JEE Main 2018 Paper 1 (B.E./B.Tech) Apr 8 Shift 1 (morning) Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2018, Annual (offline), conducted by the Central Board of Secondary Education (CBSE) in pen and paper mode on 8 April 2018, Single sitting. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$,
$x \in R - \{-1, 0, 1\}$.
If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of h(x) is [4 marks]

(A) $2\sqrt{2}$

(B) 3

(C) -3

(D) $-2\sqrt{2}$

Correct Answer: (A) $2\sqrt{2}$

Solution:

Concept:

- At a stationary point, the first derivative vanishes. A derivative changing from negative to positive gives a local minimum.
- Use $t = x - 1/x$, so $x^2 + x^{-2} = t^2 + 2$. The excluded points ensure $x \neq 0$ and $t \neq 0$.

Step 1: Rewrite the quotient The numerator and denominator have a common useful substitution. Since $t^2 = x^2 - 2 + x^{-2}$, we obtain

$$h(x) = \frac{t^2 + 2}{t} = t + \frac{2}{t}.$$

Moreover, $dt/dx = 1 + x^{-2} > 0$ on each interval of its domain. Consequently a local maximum or minimum with respect to t retains its type with respect to x.

Step 2: Find and classify the stationary values Differentiate the simpler expression:

$$\frac{dh}{dx} = \left(1 - \frac{2}{t^2}\right)\left(1 + \frac{1}{x^2}\right).$$

Thus stationary points require $t = \pm\sqrt{2}$. For positive t, the first factor is negative below $\sqrt{2}$ and positive above it, proving a local minimum. For negative t, it changes from positive to negative at $-\sqrt{2}$, proving a local maximum. Both values of t are attained: the equation $x^2 - tx - 1 = 0$ has real nonzero roots, none equal to the excluded ± 1 when $t \neq 0$.

Step 3: Evaluate and compare the options At the local minimum,

$$h = \sqrt{2} + \frac{2}{\sqrt{2}} = 2\sqrt{2}.$$

The value $-2\sqrt{2}$ belongs to the local maximum on the negative branch; it is not a local minimum. The numbers 3 and -3 are not stationary values. Notice that the question asks for a local minimum, since the function has no global minimum over the whole disconnected domain.

Step 4: Conclusion The local minimum value is $\boxed{2\sqrt{2}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a quotient involving $x^2 + x^{-2}$ and $x - x^{-1}$, first express the numerator in terms of the denominator.

2. If $\alpha, \beta \in C$ are the distinct roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to : [4 marks]

(A) 2

(B) -1

(C) 0

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- If $z^2 - z + 1 = 0$, multiplication by $z + 1$ gives $z^3 + 1 = 0$; hence $z^6 = 1$. Powers can therefore be reduced modulo six.
- Vieta's relations give $\alpha + \beta = 1$ and $\alpha\beta = 1$. Neither root is zero, so reciprocals are legitimate.

Step 1: Establish the power cycle Each of the two distinct roots satisfies

$$z^2 = z - 1, \qquad z^3 = z(z-1) = z^2 - z = -1.$$

Squaring the last identity gives $z^6 = 1$. This also shows why treating the roots as cube roots of unity would be incorrect: their cubes are -1, rather than 1. The actual roots are $(1 \pm i\sqrt{3})/2$, which is consistent with a cycle of length six.

Step 2: Reduce the exponents Both exponents have remainder five on division by six:

$$101 = 6 \cdot 16 + 5, \qquad 107 = 6 \cdot 17 + 5.$$

Therefore

$$\alpha^{101} + \beta^{107} = \alpha^5 + \beta^5 = \frac{1}{\alpha} + \frac{1}{\beta}.$$

In the last equality we used $z^6 = 1$, so $z^5 = z^{-1}$. The computation is unaffected by which root is named α.

Step 3: Use the sum and product of roots Combining the reciprocals over a common denominator gives

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{1}{1} = 1.$$

The real value is exactly one. Thus the alternatives 2, -1, and 0 all disagree with the root identities; in particular, the conjugate terms do not cancel, because their real parts add.

Step 4: Conclusion $\alpha^{101} + \beta^{107} = \boxed{1}$. Hence, the correct option is (D).

Quick Tip to Solve: For roots of $z^2 - z + 1 = 0$, use $z^3 = -1$, then reduce large powers modulo six before adding.

3. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$

then the ordered pair (A, B) is equal to : [4 marks]

(A) (4, 5)

(B) (-4, -5)

(C) (-4, 3)

(D) (-4, 5)

Correct Answer: (D) (-4, 5)

Solution:

Concept:

- A three-by-three determinant with diagonal entries a and all off-diagonal entries b equals $(a-b)^2(a+2b)$. Row operations can derive this factorisation directly.
- Compare the resulting polynomial factors with the prescribed expression $(A+Bx)(x-A)^2$, retaining signs when squaring a factor.

Step 1: Expose the repeated factor Put $a = x-4$ and $b = 2x$. Subtract the first row from each of the other two rows. These operations do not change the determinant. The new rows are $(b-a, a-b, 0)$ and $(b-a, 0, a-b)$, so factoring $a-b$ from both gives

$$D = (a-b)^2 \begin{vmatrix} a & b & b \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}.$$

This is an identity in x, including values where the factor vanishes.

Step 2: Evaluate the remaining determinant Add columns two and three to column one in the remaining determinant. Its first column becomes $(a+2b, 0, 0)^T$, while the lower two-by-two minor is the identity matrix. Hence

$$D = (a-b)^2(a+2b).$$

Substituting the actual entries yields

$$a-b = -x-4, \qquad a+2b = 5x-4,$$

and therefore

$$D = (x+4)^2(5x-4).$$

The minus sign in $-x-4$ disappears only because this factor is squared.

Step 3: Identify the two constants Comparing $(x+4)^2$ with $(x-A)^2$ gives $A = -4$. The other factor is then $-4+Bx = 5x-4$, so $B = 5$. The pair $(4, 5)$ has the wrong repeated root; $(-4, -5)$ has the wrong leading coefficient; and $(-4, 3)$ gives coefficient three instead of five. As a check, the constant term is -64, matching the determinant at $x = 0$.

Step 4: Conclusion The ordered pair is $\boxed{(A, B) = (-4, 5)}$. Hence, the correct option is (D).

Quick Tip to Solve: Equal diagonal and equal off-diagonal entries suggest the factors $(a-b)^2$ and the common row sum $a+2b$.

4. For each t $\in R$, let [t] be the greatest integer less than or equal to t.

Then $\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+.....+\left[\frac{15}{x}\right]\right)$ [4 marks]

(A) does not exist in R

(B) is equal to 0

(C) is equal to 15

(D) is equal to 120

Correct Answer: (D) is equal to 120

Solution:

Concept:

- The defining bound for the greatest-integer function is $u-1<\lfloor u\rfloor\le u$, valid for every real u.
- When multiplying an inequality by $x>0$, its direction stays unchanged. The squeeze theorem then determines the right-hand limit.

Step 1: Bound each summand For every integer k with $1\le k\le 15$, apply the floor bound to $u=k/x$. Because the limit is taken from the positive side, $x>0$, and multiplication gives

$$k-x<x\left\lfloor\frac{k}{x}\right\rfloor\le k.$$

Although the floor function jumps repeatedly as x tends to zero, the distance between this scaled summand and k is less than x. The jumps therefore do not prevent the scaled term from converging.

Step 2: Add the fifteen inequalities Summing the bounds for $k=1,2,\ldots,15$ gives

$$\sum_{k=1}^{15}k-15x<x\sum_{k=1}^{15}\left\lfloor\frac{k}{x}\right\rfloor\le\sum_{k=1}^{15}k.$$

There are exactly fifteen error contributions, each bounded in magnitude by x. The arithmetic sum is

$$\sum_{k=1}^{15}k=\frac{15\cdot 16}{2}=120.$$

Thus the complete expression lies between $120-15x$ and 120.

Step 3: Take the right-hand limit Both bounding expressions tend to 120 as $x\to 0^+$. The squeeze theorem proves that the desired limit exists and equals 120. Consequently the claim of nonexistence is false, despite the discontinuities of individual floor functions. The answer is neither zero nor fifteen: the fifteen scaled summands approach the different integers 1 through 15, rather than a common zero or one.

Step 4: Conclusion The limit is $\boxed{120}$. Hence, the correct option is (D).

Quick Tip to Solve: For a finite sum of $x\lfloor k/x\rfloor$, bound each term between $k-x$ and k, then add before taking the limit.

5. Two sets A and B are as under :

A = {$(a, b) \in R \times R : |a - 5| < 1$ and $|b - 5| < 1$};

B = {$(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \leq 36$ };

Then [4 marks]

(A) neither A $\subset$ B nor B $\subset$ A

(B) B $\subset$ A

(C) A $\subset$ B

(D) A $\cap$ B = ϕ (an empty set)

Correct Answer: (C) A $\subset$ B

Solution:

Concept:

- The inequalities $|a-5|<1$ and $|b-5|<1$ describe the open square $4<a<6$, $4<b<6$.
- To prove $A \subset B$, test an arbitrary point of A against the defining inequality of B. A single counterexample disproves the reverse inclusion.

Step 1: Bound the ellipse expression on the square Take any $(a, b) \in A$. Since $4 < a < 6$, we have $-2 < a - 6 < 0$, giving $(a-6)^2 < 4$. Similarly, $-1 < b - 5 < 1$, so $(b-5)^2 < 1$. Multiplying by the positive coefficients and adding,

$$4(a-6)^2 + 9(b-5)^2 < 4 \cdot 4 + 9 \cdot 1 = 25.$$

In particular this quantity is less than 36, so the chosen point satisfies the inequality defining B. Because the point was arbitrary, every point of A belongs to B.

Step 2: Check that the reverse inclusion fails The point $(9, 5)$ belongs to B, since

$$4(9-6)^2 + 9(5-5)^2 = 36 \leq 36.$$

It does not belong to A, because $|9 - 5| = 4$, which is not less than one. Hence B is not contained in A, and the inclusion already proved is strict. This also excludes the claim that neither inclusion holds.

Step 3: Rule out an empty intersection The point $(5, 5)$ is in A, and the ellipse expression there equals 4, so it is in B as well. Thus $A \cap B \neq \varnothing$. In fact the established inclusion gives $A \cap B = A$. The only listed statement consistent with both the universal bound and the counterexample is $A \subset B$.

Step 4: Conclusion $\boxed{A \subset B}$, with strict inclusion. Hence, the correct option is (C).

Quick Tip to Solve: For a square inside an ellipse, bound each squared coordinate difference separately; use one ellipse boundary point to test the reverse inclusion.

6. Let S = { t $\in R : f(x) = |x-\pi| \cdot \left(e^{|x|}-1\right)\sin|x|$ is not differentiable at t}, then the set S is equal to [4 marks]

(A) {0, π}

(B) ϕ (an empty set)

(C) {0}

(D) {π}

Correct Answer: (B) ϕ (an empty set)

Solution:

Concept:

- Absolute values can introduce possible corners only where their arguments vanish: here the only candidates are $x = 0$ and $x = \pi$.
- A zero in another factor can cancel a possible corner. Check such points by the derivative definition rather than assuming every absolute-value zero is nondifferentiable.

Step 1: Check all ordinary points When $x \neq 0, \pi$, both $|x|$ and $|x-\pi|$ agree locally with signed linear functions. The exponential and sine are differentiable everywhere, so their composition and the product are differentiable at all such points. Therefore any element of S would have to be one of the two candidate points.

Step 2: Evaluate the derivative at zero We have $f(0) = 0$. With $u = |h|$, write the difference quotient as

$$\frac{f(h)-f(0)}{h} = |h-\pi|\frac{(e^u-1)\sin u}{h}.$$

Since $(e^u-1)/u \to 1$ and $\sin u/u \to 1$, the product in the numerator is $u^2(1+o(1))$. But $u^2/h = h$, so this quotient tends to zero from both sides. Thus f is differentiable at zero and $f\prime(0) = 0$.

Step 3: Evaluate the derivative at pi Near π, let $x = \pi + h$; then $x > 0$, so $|x| = \pi + h$. Also $f(\pi) = 0$ and $\sin(\pi + h) = -\sin h$. Consequently

$$\frac{f(\pi+h)-f(\pi)}{h} = -\frac{|h|}{h}(e^{\pi+h}-1)\sin h \longrightarrow 0.$$

The bounded sign factor cannot stop the sine factor from tending to zero. Differentiability holds at π too. Thus neither singleton option nor the set containing both candidates is correct.

Step 4: Conclusion There are no points of nondifferentiability: $\boxed{S = \varnothing}$. Hence, the correct option is (B).

Quick Tip to Solve: Before declaring a cusp from an absolute value, check whether a neighbouring factor vanishes and makes the difference quotient tend to zero.

7. The integral

$$\int \frac{\sin^2 x\cos^2 x}{\left(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x\right)^2}dx$$

is equal to [4 marks]

(A) $\frac{-1}{1+\cot^3 x}+C$
(B) $\frac{1}{3(1+\tan^3 x)}+C$
(C) $\frac{-1}{3(1+\tan^3 x)}+C$
(D) $\frac{1}{1+\cot^3 x}+C$
Correct Answer: (C) $\frac{-1}{3(1+\tan^3 x)}+C$

Solution:

Concept:

- Use $\sin^2 x+\cos^2 x=1$ to factor the long denominator before attempting integration.
- The substitution $t=\tan x$, with $dt=\sec^2 x\,dx$, converts the simplified trigonometric expression into a rational function.

Step 1: Simplify the denominator Write $s=\sin x$ and $c=\cos x$. Group the four terms as

$$s^5+s^3c^2+c^3s^2+c^5=s^3(s^2+c^2)+c^3(s^2+c^2)=s^3+c^3.$$

Thus the given integrand is $s^2c^2/(s^3+c^3)^2$. This simplification is essential: the square applies to the entire sum, so it must be retained after factoring.

Step 2: Apply the tangent substitution On an interval where $\cos x\neq 0$, put $t=\tan x$. Dividing numerator and denominator by $\cos^6 x$ gives

$$I=\int \frac{\tan^2 x\,\sec^2 x}{(1+\tan^3 x)^2}\,dx=\int \frac{t^2}{(1+t^3)^2}\,dt.$$

Now let $u=1+t^3$, so $du=3t^2\,dt$. Therefore

$$I=\frac{1}{3}\int u^{-2}\,du=-\frac{1}{3u}+C=-\frac{1}{3(1+\tan^3 x)}+C.$$

Step 3: Verify the sign and coefficient Differentiating this answer gives

$$\frac{d}{dx}\left[-\frac{1}{3(1+\tan^3 x)}\right]=\frac{\tan^2 x\sec^2 x}{(1+\tan^3 x)^2},$$

which is the original integrand. Option (B) has the opposite sign. Rewriting $1/(1+\cot^3 x)=t^3/(1+t^3)$ shows that options (A) and (D) have derivatives respectively -3 and 3 times the integrand. The equivalent form $-\cos^3 x/[3(\sin^3 x+\cos^3 x)]$ extends across points where tangent is undefined, whenever the original denominator is nonzero.

Step 4: Conclusion The integral is $\boxed{-\frac{1}{3(1+\tan^3 x)}+C}$. Hence, the correct option is (C).

Quick Tip to Solve: When trigonometric powers occur in paired groups, factor using $\sin^2 x+\cos^2 x=1$ before choosing a substitution.

8. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is : [4 marks]

(A) $\frac{3}{4}$
(B) $\frac{3}{10}$
(C) $\frac{2}{5}$
(D) $\frac{1}{5}$

Correct Answer: (C) $\frac{2}{5}$

Solution:

Concept:

- The law of total probability adds the probabilities of disjoint routes to the desired event. Condition on the colour observed in the first draw.
- After returning the original ball and adding two balls, the bag contains twelve balls. The second-draw probability must use this new total, not ten.

Step 1: Separate the two possible first draws Let R_1 and B_1 denote a red and a black ball on the first draw, and let R_2 denote red on the second draw. The initial bag has ten balls, so $P(R_1)=4/10=2/5$ and $P(B_1)=6/10=3/5$. These events cover every first draw and cannot occur together. Returning the drawn ball means that the only net change to the original contents is the addition of the two new balls.

Step 2: Find both conditional probabilities If the first ball is red, the bag contains six red and six black balls; therefore $P(R_2 \mid R_1)=6/12=1/2$. If the first ball is black, it contains four red and eight black balls; hence $P(R_2 \mid B_1)=4/12=1/3$. The draws are dependent, so multiplying the original red fraction by itself would omit the change in composition.

Step 3: Weight the routes and compare the options Apply the total-probability formula:

$$P(R_2)=\frac{2}{5}\cdot\frac{1}{2}+\frac{3}{5}\cdot\frac{1}{3}=\frac{1}{5}+\frac{1}{5}=\frac{2}{5}.$$

The fractions $3/4$, $3/10$ and $1/5$ do not equal this sum; in particular, $1/5$ represents just one of the two red-producing routes.

Step 4: Conclusion The probability that the second ball is red is $\boxed{2/5}$. Hence, the correct option is (C).

Quick Tip to Solve: In a reinforcement draw, condition on the first colour and update both the colour count and the total before averaging.

9. Let $\overrightarrow{u}$ be a vector coplanar with the vectors $\overrightarrow{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\overrightarrow{b} = \hat{j} + \hat{k}$. If $\overrightarrow{u}$ is perpendicular to $\overrightarrow{a}$ and $\overrightarrow{u}.\overrightarrow{b} = 24$, then $|\overrightarrow{u}|^2$ is equal to [4 marks]

(A) 336

(B) 315

(C) 256

(D) 84

Correct Answer: (A) 336

Solution:

Concept:

- Two nonparallel vectors span their plane, so a coplanar vector can be written as $\vec{u} = \alpha\vec{a} + \beta\vec{b}$. The scalar coefficients are unknown real numbers.
- Perpendicular vectors have zero scalar product. Also, $|\vec{u}|^2 = \vec{u} \cdot \vec{u}$, which is the sum of squares of its Cartesian components.

Step 1: Compute the scalar products of the spanning vectors For $\vec{a} = (2, 3, -1)$ and $\vec{b} = (0, 1, 1)$, direct component multiplication gives

$$\vec{a} \cdot \vec{a} = 4 + 9 + 1 = 14, \qquad \vec{a} \cdot \vec{b} = 3 - 1 = 2, \qquad \vec{b} \cdot \vec{b} = 2.$$

These vectors are not scalar multiples, so their linear combination describes every vector in the stated plane.

Step 2: Use the two conditions to determine the coefficients Taking scalar products of $\vec{u} = \alpha\vec{a} + \beta\vec{b}$ with $\vec{a}$ and $\vec{b}$ yields

$$14\alpha + 2\beta = 0, \qquad 2\alpha + 2\beta = 24.$$

The first equation gives $\beta = -7\alpha$. Substitution into the second gives $-12\alpha = 24$, so $\alpha = -2$ and $\beta = 14$. Both constraints are needed; perpendicularity by itself does not determine the length.

Step 3: Calculate and check the squared magnitude Thus

$$\vec{u} = -2(2, 3, -1) + 14(0, 1, 1) = (-4, 8, 16).$$

Its scalar product with $\vec{a}$ is $-8 + 24 - 16 = 0$, while that with $\vec{b}$ is $8 + 16 = 24$, confirming both conditions. Consequently

$$|\vec{u}|^2 = 16 + 64 + 256 = 336.$$

None of the other values, 315, 256 or 84, equals the full sum of the three squared components.

Step 4: Conclusion The required squared magnitude is $\boxed{336}$. Hence, the correct option is (A).

Quick Tip to Solve: For a vector constrained to a plane, express it in the two given spanning vectors before applying scalar-product conditions.

10. If $\sum_{i=1}^{9}(x_i-5)=9$ **and**

$\sum_{i=1}^{9}(x_i-5)^2=45$**, then the standard deviation of the 9 items**

$x_1, x_2, \ldots\ldots, x_9$ **is** [4 marks]

(A) 3

(B) 9

(C) 4

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- For n observations, the population variance is $\sigma^2=\overline{z^2}-(\bar{z})^2$, where the bar denotes the arithmetic mean.
- Adding or subtracting the same constant from every observation changes the mean but leaves the deviations from the mean, and hence the standard deviation, unchanged.

Step 1: Shift the observations to use the supplied sums Define $z_i=x_i-5$ for $i=1,\ldots,9$. The given data become $\sum z_i=9$ and $\sum z_i^2=45$. Since there are nine observations, their shifted mean is

$$\bar{z}=\frac{\sum z_i}{9}=1.$$

In particular, the original mean is $\bar{x}=6$, so the given shift by five is not a shift to the actual mean.

Step 2: Evaluate the variance and take its square root The mean square of the shifted observations is $45/9=5$. Therefore

$$\sigma^2=\frac{45}{9}-\left(\frac{9}{9}\right)^2=5-1=4, \qquad \sigma=\sqrt{4}=2.$$

The divisor is nine because this question asks for the standard deviation of the stated nine items, rather than an unbiased estimate of an external population variance.

Step 3: Check the interpretation against the alternatives The invariance follows directly from $z_i-\bar{z}=(x_i-5)-(\bar{x}-5)=x_i-\bar{x}$. Thus the computed standard deviation is also that of the original data. Option (C), 4, is the variance before taking its square root; 9 is the number of observations; and 3 does not follow from either central moment. Only 2 satisfies the requested statistic.

Step 4: Conclusion The standard deviation of the nine observations is $\boxed{2}$. Hence, the correct option is (D).

Quick Tip to Solve: When sums about an arbitrary origin are supplied, subtract the square of the shifted mean before taking the square root.

11. Let a_1, a_2, a_3, ... ,a_{49} be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. $a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$, then m is equal to [4 marks]

(A) 33

(B) 66

(C) 68

(D) 34

Correct Answer: (D) 34

Solution:

Concept:

- If an arithmetic progression has first term a and common difference d, then $a_n = a + (n-1)d$. Symmetrically placed terms have sum twice their middle term.
- The sum of the first n positive integer squares is $n(n+1)(2n+1)/6$. A consecutive interval of squares is obtained by subtracting two such sums.

Step 1: Use the subsampled progression The terms $a_{4k+1} = a + 4kd$, for $k = 0, \ldots, 12$, form an arithmetic progression of thirteen terms. Their average is the middle term $a_{25} = a + 24d$. Hence

$$13(a + 24d) = 416 \quad \Longrightarrow \quad a + 24d = 32.$$

The endpoint count is thirteen, since both zero and twelve are included.

Step 2: Determine the first term and common difference The second condition gives

$$a_9 + a_{43} = (a + 8d) + (a + 42d) = 2a + 50d = 66,$$

or $a + 25d = 33$. Subtracting $a + 24d = 32$ yields $d = 1$, and then $a = 8$. Thus the first seventeen terms are precisely the integers from 8 through 24, inclusive.

Step 3: Sum the squares and find the parameter The required sum is

$$\sum_{j=8}^{24} j^2 = \frac{24 \cdot 25 \cdot 49}{6} - \frac{7 \cdot 8 \cdot 15}{6}$$
$$= 4900 - 140 = 4760.$$

Since this equals $140m$, division gives $m = 4760/140 = 34$. The values 33, 66 and 68 would give different sums; 33 is also the middle term inferred from the second condition, rather than the requested parameter.

Step 4: Conclusion The parameter is $\boxed{m = 34}$. Hence, the correct option is (D).

Quick Tip to Solve: Average an odd number of equally spaced AP terms to get its middle term; this often avoids expanding a long sum.

12. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x}\,dx$ is [4 marks]

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{8}$

(C) $\frac{\pi}{2}$

(D) 4π

Correct Answer: (A) $\frac{\pi}{4}$

Solution:

Concept:

- On a symmetric interval, replacing x by $-x$ leaves the integral unchanged after reversing the limits. This is useful when the denominator contains reciprocal exponential terms.
- The numerator $\sin^2 x$ is even, and $1/(1+2^x) + 1/(1+2^{-x}) = 1$. Pairing these complementary factors removes the exponential.

Step 1: Reflect the integral about the origin Write the required integral as I. Under the substitution $t = -x$, the endpoints swap and $dx = -dt$. Since $\sin^2(-t) = \sin^2 t$, relabelling the variable gives

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^{-x}}\,dx.$$

The positive denominator never vanishes, so both forms are continuous on the entire integration interval.

Step 2: Add the reflected expressions Multiplication of numerator and denominator by 2^x shows that $1/(1+2^{-x}) = 2^x/(1+2^x)$. Adding the original and reflected forms therefore gives

$$2I = \int_{-\pi/2}^{\pi/2} \sin^2 x \left(\frac{1+2^x}{1+2^x}\right) dx = \int_{-\pi/2}^{\pi/2} \sin^2 x\,dx.$$

No approximation to the exponential is involved; the simplification is exact.

