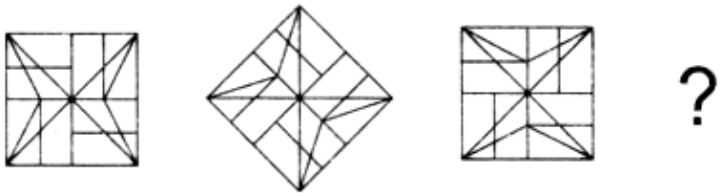
Step 1: Determine the common turn Read the three problem figures from left to right. The square frame turns through the same angle at each step, while its curved pattern turns with it as a rigid unit. The frame first tilts one way and then reaches the next tilted position. Continuing the turn brings the frame back to a horizontal-and-vertical position after a quarter turn relative to the first figure. The curves must undergo that same quarter turn, rather than being redrawn independently.

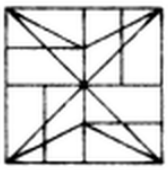
Step 2: Match the rotated crossings and gaps Track the two pointed interruptions in the curved lines and the long crossing arcs. After the next turn, the large loop occupies the upper-left portion, and the interrupted tips occur on the right. The lower crossing also retains the original connections. Option (B) preserves the full set of curves, gaps and crossings. Option (A) differs in the lower-left continuation. Option (C) omits a required upper-right continuation, and option (D) alters the lower crossing. They resemble the rotated design globally but do not preserve all its local connections.

Step 3: Conclusion The rigidly rotated pattern is shown in (B). Hence, the correct option is (B).

Quick Tip to Solve: When several options look alike after rotation, track one small gap and one crossing; both must arrive at their predicted positions.

49. Directions : (For Q. 47 to 50). Which one of the answer figure will complete the sequence of the three problem figures ? [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- A figure sequence can rotate as a single rigid pattern: the boundary, centre, internal grid and additional slanting segments must all undergo the same rotation.
- A rotation of 45° changes an upright square into a diamond; a second equal rotation restores the upright boundary but changes the positions of the internal details.

Step 1: Identify the change between successive figures The first boundary is an upright square, the second is a diamond and the third is upright again. Follow the additional bent pair of slanting lines rather than only the outline. In the first figure their junctions lie to the left and right of the centre. In the second, those junctions lie above-left and below-right. In the third, they lie above and below the centre. These successive positions show a clockwise rotation of 45° at each stage.

Step 2: Rotate the complete third pattern once more After the next clockwise turn, the outline must be a diamond. The upper junction moves above-right of the centre and the lower junction moves below-left. The two slanting connections at each junction are retained, as are all the short subdivisions of the internal grid. In particular, the top corner is joined to the upper-right junction and the bottom corner to the lower-left junction. These are actual drawn segments, so an answer that keeps the outline but omits them is incomplete.

Step 3: Compare all answer figures Option (B) has the required diamond orientation and retains both bent pairs and the complete subdivision grid. Option (A) remains an upright square, so it has the wrong orientation. Option (C) omits the connections from the top and bottom corners to the corresponding inner junctions. Option (D) retains the prominent bent connections but omits short

grid subdivisions present in the problem figures. Only (B) preserves the whole pattern under the same turn.

Step 4: Conclusion The missing figure is the complete third pattern rotated clockwise through $45°$. Hence, the correct option is (B).

Quick Tip to Solve: Track one distinctive inner junction and one short grid segment together; an outline alone cannot distinguish the three diamond options.

50. Directions : (For Q. 47 to 50). Which one of the answer figure will complete the sequence of the three problem figures ? [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- A rigid rotation preserves every connection inside a figure, including which internal line meets which side of the boundary.
- The tip of the triangular boundary turns anticlockwise by $45°$ at each stage: upward, upper-left, then left.

Step 1: Determine the direction of rotation The first triangle has its sharp tip at the top. The second has the corresponding tip toward the upper-left, and the third points left. Thus the whole triangle rotates anticlockwise by $45°$ between consecutive figures. The fourth triangle must point toward the lower-left. The internal junction travels with the triangle; it does not remain fixed relative to the page.

Step 2: Preserve the internal subdivisions Follow the long line that runs from the sharp tip through the inner junction to the opposite side. Also follow the shorter line parallel to it and the connections from the inner junction to the other two sides and the remaining corner. Rotating the third figure by

the same amount places these features in a lower-left-pointing triangle. No new segment is introduced and no existing subdivision is removed: the question is a change of orientation, not a rule that progressively adds or deletes lines.

Step 3: Eliminate the alternatives Option (A) shows the required lower-left direction and the complete rotated arrangement of the internal segments. Option (C) points in a different direction, with the sharp tip downward-right. Although (B) and (D) have the general lower-left orientation, their internal divisions do not match: (B) adds an extra short upper subdivision, whereas (D) lacks the shorter parallel subdivision. Checking the side connections, rather than merely counting the sides of the triangle, singles out (A).

Step 4: Conclusion The fourth figure is the third triangle turned anticlockwise through 45°, with every internal line preserved. Hence, the correct option is (A).

Quick Tip to Solve: For rotating triangles, identify the sharp tip first, then check the shorter internal line: it quickly eliminates otherwise similar options.

Mathematics (120 marks)

51. Let PQ be a focal chord of the parabola $y^2 = 4x$. If the centre of a circle having PQ as its diameter lies on the line $\sqrt{5}y + 4 = 0$, then the length of the chord PQ is : [4 marks]

(A) $\frac{26}{5}$

(B) $\frac{36\sqrt{5}}{5}$

(C) $\frac{26\sqrt{5}}{5}$

(D) $\frac{36}{5}$

Correct Answer: (D) $\frac{36}{5}$

Solution:

Concept:

- For $y^2 = 4ax$, a parametric point is $(at^2, 2at)$. Here $a = 1$, so take $P = (t^2, 2t)$ and $Q = (s^2, 2s)$.
- The parameters of the endpoints of a focal chord satisfy $ts = -1$. The centre of a circle having that chord as diameter is its midpoint.

Step 1: Use the given locus of the midpoint The midpoint of the chord is

$$M = \left(\frac{t^2 + s^2}{2}, t + s\right).$$

It lies on $\sqrt{5}\,y + 4 = 0$, so its ordinate gives

$$t + s = -\frac{4}{\sqrt{5}}, \qquad (t + s)^2 = \frac{16}{5}.$$

Only the ordinate is fixed by the given line; the abscissa must not be set equal to zero. The focal-chord condition supplies the second relation between the two endpoint parameters.

Step 2: Express the distance using the parameter sum Apply the distance formula, keeping the factor $t - s$ common:

$$PQ = \sqrt{(t^2 - s^2)^2 + (2t - 2s)^2}$$
$$= |t - s|\sqrt{(t + s)^2 + 4}.$$

Since $ts = -1$,

$$(t - s)^2 = (t + s)^2 - 4ts = (t + s)^2 + 4.$$

The two nonnegative factors in the distance expression are therefore equal. Hence $PQ = (t + s)^2 + 4$.

Step 3: Evaluate and distinguish the options Substitute the known square of the sum:

$$PQ = \frac{16}{5} + 4 = \frac{36}{5}.$$

The minimum focal-chord length is the latus rectum, 4, and this value is larger, as expected for a nonzero midpoint ordinate. Options (A), (B) and (C) do not equal the distance obtained from both endpoint coordinates; in particular an extra factor $\sqrt{5}$ is not retained after the equal square-root factors multiply.

Step 4: Conclusion The length of the focal chord is $\boxed{36/5}$. Hence, the correct option is (D).

Quick Tip to Solve: For $y^2 = 4ax$, a focal chord with midpoint ordinate y_M has length $4a + y_M^2/a$; derive it from $ts = -1$.

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52. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = f(x)\left(1 + x^{-6}\right)^{1/3} + C$, where C is a constant of integration, then $f(x)$ is equal to : [4 marks]

(A) $-\frac{1}{6}$
(B) $-\frac{6}{x}$
(C) $-\frac{x}{2}$
(D) $-\frac{1}{2}$

Correct Answer: (D) $-\frac{1}{2}$

Solution:

Concept:

- For an integral involving a fractional power, choose a substitution whose derivative reproduces the remaining power of the variable.
- For real $x \neq 0$, $1 + x^6 = x^6(1 + x^{-6})$, and $(x^6)^{2/3} = x^4$. Use real cube roots throughout.

Step 1: Rewrite the integrand in the required form Extract x^6 from the denominator factor:

$$\frac{1}{x^3(1 + x^6)^{2/3}} = \frac{1}{x^3 x^4 (1 + x^{-6})^{2/3}}$$
$$= x^{-7}(1 + x^{-6})^{-2/3}.$$

This identity is valid on either interval $x > 0$ or $x < 0$. The original integrand is undefined at zero, so

an antiderivative is considered separately on such intervals.

Step 2: Make the substitution and integrate Put $u = 1 + x^{-6}$. Then

$$du = -6x^{-7}\,dx.$$

Therefore

$$\begin{aligned}\int x^{-7}(1+x^{-6})^{-2/3}\,dx &= -\frac{1}{6}\int u^{-2/3}\,du \\ &= -\frac{1}{6}\cdot 3u^{1/3} + C \\ &= -\frac{1}{2}(1+x^{-6})^{1/3} + C.\end{aligned}$$

The factor 3 from the power rule is essential: $-1/6$ is the substitution factor before integration, not the final coefficient.

Step 3: Verify the result and compare the choices Differentiating the expression found gives

$$\frac{d}{dx}\left[-\frac{1}{2}(1+x^{-6})^{1/3}\right] = x^{-7}(1+x^{-6})^{-2/3},$$

which is exactly the rewritten integrand. Thus the multiplier of the prescribed cube-root factor is the constant $-1/2$. Option (A) misses the power-rule factor. The variable multipliers in (B) and (C) introduce additional product-rule terms, so neither has the required derivative.

Step 4: Conclusion One obtains $f(x) = \boxed{-\frac{1}{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: After substituting $u = 1 + x^{-6}$, keep the factor $-1/6$ separate until the power-rule integration is complete.

53. The sum of the series

$S = \frac{1}{19!} + \frac{1}{3!17!} + \frac{1}{5!15!} + \ldots..$ **to 10 terms is equal to :** [4 marks]

(A) $\frac{2^{20}}{20!}$

(B) $\frac{2^{10}}{20!}$

(C) $\frac{2^{19}}{19!}$

(D) $\frac{2^{19}}{20!}$

Correct Answer: (D) $\frac{2^{19}}{20!}$

Solution:

Concept:

- Multiplying $1/[r!(n-r)!]$ by $n!$ produces the binomial coefficient $\binom{n}{r}$.
- The odd-index coefficients of $(1+z)^n$ sum to 2^{n-1}, found by subtracting its values at $z = 1$ and $z = -1$.

