JEE Main 2018 Paper 1 (B.E./B.Tech) Apr 15 Slot 2 (afternoon) Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2018, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 15 April 2018, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. If the system of linear equations

x + ay + z = 3

x + 2y + 2z = 6

x + 5y + 3z = b

has no solution, then : [4 marks]

(A) a = − 1, b = 9

(B) a = − 1, b ≠ 9

(C) a ≠ − 1, b = 9

(D) a = 1, b ≠ 9

Correct Answer: (B) a = − 1, b ≠ 9

Solution:

Concept:

- A system has no solution when elimination produces an inconsistent equation such as $0 = c$, where $c \neq 0$. A zero coefficient determinant alone does not distinguish inconsistency from infinitely many solutions.
- Elementary row subtraction preserves the solutions. Eliminate x first, and then test whether the remaining equations in y, z have proportional coefficients and compatible constants.

Step 1: Eliminate the first variable Subtract the first equation from the second and third equations. This gives

$$(2-a)y + z = 3, \qquad (5-a)y + 2z = b - 3.$$

The first relation determines $z = 3 - (2-a)y$. Substitute this into the second relation rather than dividing by a parameter which might be zero. After collecting coefficients,

$$(5-a)y + 6 - 2(2-a)y = b - 3,$$

so the complete consistency condition reduces to

$$(a+1)y = b - 9.$$

Step 2: Distinguish the parameter cases If $a \neq -1$, then $y = (b-9)/(a+1)$ exists for every real b. The eliminated equations then give one value of z, and the first original equation gives one value of x. Thus the system is consistent in this case. If $a = -1$, the last equation becomes $0 = b - 9$. It is contradictory precisely when $b \neq 9$. When $b = 9$, it is an identity instead: y is free, $z = 3 - 3y$, and $x = 4y$, giving infinitely many solutions. Option (A) therefore describes a consistent dependent system, while (C) and (D) have $a \neq -1$ and admit unique solutions.

Step 3: Conclusion The system has no solution exactly when $a = -1$ and $b \neq 9$. Hence, the correct option is (B).

Quick Tip to Solve: Eliminate variables without dividing by a parameter; the final equation $(a+1)y = b - 9$ exposes both the singular case and the compatibility condition.

2. Let f(x) = $\begin{cases} (x-1)^{\frac{1}{2-x}}, & x>1, x\neq 2 \\ k & , x=2 \end{cases}$

Thevaue of k for which f s continuous at x = 2 is : [4 marks]

(A) 1

(B) e

(C) e^{-1}

(D) e^{-2}

Correct Answer: (C) e^{-1}

Solution:

Concept:

- Continuity at $x=2$ requires the two-sided limit of the expression for $x\neq 2$ to equal its assigned value $k=f(2)$. Direct substitution gives the indeterminate form 1^{∞}.
- For a positive base, take logarithms: $u^v=\exp(v\ln u)$. The standard limit $\lim_{h\to 0}\ln(1+h)/h=1$ evaluates the exponent and retains its sign.

Step 1: Shift the variable to the point of continuity Put $h=x-2$. Then $x-1=1+h$ and $2-x=-h$, so for $h\neq 0$

$$f(2+h)=(1+h)^{-1/h}.$$

For all sufficiently small positive or negative h, $1+h>0$; consequently the real logarithm is defined on both sides of the point. Let the logarithm of this positive expression be $g(h)$. It is

$$g(h)=-\frac{\ln(1+h)}{h}.$$

This transformation avoids treating the base tending to one as if the exponent were fixed.

Step 2: Evaluate the limit and apply continuity The logarithmic standard limit gives

$$\lim_{h\to 0} g(h)=-1.$$

Since the exponential function is continuous, exponentiating yields

$$\lim_{x\to 2}(x-1)^{1/(2-x)}=\exp(-1)=e^{-1}.$$

The same value is obtained from both sides, so it is the two-sided limit required by continuity. Assigning $k=e^{-1}$ fills the removable discontinuity. The value 1 incorrectly ignores the unbounded exponent; e would result from reversing the sign of $2-x$; and e^{-2} introduces a factor of two absent from the logarithmic limit.

Step 3: Conclusion For continuity at $x=2$, we must have $k=e^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: Write $x=2+h$ before taking logarithms: the denominator becomes $-h$, making the negative sign in the exponential limit clear.

3. If $|z - 3 + 2i| \leq 4$ then the difference between the greatest value and the least value of $|z|$ is : [4 marks]

(A) $2\sqrt{13}$

(B) 8

(C) $4 + \sqrt{13}$

(D) $\sqrt{13}$

Correct Answer: (C) $4 + \sqrt{13}$

Solution:

Concept:

- The inequality $|z - z_0| \leq R$ represents the closed disc centred at z_0 with radius R. Here $z_0 = 3 - 2i$, whose distance from the origin is $|z_0| = \sqrt{13}$.
- For points in a closed disc, the largest distance from the origin is $|z_0| + R$. The least distance is $\max(0, |z_0| - R)$: it is zero whenever the disc contains the origin.

Step 1: Locate the origin relative to the disc Rewrite the condition as $|z - (3 - 2i)| \leq 4$. The centre has distance

$$|3 - 2i| = \sqrt{3^2 + (-2)^2} = \sqrt{13} < 4$$

from the origin. Thus $z = 0$ lies inside the allowed disc, because $|0 - 3 + 2i| = \sqrt{13} \leq 4$. Since a modulus is nonnegative and zero is attainable, the least possible value of $|z|$ is exactly zero. In particular, $4 - \sqrt{13}$ is not the minimum for this inequality; it would be the minimum on the boundary circle.

Step 2: Find the farthest point and the requested difference The triangle inequality gives

$$|z| = |(z - z_0) + z_0| \leq |z - z_0| + |z_0| \leq 4 + \sqrt{13}.$$

Equality is attainable at the boundary point $z = z_0 + 4z_0/|z_0|$, since the displacement from the centre then points directly away from the origin. Therefore the greatest modulus is $4 + \sqrt{13}$, and subtracting the attained minimum zero leaves the same value. The value 8 is the diameter of the disc rather than this range of radial distances. The expression $2\sqrt{13}$ wrongly uses boundary-circle extrema despite the filled disc, and $\sqrt{13}$ is merely the distance of its centre.

Step 3: Conclusion The supplied key differs, but the printed inequality includes the origin, so the minimum is zero and the maximum is $4 + \sqrt{13}$; hence the key value does not follow from the stated question. The required difference is $(4 + \sqrt{13}) - 0 = 4 + \sqrt{13}$. Hence, the correct option is (C).

Quick Tip to Solve: Check whether the origin is inside before using a nearest-point formula. For a filled disc containing zero, the minimum modulus is zero.

4. Suppose A is any 3× 3 non-singular matrix and (A − 3I) (A − 5I) = O where I = I_3 and O = O_3. If αA + βA^{-1} = 4I, then α + β is equal to : [4 marks]

(A) 8

(B) 7

(C) 13

(D) 12

Correct Answer: (A) 8

Solution:

Concept:

- Matrix polynomial identities may be expanded because every polynomial in a fixed matrix A commutes with A and the identity matrix I. Here the given product yields a quadratic identity for A.
- Since A is nonsingular, multiply that identity by A^{-1} to express a linear combination of A and A^{-1} as a scalar multiple of I. This avoids computing any matrix entries.

Step 1: Expand the given product Use $AI = IA = A$ and $I^2 = I$ to obtain

$$(A - 3I)(A - 5I) = A^2 - 8A + 15I = O.$$

The coefficients -8 and 15 are respectively minus the sum and the product of the two scalar roots, 3 and 5. As A^{-1} exists, multiplication on either side by it is legitimate. Multiplying the quadratic relation gives

$$A - 8I + 15A^{-1} = O,$$

or equivalently $A + 15A^{-1} = 8I$.

Step 2: Scale the identity to the specified right side The desired right-hand side is $4I$, half of the $8I$ just obtained. Dividing the entire identity by two gives

$$\frac{1}{2}A + \frac{15}{2}A^{-1} = 4I.$$

Thus the coefficients which work for every allowed matrix are $\alpha = 1/2$ and $\beta = 15/2$, with sum 8. To see why these are the universal coefficients, take an allowed diagonal matrix containing both eigenvalues 3 and 5. The respective diagonal entries require $3\alpha + \beta/3 = 4$ and $5\alpha + \beta/5 = 4$, which give these same values. Their sum is not 7, 13, or 12, ruling out the other listed choices. Multiplying by the inverse, rather than replacing the matrix by a scalar root, keeps the identity valid for all the matrices in the question.

Step 3: Conclusion The identity valid for any such A has $\alpha + \beta = \frac{1}{2} + \frac{15}{2} = 8$. Hence, the correct option is (A).

Quick Tip to Solve: For $(A - rI)(A - sI) = O$ with invertible A, multiply the expanded relation by A^{-1}: $A + rsA^{-1} = (r + s)I$.

5. Tangents drawn from the point $(-8, 0)$ to the parabola $y^2 = 8x$ touch the parabola at P and Q. If F is the focus of the parabola, then the area of the triangle PFQ (in sq. units) is equal to : [4 marks]

(A) 24

(B) 32

(C) 48

(D) 64

Correct Answer: (C) 48

Solution:

Concept:

- For the standard parabola $y^2 = 4ax$, its focus is $(a, 0)$. The point with parameter t is $(at^2, 2at)$, and its tangent is $ty = x + at^2$.
- The area of a triangle is half the product of a chosen base and its perpendicular height. Symmetric contact points make their common vertical chord a convenient base.

Step 1: Determine the two contact points Comparing $y^2 = 8x$ with $y^2 = 4ax$ gives $a = 2$, so the focus is $F = (2, 0)$. A tangent at parameter t has equation

$$ty = x + 2t^2.$$

It passes through the external point $(-8, 0)$, which imposes $0 = -8 + 2t^2$. Hence $t^2 = 4$, and the two real parameters are $t = 2$ and $t = -2$. Substituting into $(2t^2, 4t)$ gives the contact points

$$P = (8, 8), \qquad Q = (8, -8).$$

They satisfy both the parabola and their respective tangents, confirming that these are the two required points.

Step 2: Use the chord as the base of the triangle The line PQ is vertical, with equation $x = 8$. Its length is $8 - (-8) = 16$. The perpendicular distance from $F = (2, 0)$ to this line is the horizontal distance $|8 - 2| = 6$. Therefore

$$[PFQ] = \frac{1}{2} \times 16 \times 6 = 48.$$

The height must be measured from the focus, not from the origin or the external point from which the tangents were drawn. A height of eight would incorrectly give 64, while halving the chord or the correct area again could give 24. The value 32 also does not satisfy the base-height calculation. Thus only one printed option equals the computed area.

Step 3: Conclusion The area of triangle PFQ is 48 square units. Hence, the correct option is (C).

Quick Tip to Solve: Use $ty = x + at^2$ to find the contact parameters, then take the symmetric chord as the base; measure the height from the focus.

6. The sides of a rhombus ABCD are parallel to the lines, x − y + 2 = 0 and 7x − y + 3 = 0. If the diagonals of the rhombus intersect P(1, 2) and the vertex A (different from the origin) is on the y-axis, then the coordinate of A is : [4 marks]

(A) $\frac{5}{2}$

(B) $\frac{7}{4}$

(C) 2

(D) $\frac{7}{2}$

Correct Answer: (A) $\frac{5}{2}$

Solution:

Concept:

- The diagonals of a rhombus bisect the angles at its vertices. Thus their directions are the two angle-bisector directions of its adjacent side lines, and both diagonals pass through their intersection point P.
- For two intersecting lines in normalized form, their angle bisectors satisfy equality of signed perpendicular distances. Only the side directions matter; parallel translates have the same angle-bisector directions.

Step 1: Find the diagonal directions from the side slopes The given side directions have slopes 1 and 7. Translate a pair of lines with these slopes to pass through $P = (1, 2)$. In coordinates $X = x - 1$, $Y = y - 2$, these lines are $X - Y = 0$ and $7X - Y = 0$. Their normal lengths are $\sqrt{2}$ and $\sqrt{50} = 5\sqrt{2}$. Therefore the two bisectors are given by

$$5(X - Y) = \pm(7X - Y).$$

With the plus sign, this reduces to $Y = -X/2$; with the minus sign, it reduces to $Y = 2X$. Consequently the two rhombus diagonal directions have slopes $-1/2$ and 2, which are perpendicular as expected.

Step 2: Intersect both possible diagonals with the vertical axis The vertex A lies on one of the diagonals through P. In the original coordinates the two possible diagonal lines are

$$y - 2 = 2(x - 1), \qquad y - 2 = -\frac{1}{2}(x - 1).$$

On the y-axis, $x = 0$. The first equation gives $y = 0$, which is the origin and is explicitly excluded. The second equation gives $y = 2 + 1/2 = 5/2$. Thus $A = (0, 5/2)$; the scalar choices in the question give its nonzero ordinate. The values $7/4$, 2, and $7/2$ satisfy neither of these permitted intersections with $x = 0$, so they cannot be the requested ordinate. Using a side slope itself for a diagonal would miss the angle-bisector condition.

Step 3: Conclusion The allowed vertex is $A = (0, 5/2)$, so its ordinate is $5/2$. Hence, the correct option is (A).

Quick Tip to Solve: Normalize the two line equations before finding angle bisectors; equal unnormalized expressions generally do not give the diagonal directions.

7. If a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. such that $a < b < c$ and $a + b + c = \frac{3}{4}$, then the value of a is : [4 marks]

(A) $\frac{1}{4} - \frac{1}{4\sqrt{2}}$

(B) $\frac{1}{4} - \frac{1}{3\sqrt{2}}$

(C) $\frac{1}{4} - \frac{1}{2\sqrt{2}}$

(D) $\frac{1}{4} - \frac{1}{\sqrt{2}}$

Correct Answer: (C) $\frac{1}{4} - \frac{1}{2\sqrt{2}}$

Solution:

Concept:

- Write three terms in arithmetic progression as $a = b - d$, b, $c = b + d$. The strict ordering $a < b < c$ means the common difference d is positive.
- Three squared terms form a geometric progression when the square of the middle squared term equals the product of the other squared terms: $b^4 = a^2c^2$. Retain both signs when taking a square root.

Step 1: Use the sum to determine the middle term Since a, b, c are in arithmetic progression, their sum is $3b$. The given sum therefore gives

$$3b = \frac{3}{4} \quad \Longrightarrow \quad b = \frac{1}{4}.$$

Write $a = 1/4 - d$ and $c = 1/4 + d$, with $d > 0$. Their product is $ac = b^2 - d^2$. Applying the geometric-progression condition to their squares now gives

$$b^4 = (b^2 - d^2)^2.$$

This equation permits $b^2 - d^2 = b^2$ or $b^2 - d^2 = -b^2$; using only the positive sign would lose the valid solution.

Step 2: Select the nonconstant progression and identify the first term The positive-sign case gives $d = 0$, contradicting $a < b < c$. Thus we must have $b^2 - d^2 = -b^2$, which yields

$$d^2 = 2b^2 = \frac{1}{8}, \qquad d = \frac{1}{2\sqrt{2}}.$$

The positive root is required by the ordering. Consequently

$$a = b - d = \frac{1}{4} - \frac{1}{2\sqrt{2}}.$$

As a check, $ac = -b^2$, so $a^2c^2 = b^4$; all squared terms are nonzero, and the common ratio of the squared progression is positive. The first term a itself is negative, which is allowed: only its square belongs to the geometric progression. The differences used in options (A), (B), and (D) have squares $1/32$, $1/18$, and $1/2$, respectively, rather than the required $1/8$.

Step 3: Conclusion The first term is $a = \frac{1}{4} - \frac{1}{2\sqrt{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use symmetric arithmetic-progression terms $b-d, b, b+d$, then keep the negative possibility $ac = -b^2$ when the squares form a geometric progression.

8. The number of four letter words that can be formed using the letters of the word BARRACK is : [4 marks]

(A) 120

(B) 144

(C) 264

(D) 270

Correct Answer: (D) 270

Solution:

Concept:

- The word BARRACK supplies five distinct letter types: B, A, R, C and K. The letters A and R are available twice each; each other letter is available only once.
- For four selected letters with multiplicities $r_1, r_2, \ldots$, the number of distinct arrangements is $4!/(r_1!r_2!\cdots)$. Identical copies do not create new words, and a word here need not have a dictionary meaning.

Step 1: Count words with four different letters Select any four of the five letter types, then arrange the selected letters. Each selected letter appears once, so there is no repetition divisor:

$$N_1 = \binom{5}{4} 4! = 5 \times 24 = 120.$$

This case includes every word in which neither A nor R is repeated, even when both occur once.

Step 2: Count words with exactly one repeated pair Choose the repeated letter as A or R, giving two choices. After that choice, select two different single letters from the other four types. These four letters have multiplicities $2, 1, 1$, so

$$N_2 = 2\binom{4}{2}\frac{4!}{2!} = 2 \times 6 \times 12 = 144.$$

Choosing the other repeatable letter once is allowed here; choosing it twice belongs to the next case.

Step 3: Count two-pair words and combine the cases Two repeated pairs can only be AARR. Its distinct arrangements number

$$N_3 = \frac{4!}{2!2!} = 6.$$

No letter is available three times, so no other repetition pattern is possible. The three cases are disjoint and exhaustive. Thus

$$N = N_1 + N_2 + N_3 = 120 + 144 + 6 = 270.$$

Options (A) and (B) give only individual cases, while (C) adds the first two cases but omits AARR arrangements.

Step 4: Conclusion The number of four-letter words is 270. Hence, the correct option is (D).

Quick Tip to Solve: Classify four-letter selections by multiplicity: all different, one pair, or two pairs. Apply the repetition divisor separately in each case.

9. If f(x) = $\sin^{-1}\left(\frac{2\times 3^x}{1+9^x}\right)$, then f'$\left(-\frac{1}{2}\right)$ equals : [4 marks]

(A) $-\sqrt{3}\log_e\sqrt{3}$

(B) $\sqrt{3}\log_e\sqrt{3}$

(C) $-\sqrt{3}\log_e 3$

(D) $\sqrt{3}\log_e 3$

Correct Answer: (B) $\sqrt{3}\log_e\sqrt{3}$

Solution:

Concept:

- The identity $\sin(2\tan^{-1}t) = 2t/(1+t^2)$ converts the given composite function into a simpler local expression, provided the principal range of the inverse sine is respected.
- The principal value of $\sin^{-1}u$ lies in $[-\pi/2, \pi/2]$. Also, $d(3^x)/dx = 3^x \ln 3$ and $d(\tan^{-1}t)/dt = 1/(1+t^2)$.

Step 1: Choose the correct local inverse-sine branch Put $t = 3^x$. Near $x = -1/2$, we have $0 < t < 1$, since $x < 0$. Therefore $0 < 2\tan^{-1}t < \pi/2$, so this angle belongs to the principal inverse-sine range. Consequently, in a neighbourhood of the required point,

$$f(x) = \sin^{-1}\left(\frac{2t}{1+t^2}\right) = 2\tan^{-1}(3^x).$$

This branch check matters: the same expression cannot be used unchanged for positive x, where the doubled angle exceeds $\pi/2$.

Step 2: Differentiate and substitute the given point By the chain rule,

$$f'(x) = \frac{2\,3^x \ln 3}{1+3^{2x}}.$$

At $x = -1/2$, the exponential is $3^{-1/2} = 1/\sqrt{3}$, and its square is $1/3$. Thus

$$f'\left(-\frac{1}{2}\right) = \frac{(2/\sqrt{3})\ln 3}{1+1/3} = \frac{\sqrt{3}}{2}\ln 3 = \sqrt{3}\ln\sqrt{3}.$$

The derivative is positive because 3^x is increasing and the local doubled-arctangent expression is increasing. This excludes (A) and (C). Option (D) is twice the computed derivative because $\ln\sqrt{3} = \frac{1}{2}\ln 3$.

Step 3: Conclusion We obtain $f'(-1/2) = \sqrt{3}\log_e\sqrt{3}$. Hence, the correct option is (B).

Quick Tip to Solve: Before using the double-angle identity inside inverse sine, check whether the doubled angle lies in its principal range; the branch determines the derivative's sign.

10. A player X has a biased coin whose probability of showing heads is p and a player Y has a fair coin. They start playing a game with their own coins and play alternately. The player who throws a head first is a winner. If X starts the game, and the probability of winning the game by both the players is equal, then the value of 'p' is : [4 marks]

(A) $\frac{1}{5}$
(B) $\frac{1}{3}$
(C) $\frac{2}{5}$
(D) $\frac{1}{4}$

Correct Answer: (B) $\frac{1}{3}$

Solution:

Concept:

- A fresh round consists of X tossing first and Y tossing next if X fails. The probability of both players obtaining tails in a round is $(1-p)/2$; after that event, the game restarts in exactly the same state.
- Winning on the first successful round produces a geometric series. For $|r|<1$, $1+r+r^2+\cdots = 1/(1-r)$, where here $r=(1-p)/2$.

Step 1: Express each player's winning probability X wins immediately with probability p, or after one, two, or more complete tail-tail rounds. Hence

$$P_X = p + \frac{1-p}{2}p + \left(\frac{1-p}{2}\right)^2 p + \cdots = \frac{p}{1-(1-p)/2} = \frac{2p}{1+p}.$$

Y can win in a round only when X first obtains a tail and Y then obtains a head, an event of probability $(1-p)/2$. The same restart ratio applies, so

$$P_Y = \frac{(1-p)/2}{1-(1-p)/2} = \frac{1-p}{1+p}.$$

Since the restart ratio is at most $1/2$, infinite play has probability zero. Indeed, the two displayed winning probabilities add to one.

Step 2: Enforce equal winning chances Set $P_X = P_Y$. Their common positive denominator cancels:

$$\frac{2p}{1+p} = \frac{1-p}{1+p} \quad \Longrightarrow \quad 2p = 1-p \quad \Longrightarrow \quad p = \frac{1}{3}.$$

Substitution gives $P_X = P_Y = 1/2$, as required. For options (A), (C), and (D), the respective pairs (P_X, P_Y) are $(1/3, 2/3)$, $(4/7, 3/7)$, and $(2/5, 3/5)$; none is equal. The first player's starting advantage explains why X's coin must have a head probability below $1/2$.

Step 3: Conclusion Equal chances of winning require $p = 1/3$. Hence, the correct option is (B).

Quick Tip to Solve: For alternating games, compare success probabilities within a single round when both players share the same restart factor; the geometric denominator cancels.

11. Let $f : A \to B$ be a function defined as f(x) = $\frac{x-1}{x-2}$, Where A = R − {2} and B = R − {1}. Then f is : [4 marks]

(A) invertible and $f^{-1}(y) = \frac{3y-1}{y-1}$

(B) invertible and $f^{-1}(y) = \frac{2y-1}{y-1}$

(C) invertible and $f^{-1}(y) = \frac{2y+1}{y-1}$

(D) not invertible

Correct Answer: (B) invertible and $f^{-1}(y) = \frac{2y-1}{y-1}$

Solution:

Concept:

- A function is invertible from its stated domain to its stated codomain when it is both one-to-one and onto. Solving $y = f(x)$ for a unique admissible x establishes these properties together.
- Excluded values must be checked in both directions. Here $x = 2$ is outside the domain and $y = 1$ is outside the codomain, so the inverse must be defined for every $y \neq 1$ and must never return 2.

Step 1: Solve the defining equation for the input Let $y = (x-1)/(x-2)$ with $x \neq 2$. Multiplying by the nonzero denominator and collecting terms gives

$$yx - 2y = x - 1, \qquad x(y-1) = 2y - 1.$$

Since $y \neq 1$, division is valid and yields the unique candidate

$$x = \frac{2y-1}{y-1}.$$

The output of the original function can never equal 1: that would imply $x - 1 = x - 2$, which is impossible. Thus the specified codomain correctly excludes 1.

Step 2: Check that the candidate is a genuine inverse For any $y \neq 1$, write the candidate as $x = 2 + 1/(y-1)$. It is well defined and cannot equal 2. Substitution gives

$$x - 1 = \frac{y}{y-1}, \qquad x - 2 = \frac{1}{y-1}, \qquad f(x) = y.$$

Hence each member of the codomain has exactly one preimage: the function is onto and one-to-one. Option (D) is therefore false. The numerator in (A) has the wrong coefficient of y, and (C) has the wrong constant sign. For example, at $y = 0$, the unique preimage is $x = 1$, which already rules out (C); at $y = 2$, the preimage is 3, ruling out (A).

Step 3: Conclusion The function is invertible, with $f^{-1}(y) = (2y-1)/(y-1)$ for $y \neq 1$. Hence, the correct option is (B).

Quick Tip to Solve: After solving a fractional linear function for its inverse, rewrite the result as a constant plus a reciprocal to check its excluded output immediately.

12. An angle between the lines whose direction cosines are gien by the equations, $l + 3m + 5n = 0$ and $5lm - 2mn + 6nl = 0$, is : [4 marks]

(A) $\cos^{-1}\left(\frac{1}{3}\right)$

(B) $\cos^{-1}\left(\frac{1}{4}\right)$

(C) $\cos^{-1}\left(\frac{1}{6}\right)$

(D) $\cos^{-1}\left(\frac{1}{8}\right)$

Correct Answer: (C) $\cos^{-1}\left(\frac{1}{6}\right)$

Solution:

Concept:

- Direction cosines (l, m, n) are proportional to direction ratios for the same line. Homogeneous equations in them can therefore be solved using convenient proportional triples, without first normalising each triple.
- For direction vectors $\mathbf{a}$ and $\mathbf{b}$, the acute angle between their lines satisfies $\cos\theta = |\mathbf{a}\cdot\mathbf{b}|/(|\mathbf{a}|\,|\mathbf{b}|)$. Reversing either vector does not change the line.

Step 1: Eliminate one direction cosine The first condition gives $l = -3m - 5n$. Substitute this into the quadratic condition:

$$\begin{aligned} 5lm - 2mn + 6nl &= 5(-3m-5n)m - 2mn + 6n(-3m-5n) \\ &= -15m^2 - 45mn - 30n^2 \\ &= -15(m+n)(m+2n) = 0. \end{aligned}$$

Thus either $m = -n$ or $m = -2n$. Here n cannot be zero: the resulting factorisation would force $m = 0$, and then $l = 0$, which is impossible for direction cosines.

Step 2: Obtain the two line directions When $m = -n$, the linear condition gives $l = -2n$; hence a direction vector is $\mathbf{a} = (-2, -1, 1)$. When $m = -2n$, it gives $l = n$; a direction vector is $\mathbf{b} = (1, -2, 1)$. Both satisfy the original homogeneous conditions, so they represent the required two lines.

Step 3: Evaluate the angle Their scalar product and magnitudes are

$$\mathbf{a}\cdot\mathbf{b} = -2 + 2 + 1 = 1, \qquad |\mathbf{a}| = |\mathbf{b}| = \sqrt{6}.$$

Therefore $\cos\theta = 1/6$. Options (A), (B), and (D) use different cosine values and do not satisfy this scalar-product calculation. In particular the denominator is the product of the two lengths, namely 6, rather than either length alone.

Step 4: Conclusion An angle between the two lines is $\cos^{-1}(1/6)$. Hence, the correct option is (C).

Quick Tip to Solve: Eliminate a variable using the linear condition, factor the remaining homogeneous quadratic, and use the two factors to obtain direction ratios before normalising.

13. The curve satifying the differeial equation, $(x^2 - y^2)\,dx + 2xydy = 0$ and passing through the point (1, 1) is : [4 marks]

(A) a circle of radius one.

(B) a hyperbola.

(C) an ellipse.

(D) a circle of radius two.

Correct Answer: (A) a circle of radius one.

Solution:

Concept:

- The substitution $z = y^2$ gives $dz/dx = 2y\,dy/dx$, converting this differential equation into a first-order linear equation in z.
- A linear equation $z' + P(x)z = Q(x)$ is solved by multiplying by the integrating factor $e^{\int P(x)\,dx}$. Completing squares then identifies the resulting conic and its radius.

