JEE Main 2017 Paper 1 (B.E./B.Tech) Apr 2 Shift 1 (morning) Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2017, Annual (offline), conducted by the Central Board of Secondary Education (CBSE) in pen and paper mode on 2 April 2017, Single sitting. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. Let a, b, c $\in R$. If $f(x) = ax^2 + bx + c$ is such that
$a + b + c = 3$ and $f(x + y) = f(x) + f(y) + xy$, $\forall x, y \in R$,
then $\sum_{n=1}^{10} f(n)$ is equal to [4 marks]

(A) 165

(B) 190

(C) 255

(D) 330

Correct Answer: (D) 330

Solution:

Concept:

- The identity holds for every pair of real numbers. Substituting zero determines the constant term, while comparing the coefficient of xy determines the quadratic coefficient.
- For the required sum, use $\sum_{n=1}^{10} n = 55$ and $\sum_{n=1}^{10} n^2 = 385$. Sum the quadratic and linear terms separately.

Step 1: Determine the polynomial coefficients Put $x = y = 0$ in the identity. It gives $c = 2c$, so $c = 0$. Expanding both sides now gives

$$a(x^2 + 2xy + y^2) + b(x + y) = ax^2 + bx + ay^2 + by + xy.$$

The coefficients of xy must agree, hence $2a = 1$ and $a = \frac{1}{2}$. The given condition $a + b + c = 3$ then yields $b = \frac{5}{2}$. Thus $f(x) = \frac{1}{2}x^2 + \frac{5}{2}x$.

Step 2: Apply the finite-sum formulae Substitute this polynomial into the required sum, preserving both contributions:

$$\sum_{n=1}^{10} f(n) = \frac{1}{2}\sum_{n=1}^{10} n^2 + \frac{5}{2}\sum_{n=1}^{10} n = \frac{385 + 275}{2} = 330.$$

As a consistency check, $f(1) = 3$, as required by $a + b + c = 3$. The alternatives 165, 190 and 255 do not equal the combined quadratic and linear sums; omitting either term cannot satisfy the polynomial already determined.

Step 3: Conclusion The required sum is 330. Hence, the correct option is (D).

Quick Tip to Solve: For a polynomial functional identity, substitute zero first and compare mixed terms next; this avoids solving for coefficients from arbitrary sample values.

2. The function $f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is [4 marks]

(A) invertible

(B) injective but not surjective.

(C) surjective but not injective

(D) neither injective nor surjective.

Correct Answer: (C) surjective but not injective

Solution:

Concept:

- A function is injective if distinct inputs always have distinct images. It is surjective when its actual range equals its stated codomain.
- To find the range of $x/(1+x^2)$, set the output equal to y and require the resulting quadratic in x to have a real solution.

Step 1: Check whether different inputs can share an output The denominator is positive for every real input, so the function is defined on all of $\mathbb{R}$. However,

$$f(2) = \frac{2}{5}, \qquad f\left(\frac{1}{2}\right) = \frac{1/2}{1+1/4} = \frac{2}{5}.$$

The inputs 2 and $1/2$ are distinct. Therefore the function is not injective. In general, nonzero reciprocal inputs have equal images, which explains why a single inverse function cannot exist on this whole domain.

Step 2: Find every attainable output Write $y = x/(1+x^2)$. For $y \neq 0$, this is the quadratic $yx^2 - x + y = 0$, whose discriminant is $1 - 4y^2$. It has a real root precisely when $|y| \leq \frac{1}{2}$. The value $y = 0$ is also attained, at $x = 0$. The endpoints are attained at $x = 1$ and $x = -1$, respectively. Thus the actual range is exactly the closed interval given as codomain. The function is surjective. Options (B) and (D) incorrectly deny surjectivity; option (A) requires injectivity as well, which the explicit counterexample disproves.

Step 3: Conclusion The function is surjective but not injective. Hence, the correct option is (C).

Quick Tip to Solve: Check reciprocal inputs to disprove injectivity quickly; then use the discriminant of the output equation to establish the complete range, including endpoints.

3. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is: [4 marks]

(A) 468

(B) 469

(C) 484

(D) 485

Correct Answer: (D) 485

Solution:

Concept:

- Selecting guests is an unordered choice, so use combinations. Since the two friend lists are disjoint, each party uniquely determines the guests selected from each list.
- Let k be the number of ladies invited from X' s friends. Both the gender total and the requirement of three guests from each list then fix all remaining group sizes.

Step 1: Express the selections in terms of one integer If X contributes k ladies, he contributes $3-k$ men because exactly three of his friends attend. To obtain three ladies altogether, Y must contribute $3-k$ ladies; consequently Y contributes k men. Therefore the number for this value of k is

$$N_k = \binom{4}{k}\binom{3}{3-k}\binom{3}{3-k}\binom{4}{k}.$$

The feasible values are $k = 0, 1, 2, 3$. These cases are disjoint and exhaustive; no ordering of the guests is to be counted.

Step 2: Evaluate and add the four cases The contributions are

$$\begin{aligned} N_0 &= 1, \\ N_1 &= 4 \cdot 3 \cdot 3 \cdot 4 = 144, \\ N_2 &= 6 \cdot 3 \cdot 3 \cdot 6 = 324, \\ N_3 &= 4 \cdot 1 \cdot 1 \cdot 4 = 16. \end{aligned}$$

Hence $N = 1 + 144 + 324 + 16 = 485$. Each factor refers to a different disjoint group of friends, so multiplication within a case and addition across cases are justified. The other options 468, 469 and 484 omit or miscount at least one of these feasible compositions; in particular, the all-men-from-X case contributes one party and must be included.

Step 3: Conclusion There are 485 possible invitation sets. Hence, the correct option is (D).

Quick Tip to Solve: Choose the number of women from one friend list as the case variable. Enforce both totals before writing any combination factors, and include the extreme cases.

4. If S is the set of distinct values of 'b' for which the following system of linear equations

x + y + z = 1

x + ay + z = 1

ax + by + z = 0

has no solution, then S is : [4 marks]

(A) an empty set

(B) an infinite set

(C) a finite set containing two or more elements

(D) a singleton

Correct Answer: (D) a singleton

Solution:

Concept:

- An inconsistent system has contradictory reduced equations. A zero determinant alone is not enough: it can also give infinitely many solutions.
- Subtracting the first equation from the second and eliminating z from the third isolates the parameter cases without dividing by a potentially zero quantity.

Step 1: Reduce the equations and separate the cases Subtract the first equation from the second to obtain $(a-1)y=0$. From the first equation, $z=1-x-y$. Insert this into the third equation:

$$(a-1)x+(b-1)y=-1.$$

If $a \neq 1$, the first reduced equation forces $y=0$. The second then gives $x=-1/(a-1)$, and $z=1-x$. These values exist for every real b, so no inconsistency is possible in this case.

Step 2: Identify the only inconsistent parameter pair If $a=1$, the first two original equations coincide and the remaining condition becomes $(b-1)y=-1$. When $b \neq 1$, take $y=-1/(b-1)$, choose any real x, and put $z=1-x-y$; solutions exist. When $b=1$, however, the reduced equation is $0=-1$, which is impossible. Equivalently, the first equation demands $x+y+z=1$ while the third demands $x+y+z=0$. Thus inconsistency occurs exactly at $(a,b)=(1,1)$. The set of distinct possible b-values is $\{1\}$, excluding an empty set, an infinite set, and a finite set of at least two elements.

Step 3: Conclusion $S=\{1\}$, which is a singleton. Hence, the correct option is (D).

Quick Tip to Solve: Whenever elimination gives a factor such as $a-1$, split into its zero and nonzero cases before dividing; otherwise the inconsistent case may disappear.

5. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$,

then adj($3A^2$ + 12A) is equal to [4 marks]

(A) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

(B) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

(C) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

(D) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

Solution:

Concept:

- For a matrix $M = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$, its adjugate is $\operatorname{adj} M = \begin{pmatrix} s & -q \\ -r & p \end{pmatrix}$. Swap diagonal entries and negate the off-diagonal entries; do not transpose the original matrix first.
- Matrix powers use row-by-column multiplication. Form $3A^2 + 12A$ completely before taking its adjugate.

Step 1: Calculate the matrix polynomial Multiplication gives

$$A^2 = \begin{pmatrix} 2 \cdot 2 + (-3)(-4) & 2(-3) + (-3)1 \\ -4 \cdot 2 + 1(-4) & (-4)(-3) + 1 \end{pmatrix} = \begin{pmatrix} 16 & -9 \\ -12 & 13 \end{pmatrix}.$$

Therefore

$$3A^2 + 12A = \begin{pmatrix} 48 & -27 \\ -36 & 39 \end{pmatrix} + \begin{pmatrix} 24 & -36 \\ -48 & 12 \end{pmatrix} = \begin{pmatrix} 72 & -63 \\ -84 & 51 \end{pmatrix}.$$

Step 2: Take the adjugate and check the signs Interchange 72 and 51 and reverse the signs of -63 and -84. This gives

$$\operatorname{adj}(3A^2 + 12A) = \begin{pmatrix} 51 & 63 \\ 84 & 72 \end{pmatrix}.$$

For a further check, the off-diagonal entry of the product of the matrix and this adjugate is $72(63) - 63(72) = 0$, as required by $M \operatorname{adj} M = (\det M)I$. Option (B) incorrectly interchanges the two off-diagonal entries. Option (C) is the matrix polynomial itself, rather than its adjugate, while option (D) also has incorrect positions and signs.

Step 3: Conclusion The adjugate is $\begin{pmatrix} 51 & 63 \\ 84 & 72 \end{pmatrix}$. Hence, the correct option is (A).

Quick Tip to Solve: For a two by two adjugate, swap only the diagonal entries and change only the signs of the off-diagonal entries; their positions stay fixed.

6. If for a positive integer n, the quadratic equation

$x(x+1)+(x+1)(x+2)+....+\left(x+\overline{n-1}\right)(x+n)=10n$

has two consecutive integral solutions, then n is equal to : [4 marks]

(A) 9

(B) 10

(C) 11

(D) 12

Correct Answer: (C) 11

Solution:

Concept:

- Expand each consecutive product using an index $k = 0, \ldots, n-1$, then apply the standard sums of k and k^2. The condition $n > 0$ permits division by n.
- For a monic quadratic, the square of the difference between its roots equals its discriminant. Consecutive integer roots therefore require discriminant one, followed by an integrality check.

Step 1: Simplify the given sum to a monic quadratic The general term is $(x+k)(x+k+1) = x^2 + (2k+1)x + k(k+1)$. Summing all n terms gives

$$nx^2 + n^2x + \frac{n(n-1)(n+1)}{3} = 10n.$$

Here $\sum(2k+1) = n^2$, and $\sum k(k+1) = n(n-1)(n+1)/3$. Divide by the positive integer n and rearrange:

$$x^2 + nx + \frac{n^2-31}{3} = 0.$$

Step 2: Enforce consecutive roots and verify them The discriminant of this monic quadratic is

$$D = n^2 - \frac{4(n^2-31)}{3} = \frac{124-n^2}{3}.$$

The two roots differ by $\sqrt{D}$. Consecutive integer roots have difference one, so $D = 1$. Thus $124 - n^2 = 3$, giving $n^2 = 121$, and positivity yields $n = 11$. Substitution gives $x^2 + 11x + 30 = (x+5)(x+6) = 0$; the roots -5 and -6 are indeed consecutive integers. For the other listed values $n = 9, 10, 12$, the discriminants are $43/3, 8, -20/3$, respectively, none equal to one.

Step 3: Conclusion The required positive integer is $n = 11$. Hence, the correct option is (C).

Quick Tip to Solve: For a monic quadratic with consecutive roots, set its discriminant to one, then explicitly verify that both roots are integers; the difference condition alone is not enough.

7. If for $x \in \left(0, \frac{1}{4}\right)$, the derivatives of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}.g\,(x)$, then $g\,(x)$ equals [4 marks]

(A) $\frac{3x\sqrt{x}}{1-9x^3}$

(B) $\frac{3x}{1-9x^3}$

(C) $\frac{3}{1+9x^3}$

(D) $\frac{9}{1+9x^3}$

Correct Answer: (D) $\frac{9}{1+9x^3}$

Solution:

Concept:

- The tangent double-angle identity is $\tan(2\theta) = 2\tan\theta/(1-\tan^2\theta)$. Its inverse form must respect the principal range $(-\pi/2, \pi/2)$ of inverse tangent.
- The chain rule gives $\frac{d}{dx}\tan^{-1}u = u'/(1+u^2)$. Differentiate the simplified expression and then factor out the prescribed $\sqrt{x}$.

Step 1: Recognise the double-angle expression and its branch Let $u = 3x^{3/2}$. The argument of the given inverse tangent is $2u/(1-u^2)$. Because $0 < x < 1/4$, we have $0 < u < 3/8 < 1$. Hence $0 < 2\tan^{-1}u < \pi/2$, so no additive branch correction is needed:

$$\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right) = 2\tan^{-1}(3x^{3/2}).$$

The restriction on x also ensures that the original denominator remains positive and nonzero.

Step 2: Differentiate and extract the required factor Apply the chain rule to the simplified expression:

$$\frac{d}{dx}\left[2\tan^{-1}(3x^{3/2})\right] = \frac{2\cdot 3\cdot \frac{3}{2}\sqrt{x}}{1+(3x^{3/2})^2} = \sqrt{x}\,\frac{9}{1+9x^3}.$$

Since $x > 0$, division by $\sqrt{x}$ is legitimate. Comparing with $\sqrt{x}\,g(x)$ therefore determines g. Options (A) and (B) retain the original minus-sign denominator and extraneous powers of x. Option (C) has the right denominator but misses a factor of three from the combined chain-rule factors. Only option (D) gives the exact derivative after removing $\sqrt{x}$.

Step 3: Conclusion $g(x) = \frac{9}{1+9x^3}$. Hence, the correct option is (D).

Quick Tip to Solve: When an inverse tangent contains $2u/(1-u^2)$, check the domain before replacing it by $2\tan^{-1}u$, then retain every chain-rule factor.

8. Let $I_n = \int \tan^n x\, dx,\ (n > 1)$.

If $I_4 + I_6 = a\tan^5 x + bx^5 + C$**, where C is a constant of integration,**

then the ordered pair (a, b) **is equal to** [4 marks]

(A) $\left(\frac{1}{5}, 0\right)$

(B) $\left(\frac{1}{5}, -1\right)$

(C) $\left(-\frac{1}{5}, 0\right)$

(D) $\left(-\frac{1}{5}, 1\right)$

Correct Answer: (A) $\left(\frac{1}{5}, 0\right)$

Solution:

Concept:

- Use linearity of indefinite integration: the sum of two antiderivatives is an antiderivative of the sum, with their constants absorbed into one constant.
- The identity $1 + \tan^2 x = \sec^2 x$ creates the differential $d(\tan x) = \sec^2 x\, dx$. Work on any interval containing no pole of the tangent.

Step 1: Combine the integrands before integrating Instead of expanding two separate reduction formulae, take the common factor $\tan^4 x$. This gives

$$I_4 + I_6 = \int \tan^4 x(1 + \tan^2 x)\, dx = \int \tan^4 x \sec^2 x\, dx.$$

The exponent six is exactly two greater than four, so the identity applies directly. No separate polynomial term in x is generated by this simplification.

Step 2: Substitute the tangent and compare coefficients Set $u = \tan x$, so that $du = \sec^2 x\, dx$. Then

$$I_4 + I_6 = \int u^4\, du = \frac{u^5}{5} + C = \frac{\tan^5 x}{5} + C.$$

Comparing with $a\tan^5 x + bx^5 + C$ yields $a = 1/5$ and $b = 0$. Differentiation verifies the result: the derivative is $\tan^4 x \sec^2 x = \tan^4 x + \tan^6 x$. Options (C) and (D) reverse the sign of this derivative; option (B) additionally introduces the unwanted term $-5x^4$. Thus none of those alternatives represents the required antiderivative.

Step 3: Conclusion The ordered pair is $(a, b) = (1/5, 0)$. Hence, the correct option is (A).

Quick Tip to Solve: When tangent powers differ by two, combine the integrands first and look for a factor of $\sec^2 x$.

9. If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and y(0) = 1,
then $y\left(\frac{\pi}{2}\right)$ is equal to : [4 marks]

(A) $-\frac{2}{3}$
(B) $-\frac{1}{3}$
(C) $\frac{4}{3}$
(D) $\frac{1}{3}$

Correct Answer: (D) $\frac{1}{3}$

Solution:

Concept:

- The product rule gives $\frac{d}{dx}[u(x)v(x)]=uv\prime+u\prime v$. A differential equation can sometimes be integrated immediately by recognizing this pattern.
- The coefficient $2+\sin x$ is always positive, so division by it does not remove or introduce a real singularity.

Step 1: Recognize an exact product derivative Write the equation using the shifted dependent variable $y+1$. Since its derivative is dy/dx, the left side is exactly

$$\frac{d}{dx}\big[(2+\sin x)(y+1)\big]=0.$$

Integrating therefore gives

$$(2+\sin x)(y+1)=K,$$

where K is constant. This avoids separating variables and taking logarithms; in particular, there is no sign ambiguity to resolve in the resulting constant.

Step 2: Apply the initial condition and evaluate At $x=0$, the prescribed value is $y=1$, not $y+1=1$. Consequently

$$K=(2+0)(1+1)=4.$$

Thus the unique solution satisfying the initial condition is

$$y(x)=\frac{4}{2+\sin x}-1.$$

At the requested argument $\sin(\pi/2)=1$, so

$$y(\pi/2)=\frac{4}{3}-1=\frac{1}{3}.$$

The value $4/3$ in option (C) is the value of $y+1$, rather than y. The negative values in (A) and (B) fail the conserved-product equation: they would make $3(y+1)$ equal to 1 and 2 respectively, instead of 4.

Step 3: Conclusion We obtain $y(\pi/2)=1/3$. Hence, the correct option is (D).

Quick Tip to Solve: Check whether the coefficient of $y+1$ is the derivative of the coefficient of dy/dx; if so, use the product rule immediately.

10. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$.
Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$,
$\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ is $30°$.
Then $\vec{a}.\vec{c}$ is equal to : [4 marks]

(A) 2

(B) 5

(C) $\frac{1}{8}$

(D) $\frac{25}{8}$

Correct Answer: (A) 2

Solution:

Concept:

- The magnitude of a cross product is $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}| \sin\theta$, where θ is the angle between the vectors.
- The squared distance identity is $|\mathbf{c} - \mathbf{a}|^2 = |\mathbf{c}|^2 + |\mathbf{a}|^2 - 2\mathbf{a} \cdot \mathbf{c}$. It determines the requested dot product once the two lengths are known.

Step 1: Find the length of the cross-product vector For $\mathbf{a} = (2, 1, -2)$ and $\mathbf{b} = (1, 1, 0)$, direct component calculation gives

$$\mathbf{a} \times \mathbf{b} = (2, -2, 1).$$

Therefore

$$|\mathbf{a} \times \mathbf{b}| = \sqrt{4 + 4 + 1} = 3, \qquad |\mathbf{a}|^2 = 4 + 1 + 4 = 9.$$

The signs in the middle component do not affect its length, but computing the full vector explicitly prevents confusion between a cross product and a dot product.

Step 2: Use the angle condition and the distance condition The angle between $\mathbf{c}$ and $\mathbf{a} \times \mathbf{b}$ is $30°$. The second magnitude condition becomes

$$3 = 3|\mathbf{c}| \sin 30° = \frac{3}{2}|\mathbf{c}|,$$

so $|\mathbf{c}| = 2$. Now square the given distance $|\mathbf{c} - \mathbf{a}| = 3$:

$$9 = 4 + 9 - 2\mathbf{a} \cdot \mathbf{c}.$$

It follows that $2\mathbf{a} \cdot \mathbf{c} = 4$, hence the dot product is 2. The other offered values 5, $1/8$ and $25/8$ would make the squared distance equal to 3, $51/4$, and $27/4$, respectively, rather than 9. Thus they contradict the distance condition even though the direction of $\mathbf{c}$ itself need not be found.

Step 3: Conclusion The required scalar product is $\mathbf{a} \cdot \mathbf{c} = 2$. Hence, the correct option is (A).

Quick Tip to Solve: For a dot product constrained by a distance, find the vector lengths first and expand the squared distance instead of solving for three components.

11. If two different numbers are taken from the set {0, 1, 2, 3, ..., 10}; then the probability that their sum as well as absolute difference are both multiple of 4, is : [4 marks]

(A) $\frac{12}{55}$
(B) $\frac{14}{45}$
(C) $\frac{7}{55}$
(D) $\frac{6}{55}$

Correct Answer: (D) $\frac{6}{55}$

Solution:

Concept:

- Two different numbers chosen from eleven form $\binom{11}{2} = 55$ equally likely unordered pairs. Counting ordered pairs instead doubles both numerator and denominator.
- An integer is a multiple of four exactly when it is congruent to zero modulo four. Zero itself is a multiple of four and must be allowed in the sum or difference condition.

Step 1: Translate both requirements into residue conditions Let the selected numbers be a and b. Divisibility of $|a-b|$ by four says that $a \equiv b \pmod 4$. If their common residue is r, the condition on their sum says $2r \equiv 0 \pmod 4$. Thus r must be 0 or 2. A pair of odd numbers with the same residue has sum congruent to 2, not 0; numbers from different residue classes fail the difference condition. Both tests are required simultaneously.

Step 2: Count distinct favourable pairs and divide The allowable residue classes in the set are

$$\{0,4,8\} \quad \text{and} \quad \{2,6,10\}.$$

Each class contributes $\binom{3}{2} = 3$ pairs. Explicitly these are $(0,4),(0,8),(4,8)$ and $(2,6),(2,10),(6,10)$, giving six favourable pairs in all. Therefore

$$P = \frac{3+3}{\binom{11}{2}} = \frac{6}{55}.$$

Option (A), $12/55$, doubles only the favourable count by treating those pairs as ordered while leaving the sample space unordered. Option (C) would require seven favourable pairs, which the exhaustive residue count excludes. Option (B) has neither the correct count nor the correct denominator for this eleven-element set.

Step 3: Conclusion The probability is $6/55$. Hence, the correct option is (D).

Quick Tip to Solve: Use equal residues for the difference condition, then test the sum within each residue class; keep ordered and unordered counting consistent.

12. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to [4 marks]

(A) 2

(B) 4

(C) -1

(D) -2

Correct Answer: (A) 2

Solution:

Concept:

- The half-angle identity $1+\cos x = 2\cos^2(x/2)$ converts this reciprocal into a multiple of $\sec^2(x/2)$.
- The antiderivative is $\tan(x/2)$, since its derivative is $\frac{1}{2}\sec^2(x/2)$. The denominator is positive throughout the given closed interval, so the integral is proper.

Step 1: Convert the integrand and integrate Apply the half-angle identity directly:

$$\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x} = \frac{1}{2}\int_{\pi/4}^{3\pi/4} \sec^2(x/2)\,dx.$$

The factor $1/2$ is cancelled by the factor 2 from substituting $u = x/2$, with $dx = 2\,du$. Hence

$$I = [\tan(x/2)]_{\pi/4}^{3\pi/4} = \tan(3\pi/8) - \tan(\pi/8).$$

Halving the arguments of the limits is essential; using the original angles inside tangent would evaluate a different antiderivative.

Step 2: Evaluate the half-angle values and check the options The standard exact values are

$$\tan(\pi/8) = \sqrt{2}-1, \qquad \tan(3\pi/8) = \sqrt{2}+1.$$

They follow from the half-angle formula and the complementary-angle identity, since $3\pi/8 = \pi/2 - \pi/8$. Their difference is

$$I = (\sqrt{2}+1) - (\sqrt{2}-1) = 2.$$

The integral is positive because its integrand is positive everywhere between the limits, immediately excluding the negative options (C) and (D). Option (B), 4, would arise from retaining an extra factor 2 despite the prefactor $1/2$ in the half-angle identity; careful substitution shows why that doubling is incorrect.

Step 3: Conclusion The definite integral equals 2. Hence, the correct option is (A).

Quick Tip to Solve: For $1/(1+\cos x)$, remember the antiderivative $\tan(x/2)$, and halve both endpoint angles before evaluation.

13. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is : [4 marks]

(A) 10

(B) 25

(C) 30

(D) 12.5

Correct Answer: (B) 25

Solution:

Concept:

- A circular sector with radius r and angle θ in radians has arc length $r\theta$ and area $A = \frac{1}{2}r^2\theta$. Its boundary consists of the arc and two radii.
- An available length of wire fixes the entire perimeter. Eliminating the angle produces a quadratic in the radius, whose vertex gives the maximum area.

Step 1: Express the area using the fencing constraint The two straight sides consume length $2r$, so the remaining wire forms an arc of length $20 - 2r$. Thus

$$2r + r\theta = 20, \qquad r\theta = 20 - 2r.$$

Multiplying the arc length by half the radius gives

$$A = \frac{1}{2}r(r\theta) = \frac{1}{2}r(20 - 2r) = 10r - r^2.$$

This formula includes both straight sides; treating the wire as just a circumference or just an arc would impose the wrong constraint. The radius and arc length must be positive for a nondegenerate sector.

Step 2: Maximize the quadratic and verify feasibility Complete the square:

$$A = 25 - (r - 5)^2 \leq 25.$$

Equality holds at $r = 5\,\text{m}$. The required arc then has length $20 - 2(5) = 10\,\text{m}$, and its angle is

$$\theta = \frac{10}{5} = 2 \text{ radians}.$$

This is a valid circular sector, with angle between zero and 2π, so the upper bound is actually attained rather than merely formal. Equivalently, $A\prime = 10 - 2r = 0$ and $A\prime\prime = -2 < 0$ confirm a maximum. Options (A) and (D) are smaller than the attained area, while option (C), 30, exceeds the exact bound $A \leq 25$ and cannot occur for any radius.

Step 3: Conclusion The maximum flower-bed area is $25\,\text{m}^2$. Hence, the correct option is (B).

Quick Tip to Solve: For a sector fenced around its full boundary, use area = half the radius times the arc length, after subtracting the two radii from the perimeter.

