JEE Main 2017 Paper 1 (B.E./B.Tech) Apr 8 Single slot Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2017, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 8 April 2017, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. Consider an ellipse, whose center is at the origin and its major axis is along the x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area (in sq. units) of the quadrilatateral inscribed in the ellipse, with the vertices as the vertices of the ellipse, is : [4 marks]

(A) 8

(B) 32

(C) 80

(D) 40

Correct Answer: (D) 40

Solution:

Concept:

- For an ellipse centred at the origin with its major axis along the x-axis, the equation is $x^2/a^2 + y^2/b^2 = 1$, where $a > b > 0$. Its foci are $(\pm c, 0)$, with $c^2 = a^2 - b^2$.
- The eccentricity is $e = c/a$. The quadrilateral joining the four axis vertices has perpendicular diagonals of lengths $2a$ and $2b$; its area is half the product of these diagonals.

Step 1: Find the semi-major axis The distance between the two foci is twice the focal distance, not the major-axis length. Thus

$$2c = 6 \quad \Rightarrow \quad c = 3.$$

Using the specified eccentricity gives

$$\frac{3}{5} = \frac{c}{a} = \frac{3}{a} \quad \Rightarrow \quad a = 5.$$

Hence the major-axis vertices are $(5, 0)$ and $(-5, 0)$.

Step 2: Determine the minor axis and the enclosed area Use the relation between the three lengths to obtain

$$b^2 = a^2 - c^2 = 25 - 9 = 16, \qquad b = 4.$$

The other vertices are therefore $(0, 4)$ and $(0, -4)$. Joining these four points gives a rhombus whose diagonals are the two axes of the ellipse. Consequently,

$$\text{Area} = \frac{1}{2}(2a)(2b) = 2ab = 2(5)(4) = 40 \text{ sq. units}.$$

The value 80 omits the factor one-half in the diagonal formula. Neither 8 nor 32 equals the area determined by the focal distance and eccentricity; they cannot satisfy this geometric calculation.

Step 3: Conclusion The quadrilateral has area $\boxed{40 \text{ sq. units}}$. Hence, the correct option is (D).

Quick Tip to Solve: Find c from half the focal separation, then obtain $a = c/e$ and $b = \sqrt{a^2 - c^2}$. The vertex quadrilateral has area $2ab$, whereas the ellipse itself has area πab.

2. Let A be any 3 × 3 invertible matrix. Then which one of the following is not always true ? [4 marks]

(A) adj (A) = $|A|.A^{-1}$

(B) adj (adj(A)) = $|A|.A$

(C) adj (adj(A)) = $|A|^2.(\text{adj}(A))^{-1}$

(D) adj (adj(A)) = $|A|.(\text{adj}(A))^{-1}$

Correct Answer: (D) adj (adj(A)) = $|A|.(\text{adj}(A))^{-1}$

Solution:

Concept:

- For an invertible square matrix A, its adjugate satisfies $\text{adj}(A) = (\det A)A^{-1}$. Write $d = \det A$; invertibility ensures $d \neq 0$.
- For a 3×3 matrix, multiplying every entry by a scalar k multiplies its determinant by k^3. Also, $\det(A^{-1}) = 1/d$. These identities determine the adjugate of the adjugate.

Step 1: Derive the repeated-adjugate formula Put $B = \text{adj}(A) = dA^{-1}$. Its determinant and inverse are

$$\det B = d^3 \det(A^{-1}) = d^2, \qquad B^{-1} = \frac{1}{d}A.$$

Applying the defining identity to B gives

$$\text{adj}(\text{adj}\, A) = \text{adj}\, B = (\det B)B^{-1} = d^2\frac{A}{d} = dA.$$

Thus option (B) holds for every invertible matrix of the specified order. The defining formula itself proves option (A).

Step 2: Test the remaining two expressions Option (C) has right-hand side

$$d^2(\text{adj}\, A)^{-1} = d^2\frac{A}{d} = dA,$$

so it is also always true. In option (D), however, the right-hand side becomes

$$d(\text{adj}\, A)^{-1} = d\frac{A}{d} = A,$$

while the left-hand side is dA. These coincide only when $d = 1$, since an invertible matrix is not the zero matrix. For a counterexample choose $A = 2I_3$. Then $d = 8$, $\text{adj}\, A = 4I_3$, and the two sides of (D) are $16I_3$ and $2I_3$, respectively.

Step 3: Conclusion Option (D) is not always true: its right-hand side equals A, but its left-hand side equals $(\det A)A$. Hence, the correct option is (D).

Quick Tip to Solve: Introduce $d = \det A$ and replace every adjugate by determinant times inverse. Remember that $\det(\text{adj}\, A) = d^2$ for order three; the determinant power is essential.

3. Let p(x) be a quadratic polynomial such that p(0)=1. If p(x) leaves remainder 4 when divided by x− 1 and it leaves remainder 6 when divided by x + 1; then : [4 marks]

(A) p(2) = 11

(B) p(2) = 19

(C) p(− 2) = 19

(D) p(− 2) = 11

Correct Answer: (C) p(− 2) = 19

Solution:

Concept:

- The remainder theorem states that the remainder on dividing $p(x)$ by $x - r$ equals $p(r)$. Therefore the two given divisions specify $p(1) = 4$ and $p(-1) = 6$.
- A general quadratic polynomial has the form $p(x) = ax^2 + bx + c$, with $a \neq 0$. Three independent values determine its three coefficients; $p(0)$ immediately supplies its constant term.

Step 1: Translate the remainders into equations Write $p(x) = ax^2 + bx + c$. Since $p(0) = 1$, the constant term is $c = 1$. Substituting $x = 1$ and $x = -1$, respectively, gives

$$a + b + 1 = 4, \qquad a - b + 1 = 6.$$

Hence

$$a + b = 3, \qquad a - b = 5.$$

Adding these equations eliminates the linear coefficient and yields $2a = 8$, so $a = 4$. Subtracting gives $2b = -2$, so $b = -1$. Thus the required quadratic is

$$p(x) = 4x^2 - x + 1.$$

Its leading coefficient is nonzero, as required.

Step 2: Evaluate both arguments appearing in the choices The options refer to the values at 2 and at -2, so calculate both rather than inferring one from the other:

$$p(2) = 4(2)^2 - 2 + 1 = 16 - 2 + 1 = 15,$$

$$p(-2) = 4(-2)^2 - (-2) + 1 = 16 + 2 + 1 = 19.$$

The linear term changes sign when the argument changes sign; the squared term does not. Therefore the claims $p(2) = 11$ and $p(2) = 19$ are false, excluding (A) and (B). The claim $p(-2) = 11$ is also false, excluding (D). As a check on the coefficients, the polynomial gives $p(1) = 4$ and $p(-1) = 6$, exactly the stated remainders.

Step 3: Conclusion We obtain $p(x) = 4x^2 - x + 1$ and $\boxed{p(-2) = 19}$. Hence, the correct option is (C).

Quick Tip to Solve: When values at 1 and -1 are given, add their equations to isolate the even-power terms and subtract to isolate the odd-power term. Keep the sign of the argument inside parentheses.

4. Let z∈C, the set of complex numbers. Then the equation, $2|z + 3i| - |z - i| = 0$ represents :

[4 marks]

(A) a circle with radius $\frac{8}{3}$.
(B) a circle with diameter $\frac{10}{3}$.
(C) an ellipse with length of major axis $\frac{16}{3}$.
(D) an ellipse with length of minor axis $\frac{16}{9}$.
Correct Answer: (A) a circle with radius $\frac{8}{3}$.

Solution:

Concept:

- For $z = x + iy$, the modulus $|z - z_0|$ is the distance from (x, y) to the point representing z_0. A fixed ratio of distances to two distinct points, unequal to one, describes an Apollonius circle.
- Both moduli are nonnegative. Thus squaring $2|z + 3i| = |z - i|$ is reversible and introduces no extra points. Complete squares in the resulting Cartesian equation to identify its centre and radius.

Step 1: Convert the modulus relation into Cartesian form Let $z = x + iy$, where x, y are real. The distances in the equation satisfy

$$|z + 3i| = \sqrt{x^2 + (y + 3)^2}, \qquad |z - i| = \sqrt{x^2 + (y - 1)^2}.$$

Move the second modulus to the right and square:

$$4\{x^2 + (y + 3)^2\} = x^2 + (y - 1)^2.$$

Expanding carefully, including the signs of the linear terms, gives

$$4x^2 + 4y^2 + 24y + 36 = x^2 + y^2 - 2y + 1,$$

or

$$3x^2 + 3y^2 + 26y + 35 = 0.$$

Step 2: Complete the square and compare the choices Divide by 3 and complete the square in y:

$$x^2 + y^2 + \frac{26}{3}y + \frac{35}{3} = 0,$$

$$x^2 + \left(y + \frac{13}{3}\right)^2 = \frac{169}{9} - \frac{105}{9} = \frac{64}{9}.$$

Therefore the centre is $(0, -13/3)$ and the radius is $8/3$. The diameter is $16/3$, so the diameter asserted in (B) is incorrect. The coefficients of x^2 and y^2 are equal, giving a circle rather than an ellipse with the unequal axis lengths described in (C) or (D).

Step 3: Conclusion The locus is the circle $x^2 + (y + 13/3)^2 = 64/9$, of radius $\boxed{8/3}$. Hence, the correct option is (A).

Quick Tip to Solve: A difference of weighted distances need not give a hyperbola. First rearrange it: a fixed distance ratio different from one gives a circle. Square the positive distances and complete the square.

5. If

$$S = \left\{ x \in [0, 2\pi] : \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0 \right\},$$

then $\sum_{x \in S} \tan\left(\frac{\pi}{3} + x\right)$ **is equal to :** [4 marks]

(A) $4 + 2\sqrt{3}$

(B) $-2 + \sqrt{3}$

(C) $-2 - \sqrt{3}$

(D) $-4 - 2\sqrt{3}$

Correct Answer: (D) $-4 - 2\sqrt{3}$

Solution:

Concept:

- A determinant can be expanded along a row containing a zero. Here writing $c = \cos x$ and $s = \sin x$ simplifies the expression to a difference of cubes.
- The identity $c^3 - s^3 = (c - s)(c^2 + cs + s^2)$, together with $c^2 + s^2 = 1$, identifies the real angles. Tangent has period π, so angles differing by π give equal summands.

Step 1: Expand and solve the determinant equation Expand along the first row, retaining the negative sign already present in its third entry:

$$\begin{vmatrix} 0 & c & -s \\ s & 0 & c \\ c & s & 0 \end{vmatrix} = -c(0 - c^2) - s(s^2 - 0) = c^3 - s^3.$$

Hence the condition becomes

$$(c - s)(1 + cs) = 0.$$

Since $-1/2 \le cs \le 1/2$, the factor $1 + cs$ is at least $1/2$ and never vanishes. We must therefore have $\cos x = \sin x$. On the specified closed interval $[0, 2\pi]$, its solutions are exactly

$$S = \{\pi/4,\ 5\pi/4\}.$$

Neither endpoint satisfies the equation.

Step 2: Evaluate the two tangent terms The arguments for the two allowed angles are $7\pi/12$ and $19\pi/12$, whose difference is π. Thus

$$\sum_{x \in S} \tan(\pi/3 + x) = 2\tan(7\pi/12).$$

Apply the tangent addition formula to $\pi/3 + \pi/4$:

$$\tan(7\pi/12) = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} = -(2 + \sqrt{3}).$$

The denominator is nonzero. Therefore the sum is $-4-2\sqrt{3}$. Option (A) has the opposite sign, while (B) and (C) have the wrong constant and do not include both identical contributions.

Step 3: Conclusion The required sum is $\boxed{-4-2\sqrt{3}}$. Both $\pi/4$ and $5\pi/4$ lie in the printed interval $[0, 2\pi]$; each contributes $-2-\sqrt{3}$. Thus the stated sum gives (D), although the supplied key selects the third choice, which counts only one contribution. Hence, the correct option is (D).

Quick Tip to Solve: After factoring a trigonometric determinant, check whether each factor can actually vanish. Then exploit tangent periodicity before doing the exact-angle arithmetic; here the two summands are identical.

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6. The number of real values of λ for which the system of linear equations

$$2x + 4y - \lambda z = 0$$

$$4x + \lambda y + 2z = 0$$

$$\lambda x + 2y + 2z = 0$$

has infinitely many solutions, is : [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Concept:

- A homogeneous system $M\mathbf{x} = \mathbf{0}$ always has the zero solution. It has infinitely many real solutions precisely when its square coefficient matrix is singular, that is, when $\det M = 0$.
- For a singular real matrix, a nonzero vector lies in its null space, and all scalar multiples of that vector are solutions. To count admissible parameters, count the distinct real roots of the determinant polynomial.

Step 1: Form the coefficient determinant Keep the signs and coefficient order exactly as given. The matrix is

$$M = \begin{pmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & 2 \\ \lambda & 2 & 2 \end{pmatrix}.$$

Expanding across its first row gives

$$\det M = 2(2\lambda - 4) - 4(8 - 2\lambda) - \lambda(8 - \lambda^2).$$

Simplify the separate terms before combining:

$$\det M = 4\lambda - 8 - 32 + 8\lambda - 8\lambda + \lambda^3 = \lambda^3 + 4\lambda - 40.$$

The required parameter values therefore solve $f(\lambda) = \lambda^3 + 4\lambda - 40 = 0$.

Step 2: Count its real zeros Differentiate the real polynomial:

$$f\prime(\lambda) = 3\lambda^2 + 4 > 0 \qquad \text{for every real } \lambda.$$

It is strictly increasing, so it has at most one real zero. It has at least one because it is continuous, with

$$f(3) = 27 + 12 - 40 = -1 < 0, \qquad f(4) = 64 + 16 - 40 = 40 > 0.$$

The intermediate value theorem puts a zero between 3 and 4, and strict monotonicity makes it unique. At this parameter the matrix has a nontrivial null space, hence infinitely many solutions. Thus zero admissible parameters is impossible, and counts of two or three are excluded by monotonicity.

Step 3: Conclusion There is exactly $\boxed{1}$ real value of λ producing infinitely many solutions. Hence, the correct option is (B).

Quick Tip to Solve: For a homogeneous square system, use the determinant directly; no separate augmented-determinant conditions are needed. A cubic need not have three real roots: a positive derivative proves uniqueness here.

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7. If all the words, with or without meaning, are written using the letters of the word QUEEN and are arranged as in English dictionary, then the position of the word QUEEN is : [4 marks]

(A) 44^{th}

(B) 45^{th}

(C) 46^{th}

(D) 47^{th}

Correct Answer: (C) 46^{th}

Solution:

Concept:

- Dictionary order compares the first unequal letters. The alphabetical order of the available distinct letters is $E < N < Q < U$; the letter E appears twice in QUEEN.
- For a multiset of n letters with a letter repeated r times, the number of distinct permutations is $n!/r!$. Count all words preceding the target, then add one for the target itself.

Step 1: Count words whose first letter is smaller than Q The target word begins with Q. Every arrangement beginning with E or N precedes it. If the first letter is E, the remaining letters are E, N, Q and U, all distinct, so there are $4! = 24$ words. If the first letter is N, the remaining letters are E, E, Q and U, giving $4!/2! = 12$ words. Thus

$$24 + 12 = 36$$

words precede every word that starts with Q. The repeat factor differs between these cases: using E leaves only one E, but using N leaves both E letters.

Step 2: Fix Q and count smaller second letters After fixing Q, the target has second letter U. The

remaining letters are E, E, N and U. A smaller second letter may be E or N. Prefix QE leaves E, N and U, giving $3! = 6$ arrangements. Prefix QN leaves E, E and U, giving $3!/2! = 3$ arrangements. Fixing the prefix QU leaves E, E and N; EEN is their first dictionary arrangement, so no additional words with prefix QU precede QUEEN. The number before the target is therefore

$$36 + 6 + 3 = 45.$$

The rank includes the target itself:

$$\text{rank} = 45 + 1 = 46.$$

The value 45 counts only preceding words, explaining why (B) is tempting. Neither 44 nor 47 follows from the exhaustive prefix counts, excluding (A) and (D).

Step 3: Conclusion The word QUEEN occupies the $\boxed{46\text{th}}$ position. Hence, the correct option is (C).

Quick Tip to Solve: At each fixed prefix, recalculate which letters are still repeated before dividing factorials. Finally add one: the number of preceding words is one less than the target word's rank.

8. Three persons P, Q and R independently try to hit a target. I the probabilities of their hitting the target are $\frac{3}{4}, \frac{1}{2}$ and $\frac{5}{8}$ respectively, then the probability that the target is hit by P or Q but not by R is : [4 marks]

(A) $\frac{21}{64}$
(B) $\frac{9}{64}$
(C) $\frac{15}{64}$
(D) $\frac{39}{64}$

Correct Answer: (A) $\frac{21}{64}$

Solution:

Concept:

- For events P and Q, the word "or" denotes the inclusive union: either event, or both, may occur. Thus $\Pr(P \cup Q) = 1 - \Pr(P^c \cap Q^c)$.
- Independent attempts allow probabilities to be multiplied. Failure of R is independent of the event involving the other two persons; $\Pr(R^c) = 1 - \Pr(R)$.

Step 1: Find the probability that at least one of P and Q hits The complement of the required union is that both P and Q miss. Their miss probabilities are $1 - 3/4 = 1/4$ and $1 - 1/2 = 1/2$. Independence therefore gives

$$\Pr(P^c \cap Q^c) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}, \qquad \Pr(P \cup Q) = 1 - \frac{1}{8} = \frac{7}{8}.$$

This includes the outcome in which both P and Q hit; excluding that outcome would incorrectly interpret "or" as "exactly one".

Step 2: Combine this event with R missing The third person misses with probability $1 - 5/8 = 3/8$.

Since this attempt is independent of both earlier attempts,

$$\Pr((P \cup Q) \cap R^c) = \frac{7}{8} \cdot \frac{3}{8} = \frac{21}{64}.$$

As a direct check, the disjoint cases P alone, Q alone and both hitting contribute $9/64$, $3/64$ and $9/64$, respectively. Their sum is $21/64$; the other listed fractions do not include precisely these three cases.

Step 3: Conclusion The probability that P or Q hits while R misses is $\boxed{21/64}$. Hence, the correct option is (A).

Quick Tip to Solve: For "P or Q", first calculate one minus the probability that both miss; then multiply by the independent probability that R misses.

9. The integral $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{8\cos 2x}{(\tan x+\cot x)^3}\,dx$ equals : [4 marks]

(A) $\frac{15}{128}$
(B) $\frac{15}{64}$
(C) $\frac{13}{32}$
(D) $\frac{13}{256}$

Correct Answer: (A) $\frac{15}{128}$

Solution:

Concept:

- The identity $\tan x + \cot x = 1/(\sin x \cos x) = 2/\sin 2x$ simplifies a reciprocal power of this sum.
- For a definite integral, substituting $u = \sin 2x$ gives $du = 2\cos 2x\,dx$, and the limits must be transformed at the same time.

Step 1: Simplify the integrand before integrating On $[\pi/12, \pi/4]$, sine and cosine are nonzero, so the reciprocal identity is valid throughout. The cube in the denominator becomes

$$(\tan x + \cot x)^3 = \frac{8}{\sin^3 2x}.$$

Cancelling the factor 8 gives

$$I = \int_{\pi/12}^{\pi/4} \sin^3 2x \cos 2x\,dx.$$

Keeping the power three is essential; expanding separate powers of tangent and cotangent would make the calculation longer without helping.

Step 2: Substitute and evaluate the endpoint values Put $u = \sin 2x$. At the lower limit $u = \sin(\pi/6) = 1/2$, while at the upper limit $u = \sin(\pi/2) = 1$. Therefore

$$I = \frac{1}{2}\int_{1/2}^{1} u^3\,du = \left[\frac{u^4}{8}\right]_{1/2}^{1} = \frac{1}{8}\left(1 - \frac{1}{16}\right) = \frac{15}{128}.$$

The factor $1/2$ from the differential rules out $15/64$, which is twice the answer. The remaining two

fractions also disagree with the exact endpoint difference; no approximate trigonometric values are required.

Step 3: Conclusion The integral equals $\boxed{15/128}$. Hence, the correct option is (A).

Quick Tip to Solve: Whenever tangent plus cotangent is in the denominator, replace it by twice cosecant of the double angle before choosing a substitution.

10. If $(27)^{999}$ is divided by 7, then the remainder is : [4 marks]

(A) 1

(B) 2

(C) 3

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- Numbers differing by a multiple of 7 have the same remainder on division by 7. In particular, $27 = 28 - 1$ is congruent to -1 modulo 7.
- In a binomial expansion of $(7k - 1)^n$, every term containing $7k$ is divisible by 7; only the constant term can determine the remainder.

Step 1: Express the base relative to a multiple of seven Write

$$27^{999} = (28 - 1)^{999}.$$

In its binomial expansion the terms with a positive power of 28 are all integer multiples of 7, because $28 = 4 \cdot 7$. The one remaining term is $(-1)^{999}$. This reduces a very large power to the parity of its exponent; there is no need to compute the power itself or its decimal digits.

Step 2: Use the odd exponent and choose a valid remainder Since 999 is odd,

$$27^{999} \equiv (-1)^{999} = -1 \pmod 7.$$

A remainder on division by a positive integer must be between 0 and 6. Thus replace the negative representative by $-1+7 = 6$. Equivalently, $27^{999}+1$ is a multiple of 7, so $27^{999} = 7m-1 = 7(m-1)+6$ for some integer m. The proposed remainders 1, 2 and 3 have differences 2, 3 and 4 from -1; none is congruent to -1 modulo 7. The remainder 1 would occur for an even exponent here, which is why the parity check matters.

Step 3: Conclusion The required nonnegative remainder is $\boxed{6}$. Hence, the correct option is (D).

Quick Tip to Solve: Reduce the base modulo the divisor first. If it becomes minus one, the parity of the exponent immediately determines the remainder.

11. The area (in sq. units) of the parallelogram whose diagonals are along the vectors $8\hat{i} - 6\hat{j}$ and $3\hat{i} + 4\hat{j} - 12\hat{k}$, is : [4 marks]

(A) 26

(B) 65

(C) 20

(D) 52

Correct Answer: (B) 65

Solution:

Concept:

- If adjacent side vectors are $\mathbf{a}$ and $\mathbf{b}$, the parallelogram area is $|\mathbf{a} \times \mathbf{b}|$. Its diagonals are $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$.
- The diagonal formula is $A = \frac{1}{2}|\mathbf{d}_1 \times \mathbf{d}_2|$, since the cross product of those two diagonal vectors has twice the magnitude of the side-vector cross product.

Step 1: Calculate the cross product of the given diagonal vectors Take $\mathbf{d}_1 = (8, -6, 0)$ and $\mathbf{d}_2 = (3, 4, -12)$. Their cross product is

$$\begin{aligned}\mathbf{d}_1 \times \mathbf{d}_2 &= \big((-6)(-12) - 0(4),\ 0(3) - 8(-12),\ 8(4) - (-6)(3)\big) \\ &= (72, 96, 50).\end{aligned}$$

These are the diagonal vectors, rather than adjacent side vectors, so their cross-product magnitude cannot be used directly as the area.

Step 2: Find its magnitude and include the diagonal factor The magnitude is

$$|\mathbf{d}_1 \times \mathbf{d}_2| = \sqrt{72^2 + 96^2 + 50^2} = \sqrt{5184 + 9216 + 2500} = \sqrt{16900} = 130.$$

Consequently

$$A = \frac{1}{2}(130) = 65 \text{ square units.}$$

A useful independent check is that $\mathbf{d}_1 \cdot \mathbf{d}_2 = 24 - 24 = 0$: the diagonals are perpendicular, and their lengths are 10 and 13. Thus the elementary formula $A = d_1 d_2/2$ also gives 65. The choices 26, 20 and 52 fail both exact area calculations.

Step 3: Conclusion The area of the parallelogram is $\boxed{65 \text{ square units}}$. Hence, the correct option is (B).

Quick Tip to Solve: Diagonal vectors give half the magnitude of their cross product. Check perpendicularity first: it may reduce the computation to half the product of their lengths.

12. The curve satisfying the differential equation, ydx $-$(x + 3y^2)dy = 0 and passing through the point (1, 1), also passes through the point : [4 marks]

(A) $\left(\frac{1}{4}, -\frac{1}{2}\right)$
(B) $\left(-\frac{1}{3}, \frac{1}{3}\right)$
(C) $\left(\frac{1}{3}, -\frac{1}{3}\right)$
(D) $\left(\frac{1}{4}, \frac{1}{2}\right)$

Correct Answer: (B) $\left(-\frac{1}{3}, \frac{1}{3}\right)$

Solution:

Concept:

- When an equation contains $y\,dx - (x + 3y^2)\,dy = 0$, regard x as a function of y. This produces a first-order linear differential equation.
- For $dx/dy + P(y)x = Q(y)$, the integrating factor is $e^{\int P(y)\,dy}$. Equivalently here, the quotient derivative $d(x/y)/dy$ immediately separates the equation.

Step 1: Solve the equation in terms of y Rearrange the differential equation as

$$y\frac{dx}{dy} - x = 3y^2.$$

For $y \neq 0$, divide by y^2 to recognize a derivative:

$$\frac{d}{dy}\left(\frac{x}{y}\right) = \frac{y(dx/dy) - x}{y^2} = 3.$$

Integrating gives $x/y = 3y + C$, or $x = 3y^2 + Cy$. Substitution into the original differential relation verifies this expression; it also describes the polynomial curve through the point where $y = 0$.

Step 2: Apply the given point and test the options At $(x, y) = (1, 1)$, $1 = 3 + C$, so $C = -2$ and

$$x = 3y^2 - 2y.$$

For option (B), $y = 1/3$ gives $x = 1/3 - 2/3 = -1/3$, exactly its stated x-coordinate. At $y = -1/2$, the curve instead has $x = 7/4$, ruling out (A). At $y = -1/3$, it has $x = 1$, ruling out (C). At $y = 1/2$, it has $x = -1/4$, ruling out (D). Only the second ordered pair lies on the required curve.

Step 3: Conclusion The curve also passes through $\boxed{(-1/3,\ 1/3)}$. Hence, the correct option is (B).

Quick Tip to Solve: If treating y as a function of x looks awkward, interchange the dependent variable. The combination y dx minus x dy suggests the derivative of x divided by y.

13. If the arithmetic mean of two numbers a and b, a > b > 0, is five times their geometric mean, then $\frac{a+b}{a-b}$ is equal to : [4 marks]

(A) $\frac{\sqrt{6}}{2}$

(B) $\frac{3\sqrt{2}}{4}$

(C) $\frac{7\sqrt{3}}{12}$

(D) $\frac{5\sqrt{6}}{12}$

Correct Answer: (D) $\frac{5\sqrt{6}}{12}$

Solution:

Concept:

- For positive numbers a, b, their arithmetic mean is $(a+b)/2$, and their geometric mean is $\sqrt{ab}$. The given ratio translates directly into $a + b = 10\sqrt{ab}$.
- The identity $(a-b)^2 = (a+b)^2 - 4ab$ relates the difference to the sum. Since $a > b > 0$, the difference is positive and the positive square root must be selected.

