JEE Main 2018 Paper 1 (B.E./B.Tech) Apr 15 Slot 1 (morning) Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2018, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 15 April 2018, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. The value of the integral

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \left(1 + \log\left(\frac{2+\sin x}{2-\sin x}\right)\right) dx$ **is :** [4 marks]

(A) 0

(B) $\frac{3}{4}$

(C) $\frac{3}{8}\,\pi$

(D) $\frac{3}{16}\,\pi$

Correct Answer: (C) $\frac{3}{8}\,\pi$

Solution:

Concept:

- On a symmetric interval, the integral of an odd continuous function is zero: $\int_{-a}^{a} g(x)\,dx = 0$ when $g(-x) = -g(x)$.
- The power-reduction identity $\sin^4 x = (3 - 4\cos 2x + \cos 4x)/8$ reduces an even fourth power to elementary cosine terms.

Step 1: Separate the even and odd contributions Put $L(x) = \log((2 + \sin x)/(2 - \sin x))$. Both numerator and denominator are positive throughout the interval, so the logarithm is defined and continuous. Since replacing x by $-x$ interchanges them, $L(-x) = -L(x)$. The factor $\sin^4 x$ is even, making $\sin^4 x L(x)$ odd. Therefore its contribution vanishes and

$$I = \int_{-\pi/2}^{\pi/2} \sin^4 x\,dx.$$

Step 2: Evaluate the remaining integral and compare options Apply power reduction before integrating:

$$I = \left[\frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32}\right]_{-\pi/2}^{\pi/2} = \frac{3\pi}{8}.$$

The sine boundary terms vanish at both endpoints. The answer cannot be zero because the remaining integrand is nonnegative and positive on an interval. Option (B) omits the factor of π, and option (D) is the value on only half the symmetric interval, rather than on the whole interval specified.

Step 3: Conclusion The integral equals $\boxed{3\pi/8}$. Hence, the correct option is (C).

Quick Tip to Solve: For a logarithm involving $(a + \sin x)/(a - \sin x)$ on symmetric limits, test oddness before expanding or integrating.

2. If tanA and tanB are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$, then the value of $3\sin^2(A + B) - 10\sin(A + B).\cos(A + B) - 25\cos^2(A + B)$ is : [4 marks]

(A) -10

(B) 10

(C) -25

(D) 25

Correct Answer: (C) -25

Solution:

Concept:

- For roots u, v of $ax^2 + bx + c = 0$, Vieta's relations give $u + v = -b/a$ and $uv = c/a$.
- The addition formula gives $\tan(A + B) = (\tan A + \tan B)/(1 - \tan A \tan B)$. Also $\sin^2\theta = \tan^2\theta/(1 + \tan^2\theta)$.

Step 1: Find the tangent of the sum Write $u = \tan A$, $v = \tan B$. From the quadratic,

$$u + v = \frac{10}{3}, \qquad uv = -\frac{25}{3}.$$

Thus $1 - uv = 28/3 \neq 0$, and

$$t = \tan(A + B) = \frac{10/3}{28/3} = \frac{5}{14}.$$

The denominator being nonzero also ensures $\cos(A + B) \neq 0$, so division by its square is legitimate. The individual angles need not be found; their possible changes by integer multiples of π do not affect the requested expression.

Step 2: Rewrite the expression in terms of the tangent For $\theta = A + B$, factor out $\cos^2\theta$. Using $\cos^2\theta = 1/(1 + t^2)$, the desired value is

$$\begin{aligned} E &= \frac{3t^2 - 10t - 25}{1 + t^2} \\ &= \frac{75 - 700 - 4900}{196 + 25} = \frac{-5525}{221} = -25. \end{aligned}$$

The positive options have the wrong sign, while -10 does not equal the explicitly evaluated quotient. The root sum alone is not the tangent of the sum: the factor $1 - uv$ is essential.

Step 3: Conclusion The required value is $\boxed{-25}$. Hence, the correct option is (C).

Quick Tip to Solve: Use Vieta and the tangent-addition formula together; then divide the quadratic trigonometric expression by $\cos^2\theta$.

3. In a triangle ABC, coordinates of A are (1, 2) and the equations of the medians through B and C are respectively, x + y = 5 and x = 4. Then area of △ ABC (in sq. units) is : [4 marks]

(A) 12

(B) 4

(C) 5

(D) 9

Correct Answer: (D) 9

Solution:

Concept:

- The three medians meet at the centroid G, whose coordinates are the arithmetic means of the three vertices.
- For vertices A, B, C, the area is half the absolute determinant of $B - A$ and $C - A$; the absolute value removes orientation.

Step 1: Locate the centroid and the remaining vertices The given medians intersect at $x = 4$, $x+y = 5$, hence $G = (4, 1)$. If $B = (b_1, b_2)$ and $C = (c_1, c_2)$, the centroid relation with $A = (1, 2)$ gives

$$b_1 + c_1 = 11, \qquad b_2 + c_2 = 1.$$

The median through C is $x = 4$, so $c_1 = 4$ and consequently $b_1 = 7$. Since B lies on its own median $x + y = 5$, $b_2 = -2$; therefore $c_2 = 3$. Thus $B = (7, -2)$ and $C = (4, 3)$. Each condition has now been used with the vertex to which its median belongs.

Step 2: Compute and check the area Subtract A to obtain $B - A = (6, -4)$ and $C - A = (3, 1)$. Their determinant gives

$$[ABC] = \frac{1}{2}\,|6(1) - (-4)(3)| = \frac{1}{2}(18) = 9.$$

As a consistency check, the midpoint of AC is $(5/2, 5/2)$, which satisfies $x + y = 5$. The midpoint of AB is $(4, 0)$, which satisfies $x = 4$. Therefore both reconstructed medians have the required equations. The values 12, 4 and 5 do not agree with the determinant and so are excluded.

Step 3: Conclusion The area is $\boxed{9 \text{ square units}}$. Hence, the correct option is (D).

Quick Tip to Solve: Intersect two medians to find the centroid first, then use the vertex average to recover the unknown coordinates.

4. If a right circular cone, having maximum volume, is inscribed in a sphere of radius 3 cm, then the curved surface area (in cm^2) of this cone is : [4 marks]

(A) $6\sqrt{2}\pi$

(B) $6\sqrt{3}\pi$

(C) $8\sqrt{2}\pi$

(D) $8\sqrt{3}\pi$

Correct Answer: (D) $8\sqrt{3}\pi$

Solution:

Concept:

- For a cone of height h inscribed in a sphere of radius R, its base radius satisfies $r^2 = R^2-(R-h)^2 = 2Rh - h^2$.
- The cone volume is $V = \pi r^2 h/3$; its curved surface area is $\pi r\ell$, where $\ell = \sqrt{r^2+h^2}$ is the slant height.

Step 1: Maximize the volume with respect to height Take the vertex at the top of the sphere and measure h down the axis to the circular base. With $R = 3$, the right triangle from the sphere centre to the base rim gives $r^2 = 6h - h^2$, where $0 \le h \le 6$. Therefore

$$V(h) = \frac{\pi}{3}(6h^2 - h^3), \qquad V'(h) = \pi h(4-h).$$

The only interior critical point is $h = 4$. The derivative is positive for $0 < h < 4$ and negative for $4 < h < 6$. Also the endpoint volumes are zero. Thus $h = 4$ gives the maximum-volume cone, rather than merely a stationary value.

Step 2: Find its curved surface area At this height,

$$r^2 = 6(4) - 4^2 = 8, \quad r = 2\sqrt{2}, \qquad \ell = \sqrt{8+16} = 2\sqrt{6}.$$

Consequently the curved surface area, which excludes the circular base, is

$$\pi r\ell = \pi(2\sqrt{2})(2\sqrt{6}) = 8\sqrt{3}\pi \text{ cm}^2.$$

This calculation uses the optimized height, not the sphere diameter or radius as the cone height. The other three offered values differ from this product; adding the base area would instead produce the total surface area, which was not requested.

Step 3: Conclusion The curved surface area is $\boxed{8\sqrt{3}\pi \text{ cm}^2}$. Hence, the correct option is (D).

Quick Tip to Solve: Optimize volume using $r^2 = 2Rh - h^2$ first; calculate the slant height only after finding the optimal height.

5. Let S = {(λ, μ) $\in$ R $\times$ R : f(t) = ($|\lambda|$ $e^{|t|} - \mu$). sin (2|t|), t $\in$ R, is a differentiable function}. Then S is a subset of : [4 marks]

(A) $\mathbf{R} \times [0, \infty)$

(B) $[0, \infty) \times \mathbf{R}$

(C) $\mathbf{R} \times (-\infty, 0)$

(D) $(-\infty, 0) \times \mathbf{R}$

Correct Answer: (A) $\mathbf{R} \times [0, \infty)$

Solution:

Concept:

- A composition involving $|t|$ is smooth away from $t = 0$; differentiability on the whole real line therefore requires checking the two one-sided derivatives at zero.
- Here $f(0) = 0$, and the standard limits $e^{|t|} \to 1$ and $\sin(2|t|)/|t| \to 2$ determine the difference quotient.

Step 1: Compare the one-sided derivatives at zero For $t > 0$, divide $f(t) - f(0)$ by t and let $t \to 0^+$. This gives

$$f'_+(0) = 2(|\lambda| - \mu).$$

For $t < 0$, $|t| = -t$, so the quotient acquires a minus sign and

$$f'_-(0) = -2(|\lambda| - \mu).$$

Equality is necessary and sufficient at the joining point, hence $\mu = |\lambda|$. In that case both derivatives are zero. There are no other exceptional points because the formulas on each open half-line are compositions and products of smooth elementary functions.

Step 2: Determine which Cartesian region contains the set The exact parameter set is therefore

$$S = \{(\lambda, |\lambda|) : \lambda \in \mathbb{R}\}.$$

Its second coordinate is always nonnegative, so $S \subset \mathbb{R} \times [0, \infty)$. The first coordinate can be negative, zero or positive: $(-1, 1) \in S$ rules out option (B), and $(1, 1) \in S$ rules out option (D). Option (C) requires the second coordinate to be strictly negative, whereas every point of S has nonnegative second coordinate. In particular $(0, 0)$ is allowed.

Step 3: Conclusion The set satisfies $\boxed{S \subset \mathbb{R} \times [0, \infty)}$. Hence, the correct option is (A).

Quick Tip to Solve: For $g(|t|)$, compare the opposite one-sided slopes at zero; smoothness away from zero does not settle differentiability there.

6. If $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$, then $\lim\limits_{x\to 0} \frac{f'(x)}{x}$ [4 marks]

(A) does not exist.

(B) exists and is equal to 2.

(C) existsand is equal to 0.

(D) exists and is equal to -2.

Correct Answer: (D) exists and is equal to -2.

Solution:

Concept:

- Subtracting one row from another preserves a determinant and can expose a simple factor when two rows share entries.
- After simplifying the determinant, ordinary differentiation and continuity of the trigonometric functions near zero evaluate the limit; $\tan x$ is defined in a neighbourhood of zero.

Step 1: Simplify the determinant before differentiating Apply $R_1 \leftarrow R_1 - R_3$. The first row becomes $(\cos x - \tan x, 0, 0)$, so expansion along it yields

$$f(x) = (\cos x - \tan x)\begin{vmatrix} x^2 & 2x \\ x & 1 \end{vmatrix}$$
$$= (\cos x - \tan x)(x^2 - 2x^2) = -x^2(\cos x - \tan x).$$

The cofactor has positive sign at the first row and first column. Keeping the negative sign from the small determinant is crucial; dropping it would give the opposite answer.

Step 2: Differentiate the product and take the limit Differentiate the simplified expression and divide by $x \neq 0$:

$$\frac{f'(x)}{x} = -2(\cos x - \tan x) + x(\sin x + \sec^2 x).$$

As $x \to 0$, the first parenthesis tends to 1 and the second parenthesis remains finite, tending to 1. Hence the term multiplied by x tends to zero, and the whole expression tends to -2. This proves that both one-sided limits agree, excluding nonexistence. It also excludes 0 and 2: the leading term of $f(x)$ is $-x^2$, whose derivative divided by x approaches -2.

Step 3: Conclusion The limit exists and equals $\boxed{-2}$. Hence, the correct option is (D).

Quick Tip to Solve: When determinant rows have identical trailing entries, subtract them before differentiating; this often reduces the determinant to a single cofactor.

7. Let S be the set of all real values of k for which the systemof linear equations

x + y + z = 2

2x + y − z = 3

3x + 2y + kz = 4

has a unique solution. Then S is : [4 marks]

(A) an empty set

(B) equal to {0}

(C) equal to **R**

(D) equal to **R** − {0}

Correct Answer: (D) equal to **R** − {0}

Solution:

Concept:

- A square system $M\mathbf{x} = \mathbf{b}$ has a unique solution exactly when the coefficient matrix has nonzero determinant.
- Elementary row operations simplify the determinant without using the right-hand side; when a determinant vanishes, consistency must be checked separately, but uniqueness is already excluded.

Step 1: Reduce the coefficient determinant The coefficient matrix is

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{pmatrix}.$$

Replace its third row by the third row minus the first and second rows. Since the first two entries cancel and the corresponding last entries add to zero, the new third row is $(0, 0, k)$. Expansion along that row gives

$$\det M = k \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -k.$$

Thus every nonzero real k gives an invertible coefficient matrix, independently of the constants on the right side.

Step 2: Check the excluded parameter and identify the set For $k = 0$, the coefficient row of the third equation is the sum of those of the first two. The first two equations would imply

$$3x + 2y = 2 + 3 = 5,$$

whereas the third equation demands $3x + 2y = 4$. The system is inconsistent at $k = 0$, so it certainly has no unique solution there. Consequently all and only the nonzero real values belong to S. The set is nonempty, is not the singleton $\{0\}$, and does not include all real numbers; these checks rule out options (A), (B) and (C), respectively.

Step 3: Conclusion The required set is $\boxed{\mathbb{R} \setminus \{0\}}$. Hence, the correct option is (D).

Quick Tip to Solve: For parameter-dependent linear systems, test the coefficient determinant first; only its zero values need separate consistency checks.

8. If $f\left(\frac{x-4}{x+2}\right) = 2x + 1$, (x ∈ R −{1, − 2}), then $\int f(x)\,dx$ is equal to :
(where C is a constant of integration) [4 marks]

(A) 12 $\log_e$ | 1 − x | + 3x + C
(B) − 12 $\log_e$ | 1 − x | − 3x + C
(C) 12 $\log_e$ | 1 − x | − 3x + C
(D) − 12 $\log_e$ | 1 − x | + 3x + C
Correct Answer: (B) − 12 $\log_e$ | 1 − x | − 3x + C

Solution:

Concept:

- To recover a function from a composite argument, introduce a new variable and solve the argument equation for the original variable.
- The logarithmic integral is $\int (1-t)^{-1}\,dt = -\ln|1-t| + C$, on any interval avoiding $t = 1$. The negative sign comes from differentiating $1 - t$.

Step 1: Invert the given argument Write $t = (x-4)/(x+2)$. Multiplying through and collecting the terms containing x gives

$$tx + 2t = x - 4, \qquad x(1-t) = 4 + 2t.$$

Therefore

$$x = \frac{4+2t}{1-t}.$$

The value $t = 1$ cannot occur in the given fractional transformation. The other stated exclusion of the original variable removes an additional isolated argument; the following formula and primitive hold on each interval of its domain.

Step 2: Express the function and integrate Substitute the inverse transformation into the given right-hand side:

$$f(t) = 2\frac{4+2t}{1-t} + 1 = \frac{9+3t}{1-t} = \frac{12}{1-t} - 3.$$

Relabel the independent variable as x and integrate term by term:

$$\int f(x)\,dx = -12\ln|1-x| - 3x + C.$$

As a direct check, differentiating the logarithm gives $12/(1-x)$, and differentiating the linear term gives -3. Option A has the wrong signs for both contributions; C has the wrong logarithmic sign; D has the wrong linear sign.

Step 3: Conclusion $\int f(x)\,dx = -12\ln|1-x| - 3x + C$, valid on intervals in the domain. Hence, the correct option is (B).

Quick Tip to Solve: Invert the fractional argument before integrating; then differentiate the proposed answer to check both signs.

9. If b is the first term of an infinite G.P. whose sum is five, then b lies in the interval : [4 marks]

(A) $(-\infty, -10]$

(B) $(-10, 0)$

(C) $(0, 10)$

(D) $[10, \infty)$

Correct Answer: (C) $(0, 10)$

Solution:

Concept:

- For a real infinite geometric progression with first term b and common ratio r, convergence requires $|r| < 1$.
- Its sum is $S = b/(1-r)$. Here $S = 5$, so the admissible first terms are obtained from $b = 5(1-r)$.

Step 1: Use the convergence condition The first term cannot be zero, since that would give a zero sum. For a nonzero first term, an infinite G.P. has a finite sum precisely when $-1 < r < 1$. In particular, neither endpoint is allowed: at $r = 1$ the partial sums do not approach a finite limit, while at $r = -1$ nonzero partial sums oscillate. The fact that the sum is positive does not require a positive ratio; a convergent alternating progression is possible.

Step 2: Find the range of the first term Using the given sum,

$$\frac{b}{1-r} = 5 \quad \Longrightarrow \quad b = 5(1-r).$$

Now $-1 < r < 1$ implies

$$0 < 1 - r < 2 \quad \Longrightarrow \quad 0 < b < 10.$$

Conversely, every b in this open interval gives $r = 1 - b/5$ in $(-1, 1)$, so the whole interval is attained. For example, $b = 5$ corresponds to $r = 0$. The negative intervals in A and B contradict $b = 5(1-r) > 0$. Option D would require $r \leq -1$, which fails convergence, and the endpoint 10 itself is also excluded.

Step 3: Conclusion The first term satisfies $b \in (0, 10)$. Hence, the correct option is (C).

Quick Tip to Solve: For a fixed positive infinite-G.P. sum S, use $b = S(1-r)$ and $-1 < r < 1$ to get $0 < b < 2S$.

10. If $\vec{a}$, $\vec{b}$, and $\vec{C}$ are unit vectors such that $\vec{a}+2\vec{b}+2\vec{c}=\vec{0}$, then $|\vec{a}\times\vec{c}|$ is equal to : [4 marks]

(A) $\frac{\sqrt{15}}{4}$

(B) $\frac{1}{4}$

(C) $\frac{15}{16}$

(D) $\frac{\sqrt{15}}{16}$

Correct Answer: (A) $\frac{\sqrt{15}}{4}$

Solution:

Concept:

- For vectors $\mathbf{u}, \mathbf{v}$, $|\mathbf{u}+\mathbf{v}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 + 2\mathbf{u}\cdot\mathbf{v}$.
- For unit vectors, $|\mathbf{a}\times\mathbf{c}|^2 = 1-(\mathbf{a}\cdot\mathbf{c})^2$. This converts a cross-product question into a dot-product calculation.

Step 1: Isolate the remaining unit vector The given relation is $\mathbf{a}+2\mathbf{b}+2\mathbf{c}=\mathbf{0}$. Move the term involving $\mathbf{b}$ to the other side:

$$\mathbf{a}+2\mathbf{c}=-2\mathbf{b}.$$

Take the squared magnitude of both sides. Since all three vectors are unit vectors,

$$1+4+4\mathbf{a}\cdot\mathbf{c}=4.$$

Consequently,

$$\mathbf{a}\cdot\mathbf{c}=-\frac{1}{4}.$$

The angle between $\mathbf{a}$ and $\mathbf{c}$ is obtuse, so the negative dot product is consistent with the vector relation.

Step 2: Calculate the cross-product magnitude Apply the dot-cross identity:

$$|\mathbf{a}\times\mathbf{c}|^2 = |\mathbf{a}|^2|\mathbf{c}|^2-(\mathbf{a}\cdot\mathbf{c})^2 = 1-\frac{1}{16}=\frac{15}{16}.$$

The requested quantity is a magnitude and is nonnegative. Therefore

$$|\mathbf{a}\times\mathbf{c}|=\frac{\sqrt{15}}{4}.$$

Option B gives the magnitude of the dot product instead of the cross product. Option C is the square of the requested magnitude. Option D incorrectly retains 16 as the denominator after taking a square root; $\sqrt{16}=4$.

Step 3: Conclusion $|\mathbf{a}\times\mathbf{c}|=\frac{\sqrt{15}}{4}$. Hence, the correct option is (A).

Quick Tip to Solve: Isolate the third unit vector, square the relation to find a dot product, and only then apply the dot-cross identity.

11. The area (in sq. units) of the region

$\{x \in R : x \geq 0, y \geq 0, y \geq x - 2$ *and* $y \leq \sqrt{x}\}$, is : [4 marks]

(A) $\frac{13}{3}$

(B) $\frac{8}{3}$

(C) $\frac{10}{3}$

(D) $\frac{5}{3}$

Correct Answer: (C) $\frac{10}{3}$

Solution:

Concept:

- Area between vertical boundaries is $\int(\text{upper ordinate} - \text{lower ordinate})\,dx$. All stated inequalities must be satisfied simultaneously.
- Here the upper boundary is $y = \sqrt{x}$, while the lower boundary is $\max(0, x-2)$. The lower boundary changes at $x = 2$.

Step 1: Determine the admissible interval For $0 \leq x \leq 2$, the condition $y \geq x - 2$ is automatically satisfied by $y \geq 0$. For $x \geq 2$, we need $x - 2 \leq \sqrt{x}$. At the intersection, let $t = \sqrt{x} \geq 0$. Then

$$t^2 - 2 = t \quad \Longrightarrow \quad (t-2)(t+1) = 0.$$

Only $t = 2$ is admissible, giving $x = 4, y = 2$. Thus the region runs from $x = 0$ to $x = 4$. Squaring the intersection equation without enforcing $x \geq 2$ would introduce an extraneous intersection.

Step 2: Integrate with the correct lower boundary Split at the change in lower boundary:

$$A = \int_0^2 \sqrt{x}\,dx + \int_2^4 \left(\sqrt{x} - (x-2)\right) dx.$$

Equivalently, take the entire area under the square-root curve and subtract the triangular portion below the line:

$$\begin{aligned} A &= \int_0^4 \sqrt{x}\,dx - \int_2^4 (x-2)\,dx \\ &= \left[\frac{2}{3}x^{3/2}\right]_0^4 - \left[\frac{(x-2)^2}{2}\right]_2^4 \\ &= \frac{16}{3} - 2 = \frac{10}{3}. \end{aligned}$$

The exact integral is $10/3$; the values $13/3$, $8/3$, and $5/3$ therefore do not equal the required area.

Step 3: Conclusion The required area is $\frac{10}{3}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Combine $y \geq 0$ and $y \geq x - 2$ as $y \geq \max(0, x-2)$; this immediately identifies the split at $x = 2$.

12. If $\lambda \in \mathbf{R}$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is : [4 marks]

(A) $4\sqrt{2}$

(B) $2\sqrt{5}$

(C) $2\sqrt{7}$

(D) 20

Correct Answer: No global minimum exists for real lambda; option B follows only from interpreting minimum as a local minimum.

Solution:

Concept:

- If α, β are the roots, Vieta's relations give $\alpha + \beta = \lambda - 2$ and $\alpha\beta = 10 - \lambda$.
- The identity $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ avoids calculating the individual roots initially. A positive second derivative identifies a local minimum.

Step 1: Find the stationary minimum of the sum Put $S = \alpha^3 + \beta^3$. Substitution gives

$$\begin{aligned} S &= (\lambda - 2)^3 - 3(10 - \lambda)(\lambda - 2) \\ &= \lambda^3 - 3\lambda^2 - 24\lambda + 52. \end{aligned}$$

Hence

$$S\prime(\lambda) = 3(\lambda - 4)(\lambda + 2), \qquad S\prime\prime(\lambda) = 6\lambda - 6.$$

The stationary point $\lambda = -2$ has negative second derivative and is a local maximum. At $\lambda = 4$, $S\prime\prime = 18 > 0$, so this is the local minimum intended by the alternatives. Strictly, the cubic is unbounded below as $\lambda \to -\infty$, so no global minimum exists over all real λ.

Step 2: Evaluate the separation of the roots At the stationary minimum $\lambda = 4$, the quadratic becomes

$$x^2 - 2x + 6 = 0, \qquad x = 1 \pm i\sqrt{5}.$$

Thus

$$\alpha - \beta = 2i\sqrt{5}, \qquad |\alpha - \beta| = 2\sqrt{5}.$$

Equivalently, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 4 - 24 = -20$, whose square root has modulus $\sqrt{20}$. The magnitude is real and positive even though the roots are complex. The squared magnitudes of options A, C and D are 32, 28 and 400, whereas the calculation requires 20; only B matches.

Step 3: Conclusion The literal question has no global minimizing real λ, because $S(\lambda) = \lambda^3 - 3\lambda^2 - 24\lambda + 52$ is unbounded below. Thus none of the options is uniquely correct under the stated global condition. If minimum is intended to mean a local minimum, then $\lambda = 4$ and $|\alpha - \beta| = 2\sqrt{5}$, corresponding conditionally to option (B).

Quick Tip to Solve: Use Vieta's relations before differentiating, and distinguish a local stationary minimum from a global minimum on an unbounded domain.

13. If the tangents drawn to the hyperbola $4y^2 = x^2 + 1$ intersect the co-ordinate axes at the distinct points A and B then the locus of the mid point of AB is : [4 marks]

(A) $x^2 - 4y^2 + 16x^2y^2 = 0$

(B) $x^2 - 4y^2 - 16x^2y^2 = 0$

(C) $4x^2 - y^2 + 16x^2y^2 = 0$

(D) $4x^2 - y^2 - 16x^2y^2 = 0$

Correct Answer: (B) $x^2 - 4y^2 - 16x^2y^2 = 0$

Solution:

Concept:

- For the hyperbola $4y^2 - x^2 = 1$, a tangent at (x_1, y_1) is $4yy_1 - xx_1 = 1$, where $4y_1^2 - x_1^2 = 1$.
- The tangent's axis intercepts determine the midpoint. Eliminating the point-of-contact coordinates gives the locus equation, with restrictions for finite distinct intercepts.

Step 1: Express the midpoint through the point of contact At a point (x_1, y_1) on the hyperbola, its tangent is

$$4yy_1 - xx_1 = 1.$$

On the x-axis, set $y = 0$, giving $A = (-1/x_1, 0)$. On the y-axis, set $x = 0$, giving $B = (0, 1/(4y_1))$. Finite distinct intercepts require $x_1 \neq 0$; the hyperbola already ensures $y_1 \neq 0$. If the midpoint is (h, k), then

$$h = -\frac{1}{2x_1}, \qquad k = \frac{1}{8y_1}.$$

Thus h and k are both nonzero.