14. If $5\left(\tan^2 x - \cos^2 x\right) = 2\cos 2x + 9$,
then the value of $\cos 4x$ is : [4 marks]

(A) $\frac{1}{3}$
(B) $\frac{2}{9}$
(C) $-\frac{7}{9}$
(D) $-\frac{3}{5}$

Correct Answer: (C) $-\frac{7}{9}$

Solution:

Concept:

- Use $\tan^2 x = (1 - \cos^2 x)/\cos^2 x$ and $\cos 2x = 2\cos^2 x - 1$ to express the entire equation in one variable.
- The tangent requires $\cos x \neq 0$, so $t = \cos^2 x$ must satisfy $0 < t \leq 1$. Algebraic roots outside this range must be rejected before evaluating the requested expression.

Step 1: Reduce the equation to a quadratic Set $t = \cos^2 x$. Then the given equation becomes

$$5\left(\frac{1-t}{t} - t\right) = 2(2t - 1) + 9 = 4t + 7.$$

Multiplication by the nonzero quantity t gives

$$5 - 5t - 5t^2 = 4t^2 + 7t,$$

or

$$9t^2 + 12t - 5 = (3t - 1)(3t + 5) = 0.$$

Thus $t = 1/3$ or $t = -5/3$. The second root is impossible because a squared cosine cannot be negative. The admissible root is therefore uniquely $\cos^2 x = 1/3$.

Step 2: Compute the quadruple angle and compare The double angle first gives

$$\cos 2x = 2\cos^2 x - 1 = \frac{2}{3} - 1 = -\frac{1}{3}.$$

Apply the same identity again:

$$\cos 4x = 2\cos^2(2x) - 1 = 2\left(\frac{1}{9}\right) - 1 = -\frac{7}{9}.$$

The sign of $\cos x$ itself need not be chosen: both double-angle expressions depend only on its square. Option (A), $1/3$, is the intermediate value of $\cos^2 x$. Option (B), $2/9$, omits the final subtraction of 1. Option (D), $-3/5$, is incompatible with the unique admissible root of the original equation. As a check, substituting $t = 1/3$ gives both sides of that equation equal to $25/3$.

Step 3: Conclusion The required value is $\cos 4x = -7/9$. Hence, the correct option is (C).

Quick Tip to Solve: In equations involving both tangent squared and cosine squared, substitute $t = \cos^2 x$, enforce $0 < t \leq 1$, and only then use double angles.

15. Let k be an integer such that the triangle with vertices (k, – 3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point : [4 marks]

(A) $\left(1, \frac{3}{4}\right)$
(B) $\left(1, -\frac{3}{4}\right)$
(C) $\left(2, \frac{1}{2}\right)$
(D) $\left(2, -\frac{1}{2}\right)$

Correct Answer: (C) $\left(2, \frac{1}{2}\right)$

Solution:

Concept:

- The coordinate determinant gives twice the unsigned area: $2\Delta = |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$. The absolute value requires checking both signs.
- The orthocentre is the intersection of altitudes. An altitude is perpendicular to the opposite side; horizontal opposite sides give vertical altitudes.

Step 1: Use the area to determine the integer Substitute the three vertices into the area determinant:

$$2\Delta = |k(k-2) + 5(2+3k) - k(-3k-k)| = |5k^2 + 13k + 10|.$$

The stated area gives an absolute value of 56. The positive case yields

$$5k^2 + 13k - 46 = (k-2)(5k+23) = 0.$$

Only $k = 2$ is an integer. The negative case gives $5k^2 + 13k + 66 = 0$, whose discriminant $169 - 1320 = -1151$ is negative, so it adds no real value.

Step 2: Intersect two perpendicular altitudes The vertices are now $A = (2, -6), B = (5, 2), C = (-2, 2)$. Since BC is horizontal, the altitude from A is $x = 2$. The slope of AB is

$$m_{AB} = \frac{2 - (-6)}{5 - 2} = \frac{8}{3}.$$

The altitude through C therefore has slope $-3/8$, giving

$$y - 2 = -\frac{3}{8}(x + 2).$$

At $x = 2$, this gives $y = 2 - 12/8 = 1/2$. The two options with first coordinate 1 fail the vertical-altitude condition; the remaining negative ordinate also fails this equation.

Step 3: Conclusion The orthocentre is $\boxed{(2, 1/2)}$. Hence, the correct option is (C).

Quick Tip to Solve: When two vertices have the same ordinate, find the vertical altitude first; it immediately fixes the orthocentre's abscissa.

16. $\lim\limits_{x\to\frac{\pi}{2}} \frac{\cot x-\cos x}{(\pi-2x)^3}$ **equals** [4 marks]

(A) $\frac{1}{16}$

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{1}{24}$

Correct Answer: (A) $\frac{1}{16}$

Solution:

Concept:

- Use a small angle $h = \pi/2 - x$ to move the limit to zero. Then $\cot x = \tan h$, $\cos x = \sin h$, and $\pi - 2x = 2h$.
- The standard radian limits are $\sin h/h \to 1$ and $(1 - \cos h)/h^2 \to 1/2$. Factoring a difference before taking limits avoids cancellation errors.

Step 1: Factor the small-angle numerator As x approaches $\pi/2$, h approaches zero from either side. Substitute the complementary angle, retaining the cubic scale factor:

$$L = \lim_{h\to 0} \frac{\tan h - \sin h}{8h^3}.$$

Since $\tan h = \sin h/\cos h$, the numerator factors exactly as

$$\tan h - \sin h = \frac{\sin h(1 - \cos h)}{\cos h}.$$

This shows that the numerator is of third order, so replacing both trigonometric functions merely by h would lose the decisive term.

Step 2: Apply the standard limits Separate the factors:

$$L = \frac{1}{8} \lim_{h\to 0} \left(\frac{\sin h}{h}\right)\left(\frac{1 - \cos h}{h^2}\right)\frac{1}{\cos h}.$$

The second factor follows from $1 - \cos h = 2\sin^2(h/2)$, and therefore tends to $1/2$. The other two factors tend to 1. Thus

$$L = \frac{1}{8} \cdot 1 \cdot \frac{1}{2} \cdot 1 = \frac{1}{16}.$$

Option B omits the factor $1/2$ from $1 - \cos h$; C and D likewise do not equal the product of the exact limiting factors. The result is positive on both sides because numerator and denominator have the same cubic sign.

Step 3: Conclusion The limit equals $\boxed{1/16}$. Hence, the correct option is (A).

Quick Tip to Solve: For a difference such as tan h minus sin h, factor out sin h and expose 1 minus cos h before using standard limits.

17. For three events A, B and C,

P(Exactly one of A or B occurs)

= P(Exactly one of B or C occurs)

= P (Exactly one of C or A occurs) = $\frac{1}{4}$

and P(All the three events occur simultaneously) = $\frac{1}{16}$.

Then the probability that at least one of the events occurs, is : [4 marks]

(A) $\frac{7}{16}$

(B) $\frac{7}{64}$

(C) $\frac{3}{16}$

(D) $\frac{7}{32}$

Correct Answer: (A) $\frac{7}{16}$

Solution:

Concept:

- Exactly one of A and B is their symmetric difference $A \triangle B$. It does not require that C be absent.
- Classify outcomes by how many of the three events occur. Disjoint classes can be added directly, and counting contributions to a sum of event indicators avoids assumptions of independence.

Step 1: Count each type of outcome Let p_j be the probability that exactly j of A, B and C occur, where $j = 0, 1, 2, 3$. Add the three given pairwise exactly-one probabilities. If no events occur, none of the three pairs differs. If all three occur, again no pair differs. If exactly one event occurs, its two pairs with absent events differ. If exactly two occur, the absent event forms two differing pairs with the two present events. Thus every outcome with exactly one or two events contributes twice, and every other outcome contributes zero.

Step 2: Add the triple intersection once Consequently,

$$P(A \triangle B) + P(B \triangle C) + P(C \triangle A) = 2(p_1 + p_2).$$

Each term on the left is $1/4$, so

$$p_1 + p_2 = \frac{1}{2}\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) = \frac{3}{8}.$$

At least one event means exactly one, exactly two, or all three, with no overlap between these classes. The triple intersection is given as $p_3 = 1/16$, hence

$$P(A \cup B \cup C) = \frac{3}{8} + \frac{1}{16} = \frac{7}{16}.$$

The value $3/16$ is too small even to include $p_1 + p_2$, while $7/64$ and $7/32$ are also smaller than $3/8$. No independence hypothesis was used or needed.

Step 3: Conclusion The required probability is $\boxed{7/16}$. Hence, the correct option is (A).

Quick Tip to Solve: Sum the three pairwise symmetric differences: outcomes in exactly one or exactly two events are each counted twice.

18. The area (in sq. units) of the region

$\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \, and \, y \leq 1 + \sqrt{x}\}$ **is** [4 marks]

(A) $\frac{3}{2}$

(B) $\frac{7}{3}$

(C) $\frac{5}{2}$

(D) $\frac{59}{12}$

Correct Answer: (C) $\frac{5}{2}$

Solution:

Concept:

- For vertical slices, the area is $\int (y_{\text{upper}} - y_{\text{lower}})\, dx$. Both upper restrictions must hold, so use the smaller upper curve.
- The lower boundary is $y = x^2/4$; the possible upper boundaries are $y = 3 - x$ and $y = 1 + \sqrt{x}$, with $x \geq 0$. Split the integral where the upper boundaries meet.

Step 1: Find the change in upper boundary The upper curves meet when $3 - x = 1 + \sqrt{x}$. Put $t = \sqrt{x} \geq 0$:

$$t^2 + t - 2 = (t + 2)(t - 1) = 0, \qquad x = 1.$$

For $0 \leq x \leq 1$, $1 + \sqrt{x}$ is lower; for $x \geq 1$, $3 - x$ is lower. The line meets the lower parabola where

$$x^2/4 = 3 - x \quad \Longrightarrow \quad (x + 6)(x - 2) = 0.$$

Thus the right endpoint is 2. On the first interval the lower parabola is below the upper curve, and on the second it remains below the line until x=2; no extra interval contributes.

Step 2: Integrate the two slice heights The desired area is

$$A = \int_0^1 \left(1 + \sqrt{x} - \frac{x^2}{4}\right) dx + \int_1^2 \left(3 - x - \frac{x^2}{4}\right) dx.$$

The first contribution is

$$\left[x + \frac{2}{3}x^{3/2} - \frac{x^3}{12}\right]_0^1 = \frac{19}{12}.$$

The second contribution is

$$\left[3x - \frac{x^2}{2} - \frac{x^3}{12}\right]_1^2 = \frac{40}{12} - \frac{29}{12} = \frac{11}{12}.$$

Their sum is $30/12 = 5/2$. The values $3/2$, $7/3$ and $59/12$ do not equal the sum of these two nonoverlapping contributions; using either upper curve over the entire interval would violate the other inequality.

Step 3: Conclusion The area is $\boxed{5/2 \text{ square units}}$. Hence, the correct option is (C).

Quick Tip to Solve: With two upper inequalities, integrate under their minimum and split exactly where those two upper curves cross.

19. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k,$$

then k is equal to : [4 marks]

(A) z

(B) -1

(C) 1

(D) -z

Correct Answer: (D) -z

Solution:

Concept:

- From $2\omega + 1 = z$ and $z^2 = -3$, squaring gives $\omega^2 + \omega + 1 = 0$. Hence $\omega^3 = 1$, with $\omega \neq 1$.
- Adding a multiple of one row to another leaves a determinant unchanged. Use the cube-root identities before expanding to reduce powers and avoid unnecessary complex arithmetic.

Step 1: Reduce the entries using the cube-root identity The relation $\omega^2 + \omega + 1 = 0$ gives $-\omega^2 - 1 = \omega$, and $\omega^7 = \omega$. Therefore the determinant becomes

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}.$$

Subtract the first row from each of the other rows. Expanding along the first column now leaves the two-by-two determinant

$$D = (\omega - 1)^2 - (\omega^2 - 1)^2.$$

The vertical bars denote a determinant, so this value is a complex number; they are not a request for its modulus.

Step 2: Factor and recover the given complex number Use the difference of two squares:

$$D = (\omega - \omega^2)(\omega + \omega^2 - 2).$$

Since $\omega + \omega^2 = -1$, the second factor is -3. Also

$$\omega - \omega^2 = \omega - (-1 - \omega) = 2\omega + 1 = z.$$

Thus $D = -3z$. Comparing with the stated $D = 3k$ gives $k = -z$. The sign is fixed by the row order: reversing the second and third rows would change the sign and incorrectly give option A. The real numbers 1 and -1 cannot equal the resulting nonzero purely imaginary value. This reasoning uses only $z^2 = -3$ and works consistently with the given square root.

Step 3: Conclusion The value is $\boxed{k = -z}$. Hence, the correct option is (D).

Quick Tip to Solve: Reduce every power of a nonreal cube root modulo 3, then subtract the all-ones row before expanding the determinant.

20. The radius of a circle, having minimum area, which touches the curve y = 4 – x^2 and the lines, y = |x| is : [4 marks]

(A) $2\left(\sqrt{2}-1\right)$

(B) $4\left(\sqrt{2}-1\right)$

(C) $4\left(\sqrt{2}+1\right)$

(D) $2\left(\sqrt{2}+1\right)$

Correct Answer: No printed option equals the literal minimum radius $(\sqrt{17}-\sqrt{2})/2$; vertex tangency alone would give option (B).

Solution:

Concept:

- A circle tangent to both rays of $y=|x|$ has centre $(0,h)$, $h>0$, and radius $r=h/\sqrt{2}$, by equality of the perpendicular distances to the two lines. The other angle bisector gives a negative-height contact on one ray and cannot touch both rays.
- At a smooth tangency, the radius is normal to the parabola. Tangency need not occur at its vertex: symmetric nonvertex contact points must also be checked when minimising the radius.

Step 1: Include every possible contact with the parabola Let a contact point on $y=4-x^2$ be $(t,4-t^2)$. For $t\neq 0$, the tangent slope is $-2t$. The radius vector from $(0,h)$ must be perpendicular to $(1,-2t)$, giving

$$t-2t(4-t^2-h)=0, \qquad h=\frac{7}{2}-t^2.$$

The squared radius is also the squared distance from the centre to the contact point. Since $4-t^2-h=1/2$,

$$\frac{h^2}{2}=t^2+\frac{1}{4}=\frac{15}{4}-h.$$

Thus $2h^2+4h-15=0$. Its only positive root and associated radius are

$$h=\frac{\sqrt{34}-2}{2}, \qquad r=\frac{\sqrt{17}-\sqrt{2}}{2}.$$

Step 2: Check tangency and compare with vertex contacts Here $t^2=(9-\sqrt{34})/2>0$, so both nonvertex contact points are real. More directly, the squared distance from the centre to a general point $(x,4-x^2)$ of the parabola satisfies

$$x^2+(4-x^2-h)^2-r^2=\left(x^2-\left(\frac{7}{2}-h\right)\right)^2.$$

The right side is nonnegative and vanishes at the two stated points, establishing genuine tangency without crossing the circle. Contacts with the two rays occur at $(\pm h/2, h/2)$, so both are on the specified rays. At the vertex $(0,4)$, the tangency condition is $|4-h|=h/\sqrt{2}$, giving radii $4(\sqrt{2}-1)$ and $4(\sqrt{2}+1)$. Both exceed the nonvertex value $r\approx 1.354$. These exhaust the vertex and nonvertex possibilities, so the nonvertex circle has the smallest area. Option A is approximately 0.828 and is not a possible tangency radius; B, C and D are also different from the minimum.

Step 3: Conclusion The literal minimum radius is $\boxed{(\sqrt{17}-\sqrt{2})/2}$, which is absent from the options. The listed value $4(\sqrt{2}-1)$, associated with option (B) and the question-sheet key, is a vertex-tangent radius but is larger than the rigorously verified nonvertex minimum. Therefore none of the printed options answers the question as stated.

Quick Tip to Solve: For minimum-radius tangency problems, test nonvertex normals as well as the axis vertex; symmetry can produce two equal off-axis contacts.

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21. The normal to the curve y(x – 2)(x – 3) = x + 6 at the point where the curve intersects the y-axis passes through the point : [4 marks]

(A) $\left(\frac{1}{2}, \frac{1}{2}\right)$
(B) $\left(\frac{1}{2}, -\frac{1}{3}\right)$
(C) $\left(\frac{1}{2}, \frac{1}{3}\right)$
(D) $\left(-\frac{1}{2}, -\frac{1}{3}\right)$

Correct Answer: (A) $\left(\frac{1}{2}, \frac{1}{2}\right)$

Solution:

Concept:

- On the y-axis, $x = 0$. Substitute this into the implicit curve before evaluating its derivative.
- Implicit differentiation of $y(x-2)(x-3)$ uses the product rule. A normal to a curve has slope $-1/m$ when its tangent has finite nonzero slope m.

Step 1: Locate the point and calculate the tangent slope The curve is $y(x-2)(x-3) = x+6$. At x=0, this gives $6y = 6$, so the relevant point is $P = (0, 1)$. Differentiate both sides with respect to x:

$$y'(x-2)(x-3) + y\big((x-3)+(x-2)\big) = 1.$$

Equivalently,

$$y'(x-2)(x-3) + y(2x-5) = 1.$$

Evaluate at P rather than treating y as a constant: $6y' - 5 = 1$, hence $y' = 1$. The tangent therefore has slope 1 at this intersection.

Step 2: Write the normal and test the options The perpendicular normal has slope -1. Through $(0, 1)$ its equation is

$$y - 1 = -(x - 0), \qquad x + y = 1.$$

For option A, $x + y = 1/2 + 1/2 = 1$, so it lies on this line. For B the sum is $1/2 - 1/3 = 1/6$; for C it is $1/2 + 1/3 = 5/6$; for D it is $-1/2 - 1/3 = -5/6$. None of these three equals 1. Substitution checks the actual line rather than merely comparing the apparent direction of a point from the origin, since the normal passes through $(0, 1)$, not through the origin.

Step 3: Conclusion The normal passes through $\boxed{(1/2, 1/2)}$. Hence, the correct option is (A).

Quick Tip to Solve: After finding the normal in the form x plus y equals a constant, test each option by its coordinate sum.

22. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to: [4 marks]

(A) $\frac{1}{4}$
(B) $\frac{2}{9}$
(C) $\frac{4}{9}$
(D) $\frac{6}{7}$

Correct Answer: (B) $\frac{2}{9}$

Solution:

Concept:

- For an angle of elevation in a right triangle, the tangent is the vertical height divided by the horizontal distance. The tower and the level ground are perpendicular.
- If two rays from the same point have inclinations θ and ϕ, their included angle is $\theta - \phi$, and $\tan(\theta - \phi) = (\tan\theta - \tan\phi)/(1 + \tan\theta\tan\phi)$.

Step 1: Express both elevations using one height Let $AB = h > 0$. Since C is the midpoint, $AC = h/2$, while the common horizontal distance is $AP = 2h$. Put $\theta = \angle BPA$ and $\phi = \angle CPA$. The two right triangles therefore give

$$\tan\theta = \frac{AB}{AP} = \frac{1}{2}, \qquad \tan\phi = \frac{AC}{AP} = \frac{1}{4}.$$

The ray towards B is steeper than the ray towards C; hence $\beta = \theta - \phi$. The height cancels because only ratios of lengths determine these angles.

Step 2: Apply the tangent subtraction formula Substitute the two tangent values, keeping the product term in the denominator:

$$\tan\beta = \frac{\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{2}\cdot\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{9}{8}} = \frac{2}{9}.$$

Option (A) gives only the difference of the tangents and misses the denominator. Option (C) is twice the required ratio, and option (D) also fails the exact substitution. The result is positive, as expected for the angle between the two upward rays.

Step 3: Conclusion $\boxed{\tan\beta = \frac{2}{9}}$. Hence, the correct option is (B).

Quick Tip to Solve: Calculate the two elevation tangents first; the tangent of their difference is not their simple difference.

23. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is x = – 4, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is : [4 marks]

(A) 2y – x = 2

(B) 4x – 2y = 1

(C) 4x + 2y = 7

(D) x + 2y = 4

Correct Answer: (B) 4x – 2y = 1

Solution:

Concept:

- For an ellipse $x^2/a^2 + y^2/b^2 = 1$ with major axis along x, $b^2 = a^2(1 - e^2)$ and its directrices are $x = \pm a/e$.
- Implicit differentiation gives the tangent slope $-b^2x/(a^2y)$. The normal is perpendicular to the tangent and passes through the specified point.

Step 1: Recover the ellipse from its directrix The vertical directrix $x = -4$, together with the centre at the origin, fixes the major-axis direction. Using $e = 1/2$,

$$\frac{a}{e} = 4 \quad \Longrightarrow \quad a = 2, \qquad b^2 = 4\left(1 - \frac{1}{4}\right) = 3.$$

Thus the ellipse is $x^2/4 + y^2/3 = 1$. The proposed point lies on it, since $1/4 + (9/4)/3 = 1$, so a normal there is well defined.

Step 2: Calculate the normal and compare the options Differentiate the ellipse equation:

$$\frac{x}{2} + \frac{2y}{3}\frac{dy}{dx} = 0 \quad \Longrightarrow \quad \frac{dy}{dx} = -\frac{3x}{4y}.$$

At $(1, 3/2)$, the tangent slope is $-1/2$, and the normal slope is 2. Its point-slope equation is therefore

$$y - \frac{3}{2} = 2(x - 1) \quad \Longrightarrow \quad 4x - 2y = 1.$$

Option (A) has slope $1/2$, (C) has slope -2, and (D) has slope $-1/2$. They do not have the required normal slope, even though passing through the point alone is a necessary check.

Step 3: Conclusion The normal is $\boxed{4x - 2y = 1}$. Hence, the correct option is (B).

Quick Tip to Solve: For an ellipse, use the directrix to find a before finding b; then check both the normal slope and the given point.

24. The following statement

$(p \to q) \to [(\sim p \to q) \to q]$ **is :** [4 marks]

(A) equivalent to $\sim p \to q$

(B) equivalent to $p \to \sim q$

(C) a fallacy

(D) a tautology

Correct Answer: (D) a tautology

Solution:

Concept:

- An implication $r \to s$ is equivalent to $\neg r \vee s$. It is false only when its antecedent is true and its consequent false.
- A tautology is true for every assignment of truth values. The laws of distribution and excluded middle simplify nested implications without guessing truth values.

Step 1: Simplify the inner implication Let $T = (p \to q) \to [(\neg p \to q) \to q]$. First, $\neg p \to q \equiv p \vee q$. Consequently, the bracket becomes

$$\begin{aligned}(p \vee q) \to q &\equiv \neg(p \vee q) \vee q \\ &\equiv (\neg p \wedge \neg q) \vee q \\ &\equiv (\neg p \vee q) \wedge (\neg q \vee q) \\ &\equiv \neg p \vee q.\end{aligned}$$

The final expression is exactly $p \to q$. Every step is an equivalence, so the simplification applies to all four truth-value assignments.

Step 2: Evaluate the resulting self-implication Writing $R = p \to q$, the original statement is now $R \to R$. Therefore

$$T \equiv \neg R \vee R \equiv \text{True}.$$

If R is true, both antecedent and consequent are true; if R is false, the implication is still true because its antecedent is false. Option (A), equivalent to $p \vee q$, is false when both p and q are false. Option (B), equivalent to $\neg p \vee \neg q$, is false when both are true. Neither is a tautology. A fallacy is always false, so option (C) is the opposite classification.

Step 3: Conclusion The statement is true for every choice of p and q, so it is a tautology. Hence, the correct option is (D).

Quick Tip to Solve: Work from the innermost implication outward; spotting the same expression on both sides reduces the final test to R implies R.

25. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is : [4 marks]

(A) 6

(B) 4

(C) $\frac{6}{25}$

(D) $\frac{12}{5}$

Correct Answer: (D) $\frac{12}{5}$

Solution:

Concept:

- With replacement, every draw has the same probability of success, and the draws are independent. The number of successes in n such trials follows a binomial distribution.
- For $X \sim \text{Bin}(n, p)$, the mean is np and the variance is $np(1-p)$. Variance concerns the count, rather than the probability on one draw.

Step 1: Identify the trials and success probability Call a green ball a success, and let X be the total number of green balls among the ten draws. There are $15 + 10 = 25$ balls, so

$$p = P(\text{green}) = \frac{15}{25} = \frac{3}{5}, \qquad 1 - p = \frac{2}{5}.$$

After each draw the ball is replaced; the composition of the box is restored and p remains unchanged. Hence $n = 10$ and $X \sim \text{Bin}(10, 3/5)$. A without-replacement model would not apply to the experiment stated here.

Step 2: Calculate the variance and eliminate the distractors Use the binomial variance formula:

$$\text{Var}(X) = 10 \cdot \frac{3}{5} \cdot \frac{2}{5} = \frac{60}{25} = \frac{12}{5}.$$

Option (A), 6, is the mean np, not the variance. Option (B), 4, is the expected count of yellow balls. Option (C), $6/25$, is the variance of one green-success indicator and omits the factor of ten. As a check, independent indicator variables have additive variances: the ten equal single-draw variances sum to $10(6/25)$, the same result.

Step 3: Conclusion The variance of the number of green balls drawn is $\boxed{12/5}$. Hence, the correct option is (D).

Quick Tip to Solve: The word replacement signals constant p: use np for the mean and np(1-p) for the variance of the success count.

26. The distance of the point (1, 3, – 7) from the plane passing through the point (1, –1, – 1), having normal perpendicular to both the lines

$$\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$$

and

$\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$ **is :** [4 marks]

(A) $\frac{10}{\sqrt{83}}$

(B) $\frac{5}{\sqrt{83}}$

(C) $\frac{10}{\sqrt{74}}$

(D) $\frac{20}{\sqrt{74}}$

Correct Answer: (A) $\frac{10}{\sqrt{83}}$

Solution:

Concept:

- The denominators of a line in symmetric form give its direction ratios. A vector perpendicular to two nonparallel line directions is their cross product.
- A plane with normal (a, b, c) through (x_0, y_0, z_0) has equation $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$. Distance from (x_1, y_1, z_1) is the absolute plane expression divided by $\sqrt{a^2 + b^2 + c^2}$.

Step 1: Find the plane normal and its equation The two line directions are $\mathbf{u} = (1, -2, 3)$ and $\mathbf{v} = (2, -1, -1)$. Their cross product is

$$\mathbf{u} \times \mathbf{v} = \big(2 + 3,\ 6 + 1,\ -1 + 4\big) = (5, 7, 3).$$

Indeed, $(5, 7, 3) \cdot (1, -2, 3) = 0$ and $(5, 7, 3) \cdot (2, -1, -1) = 0$. This normal gives the required plane through $(1, -1, -1)$:

$$5(x - 1) + 7(y + 1) + 3(z + 1) = 0,$$

so its Cartesian equation is $5x + 7y + 3z + 5 = 0$.

