JEE Main 2018 Paper 1 (B.E./B.Tech) Apr 16 Slot 1 (morning) Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2018, Annual (online CBT), conducted by the Central Board of Secondary Education (CBSE) in Computer Based Test (CBT) mode on 16 April 2018, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. If the function f defined as

$f(x) = \frac{1}{x} - \frac{k-1}{e^{2x}-1}, x \neq 0$, **is continuous at**

x = 0, then the ordered pair (k, f(0)) is equal to : [4 marks]

(A) (3, 2)

(B) (3, 1)

(C) (2, 1)

(D) $\left(\frac{1}{3}, 2\right)$

Correct Answer: (B) (3, 1)

Solution:

Concept:

- Continuity at zero requires a finite two-sided limit equal to $f(0)$. A nonzero coefficient of $1/x$ prevents such a limit.
- The standard exponential expansion is $e^{2x} = 1 + 2x + 2x^2 + O(x^3)$. It can separate the singular and constant terms.

Step 1: Cancel the singular term First multiply by x. If f has a finite limit, then $xf(x) \to 0$. But

$$xf(x) = 1 - (k-1)\frac{x}{e^{2x}-1} \longrightarrow 1 - \frac{k-1}{2}.$$

Therefore $k - 1 = 2$, giving $k = 3$. This condition is necessary: any other value leaves a nonzero multiple of $1/x$, which diverges with opposite signs from the two sides.

Step 2: Evaluate the remaining finite limit With $k = 3$, combine the fractions before taking the limit:

$$f(x) = \frac{e^{2x} - 1 - 2x}{x(e^{2x} - 1)}.$$

The numerator is $2x^2 + O(x^3)$, while the denominator is $2x^2 + O(x^3)$. Their ratio tends to 1 from either side. Hence assigning $f(0) = 1$ makes the function continuous. Option (A) has the necessary value of k but the wrong limiting value. Options (C) and (D) fail the cancellation condition itself.

Step 3: Conclusion The required ordered pair is $(k, f(0)) = (3, 1)$. Hence, the correct option is (B).

Quick Tip to Solve: Cancel the coefficient of the singular term first; only then evaluate the constant term that determines the continuous extension.

2. The number of values of k for which the system of linear equations,

(k + 2)x + 10y = k

kx + (k +3)y = k -1

has no solution, is : [4 marks]

(A) 1

(B) 2

(C) 3

(D) infinitely many

Correct Answer: (A) 1

Solution:

Concept:

- A two-variable system has a unique solution when its coefficient determinant is nonzero. Thus inconsistency can occur only at roots of that determinant.
- When the determinant vanishes, compare the actual equations: proportional left sides with unequal right sides give no solution.

Step 1: Find all candidate parameter values The coefficient determinant is

$$\Delta = (k+2)(k+3) - 10k = k^2 - 5k + 6 = (k-2)(k-3).$$

Thus every k other than 2 and 3 gives a unique solution and cannot be counted. The two roots are only candidates; a zero determinant alone does not distinguish inconsistency from infinitely many solutions.

Step 2: Test consistency at each candidate For $k = 2$, the equations become

$$4x + 10y = 2, \qquad 2x + 5y = 1.$$

The first is exactly twice the second, including its right side, so there are infinitely many solutions. For $k = 3$, the equations become

$$5x + 10y = 3, \qquad 3x + 6y = 2.$$

Dividing the first by 5 and the second by 3 requires $x + 2y = 3/5$ and $x + 2y = 2/3$ simultaneously, which is impossible. Hence only $k = 3$ is inconsistent. Counting both determinant roots would incorrectly include the dependent system at $k = 2$; three or infinitely many exceptional values are impossible because all candidates have already been exhausted.

Step 3: Conclusion Exactly one value, $k = 3$, gives no solution. Hence, the correct option is (A).

Quick Tip to Solve: A zero determinant is only a candidate for no solution: always check the constants to distinguish parallel equations from coincident ones.

3. If the length of the latus rectum of an ellipse is 4 units and the distance between a focus an its nearest vertex on the major axis is $\frac{3}{2}$ units, then its eccentricity is : [4 marks]

(A) $\frac{1}{2}$
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) $\frac{1}{9}$

Correct Answer: (B) $\frac{1}{3}$

Solution:

Concept:

- For an ellipse with semimajor axis a, semiminor axis b and eccentricity e, $b^2 = a^2(1-e^2)$, where $0 < e < 1$.
- The full latus rectum has length $2b^2/a$. The nearest major-axis vertex is at distance $a - ae = a(1-e)$ from the focus.

Step 1: Express both given lengths using the ellipse parameters The latus rectum condition gives

$$\frac{2b^2}{a} = 4 \quad \Longrightarrow \quad \frac{b^2}{a} = 2.$$

Using $b^2 = a^2(1-e^2)$, this becomes $a(1-e^2) = 2$. The nearest-vertex condition independently gives

$$a(1-e) = \frac{3}{2}.$$

The word nearest matters: the distance to the other vertex would be $a(1+e)$, which would describe a different condition.

Step 2: Eliminate the semimajor axis Divide the first relation by the second. Since $e < 1$ and $a > 0$, the divisor is nonzero:

$$\frac{a(1-e^2)}{a(1-e)} = \frac{2}{3/2}, \qquad 1+e = \frac{4}{3}.$$

Thus $e = 1/3$. As a check, $a = (3/2)/(2/3) = 9/4$ and $b^2 = 2a = 9/2$; these give both the prescribed latus rectum and focus-to-vertex distance. The other proposed eccentricities fail $1 + e = 4/3$; in particular $1/9$ is the square of the answer rather than the eccentricity itself.

Step 3: Conclusion The eccentricity is $e = 1/3$. Hence, the correct option is (B).

Quick Tip to Solve: Factor $1 - e^2 = (1-e)(1+e)$ and divide the latus-rectum relation by the nearest-vertex relation to eliminate a.

4. $\lim_{x\to 0} \frac{(27+x)^{\frac{1}{3}}-3}{9-(27+x)^{\frac{2}{3}}}$ **equals.** [4 marks]

(A) $\frac{1}{3}$

(B) $-\frac{1}{3}$

(C) $-\frac{1}{6}$

(D) $\frac{1}{6}$

Correct Answer: (C) $-\frac{1}{6}$

Solution:

Concept:

- An indeterminate $0/0$ form often becomes a removable factor after a suitable substitution. Here the denominator is a difference of squares.
- Put $t = (27+x)^{1/3}$. Then $t \to 3$ as $x \to 0$, and $(27+x)^{2/3} = t^2$ for real x near zero.

Step 1: Factor the denominator before substituting the limit In terms of t, the quotient is

$$\frac{t-3}{9-t^2} = \frac{t-3}{(3-t)(3+t)}.$$

Because $t-3 = -(3-t)$, its simplified form is

$$-\frac{1}{3+t} \qquad (t \neq 3).$$

This equality is valid in a punctured neighbourhood of $x = 0$, exactly the region used to evaluate the limit. Cancelling a common vanishing factor is legitimate there, even though the original quotient itself is undefined at zero.

Step 2: Take the limit and check the sign The simplified denominator tends to $3+3 = 6$, which is nonzero. Hence

$$\lim_{x\to 0} \frac{(27+x)^{1/3}-3}{9-(27+x)^{2/3}} = -\frac{1}{6}.$$

For a small positive x, the numerator is positive while the denominator is negative, confirming the negative sign. This rules out both positive options. The value $-1/3$ omits one of the two equal terms in $3+t$; the limiting denominator is six, not three. No differentiation or approximation of cube roots is necessary once the algebraic factor is exposed.

Step 3: Conclusion The limit equals $-1/6$. Hence, the correct option is (C).

Quick Tip to Solve: When a cube-root expression and its square occur together, rename the cube root and factor the resulting difference of squares.

5. Let $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{c} = \hat{j} - \hat{k}$ and a vector $\overrightarrow{b}$ be such that $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{c}$ and $\overrightarrow{a} . \overrightarrow{b} = 3$. Then $\left|\overrightarrow{b}\right|$ equals : [4 marks]

(A) $\frac{11}{3}$

(B) $\frac{11}{\sqrt{3}}$

(C) $\sqrt{\frac{11}{3}}$

(D) $\frac{\sqrt{11}}{3}$

Correct Answer: (C) $\sqrt{\frac{11}{3}}$

Solution:

Concept:

- For any vectors $\vec{a}, \vec{b}$, Lagrange's identity gives $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$.
- In an orthonormal basis, the squared magnitude is the sum of squares of the components. The identity combines the parallel and perpendicular information about $\vec{b}$.

Step 1: Compute the known squared magnitudes The given vectors are $\vec{a} = (1, 1, 1)$ and $\vec{c} = (0, 1, -1)$. Consequently

$$|\vec{a}|^2 = 1 + 1 + 1 = 3, \qquad |\vec{c}|^2 = 0 + 1 + 1 = 2.$$

Since $\vec{a} \times \vec{b} = \vec{c}$, the squared cross-product magnitude is 2. The dot product is 3, so its square is 9. Also $\vec{a} \cdot \vec{c} = 0$, as required for a cross product perpendicular to $\vec{a}$.

Step 2: Apply the identity and take the positive root Substituting into Lagrange's identity gives

$$3|\vec{b}|^2 = 2 + 9 = 11, \qquad |\vec{b}| = \sqrt{\frac{11}{3}}.$$

A magnitude is nonnegative, so the positive square root is required. As a component check, $\vec{b} = (5/3, 2/3, 2/3)$ has dot product 3 with $\vec{a}$, cross product $(0, 1, -1)$, and squared magnitude $33/9 = 11/3$. Option (A) reports the squared magnitude. Options (B) and (D) place the square root incorrectly and do not square to $11/3$.

Step 3: Conclusion The magnitude is $|\vec{b}| = \sqrt{11/3}$. Hence, the correct option is (C).

Quick Tip to Solve: When both the dot product and cross product are given, use their squared-magnitude identity instead of solving three component equations.

6. Let A, B and C be three events, which are pair-wise independent and $\overrightarrow{E}$ denotes the completement of an event E. If $P(A \cap B \cap C) = 0$ and $P(C) > 0$, then $P\left[\left(\overline{A} \cap \overline{B}\right) | C\right]$ is equal to : [4 marks]

(A) $P\left(\overline{A}\right) - P(B)$

(B) $P(A) + P\left(\overline{B}\right)$

(C) $P\left(\overline{A}\right) - P\left(\overline{B}\right)$

(D) $P\left(\overline{A}\right) + P\left(\overline{B}\right)$

Correct Answer: (A) $P\left(\overline{A}\right) - P(B)$

Solution:

Concept:

- Conditional probability is $P(D \mid C) = P(D \cap C)/P(C)$, valid here because $P(C) > 0$.
- Pairwise independence gives $P(A \cap C) = P(A)P(C)$ and $P(B \cap C) = P(B)P(C)$. It does not imply independence of all three events.

Step 1: Express the required event by a complement inside C The outcomes of C that lie in neither A nor B form $C \setminus [(A \cap C) \cup (B \cap C)]$. Inclusion-exclusion therefore gives

$$P(\overline{A} \cap \overline{B} \cap C) = P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

The last term is zero by the given condition; it must not be replaced by a product of three probabilities, because only pairwise independence is supplied.

Step 2: Divide by $P(C)$ and simplify Use the two pairwise-independence relations in the numerator:

$$P(\overline{A} \cap \overline{B} \cap C) = P(C)[1 - P(A) - P(B)].$$

Division by the positive $P(C)$ yields

$$P(\overline{A} \cap \overline{B} \mid C) = 1 - P(A) - P(B) = P(\overline{A}) - P(B).$$

This is precisely option (A). The other expressions simplify respectively to $1 + P(A) - P(B)$, $P(B) - P(A)$, and $2 - P(A) - P(B)$; none is the general identity just derived. Nonnegativity also implies $P(A) + P(B) \leq 1$ under the stated conditions, consistent with the final probability.

Step 3: Conclusion The probability is the complement probability of the first event minus the probability of the second event, as derived above. Hence, the correct option is (A).

Quick Tip to Solve: Apply inclusion-exclusion after intersecting every event with the conditioning event; pairwise independence then cancels the conditioning probability.

7. The mean and the standard deviation(s.d.) of five observations are9 and 0, respectively. If one of the observations is changed such that the mean of the new set of five observations becomes 10, then their s.d. is : [4 marks]

(A) 0

(B) 1

(C) 2

(D) 4

Correct Answer: (C) 2

Solution:

Concept:

- A zero standard deviation means every observation equals the mean, because a sum of nonnegative squared deviations can vanish only when each is zero.
- For n observations, the population variance is $\sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$. The standard deviation is its positive square root.

Step 1: Recover the original values and the changed observation The original mean is 9 and the standard deviation is zero, so the five original observations are all 9. Their sum is $5 \times 9 = 45$. The new mean 10 requires the new sum $5 \times 10 = 50$, an increase of five. As only one observation changes, that observation becomes $9 + 5 = 14$. The new data set is therefore $9, 9, 9, 9, 14$; the mean has indeed become ten.

Step 2: Compute the new variance about the new mean The deviations from 10 are $-1, -1, -1, -1, 4$. Squaring and averaging all five gives

$$\sigma^2 = \frac{4(1) + 16}{5} = \frac{20}{5} = 4.$$

Hence $\sigma = \sqrt{4} = 2$. Option (D) is the variance rather than the standard deviation. Zero is impossible because the new observations are unequal, and one ignores the contribution of the displaced observation. The denominator is five, as required for the standard deviation of the full set of observations; using the sample-estimation denominator four would solve a different statistical problem.

Step 3: Conclusion The new standard deviation is 2. Hence, the correct option is (C).

Quick Tip to Solve: When the original standard deviation is zero, first make all observations equal to the original mean, then use the change in total to find the altered value.

8. If an angle A of a $\triangle$ABC satiesfies 5 cosA + 3 = 0, then the roots of the quadratic equation, $9x^2$ + 27x + 20 = 0 are : [4 marks]

(A) secA, cotA

(B) sinA, secA

(C) secA, tanA

(D) tanA, cosA

Correct Answer: (C) secA, tanA

Solution:

Concept:

- For an angle of a triangle, $0 < A < \pi$, so $\sin A > 0$. The sign of the cosine determines the quadrant and hence the signs of the other trigonometric ratios.
- A factorisation of a quadratic gives both its roots. Alternatively, their sum and product are $-b/a$ and c/a for $ax^2 + bx + c = 0$.

Step 1: Determine the trigonometric ratios The given equation gives $\cos A = -3/5$. Since a triangle angle has positive sine, choose the positive square root, not the negative one:

$$\sin A = \sqrt{1 - 9/25} = 4/5.$$

Thus A is obtuse and

$$\sec A = -5/3, \qquad \tan A = \frac{4/5}{-3/5} = -4/3, \qquad \cot A = -3/4.$$

Both the secant and tangent are negative, as expected in the second quadrant.

Step 2: Factor the quadratic and compare the options Split the middle coefficient to obtain

$$9x^2 + 27x + 20 = (3x + 5)(3x + 4).$$

Consequently the two roots are $-5/3$ and $-4/3$. These agree with $\sec A$ and $\tan A$, respectively. Their sum is $-3 = -27/9$ and their product is $20/9$, independently checking the factorisation. Option (A) contains $\cot A = -3/4$, which is not a root. Option (B) contains the positive number $\sin A = 4/5$, while both roots are negative. Option (D) contains $\cos A = -3/5$, also not a root.

Step 3: Conclusion The roots are $\sec A = -5/3$ and $\tan A = -4/3$. Hence, the correct option is (C).

Quick Tip to Solve: Use the triangle-angle condition before taking a square root: a negative cosine here still gives a positive sine.

9. Two different families A and B are blessed with equal numbe of children. There are 3 tickets to be distributed amongst the children of these families so that no child gets more than one ticket. If the probability that all the tickets go to the children of the family B is $\frac{1}{12}$, then the number of children in each family is : [4 marks]

(A) 3

(B) 4

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Concept:

- If three distinct children are selected without replacement from $2n$ children, every three-child subset is equally likely. The total number of such subsets is $\binom{2n}{3}$.
- For all recipients to belong to family B, choose all three from its n children, giving $\binom{n}{3}$ favourable subsets. Here $n \geq 3$.

Step 1: Express the probability in terms of family size Let each family have n children. No child can receive two tickets, so selecting recipients is a selection without replacement. Whether the tickets are considered identical or distinct makes no difference to the probability: in the distinct case every recipient subset has the same 3! assignments. Therefore

$$P = \frac{\binom{n}{3}}{\binom{2n}{3}} = \frac{n(n-1)(n-2)}{(2n)(2n-1)(2n-2)}.$$

Using $2n-2 = 2(n-1)$, this simplifies to $P = (n-2)/[4(2n-1)]$.

Step 2: Solve the condition and check the choices Equate the expression to the stated probability:

$$\frac{n-2}{4(2n-1)} = \frac{1}{12} \quad \Longrightarrow \quad 3(n-2) = 2n-1 \quad \Longrightarrow \quad n = 5.$$

For this value, $\binom{5}{3}/\binom{10}{3} = 10/120 = 1/12$, confirming the result. For the other choices $n = 3, 4, 6$, the probabilities are respectively $1/20, 1/14, 1/11$, so none satisfies the given condition. Using $(1/2)^3$ would be inappropriate because the selections are dependent: choosing one child changes the remaining proportions.

Step 3: Conclusion Each family has 5 children. Hence, the correct option is (C).

Quick Tip to Solve: When no recipient may repeat, use combinations or successive fractions without replacement rather than independent-trial probabilities.

10. The number of numbers between 2,000 and 5,000 that can be formed with the digits 0, 1, 2, 3, 4 (repetition of digits is not allowed) and are multiple of 3 is : [4 marks]

(A) 24

(B) 30

(C) 36

(D) 48

Correct Answer: (B) 30

Solution:

Concept:

- A number is divisible by 3 exactly when its digit sum is divisible by 3. The digits may be rearranged without changing this divisibility test.
- A number between 2000 and 5000 has four digits and its thousands digit is 2, 3, or 4. The remaining three positions can be permuted in $3! = 6$ ways once the leading digit is fixed.

Step 1: Find all admissible sets of four digits The sum of all five available digits is $0+1+2+3+4 = 10$. Every permitted four-digit number omits exactly one digit, say d. Its digit sum is $10 - d$, which must be divisible by 3. Thus $d \equiv 1 \pmod 3$, so only $d = 1$ or $d = 4$ is possible. The admissible digit sets are therefore $\{0, 2, 3, 4\}$ and $\{0, 1, 2, 3\}$. Omitting 0, 2, or 3 gives sums $10, 8, 7$, respectively, and cannot produce a multiple of 3.

Step 2: Count arrangements that lie inside the interval For $\{0, 2, 3, 4\}$, the thousands digit can be $2, 3, 4$, giving $3 \cdot 3! = 18$ numbers. For $\{0, 1, 2, 3\}$, the thousands digit can only be 2 or 3, giving $2 \cdot 3! = 12$ numbers. The sets are disjoint since they omit different digits, so adding does not double count:

$$N = 18 + 12 = 30.$$

Every counted number is strictly above 2000 and below 5000; neither endpoint can occur without repeating digits or using the unavailable digit 5. The alternatives $24, 36, 48$ do not follow the required leading-digit restrictions.

Step 3: Conclusion There are 30 such numbers. Hence, the correct option is (B).

Quick Tip to Solve: For a four-digit selection from five digits, test the omitted digit first; then impose the thousands-digit restriction before arranging.

11. If $f(x) = \int_0^x t\,(\sin x - \sin t)\,dt$ **then :** [4 marks]

(A) f'''(x) + f''(x) = sinx

(B) f'''(x) + f''(x) − f'(x) = cosx

(C) f'''(x) + f'(x) = cosx − 2x sinx

(D) f'''(x) − f''(x) = cosx − 2x sinx

Correct Answer: (C) f'''(x) + f'(x) = cosx − 2x sinx

Solution:

Concept:

- In $\int_0^x t(\sin x - \sin t)\,dt$, the variable x appears both in the upper limit and in the integrand. Its dependence cannot be ignored when differentiating.
- The fundamental theorem of calculus gives $\frac{d}{dx}\int_0^x t\sin t\,dt = x\sin x$. Splitting the integral isolates this standard form.

Step 1: Separate the integral and find the first derivative Treat $\sin x$ as constant with respect to the integration variable t, and integrate the factor t:

$$f(x) = \frac{x^2}{2}\sin x - \int_0^x t\sin t\,dt.$$

Apply the product rule and the fundamental theorem to obtain

$$f\prime(x) = x\sin x + \frac{x^2}{2}\cos x - x\sin x = \frac{x^2}{2}\cos x.$$

The two $x\sin x$ terms cancel. This cancellation is the key simplification; differentiating only the upper limit would incorrectly give zero.

Step 2: Differentiate twice more and form the required identity Two further product-rule differentiations give

$$f\prime\prime(x) = x\cos x - \frac{x^2}{2}\sin x,$$

$$f\prime\prime\prime(x) = \cos x - 2x\sin x - \frac{x^2}{2}\cos x.$$

Adding the first derivative cancels the quadratic cosine term:

$$f\prime\prime\prime(x) + f\prime(x) = \cos x - 2x\sin x.$$

This is option (C). At $x = 0$, the first two derivatives vanish and the third derivative equals 1, so option (A), whose right side is zero, fails immediately. Options (B) and (D) retain uncancelled $f\prime\prime$ contributions and do not give the displayed identity for general x.

Step 3: Conclusion $f\prime\prime\prime(x) + f\prime(x) = \cos x - 2x\sin x$. Hence, the correct option is (C).

Quick Tip to Solve: Split a parameter-dependent integral first; here the upper-limit term cancels a product-rule term and leaves a very simple first derivative.

12. Let N denote the set of all natural numbers. Define two binary relations on N as $_R$ = {(x, y) ∈ N × N : 2x + y = 10} and R_2 = {(x, y) ∈ N × N : x + 2y = 10}. Then : [4 marks]

(A) Range of R_1 is {2, 4, 8).

(B) Range of R_2 is {1, 2, 3, 4}.

(C) Both R_1 and R_2 are symmetric relations.

(D) Both R_1 and R_2 are transitive relations.

Correct Answer: (B) Range of R_2 is {1, 2, 3, 4}.

Solution:

Concept:

- The range of a relation consists of all second coordinates of its ordered pairs. Natural numbers here are the positive integers $1, 2, 3, \ldots$.
- Symmetry requires (y, x) whenever (x, y) occurs. Transitivity requires (x, z) whenever both (x, y) and (y, z) occur; one counterexample disproves either property.

Step 1: List the pairs and read their ranges For R_1, write $y = 10 - 2x$. Positivity of y allows $x = 1, 2, 3, 4$, giving

$$R_1 = \{(1, 8), (2, 6), (3, 4), (4, 2)\}.$$

Its range is $\{2, 4, 6, 8\}$; option (A) omits 6. For R_2, write $x = 10 - 2y$. Taking $y = 1, 2, 3, 4$ gives

$$R_2 = \{(8, 1), (6, 2), (4, 3), (2, 4)\}.$$

Thus its second coordinates are exactly $1, 2, 3, 4$, establishing option (B). No larger second coordinate is possible because it would make $x \leq 0$.

Step 2: Test the remaining relation properties Neither relation is symmetric. For example, $(1, 8) \in R_1$ but $(8, 1) \notin R_1$; likewise $(8, 1) \in R_2$ but $(1, 8) \notin R_2$. Hence option (C) fails. Neither is transitive either: $(3, 4), (4, 2) \in R_1$, but $(3, 2) \notin R_1$. In R_2, the pairs $(6, 2), (2, 4)$ occur, but $(6, 4)$ does not. These explicit failed chains rule out option (D), without confusing a relation with its inverse.

Step 3: Conclusion The range of R_2 is $\{1, 2, 3, 4\}$. Hence, the correct option is (B).

Quick Tip to Solve: List the few allowed positive integer pairs before testing a finite relation; read the range from second coordinates and use counterexamples for properties.

13. Let $\frac{1}{x_1}, \frac{1}{x_2}, ..., \frac{1}{x_n}$ ($x_i \neq 0$ for i = 1, 2, ..., n) be in A.P. such that $x_1=4$ and $x_{21} = 20$. If n is the least positive integer for which $x_n > 50$, then $\sum_{i=1}^{n} \left(\frac{1}{x_i}\right)$ is equal to : [4 marks]

(A) $\frac{1}{8}$

(B) 3

(C) $\frac{13}{8}$

(D) $\frac{13}{4}$

Correct Answer: (D) $\frac{13}{4}$

Solution:

Concept:

- The reciprocals $a_i = 1/x_i$, rather than the numbers x_i, form an arithmetic progression. Its general term is $a_i = a_1 + (i-1)d$.
- The sum of the first n AP terms is $S_n = n(a_1 + a_n)/2$. When taking reciprocals in an inequality, the sign and possible zero values must be checked first.

Step 1: Find the reciprocal progression The first and twenty-first reciprocal terms are $a_1 = 1/4$ and $a_{21} = 1/20$. Therefore

$$20d = \frac{1}{20} - \frac{1}{4} = -\frac{1}{5}, \qquad d = -\frac{1}{100}.$$

The general reciprocal term is

$$a_i = \frac{1}{4} - \frac{i-1}{100} = \frac{26-i}{100}, \qquad x_i = \frac{100}{26-i}.$$

The formula is valid at every term used below, since $i \leq 25$ and the denominator is positive. In particular, no forbidden zero reciprocal is introduced into the required finite sequence.