Step 1: Convert the mean condition into a sum-product identity The hypothesis states

$$\frac{a+b}{2} = 5\sqrt{ab}, \qquad a + b = 10\sqrt{ab}.$$

Because both numbers are positive, squaring introduces no sign ambiguity and yields $(a+b)^2 = 100ab$. It is unnecessary to determine either number separately: multiplying both by the same positive constant leaves the requested ratio unchanged.

Step 2: Find the difference and simplify the ratio Apply the difference identity:

$$(a-b)^2 = 100ab - 4ab = 96ab.$$

The condition $a > b$ gives $a - b = \sqrt{96ab} = 4\sqrt{6}\sqrt{ab}$. Hence

$$\frac{a+b}{a-b} = \frac{10\sqrt{ab}}{4\sqrt{6}\sqrt{ab}} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}.$$

As a consistency check, its square is $25/24$, so the ratio is just greater than 1, as expected when two unequal positive numbers have a large sum relative to their geometric mean. The squares of options (A), (B) and (C) are $3/2$, $9/8$ and $49/48$, respectively; none equals $25/24$.

Step 3: Conclusion The required value is $\boxed{5\sqrt{6}/12}$. Hence, the correct option is (D).

Quick Tip to Solve: When the sum and product are related, obtain the difference from its square instead of solving separately for the two numbers.

14. If a point P has co-ordinates (0, −2) and Q is any point on the circle, $x^2 + y^2 - 5x - y + 5 = 0$, then the maximum value of $(PQ)^2$ is : [4 marks]

(A) $\frac{25+\sqrt{6}}{2}$

(B) $14 + 5\sqrt{3}$

(C) $\frac{47+10\sqrt{6}}{2}$

(D) $8 + 5\sqrt{3}$

Correct Answer: (B) $14 + 5\sqrt{3}$

Solution:

Concept:

- Completing squares in the general equation of a circle identifies its centre C and radius r. For a fixed point P, the farthest point on the circle is on the line of centres, opposite to P.
- The triangle inequality gives $PQ \le PC + CQ = PC + r$, with equality at that farthest point. Since distances are nonnegative, the maximum of PQ^2 is $(PC + r)^2$.

Step 1: Find the centre, radius and distance from P Complete the squares in the stated equation:

$$\left(x - \frac{5}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{25}{4} + \frac{1}{4} - 5 = \frac{3}{2}.$$

Thus $C = (5/2, 1/2)$ and $r = \sqrt{6}/2$. From $P = (0, -2)$,

$$PC^2 = \left(\frac{5}{2}\right)^2 + \left(\frac{1}{2} + 2\right)^2 = \frac{25}{2}, \qquad PC = \frac{5\sqrt{2}}{2}.$$

The positive radius and distance are used because the problem asks for a geometric maximum.

Step 2: Square the maximum distance and compare the choices The circle contains a point Q on the ray starting at P and going through C, beyond C. At that point, $PQ = PC + r$, so the triangle-inequality bound is attained. Therefore

$$(PQ)^2_{\max} = \left(\frac{5\sqrt{2} + \sqrt{6}}{2}\right)^2$$
$$= \frac{50 + 6 + 20\sqrt{3}}{4} = 14 + 5\sqrt{3}.$$

The cross term is essential: adding only the two squared lengths would give 14. Options (A), (C) and (D) have different constant or radical terms and do not equal this exact attained bound.

Step 3: Conclusion The maximum squared distance is $\boxed{14 + 5\sqrt{3}}$. Hence, the correct option is (B).

Quick Tip to Solve: For the farthest point on a circle, first add the centre distance and the radius, and only then square; otherwise the cross term is lost.

15. $\lim\limits_{x\to 3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$ **is equal to :** [4 marks]

(A) $\sqrt{3}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{1}{2\sqrt{2}}$

Correct Answer: (B) $\frac{1}{\sqrt{2}}$

Solution:

Concept:

- The given quotient has the indeterminate form $0/0$ at $x = 3$. Rationalising each difference removes its vanishing factor without changing the limit.
- Use the identity $\sqrt{u} - \sqrt{v} = (u - v)/(\sqrt{u} + \sqrt{v})$. Cancellation is valid for $x \neq 3$, which is precisely the punctured neighbourhood used in a limit.

Step 1: Check the form and rationalise the numerator At $x = 3$, both numerator and denominator are zero, so direct division is undefined. In a sufficiently small neighbourhood of 3, both square roots are real and their sums are positive. Write

$$\sqrt{3x} - 3 = \frac{3x - 9}{\sqrt{3x} + 3} = \frac{3(x - 3)}{\sqrt{3x} + 3}.$$

The factor $x - 3$ makes the source of the zero explicit.

Step 2: Rationalise the denominator and cancel Similarly,

$$\sqrt{2x - 4} - \sqrt{2} = \frac{2x - 6}{\sqrt{2x - 4} + \sqrt{2}} = \frac{2(x - 3)}{\sqrt{2x - 4} + \sqrt{2}}.$$

Consequently, for $x \neq 3$, the whole quotient equals

$$\frac{3}{2}\frac{\sqrt{2x - 4} + \sqrt{2}}{\sqrt{3x} + 3}.$$

This expression is continuous at 3 because its denominator there is 6.

Step 3: Evaluate the continuous expression Now substitution is legitimate:

$$L = \frac{3}{2}\frac{2\sqrt{2}}{6} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}.$$

Option (D) is half this value and results from losing a factor while rationalising; (A) and (C) do not equal the simplified ratio. As a check, the derivatives of the original numerator and denominator at 3 are $1/2$ and $1/\sqrt{2}$, giving the same quotient.

Step 4: Conclusion The required limit is $1/\sqrt{2}$. Hence, the correct option is (B).

Quick Tip to Solve: For a quotient of square-root differences, rationalise both differences first; cancel the common vanishing factor before substituting.

16. An unbiased coin is tossed eight times. The probability of obtaining at least one head and at least one tail is : [4 marks]

(A) $\frac{255}{256}$
(B) $\frac{127}{128}$
(C) $\frac{63}{64}$
(D) $\frac{1}{2}$

Correct Answer: (B) $\frac{127}{128}$

Solution:

Concept:

- Eight independent tosses of an unbiased coin give 2^8 equally likely ordered outcomes; each particular sequence has probability $(1/2)^8$.
- The complement of obtaining at least one head and at least one tail is obtaining only heads or only tails. These two excluded events are disjoint.

Step 1: Describe the required event precisely The requirement allows any number of heads from 1 through 7. It does not require equal numbers of heads and tails, a particular order, or a head followed immediately by a tail. Instead of adding seven binomial probabilities, remove the outcomes that contain only one kind of face. There are exactly two such sequences: HHHHHHHH and TTTTTTTT.

Step 2: Subtract the disjoint excluded outcomes Independence gives

$$P(\text{all heads}) = \left(\frac{1}{2}\right)^8 = \frac{1}{256}, \qquad P(\text{all tails}) = \frac{1}{256}.$$

Their intersection is empty, because a sequence of eight tosses cannot consist entirely of both heads and tails. Therefore

$$P(\text{at least one of each}) = 1 - \frac{1}{256} - \frac{1}{256} = \frac{254}{256} = \frac{127}{128}.$$

Equivalently, 254 of the 256 ordered sequences are favourable.

Step 3: Check the alternative probabilities Option (A), $255/256$, excludes just one constant sequence and therefore still includes an outcome with no head or no tail. Option (C), $63/64 = 252/256$, excludes four sequences rather than the required two. Option (D), $1/2$, is the probability of one specified result on a single toss, not this eight-toss event. The computed value is close to one, as expected after many independent tosses.

Step 4: Conclusion The probability is $127/128$. Hence, the correct option is (B).

Quick Tip to Solve: For at least one head and at least one tail in n fair tosses, subtract both constant sequences: $1 - 2^{1-n}$.

17. The locus of the point of intersection of the straight lines,

$tx - 2y - 3t = 0$

$x - 2ty + 3 = 0$ ($t \in R$), is : [4 marks]

(A) an ellipse with eccentricity $\frac{2}{\sqrt{5}}$

(B) an ellipse with the length of major axis 6

(C) a hyperbola with eccentricity $\sqrt{5}$

(D) a hyperbola with the length of conjugate axis 3

Correct Answer: (D) a hyperbola with the length of conjugate axis 3

Solution:

Concept:

- A parameter can be eliminated by expressing it from one line and substituting into the other. The resulting Cartesian equation identifies the conic containing the intersections.
- For $x^2/a^2 - y^2/b^2 = 1$, the transverse axis has length $2a$, the conjugate axis has length $2b$, and the eccentricity is $e = \sqrt{1 + b^2/a^2}$.

Step 1: Eliminate the common parameter The first line gives $t(x - 3) = 2y$, and the second gives $2ty = x + 3$. Multiplying the second equation by $x - 3$ and replacing $t(x - 3)$ by $2y$ yields

$$4y^2 = (x + 3)(x - 3) = x^2 - 9.$$

Thus every finite intersection lies on

$$x^2 - 4y^2 = 9.$$

This elimination avoids dividing by y, so the intersection at $(-3, 0)$, obtained for $t = 0$, is retained.

Step 2: Put the conic in standard form Divide by 9 to obtain

$$\frac{x^2}{9} - \frac{y^2}{9/4} = 1.$$

The opposite signs of the two squared terms show that the conic is a hyperbola, not an ellipse. Here $a^2 = 9$ and $b^2 = 9/4$, giving $a = 3$, $b = 3/2$. Hence its conjugate-axis length is $2b = 3$, whereas its transverse-axis length is 6.

Step 3: Test the choices and parameter restriction The eccentricity is

$$e = \sqrt{1 + \frac{9/4}{9}} = \frac{\sqrt{5}}{2},$$

so the eccentricity stated in (C) is incorrect. Options (A) and (B) incorrectly classify the conic as an ellipse. Strictly, $(3, 0)$ is not attained for finite t, and $t = \pm 1$ makes the lines parallel. These parameter restrictions do not change the associated hyperbola or its conjugate-axis length, the property identified in (D).

Step 4: Conclusion The intersections lie on the hyperbola $x^2/9 - y^2/(9/4) = 1$, whose conjugate axis has length 3. Hence, the correct option is (D).

Quick Tip to Solve: After eliminating a parameter, divide the Cartesian equation into standard form before reading a, b, or the axis lengths.

18. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If now the mean age of the teachers in this school is 39 years, then the age (in years) of the newly appointed teacher is : [4 marks]

(A) 25

(B) 30

(C) 35

(D) 40

Correct Answer: (C) 35

Solution:

Concept:

- The arithmetic mean equals the total of all observations divided by their number. Therefore the total age is the mean age multiplied by the number of teachers.
- Replacing one teacher keeps the number of teachers fixed at 25. Only the outgoing and incoming ages change the total; the ages of the other 24 teachers cancel.

Step 1: Calculate the old and new total ages The original 25 teachers have total age

$$T_{\text{old}} = 25 \times 40 = 1000 \text{ years.}$$

After the replacement there are still 25 teachers, but their mean is 39 years. Their new total age is therefore

$$T_{\text{new}} = 25 \times 39 = 975 \text{ years.}$$

The one-year decrease is in the average of all 25 teachers; it represents a 25-year decrease in their combined total age.

Step 2: Account for the retirement and replacement Let a years be the age of the new teacher. Removing the retiring teacher subtracts 60 years from the original total, and appointing the new teacher adds a years. Thus

$$1000 - 60 + a = 975,$$

and hence

$$a = 975 - 940 = 35 \text{ years.}$$

The new teacher is 25 years younger than the teacher who retires, which is exactly the total decrease already found.

Step 3: Verify the answer against the options With age 35, the revised mean is

$$\frac{1000 - 60 + 35}{25} = \frac{975}{25} = 39.$$

The alternative ages 25, 30 and 40 would instead give total ages 965, 970 and 980, and means 38.6, 38.8 and 39.2 years respectively. None reproduces the given mean. The teacher count must remain 25 throughout; using 24 would describe the temporary situation after retirement but before replacement.

Step 4: Conclusion The newly appointed teacher is 35 years old. Hence, the correct option is (C).

Quick Tip to Solve: When one person is replaced, multiply the change in the mean by the unchanged group size to find the change in that person's age.

19. Let f(x) = 2^{10}.x + 1 and g(x)=3^{10}.x − 1. If (fog) (x) = x, then x is equal to : [4 marks]

(A) $\frac{3^{10}-1}{3^{10}-2^{-10}}$

(B) $\frac{2^{10}-1}{2^{10}-3^{-10}}$

(C) $\frac{1-3^{-10}}{2^{10}-3^{-10}}$

(D) $\frac{1-2^{-10}}{3^{10}-2^{-10}}$

Correct Answer: (D) $\frac{1-2^{-10}}{3^{10}-2^{-10}}$

Solution:

Concept:

- The composition $(f \circ g)(x)$ means $f(g(x))$: first apply g, then use its entire output as the input of f.
- A fixed point of a linear function is obtained by equating its value to x. Dividing the numerator and denominator of a fraction by the same nonzero power produces equivalent answer forms.

Step 1: Form the function composition in the given order The two functions are $f(x) = 2^{10}x + 1$ and $g(x) = 3^{10}x - 1$. Substitution into the outer function gives

$$(f \circ g)(x) = 2^{10}(3^{10}x - 1) + 1 = 6^{10}x - 2^{10} + 1.$$

The factor 2^{10} multiplies both terms of $g(x)$. Reversing the composition would put 3^{10} in the constant term and lead to a different fixed point.

Step 2: Solve the fixed-point equation The condition in the question now becomes

$$6^{10}x - 2^{10} + 1 = x.$$

Collecting the terms containing x,

$$(6^{10} - 1)x = 2^{10} - 1.$$

Since $6^{10} - 1 \neq 0$, there is one fixed point:

$$x = \frac{2^{10} - 1}{6^{10} - 1}.$$

There is no approximation involved, so preserve this exact form when comparing the options.

Step 3: Rewrite the fraction to identify the option Divide numerator and denominator by 2^{10}:

$$x = \frac{1 - 2^{-10}}{3^{10} - 2^{-10}}.$$

This is option (D). Multiplying both parts of (A), (B), and (C) by the power that clears their negative exponent gives numerators $6^{10} - 2^{10}$, $6^{10} - 3^{10}$, and $3^{10} - 1$, respectively, over $6^{10} - 1$. None equals the required numerator $2^{10} - 1$.

Step 4: Conclusion The fixed point is $\frac{1 - 2^{-10}}{3^{10} - 2^{-10}}$. Hence, the correct option is (D).

Quick Tip to Solve: Compose in the indicated order first; compare power fractions by clearing negative exponents rather than evaluating large powers.

20. The value of $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$, $|x| < \frac{1}{2}, x \neq 0$, is equal to : [4 marks]

(A) $\frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$

(B) $\frac{\pi}{4} + \cos^{-1} x^2$

(C) $\frac{\pi}{4} - \frac{1}{2}\cos^{-1} x^2$

(D) $\frac{\pi}{4} - \cos^{-1} x^2$

Correct Answer: (A) $\frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$

Solution:

Concept:

- The principal value of $\tan^{-1} u$ lies in $(-\pi/2, \pi/2)$; an identity for its tangent identifies the angle only after its range is checked.
- For $\theta = \cos^{-1}(x^2)$, the half-angle identities give $\sqrt{1+x^2} = \sqrt{2}\cos(\theta/2)$ and $\sqrt{1-x^2} = \sqrt{2}\sin(\theta/2)$ on the stated domain.

Step 1: Choose an angle and establish its range Set $\theta = \cos^{-1}(x^2)$. Because $0 < |x| < 1/2$, we have $0 < x^2 < 1/4$, so

$$\cos^{-1}(1/4) < \theta < \pi/2.$$

In particular, $0 < \theta/2 < \pi/4$. Both its sine and cosine are positive, which justifies taking positive square roots in the half-angle formulae. Also $\sqrt{1+x^2} > \sqrt{1-x^2}$, so the denominator of the printed ratio is nonzero and positive.

Step 2: Express the ratio as a tangent Writing $u = \theta/2$, the argument of the inverse tangent becomes

$$\frac{\cos u + \sin u}{\cos u - \sin u} = \frac{1+\tan u}{1-\tan u} = \tan\left(\frac{\pi}{4} + u\right).$$

The denominator $1 - \tan u$ is positive because $u < \pi/4$. Thus the addition identity applies with no singularity. The angle $\pi/4 + u$ is between $\pi/4$ and $\pi/2$, so it belongs to the principal inverse-tangent interval.

Step 3: Apply the principal inverse and compare It follows that

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(x^2).$$

Option (B) lacks the half-angle factor. Option (C) gives an angle below $\pi/4$, although the positive argument exceeds 1. Option (D) is negative on this domain, whereas the inverse tangent of the given positive ratio is positive.

Step 4: Conclusion The value is $\pi/4 + \frac{1}{2}\cos^{-1}(x^2)$. Hence, the correct option is (A).

Quick Tip to Solve: When an inverse tangent contains sums of square roots $1 \pm z$, set $z = \cos\theta$, then check the principal-angle range.

21. The area (in sq. units) of the smaller portion enclosed between the curves, $x^2 + y^2 = 4$ and $y^2 = 3x$, is : [4 marks]

(A) $\frac{1}{2\sqrt{3}} + \frac{\pi}{3}$

(B) $\frac{1}{\sqrt{3}} + \frac{2\pi}{3}$

(C) $\frac{1}{2\sqrt{3}} + \frac{2\pi}{3}$

(D) $\frac{1}{\sqrt{3}} + \frac{4\pi}{3}$

Correct Answer: (D) $\frac{1}{\sqrt{3}} + \frac{4\pi}{3}$

Solution:

Concept:

- For horizontal strips, the area between two curves is $\int (x_{\text{right}} - x_{\text{left}})\, dy$. Symmetry about the x-axis can reduce the calculation to the upper half.
- The circle $x^2 + y^2 = 4$ has radius 2 and right boundary $x = \sqrt{4 - y^2}$. The parabola $y^2 = 3x$ gives the left boundary $x = y^2/3$ of the right-hand enclosed region.

Step 1: Find the intersections and the relevant strip Substitute $y^2 = 3x$ into the circle:

$$x^2 + 3x = 4, \qquad (x - 1)(x + 4) = 0.$$

The root $x = -4$ would require $y^2 = -12$, so it gives no real intersection. The two actual intersections are $(1, \sqrt{3})$ and $(1, -\sqrt{3})$. Between their ordinates, the right-hand enclosed region runs from the parabola to the right semicircle. Its area is

$$A = 2 \int_0^{\sqrt{3}} \left(\sqrt{4 - y^2} - \frac{y^2}{3} \right) dy.$$

Step 2: Evaluate the two integrals Using the standard circle integral,

$$\int \sqrt{4 - y^2}\, dy = \frac{y}{2}\sqrt{4 - y^2} + 2\sin^{-1}(y/2).$$

At $y = \sqrt{3}$, the radical is 1 and the inverse sine is $\pi/3$, while both terms vanish at zero. Therefore

$$A = 2\left(\frac{\sqrt{3}}{2} + \frac{2\pi}{3} - \frac{(\sqrt{3})^3}{9} \right) = \frac{\sqrt{3}}{3} + \frac{4\pi}{3}.$$

The factor 2 accounts for the equal area below the x-axis.

Step 3: Confirm that this is the smaller portion The calculated area is approximately 4.77 square units. The entire circle has area 4π, so the other portion has area $4\pi - A$, approximately 7.80 square units. Thus the region integrated is indeed the smaller one. Since $\sqrt{3}/3 = 1/\sqrt{3}$, the exact result matches (D). Options (A), (B), and (C) do not contain both the correct full circular contribution $4\pi/3$ and the correct residual algebraic term.

Step 4: Conclusion The smaller area is $\frac{1}{\sqrt{3}} + \frac{4\pi}{3}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: For a sideways parabola and a circle, horizontal strips often give one integral; reject intersection roots producing negative y^2.

22. If two parallel chords of a circle, having diameter 4units, lie on the opposite sides of the center and subtend angles $\cos^{-1}\left(\frac{1}{7}\right)$ and $\sec^{-1}(7)$ at the center respectivey, then the distance between these chords, is : [4 marks]

(A) $\frac{4}{\sqrt{7}}$

(B) $\frac{8}{\sqrt{7}}$

(C) $\frac{8}{7}$

(D) $\frac{16}{7}$

Correct Answer: (B) $\frac{8}{\sqrt{7}}$

Solution:

Concept:

- A perpendicular from the centre to a chord bisects both the chord and its subtended central angle. For radius r and central angle θ, the distance to the chord is $r\cos(\theta/2)$.
- The half-angle identity is $\cos^2(\theta/2) = (1+\cos\theta)/2$. Distances of chords on opposite sides of the centre are added.

Step 1: Identify the radius and the two angles The diameter is 4 units, so the radius is $r = 2$ units. Write the smaller central angles as α and β. Since $\alpha = \cos^{-1}(1/7)$ and $\beta = \sec^{-1}(7)$, both satisfy $\cos\alpha = \cos\beta = 1/7$. In their principal ranges these are the same acute angle. Thus the chords have equal lengths and equal perpendicular distances from the centre.

Step 2: Calculate each perpendicular distance Apply the half-angle formula with the positive root, because half of an acute angle has positive cosine:

$$d = 2\sqrt{\frac{1+1/7}{2}} = 2\sqrt{\frac{4}{7}} = \frac{4}{\sqrt{7}}.$$

This is the centre-to-chord distance for either chord, rather than the distance between them.

Step 3: Use the opposite-side arrangement The two perpendiculars lie along one common normal to the parallel chords. Since the centre lies between the chords, their separation is

$$D = d + d = \frac{8}{\sqrt{7}} \text{ units.}$$

Option (A) counts only one of the two equal distances. Options (C) and (D) do not equal this value; replacing the square root in the half-angle formula by a linear factor would give an incorrect distance.

Step 4: Conclusion The distance between the parallel chords is $\boxed{8/\sqrt{7}}$ units. Hence, the correct option is (B).

Quick Tip to Solve: For parallel chords on opposite sides of the centre, add the centre-to-chord distances; on the same side, take their difference.

23. If y = $\left[x+\sqrt{x^2-1}\right]^{15}+\left[x-\sqrt{x^2-1}\right]^{15}$,

then (x^2-1) $\frac{d^2y}{dx^2}+x\frac{dy}{dx}$ is equal to : [4 marks]

(A) 125 y

(B) 124 y^2

(C) 225 y^2

(D) 225 y

Correct Answer: (D) 225 y

Solution:

Concept:

- For $s=\sqrt{x^2-1}$, differentiation gives $s'=x/s$ when $|x|>1$. The two factors $u=x+s$ and $v=x-s$ satisfy $uv=1$.
- The chain rule gives $(u^{15})'=15u^{14}u'$. A useful first step is to multiply the first derivative by s, eliminating a denominator before differentiating again.

Step 1: Differentiate the two reciprocal expressions Take $|x|>1$ and put $s=\sqrt{x^2-1}$, $u=x+s$, and $v=x-s$. Then

$$u'=1+\frac{x}{s}=\frac{u}{s},\qquad v'=1-\frac{x}{s}=-\frac{v}{s}.$$

With $y=u^{15}+v^{15}$, the chain rule therefore gives

$$y'=\frac{15}{s}(u^{15}-v^{15}),\qquad sy'=15(u^{15}-v^{15}).$$

Keeping the difference of powers together avoids expanding a polynomial of degree 15.

Step 2: Differentiate the simplified first-derivative identity Apply the product rule to sy', and differentiate both powers on the right:

$$\frac{x}{s}y'+sy''=15\left(\frac{15u^{15}}{s}+\frac{15v^{15}}{s}\right)=\frac{225y}{s}.$$

Multiplying by s and using $s^2=x^2-1$ gives exactly

$$(x^2-1)y''+xy'=225y.$$

The positive sign between the two powers on the right arises because differentiating $-v^{15}$ cancels the negative sign in v'.

Step 3: Check the expression and endpoint cases The coefficient is $15^2=225$, which rules out option (A). There is no multiplication of two copies of y, so the result is linear in y, unlike options (B) and (C). In the original sum, expanding the two powers cancels the odd powers of s. All even powers of s are powers of x^2-1, so y is a polynomial. Thus the derived differential identity extends as a polynomial identity to the endpoints $x=\pm1$, where the intermediate division by s was unavailable. No expansion is needed to evaluate the requested operator.

Step 4: Conclusion The requested expression equals $\boxed{225y}$. Hence, the correct option is (D).

Quick Tip to Solve: For reciprocal radical powers, differentiate once, multiply by the radical, and differentiate again; this eliminates the radical denominator cleanly.

24. The integral

$$\int \sqrt{1+2\cot x(\operatorname{cosec} x+\cot x)}\ dx$$

$\left(0<x<\frac{\pi}{2}\right)$ **is equal to :**

(where C is a constant of integration) [4 marks]

(A) 4 log(sin $\frac{x}{2}$) + C

(B) 2 log(sin $\frac{x}{2}$) + C

(C) 2 log(cos $\frac{x}{2}$) + C

(D) 4 log(cos $\frac{x}{2}$) + C

Correct Answer: (B) 2 log(sin $\frac{x}{2}$) + C

Solution:

Concept:

- The identity $\csc^2 x = 1+\cot^2 x$ helps recognize a perfect square under a radical.
- On $0<x<\pi/2$, both $\csc x$ and $\cot x$ are positive. Also $\csc x+\cot x = (1+\cos x)/\sin x = \cot(x/2)$.

Step 1: Simplify the expression under the square root Expand the radicand and complete a square using the Pythagorean identity:

$$\begin{aligned} 1+2\cot x(\csc x+\cot x) &= 1+2\csc x\cot x+2\cot^2 x \\ &= \csc^2 x+2\csc x\cot x+\cot^2 x \\ &= (\csc x+\cot x)^2. \end{aligned}$$

A square root of a square is an absolute value. Here the stated interval makes $\csc x+\cot x>0$, so the integrand is exactly $\csc x+\cot x$.

Step 2: Integrate by the half-angle substitution The half-angle identity now gives

$$I=\int \cot\frac{x}{2}\,dx.$$

Put $u=x/2$, so $dx=2\,du$. Thus

$$I=2\int \cot u\,du = 2\log|\sin u|+C = 2\log\left(\sin\frac{x}{2}\right)+C.$$

The absolute-value signs may be removed because $0<x/2<\pi/4$, making the sine positive.

Step 3: Differentiate to distinguish the options The derivative of $2\log(\sin(x/2))$ is $\cot(x/2)$, which is the required integrand. Option (A), with coefficient 4, differentiates to twice that value. Options (C) and (D) differentiate to negative multiples of $\tan(x/2)$, so neither has the required positive derivative on the given interval. The constant of integration can absorb a constant additive term but cannot repair a wrong derivative.

Step 4: Conclusion The integral is $\boxed{2\log(\sin(x/2))+C}$. Hence, the correct option is (B).

Quick Tip to Solve: After recognizing a trigonometric square inside a radical, check the interval before dropping its absolute value; then try a half-angle identity.

25. The tangent at the point (2, −2) to the curve, $x^2y^2 - 2x = 4(1 - y)$ does not pass through the point : [4 marks]

(A) $\left(4, \frac{1}{3}\right)$

(B) (8, 5)

(C) (−4, −9)

(D) (−2, −7)

Correct Answer: (D) (−2, −7)

Solution:

Concept:

- For a curve given implicitly by $F(x, y) = 0$, differentiate with respect to x, treating y as a function of x. The slope of the tangent is dy/dx.
- A line with slope m through (x_0, y_0) is $y - y_0 = m(x - x_0)$. A proposed point lies on it only if it satisfies the line equation exactly.