Step 1: Identify the ten indices correctly The factorial arguments in every denominator add to 20. The first term is $1/(1!19!)$, since $1! = 1$; the next terms have first arguments $3, 5, \ldots$. Ten odd indices

are

$$1, 3, 5, 7, 9, 11, 13, 15, 17, 19.$$

The last term is consequently $1/(19!1!)$. Both end terms occur in the printed ten-term series; their equality does not mean that one should be discarded.

Step 2: Convert the series into a binomial sum Using the common factorial $20!$, write

$$S = \frac{1}{20!}\left[\binom{20}{1} + \binom{20}{3} + \cdots + \binom{20}{19}\right].$$

The upper index is 20, not 19: it is determined by the sum of the two factorial arguments. This avoids confusing the largest displayed factorial with the upper index of the binomial coefficients.

Step 3: Evaluate the odd-index sum Let E and O denote the even- and odd-index sums. The binomial theorem gives

$$E + O = (1+1)^{20} = 2^{20}, \qquad E - O = (1-1)^{20} = 0.$$

Subtracting yields $2O = 2^{20}$, hence $O = 2^{19}$. Thus

$$S = \frac{2^{19}}{20!}.$$

Option (A) counts both even and odd coefficients. Option (B) incorrectly relates the exponent to the number of terms. Option (C) uses the wrong common factorial; it is twenty times the required value.

Step 4: Conclusion The sum of the ten terms is $\boxed{2^{19}/20!}$. Hence, the correct option is (D).

Quick Tip to Solve: First add the two factorial arguments to identify n; then decide whether the series selects all even or all odd binomial coefficients.

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54. The solution of the differential equation
$\frac{ydx+xdy}{ydx-xdy} = \frac{x^2e^{xy}}{y^4}$,
satisfying $y(0) = 1$, is : [4 marks]

(A) $x^3 = 3y^3\left(1 - e^{-xy}\right)$

(B) $x^3 = 3y^3\left(-1 + e^{xy}\right)$

(C) $x^3 = 3y^3\left(1 - e^{xy}\right)$

(D) $x^3 = 3y^3\left(-1 + e^{-xy}\right)$

Correct Answer: (A) $x^3 = 3y^3\left(1 - e^{-xy}\right)$

Solution:

Concept:

- The numerator $y\,dx + x\,dy$ is $d(xy)$, while $y\,dx - x\,dy = y^2 d(x/y)$.
- The substitutions $u = xy$ and $v = x/y$ turn the equation into a separable relation. The initial value fixes the integration constant.

Step 1: Transform both differentials consistently Work in a neighbourhood of the initial point where

$y \neq 0$. With $u = xy$ and $v = x/y$,

$$du = y\,dx + x\,dy, \qquad dv = \frac{y\,dx - x\,dy}{y^2}.$$

Also $x^2/y^4 = v^2/y^2$. Substituting into the given equation gives

$$\frac{du}{y^2\,dv} = \frac{v^2 e^u}{y^2}.$$

Cancel the common factor to obtain $e^{-u}\,du = v^2\,dv$. The exponential has a negative exponent here because e^u was moved to the left.

Step 2: Integrate and apply the initial condition Integrating both sides,

$$-e^{-u} = \frac{v^3}{3} + C.$$

At $x = 0,\ y = 1$, both u and v equal zero, so $-1 = C$. Consequently

$$-e^{-u} = \frac{v^3}{3} - 1, \qquad v^3 = 3(1 - e^{-u}).$$

Returning to the original variables and multiplying by y^3 gives

$$x^3 = 3y^3(1 - e^{-xy}).$$

Step 3: Check the sign and reject the other forms Differentiating $v^3 = 3(1 - e^{-u})$ yields $3v^2\,dv = 3e^{-u}\,du$, exactly the separated equation. All four displayed alternatives vanish at the initial point, so the initial condition alone cannot identify the answer. Options (B) and (C) have e^{xy}, leading to the wrong exponential factor on differentiation. Option (D) reverses the sign of $1 - e^{-xy}$ and therefore gives the negative of the required differential relation.

Step 4: Conclusion The solution satisfying the initial value is $\boxed{x^3 = 3y^3(1 - e^{-xy})}$. Hence, the correct option is (A).

Quick Tip to Solve: When both $y\,dx + x\,dy$ and $y\,dx - x\,dy$ appear, try xy and x/y together; verify signs by differentiation.

55. If the system of linear equations :

$$\begin{aligned} x + 3y + 7z &= 0 \\ -x + 4y + 7z &= 0 \\ (\sin 3\theta)x + (\cos 2\theta)y + 2z &= 0 \end{aligned}$$

has a non-trivial solution, then the number of values of θ lying in the interval $[0, \pi]$, is : [4 marks]

(A) two

(B) three

(C) more than three

(D) one

Correct Answer: (C) more than three

Solution:

Concept:

- A homogeneous system has a non-trivial solution precisely when its equations admit a nonzero vector; elimination can find the required parameter condition without expanding a determinant.
- Use $\cos 2\theta = 1 - 2\sin^2\theta$ and $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$, then count every angle in the specified closed interval.

Step 1: Eliminate two unknowns Adding the first two equations gives $7y + 14z = 0$, so $y = -2z$. Substitution into the first equation yields $x = -z$. If $z = 0$, all three unknowns vanish; therefore a non-trivial solution requires $z \neq 0$. The third equation consequently becomes

$$-\sin 3\theta - 2\cos 2\theta + 2 = 0.$$

Step 2: Reduce the condition to a polynomial Put $s = \sin\theta$. The condition is $\sin 3\theta = 4\sin^2\theta$. Thus

$$3s - 4s^3 = 4s^2 \quad \Longleftrightarrow \quad s(4s^2 + 4s - 3) = 0.$$

Factoring the quadratic gives $s(2s-1)(2s+3) = 0$. Its candidates are $s = 0, \frac{1}{2}, -\frac{3}{2}$. The last is impossible for a real angle, independently of the interval restriction.

Step 3: Count the admissible angles On $[0, \pi]$, $\sin\theta = 0$ gives both endpoints $0, \pi$, while $\sin\theta = \frac{1}{2}$ gives $\pi/6, 5\pi/6$. Each value satisfies the original parameter condition and permits, for example, the nonzero vector $(-1, -2, 1)$. There are four distinct values, so neither one, two nor three is the count.

Step 4: Conclusion The four values are $0, \pi/6, 5\pi/6, \pi$. The number is more than three. Hence, the correct option is (C).

Quick Tip to Solve: For a homogeneous system, first find the ratios of the unknowns from the parameter-free equations; remember both endpoints when counting sine roots on a closed interval.

56. For a positive integer n, if the mean of the binomial coefficients in the expansion of $(a+b)^{2n-3}$ is 16, then n is equal to : [4 marks]

(A) 5

(B) 7

(C) 9

(D) 4

Correct Answer: (A) 5

Solution:

Concept:

- For a nonnegative integer exponent m, the expansion of $(a+b)^m$ contains $m+1$ binomial coefficients, including both endpoint coefficients.
- The sum of those coefficients is obtained by putting $a = b = 1$, giving 2^m. Their arithmetic mean is therefore $2^m/(m+1)$.

Step 1: Express the mean correctly Here $m = 2n-3$. Since this is a binomial expansion with a nonnegative integral exponent and n is an integer, we require $n \geq 2$. There are $2n-2$, rather than $2n-3$, coefficients. The stated mean gives

$$\frac{2^{2n-3}}{2n-2} = 16.$$

Equivalently, $2^{2n-3} = 16(2n-2)$. This equation uses the sum of all coefficients, not just the middle coefficient.

Step 2: Test the listed integers For $n = 5$, the exponent is 7, so the mean is $2^7/8 = 128/8 = 16$. The other proposed values give

$$n = 4 : \frac{2^5}{6} = \frac{16}{3}, \qquad n = 7 : \frac{2^{11}}{12} = \frac{512}{3}, \qquad n = 9 : \frac{2^{15}}{16} = 2048.$$

None of these equals the specified mean. Thus options (B), (C) and (D) are excluded by direct substitution.

Step 3: Check uniqueness among admissible integers If $M_n = 2^{2n-3}/(2n-2)$, then $M_{n+1}/M_n = 4(n-1)/n$. For every integer $n \geq 2$, this ratio is greater than one. Hence the means increase strictly, and the verified value $n = 5$ is the only admissible integer satisfying the condition.

Step 4: Conclusion The required positive integer is $n = 5$. Hence, the correct option is (A).

Quick Tip to Solve: Count coefficients as exponent plus one, and use the substitution $a = b = 1$ for their sum before taking their mean.

57. The negation of $A \to (A \vee \sim B)$ is : [4 marks]

(A) a tautology

(B) equivalent to $(A \vee \sim B) \to A$

(C) equivalent to $A \to (A \wedge \sim B)$

(D) a fallacy

Correct Answer: (D) a fallacy

Solution:

Concept:

- An implication $P \to Q$ is equivalent to $\sim P \vee Q$. Its negation is $P \wedge \sim Q$, not the converse implication.
- A tautology is true for every assignment of truth values; a fallacy, also called a contradiction, is false for every assignment.

Step 1: Simplify the original statement Let $F = A \to (A \vee \sim B)$. Replacing the implication gives

$$F \equiv \sim A \vee (A \vee \sim B) \equiv (\sim A \vee A) \vee \sim B \equiv \text{True}.$$

If A is false, the implication is true automatically; if A is true, its consequent contains the true term A. This explains directly why F is always true, whatever the value of B.

Step 2: Negate using the logical laws Apply the negation rule for an implication and then De Morgan's law:

$$\sim F \equiv A \wedge \sim (A \vee \sim B) \equiv A \wedge (\sim A \wedge B).$$

The simultaneous requirements A and $\sim A$ cannot hold. Therefore $\sim F$ is always false; there is no assignment that makes it true.

Step 3: Eliminate the alternative descriptions Option (A) describes the original statement, not its negation. To reject (B), take A true: $(A \vee \sim B) \to A$ is then true, whereas $\sim F$ is false. To reject (C), take A false: $A \to (A \wedge \sim B)$ is true, again unlike $\sim F$. Neither expression is equivalent to a contradiction.

Step 4: Conclusion The negation is a fallacy because it is false for all truth assignments. Hence, the correct option is (D).

Quick Tip to Solve: Before building a full truth table, check whether the consequent of an implication already contains its antecedent as a disjunct.

58. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. If the eccentricity of the hyperbola is 2, then the equation of the tangent to this hyperbola passing through the point $(4, 6)$ is : [4 marks]

(A) $3x - 2y = 0$

(B) $2x - 3y + 10 = 0$

(C) $x - 2y + 8 = 0$

(D) $2x - y - 2 = 0$

Correct Answer: (D) $2x - y - 2 = 0$

Solution:

Concept:

- For the ellipse $x^2/a_e^2 + y^2/b_e^2 = 1$, with $a_e > b_e$, the focal distance satisfies $c^2 = a_e^2 - b_e^2$.
- For a horizontal hyperbola $x^2/a^2 - y^2/b^2 = 1$, $e = c/a$, $c^2 = a^2 + b^2$, and its tangent at (x_1, y_1) is $xx_1/a^2 - yy_1/b^2 = 1$.

Step 1: Find the common focal distance The given ellipse has $a_e^2 = 25$ and $b_e^2 = 9$. Hence $c^2 = 25 - 9 = 16$, so its foci are $(4, 0)$ and $(-4, 0)$. The confocal hyperbola has the same horizontal transverse axis and the same focal distance $c = 4$. Its eccentricity is 2, so

$$a = \frac{c}{e} = 2, \qquad b^2 = c^2 - a^2 = 16 - 4 = 12.$$

Step 2: Check the point and form the tangent The hyperbola is $x^2/4 - y^2/12 = 1$. At $(4, 6)$, the left-hand side is $16/4 - 36/12 = 4 - 3 = 1$; thus this point lies on the hyperbola itself. The tangent through this point is therefore its tangent at that point:

$$\frac{4x}{4} - \frac{6y}{12} = 1 \quad \Longleftrightarrow \quad 2x - y - 2 = 0.$$

Step 3: Check against the proposed lines Implicit differentiation gives $x/2 - yy'/6 = 0$, hence the tangent slope at $(4, 6)$ is $y' = 3x/y = 2$. All the listed lines pass through the supplied point, but their slopes are respectively $3/2, 2/3, 1/2, 2$. Only (D) has the required slope. Passing through the point alone would therefore not distinguish the answer.

Step 4: Conclusion The tangent is $2x - y - 2 = 0$. Hence, the correct option is (D).

Quick Tip to Solve: For a tangent stated to pass through a point, first test whether the point lies on the conic; if it does, the point-form tangent is immediate.

59. From a point A with position vector $p(\hat{i}+\hat{j}+\hat{k})$, AB and AC are drawn perpendicular to the lines $\vec{r}=\hat{k}+\lambda(\hat{i}+\hat{j})$ and $\vec{r}=-\hat{k}+\mu(\hat{i}-\hat{j})$, respectively. A value of p is equal to : [4 marks]

(A) -1

(B) $\sqrt{2}$

(C) 2

(D) -2

Correct Answer: No unique option: the perpendicular feet exist for arbitrary p, with several nondegenerate listed choices.

Solution:

Concept:

- Every point has a unique perpendicular foot on any specified line with a nonzero direction vector; the existence of such feet alone imposes no scalar constraint on the point.
- If A has position vector $\mathbf{a}$ and a line is $\mathbf{r}=\mathbf{r}_0+t\mathbf{d}$, its perpendicular foot satisfies $(\mathbf{r}_0+t\mathbf{d}-\mathbf{a})\cdot\mathbf{d}=0$.

Step 1: Find the foot on the first line Here $A=(p,p,p)$. A point on the first line is $B=(\lambda,\lambda,1)$, with direction $(1,1,0)$. Perpendicularity of AB to that line gives

$$(\lambda-p,\lambda-p,1-p)\cdot(1,1,0)=2\lambda-2p=0.$$

Thus $\lambda=p$ and $B=(p,p,1)$. This construction works for every real p; the equation determines the foot parameter, not p.

Step 2: Find the foot on the second line A point on the second line is $C=(\mu,-\mu,-1)$, whose line direction is $(1,-1,0)$. Consequently

$$(\mu-p,-\mu-p,-1-p)\cdot(1,-1,0)=2\mu=0.$$

We obtain $\mu=0$ and $C=(0,0,-1)$, again for arbitrary p. No additional angle, length or relation between AB and AC is specified, so none may be imposed.

Step 3: Assess whether a unique option follows The nondegenerate choices $p=\sqrt{2},2,-2$ all produce the required perpendiculars, so at least three listed options satisfy the conditions. At $p=-1$, the point lies on the second line and AC has zero length; whether that degenerate case is admitted does not restore uniqueness. Hence the given information cannot single out any one option.

Step 4: Conclusion The value of p is not uniquely determined; in particular, (B), (C) and (D) all satisfy the nondegenerate conditions. The official answer key dropped this question and awarded bonus marks to all candidates.

Quick Tip to Solve: When perpendiculars are merely drawn from a point to two lines, compute their feet before assuming that the point itself is constrained.

60. If for a matrix A, $|A| = 6$ and $\operatorname{adj} A = \begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & 1 \\ -1 & k & 0 \end{bmatrix}$, then k is equal to : [4 marks]

(A) 0

(B) 1

(C) 2

(D) -1

Correct Answer: (C) 2

Solution:

Concept:

- For an $n \times n$ matrix, $\det(\operatorname{adj} A) = (\det A)^{n-1}$. In particular, a 3×3 matrix has $\det(\operatorname{adj} A) = (\det A)^2$.
- A determinant can be expanded along any row using alternating cofactor signs. Its entries are those of the adjugate supplied in the question, not those of the original matrix.

Step 1: Apply the determinant identity The displayed adjugate is a 3×3 matrix. Since $\det A = 6$, its determinant must be

$$\det(\operatorname{adj} A) = 6^{3-1} = 36.$$

This also follows by taking determinants in $A \operatorname{adj} A = 6I_3$: the left side is $6 \det(\operatorname{adj} A)$, and the right side is 6^3. Dividing by 6 gives 36, so there is no need to reconstruct A.

Step 2: Expand the given adjugate Expansion along the first row gives

$$\begin{aligned} \det(\operatorname{adj} A) &= 1(1 \cdot 0 - 1 \cdot k) - (-2)(4 \cdot 0 - 1 \cdot (-1)) \\ &\quad + 4(4k - 1 \cdot (-1)) \\ &= -k + 2 + 4(4k + 1) = 15k + 6. \end{aligned}$$

The middle contribution is positive because its cofactor sign is negative and the entry is itself -2. Retaining both signs is essential.

Step 3: Solve and check the alternatives Equate this determinant to 36: $15k + 6 = 36$, so $15k = 30$ and $k = 2$. Substituting $k = 0, 1, -1$ instead gives determinants $6, 21, -9$, respectively. None equals 36, so (A), (B) and (D) fail the required identity.

Step 4: Conclusion The required value is $k = 2$. Hence, the correct option is (C).

Quick Tip to Solve: When an adjugate is supplied with a determinant, use its determinant identity first; this avoids finding the full original matrix.

61. A box contains 5 black and 4 white balls. A ball is drawn at random and its colour is noted. The ball is then put back in the box along with two additional balls of its opposite colour. If a ball is drawn again from the box, then the probability that the ball drawn now is black, is : [4 marks]

(A) $\frac{5}{11}$
(B) $\frac{53}{99}$
(C) $\frac{48}{99}$
(D) $\frac{7}{11}$

Correct Answer: (B) $\frac{53}{99}$

Solution:

Concept:

- The law of total probability combines conditional probabilities over mutually exclusive and exhaustive first-draw outcomes. If B_1 and W_1 denote the first colour, then $P(B_2) = P(B_1)P(B_2 \mid B_1) + P(W_1)P(B_2 \mid W_1)$.
- The first ball is replaced before the two opposite-coloured balls are added. Thus the second draw always comes from eleven balls, but its black-ball count depends on the first colour.

Step 1: Separate the first-draw outcomes Initially there are five black and four white balls, so the probabilities of the two possible first colours are $P(B_1) = 5/9$ and $P(W_1) = 4/9$. These cases cover every first draw and cannot happen together. Replacement restores the initial ball before the additional balls are introduced.

Step 2: Find the conditional black probabilities After a black first draw, two white balls are added: there are five black and six white balls. After a white first draw, two black balls are added: there are seven black and four white balls. Therefore

$$P(B_2 \mid B_1) = \frac{5}{11}, \qquad P(B_2 \mid W_1) = \frac{7}{11}.$$

Each conditional denominator is eleven, rather than ten, because the drawn ball has been put back.

Step 3: Weight and add the two cases Using the probabilities of the first colours as weights gives

$$P(B_2) = \frac{5}{9}\frac{5}{11} + \frac{4}{9}\frac{7}{11} = \frac{25+28}{99} = \frac{53}{99}.$$

Options (A) and (D) give only the conditional probabilities for one first-draw outcome. Option (C), $48/99$, is neither the weighted sum nor its numerator obtained from the two cases.

Step 4: Conclusion The required probability is $\boxed{53/99}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the new black and white counts separately for each first colour; then weight those conditional probabilities by the original first-draw probabilities.

62. If $\sum_{i=1}^{n}\left(\frac{{}^{n}C_{i-1}}{{}^{n}C_{i}+{}^{n}C_{i-1}}\right)^{3}=\frac{36}{13}$, then n is equal to : [4 marks]

(A) 11

(B) 12

(C) 13

(D) 10

Correct Answer: (B) 12

Solution:

Concept:

- Pascal's identity is $\binom{n}{i}+\binom{n}{i-1}=\binom{n+1}{i}$. Factorials then give $\binom{n}{i-1}/\binom{n+1}{i}=i/(n+1)$.
- The sum of the first n positive integer cubes is $\sum_{i=1}^{n} i^3=[n(n+1)/2]^2$. Here n is a positive integer because it is the upper binomial parameter and summation limit.

Step 1: Simplify the coefficient ratio Apply Pascal's identity to the denominator, keeping the lower index i unchanged. Then cancel factorials:

$$\frac{\binom{n}{i-1}}{\binom{n}{i}+\binom{n}{i-1}}=\frac{\binom{n}{i-1}}{\binom{n+1}{i}}=\frac{n!\,i!\,(n+1-i)!}{(i-1)!\,(n-i+1)!\,(n+1)!}=\frac{i}{n+1}.$$

The cancellation is valid for all indices $1\le i\le n$.

Step 2: Evaluate the sum The denominator $(n+1)^3$ is independent of the index, so take it outside the sum:

$$\sum_{i=1}^{n}\left(\frac{i}{n+1}\right)^3=\frac{1}{(n+1)^3}\left[\frac{n(n+1)}{2}\right]^2=\frac{n^2}{4(n+1)}.$$

Equating this to the given value yields $13n^2=144(n+1)$.