Step 2: Apply the point-to-plane distance formula For the given point $(1, 3, -7)$, the numerator is

$$|5(1) + 7(3) + 3(-7) + 5| = |5 + 21 - 21 + 5| = 10.$$

The length of the normal is $\sqrt{25 + 49 + 9} = \sqrt{83}$, giving

$$d = \frac{10}{\sqrt{83}}.$$

Option (B) halves the correct numerator. Options (C) and (D) use $\sqrt{74}$, which is not the length of the normal. Reversing the cross product would reverse every plane coefficient but leave this absolute distance unchanged.

Step 3: Conclusion The required perpendicular distance is $\boxed{10/\sqrt{83}}$. Hence, the correct option is (A).

Quick Tip to Solve: Take the cross product of the two direction vectors, not of points on the lines; verify the normal with two dot products.

27. A hyperbola passes through the point P$(\sqrt{2}, \sqrt{3})$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point : [4 marks]

(A) $(2\sqrt{2}, 3\sqrt{3})$
(B) $(\sqrt{3}, \sqrt{2})$
(C) $(-\sqrt{2}, -\sqrt{3})$
(D) $(3\sqrt{2}, 2\sqrt{3})$

Correct Answer: (A) $(2\sqrt{2}, 3\sqrt{3})$

Solution:

Concept:

- For a hyperbola $x^2/a^2 - y^2/b^2 = 1$, the foci are $(\pm c, 0)$, where $c^2 = a^2 + b^2$. Both squared semiaxes must be positive.
- At a point (x_1, y_1) of this hyperbola, the tangent equation is $xx_1/a^2 - yy_1/b^2 = 1$. A proposed point lies on it precisely when this equation is satisfied.

Step 1: Determine the hyperbola parameters The foci give $c = 2$, hence $a^2 + b^2 = 4$. Put $a^2 = t$, so $b^2 = 4 - t$ and $0 < t < 4$. Substitution of $P = (\sqrt{2}, \sqrt{3})$ gives

$$\frac{2}{t} - \frac{3}{4-t} = 1 \quad \Longrightarrow \quad 8 - 5t = 4t - t^2.$$

Thus $t^2 - 9t + 8 = (t-1)(t-8) = 0$. The root 8 would make b^2 negative, so it is inadmissible. Therefore $a^2 = 1$, $b^2 = 3$, and the hyperbola is $x^2 - y^2/3 = 1$.

Step 2: Test the proposed points in the tangent The tangent at P is

$$\sqrt{2}\,x - \frac{\sqrt{3}}{3}y = 1.$$

For option (A), $(2\sqrt{2}, 3\sqrt{3})$, its left side is $4 - 3 = 1$, so the point lies on the tangent. For option (B) the left side is $\sqrt{6} - \sqrt{6}/3 = 2\sqrt{6}/3 \neq 1$. For option (C) it is $-2 + 1 = -1$, and for option (D) it is $6 - 2 = 4$. These explicit substitutions distinguish the correct point from the others without relying on the appearance or relative locations of their coordinates.

Step 3: Conclusion The tangent also passes through $\boxed{(2\sqrt{2}, 3\sqrt{3})}$. Hence, the correct option is (A).

Quick Tip to Solve: When solving for a squared semiaxis, discard any algebraic root that makes the other squared semiaxis negative before constructing the tangent.

28. The value of $\left({}^{21}C_1 - {}^{10}C_1\right) + \left({}^{21}C_2 - {}^{10}C_2\right) + \left({}^{21}C_3 - {}^{10}C_3\right)$ $\left({}^{21}C_4 - {}^{10}C_4\right) + \ldots + \left({}^{21}C_{10} - {}^{10}C_{10}\right)$ **is** [4 marks]

(A) $2^{21} - 2^{10}$

(B) $2^{20} - 2^9$

(C) $2^{20} - 2^{10}$

(D) $2^{21} - 2^{11}$

Correct Answer: The missing separator makes the expression ambiguous: literal multiplication gives no listed option; the indicated additive series gives (C).

Solution:

Concept:

- The binomial theorem at unity gives $\sum_{r=0}^{n} \binom{n}{r} = 2^n$. The symmetry identity $\binom{n}{r} = \binom{n}{n-r}$ pairs coefficients equidistant from the ends.
- For odd n there is no unpaired middle coefficient. Thus the lower half of the coefficients has exactly half of the total sum; any omitted r=0 terms must be removed consistently.

Step 1: Sum the coefficients with upper index 21 The displayed continuation indicates the differences with indices 1 through 10. Its separator between the third and fourth displayed terms is missing; treating that continuation as the indicated sum, write

$$S = \sum_{r=1}^{10} \left(\binom{21}{r} - \binom{10}{r} \right).$$

For the coefficients of order 21, the 22 terms pair as r and $21 - r$. Therefore the sum for $r = 0, \ldots, 10$ is half of 2^{21}:

$$\sum_{r=0}^{10} \binom{21}{r} = 2^{20}, \qquad \sum_{r=1}^{10} \binom{21}{r} = 2^{20} - 1.$$

Step 2: Subtract the complete nonconstant order-10 sum The indices 1 through 10 include every order-10 coefficient except its constant coefficient. Hence

$$\sum_{r=1}^{10} \binom{10}{r} = 2^{10} - 1,$$

so

$$S = (2^{20} - 1) - (2^{10} - 1) = 2^{20} - 2^{10}.$$

The two omitted constant terms cancel. Option (A) incorrectly uses the full order-21 sum, (B) incorrectly halves the order-10 sum, and (D) doubles the desired result. Only the order-21 sum is halved, because ten is its lower-half cutoff but is the final index for order 10.

Step 3: Check the expression with literal multiplication The displayed third and fourth differences are adjacent with no plus sign between them. Under the usual multiplication convention, their values are $t_3 = \binom{21}{3} - \binom{10}{3} = 1330 - 120 = 1210$ and $t_4 = \binom{21}{4} - \binom{10}{4} = 5985 - 210 = 5775$. Replacing these

two additive terms by their product gives

$$1{,}047{,}552 - 1210 - 5775 + 1210(5775) = 8{,}028{,}317.$$

This does not equal any listed option. Thus the printed notation does not support a unique listed answer without specifying that the missing separator is addition.

Step 4: Conclusion The additive continuation gives $2^{20} - 2^{10}$, option (C), whereas literal multiplication at the missing separator gives $8{,}028{,}317$, which is not listed. The expression as printed is therefore ambiguous; (C) applies only to the additive-series interpretation.

Quick Tip to Solve: Check whether the upper summation limit is halfway through a row or reaches its end; the same limit 10 plays different roles for rows 21 and 10.

29. If the image of the point P(1, –2, 3) in the plane, 2x + 3y – 4z + 22 = 0 measured parallel to the line,

$\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ **is Q, then PQ is equal to:** [4 marks]

(A) $2\sqrt{42}$

(B) $\sqrt{42}$

(C) $6\sqrt{5}$

(D) $3\sqrt{5}$

Correct Answer: (A) $2\sqrt{42}$

Solution:

Concept:

- A reflection measured parallel to a specified line joins the point and its image along that direction. The plane bisects this segment, although the segment need not be normal to the plane.
- A line through a point with direction vector $\mathbf{d}$ is $\mathbf{r} = \mathbf{r}_0 + t\mathbf{d}$. Its distance parameter is $|t|\|\mathbf{d}\|$.

Step 1: Parameterise the line through the point The direction ratios are $(1, 4, 5)$, so write a point on the line through $P(1, -2, 3)$ as $(1 + t, -2 + 4t, 3 + 5t)$. The required image lies on this line; using the plane normal instead would describe perpendicular reflection and would ignore the stated direction.

Step 2: Locate the midpoint in the plane Let M be the intersection with the plane. Substitution gives

$$2(1 + t) + 3(-2 + 4t) - 4(3 + 5t) + 22 = 0.$$

Collecting terms,

$$6 - 6t = 0, \qquad t = 1.$$

Thus $M = (2, 2, 8)$. The coefficient of t is nonzero, so the direction is not parallel to the plane and this intersection is unique.

Step 3: Double the distance to the midpoint Since M bisects PQ, the image is $Q = 2M - P = (3, 6, 13)$. Hence

$$PQ = \sqrt{(3 - 1)^2 + (6 + 2)^2 + (13 - 3)^2} = \sqrt{168} = 2\sqrt{42}.$$

Option (B) is only the distance PM, rather than the full image distance. Options (C) and (D) do not equal the length obtained with the specified direction ratios.

Step 4: Conclusion The required distance is $PQ = 2\sqrt{42}$ units. Hence, the correct option is (A).

Quick Tip to Solve: For an oblique image, find the intersection along the stated direction first, then double that distance; do not use the perpendicular point-to-plane distance.

30. For any three positive real numbers a, b and c,
$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$.
Then [4 marks]

(A) b, c and a are in G.P.

(B) b, c and a are in A.P.

(C) a, b and c are in A.P.

(D) a, b and c are in G.P.

Correct Answer: (B) b, c and a are in A.P.

Solution:

Concept:

- For real numbers x, y, z, $x^2 + y^2 + z^2 - xy - yz - zx = \frac{1}{2}[(x-y)^2 + (y-z)^2 + (z-x)^2]$. A sum of real squares vanishes only when each square is zero.
- Three terms in their specified order form an arithmetic progression when twice the middle term equals the sum of the outer terms. A geometric progression instead requires the square of the middle term to equal their product.

Step 1: Rearrange the given equality Move every term to the left and expand without changing the order of the factors:

$$225a^2 + 9b^2 + 25c^2 - 45ab - 15bc - 75ac = 0.$$

The coefficients suggest comparing $15a$, $3b$, and $5c$. This substitution makes the three cross terms precisely the negative pairwise products.

Step 2: Express the condition as squares Put $x = 15a$, $y = 3b$, $z = 5c$. The equality becomes

$$\frac{1}{2}[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2] = 0.$$

All three summands are nonnegative, so they must vanish individually. Consequently $15a = 3b = 5c$, which gives $b = 5a$ and $c = 3a$.

Step 3: Check the proposed orders In the order b, c, a, the terms are $5a, 3a, a$. Both successive differences equal $-2a$, so these terms form an A.P. For order a, b, c, the differences are $4a$ and $-2a$, which differ because $a > 0$. Neither proposed G.P. works: $c^2 = 9a^2 \neq ab = 5a^2$, and $b^2 = 25a^2 \neq ac = 3a^2$.

Step 4: Conclusion The numbers b, c, a, in that order, are in arithmetic progression. Hence, the correct option is (B).

Quick Tip to Solve: When a quadratic equality has three squared terms and three negative pairwise products, try the sum-of-squared-differences identity before testing progressions.

Physics (120 marks)

31. C_P and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$C_P - C_v$ = a for hydrogen gas

$C_P - C_v$ = b for nitrogen gas

The correct relation between a and b is [4 marks]

(A) a = 28 b

(B) a = 1/14 b

(C) a = b

(D) a = 14 b

Correct Answer: (D) a = 14 b

Solution:

Concept:

- For an ideal gas, molar heat capacities satisfy Mayer's relation $C_{P,m} - C_{V,m} = R$, where R is the universal gas constant.
- Specific heat capacities are per unit mass, so their difference is $c_P - c_V = R/M$, where M is the molar mass expressed in the same mass unit.

Step 1: Distinguish specific from molar heat capacity The question asks for specific heats, so the compared samples have equal mass rather than equal mole count. A mass m contains m/M moles; dividing the molar relation by molar mass yields the mass-specific difference R/M. Equal molar differences therefore do not imply equal specific-heat differences.

Step 2: Use the molecular masses Hydrogen and nitrogen gases are diatomic, with molar masses approximately $M_{\mathrm{H_2}} = 2\,\mathrm{g\,mol^{-1}}$ and $M_{\mathrm{N_2}} = 28\,\mathrm{g\,mol^{-1}}$. Use either grams consistently in both expressions or convert both to kilograms. Hence

$$a = \frac{R}{M_{\mathrm{H_2}}}, \qquad b = \frac{R}{M_{\mathrm{N_2}}}.$$

Step 3: Form and interpret the ratio Taking the ratio cancels R and the common mass-unit conversion:

$$\frac{a}{b} = \frac{M_{\mathrm{N_2}}}{M_{\mathrm{H_2}}} = \frac{28}{2} = 14.$$

Thus hydrogen has the larger difference. Option (A) ignores hydrogen's molecular mass of 2; option (B) reverses the ratio; option (C) would apply to molar heat capacities, which are a different quantity. The diatomic degrees of freedom affect the individual heat capacities, but cancel in their difference.

Step 4: Conclusion The correct relation is $a = 14b$. Hence, the correct option is (D).

Quick Tip to Solve: Check whether heat capacities are per mole or per unit mass: the difference is respectively R or R/M.

32. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15cm from a converging lens of magnitude of focal length 20cm. A beam of parallel light falls on the diverging lens. The final image formed is: [4 marks]

(A) real and at a distance of 6 cm from the convergent lens.

(B) real and at a distance of 40 cm from convergent lens.

(C) virtual and at a distance of 40 cm from convergent lens.

(D) real and at a distance of 40 cm from the divergent lens.

Correct Answer: (B) real and at a distance of 40 cm from convergent lens.

Solution:

Concept:

- With Cartesian sign convention for light travelling left to right, the thin-lens equation is $1/v - 1/u = 1/f$. A diverging lens has negative focal length and a converging lens has positive focal length.
- The first lens's image acts as the object for the second lens. The object distance for the second lens must include the separation between the lenses.

Step 1: Find the first image Place the diverging lens at $x = 0$ and the converging lens at $x = 15\,\text{cm}$. A parallel incident beam corresponds to an object at infinity. The diverging lens therefore produces a virtual image at its left focal point,

$$v_1 = f_1 = -25\,\text{cm}.$$

The emerging rays diverge as if they originated at $x = -25\,\text{cm}$.

Step 2: Apply the second lens equation That point lies $25 + 15 = 40\,\text{cm}$ to the left of the converging lens, so $u_2 = -40\,\text{cm}$, while $f_2 = +20\,\text{cm}$. Thus

$$\frac{1}{v_2} - \frac{1}{-40} = \frac{1}{20}, \qquad \frac{1}{v_2} = \frac{1}{40}.$$

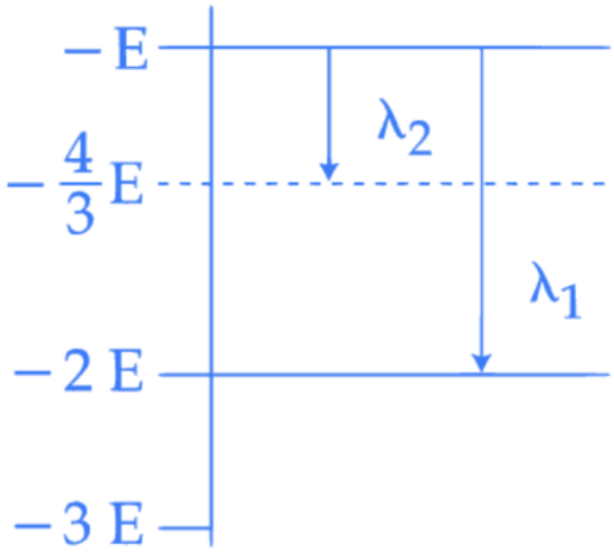
It follows that $v_2 = +40\,\text{cm}$.

Step 3: Identify the image and reference lens The positive image distance means actual convergence to the right of the second lens: the final image is real. Its coordinate is $x = 15 + 40 = 55\,\text{cm}$, so it is 55 cm from the diverging lens. Option (A) gives an incorrect distance, option (C) gives the wrong nature, and option (D) measures 40 cm from the wrong lens. As a useful check, the second lens has its object at $2f_2$, so its image must also be at $2f_2$.

Step 4: Conclusion The final image is real and 40 cm from the converging lens on its far side. Hence, the correct option is (B).

Quick Tip to Solve: For separated lenses, first locate the intermediate image on a common coordinate axis; then measure the second object distance from the second lens.

33. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths r = λ_1/λ_2, is given by: [4 marks]



(A) r = 1/3

(B) r = 4/3

(C) r = 2/3

(D) r = 3/4

Correct Answer: (A) r = 1/3

Solution:

Concept:

- A photon emitted in a downward transition has energy $\Delta E = E_{\text{initial}} - E_{\text{final}} > 0$. Its wavelength satisfies $\lambda = hc/\Delta E$.
- Wavelength is inversely proportional to the energy gap, so $\lambda_1/\lambda_2 = \Delta E_2/\Delta E_1$. The labelled level energies, rather than the drawn spacing, determine the gaps.

Step 1: Read the two labelled transitions In the figure, the arrow labelled λ_1 runs from $-E$ to $-2E$, whereas the arrow labelled λ_2 runs from $-E$ to $-4E/3$. Here E denotes a positive energy scale. Both arrows are downward emissions; the unused level at $-3E$ does not participate in either transition.

Step 2: Calculate the emitted photon energies Subtract the final level from the initial level for each arrow, retaining the negative signs of the levels:

$$\Delta E_1 = (-E) - (-2E) = E,$$

$$\Delta E_2 = (-E) - \left(-\frac{4E}{3}\right) = \frac{E}{3}.$$

The first photon thus carries three times the energy of the second. Absolute magnitudes of the final levels alone are not photon energies.

Step 3: Take the inverse energy ratio Using the photon relation separately and dividing gives

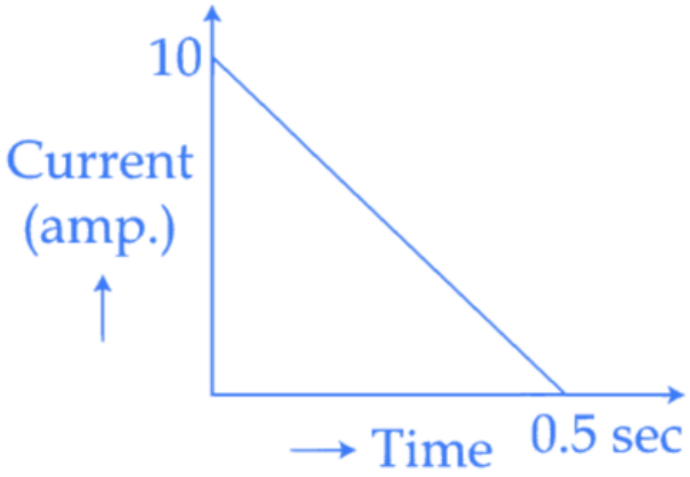
$$r = \frac{\lambda_1}{\lambda_2} = \frac{hc/E}{hc/(E/3)} = \frac{1}{3}.$$

The factors h, c, E cancel, so the result is dimensionless. Since the first energy gap is larger, its wavelength must be smaller; this rejects $4/3$. The values $2/3$ and $3/4$ do not follow from the two differences of the labelled energies.

Step 4: Conclusion The wavelength ratio is $r = 1/3$. Hence, the correct option is (A).

Quick Tip to Solve: Write both energy gaps as differences before taking their inverse ratio; negative bound-state energies should never be compared by magnitude alone.

34. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is: [4 marks]



(A) 275 Wb

(B) 200 Wb

(C) 225 Wb

(D) 250 Wb

Correct Answer: (D) 250 Wb

Solution:

Concept:

- Faraday's law relates induced emf to the rate of change of linked magnetic flux: $\mathcal{E} = -d\Phi/dt$. For resistance R, Ohm's law gives the induced current magnitude $I = |\mathcal{E}|/R$.
- For a current of one sign throughout the interval, the magnitude of the total linked-flux change is $|\Delta\Phi| = R \int I\, dt$. The integral is the area under the current-time graph.

Step 1: Read the graph limits The plotted current decreases linearly from $10\,\mathrm{A}$ at $t = 0$ to zero at

$t = 0.5\,\text{s}$. The complete shaded-by-integration region is a triangle, with base 0.5 s and height 10 A. There is no negative-current region to subtract. We need its area rather than the slope of the sloping line.

Step 2: Calculate the time integral of current The charge passing through the coil during the change is

$$\int_0^{0.5} I\,dt = \frac{1}{2}(0.5\,\text{s})(10\,\text{A}) = 2.5\,\text{C}.$$

Equivalently, the average current is $(10+0)/2 = 5\,\text{A}$, and multiplying by 0.5 s gives the same integral. Using the peak current throughout would double the result.

Step 3: Convert the area to a flux change With constant coil resistance $R = 100\,\Omega$,

$$|\Delta\Phi| = 100 \times 2.5 = 250\,\text{V s} = 250\,\text{Wb}.$$

The graph fixes the area exactly, so 275 Wb, 200 Wb and 225 Wb each correspond to an incorrect area or resistance factor. Lenz's minus sign determines the direction of the emf relative to the change; it does not make the requested magnitude negative.

Step 4: Conclusion The magnitude of the linked magnetic-flux change through the coil is $250\,\text{Wb}$. Hence, the correct option is (D).

Quick Tip to Solve: For an induced-current versus time graph, multiply its signed area by the constant resistance to obtain the linked-flux change in magnitude.

35. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is [4 marks]

(A) 15.6 mm

(B) 1.56 mm

(C) 7.8 mm

(D) 9.75 mm

Correct Answer: (C) 7.8 mm

Solution:

Concept:

- In Young's double-slit experiment, bright fringes occur at $y = m\lambda D/d$, where m is an integer order, D is the screen distance and d is the slit separation.
- Two bright fringes coincide when their path differences are equal: $m_1\lambda_1 = m_2\lambda_2$. The least nonzero coincidence uses the smallest positive integer pair satisfying this equality.

Step 1: Find the first common nonzero order Let m_1 be the order for 650 nm and m_2 the order for

520 nm. Coincidence requires

$$650m_1 = 520m_2, \qquad 5m_1 = 4m_2.$$

Since 4 and 5 are coprime, the smallest positive solution is $m_1 = 4$, $m_2 = 5$. The common central maximum corresponds to zero orders and is excluded by asking for the least distance to the next coincidence.

Step 2: Calculate the shared path difference The path difference of that common bright fringe is

$$\delta = 4(650 \times 10^{-9}) = 5(520 \times 10^{-9}) = 2.6 \times 10^{-6}\,\text{m}.$$

Convert the apparatus lengths consistently: $D = 150\,\text{cm} = 1.50\,\text{m}$, and $d = 0.5\,\text{mm} = 5.0 \times 10^{-4}\,\text{m}$.

Step 3: Locate and compare the fringe Using $\delta = dy/D$,

$$y = \frac{D\delta}{d} = \frac{1.50(2.6 \times 10^{-6})}{5.0 \times 10^{-4}} = 7.8 \times 10^{-3}\,\text{m} = 7.8\,\text{mm}.$$

The 15.6 mm value is the next common coincidence, not the first. The 1.56 mm value is only the first 520 nm bright-fringe distance and is not a 650 nm bright position; 9.75 mm is a 650 nm bright position but does not have integer order for 520 nm.

Step 4: Conclusion The least nonzero distance from the central maximum is $7.8\,\text{mm}$. Hence, the correct option is (C).

Quick Tip to Solve: Reduce the wavelength ratio to coprime integers first; reverse that ratio to get the two smallest matching bright-fringe orders.

36. Which of the following statements is false? [4 marks]

(A) Kirchhoff's second law represents energy conservation.

(B) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.

(C) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.

(D) A rheostat can be used as a potential divider.

Correct Answer: (C) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.

Solution:

Concept:

- Kirchhoff's loop law states that the algebraic sum of potential changes around a closed loop is zero; it expresses conservation of energy per unit charge.
- A balanced Wheatstone bridge satisfies $P/Q = R/S$. Its null condition depends on resistance ratios, rather than the magnitude of the applied emf.

Step 1: Test the bridge interchange statement At balance the galvanometer joins two points of equal

potential, so its current vanishes. The balance condition can be written $PS = QR$. Exchanging the source and detector diagonals gives the reciprocal bridge arrangement, for which the same product equality is the null condition. Thus interchanging the cell and galvanometer does not disturb the null point in an ideal balanced bridge. Statement (C), which asserts that it does, is false.

Step 2: Check the remaining statements Statement (A) is true: the energy gained from sources equals the energy lost in circuit elements on a complete circuit traversal. Statement (B) is true: comparable arm resistances give a sensitive bridge, whereas very unequal arms make the detector response to a small resistance change poor. Statement (D) is also true: the full resistance track of a rheostat can be connected across a supply and its slider used as a tapping point, providing a variable fraction of the supply voltage. Therefore none of these three statements answers the request for a false statement.

Step 3: Conclusion Exchanging the cell and galvanometer leaves a balanced Wheatstone bridge balanced. Hence, the correct option is (C).

Quick Tip to Solve: For a bridge interchange question, write the product balance condition first; swapping the two diagonals preserves it.

37. The following observations were taken for determining surface tension T of water by capillary method:

diameter of capillary, D = 1.25 $\times$ 10^{-2} m

rise of water, h = 1.45 $\times$ 10^{-2}m

Using g = 9.80 m/s^2 and the simplified relation T = $\frac{rhg}{2} \times 10^3 N/m$, the possible error in surface tension is closest to : [4 marks]

(A) 10 %

(B) 0.15 %

(C) 1.5 %

(D) 2.4 %

Correct Answer: (C) 1.5 %

Solution:

Concept:

- For a product of measured quantities, the maximum fractional error is the sum of their fractional errors. Since $r = D/2$, the radius and diameter have the same fractional error.
- Here $T = 10^3 Dhg/4$. The numerical factors are exact; use one unit in the last reported decimal place as the possible absolute error in each measured number.

Step 1: Read the measurement precision The diameter and height are quoted to $0.01 \times 10^{-2}\,\mathrm{m}$, so take $\Delta D = \Delta h = 10^{-4}\,\mathrm{m}$. The quoted gravitational acceleration gives $\Delta g = 0.01\,\mathrm{m\,s^{-2}}$. Dividing the diameter by two also divides its absolute uncertainty by two, leaving $\Delta r/r = \Delta D/D$. Do not add a separate radius error to the diameter error: they describe the same measurement.