Step 1: Reduce to a linear differential equation Divide the given equation by dx, using a neighbourhood of the specified point where $x \neq 0$:

$$x^2 - y^2 + 2xy\frac{dy}{dx} = 0.$$

With $z = y^2$, this becomes

$$x\frac{dz}{dx} - z = -x^2, \qquad \frac{dz}{dx} - \frac{z}{x} = -x.$$

Since the point has $x = 1 > 0$, we may use the integrating factor $1/x$ on its local interval.

Step 2: Integrate and impose the initial point Multiplication by $1/x$ gives an exact derivative:

$$\frac{d}{dx}\left(\frac{z}{x}\right) = -1.$$

Hence $z/x = -x + C$, or $x^2 + y^2 = Cx$. Substituting $(x, y) = (1, 1)$ yields $C = 2$. Thus the curve through the given point is

$$x^2 + y^2 = 2x \quad \Longleftrightarrow \quad (x-1)^2 + y^2 = 1.$$

Step 3: Identify and verify the conic The completed-square form describes a circle with centre $(1, 0)$ and radius 1. Differentiating $x^2 + y^2 = 2x$ gives $2x + 2yy' = 2$; substituting this relation and $y^2 = 2x - x^2$ into the original equation makes its left side zero. This confirms the solution directly. The equal positive squared coefficients exclude a hyperbola and a noncircular ellipse, while the right-hand side after completing the square excludes radius two.

Step 4: Conclusion The required curve is $(x-1)^2 + y^2 = 1$, a circle of radius one. Hence, the correct option is (A).

Quick Tip to Solve: When a differential equation contains both y^2 and $2y\,dy/dx$, try $z = y^2$; the new equation may be linear in z.

14. If $\int \frac{2x+5}{\sqrt{7-6x-x^2}}\,dx = A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right) + C$ **(where C is a constant of integration), then the ordered pair (A, B) is equal to :** [4 marks]

(A) (2, 1)

(B) (− 2, −1)

(C) (− 2, 1)

(D) (2, −1)

Correct Answer: (B) (− 2, −1)

Solution:

Concept:

- Completing the square gives $7 - 6x - x^2 = 16 - (x+3)^2$, which has the standard inverse-sine integral form. For a real-valued integrand, the open interval is $-7 < x < 1$.
- If $R = 7 - 6x - x^2$, then $d\sqrt{R}/dx = -(x+3)/\sqrt{R}$. Also, $d[\sin^{-1}((x+3)/4)]/dx = 1/\sqrt{R}$ on the stated interval.

Step 1: Rewrite the numerator to match the radical derivative The derivative of the radicand is $-6 - 2x = -2(x+3)$, whereas the numerator is $2x+5$. Express it as $2(x+3) - 1$, so

$$\int \frac{2x+5}{\sqrt{7-6x-x^2}}\,dx = 2\int \frac{x+3}{\sqrt{16-(x+3)^2}}\,dx - \int \frac{dx}{\sqrt{16-(x+3)^2}}.$$

The first term is a radical derivative and the second is the standard inverse-sine integral; no trigonometric substitution is necessary.

Step 2: Integrate the two terms and identify coefficients Using the two derivative identities gives

$$\int \frac{2x+5}{\sqrt{7-6x-x^2}}\,dx = -2\sqrt{7-6x-x^2} - \sin^{-1}\left(\frac{x+3}{4}\right) + C.$$

Comparing with the required form gives $A = -2$ and $B = -1$. The negative signs come respectively from the decreasing quadratic radicand and from the residual constant -1 in the numerator.

Step 3: Check both signs by differentiating The derivative of the proposed answer is

$$\frac{2(x+3)}{\sqrt{7-6x-x^2}} - \frac{1}{\sqrt{7-6x-x^2}} = \frac{2x+5}{\sqrt{7-6x-x^2}},$$

exactly the printed integrand. More generally its numerator would be $-A(x+3) + B$. Matching the coefficient of x forces $A = -2$, eliminating (A) and (D); matching the constant then forces $B = -1$, eliminating (C).

Step 4: Conclusion The ordered pair is $(A, B) = (-2, -1)$. Hence, the correct option is (B).

Quick Tip to Solve: For a linear numerator over a quadratic radical, split the numerator into a multiple of the radicand's derivative plus a constant; check signs by differentiation.

15. The foot of the perpendicular drawn from the origin, on the line, 3x + y = λ ($\lambda \neq 0$) is P. If the line meets x-axis at A and y-axis at B, then the ratio BP : PA is : [4 marks]

(A) 1 : 3

(B) 3 : 1

(C) 1 : 9

(D) 9 : 1

Correct Answer: (D) 9 : 1

Solution:

Concept:

- The perpendicular from the origin to the line $ax+by=c$ has its foot at $\big(ac/(a^2+b^2), bc/(a^2+b^2)\big)$, since its direction is the normal vector (a, b).
- For three collinear points, lengths can be compared using their coordinate differences. The nonzero parameter λ cancels from the ratio, so its sign cannot change the answer.

Step 1: Locate the intercepts and the perpendicular foot Putting $y = 0$ and $x = 0$, respectively, gives

$$A = (\lambda/3, 0), \qquad B = (0, \lambda).$$

The normal direction to $3x + y = \lambda$ is $(3, 1)$, so the perpendicular through the origin is $y = x/3$. Solving this together with the given line gives

$$3x + x/3 = \lambda, \qquad P = (3\lambda/10, \lambda/10).$$

These coordinates satisfy the given line, confirming that the point lies on AB.

Step 2: Compare the two segments Compute the vectors along the same straight line:

$$\overrightarrow{BP} = (3\lambda/10, -9\lambda/10), \qquad \overrightarrow{PA} = (\lambda/30, -\lambda/10).$$

The first vector is nine times the second, so their magnitudes have ratio $9 : 1$. In particular, the coefficient is positive even if $\lambda < 0$, and the foot lies between the intercepts. The choices $1 : 3$ and $3 : 1$ confuse the line coefficients with segment lengths; $1 : 9$ reverses the requested order $BP : PA$.

Step 3: Conclusion The required ratio is $BP : PA = 9 : 1$. Hence, the correct option is (D).

Quick Tip to Solve: For a line through two axis intercepts, find the normal projection first; compare directed coordinate differences in the order the ratio asks for.

16. If $I_1 = \int_0^1 e^{-x}\cos^2 x\,dx$;
$I_2 = \int_0^1 e^{-x^2}\cos^2 x\,dx$ **and**
$I_3 = \int_0^1 e^{-x^3}\,dx$; **then** [4 marks]

(A) $I_2 > I_3 > I_1$

(B) $I_2 > I_1 > I_3$

(C) $I_3 > I_2 > I_1$

(D) $I_3 > I_1 > I_2$

Correct Answer: (C) $I_3 > I_2 > I_1$

Solution:

Concept:

- If two continuous functions satisfy $f(x) > g(x)$ throughout an interval except possibly its endpoints, then the integral of f over that interval is strictly larger.
- The exponential function is strictly increasing. On $0 < x < 1$, the powers satisfy $x^3 < x^2 < x$; also $0 < \cos^2 x < 1$.

Step 1: Compare the first two integrands For every interior point $0 < x < 1$, we have $-x^2 > -x$, hence

$$e^{-x^2} > e^{-x}.$$

Since $\cos^2 x$ is positive on this interval, multiplication preserves the strict inequality:

$$e^{-x^2}\cos^2 x > e^{-x}\cos^2 x.$$

Integrating over the common limits 0 and 1 yields $I_2 > I_1$. Equality at an isolated endpoint does not affect the strict comparison of the integrals.

Step 2: Compare the third integrand and select the ordering The inequality $x^3 < x^2$ similarly gives $e^{-x^3} > e^{-x^2}$. The factor $\cos^2 x$ in I_2 reduces its integrand further, so

$$e^{-x^3} > e^{-x^2} > e^{-x^2}\cos^2 x.$$

Therefore $I_3 > I_2$, and combining both comparisons gives $I_3 > I_2 > I_1$. Option (A) wrongly places I_2 above I_3. Option (B) also places I_3 below both other integrals. Option (D) reverses the already proved inequality between I_1 and I_2. No antiderivative or approximate numerical integration is needed.

Step 3: Conclusion The strict ordering is $I_3 > I_2 > I_1$. Hence, the correct option is (C).

Quick Tip to Solve: When integral limits agree, compare the integrands before trying to integrate; on $(0, 1)$, larger positive powers of x are smaller.

17. If f(x) is a quadratic expression such that f (1) + f (2) = 0, and $-$ 1 is a root of f (x) = 0, then the other root of f(x) = 0 is : [4 marks]

(A) $-\frac{5}{8}$

(B) $-\frac{8}{5}$

(C) $\frac{5}{8}$

(D) $\frac{8}{5}$

Correct Answer: (D) $\frac{8}{5}$

Solution:

Concept:

- If a quadratic polynomial has roots -1 and r, the factor theorem writes it as $f(x) = a(x+1)(x-r)$, with $a \neq 0$. The nonzero leading coefficient does not affect its roots.
- A relation involving polynomial values, such as $f(1) + f(2) = 0$, becomes an algebraic equation for the unknown root after substitution.

Step 1: Express the quadratic using its known root Let r denote the root other than -1. Because the polynomial is genuinely quadratic, write

$$f(x) = a(x+1)(x-r), \qquad a \neq 0.$$

Substitute $x = 1$ and $x = 2$ separately:

$$f(1) = 2a(1-r), \qquad f(2) = 3a(2-r).$$

The multipliers 2 and 3 arise from the known factor $x + 1$; retaining them is essential. The problem gives the sum of these values, not equality of the two values.

Step 2: Apply the relation and check the options Using $f(1)+f(2) = 0$, divide by the nonzero number a:

$$2(1-r) + 3(2-r) = 0.$$

Collecting terms gives

$$2 - 2r + 6 - 3r = 0, \qquad 8 - 5r = 0, \qquad r = \frac{8}{5}.$$

As a check, for this root the two values are $f(1) = -6a/5$ and $f(2) = 6a/5$, whose sum is zero. The negative choices $-5/8$ and $-8/5$ cannot satisfy $8 - 5r = 0$, and the positive reciprocal $5/8$ also fails it. Thus neither a sign change nor a reciprocal is permissible.

Step 3: Conclusion The other root is $8/5$. Hence, the correct option is (D).

Quick Tip to Solve: When one root is known and values of the polynomial are related, use its factored form; the common leading coefficient usually cancels immediately.

18. The value of integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}dx$ is : [4 marks]

(A) $\pi\sqrt{2}$

(B) $\pi\left(\sqrt{2}-1\right)$

(C) $\frac{\pi}{2}\left(\sqrt{2}+1\right)$

(D) $2\pi\left(\sqrt{2}-1\right)$

Correct Answer: (B) $\pi\left(\sqrt{2}-1\right)$

Solution:

Concept:

- For fixed limits a, b, the substitution $x \mapsto a+b-x$ preserves the integral: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$. Here the sum of the limits is π.
- The identity $\sin(\pi - x) = \sin x$ makes the denominator symmetric. Pairing the original and reflected integrals replaces the numerator x by the constant $\pi/2$.

Step 1: Use the reflection to remove the factor x Write

$$I = \int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x}\,dx.$$

The reflection $x = \pi - t$, with the reversed limits put back in increasing order, gives

$$I = \int_{\pi/4}^{3\pi/4} \frac{\pi - x}{1+\sin x}\,dx.$$

Adding these equal integrals yields

$$2I = \pi J, \qquad J = \int_{\pi/4}^{3\pi/4} \frac{dx}{1+\sin x}.$$

The denominator is positive throughout this interval, so all these integrals are well defined.

Step 2: Evaluate the remaining integral without a singular expression A convenient antiderivative, valid throughout the interval including $x = \pi/2$, is

$$\frac{d}{dx}\left(\frac{-\cos x}{1+\sin x}\right) = \frac{1}{1+\sin x}.$$

At the lower limit this expression is $-(\sqrt{2}-1)$, and at the upper limit it is $\sqrt{2}-1$. Consequently,

$$J = 2(\sqrt{2}-1), \qquad I = \pi(\sqrt{2}-1).$$

The other options have an extra factor or the wrong endpoint combination. In particular, (D) is twice the required value and would result from forgetting the factor $2I$ when adding the integrals.

Step 3: Conclusion The integral equals $\pi(\sqrt{2}-1)$. Hence, the correct option is (B).

Quick Tip to Solve: For a symmetric denominator, pair x with $a+b-x$. Use $-\cos x/(1+\sin x)$ here to avoid artificial singularities at $\pi/2$.

19. If the mean of the data : 7, 8, 9, 7, 8, 7, λ, 8 is 8, then the variance of this data is : [4 marks]

(A) $\frac{7}{8}$

(B) 1

(C) $\frac{9}{8}$

(D) 2

Correct Answer: (B) 1

Solution:

Concept:

- The arithmetic mean of n observations is $\bar{x} = (\sum x_i)/n$. A known mean determines a missing observation from the total sum.
- The variance of the given data is $\sigma^2 = (1/n)\sum(x_i - \bar{x})^2$. This is the variance of the complete data set, so its denominator is n, not $n-1$.

Step 1: Determine the missing observation There are eight observations and the mean is 8, so their sum must be

$$\sum x_i = 8 \times 8 = 64.$$

The seven specified observations add to

$$7+8+9+7+8+7+8 = 54.$$

Thus $54 + \lambda = 64$, giving $\lambda = 10$. The complete ordered list is therefore $7, 8, 9, 7, 8, 7, 10, 8$. The repeated observations must each be retained when forming the sum or the variance.

Step 2: Calculate squared deviations and average them Subtracting the mean 8 from each observation gives

$$-1,\ 0,\ 1,\ -1,\ 0,\ -1,\ 2,\ 0.$$

Their squared deviations are $1, 0, 1, 1, 0, 1, 4, 0$, with total 8. Hence

$$\sigma^2 = \frac{1+0+1+1+0+1+4+0}{8} = 1.$$

The alternatives $7/8$, $9/8$, and 2 do not equal this average. In particular, using unsquared signed deviations would give zero, while using a sample-variance denominator would give $8/7$; neither procedure is the requested variance of these eight data values.

Step 3: Conclusion The variance is 1. Hence, the correct option is (B).

Quick Tip to Solve: Find the missing value from the mean first, then count squared deviations by frequency; here three values of 7, one 9 and one 10 supply all the nonzero terms.

20. If the position vectors of the vertices A, B and C of a Δ ABC are respectively $4\hat{i}+7\hat{j}+8\hat{k}$, $2\hat{i}+3\hat{j}+4\hat{k}$, and $2\hat{i}+5\hat{j}+7\hat{k}$, then the position vectors of the point, where the bisector of $\angle$A meets BC is : [4 marks]

(A) $\frac{1}{2}\left(4\hat{i}+8\hat{j}+11\hat{k}\right)$

(B) $\frac{1}{3}\left(6\hat{i}+11\hat{j}+15\hat{k}\right)$

(C) $\frac{1}{3}\left(6\hat{i}+13\hat{j}+18\hat{k}\right)$

(D) $\frac{1}{4}\left(8\hat{i}+14\hat{j}+19\hat{k}\right)$

Correct Answer: (C) $\frac{1}{3}\left(6\hat{i}+13\hat{j}+18\hat{k}\right)$

Solution:

Concept:

- The internal angle bisector from a vertex A of a triangle meets the opposite side BC at a point D satisfying $BD/DC = AB/AC$. Distances must be found from vector differences.
- If $BD : DC = m : n$, the position vector of D is $\mathbf{d} = (n\mathbf{b} + m\mathbf{c})/(m+n)$. Each endpoint receives the weight corresponding to the opposite segment.

Step 1: Find the two side lengths adjacent to angle A Use the stated position vectors $\mathbf{a} = (4, 7, 8)$, $\mathbf{b} = (2, 3, 4)$, and $\mathbf{c} = (2, 5, 7)$. Their differences are

$$\overrightarrow{AB} = (-2, -4, -4), \qquad \overrightarrow{AC} = (-2, -2, -1).$$

Therefore

$$AB = \sqrt{4 + 16 + 16} = 6, \qquad AC = \sqrt{4 + 4 + 1} = 3.$$

The angle bisector theorem gives $BD : DC = 6 : 3 = 2 : 1$. This is not a midpoint construction: the two adjacent sides have unequal lengths.

Step 2: Apply internal division and distinguish the options For the ratio $2 : 1$, the internal section formula yields

$$\mathbf{d} = \frac{\mathbf{b} + 2\mathbf{c}}{3} = \frac{(2, 3, 4) + (4, 10, 14)}{3} = \frac{(6, 13, 18)}{3}.$$

Thus the point is closer to C, as expected because BD is twice DC. Option (A) gives the midpoint and would require equal adjacent sides. Option (B) instead uses $(2\mathbf{b} + \mathbf{c})/3$, reversing the ratio. Option (D) has coordinates $(2, 7/2, 19/4)$, which do not give the required internal ratio. Only option (C) satisfies the angle bisector condition with these side lengths.

Step 3: Conclusion The position vector is $\frac{1}{3}(6\hat{i} + 13\hat{j} + 18\hat{k})$. Hence, the correct option is (C).

Quick Tip to Solve: Calculate the lengths adjacent to the bisected angle, then cross-weight the opposite endpoints in the section formula; the longer segment lies opposite the larger endpoint weight.

21. $\lim_{x\to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ **equals :** [4 marks]

(A) $\frac{1}{4}$

(B) 1

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

Correct Answer: (C) $\frac{1}{2}$

Solution:

Concept:

- The double-angle identity $\tan 2x = 2\tan x/(1-\tan^2 x)$ allows the cancellation in $\tan 2x - 2\tan x$ to be handled exactly.
- Use $1-\cos 2x = 2\sin^2 x$ and the standard radian limit $\lim_{x\to 0} \sin x/x = 1$. The leading linear terms in the two tangents cancel, so replacing each tangent by only its first approximation is insufficient.

Step 1: Simplify the numerator before taking the limit For sufficiently small nonzero x, all denominators below are nonzero. Setting $t = \tan x$, the numerator becomes

$$x\tan 2x - 2x\tan x = x\left(\frac{2t}{1-t^2} - 2t\right) = \frac{2xt^3}{1-t^2}.$$

Meanwhile, $(1-\cos 2x)^2 = 4\sin^4 x$. Therefore the original quotient is exactly

$$\frac{x\tan^3 x}{2(1-\tan^2 x)\sin^4 x}.$$

This retains the first nonvanishing contribution after cancellation, rather than incorrectly treating the numerator as zero.

Step 2: Reduce to a standard sine limit Since $\tan^3 x = \sin^3 x/\cos^3 x$ and $1-\tan^2 x = \cos 2x/\cos^2 x$, cancellation gives

$$\frac{x}{2\sin x\cos x\cos 2x} = \frac{1}{2}\frac{x}{\sin x}\frac{1}{\cos x\cos 2x}.$$

As $x \to 0$, the three variable factors tend to $1, 1, 1$. The two-sided limit is therefore $1/2$. The sign is positive on either side near zero. Option (D) has the wrong sign, and options (A) and (B) lose one of the factors of two in the double-angle identities.

Step 3: Conclusion The limit equals $1/2$. Hence, the correct option is (C).

Quick Tip to Solve: When tangent terms cancel at first order, simplify with the double-angle identity before taking a limit; then reduce the remaining expression to $\sin x/x$.

22. Let f(x) be a polynomial of degree 4 having extreme values at $x = 1$ and $x = 2$.
If $\lim_{x\to 0}\left(\frac{f(x)}{x^2}+1\right) = 3$ then f(−1) is equal to : [4 marks]

(A) $\frac{9}{2}$

(B) $\frac{5}{2}$

(C) $\frac{3}{2}$

(D) $\frac{1}{2}$

Correct Answer: (A) $\frac{9}{2}$

Solution:

Concept:

- A finite limit of $f(x)/x^2$ at zero forces the constant and linear coefficients of the polynomial to vanish. The limit then equals its quadratic coefficient.
- At an interior local maximum or minimum of a differentiable function, its first derivative is zero. Apply this necessary condition at the two stated extreme points.

Step 1: Use the limit to determine the low-degree coefficients Write the general quartic as $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, where $a \neq 0$. Dividing by x^2 gives

$$\frac{f(x)}{x^2} = ax^2 + bx + c + \frac{d}{x} + \frac{e}{x^2}.$$

For the two-sided finite limit to exist, first $e = 0$, then $d = 0$. The stated limit is therefore $c + 1 = 3$, giving $c = 2$. Thus $f(x) = ax^4 + bx^3 + 2x^2$.

Step 2: Apply the stationary-point conditions Differentiate this polynomial:

$$f'(x) = 4ax^3 + 3bx^2 + 4x.$$

The extreme values at $x = 1$ and $x = 2$ imply

$$4a + 3b + 4 = 0, \qquad 32a + 12b + 8 = 0.$$

Dividing the second equation by four and subtracting the first gives $4a - 2 = 0$. Hence $a = \frac{1}{2}$ and $b = -2$. This is indeed a quartic; also $f'(x) = 2x(x-1)(x-2)$, which changes sign at each of the stated points.

Step 3: Evaluate the requested value and compare the choices Substitute $x = -1$ in the determined polynomial:

$$f(-1) = \frac{1}{2}(-1)^4 - 2(-1)^3 + 2(-1)^2 = \frac{1}{2} + 2 + 2 = \frac{9}{2}.$$

The cubic term contributes positively because both its coefficient and $(-1)^3$ are negative. The values $5/2$, $3/2$ and $1/2$ therefore do not satisfy the evaluation; retaining only the leading term would incorrectly produce the last choice.

Step 4: Conclusion The required value is $f(-1) = \frac{9}{2}$. Hence, the correct option is (A).

Quick Tip to Solve: For a finite polynomial limit after division by x^2, eliminate its constant and linear terms before using the derivative conditions.

23. A normal to the hyperbola, $4x^2 - 9y^2 = 36$ meets the co-ordinate axes x and y at A and B, respectively. If the parallelogram OABP (O being the origin) is formed, then the ocus of P is :

[4 marks]

(A) $4x^2 + 9y^2 = 121$

(B) $9x^2 + 4y^2 = 169$

(C) $4x^2 - 9y^2 = 121$

(D) $9x^2 - 4y^2 = 169$

Correct Answer: (D) $9x^2 - 4y^2 = 169$

Solution:

Concept:

- Implicit differentiation gives the tangent slope, and the normal slope is its negative reciprocal at a nonvertex point.
- A point on the hyperbola satisfies $4u^2 - 9v^2 = 36$. Express its normal intercepts in terms of u, v, then eliminate these parameters using that equation.

Step 1: Find the normal at a general point Let (u, v) be the point of contact. Differentiating $4x^2 - 9y^2 = 36$ yields

$$8x - 18y\frac{dy}{dx} = 0, \qquad m_{\text{tangent}} = \frac{4u}{9v}.$$

Consequently the normal has slope $-9v/(4u)$, so its equation is

$$y - v = -\frac{9v}{4u}(x - u),$$

which can be rearranged without fractions as

$$9vx + 4uy = 13uv.$$

For the nondegenerate parallelogram the intercepts are distinct and nonzero, so this calculation uses $v \neq 0$.

Step 2: Relate the intercepts to the fourth vertex Putting $y = 0$ and $x = 0$, respectively, gives

$$A = \left(\frac{13u}{9}, 0\right), \qquad B = \left(0, \frac{13v}{4}\right).$$

For vertices named consecutively as O, A, B, P, opposite vertices satisfy $O + B = A + P$. Thus, if $P = (X, Y)$,

$$X = -\frac{13u}{9}, \qquad Y = \frac{13v}{4}.$$

In particular $u^2 = 81X^2/169$ and $v^2 = 16Y^2/169$. The sign of the first coordinate will disappear in the locus equation.

Step 3: Eliminate the contact point and select the equation Use $4u^2 - 9v^2 = 36$ to obtain

$$4\frac{81X^2}{169} - 9\frac{16Y^2}{169} = 36.$$

Multiplication by $169/36$ simplifies this to

$$9X^2 - 4Y^2 = 169.$$

Both plus-sign choices describe ellipses and cannot be the result of this elimination. The other hyperbola choice has the wrong coefficients and constant. Any limiting degenerate vertices are excluded if a genuine parallelogram is required.

Step 4: Conclusion The locus equation is $9x^2 - 4y^2 = 169$. Hence, the correct option is (D).

Quick Tip to Solve: Find the normal in intercept form first. The factors $a^2 + b^2$ generate its intercepts and make parameter elimination straightforward.

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24. The tangent to the circle $C_1 : x^2 + y^2 - 2x - 1 = 0$ at the point (2, 1) cuts off a chord of length 4 from a circle C_2 whose center is $(3, -2)$. The radius of C_2 is : [4 marks]

(A) 2

(B) $\sqrt{2}$

(C) 3

(D) $\sqrt{6}$

Correct Answer: (D) $\sqrt{6}$

Solution:

Concept:

- The tangent at a point of a circle is perpendicular to the radius drawn to that point. Complete the square to find the centre of the first circle.
- A perpendicular from the centre of a circle bisects a chord. If the chord length is L and its distance from the centre is d, the radius satisfies $r^2 = d^2 + (L/2)^2$.

Step 1: Determine the tangent line to the first circle Complete the square in the equation of C_1:

$$x^2 + y^2 - 2x - 1 = 0 \quad \Longleftrightarrow \quad (x-1)^2 + y^2 = 2.$$

Its centre is $(1, 0)$, and the given point $(2, 1)$ is on the circle because $(2-1)^2 + 1^2 = 2$. The radius to this point has direction vector $(1, 1)$, which is a normal vector to the tangent. Hence the tangent is

$$(x-2) + (y-1) = 0, \quad \text{or} \quad x + y - 3 = 0.$$

Step 2: Measure the distance from the second centre For C_2, the centre is $(3, -2)$. The perpendicular distance from this centre to the chord line is

$$d = \frac{|3 + (-2) - 3|}{\sqrt{1^2 + 1^2}} = \frac{2}{\sqrt{2}} = \sqrt{2}.$$

This distance is measured to the tangent of C_1, which cuts a chord of C_2; the same line need not be tangent to C_2. The sign of the centre's second coordinate must be retained when substituting.

Step 3: Use the half-chord right triangle The given chord has length 4, so each half has length 2. Joining the second centre to a chord endpoint gives a right triangle with legs $\sqrt{2}$ and 2, and hypotenuse equal to its radius r:

$$r^2 = (\sqrt{2})^2 + 2^2 = 2 + 4 = 6.$$

Thus $r = \sqrt{6}$, taking the positive square root. A radius of 2 would make the chord too short at this nonzero distance, $\sqrt{2}$ would only touch the line, and 3 would give a chord of length $2\sqrt{7}$, not 4.

Step 4: Conclusion The radius of C_2 is $\sqrt{6}$. Hence, the correct option is (D).

Quick Tip to Solve: When a line cuts a known chord from another circle, use half the chord in $r^2 = d^2 + (L/2)^2$, not its full length.

25. The coefficien of x^{10} in the expansion of $(1 + x)^2(1 + x^2)^3(1 + x^3)^4$ is equal to : [4 marks]

(A) 52

(B) 56

(C) 50

(D) 44

Correct Answer: (A) 52

Solution:

Concept:

- Choosing the terms x^a, x^{2b} and x^{3c} from the three factors contributes a power $x^{a+2b+3c}$.
- By the binomial theorem, the coefficient contributed by one such choice is $\binom{2}{a}\binom{3}{b}\binom{4}{c}$. Add it over all allowed choices yielding the target exponent.

Step 1: Set up the bounded exponent equation In the three binomial expansions the permitted indices are

$$0 \le a \le 2, \qquad 0 \le b \le 3, \qquad 0 \le c \le 4.$$

The coefficient of interest is therefore the sum of $\binom{2}{a}\binom{3}{b}\binom{4}{c}$ over integer solutions of

$$a + 2b + 3c = 10.$$

These bounds matter: they express how many terms can be selected from each factor, and prevent counting powers that a factor cannot supply.

