# JEE Main 2019 Paper 1 (B.E./B.Tech) Apr 9 Shift 1 Question Paper with Solutions PDF

## Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

**Time Allowed: 3 Hours** **Maximum Marks: 360**

### General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 9 April 2019, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, $-1$ for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

## Mathematics (120 marks)

**1. Let f(x) = 15 – |x – 10|; x ∈ R. Then the set of all values of x, at which the function, g(x) = f(f(x)) is not differentiable, is :** [4 marks]

(A) {10,15}

(B) {5,10,15,20}

(C) {10}

(D) {5,10,15}

**Correct Answer:** (D) {5,10,15}

**Solution:**

**Concept:**

- An absolute-value function can change its derivative where the expression inside its modulus vanishes. For a composition, inspect both the inner corner and preimages of the outer corner.
- For $f(x) = 15 - |x - 10|$, the outer function has its corner at input 10. A candidate point must still be checked using one-sided derivatives; a corner can sometimes disappear in a composition.

**Step 1:** Write the composition explicitly Substituting the complete inner function gives

$$g(x) = 15 - |5 - |x - 10||\,.$$

Thus possible changes of formula occur at $x = 10$, or when $|x - 10| = 5$. The latter equation gives $x = 5, 15$. There are no further candidates because on every interval between these numbers both moduli have fixed signs.

**Step 2:** Remove the moduli on the four intervals The formulas are

$$g(x) = \begin{cases} x + 10, & x < 5, \\ 20 - x, & 5 < x < 10, \\ x, & 10 < x < 15, \\ 30 - x, & x > 15. \end{cases}$$

Their slopes, in increasing order of the intervals, are $1, -1, 1, -1$. The formulas agree in value at the endpoints, so the function remains continuous there. However, continuity alone does not imply differentiability.

**Step 3:** Compare the one-sided slopes At 5 the slopes change from 1 to -1; at 10 they change from -1 to 1; at 15 they change from 1 to -1. Every change is a genuine corner. In contrast, 20 lies strictly inside the final interval and has derivative -1. Options (A) and (C) omit genuine corners, while option (B) unnecessarily includes 20.

**Step 4:** Conclusion The points of non-differentiability are exactly $\{5, 10, 15\}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For a nested modulus, list zeros of every inner expression, then verify the slope change at each candidate.

**2. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to** [4 marks]

(A) $3\hat{i} - 9\hat{j} - 5\hat{k}$

(B) $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$

(C) $-3\hat{i} + 9\hat{j} + 5\hat{k}$

(D) $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$

**Correct Answer:** (B) $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$

**Solution:**

**Concept:**

- The component of $\boldsymbol{\beta}$ parallel to a nonzero vector $\boldsymbol{\alpha}$ is its projection $\dfrac{\boldsymbol{\beta} \cdot \boldsymbol{\alpha}}{|\boldsymbol{\alpha}|^2}\boldsymbol{\alpha}$.
- Here the decomposition is a difference: $\boldsymbol{\beta} = \boldsymbol{\beta}_1 - \boldsymbol{\beta}_2$. Therefore $\boldsymbol{\beta}_2 = \boldsymbol{\beta}_1 - \boldsymbol{\beta}$, which has the opposite sign to the usual perpendicular remainder.

**Step 1:** Find the parallel component Using $\boldsymbol{\alpha} = (3, 1, 0)$ and $\boldsymbol{\beta} = (2, -1, 3)$,

$$\boldsymbol{\beta} \cdot \boldsymbol{\alpha} = 6 - 1 = 5, \qquad |\boldsymbol{\alpha}|^2 = 9 + 1 = 10.$$

Taking a dot product of the stated decomposition with $\boldsymbol{\alpha}$ removes the perpendicular term. Consequently,

$$\boldsymbol{\beta}_1 = \tfrac{1}{2}\boldsymbol{\alpha} = (\tfrac{3}{2}, \tfrac{1}{2}, 0).$$

**Step 2:** Use the stated subtraction convention The other vector is

$$\boldsymbol{\beta}_2 = (\tfrac{3}{2}, \tfrac{1}{2}, 0) - (2, -1, 3) = (-\tfrac{1}{2}, \tfrac{3}{2}, -3).$$

As a check, its dot product with $(3, 1, 0)$ is $-3/2 + 3/2 = 0$, so it is indeed perpendicular.

**Step 3:** Evaluate the cross product in the requested order Componentwise expansion gives

$$\begin{aligned}\boldsymbol{\beta}_1 \times \boldsymbol{\beta}_2 &= \left(\tfrac{1}{2}(-3), -\tfrac{3}{2}(-3), \tfrac{3}{2}\tfrac{3}{2} - \tfrac{1}{2}(-\tfrac{1}{2})\right) \\ &= (-\tfrac{3}{2}, \tfrac{9}{2}, \tfrac{5}{2}).\end{aligned}$$

Thus the factor one-half in option (B) is essential. Option (C) doubles the vector, while (A) and (D) have incorrect component signs.