Step 2: Locate the first qualifying term and sum the AP For positive denominators, $x_i > 50$ means

$$\frac{100}{26-i} > 50 \quad \Longleftrightarrow \quad 0 < 26 - i < 2.$$

The first positive integer index satisfying this is $i = 25$. Indeed, $x_{24} = 50$, which fails the strict inequality, while $x_{25} = 100$. For $i < 24$, the denominator is larger, so $x_i < 50$. Thus $n = 25$ and $a_{25} = 1/100$. Sum the reciprocal terms, not the x_i:

$$\sum_{i=1}^{25} \frac{1}{x_i} = \frac{25}{2}\left(\frac{1}{4} + \frac{1}{100}\right) = \frac{25}{2} \cdot \frac{26}{100} = \frac{13}{4}.$$

The other proposed sums $1/8, 3, 13/8$ differ from this direct AP sum.

Step 3: Conclusion The least index is $n = 25$, and the required sum is $13/4$. Hence, the correct option is (D).

Quick Tip to Solve: Introduce a new AP variable for the reciprocal, and check equality separately when the threshold is strict: here the preceding term is exactly 50.

14. Let M and m be respectively the absolute maximum and the absolute minimum values of the function, $f(x) = 2x^3 - 9x^2 + 12x + 5$ in the interval [0, 3]. Then M $-$m is equal to : [4 marks]

(A) 5

(B) 9

(C) 4

(D) 1

Correct Answer: (B) 9

Solution:

Concept:

- A continuous function on a closed interval attains its absolute maximum and minimum. For a differentiable polynomial, compare all interior stationary points with both endpoints.
- Stationary points satisfy $f'(x) = 0$. A local maximum or minimum alone need not be the absolute extremum on the whole specified interval.

Step 1: Find the interior candidates Differentiate the given cubic:

$$f'(x) = 6x^2 - 18x + 12 = 6(x-1)(x-2).$$

The stationary points are $x = 1$ and $x = 2$, both lying inside $[0, 3]$. The complete candidate set is therefore $\{0, 1, 2, 3\}$, including the endpoints. The derivative is positive before 1, negative between 1 and 2, and positive after 2. Thus 1 is a local maximum and 2 a local minimum, but their values must still be compared with those at 0 and 3.

Step 2: Evaluate and compare all candidates Substitution gives

$$\begin{aligned} f(0) &= 5, \\ f(1) &= 2 - 9 + 12 + 5 = 10, \\ f(2) &= 16 - 36 + 24 + 5 = 9, \\ f(3) &= 54 - 81 + 36 + 5 = 14. \end{aligned}$$

The largest of these values is 14, attained at $x = 3$, while the smallest is 5, attained at $x = 0$. Hence

$$M - m = 14 - 5 = 9.$$

Option (D), namely 1, is only the difference between the two stationary values 10 and 9; it incorrectly ignores the endpoints. Options (A) and (C) likewise are not the difference between the absolute extrema. Including both endpoints is essential because both absolute extrema occur there in this example.

Step 3: Conclusion $M = 14$, $m = 5$, and $M - m = 9$. Hence, the correct option is (B).

Quick Tip to Solve: For absolute extrema on a closed interval, make a candidate list containing the endpoints before evaluating the stationary points.

15. The locus of the point of intersection of the lines, $\sqrt{2}x - y + 4\sqrt{2}k = 0$ and $\sqrt{2}k\,x + k\,y - 4\sqrt{2} = 0$ (k is any non-zero real parameter), is : [4 marks]

(A) an ellipse whose eccentricity is $\frac{1}{\sqrt{3}}$.

(B) an ellipse with length of its major axis $8\sqrt{2}$.

(C) a hyperbola whose eccentricity is $\sqrt{3}$.

(D) a hyperbola with length of its transverse axis $8\sqrt{2}$.

Correct Answer: (D) a hyperbola with length of its transverse axis $8\sqrt{2}$.

Solution:

Concept:

- Eliminate the real parameter between the two line equations to obtain a Cartesian equation for their intersection.
- A hyperbola of the form $y^2/a^2 - x^2/b^2 = 1$ has transverse-axis length $2a$ and eccentricity $e = \sqrt{1 + b^2/a^2}$.

Step 1: Eliminate the parameter The first line gives $4\sqrt{2}k = y - \sqrt{2}x$. The second gives $k(\sqrt{2}x + y) = 4\sqrt{2}$. Substitute without changing either sign:

$$\frac{(y - \sqrt{2}x)(y + \sqrt{2}x)}{4\sqrt{2}} = 4\sqrt{2}.$$

Thus

$$y^2 - 2x^2 = 32, \qquad \frac{y^2}{32} - \frac{x^2}{16} = 1.$$

Both factors in the product are nonzero because their product is 32. Consequently the eliminated relation does not introduce a point requiring the forbidden parameter value zero.

Step 2: Identify the conic and test its properties The squared terms have opposite signs, so this is a hyperbola, with its transverse axis along the y-axis. Comparing with the standard equation gives $a^2 = 32$, $b^2 = 16$. Hence

$$2a = 2\sqrt{32} = 8\sqrt{2}, \qquad e = \sqrt{1 + 16/32} = \sqrt{3/2}.$$

Options (A) and (B) incorrectly describe an ellipse. Option (C) names the correct conic but gives the wrong eccentricity: $\sqrt{3}$ would require $b^2/a^2 = 2$, whereas here that ratio is $1/2$.

Step 3: Conclusion The locus is a hyperbola with transverse-axis length $8\sqrt{2}$. Hence, the correct option is (D).

Quick Tip to Solve: After eliminating a parameter, divide by the constant before reading off the semi-axes; the positive squared term determines the transverse axis.

16. If $\int \frac{\tan x}{1+\tan x+\tan^2 x}dx = x - \frac{K}{\sqrt{A}}\tan^{-1}\left(\frac{K\tan x+1}{\sqrt{A}}\right)+C$, ($C$ is a constant of integration) then the ordered pair (K, A) is equal to : [4 marks]

(A) (2, 1)

(B) (−2, 3)

(C) (2, 3)

(D) (−2, 1)

Correct Answer: (C) (2, 3)

Solution:

Concept:

- Use $1+\tan^2 x = \sec^2 x$ to split the integrand into a constant minus a term suitable for $t = \tan x$.
- Complete the square and apply $\int du/(u^2+a^2) = a^{-1}\tan^{-1}(u/a)+C$, with $a > 0$.

Step 1: Separate the easily integrated term Write $D = 1+\tan x+\tan^2 x$. Since $D - \sec^2 x = \tan x$, the required integral becomes

$$I = \int \left(1 - \frac{\sec^2 x}{1+\tan x+\tan^2 x}\right) dx.$$

Set $t = \tan x$, so $dt = \sec^2 x\, dx$. On any interval where tangent is defined,

$$I = x - \int \frac{dt}{t^2+t+1}.$$

This split keeps the minus sign outside the remaining integral; replacing the numerator by just dt in the original integral would be incorrect.

Step 2: Complete the square and compare coefficients The denominator is $t^2+t+1 = (t+1/2)^2+3/4$. Therefore

$$\int \frac{dt}{t^2+t+1} = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right).$$

Substituting back yields

$$I = x - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x+1}{\sqrt{3}}\right)+C.$$

Both the outside coefficient and the numerator inside the inverse tangent agree with $K = 2$, while the completed-square constant requires $A = 3$. The pairs with $A = 1$ give the wrong denominator after differentiation, and a negative K reverses the linear tangent term inside the inverse tangent.

Step 3: Conclusion The ordered pair is $(K, A) = (2, 3)$. Hence, the correct option is (C).

Quick Tip to Solve: When a tangent integral has a quadratic denominator, try writing its numerator as the denominator minus $\sec^2 x$ before substituting.

17. If a circle C, whose radius is 3, touches externally the circle, $x^2 + y^2 + 2x - 4y - 4 = 0$ at the point (2, 2), then the length of the intercept cut by this circle C, on the x-axis is equal to : [4 marks]

(A) $2\sqrt{5}$

(B) $3\sqrt{2}$

(C) $\sqrt{5}$

(D) $2\sqrt{3}$

Correct Answer: (A) $2\sqrt{5}$

Solution:

Concept:

- Complete squares to find the centre and radius of a circle in general form.
- For external tangency, the centres and contact point are collinear, and the contact point lies between the centres.
- A chord at perpendicular distance d from the centre of a circle of radius R has length $2\sqrt{R^2 - d^2}$.

Step 1: Locate the centre of the required circle The given circle can be written as

$$(x + 1)^2 + (y - 2)^2 = 9.$$

Its centre is $O = (-1, 2)$ and radius is 3. The contact point is $T = (2, 2)$, exactly three units to the right of O. The new circle also has radius 3. Because the circles touch externally, its centre must lie three units beyond T along the same line, giving

$$O_C = (5, 2).$$

Choosing the centre on the other side of T would reproduce the given circle rather than produce external tangency.

Step 2: Find the x-axis intercept The required circle is

$$(x - 5)^2 + (y - 2)^2 = 9.$$

Its centre lies at perpendicular distance $d = 2$ from the x-axis. Setting $y = 0$ gives

$$(x - 5)^2 = 9 - 4 = 5, \qquad x = 5 \pm \sqrt{5}.$$

The distance between these two intercept points is

$$(5 + \sqrt{5}) - (5 - \sqrt{5}) = 2\sqrt{5}.$$

Option (C) gives only the half-chord. Options (B) and (D) would require different centre-to-axis distances and do not satisfy the circle equation.

Step 3: Conclusion The length of the intercept on the x-axis is $2\sqrt{5}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the new centre using the contact point first; then use the perpendicular distance to the axis to calculate the full chord.

18. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is : [4 marks]

(A) 210

(B) 211

(C) 231

(D) 251

Correct Answer: (C) 231

Solution:

Concept:

- If $A = I + N$ and $N^3 = 0$, the binomial expansion terminates: $A^m = I + mN + \binom{m}{2}N^2$.
- The identity matrix commutes with every matrix, so the usual binomial coefficients apply to $(I + N)^m$.

Step 1: Separate the identity and calculate the nilpotent powers Write

$$N = A - I = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}.$$

Direct multiplication gives

$$N^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad N^3 = 0.$$

Thus no powers beyond the square can contribute. This avoids twenty successive matrix multiplications while retaining the contribution from the lower-left entry of N^2.

Step 2: Expand the twentieth power and sum the first column The finite binomial expansion is

$$B = (I + N)^{20} = I + 20N + 190N^2.$$

Its first column is consequently

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 20 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + 190 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 20 \\ 210 \end{pmatrix}.$$

The required sum is

$$1 + 20 + 210 = 231.$$

The number 210 in option (A) is just the last entry of the column, not its sum. Option (B) omits the middle entry 20; option (D) adds an extra 20 beyond the actual column total. For a general power m, the same column is $(1, m, m(m + 1)/2)^T$, confirming the twentieth-power calculation.

Step 3: Conclusion The sum of the entries in the first column is 231. Hence, the correct option is (C).

Quick Tip to Solve: For a triangular matrix with ones on its diagonal, subtract the identity and check whether the remaining matrix has a low power equal to zero.

19. The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1$, is : [4 marks]

(A) 2

(B) 3

(C) 5

(D) 6

Correct Answer: (B) 3

Solution:

Concept:

- Write a nonzero complex number as $r(\cos\theta + i\sin\theta)$; arguments subtract in a quotient.
- By De Moivre's theorem, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$. This equals 1 exactly when $n\theta$ is an integral multiple of 2π.

Step 1: Express the quotient in polar form The numerator and denominator both have modulus $\sqrt{1+3} = 2$. Their arguments are respectively $\pi/3$ and $-\pi/3$, because their real parts are positive and their imaginary parts have opposite signs. Hence

$$z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}} = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}.$$

The argument is $2\pi/3$, not $\pi/3$: the quotient subtracts the negative argument of its denominator.

Step 2: Find the first power returning to unity For a positive integer n, the condition $z^n = 1$ requires

$$\frac{2\pi n}{3} = 2\pi m \quad (m \in \mathbb{Z}), \qquad n = 3m.$$

The least positive choice is $m = 1$, giving $n = 3$. Powers 1 and 2 have arguments $2\pi/3$ and $4\pi/3$, so neither equals unity. Thus option (A) fails; option (C), with exponent 5, leaves the same argument as exponent 2 modulo 2π. Exponent 6 in option (D) does give unity, but it is twice the least positive exponent and is therefore excluded by the word least.

Step 3: Conclusion The least positive integer is $n = 3$. Hence, the correct option is (B).

Quick Tip to Solve: For a unit complex number with argument $2\pi a/b$ in lowest terms, the least positive exponent giving unity is b.

20. Let p, q and r be real numbers ($p \neq q$, $r \neq 0$), such that the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to : [4 marks]

(A) $\frac{p^2+q^2}{2}$

(B) $p^2 + q^2$

(C) $2(p^2 + q^2)$

(D) $p^2 + q^2 + r^2$

Correct Answer: (B) $p^2 + q^2$

Solution:

Concept:

- Clear denominators only on the domain $x \neq -p, -q$, then apply the sum and product formulas for a monic quadratic.
- Roots equal in magnitude and opposite in sign have sum zero. For roots α, β, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

Step 1: Obtain the quadratic and impose the zero root sum The rational equation is

$$\frac{2x + p + q}{(x + p)(x + q)} = \frac{1}{r}.$$

Because $r \neq 0$, multiplication gives

$$x^2 + (p + q - 2r)x + pq - r(p + q) = 0.$$

Let its roots be α and $-\alpha$. Their sum vanishes, so the coefficient of x must be zero:

$$p + q - 2r = 0, \qquad r = \frac{p + q}{2}.$$

This relation also shows that $p + q \neq 0$, since the question excludes $r = 0$.

Step 2: Use the product to evaluate the square sum The product is

$$\alpha\beta = pq - \frac{(p + q)^2}{2} = -\frac{p^2 + q^2}{2}.$$

Therefore

$$\alpha^2 + \beta^2 = 0^2 - 2\left(-\frac{p^2 + q^2}{2}\right) = p^2 + q^2.$$

No extraneous root has been counted: at $x = -p$ the cross-multiplied equation would require $r(q - p) = 0$, impossible under the stated assumptions, and similarly for $x = -q$. Option (A) gives the square of just one of the opposite roots; option (C) doubles the required sum, while option (D) introduces an additional r^2 not present in the root identity.

Step 3: Conclusion The sum of the squares of the roots is $p^2 + q^2$. Hence, the correct option is (B).

Quick Tip to Solve: Opposite roots make the linear coefficient vanish. Use that relation before simplifying the constant term of the quadratic.

21. If the angle between the lines, $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{5-x}{-2}=\frac{7y-14}{p}=\frac{z-3}{4}$ is $\cos^{-1}\left(\frac{2}{3}\right)$, then p is equal to : [4 marks]

(A) $\frac{7}{2}$

(B) $\frac{2}{7}$

(C) $-\frac{7}{4}$

(D) $-\frac{4}{7}$

Correct Answer: (A) $\frac{7}{2}$

Solution:

Concept:

- The direction ratios of a line are the coefficients of its common parameter after each coordinate is solved explicitly.
- The acute angle between directions $\mathbf{u}$ and $\mathbf{v}$ satisfies $\cos\theta = |\mathbf{u}\cdot\mathbf{v}|/(|\mathbf{u}||\mathbf{v}|)$.

Step 1: Read the direction ratios carefully The first line has direction vector $\mathbf{u}=(2,2,1)$, of length 3. Set the common ratio of the second line equal to t. Then

$$x = 5+2t, \qquad y = 2+\frac{p}{7}t, \qquad z = 3+4t.$$

Its direction vector is thus $\mathbf{v}=(2,p/7,4)$. In particular the x component is positive 2 because both $5-x$ and its denominator carry a minus sign. The y component is $p/7$, not p, because the numerator is $7y-14$.

Step 2: Apply the given cosine and solve Put $s=p/7$. The given angle has cosine $2/3$, so

$$\frac{|8+2s|}{3\sqrt{20+s^2}}=\frac{2}{3}.$$

Squaring gives

$$(8+2s)^2=4(20+s^2), \qquad (4+s)^2=20+s^2.$$

After cancellation, $8s=4$, hence $s=1/2$ and

$$p=7s=\frac{7}{2}.$$

The unsquared expression checks this result: the dot product is 9, the second length is $9/2$, and $9/(3\cdot 9/2)=2/3$. Each of options (B), (C) and (D) yields a different s and fails the necessary equation $8s=4$, so none can give the stated angle.

Step 3: Conclusion The parameter is $p=7/2$. Hence, the correct option is (A).

Quick Tip to Solve: Introduce a common parameter before reading direction ratios when a symmetric line contains scaled coordinates or a reversed numerator.

22. If the area of the region bounded by the curves, $y = x^2, y = \frac{1}{x}$ and the lines y = 0 and x= t (t >1) is 1 sq. unit, then t is equal to : [4 marks]

(A) $e^{\frac{3}{2}}$

(B) $\frac{4}{3}$

(C) $\frac{3}{2}$

(D) $e^{\frac{2}{3}}$

Correct Answer: (D) $e^{\frac{2}{3}}$

Solution:

Concept:

- The area between a nonnegative curve and the x-axis is its definite integral. Where the bounding curve changes, split the integral at the intersection.
- The curves intersect when $x^2 = 1/x$, so their positive intersection is $(1, 1)$. For $t > 1$, the bounded region has upper boundary x^2 on $[0, 1]$ and $1/x$ on $[1, t]$.

Step 1: Identify the enclosed region The parabola meets the x-axis at the origin, while the hyperbola has no finite x-intercept. Thus the finite closed boundary travels from the origin along the parabola to $(1, 1)$, then along the hyperbola to $(t, 1/t)$, vertically down to $(t, 0)$, and back along the x-axis. Integrating the difference between the two curves over the entire interval would describe a different region.

Step 2: Compute the two contributions Using vertical strips gives the area in square units as

$$A = \int_0^1 x^2\,dx + \int_1^t \frac{1}{x}\,dx = \frac{1}{3} + \ln t.$$

The logarithm is valid because both limits are positive. The stipulated area is one, hence

$$\frac{1}{3} + \ln t = 1, \qquad \ln t = \frac{2}{3}, \qquad t = e^{2/3}.$$

This is greater than one as required. The area is strictly increasing for $t > 1$, since $dA/dt = 1/t > 0$, so this solution is unique.

Step 3: Check the alternatives Option (A) gives $A = 1/3 + 3/2 = 11/6$, not one. The values $4/3$ and $3/2$ give $1/3 + \ln(4/3)$ and $1/3 + \ln(3/2)$, both less than one. Only the logarithm $2/3$ supplies the missing area.

Step 4: Conclusion Therefore $t = \boxed{e^{2/3}}$. Hence, the correct option is (D).

Quick Tip to Solve: Trace the entire closed boundary first; the point where the upper boundary switches is the split point of the area integral.

23. If $x = \sqrt{2^{\cos ec^{-1}}}$ and $y = \sqrt{2^{sec^{-1}t}}$ $(|t| \geq 1)$, then $\frac{dy}{dx}$ is equal to : [4 marks]

(A) $\frac{y}{x}$

(B) $\frac{x}{y}$

(C) $-\frac{y}{x}$

(D) $-\frac{x}{y}$

Correct Answer: The first inverse cosecant has no argument, so x(t) and hence dy/dx are not defined uniquely as printed.

Solution:

Concept:

- Parametric differentiation uses $dy/dx = (dy/dt)/(dx/dt)$ on intervals where both functions are defined and $dx/dt \neq 0$.
- An inverse trigonometric function needs an argument: $\csc^{-1} t$ is a value, whereas $\csc^{-1}$ alone is an operator without a specified input.

Step 1: Inspect the first parameter expression The printed expression for y contains $\sec^{-1} t$, but the exponent in the expression for x is printed as an inverse cosecant without its argument. Therefore the first displayed formula does not specify x as a function of t. The domain condition $|t| \geq 1$ supplies a domain for the parameter; it does not supply the missing input to the inverse cosecant.

Step 2: Test whether a derivative follows To take the ratio of parameter derivatives we require dx/dt. There is no defined expression to differentiate in the first formula, so that derivative cannot be obtained. Treating the omitted expression as an arbitrary constant does not help: then $dx/dt = 0$, and it still does not yield any of the four proposed finite ratios. Thus neither a positive nor a negative ratio of x and y has been established by the supplied data.

Step 3: Explain the ambiguity without changing the question For example, an additional specification $\csc^{-1} t$ would permit use of $\csc^{-1} t + \sec^{-1} t = \pi/2$ and lead to a negative logarithmic derivative ratio. A different input to the inverse cosecant would generally yield a different derivative. This illustrates why the missing input matters; it is not a licence to insert that input into the printed question. Consequently options (A), (B), (C) and (D) cannot be selected uniquely from the literal statement. A complete parameter expression is necessary before any one of those choices can be proved.

Step 4: Conclusion As printed, the first inverse cosecant has no argument, so $x(t)$ is not defined and dy/dx has no uniquely determined value. No unique option follows; the missing argument must be supplied to determine an answer.

Quick Tip to Solve: Before using a parametric derivative ratio, check that every function has an argument and both parameter formulas are fully defined.

24. The differential equation representing the family of ellipse having foci eith on the x-axis or on the y-axis, center at the origin and passing through the point (0, 3) is : [4 marks]

(A) xy y" + x $(y')^2$ − y y' = 0

(B) x + y y" = 0

(C) xy y'+ y^2 − 9 = 0

(D) xy y' − y^2 + 9 = 0

Correct Answer: (D) xy y' − y^2 + 9 = 0

Solution:

Concept:

- An ellipse centered at the origin with foci on a coordinate axis has equation $x^2/a^2 + y^2/b^2 = 1$, with positive semiaxes.
- Passing through $(0, 3)$ fixes $b^2 = 9$. The family therefore has just one free parameter, a, which can be eliminated using one differentiation.

Step 1: Apply the given point Substituting the point into the standard equation gives $9/b^2 = 1$. Hence every ellipse in the specified family satisfies

$$\frac{x^2}{a^2} + \frac{y^2}{9} = 1.$$

If $a > 3$, its foci are on the x-axis; if $a < 3$, they are on the y-axis. Thus this single expression covers both orientations without introducing a second free semiaxis.

Step 2: Differentiate and eliminate the parameter Differentiate along a locally differentiable arc, treating a as constant:

$$\frac{2x}{a^2} + \frac{2yy\prime}{9} = 0, \qquad \frac{x}{a^2} = -\frac{yy\prime}{9}.$$

Multiply this relation by x. The original ellipse gives $x^2/a^2 = 1 - y^2/9$, so

$$1 - \frac{y^2}{9} = -\frac{xyy\prime}{9}, \qquad xyy\prime - y^2 + 9 = 0.$$

No division by x was required, and the relation is consistent at the given point.

Step 3: Compare the proposed equations Option (C) reverses the sign needed in parameter elimination. For example at $y^2 = 9/2$, the family has $xyy\prime = -9/2$, which satisfies (D) but not (C). Option (B) imposes a different second-derivative constraint. Option (A) is a second-order consequence for the broader two-parameter centered conic family; it does not encode the requirement of passing through $(0, 3)$. The required one-parameter family is represented by (D).

Step 4: Conclusion The required differential equation is $\boxed{xyy\prime - y^2 + 9 = 0}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the fixed point before differentiating: it removes one semiaxis parameter and immediately tells you the required order.

25. If p $\rightarrow$ ($\sim$ p$\vee$ $\sim$ q) is false, then the truth values of p and q are respectively : [4 marks]

(A) F, F

(B) T, F

(C) F, T

(D) T, T

Correct Answer: (D) T, T

Solution:

Concept:

- An implication $p \rightarrow r$ is false only when its antecedent p is true and its consequent r is false.
- A disjunction $u \vee v$ is false only when both u and v are false. Negation reverses the truth value.

Step 1: Apply the false-implication condition Here the consequent is the compound statement $r = (\sim p \vee \sim q)$. The question says that $p \rightarrow r$ is false. Consequently p must be true, and $\sim p \vee \sim q$ must be false. This first observation already rules out both options (A) and (C), whose antecedent is false: an implication with a false antecedent is always true.

Step 2: Resolve the disjunction For $\sim p \vee \sim q$ to be false, each entry of the disjunction must be false. Therefore $\sim p$ is false and $\sim q$ is false, giving $p = T$ and $q = T$. Check directly:

$$T \rightarrow (F \vee F) = T \rightarrow F = F.$$

This reproduces exactly the truth value required in the question.

Step 3: Reject the remaining pair For option (B), $p = T$ and $q = F$. Then $\sim q = T$, so the consequent is $F \vee T = T$, and the implication becomes $T \rightarrow T = T$, not false. We have therefore checked all four pairs. Equivalently, the negation of the whole implication is $p \wedge \neg(\neg p \vee \neg q) = p \wedge p \wedge q = p \wedge q$; making the implication false forces both original statements to be true.

Step 4: Conclusion The truth values of p and q, respectively, are $\boxed{T, T}$. Hence, the correct option is (D).

Quick Tip to Solve: For a false implication, write "true antecedent, false consequent" before expanding any compound consequent.

26. The sum of the intercepts on the coordinate axes of the plane passing through the point $(-2, -2, 2)$ and containing the line joining the points $(1, -1, 2)$ and $(1, 1, 1)$ is : [4 marks]

(A) 4

(B) -4

(C) -8

(D) 12

Correct Answer: (B) -4

Solution:

Concept:

- Two nonparallel direction vectors in a plane have a cross product normal to that plane. A normal (l, m, n) gives the equation $lx + my + nz = d$.
- The signed intercepts are obtained by setting the other two coordinates to zero. Their algebraic sum need not equal the sum of their absolute lengths.