Step 2: List every admissible combination If $c = 0$, then $a + 2b = 10$, impossible because its maximum is 8. If $c = 1$, the remaining exponent is 7, and only $(a, b) = (1, 3)$ is allowed. If $c = 2$, the remaining exponent is 4, giving $(a, b) = (0, 2)$ and $(2, 1)$. If $c = 3$, the remaining exponent is 1, giving only $(1, 0)$. Finally $c = 4$ already supplies exponent 12, so contributes nothing. This exhausts all possible powers from the last factor.

Step 3: Compute and add their binomial weights The four nonzero contributions are

$$(1,3,1): \quad \binom{2}{1}\binom{3}{3}\binom{4}{1} = 2\cdot 1\cdot 4 = 8,$$
$$(0,2,2): \quad \binom{2}{0}\binom{3}{2}\binom{4}{2} = 1\cdot 3\cdot 6 = 18,$$
$$(2,1,2): \quad \binom{2}{2}\binom{3}{1}\binom{4}{2} = 1\cdot 3\cdot 6 = 18,$$
$$(1,0,3): \quad \binom{2}{1}\binom{3}{0}\binom{4}{3} = 2\cdot 1\cdot 4 = 8.$$

Their sum is $8+18+18+8=52$. The alternatives 56, 50 and 44 fail this exhaustive weighted count; simply counting exponent combinations would also miss their unequal binomial weights.

Step 4: Conclusion The coefficient of x^{10} is 52. Hence, the correct option is (A).

Quick Tip to Solve: For a product of binomials with powers x, x^2, x^3, solve the bounded exponent equation before calculating binomial coefficients.

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26. Let $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots\ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$.
Then, the least dd natural numbr p, so that $B_n > A_n$, for all n$\geq$ p, is : [4 marks]

(A) 9

(B) 7

(C) 11

(D) 5

Correct Answer: (B) 7

Solution:

Concept:

- The finite geometric sum with first term a and ratio $r \neq 1$ is $a(1-r^n)/(1-r)$. Here the common ratio is negative.
- Since $B_n = 1 - A_n$, the required comparison is equivalent to $A_n < 1/2$. Treat even and odd indices separately before finding the final threshold.

Step 1: Evaluate the geometric sum The first term of A_n is $3/4$ and its common ratio is $-3/4$. Therefore

$$A_n = \frac{(3/4)\{1-(-3/4)^n\}}{1+3/4} = \frac{3}{7}\left\{1-\left(-\frac{3}{4}\right)^n\right\}.$$

The condition $B_n > A_n$ becomes $1 - A_n > A_n$, or $A_n < 1/2$. It is a strict inequality, so equality at a proposed threshold would not suffice.

Step 2: Separate the even and odd cases For even n, the power $(-3/4)^n$ is positive, giving $A_n < 3/7 < 1/2$. Thus all positive even indices already satisfy the comparison. For odd n, the sum instead

becomes

$$A_n = \frac{3}{7}\left\{1 + \left(\frac{3}{4}\right)^n\right\}.$$

Hence the odd-index condition is

$$\frac{3}{7}\left\{1 + \left(\frac{3}{4}\right)^n\right\} < \frac{1}{2} \quad \Longleftrightarrow \quad \left(\frac{3}{4}\right)^n < \frac{1}{6}.$$

The positive powers of $3/4$ decrease strictly as n increases.

Step 3: Locate the least odd threshold and check all larger indices At the adjacent odd candidates,

$$\left(\frac{3}{4}\right)^5 = \frac{243}{1024} > \frac{1}{6}, \qquad \left(\frac{3}{4}\right)^7 = \frac{2187}{16384} < \frac{1}{6}.$$

The comparisons follow exactly from $1458 > 1024$ and $13122 < 16384$. Thus $n = 5$ fails, while $n = 7$ and every later odd index pass; every even index passes as shown above. The least odd p making the condition hold for all $n \geq p$ is consequently 7. The choices 9 and 11 work but are not least, and 5 itself fails.

Step 4: Conclusion The least odd natural number is $p = 7$. Hence, the correct option is (B).

Quick Tip to Solve: For an alternating geometric sum, the even indices may already satisfy the inequality. Find the first passing odd index using exact neighboring powers.

27. A plane bisects the line segment joining the points (1, 2, 3) and (− 3, 4, 5) at rigt angles. Then this plane also passes through the point : [4 marks]

(A) (− 3, 2, 1)

(B) (3, 2, 1)

(C) (− 1, 2, 3)

(D) (1, 2, − 3)

Correct Answer: (A) (− 3, 2, 1)

Solution:

Concept:

- A plane bisecting a segment at right angles passes through its midpoint and has a normal vector parallel to the segment.
- The plane through $M = (x_0, y_0, z_0)$ with normal (a, b, c) has equation $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$. A point lies on it exactly when its coordinates satisfy the equation.

Step 1: Find the midpoint and a normal vector Call the endpoints $U = (1, 2, 3)$ and $V = (-3, 4, 5)$. Their midpoint is

$$M = \left(\frac{1-3}{2}, \frac{2+4}{2}, \frac{3+5}{2}\right) = (-1, 3, 4).$$

A direction vector along the segment is

$$\overrightarrow{UV} = (-3-1, 4-2, 5-3) = (-4, 2, 2).$$

Because the desired plane is perpendicular to this segment, we may divide this vector by two and use $(-2, 1, 1)$ as its normal. Scaling a nonzero normal vector leaves the plane unchanged.

Step 2: Write the perpendicular bisector plane Using the midpoint and the normal vector gives

$$-2(x+1) + (y-3) + (z-4) = 0.$$

Collecting constants carefully,

$$-2x + y + z = 9.$$

The midpoint itself gives $2 + 3 + 4 = 9$, confirming the constant. This plane is the locus of points equidistant from the two endpoints, so the midpoint and perpendicularity conditions determine it uniquely.

Step 3: Test all four listed points Evaluate $-2x + y + z$ at each option:

$$\begin{aligned}(-3, 2, 1): &\quad 6 + 2 + 1 = 9,\\ (3, 2, 1): &\quad -6 + 2 + 1 = -3,\\ (-1, 2, 3): &\quad 2 + 2 + 3 = 7,\\ (1, 2, -3): &\quad -2 + 2 - 3 = -3.\end{aligned}$$

Only the first value equals the required constant 9; the other points therefore lie outside the plane. As a direct distance check for the first point, its squared distances from U and V are $16 + 0 + 4 = 20$ and $0 + 4 + 16 = 20$, respectively, agreeing with the perpendicular-bisector interpretation.

Step 4: Conclusion The plane passes through $(-3, 2, 1)$. Hence, the correct option is (A).

Quick Tip to Solve: Use the segment direction as the plane normal, then insert its midpoint. Checking equidistance offers an independent quick test of the selected point.

28. The number of solutions of sin3x = cos 2x, in the interval $\left(\frac{\pi}{2}, \pi\right)$ is : [4 marks]

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (A) 1

Solution:

Concept:

- Convert the cosine to a sine using $\cos\theta = \sin(\pi/2 - \theta)$. The equation $\sin A = \sin B$ has families $A = B + 2k\pi$ and $A = \pi - B + 2k\pi$, where k is an integer.

- After finding the general solutions, apply the stated open interval strictly. Endpoints are excluded even when they solve the original equation.

Step 1: Convert to equal sines and obtain both families The equation can be written as

$$\sin 3x = \sin\left(\frac{\pi}{2} - 2x\right).$$

The first equality family gives

$$3x = \frac{\pi}{2} - 2x + 2k\pi \quad \Longrightarrow \quad x = \frac{\pi}{10} + \frac{2k\pi}{5}.$$

The second gives

$$3x = \pi - \left(\frac{\pi}{2} - 2x\right) + 2k\pi \quad \Longrightarrow \quad x = \frac{\pi}{2} + 2k\pi.$$

Keeping both families is essential because the sine function takes the same value at supplementary angles as well as at coterminal angles.

Step 2: Restrict the first family to the specified interval For $\pi/2 < x < \pi$, substitute the first expression and divide by the positive number π:

$$\frac{1}{2} < \frac{1}{10} + \frac{2k}{5} < 1.$$

Subtracting $1/10$ and multiplying by $5/2$ yields

$$1 < k < \frac{9}{4}.$$

The only integer here is $k = 2$, giving $x = 9\pi/10$. The preceding value $k = 1$ gives $x = \pi/2$, exactly the excluded lower endpoint; the next integer gives a value exceeding π.

Step 3: Check the second family and count distinct roots The second family must satisfy

$$\frac{1}{2} < \frac{1}{2} + 2k < 1, \quad \text{or} \quad 0 < k < \frac{1}{4}.$$

It contains no integer k, and hence gives no admissible root. The surviving root verifies the original equation: $\sin(27\pi/10) = \sin(7\pi/10) = \cos(\pi/5)$, while $\cos(18\pi/10) = \cos(\pi/5)$. There is therefore exactly one root. Counts of two, three or four would require including excluded endpoints or counting values outside the interval.

Step 4: Conclusion There is exactly 1 solution in $(\pi/2, \pi)$. Hence, the correct option is (A).

Quick Tip to Solve: Write both sine-equality families, then bound the integer parameter. The strict inequality discards $x = \pi/2$ in this problem.

29. A tower T_1 of height 60 m is located exactly opposite to a tower T_2 of height 80 m on a straight road. Fromthe top of T_1, if the angle of depression of the foot of T_2 is twice the angle of elevation of the top of T_2, then the width (in m) of the road between the feetof the towers T_1 and T_2 is : [4 marks]

(A) $10\sqrt{2}$

(B) $10\sqrt{3}$

(C) $20\sqrt{3}$

(D) $20\sqrt{2}$

Correct Answer: (C) $20\sqrt{3}$

Solution:

Concept:

- The angle of elevation is measured above the horizontal and the angle of depression below it. For a right triangle, the tangent is the vertical separation divided by the horizontal distance.
- The double-angle identity is $\tan 2\alpha = 2\tan\alpha/(1-\tan^2\alpha)$. Here the two sight lines have the same horizontal separation, equal to the road width.

Step 1: Express the two viewing angles Let the road width be $x > 0$ metres and the angle of elevation from the top of the shorter tower be α. The tops differ in height by $80 - 60 = 20\,\text{m}$, whereas the foot of the taller tower is $60\,\text{m}$ below the observer. Thus

$$\tan\alpha = \frac{20}{x}, \qquad \tan 2\alpha = \frac{60}{x}.$$

The angle of depression is less than 90°, so $0 < \alpha < 45^\circ$. This fixes the physically relevant branch of the tangent identity.

Step 2: Solve for the horizontal separation Put $t = \tan\alpha = 20/x$. The second equation becomes

$$\frac{2t}{1-t^2} = 3t.$$

Since x is finite and positive, $t > 0$, and division by t is valid. Hence

$$2 = 3(1-t^2), \qquad t^2 = \frac{1}{3}, \qquad t = \frac{1}{\sqrt{3}}.$$

Consequently $x = 20\sqrt{3}\,\text{m}$. As a check, the elevation is 30° and the depression is 60°. The other proposed widths fail the required relation $\tan 2\alpha = 3\tan\alpha$; the height difference, rather than the full taller-tower height, must be used for the elevation.

Step 3: Conclusion The road width is $\boxed{20\sqrt{3}\,\text{m}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the difference of tower heights for the upward sight line and the observer's full height for the downward sight line; both share the road width.

30. Consider the following two statements :

Statement p :

The value of sin 120° can be derived by taking $\theta = 240^o$ in the equation

$2\sin\frac{\theta}{2} = \sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}$

Statement q :

The angles A, B, C and D of any quadrilateral ABCD satisfy the equation

$\cos\left(\frac{1}{2}(A+C)\right) + \cos\left(\frac{1}{2}(B+D)\right) = 0$

Then the truth values of p and q are respectively : [4 marks]

(A) F, T

(B) T, F

(C) T, T

(D) F, F

Correct Answer: (A) F, T

Solution:

Concept:

- Principal square roots are nonnegative. Identities obtained by removing absolute values from half-angle expressions need quadrant restrictions.
- The interior angles of a quadrilateral sum to $360°$. Supplementary opposite half-sums therefore add to $180°$, and $\cos(180° - u) = -\cos u$.

Step 1: Test statement p at the stated angle For $\theta = 240°$, the left side of the proposed equation is

$$2\sin 120° = \sqrt{3} > 0.$$

But $\sin 240° = -\sqrt{3}/2$, so the right side is

$$\sqrt{1-\sqrt{3}/2} - \sqrt{1+\sqrt{3}/2} < 0.$$

The first radical is smaller than the second, which alone proves that the two sides cannot be equal. In fact their difference is -1. Thus the displayed equation is not valid at the angle proposed, and statement p is false. One cannot apply a sign-restricted half-angle formula blindly in another quadrant.

Step 2: Test statement q using the angle sum Set $u = (A+C)/2$. Since $A + B + C + D = 360°$,

$$\frac{B+D}{2} = 180° - u.$$

Therefore

$$\cos\frac{A+C}{2} + \cos\frac{B+D}{2} = \cos u + \cos(180° - u) = 0.$$

This argument uses only the angle sum and does not require a cyclic or rectangular quadrilateral. Statement q is true. Thus the pairs in options (B) and (C) wrongly assert p, while (D) wrongly denies q.

Step 3: Conclusion The truth values of p and q, in order, are $\boxed{F, T}$. Hence, the correct option is (A).

Quick Tip to Solve: Before evaluating nested radicals exactly, compare their sizes and signs. For quadrilateral angle identities, start with the total $360°$.

Physics (120 marks)

31. Muon (μ^-) is a negatively charged ($|q| = |e|$) particle with a mass $m_\mu = 200\ m_e$, where m_e is the mass of the electron and e is the electronic charge. If μ^- is bond to a proton to form a hydrogen like atom, identify the correct statements.

(A) Radis of the muonic orbit is 200 times smaller than that of the electron.

(B) The speed of the μ^- in the n^{th} orbit is $\frac{1}{200}$ times that of the electron in the n^{th} orbit.

(C) The ionization energy of muonic atom is 200 timesmore than of an hydroen atom.

(D) The momentum of the muon in the n^{th} orbit is 200 times more than that of the electron. [4 marks]

(A) (A), (B), (D)

(B) (A), (C), (D)

(C) (B), (D)

(D) (C), (D)

Correct Answer: (B) (A), (C), (D)

Solution:

Concept:

- In the Bohr model for a particle of charge magnitude e orbiting a proton, $mvr = n\hbar$ and $mv^2/r = ke^2/r^2$, where $k = 1/(4\pi\varepsilon_0)$.
- For the same nuclear charge and orbit number, the resulting scalings are $r_n \propto 1/m$, $v_n \propto m^0$, $|E_n| \propto m$, and $p_n = mv_n \propto m$. Use the usual fixed-proton Bohr approximation intended by the listed ratios.

Step 1: Compare the orbital radius and speed Combining angular-momentum quantization with Coulomb attraction gives

$$r_n = \frac{n^2\hbar^2}{mke^2}, \qquad v_n = \frac{ke^2}{n\hbar}.$$

Replacing the electron by a muon of mass $200m_e$ therefore gives

$$\frac{r_{\mu,n}}{r_{e,n}} = \frac{1}{200}, \qquad \frac{v_{\mu,n}}{v_{e,n}} = 1.$$

Statement (A) is true. Statement (B) is false: although the orbit shrinks, the quantized orbital speed at a given n is independent of the orbiting particle mass.

Step 2: Compare the energy and momentum The bound-state energy is

$$E_n = -\frac{mk^2e^4}{2n^2\hbar^2}.$$

The energy required to remove the particle from its ground state is proportional to m, so the muonic ionization energy is 200 times the hydrogen value. Statement (C) is true. Also $p_n = mv_n$, and the equal speeds imply $p_{\mu,n}/p_{e,n} = 200$; statement (D) is true. Hence the correct set contains (A), (C), and (D). Option (A) and option (C) include the false speed statement, while option (D) omits the true radius statement.

Step 3: Conclusion The radius decreases by a factor of 200; ionization energy and momentum increase by a factor of 200, while speed stays unchanged. Hence, the correct option is (B).

Quick Tip to Solve: Derive the mass scalings before inserting 200: radius varies inversely with mass, speed stays fixed, and binding energy and momentum increase with mass.

32. An unstable heavy nucleus at rest breaks into two nuclei which move away with velocities in the ratio of 8 : 27. The ratio of the radii of the nuclei (assumed to be spherical) is : [4 marks]

(A) 8 : 27

(B) 4 : 9

(C) 3 : 2

(D) 2 : 3

Correct Answer: (C) 3 : 2

Solution:

Concept:

- When a nucleus initially at rest splits without an external impulse, the vector sum of the fragment momenta remains zero. The two momentum magnitudes are therefore equal.
- For nuclei of approximately equal nuclear density, $R = R_0 A^{1/3}$. Since nuclear mass is approximately proportional to mass number A, radius ratios are cube roots of mass ratios.

Step 1: Use the inverse relation between mass and speed Label the fragments so that their speed ratio is $v_1 : v_2 = 8 : 27$. Their velocities point in opposite directions because the parent was at rest. Conservation of linear momentum gives

$$m_1 v_1 = m_2 v_2, \qquad \frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{27}{8}.$$

The slower fragment is the heavier one. Equating masses in the same ratio as speeds would violate the zero-momentum condition.

Step 2: Convert the mass ratio to a radius ratio The nuclear-radius law yields

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} \simeq \left(\frac{m_1}{m_2}\right)^{1/3} = \left(\frac{27}{8}\right)^{1/3} = \frac{3}{2}.$$

Thus the first, slower nucleus has the larger radius. Option (A) repeats the speed ratio instead of using momentum and the radius law. Option (B) resembles a squared ratio rather than the required cube-root mass ratio. Option (D) reverses the fragments and would incorrectly make the faster fragment larger. No kinetic-energy equality is needed: the two momenta, rather than the two energies, have equal magnitudes.

Step 3: Conclusion The radii are in the ratio $\boxed{3 : 2}$, corresponding to the stated order of fragment speeds. Hence, the correct option is (C).

Quick Tip to Solve: For a breakup from rest, first invert the speed ratio to obtain the mass ratio, then take its cube root to obtain the nuclear-radius ratio.

33. A convergent doublet of separated lenses, corrected for spherical aberration, has resultant focal length of 10 cm. The separation between the two lenses is 2 cm. The focal lengths of the component lenses are : [4 marks]

(A) 10 cm, 12 cm

(B) 12 cm, 14 cm

(C) 16 cm, 18 cm

(D) 18 cm, 20cm

Correct Answer: (D) 18 cm, 20cm

Solution:

Concept:

- Two thin lenses of focal lengths f_1 and f_2, separated by distance d in air, have equivalent power $1/F = 1/f_1 + 1/f_2 - d/(f_1 f_2)$.
- Thus $F = f_1 f_2/(f_1 + f_2 - d)$. All lengths must be in the same units, and the separation term must be retained; the lenses are not in contact.

Step 1: Write the necessary equivalent-focal-length equation The required resultant focal length is $F = 10\,\text{cm}$, with separation $d = 2\,\text{cm}$. Hence any possible component pair must satisfy

$$\frac{1}{10} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{2}{f_1 f_2},$$

or, equivalently,

$$f_1 f_2 = 10(f_1 + f_2 - 2).$$

This necessary condition already distinguishes the listed choices, so no additional aberration formula is needed to select between them. The correction for aberration does not remove the requirement to have the stated resultant power.

Step 2: Check each listed component pair For the pairs in options (A), (B), (C), and (D), respectively, the equivalent focal lengths are

$$F_A = \frac{10 \times 12}{10 + 12 - 2} = 6\,\text{cm},$$
$$F_B = \frac{12 \times 14}{12 + 14 - 2} = 7\,\text{cm},$$
$$F_C = \frac{16 \times 18}{16 + 18 - 2} = 9\,\text{cm},$$
$$F_D = \frac{18 \times 20}{18 + 20 - 2} = 10\,\text{cm}.$$

Only the last pair has the specified resultant focal length. Adding the two focal lengths, or using only the contact-lens expression, would give an incorrect check because propagation between the lenses changes the combined power.

Step 3: Conclusion The component focal lengths are $\boxed{18\,\text{cm and } 20\,\text{cm}}$. Hence, the correct option is (D).

Quick Tip to Solve: In a separated-lens MCQ, test the equivalent power before invoking an aberration condition; a necessary focal-length condition may eliminate every wrong pair.

34. A constant voltages is applied between two ends of a metallic wire. If the length is halved and the radius of the wire is doubled, the rate of heat developed in the wire will be : [4 marks]

(A) Doubled

(B) Halved

(C) Unchanged

(D) Increased 8 times

Correct Answer: (D) Increased 8 times

Solution:

Concept:

- The resistance of a uniform wire is $R = \rho L/A = \rho L/(\pi r^2)$, where ρ is resistivity, L is length and r is radius.
- At a fixed potential difference V, the heating rate is the electric power $P = V^2/R$. Assume the metallic material and its resistivity remain the same during the comparison.

Step 1: Calculate the change in resistance The new dimensions are $L' = L/2$ and $r' = 2r$. Since area depends on radius squared,

$$A' = \pi(2r)^2 = 4A.$$

Consequently

$$R' = \frac{\rho(L/2)}{4A} = \frac{1}{8}\frac{\rho L}{A} = \frac{R}{8}.$$

The length reduction halves the resistance, and the area increase introduces a further factor of one-fourth. These are independent changes in the dimensions; the problem does not impose conservation of the original wire volume.

Step 2: Use the fixed-voltage heating law The applied voltage is unchanged, so

$$\frac{P'}{P} = \frac{V^2/R'}{V^2/R} = \frac{R}{R'} = 8.$$

Equivalently, the current increases from V/R to $8V/R$, and $P = VI$ increases eightfold. Option (A) accounts for the length change alone and misses the larger cross-section. Option (B) has the opposite resistance-power dependence for fixed voltage. Option (C) would require no resistance change. Using I^2R while incorrectly holding current fixed would also give the wrong conclusion: it is voltage, not current, that is specified as constant.

Step 3: Conclusion The rate of heat production becomes $\boxed{8}$ times its original value. Hence, the correct option is (D).

Quick Tip to Solve: Identify what is held fixed first. With constant voltage, $P \propto r^2/L$, so doubling radius and halving length gives $4 \times 2 = 8$.

35. A plane polarized monochromatic EM wave is traveling in vacuum along z direction such that at $t = t_1$ it is found that the electric field is zero at a spatial point z_1. The next zero that occurs in its neighbourhood is at z_2. The frequency of the electroagnetic wave is : [4 marks]

(A) $\frac{3\times10^8}{|z_2-z_1|}$

(B) $\frac{1.5\times10^8}{|z_2-z_1|}$

(C) $\frac{6\times10^8}{|z_2-z_1|}$

(D) $\frac{1}{t_1+\frac{|z_2-z_1|}{3\times10^8}}$

Correct Answer: (B) $\frac{1.5\times10^8}{|z_2-z_1|}$

Solution:

Concept:

- For a monochromatic plane wave, one electric-field component can be written $E(z,t) = E_0\sin(kz-\omega t+\phi)$, where $k = 2\pi/\lambda$.
- At a fixed instant, adjacent zeros of a sinusoid differ in phase by π and are separated by $\lambda/2$. In vacuum, the wave speed is $c = \nu\lambda = 3\times10^8\,\mathrm{m\,s^{-1}}$.

Step 1: Relate the two spatial zeros to wavelength At the specified time t_1, a zero at z_1 satisfies

$$kz_1 - \omega t_1 + \phi = n\pi$$

for an integer n. The next zero along the spatial profile has the next integer phase, so subtracting the phase conditions gives

$$k|z_2 - z_1| = \pi.$$

With $k = 2\pi/\lambda$, this means

$$\lambda = 2|z_2 - z_1|.$$

The word next is essential: the two positions are adjacent zeros, not two successive crests, whose separation would instead be a full wavelength.

Step 2: Find the frequency and compare the options Let $\Delta z = |z_2 - z_1|$, in metres. Then

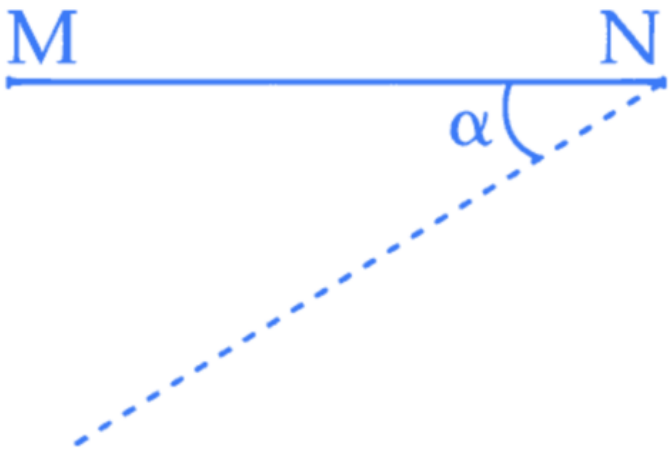
$$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{2\Delta z} = \frac{1.5\times10^8}{|z_2 - z_1|}\,\mathrm{Hz}.$$

Option (A) treats the adjacent-zero spacing as a full wavelength. Option (C) makes the opposite factor-of-two error. Option (D) incorrectly introduces the observation time t_1: shifting the time changes the phase of the spatial pattern but cannot change the frequency of a monochromatic wave. The ratio of a speed to a distance correctly has dimensions of inverse time.

Step 3: Conclusion The frequency is $\boxed{\nu = \frac{1.5\times10^8}{|z_2 - z_1|}\,\mathrm{Hz}}$, with the separation in metres. Hence, the correct option is (B).

Quick Tip to Solve: Adjacent zeros of a sinusoidal spatial profile are half a wavelength apart. Double their separation before using $\nu = c/\lambda$.

36. A thin rod MN, free to rotate in the vertical plane aboutthe fixed end N, is held horizontal. When the end M is released the speed of this end, when the rod makes an angle α with the horizontal, will be proportional to : (see figure) [4 marks]



(A) $\sqrt{\sin\alpha}$

(B) $\sin\alpha$

(C) $\sqrt{\cos\alpha}$

(D) $\cos\alpha$

Correct Answer: (A) $\sqrt{\sin\alpha}$

Solution:

Concept:

- A rigid rod rotating about a fixed end has kinetic energy $K = \frac{1}{2}I_N\omega^2$. For a uniform thin rod of mass m and length L, $I_N = mL^2/3$.
- The centre of mass lies at the midpoint. Its loss in gravitational potential energy is mg times its vertical descent, and a point at distance L from the pivot has speed $v = L\omega$.

Step 1: Find the fall of the centre of mass Initially the rod is horizontal and at rest. After release it rotates downward through angle α. The centre of mass is only $L/2$ from the fixed end, so its vertical descent is

$$h = \frac{L}{2}\sin\alpha.$$

The fixed pivot does no work because its point of application remains at rest. Thus gravity supplies all of the rod's kinetic energy.

Step 2: Apply conservation of mechanical energy Equate the loss of potential energy to the rotational kinetic energy:

$$mg\frac{L}{2}\sin\alpha = \frac{1}{2}\left(\frac{mL^2}{3}\right)\omega^2.$$

Cancelling the mass and simplifying gives

$$\omega^2 = \frac{3g}{L}\sin\alpha.$$

It is the speed squared, rather than the speed itself, that varies linearly with the fall.

Step 3: Convert angular speed to the speed of M The free end M is a distance L from N, hence

$$v_M = L\omega = \sqrt{3gL\sin\alpha}.$$

The factor $\sqrt{3gL}$ is independent of the angle. Option B misses the square root. Options C and D would predict a nonzero speed at the initial horizontal position $\alpha = 0$, contrary to release from rest.

Step 4: Conclusion The speed of the released end is $v_M = \sqrt{3gL}\sqrt{\sin\alpha}$, so $v_M \propto \sqrt{\sin\alpha}$. Hence, the correct option is (A).