Step 2: Eliminate the point-of-contact coordinates Invert the midpoint relations and substitute into the hyperbola equation:

$$x_1 = -\frac{1}{2h}, \qquad y_1 = \frac{1}{8k},$$

$$4\left(\frac{1}{8k}\right)^2 - \left(-\frac{1}{2h}\right)^2 = 1.$$

Simplifying and multiplying by $16h^2k^2$ gives

$$\frac{1}{16k^2} - \frac{1}{4h^2} = 1 \quad \Longrightarrow \quad h^2 - 4k^2 - 16h^2k^2 = 0.$$

Conversely, a point satisfying this equation with $hk \neq 0$ determines a real contact point and hence such a tangent. Option A has the wrong sign for the product term; C and D interchange the unequal axis coefficients. Only B follows from the contact-point equation.

Step 3: Conclusion The locus is $x^2 - 4y^2 - 16x^2y^2 = 0$, restricted to $xy \neq 0$ because the intercepts must be finite and distinct. Hence, the correct option is (B).

Quick Tip to Solve: Write the tangent in contact-point form, find its two axis intercepts, and substitute the midpoint relations into the original conic.

14. If $x_1, x_2, \ldots, x_n$ and $\frac{1}{h_1}, \frac{1}{h_2}, \ldots, \frac{1}{h_n}$ are two A.P..s such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5.h_{10}$ equals : [4 marks]

(A) 2560

(B) 2650

(C) 3200

(D) 1600

Correct Answer: (A) 2560

Solution:

Concept:

- An arithmetic progression satisfies $x_n = x_p + (n-p)d$, where d is its common difference.
- The reciprocals $u_n = 1/h_n$ form the second A.P.; the sequence h_n itself is not an A.P. Compute in reciprocals and invert only at the end.

Step 1: Calculate the fifth term of the first A.P. The data give $x_3 = 8$ and $x_8 = 20$. Since these terms are five places apart, the common difference is

$$d_x = \frac{x_8 - x_3}{8-3} = \frac{20-8}{5} = \frac{12}{5}.$$

The fifth term is two places after the third, so

$$x_5 = x_3 + 2d_x = 8 + \frac{24}{5} = \frac{64}{5}.$$

Keeping this value as an exact fraction avoids unnecessary rounding before multiplication.

Step 2: Calculate the tenth harmonic term Put $u_n = 1/h_n$. The given values $h_2 = 8$ and $h_7 = 20$ mean $u_2 = 1/8$ and $u_7 = 1/20$. Therefore

$$d_u = \frac{u_7 - u_2}{7-2} = \frac{1/20 - 1/8}{5} = -\frac{3}{200}.$$

Now

$$u_{10} = u_2 + 8d_u = \frac{1}{8} - \frac{24}{200} = \frac{1}{200}, \qquad h_{10} = 200.$$

The required product is

$$x_5 h_{10} = \frac{64}{5} \times 200 = 2560.$$

Option B merely transposes digits; neither 3200 in C nor 1600 in D equals this exact product. In particular, using an A.P. formula directly for h_n would ignore that it is the reciprocals which progress arithmetically.

Step 3: Conclusion $x_5 h_{10} = 2560$. Hence, the correct option is (A).

Quick Tip to Solve: For the second progression, interpolate or extrapolate $1/h_n$, not h_n, and take the reciprocal only after finding the desired term.

15. Let A be a matrix such that $A.\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ is a scalar matrix and |3A| = 108.

Then A^2 equals : [4 marks]

(A) $\begin{bmatrix}4 & -32\\ 0 & 36\end{bmatrix}$

(B) $\begin{bmatrix}36 & 0\\ -32 & 4\end{bmatrix}$

(C) $\begin{bmatrix}4 & 0\\ -32 & 36\end{bmatrix}$

(D) $\begin{bmatrix}36 & -32\\ 0 & 4\end{bmatrix}$

Correct Answer: (D) $\begin{bmatrix}36 & -32\\ 0 & 4\end{bmatrix}$

Solution:

Concept:

- A scalar matrix has the form kI. If $AB = kI$ and B is invertible, then $A = kB^{-1}$.
- For a matrix of order two, $\det(3A) = 3^2 \det A$. The scalar multiplying a matrix must therefore be squared in its determinant.

Step 1: Express the matrix using its inverse Put $B = \begin{pmatrix}1 & 2\\ 0 & 3\end{pmatrix}$. Since $\det B = 3 \neq 0$, its inverse exists.

The scalar-matrix condition gives

$$A = kB^{-1} = k\begin{pmatrix}1 & -2/3\\ 0 & 1/3\end{pmatrix}.$$

We need not find the sign of k, because the required square depends only on k^2.

Step 2: Use the determinant and square the matrix The determinant condition becomes

$$108 = \det(3A) = 9\det A = 9\frac{k^2}{3} = 3k^2, \qquad k^2 = 36.$$

Now multiply B^{-1} by itself rather than squaring its entries:

$$(B^{-1})^2 = \begin{pmatrix}1 & -8/9\\ 0 & 1/9\end{pmatrix}, \qquad A^2 = \begin{pmatrix}36 & -32\\ 0 & 4\end{pmatrix}.$$

The upper-right entry is the sum $-2/3 - 2/9 = -8/9$. Options (B) and (C) have a nonzero lower-left entry, impossible for this upper-triangular square. Option (A) reverses the diagonal entries.

Step 3: Conclusion $A^2 = \begin{pmatrix}36 & -32\\ 0 & 4\end{pmatrix}$. Hence, the correct option is (D).

Quick Tip to Solve: Write the scalar product as $AB = kI$, and use the determinant to find k^2 directly when only A^2 is required.

16. The set of all $\alpha \in$ R, for which w = $\frac{1+(1-8\alpha)z}{1-z}$ is purely imaginary number, for all z $\in$ C satisfying |z| = 1 and Re z $\neq$ 1, is : [4 marks]

(A) an empty set

(B) {0}

(C) $\left\{0, \frac{1}{4}, -\frac{1}{4}\right\}$

(D) equal to **R**

Correct Answer: (B) {0}

Solution:

Concept:

- On the unit circle, $\overline{z} = 1/z$. For $z \neq 1$, the expression $(1+z)/(1-z)$ has zero real part.
- A complex number has zero real part exactly when it and its conjugate sum to zero. A real constant added to such a number changes only the real part.

Step 1: Separate the real constant Rewrite the numerator using $1 - z$:

$$1 + (1 - 8\alpha)z = (1 - 4\alpha)(1 + z) + 4\alpha(1 - z).$$

Consequently,

$$w = (1 - 4\alpha)\frac{1+z}{1-z} + 4\alpha.$$

The hypothesis $\operatorname{Re} z \neq 1$, together with $|z| = 1$, excludes $z = 1$, so the denominator does not vanish.

Step 2: Check necessity and sufficiency Let $u = (1+z)/(1-z)$. Its conjugate is

$$\overline{u} = \frac{1 + 1/z}{1 - 1/z} = \frac{z+1}{z-1} = -u.$$

Thus $\operatorname{Re} u = 0$. Since α is real, the decomposition proves $\operatorname{Re} w = 4\alpha$ for every permitted z. This vanishes precisely when $\alpha = 0$, which is sufficient as well as necessary. In particular $z = -1$ already forces $w = 4\alpha = 0$. Therefore the set is not empty; the extra values $\pm 1/4$ in (C) fail, and arbitrary real values in (D) fail too.

Step 3: Conclusion The required set is $\{0\}$. Hence, the correct option is (B).

Quick Tip to Solve: For a quotient involving a unit complex number, substitute $\overline{z} = 1/z$; alternatively, test $z = -1$ first to force the parameter.

17. The mean of set of 30 observations is 75. If each observation is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. Then λ is equal to : [4 marks]

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $\frac{4}{3}$
(D) $\frac{10}{3}$

Correct Answer: (C) $\frac{4}{3}$

Solution:

Concept:

- For n observations x_i, their arithmetic mean is $\overline{x} = (\sum x_i)/n$.
- Under a common affine transformation $u_i = \lambda x_i - c$, linearity of summation gives $\overline{u} = \lambda\overline{x} - c$.

Step 1: Find the total before and after the transformation Let the original observations be $x_1, \ldots, x_{30}$. Their stated mean gives

$$\sum_{i=1}^{30} x_i = 30 \times 75 = 2250.$$

Each new observation is $\lambda x_i - 25$: the subtraction follows the multiplication. Hence the new total is

$$\sum_{i=1}^{30} (\lambda x_i - 25) = 2250\lambda - 30 \times 25 = 2250\lambda - 750.$$

The same constant is subtracted from every observation, so the total decreases by 750, not merely by 25.

Step 2: Impose the unchanged mean and check the choices Divide the transformed total by the unchanged number of observations:

$$\overline{u} = \frac{2250\lambda - 750}{30} = 75\lambda - 25.$$

The mean is still 75, so

$$75\lambda - 25 = 75, \qquad 75\lambda = 100, \qquad \lambda = \frac{4}{3}.$$

This is nonzero, as required, and substitution gives $75(4/3) - 25 = 100 - 25 = 75$. The choices $1/3$, $2/3$, and $10/3$ instead give means 0, 25, and 225, respectively; none preserves the original mean. The individual observations are unnecessary because only their sum is involved.

Step 3: Conclusion $\lambda = 4/3$. Hence, the correct option is (C).

Quick Tip to Solve: Apply the same multiply-then-subtract operation directly to the mean; the number of observations cancels from an affine transformation.

18. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} + 2y = f(x)$,

where $f(x) = \begin{cases} 1, & x \in [0,1] \\ 0, & otherwise \end{cases}$

If y(0) = 0, then $y\left(\frac{3}{2}\right)$ is : [4 marks]

(A) $\frac{e^2+1}{2e^4}$

(B) $\frac{1}{2e}$

(C) $\frac{e^2-1}{e^3}$

(D) $\frac{e^2-1}{2e^3}$

Correct Answer: (D) $\frac{e^2-1}{2e^3}$

Solution:

Concept:

- The integrating factor of $y' + 2y = f(x)$ is e^{2x}, giving $(e^{2x}y)' = e^{2x}f(x)$.
- When the forcing changes at $x = 1$, integrate separately on the two intervals and carry the continuous value of y across the switching point.

Step 1: Solve on the interval with nonzero forcing For $0 \le x \le 1$, use the initial condition to integrate:

$$e^{2x}y(x) - y(0) = \int_0^x e^{2t}\,dt = \frac{e^{2x}-1}{2}.$$

Since $y(0) = 0$, this yields

$$y(x) = \frac{1-e^{-2x}}{2}, \qquad y(1) = \frac{1-e^{-2}}{2}.$$

The finite jump in the forcing changes the derivative across the switch, while the solution itself remains continuous.

Step 2: Propagate the value after the switch For $x > 1$, the equation is homogeneous: $y' + 2y = 0$. Thus its value decays from the value at 1:

$$y(x) = y(1)e^{-2(x-1)}.$$

At the required point,

$$y(3/2) = \frac{1-e^{-2}}{2}\,e^{-1} = \frac{e^2-1}{2e^3}.$$

Option (C) is twice this value and omits the factor arising from integrating e^{2x}. Option (B) would replace $1 - e^{-2}$ by 1, losing the finite initial growth interval. Option (A) has an incorrect sign and decay factor; it does not equal the propagated value.

Step 3: Conclusion $y(3/2) = (e^2-1)/(2e^3)$. Hence, the correct option is (D).

Quick Tip to Solve: For a piecewise forcing, solve up to the switching point first and then use that value as the initial value for the next interval.

19. A box 'A' contains 2 white, 3 red and 2 black balls. Another box 'B' contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without eplacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box 'B' is : [4 marks]

(A) $\frac{9}{16}$
(B) $\frac{7}{16}$
(C) $\frac{9}{32}$
(D) $\frac{7}{8}$

Correct Answer: (B) $\frac{7}{16}$

Solution:

Concept:

- Bayes theorem updates a prior choice of box using the likelihood of the observed colours: $P(B \mid E) = P(B)P(E \mid B)/P(E)$.
- Without replacement, an unordered pair is sampled from $\binom{n}{2}$ pairs. The number containing one white and one red ball is the product of the two colour counts.

Step 1: Calculate the observation likelihood for each box Let E denote drawing one white and one red ball, in either order. Box A contains 7 balls, of which 2 are white and 3 red. Therefore

$$P(E \mid A) = \frac{2 \times 3}{\binom{7}{2}} = \frac{6}{21} = \frac{2}{7}.$$

Box B contains 9 balls, including 4 white and 2 red, giving

$$P(E \mid B) = \frac{4 \times 2}{\binom{9}{2}} = \frac{8}{36} = \frac{2}{9}.$$

The black balls enter the total pair counts even though a favourable pair contains none of them.

Step 2: Condition on the observed pair Random selection of a box gives equal prior probabilities $P(A) = P(B) = 1/2$. Hence

$$P(B \mid E) = \frac{(1/2)(2/9)}{(1/2)(2/7) + (1/2)(2/9)} = \frac{1/9}{1/7 + 1/9} = \frac{7}{16}.$$

The complementary probability $9/16$, option (A), belongs to box A. Option (C) fails to normalize the two possible box contributions correctly, while (D) exceeds the posterior obtained from the equal priors. Since the observed colours are more likely in A than in B, the answer for B should be below $1/2$, consistent with $7/16$.

Step 3: Conclusion The probability that the selected box was B is $7/16$. Hence, the correct option is (B).

Quick Tip to Solve: With equally likely boxes, normalize their colour-pair likelihoods directly: posterior for B equals its likelihood divided by the sum of both likelihoods.

20. If $x^2 + y^2 + \sin y = 4$, then the value of $\frac{d^2y}{dx^2}$ at the point $(-2,0)$ is : [4 marks]

(A) -34

(B) -32

(C) 4

(D) -2

Correct Answer: (A) -34

Solution:

Concept:

- Implicit differentiation treats y as a function of x. Thus $d(\sin y)/dx = \cos y\, y'$.
- To find y'', differentiate the first-derivative equation again, applying both the product rule and the chain rule before substituting the point.

Step 1: Calculate the first derivative at the point Differentiate the given relation:

$$2x + 2y\,y' + \cos y\,y' = 0, \qquad (2y + \cos y)y' = -2x.$$

At $(x, y) = (-2, 0)$, the coefficient $2y + \cos y$ equals 1, so

$$y'(-2) = 4.$$

The point satisfies the original relation because $(-2)^2 + 0 + \sin 0 = 4$. The nonzero coefficient also permits a local differentiable branch through this point.

Step 2: Differentiate again and substitute Differentiate $2x + (2y + \cos y)y' = 0$. The derivative of the coefficient is $(2 - \sin y)y'$, so

$$2 + (2 - \sin y)(y')^2 + (2y + \cos y)y'' = 0.$$

Now substitute $y = 0$ and $y' = 4$:

$$2 + 2(4)^2 + y'' = 0, \qquad y'' = -34.$$

Option (B), -32, misses the derivative 2 of $2x$. Option (C), 4, is the first derivative rather than the second. Option (D), -2, omits the chain-rule term involving $(y')^2$.

Step 3: Conclusion $d^2y/dx^2\big|_{(-2,0)} = -34$. Hence, the correct option is (A).

Quick Tip to Solve: Keep the first-derivative equation implicit until the second differentiation; then substitute both the point and the already computed slope.

21. Consider the following two binary relations on the set A = {a, b, c} :

R_1 = {(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)} and

R_2 = {(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c)}.

Then : [4 marks]

(A) both R_1 and R_2 are not symmetric.

(B) R_1 is not symmetric but it is transitive.

(C) R_2 is symmetric but it is not transitive.

(D) both R_1 and R_2 are transitive.

Correct Answer: (C) R_2 is symmetric but it is not transitive.

Solution:

Concept:

- A relation is symmetric when every pair (u, v) has its reverse (v, u) in the relation.
- Transitivity requires (u, w) whenever (u, v) and (v, w) are present. One counterexample is enough to disprove either property.

Step 1: Test the first relation In R_1, the pair (b, c) is present but (c, b) is absent, so R_1 is not symmetric. It is also not transitive: both (b, c) and (c, a) belong to R_1, whereas (b, a) does not. Thus the chain from b through c to a fails the transitivity requirement. This explicitly rules out option (B), which incorrectly attributes transitivity to R_1, as well as option (D).

Step 2: Test the second relation and identify the statement The off-diagonal pairs of R_2 occur in reverse pairs:

$$(a, b) \leftrightarrow (b, a), \qquad (c, a) \leftrightarrow (a, c).$$

The remaining listed pairs (a, a), (b, b), and (c, c) reverse to themselves. Therefore R_2 is symmetric. But (b, a) and (a, c) are present while (b, c) is absent. This proves that R_2 is not transitive. Option (A) is false because it says neither relation is symmetric. Option (D) is false for both relations. The description of R_2 in option (C) is exactly the property pair established by these direct membership checks.

Step 3: Conclusion R_2 is symmetric but is not transitive. Hence, the correct option is (C).

Quick Tip to Solve: For symmetry inspect reversed pairs; for transitivity search for a two-link chain whose direct pair is missing. Record the actual pairs as a decisive counterexample.

22. n−digit numbers are formed using only three digits 2, 5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is : [4 marks]

(A) 6

(B) 7

(C) 8

(D) 9

Correct Answer: (B) 7

Solution:

Concept:

- If each of n positions has three independent choices, the product rule gives 3^n different strings. Here each string is a number because none of the allowed digits is zero.
- The condition that 900 distinct numbers can be formed means the available total must be at least 900. We seek the least positive integer n satisfying $3^n \geq 900$.

Step 1: Count the choices in each position The first digit can be 2, 5 or 7, and the same three choices are available for every remaining digit. Repetition is permitted: the question restricts the digits used but does not require them to occur at most once. For example, using the digit 2 twice still gives a valid number. Consequently the total is

$$N_n = \underbrace{3 \times 3 \times \cdots \times 3}_{n \text{ factors}} = 3^n.$$

Distinct strings correspond to distinct numbers, so there is no further division by a factorial.

Step 2: Compare consecutive powers around 900 Calculate the powers at the boundary:

$$3^5 = 243, \qquad 3^6 = 729, \qquad 3^7 = 2187.$$

Six digits provide only 729 numbers, which is fewer than the required 900. Seven digits provide 2187 numbers, so a collection of 900 distinct such numbers can be chosen. Since multiplication by 3 makes the count strictly increase with each additional digit, every smaller length also fails.

Step 3: Check the listed lengths Option (A), 6, fails the lower bound. Options (C) and (D), 8 and 9, would also provide enough numbers, but cannot be the smallest length because 7 already works. The problem does not ask for the total to equal exactly 900; no integer power of 3 equals 900.

Step 4: Conclusion The smallest permissible length is $\boxed{n = 7}$. Hence, the correct option is (B).

Quick Tip to Solve: For a smallest-length counting problem, compare two consecutive powers bracketing the target; "can be formed" requires at least the target count.

23. If n is the degree of the polynomial,

$\left[\frac{2}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8+\left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$

and m is the coefficient of x^n in it, then the ordered pair (n, m) is equal to : [4 marks]

(A) (24, $(10)^8$)

(B) (8, $5(10)^4$)

(C) (12, $(20)^4$)

(D) (12, $8(10)^4$)

Correct Answer: (C) (12, $(20)^4$)

Solution:

Concept:

- Conjugate radicals satisfy $(a-b)(a+b)=a^2-b^2$. This simplifies reciprocals before applying the binomial theorem.
- In $(a+b)^8+(a-b)^8$, the terms with odd powers of b cancel and the terms with even powers double. The remaining expression is a polynomial in a^2 and b^2.

Step 1: Rationalise the two denominators Put $a=\sqrt{5x^3+1}$ and $b=\sqrt{5x^3-1}$. Then $a^2-b^2=2$, and hence

$$\frac{2}{a-b}=a+b, \qquad \frac{2}{a+b}=a-b.$$

The required expression is therefore $P(x)=(a+b)^8+(a-b)^8$. This removes the denominators without approximating either radical.

Step 2: Identify the highest possible power of x Adding the two binomial expansions gives

$$P(x)=2\sum_{j=0}^{4}\binom{8}{2j}(a^2)^{4-j}(b^2)^j.$$

Substitute $a^2=5x^3+1$ and $b^2=5x^3-1$. Each summand is a product of four factors of degree 3, so its degree is at most 12. Its coefficient of x^{12} is $\binom{8}{2j}5^4$, before the common factor 2. All these leading coefficients are positive, so cancellation cannot reduce the degree.

Step 3: Sum the leading coefficients and select the pair The sum of the even binomial coefficients in row 8 is

$$\sum_{j=0}^{4}\binom{8}{2j}=\frac{(1+1)^8+(1-1)^8}{2}=128.$$

Thus

$$m=2\cdot 128\cdot 5^4=256\cdot 625=160000=20^4.$$

The nonzero leading coefficient confirms $n=12$. Options (A) and (B) have incorrect degrees, while option (D) has the correct degree but half the required coefficient.

Step 4: Conclusion The ordered pair is $\boxed{(n,m)=(12,20^4)}$. Hence, the correct option is (C).

Quick Tip to Solve: First use the conjugate identity; then extract only the leading coefficient from the even terms instead of expanding the entire polynomial.

24. Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3, then the equation of the common tangent to the two parabolas is : [4 marks]

(A) 4(x + y) + 3 = 0

(B) 3(x + y) + 4 = 0

(C) 8(2x + y) + 3 = 0

(D) x + 2y + 3 = 0

Correct Answer: (A) 4(x + y) + 3 = 0

Solution:

Concept:

- A parabola $y^2 = 4ax$ has latus rectum length $4a$ and slope-form tangent $y = mx + a/m$, for $m \neq 0$.
- For $x^2 = 4ay$, the slope-form tangent is $y = mx - am^2$. A common tangent must have both the same slope and the same intercept.

Step 1: Write the equations of the parabolas The common vertex is the intersection of their coordinate-axis axes, namely the origin. Both open in the positive coordinate directions because they intersect in the first quadrant. Since the latus rectum length is 3, $4a = 3$ and $a = 3/4$. Their equations are therefore

$$y^2 = 3x, \qquad x^2 = 3y.$$

The negative-opening choices would not have an intersection with both coordinates positive.

Step 2: Impose tangency to both curves Let the common tangent have slope m. The vertical line through the vertex is not tangent to both curves, and neither is the horizontal line, so we may use finite nonzero m. Tangency to the first and second parabola respectively requires

$$y = mx + \frac{3}{4m}, \qquad y = mx - \frac{3m^2}{4}.$$

For one line to satisfy both equations, equate the intercepts:

$$\frac{3}{4m} = -\frac{3m^2}{4} \quad \Longrightarrow \quad m^3 = -1.$$

The real slope is $m = -1$.

Step 3: Form the line and check the alternatives Its intercept is $-3/4$, giving

$$y = -x - \frac{3}{4} \quad \Longleftrightarrow \quad 4(x + y) + 3 = 0.$$

Substituting this line into either parabola gives a quadratic with zero discriminant, confirming tangency. Option (B) has intercept $-4/3$, which fails the required intercept for slope -1. Options (C) and (D) have slopes -2 and $-1/2$, neither of which satisfies $m^3 = -1$.

Step 4: Conclusion The common tangent is $\boxed{4(x + y) + 3 = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: For common tangents to two parabolas, express each tangent using the same slope and equate the intercepts.

25. If $(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$ is false, then the truth values of p, q and r are, respectively :

[4 marks]

(A) F, T, F

(B) T, F, T

(C) T, T, T

(D) F, F, F

Correct Answer: (B) T, F, T

Solution:

Concept:

- An implication $A \rightarrow B$ is false only when its antecedent A is true and its consequent B is false.
- A conjunction is true precisely when all its components are true. A disjunction is false precisely when all its components are false; negation reverses the truth value.

Step 1: Separate the implication into two parts Use the antecedent and consequent

$$A = (p \wedge \neg q) \wedge (p \wedge r), \qquad B = \neg p \vee q.$$

The premise states that $A \rightarrow B$ is false. Therefore we must impose both $A = \mathrm{T}$ and $B = \mathrm{F}$. It would be incorrect to make both parts false: a false antecedent makes an implication true, regardless of the consequent.

Step 2: Use the false consequent For $\neg p \vee q$ to be false, neither of its two components can be true. Thus

$$\neg p = \mathrm{F}, \qquad q = \mathrm{F} \quad \Longrightarrow \quad p = \mathrm{T}, \quad q = \mathrm{F}.$$

These values also make $p \wedge \neg q$ true, as the antecedent requires.

Step 3: Determine r and verify all options The remaining conjunction $p \wedge r$ must be true. Since $p = \mathrm{T}$, this forces $r = \mathrm{T}$. Substituting the full triple gives

$$(\mathrm{T} \wedge \mathrm{T}) \wedge (\mathrm{T} \wedge \mathrm{T}) = \mathrm{T}, \qquad \neg \mathrm{T} \vee \mathrm{F} = \mathrm{F}.$$

The implication is indeed $\mathrm{T} \rightarrow \mathrm{F}$, so it is false. Options (A) and (D) have $p = \mathrm{F}$, making the antecedent false. Option (C) has $q = \mathrm{T}$, making the consequent true. None of those can produce a false implication.

Step 4: Conclusion The truth values are $\boxed{p = \mathrm{T},\ q = \mathrm{F},\ r = \mathrm{T}}$. Hence, the correct option is (B).

Quick Tip to Solve: When an implication is declared false, write antecedent true and consequent false before simplifying its logical components.

26. A circle passes through the points (2, 3) and (4, 5). If its centre lies on the line, $y - 4x + 3 = 0$, then its radius is equal to : [4 marks]

(A) 2

(B) $\sqrt{5}$

(C) $\sqrt{2}$

(D) 1

Correct Answer: (A) 2

Solution:

Concept:

- The centre of a circle is equidistant from any two points on it. Equating squared distances gives the perpendicular bisector of their chord.
- The centre also satisfies the specified straight-line equation. Their intersection determines the centre, after which the distance formula gives the radius.

Step 1: Express the equal-distance condition Let the centre be $C(h, k)$, and let the given points be $P(2, 3)$ and $Q(4, 5)$. Because both are on the circle,

$$(h-2)^2 + (k-3)^2 = (h-4)^2 + (k-5)^2.$$

Expanding each side and cancelling h^2 and k^2 yields

$$-4h - 6k + 13 = -8h - 10k + 41,$$

so

$$h + k = 7.$$

This is the perpendicular bisector condition; it does not assert that the chord midpoint is necessarily the centre.

Step 2: Find the centre from the prescribed line The centre lies on $y - 4x + 3 = 0$, so $k = 4h - 3$. Combine this with the equal-distance equation:

$$h + 4h - 3 = 7 \quad \Longrightarrow \quad 5h = 10 \quad \Longrightarrow \quad h = 2.$$

Then $k = 5$, and the centre is $C(2, 5)$. It satisfies the given line since $5 - 8 + 3 = 0$. The midpoint $(3, 4)$ of the chord does not satisfy that line, so the chord is not a diameter.