Step 1: Differentiate the given curve The point $(2, -2)$ satisfies the curve because both sides equal 12. Differentiate $x^2y^2 - 2x = 4(1 - y)$, applying the product and chain rules:

$$2xy^2 + 2x^2y\frac{dy}{dx} - 2 = -4\frac{dy}{dx}.$$

Collect the derivative terms to obtain

$$\frac{dy}{dx} = \frac{2 - 2xy^2}{2x^2y + 4}.$$

The derivative of y^2 includes the factor dy/dx; omitting it would change the tangent.

Step 2: Find the tangent at the specified point At $x = 2$ and $y = -2$, the numerator is $2 - 2(2)(4) = -14$, and the denominator is $2(4)(-2) + 4 = -12$. Therefore the slope is $7/6$, and the tangent is

$$y + 2 = \frac{7}{6}(x - 2), \qquad 6y - 7x + 26 = 0.$$

This exact integer-coefficient form makes checking each candidate straightforward.

Step 3: Test all four candidate points Substitute each point into $6y - 7x + 26$. For option (A), $(4, 1/3)$, the result is $2 - 28 + 26 = 0$. For option (B), $(8, 5)$, it is $30 - 56 + 26 = 0$. For option (C), $(-4, -9)$, it is $-54 + 28 + 26 = 0$. For option (D), $(-2, -7)$, it is $-42 + 14 + 26 = -2 \neq 0$. Thus the first three points lie on the tangent, and only the fourth fails its equation. The question asks for the point that does not lie on the tangent.

Step 4: Conclusion The tangent does not pass through $\boxed{(-2, -7)}$. Hence, the correct option is (D).

Quick Tip to Solve: Write an implicitly derived tangent in integer-coefficient form and substitute every option; pay attention to the word "does not.

26. If the common tangents to the parabola, $x^2 = 4y$ and the circle, $x^2 + y^2 = 4$ intersect at the point P, then the distance of P from the origin, is : [4 marks]

(A) $\sqrt{2}+1$

(B) $2(3 + 2\sqrt{2})$

(C) $2(\sqrt{2} + 1)$

(D) $3 + 2\sqrt{2}$

Correct Answer: (C) $2(\sqrt{2} + 1)$

Solution:

Concept:

- For the parabola $x^2 = 4ay$, a nonvertical tangent of slope m has equation $y = mx - am^2$. Here $a = 1$.
- A line is tangent to a circle when its perpendicular distance from the centre equals the radius. Symmetric tangents to these two curves have slopes m and $-m$.

Step 1: Write the tangent to the parabola The parabola $x^2 = 4y$ has $a = 1$, so a tangent with finite slope m is $y = mx - m^2$. This also follows by substituting the line into the parabola: the resulting quadratic is $x^2 - 4mx + 4m^2 = (x - 2m)^2 = 0$, giving one repeated intersection. A vertical line is not tangent to this upward-opening parabola at any finite point, so the slope form covers the required common tangents.

Step 2: Impose tangency to the circle The circle $x^2 + y^2 = 4$ has centre $(0, 0)$ and radius 2. The line can be written $mx - y - m^2 = 0$, whose distance from the origin is $m^2/\sqrt{m^2 + 1}$. Hence

$$\frac{m^2}{\sqrt{m^2+1}} = 2, \qquad m^4 = 4m^2 + 4.$$

Set $u = m^2 \geq 0$. Solving $u^2 - 4u - 4 = 0$ gives $u = 2 \pm 2\sqrt{2}$. The negative root is impossible, so $m^2 = 2 + 2\sqrt{2}$.

Step 3: Find their intersection and its distance The two tangent equations are $y = mx - u$ and $y = -mx - u$, with nonzero m. Subtracting gives $x = 0$, and then $y = -u$. Thus $P = (0, -2 - 2\sqrt{2})$, and

$$OP = |y| = 2 + 2\sqrt{2} = 2(\sqrt{2} + 1).$$

Option (A) is half this distance. Options (B) and (D) involve $3 + 2\sqrt{2} = (1 + \sqrt{2})^2$, which is not the positive value of u obtained from the tangency equation.

Step 4: Conclusion The distance of P from the origin is $\boxed{2(\sqrt{2} + 1)}$ units. For the curves as printed, the tangency equation and the intersection give this value, rather than $2(3 + 2\sqrt{2})$. Hence, the correct option is (C).

Quick Tip to Solve: For symmetric common tangents, first solve for the square of the slope; their common y-intercept then gives the intersection directly.

27. The proposition $(\sim p) \vee (p \wedge \sim q)$ is equivalent to : [4 marks]

(A) p $\vee \sim$ q

(B) p $\rightarrow \sim$ q

(C) p $\wedge \sim$ q

(D) q $\rightarrow$ p

Correct Answer: (B) p $\rightarrow \sim$ q

Solution:

Concept:

- Distributivity gives $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$. The statement $p \vee \neg p$ is always true.
- The implication $p \rightarrow r$ is equivalent to $\neg p \vee r$; it is false only when its premise is true and its conclusion is false.

Step 1: Distribute the disjunction Write the given proposition using $\neg$ for negation:

$$E = \neg p \vee (p \wedge \neg q).$$

Applying distributivity, rather than interchanging a conjunction and disjunction without a rule, gives

$$E = (\neg p \vee p) \wedge (\neg p \vee \neg q).$$

The first parenthesis is a tautology. Conjoining a statement with a tautology leaves that statement unchanged, so $E = \neg p \vee \neg q$.

Step 2: Recognize the correct implication In the implication identity take the conclusion to be $\neg q$. Then

$$p \rightarrow \neg q \equiv \neg p \vee \neg q.$$

Thus the simplified expression is precisely option (B). In particular, it is false only if p is true and q is true; for any other assignment at least one of $\neg p$ and $\neg q$ is true.

Step 3: Exclude each remaining expression For $p =$ true and $q =$ true, the given proposition is false, whereas option (A), $p \vee \neg q$, is true. For $p =$ false and $q =$ false, the given proposition is true, whereas option (C), $p \wedge \neg q$, is false. Finally, for $p =$ false and $q =$ true, the given proposition is true, whereas option (D), $q \rightarrow p$, is false. Each mismatch rules out logical equivalence; one counterexample is sufficient to reject an option.

Step 4: Conclusion The equivalent proposition is $\boxed{p \rightarrow \neg q}$. Hence, the correct option is (B).

Quick Tip to Solve: To compare a formula with an implication, replace the implication by "negated premise OR conclusion and simplify both expressions.

28. If the sum of the first n terms of the series $\sqrt{3}+\sqrt{75}+\sqrt{243}+\sqrt{507}+.....$ is $435\sqrt{3}$, then n equals : [4 marks]

(A) 18

(B) 15

(C) 13

(D) 29

Correct Answer: (B) 15

Solution:

Concept:

- Factor out square factors from each radical before identifying a sequence. Here $\sqrt{3k^2}=k\sqrt{3}$ for positive k.
- An arithmetic progression with first term a and common difference d has sum $S_n=\frac{n}{2}[2a+(n-1)d]$, where n is a positive integer.

Step 1: Recognize the arithmetic progression Simplify each term exactly:

$$\sqrt{3}=1\sqrt{3},\quad \sqrt{75}=5\sqrt{3},\quad \sqrt{243}=9\sqrt{3},\quad \sqrt{507}=13\sqrt{3}.$$

The coefficients are $1,5,9,13,\ldots$, whose consecutive differences are all 4. Hence the original terms form an arithmetic progression with $a=\sqrt{3}$ and $d=4\sqrt{3}$. The radicands themselves need not form an arithmetic progression; the terms after taking square roots do.

Step 2: Use the prescribed sum Apply the sum formula and set it equal to the given value:

$$S_n=\frac{n}{2}[2\sqrt{3}+4(n-1)\sqrt{3}]=n(2n-1)\sqrt{3}=435\sqrt{3}.$$

Since $\sqrt{3}\neq 0$, division gives $2n^2-n-435=0$. Factoring,

$$(n-15)(2n+29)=0.$$

The roots are 15 and $-29/2$; only $n=15$ is a permissible positive integral number of terms.

Step 3: Check the answer against every option For $n=15$, the coefficient of $\sqrt{3}$ in the sum is $15(30-1)=435$, as required. The other listed values give different coefficients: $n=18$ gives $18\times 35=630$, $n=13$ gives $13\times 25=325$, and $n=29$ gives $29\times 57=1653$. Thus none of options (A), (C), or (D) satisfies the sum equation. Notice that 29 is the factor $2n-1$ at the correct value of n; it is not the number of terms.

Step 4: Conclusion The number of terms is $\boxed{15}$. Hence, the correct option is (B).

Quick Tip to Solve: Simplify radical terms to expose an arithmetic progression, then cancel the common radical from the sum equation before solving for n.

29. The coordinates of the foot of the perpendicular from the point (1, −2, 1) on the plane containing the lines, $\frac{x+1}{6} = \frac{y-1}{7} = \frac{z-3}{8}$ and $\frac{x-1}{3} = \frac{y-2}{5} = \frac{z-3}{7}$, is : [4 marks]

(A) (2, −4, 2)

(B) (− 1, 2, −1)

(C) (0, 0, 0)

(D) (1, 1, 1)

Correct Answer: (C) (0, 0, 0)

Solution:

Concept:

- The direction vectors of two intersecting lines lying in a plane give its normal through their cross product. A plane through a point $\mathbf{a}$ with normal $\mathbf{n}$ satisfies $\mathbf{n} \cdot (\mathbf{r} - \mathbf{a}) = 0$.
- The perpendicular from $\mathbf{p}$ meets the plane $\mathbf{n} \cdot \mathbf{r} = d$ at $\mathbf{h} = \mathbf{p} - [(\mathbf{n} \cdot \mathbf{p} - d)/(\mathbf{n} \cdot \mathbf{n})]\mathbf{n}$.

Step 1: Find the common plane The line directions are $\mathbf{u} = (6, 7, 8)$ and $\mathbf{v} = (3, 5, 7)$. Their cross product is

$$\mathbf{u} \times \mathbf{v} = (49 - 40, 24 - 42, 30 - 21) = 9(1, -2, 1).$$

Thus take $\mathbf{n} = (1, -2, 1)$. The first line passes through $(-1, 1, 3)$, so its plane has equation

$$x - 2y + z = -1 - 2 + 3 = 0.$$

The second base point $(1, 2, 3)$ also satisfies this equation, confirming both lines lie in it.

Step 2: Project the given point and test the options For $\mathbf{p} = (1, -2, 1)$, the plane expression is $1 + 4 + 1 = 6$, while $\|\mathbf{n}\|^2 = 6$. Therefore

$$\mathbf{h} = (1, -2, 1) - \frac{6}{6}(1, -2, 1) = (0, 0, 0).$$

The displacement from the point to this foot is normal to the plane. Options (A) and (B) do not lie in the plane. Option (D) lies in it, but its displacement from the given point is $(0, 3, 0)$, which is not parallel to the normal.

Step 3: Conclusion The foot of the perpendicular is $(0, 0, 0)$. Hence, the correct option is (C).

Quick Tip to Solve: After finding a candidate foot, check both requirements: it lies in the plane and its displacement from the given point is parallel to the plane normal.

30. The line of intersection of the planes $\vec{r}.\left(3\hat{i}-\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(\hat{i}+4\hat{j}-2\hat{k}\right)=2$, is : [4 marks]

(A) $\frac{x-\frac{4}{7}}{-2}=\frac{y}{7}=\frac{z-\frac{5}{7}}{13}$

(B) $\frac{x-\frac{4}{7}}{2}=\frac{y}{-7}=\frac{z+\frac{5}{7}}{13}$

(C) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{-7}=\frac{z}{-13}$

(D) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{7}=\frac{z}{-13}$

Correct Answer: (C) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{-7}=\frac{z}{-13}$

Solution:

Concept:

- The intersection line is perpendicular to both plane normals. Its direction is therefore the cross product of the normals.
- A symmetric line equation needs both a point satisfying the two plane equations and the correct direction ratios. Either the direction vector or its negative represents the same line.

Step 1: Find direction ratios The plane normals are $\mathbf{n}_1=(3,-1,1)$ and $\mathbf{n}_2=(1,4,-2)$. Hence

$$\mathbf{n}_1\times\mathbf{n}_2=(-2,7,13).$$

We may equally choose direction ratios $(2,-7,-13)$. Dot products confirm $3(2)-(-7)+(-13)=0$ and $2+4(-7)-2(-13)=0$, so this direction lies within each plane.

Step 2: Locate a point on the intersection Set $z=0$ in the equations $3x-y+z=1$ and $x+4y-2z=2$. Then $y=3x-1$ gives

$$x+4(3x-1)=2,\qquad x=\frac{6}{13},\quad y=\frac{5}{13}.$$

Thus the line is

$$\frac{x-6/13}{2}=\frac{y-5/13}{-7}=\frac{z}{-13}.$$

Option (A) has a valid direction but its stated point $(4/7,0,5/7)$ fails the planes. In (B), the stated direction $(2,-7,13)$ is not perpendicular to the first normal. Option (D) also has an incorrect sign in its direction ratios: $(2,7,-13)$ has nonzero dot product with the first normal.

Step 3: Conclusion The intersection is $\frac{x-6/13}{2}=\frac{y-5/13}{-7}=\frac{z}{-13}$. Hence, the correct option is (C).

Quick Tip to Solve: For a proposed intersection line, separately test its base point in both planes and its direction against both plane normals.

Physics (120 marks)

31. Two deuterons undergo nuclear fusion to form a Helium nucleus. Energy released in this process is : (given binding energy per nucleon for deuteron = 1.1 MeV and for helium = 7.0 MeV) [4 marks]

(A) 30.2 MeV

(B) 32.4 MeV

(C) 23.6 MeV

(D) 25.8 MeV

Correct Answer: (C) 23.6 MeV

Solution:

Concept:

- Binding energy is the energy needed to separate a nucleus into free nucleons. A nucleus with mass number A and binding energy per nucleon b has total binding energy $B = Ab$.
- For a reaction conserving the total numbers of protons and neutrons, the released energy is the increase in total nuclear binding energy: $Q = B_{\text{products}} - B_{\text{reactants}}$.

Step 1: Calculate the initial binding energy A deuteron has two nucleons: one proton and one neutron. The stated binding energy per nucleon is $1.1\,\text{MeV}$, so each deuteron has total binding energy

$$B_D = 2(1.1) = 2.2\,\text{MeV}.$$

There are two deuterons initially; hence their combined binding energy is

$$B_{\text{initial}} = 2(2.2) = 4.4\,\text{MeV}.$$

Multiplying by both the nucleons per nucleus and the number of nuclei is essential; the given number is not the binding energy of an entire deuteron.

Step 2: Compare with the final nucleus The helium nucleus contains four nucleons, so its total binding energy is

$$B_{\text{final}} = 4(7.0) = 28.0\,\text{MeV}.$$

Therefore

$$Q = 28.0 - 4.4 = 23.6\,\text{MeV}.$$

The positive sign means the final nucleus is more tightly bound, with a smaller rest mass, and the mass difference is released as energy. Option (B), $32.4\,\text{MeV}$, adds the two binding energies rather than subtracting. Option (D), $25.8\,\text{MeV}$, subtracts the binding of only one deuteron. Option (A) is also inconsistent with the required initial total and final total.

Step 3: Conclusion The fusion releases $23.6\,\text{MeV}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert every binding energy per nucleon into the total for that nucleus before taking final binding energy minus initial binding energy.

32. Magnetic field in a plane electromagnetic wave is given by

$\vec{B}$ = B_0 sin (k x + ωt) $\hat{j}\,T$

Expression for corresponding electric field will be :

Where c is speed of light. [4 marks]

(A) $\vec{E}$ = B_0 c sin (k x + ωt) $\hat{k}$ V/m

(B) $\vec{E}$ = $\frac{B_0}{c}$ sin (k x + ωt) $\hat{k}$ V/m

(C) $\vec{E}$ = − B_0 c sin (kx +ωt) $\hat{k}$ V/m

(D) $\vec{E}$ = B_0 c sin (kx −ωt) $\hat{k}$ V/m

Correct Answer: (A) $\vec{E}$ = B_0 c sin (k x + ωt) $\hat{k}$ V/m

Solution:

Concept:

- In a plane electromagnetic wave in vacuum, the electric and magnetic fields have the same phase and amplitudes related by $E_0 = cB_0$. Both are perpendicular to the propagation direction.
- The energy travels in the direction of $\mathbf{E} \times \mathbf{B}$. A phase $kx + \omega t$ propagates in the negative x-direction because a fixed phase obeys $dx/dt = -\omega/k$.

Step 1: Determine the direction of propagation The given magnetic field is

$$\mathbf{B} = B_0 \sin(kx + \omega t)\,\hat{\mathbf{j}}.$$

Keeping the phase constant gives $kx + \omega t = \text{constant}$, hence $x = -\omega t/k + \text{constant}$. Therefore the wave travels along $-\hat{\mathbf{i}}$. The electric field must be perpendicular to the magnetic field, along the z-axis, with its sign determined by the propagation direction.

Step 2: Choose the electric field and exclude the alternatives For an electric field along $+\hat{\mathbf{k}}$, the cross product satisfies

$$\hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}},$$

exactly the required direction. The amplitude and phase conditions then give

$$\mathbf{E} = cB_0 \sin(kx + \omega t)\,\hat{\mathbf{k}}\ \text{V/m}.$$

Option (B) uses B_0/c instead of cB_0. Option (C) reverses the relative field sign, so its energy flow would point the wrong way. Option (D) changes the phase to $kx - \omega t$, which describes the opposite propagation direction and is not in phase with the specified magnetic field.

Step 3: Conclusion The required field is $\mathbf{E} = cB_0 \sin(kx + \omega t)\hat{\mathbf{k}}\ \text{V/m}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the phase sign to determine propagation first, then check the right-hand rule for the ordered cross product of electric field with magnetic field.

33. In a physical balance working on the principle of moments, when 5 mg weight is placed on the left pan, the beam becomes horizontal. Both the empty pans of the balance are of equal mass. Which of the following statements is correct ? [4 marks]

(A) Left arm is longer than the right arm

(B) Both the arms are of same length

(C) Left arm is shorter than the right arm

(D) Every object that is weighed using this balance appears lighter than its actual weight.

Correct Answer: (C) Left arm is shorter than the right arm

Solution:

Concept:

- A balance beam is horizontal at equilibrium when the clockwise and anticlockwise torques about its pivot are equal. A vertical weight produces torque equal to weight multiplied by its horizontal arm length.
- The empty pans have equal mass. If an extra positive mass is needed on one pan to balance them, that pan must be attached to the shorter arm.

Step 1: Write the torque balance Let each empty pan have mass $m > 0$, and let the left and right arm lengths be l_L and l_R. The extra mass on the left is $\delta m = 5\,\text{mg} > 0$. For the stated horizontal equilibrium, the principle of moments gives

$$(m + \delta m)gl_L = mgl_R.$$

Cancelling gravity and dividing by ml_L yields

$$\frac{l_R}{l_L} = 1 + \frac{\delta m}{m} > 1.$$

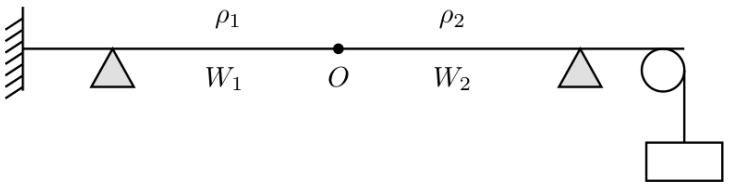
Thus the right arm is longer, and the left arm is shorter.

Step 2: Interpret the options Option (C) states this directly. If the left arm were longer, equal empty pans would already give a greater left torque and adding mass on the left would worsen the imbalance; option (A) is therefore false. Equal arms would balance the equal empty pans without an extra mass, contradicting the observation, so (B) is false. Option (D) makes an unconditional claim about every measured object, but the reading error depends on which pan holds the object and which holds the standards. Swapping the object and standards changes the direction of the unequal-arm error. Thus the stated observation determines the relative lengths, not a universal lighter reading.

Step 3: Conclusion The left arm is shorter than the right arm. Hence, the correct option is (C).

Quick Tip to Solve: An added mass compensates for a shorter lever arm: the equilibrium comparison must use total mass on each pan, including the pan itself.

34. Two wires W_1 and W_2 have the same radius r and respective densities ρ_1 and ρ_2 such that $\rho_2 = 4\rho_1$. They are joined together at the point O, as shown in the figure. The combination is used as a sonometer wire and kept under tension T. The point O is midway between the two bridges. When a stationary wave is set up in the composite wire, the joint is found to be a node. The ratio of the number of antinodes formed in W_1 to W_2 is : [4 marks]



(A) 1 : 1

(B) 1 : 2

(C) 1 : 3

(D) 4 : 1

Correct Answer: (B) 1 : 2

Solution:

Concept:

- For a stretched wire, transverse-wave speed is $v = \sqrt{T/\mu}$, where T is tension and $\mu = \rho\pi r^2$ is mass per unit length.
- Between two nodes a standing-wave segment of length L containing n antinodes satisfies $L = n\lambda/2$, and therefore $f = nv/(2L)$. Joined sections oscillate at one common frequency.

Step 1: Compare the wave speeds The wires have the same radius, so their cross-sectional areas are equal. Their density relation gives

$$\mu_2 = 4\mu_1, \qquad \frac{v_1}{v_2} = \sqrt{\frac{\mu_2}{\mu_1}} = 2.$$

The lighter first wire therefore carries waves twice as fast as the second wire under the common tension. Density affects the speed through its square root, not in direct proportion.

Step 2: Use the node at the joint The joint is midway between the bridges, so both wire segments have equal length L. Each bridge and the joint are nodes, allowing $L = n_1\lambda_1/2 = n_2\lambda_2/2$. At the common frequency,

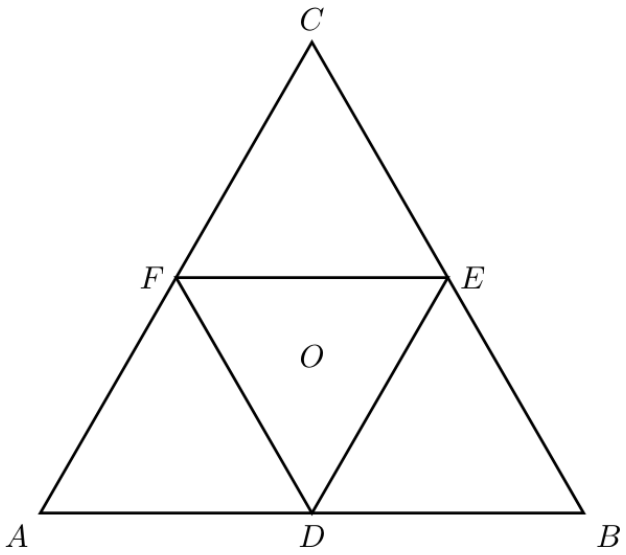
$$\frac{n_1 v_1}{2L} = \frac{n_2 v_2}{2L}, \qquad \frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{1}{2}.$$

Thus the first wire has half as many antinodes as the second; a compatible smallest pair is one and two. Option (A) would require equal speeds. Option (C) would require a speed ratio of three instead of two. Option (D) both reverses the trend and uses a factor of four where the speed relation supplies a factor of two.

Step 3: Conclusion The ratio of antinodes in the first and second wires is $1 : 2$. Hence, the correct option is (B).

Quick Tip to Solve: For equal-length segments at one frequency, antinode number is inversely proportional to wave speed and hence proportional to the square root of linear density.

35. Moment of inertia of an equilateral triangular lamina ABC, about the axis passing through its centre O and perpendicular to its plane is I_o as shown in the figure. A cavity DEF is cut out from the lamina, where D, E, F are the mid points of the sides. Moment of inertia of the remaining part of lamina about the same axis is : [4 marks]



(A) $\frac{7}{8}$ I_o

(B) $\frac{15}{16}$ I_o

(C) $\frac{3I_o}{4}$

(D) $\frac{31I_o}{32}$

Correct Answer: (B) $\frac{15}{16}$ I_o

Solution:

Concept:

- For a uniform lamina, mass is proportional to area. Similar planar shapes scaled by a linear factor s have mass ratio s^2 and moment-of-inertia ratio s^4 about corresponding perpendicular centroidal axes.
- The moment of inertia is additive over mass elements. Removing a cavity subtracts the inertia of the material formerly occupying that cavity about the same axis.

Step 1: Identify the size and centre of the cavity Joining the midpoints of the sides of an equilateral triangle produces a smaller equilateral triangle with side length one-half the original. The medial triangle shares the same centroid O as the original triangle, although its orientation is reversed. Thus both inertias are taken about the same perpendicular centroidal axis. The area and mass ratios are

$$\frac{A_{DEF}}{A_{ABC}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}, \qquad \frac{M_{DEF}}{M_{ABC}} = \frac{1}{4}.$$

No parallel-axis shift is needed because their centroids coincide.

Step 2: Subtract the inertia of the removed material All distances from the common centroid in the

smaller triangle are scaled by $1/2$. Since inertia contains the square of distance,

$$\frac{I_{DEF}}{I_o} = \frac{1}{4}\left(\frac{1}{2}\right)^2 = \frac{1}{16}.$$

Consequently

$$I_{\text{remaining}} = I_o - I_{DEF} = I_o - \frac{I_o}{16} = \frac{15I_o}{16}.$$

Option (C) subtracts only the mass fraction and ignores the smaller radius scale. Option (A) would require removing $I_o/8$, and (D) would require removing $I_o/32$; neither follows the required mass-times-distance-squared scaling.

Step 3: Conclusion The remaining lamina has moment of inertia $\frac{15}{16}I_o$. Hence, the correct option is (B).

Quick Tip to Solve: For similar uniform laminae about corresponding centroidal axes, scale moment of inertia by the fourth power of the linear scale, not merely by area.

36. In an experiment, a sphere of aluminium of mass 0.20 kg is heated upto 150°C. Immediately, it is put into water of volume 150 cc at 27°C kept in a calorimeter of water equivalent to 0.025 kg. Final temperature of the system is 40°C. The specific heat of aluminium is : (take 4.2 Joule = 1 calorie) [4 marks]

(A) 378 J/kg $-$°C

(B) 315 J/kg $-$°C

(C) 476 J/kg $-$°C

(D) 434 J/kg $-$°C

Correct Answer: (D) 434 J/kg $-$°C

Solution:

Concept:

- In an insulated calorimeter, heat lost by the hot metal equals heat gained by the water and the calorimeter. Use $Q = mc\Delta T$, where c is the specific heat capacity.
- The water equivalent is the mass of water having the same heat capacity as the calorimeter; it must be added to the actual water mass. Water has $c_w = 4200\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$.

Step 1: Find the effective cold mass The density of water is $1\,\text{g}\,\text{cm}^{-3}$, so $150\,\text{cc}$ corresponds to $150\,\text{g} = 0.150\,\text{kg}$. Including the calorimeter gives an effective water mass $m_{\text{eff}} = 0.150 + 0.025 = 0.175\,\text{kg}$. Both begin at the same temperature, so their temperature rise is $40 - 27 = 13\,\text{K}$.