Quick Tip to Solve: For a rod released from rest, find the vertical drop of its midpoint first; energy then gives the square of the free-end speed.

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37. A solid ball of radius R has a charge density ρ given by $\rho = \rho_o\,(1 - {}^{r}/_{R})$ for $0 \le r \le R$. The electric field outside the ball is : [4 marks]

(A) $\frac{\rho_o R^3}{\epsilon_o r^2}$

(B) $\frac{\rho_o R^3}{12\epsilon_o r^2}$

(C) $\frac{4\rho_o R^3}{3\epsilon_o r^2}$

(D) $\frac{3\rho_o R^3}{4\epsilon_o r^2}$

Correct Answer: (B) $\frac{\rho_o R^3}{12\epsilon_o r^2}$

Solution:

Concept:

- For spherical symmetry, Gauss's law outside the distribution gives $E(4\pi r^2) = Q/\varepsilon_0$, where $r \ge R$ is the distance from the centre.
- A thin spherical shell of radius s and thickness ds has volume $4\pi s^2 ds$. Its charge is its local volume charge density multiplied by this volume.

Step 1: Integrate the charge density over the ball Use s for the integration variable so it is distinct from the external observation distance r. The given density is $\rho(s) = \rho_0(1 - s/R)$. Therefore

$$\begin{aligned} Q &= 4\pi\rho_0 \int_0^R \left(1 - \frac{s}{R}\right) s^2\, ds \\ &= 4\pi\rho_0 \left[\frac{s^3}{3} - \frac{s^4}{4R}\right]_0^R \\ &= 4\pi\rho_0 R^3 \left(\frac{1}{3} - \frac{1}{4}\right) = \frac{\pi\rho_0 R^3}{3}. \end{aligned}$$

The density is not constant, so replacing it by its central value throughout the sphere would overestimate this charge.

Step 2: Apply Gauss law outside the sphere Choose a concentric spherical Gaussian surface of radius

$r \geq R$. The field has equal magnitude everywhere on this surface and is radial. It encloses the entire charge just calculated:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\rho_0 R^3}{12\varepsilon_0 r^2}.$$

For positive ρ_0 the field points outward. The inverse-square dependence is required outside any spherically symmetric distribution. Options A, C and D have coefficients $1, 4/3, 3/4$, respectively, while integration gives $1/12$; all three overestimate the field.

Step 3: Conclusion The electric field outside the ball is $E = \rho_0 R^3/(12\varepsilon_0 r^2)$, directed radially outward for positive charge density. Hence, the correct option is (B).

Quick Tip to Solve: For a nonuniform sphere, integrate $4\pi s^2 \rho(s)\, ds$ to find its total charge before using the point-charge field outside.

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38. A body of mass 2 kg slides down with an acceleration of 3 m/s^2 on a rough inclined plane having a slope of 30^o. The external force required to take the same body up the plane with the same acceleration will be : (g = 10 m/s^2) [4 marks]

(A) 14 N

(B) 20 N

(C) 6 N

(D) 4 N

Correct Answer: (B) 20 N

Solution:

Concept:

- Kinetic friction opposes the direction of sliding. For a force parallel to the slope, the normal reaction remains $N = mg\cos\theta$, so the friction magnitude remains the same when the motion reverses.
- Newton's second law along the plane is $\sum F_{\parallel} = ma$. The downhill component of gravity is $mg\sin\theta$.

Step 1: Determine the friction from the downward motion Take downhill as positive while the body slides down. Gravity pulls downhill and friction acts uphill:

$$mg\sin 30^\circ - f = ma.$$

Using $m = 2\,\text{kg}$, $g = 10\,\text{m s}^{-2}$ and $a = 3\,\text{m s}^{-2}$,

$$f = 2(10)\left(\frac{1}{2}\right) - 2(3) = 10 - 6 = 4\,\text{N}.$$

The normal reaction is $10\sqrt{3}\,\text{N}$, so the corresponding kinetic friction coefficient is $2/(5\sqrt{3})$. We only need the friction force itself.

Step 2: Reverse the motion and write the new force balance For upward motion, both gravity and friction act down the plane. The applied force F acts up the plane, and the required acceleration is

also upward. Taking uphill as positive gives

$$F - mg\sin 30^\circ - f = ma.$$

The friction force has reversed direction compared with the first motion, but its magnitude is still $4\,\mathrm{N}$. Thus

$$F = 6 + 10 + 4 = 20\,\mathrm{N}.$$

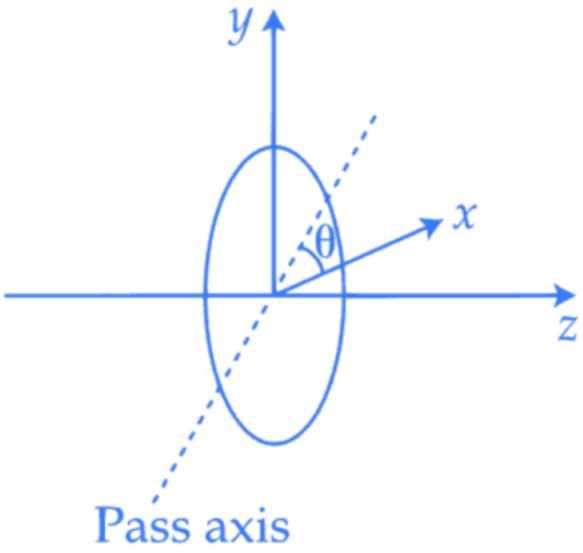
Step 3: Check the other proposed forces A force of $14\,\mathrm{N}$ only balances the two opposing forces and produces zero acceleration. A force of $6\,\mathrm{N}$ is merely ma, neglecting gravity and friction, and $4\,\mathrm{N}$ is only the friction magnitude. None produces the requested upward acceleration.

Step 4: Conclusion The required external force parallel to the incline is $20\,\mathrm{N}$, directed up the plane. Hence, the correct option is (B).

Quick Tip to Solve: When a sliding body reverses direction, reverse the friction arrow before writing the second force equation; do not reuse the old net force.

39. A plane polarized light is incident on a polariser with its pass axis aking angle θ with x-axis, as shown in the figure. At four different values of θ, θ = 8°, 38°, 188° and 218°, the observed intensities are same.

What is the angle between the direction of polarization and x-axis ? [4 marks]



(A) 98°

(B) 128°

(C) 203°

(D) 45°

Correct Answer: (C) 203°

Solution:

Concept:

- Malus's law states that transmitted intensity is $I(\theta) = I_0 \cos^2(\theta - \phi)$, where ϕ is the angle of the incident polarization direction with the x-axis.
- A linear polarization axis is unchanged by a rotation of $180°$. Equal intensities can also occur at analyzer angles symmetric about a maximum or minimum of $\cos^2(\theta - \phi)$.

Step 1: Use the two distinct analyzer orientations The settings $188°$ and $218°$ repeat $8°$ and $38°$ after adding $180°$, so the independent condition is

$$\cos^2(8° - \phi) = \cos^2(38° - \phi).$$

Since these two analyzer orientations differ by $30°$, they are not identical axes. Their equal intensities mean they lie symmetrically about a transmission extremum.

Step 2: Find the possible polarization axes Using $\cos^2 u = \cos^2 v$, the nontrivial case here is $u = -v + n180°$. Hence

$$8° - \phi = -(38° - \phi) + n180°,$$

which yields

$$\phi = 23° - n90°.$$

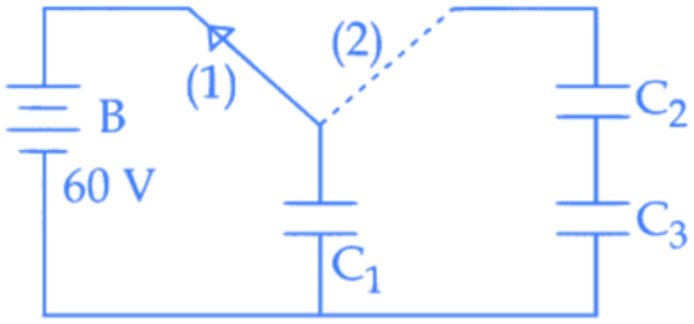
Modulo $180°$, the two possible axes are $23°$ and $113°$. The given intensity equality alone does not distinguish those perpendicular axes.

Step 3: Identify the orientation represented by a listed option Among the proposed angles, $203° = 23° + 180°$ is the same linear polarization direction as $23°$, so it satisfies the condition. It gives relative analyzer angles $-195°, -165°, -15°, 15°$, all with equal squared cosines. For $98°$, the first two relative angles are $-90°, -60°$, giving unequal intensities; for $128°$ they are $-120°, -90°$, also unequal. At $45°$, the differences $-37°, -7°$ likewise give unequal values.

Step 4: Conclusion The listed angle consistent with all four equal intensities is $203°$, representing the polarization axis $23°$ modulo $180°$. Hence, the correct option is (C).

Quick Tip to Solve: Reduce polarizer angles modulo $180°$, then bisect the two distinct equal-intensity orientations; also consider the perpendicular bisector axis.

40. A capacitor C_1 = 1.0 μF is charged up to a voltage V = 60 V by connecting it to battery B through switch (1). Now C_1 is disconnected from battery and connected to a circuit consisting of two uncharged capacitors $C_2 = 3.0\mu F$ and C_3 = 6.0 μF through switch (2), as shown in the figure. The sum of final charges on C_2 and C_3 is : [4 marks]



(A) 40 μC

(B) 36 μC

(C) 20 μC

(D) 54 μC

Correct Answer: The literal sum is $80\,\mu$C, absent from the options; $40\,\mu$C is the charge on each series capacitor.

Solution:

Concept:

- The charge on a capacitor is $Q = CV$. Two capacitors in series have equal charge magnitudes, and their equivalent capacitance is $C_s = C_2C_3/(C_2 + C_3)$.
- After the battery is disconnected, the total charge on each isolated conducting node is conserved. The series combination is connected in parallel with C_1, so both branches share the same final terminal voltage.

Step 1: Find the initial charge and series capacitance Before switching, the initially charged capacitor stores

$$Q_0 = C_1V_0 = (1.0\,\mu\text{F})(60\,\text{V}) = 60\,\mu\text{C}.$$

The initially uncharged capacitors C_2 and C_3 are in series in the diagram, with an isolated neutral conductor between them. Their equivalent capacitance is

$$C_s = \frac{(3.0)(6.0)}{3.0 + 6.0}\,\mu\text{F} = 2.0\,\mu\text{F}.$$

Step 2: Calculate the final charge on each capacitor The total capacitance across the charged outer nodes is $C_1 + C_s = 3.0\,\mu$F. Charge conservation gives

$$V_f = \frac{60\,\mu\text{C}}{3.0\,\mu\text{F}} = 20\,\text{V}.$$

Consequently, $Q_1 = 20\,\mu\text{C}$, and the series branch carries

$$Q_s = C_s V_f = (2.0)(20)\,\mu\text{C} = 40\,\mu\text{C}.$$

Since the middle node remains neutral, the individual charge magnitudes are $Q_2 = Q_3 = 40\,\mu\text{C}$. Their voltages are $40/3\,\text{V}$ and $20/3\,\text{V}$, which correctly add to $20\,\text{V}$.

Step 3: Distinguish the requested sum from the branch charge The literal sum of the individual capacitor charge magnitudes is

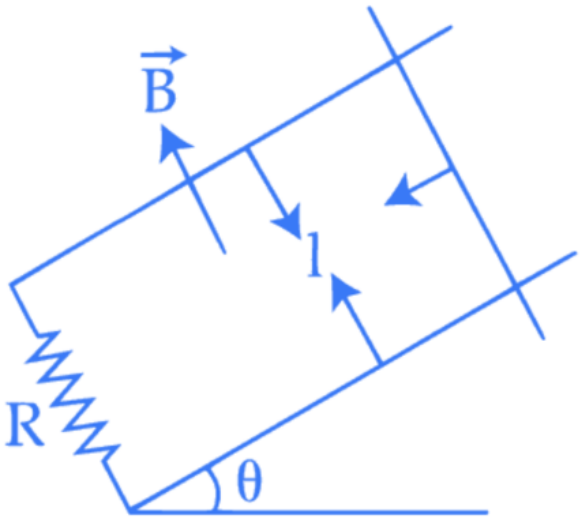
$$Q_2 + Q_3 = 40 + 40 = 80\,\mu\text{C}.$$

This is absent from the options. Option A, $40\,\mu\text{C}$, is the charge on each series capacitor or the charge of the equivalent series branch, not their sum. Options B, C and D also do not equal the sum; $20\,\mu\text{C}$ is instead the charge remaining on C_1.

Step 4: Conclusion Each of C_2 and C_3 has charge magnitude $40\,\mu\text{C}$, so their literal sum is $80\,\mu\text{C}$; none of the printed options gives it. The answer-key choice (A), $40\,\mu\text{C}$, describes the charge on each series capacitor. It does not equal the sum requested by the printed wording.

Quick Tip to Solve: Count a series branch once in charge conservation at the outer node, but count both individual capacitor magnitudes when their sum is explicitly requested.

41. A copper rod of mass m slides under gravity on two smooth parallel rails, with separation *l* and set at an angle of θ with the horizontal. At the bottom rails are joined by a resistance R. There is a uniform magnetic field B normal to the plane of the rails, as shown in the igure. The terminal speed of the copper rod is : [4 marks]



(A) $\frac{mgR\tan\theta}{B^2l^2}$

(B) $\frac{mgR\cot\theta}{B^2l^2}$

(C) $\frac{mgR\sin\theta}{B^2l^2}$

(D) $\frac{mgR\cos\theta}{B^2l^2}$

Correct Answer: (C) $\frac{mgR\sin\theta}{B^2l^2}$

Solution:

Concept:

- A rod of length l moving perpendicular to itself at speed v in a magnetic field normal to the circuit plane has motional emf $\mathcal{E} = Blv$.
- With circuit resistance R, the induced current is $I = \mathcal{E}/R$. Lenz's law makes the magnetic force oppose the rod's motion; terminal speed occurs when the net force along the rails vanishes.

Step 1: Find the induced current and retarding force The rod slides down the smooth rails and changes the circuit area at rate lv. Since the field is normal to the plane of the rails, the flux changes at rate Blv. Thus

$$\mathcal{E} = Blv, \qquad I = \frac{Blv}{R}.$$

There is no additional factor $\cos\theta$ in this expression: the supplied field already points normal to the circuit plane. The magnetic force on the rod is along the rails, opposing its downward motion:

$$F_B = IlB = \frac{B^2 l^2 v}{R}.$$

Step 2: Balance the forces along the incline The gravitational force parallel to the rails is $mg\sin\theta$. The normal reaction balances the perpendicular component, and the rails exert no friction. Therefore the equation of motion is

$$m\frac{dv}{dt} = mg\sin\theta - \frac{B^2 l^2 v}{R}.$$

At terminal speed the acceleration is zero, giving

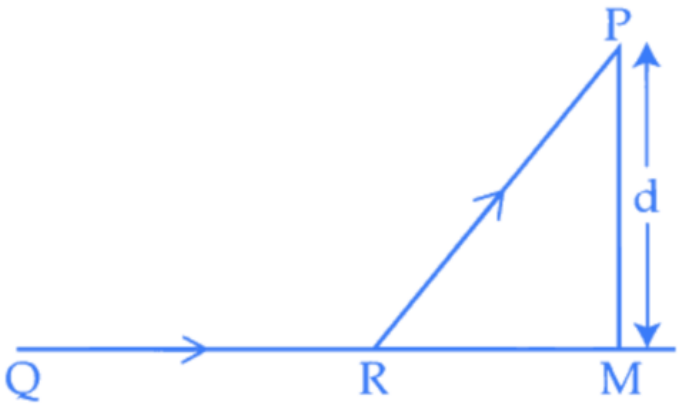
$$mg\sin\theta = \frac{B^2 l^2 v_t}{R}, \qquad v_t = \frac{mgR\sin\theta}{B^2 l^2}.$$

Step 3: Check the angular dependence For horizontal rails, $\theta = 0$ and there is no downhill gravity, so the terminal speed must vanish. This excludes the cosine and cotangent expressions. The tangent expression also vanishes there, but incorrectly introduces an extra inverse cosine; it diverges for vertical rails even though both the driving force and magnetic damping remain finite. Only the sine dependence follows the force balance.

Step 4: Conclusion The terminal speed down the rails is $v_t = mgR\sin\theta/(B^2 l^2)$. Hence, the correct option is (C).

Quick Tip to Solve: Check whether the field is normal to the rails or vertical before resolving it; here only gravity requires the factor $\sin\theta$.

42. A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from the highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM, so that the time taken to reach P is minimum ? [4 marks]



(A) d

(B) $\frac{d}{\sqrt{2}}$

(C) $\frac{d}{2}$

(D) $\frac{d}{\sqrt{3}}$

Correct Answer: (D) $\frac{d}{\sqrt{3}}$

Solution:

Concept:

- Travel time on each straight segment equals its length divided by its speed. The total time is minimized by varying the point where the car leaves the highway.
- For a differentiable travel-time function $t(x)$, an interior minimum satisfies $dt/dx = 0$; a positive second derivative confirms that the stationary point is a minimum.

Step 1: Express the travel time in terms of RM Let $QM = L$, $RM = x$ and $PM = d$. The diagram places R between Q and M, so the highway distance is $QR = L - x$. The straight field distance is $RP = \sqrt{x^2 + d^2}$. Since the highway speed is v and the field speed is $v/2$, the total time is

$$t(x) = \frac{L - x}{v} + \frac{2\sqrt{x^2 + d^2}}{v}.$$

For a fixed exit point, a straight field path is fastest because its speed there is constant.

Step 2: Differentiate and solve the stationary condition Differentiation gives

$$\frac{dt}{dx} = \frac{1}{v}\left(-1 + \frac{2x}{\sqrt{x^2 + d^2}}\right).$$

Set the derivative to zero and use $x \geq 0$:

$$2x = \sqrt{x^2 + d^2}, \qquad 4x^2 = x^2 + d^2.$$

Thus $3x^2 = d^2$, and the positive distance is $x = d/\sqrt{3}$. The fixed distance L disappears on differentiation, so it does not affect the optimum when the displayed interior exit point is available.

Step 3: Confirm the minimum and test the choices The second derivative is

$$\frac{d^2t}{dx^2} = \frac{2d^2}{v(x^2+d^2)^{3/2}} > 0.$$

Therefore the stationary point is a strict minimum. At this point $RP = 2x$, so the derivative condition is satisfied exactly. Options A, B and C give $x = d, d/\sqrt{2}, d/2$; none satisfies $2x/\sqrt{x^2+d^2} = 1$. If Q were closer to M than the calculated optimum, the constrained path would instead leave the road immediately, but the figure depicts the interior optimum asked for.

Step 4: Conclusion The time is minimum when $RM = d/\sqrt{3}$, for the interior exit point shown. Hence, the correct option is (D).

Quick Tip to Solve: Write both segment times before differentiating. The slower field segment carries a factor of two, which fixes the optimal exit angle.

43. A copper rod of cross-sectional area A carries a uniform current I through it. At temperature T, if the volume charge density of the rod is ρ, how long will the changes take to travel a distance d ? [4 marks]

(A) $\frac{2\rho\, d\, A}{\mathrm{I}}$
(B) $\frac{2\rho\, d\, A}{\mathrm{I}T}$
(C) $\frac{\rho\, d\, A}{\mathrm{I}}$
(D) $\frac{\rho\, d\, A}{\mathrm{I}T}$

Correct Answer: (C) $\frac{\rho\, d\, A}{\mathrm{I}}$

Solution:

Concept:

- Current is the amount of charge crossing a transverse section per unit time: $I = Q/t$. Here ρ denotes the magnitude of mobile charge per unit volume.
- For a conductor of cross-sectional area A, the charges drifting through distance d occupy a cylinder of volume Ad. Thus $Q = \rho Ad$, and the drift-speed relation is $I = \rho A v_d$.

Step 1: Identify the charge transported Follow the mobile charges initially contained in a length d of the rod. After these charges drift through that distance, the amount passing a chosen cross-section equals the charge in that cylindrical segment. Its volume and charge are therefore

$$V = Ad, \qquad Q = \rho V = \rho Ad.$$

Use the carrier charge-density magnitude; the electrically neutral copper lattice as a whole is not the charge density relevant to this current formula.

Step 2: Calculate the travel time Since the current is uniform, the travel time follows directly from

charge divided by current:

$$t = \frac{Q}{I} = \frac{\rho A d}{I}.$$

Equivalently, the drift speed is $v_d = I/(\rho A)$, so $t = d/v_d$ gives exactly the same expression. This is a drift time, rather than the propagation time of an electrical signal.

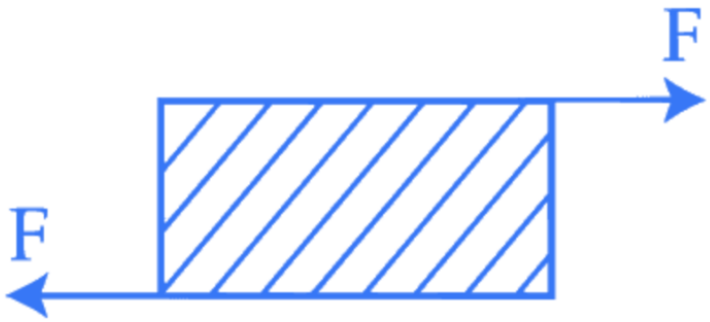
Step 3: Check the dimensions and eliminate the alternatives The dimensions of $\rho A d$ are charge, because $[\rho] = \mathrm{C\,m^{-3}}$ and $[Ad] = \mathrm{m^3}$. Dividing by current gives seconds. Option (A) introduces an unjustified factor of two. Options (B) and (D) additionally divide by the absolute temperature, which has no place when the current and charge density are already specified. Temperature may affect resistivity, but that does not change this charge-continuity relation at the stated current.

Step 4: Conclusion The charges take $\boxed{t = \rho A d/I}$ to drift through distance d. Hence, the correct option is (C).

Quick Tip to Solve: When both the current and carrier charge density are given, use $I = \rho A v_d$ before introducing resistance or temperature.

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44. As shown in the figure, forces of 10^5 N each are applied in opposite directions, on the upper and lower faces of a cube of side 10 cm, shifting the upper face parallel to itself by 0.5 cm. If the side of another cube of the same material is 20 cm, then under similar conditions as above, the displacement will be : [4 marks]



(A) 0.25 cm

(B) 0.37 cm

(C) 0.75 cm

(D) 1.00 cm

Correct Answer: (A) 0.25 cm

Solution:

Concept:

- Shear modulus is tangential stress divided by shear strain: $G = (F/A)/(x/L)$, where x is the lateral displacement and L is the separation of the loaded faces.
- For a cube of side L, the face area is L^2, giving $G = F/(Lx)$. The same material has the same shear modulus under the stated conditions.

Step 1: Write the deformation relation for a cube The opposing forces shear the cube: its upper face moves parallel to the lower face. The small shear strain is the displacement divided by the cube height, x/L. Since the force acts on a face of area L^2,

$$G = \frac{F/L^2}{x/L} = \frac{F}{Lx}.$$

Rearranging gives $x = F/(GL)$. Thus at a fixed applied force, displacement varies inversely with the side length; it is not proportional to the height alone.

Step 2: Compare the two cubes The two cubes are of the same material and are subjected to the same pair of forces, so G and F cancel on taking the ratio:

$$\frac{x_2}{x_1} = \frac{L_1}{L_2} = \frac{10}{20} = \frac{1}{2}.$$

Consequently,

$$x_2 = 0.5 \times \frac{1}{2}\ \text{cm} = 0.25\ \text{cm}.$$

There is no need to calculate the numerical modulus or convert both lengths to metres when forming this dimensionless ratio.

Step 3: Check the physical scaling and the options Doubling the side makes the loaded area four times larger, so the shear stress becomes one quarter of its original value. With the same modulus, shear strain is also one quarter. Multiplying this strain by a height twice as large gives half the original displacement. This agrees with 0.25 cm. The values 0.37 cm and 0.75 cm follow neither the stress nor the strain scaling, while 1.00 cm would incorrectly hold the stress constant despite the unchanged force.

Step 4: Conclusion The displacement of the larger cube is $\boxed{0.25\ \text{cm}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a cube sheared by a fixed force, combine the changing area and height first: $x = F/(GL)$, so doubling the side halves the displacement.

45. A disc rotates about its axis of symmetry in a horizontal plane at a steady rate of 3.5 revolutions per second. A coin placed at a distnce of 1.25 cm from the axis of rotation remains at rest on the disc. The coefficient of friction between the coin and the disc is : (g = 10 m/s^2) [4 marks]

(A) 0.5

(B) 0.3

(C) 0.7

(D) 0.6

Correct Answer: As printed, $\mu_s \geq 0.06045$, and all four listed coefficients permit the coin to remain at rest. There is no unique correct option. The supplied key selects (D), but the printed radius and the no-slip inequality do not single out that option.

Solution:

Concept:

- A coin fixed relative to a rotating horizontal disc moves in a circle in the ground frame. Static friction supplies its centripetal force, $f_s = m\omega^2 r$.
- The normal force is $N = mg$, and static friction can take any value up to $\mu_s N$. Therefore the no-slip condition is $\mu_s \geq \omega^2 r/g$; equality applies only at impending slipping.

Step 1: Convert the printed quantities The stated rotation frequency is $f = 3.5\ \mathrm{s}^{-1}$, so

$$\omega = 2\pi f = 7\pi\ \mathrm{rad\,s}^{-1}.$$

The printed radius is $1.25\ \mathrm{cm} = 0.0125\ \mathrm{m}$. The factor converting centimetres to metres is essential because the given acceleration due to gravity is in $\mathrm{m\,s}^{-2}$.

Step 2: Find the minimum coefficient needed for rest The friction required is $m\omega^2 r$. It must not exceed the maximum available static friction, giving

$$\mu_s mg \geq m\omega^2 r, \qquad \mu_s \geq \frac{(7\pi)^2(0.0125)}{10}.$$

The mass cancels, and the threshold is

$$\mu_{\min} = 0.006125\pi^2 \simeq 0.06045.$$

A measured coefficient cannot be uniquely inferred merely because the coin stays at rest; only this lower bound is established.

Step 3: Test every listed coefficient Option (A), 0.5; option (B), 0.3; option (C), 0.7; and option (D), 0.6 are all greater than 0.06045. Each therefore permits sufficient static friction at the printed radius. None equals the minimum coefficient if the question is interpreted as asking for the limiting value. In particular, the commonly selected value 0.6 is ten times the threshold for 1.25 cm, so it does not follow from equating friction to its limiting value here.

Step 4: Conclusion As printed, $\mu_s \geq 0.06045$, and all four listed coefficients permit the coin to remain at rest. There is no unique correct option. The supplied key selects (D), but the printed radius and the no-slip inequality do not single out that option.

Quick Tip to Solve: For a resting coin on a rotating disc, distinguish the required friction from its maximum: no slipping gives a lower bound on μ_s, not an equality.

46. At the center of a fixed large circular coil of radius R, a much smaller circular coil of radius r is placed. The two coils are concentric and are in the same plane. The larger coil carries a current I. The smaller coil is set to rotate with a constant angular velocity ω about an axis along their common diameter. Calculate the emf induced in their smaller coil after a time t of its start of rotation. [4 marks]

(A) $\frac{\mu_o I}{2R}\ \omega\ \pi\ r^2\ \sin\omega\ t$
(B) $\frac{\mu_o I}{4R}\ \omega\ \pi\ r^2\ \sin\omega\ t$
(C) $\frac{\mu_o I}{4R}\ \omega\ r^2\ \sin\omega\ t$
(D) $\frac{\mu_o I}{2R}\ \omega\ r^2\ \sin\omega\ t$

Correct Answer: (A) $\frac{\mu_o I}{2R}\ \omega\ \pi\ r^2\ \sin\omega\ t$

Solution:

Concept:

- The magnetic field at the centre of a circular single-turn coil is $B = \mu_0 I/(2R)$. Since $r \ll R$, this field can be treated as uniform over the smaller coil.
- Magnetic flux through a plane coil is $\Phi = BA\cos\theta$. Faraday's law gives the induced emf $\mathcal{E} = -d\Phi/dt$, where θ is the angle between the field and the coil normal.