Step 3: Calculate and check the radius Taking the distance to P,

$$r = \sqrt{(2-2)^2 + (5-3)^2} = \sqrt{4} = 2.$$

As an independent check, the distance from C to Q is also $\sqrt{(2-4)^2 + (5-5)^2} = 2$. Thus the circle has equation $(x-2)^2 + (y-5)^2 = 4$. The other listed radii $\sqrt{5}$, $\sqrt{2}$ and 1 do not agree with either of these two equal centre-to-point distances.

Step 4: Conclusion The radius is $\boxed{2}$ coordinate units. Hence, the correct option is (A).

Quick Tip to Solve: Find the centre by intersecting the perpendicular bisector with the stated centre line; do not assume a given chord is a diameter.

27. An angle between the plane, x + y + z = 5 and the line of intersection of the planes, 3x + 4y + z − 1 = 0 and 5x + 8y + 2z + 14 =0, is : [4 marks]

(A) $\sin^{-1}\left(\sqrt{3/17}\right)$

(B) $\cos^{-1}\left(\sqrt{3/17}\right)$

(C) $\cos^{-1}(3/17)$

(D) $\sin^{-1}(3/17)$

Correct Answer: (A) $\sin^{-1}\left(\sqrt{3/17}\right)$

Solution:

Concept:

- The intersection line of two planes is perpendicular to both plane normals, so a direction vector is their cross product.
- If θ is the acute angle between a line of direction $\mathbf{d}$ and a plane of normal $\mathbf{n}$, then $\sin\theta = |\mathbf{d}\cdot\mathbf{n}|/(|\mathbf{d}|\,|\mathbf{n}|)$. This is the complement of the angle with the normal.

Step 1: Find a direction vector of the intersection line The planes $3x + 4y + z - 1 = 0$ and $5x + 8y + 2z + 14 = 0$ have normals $\mathbf{n}_1 = (3, 4, 1)$ and $\mathbf{n}_2 = (5, 8, 2)$. Hence

$$\mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2 = (8 - 8,\ 5 - 6,\ 24 - 20) = (0, -1, 4).$$

Its magnitude is $\sqrt{0^2 + (-1)^2 + 4^2} = \sqrt{17}$. Either sign of this direction vector describes the same line and gives the same acute angle.

Step 2: Use the normal of the target plane The plane $x + y + z = 5$ has normal $\mathbf{n} = (1, 1, 1)$, of magnitude $\sqrt{3}$. The scalar product is

$$\mathbf{d}\cdot\mathbf{n} = 0 - 1 + 4 = 3.$$

The constant terms of the plane equations affect the positions of the planes but not their normals or the direction of the intersection line, so they do not enter this angle calculation.

Step 3: Calculate the line-plane angle The desired angle therefore satisfies

$$\sin\theta = \frac{3}{\sqrt{17}\sqrt{3}} = \sqrt{\frac{3}{17}}, \qquad \theta = \sin^{-1}\sqrt{\frac{3}{17}}.$$

Option (B) gives the complementary angle, the angle with the normal rather than the plane. Options (C) and (D) use $3/17$ instead of $\sqrt{3/17}$, incorrectly handling the product of magnitudes. The sine form and its square-root argument are both necessary.

Step 4: Conclusion The angle is $\boxed{\sin^{-1}\sqrt{3/17}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use a cross product for the intersection direction, then use sine for the angle with a plane and cosine for the angle with its normal.

28. An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}$ kmabove it, is obsered at an elevation of 60^o from a point on the ground. If, after five seconds, its elevation from the same point, is 30^o, then the speed (in km / hr) of the aeroplane, is : [4 marks]

(A) 1500

(B) 1440

(C) 750

(D) 720

Correct Answer: (B) 1440

Solution:

Concept:

- For a right triangle with height h, horizontal distance x, and elevation θ, $\tan\theta = h/x$, so $x = h\cot\theta$.
- At constant height, the difference of the horizontal distances gives the distance travelled in the usual vertical section of the flight. Speed equals distance divided by elapsed time, with consistent units.

Step 1: Find the first horizontal distance Let the observation point be O, and let the perpendicular projections of the two positions on the ground be P and Q. The height is $h = \sqrt{3}$ km at both instants. At the first position the elevation is $60°$, so

$$OP = h\cot 60° = \sqrt{3}\cdot\frac{1}{\sqrt{3}} = 1 \text{ km}.$$

Use the horizontal distance, rather than the slant distance from the observer, because the aeroplane flies parallel to the ground.

Step 2: Find the distance travelled in five seconds After five seconds the elevation is $30°$, giving

$$OQ = h\cot 30° = \sqrt{3}\cdot\sqrt{3} = 3 \text{ km}.$$

The decreasing elevation corresponds to the aeroplane receding from the observation point in the vertical section. The horizontal distance travelled between the observations is therefore

$$PQ = OQ - OP = 3 - 1 = 2 \text{ km}.$$

The constant height means the aeroplane travels the same distance as its ground projection.

Step 3: Convert the speed to kilometres per hour The elapsed time is $5/3600$ hour. Consequently

$$v = \frac{2}{5/3600} = \frac{7200}{5} = 1440 \text{ km h}^{-1}.$$

Equivalently, its speed is 0.4 km per second and multiplication by 3600 gives the same result. Options (A), (C) and (D) do not equal the quotient of the 2 km displacement and the five-second interval; using the first horizontal distance alone would incorrectly give 720.

Step 4: Conclusion The aeroplane speed is $\boxed{1440 \text{ km h}^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Compute both horizontal distances using height times cotangent, subtract them, and convert the time to hours only at the final step.

29. If β is one of the angles between the normals to the ellipse, $x^2 + 3y^2 = 9$ at the points $(3\cos\theta, \sqrt{3}\sin\theta)$ and $(-3\sin\theta, \sqrt{3}\cos\theta)$; $\theta \in \left(0, \frac{\pi}{2}\right)$; then $\frac{2\cot\beta}{\sin 2\theta}$ is equal to : [4 marks]

(A) $\frac{2}{\sqrt{3}}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\sqrt{2}$

(D) $\frac{\sqrt{3}}{4}$

Correct Answer: (A) $\frac{2}{\sqrt{3}}$

Solution:

Concept:

- For an implicit curve, the normal is perpendicular to the tangent. For $x^2 + 3y^2 = 9$, differentiation gives tangent slope $-x/(3y)$, so the normal slope is $3y/x$.
- If two nonvertical lines have slopes m_1, m_2, the acute angle between them satisfies $\tan\beta = |(m_1 - m_2)/(1 + m_1 m_2)|$. Use $\sin 2\theta = 2\sin\theta\cos\theta$.

Step 1: Find the two normal slopes Implicitly differentiate the given ellipse, keeping the factor 3 multiplying y^2:

$$2x + 6y\frac{dy}{dx} = 0.$$

At the first point $P = (3\cos\theta, \sqrt{3}\sin\theta)$, the normal therefore has slope

$$m_1 = \frac{3\sqrt{3}\sin\theta}{3\cos\theta} = \sqrt{3}\tan\theta.$$

At the second point $Q = (-3\sin\theta, \sqrt{3}\cos\theta)$, it has slope

$$m_2 = \frac{3\sqrt{3}\cos\theta}{-3\sin\theta} = -\sqrt{3}\cot\theta.$$

Both slopes are finite since the given interval excludes its endpoints.

Step 2: Evaluate the angle and simplify the expression The product of the slopes is $m_1 m_2 = -3$. For the acute angle selected by the positive options,

$$\tan\beta = \frac{\sqrt{3}(\tan\theta + \cot\theta)}{2} = \frac{\sqrt{3}}{2\sin\theta\cos\theta} = \frac{\sqrt{3}}{\sin 2\theta}.$$

Taking reciprocals and substituting in the requested quantity gives

$$\frac{2\cot\beta}{\sin 2\theta} = \frac{2}{\sin 2\theta}\frac{\sin 2\theta}{\sqrt{3}} = \frac{2}{\sqrt{3}}.$$

The value is independent of θ. Option (B) misses the factor 2; options (C) and (D) do not equal this value. The supplementary angle has the opposite cotangent; the conventional angle between lines here is the acute one.

Step 3: Conclusion The required value is $2/\sqrt{3}$. Hence, the correct option is (A).

Quick Tip to Solve: Compute the product of the two normal slopes first: here it is the constant -3, so the angle formula simplifies immediately.

30. A variable plane passes through a fixed point (3,2,1) and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to yz -plane through A, a second plane is drawn parallel zx-plane through B and a third plane is drawn parallel to xy-plane through C. Then the locus of the point of intersection of these three planes, is : [4 marks]

(A) $\frac{x}{3}+\frac{y}{2}+\frac{z}{1}=1$

(B) x + y + z = 6

(C) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{6}$

(D) $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

Correct Answer: (D) $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

Solution:

Concept:

- A plane with finite nonzero axis intercepts a, b, c has equation $X/a + Y/b + Z/c = 1$. A fixed point on it imposes a relation among these three intercepts.
- A plane parallel to a coordinate plane keeps the remaining coordinate constant. Thus planes parallel to yz, zx, xy have equations $X = a, Y = b, Z = c$, respectively.

Step 1: Express the condition on the variable plane Let the intercepts be $A = (a, 0, 0)$, $B = (0, b, 0)$, and $C = (0, 0, c)$. Use capital letters X, Y, Z for a general point on the variable plane so that the intercept parameters remain distinct:

$$\frac{X}{a}+\frac{Y}{b}+\frac{Z}{c}=1.$$

Since the plane must pass through the fixed point $(3, 2, 1)$, these parameters obey

$$\frac{3}{a}+\frac{2}{b}+\frac{1}{c}=1.$$

The fixed point is a condition on a family of planes, rather than the intercept triple of every member of the family.

Step 2: Identify the intersection point and eliminate the intercepts The three newly drawn planes have equations $X = a$, $Y = b$, and $Z = c$, because each passes through its stated intercept and is parallel to the corresponding coordinate plane. Their common point is therefore $P = (a, b, c)$. If its moving coordinates are (x, y, z), then $a = x, b = y, c = z$. Consequently,

$$\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1, \qquad xyz \neq 0.$$

Conversely, every finite nonzero triple satisfying this equation defines the required intercept plane, so the condition describes the locus. Option (A) incorrectly treats the fixed point as the intercepts. Option (B) replaces a reciprocal relation by an unsupported sum, and option (C) discards the different fixed coordinates.

Step 3: Conclusion The locus is $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$. Hence, the correct option is (D).

Quick Tip to Solve: For an intercept-locus question, name the intercepts first; the intersection of the three coordinate-parallel planes is simply their ordered triple.

Physics (120 marks)

31. The energy required to remove the electron from a singly ionized Helium atom is 2.2 times the energies required to remove an electron from Helium atom. The total energy required to ionize the Helium atom completely is : [4 marks]

(A) $20\ eV$

(B) $34\ eV$

(C) $79\ eV$

(D) $109\ eV$

Correct Answer: (C) $79\ eV$

Solution:

Concept:

- The singly ionized helium ion He^+ has one electron and is hydrogen-like. Its ground-state binding energy is $13.6Z^2\,\mathrm{eV}$, with nuclear charge $Z = 2$.
- Complete ionization of a neutral helium atom removes its two electrons successively. The required total is the sum of the first and second ionization energies, not twice either one.

Step 1: Calculate the second ionization energy After the first electron has been removed, the remaining electron belongs to the one-electron ion He^+. The Bohr ground-state energy is

$$E_1 = -13.6\frac{Z^2}{1^2}\,\mathrm{eV} = -13.6(2)^2\,\mathrm{eV} = -54.4\,\mathrm{eV}.$$

The zero of energy is the free electron at infinite separation, so the positive energy required for the second removal is

$$I_2 = 0 - (-54.4) = 54.4\,\mathrm{eV}.$$

The neutral atom is a two-electron system, so the same hydrogen-like formula cannot directly be applied to its first ionization.

Step 2: Use the given ratio and add the two energies The question states that removing the electron from the singly ionized atom requires 2.2 times the energy of the first removal. Thus

$$I_2 = 2.2I_1, \qquad I_1 = \frac{54.4}{2.2} = 24.727\ldots\ \mathrm{eV}.$$

The total energy supplied for the successive processes is

$$I_{\text{total}} = I_1 + I_2 = 24.727\ldots + 54.4 = 79.127\ldots\ \mathrm{eV}.$$

This agrees with $79\,\mathrm{eV}$ to the precision of the options and the supplied ratio. Neither 20 nor $34\,\mathrm{eV}$ even exceeds the second ionization energy. The $109\,\mathrm{eV}$ option is approximately $2I_2$, which incorrectly assigns the same binding energy to both removals.

Step 3: Conclusion The total energy required is approximately $79\,\mathrm{eV}$. Hence, the correct option is (C).

Quick Tip to Solve: Apply the Z^2 Bohr scaling only after helium has become a one-electron ion, then recover the first ionization energy from the stated ratio.

32. One mole of an ideal monoatomic gas is compressed isothermally in a rigid vessel to double its pressure at room temperature, $27^{\circ}C$. The work done on the gas will be : [4 marks]

(A) $300\ R$

(B) $300\ R\ ln\ 6$

(C) $300\ R\ ln\ 2$

(D) $300\ R\ ln\ 7$

Correct Answer: (C) $300\ R\ ln\ 2$

Solution:

Concept:

- For a fixed amount of ideal gas, $PV = nRT$. During an isothermal change, PV remains constant and the internal energy does not change.
- For reversible isothermal compression, the positive work done on the gas is $W_{\text{on}} = nRT\ln(V_i/V_f) = nRT\ln(P_f/P_i)$. Temperature must be expressed in kelvin.

Step 1: Convert the temperature and determine the compression ratio The gas amount is $n = 1\,\text{mol}$, and its temperature is

$$T = 27 + 273 = 300\,\text{K}.$$

At constant temperature, doubling the pressure gives

$$P_iV_i = P_fV_f, \qquad P_f = 2P_i, \qquad V_f = \frac{V_i}{2}.$$

The fact that the gas is monoatomic does not introduce a factor $3/2$ into the isothermal work. That factor concerns internal energy, whose change is zero at fixed temperature.

Step 2: Integrate the pressure-volume work and compare the options For the usual reversible isothermal compression represented by the options, use $P = nRT/V$:

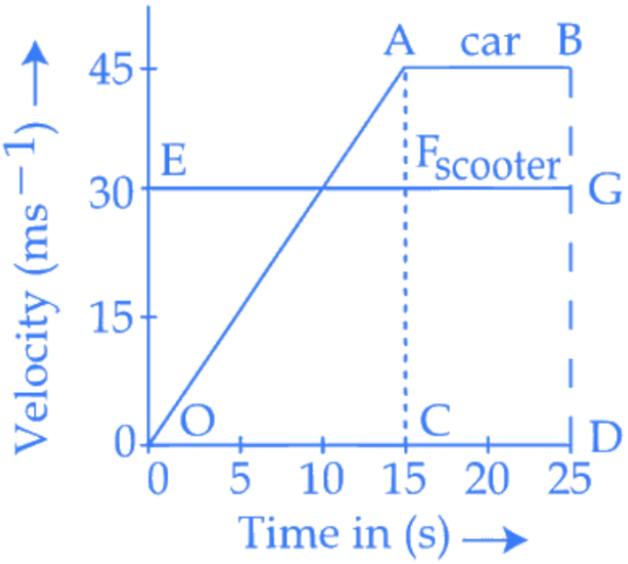
$$W_{\text{on}} = -\int_{V_i}^{V_f} P\,dV = nRT\ln\frac{V_i}{V_f} = 300R\ln 2.$$

This work is positive because the surroundings compress the gas; an equal amount of heat leaves it to maintain the temperature. Option (A) omits the logarithm. Options (B) and (D) would require volume ratios 6 and 7, whereas the pressure ratio here is only 2. There is a wording inconsistency: a genuinely rigid vessel has fixed volume, and a fixed mole of ideal gas cannot double its pressure isothermally in it. Thus the printed compression and pressure change use the standard variable-volume interpretation; a literal rigid-volume process would not realize the stated change.

Step 3: Conclusion For the stated isothermal compression, the intended work is $300R\ln 2\,\text{J}$, with R in SI units. Hence, the correct option is (C). A strictly rigid vessel would make the described change impossible.

Quick Tip to Solve: In isothermal work, use the logarithm of the pressure ratio and kelvin temperature; also check that the described volume change is physically possible.

33. The velocity-time graphs of a car and a scooter are shown in the figure. (i) The difference between the distance travelled by the car and the scooter in $15\ s$ and (ii) the time at which the car will catch up with the scooter are, respectively. [4 marks]



(A) $112.5\ m$ and $22.5\ s$

(B) $337.5\ m$ and $25\ s$

(C) $112.5\ m$ and $15\ s$

(D) $225.5\ m$ and $10\ s$

Correct Answer: (A) $112.5\ m$ and $22.5\ s$

Solution:

Concept:

- Distance travelled is the area under a nonnegative velocity-time graph. A uniformly accelerating segment gives a triangular area, while constant velocity gives a rectangular area.
- The car catches the scooter when their total distances from their common starting point become equal. After 15 s, use the remaining separation divided by the relative speed.

Step 1: Read the graph and calculate the distances after 15 seconds The scooter moves at $30\,\mathrm{m\,s^{-1}}$ throughout. The car starts from rest and reaches $45\,\mathrm{m\,s^{-1}}$ at $t = 15\,\mathrm{s}$, then continues at that speed. The car distance during the initial interval is the triangular area:

$$s_c(15) = \frac{1}{2}(15)(45) = 337.5\,\mathrm{m}.$$

The scooter distance is the rectangular area:

$$s_s(15) = (15)(30) = 450\,\mathrm{m}.$$

Hence the scooter is ahead by

$$450 - 337.5 = 112.5\,\mathrm{m}.$$

Although the car becomes faster than the scooter at 10 s, equality of instantaneous velocities does not mean equality of their distances.

Step 2: Find when the car closes the remaining gap For times after 15 s, the relative speed is constant:

$$v_c - v_s = 45 - 30 = 15\,\mathrm{m\,s^{-1}}.$$

The additional time to remove the separation is

$$\Delta t = \frac{112.5}{15} = 7.5\,\mathrm{s}.$$

Thus the catch-up time measured from the common start is $15 + 7.5 = 22.5\,\mathrm{s}$. As a check, both distances at this instant are $675\,\mathrm{m}$: the car travels $337.5 + 45(7.5)$, and the scooter travels $30(22.5)$. Option (B) confuses the car distance with the difference; (C) gives no time to close the existing gap; (D) incorrectly uses the velocity-intersection time.

Step 3: Conclusion The distance difference is $112.5\,\mathrm{m}$, and the car catches the scooter at $22.5\,\mathrm{s}$. Hence, the correct option is (A).

Quick Tip to Solve: A crossing of velocity-time lines means equal speeds. For catching up, compare the accumulated areas under the two graphs instead.

34. A planoconvex lens becomes an optical system of $28\ cm$ focal length when its plane surface is silvered and illuminated from left to right as shown in Fig-A.

If the same lens is instead silvered on the curved surface and illuminated from other side as in Fig-B, it acts like an optical system of focal length $10\ cm$. The refractive index of the material of lens is : [4 marks]



Fig. A Fig. B

(A) 1.50

(B) 1.55

(C) 1.75

(D) 1.51

Correct Answer: (B) 1.55

Solution:

Concept:

- For a thin plano-convex lens of refractive index μ and curved-surface radius magnitude R, its lens power is $(\mu - 1)/R$. Light traverses the lens twice when its rear surface is silvered.

- A silvered thin lens has equivalent converging power $2/f_L + 1/f_M$. The plane mirror has zero power; for the curved rear mirror in Fig. B, the converging mirror power is $2/R$.

Step 1: Use the plane-silvered arrangement to relate radius and index In Fig. A, reflection occurs at the plane surface, so the mirror itself has infinite focal length and contributes no power. The double passage through the refracting lens gives

$$\frac{1}{F_A} = \frac{2(\mu - 1)}{R}.$$

The given equivalent focal length is $F_A = 28\,\text{cm}$, hence

$$R = 56(\mu - 1)\,\text{cm}.$$

Using $28\,\text{cm}$ as the focal length of the unsilvered lens would miss the double traversal and produce the wrong index.

Step 2: Include the curved mirror power in the second arrangement In Fig. B, the curved silvered rear surface adds positive mirror power $2/R$ to the two lens traversals. Therefore

$$\frac{1}{F_B} = \frac{2(\mu - 1)}{R} + \frac{2}{R} = \frac{2\mu}{R}.$$

Putting $F_B = 10\,\text{cm}$ gives $R = 20\mu\,\text{cm}$. Eliminate the common radius:

$$56(\mu - 1) = 20\mu, \qquad 36\mu = 56, \qquad \mu = \frac{14}{9} = 1.555\ldots.$$

The listed approximate value is 1.55. As a numerical check, $R = 280/9\,\text{cm}$ yields $R/[2(\mu - 1)] = 28\,\text{cm}$ and $R/(2\mu) = 10\,\text{cm}$. Values 1.50, 1.75, and 1.51 do not satisfy both measured focal lengths simultaneously; 1.55 is the closest available choice.

Step 3: Conclusion The refractive index is $\mu = 14/9 \approx 1.55$ among the given choices. Hence, the correct option is (B).

Quick Tip to Solve: Write the two equivalent powers with the same radius. Plane silvering gives double lens power; curved silvering adds the mirror power as well.

35. Take the mean distance of the moon and the sun from the earth to be 0.4×10^6 km and 150×10^6 km respectively. Their masses are 8×10^{22} kg and 2×10^{30} kg respectively. The radius of the earth is 6400 km. Let ΔF_1 be the difference in the forces exerted by the moon at the nearest and farthest points on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest points on the earth. Then, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is : [4 marks]

(A) 2

(B) 10^{-2}

(C) 0.6

(D) 6

Correct Answer: (A) 2

Solution:

Concept:

- The gravitational force on the same test mass m, due to an external body of mass M at distance r, is GMm/r^2. Compare equal test masses at the near and far surface points.
- When the Earth radius R_E is small compared with the external distance d, the near-far force difference is proportional to M/d^3, rather than the ordinary attraction scale M/d^2.

Step 1: Derive the near-far difference The two distances from an external body to the nearest and farthest points are $d - R_E$ and $d + R_E$. Subtract the force magnitudes:

$$\Delta F = GMm\left[\frac{1}{(d-R_E)^2} - \frac{1}{(d+R_E)^2}\right] = \frac{4GMm\,dR_E}{(d^2-R_E^2)^2}.$$

For the Moon, $R_E/d = 6400/400000 = 0.016$, and the ratio is even smaller for the Sun. Thus $(d^2 - R_E^2)^2 \approx d^4$, giving $\Delta F \approx 4GMmR_E/d^3$. The same G, m, R_E factors cancel in the requested ratio.

Step 2: Substitute the two masses and distances Keep both distances in the same units so their quotient is dimensionless:

$$\frac{\Delta F_1}{\Delta F_2} \approx \frac{8 \times 10^{22}}{2 \times 10^{30}}\left(\frac{150 \times 10^6}{0.4 \times 10^6}\right)^3 = 4 \times 10^{-8}(375)^3 = 2.109375.$$

The exact finite-radius correction multiplies this by

$$\frac{[1-(6400/(150\times10^6))^2]^2}{[1-(6400/(0.4\times10^6))^2]^2},$$

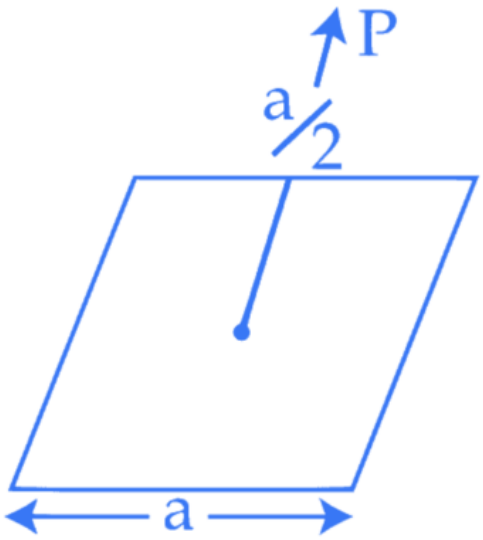
which changes it by less than 0.1 percent. The nearest listed number remains 2. Option (C) results from an incorrect distance scaling; 10^{-2} and 6 are also far from the computed ratio. The closer Moon produces a larger tidal difference even though the Sun produces the greater ordinary attraction.

Step 3: Conclusion The ratio is approximately 2.11, whose closest listed value is 2. Hence, the correct option is (A).

Quick Tip to Solve: Use M/d^3 for a near-far gravitational difference and M/d^2 for ordinary attraction; confusing these gives a very different Moon-Sun comparison.

36. A charge Q is placed at a distance $a/2$ above the center of the square surface of edge a as shown in the figure.

The electric flux through the square surface is [4 marks]



(A) $\frac{Q}{\epsilon_0}$

(B) $\frac{Q}{2\epsilon_0}$

(C) $\frac{Q}{3\epsilon_0}$

(D) $\frac{Q}{6\epsilon_0}$

Correct Answer: (D) $\frac{Q}{6\epsilon_0}$

Solution:

Concept:

- Gauss law relates the total outward flux through a closed surface to its enclosed charge: $\Phi_{\text{closed}} = Q/\varepsilon_0$. It cannot be applied directly to just one open square.
- When a point charge is at the centre of a cube, symmetry makes the flux through each of its six identical faces equal. The given height $a/2$ is half the edge of just such a cube.

Step 1: Complete the square into a symmetric closed surface Take the given square as one face of a cube of edge a, extending the cube toward the charge. The distance from the centre of any face to the cube centre is $a/2$. Since the charge lies above the centre of the given face at precisely this distance, it is at the cube centre. All six faces are therefore geometrically equivalent as viewed from the charge; none receives a preferential share of the flux.

Step 2: Apply Gauss law and divide among the faces The cube encloses the charge Q, so its total outward electric flux is

$$6\Phi_{\text{square}} = \frac{Q}{\varepsilon_0}.$$

Consequently,

$$\Phi_{\text{square}} = \frac{Q}{6\varepsilon_0}.$$

This is the flux magnitude through the square (or its outward flux as a cube face). Option (A) assigns the entire closed-surface flux to one face. Options (B) and (C) would divide it among two or three equivalent faces, whereas the completed cube has six.

Step 3: Conclusion The electric flux through the square is $Q/(6\varepsilon_0)$. Hence, the correct option is (D).

Quick Tip to Solve: For a charge a half-side above a square centre, complete a cube: one face receives one-sixth of the total flux.

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37. A body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\frac{R}{2}$, and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is : [4 marks]

(A) $-\frac{GMm}{2R}$
(B) $+\frac{GMm}{6R}$
(C) $\frac{GMm}{2R}$
(D) $-\frac{GMm}{6R}$

Correct Answer: (D) $-\frac{GMm}{6R}$

Solution:

Concept:

- The total mechanical energy of a mass μ in a circular orbit of radius s around mass M is $E = -GM\mu/(2s)$, with zero potential energy at infinity.
- Total energy is the sum of the energies of both fragments. The requested change is final minus initial, so the negative sign of each bound-orbit energy must be preserved.