Step 3: Evaluate the elementary remaining integral Use $\sin^2 x = (1-\cos 2x)/2$:

$$2I = \left[\frac{x}{2} - \frac{\sin 2x}{4}\right]_{-\pi/2}^{\pi/2} = \frac{\pi}{2}.$$

Hence $I = \pi/4$. The alternative $\pi/2$ forgets that the paired integral is $2I$; $\pi/8$ halves the result once too often, while 4π is incompatible with the bounded integrand.

Step 4: Conclusion The integral equals $\boxed{\pi/4}$. Hence, the correct option is (A).

Quick Tip to Solve: On symmetric limits, pair an even numerator over $1 + a^x$ with its reflected form before attempting direct integration.

13. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is : [4 marks]

(A) 3x + 2y = 6xy

(B) 3x + 2y = 6

(C) 2x + 3y = xy

(D) 3x + 2y = xy

Correct Answer: (D) 3x + 2y = xy

Solution:

Concept:

- A line with nonzero intercepts p and q on the coordinate axes has equation $X/p + Y/q = 1$, where (X, Y) is a variable point on the line.
- In the rectangle with adjacent vertices $O = (0, 0)$, $P = (p, 0)$ and $Q = (0, q)$, the opposite vertex is $R = (p, q)$. The two intercepts therefore become the coordinates of the locus point.

Step 1: Parameterise the intercepts and the opposite vertex Let the line meet the horizontal axis at $P = (p, 0)$ and the vertical axis at $Q = (0, q)$. A nondegenerate rectangle requires $p \neq 0$ and $q \neq 0$. The sides through P and Q parallel to the other coordinate axis intersect at $R = (p, q)$. Thus if $R = (x, y)$, its coordinates are exactly the line intercepts, rather than the coordinates of a general point of the moving line.

Step 2: Impose passage through the fixed point The intercept equation of PQ is $X/p + Y/q = 1$. Since the fixed point $(2, 3)$ lies on it,

$$\frac{2}{p} + \frac{3}{q} = 1.$$

Replacing p, q by x, y and multiplying by xy gives

$$2y + 3x = xy.$$

The condition $xy \neq 0$ remains understood because multiplying denominators can otherwise introduce the degenerate origin.

Step 3: Check the coefficients and the choices For example, intercepts $p = 4$, $q = 6$ give $2/4 + 3/6 = 1$, and the opposite corner $(4, 6)$ satisfies $3x + 2y = xy$. Interchanging the coefficients as in (C) fails this example. Option (B) is linear rather than the intercept relation; option (A) introduces an unsupported factor of six multiplying the product.

Step 4: Conclusion The locus equation is $\boxed{3x + 2y = xy}$, with the degenerate origin excluded. Hence, the correct option is (D).

Quick Tip to Solve: For a rectangle completed from axis intercepts, identify the opposite corner with the pair of intercepts before eliminating parameters.

14. Let y = y(x) be the solution of the differential equation

$\sin x \frac{dy}{dx} + y \cos x = 4x$, $x \in (0, \pi)$.

If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to : [4 marks]

(A) $-\frac{4}{9}\pi^2$

(B) $\frac{4}{9\sqrt{3}}\pi^2$

(C) $-\frac{8}{9\sqrt{3}}\pi^2$

(D) $-\frac{8}{9}\pi^2$

Correct Answer: (D) $-\frac{8}{9}\pi^2$

Solution:

Concept:

- The product rule gives $d(y \sin x)/dx = \sin x \, dy/dx + y \cos x$, exactly the left side of the differential equation.
- Integrating determines the product up to a constant. The initial condition fixes that constant, and $\sin x \neq 0$ on $(0, \pi)$, so division by $\sin x$ is valid throughout the given domain.

Step 1: Recognise and integrate the exact product derivative The equation can be rewritten as

$$\frac{d}{dx}(y \sin x) = 4x.$$

Integrating with respect to x, including a single arbitrary constant, yields

$$y \sin x = 2x^2 + C.$$

This also agrees with the integrating-factor method after division by $\sin x$: the coefficient of y is $\cot x$, whose integrating factor on this interval is $\sin x$.

Step 2: Apply the initial condition before evaluation At $x = \pi/2$, the prescribed value $y = 0$ gives

$$0 = 2\left(\frac{\pi}{2}\right)^2 + C = \frac{\pi^2}{2} + C.$$

Therefore $C = -\pi^2/2$, and the unique solution on the stated interval is

$$y(x) = \frac{2x^2 - \pi^2/2}{\sin x}.$$

Substitution at the initial point indeed gives zero, confirming the sign of the constant.

Step 3: Substitute the requested argument and compare Since $\sin(\pi/6) = 1/2$,

$$y(\pi/6) = \frac{\pi^2/18 - \pi^2/2}{1/2}$$
$$= 2\left(-\frac{4\pi^2}{9}\right) = -\frac{8\pi^2}{9}.$$

Option (A) misses the final division by one-half. Options (B) and (C) contain $\sqrt{3}$, which is inconsistent

with the sine value at $\pi/6$; (B) also has the wrong sign.

Step 4: Conclusion The requested value is $\boxed{-8\pi^2/9}$. Hence, the correct option is (D).

Quick Tip to Solve: Look for a product derivative before finding an integrating factor; apply the initial condition to the integrated product first.

15. Let g(x) = cosx^2, f(x) = $\sqrt{x}$ and $\alpha, \beta\,(\alpha < \beta)$ be the roots of the quadratic equation 18x^2 - 9πx + π^2 = 0. Then the area (in sq. units) bounded by the curve y = (gof)(x) and the lines $x = \alpha$, $x = \beta$ and y = 0 is : [4 marks]

(A) $\frac{1}{2}\left(\sqrt{2}-1\right)$
(B) $\frac{1}{2}\left(\sqrt{3}-1\right)$
(C) $\frac{1}{2}\left(\sqrt{3}+1\right)$
(D) $\frac{1}{2}\left(\sqrt{3}-\sqrt{2}\right)$

Correct Answer: (B) $\frac{1}{2}\left(\sqrt{3}-1\right)$

Solution:

Concept:

- For two functions, composition means substituting the complete inner function into the argument of the outer one. Here $g(u) = \cos(u^2)$ and $f(x) = \sqrt{x}$.
- The area between a curve and the horizontal axis on an interval is $\int |y|\,dx$. If the curve is nonnegative throughout that interval, this is its ordinary definite integral.

Step 1: Find the two vertical boundaries The quadratic factors without any decimal approximation:

$$18x^2 - 9\pi x + \pi^2 = (6x - \pi)(3x - \pi).$$

Its roots in increasing order are therefore $\alpha = \pi/6$ and $\beta = \pi/3$. These boundaries both lie in the first quadrant, where the cosine function is positive.

Step 2: Compose the functions before integrating Substitute $f(x)$ for the argument of g, rather than multiplying the functions:

$$(g \circ f)(x) = \cos\left((\sqrt{x})^2\right) = \cos x.$$

The domain restriction from the square root is satisfied because the entire interval has positive x.

Step 3: Evaluate the definite integral and compare Since $\cos x > 0$ between the two boundaries, no sign reversal or splitting of the area is required:

$$A = \int_{\pi/6}^{\pi/3} \cos x\,dx = \sin(\pi/3) - \sin(\pi/6) = \frac{\sqrt{3}-1}{2}.$$

The other options use the wrong endpoint value, add the endpoint sines, or introduce $\sqrt{2}$, which does not arise at these limits.

Step 4: Conclusion The required area is $\frac{\sqrt{3}-1}{2}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: Compose first: squaring the inner square root turns this apparently composite-function integral into the integral of $\cos x$.

16. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is : [4 marks]

(A) at least 750 but less than 1000

(B) at least 1000

(C) less than 500

(D) at least 500 but less than 750

Correct Answer: (B) at least 1000

Solution:

Concept:

- Use combinations to select the books and permutations to place distinct selected books into specified positions. The multiplication principle combines independent choices.
- Five books have a unique middle position, namely position 3. Fixing the dictionary there leaves four distinct shelf positions for the four selected novels.

Step 1: Select the novels and dictionary The six novels are all different, so the set of four novels can be chosen in

$$\binom{6}{4} = \frac{6!}{4!\,2!} = 15$$

ways. Independently, exactly one of the three different dictionaries can be chosen in $\binom{3}{1} = 3$ ways. At this stage we have chosen the books, but have not ordered the novels.

Step 2: Fix the dictionary and order the novels Place the chosen dictionary in the third shelf position. It has only one allowed position, so it contributes no additional position factor. Fill positions 1, 2, 4 and 5 with the four distinct selected novels. There are $4! = 24$ bijections between these novels and the available positions. Their separation into two positions on each side is already counted in this factorial.

Step 3: Multiply and identify the stated range Every valid arrangement has one selected dictionary and one ordered list of four novels, so it is counted exactly once by

$$N = \binom{6}{4}\binom{3}{1}4! = 15 \times 3 \times 24 = 1080.$$

This is at least 1000. It is not below 500, not between 500 and 750, and not below 1000; thus the other three intervals do not contain the result.

Step 4: Conclusion There are 1080 valid arrangements, which is at least 1000. Hence, the correct option is (B).

Quick Tip to Solve: Once the dictionary occupies the middle, use ${}^{6}P_{4} \times 3$; selecting and ordering the novels can be combined into one factor.

17. If the system of linear equations

x + ky + 3z = 0

3x + ky - 2z = 0

2x + 4y - 3z = 0

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to [4 marks]

(A) 30

(B) -10

(C) 10

(D) -30

Correct Answer: (C) 10

Solution:

Concept:

- A homogeneous linear system always has the zero solution. A nonzero solution forces compatibility of its equations; elimination can reveal the ratios without expanding a determinant.
- The requested ratio xz/y^2 is unchanged when all three coordinates are multiplied by the same nonzero constant. Find their relative values and verify the denominator is nonzero.

Step 1: Eliminate the parameter term Subtract the first equation from the second. The two terms ky cancel exactly:

$$(3x + ky - 2z) - (x + ky + 3z) = 0 \quad \Longrightarrow \quad 2x - 5z = 0.$$

Consequently $x = 5z/2$. This relation holds for any value of k that admits the stipulated solution.

Step 2: Use the third equation to obtain the remaining ratio Substitute this value of x into the third equation:

$$2\left(\frac{5z}{2}\right) + 4y - 3z = 0 \quad \Longrightarrow \quad 2z + 4y = 0.$$

Thus $y = -z/2$. If $z = 0$, these relations force $x = y = 0$, contradicting the nonzero-solution condition. Hence $z \neq 0$ and also $y \neq 0$. As a consistency check, the first equation then gives $(11 - k)z/2 = 0$, so $k = 11$.

Step 3: Evaluate the scale-independent expression Insert the ratios directly:

$$\frac{xz}{y^2} = \frac{(5z/2)z}{(-z/2)^2} = \frac{5z^2/2}{z^2/4} = 10.$$

The square in the denominator makes the value positive, excluding the negative options. The cancellation gives 10 rather than 30, excluding the remaining positive distractor.

Step 4: Conclusion The required value is $\frac{xz}{y^2} = 10$. Hence, the correct option is (C).

Quick Tip to Solve: When two equations contain the same parameter term, subtract them first; homogeneous ratios often become accessible without computing a determinant.

18. Let S = { $x \in \mathbf{R} : x \geq 0$ and $2\left|\sqrt{x}-3\right|+\sqrt{x}\left(\sqrt{x}-6\right)+6=0$}. Then S [4 marks]

(A) contains exactly four elements

(B) is an empty set

(C) contains exactly one element

(D) contains exactly two elements

Correct Answer: (D) contains exactly two elements

Solution:

Concept:

- Introduce $t=\sqrt{x}$, with the essential restriction $t \geq 0$. Squaring back to x is one-to-one on this restricted domain.
- Complete the square around the same centre that appears in the absolute value. Then use $u^2=|u|^2$ to convert the equation into a quadratic in a nonnegative variable.

Step 1: Rewrite the equation in terms of the square root With $t=\sqrt{x}$, the given equation is

$$2|t-3|+t(t-6)+6=0.$$

Because $t^2-6t+6=(t-3)^2-3$, this becomes

$$(t-3)^2+2|t-3|-3=0.$$

This rearrangement preserves every real solution and avoids separately expanding two sign cases at the outset.

Step 2: Solve the quadratic with its sign restriction Set $v=|t-3|$, so $v \geq 0$. The equation factors as

$$v^2+2v-3=(v-1)(v+3)=0.$$

The root $v=-3$ is impossible for an absolute value. Thus $v=1$, giving $t-3=1$ or $t-3=-1$. Both values $t=4$ and $t=2$ satisfy $t \geq 0$.

Step 3: Return to the original variable and count Since $x=t^2$, the two values are $x=16$ and $x=4$. Checking in the original equation gives respectively $2+4(-2)+6=0$ and $2+2(-4)+6=0$. These are distinct and exhaust the admissible roots of the factored equation. Therefore the set is neither empty nor a singleton and has no additional pair of elements.

Step 4: Conclusion We obtain $S=\{4,16\}$, which contains exactly two elements. Hence, the correct option is (D).

Quick Tip to Solve: When a quadratic and an absolute value share the centre $t=3$, complete the square and solve in $|t-3|$, remembering that it cannot be negative.

19. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is : [4 marks]

(A) $\frac{9}{2}$

(B) 6

(C) $\frac{7}{2}$

(D) 4

Correct Answer: (A) $\frac{9}{2}$

Solution:

Concept:

- Two differentiable curves intersect at right angles when the product of their finite tangent slopes at a common point equals -1.
- Implicit differentiation gives the tangent slopes without solving either curve explicitly for y. The common-point relation $y^2 = 6x$ simplifies their product.

Step 1: Differentiate both curve equations For the parabola, differentiating $y^2 = 6x$ with respect to x gives $2y\,dy/dx = 6$, so its tangent slope is $m_1 = 3/y$. For the second curve,

$$18x + 2by\frac{dy}{dx} = 0 \quad \Longrightarrow \quad m_2 = -\frac{9x}{by}.$$

The point $(0, 0)$ is not on the second curve because its equation has right side 16. Any common point consequently has $x > 0$ and $y \neq 0$.

Step 2: Impose orthogonality at a common point Multiply the slopes and substitute the parabola equation:

$$m_1 m_2 = -\frac{27x}{by^2} = -\frac{27x}{6bx} = -\frac{9}{2b}.$$

For perpendicular tangents this must equal -1, giving $b = 9/2$. The option $b = 0$ is not among the choices; all printed options allow the finite-slope calculation.

Step 3: Check existence and exclude the other values With $b = 9/2$, simultaneous substitution gives $9x^2 + 27x = 16$, which has a positive root because its left side is 0 at $x = 0$ and grows without bound. Thus real intersections exist. For $b = 6, 7/2, 4$, the slope products are respectively $-3/4, -9/7, -9/8$, none equal to -1.

Step 4: Conclusion The required parameter is $b = \frac{9}{2}$. Hence, the correct option is (A).

Quick Tip to Solve: At an orthogonal intersection, multiply the two implicit slopes and simplify using a curve equation before attempting to find the intersection coordinates.

20. The length of the projection of the line segment joining the points (5, -1, 4) and (4, -1, 3) on the plane, x + y + z = 7 is : [4 marks]

(A) $\sqrt{\frac{2}{3}}$

(B) $\frac{2}{\sqrt{3}}$

(C) $\frac{2}{3}$

(D) $\frac{1}{3}$

Correct Answer: (A) $\sqrt{\frac{2}{3}}$

Solution:

Concept:

- A plane with equation $ax + by + cz = d$ has normal vector $\boldsymbol{n} = (a, b, c)$. Its constant term locates the plane but does not affect a projected segment length.
- Decompose the segment vector $\boldsymbol{v}$ into perpendicular normal and in-plane components. The in-plane projection length is $\sqrt{|\boldsymbol{v}|^2 - (\boldsymbol{v} \cdot \boldsymbol{n})^2/|\boldsymbol{n}|^2}$.

Step 1: Form the segment and normal vectors Take the displacement from $A = (5, -1, 4)$ to $B = (4, -1, 3)$:

$$\boldsymbol{v} = B - A = (-1, 0, -1), \qquad |\boldsymbol{v}|^2 = 2.$$

The normal to $x + y + z = 7$ is $\boldsymbol{n} = (1, 1, 1)$, with $|\boldsymbol{n}|^2 = 3$. Reversing the segment direction would change its signs but not the projection length.

Step 2: Remove the normal component The scalar product is $\boldsymbol{v} \cdot \boldsymbol{n} = -2$. Thus the squared length perpendicular to the plane is

$$\frac{(\boldsymbol{v} \cdot \boldsymbol{n})^2}{|\boldsymbol{n}|^2} = \frac{4}{3}.$$

By Pythagoras, the squared length parallel to the plane is $2 - 4/3 = 2/3$. This subtraction is necessary because the requested projection is onto the plane, rather than onto its normal line.

Step 3: Take the positive square root and compare The length is therefore $\sqrt{2/3}$. The option $2/\sqrt{3}$ is the normal component length, not the plane projection. The value $2/3$ is the squared plane-projection length and cannot be used without a square root; $1/3$ is neither component length. The result is less than the original segment length $\sqrt{2}$, as a projection must be.

Step 4: Conclusion The projected segment has length $\sqrt{\frac{2}{3}}$. Hence, the correct option is (A).

Quick Tip to Solve: The dot product with the unit normal gives the perpendicular component; subtract its square from the segment length squared to obtain the plane projection.

21. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $(x>1)$ is [4 marks]

(A) 2

(B) -1

(C) 0

(D) 1

Correct Answer: (A) 2

Solution:

Concept:

- Adding equal powers of $x+s$ and $x-s$ cancels all terms with an odd power of s. Here the surviving even powers of the radical become polynomial expressions.
- For a polynomial $P(x)$, the sum of coefficients of odd powers is $[P(1)-P(-1)]/2$. Use this identity only after obtaining the polynomial; the original radical expression is stated for $x>1$.

Step 1: Cancel the odd radical terms Write $s=\sqrt{x^3-1}$. The binomial theorem gives

$$P(x)=(x+s)^5+(x-s)^5=2\left(x^5+10x^3s^2+5xs^4\right).$$

The terms involving s, s^3, s^5 cancel in pairs, while those involving $1, s^2, s^4$ double. Thus no radical remains after replacing s^2 by x^3-1.

Step 2: Expand into ordinary integer powers Substitute $s^2=x^3-1$ and $s^4=(x^3-1)^2$:

$$\begin{aligned}P(x)&=2x^5+20x^3(x^3-1)+10x(x^3-1)^2\\&=10x^7+20x^6+2x^5-20x^4-20x^3+10x.\end{aligned}$$

The degree-seven term is present because the original radical contains x^3, rather than x^2.

Step 3: Collect the coefficients of odd degrees The odd-degree terms have powers 7, 5, 3 and 1. Their coefficients sum to

$$10+2-20+10=2.$$

As a check, the expanded polynomial has $P(1)=2$ and $P(-1)=-2$, so $[P(1)-P(-1)]/2=2$. These evaluations concern the polynomial identity and do not substitute outside the domain of the original radical expression. The alternatives $-1, 0, 1$ do not equal the collected sum.

Step 4: Conclusion The sum of the coefficients of all odd-degree terms is 2. Hence, the correct option is (A).

Quick Tip to Solve: First pair the conjugate binomial expressions to eliminate odd radical powers; then select the odd powers of x in the resulting polynomial.

22. If L_1 is the line of intersection of the planes 2x - 2y + 3z - 2 = 0, x - y + z + 1 = 0 and L_2 is the line of intersection of the planes x + 2y - z - 3 = 0, 3x - y + 2z - 1 = 0, then the distance of the origin from the plane, containing the lines L_1 and L_2, is : [4 marks]

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{1}{4\sqrt{2}}$

(C) $\frac{1}{3\sqrt{2}}$

(D) $\frac{1}{2\sqrt{2}}$

Correct Answer: (C) $\frac{1}{3\sqrt{2}}$

Solution:

Concept:

- A plane containing a line of intersection is a linear combination of the two plane equations. It must vanish at every point of that line.
- For a plane $ax + by + cz + d = 0$, the perpendicular distance of the origin is $|d|/\sqrt{a^2 + b^2 + c^2}$. Its normal is perpendicular to every direction lying in the plane.

Step 1: Simplify the first line Subtract twice the second defining equation from the first. This gives $z - 4 = 0$. Substituting back gives $x - y + 5 = 0$. Hence every plane through L_1 has the form

$$a(x - y + 5) + b(z - 4) = 0.$$

Its normal vector is $(a, -a, b)$; this automatically makes it perpendicular to the direction $(1, 1, 0)$ of L_1.

Step 2: Impose the second line The direction of L_2 is the cross product of its plane normals:

$$(1, 2, -1) \times (3, -1, 2) = (3, -5, -7).$$

Thus $8a - 7b = 0$. Take $a = 7, b = 8$, obtaining $7x - 7y + 8z + 3 = 0$. This plane really contains the second line: its equation equals $2(x + 2y - z - 3) - 3(3x - y + 2z - 1) = 0$, multiplied by -1.

Step 3: Compute and compare the distance The constant term must be retained when computing a distance. Therefore

$$d = \frac{3}{\sqrt{7^2 + (-7)^2 + 8^2}} = \frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}.$$

The other three listed values have different denominators and cannot equal this distance; the result is positive as required.

Step 4: Conclusion The required distance is $\boxed{1/(3\sqrt{2})}$. Hence, the correct option is (C).

Quick Tip to Solve: Find each line direction first, then use a point to determine the constant in their common plane; normal vectors alone do not give the distance.

23. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series
$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2$ .....
If B - 2A = 100λ, then λ is equal to [4 marks]

(A) 496

(B) 232

(C) 248

(D) 464

Correct Answer: (C) 248

Solution:

Concept:

- The odd-indexed term is the square of its index, whereas the even-indexed term is twice that square. The dots between factors denote multiplication.
- The sum of the first m squares is $m(m+1)(2m+1)/6$. Separate the extra copy of each even square before applying this formula.

Step 1: Express an even partial sum For $2n$ terms, start with one copy of every square from 1^2 to $(2n)^2$. Each even square needs one additional copy, so

$$S_{2n} = \sum_{j=1}^{2n} j^2 + \sum_{j=1}^{n} (2j)^2 = \frac{2n(2n+1)(4n+1)}{6} + 4\frac{n(n+1)(2n+1)}{6}.$$

This construction gives coefficient two, rather than four, to each even-indexed term.

Step 2: Evaluate both required sums For the first 20 terms, $n = 10$, while the first 40 terms require $n = 20$. Hence

$$A = \frac{20 \cdot 21 \cdot 41}{6} + 4\frac{10 \cdot 11 \cdot 21}{6} = 2870 + 1540 = 4410.$$

Similarly,

$$B = \frac{40 \cdot 41 \cdot 81}{6} + 4\frac{20 \cdot 21 \cdot 41}{6} = 22140 + 11480 = 33620.$$

The indices refer to numbers of terms, not numbers of odd-even pairs.

Step 3: Use the defining equation for lambda Now calculate

$$100\lambda = B - 2A = 33620 - 8820 = 24800, \qquad \lambda = 248.$$

Substitution checks the result directly: $2(4410) + 100(248) = 33620$. Thus 496, 232 and 464 fail the stated equation and are excluded.

Step 4: Conclusion $\boxed{\lambda = 248}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the series as all squares plus one extra copy of the even squares; this avoids separately summing odd squares.

24. Let the orthocentre and centroid of a triangle be A(-3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is : [4 marks]

(A) $\frac{3\sqrt{5}}{2}$

(B) $\sqrt{10}$

(C) $2\sqrt{10}$

(D) $3\sqrt{\frac{5}{2}}$

Correct Answer: (D) $3\sqrt{\frac{5}{2}}$

Solution:

Concept:

- The orthocentre, centroid and circumcentre lie on the Euler line. The centroid divides the segment from orthocentre to circumcentre internally in the ratio $2:1$.
- If H, G, O denote these three centres, their position vectors satisfy $3G = H + 2O$. The radius of a circle whose diameter is a specified segment is half the length of that segment.

Step 1: Locate the circumcentre Here $H = A = (-3, 5)$ and $G = B = (3, 3)$. The circumcentre is the point called C, so

$$C = \frac{3B - A}{2} = \left(\frac{9+3}{2}, \frac{9-5}{2}\right) = (6, 2).$$

The ratio must be used in the correct direction: the centroid is nearer the circumcentre than the orthocentre. Indeed, $\overrightarrow{AB} = (6, -2)$ and $\overrightarrow{BC} = (3, -1)$.

Step 2: Calculate the requested diameter The circle in the question has diameter AC. It is not necessarily the original triangle circumcircle. Using the distance formula gives

$$AC = \sqrt{(6-(-3))^2 + (2-5)^2} = \sqrt{81+9} = 3\sqrt{10}.$$

Both coordinate differences are squared, so the negative vertical difference contributes positively.

Step 3: Halve the length and match the option The required radius is therefore

$$r = \frac{AC}{2} = \frac{3\sqrt{10}}{2} = 3\sqrt{\frac{5}{2}}.$$

The last equality follows since $\sqrt{10}/2 = \sqrt{10/4}$. The alternative values $3\sqrt{5}/2$, $\sqrt{10}$ and $2\sqrt{10}$ are different; none equals half of the computed diameter.