Step 1: Establish the initial flux and rotation angle The field of the large coil is perpendicular to its plane. Initially both coils lie in the same plane, so the small coil normal is parallel to that field and its initial flux is maximum. Rotation about a common diameter tilts the normal through $\theta = \omega t$. The field and area are

$$B = \frac{\mu_0 I}{2R}, \qquad A_s = \pi r^2.$$

The large coil is fixed and its current is constant, so the change of flux comes entirely from the changing orientation.

Step 2: Differentiate the magnetic flux At time t,

$$\Phi(t) = \frac{\mu_0 I}{2R}\pi r^2 \cos(\omega t).$$

Choosing the initial positive normal to point along the magnetic field, differentiation yields

$$\mathcal{E}(t) = -\frac{d\Phi}{dt} = \frac{\mu_0 I \pi r^2 \omega}{2R}\sin(\omega t).$$

The sign reverses periodically according to the direction in which the flux changes; the expression is the signed emf with this normal convention.

Step 3: Check the amplitude and compare the choices At $t = 0$, the flux has a maximum and its rate of change is zero, agreeing with the sine factor. The peak emf is $B\pi r^2\omega$. Option (B) incorrectly halves the central field by using $4R$. Option (C) makes that error and also omits the circular-area factor π. Option (D) has the right central field but omits π, so only option (A) contains both the correct field and the full small-coil area.

Step 4: Conclusion The induced emf is $\boxed{\mathcal{E}(t) = \frac{\mu_0 I \pi r^2 \omega}{2R}\sin(\omega t)}$. Hence, the correct option is (A).

Quick Tip to Solve: For a rotating coil, write $\Phi = BA\cos\omega t$ first; the emf amplitude is $BA\omega$, with the circular area πr^2.

47. When an air bubble of radius r rises from the bottom to the surface of a lake its radius becomes $\frac{5r}{4}$. Taking the atmospheric pressure to be equal to 10 m height of water column, the depth of the lake would approximately be (ignore the surface tension and the effect of temperature) : [4 marks]

(A) 11.2 m

(B) 8.7 m

(C) 9.5 m

(D) 10.5 m

Correct Answer: (C) 9.5 m

Solution:

Concept:

- At constant temperature, a fixed amount of gas obeys Boyle's law, $P_bV_b = P_sV_s$, with absolute pressures at the bottom and surface.
- Ignoring surface tension, the bubble pressure equals the surrounding water pressure. At depth H, $P_b = P_0 + \rho_w gH$, whereas at the surface $P_s = P_0$.

Step 1: Calculate the volume expansion The bubble is spherical, so its volume is proportional to the cube of its radius. The radius grows from r to $5r/4$, hence

$$\frac{V_s}{V_b} = \left(\frac{5r/4}{r}\right)^3 = \frac{125}{64}.$$

The gas expands on rising because the external pressure falls. The relevant expansion is a volume ratio, not the radius ratio or the cross-sectional-area ratio.

Step 2: Relate the bottom pressure to the lake depth Boyle's law gives

$$\frac{P_b}{P_s} = \frac{V_s}{V_b} = \frac{125}{64}.$$

The atmospheric pressure is equivalent to a water head of 10 m, so $P_0 = \rho_w g(10\ \text{m})$. Therefore,

$$\frac{P_b}{P_0} = \frac{\rho_w g(10 + H)}{\rho_w g\, 10} = \frac{10 + H}{10}.$$

The density and gravitational acceleration cancel when the pressure is expressed as a water-column height.

Step 3: Solve for the depth and inspect the choices Equating the two pressure ratios gives

$$\frac{10 + H}{10} = \frac{125}{64}, \qquad H = 10\left(\frac{125}{64} - 1\right) = \frac{610}{64}\ \text{m}.$$

$$H = 9.53125\ \text{m} \simeq 9.5\ \text{m}.$$

This is option (C). The other depths, 11.2 m, 8.7 m and 10.5 m, do not produce the required pressure ratio 125/64. Using only the excess water pressure in Boyle's law would be incorrect because the gas law requires absolute pressure.

Step 4: Conclusion The depth of the lake is approximately $\boxed{9.5\ \text{m}}$. Hence, the correct option is (C).

Quick Tip to Solve: When a gas bubble rises, cube the radius ratio and use absolute pressure: $(10 + H)/10 = (r_s/r_b)^3$ for a 10 m atmospheric head.

48. The characteristic distance at which quantum gravitational effects are significant, the Planck length, can be determined from a suitable combination of the fundamental physical constants G, h and c.

Which of the following correctly gives the Planck length ? [4 marks]

(A) G $\hbar^2$ c^3

(B) G^2 $\hbar$ c

(C) $G^{1/2}\hbar^2 c$

(D) $\left(\frac{G\hbar}{c^3}\right)^{1/2}$

Correct Answer: (D) $\left(\frac{G\hbar}{c^3}\right)^{1/2}$

Solution:

Concept:

- The gravitational constant, reduced Planck constant and speed of light have dimensions $[G] = M^{-1}L^3T^{-2}$, $[\hbar] = ML^2T^{-1}$ and $[c] = LT^{-1}$.
- If a length is built as $G^a\hbar^b c^d$, its mass and time exponents must be zero and its length exponent must be one. Numerical dimensionless factors cannot be fixed by dimensional analysis.

Step 1: Form a general product of the constants Let the required length be proportional to $G^a\hbar^b c^d$. Multiplying the dimensions gives

$$[G^a\hbar^b c^d] = M^{-a+b}L^{3a+2b+d}T^{-2a-b-d}.$$

Since a length has dimensions $M^0L^1T^0$, equality requires three equations:

$$-a+b=0, \qquad 3a+2b+d=1, \qquad -2a-b-d=0.$$

Using h instead of $\hbar$ would not alter these equations, since they have the same dimensions.

Step 2: Determine the three exponents The mass equation gives $b = a$. The time equation then gives $d = -3a$. Substituting these into the length equation,

$$3a+2a-3a=1 \quad \Rightarrow \quad a=\frac{1}{2}.$$

Consequently,

$$b=\frac{1}{2}, \qquad d=-\frac{3}{2}, \qquad \ell_P = G^{1/2}\hbar^{1/2}c^{-3/2} = \sqrt{\frac{G\hbar}{c^3}}.$$

This is the conventional Planck-length combination shown among the options.

Step 3: Verify the result and reject the other products The dimensions inside the square root are

$$\left[\frac{G\hbar}{c^3}\right] = \frac{(M^{-1}L^3T^{-2})(ML^2T^{-1})}{L^3T^{-3}} = L^2.$$

Its square root is therefore a length. Option (A) has dimensions $ML^{10}T^{-7}$, and option (B) has $M^{-1}L^9T^{-6}$. Option (C), $G^{1/2}\hbar^2 c$, has dimensions $M^{3/2}L^{13/2}T^{-4}$. None is a pure length, leaving only

option (D).

Step 4: Conclusion The Planck length is $\boxed{\ell_P = \sqrt{G\hbar/c^3}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a length made from $G, \hbar, c$, cancel mass first to get equal powers of G and $\hbar$; then cancel time using c^{-3} inside a square root.

49. Two simple harmonic motions, as shown below, are at right angles. They are combined to form Lissajous figures.

x(t) = A sin (at + δ)

y(t) = B sin (bt)

Identify the correct match below. [4 marks]

(A) **Parameters** A $\neq$ B, a = b; $\delta = 0$;

Curve Parabola

(B) **Parameters** A = B, a = b; $\delta = \pi/2$

Curve Line

(C) **Parameters** A $\neq$ B, a = b; $\delta = \pi/2$

Curve Ellipse

(D) **Parameters** A = B, a = 2b; $\delta = \pi/2$

Curve Circle

Correct Answer: (C) **Parameters** A $\neq$ B, a = b; $\delta = \pi/2$

Curve Ellipse

Solution:

Concept:

- A Lissajous figure is the path obtained by eliminating time between two perpendicular oscillations. Its shape depends on the amplitude ratio, the frequency ratio and the phase difference.
- For equal angular frequencies and a phase difference of $\pi/2$, the displacements become $x = A\cos(bt)$ and $y = B\sin(bt)$, so $x^2/A^2 + y^2/B^2 = 1$.

Step 1: Examine equal frequencies in phase If $a = b$ and $\delta = 0$, the equations are $x = A\sin(bt)$ and $y = B\sin(bt)$. Eliminating the common sine gives

$$y = \frac{B}{A}x.$$

The particle moves back and forth along a straight line segment, whether or not $A = B$. Thus the unequal-amplitude, zero-phase case in option (A) is a line rather than a parabola.

Step 2: Examine a quarter-cycle phase difference For $a = b$ and $\delta = \pi/2$, use $\sin(bt + \pi/2) = \cos(bt)$. Squaring the normalized displacements and adding gives

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = \cos^2(bt) + \sin^2(bt) = 1.$$

When $A \neq B$, this is an ellipse with unequal semiaxes, exactly as in option (C). If $A = B$, it reduces to a circle, not the line claimed in option (B).

Step 3: Check the double-frequency alternative For option (D), put $A = B$, $a = 2b$ and $\delta = \pi/2$. Then

$$x = A\cos(2bt), \qquad y = A\sin(bt).$$

Using $\cos 2u = 1 - 2\sin^2 u$ gives

$$\frac{x}{A} = 1 - 2\left(\frac{y}{A}\right)^2.$$

This is a portion of a parabola traversed back and forth, so it is not a circle. The frequency ratio cannot be ignored: equal amplitudes alone do not ensure a circular path. All four proposed matches have now been checked directly against their time equations.

Step 4: Conclusion For $A \neq B$, $a = b$ and $\delta = \pi/2$, the path is the ellipse $x^2/A^2 + y^2/B^2 = 1$. Hence, the correct option is (C).

Quick Tip to Solve: Equal frequencies with zero phase difference give a line; a quarter-cycle phase difference gives an ellipse, becoming a circle only when the amplitudes are equal.

50. A current of 1 A is flowing on the sides of an equilateral triangle of side 4.5 $\times$ 10^{-2} m. The magnetic field at the center of the triangle will be : [4 marks]

(A) 2 $\times$ 10^{-5} Wb/m^2

(B) Zero

(C) 8 $\times$ 10^{-5} Wb/m^2

(D) 4 $\times$ 10^{-5} Wb/m^2

Correct Answer: (D) 4 $\times$ 10^{-5} Wb/m^2

Solution:

Concept:

- For a straight wire segment at perpendicular distance r, the Biot–Savart result is $B = \frac{\mu_0 I}{4\pi r}(\sin\theta_1 + \sin\theta_2)$, where the angles are measured from the perpendicular to the wire.
- The centre of an equilateral triangle is equidistant from its three sides. Their perpendicular magnetic fields have the same sense by the right-hand rule, so their magnitudes add.

Step 1: Find the distance and endpoint angles Let the side length be $a = 4.5 \times 10^{-2}\,\text{m}$. The altitude is $\sqrt{3}a/2$, and the centre is one third of this altitude from a side. Thus $r = a/(2\sqrt{3})$. The perpendicular bisects each side, so $\tan\theta = (a/2)/r = \sqrt{3}$; both endpoint angles are 60°.

Step 2: Add the contributions of all three sides Each side carries the same current $I = 1\,\text{A}$. Therefore

$$B_{\text{total}} = 3\frac{\mu_0 I}{4\pi r}(2\sin 60^\circ) = \frac{18 \times 10^{-7} I}{a}.$$

Substituting the given length gives

$$B_{\text{total}} = \frac{18 \times 10^{-7}}{4.5 \times 10^{-2}} = 4.0 \times 10^{-5}\,\text{T}.$$

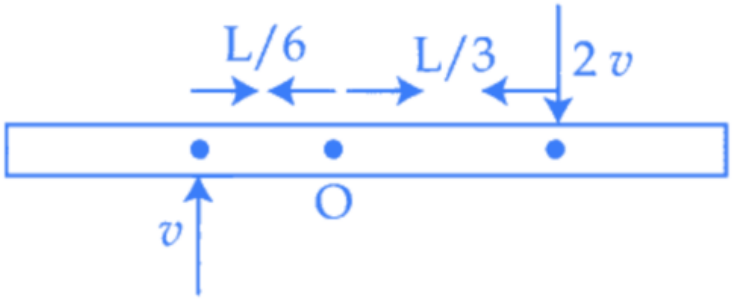
The field is perpendicular to the triangle; its sense depends on the current direction.

Step 3: Check the listed alternatives The unit $\text{Wb}\,\text{m}^{-2}$ equals tesla. The contributions reinforce, so zero is impossible. The value 2×10^{-5} is half the calculated field, while 8×10^{-5} is twice it; neither includes the three finite-side contributions correctly.

Step 4: Conclusion The magnetic-field magnitude at the centre is $4 \times 10^{-5}\,\text{Wb}\,\text{m}^{-2}$. Hence, the correct option is (D).

Quick Tip to Solve: For an equilateral current loop, use the inradius, not the circumradius, as the wire distance and add three identical finite-wire fields.

51. A thin uniform bar of length L and mass 8 m lies on a smooth horizontal table. Two point masses m and 2 m are moving in the same horizontal plane from opposite sides of the bar with speeds $2v$ and v respectively. The masses stick to the bar after collision at a distance $\frac{L}{3}$ and $\frac{L}{6}$ respectively from the center of the bar. If the br starts rotating about its center of mass as a result of collision, the angular speed of the bar will be : [4 marks]



(A) $\frac{v}{5L}$
(B) $\frac{6v}{5L}$
(C) $\frac{3v}{5L}$
(D) $\frac{v}{6L}$

Correct Answer: (B) $\frac{6v}{5L}$

Solution:

Concept:

- During the short collision, the smooth table exerts no external torque about the vertical axis through the system centre of mass. Angular momentum is conserved even though the masses stick and kinetic energy is not conserved.
- A thin uniform bar has moment of inertia $ML^2/12$ about its centre. A point mass contributes mr^2, and the final angular momentum about the same axis is $I\omega$.

Step 1: Locate the system centre of mass Take the bar centre O as origin, with the mass m striking at $+L/3$ and the mass $2m$ at $-L/6$, as in the figure. Their position moments cancel:

$$m(L/3) + 2m(-L/6) = 0.$$

Hence the combined centre of mass remains at O. Their incoming momenta also cancel, since their magnitudes are $m(2v) = 2mv$, in opposite directions, so there is no translational motion after impact.

Step 2: Compute the incoming angular momentum The particles strike on opposite sides of O with opposite velocities, so both torques have the same rotational sense. Thus their angular momenta add:

$$H = m(2v)\frac{L}{3} + (2m)v\frac{L}{6} = mvL.$$

The initially stationary bar contributes no angular momentum.

Step 3: Include both attached masses in the final inertia After impact the bar and particles rotate together. Their combined moment of inertia is

$$I = \frac{8mL^2}{12} + m\left(\frac{L}{3}\right)^2 + 2m\left(\frac{L}{6}\right)^2 = \frac{5}{6}mL^2.$$

Therefore $\omega = H/I = 6v/(5L)$. The other three options do not satisfy $I\omega = mvL$; in particular, omitting either attached mass gives an incorrect inertia.

Step 4: Conclusion The angular speed is $\omega = \dfrac{6v}{5L}$. Hence, the correct option is (B).

Quick Tip to Solve: Check the final centre of mass before choosing the rotation axis; then add angular momenta with their signs and include all stuck masses in the inertia.

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52. A parallel plate capacitor with area 200 cm^2 and separation between the plates 1.5 cm, is connected across a battery of emf V. If the force of attraction between the plates is $25 \times 10^{-6} N$, the value of V is approximately : $\left(\in_o = 8.85 \times 10^{-12} \frac{C^2}{N.m^2}\right)$ [4 marks]

(A) 250 V

(B) 100 V

(C) 300 V

(D) 150 V

Correct Answer: (A) 250 V

Solution:

Concept:

- The force on one charged capacitor plate is due to the field of the other plate. The electrostatic pressure is $p = \frac{1}{2}\varepsilon_0 E^2$, so $F = \frac{1}{2}\varepsilon_0 A E^2$.
- For parallel plates with separation d and maintained potential difference V, the uniform interior field is $E = V/d$, neglecting edge effects. Areas and distances must be converted to SI units before substitution.

Step 1: Express the force in terms of voltage Substitute $E = V/d$ into the pressure formula:

$$F = \frac{\varepsilon_0 A V^2}{2d^2}, \qquad V = d\sqrt{\frac{2F}{\varepsilon_0 A}}.$$

The factor $1/2$ is essential: using the total interior field as the field acting on a plate would incorrectly include that plate's own electric field.

Step 2: Convert the data and substitute The plate area is $A = 200 \times 10^{-4} = 2.00 \times 10^{-2}\,\mathrm{m}^2$, and $d = 1.5 \times 10^{-2}\,\mathrm{m}$. With $F = 25 \times 10^{-6}\,\mathrm{N}$,

$$V^2 = \frac{2(25 \times 10^{-6})(1.5 \times 10^{-2})^2}{(8.85 \times 10^{-12})(2.00 \times 10^{-2})} = 6.36 \times 10^4\,\mathrm{V}^2.$$

Thus $V \simeq 252\,\mathrm{V}$, which is approximately $250\,\mathrm{V}$.

Step 3: Compare the possible voltages The force varies as the square of the voltage. The choices $100\,\mathrm{V}$ and $150\,\mathrm{V}$ would give substantially smaller forces, whereas $300\,\mathrm{V}$ would give a larger force. Only $250\,\mathrm{V}$ lies near the positive square root of the measured-force expression.

Step 4: Conclusion The required battery emf is approximately $250\,\mathrm{V}$. Hence, the correct option is (A).

Quick Tip to Solve: For capacitor-plate attraction use $F = \varepsilon_0 A V^2/(2d^2)$, and remember that $1\,\mathrm{cm}^2 = 10^{-4}\,\mathrm{m}^2$.

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53. The value closest to the thermal velocity of a Helium atom at room temperature (300 K) in ms^{-1} is :

[k_B =1.4 $\times$ 10^{-23} J/K; m_{He} = 7 $\times$ 10^{-27} kg] [4 marks]

(A) 1.3×10^4

(B) 1.3×10^3

(C) 1.3×10^5

(D) 1.3×10^2

Correct Answer: (B) 1.3×10^3

Solution:

Concept:

- For an ideal monatomic gas in thermal equilibrium, the mean translational kinetic energy per atom is $\frac{1}{2}mv_{\mathrm{rms}}^2 = \frac{3}{2}k_B T$. Thus the root-mean-square thermal speed is $v_{\mathrm{rms}} = \sqrt{3k_B T/m}$.
- The mass entering this formula is the mass of one atom, not the molar mass. The result is an order of magnitude estimate of the random thermal motion; it is not the mean vector velocity, which vanishes in equilibrium.

Step 1: Calculate the squared thermal speed Use the given values $T = 300\,\mathrm{K}$, $k_B = 1.4 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

and $m_{\mathrm{He}} = 7 \times 10^{-27}\,\mathrm{kg}$:

$$v_{\mathrm{rms}}^2 = \frac{3(1.4 \times 10^{-23})(300)}{7 \times 10^{-27}} = 1.8 \times 10^6\,\mathrm{m^2\,s^{-2}}.$$

The kelvin cancels, and $\mathrm{J/kg} = \mathrm{m^2/s^2}$, confirming the units.

Step 2: Take the square root and identify the scale Taking the positive square root gives

$$v_{\mathrm{rms}} = \sqrt{1.8} \times 10^3 \simeq 1.34 \times 10^3\,\mathrm{m\,s^{-1}}.$$

The nearest listed value is 1.3×10^3. The options with powers 10^4 and 10^5 are far too large, while 10^2 is too small. Squaring any of them gives a kinetic-energy scale inconsistent with $k_B T$.

Step 3: Check the meaning of thermal velocity Other standard speed measures, such as the most probable and mean speeds, differ from the rms value only by factors of order unity. All place helium atoms at this temperature on the $10^3\,\mathrm{m\,s^{-1}}$ scale, so the widely spaced choices lead to the same selection.

Step 4: Conclusion The thermal speed is closest to $1.3 \times 10^3\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Estimate $k_B T/m$ first, then take its square root; do not carry the 10^6 power unchanged through the square root.

54. A proton of mass m collides elastically with a particle of unknown mass at rest. After the collision, the proton and the unknown particle are seen moving at an angle of 90° with respect to each other. The mass of unknown particle is : [4 marks]

(A) $\frac{m}{2}$

(B) m

(C) $\frac{m}{\sqrt{3}}$

(D) 2 m

Correct Answer: (B) m

Solution:

Concept:

- An elastic collision conserves both total linear momentum and total kinetic energy. Because the target is initially at rest, the initial momentum and energy belong entirely to the incoming proton.
- If the two outgoing momenta are perpendicular, the squared magnitude of their vector sum equals the sum of their squared magnitudes. This relates the geometry of the outgoing paths to the masses.

Step 1: Write momentum conservation using the right angle Let the target mass be M, the incident proton speed be u, and the two final speeds be v and V. The momenta satisfy $m\mathbf{u} = m\mathbf{v} + M\mathbf{V}$. Since $\mathbf{v} \cdot \mathbf{V} = 0$, squaring gives

$$m^2u^2 = m^2v^2 + M^2V^2.$$

There is no cross term precisely because the outgoing paths meet at 90°.

Step 2: Combine this equation with energy conservation Elasticity gives

$$\tfrac{1}{2}mu^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}MV^2.$$

Multiply by $2m$ to obtain

$$m^2u^2 = m^2v^2 + mMV^2.$$

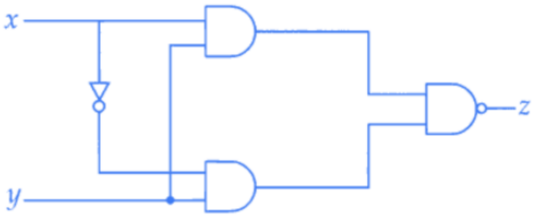
Comparing with the momentum equation yields $M^2V^2 = mMV^2$, or $M(M-m)V^2 = 0$. The target is observed moving and has positive mass, so $V \neq 0$ and $M \neq 0$; consequently $M = m$.

Step 3: Rule out unequal masses For $M = m/2$, $m/\sqrt{3}$, or $2m$, the two squared equations would require a nonzero cross term or a stationary target. They therefore cannot describe the stated perpendicular, nonzero final velocities in an elastic collision. Equal masses make the momentum triangle and kinetic-energy relation consistent.

Step 4: Conclusion The unknown particle has mass $M = m$, equal to the proton mass. Hence, the correct option is (B).

Quick Tip to Solve: When an initially stationary target recoils at right angles to the projectile, square the momentum equation and compare it directly with energy conservation.

55. Truth table for the following digital circuit will be : [4 marks]



(A)

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

(B)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	0

(C)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	1

(D)

x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

Correct Answer: (C)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	1

Solution:

Concept:

- An AND gate outputs 1 only when both inputs are 1. A NOT gate complements its input, and a NAND gate complements the AND of its two inputs.
- The Boolean identities $x\overline{x} = 0$ and $y^2 = y$ simplify circuits. Signals connected to the same branching wire retain the same logical value.

Step 1: Read the two intermediate AND outputs The upper AND gate receives x directly and y from the vertical branch, so its output is $P = xy$. The lower AND gate receives the inverted value $\overline{x}$ and the direct value y, so its output is $Q = \overline{x}y$. The inverter acts on x, not on the common y input.

Step 2: Apply the final NAND operation The output bubble on the final gate indicates negation. Hence

$$z = \overline{PQ} = \overline{(xy)(\overline{x}y)} = \overline{x\overline{x}y^2} = \overline{0} = 1.$$

For $x = 0$, the upper intermediate output is always 0; for $x = 1$, the lower one is always 0. The final NAND therefore never receives two simultaneous 1 inputs.

Step 3: Construct the four-row truth table The outputs for the conventional input order are

x	y	P	Q	z
0	0	0	0	1
0	1	0	1	1
1	0	0	0	1
1	1	1	0	1

Thus z is 1 in all four rows. Option (A) is the AND table, option (B) is the ordinary NAND table for the original inputs, and option (D) is the OR table. None describes this composite circuit; only option (C) gives the constant high output.

Step 4: Conclusion The output sequence is $1, 1, 1, 1$ for inputs $00, 01, 10, 11$. Hence, the correct option is (C).

Quick Tip to Solve: Before making a truth table, look for complementary signals in a product: $x\overline{x} = 0$ makes the final NAND output permanently high.

56. If the de Broglie wavelengths associated with a proton and an α-particle are equal, then the ratio of velocities of the proton and the α-particle will be : [4 marks]

(A) 4 : 1

(B) 2 : 1

(C) 1 : 2

(D) 1 : 4

Correct Answer: (A) 4 : 1

Solution:

Concept:

- A particle with momentum magnitude p has de Broglie wavelength $\lambda = h/p$. Equal wavelengths therefore imply equal momenta, since Planck's constant is the same for both particles.
- For nonrelativistic particles, $p = mv$. An alpha particle is a helium nucleus containing two protons and two neutrons, with mass approximately four times the proton mass: $m_\alpha \simeq 4m_p$.

Step 1: Set the particle momenta equal The premise gives

$$\lambda_p = \lambda_\alpha \quad \Rightarrow \quad \frac{h}{m_p v_p} = \frac{h}{m_\alpha v_\alpha}.$$

Cancel the common nonzero constant h and cross-multiply to obtain $m_p v_p = m_\alpha v_\alpha$. The proton must move faster because it has less mass but the same momentum.

Step 2: Insert the alpha-particle mass ratio Dividing the momentum equation by $m_p v_\alpha$ gives

$$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = 4.$$

Thus the ordered ratio requested, proton speed to alpha-particle speed, is $4 : 1$. It is not the inverse ratio; maintaining the order of the named particles is essential.

Step 3: Check the other proposed ratios The ratio $2 : 1$ would follow from equal kinetic energies, because $p^2 = 2mK$, but equal energies were not specified. The ratios $1 : 2$ and $1 : 4$ would make the lighter proton slower, contradicting equal momenta. Charge does not enter the de Broglie relation here; the alpha particle's charge of $+2e$ cannot replace its mass ratio.

Step 4: Conclusion The velocity-magnitude ratio is $v_p : v_\alpha = 4 : 1$. Hence, the correct option is (A).

Quick Tip to Solve: Equal wavelength means equal momentum, so speeds vary inversely with mass; use a square-root mass ratio only when the kinetic energies are equal.

57. 5 beats / econd are heard when a tuning fork is sounded with a sonometer wire under tension, when the length of the sonometer wire is either 0.95 m or 1 m. The frequency of the fork will be : [4 marks]

(A) 195 Hz

(B) 150 Hz

(C) 300 Hz

(D) 251 Hz

Correct Answer: (A) 195 Hz

Solution:

Concept:

- For a sonometer wire at fixed tension and linear mass density, its fundamental frequency is inversely proportional to vibrating length: $f = K/L$, where K is constant.
- The beat frequency is the magnitude of the difference between the two frequencies: $f_b = |f - f_0|$, with f_0 the tuning-fork frequency.

Step 1: Order the two wire frequencies The shorter wire, of length $0.95\,\mathrm{m}$, has the higher frequency. Two different wire frequencies give the same nonzero beat rate with one fork, so the fork frequency lies between them. Therefore the shorter wire has frequency $f_0 + 5$, while the longer wire, of length $1.00\,\mathrm{m}$, has frequency $f_0 - 5$. They cannot both lie on the same side of the fork frequency, since their equal distance from it would then make their frequencies equal.

Step 2: Use the inverse-length relation Equating frequency times length for the same wire gives

$$0.95(f_0 + 5) = 1.00(f_0 - 5).$$

Thus $0.05 f_0 = 9.75$, and hence $f_0 = 195\,\mathrm{Hz}$. As a check, the wire frequencies are $200\,\mathrm{Hz}$ and $190\,\mathrm{Hz}$; each differs from the fork by $5\,\mathrm{Hz}$, and $200(0.95) = 190(1.00)$.