Step 3: Select the admissible integer Rearrange and factor the resulting quadratic:

$$13n^2-144n-144=(n-12)(13n+12)=0.$$

Thus $n=12$ or $n=-12/13$. The latter is inadmissible. For $n=12$, the sum is $144/(4\cdot 13)=36/13$, as required. The other offered integers 11, 13 and 10 are not roots of the factored equation, so options (A), (C) and (D) fail.

Step 4: Conclusion The positive integral value is $\boxed{n=12}$. Hence, the correct option is (B).

Quick Tip to Solve: Use Pascal's identity before expanding factorials: the adjacent-coefficient ratio reduces to a linear expression in the summation index.

63. A code word of length 4 consists of two distinct consonants in the English alphabet followed by two digits from 1 to 9, with repetition allowed in digits. If the number of code words so formed ending with an even digit is $432k$, then k is equal to : [4 marks]

(A) 5

(B) 49

(C) 35

(D) 7

Correct Answer: (C) 35

Solution:

Concept:

- For a code with ordered positions, multiply the number of choices available at each position. Distinct consonants mean that the second consonant cannot repeat the first.
- The English alphabet has 26 letters, including five vowels, hence 21 consonants. Among the allowed digits 1 through 9, precisely four, namely 2, 4, 6 and 8, are even.

Step 1: Count the consonant positions The first position can contain any of the 21 consonants. Having chosen it, the second can contain any of the remaining 20 consonants. Order matters: interchanging the two consonants changes the code word. Thus the consonant prefix has

$$21 \times 20 = 420$$

possibilities. We do not divide by $2!$, because these are positions in a word, not an unordered selection.

Step 2: Count the permitted digit endings The third position can contain any digit from 1 to 9, giving nine choices. The final digit must be even, leaving the four choices 2, 4, 6 and 8. Digit repetition is allowed, so choosing the third digit does not reduce the choices for the fourth, even when the third digit is even. The digit suffix therefore has $9 \times 4 = 36$ possibilities.

Step 3: Compare the total with the given form Multiply the independent counts and equate to the specified expression:

$$N = 21 \times 20 \times 9 \times 4 = 15120 = 432k,$$

$$k = \frac{15120}{432} = 35.$$

As a direct check, $432 \times 35 = 15120$. Options (A), (B) and (D) would give respectively 2160, 21168 and 3024 code words; none agrees with the position-by-position count.

Step 4: Conclusion The required constant is $\boxed{k = 35}$. Hence, the correct option is (C).

Quick Tip to Solve: For an even-ending code, fill the restricted final digit separately. Treat distinct letters without replacement but the repeatable digits with replacement.

64. $\lim_{x\to 1}((1-x)+[x-1]+|1-x|)$**, where** $[x]$ **denotes the greatest integer less than or equal to** x **:** [4 marks]

(A) is equal to 0

(B) is equal to 1

(C) does not exist

(D) is equal to -1

Correct Answer: (C) does not exist

Solution:

Concept:

- A two-sided limit at a point exists only when its left-hand and right-hand limits both exist and are equal. The actual value at the point does not decide the limit.
- The floor function satisfies $[t]=-1$ when $-1<t<0$, but $[t]=0$ when $0\le t<1$. The absolute value also changes its expression as x crosses 1.

Step 1: Approach 1 from the left For $0<x<1$, we have $-1<x-1<0$, and therefore $[x-1]=-1$. Also $|1-x|=1-x$. On this entire left neighbourhood the expression is

$$f(x)=(1-x)-1+(1-x)=1-2x.$$

Consequently,

$$\lim_{x\to 1^-} f(x)=1-2=-1.$$

The floor of a small negative number is negative one, not zero; replacing it by zero would erase the jump.

Step 2: Approach 1 from the right For $1<x<2$, the floor is $[x-1]=0$, while $|1-x|=x-1$. Hence throughout the right neighbourhood,

$$f(x)=(1-x)+0+(x-1)=0,$$

so $\lim_{x\to 1^+} f(x)=0$. In particular the cancellation on the right is exact, rather than merely an approximation close to the limiting point.

Step 3: Compare the two one-sided limits The left limit is -1 and the right limit is 0. Since these are unequal, the two-sided limit does not exist. Option (A) reports only the right limit, and option (D) reports only the left limit. Option (B) gives 1, which is neither of them. Although the expression at $x=1$ itself is zero, that isolated value cannot make the unequal one-sided limits agree.

Step 4: Conclusion The stated two-sided limit does not exist. Hence, the correct option is (C).

Quick Tip to Solve: Whenever the argument of a floor function crosses an integer, evaluate both sides explicitly; direct substitution can hide a jump.

65. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ **is :** [4 marks]

(A) $\frac{20}{19}$

(B) $\frac{19}{21}$

(C) $\frac{21}{19}$

(D) $\frac{19}{20}$

Correct Answer: (C) $\frac{21}{19}$

Solution:

Concept:

- The inner arithmetic sum is $\sum_{p=1}^{n}2p=n(n+1)$. Thus the inverse-cotangent argument becomes n^2+n+1.
- For positive integers n, the difference $\tan^{-1}(1/n)-\tan^{-1}(1/(n+1))$ is acute and has tangent $1/(n^2+n+1)$. It therefore equals the principal value $\cot^{-1}(n^2+n+1)$.

Step 1: Find a telescoping representation Let $a=\tan^{-1}(1/n)$ and $b=\tan^{-1}(1/(n+1))$. The tangent subtraction formula gives

$$\tan(a-b)=\frac{1/n-1/(n+1)}{1+1/[n(n+1)]}=\frac{1}{n^2+n+1}.$$

Since $0<a-b<\pi/2$, no multiple of π is needed. Therefore the inverse cotangent of n^2+n+1 is exactly $a-b$.

Step 2: Sum and cancel the intermediate angles Writing S for the angle inside the outer cotangent, we obtain

$$S=\sum_{n=1}^{19}\left[\tan^{-1}\frac{1}{n}-\tan^{-1}\frac{1}{n+1}\right]=\tan^{-1}1-\tan^{-1}\frac{1}{20}.$$

Every term from $\tan^{-1}(1/2)$ through $\tan^{-1}(1/19)$ occurs once with each sign, leaving just the first and last endpoints.

Step 3: Evaluate the outer cotangent Apply the subtraction formula once more:

$$\tan S=\frac{1-1/20}{1+1/20}=\frac{19}{21},\qquad \cot S=\frac{21}{19}.$$

Option (B) is the tangent of the sum, not the requested cotangent. Options (A) and (D) miss the addition of the product term in the subtraction-formula denominator; they do not equal the computed ratio.

Step 4: Conclusion The required value is $\boxed{21/19}$. Hence, the correct option is (C).

Quick Tip to Solve: An inverse cotangent with argument n^2+n+1 suggests the difference of inverse tangents of $1/n$ and $1/(n+1)$; check the acute branch before telescoping.

66. The abscissa of a point, tangent at which to the curve $y = e^x \sin x, x \in [0, \pi]$, has maximum slope, is : [4 marks]

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) π

(D) 0

Correct Answer: (B) $\frac{\pi}{2}$

Solution:

Concept:

- The slope of a tangent to $y = f(x)$ is $f'(x)$. To maximize the slope, optimize this derivative as a function of x, rather than optimizing the height $f(x)$.
- On a closed interval, an interior critical point of a differentiable function is a maximum when the function increases before it and decreases after it. Endpoint comparisons provide an additional check.

Step 1: Express the tangent slope Use the product rule on $y = e^x \sin x$. If $m(x)$ denotes its tangent slope, then

$$m(x) = \frac{dy}{dx} = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x).$$

This function, rather than y, must be maximized on the specified interval $[0, \pi]$.

Step 2: Locate and classify the critical point Differentiate the slope function, again using the product rule:

$$m'(x) = e^x(\sin x + \cos x) + e^x(\cos x - \sin x) = 2e^x \cos x.$$

Since $e^x > 0$, the sign depends entirely on $\cos x$. It is positive for $0 < x < \pi/2$, zero at $\pi/2$, and negative for $\pi/2 < x < \pi$. Thus the slope increases up to $\pi/2$ and decreases thereafter.

Step 3: Check the interval and the alternatives The maximum is therefore the unique interior point $x = \pi/2$. At the endpoints and the critical point,

$$m(0) = 1, \qquad m(\pi/2) = e^{\pi/2}, \qquad m(\pi) = -e^{\pi}.$$

These confirm the maximum slope is achieved at the middle value. Option (A), $\pi/4$, lies in the interval where the slope is still increasing. Options (C) and (D) are endpoints with slopes strictly smaller than $e^{\pi/2}$.

Step 4: Conclusion The abscissa of the required point is $\boxed{x = \pi/2}$. Hence, the correct option is (B).

Quick Tip to Solve: To maximize a tangent slope, first differentiate the curve to obtain the slope function, then differentiate that function to locate its maximum.

67. The integral $\int_0^2 [x^2]\,dx$ ($[t]$ denotes the greatest integer less than or equal to t) is equal to :

[4 marks]

(A) $5 - 2\sqrt{3}$

(B) $5 - \sqrt{2} - \sqrt{3}$

(C) $6 - \sqrt{2} - \sqrt{3}$

(D) $3 - \sqrt{2}$

Correct Answer: (B) $5 - \sqrt{2} - \sqrt{3}$

Solution:

Concept:

- The greatest-integer function $\lfloor t \rfloor$ equals the integer k whenever $k \le t < k+1$. Thus $\lfloor x^2 \rfloor$ is constant between consecutive square roots of integers.
- Integrate a piecewise constant function by adding the areas of its rectangles. Values at individual endpoints do not affect a definite integral.

Step 1: Locate every jump inside the interval For $0 \le x \le 2$, the quantity x^2 runs from 0 to 4. It reaches the integers 1, 2 and 3 at $x = 1, \sqrt{2}, \sqrt{3}$, respectively. Therefore the four relevant intervals are $[0, 1)$, $[1, \sqrt{2})$, $[\sqrt{2}, \sqrt{3})$ and $[\sqrt{3}, 2)$. The corresponding heights of the integrand are 0, 1, 2 and 3. Although its value at $x = 2$ is 4, that single point contributes no area.

Step 2: Add the four rectangular areas Using these intervals rather than integrating x^2 itself gives

$$\begin{aligned} I &= 0(1-0) + 1(\sqrt{2}-1) + 2(\sqrt{3}-\sqrt{2}) + 3(2-\sqrt{3}) \\ &= \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3} \\ &= 5 - \sqrt{2} - \sqrt{3}. \end{aligned}$$

Both radical terms have negative coefficients after collection. Option (A) incorrectly removes the $\sqrt{2}$ contribution; option (C) has constant 6 instead of 5; option (D) omits the change at $\sqrt{3}$. None equals the sum of the four areas.