Step 4: Conclusion The radius is $\boxed{3\sqrt{5/2}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use $3G = H + 2O$ to prevent reversal of the Euler-line ratio, then distinguish the requested diameter circle from the triangle circumcircle.

25. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is : [4 marks]

(A) $\frac{4}{3}$

(B) $\frac{1}{2}$

(C) 2

(D) 3

Correct Answer: (C) 2

Solution:

Concept:

- The derivative of $y^2 = 16x$ is $dy/dx = 8/y$. A normal has slope equal to the negative reciprocal of the tangent slope.
- A circle centre lies on the perpendicular bisector of each chord. The tangent of the angle between two vectors is the absolute cross product divided by their dot product when the angle is acute.

Step 1: Find the intercepts of tangent and normal At $P = (16, 16)$, the tangent slope is $1/2$, so

$$y - 16 = \tfrac{1}{2}(x - 16).$$

Setting $y = 0$ gives $A = (-16, 0)$. The normal has slope -2, so

$$y - 16 = -2(x - 16),$$

giving $B = (24, 0)$. These intercepts belong to the horizontal axis of the parabola.

Step 2: Determine the circle centre The midpoint of AB is $(4, 0)$, so the perpendicular bisector is $x = 4$. Write $C = (4, k)$ and impose $CA = CP$:

$$400 + k^2 = 144 + (k - 16)^2 = 400 + k^2 - 32k.$$

Therefore $k = 0$, and $C = (4, 0)$. This also agrees with the right angle between the tangent PA and normal PB: AB is a diameter of their circumcircle.

Step 3: Evaluate the angle at P The vectors from P are $\overrightarrow{PC} = (-12, -16)$ and $\overrightarrow{PB} = (8, -16)$. Their dot product is $-96 + 256 = 160 > 0$, establishing an acute angle. Thus

$$\tan\theta = \frac{|(-12)(-16) - (-16)(8)|}{160} = \frac{320}{160} = 2.$$

Neither $4/3$, $1/2$, nor 3 gives this cross-product to dot-product ratio.

Step 4: Conclusion $\boxed{\tan\theta = 2}$. Hence, the correct option is (C).

Quick Tip to Solve: The tangent and normal are perpendicular, making their two axis intercepts the endpoints of a diameter of the circle through the contact point.

26. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q.
If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is : [4 marks]

(A) $36\sqrt{5}$
(B) $45\sqrt{5}$
(C) $54\sqrt{3}$
(D) $60\sqrt{3}$

Correct Answer: (B) $45\sqrt{5}$

Solution:

Concept:

- For $4x^2 - y^2 = 36$, the tangent at (x_1, y_1) is $4xx_1 - yy_1 = 36$. Substituting an external point imposes a condition on the contact point.
- The two contact points of tangents from a point form its chord of contact. Once that chord is horizontal, the triangle area is half its length times the perpendicular distance of the external point.

Step 1: Find the chord of contact Let either point of contact be (x_1, y_1). Since the tangent passes through $T = (0, 3)$, its equation yields

$$4(0)x_1 - 3y_1 = 36, \qquad y_1 = -12.$$

Thus both contact points have the same ordinate. The positive ordinate of T does not imply positive ordinates of the contact points: the minus sign in the hyperbola tangent equation is essential.

Step 2: Locate both points on the hyperbola Substitute $y = -12$ into the curve:

$$4x^2 - 144 = 36, \qquad x^2 = 45, \qquad x = \pm 3\sqrt{5}.$$

Hence we can name the points $P = (3\sqrt{5}, -12)$ and $Q = (-3\sqrt{5}, -12)$. Their horizontal separation, and therefore the base length, is $PQ = 6\sqrt{5}$. The two symmetric contacts are distinct, as expected.

Step 3: Calculate the triangular area The distance from $T = (0, 3)$ to the line $y = -12$ is $3-(-12) = 15$. Consequently,

$$[PTQ] = \frac{1}{2}(6\sqrt{5})(15) = 45\sqrt{5}.$$

Using 12 for the height would wrongly measure from the origin instead of T. The values $36\sqrt{5}$, $54\sqrt{3}$, and $60\sqrt{3}$ do not equal the computed base-height product.

Step 4: Conclusion The area is $\boxed{45\sqrt{5} \text{ square units}}$. Hence, the correct option is (B).

Quick Tip to Solve: Substitute the intersection of the tangents into the general tangent equation to obtain the chord of contact immediately.

27. If sum of all the solutions of the equation

$$8\cos x.\left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$$

in [0, π] is kπ, then k is equal to [4 marks]

(A) $\frac{20}{9}$
(B) $\frac{2}{3}$
(C) $\frac{13}{9}$
(D) $\frac{8}{9}$

Correct Answer: (C) $\frac{13}{9}$

Solution:

Concept:

- The product identity $\cos(a+x)\cos(a-x)=\cos^2 x-\sin^2 a$ reduces the shifted cosine factors to a polynomial in $\cos x$.
- The triple-angle identity is $\cos 3x=4\cos^3 x-3\cos x$. The interval for the multiplied angle must also be multiplied before listing roots.

Step 1: Reduce the equation With $a=\pi/6$, we have $\sin^2 a=1/4$. Thus the quantity in parentheses is $\cos^2 x-3/4$, and the printed equation becomes

$$8\cos^3 x-6\cos x=1.$$

Its left side is $2\cos 3x$. Hence the equivalent equation is

$$\cos 3x=\frac{1}{2}.$$

No division by a variable expression was used, so no possible solution has been lost.

Step 2: List every root in the required interval Set $t=3x$. Since $0\le x\le\pi$, we require $0\le t\le 3\pi$. The cosine equals $1/2$ at

$$t=\frac{\pi}{3},\quad\frac{5\pi}{3},\quad\frac{7\pi}{3}.$$

The next value, $11\pi/3$, lies beyond the upper bound, and neither endpoint is a solution. Dividing by three gives $x=\pi/9,5\pi/9,7\pi/9$, three distinct solutions.

Step 3: Sum the solutions The sum of all roots is

$$\frac{\pi}{9}+\frac{5\pi}{9}+\frac{7\pi}{9}=\frac{13\pi}{9}.$$

Comparing with $k\pi$ gives $k=13/9$. The fractions $20/9$, $2/3$, and $8/9$ fail this complete root sum; in particular, retaining only roots from one cosine period would miss a permitted root.

Step 4: Conclusion $\boxed{k=13/9}$. Hence, the correct option is (C).

Quick Tip to Solve: After applying a multiple-angle identity, enlarge the angle interval before enumerating roots; otherwise the final partial period is easily missed.

28. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is : [4 marks]

(A) $50\sqrt{2}$

(B) 100

(C) 50

(D) $100\sqrt{3}$

Correct Answer: (B) 100

Solution:

Concept:

- For a vertical tower of height h, the angle of elevation α at a point at horizontal distance d satisfies $\tan\alpha = h/d$.
- The median from the vertex of an isosceles triangle to its base is perpendicular to the base. Pythagoras therefore relates the two horizontal distances to the equal side length.

Step 1: Relate the ground distances to the height Let M be the midpoint of QR, which is the foot of the tower, and let its height be $h > 0$. The angle of elevation at P gives

$$\tan 45^\circ = \frac{h}{PM}, \qquad PM = h.$$

At Q, similarly,

$$\tan 30^\circ = \frac{h}{QM}, \qquad QM = \sqrt{3}h.$$

The angle at R gives the same distance, consistent with M being the midpoint.

Step 2: Use the horizontal right triangle Since $PQ = PR$, the segment joining P to the midpoint M is perpendicular to QR. Hence $\triangle PMQ$ is right angled at M, with hypotenuse $PQ = 200$ m. Pythagoras gives

$$200^2 = PM^2 + QM^2 = h^2 + 3h^2 = 4h^2.$$

Both lengths here are ground distances; the tower height enters through the elevation equations.

Step 3: Choose the positive height and check It follows that $h^2 = 10000$, so $h = 100$ m, taking the positive root for a physical height. Then $PM = 100$ m and $QM = 100\sqrt{3}$ m, whose squared sum is 40000, checking the given side. None of $50\sqrt{2}$, 50, or $100\sqrt{3}$ satisfies $4h^2 = 40000$.

Step 4: Conclusion The height of the tower is $\boxed{100 \text{ m}}$. Hence, the correct option is (B).

Quick Tip to Solve: Express both horizontal legs as multiples of the same tower height, then apply Pythagoras in the ground triangle.

29. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$, then the value of c is : [4 marks]

(A) 95

(B) 195

(C) 185

(D) 85

Correct Answer: (A) 95

Solution:

Concept:

- The derivative gives the slope of the tangent: for $y = x^2 + 6$, $dy/dx = 2x$. A tangent line has the same instantaneous slope as the curve at the point of contact.
- A line touches a real circle when the perpendicular distance of its centre from the line equals its radius. Completing squares in the circle equation identifies both its centre and squared radius.

Step 1: Find the tangent at the given point At $(1, 7)$, the slope is $2(1) = 2$. Using the point-slope form of a straight line,

$$y - 7 = 2(x - 1) \quad \Longrightarrow \quad 2x - y + 5 = 0.$$

The given point satisfies the parabola because $1^2 = 7 - 6$; thus the derivative has been evaluated on the curve, as required. The tangent equation is fixed before imposing any condition involving the circle.

Step 2: Apply the centre-to-tangent distance condition Complete the squares in the circle equation:

$$(x + 8)^2 + (y + 6)^2 = 100 - c.$$

The centre is $(-8, -6)$, and the radius squared is $100 - c$. Its distance from the tangent is

$$d = \frac{|2(-8) - (-6) + 5|}{\sqrt{2^2 + (-1)^2}} = \frac{5}{\sqrt{5}} = \sqrt{5}.$$

Tangency therefore requires $100 - c = d^2 = 5$, giving $c = 95$. This gives a positive squared radius. The alternatives 195 and 185 give negative squared radii, while 85 gives radius $\sqrt{15}$, which does not equal the distance.

Step 3: Conclusion The circle touches the specified tangent when $c = 95$. Hence, the correct option is (A).

Quick Tip to Solve: Complete the squares first, then equate the squared radius to the squared perpendicular distance; this avoids unnecessary radicals.

30. The Boolean expression

$\sim (p \vee q) \vee (\sim p \wedge q)$ **is equvalent to :** [4 marks]

(A) $\sim q$

(B) $\sim p$

(C) p

(D) q

Correct Answer: (B) $\sim p$

Solution:

Concept:

- De Morgan's law states $\sim (p \vee q) = (\sim p) \wedge (\sim q)$: the negation of an OR requires both propositions to be false.
- The distributive law gives $(a \wedge b) \vee (a \wedge c) = a \wedge (b \vee c)$. Also, $(\sim q) \vee q$ is always true, and conjoining a proposition with a tautology leaves that proposition unchanged.

Step 1: Expand the negated disjunction Let the given expression be E. Apply De Morgan's law only to its first term; the second term retains its original parentheses:

$$E = ((\sim p) \wedge (\sim q)) \vee ((\sim p) \wedge q).$$

Both alternatives now contain $\sim p$. This means either way for E to be true requires p to be false. The two alternatives differ only in whether q is false or true.

Step 2: Factor the common proposition and test the alternatives Use distributivity to factor $\sim p$:

$$E = (\sim p) \wedge ((\sim q) \vee q) = (\sim p) \wedge \text{True} = \sim p.$$

Thus when p is false, exactly one of the two terms is true, regardless of q; when p is true, both terms are false. For example $p = \text{False}, q = \text{True}$ makes E true but $\sim q$ false, rejecting (A). Taking both p and q false makes E true but both p and q false, rejecting (C) and (D). These checks confirm that the simplified result is independent of q.

Step 3: Conclusion The expression is equivalent to $\sim p$. Hence, the correct option is (B).

Quick Tip to Solve: After using De Morgan's law, look for a common proposition and the complementary pair $q \vee \sim q$.

Physics (120 marks)

31. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant K = 5/3 is inserted between the plates, the magnitude of the induced charge will be : [4 marks]

(A) 0.9 n C

(B) 1.2 n C

(C) 0.3 n C

(D) 2.4 n C

Correct Answer: (B) 1.2 n C

Solution:

Concept:

- For an initially empty parallel-plate capacitor, $Q_0 = C_0V$. With a dielectric completely filling the gap, the capacitance becomes $C = KC_0$, where K is the relative permittivity.
- When the battery remains connected, its voltage is fixed and the free charge on the plates becomes $Q = KC_0V$. The magnitude of bound polarization charge on either dielectric face is $Q_{\text{ind}} = Q(1 - 1/K)$.

Step 1: Find the free charges before and after insertion The given capacitance is the initial value $C_0 = 90\,\text{pF}$, and the voltage remains $20\,\text{V}$ because the battery is connected. Initially,

$$Q_0 = 90 \times 20\,\text{pC} = 1800\,\text{pC} = 1.8\,\text{nC}.$$

After insertion,

$$C = \frac{5}{3}(90) = 150\,\text{pF}, \qquad Q = 150 \times 20 = 3000\,\text{pC} = 3.0\,\text{nC}.$$

The free charge rises because the battery supplies additional charge to maintain the potential difference.

Step 2: Calculate the polarization charge The bound surface charge reduces the electric field produced by the free charge by a factor K. Thus its magnitude on a dielectric face is

$$Q_{\text{ind}} = Q\left(1 - \frac{1}{K}\right) = 3.0\left(1 - \frac{3}{5}\right) = 1.2\,\text{nC}.$$

Equivalently, this equals $(K - 1)C_0V = Q - Q_0$ for the fixed-voltage arrangement. The two faces carry opposite signs, with equal magnitudes; the question asks the magnitude, not the net charge of the neutral dielectric. The listed values 0.9, 0.3, and $2.4\,\text{nC}$ do not satisfy the polarization relation.

Step 3: Conclusion The induced charge magnitude is $1.2\,\text{nC}$. Hence, the correct option is (B).

Quick Tip to Solve: Check whether the battery stays connected: at fixed voltage, the induced bound charge is $(K - 1)C_0V$.

32. In an a.c. circuit, the instantaneous e.m.f. and current are given by

e = 100 sin 30 t

i = 20 sin $\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively [4 marks]

(A) 50, 0

(B) 50, 10

(C) $\frac{1000}{\sqrt{2}}, 10$

(D) $\frac{50}{\sqrt{2}}$

Correct Answer: (C) $\frac{1000}{\sqrt{2}}, 10$

Solution:

Concept:

- For sinusoidal voltage and current, $V_{\text{rms}} = V_0/\sqrt{2}$ and $I_{\text{rms}} = I_0/\sqrt{2}$; the coefficients in the instantaneous equations are peak values.
- The average real power over a cycle is $P = V_{\text{rms}} I_{\text{rms}} \cos\phi$, where ϕ is their phase difference. The wattless component of current is the rms quadrature component $I_{\text{rms}} \sin\phi$.

Step 1: Identify the amplitudes and phase difference Compare the given equations with $e = V_0 \sin\omega t$ and $i = I_0 \sin(\omega t - \phi)$. They give $V_0 = 100\,\text{V}$, $I_0 = 20\,\text{A}$, and $\phi = \pi/4$. The current lags the voltage; the common angular frequency is $30\,\text{rad}\,\text{s}^{-1}$. Therefore,

$$V_{\text{rms}} = \frac{100}{\sqrt{2}}\,\text{V}, \qquad I_{\text{rms}} = \frac{20}{\sqrt{2}}\,\text{A}.$$

Averaging over a complete cycle eliminates the oscillating double-frequency term in the instantaneous power.

Step 2: Evaluate real power and the quadrature current The power factor is $\cos(\pi/4) = 1/\sqrt{2}$. Hence

$$P = \frac{100}{\sqrt{2}}\frac{20}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{1000}{\sqrt{2}}\,\text{W}.$$

The wattless current is

$$I_{\text{w}} = \frac{20}{\sqrt{2}}\sin\frac{\pi}{4} = 10\,\text{A}.$$

It contributes no average power because it is perpendicular in phase to the voltage. Option (A) wrongly sets this component to zero; (B) has the correct component but incorrect power. Option (D) also has an incorrect power value. Only the pair in (C) agrees with both calculations.

Step 3: Conclusion The average power and wattless current are $1000/\sqrt{2}\,\text{W}$ and $10\,\text{A}$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Convert both amplitudes to rms values before finding average power; wattless current uses the sine of the phase angle.

33. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances? [4 marks]

(A) 910 Ω

(B) 990 Ω

(C) 505 Ω

(D) 550 Ω

Correct Answer: (D) 550 Ω

Solution:

Concept:

- For a balanced uniform metre bridge, the ratio of gap resistances equals the ratio of wire lengths: $R/S = l/(100-l)$, with l measured in centimetres from the left end.
- Interchanging the two resistances inverts the ratio, so the new balance length measured from the same end is $l\prime = 100 - l$. The series resistance provides the independent relation $R + S = 1000\,\Omega$.

Step 1: Determine the original balance length Write the original and interchanged balance conditions:

$$\frac{R}{S} = \frac{l}{100-l}, \qquad \frac{S}{R} = \frac{l\prime}{100-l\prime}.$$

Inverting the first expression and comparing with the second gives $l\prime = 100 - l$. The balance point shifts to the left by $10\,\text{cm}$, so $l\prime = l - 10$. Equating these two expressions,

$$100 - l = l - 10 \quad \Longrightarrow \quad l = 55\,\text{cm}, \qquad l\prime = 45\,\text{cm}.$$

The direction of the displacement fixes which of the two resistances was originally on the left.

Step 2: Use the series total to find the left resistance Originally $R/S = 55/45 = 11/9$. Put $R = 11a$, $S = 9a$; their series combination gives

$$20a = 1000\,\Omega \quad \Longrightarrow \quad a = 50\,\Omega.$$

Thus $R = 550\,\Omega$ and $S = 450\,\Omega$. As a direct check, swapping them changes the ratio to $450/550$, which balances at $45\,\text{cm}$, exactly $10\,\text{cm}$ left of the original point. The alternative values 910, 990, and $505\,\Omega$, together with the required total, would produce different balance displacements and therefore fail the stated condition.

Step 3: Conclusion The resistance originally in the left slot was $550\,\Omega$. Hence, the correct option is (D).

Quick Tip to Solve: After swapping the two gaps, the balance lengths add to 100 cm; combine this with the signed shift before using the ratio.

34. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7 $\times$ 10^3 kg/m^3 and its Young's modulus is 9.27 $\times$ 10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations ? [4 marks]

(A) 7.5 kHz

(B) 5 kHz

(C) 2.5 kHz

(D) 10 kHz

Correct Answer: (B) 5 kHz

Solution:

Concept:

- The speed of a longitudinal elastic wave in a thin rod is $v = \sqrt{Y/\rho}$, where Y is Young's modulus and ρ is the density.
- The clamped midpoint is a displacement node and the two free ends are displacement antinodes. In the fundamental symmetric mode, each half of the rod is a quarter wavelength, so $L = \lambda/2$ and $f = v/(2L)$.

Step 1: Identify the fundamental wavelength The total length is $L = 60\,\text{cm} = 0.60\,\text{m}$; each half has length $0.30\,\text{m}$. The midpoint cannot move because it is clamped, whereas a free end supports maximum longitudinal displacement. The shortest node-to-antinode distance is one quarter wavelength. Therefore,

$$\frac{L}{2} = \frac{\lambda}{4} \quad \Longrightarrow \quad \lambda = 2L = 1.20\,\text{m}.$$

Treating the entire rod length as a quarter wavelength would impose a node at an end rather than at the stated midpoint.

Step 2: Calculate the wave speed and frequency Use the given material constants in SI units:

$$v = \sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^{3}}} = \sqrt{3.4333 \times 10^{7}} \approx 5859\,\text{m}\,\text{s}^{-1}.$$

Then

$$f = \frac{5859}{1.20}\,\text{Hz} \approx 4883\,\text{Hz} = 4.88\,\text{kHz}.$$

The listed value corresponding to this result is $5\,\text{kHz}$. Neither $2.5\,\text{kHz}$, $7.5\,\text{kHz}$, nor $10\,\text{kHz}$ agrees with the fundamental mode and the supplied constants. The small difference from $5\,\text{kHz}$ reflects the rounding of the offered choices.

Step 3: Conclusion The fundamental frequency is approximately $4.88\,\text{kHz}$, corresponding to $5\,\text{kHz}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the clamp as a displacement node: from the midpoint to either free end is one quarter wavelength in the fundamental mode.

35. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avogadro number = 6.02×10^{23} gm mole^{-1}) [4 marks]

(A) 5.5 N/m

(B) 6.4 N/m

(C) 7.1 N/m

(D) 2.2 N/m

Correct Answer: (C) 7.1 N/m

Solution:

Concept:

- A harmonic oscillator of effective mass m and restoring force constant k satisfies $\omega = \sqrt{k/m}$. Its angular frequency is $\omega = 2\pi f$, not the ordinary frequency f.
- The mass of one atom is the molar mass divided by Avogadro's number. Convert the molar mass from grams to kilograms before calculating k in $\mathrm{N\,m^{-1}}$.

Step 1: Calculate the mass and angular frequency The molar mass of silver is $108\,\mathrm{g\,mol^{-1}} = 0.108\,\mathrm{kg\,mol^{-1}}$. Hence one atom has mass

$$m = \frac{0.108}{6.02 \times 10^{23}}\,\mathrm{kg} \approx 1.794 \times 10^{-25}\,\mathrm{kg}.$$

The given frequency is $f = 10^{12}\,\mathrm{s^{-1}}$, so

$$\omega = 2\pi \times 10^{12}\,\mathrm{rad\,s^{-1}}.$$

The force constant sought is the effective restoring constant for the atomic oscillation described; it is the constant relating restoring force to that atom's displacement.

Step 2: Find the effective restoring force constant Rearrange the harmonic-oscillator relation as $k = m\omega^2 = 4\pi^2 m f^2$. Substitution gives

$$k = 4\pi^2 \left(\frac{0.108}{6.02 \times 10^{23}}\right)(10^{12})^2 \approx 7.08\,\mathrm{N\,m^{-1}}.$$

The dimensions are $\mathrm{kg\,s^{-2}} = \mathrm{N\,m^{-1}}$, as required. Rounded to the precision of the choices, this is $7.1\,\mathrm{N/m}$. The values 5.5, 6.4, and $2.2\,\mathrm{N/m}$ do not agree with the mass and frequency. Omitting the factor $4\pi^2$ would confuse frequency with angular frequency and would not describe the specified oscillation.

Step 3: Conclusion The effective force constant is approximately $7.1\,\mathrm{N/m}$. Hence, the correct option is (C).

Quick Tip to Solve: For atomic SHM, first divide the SI molar mass by Avogadro's number, then use $k = 4\pi^2 m f^2$.

36. Two batteries with e.m.f 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between : [4 marks]

(A) 11.7 V and 11.8 V

(B) 11.6 V and 11.7 V

(C) 11.5 V and 11.6 V

(D) 11.4 V and 11.5 V

Correct Answer: (C) 11.5 V and 11.6 V

Solution:

Concept:

- The terminal voltage V is common to both parallel batteries and the load. Each battery supplies a current $I = (E - V)/r$, where E is its emf and r its internal resistance.
- Kirchhoff's junction law requires the sum of the battery currents to equal the load current V/R. The emf values must not simply be averaged, because the internal resistances are unequal.

Step 1: Express the branch currents Let the common positive terminals feed the load. The current supplied by the 12 V battery is $(12 - V)/1$, while that supplied by the 13 V battery is $(13 - V)/2$, in amperes. The load draws $V/10$. Thus the current balance at the junction is

$$\frac{12 - V}{1} + \frac{13 - V}{2} = \frac{V}{10}.$$

Both emfs exceed the eventual terminal voltage, so both branch currents have the assumed outward direction.

Step 2: Solve for the terminal voltage Multiply the equation by 10 and collect the voltage terms:

$$120 - 10V + 65 - 5V = V, \qquad 185 = 16V.$$

Therefore $V = 185/16 = 11.5625\,\text{V}$. As a check, the branch currents are $0.4375\,\text{A}$ and $0.71875\,\text{A}$; their sum is $1.15625\,\text{A}$, exactly the load current. The terminal voltage is lower than either emf because there are voltage drops inside the batteries.

Step 3: Compare the given intervals The computed voltage lies strictly between 11.5 V and 11.6 V. It lies below both intervals in (A) and (B), and above the interval in (D). This also rules out treating the batteries as ideal sources with different fixed voltages in parallel; their internal resistances are essential to this calculation.

Step 4: Conclusion The load voltage is $11.5625\,\text{V}$, between 11.5 V and 11.6 V. Hence, the correct option is (C).

Quick Tip to Solve: For unequal cells in parallel, write each branch current as $(E - V)/r$ and use one junction equation for the common voltage.

37. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$ Its total energy is: [4 marks]

(A) $-\frac{3}{2}\frac{k}{a^2}$

(B) Zero

(C) $-\frac{k}{4a^2}$

(D) $\frac{k}{2a^2}$

Correct Answer: (B) Zero

Solution:

Concept:

- For a central potential $U(r)$, the radial force is $F_r = -dU/dr$; a negative radial component means attraction towards the centre.
- In a circular orbit of radius a, the inward force equals mv^2/a. The total mechanical energy is $E = \frac{1}{2}mv^2 + U(a)$.

Step 1: Differentiate the potential Write the given potential as $U(r) = -(k/2)r^{-2}$, with $k > 0$ for the stated attractive interaction. Differentiating gives

$$\frac{dU}{dr} = \frac{k}{r^3}, \qquad F_r = -\frac{k}{r^3}.$$

Thus the force magnitude at the orbital radius is k/a^3. Retaining this extra power of radius is important: the force is inverse-cube, rather than the inverse-square force of ordinary gravitation.

Step 2: Use the circular-orbit condition The speed is constant in the circular path, but its direction changes, requiring centripetal acceleration. Hence

$$\frac{mv^2}{a} = \frac{k}{a^3}, \qquad mv^2 = \frac{k}{a^2}.$$

The orbital kinetic energy is therefore $K = k/(2a^2)$. It is positive and exactly equal to the magnitude of the given negative potential energy at that radius. The particle mass cancels when finding this energy in terms of k and a.

Step 3: Add kinetic and potential energies At $r = a$, the potential energy is $U(a) = -k/(2a^2)$, so

$$E = K + U(a) = \frac{k}{2a^2} - \frac{k}{2a^2} = 0.$$

Option (D) is the kinetic energy alone and omits the potential energy. The negative expressions in (A) and (C) fail to use the kinetic energy required by the circular-motion condition. Their nonzero values cannot equal the sum found above.

Step 4: Conclusion The kinetic and potential energies cancel, giving total energy $E = 0$. Hence, the correct option is (B).

Quick Tip to Solve: For a circular orbit in a power-law potential, find the force by differentiation before using centripetal balance; do not import the gravitational energy formula.

38. Two moles of an ideal monatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2 V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

[4 marks]

(A) (a) 195 K (b) 2.7 kJ

(B) (a) 189 K (b) 2.7 kJ

(C) (a) 195 K (b) –2.7 kJ

(D) (a) 189 K (b) – 2.7 kJ

Correct Answer: (D) (a) 189 K (b) – 2.7 kJ

Solution:

Concept:

- For an ideal monatomic gas, $C_V = 3R/2$ and $\gamma = C_P/C_V = 5/3$. In the standard reversible adiabatic expansion, $TV^{\gamma-1}$ remains constant.
- The internal energy of an ideal gas depends only on absolute temperature: $\Delta U = nC_V(T_f - T_i)$. An expanding adiabatic gas cools because it does work without absorbing heat.

Step 1: Calculate the final temperature Convert the initial temperature to kelvin before using the adiabatic relation: $T_i = 27 + 273 = 300\,\text{K}$. With $V_f = 2V_i$,

$$T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = 300\left(\frac{1}{2}\right)^{2/3} \simeq 188.99\,\text{K}.$$

Thus the final temperature is approximately $189\,\text{K}$. The volume ratio is dimensionless; using the Celsius temperature in this power law would give an incorrect result.

Step 2: Find the change in internal energy Here $n = 2\,\text{mol}$, so the total constant-volume heat capacity is $nC_V = 3R$. Therefore

$$\Delta U = 3(8.314)(188.99 - 300)\,\text{J} \simeq -2.77 \times 10^3\,\text{J}.$$

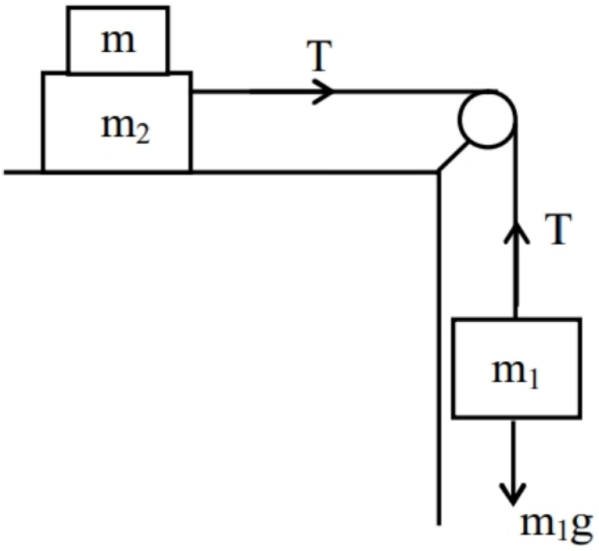
The decrease is about $2.8\,\text{kJ}$; the supplied choices quote it approximately as $-2.7\,\text{kJ}$. The negative sign follows independently from $Q = 0$ and positive work done by the gas in expansion, using $\Delta U = Q - W$.

Step 3: Identify the matching choice Options (A) and (B) show a positive energy change, inconsistent with adiabatic cooling. Option (C) has the correct sign but uses 195 K, which does not follow from $300/2^{2/3}$. Option (D) has both the correct final temperature and the negative energy change, with the approximate numerical energy stated in the choices.

Step 4: Conclusion The final temperature is $189\,\text{K}$, and $\Delta U \simeq -2.77\,\text{kJ}$, represented approximately as $-2.7\,\text{kJ}$ in the choices. Hence, the correct option is (D).

Quick Tip to Solve: For a monatomic gas use $\gamma - 1 = 2/3$; then check the energy sign from the fact that adiabatic expansion cools the gas.

39. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is : [4 marks]



(A) 10.3 kg

(B) 18.3 kg

(C) 27.3 kg

(D) 43.3 kg

Correct Answer: The printed values give a limiting mass of $23.33\,\text{kg}$, which is absent; (C) is only the smallest listed sufficient load.

Solution:

Concept:

- The extra mass increases the normal force on the horizontal surface to $N = (m_2 + m)g$. The available friction is $f = \mu(m_2 + m)g$, opposite the motion.
- The downward driving force of the hanging block is $m_1 g$. The limiting load for preventing further acceleration is obtained by balancing this force against friction; a greater load produces deceleration.

Step 1: Write the force balance for the whole system Take the hanging block's downward motion and the horizontal block's rightward motion as positive. The string tension is internal to the combined system and cancels. Thus, for a load moving with the horizontal block,

$$(m_1 + m_2 + m)a = m_1 g - \mu(m_2 + m)g.$$

The coefficient is stated as 0.15, and the original masses are $m_1 = 5\,\text{kg}$ and $m_2 = 10\,\text{kg}$. The added load does not increase the hanging driving weight.

Step 2: Find the limiting added mass At the threshold $a = 0$, the balance becomes

$$5g = 0.15(10 + m)g, \qquad m = \frac{5}{0.15} - 10 = 23.33\,\text{kg}.$$

The factor g cancels, so the answer does not depend on choosing 9.8 or 10 for gravitational acceleration. For the already moving system, a mass strictly above this threshold supplies a net retarding force; equality would maintain constant speed in the ideal kinetic-friction model.

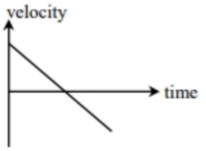
Step 3: Check the choices and the wording The listed masses 10.3 kg and 18.3 kg give friction below $5g$, so they cannot stop the stated motion. Both 27.3 kg and 43.3 kg give friction above $5g$, and can decelerate it. Therefore 27.3 kg, option (C), is the smallest sufficient mass among the supplied choices, but the actual limiting mass 23.33 kg is absent. A strict claim that 27.3 kg is the physical minimum would contradict the printed data.

Step 4: Conclusion The limiting added mass is $23.33\,\text{kg}$, absent from the options; a larger mass stops the moving system. Option (C), $27.3\,\text{kg}$, is the smallest listed sufficient load, but no option states the exact minimum.

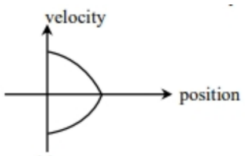
Quick Tip to Solve: Cancel the internal tension by treating both blocks as one system, and distinguish the actual threshold from the smallest mass merely offered in an MCQ.

40. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. [4 marks]

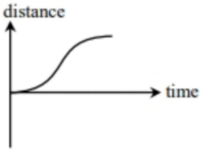
(A)



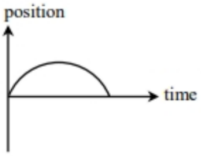
(B)



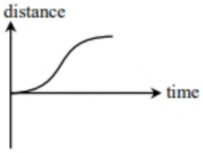
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- The slope of a position–time graph is velocity, while the slope of a velocity–time graph is acceleration. Distance travelled has slope equal to speed $|v|$, and can never decrease.
- A straight velocity–time line of negative slope describes constant negative acceleration. Position then follows a concave-down parabola, while the speed falls to zero at the turning point and subsequently increases.

Step 1: Read the velocity and position graphs Option (A) begins with positive velocity, crosses zero, and becomes negative with constant negative slope. Write this as

$$v = u - at, \qquad x = ut - \frac{1}{2}at^2, \qquad a > 0.$$

The position initially increases, reaches a maximum when $v = 0$, then decreases. This agrees with the concave-down position–time curve in (D). The object turns around; it does not remain at rest after reaching the maximum position.

Step 2: Check velocity against position Eliminating time gives

$$v^2 = u^2 - 2ax, \qquad x = \frac{u^2 - v^2}{2a}.$$

For a plot with velocity vertically and position horizontally, this is a sideways parabola opening towards smaller position. Its upper branch has positive velocity before the turn, and its lower branch negative velocity afterwards. This is the shape in (B), with zero velocity at the rightmost point.

Step 3: Test the distance–time graph Distance travelled s obeys $ds/dt = |v| = |u - at|$. Its slope must start positive, decrease to zero at the turnaround, and then increase as the object speeds up in the opposite direction. The S-shaped curve in (C) instead starts almost flat, becomes steeper, and then flattens again. That would require the speed first to rise and then fall. It cannot describe the same motion as (A), whereas (B) and (D) follow directly from (A).

Step 4: Conclusion The distance–time curve in (C) has the wrong variation of slope and hence the wrong speed history. Hence, the correct option is (C).

Quick Tip to Solve: On a distance–time graph compare the slope with $|v|$, not with signed velocity; at a reversal the distance keeps increasing but its slope momentarily vanishes.

41. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the nth power of R. If the period of rotation of the particle is T, then : [4 marks]

(A) T $\propto$ R$^{n/2}$

(B) T $\propto$ R$^{3/2}$ for any n

(C) T $\propto$ R$^{n/2\,+1}$

(D) T $\propto$ R$^{(n+1)/2}$

Correct Answer: (D) T $\propto$ R$^{(n+1)/2}$

Solution:

Concept:

- For circular motion, the required inward force is mv^2/R. If the central force has magnitude C/R^n, then C and the particle mass m are fixed when comparing different orbital radii.
- The orbital period is circumference divided by speed: $T = 2\pi R/v$. Equivalently, centripetal force is $4\pi^2 mR/T^2$.

Step 1: Equate the central and centripetal forces Let $F = CR^{-n}$, where $C > 0$. The uniform speed means the tangential acceleration is zero, but a radial acceleration is still needed to bend the path into a circle. Therefore

$$\frac{mv^2}{R} = \frac{C}{R^n}, \qquad v^2 = \frac{C}{m}R^{1-n}.$$

It follows that $v = \sqrt{C/m}\,R^{(1-n)/2}$. Keeping the factor of R in the centripetal-force expression is essential when finding the exponent.

Step 2: Convert the speed scaling to period scaling Substitute the speed into $T = 2\pi R/v$:

$$T = 2\pi\sqrt{\frac{m}{C}}\,R^{1-(1-n)/2} = 2\pi\sqrt{\frac{m}{C}}\,R^{(n+1)/2}.$$

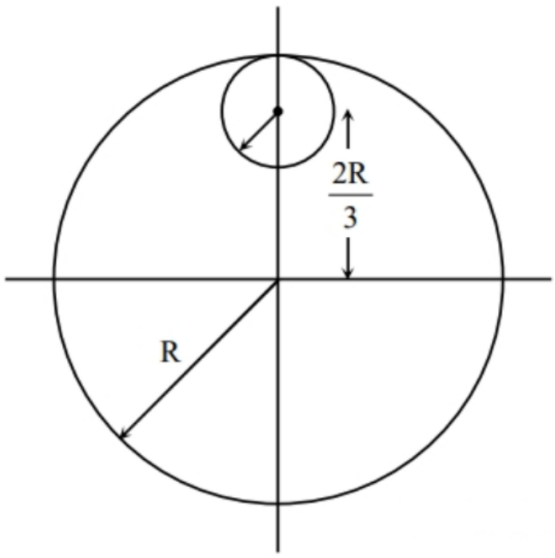
Thus T^2 is proportional to R^{n+1}, and the requested relation is $T \propto R^{(n+1)/2}$. The radius and speed both enter the period, so force scaling alone cannot supply its exponent.

Step 3: Use special cases to reject alternatives For $n = 2$, the result gives $T \propto R^{3/2}$, the familiar inverse-square-force law. This does not make (B) valid for every n: for example, $n = 1$ gives $T \propto R$. Options (A) and (C) have exponents respectively one-half too small and one-half too large compared with the derived exponent. Only (D) works for general n.

Step 4: Conclusion The orbital period varies as $T \propto R^{(n+1)/2}$. Hence, the correct option is (D).

Quick Tip to Solve: Use $F = 4\pi^2 mR/T^2$ directly: a force proportional to R^{-n} immediately gives $T^2 \propto R^{n+1}$.

42. From a uniform circular disc of radius R and mass 9M, a small disc of radius R/3 is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is : [4 marks]



(A) $\frac{37}{9}MR^2$

(B) $4MR^2$

(C) $\frac{40}{9}MR^2$

(D) $10MR^2$

Correct Answer: (B) $4MR^2$

Solution:

Concept:

- For a uniform disc of mass m and radius r, the moment of inertia about its central perpendicular axis is $I_c = mr^2/2$. Mass is proportional to area when thickness and density are uniform.
- The parallel-axis theorem gives $I = I_c + md^2$ about an axis displaced by d. The moment of inertia of the remaining plate equals the original disc's moment minus the removed part's moment about the same axis.

Step 1: Find the removed disc's mass and displacement The original disc has mass $9M$ and radius R. Since the small radius is $R/3$, its area is one-ninth of the original area. Thus

$$m_{\text{hole}} = 9M\frac{(R/3)^2}{R^2} = M.$$

The figure places the small disc internally tangent to the rim, with its centre a distance $d = 2R/3$ from the original centre. The requested axis passes through that original centre, rather than through the centre of mass of the remaining plate.

Step 2: Calculate both moments about the requested axis For the full disc,

$$I_{\text{full}} = \frac{1}{2}(9M)R^2 = \frac{9}{2}MR^2.$$

For the removed disc, transfer its central moment to the original central axis:

$$I_{\text{hole}} = \frac{1}{2}M\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2 = \left(\frac{1}{18} + \frac{4}{9}\right)MR^2 = \frac{1}{2}MR^2.$$

The displacement term is necessary because the two disc centres do not coincide.

Step 3: Subtract and compare with the choices Removing material subtracts its contribution about the same fixed axis:

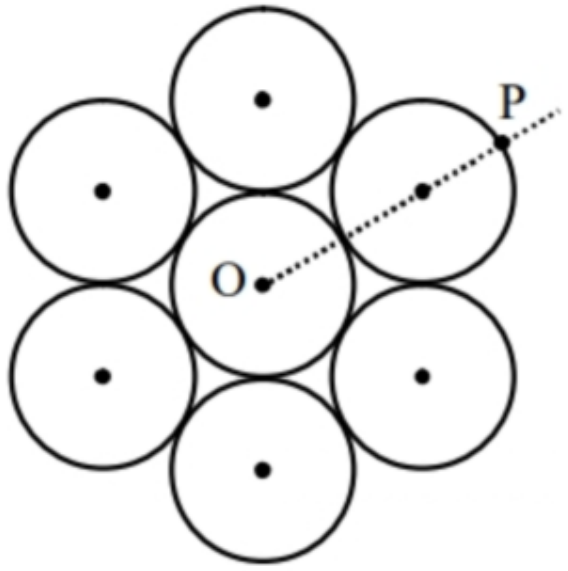
$$I_{\text{remaining}} = \left(\frac{9}{2} - \frac{1}{2}\right)MR^2 = 4MR^2.$$

The value $40MR^2/9$ in (C) results from subtracting only the hole's central moment and omitting its displacement. Option (A) also differs from the full parallel-axis subtraction. Option (D) exceeds the full disc's moment, which is impossible after removing material at fixed axis.

Step 4: Conclusion The remaining disc has moment of inertia $I = 4MR^2$ about the stated perpendicular axis. Hence, the correct option is (B).

Quick Tip to Solve: When subtracting a hole, first calculate its moment about the original requested axis, including the parallel-axis term; never subtract moments about different centres.

43. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is : [4 marks]



(A) $\frac{181}{2}MR^2$

(B) $\frac{55}{2}MR^2$

(C) $\frac{19}{2}MR^2$

(D) $\frac{73}{2}MR^2$

Correct Answer: (A) $\frac{181}{2}MR^2$

Solution:

Concept:

- A planar circular disk has moment of inertia $MR^2/2$ about the normal through its own centre. Translating this axis a distance d adds Md^2.
- The seven-disk assembly has its centre of mass at the central disk centre O, by sixfold symmetry. The parallel-axis theorem applies to the entire mass $7M$.

Step 1: Find the moment about the central normal The six surrounding centres are each $2R$ from O, because adjacent disks touch. Include both each disk's own spin contribution and the displacement of its centre. Thus

$$I_O = \frac{1}{2}MR^2 + 6\left(\frac{1}{2}MR^2 + M(2R)^2\right) = \frac{55}{2}MR^2.$$

The central disk contributes only its own central moment. The six outer disks each contribute the same amount, so no individual coordinate calculation is needed.

Step 2: Shift the axis to the marked point The dotted radius runs through the centre of the upper-right outer disk and ends at its far circumference. Hence $OP = 2R + R = 3R$. The two normal axes are parallel, and the total assembly mass is $7M$. Therefore

$$I_P = I_O + (7M)(3R)^2 = \left(\frac{55}{2} + 63\right)MR^2 = \frac{181}{2}MR^2.$$

Option (B) is the moment about O, not about P. Options (C) and (D) do not include the required displacement contribution for all seven masses; in particular the added contribution alone is $63MR^2$.

Step 3: Conclusion The required moment of inertia is $I_P = \frac{181}{2}MR^2$. Hence, the correct option is (A).

Quick Tip to Solve: For a symmetric cluster, first find the moment about its centre of mass, then shift the whole mass to the requested parallel axis; read the final extra radius to the edge carefully.

44. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is: [4 marks]

(A) 6%

(B) 2.5%

(C) 3.5%

(D) 4.5%

Correct Answer: (D) 4.5%

Solution:

Concept:

- Density is mass divided by volume: $\rho = m/(l_1 l_2 l_3)$. For a cube all three nominal edge lengths equal l, so $\rho = m/l^3$.

- To first order, maximum fractional errors add for products and quotients. An exponent multiplies the fractional error of its measured base.

Step 1: Write the fractional-error relation Logarithmic differentiation gives the signed change

$$\frac{d\rho}{\rho} = \frac{dm}{m} - \frac{dl_1}{l_1} - \frac{dl_2}{l_2} - \frac{dl_3}{l_3}.$$

For a maximum error we bound its absolute magnitude. The denominator does not cause the uncertainties to cancel: the extreme errors can act together, with an overestimated mass and underestimated lengths. Therefore

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max} = \frac{\Delta m}{m} + \sum_{i=1}^{3} \frac{\Delta l_i}{l_i}.$$

Step 2: Substitute the measurement errors Each of the three length measurements has a relative error of 1%, while the mass measurement has relative error 1.5%. Hence

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max} \times 100 = 1.5 + 1 + 1 + 1 = 4.5\%.$$

Equivalently, for m/l^3, the result is $1.5\% + 3(1\%)$. The 2.5% option counts length only once, 3.5% counts only two lengths, and 6% is not the sum of the stated contributions. Quadrature would describe a statistical uncertainty under additional assumptions, not the maximum-error rule requested here.

Step 3: Conclusion The maximum percentage error in density is 4.5%. Hence, the correct option is (D).

Quick Tip to Solve: In a maximum percentage-error question, write the measured quantity as powers first: the exponent 3 in volume multiplies the edge-length percentage error.

45. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$ is: [4 marks]

(A) $\frac{mg}{Ka}$
(B) $\frac{Ka}{mg}$
(C) $\frac{Ka}{3mg}$
(D) $\frac{mg}{3Ka}$

Correct Answer: (D) $\frac{mg}{3Ka}$

Solution:

Concept:

- The bulk modulus relates a small increase in pressure to fractional volume decrease: $K = \Delta P/(\Delta V_{\text{dec}}/V)$, with both increments written as positive magnitudes.
- For a sphere $V = 4\pi r^3/3$, so a small fractional radius decrement dr/r produces a fractional volume

decrement $3dr/r$.

Step 1: Find the added pressure on the sphere The added mass has weight mg, and the massless piston distributes this force over its area a. Mechanical equilibrium therefore requires an additional liquid pressure

$$\Delta P = \frac{mg}{a}.$$

By Pascal's law this added pressure is transmitted throughout the liquid and acts on the sphere. The initial hydrostatic pressure is already part of the initial state; the requested decrement is caused by the additional mass, so it is the pressure increment that enters the elastic relation.

Step 2: Relate the volume change to the radius change Differentiate the spherical volume to first order in the small deformation:

$$\frac{\Delta V_{\text{dec}}}{V} = \frac{4\pi r^2 dr}{(4/3)\pi r^3} = 3\frac{dr}{r}.$$

Combining this geometric result with the bulk-modulus definition gives

$$K = \frac{mg/a}{3dr/r}, \qquad \frac{dr}{r} = \frac{mg}{3Ka}.$$

Option (A) gives the fractional volume decrease rather than the radius decrease. Options (B) and (C) invert the physical dependence: a stiffer sphere must deform less as K increases, while a larger applied mass must deform it more.

Step 3: Conclusion The fractional decrement, as a positive magnitude, is $\frac{dr}{r} = \frac{mg}{3Ka}$. Hence, the correct option is (D).

Quick Tip to Solve: For any spherical body under a small uniform compression, divide the fractional volume decrease by three to obtain the fractional radius decrease.

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46. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is: [4 marks]

(A) $r_e < r_\alpha < r_p$

(B) $r_e > r_p = r_\alpha$

(C) $r_e < r_p = r_\alpha$

(D) $r_e < r_p < r_\alpha$

Correct Answer: (C) $r_e < r_p = r_\alpha$

Solution:

Concept:

- For motion perpendicular to a uniform magnetic field, the magnetic force supplies the centripetal force: $|q|vB = mv^2/r$, so $r = mv/(|q|B)$.
- At equal nonrelativistic kinetic energy E, momentum is $p = \sqrt{2mE}$. Thus the radius is proportional to $\sqrt{m}/|q|$, not just to mass.

Step 1: Express the radius using the common energy Eliminate speed from $E = mv^2/2$ and the magnetic-radius expression:

$$r = \frac{\sqrt{2mE}}{|q|B}.$$

All three particles share E and B. The negative charge of an electron changes the direction of its circular motion but does not make its radius negative. We therefore compare magnitudes of charge. The electron and proton each have charge magnitude e, giving

$$\frac{r_e}{r_p} = \sqrt{\frac{m_e}{m_p}} \ll 1.$$

Step 2: Compare the proton and alpha particle An alpha particle has mass approximately $4m_p$ and charge $+2e$, whereas a proton has mass m_p and charge $+e$. Consequently

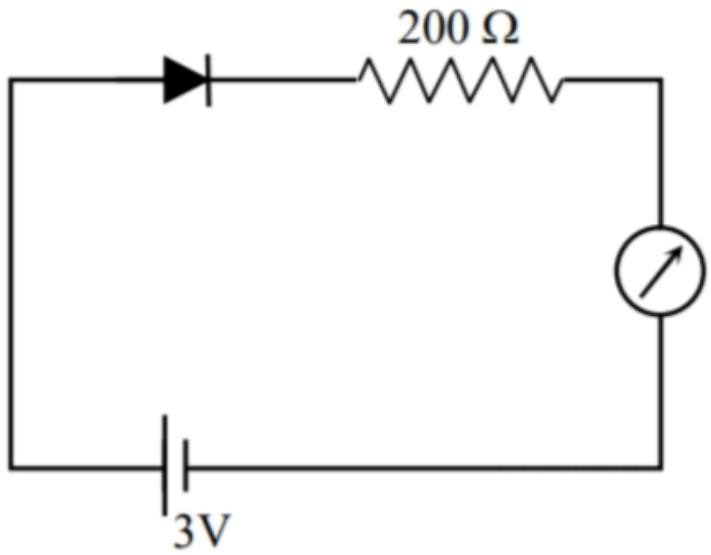
$$\frac{r_\alpha}{r_p} = \frac{\sqrt{4m_p}/(2e)}{\sqrt{m_p}/e} = 1.$$

Its larger momentum at the same kinetic energy is exactly offset, in this approximation, by its doubled charge. Thus $r_p = r_\alpha$, both much larger than r_e. Option (B) reverses the electron comparison, while (A) and (D) incorrectly assign unequal radii to the proton and alpha particle. Treating equal energy as equal speed would also give an incorrect mass dependence.