Step 3: Reject the other proposed frequencies In this arrangement the required ratio is $(f_0 + 5)/(f_0 - 5) = 1/0.95 = 20/19$. Substitution of 150, 300 or $251\,\mathrm{Hz}$ does not satisfy this ratio, so those alternatives cannot give both of the stated beat observations at the two given lengths.

Step 4: Conclusion The tuning fork has frequency $195\,\mathrm{Hz}$. Hence, the correct option is (A).

Quick Tip to Solve: For equal beats at two different lengths, put the fork frequency between the two wire frequencies before using $fL =$ constant.

58. Two Carnot engines A and B are operated in series. Engine A receives heat from a reservoir at 600 K and rejects heat to a reservoir at temperature T. Engine B receives heat rejected by engine A and in turn rejects it to a reservoir at 100 K. If the efficiencies of the two energies A and B are represented by η_A and η_B, respectively, then what is the value of $\frac{\eta_B}{\eta_A}$? [4 marks]

(A) $\frac{12}{7}$

(B) $\frac{7}{12}$

(C) $\frac{12}{5}$

(D) $\frac{5}{12}$

Correct Answer: The intermediate temperature T is unspecified, so no unique numerical efficiency ratio follows.

Solution:

Concept:

- A reversible Carnot engine operating between absolute temperatures T_h and T_c has efficiency $\eta = 1 - T_c/T_h$.
- In series operation, the heat rejected by the first engine supplies the second; this heat-flow relation does not by itself determine the intermediate reservoir temperature.

Step 1: Write both efficiencies in terms of the unknown temperature For engine A the hot and cold temperatures are $600\,\mathrm{K}$ and T, respectively. For engine B they are T and $100\,\mathrm{K}$. Therefore

$$\eta_A = \frac{600 - T}{600}, \qquad \eta_B = \frac{T - 100}{T}.$$

Both act as heat engines for $100 < T < 600\,\mathrm{K}$. The requested ratio is consequently

$$\frac{\eta_B}{\eta_A} = \frac{600(T - 100)}{T(600 - T)}.$$

Step 2: Test whether series operation fixes the ratio Take a heat input Q to engine A. Reversibility makes its rejected heat $QT/600$; engine B then rejects $(QT/600)(100/T) = Q/6$. This is consistent for every admissible T, so the series condition supplies no further equation for it. For instance, $T = 300\,\mathrm{K}$ gives a ratio $4/3$, whereas $T = 200\,\mathrm{K}$ gives $3/4$. The same external reservoir temperatures thus permit different answers.

Step 3: Assess the listed numerical choices No value of T, equality of work outputs, or other condition fixing T is stated. It would be unjustified to insert such a condition. The supplied answer key selects (A), $12/7$, but that number is not implied by the printed data; the explicit expression above and the two counterexamples show why a unique numerical option cannot be established.

Step 4: Conclusion As printed, the question is underdetermined: $\eta_B/\eta_A = 600(T-100)/[T(600-T)]$. The key lists option (A), but the unspecified intermediate temperature prevents selecting a unique numerical answer from the given information.

Quick Tip to Solve: For cascaded Carnot engines, distinguish the overall efficiency, fixed by the outer reservoirs, from the individual efficiencies, which depend on the intermediate temperature.

59. The carrier frequency of a transmitter is provided by a tank circuit of a coil of inductance 49 μH and a capacitance of 2.5 nF. It is modulated by an audio signal of $12\ kHz$. The frequency range occupied by the side bands is : [4 marks]

(A) 13482 kHz – 13494 kHz

(B) 442 kHz – 466 kHz

(C) 63 kHz – 75 kHz

(D) 18 kHz – 30 kHz

Correct Answer: (B) 442 kHz – 466 kHz

Solution:

Concept:

- An ideal LC tank circuit oscillates at $f_c = 1/(2\pi\sqrt{LC})$, with L in henries and C in farads.
- Amplitude modulation by a single audio frequency f_m produces lower and upper sidebands at $f_c - f_m$ and $f_c + f_m$. Their separation, or transmission bandwidth, is $2f_m$.

Step 1: Calculate the carrier frequency Convert the component values before substituting:

$$L = 49 \times 10^{-6}\,\text{H}, \qquad C = 2.5 \times 10^{-9}\,\text{F}.$$

Their product is $1.225 \times 10^{-13}\,\text{s}^2$, so

$$\sqrt{LC} = 3.5 \times 10^{-7}\,\text{s}, \qquad f_c = \frac{1}{2\pi(3.5 \times 10^{-7})} \simeq 454.7\,\text{kHz}.$$

Using the usual examination approximation $\pi = 22/7$ gives about $454.5\,\text{kHz}$.

Step 2: Find the two sideband frequencies The modulating frequency is $f_m = 12\,\text{kHz}$. Thus the sidebands lie at approximately

$$f_L = 454.7 - 12 = 442.7\,\text{kHz}, \qquad f_U = 454.7 + 12 = 466.7\,\text{kHz}.$$

The given limits 442–$466\,\text{kHz}$ are the approximate pair corresponding to these values. In particular the carrier must lie midway between the limits, and their separation must be $24\,\text{kHz}$.

Step 3: Compare the other ranges The range in (A) is centred near $13.5\,\text{MHz}$, wholly inconsistent with the tank resonance, and has only $12\,\text{kHz}$ separation. Options (C) and (D) are also centred far below $455\,\text{kHz}$ and each has only $12\,\text{kHz}$ separation. Only (B) matches both the carrier scale and twice the audio frequency.

Step 4: Conclusion The occupied sideband range is approximately 442–$466\,\text{kHz}$. Hence, the correct option is (B).

Quick Tip to Solve: Check both the midpoint and the width of a proposed AM range: the midpoint is f_c, and the total width is $2f_m$.

60. A body takes 10 minutes to cool from 60°C to 50°C. The tempertature of surroundings is constant at 25°C. Then, the temperature of the body after next 10 minutes will be approximately : [4 marks]

(A) 47°C

(B) 41°C

(C) 45°C

(D) 43°C

Correct Answer: (D) 43°C

Solution:

Concept:

- At constant surrounding temperature T_s, Newton's law gives $d(T - T_s)/dt = -k(T - T_s)$, with positive cooling constant k.
- The excess temperature $\theta = T - T_s$ decreases exponentially. Therefore it is multiplied by the same factor during every equal time interval.

Step 1: Find the excess-temperature ratio for ten minutes The surroundings are at $25°\text{C}$. Initially the excess temperature is $60 - 25 = 35°\text{C}$; after ten minutes it is $50 - 25 = 25°\text{C}$. For the same body and surroundings the multiplying factor for this time interval is therefore

$$r = e^{-10k} = \frac{25}{35} = \frac{5}{7}.$$

The ratio applies to the excess above the surroundings, not to Celsius temperatures themselves.

Step 2: Apply the same factor over the next ten minutes After the second interval the excess becomes

$$\theta_2 = 25\left(\frac{5}{7}\right) = \frac{125}{7} \simeq 17.86°\text{C}.$$

Adding the ambient temperature restores the actual body temperature:

$$T_2 = 25 + 17.86 \simeq 42.86°\text{C}.$$

Hence the nearest given temperature is $43°\text{C}$. The second fall is about $7.14°\text{C}$, smaller than the initial $10°\text{C}$, as cooling slows when the body approaches its surroundings.

Step 3: Check the alternatives against the decay factor The temperatures 47, 41, and $45°\text{C}$ would leave excesses 22, 16, and $20°\text{C}$, respectively. Relative to the excess of $25°\text{C}$ at ten minutes, none gives the required factor $5/7$. A constant temperature fall would also be incorrect because the cooling rate depends on the current excess temperature.

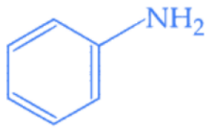
Step 4: Conclusion After the next ten minutes the temperature is approximately $43°\text{C}$. Hence, the correct option is (D).

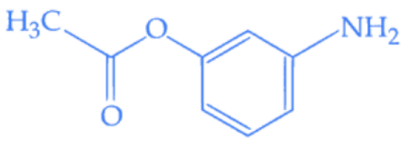
Quick Tip to Solve: For equal cooling intervals, form a geometric progression of $T - T_s$; add T_s back only after finding the new excess.

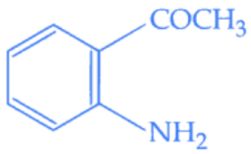
Chemistry (120 marks)

61. The increasing order of diazotisation of the following compounds is : [4 marks]

(a) 

(b) 

(c) 

(d) 

(A) (a) < (b) < (c) < (d)

(B) (a) < (d) < (b) < (c)

(C) (a) < (d) < (c) < (b)

(D) (d) < (c) < (b) < (a)

Correct Answer: (B) (a) < (d) < (b) < (c)

Solution:

Concept:

- Diazotisation of a primary aromatic amine with nitrous acid in cold mineral acid produces an arenediazonium salt; the nitrosating electrophile is represented by NO^+.
- The comparison concerns diazotisation in acidic medium, so protonation, substituent effects and steric accessibility all matter. Stability of the resulting diazonium species also distinguishes aromatic from aliphatic amines.

Step 1: Distinguish the aliphatic amino acid from aromatic amines In (a), NH_2 is attached to a saturated carbon of a cyclohexane ring, the same carbon bearing COOH. It is not an aniline derivative. Any aliphatic diazonium species formed is unstable and readily loses N_2, unlike the resonance-stabilised arenediazonium salts from (b), (c) and (d). Thus (a) is placed first in the stated increasing order of diazotisation.

Step 2: Compare the three aromatic structures Compound (b) is aniline. Compound (d) has an acetyl group ortho to the amino group; its neighbouring carbonyl and steric crowding inhibit ready nitrosation at the amino nitrogen, placing it below aniline. Compound (c) bears an acetoxy substituent meta to the amino group. At this position its inductive withdrawal reduces amine basicity; direct resonance donation to the amino-bearing carbon is absent. The reactive species for nitrosation is free

amine B, not its protonated form. For $BH^+ \rightleftharpoons B + H^+$,

$$[B] \simeq K_a \frac{[BH^+]}{[H^+]}, \qquad v = k_{nit}[B][NO^+].$$

Reduced basicity increases K_a, leaving a greater free-amine fraction in acid. For this comparison that reduced protonation favours (c) over aniline: it outweighs the decrease in intrinsic nitrogen nucleophilicity. The ortho-carbonyl compound additionally suffers steric inhibition and intramolecular hydrogen bonding between its amino group and carbonyl oxygen, so the same simple meta-substituent comparison cannot be applied to it. These competing effects explain why basicity alone is not a rate ranking.

Step 3: Assemble the sequence and compare the choices The resulting increasing sequence is

$$(a) < (d) < (b) < (c).$$

Option (A) incorrectly places the ortho-acetyl compound above both of the other aromatic amines. Option (C) interchanges the final two aromatic compounds, and option (D) reverses the principal aliphatic-versus-aromatic comparison. These alternatives do not give the stated reaction order.

Step 4: Conclusion The aliphatic amino acid is followed by the ortho-acetyl aromatic amine, aniline, and the acetoxy-substituted aromatic amine. Hence, the correct option is (B).

Quick Tip to Solve: For diazotisation, first identify whether the amino group is on an aromatic carbon; then account for ortho crowding and the acidic reaction medium.

62. For a first order reaction, A → P, $t_{1/2}$ (half-life) is 10 days The time required for $\frac{1}{4}^{th}$ conversion of A (in days) is : (ln 2 = 0.693, ln 3 = 1.1) [4 marks]

(A) 5

(B) 3.2

(C) 4.1

(D) 2.5

Correct Answer: (C) 4.1

Solution:

Concept:

- For a first-order reaction, the integrated rate law is $kt = \ln([A]_0/[A]_t)$, where k is the rate constant and $[A]_0, [A]_t$ are reactant concentrations.
- The first-order half-life satisfies $t_{1/2} = \ln 2/k$. A specified conversion is the fraction consumed, whereas the logarithm uses the fraction remaining.

Step 1: Calculate the first-order rate constant The half-life is 10 days, and the given value is $\ln 2 = 0.693$. Hence

$$k = \frac{0.693}{10} = 0.0693\,\text{day}^{-1}.$$

The time unit is days because the half-life was supplied in days. This relation uses only the order of the reaction and its measured half-life; the initial amount of A cancels from the calculation.

Step 2: Convert the extent reacted into the fraction remaining One-fourth conversion means that $[A]_0/4$ has reacted, leaving $[A]_t = 3[A]_0/4$. It does not mean that one-fourth of A remains. Therefore

$$kt = \ln\left(\frac{4}{3}\right) = 2\ln 2 - \ln 3 = 2(0.693) - 1.1 = 0.286.$$

Using the supplied logarithms gives

$$t = \frac{0.286}{0.0693} \simeq 4.13\,\text{days}.$$

Step 3: Check the proposed times The nearest listed value is 4.1 days. For a quarter conversion the required exponent is about 0.286. The other proposed times 5, 3.2, and 2.5 days would give exponents 0.3465, 0.2218, and 0.1733, respectively, so none matches the required remaining fraction. Also the result is less than the half-life, consistent with less than half the reactant having been consumed.

Step 4: Conclusion The required time is approximately $4.1\,\text{days}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the remaining fraction before taking logarithms: one-fourth conversion means $3/4$ remains, so use $\ln(4/3)$.

63. The correct order of electron affinity is : [4 marks]

(A) F > Cl > O

(B) F > O > Cl

(C) Cl > F > O

(D) O > F > Cl

Correct Answer: (C) Cl > F > O

Solution:

Concept:

- Electron affinity measures the energy released when an isolated gaseous atom gains an electron. A greater positive affinity means a more favourable electron-gain process.
- Across a period, increasing effective nuclear attraction usually favours electron gain, but very compact shells can increase repulsion against the incoming electron.

Step 1: Compare chlorine with fluorine Fluorine is the most electronegative element, but electronegativity in a bond is not the same quantity as atomic electron affinity. An additional electron enters the very compact $2p$ subshell of F, where electron-electron repulsions are strong. In Cl it enters the more diffuse $3p$ subshell, with less crowding. Chlorine therefore releases more energy on electron gain than fluorine, despite its greater atomic size.

Step 2: Compare oxygen with the halogens Oxygen has configuration $1s^2 2s^2 2p^4$; addition of one electron gives $2p^5$, whereas a halogen gains an electron to complete its outer p subshell. The halogens

have a stronger effective attraction for the added electron and a more favourable first electron-gain process. Oxygen's small $2p$ region also produces appreciable repulsion. Consequently its first electron affinity is lower than those of both F and Cl.

Step 3: Arrange the affinities and eliminate the other orders The combined order is

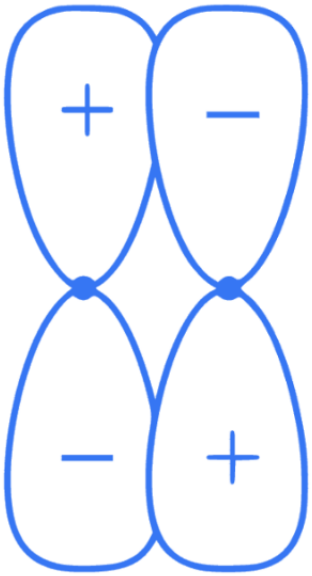
$$\mathrm{Cl} > \mathrm{F} > \mathrm{O}.$$

As an approximate magnitude check, the released energies are about 349, 328, and $141\,\mathrm{kJ\,mol^{-1}}$, respectively. Options (A) and (B) incorrectly put F above Cl. Option (D) incorrectly puts O above both halogens. Be careful that electron-gain enthalpies would be negative for these exothermic processes; the question compares affinity magnitudes.

Step 4: Conclusion The correct order of electron affinity is $\mathrm{Cl} > \mathrm{F} > \mathrm{O}$. Hence, the correct option is (C).

Quick Tip to Solve: Remember the compact-shell exception: chlorine has greater electron affinity than fluorine, although fluorine has greater electronegativity.

64. Which of the following best describes the diagram below of a molecular orbital ? [4 marks]



(A) A non-bonding orbital

(B) An antibonding σ orbital

(C) A bonding π orbital

(D) An antibonding π orbital

Correct Answer: (D) An antibonding π orbital

Solution:

Concept:

- A molecular orbital formed by lateral overlap of two parallel p orbitals has π symmetry about the internuclear axis. The signs shown on lobes represent orbital phase, not electric charge.
- Overlap of lobes of the same phase is constructive and bonding; overlap of opposite phases is destructive and gives an antibonding orbital, denoted by an asterisk.

Step 1: Identify the direction of overlap The two nuclear positions are side by side, while the lobes of each p orbital extend above and below the line joining the nuclei. Thus the orbitals approach laterally rather than head on. This geometry identifies a π-type interaction. A σ orbital instead arises from overlap directed along the internuclear axis, with cylindrical symmetry around that axis.

Step 2: Compare the phases on neighbouring lobes Above the axis, the left lobe is marked $+$ and the right lobe $-$; below it the signs are reversed. Each pair of lobes facing across the internuclear region therefore has opposite phase. Their wavefunctions subtract in the region between the atoms, producing a node and reducing the electron density that would otherwise bind the nuclei. This is the antibonding combination π^*, rather than the bonding π combination.

Step 3: Eliminate the remaining descriptions Option (A) does not describe this interaction because the diagram explicitly represents overlapping orbitals on two atoms with destructive interference. Option (B) has the wrong symmetry: the overlap is lateral, not axial. Option (C) would require matching signs on the upper pair and matching signs on the lower pair; the diagram shows the opposite arrangement.

Step 4: Conclusion The diagram represents an antibonding π molecular orbital, π^*. Hence, the correct option is (D).

Quick Tip to Solve: First use the orientation to distinguish σ from π; then compare the phases of the lobes facing one another to decide bonding or antibonding.

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65. The dipeptide, Gln-Gly, on treatment with CH_3COCl followed y aqueous work up gives : [4 marks]

(A) $CH_3CONHCH(CH_2CH_2CONHCOCH_3)CONHCH_2COOH$

(B) $CH_3CONHCH_2CONHCH(CH_2CH_2CONHCOCH_3)COOH$

(C) $CH_3CONHCH(CH_2CH_2CONH_2)CONHCH_2COOH$

(D) $CH_3CONHCH_2CONHCH(CH_2CH_2CONH_2)COOH$

Correct Answer: (C) $CH_3CONHCH(CH_2CH_2CONH_2)CONHCH_2COOH$

Solution:

Concept:

- In Gln-Gly, glutamine is the N-terminal residue and glycine is the C-terminal residue. The free amino group is on glutamine; the linkage between the residues is already an amide.
- Acetyl chloride, CH_3COCl, acetylates the free amino group to give $-NHCOCH_3$. Amide nitrogens are much less nucleophilic because their lone pairs are delocalised into a carbonyl.

Step 1: Write the connectivity of the original dipeptide Glutamine has the side chain $-CH_2CH_2CONH_2$, whereas glycine has hydrogen as its side chain. Writing the sequence from its free amino terminus gives

$$H_2NCH(CH_2CH_2CONH_2)CONHCH_2COOH.$$

The central $-CONH-$ group is the peptide linkage, and the terminal $-COOH$ belongs to glycine. The side-chain $-CONH_2$ is a carboxamide, not an additional free amino group.

Step 2: Acetylate the free amino terminus The free $-NH_2$ attacks the acetyl carbonyl, chloride is displaced, and loss of a proton produces an N-acetyl amide. After aqueous work-up the product is

$$CH_3CONHCH(CH_2CH_2CONH_2)CONHCH_2COOH.$$

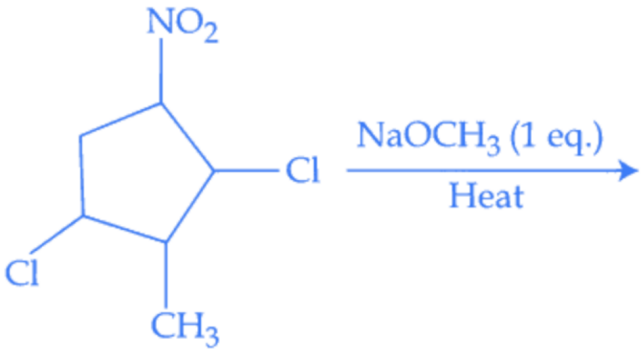
The peptide sequence remains Gln-Gly: neither the peptide bond nor the side-chain amide is rearranged by this protection reaction.

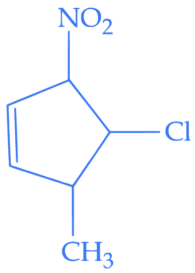
Step 3: Compare the four structures Option (C) acetylates the N-terminal amino group while preserving the glutamine side-chain amide and the Gln-Gly order. Option (A) additionally acetylates the side-chain amide nitrogen, which is not the ordinary amino-group acetylation product. Options (B) and (D) place glycine before glutamine, reversing the given dipeptide sequence; (B) also has the extra side-chain acetyl group. They therefore do not give the required product.

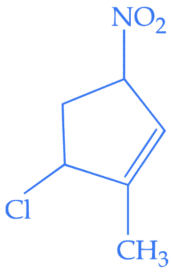
Step 4: Conclusion The product is N-acetyl-Gln-Gly, with the glutamine side-chain amide and the peptide sequence unchanged. Hence, the correct option is (C).

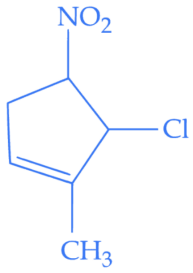
Quick Tip to Solve: For peptide acetylation, identify the free N-terminus first. Do not mistake an amide $-CONH_2$ side chain for a free amine.

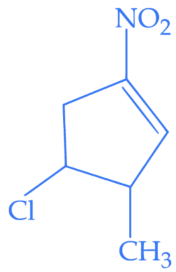
66. **The major product formed in the following reaction is :** [4 marks]



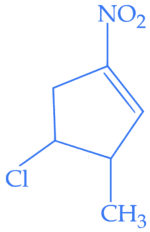
(A)


(B)


(C)


(D)


Correct Answer: (D)



Solution:

Concept:

- Sodium methoxide is a base, and heating promotes elimination of hydrogen chloride from the chlorinated ring. A hydrogen on a carbon bearing $-NO_2$ is especially acidic because the corresponding carbanion is resonance-stabilised.
- Eliminating the adjacent chloride forms an alkene conjugated with the nitro group. The nitro substituent therefore controls which hydrogen is removed, rather than alkene substitution alone.

Step 1: Locate the activated hydrogen and leaving group For clarity, call the ring carbon bearing $-NO_2$ C1 and follow the ring clockwise: C2 bears chlorine, C3 bears methyl, C4 bears the second chlorine, and C5 is the remaining carbon. C1 carries one hydrogen. Removing that hydrogen gives a carbanion whose negative charge can delocalise into the nitro group as a nitronate. The hydrogens at the other ring positions do not receive this direct stabilisation.

Step 2: Form the favoured double bond Methoxide removes the acidic C1 hydrogen. The adjacent C2 carbon carries a leaving group, so loss of its chloride forms the C1=C2 double bond. This is the base-induced elimination pathway favoured by the nitro substituent, and the product is a conjugated nitroalkene. The chlorine on C4 and the methyl on C3 remain attached to their original carbons. One equivalent of base corresponds to the single elimination drawn in the answer.

Step 3: Match the product to the displayed options Option (D) shows exactly the C1=C2 double bond, retains the C4 chlorine and C3 methyl, and keeps the nitro group on an alkene carbon. Option (B) instead forms C2=C3, while options (A) and (C) eliminate the other chlorine to form double bonds on the opposite side of the ring. Those alternatives do not exploit removal of the nitro-activated C1 hydrogen and formation of the conjugated nitroalkene.

Step 4: Conclusion The major product has a double bond between the nitro-bearing carbon and the adjacent carbon from which chloride is lost, as drawn in option (D). Hence, the correct option is (D).

Quick Tip to Solve: Before applying a simple more-substituted-alkene rule, check for an acidic hydrogen next to a strong electron-withdrawing group such as $-NO_2$.

67. Two 5 molal solutions are prepared by dissolving a non-electrolyte non-volatile solute separately in the solvents X and Y. The molecular weights of the solvents are M_x and M_y, respectively where $M_x = \frac{3}{4} M_y$. The relative lowering of vapor pressure of the solution in X is "m" times that of the solution in Y. Given that the number of moles of solute is very small in comparison to that of the solvent, the value of "m" is : [4 marks]

(A) $\frac{4}{3}$
(B) $\frac{3}{4}$
(C) $\frac{1}{2}$
(D) $\frac{1}{4}$

Correct Answer: (B) $\frac{3}{4}$

Solution:

Concept:

- For a nonvolatile nonelectrolyte solute, Raoult's law gives relative lowering $\Delta p/p^0 = x_2$, where x_2 is the solute mole fraction. For a dilute solution, $x_2 \simeq n_2/n_1$.
- Molality is moles of solute per kilogram of solvent. Equal molality fixes the solute amount for equal solvent masses, but the number of solvent moles still depends on its molar mass.

Step 1: Choose the same solvent mass for both solutions Take $1000\,\mathrm{g}$ of each solvent. Since both solutions are 5 molal, each contains $n_2 = 5\,\mathrm{mol}$ of solute. If the solvent molar masses are M_x and M_y, expressed in $\mathrm{g\,mol^{-1}}$, the respective solvent amounts are

$$n_{1,X} = \frac{1000}{M_x}, \qquad n_{1,Y} = \frac{1000}{M_y}.$$

Thus a larger solvent molar mass means fewer solvent molecules for the chosen mass.

Step 2: Use the stated dilute-solution approximation Using the explicit assumption that solute moles are small compared with solvent moles,

$$\left(\frac{\Delta p}{p^0}\right)_X \simeq \frac{5M_x}{1000}, \qquad \left(\frac{\Delta p}{p^0}\right)_Y \simeq \frac{5M_y}{1000}.$$

Their ratio is consequently

$$m = \frac{M_x}{M_y} = \frac{3}{4}.$$

The solvent vapour pressures themselves cancel because the quantity compared is relative lowering, rather than the absolute pressure change.

Step 3: Check the direction and reject the other ratios Since $M_x < M_y$, the same mass of X contains more solvent molecules, so its solute mole fraction and relative lowering must be smaller. The reciprocal $4/3$ in (A) gives the wrong direction. The ratios $1/2$ and $1/4$ in (C) and (D) do not follow from the specified molar-mass ratio. The common molality cancels rather than being multiplied into the ratio.

Step 4: Conclusion The relative lowering in X is $m = 3/4$ times that in Y. Hence, the correct option is (B).

Quick Tip to Solve: At equal molality and in the dilute limit, relative lowering of vapour pressure is proportional to the solvent molar mass; compare solvent moles before taking a ratio.

68. At a certain temperature in a $5\ L$ vessel, 2 moles of carbon monoxide and 3 moles of chlorine were allowed to reach equilibrium according to the reaction,

$CO + Cl_2 \rightleftharpoons COCl_2$

At equilibrium, if one mole of CO is present then equilibrium constant (K_c) for the reaction is :

[4 marks]

(A) 2

(B) 2.5

(C) 3

(D) 4

Correct Answer: (B) 2.5

Solution:

Concept:

- For $CO + Cl_2 \rightleftharpoons COCl_2$, the concentration equilibrium constant is $K_c = [COCl_2]/([CO][Cl_2])$. Use equilibrium concentrations, not starting mole amounts.
- The reaction consumes carbon monoxide and chlorine in a 1:1 ratio and produces one mole of phosgene for each mole of carbon monoxide consumed.