Step 1: Find independent directions Let $A = (1, -1, 2)$, $B = (1, 1, 1)$, and $C = (-2, -2, 2)$. Since the plane contains the whole line through A, B and the point C, it contains the directions

$$\overrightarrow{AB} = (0, 2, -1), \qquad \overrightarrow{AC} = (-3, -1, 0).$$

Their cross product is

$$\overrightarrow{AB} \times \overrightarrow{AC} = (-1, 3, 6).$$

It is nonzero, so these data determine a unique plane.

Step 2: Write and check the plane equation Using the normal and point A gives

$$-(x - 1) + 3(y + 1) + 6(z - 2) = 0,$$

or

$$-x + 3y + 6z = 8.$$

Substitution gives eight at all three defining points: at A, $-1 - 3 + 12 = 8$; at B, $-1 + 3 + 6 = 8$; at C, $2 - 6 + 12 = 8$. This checks both orientation and constant.

Step 3: Add the signed intercepts The x-intercept is -8, the y-intercept is $8/3$, and the z-intercept is $8/6 = 4/3$. Thus

$$a + b + c = -8 + \frac{8}{3} + \frac{4}{3} = -8 + 4 = -4.$$

The value four in (A) keeps only the positive intercepts; negative eight in (C) keeps only the x-intercept. Twelve in (D) is the sum of absolute intercept lengths, whereas the requested intercept sum is algebraic.

Step 4: Conclusion The sum of the intercepts is $\boxed{-4}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep the signs of axis intercepts. A plane may cut a negative half-axis even when the other intercepts are positive.

27. The sum of the first 20 terms of the series

$1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + \ldots$, **is :** [4 marks]

(A) $38 + \frac{1}{2^{19}}$

(B) $38 + \frac{1}{2^{20}}$

(C) $39 + \frac{1}{2^{20}}$

(D) $39 + \frac{1}{2^{19}}$

Correct Answer: (A) $38 + \frac{1}{2^{19}}$

Solution:

Concept:

- The nth numerator is $2^n - 1$, while the nth denominator is 2^{n-1}. Thus the nth term is $a_n = 2 - 2^{-(n-1)}$.
- The sum of a finite geometric progression with first term one and ratio $1/2$ is $(1 - 2^{-N})/(1 - 1/2)$.

Step 1: Express the terms as a constant minus a geometric term The first terms can be rewritten as

$$1 = 2 - 1, \quad \frac{3}{2} = 2 - \frac{1}{2}, \quad \frac{7}{4} = 2 - \frac{1}{4}, \quad \frac{15}{8} = 2 - \frac{1}{8}.$$

This identifies the pattern without treating the original series as a geometric progression: its consecutive ratios are not constant. Each of the twenty terms contributes a constant two, followed by a correction whose exponents run from zero to nineteen.

Step 2: Sum the correction accurately Therefore

$$S_{20} = 40 - \left(1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{19}}\right).$$

The bracket has twenty terms, so its sum is

$$\frac{1 - (1/2)^{20}}{1 - 1/2} = 2 - \frac{1}{2^{19}}.$$

Subtracting this bracket gives

$$S_{20} = 40 - 2 + \frac{1}{2^{19}} = 38 + \frac{1}{2^{19}}.$$

Notice that multiplying the last geometric remainder by two changes its exponent from twenty to nineteen.

Step 3: Check the size and alternatives The infinite correction sum is two, while the finite correction is just smaller than two. Hence the answer must be just larger than thirty-eight, excluding (C) and (D), which are near thirty-nine. Option (B) uses half the correct geometric remainder, from forgetting the division by $1 - 1/2$. A small check with two terms gives $4 - (1 + 1/2) = 5/2$, agreeing with the general pattern.

Step 4: Conclusion The sum is $\boxed{38 + 2^{-19}}$. Hence, the correct option is (A).

Quick Tip to Solve: When numerators are one less than powers of two, divide each term first; the series usually becomes a constant sum minus a GP.

28. A man on the top of a vertical tower observes a car moving at a uniform speed towards the tower on a horizontal road. If it takes 18 min. for the angle of depression of the car to change from 30° to 45° ; then after this, the time taken (in min.) by the car to reach the foot of the tower, is : [4 marks]

(A) $9\left(1+\sqrt{3}\right)$
(B) $18\left(1+\sqrt{3}\right)$
(C) $18\left(\sqrt{3}-1\right)$
(D) $\frac{9}{2}\left(\sqrt{3}-1\right)$

Correct Answer: (A) $9\left(1+\sqrt{3}\right)$

Solution:

Concept:

- The angle of depression from the tower equals the angle of elevation from the car because the horizontal lines are parallel.
- For tower height h and horizontal distance d, $\tan\theta = h/d$. Uniform speed makes time proportional to distance travelled.

Step 1: Translate both angles into horizontal distances Let the tower height be $h > 0$. At the first observation the angle is $30°$, so

$$d_1 = \frac{h}{\tan 30°} = \sqrt{3}\,h.$$

At the second observation the angle is $45°$, so

$$d_2 = \frac{h}{\tan 45°} = h.$$

The car approaches the tower, consistent with the angle increasing and the distance decreasing. We use horizontal road distances rather than slant distances from the observer.

Step 2: Find the speed and remaining time The car travels $d_1 - d_2 = (\sqrt{3}-1)h$ in eighteen minutes. Its constant speed in distance units per minute is

$$v = \frac{(\sqrt{3}-1)h}{18}.$$

After the second observation it still has the horizontal distance h to travel. Therefore

$$T = \frac{h}{v} = \frac{18}{\sqrt{3}-1} = \frac{18(\sqrt{3}+1)}{3-1} = 9(\sqrt{3}+1)\ \text{min}.$$

The unknown tower height cancels, as it should.

Step 3: Check the alternatives Numerically the remaining time is about 24.59 minutes. It exceeds eighteen minutes because the remaining distance h exceeds the previously covered distance $(\sqrt{3}-1)h$. Options (C) and (D) give less than eighteen minutes and contradict this distance comparison. Option (B) is twice the computed time, arising from omitting the factor two in rationalising the denominator.

Step 4: Conclusion The time after the angle becomes $45°$ is $\boxed{9(1+\sqrt{3})\ \text{min}}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the two ground distances as multiples of the tower height, then take their time ratio; no numerical height is needed.

29. The coefficient of x^2 in the expansion of the product $(2-x^2)\cdot((1+2x+3x^2)^6+(1-4x^2)^6)$ is : [4 marks]

(A) 107

(B) 106

(C) 108

(D) 155

Correct Answer: (B) 106

Solution:

Concept:

- To obtain a specified power in a product, combine only terms whose degrees add to that power. Higher-degree terms cannot contribute to the coefficient of x^2.
- In $(1+2x+3x^2)^6$, degree two can arise by choosing one quadratic term or two linear terms; the remaining factors contribute their constant terms.

Step 1: Find the quadratic term in the first sixth power There are six choices for the factor supplying $3x^2$, giving $6(3)=18$. Alternatively, choose two of the six factors to supply $2x$, giving $\binom{6}{2}(2)^2=60$. Hence

$$[x^2](1+2x+3x^2)^6=18+60=78.$$

These are the only possibilities because choosing any additional nonconstant factor would raise the degree above two.

Step 2: Combine both powers and multiply by the outer factor The second power has quadratic coefficient $\binom{6}{1}(-4)=-24$. Thus the expression inside the large parentheses has quadratic coefficient $78-24=54$ and constant coefficient $1+1=2$. Multiplication by $2-x^2$ therefore gives

$$[x^2]\{(2-x^2)[(1+2x+3x^2)^6+(1-4x^2)^6]\}=2(54)-2=106.$$

The subtraction of the constant term is essential. The value 108 omits it, while 107 and 155 do not follow the complete coefficient calculation.

Step 3: Conclusion The required coefficient is 106. Hence, the correct option is (B).

Quick Tip to Solve: When multiplying by $a-bx^2$, use $a[x^2]f-b[x^0]f$; there is no need to expand the full polynomial.

30. Let P be a point on the parabola, $x^2 = 4y$. If the distance of P from the center of the circle, $x^2 + y^2 + 6x + 8 = 0$ is minimum, then the equation of the tangent to the parabola at P, is : [4 marks]

(A) $x + 4y - 2 = 0$

(B) $x - y + 3 = 0$

(C) $x + y + 1 = 0$

(D) $x + 2y = 0$

Correct Answer: (C) $x + y + 1 = 0$

Solution:

Concept:

- Complete squares to identify the fixed circle centre. Minimising distance is equivalent to minimising its square because square root is increasing for nonnegative arguments.
- A point on $x^2 = 4y$ can be written as $(x, x^2/4)$. Differentiation finds the nearest point, and implicit differentiation gives the tangent slope there.

Step 1: Locate the centre and minimise squared distance The circle is $(x + 3)^2 + y^2 = 1$, so its centre is $(-3, 0)$. For a point on the parabola,

$$D^2 = (x + 3)^2 + \frac{x^4}{16}.$$

Differentiating gives

$$\frac{d(D^2)}{dx} = 2(x + 3) + \frac{x^3}{4} = 0, \qquad x^3 + 8x + 24 = 0.$$

Factorisation yields $(x + 2)(x^2 - 2x + 12) = 0$. The quadratic has negative discriminant, leaving the sole real stationary point $x = -2$, and consequently $y = 1$.

Step 2: Verify the minimum and construct the tangent The second derivative is $2 + 3x^2/4 > 0$ for all real x, and squared distance grows without bound as $|x|$ grows. Thus $(-2, 1)$ is the unique global minimum. From $2x = 4\,dy/dx$, the tangent slope there is -1. Hence

$$y - 1 = -(x + 2), \qquad x + y + 1 = 0.$$

The line passes through the nearest point and has the required slope. Option A has slope $-1/4$, option B has slope 1, and option D has slope $-1/2$; none matches the tangent at this point.

Step 3: Conclusion The tangent is $x + y + 1 = 0$. Hence, the correct option is (C).

Quick Tip to Solve: Minimise squared distance instead of distance; then check the resulting tangent using both its slope and the point of contact.

Physics (120 marks)

31. A galvanometer with its coil resistance 25 Ω requires a current of 1 mA for its full deflection. In order to construct an ammeter to read upto a current of 2 A, the approximate value of the shunt resistance should be : [4 marks]

(A) $2.5 \times 10^{-3}\,\Omega$

(B) $1.25 \times 10^{-2}\Omega$

(C) $1.25 \times 10^{-3}\Omega$

(D) $2.5 \times 10^{-2}\Omega$

Correct Answer: (B) $1.25 \times 10^{-2}\Omega$

Solution:

Concept:

- A low-resistance shunt is connected in parallel with a galvanometer so that most of the total current bypasses the sensitive coil.
- At full-scale deflection, equal parallel potential differences give $I_g G = (I - I_g)S$, where G is coil resistance and S is shunt resistance.

Step 1: Identify the coil and shunt currents The coil resistance is $G = 25\,\Omega$, the full-scale coil current is $I_g = 1\,\text{mA} = 0.001\,\text{A}$, and the required ammeter range is $I = 2\,\text{A}$. At the upper end of the range the galvanometer must still carry only its own full-scale current. The shunt therefore carries

$$I_s = I - I_g = 2 - 0.001 = 1.999\,\text{A}.$$

The voltage across the coil, and hence across the parallel shunt, is $I_g G = 0.025\,\text{V}$.

Step 2: Calculate the shunt and compare the options Using the common voltage across the parallel branches,

$$S = \frac{I_g G}{I - I_g} = \frac{0.001 \times 25}{1.999} = 0.012506\ldots\,\Omega \simeq 1.25 \times 10^{-2}\,\Omega.$$

Since $I_g \ll I$, replacing 1.999 A by 2 A gives the same requested approximation. Option A is five times too small, option C is ten times too small, and option D is twice the required value. A shunt this small is physically appropriate: it carries nearly all the current while the coil carries only one part in two thousand.

Step 3: Conclusion The approximate shunt resistance is $1.25 \times 10^{-2}\,\Omega$. Hence, the correct option is (B).

Quick Tip to Solve: For an ammeter shunt use the bypass current $I - I_g$, not the full-scale coil current, in the denominator.

32. Two identical conducting spheres A and B, carry equal charge. They are separated by a distance much larger than their diameters, and the force between theis F. A third identical conducting sphere, C, is uncharged. Sphere C is first touhed to A, then to B, and then removed. As a result, the force between A and B would be equal to : [4 marks]

(A) F

(B) $\frac{3F}{4}$

(C) $\frac{3F}{8}$

(D) $\frac{F}{2}$

Correct Answer: (C) $\frac{3F}{8}$

Solution:

Concept:

- Identical conducting spheres have equal capacitances, so on contact they share their combined charge equally. Total charge is conserved in every contact.
- At separations large compared with sphere size, Coulomb force is proportional to the product of the two charges: $F = kq_Aq_B/r^2$.

Step 1: Track the first and second contacts Let each initially charged sphere carry charge q. Initially $q_A = q_B = q$, $q_C = 0$, and $F = kq^2/r^2$. When C touches A, their combined charge q divides equally:

$$q_A = q_C = \frac{q}{2}, \qquad q_B = q.$$

C then touches B. Their combined charge is now $q/2 + q = 3q/2$, so after sharing,

$$q_B = q_C = \frac{3q}{4}.$$

A is no longer in contact and retains its charge $q/2$.

Step 2: Compare the final force with the initial force After C is removed, A and B remain at their original separation, carrying $q/2$ and $3q/4$. Thus

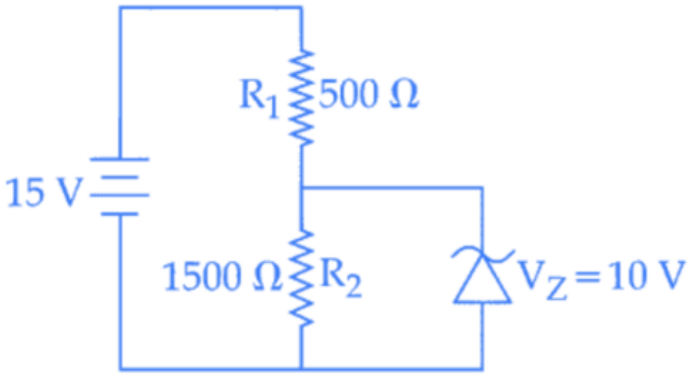
$$F_{\text{final}} = \frac{k}{r^2}\left(\frac{q}{2}\right)\left(\frac{3q}{4}\right) = \frac{3}{8}\frac{kq^2}{r^2} = \frac{3F}{8}.$$

The force remains repulsive because the two remaining charges still have the same sign. Option A ignores redistribution. Option B forgets the halving of A, and option D overlooks the second contact, which changes B. As a conservation check, all three final charges add to $q/2 + 3q/4 + 3q/4 = 2q$, equal to the initial total.

Step 3: Conclusion The final force is $3F/8$. Hence, the correct option is (C).

Quick Tip to Solve: Update the travelling sphere after each contact; its second contact starts with the charge acquired during the first.

33. In the given circuit, the current through zener diode is : [4 marks]



(A) 5.5 mA

(B) 6.7 mA

(C) 2.5 mA

(D) 3.3 mA

Correct Answer: (D) 3.3 mA

Solution:

Concept:

- A reverse-biased zener in breakdown maintains its rated voltage across the parallel load. The series resistor carries the sum of load current and zener current.
- Kirchhoff's current law at the load node gives $I_Z = I_{R_1} - I_{R_2}$; the zener current is not the current through the load resistor.

Step 1: Check that breakdown is possible The supply is 15 V, the series resistor is $R_1 = 500\,\Omega$, the load is $R_2 = 1500\,\Omega$, and the zener rating is 10 V. With the zener initially omitted, the voltage divider would put

$$V_{R_2} = 15\frac{1500}{500 + 1500} = 11.25\,\text{V}$$

across its terminals. This exceeds 10 V, so reverse breakdown is reached and the regulated load voltage is 10 V. The series resistor consequently drops $15 - 10 = 5\,\text{V}$.

Step 2: Subtract the load current from the supplied current The current entering the parallel combination is

$$I_{R_1} = \frac{15 - 10}{500} = 0.010\,\text{A} = 10\,\text{mA}.$$

The load current is

$$I_{R_2} = \frac{10}{1500} = 6.666\ldots\,\text{mA}.$$

Therefore

$$I_Z = 10 - 6.666\ldots = 3.333\ldots\,\text{mA} \simeq 3.3\,\text{mA}.$$

The positive result confirms that the assumed breakdown state is self-consistent. Option B approximately equals the load current rather than the zener current; options A and C do not satisfy current conservation with the shown resistance values.

Step 3: Conclusion The current through the zener diode is approximately $3.3\,\mathrm{mA}$. Hence, the correct option is (D).

Quick Tip to Solve: In a zener regulator, first calculate series-resistor current, then subtract the parallel load current to obtain the diode current.

34. A coil of cross-sectional area A having n turns is placed in a uniform magnetic field B. When it is rotated with an angular velocity ω, the maxium e.m.f. induced in the coil will be: [4 marks]

(A) 3 nBAω

(B) $\frac{3}{2}$ nBAω

(C) nBAω

(D) $\frac{1}{2}$ nBAω

Correct Answer: (C) nBAω

Solution:

Concept:

- Faraday's law gives induced emf as the negative time derivative of magnetic flux linkage: $\mathcal{E} = -d(N\Phi)/dt$.
- For a uniformly rotating coil in a uniform field, flux varies sinusoidally. Its amplitude is BA per turn, while differentiation introduces the angular speed ω.

Step 1: Express the changing flux linkage Let θ be the angle between the magnetic field and the area normal of the coil. The flux through one turn is $\Phi = BA\cos\theta$. For n turns rotating with angular speed ω, choose the time origin so that $\theta = \omega t$. The total flux linkage is then

$$n\Phi = nBA\cos(\omega t).$$

Choosing a different initial angle would merely shift the phase and would not change the maximum emf. The magnetic field, coil area and number of turns are constant during rotation.

Step 2: Differentiate and take the maximum magnitude Applying Faraday's law gives

$$\mathcal{E} = -\frac{d}{dt}[nBA\cos(\omega t)] = nBA\omega\sin(\omega t).$$

Since $|\sin(\omega t)| \le 1$, its peak magnitude is

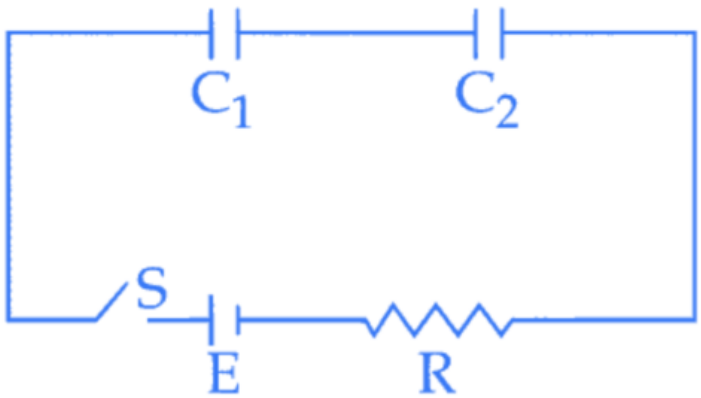
$$\mathcal{E}_{\max} = nBA\omega.$$

The maximum occurs when the normal is perpendicular to the field, where the flux changes fastest, not where the flux itself is largest. Options A and B add unsupported factors 3 and 3/2. Option D halves the peak without justification; it is not the requested maximum. The expression has units $\mathrm{Wb\,s^{-1}} = \mathrm{V}$.

Step 3: Conclusion The maximum induced emf is $nBA\omega$. Hence, the correct option is (C).

Quick Tip to Solve: For a rotating coil, differentiate the flux before finding the maximum: peak flux and peak induced emf occur a quarter-cycle apart.

35. In the following circuit, the switch S is closed at t = 0. The charge on the capacitor C_1 as a function of time will be given by $\left(C_{eq} = \frac{C_1 C_2}{C_1 + C_2}\right)$ [4 marks]



(A) $C_1 E\left[1 - \exp\left(-tR/C_1\right)\right]$

(B) $C_2 E\left[1 - \exp\left(-t/RC_2\right)\right]$

(C) $C_{eq} E\left[1 - \exp\left(-t/RC_{eq}\right)\right]$

(D) $C_{eq} E\ \exp\left(-t/RC_{eq}\right)$

Correct Answer: (C) $C_{eq} E\left[1 - \exp\left(-t/RC_{eq}\right)\right]$

Solution:

Concept:

- Capacitors in series carry equal charge magnitudes, and their equivalent capacitance is $C_{\text{eq}} = C_1 C_2/(C_1 + C_2)$.
- A series RC circuit connected to a constant emf charges with time constant $\tau = RC_{\text{eq}}$. The charge grows from zero to $C_{\text{eq}} E$.

Step 1: Reduce the circuit and write its charging equation The two capacitors lie in the same series loop with resistance R, switch S and battery emf E. Their intermediate conductor has no external connection, so the same charge magnitude q develops on each capacitor. Their combined voltage is

$$V_1 + V_2 = \frac{q}{C_1} + \frac{q}{C_2} = \frac{q}{C_{\text{eq}}}.$$

After the switch closes, Kirchhoff's loop law becomes

$$E = R\frac{dq}{dt} + \frac{q}{C_{\text{eq}}}.$$

The charging condition is $q(0) = 0$.

Step 2: Solve and check the initial and final values Separating variables, or using the standard first-order charging solution, gives

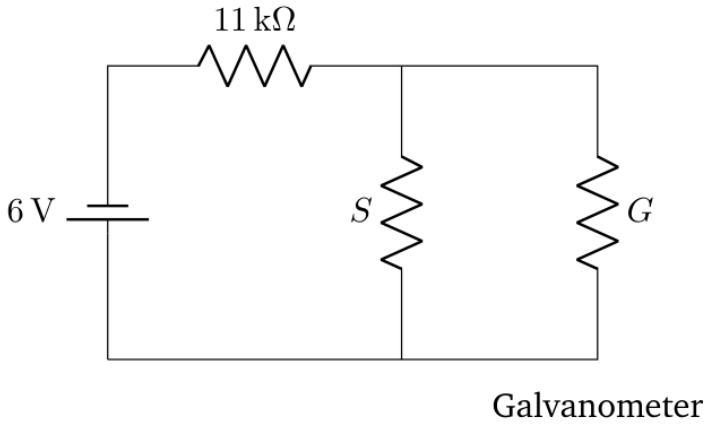
$$q(t) = C_{\text{eq}} E\left[1 - \exp\left(-\frac{t}{RC_{\text{eq}}}\right)\right].$$

This is also the charge on C_1, since series capacitors carry equal charges rather than equal voltages. At $t = 0$ it gives zero, and as $t \to \infty$ it tends to $C_{\text{eq}}E$, as required. Options A and B incorrectly use an individual capacitance in place of the series equivalent. Option D starts with maximum charge and decays to zero, describing discharge rather than charging. The exponential argument is dimensionless because RC_{eq} has units of time.

Step 3: Conclusion The charge is $q_1(t) = C_{\text{eq}}E[1 - \exp(-t/(RC_{\text{eq}}))]$. Hence, the correct option is (C).

Quick Tip to Solve: For series capacitors, calculate the equivalent capacitance first; its RC time constant governs the common charge on every capacitor.

36. In a circuit for finding the resistance of a galvanometer by half deflection method, a 6 V battery and a high resistance of 11 kΩ are used. The figure of merit of the galvanometer is 60 μA/division. In the absence of shunt resistance, the galvanometer produces a deflection of θ = 9 divisions when current flows in the circuit. The value of the shunt resistance that can cause the deflection of $\theta/2$, is closest to : [4 marks]



(A) 500 Ω

(B) 220 Ω

(C) 55 Ω

(D) 110 Ω

Correct Answer: (D) 110 Ω

Solution:

Concept:

- The figure of merit converts a deflection into current: $I_g = k\theta$, where k is current per division.
- The high resistance R is in series with the galvanometer of resistance G. Adding a shunt S changes the total current, so use the exact parallel-circuit relation.

Step 1: Find the galvanometer resistance Before connecting the shunt, the same current flows through

the series resistance and galvanometer. The given deflection therefore gives

$$I_0 = 60 \times 10^{-6} \times 9 = 540 \times 10^{-6}\,\text{A}.$$

Using the battery voltage,

$$R + G = \frac{6}{540 \times 10^{-6}} = 11111.11\,\Omega.$$

Since $R = 11000\,\Omega$, we obtain $G = 111.11\,\Omega$.

Step 2: Apply the half-deflection condition Half deflection means half the original galvanometer current, $I_g = I_0/2 = 270\,\mu\text{A}$. The voltage across the galvanometer and shunt is then

$$V_g = I_g G = 0.030\,\text{V}.$$

Thus the current supplied through the high resistance is

$$I = \frac{6 - 0.030}{11000} = 542.727\,\mu\text{A}.$$

The shunt carries the difference, $I_s = 272.727\,\mu\text{A}$, so

$$S = \frac{0.030}{272.727 \times 10^{-6}} = 110\,\Omega.$$

Step 3: Check the alternatives The high series resistance is much larger than G, so the usual approximation $S \simeq G$ predicts a value near $111\,\Omega$. This supports $110\,\Omega$; $55\,\Omega$ would divert too much current, while $220\,\Omega$ and $500\,\Omega$ would divert too little.

Step 4: Conclusion The required shunt resistance is $110\,\Omega$. Hence, the correct option is (D).

Quick Tip to Solve: Find G from the unshunted current first; for an exact answer include the small change in the current through the high series resistance.