Step 1: Calculate the energy before separation For the original body, the orbiting mass is m and the radius is R. Its kinetic energy is $GMm/(2R)$, while its gravitational potential energy is $-GMm/R$. Their sum is therefore

$$E_i = -\frac{GMm}{2R}.$$

The formula is for total orbital energy, rather than potential energy alone; using the latter would introduce an erroneous factor of two.

Step 2: Add the final energies and subtract the initial energy Each fragment has mass $m/2$. At radii $R/2$ and $3R/2$, their respective circular-orbit energies are

$$E_1 = -\frac{GM(m/2)}{2(R/2)} = -\frac{GMm}{2R}, \qquad E_2 = -\frac{GM(m/2)}{2(3R/2)} = -\frac{GMm}{6R}.$$

Hence

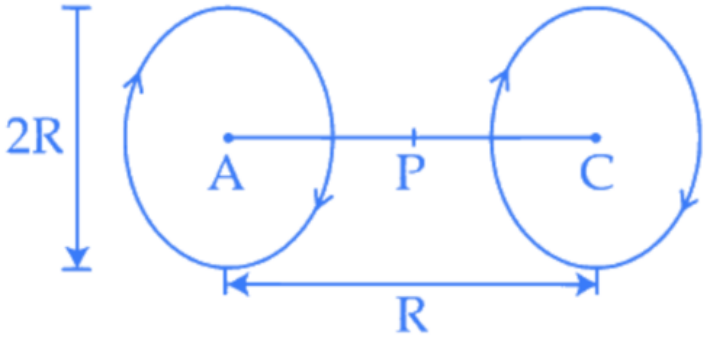
$$E_f = -\frac{2GMm}{3R}, \qquad E_f - E_i = -\frac{2GMm}{3R} + \frac{GMm}{2R} = -\frac{GMm}{6R}.$$

The final state is more tightly bound. Thus the positive changes in (B) and (C) have the wrong sign, and (A) is the original energy rather than the energy change.

Step 3: Conclusion The difference between final and initial total energies is $-GMm/(6R)$. Hence, the correct option is (D).

Quick Tip to Solve: Apply the circular-orbit energy formula to each fragment with its own mass and radius before adding the energies.

38. A Helmholtz coil has a pair of loops, each with N turns and radius R. They are placed coaxially at distance R and the same current I flows through the loops in the same direction. P, midway between the centers A and C, is given by [Refer to figure given below] : [4 marks]



(A) $\frac{8N\mu_0 I}{5^{1/2}R}$
(B) $\frac{8N\mu_0 I}{5^{3/2}R}$
(C) $\frac{4N\mu_0 I}{5^{1/2}R}$
(D) $\frac{4N\mu_0 I}{5^{3/2}R}$

Correct Answer: (B) $\frac{8N\mu_0 I}{5^{3/2}R}$

Solution:

Concept:

- The axial magnetic field at distance x from the centre of a circular coil of radius R, N turns and current I is $B = \mu_0 NIR^2/[2(R^2+x^2)^{3/2}]$.
- Fields obey vector superposition. For coaxial coils carrying current in the same sense, the two axial fields at their midpoint point in the same direction and add.

Step 1: Find the field due to either coil at the midpoint The centres A and C are separated by R, so the midpoint P is at axial distance $x = R/2$ from each coil. Substituting this into the axial-field expression gives

$$B_1 = \frac{\mu_0 NIR^2}{2[R^2+(R/2)^2]^{3/2}}.$$

The denominator contains the power $3/2$, because this is an axial point rather than the centre of the loop.

Step 2: Add the equal fields and simplify Both coils contribute the same field magnitude and the

right-hand rule gives the same direction. Therefore

$$B_P = 2B_1 = \frac{\mu_0 N I R^2}{(5R^2/4)^{3/2}} = \frac{8\mu_0 N I}{5^{3/2} R}.$$

In the simplification, $(5R^2/4)^{3/2} = 5^{3/2}R^3/8$. Option (D) is the field of only one coil and omits the addition. Options (A) and (C) use $5^{1/2}$, which misses the full axial-distance dependence. The result has the expected dimensions of magnetic field and varies inversely with coil radius.

Step 3: Conclusion The magnetic field at P is $8N\mu_0 I/(5^{3/2}R)$. Hence, the correct option is (B).

Quick Tip to Solve: At the midpoint of Helmholtz coils, substitute axial distance R/2 for one coil and then double its field.

---

39. Two electrons are moving with non-relativistic speed perpendicular to each other. If corresponding de Broglie wavelength are λ_1 and λ_2, their de Broglie wavelength in the frame of reference attached to their center of masses : [4 marks]

(A) $\lambda_{CM} = \lambda_1 = \lambda_2$

(B) $\lambda_{CM} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$

(C) $\frac{1}{\lambda_{CM}} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

(D) $\lambda_{CM} = \left(\frac{\lambda_1+\lambda_2}{2}\right)$

Correct Answer: (B) $\lambda_{CM} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$

Solution:

Concept:

- For a non-relativistic electron, the de Broglie wavelength is $\lambda = h/p$, where p is the magnitude of its momentum in the frame being considered.
- For two equal masses, the centre-of-mass velocity is their average velocity. Each electron has equal and opposite momentum in that frame, found by subtracting half the total laboratory momentum.

Step 1: Transform the momenta to the centre-of-mass frame Write the perpendicular laboratory momenta as $\mathbf{p}_1$ and $\mathbf{p}_2$. Since both electrons have mass m_e,

$$\mathbf{V}_{\text{CM}} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2m_e}, \qquad \mathbf{p}_1' = \mathbf{p}_1 - m_e\mathbf{V}_{\text{CM}} = \frac{\mathbf{p}_1 - \mathbf{p}_2}{2}.$$

The second electron has $\mathbf{p}_2' = -\mathbf{p}_1'$, so both centre-of-mass wavelengths are equal.

Step 2: Use perpendicularity and express the result in wavelengths Because the original momenta are perpendicular,

$$p_{\text{CM}} = \frac{1}{2}\sqrt{p_1^2 + p_2^2} = \frac{h}{2}\sqrt{\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}}.$$

It follows that

$$\lambda_{\mathrm{CM}} = \frac{h}{p_{\mathrm{CM}}} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}.$$

The scalar reciprocal addition in (C) ignores the perpendicular vectors. The arithmetic mean in (D) has no momentum-transformation basis. Option (A) incorrectly leaves the laboratory wavelengths unchanged; even when they are equal, the transformed wavelength is $\sqrt{2}$ times either laboratory wavelength.

Step 3: Conclusion Each electron has centre-of-mass wavelength $2\lambda_1\lambda_2/\sqrt{\lambda_1^2+\lambda_2^2}$. Hence, the correct option is (B).

Quick Tip to Solve: For two equal masses, subtract half the total momentum first; combine perpendicular momentum components only afterward.

40. An ideal capacitor of capacitance $0.2\,\mu F$ is charged to a potential difference of $10\ V$. The charging battery is then disconnected. The capacitor is then connected to an ideal inductor of self inductance $0.5\ mH$. The current at a time when the potential difference across the capacitor is $5\ V$, is : [4 marks]

(A) $0.34\ A$

(B) $0.25\ A$

(C) $0.17\ A$

(D) $0.15\ A$

Correct Answer: (C) $0.17\ A$

Solution:

Concept:

- An ideal isolated LC circuit conserves its total electromagnetic energy. The capacitor stores $CV^2/2$ and the inductor stores $LI^2/2$.
- Initially the charged capacitor has voltage $V_0 = 10\,\mathrm{V}$ and the inductor current is zero. At any later instant the lost electric energy becomes magnetic energy.

Step 1: Write the energy balance at the specified voltage The battery is disconnected, so it supplies no further energy during the oscillation. With no resistance there is also no dissipative loss. Therefore

$$\frac{1}{2}CV_0^2 = \frac{1}{2}CV^2 + \frac{1}{2}LI^2.$$

Rearranging gives

$$I^2 = \frac{C}{L}(V_0^2 - V^2).$$

This determines the magnitude of current; its direction depends on the stage of oscillation, which the question does not specify.

Step 2: Convert the units and evaluate the current Use $C = 0.2\times10^{-6}\,\mathrm{F}$, $L = 0.5\times10^{-3}\,\mathrm{H}$, and $V = 5\,\mathrm{V}$. Then

$$I^2 = \frac{0.2\times10^{-6}}{0.5\times10^{-3}}(10^2 - 5^2) = 4\times10^{-4}\times75 = 0.030\,\mathrm{A}^2.$$

Thus

$$|I| = \sqrt{0.030}\,\text{A} = 0.1732\,\text{A} \simeq 0.17\,\text{A}.$$

Squaring voltages is essential because stored electric energy depends on V^2, not V. The values in (A), (B), and (D) do not satisfy the same energy balance; substituting their squares would give magnetic energies inconsistent with the capacitor energy decrease.

Step 3: Conclusion The current magnitude is approximately $0.17\,\text{A}$. Hence, the correct option is (C).

Quick Tip to Solve: When an LC problem gives the capacitor voltage rather than time, conserve energy instead of finding the oscillation phase.

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41. A thin uniform tube is bent into a circle of radius r in the vertical plane. Equal volumes of two immiscible liquids, whose densities are ρ_1 and ρ_2 $(\rho_1 > \rho_2)$, fill half the circle. The angle θ between the radius vector passing through the common interface and the vertical is : [4 marks]

(A) $\theta = \tan^{-1}\pi\left(\frac{\rho_1}{\rho_2}\right)$

(B) $\theta = \tan^{-1}\frac{\pi}{2}\left(\frac{\rho_1}{\rho_2}\right)$

(C) $\theta = \tan^{-1}\left(\frac{\rho_1-\rho_2}{\rho_1+\rho_2}\right)$

(D) $\theta = \tan^{-1}\frac{\pi}{2}\left(\frac{\rho_1+\rho_2}{\rho_1-\rho_2}\right)$

Correct Answer: (C) $\theta = \tan^{-1}\left(\frac{\rho_1-\rho_2}{\rho_1+\rho_2}\right)$

Solution:

Concept:

- In a static liquid, the pressure increase over a vertical depth h is $\rho g h$. The pressure at the common interface must agree when calculated through either liquid.
- Equal volumes in a uniform tube mean equal arc lengths. Together the liquids occupy half the circumference, so each liquid occupies a quarter-circle arc with central angle $\pi/2$.

Step 1: Express the vertical heights of the two columns Choose the common interface in the lower part of the circle and measure its displacement θ from the downward vertical toward the denser-liquid side. Its vertical coordinate relative to the centre is $-r\cos\theta$. Each free surface is a quarter-turn away from it, with vertical coordinates $-r\sin\theta$ on the denser side and $+r\sin\theta$ on the lighter side. Hence the heights above the interface are

$$h_1 = r(\cos\theta - \sin\theta), \qquad h_2 = r(\cos\theta + \sin\theta).$$

The denser liquid needs the shorter column.

Step 2: Equate pressures and solve for the angle Both free surfaces experience the same external pressure. Equating interface pressures and cancelling gr gives

$$\rho_1(\cos\theta - \sin\theta) = \rho_2(\cos\theta + \sin\theta).$$

Thus

$$(\rho_1 - \rho_2)\cos\theta = (\rho_1 + \rho_2)\sin\theta, \qquad \tan\theta = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}.$$

For equal densities the angle tends to zero, as symmetry requires. Options (A) and (B) fail this limit. Option (D), involving the reciprocal density combination, also does not approach zero. Here $0 < \theta < \pi/4$ for $\rho_1 > \rho_2 > 0$.

Step 3: Conclusion The angle is $\theta = \tan^{-1}[(\rho_1 - \rho_2)/(\rho_1 + \rho_2)]$. Hence, the correct option is (C).

Quick Tip to Solve: In a bent liquid tube, use vertical height differences for pressure, never the liquid arc lengths themselves.

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42. In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of $0.25\ cm$. There are 100 circular scale divisions. The thickness of a wire measured by this screw gauge gives a reading of 4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, the thickness of the wire is : [4 marks]

(A) $0.4300\ cm$

(B) $0.2150\ cm$

(C) $0.3150\ cm$

(D) $0.0430\ cm$

Correct Answer: (B) $0.2150\ cm$

Solution:

Concept:

- The pitch of a screw gauge is its axial advance in one full revolution. Its least count is the pitch divided by the number of circular-scale divisions.
- With negligible zero error, the measured thickness is the main-scale reading plus the circular-scale reading times the least count. Here each main-scale pitch division corresponds to one revolution.

Step 1: Determine the pitch and least count Five complete revolutions advance the screw by $0.25\,\text{cm}$. Thus one complete revolution advances it by

$$p = \frac{0.25}{5}\,\text{cm} = 0.05\,\text{cm}.$$

The circular scale has 100 equal divisions, so one circular division represents

$$\text{LC} = \frac{p}{100} = \frac{0.05}{100}\,\text{cm} = 0.0005\,\text{cm}.$$

Dividing by five first is essential; the stated travel is not the pitch of a single rotation.

Step 2: Combine the main and circular scale readings The four main-scale divisions contribute

$$\text{MSR} = 4(0.05)\,\text{cm} = 0.20\,\text{cm}.$$

The circular-scale reading of 30 divisions contributes

$$\text{CSR} = 30(0.0005)\,\text{cm} = 0.0150\,\text{cm}.$$

No zero correction is required, so

$$d = 0.20 + 0.0150 = 0.2150\,\text{cm}.$$

Option (D) would arise from a smaller pitch and is inconsistent with the given advance. Option (A) doubles the correct thickness. Option (C) likewise does not equal the sum of the two properly calibrated readings. All readings and the least count have been kept in centimetres, avoiding a millimetre conversion error.

Step 3: Conclusion The thickness of the wire is $0.2150\,\text{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the travel per revolution before the least count, and keep both scale contributions in the same length unit.

---

43. A monochromatic beam of light has a frequency $v = \frac{3}{2\pi} \times 10^{12} Hz$ and is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$.

It is polarized along the $\hat{k}$ direction. The acceptable form for the magnetic field is : [4 marks]

(A) $\frac{E_0}{c}\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left[10^4\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cdot\vec{r}-(3\times10^{12})t\right]$

(B) $\frac{E_0}{c}\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left[10^4\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cdot\vec{r}-(3\times10^{12})t\right]$

(C) $\frac{E_0}{c}\hat{k}\cos\left[10^4\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cdot\vec{r}+(3\times10^{12})t\right]$

(D) $\frac{E_0}{c}\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}\cos\left[10^4\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cdot\vec{r}+(3\times10^{12})t\right]$

Correct Answer: (A) $\frac{E_0}{c}\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left[10^4\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cdot\vec{r}-(3\times10^{12})t\right]$

Solution:

Concept:

- In vacuum, the electric field, magnetic field and propagation direction form a right-handed orthogonal triad: $\vec{B} = c^{-1}\hat{n}\times\vec{E}$.
- For a travelling wave the phase is $\vec{k}\cdot\vec{r}-\omega t$, where $\vec{k} = (\omega/c)\hat{n}$, $\omega = 2\pi\nu$ and $B_0 = E_0/c$.

Step 1: Calculate the angular frequency and wave number The given frequency is the number of cycles per second, so it must first be converted to angular frequency. Using $c = 3\times10^8\,\text{m s}^{-1}$,

$$\omega = 2\pi\left(\frac{3}{2\pi}\times10^{12}\right) = 3\times10^{12}\,\text{s}^{-1}, \qquad k = \frac{\omega}{c} = 10^4\,\text{m}^{-1}.$$

The wave vector points along $(\hat{i}+\hat{j})/\sqrt{2}$, independently of the direction of the magnetic field.

Step 2: Determine the magnetic-field direction and test the phases The electric polarization is along $\hat{k}$. Hence

$$\hat{n}\times\hat{k}=\frac{\hat{i}\times\hat{k}+\hat{j}\times\hat{k}}{\sqrt{2}}=\frac{\hat{i}-\hat{j}}{\sqrt{2}}.$$

Option (A) has this polarization, the required amplitude, and the correct wave vector in its phase. Option (B) instead has a wave vector along $\hat{i}-\hat{j}$. Option (C) incorrectly places the magnetic field along the electric polarization and reverses propagation through its plus time sign. Option (D) is not transverse to the given direction and also has the wrong time sign.

Step 3: Conclusion The acceptable field is

$$\vec{B}=\frac{E_0}{c}\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left[10^4\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cdot\vec{r}-(3\times10^{12})t\right].$$

Hence, the correct option is (A).

Quick Tip to Solve: Check the vector multiplying the cosine separately from the wave vector inside it: one gives polarization, the other propagation.

44. Light of wavelength $550\ nm$ falls normally on a slit of width $22.0\times10^{-5}\ cm$. The angular position of the second minima from the central maximum will (in radians) : [4 marks]

(A) $\frac{\pi}{12}$

(B) $\frac{\pi}{8}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$

Correct Answer: (C) $\frac{\pi}{6}$

Solution:

Concept:

- The dark fringes of a single slit satisfy $a\sin\theta_n=n\lambda$, with $n=1,2,\ldots$. Here a is the full slit width.
- The second minimum corresponds to $n=2$. The exact inverse sine is needed when the resulting angle is not small.

Step 1: Convert the width and wavelength to the same unit The slit width is quoted in centimetres, whereas the wavelength is in nanometres. Converting both to metres gives

$$a=22.0\times10^{-5}\times10^{-2}=2.20\times10^{-6}\,\text{m},\qquad\lambda=550\times10^{-9}=5.50\times10^{-7}\,\text{m}.$$

The width is only four wavelengths, so the second dark fringe need not lie at a very small angle. Keeping centimetres in the width while using metres for the wavelength would give a wrong ratio by a factor of one hundred.

Step 2: Apply the second-minimum condition For the second minimum, the path difference across

the slit is twice the wavelength. Therefore

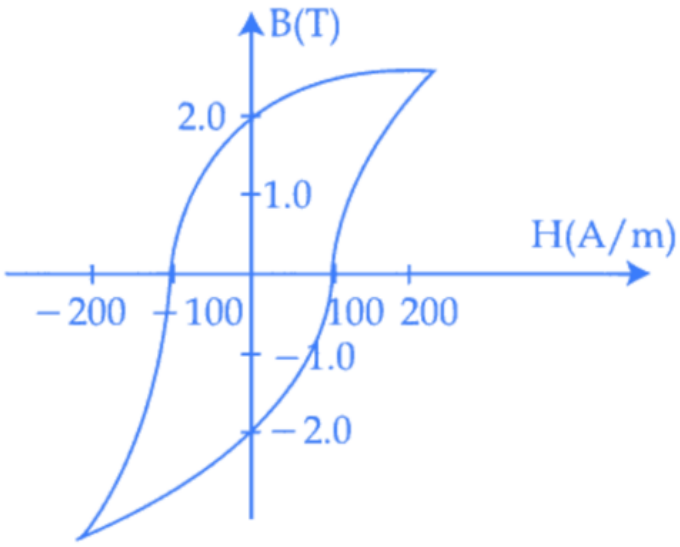
$$\sin\theta_2 = \frac{2\lambda}{a} = \frac{2(5.50\times10^{-7})}{2.20\times10^{-6}} = \frac{1}{2}.$$

In the forward diffraction pattern, $0 < \theta_2 < \pi/2$, so $\theta_2 = \sin^{-1}(1/2) = \pi/6$. The negative-angle minimum is symmetric about the central maximum. Options (A), (B) and (D) have sines different from one half and do not satisfy the required path difference. In particular, replacing sine by its argument would give 0.5 rad, an approximation rather than the exact listed angle.

Step 3: Conclusion The angular position of the second minimum has magnitude $\theta_2 = \pi/6\,\text{rad}$, approximately $0.524\,\text{rad}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert the slit width before calculating $n\lambda/a$, and use inverse sine rather than the small-angle approximation when the ratio is large.

45. The B-H curve for a ferromagnet is shown in the figure. The ferromagnet is placed inside a long solended with 1000 turns/cm. The current that should be passed in the solended to demagnetise the ferromagnet completely is : [4 marks]



(A) $1\ mA$

(B) $2\ mA$

(C) $20\,\mu A$

(D) $40\,\mu A$

Correct Answer: (A) $1\ mA$

Solution:

Concept:

- The coercive field H_c is the magnitude of the reverse magnetic-field intensity needed to bring the flux density B to zero on a hysteresis loop.
- Inside a long solenoid, $H = nI$, where n is the number of turns per metre and I is the current.

Step 1: Read the required field from the hysteresis loop To remove the remanent flux density, follow the branch from its nonzero intercept on the vertical axis to its intercept on the horizontal axis. The graph has $B = 0$ at $H = -100\,\mathrm{A\,m^{-1}}$ on one branch, or $+100\,\mathrm{A\,m^{-1}}$ on the opposite branch. Thus the required coercive-field magnitude is $100\,\mathrm{A\,m^{-1}}$. The current must produce a field opposing the initial magnetization; its magnitude is what the options specify. The vertical-axis value $2.0\,\mathrm{T}$ is remanence, not coercivity.

Step 2: Convert the turn density and calculate the current There are one hundred centimetres in one metre, so

$$n = 1000\,\mathrm{cm^{-1}} = 1000 \times 100 = 10^5\,\mathrm{m^{-1}}.$$

Using the solenoid field intensity rather than the vacuum flux-density formula gives

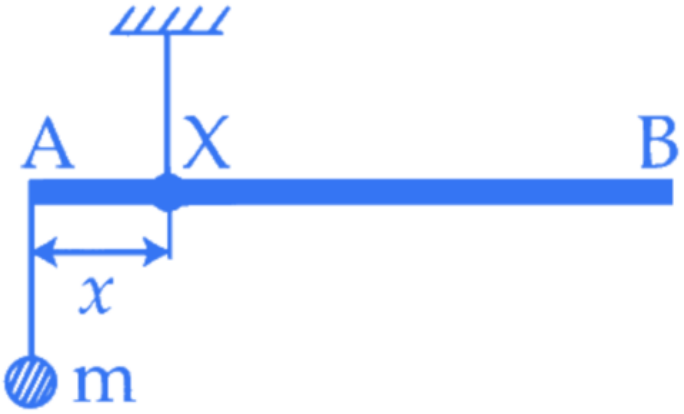
$$I = \frac{H_c}{n} = \frac{100}{10^5} = 10^{-3}\,\mathrm{A} = 1\,\mathrm{mA}.$$

A current of $2\,\mathrm{mA}$ would produce $H = 200\,\mathrm{A\,m^{-1}}$, overshooting the zero-flux intercept. The two microampere options yield only 2 and $4\,\mathrm{A\,m^{-1}}$, far below the coercive intensity. No permeability factor belongs in $H = nI$.

Step 3: Conclusion A reverse current of magnitude $1\,\mathrm{mA}$ supplies the coercive field required for zero flux density. Hence, the correct option is (A).

Quick Tip to Solve: On a hysteresis graph, read coercivity from the horizontal-axis intercept and remanence from the vertical-axis intercept; then use $H = nI$.

46. A uniform rod AB is suspended from a point X, at a variable distance x from A, as shown. To make the rod horizontal, a mass m is suspended from its end A. A set of (m, x) values is recorded. The appropriate variables that give a straight line, when plotted, are: [4 marks]



(A) m, x

(B) $m, \frac{1}{x}$

(C) $m, \frac{1}{x^2}$

(D) m, x^2

Correct Answer: (B) $m, \frac{1}{x}$

Solution:

Concept:

- A horizontal rigid body in equilibrium has zero net torque about any point. Taking moments about the suspension point removes the unknown support tension.
- The weight of a uniform rod of mass M_r and length L acts at its midpoint, a distance $L/2$ from end A.

Step 1: Balance the torques about the suspension point Let the rod mass be M_r, so it is not confused with the suspended mass m. The support is at distance x from A. The load at A has lever arm x, and the rod weight has lever arm $L/2 - x$ on the other side of the support. In the arrangement shown, $x < L/2$, allowing the two downward forces to produce opposite torques. Hence

$$mgx = M_r g \left(\frac{L}{2} - x \right).$$

The support force acts through the chosen pivot and has zero moment, so its magnitude is not needed.

Step 2: Rearrange the equation into straight-line form Cancel g, expand the right side and divide by x:

$$mx = \frac{M_r L}{2} - M_r x, \qquad m = \frac{M_r L}{2} \left(\frac{1}{x} \right) - M_r.$$

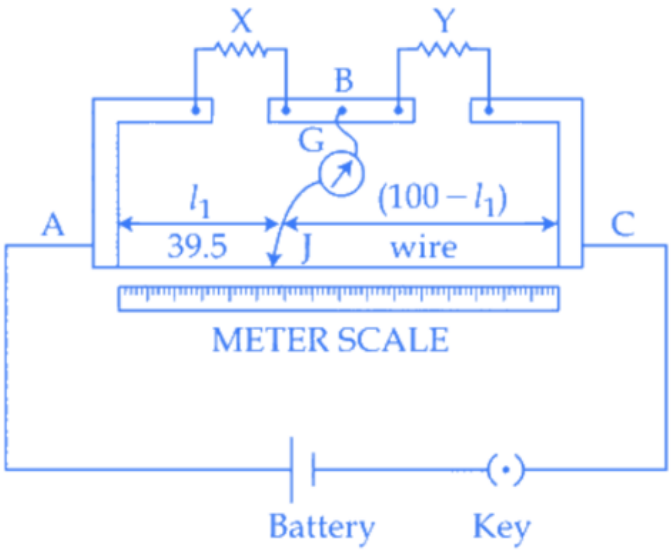
This is $Y = sX + b$ with $Y = m$, $X = 1/x$, slope $M_r L/2$, and intercept $-M_r$. Plotting m against x gives a reciprocal curve, while replacing the horizontal variable by x^2 or $1/x^2$ does not remove the reciprocal dependence. As a physical check, the required suspended mass tends to zero as the support

approaches the rod midpoint, exactly as the torque equation predicts.

Step 3: Conclusion The straight-line variables are m and $1/x$. Their relation is $m = (M_r L/2)(1/x) - M_r$. Hence, the correct option is (B).

Quick Tip to Solve: Choose the support as pivot, then rearrange the torque equation to the form $y = ax + b$ before selecting graph variables.

47. In a meter bridge, as shown in the figure, it is given that resistance $Y = 12.5\ \Omega$ and that the balance is obtained at a distance $39.5\ cm$ from end A (by Jockey J). After interchanging the resistances X and Y, a new balance point is found at a distance l_2 from end A. What are the value of X and l_2 ? [4 marks]



(A) $8.16\ \Omega$ and $60.5\ cm$

(B) $19.15\ \Omega$ and $39.5\ cm$

(C) $8.16\ \Omega$ and $39.5\ cm$

(D) $19.15\ \Omega$ and $60.5\ cm$

Correct Answer: (A) $8.16\ \Omega$ and $60.5\ cm$

Solution:

Concept:

- At balance in an ideal metre bridge, the resistance ratio equals the ratio of the two wire lengths: $X/Y = l/(100 - l)$, with lengths in centimetres.
- Interchanging the two resistances inverts the ratio, making the new balance length complementary to the old length: $l_2 = 100 - l_1$.