Step 1: Determine the extent of reaction Initially there are $2\,\text{mol}$ of CO and $3\,\text{mol}$ of Cl_2, with no phosgene initially added. The equilibrium amount of carbon monoxide is $1\,\text{mol}$, so

$$\xi = 2 - 1 = 1\,\text{mol}.$$

Stoichiometry therefore gives the equilibrium amounts

$$n(CO) = 1, \quad n(Cl_2) = 3 - 1 = 2, \quad n(COCl_2) = 1$$

in moles. Each change uses the coefficients of the balanced equation.

Step 2: Convert each amount to concentration The common vessel volume is $5\,\text{L}$. Therefore

$$[CO] = \frac{1}{5} = 0.2\,\text{mol}\,\text{L}^{-1},$$

$$[Cl_2] = \frac{2}{5} = 0.4\,\text{mol}\,\text{L}^{-1}, \qquad [COCl_2] = \frac{1}{5} = 0.2\,\text{mol}\,\text{L}^{-1}.$$

All three concentrations must be evaluated at equilibrium, using this same volume.

Step 3: Substitute into the equilibrium expression Now

$$K_c = \frac{0.2}{(0.2)(0.4)} = \frac{0.2}{0.08} = 2.5.$$

In the conventional concentration-unit expression this has units $\text{L}\,\text{mol}^{-1}$. Direct use of mole numbers would yield $1/(1 \times 2) = 0.5$, omitting the necessary volume factor. The values 2, 3 and 4 in the other options do not satisfy the concentration quotient for the calculated equilibrium composition.

Step 4: Conclusion The equilibrium constant has numerical value $K_c = 2.5$ in the stated molar-

concentration convention. Hence, the correct option is (B).

Quick Tip to Solve: When one product forms from two gaseous reactants, the mole quotient must be multiplied by the vessel volume to obtain the concentration quotient.

69. Two compounds I and II are eluted by column chromatography (adsorption of I > II). Which one of following is a correct statement ? [4 marks]

(A) I moves faster and has higher R_f value than II

(B) II movesfaster and has higher R_f value than I

(C) I moves slower and has higher R_f value than II

(D) II moves slower and has higher R_f value than I

Correct Answer: (B) II movesfaster and has higher R_f value than I

Solution:

Concept:

- In adsorption chromatography, a compound repeatedly partitions between the moving mobile phase and the stationary adsorbent. Stronger adsorption means a longer residence time on the stationary phase.
- The retention factor R_f is the distance travelled by a compound divided by the distance travelled by the solvent front in planar chromatography. Greater retention on an adsorbent gives smaller R_f under comparable conditions.

Step 1: Translate the adsorption comparison into mobility The statement adsorption of I greater than II means compound I has a stronger attraction for the stationary adsorbent. It consequently remains attached to the stationary phase for a greater fraction of the time. Compound II is less strongly retained and spends a greater fraction of its time in the moving solvent. In a column, II therefore travels down the column faster and elutes before I.

Step 2: Relate mobility to the retention factor For an analogous chromatographic separation with the same phases, the compound that moves farther with the mobile phase has the larger R_f. In symbols,

$$R_f = \frac{\text{distance travelled by compound}}{\text{distance travelled by solvent front}}.$$

At a fixed solvent-front distance, the weaker adsorption of II gives

$$d_{II} > d_I \quad \Rightarrow \quad R_{f,II} > R_{f,I}.$$

Here the denominator is common, so the comparison depends only on compound movement.

Step 3: Check both clauses of each statement Option (B) correctly states both faster movement of II and its higher R_f. Option (A) reverses the mobility and retention-factor ordering. Option (C) correctly makes I slower but incorrectly assigns it the higher R_f. Option (D) correctly gives II the higher R_f but incorrectly says it is slower. A statement is acceptable only if its speed comparison and retention-factor comparison agree with the adsorption strength.

Step 4: Conclusion Compound II moves faster and has a higher R_f value than compound I. Hence,

the correct option is (B).

Quick Tip to Solve: Use the chain: stronger adsorption → longer retention → slower motion → smaller R_f.

70. For per gram of reactant, the maximum quantity of N_2 gas is produced in which of the following thermal decomposition reactions ?

(Given : Atomic wt. - Cr = 52 u, Ba = 137 u) [4 marks]

(A) $(NH_4)_2Cr_2O_7(s) \rightarrow N_2(g) + 4H_2O(g) + Cr_2O_3(s)$

(B) $2NH_4NO_3(s) \rightarrow 2\,N_2(g) + 4H_2O(g) + O_2(g)$

(C) $Ba(N_3)_2(s) \rightarrow Ba(s) + 3N_2(g)$

(D) $2NH_3(g) \rightarrow N_2(g) + 3H_2(g)$

Correct Answer: (D) $2NH_3(g) \rightarrow N_2(g) + 3H_2(g)$

Solution:

Concept:

- To compare the amount of N_2 from one gram of each reactant, divide the moles of nitrogen produced in the balanced equation by the corresponding mass of reactant consumed.
- Because the product is the same gas in all four cases, ranking moles per gram also ranks mass per gram and volume per gram at the same temperature and pressure.

Step 1: Calculate the yield for ammonium dichromate and nitrate The molar mass of $(NH_4)_2Cr_2O_7$ is

$$2(14) + 8(1) + 2(52) + 7(16) = 252\,\text{g mol}^{-1}.$$

One mole gives one mole of N_2, so option (A) yields $1/252 = 0.00397\,\text{mol g}^{-1}$. For NH_4NO_3, the molar mass is $28 + 4 + 48 = 80\,\text{g mol}^{-1}$. The printed equation gives two moles of nitrogen from two moles of nitrate, so option (B) yields $2/(2 \times 80) = 0.0125\,\text{mol g}^{-1}$.

Step 2: Calculate the yield for barium azide and ammonia For $Ba(N_3)_2$, the molar mass is $137+6(14) = 221\,\text{g mol}^{-1}$. Its one mole gives three moles of nitrogen, making option (C)

$$\frac{3}{221} = 0.01357\,\text{mol g}^{-1}.$$

For ammonia, $M = 14 + 3 = 17\,\text{g mol}^{-1}$. Two moles, or $34\,\text{g}$, produce one mole of nitrogen, making option (D)

$$\frac{1}{34} = 0.02941\,\text{mol g}^{-1}.$$

Step 3: Compare yields on the common one-gram basis The yields obey

$$0.00397 < 0.0125 < 0.01357 < 0.02941.$$

Thus ammonia gives more nitrogen per gram than any of the three salts. Simply choosing barium azide because its equation shows three nitrogen molecules would overlook its much greater reactant mass. The smaller yields calculated explicitly for (A), (B) and (C) rule them out. Multiplying all yields

by $28\,\mathrm{g\,mol^{-1}}$ would leave their ordering unchanged.

Step 4: Conclusion Thermal decomposition of ammonia gives the maximum quantity of N_2 per gram of reactant. Hence, the correct option is (D).

Quick Tip to Solve: Compare the ratio $\nu(N_2)/[\nu(\text{reactant})M(\text{reactant})]$; the largest nitrogen coefficient alone does not identify the best mass yield.

71. The total number of optically active compounds formed in the following reactio is : [4 marks]



(A) Two

(B) Four

(C) Six

(D) Zero

Correct Answer: (B) Four

Solution:

Concept:

- An enol ether contains an alkoxy group directly bonded to a carbon of the double bond. Its oxygen lone pair stabilises a neighbouring carbocation by resonance, directing electrophilic addition of HBr.
- A tetrahedral carbon is stereogenic when its four substituents are different. With two independent stereogenic centres and no internal symmetry, there are $2^2 = 4$ configurational stereoisomers.

Step 1: Identify the favoured addition pathway The reacting unit is

$$CH_3CH_2C(CH_3){=}CH{-}O{-}CH(CH_3)_2.$$

Proton addition to the alkene carbon bearing methyl and ethyl groups leaves the positive charge at the carbon next to oxygen. Oxygen donation stabilises that intermediate as an oxocarbenium ion. Attack of Br^- at this carbon gives the addition product

$$CH_3CH_2CH(CH_3)CH(Br)OCH(CH_3)_2.$$

The ether oxygen is therefore relevant to the orientation; treating this as an isolated alkene would miss its stabilising effect.

Step 2: Count the stereogenic centres The carbon now bearing the new hydrogen has four different groups: hydrogen, methyl, ethyl and the brominated alkoxy-bearing carbon chain. The adjacent carbon has hydrogen, bromine, an isopropoxy group and the substituted carbon chain, so it is also

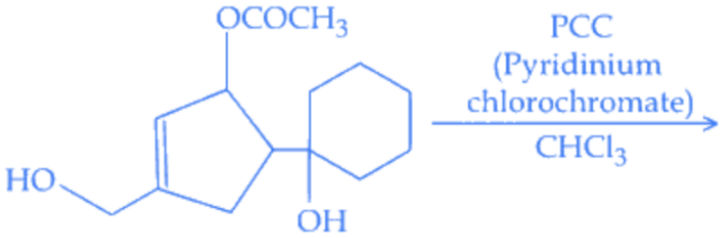
stereogenic. The carbon of the isopropyl group has two identical methyl groups and is not stereogenic. Thus exactly two stereogenic centres are produced.

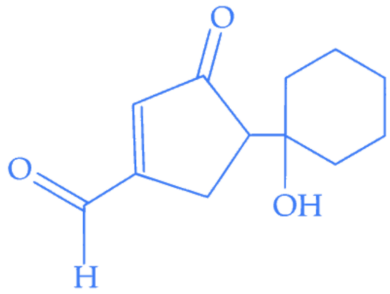
Step 3: Exclude meso forms and compare the counts The two stereogenic centres have different substituent sets, so no internal plane of symmetry can generate a meso form. The four configurations are $(R,R),(S,S),(R,S),(S,R)$, arranged into two enantiomeric pairs. Each individual stereoisomer is optically active, although an equal mixture of an enantiomeric pair has no net rotation. Two would count only one enantiomeric pair; six has no basis in the two-centre count, and zero would confuse a racemic mixture with its chiral components.

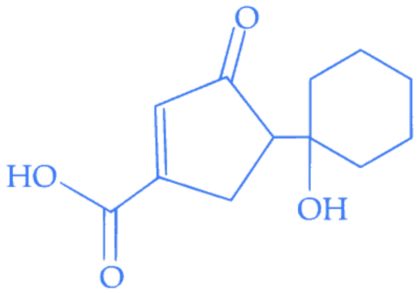
Step 4: Conclusion Four individually optically active stereoisomers can form. Hence, the correct option is (B).

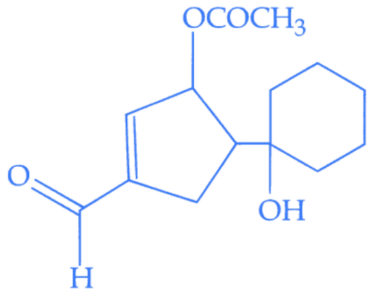
Quick Tip to Solve: For an enol ether, first locate the oxygen-stabilised carbocation, then count new stereogenic centres in the addition product; check for a meso form before using 2^n.

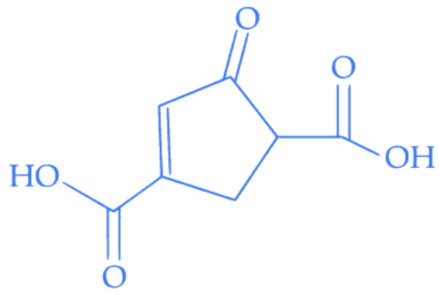
72. The major product formed in the following reaction is : [4 marks]



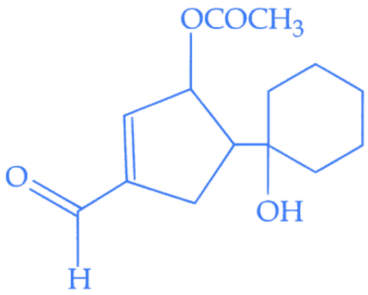
(A)


(B)


(C)


(D)


Correct Answer: (C)



Solution:

Concept:

- Pyridinium chlorochromate (PCC) oxidises a free primary alcohol to an aldehyde in a nonaqueous solvent. It normally stops at the aldehyde instead of forming a carboxylic acid.
- A tertiary alcohol has no hydrogen on its hydroxyl-bearing carbon, so ordinary chromium(VI) alcohol oxidation cannot form a carbonyl there without breaking a carbon–carbon bond. An acetate ester is a protected alcohol and remains intact under these conditions.

Step 1: Classify the oxygen-containing groups The substrate has a side-chain primary alcohol, $-CH_2OH$, attached to the unsaturated five-membered ring. A different ring carbon bears $-OCOCH_3$, an acetate ester rather than a free secondary alcohol. The hydroxyl group on the six-membered ring belongs to a tertiary alcohol: that carbon is bonded to two ring carbons and to the five-membered ring, leaving no carbon-bound hydrogen at that site.

Step 2: Apply the PCC selectivity In $CHCl_3$, PCC converts the free primary alcohol into $-CHO$. The ring double bond remains, the acetate substituent remains $-OCOCH_3$, and the tertiary alcohol remains $-OH$. No hydrolysis of the acetate or oxidative cleavage of the carbon skeleton is required. Thus the product differs from the reactant only by oxidation of the side-chain $-CH_2OH$ group to an aldehyde.

Step 3: Match all features against the options Option (C) has precisely the aldehyde, the unchanged acetate and the intact tertiary hydroxyl group. Option (A) additionally replaces the acetate-bearing site by a ketone, which would require removing the protecting group first. Option (B) has that extra ketone and also overoxidises the side chain to a carboxylic acid. Option (D) involves carboxylic acids and destruction of the six-membered carbon framework. None of these additional changes is the selective PCC oxidation shown.

Step 4: Conclusion The product is the structure in (C), with $-CH_2OH$ oxidised to $-CHO$ and the acetate and tertiary alcohol preserved. Hence, the correct option is (C).

Quick Tip to Solve: Before applying PCC, circle only free primary or secondary alcohols; an acetate-protected group and a tertiary alcohol are not oxidised in this standard transformation.

73. When 2-butyne is treate with H_2/Lindlar's catalyst, compound X is produced as the major product and when treated with Na/liq. NH_3 it produces Y as the major product Which of the following statements is correct ? [4 marks]

(A) X will have higher dipole moment and higher boiling point than Y.

(B) Y will have higher dipole moment and higher boiling point than X.

(C) X will have lower dipole moment and lower boiling point than Y.

(D) Y will have higher dipole momet and lower boiling point than X.

Correct Answer: (A) X will have higher dipole moment and higher boiling point than Y.

Solution:

Concept:

- Partial catalytic hydrogenation of an alkyne with H_2 and Lindlar's catalyst gives the cis alkene by syn addition. Dissolving-metal reduction with sodium in liquid ammonia gives the trans alkene.
- Molecular dipole moment is the vector sum of bond contributions. In symmetric trans-2-butene these contributions cancel; in cis-2-butene they do not cancel completely, giving stronger dipolar intermolecular attraction.

Step 1: Identify the two alkene products Both treatments partially reduce the carbon–carbon triple bond of 2-butyne, $CH_3C{\equiv}CCH_3$, to a double bond. Lindlar's catalyst delivers the hydrogen atoms from the same side, so X is cis-2-butene. Sodium in liquid ammonia gives net anti addition, so Y is trans-2-butene. The comparison is between these geometrical isomers, not between a butene and fully hydrogenated butane.

Step 2: Compare the molecular dipoles In trans-2-butene the identical methyl groups lie on opposite sides of the double bond. The molecule has a centre of symmetry, and the relevant bond-dipole contributions cancel. Its net dipole moment is zero. In cis-2-butene the methyl groups are on the same side, so the dipole vectors do not completely cancel. Consequently,

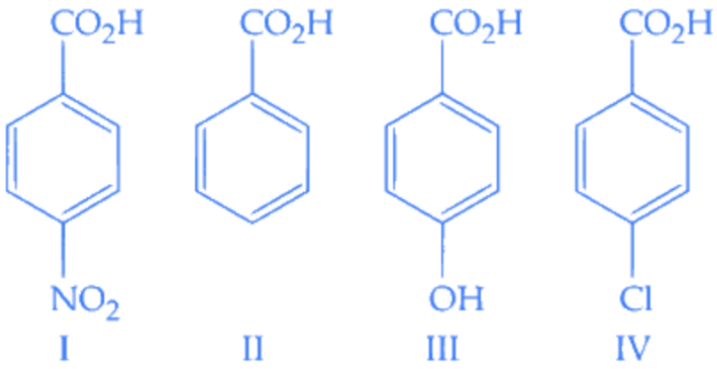
$$\mu_X > \mu_Y .$$

Step 3: Compare boiling points and eliminate alternatives The isomers have the same molecular formula and molar mass, so their dispersion forces are comparable. The nonzero dipole of the cis isomer adds dipole–dipole attraction, making its boiling point higher than that of trans-2-butene. Hence X has both the higher dipole moment and the higher boiling point. Options (B) and (D) incorrectly give Y the larger dipole moment, while option (C) reverses both trends. Melting-point arguments based on crystal packing do not determine this boiling-point comparison.

Step 4: Conclusion X, cis-2-butene, has a higher dipole moment and a higher boiling point than Y, trans-2-butene. Hence, the correct option is (A).

Quick Tip to Solve: Remember Lindlar gives cis and sodium/liquid ammonia gives trans; for these two butenes, compare dipole cancellation before comparing boiling points.

74. The increasing order of the acidity of the following carboxylic acids is : [4 marks]



(A) I < III < II < IV

(B) IV < II < III < I

(C) II < IV < III < I

(D) III < II < IV < I

Correct Answer: (D) III < II < IV < I

Solution:

Concept:

- Acidity increases when a substituent stabilises the conjugate carboxylate anion by withdrawing electron density. Electron-donating substituents generally reduce the acidity of a substituted benzoic acid.
- At the para position, nitro withdraws by both induction and resonance; hydroxy has a dominant resonance-donating effect. Chlorine has competing effects, but its inductive withdrawal makes para-chlorobenzoic acid more acidic than benzoic acid.

Step 1: Identify the four acids from the structures The labels denote (I) para-nitrobenzoic acid, (II) unsubstituted benzoic acid, (III) para-hydroxybenzoic acid and (IV) para-chlorobenzoic acid. Each compound loses the proton of its carboxyl group to form the corresponding benzoate ion. Therefore the comparison concerns how the para substituent affects that carboxylate, rather than the acidity of the phenolic hydroxyl group in (III).

Step 2: Compare donation and withdrawal relative to benzoic acid The $-OH$ group in (III) donates an oxygen lone pair into the aromatic ring by resonance. At the para position this donation outweighs its inductive withdrawal, making the carboxyl group less acidic than that in (II). Chlorine in (IV) withdraws inductively; although it can donate by resonance, the net effect here increases carboxylic-acid strength relative to (II). Thus

$$\text{III} < \text{II} < \text{IV}.$$

Step 3: Place the nitro derivative and check the alternatives The nitro group in (I) is a strong electron-withdrawing substituent through both its inductive and resonance effects. It stabilises the conjugate base most strongly among these groups, so (I) is the strongest acid. Appending it gives

$$\text{III} < \text{II} < \text{IV} < \text{I}.$$

Option (A) wrongly places the strongly withdrawing nitro derivative first. Option (B) puts the chloro derivative below benzoic acid, and option (C) places the hydroxy derivative above both benzoic and chloro acids. These all contradict the net substituent effects.

Step 4: Conclusion The increasing acidity order is III $<$ II $<$ IV $<$ I. Hence, the correct option is (D).

Quick Tip to Solve: Use benzoic acid as the reference: para-hydroxy lowers its acidity, para-chloro raises it moderately, and para-nitro raises it most strongly.

75. The correct order of spin-only magnetic moments among the following is :
(Atomic number : Mn = 25, Co = 27, Ni = 28, Zn = 30) [4 marks]

(A) $[ZnCl_4]^{2-} > [NiCl_4]^{2-} > [CoCl_4]^{2-} > [MnCl_4]^{2-}$
(B) $[CoCl_4]^{2-} > [MnCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$
(C) $[NiCl_4]^{2-} > [CoCl_4]^{2-} > [MnCl_4]^{2-} > [ZnCl_4]^{2-}$
(D) $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$
Correct Answer: (D) $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons and μ_B is the Bohr magneton. For nonnegative n, the moment increases with the number of unpaired electrons.
- In each $[MCl_4]^{2-}$ ion, four Cl^- ligands require oxidation state $+2$ on the metal. These chloride complexes are tetrahedral; their small tetrahedral splitting gives the high-spin electron arrangements.

Step 1: Find the metal-ion configurations Charge balance gives $x + 4(-1) = -2$, hence $x = +2$ for every metal. Removing the two outer $4s$ electrons gives $Mn^{2+} : 3d^5$, $Co^{2+} : 3d^7$, $Ni^{2+} : 3d^8$, and $Zn^{2+} : 3d^{10}$. The configurations of the ions, rather than the neutral atoms, must be used in the complexes.

Step 2: Count unpaired electrons and evaluate the moments For the tetrahedral high-spin configurations, the numbers of unpaired electrons are respectively $5, 3, 2, 0$. Therefore

$$\mu(Mn^{2+}) = \sqrt{35}\,\mu_B \approx 5.92\,\mu_B,$$
$$\mu(Co^{2+}) = \sqrt{15}\,\mu_B \approx 3.87\,\mu_B,$$
$$\mu(Ni^{2+}) = \sqrt{8}\,\mu_B \approx 2.83\,\mu_B,$$
$$\mu(Zn^{2+}) = 0.$$

Zinc is diamagnetic because its d shell is full.

Step 3: Arrange the complexes and reject the alternatives The required descending order follows $5 > 3 > 2 > 0$ unpaired electrons. Option (A) incorrectly makes diamagnetic zinc the largest. Option (B) puts cobalt above manganese even though manganese has more unpaired electrons. Option (C)

incorrectly places nickel above both cobalt and manganese. Only option (D) follows the spin-only formula for all four complexes; no orbital contribution needs to be added because the question specifically asks for spin-only moments.

Step 4: Conclusion The order is $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$. Hence, the correct option is (D).

Quick Tip to Solve: For tetrahedral chloride complexes, first find the metal oxidation state, then count high-spin unpaired d electrons; ordering n also orders the spin-only moment.

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76. Given

(i) $2Fe_2O_3(s) \rightarrow 4Fe(s) + 3O_2(g)$;

$\Delta_rG^o = +1487.0$ kJ mol^{-1}

(ii) $2CO(g) + O_2(g) \rightarrow 2CO_2(g)$;

$\Delta_rG^o = -514.4$ kJ mol^{-1}

Free energy change, Δ_rG^o for the reaction

$2Fe_2O_3(s) + 6CO(g) \rightarrow 4Fe(s) + 6CO_2(g)$ will be : [4 marks]

(A) -112.4 kJ mol^{-1}

(B) -56.2 kJ mol^{-1}

(C) -168.2 kJ mol^{-1}

(D) -208.0 kJ mol^{-1}

Correct Answer: (B) -56.2 kJ mol^{-1}

Solution:

Concept:

- Gibbs energy is a state function. When balanced reactions are added, their standard reaction Gibbs energies add with the same multipliers used for the stoichiometric equations.
- Multiplying a reaction by a factor multiplies Δ_rG° by that factor. Reversing a reaction changes the sign. The quoted reaction energy refers to the complete balanced equation as written.

Step 1: Choose the reaction multipliers Reaction (i) already has the required $2Fe_2O_3$ on the left and $4Fe$ on the right, so keep it unchanged. It produces $3O_2$. To consume all this oxygen and the required $6CO$, multiply reaction (ii) by three:

$$6\,CO\,(g) + 3\,O_2(g) \longrightarrow 6\,CO_2(g).$$

The corresponding Gibbs energy is $3(-514.4) = -1543.2\,kJ\,mol^{-1}$. Neither reaction is reversed.

Step 2: Add the equations and cancel the intermediate Adding reaction (i) and three times reaction (ii) places $3O_2$ on both sides, so oxygen cancels. The net equation is exactly

$$2\,Fe_2O_3(s) + 6\,CO\,(g) \longrightarrow 4\,Fe\,(s) + 6\,CO_2(g).$$

This cancellation checks the multipliers independently of the numerical calculation and ensures that

the energy corresponds to the requested stoichiometry.

Step 3: Calculate and compare with the options The standard reaction Gibbs energy is

$$\begin{aligned}\Delta_r G^\circ &= 1487.0 + 3(-514.4)\\ &= 1487.0 - 1543.2\\ &= -56.2\,\text{kJ mol}^{-1}.\end{aligned}$$

The negative sign means the written overall reaction is thermodynamically favourable under standard conditions. Option (A) doubles the correct energy without doubling the requested reaction. Option (C) triples it, and option (D) is also inconsistent with the balanced sum. The unit is retained for the reaction as written; there is no extra division by the coefficient of iron oxide or carbon monoxide.

Step 4: Conclusion For the reaction as written, $\Delta_r G^\circ = -56.2\,\text{kJ mol}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Balance and cancel the equations before adding Gibbs energies; apply the same multiplier to each reaction and its energy, without renormalising the requested equation.

77. In KO_2, the nature of oxygen species and the oxydation state of oxygen atom are, respectively :

[4 marks]

(A) Oxide and -2

(B) Superoxide and $-1/2$

(C) Peroxide and $-1/2$

(D) Superoxide and -1

Correct Answer: (B) Superoxide and $-1/2$

Solution:

Concept:

- Potassium has oxidation state $+1$ in its ionic compounds. A neutral formula unit of KO_2 must therefore contain an oxygen species with total charge -1.
- Oxide is O^{2-}, peroxide is $O_2{}^{2-}$, and superoxide is $O_2{}^{-}$. For an equivalent pair of oxygen atoms, each atom has half the oxidation state assigned to the entire dioxygen unit.

Step 1: Determine the charge on the oxygen unit Write the neutral compound as K^+ and a dioxygen unit. Since one potassium contributes $+1$, the two oxygen atoms together must contribute -1. Thus the anion is $O_2{}^{-}$, the superoxide ion. It cannot be oxide, because an oxide ion has charge -2 on a single oxygen atom; it cannot be peroxide, because a peroxide unit has total charge -2.

Step 2: Calculate the oxidation state per oxygen atom Let the oxidation state of each oxygen atom be x. Charge neutrality of KO_2 gives

$$+1 + 2x = 0, \qquad x = -\frac{1}{2}.$$

This fractional value is the average oxidation state of an oxygen atom in the symmetric dioxygen ion.

It does not mean that the superoxide unit has charge $-\frac{1}{2}$; its total charge is -1, distributed over the two equivalent oxygen atoms.

Step 3: Check the species and number together Option (B) correctly gives both superoxide and $-\frac{1}{2}$ per oxygen atom. Option (A) describes an ordinary oxide, whose potassium salt would have the formula K_2O. Option (C) names peroxide but assigns the superoxide oxidation state: oxygen in a peroxide has oxidation state -1, as in K_2O_2. Option (D) names superoxide but uses the peroxide oxidation state. Both the identity of the ion and the oxidation state per atom must agree with the one-to-two potassium-to-oxygen ratio.

Step 4: Conclusion The oxygen species is superoxide, $O_2{}^-$, and each oxygen atom has oxidation state $-\frac{1}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish the charge on the whole oxygen ion from the oxidation state per atom: divide the charge of $O_2{}^-$ by two, giving $-\frac{1}{2}$ for superoxide oxygen.

78. Following four solutions are prepared by mixing different volumes of NaOH and HCl of different concentrations, pH of which one of them will be equal to 1 ? [4 marks]

(A) 100 mL $\frac{M}{10}$ HCl + 100 mL $\frac{M}{10}$ NaOH
(B) 75 mL $\frac{M}{5}$ HCl + 25 mL $\frac{M}{5}$ NaOH
(C) 60 mL $\frac{M}{10}$ HCl + 40 mL $\frac{M}{10}$ NaOH
(D) 55 mL $\frac{M}{10}$ HCl + 45 mL $\frac{M}{10}$ NaOH

Correct Answer: (B) 75 mL $\frac{M}{5}$ HCl + 25 mL $\frac{M}{5}$ NaOH

Solution:

Concept:

- Both HCl and NaOH dissociate completely. Their reaction is $H^+ + OH^- \longrightarrow H_2O$, so equal amounts in moles neutralise one another.
- For an acidic mixture, $[H^+] = (C_aV_a - C_bV_b)/(V_a + V_b)$. Use volumes in litres when finding moles; pH is $-\log_{10}[H^+]$.