37. The de-Broglie wavelength (λ_B) associated with the electron orbiting in the second excited state of hydrogen atom is related to that in the ground state (λ_G) by : [4 marks]

(A) $\lambda_B = 2\lambda_G$

(B) $\lambda_B = 3\lambda_G$

(C) $\lambda_B = \lambda_{G/2}$

(D) $\lambda_B = \lambda_{G/3}$

Correct Answer: (B) $\lambda_B = 3\lambda_G$

Solution:

Concept:

- The de Broglie wavelength of an electron is $\lambda = h/(m_e v)$, where v is its orbital speed.
- For a hydrogen atom in the Bohr model, $v_n = v_1/n$, so its matter wavelength is proportional to the principal quantum number n.

Step 1: Identify the two energy levels The ground state is the lowest allowed orbit and has principal quantum number $n = 1$. The first excited state is $n = 2$, and the second excited state is $n = 3$. Thus the wavelength denoted by λ_B belongs to the third orbit, not the second orbit. The word excited counts levels above the ground state.

Step 2: Form the wavelength ratio For the same electron, Planck's constant and its mass cancel in the ratio. Since $v_3 = v_1/3$,

$$\frac{\lambda_B}{\lambda_G} = \frac{h/(m_e v_3)}{h/(m_e v_1)} = \frac{v_1}{v_3} = 3.$$

Equivalently, the standing-wave condition $2\pi r_n = n\lambda_n$, together with $r_n = n^2 r_1$, gives $\lambda_n = n\lambda_1$. The radius grows as n^2, but the wavelength grows only as n.

Step 3: Reject the other scalings The relation $\lambda_B = 2\lambda_G$ would describe the first excited state. The inverse ratios in (C) and (D) incorrectly imply a smaller wavelength in the excited state. An electron in a higher hydrogen orbit has lower speed and momentum; consequently its de Broglie wavelength must increase, rather than decrease.

Step 4: Conclusion For the second excited state, $\lambda_B = 3\lambda_G$. Hence, the correct option is (B).

Quick Tip to Solve: Convert the excitation label to n first: second excited means $n = 3$; then use $\lambda_n \propto n$.

38. Both the nucleus and the atom of some element arein their respective first excited states. They get de-excted by emitting photons of wavelengths $^{\lambda}$N, $^{\lambda}$A respectively. The ratio $\frac{\lambda_N}{\lambda_A}$ is closest to : [4 marks]

(A) 10^{-6}

(B) 10

(C) 10^{-10}

(D) 10^{-1}

Correct Answer: (A) 10^{-6}

Solution:

Concept:

- A photon emitted in a transition has energy $\Delta E = hc/\lambda$; a larger transition energy means a shorter wavelength.
- Typical nuclear excitation energies are on the MeV scale, whereas electronic atomic excitation energies are on the eV scale. This question asks for the closest order of magnitude.

Step 1: Compare the transition energies Let ΔE_N be the energy lost by the nucleus and ΔE_A that lost by the atom. Nuclear states involve the strong nuclear interaction and characteristically have much larger energy separations than the electronic states governed by electromagnetic binding. In the standard scale estimate, $\Delta E_N \sim 10^6\,\text{eV}$ while $\Delta E_A \sim 1\,\text{eV}$. The unspecified element prevents an exact numerical ratio, so these characteristic scales are the relevant comparison.

Step 2: Convert the energy ratio into a wavelength ratio Apply the photon energy relation separately to both emissions:

$$\lambda_N = \frac{hc}{\Delta E_N}, \qquad \lambda_A = \frac{hc}{\Delta E_A}.$$

Dividing cancels h and c, giving

$$\frac{\lambda_N}{\lambda_A} = \frac{\Delta E_A}{\Delta E_N} \sim \frac{1}{10^6} = 10^{-6}.$$

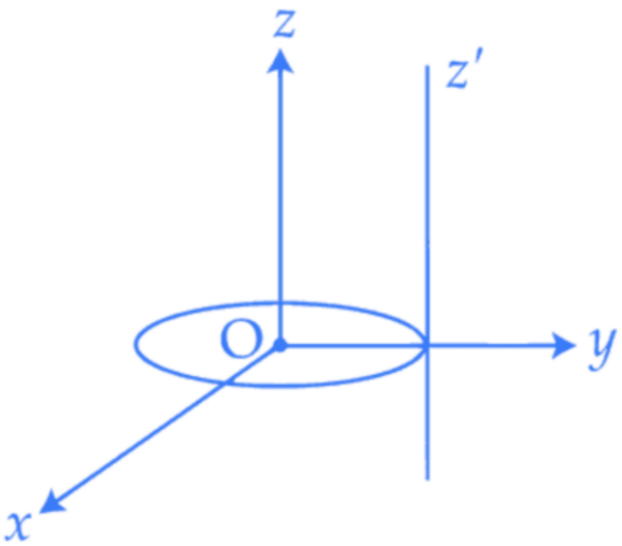
It is essential to invert the energy ratio because wavelength and energy vary inversely.

Step 3: Compare the offered orders of magnitude Option (B) would make the nuclear wavelength larger than the atomic wavelength and reverses the physical energy hierarchy. Option (D) allows only a factor of ten in energy, far too small for nuclear versus electronic transitions. Option (C) would require a separation of ten orders of magnitude in energy, far larger than the conventional MeV-to-eV comparison used here.

Step 4: Conclusion The closest characteristic ratio is $\lambda_N/\lambda_A \sim 10^{-6}$. Hence, the correct option is (A).

Quick Tip to Solve: For nuclear versus atomic photons, compare MeV with eV and then invert: the higher-energy photon has the shorter wavelength.

39. A thin circular disk is in the xy plane as shown in the figure. The ratio of its moment of inertia about z and z' axes will be : [4 marks]



(A) 1 : 3

(B) 1 : 4

(C) 1 : 5

(D) 1 : 2

Correct Answer: (A) 1 : 3

Solution:

Concept:

- A uniform thin disc of mass M and radius a has moment of inertia $I_z = \frac{1}{2}Ma^2$ about its central axis perpendicular to its plane.
- The parallel-axis theorem gives $I_{z'} = I_z + Md^2$, where d is the perpendicular distance between parallel axes.

Step 1: Read the axes from the figure The disc lies in the xy-plane. The z-axis passes through its centre O and is perpendicular to that plane. The axis z' is parallel to z and passes through the rim on the positive y side. Their separation is therefore one disc radius, $d = a$. The apparent ellipse is a perspective drawing of the circular disc; it does not change the distance from the centre to its rim.

Step 2: Translate the central moment to the rim axis Use the parallel-axis theorem with the central perpendicular axis as the reference:

$$I_{z'} = \frac{1}{2}Ma^2 + Ma^2 = \frac{3}{2}Ma^2.$$

Both axes are perpendicular to the disc, so no moment about an in-plane diameter is involved. The added term depends on the full disc mass and the squared separation between the axes.

Step 3: Form the requested ratio and check it The question asks for the central-axis value first:

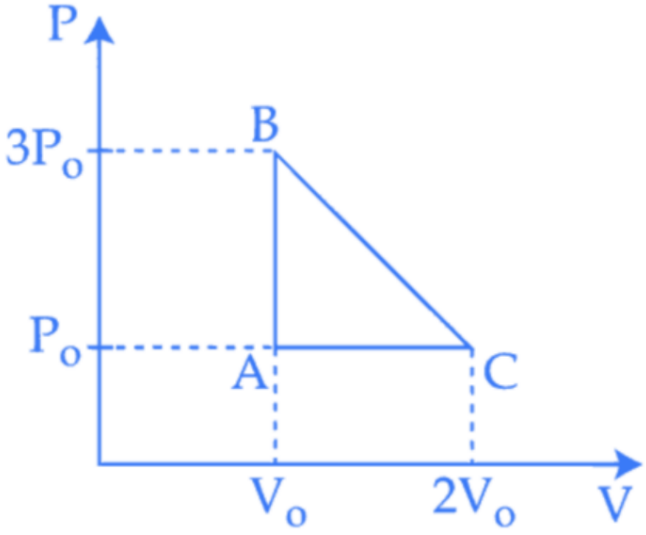
$$\frac{I_z}{I_{z'}} = \frac{\frac{1}{2}Ma^2}{\frac{3}{2}Ma^2} = \frac{1}{3}.$$

The common mass and radius factors cancel. The ratio must be smaller than one because moving a parallel axis away from the centre of mass increases the moment of inertia. The values $1:4$, $1:5$, and $1:2$ do not follow from adding Ma^2 to the central disc value.

Step 4: Conclusion The ratio of the moments about z and z' is $1:3$. Hence, the correct option is (A).

Quick Tip to Solve: For a rim axis parallel to a disc's central normal, add Ma^2 to $Ma^2/2$, then preserve the requested ratio order.

40. One mole of an ideal monoatomic gas is taken along the path ABCA as show in the PV diagram. The maximum temperature attained by the gas along the path BC is given by : [4 marks]



(A) $\frac{25}{16}\frac{P_oV_o}{R}$
(B) $\frac{25}{8}\frac{P_oV_o}{R}$
(C) $\frac{25}{4}\frac{P_oV_o}{R}$
(D) $\frac{5}{8}\frac{P_oV_o}{R}$

Correct Answer: (B) $\frac{25}{8}\frac{P_oV_o}{R}$

Solution:

Concept:

- For one mole of ideal gas, $T = PV/R$. Maximizing temperature therefore means maximizing the product PV, not pressure or volume separately.
- On a straight segment of a pressure-volume graph, pressure is a linear function of volume. The resulting temperature is quadratic in volume.

Step 1: Write the equation of the path The plotted endpoints are $B = (V_0, 3P_0)$ and $C = (2V_0, P_0)$. Hence the slope of BC is

$$\frac{P_0 - 3P_0}{2V_0 - V_0} = -\frac{2P_0}{V_0}.$$

Its equation is therefore

$$P - 3P_0 = -\frac{2P_0}{V_0}(V - V_0), \qquad P = 5P_0 - \frac{2P_0V}{V_0}.$$

These coefficients come directly from the two endpoints.

Step 2: Maximize the temperature along BC Put $x = V/V_0$, with $1 \le x \le 2$. The ideal-gas equation gives

$$T = \frac{P_0V_0}{R}(5x - 2x^2).$$

Differentiate the bracket:

$$\frac{dT}{dx} = \frac{P_0V_0}{R}(5 - 4x) = 0 \quad \Rightarrow \quad x = \frac{5}{4}.$$

The stationary point lies on BC. The second derivative is negative, $-4P_0V_0/R$, so it gives a maximum.

Step 3: Evaluate and compare the result At $x = 5/4$,

$$T_{\max} = \frac{P_0V_0}{R}\left(\frac{25}{4} - \frac{25}{8}\right) = \frac{25P_0V_0}{8R}.$$

At B and C the temperatures are respectively $3P_0V_0/R$ and $2P_0V_0/R$; the interior value $25/8 = 3.125$ is larger than both. The other coefficients in the options fail this quadratic maximum. The monoatomic nature affects internal energy, but is not needed for $PV = RT$.

Step 4: Conclusion The maximum temperature along BC is $\frac{25P_0V_0}{8R}$. Hence, the correct option is (B).

Quick Tip to Solve: On a sloping straight PV segment, maximize the quadratic $P(V)V$; the hottest point need not be an endpoint.

41. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns, giving the output power at 230 V. If the current in the primary of the transformer is 5 A, and its efficiency is 90%, the output current would be ; [4 marks]

(A) 50 A

(B) 45 A

(C) 25 A

(D) 20 A

Correct Answer: (B) 45 A

Solution:

Concept:

- Transformer efficiency is the ratio of useful output power to input power: $\eta = P_{\text{out}}/P_{\text{in}}$.
- For the voltage and current values used in this transformer problem, $P_{\text{in}} = V_pI_p$ and $P_{\text{out}} = V_sI_s$. The stated efficiency must be included when finding the secondary current.

Step 1: Calculate the input power The primary receives $V_p = 2300\,\text{V}$ and carries $I_p = 5\,\text{A}$. Its input power is

$$P_{\text{in}} = 2300 \times 5 = 11500\,\text{W}.$$

The secondary voltage is $V_s = 230\,\text{V}$, ten times smaller. A step-down transformer raises the current, but its output power is less than its input power when its efficiency is below unity.

Step 2: Allow for the efficiency and find the current The given efficiency is $90\% = 0.90$. Thus

$$P_{\text{out}} = 0.90 \times 11500 = 10350\,\text{W}.$$

Dividing this useful output power by the secondary voltage gives

$$I_s = \frac{10350}{230} = 45\,\text{A}.$$

In a single calculation, the same result is $I_s = \eta(V_p/V_s)I_p = 0.90 \times 10 \times 5$. The 4000 primary turns need not be used because both terminal voltages are already specified.

Step 3: Check the options against power balance An output current of $50\,\text{A}$ would give $11500\,\text{W}$, corresponding to an ideal transformer with no loss. The options $25\,\text{A}$ and $20\,\text{A}$ give only $5750\,\text{W}$ and $4600\,\text{W}$, so they imply efficiencies of 50 percent and 40 percent. Only $45\,\text{A}$ is consistent with the specified 90 percent efficiency.

Step 4: Conclusion The output current is $45\,\text{A}$. Hence, the correct option is (B).

Quick Tip to Solve: First multiply the input power by efficiency; then divide by the output voltage. The ideal turns-ratio current alone would overestimate the answer.

42. Two moles of helium are mixed with n moles of hydrogen. If $\frac{Cp}{Cv} = \frac{3}{2}$ for the mixture, then the value of n is : [4 marks]

(A) 1

(B) 3

(C) 2

(D) 3 / 2

Correct Answer: (C) 2

Solution:

Concept:

- Monatomic helium has molar heat capacities $C_{V,\text{He}} = 3R/2$ and $C_{P,\text{He}} = 5R/2$.
- Diatomic hydrogen, with rotational modes active and vibrational modes neglected, has $C_{V,\text{H}_2} = 5R/2$ and $C_{P,\text{H}_2} = 7R/2$. Extensive heat capacities add when gases are mixed.

Step 1: Add the heat capacities of the components There are two moles of helium and n moles of hydrogen. For a common temperature rise, the required heat is the sum of that for the two species, so the total constant-volume heat capacity is

$$C_V = 2\left(\frac{3}{2}R\right) + n\left(\frac{5}{2}R\right) = \frac{R}{2}(6 + 5n).$$

Similarly, the total constant-pressure heat capacity is

$$C_P = 2\left(\frac{5}{2}R\right) + n\left(\frac{7}{2}R\right) = \frac{R}{2}(10 + 7n).$$

Dividing either by the total number of moles would not change their ratio.

Step 2: Use the specified ratio to solve for n The common factor $R/2$ cancels, leaving

$$\frac{10+7n}{6+5n}=\frac{3}{2}.$$

Cross-multiplication gives

$$20+14n=18+15n, \qquad n=2.$$

This result is positive, as required for the amount of hydrogen added to the helium.

Step 3: Verify the mixture and eliminate alternatives For $n=2$, $C_P=12R$ and $C_V=8R$, so the ratio is exactly $3/2$. For $n=1$, the ratio would be $17/11$; for $n=3$, it would be $31/21$; and for $n=3/2$, it would be $41/27$. None of these equals $3/2$. One must combine heat capacities rather than directly average the individual adiabatic indices.

Step 4: Conclusion The required amount of hydrogen is $n=2$ moles. Hence, the correct option is (C).

Quick Tip to Solve: For an ideal-gas mixture, add $n_iC_{P,i}$ and $n_iC_{V,i}$ separately before taking their ratio; do not average the two gases' values of γ.

43. The end correction of a resonance column is 1 cm. If the shortest length resonating with the tunning fork is 10 cm, the next resonating length should be : [4 marks]

(A) 28 cm

(B) 32 cm

(C) 36 cm

(D) 40 c

Correct Answer: (B) 32 cm

Solution:

Concept:

- A resonance column closed by water has a displacement node at the water surface and a displacement antinode slightly beyond the open end.
- The effective length is $L+e$, where L is the measured air-column length and e is its end correction. Successive resonances have effective lengths $\lambda/4, 3\lambda/4, 5\lambda/4, \ldots$.

Step 1: Find the wavelength from the first resonance The shortest resonating length is the fundamental, so the end correction must be added before using the quarter-wavelength condition. With both lengths in centimetres,

$$L_1+e=10+1=11\ \text{cm}=\frac{\lambda}{4}.$$

Therefore $\lambda=44$ cm. The tuning fork fixes the frequency, and the speed of sound in the column is unchanged, so this same wavelength applies to the next resonance.

Step 2: Apply the next allowed resonance condition A closed column supports odd quarter-

wavelengths, rather than every integer quarter-wavelength. Thus the next resonance satisfies

$$L_2 + e = \frac{3\lambda}{4} = 33 \text{ cm}.$$

Subtract the end correction to recover the measured length:

$$L_2 = 33 - 1 = 32 \text{ cm}.$$

Equivalently, consecutive measured lengths differ by $\lambda/2 = 22$ cm. The proposed lengths 28, 36 and 40 cm give effective lengths 29, 37 and 41 cm, respectively; none equals the required 33 cm. Multiplying the measured first length by three would wrongly apply the end correction three times.

Step 3: Conclusion The next resonating air-column length is $\boxed{32 \text{ cm}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use $L_2 = 3(L_1 + e) - e = 3L_1 + 2e$: multiply the effective length, then subtract one end correction.

44. A charge q is spread uniformly over an insulated loop of radius r. If it is rotated with an angular velocity ω with resect to normal axis then the magnetic moment of the loop is : [4 marks]

(A) q ωr^2

(B) $\frac{4}{3}$ q ωr^2

(C) $\frac{3}{2}$ q ωr^2

(D) $\frac{1}{2}$ q ωr^2

Correct Answer: (D) $\frac{1}{2}$ q ωr^2

Solution:

Concept:

- A charge completing one revolution in time T constitutes a current $I = q/T$, even when it is carried by an insulating ring rather than moving through a conducting wire.
- The magnetic dipole moment of a planar current loop is $\boldsymbol{\mu} = I\mathbf{A}$. A circular ring has area πr^2, and its rotation period is $T = 2\pi/\omega$.

Step 1: Convert the circulating charge into an equivalent current Uniform charge density means the same total charge passes any fixed radial half-plane once during a revolution. Therefore

$$I = \frac{q}{T} = \frac{q\omega}{2\pi}.$$

The normal axis passes through the centre perpendicular to the plane, so every charge element has the same period and traces the same circular path. There is no integration over different radii as there would be for a charged disc.

Step 2: Multiply the current by the enclosed area Using the area of the circular loop gives

$$\mu = I\pi r^2 = \frac{q\omega}{2\pi}\pi r^2 = \frac{1}{2}q\omega r^2.$$

For positive charge the direction follows the right-hand rule for the rotation; for negative charge it reverses. The magnitude uses $|q\omega|$. The factor 2π in the period cancels only one factor of π, leaving the factor one-half. Option (A) misses this factor, while the coefficients in (B) and (C) do not follow from the current-period relation. As a dimensional check, charge times angular frequency times area has units $\mathrm{A\,m^2}$, the units of magnetic moment.

Step 3: Conclusion The magnetic moment is $\boxed{\mu = \frac{1}{2}q\omega r^2}$, with its sign determining the normal direction. Hence, the correct option is (D).

Quick Tip to Solve: For a rotating charged ring, first write $I = q\omega/(2\pi)$, then multiply by πr^2; do not substitute a disc formula.

45. A small soap bubble of radius 4 cm is trapped inside another bubble of radius 6 cm without any contact. Let P_2 be the pressure inside the inner bubble and P_0, the pressure outside the outer bubble. Radius of another bubble with pressure difference $P_2 - P_0$ between its inside and outside would be : [4 marks]

(A) 12 cm

(B) 2.4 cm

(C) 6 cm

(D) 4.8 cm

Correct Answer: (B) 2.4 cm

Solution:

Concept:

- A soap bubble has two liquid-air surfaces. Its excess pressure is $\Delta P = 4\gamma/R$, where γ is surface tension and R is the bubble radius.
- For two separate nested bubbles, the intermediate air pressure is the outside pressure for the smaller bubble. Pressure differences add along the path from the external air to the innermost air.

Step 1: Write the pressure jumps at both bubbles Let P_1 be the pressure in the space between the two bubbles. For the outer bubble of radius $6\ \mathrm{cm}$,

$$P_1 - P_0 = \frac{4\gamma}{6\ \mathrm{cm}}.$$

For the inner bubble of radius $4\ \mathrm{cm}$,

$$P_2 - P_1 = \frac{4\gamma}{4\ \mathrm{cm}}.$$

Each jump is positive on going inward. The fact that the bubbles do not touch permits treating their spherical films separately.

Step 2: Replace the sum by one equivalent pressure jump Adding cancels the unknown intermediate pressure:

$$P_2 - P_0 = 4\gamma\left(\frac{1}{4\text{ cm}} + \frac{1}{6\text{ cm}}\right).$$

If a single soap bubble of radius R has this excess pressure,

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}\text{ cm}^{-1}, \qquad R = \frac{12}{5}\text{ cm} = 2.4\text{ cm}.$$

A radius smaller than 4 cm is physically consistent: the combined pressure jump exceeds the jump across the smaller bubble alone. Radii 12, 6 and 4.8 cm would each give a smaller excess pressure than that smaller bubble, and cannot represent the sum.

Step 3: Conclusion The required radius is $\boxed{2.4\text{ cm}}$. Hence, the correct option is (B).

Quick Tip to Solve: For nested soap bubbles with equal surface tension, add reciprocal radii: $1/R_{\text{eq}} = 1/R_1 + 1/R_2$.

46. Let $\vec{A} = \left(\hat{i} + \hat{j}\right)$ and, $\vec{B} = \left(2\hat{i} - \hat{j}\right)$. The magnitude of a coplanar vector $\vec{C}$ such that $\vec{A}.\vec{C} = \vec{B}.\vec{C} = \vec{A}.\vec{B}$, is given by : [4 marks]

(A) $\sqrt{\frac{10}{9}}$

(B) $\sqrt{\frac{5}{9}}$

(C) $\sqrt{\frac{20}{9}}$

(D) $\sqrt{\frac{9}{12}}$

Correct Answer: (B) $\sqrt{\frac{5}{9}}$

Solution:

Concept:

- The scalar product of Cartesian vectors is the sum of products of their corresponding components: $(a_x, a_y) \cdot (c_x, c_y) = a_x c_x + a_y c_y$.
- Coplanarity with the two nonparallel vectors restricts $\mathbf{C}$ to their plane, so write $\mathbf{C} = x\hat{i} + y\hat{j}$. Its magnitude is $\sqrt{x^2 + y^2}$.

Step 1: Translate both scalar-product conditions into equations First calculate the fixed scalar product:

$$\mathbf{A} \cdot \mathbf{B} = (1)(2) + (1)(-1) = 1.$$

Thus $\mathbf{A} \cdot \mathbf{C} = 1$ gives $x + y = 1$, while $\mathbf{B} \cdot \mathbf{C} = 1$ gives $2x - y = 1$. These are two separate constraints; equal scalar products do not mean the vectors themselves are equal or parallel.

Step 2: Solve for the components and calculate the norm Adding the equations eliminates y:

$$3x = 2, \qquad x = \frac{2}{3}, \qquad y = 1 - x = \frac{1}{3}.$$

Therefore

$$\mathbf{C} = \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j}, \qquad |\mathbf{C}| = \sqrt{\frac{4}{9} + \frac{1}{9}} = \sqrt{\frac{5}{9}}.$$

Checking the result in both conditions gives $2/3 + 1/3 = 1$ and $4/3 - 1/3 = 1$. This confirms both projections, not just one. The squared magnitude must consequently be $5/9$; the squared values $10/9$, $20/9$ and $9/12$ in the other options do not agree. Without the coplanarity condition an unconstrained perpendicular component could change the norm, which is why that word matters.

Step 3: Conclusion The magnitude is $\boxed{\sqrt{5/9} = \sqrt{5}/3}$. Hence, the correct option is (B).

Quick Tip to Solve: Introduce the unknown vector components immediately; scalar-product constraints then become simultaneous linear equations.

---

47. A body of mass m starts moving from rest along x-axis so that its velocity varies as $v = a\sqrt{s}$ where a is a constant and s is the distance covered by the body. The total work done by all the forces acting on the body in the first t seconds after the start of the motion is : [4 marks]

(A) $\frac{1}{8}$ m a^4 t^2

(B) 8 m a^4 t^2

(C) 4 m a^4 t^2

(D) $\frac{1}{4}$ m a^4 t^2

Correct Answer: (A) $\frac{1}{8}$ m a^4 t^2

Solution:

Concept:

- The total work of all forces equals the change in kinetic energy: $W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$. Here the body starts from rest.
- When speed depends on distance, acceleration can be written $dv/dt = v\,dv/ds$, using $ds/dt = v$ for the stated forward motion.

Step 1: Determine the acceleration and the speed at time t From $v = a\sqrt{s}$, differentiation with respect to distance gives

$$\frac{dv}{ds} = \frac{a}{2\sqrt{s}}, \qquad \frac{dv}{dt} = v\frac{dv}{ds} = \frac{a^2}{2}.$$

This is constant during the motion. Taking the continuous moving branch from rest at $t = 0$,

$$v(t) = \frac{a^2 t}{2}.$$

The corresponding distance is $s(t) = a^2t^2/4$, which indeed satisfies the original speed-distance relation.

