JEE Main 2017 Paper 1 (B.E./B.Tech) Apr 9 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2017, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 9 April 2017, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. The eccentricity of an ellipse having centre at the origin, axes along the co-ordinate axes and passing through the points (4, −1) and (−2, 2) is : [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{2}{\sqrt{5}}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{\sqrt{3}}{4}$

Correct Answer: (C) $\frac{\sqrt{3}}{2}$

Solution:

Concept:

- For an ellipse centred at the origin with axes along the coordinate axes, write $ux^2 + vy^2 = 1$, where $u, v > 0$ are the reciprocals of the squared semiaxes.
- If the major and minor semiaxes are a and b, respectively, its eccentricity is $e = \sqrt{1 - b^2/a^2}$. Identify the larger squared semiaxis before using this formula.

Step 1: Use both given points Substitute the coordinates into the ellipse equation. Since coordinates are squared, the negative signs do not change either substitution. The two independent conditions determine both coefficients:

$$16u + v = 1, \qquad 4u + 4v = 1.$$

The second equation gives $u + v = 1/4$. Subtract this from the first to eliminate v:

$$15u = \frac{3}{4}, \qquad u = \frac{1}{20}, \qquad v = \frac{1}{4} - \frac{1}{20} = \frac{1}{5}.$$

Step 2: Identify the major axis and calculate eccentricity The ellipse is therefore

$$\frac{x^2}{20} + \frac{y^2}{5} = 1.$$

The larger denominator belongs to the major axis, so $a^2 = 20$ and $b^2 = 5$. Consequently,

$$e^2 = 1 - \frac{5}{20} = \frac{3}{4}, \qquad e = \frac{\sqrt{3}}{2}.$$

The positive square root is required because eccentricity is nonnegative. As a check, the first point gives $16/20 + 1/5 = 1$, and the second gives $4/20 + 4/5 = 1$. Thus both points lie on this ellipse. The other listed eccentricities have different squares and cannot satisfy the determined axis ratio.

Step 3: Conclusion The eccentricity is $\boxed{\sqrt{3}/2}$. Hence, the correct option is (C).

Quick Tip to Solve: Solve first for the reciprocals of the squared semiaxes; choose the larger denominator as a^2 only after solving.

2. From a group of 10 men and 5 women, four member committees are to be formed each of which must contain at least one woman. Then the probability for these committees to have more women than men, is : [4 marks]

(A) $\frac{21}{220}$
(B) $\frac{3}{11}$
(C) $\frac{1}{11}$
(D) $\frac{2}{23}$

Correct Answer: (C) $\frac{1}{11}$

Solution:

Concept:

- An unordered committee is counted by a combination. Selecting r people from n distinct people gives $\binom{n}{r}$ committees, without distinguishing selection order.
- The sample space already excludes all-male committees. Use the number of permitted committees in the denominator, since the desired probability is conditional on having at least one woman.

Step 1: Count the permitted committees There are 15 people altogether. Count all four-person committees, then subtract those selected entirely from the 10 men:

$$N = \binom{15}{4} - \binom{10}{4} = 1365 - 210 = 1155.$$

Every remaining committee contains at least one woman, exactly as required. Using $\binom{15}{4}$ alone would include forbidden committees and produce a different probability.

Step 2: Count committees with a female majority With four members, more women than men means either three women and one man or four women and no man. A two-woman, two-man committee is a tie and does not qualify. The two favourable cases are disjoint:

$$F = \binom{5}{3}\binom{10}{1} + \binom{5}{4}\binom{10}{0} = 10 \cdot 10 + 5 = 105.$$

Therefore

$$P = \frac{F}{N} = \frac{105}{1155} = \frac{1}{11}.$$

There is no additional ordering factor because committees are sets of people. The alternatives $21/220$, $3/11$, and $2/23$ do not equal this ratio. The count can also be checked by partitioning the permitted sample space into committees containing one, two, three, and four women.

Step 3: Conclusion The required probability is $\boxed{1/11}$. Hence, the correct option is (C).

Quick Tip to Solve: For a conditional committee question, remove excluded committees from the denominator before counting the favourable composition.

3. The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is : [4 marks]

(A) $5 \times 6!$

(B) $6 \times 6!$

(C) $7!$

(D) $5 \times 7!$

Correct Answer: (A) $5 \times 6!$

Solution:

Concept:

- For n distinct people around a round table, rotations represent the same seating, so the number of circular arrangements is $(n-1)!$. Reflected arrangements remain distinct.
- To count arrangements where two specified people are adjacent, treat them as one block and multiply by two for their two internal orders. Subtract these arrangements from the unrestricted total.

Step 1: Count all circular seatings The five boys and three girls are eight distinct people. Fixing one person as a reference removes the rotational repetition, leaving seven people to arrange:

$$N_{\text{all}} = 7!.$$

No restriction is imposed on any other boy-girl pair. In particular, the question does not ask for all boys together, all girls together, or alternating genders.

Step 2: Subtract seatings with the forbidden adjacency Treat the specified boy B_1 and girl G_1 as a single block. Together with the other six people, there are seven units around the table. Their circular arrangements number $6!$. Within the block the order can be B_1G_1 or G_1B_1, so

$$N_{\text{adjacent}} = 2 \cdot 6!.$$

The complement gives

$$N_{\text{allowed}} = 7! - 2 \cdot 6! = (7-2)6! = 5 \cdot 6!.$$

This subtraction includes adjacency across the apparent beginning and end of a written seating because the arrangements were circular throughout. Option (B) removes only $6!$, overlooking one block orientation; option (C) applies no restriction, and option (D) exceeds even the unrestricted count.

Step 3: Conclusion The number of permissible seatings is $\boxed{5 \times 6!}$. Hence, the correct option is (A).

Quick Tip to Solve: For one forbidden pair around a round table, subtract the two internal orders of its adjacency block from the unrestricted circular count.

4. The function f : N $\to$ N defined by f (x) = x $-$ 5 $\left[\frac{x}{5}\right]$, Where N is the set of natural numbers and [x] denotes the greatest integer less than or equal to x, is : [4 marks]

(A) one-one and onto

(B) one-one but not onto.

(C) onto but not one-one.

(D) neither one-one nor onto.

Correct Answer: (D) neither one-one nor onto.

Solution:

Concept:

- The expression $x - 5\lfloor x/5 \rfloor$ is the remainder on dividing the integer x by 5. The division algorithm writes $x = 5q + r$, with $0 \leq r < 5$.
- A map is one-one if equal outputs force equal inputs. It is onto its stated codomain if every element of that codomain occurs as an output. Repeated residues and the finite range decide these properties.

Step 1: Determine the possible outputs For $x = 5q + r$, where q is an integer and $r \in \{0, 1, 2, 3, 4\}$, we have

$$\left\lfloor \frac{x}{5} \right\rfloor = q, \qquad f(x) = 5q + r - 5q = r.$$

Thus the formula produces only five possible values. For example, $f(1) = 1$, $f(2) = 2$, and $f(5) = 0$. In particular, increasing the input by 5 leaves the output unchanged.

Step 2: Test injectivity and surjectivity separately Two distinct natural inputs already disprove injectivity:

$$f(1) = 1 = f(6), \qquad 1 \neq 6.$$

Surjectivity onto the natural numbers also fails: the natural number 5 is never a remainder modulo 5, so no input satisfies $f(x) = 5$. The same is true of every natural number larger than 4. Consequently, neither required property holds. If natural numbers are defined as positive integers only, the printed formula also has a codomain defect because multiples of 5 produce 0. Under the convention that 0 is included, it is a well-defined map and still has exactly the same non-injectivity and non-surjectivity. Either conventional reading explains why the listed classification intended here is neither one-one nor onto.

Step 3: Conclusion The residue rule is neither one-one nor onto **N**. Hence, the correct option is (D).

Quick Tip to Solve: For a remainder function, compare inputs differing by the divisor, then check a codomain value at least as large as the divisor.

5. A value of x satisfying the equation $\sin[\cot^{-1}(1+x)] = \cos[\tan^{-1}x]$, is : [4 marks]

(A) $-\frac{1}{2}$

(B) -1

(C) 0

(D) $\frac{1}{2}$

Correct Answer: (A) $-\frac{1}{2}$

Solution:

Concept:

- For the principal angle $\theta = \cot^{-1}t \in (0,\pi)$, $\sin\theta > 0$ and $\sin\theta = 1/\sqrt{1+t^2}$. The sign remains positive even when t is negative.
- For $\phi = \tan^{-1}x \in (-\pi/2, \pi/2)$, $\cos\phi > 0$ and $\cos\phi = 1/\sqrt{1+x^2}$. These positive-root identities convert the equation into algebra without a quadrant ambiguity.

Step 1: Replace the trigonometric expressions Apply the two principal-value identities with $t = 1+x$. The given equation becomes

$$\frac{1}{\sqrt{1+(1+x)^2}} = \frac{1}{\sqrt{1+x^2}}.$$

Both denominators are strictly positive for every real x, so taking reciprocals and squaring preserves equivalence. It follows that

$$1+(1+x)^2 = 1+x^2.$$

Step 2: Solve and check the original equality Expand only after the radical equation has been reduced:

$$1+2x+x^2 = x^2, \qquad 2x+1=0, \qquad x = -\frac{1}{2}.$$

At this value, $1+x = 1/2$, and both sides of the original equation equal

$$\frac{1}{\sqrt{1+1/4}} = \frac{2}{\sqrt{5}}.$$

Thus the candidate satisfies the unsquared equation, not merely a necessary squared condition. For $x = -1, 0, 1/2$, the quantities $(1+x)^2$ and x^2 are unequal, so none of the other listed values works. The principal ranges used above also show that no sign choice was lost when the inverse trigonometric functions were replaced.

Step 3: Conclusion A satisfying value is $\boxed{x = -1/2}$. Hence, the correct option is (A).

Quick Tip to Solve: Write sine of inverse cotangent and cosine of inverse tangent as positive reciprocal square roots before comparing their arguments.

6. If $f\left(\frac{3x-4}{3x+4}\right) = x + 2, x \neq -\frac{4}{3}$, and $\int f(x)\,dx = A \log|1 - x| + Bx + C$, then the ordered pair (A, B) is equal to : (where C is a constant of integration) [4 marks]

(A) $\left(\frac{8}{3}, \frac{2}{3}\right)$
(B) $\left(-\frac{8}{3}, \frac{2}{3}\right)$
(C) $\left(-\frac{8}{3}, -\frac{2}{3}\right)$
(D) $\left(\frac{8}{3}, -\frac{2}{3}\right)$

Correct Answer: (B) $\left(-\frac{8}{3}, \frac{2}{3}\right)$

Solution:

Concept:

- To recover a function from a fractional-linear input, introduce a new variable for that input and solve back for the original variable. The excluded denominator value must be retained in the domain.
- The antiderivative $\int (1-x)^{-1}\,dx = -\log|1-x| + C$ has a minus sign from the derivative of $1 - x$. Separating a rational function into a constant and a reciprocal term makes coefficient comparison direct.

Step 1: Invert the input transformation Let $t = (3u - 4)/(3u + 4)$, using u for the original input to avoid confusing it with the eventual integration variable. Cross-multiplication gives

$$3tu + 4t = 3u - 4, \qquad 3u(1 - t) = 4(1 + t).$$

Hence $u = 4(1 + t)/[3(1 - t)]$, and

$$f(t) = u + 2 = \frac{10 - 2t}{3(1 - t)} = \frac{2}{3} + \frac{8}{3(1 - t)}.$$

The transformed variable cannot equal 1, consistently with this expression.

Step 2: Integrate and compare coefficients Rename t as x and integrate on an interval excluding 1:

$$\int f(x)\,dx = \frac{2}{3}x - \frac{8}{3}\log|1 - x| + C.$$

Comparison with $A\log|1 - x| + Bx + C$ gives $A = -8/3$ and $B = 2/3$. Check by differentiation: the logarithmic term contributes $8/[3(1 - x)]$, while the linear term contributes $2/3$, recovering $f(x)$. A positive A would reverse the reciprocal term, and a negative B would reverse the constant term; these sign checks eliminate all other listed pairs.

Step 3: Conclusion The ordered pair is $\boxed{(A, B) = (-8/3,\ 2/3)}$. Hence, the correct option is (B).

Quick Tip to Solve: Use a fresh variable to invert the fractional input, then check the logarithmic coefficient by differentiation to catch its sign.

7. The sum of 100 observations and the sum of their squares are 400 and 2475, respectively. Later on, three observations, 3, 4 and 5, were found to be incorrect. If the incorrect observations are omitted, then the variance of the remaining observations is : [4 marks]

(A) 8.25

(B) 8.50

(C) 8.00

(D) 9.00

Correct Answer: (D) 9.00

Solution:

Concept:

- For n observations, the variance is $\sigma^2 = (\sum x_i^2)/n - (\sum x_i/n)^2$. Both sums and the number of observations must refer to the same retained data set.
- When observations are omitted rather than replaced, subtract their values from the sum, subtract their squares from the sum of squares, and reduce the count by the number deleted.

Step 1: Update all three data summaries Three observations are removed from the original 100, leaving

$$n = 100 - 3 = 97.$$

Their sum is $3 + 4 + 5 = 12$, so the retained observations have sum

$$S = 400 - 12 = 388.$$

Their squared values sum to $3^2 + 4^2 + 5^2 = 9 + 16 + 25 = 50$, so

$$Q = 2475 - 50 = 2425.$$

Subtracting 12^2 here would be incorrect: the given summary is the sum of individual squares, not the square of the sum.

Step 2: Calculate the retained mean and variance The retained mean is

$$\bar{x} = \frac{388}{97} = 4.$$

The retained mean square is

$$\frac{Q}{n} = \frac{2425}{97} = 25.$$

Therefore the required variance is

$$\sigma^2 = 25 - 4^2 = 25 - 16 = 9.$$

The denominator is 97 throughout because the incorrect observations are omitted completely. This question asks for the variance of the listed observations, so use the population-data formula, rather than an unbiased sample estimator with denominator 96. Values 8.25, 8.50, and 8.00 result from different summaries and do not equal the variance of this retained data set. The unchanged mean is

consistent with deleting 3, 4, and 5, whose own mean is also 4.

Step 3: Conclusion The variance of the remaining observations is $\boxed{9.00}$. Hence, the correct option is (D).

Quick Tip to Solve: After deletions, update count, sum, and sum of squares together; only then form the mean and the mean square.

8. The function f defined by

$f(x) = x^3 - 3x^2 + 5x + 7$, is : [4 marks]

(A) increasing in **R**.

(B) decreasing in **R**.

(C) decreasing in $(0, \infty)$ and increasing in $(-\infty, 0)$

(D) increasing in $(0, \infty)$ and decreasing in $(-\infty, 0)$

Correct Answer: (A) increasing in **R**.

Solution:

Concept:

- A differentiable function is strictly increasing on an interval when its derivative is positive throughout that interval.
- Completing a square gives a global lower bound for a quadratic derivative; the sign must be checked on the entire real line.

Step 1: Differentiate the polynomial The derivative controls the direction of change, whereas the constant term only shifts the graph vertically. Differentiating each power separately gives

$$f\prime(x) = 3x^2 - 6x + 5.$$

The sign cannot be inferred merely from the positive leading coefficient: we must also inspect the minimum value of this quadratic.

Step 2: Complete the square and determine its sign Rearrange the derivative about its vertex at $x = 1$:

$$f\prime(x) = 3(x^2 - 2x + 1) + 2 = 3(x - 1)^2 + 2.$$

For every real x, the squared term is nonnegative, so $f\prime(x) \geq 2 > 0$. Thus the derivative never vanishes and never becomes negative. More formally, for any $a < b$, the mean value theorem gives $f(b) - f(a) = f\prime(c)(b - a) > 0$ for some $c \in (a, b)$. The function therefore increases across any two real arguments, including arguments on opposite sides of zero. Option (B) would require the opposite sign; options (C) and (D) incorrectly assert a change in monotonicity at zero.

Step 3: Conclusion The function is strictly increasing on $\mathbb{R}$. Hence, the correct option is (A).

Quick Tip to Solve: For a cubic, complete the square in its quadratic derivative before looking for intervals of increase or decrease.

9. Let f be a polynomial function such that
f (3x) = f ' (x) . f '' (x), for all x $\in$ R. Then : [4 marks]

(A) f (2) + f ' (2) = 28

(B) f '' (2) − f ' (2) = 0

(C) f '' (2) − f (2) = 4

(D) f (2) − f ' (2) + f '' (2) = 10

Correct Answer: (B) f '' (2) − f ' (2) = 0

Solution:

Concept:

- A polynomial identity valid for every real argument has equal coefficients of every power on its two sides.
- For a nonzero polynomial of degree $n \geq 2$, the degrees of $f(3x)$ and $f\prime(x)f''(x)$ are n and $2n-3$.

Step 1: Determine the possible degree The printed second derivative is denoted by f''. If f has degree zero or one, then $f'' = 0$, so the identity forces $f \equiv 0$. Otherwise degree comparison gives

$$n = 2n - 3 \quad \Rightarrow \quad n = 3.$$

Thus it suffices to inspect a general cubic, as well as retaining the zero solution.

Step 2: Compare coefficients of the cubic Write $f(x) = ax^3 + bx^2 + cx + d$, with $a \neq 0$. Then

$$f\prime(x)f''(x) = 18a^2x^3 + 18abx^2 + (4b^2 + 6ac)x + 2bc.$$

Comparing with $f(3x) = 27ax^3 + 9bx^2 + 3cx + d$ yields $a = 3/2$, then $b = 0$, $c = 0$ and $d = 0$. For this cubic,

$$f(2) = 12, \qquad f\prime(2) = 18, \qquad f''(2) = 18.$$

Therefore the difference in option (B) is zero. Options (A), (C) and (D) instead give 30, 6 and 12, respectively, rather than their stated values. For the zero polynomial option (B) also remains true, while all the other asserted nonzero values are false.

Step 3: Conclusion In every permitted case, $f''(2) - f\prime(2) = 0$. Hence, the correct option is (B).

Quick Tip to Solve: Compare degrees before expanding a polynomial functional equation, but check the zero polynomial separately.

10. If $\int_1^2 \frac{dx}{(x^2-2x+4)^{\frac{3}{2}}} = \frac{k}{k+5}$, **then k is equal to :** [4 marks]

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (A) 1

Solution:

Concept:

- Completing the square reduces $x^2 - 2x + 4$ to $(x-1)^2 + 3$.
- The identity $\frac{d}{du}\left(\frac{u}{\sqrt{u^2+a^2}}\right) = \frac{a^2}{(u^2+a^2)^{3/2}}$ evaluates this integral directly.

Step 1: Shift the variable and find a primitive Put $u = x - 1$, so $dx = du$; the bounds $x = 1, 2$ become $u = 0, 1$. The integral is

$$I = \int_0^1 \frac{du}{(u^2+3)^{3/2}}.$$

The denominator remains positive, so the substitution crosses no singularity. Differentiating the useful quotient verifies the constant factor explicitly:

$$\frac{d}{du}\left(\frac{u}{3\sqrt{u^2+3}}\right) = \frac{1}{(u^2+3)^{3/2}}.$$

Step 2: Evaluate the endpoints and solve for k Using the primitive at the transformed endpoints gives

$$I = \left[\frac{u}{3\sqrt{u^2+3}}\right]_0^1 = \frac{1}{3\cdot 2} - 0 = \frac{1}{6}.$$

The given relation therefore reads

$$\frac{k}{k+5} = \frac{1}{6} \quad \Rightarrow \quad 6k = k+5 \quad \Rightarrow \quad k = 1.$$

This value is allowed because $k + 5 = 6 \neq 0$, and substitution returns $1/6$. The other listed values produce $2/7$, $3/8$ and $4/9$, none equal to the evaluated integral. The positive integral also agrees with the positive value found.

Step 3: Conclusion The integral is $1/6$, and $k = 1$. Hence, the correct option is (A).

Quick Tip to Solve: After completing the square, look for the derivative of a linear numerator divided by the square root of the quadratic.

11. The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$ is : [4 marks]

(A) 16

(B) 14

(C) -4

(D) -5

Correct Answer: (C) -4

Solution:

Concept:

- For a real exponent t, $2^t = 1$ holds exactly when $t = 0$, because the exponential function with base 2 is strictly increasing.
- Once the exponent vanishes, factor its polynomial and include every distinct real root in the required sum.

Step 1: Set the exponent equal to zero The base is the fixed positive number 2, not an expression involving x. Since $2^0 = 1$ and no other real exponent gives 1, the equation is equivalent to

$$(x-1)(x^2+5x-50) = 0.$$

This step is valid for all real x, with no logarithmic domain restriction on x itself: every real exponent defines a positive power of 2.

Step 2: Factor and sum all real solutions The quadratic factors because the two numbers 10 and -5 multiply to -50 and add to 5. Hence

$$(x-1)(x+10)(x-5) = 0.$$

The complete real solution set is $\{1, -10, 5\}$. Each root makes the exponent zero and therefore satisfies the original exponential equation; there are no other roots because the product exhausts all factors of the exponent. Their sum is

$$1 + (-10) + 5 = -4.$$

Equivalently, the quadratic roots sum to -5 by Vieta, and the remaining linear root contributes $+1$. Option (D) omits this extra root, while the positive sums in options (A) and (B) do not include the negative root correctly.

Step 3: Conclusion The sum of all real solutions is -4. Hence, the correct option is (C).

Quick Tip to Solve: For a fixed base other than 1, first reduce an exponential equation with right side 1 to a zero-exponent equation.

12. A line drawn through the point P(4, 7) cuts the circle $x^2 + y^2 = 9$ at the points A and B. Then PA·PB is equal to : [4 marks]

(A) 53

(B) 56

(C) 74

(D) 65

Correct Answer: (B) 56

Solution:

Concept:

- For a circle with centre O and radius r, the power of an external point P is $OP^2 - r^2$.
- For any secant through an external point meeting the circle at A, B, the product of the two secant distances is $PA \cdot PB = OP^2 - r^2$.

Step 1: Find the centre, radius and position of P The circle $x^2 + y^2 = 9$ has centre $O = (0, 0)$ and radius $r = 3$. For $P = (4, 7)$,

$$OP^2 = 4^2 + 7^2 = 16 + 49 = 65.$$

Since $65 > 9$, the point lies outside the circle. Thus the two intersections of a secant from P lie along the same ray toward the circle, and the product of their ordinary positive distances is the positive power of the point.

Step 2: Apply the secant product relation The power theorem gives

$$PA \cdot PB = OP^2 - r^2 = 65 - 9 = 56.$$

The result is independent of which intersecting line through P is drawn. To see this algebraically, parameterise such a line by a unit direction vector $\mathbf{u}$: a point is $P + t\mathbf{u}$. Substitution into the circle gives $t^2 + 2(P \cdot \mathbf{u})t + 56 = 0$. The two signed intersection parameters have product 56, which is also the product of their distances because they have the same sign. Option (D) gives only OP^2 and neglects the radius correction; the other listed products likewise fail the secant identity.

Step 3: Conclusion The required product is $PA \cdot PB = 56$. Hence, the correct option is (B).

Quick Tip to Solve: For a product of two secant lengths from one external point, compute the point power instead of finding either intersection.

13. The coefficient of x^{-5} in the binomial expansion of $\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}$, where $x \neq 0, 1$, is : [4 marks]

(A) 1

(B) 4

(C) -4

(D) -1

Correct Answer: (A) 1

Solution:

Concept:

- The sum-of-cubes factorisation $t^3+1=(t+1)(t^2-t+1)$ simplifies the first fraction.
- In $(a+b)^{10}$, the term containing b^r is $\binom{10}{r}a^{10-r}b^r$; match its exponent to the requested power.

Step 1: Simplify the expression inside the power Put $t=x^{1/3}$ in the first quotient. Then

$$\frac{x+1}{x^{2/3}-x^{1/3}+1}=x^{1/3}+1.$$

For the second quotient factor $x-1=(\sqrt{x}-1)(\sqrt{x}+1)$ and $x-\sqrt{x}=\sqrt{x}(\sqrt{x}-1)$. For the real expression its domain has $x>0$, $x\neq 1$, so the cancellation is valid, giving

$$\frac{x-1}{x-\sqrt{x}}=1+x^{-1/2}.$$

The expression to expand is consequently $(x^{1/3}-x^{-1/2})^{10}$.

Step 2: Match the exponent in the general term Choosing the second term r times produces

$$T_{r+1}=\binom{10}{r}(-1)^r x^{(10-r)/3-r/2}=\binom{10}{r}(-1)^r x^{(20-5r)/6}.$$

To obtain x^{-5}, solve

$$(20-5r)/6=-5 \quad \Rightarrow \quad r=10.$$

This is an allowed index and is unique, since the exponent decreases strictly with r. Its coefficient is $\binom{10}{10}(-1)^{10}=1$. There is no factor 4: this is the last term of the expansion, and its even number of negative factors makes the sign positive. Thus none of the other three coefficients can apply.

Step 3: Conclusion The coefficient of x^{-5} is 1. Hence, the correct option is (A).

Quick Tip to Solve: Simplify fractional algebraic expressions before applying the binomial theorem; the desired negative power may be an endpoint term.

14. A square, of each side 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x-axis, then the sum of the x-coordinates of the vertices of the square is : [4 marks]

(A) $2\sqrt{3}-1$

(B) $2\sqrt{3}-2$

(C) $\sqrt{3}-2$

(D) $\sqrt{3}-1$

Correct Answer: (B) $2\sqrt{3}-2$

Solution:

Concept:

- Adjacent sides of a square are perpendicular vectors of equal length.
- A segment of length 2 at angle θ has coordinate vector $(2\cos\theta, 2\sin\theta)$; the fourth vertex is the sum of the two adjacent side vectors.

Step 1: Choose the two side vectors above the axis Let the origin be O, and let the given side terminate at A. Its length and inclination give

$$A = (2\cos 30^\circ, 2\sin 30^\circ) = (\sqrt{3}, 1).$$

The other side through O must have inclination 120° or -60°, because it is perpendicular. The latter would put its endpoint below the x-axis, contrary to the placement of the square. Therefore its endpoint is

$$B = (2\cos 120^\circ, 2\sin 120^\circ) = (-1, \sqrt{3}).$$

Step 2: Locate the remaining vertex and add x-coordinates The opposite vertex C is obtained by adding the adjacent side vectors:

$$C = A + B = (\sqrt{3}-1, 1+\sqrt{3}).$$

All three non-origin vertices have positive y-coordinates, checking the required position. The four x-coordinates are $0, \sqrt{3}, -1, \sqrt{3}-1$, whose sum is

$$0 + \sqrt{3} - 1 + (\sqrt{3}-1) = 2\sqrt{3} - 2.$$

The negative x-coordinate of B is essential; lying above the x-axis does not require lying to the right of the y-axis. Options (A), (C) and (D) omit part of the contributions or count only some vertices. The square centre also has x-coordinate $(\sqrt{3}-1)/2$, and four times that value confirms the sum.