Step 2: Apply the heat balance The aluminium cools through $150 - 40 = 110\,\text{K}$, whereas the effective water mass gains heat. Thus

$$0.20\,c_{\text{Al}}(110) = 0.175(4200)(13).$$

Solving for the unknown gives

$$c_{\mathrm{Al}} = \frac{9555}{22} = 434.3\,\mathrm{J\,kg^{-1}K^{-1}}.$$

A temperature interval of one kelvin is the same as one degree Celsius.

Step 3: Check the alternatives Only 434 agrees with this energy balance to the precision of the given options. The values 378, 315 and 476 would predict a different heat loss for the metal at the stated final temperature. Omitting the calorimeter would underestimate the heat gained and does not represent this experiment.

Step 4: Conclusion The specific heat capacity is approximately $434\,\mathrm{J\,kg^{-1}\,^{\circ}C^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: Add the water equivalent to the water mass before writing the heat balance; use the final temperature separately in each temperature change.

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37. The energy stored in the electric field produced by a metal sphere is 4.5 J. If the sphere contains 4 μC charge, its radius will be :

[Take : $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ N $-$ m^2/C^2] [4 marks]

(A) 20 mm

(B) 32 mm

(C) 28 mm

(D) 16 mm

Correct Answer: (D) 16 mm

Solution:

Concept:

- A metal sphere in electrostatic equilibrium has its charge on its surface and no electric field inside. Its capacitance relative to infinity is $C = 4\pi\varepsilon_0 R = R/k$.
- The field energy is $U = Q^2/(2C) = kQ^2/(2R)$, where $k = 1/(4\pi\varepsilon_0)$. The factor $1/2$ accounts for building the charge gradually.

Step 1: Express the radius in terms of the energy The given $4.5\,\mathrm{J}$ is the total energy stored in the electric field, not the work on a separate test charge. Therefore use the self-energy formula and rearrange:

$$U = \frac{kQ^2}{2R} \quad \Longrightarrow \quad R = \frac{kQ^2}{2U}.$$

There is no interior field-energy contribution for a conductor.

Step 2: Substitute the charge in SI units Convert $Q = 4\,\mu\mathrm{C} = 4 \times 10^{-6}\,\mathrm{C}$. Then

$$R = \frac{(9 \times 10^9)(4 \times 10^{-6})^2}{2(4.5)} = \frac{0.144}{9}\,\mathrm{m} = 0.016\,\mathrm{m}.$$

Hence $R = 16\,\mathrm{mm}$. Squaring the charge also squares the power of ten in its conversion.

Step 3: Eliminate the remaining values At fixed charge the energy varies inversely with radius. A radius of $32\,\text{mm}$ would give half the required energy, and often results from omitting the factor $1/2$. The $20\,\text{mm}$ and $28\,\text{mm}$ radii also give energies below $4.5\,\text{J}$; only $16\,\text{mm}$ reproduces the specified value.

Step 4: Conclusion The sphere has radius $16\,\text{mm}$. Hence, the correct option is (D).

Quick Tip to Solve: For an isolated conducting sphere use $U = kQ^2/(2R)$; omitting the half doubles the inferred radius.

38. A magnetic dipole in a constant magnetic field has : [4 marks]

(A) maximum potential energy when the torque is maximum.

(B) zero potential energy when the torque is minimum.

(C) zero potential energy when the torque is maximum.

(D) minimum potential energy when the torque is maximum.

Correct Answer: (C) zero potential energy when the torque is maximum.

Solution:

Concept:

- For a dipole of magnetic moment magnitude m at angle θ to a uniform field of magnitude B, the potential energy is $U = -\boldsymbol{m} \cdot \boldsymbol{B} = -mB\cos\theta$.
- The torque magnitude is $\tau = mB|\sin\theta|$. The conventional energy reference is $U = 0$ for a dipole perpendicular to the field.

Step 1: Locate the maximum torque The moment and field are fixed, so the maximum possible torque is mB. This occurs when the dipole axis is perpendicular to the field:

$$\theta = 90^\circ, \qquad |\sin\theta| = 1.$$

At this orientation the field has its greatest tendency to turn the dipole. A large turning tendency does not by itself mean a largest potential energy.

Step 2: Evaluate the energy at that orientation Substitute the same angle into the energy expression, rather than using a separate angle for the two quantities:

$$U = -mB\cos 90^\circ = 0.$$

Thus zero energy and maximum torque occur together. In contrast, the energy is minimum, $-mB$, for parallel alignment and maximum, $+mB$, for antiparallel alignment.

Step 3: Test each rejected statement Both parallel and antiparallel orientations have zero torque, the minimum torque magnitude, but their energies are $-mB$ and $+mB$, not zero. Therefore (B) is false. At maximum torque the energy is neither its maximum nor its minimum, rejecting (A) and (D). The perpendicular configuration establishes precisely the statement in (C).

Step 4: Conclusion The dipole has zero potential energy when the torque magnitude is maximum. Hence, the correct option is (C).

Quick Tip to Solve: Use sine for torque and negative cosine for energy, evaluating both at the same dipole orientation.

39. A small circular loop of wire of radius a is located at the centre of a much larger circular wire loop of radius b. The two loops are in the same plane. The outer loop of radius b carries an alternating current $I = I_o \cos(\omega t)$. The emf induced in the smaller inner loop is nearly : [4 marks]

(A) $\frac{\pi\mu_o I_o}{2}.\frac{a^2}{b}\,\omega \sin(\omega t)$

(B) $\frac{\pi\mu_o I_o}{2}.\frac{a^2}{b}\,\omega \cos(\omega t)$

(C) $\pi\mu_o I_o\,\frac{a^2}{b}\omega \sin(\omega t)$

(D) $\frac{\pi\mu_o I_o\, b^2}{a}\omega \cos(\omega t)$

Correct Answer: (A) $\frac{\pi\mu_o I_o}{2}.\frac{a^2}{b}\,\omega \sin(\omega t)$

Solution:

Concept:

- The magnetic field at the centre of a circular loop of radius b carrying current I is $B = \mu_0 I/(2b)$, directed normal to its plane.
- For $a \ll b$, this field is nearly uniform over the small loop of area πa^2. Faraday's law gives the induced emf $\mathcal{E} = -d\Phi/dt$, with $\Phi = BA$.

Step 1: Find the flux through the inner loop The loops are coplanar, so the field at their common centre is perpendicular to the small loop. Their area vector and the field can therefore be chosen in the same direction. Since $I(t) = I_0 \cos(\omega t)$,

$$\Phi(t) = \frac{\mu_0 I_0}{2b}\cos(\omega t)\,\pi a^2.$$

The small area is πa^2, while the source-loop radius in the field formula is b.

Step 2: Differentiate the time-dependent flux Taking the positive circulation associated with that area normal, Faraday's law gives

$$\mathcal{E} = -\frac{\pi\mu_0 I_0 a^2}{2b}\frac{d}{dt}\cos(\omega t) = \frac{\pi\mu_0 I_0 a^2}{2b}\,\omega \sin(\omega t).$$

The two minus signs cancel. Reversing the chosen circulation would reverse the emf sign, without changing the amplitude or phase relation physically.

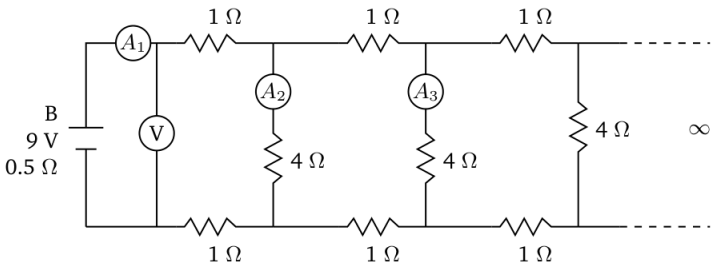
Step 3: Compare the coefficients and phase Option (B) retains a cosine instead of differentiating it. Option (C) misses the factor $1/2$ in the field at a circular loop's centre. Option (D) interchanges the relevant radii and has the wrong time dependence. The field-uniformity approximation is justified by the stated much smaller inner loop, which is why the result is only nearly exact.

Step 4: Conclusion The induced emf is $\mathcal{E} = \frac{\pi\mu_0 I_0 a^2}{2b}\omega \sin(\omega t)$. Hence, the correct option is (A).

Quick Tip to Solve: Use the large loop to calculate the field, the small loop to calculate the flux area, and then differentiate once.

40. A 9 V battery with internal resistance of 0.5 Ω is connected across an infinite network as shown in the figure. All ammeters A_1, $A_{2,3}$ and voltmeter V are ideal.

Choose correct statement. [4 marks]



(A) Reading of A_1 is 2 A

(B) Reading of A_1 is 18 A

(C) Reading of V is 9 V

(D) Reading of V is 7 V

Correct Answer: (A) Reading of A_1 is 2 A

Solution:

Concept:

- An infinite repeating ladder has the same equivalent resistance after its first repeating section is removed. This self-similarity gives an algebraic equation for its resistance.
- Ideal ammeters have zero resistance and ideal voltmeters have infinite resistance. The battery's internal resistance must still be included when calculating the current supplied.

Step 1: Reduce the repeating part Let X be the resistance between the upper and lower nodes at a vertical $4\,\Omega$ rung. Beyond this rung, the next identical part is reached through one upper and one lower $1\,\Omega$ resistor. Thus the direct $4\,\Omega$ path is in parallel with a path of resistance $2 + X$:

$$X = \frac{4(2+X)}{4+2+X}.$$

Consequently $X^2 + 2X - 8 = 0$, giving $X = 2\,\Omega$; the root $-4\,\Omega$ is inadmissible for this passive resistor network.

Step 2: Include the entrance resistors and the battery There is an additional $1\,\Omega$ resistor on each rail before the first rung. The external equivalent is therefore

$$R_{\text{ext}} = 1 + X + 1 = 4\,\Omega.$$

With internal resistance $r = 0.5\,\Omega$, the total current through the entrance ammeter is

$$I_{A_1} = \frac{9}{4 + 0.5} = 2\,\text{A}.$$

The ideal voltmeter draws no branch current, and ideal ammeters introduce no extra voltage drop.

Step 3: Check the voltmeter alternatives The voltmeter is connected across the supply rails at the

entrance, so it measures the terminal potential difference

$$V = 9 - Ir = 9 - 2(0.5) = 8\,\text{V}.$$

This rejects both $9\,\text{V}$ and $7\,\text{V}$. A current of $18\,\text{A}$ would correspond to a short across the battery terminals, ignoring the ladder resistance, so (B) is also false.

Step 4: Conclusion The reading of A_1 is $2\,\text{A}$. Hence, the correct option is (A).

Quick Tip to Solve: Define the repeating resistance at a rung first; then add the two entrance resistors and finally the internal resistance.

---

41. If the Earth has no rotational motion, the weight of a person on the equator is W. Determine the speed with which the earth would have to rotate about its axis so that the person at the equator will weigh $\frac{3}{4}$ W. Radius of the Earth is 6400 km and g=10 m/s^2. [4 marks]

(A) 1.1×10^{-3} rad/s

(B) 0.83×10^{-3} rad/s

(C) 0.63×10^{-3} rad/s

(D) 0.28×10^{-3} rad/s

Correct Answer: (C) 0.63×10^{-3} rad/s

Solution:

Concept:

- At the equator a person at rest relative to Earth moves in a circle of radius R, requiring inward centripetal force $m\omega^2 R$.
- The measured weight is the normal reaction N, not the gravitational force itself. Radially inward, $mg - N = m\omega^2 R$, whereas the nonrotating weight is $W = mg$.

Step 1: Write the required normal reaction Rotation does not replace the gravitational force; rather gravity and the support force together provide the needed inward resultant. For the requested apparent weight,

$$N = \frac{3}{4}W = \frac{3}{4}mg.$$

Substitute into the radial force equation to obtain

$$mg - \frac{3}{4}mg = m\omega^2 R.$$

The mass cancels, so the angular speed is independent of the person's mass.

Step 2: Calculate the angular speed The required reduction in effective gravitational acceleration is $g/4$, so

$$\omega = \sqrt{\frac{g}{4R}}.$$

Convert $R = 6400\,\text{km} = 6.4 \times 10^6\,\text{m}$. With the specified $g = 10\,\text{m s}^{-2}$,

$$\omega = \sqrt{\frac{10}{4(6.4 \times 10^6)}} = \sqrt{3.90625 \times 10^{-7}} = 6.25 \times 10^{-4}\,\text{rad s}^{-1}.$$

Step 3: Round and compare with the options This is $0.625 \times 10^{-3}\,\text{rad s}^{-1}$, which rounds to 0.63×10^{-3}. The other speeds produce different centrifugal reductions because the reduction depends on the square of angular speed. In particular, using $3g/4$ for the reduction would confuse the remaining weight with the lost weight.

Step 4: Conclusion The required angular speed is approximately $0.63 \times 10^{-3}\,\text{rad s}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: First calculate the fraction of weight lost: a remaining weight of $3W/4$ means the centrifugal reduction is $W/4$.

42. Let the refractive index of a denser medium with respect to a rarer medium be n_{12} and its critical angle be θ_C . At an angle of incidence A when light is travelling from denser medium to rarer medium, a part of the light is reflected and the rest is refracted and the angle between reflected and refracted rays is 90°. Angle A is given by : [4 marks]

(A) $\frac{1}{\cos^{-1}(\sin\theta_C)}$

(B) $\frac{1}{\tan^{-1}(\sin\theta_C)}$

(C) $\cos^{-1}(\sin\theta_C)$

(D) $\tan^{-1}(\sin\theta_C)$

Correct Answer: (D) $\tan^{-1}(\sin\theta_C)$

Solution:

Concept:

- For light going from a denser medium of index n_d to a rarer medium of index n_r, Snell's law is $n_d \sin A = n_r \sin r$.
- The critical angle satisfies $\sin\theta_C = n_r/n_d = 1/n_{12}$. Perpendicular reflected and refracted rays give the Brewster condition $A + r = 90°$.

Step 1: Use the perpendicular-ray geometry The reflected ray makes the same angle A with the normal as the incident ray. When the reflected and refracted rays are perpendicular, the refraction angle is $r = 90° - A$. Hence $\sin r = \cos A$. This condition is about the two outgoing directions and does not mean the incident angle equals the critical angle.

Step 2: Combine the geometry with Snell's law Insert $\sin r = \cos A$ into the refraction law:

$$n_d \sin A = n_r \cos A \quad \Longrightarrow \quad \tan A = \frac{n_r}{n_d}.$$

Replacing the index ratio using the critical angle then gives

$$\tan A = \sin\theta_C, \qquad A = \tan^{-1}(\sin\theta_C).$$

The physical incident angle lies between zero and 90°, so the principal inverse tangent is the relevant branch.

Step 3: Distinguish the alternative expressions Option (C) would imply a cosine relation instead of the tangent relation obtained directly from Snell's law. Options (A) and (B) are reciprocals of inverse-angle values and do not represent the required inverse trigonometric operation. Since $n_r < n_d$, the tangent is less than one and the resulting Brewster angle is below 45°, consistent with incidence from the denser side.

Step 4: Conclusion The required angle is $A = \tan^{-1}(\sin\theta_C)$. Hence, the correct option is (D).

Quick Tip to Solve: For Brewster incidence use $\tan A = n_{\text{transmitted}}/n_{\text{incident}}$; express that same ratio as $\sin\theta_C$.

43. Time (T), velocity (C) and angular momentum (h) are chosen as fundamentalquantities instead of mass, length and time. In terms of these, the dimensions of mass would be : [4 marks]

(A) [M] = [T^{-1} C^{-2} h]

(B) [M] = [T^{-1} C^{2} h]

(C) [M] = [T^{-1} C^{-2} h^{-1}]

(D) [M] = [T C^{-2} h]

Correct Answer: (A) [M] = [T^{-1} C^{-2} h]

Solution:

Concept:

- Dimensions describe the physical nature of a quantity independently of the chosen units. Angular momentum has dimensions $[h] = ML^2T^{-1}$, while velocity has $[C] = LT^{-1}$.
- To express mass in the new fundamental quantities, write $[M] = [T^aC^bh^c]$ and equate the powers of the original mass, length and time dimensions.

Step 1: Expand the proposed dimensions The symbol T in the new basis still represents time; C represents velocity, not an additional length. Substituting their dimensions gives

$$[T^aC^bh^c] = T^a(LT^{-1})^b(ML^2T^{-1})^c = M^cL^{b+2c}T^{a-b-c}.$$

The required result is pure mass, so its powers of length and time must both vanish.

Step 2: Equate and solve the exponents Comparing with $M^1L^0T^0$, the mass equation gives $c = 1$. The length equation then gives $b + 2c = 0$, hence $b = -2$. Finally, the time equation is $a - b - c = 0$, so $a + 2 - 1 = 0$ and $a = -1$. Thus

$$[M] = [T^{-1}C^{-2}h].$$

As a direct check, h/C^2 has dimensions MT; dividing once more by time leaves M.

Step 3: Check the other proposed combinations Option B uses C^2, which increases rather than cancels the length power. Option C has h^{-1}, producing inverse mass instead of mass. Option D has T rather than T^{-1}, leaving an extra factor of time squared. Only option A cancels both unwanted dimensions.

Step 4: Conclusion The dimensions of mass in the chosen basis are $\boxed{[M] = [T^{-1}C^{-2}h]}$. Hence, the correct option is (A).

Quick Tip to Solve: Fix the angular-momentum exponent from the mass power first; then cancel length with velocity and time last.

44. An ideal gas has molecules with 5 degrees of freedom. The ratio of specific heats at constant pressure (C_p) and at constant volume (C_v) is : [4 marks]

(A) 6

(B) $\frac{7}{2}$

(C) $\frac{5}{2}$

(D) $\frac{7}{5}$

Correct Answer: (D) $\frac{7}{5}$

Solution:

Concept:

- For an ideal gas with f active degrees of freedom, equipartition gives molar internal energy $U = \frac{f}{2}RT$, where R is the molar gas constant.
- The molar heat capacities obey $C_V = \frac{f}{2}R$ and Mayer's relation $C_P - C_V = R$. The ratio $\gamma = C_P/C_V$ is dimensionless.

Step 1: Find the constant-volume heat capacity All five stated degrees of freedom contribute equally to the thermal energy in the ideal-gas model. For one mole,

$$U = \frac{5}{2}RT.$$

At constant volume no expansion work is done, so the heat supplied changes the internal energy alone. Differentiating with respect to temperature gives

$$C_V = \left(\frac{dU}{dT}\right)_V = \frac{5}{2}R.$$

This expression is a heat capacity, not the ratio asked for.

Step 2: Include the expansion work at constant pressure At constant pressure the gas expands as its temperature rises. It therefore requires additional heat compared with constant volume. Using Mayer's relation,

$$C_P = C_V + R = \frac{5}{2}R + R = \frac{7}{2}R.$$

Taking the ratio cancels the gas constant:

$$\gamma = \frac{C_P}{C_V} = \frac{(7/2)R}{(5/2)R} = \frac{7}{5}.$$

Equivalently, the general formula is $\gamma = 1 + 2/f$.

Step 3: Distinguish a ratio from a heat-capacity coefficient The numbers $7/2$ and $5/2$ are the coefficients of R in the individual molar heat capacities, so options B and C do not give their ratio. Option A, 6, also fails the formula $1 + 2/5$. The result $7/5 = 1.4$ is greater than unity, consistent with $C_P > C_V$.

Step 4: Conclusion For five degrees of freedom, $\boxed{C_P/C_V = 7/5}$. Hence, the correct option is (D).

Quick Tip to Solve: For an ideal gas, use $\gamma = 1 + 2/f$; do not confuse C_P/R or C_V/R with their ratio.

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45. The ratio of maximum acceleration to maximum velocity in a simple harmonic motion is 10 s^{-1} . At, t = 0 the displacement is 5 m. What is the maximum acceleration ? The initial phase is $\frac{\pi}{4}$. [4 marks]

(A) 500 m/s^2

(B) 500 $\sqrt{2}m/$ s^2

(C) 750 m/s^2

(D) 750 $\sqrt{2}$m / s^2

Correct Answer: (B) 500 $\sqrt{2}m/$ s^2

Solution:

Concept:

- For displacement $x = A\sin(\omega t + \phi)$, the maximum speed is $v_{\max} = A\omega$ and the maximum acceleration magnitude is $a_{\max} = A\omega^2$.
- The ratio $a_{\max}/v_{\max} = \omega$ determines angular frequency. Initial displacement equals amplitude multiplied by the sine of the initial phase.

Step 1: Determine the angular frequency Both maxima depend on the same amplitude, so it cancels in their ratio:

$$\frac{a_{\max}}{v_{\max}} = \frac{A\omega^2}{A\omega} = \omega.$$

The specified ratio is $10\,\mathrm{s}^{-1}$, therefore $\omega = 10\,\mathrm{rad\,s}^{-1}$. This is angular frequency; introducing an extra 2π would confuse it with ordinary frequency.

Step 2: Use the initial phase to obtain the amplitude At $t = 0$, the displacement is $5\,\mathrm{m}$ and the phase is $\pi/4$. Hence

$$5 = A\sin\frac{\pi}{4} = \frac{A}{\sqrt{2}}, \qquad A = 5\sqrt{2}\,\mathrm{m}.$$

A cosine convention gives the same amplitude here because $\cos(\pi/4) = \sin(\pi/4)$. The displacement at the initial instant is not the amplitude, since the particle is not at an extreme position.

Step 3: Calculate and compare the acceleration The required maximum is

$$a_{\max} = \omega^2 A = (10)^2(5\sqrt{2}) = 500\sqrt{2}\,\mathrm{m\,s}^{-2}.$$

Option A would follow from incorrectly taking $A = 5\,\mathrm{m}$. The coefficient 750 in options C and D cannot follow from the measured angular frequency and the stated initial displacement. As a consistency

check, $v_{\max} = 50\sqrt{2}\,\mathrm{m\,s^{-1}}$, whose ratio with the acceleration is indeed $10\,\mathrm{s^{-1}}$.

Step 4: Conclusion The maximum acceleration is $\boxed{500\sqrt{2}\,\mathrm{m\,s^{-2}}}$. Hence, the correct option is (B).

Quick Tip to Solve: First divide the SHM maxima to find ω; use the initial phase separately to recover the amplitude.

46. What is the conductivity of a semiconductor sale having electron concentration of $5 \times 10^{18}\ m^{-3}$, hole concentration of $5 \times 10^{19}\ m^{-3}$, electron mobility of 2.0 $m^2\ V^{-1}\ s^{-1}$ and hole mobility of 0.01 $m^2\ V^{-1}\ s^{-1}$?

(Take charge of electronas 1.6 $\times$ 10 $^{-19}$ c) [4 marks]

(A) 1.68 $(\Omega\text{-m})^{-1}$

(B) 1.83 $(\Omega\text{-m})^{-1}$

(C) 0.59 $(\Omega\text{-m})^{-1}$

(D) 1.20 $(\Omega\text{-m})^{-1}$

Correct Answer: (A) 1.68 $(\Omega\text{-m})^{-1}$

Solution:

Concept:

- Electrons and holes both carry current in a semiconductor. With concentrations n, p and positive mobilities μ_e, μ_h, the conductivity is $\sigma = e(n\mu_e + p\mu_h)$.
- Conductivity contributions add because conventional electron and hole currents point along the electric field. The sign of the electron charge does not require subtracting its contribution.

Step 1: Evaluate the two carrier contributions The given electron concentration and mobility are $n = 5 \times 10^{18}\,\mathrm{m^{-3}}$ and $\mu_e = 2.0\,\mathrm{m^2\,V^{-1}\,s^{-1}}$. Their product is

$$n\mu_e = (5 \times 10^{18})(2.0) = 1.0 \times 10^{19}.$$

For holes, $p = 5 \times 10^{19}\,\mathrm{m^{-3}}$ and $\mu_h = 0.01\,\mathrm{m^2\,V^{-1}\,s^{-1}}$, giving

$$p\mu_h = (5 \times 10^{19})(0.01) = 5 \times 10^{17}.$$

Although holes are more numerous, their much smaller mobility makes their current contribution smaller.

Step 2: Multiply the sum by the elementary charge Using $e = 1.6 \times 10^{-19}\,\mathrm{C}$,

$$\begin{aligned}\sigma &= (1.6 \times 10^{-19})(1.0 \times 10^{19} + 5 \times 10^{17}) \\ &= (1.6 \times 10^{-19})(1.05 \times 10^{19}) \\ &= 1.68\,\mathrm{S\,m^{-1}}.\end{aligned}$$

Separately, the electron and hole contributions are 1.60 and $0.08\,\mathrm{S\,m^{-1}}$. Their sum checks the arithmetic.

Step 3: Check the units and remaining options Since $\mathrm{C/(V\,s)} = \mathrm{A/V} = \mathrm{S}$, the formula has units $\mathrm{S\,m^{-1}} = (\Omega\,\mathrm{m})^{-1}$, exactly those in the options. The value 0.59 in option C is close to the reciprocal and would correspond to resistivity, not conductivity. Neither 1.83 nor 1.20 equals the sum of the two carrier conductivities.

Step 4: Conclusion The conductivity is $\boxed{1.68\,(\Omega\,\mathrm{m})^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Compute $en\mu_e$ and $ep\mu_h$ separately and add; a large concentration does not ensure a large current contribution.

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47. There is a uniform electrostatic field in a region. The potential at various points on a small sphere centred at P, in the region, is found to vary between the limits 589.0 V to 589.8 V. What is the potential at a point on the sphere whose radius vector makes an angle of 60° with the direction of the field ? [4 marks]

(A) 589.5 V

(B) 589.2 V

(C) 589.4 V

(D) 589.6 V

Correct Answer: (B) 589.2 V

Solution:

Concept:

- For a uniform electric field, the potential at radius vector $\mathbf{r}$ from the centre is $V(\mathbf{r}) = V_P - \mathbf{E}\cdot\mathbf{r} = V_P - Er\cos\theta$.
- On a sphere of fixed radius, the extrema are $V_{\min} = V_P - Er$ and $V_{\max} = V_P + Er$. The centre potential is their average and Er is half their difference.

Step 1: Find the central potential and the field-radius product The two extreme potentials are $589.0\,\mathrm{V}$ and $589.8\,\mathrm{V}$. They occur at opposite ends of the diameter parallel to the field, because the scalar product has its largest positive and negative values there. Consequently,

$$V_P = \frac{589.0 + 589.8}{2} = 589.4\,\mathrm{V}, \qquad Er = \frac{589.8 - 589.0}{2} = 0.4\,\mathrm{V}.$$

Neither the sphere radius nor the field magnitude is needed individually.

Step 2: Project the radius along the field The required radius vector makes 60° with the field, so its component along the field is $r\cos 60^\circ = r/2$. Potential decreases in the field direction. Therefore

$$\begin{aligned} V &= V_P - Er\cos 60^\circ \\ &= 589.4 - (0.4)\left(\frac{1}{2}\right) \\ &= 589.2\,\mathrm{V}. \end{aligned}$$

The minus sign is essential: the requested point lies on the forward-field side of the plane through the

centre perpendicular to the field.