Step 3: Conclusion The integral equals $\boxed{5 - \sqrt{2} - \sqrt{3}}$. Hence, the correct option is (B).

Quick Tip to Solve: For $\lfloor x^2 \rfloor$, split at square roots of integers, not at equally spaced integer values of x. Ignore isolated endpoint values when adding areas.

68. Two vertices of a triangle are $(3,-2)$ and $(-2,3)$, and its orthocentre is $(-6,1)$. Then the third vertex of this triangle can NOT lie on the line : [4 marks]

(A) $4x+y=2$

(B) $5x+y=2$

(C) $3x+y=3$

(D) $6x+y=0$

Correct Answer: (B) $5x+y=2$

Solution:

Concept:

- The orthocentre is the intersection of the three altitudes. The altitude through each known vertex is perpendicular to the opposite side.
- For nonvertical perpendicular lines, the product of their slopes is -1. Two side equations determine the third vertex uniquely.

Step 1: Use the altitude through the first vertex Let $A=(3,-2)$, $B=(-2,3)$, $H=(-6,1)$, and let the third vertex be C. The altitude AH has slope

$$m_{AH}=\frac{1-(-2)}{-6-3}=-\frac{1}{3}.$$

Hence BC, perpendicular to AH, has slope 3. As it passes through B, its equation is

$$y-3=3(x+2), \qquad y=3x+9.$$

Step 2: Use the second altitude to determine the vertex The altitude BH has slope

$$m_{BH}=\frac{1-3}{-6-(-2)}=\frac{1}{2}.$$

Therefore AC has slope -2. Its equation through A is

$$y+2=-2(x-3), \qquad y=-2x+4.$$

At their intersection, $3x+9=-2x+4$, so $x=-1$ and $y=6$. Thus $C=(-1,6)$. This is noncollinear with A, B, whose joining line is $x+y=1$. Also CH has slope 1, perpendicular to the slope -1 of AB, confirming the orthocentre condition.

Step 3: Check all four proposed lines Substitute $C=(-1,6)$ in the left sides:

$$4x+y=2, \quad 5x+y=1,$$
$$3x+y=3, \quad 6x+y=0.$$

Options (A), (C) and (D) contain this vertex. In option (B), the required right side is 2 whereas the actual value is 1, so that line cannot contain the third vertex.

Step 4: Conclusion The third vertex is $(-1,6)$, which does not lie on $\boxed{5x+y=2}$. Hence, the correct option is (B).

Quick Tip to Solve: Join each given vertex to the orthocentre first. The perpendicular opposite sides then locate the unknown vertex; check every proposed line by substitution.

69. Let $p(x)$ be a real polynomial of degree 4 having extreme values at $x = 1$ and $x = 2$. If $\lim_{x\to 0} \frac{p(x)}{x^2} = 1$, then $p(4)$ is equal to : [4 marks]

(A) 16

(B) 32

(C) 64

(D) 8

Correct Answer: (A) 16

Solution:

Concept:

- A finite limit $p(x)/x^2 \to 1$ forces the constant and linear terms of the polynomial to vanish and its quadratic coefficient to equal 1.
- At an interior local maximum or minimum of a differentiable polynomial, $p\prime(x) = 0$. Apply this condition at both specified points.

Step 1: Write the polynomial using the limit A general quartic has constant, linear, quadratic, cubic and quartic terms. A nonzero constant or linear coefficient would make $p(x)/x^2$ unbounded near zero. The stated limit is therefore equivalent to writing

$$p(x) = ax^4 + bx^3 + x^2, \qquad a \neq 0.$$

Indeed $p(x)/x^2 = ax^2 + bx + 1$, whose limit is 1. This uses the limit before the stationary-point information, leaving only two unknown coefficients.

Step 2: Impose the two stationary-point conditions Differentiate and use the extreme points:

$$p\prime(x) = 4ax^3 + 3bx^2 + 2x.$$

$$p\prime(1) = 0 \;\Rightarrow\; 4a + 3b + 2 = 0,$$
$$p\prime(2) = 0 \;\Rightarrow\; 32a + 12b + 4 = 0.$$

Divide the second equation by 2 and subtract the first to obtain $12a + 3b = 0$, hence $b = -4a$. Substitution into the first gives $-8a+2 = 0$, so $a = 1/4$, $b = -1$. The derivative becomes $x(x-1)(x-2)$; its sign changes at both 1 and 2, so these points really are extrema.

Step 3: Evaluate the polynomial at four The polynomial is $p(x) = x^4/4 - x^3 + x^2$. Consequently,

$$p(4) = \frac{4^4}{4} - 4^3 + 4^2 = 64 - 64 + 16 = 16.$$

Thus 32, 64 and 8, offered in (B), (C) and (D), are inconsistent with the uniquely determined polynomial; 64 in particular is only the quartic contribution before the other terms are added.

Step 4: Conclusion $\boxed{p(4) = 16}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert a polynomial-over-x^2 limit into coefficient conditions first; the two derivative zeros then give a small linear system.

70. The plane through the intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and parallel to y-axis, also passes through the point : [4 marks]

(A) $(3,0,1)$

(B) $(-3,0,1)$

(C) $(3,0,-1)$

(D) $(-3,0,-1)$

Correct Answer: (A) $(3,0,1)$

Solution:

Concept:

- Every plane of the form $F+\lambda G=0$, where $F=0$ and $G=0$ are the given planes, contains their intersection line.
- A plane parallel to the y-axis has normal vector perpendicular to $(0,1,0)$, so its coefficient of y must be zero.

Step 1: Form the family and impose the direction condition Write the given planes as $F=x+y+z-1=0$ and $G=2x+3y-z+4=0$. A member through their intersection is

$$(x+y+z-1)+\lambda(2x+3y-z+4)=0.$$

Its normal is $(1+2\lambda, 1+3\lambda, 1-\lambda)$. Parallelism to the y-axis means its dot product with $(0,1,0)$ vanishes. Hence $1+3\lambda=0$, giving $\lambda=-1/3$. A zero y coefficient is essential: setting $y=0$ alone would merely find a trace, not enforce parallelism.

Step 2: Simplify the required plane Multiply the resulting equation by 3:

$$\begin{aligned} 3(x+y+z-1)-(2x+3y-z+4)&=0,\\ x+4z-7&=0. \end{aligned}$$

There is no restriction on y. Changing y while keeping x, z fixed therefore stays within the plane, which verifies the required direction. The equation is a linear combination of the originals, so every point common to the original two planes also lies in it.

Step 3: Test each proposed point The condition is $x+4z=7$. At the four points in their printed order, the left side is

$$3+4=7, \quad -3+4=1, \quad 3-4=-1, \quad -3-4=-7.$$

Only $(3,0,1)$ satisfies the plane equation. Options (B), (C) and (D) fail regardless of their common y-coordinate zero, since y does not occur in this equation.

Step 4: Conclusion The required plane is $x+4z=7$, and it passes through $\boxed{(3,0,1)}$. Hence, the correct option is (A).

Quick Tip to Solve: In a family of planes, cancel the coefficient of the coordinate axis to which the new plane must be parallel, then test the points in the simplified equation.

71. A line passing through the point $P(1,2)$ meets the line $x+y=7$ at the distance of 3 units from P. Then the slope of this line satisfies the equation : [4 marks]

(A) $7x^2-18x+7=0$

(B) $16x^2-39x+16=0$

(C) $7x^2-6x-7=0$

(D) $8x^2-9x+1=0$

Correct Answer: (A) $7x^2-18x+7=0$

Solution:

Concept:

- A line of slope m through $P(1,2)$ can be parameterised as $(1+t,2+mt)$. The parameter t is its horizontal displacement.
- The distance formula gives $PQ^2=t^2(1+m^2)$. Combine it with the equation of the line containing Q to eliminate t.

Step 1: Express the intersection using the slope Let Q be the intersection with $x+y=7$. For a finite slope m, write $Q=(1+t,2+mt)$. Substitution gives

$$1+t+2+mt=7, \qquad t(1+m)=4.$$

Thus $m\neq -1$ and $t=4/(1+m)$. The excluded slope -1 would give a parallel line $x+y=3$. A vertical line through P meets the target at $(1,6)$, distance 4 rather than 3, so there is no omitted vertical solution.

Step 2: Apply the distance requirement and eliminate the parameter The horizontal and vertical displacements are t and mt. Since $PQ=3$,

$$t^2(1+m^2)=9, \qquad \frac{16(1+m^2)}{(1+m)^2}=9.$$

Multiply by the nonzero denominator and collect terms:

$$16+16m^2=9+18m+9m^2, \qquad 7m^2-18m+7=0.$$

The discriminant is $128>0$, so two real slopes satisfy the distance condition. Neither is -1; the division above has not introduced an invalid intersection.

Step 3: Match the equation with the options The question uses x as the variable representing the slope in its alternatives. Replacing m by that variable gives exactly option (A). The other proposed quadratics have different ratios of coefficients, so they are not scalar multiples of the derived equation and cannot describe this same pair of slopes.

Step 4: Conclusion The slope satisfies $\boxed{7m^2-18m+7=0}$, written as $7x^2-18x+7=0$ in the options. Hence, the correct option is (A).

Quick Tip to Solve: Parameterise the intersection as the given point plus $t(1,m)$; a linear constraint determines t, and the distance formula produces the slope equation.

72. Let a, b, c, d and e be distinct positive numbers. If a, b, c and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ both are in A.P. and b, c, d are in G.P. then : [4 marks]

(A) a, b, e are in G.P.

(B) a, b, e are in A.P.

(C) a, c, e are in A.P.

(D) a, c, e are in G.P.

Correct Answer: (D) a, c, e are in G.P.

Solution:

Concept:

- Three numbers u, v, w are in arithmetic progression when $2v = u + w$, and positive numbers are in geometric progression when $v^2 = uw$.
- Use a positive ratio to express all the numbers in terms of the common central quantity c. Positivity permits division and excludes spurious zero solutions.

Step 1: Parameterise the first arithmetic and the geometric progression Put $t = b/c > 0$, so $b = ct$. Since a, b, c are in A.P.,

$$a = 2b - c = c(2t - 1).$$

The G.P. condition on b, c, d gives $c^2 = bd$, hence

$$d = \frac{c^2}{b} = \frac{c}{t}.$$

Positivity of a implies $2t - 1 > 0$. Also $t \neq 1$, since the five numbers are distinct and $b \neq c$. These restrictions will be useful when rejecting the other possibilities.