Step 2: Use the change in kinetic energy The initial kinetic energy vanishes, so the net work in the first t seconds is

$$W_{\text{net}} = \frac{1}{2}m\left(\frac{a^2t}{2}\right)^2 = \frac{1}{8}ma^4t^2.$$

The same result follows from constant net force $ma^2/2$ acting through distance $a^2t^2/4$. The coefficient is therefore fixed by two factors: one-half in kinetic energy and one-quarter from squaring the speed. Options (B) and (C) are much too large; option (D) omits the kinetic-energy factor one-half. Dimensionally $[a] = \mathrm{m}^{1/2}\mathrm{s}^{-1}$, so ma^4t^2 has energy units as required.

Step 3: Conclusion The total work done is $\boxed{\frac{1}{8}ma^4t^2}$. Hence, the correct option is (A).

Quick Tip to Solve: For $v = a\sqrt{s}$, use $v\,dv/ds$ to find constant acceleration, then apply the work-energy theorem.

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48. The percentage errors in quantities P, Q, R and S are 0.5%, 1%, 3% and 1.5% respectively in the measurement of a physical quantity A = $\frac{P^3Q^2}{\sqrt{RS}}$.
The maximum percentage error in the value of A will be : [4 marks]

(A) 6.0%

(B) 7.5%

(C) 8.5%

(D) 6.5%

Correct Answer: (D) 6.5%

Solution:

Concept:

- For $A = P^pQ^qR^rS^s$, the maximum fractional error to first order is the sum of absolute exponent-weighted fractional errors.
- The printed expression is $A = P^3Q^2/(\sqrt{R}\,S) = P^3Q^2R^{-1/2}S^{-1}$. Only R is under the square root.

Step 1: Identify the weights multiplying the measurement errors Taking a logarithmic differential gives

$$\frac{dA}{A} = 3\frac{dP}{P} + 2\frac{dQ}{Q} - \frac{1}{2}\frac{dR}{R} - \frac{dS}{S}.$$

For a maximum error estimate the signs of measurement errors are unknown and can combine unfavourably. Consequently denominator contributions must be added in magnitude, rather than subtracted. The relevant weights are $3, 2, 1/2, 1$, respectively.

Step 2: Substitute the four percentage uncertainties Using the given percentage errors directly,

$$\begin{aligned}\frac{\Delta A}{A}\,100\% &= 3(0.5\%) + 2(1\%) + \frac{1}{2}(3\%) + 1(1.5\%)\\ &= 1.5\% + 2\% + 1.5\% + 1.5\%\\ &= 6.5\%.\end{aligned}$$

Every term is dimensionless and already expressed in percent, so no further multiplication by 100 is needed. A result of 6.0, 7.5 or 8.5 percent cannot be obtained with these four correct exponent weights. In particular, treating both R and S as square-rooted would incorrectly halve the contribution from S, while retaining the negative denominator signs would estimate a particular signed change rather than the maximum error.

Step 3: Conclusion The maximum percentage error is $\boxed{6.5\%}$. Hence, the correct option is (D).

Quick Tip to Solve: Rewrite radicals as powers first, then multiply every percentage error by the absolute value of its exponent.

49. An oscillator of mass M is at rest in its equilibrium position in a potential $V = \frac{1}{2} k(x - X)^2$. A particle of mass m comes from right with speed u and collides completely inelastically with M and sticks to it. This process repeats every time the oscillator crosses its equilibrium position. The amplitude of oscillations after 13 collisions is : (M = 10, m = 5, u = 1, k = 1) [4 marks]

(A) $\frac{1}{\sqrt{3}}$
(B) $\frac{1}{2}$
(C) $\frac{2}{3}$
(D) $\frac{3}{\sqrt{5}}$

Correct Answer: (A) $\frac{1}{\sqrt{3}}$

Solution:

Concept:

- At the equilibrium position $x = X$, the spring potential energy $\frac{1}{2}k(x - X)^2$ is zero. The oscillator therefore has its maximum speed there.
- Momentum is conserved during each brief completely inelastic collision. Between collisions the attached mass undergoes SHM, with $\omega_n = \sqrt{k/(M + nm)}$ after n particles have stuck.

Step 1: Track signed momentum across successive collisions Take rightward momentum as positive. Each incoming particle comes from the right, so its momentum is $-mu$. Initially the oscillator is at rest. The first collision gives $p_1 = -mu$, followed by a half-oscillation back to equilibrium with momentum $+mu$. The second incoming particle has momentum $-mu$, so momentum conservation gives

$$p_2 = mu - mu = 0.$$

At this instant the oscillator is also at equilibrium, where the spring force is zero. It remains there at rest and does not cross equilibrium again. Consequently the rule that each subsequent impact occurs when the oscillator crosses equilibrium cannot produce even the third collision. The literal wording therefore does not define an amplitude after thirteen collisions.

Step 2: Calculate the formal intended result and distinguish it from the literal motion If later impacts

are independently imposed at equilibrium despite that stopping state, the formal recurrence would be

$$p_n = -p_{n-1} - mu, \qquad p_0 = 0.$$

It gives $p_n = -mu$ for odd n, and zero for even n. Under this additional prescription, $M_{13} = M + 13m = 75$ and $|p_{13}| = mu = 5$. Converting equilibrium kinetic energy to turning-point spring energy gives

$$\frac{p_{13}^2}{2M_{13}} = \frac{1}{2}kA_{13}^2, \qquad A_{13} = \frac{5}{\sqrt{75}} = \frac{1}{\sqrt{3}}.$$

This explains option (A), the intended formal answer, whereas the other listed values do not satisfy that formal energy calculation. However, this calculation cannot supply the missing physical equilibrium crossings under the collision rule actually printed.

Step 3: Conclusion The literal collision sequence stops after the second impact, so its thirteenth-collision amplitude is undefined. The answer key's option (A), $1/\sqrt{3}$, is the formal intended result only if further impacts at equilibrium are imposed. Hence, the correct option is (A) under that additional interpretation; the printed process itself is inconsistent.

Quick Tip to Solve: Check that a repeated-event rule can actually continue: zero speed at equilibrium stops this oscillator, even though a formal momentum recurrence can still be iterated.

50. Suppose that the angular velocity of rotation of earth is increased. Then, as a consequence :

[4 marks]

(A) Weight of the object, everywhere on the earth, will increase.

(B) Weight of the object, everywhere on the earth, will decrease.

(C) There will be no change in weight anywhere on the earth.

(D) Except at poles, weight of the object on the earth will decrease.

Correct Answer: (D) Except at poles, weight of the object on the earth will decrease.

Solution:

Concept:

- Apparent weight is the support force on an object stationary relative to the rotating Earth; it includes the effect of rotation.
- The centrifugal acceleration is directed away from the rotation axis and has magnitude $\omega^2\rho$, where ρ is distance from that axis.

Step 1: Locate the rotational correction At latitude ϕ, an object on a spherical Earth of radius R is at distance $\rho = R\cos\phi$ from the rotation axis. Rotation therefore reduces the effective gravity. Its component along the local outward radius is $\omega^2 R\cos^2\phi$. In the usual terrestrial approximation, the apparent weight is consequently

$$W_{\text{app}} = m\left(g - \omega^2 R\cos^2\phi\right).$$

Here g denotes the gravitational acceleration before allowing for rotation; the change under discussion

is the rotational correction.

Step 2: Compare the poles with other latitudes Increasing ω increases the subtractive term wherever $\cos\phi \neq 0$. Thus the apparent weight decreases at the equator and at all intermediate latitudes. At either pole, $\phi = \pm 90^\circ$, so $\rho = 0$ and the centrifugal effect vanishes. Increasing the rotation rate therefore leaves the polar weight unchanged. Option (A) has the wrong direction of change; option (B) incorrectly includes the poles; option (C) ignores the centrifugal correction at other latitudes.

Step 3: Conclusion Except at the poles, the weight decreases as the angular speed of Earth increases. Hence, the correct option is (D).

Quick Tip to Solve: Check the poles first: their distance from the rotation axis is zero, so rotation cannot change their centrifugal correction.

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51. The relative uncertainly in the period of a satellite orbiting around the earth is 10^{-2}. If the relative uncertainty in the radius of the orbit is negligible, the relative uncertainty in the mass of the earth is : [4 marks]

(A) 10^{-2}

(B) 2×10^{-2}

(C) 3×10^{-2}

(D) 6×10^{-2}

Correct Answer: (B) 2×10^{-2}

Solution:

Concept:

- For a circular satellite orbit, gravitational attraction supplies centripetal acceleration: $GM/r^2 = 4\pi^2 r/T^2$.
- For products and powers, small relative uncertainties add with the absolute values of the exponents; a negative power does not make an uncertainty negative.

Step 1: Express the mass through measurable quantities Let M be the mass of Earth, r the orbital radius and T the orbital period. Rearranging the orbital equation gives

$$M = \frac{4\pi^2 r^3}{GT^2}.$$

The constants $4\pi^2$ and G are treated as exact for the uncertainty requested here. The radius enters with exponent three, whereas the period enters with exponent minus two. We must propagate uncertainties in the inferred mass, rather than in the orbital speed separately.

Step 2: Propagate the stated uncertainty The first-order relative uncertainty is

$$\frac{\Delta M}{M} = 3\frac{\Delta r}{r} + 2\frac{\Delta T}{T}.$$

Since the uncertainty in radius is negligible and $\Delta T/T = 10^{-2}$,

$$\frac{\Delta M}{M} = 2 \times 10^{-2}.$$

The minus sign in the logarithmic derivative describes an anticorrelation between mass and period; the magnitude of the uncertainty remains positive. Option (A) misses the factor two. Option (C) uses the radius exponent despite negligible radius error, and option (D) incorrectly combines the two exponents multiplicatively.

Step 3: Conclusion The relative uncertainty in the mass of Earth is 2×10^{-2}, or two percent. Hence, the correct option is (B).

Quick Tip to Solve: Write the target as a power law first: with radius fixed, $M \propto T^{-2}$, so its fractional uncertainty is twice that of the period.

52. A heating element has a resistance of 100 Ω at room temperature. When it is connected to a supply of 220 V, a steady current of 2 A passes in it and temperature is 500°C more than room temperature. what is the temperature coefficient of resistance of the heating element ? [4 marks]

(A) $0.5 \times 10^{-4}\ ^{o}C^{-1}$

(B) $5 \times 10^{-4}\ ^{o}C^{-1}$

(C) $1 \times 10^{-4}\ ^{o}C^{-1}$

(D) $2 \times 10^{-4}\ ^{o}C^{-1}$

Correct Answer: (D) $2 \times 10^{-4}\ ^{o}C^{-1}$

Solution:

Concept:

- Ohm's law gives the operating resistance from steady voltage and current: $R = V/I$.
- The linear temperature law relative to room temperature is $R = R_0(1 + \alpha\Delta T)$, where R_0 is the room-temperature resistance.

Step 1: Find the hot resistance The element has reached a steady operating state, so the supplied voltage and current determine its hot resistance, not its room-temperature resistance. Thus

$$R = \frac{220}{2} = 110\,\Omega.$$

The resistance increase is $\Delta R = 110 - 100 = 10\,\Omega$. This positive change is consistent with a positive temperature coefficient for the heating element. The stated temperature rise is $500\,^{\circ}\mathrm{C}$; an absolute room-temperature value is unnecessary.

Step 2: Calculate and check the coefficient Rearrange the linear resistance relation to obtain

$$\alpha = \frac{R - R_0}{R_0 \Delta T} = \frac{10}{100 \times 500} = 2 \times 10^{-4}\,^{\circ}\mathrm{C}^{-1}.$$

Substitution checks the result: $100[1+(2\times10^{-4})(500)] = 110\,\Omega$, giving the observed current $220/110 =$

2 A. Options (A), (B) and (C) would instead predict resistance increases of 2.5, 25 and $5\,\Omega$, respectively, so none reproduces the stated steady current.

Step 3: Conclusion The temperature coefficient is $2 \times 10^{-4}\,^{\circ}\mathrm{C}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the temperature rise directly and divide the fractional resistance increase, 0.10, by 500; do not substitute the hot resistance for R_0.

53. A ray of light is incident at an angle of 60° on one face of a prism of angle 30°. The emergent ray of light makes an angle of 30° with incident ray. The angle made by the emergent ray with second face of prism will be : [4 marks]

(A) 0°

(B) 90°

(C) 45°

(D) 30°

Correct Answer: (B) 90°

Solution:

Concept:

- For a prism, the angle of deviation satisfies $\delta = i + e - A$, with incidence and emergence measured from the respective surface normals.
- The angle a ray makes with a plane face is complementary to its angle with the normal to that face.

Step 1: Identify the angular quantities The incident ray makes $i = 60^{\circ}$ with the normal to the first face, and the prism angle is $A = 30^{\circ}$. The angle between the incident and emergent directions is the deviation, $\delta = 30^{\circ}$. The prism relation follows by adding the deviations at its two faces: $(i - r_1) + (e - r_2)$, where the internal refraction angles obey $r_1 + r_2 = A$. No refractive index is needed because the external angular data already specify the emergence angle.

Step 2: Find emergence and convert to the face angle Substitute the given values into the deviation equation:

$$30^{\circ} = 60^{\circ} + e - 30^{\circ}, \qquad e = 0^{\circ}.$$

Thus the emergent ray is along the normal to the second face. Its angle with that face is consequently

$$90^{\circ} - e = 90^{\circ}.$$

Option (A) gives the angle with the normal rather than the requested face. Options (C) and (D) neither satisfy the prism deviation relation nor give the complement of the calculated emergence angle. The distinction between a face and its normal is decisive.

Step 3: Conclusion The emergent ray makes an angle of 90° with the second face of the prism. Hence, the correct option is (B).

Quick Tip to Solve: Apply prism formulae using angles with normals, then take the complement if the question asks for an angle with a face.

54. Two particles of the same mass m are moving in circular orbits because of force, given by $F(r) = \frac{-16}{r} - r^3$

The first particle is at a distance r = 1, and the second, at r = 4. The best estimate for the ratio of kinetic energies of the first and the second particle is closest to : [4 marks]

(A) 6×10^{-2}

(B) 3×10^{-3}

(C) 10^{-1}

(D) 6×10^{2}

Correct Answer: (A) 6×10^{-2}

Solution:

Concept:

- For a circular orbit, the inward force magnitude equals mv^2/r; the negative sign in a radial force denotes inward direction.
- The kinetic energy is $K = mv^2/2 = r|F(r)|/2$. The two masses cancel from the energy ratio.

Step 1: Convert the radial force to kinetic energy The given force is $F(r) = -16/r - r^3$. For positive radii both terms point inward, so its magnitude is $16/r + r^3$. Equating this to the centripetal force gives

$$\frac{mv^2}{r} = \frac{16}{r} + r^3.$$

Multiplication by $r/2$ yields

$$K(r) = \frac{1}{2}mv^2 = \frac{16 + r^4}{2}.$$

It would be incorrect to use the signed inward force as a negative kinetic energy, or to identify the ratio of forces directly with the ratio of kinetic energies.

Step 2: Evaluate the two radii and select the nearest estimate At $r_1 = 1$, $K_1 = (16 + 1)/2 = 17/2$. At $r_2 = 4$, $K_2 = (16 + 4^4)/2 = 272/2$. Therefore

$$\frac{K_1}{K_2} = \frac{17}{272} = \frac{1}{16} = 0.0625 = 6.25 \times 10^{-2}.$$

This is closest to 6×10^{-2}. The value 3×10^{-3} is much too small, 10^{-1} is a poorer approximation, and 6×10^{2} contradicts the fact that the outer orbit has the larger kinetic energy. The ratio is dimensionless.

Step 3: Conclusion The kinetic-energy ratio is $1/16$, closest to 6×10^{-2}. Hence, the correct option is (A).

Quick Tip to Solve: For any circular orbit under a central force, compute $r|F|/2$ directly; this avoids finding the speed or mass separately.

55. Unpolarized light of intensity I is incident on a system of two polarizers, A followed by B. The intensity of emergent light is I/2. If a third polarizer C is placed between A and B, the intensity of emergent light is reduced to I/3. The angle between the polarizers A and C is θ. Then : [4 marks]

(A) $\cos\theta = \left(\frac{2}{3}\right)^{1/2}$

(B) $\cos\theta = \left(\frac{2}{3}\right)^{1/4}$

(C) $\cos\theta = \left(\frac{1}{3}\right)^{1/2}$

(D) $\cos\theta = \left(\frac{1}{3}\right)^{1/4}$

Correct Answer: (B) $\cos\theta = \left(\frac{2}{3}\right)^{1/4}$

Solution:

Concept:

- An ideal first polarizer transmits half the intensity of unpolarized incident light.
- Malus law gives $I_{\text{out}} = I_{\text{in}} \cos^2\theta$ for linearly polarized light and an analyzer at relative angle θ.

Step 1: Determine the original alignment After polarizer A, the intensity is $I_A = I/2$. If B has angle ϕ relative to A, the original transmitted intensity is $(I/2)\cos^2\phi$. Since this is given as $I/2$, $\cos^2\phi = 1$: A and B have parallel transmission axes, with axes understood modulo 180°. Their common orientation is essential; one cannot treat the two angles introduced by the middle polarizer as independent.

Step 2: Apply Malus law at both subsequent polarizers With C between the parallel polarizers, its angle to A is θ, and its angle to B has the same squared cosine. Thus

$$I_C = \frac{I}{2}\cos^2\theta, \qquad I_{\text{final}} = \frac{I}{2}\cos^4\theta.$$

Equating the latter to $I/3$ gives

$$\cos^4\theta = \frac{2}{3}, \qquad \cos\theta = \left(\frac{2}{3}\right)^{1/4},$$

using the acute angle between transmission axes. Option (A) applies only one cosine-squared factor. Options (C) and (D) also miss the initial halving by A, producing the wrong fraction. Substitution of the fourth root indeed gives $(I/2)(2/3) = I/3$.

Step 3: Conclusion For the acute angle between A and C, $\cos\theta = (2/3)^{1/4}$. Hence, the correct option is (B).

Quick Tip to Solve: When a polarizer is inserted between two parallel axes, two Malus-law factors occur, so the final intensity contains $\cos^4\theta$.

56. A plane electromagnetic wave of wavelength λ has an intensity I. It is propagatting along the positive Y-direction. The allowed expressions for the electric and magnetic fields are givn by :

[4 marks]

(A) $\vec{E} = \sqrt{\frac{2I}{\varepsilon_0 c}} \cos\left[\frac{2\pi}{\lambda}(y - ct)\right] \hat{k}; \quad \vec{B} = +\frac{E}{c}\hat{i}$

(B) $\vec{E} = \sqrt{\frac{2I}{\varepsilon_0 c}} \cos\left[\frac{2\pi}{\lambda}(y + ct)\right] \hat{k}; \quad \vec{B} = \frac{E}{c}\hat{i}$

(C) $\vec{E} = \sqrt{\frac{I}{\varepsilon_0 c}} \cos\left[\frac{2\pi}{\lambda}(y - ct)\right] \hat{k}; \quad \vec{B} = \frac{E}{c}\hat{i}$

(D) $\vec{E} = \sqrt{\frac{I}{\varepsilon_0 c}} \cos\left[\frac{2\pi}{\lambda}(y - ct)\right] \hat{i}; \quad \vec{B} = \frac{E}{c}\hat{k}$

Correct Answer: (A) $\vec{E} = \sqrt{\frac{2I}{\varepsilon_0 c}} \cos\left[\frac{2\pi}{\lambda}(y - ct)\right] \hat{k}; \quad \vec{B} = +\frac{E}{c}\hat{i}$

Solution:

Concept:

- A wave with phase $k(y - ct)$ propagates along positive Y; $k = 2\pi/\lambda$ is the wave number.
- For a sinusoidal electromagnetic wave in vacuum, $I = \frac{1}{2}\varepsilon_0 c E_0^2$, $B_0 = E_0/c$, and energy propagates along $\vec{E} \times \vec{B}$.

Step 1: Fix the phase and amplitude For positive Y propagation, a constant-phase point obeys $y - ct =$ constant, so $dy/dt = c > 0$. The required phase is therefore $(2\pi/\lambda)(y - ct)$. The time-averaged intensity fixes the peak electric field as

$$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}.$$

The factor two follows from averaging the square of a sinusoid. Options (C) and (D) use $\sqrt{I/(\varepsilon_0 c)}$, whose peak field would yield average intensity $I/2$, not the specified I.

Step 2: Check the field directions and reject the alternatives In option (A), the electric field is along $\hat{k}$ and the magnetic field is along $+\hat{i}$, with the same phase and scalar relation $B = E/c$. The cross product is

$$\hat{k} \times \hat{i} = \hat{j},$$

which is positive Y as required. Both fields are transverse and mutually perpendicular. Option (B) has the correct peak amplitude but phase $y + ct$, representing negative Y motion, and thus fails the propagation requirement. Option (D) also has $\hat{i} \times \hat{k} = -\hat{j}$, inconsistent with its positive Y phase. Hence only the first pair simultaneously satisfies the intensity, phase and right-handed direction conditions.

Step 3: Conclusion The fields in option (A), with peak electric field $\sqrt{2I/(\varepsilon_0 c)}$, phase $(2\pi/\lambda)(y - ct)$, and $\vec{B} = (E/c)\hat{i}$, meet all conditions. Hence, the correct option is (A).

Quick Tip to Solve: Check an electromagnetic-wave option in three passes: phase direction, $\vec{E} \times \vec{B}$ direction, then the factor two in average intensity.

57. A particle executes simple harmonic motion and is located at x = a, b and c at times t_0, $2t_0$ and $3t_0$ respectively. The freqquency of the oscillation is : [4 marks]

(A) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$
(B) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{2c}\right)$
(C) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{2a+3c}{b}\right)$
(D) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+2b}{3c}\right)$

Correct Answer: (A) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$

Solution:

Concept:

- An SHM displacement can be written as $x(t) = A\cos(\omega t + \phi)$, where A is the amplitude, ω the angular frequency and ϕ the initial phase.
- For three equally spaced times, the identity $\cos(u-v) + \cos(u+v) = 2\cos u \cos v$ eliminates the unknown amplitude and phase. Ordinary frequency is $f = \omega/(2\pi)$.

Step 1: Write the phases about the middle observation Put $u = 2\omega t_0 + \phi$ and $v = \omega t_0$. Then the three measured displacements become

$$a = A\cos(u-v), \qquad b = A\cos u, \qquad c = A\cos(u+v).$$

The observations are equally spaced in time, so their phases differ by the same amount. The value of the initial phase is therefore unnecessary.

Step 2: Add the first and third displacements Apply the cosine identity to obtain

$$a + c = 2A\cos u\cos v = 2b\cos(\omega t_0).$$

Thus, for $b \neq 0$,

$$\cos(\omega t_0) = \frac{a+c}{2b}.$$

Taking the principal inverse cosine gives the frequency expression offered in the options. The sum must involve the two outer observations and the denominator the middle one; the other three options do not satisfy this identity.

Step 3: Interpret the inverse cosine The conventional MCQ expression uses the principal phase increment $0 \le \omega t_0 \le \pi$. More generally, discrete position observations allow phase increments $2\pi k \pm \cos^{-1}((a+c)/(2b))$; they alone do not distinguish higher-frequency aliases. If $b = 0$, the displayed ratio cannot be used and the data need additional information.

Step 4: Conclusion With the usual principal-phase interpretation,

$$f = \frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right).$$

Hence, the correct option is (A).

Quick Tip to Solve: For three equally spaced SHM observations, add the outer displacements: their sum is twice the middle displacement times the cosine of the phase increment.

58. Two sitar strings, A and B, playing the note 'Dha' are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string B s slightly increased and the beat frequency is found to decrease by 3Hz. If the frequency of A is 425 Hz, the original frequency of B is : [4 marks]

(A) 430 Hz

(B) 420 Hz

(C) 428 Hz

(D) 422 Hz

Correct Answer: (B) 420 Hz

Solution:

Concept:

- The beat frequency of two nearly equal tones is $f_b = |f_A - f_B|$. It measures a frequency difference, not either string frequency separately.
- For the same vibrating length and linear mass density, $f = (2L)^{-1}\sqrt{T/\mu}$. Increasing tension T therefore increases the frequency of string B.

Step 1: Find the two initial possibilities The initial beat frequency is $5\,\text{Hz}$, and $f_A = 425\,\text{Hz}$. Hence

$$|425 - f_B| = 5 \quad \Rightarrow \quad f_B = 420 \text{ or } 430\,\text{Hz}.$$

The values 428 Hz and 422 Hz each differ from 425 Hz by only 3 Hz, so neither matches the initial observation. The reported decrease of 3 Hz is a change in beat frequency, rather than the original frequency difference.

Step 2: Use the direction of the tension change An increase in B's tension raises its natural frequency. If B initially had 430 Hz, it would move farther above 425 Hz, increasing the beat frequency. That contradicts the observed decrease. If B initially had 420 Hz, a small increase takes it toward 425 Hz, decreasing the beat frequency as required. This selects the lower initial frequency.

Step 3: Check against the new beats The new beat frequency is $5 - 3 = 2\,\text{Hz}$. Before crossing A's frequency, B can therefore rise from 420 Hz to 423 Hz. This yields $|425 - 423| = 2\,\text{Hz}$, exactly the measured new difference. Even if a slightly larger increase passes through unison and reaches 427 Hz, the original frequency still has to have been 420 Hz; an initial 430 Hz cannot give a decrease when raised.

Step 4: Conclusion The original frequency of string B was $420\,\text{Hz}$. Hence, the correct option is (B).

Quick Tip to Solve: First write both frequencies allowed by the beats; then use whether increasing tension makes the beat frequency grow or shrink to choose the correct side of the known tone.