Step 1: Find the required excess acid A pH of 1 requires $[H^+] = 10^{-1} = 0.1\,\text{mol}\,L^{-1}$. The final concentration depends on the excess moles after neutralisation, divided by the combined volume. Comparing just the initial acid concentrations would ignore both dilution and consumption by the base.

Step 2: Calculate the mixture in option B Here both solutions are $M/5 = 0.2\,\text{mol}\,L^{-1}$. The acid supplies $0.075 \times 0.2 = 0.015$ mol and the base supplies $0.025 \times 0.2 = 0.005$ mol. Thus

$$n_{\text{excess}} = 0.015 - 0.005 = 0.010\,\text{mol}, \qquad V = 0.100\,\text{L}.$$

$$[H^+] = \frac{0.010}{0.100} = 0.1\,\text{mol}\,L^{-1}, \qquad \text{pH} = 1.$$

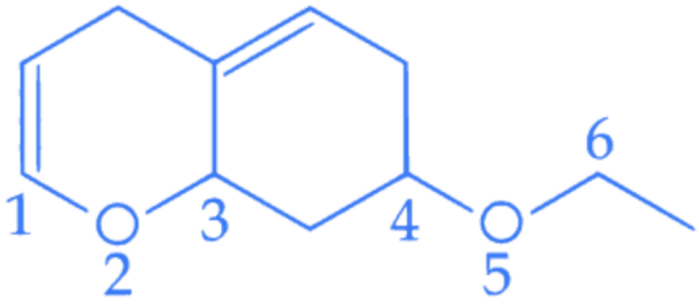
Step 3: Check the other three mixtures In A the equal concentrations and equal volumes give complete neutralisation, so its pH is approximately 7 at room temperature. In C the excess acid is

$(0.060 - 0.040) \times 0.1 = 0.002$ mol in 0.100 L, giving $[H^+] = 0.020\,M$ and pH about 1.70. In D the excess is 0.001 mol in 0.100 L, giving 0.010 M and pH 2. Neither C nor D reaches the required concentration.

Step 4: Conclusion Only 75 mL of $M/5$ HCl mixed with 25 mL of $M/5$ NaOH gives pH 1. Hence, the correct option is (B).

Quick Tip to Solve: For mixtures of a strong acid and a strong base, subtract moles before dividing by total volume; never average their pH values.

79. On treatment of the following compound with a strong acid, the most susceptible site for bond cleavage is : [4 marks]



(A) Cl – O2

(B) O2 – C3

(C) C4 – O5

(D) O5 – C6

Correct Answer: (D) O5 – C6

Solution:

Concept:

- Acid cleavage of ethers starts with oxygen protonation. Resonance delocalisation lowers the basicity of an oxygen directly bonded to a vinylic carbon compared with an ordinary dialkyl ether oxygen.
- In a protonated unsymmetrical alkyl ether with primary and secondary carbon centres, nucleophilic cleavage is favoured at the less hindered primary carbon through an S_N2 pathway.

Step 1: Compare the two oxygen atoms The ring oxygen O2 is bonded directly to the vinylic carbon C1. Its lone pair is conjugated with the adjacent double bond, so this is an enol-ether linkage. By contrast, O5 is bonded to two saturated carbons, C4 and C6, and its lone pairs are more available for protonation. The ordinary alkyl ether at O5 is therefore the relevant site for the ether-cleavage comparison.

Step 2: Choose the susceptible bond at protonated O5 Once O5 is protonated, nucleophilic attack at an attached carbon can expel the other fragment as a neutral alcohol. C6 is the primary carbon of the ethyl group, whereas C4 is a secondary carbon within the ring. Backside attack is less hindered at C6. Thus cleavage of O5–C6 is favoured over C4–O5 by the substitution pathway. This reasoning does not

require formation of an unstable free primary ethyl carbocation.

Step 3: Reject the other proposed sites C1–O2 is a vinylic carbon–oxygen bond; attack at the vinylic carbon is unsuitable for an ordinary S_N2 cleavage, and direct ionisation would demand an unstable vinyl cation. O2–C3 lies at the less readily protonated, conjugated oxygen. Although a cation-centred argument may appear to favour its allylic carbon, it neglects both oxygen basicity and the facile primary-carbon substitution available at protonated O5. C4–O5 is more sterically hindered than O5–C6. Therefore the primary ethyl-side bond gives the preferred ether cleavage.

Step 4: Conclusion The most susceptible bond is O5–C6, at the primary ethyl carbon of the protonated alkyl ether. Hence, the correct option is (D).

Quick Tip to Solve: For acid cleavage of an unsymmetrical ether, compare oxygen basicity first, then favour nucleophilic attack at a less hindered primary carbon.

80. The total number of possible isomers for square-planar $[Pt(Cl)(NO_2)(NO_3)(SCN)]^{2-}$ is : [4 marks]

(A) 8

(B) 12

(C) 16

(D) 24

Correct Answer: (B) 12

Solution:

Concept:

- Linkage isomerism occurs when an ambidentate ligand coordinates through different donor atoms. NO_2^- can bind through N or O, and SCN^- can bind through S or N.
- A square-planar complex containing four different monodentate ligands has three geometrical arrangements, distinguished by which ligand lies trans to a chosen reference ligand.

Step 1: Count the donor-atom choices For the nitrite ligand, the two possibilities are nitro (N-bonded) and nitrito (O-bonded). For thiocyanate, the two possibilities are S-bonded thiocyanato and N-bonded isothiocyanato. Chloride has only the Cl donor, while the nitrate ligand coordinates through oxygen in this square-planar monodentate description; equivalent nitrate oxygen atoms do not give distinct linkage isomers. The two independent ambidentate choices therefore give

$$N_{\text{linkage}} = 2 \times 2 = 4.$$

Step 2: Count arrangements for each linkage choice Fix the chloride ligand at one vertex of the square. Any one of the other three distinct ligands may occupy the opposite (trans) vertex. Once the trans ligand is chosen, interchanging the two remaining adjacent ligands gives the same arrangement upon a rotation of the square in space. Consequently there are three distinct geometrical isomers for each donor-atom combination, rather than six arrangements obtained by counting equivalent positions separately.

Step 3: Combine the independent counts Every one of the four linkage combinations still contains

four distinct ligands and hence has the same three geometrical possibilities. Thus

$$N_{\text{total}} = 4 \times 3 = 12.$$

The square-planar arrangements are planar and do not supply an additional independent pair of optical isomers. The value 8 misses the correct geometrical count, while 16 or 24 counts extra arrangements that are not distinct in this geometry.

Step 4: Conclusion The total number of linkage and geometrical isomers is 12. Hence, the correct option is (B).

Quick Tip to Solve: For a square-planar complex with four different ligands, count three trans-pair arrangements and multiply by independent ambidentate donor choices.

81. $\Delta_f G^o$ at 500 K for substance 'S' in liquid state and gaseous state are +100.7 kcl mol^{-1} and +103 kcal mol^{-1}, respectively. Vapour pressure of liquid 'S' at 500 K is approximately equal to : (R = 2 cal K^{-1} mol^{-1}) [4 marks]

(A) 0.1 atm

(B) 1 atm

(C) 10 atm

(D) 100 atm

Correct Answer: (A) 0.1 atm

Solution:

Concept:

- For vaporisation, the standard Gibbs-energy change is the gaseous standard Gibbs energy of formation minus that of the liquid.
- At liquid–vapour equilibrium, $\Delta G = \Delta G^\circ + RT\ln(p/p^\circ) = 0$, taking the pure liquid activity as unity and the vapour as ideal.

Step 1: Calculate the standard vaporisation energy The formation energies are both given at 500 K, so their difference applies directly at this temperature:

$$\Delta G^\circ_{\text{vap}} = 103.0 - 100.7 = 2.3\,\text{kcal mol}^{-1}.$$

Convert kilocalories to calories to match the given gas constant:

$$\Delta G^\circ_{\text{vap}} = 2300\,\text{cal mol}^{-1}, \quad RT = 2 \times 500 = 1000\,\text{cal mol}^{-1}.$$

Step 2: Impose equality of chemical potentials The liquid and its saturated vapour have equal chemical potentials. Relative to the standard gaseous state, the pressure-dependent gas contribution is $RT\ln(p/p^\circ)$. Therefore

$$\ln\frac{p}{p^\circ} = -\frac{\Delta G^\circ_{\text{vap}}}{RT} = -\frac{2300}{1000} = -2.3.$$

Since $\ln 10 \simeq 2.303$, the pressure ratio is $e^{-2.3} \simeq 0.100$. Taking the question's standard pressure convention as 1 atm gives approximately 0.1 atm.

Step 3: Check the sign against the alternatives The positive standard vaporisation free energy means vaporisation is unfavourable at unit gas pressure. Equilibrium is reached at a lower vapour pressure, so 10 atm and 100 atm have the wrong direction. A pressure of 1 atm would require a zero standard vaporisation free energy at 500 K, which is inconsistent with the 2.3 kcal difference.

Step 4: Conclusion The vapour pressure is approximately $0.1\,\text{atm}$. Hence, the correct option is (A).

Quick Tip to Solve: Use gas minus liquid for vaporisation, match the energy units to R, and retain the minus sign in $\ln(p/p^\circ) = -\Delta G^\circ/(RT)$.

82. The de-Broglie's wavelength of electron present in first Bohr orbit of 'H' atom is : [4 marks]

(A) 0.529 $\mathring{\text{A}}$

(B) $2\pi \times 0.529$ $\mathring{\text{A}}$

(C) $\frac{0.529}{2\pi}$ $\mathring{\text{A}}$

(D) 4×0.529 $\mathring{\text{A}}$

Correct Answer: (B) $2\pi \times 0.529$ $\mathring{\text{A}}$

Solution:

Concept:

- The electron de Broglie wavelength is $\lambda = h/p = h/(m_e v)$, where p is its momentum.
- Bohr angular-momentum quantisation gives $m_e v r_n = nh/(2\pi)$. Equivalently, an allowed orbit contains an integer number of electron wavelengths around its circumference.

Step 1: Relate wavelength to orbital radius Start with $m_e v r_n = nh/(2\pi)$ and solve for the electron momentum:

$$m_e v = \frac{nh}{2\pi r_n}.$$

Substitute this into the de Broglie relation:

$$\lambda_n = \frac{h}{m_e v} = \frac{h}{nh/(2\pi r_n)} = \frac{2\pi r_n}{n}.$$

The circumference is therefore $2\pi r_n = n\lambda_n$, which expresses the standing-wave condition without treating the radius itself as a wavelength.

Step 2: Use the first hydrogen orbit For hydrogen in its first Bohr orbit, $n = 1$ and $r_1 = a_0 = 0.529\,\text{Å}$. Hence

$$\lambda_1 = 2\pi a_0 = 2\pi \times 0.529\,\text{Å} \simeq 3.32\,\text{Å}.$$

The angstrom is a length unit, so the result may also be written as approximately $3.32 \times 10^{-10}\,\text{m}$. Exactly one de Broglie wavelength fits around this first orbit.

Step 3: Eliminate radius and inverse-factor answers Option A gives only the Bohr radius and omits the circumference factor 2π. Option C divides the radius by 2π, reversing the factor required by

quantisation. Option D replaces the circumference factor by 4, which has no basis in either $\lambda = h/p$ or the Bohr angular-momentum condition. Only B agrees with the integer-wave condition at $n = 1$.

Step 4: Conclusion The electron wavelength is $2\pi \times 0.529\,\text{Å}$. Hence, the correct option is (B).

Quick Tip to Solve: Remember $2\pi r_n = n\lambda_n$: in the first orbit, the wavelength equals the circumference, not the radius.

83. In the leaching method, bauxite ore is digested with a concentrated solution of NaOH that produces 'X'. When CO_2 gas is passed through the aqueous solution of 'X', a hydrated compound 'Y' is precipitated. 'X' and 'Y' respectively are : [4 marks]

(A) $NaAlO_2$ and $Al_2(CO_3)_3.x\ H_2O$

(B) $A(OH)_3$ and $Al_2O_3.x\ H_2O$

(C) $Na[Al(OH)_4]$ and $Al_2O_3.x\ H_2O$

(D) $Na[Al(OH)_4]$ and $Al_2(CO_3)_3.x\ H_2O$

Correct Answer: (C) $Na[Al(OH)_4]$ and $Al_2O_3.x\ H_2O$

Solution:

Concept:

- Alumina is amphoteric: hot concentrated NaOH dissolves it as soluble sodium tetrahydroxoaluminate, $Na[Al(OH)_4]$.
- Passing CO_2 into the alkaline aluminate solution reduces its alkalinity and causes hydrated alumina to precipitate; it does not isolate aluminium carbonate from water.

Step 1: Identify the soluble leaching product X The alumina portion of bauxite reacts with the concentrated aqueous alkali. A useful hydrated representation is

$$Al_2O_3 + 2\,NaOH + 3\,H_2O \longrightarrow 2\,Na[Al(OH)_4].$$

The aluminium-containing product remains in solution while insoluble impurities can be separated. Thus X is sodium tetrahydroxoaluminate, $Na[Al(OH)_4]$, rather than precipitated $Al(OH)_3$. The latter would not represent the soluble product of alkaline digestion.

Step 2: Identify the precipitate after carbonation Carbon dioxide reacts with the alkaline solution and allows aluminium hydroxide, conventionally described as hydrated alumina, to separate. One balanced representation is

$$2\,Na[Al(OH)_4] + CO_2 \longrightarrow 2\,Al(OH)_3\downarrow + Na_2CO_3 + H_2O.$$

Hydroxide $Al(OH)_3$ corresponds compositionally to $Al_2O_3 \cdot 3\,H_2O$; hydrated alumina is generally written $Al_2O_3.xH_2O$. Thus the given general formula for Y is hydrated oxide, not a hydrated aluminium carbonate.

Step 3: Compare all the pairs Option C names both the soluble hydroxoaluminate and the hydrated alumina precipitate correctly. Options A and D make the aluminium precipitate a carbonate, which

is not the product of this aqueous purification step. Option B wrongly identifies the alkaline leach product as a hydroxide rather than the soluble aluminate complex. Sodium carbonate formed in solution must not be confused with an aluminium carbonate precipitate.

Step 4: Conclusion $X = \mathrm{Na[Al(OH)_4]}$ and $Y = \mathrm{Al_2O_3}.x\mathrm{H_2O}$. Hence, the correct option is (C).

Quick Tip to Solve: In bauxite purification, separate the soluble aluminate made by excess alkali from the hydrated alumina precipitated when alkalinity is lowered.

84. The number of P − O bonds in P_4O_6 is : [4 marks]

(A) 6

(B) 9

(C) 12

(D) 18

Correct Answer: (C) 12

Solution:

Concept:

- P_4O_6 has a cage structure based on four phosphorus atoms at the vertices of a tetrahedron. An oxygen atom bridges each pair of neighbouring phosphorus vertices.
- Each bridging oxygen contributes two single P–O bonds, one to each phosphorus. There are no terminal P=O bonds in P_4O_6.

Step 1: Count the oxygen bridges A tetrahedron has four vertices and six edges. In the P_4O_6 cage, each of those six edges is replaced by a P–O–P bridge. The six oxygen atoms in the formula therefore account for all six bridges. The cage contains no direct P–P bond along these edges; each connection goes through oxygen.

Step 2: Convert bridges into individual bonds One oxygen bridge contains two bonds, not one: one phosphorus–oxygen bond enters the bridging O atom and another leaves it. Consequently

$$N_{\mathrm{P-O}} = 6 \times 2 = 12.$$

As an independent check, each of the four phosphorus atoms is bonded to three oxygen neighbours, so

$$N_{\mathrm{P-O}} = 4 \times 3 = 12.$$

In this second count each bond is counted once at its phosphorus end; there is no need to divide by two because its other end is oxygen, outside the set of four counted centres.

Step 3: Reject the alternative counts The value 6 counts oxygen bridges rather than individual P–O bonds. The values 9 and 18 agree with neither the threefold oxygen coordination around every phosphorus nor the twofold phosphorus coordination of every oxygen. Confusing this compound with P_4O_{10}, which also has terminal oxygen atoms, would add bonds that are absent from the specified formula and cage.

Step 4: Conclusion There are 12 phosphorus–oxygen bonds in P_4O_6. Hence, the correct option is (C).

Quick Tip to Solve: For P_4O_6, count either four phosphorus atoms times three bonds or six bridging oxygen atoms times two bonds; both give twelve.

85. All of the following share the same crystal structure except : [4 marks]

(A) LiCl

(B) NaCl

(C) RbCl

(D) CsCl

Correct Answer: (D) CsCl

Solution:

Concept:

- The rock-salt structure has coordination number $6:6$: each cation is surrounded octahedrally by six anions and each anion by six cations.
- The caesium-chloride structure has coordination number $8:8$. Its two ions occupy the body centre and corners of a cubic arrangement; coordination depends on ionic size, not just the formula MX.

Step 1: Identify the three rock-salt salts Under ordinary conditions, LiCl, NaCl and RbCl adopt the sodium-chloride or rock-salt structure. In this structure one ion forms a face-centred cubic array and the oppositely charged ions occupy all octahedral holes. There are four formula units in the conventional unit cell. Thus the first three salts share the same arrangement even though their cation sizes, lattice parameters and lattice energies differ.

Step 2: Compare caesium chloride The larger Cs^+ ion permits eight nearest-neighbour Cl^- ions rather than six. In CsCl, one species is at the cube corners and the other at the body centre, giving one formula unit per conventional cubic cell and $8:8$ coordination. The centre and corner sites contain different species, so this must not be confused with a body-centred cubic lattice of identical atoms. Options (A), (B) and (C) are excluded because their ordinary structures are all rock salt; only (D) changes the coordination pattern.

Step 3: Conclusion CsCl has the caesium-chloride structure, whereas LiCl, NaCl and RbCl have the rock-salt structure. Hence, the correct option is (D).

Quick Tip to Solve: For alkali-metal chlorides, distinguish CsCl by its $8:8$ coordination; the usual rock-salt examples have $6:6$ coordination.

86. If x gram of gas is adsorbed by m gram of adsorbent at pressure P, the plot of log $\frac{x}{m}$ versus log P is linear. The slope of the plot is :

(n and k are constants and n $>$ 1) [4 marks]

(A) 2 k

(B) log k

(C) n

(D) $\frac{1}{n}$

Correct Answer: (D) $\frac{1}{n}$

Solution:

Concept:

- At fixed temperature the Freundlich adsorption isotherm is $x/m = kP^{1/n}$, where x/m is the mass of gas adsorbed per unit mass of adsorbent and k, n are constants.
- Taking logarithms converts a power law into a straight-line relation. In $Y = c + sX$, s is the slope and c is the vertical intercept.

Step 1: Take logarithms of the isotherm Start with the dependence of adsorption on pressure, keeping temperature fixed. Taking the same logarithm on both sides gives

$$\log\left(\frac{x}{m}\right) = \log\left(kP^{1/n}\right) = \log k + \frac{1}{n}\log P.$$

The product law produces the additive term $\log k$, while the power law brings the exponent $1/n$ down as a multiplier. The result applies using either common or natural logarithms, provided the same base is used on both axes.

Step 2: Identify the coefficient of the horizontal coordinate Compare this equation with $Y = c + sX$, with $Y = \log(x/m)$ and $X = \log P$. The coefficient multiplying X is $1/n$, so that is the slope. The term $\log k$ is the intercept, excluding (B). Neither $2k$ in (A) nor n in (C) multiplies $\log P$. Since $n > 1$, the required slope lies between zero and one; this also rules out n itself and agrees with sublinear adsorption in the Freundlich range.

Step 3: Conclusion The log-log plot has slope $1/n$ and vertical intercept $\log k$. Hence, the correct option is (D).

Quick Tip to Solve: For any power law $y = ax^b$, a plot of $\log y$ against $\log x$ has slope b, not a or $\log a$.

87. Which of the following statements is not true ? [4 marks]

(A) Step growth polymerisation requires a bifunctional monomer.

(B) Nylon 6 is an example of step-growth polymerisation.

(C) Chain growth polymerisation includes both homopolymerisation and copylymerisation.

(D) Chain growth polymerisation involves homopolymerisation only.

Correct Answer: (D) Chain growth polymerisation involves homopolymerisation only.

Solution:

Concept:

- In chain-growth polymerisation, monomer units add successively to an active chain end. The reacting monomers may be of one kind or of two or more kinds.
- A homopolymer contains repeating units derived from one monomer species; a copolymer contains units derived from more than one monomer species. This distinction does not by itself determine the growth mechanism.

Step 1: Check the step-growth statements Step-growth polymerisation requires reactive functional groups that can join molecules repeatedly; a bifunctional monomer is the standard requirement for a linear chain, so (A) states the usual textbook condition. In the NCERT treatment, nylon 6 is obtained by heating caprolactam with water. Ring opening gives the amino-acid functionality needed for formation of successive amide links, and nylon 6 is classified with the condensation polyamides. Thus (B) is accepted in the textbook classification used here.

Step 2: Test the word only in the chain-growth statements Chain growth is not restricted to a single monomer species. Ethene can undergo chain-growth homopolymerisation to give polyethene, while styrene and 1,3-butadiene can undergo chain-growth copolymerisation to give Buna-S. Both proceed by growth at an active chain end, even though their monomer compositions differ. Therefore (C), which allows both homopolymerisation and copolymerisation, is true. Statement (D) incorrectly excludes chain-growth copolymerisation; the example of Buna-S directly disproves its claim.

Step 3: Conclusion Statement (D) is not true: chain-growth polymerisation can produce both homopolymers and copolymers. Hence, the correct option is (D).

Quick Tip to Solve: Keep the two classifications separate: chain versus step describes growth, whereas homo versus co describes the number of monomer species.

88. Biochemical Oxygen Demand (BOD) value can be a measure of water pollution caused by the organic matter. Which of the following statements is correct ? [4 marks]

(A) Aerobic bacteria decrease the BOD value.

(B) Anaerobic bacteria increase the BOD value.

(C) Clean water has BOD value higher than 10 ppm.

(D) Polluted water has BOD value higher than 10 ppm.

Correct Answer: (D) Polluted water has BOD value higher than 10 ppm.

Solution:

Concept:

- Biochemical oxygen demand (BOD) is the amount of dissolved oxygen required by aerobic microorganisms to oxidise biodegradable organic material in a water sample under specified test conditions.
- A greater biodegradable organic load generally causes a greater oxygen demand. In the usual textbook comparison, clean water has a low BOD, whereas heavily polluted water can have BOD above $10\,\text{ppm}$.

Step 1: Relate organic loading to oxygen demand When sewage or other biodegradable organic material enters water, microorganisms use it as food. Aerobic decomposition consumes dissolved oxygen. The oxygen needed for this biological oxidation is measured as BOD, so a large value indicates a substantial organic-pollution load. BOD is a demand measured over a specified incubation period; it is not the amount of oxygen initially present in the water. Consequently a high BOD can accompany low dissolved oxygen.

Step 2: Compare the statements with the BOD criterion Option (D) gives the standard pollution criterion: polluted water has BOD higher than $10\,\text{ppm}$. Option (C) reverses the clean-water comparison. Option (B) incorrectly makes anaerobic bacteria the direct basis of an oxygen-demand measurement. Option (A) is not a general statement identifying polluted water: aerobic bacteria exert the measured oxygen demand while oxidising the organic load. Prolonged biological treatment can reduce the residual BOD by removing that load, but bacterial presence alone does not imply a decrease in the measured demand; the treatment qualification is absent here.

Step 3: Conclusion The stated indicator of organic pollution is BOD higher than $10\,\text{ppm}$. Hence, the correct option is (D).

Quick Tip to Solve: High BOD means a high requirement for oxygen during organic decomposition; do not confuse it with high dissolved-oxygen content.

89. In XeO_3F_2, the number of bond pair(s), π-bond(s) and lone pair(s) on Xe atom respectively are : [4 marks]

(A) 5, 2, 0

(B) 4, 2, 2

(C) 5, 3, 0

(D) 4, 4, 0

Correct Answer: (C) 5, 3, 0

Solution:

Concept:

- For the conventional Lewis structure of XeO_3F_2, each oxygen forms a double bond to xenon and each fluorine forms a single bond. A double bond consists of one σ bond and one π bond.
- For VSEPR counting, each atom attached to the central atom supplies one bonding domain, irrespective of whether the bond is single or multiple. Lone pairs must be counted on the central atom, not on the terminal atoms.

Step 1: Construct the central-atom bonding arrangement Xenon has eight valence electrons. The three oxygen atoms contribute eighteen electrons and the two fluorine atoms contribute fourteen, giving forty valence electrons altogether. The conventional arrangement has three Xe=O double bonds and two Xe−F single bonds. The bonds use sixteen electrons; completing the terminal oxygen and fluorine octets uses the remaining twenty-four. No electrons remain as a lone pair on xenon. This also gives zero formal charge on the atoms in the conventional expanded-octet Lewis representation.

Step 2: Count the requested quantities separately There are five atoms directly bonded to Xe, so there are five σ-bonding domains. The three double bonds supply three π bonds, and Xe has zero lone pairs. Thus the requested tuple is $(5, 3, 0)$, corresponding to (C). Option (A) misses one π bond; (B) incorrectly gives four bonding domains and two xenon lone pairs; (D) gives both the wrong domain count and the wrong π-bond count. In this question the first count is the five bonding domains or σ bonds, with the π bonds counted separately.

Step 3: Conclusion XeO_3F_2 has five σ-bonding domains, three π bonds and no lone pair on Xe: $(5, 3, 0)$. Hence, the correct option is (C).

Quick Tip to Solve: Count one σ bond for each atom attached to Xe; then count the extra π bond in each Xe=O separately.

90. Lithium aluminium hydride reacts with silicon tetrachloride to form : [4 marks]

(A) LiCl, AlH_3 and SiH_4

(B) LiCl, $AlCl_3$ and SiH_4

(C) LiH, $AlCl_3$ and $SiCl_2$

(D) LiH, AlH_3 and SiH_4

Correct Answer: (B) LiCl, $AlCl_3$ and SiH_4

Solution:

Concept:

- Lithium aluminium hydride, $LiAlH_4$, is a hydride donor. It replaces the chlorine atoms in silicon tetrachloride with hydrogen to produce silane, SiH_4.
- Write the complete balanced reaction to identify the accompanying lithium and aluminium products. Atom conservation checks the products as well as the molecular formula of the hydride.

Step 1: Identify the silicon-containing product Silicon tetrachloride contains four chlorine atoms bonded to silicon. Replacement of these chlorine atoms by hydrogen gives SiH_4, not the lower chloride $SiCl_2$. The four hydrogens needed are supplied by one formula unit of $LiAlH_4$. This immediately excludes option (C), whose silicon product still contains chlorine and whose lithium product also retains a hydride.

Step 2: Balance all atoms and compare the remaining choices The complete reaction is

$$SiCl_4 + LiAlH_4 \longrightarrow SiH_4 + LiCl + AlCl_3.$$

There is one Si, one Li, one Al, four H and four Cl on each side. Lithium takes one chloride and aluminium takes the other three; all four hydrogen atoms appear in silane. This set of products is (B). Option (A), with AlH_3 as well as SiH_4, does not account for the four chlorines in the standard reduction and would require additional hydrogen. Option (D) retains both lithium and aluminium hydrides and supplies no chloride-containing product, so it cannot represent the reduction of $SiCl_4$. The balanced equation selects the unique complete product set.

Step 3: Conclusion The products are LiCl, $AlCl_3$ and SiH_4. Hence, the correct option is (B).

Quick Tip to Solve: For $SiCl_4$ + $LiAlH_4$, allocate all four H atoms to SiH_4; the four Cl atoms then divide as LiCl + $AlCl_3$.