Step 2: Use the arithmetic progression of reciprocals The second A.P. gives

$$\frac{2}{d} = \frac{1}{c} + \frac{1}{e}.$$

Insert $d = c/t$:

$$\frac{2t}{c} = \frac{1}{c} + \frac{1}{e}, \qquad e = \frac{c}{2t - 1}.$$

Consequently,

$$ae = c(2t - 1)\frac{c}{2t - 1} = c^2.$$

Thus the middle number c is the geometric mean of a and e, which is exactly the G.P. test for a, c, e.

Step 3: Exclude the other proposed progressions For option (A), a, b, e in G.P. would require $b^2 = ae = c^2$; positivity then gives $b = c$, forbidden by distinctness. For option (B), $2b = a + e$ would give $e = 2b - a = c$, also forbidden. For option (C), put $u = 2t - 1 > 0$. The condition $a + e = 2c$ becomes $u + 1/u = 2$, or $(u - 1)^2 = 0$. Then $t = 1$, again contradicting distinctness. Only option (D) is consistent with the given conditions.

Step 4: Conclusion Since $ae = c^2$ and the numbers are positive, $\boxed{a, c, e \text{ are in G.P.}}$. Hence, the correct option is (D).

Quick Tip to Solve: When one A.P. involves reciprocals, clear its denominators or introduce a ratio; aim for a product identity such as $ae = c^2$.

73. If the line $x = a$ bisects the area under the curve $y = \frac{1}{x^2}, 1 \le x \le 9$, then '$a$' is equal to : [4 marks]

(A) $\frac{9}{5}$

(B) $\frac{5}{9}$

(C) $\frac{9}{4}$

(D) $\frac{4}{9}$

Correct Answer: (A) $\frac{9}{5}$

Solution:

Concept:

- The area beneath a nonnegative curve on an interval is its definite integral. Here $y = x^{-2} > 0$ throughout $[1, 9]$, so no absolute-value adjustment is required.
- A vertical bisector $x = a$ divides this total area into equal left and right areas. Consequently, the integral from 1 to a must equal half the integral from 1 to 9.

Step 1: Calculate the entire area The antiderivative of x^{-2} is $-x^{-1}$. Evaluating it at the two endpoints gives

$$A = \int_1^9 \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^9 = 1 - \frac{1}{9} = \frac{8}{9}.$$

The area on each side of the required vertical line must therefore be $A/2 = 4/9$. We halve the area, rather than the horizontal length, because the height of the curve varies with x.

Step 2: Locate the line that encloses half the area The left-hand area is

$$\int_1^a \frac{1}{x^2}\,dx = 1 - \frac{1}{a}.$$

Equating this with $4/9$ yields

$$1 - \frac{1}{a} = \frac{4}{9}, \qquad \frac{1}{a} = \frac{5}{9}, \qquad a = \frac{9}{5}.$$

This lies between the stated endpoints. The remaining area is $1/a - 1/9 = 5/9 - 1/9 = 4/9$, independently checking the bisection.

Step 3: Compare the proposed values Option (A) gives the value just obtained. Options (B) and (D) lie below 1, so they cannot bisect the region inside the given interval. Option (C) gives left-hand area $1 - 4/9 = 5/9$, exceeding the required $4/9$, so it also fails.

Step 4: Conclusion The area-bisecting line is $\boxed{x = 9/5}$, so $a = 9/5$. Hence, the correct option is (A).

Quick Tip to Solve: For a vertical area bisector, equate the cumulative integral to half the total; the midpoint of the horizontal interval usually does not work.

74. Let $S = \{z \in \mathbf{C} : z\,(iz_1 - 1) = z_1 + 1, |z_1| < 1\}$. Then, for all $z \in S$, which one of the following is always true ? [4 marks]

(A) $\operatorname{Re} z + \operatorname{Im} z < 0$

(B) $\operatorname{Re} z < 0$

(C) $\operatorname{Re} z - \operatorname{Im} z > -1$

(D) $\operatorname{Re} z - \operatorname{Im} z < 0$

Correct Answer: (D) $\operatorname{Re} z - \operatorname{Im} z < 0$

Solution:

Concept:

- For complex numbers u, v with $v \neq 0$, the condition $|u/v| < 1$ is equivalent to $|u| < |v|$. Squaring preserves this inequality because both moduli are nonnegative.
- Write $z = x + iy$, where $x = \operatorname{Re} z$ and $y = \operatorname{Im} z$. Then $|z + 1|^2 = (x + 1)^2 + y^2$, while $|iz - 1|^2 = x^2 + (y + 1)^2$.

Step 1: Express the parameter in terms of z Rearrange the defining equation without replacing the parameter by its conjugate:

$$izz_1 - z = z_1 + 1, \qquad z_1(iz - 1) = z + 1.$$

The denominator $iz - 1$ cannot vanish for a solution: it vanishes at $z = -i$, but the equation would then require $0 = 1 - i$. Thus every admissible z satisfies

$$z_1 = \frac{z + 1}{iz - 1}.$$

Step 2: Convert the unit-disc condition to Cartesian form Using $|z_1| < 1$, we obtain

$$|z + 1|^2 < |iz - 1|^2.$$

Substitute $z = x + iy$ and cancel the identical quadratic terms:

$$(x + 1)^2 + y^2 < x^2 + (y + 1)^2, \qquad 2x < 2y.$$

Therefore $x - y < 0$, a strict inequality because the parameter lies inside, rather than on, the unit circle.

Step 3: Check why the other statements need not hold The point $z = 1 + 2i$ satisfies $x < y$ but has positive real part and positive $x + y$, disproving (A) and (B). For (C), take $z = -2$: its associated parameter has modulus $1/\sqrt{5} < 1$, yet $x - y = -2$, which is not greater than -1. Thus none of those three statements is universal.

Step 4: Conclusion Every member of S obeys $\boxed{\operatorname{Re} z - \operatorname{Im} z < 0}$. Hence, the correct option is (D).

Quick Tip to Solve: For a fractional complex transformation, isolate the original parameter first, then square the modulus inequality; quadratic terms often cancel to give a half-plane.

75. If f is a function of real variable x satisfying $f(x+4)-f(x+2)+f(x)=0$, then f is a periodic function with period : [4 marks]

(A) 8

(B) 10

(C) 12

(D) 6

Correct Answer: (C) 12

Solution:

Concept:

- A positive number T is a period of f when $f(x+T)=f(x)$ for every real x. A shifted identity can be substituted into another copy of the same identity.
- The relation in the question is $f(x+4)=f(x+2)-f(x)$. Tracking successive shifts by 2 reveals a sign change after three shifts and repetition after six.

Step 1: Derive the sign change after a shift of six Replace x by $x+2$ in the given identity:

$$f(x+6)-f(x+4)+f(x+2)=0.$$

Hence

$$f(x+6)=f(x+4)-f(x+2).$$

From the original equation, $f(x+4)=f(x+2)-f(x)$. Substituting gives

$$f(x+6)=-f(x).$$

Thus a shift of six generally reverses the value; it is not necessarily a period.

Step 2: Apply the sign change twice The last identity holds for every real input. Apply it at $x+6$:

$$f(x+12)=-f(x+6)=-[-f(x)]=f(x).$$

This proves that 12 is always a period of a function satisfying the equation. No continuity or differentiability assumption was needed; the conclusion follows entirely from the translated functional identities.

Step 3: Distinguish a guaranteed period from a least period The test function $f(x)=\cos(\pi x/6)$ satisfies the equation, using the identity $\cos(t+2\pi/3)-\cos(t+\pi/3)+\cos t=0$. Its least positive period is 12, so shifts of 8, 10 and 6 are not periods in general. Special functions, including the zero function, may have smaller periods; this does not invalidate the guaranteed period asked for here.

Step 4: Conclusion For every solution of the stated relation, $\boxed{f(x+12)=f(x)}$. Thus 12 is a period. Hence, the correct option is (C).

Quick Tip to Solve: When a shifted recurrence gives $f(x+a)=-f(x)$, apply the same shift again to obtain the guaranteed period $2a$.

76. For all values of $\theta \in \left(0, \frac{\pi}{2}\right)$, the determinant of the matrix

$$\begin{bmatrix} -2 & \tan\theta + \sec^2\theta & 3 \\ -\sin\theta & \cos\theta & \sin\theta \\ -3 & -4 & 3 \end{bmatrix}$$

always lies in the interval : [4 marks]

(A) $[3, 5]$

(B) $(4, 6)$

(C) $\left(\frac{5}{2}, \frac{19}{4}\right)$

(D) $\left[\frac{7}{2}, \frac{21}{4}\right]$

Correct Answer: (A) $[3, 5]$

Solution:

Concept:

- Adding one column to another leaves a determinant unchanged. A column with two zero entries then makes cofactor expansion especially short.
- A combination $a\cos\theta + b\sin\theta$ is at most $\sqrt{a^2 + b^2}$. Its lower behaviour on a restricted interval must be checked separately, rather than using the unrestricted range.

Step 1: Simplify the first column Let the determinant be D. Apply $C_1 \leftarrow C_1 + C_3$. The transformed first column is $(1, 0, 0)^T$, so expansion along it gives

$$D = \begin{vmatrix} \cos\theta & \sin\theta \\ -4 & 3 \end{vmatrix} = 3\cos\theta + 4\sin\theta.$$

The term $\tan\theta + \sec^2\theta$ contributes nothing after this operation because its cofactor includes zeros. Thus there is no need to expand that potentially large expression.

Step 2: Bound the reduced expression The Cauchy–Schwarz inequality gives

$$D \leq \sqrt{3^2 + 4^2}\sqrt{\cos^2\theta + \sin^2\theta} = 5.$$

Equality is attained when $\cos\theta = 3/5$ and $\sin\theta = 4/5$, an allowed first-quadrant angle. For the lower bound, $\sin\theta + \cos\theta > 1$ and $\sin\theta > 0$ on the given open interval, giving

$$D = 3(\sin\theta + \cos\theta) + \sin\theta > 3.$$

As θ tends to zero from above, D tends to 3.

Step 3: Choose an interval containing every value The actual range is $(3, 5]$, which is contained in option (A), $[3, 5]$. Option (B) excludes values close to 3; option (C) excludes the attained maximum 5; option (D) excludes values between 3 and $7/2$. The question asks for an interval in which the determinant always lies, so inclusion of an unattained endpoint in (A) is harmless.