Step 3: Conclusion The radii satisfy $r_e < r_p = r_\alpha$. Hence, the correct option is (C).

Quick Tip to Solve: At equal kinetic energy compare $\sqrt{m}/|q|$; at equal speed compare $m/|q|$. Distinguish these two conditions before comparing magnetic radii.

47. The reading of the ammeter for a silicon diode in the given circuit is : [4 marks]



(A) 13.5 mA

(B) 0

(C) 15 mA

(D) 11.5 mA

Correct Answer: (D) 11.5 mA

Solution:

Concept:

- A conducting silicon diode is modelled with a forward voltage drop of approximately $0.7\,\mathrm{V}$. Its current is set by the remaining circuit resistance.
- In a single series loop, the diode, resistor and ideal ammeter carry the same current. Kirchhoff's voltage law makes the source voltage equal the sum of the component drops.

Step 1: Determine the bias of the diode The longer battery plate is on the left, so the left side of the circuit is connected to the positive terminal of the $3\,\mathrm{V}$ source. This terminal connects to the diode anode, while its cathode, marked by the vertical bar, connects towards the resistor and negative terminal. The diode is therefore forward biased. Since the source exceeds the silicon forward drop, a nonzero clockwise conventional current flows; the zero-current option is not appropriate for this connection.

Step 2: Calculate the series current The ideal ammeter has negligible resistance. Allowing $0.7\,\mathrm{V}$ across the silicon diode leaves

$$V_R = 3.0 - 0.7 = 2.3\,\mathrm{V}$$

across the $200\,\Omega$ resistor. Ohm's law then gives

$$I = \frac{V_R}{R} = \frac{2.3}{200}\,\mathrm{A} = 0.0115\,\mathrm{A} = 11.5\,\mathrm{mA}.$$

The $15\,\mathrm{mA}$ option ignores the diode drop altogether. The $13.5\,\mathrm{mA}$ option corresponds to a $0.3\,\mathrm{V}$ drop,

characteristic of the usual germanium model rather than silicon. This checks both the component identification and the arithmetic against the options.

Step 3: Conclusion The ammeter reads $11.5\,\text{mA}$. Hence, the correct option is (D).

Quick Tip to Solve: Identify forward bias from the battery polarity and the cathode bar first; then subtract the silicon drop before dividing by the series resistance.

48. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is : [4 marks]

(A) $\frac{v_0}{\sqrt{2}}$

(B) $\frac{v_0}{4}$

(C) $\sqrt{2}v_0$

(D) $\frac{v_0}{2}$

Correct Answer: (C) $\sqrt{2}v_0$

Solution:

Concept:

- In an isolated collinear collision, total linear momentum is conserved even when kinetic energy changes. The stated increase in kinetic energy describes a superelastic collision.
- For two equal masses, $(v_1 - v_2)^2 = 2(v_1^2 + v_2^2) - (v_1 + v_2)^2$. This identity gives their relative speed without solving separately for each final velocity.

Step 1: Use momentum and the stated kinetic energy Let each particle have mass m, and let v_1, v_2 be signed final velocities along the initial direction of motion. Conservation of momentum gives

$$mv_0 = mv_1 + mv_2, \qquad v_1 + v_2 = v_0.$$

The initial kinetic energy is $K_i = mv_0^2/2$. Since the final energy is 50% greater, it is $1.5K_i$, not $0.5K_i$. Hence

$$\frac{1}{2}m(v_1^2 + v_2^2) = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right), \quad v_1^2 + v_2^2 = \frac{3}{2}v_0^2.$$

Step 2: Extract the relative speed Apply the sum-and-difference identity to these two constraints:

$$\begin{aligned}(v_1 - v_2)^2 &= 2(v_1^2 + v_2^2) - (v_1 + v_2)^2 \\ &= 2\left(\frac{3}{2}v_0^2\right) - v_0^2 = 2v_0^2.\end{aligned}$$

Taking the positive root gives $|v_1 - v_2| = \sqrt{2}v_0$. A relative speed larger than the incident speed is consistent with the specified energy release. The other three options are all less than v_0, and would give a final energy smaller than the initial energy when inserted into $v_1^2 + v_2^2 = [v_0^2 + (v_1 - v_2)^2]/2$; they therefore cannot represent this collision.

Step 3: Conclusion The magnitude of the final relative velocity is $\sqrt{2}v_0$. Hence, the correct option is

(C).

Quick Tip to Solve: For equal masses, combine the known sum of velocities with the sum of their squares; the identity for the squared difference avoids unnecessary quadratic equations.

49. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly: [4 marks]

(A) 2.35×10^3 N m^{-2}

(B) 4.70×10^3 N m^{-2}

(C) 2.35×10^2 N m^{-2}

(D) 4.70×10^2 N m^{-2}

Correct Answer: (A) 2.35×10^3 N m^{-2}

Solution:

Concept:

- Pressure is the normal force per unit area. The force on a wall equals the rate at which molecules transfer normal momentum to it.
- In an elastic rebound from a smooth stationary wall, the normal velocity reverses and the tangential velocity is unchanged. The normal impulse magnitude per molecule is $2mv\cos\theta$, where θ is measured from the normal.

Step 1: Calculate the force from the molecular impacts Let the stated collision rate be $\dot{N} = 10^{23}\,\mathrm{s}^{-1}$. The normal speed of each molecule is $v\cos 45^\circ$. Reversal on rebound makes its normal momentum change twice the incoming normal momentum, so the force magnitude is

$$F = \dot{N}(2mv\cos 45^\circ).$$

Substitution, with $m = 3.32 \times 10^{-27}\,\mathrm{kg}$ and $v = 10^3\,\mathrm{m\,s}^{-1}$, gives

$$F = 10^{23} \times 2 \times 3.32 \times 10^{-27} \times 10^3 \times \frac{1}{\sqrt{2}} \simeq 0.4695\,\mathrm{N}.$$

Step 2: Convert the area and obtain the pressure The wall area must be converted using the square of the centimetre conversion:

$$A = 2\,\mathrm{cm}^2 = 2 \times 10^{-4}\,\mathrm{m}^2.$$

Therefore

$$P = \frac{F}{A} = \frac{0.4695}{2 \times 10^{-4}} \simeq 2.35 \times 10^3\,\mathrm{N\,m}^{-2}.$$

The given number already counts impacts per second, so no further angular factor is needed for the collision rate. Only the momentum component contributes $\cos 45^\circ$. Option (B) doubles the computed pressure; options (C) and (D) have a power of ten too small. Recoil requires the factor two in impulse, while pressure requires division by the actual wall area.

Step 3: Conclusion The pressure is approximately $2.35 \times 10^3\,\mathrm{N\,m^{-2}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the normal momentum reversal $2mv\cos\theta$ per impact, multiply by impacts per second, and convert square centimetres to square metres before dividing by area.

50. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is: [4 marks]

(A) $\nu_L/25$

(B) $25\nu_L$

(C) $16\nu_L$

(D) $\nu_L/16$

Correct Answer: (A) $\nu_L/25$

Solution:

Concept:

- The Rydberg formula for a hydrogen transition is $\nu = cR_H(1/n_f^2 - 1/n_i^2)$, where n_i and n_f are the initial and final principal quantum numbers.
- A series limit corresponds to the largest photon energy within that series: the initial level tends to infinity while the final level stays fixed.

Step 1: Identify the final levels The Lyman series contains transitions ending at the ground level, so $n_f = 1$. The Pfund series ends at $n_f = 5$. These final levels identify the series, independently of the particular excited level from which the electron starts. The levels 2, 3 and 4 instead define the Balmer, Paschen and Brackett series.

Step 2: Take the series limits At either limit, $n_i \to \infty$, so $1/n_i^2 \to 0$. Thus the two limit frequencies are

$$\nu_L = cR_H, \qquad \nu_P = \frac{cR_H}{5^2} = \frac{cR_H}{25}.$$

Both frequencies have the same factor cR_H, which has units of inverse time. Dividing removes the constants and gives $\nu_P/\nu_L = 1/25$.

Step 3: Check the alternatives The Pfund final level is less tightly bound than the ground level, so a transition from the ionization threshold to that level releases less energy and has a smaller frequency. This excludes $25\nu_L$ and $16\nu_L$. The factor $1/16$ would correspond to the Brackett limit, whose final level is 4, rather than the Pfund limit.

Step 4: Conclusion The Pfund series limit frequency is $\boxed{\nu_L/25}$. Hence, the correct option is (A).

Quick Tip to Solve: For a hydrogen series limit ending at level n_f, divide the Lyman limit frequency by n_f^2; do not confuse frequency with wavelength.

51. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is: [4 marks]

(A) 2

(B) $\sqrt{3}$

(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{2}}$

Correct Answer: (C) $\sqrt{2}$

Solution:

Concept:

- A single circular loop of radius r carrying current I has magnetic dipole moment $m = I\pi r^2$.
- The magnetic field magnitude at its centre is $B = \mu_0 I/(2r)$. At fixed current the moment increases as area, whereas the central field decreases as radius.

Step 1: Relate the two radii Let the original and new radii be r_1 and r_2. The current remains the same; hence doubling the moment means doubling the area rather than doubling the current. Applying the moment formula to both loops gives

$$\frac{m_2}{m_1} = \frac{I\pi r_2^2}{I\pi r_1^2} = 2.$$

Since radii are positive, $r_2 = \sqrt{2}\, r_1$.

Step 2: Compare the central fields Use the field expression separately for the original and enlarged loops:

$$B_1 = \frac{\mu_0 I}{2r_1}, \qquad B_2 = \frac{\mu_0 I}{2r_2}.$$

Taking their ratio in the order explicitly requested,

$$\frac{B_1}{B_2} = \frac{r_2}{r_1} = \sqrt{2}.$$

The larger loop has the smaller central field, so the ratio must exceed unity.

Step 3: Eliminate the other values A ratio of 2 incorrectly treats the moment as proportional to radius instead of radius squared. A ratio of $\sqrt{3}$ would require tripling the area and moment. The reciprocal $1/\sqrt{2}$ is the value of B_2/B_1, not the requested B_1/B_2. The fixed-current condition is essential: doubling current at fixed radius would give a different result.

Step 4: Conclusion $\boxed{B_1/B_2 = \sqrt{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: At constant current eliminate the radius: $B \propto m^{-1/2}$. Check the order of the requested ratio before choosing the reciprocal.

52. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the nth state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the nth state to the ground state. For large n, (A, B are constants) [4 marks]

(A) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$

(B) $\Lambda_n \approx A + B\lambda_n$

(C) $\Lambda_n^2 \approx A + B\lambda_n^2$

(D) $\Lambda_n^2 \approx \lambda$

Correct Answer: (A) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$

Solution:

Concept:

- In the Bohr model of hydrogen, the electron speed varies as $v_n = v_1/n$. Its de Broglie wavelength is $\lambda_n = h/(m_e v_n) = n\lambda_g$.
- For a direct transition from level n to the ground state, the photon obeys $1/\Lambda_n = R_H(1 - 1/n^2)$. Electron and photon wavelengths refer to different particles.

Step 1: Express the level through the electron wavelength The orbital momentum quantization gives $m_e v_n r_n = n\hbar$, with $r_n \propto n^2$. Consequently $v_n \propto 1/n$, and the electron wavelength increases linearly with n. Thus

$$n = \frac{\lambda_n}{\lambda_g}, \qquad \frac{1}{n^2} = \frac{\lambda_g^2}{\lambda_n^2}.$$

This substitution is needed because the options express the photon wavelength using the electron wavelength.

Step 2: Expand the photon wavelength at large level number The Rydberg relation becomes

$$\Lambda_n = \frac{1}{R_H}\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)^{-1}.$$

For large n, the ratio $x = \lambda_g^2/\lambda_n^2$ is small. Using $(1 - x)^{-1} = 1 + x + O(x^2)$,

$$\Lambda_n \approx \frac{1}{R_H} + \frac{\lambda_g^2/R_H}{\lambda_n^2}.$$

Hence the constants can be identified as $A = 1/R_H$ and $B = \lambda_g^2/R_H$.

Step 3: Check the asymptotic dependence The photon wavelength approaches the finite Lyman limit from above as n increases. Its leading correction must therefore decrease as $1/n^2$, equivalently $1/\lambda_n^2$. An increasing linear correction or one proportional to λ_n^2 does not have this behaviour. The final displayed alternative gives no such inverse-square correction and does not express the required large-level dependence.

Step 4: Conclusion $\boxed{\Lambda_n \approx A + B/\lambda_n^2}$. Hence, the correct option is (A).

Quick Tip to Solve: First use $\lambda_n = n\lambda_g$, then expand the reciprocal Rydberg expression; the photon wavelength approaches a constant while the electron wavelength grows.

53. An EM wave from air enters a medium. The electric fields are

$\overrightarrow{E_1} = E_{01}\hat{x}\cos\left[2\pi v\left(\frac{z}{c} - t\right)\right]$ **in air and**

$\overrightarrow{E_2} = E_{02}\hat{x}\cos\left[k\left(2z - ct\right)\right]$ **in medium,**

where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic. If ε_{r_1} and ε_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct ? [4 marks]

(A) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$

(B) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$

(C) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$

(D) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$

Correct Answer: (C) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$

Solution:

Concept:

- For a travelling wave with phase $Kz - \omega t$, the phase velocity is $v = \omega/K$, obtained by holding the phase constant.
- In a non-magnetic medium $v = 1/\sqrt{\mu_0\varepsilon}$. Therefore the relative permittivity is inversely proportional to the square of the propagation speed.

Step 1: Read the speeds from the phases The air-wave phase is $2\pi\nu z/c - 2\pi\nu t$. Its spatial coefficient is $2\pi\nu/c$, and its temporal coefficient is $2\pi\nu$; their ratio gives $v_1 = c$. The medium phase is $2kz - kct$, so its spatial and temporal coefficients are $2k$ and kc, respectively. Thus

$$v_2 = \frac{kc}{2k} = \frac{c}{2}.$$

The amplitude factors do not determine the speed.

Step 2: Relate speed and permittivity Because both media are non-magnetic, the permeability factors cancel when the squared wave-speed expressions are divided:

$$\frac{v_2^2}{v_1^2} = \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}}.$$

Substituting the speeds read directly from the phases,

$$\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \left(\frac{c/2}{c}\right)^2 = \frac{1}{4}.$$

Equivalently, the medium has four times the permittivity of air.

Step 3: Check the direction and power The reduced wave speed requires increased permittivity, so the requested air-to-medium ratio must be smaller than one; this excludes 4 and 2. The value $1/2$ is the speed ratio itself. The permittivity ratio is its square, not its first power. No comparison of electric-field amplitudes is needed for this question.

Step 4: Conclusion $\boxed{\varepsilon_{r_1}/\varepsilon_{r_2} = 1/4}$. Hence, the correct option is (C).

Quick Tip to Solve: Read wave speed as the time coefficient divided by the position coefficient in the phase, then square the speed ratio for non-magnetic media.

54. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e. distance between the centres of each slit.) [4 marks]

(A) 100 μm

(B) 25 μm

(C) 50 μm

(D) 75 μm

Correct Answer: (B) 25 μm

Solution:

Concept:

- For a slit of width a, the first diffraction minima obey $a\sin\theta = \lambda$. The full angular width of the central maximum is 2θ.
- For two coherent slits separated by centre-to-centre distance d, the small-angle fringe width at screen distance D is $\beta = \lambda D/d$.

Step 1: Find the wavelength from diffraction The stated 60° is the full angle between the first minima on opposite sides of the central peak. The angle from the axis to either first minimum is therefore $\theta = 30^\circ$. With $a = 1\,\mu$m,

$$\lambda = a\sin 30^\circ = (1\,\mu\text{m})\frac{1}{2} = 0.5\,\mu\text{m} = 5\times10^{-7}\,\text{m}.$$

Using the sine is necessary because this diffraction angle is not small.

Step 2: Use the interference fringe width The same monochromatic light illuminates the two slits, so use the wavelength just calculated. Convert the other quantities consistently: $D = 50\,\text{cm} = 0.50\,\text{m}$ and $\beta = 1\,\text{cm} = 0.010\,\text{m}$. Rearranging the fringe formula gives

$$d = \frac{\lambda D}{\beta} = \frac{(5\times10^{-7})(0.50)}{0.010}\,\text{m} = 2.5\times10^{-5}\,\text{m} = 25\,\mu\text{m}.$$

Step 3: Verify the scale and alternatives The separation is 25 times the slit width and much smaller than the screen distance, consistent with distinct narrow slits and the Young fringe approximation. The ratio $\beta/D = 0.02$ is small. A separation of $50\,\mu$m would give only half the observed fringe width; 75 and $100\,\mu$m would give still smaller widths. Thus none matches the specified screen pattern.

Step 4: Conclusion The centre-to-centre slit separation is $\boxed{25\,\mu\text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: Halve the full diffraction angular width before applying the first-minimum condition; then use that wavelength in the two-slit fringe formula.

55. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively : [4 marks]

(A) (0, 1)

(B) (0.89, 0.28)

(C) (0.28, 0.89)

(D) (0, 0)

Correct Answer: (B) (0.89, 0.28)

Solution:

Concept:

- In a head-on elastic collision, both total momentum and total kinetic energy are conserved. An initially stationary target can gain energy even though the total energy is unchanged.
- For a projectile of mass m striking a stationary target of mass M, its final velocity is $v = (m - M)u/(m + M)$, where u is its initial velocity.

Step 1: Obtain the fractional projectile energy loss The initial neutron kinetic energy is $K_i = mu^2/2$. After collision, its kinetic energy is $K_f = mv^2/2$. Hence its fractional loss is

$$p = \frac{K_i - K_f}{K_i} = 1 - \frac{v^2}{u^2} = 1 - \left(\frac{m - M}{m + M}\right)^2 = \frac{4mM}{(m + M)^2}.$$

The sign of the final neutron velocity does not affect its kinetic energy. This loss is energy transferred to the target, not energy destroyed.

Step 2: Substitute the two target masses Take the deuterium nucleus mass as $2m$, where m is the neutron mass. Then

$$p_d = \frac{4m(2m)}{(m + 2m)^2} = \frac{8}{9} = 0.888\ldots \approx 0.89.$$

For a carbon nucleus, use $M = 12m$:

$$p_c = \frac{4m(12m)}{(m + 12m)^2} = \frac{48}{169} = 0.2840\ldots \approx 0.28.$$

These mass-number approximations are adequate for the precision of the listed alternatives.

Step 3: Check the order and physical meaning A lighter target whose mass is closer to the neutron mass receives a greater fraction of the neutron energy in a head-on elastic collision. Thus deuterium must have the larger loss, excluding the reversed pair. Zero projectile loss is not a consequence of elasticity: elasticity conserves the sum of both kinetic energies. Consequently the pairs containing zeros are also excluded.

Step 4: Conclusion $\boxed{(p_d, p_c) = (0.89, 0.28)}$. Hence, the correct option is (B).

Quick Tip to Solve: Use $p = 4A/(1 + A)^2$, with target-to-neutron mass ratio A; retain the requested deuterium-then-carbon order.

56. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be I/2. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be I/8. The angle between polarizer A and C is : [4 marks]

(A) 60°

(B) 30°

(C) 45°

(D) 0°

Correct Answer: (C) 45°

Solution:

Concept:

- An ideal polarizer transmits half the intensity of unpolarized light. After the first polarizer the beam is linearly polarized along its transmission axis.
- Malus law states $I_{\text{out}} = I_{\text{in}} \cos^2 \phi$, where ϕ is the angle between the incident polarization and the next polarizer axis.

Step 1: Determine the relation between the original axes After polarizer A the intensity is $I_A = I/2$. Without C, the observed intensity after B is also $I/2$. If the angle between A and B is α, Malus law gives

$$\frac{I}{2}\cos^2\alpha = \frac{I}{2}, \qquad \cos^2\alpha = 1.$$

Therefore A and B have parallel transmission axes, with angles equivalent modulo $180°$.

Step 2: Apply Malus law at both added interfaces Let the acute angle between A and C be θ. Since B is parallel to A, the angle between C and B has the same squared cosine. The intensity after C is $(I/2)\cos^2\theta$; on passing through B it becomes

$$I_B = \frac{I}{2}\cos^4\theta.$$

The stated new intensity is $I/8$, so $\cos^4\theta = 1/4$. Since $\cos^2\theta$ is nonnegative, this gives $\cos^2\theta = 1/2$, and the angle between the axes is $45°$.

Step 3: Check all listed angles For $\theta = 60°$, the final intensity would be $I/32$. For $30°$, it would be $9I/32$. For $0°$, it would remain $I/2$. Only $45°$ gives $(I/2)(1/2)(1/2) = I/8$. The reduction must be applied at both C and B, not only once when C is inserted.

Step 4: Conclusion The angle between A and C is $\boxed{45°}$. Hence, the correct option is (C).

Quick Tip to Solve: First infer the original polarizers are parallel. Inserting an intermediate axis then produces two equal Malus factors, giving $\cos^4\theta$.

57. Three concentric metal shells A, B and C of respective radii a, b and c (a < b < c) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is : [4 marks]

(A) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

(B) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$

(C) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

(D) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$

Correct Answer: (C) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

Solution:

Concept:

- A spherical shell of radius R and total charge Q produces potential kQ/r outside it and the constant potential kQ/R inside it, where $k = 1/(4\pi\varepsilon_0)$.
- Potential is a scalar, so the signed potentials of all three shells must be added. A specified surface charge density gives total charge $Q = 4\pi R^2\sigma$.

Step 1: Calculate the charges on the shells The areas differ, so equal magnitudes of surface charge density do not imply equal magnitudes of total charge. Using the stated signs gives

$$Q_A = 4\pi a^2\sigma, \qquad Q_B = -4\pi b^2\sigma, \qquad Q_C = 4\pi c^2\sigma.$$

The required point is at radius b, outside A, on B, and inside C.

Step 2: Add the three potentials at radius b The inner shell acts like a point charge at the centre. Shell B contributes its own surface potential, while the outer shell contributes its constant interior potential:

$$\begin{aligned} V_B &= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_A}{b}+\frac{Q_B}{b}+\frac{Q_C}{c}\right) \\ &= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{b}-b+c\right). \end{aligned}$$

In particular, the denominator for the contribution of A is the observation radius b, rather than its own radius a.

Step 3: Compare the printed expressions Combine the first two terms to get $(a^2-b^2)/b$. This is the bracket in option C. Option B uses the wrong denominator for the inner-shell contribution. Options A and D put the squared radii of the outer two shells into the numerator and omit the required a^2/b contribution.

Step 4: Conclusion

$$V_B = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right].$$

Hence, the correct option is (C).

Quick Tip to Solve: At the observation radius, classify each shell as inside or outside before choosing the denominator in its potential.

58. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell. [4 marks]

(A) 2.5 Ω

(B) 1 Ω

(C) 1.5 Ω

(D) 2 Ω

Correct Answer: (C) 1.5 Ω

Solution:

Concept:

- At a potentiometer balance there is no galvanometer current, and the measured voltage equals the potential gradient k times the balancing length l.
- For a cell with emf E, internal resistance r, and external load R, its terminal voltage is $V = ER/(R+r)$. The same potential gradient must be used for both balance lengths.

Step 1: Identify the unloaded and loaded voltages Without the shunt the balance measures the emf, since no current is drawn from the cell at null balance. Thus $E = k(52\,\text{cm})$. With the $5\,\Omega$ resistor connected across the cell, current flows through that resistor and the cell has terminal voltage $V = k(40\,\text{cm})$. The smaller length is consistent with a voltage drop inside the cell.

Step 2: Eliminate the potential gradient Take the ratio rather than trying to determine k:

$$\frac{E}{V} = \frac{52}{40} = \frac{R+r}{R} = 1 + \frac{r}{R}.$$

Hence the internal resistance is

$$r = R\left(\frac{52}{40} - 1\right) = 5\left(\frac{12}{40}\right) = 1.5\,\Omega.$$

Both lengths are in centimetres, so their ratio is dimensionless and no length conversion is required.

Step 3: Check the voltage ratio and alternatives With $r = 1.5\,\Omega$, the terminal-voltage fraction is $R/(R+r) = 5/6.5 = 40/52$, exactly the observed length ratio. The alternatives $2.5\,\Omega$, $1\,\Omega$, and $2\,\Omega$ would give emf-to-terminal-voltage ratios 1.5, 1.2, and 1.4, respectively, instead of 1.3.

Step 4: Conclusion The cell has internal resistance $\boxed{1.5\,\Omega}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $r = R(l_{\text{open}} - l_{\text{load}})/l_{\text{load}}$; the loaded balance length belongs in the denominator.

59. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz ? [4 marks]

(A) 2×10^6

(B) 2×10^3

(C) 2×10^5

(D) 2×10^4

Correct Answer: (C) 2×10^5

Solution:

Concept:

- The number of separate channels is the available transmission bandwidth divided by the bandwidth required per channel, assuming the full stated allocation is usable.
- Convert all frequencies to the same unit before division: $1\,\text{GHz} = 10^9\,\text{Hz}$ and $1\,\text{kHz} = 10^3\,\text{Hz}$. A percentage must also be converted to a fraction.

Step 1: Calculate the available frequency band The stated carrier frequency is $10\,\text{GHz} = 10 \times 10^9\,\text{Hz}$. The problem allocates only ten percent of this value for transmission, so the bandwidth available to all channels together is

$$B_{\text{available}} = \frac{10}{100}\,(10 \times 10^9) = 10^9\,\text{Hz}.$$

The remaining ninety percent does not enter the channel count.

Step 2: Divide by the bandwidth of one channel Each channel occupies $5\,\text{kHz} = 5 \times 10^3\,\text{Hz}$. Therefore

$$N = \frac{B_{\text{available}}}{B_{\text{channel}}} = \frac{10^9}{5 \times 10^3} = \frac{10^6}{5} = 2 \times 10^5.$$

The hertz units cancel, leaving a dimensionless count of channels. No extra factor of two is specified: the given bandwidth is already the requirement of a complete channel.

Step 3: Check the scale of the alternatives Multiplying 2×10^5 channels by $5 \times 10^3\,\text{Hz}$ per channel returns $10^9\,\text{Hz}$, the available allocation. Option A would consume the entire $10\,\text{GHz}$, ignoring the ten-percent limit. Options B and D use only 10^7 and $10^8\,\text{Hz}$, so they undercount by factors of one hundred and ten, respectively.

Step 4: Conclusion $\boxed{2 \times 10^5}$ telephonic channels can be transmitted simultaneously. Hence, the correct option is (C).

Quick Tip to Solve: Apply the usable percentage first, then convert GHz and kHz to Hz before dividing the two bandwidths.

60. For an RLC circuit driven with voltage of amplitude vm and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by : [4 marks]

(A) $\frac{CR}{\omega_0}$
(B) $\frac{\omega_0 L}{R}$
(C) $\frac{\omega_0 R}{L}$
(D) $\frac{R}{(\omega_0 C)}$

Correct Answer: (B) $\frac{\omega_0 L}{R}$

Solution:

Concept:

- For a series RLC circuit, resonance occurs when the inductive and capacitive reactances cancel: $\omega_0 L = 1/(\omega_0 C)$, so $\omega_0 = 1/\sqrt{LC}$.
- The quality factor measures resonance sharpness. For a series circuit, Q equals the voltage amplitude across either reactive component at resonance divided by the applied voltage amplitude.

Step 1: Find the current at resonance The net reactance is zero at the stated angular frequency, so the impedance is the resistance alone. With applied voltage amplitude v_m, the current amplitude is

$$I_m = \frac{v_m}{R}.$$

Although the inductor and capacitor voltages cancel in the total circuit voltage, each can individually be much larger than the supply voltage.

Step 2: Form the voltage magnification ratio The inductor voltage amplitude at resonance is $V_{Lm} = I_m \omega_0 L$. Divide by the applied amplitude:

$$Q = \frac{V_{Lm}}{v_m} = \frac{(v_m/R)\omega_0 L}{v_m} = \frac{\omega_0 L}{R}.$$

Using the resonance relation gives the equivalent forms

$$Q = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

Thus the drive amplitude cancels, as expected for the quality factor of a linear circuit.

Step 3: Check dimensions and resistance dependence Inductive reactance $\omega_0 L$ and resistance both have units of ohms, making option B dimensionless. Raising resistance increases damping and must lower Q, consistent with the inverse R factor. Options A, C and D do not have the required dimensionless ratio of reactance to resistance and also have the wrong dependence on resistance.

Step 4: Conclusion The quality factor is $\boxed{Q = \omega_0 L/R}$. Hence, the correct option is (B).

Quick Tip to Solve: For series resonance, remember $Q = X_L/R = X_C/R$, with both reactances evaluated at ω_0.

Chemistry (120 marks)

61. The compound that does not produce nitrogen gas by the thermal decomposition is : [4 marks]

(A) $(NH_4)_2SO_4$

(B) $Ba(N_3)_2$

(C) $(NH_4)_2Cr_2O_7$

(D) NH_4NO_2

Correct Answer: (A) $(NH_4)_2SO_4$

Solution:

Concept:

- Heating a nitrogen-containing compound does not necessarily liberate elemental nitrogen. The oxidation states and the nature of the counter-ion determine the gaseous products.
- Ammonium salts containing oxidising anions such as nitrite and dichromate can undergo internal redox decomposition to N_2; an azide also readily gives nitrogen gas on heating.

Step 1: Examine ammonium sulfate Ammonium sulfate initially loses ammonia on heating, forming ammonium hydrogen sulfate:

$$(NH_4)_2SO_4 \xrightarrow{\Delta} NH_4HSO_4 + NH_3.$$

Further heating can produce ammonia, sulfur oxides and water. This is not the usual nitrogen-gas-producing decomposition. Its nitrogen remains in ammonia in the characteristic decomposition, making sulfate the required exception.

Step 2: Check the azide and dichromate Barium azide decomposes to the metal and nitrogen:

$$Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3\,N_2.$$

Ammonium dichromate undergoes the familiar exothermic internal redox reaction:

$$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} Cr_2O_3 + N_2 + 4\,H_2O.$$

In the latter, nitrogen in ammonium is oxidised while chromium(VI) is reduced to chromium(III). Therefore options B and C both explicitly yield elemental nitrogen.

Step 3: Check ammonium nitrite Ammonium nitrite is a standard laboratory source of nitrogen gas:

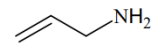
$$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2\,H_2O.$$

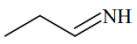
The negative oxidation state of nitrogen in ammonium and positive oxidation state in nitrite meet at oxidation state zero in N_2. Option D therefore also produces nitrogen, leaving only option A as the exception.

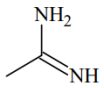
Step 4: Conclusion $(NH_4)_2SO_4$ does not give nitrogen gas in its characteristic thermal decomposition. Hence, the correct option is (A).

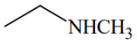
Quick Tip to Solve: Distinguish ammonium nitrite from sulfate: nitrite gives N_2, whereas sulfate characteristically releases NH_3.

62. **The increasing order of basicity of the following compounds is :** [4 marks]

(a) 

(b) 

(c) 

(d) 

(A) (d) $<$ (b) $<$ (a) $<$ (c)

(B) (a) $<$ (b) $<$ (c) $<$ (d)

(C) (b) $<$ (a) $<$ (c) $<$ (d)

(D) (b) $<$ (a) $<$ (d) $<$ (c)

Correct Answer: (D) (b) $<$ (a) $<$ (d) $<$ (c)

Solution:

Concept:

- Basicity depends on the availability of the nitrogen lone pair and on the stability of the conjugate acid formed after protonation.
- An imine has its basic lone pair on an sp^2-hybridised nitrogen. An aliphatic amine has an sp^3 nitrogen, while an amidine forms a strongly resonance-stabilised amidinium ion.

Step 1: Identify the four functional groups Compound (a) is allylamine, $CH_2{=}CHCH_2NH_2$; the methylene group separates its nitrogen from the double bond, so this is not an aniline-like conjugated lone pair. Compound (b) is an aldimine, $CH_3CH_2CH{=}NH$. Compound (c) is an amidine, $CH_3C(=NH)NH_2$. Compound (d) is the secondary aliphatic amine $CH_3CH_2NHCH_3$.

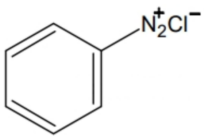
Step 2: Compare the imine with the ordinary amines The sp^2 lone pair of (b) is held more tightly because it has greater s character, making (b) the weakest base here. The primary amine (a) has a more available sp^3 lone pair. The two alkyl groups of secondary amine (d) donate electron density toward nitrogen and stabilise its protonated form, so (d) is more basic than (a). Thus the first three positions are $(b) < (a) < (d)$.

Step 3: Place the amidine and eliminate the alternatives Protonation of amidine (c) gives an amidinium ion whose positive charge is delocalised over two nitrogen atoms. This especially strong conjugate-acid stabilisation makes (c) the strongest base, above the secondary amine. Option A wrongly puts (d) last in basicity; option B makes the imine stronger than (a) and places (d) above (c); option C also incorrectly puts (d) above the amidine.

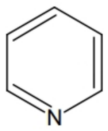
Step 4: Conclusion The increasing order is $\boxed{\text{b} < \text{a} < \text{d} < \text{c}}$. Hence, the correct option is (D).

Quick Tip to Solve: An amidine is unusually basic because its conjugate acid shares the positive charge between two nitrogen atoms.

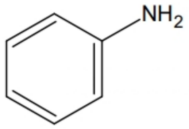
63. **Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?** [4 marks]

(A) Benzenediazonium chloride 

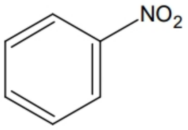
(B) Pyridine



(C) Aniline

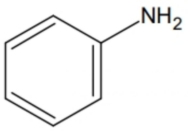


(D) Nitrobenzene



Correct Answer: (C)

Aniline



Solution:

Concept:

- Kjeldahl's method converts suitable organically bound nitrogen into ammonium sulfate by digestion with concentrated sulfuric acid, then releases ammonia by treatment with alkali.
- The standard method is unsuitable for nitro and diazonium nitrogen and for nitrogen incorporated in a heteroaromatic ring. An amino group attached to an aromatic carbon is suitable.

Step 1: Recall what is actually measured After sulfuric-acid digestion, nitrogen must be quantitatively retained as ammonium ions. On adding excess alkali, ammonia is liberated, absorbed in a measured quantity of standard acid, and estimated by titration. The essential requirement is conversion of all sample nitrogen into the ammonium form; merely containing nitrogen is insufficient.

Step 2: Identify the suitable amino compound Option C is aniline, $C_6H_5NH_2$. Its nitrogen is in an amino substituent outside the benzene ring, so it undergoes the required conversion during Kjeldahl digestion. On alkalisation the ammonium sulfate formed releases ammonia:

$$(NH_4)_2SO_4 + 2\,NaOH \longrightarrow 2\,NH_3 + Na_2SO_4 + 2\,H_2O.$$

The amount of ammonia therefore measures the nitrogen originally present in aniline.

Step 3: Reject the three unsuitable nitrogen environments Option A is benzenediazonium chloride: diazonium nitrogen is not quantitatively converted into ammonium sulfate by this method. Option B is pyridine, whose nitrogen is an integral part of the aromatic ring and is unsuitable for the standard digestion. Option D is nitrobenzene, whose nitro nitrogen also requires a modified procedure rather than the ordinary Kjeldahl method. Consequently only the amino group in option C meets the stated requirement.

Step 4: Conclusion Aniline, $C_6H_5NH_2$, is suitable for nitrogen estimation by the standard Kjeldahl

method. Hence, the correct option is (C).

Quick Tip to Solve: For Kjeldahl questions, first exclude nitro, diazonium and ring nitrogen; an ordinary amino nitrogen is suitable.

64. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point? [4 marks]

(A) $[Co(H_2O)_3\ Cl_3].3H_2O$

(B) $[Co(H_2O)_6]\ Cl_3$

(C) $[Co(H_2O)_5\ Cl]\ Cl_2.H_2O$

(D) $[Co(H_2O)_4\ Cl_2]\ Cl.2H_2O$

Correct Answer: (A) $[Co(H_2O)_3\ Cl_3].3H_2O$

Solution:

Concept:

- Freezing-point depression is a colligative property: $\Delta T_f = iK_f m$, where i counts the independently moving solute particles, K_f is the cryoscopic constant of water and m is molality.
- Ligands inside the coordination sphere remain attached to cobalt. Only chloride ions outside the square brackets dissociate as counter-ions; water of crystallisation does not supply extra solute ions.

Step 1: Count particles after dissolution The complex in (A), $[Co(H_2O)_3Cl_3]$, has no chloride counter-ion and remains one neutral solute species, so $i \simeq 1$. For (B), $[Co(H_2O)_6]Cl_3$ gives one complex cation and three chloride anions, so $i \simeq 4$. The corresponding counts for (C) and (D) are three and two particles respectively.

Step 2: Compare the freezing points All four solutions have the same solvent and molality, so their depressions are proportional to i. Thus

$$\Delta T_f(A) : \Delta T_f(B) : \Delta T_f(C) : \Delta T_f(D) = 1 : 4 : 3 : 2.$$

Since $T_f = T_f^\circ - \Delta T_f$, the smallest depression gives the highest freezing point. The waters written after the dot join the solvent; counting them as independently dissociated salt ions would give a wrong ordering. Options (B), (C) and (D) all lower the freezing point more strongly than (A).

Step 3: Conclusion $[Co(H_2O)_3Cl_3] \cdot 3\,H_2O$ produces the fewest solute particles and therefore has the highest freezing point. Hence, the correct option is (A).

Quick Tip to Solve: For coordination salts, count one complex species plus the counter-ions outside the brackets; highest freezing point means smallest particle count.

65. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination? [4 marks]

(A)

Base	Acid	End point
Weak	Strong	Colourless to pink

(B)

Base	Acid	End point
Strong	Strong	Pinkish red to yellow

(C)

Base	Acid	End point
Weak	Strong	Yellow to pinkish red

(D)

Base	Acid	End point
Strong	Strong	Pink to colourless

Correct Answer: (C)

Base	Acid	End point
Weak	Strong	Yellow to pinkish red

Solution:

Concept:

- Methyl orange is yellow in alkaline solution and pinkish red in acidic solution. Its transition range lies on the acidic side, approximately pH 3.1–4.4.
- A weak base titrated with a strong acid has an acidic equivalence region because the salt contains the conjugate acid of the weak base. A suitable indicator changes colour within the steep pH fall near equivalence.

Step 1: Identify the suitable acid-base pair For a weak alkali such as ammonia, a strong acid neutralises the base and produces an ammonium salt. At equivalence the conjugate acid hydrolyses, so the solution is acidic. Methyl orange can therefore signal a weak-base/strong-acid titration. This establishes that the weak-base/strong-acid row must be checked together with its stated colour change; choosing only from the strength labels is insufficient.

Step 2: Follow the colour as acid is added The flask initially contains alkali and the indicator is yellow. Addition of acid lowers the pH until the indicator enters its acidic colour range, becoming pinkish red. Thus (C) supplies both the suitable pair and the correct colour direction. Option (A) gives colourless to pink, which is not the methyl-orange transition. In (B), pinkish red to yellow reverses the direction for adding acid to alkali, even though methyl orange can also be used for strong-acid/strong-base titrations. Option (D), pink to colourless, is associated with phenolphthalein rather than methyl orange.

Step 3: Conclusion The correct combination is weak base, strong acid, and a yellow-to-pinkish-red end point. Hence, the correct option is (C).

Quick Tip to Solve: Write the initial and final pH conditions before recalling an indicator colour: methyl orange changes yellow to red as acid is added to alkali.

66. Consider the following reaction and statements:

$[Co(NH_3)_4Br_2]^+ + Br^- \rightarrow [Co(NH_3)_3Br_3] + NH_3$

(I) Two isomers are produced if the reactant complex ion is a cis-isomer

(II) Two isomers are produced if the reactant complex ion is a trans-isomer

(III) Only one isomer is produced if the reactant complex ion is a trans-isomer

(IV) Only one isomer is produced if the reactant complex ion is a cis – isomer

The correct statements are [4 marks]

(A) (II) and (IV)

(B) (I) and (II)

(C) (I) and (III)

(D) (III) and (IV)

Correct Answer: (C) (I) and (III)

Solution:

Concept:

- An octahedral MA_4B_2 complex has cis and trans arrangements. Replacing one A ligand by B gives MA_3B_3, whose geometrical isomers are facial (fac) and meridional (mer).
- In the fac isomer all three B ligands are mutually cis and occupy one triangular face. In the mer isomer one pair of B ligands is trans. Count inequivalent substitution sites while retaining the positions of the original ligands.

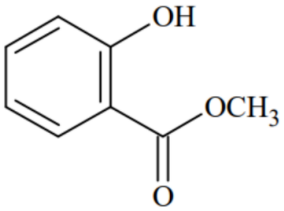
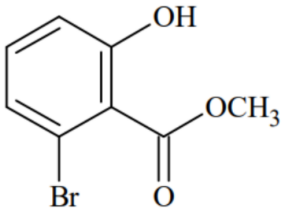
Step 1: Start from the cis reactant Place the two original bromides at the $+x$ and $+y$ octahedral sites. The remaining four sites carry ammonia. If the incoming bromide replaces ammonia at $+z$ or $-z$, all three bromides are mutually cis, giving fac product. If it replaces ammonia at $-x$ or $-y$, a trans bromide pair is created, giving mer product. These are two distinct geometrical isomers, so statement (I) is true and statement (IV) is false.

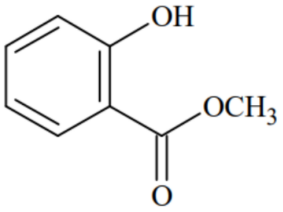
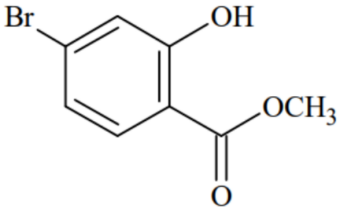
Step 2: Start from the trans reactant Now put the original bromides at $+z$ and $-z$. Whichever equatorial ammonia is replaced, those bromides remain trans. Every product therefore has the mer arrangement; the four equatorial choices are equivalent by rotation. A fac product cannot contain that original trans pair, so it is not formed by this single substitution without rearrangement. Consequently statement (III) is true and (II) is false. Options (A) and (B) include the false statement (II), while (D) includes the false statement (IV).

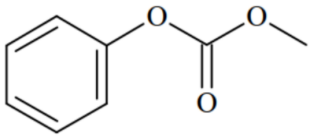
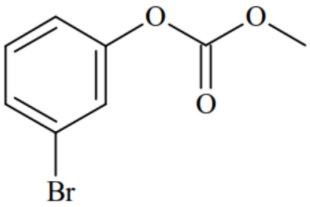
Step 3: Conclusion The cis reactant gives fac and mer products, whereas the trans reactant gives only mer product. The correct statements are (I) and (III). Hence, the correct option is (C).

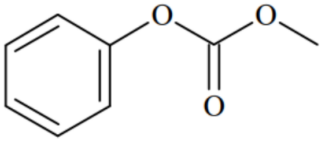
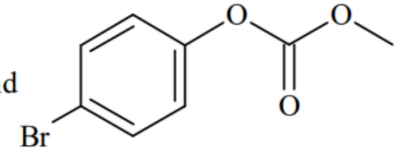
Quick Tip to Solve: Track the original two bromide positions: a preserved trans pair forces mer; an original cis pair permits both fac and mer.

67. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively : [4 marks]

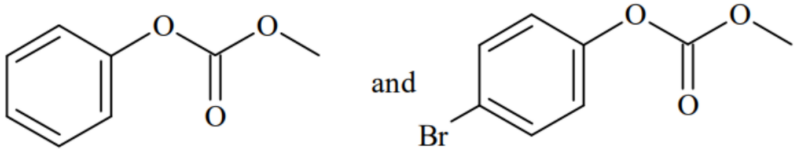
(A)  and 

(B)  and 

(C)  and 

(D)  and 

Correct Answer: (D)



Solution:

Concept:

- In basic medium phenol forms phenoxide, $C_6H_5O^-$, a nucleophile that attacks the carbonyl carbon of methyl chloroformate, $ClCOOCH_3$. Chloride is displaced in nucleophilic acyl substitution.
- In the resulting aryl carbonate the atom directly bonded to the ring is oxygen. Its resonance donation directs electrophilic bromination towards ortho and para positions; steric crowding favours the para product.

Step 1: Form the carbonate ester A First, $C_6H_5OH + OH^- \longrightarrow C_6H_5O^- + H_2O$. Phenoxide then attacks methyl chloroformate and the tetrahedral intermediate loses chloride:

$$C_6H_5O^- + ClCOOCH_3 \longrightarrow C_6H_5OCOOCH_3 + Cl^-.$$

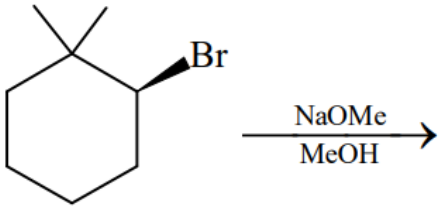
Thus A is methyl phenyl carbonate, with the ring attached through oxygen as $\mathrm{Ph{-}O{-}C(=O){-}OCH_3}$. It is not methyl salicylate: this reagent acylates oxygen rather than introducing a carbonyl group directly onto an ortho ring carbon.

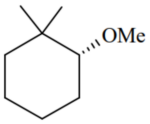
Step 2: Locate bromination in B The $-\mathrm{OCOOCH_3}$ substituent retains oxygen-to-ring resonance donation and is ortho/para directing. Bromination therefore gives predominantly the less crowded para-bromo derivative, $\mathrm{p\text{-}BrC_6H_4OCOOCH_3}$. Options (A) and (B) already have an incorrect carbon-bonded ester structure for A, so neither can be the answer. Option (C) has the correct carbonate A but places bromine meta to the oxygen substituent; this conflicts with the directing effect. The pair in (D) has both the oxygen-acylated carbonate and its para-bromo product.

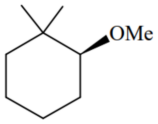
Step 3: Conclusion A is methyl phenyl carbonate and B is its para-bromo derivative, as shown in (D). Hence, the correct option is (D).

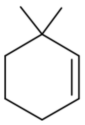
Quick Tip to Solve: Check which atom connects the substituent to the ring: an oxygen-linked carbonate directs ortho/para, unlike a carbonyl carbon linked directly to benzene.

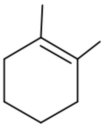
68. The major product of the following reaction is: [4 marks]



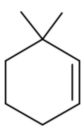
(A) 

(B) 

(C) 

(D) 

Correct Answer: (C)



Solution:

Concept:

- Methoxide, CH_3O^-, is a strong base. With a sterically hindered secondary alkyl bromide it favours bimolecular elimination (E2), especially when backside substitution is crowded.
- E2 elimination removes a beta hydrogen while the carbon–bromine bond breaks. In a cyclohexane chair, the reacting hydrogen and leaving group must have an anti-periplanar, trans-diaxial arrangement.

Step 1: Find the beta carbon that can lose hydrogen Call the bromine-bearing ring carbon C1. One adjacent carbon is the gem-dimethyl carbon: it is bonded to two ring carbons and two methyl groups, so it has no hydrogen. Elimination towards that beta carbon is therefore impossible. The other adjacent ring carbon is a methylene group and does possess beta hydrogen. Ring inversion can place bromine axial; an axial hydrogen on this methylene is then anti to bromine and can be abstracted by methoxide.

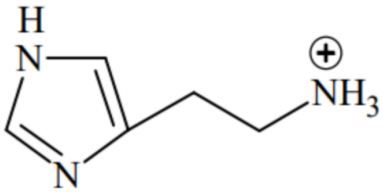
Step 2: Form the allowed double bond and compare the options The base removes that beta hydrogen, the C–H electrons form a double bond to C1, and bromide leaves in the same concerted step. The two methyl groups remain on the same saturated carbon, giving 3,3-dimethylcyclohex-1-ene, the structure in (C). Options (A) and (B) are substitution products with methoxy replacing bromine; steric hindrance disfavors that pathway relative to E2 here. Option (D) has methyl groups on different carbons and a rearranged double-bond framework. Such a skeletal rearrangement is not part of the concerted E2 mechanism, and cannot rescue the absence of a beta hydrogen on the original quaternary carbon.

Step 3: Conclusion The major product is the unrearranged alkene formed on the side containing a beta hydrogen, 3,3-dimethylcyclohex-1-ene. Hence, the correct option is (C).

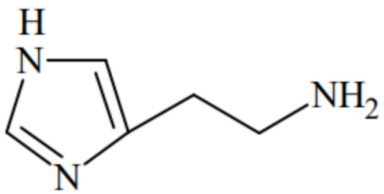
Quick Tip to Solve: Before applying a substitution or alkene-stability rule, count beta hydrogens: the gem-dimethyl ring carbon is quaternary and cannot supply one.