Step 3: Conclusion The sum is $2\sqrt{3}-2$. Hence, the correct option is (B).

Quick Tip to Solve: For a square based at the origin, write its adjacent side vectors; the four vertices are zero, each vector, and their sum.

15. For two 3 × 3 matrices A and B, let A + B = $2B^T$ and 3A + 2B = I_3, where B^T is the transpose of B and I_3 is 3 × 3 identity matrix. Then : [4 marks]

(A) 5A + 10B = $2I_3$

(B) 10A + 5B = $3I_3$

(C) B + 2A = I_3

(D) 3A + 6B = $2I_3$

Correct Answer: (B) 10A + 5B = $3I_3$

Solution:

Concept:

- Transpose is linear: $(A+B)^T = A^T + B^T$, and $(B^T)^T = B$. A symmetric matrix equals its transpose.
- The identity matrix I_3 satisfies $I_3^T = I_3$. Linear equations in matrices may be combined using scalar multiplication and addition, preserving their order.

Step 1: Eliminate the first matrix From the first given equation, express A in terms of B and its transpose:

$$A = 2B^T - B.$$

Substitute this expression in the second equation. This gives

$$3(2B^T - B) + 2B = I_3, \qquad 6B^T - B = I_3.$$

No product of matrices occurs here, so no commutativity assumption is involved.

Step 2: Use the transposed equation Transpose the last equality, noting that the transpose of the identity is unchanged:

$$6B - B^T = I_3.$$

Subtract the transposed equality from the original one. The right sides cancel and the left sides give

$$7B^T - 7B = 0.$$

Thus $B^T = B$. Returning to $6B^T - B = I_3$, obtain

$$5B = I_3, \qquad B = \frac{1}{5}I_3.$$

The first relation then gives $A = B = I_3/5$.

Step 3: Check the proposed identities For these values,

$$10A + 5B = \frac{10+5}{5}I_3 = 3I_3.$$

In contrast, $5A + 10B = 3I_3$, not $2I_3$; $B + 2A = 3I_3/5$, not I_3; and $3A + 6B = 9I_3/5$, not $2I_3$. Hence only the second identity follows from both given conditions.

Step 4: Conclusion $10A + 5B = 3I_3$. Hence, the correct option is (B).

Quick Tip to Solve: When a matrix and its transpose occur together, transpose the equation and subtract; this often forces the matrix to be symmetric.

16. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60°. If the area of the quadrilateral is $4\sqrt{3}$, then the perimeter of the quadrilateral is : [4 marks]

(A) 12.5

(B) 13.2

(C) 12

(D) 13

Correct Answer: (C) 12

Solution:

Concept:

- Opposite angles of a cyclic quadrilateral sum to $180°$. A diagonal separates a convex cyclic quadrilateral into two triangles.
- For sides a, b enclosing angle θ, the triangle area is $ab\sin\theta/2$, and the opposite side satisfies the cosine rule $d^2 = a^2 + b^2 - 2ab\cos\theta$.

Step 1: Find the diagonal and the first area Let the known adjacent sides be $AB = 2$ and $BC = 5$, with $\angle ABC = 60°$. Draw the diagonal AC conceptually and apply the cosine rule in triangle ABC:

$$AC^2 = 2^2 + 5^2 - 2(2)(5)\cos 60° = 4 + 25 - 10 = 19.$$

Its area is

$$[ABC] = \frac{1}{2}(2)(5)\sin 60° = \frac{5\sqrt{3}}{2}.$$

Step 2: Use the area of the remaining triangle The opposite angle ADC is $120°$ because the quadrilateral is cyclic. Let $AD = u$ and $CD = v$. Subtracting the first triangle area from the given total gives

$$[ADC] = 4\sqrt{3} - \frac{5\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}.$$

Consequently,

$$\frac{1}{2}uv\sin 120° = \frac{3\sqrt{3}}{2}, \qquad \frac{uv\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}, \qquad uv = 6.$$

Step 3: Determine the sum of the unknown sides Apply the cosine rule to the same diagonal in triangle ADC:

$$19 = u^2 + v^2 - 2uv\cos 120° = u^2 + v^2 + uv.$$

Since $uv = 6$, we have $u^2 + v^2 = 13$. Thus

$$(u + v)^2 = 13 + 2(6) = 25.$$

Both lengths are positive, so $u + v = 5$. The perimeter is therefore $2 + 5 + 5 = 12$. The other listed values do not satisfy this sum obtained simultaneously from the area and the diagonal.

Step 4: Conclusion The perimeter of the cyclic quadrilateral is 12. Hence, the correct option is (C).

Quick Tip to Solve: Use the area to find the product of the two unknown sides, then the cosine rule to find their sum without finding each side separately.

17. If the vector $\overrightarrow{b} = 3\hat{j} + 4\hat{k}$ is written as the sum of a vector $\overrightarrow{b_1}$, paralel to $\overrightarrow{a} = \hat{i} + \hat{j}$ and a vector $\overrightarrow{b_2}$, perpendicular to $\overrightarrow{a}$, then $\overrightarrow{b_1} \times \overrightarrow{b_2}$ is equal to : [4 marks]

(A) $-3\hat{i} + 3\hat{j} - 9\hat{k}$

(B) $6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}$

(C) $-6\hat{i} + 6\hat{j} - \frac{9}{2}\hat{k}$

(D) $3\hat{i} - 3\hat{j} + 9\hat{k}$

Correct Answer: (B) $6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}$

Solution:

Concept:

- The component of $\vec{b}$ parallel to a nonzero vector $\vec{a}$ is $\vec{b}_1 = ((\vec{b}\cdot\vec{a})/(\vec{a}\cdot\vec{a}))\vec{a}$. The remaining component $\vec{b}_2 = \vec{b} - \vec{b}_1$ is perpendicular to $\vec{a}$.
- The cross product is bilinear and $\vec{v} \times \vec{v} = 0$. Reversing its order reverses its sign.

Step 1: Calculate the parallel projection Here $\vec{a} = (1, 1, 0)$ and $\vec{b} = (0, 3, 4)$. Their dot products give

$$\vec{b} \cdot \vec{a} = 3, \qquad \vec{a} \cdot \vec{a} = 2.$$

Hence the required parallel component is

$$\vec{b}_1 = \frac{3}{2}(\hat{i} + \hat{j}).$$

It is important to divide by the squared magnitude of $\vec{a}$, rather than just its magnitude, since the result must be a vector projection.

Step 2: Find and check the perpendicular component Subtract the projection from the original vector:

$$\vec{b}_2 = -\frac{3}{2}\hat{i} + \frac{3}{2}\hat{j} + 4\hat{k}.$$

This decomposition has the required sum, since the first coordinates cancel and the second coordinates add to 3. Also

$$\vec{b}_2 \cdot \vec{a} = -\frac{3}{2} + \frac{3}{2} = 0,$$

confirming the stated perpendicularity.

Step 3: Evaluate the ordered cross product Using the components in the prescribed order,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3/2 & 3/2 & 0 \\ -3/2 & 3/2 & 4 \end{vmatrix} = 6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}.$$

Option (C) is the negative of this vector and corresponds to reversing the cross-product order. Options (A) and (D) have different component magnitudes and do not result from the projection calculated above.

Step 4: Conclusion $\vec{b}_1 \times \vec{b}_2 = 6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}$. Hence, the correct option is (B).

Quick Tip to Solve: After projecting, use $\vec{b}_1 \times (\vec{b} - \vec{b}_1) = \vec{b}_1 \times \vec{b}$ to shorten the determinant calculation.

18. The equation

$\text{Im}\left(\frac{iz-2}{z-i}\right) + 1 = 0, z \in C, z \neq i$

represents a part of a circle having radius equal to : [4 marks]

(A) 2

(B) 1

(C) $\frac{3}{4}$

(D) $\frac{1}{2}$

Correct Answer: (C) $\frac{3}{4}$

Solution:

Concept:

- Write a complex number as $z = x + iy$, where x, y are real. Multiplying a quotient by the conjugate of its denominator separates its real and imaginary parts.
- A circle in Cartesian form is $(x-h)^2 + (y-k)^2 = r^2$. Completing the square gives its centre and radius; a forbidden value of z remains excluded from the locus.

Step 1: Extract the imaginary part Put $z = x + iy$. Then $iz - 2 = -y - 2 + ix$ and $z - i = x + i(y-1)$. Since $z \neq i$, the real denominator $D = x^2 + (y-1)^2$ is positive. Rationalising gives

$$\frac{iz-2}{z-i} = \frac{(-y-2+ix)(x-i(y-1))}{D}.$$

The imaginary coefficient of the numerator is

$$(-y-2)(-(y-1)) + x^2 = x^2 + y^2 + y - 2.$$

Step 2: Convert the condition to a circle equation The given condition says the imaginary part is -1, so

$$\frac{x^2 + y^2 + y - 2}{x^2 + (y-1)^2} = -1.$$

Multiplication is legitimate because the denominator is nonzero. Collecting the terms on one side gives

$$2x^2 + 2y^2 - y - 1 = 0, \qquad x^2 + y^2 - \frac{y}{2} = \frac{1}{2}.$$

Complete the square in y:

$$x^2 + \left(y - \frac{1}{4}\right)^2 = \frac{1}{2} + \frac{1}{16} = \frac{9}{16}.$$

Step 3: Read the radius and retain the exclusion The centre is $(0, 1/4)$, and the positive square root of $9/16$ is $3/4$. The point $(0, 1)$, corresponding to $z = i$, lies on this circle but is excluded by the original quotient. Removing that point explains the wording "part of a circle" and does not alter the radius. The other options are not the square root of the completed-square constant.

Step 4: Conclusion The radius is $3/4$. Hence, the correct option is (C).

Quick Tip to Solve: Keep the original denominator restriction when converting a complex quotient into a Cartesian locus; cross-multiplication can otherwise add an excluded point.

19. If $2x = y^{\frac{1}{5}} + y^{-\frac{1}{5}}$ and
$(x^2 - 1)\frac{d^2y}{dx^2} + \lambda x \frac{dy}{dx} + ky = 0$,
then $\lambda + k$ is equal to : [4 marks]

(A) -23

(B) -24

(C) 26

(D) -26

Correct Answer: (B) -24

Solution:

Concept:

- A substitution can simplify an implicit relation before differentiation. Here $t = y^{1/5} \neq 0$ gives $x = (t + t^{-1})/2$ and $y = t^5$.
- For parametric functions, $dy/dx = (dy/dt)/(dx/dt)$, and the second derivative is obtained by differentiating this quotient with respect to t and dividing by dx/dt.

Step 1: Express the first derivative parametrically Let $t = y^{1/5}$. The reciprocal in the question requires $t \neq 0$. Differentiation yields

$$\frac{dx}{dt} = \frac{t^2 - 1}{2t^2}, \qquad \frac{dy}{dt} = 5t^4.$$

On a differentiable branch with $t^2 \neq 1$,

$$y' = \frac{10t^6}{t^2 - 1}.$$

Also the implicit relation gives

$$x^2 - 1 = \frac{(t^2 - 1)^2}{4t^2}.$$

Step 2: Calculate the second derivative combination Differentiate the first derivative with the quotient rule and divide by dx/dt:

$$y'' = \frac{40t^7(2t^2 - 3)}{(t^2 - 1)^3}.$$

Therefore

$$(x^2 - 1)y'' = \frac{10t^5(2t^2 - 3)}{t^2 - 1}, \qquad xy' = \frac{5t^5(t^2 + 1)}{t^2 - 1}.$$

Their sum simplifies because the numerator contains $25t^5(t^2 - 1)$:

$$(x^2 - 1)y'' + xy' = 25t^5 = 25y.$$

Step 3: Identify the coefficients The resulting differential equation is

$$(x^2 - 1)y'' + xy' - 25y = 0.$$

Comparing it with the equation in the question yields $\lambda = 1$ and $k = -25$, hence $\lambda + k = -24$. The calculation holds on either smooth branch of the implicit relation; division by $t^2 - 1$ concerns the derivative parametrisation rather than a choice of the answer. The other options do not equal the

coefficient sum of this identity.

Step 4: Conclusion $\lambda + k = 1 - 25 = -24$. Hence, the correct option is (B).

Quick Tip to Solve: For relations of the form $2x = t + t^{-1}$, compute $x^2 - 1 = (t - t^{-1})^2/4$ before taking a second derivative.

20. Let E and F be two independent events. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then a value of $\frac{P(E)}{P(F)}$ is : [4 marks]

(A) $\frac{4}{3}$
(B) $\frac{3}{2}$
(C) $\frac{1}{3}$
(D) $\frac{5}{12}$

Correct Answer: (A) $\frac{4}{3}$

Solution:

Concept:

- For independent events E, F, the probability of their intersection is the product: $P(E \cap F) = P(E)P(F)$. Their complements are independent as well.
- If two numbers have known sum s and product p, they are the roots of $u^2 - su + p = 0$. The unordered pair can give two reciprocal ratios.

Step 1: Translate both probability statements Let $p = P(E)$ and $q = P(F)$. The first statement gives

$$pq = \frac{1}{12}.$$

The probability that neither event happens is

$$(1 - p)(1 - q) = \frac{1}{2}.$$

Expanding the latter equality and using the product yields

$$1 - p - q + \frac{1}{12} = \frac{1}{2}, \qquad p + q = \frac{7}{12}.$$

This uses independence; without it the intersection need not equal the product.

Step 2: Recover the possible probabilities The two probabilities are the roots of

$$u^2 - \frac{7}{12}u + \frac{1}{12} = 0.$$

Multiply through by 12 and factor:

$$12u^2 - 7u + 1 = (3u - 1)(4u - 1) = 0.$$

Thus their unordered values are $1/3$ and $1/4$. Both are valid probabilities. Their product is $1/12$, and the complementary product is $(2/3)(3/4) = 1/2$, checking both given conditions.

Step 3: Compare the possible ratios with the options If $p = 1/3$ and $q = 1/4$, then

$$\frac{P(E)}{P(F)} = \frac{1/3}{1/4} = \frac{4}{3}.$$

Interchanging the event labels gives the other possible ratio $3/4$. The question asks for a value, so it does not need to distinguish these assignments. Among the listed options, only $4/3$ occurs; $3/2$, $1/3$, and $5/12$ are neither of the admissible ratios.

Step 4: Conclusion A possible value of the requested probability ratio is $4/3$. Hence, the correct option is (A).

Quick Tip to Solve: For independent events, use both intersection and neither-event probabilities to obtain the sum and product of the individual probabilities.

21. The value of k for which the function

$$f(x) = \begin{cases} \left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}, & 0 < x < \frac{\pi}{2} \\ k + \frac{2}{5}, & x = \frac{\pi}{2} \end{cases}$$

is continuous at x = $\frac{\pi}{2}$, is : [4 marks]

(A) $\frac{17}{20}$
(B) $\frac{2}{5}$
(C) $\frac{3}{5}$
(D) $-\frac{2}{5}$

Correct Answer: (C) $\frac{3}{5}$

Solution:

Concept:

- Continuity at an included endpoint means the function value equals its one-sided limit from within the domain.
- For a fixed positive base a, the function a^u is continuous in u; if the exponent tends to zero, the power tends to 1. Trigonometric limits must be evaluated at the stated endpoint rather than at zero.

Step 1: Rewrite the exponent near the endpoint Let $h = x - \pi/2$, so $h \to 0^-$. By the periodicity and shift identities for tangent,

$$\tan 4x = \tan(2\pi + 4h) = \tan 4h,$$

while

$$\tan 5x = \tan(5\pi/2 + 5h) = -\cot 5h.$$

Thus, for nearby points where the expression is defined,

$$\frac{\tan 4x}{\tan 5x} = -\tan 4h \tan 5h.$$

Both tangents tend to zero as $h \to 0$, so this exponent tends to zero. In fact it behaves like $-20h^2$.

Step 2: Find the function limit The base $4/5$ is positive and fixed, so the continuity of the exponential function gives

$$\lim_{x \to (\pi/2)^-} \left(\frac{4}{5}\right)^{\tan 4x/\tan 5x} = \left(\frac{4}{5}\right)^{0} = 1.$$

A direct use of $\tan 4x/\tan 5x \to 4/5$ would be incorrect here: that small-angle ratio applies as $x \to 0$, whereas the endpoint here is $\pi/2$. The shift calculation accounts for the pole in the denominator.

Step 3: Match the endpoint value At the included endpoint the given value is $k + 2/5$. Consequently continuity there requires

$$k + \frac{2}{5} = 1, \qquad k = \frac{3}{5}.$$

This choice makes the endpoint value exactly the one-sided limit. The other options give endpoint values different from 1, and therefore cannot produce continuity at the specified endpoint.

Step 4: Conclusion $k = 3/5$. Hence, the correct option is (C).

Quick Tip to Solve: At a nonzero trigonometric endpoint, shift the variable to that endpoint before applying standard small-angle limits.

22. If three positive numbers a, b and c are in A.P. such that abc = 8, then the minimum possible value of b is : [4 marks]

(A) 2

(B) $4^{\frac{1}{3}}$

(C) $4^{\frac{2}{3}}$

(D) 4

Correct Answer: (A) 2

Solution:

Concept:

- For three numbers in arithmetic progression, the middle term is their arithmetic mean: $a + c = 2b$. Positivity permits the arithmetic mean–geometric mean inequality.
- For positive numbers, $(a + b + c)/3 \geq (abc)^{1/3}$, with equality exactly when all three numbers are equal. A lower bound is a minimum only if it can actually be attained.

Step 1: Use the arithmetic progression condition Because a, b, c are in A.P., their sum is $a + b + c = 3b$. Their geometric mean is fixed by the product:

$$(abc)^{1/3} = 8^{1/3} = 2.$$

Therefore the arithmetic mean–geometric mean inequality immediately gives

$$b = \frac{a+b+c}{3} \geq 2.$$

This inequality applies to every allowed positive triple; it is not restricted to increasing progressions.

Step 2: Check that the bound is attainable Choose $a = b = c = 2$. This is an arithmetic progression of common difference zero, all its terms are positive, and its product is $2 \cdot 2 \cdot 2 = 8$. Hence the lower bound is achieved. Equivalently, writing the terms as $b - d, b, b + d$ gives

$$8 = b(b^2 - d^2) \leq b^3,$$

again implying $b \geq 2$, with equality at $d = 0$. The value $4^{1/3} < 2$ violates the bound, while $4^{2/3} > 2$ and $4 > 2$ cannot be the minimum because the equality triple already gives a smaller value.

Step 3: Conclusion The minimum possible value of the middle term is $\boxed{2}$. Hence, the correct option is (A).

Quick Tip to Solve: For three A.P. terms with fixed positive product, replace their arithmetic mean by the middle term and test the equal-term case.

23. A tangent to the curve, y = f(x) at P(x, y) meets x-axis at A and y-axis at B. If AP : BP = 1 : 3 and f(1) = 1, then the curve also passes through the point : [4 marks]

(A) $\left(\frac{1}{3}, 24\right)$
(B) $\left(\frac{1}{2}, 4\right)$
(C) $\left(2, \frac{1}{8}\right)$
(D) $\left(3, \frac{1}{28}\right)$

Correct Answer: (C) $\left(2, \frac{1}{8}\right)$

Solution:

Concept:

- The tangent at $P(x, y)$ has slope dy/dx. A line with nonzero intercepts α and β on the axes has slope $-\beta/\alpha$.
- If P divides AB internally in the ratio $AP : PB = 1 : 3$, the section formula gives $P = (3A + B)/4$. This relates the intercepts to the coordinates of the point of tangency.

Step 1: Express the tangent slope using the ratio Let $A = (\alpha, 0)$ and $B = (0, \beta)$. The given ratio yields

$$x = \frac{3\alpha}{4}, \qquad y = \frac{\beta}{4}.$$

Thus $\alpha = 4x/3$ and $\beta = 4y$. Substituting these intercepts into the tangent slope gives

$$\frac{dy}{dx} = -\frac{\beta}{\alpha} = -\frac{3y}{x}.$$

The minus sign is essential: the line joins its two axis intercepts, so its slope is the negative ratio of the intercepts.

Step 2: Solve and test the listed points Separate variables in the neighbourhood containing $(1, 1)$:

$$\frac{dy}{y} = -3\frac{dx}{x}.$$

Integration gives $\ln|y| = -3\ln|x| + C$, hence $y = Kx^{-3}$. The condition $f(1) = 1$ fixes $K = 1$, so

$$y = \frac{1}{x^3}.$$

At $x = 2$, this gives $y = 1/8$. The other proposed points fail direct substitution: at $x = 1/3$ the ordinate is 27, not 24; at $x = 1/2$ it is 8, not 4; and at $x = 3$ it is $1/27$, not $1/28$.

Step 3: Conclusion The curve passes through $\boxed{(2, 1/8)}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert the intercept division ratio into intercept coordinates before writing the tangent slope; then the equation separates immediately.

---

24. Contrapositive of the statement

'If two numbers are not equal, then their squares are not equal', is : [4 marks]

(A) If the squares of two numbers are equal, then the numbers are equal.

(B) If the squares of two numbers are equal, then the numbers are not equal.

(C) If the squares of two numbers are not equal, then the numbers are not equal.

(D) If the squares of two numbers are not equal, then the numbers are equal.

Correct Answer: (A) If the squares of two numbers are equal, then the numbers are equal.

Solution:

Concept:

- The contrapositive of an implication $p \Rightarrow q$ is $\neg q \Rightarrow \neg p$. It reverses the order and negates both statements.
- The negation of "not equal" is "equal". Forming a contrapositive is a logical operation on the given statement; it does not require proving the original statement true for all real numbers.

Step 1: Identify the hypothesis and conclusion Let the two numbers be u and v. The hypothesis is

$$p: \ u \neq v,$$

while the conclusion is

$$q: \ u^2 \neq v^2.$$

We must negate these complete propositions, preserving which one is the hypothesis and which one is the conclusion. Their negations are $\neg p: \ u = v$ and $\neg q: \ u^2 = v^2$. In particular, negating the square inequality does not introduce a minus sign or change either number.

Step 2: Reverse and negate both propositions The required implication is

$$\neg q \Rightarrow \neg p, \qquad \text{that is,} \qquad u^2 = v^2 \Rightarrow u = v.$$

In words: if the squares of two numbers are equal, then the numbers are equal. Option (B) reverses the implication but does not negate its conclusion correctly. Option (C) is the converse of the original implication, since it reverses the two propositions without negating them. Option (D) negates the original hypothesis alone and does not give the contrapositive. The fact that opposite real numbers can have equal squares concerns the truth of the statement, not the requested formal transformation.

Step 3: Conclusion The contrapositive is: "If the squares of two numbers are equal, then the numbers are equal." Hence, the correct option is (A).

Quick Tip to Solve: Write the statement as $p \Rightarrow q$, then use $\neg q \Rightarrow \neg p$; do not confuse a contrapositive with a converse.

---

25. If x = a, y = b, z = c is a solution of the system of linear equations

x + 8y + 7z = 0

9x + 2y + 3z = 0

x + y + z = 0

such that the point (a, b, c) lies on the plane x + 2y + z = 6, then 2a + b + c equals : [4 marks]

(A) -1

(B) 0

(C) 1

(D) 2

Correct Answer: (C) 1

Solution:

Concept:

- A solution (a, b, c) must satisfy all three homogeneous equations and the additional nonhomogeneous plane equation. Subtracting equations can eliminate a variable without dividing by an unknown.
- The equation $a + b + c = 0$ simplifies both the plane condition and the requested expression: $a + 2b + c = b$ and $2a + b + c = a$.

Step 1: Use the plane condition to find the middle coordinate At the solution point we have

$$a + b + c = 0, \qquad a + 2b + c = 6.$$

Subtract the first relation from the second to obtain $b = 6$. Next subtract $a + b + c = 0$ from $a + 8b + 7c = 0$:

$$7b + 6c = 0.$$

With $b = 6$, this becomes $42 + 6c = 0$, so $c = -7$. Finally, the third homogeneous equation gives

$a = -b - c = -6 + 7 = 1$.

Step 2: Check consistency and evaluate the expression The resulting point is $(1, 6, -7)$. It satisfies the equation not used in the elimination:

$$9a + 2b + 3c = 9 + 12 - 21 = 0.$$

It also satisfies the first equation, since $1 + 48 - 49 = 0$, and the plane, since $1 + 12 - 7 = 6$. Thus no extraneous point has been introduced. The required value is

$$2a + b + c = 2(1) + 6 - 7 = 1.$$

Alternatively, $b + c = -a$ makes the requested expression equal to a directly. The proposed values $-1, 0, 2$ are incompatible with this uniquely fixed point; the zero solution of the homogeneous system is excluded by the additional plane condition.

Step 3: Conclusion We obtain $\boxed{2a + b + c = 1}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the shortest equation twice: subtract it from the plane condition, and use it to simplify the target expression.

26. If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of ΔABC is : [4 marks]

(A) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 1$
(B) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 3$
(C) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{9}$
(D) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 9$

Correct Answer: (A) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 1$

Solution:

Concept:

- A plane with finite nonzero coordinate intercepts a, b, c has equation $X/a + Y/b + Z/c = 1$. Its distance from the origin is the absolute constant divided by the magnitude of its normal.
- The centroid of the triangle with vertices $(a, 0, 0), (0, b, 0), (0, 0, c)$ is $(a/3, b/3, c/3)$. The plane intercepts are therefore three times the centroid coordinates.

Step 1: Apply the distance condition to the intercept form Let the axis intersections be $A = (a, 0, 0)$, $B = (0, b, 0)$ and $C = (0, 0, c)$. The plane is

$$\frac{X}{a} + \frac{Y}{b} + \frac{Z}{c} - 1 = 0.$$

Its distance from the origin is

$$d = \frac{1}{\sqrt{a^{-2} + b^{-2} + c^{-2}}}.$$

Since $d = 3$, squaring and rearranging gives

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9}.$$

The absolute value in the distance formula makes this relation valid for positive or negative intercepts.

Step 2: Replace intercepts by centroid coordinates Write the centroid as (x, y, z). By averaging the three vertex coordinates,

$$x = \frac{a}{3}, \qquad y = \frac{b}{3}, \qquad z = \frac{c}{3}.$$

Substitute $a = 3x, b = 3y, c = 3z$ into the previous relation:

$$\frac{1}{9x^2} + \frac{1}{9y^2} + \frac{1}{9z^2} = \frac{1}{9}.$$

Multiplication by 9 gives a right-hand side of 1, yielding the locus. Conversely, any point with nonzero coordinates satisfying this locus defines finite intercepts $3x, 3y, 3z$ and a plane at distance 3, so the condition is sufficient as well as necessary. The other options miss either the factor of three in the centroid or its square in the reciprocals.