Step 2: Add the percentage contributions Applying the product rule gives

$$\frac{\Delta T}{T} \times 100 = \left(\frac{0.01}{1.25} + \frac{0.01}{1.45} + \frac{0.01}{9.80}\right) 100$$
$$= 0.800 + 0.690 + 0.102$$
$$\simeq 1.59\%.$$

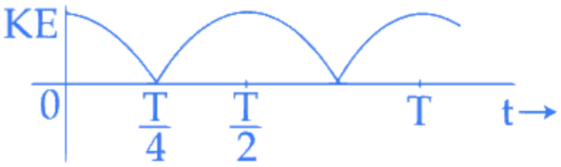
Even if g is treated as an exact supplied constant, the first two terms give 1.49%, leading to the same closest choice. The values 10% and 2.4% are too large, while 0.15% is ten times too small.

Step 3: Conclusion The possible percentage error is closest to 1.5%. Hence, the correct option is (C).

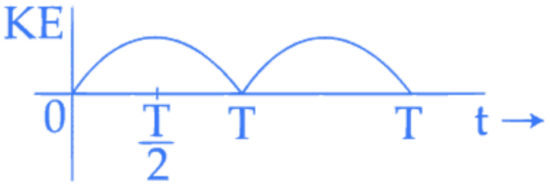
Quick Tip to Solve: Convert each last-place uncertainty into a percentage before adding; halving a measured diameter does not halve its percentage error.

38. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy – time graph of the particle will look like: [4 marks]

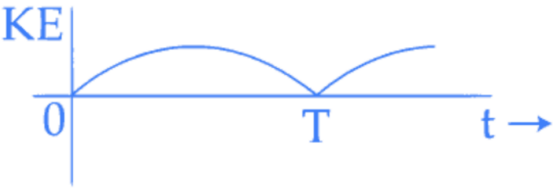
(A)



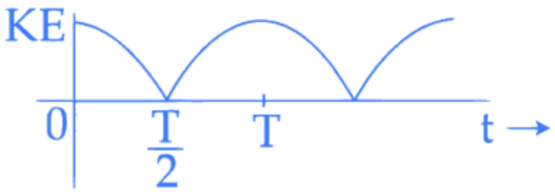
(B)



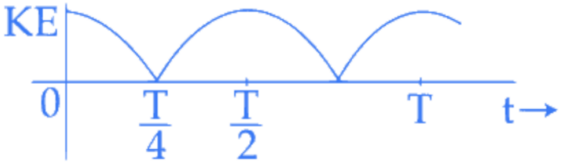
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- For SHM with equilibrium crossing at $t = 0$, choose $x = A\sin\omega t$, where $\omega = 2\pi/T$. Reversing the initial velocity changes its sign but not the kinetic energy.
- The speed follows $v = A\omega\cos\omega t$, so $K = \frac{1}{2}mA^2\omega^2\cos^2\omega t$. Kinetic energy is nonnegative and repeats after $T/2$.

Step 1: Determine the initial value and zeroes The equilibrium point is where the restoring potential energy is smallest. Thus the kinetic energy is maximum at the initial instant, rather than zero. From the squared cosine expression,

$$K(0) = K_{\max}, \qquad K(T/4) = 0, \qquad K(T/2) = K_{\max}.$$

The next zero is at $3T/4$, followed by another maximum at T. The graph never enters negative kinetic energy, and its energy oscillation frequency is twice the displacement frequency.

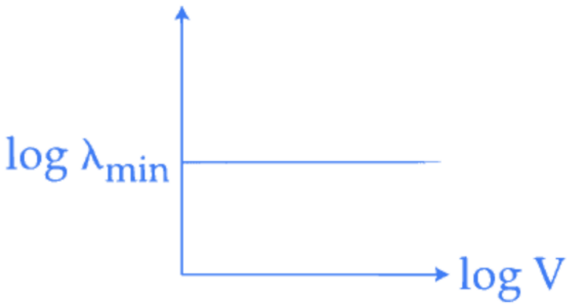
Step 2: Compare the four plotted options Graph (A) starts at a maximum, reaches zero at $T/4$, and returns to a maximum at $T/2$, so it has the required timing. Graphs (B) and (C) start from zero energy, contradicting the initial equilibrium crossing. Graph (D) starts at a maximum but places its first zero at $T/2$, which is too late by a factor of two. At that instant the particle has crossed equilibrium again and must have maximum kinetic energy. These checks distinguish the plotted curves without needing the numerical amplitude or particle mass.

Step 3: Conclusion The kinetic energy varies as $K_{\max}\cos^2(2\pi t/T)$, represented by graph (A). Hence, the correct option is (A).

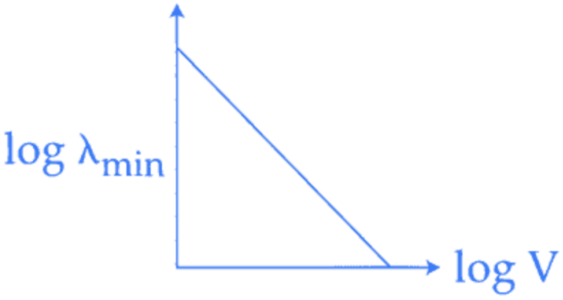
Quick Tip to Solve: Mark the energy at $0, T/4, T/2$: maximum, zero, maximum. Energy repeats twice during one complete displacement cycle.

39. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X–rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of $\log\lambda_{min}$ with log V is correctly represented in: [4 marks]

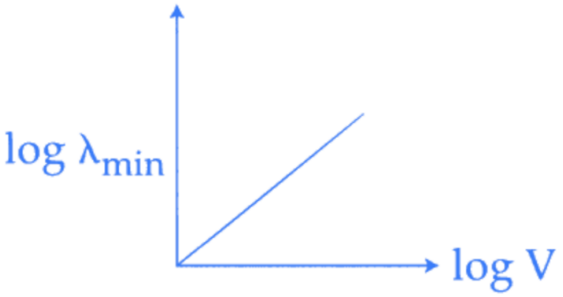
(A)



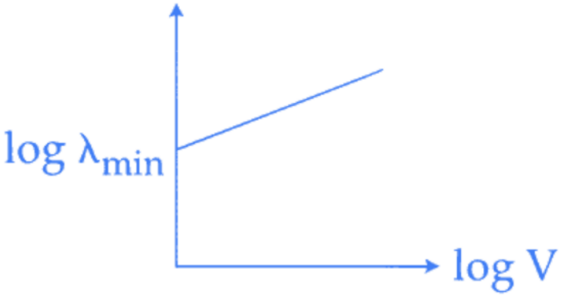
(B)



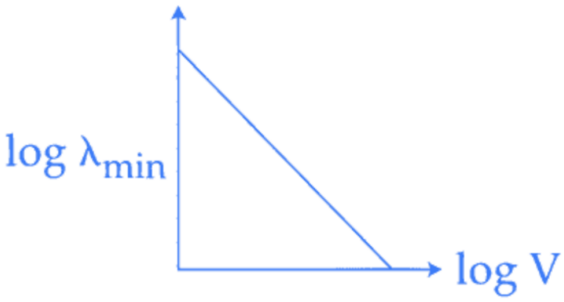
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- An electron accelerated from rest through potential difference V gains kinetic energy eV. The highest-energy X-ray photon can carry this entire energy.
- Photon energy is $E = hc/\lambda$, so the short-wavelength cutoff satisfies $hc/\lambda_{\min} = eV$. The cutoff depends on accelerating voltage, unlike the target-dependent characteristic lines.

Step 1: Obtain the voltage dependence The minimum wavelength corresponds to maximum photon

energy, not to an average electron collision. Energy conservation gives

$$\lambda_{\min} = \frac{hc}{eV}.$$

Here h, c, e are constants. Thus doubling the accelerating voltage halves the minimum wavelength; a voltage-independent or increasing cutoff would violate the maximum available energy.

Step 2: Take logarithms and identify the slope Using the same logarithm base for the two axes,

$$\log \lambda_{\min} = \log\left(\frac{hc}{e}\right) - \log V.$$

This is a straight-line equation with slope -1. The intercept depends on the wavelength and voltage units used, but the negative slope does not. Option (B) shows the decreasing straight line. The horizontal line (A) would imply a cutoff unaffected by voltage, while the positive slopes in (C) and (D) imply that a more energetic beam produces a longer minimum wavelength. All three conflict with the inverse dependence. The continuous spectrum and characteristic lines do not change this energy limit.

Step 3: Conclusion The plot of $\log \lambda_{\min}$ against $\log V$ is a straight line of slope -1. Hence, the correct option is (B).

Quick Tip to Solve: For a log-log plot, first write the power law: $\lambda_{\min} \propto V^{-1}$. Its exponent is the graph's slope.

40. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0 – 10V is: [4 marks]

(A) $4.005 \times 10^3\ \Omega$

(B) $1.985 \times 10^3\ \Omega$

(C) $2.535 \times 10^3\ \Omega$

(D) $2.045 \times 10^3\ \Omega$

Correct Answer: (B) $1.985 \times 10^3\ \Omega$

Solution:

Concept:

- A galvanometer becomes a voltmeter by adding a large resistance in series. At the desired full-scale voltage, the current must equal the galvanometer's full-scale current I_g.
- The voltage across the series combination is $V = I_g(R + G)$, where G is coil resistance and R is the added resistance. Thus $R = V/I_g - G$.

Step 1: Calculate the required total resistance Convert the current before using Ohm's law:

$$I_g = 5\,\text{mA} = 5 \times 10^{-3}\,\text{A}.$$

The full-scale voltage is $10\,\text{V}$, so the total resistance required is

$$R + G = \frac{10}{5 \times 10^{-3}} = 2000\,\Omega.$$

This total includes the galvanometer coil; it is not the resistance of the external multiplier alone.

Step 2: Subtract the coil resistance and check With $G = 15\,\Omega$, the additional series resistance is

$$R = 2000 - 15 = 1985\,\Omega = 1.985 \times 10^3\,\Omega.$$

As a direct check, the coil drop is $0.005 \times 15 = 0.075\,\text{V}$, and the series resistor drops $0.005 \times 1985 = 9.925\,\text{V}$; together they give $10.000\,\text{V}$. The other options would give different total resistances and hence would not produce full-scale deflection at $10\,\text{V}$. A parallel shunt is used for an ammeter conversion, rather than this voltage-range extension.

Step 3: Conclusion The required series resistance is $1.985 \times 10^3\,\Omega$. Hence, the correct option is (B).

Quick Tip to Solve: Compute V/I_g first, then subtract the coil resistance; the voltmeter range applies to both resistances together.

41. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is: [4 marks]

(A) 2

(B) 16

(C) 32

(D) 24

Correct Answer: (C) 32

Solution:

Concept:

- Identical capacitors in series share the applied voltage equally. A string of n capacitors, each of capacitance C, has equivalent capacitance C/n.
- Parallel strings have the same applied voltage and their capacitances add. The design must satisfy the voltage rating as well as the required total capacitance.

Step 1: Meet the working-voltage requirement The supply is $1.0\,\text{kV} = 1000\,\text{V}$. If a series string contains n identical capacitors, each experiences $1000/n$ volts. Therefore

$$\frac{1000}{n} \le 300 \quad \Rightarrow \quad n \ge \frac{10}{3}.$$

The smallest integer is $n = 4$, giving $250\,\text{V}$ across each capacitor. Three in series would give approximately $333\,\text{V}$ each, exceeding the permitted value.

Step 2: Build the required capacitance with parallel strings A four-capacitor string has capacitance

$$C_s = \frac{1}{4}\,\mu\text{F}.$$

To obtain $2\,\mu$F, connect m such strings in parallel:

$$m = \frac{2}{1/4} = 8, \qquad N = mn = 8 \times 4 = 32.$$

Using more capacitors per identical series string lowers its capacitance, requiring still more parallel strings: in general $m = 2n$ and $N = 2n^2$. Its smallest allowed value occurs at $n = 4$. Two capacitors cannot withstand the supply, while sixteen or twenty-four do not provide both the required capacitance and the voltage rating in the series-parallel arrangement.

Step 3: Conclusion Eight parallel strings, each containing four capacitors in series, require 32 capacitors. Hence, the correct option is (C).

Quick Tip to Solve: Choose the series count from the voltage rating first; only then choose the parallel count from the required capacitance.

42. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of : [4 marks]

(A) $\frac{1}{81}$
(B) 9
(C) $\frac{1}{9}$
(D) 81

Correct Answer: (B) 9

Solution:

Concept:

- Compressive stress is the supported force divided by load-bearing cross-sectional area: $\sigma = F/A$. For a standing person, the relevant force scales with body weight.
- For geometrically similar bodies enlarged by a linear factor k, volume scales as k^3 while area scales as k^2. At unchanged density, mass is proportional to volume.

Step 1: Scale the supporting force Let the original volume, density and mass be V, ρ, M. Every linear dimension becomes nine times its former value, so

$$V' = 9^3 V = 729V.$$

Since density is unchanged, $M' = \rho V' = 729M$. The gravitational acceleration has not changed, so weight and the load carried by the legs both grow by the same factor 729. If the person stands symmetrically, each leg supports a fixed fraction of the weight before and after enlargement; that fraction cancels in the ratio.

Step 2: Scale the area and form the stress ratio The leg's cross-sectional area involves two linear dimensions. Thus

$$A' = 9^2 A = 81A.$$

Combining the force and area changes,

$$\frac{\sigma'}{\sigma} = \frac{F'/F}{A'/A} = \frac{729}{81} = 9.$$

The stress increases even though the leg becomes thicker, because its load grows as volume, faster than its area. Option (D), 81, is the area factor rather than the stress factor. The reciprocal factors in (A) and (C) predict a decrease and reverse the force-to-area dependence. No elastic modulus is needed because the question asks for stress, not strain.

Step 3: Conclusion The stress in the leg becomes 9 times its original value. Hence, the correct option is (B).

Quick Tip to Solve: For similar bodies of the same density, supporting stress scales as volume divided by area, hence directly as the linear scale.

---

43. A body of mass m = 10^{-2} kg is moving in a medium and experiences a frictional force F = –kv^2. Its initial speed is v_0 = 10 ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be: [4 marks]

(A) 10^{-1} kg m^{-1} s^{-1}

(B) 10^{-3} kg m^{-1}

(C) 10^{-3} kg s^{-1}

(D) 10^{-4} kg m^{-1}

Correct Answer: (D) 10^{-4} kg m^{-1}

Solution:

Concept:

- Newton's second law along the motion gives $m\,dv/dt = -kv^2$. The minus sign expresses deceleration; k is positive.
- Kinetic energy is $K = mv^2/2$. Since the force depends on speed, acceleration is not constant and a constant-acceleration formula cannot be used.

Step 1: Infer the final speed from the stated energy The energy after ten seconds is one quarter of the initial kinetic energy, rather than one eighth of it: the initial energy already contains a factor of one half. Thus

$$\frac{1}{2}mv^2 = \frac{1}{8}mv_0^2 \quad \Longrightarrow \quad v = \frac{v_0}{2} = 5\,\mathrm{m\,s^{-1}}.$$

The positive root is appropriate because drag slows the body without reversing its motion.

Step 2: Integrate the speed-dependent equation Separate the variables and integrate from the initial speed to the final speed:

$$\int_{v_0}^{v} \frac{dv}{v^2} = -\frac{k}{m}\int_0^t dt.$$

Consequently, $1/v - 1/v_0 = kt/m$. Substitution gives

$$k = \frac{10^{-2}}{10}\left(\frac{1}{5} - \frac{1}{10}\right) = 10^{-4}\,\mathrm{kg\,m^{-1}}.$$

As a check, $kv_0t/m = 1$, so the exact formula $v = v_0/(1 + kv_0t/m)$ indeed halves the speed.

Step 3: Check dimensions and eliminate the other choices From $F = kv^2$, the dimensions of k are force divided by speed squared, namely $\mathrm{kg\,m^{-1}}$. This eliminates (A), which has an extra inverse second, and (C), which has the wrong length and time dimensions. Option (B) has the right dimensions but is ten times too large and would reduce the speed to $v_0/11$, not $v_0/2$.

Step 4: Conclusion The drag coefficient is $k = 10^{-4}\,\mathrm{kg\,m^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert the energy ratio to a speed ratio first; for quadratic drag, the reciprocal speed increases linearly with time.

44. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by: (Given : room temperature = 30°C, specific heat of copper = 0.1 cal/gm°C) [4 marks]

(A) 825°C

(B) 800°C

(C) 885°C

(D) 1250°C

Correct Answer: (C) 885°C

Solution:

Concept:

- In an insulated mixing experiment, heat lost by the hot ball equals the heat gained by both the water and the calorimeter.
- Sensible heat is $Q = mc\Delta T$. Here $c_{\mathrm{Cu}} = 0.1\,\mathrm{cal\,g^{-1}\,^\circ C^{-1}}$ and $c_{\mathrm{water}} = 1\,\mathrm{cal\,g^{-1}\,^\circ C^{-1}}$.

Step 1: Calculate the heat absorbed by the cold contents The copper vessel and water begin at $30^\circ\mathrm{C}$ and reach the common equilibrium temperature $75^\circ\mathrm{C}$. Their rise is therefore $45^\circ\mathrm{C}$. The two contributions must both be included:

$$Q_{\mathrm{water}} = 170(1)(45) = 7650\,\mathrm{cal},$$

$$Q_{\mathrm{vessel}} = 100(0.1)(45) = 450\,\mathrm{cal}.$$

Thus the total heat gained is $8100\,\mathrm{cal}$.

Step 2: Equate the loss of the ball to this gain The ball has heat capacity $100(0.1) = 10\,\mathrm{cal\,^\circ C^{-1}}$. It

cools from T to 75°C, so energy conservation gives

$$100(0.1)(T - 75) = 8100.$$

Dividing by ten yields $T - 75 = 810$, and hence $T = 885^\circ\text{C}$. The ball must start well above the final temperature because its heat capacity is much smaller than that of the cold water and vessel together.

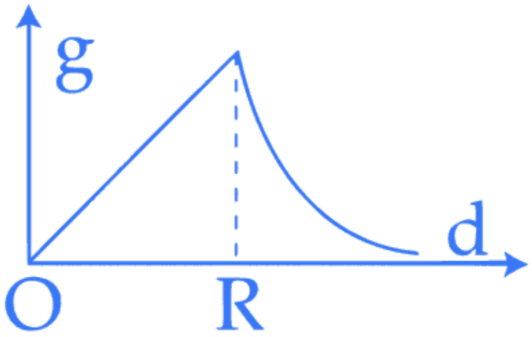
Step 3: Test the numerical alternatives At 885°C, the heat loss is exactly $10(885 - 75) = 8100\,\text{cal}$, matching the required gain. Options (A), (B) and (D) would give losses of 7500, 7250 and $11750\,\text{cal}$, respectively, so none satisfies the heat balance. No latent-heat term is required because the stated final water temperature is below its boiling point.

Step 4: Conclusion The initial ball temperature is 885°C. Hence, the correct option is (C).

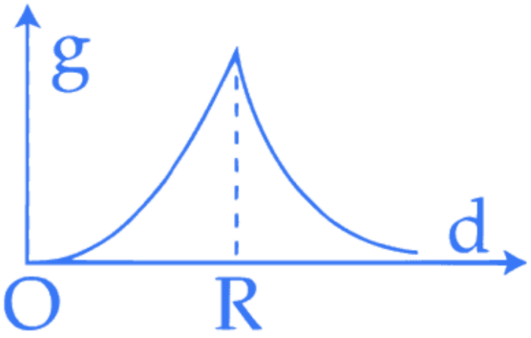
Quick Tip to Solve: Treat the calorimeter as another cold body: add its heat capacity to that of the water before solving for the hot body temperature.

45. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius): [4 marks]

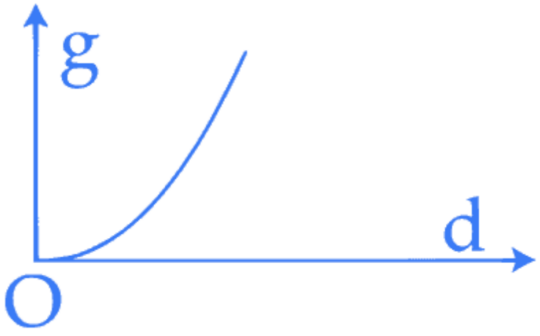
(A)



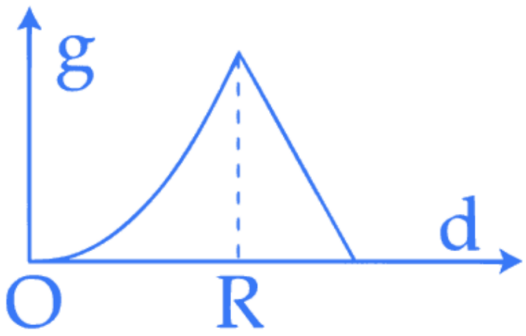
(B)



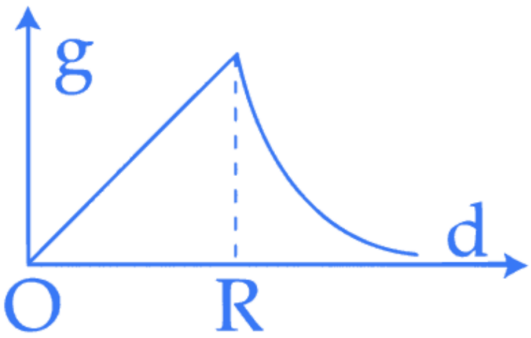
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- For the standard spherical, uniform-density Earth model, only the mass enclosed within radius d contributes to the net gravitational field there.
- Outside a spherical Earth, the field is the same as that of its entire mass concentrated at its centre: $g = GM/d^2$.

Step 1: Find the interior dependence Let the Earth have total mass M, radius R, and uniform density. At a point with $0 < d < R$, the enclosed mass is proportional to the volume:

$$M_d = M\frac{d^3}{R^3}.$$

The spherical shells outside that point exert zero resultant gravitational force. Therefore

$$g(d) = \frac{GM_d}{d^2} = \frac{GM}{R^3}d.$$

The graph starts at zero at the centre and rises as a straight line up to the surface; it does not rise quadratically.

Step 2: Find the exterior dependence and match at the surface For $d \geq R$, the enclosed mass is the fixed total M, so

$$g(d) = \frac{GM}{d^2}.$$

Both formulae give GM/R^2 at $d = R$, so the graph is continuous and reaches its maximum there. Beyond the surface it falls with a curved inverse-square tail, approaching zero only as the distance

becomes arbitrarily large.

Step 3: Compare all four plotted options Option (A) alone shows the required straight interior rise followed by a curved exterior decrease. Option (B) incorrectly curves upward inside the Earth. Option (C) continues increasing with distance instead of decreasing outside the surface. Option (D) has both a curved interior rise and a straight exterior fall that reaches zero at a finite distance; neither feature agrees with the gravitational-field formulae. The sharp change of slope at the surface in (A) is consistent with the ideal uniform-density model.

Step 4: Conclusion The acceleration increases linearly from the centre to R, then decreases as $1/d^2$. Hence, the correct option is (A).

Quick Tip to Solve: Check three features of a gravity graph: zero at the centre, a straight interior rise, and an inverse-square exterior tail.

46. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains 1 $\times$ 10^5 Pa. If N_i and N_f are the number of molecules in the room before and after heating, then $N_f - N_i$ will be : [4 marks]

(A) - 1.61 $\times$ 10^{23}

(B) 1.38 $\times$ 10^{23}

(C) 2.5 $\times$ 10^{25}

(D) - 2.5 $\times$ 10^{25}

Correct Answer: (D) - 2.5 $\times$ 10^{25}

Solution:

Concept:

- The molecular form of the ideal-gas equation is $PV = Nk_BT$, where N counts molecules and $k_B = 1.38 \times 10^{-23}\,J\,K^{-1}$.
- An open room exchanges molecules with the surroundings. At fixed pressure and volume, N is inversely proportional to its absolute temperature.

Step 1: Convert the temperatures and express the change The relevant temperatures are absolute temperatures, not Celsius readings:

$$T_i = 17 + 273 = 290\,K, \qquad T_f = 27 + 273 = 300\,K.$$

Applying the gas law to the same room at the two temperatures gives

$$N_f - N_i = \frac{PV}{k_B}\left(\frac{1}{T_f} - \frac{1}{T_i}\right).$$

Since the final temperature is higher, the bracket is negative. Thus some air leaves the room rather than its molecule count increasing.

Step 2: Substitute the pressure and room volume The product PV is $10^5 \times 30 = 3.0 \times 10^6\,J$.

Consequently,

$$N_f - N_i = \frac{3.0 \times 10^6}{1.38 \times 10^{-23}} \left(\frac{1}{300} - \frac{1}{290} \right)$$
$$= \frac{3.0 \times 10^6}{1.38 \times 10^{-23}} \frac{-10}{87000}$$
$$\simeq -2.50 \times 10^{25}.$$

This is a change in the number of molecules, so the numerical count has no physical unit.

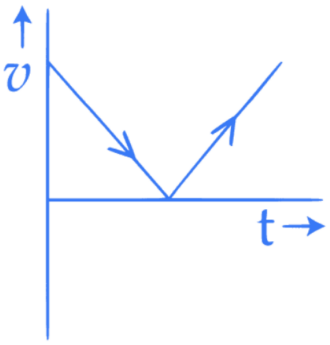
Step 3: Reject alternatives using sign and scale Options (B) and (C) have a positive sign and contradict the loss of air required to maintain atmospheric pressure during heating. Option (A) has the negative sign but its magnitude is too small. For another check, the initial molecule count is about 7.50×10^{26}, and the fraction lost is $1 - 290/300 = 1/30$; their product is again about 2.50×10^{25}.

Step 4: Conclusion The molecule-count change is $N_f - N_i \simeq -2.5 \times 10^{25}$. Hence, the correct option is (D).

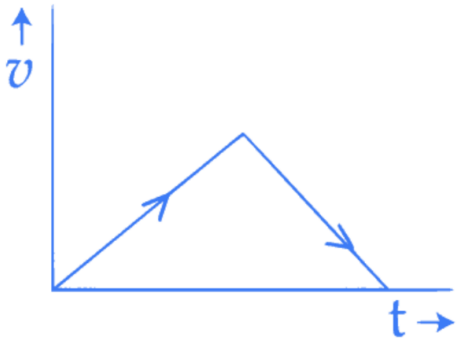
Quick Tip to Solve: For an open room at fixed pressure, use $N_f/N_i = T_i/T_f$; heating expels molecules, so check the sign before calculating.

47. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time? [4 marks]

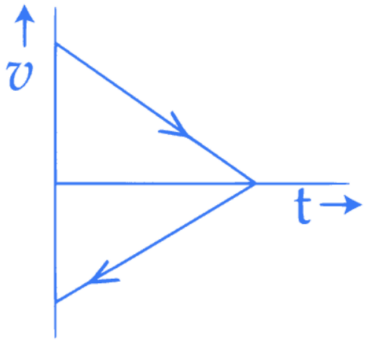
(A)



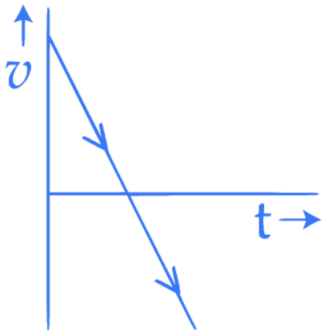
(B)



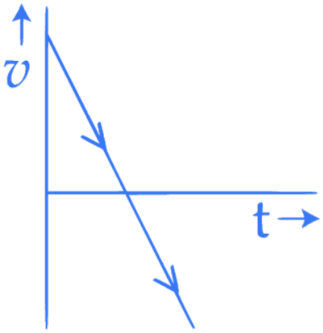
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- Take upward as the positive direction. A vertically projected body has initial velocity $u > 0$ and constant acceleration $a = -g$, neglecting air resistance.
- Constant acceleration implies $v = u - gt$, so the velocity-time graph is a single straight line with negative slope throughout the flight.

Step 1: Follow the velocity during ascent At the instant of projection, $t = 0$, the velocity is the positive value u. Gravity reduces the velocity at a constant rate, giving

$$v(t) = u - gt.$$

At the highest point the velocity becomes zero at $t = u/g$. The acceleration has not become zero there: gravity still acts downward, so the slope of the graph remains $-g$ rather than changing sign.

Step 2: Continue the same relation during descent For times greater than u/g, the expression $u - gt$ is negative. This negative velocity describes downward motion. It is not a negative speed: the magnitude of velocity is positive but its direction has reversed. The graph therefore crosses the time axis and continues into the negative-velocity region along the same straight line, as shown in option (D).

Step 3: Distinguish the other three graphs Option (A) is V-shaped: it represents the magnitude of velocity decreasing and then increasing, and so resembles a speed-time graph rather than the signed velocity-time graph. Option (B) begins at zero and has an increasing segment, contradicting the nonzero upward launch velocity and downward acceleration. Option (C) is not a single-valued velocity function of time over its branches; it also reverses the time direction along the lower branch. Only (D) gives one velocity for each instant and keeps the gravitational acceleration constant across the highest point.

Step 4: Conclusion The correct graph is the straight line decreasing from positive to negative velocity with slope $-g$. Hence, the correct option is (D).

Quick Tip to Solve: Velocity changes sign at the highest point, while acceleration stays downward; extend the same straight line through the time axis.

48. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be: [4 marks]

(A) 18 J

(B) 4.5 J

(C) 22 J

(D) 9 J

Correct Answer: (B) 4.5 J

Solution:

Concept:

- Newton's law gives $a(t) = F(t)/m$. When force depends on time, integrate acceleration to obtain velocity before applying the work-energy theorem.
- The net work is $W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$. The particle starts from rest, so its initial kinetic energy vanishes.

Step 1: Obtain the velocity after one second The mass is $1\,\text{kg}$ and the force is $F(t) = 6t$ in SI units. Therefore

$$a(t) = 6t, \qquad v(t) - v(0) = \int_0^t 6\tau\, d\tau = 3t^2.$$

The initial velocity is zero, and hence

$$v(1) = 3\,\text{m}\,\text{s}^{-1}.$$

Using the final force as a constant force over the entire interval would give the wrong impulse and therefore the wrong final speed.

Step 2: Apply the work-energy theorem The applied force supplies the change in kinetic energy:

$$W = \frac{1}{2}(1)(3)^2 = 4.5\,\text{J}.$$

This value also follows directly from the power. Since instantaneous power is Fv,

$$W = \int_0^1 (6t)(3t^2)\,dt = 18\int_0^1 t^3\,dt = \frac{18}{4} = 4.5\,\text{J}.$$

Both methods account for the continuously changing force and velocity and give the same energy transfer.

Step 3: Check the alternatives and the interpretation Option (B) is the only value equal to the final kinetic energy. The values $18\,\text{J}$, $22\,\text{J}$, and $9\,\text{J}$ in (A), (C), and (D) would require final speeds different from the $3\,\text{m s}^{-1}$ fixed by the impulse integral. In particular, omitting the one-half in kinetic energy gives $9\,\text{J}$, explaining the tempting distractor (D).

Step 4: Conclusion The work during the first second is $4.5\,\text{J}$. Hence, the correct option is (B).

Quick Tip to Solve: For $F(t)$, integrate F/m to find the final speed, then use the kinetic-energy change; do not multiply the final force by a displacement.

49. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio $\frac{l}{R}$ such that the moment of inertia is minimum? [4 marks]

(A) $\frac{3}{\sqrt{2}}$

(B) $\sqrt{\frac{3}{2}}$

(C) $\frac{\sqrt{3}}{2}$

(D) 1

Correct Answer: (B) $\sqrt{\frac{3}{2}}$

Solution:

Concept:

- For a uniform solid cylinder of mass M, radius R, and length l, the moment of inertia about a central axis perpendicular to its symmetry axis is $I = M(3R^2 + l^2)/12$.
- The standard shape optimisation compares cylinders made from the same fixed quantity of material: mass and density are fixed, so their volume $\pi R^2 l$ is constant. Without such a constraint an overall size change would not select a shape ratio.

Step 1: Use the volume constraint to retain one variable Write $C = R^2 l$, a positive constant fixed by the cylinder volume. Then $l = C/R^2$, and the transverse moment becomes

$$I(R) = \frac{M}{12}\left(3R^2 + \frac{C^2}{R^4}\right).$$

A larger radius increases the radial contribution but, at fixed volume, shortens the cylinder and

decreases the length contribution. The minimum comes from balancing these two changes rather than varying only one dimension while holding the other fixed.

Step 2: Differentiate and test the stationary point With M and C fixed,

$$\frac{dI}{dR} = \frac{M}{12}\left(6R - \frac{4C^2}{R^5}\right).$$

Setting the derivative to zero gives $6R^6 = 4C^2$. Since $C^2 = R^4 l^2$, this reduces to

$$6R^2 = 4l^2 \quad \Longrightarrow \quad \frac{l}{R} = \sqrt{\frac{3}{2}}.$$

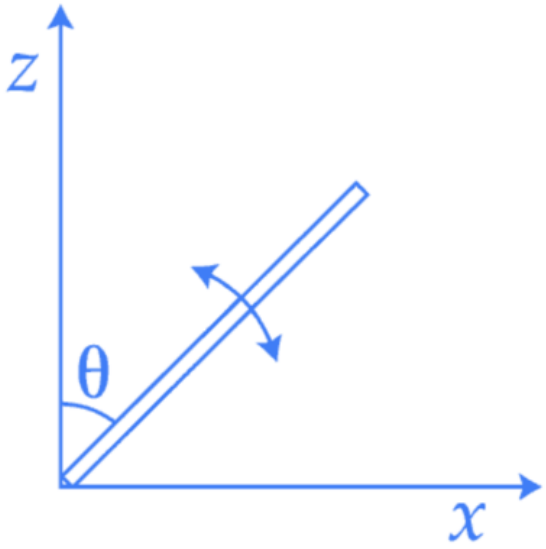
The second derivative is $M(6 + 20C^2/R^6)/12 > 0$; thus the stationary point is a minimum. Moreover, the inertia diverges at either extreme $R \to 0$ or $R \to \infty$, so it is the unique minimum under this constraint.

Step 3: Compare the proposed ratios The condition is $(l/R)^2 = 3/2$. Squaring the ratios in (A), (C), and (D) gives $9/2$, $3/4$, and 1, respectively, so none satisfies the stationary condition. Option (B) gives precisely $3/2$ on squaring and therefore selects the minimum for a cylinder of fixed volume and mass.

Step 4: Conclusion For a fixed quantity of uniform cylinder material, the minimum occurs at $l/R = \sqrt{3/2}$. Hence, the correct option is (B).

Quick Tip to Solve: Before minimising an inertia with two dimensions, enforce the fixed-volume relation $R^2 l =$ constant; otherwise no unique shape ratio follows.

50. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is [4 marks]



(A) $\frac{2g}{3l}\cos\theta$

(B) $\frac{3g}{2l}\sin\theta$

(C) $\frac{2g}{3l}\sin\theta$

(D) $\frac{3g}{3l}\sin\theta$

Correct Answer: (B) $\frac{3g}{2l}\sin\theta$

Solution:

Concept:

- A rigid body rotating about a fixed axis satisfies $\tau = I\alpha_{\text{ang}}$, where I is its moment of inertia about that axis.
- For a uniform rod pivoted at an end, $I = Ml^2/3$; its weight acts at the centre of mass, a distance $l/2$ from the pivot.

Step 1: Find the moment arm of the weight The rod makes angle θ with the upward vertical. Gravity acts vertically downward through the midpoint. The perpendicular distance from the pivot to this downward line of action is therefore $(l/2)\sin\theta$, not $(l/2)\cos\theta$. The force at the pivot has zero moment about the pivot, and friction there is negligible. Thus the magnitude of the gravitational torque is

$$\tau = Mg\frac{l}{2}\sin\theta.$$

It acts to increase the displacement from the upright position.

Step 2: Apply the equation for angular acceleration Use the moment of inertia about the fixed end,

rather than the value about the centre of the rod. Dividing the torque by this inertia gives

$$\alpha_{\text{ang}} = \frac{Mg(l/2)\sin\theta}{Ml^2/3} = \frac{3g}{2l}\sin\theta.$$

The mass cancels because both the torque and rotational inertia are proportional to it. At $\theta = 0$, this expression vanishes, as it must when the centre of mass is directly above the pivot. The exactly upright position is unstable: an arbitrarily small displacement initiates the falling rotation.

Step 3: Compare the four choices Option (A) has the wrong trigonometric dependence and incorrectly predicts a nonzero torque in the exactly vertical position. Options (C) and (D) have the sine dependence but incorrect coefficients, caused by using an incorrect moment arm or moment of inertia. Only option (B) agrees with both the torque calculation and the dimensions g/l of angular acceleration.

Step 4: Conclusion The angular acceleration has magnitude $\frac{3g}{2l}\sin\theta$, directed away from the unstable upright position. Hence, the correct option is (B).

Quick Tip to Solve: When finding the angular acceleration of a hinged rod, use its centre of mass for the torque and its fixed-end moment of inertia for the rotation.

51. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by: [4 marks]

(A) $\frac{P}{3\alpha K}$
(B) $\frac{P}{\alpha K}$
(C) $\frac{3\alpha}{PK}$
(D) 3PKα

Correct Answer: (A) $\frac{P}{3\alpha K}$

Solution:

Concept:

- The bulk modulus is $K = -\Delta P/(\Delta V/V)$, so an applied pressure P produces volume strain $-P/K$.
- For a freely expanding isotropic solid, the volume expansion coefficient is $\beta = 3\alpha$, to first order in the small strain.

Step 1: Calculate the compression caused by pressure Let V_0 denote the original volume at 0°C. The pressure acts equally from all sides, so the appropriate elastic constant is the bulk modulus, not Young's modulus. Within linear elasticity,

$$\frac{\Delta V_{\text{pressure}}}{V_0} = -\frac{P}{K}.$$

The minus sign records a decrease in volume. Since all three lengths contract equally, returning the

volume to V_0 also restores the cube's original side length.

Step 2: Balance compression with thermal expansion Let the required temperature increase be ΔT, keeping the applied pressure in place. Heating produces the positive volume strain $3\alpha\Delta T$. To restore the original size, the net strain must vanish:

$$-\frac{P}{K} + 3\alpha\Delta T = 0.$$

Consequently,

$$\Delta T = \frac{P}{3\alpha K}.$$

This uses the usual small-strain approximation, ignoring products of thermal and pressure strains and treating the material constants as constant over the temperature change.

Step 3: Check units and reject the remaining options The ratio P/K is dimensionless, while α has units of inverse temperature; the result is therefore a temperature interval. Option (B) omits the factor of three that converts linear expansion into volume expansion. Option (C) has an inverse pressure dependence and does not have temperature units. Option (D) also has incorrect dimensions and would increase with the bulk modulus, whereas a less compressible material needs less compensating heating. Temperature intervals have the same numerical value in kelvin and degrees Celsius.

Step 4: Conclusion The temperature should be raised by $\Delta T = \frac{P}{3\alpha K}$. Hence, the correct option is (A).

Quick Tip to Solve: Match the elastic strain with the same type of thermal strain: bulk modulus concerns volume, so pair it with 3α, not α.

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52. A particle A of mass m and initial velocity v collides with a particle B of mass m/2 which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is: [4 marks]

(A) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$
(B) $\frac{\lambda_A}{\lambda_B} = 2$
(C) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$
(D) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

Correct Answer: (B) $\frac{\lambda_A}{\lambda_B} = 2$

Solution:

Concept:

- In a head-on elastic collision, linear momentum and kinetic energy are conserved; the relative speed of separation equals the relative speed of approach.
- The de Broglie wavelength is $\lambda = h/p$, where p is the magnitude of the particle's momentum after the collision.

Step 1: Find the velocities after the elastic collision Write u_A, u_B for the final velocities in the direction

of A's original motion. Conservation of momentum gives

$$mv = mu_A + \frac{m}{2}u_B, \qquad v = u_A + \frac{u_B}{2}.$$

For a head-on elastic collision, the relative speed reverses, so

$$u_B - u_A = v.$$

Substituting $u_B = u_A + v$ into the momentum equation yields $v = 3u_A/2 + v/2$. Hence

$$u_A = \frac{v}{3}, \qquad u_B = \frac{4v}{3}.$$

Both velocities are positive: the lighter target can move faster than the incoming particle without violating energy conservation.

Step 2: Calculate the momenta and wavelength ratio The final momenta, which determine the wavelengths, are

$$p_A = m\frac{v}{3} = \frac{mv}{3}, \qquad p_B = \frac{m}{2}\frac{4v}{3} = \frac{2mv}{3}.$$

Therefore

$$\frac{\lambda_A}{\lambda_B} = \frac{h/p_A}{h/p_B} = \frac{p_B}{p_A} = 2.$$

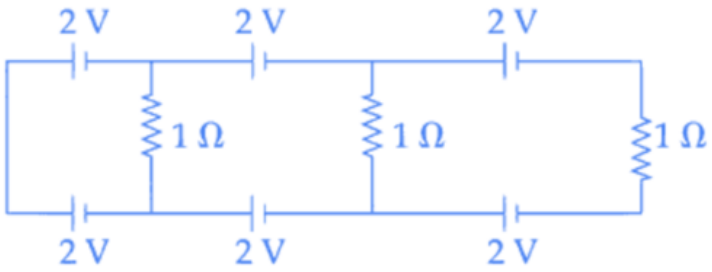
The factor h cancels. Comparing inverse speeds alone would be wrong because the masses of the two particles differ.

Step 3: Check the conservation laws and the choices The momenta add to mv, as required. The final kinetic energies are $mv^2/18$ and $4mv^2/9$, whose sum is $mv^2/2$, the initial kinetic energy. Thus the collision calculation is consistent. Option (D) is the reciprocal wavelength ratio. Option (A) confuses the speed reduction of A with a wavelength ratio. Option (C) is also inconsistent with the final momentum ratio. Only option (B) gives the larger wavelength for A, whose final momentum is the smaller one.

Step 4: Conclusion The ratio of the final de Broglie wavelengths is $\lambda_A/\lambda_B = 2$. Hence, the correct option is (B).

Quick Tip to Solve: For unequal masses, first find the final momenta; wavelength ratios are inverse momentum ratios, not simply inverse velocity ratios.

53. In the given circuit, the current in each resistance is: [4 marks]



(A) 0 A

(B) 1 A

(C) 0.25 A

(D) 0.5 A

Correct Answer: (A) 0 A

Solution:

Concept:

- An ideal cell fixes the potential difference between its terminals according to its emf and polarity.
- Ohm's law gives $I = \Delta V/R$; a resistor joining equipotential points carries no current even when cells are present elsewhere in the circuit.

Step 1: Assign potentials along the upper and lower branches Take the common left-hand vertical wire as the reference node of potential zero. Every cell in each horizontal branch has emf $2\,\mathrm{V}$, and the corresponding upper and lower cells have the same polarity. Following either branch to the right therefore produces the same sequence of potential changes. With the positive plates drawn on the left, the potentials after successive cells are

$$V_1 = -2\,\mathrm{V}, \qquad V_2 = -4\,\mathrm{V}, \qquad V_3 = -6\,\mathrm{V}.$$

The important point is equality between the upper and lower nodes at each position, rather than the chosen zero or sign convention.

Step 2: Find the voltage across each resistor Each vertical $1\,\Omega$ resistor joins an upper node to the lower node after exactly the same number of equally oriented cells. Its endpoints therefore have equal potentials. Explicitly,

$$\Delta V_1 = (-2) - (-2) = 0,$$

$$\Delta V_2 = (-4) - (-4) = 0, \qquad \Delta V_3 = (-6) - (-6) = 0.$$

Applying Ohm's law separately to the three resistors gives

$$I_1 = I_2 = I_3 = \frac{0}{1} = 0\,\mathrm{A}.$$

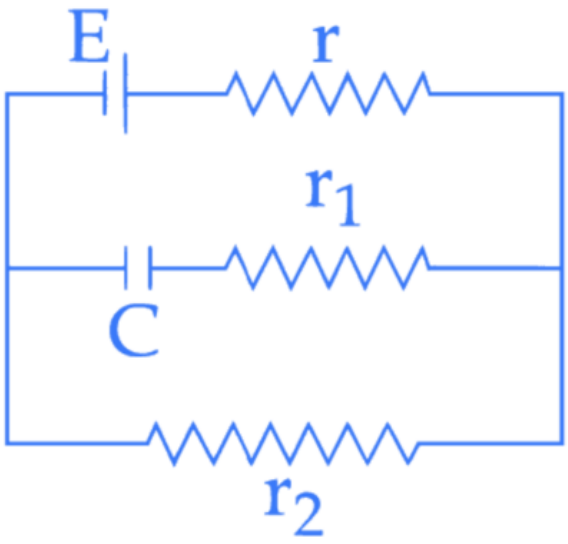
There is no source of net emf around a loop containing corresponding upper and lower cells, since their contributions oppose on traversing the closed loop.

Step 3: Rule out nonzero current choices Options (B), (C) and (D) would require voltage drops of 1, 0.25 and $0.5\,\text{V}$, respectively, across a $1\,\Omega$ resistor. None of these is compatible with the node potentials fixed by the ideal cells. Simply adding all the cell emfs and dividing by a sum of resistances treats the network as one series loop, which it is not.

Step 4: Conclusion Each resistance has zero potential difference across it, so the current in each is $0\,\text{A}$. Hence, the correct option is (A).

Quick Tip to Solve: Before combining resistances, label node potentials fixed by ideal cells; matching cell chains often make the connecting resistors equipotential.

54. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be: [4 marks]



(A) $CE\frac{r_1}{(r_1+r)}$
(B) CE
(C) $CE\frac{r_1}{(r_2+r)}$
(D) $CE\frac{r_2}{(r+r_2)}$
Correct Answer: (D) $CE\frac{r_2}{(r+r_2)}$

Solution:

Concept:

- At direct-current steady state, the current through an ideal capacitor is zero; its branch behaves as an open circuit.
- The charge magnitude is $Q = CV_C$. A series resistor in the open capacitor branch has no voltage drop when its branch current is zero.

Step 1: Identify the conducting loop at steady state The capacitor C and resistor r_1 form the middle branch between the left and right nodes. Once the capacitor is fully charged, no steady current flows

through that branch. The remaining closed loop contains the cell of emf E, resistor r and the lower resistor r_2. Those two resistors carry the same loop current, so Kirchhoff's loop law gives

$$E - Ir - Ir_2 = 0, \qquad I = \frac{E}{r + r_2}.$$

The resistor r_1 can affect how quickly the capacitor charges, but it does not enter this steady loop current.

Step 2: Find the capacitor voltage and charge The voltage magnitude between the left and right nodes is the drop across r_2:

$$V_{\text{nodes}} = Ir_2 = \frac{Er_2}{r + r_2}.$$

Because the middle-branch current is zero, the voltage across r_1 is $0 \times r_1 = 0$. Thus the entire node-to-node voltage appears across the capacitor, giving

$$V_C = \frac{Er_2}{r + r_2}, \qquad Q = \frac{CEr_2}{r + r_2}.$$

The expression is the magnitude of the charge on either plate; the plates carry equal and opposite charges.

Step 3: Test the alternatives with physical limits Options (A) and (C) incorrectly retain r_1, despite there being no steady voltage drop across it. Option (B), CE, is true only in the special limit $r = 0$, rather than for arbitrary r. In the limit $r_2 \to 0$, the two nodes are shorted and the capacitor voltage must vanish; option (D) has precisely this limit. For r_2 much greater than r, it tends to CE, as expected when almost no voltage is lost in r.

Step 4: Conclusion The steady-state charge is $Q = \frac{CEr_2}{r + r_2}$. Hence, the correct option is (D).

Quick Tip to Solve: For a steady-state capacitor, first open its branch, solve the remaining resistor circuit, and then find the potential difference between its terminals.

55. A magnetic needle of magnetic moment 6.7 $\times$ 10^{-2} A m^2 and moment of inertia 7.5 $\times$ 10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is: [4 marks]

(A) 8.76 s

(B) 6.65 s

(C) 8.89 s

(D) 6.98 s

Correct Answer: (B) 6.65 s

Solution:

Concept:

- A magnetic dipole of moment μ in a uniform field B experiences restoring torque $\tau = -\mu B \sin\theta$.

- For small angular displacement, $\sin\theta \simeq \theta$, so the oscillation period is $T = 2\pi\sqrt{I/(\mu B)}$, where I is rotational inertia.

Step 1: Write the angular simple harmonic motion equation The problem specifies simple harmonic oscillations, so use the small-angle restoring torque. With positive displacement measured away from the stable field direction, the equation of rotational motion is

$$I\ddot{\theta} = -\mu B\theta.$$

Comparison with $\ddot{\theta} = -\omega^2\theta$ gives

$$\omega = \sqrt{\frac{\mu B}{I}}, \qquad T = 2\pi\sqrt{\frac{I}{\mu B}}.$$

The magnetic moment is distinct from mass; its product with the magnetic field supplies the coefficient of the restoring torque.

Step 2: Substitute the data carefully Here $\mu = 6.7 \times 10^{-2}\,\mathrm{A\,m^2}$, $B = 0.01\,\mathrm{T}$, and $I = 7.5 \times 10^{-6}\,\mathrm{kg\,m^2}$. Their ratio is

$$\frac{I}{\mu B} = \frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 10^{-2}} = \frac{7.5}{6.7} \times 10^{-2}\,\mathrm{s^2}.$$

Thus

$$T = 2\pi\sqrt{0.011194}\,\mathrm{s} \simeq 0.665\,\mathrm{s}.$$

Taking the square root of the power of ten as well as the numerical factor avoids a factor-of-ten error.

Step 3: Convert the period into the time for ten oscillations The period is the time for one complete oscillation, whereas the question asks for ten. Therefore

$$t_{10} = 10T \simeq 10(0.665) = 6.65\,\mathrm{s}.$$

The other offered times, 8.76, 8.89 and $6.98\,\mathrm{s}$, do not follow from the stated moment, field and inertia. In particular, there is no additional factor of two for a complete oscillation: the factor 2π in the period already accounts for the full cycle.

Step 4: Conclusion The time for ten complete oscillations is approximately $6.65\,\mathrm{s}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep magnetic moment and moment of inertia separate, compute one period first, and multiply by the specified number of complete oscillations only at the end.

56. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E_1} = E\hat{i}$, it experiences a torque $\vec{T_1} = \tau\hat{k}$. When subjected to another electric field $\vec{E_2} = \sqrt{3}E_1\hat{j}$ it experiences a torque $\vec{T_2} = -\vec{T_1}$. The angle θ is: [4 marks]

(A) 90°

(B) 45°

(C) 30°

(D) 60°

Correct Answer: (D) 60°

Solution:

Concept:

- The torque on an electric dipole is $\vec{T} = \vec{p} \times \vec{E}$, so both field direction and the sign of the cross product matter.
- For a dipole in the xy-plane, write $\vec{p} = p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}$, with $\hat{i} \times \hat{j} = \hat{k}$.

Step 1: Calculate the torque in the first field The first field is $\vec{E}_1 = E\hat{i}$. The component of the dipole parallel to $\hat{i}$ contributes no torque; only its y-component does. Since $\hat{j} \times \hat{i} = -\hat{k}$,

$$\vec{T}_1 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}) \times E\hat{i} = -pE\sin\theta\,\hat{k}.$$

The sign is fixed by the right-hand rule. The coefficient denoted by τ in the question must be interpreted with this vector sign convention; equating torque magnitudes without identifying the directions can lead to a reciprocal tangent.

Step 2: Use the equal and opposite second torque The second field has magnitude $\sqrt{3}E$ and direction $\hat{j}$. Now only the x-component of the dipole contributes:

$$\vec{T}_2 = \sqrt{3}pE\cos\theta\,\hat{k}.$$

The condition $\vec{T}_2 = -\vec{T}_1$ therefore becomes

$$\sqrt{3}pE\cos\theta = pE\sin\theta.$$

Cancelling the common nonzero factor pE gives

$$\tan\theta = \sqrt{3}.$$

Among the listed positive acute angles, this is $\theta = 60°$.

Step 3: Check the proposed angle and the other choices At $60°$, the two torque magnitudes are $pE\sin 60° = \sqrt{3}pE/2$ and $\sqrt{3}pE\cos 60° = \sqrt{3}pE/2$; their cross products point in opposite directions. At $90°$, the second torque vanishes but the first does not. At $45°$, the second torque is $\sqrt{3}$ times the first in magnitude. At $30°$, the second torque is three times the first. Hence none of options (A), (B) or (C) satisfies the stated vector relation.

Step 4: Conclusion The angle is $\theta = 60°$, for which the two torques have equal magnitudes and opposite directions. Hence, the correct option is (D).

Quick Tip to Solve: Resolve the dipole along the coordinate axes: an x-directed field uses its y-component, while a y-directed field uses its x-component.

57. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = 3×10^8 ms^{-1}) [4 marks]

(A) 15.3 GHz

(B) 10.1 GHz

(C) 12.1 GHz

(D) 17.3 GHz

Correct Answer: (D) 17.3 GHz

Solution:

Concept:

- For electromagnetic waves in vacuum, the measured speed is c in every inertial frame. At an observer speed comparable with c, use the relativistic longitudinal Doppler formula.
- For an observer approaching a stationary source, $f_{\text{obs}} = f\sqrt{(1+\beta)/(1-\beta)}$, where $\beta = v/c$. Approach gives a blue shift, so the observed frequency exceeds the emitted frequency.

Step 1: Identify the relative speed The source frequency is $f = 10\,\text{GHz}$, and the observer moves directly towards it with $v = c/2$. Hence $\beta = 1/2$. The observer encounters the advancing wavefronts more frequently. Because this speed is not small compared with light speed, the ordinary sound-wave observer formula would omit relativistic time dilation.

Step 2: Evaluate the frequency shift Substitute the given speed into the electromagnetic-wave formula:

$$f_{\text{obs}} = 10\sqrt{\frac{1+1/2}{1-1/2}} = 10\sqrt{3} = 17.32\,\text{GHz}.$$

The factor $\sqrt{3}$ includes both the changed rate of encountering wavefronts and the observer clock transformation. The choices 10.1 and 12.1 GHz give shifts too small for $v = c/2$; 15.3 GHz also differs from the relativistic value. Using $f(1+v/c)$ would give 15 GHz, but that classical sound-wave formula is inappropriate here.

Step 3: Conclusion The observer measures approximately $17.3\,\text{GHz}$. Hence, the correct option is (D).

Quick Tip to Solve: When a light-wave Doppler problem gives $v = c/2$, the approaching frequency multiplier is immediately $\sqrt{3}$, not $3/2$.

58. A radioactive nucleus A with a half life T, decays into a nucleus B. At t = 0, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by : [4 marks]

(A) $t = \frac{T}{\log(1.3)}$

(B) $t = T\log(1.3)$

(C) $t = \frac{T}{2}\frac{\log 2}{\log 1.3}$

(D) $t = T\frac{\log 1.3}{\log 2}$

Correct Answer: (D) $t = T\frac{\log 1.3}{\log 2}$

Solution:

Concept:

- Radioactive parent nuclei obey $N_A = N_0 e^{-\lambda t}$, with decay constant $\lambda = \ln 2/T$ for half-life T.
- With no daughter nuclei initially and one daughter produced per parent decay, $N_B = N_0 - N_A$. Therefore the daughter-to-parent ratio measures the fraction transformed relative to the parents remaining.

Step 1: Use conservation of the nuclear count The given ratio refers to the daughter and the remaining parent at the same instant, not to the initial population. Thus

$$\frac{N_B}{N_A} = \frac{N_0 - N_A}{N_A} = 0.3.$$

Adding one gives $N_0/N_A = 1.3$, or $N_A = N_0/1.3$. About 76.9 percent of the initial parents remain, so the elapsed time must be less than one half-life.

Step 2: Insert the exponential decay law Using $N_0/N_A = e^{\lambda t}$, we obtain

$$e^{\lambda t} = 1.3, \qquad \lambda t = \ln 1.3.$$

Since $\lambda = \ln 2/T$,

$$t = T\frac{\ln 1.3}{\ln 2} \simeq 0.379T.$$

Any common logarithm base gives the same ratio. Option (A) has the wrong reciprocal dependence, and (B) omits the conversion between half-life and decay constant. Option (C) inverts the logarithmic ratio and includes an unjustified factor of one half; it also conflicts with the substantial parent population that remains.

Step 3: Conclusion The elapsed time is $t = T\frac{\log 1.3}{\log 2}$. Hence, the correct option is (D).

Quick Tip to Solve: For an initially pure parent sample, set $1 + N_B/N_A = 2^{t/T}$; this avoids confusing the daughter fraction with the daughter-to-parent ratio.

59. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be: [4 marks]

(A) 180°

(B) 45°

(C) 90°

(D) 135°

Correct Answer: (A) 180°

Solution:

Concept:

- An n-p-n transistor biased in the active region has a collector current that increases when its base-emitter input voltage increases.
- In a common-emitter amplifier, the collector output voltage is $V_C = V_{CC} - I_C R_C$. An increased collector current increases the drop across R_C and therefore decreases the collector potential.

Step 1: Track a positive input excursion Consider a small positive change in the base-emitter input voltage around the operating point. Forward bias of the base-emitter junction increases, giving a positive change in base current. In the active region, $\Delta I_C = \beta \Delta I_B$, so collector current also increases. This is the current-amplifying action; it does not itself introduce an opposite sign between base and collector currents.

Step 2: Determine the output voltage polarity The output is the collector voltage relative to the common emitter. Since the supply voltage is fixed,

$$\Delta V_{\text{out}} = -R_C \Delta I_C.$$

A positive input excursion consequently produces a negative output excursion. A negative input excursion reverses these changes, producing a positive output excursion. Thus a sinusoidal output has the opposite sign throughout each cycle, corresponding to half a cycle or $180°$. The values $45°$, $90°$, and $135°$ do not represent complete inversion in the usual amplifier operating range.

Step 3: Conclusion The common-emitter amplifier inverts the input voltage, giving a phase difference of $180°$. Hence, the correct option is (A).

Quick Tip to Solve: For a common-emitter stage, remember the minus sign in $\Delta V_C = -R_C \Delta I_C$; it directly establishes voltage phase inversion.

60. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave? [4 marks]

(A) ω_m

(B) ω_c

(C) ω_m + ω_c

(D) ω_c - ω_m

Correct Answer: (A) ω_m

Solution:

Concept:

- In ordinary amplitude modulation, the amplitude of a high-frequency carrier changes in proportion to the message: $s(t) = A_c[1 + \mu\cos(\omega_m t)]\cos(\omega_c t)$, where μ is the modulation index.
- The identity $2\cos a\cos b = \cos(a+b) + \cos(a-b)$ resolves the product into upper and lower sideband frequencies.

Step 1: Expand the modulated signal Multiplying the envelope and carrier gives

$$s(t) = A_c\cos(\omega_c t) + \mu A_c\cos(\omega_m t)\cos(\omega_c t).$$

The first term is the carrier. Apply the product-to-sum identity to the second term:

$$\begin{aligned} s(t) = A_c\cos(\omega_c t) &\\ &+ \frac{\mu A_c}{2}\cos[(\omega_c+\omega_m)t] \\ &+ \frac{\mu A_c}{2}\cos[(\omega_c-\omega_m)t]. \end{aligned}$$

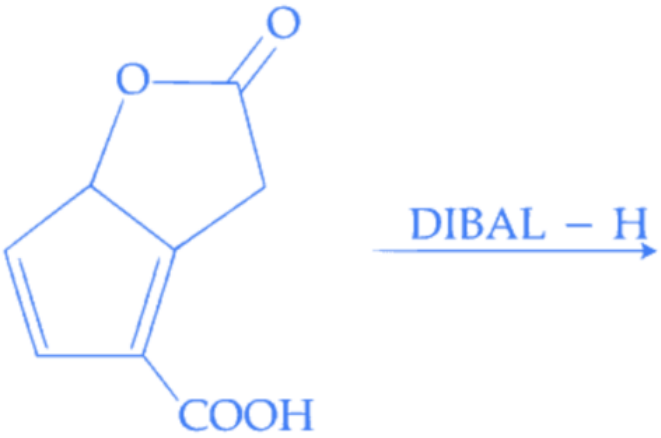
Step 2: Read the frequency components The expansion contains only the carrier ω_c, upper sideband $\omega_c + \omega_m$, and lower sideband $\omega_c - \omega_m$. Therefore options (B), (C), and (D) are all present in the transmitted AM wave. There is no separate term proportional to $\cos(\omega_m t)$, so the message frequency by itself is absent from this radio-frequency spectrum. The message is encoded in the envelope and can be recovered by detection; that recovery does not mean it was an independent frequency component of the transmitted wave.

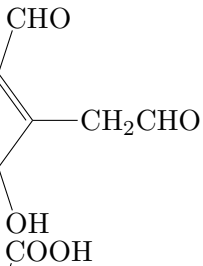
Step 3: Conclusion The frequency not contained as a separate component in the modulated wave is ω_m. Hence, the correct option is (A).

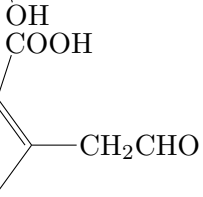
Quick Tip to Solve: Expand the AM product before choosing: the spectrum has carrier and two sidebands; the message alone appears after demodulation.

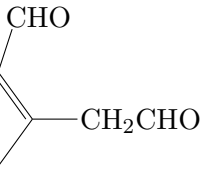
Chemistry (120 marks)

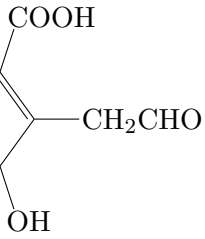
61. **The major product obtained in the following reaction is :** [4 marks]

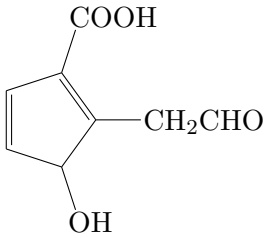


(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- DIBAL-H is a controlled hydride reducing reagent. Partial reduction of an ester carbonyl gives an aldehyde after work-up; for a cyclic ester, the corresponding product can be written as a hydroxy aldehyde.
- The substrate contains a lactone ring fused to a cyclopentadiene ring and a separate carboxylic acid group. The product options must be compared for retention of the ring skeleton, double bonds, and carboxylic acid.

Step 1: Reduce the lactone carbonyl The carbonyl in the oxygen-containing five-membered ring is an ester carbonyl. DIBAL-H transfers hydride to this carbonyl. On work-up, partial ester reduction gives an aldehyde at the former acyl carbon. Opening the cyclic ester produces a side chain $-CH_2CHO$ attached to the cyclopentadiene skeleton. The oxygen originally bonded to the saturated ring carbon becomes an $-OH$ substituent at that carbon; it is not simply removed from the molecule.

Step 2: Match all functional groups The selective product therefore retains the two carbon-carbon double bonds and the carboxylic acid substituent, while the lactone becomes an alcohol plus aldehyde. Option (D) has exactly this combination: a ring $-OH$, the $-CH_2CHO$ side chain, and an unchanged $-COOH$ group. Option (A) additionally converts the acid substituent into $-CHO$. Options (B) and (C) both omit the hydroxyl group derived from the lactone oxygen; (C) also changes the acid to an aldehyde. These do not describe the controlled lactone reduction shown.

Step 3: Conclusion The major product is the hydroxy aldehyde with the carboxylic acid group retained, shown in option (D). Hence, the correct option is (D).

Quick Tip to Solve: For a lactone reduced by DIBAL-H, track both ester oxygens: the carbonyl becomes an aldehyde and the ring oxygen gives the alcohol end after opening.

62. Which of the following reactions is an example of a redox reaction? [4 marks]

(A) $XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$

(B) $XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$

(C) $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$

(D) $XeF_2 + PF_5 \rightarrow [XeF]^+PF_6^-$

Correct Answer: (C) $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$

Solution:

Concept:

- A redox reaction involves changes in oxidation numbers: oxidation raises an oxidation state and reduction lowers another. Bond rearrangement alone does not establish redox.
- Fluorine is assigned oxidation state -1; oxygen is normally -2, but is $+1$ in O_2F_2. An element in its free form, such as O_2, has oxidation state zero.

Step 1: Check the reactions without electron transfer In option (A), xenon is $+6$ in both XeF_6 and $XeOF_4$: the latter has one oxygen at -2 and four fluorines at -1. In option (B), XeO_2F_2 likewise

has xenon at $+6$. These are hydrolysis reactions without oxidation-number changes. In (D), xenon is $+2$ in XeF_2 and remains $+2$ in $[XeF]^+$; phosphorus remains $+5$ when PF_5 becomes PF_6^-. Fluoride transfer there is not redox.

Step 2: Identify the coupled oxidation and reduction For option (C),

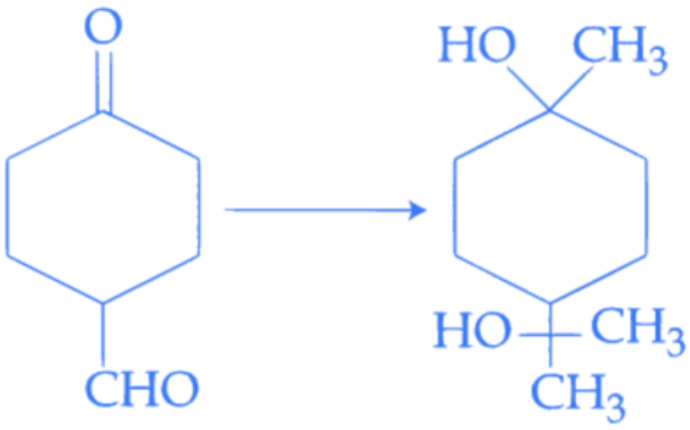
$$XeF_4 + O_2F_2 \longrightarrow XeF_6 + O_2.$$

Xenon changes from $+4$ to $+6$, so it is oxidised and loses two electrons formally. In O_2F_2, the two fluorines contribute -2 in total, forcing the two equivalent oxygens to total $+2$, or $+1$ each. They become oxygen at zero in O_2, so the two oxygen atoms gain two electrons in total. The electron losses and gains balance; fluorine stays at -1.

Step 3: Conclusion Only $XeF_4 + O_2F_2 \longrightarrow XeF_6 + O_2$ involves oxidation and reduction. Hence, the correct option is (C).

Quick Tip to Solve: In xenon fluoride reactions, compute oxidation states first; oxygen bonded to fluorine can have a positive oxidation state, unlike ordinary oxides.

63. The correct sequence of reagents for the following conversion will be : [4 marks]



(A) CH_3MgBr, $H^+/\ CH_3OH$, $[Ag(NH_3)_2]^+OH^-$

(B) $[Ag(NH_3)_2]^+OH^-$, CH_3MgBr, $H^+/\ CH_3OH$

(C) $[Ag(NH_3)_2]^+OH^-$, $H^+/\ CH_3OH$, CH_3MgBr

(D) CH_3MgBr, $[Ag(NH_3)_2]^+OH^-$, $H^+/\ CH_3OH$

Correct Answer: (C) $[Ag(NH_3)_2]^+OH^-$, $H^+/\ CH_3OH$, CH_3MgBr

Solution:

Concept:

- Tollens reagent selectively oxidises an aldehyde to a carboxylate, giving the corresponding carboxylic acid on work-up, while an ordinary ketone remains unchanged.
- Acidic methanol esterifies the acid. A Grignard reagent adds once to a ketone and twice to an ester, producing tertiary alcohols after hydrolytic work-up; an unprotected acid instead consumes

Grignard reagent by proton transfer.

Step 1: Prepare the aldehyde group for two methyl additions The starting molecule has a ring ketone and an aldehyde substituent. The target ring carbon bearing oxygen also bears one added methyl group, while the side-chain carbon bearing oxygen bears two methyl groups. Direct methylmagnesium bromide addition to the starting aldehyde would add only one methyl group there and give a secondary alcohol, so the aldehyde must first be converted into an ester. Tollens reagent oxidises $-\mathrm{CHO}$ to $-\mathrm{COOH}$ after work-up without changing the ring ketone.

Step 2: Esterify, then carry out Grignard addition Use $\mathrm{CH_3OH}$ in acid to convert the carboxylic acid group to $-\mathrm{COOCH_3}$. Now excess $\mathrm{CH_3MgBr}$, followed by hydrolytic work-up, adds once to the cyclohexanone carbonyl and twice to the ester carbonyl. The ring thus acquires $-\mathrm{OH}$ and $-\mathrm{CH_3}$ at its original carbonyl carbon, and the substituent becomes $-\mathrm{C(OH)(CH_3)_2}$, as shown. Options (A) and (D) start with Grignard reagent and therefore give the wrong aldehyde-derived alcohol. Option (B) exposes the acid to Grignard reagent before esterification, causing acid-base consumption instead of the required ester addition.

Step 3: Conclusion The required order is Tollens reagent, acidic methanol, and then $\mathrm{CH_3MgBr}$, with the usual work-ups. Hence, the correct option is (C).

Quick Tip to Solve: Count methyl groups gained at each original carbonyl carbon: two methyl groups indicate ester-to-tertiary-alcohol conversion before choosing the reagent order.

64. On treatment of 100 mL of 0.1 M solution of $CoCl_3.6H_2O$ with excess $AgNO_3$; 1.2×10^{22} ions are precipitated. The complex is : [4 marks]

(A) $[Co(H_2O)_3Cl_3].3H_2O$

(B) $[Co(H_2O)_6]Cl_3$

(C) $[Co(H_2O)_5Cl]Cl_2.H_2O$

(D) $[Co(H_2O)_4Cl_2]Cl.2H_2O$

Correct Answer: (C) $[Co(H_2O)_5Cl]Cl_2.H_2O$

Solution:

Concept:

- Only chloride ions outside the coordination sphere precipitate immediately with excess $AgNO_3$, according to $Ag^+ + Cl^- \longrightarrow AgCl\,(s)$.
- The number of ionisable chlorides per complex is the mole ratio of precipitated chloride to the original cobalt salt. Coordinated water and chloride together give coordination number six.

Step 1: Calculate the amount of cobalt salt Convert the volume to litres before using molarity:

$$n_{\text{complex}} = MV = 0.1 \times 0.100 = 0.0100 \text{ mol.}$$

Each formula unit contains one cobalt atom, so this is also the number of moles of complex units available. Excess silver nitrate ensures that the free chloride is completely precipitated; its amount is not limited by the silver reagent.

Step 2: Count the chlorides outside the sphere The reported number of precipitated ions corresponds to chloride ions removed as silver chloride:

$$n_{Cl^-} = \frac{1.2 \times 10^{22}}{6.022 \times 10^{23}} \simeq 0.0200 \text{ mol.}$$

Therefore,

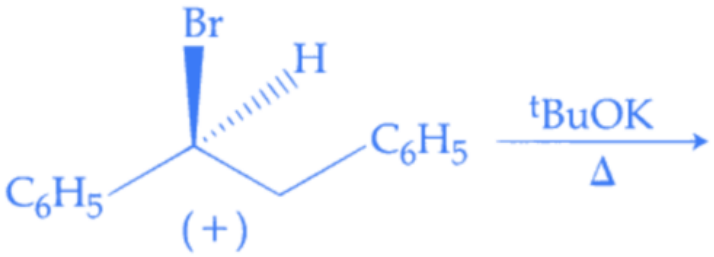
$$\frac{n_{Cl^-}}{n_{complex}} \simeq \frac{0.0200}{0.0100} = 2.$$

There must be two counter-ion chlorides per cobalt. Of the three chlorides in the empirical formula, one is coordinated. Five water molecules complete the six-coordinate sphere and the remaining water is water of crystallisation. Option (A) would give no free chloride, (B) gives three, and (D) gives one; none gives the measured ratio.

Step 3: Conclusion The complex is $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$, containing two ionisable chloride ions per formula unit. Hence, the correct option is (C).

Quick Tip to Solve: Divide moles of AgCl by moles of cobalt complex first; this directly counts the chlorides outside the square brackets.

65. The major product obtained in the following reaction is : [4 marks]



(A) $C_6H_5CH = CHC_6H_5$

(B) (+) - $C_6H_5CH(Ot\text{-}Bu)CH_2C_6H_5$

(C) (-) - $C_6H_5CH(Ot\text{-}Bu)CH_2C_6H_5$

(D) (±) - $C_6H_5CH(Ot\text{-}Bu)CH_2C_6H_5$

Correct Answer: (A) $C_6H_5CH = CHC_6H_5$

Solution:

Concept:

- Potassium tert-butoxide is a strong, bulky base. With a secondary alkyl halide and heat it favours bimolecular elimination, E2, over substitution.
- An E2 reaction removes a beta hydrogen while the carbon–halogen bond breaks, creating a double bond between the alpha and beta carbon atoms.

Step 1: Locate the leaving group and beta hydrogen The substrate is $C_6H_5CH(Br)CH_2C_6H_5$. Bromine is attached to the benzylic alpha carbon, and the neighbouring CH_2 group supplies a beta hydrogen.

Rotation about the intervening single bond can place the C–H and C–Br bonds in an antiperiplanar arrangement. Removal of this hydrogen produces the carbon–carbon double bond linking the two phenyl-substituted carbon atoms.

Step 2: Apply the reagent and temperature conditions The bulky tert-butoxide ion readily abstracts a proton but approaches the crowded secondary carbon less readily as a nucleophile. Heating further favours elimination. The net transformation can be written as

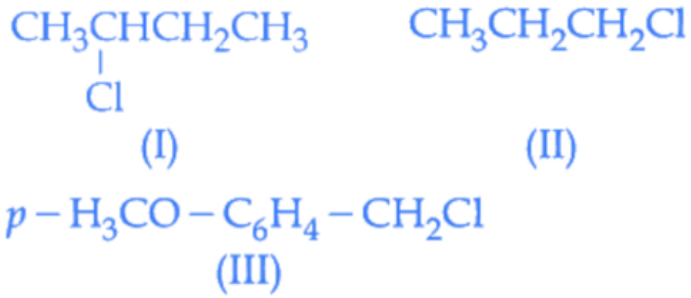
$$C_6H_5CH(Br)CH_2C_6H_5 \xrightarrow[\Delta]{\text{t-BuOK}} C_6H_5CH{=}CHC_6H_5.$$

The alkene is additionally stabilised by conjugation with the phenyl groups. Options (B), (C) and (D) all represent substitution products in which bromine has been replaced by a tert-butoxy group. Their different optical descriptions do not overcome the decisive reagent preference for elimination. The optical sign of the initial halide therefore does not select any of these ether products. No particular alkene geometry needs to be assigned because the listed alkene option does not distinguish it.

Step 3: Conclusion The major product is stilbene, $C_6H_5CH{=}CHC_6H_5$. Hence, the correct option is (A).

Quick Tip to Solve: For a secondary halide, the combination of bulky tert-butoxide and heat is a strong cue to remove a beta hydrogen and form an alkene.

66. The increasing order of the reactivity of the following halides for the S_N1 reaction is : [4 marks]



(A) (II) < (I) < (III)

(B) (I) < (III) < (II)

(C) (II) < (III) < (I)

(D) (III) < (II) < (I)

Correct Answer: (A) (II) < (I) < (III)

Solution:

Concept:

- The rate-determining step of an S_N1 reaction is heterolytic cleavage of the carbon–halogen bond to form a carbocation.
- For comparable leaving groups, a more stable carbocation forms more readily. Resonance stabilisation and electron donation can outweigh ordinary alkyl hyperconjugation.

Step 1: Identify the carbocation from each halide All three compounds have chloride as their leaving group, so compare the carbon framework that would bear positive charge. Compound (I), $CH_3CH(Cl)CH_2CH_3$, gives a secondary butyl carbocation. Compound (II), $CH_3CH_2CH_2Cl$, would give a primary propyl carbocation. Compound (III), para-methoxybenzyl chloride, gives a benzylic carbocation. The aromatic ring can delocalise its positive charge, unlike the purely aliphatic cations.

Step 2: Rank their stabilisation and reaction rates The primary cation from (II) has the least alkyl stabilisation and no resonance with a ring, so its formation is least favourable. The secondary cation from (I) is more stable through hyperconjugation and the electron-releasing inductive effect of two alkyl groups. The cation from (III) is resonance-stabilised over the benzylic carbon and ring; the para methoxy group donates a lone pair by resonance and stabilises a contributor carrying positive charge in the ring. Thus (III) ionises most readily:

$$\text{(II)} < \text{(I)} < \text{(III)}.$$

Option (B) incorrectly makes the primary chloride fastest; option (C) places the secondary chloride above the resonance-stabilised benzyl chloride, while (D) makes the latter slowest. These reverse the relevant stability comparisons.

Step 3: Conclusion The increasing order of S_N1 reactivity is (II) $<$ (I) $<$ (III). Hence, the correct option is (A).

Quick Tip to Solve: In an SN1 comparison with the same leaving group, draw the incipient carbocation and check for benzylic resonance before counting alkyl groups.

67. Given

$E^o_{Cl_2/Cl^-}$ **= 1.36 V,** $E^o_{Cr^{3+}/Cr}$ **= - 0.74 V**

$E^o_{Cr_2O_7{}^{2-}/Cr^{3+}}$ **= 1.33 V,** $E^o_{MnO_4{}^-/Mn^{2+}}$ **= 1.51 V**

Among the following, the strongest reducing agent is : [4 marks]

(A) Mn^{2+}

(B) Cr^{3+}

(C) Cl^-

(D) Cr

Correct Answer: (D) Cr

Solution:

Concept:

- A reducing agent donates electrons and is itself oxidised. Compare oxidation tendencies of the reduced members of the given redox couples.
- Reversing a standard reduction half-reaction reverses the sign of its electrode potential: $E^\circ_{ox} = -E^\circ_{red}$. Electrode potentials are not multiplied by stoichiometric coefficients.

Step 1: Reverse the relevant reduction couples The given reduced species are Cl^-, Cr, Cr^{3+}, and

Mn^{2+}. Their oxidation potentials, obtained by reversing their respective listed reduction reactions, are

$$E^{\circ}_{Cl-/Cl_2} = -1.36\ V,$$
$$E^{\circ}_{Cr/Cr^{3+}} = +0.74\ V,$$
$$E^{\circ}_{Cr^{3+}/Cr_2O_7{}^{2-}} = -1.33\ V,$$
$$E^{\circ}_{Mn^{2+}/MnO_4{}^{-}} = -1.51\ V.$$

The chromium-metal couple is the only one here with a positive standard oxidation potential.

Step 2: Select the most readily oxidised species Chromium metal has the greatest oxidation tendency among the alternatives under the standard conditions of the listed couples:

$$Cr \longrightarrow Cr^{3+} + 3\,e^{-}.$$

It can therefore supply electrons most readily and is the strongest reducing agent. In contrast, the large positive reduction potentials for permanganate, dichromate and chlorine show that their oxidised forms readily accept electrons; their reduced partners Mn^{2+}, Cr^{3+}, and Cl^{-} are relatively reluctant to lose electrons in the corresponding reverse reactions. Consequently options (A), (B) and (C) are weaker reducing agents than chromium metal.

Step 3: Conclusion Chromium metal, Cr, is the strongest reducing agent. Hence, the correct option is (D).

Quick Tip to Solve: To find the strongest reducing agent, compare the reduced species: the smallest listed reduction potential gives the largest reverse oxidation potential.