69. The predominant form of histamine present in human blood is (pK_a, Histidine = 6.0) [4 marks]

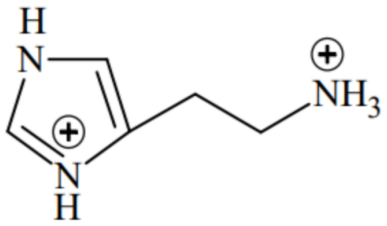
(A)



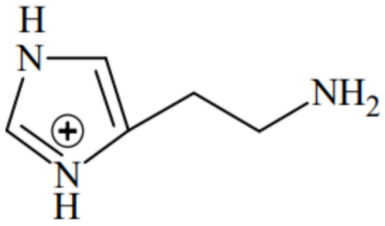
(B)



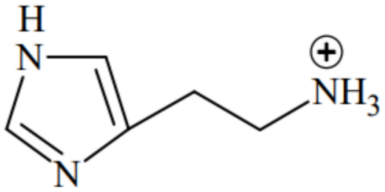
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- Human blood is mildly alkaline, approximately pH 7.4. For a conjugate-acid/base pair, $\mathrm{pH} = \mathrm{p}K_a + \log([\text{base}]/[\text{acid}])$, so the less protonated form dominates when pH exceeds its $\mathrm{p}K_a$.
- Histamine has two different basic sites: the pyridine-like imidazole nitrogen and the side-chain primary amine. The ring conjugate-acid dissociation has $\mathrm{p}K_a$ near 6, while the aliphatic ammonium group is a substantially weaker acid and stays predominantly protonated at blood pH.

Step 1: Determine the imidazole-ring state Use the stated ring $\mathrm{p}K_a = 6.0$. At blood pH,

$$\frac{[\text{neutral imidazole}]}{[\text{imidazolium}]} = 10^{7.4-6.0} = 10^{1.4} \approx 25.$$

Hence the ring is predominantly uncharged. The ordinary pyrrole-like ring nitrogen still carries its original hydrogen; neutral ring here does not mean removing that hydrogen. It means that the other, pyridine-like nitrogen is not additionally protonated.

Step 2: Determine the side-chain state and select the structure The primary aliphatic amine is more basic than the imidazole site. Its conjugate acid has a $\mathrm{p}K_a$ well above blood pH, so its side chain is predominantly $-CH_2CH_2NH_3^+$, rather than $-CH_2CH_2NH_2$. Combining the two sites gives an uncharged imidazole ring and a protonated side-chain amine, carrying net charge +1. That is option (A). Option (B) incorrectly leaves the side-chain amine neutral; (C) additionally protonates the ring; (D) protonates the less basic ring site while leaving the more basic side-chain site neutral. None represents the predominant population.

Step 3: Conclusion Histamine predominantly has a neutral imidazole ring and a terminal $-NH_3^+$ group in blood. Hence, the correct option is (A).

Quick Tip to Solve: Apply the pH–pKa comparison separately to each nitrogen site; the ring and the aliphatic amine do not have the same basicity.

70. Glucose on prolonged heating with HI gives [4 marks]

(A) 6-iodohexanal

(B) n-Hexane

(C) 1-Hexene

(D) Hexanoic acid

Correct Answer: (B) n-Hexane

Solution:

Concept:

- On prolonged heating, concentrated hydrogen iodide reduces the oxygen-containing functional groups of glucose. The aldehyde group and the alcohol groups are ultimately replaced by carbon–hydrogen bonds.
- The carbon skeleton survives this exhaustive reduction. Obtaining a straight-chain six-carbon alkane is therefore evidence that all six carbon atoms of glucose form one unbranched chain.

Step 1: Track the six-carbon framework The open-chain representation of glucose is

$$CHO-(CHOH)_4-CH_2OH.$$

It contains six carbons in a continuous chain: one aldehyde carbon, four secondary-alcohol carbons and one terminal primary-alcohol carbon. Prolonged heating with HI removes oxygen from all these positions. The terminal aldehyde carbon becomes a methyl carbon; the internal carbon atoms become methylene groups, and the other terminus also becomes methyl. No carbon is added or removed.

Step 2: Identify the product and reject incomplete changes The resulting saturated hydrocarbon is

$$CH_3-CH_2-CH_2-CH_2-CH_2-CH_3,$$

namely n-hexane. Option (A), 6-iodohexanal, retains the aldehyde group and corresponds to an incomplete transformation rather than exhaustive reduction. Option (C), 1-hexene, retains a carbon–carbon double bond, whereas the final product under the specified prolonged reducing conditions is the saturated alkane. Option (D), hexanoic acid, contains an oxidised carboxyl group; formation of this oxygen-rich product contradicts the reducing action of HI. The word prolonged is therefore decisive: the question asks for the final fully reduced carbon framework, not an iodinated intermediate.

Step 3: Conclusion Glucose gives the straight-chain saturated hydrocarbon n-hexane on prolonged heating with HI. Hence, the correct option is (B).

Quick Tip to Solve: Use the HI test to read the sugar carbon skeleton: complete deoxygenation of glucose leaves the unbranched six-carbon chain as n-hexane.

71. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane?

(Atomic weight of B = 10.8 u) [4 marks]

(A) 1.6 hours

(B) 6.4 hours

(C) 0.8 hours

(D) 3.2 hours

Correct Answer: (D) 3.2 hours

Solution:

Concept:

- Faraday's law connects the charge passed to the amount of electrode product: $Q = n_e F = It$, where $F = 96485\,\mathrm{C\,mol^{-1}}$.
- Complete combustion of diborane gives boron oxide and water. Four moles of electrons are transferred for each mole of oxygen liberated by water electrolysis.

Step 1: Calculate the amount of diborane Diborane is $\mathrm{B_2H_6}$. Using the specified atomic mass of boron and the usual hydrogen atomic mass gives

$$M = 2(10.8) + 6(1) = 27.6\,\mathrm{g\,mol^{-1}}, \qquad n = \frac{27.66}{27.6} \simeq 1.0022\,\mathrm{mol}.$$

The small difference between 27.66 g and one mole does not affect the approximate option.

Step 2: Balance combustion and find the charge The balanced reaction is

$$\mathrm{B_2H_6 + 3\,O_2 \longrightarrow B_2O_3 + 3\,H_2O}.$$

Hence the oxygen required is $3(1.0022) = 3.0065\,\mathrm{mol}$. At the anode,

$$\mathrm{2\,H_2O \longrightarrow O_2 + 4\,H^+ + 4\,e^-}.$$

Thus $n_e = 4(3.0065) = 12.026\,\mathrm{mol}$ and $Q = 12.026(96485) \simeq 1.1603 \times 10^6\,\mathrm{C}$. It would be incorrect to use two electrons per oxygen molecule: that is the electron count per oxygen atom.

Step 3: Convert charge to electrolysis time At $I = 100\,\mathrm{A} = 100\,\mathrm{C\,s^{-1}}$,

$$t = \frac{1.1603 \times 10^6}{100} = 1.1603 \times 10^4\,\mathrm{s} = \frac{1.1603 \times 10^4}{3600}\,\mathrm{h} \simeq 3.22\,\mathrm{h}.$$

The other times, 0.8 h, 1.6 h and 6.4 h, correspond to incorrect factors in the oxygen requirement or electron stoichiometry.

Step 4: Conclusion The required time is approximately 3.2 hours. Hence, the correct option is (D).

Quick Tip to Solve: First balance combustion to count moles of oxygen; only then use four Faradays per mole of $\mathrm{O_2}$.

72. The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$ and $K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$ respectively are :

[4 marks]

(A) +3, 0 and +4

(B) +3, +4 and +6

(C) +3, +2 and +4

(D) +3, 0 and +6

Correct Answer: (D) +3, 0 and +6

Solution:

Concept:

- The sum of the oxidation-state contributions of a metal and its ligands equals the overall charge of the coordination entity.
- Water, benzene and ammonia are neutral ligands. Cyanide has charge -1, an oxido ligand O^{2-} has charge -2, and a peroxido ligand ${O_2}^{2-}$ has total charge -2.

Step 1: Find chromium in the hydrated chloride In $[Cr(H_2O)_6]Cl_3$, the three chloride counterions each have charge -1. Consequently the complex ion has charge $+3$. All six coordinated water molecules are neutral, so

$$x + 6(0) = +3 \quad \Longrightarrow \quad x = +3.$$

The oxidation state is not the coordination number: six water molecules do not imply chromium(VI).

Step 2: Identify the neutral sandwich complex In $[Cr(C_6H_6)_2]$, each benzene ring is a neutral ligand and no counterions are present. Therefore the entire complex is neutral and

$$x + 2(0) = 0 \quad \Longrightarrow \quad x = 0.$$

Benzene coordination does not make the rings formally negatively charged. This eliminates options (B) and (C), which assign a positive oxidation state to chromium here.

Step 3: Balance the final complex charge The two potassium ions in the final compound make the complex anion carry charge -2. There are two cyanide ligands, two individual oxido ligands, one peroxido group and neutral ammonia:

$$x + 2(-1) + 2(-2) + (-2) + 0 = -2.$$

Thus $x - 8 = -2$, giving $x = +6$. Treating the O_2 group as neutral would give the erroneous value $+4$ in option (A); its identity as peroxido is essential.

Step 4: Conclusion The oxidation states are $+3,\ 0,\ +6$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: For oxygen-containing complexes, count an oxido ligand as -2 and an entire peroxido group as -2, not each oxygen in it.

73. Which of the following salts is the most basic in aqueous solution? [4 marks]

(A) $Pb(CH_3COO)_2$

(B) $Al(CN)_3$

(C) CH_3COOK

(D) $FeCl_3$

Correct Answer: (C) CH_3COOK

Solution:

Concept:

- A salt of a weak acid and strong base is basic because its anion accepts a proton from water and produces OH^-.
- Highly charged hydrated metal cations can act as acids. For an acetate salt with a non-hydrolysing alkali-metal cation, $K_b(CH_3COO^-) = K_w/K_a(CH_3COOH)$.

Step 1: Identify the salt with unopposed basic hydrolysis Potassium acetate, CH_3COOK, dissociates into K^+ and CH_3COO^-. Potassium is the cation of a strong base and does not produce appreciable acidity through hydrolysis. Acetate is the conjugate base of weak acetic acid:

$$CH_3COO^- + H_2O \rightleftharpoons CH_3COOH + OH^-.$$

The resulting hydroxide ions make its aqueous solution basic. There is no competing acidic metal-ion hydrolysis in this option.

Step 2: Compare the salts containing metal cations In $FeCl_3$, chloride is the conjugate base of a strong acid and contributes negligible basicity, whereas hydrated Fe^{3+} acidifies water. Thus (D) is acidic. In $Pb(CH_3COO)_2$, acetate tends to produce basicity, but hydrolysis of Pb^{2+} introduces competing acidity; it is not the simple strong-base/weak-acid salt represented by potassium acetate. Hence (A) is not the best basic salt here.

Step 3: Check why cyanide alone does not decide the answer Although CN^- is a basic anion, $Al(CN)_3$ also contains strongly hydrolysing Al^{3+}. Its behaviour cannot be classified by the cyanide ion alone. The strong tendency of aluminium(III) to hydrolyse, including formation of hydroxide, prevents using this formula as an uncomplicated soluble basic salt. Option (C) gives the clear, stable aqueous example of basic anionic hydrolysis without an acidic cation.

Step 4: Conclusion Potassium acetate, CH_3COOK, is the most basic salt among the listed choices. Hence, the correct option is (C).

Quick Tip to Solve: Inspect both ions: a basic anion is most effective when the accompanying cation, such as K^+, does not hydrolyse appreciably.

74. Which of the following are Lewis acids? [4 marks]

(A) BCl_3 and $AlCl_3$

(B) PH_3 and BCl_3

(C) $AlCl_3$ and $SiCl_4$

(D) PH_3 and $SiCl_4$

Correct Answer: (A) BCl_3 and $AlCl_3$

Solution:

Concept:

- A Lewis acid accepts an electron pair to form a coordinate bond. A Lewis base donates a lone pair; this definition does not require transfer of a proton.
- Electron-deficient BCl_3 and $AlCl_3$ are standard Lewis acids, whereas PH_3 normally acts as an electron-pair donor.

Step 1: Test the electron-deficient chlorides In monomeric BCl_3 and $AlCl_3$, the central atom is surrounded by three bonding pairs and has an incomplete octet. Each can accept a donor lone pair; for example,

$$BCl_3 + NH_3 \longrightarrow BCl_3 \cdot NH_3.$$

Similarly aluminium chloride accepts chloride to form $AlCl_4^-$. Thus both species in option (A) meet the Lewis-acid definition, which explains the listed answer key.

Step 2: Eliminate pairs containing phosphine Phosphine has a nonbonding electron pair on phosphorus and normally functions as a Lewis base by donating that pair to an acceptor. Thus a pair including PH_3 does not consist entirely of Lewis acids in the intended classification. This excludes options (B) and (D), regardless of whether the other member accepts an electron pair.

Step 3: Recognise the limitation of the single-answer choices There is a genuine ambiguity in the wording: $SiCl_4$ can also accept electron pairs and form higher-coordinate species, for example hexachlorosilicate:

$$SiCl_4 + 2\,Cl^- \longrightarrow [SiCl_6]^{2-}.$$

Therefore the fact that silicon already has an octet in tetrahedral $SiCl_4$ does not exclude Lewis acidity. Option (C), containing $AlCl_3$ and $SiCl_4$, is also defensible under the general Lewis definition. No assertion that an octet-complete molecule can never be a Lewis acid should be used to force uniqueness.

Step 4: Conclusion The supplied key selects option (A), the pair BCl_3 and $AlCl_3$. Hence, the correct option is (A) as keyed. However, $SiCl_4$ is also a Lewis acid, so option (C) is chemically defensible and the printed single-answer question is ambiguous.

Quick Tip to Solve: Electron deficiency is sufficient for Lewis acidity, but it is not necessary: the defining test is acceptance of an electron pair.

75. At 518°C the rate of decomposition of a sample of gaseous acetaldehyde initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% had reacted and 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is [4 marks]

(A) 0

(B) 2

(C) 3

(D) 1

Correct Answer: (B) 2

Solution:

Concept:

- For a gaseous reactant at fixed temperature and volume, partial pressure is proportional to concentration. Hence a rate law can be written $r = kP^n$, where P is the unreacted acetaldehyde pressure.
- At the same temperature the rate constant is unchanged, so dividing two instantaneous rate laws removes k. The percentage reacted must first be converted to the fraction remaining.

Step 1: Calculate both remaining reactant pressures The initial acetaldehyde pressure is $P_0 = 363\,\text{Torr}$. When 5 percent has reacted, 95 percent remains; when 33 percent has reacted, 67 percent remains:

$$P_1 = 0.95(363) = 344.85\,\text{Torr}, \qquad P_2 = 0.67(363) = 243.21\,\text{Torr}.$$

These are reactant partial pressures, not total pressures of the reacting mixture. Using the percentages consumed would reverse the physical comparison.

Step 2: Divide the two rate laws The measured rates are $r_1 = 1.00\,\text{Torr}\,\text{s}^{-1}$ and $r_2 = 0.50\,\text{Torr}\,\text{s}^{-1}$. Therefore

$$\frac{r_1}{r_2} = \left(\frac{P_1}{P_2}\right)^n \quad \Longrightarrow \quad 2 = \left(\frac{0.95}{0.67}\right)^n.$$

Taking logarithms gives

$$n = \frac{\ln 2}{\ln(0.95/0.67)} \simeq 1.99 \simeq 2.$$

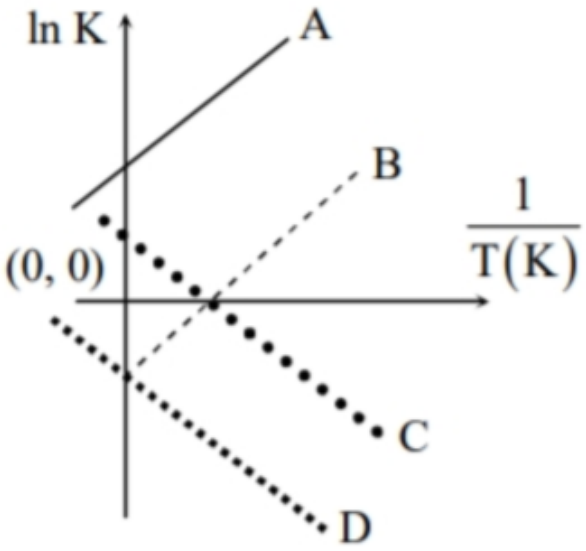
The rounded percentages account for the slight departure from an exact integer.

Step 3: Check the integer choices directly The pressure ratio is approximately 1.418. A zero-order law predicts a rate ratio of 1, a first-order law predicts 1.418, and a third-order law predicts approximately 2.85. None agrees with the observed ratio 2. A second-order law predicts $(1.418)^2 \simeq 2.01$, matching the observations within their stated precision. The common initial pressure cancels, so its numerical value is not needed after forming the ratio.

Step 4: Conclusion The decomposition is approximately second order. Hence, the correct option is (B).

Quick Tip to Solve: For percentage-conversion data, use the remaining fractions in the rate ratio; the initial pressure then cancels automatically.

76. Which of the following lines correctly show the temperature dependence of equilibrium constant K, for an exothermic reaction? [4 marks]



(A) A and D

(B) A and B

(C) B and C

(D) C and D

Correct Answer: (B) A and B

Solution:

Concept:

- The integrated van't Hoff equation is $\ln K = -\Delta H^\circ/(RT) + \Delta S^\circ/R$, assuming approximately constant standard enthalpy and entropy over the temperature interval.
- For a plot of $\ln K$ against $1/T$, the slope is $-\Delta H^\circ/R$. An exothermic reaction has $\Delta H^\circ < 0$, so this slope is positive.

Step 1: Identify the variables actually plotted The vertical axis in the figure is $\ln K$, whereas the horizontal axis is reciprocal absolute temperature, $1/T$. Thus increasing the horizontal coordinate means cooling, not heating. Rewriting the van't Hoff relation as a straight-line equation gives

$$y = mx + c, \qquad y = \ln K, \quad x = 1/T, \quad m = -\frac{\Delta H^\circ}{R}.$$

This distinction is crucial because the sign of the slope against T would be different.

Step 2: Apply the exothermic sign condition For an exothermic reaction ΔH° is negative, while R is positive. Hence $-\Delta H^\circ/R > 0$. The correct lines must therefore rise from left to right. In the figure, both A and B have positive slopes, while C and D have negative slopes. Consequently C and D would describe an endothermic enthalpy sign in this representation.

Step 3: Explain why either intercept is allowed The intercept is $\Delta S^\circ/R$, and exothermicity alone places no restriction on the sign of the reaction entropy. Thus the positive intercept of A and the

negative intercept of B are both allowed. An equilibrium constant may be greater or less than one, so $\ln K$ may likewise be positive or negative. Options (A) and (C) each include a negative-slope line; option (D) contains only negative-slope lines. Only the pair A and B satisfies the required condition throughout.

Step 4: Conclusion The positive-slope lines A and B represent an exothermic reaction. Hence, the correct option is (B).

Quick Tip to Solve: Before deciding a thermodynamic graph's slope, read the horizontal axis: $1/T$ reverses the direction of increasing temperature.

77. Total number of lone pair of electrons in I_3^- ion is [4 marks]

(A) 3

(B) 9

(C) 6

(D) 12

Correct Answer: (B) 9

Solution:

Concept:

- Each iodine atom contributes seven valence electrons, and the negative charge adds one electron. Thus I_3^- contains $3(7)+1=22$ valence electrons.
- A Lewis structure uses two I–I single bonds and completes the terminal octets. The remaining electrons lie on the central iodine; total lone pairs include those on all three atoms.

Step 1: Allocate the bonding electrons Place the three iodine atoms in an I–I–I chain. Two single bonds require four electrons, leaving

$$22-2(2)=18 \text{ nonbonding electrons.}$$

Every lone pair consists of two electrons, so the total count is already $18/2=9$. Counting only the central atom would answer a different question from the total asked here.

Step 2: Check the distribution atom by atom Each terminal iodine has one bond and needs three lone pairs to complete its octet. The two terminal atoms therefore use $2(3)=6$ lone pairs, or 12 electrons. Of the 18 nonbonding electrons, the remaining six form three lone pairs on the central iodine. Thus

$$N_{\text{lone pairs}}=3+3+3=9.$$

The central atom has five electron-pair domains, two bonding and three nonbonding, corresponding to the familiar linear molecular shape.

Step 3: Reject the tempting partial counts Option (A), 3, counts only the central iodine's lone pairs. Option (C), 6, counts the lone pairs of the terminal atoms alone. Option (D), 12, would require 24 nonbonding electrons even before adding bonding electrons, exceeding the entire 22-electron budget.

The count of nine is consistent both with the total electron inventory and with the individual atoms' octets. The formal negative charge is on the central iodine in this Lewis description.

Step 4: Conclusion The total number of lone pairs in I_3^- is 9. Hence, the correct option is (B).

Quick Tip to Solve: Subtract bonding electrons from the total valence-electron count, then divide by two to count all lone pairs in the ion.

78. Which of the following compounds contain(s) no covalent bond(s)? KCl, PH_3, O_2, B_2H_6, H_2SO_4

[4 marks]

(A) KCl, B_2H_6

(B) KCl, B_2H_6, PH_3

(C) KCl, H_2SO_4

(D) KCl

Correct Answer: (D) KCl

Solution:

Concept:

- An ionic solid is held together by electrostatic attraction between oppositely charged ions; a covalent bond involves sharing electron density between atoms.
- Ordinary two-centre bonds and electron-deficient three-centre bonds are both covalent. Ionisation of a molecular acid in water does not remove the covalent bonds within its molecule.

Step 1: Identify the ionic compound Potassium is an electropositive metal and chlorine is an electronegative non-metal. Transfer of the outer electron of potassium produces K^+ and Cl^-. The crystal of KCl consists of these ions and is classified as an ionic compound in the usual bonding model. Thus it is the candidate containing no covalent bond.

Step 2: Check every other compound In PH_3, phosphorus shares electron pairs with hydrogen in three P–H bonds. The two oxygen atoms in O_2 are joined by a covalent double bond. Diborane, B_2H_6, has ordinary terminal B–H bonds and electron-deficient bridging B–H–B bonds: the latter are three-centre two-electron covalent bonds, not ionic bonds. Molecular H_2SO_4 contains covalent S–O and O–H bonds, even though it produces ions in aqueous solution.

Step 3: Eliminate the combinations Option (A) wrongly includes diborane. Option (B) includes both diborane and phosphine, each of which has covalent bonds. Option (C) wrongly includes sulfuric acid. Only option (D) contains the ionic compound alone.

Step 4: Conclusion Only KCl contains no covalent bond in the standard ionic model used here. Hence, the correct option is (D).

Quick Tip to Solve: Do not confuse electron-deficient bonding or acid ionisation with the absence of covalent bonds; inspect the bonds inside each species.

79. According to molecular orbital theory, which of the following will not be a viable molecule?

[4 marks]

(A) He_2^{2+}

(B) He_2^{+}

(C) H_2^{-}

(D) H_2^{2-}

Correct Answer: (D) H_2^{2-}

Solution:

Concept:

- For the species formed from 1s atomic orbitals, electrons occupy the bonding σ_{1s} orbital before the antibonding σ_{1s}^{*} orbital.
- The bond order is $\text{BO} = (N_b - N_a)/2$, where N_b and N_a are the numbers of bonding and antibonding electrons. A zero bond order indicates no net stabilising bond in this model.

Step 1: Count the electrons in each ion Two helium nuclei contribute four electrons before ionisation, so $He_2{}^{2+}$ has two electrons and $He_2{}^{+}$ has three. Two hydrogen nuclei initially contribute two electrons, so $H_2{}^{-}$ has three electrons and $H_2{}^{2-}$ has four. It is the total electron count, rather than the sign of the ion, that determines orbital filling.

Step 2: Calculate the bond orders The two-electron species has configuration $(\sigma_{1s})^2$, giving bond order $(2-0)/2 = 1$. Each three-electron species has configuration $(\sigma_{1s})^2(\sigma_{1s}^{*})^1$, giving bond order $(2-1)/2 = 1/2$. For the four-electron species, both orbitals are filled:

$$(\sigma_{1s})^2(\sigma_{1s}^{*})^2, \qquad \text{BO} = \frac{2-2}{2} = 0.$$

Step 3: Choose the non-viable species Options (A), (B) and (C) have positive bond orders of 1, one-half and one-half respectively, so the elementary molecular-orbital criterion allows bonding. Option (D) has equal bonding and antibonding populations, whose effects cancel; it therefore has no net bond by the stated criterion.

Step 4: Conclusion $H_2{}^{2-}$ has bond order zero and is not a viable molecule according to this molecular-orbital model. Hence, the correct option is (D).

Quick Tip to Solve: Count total electrons first, including the charge, then fill the bonding and antibonding 1s orbitals; four electrons give zero bond order.

80. The ratio of mass percent of C and H of an organic compound ($C_XH_YO_Z$) is 6 : 1. If one molecule of the above compound ($C_XH_YO_Z$) contains half as much oxygen as required to burn one molecule of compound C_XH_Y completely to CO_2 and H_2O. The empirical formula of compound $C_XH_YO_Z$ is [4 marks]

(A) $C_2H_4O_3$

(B) $C_3H_6O_3$

(C) C_2H_4O

(D) $C_3H_4O_2$

Correct Answer: (A) $C_2H_4O_3$

Solution:

Concept:

- For $C_xH_yO_z$, the carbon-to-hydrogen mass ratio is $12x : y$, taking atomic masses of carbon and hydrogen as 12 and 1.
- Complete combustion of C_xH_y produces x molecules of CO_2 and $y/2$ molecules of H_2O; their total oxygen requirement is $2x + y/2$ oxygen atoms.