Step 3: Conclusion The locus is $\boxed{\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 1}$, with nonzero coordinates. Hence, the correct option is (A).

Quick Tip to Solve: First impose the distance condition on the intercepts, then substitute intercept $= 3\times$ centroid coordinate; square this factor carefully.

27. If the line, $\frac{x-3}{1} = \frac{y+2}{-1} = \frac{z+\lambda}{-2}$ lies in the plane, 2x−4y+3z=2, then the shortest distance between this line and the line, $\frac{x-1}{12} = \frac{y}{9} = \frac{z}{4}$ is : [4 marks]

(A) 2

(B) 1

(C) 0

(D) 3

Correct Answer: (C) 0

Solution:

Concept:

- A line lying in a plane has its direction vector perpendicular to the plane normal, and every point on the line satisfies the plane equation. The latter condition determines the unknown parameter.
- For nonparallel lines through points P, Q with direction vectors $\mathbf{u}, \mathbf{v}$, their shortest distance is $|(Q - P) \cdot (\mathbf{u} \times \mathbf{v})|/|\mathbf{u} \times \mathbf{v}|$. A zero numerator means the lines are coplanar and hence intersect.

Step 1: Find the parameter from the plane condition The first line can be parametrised as

$$(x, y, z) = (3, -2, -\lambda) + t(1, -1, -2).$$

Its direction satisfies $(2,-4,3)\cdot(1,-1,-2)=2+4-6=0$, as required for a line lying in the plane. Substitute the point $(3,-2,-\lambda)$ into the plane:

$$2(3)-4(-2)+3(-\lambda)=2.$$

Thus $14-3\lambda=2$, giving $\lambda=4$. Take $P=(3,-2,-4)$, $Q=(1,0,0)$, $\mathbf{u}=(1,-1,-2)$ and $\mathbf{v}=(12,9,4)$.

Step 2: Compute the distance and confirm intersection The cross product is

$$\mathbf{u}\times\mathbf{v}=(14,-28,21), \qquad |\mathbf{u}\times\mathbf{v}|=35.$$

Since $Q-P=(-2,2,4)$, the scalar triple product is

$$(-2)(14)+2(-28)+4(21)=0.$$

Therefore the shortest distance is zero. For a direct check, the first line at $t=-2$ gives $(1,0,0)$, which is the second line at parameter zero. Thus the lines actually intersect at $(1,0,0)$; they are not skew or merely parallel. Every positive distance among the other options is excluded by this common point.

Step 3: Conclusion The shortest distance is $\boxed{0}$. Hence, the correct option is (C).

Quick Tip to Solve: After finding a parameter, check whether a simple point on one line also lies on the other; an intersection makes the shortest distance zero.

28. If $\lim\limits_{n\to\infty}\frac{1^a+2^a+.....+n^a}{(n+1)^{a-1}[(na+1)+(na+2)+.....+(na+n)]}=\frac{1}{60}$ **for some positive real number a, then a is equal to :** [4 marks]

(A) 7

(B) 8

(C) $\frac{15}{2}$

(D) $\frac{17}{2}$

Correct Answer: (A) 7

Solution:

Concept:

- For $a>0$, the continuous function t^a on $[0,1]$ gives the Riemann-sum limit $n^{-a-1}\sum_{k=1}^{n}k^a\to\int_0^1 t^a\,dt=1/(a+1)$.
- The denominator contains an arithmetic sum: $\sum_{k=1}^{n}(na+k)=n^2a+n(n+1)/2$. Normalize numerator and denominator by the same power n^{a+1} before taking limits.

Step 1: Normalize the numerator and denominator The numerator divided by n^{a+1} is

$$\frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^a\longrightarrow\frac{1}{a+1}.$$

Meanwhile,

$$\sum_{k=1}^{n}(na+k) = n^2\left(a+\frac{1}{2}+\frac{1}{2n}\right).$$

Consequently the denominator divided by n^{a+1} becomes

$$\left(\frac{n+1}{n}\right)^{a-1}\left(a+\frac{1}{2}+\frac{1}{2n}\right) \longrightarrow a+\frac{1}{2}.$$

The first factor tends to one for the fixed real number a.

Step 2: Use the prescribed limit to solve for the exponent The entire limit is therefore

$$\frac{1}{(a+1)(a+1/2)} = \frac{1}{60}.$$

Since $a > 0$, the denominator is nonzero, so we may cross-multiply:

$$(a+1)\left(a+\frac{1}{2}\right) = 60.$$

Expanding and multiplying by two gives

$$2a^2+3a-119 = 0 = (a-7)(2a+17).$$

The roots are 7 and $-17/2$. The negative root violates the stated positivity, leaving $a = 7$. A substitution check gives $(7+1)(7+1/2) = 8\cdot 7.5 = 60$. The other positive options do not satisfy this equation and hence cannot give the required limit.

Step 3: Conclusion The required positive real number is $\boxed{a=7}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert the power sum to a Riemann sum and simplify the arithmetic sum exactly; both scale as n^{a+1}.

29. If y = mx + c is the normal at a point on the parabola $y^2 = 8x$ whose focal distance is 8 units, then $|c|$ is equal to : [4 marks]

(A) $2\sqrt{3}$

(B) $8\sqrt{3}$

(C) $10\sqrt{3}$

(D) $16\sqrt{3}$

Correct Answer: (C) $10\sqrt{3}$

Solution:

Concept:

- For the parabola $y^2 = 4ax$, a general point is $P(at^2, 2at)$, where t is a real parameter and the focus is $S(a, 0)$.
- The focal distance is $SP = a(1+t^2)$. The normal at parameter t has equation $y = -tx + 2at + at^3$;

its constant term is the required intercept.

Step 1: Determine the parameter from the focal distance Comparing $y^2 = 8x$ with the standard form gives $a = 2$. Thus the focus is $(2, 0)$. At $P = (2t^2, 4t)$, calculate the distance directly:

$$SP = \sqrt{(2t^2 - 2)^2 + (4t)^2} = 2\sqrt{(t^2 + 1)^2} = 2(t^2 + 1).$$

The given distance is eight units, so

$$2(t^2 + 1) = 8 \quad \Rightarrow \quad t^2 = 3, \qquad t = \pm\sqrt{3}.$$

Both signs must be retained because there are symmetric points above and below the axis.

Step 2: Find the constant term of the normal Differentiating the parabola gives $2y\,dy/dx = 8$, so at P the tangent slope is $1/t$ and the normal slope is $-t$. Therefore

$$y - 4t = -t(x - 2t^2), \qquad y = -tx + 4t + 2t^3.$$

Compare with $y = mx + c$:

$$c = 4t + 2t^3 = 2t(2 + t^2) = 10t.$$

Consequently $c = \pm 10\sqrt{3}$, and the absolute value removes the choice of the symmetric point. The values $2\sqrt{3}$, $8\sqrt{3}$, and $16\sqrt{3}$ do not equal this intercept; in particular, the focal distance alone is not the intercept.

Step 3: Conclusion

$$|c| = 10\sqrt{3}.$$

Hence, the correct option is (C).

Quick Tip to Solve: Use the focal distance to find t^2 first, and substitute it in the normal intercept $at(2 + t^2)$.

30. Let

$S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} + \ldots\ldots\ldots + \frac{1+2+\ldots\ldots+n}{1^3+2^3+\ldots\ldots+n^3}$.

If 100 S_n = n, then n is equal to : [4 marks]

(A) 199

(B) 99

(C) 200

(D) 19

Correct Answer: (A) 199

Solution:

Concept:

- The sum of the first k natural numbers is $k(k + 1)/2$, while their cubes sum to $[k(k + 1)/2]^2$.
- A rational term proportional to $1/[k(k + 1)]$ telescopes because $1/[k(k + 1)] = 1/k - 1/(k + 1)$.

Step 1: Simplify the general summand For each positive integer k, write the numerator and denominator of the corresponding fraction using their standard sum formulae. The denominator is the square of the numerator, so

$$\frac{1+2+\cdots+k}{1^3+2^3+\cdots+k^3} = \frac{k(k+1)/2}{[k(k+1)/2]^2} = \frac{2}{k(k+1)}.$$

This formula also gives the first term as $2/(1\cdot 2) = 1$, so it covers the entire sum, including its initial separately displayed term.

Step 2: Telescope and impose the equation Splitting each term into two fractions gives

$$S_n = 2\sum_{k=1}^{n}\left(\frac{1}{k} - \frac{1}{k+1}\right) = 2\left(1 - \frac{1}{n+1}\right) = \frac{2n}{n+1}.$$

All intermediate reciprocals cancel, leaving only the first and last. Substitute this exact result into the stated condition:

$$100\frac{2n}{n+1} = n.$$

Here n is the positive number of summands, so division by n is valid. It gives

$$n+1 = 200, \qquad n = 199.$$

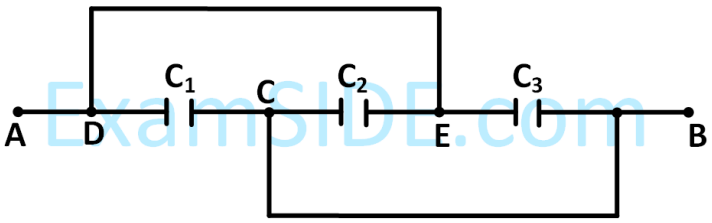
As a check, $S_{199} = 398/200 = 1.99$, and multiplying by 100 gives 199. The choices 99, 200 and 19 do not satisfy $n+1 = 200$; 200 results from forgetting the final subtraction of one.

Step 3: Conclusion The number of terms is $n = 199$. Hence, the correct option is (A).

Quick Tip to Solve: Whenever a ratio contains a sum of cubes, replace the cube sum by the square of the arithmetic sum before attempting addition.

Physics (120 marks)

31. A combination of parallel plate capacitors is maintained at a certain potential difference. When a 3 mm thick slab is introduced between all the plates, in order to maintain the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant of the slab. [4 marks]



ExamSIDE.com

(A) 3

(B) 4

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Concept:

- A slab of thickness t and dielectric constant K, inserted across a parallel-plate capacitor, changes its effective air separation from d to $d - t + t/K$.
- For unchanged stored charge, $V = Q/C$; restoring the same potential difference restores the original capacitance. Increasing the physical gap compensates the dielectric insertion.

Step 1: Express the compensated effective gap Let the initial plate separation be d, and let the increase be x. The new physical separation is $d + x$, consisting of air thickness $d + x - t$ and dielectric thickness t. Their series-layer capacitance is

$$C_{\text{new}} = \frac{\varepsilon_0 A}{d + x - t + t/K}, \qquad C_{\text{old}} = \frac{\varepsilon_0 A}{d}.$$

The stated compensation means these capacitances are equal. The connecting wires in the figure identify common nodes, so the capacitors are in parallel. Applying the same change between every pair of plates gives the same compensation condition for the combination.

Step 2: Solve for the dielectric constant Equating the two effective separations removes the unknown initial separation and area:

$$d + x - t + \frac{t}{K} = d, \qquad x = t\left(1 - \frac{1}{K}\right).$$

Use the same length unit for the two measured distances:

$$2.4 = 3\left(1 - \frac{1}{K}\right), \qquad \frac{1}{K} = 1 - 0.8 = 0.2, \qquad K = 5.$$

The dielectric reduces the effective separation by $3 - 3/5 = 2.4\,\text{mm}$, exactly the amount offset by moving the plates apart. The alternatives $K = 3, 4, 6$ would require increases of $2, 2.25, 2.5\,\text{mm}$, respectively, so none fits the given compensation.

Step 3: Conclusion The slab has dielectric constant $K = 5$. Hence, the correct option is (C).

Quick Tip to Solve: Use $K = t/(t - x)$, where x is the increase in physical separation needed to offset a slab of thickness t.

---

32. The electric field component of a monochromatic radiation is given by
$\vec{E}$ **= 2 E$_0$** $\hat{i}$ **cos kz cos** ω**t**
Its magnetic field $\vec{B}$ **is then given by :** [4 marks]

(A) $\frac{2E_0}{c}\,\hat{j}$ sin kz cos ωt
(B) $-\frac{2E_0}{c}\,\hat{j}$ sin kz sin ωt
(C) $\frac{2E_0}{c}\,\hat{j}$ sin kz sin ωt
(D) $\frac{2E_0}{c}\,\hat{j}$ cos kz cos ωt

Correct Answer: (C) $\frac{2E_0}{c}\,\hat{j}$ sin kz sin ωt

Solution:

Concept:

- Faraday's law relates the fields by $\nabla \times \vec{E} = -\partial\vec{B}/\partial t$. For a vacuum monochromatic wave, $\omega = ck$.
- In a standing wave, electric and magnetic fields need not peak at the same position or time. Their spatial and temporal sine/cosine factors follow from the field derivatives.

Step 1: Calculate the curl of the electric field The given field has only an x component, $E_x = 2E_0\cos(kz)\cos(\omega t)$, which varies with z and t. The relevant component of its curl is

$$(\nabla \times \vec{E})_y = \frac{\partial E_x}{\partial z} = -2kE_0\sin(kz)\cos(\omega t).$$

Faraday's law therefore requires

$$\frac{\partial B_y}{\partial t} = 2kE_0\sin(kz)\cos(\omega t).$$

The sign here is positive because the minus sign in Faraday's law cancels the minus sign obtained on differentiating the spatial cosine.

Step 2: Integrate with respect to time The oscillating magnetic field is obtained by integrating the last equation:

$$B_y = \frac{2kE_0}{\omega}\sin(kz)\sin(\omega t) = \frac{2E_0}{c}\sin(kz)\sin(\omega t).$$

There is no additional static field in the monochromatic radiation specified in the question. Its direction is $\hat{j}$. Differentiating this result in time reproduces the required expression, providing a direct sign and phase check. Option (B) has the opposite sign; (A) has a time cosine whose derivative has the wrong phase; (D) has both the wrong spatial dependence and the wrong temporal factor. Applying

the travelling-wave relation to the instantaneous electric amplitude would miss the standing-wave phase difference.

Step 3: Conclusion

$$\vec{B} = \frac{2E_0}{c}\,\hat{j}\,\sin(kz)\sin(\omega t).$$

Hence, the correct option is (C).

Quick Tip to Solve: For a standing electromagnetic wave, derive the magnetic field from the curl equation rather than copying the electric field phase.

33. In an experiment to determine the period of a simple pendulum of length 1 m, it is attached to different spherical bobs of radii r_1 and r_2 . The two spherical bobs have uniform mass distribution. If the relative difference in the periods, is found to be 5×10^{-4} s, the difference in radii, $|r_1 - r_2|$ is best given by : [4 marks]

(A) 1 cm

(B) 0.05 cm

(C) 0.5 cm

(D) 0.01 cm

Correct Answer: (B) 0.05 cm

Solution:

Concept:

- The simple-pendulum period is $T = 2\pi\sqrt{L/g}$, where L is measured to the bob's centre of mass. For a uniform spherical bob, that centre is its geometrical centre.
- For a small change in effective length, $\Delta T/T \simeq \Delta L/(2L)$. With the same suspension length, the difference in effective lengths is the difference in bob radii.

Step 1: Identify the time change and effective length The numerical difference in the question is accompanied by seconds, so use $|\Delta T| = 5 \times 10^{-4}\,\text{s}$ as the small measured time difference. The stated one-metre pendulum has a period close to

$$T = 2\pi\sqrt{\frac{1}{9.8}} \simeq 2.0\,\text{s}.$$

Replacing one small spherical bob by another shifts the centre of mass relative to its attachment by the change in radius. Thus $|\Delta L| = |r_1 - r_2|$. We use the first-order small-change approximation, appropriate because the timing change is tiny compared with the period.

Step 2: Relate the timing change to the radii Differentiate the period formula at fixed g:

$$\frac{|\Delta T|}{T} \simeq \frac{|\Delta L|}{2L}, \qquad |r_1 - r_2| \simeq \frac{2L|\Delta T|}{T}.$$

Substitution gives

$$|r_1 - r_2| \simeq \frac{2(1)(5 \times 10^{-4})}{2}\,\mathrm{m} = 5 \times 10^{-4}\,\mathrm{m} = 0.05\,\mathrm{cm}.$$

The other listed distances are either substantially larger or smaller than this value. The term relative is used loosely in the statement: a true fractional difference is dimensionless. If 5×10^{-4} were instead a fractional difference, it would give $0.1\,\mathrm{cm}$, which is absent from the options. The given seconds and the offered values select the absolute-time interpretation above.

Step 3: Conclusion Using the stated time difference in seconds, $|r_1 - r_2| \simeq 0.05\,\mathrm{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: Check the units before using $\Delta T/T$: a value in seconds is ΔT, whereas a relative change has no unit.

---

34. A single slit of width 0.1 mm is illuminated by a parallel beam of light of wavelength 6000 $\mathring{A}$ and diffraction bands are observed on a screen 0.5 m from the slit. The distance of the third dark band from the central bright band is : [4 marks]

(A) 3 mm

(B) 9 mm

(C) 4.5 mm

(D) 1.5 mm

Correct Answer: (B) 9 mm

Solution:

Concept:

- For a slit of width a, dark bands satisfy $a \sin\theta_m = m\lambda$, with $m = 1, 2, 3, \ldots$. The third dark band corresponds to $m = 3$.
- For a distant screen and a small diffraction angle, $y_m = D \tan\theta_m \simeq m\lambda D/a$. Here y_m is measured from the centre of the central bright band.

Step 1: Convert the given quantities and choose the order The slit width and wavelength must be expressed in compatible units. Their SI values are

$$a = 0.1\,\mathrm{mm} = 1.0 \times 10^{-4}\,\mathrm{m}, \qquad \lambda = 6000 \times 10^{-10}\,\mathrm{m} = 6.0 \times 10^{-7}\,\mathrm{m}.$$

The screen distance is $D = 0.5\,\mathrm{m}$. Dark-band order starts with one for the first minimum on either side of the central maximum, so the requested third dark band has $m = 3$. No half-integer factor belongs in the minimum condition for a single slit.

Step 2: Find the transverse position of the third minimum First calculate its angle factor:

$$\sin\theta_3 = \frac{3\lambda}{a} = \frac{3(6.0 \times 10^{-7})}{1.0 \times 10^{-4}} = 0.018.$$

This is small enough to use $\tan\theta_3 \simeq \sin\theta_3$. Therefore

$$y_3 = \frac{3\lambda D}{a} = \frac{3(6.0\times10^{-7})(0.5)}{1.0\times10^{-4}}\,\mathrm{m} = 9.0\times10^{-3}\,\mathrm{m} = 9\,\mathrm{mm}.$$

The first minimum would be at $3\,\mathrm{mm}$, so option (A) uses the wrong order. The values $4.5\,\mathrm{mm}$ and $1.5\,\mathrm{mm}$ introduce an inappropriate half factor. The result is the one-sided distance from the central bright-band centre, not the separation between minima on opposite sides.

Step 3: Conclusion The third dark band is $9\,\mathrm{mm}$ from the centre. Hence, the correct option is (B).

Quick Tip to Solve: For single-slit dark bands, use integer orders $m\lambda D/a$; reserve half-integer approximations for secondary bright-band positions.

35. A uniform wire of length 1 and radius r has a resistance of 100 Ω. It is recast into a wire of radius $\frac{r}{2}$. The resistance of new wire will be : [4 marks]

(A) 1600 Ω

(B) 400 Ω

(C) 200 Ω

(D) 100 Ω

Correct Answer: (A) 1600 Ω

Solution:

Concept:

- A uniform wire has resistance $R = \rho L/A$, where ρ is resistivity, L length and $A = \pi r^2$ cross-sectional area.
- Recasting the same material without loss conserves volume: $AL = A\prime L\prime$. At the same temperature, resistivity remains unchanged, so both geometry changes must be included.

Step 1: Use volume conservation to obtain the new length Let the original length be L; its particular numerical value is not required. Reducing the radius to half makes the new area

$$A\prime = \pi\left(\frac{r}{2}\right)^2 = \frac{\pi r^2}{4} = \frac{A}{4}.$$

Since the amount of metal is unchanged,

$$AL = A\prime L\prime, \qquad L\prime = \frac{A}{A\prime}L = 4L.$$

Thus the new wire is four times as long as well as having one-quarter the original area. Ignoring the length increase would underestimate the final resistance.

Step 2: Compare the resistances Write the ratio so that the unchanged resistivity cancels:

$$\frac{R\prime}{R} = \frac{\rho L\prime/A\prime}{\rho L/A} = \frac{L\prime}{L}\frac{A}{A\prime} = 4\times4 = 16.$$

Using the original resistance,

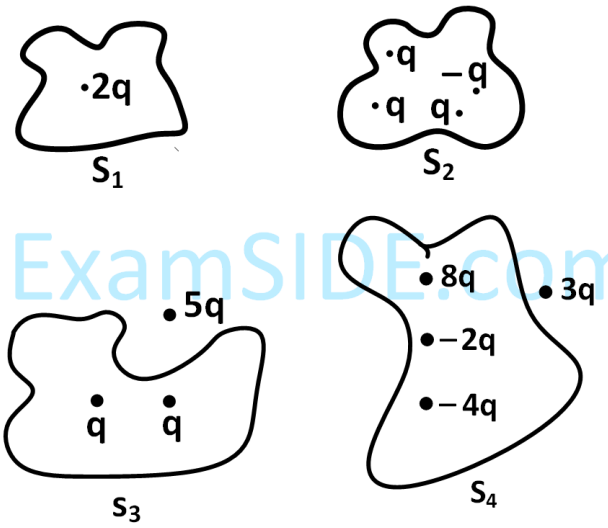
$$R\prime = 16(100\,\Omega) = 1600\,\Omega.$$

The $400\,\Omega$ choice accounts for only one of the two factors of four. The $200\,\Omega$ choice scales with radius rather than area, and $100\,\Omega$ incorrectly treats resistance as a property of material alone. Resistivity is the material property; resistance depends on the changed shape. Equivalently, at fixed volume the resistance is proportional to $1/r^4$, so halving the radius multiplies it by sixteen.

Step 3: Conclusion The recast wire has resistance $1600\,\Omega$. Hence, the correct option is (A).

Quick Tip to Solve: For recasting at constant volume, combine $L = V/A$ with $R = \rho L/A$ to get $R \propto 1/A^2 \propto r^{-4}$.

36. Four closed surfaces and corresponding charge distributions are shown below.
Let the respective electric fluxes through the surfaces be Φ_1, Φ_2, Φ_3 and Φ_4. Then : [4 marks]



ExamSIDE.com

(A) $\Phi_1 < \Phi_2 = \Phi_3 > \Phi_4$
(B) $\Phi_1 > \Phi_2 > \Phi_3 > \Phi_4$
(C) $\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$
(D) $\Phi_1 > \Phi_3$; $\Phi_2 < \Phi_4$

Correct Answer: (C) $\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$

Solution:

Concept:

- For a closed surface, Gauss's law gives $\Phi = Q_{\text{enclosed}}/\varepsilon_0$. The enclosed charge is the algebraic sum of charges inside the surface.
- Charges outside a closed surface may change the electric field locally, but their net contribution to its outward electric flux is zero. Surface shape and charge position do not affect this total.

Step 1: Count the charges inside each boundary The first surface encloses $2q$. Inside the second boundary the three positive charges and the one negative charge give $q + q + q - q = 2q$. The third

boundary encloses two charges q and q; the charge $5q$ lies outside it, so its enclosed total is also $2q$.

Step 2: Apply Gauss law and compare the fluxes The fourth boundary encloses $8q$, $-2q$, and $-4q$, whereas $3q$ lies outside. Hence its total is $8q - 2q - 4q = 2q$. Consequently,

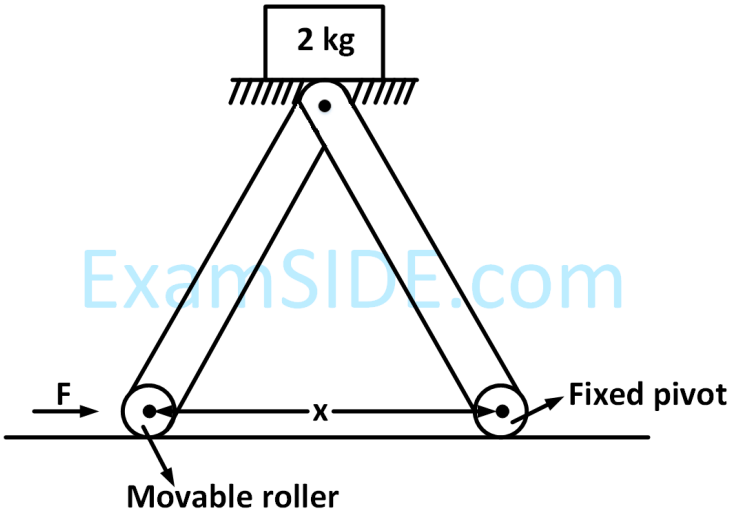
$$\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4 = \frac{2q}{\varepsilon_0}.$$

The different-looking distributions therefore do not imply different net fluxes. Options (A), (B), and (D) each assert a strict inequality between at least two surfaces, contradicting the equal enclosed totals. Adding an external charge to an enclosed sum would incorrectly change this result.

Step 3: Conclusion All four closed surfaces have equal electric flux, $2q/\varepsilon_0$. Hence, the correct option is (C).

Quick Tip to Solve: First trace the boundary and add only the charges inside it, with their signs; ignore outside charges when comparing total closed-surface flux.

37. The machine as shown has 2 rods of length1 m connected by a pivot at the top. The end of one rod is connected to the floor by a stationary pivot and the end of the other rod has a roller that rolls along the floor in a slot. As the roller goes back and forth, a 2 kg weight moves up and down. If the roller is moving towards right at a constant speed, the weight moves up with a : [4 marks]



ExamSIDE.com

(A) Constant speed

(B) decreasing speed

(C) increasing speed

(D) speed which is $\frac{3}{4}$th of that of the roller when the weight is 0.4 m above the ground

Correct Answer: (B) decreasing speed

Solution:

Concept:

- The two equal rods and the floor form an isosceles triangle. If the base separation is x, the height of the upper joint is $h = \sqrt{L^2 - x^2/4}$, with $L = 1\,\text{m}$.
- A constant roller speed does not imply a constant vertical speed: the geometric relation between the base and height must be differentiated with respect to time.