68. The freezing point of benzene decreases by 0.45^{0}C when 0.2 g of acetic acid is added to 20g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be: (K_f for benzene = 5.12 K kg mol^{-1}) [4 marks]

(A) 80.4 %

(B) 74.6 %

(C) 94.6 %

(D) 64.6 %

Correct Answer: (C) 94.6 %

Solution:

Concept:

- The freezing-point depression is $\Delta T_f = iK_f m$, where m is the molality calculated from the initial monomer amount and i accounts for association.
- For dimerisation $2A \rightleftharpoons A_2$, if a fraction α of initial molecules associates, $i = 1 - \alpha/2$. Association lowers the number of solute particles.

Step 1: Calculate the molality and van't Hoff factor Acetic acid has molar mass $60\ g\,mol^{-1}$, and the

mass of solvent is $20\ \text{g} = 0.020\ \text{kg}$. Hence

$$m = \frac{0.2/60}{0.020} = \frac{1}{6}\ \text{mol kg}^{-1}.$$

Without association the depression would be

$$K_f m = 5.12 \times \frac{1}{6} = 0.8533\ \text{K}.$$

The observed depression is only $0.45\ \text{K}$, giving

$$i = \frac{0.45}{0.8533} = 0.52734.$$

A temperature difference has the same numerical value in kelvin as in degrees Celsius.

Step 2: Relate the particle reduction to association Start with one mole of acid molecules. If α mole of those molecules joins into dimers, $1 - \alpha$ mole of monomer remains and $\alpha/2$ mole of dimers forms. The total solute-particle amount is therefore $1 - \alpha/2$, so

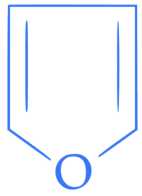
$$\alpha = 2(1 - i) = 2(1 - 0.52734) = 0.94531.$$

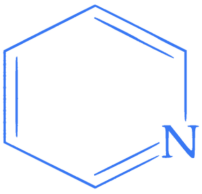
Thus the percentage association is about 94.53%, matching 94.6% at the precision of the options. The other percentages would give van't Hoff factors 0.598, 0.627, and 0.677, respectively, all larger than the observed factor, and hence all predict too large a freezing-point depression.

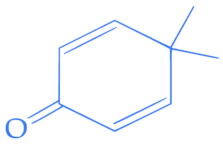
Step 3: Conclusion The percentage association is approximately 94.6%. Hence, the correct option is (C).

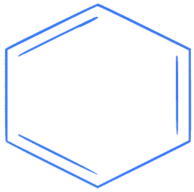
Quick Tip to Solve: For dimerisation, first calculate i from the measured depression; then use percentage association equal to $200(1 - i)$, rather than $100(1 - i)$.

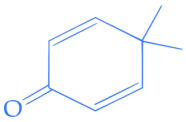
69. Which of the following molecules is least resonance stabilized? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- Aromatic stabilisation requires a cyclic, conjugated array of p orbitals containing $4n+2$ pi electrons, with suitable planarity for overlap.
- Ordinary conjugation can provide resonance stabilisation without aromaticity. A tetrahedral sp^3 carbon interrupts continuous cyclic p-orbital overlap.

Step 1: Examine the three aromatic alternatives Option (A) is furan. Its two carbon–carbon double bonds supply four pi electrons and one oxygen lone pair supplies two more to the cyclic conjugated system. Thus it has an aromatic six-pi-electron framework. Option (B) is pyridine: the ring has six pi electrons, while the nitrogen lone pair lies outside that pi system. Option (D) is benzene, the standard six-pi-electron aromatic ring. Although their resonance stabilisations are not identical, all three benefit from cyclic aromatic delocalisation.

Step 2: Identify the interrupted ring in option C Option (C) is the gem-dimethyl-substituted cyclohexadienone shown in the question. The carbon carrying two methyl groups also makes two single bonds within the ring, so it has four sigma bonds and is sp^3-hybridised. It cannot supply a p orbital to complete a conjugated loop around the six-membered ring. The carbonyl and the adjacent double bonds do allow ordinary enone-type resonance, so the molecule is not devoid of resonance; however, it lacks the extra aromatic stabilisation possessed by furan, pyridine and benzene. Counting the apparent double bonds alone would incorrectly overlook the saturated ring carbon. That interrupted conjugation makes this the least resonance-stabilised alternative among those listed.

Step 3: Conclusion The non-aromatic cyclohexadienone structure in option (C) is least resonance stabilised. Hence, the correct option is (C).

Quick Tip to Solve: Before applying the $4n+2$ rule, check that every ring atom has a usable p orbital; the gem-dimethyl ring carbon here breaks the aromatic circuit.

70. The group having isoelectronic species is: [4 marks]

(A) O^{2-} , F^{-} , Na^{+}, Mg^{2+}

(B) O^{2-} , F^{-} , Na, Mg^{2+}

(C) O^{-} , F^{-} , Na^{+}, Mg^{2+}

(D) O^{-} , F^{-} , Na, Mg^{+}

Correct Answer: (A) O^{2-} , F^{-} , Na^{+}, Mg^{2+}

Solution:

Concept:

- Isoelectronic species have the same total number of electrons, even though they may have different nuclei and ionic charges.
- For atomic number Z, add electrons for a negative ionic charge and subtract electrons for a positive charge: an ion of signed charge q has $Z - q$ electrons.

Step 1: Count the electrons in option A The atomic numbers of oxygen, fluorine, sodium and magnesium are 8, 9, 11 and 12 respectively. Apply the ionic charges separately:

$$\begin{aligned} O^{2-} &: 8 + 2 = 10, \\ F^{-} &: 9 + 1 = 10, \\ Na^{+} &: 11 - 1 = 10, \\ Mg^{2+} &: 12 - 2 = 10. \end{aligned}$$

All four species therefore have the same ten-electron configuration, $1s^2\,2s^2\,2p^6$, corresponding to neon. Their different charges do not prevent them from being isoelectronic.

Step 2: Eliminate the remaining groups In option (B), neutral sodium has 11 electrons, whereas the other three species each have 10. Thus the group is not isoelectronic. In option (C), O^{-} has only $8 + 1 = 9$ electrons, whereas the remaining species have 10. In option (D), the counts in the displayed order are $9, 10, 11, 11$; equality of the last two does not make the whole group isoelectronic. These direct counts rule out every other choice. Isoelectronic should not be confused with having equal atomic number, equal mass number, or equal ionic radius: none of those is required by the question, and equal electron count does not imply equal radii.

Step 3: Conclusion The group $O^{2-}, F^{-}, Na^{+}, Mg^{2+}$ consists entirely of ten-electron species. Hence, the correct option is (A).

Quick Tip to Solve: Write Z next to each element, then add the magnitude of an anion charge or subtract a cation charge; compare the resulting electron counts.

71. 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is : [4 marks]

(A) Zero

(B) Two

(C) Four

(D) Six

Correct Answer: (C) Four

Solution:

Concept:

- In the presence of a peroxide, HBr adds to an unsymmetrical alkene by a free-radical chain mechanism. Bromine attaches to the carbon that gives the more stable carbon radical.
- A tetrahedral carbon is stereogenic when its four substituents differ. With n independent stereogenic centres the maximum number of stereoisomers is 2^n; internal symmetry can reduce this count.

Step 1: Locate the bromine atom The starting alkene is $CH_3CH{=}C(CH_3)CH_2CH_3$. A bromine radical adds to carbon 2, placing the radical on the more substituted carbon 3. This tertiary radical is favoured over the secondary radical that the opposite orientation would produce. Hydrogen abstraction from HBr therefore gives $CH_3CH(Br)CH(CH_3)CH_2CH_3$, namely 2-bromo-3-methylpentane.

Step 2: Check both new tetrahedral centres Carbon 2 is attached to hydrogen, bromine, methyl and the substituted chain through carbon 3: all four groups differ. Carbon 3 is attached to hydrogen, methyl, ethyl and the bromine-containing chain through carbon 2: these are also four different groups. Thus both carbon 2 and carbon 3 are stereogenic.

Step 3: Count configurations and exclude symmetry The two centres can independently have R or S configuration, giving $(2R, 3R)$, $(2S, 3S)$, $(2R, 3S)$ and $(2S, 3R)$. There is no internal plane of symmetry because the two centres have different substituent sets; consequently no meso reduction occurs. Zero or two overlooks stereogenic centres, while six exceeds the 2^2 maximum.

Step 4: Conclusion The addition product has two stereogenic centres and no meso form, so it has $2^2 = 4$ possible stereoisomers. Hence, the correct option is (C).

Quick Tip to Solve: First write the anti-Markovnikov addition product, then inspect each formerly double-bonded carbon for four distinct groups before applying 2^n.

72. Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4. 'X' reacts with the acidified aqueous $CaCl_2$ solution to give a white precipitate which decolourises acidic solution of $KMnO_4$. 'X' is : [4 marks]

(A) CH_3COONa

(B) $Na_2C_2O_4$

(C) C_6H_5COONa

(D) HCOONa

Correct Answer: (B) $Na_2C_2O_4$

Solution:

Concept:

- Oxalate ions give a white precipitate of sparingly soluble calcium oxalate with calcium ions. Oxalate is also a reducing agent and is oxidised to carbon dioxide by acidified permanganate.
- The identity must satisfy the complete sequence of observations. A gas with concentrated acid alone is insufficient, because several organic salts react on acidification.

Step 1: Use the calcium-salt test Sodium oxalate supplies $C_2O_4{}^{2-}$, which precipitates with Ca^{2+}:

$$Ca^{2+} + C_2O_4{}^{2-} \longrightarrow CaC_2O_4(s).$$

The calcium salt is white. Calcium acetate and calcium formate do not give this characteristic insoluble oxalate precipitate; a benzoate precipitate, if present under suitable conditions, does not account for the subsequent reducing test.

Step 2: Check the permanganate reaction In acidic solution the oxalate portion is oxidised to CO_2, while purple permanganate is reduced to nearly colourless Mn^{2+}:

$$2\,MnO_4{}^- + 5\,C_2O_4{}^{2-} + 16\,H^+ \longrightarrow 2\,Mn^{2+} + 10\,CO_2 + 8\,H_2O.$$

This balanced redox equation explains the loss of colour. The calcium salt provides the same oxalate species on acid treatment.

Step 3: Check the concentrated-acid observation Concentrated H_2SO_4 first liberates oxalic acid from sodium oxalate. Oxalic acid decomposes in these conditions, giving CO, CO_2 and water:

$$H_2C_2O_4 \longrightarrow CO + CO_2 + H_2O.$$

The gases explain the effervescence. Acetate and benzoate lack the required oxalate redox behaviour; formate can reduce oxidants but fails the calcium-precipitate requirement.

Step 4: Conclusion The salt satisfying all the observations is sodium oxalate, $Na_2C_2O_4$. Hence, the correct option is (B).

Quick Tip to Solve: Use the conjunction of tests: a white calcium salt that reduces acidified permanganate is the characteristic oxalate clue.

73. Two reactions R_1 and R_2 have identical pre-exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1}. If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K, then $\ln(k_2/k_1)$ is equal to :

($R = 8.314$ J mol^{-1} K^{-1}) [4 marks]

(A) 12

(B) 6

(C) 4

(D) 8

Correct Answer: (C) 4

Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where A is the pre-exponential factor, E_a the activation energy and T the absolute temperature.
- At a common temperature, equal pre-exponential factors cancel in a ratio. Energies must have the same unit as R, here joules per mole.

Step 1: Form the ratio of rate constants For the two reactions, $k_1 = Ae^{-E_{a1}/RT}$ and $k_2 = Ae^{-E_{a2}/RT}$. Division gives

$$\frac{k_2}{k_1} = \exp\left(\frac{E_{a1} - E_{a2}}{RT}\right).$$

Taking the natural logarithm therefore yields

$$\ln\left(\frac{k_2}{k_1}\right) = \frac{E_{a1} - E_{a2}}{RT}.$$

Since reaction 1 has the higher barrier, $k_1 < k_2$, so this logarithm must be positive.

Step 2: Substitute consistent units The stated difference, converted to joules per mole, is

$$E_{a1} - E_{a2} = 10\,\text{kJ mol}^{-1} = 10000\,\text{J mol}^{-1}.$$

At the given temperature $T = 300\,\text{K}$,

$$\ln\left(\frac{k_2}{k_1}\right) = \frac{10000}{8.314 \times 300} = \frac{10000}{2494.2} = 4.009 \approx 4.$$

The logarithm is dimensionless because the numerator and denominator both have units of energy per mole.

Step 3: Compare the alternatives Values 6, 8 and 12 would respectively require barrier differences about 15, 20 and 30 kJ mol^{-1} at this temperature, rather than the given 10. No factor of 2.303 is needed because the question uses the natural logarithm, not the common logarithm. The pre-exponential factor is not an additional unknown because it is identical for both reactions.

Step 4: Conclusion $\ln(k_2/k_1) \approx 4$. Hence, the correct option is (C).

Quick Tip to Solve: With equal Arrhenius factors, take the logarithm of the ratio directly; convert the activation-energy difference to joules before dividing by RT.

74. **Given,** $C_{(graphite)} + O_2 \rightarrow CO_2(g)$;

$\Delta_r H^o$ **= - 393.5 kJ mol^{-1}**

$H_2(g)$ **+** $\frac{1}{2}O_2(g) \rightarrow H_2O(l)$

$\Delta_r H^o$ **= - 285.8 kJ mol^{-1}**

$CO_2(g)$ **+** $2H_2O(l) \rightarrow CH_4(g)$ **+** $2O_2(g)$

$\Delta_r H^o$ **= + 890.3 kJ mol^{-1}**

Based on the above thermochemical equations, the value of $\Delta_r H^o$ at 298 K for the reaction $C_{(graphite)}$ **+** $2H_2(g) \rightarrow CH_4(g)$ **will be :** [4 marks]

(A) +144.0 kJ mol^{-1}

(B) – 74.8 kJ mol^{-1}

(C) -144.0 kJ mol^{-1}

(D) + 74.8 kJ mol^{-1}

Correct Answer: (B) – 74.8 kJ mol^{-1}

Solution:

Concept:

- Enthalpy is a state function. Hess's law permits thermochemical equations to be added algebraically; the enthalpy changes are added with the same coefficients.
- Multiplying an equation multiplies its enthalpy change. Reversing it changes the sign. Species appearing equally on both sides cancel without contributing to the net reaction.

Step 1: Combine the elemental combustion equations Use the graphite combustion equation once and the hydrogen combustion equation twice. Their sum is

$$\mathrm{C(graphite) + 2\,H_2(g) + 2\,O_2(g) \longrightarrow CO_2(g) + 2\,H_2O\,(l).}$$

The corresponding enthalpy change is

$$\Delta H_1 = -393.5 + 2(-285.8) = -965.1\,\mathrm{kJ\,mol^{-1}}.$$

Doubling the water-formation equation is essential because the desired methane contains four hydrogen atoms.

Step 2: Add the given methane-producing equation The third supplied equation is already in the required direction:

$$\mathrm{CO_2(g) + 2\,H_2O\,(l) \longrightarrow CH_4(g) + 2\,O_2(g),} \qquad \Delta H_2 = +890.3\,\mathrm{kJ\,mol^{-1}}.$$

Add this to the previous sum. Carbon dioxide and two water molecules cancel, and the two oxygen molecules also cancel from the overall equation, leaving precisely $\mathrm{C(graphite) + 2\,H_2(g) \longrightarrow CH_4(g)}$.

Step 3: Add the enthalpy changes with their signs Thus

$$\Delta_r H^\circ = -965.1 + 890.3 = -74.8\,\mathrm{kJ\,mol^{-1}}.$$

The negative sign says the formation is exothermic. The positive 74.8 option reverses the net reaction;

the values ± 144.0 do not follow from the supplied equations with their stoichiometric multipliers. No extra reversal of the third equation is needed.

Step 4: Conclusion For methane formation at 298 K, $\Delta_r H^\circ = -74.8\,\text{kJ}\,\text{mol}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the desired net equation first and cancel species explicitly; apply each multiplier or reversal to its enthalpy at the same time.

75. ΔU is equal to : [4 marks]

(A) Isobaric work

(B) Adiabatic work

(C) Isothermal work

(D) Isochoric work

Correct Answer: (B) Adiabatic work

Solution:

Concept:

- With the chemistry sign convention, the first law is $\Delta U = q + w$: heat supplied to a system and work done on it increase its internal energy.
- An adiabatic process has no heat exchange, $q = 0$. The names isobaric, isothermal and isochoric instead constrain pressure, temperature and volume respectively.

Step 1: Apply the adiabatic condition For an adiabatic change the system is thermally insulated, or the change is so fast that heat exchange is negligible. Therefore the heat term vanishes in the energy balance:

$$\Delta U = q + w = 0 + w = w_{\text{adiabatic}}.$$

This statement uses work done on the system. In an adiabatic compression work is positive and internal energy rises; in an adiabatic expansion work is negative and internal energy falls.

Step 2: Check the other process labels At constant pressure, heat need not vanish, so isobaric work alone is generally not the complete internal-energy change. At constant volume, pressure-volume work is zero, but energy can enter as heat: $\Delta U = q_V$, not isochoric work. At constant temperature for an ideal gas, internal energy is unchanged even though the gas can do nonzero work; heat then balances that work.

Step 3: Keep the work convention consistent The first law can alternatively be written $\Delta U = Q - W$, where W denotes work done by the system. Under that convention the adiabatic result is $\Delta U = -W$. The two descriptions are identical because $w = -W$. Among the printed choices, adiabatic work is the one associated with the internal-energy change when the heat term is absent; the other restrictions do not eliminate heat in general.

Step 4: Conclusion Using the chemistry convention $\Delta U = q + w$, internal-energy change equals adiabatic work. Hence, the correct option is (B).

Quick Tip to Solve: Identify which term vanishes in $\Delta U = q + w$: adiabatic removes heat, whereas isochoric removes pressure-volume work.

76. Which of the following, upon treatment with *tert*-BuONa followed by addition of bromine water, fails to decolorize the colour of bromine? [4 marks]

(A) $C_6H_{11}-O-CH_2Br$ (C_6H_{11} = cyclohexyl)

(B) $C_6H_5-CH_2-CH(Br)-C_6H_5$

(C) $C_6H_5-O-CH(Br)-CH_3$

(D) $C_6H_{11}-CO-CH(Br)-CH_3$ (C_6H_{11} = cyclohexyl)

Correct Answer: (A) $C_6H_{11}-O-CH_2Br$ (C_6H_{11} = cyclohexyl)

Solution:

Concept:

- Sodium tert-butoxide is a bulky strong base. Alkyl bromides with an accessible beta hydrogen generally undergo dehydrohalogenation to give a carbon-carbon double bond.
- Alkenes consume bromine by addition across $C=C$, decolourising bromine water. A substrate that cannot furnish such an alkene under the stated treatment gives the exception.

Step 1: Inspect the bromomethyl ether In option (A), the structure is cyclohexyl$-O-CH_2Br$. The carbon bearing bromine is bonded to oxygen rather than directly to another carbon. There is consequently no adjacent carbon carrying the beta hydrogen required for ordinary carbon-carbon alkene formation by E2 elimination. The cyclohexyl ring is separated from the bromomethyl group by oxygen, so its hydrogens cannot supply the required beta hydrogen.

Step 2: Check the two other ether or benzylic substrates In (B), $C_6H_5CH_2CH(Br)C_6H_5$ has beta hydrogens on its benzylic CH_2 group and eliminates to $C_6H_5CH=CHC_6H_5$. In (C), $C_6H_5OCH(Br)CH_3$ has beta hydrogens on the methyl group and eliminates to the vinyl ether $C_6H_5OCH=CH_2$. Both products possess a non-aromatic double bond that consumes bromine.

Step 3: Check the carbonyl compound In (D), cyclohexyl$-CO-CH(Br)-CH_3$ eliminates a hydrogen from the terminal methyl group to form cyclohexyl$-CO-CH=CH_2$, an unsaturated ketone. Its carbon-carbon double bond also adds bromine. Thus (B), (C) and (D) all provide the unsaturation needed for the test, whereas (A) does not. Aromatic rings in the alternatives are not themselves the ordinary bromine-water addition site.

Step 4: Conclusion The cyclohexyl bromomethyl ether in (A) cannot undergo the required beta elimination to an alkene and fails to decolourise bromine water after base treatment. Hence, the correct option is (A).

Quick Tip to Solve: Locate the carbon directly bonded to bromine, then look only one bond away for a carbon bearing beta hydrogen; an intervening oxygen breaks that E2 pathway.

77. Which of the following compounds will form significant amount of meta product during mono-nitration reaction?

[4 marks]

(A) $C_6H_5-O-CO-CH_3$

(B) $C_6H_5-NH_2$

(C) $C_6H_5-NH-CO-CH_3$

(D) C_6H_5-OH

Correct Answer: (B) $C_6H_5-NH_2$

Solution:

Concept:

- Aromatic nitration uses an acidic medium that generates the nitronium electrophile, $NO_2{}^+$. Directing effects must be assessed for the substituent actually present in that medium.
- An unprotonated amino group is strongly ortho/para directing through lone-pair donation, but its protonated anilinium form is electron withdrawing and meta directing.

Step 1: Account for protonation of aniline Option (B) is aniline, $C_6H_5NH_2$. In the strongly acidic nitrating mixture its nitrogen lone pair accepts a proton:

$$C_6H_5NH_2 + H^+ \rightleftharpoons C_6H_5NH_3{}^+.$$

Protonation removes the free lone pair that would otherwise donate into the ring. The resulting positively charged $-NH_3{}^+$ group strongly withdraws electron density and directs incoming electrophiles towards the meta position.

Step 2: Explain why a significant meta fraction appears Ortho and para attack on an anilinium ring lead to particularly unfavourable resonance contributors with positive charge near the positively charged nitrogen substituent. Meta attack avoids those especially destabilised contributors. Direct nitration of aniline therefore gives a significant amount of meta-nitroaniline after work-up, alongside products associated with the small unprotonated fraction. It is incorrect to apply the neutral amino group's directing effect without considering the acidic conditions.

Step 3: Rule out the protected and oxygen substituents Option (A) has $-OCOCH_3$, with oxygen directly attached to the ring, and retains ortho/para direction by resonance donation. Option (C) is acetanilide: acetyl protection reduces nitrogen basicity and gives predominantly ortho/para nitration rather than the marked anilinium-induced meta contribution. Option (D), phenol, has a resonance-donating hydroxyl group and likewise favours ortho/para substitution. Aniline is the distinctive case among the four.

Step 4: Conclusion Aniline forms anilinium ions in the nitrating acid and consequently gives a significant meta product. Hence, the correct option is (B).

Quick Tip to Solve: Before applying aromatic directing rules, check whether the reaction medium protonates the substituent; direct aniline nitration is the standard warning case.

78. In the following reactions, ZnO is respectively acting as a/an :

(a) $ZnO + Na_2O \rightarrow Na_2ZnO_2$

(b) $ZnO + CO_2 \rightarrow ZnCO_3$ [4 marks]

(A) base and base

(B) acid and acid

(C) acid and base

(D) base and acid

Correct Answer: (C) acid and base

Solution:

Concept:

- An amphoteric oxide can react with both acidic and basic oxides. Its role in a particular reaction depends on the oxide with which it combines.
- An acidic oxide reacts with a basic oxide to give a salt. In such a reaction, identify the known partner first and then assign the complementary role to ZnO.

Step 1: Identify the role in reaction (a) Sodium oxide, Na_2O, is a strongly basic oxide. Zinc oxide combines with it to form sodium zincate, so zinc oxide supplies the acidic component of the salt-forming reaction.

$$ZnO + Na_2O \longrightarrow Na_2ZnO_2$$

Thus ZnO acts as an acid in reaction (a); calling it a base here would make both reactants basic and would miss its amphoteric behaviour.

Step 2: Identify the role in reaction (b) Carbon dioxide is an acidic oxide. Its combination with zinc oxide produces zinc carbonate.

$$ZnO + CO_2 \longrightarrow ZnCO_3$$

The oxide reacting with acidic CO_2 is acting as the base. Therefore ZnO has the opposite role to its role in reaction (a).

Step 3: Compare the ordered pair The question asks for the roles respectively, so retain the order (a), then (b). The required pair is acid followed by base. Options (A) and (B) incorrectly give the same role in both reactions, while (D) reverses the roles.

Step 4: Conclusion ZnO acts as an acid with Na_2O and as a base with CO_2. Hence, the correct option is (C).

Quick Tip to Solve: For an amphoteric oxide, label its reaction partner first: a basic partner makes it act acidic, and an acidic partner makes it act basic.

79. pK_a of a weak acid (HA) and pK_b of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is : [4 marks]

(A) 6.9

(B) 7.0

(C) 1.0

(D) 7.2

Correct Answer: (A) 6.9

Solution:

Concept:

- A salt of a weak acid and a weak base undergoes hydrolysis through both ions. Its pH depends on the relative strengths of the parent acid and base.
- At 25°C, $pK_w = 14$, and the usual weak-acid/weak-base salt relation is $\text{pH} = \frac{1}{2}(pK_w + pK_a - pK_b)$.

Step 1: Compare the acid and base strengths The given constants are logarithmic measures, with $K_a = 10^{-3.2}$ and $K_b = 10^{-3.4}$. The acid therefore has a slightly larger dissociation constant than the base. Its conjugate anion has a slightly weaker basic effect than the acidic effect of the cation derived from the weak base. The salt solution should consequently be slightly acidic, rather than exactly neutral or strongly acidic.

Step 2: Substitute in the hydrolysis relation For the salt of these two weak electrolytes,

$$\begin{aligned}\text{pH} &= 7 + \frac{1}{2}(pK_a - pK_b) \\ &= 7 + \frac{1}{2}(3.2 - 3.4) \\ &= 7 - 0.1 = 6.9.\end{aligned}$$

The sign is important: subtracting pK_a from pK_b would incorrectly predict a basic solution.

Step 3: Check the available values The computed value is only 0.1 unit below neutral pH, consistent with the small difference between the two given constants. A pH of 7.0 would require equal parent strengths. A pH of 1.0 is incompatible with this small hydrolysis imbalance; 7.2 is basic and has the wrong direction of change.