59. At some instant, a radioactive sample S_1 having an activity 5 μCi has twice the number of nuclei as another sample S_2 which has an activity of 10 μCi. The half lives of S_1 and S_2 are : [4 marks]

(A) 20 years and 5 years, respectively

(B) 20 years and 10years, respectively

(C) 5 years and 20 years, respectively

(D) 10 years and 20 years, respectively

Correct Answer: (A) 20 years and 5 years, respectively

Solution:

Concept:

- The activity of a radioactive sample is $R = \lambda N$, where N is its number of undecayed nuclei and λ its decay constant.
- The half-life is $T_{1/2} = \ln 2/\lambda$. Thus two samples with different numbers of nuclei cannot have their half-lives compared using activity alone.

Step 1: Relate the decay constants The given data are $R_1 = 5\,\mu$Ci, $R_2 = 10\,\mu$Ci and $N_1 = 2N_2$. Since both activities are in the same unit, their ratio needs no conversion to decays per second. Dividing the two activity equations gives

$$\frac{R_1}{R_2} = \frac{\lambda_1 N_1}{\lambda_2 N_2} = 2\frac{\lambda_1}{\lambda_2} = \frac{1}{2}.$$

Therefore $\lambda_1/\lambda_2 = 1/4$. Each nucleus in sample 1 has only one quarter the decay probability per unit time of a nucleus in sample 2.

Step 2: Convert the decay-constant ratio to a half-life ratio Half-life is inversely proportional to the decay constant, so

$$\frac{T_1}{T_2} = \frac{\lambda_2}{\lambda_1} = 4.$$

Sample 1 must therefore have the longer half-life. A smaller activity despite twice as many nuclei is consistent with this smaller decay constant. Reversing the ratio would incorrectly predict the first sample to be more rapidly decaying.

Step 3: Test every listed pair The ratio for option (A) is $20/5 = 4$, as required. Option (B) gives $20/10 = 2$; option (C) gives $5/20 = 1/4$; option (D) gives $10/20 = 1/2$. Only the first pair matches. The data establish a ratio, not an absolute half-life in years; the offered alternatives make the pair selectable without determining either absolute value independently.

Step 4: Conclusion Among the listed pairs, $T_1 = 20$ years and $T_2 = 5$ years satisfy $T_1/T_2 = 4$. Hence, the correct option is (A).

Quick Tip to Solve: Use $T_1/T_2 = (N_1/N_2)(R_2/R_1)$: the number ratio and inverse activity ratio multiply, so activity alone can be misleading.

60. A carrier wave of peak voltage 14 V is used for transmitting a message signal. The peak voltage of modulating signal given to achieve a modulation index of 80% will be : [4 marks]

(A) 7 V

(B) 28 V

(C) 11.2 V

(D) 22.4 V

Correct Answer: (C) 11.2 V

Solution:

Concept:

- The amplitude-modulation index is $m = V_m/V_c$, where V_m is the peak voltage of the modulating signal and V_c is the peak carrier voltage.
- A percentage modulation must be divided by 100 before substitution. The ratio uses matching amplitude definitions: both peak values here, rather than a mixture of peak and rms values.

Step 1: Convert the specified modulation depth The carrier peak voltage is $V_c = 14\,\text{V}$. A modulation index of 80% corresponds to

$$m = \frac{80}{100} = 0.80.$$

This is dimensionless because it is a voltage ratio. It describes how strongly the carrier amplitude varies about its unmodulated value; it does not mean that the signal voltage is 80 times the carrier voltage.

Step 2: Calculate the required signal amplitude Rearrange the definition without changing the voltage convention:

$$V_m = mV_c = (0.80)(14\,\text{V}) = 11.2\,\text{V}.$$

Both given and required voltages are peaks, so no factor of $\sqrt{2}$ or 2 enters. Because $m < 1$, the envelope remains in the ordinary, non-overmodulated regime. Its limiting amplitudes are $V_c(1-m) = 2.8\,\text{V}$ and $V_c(1+m) = 25.2\,\text{V}$; these envelope values are not the required signal amplitude.

Step 3: Compare all the choices A 7 V signal would give $m = 7/14 = 0.50$, or 50 percent. A 28 V signal would give $m = 2$, or 200 percent. A 22.4 V signal would give $m = 1.6$, or 160 percent. Only 11.2 V gives $11.2/14 = 0.80$. The larger choices would overmodulate the carrier and are inconsistent with the specified 80 percent modulation.

Step 4: Conclusion The modulating signal must have a peak voltage of $11.2\,\text{V}$. Hence, the correct option is (C).

Quick Tip to Solve: For amplitude modulation, multiply the carrier peak voltage by the fractional modulation depth; do not confuse the signal peak with either extreme of the envelope.

Chemistry (120 marks)

61. In Wilkinson's catalyst, the hybridization of central metal ion and its shape are respectively :

[4 marks]

(A) sp^3d, trigonal bipyramidal

(B) sp^3, tetrahedral

(C) dsp^2, square planar

(D) d^2sp^3, octahedral

Correct Answer: (C) dsp^2, square planar

Solution:

Concept:

- Wilkinson's catalyst is $[\mathrm{RhCl(PPh_3)_3}]$, with three neutral triphenylphosphine ligands and one chloride ligand bound to rhodium.
- The central ion is $\mathrm{Rh(I)}$, a d^8 metal ion. Its four-coordinate square-planar arrangement is described by dsp^2 hybridisation in the valence-bond model.

Step 1: Determine oxidation state and coordination number Each $\mathrm{PPh_3}$ is neutral, whereas the chloride ligand carries charge -1. Because the complex is neutral, rhodium has oxidation state $+1$:

$$x + (-1) + 3(0) = 0 \quad \Rightarrow \quad x = +1.$$

There are four directly bonded donor atoms, one chlorine and three phosphorus atoms. Thus the coordination number is four, regardless of the number of phenyl rings attached to each phosphorus.

Step 2: Assign the orbital arrangement Rhodium is a group-9 metal, and the formal $+1$ state gives a d^8 configuration. Four-coordinate d^8 Rh(I) in this phosphine complex favours a square-planar ligand arrangement. In the conventional valence-bond description, one d orbital, one s orbital and two p orbitals combine to provide four coplanar dsp^2 hybrid orbitals directed towards the four ligands. This identifies the hybridisation and geometry as a consistent pair.

Step 3: Eliminate the other geometry pairs Trigonal-bipyramidal sp^3d requires five donor directions, and octahedral d^2sp^3 requires six; both conflict with this catalyst's coordination number. Tetrahedral sp^3 has four directions but does not describe the known square-planar d^8 Rh(I) phosphine complex. A coordination number of four alone does not distinguish tetrahedral from square planar; the metal configuration and ligand environment are decisive.

Step 4: Conclusion The central metal has dsp^2 hybridisation and square-planar geometry. Hence, the correct option is (C).

Quick Tip to Solve: For Wilkinson's catalyst, remember $\mathrm{RhCl(PPh_3)_3}$: four donors and d^8 Rh(I) identify the square-planar dsp^2 description.

62. Which of the following conversions involves change in both shape and hybridisation ? [4 marks]

(A) $NH_3 \rightarrow NH_4^+$

(B) $CH_4 \rightarrow C_2H_6$

(C) $H_2O \rightarrow H_3O^+$

(D) $BF_3 \rightarrow BF_4^-$

Correct Answer: (D) $BF_3 \rightarrow BF_4^-$

Solution:

Concept:

- In the usual hybridisation model, three electron domains about an atom correspond to sp^2 hybridisation, while four correspond to sp^3. A domain can be a bond or a lone pair.
- Molecular shape describes the positions of atoms, whereas electron-domain geometry also counts lone pairs. A change of shape need not imply a change of hybridisation.

Step 1: Check ammonia and water conversions In NH_3, nitrogen has three bond pairs and one lone pair: it is sp^3 with trigonal-pyramidal molecular shape. In NH_4^+, four bond pairs give tetrahedral shape, but nitrogen remains sp^3. Similarly, H_2O has two bond pairs and two lone pairs, giving bent shape with sp^3 oxygen; H_3O^+ has three bond pairs and one lone pair, giving trigonal-pyramidal shape but still sp^3 oxygen. Thus (A) and (C) change shape only.

Step 2: Check the carbon conversion Carbon in CH_4 forms four sigma bonds and is tetrahedral with sp^3 hybridisation. Each carbon in C_2H_6 also forms four sigma bonds, three to hydrogen and one to the other carbon. Each carbon remains locally tetrahedral and sp^3. Changing from one carbon atom to two does not by itself change the hybridisation of the carbon centres, so (B) does not meet the condition.

Step 3: Identify the change at boron In BF_3, boron has three bond pairs and no lone pair. Its three electron domains give trigonal-planar shape and sp^2 hybridisation. Acceptance of a lone pair from F^- produces BF_4^-, with four B–F sigma bonds, tetrahedral shape and sp^3 hybridisation. Both the number of electron domains and the atomic arrangement change in this conversion.

Step 4: Conclusion $BF_3 \longrightarrow BF_4^-$ changes trigonal-planar sp^2 boron into tetrahedral sp^3 boron. Hence, the correct option is (D).

Quick Tip to Solve: Count bonds plus lone pairs before assigning hybridisation; then count bonded atoms to name the shape. Protonating a lone pair often changes shape while keeping sp^3.

63. The major product of the following reaction is:

$$C_6H_5CH{=}CHCH_3 \xrightarrow{HBr} \textbf{?}$$

[4 marks]

(A) $C_6H_5CH(Br)CH_2CH_3$

(B) $C_6H_5CH_2CH(Br)CH_3$

(C) $C_6H_5CH_2CH_2CH_2Br$

(D) p-$BrC_6H_4CH{=}CHCH_3$

Correct Answer: (A) $C_6H_5CH(Br)CH_2CH_3$

Solution:

Concept:

- In the absence of peroxide conditions, HBr adds to an alkene by electrophilic protonation followed by nucleophilic attack of Br^-.
- The preferred protonation produces the more stable carbocation. A benzylic carbocation is resonance-stabilised by the adjacent benzene ring, whereas an ordinary secondary carbocation lacks that stabilisation.

Step 1: Compare the two protonation directions The substrate is $C_6H_5CH{=}CHCH_3$, an alkene with a phenyl group directly attached to one double-bond carbon. Addition of a proton to the carbon nearer the terminal methyl group leaves positive charge on the benzylic carbon:

$$C_6H_5CH{=}CHCH_3 + H^+ \longrightarrow C_6H_5CH^+CH_2CH_3.$$

The benzylic cation can delocalise its charge into the aromatic ring. Protonation in the opposite direction would give $C_6H_5CH_2CH^+CH_3$, whose charged carbon is separated from the ring by a saturated carbon and is not benzylic.

Step 2: Capture the stabilised carbocation Bromide attacks the positively charged benzylic carbon to give

$$C_6H_5CH^+CH_2CH_3 + Br^- \longrightarrow C_6H_5CH(Br)CH_2CH_3.$$

The carbon skeleton is retained, and the double bond becomes a single bond. The preferred regioisomer therefore has bromine on the carbon directly attached to the benzene ring. No peroxide or radical initiator is specified, so an anti-Markovnikov radical pathway is not the reaction to apply.

Step 3: Eliminate the remaining structures Option (B), $C_6H_5CH_2CH(Br)CH_3$, would arise from the less stable non-benzylic carbocation direction. Option (C) places bromine at the remote terminal carbon, which is not obtained by direct addition across the given double bond. Option (D) retains the side-chain double bond and brominates the ring; HBr alone does not provide the electrophilic bromination conditions needed for that substitution.

Step 4: Conclusion The major product is $C_6H_5CH(Br)CH_2CH_3$, the benzylic bromide. Hence, the correct option is (A).

Quick Tip to Solve: For HBr addition to an alkene beside benzene, compare carbocation resonance stabilisation before applying a simple alkyl-substitution count.

64. Which of the following compounds will most readily be dehydrated to give alkene under acidic condition ? [4 marks]

(A) 1- Pentanol

(B) 4-Hydroxypentan - 2-one

(C) 3-Hydroxypentan - 2-one

(D) 2-Hydroxycyclopentanone

Correct Answer: (B) 4-Hydroxypentan - 2-one

Solution:

Concept:

- Acid-catalysed dehydration removes water from an alcohol to form an alkene. Protonation makes the hydroxyl group a better leaving group.
- A beta-hydroxy carbonyl compound readily loses water to form an alpha,beta-unsaturated carbonyl compound. Conjugation between the new double bond and the carbonyl stabilises the product.

Step 1: Locate the hydroxyl group relative to the carbonyl In 4-hydroxypentan-2-one the carbonyl is at C2, the alpha carbon is C3 and the hydroxyl-bearing beta carbon is C4. Its condensed structure is $CH_3COCH_2CH(OH)CH_3$. It therefore has the beta-hydroxy carbonyl arrangement produced in an aldol addition.

Step 2: Eliminate water and identify the stabilisation Loss of the hydroxyl group from C4 and hydrogen from C3 gives

$$CH_3COCH_2CH(OH)CH_3 \xrightarrow[-H2O]{H^+} CH_3COCH{=}CHCH_3.$$

The new double bond is directly conjugated with the carbonyl. The favourable formation of this conjugated enone makes dehydration particularly ready under acidic conditions.

Step 3: Compare the remaining alcohols 1-Pentanol is a primary alcohol and lacks a carbonyl group that could favour this aldol-type dehydration. The hydroxyl groups of 3-hydroxypentan-2-one and 2-hydroxycyclopentanone are alpha, rather than beta, to their carbonyls. They do not have the same beta-hydroxy carbonyl motif with the favourable elimination pathway. Thus simple alcohol substitution alone does not decide this comparison.

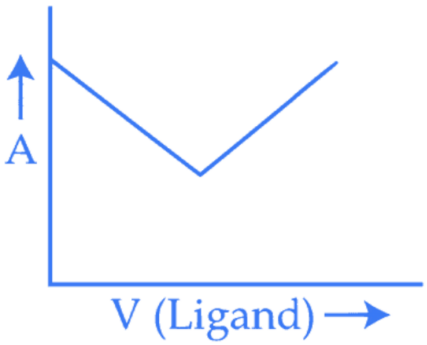
Step 4: Conclusion 4-Hydroxypentan-2-one most readily loses water to give a conjugated enone. Hence, the correct option is (B).

Quick Tip to Solve: For hydroxy ketones, number from the carbonyl first: an OH on the beta carbon signals easy dehydration to a conjugated enone.

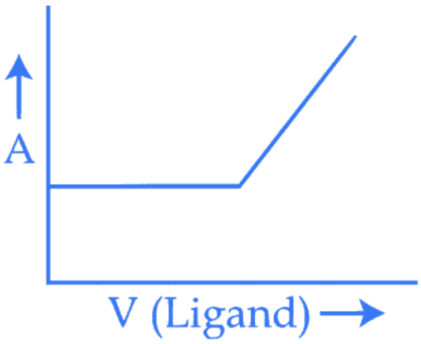
65. In a complexometric titration of metal ion with ligand

M(Metal ion) + L(Ligand) → C(Complex) end point is estimated spectrophoto - metrically (through light absorption). If 'M' and 'C' do not absorb light and only 'L' absorbs, then the titration plot between absorbed light (A) versus volume of ligand 'L' (V) would look like : [4 marks]

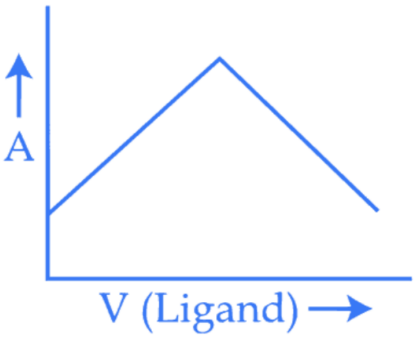
(A)



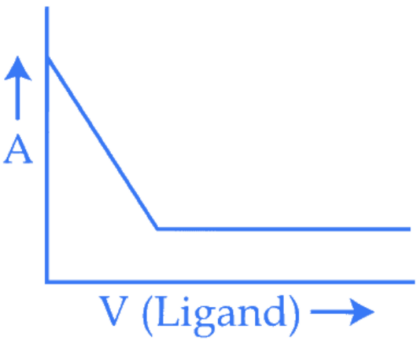
(B)



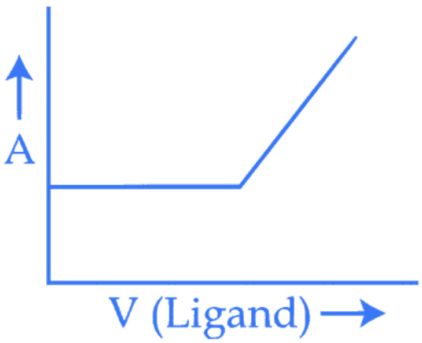
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- Beer–Lambert law gives absorbance $A = \varepsilon l c$, where c is the concentration of the absorbing species, l is optical path length and ε is molar absorptivity.
- Only free ligand L absorbs in this question. Metal M and complex C make no contribution to the measured signal.

Step 1: Follow ligand addition before equivalence Initially there is no free ligand. As ligand is added to a solution containing uncomplexed metal ions, it is consumed in forming the complex, $\mathrm{M} + \mathrm{L} \longrightarrow \mathrm{C}$. Before the equivalence point, free-ligand concentration remains negligible for the strongly forming complex used in the titration. Consequently the absorbance remains at its baseline rather than growing with the amount of complex present.

Step 2: Follow ligand addition after equivalence At equivalence all available metal has been complexed. Additional ligand can no longer be consumed by metal ions and accumulates as free L. Its concentration, and therefore the measured absorbance, increases with further ligand addition. The idealised titration curve has a nearly horizontal initial part followed by a rising part; their intersection locates the equivalence volume.

Step 3: Match the change of slope to the graphs Graph (B) has precisely the required horizontal-then-rising shape. A decreasing initial limb, as in (A) or (D), would require an initially absorbing species being removed, which is excluded because M does not absorb. The rise followed by a fall in (C) would instead require an absorbing product or another change in the absorbing species. A small baseline offset drawn above the axis does not change the diagnostic slope pattern.

Step 4: Conclusion The absorbance is initially constant and then increases when free ligand is in excess. Hence, the correct option is (B).

Quick Tip to Solve: Identify which species absorbs before sketching the curve: an absorbing titrant gives a rising limb only after it begins to remain unreacted.

66. The incorrect statement is : [4 marks]

(A) Cu^{2+} salts give red coloured borax bead test in reducing flame.

(B) $Cu2^{+}$ and Ni^{2+} ions give black precipitate with H_2S in presence of HCl solution.

(C) Ferric ion gives blood red color with potasium thiocyanate.

(D) Cu^{2+} ion gives chocolate coloured preciitate with potassium ferrocyanide solution.

Correct Answer: (B) $Cu2^{+}$ and Ni^{2+} ions give black precipitate with H_2S in presence of HCl solution.

Solution:

Concept:

- The acidity of a solution controls sulfide-ion concentration because $\mathrm{H_2S} \rightleftharpoons 2\,\mathrm{H^+} + \mathrm{S^{2-}}$. Added acid suppresses formation of $\mathrm{S^{2-}}$.
- Only sufficiently insoluble metal sulfides precipitate in the low sulfide concentration of acidic group-II analysis. Nickel sulfide requires the higher sulfide concentration obtained in ammoniacal

medium.

Step 1: Test the sulfide statement Copper(II) gives black CuS with hydrogen sulfide in dilute hydrochloric acid. Nickel(II), however, is not precipitated as black NiS under that same acidic condition. Nickel is separated in the later sulfide group in ammoniacal medium. Therefore the assertion that both ions give black precipitates in the presence of HCl is incorrect. The condition of precipitation, rather than the colour of isolated nickel sulfide, is decisive.

Step 2: Verify the other colour tests Copper salts give a red, opaque borax bead in the reducing flame owing to reduced copper species, so (A) is a recognised positive test. Ferric ions form a blood-red thiocyanate complex with potassium thiocyanate, supporting (C). Copper(II) ions react with potassium ferrocyanide to give chocolate-brown copper(II) ferrocyanide, supporting (D).

Step 3: Relate acidity to selective precipitation For a divalent metal sulfide the precipitation condition is that $[\mathrm{M}^{2+}][\mathrm{S}^{2-}]$ exceed its solubility product. The very small sulfide concentration in acid is enough for copper sulfide but insufficient for the nickel test. The remaining statements need no such correction and are not the requested incorrect one.

Step 4: Conclusion Nickel(II) does not give its black sulfide precipitate with hydrogen sulfide in the stated acidic medium. Hence, the correct option is (B).

Quick Tip to Solve: In sulfide-group analysis always check the medium: black CuS precipitates in acid, whereas black NiS needs ammoniacal conditions.

67. When 9.65 ampere current was passed for 1.0 hour into nitrobenzene in acidic medium, the amount of p-aminophenol produced is : [4 marks]

(A) 9.81 g

(B) 10.9 g

(C) 98.1 g

(D) 109.0 g

Correct Answer: (A) 9.81 g

Solution:

Concept:

- Faraday law relates charge to electrons transferred: $n(e^-) = Q/F$, with $Q = It$ and $F = 96500\,\mathrm{C\,mol^{-1}}$.
- In acidic electrolytic reduction, nitrobenzene is reduced to phenylhydroxylamine using four electrons; acid rearrangement then gives para-aminophenol without further electron transfer.

Step 1: Identify the electron requirement The four-electron reduction is

$$\mathrm{C_6H_5NO_2 + 4\,H^+ + 4\,e^- \longrightarrow C_6H_5NHOH + H_2O.}$$

Phenylhydroxylamine undergoes the Bamberger rearrangement in acid to para-aminophenol. Thus one mole of the final product corresponds to four moles of electrons, rather than the six-electron conversion of nitrobenzene all the way to aniline.

Step 2: Calculate the delivered charge and product amount Convert the time before multiplying by current:

$$t = 1.0 \times 3600 = 3600\,\text{s}, \qquad Q = 9.65 \times 3600 = 34740\,\text{C}.$$

Hence

$$n(e^-) = \frac{34740}{96500} = 0.360\,\text{mol}, \qquad n(\text{product}) = \frac{0.360}{4} = 0.0900\,\text{mol}.$$

This uses the stated electrolytic conversion with quantitative current efficiency.

Step 3: Convert moles to mass and compare Para-aminophenol has formula C_6H_7NO, so its molar mass is

$$M = 6(12) + 7(1) + 14 + 16 = 109\,\text{g mol}^{-1}.$$

The mass produced is

$$m = 0.0900 \times 109 = 9.81\,\text{g}.$$

The value 10.9 g would require 0.100 mol, inconsistent with the charge; 98.1 g and 109.0 g are also too large by about an order of magnitude.

Step 4: Conclusion The amount of para-aminophenol produced is $9.81\,\text{g}$. Hence, the correct option is (A).

Quick Tip to Solve: Determine the actual reduction product before using Faraday law: the hydroxylamine-to-aminophenol route needs four electrons per molecule.

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68. An unknown chlorohydrocarbon has 3.55% of chlorine. If each molecule of the hydrocarbon has one chlorine atom only; chlorine atoms present in 1 g of chlorohydrocarbon are :
(Atomic wt. of Cl = 35.5 u; Avogadro constant = 6.023 $\times$ 10^{23} mol^{-1}) [4 marks]

(A) 6.023×10^{20}

(B) 6.023×10^{9}

(C) 6.023×10^{21}

(D) 6.023×10^{23}

Correct Answer: (A) 6.023×10^{20}

Solution:

Concept:

- A mass percentage of chlorine specifies grams of chlorine per 100 g of the compound. It must first be divided by 100 to obtain the mass fraction.
- The number of chlorine atoms is $N = (m_{\text{Cl}}/M_{\text{Cl}})N_A$, where $M_{\text{Cl}} = 35.5\,\text{g mol}^{-1}$.

Step 1: Find the chlorine mass in the sample The sample mass is 1 g and its chlorine content is 3.55 percent. Therefore

$$m_{\text{Cl}} = 1 \times \frac{3.55}{100} = 0.0355\,\text{g}.$$

This is the mass of chlorine atoms combined in the chlorohydrocarbon, not the mass of the whole compound. Counting chlorine atoms uses the atomic molar mass, not the molar mass of chlorine gas.

Step 2: Convert the chlorine mass to atoms The amount of chlorine atoms is

$$n_{\mathrm{Cl}} = \frac{0.0355}{35.5} = 1.00 \times 10^{-3}\,\mathrm{mol}.$$

Multiplication by the given Avogadro constant gives

$$N_{\mathrm{Cl}} = (1.00 \times 10^{-3})(6.023 \times 10^{23}) = 6.023 \times 10^{20}.$$

The powers multiply as $10^{-3}10^{23} = 10^{20}$.

Step 3: Check using the one-chlorine condition With exactly one chlorine atom per molecule, the molar mass of the compound also follows from $35.5/M = 0.0355$, giving $M = 1000\,\mathrm{g\,mol^{-1}}$. One gram is therefore 0.001 mol of molecules and contains the same number of chlorine atoms. Option (C) is ten times too large, (D) would require a full mole of chlorine atoms, and (B) is incompatible with a macroscopic sample at the stated mass percentage.

Step 4: Conclusion There are 6.023×10^{20} chlorine atoms in 1 g of the chlorohydrocarbon. Hence, the correct option is (A).

Quick Tip to Solve: Count an element directly from its mass percentage: sample mass times mass fraction, divided by atomic molar mass, then multiplied by Avogadro constant.

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69. The gas phase reaction $2NO_2(g) \rightarrow N_2O_4(g)$ is an exothermic reaction. The decomposition of N_2O_4, in equilibrium mixture of $NO_2(g)$ and $N_2O_4(g)$, can be increased by : [4 marks]

(A) lowering the temperature.