Step 1: Relate the height to the roller position The upper pivot lies above the midpoint of the two lower ends because the rods have equal lengths. The right triangle formed by one rod therefore gives

$$h^2 + \frac{x^2}{4} = L^2.$$

The fixed pivot is on the right and the movable roller is on the left. As the roller moves right toward the fixed pivot, x decreases. Write $dx/dt = -u$, where the constant $u > 0$ is the roller speed.

Step 2: Differentiate and inspect the speed Differentiating the constraint gives

$$2h\dot{h} + \frac{x}{2}\dot{x} = 0, \qquad \dot{h} = \frac{ux}{4h}.$$

During the upward motion, x decreases and h increases, so the ratio x/h, and therefore the upward speed, decreases. More explicitly,

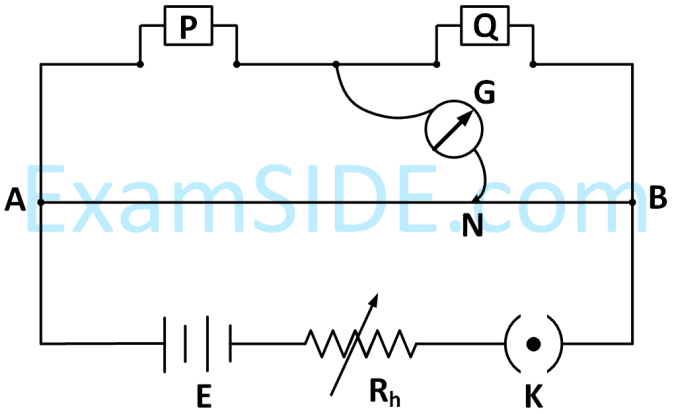
$$\ddot{h} = -\frac{u^2L^2}{4h^3} < 0.$$

The mass affects forces, not this kinematic constraint. At $h = 0.4\,\text{m}$, $x = 2\sqrt{1 - 0.4^2}\,\text{m}$ and $\dot{h}/u = \sqrt{0.84}/0.8 \approx 1.15$, not $3/4$. This also rules out the numerical claim in (D), while (A) and (C) contradict the decreasing ratio.

Step 3: Conclusion The weight rises with decreasing speed. Hence, the correct option is (B).

Quick Tip to Solve: For linked rods, write the fixed-length constraint before differentiating; track whether the base separation increases or decreases.

38. In a meter bridge experiment resistances are connected as shown in the figure. Initially resistance P = 4 Ω and the neutral point N is at 60 cm from A. Now an unknown resistance R is connected in series to P and the new position of the neutral point is at 80 cm from A. The value of unknown resistance R is : [4 marks]



ExamSIDE.com

(A) $\frac{33}{5}\ \Omega$

(B) 6 Ω

(C) 7 Ω

(D) $\frac{20}{3}\ \Omega$

Correct Answer: (D) $\frac{20}{3}\ \Omega$

Solution:

Concept:

- At the balance point of a metre bridge, no current flows through the galvanometer. The resistance ratio equals the ratio of the corresponding uniform wire lengths.
- With P in the left gap and Q in the right gap, $P/Q = l/(100-l)$, where l is measured in centimetres from end A. Series resistances add directly.

Step 1: Determine the resistance in the right gap The figure places P on the A side and Q on the B side. Initially $P = 4\,\Omega$ and $l = 60\,\text{cm}$. Thus

$$\frac{4}{Q} = \frac{60}{40} = \frac{3}{2}, \qquad Q = \frac{8}{3}\,\Omega.$$

The remaining wire length is $40\,\text{cm}$, since the full wire has length $100\,\text{cm}$. The supply and rheostat establish a current but do not enter this balance ratio.

Step 2: Use the new null point after adding the resistor After placing R in series with P, the left-gap resistance becomes $4 + R$, while Q stays unchanged. The new balance lengths are $80\,\text{cm}$ and $20\,\text{cm}$. Therefore

$$\frac{4+R}{8/3} = \frac{80}{20} = 4,$$

$$4 + R = \frac{32}{3}, \qquad R = \frac{32}{3} - 4 = \frac{20}{3}\,\Omega.$$

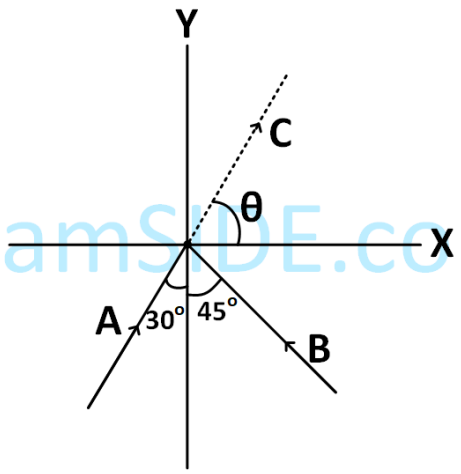
As a check, increasing the left-gap resistance increases its balancing length, consistent with the shift

from 60 to 80 cm. The values in (A), (B), and (C) do not give the exact required ratio of four with $Q = 8/3\,\Omega$.

Step 3: Conclusion The unknown series resistance is $R = \dfrac{20}{3}\,\Omega$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the left/right resistance order tied to the wire ends, and subtract the original resistance after finding the new series total.

39. Two particles A and B of equal mass M are moving with the same speed v as shown in the figure. They collide completely inelastically and move as a single particle C. The angle θ that the path of C makes with the X-axis is given by : [4 marks]



ExamSIDE.com

(A) $\tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$

(B) $\tan\theta = \frac{\sqrt{3}-\sqrt{2}}{1-\sqrt{2}}$

(C) $\tan\theta = \frac{1-\sqrt{2}}{\sqrt{2}(1+\sqrt{3})}$

(D) $\tan\theta = \frac{1-\sqrt{3}}{1+\sqrt{2}}$

Correct Answer: (A) $\tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$

Solution:

Concept:

- In a completely inelastic collision the particles stick together. Total vector momentum is conserved during the collision, although kinetic energy need not be conserved.
- The direction of the final velocity is the direction of the vector sum of the initial momenta. Thus $\tan\theta = p_y/p_x$, with both components carrying their algebraic signs.

Step 1: Resolve the two incident momenta Particle A approaches the origin from the lower left, making 30° with the upward direction parallel to the Y-axis. Its velocity components are $v\sin 30^\circ = v/2$ along positive X and $v\cos 30^\circ = \sqrt{3}v/2$ along positive Y. Particle B approaches from the lower right at

45° to the vertical, so its X component is negative and its Y component is positive:

$$p_{Ax} = \frac{Mv}{2}, \quad p_{Ay} = \frac{\sqrt{3}Mv}{2}, \quad p_{Bx} = -\frac{Mv}{\sqrt{2}}, \quad p_{By} = \frac{Mv}{\sqrt{2}}.$$

Step 2: Form the ratio of the conserved components The combined mass is $2M$, but this common factor cancels when calculating the direction. Summing components,

$$p_x = \frac{Mv}{2}(1-\sqrt{2}), \qquad p_y = \frac{Mv}{2}(\sqrt{3}+\sqrt{2}).$$

Hence

$$\tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}.$$

Here $p_x < 0$ and $p_y > 0$, so the final direction is in the second quadrant. The illustrative outgoing dashed ray must not override the signed incoming momenta. In particular, subtracting the positive Y components as in (B) would be incorrect; (C) and (D) likewise fail the component ratio.

Step 3: Conclusion Conservation of momentum gives $\tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$. Hence, the correct option is (A).

Quick Tip to Solve: Resolve each incoming arrow with its actual sign; in a sticking collision the total mass cancels from the final direction.

40. A standing wave is formed by the superposition of two waves travelling in opposite directions. The transverse displacement is given by

y(x, t) = 0.5 sin $\left(\frac{5\pi}{4}x\right)$ cos(200 πt).

What is the speed of the travelling wave moving in the positive x direction ?

(x and t are in meter and second, respectively.) [4 marks]

(A) 160 m/s

(B) 90 m/s

(C) 180 m/s

(D) 120 m/s

Correct Answer: (A) 160 m/s

Solution:

Concept:

- A standing wave of the form $y = A\sin(kx)\cos(\omega t)$ is the sum of two equal-amplitude waves travelling in opposite directions.
- Both constituent waves have speed $v = \omega/k$. The coefficient of x is the angular wave number and the coefficient of t is the angular frequency.

Step 1: Identify the wave number and angular frequency Compare the given displacement with the

standard standing-wave form:

$$y = 0.5 \sin\left(\frac{5\pi}{4}x\right)\cos(200\pi t).$$

Thus

$$k = \frac{5\pi}{4}\,\text{rad}\,\text{m}^{-1}, \qquad \omega = 200\pi\,\text{rad}\,\text{s}^{-1}.$$

The factor 0.5 sets the maximum standing-wave amplitude and has no role in the propagation speed. A standing-wave pattern does not itself travel, but the two waves that produce it do.

Step 2: Recover a constituent wave and its speed The identity $\sin A\cos B = [\sin(A+B)+\sin(A-B)]/2$ gives

$$y = 0.25\sin(kx+\omega t) + 0.25\sin(kx-\omega t).$$

The second term travels in the positive X direction since its constant phase satisfies $x = (\omega/k)t +$ constant. Its speed is

$$v = \frac{200\pi}{5\pi/4} = 160\,\text{m}\,\text{s}^{-1}.$$

Equivalently, $\lambda = 2\pi/k = 1.6\,\text{m}$ and $f = \omega/(2\pi) = 100\,\text{Hz}$, so $f\lambda = 160\,\text{m}\,\text{s}^{-1}$. The three other listed speeds do not equal either equivalent calculation; using the amplitude would be dimensionally inappropriate.

Step 3: Conclusion The travelling wave moving along positive X has speed $160\,\text{m}\,\text{s}^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: Read angular frequency and angular wave number directly from the sine/cosine arguments; their ratio gives the travelling-wave speed.

41. A physical quantity P is described by the relation

$P = a^{1/2}\, b^2\, c^3\, d^{-4}$

If the relative errors in the measurement of a, b, c and d respectively, are 2%, 1%, 3% and 5%, then the relative error in P will be : [4 marks]

(A) 8%

(B) 12%

(C) 32%

(D) 25%

Correct Answer: (C) 32%

Solution:

Concept:

- For a product of powers $P = a^\alpha b^\beta c^\gamma d^\delta$, the maximum fractional error is the sum of the fractional errors multiplied by the absolute powers.
- The error contributions are positive bounds. A negative exponent changes a variable into a denominator but does not subtract its contribution to the maximum uncertainty.

Step 1: Write the fractional-error relation The given relation is $P = a^{1/2}b^2c^3d^{-4}$. Taking logarithms and differentials gives

$$\frac{dP}{P} = \frac{1}{2}\frac{da}{a} + 2\frac{db}{b} + 3\frac{dc}{c} - 4\frac{dd}{d}.$$

For the greatest possible error, independent measurement deviations can have signs that reinforce each other, so the magnitudes of all four contributions must be added. This is the conventional maximum-relative-error rule used here, rather than a statistical root-sum-square estimate.

Step 2: Substitute the four percentage errors Using the supplied percentage uncertainties,

$$\frac{\Delta P}{|P|} \times 100\% = \frac{1}{2}(2\%) + 2(1\%) + 3(3\%) + 4(5\%).$$

The separate contributions are 1%, 2%, 9%, and 20%. Their sum is

$$1\% + 2\% + 9\% + 20\% = 32\%.$$

The half-power of a halves its relative error, while the fourth inverse power of d multiplies its contribution by four. Subtracting the denominator contribution would falsely allow cancellation of worst-case error bounds. The other choices, 8%, 12%, and 25%, omit or incorrectly combine at least one of these required contributions.

Step 3: Conclusion The relative error in P is 32%. Hence, the correct option is (C).

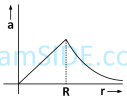
Quick Tip to Solve: For maximum percentage error in products and quotients, multiply each error by the absolute exponent and then add all contributions.

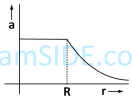
42. The mass density of a spherical body is given by

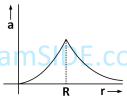
ρ (r) = $\frac{k}{r}$ for r $\leq$ R and ρ (r) = 0 for r $>$ R,

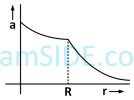
where r is the distance from the centre.

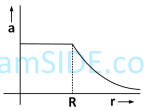
The correct graph that describes qualitatively the acceleration, a, of a test particle as a function of r is : [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- By spherical symmetry and the shell theorem, only mass enclosed within radius r contributes to the gravitational acceleration there: $a(r) = GM(r)/r^2$.

- A thin spherical shell has mass $dM = 4\pi r^2 \rho(r)\, dr$. The density is nonuniform, so the enclosed mass must be integrated rather than replaced by a uniform-density formula.

Step 1: Find the acceleration inside the sphere For $0 < r \le R$, use s as the integration variable to avoid confusing the shell radius with the upper limit:

$$M(r) = \int_0^r 4\pi s^2 \frac{k}{s}\, ds = 4\pi k \int_0^r s\, ds = 2\pi k r^2.$$

Although the density diverges as s tends to zero, the shell mass is proportional to $s\, ds$, so the enclosed mass is finite. Therefore

$$a(r) = \frac{G(2\pi k r^2)}{r^2} = 2\pi G k.$$

Its magnitude is constant throughout the interior at every positive radius.

Step 2: Match the exterior behaviour and the boundary The full mass is $M = 2\pi k R^2$. Outside the sphere,

$$a(r) = \frac{GM}{r^2} = 2\pi G k \frac{R^2}{r^2}, \qquad r > R.$$

At $r = R$, this matches the interior constant continuously, then decreases toward zero by the inverse-square law. Thus the correct graph has a horizontal interior segment followed by a decreasing curved exterior segment: option (B). Options (A) and (C) incorrectly show an increasing interior acceleration, while (D) shows a decreasing interior acceleration. The singular central density means the formula is interpreted for $r > 0$; its constant limiting magnitude determines the qualitative interior graph.

Step 3: Conclusion The acceleration magnitude is constant for $0 < r \le R$, then decreases as $1/r^2$. Hence, the correct option is (B).

Quick Tip to Solve: Integrate the density first: when $\rho \propto 1/r$, enclosed mass grows as r^2, cancelling the r^2 in the field denominator.

43. A Laser light of wavelength 660 nm is used to weld Retina detachment. If a Laser pulse of width 60 ms and power 0.5 kW is used the approximate number of photons in the pulse are :

[Take Planck's constant h = 6.62 × 10^{-34} Js] [4 marks]

(A) 10^{20}

(B) 10^{18}

(C) 10^{22}

(D) 10^{19}

Correct Answer: (A) 10^{20}

Solution:

Concept:

- A photon of wavelength λ has energy $E_\gamma = hc/\lambda$, where h is Planck's constant and c is the speed of light.
- The energy delivered by a constant-power pulse is $E = P\Delta t$; consequently its photon count is

$N = P\Delta t\lambda/(hc)$.

Step 1: Convert the pulse data The power and duration must be expressed in SI units before multiplying. Here $P = 0.5 \times 10^3 = 500\,\text{W}$, $\Delta t = 60 \times 10^{-3} = 0.060\,\text{s}$, and $\lambda = 660 \times 10^{-9}\,\text{m}$. The total energy, rather than the power alone, determines how many photons the pulse contains. Thus

$$E = 500(0.060) = 30\,\text{J}.$$

Step 2: Divide by one photon's energy Using the supplied Planck constant and $c = 3.0 \times 10^8\,\text{m s}^{-1}$,

$$E_\gamma = \frac{(6.62 \times 10^{-34})(3.0 \times 10^8)}{660 \times 10^{-9}} \simeq 3.01 \times 10^{-19}\,\text{J}.$$

Therefore

$$N = \frac{30}{3.01 \times 10^{-19}} \simeq 9.97 \times 10^{19}.$$

This is approximately 10^{20} photons. The alternatives 10^{18}, 10^{19}, and 10^{22} differ by factors of about 100, 10, and 100 respectively and do not represent the computed count.

Step 3: Conclusion The pulse contains approximately 1.0×10^{20} photons. Hence, the correct option is (A).

Quick Tip to Solve: Calculate pulse energy in joules first, then divide by hc/λ; a millisecond-to-second error changes the photon count by a thousand.

---

44. In an experiment a convex lens of focal length 15 cm is placed coaxially on an optical bench in front of a convex mirror at a distance of 5 cm from it. It is found that an object and its image coincide, if the object is placed at a distance of 20 cm from the lens. The focal length of the convex mirror is : [4 marks]

(A) 27.5 cm

(B) 20.0 cm

(C) 25.0 cm

(D) 30.5 cm

Correct Answer: (A) 27.5 cm

Solution:

Concept:

- A convex lens forms an image according to $1/f = 1/v - 1/u$, with distances measured using the Cartesian sign convention.
- For a ray bundle reflected back along its incoming paths, its convergence point is the mirror's centre of curvature. A spherical mirror has $|f_m| = |R|/2$.

Step 1: Locate the image formed by the lens Take the lens at $x = 0$, with the object to its left and the convex mirror at $x = 5\,\text{cm}$. For the first passage through the lens, $u = -20\,\text{cm}$ and $f = +15\,\text{cm}$.

Hence

$$\frac{1}{v} = \frac{1}{15} - \frac{1}{20} = \frac{1}{60}, \qquad v = 60\,\text{cm}.$$

In the absence of the mirror the rays would converge 60 cm to the right of the lens. They encounter the mirror before reaching this point.

Step 2: Apply the coincidence condition The final image coincides with the object when the reflected bundle retraces the first-pass rays. Every incident ray then lies along a normal of the spherical mirror, so the lens's prospective image is at its centre of curvature. For a convex mirror this centre is behind the reflecting surface, in agreement with the geometry here. Its distance from the mirror is

$$R = 60 - 5 = 55\,\text{cm}.$$

Therefore

$$f_m = \frac{55}{2} = 27.5\,\text{cm}.$$

The 5 cm separation must be subtracted from the lens-image distance; simply halving 60 would give a wrong result. None of the other offered focal lengths corresponds to the required 55 cm radius.

Step 3: Conclusion The focal length of the convex mirror is $27.5\,\text{cm}$. Hence, the correct option is (A).

Quick Tip to Solve: When the returning image coincides with the object, locate the lens-only image first: it is the mirror's centre of curvature.

45. A car is standing 200 m behind a bus, which is also at rest. The two start moving at the same instant but with different forward accelerations. The bus has acceleration 2 m/s^2 and the car has acceleration 4 m/s^2 . The car will catch up with the bus after a time of : [4 marks]

(A) $\sqrt{110}\,s$

(B) $\sqrt{120}\,s$

(C) $10\sqrt{2}\,s$

(D) 15 s

Correct Answer: (C) $10\sqrt{2}\,s$

Solution:

Concept:

- For constant acceleration, displacement is $s = ut + \frac{1}{2}at^2$. Both vehicles start with zero initial velocity.
- The car gains on the bus with relative acceleration $a_{\text{rel}} = a_c - a_b$; the distance it must gain is their initial separation.

Step 1: Write the positions from one origin Choose the car's initial position as $x = 0$ and the forward direction as positive. The bus starts at $x = 200\,\text{m}$. At elapsed time t, the two positions are

$$x_c = \tfrac{1}{2}(4)t^2 = 2t^2, \qquad x_b = 200 + \tfrac{1}{2}(2)t^2 = 200 + t^2.$$

The initial 200 m is added only to the bus position. Although the bus starts ahead, its smaller acceleration allows the car eventually to catch it.

Step 2: Equate positions and select the positive time At the catching instant both vehicles occupy the same position, so

$$2t^2 = 200 + t^2 \quad \Rightarrow \quad t^2 = 200.$$

Thus

$$t = \sqrt{200} = 10\sqrt{2}\,\text{s} \simeq 14.14\,\text{s}.$$

The negative square root describes no physical elapsed time and is discarded. As a check, at this time the car has travelled 400 m and the bus has travelled 200 m from its own starting position; both are 400 m from the chosen origin. The options $\sqrt{110}$ and $\sqrt{120}$ are too short, while 15 s would place the car 25 m ahead. Using only the car's acceleration would ignore that the bus also moves.

Step 3: Conclusion The car catches the bus after $10\sqrt{2}\,\text{s}$. Hence, the correct option is (C).

Quick Tip to Solve: For two bodies starting together from rest, use their acceleration difference to close the initial gap: $d = \frac{1}{2}(a_c - a_b)t^2$.

46. Two tubes of radii r_1 and r_2, and lengths l_1 and l_2 , respectively, are connected in series and a liquid flows through each of them in stream line conditions. P_1 and P_2 are pressure differences across the two tubes.

If P_2 is $4P_1$ and l_2 is $\frac{l_1}{4}$, then the radius r_2 will be equal to : [4 marks]

(A) r_1

(B) $2r_1$

(C) $4r_1$

(D) $\frac{r_1}{2}$

Correct Answer: (D) $\frac{r_1}{2}$

Solution:

Concept:

- Poiseuille's law gives steady laminar volume flow as $Q = \pi r^4 \Delta P/(8\eta l)$, where η is the liquid's viscosity.
- Tubes connected in series carry the same volume flow rate for an incompressible liquid. Their pressure drops therefore scale as $\Delta P \propto l/r^4$.

Step 1: Equate the flow through the two tubes No liquid accumulates at the junction, so $Q_1 = Q_2$. The common liquid also has the same viscosity in both tubes. Cancelling $\pi/(8\eta)$ from Poiseuille's law gives

$$\frac{P_1 r_1^4}{l_1} = \frac{P_2 r_2^4}{l_2}.$$

Here each P_i denotes the pressure difference across that tube, rather than the pressure at a single end. Thus the two drops need not be equal even though the flow rate is the same.

Step 2: Use the pressure and length ratios The supplied relations are $P_2/P_1 = 4$ and $l_2/l_1 = 1/4$. Rearranging the flow equality yields

$$\left(\frac{r_2}{r_1}\right)^4 = \frac{P_1}{P_2}\frac{l_2}{l_1} = \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}.$$

Taking the positive fourth root, since a radius is positive, gives

$$r_2/r_1 = 1/2.$$

As a direct check, halving the radius increases resistance by 16, and shortening the tube to one-quarter reduces it by 4: the net resistance is four times as large, exactly matching the pressure-drop ratio. Equal or larger radii would instead produce a smaller pressure drop and cannot satisfy the data.

Step 3: Conclusion The required radius is $r_2 = r_1/2$. Hence, the correct option is (D).

Quick Tip to Solve: In series capillaries the flow rate is common, so compare l/r^4, remembering to take a fourth root when recovering the radius.

47. The acceleration of an electron in the first orbit of the hydrogen atom (n = 1) is : [4 marks]

(A) $\frac{h^2}{\pi^2 m^2 r^3}$
(B) $\frac{h^2}{8\pi^2 m^2 r^3}$
(C) $\frac{h^2}{4\pi^2 m^2 r^3}$
(D) $\frac{h^2}{4\pi m^2 r^3}$

Correct Answer: (C) $\frac{h^2}{4\pi^2 m^2 r^3}$

Solution:

Concept:

- Bohr's angular-momentum condition is $mvr = nh/(2\pi)$, where m is the electron mass, v its speed and r its orbit radius.
- The electron's speed is constant in a circular orbit but its velocity changes direction. Its centripetal acceleration has magnitude $a = v^2/r$.

Step 1: Find the speed in the first orbit For the first orbit $n = 1$, the angular-momentum condition becomes

$$mvr = \frac{h}{2\pi}.$$

Solving for the speed gives

$$v = \frac{h}{2\pi mr}.$$

The radius in this expression is that of the same first orbit. The factor 2π comes from using Planck's constant h, rather than the reduced constant $\hbar = h/(2\pi)$. Keeping this distinction is essential because it is squared in the acceleration.

Step 2: Substitute into the centripetal acceleration Using the circular-motion relation,

$$a = \frac{1}{r}\left(\frac{h}{2\pi mr}\right)^2 = \frac{h^2}{4\pi^2 m^2 r^3}.$$

The direction is radially inward towards the proton. Its dimensions are $(\mathrm{kg\,m^2\,s^{-1}})^2/(\mathrm{kg^2\,m^3}) = \mathrm{m\,s^{-2}}$, as required. Option (A) omits the factor 4 from squaring 2, option (B) introduces an extra factor 2, and option (D) fails to square π. The electron does not have zero acceleration merely because its speed is constant: its direction continually changes around the orbit.

Step 3: Conclusion The acceleration magnitude is $h^2/(4\pi^2 m^2 r^3)$, directed towards the nucleus. Hence, the correct option is (C).

Quick Tip to Solve: Write $mvr = \hbar$ for the first Bohr orbit, then square the resulting speed before dividing by r.

---

48. A negative test charge is moving near a long straight wire carrying a current. The force acting on the test charge is parallel to the direction of the current. The motion of the charge is : [4 marks]

(A) away from the wire

(B) towards the wire

(C) parallel to the wire along the current

(D) parallel to the wire opposite to the current

Correct Answer: (B) towards the wire

Solution:

Concept:

- The field of a long straight wire circles the wire according to the right-hand grip rule, with magnitude $B = \mu_0 I/(2\pi r)$.
- The magnetic force is $\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$. For a negative charge its direction is opposite to the right-hand cross product.

Step 1: Fix a convenient coordinate system Let the current run in the $+z$ direction and place the charge on the $+x$ side of the wire. The right-hand rule then gives the local magnetic field along $+y$. A force parallel to the current must point along $+z$. For a negative charge the velocity must therefore make $\mathbf{v} \times \mathbf{B}$ point along $-z$, because multiplication by the negative charge reverses that direction.

Step 2: Test the offered directions Motion towards the wire is $\mathbf{v} = -v\hat{\mathbf{x}}$. Consequently

$$\mathbf{v} \times \mathbf{B} = (-v\hat{\mathbf{x}}) \times (B\hat{\mathbf{y}}) = -vB\hat{\mathbf{z}},$$

and hence $\mathbf{F} = |q|vB\hat{\mathbf{z}}$, along the current. Motion away from the wire would reverse this result and give force opposite to the current. If the velocity were parallel to the wire, either along or opposite to the current, its cross product with the azimuthal field would be radial rather than axial. Therefore neither of the two parallel-motion alternatives satisfies the specified force direction. Rotating our coordinate system around the wire does not change this conclusion.

Step 3: Conclusion Among the stated alternatives, the negative charge must move towards the wire. Hence, the correct option is (B).

Quick Tip to Solve: First find $\mathbf{v} \times \mathbf{B}$ for a positive charge, then reverse it once for a negative charge; forgetting that reversal swaps towards and away.