Step 4: Conclusion The determinant always satisfies $3 < D \leq 5$, and therefore always lies in $\boxed{[3, 5]}$. Hence, the correct option is (A).

Quick Tip to Solve: Look for proportional entries before expanding a determinant: a column addition can eliminate a complicated trigonometric entry altogether.

77. **If** $y(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 23 & 17 & 13 \\ 1 & 1 & 1 \end{vmatrix}$, $x \in \mathbf{R}$, **then** $\frac{d^2y}{dx^2} + y$ **is equal to :** [4 marks]

(A) 4

(B) -10

(C) 0

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- Expansion along the first row expresses the determinant as a linear combination of its three first-row entries; the other rows contribute constant cofactors.
- Both $\sin x$ and $\cos x$ satisfy $u'' + u = 0$. Therefore, in a linear combination of these functions and a constant, the expression $y'' + y$ retains only the constant.

Step 1: Evaluate the cofactors explicitly Expanding along the first row, with cofactor signs $+, -, +$, gives

$$y = \sin x(17 - 13) - \cos x(23 - 13) + (\sin x + \cos x + 1)(23 - 17).$$

The three numerical factors are 4, -10 and 6, respectively. Keeping the minus sign on the middle cofactor is essential. Consequently,

$$y = 4\sin x - 10\cos x + 6(\sin x + \cos x + 1).$$

Step 2: Collect terms and differentiate twice Combine like terms before differentiation:

$$y = 10\sin x - 4\cos x + 6.$$

Differentiation gives

$$y' = 10\cos x + 4\sin x,$$
$$y'' = -10\sin x + 4\cos x.$$

The constant 6 contributes zero to both derivatives, while each trigonometric term changes sign after the second derivative.

Step 3: Add the original function and check the choices Now the trigonometric terms cancel exactly:

$$y'' + y = (-10 + 10)\sin x + (4 - 4)\cos x + 6 = 6.$$

The value is independent of x. Option (A), 4, is only the first minor; option (B), -10, is only the cosine cofactor. Option (C) would apply if there were no constant term, but the extra 1 in the third entry produces the surviving constant 6.

Step 4: Conclusion For every real x, $\boxed{\frac{d^2y}{dx^2} + y = 6}$. Hence, the correct option is (D).

Quick Tip to Solve: For $y = A\sin x + B\cos x + C$, immediately use $y'' + y = C$; compute only the constant coefficient accurately.

78. The number of integral values of m for which the equation, $(1+m^2)\,x^2-2(1+3m)x+(1+8m)=0$, has no real root, is : [4 marks]

(A) 2

(B) 3

(C) infinitely many

(D) 1

Correct Answer: (C) infinitely many

Solution:

Concept:

- A real quadratic $ax^2+bx+c=0$, with $a\neq 0$, has no real root exactly when its discriminant $\Delta=b^2-4ac$ is negative.
- Here the leading coefficient is $1+m^2>0$ for every real m, so the equation never degenerates into a linear equation. Factoring the discriminant reduces the parameter condition to a sign test.

Step 1: Form and expand the discriminant The coefficients are $a=1+m^2$, $b=-2(1+3m)$ and $c=1+8m$. Therefore

$$\frac{\Delta}{4}=(1+3m)^2-(1+m^2)(1+8m).$$

Expand each product, retaining the cubic term from the second product:

$$\begin{aligned}\frac{\Delta}{4}&=1+6m+9m^2\\&\quad-(1+8m+m^2+8m^3)\\&=-8m^3+8m^2-2m.\end{aligned}$$

Step 2: Factor and determine its sign The cubic factorises as

$$\frac{\Delta}{4}=-2m(4m^2-4m+1)=-2m(2m-1)^2.$$

Hence $\Delta=-8m(2m-1)^2$. For an integer m, the squared factor cannot vanish, because its zero is $m=1/2$, which is not integral. It is therefore strictly positive for all integers, and the sign of Δ is the opposite of the sign of m.

Step 3: Count the admissible integers Every positive integer $m=1,2,3,\ldots$ gives $\Delta<0$, so the equation has no real root for each such value. At $m=0$, the discriminant is zero and the root is repeated and real. For negative integers the discriminant is positive and there are two real roots. Thus the complete admissible integer set is the set of all positive integers, which is infinite; none of the finite counts in (A), (B) or (D) describes it.

Step 4: Conclusion The equation has no real root for every positive integer m. There are infinitely many such integral values. Hence, the correct option is (C).

Quick Tip to Solve: Factor the discriminant before counting parameters; a squared factor usually fixes most of its sign and exposes an infinite integer family.

79. For all $d, 0 < d < 1$, which one of the following points is the reflection of the point $(d, 2d, 3d)$ in the plane passing through the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$? [4 marks]

(A) $\left(-\frac{1}{3} + 3d, 2d, \frac{1}{3} + d\right)$

(B) $(3d, 2d, d)$

(C) $\left(\frac{1}{3} + d, \frac{2}{3} - 2d, -\frac{1}{3} + d\right)$

(D) $\left(\frac{2}{3} - 3d, \frac{2}{3} - 2d, \frac{2}{3} - d\right)$

Correct Answer: (D) $\left(\frac{2}{3} - 3d, \frac{2}{3} - 2d, \frac{2}{3} - d\right)$

Solution:

Concept:

- The plane with unit intercepts on the three coordinate axes has equation $x + y + z = 1$. Its normal vector is $\mathbf{n} = (1, 1, 1)$, with $\mathbf{n} \cdot \mathbf{n} = 3$.
- To reflect a point P in a plane, move along the normal through twice its signed perpendicular displacement. For $\mathbf{n} \cdot \mathbf{r} = 1$, the reflected vector is $P' = P - 2(\mathbf{n} \cdot P - 1)\mathbf{n}/3$.

Step 1: Find the normal displacement Each of the three given intercept points satisfies $x + y + z = 1$, so this is the required plane. Put $P = (d, 2d, 3d)$. Its coordinate sum is $6d$, and the normal-line foot is $H = P - t(1, 1, 1)$. Requiring the foot to lie in the plane gives

$$6d - 3t = 1, \qquad t = \frac{6d - 1}{3}.$$

The signed value of t automatically handles points on either side of the plane; no separate case is needed for the allowed interval of d.

Step 2: Reflect and check the candidate Since H is the midpoint of the point and its image, subtract twice the displacement from P:

$$\begin{aligned} P' &= (d, 2d, 3d) - \frac{2(6d - 1)}{3}(1, 1, 1) \\ &= \left(\frac{2}{3} - 3d, \frac{2}{3} - 2d, \frac{2}{3} - d\right). \end{aligned}$$

The difference $P - P'$ has three equal components, so it is perpendicular to the plane. The midpoint has coordinate sum 1, providing an independent reflection check. Options (A), (B) and (C) do not satisfy these reflection conditions for all $0 < d < 1$; occasional equality at a special value would not meet the question's requirement. In particular, option (B) merely reverses coordinates and retains coordinate sum $6d$, rather than reflecting that sum about 1.

Step 3: Conclusion The image is $\left(\frac{2}{3} - 3d, \frac{2}{3} - 2d, \frac{2}{3} - d\right)$. Hence, the correct option is (D).

Quick Tip to Solve: For reflection in a plane, check both that the joining segment is normal to the plane and that its midpoint lies on the plane.

80. If the function $f : [1, \infty[\to [1, \infty[$ is defined by $f(x) = 3^{x(x-1)}$; then $f^{-1}(x)$ is : [4 marks]

(A) $\frac{1}{2}\left(1 - \sqrt{1 + 4\log_3 x}\right)$

(B) $\frac{1}{2}\left(1 + \sqrt{1 + 4\log_3 x}\right)$

(C) not defined

(D) $\left(\frac{1}{3}\right)^{x(x-1)}$

Correct Answer: (B) $\frac{1}{2}\left(1 + \sqrt{1 + 4\log_3 x}\right)$

Solution:

Concept:

- An inverse exists when a function is one-to-one and onto its stated codomain. On $[1, \infty)$, the exponent $t(t-1)$ increases continuously from 0 to infinity, so $3^{t(t-1)}$ increases from 1 to infinity.
- Taking logarithms to base 3 reverses exponentiation with base 3. The quadratic equation obtained after taking logarithms has two algebraic roots, but the inverse must select the root lying in the original domain.

Step 1: Establish invertibility and solve for the input Write $y = 3^{t(t-1)}$, where $t \geq 1$. If $t_2 > t_1 \geq 1$, then

$$t_2(t_2 - 1) - t_1(t_1 - 1) = (t_2 - t_1)(t_2 + t_1 - 1) > 0.$$

Thus the function is strictly increasing. Continuity, its value 1 at $t = 1$, and its unbounded growth show that it covers the whole stated codomain. An inverse is therefore defined. Taking logarithms yields

$$\log_3 y = t^2 - t, \qquad t^2 - t - \log_3 y = 0.$$

Step 2: Choose the admissible quadratic root The quadratic formula gives

$$t = \frac{1 \pm \sqrt{1 + 4\log_3 y}}{2}.$$

For $y \geq 1$, the logarithm is nonnegative and the square root is at least 1. The positive-sign root is consequently at least 1, whereas the negative-sign root is at most 0. Only the positive-sign root belongs to the original domain. Renaming the inverse input y as x gives option (B). Option (A) chooses the excluded root. Option (C) is false because invertibility was established. Option (D) gives the reciprocal of the original output at the same input, which is not the inverse function. As an endpoint check, the chosen formula gives $f^{-1}(1) = 1$, consistent with $f(1) = 1$.

Step 3: Conclusion For $x \geq 1$, $f^{-1}(x) = \frac{1 + \sqrt{1 + 4\log_3 x}}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: After solving an inverse relation algebraically, use the original domain to choose the correct root; a reciprocal is not a functional inverse.

Drawing Test (70 marks)

81(a). Draw an artistic layout with 1 Pentagon, 3 Triangle, 1 Circle..... & some more geometrical shapes using hatching method for coloring. [40 marks total]

Solution:

Concept:

- A geometric composition balances a limited set of shapes through placement, scale, overlap and negative space. The required pentagon, three triangles and circle must remain identifiable.
- Hatching creates tonal differences with controlled parallel strokes; closer strokes produce darker values. Cross-hatching can deepen selected areas without replacing the geometric outlines.

Step 1: Plan the bounding rectangle and dominant shape Lightly establish a rectangular working area with a clear margin. Place one medium-to-large pentagon slightly left of centre as the visual anchor. Avoid centring every element on the same axis: an asymmetric arrangement can still balance when the weight of smaller shapes on the right offsets the pentagon. Reserve clean background spaces around the main cluster so that the layout does not become a collection of disconnected outlines.