Step 1: Use the first balance point to determine the unknown resistance The diagram places X in the left gap and Y in the right gap. The left wire segment therefore corresponds to X, not Y. With

$l_1 = 39.5\,\text{cm}$, the other segment has length $100 - 39.5 = 60.5\,\text{cm}$. Thus

$$\frac{X}{12.5} = \frac{39.5}{60.5}, \qquad X = 12.5\frac{39.5}{60.5} = 8.161\ldots\,\Omega.$$

To the precision of the options, $X = 8.16\,\Omega$. Since the balance lies to the left of the midpoint, it is consistent that the left resistance is smaller than the right resistance.

Step 2: Find the balance point after exchanging the gaps After interchange, Y is on the left and X is on the right. The new balance condition is

$$\frac{12.5}{8.161\ldots} = \frac{l_2}{100 - l_2} = \frac{60.5}{39.5}.$$

Cross-multiplying gives $39.5l_2 = 60.5(100 - l_2)$, so $100l_2 = 6050$ and $l_2 = 60.5\,\text{cm}$. A resistance of $19.15\,\Omega$ arises from inverting the first ratio incorrectly. Keeping the balance length at $39.5\,\text{cm}$ after swapping unequal resistances also contradicts the bridge condition; this excludes the other three pairs.

Step 3: Conclusion The required values are $X = 8.16\,\Omega$ and $l_2 = 60.5\,\text{cm}$, measured from A. Hence, the correct option is (A).

Quick Tip to Solve: Keep the left-gap resistance over the right-gap resistance in the same order as left wire length over right wire length; swapping gaps complements the balance length.

48. A tuning fork vibrates with frequency $256\ Hz$ and gives one beat per second with the third normal mode of vibration of an open pipe. What is the length of the pipe ? (Speed of sound in air is $340\,ms^{-1}$) [4 marks]

(A) $220\ cm$

(B) $190\ cm$

(C) $180\ cm$

(D) $200\ cm$

Correct Answer: (D) $200\ cm$

Solution:

Concept:

- An open pipe supports every harmonic, with $f_n = nv/(2L)$. Its third normal mode is $n = 3$, not the third odd harmonic of a closed pipe.
- The beat frequency is the absolute difference: $f_b = |f_{\text{pipe}} - f_{\text{fork}}|$.

Step 1: Find the permitted third-mode frequencies The fork frequency is $256\,\text{Hz}$, and the observed beat frequency is $1\,\text{Hz}$. Therefore

$$|f_3 - 256| = 1, \qquad f_3 = 255\,\text{Hz or } 257\,\text{Hz}.$$

The beat information alone does not say whether the pipe frequency is higher or lower than the fork

frequency, so both possibilities should be checked against the listed lengths. For an open pipe, both ends are displacement antinodes, giving $L = 3\lambda_3/2$ in the third mode.

Step 2: Calculate the lengths and compare the options From $f_3 = 3v/(2L)$,

$$L = \frac{3(340)}{2f_3}.$$

For $f_3 = 255\,\text{Hz}$, this gives $L = 1020/510 = 2.00\,\text{m} = 200\,\text{cm}$. The other possible frequency gives $L = 1020/514 \approx 1.984\,\text{m}$, which is not separately listed. Directly testing $L = 2.00\,\text{m}$ gives $f_3 = 255\,\text{Hz}$, exactly one beat per second with the fork. The lengths 2.20, 1.90 and $1.80\,\text{m}$ instead give approximately 231.8, 268.4, and $283.3\,\text{Hz}$; none gives a frequency difference of one hertz.

Step 3: Conclusion The listed length satisfying the beat condition is $L = 200\,\text{cm}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the absolute frequency difference for beats, retain both signs initially, and distinguish an open pipe's third harmonic from a closed pipe's third allowed mode.

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49. A body of mass M and charge q is connected to spring of spring constant k. It is oscillating along x-direction about its equilibrium position, taken to be at $x = 0$, with an amplitude A. An electric field E is applied along the x-direction. Which of the following statements is correct ? [4 marks]

(A) The new equilibrium position is at a distance $\frac{qE}{2k}$ from $x = 0$.

(B) The total energy of the system is $\frac{1}{2}m\omega^2A^2 + \frac{1}{2}\frac{q^2E^2}{k}$.

(C) The total energy of the system is $\frac{1}{2}m\omega^2A^2 - \frac{1}{2}\frac{q^2E^2}{k}$.

(D) The new equilibrium position is at a distance $\frac{2qE}{k}$ from $x = 0$.

Correct Answer: The switching phase and energy reference are unspecified. The intended answer is (B) for switching at the old mean position and measuring energy above the new potential minimum.

Solution:

Concept:

- A constant electric field gives the charge a force qE. Combined with Hooke's law, the restoring force is $F = -kx + qE$.
- With electric potential zero at $x = 0$, the potential energy is $U(x) = kx^2/2 - qEx$. Completing the square identifies the shifted minimum.

Step 1: Locate the new equilibrium and identify the frequency Equilibrium requires zero resultant force:

$$-kx_0 + qE = 0, \qquad x_0 = \frac{qE}{k}.$$

The signed shift follows the electric force, and its distance is $|qE|/k$. Neither half nor twice this value, as in options (A) and (D), balances the forces. Introducing displacement $y = x - x_0$ gives $M\ddot{y} = -ky$, so the angular frequency remains $\omega = \sqrt{k/M}$. The constant force changes the centre of oscillation, not the spring curvature or its frequency.

Step 2: Express the energy relative to the shifted minimum Let the field be switched on when the

original oscillator is at x_s, with velocity v_s. Before switching, conservation of oscillator energy gives $Mv_s^2/2 + kx_s^2/2 = kA^2/2$. Immediately after switching, position and velocity are unchanged. The energy above the new potential minimum is therefore

$$\begin{aligned}\mathcal{E}_{\text{osc}} &= \frac{1}{2}Mv_s^2 + \frac{1}{2}k(x_s - x_0)^2 \\ &= \frac{1}{2}kA^2 - qEx_s + \frac{q^2E^2}{2k}.\end{aligned}$$

If switching occurs at the old mean position, $x_s = 0$, this becomes $\mathcal{E}_{\text{osc}} = M\omega^2A^2/2 + q^2E^2/(2k)$, the expression in option (B). The new amplitude then satisfies $A_{\text{new}}^2 = A^2 + x_0^2$. However, the printed question does not specify the switching phase. Moreover, using the potential reference $U(0) = 0$ gives total energy $\mathcal{E} = \mathcal{E}_{\text{osc}} - q^2E^2/(2k) = kA^2/2 - qEx_s$. Option (C) would instead describe an amplitude A measured about the new centre with this latter reference, rather than the stated original amplitude. Thus no phase-independent energy option follows from the literal wording.

Step 3: Conclusion The new equilibrium is always $x_0 = qE/k$, ruling out the two proposed displacements. The intended answer is option (B), $\frac{1}{2}M\omega^2A^2 + q^2E^2/(2k)$, when switching at the old mean position and measuring energy above the new minimum. Without those conventions, the printed question has no unique energy answer.

Quick Tip to Solve: For a suddenly applied constant force, keep position and velocity continuous, shift the equilibrium, and state both the switching phase and the zero of energy.

50. An automobile, travelling at 40 km/h, can be stopped at a distance of 40 m by applying brakes. If the same automobile is travelling at 80 km/h, the minimum stopping distance, in metres, is (assume no skidding) : [4 marks]

(A) 45 m

(B) 100 m

(C) 150 m

(D) 160 m

Correct Answer: (D) 160 m

Solution:

Concept:

- For uniform braking, the kinematic relation is $v^2 = u^2 + 2as$, where u is the initial speed, v the final speed, a the signed acceleration and s the stopping distance.
- The same automobile under the same limiting braking conditions has the same magnitude of retardation. Consequently, stopping distance is proportional to the square of the initial speed, rather than to the speed itself.

Step 1: Write the stopping condition At the instant the car stops, its final speed is zero. Write the

braking acceleration as $-b$, with $b > 0$. The equation of motion then gives

$$0 = u^2 - 2bs, \qquad s = \frac{u^2}{2b}.$$

The minus sign is essential because the acceleration opposes the direction of motion. No reaction-time distance is included in the question.

Step 2: Compare the two speeds Let $s_1 = 40\,\text{m}$ correspond to $u_1 = 40\,\text{km h}^{-1}$, and let s_2 correspond to $u_2 = 80\,\text{km h}^{-1}$. Dividing the two stopping-distance equations cancels the common retardation:

$$\frac{s_2}{s_1} = \left(\frac{u_2}{u_1}\right)^2 = \left(\frac{80}{40}\right)^2 = 4.$$

Both speeds have the same units, so their ratio does not require conversion to metres per second.

Step 3: Calculate and check the choices Therefore $s_2 = 4 \times 40 = 160\,\text{m}$. Doubling the speed quadruples the kinetic energy, while the available braking force remains the same; four times the distance is thus needed to remove that energy. The values 45, 100 and $150\,\text{m}$ do not obey the required factor of four and are rejected.

Step 4: Conclusion The minimum stopping distance is $\boxed{160\,\text{m}}$. Hence, the correct option is (D).

Quick Tip to Solve: For unchanged braking force, use $s \propto u^2$: doubling speed requires four times the braking distance.

51. The relative error in the determination of the surface area of sphere is α. Then the relative error in the determination of its volume is : [4 marks]

(A) $\frac{3}{2}\alpha$

(B) $\frac{2}{3}\alpha$

(C) $\frac{5}{2}\alpha$

(D) α

Correct Answer: (A) $\frac{3}{2}\alpha$

Solution:

Concept:

- The surface area and volume of a sphere of radius r are $S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$. Numerical constants have no measurement uncertainty.
- For a quantity proportional to r^p, its small relative error has magnitude $p\,\Delta r/r$. Relative errors are dimensionless; the power multiplying the radius controls their propagation.

Step 1: Extract the relative error in radius Differentiate the surface-area formula to first order:

$$\Delta S \simeq 8\pi r\,\Delta r.$$

Dividing by $S = 4\pi r^2$ gives

$$\frac{\Delta S}{S} = 2\frac{\Delta r}{r} = \alpha.$$

Thus the relative uncertainty in the measured radius is $\Delta r/r = \alpha/2$. This is the usual small-error approximation used for error propagation.

Step 2: Propagate that error to volume The volume depends on the cube of the same radius. Its first-order change is $\Delta V \simeq 4\pi r^2 \Delta r$, so

$$\frac{\Delta V}{V} = 3\frac{\Delta r}{r} = \frac{3}{2}\alpha.$$

Equivalently, eliminating the radius gives $V \propto S^{3/2}$; taking a logarithmic differential immediately yields the same factor. The errors must refer to the same radius measurement, rather than independent measurements of area and volume.

Step 3: Check the alternatives The volume is more sensitive than the area to a fractional radius error because its exponent is three instead of two. The factor $2/3$ would reverse the conversion, while $5/2$ would incorrectly add unrelated exponents. The answer α would require both quantities to have the same power of radius. Only the factor $3/2$ follows from the two sphere formulae.

Step 4: Conclusion The relative error in volume is $\boxed{\frac{3}{2}\alpha}$. Hence, the correct option is (A).

Quick Tip to Solve: For quantities depending on the same measured length, the ratio of relative errors equals the ratio of their powers.

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52. A given object takes n times more time to slide down a 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is : [4 marks]

(A) $\frac{1}{2-n^2}$

(B) $1 - \frac{1}{n^2}$

(C) $\sqrt{1 - \frac{1}{n^2}}$

(D) $\sqrt{\frac{1}{1-n^2}}$

Correct Answer: (B) $1 - \frac{1}{n^2}$

Solution:

Concept:

- Down an incline of angle θ, the gravitational component is $mg\sin\theta$ and the normal reaction is $N = mg\cos\theta$. Kinetic friction $\mu_k N$ acts up the plane.
- For motion from rest through the same distance s, $s = \frac{1}{2}at^2$. Thus the acceleration is inversely proportional to the square of the travel time.

Step 1: Write the accelerations on both planes On the perfectly smooth plane no friction acts, so

$$a_s = g\sin 45^\circ = \frac{g}{\sqrt{2}}.$$

On the rough plane Newton's second law along the slope gives

$$ma_r = mg\sin 45^\circ - \mu_k mg\cos 45^\circ, \qquad a_r = \frac{g}{\sqrt{2}}(1-\mu_k).$$

The object is sliding downward, so friction opposes its motion and reduces the acceleration. The mass cancels from the result.

Step 2: Use the stated time ratio Let the smooth-plane travel time be t_s, with rough-plane time $t_r = nt_s$. For equal lengths and release from rest,

$$\frac{1}{2}a_s t_s^2 = \frac{1}{2}a_r t_r^2, \qquad \frac{a_r}{a_s} = \frac{t_s^2}{t_r^2} = \frac{1}{n^2}.$$

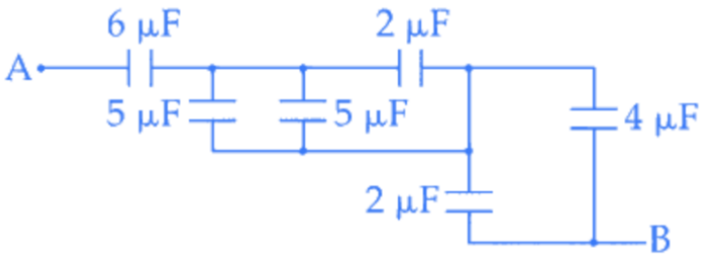
Substituting the accelerations gives $1-\mu_k = 1/n^2$, hence $\mu_k = 1 - 1/n^2$.

Step 3: Test the limiting case and options If $n = 1$, the travel times are equal and the formula gives zero friction, as required. For $n > 1$, it gives $0 < \mu_k < 1$, consistent with downward sliding at this angle. Option (A) gives a nonzero value at $n = 1$; option (D) is not real for $n > 1$. Option (C) introduces an unwarranted square root: the time square has already been included in the acceleration ratio.

Step 4: Conclusion The coefficient of kinetic friction is $\boxed{\mu_k = 1 - \frac{1}{n^2}}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal-distance runs from rest, square the time ratio before comparing accelerations; at 45°, sine and cosine cancel.

53. The equivalent capacitance between A and B in the circuit given below, is : [4 marks]



(A) $2.4\,\mu F$

(B) $4.9\,\mu F$

(C) $3.6\,\mu F$

(D) $5.4\,\mu F$

Correct Answer: (A) $2.4\,\mu F$

Solution:

Concept:

- Capacitors connected between the same two nodes are in parallel and add directly: $C_p = C_1 + C_2 + \cdots$. A node includes every point joined by an uninterrupted ideal wire.
- Capacitors in series carry the same charge, with reciprocal equivalent capacitance $1/C_s = 1/C_1 + 1/C_2 + \cdots$. Circuit topology, rather than the geometric placement of symbols, decides the reduction.

Step 1: Identify the parallel groups Call the node immediately after the leftmost $6\,\mu$F capacitor P. Call the junction to the right of the upper $2\,\mu$F capacitor Q. The lower horizontal connecting wire joins the lower plates of both $5\,\mu$F capacitors to Q. Their upper plates are both at P. Therefore both $5\,\mu$F capacitors and the upper $2\,\mu$F capacitor lie between P and Q:

$$C_{PQ} = 5 + 5 + 2 = 12\,\mu\text{F}.$$

Step 2: Reduce the right-hand group Between Q and the terminal B, the lower $2\,\mu$F and rightmost $4\,\mu$F capacitors share both end nodes. They are in parallel:

$$C_{QB} = 2 + 4 = 6\,\mu\text{F}.$$

The circuit has now reduced to three successive capacitors: $6\,\mu$F from A to P, $12\,\mu$F from P to Q, and $6\,\mu$F from Q to B.

Step 3: Combine the series chain The intermediate nodes of the reduced chain have no external connection, so the three groups are in series:

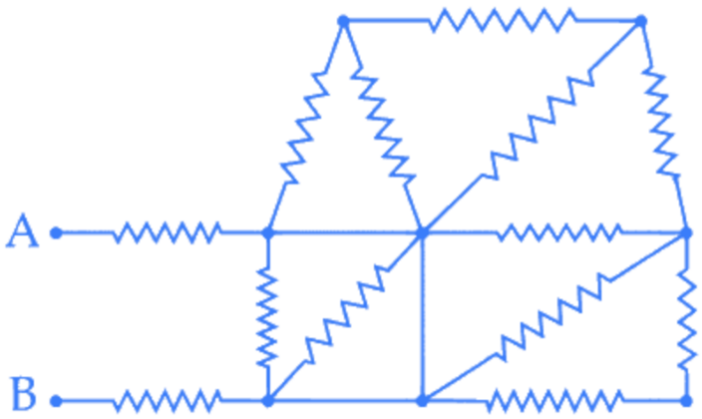
$$\frac{1}{C_{AB}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12} \quad (\mu\text{F})^{-1}.$$

Hence $C_{AB} = 12/5 = 2.4\,\mu$F. A series result must be smaller than every capacitance in the chain; the reciprocal sum fixes the value uniquely and rules out 4.9, 3.6 and $5.4\,\mu$F.

Step 4: Conclusion The equivalent capacitance is $\boxed{2.4\,\mu\text{F}}$. Hence, the correct option is (A).

Quick Tip to Solve: Mark all wire-connected points as one node before grouping capacitors; the two vertical $5\,\mu$F capacitors are parallel with the upper $2\,\mu$F.

54. In the given circuit all resistances are of value $R\ ohm$ each. The equivalent resistance between A and B is : [4 marks]



(A) $2R$

(B) $3R$

(C) $\frac{5R}{3}$

(D) $\frac{5R}{2}$

Correct Answer: (A) $2R$

Solution:

Concept:

- Every point joined by an uninterrupted ideal wire has the same electric potential. A resistor whose ends belong to the same node is short-circuited and has zero potential difference.
- For series resistors traversed by the same current, $R_{\text{eq}} = R_1 + R_2$. A subnetwork attached to the rest of a circuit at only one node cannot provide a conducting path between distinct external terminals.

Step 1: Trace the central conducting node Let P be the point just after the resistor connected to terminal A, and let Q be the point just after the resistor connected to terminal B. The diagram contains an ideal horizontal wire from P to the centre, a vertical wire from the centre to the bottom-middle junction, and a bottom horizontal wire from that junction to Q. Thus

$$V_P = V_{\text{centre}} = V_{\text{bottom middle}} = V_Q.$$

These points form a single node despite being drawn at different positions.

Step 2: Remove electrically inactive loops The resistor directly joining P to Q, and the diagonal

resistor from Q to the centre, are bypassed by wires. The upper and right-hand collections of resistors connect back only to this same common node; they have no second terminal linked independently to A or B. With no source inside those loops, their currents vanish. Their shapes and the number of resistors do not alter the two-terminal resistance.

Step 3: Add the remaining series resistors Current entering from A must pass through its adjacent resistor R, reach the common central node, then pass through the adjacent resistor R to leave at B. Therefore

$$R_{AB} = R + 0 + R = 2R.$$

Both terminal resistors are unavoidable; no branch bypasses either of them. The alternative values $3R$ and $5R/2$ retain an inactive internal resistance, while $5R/3 < 2R$ is impossible because those two unavoidable resistors are already in series.

Step 4: Conclusion The equivalent resistance between the terminals is $\boxed{2R}$. Hence, the correct option is (A).

Quick Tip to Solve: Collapse every uninterrupted wire into a single node first: here that operation shorts the whole internal network between the two terminal resistors.

55. A particle is oscillating on the X-axis with an amplitude $2\ cm$ about the point $x_0 = 10\,cm$, with a frequency ω. A concave mirror of local length $5\ cm$ is placed at the origin (see figure). Identify the correct statements. [4 marks]



(A) The image executs periodic motoin.

(B) The image executes non-periodic motion.

(C) The turning points of the image are asymmetric w.r.t. the image of the point at $x = 10\ cm$.

(D) The distance between the turning points of the oscillation of the image is $\frac{100}{21}\ cm$.

Correct Answer: Statements (A), (C) and (D) are simultaneously true; the printed choices do not provide a unique correct option.

Solution:

Concept:

- Using positive object and real-image distances from the mirror, the mirror equation is $1/f = 1/u + 1/v$, so $v = fu/(u - f)$. Here $f = 5\,\text{cm}$ and the object stays beyond the focus.
- A single-valued function of a periodic object position is periodic if it remains finite. The nonlinear mirror mapping need not preserve symmetric excursions or simple harmonic motion.

Step 1: Find the image position as a function of time The object distance ranges from 8 to $12\,\text{cm}$. If

its position is written as $u(t) = 10 + 2\cos(\Omega t)$, with Ω its angular frequency, then

$$v(t) = \frac{5[10 + 2\cos(\Omega t)]}{5 + 2\cos(\Omega t)} \text{ cm}.$$

The denominator never vanishes in this interval. The image therefore repeats after every object period: statement (A) is true and statement (B) is false. The conclusion is unchanged if the stated frequency is ordinary rather than angular frequency.

Step 2: Calculate the image turning points The derivative $dv/du = -25/(u-5)^2$ is negative, so the endpoints of the object interval map to opposite endpoints of the image interval. Specifically,

$$v(8) = \frac{40}{3}\text{ cm}, \qquad v(12) = \frac{60}{7}\text{ cm}, \qquad v(10) = 10\,\text{cm}.$$

The displacements from the image of the mean object position are $10/3\,\text{cm}$ and $-10/7\,\text{cm}$. They have unequal magnitudes; hence statement (C) is true.

Step 3: Check the separation and all statements The separation between turning points is

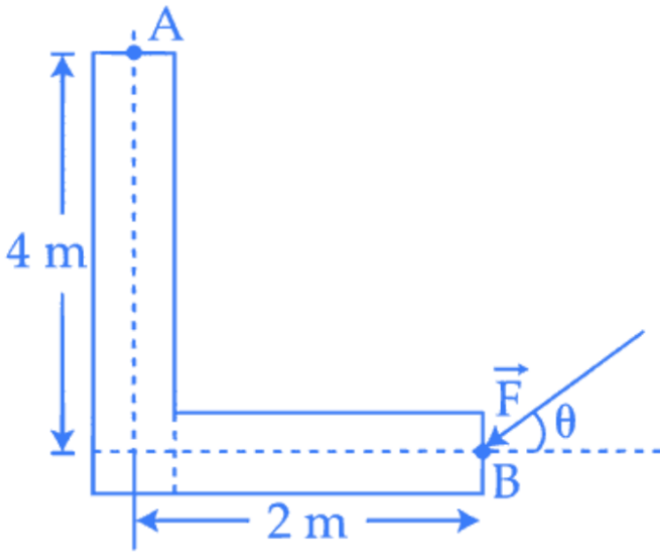
$$\frac{40}{3} - \frac{60}{7} = \frac{280 - 180}{21} = \frac{100}{21}\text{ cm}.$$

Thus statement (D) is also true. The image is periodic but its displacement is not a sinusoid centred at $v(10)$. All four statements have now been tested directly using the mirror equation, rather than inferring periodicity from the unequal endpoint displacements.

Step 4: Conclusion The correct statements are (A), (C) and (D); (B) is false. There is no unique correct option among these individual statements. The question-sheet key selects (B), but the finite expression for $v(t)$ is explicitly periodic and disproves that selection.

Quick Tip to Solve: For mirror-image oscillations, map both object endpoints with the mirror formula; periodicity survives even when symmetric excursions do not.

56. A force of $40\ N$ acts on a point B at the end of an L-shaped object, as shown in the figure. The angle θ that will produce maximum moment of the force about point A is given by : [4 marks]



(A) $\tan\theta = \frac{1}{2}$

(B) $\tan\theta = 2$

(C) $\tan\theta = 4$

(D) $\tan\theta = \frac{1}{4}$

Correct Answer: (A) $\tan\theta = \frac{1}{2}$

Solution:

Concept:

- The moment of a force about A is $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ points from A to the force application point B. For fixed force magnitude, its largest magnitude occurs when $\mathbf{F} \perp \mathbf{r}$.
- The figure gives a horizontal separation of $2\,\mathrm{m}$ and a vertical separation of $4\,\mathrm{m}$ between A and B. The moment arm is determined by their straight-line separation, not the total length of the bent object.

Step 1: Resolve the position and force vectors Take the positive x-axis to the right and the positive y-axis upward. The displacement from A to B is

$$\mathbf{r} = (2\hat{\mathbf{i}} - 4\hat{\mathbf{j}})\,\mathrm{m}.$$

The force arrow points down and left along a line making the angle θ with the horizontal, so

$$\mathbf{F} = -40\cos\theta\,\hat{\mathbf{i}} - 40\sin\theta\,\hat{\mathbf{j}}\ \mathrm{N}.$$

For the acute angle shown, both components produce clockwise moment about A.

Step 2: Form the moment magnitude The perpendicular component of the vector product is

$$\tau_z = r_x F_y - r_y F_x = -80\sin\theta - 160\cos\theta.$$

Its magnitude is thus $80\sin\theta + 160\cos\theta$ in newton metres. Maximising this expression gives

$$\frac{d|\tau_z|}{d\theta} = 80\cos\theta - 160\sin\theta = 0, \qquad \tan\theta = \frac{1}{2}.$$

The second derivative is negative at this acute-angle stationary point, so it is a maximum.

Step 3: Check the geometric condition The perpendicular-force criterion gives the same result directly: $\mathbf{r}\cdot\mathbf{F} = 0$ implies $-80\cos\theta + 160\sin\theta = 0$. The other choices 2, 4 and $1/4$ do not make this dot product zero and therefore cannot give the full value $F|\mathbf{r}|$. The force magnitude affects the maximum torque but not the angle that produces it.

Step 4: Conclusion The required angle satisfies $\boxed{\tan\theta = \frac{1}{2}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a fixed force at a fixed point, maximise torque by making its line perpendicular to the straight segment joining the pivot and application point.

57. A Carnot's engine works as a refrigerator between 250 K and 300 K. It receives 500 cal heat from the reservoir at the lower temperature. The amount of work done in each cycle to operate the refrigerator is : [4 marks]

(A) 420 J

(B) 772 J

(C) 2100 J

(D) 2520 J

Correct Answer: (A) 420 J

Solution:

Concept:

- A reversible refrigerator transfers heat from a cold reservoir at T_c to a hot reservoir at T_h, using external work W. Its coefficient of performance is $K = Q_c/W = T_c/(T_h - T_c)$.
- For a reversible cycle, $Q_h/T_h = Q_c/T_c$, and energy conservation gives $W = Q_h - Q_c$. Temperatures in these relations must be absolute temperatures.