49. A sinusoidal voltage of peak value 283 V and angular frequency 320/s is applied to a series LCR circuit. Given that R=5 Ω, L=25 mH and C=1000 μF. The total impedance, and phase difference between the voltage across the source and the current will respectively be : [4 marks]

(A) 10 Ω and $\tan^{-1}\left(\frac{5}{3}\right)$

(B) 7 Ω and 45°

(C) 10 Ω and $\tan^{-1}\left(\frac{8}{3}\right)$

(D) 7 Ω and $\tan^{-1}\left(\frac{5}{3}\right)$

Correct Answer: (B) 7 Ω and 45°

Solution:

Concept:

- For a series LCR circuit, $X_L = \omega L$, $X_C = 1/(\omega C)$, and the impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$.
- The source-voltage phase relative to current obeys $\tan\phi = (X_L - X_C)/R$. Positive net reactance means that voltage leads current.

Step 1: Calculate the two reactances Convert the component values to SI units: $L = 25 \times 10^{-3} = 0.025\,\text{H}$ and $C = 1000 \times 10^{-6} = 0.001\,\text{F}$. With $\omega = 320\,\text{s}^{-1}$,

$$X_L = 320(0.025) = 8\,\Omega, \qquad X_C = \frac{1}{320(0.001)} = 3.125\,\Omega.$$

The net reactance is therefore $X = 4.875\,\Omega$, which is inductive. The quoted peak voltage affects the current amplitude but not the impedance or phase angle, so it need not enter these calculations.

Step 2: Find the impedance and phase angle With $R = 5\,\Omega$,

$$Z = \sqrt{5^2 + 4.875^2} = \sqrt{48.765625} \simeq 6.98\,\Omega.$$

Also,

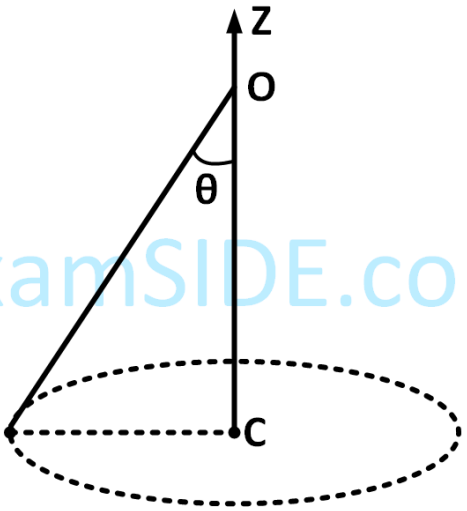
$$\tan\phi = \frac{4.875}{5} = 0.975, \qquad \phi \simeq 44.3^\circ.$$

Thus the rounded pair in the choices is $7\,\Omega$ and 45°. Voltage leads current because the inductive reactance exceeds the capacitive reactance. The two options listing $10\,\Omega$ disagree with the computed magnitude. The remaining $7\,\Omega$ option has $\tan\phi = 5/3$, which is inconsistent with the calculated ratio 0.975. The numerical values are approximate, so 45 degrees represents the intended rounded phase rather than an exact equality.

Step 3: Conclusion The impedance is approximately $7\,\Omega$ and the source voltage leads the current by approximately 45°. Hence, the correct option is (B).

Quick Tip to Solve: Compute the difference $X_L - X_C$, not their sum, and use it in both the impedance triangle and the phase-angle formula.

50. A conical pendulum of length 1 m makes an angle θ = 45° w.r.t. Z-axis and moves in a circle in the XY plane. The radius of the circle is 0.4 m and its center is vertically below O. The speed of the pendulum, in its circular path, will be: (Take g = 10 ms^{-2}) [4 marks]



ExamSIDE.com

(A) 0.4 m/s

(B) 4 m/s

(C) 0.2 m/s

(D) 2 m/s

Correct Answer: (D) 2 m/s

Solution:

Concept:

- The tension has a vertical component balancing weight and a horizontal component providing centripetal force. The bob has no vertical acceleration.
- For a conical pendulum of orbit radius r, $v^2 = rg\tan\theta$, where θ is measured from the vertical.

Step 1: Resolve the tension Write the force equations using tension F and bob mass m. The vertical and horizontal directions are independent:

$$F\cos\theta = mg, \qquad F\sin\theta = \frac{mv^2}{r}.$$

Dividing eliminates both the unknown tension and mass. Therefore

$$\tan\theta = \frac{v^2}{rg}.$$

This is a dynamical relation; the speed is not simply the radius or its square.

Step 2: Use the given orbit radius Using the explicitly stated circle radius and inclination gives

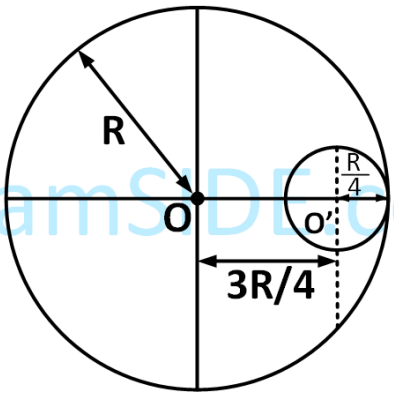
$$v = \sqrt{0.4 \times 10 \times \tan 45^\circ} = \sqrt{4} = 2\,\mathrm{m\,s^{-1}}.$$

The alternatives 0.4, 4 and 0.2 do not obey this relation: in particular, 4 is the value of v^2, not of v. The stated length of 1 m is geometrically inconsistent with the other two data, since $r = L\sin 45^\circ$ would instead give about 0.707 m. The listed answer follows from the expressly specified radius 0.4 m and angle; those are the data entering the force balance.

Step 3: Conclusion Using the given orbit radius and angle, the speed is $2\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: First eliminate tension by dividing its components; take the square root only after calculating $rg\tan\theta$.

51. A circular hole of radius $\frac{R}{4}$ is made in a thin uniform disc having mass M and radius R, as shown in figure. The moment of inertia of the remaining portion of the disc about an axis passing through the point O and perpendicular to the plane of the disc is : [4 marks]



ExamSIDE.com

(A) $\frac{219\,MR^2}{256}$
(B) $\frac{237\,MR^2}{512}$
(C) $\frac{19\,MR^2}{512}$
(D) $\frac{197\,MR^2}{256}$

Correct Answer: (B) $\frac{237\,MR^2}{512}$

Solution:

Concept:

- For a uniform lamina, mass is proportional to area. A circular disc has moment of inertia $I = \frac{1}{2}mr^2$ about its centre perpendicular to its plane.
- The parallel-axis theorem is $I_O = I_{\mathrm{cm}} + md^2$. Subtract the inertia of the removed material about the same axis as the original disc.

Step 1: Find the mass and position of the hole Here M denotes the mass of the original complete disc. The hole radius is $a = R/4$, so its removed mass is

$$m = M\frac{\pi(R/4)^2}{\pi R^2} = \frac{M}{16}.$$

The figure places its centre $O\prime$ at distance $d = 3R/4$ from O. The inertia to remove is therefore not

just its central inertia: the displacement of this small disc matters.

Step 2: Subtract the displaced small disc Apply the parallel-axis theorem to the removed piece:

$$I_{\text{hole,O}} = \frac{1}{2}\frac{M}{16}\left(\frac{R}{4}\right)^2 + \frac{M}{16}\left(\frac{3R}{4}\right)^2$$
$$= \frac{MR^2}{512} + \frac{9MR^2}{256} = \frac{19MR^2}{512}.$$

Hence the remaining inertia is

$$I = \frac{1}{2}MR^2 - \frac{19MR^2}{512} = \frac{237MR^2}{512}.$$

Option (C) gives the inertia of the removed material rather than of what remains. Options (A) and (D) exceed the original disc inertia $MR^2/2$, which removing matter cannot do about this fixed axis.

Step 3: Conclusion The remaining moment of inertia is $237MR^2/512$. Hence, the correct option is (B).

Quick Tip to Solve: For a hole, calculate its mass by its area ratio, shift its inertia to the required axis, and only then subtract.

52. A steel rail of length 5 m and area of cross section 40cm^2 is prevented from expanding along its length while the temperature rises by 10°C. If coefficient of linear expansion and Young's modulus of steel are 1.2×10^{-5} K^{-1} and 2×10^{11} Nm^{-2} respectively, the force developed in the rail is approximately : [4 marks]

(A) 2×10^7 N

(B) 1×10^5 N

(C) 2×10^9 N

(D) 3×10^{-5} N

Correct Answer: (B) 1×10^5 N

Solution:

Concept:

- Free thermal strain for a temperature rise ΔT is $\alpha\Delta T$, where α is the linear expansion coefficient.
- If expansion is completely prevented, elastic compression cancels that strain. The stress magnitude is $Y\alpha\Delta T$, and the force magnitude is $F = YA\alpha\Delta T$.

Step 1: Balance thermal and elastic changes If the rail were free, its increase in length would be

$$\Delta L_{\text{th}} = \alpha L\Delta T = (1.2 \times 10^{-5})(5)(10) = 6.0 \times 10^{-4}\,\text{m}.$$

The supports prevent this increase, producing a compressive elastic change of the same magnitude.

From Young's modulus,

$$Y = \frac{F/A}{\Delta L_{\text{th}}/L}, \qquad F = YA\alpha\Delta T.$$

The original length cancels; it must not be multiplied into the final stress formula.

Step 2: Convert the area and calculate the force The cross-sectional area in SI units is

$$A = 40\,\text{cm}^2 = 40 \times 10^{-4}\,\text{m}^2 = 4.0 \times 10^{-3}\,\text{m}^2.$$

Thus

$$\sigma = (2 \times 10^{11})(1.2 \times 10^{-5})(10) = 2.4 \times 10^{7}\,\text{Pa},$$
$$F = (2.4 \times 10^{7})(4.0 \times 10^{-3}) = 9.6 \times 10^{4}\,\text{N}.$$

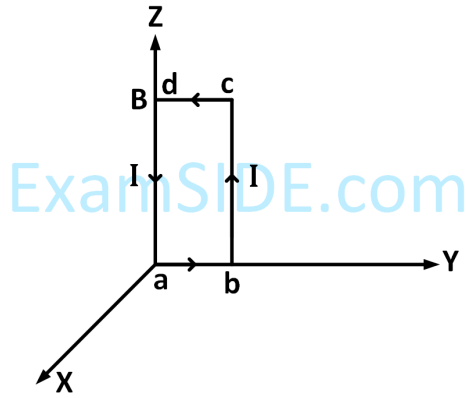
This is approximately 10^5 N. Option (A) is near the stress magnitude rather than the force; the other options are incompatible with the SI area conversion and the strain 1.2×10^{-4}.

Step 3: Conclusion The rail develops a compressive force of about 1×10^5 N. Hence, the correct option is (B).

Quick Tip to Solve: For completely blocked thermal expansion use $F = YA\alpha\Delta T$; convert square centimetres using 10^{-4}, not 10^{-2}.

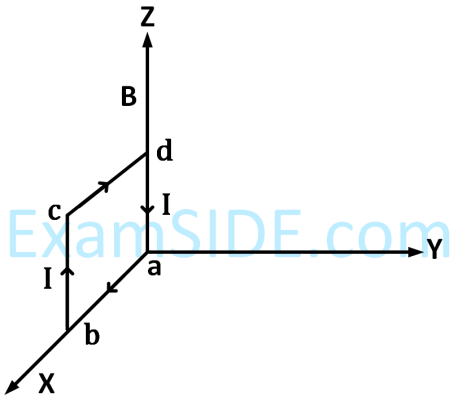
53. A uniform magnetic field B of 0.3 T is along the positive Z-direction. A rectangular loop (abcd) of sides 10 cm × 5 cm carries a current I of 12 A. Out of the following different orientations which one corresponds to stable equilibrium ? [4 marks]

(A)



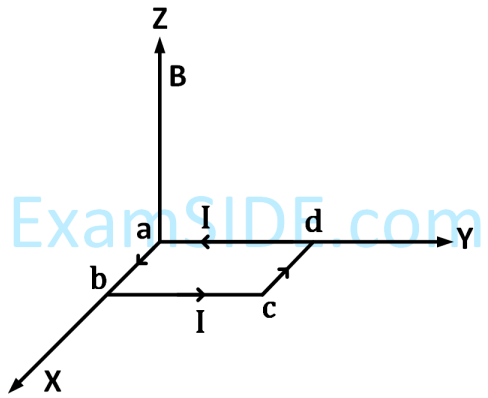
ExamSIDE.com

(B)

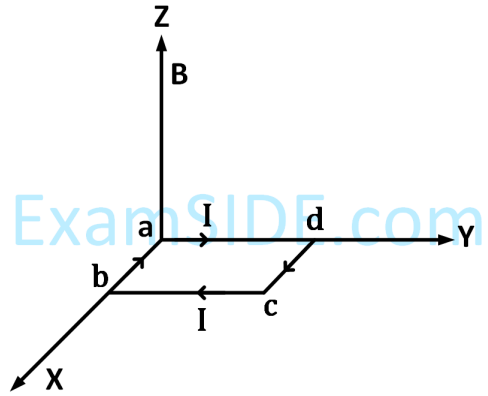


ExamSIDE.com

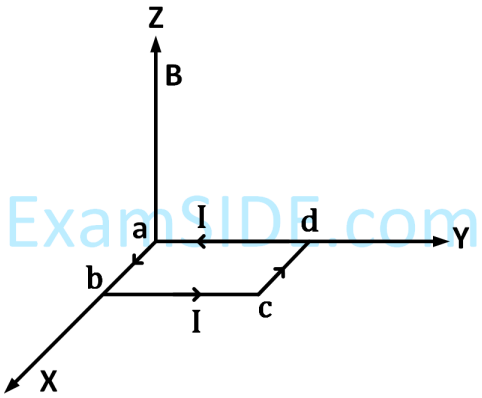
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- A current loop has magnetic moment $\boldsymbol{\mu} = IA\hat{\mathbf{n}}$; its direction follows the right-hand rule for the current circulation.
- The torque is $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$, and potential energy is $U = -\mu B \cos\phi$. Stable equilibrium requires the moment parallel to the field.

Step 1: Distinguish zero torque from stable equilibrium The loop area and moment magnitude are

$$A = (0.10)(0.05) = 5.0 \times 10^{-3}\,\mathrm{m}^2, \qquad \mu = 12A = 0.060\,\mathrm{A\,m}^2.$$

Thus $\mu B = 0.018\,\text{J}$. Zero torque occurs both for $\phi = 0$ and $\phi = \pi$, but their energies are different: $-0.018\,\text{J}$ and $+0.018\,\text{J}$, respectively. Only the parallel state is an energy minimum. A small tilt there raises the energy and produces a restoring torque.

Step 2: Inspect the current direction in each diagram In (C), the loop lies in the XY plane and its current travels from a to b along positive X, then from b to c along positive Y. The oriented normal is consequently $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$, parallel to the applied field. In (D), the XY-plane current runs in the reverse sense, giving a moment along negative Z: that zero-torque equilibrium is unstable. Loops (A) and (B) lie in the YZ and XZ planes, respectively, so their moments are perpendicular to Z and experience a nonzero torque. They are not equilibrium orientations.

Step 3: Conclusion Orientation (C) has $\boldsymbol{\mu} \parallel \mathbf{B}$ and minimum magnetic potential energy. Hence, the correct option is (C).

Quick Tip to Solve: Check the current circulation with the right-hand rule; a loop parallel to the XY plane can be stable or unstable depending on that circulation.

54. Imagine that a reactor converts all given mass into energy and that it operates at a power level of 10^9 watt. The mass of the fuel consumed per hour in the reactor will be : (velocity of light, c is 3×10^8 m/s) [4 marks]

(A) 0.96 gm

(B) 0.8 gm

(C) 4×10^{-2} gm

(D) 6.6×10^{-5} gm

Correct Answer: (C) 4×10^{-2} gm

Solution:

Concept:

- Complete conversion of rest mass to energy obeys $E = mc^2$. The mass consumption is the energy output divided by c^2.
- Power is energy per unit time: $E = Pt$ for constant power. Convert the requested one-hour interval to seconds before using watts.

Step 1: Find the energy delivered in an hour The power is $10^9\,\text{W} = 10^9\,\text{J}\,\text{s}^{-1}$. One hour contains 3600 seconds, so

$$E = Pt = (10^9)(3600) = 3.6 \times 10^{12}\,\text{J}.$$

Using a time of 1 directly with power in watts would calculate the energy in one second, not the energy in an hour. Since the problem specifies total mass conversion, no efficiency factor or fractional nuclear mass defect is to be inserted.

Step 2: Convert energy into consumed mass Use the given speed of light:

$$c^2 = (3 \times 10^8)^2 = 9 \times 10^{16}\,\text{m}^2\,\text{s}^{-2}.$$

Therefore

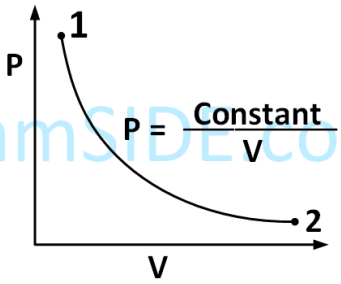
$$m = \frac{3.6 \times 10^{12}}{9 \times 10^{16}} = 4.0 \times 10^{-5}\,\text{kg} = 4.0 \times 10^{-2}\,\text{g}.$$

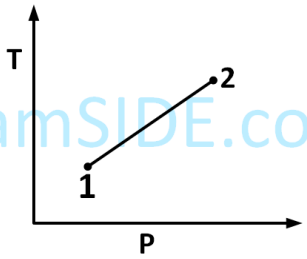
The units reduce correctly because a joule is $\text{kg}\,\text{m}^2\,\text{s}^{-2}$. Options (A) and (B) are far larger than the mass equivalent of the hourly energy. Option (D) is also incorrect; it does not equal Pt/c^2 after both the time and kilogram-to-gram conversions. The small mass follows from the very large energy equivalent per kilogram.

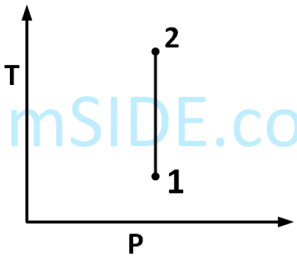
Step 3: Conclusion The reactor consumes $4 \times 10^{-2}\,\text{g}$ of fuel per hour. Hence, the correct option is (C).

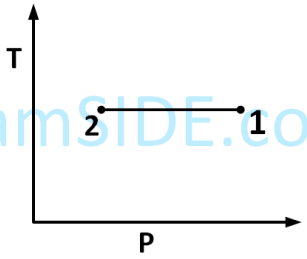
Quick Tip to Solve: Calculate Pt/c^2 in kilograms with time in seconds, then multiply by 1000 to express the result in grams.

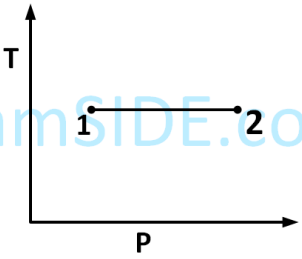
55. For the P-V diagram given for an ideal gas,
out of the following which one correctly represents the T-P diagram ? [4 marks]



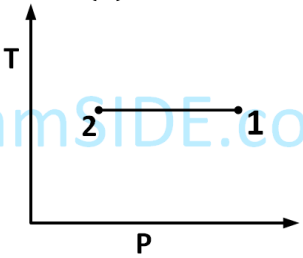
(A)


(B)


(C)


(D)


Correct Answer: (C)



Solution:

Concept:

- For a fixed amount of ideal gas, $PV = \nu RT$, with T the absolute temperature and ν the number of moles.
- The given hyperbola is labelled $P = \text{constant}/V$. It therefore represents constant PV, and hence constant temperature.

Step 1: Identify the thermodynamic constraint The initial state 1 lies at high pressure and smaller volume, while final state 2 lies at lower pressure and larger volume. Along the entire curve,

$$P = \frac{K}{V} \quad \Longrightarrow \quad PV = K.$$

Using the ideal-gas equation gives

$$T = \frac{PV}{\nu R} = \frac{K}{\nu R}.$$

Thus the temperature has the same value at states 1 and 2 and at all intermediate states. This is an isothermal expansion, not a constant-pressure process. A falling pressure alone does not imply a falling temperature, because the volume increases at the same time.

Step 2: Transfer the states to the T-P axes In a T-P plot, pressure is the horizontal coordinate and temperature is the vertical coordinate. Constant temperature must be drawn as a horizontal line. Because $P_1 > P_2$, state 1 belongs to the right of state 2, so the path goes leftward from 1 to 2. Diagram (C) has exactly this geometry. Diagram (D) also has a horizontal line but reverses the pressure ordering of the states. Diagram (A) changes temperature with pressure and diagram (B) keeps pressure constant while changing temperature. Neither can describe the specified hyperbola.

Step 3: Conclusion The temperature remains constant while the pressure decreases from state 1 to state 2. Hence, the correct option is (C).

Quick Tip to Solve: Use the equation of the whole P-V curve, then carry the state labels across: on a T-P graph an isotherm is horizontal, with the higher-pressure state farther right.

56. N moles of a diatomic gas in a cylinder are at a temperature T. Heat is supplied to the cylinder such that the temperature remains constant but n moles of the diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy of the gas ? [4 marks]

(A) $\frac{1}{2}$ nRT

(B) 0

(C) $\frac{3}{2}$ nRT

(D) $\frac{5}{2}$ nRT

Correct Answer: (A) $\frac{1}{2}$ nRT

Solution:

Concept:

- At the usual temperature where rotational motion is active and vibration is neglected, a diatomic molecule has five kinetic degrees of freedom; a monatomic particle has three.
- Equipartition assigns $RT/2$ per mole per degree of freedom. Dissociating one mole of diatomic molecules produces two moles of monatomic atoms.

Step 1: Count molecules before and after dissociation Initially there are N moles of diatomic gas, so the total translational plus rotational kinetic energy is

$$K_i = \frac{5}{2}NRT.$$

After n moles dissociate, the undissociated diatomic gas contains $N - n$ moles. Each molecule yields two separate atoms, so the monatomic component contains $2n$ moles, not n. The temperature of both components remains T; it is the particle count and the available degrees of freedom that change.

Step 2: Subtract the initial kinetic energy The final kinetic energy is

$$\begin{aligned} K_f &= \frac{5}{2}(N-n)RT + \frac{3}{2}(2n)RT \\ &= \frac{5}{2}NRT + \frac{1}{2}nRT. \end{aligned}$$

Consequently

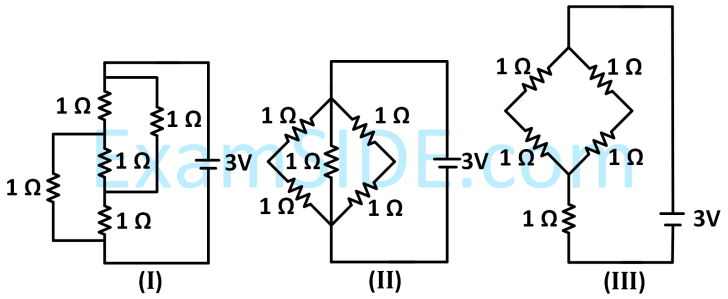
$$\Delta K = K_f - K_i = \frac{1}{2}nRT.$$

Option (B) incorrectly assumes that unchanged temperature guarantees unchanged total kinetic energy despite dissociation. Option (C) corresponds to the change of translational kinetic energy alone; the lost molecular rotational contribution must also be included in the total. Option (D) gives the original kinetic energy of the dissociated diatomic portion, rather than the net change. Bond dissociation also requires energy, but that change of internal potential energy is distinct from the total kinetic energy asked here.

Step 3: Conclusion The total kinetic energy increases by $\frac{1}{2}nRT$. Hence, the correct option is (A).

Quick Tip to Solve: For dissociation, count two moles of atoms for every mole of diatomic molecules and include the loss of rotational kinetic energy.

57. The figure shows three circuits I, II and III which are connected to a 3V battery. If the powers dissipated by the configurations I, II and III are P_1 , P_2 and P_3 respectively, then : [4 marks]



(A) $P_1 > P_2 > P_3$

(B) $P_1 > P_3 > P_2$

(C) $P_2 > P_1 > P_3$

(D) $P_3 > P_2 > P_1$

Correct Answer: (C) $P_2 > P_1 > P_3$

Solution:

Concept:

- For an ideal battery maintaining voltage V, the total resistor power is $P = V^2/R_{\text{eq}}$. Thus smaller equivalent resistance means larger total power.
- A balanced Wheatstone bridge has equal potentials at its two middle junctions. Its connecting resistor then carries no current and can be omitted when calculating the equivalent resistance.

Step 1: Reduce configuration I Label the battery terminals A and B, and the two internal junctions C and D. The four outside arms A–C, C–B, A–D and D–B each have resistance $1\,\Omega$. The remaining resistor joins C and D. Both arms have equal resistance ratios, so C and D are at the same potential. Consequently the middle resistor does not dissipate power. The two remaining branches each contain two unit resistors in series, and these branches are parallel:

$$R_1 = (1+1) \parallel (1+1) = 1\,\Omega.$$

Step 2: Reduce configurations II and III In II the left and right paths each have resistance $2\,\Omega$, while the central path has resistance $1\,\Omega$. These are three parallel paths across the battery. In III the two paths of the diamond each have resistance $2\,\Omega$, and their parallel combination is in series with the bottom unit resistor. Therefore

$$\frac{1}{R_2} = \frac{1}{2} + \frac{1}{2} + 1 = 2, \qquad R_2 = 0.5\,\Omega,$$

$$R_3 = (2 \parallel 2) + 1 = 2\,\Omega.$$

Step 3: Compare the powers The same potential difference of $3\,\text{V}$ is applied to all three circuits. Hence

$$P_1 = 9\,\text{W}, \qquad P_2 = 18\,\text{W}, \qquad P_3 = 4.5\,\text{W}.$$

In particular II has the most parallel conducting paths and the greatest power; III contains an extra series resistance and has the least power. The other three proposed rankings contradict these equivalent resistances.

Step 4: Conclusion $P_2 > P_1 > P_3$. Hence, the correct option is (C).

Quick Tip to Solve: For equal battery voltages compare equivalent resistances first, then reverse their order to obtain the power order.

58. The current gain of a common emitter amplifier is 69. If the emitter current is 7.0 mA, collector current is : [4 marks]

(A) 9.6 mA

(B) 6.9 mA

(C) 0.69 mA

(D) 69 mA

Correct Answer: (B) 6.9 mA

Solution:

Concept:

- The common-emitter current gain is $\beta = I_C/I_B$, where I_C is collector current and I_B is base current.
- At the transistor junction, the emitter supplies both base and collector currents: $I_E = I_C + I_B$. The emitter current must therefore exceed the collector current.