Step 3: Use the angular positions to reject alternatives The central value 589.4 V would occur at 90°, so option C cannot answer the stated angle. The value 589.6 V would correspond to 120°, with the radius partly opposite the field, ruling out option D. Option A is also above the centre potential and therefore has the wrong sign of the potential change for an acute angle with the field.

Step 4: Conclusion The given extrema imply a centre potential of 589.4 V and a 0.2 V decrease at the stated angle, so the printed question gives 589.2 V rather than the key's 589.4 V. The potential at the specified point is $\boxed{589.2\,\text{V}}$. Hence, the correct option is (B).

Quick Tip to Solve: Take the mean of the extreme potentials, then subtract half their separation multiplied by $\cos\theta$.

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48. A 1 kg block attached to a spring vibrates with a frequency of 1 Hz on a frictionless horizontal table. Two springs identical to the original spring are attached in parallel to an 8 kg block placed on the same table. So, the frequency of vibration of the 8 kg block is : [4 marks]

(A) $\frac{1}{4}Hz$

(B) $\frac{1}{2\sqrt{2}}Hz$

(C) $\frac{1}{2}Hz$

(D) $2\ Hz$

Correct Answer: (C) $\frac{1}{2}Hz$

Solution:

Concept:

- The oscillation frequency of a mass-spring system is $f = (2\pi)^{-1}\sqrt{k/m}$, where k is the effective spring constant and m the oscillating mass.
- For identical springs in parallel, both undergo the same displacement and their restoring forces add. Two springs of stiffness k therefore have effective stiffness $2k$.

Step 1: Express the first oscillator frequency Let each original spring have spring constant k. For the first block, $m_1 = 1\,\text{kg}$ and $f_1 = 1\,\text{Hz}$, so

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m_1}}.$$

There is no need to calculate k numerically because it will cancel in a frequency ratio. The frictionless horizontal table ensures there is no damping or changing gravitational component along the motion.

Step 2: Form the frequency ratio for the parallel pair The second block has mass $m_2 = 8m_1$ and its two identical springs act in parallel, so $k_2 = 2k$. Consequently,

$$\frac{f_2}{f_1} = \sqrt{\frac{k_2/m_2}{k/m_1}} = \sqrt{\frac{2k/(8m_1)}{k/m_1}} = \sqrt{\frac{2}{8}} = \frac{1}{2}.$$

Multiplying by the original frequency yields $f_2 = (1/2)(1\,\text{Hz}) = 0.5\,\text{Hz}$. The increased stiffness partly

offsets, but does not overcome, the eightfold increase in mass.

Step 3: Check the alternative spring arrangements Option A, $1/4\,\text{Hz}$, would result from using the series stiffness $k/2$ instead of the parallel stiffness. Option B, $1/(2\sqrt{2})\,\text{Hz}$, ignores the doubling of stiffness and changes only the mass. Option D predicts an increase even though the stiffness-to-mass ratio decreases by a factor of four. Only option C matches the correct effective spring constant.

Step 4: Conclusion The new oscillation frequency is $\boxed{\frac{1}{2}\,\text{Hz}}$. Hence, the correct option is (C).

Quick Tip to Solve: For spring-frequency comparisons, take the square root of the stiffness ratio divided by the mass ratio; parallel springs add stiffness.

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49. The maximum velocity of the photoelectrons emitted from the surface is v when light of frequency n falls on a metal surface. If the incident frequency is increased to 3n, the maximum velocity of the ejected photoelectrons will be : [4 marks]

(A) less than $\sqrt{3}$ v

(B) v

(C) more than $\sqrt{3}\,v$

(D) equal to $\sqrt{3}\,v$

Correct Answer: (C) more than $\sqrt{3}\,v$

Solution:

Concept:

- For incident frequency n, Einstein's photoelectric equation is $\frac{1}{2}mv^2 = hn - W$, where $W > 0$ is the metal work function and h is Planck's constant.
- After changing the frequency to $3n$, the same metal retains the same work function. Compare kinetic energies before taking the square root to compare maximum speeds.

Step 1: Write the two maximum kinetic energies Initially, the maximum electron kinetic energy is

$$K_1 = \frac{1}{2}mv^2 = hn - W.$$

Emission with nonzero speed means $hn > W$. Calling the new maximum speed v_2, the higher incident frequency gives

$$K_2 = \frac{1}{2}mv_2^2 = 3hn - W.$$

The photon energy triples, but the subtraction of the fixed work function occurs only once in each experiment; it does not triple with frequency.

Step 2: Compare the second energy with three times the first Three times the first kinetic energy is $3K_1 = 3hn - 3W$. Hence

$$K_2 - 3K_1 = (3hn - W) - (3hn - 3W) = 2W > 0.$$

It follows that

$$\frac{1}{2}mv_2^2 > 3\left(\frac{1}{2}mv^2\right), \qquad v_2 > \sqrt{3}\,v.$$

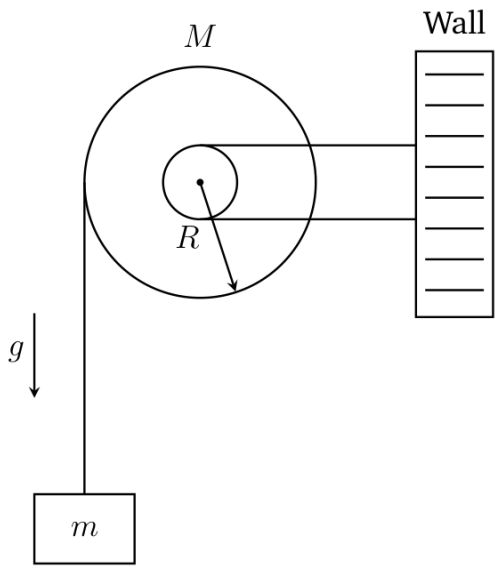
Equivalently, $v_2^2 = 3v^2 + 4W/m$, which displays the positive extra contribution explicitly. Both speeds are nonnegative magnitudes, so taking their positive square roots preserves the inequality.

Step 3: Reject the competing speed relations Equality with $\sqrt{3}\,v$, option D, would require zero work function, contrary to the positive energy needed to remove an electron from a metal. Option A reverses the established inequality. Option B ignores the extra photon energy and would imply unchanged maximum kinetic energy despite increasing the frequency. The comparison does not require knowing the numerical value of the work function, only its positivity.

Step 4: Conclusion The new maximum photoelectron speed is $\boxed{v_2 > \sqrt{3}\,v}$. Hence, the correct option is (C).

Quick Tip to Solve: When photon frequency is multiplied, compare K_2 with the same multiple of K_1; the unchanged work function decides the inequality.

50. A uniform disc of radius R and mass M is free to rotate only about its axis. A string is wrapped over its rim and a body of mass m is tied to the free end of the string as shown in the figure. The body is released from rest. Then the acceleration of the body is : [4 marks]



(A) $\frac{2\,mg}{2\,m+M}$
(B) $\frac{2\,Mg}{2\,m+M}$
(C) $\frac{2\,mg}{2\,M+m}$
(D) $\frac{2\,Mg}{2\,M+M}$

Correct Answer: (A) $\frac{2\,mg}{2\,m+M}$

Solution:

Concept:

- For the hanging body, Newton's second law gives $mg - T = ma$, taking downward as positive. The tension T is smaller than its weight because the body accelerates downward.
- A uniform disc about its symmetry axis has moment of inertia $I = MR^2/2$. The string exerts torque TR, and no slipping gives $a = R\alpha$.

Step 1: Relate the tension to the angular acceleration The fixed axle supports the disc but contributes no torque about its own axis. The tension is the tangential force that turns the disc, so

$$TR = I\alpha = \frac{MR^2}{2}\frac{a}{R}.$$

Cancelling the nonzero radius gives $T = Ma/2$. The disc therefore behaves as an additional effective translational mass $M/2$, rather than M.

Step 2: Solve the translational equation For the suspended mass, weight acts downward and tension upward. Substitute the tension found above into its equation:

$$mg - \frac{Ma}{2} = ma.$$

Therefore

$$mg = \left(m + \frac{M}{2}\right)a, \qquad a = \frac{2mg}{2m + M}.$$

The radius cancels because both the rotational inertia and the torque-to-acceleration relation contain the corresponding powers of R.

Step 3: Check the limiting cases and the alternatives As $M \to 0$, the disc has negligible rotational inertia and $a \to g$. For $M > 0$, $0 < a < g$, as expected. Options (B) and (D) incorrectly put the disc mass in the driving gravitational term; the hanging mass supplies the weight. Option (C) weights the two inertial contributions incorrectly and does not give the free-fall limit when $M \to 0$.

Step 4: Conclusion The body accelerates downward with $a = 2mg/(2m + M)$. Hence, the correct option is (A).

Quick Tip to Solve: Use $a = mg/(m + I/R^2)$ for a hanging mass driving a fixed pulley without slipping; for a uniform disc, $I/R^2 = M/2$.

51. An object is dropped from a height h from the ground. Every time it hits the ground it looses 50% of its kinetic energy. The total distance covered as t $\to \infty$ is : [4 marks]

(A) 3h

(B) ∞

(C) $\frac{5}{3}$h

(D) $\frac{8}{3}$h

Correct Answer: (A) 3h

Solution:

Concept:

- At the ground the kinetic energy before a bounce is converted into the upward kinetic energy retained after impact. The maximum rebound height is proportional to that retained energy.
- If half the kinetic energy is retained at each collision, successive rebound heights form a geometric progression with ratio $1/2$. Each rebound height contributes twice to the distance.

Step 1: Find the first rebound height The object initially falls a distance h, gaining kinetic energy mgh. Immediately after the first collision it retains half of that energy, $mgh/2$. At the top of the next rise, this has become gravitational potential energy:

$$mgh_1 = \frac{1}{2}mgh, \qquad h_1 = \frac{h}{2}.$$

The energy fraction is a height fraction, not a speed fraction. The rebound speed is multiplied by $1/\sqrt{2}$.

Step 2: Sum all the rises and subsequent falls After the next impact the rebound height is $h_2 = h/4$, and in general $h_k = h/2^k$. There is only one initial fall from h. Every subsequent rebound includes both an ascent and a descent, so

$$S = h + 2\left(\frac{h}{2} + \frac{h}{4} + \frac{h}{8} + \cdots\right).$$

The bracket is a convergent geometric series:

$$\sum_{k=1}^{\infty} \frac{h}{2^k} = \frac{h/2}{1 - 1/2} = h.$$

Thus $S = h + 2h = 3h$.

Step 3: Distinguish distance from the number of impacts There may be infinitely many idealised bounces, but their travel distances decrease geometrically, so the total distance is finite. This rules out (B). The fractions in (C) and (D) do not result from a retained-energy fraction of one half together with the required two-way travel of every rebound. Counting just the upward distances would omit the intervening downward journeys.

Step 4: Conclusion The total distance covered is $3h$. Hence, the correct option is (A).

Quick Tip to Solve: For a retained kinetic-energy fraction r, rebound heights have ratio r, and the total distance is $h + 2hr/(1 - r)$.

52. A single slit of width b is illuminated by a coherent monochromatic light of wavelength λ. If the second and fourthminima in the diffraction pattern at a distance 1 m from the slit are at 3 cm and 6 cm respectively from the central maximum, what is the width of the central maximum ? (i.e. distance between first minimum on either side of the central maximum) [4 marks]

(A) 1.5 cm

(B) 3.0 cm

(C) 4.5 cm

(D) 6.0 cm

Correct Answer: (B) 3.0 cm

Solution:

Concept:

- For a slit of width b, diffraction minima satisfy $b\sin\theta_n = n\lambda$, with $n = 1, 2, 3, \ldots$. Near the axis, $y_n = nD\lambda/b$.
- The central bright maximum extends from the first minimum on one side to the first minimum on the other side. Its full width is $W = 2D\lambda/b = 2y_1$.

Step 1: Obtain the spacing of successive minima The stated second minimum lies $3\,\text{cm}$ from the centre and the fourth lies $6\,\text{cm}$ from it. Their ratio agrees with $y_n \propto n$ in the small-angle diffraction pattern. Thus

$$\frac{D\lambda}{b} = \frac{y_2}{2} = \frac{3\,\text{cm}}{2} = 1.5\,\text{cm}.$$

The fourth minimum gives the same result, $y_4/4 = 6/4 = 1.5\,\text{cm}$. At a screen distance of one metre these angles are small, justifying the usual screen-coordinate formula.

Step 2: Measure across the central maximum The first minima occur at $y = +1.5\,\text{cm}$ and $y = -1.5\,\text{cm}$. The required width is the difference of these coordinates, not the distance to just one minimum:

$$W = (1.5) - (-1.5) = 3.0\,\text{cm}.$$

Equivalently, the central maximum is twice as wide as the separation of neighbouring minima on one side.

Step 3: Compare the four proposed widths Option (A), $1.5\,\text{cm}$, is only the half-width. Option (C), $4.5\,\text{cm}$, would span three one-sided minimum spacings. Option (D), $6.0\,\text{cm}$, is the distance of the fourth minimum from the centre and would double the actual central width. Only (B) measures between the first dark fringes on opposite sides.

Step 4: Conclusion The full width of the central maximum is $3.0\,\text{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: Find y_n/n from any minimum; multiply by two for the full central width. Do not mistake the first-minimum distance for the full width.

53. According to Bohr's theory, the time averaged magnetic field at the centre (i.e. nucleus) of a hydrogen atom due to the motion of electrons in the n^{th} orbit is proportional to : (n = principal quantum number) [4 marks]

(A) n^{-4}

(B) n^{-5}

(C) n^{-3}

(D) n^{-2}

Correct Answer: (B) n^{-5}

Solution:

Concept:

- In the Bohr model for hydrogen, the radius and speed scale as $r_n = a_0 n^2$ and $v_n = v_1/n$, where n is the principal quantum number.
- The time-averaged current of an orbiting electron has magnitude $I = e/T = ev_n/(2\pi r_n)$. At the centre of a circular current loop, $B = \mu_0 I/(2r_n)$.

Step 1: Express the orbital current in terms of speed and radius One electron passes a fixed point on its circular orbit once per period $T = 2\pi r_n/v_n$. Its charge magnitude divided by this period gives the time-averaged current. Hence

$$I_n = \frac{ev_n}{2\pi r_n} \propto \frac{n^{-1}}{n^2} = n^{-3}.$$

The conventional current is opposite to the electron motion, but this changes the field direction rather than the dependence of its magnitude on n.

Step 2: Include the magnetic-field dependence on loop radius The nucleus is at the centre of the electron orbit. Treat the time-averaged current as a circular loop and substitute into the centre-field formula:

$$B_n = \frac{\mu_0 I_n}{2r_n} = \frac{\mu_0 ev_n}{4\pi r_n^2}.$$

Since $v_n \propto n^{-1}$ and $r_n^2 \propto n^4$,

$$B_n \propto \frac{n^{-1}}{n^4} = n^{-5}.$$

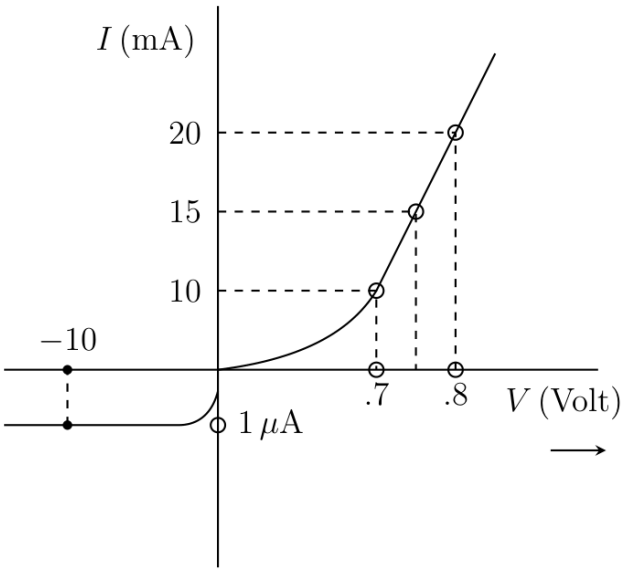
Thus both the slower orbital motion and the larger orbital radius reduce the central field.

Step 3: Identify the missing factors in the other powers The power n^{-3} in (C) describes the current alone and misses the additional inverse radius in the loop field. The power n^{-4} in (A) describes r_n^{-2} but omits the velocity factor. The power n^{-2} in (D) is only an inverse-radius scaling and does not account for the change in current. Including all factors gives only (B).

Step 4: Conclusion The time-averaged magnetic field at the nucleus is proportional to n^{-5}. Hence, the correct option is (B).

Quick Tip to Solve: Combine the loop formula and orbital period first: $B = \mu_0 ev/(4\pi r^2)$. Then insert $v \propto n^{-1}$ and $r \propto n^2$.

54. The V-I characteristic of a diode is shown in the figure. The ratio of forward to reverse bias resistance is : [4 marks]



(A) 10

(B) 10^{-6}

(C) 10^{6}

(D) 100

Correct Answer: (B) 10^{-6}

Solution:

Concept:

- The resistance inferred from a segment of a diode characteristic is $R = |\Delta V/\Delta I|$; it is the reciprocal of the slope of the plotted current against voltage.
- The forward current scale in the diagram is milliamperes, whereas the reverse current scale is microamperes. Convert both to amperes before comparing resistances.

Step 1: Read the forward conducting segment On the forward branch, the voltage changes from approximately $0.7\,\mathrm{V}$ to $0.8\,\mathrm{V}$, while the current changes from $10\,\mathrm{mA}$ to $20\,\mathrm{mA}$. The incremental forward resistance is therefore

$$R_f = \frac{0.8 - 0.7}{(20 - 10) \times 10^{-3}} = \frac{0.1}{0.01} = 10\,\Omega.$$

Using differences on the marked conducting segment avoids including the turn-on offset in the resistance calculation.

Step 2: Read the reverse branch and take the ratio The reverse-bias scale indicates a voltage change of magnitude $10\,\mathrm{V}$ for a reverse-current change of magnitude $1\,\mu\mathrm{A}$. The resistance is a positive ratio of magnitudes; the negative plotted coordinates merely denote reverse bias. Thus

$$R_r = \frac{10}{1 \times 10^{-6}} = 10^{7}\,\Omega, \qquad \frac{R_f}{R_r} = \frac{10}{10^{7}} = 10^{-6}.$$

The small ratio expresses the much easier conduction in forward bias.

Step 3: Check the scale and the direction of the requested ratio The question requests forward resistance divided by reverse resistance. Because $R_f \ll R_r$, this ratio must be much less than one. This immediately rejects (A), (C) and (D), which are all larger than one. In particular 10^6 in (C) is the reciprocal ratio. Failing to distinguish milliampere and microampere scales would give an incorrect power of ten.

Step 4: Conclusion The forward-to-reverse resistance ratio is 10^{-6}. Hence, the correct option is (B).

Quick Tip to Solve: Use voltage and current differences on each marked branch, convert mA and microampere scales, and check that R_f/R_r is far below one.

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55. A compressive force, F is applied at the two ends of a long thin steel rod. It is heated, simultaneously, such that its temperature increases by ΔT. The net change in its length is zero. Let ℓ be the length of the rod, A its area of cross-section,Y its Young's modulus, and α its coefficient of linear expansion. Then, F is equal to : [4 marks]

(A) ℓ^2 Yα ΔT

(B) ℓA Yα ΔT

(C) A Yα ΔT

(D) $\frac{AY}{\alpha\,\Delta T}$

Correct Answer: (C) A Yα ΔT

Solution:

Concept:

- For a rod of original length ℓ, free heating through ΔT causes expansion $\Delta\ell_{\text{th}} = \alpha\ell\Delta T$, where α is the linear expansion coefficient.
- Young's modulus is stress divided by longitudinal strain: $Y = (F/A)/(\Delta\ell_{\text{el}}/\ell)$. A compressive force gives a shortening of magnitude $F\ell/(AY)$.

Step 1: Write the competing length changes The temperature rise tends to increase the rod length, whereas compression tends to decrease it. In the small-strain regime these two contributions add algebraically. Taking extension as positive,

$$\Delta\ell_{\text{net}} = \alpha\ell\Delta T - \frac{F\ell}{AY}.$$

The minus sign records compression. Zero net length change does not mean zero strain from each cause; rather the thermal extension is cancelled by the elastic shortening.

Step 2: Enforce the condition of no net extension Set the total change equal to zero and solve for the force magnitude:

$$\alpha\ell\Delta T = \frac{F\ell}{AY}.$$

The positive nonzero original length cancels from both sides, giving

$$F = AY\alpha\Delta T.$$

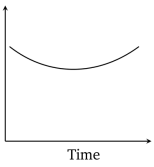
Equivalently, the compressive stress must be $F/A = Y\alpha\Delta T$. For a temperature rise this stress opposes expansion, consistent with the directions stated in the question.

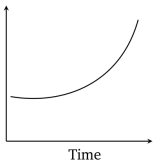
Step 3: Check dimensions and eliminate the alternatives The factor $\alpha\Delta T$ is dimensionless and AY has dimensions of force, so (C) has the correct units and dependence. Options (A) and (B) retain an incorrect length dependence; (B) also has units of force times length. Although (D) has force units, its inverse temperature dependence predicts an unbounded force as $\Delta T \to 0$, whereas the required force must tend to zero. Option (A) also incorrectly replaces the area dependence by the square of the rod length.

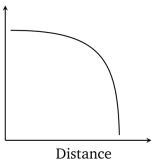
Step 4: Conclusion The compressive force needed to prevent a net change in length is $F = AY\alpha\Delta T$. Hence, the correct option is (C).

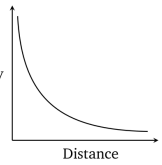
Quick Tip to Solve: When thermal expansion is exactly suppressed, the thermal strain $\alpha\Delta T$ equals the elastic compressive strain, so the thermal stress is $Y\alpha\Delta T$.

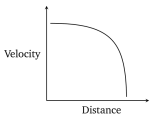
56. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity ? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- With constant acceleration a, speed and displacement obey $v^2 = u^2 + 2as$. Positive velocity means that the object continues moving in the positive direction over the shown interval.
- For a velocity against distance graph, acceleration is $a = v\,dv/ds$, rather than just the graph slope. Negative constant acceleration gives a decreasing curve with increasing steepness as v approaches zero.

Step 1: Distinguish the axes of the given graphs Options (A) and (B) plot velocity against time. For constant acceleration, $dv/dt = a$ is constant, so a velocity-time graph must be a straight line. With

negative acceleration it must have a constant negative slope while remaining above the time axis for positive velocity. Neither curved velocity-time graph satisfies this: (A) bends upward and changes slope, and (B) rises with a changing positive slope.

Step 2: Derive the shape for velocity against distance Options (C) and (D) plot velocity against distance. Let $k = |a| > 0$ and choose $s = 0$ at the initial point. Then

$$v(s) = \sqrt{u^2 - 2ks}, \qquad \frac{dv}{ds} = -\frac{k}{v}.$$

The velocity stays positive before the stopping point. As the object slows down, v decreases, so the negative slope becomes larger in magnitude. The curvature confirms this:

$$\frac{d^2v}{ds^2} = -\frac{k^2}{v^3} < 0.$$

Thus the required curve bends downward and becomes steeper to the right.

Step 3: Select the curve with the required curvature Graph (C) shows positive velocity decreasing with distance and a progressively steeper downward slope, exactly as the square-root relation predicts. Graph (D) also decreases but flattens out, so its acceleration magnitude decreases rather than remaining constant. A negative slope alone is insufficient on a velocity-distance plot: the product of velocity and slope must remain the same negative constant.

Step 4: Conclusion Graph (C) represents positive velocity with constant negative acceleration over the interval before stopping. Hence, the correct option is (C).

Quick Tip to Solve: Read the horizontal axis first: constant acceleration gives a straight line on a velocity-time plot but $v = \sqrt{u^2 - 2|a|s}$ on a velocity-distance plot.

57. In a certain region static electric and magnetic fields exist. The magnetic field is given by $\vec{B} = B_0\left(\hat{i} + 2\hat{j} - 4\hat{k}\right)$. If a test charge moving with a velocity $\vec{v} = v_0\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ experiences no force in that region, then the electric field in the region, in SI units, is : [4 marks]

(A) $\vec{E} = -v_0 B_0\left(3\hat{i} - 2\hat{j} - 4\hat{k}\right)$

(B) $\vec{E} = -v_0 B_0\left(\hat{i} + \hat{j} + 7\hat{k}\right)$

(C) $\vec{E} = v_0 B_0\left(14\hat{j} + 7\hat{k}\right)$

(D) $\vec{E} = -v_0 B_0\left(14\hat{j} + 7\hat{k}\right)$

Correct Answer: (D) $\vec{E} = -v_0 B_0\left(14\hat{j} + 7\hat{k}\right)$

Solution:

Concept:

- A charge q moving with velocity $\vec{v}$ in electric and magnetic fields experiences $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. The vector order in the magnetic term is essential.
- For a nonzero test charge to have zero total force, the electric field must satisfy $\vec{E} = -\vec{v} \times \vec{B}$. This

condition fixes both its magnitude and direction for the stated velocity.

Step 1: Apply the zero-force condition The magnetic force need not vanish individually. Its cancellation by the electric force requires

$$\vec{E} + \vec{v} \times \vec{B} = \vec{0}.$$

Factor the common scalars v_0 and B_0 out of the cross product; the remaining calculation uses only the numerical components $(3, -1, 2)$ and $(1, 2, -4)$.

Step 2: Evaluate the cross product component by component Using the right-handed Cartesian basis gives

$$\begin{aligned}\vec{v} \times \vec{B} = v_0 B_0 \big[&\{(-1)(-4) - 2(2)\}\hat{i} \\ &+ \{2(1) - 3(-4)\}\hat{j} \\ &+ \{3(2) - (-1)(1)\}\hat{k}\big] \\ = v_0 B_0 &(14\hat{j} + 7\hat{k}).\end{aligned}$$

The two contributions to the $\hat{i}$ component cancel exactly, while both other components are positive.

Step 3: Reverse the magnetic term and compare the options The required electric field is the negative of this cross product, so it has no x component and negative y and z components. Options (A) and (B) incorrectly include an x component. Option (C) has the correct component ratio but the opposite sign; it would reinforce the magnetic force rather than cancel it. The unit of $v_0 B_0$ is $\mathrm{V\,m^{-1}}$, as required for an electric field.

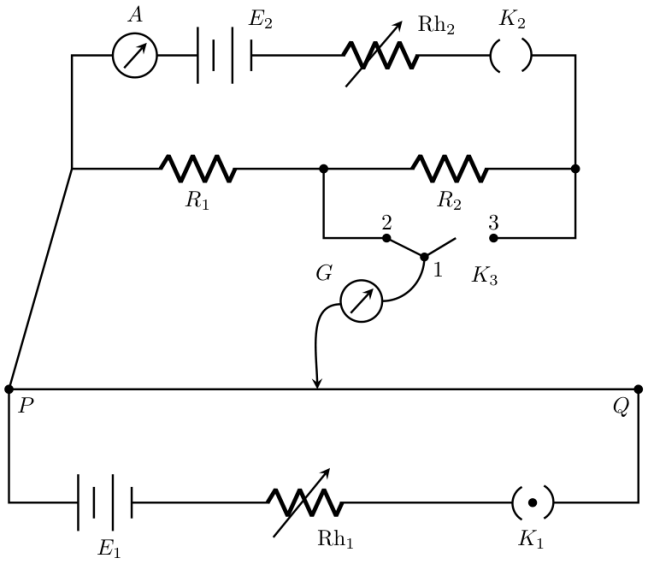
Step 4: Conclusion

$$\boxed{\vec{E} = -v_0 B_0 (14\hat{j} + 7\hat{k})\ \mathrm{V\,m^{-1}}}$$

Hence, the correct option is (D).