Step 1: Identify the absorbed heat and temperatures The refrigerator receives heat from the colder reservoir, so the specified $500\,\text{cal}$ is Q_c, not the heat rejected Q_h. The reservoir temperatures are already in kelvin: $T_c = 250\,\text{K}$ and $T_h = 300\,\text{K}$. Thus the temperature difference is $50\,\text{K}$.

Step 2: Calculate the work supplied in one cycle First determine the coefficient of performance, then divide the extracted heat by it.

$$K = \frac{250}{300 - 250} = 5, \qquad W = \frac{Q_c}{K} = \frac{500}{5} = 100\,\text{cal}.$$

Using $1\,\text{cal} = 4.2\,\text{J}$,

$$W = 100(4.2) = 420\,\text{J}.$$

As a check, the rejected heat is $600\,\text{cal}$, and $600/300 = 500/250$, as required for reversibility.

Step 3: Compare the available choices Option (A) equals the work input. Option (C), $2100\,\text{J}$, is the heat extracted, not the work. Option (D), $2520\,\text{J}$, is the heat rejected to the hot reservoir. The remaining value $772\,\text{J}$ does not satisfy the reversible heat ratio and energy balance together.

Step 4: Conclusion The external work required per cycle is $\boxed{420\,\text{J}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a Carnot refrigerator, multiply the cold-side heat by $(T_h - T_c)/T_c$; do not use the heat-engine efficiency directly.

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58. In a common emitter configuration with suitable bias, it is given that R_L is the load resistance and R_{BE} is small signal dynamic resistance (input side). Then, voltage gain, current gain and power gain are given, respectively, by :

β is curret gain, I_B, I_C and I_E are respectively base, collector and emitter currents. [4 marks]

(A) $\beta\frac{R_L}{R_{BE}}, \frac{\Delta\text{I}_C}{\Delta\text{I}_B}, \beta^2\frac{R_L}{R_{BE}}$

(B) $\beta\frac{R_L}{R_{BE}}, \frac{\Delta\text{I}_E}{\Delta\text{I}_B}, \beta^2\frac{R_L}{R_{BE}}$

(C) $\beta^2\frac{R_L}{R_{BE}}, \frac{\Delta\text{I}_C}{\Delta\text{I}_E}, \beta^2\frac{R_L}{R_{BE}}$

(D) $\beta^2\frac{R_L}{R_{BE}}, \frac{\Delta\text{I}_C}{\Delta\text{I}_B}, \beta\frac{R_L}{R_{BE}}$

Correct Answer: (A) $\beta\frac{R_L}{R_{BE}}, \frac{\Delta\text{I}_C}{\Delta\text{I}_B}, \beta^2\frac{R_L}{R_{BE}}$

Solution:

Concept:

- For a suitably biased common-emitter transistor, the small-signal current gain is $\beta = \Delta I_C/\Delta I_B$. The input signal voltage is $\Delta V_{BE} = \Delta I_B R_{BE}$.
- The output voltage magnitude across the load is $|\Delta V_o| = \Delta I_C R_L$. Voltage and power gains in the options are positive magnitudes; the actual common-emitter output voltage is phase-inverted.

Step 1: Calculate the voltage-gain magnitude The same small change of base current produces an input voltage across the dynamic input resistance and an amplified collector-current change through the load. Therefore,

$$|A_v| = \frac{|\Delta V_o|}{|\Delta V_{BE}|} = \frac{\Delta I_C R_L}{\Delta I_B R_{BE}} = \beta\frac{R_L}{R_{BE}}.$$

The signed voltage gain has a minus sign because increasing base current lowers the collector voltage, but the question lists gain magnitudes.

Step 2: Write the current and power gains The output current is the collector signal current, while the input current is the base signal current. Thus

$$A_i = \frac{\Delta I_C}{\Delta I_B} = \beta.$$

Power is voltage times current, so the ratio of signal powers is the product of voltage- and current-gain

magnitudes:

$$A_p = |A_v|A_i = \beta^2 \frac{R_L}{R_{BE}}.$$

This also follows from dividing $(\Delta I_C)^2 R_L$ by $(\Delta I_B)^2 R_{BE}$.

Step 3: Eliminate the incorrect gain combinations Option (B) uses emitter current in the output-current ratio, giving $\Delta I_E/\Delta I_B = \beta + 1$, not the common-emitter current gain. Options (C) and (D) incorrectly put β^2 into the voltage gain; (C) also uses the common-base current ratio. Only (A) contains all three derived quantities in the requested order.

Step 4: Conclusion The voltage, current and power gains are $\beta R_L/R_{BE}$, $\Delta I_C/\Delta I_B$, and $\beta^2 R_L/R_{BE}$, respectively. Hence, the correct option is (A).

Quick Tip to Solve: Derive voltage gain from the two resistive voltage drops; multiply it by the collector-to-base current gain to obtain power gain.

59. The number of amplitude modulted broadcast stations that can be accomodated in a $300\ kHz$ band width for the highest modulating frequency $15\ kHz$ will be : [4 marks]

(A) 20

(B) 15

(C) 10

(D) 8

Correct Answer: (C) 10

Solution:

Concept:

- An amplitude-modulated wave has an upper sideband at $f_c + f_m$ and a lower sideband at $f_c - f_m$, where f_c is the carrier frequency and f_m is a modulating frequency.
- For a highest modulating frequency $f_{m,\max}$, the transmitted AM channel requires bandwidth $B_{\text{ch}} = 2f_{m,\max}$. Both sidebands must fit inside the allocated frequency band.

Step 1: Determine the bandwidth of one station The audio information extends up to $15\,\text{kHz}$, so each station produces sidebands extending $15\,\text{kHz}$ below and $15\,\text{kHz}$ above its carrier. The full occupied width is consequently

$$B_{\text{ch}} = (f_c + 15) - (f_c - 15) = 30\,\text{kHz}.$$

The carrier lies in the middle of this interval and does not require an additional separate band.

Step 2: Divide the available band into channels For the maximum ideal number, take non-overlapping channels next to one another. No additional guard-band width is specified in the question, so none is added. With an available bandwidth of $300\,\text{kHz}$,

$$N = \frac{B_{\text{available}}}{B_{\text{ch}}} = \frac{300\,\text{kHz}}{30\,\text{kHz}} = 10.$$

The units cancel, leaving the integer number of stations.

Step 3: Check the other proposed numbers Twenty stations would allot only $15\,\text{kHz}$ per station and overlook one of the sidebands. Fifteen stations would allow $20\,\text{kHz}$ each, which is also insufficient. Eight stations would fit, but would leave $60\,\text{kHz}$ unused, so it is not the maximum number that can be accommodated under the stated ideal allocation.

Step 4: Conclusion The band can accommodate $\boxed{10}$ AM broadcast stations. Hence, the correct option is (C).

Quick Tip to Solve: For ordinary AM allocation, double the highest audio frequency before dividing the total available bandwidth by the channel width.

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60. A solution containing active cobalt $^{60}_{27}Co$ having activity of $0.8\ \mu Ci$ and decay constant λ is injected in an animal's body. If $1\ cm^3$ of blood is drawn from the animal's body after 10 hrs of injection, the activity found was 300 decays per minute What is the volume of blood that is flowing in the body ? $\left(\ Ci = 3.7 \times 10^{10}\ \text{decays per second and at } t = 10 \text{ hrs } e^{-\lambda t} = 0.84\right)$ [4 marks]

(A) 6 liters

(B) 7 liters

(C) 4 liters

(D) 5 liters

Correct Answer: (D) 5 liters

Solution:

Concept:

- Radioactive activity follows $A(t) = A_0 e^{-\lambda t}$, where A_0 is the injected activity and λ is the decay constant. Use the supplied survival factor at the sampling time.
- After uniform mixing, activity per unit volume is the same throughout the blood. Thus total blood volume equals the remaining total activity divided by the activity concentration in the sample.

Step 1: Convert and decay-correct the injected activity The initial activity is $0.8\,\mu\text{Ci} = 0.8 \times 10^{-6}\,\text{Ci}$. Using the conversion given in the question,

$$A_0 = 0.8 \times 10^{-6} \times 3.7 \times 10^{10} = 2.96 \times 10^{4}\,\text{s}^{-1}.$$

Ten hours later the remaining activity is

$$A = 2.96 \times 10^{4}(0.84) = 24864\,\text{s}^{-1}.$$

It is essential to use the activity at the sampling time, rather than the initial injection activity.

Step 2: Use the measured activity concentration The $1\,\text{cm}^3$ sample gives 300 decays per minute, or $300/60 = 5$ decays per second. Its activity concentration is therefore $5\,\text{s}^{-1}\text{cm}^{-3}$. Uniform mixing gives

$$V = \frac{24864\,\text{s}^{-1}}{5\,\text{s}^{-1}\text{cm}^{-3}} = 4972.8\,\text{cm}^3 = 4.9728\,\text{L}.$$

This tracer-dilution calculation presumes that the tracer stays in the circulating blood, as intended by

the question.

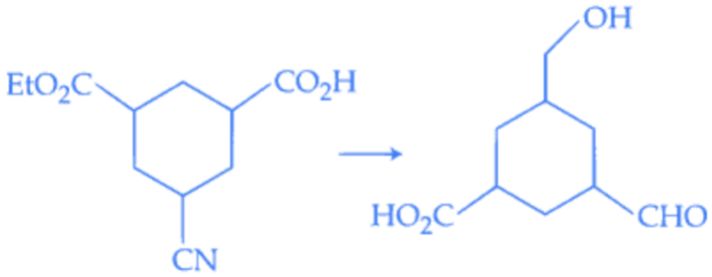
Step 3: Select the consistent rounded volume The result is approximately $5\,\mathrm{L}$, matching option (D). Volumes of 6, 7, or $4\,\mathrm{L}$ would predict activity concentrations of about 4.14, 3.55, or $6.22\,\mathrm{s^{-1}cm^{-3}}$, respectively, rather than the observed value of 5.

Step 4: Conclusion The circulating blood volume is approximately $\boxed{5\,\mathrm{L}}$. Hence, the correct option is (D).

Quick Tip to Solve: First decay-correct the total tracer activity, then divide by sample activity per cubic centimetre; convert minutes to seconds consistently.

Chemistry (120 marks)

61. The reagent(s) required for the following conversion are : [4 marks]



(A) (i) B_2H_6; (ii) $SnCl_2/HCl$; (iii) H_3O^+

(B) (i) $LiAlH_4$; (ii) H_3O^+

(C) (i) B_2H_6; (ii) DIBALH; (iii) H_3O^+

(D) (i) $NaBH_4$; (ii) Raney Ni/H_2; (iii) H_3O^+

Correct Answer: (A) (i) B_2H_6; (ii) $SnCl_2/HCl$; (iii) H_3O^+

Solution:

Concept:

- A molecule with several functional groups requires chemoselective reagents: reducing one group must not destroy a group needed in the final product.
- Diborane selectively reduces a carboxylic acid to a primary alcohol. Stephen reduction uses $SnCl_2/HCl$ to convert a nitrile to an iminium salt, which gives an aldehyde on hydrolysis.

Step 1: Identify the changes around the ring The initial ring bears an ethyl ester, a carboxylic acid and a nitrile. The product bears a carboxylic acid, a hydroxymethyl group and an aldehyde. Since the three positions alternate around the cyclohexane ring, rotating the drawing does not change their connectivity. The required transformations are $-CO_2H \longrightarrow -CH_2OH$, $-CN \longrightarrow -CHO$, and hydrolysis of $-CO_2Et$ to $-CO_2H$.

Step 2: Apply the selective reagent sequence First, B_2H_6 preferentially reduces the free acid to $-CH_2OH$, leaving the ester available for later hydrolysis. Next, $SnCl_2/HCl$ partially reduces the nitrile in the Stephen reaction rather than reducing it all the way to an amine. Finally, acidic aqueous

work-up hydrolyses the iminium intermediate to $-CHO$ and converts the ethyl ester to the acid. This yields precisely the three functional groups shown in the product.

Step 3: Reject the nonselective alternatives $LiAlH_4$ in (B) would reduce the ester and acid to alcohols and the nitrile to an amine, destroying the desired pattern. DIBALH in (C) is used for partial ester/nitrile reduction and would not preserve the ester as the acid precursor under the required sequence. $NaBH_4$ in (D) does not carry out the necessary acid-to-alcohol reduction, and hydrogenation of the nitrile with Raney nickel leads towards an amine rather than the required aldehyde.

Step 4: Conclusion The suitable sequence is (i) B_2H_6, (ii) $SnCl_2/HCl$, and (iii) H_3O^+. Hence, the correct option is (A).

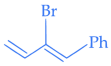
Quick Tip to Solve: Track the acid, ester and nitrile separately: use diborane for the acid and Stephen reduction for the nitrile-to-aldehyde conversion.

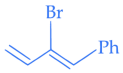
62. Which of the following will most readily give the dehydrohalogenation product ? [4 marks]

(A) $CH_2=CH-C(Br)=CH-CH_3$ 

(B) Bromobenzene 

(C) 1-Bromocyclohex-1-ene 

(D) $CH_2=CH-C(Br)=CH-Ph$ 

Correct Answer: (D) $CH_2=CH-C(Br)=CH-Ph$ 

Solution:

Concept:

- Dehydrohalogenation removes a hydrogen from a carbon adjacent to the carbon bearing the halogen, forming an additional carbon-carbon pi bond.
- Elimination from a vinylic halide can form an alkyne. Among comparable substrates, a product with extended conjugation is favoured, whereas a highly strained small-ring alkyne is difficult to form.

Step 1: Compare the open-chain vinylic bromides In (A) and (D), bromine is attached to the central vinylic carbon of a conjugated diene. Removing that bromine and the hydrogen on the adjacent alkene carbon can generate a carbon-carbon triple bond. For (A), the resulting chain is $CH_2=CH-C\equiv C-CH_3$. For (D), it is $CH_2=CH-C\equiv C-Ph$, where Ph denotes a phenyl group. Both avoid the severe ring-strain penalty of a small cyclic alkyne.

Step 2: Identify the extra stabilisation in option D The elimination product from (D) has the triple bond conjugated with the vinyl group on one side and the phenyl ring on the other. The phenyl group provides additional delocalisation compared with the methyl group in (A). It also stabilises the developing unsaturated, carbanion-like character at the beta carbon during base-promoted elimination. Consequently (D) most readily gives the dehydrohalogenation product among the listed compounds.

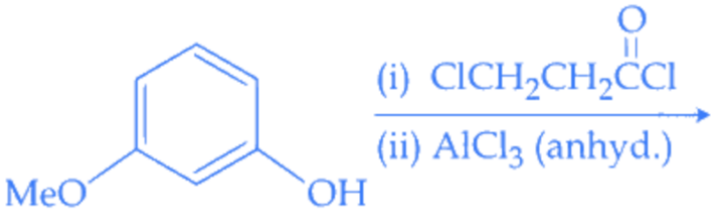
Step 3: Rule out the cyclic alternatives Option (B) is bromobenzene, whose aryl carbon-bromine bond is strong; eliminating hydrogen bromide would generate a high-energy benzyne intermediate

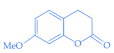
and requires harsh conditions. Option (C) is a vinylic bromide in a six-membered ring, so elimination towards an alkyne would demand a severely strained cyclohexyne. These are less favourable than the open-chain, conjugation-stabilised product of (D). Option (A) lacks the additional phenyl conjugation that distinguishes (D).

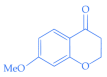
Step 4: Conclusion The phenyl-substituted open-chain vinylic bromide undergoes dehydrohalogenation most readily. Hence, the correct option is (D).

Quick Tip to Solve: For vinylic-halide elimination, draw the prospective alkyne mentally and compare conjugation and ring strain before applying ordinary alkyl-halide trends.

63. The major product of the following reaction is : [4 marks]

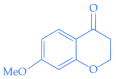


(A) 7-Methoxychroman-2-one 

(B) 7-Methoxychroman-4-one 

(C) 5-Methoxychroman-2-one 

(D) 5-Methoxychroman-4-one 

Correct Answer: (B) 7-Methoxychroman-4-one 

Solution:

Concept:

- A phenolic hydroxyl group reacts with an acid chloride to form an aryl ester. Anhydrous $AlCl_3$ promotes Fries rearrangement, transferring the acyl group onto the aromatic ring.
- The phenolic oxygen can subsequently close a ring by displacement of a terminal chloride. The position of the carbonyl distinguishes a chroman-4-one from a chroman-2-one lactone.

Step 1: Follow the three-carbon acyl side chain The reagent is $ClCH_2CH_2COCl$, 3-chloropropanoyl chloride. Acylation first gives the phenolic ester $Ar-O-COCH_2CH_2Cl$. In the presence of anhydrous $AlCl_3$, Fries rearrangement places the acyl group at an ortho position of the phenolic ring, regenerating the phenolic hydroxyl group and giving an ortho $-COCH_2CH_2Cl$ side chain.

Step 2: Choose the less hindered activated ring position The methoxy group is meta to the original hydroxyl group. Of the two positions ortho to hydroxyl, the site lying between hydroxyl and methoxy is more crowded. Acylation at the other ortho site is para to methoxy and is strongly activated without that steric crowding. This determines the substitution pattern corresponding to the 7-methoxy product rather than the 5-methoxy isomer.

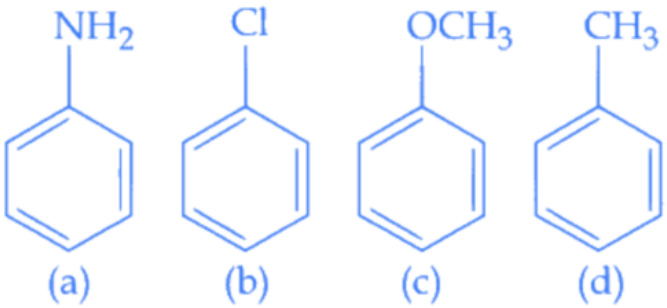
Step 3: Close the heterocyclic ring and compare options The adjacent phenolic oxygen attacks the terminal $-CH_2Cl$ carbon intramolecularly. Displacement of chloride closes a six-membered oxygen

heterocycle, with connectivity $Ar-O-CH_2-CH_2-CO-Ar$. Thus the carbonyl lies at position 4, bonded to the aromatic ring, rather than at position 2 next to oxygen. Options (A) and (C) have the lactone carbonyl connectivity and are rejected. Option (D) is the more hindered 5-methoxy regioisomer. The less hindered 7-methoxychroman-4-one is shown in (B).

Step 4: Conclusion The major product is 7-methoxychroman-4-one, with the ketone carbonyl adjacent to the aromatic ring. Hence, the correct option is (B).

Quick Tip to Solve: Trace the acid-chloride carbonyl through Fries rearrangement: it becomes the ring ketone carbonyl, while the phenolic oxygen closes onto the terminal chloroalkyl carbon.

64. The increasing order of nitration of the following compounds is : [4 marks]



(A) (b) < (a) < (c) < (d)

(B) (a) < (b) < (c) < (d)

(C) (b) < (a) < (d) < (c)

(D) (a) < (b) < (d) < (c)

Correct Answer: (D) (a) < (b) < (d) < (c)

Solution:

Concept:

- Nitration is an electrophilic aromatic substitution; its rate depends on the electron density available to attack the nitronium ion, NO_2^+.
- The strongly acidic nitrating mixture protonates aniline to the anilinium ion. Compare substituent effects in that reaction medium, rather than using the activating effect of free $-NH_2$.

Step 1: Identify the four substituents The drawings represent (a) aniline, (b) chlorobenzene, (c) anisole and (d) toluene. The amino group of aniline accepts a proton in the nitric-acid/sulphuric-acid medium:

$$C_6H_5NH_2 + H^+ \rightleftharpoons C_6H_5NH_3^+.$$

The resulting positively charged group strongly withdraws electron density from the ring and makes nitration difficult.

Step 2: Compare the remaining ring reactivities Chlorine withdraws by its inductive effect, so chlorobenzene is deactivated, but the anilinium ion is still less reactive. The methyl group activates toluene through electron donation and hyperconjugation. The methoxy group of anisole supplies

electron density from an oxygen lone pair by resonance and is more activating than methyl. Hence the increasing order is

$$\text{aniline} < \text{chlorobenzene} < \text{toluene} < \text{anisole}.$$

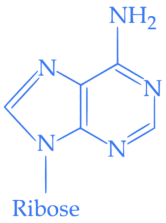
Options A and C put chlorobenzene below protonated aniline; options A and B also put toluene above anisole. These comparisons eliminate them.

Step 3: Conclusion The increasing nitration rate is aniline, chlorobenzene, toluene, anisole. Hence, the correct option is (D).

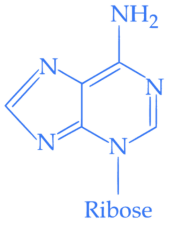
Quick Tip to Solve: For nitration of aniline, check protonation in the strongly acidic medium before applying the usual activating-group order.

65. Which of the following is the correct structure of Adenosine? [4 marks]

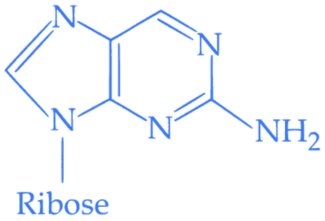
(A)



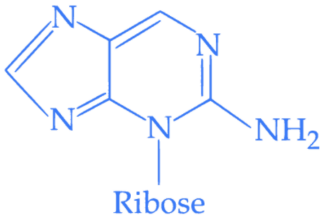
(B)



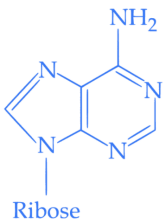
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- A nucleoside consists of a nitrogenous base joined to a pentose sugar; it contains no phosphate group, unlike a nucleotide. Adenosine is the ribose nucleoside of adenine.
- Adenine is 6-aminopurine. In its natural ribonucleoside, the anomeric carbon $C1'$ of ribose is bonded to $N9$ of the purine ring through a β-N-glycosidic bond.

Step 1: Locate the amino group of adenine The purine skeleton is a fused five-membered and six-membered nitrogen-containing ring. In adenine, the exocyclic NH_2 group is at position 6, on the upper carbon of the six-membered ring in option A. The amino groups in C and D occur at the other illustrated carbon, so those bases are not the required 6-aminopurine base.

Step 2: Check the ribose attachment The sugar must be attached to the nitrogen of the five-membered ring at position 9. Option A shows ribose bonded to this nitrogen and preserves the adenine amino-group position. Option B attaches ribose to a nitrogen in the six-membered ring, so it is not the normal adenosine linkage. Option D also has that wrong attachment, in addition to the wrong amino position. Thus A alone satisfies both structural requirements. The label Ribose represents the sugar component, while the depicted purine supplies the base; together they define the nucleoside rather than a nucleotide.

Step 3: Conclusion Adenosine is adenine linked through $N9$ to ribose, as shown in option A. Hence, the correct option is (A).

Quick Tip to Solve: Check two features in a purine nucleoside: the base substituent position and the N9-to-ribose linkage.

66. Identify the pair in which the geometry of the species is T-shape and square - pyramidal, respectively : [4 marks]

(A) ClF_3 and IO_4^-

(B) ICl_2^- and ICl_5

(C) $XeOF_2$ and $XeOF_4$

(D) IO_3^- and $IO_2F_2^-$

Correct Answer: (C) $XeOF_2$ and $XeOF_4$

Solution:

Concept:

- In VSEPR theory, each bond direction counts as one electron domain, even if the bond is multiple. Lone pairs occupy domains but are omitted when naming the molecular shape.
- An AX_3E_2 arrangement has a trigonal-bipyramidal electron-domain geometry and a T-shaped molecular geometry. An AX_5E arrangement is square-pyramidal.

Step 1: Count domains for the xenon pair In $XeOF_2$, xenon has three bonded atoms: one oxygen and two fluorines. Two lone pairs remain on xenon, so it is AX_3E_2. The lone pairs take equatorial sites and the three atom directions form a T. In $XeOF_4$, xenon has five bonded atoms and one lone

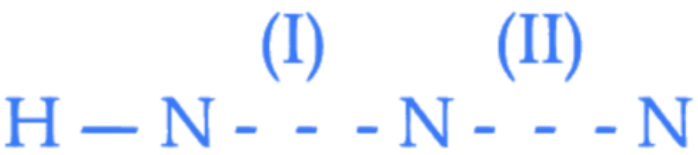
pair: AX_5E. Its six electron domains are octahedral; removing the lone-pair direction gives a square pyramid.

Step 2: Eliminate the other pairs Although ClF_3 is T-shaped, IO_4^- is tetrahedral, so A fails its second requirement. In B, ICl_2^- is AX_2E_3 and therefore linear, even though ICl_5 is square-pyramidal. In D, IO_3^- has three bonds and a lone pair and is trigonal-pyramidal, not T-shaped; $IO_2F_2^-$ has four bonded atoms and a lone pair and is seesaw-shaped. Thus only the xenon pair gives both requested geometries in the specified order.

Step 3: Conclusion $XeOF_2$ is T-shaped and $XeOF_4$ is square-pyramidal. Hence, the correct option is (C).

Quick Tip to Solve: Count bonded atoms and lone pairs separately; a double bond occupies one direction in VSEPR shape counting.

67. In hydrogen azide (above) the bond orders of bond (I) and (II) are : [4 marks]



(A) Bond (I): < 2; bond (II): > 2.

(B) Bond (I): > 2; bond (II): < 2.

(C) Bond (I): > 2; bond (II): > 2.

(D) Bond (I): < 2; bond (II): < 2.

Correct Answer: (A) Bond (I): < 2; bond (II): > 2.

Solution:

Concept:

- A resonance hybrid does not have the integral bond orders of a single Lewis structure. A bond shared between single- and double-bond contributors has bond order below two.
- For hydrogen azide, the neutral molecule has the connectivity $H{-}N{-}N{-}N$. Octet-satisfying resonance contributors redistribute the multiple bonding and formal charges over its nitrogen atoms.

Step 1: Write the important resonance contributors With the hydrogen-bearing nitrogen written on the left, two useful contributors are

$$H{-}N{=}N^+{=}N^- \longleftrightarrow H{-}N^-{-}N^+ \equiv N.$$

Both satisfy nitrogen octets. In the first, bonds (I) and (II) are both double bonds. In the second, bond (I) is a single bond and bond (II) is a triple bond. The second contributor is significant because its negative charge is on the nitrogen carrying hydrogen, whereas a less important contributor with the opposite extreme charge separation is disfavoured.

Step 2: Compare each bond with a double bond The hybrid therefore gives bond (I) partial single-bond character and bond (II) partial triple-bond character. Its bond orders satisfy

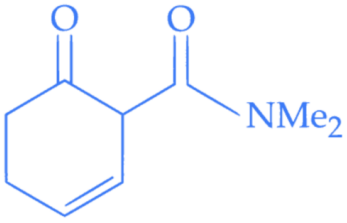
$$1 < \mathrm{BO(I)} < 2, \qquad 2 < \mathrm{BO(II)} < 3.$$

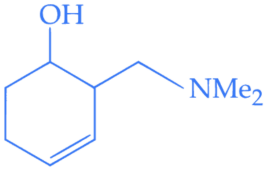
Equal weighting is not required and no exact fractional values need be assigned to choose the option. B reverses the two inequalities. C would make both nitrogen-nitrogen bonds stronger than double bonds, whereas D would make both weaker; neither represents the redistribution between these contributors.