Step 1: Express the emitter current in terms of base current The given gain is $\beta = 69$, so the collector current is 69 times the base current. Use current conservation before inserting the numerical emitter current:

$$I_C = 69I_B, \qquad I_E = 69I_B + I_B = 70I_B.$$

The extra one is essential: a common-emitter gain relates collector current to base current, rather than collector current to emitter current. Using 69 directly as the ratio I_C/I_E would violate the transistor current relation.

Step 2: Calculate the collector current With $I_E = 7.0\,\text{mA}$, the base current and collector current are

$$I_B = \frac{7.0}{70} = 0.10\,\text{mA},$$

$$I_C = 69(0.10) = 6.9\,\text{mA}.$$

Equivalently the common-base current gain is $\alpha = \beta/(\beta+1) = 69/70$, so $I_C = \alpha I_E$. This provides the same answer without separately calculating the base current.

Step 3: Check the options against conservation of current The result is slightly smaller than the emitter current, as expected for a high-gain transistor. Currents of $9.6\,\text{mA}$ and $69\,\text{mA}$ would both exceed the supplied emitter current and are impossible here. A collector current of $0.69\,\text{mA}$ would leave a base current of $6.31\,\text{mA}$, giving a gain close to 0.11 rather than 69. Thus only the second

option satisfies both equations.

Step 4: Conclusion The collector current is $6.9\,\mathrm{mA}$. Hence, the correct option is (B).

Quick Tip to Solve: When emitter current and common-emitter gain are given, use $I_C = \beta I_E/(1+\beta)$, keeping the extra one in the denominator.

59. A signal is to be transmitted through a wave of wavelength λ, using a linear antenna. The length l of the antenna and effective power radiated P_{eff} will be given respectively as :
(K is a constant of proportionality) [4 marks]

(A) λ, $P_{eff} = K\left(\frac{1}{\lambda}\right)^2$
(B) $\frac{\lambda}{8}$, $P_{eff} = K\left(\frac{1}{\lambda}\right)$
(C) $\frac{\lambda}{16}$, $P_{eff} = K\left(\frac{1}{\lambda}\right)^3$
(D) $\frac{\lambda}{5}$, $P_{eff} = K\left(\frac{1}{\lambda}\right)\left(\frac{1}{\lambda}\right)^{\frac{1}{2}}$
Correct Answer: (A) λ, $P_{eff} = K\left(\frac{1}{\lambda}\right)^2$

Solution:

Concept:

- An efficient linear antenna should have a length comparable with the wavelength of the wave it radiates. Practical resonant antennas often use a simple fraction of that wavelength.
- For fixed antenna length and fixed current conditions, the elementary radiation-power dependence is $P_{eff} \propto (l/\lambda)^2$. Absorbing the fixed factors into K gives $P_{eff} = K/\lambda^2$.

Step 1: Identify the wavelength dependence of radiation The radiation efficiency increases as the antenna becomes larger in comparison with the wavelength. For a specified linear antenna, the relevant ratio is l/λ, and the radiated power has a square dependence on this ratio:

$$P_{eff} \propto \left(\frac{l}{\lambda}\right)^2.$$

Keeping the antenna and its excitation fixed, put the length and the remaining fixed factors into the proportionality constant. The resulting comparison is

$$P_{eff} = K\left(\frac{1}{\lambda}\right)^2.$$

This is the inverse-square wavelength dependence used in the options.

Step 2: Compare the listed antenna lengths and power laws The first choice pairs a length of the order of the wavelength, $l = \lambda$, with the inverse-square dependence. The other choices give the powers λ^{-1}, λ^{-3} and $\lambda^{-3/2}$, respectively. None agrees with the required square dependence. Their much smaller listed antenna fractions also do not improve the match to the elementary efficient-antenna criterion. The power law therefore selects the first pair without needing a numerical value for K.

Step 3: Interpret the proportionality correctly The statement about varying wavelength compares a fixed antenna under fixed excitation. It should not be read as simultaneously resizing the antenna to

keep l/λ constant in that comparison. The length criterion and the wavelength dependence answer the two parts of the question separately. A quarter-wave or half-wave design is common in practice, but neither is among the supplied choices, and the inverse-square power law remains the decisive feature here.

Step 4: Conclusion The listed pair is $l = \lambda$ and $P_{\text{eff}} = K/\lambda^2$. Hence, the correct option is (A).

Quick Tip to Solve: Check the wavelength exponent in the power law first: for a fixed linear antenna it is the inverse-square dependence.

---

60. A block of mass 0.1 kg is connected to an elastic spring of spring constant 640 Nm^{-1} and oscillates in a damping medium of damping constant 10^{-2} kg s^{-1} . The system dissipates its energy gradually. The time taken for its mechanical energy of vibration to drop to half of its initial value, is closest to : [4 marks]

(A) 2 s

(B) 3.5 s

(C) 5 s

(D) 7 s

Correct Answer: (D) 7 s

Solution:

Concept:

- For viscous damping $F_d = -b\dot{x}$, the displacement amplitude has envelope $A(t) = A_0 e^{-bt/(2m)}$.
- Mechanical oscillation energy is proportional to the square of amplitude. Its envelope therefore decays as $E(t) = E_0 e^{-bt/m}$, twice as fast in its exponential rate as the amplitude.

Step 1: Determine which decay law applies The question asks for the mechanical energy to halve, rather than the amplitude to halve. With $m = 0.1\,\text{kg}$ and $b = 10^{-2}\,\text{kg}\,\text{s}^{-1}$, the energy decay constant is

$$\frac{b}{m} = \frac{10^{-2}}{0.1} = 0.1\,\text{s}^{-1}.$$

The spring constant fixes the oscillation frequency, but not the exponential rate of the energy envelope. In fact $\omega_0 = \sqrt{k/m} = 80\,\text{s}^{-1}$, whereas $b/(2m) = 0.05\,\text{s}^{-1}$, so the damping is very weak.

Step 2: Apply the half-energy condition Write the energy ratio and take natural logarithms:

$$\frac{E(t)}{E_0} = e^{-0.1t} = \frac{1}{2},$$

$$-0.1t = \ln(1/2) = -\ln 2.$$

Therefore

$$t = \frac{m}{b}\ln 2 = 10(0.693) = 6.93\,\text{s}.$$

The initial displacement or initial energy cancels from this ratio, so no amplitude information is needed.

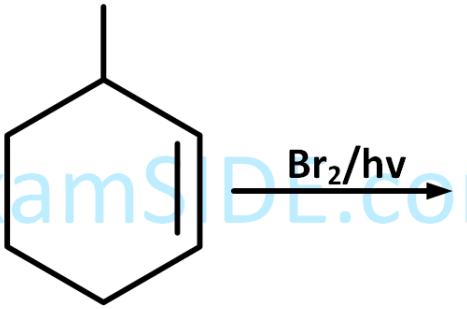
Step 3: Select the nearest time and check the factor of two The nearest listed value is $7\,\mathrm{s}$. The alternatives $2\,\mathrm{s}$, $3.5\,\mathrm{s}$ and $5\,\mathrm{s}$ are all too short: their envelope energy ratios are approximately 0.82, 0.70 and 0.61. Using the amplitude decay exponent for energy would incorrectly double the calculated time. Since the damping is weak, the rapid small variations within a cycle do not affect the nearest-second choice.

Step 4: Conclusion The half-energy time is approximately $6.93\,\mathrm{s}$, closest to $7\,\mathrm{s}$. Hence, the correct option is (D).

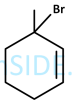
Quick Tip to Solve: Square the amplitude envelope before finding an energy half-life; this changes the decay exponent from $-bt/(2m)$ to $-bt/m$.

Chemistry (120 marks)

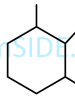
61. The major product of the following reaction is : [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- Bromine under ultraviolet light generates bromine radicals. In an alkene, radical substitution preferentially occurs at an allylic carbon because the allylic radical is resonance stabilised.
- Bromination is selective: among comparable allylic sites, the radical with greater alkyl substitution is more stable and leads to the major substitution product.

Step 1: Locate the allylic hydrogen atoms The ring contains a double bond on its right-hand side. The methyl-bearing ring carbon immediately above that double bond is allylic and has one hydrogen. The ring carbon immediately below the double bond is also allylic, but it is a methylene carbon. Removing a hydrogen at the methyl-bearing ring site forms a tertiary allylic radical; removing one at the lower

site forms a secondary allylic radical. Both are resonance stabilised, but the former benefits from greater alkyl substitution.

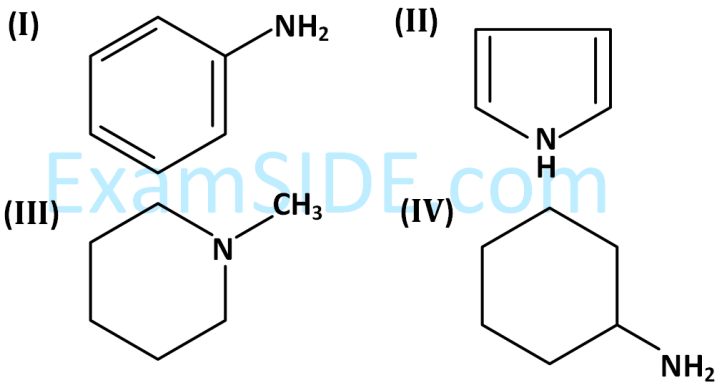
Step 2: Form the favoured radical and trap it with bromine Under the indicated $h\nu$ conditions, homolysis starts the radical chain. A bromine radical abstracts the allylic hydrogen at the methyl-bearing ring carbon, giving the more stable substituted allylic radical. That radical reacts with Br_2 to form a carbon–bromine bond and regenerate a bromine radical. The net substitution replaces the hydrogen on the carbon already carrying the methyl group with bromine; the displayed double bond is retained in the corresponding product.

Step 3: Match the product with the structures Option (B) has bromine attached to the methyl-bearing allylic ring carbon and is therefore the required major product. Option (A) represents bromination at the less substituted allylic site. Option (C) brominates the methyl substituent, whose carbon is not directly adjacent to the double bond and does not give the preferred allylic radical. Option (D) is the vicinal dibromide expected from addition across the double bond, rather than the selective allylic substitution represented under the radical conditions.

Step 4: Conclusion The major product is the tertiary allylic bromide shown in (B). Hence, the correct option is (B).

Quick Tip to Solve: For $Br_2/h\nu$, first mark allylic C–H bonds, then compare the stability of the radicals formed by their removal.

62. Among the following compounds, the increasing order of their basic strength is : [4 marks]



ExamSIDE.com

(A) (I) < (II) < (IV) < (III)

(B) (I) < (II) < (III) < (IV)

(C) (II) < (I) < (IV) < (III)

(D) (II) < (I) < (III) < (IV)

Correct Answer: (C) (II) < (I) < (IV) < (III)

Solution:

Concept:

- Basic strength depends on how readily the nitrogen lone pair can accept a proton. Delocalisation of that pair lowers its availability; electron-releasing alkyl groups increase it.
- In pyrrole the nitrogen lone pair supplies two electrons to the aromatic sextet. In aniline it is conjugated with benzene, whereas in the saturated aliphatic amines it remains localised.

Step 1: Identify the four nitrogen environments Compound (I) is aniline, (II) is pyrrole, (III) is N-methylpiperidine and (IV) is cyclohexylamine. The first two have nitrogen lone pairs affected by conjugation with a ring. The latter two are saturated aliphatic amines, so their nitrogen lone pairs are not delocalised into an aromatic system. This immediately places the aliphatic amines above the aromatic nitrogen compounds in the basicity comparison used here.

Step 2: Compare pyrrole with aniline In pyrrole, the two nitrogen lone-pair electrons complete the six-electron aromatic system. Protonating nitrogen would make this pair unavailable for that stabilisation, so pyrrole is an exceptionally weak nitrogen base. In aniline the lone pair is conjugated with benzene and is less available than in an aliphatic amine, but the benzene ring already has its own aromatic sextet. Consequently aniline is more basic than pyrrole:

$$(\mathrm{II}) < (\mathrm{I}).$$

Any option starting with the reverse order is excluded.

Step 3: Compare the saturated amines and assemble the order In the structural electron-donation comparison, the tertiary cyclic amine (III) has three carbon groups bonded to nitrogen, whereas the primary amine (IV) has only one. The alkyl groups donate electron density through the inductive effect and favour proton acceptance, placing (III) above (IV) in the intended order. Combining the comparisons gives

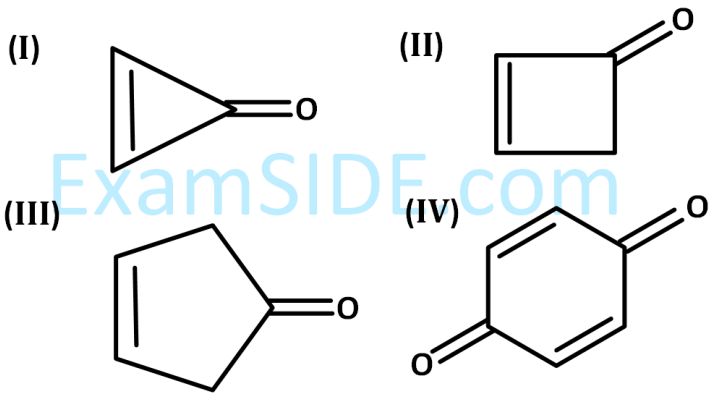
$$(\mathrm{II}) < (\mathrm{I}) < (\mathrm{IV}) < (\mathrm{III}).$$

Option (C) is the only sequence containing both the correct pyrrole–aniline order and this aliphatic-amine order. Detailed solvent-specific basicity comparisons require solvation information, which is not specified in the question.

Step 4: Conclusion The required increasing order is $(\mathrm{II}) < (\mathrm{I}) < (\mathrm{IV}) < (\mathrm{III})$. Hence, the correct option is (C).

Quick Tip to Solve: First separate nitrogen lone pairs that maintain aromaticity, those conjugated with a ring, and those localised on saturated amines.

63. **Which of the following compounds will show highest dipole moment ?** [4 marks]



ExamSIDE.com

(A) (I)

(B) (II)

(C) (III)

(D) (IV)

Correct Answer: (A) (I)

Solution:

Concept:

- The molecular dipole moment is a vector sum of bond and charge-separation contributions. Resonance structures with separated charges can strengthen the carbonyl dipole when strongly stabilised.
- A cyclic conjugated cation with $4n + 2$ pi electrons is aromatic. An aromatic charge-separated contributor can become especially important in the resonance hybrid.

Step 1: Examine the resonance form of compound I Compound (I) is cyclopropenone. Move the carbonyl pi electrons towards oxygen to obtain a charge-separated contributor with negative charge on oxygen and positive charge on the three-membered carbon ring. The ring then has a cyclic delocalised pair of pi electrons, corresponding to $4n + 2 = 2$ with $n = 0$. It is a cyclopropenyl-type aromatic cation. This aromatic stabilisation makes the polar contributor unusually important and produces strong separation of positive and negative charge.

Step 2: Compare the other carbonyl compounds Compound (II) does not obtain the same continuous aromatic three-membered cation arrangement on polarising its carbonyl group. Compound (III) is a larger ring ketone whose isolated unsaturation likewise does not generate the strongly stabilised aromatic cation found in (I). Thus ordinary carbonyl polarity in these structures does not receive the same enhancement. Merely counting a carbonyl bond is insufficient: its resonance environment determines how large the molecular dipole can be.

Step 3: Use symmetry to check compound IV Compound (IV) is the symmetric para-quinone structure. Its two carbonyl dipoles point in opposite directions and cancel by molecular symmetry, so having two oxygen atoms does not make it the most polar molecule. Among the listed choices, compound (I) combines a substantial oxygen-negative/ring-positive resonance contribution with no equivalent opposing carbonyl dipole. This rules out (II), (III) and the symmetric (IV), leaving the cyclopropenone

structure as the largest-dipole choice.

Step 4: Conclusion Compound (I) has the highest dipole moment. Hence, the correct option is (A).

Quick Tip to Solve: For cyclic unsaturated ketones, inspect whether the carbonyl charge-separated form creates an aromatic ring, then check cancellation by symmetry.

64. The incorrect statement among the following is : [4 marks]

(A) α-D-glucose and β-D-glucose are anomers.

(B) α-D-glucose and β-D-glucose are enantiomers.

(C) Cellulose is a straight chain polysaccharide made up of only β-D-glucose units.

(D) The penta acetate of glucose does not react with hydroxyl amine

Correct Answer: (B) α-D-glucose and β-D-glucose are enantiomers.

Solution:

Concept:

- Anomers are cyclic stereoisomers differing only in configuration at the anomeric carbon. In glucose this is carbon 1, formed when the aldehyde group cyclises to a hemiacetal.
- Enantiomers are non-superimposable mirror images. Changing just one stereocentre of a molecule containing several stereocentres normally produces diastereomers, rather than its mirror image.

Step 1: Compare the two glucose forms Both forms are D-glucose and retain the same configurations at the other stereogenic carbons. They differ only in the orientation of the anomeric hydroxyl group at C1. Thus they are anomers, a special class of diastereomers. The mirror image of an alpha-D form would instead be the corresponding alpha-L form. Therefore statement (A) is correct, but statement (B) incorrectly calls the two D forms enantiomers.

Step 2: Check the remaining statements Cellulose contains unbranched chains of beta-D-glucose residues joined through beta-(1,4)-glycosidic bonds, so (C) is correct. In glucose pentaacetate the anomeric hydroxyl group is acetylated along with the other hydroxyl groups. The cyclic acetal structure cannot open to expose a free aldehyde group under the usual hydroxylamine test conditions. Consequently it does not form an oxime, making (D) correct as well. The question asks for the incorrect statement, not for a true property.

Step 3: Conclusion The incorrect statement is that alpha-D-glucose and beta-D-glucose are enantiomers. They are anomers. Hence, the correct option is (B).

Quick Tip to Solve: For sugar stereoisomers, first compare the anomeric carbon; a D/L comparison is different from an alpha/beta comparison.

65. The electronic configuration with the highest ionization enthalpy is : [4 marks]

(A) [Ne] $3s^2\ 3p^1$

(B) [Ne] $3s^2\ 3p^2$

(C) [Ne] $3s^2\ 3p^3$

(D) [Ar] $3d^{10}\ 4s^2\ 4p^3$

Correct Answer: (C) [Ne] $3s^2\ 3p^3$

Solution:

Concept:

- First ionization enthalpy is the energy required to remove an electron from an isolated gaseous atom. Greater effective nuclear attraction and smaller atomic size generally increase it.
- A half-filled p subshell has one electron in each of its three orbitals and gains exchange stability. Across a period ionization enthalpy generally increases; down a group increased size and shielding usually lower it.

Step 1: Identify the configurations The configurations in (A), (B), and (C) are those of aluminium, silicon, and phosphorus respectively. They have their outer electrons in the third shell. Aluminium ends in 3p1, silicon in 3p2, and phosphorus in 3p3. Increasing nuclear charge across these atoms contracts the valence shell and strengthens the attraction of the nucleus for the electron being removed.

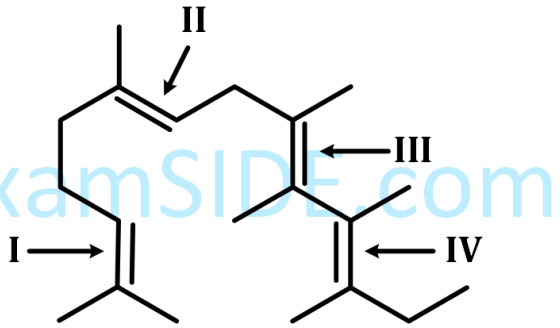
Step 2: Compare the half-filled configurations Phosphorus has a half-filled 3p subshell, which provides additional stability against removing an electron. Option (D) is arsenic, with a half-filled 4p subshell. Although both phosphorus and arsenic have p3 valence arrangements, the arsenic electron occupies the larger fourth shell and is more shielded by inner electrons. Its first ionization enthalpy is therefore lower than that of phosphorus. The 3d10 core in arsenic does not reverse this comparison. Neither aluminium nor silicon has the same combination of strong attraction and half-filled p-shell stability as phosphorus.

Step 3: Conclusion The highest ionization enthalpy is for $[\mathrm{Ne}]\,3s^2 3p^3$, phosphorus. Hence, the correct option is (C).

Quick Tip to Solve: When two choices both end in p3, compare their principal shells: the smaller, upper-group atom usually holds its electron more strongly.

66. In the following structure, the double bonds are marked as I, II, III and IV

Geometrical isomerism is not possible at site (s) : [4 marks]



ExamSIDE.com

(A) III

(B) I

(C) I and III

(D) III and IV

Correct Answer: (B) I

Solution:

Concept:

- An alkene can show geometrical isomerism only if each carbon of the double bond carries two different substituents. Restricted rotation alone is not sufficient.
- Compare complete substituent paths attached to each alkene carbon. Two carbon-containing groups can still be different if their connectivity differs beyond the first atom.

Step 1: Test site I At the lower carbon of double bond I, the two single bonds terminate at two methyl groups. These substituents are identical. Interchanging them cannot create a distinct configuration across the double bond, so geometrical isomerism is impossible at I, irrespective of the groups attached to its other carbon.

Step 2: Test the other three sites At II, one carbon has a methyl group and the continuing chain, while its partner has hydrogen and a carbon chain. Thus both carbons have two different groups. At III, the upper carbon has a methyl group and the chain continuing to the left; the lower carbon has a methyl group and the chain continuing towards IV. The two substituents on either carbon are therefore different, so III does permit geometrical isomerism. At IV, the upper carbon has a methyl group and the chain towards III, while the lower carbon has methyl and ethyl groups. Site IV also satisfies the requirement. Hence choices containing III as an impossible site misidentify unlike chain substituents as identical.

Step 3: Conclusion Geometrical isomerism is not possible only at site I, because one doubly bonded carbon bears two methyl groups. Hence, the correct option is (B).

Quick Tip to Solve: Before assigning cis/trans or E/Z, reject any double bond with two identical groups attached to the same carbon.

67. **[$Co_2(CO)_8$] displays :** [4 marks]

(A) one Co−Co bond, six terminal CO and two bridging CO

(B) one Co−Co bond, four terminal CO and four bridging CO

(C) no Co−Co bond, six terminal CO and two bridging CO

(D) no Co−Co bond, four terminal CO and four bridging CO

Correct Answer: (A) one Co−Co bond, six terminal CO and two bridging CO

Solution:

Concept:

- In the usual bridged structure of dicobalt octacarbonyl, carbon monoxide ligands can be terminal, attached to one cobalt, or bridging, attached to both cobalt atoms.
- Cobalt is a group-9 metal. In neutral electron counting, each terminal CO contributes two electrons to its metal, each bridging CO contributes one electron to each metal, and a metal-metal bond contributes one electron to each metal.

Step 1: Count the carbon monoxide ligands The bridged structure of $Co_2(CO)_8$ contains three terminal CO ligands on each cobalt and two CO ligands bridging the pair. Therefore the terminal total is six and the bridging total is two. Their sum is eight, as required by the molecular formula. The four-terminal, four-bridging descriptions in (B) and (D) do not match this standard structure.

Step 2: Confirm the cobalt-cobalt bond For each cobalt centre, the neutral electron count from the metal and the listed ligands is

$$9 + 3(2) + 2(1) = 17.$$

A single cobalt-cobalt bond provides one further electron to each centre, yielding

$$17 + 1 = 18.$$

Thus the bridged structure contains one cobalt-cobalt bond. Option (C) has the correct distribution of CO ligands but omits this bond and fails to describe the closed-shell bridged dimer. Metal carbonyls can have different structural forms under different conditions, but the bridged form specified by the available choices has six terminal and two bridging ligands.

Step 3: Conclusion $Co_2(CO)_8$ displays one Co–Co bond, six terminal CO ligands and two bridging CO ligands. Hence, the correct option is (A).

Quick Tip to Solve: For the bridged dicobalt carbonyl, count three terminal CO per cobalt, then use the 18-electron count to check the metal-metal bond.

68. The group having triangular planar structures is : [4 marks]

(A) BF_3, NF_3, CO_3^{2-}

(B) CO_3^{2-}, NO_3^-, SO_3

(C) NH_3, SO_3, CO_3^{2-}

(D) NCl_3, BCl_3, SO_3

Correct Answer: (B) CO_3^{2-}, NO_3^-, SO_3

Solution:

Concept:

- VSEPR theory counts electron-density regions around the central atom. A multiple bond counts as one region for determining the electron-domain geometry.
- Three bonding regions with no lone pair on the central atom give trigonal planar molecular geometry, with ideal bond angles of about $120°$. Three bonds and one lone pair instead give a trigonal pyramidal molecule.

Step 1: Examine carbonate, nitrate and sulfur trioxide In $CO_3{}^{2-}$, carbon is bonded to three oxygen atoms and has no lone pair; resonance does not create a fourth bonding direction. In $NO_3{}^-$, nitrogen likewise has three bonding directions and no central lone pair. In SO_3, sulfur is surrounded by three oxygen atoms in a planar arrangement. All three species in (B) therefore have trigonal planar structures. Their resonance descriptions are consistent with equivalent bonds within each species.

Step 2: Eliminate the groups containing nitrogen trihalides or ammonia NF_3, NH_3, and NCl_3 each have three sigma bonds and one lone pair on nitrogen. Their electron-domain arrangements are approximately tetrahedral, but their molecular shapes are trigonal pyramidal. Thus (A) fails because of NF_3, (C) because of NH_3, and (D) because of NCl_3. Although BF_3 and BCl_3 themselves are planar, every member of a proposed group must satisfy the stated geometry.

Step 3: Conclusion The completely trigonal planar group is $CO_3{}^{2-}$, $NO_3{}^-$, SO_3. Hence, the correct option is (B).

Quick Tip to Solve: For three-coordinate species, look for a central lone pair: it changes the molecular shape from planar to pyramidal.

69. The increasing order of the boiling points for the following compounds is : [4 marks]

EkaiSIDE.com

C_2H_5OH (I) C_2H_5Cl (II) $C_2H_5CH_3$ (III) $C_2H_5OCH_3$ (IV)

(A) (III) < (IV) < (II) < (I)

(B) (IV) < (III) < (I) < (II)

(C) (II) < (III) < (IV) < (I)

(D) (III) < (II) < (I) < (IV)

Correct Answer: (A) (III) < (IV) < (II) < (I)

Solution:

Concept:

- Boiling requires overcoming intermolecular attractions. Hydrogen bonding generally raises the boiling point strongly for small molecules of comparable size.
- London dispersion forces occur in all molecules; permanent dipoles add attractive interactions in polar molecules. An ether oxygen can accept hydrogen bonds but a pure ether lacks an O–H donor for self hydrogen bonding.