Step 2: Arrange the required shapes Place exactly three distinct triangles around and partly across the pentagon: a tall triangle toward the upper right, a smaller triangle near the lower left and a broader triangle toward the lower right. Vary their orientation and size while keeping each triangle readable through its three straight sides. Put one circle near the right of the pentagon to contrast curved and angular edges. Keep overlaps shallow enough that neither the circle nor the pentagon is lost. Use a small rectangle or short geometric bands only as secondary links, since additional shapes should support the required forms rather than compete with them.

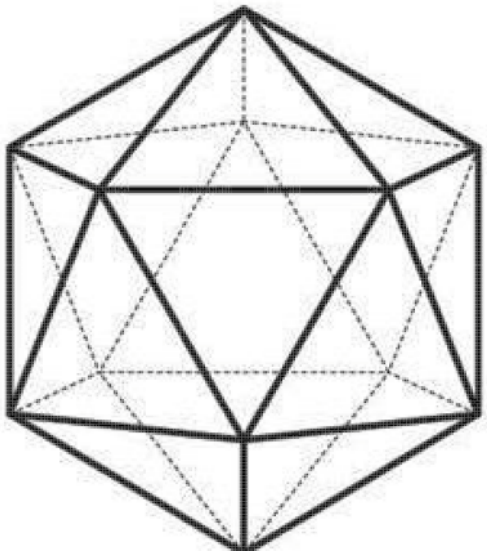
Step 3: Establish rhythm and tonal hierarchy Repeat a few edge directions to guide the eye from the pentagon through the triangles to the circle. Let one diagonal act as the principal movement, with shorter opposing diagonals stabilising it. Separate adjacent overlapping regions by tone: leave the circle nearly white, hatch the pentagon with evenly spaced diagonal strokes and give the largest triangle a darker, denser pattern. Use a medium value for the other triangles. Keep a small set of consistent stroke angles instead of changing direction randomly in every patch.

Step 4: Finish the hatching and review the layout Draw hatch strokes within each boundary, using regular spacing and a steady line weight. Cross-hatch only the darkest overlap or accent; do not smudge the whole surface into an even grey. Strengthen important outlines after the tonal areas are complete, erase construction lines and keep the margin clean. Check the count of one pentagon, three triangles and one circle, the clarity of each silhouette, the contrast between adjacent areas and the balance of filled and empty space. The result should read as a single designed layout rather than a row of shape exercises.

Step 5: Conclusion A suitable composition uses the pentagon as the main anchor, three differently oriented triangles for movement and one circle as the curved counterpoint, with subordinate geometric links. Controlled hatching supplies a clear light-to-dark hierarchy while every required shape remains legible.

Quick Tip to Solve: Count and outline the compulsory shapes before adding secondary forms; then assign three or four distinct hatch densities so overlaps stay readable.

81(b). A shapes was given and it was asked to draw the same shapes '2 times ' bigger.



Solution:

Concept:

- An enlargement with linear scale factor 2 doubles every distance from a chosen reference point while preserving angles, parallel directions and the relative positions of vertices.
- The given polyhedral wireframe is copied as a projected drawing: its visible solid edges and faint dashed hidden edges must retain their original connections. Do not substitute a different three-dimensional view.

Step 1: Measure the reference framework Identify the top and bottom apex, the left and right outer corners, and the intermediate vertices forming the central triangular network. Lightly enclose the original in a bounding rectangle and mark its vertical centreline. Record the width and height as W and H; the enlarged bounding rectangle must have width 2W and height 2H. Compare the heights of the left and right corner pairs and the central junctions before drawing any dark edge.

Step 2: Transfer the vertices at twice their offsets Choose the upper-left corner of the reference rectangle as the origin. If a vertex is x units to the right and y units down, place its counterpart 2x units right and 2y units down from the new origin. A light grid may be used: double the cell size in the new rectangle and transfer each vertex to the same relative position in its cell. Locate all the vertices first. This keeps the enlarged object equally wide and tall in proportion and prevents the common mistake of doubling its width but only estimating its height.

Step 3: Join the visible edges in the original arrangement Join the enlarged top apex to the same outer and inner vertices as in the reference. Reproduce the upper shoulder outline, the vertical outer side edges, the lower outline and the diagonals meeting at the central network. Follow each line from endpoint to endpoint instead of interpreting an apparent intersection as an additional vertex. Maintain the positions of the central horizontal edge and the lower junction relative to the overall height. Straight lines that shared a direction in the reference must retain that direction in the copy.

Step 4: Reproduce hidden edges and check the scale Transfer the faint dashed edges by the same

coordinate scaling, including their common internal junctions. Keep hidden edges lighter and dashed, and make visible edges continuous and stronger. Do not turn a hidden edge into a solid boundary. Finally compare several corresponding distances, such as overall height, overall width and the span of the central horizontal segment: each must be twice its original length. Check the apex alignment, edge connections and symmetry of the projected figure, then remove the grid and construction rectangle without erasing the required dashed lines.

Step 5: Conclusion The completed plan is an exact 2:1 linear enlargement of the supplied wireframe: all vertex offsets and edge lengths in the projected drawing are doubled, angles and connections are preserved, and visible and hidden edges remain distinguishable.

Quick Tip to Solve: Plot every vertex in a doubled bounding grid before connecting edges; enlarging by eye one edge at a time lets small proportion errors accumulate.

82(a). Draw the Cover page of any architecture test book with its name, author name & some more details. [30 marks]

Solution:

Concept:

- A book cover combines an informative typographic hierarchy with imagery that signals its subject. The title should be the strongest text, followed by the author and supporting publication details.
- For this open design task, a proposed title and author are design content. Use a limited visual language, alignment and contrast so the cover remains readable at a small size.

Step 1: Choose a complete cover concept Plan a portrait cover for a proposed textbook titled FUNDAMENTALS OF ARCHITECTURE, with the author line BY A. SHARMA. Use a smaller subtitle, Form, Space and Construction, and include a modest publisher line, Studio Learning Press, near the bottom. These are proposed cover details rather than a claim about an existing publication. Keep the text concise and leave a visible margin on all four sides. Lightly divide the rectangle into a top title area, a large central image area and a bottom information band.

Step 2: Build the architectural image Use the central area for a restrained perspective study of an entrance: a framed doorway, steps, a rectangular wall plane and a column or pier. Show a few receding floor lines meeting at a consistent vanishing point, while vertical edges remain vertical. Add a faint geometric construction grid behind the entrance to connect artistic design with architectural drafting. Let the main image occupy roughly the central half of the cover and keep it clear of the author line. Avoid a collage of unrelated buildings that would weaken the focal point.

Step 3: Set the lettering hierarchy and alignments Place FUNDAMENTALS above OF ARCHITECTURE in bold, carefully spaced uppercase lettering. Make the second line broad enough to balance the width of the image without touching the margins. Put the subtitle beneath the title in smaller lettering and the author line below the image. Align the title, subtitle and publisher to a common left edge or to one consistent central axis. Draw light baselines for lettering so letters do not drift upward or change height unintentionally. Keep a clear separation between text and image outlines.

Step 4: Apply colour or tonal contrast and finish Choose a restrained palette such as deep blue for the

title, warm grey for the building planes and a small ochre accent for the entrance. If working in pencil, replace colour contrast with dark title lettering, medium-value architecture and a pale background grid. Shade the recessed opening darkest and let one illuminated wall remain light, giving the image depth. Keep secondary text high in contrast against an uncluttered ground. Review the final cover for readable title and author, balanced margins, consistent perspective, clean edges and sufficient empty space around the main information.

Step 5: Conclusion A complete cover proposal presents FUNDAMENTALS OF ARCHITECTURE, the author A. Sharma, a subtitle and publisher details around a coherent architectural perspective image. Clear typographic hierarchy, controlled tone or colour and aligned margins make the design both informative and visually unified.

Quick Tip to Solve: Thumbnail the cover first: if the title and architectural subject are not immediately clear at that size, simplify the image and strengthen the title hierarchy.

OR

82(b). Draw a view of a house with right proportional, right perspective. [30 marks]

Solution:

Concept:

- A perspective view represents depth through receding parallel edges that converge to vanishing points on the eye-level horizon. In a normal upright view, vertical building edges remain vertical.
- Plausible proportion relates the door, windows, storey height, roof and steps to a common human scale. Nearer features appear larger, while repeated features reduce consistently with distance.

Step 1: Choose the viewpoint and establish the perspective Plan a two-point perspective view showing the front and one side of a simple one-storey house. Draw a light horizon near a standing observer's eye level and place two widely separated vanishing points on it, preferably beyond the final picture margins to avoid exaggerated distortion. Position the nearest vertical corner slightly off-centre. Draw the base and eaves edges from that corner toward the appropriate left and right vanishing points; these lines determine the two visible wall planes.

Step 2: Set the main proportions before adding detail Choose a house width and depth that form a believable domestic volume, with a door height comfortably below the eaves and enough wall above the openings for a lintel. Mark the near vertical corner, then locate the rear corners along the receding directions and raise their verticals. Use a door of familiar human scale as the reference for windows and steps. Make windows proportionate to the door and keep their heads consistently aligned. All corresponding edges on the same wall must follow that wall's vanishing direction rather than remain horizontally parallel on the page.

Step 3: Construct the roof and openings accurately For a simple pitched roof, find the centre of each relevant end wall using its diagonals, raise the gable peak on that centreline and join the roof edges to the appropriate corners. Make the roof overhang consistent with the wall perspective. Place one door and a pair of windows on the front, and a smaller visible window on the side. Draw their vertical jambs upright and their sills and lintels toward the correct vanishing point. Recess each opening

slightly with a narrow inner edge. Use the same perspective framework for the porch slab, path and steps.

Step 4: Show depth, materials and setting Choose light from the upper left. Keep the lit wall pale, shade the other wall moderately and make the doorway and underside of the eaves darkest. Lay a ground shadow in one coherent direction. Add a few roof or wall texture lines that follow the surface rather than flatten it. Include restrained shrubs or a small tree beside the house and a path leading toward the entrance, keeping them subordinate. Finish by checking verticals, roof symmetry in perspective, consistent window sizes with distance, the door-to-storey relationship and the attachment of the house to its ground plane.

Step 5: Conclusion The planned view shows a proportionate house in consistent two-point perspective, with upright verticals, correctly converging wall and roof edges, human-scaled openings and coherent lighting. A clear ground plane and modest landscaping complete the scene without distracting from the building.

Quick Tip to Solve: Complete the light perspective box and check the door-to-wall proportion before drawing roof tiles, plants or other details; surface decoration cannot correct a distorted structure.