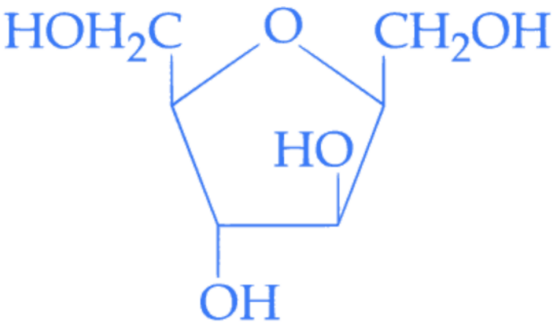
Step 4: Conclusion The salt solution has $\text{pH} = 6.9$, so it is slightly acidic. Hence, the correct option is (A).

Quick Tip to Solve: Write $\text{pH} = 7 + \frac{1}{2}(pK_a - pK_b)$, then check that a stronger parent acid gives a value below 7.

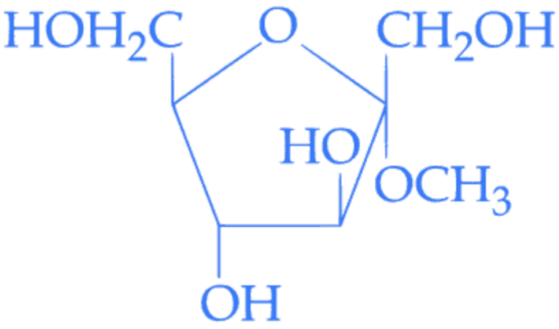
80. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?

[4 marks]

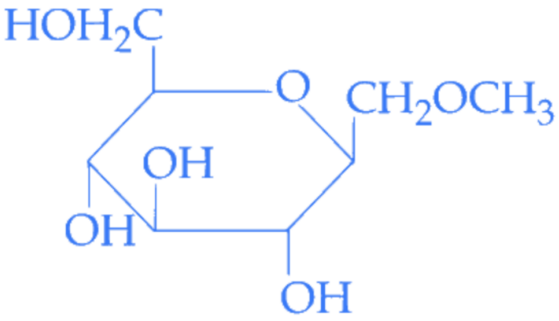
(A)



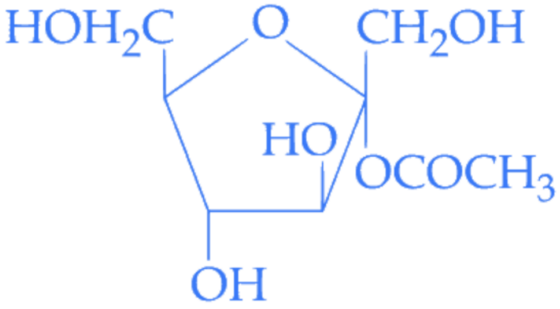
(B)



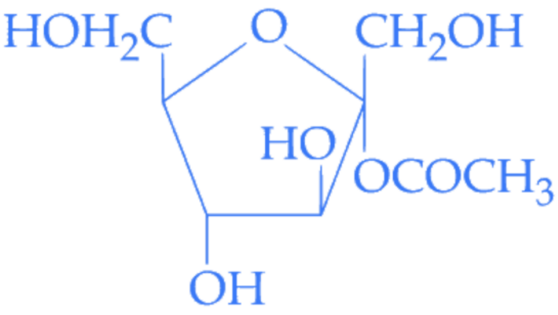
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- A reducing sugar can generate a free carbonyl group in solution. A cyclic hemiacetal or hemiketal can open to the carbonyl form, but a stable glycosidic acetal or ketal cannot do so in alkali.
- Aqueous KOH hydrolyses an acetate ester, $\mathrm{R{-}OCOCH_3}$, to an alcohol and acetate. A ketose with a free carbonyl can also form reducing aldose species through an enediol in alkaline solution.

Step 1: Inspect the anomeric position In the structures, examine the ring carbon next to the ring oxygen at the right-hand side. Option (A) has no hydroxyl group at this position; the cyclic ether therefore cannot simply open as a hemiketal. Option (B) bears $\mathrm{OCH_3}$ at the corresponding position. This locks the carbonyl-derived centre as a ketal, and ordinary aqueous alkali does not hydrolyse such a glycosidic ether.

Step 2: Use the action of the specified reagent Option (D) has an acetate ester, $-\mathrm{OCOCH_3}$, attached to the carbonyl-derived ring carbon. Hydroxide hydrolyses the ester and restores the free anomeric hydroxyl group:

$$\mathrm{R{-}OCOCH_3 + OH^- \longrightarrow R{-}OH + CH_3COO^-}.$$

Here $\mathrm{R{-}OH}$ represents the corresponding cyclic hemiketal. It can open to the ketose, which gives reducing behaviour in alkaline medium through enediol tautomerisation.

Step 3: Eliminate the remaining structure Option (C) contains an ether substituent, $\mathrm{CH_2OCH_3}$, and no free anomeric hydroxyl group. It does not possess the hydrolysable anomeric acetate that makes (D) reactive under these conditions. Thus neither the presence of several ordinary hydroxyl groups nor a ring oxygen alone is enough; the carbonyl-derived centre must become available.

Step 4: Conclusion The anomeric acetate in structure (D) is hydrolysed by aqueous KOH, giving a sugar capable of reducing behaviour. Hence, the correct option is (D).

Quick Tip to Solve: When alkali is specified, distinguish an anomeric acetate ester from a methyl glycoside: alkali removes the acetate but leaves the glycosidic ether intact.

81. The most abundant elements by mass in the body of a healthy human adult are: Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0%); and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ^{1}H atoms are replaced by ^{2}H atoms is: [4 marks]

(A) 37.5 kg

(B) 7.5 kg

(C) 10 kg

(D) 15 kg

Correct Answer: (B) 7.5 kg

Solution:

Concept:

- Mass percentages refer to the initial total mass. Hydrogen initially accounts for 10.0 percent of the person's mass.
- Replacing each ^{1}H atom by ^{2}H keeps the number of hydrogen atoms unchanged but approximately doubles the mass contributed by those atoms. All other atoms retain their original masses.

Step 1: Calculate the initial hydrogen mass The initial mass is $75\,\text{kg}$, and the stated hydrogen content is a mass fraction of $10.0/100 = 0.100$. Therefore

$$m_{\text{H}} = 0.100 \times 75 = 7.5\,\text{kg}.$$

The other element percentages are not needed separately; their combined mass remains unchanged during the hypothetical replacement.

Step 2: Compute the increase from replacement Each original hydrogen atom has approximately one atomic mass unit, while its replacement has approximately two. Thus the hydrogen contribution becomes

$$m_{\text{D}} = 2m_{\text{H}} = 15.0\,\text{kg}.$$

The question asks for the gain, rather than the final mass of hydrogen. Hence

$$\Delta m = m_{\text{D}} - m_{\text{H}} = 15.0 - 7.5 = 7.5\,\text{kg}.$$

The person's new total mass would be $82.5\,\text{kg}$.

Step 3: Rule out the misleading values The value 15 kg is the final isotopic-hydrogen contribution, not the increase. The value 10 kg confuses the numerical percentage with a mass, while 37.5 kg would require half the original total body mass to be added. Only 7.5 kg follows from changing the isotopic mass of the original hydrogen atoms.

Step 4: Conclusion The gain in mass is $7.5\,\text{kg}$. Hence, the correct option is (B).

Quick Tip to Solve: If an isotope doubles the mass of one constituent, the total mass gain equals that constituent's original mass, not its doubled mass.

82. Which of the following species is not paramagnetic? [4 marks]

(A) CO

(B) O_2

(C) B_2

(D) NO

Correct Answer: (A) CO

Solution:

Concept:

- A paramagnetic species has one or more unpaired electrons. A species with all electrons paired is diamagnetic and therefore is not paramagnetic.
- Molecular-orbital filling, including Hund's rule in degenerate orbitals, decides the number of unpaired electrons; merely having an even electron count is not sufficient.

Step 1: Check carbon monoxide Carbon monoxide has $6 + 8 = 14$ electrons, so it is isoelectronic with N_2. Its ground-state molecular orbitals are occupied by electron pairs. In the valence-orbital description, the six sigma-type electrons and four pi-bonding electrons are all paired. Consequently CO is diamagnetic. This satisfies the negative demand in the question: identify the species that is not paramagnetic.

Step 2: Check oxygen and boron molecules For O_2, the two highest-energy electrons occupy separate degenerate antibonding π^* orbitals with parallel spins. It has two unpaired electrons and is paramagnetic. For B_2, after filling the $2s$ bonding and antibonding orbitals, the two remaining valence electrons singly occupy the two degenerate bonding $\pi(2p)$ orbitals. It too has two unpaired electrons.

Step 3: Check nitric oxide NO has $7 + 8 = 15$ electrons, an odd total. Its ground state contains an unpaired electron in a pi-antibonding orbital and is paramagnetic. Thus all three alternatives to carbon monoxide contain unpaired electrons; their molecular-orbital configurations exclude them from the required answer.

Step 4: Conclusion All electrons in CO are paired, so it is diamagnetic and not paramagnetic. Hence, the correct option is (A).

Quick Tip to Solve: For these choices, remember that CO is isoelectronic with N_2; check B_2 and O_2 using Hund's rule, not electron-count parity alone.

83. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is: [4 marks]

(A) 84.3

(B) 118.6

(C) 11.86

(D) 1186

Correct Answer: (A) 84.3

Solution:

Concept:

- An acid reacts with one carbonate ion to liberate one molecule of carbon dioxide. The balanced equation establishes the mole ratio before any mass calculation.
- Molar mass is the sample mass divided by the amount of substance: $M = m/n$, with m in grams and n in moles to obtain $\mathrm{g\,mol^{-1}}$.

Step 1: Write the balanced reaction The carbonate reacts completely because hydrochloric acid is present in excess:

$$\mathrm{M_2CO_3 + 2\,HCl \longrightarrow 2\,MCl + H_2O + CO_2}.$$

One mole of $\mathrm{M_2CO_3}$ yields one mole of $\mathrm{CO_2}$. The coefficient 2 belongs to hydrochloric acid and metal chloride, not to the carbonate-to-carbon-dioxide relation. Therefore the gas amount directly gives the carbonate amount.

Step 2: Find the molar mass The given amount of gas is $0.01186\,\mathrm{mol}$, hence

$$n(\mathrm{M_2CO_3}) = 0.01186\,\mathrm{mol}.$$

Using the sample mass of $1\,\mathrm{g}$,

$$M(\mathrm{M_2CO_3}) = \frac{1}{0.01186} = 84.317\ldots\ \mathrm{g\,mol^{-1}}.$$

To the precision of the alternatives, this is $84.3\,\mathrm{g\,mol^{-1}}$.

Step 3: Check the scale and alternatives A sample of about 0.012 mole weighing 1 g should have a molar mass a little over 80 grams per mole. Multiplying 84.3 by 0.01186 gives approximately 1.00 g, which verifies the result. The other choices fail this reverse mass check; neither the identity nor the atomic mass of the metal needs to be assumed.

Step 4: Conclusion The molar mass of $\mathrm{M_2CO_3}$ is $84.3\,\mathrm{g\,mol^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: For an ordinary carbonate, one carbonate ion gives one $\mathrm{CO_2}$; use that 1:1 mole ratio before dividing mass by moles.

84. The radius of the second Bohr orbit for hydrogen atom is:

(Planck's Const. h = 6.6262 $\times$ 10^{-34} Js; mass of electron = 9.1091 $\times$ 10^{-31} kg; charge of electron (e) = 1.60210 $\times$ 10^{-19} C; permittivity of vacuum (ε_0) = 8.854185 $\times$ 10^{-12} kg^{-1} m^{-3} A^2) [4 marks]

(A) 4.76 $\mathring{A}$

(B) 2.12 $\mathring{A}$

(C) 0.529 $\mathring{A}$

(D) 1.65 $\mathring{A}$

Correct Answer: (B) 2.12 $\mathring{A}$

Solution:

Concept:

- For a hydrogen-like atom, the Bohr radius of orbit n is $r_n = \varepsilon_0 h^2 n^2/(\pi m_e Z e^2)$. Hydrogen has nuclear charge number $Z = 1$.
- The radius varies as n^2, and $1\,\text{Å} = 10^{-10}\,\text{m}$. The second orbit therefore has four times the first-orbit radius.

Step 1: Insert the orbit and atomic numbers The required orbit is $n = 2$, so $n^2 = 4$. With $Z = 1$, the formula becomes

$$r_2 = \frac{4\varepsilon_0 h^2}{\pi m_e e^2}.$$

Use h, rather than $\hbar$, in this version of the formula. If one instead uses $\hbar$, the corresponding form contains $4\pi\varepsilon_0\hbar^2$, and mixing the two versions would give an incorrect factor.

Step 2: Evaluate with the given constants First calculate the radius scale for $n = 1$ from the provided values:

$$a_0 = \frac{(8.854185 \times 10^{-12})(6.6262 \times 10^{-34})^2}{\pi(9.1091 \times 10^{-31})(1.60210 \times 10^{-19})^2}$$
$$\simeq 5.292 \times 10^{-11}\,\text{m}.$$

Consequently

$$r_2 = 4a_0 \simeq 2.117 \times 10^{-10}\,\text{m} = 2.117\,\text{Å}.$$

Step 3: Compare with the listed radii Rounding gives $2.12\,$Å. The value $0.529\,$Å describes the first orbit, while $4.76\,$Å is approximately the third-orbit radius. The remaining value, $1.65\,$Å, does not follow the second-orbit n^2 scaling. These checks select a unique listed answer.

Step 4: Conclusion The second Bohr-orbit radius is $2.12\,$Å, or approximately $2.12 \times 10^{-10}\,\text{m}$. Hence, the correct option is (B).

Quick Tip to Solve: For hydrogen, start from $a_0 = 0.529\,$Å and multiply by n^2: the second orbit is four times the first.

85. Both lithium and magnesium display several similar properties due to the diagonal relationship; however, the one which is incorrect, is : [4 marks]

(A) both form soluble bicarbonates

(B) both from nitrides

(C) nitrates of both Li and Mg yield NO_2 and O_2 on heating

(D) both form basic carbonates

Correct Answer: (D) both form basic carbonates

Solution:

Concept:

- Lithium and magnesium show a diagonal relationship because their ions have comparable polarising tendencies. Many of their salts therefore behave similarly, but the relationship does not make every compound identical.
- The question asks for the incorrect similarity. Check each proposed statement against the characteristic compounds of lithium and magnesium rather than selecting a familiar true property.

Step 1: Check bicarbonates and nitrides Both elements form bicarbonates that are soluble in water, so statement (A) is a valid similarity. Both also combine directly with nitrogen to form nitrides: Li_3N and Mg_3N_2. Thus statement (B), read as formation of nitrides, is also correct. These reactions distinguish lithium from the other alkali metals, which do not ordinarily form stable nitrides by direct combination.

Step 2: Check the thermal decomposition of nitrates On heating, the nitrates of both elements give the corresponding oxide, nitrogen dioxide and oxygen. The balanced equations are

$$4\,LiNO_3 \longrightarrow 2\,Li_2O + 4\,NO_2 + O_2$$

$$2\,Mg(NO_3)_2 \longrightarrow 2\,MgO + 4\,NO_2 + O_2.$$

Hence statement (C) correctly identifies the gaseous products for both salts.

Step 3: Distinguish normal and basic carbonates Magnesium can form basic magnesium carbonate, which contains hydroxide as well as carbonate. Lithium forms the normal carbonate Li_2CO_3; it does not share the characteristic formation of a basic carbonate. Consequently the assertion that both form basic carbonates is the exception. The fact that both normal carbonates decompose on heating is a different similarity and does not make statement (D) correct.

Step 4: Conclusion The incorrect statement is that both lithium and magnesium form basic carbonates. Hence, the correct option is (D).

Quick Tip to Solve: When a diagonal-relationship question asks for the exception, check the actual salt formula: a basic carbonate contains hydroxide as well as carbonate.

86. The formation of which of the following polymers involves hydrolysis reaction? [4 marks]

(A) Bakelite

(B) Nylon 6, 6

(C) Terylene

(D) Nylon 6

Correct Answer: (D) Nylon 6

Solution:

Concept:

- Nylon-6 is a polyamide prepared from caprolactam, a cyclic amide. Water initiates opening of the lactam ring by hydrolysis, after which the opened units form the polymer.
- Hydrolysis consumes water to cleave a bond. It must be distinguished from condensation, in which water is eliminated when two functional groups combine.

Step 1: Identify the hydrolysed monomer Caprolactam contains an amide linkage within a seven-membered ring. Heating it with water opens this linkage and produces an amino-acid-type unit, $H_2N(CH_2)_5COOH$, called 6-aminohexanoic acid. Thus the preparation involves a hydrolysis step before the amino and carboxyl groups join into long chains. The repeat unit of the resulting nylon-6 is $[-NH-(CH_2)_5-CO-]_n$. The number 6 counts the carbon atoms in the monomer unit.

Step 2: Exclude the other named polymers Bakelite is formed by condensation of phenol with formaldehyde, followed by development of a cross-linked network. Nylon-6,6 is obtained by condensation of hexamethylenediamine with adipic acid; its two monomers supply six carbon atoms each. Terylene is the polyester obtained by condensation of ethylene glycol with terephthalic acid. These preparations characteristically eliminate small molecules such as water rather than requiring hydrolysis of a lactam monomer.

Step 3: Separate preparation from degradation Nylon-6,6 and Terylene can themselves undergo hydrolytic cleavage under suitable conditions, because they contain amide and ester bonds respectively. That is degradation of an existing polymer, whereas the question asks about polymer formation. The relevant water-consuming ring-opening step belongs to caprolactam in the preparation of nylon-6, so options (A), (B) and (C) do not answer the stated question.

Step 4: Conclusion The formation of nylon-6 from caprolactam involves hydrolysis and ring opening. Hence, the correct option is (D).

Quick Tip to Solve: Associate nylon-6 with one cyclic monomer, caprolactam, and water-assisted ring opening; associate nylon-6,6 with two monomers and condensation.

87. A water sample has ppm level concentration of following anions

F^- = 10; SO_4^{2-} = 100; NO_3^- = 50

The anion/anions that make/makes the water sample unsuitable for drinking is/are : [4 marks]

(A) both SO_4^{2-} and NO_3^-

(B) only F^-

(C) only SO_4^{2-}

(D) only NO_3^-

Correct Answer: (B) only F^-

Solution:

Concept:

- For dilute aqueous samples, a concentration of 1 ppm is approximately $1\,\text{mg}\,\text{L}^{-1}$. Compare each stated concentration with the textbook drinking-water threshold used in this question.
- Excess fluoride causes fluorosis, excess nitrate can cause methemoglobinaemia, and sufficiently high sulphate produces a laxative effect. The identity of an ion alone does not determine whether its stated concentration is excessive.

Step 1: Evaluate the fluoride concentration The fluoride concentration given is $10\,\text{ppm}$. The desirable fluoride level is about $1\,\text{ppm}$, while concentrations above about $2\,\text{ppm}$ are associated with harmful effects on teeth and higher levels can affect bones. The value in this sample is therefore clearly excessive. Fluoride alone is sufficient to make the sample unsuitable under the textbook comparison.

Step 2: Compare sulphate and nitrate with their limits The sulphate concentration is $100\,\text{ppm}$, well below the approximately $500\,\text{ppm}$ level associated with a laxative effect in the relevant textbook discussion. It is therefore not the offending ion here. The nitrate concentration is $50\,\text{ppm}$, equal to the stated upper limit of $50\,\text{ppm}$, rather than greater than that limit. It must not be classified as exceeding the limit simply because it lies at the boundary.

Step 3: Select the option that contains only the offending ion Only F^- exceeds the relevant concentration threshold among the three entries. Option (A) wrongly identifies both sulphate and nitrate while omitting fluoride. Option (C) selects sulphate alone even though its concentration is low enough, and option (D) treats the boundary nitrate value as excessive. Option (B) is the only choice consistent with all three comparisons.

Step 4: Conclusion At the stated concentrations, only fluoride makes the water sample unsuitable for drinking. Hence, the correct option is (B).

Quick Tip to Solve: For concentration-limit MCQs, distinguish a value equal to the upper limit from a value above it; here nitrate is at the boundary but fluoride is far above it.

88. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are :

[4 marks]

(A) Cl^- and ClO_2^-

(B) ClO^- and ClO_3^-

(C) Cl^- and ClO^-

(D) ClO_2^- and ClO_3^-

Correct Answer: (C) Cl^- and ClO^-

Solution:

Concept:

- Chlorine undergoes disproportionation in aqueous alkali: part of the chlorine is reduced below oxidation state zero and another part is oxidised above zero.
- The products depend on conditions. Cold, dilute alkali gives chloride and hypochlorite, whereas hot, concentrated alkali favours chloride and chlorate.

Step 1: Write the reaction under the specified conditions The question specifies cold and dilute aqueous sodium hydroxide. Under these conditions, chlorine reacts according to

$$\mathrm{Cl_2 + 2\,NaOH \longrightarrow NaCl + NaClO + H_2O.}$$

The sodium salts dissociate in water, so the chlorine-containing anions formed are chloride, $\mathrm{Cl^-}$, and hypochlorite, $\mathrm{ClO^-}$. Sodium ions are spectators and do not affect identification of the requested anions.

Step 2: Verify the disproportionation In $\mathrm{Cl_2}$, chlorine has oxidation state zero. In chloride its oxidation state is -1, while in hypochlorite oxygen is -2 and the ion has charge -1, so chlorine has oxidation state $+1$. One chlorine atom is reduced by gaining an electron and the other is oxidised by losing an electron. This verifies both products and gives the net ionic equation

$$\mathrm{Cl_2 + 2\,OH^- \longrightarrow Cl^- + ClO^- + H_2O.}$$

Step 3: Reject products belonging to different oxidation levels Chlorite, $\mathrm{ClO_2^-}$, contains chlorine in oxidation state $+3$ and is not a product of the stated cold dilute reaction, eliminating (A) and (D). Chlorate, $\mathrm{ClO_3^-}$, contains chlorine in oxidation state $+5$; chloride and chlorate are obtained with hot concentrated alkali. Option (B) also omits chloride and pairs two oxidised species, so it cannot describe this disproportionation. The stated conditions select (C).

Step 4: Conclusion The products are chloride and hypochlorite ions, $\mathrm{Cl^-}$ and $\mathrm{ClO^-}$. Hence, the correct option is (C).

Quick Tip to Solve: Read the alkali conditions first: cold and dilute gives hypochlorite; hot and concentrated gives chlorate, with chloride in both cases.

89. A metal crystallises in a face centred cubic structure. If the edge length of its unit cell is 'a', the closest approach between two atoms in metallic crystal will be : [4 marks]

(A) $2\sqrt{2}a$

(B) $\sqrt{2}a$

(C) $\frac{a}{\sqrt{2}}$

(D) 2a

Correct Answer: (C) $\frac{a}{\sqrt{2}}$

Solution:

Concept:

- In a face-centred cubic lattice, atoms occupy the eight corners and the centres of the six faces. The closest neighbouring centres lie along a face diagonal.
- For hard spheres in contact in an FCC unit cell, the face diagonal is $\sqrt{2}a = 4r$, where a is the edge length and r is the atomic radius. The nearest-neighbour distance is $2r$, not r.

Step 1: Locate a nearest-neighbour pair Take a corner atom at coordinates $(0, 0, 0)$ and the atom at the centre of the same face, $(a/2, a/2, 0)$. Their centre-to-centre separation is

$$d = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}}.$$

This pair is closer than two corner atoms on the same edge, whose separation is a. The face-centre atom lies halfway along the face diagonal joining two opposite corner atoms.

Step 2: Check using the contact relation Along one face diagonal there are a corner atom, a face-centre atom and the opposite corner atom. Two nearest-neighbour separations fill the diagonal, so

$$2d = a\sqrt{2}.$$

Equivalently, four radii fill the diagonal: $4r = a\sqrt{2}$. Therefore

$$d = 2r = \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}.$$

This confirms that the closest approach refers to the distance between atomic centres, which equals the diameter for touching atoms.

Step 3: Compare the alternative distances Option (B), $\sqrt{2}a$, is the full face diagonal and skips the atom at its centre. Options (A) and (D) are even larger than a unit-cell edge and cannot represent the nearest pair when a corner-to-face-centre distance is available. Only option (C) gives half the face diagonal, consistent with both the coordinate calculation and the FCC contact geometry.

Step 4: Conclusion The closest approach between two atoms is $a/\sqrt{2}$. Hence, the correct option is (C).

Quick Tip to Solve: For FCC remember that two nearest-neighbour distances, or four radii, span a face diagonal; divide $a\sqrt{2}$ by two to obtain the distance.

90. The Tyndall effect is observed only when following conditions are satisfied:

(a) The diameter of the dispersed particles is much smaller than the wavelength of the light used.

(b) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.

(c) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude.

(d) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

[4 marks]

(A) (b) and (d)

(B) (a) and (c)

(C) (b) and (c)

(D) (a) and (d)

Correct Answer: (A) (b) and (d)

Solution:

Concept:

- The Tyndall effect is the visible scattering of a beam of light by colloidal particles, making the path of the beam observable from the side.
- The textbook conditions require particles whose diameter is not much smaller than the wavelength of the incident light and an appreciable difference between the refractive indices of the dispersed phase and the medium.

Step 1: Assess the particle-size statements Statement (b) gives the relevant size condition: the particle diameter must not be much smaller than the wavelength of the light used. Such particles scatter enough light to make the beam path visible. Statement (a) instead describes particles much smaller than the wavelength, for which scattering is weak and the characteristic visible Tyndall cone is not obtained in this comparison. Thus select (b), rather than (a).

Step 2: Assess the refractive-index statements Light must encounter a sufficient optical contrast between the dispersed particles and the surrounding dispersion medium. A large difference in refractive index makes this contrast appreciable and allows strong scattering at the particle-medium boundary. This is statement (d). If the refractive indices are almost the same, as in statement (c), the optical contrast is greatly reduced, so that statement does not supply the required condition.

Step 3: Combine the two independently required conditions The size requirement and the optical-contrast requirement must both hold. The required pair is therefore (b) and (d). Option (B) contains neither accepted condition. Option (C) has the suitable size condition but incorrectly pairs it with nearly matching refractive indices. Option (D) has the refractive-index contrast but selects the unsuitable size statement. Option (A) alone combines the two textbook conditions stated in the question.

Step 4: Conclusion The required conditions are suitable particle size and a substantial refractive-index difference. Hence, the correct option is (A).

Quick Tip to Solve: Check Tyndall conditions in two separate steps: particle size controls scattering strength, and refractive-index contrast distinguishes the particles optically from the medium.