Step 1: Identify the compounds and the weakest attraction The structures are (I) ethanol, C_2H_5OH; (II) chloroethane, C_2H_5Cl; (III) propane, $C_2H_5CH_3$; and (IV) methoxyethane, $C_2H_5OCH_3$. Propane is nonpolar and has only dispersion attractions, so it boils first. Methoxyethane is polar but cannot form the extended self hydrogen-bonded network characteristic of ethanol.

Step 2: Order the remaining compounds Chloroethane has stronger overall intermolecular attractions than methoxyethane in this comparison, including the contribution of the polarizable chlorine atom. Its boiling point is therefore above that of the ether. Ethanol has an O–H bond and forms intermolecular hydrogen bonds, making its boiling point the highest despite its modest molecular mass. The approximate normal boiling points provide a useful check: propane about $-42^\circ C$, methoxyethane about $11^\circ C$, chloroethane about $12^\circ C$, and ethanol about $78^\circ C$. Choices placing the ether above ethanol, or propane above a polar compound, fail these comparisons.

Step 3: Conclusion The increasing order is

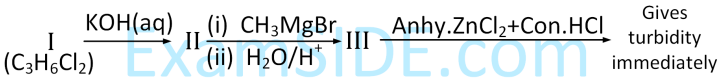
$$(III) < (IV) < (II) < (I).$$

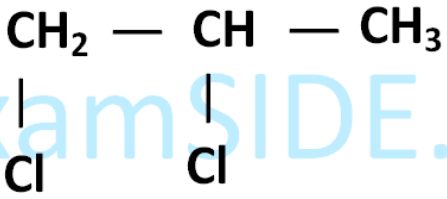
Hence, the correct option is (A).

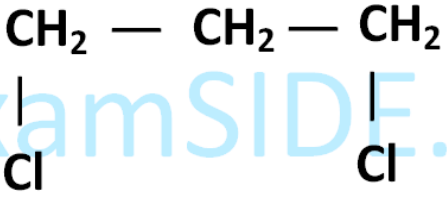
Quick Tip to Solve: Identify hydrogen-bond donors first; then compare the smaller differences in dipole and polarizability among compounds lacking an O–H bond.

70. In the following reaction sequence :

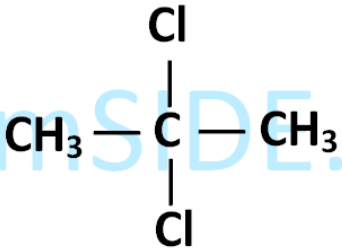
The compound I is : [4 marks]



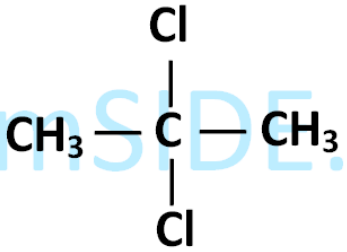
(A) 

(B) 

(C) 

(D) 

Correct Answer: (D)



ExamSIDE.com

Solution:

Concept:

- A geminal dihalide has both halogens on the same carbon. Hydrolysis gives a geminal diol, which loses water to produce an aldehyde or ketone.
- A Grignard reagent adds an alkyl group to a carbonyl carbon. After acidic hydrolysis, an aldehyde normally gives a secondary alcohol and a ketone gives a tertiary alcohol. Tertiary alcohols give immediate turbidity with Lucas reagent.

Step 1: Work backwards from the Lucas test The reagent anhydrous $ZnCl_2$ with concentrated HCl is Lucas reagent. Immediate turbidity indicates rapid formation of an insoluble alkyl chloride from a tertiary alcohol. Thus III must be a tertiary alcohol. Addition of CH_3MgBr to II followed by acidic work-up can give a tertiary alcohol only when II is a ketone, rather than an aldehyde.

Step 2: Choose the dihalide that produces the ketone Option (D), $CH_3CCl_2CH_3$, is 2,2-dichloropropane. Hydrolysis replaces its chlorine atoms by hydroxyl groups; the unstable geminal diol yields propanone. The sequence is

$$CH_3CCl_2CH_3 \xrightarrow{\text{aq} \cdot \text{KOH}} CH_3COCH_3$$

followed by methyl addition and hydrolysis to $(CH_3)_3COH$, tert-butanol. This tertiary alcohol gives immediate Lucas turbidity. Option (C) instead yields propanal and then a secondary alcohol. Options (A) and (B) are vicinal and separated dihalides respectively; their straightforward aqueous hydrolysis products are diols, not the ketone needed for this sequence.

Step 3: Conclusion Compound I is 2,2-dichloropropane, $CH_3CCl_2CH_3$, which leads through propanone to tert-butanol. Hence, the correct option is (D).

Quick Tip to Solve: Immediate Lucas turbidity suggests a tertiary alcohol; before a Grignard addition, look backwards for a ketone precursor.

71. The following reaction occurs in the Blast Furnace where iron ore is reduced to iron metal :

$Fe_2O_3(s) + 3\ CO(g) \rightleftharpoons 2\ Fe(1) + 3\ CO_2(g)$

Using the Le Chatelier's principle, predict which one of the following will not disturb the equilibrium ?

[4 marks]

(A) Removal of CO

(B) Removal of CO_2

(C) Addition of CO_2

(D) Addition of Fe_2O_3

Correct Answer: (D) Addition of Fe_2O_3

Solution:

Concept:

- In a heterogeneous equilibrium, the activities of pure solids and pure liquids are taken as unity while those phases remain present. Their amounts therefore do not appear in the equilibrium quotient.
- Le Chatelier's principle predicts a shift that opposes a change in the partial pressure of a gaseous reactant or product. Compare the reaction quotient Q_p with its equilibrium value K_p.

Step 1: Write the equilibrium quotient For the reduction reaction, the oxide and metallic iron are condensed phases. The gaseous species are carbon monoxide and carbon dioxide, so the quotient is

$$Q_p = \frac{(p_{CO_2}/p^\circ)^3}{(p_{CO}/p^\circ)^3}.$$

At equilibrium this ratio equals K_p. Adding more solid oxide, with the other conditions unchanged and solid already present, does not change either gas partial pressure or the activity of the solid.

Step 2: Test the listed changes Removing CO reduces the denominator and raises Q_p, so the equilibrium shifts towards reactants. Removing CO_2 lowers the numerator and causes a shift towards products. Adding CO_2 raises the numerator and causes a shift towards reactants. Thus options (A), (B) and (C) each disturb the gas ratio, whereas adding Fe_2O_3 only increases the amount of an already present pure solid.

Step 3: Conclusion Addition of Fe_2O_3, while the solid phase remains present, does not disturb the equilibrium. Hence, the correct option is (D).

Quick Tip to Solve: In a heterogeneous equilibrium, first omit pure condensed phases from the quotient; changing their amount alone does not shift equilibrium.

72. Which one of the following is an oxide? [4 marks]

(A) KO_2

(B) BaO_2

(C) SiO_2

(D) CsO_2

Correct Answer: (C) SiO_2

Solution:

Concept:

- A normal oxide has oxygen in oxidation state -2; a peroxide contains the ${O_2}^{2-}$ unit with each oxygen at -1. A superoxide contains ${O_2}^{-}$ with average oxygen oxidation state $-\frac{1}{2}$.
- Use the usual oxidation states of the other element and electrical neutrality to classify the oxygen compound. The presence of two oxygen atoms in a formula does not by itself mean that it is a peroxide.

Step 1: Classify the metal oxygen compounds Potassium and caesium each form a $+1$ cation. In KO_2 and CsO_2, the pair of oxygen atoms therefore carries total charge -1; these compounds are superoxides. Barium forms Ba^{2+}, so the oxygen pair in BaO_2 carries charge -2. Barium peroxide contains the peroxide ion, rather than normal oxide oxygen. Options (A), (B) and (D) belong to these separate classes.

Step 2: Identify the normal oxide Silicon in SiO_2 has oxidation state $+4$. Neutrality requires the two oxygen atoms to contribute -4, giving -2 per oxygen:

$$+4 + 2x = 0 \quad \Rightarrow \quad x = -2.$$

Silica is a covalent network oxide; it need not contain discrete free oxide ions to be classified as a normal oxide. Its formula reflects the silicon-to-oxygen ratio in that network, rather than a peroxide or superoxide ion.

Step 3: Conclusion SiO_2 is the normal oxide among the choices. Hence, the correct option is (C).

Quick Tip to Solve: Check the oxidation state of oxygen before classifying an MO_2 compound: the metal's charge decides whether it is an oxide, peroxide or superoxide.

73. A solution is prepared by mixing 8.5 g of CH_2Cl_2 and 11.95 g of $CHCl_3$. If vapour pressure of CH_2Cl_2 and $CHCl_3$ at 298 K are 415 and 200 mmHg respectively, the mole fraction of $CHCl_3$ in vapour form is : (Molar mass of Cl = 35.5 g mol^{-1}) [4 marks]

(A) 0.162

(B) 0.675

(C) 0.325

(D) 0.486

Correct Answer: (C) 0.325

Solution:

Concept:

- For an ideal liquid solution, Raoult's law gives $p_i = x_i p_i^\circ$, where x_i is the liquid mole fraction and p_i° the pure liquid vapour pressure at the same temperature.
- Dalton's law gives the vapour mole fraction $y_i = p_i/P$, where P is the sum of the component partial pressures. Liquid and vapour mole fractions are generally different.

Step 1: Calculate the liquid composition Using carbon and hydrogen molar masses of 12 and 1, the molar masses are

$$M_{CH_2Cl_2} = 12 + 2 + 2(35.5) = 85 \text{ g mol}^{-1},$$

$$M_{CHCl_3} = 12 + 1 + 3(35.5) = 119.5 \text{ g mol}^{-1}.$$

Thus the respective amounts are $8.5/85 = 0.100$ mol and $11.95/119.5 = 0.100$ mol. Both liquid mole fractions are 0.500; equal masses were not required, but equal numbers of moles have resulted.

Step 2: Convert partial pressures into vapour composition Apply Raoult's law at 298 K:

$$p_{CH_2Cl_2} = 0.5(415) = 207.5 \text{ mmHg},$$

$$p_{CHCl_3} = 0.5(200) = 100 \text{ mmHg}.$$

The total pressure is 307.5 mmHg. Therefore

$$y_{CHCl_3} = \frac{100}{307.5} = 0.3252 \approx 0.325.$$

The value 0.675 describes the other vapour component; 0.486 and 0.162 do not follow from the partial-pressure ratio. The less volatile chloroform has a smaller vapour fraction than its liquid fraction, as expected.

Step 3: Conclusion The mole fraction of $CHCl_3$ in the vapour is approximately 0.325. Hence, the correct option is (C).

Quick Tip to Solve: For an equimolar binary liquid, the vapour mole fraction is the component's pure vapour pressure divided by the sum of the two pure vapour pressures.

74. The rate of a reaction quadruples when the temperature changes from 300 to 310 K. The activation energy of this reaction is :

(Assume activation energy and preexponential factor are independent of temperature; ln 2 = 0.693; R = 8.314 J mol^{-1} K^{-1}) [4 marks]

(A) 107.2 kJ mol^{-1}

(B) 53.6 kJ mol^{-1}

(C) 26.8 kJ mol^{-1}

(D) 214.4 kJ mol^{-1}

Correct Answer: (A) 107.2 kJ mol^{-1}

Solution:

Concept:

- The Arrhenius equation is $k = Ae^{-E_a/(RT)}$, with activation energy E_a, gas constant R and absolute temperature T. With temperature-independent A and E_a, two temperatures can be compared directly.
- For unchanged reactant concentrations, the reaction-rate ratio equals the rate-constant ratio. Taking natural logarithms gives $\ln(k_2/k_1) = (E_a/R)(1/T_1 - 1/T_2)$.

Step 1: Insert the temperature and rate ratios The rate quadruples, so $k_2/k_1 = 4$, not 2. Also $T_1 = 300$ K and $T_2 = 310$ K. Thus

$$\ln 4 = 2\ln 2 = 2(0.693) = 1.386,$$

$$\frac{1}{300} - \frac{1}{310} = \frac{10}{300(310)} = \frac{1}{9300}\ \text{K}^{-1}.$$

This reciprocal-temperature difference is positive, consistent with a positive activation energy and an increasing rate constant as the temperature rises.

Step 2: Calculate and convert the activation energy Rearrange before substituting:

$$E_a = \frac{R\ln 4}{1/300 - 1/310}$$

$$= 8.314(1.386)(9300)\ \text{J mol}^{-1}$$

$$\approx 1.072 \times 10^5\ \text{J mol}^{-1} = 107.2\ \text{kJ mol}^{-1}.$$

The unit conversion divides joules by 1000. Using $\ln 2$ instead of $\ln 4$ would produce 53.6, the value for doubling rather than quadrupling; the values 26.8 and 214.4 introduce extra factors of one quarter and two relative to the required value. The required ratio fixes the value uniquely.

Step 3: Conclusion The activation energy is $107.2\ \text{kJ mol}^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the full rate ratio inside the logarithm first; quadrupling means $\ln 4 = 2\ln 2$, even for a temperature rise of only 10 K.

75. The electron in the hydrogen atom undergoes transition from higher orbitals to orbital of radius 211.6 pm. This transition is associated with : [4 marks]

(A) Lyman series

(B) Balmer series

(C) Paschen series

(D) Brackett series

Correct Answer: (B) Balmer series

Solution:

Concept:

- For the hydrogen atom, the Bohr radius of the nth stationary orbit is $r_n = a_0 n^2$, where $a_0 = 52.9$ pm and n is a positive integer.
- An emission spectral series is determined by the final principal quantum number, not by the starting orbit. Lyman, Balmer, Paschen and Brackett series terminate at $n_f = 1, 2, 3, 4$, respectively.

Step 1: Determine the final principal quantum number The given radius belongs to the final orbit reached by the electron. Substitute it into the hydrogen radius formula:

$$211.6 = 52.9 n_f^2,$$

$$n_f^2 = \frac{211.6}{52.9} = 4, \qquad n_f = 2.$$

Only the positive square root is allowed for a principal quantum number. The radius is four times the ground-state radius, so it corresponds to the second orbit, not the fourth orbit. Radius scales as the square of n, rather than linearly with n.

Step 2: Identify the series from its lower orbit The transition is from a higher orbital to $n_f = 2$, so the initial value can be $n_i = 3, 4, 5, \ldots$. Every such downward transition belongs to the Balmer series. In the Rydberg expression this appears as

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n_i^2} \right).$$

Lyman lines would end at radius 52.9 pm; Paschen and Brackett lines would end at $9a_0$ and $16a_0$, respectively. None has the stated terminal radius, so options (A), (C) and (D) are excluded.

Step 3: Conclusion The transition terminates at $n_f = 2$, and therefore belongs to the Balmer series. Hence, the correct option is (B).

Quick Tip to Solve: Convert the terminal radius into n_f using the square root of r_f/a_0; then name the series from that final quantum number.

76. An ideal gas undergoes isothermal expansion at constant pressure. During the process : [4 marks]

(A) enthalpy increases but entropy decreases.

(B) enthalpy remains constant but entropy increases.

(C) enthalpy decreases but entropy increases.

(D) Both enthalpy and entropy remain constant.

Correct Answer: (B) enthalpy remains constant but entropy increases.

Solution:

Concept:

- For a fixed amount of ideal gas, internal energy and enthalpy depend only on temperature. Thus an isothermal process has $\Delta H = nC_p\Delta T = 0$, irrespective of its reversible or irreversible path.
- Entropy is a state function. For ideal-gas states at the same temperature, $\Delta S = nR\ln(V_2/V_1)$. Expansion has $V_2 > V_1$, so its entropy change is positive.

Step 1: Apply the ideal-gas enthalpy relation Isothermal means the initial and final temperatures are equal. The ideal-gas enthalpy change is therefore

$$\Delta H = nC_p(T_2 - T_1) = 0.$$

There is no intermolecular potential-energy contribution that could make ideal-gas enthalpy depend on volume at fixed temperature. Options (A) and (C), which require an enthalpy increase or decrease, are excluded immediately. A constant opposing external pressure can govern an irreversible expansion without changing this state-function result.

Step 2: Evaluate the entropy change and interpret the pressure wording Use a reversible isothermal reference path to evaluate the entropy difference between the same endpoints:

$$\Delta S = \int_{V_1}^{V_2} \frac{nR}{V}\,dV = nR\ln\frac{V_2}{V_1} > 0.$$

Consequently option (D) is also excluded. The phrase constant pressure can describe constant external pressure during expansion. If it instead meant a fixed equilibrium pressure of a closed gas throughout, $PV = nRT$ would forbid any isothermal volume change. Under the physically consistent external-pressure reading, the entropy increases while enthalpy remains unchanged.

Step 3: Conclusion For the stated isothermal expansion, enthalpy remains constant and entropy increases; constant pressure is understood as the opposing external pressure. Hence, the correct option is (B).

Quick Tip to Solve: Separate state functions from process conditions: ideal-gas enthalpy follows temperature, while isothermal entropy follows the volume ratio.

77. What quantity (in mL) of a 45% acid solution of a mono-protic strong acid must be mixed with a 20% solution of the same acid to produce 800 mL of a 29.875% acid solution ? [4 marks]

(A) 320

(B) 325

(C) 316

(D) 330

Correct Answer: (C) 316

Solution:

Concept:

- For solutions of the same solute with additive volumes and percentage concentration on the same basis, conserve the total acid content: $c_1V_1 + c_2V_2 = c_f(V_1 + V_2)$.
- A weighted average concentration must lie between the two original concentrations. The volume of the stronger solution is fixed by the target's fractional position between 20 percent and 45 percent.

Step 1: Set up the acid-content balance Let x mL be the required volume of the 45 percent solution. The other solution must supply $(800 - x)$ mL to make the stated final volume. Using the same percentage scale throughout,

$$45x + 20(800 - x) = 29.875(800).$$

Equivalently, one could divide every concentration by 100 and use 0.45, 0.20 and 0.29875. The common factor cancels; it must not be applied to just one term. The strong acid being monoprotic does not introduce an additional stoichiometric factor when the same acid is mixed with itself.

Step 2: Solve and check the mixture Expand and collect the terms in x:

$$25x + 16000 = 23900,$$

$$25x = 7900, \qquad x = 316 \text{ mL}.$$

The weaker solution volume is 484 mL. The direct check is

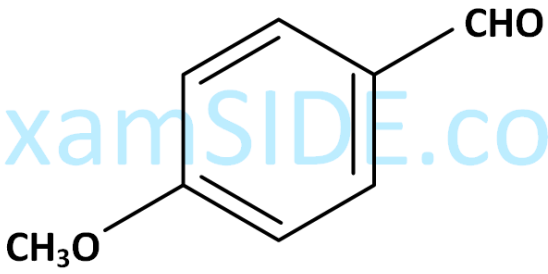
$$\frac{45(316) + 20(484)}{800} = 29.875.$$

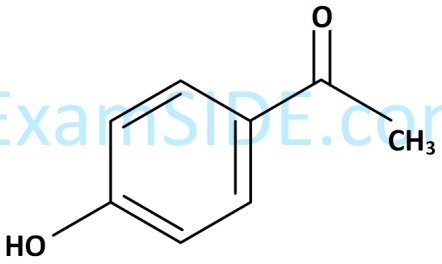
The stronger-solution fraction is $316/800 = 0.395$, consistent with a target closer to 20 percent than to 45 percent. Substituting 320, 325 or 330 mL raises the weighted concentration above the stated target, excluding options (A), (B) and (D).

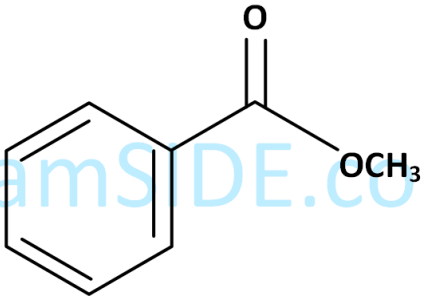
Step 3: Conclusion Mix 316 mL of the 45 percent solution with 484 mL of the 20 percent solution. Hence, the correct option is (C).

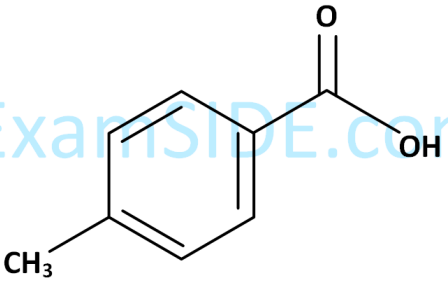
Quick Tip to Solve: Use alligation as a check: stronger to weaker volume is $(29.875 - 20) : (45 - 29.875) = 9.875 : 15.125$, then scale to 800 mL.

78. **A compound of molecular formula $C_8H_8O_2$ reacts with acetophenone to form a single cross-aldol product in the presence of base. The same compound on reaction with conc. NaOH forms benzyl alcohol as one of the products. The structure of the compound is :** [4 marks]

(A) 

(B) 

(C) 

(D) 

ExamSIDE.com

Correct Answer: None of the structures gives unsubstituted benzyl alcohol; option A is the intended aldehyde but gives p-methoxybenzyl alcohol.

Solution:

Concept:

- A crossed aldol condensation is selective when only one carbonyl partner can form the enolate. Acetophenone has alpha hydrogen atoms; an aromatic aldehyde directly attached to the ring does not.

- A non-enolisable aldehyde undergoes Cannizzaro disproportionation with concentrated alkali: one molecule is reduced to the corresponding alcohol while another is oxidised to carboxylate. Ring substituents remain in place.

Step 1: Use the aldol requirement Option A is p-methoxybenzaldehyde, $\text{p-CH}_3\text{OC}_6\text{H}_4\text{CHO}$, with molecular formula $\text{C}_8\text{H}_8\text{O}_2$. Its aldehyde group has no alpha hydrogen. Acetophenone supplies the enolate, which adds to this aldehyde and dehydrates to the single crossed condensation product $\text{p-CH}_3\text{OC}_6\text{H}_4\text{CH=CHCOC}_6\text{H}_5$. Thus A satisfies the molecular formula and the crossed-aldol test.

Step 2: Check the alcohol identity literally For option A, write $R = \text{p-CH}_3\text{OC}_6\text{H}_4$. The Cannizzaro equation is

$$2\,\text{RCHO} + \text{NaOH} \longrightarrow \text{RCH}_2\text{OH} + \text{RCOONa}.$$

The alcohol is p-methoxybenzyl alcohol, not benzyl alcohol $\text{C}_6\text{H}_5\text{CH}_2\text{OH}$. Concentrated aqueous alkali does not remove the methoxy substituent from the benzene ring. This distinction is essential because the question names the unsubstituted alcohol.

Step 3: Exclude the remaining functional groups Option B is p-hydroxyacetophenone, an enolisable ketone; it does not give benzyl alcohol by Cannizzaro reaction. Option C is methyl benzoate and gives benzoate plus methanol on alkaline hydrolysis. Option D is p-methylbenzoic acid and merely forms its carboxylate salt. Neither C nor D is the aldehyde needed for the stated aldol test.

Step 4: Conclusion The key selects (A), which is the intended non-enolisable aldehyde. Literally, however, no option satisfies both statements: (A) produces p-methoxybenzyl alcohol rather than benzyl alcohol. The printed question therefore has no fully correct option.

Quick Tip to Solve: In a Cannizzaro reaction replace CHO by CH2OH on the same carbon skeleton; never discard a ring substituent.

79. Which of the following compounds is most reactive to an aqueous solution of sodium carbonate ?

[4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- Carbonate acts as a base, so compare the ease with which each hydrocarbon loses a proton and the stability of the conjugate anion.
- A planar, cyclic, fully conjugated anion containing $4n + 2$ pi electrons is aromatic. Aromatic stabilisation can make a hydrocarbon much more acidic than its apparently similar non-aromatic alternatives.

Step 1: Identify the special proton in option C Option C is cyclopentadiene. Its saturated ring carbon

bears hydrogen atoms; removing one proton converts that carbon into a carbon with a p orbital and a lone pair. The remaining five-membered ring can become planar and continuously conjugated. Thus the negative charge is delocalised over all five ring carbon atoms rather than remaining at one localised carbanion centre.

Step 2: Count the electrons in the conjugate base The two double bonds contribute four pi electrons and the carbanion lone pair contributes two. The cyclopentadienyl anion therefore has

$$N_\pi = 4 + 2 = 6 = 4(1) + 2.$$

It satisfies the aromatic electron count and gains a large stabilisation unavailable to the neutral non-aromatic diene. This makes the methylene proton unusually acidic for a hydrocarbon and makes C the most responsive to a basic carbonate solution.

Step 3: Compare the other rings Deprotonating cyclopropene (A), cyclobutene (B), or the cyclohexadiene structure (D) does not create the same planar five-membered six-pi-electron aromatic anion. Ordinary alkene or allylic carbanion stabilisation is weaker. The question asks for the most reactive member comparatively; carbonate is a weak base, so this comparison should not be mistaken for a claim of quantitative hydrocarbon deprotonation in water.

Step 4: Conclusion Cyclopentadiene gives the exceptionally resonance-stabilised aromatic cyclopentadienyl anion and is the most acidic of these hydrocarbons. Hence, the correct option is (C).

Quick Tip to Solve: For hydrocarbon acidity, draw the conjugate anion first and count its pi electrons; cyclopentadiene gains aromaticity on losing a proton.

80. Which of the following ions does not liberate hydrogen gas on reaction with dilute acids ?

[4 marks]

(A) Ti^{2+}

(B) V^{2+}

(C) Cr^{2+}

(D) Mn^{2+}

Correct Answer: (D) Mn^{2+}

Solution:

Concept:

- Hydrogen gas is liberated when a dissolved metal ion transfers electrons to acid protons: $2\,H^+ + 2\,e^- \longrightarrow H_2$. The metal ion must itself be readily oxidised.
- The reducing strength of a divalent transition-metal ion depends on the stability of its oxidation products and its electronic configuration. A half-filled $3d^5$ configuration is particularly stable.

Step 1: Write the relevant oxidation test For a divalent ion, the process competing with proton reduction is $M^{2+} \longrightarrow M^{3+} + e^-$. Combining two such oxidations with proton reduction gives

$$2\,M^{2+} + 2\,H^+ \longrightarrow 2\,M^{3+} + H_2.$$

This reaction proceeds for a sufficiently strong reducing ion, whereas simply being a transition-metal ion does not guarantee hydrogen evolution.

Step 2: Compare the listed ions Ti^{2+}, V^{2+}, and Cr^{2+} have configurations $3d^2$, $3d^3$, and $3d^4$, respectively. They are reducing in acidic aqueous media and can be oxidised to higher oxidation states while reducing protons. In particular, oxidation of Cr^{2+} gives the comparatively stable $3d^3$ chromium(III) ion.