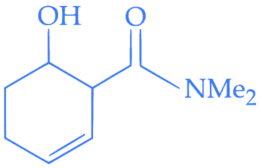
Step 3: Conclusion Bond (I) has bond order less than 2 and bond (II) has bond order greater than 2. Hence, the correct option is (A).

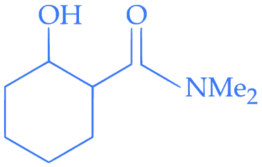
Quick Tip to Solve: Keep the hydrogen-bearing nitrogen fixed when comparing azide resonance forms; swapping the ends reverses the labelled bonds.

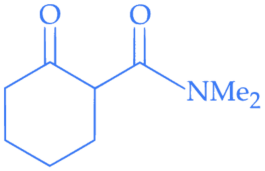
68. The main reduction product of the following compound with $NaBH_4$ in methanol is : [4 marks]



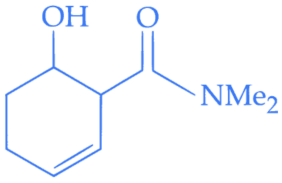
(A) 

(B) 

(C) 

(D) 

Correct Answer: (B)



Solution:

Concept:

- Sodium borohydride, $NaBH_4$, is a mild hydride reducing agent that normally converts aldehydes and ketones into alcohols in methanol.

- Under these conditions, an amide carbonyl is much less reactive because of nitrogen-to-carbonyl resonance donation. An isolated carbon-carbon double bond is also retained.

Step 1: Identify the functional groups in the substrate The molecule contains a cyclic ketone, an alkene within the ring, and an N,N-dimethylamide substituent, $-\mathrm{CON(CH_3)_2}$. The ring ketone is the carbonyl that reacts readily with borohydride. Hydride adds to its carbonyl carbon, giving an alkoxide; methanol protonates that alkoxide to produce a secondary alcohol at the same ring carbon.

Step 2: Preserve the groups that are not reduced The ring carbon-carbon double bond is not hydrogenated by $\mathrm{NaBH_4}$ in methanol. The amide group remains $-\mathrm{CON(CH_3)_2}$, since this reagent is not strong enough for ordinary amide-to-amine reduction. Option B alone shows the ring ketone replaced by $-\mathrm{CHOH}-$ with both the alkene and amide carbonyl intact. A incorrectly reduces the amide carbonyl to a methylene group. C also saturates the alkene. D retains the ketone while saturating the alkene, which is the opposite of the expected selectivity.

Step 3: Conclusion The product is the unsaturated cyclic alcohol with its dimethylamide carbonyl unchanged, shown in option B. Hence, the correct option is (B).

Quick Tip to Solve: With NaBH4 in methanol, mark the aldehyde or ketone carbonyl first; leave ordinary amides and isolated C=C bonds unchanged.

69. The correct match between items of List-I and List-II is:

List-I	List-II
(A) Coloured impurity	(P) Steam distillation
(B) Mixture of o-nitrophenol and p-nitrophenol	(Q) Fractional distillation
(C) Crude Naphtha	(R) Charcoal treatment
(d) Mixture of glycerol and sugars	(S) Distillation under reduced pressure

[4 marks]

(A) (A)-(R), (B)-(S), (C)-(P), (D)-(Q)

(B) (A)-(R), (B)-(P), (C)-(S), (D)-(Q)

(C) (A)-(R), (B)-(P), (C)-(Q), (D)-(S)

(D) (A)-(P), (B)-(S), (C)-(R), (D)-(Q)

Correct Answer: (C) (A)-(R), (B)-(P), (C)-(Q), (D)-(S)

Solution:

Concept:

- A purification method exploits a physical-property difference: adsorption removes coloured impurities, volatility separates liquids, and reduced pressure lowers boiling temperature.
- Steam distillation separates a steam-volatile component from a less volatile one; fractional distillation resolves a mixture containing components of different boiling ranges.

Step 1: Match the coloured impurity and nitrophenols Activated charcoal adsorbs many coloured impurities on its large surface area. Thus (A) matches (R), charcoal treatment. In o-nitrophenol,

intramolecular hydrogen bonding reduces intermolecular association, making it steam volatile. The para isomer forms intermolecular hydrogen bonds and is less volatile. Steam distillation therefore separates their mixture, giving (B)-(P).

Step 2: Match the petroleum fraction and glycerol mixture Crude naphtha contains hydrocarbons with different boiling points, so fractional distillation provides the required separation: (C)-(Q). Glycerol has a high boiling point and may decompose on strong heating at atmospheric pressure. Distillation under reduced pressure lowers its boiling temperature and separates it from nonvolatile sugars: (D)-(S). Together the four matches are

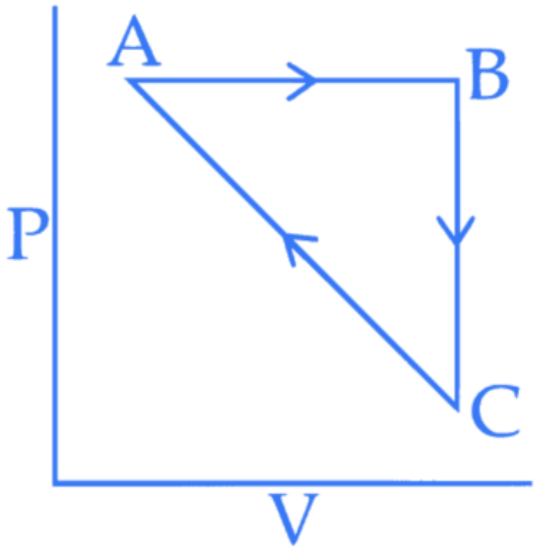
$$(A)-(R), \quad (B)-(P), \quad (C)-(Q), \quad (D)-(S).$$

A assigns steam distillation to naphtha; B interchanges the final two methods; D fails the charcoal match as well. Only C preserves every physical basis of separation.

Step 3: Conclusion Charcoal removes colour, steam separates nitrophenols, fractional distillation separates naphtha, and reduced pressure isolates glycerol. Hence, the correct option is (C).

Quick Tip to Solve: Anchor matching questions with charcoal for colour and reduced pressure for high-boiling, heat-sensitive liquids before comparing the remaining choices.

70. An ideal gas undergoes a cyclic process as shown in Figure.
$\Delta U_{BC} = -5$ kJ mol^{-1}, $q_{AB} = 2$ kJ mol^{-1}, $W_{AB} = -5$ kJ mol^{-1}, $W_{CA} = 3$ kJ mol^{-1}. Heat absorbed by the system during process CA is : [4 marks]



(A) -5 kJ mol^{-1}
(B) $+5$ kJ mol^{-1}
(C) 18 kJ mol^{-1}
(D) -18 kJ mol^{-1}
Correct Answer: (B) $+5$ kJ mol^{-1}

Solution:

Concept:

- Using the chemistry sign convention, the first law is $\Delta U = q + w$, where heat absorbed and work done on the system are positive.
- Internal energy is a state function. Its changes sum to zero around the complete cycle: $\Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$.

Step 1: Find the energy change along AB The stated values are $q_{AB} = +2\,\text{kJ mol}^{-1}$ and $w_{AB} = -5\,\text{kJ mol}^{-1}$. Therefore

$$\Delta U_{AB} = q_{AB} + w_{AB} = 2 - 5 = -3\,\text{kJ mol}^{-1}.$$

The negative work agrees with the expansion shown along AB. Heat input does not by itself imply a rise in internal energy, because the gas performs more work than the heat it receives.

Step 2: Close the energy cycle and calculate the heat For BC, $\Delta U_{BC} = -5\,\text{kJ mol}^{-1}$. Hence

$$\Delta U_{CA} = -[(-3) + (-5)] = +8\,\text{kJ mol}^{-1}.$$

The compression CA has $w_{CA} = +3\,\text{kJ mol}^{-1}$, so

$$q_{CA} = \Delta U_{CA} - w_{CA} = 8 - 3 = +5\,\text{kJ mol}^{-1}.$$

The sign is positive: the system absorbs heat. A reverses this sign, while C and D have a magnitude inconsistent with the first law and the zero net energy change of a cycle. No value of the gas heat capacity or temperature is needed.

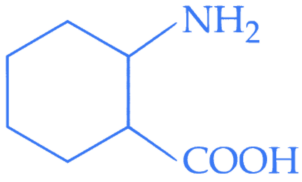
Step 3: Conclusion The heat absorbed during CA is $+5\,\text{kJ mol}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: For a cyclic process, find the missing internal-energy change first; then use q = Delta U - w with one consistent work convention.

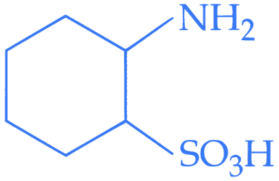
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71. Which of the following will not exist in zwitter ionic form at pH=7 ? [4 marks]

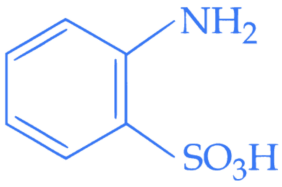
(A) 2-Aminocyclohexane-1-carboxylic acid (adjacent amino and carboxyl groups on a saturated six-membered ring).



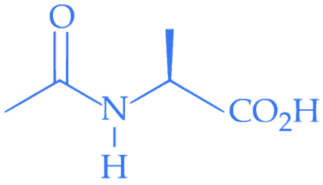
(B) 2-Aminocyclohexane-1-sulfonic acid (adjacent amino and sulfonic acid groups on a saturated six-membered ring).



(C) 2-Aminobenzenesulfonic acid (adjacent amino and sulfonic acid groups on a benzene ring).



(D) $CH_3C(=O)NHCH(CH_3)CO_2H$, with the methyl group on the wedge as drawn.



Correct Answer: Expected answer: D. At literal pH 7, C is also predominantly anionic, so the predominant-form interpretation is ambiguous.

Solution:

Concept:

- A zwitterion contains both positive and negative formal charges within one molecule. An amino acid can form $-NH_3{}^+$ and $-COO-$ by proton transfer.
- Protonation at a specified aqueous pH depends on the basicity of nitrogen. A zwitterionic representation used for an isolated solid does not by itself establish the predominant dissolved species at pH 7; a free aliphatic amino group, an aromatic amino group and an amide nitrogen have different basicities.

Step 1: Identify the nitrogen in each structure In (A) and (B), the amino group is bonded to a saturated ring. At neutral pH it can be protonated while the carboxylic or sulfonic acid group is ionised. Their usual ionic forms therefore contain both an ammonium group and a negative acid residue. In (D), however, nitrogen is directly bonded to an acetyl carbonyl: it is an amide, not a free amino group. Its lone pair is delocalised towards the carbonyl and does not give the usual ammonium-carboxylate zwitterion at pH 7.

Step 2: Check the literal pH condition for the aromatic option Option (C) is an aminobenzenesulfonic acid. Its amino lone pair is conjugated with the aromatic ring, and its anilinium conjugate acid is much more acidic than an aliphatic ammonium ion. At pH 7 its sulfonic acid is ionised and its amino group is predominantly unprotonated, so the predominant species is an anion rather than a zwitterion. Merely having both acidic and basic groups does not guarantee a zwitterionic predominant form at every pH. Thus (D) is the expected answer based on the amide distinction, but the stated pH makes a strictly unique choice problematic.

Step 3: Conclusion The expected structural answer is (D), the N-acetyl amino acid. Taken literally at pH 7, (C) also is not predominantly zwitterionic; hence the pH-specific wording does not give a unique predominant-form answer.

Quick Tip to Solve: Check whether nitrogen is a free amine, an aromatic amine or an amide before assigning an ammonium group at neutral pH.

72. In the molecular orbital diagram for the molecular ion, N_2^+, the number of electrons in the σ_{2p_z} molecular orbital is : [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Concept:

- Neutral N_2 has fourteen electrons; N_2^+ has thirteen. Ionisation removes one electron from the highest occupied molecular orbital.
- For nitrogen the bonding π_{2p_x} and π_{2p_y} orbitals lie below the bonding σ_{2p_z} orbital because of s-p mixing.

Step 1: Fill the orbitals of neutral nitrogen The occupied molecular orbitals of neutral nitrogen have the configuration

$$(\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2(\pi_{2p_x})^2(\pi_{2p_y})^2(\sigma_{2p_z})^2.$$

The two degenerate pi orbitals are completely filled before the final pair enters the sigma orbital. The highest occupied orbital is therefore σ_{2p_z}, not either pi orbital.

Step 2: Remove the electron and count the remaining occupancy The positive charge means that one electron has been removed, not added. Removal from the highest occupied orbital changes its occupancy as follows:

$$(\sigma_{2p_z})^2 \longrightarrow (\sigma_{2p_z})^1.$$

Thus exactly one electron remains in this orbital. Zero would require removal of both sigma electrons; two would describe neutral nitrogen in this orbital. Three electrons cannot occupy any one molecular orbital because an orbital holds at most two electrons with opposite spins.

Step 3: Conclusion The σ_{2p_z} molecular orbital of N_2^+ contains 1 electron. Hence, the correct option is (B).

Quick Tip to Solve: For nitrogen, remember that the pi bonding pair lies below the 2p sigma level; ionisation removes a sigma electron first.

73. Ejection of the photoelectron from metal in the photoelectric experiment can be stopped by applying 0.5 V when the radiation of 250 nm is used. The work function of the metal is : [4 marks]

(A) 4 eV

(B) 4.5 eV

(C) 5 eV

(D) 5.5 eV

Correct Answer: (B) 4.5 eV

Solution:

Concept:

- Einstein's photoelectric equation is $h\nu = \phi + K_{\max}$, where ϕ is the work function and $K_{\max}$ is the largest electron kinetic energy.
- A stopping potential V_s removes kinetic energy eV_s. Photon energy in electron volts is conveniently $1240/\lambda$, when λ is in nanometres.

Step 1: Calculate the incident photon energy For the stated wavelength $\lambda = 250\,\text{nm}$, the energy carried by each photon is

$$E = \frac{1240\,\text{eV nm}}{250\,\text{nm}} = 4.96\,\text{eV}.$$

The stopping potential of $0.5\,\text{V}$ corresponds to a maximum photoelectron kinetic energy of $0.50\,\text{eV}$. A volt is a potential difference; multiplying it by one electronic charge gives the energy used here.

Step 2: Subtract the energy retained by the photoelectron The metal supplies the binding barrier, so the work function is the photon energy minus the maximum kinetic energy:

$$\phi = 4.96 - 0.50 = 4.46\,\text{eV} \simeq 4.5\,\text{eV}.$$

The options use rounded values, so 4.5 eV is the matching answer. The 5 eV choice approximately equals the incident photon energy and ignores the emitted electron's kinetic energy. Adding the stopping energy would instead approach 5.5 eV; subtracting a full electron volt would approach 4 eV, neither of which obeys the stated stopping potential.

Step 3: Conclusion The work function is approximately $4.46\,\text{eV}$, represented by $4.5\,\text{eV}$. Hence, the correct option is (B).

Quick Tip to Solve: In electron volts, subtract the numerical stopping potential directly from the photon energy expressed in electron volts.

74. The minimum volume of water required to dissolve 0.1 g lead (II) chloride to get a saturated solution (K_{sp} of $PbCl_2$ = 3.2 $\times$ 10^{-8} atomic mass of Pb = 207 u) is : [4 marks]

(A) 0.36 L

(B) 17.98 L

(C) 0.18 L

(D) 1.798 L

Correct Answer: (C) 0.18 L

Solution:

Concept:

- For $\mathrm{PbCl_2(s)} \rightleftharpoons \mathrm{Pb^{2+}(aq)} + 2\,\mathrm{Cl^-(aq)}$, let the molar solubility in pure water be $s\,\mathrm{mol\,L^{-1}}$. Then $[\mathrm{Pb^{2+}}] = s$ and $[\mathrm{Cl^-}] = 2s$.
- The solubility product is $K_{\mathrm{sp}} = [\mathrm{Pb^{2+}}][\mathrm{Cl^-}]^2 = 4s^3$. The minimum volume dissolving a fixed amount is reached when the solution is saturated.

Step 1: Calculate the molar solubility Substitute the given product into the equilibrium expression:

$$4s^3 = 3.2 \times 10^{-8}, \qquad s^3 = 8.0 \times 10^{-9}.$$

$$s = 2.0 \times 10^{-3}\,\mathrm{mol\,L^{-1}}.$$

The factor of four comes from the two chloride ions formed per formula unit. Using s^2 would incorrectly treat the salt as a one-to-one electrolyte.

Step 2: Convert the mass to moles and find the volume The molar mass is $207 + 2(35.5) = 278\,\mathrm{g\,mol^{-1}}$. Thus

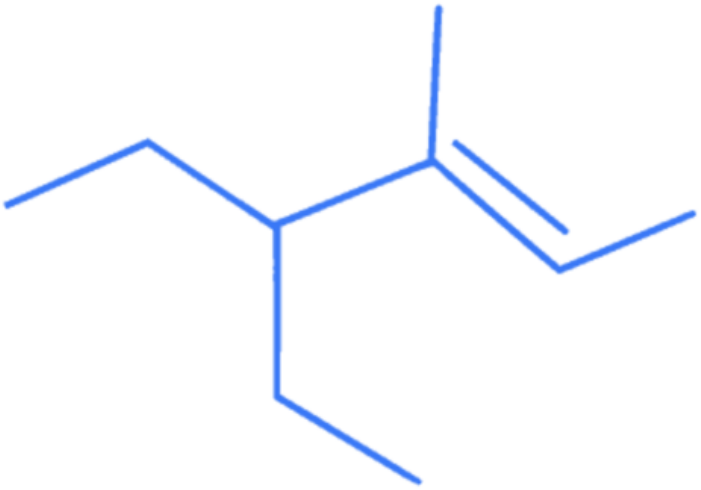
$$n = \frac{0.1}{278} = 3.597 \times 10^{-4}\,\mathrm{mol}, \qquad V = \frac{n}{s} = 0.1799\,\mathrm{L}.$$

To the precision of the options this is 0.18 L. A smaller volume cannot hold the entire mass at this solubility. The alternatives 0.36 L, 1.798 L and 17.98 L are larger than the saturation volume, corresponding to unnecessary dilution rather than the minimum required amount of water.

Step 3: Conclusion The minimum required volume is $0.18\,\mathrm{L}$, approximately $180\,\mathrm{mL}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the dissolution stoichiometry first: for a salt of the form AB2, the pure-water expression is 4s cubed, not s squared.

75. The IUPAC name of the following compound is : [4 marks]



(A) 4-methyl-3-ethylhex-4-ene

(B) 3-ethyl-4-methylhex-4-ene

(C) 4-ethyl-3-methylhex-2-ene

(D) 4, 4- diethyl-3-methylbut-2-ene

Correct Answer: (C) 4-ethyl-3-methylhex-2-ene

Solution:

Concept:

- The parent hydrocarbon must contain the double bond. Among eligible continuous chains, choose the longest and number it to give the double bond the lowest locant.
- After numbering the parent, locate substituents and place their names alphabetically. Ethyl precedes methyl regardless of their numerical positions.

Step 1: Choose and number the parent chain Trace a continuous chain from the right-hand terminal methyl group, through both carbon atoms of the double bond and the central branching carbon, and then along either of its two equivalent ethyl arms. This gives six carbon atoms, so the parent is hexene. Numbering from the right places the double bond between carbons 2 and 3, giving hex-2-ene. Numbering from the other end gives 4 for the double bond and must be rejected because the multiple-bond locant has priority over substituent locants.

Step 2: Identify the two remaining substituents With this numbering, the short upward branch on carbon 3 is methyl. The ethyl arm not included in the parent is attached to carbon 4. Writing ethyl before methyl therefore gives 4-ethyl-3-methylhex-2-ene. The first and second options both use hex-4-ene and violate the lowest double-bond locant. The fourth option chooses a four-carbon parent even though a six-carbon chain containing the same double bond is available. The drawing does not require changing the selected connectivity or choosing a shorter chain merely to count more branches.

Step 3: Conclusion The compound is 4-ethyl-3-methylhex-2-ene. Hence, the correct option is (C).

Quick Tip to Solve: First minimise the double-bond locant; only then assign branch positions and alphabetise ethyl before methyl.

76. Which of the following is a Lewis acid? [4 marks]

(A) PH_3

(B) $B(CH_3)_3$

(C) NaH

(D) NF_3

Correct Answer: (B) $B(CH_3)_3$

Solution:

Concept:

- A Lewis acid accepts an electron pair to form a coordinate bond; a Lewis base donates an electron pair. This definition does not require a transferable proton.
- Boron with three ordinary covalent bonds has only six electrons in its valence shell and an empty orbital available for accepting a donor pair.

Step 1: Locate the electron-deficient centre In trimethylborane, $B(CH_3)_3$, boron is bonded to three methyl carbon atoms. The three bond pairs provide only six electrons around boron, leaving it electron-deficient. Its vacant orbital can accept a lone pair from a donor : L, forming a four-coordinate adduct:

$$B(CH_3)_3 + L \longrightarrow (CH_3)_3BL.$$

Thus this substance satisfies the electron-pair-acceptor definition directly.

Step 2: Compare the other candidates Phosphine, PH_3, has a lone pair on phosphorus and normally acts as a donor rather than an electron-deficient acceptor. Nitrogen trifluoride, NF_3, also has a lone pair on nitrogen and a complete octet; its low basicity does not make it the boron-like Lewis acid sought here. Sodium hydride is an ionic source of H^-, whose filled electron pair produces strongly basic behaviour. Hence the presence of fluorine or an ionic metal alone is not the appropriate test; the accessible vacant orbital on boron is decisive.

Step 3: Conclusion $B(CH_3)_3$ is the Lewis acid. Hence, the correct option is (B).

Quick Tip to Solve: Look for an electron-deficient atom with a vacant orbital; three-coordinate boron is a standard Lewis-acid centre.

77. In graphite and diamond, the percentage of p-characters of the hybrid orbitals in hybridisation are respectively : [4 marks]

(A) 33 and 25

(B) 33 and 75

(C) 50 and 75

(D) 67 and 75

Correct Answer: (D) 67 and 75

Solution:

Concept:

- In an sp^n hybrid set, one s orbital mixes with n p orbitals. Its p character is $n/(n+1) \times 100\%$.
- Graphite has planar, three-coordinate carbon with sp^2 hybridisation; diamond has tetrahedral, four-coordinate carbon with sp^3 hybridisation.

Step 1: Calculate the graphite percentage Each graphite carbon forms three sigma bonds within a plane, using three sp^2 hybrid orbitals. One s orbital and two p orbitals contribute to this hybrid set, so

$$\text{p character in graphite} = \frac{2}{1+2} \times 100\% = 66.67\% \simeq 67\%.$$

The remaining unhybridised p orbital participates in the delocalised pi system. It is not included as an additional orbital in the sp^2 hybridisation percentage.

Step 2: Calculate the diamond percentage and compare Each diamond carbon forms four equivalent sigma bonds in a tetrahedral arrangement. One s and three p orbitals mix, giving

$$\text{p character in diamond} = \frac{3}{1+3} \times 100\% = 75\%.$$

The pair is therefore 67 and 75 in the stated order. The 33 and 25 values in the first choice are the corresponding s-character percentages, while the next choice confuses graphite's s character with its p character. A 50 percent p contribution would correspond to sp, which is not the hybridisation of carbon in graphite.

Step 3: Conclusion The percentages of p character in graphite and diamond are respectively 67% and 75%. Hence, the correct option is (D).

Quick Tip to Solve: Count the p orbitals within the hybrid label: sp2 has two out of three, while sp3 has three out of four.

78. For Na^+, Mg^{2+}, F^- and O^{2-}; the correct order of increasing ionic radii is : [4 marks]

(A) $O^{2-} < F^- < Na^+ < Mg^{2+}$

(B) $Na^+ < Mg^{2+} < F^- < O^{2-}$

(C) $Mg^{2+} < Na^+ < F^- < O^{2-}$

(D) $Mg^{2+} < O^{2-} < Na^+ < F^-$

Correct Answer: (C) $Mg^{2+} < Na^+ < F^- < O^{2-}$

Solution:

Concept:

- An isoelectronic series contains species with the same number of electrons. Its members therefore have comparable shielding and occupy the same principal shells.
- For such a series, increasing nuclear charge Z increases the attraction on the common electron cloud and decreases the ionic radius. Compare atomic numbers, rather than comparing the neutral atoms.

Step 1: Count the electrons in each ion Use the atomic numbers oxygen 8, fluorine 9, sodium 11 and magnesium 12. Removing an electron decreases the electron count, while adding one increases it. Thus O^{2-} has $8 + 2 = 10$, F^- has $9 + 1 = 10$, Na^+ has $11 - 1 = 10$, and Mg^{2+} has $12 - 2 = 10$ electrons. All four ions have the closed-shell configuration $1s^2 2s^2 2p^6$.

Step 2: Apply the nuclear-charge trend The common ten-electron cloud is held most strongly by magnesium, whose nucleus contains twelve protons. Sodium has eleven protons, fluorine nine and oxygen eight, so their radii increase in that order. Consequently,

$$r(Mg^{2+}) < r(Na^+) < r(F^-) < r(O^{2-}).$$

Option A reverses the whole trend. Option B incorrectly makes sodium smaller than magnesium. Option D incorrectly places oxide below sodium and fluoride, although oxide has the weakest nuclear attraction in this series.

Step 3: Conclusion The increasing order is $Mg^{2+} < Na^+ < F^- < O^{2-}$. Hence, the correct option is (C).

Quick Tip to Solve: First establish equal electron counts; within an isoelectronic series, the ion with the highest atomic number has the smallest radius.

79. When an electric currents passed through acidified water, 112 mL of hydrogen gas at N.T.P. was collected at the cathode in 965 seconds. The current passed, in ampere, is : [4 marks]

(A) 1.0

(B) 0.5

(C) 0.1

(D) 2.0

Correct Answer: (A) 1.0

Solution:

Concept:

- At the cathode, hydrogen ions gain electrons according to $2\,H^+ + 2\,e^- \longrightarrow H_2$. Therefore one mole of hydrogen requires two moles of electrons.
- The electric charge is $Q = n_e F$, where n_e is the amount of electrons and $F = 96500\,C\,mol^{-1}$. Current is $I = Q/t$.

Step 1: Convert the hydrogen volume into moles At the stated normal conditions, one mole of gas occupies $22400\,mL$. The collected hydrogen therefore represents

$$n(H_2) = \frac{112}{22400} = 5.00 \times 10^{-3}\,mol.$$

Use the gas volume at N.T.P. directly in this molar-volume relation. The time is already in seconds, so no time conversion is needed.