Step 1: Use the mass-percent ratio The common molecular-mass denominator cancels when the two mass percentages are divided. Hence

$$\frac{12x}{y} = 6 \quad \Rightarrow \quad y = 2x.$$

This is an atom-number ratio, not a mass ratio: there are twice as many hydrogen atoms as carbon atoms.

Step 2: Apply the oxygen condition The corresponding hydrocarbon would burn according to

$$C_xH_y + \left(x + \frac{y}{4}\right) O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O.$$

It requires $2x + y/2$ oxygen atoms. The compound contains half that amount of oxygen, so

$$z = \frac{1}{2}\left(2x + \frac{y}{2}\right) = x + \frac{y}{4} = \frac{3x}{2}.$$

The factor of two between oxygen molecules and oxygen atoms must be retained.

Step 3: Reduce and compare the formulae Combining the two conditions gives $x : y : z = 1 : 2 : 3/2 = 2 : 4 : 3$, the smallest whole-number ratio. Option (A) satisfies it. Option (B) has oxygen-to-carbon ratio 1 instead of 3/2; option (C) has ratio 1/2 instead of 3/2. Option (D) already fails the carbon-to-hydrogen mass ratio because $36 : 4 = 9 : 1$.

Step 4: Conclusion The empirical formula is $C_2H_4O_3$. Hence, the correct option is (A).

Quick Tip to Solve: Write the combustion oxygen requirement in oxygen atoms before comparing it with the oxygen subscript in an empirical formula.

81. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a 1 M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1×10^{-10}. What is the original concentration of Ba^{2+}? [4 marks]

(A) 1.0×10^{-10} M

(B) 5×10^{-9} M

(C) 2×10^{-9} M

(D) 1.1×10^{-9} M

Correct Answer: (D) 1.1×10^{-9} M

Solution:

Concept:

- At the onset of precipitation, the ionic product equals the solubility product: $K_{sp} = [Ba^{2+}][SO_4{}^{2-}]$. Before this threshold essentially no barium has been removed as precipitate.
- The added sulfate amount is concentration times volume. Dilution conserves the initial moles of barium, so its original and final concentrations obey $C_iV_i = C_fV_f$.

Step 1: Find sulfate concentration in the mixture The added sodium sulfate supplies one sulfate ion per formula unit. Its amount is

$$n(SO_4{}^{2-}) = 1 \times 0.050 = 0.050 \text{ mol}.$$

In the final volume of 0.500 L, the sulfate concentration is $0.050/0.500 = 0.100 \text{ mol L}^{-1}$. Using the stock concentration of 1 M directly would overlook dilution.

Step 2: Use the precipitation threshold Since the solution has only just reached saturation with respect to barium sulfate,

$$[Ba^{2+}]_f = \frac{K_{sp}}{[SO_4{}^{2-}]} = \frac{1.0 \times 10^{-10}}{0.100} = 1.0 \times 10^{-9} \text{ mol L}^{-1}.$$

This is the barium concentration after adding the sulfate solution, rather than the original concentration asked for.

Step 3: Undo the dilution and select the option The initial volume was $500 - 50 = 450$ mL. Conservation of dissolved barium at the threshold gives

$$C_i = (1.0 \times 10^{-9})\frac{500}{450} = 1.11 \times 10^{-9} \text{ mol L}^{-1}.$$

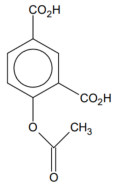
Option (D) is this value rounded to the precision of the choices. Options (A), (B) and (C) do not satisfy both the saturation equation and the 450-to-500 mL dilution.

Step 4: Conclusion The original barium-ion concentration is approximately 1.1×10^{-9} M. Hence, the correct option is (D).

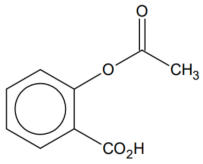
Quick Tip to Solve: At first precipitation use the final-volume sulfate concentration in the solubility product, then undo dilution to recover the original metal-ion concentration.

82. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces: [4 marks]

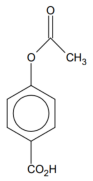
(A) 4-Acetoxybenzene-1,3-dicarboxylic acid



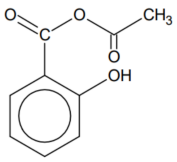
(B) 2-Acetoxybenzoic acid



(C) 4-Acetoxybenzoic acid

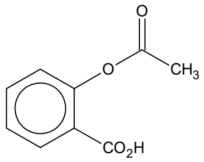


(D) Mixed anhydride of 2-hydroxybenzoic acid and acetic acid



Correct Answer: (B)

2-Acetoxybenzoic acid



Solution:

Concept:

- In the Kolbe–Schmitt reaction, sodium phenoxide reacts with CO_2 to introduce a carboxyl group predominantly ortho to the phenoxide oxygen; acidification gives salicylic acid.
- Acetic anhydride acetylates the phenolic oxygen of salicylic acid in acid catalysis, converting $-OH$ into $-OCOCH_3$ while retaining the carboxylic acid group.

Step 1: Identify the intermediate X First $NaOH$ deprotonates phenol to sodium phenoxide. Carboxylation with carbon dioxide forms sodium salicylate as the major ortho product; acidification restores the carboxylic acid. Thus X is 2-hydroxybenzoic acid, with $-OH$ and $-CO_2H$ on adjacent ring carbons. The sequence introduces one carboxyl group, not two.

Step 2: Acetylate the phenolic group The phenolic oxygen attacks activated acetic anhydride

and receives an acetyl group. Its O–H hydrogen is replaced by $COCH_3$, giving the ester group $-O-C(=O)-CH_3$. The overall acetylation is

$$C_7H_6O_3 + (CH_3CO)_2O \longrightarrow C_9H_8O_4 + CH_3COOH.$$

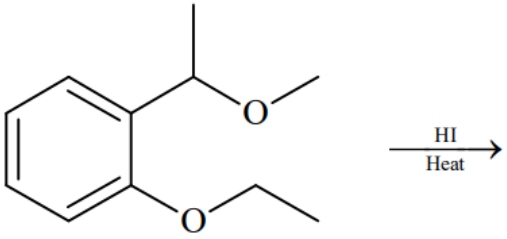
The product is acetylsalicylic acid, commonly called aspirin; the original ortho relation of the two ring substituents is preserved.

Step 3: Compare the depicted structures Option (B) shows the required adjacent $-CO_2H$ and $-OCOCH_3$ groups. Option (A) contains an additional carboxyl group not produced by this sequence. Option (C) puts the groups para rather than ortho. Option (D) depicts a mixed anhydride at the carboxyl group and leaves the phenolic OH free, instead of the phenolic acetate formed here.

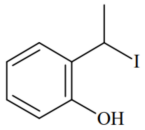
Step 4: Conclusion The product is 2-acetoxybenzoic acid (acetylsalicylic acid, aspirin), the structure shown in (B). Hence, the correct option is (B).

Quick Tip to Solve: Track the two operations separately: Kolbe carboxylation sets the ortho position, then acetic anhydride acetylates the phenolic oxygen.

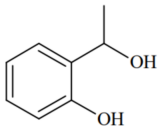
83. The major product formed in the following reaction is : [4 marks]



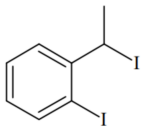
(A) 2-HO$-C_6H_4-$CH(I)CH_3



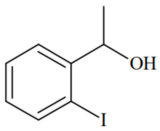
(B) 2-HO$-C_6H_4-$CH(OH)CH_3



(C) 2-I$-C_6H_4-$CH(I)CH_3

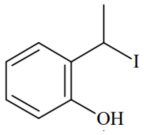


(D) 2-I$-C_6H_4-$CH(OH)CH_3



Correct Answer: (A)

2-HO$-C_6H_4-$CH(I)CH_3



Solution:

Concept:

- Hot hydrogen iodide protonates ether oxygen, allowing cleavage of a carbon–oxygen bond. A secondary benzylic centre can form a resonance-stabilised carbocation and then react with iodide.
- In an aryl–alkyl ether, the aryl carbon–oxygen bond resists ordinary substitution; cleavage occurs on the alkyl side, leaving the oxygen attached to the ring as phenolic OH.

Step 1: Examine the benzylic methyl ether The side chain attached directly to the ring is $-CH(CH_3)-O-CH_3$. After protonation of its oxygen, cleavage at the secondary benzylic carbon is favoured by resonance stabilisation of the resulting benzylic cation. Capture by I^- gives the side chain $-CH(I)CH_3$. Under hot HI any corresponding benzylic alcohol also readily undergoes conversion to the benzylic iodide; an alcohol is therefore not the final major side chain.

Step 2: Cleave the ethoxy substituent The neighbouring ring substituent is $-O-CH_2CH_3$. Protonation followed by iodide attack at the primary ethyl carbon cleaves the O–ethyl bond, yielding ethyl iodide and the phenolic group $Ar-OH$. The aryl C–O bond is retained: an aryl carbon does not undergo the ordinary backside substitution that would replace the oxygen by iodine.

Step 3: Match both transformations to the choices The ring therefore bears adjacent $-OH$ and $-CH(I)CH_3$ groups, exactly as in option (A). Option (B) leaves the benzylic group as an alcohol despite hot HI. Option (C) replaces the aryl oxygen with iodine, which ether cleavage does not do. Option (D) makes both errors: aryl iodide and an unconverted benzylic alcohol.

Step 4: Conclusion The major organic product shown is ortho-$HO-C_6H_4-CH(I)CH_3$, represented in (A). Hence, the correct option is (A).

Quick Tip to Solve: For hot-HI ether cleavage, distinguish a resonance-stabilised benzylic carbon from an aryl carbon: the former can become an iodide, while aryl oxygen stays as OH.

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84. The trans-alkenes are formed by the reduction of alkynes with [4 marks]

(A) Sn - HCl

(B) H_2 – Pd/C, $BaSO_4$

(C) $NaBH_4$

(D) Na/liq. NH_3

Correct Answer: (D) Na/liq. NH_3

Solution:

Concept:

- Dissolving-metal reduction of an internal alkyne with sodium in liquid ammonia proceeds through electron transfer and protonation steps, giving predominantly an anti addition product.
- Anti addition of two hydrogen atoms across the triple bond produces a trans alkene, whereas catalytic partial hydrogenation using a poisoned palladium catalyst gives a cis alkene.

Step 1: Identify the reagent with anti selectivity Sodium supplies an electron to the alkyne, producing

a radical anion. Proton transfer from ammonia, followed by another electron transfer and another proton transfer, converts the triple bond into a double bond. The favoured arrangement of the intermediates places the developing substituents apart, so the net addition of the two hydrogens is anti.

Step 2: Relate anti addition to trans geometry For an internal alkyne $\mathrm{R{-}C{\equiv}C{-}R\prime}$, the two added hydrogen atoms end up on opposite sides of the double bond. Consequently the original carbon substituents are trans. Sodium in liquid ammonia is the standard dissolving-metal reagent for this stereoselective reduction; it stops at the alkene in the usual reaction conditions.

Step 3: Rule out the other reagents Option (A), tin and hydrochloric acid, is commonly used for reduction of nitro groups and is not the standard reagent for selective conversion of alkynes into trans alkenes. The palladium-based partial hydrogenation reagent in (B) delivers hydrogens from the catalyst surface by syn addition, favouring cis alkene rather than trans; ordinary unpoisoned Pd can reduce further. Sodium borohydride in (C) reduces polar carbonyl groups and does not ordinarily perform this reduction of an isolated carbon–carbon triple bond. Thus only (D) has the required combination of reactivity and stereoselectivity.

Step 4: Conclusion Trans alkenes are obtained by dissolving-metal reduction with Na in liquid $\mathrm{NH_3}$. Hence, the correct option is (D).

Quick Tip to Solve: Remember the stereochemical pair: poisoned palladium gives cis by syn addition; sodium in liquid ammonia gives trans by anti addition.

85. An aqueous solution contains 0.10 M H_2S and 0.20 M HCl. If the equilibrium constants for the formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is : [4 marks]

(A) 5×10^{-19}

(B) 5×10^{-8}

(C) 3×10^{-20}

(D) 6×10^{-21}

Correct Answer: (C) 3×10^{-20}

Solution:

Concept:

- The strong acid HCl supplies approximately $0.20\,\mathrm{mol\,L^{-1}}$ hydrogen ions; this common ion suppresses both ionisations of $\mathrm{H_2S}$.
- For successive dissociations, $K_{a1} = [\mathrm{H^+}][\mathrm{HS^-}]/[\mathrm{H_2S}]$ and $K_{a2} = [\mathrm{H^+}][\mathrm{S^{2-}}]/[\mathrm{HS^-}]$. Concentrations are equilibrium molar concentrations.

Step 1: Obtain the intermediate hydrogensulfide concentration The first dissociation is $\mathrm{H_2S} \rightleftharpoons \mathrm{H^+ + HS^-}$. Since the hydrogen-ion concentration is fixed overwhelmingly by the strong acid, the

undissociated weak-acid concentration remains nearly $0.10\,\mathrm{M}$. Hence

$$[\mathrm{HS^-}] = \frac{K_{a1}[\mathrm{H_2S}]}{[\mathrm{H^+}]} = \frac{(1.0\times10^{-7})(0.10)}{0.20} = 5.0\times10^{-8}\,\mathrm{M}.$$

This is negligible beside $0.20\,\mathrm{M}$, validating the common-ion approximation.

Step 2: Apply the second equilibrium and compare the options For $\mathrm{HS^-} \rightleftharpoons \mathrm{H^+} + \mathrm{S^{2-}}$, divide by the hydrogen-ion concentration once more:

$$[\mathrm{S^{2-}}] = \frac{(1.2\times10^{-13})(5.0\times10^{-8})}{0.20} = 3.0\times10^{-20}\,\mathrm{M}.$$

Option (B) is the intermediate $\mathrm{HS^-}$ concentration, not the sulfide concentration. Options (A) and (D) do not satisfy both equilibrium expressions with the stated hydrogen-ion concentration.

Step 3: Conclusion The sulfide-ion concentration is $3.0\times10^{-20}\,\mathrm{mol\,L^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Multiply the two acid constants first: $[\mathrm{S^{2-}}] = K_{a1}K_{a2}[\mathrm{H_2S}]/[\mathrm{H^+}]^2$. The strong-acid concentration enters squared.

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86. The combustion of benzene(l) gives CO_2(g) and H_2O(l). Given that heat of combustion of benzene at constant volume is –3263.9 kJ mol^{-1} at 25°C; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be :

(R = 8.314 JK^{-1} mol^{-1}) [4 marks]

(A) –3267.6

(B) 4152.6

(C) –452.46

(D) 3260

Correct Answer: (A) –3267.6

Solution:

Concept:

- At constant volume the reaction heat is $q_V = \Delta U$, whereas at constant pressure it is $q_P = \Delta H$. Both are negative for this exothermic combustion.
- For ideal gaseous reactants and products, $\Delta H = \Delta U + \Delta n_g RT$, where Δn_g counts only gaseous stoichiometric amounts per mole of benzene burned.

Step 1: Balance combustion and count only the gases The combustion equation per mole of liquid benzene is

$$\mathrm{C_6H_6(l)} + \frac{15}{2}\,\mathrm{O_2(g)} \longrightarrow 6\,\mathrm{CO_2(g)} + 3\,\mathrm{H_2O\,(l)}.$$

Water is explicitly liquid, so it contributes nothing to the gaseous mole count. Benzene is also liquid.

Therefore

$$\Delta n_g = 6 - \frac{15}{2} = -\frac{3}{2}.$$

Counting the three water molecules as gas would reverse this correction incorrectly.

Step 2: Calculate the pressure correction in consistent units Use $T = 25 + 273 = 298\,\text{K}$ and convert joules to kilojoules:

$$\Delta n_g RT = -\frac{3}{2}\frac{8.314 \times 298}{1000} = -3.716\,\text{kJ mol}^{-1}.$$

Thus

$$\Delta H = -3263.9 - 3.716 = -3267.616\,\text{kJ mol}^{-1}.$$

The change must be slightly more negative than ΔU, since fewer gas moles remain. Positive options (B) and (D) contradict exothermic combustion; option (C) has the wrong magnitude and cannot result from this small gas correction.

Step 3: Conclusion The heat of combustion at constant pressure is approximately $-3267.6\,\text{kJ mol}^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: Before using $\Delta H - \Delta U = \Delta n_g RT$, circle the phase labels. Liquid water contributes zero to Δn_g.

87. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is [4 marks]

(A) Fe

(B) Zn

(C) Ca

(D) Al

Correct Answer: (D) Al

Solution:

Concept:

- Aluminium hydroxide, Al(OH)_3, is a white gelatinous precipitate and is amphoteric: it dissolves in excess aqueous alkali to form soluble aluminate.
- Strong heating dehydrates Al(OH)_3 to alumina, Al_2O_3. Activated alumina has a large adsorbing surface and is used as a stationary-phase adsorbent in chromatography.

Step 1: Identify the hydroxide from its behaviour in alkali The analytical clues identify the precipitated compound X as aluminium hydroxide. Its precipitation from aluminium ions and dissolution in excess hydroxide are represented by

$$\text{Al}^{3+} + 3\,\text{OH}^- \longrightarrow \text{Al(OH)}_3(\text{s})$$

$$\text{Al(OH)}_3(\text{s}) + \text{OH}^- \longrightarrow [\text{Al(OH)}_4]^-\,(\text{aq}).$$

The first reaction explains the white gelatinous solid; the second accounts for its amphoteric nature

and solubility in excess NaOH. The identity of the metal must also satisfy the oxide clue.

Step 2: Use the chromatography clue to distinguish aluminium from zinc On strong heating the hydroxide loses water:

$$2\,\mathrm{Al(OH)_3} \xrightarrow{\Delta} \mathrm{Al_2O_3} + 3\,\mathrm{H_2O}.$$

The resulting alumina is the familiar chromatographic adsorbent. Zinc hydroxide is also white and amphoteric, but its heating product is ZnO, rather than the alumina identified here. Iron hydroxides give coloured precipitates and do not show this excess-alkali dissolution; calcium hydroxide does not give the amphoteric-hydroxide sequence. The combined observations therefore select aluminium rather than relying on colour alone.

Step 3: Conclusion The metal is aluminium, Al, and X is $\mathrm{Al(OH)_3}$, whose thermal decomposition gives alumina. Hence, the correct option is (D).

Quick Tip to Solve: White hydroxide plus excess-alkali solubility narrows the choices; the oxide used in chromatography singles out alumina and aluminium.

88. Which type of 'defect' has the presence of cations in the interstitial sites? [4 marks]

(A) Metal deficiency defect

(B) Schottky defect

(C) Vacancy defect

(D) Frenkel defect

Correct Answer: (D) Frenkel defect

Solution:

Concept:

- A Frenkel defect occurs when a small ion, usually a cation, leaves its regular lattice site and occupies an interstitial site within the same crystal.
- The displaced cation creates a vacancy–interstitial pair. No ion is lost from the crystal, so its composition, electrical neutrality and mass remain unchanged.

Step 1: Relate the interstitial cation to its original lattice site In an ionic solid the normal lattice sites are the ordered positions assigned to cations and anions. An interstitial site is a normally unoccupied space between these positions. If a cation moves into such a space, its original site becomes vacant. The linked pair of a vacant normal cation site and an interstitial cation is the characteristic Frenkel defect. This displacement is favoured when the cation is small enough to fit into the available interstitial space; familiar examples include AgCl, AgBr and ZnS.

Step 2: Eliminate the vacancy-based alternatives A Schottky defect consists of missing cations and anions in the stoichiometric proportion, with no required interstitial cation; it lowers density because ions are absent. A simple vacancy defect similarly describes an unoccupied normal site, not the relocated ion specified here. Metal deficiency defects involve too few metal ions, often compensated by higher oxidation states of some remaining cations. These features distinguish options (A), (B) and (C) from the cation displacement into an interstitial site. In a Frenkel defect the cation stays inside

the crystal, so there is no loss of mass.

Step 3: Conclusion The defect associated with cations displaced into interstitial sites is the Frenkel defect. Hence, the correct option is (D).

Quick Tip to Solve: Pair the two sites mentally: an ion missing from its normal position but present between sites indicates Frenkel displacement; ions missing from the crystal indicate a vacancy defect.

89. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively : [4 marks]

(A) H_2O and $(H_2O + OH^-)$

(B) $(H_2O + O_2)$ and H_2O

(C) $(H_2O + O_2)$ and $(H_2O + OH^-)$

(D) H_2O and $(H_2O + O_2)$

Correct Answer: (D) H_2O and $(H_2O + O_2)$

Solution:

Concept:

- In H_2O_2, oxygen has oxidation state -1, so peroxide can be reduced to water or oxidised to oxygen depending on its reaction partner and medium.
- Oxidation of $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ releases one electron per complex; the reverse change consumes one electron per complex.

Step 1: Identify the peroxide product in acidic medium Peroxide acts as the oxidising agent when it oxidises ferrocyanide to ferricyanide. It therefore accepts the electrons released by the iron complexes:

$$H_2O_2 + 2\,H^+ + 2\,e^- \longrightarrow 2\,H_2O.$$

Combining this with oxidation of two ferrocyanide ions gives

$$2\,[Fe(CN)_6]^{4-} + H_2O_2 + 2\,H^+ \longrightarrow 2\,[Fe(CN)_6]^{3-} + 2\,H_2O.$$

Thus water is the other product in acidic medium; oxygen is not evolved in this oxidation reaction.

Step 2: Balance the reducing action in alkaline medium Here ferricyanide accepts electrons, while peroxide is oxidised:

$$H_2O_2 + 2\,OH^- \longrightarrow O_2 + 2\,H_2O + 2\,e^-.$$

Adding the complex-ion reduction gives

$$2\,[Fe(CN)_6]^{3-} + H_2O_2 + 2\,OH^- \longrightarrow 2\,[Fe(CN)_6]^{4-} + O_2 + 2\,H_2O.$$

Water and oxygen are the other products in alkaline medium; hydroxide is consumed. Option (A)

omits oxygen in this medium, while (B) and (C) incorrectly include oxygen among the acidic-medium products.

Step 3: Conclusion The products are respectively H_2O in acidic medium and $H_2O + O_2$ in alkaline medium. Hence, the correct option is (D).

Quick Tip to Solve: When peroxide oxidises another species it becomes water; when it reduces another species it releases oxygen. Balance hydrogen and charge in the stated medium.

90. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting

$[3Ca_3(PO_4)_2.Ca(OH)_2]$ to [4 marks]

(A) $[3\{Ca(OH)_2\}.\ CaF_2]$

(B) $[CaF_2]$

(C) $[3(CaF_2).\ Ca(OH)_2]$

(D) $[3Ca_3(PO_4)_2.\ CaF_2]$

Correct Answer: (D) $[3Ca_3(PO_4)_2.\ CaF_2]$

Solution:

Concept:

- Tooth enamel contains hydroxyapatite, commonly represented as $Ca_{10}(PO_4)_6(OH)_2$, equivalently $3\,Ca_3(PO_4)_2 \cdot Ca(OH)_2$.
- Fluoride ions replace hydroxide ions in the apatite structure, forming fluorapatite, $Ca_{10}(PO_4)_6F_2$. Its greater resistance to acid attack helps protect enamel.

Step 1: Preserve the phosphate framework during substitution The starting formula contains three calcium-phosphate units together with a calcium-hydroxide unit. Fluoride treatment replaces the two hydroxide ions by two fluoride ions; it does not remove the phosphate groups or replace every calcium environment by calcium fluoride. In condensed apatite notation the substitution is

$$Ca_{10}(PO_4)_6(OH)_2 + 2\,F^- \longrightarrow Ca_{10}(PO_4)_6F_2 + 2\,OH^-.$$

Both substituting ions are monovalent, so this exchange preserves charge balance while retaining the calcium-phosphate framework.

Step 2: Express the product in the option notation Rewrite the resulting fluorapatite in the same component notation used in the question:

$$Ca_{10}(PO_4)_6F_2 \equiv 3\,Ca_3(PO_4)_2 \cdot CaF_2.$$

This contains ten calcium atoms, six phosphate groups and two fluoride ions, exactly as the substituted apatite requires. Option (B), pure CaF_2, loses the phosphate framework. Options (A) and (C) also omit the phosphate groups and retain hydroxide, so neither represents fluorapatite. The question's low recommended fluoride concentration refers to beneficial controlled incorporation; excessive fluoride

is harmful and is not implied by this conversion.

Step 3: Conclusion The enamel mineral is converted to fluorapatite, represented as $3\,Ca_3(PO_4)_2 \cdot CaF_2$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the three calcium-phosphate units unchanged and replace only the terminal $Ca(OH)_2$ component with CaF_2.