Step 3: Apply the half-filled-shell stability Mn^{2+} is $3d^5$, with one electron in each of the five d orbitals. Oxidising it to Mn^{3+}, which is $3d^4$, removes this half-filled-shell stability and is unfavourable with dilute acid alone. Consequently manganese(II) is not a sufficiently strong reducing agent to liberate hydrogen under the stated conditions. This is different from metallic manganese, which can dissolve in acid to produce manganese(II) and hydrogen.

Step 4: Conclusion Mn^{2+} does not liberate hydrogen gas with dilute acid because its half-filled $3d^5$ state resists further oxidation. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish a metal from its divalent ion: Mn metal reacts with acid, while stable Mn(II) need not reduce hydrogen ions.

81. A gas undergoes change from state A to state B. In this process, the heat absorbed and work done by the gas is 5 J and 8 J, respectively. Now gas is brought back to A by another process during which 3 J of heat is evolved. In this reverse process of B to A : [4 marks]

(A) 10 J of the work will be done by the gas.

(B) 6 J of the work will be done by the gas.

(C) 10 J of the work will be done by the surrounding on gas.

(D) 6 J of the work will be done by the surrounding on gas.

Correct Answer: (D) 6 J of the work will be done by the surrounding on gas.

Solution:

Concept:

- Using work done by the gas, the first law is $\Delta U = q - W_{\text{by}}$. Heat absorbed is positive and heat evolved is negative.
- Internal energy is a state function. Reversing the initial and final states reverses the sign of ΔU, even when the return process follows another path. Heat and work individually need not reverse.

Step 1: Find the forward internal-energy change In the process from A to B, the gas absorbs $q_{AB} = +5\,\text{J}$ and performs $W_{AB} = +8\,\text{J}$ of work. Therefore

$$\Delta U_{AB} = q_{AB} - W_{AB} = 5 - 8 = -3\,\text{J}.$$

The gas loses internal energy because the energy delivered as work exceeds the energy entering as heat.

Step 2: Reverse the state-function change For the return journey from B to A,

$$\Delta U_{BA} = -\Delta U_{AB} = +3\,\text{J}.$$

The gas evolves heat, so $q_{BA} = -3\,\text{J}$. It is essential to use the word evolved to assign this negative sign; treating the given magnitude 3 J as absorbed heat would describe a different process.

Step 3: Solve for work and interpret its sign Apply the same sign convention on the return path:

$$3 = -3 - W_{BA}, \qquad W_{BA} = -6\,\text{J}.$$

Negative work done by the gas means positive work done on it by the surroundings. Thus the surroundings supply 6 J of work; 3 J escapes as heat and the remaining 3 J raises the internal energy. The 10 J choices violate the first law, and 6 J done by the gas has the wrong direction. No assumption about a reversible path or constant pressure is required.

Step 4: Conclusion The surroundings do $6\,\text{J}$ of work on the gas during the return process. Hence, the correct option is (D).

Quick Tip to Solve: Write one sign convention at the start: with work by the gas, use delta U = q - W; reverse only the state-function change.

82. 50 mL of 0.2 M ammonia solution is treated with 25 mL of 0.2 M HCl. If pK_b of ammonia solution is 4.75, the pH of the mixture will be : [4 marks]

(A) 3.75

(B) 4.75

(C) 8.25

(D) 9.25

Correct Answer: (D) 9.25

Solution:

Concept:

- A strong acid first neutralises ammonia stoichiometrically: $\text{NH}_3 + \text{H}^+ \longrightarrow \text{NH}_4{}^+$. Only after this reaction should the remaining weak base and its conjugate acid be used in a buffer calculation.
- For an ammonia/ammonium buffer, $\text{pOH} = pK_b + \log([\text{NH}_4{}^+]/[\text{NH}_3])$. At the usual aqueous temperature of 298 K, $\text{pH} + \text{pOH} = 14$.

Step 1: Calculate the initial reacting amounts Concentration in mol per litre multiplied by volume in millilitres directly gives millimoles. Initially,

$$n(\text{NH}_3) = 0.2 \times 50 = 10\,\text{mmol}, \qquad n(\text{HCl}) = 0.2 \times 25 = 5\,\text{mmol}.$$

The strong acid is the limiting reagent. It cannot leave the final mixture strongly acidic because there is twice as much ammonia available to consume it.

Step 2: Find the buffer composition The 1:1 neutralisation consumes 5 mmol ammonia and produces

5 mmol ammonium ion. Thus the mixture contains 5 mmol NH_3 and 5 mmol NH_4^+ in a total volume of 75 mL. Both concentrations are $5/75\,\text{mol L}^{-1}$, so their ratio is exactly one. Dilution affects both members equally and cancels from this ratio.

Step 3: Evaluate pOH and convert to pH Substituting the equal amounts into the buffer expression gives

$$\text{pOH} = 4.75 + \log 1 = 4.75.$$

Therefore

$$\text{pH} = 14 - 4.75 = 9.25.$$

The options 3.75 and 4.75 would incorrectly describe an acidic final mixture; 4.75 also confuses pOH with pH. A result of 8.25 would require a tenfold change in the salt/base ratio, which the neutralisation amounts do not provide.

Step 4: Conclusion The equal-ammonia/equal-ammonium buffer has $\text{pH} = 9.25$. Hence, the correct option is (D).

Quick Tip to Solve: At half-neutralisation of a weak base, base and conjugate acid are equal, so pOH equals pKb before conversion to pH.

---

83. To find the standard potential of M^{3+}/M electrode,the following cell is constituted : Pt/M/M^{3+}(0.001 mol L^{-1})/Ag^+(0.01 mol L^{-1})/Ag

The emf of the cell is found to be 0.421 volt at 298 K. The standard potential of half reaction $M3^+$ + $3e^- \rightarrow$ M at 298 K will be :

(Given $E^-_{Ag^+\,/\,Ag}$ at 298 K = 0.80 Volt) [4 marks]

(A) 0.38 Volt

(B) 0.32 Volt

(C) 1.28 Volt

(D) 0.66 Volt

Correct Answer: (B) 0.32 Volt

Solution:

Concept:

- The standard cell potential is the cathode reduction potential minus the anode reduction potential: $E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}$. Electrode potentials are never multiplied by balancing coefficients.
- At 298 K, the Nernst equation is $E = E^\circ - (0.0591/n)\log Q$, where n is the number of transferred electrons and activities of pure solids equal one.

Step 1: Balance the cell reaction The left electrode undergoes oxidation $\text{M} \longrightarrow \text{M}^{3+} + 3\,\text{e}^-$; at the right electrode $\text{Ag}^+ + \text{e}^- \longrightarrow \text{Ag}$. Therefore the balanced overall reaction is

$$\text{M} + 3\,\text{Ag}^+ \longrightarrow \text{M}^{3+} + 3\,\text{Ag}.$$

Three electrons are transferred, so $n = 3$. The metal and silver solids do not appear in the reaction

quotient.

Step 2: Calculate the nonstandard correction Using the given concentrations,

$$Q = \frac{[M^{3+}]}{[Ag^{+}]^3} = \frac{0.001}{(0.01)^3} = 10^3.$$

Hence

$$0.421 = E^\circ_{cell} - \frac{0.0591}{3}\log(10^3) = E^\circ_{cell} - 0.0591.$$

Thus $E^\circ_{cell} = 0.4801\,V$. The measured emf is lower than the standard emf because the reaction quotient exceeds unity.

Step 3: Recover the metal reduction potential The silver standard reduction potential is $+0.80\,V$. Therefore

$$0.4801 = 0.80 - E^\circ_{M^{3+}/M}, \qquad E^\circ_{M^{3+}/M} = 0.3199\,V.$$

Rounding to the stated options gives 0.32 V. The value near 0.38 V comes from subtracting the measured emf directly from 0.80 V and neglecting the concentration correction. The larger choices have the incorrect potential combination or sign.

Step 4: Conclusion The standard reduction potential for $M^{3+} + 3\,e^- \longrightarrow M$ is $+0.32\,V$. Hence, the correct option is (B).

Quick Tip to Solve: Balance electrons before writing Q: the silver concentration must be cubed because three silver ions accept the three electrons.

84. Which of the following is a set of greenhouse gases? [4 marks]

(A) CH_4, O_3, N_2, SO_2

(B) O_3, N_2, CO_2, NO_2

(C) O_3, NO_2, SO_2, Cl_2

(D) CO_2, CH_4, N_2O, O_3

Correct Answer: (D) CO_2, CH_4, N_2O, O_3

Solution:

Concept:

- Greenhouse gases absorb some of the infrared radiation emitted by Earth and contribute to warming of the lower atmosphere. The relevant criterion is infrared absorption, not simply whether a gas is a pollutant.
- Important greenhouse gases include carbon dioxide, methane, nitrous oxide, ozone and water vapour. Homonuclear nitrogen is not a significant infrared-active greenhouse gas.

Step 1: Check every gas in option D CO_2 has infrared-active bending and asymmetric stretching vibrations. CH_4 absorbs strongly in infrared vibrational bands, and N_2O also absorbs terrestrial infrared radiation. Ozone, O_3, has infrared absorption and contributes to the greenhouse effect. Thus all four members in D satisfy the required classification, despite having different molecular structures

and atmospheric abundances.

Step 2: Eliminate the sets containing nitrogen Options A and B both contain N_2. Molecular nitrogen is a homonuclear diatomic molecule; its ordinary vibration does not produce a changing electric dipole and does not strongly absorb terrestrial infrared radiation. Consequently it is not classified as a greenhouse gas in this context. Its large abundance in the atmosphere does not change this spectroscopic distinction, so a set containing it fails the all-members test.

Step 3: Separate greenhouse action from other pollution Option C contains Cl_2, another homonuclear diatomic molecule, and sulfur dioxide. Chlorine is not a greenhouse gas in this classification. Sulfur dioxide is primarily associated with acid rain and sulfate aerosol formation, whose cooling effects differ from the greenhouse mechanism. The appearance of ozone or nitrogen dioxide among other options does not rescue a set containing a disqualifying member. Each gas must be checked; a list with one familiar greenhouse gas is insufficient.

Step 4: Conclusion CO_2, CH_4, N_2O, and O_3 form the greenhouse-gas set. Hence, the correct option is (D).

Quick Tip to Solve: For a proposed greenhouse-gas set, check every member; nitrogen gas and chlorine gas are useful immediate eliminators.

---

85. The correct sequence of decreasing number of π-bonds in the structures of H_2SO_3, H_2SO_4 and $H_2S_2O_7$ is : [4 marks]

(A) $H_2SO_3 > H_2SO_4 > H_2S_2O_7$

(B) $H_2SO_4 > H_2S_2O_7 > H_2SO_3$

(C) $H_2S_2O_7 > H_2SO_4 > H_2SO_3$

(D) $H_2S_2O_7 > H_2SO_3 > H_2SO_4$

Correct Answer: (C) $H_2S_2O_7 > H_2SO_4 > H_2SO_3$

Solution:

Concept:

- A double bond contains one sigma bond and one π-bond. Count the terminal $S{=}O$ bonds in the conventional structures of the sulfur oxoacids.
- An $S{-}OH$ bond and each bond to a bridging oxygen are single bonds; neither contributes a π-bond. The question compares the total per molecule, not the count per sulfur atom.

Step 1: Count the bonds in sulfurous and sulfuric acids Sulfurous acid has the conventional structure $HO{-}S(=O){-}OH$. Its single terminal $S{=}O$ double bond contributes one pi bond. Sulfuric acid is represented as $HO{-}S(=O)_2{-}OH$, with two terminal double bonds and hence two pi bonds. The two hydroxyl groups in each formula contribute only single sulfur-oxygen bonds; the hydrogen atoms do not add any pi bonding.

Step 2: Count the bonds in disulfuric acid and compare Disulfuric acid has two sulfur centres joined through an oxygen: $HO{-}S(=O)_2{-}O{-}S(=O)_2{-}OH$. Each sulfur has two terminal oxo groups. The total is therefore $2 + 2 = 4$ pi bonds. The bridging $S{-}O{-}S$ unit has two single bonds and adds no pi

bond. Thus the counts in the order listed in the question are 1, 2 and 4. Option A reverses the actual trend; B incorrectly puts sulfuric acid above disulfuric acid; D incorrectly puts sulfurous acid above sulfuric acid.

Step 3: Conclusion The decreasing order is $H_2S_2O_7 > H_2SO_4 > H_2SO_3$, corresponding to $4 > 2 > 1$ pi bonds. Hence, the correct option is (C).

Quick Tip to Solve: Write the terminal oxo groups explicitly before counting: bridging oxygen and hydroxyl oxygen do not supply pi bonds.

86. At 300 K, the density of a certain gaseous molecule at 2 bar is double to that of dinitrogen (N_2) at 4 bar. The molar mass of gaseous molecule is : [4 marks]

(A) 28 g mol^{-1}

(B) 56 g mol^{-1}

(C) 112 g mol^{-1}

(D) 224 g mol^{-1}

Correct Answer: (C) 112 g mol^{-1}

Solution:

Concept:

- For an ideal gas, $pV = nRT = (m/M)RT$, where M is molar mass and m is gas mass. With density $d = m/V$, this gives $d = pM/(RT)$.
- At the same temperature, density is proportional to the product of pressure and molar mass. Pressure must be included when comparing gases measured under different conditions.

Step 1: Form the density ratio Let the unknown gas have molar mass M, and denote the density of nitrogen by d_N. Both measurements are at 300 K, so R and T cancel in their density ratio. Nitrogen is diatomic, with molar mass $M_N = 28\,\text{g mol}^{-1}$. Using the two stated pressures gives

$$\frac{d}{d_N} = \frac{(2\,\text{bar})M}{(4\,\text{bar})(28\,\text{g mol}^{-1})}.$$

Because the pressure units match, no pressure conversion is needed.

Step 2: Use the given factor of two The unknown gas density is twice that of nitrogen, so substitute $d/d_N = 2$:

$$2 = \frac{2M}{4 \times 28}, \qquad M = 112\,\text{g mol}^{-1}.$$

As a physical check, the unknown gas is measured at half the pressure. To still have double the density, its molar mass must be four times that of nitrogen. A molar mass of 28 would give half the nitrogen density, 56 would give equal density, and 224 would give four times the density. Only 112 gives the specified factor of two.

Step 3: Conclusion The molar mass of the gaseous molecule is $112\,\text{g mol}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: At equal temperatures use $M_1/M_2 = (d_1/d_2)(p_2/p_1)$; a density ratio alone gives the wrong answer when pressures differ.

87. XeF_6 on partial hydrolysis with water produces a compound 'X'. The same compound 'X' is formed when XeF_6 reacts with silica. The compound 'X' is : [4 marks]

(A) XeF_2

(B) XeF_4

(C) $XeOF_4$

(D) XeO_3

Correct Answer: (C) $XeOF_4$

Solution:

Concept:

- Controlled hydrolysis of XeF_6 replaces pairs of fluorine atoms by oxygen, initially forming xenon oxytetrafluoride, $XeOF_4$.
- Silica supplies oxygen to XeF_6, while silicon forms the stable fluoride SiF_4. Partial hydrolysis must be distinguished from complete hydrolysis.

Step 1: Write the first hydrolysis stage With a limited amount of water, the first hydrolysis reaction is

$$XeF_6 + H_2O \longrightarrow XeOF_4 + 2\,HF.$$

One oxygen atom replaces two fluorine atoms on xenon. The equation conserves one xenon, six fluorines, two hydrogens and one oxygen. Xenon remains in oxidation state +6: in the product, oxygen contributes -2 and four fluorines contribute -4. Thus this is an oxygen-for-fluorine replacement rather than a reduction to a lower xenon fluoride.

Step 2: Check the reaction with silica and eliminate alternatives The reaction with silica yields the same oxyfluoride:

$$2\,XeF_6 + SiO_2 \longrightarrow 2\,XeOF_4 + SiF_4.$$

The two oxygen atoms from silica supply one oxygen to each xenon, and four fluorines are transferred to silicon. Both reactions therefore identify X as the same compound. The simple fluorides XeF_2 and XeF_4 contain no oxygen and are not these hydrolysis products. XeO_3 is the product of complete hydrolysis, $XeF_6 + 3\,H_2O \longrightarrow XeO_3 + 6\,HF$, so choosing it would ignore the word partial and the silica reaction.

Step 3: Conclusion The common product X is xenon oxytetrafluoride, $XeOF_4$. Hence, the correct option is (C).

Quick Tip to Solve: Remember the first hydrolysis replacement as two F atoms out and one O atom in; complete hydrolysis replaces all six F atoms.

88. The number of P−OH bonds and the oxidation state of phosphorus atom in pyrophosphoric acid ($H_4P_2O_7$) respectively are : [4 marks]

(A) four and four

(B) five and four

(C) five and five

(D) four and five

Correct Answer: (D) four and five

Solution:

Concept:

- Pyrophosphoric acid, $H_4P_2O_7$, forms by condensation of two orthophosphoric acid molecules with loss of one water molecule.
- Count $P-OH$ groups from the structure. For oxidation states in this neutral oxoacid, hydrogen is $+1$ and oxygen is -2, and the total must be zero.

Step 1: Count the hydroxyl groups The condensation can be written as

$$2\,H_3PO_4 \longrightarrow H_4P_2O_7 + H_2O.$$

The product contains a bridging $P-O-P$ oxygen. Its conventional condensed structure is $(HO)_2P(=O)-O-P(=O)(OH)_2$. Each phosphorus is bonded to two hydroxyl oxygens, one terminal oxo oxygen and the bridging oxygen. Therefore the whole molecule has $2+2=4$ phosphorus-hydroxyl bonds. The bridge is not an OH group, because its oxygen is bonded to two phosphorus atoms and carries no hydrogen.

Step 2: Calculate the phosphorus oxidation state The two phosphorus atoms have equivalent environments, so let each have oxidation state x. Charge balance for the neutral molecule gives

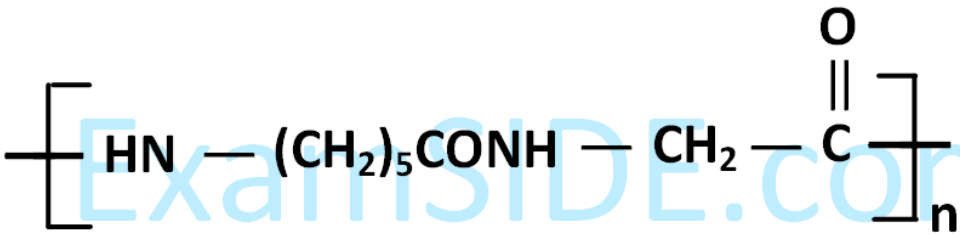
$$4(+1) + 2x + 7(-2) = 0.$$

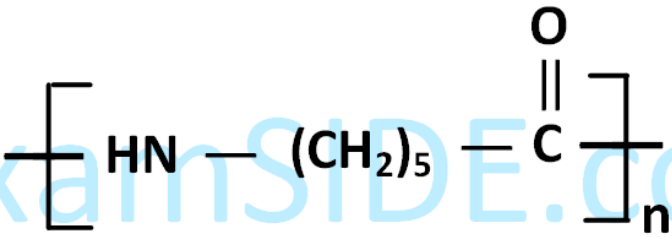
Hence $2x = 10$ and $x = +5$. The number 10 is the sum of the oxidation states of both phosphorus atoms, not the oxidation state of one atom. The required ordered pair is consequently four and five. Options B and C incorrectly count five OH bonds, while A gives an oxidation state that would fail the neutral charge balance.

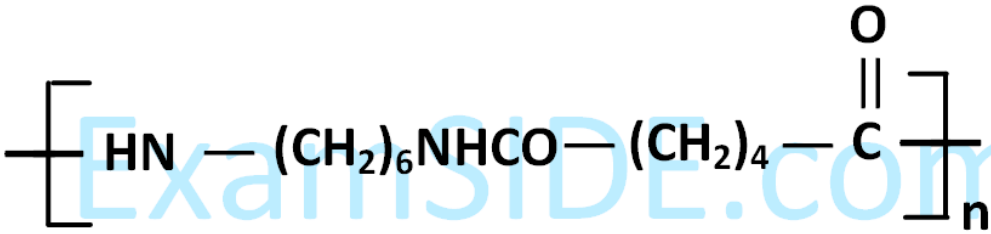
Step 3: Conclusion Pyrophosphoric acid has four $P-OH$ bonds, and each phosphorus atom is in oxidation state $+5$. Hence, the correct option is (D).

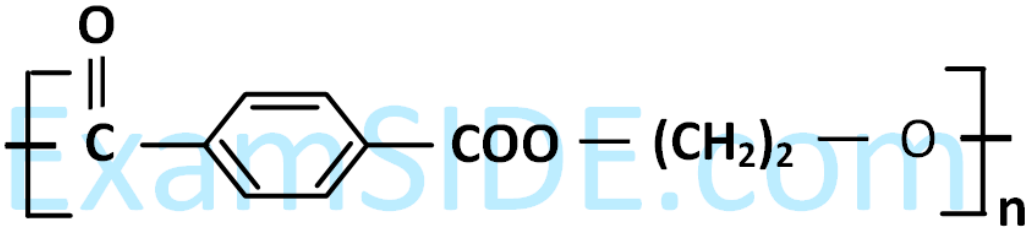
Quick Tip to Solve: In condensed phosphoric acids count hydroxyl groups after forming the P-O-P bridge; use charge balance separately for the oxidation state.

89. **Which of the following is a biodegradable polymer ?** [4 marks]

(A) 

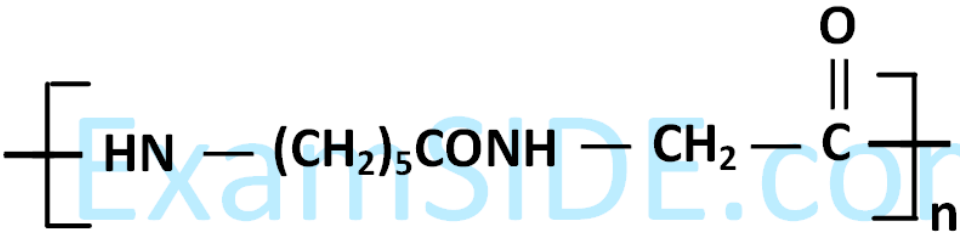
(B) 

(C) 

(D) 

ExamSIDE.com

Correct Answer: (A)



ExamSIDE.com

Solution:

Concept:

- Biodegradable polymers can be broken down by biological processes. Nylon-2-nylon-6 is a biodegradable copolyamide derived from glycine and 6-aminohexanoic acid.
- Identify a polymer from its complete repeat unit rather than assuming that every polymer containing an amide or ester linkage has the same biodegradability.

Step 1: Identify the copolyamide in option A The repeat unit in A is

$$[-\mathrm{NH}-(\mathrm{CH_2})_5-\mathrm{CO}-\mathrm{NH}-\mathrm{CH_2}-\mathrm{CO}-]_n$$

. It contains a 6-aminohexanoic-acid residue, $-\mathrm{NH}-(\mathrm{CH_2})_5-\mathrm{CO}-$, and a glycine residue, $-\mathrm{NH}-\mathrm{CH_2}-\mathrm{CO}-$. These correspond to the nylon-6 and nylon-2 components, respectively. Thus A is nylon-2-nylon-6, the biodegradable copolyamide containing the amino-acid-derived segment. The two different residues in a single repeating sequence distinguish this copolymer from either ordinary nylon alone.

Step 2: Identify the remaining repeat units Option B shows

$$[-\mathrm{NH}-(\mathrm{CH_2})_5-\mathrm{CO}-]_n$$

, which is nylon-6. Option C shows the hexamethylenediamine and adipic-acid residues

$$[-\mathrm{NH}-(\mathrm{CH_2})_6-\mathrm{NH}-\mathrm{CO}-(\mathrm{CH_2})_4-\mathrm{CO}-]_n$$

, identifying nylon-6,6. Option D contains a para-disubstituted benzene ring with ethylene glycol ester links; it is polyethylene terephthalate, or Terylene. These are conventional durable synthetic polymers, rather than the biodegradable copolyamide requested here. The presence of an amide group alone cannot select B or C, and the ester linkage in D does not make it the biodegradable example in this set.

Step 3: Conclusion The biodegradable polymer is nylon-2-nylon-6, represented by the repeat unit in option A. Hence, the correct option is (A).

Quick Tip to Solve: Look for the glycine segment $-\mathrm{NH}-\mathrm{CH_2}-\mathrm{CO}-$ paired with the nylon-6 segment to recognise biodegradable nylon-2-nylon-6.

90. Adsorption of a gas on a surface follows Freundlich adsorption isotherm. Plot of log $\frac{x}{m}$ versus log p gives a straight line with slope equal to 0.5, then:

($\frac{x}{m}$ is the mass of the gas adsorbed per gram of adsorbent) [4 marks]

(A) Adsorption is independent of pressure.

(B) Adsorption is proportional to the pressure.

(C) Adsorption is proportional to the square root of pressure.

(D) Adsorption is proportional to the square of pressure.

Correct Answer: (C) Adsorption is proportional to the square root of pressure.

Solution:

Concept:

- The Freundlich adsorption isotherm is $x/m = kp^{1/n}$, where x/m is the mass adsorbed per unit mass of adsorbent, p is pressure, and k, n are constants at a fixed temperature.
- Taking logarithms converts a power law to a straight line: $\log(x/m) = \log k + (1/n)\log p$. The slope on a log-log plot is the pressure exponent.

Step 1: Read the exponent from the straight-line slope Compare the logarithmic isotherm with $y = c + sx$, using vertical coordinate $y = \log(x/m)$ and horizontal coordinate $x = \log p$. The intercept is $\log k$, while the slope is $1/n$. Since the stated slope is 0.5,

$$\frac{1}{n} = 0.5 = \frac{1}{2}, \qquad n = 2.$$

The value of the intercept is not needed to determine the proportionality with pressure; it only fixes the multiplicative constant k.

Step 2: Return to the original adsorption variables Substitute the exponent into the original, unlogged equation:

$$\frac{x}{m} = kp^{1/2} = k\sqrt{p}.$$

Hence adsorption per gram of adsorbent is proportional to the square root of pressure. At a fixed mass of adsorbent this is also the pressure dependence of the adsorbed mass. As a check, multiplying pressure by four doubles adsorption, rather than multiplying it by four or sixteen. Independence of pressure would require slope zero, direct proportionality would require slope one, and proportionality to pressure squared would require slope two. None agrees with the measured slope 0.5.

Step 3: Conclusion Adsorption is proportional to $\sqrt{p}$, the square root of pressure. Hence, the correct option is (C).

Quick Tip to Solve: On a log-log plot the slope is the exponent itself: write $x/m \propto p^{\text{slope}}$ before interpreting the options.