Step 2: Find the charge and divide by the time The cathode equation requires two electrons per hydrogen molecule, not one. Hence

$$n_e = 2(5.00 \times 10^{-3}) = 1.00 \times 10^{-2}\,mol,$$

$$Q = (1.00 \times 10^{-2})(96500) = 965\,C.$$

It follows that

$$I = \frac{965\,C}{965\,s} = 1.0\,A.$$

A current of 0.5 A would supply only half the required charge and yield 56 mL; 0.1 A would yield 11.2 mL; 2.0 A would yield 224 mL. Thus none of the other stated currents can produce the measured 112 mL in the given time.

Step 3: Conclusion The current passed is $1.0\,A$. Hence, the correct option is (A).

Quick Tip to Solve: For hydrogen evolution, attach the factor of two to the electron amount before using Faraday's constant; one mole of hydrogen consumes two faradays.

80. The decreasing order of bond angles in BF_3, NH_3, PF_3 and I_3^- is : [4 marks]

(A) $I_3^- > NH_3 > PF_3 > BF_3$

(B) $I_3^- > BF_3 > NH_3 > PF_3$

(C) $BF_3 > I_3^- > PF_3 > NH_3$

(D) $BF_3 > NH_3 > PF_3 > I_3^-$

Correct Answer: (B) $I_3^- > BF_3 > NH_3 > PF_3$

Solution:

Concept:

- VSEPR theory predicts molecular shape by arranging electron domains to minimize repulsion. The measured bond angle is between bonds, so lone-pair positions must be distinguished from atom positions.
- The relevant bond angles are $180°$ for linear I_3^-, $120°$ for trigonal-planar BF_3, about $107°$ for NH_3, and about $98°$ for PF_3.

Step 1: Identify the linear and planar species The central iodine in I_3^- has two bond pairs and three lone pairs. Its trigonal-bipyramidal electron-domain arrangement places the three lone pairs equatorially, leaving the iodine atoms on opposite axial sites. Thus the I–I–I angle is $180°$. Boron in BF_3 has three bonding domains and no lone pair, giving a trigonal-planar molecule with F–B–F angles of $120°$.

Step 2: Compare the two pyramidal molecules Both NH_3 and PF_3 are pyramidal, each having three bonds and one lone pair on its central atom. Their bond angles are smaller than the planar $120°$ angle. The H–N–H angle in ammonia is about $107°$, while the F–P–F angle is about $98°$. Bonding in the heavier phosphorus hydride/halide family has greater p character, and electronegative fluorine withdraws bonding density from phosphorus. These effects give the smaller angle in PF_3. Option A wrongly puts BF_3 last, while C and D wrongly put the linear triiodide below a smaller-angle species.

Step 3: Conclusion The decreasing order is $I_3^- > BF_3 > NH_3 > PF_3$. Hence, the correct option is (B).

Quick Tip to Solve: Fix the unmistakable angles first: linear triiodide is 180 degrees and planar boron trifluoride is 120 degrees; then compare the two pyramidal species.

81. For which of the following reactions, ΔH is equal to ΔU? [4 marks]

(A) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$

(B) $2HI(g) \rightarrow H_2(g) + I_2(g)$

(C) $2NO_2(g) \rightarrow N_2O_4(g)$

(D) $2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$

Correct Answer: (B) $2HI(g) \rightarrow H_2(g) + I_2(g)$

Solution:

Concept:

- Enthalpy is defined by $H = U + pV$. For reactions involving ideal gases at a fixed temperature, $\Delta H = \Delta U + \Delta n_g RT$.
- Here Δn_g is the sum of stoichiometric coefficients of gaseous products minus that of gaseous reactants. Thus equality requires no net change in gaseous mole number.

Step 1: Count gaseous moles in the first two reactions For ammonia formation, $N_2(g) + 3\,H_2(g) \longrightarrow 2\,NH_3(g)$, four moles of gas become two, so $\Delta n_g = 2 - (1 + 3) = -2$. Its enthalpy change is therefore $\Delta U - 2RT$, rather than ΔU. For $2\,HI\,(g) \longrightarrow H_2(g) + I_2(g)$, two moles of gaseous reactant give two moles of gaseous products. Here

$$\Delta n_g = (1 + 1) - 2 = 0.$$

The gas-volume contribution to the enthalpy change vanishes.

Step 2: Check the remaining reactions For $2\,NO_2(g) \longrightarrow N_2O_4(g)$, the gas mole change is $1 - 2 = -1$, giving $\Delta H = \Delta U - RT$. For $2\,SO_2(g) + O_2(g) \longrightarrow 2\,SO_3(g)$, it is $2 - (2 + 1) = -1$, also giving a nonzero correction. Therefore neither C nor D satisfies the requested equality. The test concerns numbers of gaseous molecules, not the conservation of atoms: all balanced reactions conserve atoms, but they need not conserve the number of gas molecules.

Step 3: Conclusion Only $2\,HI\,(g) \longrightarrow H_2(g) + I_2(g)$ has $\Delta n_g = 0$, so $\Delta H = \Delta U$. Hence, the correct option is (B).

Quick Tip to Solve: Count only gaseous stoichiometric coefficients on each side; equal gas totals make the ideal-gas enthalpy correction zero.

82. N_2O_5 decomposes to NO_2 and O_2 and follows first order kinetics. After 50 minutes, the pressure inside the vessel increases from 50 mmHg to 87.5 mmHg. The pressure of the gaseous mixture after 100 minute at constant temperature will be : [4 marks]

(A) 175.0 mmHg

(B) 116.25 mmHg

(C) 136.25 mmHg

(D) 106.25 mmHg

Correct Answer: (D) 106.25 mmHg

Solution:

Concept:

- The balanced decomposition is $2\,N_2O_5(g) \longrightarrow 4\,NO_2(g) + O_2(g)$. At fixed temperature and vessel volume, total pressure is proportional to the total gaseous mole number.
- For first-order kinetics, the fraction of reactant remaining is e^{-kt}. Equal time intervals leave the same fraction of the reactant present at the beginning of that interval.

Step 1: Express total pressure in terms of decomposition Let α be the fraction of the initial N_2O_5 that has decomposed. Starting with one mole gives $1-\alpha$ mole of reactant, 2α mole of NO_2, and $\alpha/2$ mole of oxygen. The total is $1+3\alpha/2$. Thus

$$P_t = 50\left(1+\frac{3\alpha}{2}\right).$$

At 50 minutes,

$$87.5 = 50(1+1.5\alpha), \qquad \alpha = 0.50.$$

Half the reactant remains, so 50 minutes is the half-life.

Step 2: Apply a second half-life and calculate pressure After 100 minutes, two half-lives have elapsed. The remaining fraction is $(1/2)^2 = 1/4$, so the decomposed fraction is $\alpha = 3/4$. Consequently,

$$P_{100} = 50\left(1+\frac{3}{2}\cdot\frac{3}{4}\right) = 50\left(\frac{17}{8}\right) = 106.25\,\text{mmHg}.$$

At complete decomposition the limiting pressure would be $2.5(50) = 125\,\text{mmHg}$, which immediately excludes 175.0 and 136.25 mmHg. The value 116.25 mmHg also fails the remaining-fraction calculation; total pressure must not be doubled merely because time doubles.

Step 3: Conclusion The pressure after 100 minutes is $106.25\,\text{mmHg}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert total pressure to the fraction of reactant remaining first; first-order decay applies to reactant pressure, not directly to total mixture pressure.

83. In which of the following reactions, an increase in the volume of the container will favour the formation of products? [4 marks]

(A) $2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$

(B) $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

(C) $4NH_3(g) + 5O_2(g) \rightleftharpoons 4NO(g) + 6H_2O(1)$

(D) $3O_2\ (g) \rightleftharpoons 2O_3(g)$

Correct Answer: (A) $2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$

Solution:

Concept:

- At constant temperature, increasing the container volume lowers the partial pressures of all gases. An equilibrium responds by favouring the direction that produces more gaseous particles.
- Count only gaseous species when comparing the two sides. A pure liquid has essentially fixed activity and does not contribute to the gas-mole count that governs the volume response.

Step 1: Compare gas totals in the first two equilibria For $2\,NO_2(g) \rightleftharpoons 2\,NO\,(g) + O_2(g)$, the reactant side has two gaseous moles and the product side has three. Expansion therefore favours the forward direction, opposing the pressure decrease by generating more gas. For $H_2(g) + I_2(g) \rightleftharpoons 2\,HI\,(g)$, two gaseous moles occur on each side. Both sides are diluted by the same overall power of volume, so expansion alone gives no equilibrium shift.

Step 2: Include the printed phases in the other equilibria In $4\,NH_3(g) + 5\,O_2(g) \rightleftharpoons 4\,NO\,(g) + 6\,H_2O\,(l)$, there are nine gaseous moles on the left but only four on the right. The water is liquid, so its coefficient must not be added to the product gas total. Increasing volume favours the reactants, not products. Finally, $3\,O_2(g) \rightleftharpoons 2\,O_3(g)$ has three gaseous moles on the left and two on the right; it too shifts towards reactants on expansion. Hence the first equilibrium alone has a larger product-side gas count. The equilibrium constant remains unchanged because the temperature is fixed.

Step 3: Conclusion Expansion favours products for $2\,NO_2(g) \rightleftharpoons 2\,NO\,(g) + O_2(g)$. Hence, the correct option is (A).

Quick Tip to Solve: Write the gaseous mole totals under both sides before deciding the shift, and cross out pure liquids and solids from this count.

84. A sample of $NaClO_3$ is converted by heat to $NaCl$ with a loss of $0.16\ g$ of oxygen. The residue is dissolved in water and precipitated as $AgCl$. The mass of $AgCl$ (in g) obtained will be : (Given : Molar mass of $AgCl = 143.5\ g\ mol^{-1}$) [4 marks]

(A) 0.35

(B) 0.41

(C) 0.48

(D) 0.54

Correct Answer: (C) 0.48

Solution:

Concept:

- Thermal decomposition follows $2\,\mathrm{NaClO_3} \longrightarrow 2\,\mathrm{NaCl} + 3\,\mathrm{O_2}$. The oxygen loss fixes the amount of chlorate decomposed and hence the amount of chloride left.
- Each mole of NaCl gives one mole of AgCl on quantitative precipitation: $\mathrm{Ag^+ + Cl^- \longrightarrow AgCl(s)}$. Use stoichiometric mole ratios before converting to mass.

Step 1: Calculate the oxygen amount and chloride amount The lost mass is the oxygen evolved, not the original mass of sodium chlorate. With $M(\mathrm{O_2}) = 32\,\mathrm{g\,mol^{-1}}$,

$$n(\mathrm{O_2}) = \frac{0.16}{32} = 0.00500\,\mathrm{mol}.$$

The balanced equation produces two moles of sodium chloride for every three moles of oxygen, so

$$n(\mathrm{NaCl}) = \frac{2}{3}(0.00500) = \frac{1}{300}\,\mathrm{mol}.$$

The same result follows because one mole of chlorate loses 48 grams of oxygen and gives one mole of chloride.

Step 2: Convert the precipitate amount to its mass All the chloride in the dissolved residue is precipitated, and the sodium-chloride to silver-chloride mole ratio is one to one. Therefore

$$n(\mathrm{AgCl}) = \frac{1}{300}\,\mathrm{mol}, \qquad m(\mathrm{AgCl}) = \frac{143.5}{300} = 0.4783\,\mathrm{g}.$$

To the precision of the options, this is $0.48\,\mathrm{g}$. The values 0.35, 0.41 and 0.54 g imply different chloride amounts and do not satisfy the fixed $2:3$ sodium-chloride to oxygen ratio. In particular, equating oxygen moles directly with silver-chloride moles would miss the stoichiometric factor.

Step 3: Conclusion The mass of AgCl obtained is approximately $0.48\,\mathrm{g}$. Hence, the correct option is (C).

Quick Tip to Solve: For chlorate decomposition, 48 g of oxygen loss corresponds to one mole of chloride and therefore one mole of silver chloride precipitate.

85. The correct combination is : [4 marks]

(A) $[Ni(CN)_4]^{2-}$ – tetrahedral ;
$[Ni(CO)_4]$ – paramagnetic
(B) $[Ni(Cl)_4]^{2-}$ – paramagnetic ;
$[Ni(CO)_4]$ – tetrahedral
(C) $[Ni(Cl)_4]^{2-}$ – square-planar ;
$[Ni(CN)_4]^{2-}$ – paramagnetic
(D) $[Ni(Cl)_4]^{2-}$ – diamagnetic ;
$[Ni(CO)_4]$ –square-planar

Correct Answer: (B) $[Ni(Cl)_4]^{2-}$ – paramagnetic ;
$[Ni(CO)_4]$ – tetrahedral

Solution:

Concept:

- The oxidation state determines the metal-ion electron count. Nickel is $[Ar]3d^8 4s^2$; Ni^{2+} is $3d^8$, whereas nickel in $Ni(CO)_4$ has oxidation state zero.
- A complex is paramagnetic when it contains unpaired electrons. Strong-field and weak-field ligands can give different geometries and spin arrangements for the same metal ion.

Step 1: Examine the chloride complex In $[NiCl_4]^{2-}$, four chloride ligands contribute charge -4, so nickel has oxidation state $+2$. Chloride is a weak-field ligand and does not force the $3d$ electrons to pair. The four-coordinate complex uses outer-orbital sp^3 hybridisation and is tetrahedral. Its d^8 configuration has two unpaired electrons, so the complex is paramagnetic. Thus the chloride statement in option (B) is correct, while the square-planar and diamagnetic descriptions in (C) and (D) are false.

Step 2: Check the carbonyl and cyanide complexes Carbon monoxide is neutral, so nickel in $Ni(CO)_4$ is in oxidation state zero. Its electrons pair to give a filled $3d^{10}$ set; bonding uses four sp^3 orbitals. Therefore nickel tetracarbonyl is tetrahedral and diamagnetic. In contrast, strong-field cyanide pairs the electrons in $[Ni(CN)_4]^{2-}$, producing a square-planar, dsp^2, diamagnetic complex. Option (A) misidentifies both cyanide geometry and carbonyl magnetism; (C) misidentifies both listed properties; (D) also gives two incorrect properties.

Step 3: Conclusion $[NiCl_4]^{2-}$ is paramagnetic and $Ni(CO)_4$ is tetrahedral. Hence, the correct option is (B).

Quick Tip to Solve: For nickel complexes, first distinguish d^8 nickel(II) from d^{10} nickel(0), then use ligand strength to decide pairing.

86. Which of the following arrangements shows the schematic alignment of magnetic moments of antiferromagnetic substance ? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C)



Solution:

Concept:

- Magnetic moments behave as vectors. Equal moments pointing in opposite directions cancel, while parallel moments add to give a resultant magnetisation.
- An antiferromagnetic substance has an ordered antiparallel arrangement of neighbouring moments of equal magnitude; its two opposing sublattices have equal magnetisation.

Step 1: Identify equal and opposite neighbours In option (C), the six arrows have the same length and alternate upward and downward. Every neighbouring pair has opposite orientation, and there are three arrows in each direction. If the magnitude of one moment is μ, the sum along the common axis is

$$\mu_{\text{net}} = 3\mu - 3\mu = 0.$$

This is ordered cancellation, rather than a random arrangement. Both the antiparallel pattern and the equality of the opposing contributions are essential to the schematic representation of antiferromagnetism.

Step 2: Eliminate the other patterns Option (A) has all six moments parallel. They reinforce one another, giving 6μ, the schematic ferromagnetic alignment rather than antiferromagnetic alignment. Option (B) has two upward arrows and four downward arrows; equal magnitudes therefore leave a nonzero downward resultant of magnitude 2μ. Option (D) has four upward arrows and two downward arrows, leaving a nonzero upward resultant of magnitude 2μ. Neither (B) nor (D) has the alternating equal-opposition pattern. The deciding feature is the alignment of the moments, not merely the fact that some arrows point down.

Step 3: Conclusion The equal, alternating antiparallel moments in option (C) represent an antiferromagnetic substance. Hence, the correct option is (C).

Quick Tip to Solve: Count arrows in each direction and check their lengths: antiferromagnetic order requires equal opposing contributions and alternating alignment.

87. Which of the following statements about colloids is False ? [4 marks]

(A) Freezing point of colloidal solution is lower than true solution at same concentration of a solute.

(B) Colloidal particles can pass through ordinary filter paper.

(C) When silver nitrate solution is added to potassium iodide solution, a negatively charged colloidal solution is formed.

(D) When excess of electolyte is added to colloidal solution, colloidal particle will be precipitated.

Correct Answer: (A) Freezing point of colloidal solution is lower than true solution at same concentration of a solute.

Solution:

Concept:

- Freezing-point depression is a colligative property: $\Delta T_f = K_f m$ for a dilute nonelectrolyte solution, where m counts independently dispersed particles per kilogram of solvent.
- Colloidal aggregates are much larger than molecules. For the same amount of dispersed material, aggregation gives fewer independently dispersed particles than a true molecular solution.

Step 1: Compare the freezing-point depressions A colloidal particle contains many molecules or formula units. The number of independently moving particles in a colloidal dispersion is therefore smaller than in the corresponding true solution at the same concentration of solute material. Consequently, its colligative effects, including freezing-point depression, are much smaller. Its freezing point lies closer to that of the pure solvent, rather than lower than the freezing point of the true solution. Statement (A) reverses this comparison and is the false statement requested.

Step 2: Verify the remaining statements Ordinary filter paper has pores large enough for colloidal particles to pass through, so (B) is true; separating colloids requires finer membranes or other specialised methods. Adding $\mathrm{AgNO_3}$ solution to a solution of KI forms AgI in the presence of excess iodide. Preferential adsorption of $\mathrm{I^-}$ on the particles produces a negatively charged sol, so (C) is true in the stated addition order. An excess of a suitable electrolyte neutralises the stabilising charge and causes coagulation or precipitation, making (D) true. Thus these three alternatives do not answer the request for a false statement.

Step 3: Conclusion A colloidal dispersion has a smaller freezing-point depression than the corresponding true solution, so statement (A) is false. Hence, the correct option is (A).

Quick Tip to Solve: For colligative effects, compare the number of independent particles, not their size: aggregation sharply reduces that number.

88. Xenon hexafluoride on partial hydrolysis produces compounds 'X' and 'Y' . Compounds 'X' and 'Y' and the oxidation state of Xe are respectively : [4 marks]

(A) $XeO_2(+4)$ and $XeO_3(+6)$

(B) $XeOF_4(+6)$ and $XeO_3(+6)$

(C) $XeO_2F_2(+6)$ and $XeO_2(+4)$

(D) $XeOF_4(+6)$ and $XeO_2F_2(+6)$

Correct Answer: (D) $XeOF_4(+6)$ and $XeO_2F_2(+6)$

Solution:

Concept:

- Hydrolysis of XeF_6 replaces two fluorine atoms by one oxygen atom at each stage and releases hydrogen fluoride. Controlled hydrolysis stops before all fluorine atoms are replaced.
- Use -1 for fluorine and -2 for oxygen when finding xenon oxidation states. The total oxidation-state sum is zero in each neutral molecule.

Step 1: Write the successive partial-hydrolysis reactions With a limited quantity of water, xenon hexafluoride first forms xenon oxytetrafluoride:

$$XeF_6 + H_2O \longrightarrow XeOF_4 + 2\,HF.$$

Further controlled hydrolysis forms xenon dioxydifluoride:

$$XeOF_4 + H_2O \longrightarrow XeO_2F_2 + 2\,HF.$$

Thus the two partial-hydrolysis products are $XeOF_4$ and XeO_2F_2. A further water molecule would give XeO_3, but this is the final product of complete hydrolysis, rather than the second intermediate required here.

Step 2: Calculate the oxidation states and compare options For $XeOF_4$, write the xenon oxidation state as x:

$$x - 2 - 4 = 0 \quad \Rightarrow \quad x = +6.$$

For XeO_2F_2,

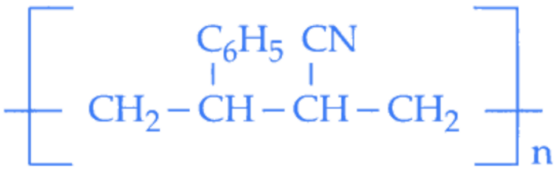
$$x - 4 - 2 = 0 \quad \Rightarrow \quad x = +6.$$

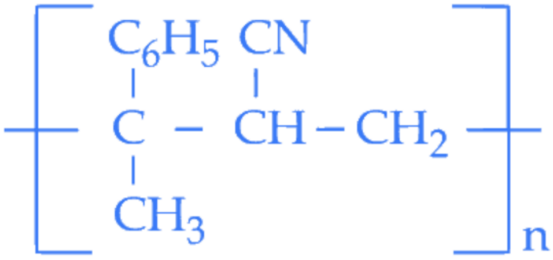
The hydrolysis is substitution without a change in the xenon oxidation state. Options (A) and (C) introduce XeO_2 with xenon(IV), which is not one of these partial-hydrolysis products. Option (B) includes XeO_3, whose xenon oxidation state is indeed $+6$, but it replaces the second intermediate with the complete-hydrolysis product. Only (D) names both intermediates correctly.

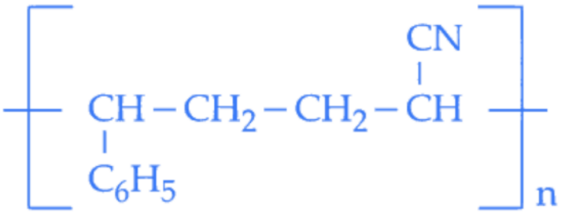
Step 3: Conclusion The products are $XeOF_4$ and XeO_2F_2, with xenon in oxidation state $+6$ in both. Hence, the correct option is (D).

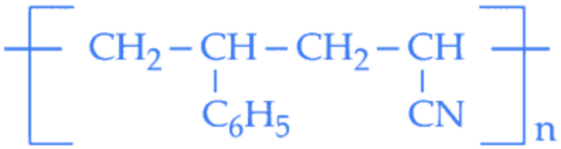
Quick Tip to Solve: In each hydrolysis step replace two F atoms by one O atom; this preserves xenon in the $+6$ oxidation state.

89. The copolymer formed by addition polymerization of styrene and acrylonitrile in the presence of peroxide is : [4 marks]

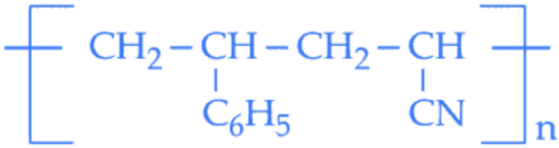
(A) 

(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- Addition polymerisation opens the carbon-carbon double bonds of vinyl monomers while retaining their side groups. A peroxide supplies radicals that initiate chain growth.
- Styrene is $CH_2{=}CH(C_6H_5)$, whereas acrylonitrile is $CH_2{=}CH(CN)$. Their normal head-to-tail units are $-CH_2{-}CH(C_6H_5){-}$ and $-CH_2{-}CH(CN){-}$, respectively.

Step 1: Construct the copolymer backbone Opening the double bond of styrene gives two saturated backbone carbon atoms, with the phenyl group still bonded to the substituted carbon. Opening the double bond of acrylonitrile likewise gives two backbone carbon atoms, with the nitrile group retained

as a pendant group. Combining their normal head-to-tail units gives the displayed segment

$$\left[-\mathrm{CH_2}-\mathrm{CH(C_6H_5)}-\mathrm{CH_2}-\mathrm{CH(CN)}-\right]_n.$$

The peroxide initiator does not convert the nitrile group into part of the backbone and does not introduce a methyl substituent into the repeat segment.

Step 2: Check the four displayed structures Option (D) retains both monomer substituents and joins the normal vinyl units head to tail. Option (B) has a methyl group on a backbone carbon and only three backbone carbons in its displayed pair of units, so it cannot follow by simply opening the two stated double bonds. Option (A) puts the two substituted carbons adjacent, giving a head-to-head junction. Option (C) puts the two methylene carbons adjacent, giving a tail-to-tail junction. Those are not the conventional regular head-to-tail copolymer segment asked for here. The subscript does not require a real copolymer to have strictly alternating monomers; the diagram identifies the linkage and retained groups.

Step 3: Conclusion The head-to-tail styrene-acrylonitrile copolymer has the segment displayed in option (D). Hence, the correct option is (D).

Quick Tip to Solve: Open each vinyl double bond without moving its side group, then check for the normal head-to-tail sequence along the carbon backbone.

90. A white sodium salt dissolves readily in water to give a solution which is neutral to litmus. When silver nitrate solution is added to the aformentioned solution, a white precipitate is obtained which does not dissolve in dil. nitric acid. The anion is : [4 marks]

(A) CO_3^{2-}

(B) SO_4^{2-}

(C) Cl^-

(D) S^{2-}

Correct Answer: (C) Cl^-

Solution:

Concept:

- Silver nitrate identifies several anions by precipitation. The precipitate colour and its behaviour toward dilute nitric acid distinguish chloride from carbonate and other possible anions.
- A salt of a strong acid and strong base, such as sodium chloride, gives a nearly neutral aqueous solution. Carbonate and sulfide ions undergo hydrolysis and make their sodium salt solutions alkaline.

Step 1: Apply the neutral-solution and precipitation tests Aqueous sodium chloride is neutral to litmus because neither $\mathrm{Na^+}$ nor $\mathrm{Cl^-}$ undergoes appreciable hydrolysis. With silver nitrate it gives the white precipitate silver chloride:

$$\mathrm{Ag^+(aq)} + \mathrm{Cl^-(aq)} \longrightarrow \mathrm{AgCl(s)}.$$

Silver chloride does not dissolve in dilute nitric acid. Therefore chloride satisfies all three observations together: a soluble white sodium salt, a neutral aqueous solution, and a white silver precipitate resistant to the acid test. The relevant test is resistance to dilute nitric acid; silver chloride can dissolve in aqueous ammonia by complex formation.

Step 2: Exclude carbonate, sulfate and sulfide Carbonate solution is alkaline, and silver carbonate is pale yellow rather than the characteristic white chloride precipitate. Carbonate precipitates also dissolve in dilute acid with carbon dioxide evolution. Sulfide solution is alkaline and gives black Ag_2S, so it is ruled out immediately by the observed colour. Although sodium sulfate is neutral, silver sulfate is sufficiently soluble that sulfate is not identified by the characteristic chloride test under ordinary qualitative-analysis conditions; its white salt also dissolves on acidification. Sulfate is instead normally tested with barium ions. The complete set of observations therefore selects chloride, rather than a choice based on white colour alone.

Step 3: Conclusion The anion is Cl^-, which produces white $AgCl$ insoluble in dilute nitric acid. Hence, the correct option is (C).

Quick Tip to Solve: Combine colour with acid solubility: white $AgCl$ survives dilute nitric acid, whereas carbonate is removed by acid and sulfide gives a black precipitate.