Quick Tip to Solve: For zero Lorentz force, first compute $\vec{v} \times \vec{B}$ in that order, then reverse its direction to obtain $\vec{E}$.

58. A potentiometer PQ is set up to compare two resistances as shown in the figure. The ammeter A in the circuit reads 1.0 A when two way key K_3 is open. The balance point is at a length ℓ_1 cm from P when two way key K_3 is plugged in between 2 and 1, while the balance point is at a length ℓ_2 cm from P when key K_3 is plugged in between 3 and 1. The ratio of two resistances $\frac{R_1}{R_2}$, is found to be : [4 marks]



(A) $\frac{l_1}{l_1+l_2}$
(B) $\frac{l_2}{l_2-l_1}$
(C) $\frac{l_1}{l_1-l_2}$
(D) $\frac{l_1}{l_2-l_1}$

Correct Answer: (D) $\frac{l_1}{l_2-l_1}$

Solution:

Concept:

- At a potentiometer null point, no current flows in the galvanometer branch. The selected potential difference equals $k\ell$, where k is the constant potential gradient along PQ.
- The resistors R_1 and R_2 carry the same series current I. Thus their individual drops are IR_1 and IR_2, and the combined drop is $I(R_1 + R_2)$.

Step 1: Identify the two selected voltage drops With key K_3 joining terminals 2 and 1, the circuit selects the potential difference across R_1. The first balance therefore gives

$$IR_1 = k\ell_1.$$

Joining terminals 3 and 1 instead selects the endpoints of the complete series combination. The second balance gives

$$I(R_1 + R_2) = k\ell_2.$$

In particular, the second length represents the sum of the two resistor drops; it is not the drop across R_2 alone.

Step 2: Subtract the balance equations The primary potentiometer circuit is unchanged, so the same gradient k applies to both measurements. At balance there is no loading through the galvanometer, so subtraction isolates the second resistor:

$$IR_2 = k(\ell_2 - \ell_1).$$

Divide the first balance relation by this difference to eliminate both the common current and the potential gradient:

$$\frac{R_1}{R_2} = \frac{IR_1}{IR_2} = \frac{k\ell_1}{k(\ell_2 - \ell_1)}.$$

The stated ammeter reading $1.0\,\text{A}$ is consistent with the method but cancels from the ratio.

Step 3: Check the sign and the alternative expressions For positive resistances, the combined drop exceeds the drop across R_1, so $\ell_2 > \ell_1$. Option (C) would produce a negative resistance ratio. Option (A) incorrectly adds the lengths although ℓ_2 already includes both drops. Option (B) has the correct denominator but uses the total drop rather than the drop across R_1 in the numerator.

Step 4: Conclusion

$$\boxed{\frac{R_1}{R_2} = \frac{\ell_1}{\ell_2 - \ell_1}}$$

Hence, the correct option is (D).

Quick Tip to Solve: Identify which resistor endpoints each key position selects; subtract a single-resistor balance length from the total length to obtain the other resistor drop.

59. A signal of frequency 20 kHz and peak voltage of 5 Volt is used to modulate a carrier wave of frequency 1.2 MHz and peak voltage 25 Volts. Choose the correct statement. [4 marks]

(A) Modulation index = 5, side frequency bands are at 1400 kHz and 1000 kHz

(B) Modulation index = 5, side frequency bands are at 21.2 kHz and 18.8 kHz

(C) Modulation index = 0.8, side frequency bands are at 1180 kHz and 1220 kHz

(D) Modulation index = 0.2, side frequency bands are at 1220 kHz and 1180 kHz

Correct Answer: (D) Modulation index = 0.2, side frequency bands are at 1220 kHz and 1180 kHz

Solution:

Concept:

- For ordinary amplitude modulation, the modulation index is $m = A_m/A_c$, where A_m is the peak signal voltage and A_c is the peak carrier voltage.
- A sinusoidal signal of frequency f_m produces upper and lower side frequencies $f_c + f_m$ and $f_c - f_m$, respectively. All frequencies must first be written in the same unit.

Step 1: Calculate the modulation index The modulating voltage has peak value $A_m = 5\,\text{V}$, whereas the carrier has peak value $A_c = 25\,\text{V}$. Therefore

$$m = \frac{5}{25} = 0.2.$$

This dimensionless value represents 20 percent modulation. It is less than unity, so the stated amplitudes do not give overmodulation. Taking the reciprocal ratio would incorrectly give 5; subtracting the ratio from unity would incorrectly give 0.8.

Step 2: Calculate the two side frequencies Convert the carrier frequency before adding or subtracting the signal frequency:

$$f_c = 1.2\,\text{MHz} = 1200\,\text{kHz}, \qquad f_m = 20\,\text{kHz}.$$

The two side frequencies are consequently

$$f_{\text{USB}} = 1200 + 20 = 1220\,\text{kHz},$$

$$f_{\text{LSB}} = 1200 - 20 = 1180\,\text{kHz}.$$

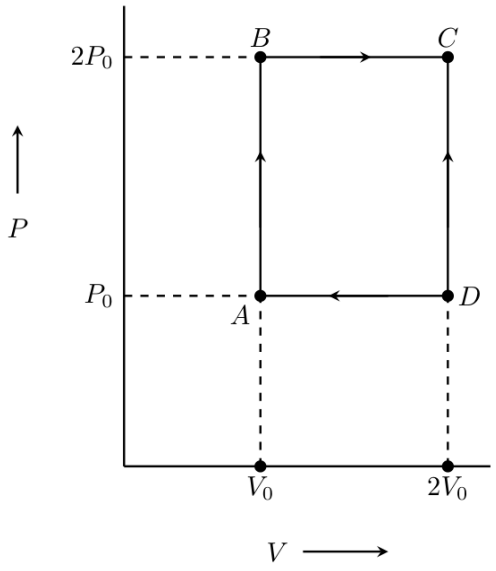
They lie symmetrically about the carrier. Equivalently, the AM expansion contains cosine terms at $\omega_c + \omega_m$ and $\omega_c - \omega_m$, which explains the sum and difference rule.

Step 3: Compare both parts of every statement Options (A) and (B) fail already because their modulation index is 5, and their frequency values also do not follow the sum and difference calculation with consistent units. Option (C) gives the correct side-frequency pair but an incorrect index of 0.8. Option (D) alone gives both the correct index and the correct side frequencies; listing the upper frequency first has no effect on correctness.

Step 4: Conclusion The modulation index is $\boxed{0.2}$, and the side frequencies are $\boxed{1220\,\text{kHz and } 1180\,\text{kHz}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use signal amplitude divided by carrier amplitude for the index; convert MHz to kHz before calculating $f_c \pm f_m$.

60. An engine operates by taking n moles of an ideal gas through the cycle ABCDA shown in figure. The thermal efficiency of the engine is : (Take C_v = 1.5 R, where R is gas constant) [4 marks]



(A) 0.24

(B) 0.15

(C) 0.32

(D) 0.08

Correct Answer: (B) 0.15

Solution:

Concept:

- The thermal efficiency of a cyclic heat engine is $\eta = W_{\text{net}}/Q_{\text{in}}$. The numerator is the enclosed area of its clockwise pressure-volume cycle.
- For the ideal gas, $\Delta U = nC_v\Delta T$ and $PV = nRT$. At constant volume $Q = nC_v\Delta T$; at constant pressure $Q = nC_p\Delta T$, with $C_p = C_v + R = 2.5R$.

Step 1: Read the corner states and the net work The diagram gives $A = (V_0, P_0)$, $B = (V_0, 2P_0)$, $C = (2V_0, 2P_0)$, and $D = (2V_0, P_0)$. Let $T_0 = P_0V_0/(nR)$. Their temperatures are $T_A = T_0$, $T_B = 2T_0$, $T_C = 4T_0$, and $T_D = 2T_0$. The cycle is clockwise; therefore the positive net output is the rectangle area:

$$W_{\text{net}} = (2P_0 - P_0)(2V_0 - V_0) = P_0V_0.$$

The vertical branches do no work.

Step 2: Add only the heat absorbed on the heating branches Heat enters during the isochoric rise $A \to B$ and the isobaric expansion $B \to C$. Their contributions are

$$Q_{AB} = n(1.5R)(2T_0 - T_0) = 1.5P_0V_0,$$

$$Q_{BC} = n(2.5R)(4T_0 - 2T_0) = 5P_0V_0.$$

Thus

$$Q_{\text{in}} = 6.5P_0V_0.$$

The remaining branches cool the gas and reject heat, so their magnitudes must not be included in this input sum. Using heat from just one of the heating branches would give an incorrect efficiency.

Step 3: Form the ratio and check energy conservation The efficiency is

$$\eta = \frac{P_0V_0}{6.5P_0V_0} = \frac{2}{13} \simeq 0.154.$$

As a check, $Q_{CD} = -3P_0V_0$ and $Q_{DA} = -2.5P_0V_0$; hence the net heat is $(6.5 - 5.5)P_0V_0 = P_0V_0$, equal to the net work. Among the choices, 0.15 matches the result. Values 0.24, 0.32 and 0.08 do not follow from the stated heat capacities and both heat-input branches.

Step 4: Conclusion

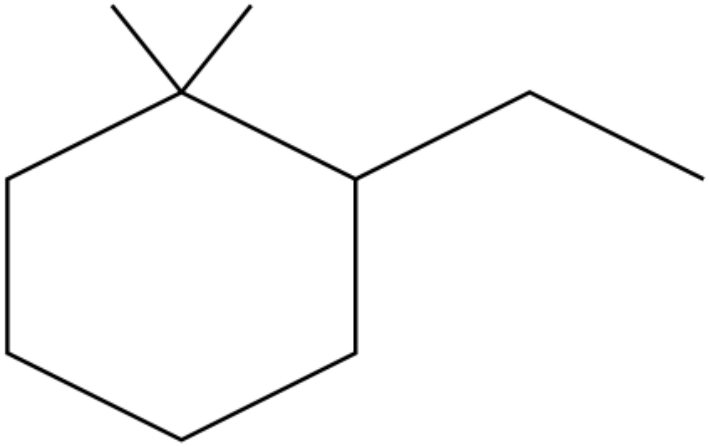
$$\boxed{\eta = \frac{2}{13} \approx 0.15}$$

Hence, the correct option is (B).

Quick Tip to Solve: For a rectangular PV engine, use its area for net work, but add heat on every heating branch when forming the efficiency denominator.

Chemistry (120 marks)

61. The IUPAC name of the following compound is : [4 marks]



(A) 1, 1-Dimethyl-2-ethylcylohexane

(B) 2-Ethyl-1, 1-dimethylcyclohexane

(C) 1-Ethyl-2,2-dimethylcyclohexane

(D) 2, 2-Dimethyl-1-ethylcycohexane

Correct Answer: (B) 2-Ethyl-1, 1-dimethylcyclohexane

Solution:

Concept:

- A saturated six-membered ring is named as cyclohexane when it is the parent framework. Alkyl groups attached to the ring are cited as prefixes with their ring-carbon locants.
- Choose the numbering that gives the lowest set of locants at the first point of difference. Prefixes are then arranged alphabetically, ignoring multiplicative prefixes such as di-.

Step 1: Identify the parent and the substituents The skeletal structure contains a cyclohexane ring. Two terminal single-bond lines leave the same ring carbon, so these represent two methyl groups on one carbon. A two-carbon side chain leaves the adjacent ring carbon and is an ethyl substituent. Thus the name must describe one ethyl group and two methyl groups on adjacent ring positions; the ring is larger than any side chain and supplies the parent name.

Step 2: Select the lowest set of locants Number the ring from the carbon bearing both methyl groups and proceed towards the ethyl group. This gives the complete substituent-locant set $(1, 1, 2)$. Starting at the ethyl-bearing carbon instead gives $(1, 2, 2)$. Comparing the two ordered sets, the first entries tie, while the second entry is 1 in the first set and 2 in the second. Therefore $(1, 1, 2)$ is required. Alphabetical precedence of ethyl does not override this lowest-locant comparison; it is used for naming order after the locants have been selected.

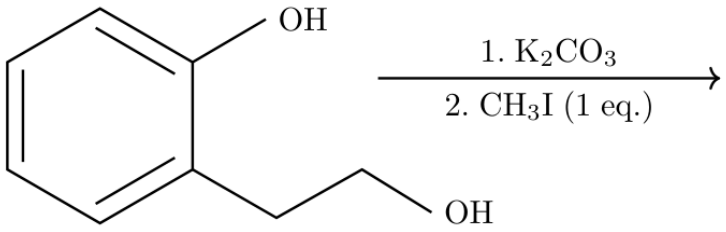
Step 3: Arrange the prefixes alphabetically Ethyl precedes methyl alphabetically; the prefix di- is ignored in this ordering. Write the ethyl prefix first, with locant 2, followed by 1,1-dimethyl and the parent cyclohexane. Option (A) reverses the alphabetical prefix order. Option (C) uses the

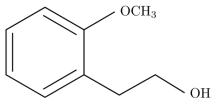
inferior locant set $(1, 2, 2)$. Option (D) combines that inferior numbering with the wrong alphabetical sequence. Only option (B) satisfies both the numbering and ordering requirements.

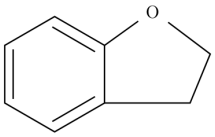
Step 4: Conclusion The IUPAC name is $\boxed{\text{2-ethyl-1,1-dimethylcyclohexane}}$. Hence, the correct option is (B).

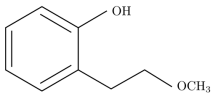
Quick Tip to Solve: First compare complete locant sets, then alphabetize substituent names; ignore di-, tri- and similar multipliers when deciding alphabetical order.

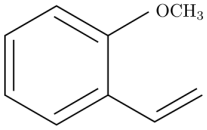
62. The major product of the following reaction is : [4 marks]

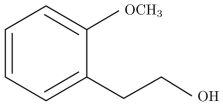


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- Phenols are appreciably more acidic than ordinary primary alcohols because a phenoxide ion is resonance stabilized. Potassium carbonate therefore preferentially generates the phenoxide nucleophile under these methylation conditions.
- An oxygen anion attacks methyl iodide by an S_N2 displacement, giving an O-methyl ether and iodide. One equivalent of methyl iodide supplies one methyl group per substrate molecule.

Step 1: Identify the two oxygen-containing groups The substrate has a phenolic $-OH$ directly attached to the benzene ring and a primary alcohol $-CH_2CH_2OH$ on the adjacent substituent. The phenolic proton is substantially more acidic than the terminal alcoholic proton. Consequently, treatment with K_2CO_3 preferentially supplies the phenoxide form rather than a comparable concentration of the side-chain alkoxide. This difference in acidity controls which oxygen reacts first.

Step 2: Methylate the phenoxide oxygen On adding one equivalent of CH_3I, the phenoxide oxygen

attacks its methyl carbon and displaces iodide:

$$\mathrm{ArO^- + CH_3I \longrightarrow ArOCH_3 + I^-}.$$

The electrophile is unhindered, so this is the standard Williamson ether-forming step. The aromatic oxygen thus becomes $-\mathrm{OCH_3}$, while the terminal side-chain group remains $-\mathrm{OH}$. No carbon skeleton rearrangement or change in aromatic bonding is required.

Step 3: Match the product structures and reject alternatives Option (A) shows exactly this phenolic O-methylation with the terminal alcohol retained. Option (C) methylates the less acidic side-chain alcohol while leaving the phenol unchanged, contrary to the selective generation of phenoxide. Option (D) shows an alkene side chain, which would require elimination of the terminal hydroxyl group; these mild basic methylation conditions do not promote that dehydration. Option (B) is a cyclic ether and would require a suitable intramolecular substitution or dehydration pathway; neither a leaving group on the side chain nor an acid-catalysed dehydration condition is supplied here.

Step 4: Conclusion The product is the aromatic methyl ether, with the $-\mathrm{CH_2CH_2OH}$ group unchanged. Hence, the correct option is (A).

Quick Tip to Solve: With a phenol and a primary alcohol in one molecule, mild carbonate base plus one equivalent of methyl iodide selectively methylates the phenolic oxygen.

63. For a reaction, A(g) $\rightarrow$ A(ℓ); ΔH= $-$ 3RT.

The correct statement for the reaction is : [4 marks]

(A) $\Delta \mathrm{H} = \Delta \mathrm{U} \neq \mathrm{O}$

(B) $\Delta \mathrm{H} = \Delta \mathrm{U} = \mathrm{O}$

(C) $|\Delta \mathrm{H}| < |\Delta \mathrm{U}|$

(D) $|\Delta \mathrm{H}| > |\Delta \mathrm{U}|$

Correct Answer: (D) $|\Delta \mathrm{H}| > |\Delta \mathrm{U}|$

Solution:

Concept:

- For an ideal-gas reaction at temperature T, neglecting the small condensed-phase volume, $\Delta H = \Delta U + \Delta n_g RT$, where Δn_g counts gaseous products minus gaseous reactants.
- Condensation removes one mole from the gaseous phase, so for $\mathrm{A\,(g) \longrightarrow A\,(l)}$, $\Delta n_g = 0 - 1 = -1$. A liquid is not counted as one gaseous product.

Step 1: Apply the gas-mole correction The relation follows from $H = U + PV$: the gas contributes RT per mole to PV, whereas the liquid contribution is negligible in this treatment. For the stated condensation, the gaseous mole count decreases by one, hence

$$\Delta H = \Delta U - RT.$$

The sign is negative because the final state has less gaseous PV than the initial state. Treating the

liquid as a gas would incorrectly make this correction vanish.

Step 2: Calculate the internal-energy change Substitute the given enthalpy change $\Delta H = -3RT$:

$$-3RT = \Delta U - RT.$$

Rearranging gives

$$\Delta U = -2RT.$$

Both changes are negative, as appropriate for an exothermic condensation, but their magnitudes differ. The enthalpy change includes the additional negative gas-volume contribution, so it is more negative than the internal-energy change.

Step 3: Compare magnitudes rather than signed values At a positive absolute temperature,

$$|\Delta H| = 3RT, \qquad |\Delta U| = 2RT,$$

and therefore $|\Delta H| > |\Delta U|$. Although the signed quantity $-3RT$ is smaller than $-2RT$, its absolute magnitude is larger. Options (A) and (B) incorrectly equate the two changes; they neglect the nonzero gas-mole correction. Option (B) also incorrectly sets both changes to zero. Option (C) reverses the magnitude inequality. The condensation relation and the given value select option (D).

Step 4: Conclusion $\Delta U = -2RT$ and $\boxed{|\Delta H| > |\Delta U|}$. Hence, the correct option is (D).

Quick Tip to Solve: Count only gaseous species in Δn_g, then remove signs before comparing the magnitudes of negative thermodynamic changes.

64. Which of the following statements is not true about partition chromatography? [4 marks]

(A) Mobile phase can be a gas

(B) Stationary phase is a finely divided solid adsorbent

(C) Separation depends upon equilibration of solute between a mobile and a stationary phase

(D) Paper chromatography is an example of partition chromatography

Correct Answer: (B) Stationary phase is a finely divided solid adsorbent

Solution:

Concept:

- Partition chromatography separates components by their different distributions between two phases; the stationary phase is a liquid held on a support.
- The distribution coefficient is $K = C_s/C_m$, where C_s and C_m are the equilibrium solute concentrations in the stationary and mobile phases.

Step 1: Identify the mechanism of separation A solute repeatedly equilibrates between the moving phase and the stationary liquid. A component with a greater affinity for the stationary phase spends more time there and travels more slowly. Components therefore separate because their partition coefficients differ, not because a bare solid surface adsorbs them. The support holds the stationary

liquid in place; it is not itself the defining separating phase.

Step 2: Check each statement The mobile phase can be a gas, as in gas-liquid chromatography, so the first statement is true. A finely divided solid adsorbent describes adsorption chromatography, making the second statement false for partition chromatography. Equilibration between the two phases is precisely the partition mechanism, so the third statement is true. Paper chromatography is a standard example: water held in the cellulose fibres acts as the stationary liquid and the developing solvent is the mobile phase. Thus the fourth statement is also true. The word "not" asks for the false description rather than an example of a valid method.

Step 3: Conclusion The statement that the stationary phase is a finely divided solid adsorbent describes adsorption, not partition chromatography. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish the stationary separating phase from its support: solid adsorption and liquid-liquid partition are different mechanisms.

65. The major product of the following reaction is : [4 marks]

$$CH_3\underset{\underset{Br}{|}}{CH}CH_2\underset{\underset{Br}{|}}{CH}CH_2CH_3 \xrightarrow[\text{heat}]{KOH,\ CH_3OH}$$

(A) $CH_2 = CHCH_2CH = CHCH_3$

(B) $CH_2 = CHCH = CHCH_2CH_3$

(C) $CH_3CH = C = CHCH_2CH_3$

(D) $CH_3CH = CH - CH = CHCH_3$

Correct Answer: (D) $CH_3CH = CH - CH = CHCH_3$

Solution:

Concept:

- Alcoholic KOH with heat promotes elimination of HBr from an alkyl bromide by removal of a beta hydrogen.
- Among accessible diene products, conjugation and greater alkene substitution generally favour the more stable major product.

Step 1: Locate the two elimination sites The starting chain is $CH_3CH(Br)CH_2CH(Br)CH_2CH_3$, namely 2,4-dibromohexane. Removal of two molecules of HBr creates two carbon-carbon double bonds while retaining the six-carbon skeleton. The bromines are on carbons 2 and 4, so each elimination must use a hydrogen on a carbon adjacent to the respective brominated carbon. Alcoholic base and heat favour this elimination process rather than replacing both bromines with hydroxyl groups.

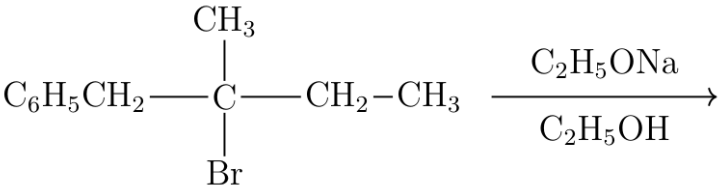
Step 2: Select the most stable accessible diene Elimination across C2-C3 and C4-C5 produces $CH_3CH{=}CHCH{=}CHCH_3$, hexa-2,4-diene. Its alternating double and single bonds form a conjugated system, and both double bonds are internal. The first candidate is an isolated diene and lacks conjugative stabilization. The second candidate is conjugated but has a less substituted terminal

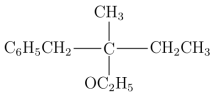
double bond. The third candidate is a cumulated diene, with adjacent double bonds sharing one carbon; it is less stable than the conjugated product under these conditions. These comparisons favour the internal conjugated diene rather than simply choosing any product with two double bonds.

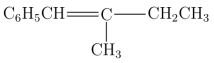
Step 3: Conclusion The major product is hexa-2,4-diene, $CH_3CH{=}CH{-}CH{=}CHCH_3$. Hence, the correct option is (D).

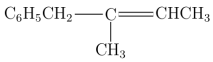
Quick Tip to Solve: For a dihalide with alcoholic base, first form the allowable double bonds, then compare conjugation and alkene substitution.

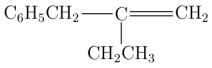
66. The major product of the following reaction is : [4 marks]

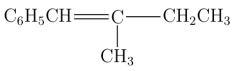


(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- Sodium ethoxide, C_2H_5ONa, in ethanol is a strong base that promotes E2 elimination of a beta hydrogen and bromide to form an alkene.
- A double bond directly attached to a phenyl ring is conjugated with the aromatic pi system. This can outweigh the stability of an isolated alternative.

Step 1: Identify the beta carbons The substrate is $C_6H_5CH_2C(Br)(CH_3)CH_2CH_3$. The carbon bearing bromine is tertiary. Its three beta positions are the benzylic CH_2, the ethyl-side CH_2, and the methyl substituent. In E2 elimination, ethoxide removes a beta hydrogen as bromide leaves, giving three possible alkene connectivities. The benzylic beta carbon offers the route to a double bond directly joined to the phenyl ring.

Step 2: Compare elimination with the other products Loss of a benzylic hydrogen gives $C_6H_5CH{=}C(CH_3)CH_2CH_3$. This alkene is both substituted and conjugated with the benzene ring, allowing delocalization beyond the new double bond. Elimination towards the ethyl side gives an isolated internal alkene, whereas elimination towards the methyl group gives a terminal methylene alkene; neither has the same direct conjugation with the ring. The ethoxy product shown in the first candidate is a substitution product, but the tertiary substrate hinders direct displacement while the strong ethoxide base favours elimination. Reading the drawings by connectivity identifies the conjugated elimination product as the second structure, not the ethoxy derivative or either

nonconjugated alkene.

Step 3: Conclusion The major product is $C_6H_5CH{=}C(CH_3)CH_2CH_3$, the alkene conjugated with the phenyl ring. Hence, the correct option is (B).

Quick Tip to Solve: Check whether the new double bond touches the aromatic ring directly; a separating methylene group interrupts conjugation.

67. Which of the following is paramagnetic ? [4 marks]

(A) NO^+

(B) CO

(C) O_2^{2-}

(D) B_2

Correct Answer: (D) B_2

Solution:

Concept:

- Paramagnetism requires at least one unpaired electron. A species in which every occupied orbital is doubly occupied is diamagnetic.
- For B_2, the bonding $\pi(2p)$ orbitals lie below $\sigma(2p)$, and Hund's rule places electrons separately in degenerate pi orbitals.

Step 1: Fill the orbitals of diboron Each boron atom supplies three valence electrons, giving six valence electrons for B_2. Four fill $\sigma(2s)$ and $\sigma^*(2s)$. The remaining two occupy the two degenerate bonding pi orbitals separately. Thus its valence configuration is

$$(\sigma 2s)^2(\sigma^* 2s)^2(\pi 2p_x)^1(\pi 2p_y)^1.$$

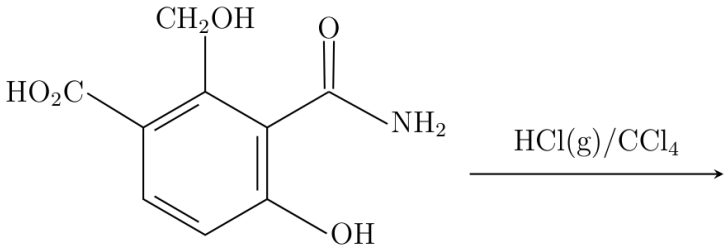
There are two unpaired electrons, so diboron is paramagnetic. Pairing these last two electrons in one pi orbital would incorrectly violate Hund's rule.