(B) increasing the pressure.

(C) addition of an inert gas at constant volume.

(D) addition of an inert gas at constant pressure.

Correct Answer: (D) addition of an inert gas at constant pressure.

Solution:

Concept:

- The decomposition equilibrium is $\mathrm{N_2O_4(g)} \rightleftharpoons 2\,\mathrm{NO_2(g)}$. Its forward direction increases the number of gas molecules and is endothermic.
- An inert gas changes equilibrium only when the imposed condition changes the partial pressures of reacting gases. Total pressure alone is not the quantity in the equilibrium expression.

Step 1: Test the temperature and pressure changes The association $2\,\mathrm{NO_2} \longrightarrow \mathrm{N_2O_4}$ is exothermic, so lowering temperature favours association and decreases decomposition. Increasing pressure favours the side with fewer gas molecules, again $\mathrm{N_2O_4}$. Thus neither (A) nor (B) increases the dissociation of the dimer.

Step 2: Distinguish constant volume from constant pressure At constant volume and temperature, adding an inert gas leaves each reactive partial pressure unchanged because $p_i = n_i RT/V$. Therefore it causes no equilibrium shift, excluding (C). At constant pressure, adding inert gas makes the mixture

expand. The partial pressures of both reacting species decrease, so the system favours the side with more gas molecules: two molecules of NO_2 rather than one of N_2O_4.

Step 3: Confirm with the reaction quotient For decomposition, $Q_p = p^2_{NO_2}/p_{N_2O_4}$. If expansion immediately scales each reactive partial pressure by $f < 1$, then

$$Q_p^{\text{new}} = \frac{(f p_{NO_2})^2}{f p_{N_2O_4}} = f K_p < K_p.$$

The reaction consequently proceeds toward more NO_2 until equilibrium is restored. This explicitly confirms an increase in decomposition rather than simply an increase in total pressure.

Step 4: Conclusion Adding an inert gas at constant pressure increases the decomposition of N_2O_4. Hence, the correct option is (D).

Quick Tip to Solve: For an inert-gas question ask whether volume changes: no shift at fixed volume, but at fixed pressure expansion favours the side with more gas moles.

70. For which of the following processes, ΔS is negative ? [4 marks]

(A) $H_2(g) \rightarrow 2H(g)$

(B) $N_2(g, 1\text{ atm}) \rightarrow N_2(g, 5\text{ atm})$

(C) C(diamond) $\rightarrow$ C(graphite)

(D) $N_2(g, 273\text{ K}) \rightarrow N_2(g, 300\text{ K})$

Correct Answer: (B) $N_2(g, 1\text{ atm}) \rightarrow N_2(g, 5\text{ atm})$

Solution:

Concept:

- Entropy measures the number of accessible microscopic arrangements. Gas expansion, heating and an increase in gas-particle number usually increase entropy.
- For isothermal compression of an ideal gas, $\Delta S = nR\ln(V_2/V_1) = -nR\ln(P_2/P_1)$. An increase in pressure therefore gives a negative entropy change.

Step 1: Evaluate the pressure change For N_2 compressed from 1 atm to 5 atm at the same temperature,

$$\Delta S = -nR\ln\left(\frac{5}{1}\right) = -nR\ln 5 < 0.$$

The available volume decreases to one fifth of its initial value. Fewer positional arrangements are accessible to the gas molecules, which explains the negative sign rather than merely relying on a formula.

Step 2: Compare dissociation and heating In (A), one mole of gaseous hydrogen molecules produces two moles of gaseous atoms. The increased number of independently moving gas particles gives a positive entropy change. In (D), warming nitrogen from 273 K to 300 K increases its accessible thermal states. At fixed pressure, $\Delta S = nC_p\ln(300/273) > 0$; the same positive sign follows using C_V at fixed volume.

Step 3: Compare the solid allotropes Graphite has a higher standard molar entropy than diamond. Therefore the conversion $\mathrm{C(diamond)} \longrightarrow \mathrm{C(graphite)}$ in (C) has positive, rather than negative, entropy change. The fact that both phases are solids does not make their entropies equal. Combining these comparisons leaves only the compression process with the required negative sign.

Step 4: Conclusion Compression of $\mathrm{N_2(g)}$ from 1 atm to 5 atm has $\Delta S < 0$. Hence, the correct option is (B).

Quick Tip to Solve: For the same amount of gas at constant temperature, use $\Delta S = -nR\ln(P_2/P_1)$: a larger final pressure immediately gives a negative sign.

71. If 50% of a reaction occurs in 100 second and 75% of the reaction occurs in 200 secod, the order of this reaction is : [4 marks]

(A) Zero

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Concept:

- The order of a reaction is identified from how its rate depends on concentration. Equal half-lives are characteristic of a first-order reaction.
- For a first-order reaction, the remaining fraction is $a_t/a_0 = e^{-kt}$, so $t_{1/2} = \ln 2/k$, independent of initial concentration.

Step 1: Translate reaction progress into concentration Let the initial concentration be a_0. At $100\,\mathrm{s}$, 50% has reacted and $a_{100} = a_0/2$. At $200\,\mathrm{s}$, 75% has reacted and $a_{200} = a_0/4$. Thus, during the second interval of 100 seconds, half of the material left after the first interval reacts. The two successive half-lives are equal, although the concentration at the beginning of the second interval is smaller.

Step 2: Check the integrated rate law and eliminate alternatives Using the first measurement gives

$$k = \frac{\ln 2}{100} = 6.93 \times 10^{-3}\,\mathrm{s}^{-1}.$$

At twice this time, the predicted fraction left is

$$e^{-k(200)} = e^{-2\ln 2} = \frac{1}{4},$$

exactly as required. A zero-order reaction removes equal amounts in equal intervals, so it would finish at 200 seconds. For second order, the time to reach one-quarter concentration is three times the first half-life, or 300 seconds. For third order it is five times the first half-life, or 500 seconds. Neither matches the stated 200 seconds.

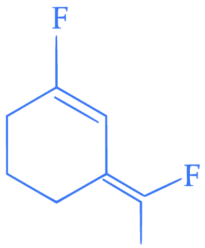
Step 3: Conclusion The constant successive half-life identifies a first-order reaction. Hence, the

correct option is (B).

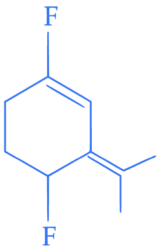
Quick Tip to Solve: Compare the fraction remaining, not the percentage reacted: 50% followed by 25% remaining means two successive halvings.

72. The most polar compound among the following is : [4 marks]

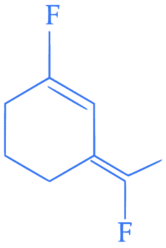
(A)



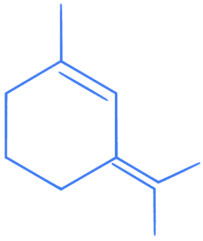
(B)



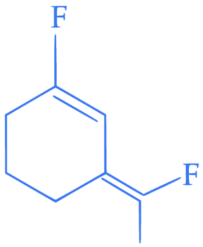
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- The molecular dipole moment is the vector sum of bond dipoles and other charge-displacement contributions; counting polar bonds alone does not determine polarity.
- A carbon–fluorine bond has a strong dipole directed toward fluorine. The relative directions of the two fluorine-bearing bonds in the displayed structures decide whether they reinforce or oppose one another.

Step 1: Compare the fluorine positions in the structures In structure (A), the fluorine on the ring points upward, and the fluorine on the exocyclic carbon points toward the upper right. Their dipoles therefore have a common upward component and reinforce one another. The acute angle between these drawn bond directions favours a larger vector resultant. In structure (C), the corresponding exocyclic fluorine points downward, so its dipole opposes much of the contribution from the ring fluorine. Interchanging fluorine and methyl at that carbon changes vector addition even though the number of fluorine atoms remains the same.

Step 2: Eliminate the less polar arrangements In (B), the fluorines are drawn on opposite positions of the ring, with their carbon–fluorine bond directions opposed. Their large individual contributions substantially cancel instead of adding in the same direction. Structure (D) contains no fluorine: its carbon–carbon and carbon–hydrogen framework lacks the strong reinforcing carbon–fluorine dipoles present in (A). Thus the relevant comparison is between the arrangement of polar bonds, not merely the presence of a conjugated diene or the total number of carbon atoms. The reinforcing disposition in (A) gives the largest resultant dipole among the options.

Step 3: Conclusion Structure (A) has the most strongly reinforcing polar bond arrangement and is the most polar compound. Hence, the correct option is (A).

Quick Tip to Solve: Draw an arrow toward each F atom and add the arrows geometrically; opposed arrows cancel while arrows with a common component reinforce.

73. The incorrect geometry is represented by : [4 marks]

(A) BF_3 - trigonal planar

(B) H_2O - bent

(C) NF_3 - trigonal planar

(D) AsF_5 - trigonal bipyramidal

Correct Answer: (C) NF_3 - trigonal planar

Solution:

Concept:

- VSEPR theory counts bonding pairs and lone pairs on the central atom. Molecular geometry describes atom positions, whereas electron-pair geometry includes lone pairs.
- Three bonding pairs without a lone pair give trigonal planar geometry; three bonds and one lone pair give a trigonal pyramidal molecular shape.

Step 1: Identify the incorrect nitrogen geometry Nitrogen has five valence electrons. In NF_3, it forms three single bonds to fluorine atoms and retains one lone pair. There are four electron domains around nitrogen:

$$NF_3: \quad AX_3E.$$

Their electron-pair arrangement is approximately tetrahedral, but only the three bonded atoms determine the molecular shape. Those atoms form a trigonal pyramid with nitrogen above the triangular fluorine base. The lone pair prevents all four atoms from lying in a trigonal planar arrangement. Thus calling this molecule trigonal planar is incorrect.

Step 2: Verify the other three descriptions In BF_3, boron forms three bonds and has no lone pair, giving AX_3 and a trigonal planar shape; (A) is correct. In H_2O, oxygen has two bonds and two lone pairs, giving AX_2E_2; the molecular shape is bent, so (B) is correct. In AsF_5, arsenic has five bonding domains and no lone pair, giving AX_5 and trigonal bipyramidal geometry, so (D) is also correct. These checks leave only the nitrogen trifluoride statement as the incorrect pairing.

Step 3: Conclusion NF_3 is trigonal pyramidal, not trigonal planar. Hence, the correct option is (C).

Quick Tip to Solve: Before assigning a molecular shape, count the central atom's lone pairs; three attached atoms do not automatically imply a planar molecule.

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74. When XO_2 is fused with an alkali metal hydroxide in presence of an oxidizing agent such as KNO_3 ; a dark green product is formed which disproportioates in acidic solution to afforda dark purple solution. X is : [4 marks]

(A) Ti

(B) V

(C) Cr

(D) Mn

Correct Answer: (D) Mn

Solution:

Concept:

- Manganese forms green manganate(VI), MnO_4^{2-}, and purple permanganate(VII), MnO_4^-. Their distinctive colours identify the element.
- Disproportionation is simultaneous oxidation and reduction of the same oxidation state. Acidification converts manganate(VI) into manganese(VII) and manganese(IV) products.

Step 1: Identify the green oxidation product The dioxide must be MnO_2, with manganese in oxidation state +4. Fusion with an alkali hydroxide in the presence of nitrate oxidizes it to manganate, where manganese is +6. A balanced equation using potassium salts is

$$MnO_2 + 2\,KOH + KNO_3 \longrightarrow K_2MnO_4 + KNO_2 + H_2O.$$

Potassium manganate is dark green, matching the first observation. Nitrate acts as the oxidizing agent and is reduced to nitrite in this equation.

Step 2: Explain the purple product and reject the other metals On acidification the green manganate ion disproportionates according to

$$3\,\mathrm{MnO_4}^{2-} + 4\,\mathrm{H}^{+} \longrightarrow 2\,\mathrm{MnO_4}^{-} + \mathrm{MnO_2} + 2\,\mathrm{H_2O}.$$

Two manganese atoms are oxidized from +6 to +7 while one is reduced from +6 to +4. The $\mathrm{MnO_4}^{-}$ ion gives the dark purple solution. Chromium would instead give chromate or dichromate species, characteristically yellow or orange rather than this green-to-purple sequence. Titanium and vanadium likewise do not produce the stated manganate–permanganate pair. Both the colours and the disproportionation therefore identify manganese uniquely.

Step 3: Conclusion X is manganese, Mn. Hence, the correct option is (D).

Quick Tip to Solve: Remember the colour sequence for manganese: dioxide is dark, manganate is green, and permanganate is purple.

75. At 320 K, a gas A_2 is 20% dissociated to A(g). The standard free energy change at 320 K and 1 atm in J mol^{-1} is approximately : (R = 8.314 JK^{-1} mol^{-1}; ln2 = 0.693; ln 3 = 1.098) [4 marks]

(A) 4763

(B) 2068

(C) 1844

(D) 4281

Correct Answer: (A) 4763

Solution:

Concept:

- For $\mathrm{A_2(g)} \rightleftharpoons 2\,\mathrm{A(g)}$, the equilibrium constant in terms of dimensionless pressures is $K_p = (p_A/p^\circ)^2/(p_{A_2}/p^\circ)$.
- The standard reaction Gibbs energy is $\Delta_r G^\circ = -RT\ln K_p$. It is not generally zero at equilibrium; the actual reaction Gibbs energy is zero there.

Step 1: Calculate equilibrium partial pressures Start with 1 mole of $\mathrm{A_2}$. A degree of dissociation $\alpha = 0.20$ leaves 0.80 mole of $\mathrm{A_2}$ and forms 0.40 mole of A. Total moles are therefore 1.20. At total pressure $P = 1\,\mathrm{atm}$,

$$p_{A_2} = \frac{0.80}{1.20}P = \frac{2}{3}\,\mathrm{atm}, \qquad p_A = \frac{0.40}{1.20}P = \frac{1}{3}\,\mathrm{atm}.$$

The factor of two in the amount of atomic gas is essential because each dissociated molecule yields two atoms.

Step 2: Evaluate the equilibrium constant and free energy With the stated 1 atm reference,

$$K_p = \frac{(1/3)^2}{2/3} = \frac{1}{6}.$$

Hence

$$\begin{aligned}\Delta_r G^\circ &= RT\ln 6\\ &= 8.314\times 320\times(\ln 2+\ln 3)\\ &= 8.314\times 320\times(0.693+1.098)\\ &\simeq 4765\,\mathrm{J\,mol^{-1}}.\end{aligned}$$

The closest printed value is 4763. The other values do not follow from the equilibrium mole fractions and logarithm. The positive sign is consistent with $K_p < 1$, meaning that the standard-state equilibrium favours undissociated gas.

Step 3: Conclusion The standard free energy change is approximately $4.76\times10^3\,\mathrm{J\,mol^{-1}}$, corresponding to 4763. Although the supplied key gives 4281, the printed dissociation gives $K_p = 1/6$, so 4281 is inconsistent with the stated equilibrium. Hence, the correct option is (A).

Quick Tip to Solve: For a dissociation $\mathrm{A_2} \longrightarrow 2\,\mathrm{A}$, use total equilibrium moles $1+\alpha$; treating α itself as a mole fraction gives the wrong constant.

76. Among the following, the incorrect statement is : [4 marks]

(A) Maltose and lactose has 1, 4 - glycosidic linkage.

(B) Sucrose and amylose has 1, 2 - glycosidic linkage

(C) Cellulose and amylose has 1, 4 - glycosidic linkage.

(D) Lactose contains β-D-galactose and β-D-glucose.

Correct Answer: (B) Sucrose and amylose has 1, 2 - glycosidic linkage

Solution:

Concept:

- A glycosidic linkage joins specified carbon atoms of monosaccharide units. The carbon numbers and the alpha or beta configuration are separate features of that bond.
- Amylose is an unbranched polymer of alpha-D-glucose with $\alpha(1\to4)$ bonds. Sucrose instead joins the anomeric carbons C1 of glucose and C2 of fructose.

Step 1: Check the sucrose and amylose comparison Sucrose consists of glucose and fructose joined through a $1\to2$ glycosidic linkage, involving the anomeric carbon of each unit. Amylose has successive glucose units linked from C1 of one unit to C4 of the next. Its linkage is $\alpha(1\to4)$, not $1\to2$. Thus the statement that both sucrose and amylose have a 1,2 linkage is false: its sucrose part is right but its amylose part is wrong. Only one incorrect component is needed to invalidate the combined statement.

Step 2: Verify the remaining linkage statements Maltose contains two glucose units connected by an $\alpha(1\to4)$ bond. Lactose contains a galactose unit linked through its beta C1 to C4 of glucose, so it also has a $1\to4$ bond; (A) is correct. Cellulose has $\beta(1\to4)$ links and amylose has $\alpha(1\to4)$ links, so (C) correctly gives the carbon numbers even though their configurations differ. Lactose is conventionally described in this context as yielding beta-D-galactose and beta-D-glucose units; its reducing glucose end can mutarotate in solution. That does not alter the 1,4 bond or make (D) the

intended incorrect linkage statement.

Step 3: Conclusion The incorrect statement is (B), because amylose has $\alpha(1 \rightarrow 4)$ linkages. Hence, the correct option is (B).

Quick Tip to Solve: Associate amylose with alpha-1,4 and cellulose with beta-1,4; sucrose is the glucose-C1 to fructose-C2 exception.

77. Which of the following complexes will shows geometrical isomerism ? [4 marks]

(A) aquachlorobis (ethylenediamine) cobalt (II) chloride

(B) pentaaquachlorohromium (III) chloride

(C) potassium amminetrichloropltinate (II)

(D) potassium tris(oxalato) chromate (III)

Correct Answer: (A) aquachlorobis (ethylenediamine) cobalt (II) chloride

Solution:

Concept:

- Geometrical isomers have the same metal–ligand connectivity but differ in the relative positions of ligands. Octahedral complexes may place two distinct monodentate ligands cis or trans.
- Ethylenediamine, en, and oxalate are bidentate ligands. A ligand's denticity determines coordination number; counting ligand molecules alone can give the wrong geometry.

Step 1: Examine the bis(ethylenediamine) complex The coordination entity in (A) is

$$[\mathrm{Co(en)_2(H_2O)Cl}]^{+}.$$

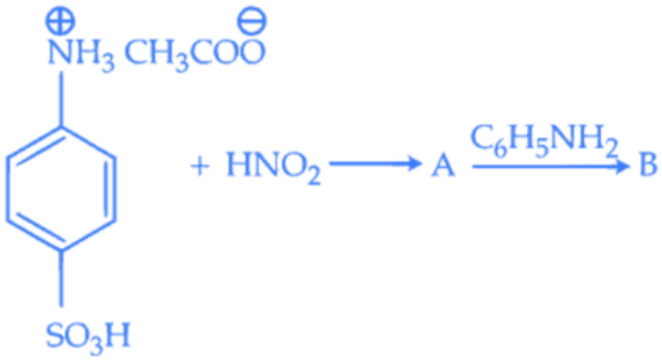
The two en ligands occupy four sites, while water and chloride occupy the remaining two sites of an octahedron. Water and chloride can be adjacent, at 90°, or opposite, at 180°. These are distinct cis and trans forms. Both arrangements retain two chelating en ligands, so the geometrical alternatives are feasible and correspond to the same composition.

Step 2: Exclude the other ligand arrangements Option (B), $[\mathrm{Cr(H_2O)_5Cl}]^{2+}$, has five identical ligands and one different ligand. Every position for the single chloride is equivalent in an octahedron, so there is no cis–trans pair. In (C), the square-planar $[\mathrm{Pt(NH_3)Cl_3}]^{-}$ has only one ammonia and three identical chlorides; all choices of the ammonia site are equivalent. Option (D), $[\mathrm{Cr(C_2O_4)_3}]^{3-}$, has three identical bidentate ligands and can exhibit optical isomerism, but not the geometrical isomerism asked for. Thus only (A) has the required distinct relative ligand positions.

Step 3: Conclusion Aquachlorobis(ethylenediamine)cobalt(II) chloride exhibits cis–trans geometrical isomerism. Hence, the correct option is (A).

Quick Tip to Solve: For octahedral $[M(AA)_2ab]$, test whether the two monodentate ligands can be cis and trans; distinguish this from optical isomerism of $[M(AA)_3]$.

78. Products A and B formed in the following reactions are respectively : [4 marks]



(A) p-$HO_3S-C_6H_4-N=N-COCH_3$ and p-$HO_3S-C_6H_4-N=N-C_6H_5$.

(B) p-$HO_3S-C_6H_4-N=N-COCH_3$ and p-$HO_3S-C_6H_4-NH-C_6H_4-NH_2$, with the amino group para to the bridging nitrogen.

(C) p-$HO_3S-C_6H_4-N=N-OCOCH_3$ and p-$HO_3S-C_6H_4-N=N-NH-C_6H_5$.

(D) p-$HO_3S-C_6H_4-N=N-OCOCH_3$ and p-$HO_3S-C_6H_4-N=N-C_6H_4-NH_2$, with the amino group para to the azo linkage.

Correct Answer: (D) p-$HO_3S-C_6H_4-N=N-OCOCH_3$ and p-$HO_3S-C_6H_4-N=N-C_6H_4-NH_2$, with the amino group para to the azo linkage.

Solution:

Concept:

- A primary aromatic amine reacts with nitrous acid to give an arenediazonium salt. The counter-ion is not a carbon substituent on nitrogen.
- Aniline activates its aromatic ring for electrophilic azo coupling, predominantly at the para position when that position is free.

Step 1: Identify the diazonium intermediate The starting material is the acetate salt of sulfanilic acid. Nitrosation and dehydration convert its protonated amino group into the diazonium group, while the para sulfonic acid group stays on the same ring. With acetate as counter-ion, A is represented in the options as p-$HO_3S-C_6H_4-N=N-OCOCH_3$. The acetate belongs to the salt representation; it must contain its oxygen linkage and must not become a direct nitrogen–acetyl carbon bond.

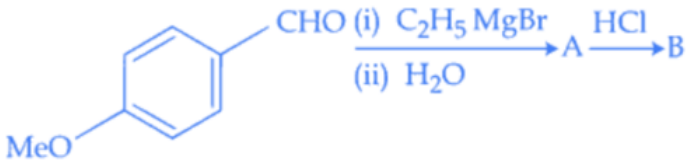
Step 2: Couple with the aniline ring The diazonium species acts as the electrophile. The $-NH_2$ group of aniline donates electron density to its ring and directs substitution mainly to the free para carbon. Loss of a proton restores aromaticity and gives p-$HO_3S-C_6H_4-N=N-C_6H_4-NH_2$. Both nitrogen atoms of the azo bridge are retained. This is aromatic carbon coupling, not replacement of the amino group and not a bridge containing only $-NH-$.

Step 3: Compare both products in each option Options (A) and (B) show the wrong direct acetyl connection in A. Option (A) also omits the amino substituent from the second ring, and (B) lacks the required azo bridge. Option (C) gives nitrogen coupling to a diazoamino structure rather than the para carbon-coupled azo dye. Only (D) contains the appropriate diazonium acetate representation together with the para-amino azo product.

Step 4: Conclusion A is the diazonium acetate of sulfanilic acid, and B is the para-amino azo-coupling product. Hence, the correct option is (D).

Quick Tip to Solve: Check both halves of a paired-product option: diazotisation retains two nitrogens, and aniline couples at its free para carbon.

79. The major product B formed in the following reaction sequence is : [4 marks]



(A) p-$Cl-C_6H_4-CH_2-CHCl-CH_3$.

(B) p-$CH_3O-C_6H_4-CHCl-CH_2CH_3$.

(C) p-$CH_3O-C_6H_4-CH=CH-CH_3$.

(D) p-$CH_3O-C_6H_4-CH_2-CH=CH_2$.

Correct Answer: (B) p-$CH_3O-C_6H_4-CHCl-CH_2CH_3$.

Solution:

Concept:

- A Grignard reagent adds its alkyl group to an aldehyde carbonyl carbon; aqueous work-up gives a secondary alcohol.
- Protonation of a benzylic alcohol allows water to leave. Chloride replaces the hydroxyl group at the same benzylic carbon.

Step 1: Add the ethyl group to the aldehyde Let Ar denote p-$CH_3OC_6H_4$. The reagent C_2H_5MgBr supplies a nucleophilic ethyl group to ArCHO, producing the magnesium alkoxide $ArCH(OMgBr)CH_2CH_3$. The aqueous work-up protonates oxygen, so A is $ArCH(OH)CH_2CH_3$. The aldehydic carbon remains bonded directly to the aromatic ring and becomes the alcohol-bearing carbon.

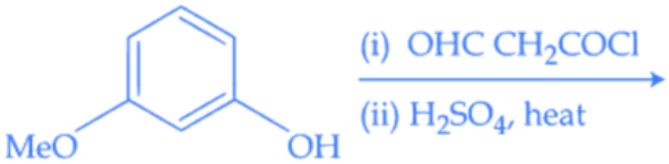
Step 2: Replace the activated hydroxyl group On treatment with HCl, the hydroxyl group is protonated and can depart as water. The resulting secondary benzylic cation is resonance stabilised; the para methoxy group further supports this stabilisation through the ring. Chloride capture gives $ArCH(Cl)CH_2CH_3$. No carbon skeleton rearrangement is required, since moving the charge away from the benzylic position would lose its stabilisation.

Step 3: Eliminate the competing structures Option (B) preserves the methoxy group and places chlorine exactly at the former carbonyl carbon. Option (A) incorrectly replaces the aryl methoxy substituent by chlorine and moves the side-chain chlorine away from the benzylic site. Options (C) and (D) are alkenes, corresponding to loss of water rather than the specified major chloride product with hydrochloric acid. The terminal alkene in (D) also fails to retain conjugation with the ring.