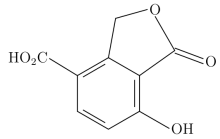
Step 2: Exclude the closed-shell species Both NO^+ and CO have 14 total electrons and are isoelectronic with N_2; their occupied molecular orbitals contain paired electrons. The peroxide ion $O_2{}^{2-}$ has two additional electrons compared with neutral oxygen. These fill the two antibonding pi orbitals completely, eliminating the unpaired electrons responsible for the paramagnetism of O_2. Therefore peroxide is diamagnetic even though oxygen itself is paramagnetic. Electron count and orbital occupation, rather than the mere presence of oxygen or a charge, settle this comparison.

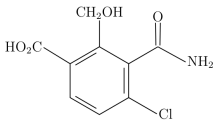
Step 3: Conclusion B_2 has two unpaired electrons in its degenerate bonding pi orbitals. Hence, the correct option is (D).

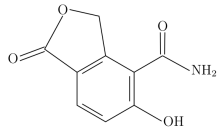
Quick Tip to Solve: For second-period diatomics, fill the pi pair with one electron in each orbital before pairing; distinguish oxygen from peroxide.

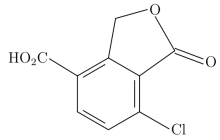
68. The major product expected from the following reaction is : [4 marks]

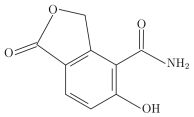


(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- Dry HCl provides acid catalysis. An ortho hydroxymethyl group and a carboxylic acid can cyclize to a five-membered lactone.
- Intramolecular formation of a favourable five-membered ring is preferred over acyl substitution at a much less reactive amide carbonyl.

Step 1: Identify the neighbouring functional groups The benzene ring bears adjacent $-COOH$ and $-CH_2OH$ groups on its left and upper positions. These can react with each other by esterification. The carbonyl oxygen of the carboxylic acid is protonated by the acid catalyst, increasing the electrophilicity of its carbonyl carbon. The oxygen of the neighbouring hydroxymethyl group then attacks intramolecularly. Counting the carboxyl carbon, ester oxygen, methylene carbon and the two aromatic junction carbons gives a five-membered ring.

Step 2: Complete cyclization and compare the drawings Proton transfer followed by loss of water produces the fused lactone, a substituted phthalide. The amide $-CONH_2$ and the phenolic $-OH$ remain attached to their original aromatic positions. The third drawing shows exactly this connectivity: lactonization consumes the carboxylic acid and hydroxymethyl groups while leaving the amide and phenol intact. The first drawing instead consumes the amide-side carbonyl and retains the carboxyl group, which is not the favoured esterification. The second drawing replaces the phenolic hydroxyl by chlorine, a transformation not caused by these conditions. The fourth combines that incorrect aryl chlorination with the wrong cyclization. Neither aromatic substitution is required for ring closure.

Step 3: Conclusion The product is the five-membered lactone formed from the neighbouring carboxylic acid and hydroxymethyl groups, with the amide and phenolic hydroxyl unchanged. Hence, the correct option is (C).

Quick Tip to Solve: Before predicting substitution by HCl, look for a nearby alcohol and carboxylic acid that can close into a five-membered lactone.

69. A solution containing a group-IV cation gives a precipitate on passing H_2S. A solution of this precipitate in dil.HCl produces a white precipitate with NaOH solution and bluish-white precipitate with basic potassium ferrocyanide. The cation is : [4 marks]

(A) Co^{2+}

(B) Ni^{2+}

(C) Mn^{2+}

(D) Zn^{2+}

Correct Answer: (D) Zn^{2+}

Solution:

Concept:

- Group-IV cations precipitate as sulphides when H_2S is passed in the appropriate basic medium; the subsequent tests identify the cation.
- Zn^{2+} gives white $Zn(OH)_2$ with alkali and a characteristic pale or bluish-white ferrocyanide precipitate.

Step 1: Use the hydroxide colour Adding sodium hydroxide to a zinc salt precipitates white zinc hydroxide:

$$Zn^{2+} + 2\,OH^- \longrightarrow Zn(OH)_2\downarrow.$$

Zinc hydroxide is amphoteric and dissolves in excess strong alkali to form zincate, although the question only requires its initial white appearance. Nickel(II) hydroxide is green, and cobalt(II) hydroxide is characteristically coloured rather than the zinc-like white precipitate described. Manganese(II) hydroxide can initially appear pale, but it oxidizes readily in air to a brown product, so the corroborating test is important.

Step 2: Confirm with ferrocyanide Zinc gives the stated bluish-white precipitate with basic potassium ferrocyanide. In the customary potassium-containing form the precipitate is represented as $K_2Zn_3[Fe(CN)_6]_2$. This second observation supports the zinc assignment rather than relying on a single hydroxide colour. The initial group precipitate is ZnS; it dissolves in dilute hydrochloric acid to regenerate a zinc salt and evolve hydrogen sulphide:

$$ZnS + 2\,HCl \longrightarrow ZnCl_2 + H_2S\uparrow.$$

Thus the sulphide formation, acid dissolution and both confirmatory observations are consistent with the same ion. None of cobalt, nickel or manganese matches the full set of zinc tests.

Step 3: Conclusion The cation is Zn^{2+}. Hence, the correct option is (D).

Quick Tip to Solve: Use the combination of a white amphoteric hydroxide and pale ferrocyanide precipitate to distinguish zinc from coloured transition-metal ions.

70. sp^3d^2 hybridization is not displayed by : [4 marks]

(A) BrF_5

(B) SF_6

(C) $[CrF_6]^{3-}$

(D) PF_5

Correct Answer: (D) PF_5

Solution:

Concept:

- In the usual main-group hybridization model, six electron-pair domains correspond to sp^3d^2, whereas five correspond to sp^3d.
- For transition-metal complexes, d^2sp^3 and sp^3d^2 both describe octahedral orbital sets but use different d shells; their order must not be treated as interchangeable.

Step 1: Compare the main-group examples In BrF_5, bromine has five bond pairs and one lone pair, giving six domains and a square-pyramidal molecular shape derived from an octahedral arrangement. Its conventional hybridization is sp^3d^2. In SF_6, six bond pairs and no lone pair likewise give six domains and octahedral geometry. In contrast, PF_5 has five bond pairs, no lone pair and trigonal-bipyramidal geometry; its conventional hybridization is sp^3d. This is the clear five-coordinate exception and the answer accepted by the question key.

Step 2: Check the literal complex alternative For $[CrF_6]^{3-}$, chromium is in oxidation state +3 and has a $3d^3$ configuration. The three electrons can remain unpaired in three 3d orbitals, leaving two 3d orbitals vacant for inner-orbital d^2sp^3 hybridization with 4s and 4p orbitals. No forced electron pairing is required. Consequently, under strict valence-bond nomenclature this complex is described as d^2sp^3, not sp^3d^2. The ligand being weak-field does not change the fact that a d-three ion already has two vacant inner d orbitals. Thus the literal wording also raises an objection to the third candidate, despite both octahedral descriptions involving six hybrid orbitals.

Step 3: Conclusion The accepted key selects (D), PF_5, because it is sp^3d and trigonal bipyramidal. Strictly, (C) is d^2sp^3 in valence-bond theory, so the printed question is not unique if the distinction between inner- and outer-orbital hybridization is enforced.

Quick Tip to Solve: Count domains for main-group molecules, but check the metal d-electron configuration before labelling a complex inner- or outer-orbital.

71. 5 g of Na_2SO_4 was dissolved in x g of H_2O. The change in freezing point was found to be 3.82°C. If Na_2SO_4 is 81.5% ionised, the value of x
(K_f for water=1.86°C kg mol^{-1}) is approximately :
(molar mass of S = 32 g mol^{-1} and that of Na = 23 g mol^{-1}) [4 marks]

(A) 15 g
(B) 25 g
(C) 45 g
(D) 65 g

Correct Answer: (C) 45 g

Solution:

Concept:

- Freezing-point depression counts solute particles: $\Delta T_f = iK_f m$, where m is the number of moles of solute formula units per kilogram of solvent.
- For a solute producing v ions per formula unit with degree of dissociation α, the van't Hoff factor is $i = 1 + (v-1)\alpha$.

Step 1: Calculate the effective particle factor Sodium sulfate dissociates as $\mathrm{Na_2SO_4} \longrightarrow 2\,\mathrm{Na^+} + \mathrm{SO_4}^{2-}$. Thus one dissociated formula unit produces three particles, whereas an undissociated formula unit remains one particle. With $\alpha = 0.815$,

$$i = (1 - 0.815) + 3(0.815) = 1 + 2(0.815) = 2.63.$$

Using three directly would incorrectly treat ionization as complete.

Step 2: Express molality using the unknown solvent mass The molar mass of sodium sulfate is $2(23) + 32 + 4(16) = 142\,\mathrm{g\,mol^{-1}}$. The amount dissolved is $5/142$ mol. Since x is in grams, its mass in kilograms is $x/1000$, giving

$$m = \frac{5/142}{x/1000} = \frac{5000}{142x}\,\mathrm{mol\,kg^{-1}}.$$

The denominator uses solvent mass, not the mass of the final solution.

Step 3: Substitute and compare the options Insert the measured depression and water's freezing constant:

$$3.82 = 2.63(1.86)\frac{5000}{142x}.$$

Therefore

$$x = \frac{2.63(1.86)(5000)}{142(3.82)} = 45.09\,\mathrm{g}.$$

The nearest offered mass is 45 g. The values 15, 25 and 65 g do not satisfy the measured depression for the specified degree of ionization.

Step 4: Conclusion The required solvent mass is approximately $45\,\mathrm{g}$. Hence, the correct option is (C).

Quick Tip to Solve: First replace complete dissociation by $i = 1 + 2\alpha$; then convert the solvent mass from grams to kilograms in molality.

72. Consider the following ionization enthalpies of two elements 'A' and 'B' .

Element	Ionization enthalpy (kJ/mol)		
	1st	2nd	3rd
A	899	1757	14847
B	737	1450	7731

Which of the following statements is correct ? [4 marks]

(A) Both 'A' and 'B' belong to group-1where 'B' comes below 'A'.

(B) Both 'A' and 'B' belong to group-1 where 'A' comes below 'B'.

(C) Both 'A' and 'B' belong to group-2 where 'B' comes below 'A'.

(D) Both 'A' and 'B' belong to group-2 where 'A' comes below 'B'.

Correct Answer: (C) Both 'A' and 'B' belong to group-2 where 'B' comes below 'A'.

Solution:

Concept:

- Successive ionization enthalpies measure the energy needed to remove electrons one at a time from gaseous atoms and ions. A very large jump signals that removal has begun from a closed inner shell.
- For a group-2 atom, two outer electrons can be removed before the third removal breaks the noble-gas core. Down a group, increased size and shielding generally lower ionization enthalpy.

Step 1: Locate the large jump for both elements For A, the first two values are 899 and 1757, whereas the third rises to 14847, all in $\mathrm{kJ\,mol^{-1}}$. For B, the first two values are 737 and 1450, followed by 7731. Thus the especially difficult removal is the third in both cases, after two outer electrons have already been lost.

$$\frac{IE_3}{IE_2}(A) = \frac{14847}{1757} \approx 8.45, \qquad \frac{IE_3}{IE_2}(B) = \frac{7731}{1450} \approx 5.33.$$

This is the signature of two valence electrons, so both elements belong to group 2. A group-1 element would instead show its characteristic large jump between the first and second values.

Step 2: Use the lower enthalpies to place B below A Both the first and second ionization enthalpies of B are smaller than those of A: $737 < 899$ and $1450 < 1757$. The outer electrons of B are therefore less strongly held. Within group 2 this corresponds to a larger atom with greater shielding, so B lies below A. The data are consistent with A being beryllium and B being magnesium, although identifying their names is not needed. Options (A) and (B) give the wrong group; option (D) reverses the vertical order.

Step 3: Conclusion Both A and B belong to group 2, and B comes below A. Hence, the correct option is (C).

Quick Tip to Solve: First locate the sharp jump to count valence electrons; only then compare the earlier ionization enthalpies to decide the order down the group.

73. The pair of compounds having metals in their highest oxidation state is : [4 marks]

(A) MnO_2 and CrO_2Cl_2

(B) $[NiCl_4]^{2-}$ and $[CoCl_4]^{2-}$

(C) $[Fe(CN)_6]^{3-}$ and $[Cu(CN)_4]^{2-}$

(D) $[FeCl_4]^{-}$ and Co_2O_3

Correct Answer: No listed pair has both metals in their highest oxidation states; option A has Mn(IV), not Mn(VII).

Solution:

Concept:

- The sum of oxidation numbers in a species equals its overall charge. Oxygen is -2, and chloride and cyanide ligands each have charge -1 in these species.
- To test a pair, calculate both metal oxidation numbers and then check whether a higher oxidation state is known for either metal. A pair fails when even one metal is not at its maximum.

Step 1: Test the manganese and chromium pair For MnO_2, $x + 2(-2) = 0$, so manganese is $+4$. For CrO_2Cl_2, $x + 2(-2) + 2(-1) = 0$, so chromium is $+6$. Chromium is at its highest usual oxidation state, but manganese reaches $+7$ in MnO_4^-. Therefore the literal option (A) does not contain two metals at their highest oxidation states.

Step 2: Calculate the states in the remaining pairs In option (B), $x - 4 = -2$ gives $+2$ for both nickel and cobalt; cobalt in Co_2O_3 already provides a higher state, $+3$. In option (C), $x - 6 = -3$ gives iron $+3$, and $x - 4 = -2$ gives copper $+2$. In option (D), $x - 4 = -1$ gives iron $+3$, while $2x - 6 = 0$ gives cobalt $+3$.

Step 3: Distinguish a common state from the highest state Iron(III) is common, but it is not iron's highest oxidation state: ferrate, FeO_4^{2-}, contains iron in $+6$, since $x - 8 = -2$. Thus iron rules out both (C) and (D), while cobalt rules out (B). All four options fail the stated requirement. Selecting a familiar stable oxidation state in place of the highest oxidation state would change the question being answered.

Step 4: Conclusion As printed, none of the four pairs satisfies the condition. The supplied key selects (A), but MnO_2 contains Mn(IV), below Mn(VII) in permanganate, so that key is incompatible with the literal formula. No unique correct listed option exists.

Quick Tip to Solve: For a highest-oxidation-state question, test both members; one counterexample such as Mn(VII) or Fe(VI) is enough to reject a pair.

74. The rate of a reaction A doubles on increasing the temperature from 300 to 310 K. By how much, the temperature of reaction B should be Increased from 300 K so that rate doubles if activation energy of the reaction B is twice to that of reaction A. [4 marks]

(A) 9.84 K

(B) 4.92 K

(C) 2.45 K

(D) 19.67 K

Correct Answer: (B) 4.92 K

Solution:

Concept:

- The Arrhenius equation is $k = A\exp(-E_a/RT)$. Comparing two temperatures for one reaction gives $\ln(k_2/k_1) = (E_a/R)(1/T_1 - 1/T_2)$.
- With concentration conditions unchanged, doubling the reaction rate means doubling its rate constant. For both reactions the logarithmic increase is consequently $\ln 2$.

Step 1: Use the temperature change of reaction A Let the activation energy of reaction A be E. Its rate doubles between 300 K and 310 K, so

$$\ln 2 = \frac{E}{R}\left(\frac{1}{300} - \frac{1}{310}\right).$$

The reciprocal-temperature difference is $10/(300 \times 310)$. We need not calculate E separately because it cancels when the two reactions are compared.

Step 2: Write the corresponding equation for reaction B Let B reach a doubled rate at temperature T. Its activation energy is $2E$, and its starting temperature remains 300 K:

$$\ln 2 = \frac{2E}{R}\left(\frac{1}{300} - \frac{1}{T}\right).$$

Equating the two expressions for $\ln 2$ yields

$$\frac{1}{300} - \frac{1}{T} = \frac{1}{2}\left(\frac{1}{300} - \frac{1}{310}\right).$$

Step 3: Solve for the increase rather than the final temperature Rearranging,

$$\frac{1}{T} = \frac{1}{2}\left(\frac{1}{300} + \frac{1}{310}\right) = \frac{61}{18600}.$$

Thus $T = 18600/61 = 304.918\,\text{K}$, and $T - 300 = 4.918\,\text{K}$. Options 9.84 K and 19.67 K overestimate the necessary change, whereas 2.45 K underestimates it. A larger activation energy makes a rate more temperature-sensitive, so B should require less than A's 10 K increase.

Step 4: Conclusion The required temperature increase is approximately $4.92\,\text{K}$. Hence, the correct option is (B).

Quick Tip to Solve: For the same rate ratio, the reciprocal-temperature change is inversely proportional to activation energy; solve exactly before subtracting 300 K.

75. Among the following, the essential amino acid is : [4 marks]

(A) Alanine

(B) Valine

(C) Aspartic acid

(D) Serine

Correct Answer: (B) Valine

Solution:

Concept:

- An essential amino acid is one that the human body cannot synthesize in sufficient quantity to meet its needs. It must therefore be supplied through dietary proteins.
- Essentiality is a nutritional classification; it is not determined merely by the presence of an amino group, a carboxyl group, or the ability to form peptide bonds.

Step 1: Identify the essential amino acid among the choices Valine is an essential amino acid in humans. It is one of the branched-chain amino acids, together with leucine and isoleucine. Its side chain is an isopropyl group, $-\mathrm{CH}(\mathrm{CH}_3)_2$, attached to the alpha carbon. Human metabolism cannot construct its carbon skeleton in the amounts needed for normal protein synthesis, so an adequate dietary supply is required.

Step 2: Check why the other three choices do not qualify Alanine is non-essential: it can be formed by transamination of pyruvate. Aspartic acid is also non-essential: its carbon skeleton is obtainable from oxaloacetate by transamination. Serine can be synthesized from an intermediate of glycolysis, 3-phosphoglycerate. These biosynthetic routes distinguish options (A), (C) and (D) from valine; their importance in proteins does not make them nutritionally essential.

Step 3: Apply the nutritional definition consistently All four compounds are amino acids involved in biological chemistry, and all can occur in proteins. Consequently, selecting on the basis of protein occurrence alone would not distinguish the choices. The decisive feature is whether a sufficient supply can be made within the body. Among the listed amino acids, valine alone satisfies the requirement of having to be obtained from the diet.

Step 4: Conclusion Valine is the essential amino acid in the list. Hence, the correct option is (B).

Quick Tip to Solve: Remember valine with leucine and isoleucine as the essential branched-chain trio; alanine, aspartate and serine are synthesized metabolically.

76. If the shortest wavelength in Lyman series of hydrogen atom is A, then the longest wavelength in Paschen series of He^+ is : [4 marks]

(A) $\frac{5A}{9}$
(B) $\frac{9A}{5}$
(C) $\frac{36A}{5}$
(D) $\frac{36A}{7}$

Correct Answer: (D) $\frac{36A}{7}$

Solution:

Concept:

- For a one-electron species, the Rydberg relation is $1/\lambda = RZ^2(1/n_1^2 - 1/n_2^2)$, with $n_2 > n_1$. Hydrogen has $Z = 1$, while He^+ has $Z = 2$.
- The shortest wavelength in a series is its series limit, $n_2 \to \infty$. The longest wavelength arises from the smallest available energy gap, $n_2 = n_1 + 1$.

Step 1: Express the given Lyman limit in terms of R Lyman transitions terminate at $n_1 = 1$. For the shortest hydrogen wavelength, the initial level tends to infinity, so

$$\frac{1}{A} = R\,(1 - 0) = R.$$

Therefore the given symbol A represents $1/R$, a wavelength. It is a variable defined by the problem, not an additional atomic number.

Step 2: Identify the longest Paschen transition for helium ion Paschen transitions terminate at $n_1 = 3$. The longest wavelength is produced by $n_2 = 4$ rather than by the series limit. Since helium ion is hydrogen-like and has nuclear charge two,

$$\frac{1}{\lambda} = 4R\left(\frac{1}{9} - \frac{1}{16}\right) = 4R\frac{7}{144} = \frac{7R}{36}.$$

Both the correct pair of quantum numbers and the factor $Z^2 = 4$ are essential.

Step 3: Convert to the specified wavelength variable and test the choices Inverting the result and using $1/R = A$,

$$\lambda = \frac{36}{7R} = \frac{36A}{7}.$$

The fractions containing a denominator five do not follow from the $4 \to 3$ transition, whose squared-level difference produces seven. Omitting the helium nuclear-charge factor would instead give $144A/7$, which is also incorrect. Thus only one of the offered expressions has the proper series transition and charge dependence.

Step 4: Conclusion The longest Paschen wavelength of He^+ is $\frac{36A}{7}$. Hence, the correct option is (D).

Quick Tip to Solve: Use infinity for a series minimum wavelength and the adjacent upper level for its maximum; apply the Z^2 factor before inverting.

77. Additin of sodium hydroxide solution to a weak acid (HA)results in a buffer of pH 6. If ionition constant of HA is 10^{-5}, the ratio of salt to acid concentration in the buffer solution will be : [4 marks]

(A) 4 : 5

(B) 1 : 10

(C) 10 : 1

(D) 5 : 4

Correct Answer: (C) 10 : 1

Solution:

Concept:

- Partial neutralization of a weak acid by a strong base produces a mixture of the weak acid and its conjugate-base salt: $\mathrm{HA} + \mathrm{OH^-} \longrightarrow \mathrm{A^-} + \mathrm{H_2O}$. This mixture is an acidic buffer.
- The Henderson–Hasselbalch relation is $\mathrm{pH} = \mathrm{p}K_a + \log_{10}([\mathrm{A^-}]/[\mathrm{HA}])$. The requested salt-to-acid ratio has the conjugate base in the numerator.

Step 1: Calculate the acid constant in logarithmic form The stated ionization constant is $K_a = 10^{-5}$, so

$$\mathrm{p}K_a = -\log_{10}(10^{-5}) = 5.$$

The buffer has pH 6, one unit above the acid's pKa. It should therefore contain more conjugate base than undissociated acid. This qualitative check fixes the direction of the concentration ratio.

Step 2: Solve the buffer equation for the ratio Let $r = [\text{salt}]/[\text{acid}] = [\mathrm{A^-}]/[\mathrm{HA}]$. Substitution gives

$$6 = 5 + \log_{10} r.$$

Hence $\log_{10} r = 1$, and taking the base-ten antilogarithm gives $r = 10$. Equivalently, the equilibrium equation $K_a = [\mathrm{H^+}][\mathrm{A^-}]/[\mathrm{HA}]$, with $[\mathrm{H^+}] = 10^{-6}$, gives $r = 10^{-5}/10^{-6} = 10$.

Step 3: Compare each numerical alternative The required ratio is 10:1, matching option (C). The inverse ratio 1:10 would give pH 4, not pH 6. A ratio of 4:5 would give a pH slightly below 5, and 5:4 a pH only slightly above 5, so neither can account for a full one-unit increase above pKa. The ratio is dimensionless because both concentrations are measured in the same units.

Step 4: Conclusion The salt-to-acid concentration ratio is $10 : 1$. Hence, the correct option is (C).

Quick Tip to Solve: One pH unit above pKa means ten times as much conjugate base as acid; one unit below means the inverse ratio.

78. Excess of NaOH (aq) was added to 100 mL of $FeCl_3$ (aq) resulting into 2.14 g of $Fe(OH)_3$. The molarity of $FeCl_3$ (aq) is :

(Given molar mass of Fe = 56 g mol^{-1} and molar mass of Cl = 35.5 g mol_{-1}) [4 marks]

(A) 0.2 M

(B) 03 M

(C) 0.6 M

(D) 1.8 M

Correct Answer: (A) 0.2 M

Solution:

Concept:

- An excess of hydroxide precipitates ferric ions quantitatively as ferric hydroxide. The balanced reaction fixes the mole ratio, not the ratio of masses.
- Molarity is moles of solute per litre of solution: $c = n/V$. For the precipitate, $n = m/M_{\text{molar}}$.

Step 1: Balance the precipitation reaction The aqueous ferric chloride reacts with sodium hydroxide according to

$$\mathrm{FeCl_3(aq) + 3\,NaOH\,(aq) \longrightarrow Fe(OH)_3(s) + 3\,NaCl\,(aq).}$$

One mole of ferric chloride yields one mole of ferric hydroxide. Since sodium hydroxide is in excess, ferric chloride is the limiting reagent; no correction for incomplete precipitation is required in this stoichiometric calculation.

Step 2: Convert the precipitate mass into moles Using atomic masses 56, 16 and 1 for Fe, O and H, respectively, the molar mass of the precipitate is

$$M_{\mathrm{Fe(OH)_3}} = 56 + 3(16 + 1) = 107\,\mathrm{g\,mol^{-1}}.$$

Consequently,

$$n_{\mathrm{Fe(OH)_3}} = \frac{2.14}{107} = 0.0200\,\mathrm{mol}.$$

The original sample therefore contained $0.0200\,\mathrm{mol}$ of $\mathrm{FeCl_3}$, rather than three times this quantity; the coefficient three belongs to hydroxide.

Step 3: Use the volume of the original solution The solution volume before adding the excess reagent is $100\,\mathrm{mL} = 0.100\,\mathrm{L}$. Thus

$$c_{\mathrm{FeCl_3}} = \frac{0.0200}{0.100} = 0.200\,\mathrm{mol\,L^{-1}}.$$

The ferric chloride molar mass is unnecessary because the measured mass is that of the hydroxide. Options (B), (C) and (D) do not reproduce the measured precipitate mass for this volume.

Step 4: Conclusion The molarity of the original ferric chloride solution is $0.20\,\mathrm{M}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the formula of the weighed precipitate to find moles, then apply the balanced mole ratio to the original solute.

79. What is the standard reduction potential (E^o) for $Fe^{3+} \rightarrow Fe$?

Given that :

$Fe^{2+} + 2e^- \rightarrow Fe$; $E^o_{Fe^{2+}/Fe} = -0.47$ V

$Fe^{3+} + e^- \rightarrow Fe^{2+}$; $E^o_{Fe^{3+}/Fe^{2+}} = +0.77$ V [4 marks]

(A) − 0.057 V

(B) + 0.057 V

(C) + 0.30 V

(D) − 0.30 V

Correct Answer: (A) − 0.057 V

Solution:

Concept:

- Standard potentials are intensive quantities and cannot be added directly. The additive thermodynamic quantity is standard Gibbs energy: $\Delta G^\circ = -nFE^\circ$.
- For consecutive reductions, add the balanced half-reactions and their Gibbs energies; then divide the total by the total electron count.

Step 1: Write the two consecutive reductions The first reduction changes ferric to ferrous iron, and the second produces the metal:

$$\mathrm{Fe^{3+} + e^- \longrightarrow Fe^{2+}}, \qquad E^\circ_1 = +0.77\,\mathrm{V},$$

$$\mathrm{Fe^{2+} + 2\,e^- \longrightarrow Fe}, \qquad E^\circ_2 = -0.47\,\mathrm{V}.$$

Adding them cancels the ferrous intermediate and gives $\mathrm{Fe^{3+} + 3\,e^- \longrightarrow Fe}$. Both given reactions are already reductions in the required direction.

Step 2: Add their standard Gibbs energies For the first reaction, one mole of electrons is transferred, while the second transfers two. Hence

$$\Delta G^\circ_1 = -F(0.77) = -0.77F,$$

$$\Delta G^\circ_2 = -2F(-0.47) = +0.94F.$$

The net change is therefore

$$\Delta G^\circ_{\mathrm{net}} = (-0.77 + 0.94)F = +0.17F.$$

The positive result already shows that the overall reduction potential must be negative.