Step 4: Conclusion B is p-$CH_3O-C_6H_4-CH(Cl)-CH_2CH_3$. Hence, the correct option is (B).

Quick Tip to Solve: Track the original aldehyde carbon through both stages: it receives ethyl first and chlorine replaces its hydroxyl group next.

80. The major product of the following reaction is : [4 marks]



(A) 7-Methoxy-2H-chromen-2-one (7-methoxycoumarin).

(B) 7-Methoxy-4H-chromen-4-one (7-methoxychromone).

(C) 5-Methoxy-2H-chromen-2-one (5-methoxycoumarin).

(D) 5-Methoxy-4H-chromen-4-one (5-methoxychromone).

Correct Answer: (A) 7-Methoxy-2H-chromen-2-one (7-methoxycoumarin).

Solution:

Concept:

- Phenolic oxygen can be acylated by an acid chloride to form an ester, retaining the acid-chloride carbonyl.
- Acid-catalysed intramolecular condensation at an activated ortho carbon, followed by dehydration, produces the conjugated coumarin ring.

Step 1: Form the phenolic ester The substrate is meta-methoxyphenol, and the reagent is $OHCCH_2COCl$. Acylation of the free phenolic oxygen gives $ArOCOCH_2CHO$. The methoxy group remains an ether substituent on the aromatic ring. The oxygen originally in the phenolic hydroxyl therefore becomes the ring oxygen of the eventual lactone, while the ester carbonyl stays adjacent to it.

Step 2: Close the ring at the favoured position Under H_2SO_4 and heat, activation of the terminal aldehyde permits attack by an ortho carbon of the phenolic ring. The preferred site is the less crowded site para to methoxy, rather than the site squeezed between methoxy and the ester-bearing oxygen. Ring closure gives a six-membered oxygen heterocycle, and dehydration establishes the conjugated double bond of the coumarin system.

Step 3: Assign the carbonyl and methoxy positions The retained ester carbonyl is at position 2 of the heterocycle: this is a coumarin, not a chromone with its carbonyl at position 4. The selected aromatic attack site places the unchanged methoxy substituent at position 7. Thus (A) has both required features. Option (C) is the alternative methoxy regioisomer, whereas (B) and (D) have the wrong carbonyl position and would require a different connectivity from the initially formed ester.

Step 4: Conclusion The major product is 7-methoxycoumarin, namely 7-methoxy-2H-chromen-2-one. Hence, the correct option is (A).

Quick Tip to Solve: Follow the phenolic oxygen and the acid-chloride carbonyl: their preserved ester linkage identifies a coumarin, then locate methoxy after ring closure.

81. Which of the following statements is false ? [4 marks]

(A) Photon has momentum as well as wavelength.

(B) Splitting of spectral lines in electrical field is called Stark effect.

(C) Rydberg constant has unit of energy.

(D) Frequency of emitted radiation from a black body goes from a lower wavelength to higher wavelength as the temperature increases.

Correct Answer: (D) Frequency of emitted radiation from a black body goes from a lower wavelength to higher wavelength as the temperature increases.

Solution:

Concept:

- A photon obeys $E = h\nu = hc/\lambda$ and has momentum $p = h/\lambda$, despite zero rest mass.
- Wien's displacement law is $\lambda_{\max} T = b$; increasing absolute temperature shifts the emission peak to shorter wavelength.

Step 1: Check the photon and Stark-effect statements Option (A) is true: a photon carries momentum h/λ, and its wavelength is related to frequency by $c = \nu\lambda$. Rest mass is not required for a photon to transport momentum. Option (B) is also true: an external electric field shifts and splits suitable atomic spectral lines, which is the Stark effect. The corresponding magnetic-field phenomenon is the Zeeman effect.

Step 2: Distinguish the two Rydberg conventions In the energy-level expression $E_n = -R_E/n^2$, the Rydberg energy R_E has units of joules and equals approximately $2.18 \times 10^{-18}\,\text{J}$ for hydrogen. In the wavenumber expression, the Rydberg constant R_∞ instead has units m^{-1}, with $R_E = hcR_\infty$. Statement (C) uses the energy convention common in atomic-energy formulae; the two constants must not be confused.

Step 3: Apply the temperature dependence of the peak For temperatures $T_2 > T_1$, Wien's law gives $\lambda_{\max,2}/\lambda_{\max,1} = T_1/T_2 < 1$. The peak moves from higher wavelength to lower wavelength, and its associated frequency increases. Statement (D) explicitly claims a shift from lower wavelength to higher wavelength, which reverses this result. A black body still emits a distribution of wavelengths: the temperature trend concerns its characteristic peak, rather than a single monochromatic photon.

Step 4: Conclusion The false temperature-shift statement is (D): heating shifts the black-body peak towards shorter wavelengths. Hence, the correct option is (D).

Quick Tip to Solve: Use the inverse product in Wien's law: if temperature rises, the peak wavelength falls; keep Rydberg energy distinct from Rydberg wavenumber.

82. The mass of a non-volatile, non-electrolyte solute (molar mass = 50 g mol^{-1}) needed to be dissolved in 114 g octane to reduce its vapour pressure to 75%, is : [4 marks]

(A) 37.5 g

(B) 75 g

(C) 150 g

(D) 50 g

Correct Answer: As worded, the required mass is $50/3\,\mathrm{g}$, and no listed option is correct. The supplied key (C), $150\,\mathrm{g}$, corresponds to a reduction by 75%, a different demand.

Solution:

Concept:

- For a non-volatile solute in an ideal solution, $p = x_1 p^0$, where x_1 is the solvent mole fraction.
- Octane is C_8H_{18}, with molar mass $114\,\mathrm{g\,mol^{-1}}$. Thus the stated solvent mass is one mole.

Step 1: Use the stated remaining vapour pressure The wording says reduce the vapour pressure *to* 75%, so $p/p^0 = 0.75$. This specifies the pressure remaining, rather than a 75% reduction. The solvent amount is

$$n_1 = \frac{114}{114} = 1\,\mathrm{mol}.$$

The solute is a non-electrolyte, so it does not dissociate and there is no van't Hoff multiplication of its number of moles.

Step 2: Solve the exact mole-fraction equation Let the added solute amount be n_2. Raoult's law gives

$$\frac{1}{1+n_2} = \frac{3}{4}.$$

Hence $4 = 3 + 3n_2$, so $n_2 = 1/3\,\mathrm{mol}$. With molar mass $50\,\mathrm{g\,mol^{-1}}$, the mass is

$$m_2 = \frac{1}{3} \times 50 = \frac{50}{3}\,\mathrm{g} \approx 16.67\,\mathrm{g}.$$

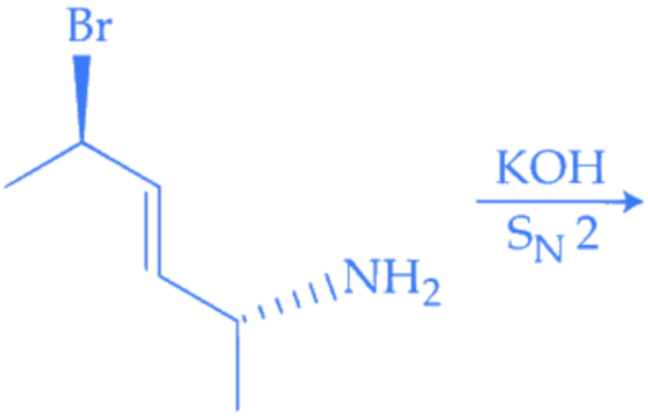
The exact solvent mole fraction must be used here; the solute fraction is not small enough to replace its denominator by solvent moles.

Step 3: Test the options and the keyed interpretation The four offered masses give remaining pressure fractions $4/7$, $2/5$, $1/4$, and $1/2$, respectively; none equals $3/4$. The supplied key selects (C), $150\,\mathrm{g}$. That is three moles of solute, making $x_1 = 1/(1+3) = 1/4$: it reduces pressure *by* 75%, not *to* 75%. This calculation establishes the wording discrepancy without replacing the printed demand.

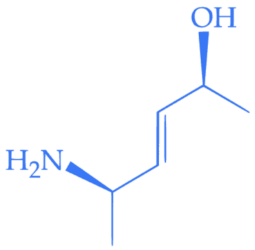
Step 4: Conclusion As worded, the required mass is $50/3\,\mathrm{g}$, and no listed option is correct. The supplied key (C), $150\,\mathrm{g}$, corresponds to a reduction by 75%, a different demand.

Quick Tip to Solve: Translate "to 75 percent" into remaining fraction 0.75 before applying Raoult's law; "by 75 percent" would leave 0.25.

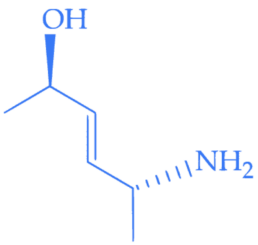
83. **The major product of the following reaction is :** [4 marks]



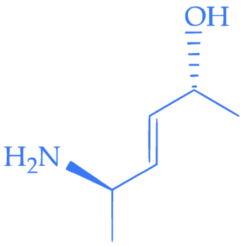
(A)



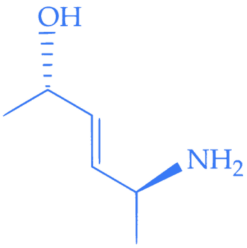
(B)



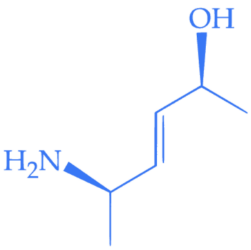
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- An S_N2 reaction replaces a leaving group by backside attack and gives inversion at the reacting tetrahedral carbon.
- A stereocentre whose bonds are not broken retains its spatial configuration. Wedges must be compared after aligning the carbon skeleton.

Step 1: Locate the reacting stereocentre The displayed reagent specifies S_N2 substitution with KOH. Hydroxide attacks the carbon bearing bromine, and the carbon–bromine bond breaks as the carbon–oxygen bond forms. Thus the brominated centre becomes an alcohol-bearing centre. The double bond and the remote amino-bearing stereocentre remain part of the same carbon skeleton; neither an alkene addition nor a second substitution is indicated.

Step 2: Apply inversion only at the brominated carbon Backside approach gives Walden inversion at this carbon. It is essential to compare the three-dimensional arrangements rather than simply to demand a dashed hydroxyl bond wherever a solid bromine bond appeared: some answer drawings reverse the left-to-right orientation of the entire skeleton. Turning a drawing around in its own plane is allowed, but reflecting its skeleton changes the relationship of its wedge bond to the in-plane bonds.

Step 3: Compare both stereocentres in the proposed products In option (A), the in-plane bonds at the alcohol carbon are reflected relative to the original orientation, while its solid wedge is retained in the drawing. Together these changes represent inversion at the substituted carbon. At the amino carbon, reflection of the in-plane skeleton is accompanied by changing the dashed amino bond to a solid wedge, which preserves that centre's spatial configuration. Option (B) simply retains the original bromine-centre configuration on replacing bromine by hydroxyl. Option (C) gives the wrong alcohol-centre configuration after skeleton alignment. Option (D) changes the remote amino configuration as well as replacing bromine, which the specified substitution cannot do.

Step 4: Conclusion The product with inversion at the bromine-bearing carbon and retention at the amino-bearing carbon is (A). Hence, the correct option is (A).

Quick Tip to Solve: Align the chain before judging wedges: inversion is required at the substitution site, while the remote amino stereocentre must retain configuration.

84. For standardizing NaOH solution, which of the following is used as a primary standard ? [4 marks]

(A) Ferrous Ammonium Sulfate

(B) dil. HCl

(C) Oxalic acid

(D) Sodium tetraborate

Correct Answer: (C) Oxalic acid

Solution:

Concept:

- A primary standard is sufficiently pure, stable and of known composition that an accurately weighed mass determines its amount of substance.
- A base such as sodium hydroxide is standardised against a suitable primary-standard acid through a known neutralisation stoichiometry.

Step 1: Choose an acid with reliable composition Oxalic acid, normally used as its crystalline dihydrate $H_2C_2O_4 \cdot 2\,H_2O$, is suitable for preparing a solution of accurately known concentration by weighing. Its formula mass includes both waters of crystallisation. It supplies two acidic protons per molecule, so its reaction with sodium hydroxide has a fixed two-to-one mole ratio of base to acid.

Step 2: Use the neutralisation relationship The reaction is

$$H_2C_2O_4 + 2\,NaOH \longrightarrow Na_2C_2O_4 + 2\,H_2O.$$

For an accurately prepared oxalic-acid solution of molarity M_a, an aliquot V_a consumes V_b of the sodium hydroxide solution according to

$$M_bV_b = 2M_aV_a.$$

Volumes must be expressed in the same units. This gives the actual base concentration, which cannot be trusted from simple weighing of sodium hydroxide because it absorbs moisture and carbon dioxide.

Step 3: Reject the alternatives for this titration Ferrous ammonium sulfate is a reducing primary standard used in suitable redox titrations; it does not provide the required standard acid neutralisation here. Dilute hydrochloric acid can titrate a base only once its own concentration has been established, and is not prepared as a primary standard merely by dilution of volatile aqueous acid. Sodium tetraborate is a basic salt and serves to standardise acids such as hydrochloric acid, so it is unsuitable as the acid standard for sodium hydroxide. Oxalic acid is the matching choice.

Step 4: Conclusion Oxalic acid is used as the primary standard for standardising $NaOH$. Hence, the correct option is (C).

Quick Tip to Solve: For standardising a base, first choose a stable primary-standard acid, then include its number of acidic protons in the titration equation.

85. Assuming ideal gas behaviour, the ratio of density of ammonia to that of hydrogen chloride at same temperature and pressure is : (Atomic wt. of Cl = 35.5 u) [4 marks]

(A) 1.46

(B) 0.46

(C) 1.64

(D) 0.64

Correct Answer: (B) 0.46

Solution:

Concept:

- For an ideal gas, $PV = nRT = (m/M)RT$, where M is molar mass, m is mass and R is the gas constant.
- Density is $d = m/V$, so $d = PM/(RT)$. At common pressure and temperature the density ratio equals the molar-mass ratio, not its reciprocal.

Step 1: Calculate the molar masses Ammonia is NH_3, containing one nitrogen and three hydrogen atoms. Hydrogen chloride is HCl, containing one hydrogen and one chlorine atom. Using the stated chlorine atomic mass,

$$M_{NH_3} = 14 + 3(1) = 17\,\text{g mol}^{-1}, \qquad M_{HCl} = 1 + 35.5 = 36.5\,\text{g mol}^{-1}.$$

Both masses must be expressed in the same units; their units cancel in the requested ratio.

Step 2: Form the density ratio in the stated order Because both gases have the same P and T, the common factor $P/(RT)$ cancels:

$$\frac{d_{NH_3}}{d_{HCl}} = \frac{17}{36.5} = \frac{34}{73} = 0.46575\ldots.$$

This corresponds to 0.46 among the given choices. Ammonia is lighter, so a ratio exceeding one, as in 1.46 or 1.64, cannot be correct. The remaining value 0.64 is below one but does not equal the molar-mass ratio.

Step 3: Conclusion The required density ratio is approximately 0.46. Hence, the correct option is (B).

Quick Tip to Solve: Write the numerator gas first in both density and molar-mass ratios. At equal pressure and temperature, the lighter gas must have the lower density.

86. In the extraction of copper from its sulphate ore, metal is finally obtained by the oxidation of cuprous sulphide with : [4 marks]

(A) Fe_2O_3

(B) Cu_2O

(C) SO_2

(D) CO

Correct Answer: (B) Cu_2O

Solution:

Concept:

- The final conversion of cuprous sulphide to copper is a self-reduction reaction involving cuprous oxide, Cu_2O.
- The sulphur of Cu_2S is oxidised to SO_2, while copper(I) in the sulphide and oxide is reduced to copper metal.

Step 1: Identify the oxide that reacts with the sulphide During copper extraction, controlled oxidation converts part of the cuprous sulphide into cuprous oxide. The remaining sulphide then reacts with that oxide. The decisive final reaction is

$$Cu_2S + 2\,Cu_2O \longrightarrow 6\,Cu + SO_2.$$

It contains six copper atoms, one sulphur atom and two oxygen atoms on each side. Thus the oxygen required to oxidise sulphur comes from cuprous oxide, not from an iron oxide or carbon monoxide.

Step 2: Check the redox roles and the alternatives Sulphur changes from oxidation state -2 in Cu_2S to $+4$ in SO_2, losing six electrons overall. Six copper(I) atoms each gain one electron to become copper metal. This explains why the reaction is called self-reduction: no separate carbon reducing agent is needed. Fe_2O_3 is not the oxide used in this final copper-producing step. SO_2 is a product, while CO is a reducing agent and cannot serve as the oxidant specified in the question.

Step 3: Conclusion Cuprous sulphide reacts with Cu_2O to yield copper and sulphur dioxide. Hence, the correct option is (B).

Quick Tip to Solve: Remember the balanced self-reduction equation: one Cu_2S plus two Cu_2O produces six copper atoms. It identifies the required oxide immediately.

87. Which one of the following is not a property of physical adsorption? [4 marks]

(A) Higher the pressure, mre the adsorption

(B) Lower the temperature, more the adsorption

(C) Greater the surface area, more the adsorption

(D) Unilayer adsorption ocurs

Correct Answer: (D) Unilayer adsorption ocurs

Solution:

Concept:

- Physical adsorption holds adsorbate molecules to a surface mainly through weak van der Waals forces; it does not normally require formation of specific chemical bonds.
- Physisorption is generally exothermic and can form multiple molecular layers. Pressure, temperature and available surface area affect its extent.

Step 1: Check pressure, temperature and area Increasing gas pressure increases the number of molecules available near the surface, so adsorption generally increases at fixed temperature before approaching saturation. Lowering temperature favours the exothermic adsorption process and decreases the tendency of adsorbed molecules to escape. Increasing the accessible surface area provides more sites for adsorption, increasing the amount that a given mass of adsorbent can retain. Thus the trends described in options (A), (B) and (C) are characteristic physical-adsorption trends.

Step 2: Distinguish monolayer and multilayer adsorption Once a first layer of molecules is held on a surface, further molecules can be attracted to that adsorbed layer through intermolecular forces. Consequently physical adsorption is not restricted to a single layer: multilayer adsorption is possible. By contrast, the simplest chemical-adsorption picture involves a monolayer because chemical bonding uses particular surface sites. The assertion that unilayer adsorption occurs as the characteristic restriction of physical adsorption is therefore the statement to reject. The question asks for the exception, rather than for a true trend.

Step 3: Conclusion Physical adsorption can be multilayer, so unilayer adsorption is not its characteristic property. Hence, the correct option is (D).

Quick Tip to Solve: For a question asking what is NOT a physisorption property, check for a monolayer-only claim after verifying the usual high-pressure and low-temperature trends.

88. A group 13 element 'X' reacts with chlorine gas to produce a compound XCl_3. XCl_3 is electron deficient and easily reacts with NH_3 to form $Cl_3X \leftarrow NH_3$ adduct ; however, XCl_3 does not dimerize X is : [4 marks]

(A) B

(B) Al

(C) Ga

(D) In

Correct Answer: (A) B

Solution:

Concept:

- A group 13 atom has three valence electrons. In a simple covalent trichloride it forms three bonds and has only six electrons around its central atom.
- An electron-deficient trichloride acts as a Lewis acid, accepting a lone pair from a Lewis base such as ammonia. Whether it dimerises also depends on the size and bonding of its central atom.

Step 1: Use the ammonia adduct to identify Lewis acidity The lone pair on nitrogen in ammonia can be donated into an empty orbital on the central atom:

$$BCl_3 + NH_3 \longrightarrow Cl_3B \leftarrow NH_3.$$

The coordinate bond supplies an additional electron pair to boron. Therefore the adduct is fully consistent with electron deficiency of BCl_3. Adduct formation alone does not uniquely distinguish the choices, since several group 13 chlorides can act as Lewis acids; the absence of dimerisation is the decisive extra condition.

Step 2: Apply the non-dimerisation condition Boron is the smallest atom among the listed elements, and BCl_3 remains a trigonal-planar monomer rather than forming a stable chlorine-bridged dimer. Its bonding is also stabilised by donation from chlorine lone pairs into the vacant boron orbital. Aluminium chloride is a familiar contrast: it can form the bridged dimer Al_2Cl_6. The heavier gallium and indium chlorides do not supply the characteristic small-boron explanation for the complete combination of clues. Thus the species intended is boron trichloride and the element is B.

Step 3: Conclusion The element is boron, B, whose trichloride forms an ammonia adduct without dimerising. Hence, the correct option is (A).

Quick Tip to Solve: Use both clues together: electron deficiency suggests a Lewis-acid trichloride, while the monomeric trichloride points specifically to small boron.

89. Among the oxides of nitrogen : N_2O_3, N_2O_4 and N_2O_5; the molecule(s) having nitrogen-nitrogen bond is / are : [4 marks]

(A) Only N_2O_5

(B) N_2O_3 and N_2O_5

(C) N_2O_4 and N_2O_5

(D) N_2O_3 and N_2O_4

Correct Answer: (D) N_2O_3 and N_2O_4

Solution:

Concept:

- Molecular formula alone does not reveal whether two nitrogen atoms are directly bonded. The molecular connectivity must distinguish an N–N link from an N–O–N bridge.
- Molecular N_2O_3 and N_2O_4 have a direct nitrogen–nitrogen bond, whereas molecular N_2O_5 joins its nitrogen centres through an oxygen atom.

Step 1: Inspect dinitrogen trioxide and tetroxide Dinitrogen trioxide has molecular connectivity represented by $O{=}N{-}NO_2$: the nitrogen of the nitrosyl part is directly linked to the nitrogen of the nitro part. Dinitrogen tetroxide is formed by association of two NO_2 units and has connectivity $O_2N{-}NO_2$, again with a direct N–N bond. The oxygen atoms bonded to either nitrogen do not remove this central nitrogen–nitrogen linkage. Thus both of the first two listed oxides satisfy the requirement.

Step 2: Compare dinitrogen pentoxide and select the pair Molecular dinitrogen pentoxide has connectivity $O_2N{-}O{-}NO_2$. Here an oxygen atom lies between the nitrogen centres, giving an N–O–N bridge rather than an N–N bond. Its solid ionic description as nitronium nitrate also supplies no direct N–N bond. Consequently every answer including N_2O_5 is unsuitable: this excludes options (A), (B) and (C). The remaining combination contains exactly the two oxides already found to have the direct bond.

Step 3: Conclusion The oxides with a nitrogen–nitrogen bond are N_2O_3 and N_2O_4. Hence, the correct option is (D).

Quick Tip to Solve: For nitrogen-oxide bond questions, write the skeletons first: $O{=}N{-}NO_2$, $O_2N{-}NO_2$, and $O_2N{-}O{-}NO_2$. The bridging atom decides the answer.

90. The correct match between items of List - I and List - II is :

List - I		List - II	
(A)	Phenelzine	(P)	Pyrimidine
(B)	Chloroxylenol	(Q)	Furan
(C)	Uracil	(R)	Hydrazine
(D)	Ranitidine	(S)	Phenol

[4 marks]

(A) (A) - (R), (B) - (S), (C) - (P), (D) - (Q)

(B) (A) - (s), (B) - (R), (C) - (P), (D) - (Q)

(C) (A) - (S), (B) - (R), (C) - (Q), (D) - (P)

(D) (A) - (R), (B) - (S), (C) - (Q), (D) - (P)

Correct Answer: (A) (A) - (R), (B) - (S), (C) - (P), (D) - (Q)

Solution:

Concept:

- Identify the characteristic structural unit of each medicinal or biological compound. The matching concerns chemical structure, rather than the therapeutic use of the compound.
- Hydrazine has an N–N linkage, phenol has an OH group attached directly to a benzene ring, pyrimidine is a six-membered nitrogen heterocycle, and furan is a five-membered oxygen heterocycle.

Step 1: Match phenelzine and chloroxylenol Phenelzine is a hydrazine derivative: its side chain contains the $-NH-NH_2$ unit. Therefore phenelzine, item (A), matches hydrazine, item (R). Chloroxylenol is a substituted phenol, with chlorine and methyl substituents on the benzene ring bearing an OH group. Consequently item (B) matches phenol, item (S). These two assignments already exclude options (B) and (C), which interchange the phenol and hydrazine matches.

Step 2: Distinguish the two heterocyclic rings Uracil is a pyrimidine base. Its ring is a six-membered heterocycle containing two nitrogen atoms, with carbonyl groups at the 2 and 4 positions; hence item (C) matches pyrimidine, item (P). Ranitidine contains a furan ring, a five-membered ring with one oxygen atom, so item (D) matches furan, item (Q). The carbonyl oxygen atoms in uracil do not make its nitrogen-containing ring a furan ring. This distinction rules out option (D), which reverses the last two assignments.

Step 3: Compare the complete matching with the options Collecting all four independently identified structural units gives

$$(A)-(R), \qquad (B)-(S), \qquad (C)-(P), \qquad (D)-(Q).$$

Only the first option retains both the hydrazine/phenol assignments and the pyrimidine/furan assignments. A correct partial match is insufficient because every entry in the proposed list must be correct.

Step 4: Conclusion Phenelzine matches hydrazine, chloroxylenol matches phenol, uracil matches pyrimidine, and ranitidine matches furan. Hence, the correct option is (A).

Quick Tip to Solve: First fix chloroxylenol as a phenol and uracil as a pyrimidine base; then check the N–N unit in phenelzine and the oxygen-containing five-membered ring in ranitidine.