Step 3: Recover the potential for the three-electron reduction Using three electrons for the net process,

$$E^\circ_{\mathrm{Fe^{3+}/Fe}} = -\frac{\Delta G^\circ_{\mathrm{net}}}{3F} = -\frac{0.17}{3} = -0.0567\,\mathrm{V}.$$

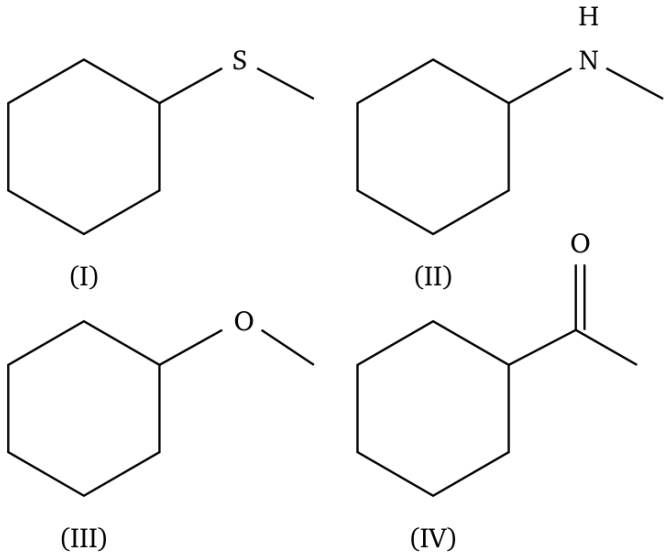
The value rounds to $-0.057\,\mathrm{V}$. Option (B) reverses its sign. The value $+0.30\,\mathrm{V}$ in (C) comes from directly adding potentials, which is invalid; (D) is not the electron-weighted result either.

Step 4: Conclusion The standard reduction potential for $\mathrm{Fe^{3+} \longrightarrow Fe}$ is approximately $-0.057\,\mathrm{V}$.

Hence, the correct option is (A).

Quick Tip to Solve: For sequential electrode reactions use an electron-weighted average of reduction potentials, after checking their directions.

80. A mixture containing the following four compounds is extracted with $1M\ HCl$. The compound that goes to aqueous layer is : [4 marks]



(I) (II) (III) (IV)

(A) (I)

(B) (II)

(C) (III)

(D) (IV)

Correct Answer: (B) (II)

Solution:

Concept:

- Aqueous hydrochloric acid extracts an organic base by converting it into an ionic hydrochloride salt. The charged species is much more soluble in the aqueous phase than its neutral parent.
- An amine nitrogen has a lone pair that accepts a proton from hydrochloric acid. Neutral ethers, thioethers and ketones do not form the ordinary persistent ammonium salts used in this separation.

Step 1: Identify the amine and form its salt Compound (II) is a secondary amine, represented as $\mathrm{RNHCH_3}$, where R is the six-membered carbon group shown in the figure. Its nitrogen lone pair can accept a proton from hydrochloric acid. Aqueous $1\,\mathrm{M}$ hydrochloric acid protonates it sufficiently for acid-base extraction:

$$\mathrm{RNHCH_3 + HCl \longrightarrow [RNH_2CH_3]^+\,Cl^-}.$$

The resulting ammonium chloride salt is ionic and enters the aqueous phase.

Step 2: Compare the sulfur and oxygen compounds Compound (I) contains the thioether group

$R-S-CH_3$, and compound (III) contains the ether group $R-O-CH_3$. They are a thioether and an ether, respectively. Their heteroatom lone pairs do not make them ordinary amine bases; under the stated extraction conditions they do not produce the stable, water-soluble ammonium hydrochloride obtained from compound (II). Thus options (A) and (C) do not provide the selective aqueous-layer component.

Step 3: Exclude the neutral ketone Compound (IV) contains a ketone group, $RCOCH_3$. Its carbonyl oxygen may be protonated transiently in acid-catalysed reactions, but this ketone does not form the persistent amine hydrochloride needed for this separation. It remains a neutral organic component under the extraction criterion, so option (D) fails. Comparing all four structures, only the secondary amine in (II) is converted efficiently into a water-soluble ionic salt by dilute aqueous hydrochloric acid.

Step 4: Conclusion Compound (II), the secondary amine, enters the aqueous layer as ammonium chloride salt. Hence, the correct option is (B).

Quick Tip to Solve: In acid extraction, identify an amine nitrogen: protonation converts the neutral amine into an ionic, water-soluble hydrochloride.

81. A metal 'M' reacts with nitrogen gas to afford 'M_3N'. 'M_3N' on heating at high temperature gives back 'M' and on reaction with water produces a gas 'B'. Gas 'B' reacts with aqueous solution of $CuSO_4$ to form a deep blue compound. 'M' and 'B' respectively are : [4 marks]

(A) Li and NH_3

(B) Ba and N_2

(C) Na and NH_3

(D) Al and N_2

Correct Answer: (A) Li and NH_3

Solution:

Concept:

- A nitride ion has charge -3. The formula M_3N therefore requires a monovalent metal. Lithium is the alkali metal that combines directly with nitrogen readily.
- Metal nitrides hydrolyse to release ammonia, which acts as a ligand for aqueous copper(II), producing its characteristic deep-blue ammine complex.

Step 1: Use the nitride formula and direct reaction with nitrogen Three atoms of metal balance one nitride ion in M_3N, so each metal atom supplies one positive charge. Lithium reacts directly with nitrogen:

$$6\,Li + N_2 \longrightarrow 2\,Li_3N.$$

This identifies a monovalent metal with the required special reactivity. Barium would form Ba_3N_2, and aluminium would form AlN, so their proposed identities are inconsistent with the stated formula.

Step 2: Identify the gas formed by hydrolysis Lithium nitride reacts with water to give lithium

hydroxide and ammonia:

$$Li_3N + 3\,H_2O \longrightarrow 3\,LiOH + NH_3\uparrow.$$

The hydrolysis equation conserves Li, N, O and H. The gas B is thus NH_3, not molecular nitrogen. Under the high-temperature decomposition condition stated in the question, the reverse formation process regenerates lithium and nitrogen.

Step 3: Confirm the gas by the copper(II) test In excess ammonia, aqueous copper(II) forms a deep-blue ammine complex, commonly represented as

$$[Cu(H_2O)_6]^{2+} + 4\,NH_3 \rightleftharpoons [Cu(NH_3)_4(H_2O)_2]^{2+} + 4\,H_2O.$$

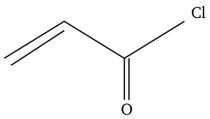
This is the diagnostic observation for ammonia. Options (B) and (D) incorrectly give nitrogen, which does not produce this complex. Option (C) gives ammonia but assigns sodium, which does not share lithium's direct nitrogen reaction under the conditions tested.

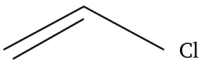
Step 4: Conclusion The metal is lithium and the gas is ammonia: $M = Li$, $B = NH_3$. Hence, the correct option is (A).

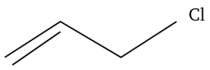
Quick Tip to Solve: Combine two clues: the nitride formula fixes the metal valency, while the deep-blue copper(II) complex identifies ammonia.

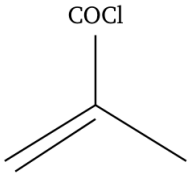
82. Which of the following compounds will not undergo Friedel Craft's reaction with benzene ?

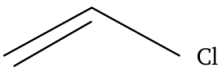
[4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- Friedel-Crafts substitution needs an electrophile generated from an alkyl halide or acyl halide in the presence of a Lewis acid such as $AlCl_3$.
- A vinylic carbon-halogen bond does not ionize like an ordinary alkyl-halide bond. Its stronger, partly double-bond character and the instability of a vinyl cation prevent ordinary Friedel-Crafts alkylation.

Step 1: Distinguish vinyl chloride from allyl chloride In option (B), the chlorine atom is bonded directly to a carbon of the carbon-carbon double bond: this compound is vinyl chloride, $CH_2{=}CHCl$.

Generating the usual alkylating electrophile would require cleavage of a vinylic $C-Cl$ bond and formation of a highly unstable vinyl cation. It therefore does not undergo the normal Friedel-Crafts reaction with benzene.

Step 2: Check the other unsaturated halide Option (C) has chlorine on the carbon next to the double bond, giving allyl chloride, $CH_2=CHCH_2Cl$. This is an allylic rather than a vinylic halide. The allylic electrophile is resonance-stabilized, so this compound can alkylate benzene under Friedel-Crafts conditions. The mere presence of a carbon-carbon double bond does not make both compounds unreactive; its position relative to chlorine matters.

Step 3: Check both acyl chlorides Option (A) is acryloyl chloride, $CH_2=CHCOCl$, and option (D) is methacryloyl chloride, $CH_2=C(CH_3)COCl$. In both cases chlorine is attached to a carbonyl carbon, so a Lewis acid can generate a resonance-stabilized acylium electrophile, represented by RCO^+. They undergo Friedel-Crafts acylation of benzene. The additional carbon-carbon double bond in each acyl group does not convert the acyl chloride into a vinyl chloride: the chlorine-bearing atom is still the carbonyl carbon. Thus (A), (C) and (D) can react; only (B) lacks the required normal electrophile-generation pathway.

Step 4: Conclusion Vinyl chloride, shown in option (B), will not undergo the ordinary Friedel-Crafts reaction with benzene. Hence, the correct option is (B).

Quick Tip to Solve: Locate the carbon bonded to chlorine: vinyl halides resist Friedel-Crafts alkylation, whereas allyl halides and acyl chlorides can react.

83. The enthalpy change on freezing of 1 mol of water at 5°C to ice at -5°C is :
(Given $\Delta_{fus}H$ = 6 kJ mol^{-1} at 0°C,
$C_p(H_2O, \ell$ = 75.3J mol^{-1} K^{-1})
$C_p(H_2O$ s) =36.8 J mol^{-1} K^{-1}) [4 marks]

(A) 5.44 kJ mol^{-1}
(B) 5.81 kJ mol^{-1}
(C) 6.56 kJ mol^{-1}
(D) 6.00 kJ mol^{-1}

Correct Answer: (C) 6.56 kJ mol^{-1}

Solution:

Concept:

- Enthalpy is a state function, so cooling liquid water, freezing it, and cooling the ice may be combined into a convenient three-stage path.
- For cooling within one phase, $\Delta H = nC_p\Delta T$. Freezing is the reverse of fusion, hence its enthalpy is $-\Delta_{fus}H$, not positive.

Step 1: Cool liquid water from five degrees to zero For one mole of liquid water, the temperature

decreases by $5\,\mathrm{K}$. Using the given liquid heat capacity,

$$\Delta H_1 = (1)(75.3)(0-5) = -376.5\,\mathrm{J}.$$

A decrease in temperature at constant pressure removes heat from the water, so the enthalpy contribution is negative.

Step 2: Freeze the water at zero degrees The supplied molar enthalpy of fusion is $+6\,\mathrm{kJ\,mol^{-1}}$. Reversing that process produces ice and reverses its sign:

$$\Delta H_2 = -6000\,\mathrm{J}.$$

No temperature change occurs during this phase-change stage. The latent heat must be included even though the temperature remains fixed.

Step 3: Cool the ice and add all contributions The newly formed ice is cooled from $0^\circ\mathrm{C}$ to $-5^\circ\mathrm{C}$:

$$\Delta H_3 = (1)(36.8)(-5) = -184.0\,\mathrm{J}.$$

Therefore

$$\Delta H = -376.5 - 6000 - 184.0 = -6560.5\,\mathrm{J} = -6.5605\,\mathrm{kJ}.$$

For one mole, the molar change is $-6.56\,\mathrm{kJ\,mol^{-1}}$. The positive options list magnitudes: (D) omits both sensible-cooling terms, while (A) and (B) do not correctly add the heat removed in all three stages.

Step 4: Conclusion The enthalpy change is $-6.56\,\mathrm{kJ\,mol^{-1}}$: heat is released. Its magnitude is $6.56\,\mathrm{kJ\,mol^{-1}}$, so the listed/keyed magnitude corresponds to (C). None of the printed positive choices gives the signed enthalpy change literally. Hence, the correct option is (C) when the choices are read as heat-released magnitudes.

Quick Tip to Solve: Draw a three-stage temperature path mentally: cool the liquid, freeze at zero, then cool the solid. Every term is negative for this process.

84. Consider the following standard electrode potentials (E^o in volts) in aqueous solution :

Element	M^{3+} /M	M^{+} /M
A1	-1.66	+ 0.55
T1	+1.26	- 0.34

Based on these data, which of the following statements is correct ? [4 marks]

(A) $T1^{+}$ is more stable than $A1^{3+}$

(B) $A1^{+}$ is more stable than $A1^{3+}$

(C) $T1^{+}$ is more stable than $A1^{+}$

(D) $T1^{3+}$ is more stable than $A1^{3+}$

Correct Answer: (C) $T1^{+}$ is more stable than $A1^{+}$

Solution:

Concept:

- Standard reduction potentials describe the tendency of ions to accept electrons in aqueous solution. More positive reduction potential means more favourable reduction to the metal under standard conditions.
- Potentials cannot be added directly when electron counts differ. Use $\Delta G^\circ = -nFE^\circ$, where n is the electron number and F is the Faraday constant, to compare oxidation states.

Step 1: Calculate the potentials between the two ionic states Denote the table's two elements by their chemical symbols Al and Tl. Combining the three-electron reduction to the metal with the reverse of the one-electron reduction gives the two-electron reduction from the tripositive ion to the unipositive ion. Free-energy addition gives

$$E^\circ_{3+/+} = \frac{3E^\circ_{3+/0} - E^\circ_{+/0}}{2}.$$

Therefore,

$$E^\circ_{3+/+}(\mathrm{Al}) = \frac{3(-1.66) - 0.55}{2} = -2.765\ \mathrm{V},$$
$$E^\circ_{3+/+}(\mathrm{Tl}) = \frac{3(1.26) - (-0.34)}{2} = 2.06\ \mathrm{V}.$$

Reduction of aluminium(III) to aluminium(I) is unfavourable, whereas reduction of thallium(III) to thallium(I) is favourable. Thus the stable common oxidation states are aluminium(III) and thallium(I).

Step 2: Compare the unipositive ions and eliminate the alternatives Aluminium(I) also tends to disproportionate into aluminium metal and aluminium(III). For $3M^{+} \longrightarrow M^{3+} + 2M$, the cell potential is $E^\circ_{+/0} - E^\circ_{3+/+}$. It is $0.55 - (-2.765) = 3.315$ V for aluminium, but $-0.34 - 2.06 = -2.40$ V for thallium. Consequently the unipositive thallium ion resists this disproportionation while the unipositive aluminium ion does not. Option (B) reverses aluminium's stability; (D) reverses the stability of the tripositive ions. Option (A) compares two favoured states and does not follow from these data as a stronger stability ordering. The supported comparison is (C).

Step 3: Conclusion The unipositive thallium ion, printed as $T1^{+}$, is more stable than the unipositive aluminium ion, printed as $A1^{+}$. Hence, the correct option is (C).

Quick Tip to Solve: Combine electrode data through electron-weighted free energies, then test disproportionation; never average reduction potentials without their electron counts.

85. Among the following, the incorrect statement is : [4 marks]

(A) At low pressure, real gases show ideal behaviour.

(B) At very low temperature, real gases show ideal behaviour.

(C) At very large volume, real gases show ideal behaviour.

(D) At Boyle's temperature, real gases show ideal behaviour.

Correct Answer: (B) At very low temperature, real gases show ideal behaviour.

Solution:

Concept:

- A gas approaches ideal behaviour when intermolecular attractions and the volume occupied by its molecules are negligible compared with its thermal motion and container volume.
- The compressibility factor is $Z = PV_m/(RT)$, where V_m is molar volume. Ideal behaviour corresponds to $Z = 1$.

Step 1: Test pressure, volume and temperature At low pressure the molecules are far apart, so both attractions and their excluded volume have little effect. A very large molar volume has the same consequence. Thus (A) and (C) describe the dilute-gas limit correctly. Very low temperature does the opposite: molecular kinetic energy decreases, attractive interactions become relatively important, and the gas can approach condensation. It is therefore incorrect to claim that very low temperature generally produces ideal behaviour.

Step 2: Interpret the Boyle temperature For a van der Waals gas, expansion at large molar volume gives

$$Z = 1 + \frac{b - a/(RT)}{V_m} + \cdots .$$

Here a measures attractions and b the excluded molar volume. At the Boyle temperature $T_B = a/(Rb)$, the leading correction vanishes because the attractive and repulsive corrections compensate. Statement (D) describes this familiar approximately ideal regime; it is not a claim of exact ideality at arbitrarily high pressure. Consequently the statement that fails the usual conditions for ideal behaviour is (B).

Step 3: Conclusion At very low temperature, intermolecular attractions become significant and real gases do not generally behave ideally. Hence, the correct option is (B).

Quick Tip to Solve: For ideal behaviour look for low density and sufficiently high temperature; low temperature favours attraction and liquefaction.

86. The number of S = O and S − OH bonds present in peroxodisulphuric acid and pyrosulphuric acid respectively are : [4 marks]

(A) (2 and 2) and (2 and 2)

(B) (2 and 4) and (2 and 4)

(C) (4 and 2) and (2 and 4)

(D) (4 and 2) and (4 and 2)

Correct Answer: (D) (4 and 2) and (4 and 2)

Solution:

Concept:

- Count bonds from the structural formula, rather than from the molecular formula alone. An $S-OH$ bond means a sulphur atom attached to a hydroxyl oxygen.
- Peroxodisulphuric acid contains an $O-O$ peroxide bridge, whereas pyrosulphuric acid contains an $S-O-S$ bridge.

Step 1: Write the peroxoacid connectivity Peroxodisulphuric acid is $H_2S_2O_8$, with connectivity

$$HO-S(=O)_2-O-O-S(=O)_2-OH.$$

Each sulphur has two terminal double-bonded oxygens and one hydroxyl group. Summing over the two sulphur atoms therefore gives four $S=O$ bonds and two $S-OH$ bonds. The two oxygens in the central peroxide linkage do not carry hydrogen, so neither adds to the hydroxyl count.

Step 2: Count the bonds in the pyroacid Pyrosulphuric acid, $H_2S_2O_7$, has connectivity

$$HO-S(=O)_2-O-S(=O)_2-OH.$$

Again each sulphur contributes two double-bonded oxygens and one hydroxyl group. Its counts are also four and two. The bridging oxygen is bonded to two sulphur atoms, but it is not a hydroxyl oxygen. Option (A) undercounts the double bonds in both acids, while (B) undercounts double bonds and overcounts hydroxyl groups. Option (C) has the first pair right but reverses the second acid's required counts.

Step 3: Conclusion The counts $(S=O, S-OH)$ are $(4,2)$ for peroxodisulphuric acid and $(4,2)$ for pyrosulphuric acid. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish an O–O bridge from an S–O–S bridge, and count only terminal OH groups as S–OH bonds.

87. Among the following, correct statement is : [4 marks]

(A) Brownian movement is more pronounced for smaller particles than for bigger–particles

(B) Sols of metal sulphides are lyophilic.

(C) Hardy Schulze law states that bigger the size of the ions, the greater is its coagulating power.

(D) One would expect charcoal to adsorb chlorine more than hydrogen sulphide.

Correct Answer: (D) One would expect charcoal to adsorb chlorine more than hydrogen sulphide.

Solution:

Concept:

- Brownian movement results from unequal instantaneous impacts of dispersion-medium molecules on colloidal particles. Smaller particles generally show more conspicuous random motion.
- For physical adsorption on charcoal under comparable conditions, greater ease of liquefaction generally favours adsorption; critical temperature is a useful qualitative guide.

Step 1: Check the colloid statements Statement (A) is correct as written: smaller particles respond more strongly to random molecular bombardment, so their Brownian movement is more pronounced. Metal sulphide sols are normally lyophobic rather than lyophilic, making (B) incorrect. The Hardy–Schulze rule relates coagulating power chiefly to the valency of the ion whose charge is opposite to that of the colloidal particles. It does not state that larger ionic size alone gives greater coagulating power, so (C) is incorrect.

Step 2: Compare adsorption and identify the overlap Chlorine is more readily liquefied than hydrogen sulphide: their critical temperatures are approximately $417\,\mathrm{K}$ and $373\,\mathrm{K}$, respectively. The standard qualitative physical-adsorption trend therefore supports stronger adsorption of chlorine by charcoal under comparable conditions, as stated in (D). This is the keyed answer. However, that reasoning does not make (A) false: its Brownian-motion statement is independently correct. Thus the literal wording does not yield a unique correct statement. Retaining both observations avoids rejecting a valid colloid principle merely to force a single answer.

Step 3: Conclusion The supplied key selects (D), supported by chlorine being more readily liquefied and generally more strongly physisorbed than hydrogen sulphide. Statement (A) is also true as printed, so the question has more than one defensible statement. The keyed option is (D).

Quick Tip to Solve: Test every statement independently: a correct adsorption trend cannot invalidate the usual stronger Brownian motion of smaller particles.

88. Identify the pollutant gases largely responsible for the discoloured and lustreless nature of marble of the Taj Mahal. [4 marks]

(A) O_3 and CO_2

(B) CO_2 and NO_2

(C) SO_2 and NO_2

(D) SO_2 and O_3

Correct Answer: (C) SO_2 and NO_2

Solution:

Concept:

- Marble consists chiefly of calcium carbonate, $CaCO_3$, which reacts with acids. Acid deposition can corrode its surface and reduce its lustre.
- Sulphur dioxide and nitrogen oxides are major precursors of sulphuric and nitric acids in the atmosphere.

Step 1: Identify the acid-forming pollutants Combustion of sulphur-containing fuels releases SO_2, while high-temperature combustion produces nitrogen oxides, including NO_2. Oxidation and reaction with atmospheric moisture generate strong acids. Representative overall changes are

$$2\,SO_2 + O_2 + 2\,H_2O \longrightarrow 2\,H_2SO_4,$$

$$4\,NO_2 + O_2 + 2\,H_2O \longrightarrow 4\,HNO_3.$$

These equations describe the net acid formation; actual atmospheric pathways can involve intermediate oxidants and dissolved species.

Step 2: Connect acid formation to marble damage The acids attack carbonate, releasing carbon dioxide and forming calcium salts:

$$CaCO_3 + H_2SO_4 \longrightarrow CaSO_4 + CO_2 + H_2O.$$

Surface conversion and erosion make the marble weathered and lustreless; deposited particles can also contribute to discolouration. Carbon dioxide forms only weak carbonic acid and is not the principal industrial acid-rain precursor sought here. Ozone is an atmospheric oxidant, but replacing either sulphur dioxide or nitrogen dioxide with ozone omits one of the two principal acid-forming pollutants. Hence (A), (B) and (D) do not give the required pair.

Step 3: Conclusion The pollutant gases chiefly responsible are SO_2 and NO_2, through the acids they form and their attack on calcium carbonate. Hence, the correct option is (C).

Quick Tip to Solve: Link marble damage to acid rain: sulphur and nitrogen oxides form strong acids that consume the carbonate surface.

89. The reason for "drug induced poisoning" is : [4 marks]

(A) Binding reversibly at the active site of the enzyme

(B) Bringing conformational change in the binding site of enzyme

(C) Binding irreversibly to the active site of the enzyme

(D) Binding at the allosteric sites of the enzyme

Correct Answer: (C) Binding irreversibly to the active site of the enzyme

Solution:

Concept:

- Enzymes catalyse biochemical reactions through specific binding and catalytic sites. A drug may inhibit an enzyme by occupying its active site or by altering its shape.
- Reversible inhibition permits recovery when the inhibitor dissociates; irreversible active-site binding can permanently inactivate the affected enzyme molecules.

Step 1: Distinguish temporary inhibition from inactivation When a drug binds reversibly at the active site, it competes with the normal substrate. Once the drug dissociates, the substrate can bind again and the enzyme can recover its catalytic activity. Such controlled, reversible inhibition is a common therapeutic mechanism, so (A) does not identify the specific cause of poisoning asked here. Binding at an allosteric site can change the active-site conformation and reduce activity without necessarily permanently destroying enzyme function.

Step 2: Explain why irreversible binding is dangerous If the drug binds irreversibly to the active site, simple dilution or removal of free drug does not restore that enzyme molecule. Substrate can no longer use the blocked catalytic site, and an essential metabolic reaction may stop until new functional enzyme is made. Sustained loss of an essential enzyme's activity can therefore produce drug-induced poisoning. This is the decisive feature of (C). Options (B) and (D) describe conformational or allosteric regulation, which can be reversible and therapeutically useful; neither by itself entails permanent active-site inactivation. The distinction concerns binding persistence, not merely the location at which any drug first associates.

Step 3: Conclusion Irreversible binding to the enzyme's active site can permanently block its activity and cause drug-induced poisoning. Hence, the correct option is (C).

Quick Tip to Solve: In enzyme-inhibition questions, ask whether activity returns after the inhibitor leaves: irreversible active-site blockage signals persistent toxicity.

90. In which of the following reactions, hydrogen peroxide acts as an oxidizing agent ? [4 marks]

(A) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(B) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

(C) $2MnO_4^- + 3H_2O_2 \rightarrow 2\ MnO_2 + 3O_2 + 2H_2O + 2OH^-$

(D) $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$

Correct Answer: (D) $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$

Solution:

Concept:

- An oxidizing agent accepts electrons and is itself reduced. Oxygen in H_2O_2 has oxidation number -1, intermediate between 0 in O_2 and -2 in water.
- Hydrogen peroxide is a reducing agent when its oxygen becomes O_2, but an oxidizing agent when its oxygen is reduced to water or hydroxide.

Step 1: Eliminate the oxygen-evolving reactions In (A), peroxide produces O_2, so its oxygen changes from -1 to 0. It is oxidized and acts as the reducing agent while chlorine is reduced. In (B), iodine changes from 0 in I_2 to -1 in iodide; peroxide again produces oxygen and supplies the reducing action. In (C), manganese is reduced from $+7$ in permanganate to $+4$ in MnO_2, and peroxide oxygen is oxidized to O_2. Thus peroxide acts as a reducing agent in all three reactions.

Step 2: Check oxidation of sulphide in the remaining reaction In (D), sulphur changes from -2 in PbS to $+6$ in sulphate, losing eight electrons per sulphur atom. Four peroxide molecules can accept these eight electrons because each is reduced by two electrons:

$$H_2O_2 + 2\,H^+ + 2\,e^- \longrightarrow 2\,H_2O.$$

The balanced overall reaction is

$$PbS + 4\,H_2O_2 \longrightarrow PbSO_4 + 4\,H_2O.$$

Lead remains $+2$. The oxidation of sulphide and simultaneous reduction of peroxide establish that peroxide is the oxidizing agent in (D), independently of merely recognizing the product formula.

Step 3: Conclusion In $PbS + 4\,H_2O_2 \longrightarrow PbSO_4 + 4\,H_2O$, hydrogen peroxide oxidizes sulphide to sulphate and is itself reduced. Hence, the correct option is (D).

Quick Tip to Solve: Track peroxide oxygen first: formation of O2 means reducing action, while reduction toward water means oxidizing action.