**Step 4:** Conclusion $\boldsymbol{\beta}_1 \times \boldsymbol{\beta}_2 = \dfrac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$. Hence, the correct option is (B).

**Quick Tip to Solve:** Write the perpendicular vector from the given decomposition before crossing; a minus decomposition reverses its sign.

**3. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is :** [4 marks]

(A) 24

(B) 20

(C) 25

(D) 22

**Correct Answer:** (C) 25

**Solution:**

**Concept:**

- The standard parabola $y^2 = 4ax$ has focus $(a, 0)$. A focal chord is a chord whose supporting line passes through this focus.
- Intersect the line through the known endpoint and focus with the parabola. One intersection is the given endpoint; the other supplies the full chord length through the distance formula.

**Step 1:** Find the focus and chord line Comparing $y^2 = 16x$ with $y^2 = 4ax$ gives $a = 4$, so the focus is $F = (4, 0)$. The supplied endpoint $P = (1, 4)$ satisfies the parabola since $4^2 = 16(1)$. The line joining $P$ and $F$ has slope $(0 - 4)/(4 - 1) = -4/3$. Its equation is

$$y = -\frac{4}{3}(x - 4), \qquad x = 4 - \frac{3}{4}y.$$

**Step 2:** Find the second intersection Substitute this linear expression for $x$ into the parabola:

$$y^2 = 16\left(4 - \frac{3}{4}y\right) = 64 - 12y.$$

Therefore

$$y^2 + 12y - 64 = (y - 4)(y + 16) = 0.$$

The root $y = 4$ belongs to the known endpoint. The second root is $y = -16$, and its corresponding abscissa is $x = 4 + 12 = 16$. Thus the other endpoint is $Q = (16, -16)$.

**Step 3:** Calculate the whole chord length The coordinate changes from $P$ to $Q$ are 15 horizontally and -20 vertically. Hence

$$PQ = \sqrt{15^2 + (-20)^2} = \sqrt{625} = 25.$$

The focus lies between the endpoints, consistent with a focal chord. The alternatives 24, 20 and 22 fail this exact distance calculation; in particular, 20 is merely the vertical separation, not the slant length.

**Step 4:** Conclusion The length of the focal chord is 25 coordinate units. Hence, the correct option is (C).

**Quick Tip to Solve:** Use the focus to determine the chord line; the known endpoint then factors out of the intersection equation.

**4. The value of $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$ is** [4 marks]

(A) $\frac{3}{2} + \cos 20^o$

(B) $\frac{3}{4}$

(C) $\frac{3}{2}(1 + \cos 20^o)$

(D) $\frac{3}{2}$

**Correct Answer:** (B) $\frac{3}{4}$

**Solution:**

**Concept:**

- Use $\cos^2\theta = (1 + \cos 2\theta)/2$ to combine the two squared terms before simplifying the product.
- The sum-to-product identity is $\cos u + \cos v = 2\cos((u+v)/2)\cos((u-v)/2)$, while $2\cos u \cos v = \cos(u+v) + \cos(u-v)$.

**Step 1:** Combine the squared cosine terms Let $E = \cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$. All angles are in degrees. First,

$$\cos^2 10^\circ + \cos^2 50^\circ = 1 + \frac{\cos 20^\circ + \cos 100^\circ}{2}.$$

The angles 20 and 100 have mean 60 and half-difference -40. Therefore

$$\cos 20^\circ + \cos 100^\circ = 2\cos 60^\circ \cos 40^\circ = \cos 40^\circ.$$

The sum of squares is consequently $1 + \frac{1}{2}\cos 40^\circ$.

**Step 2:** Expand the remaining product The product-to-sum identity applied to 10 and 50 degrees gives

$$\cos 10^\circ \cos 50^\circ = \frac{\cos 60^\circ + \cos 40^\circ}{2} = \frac{1}{4} + \frac{1}{2}\cos 40^\circ.$$

Subtract this entire expression, retaining the minus sign on both terms. The same variable cosine term occurs in the sum of squares and in the product.

**Step 3:** Cancel the matching terms and compare options We obtain

$$E = 1 + \frac{1}{2}\cos 40^\circ - \frac{1}{4} - \frac{1}{2}\cos 40^\circ = \frac{3}{4}.$$

This exact cancellation explains why no residual angle appears in the answer. Options (A) and (C) leave a positive cosine contribution, and option (D) gives twice the resulting constant; none equals the evaluated expression.

**Step 4:** Conclusion The given expression equals $\frac{3}{4}$. Hence, the correct option is (B).

**Quick Tip to Solve:** When two squared cosines flank a product, convert the squares and product separately; their cosine terms may cancel.

**5. If the fourth term in the binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ (x > 0) is $20 \times 8^7$, then a value of x is :** [4 marks]

(A) $8^{-2}$

(B) $8^2$

(C) $8^3$

(D) 8

**Correct Answer:** (B) $8^2$

**Solution:**

**Concept:**

- The fourth term of $(a+b)^6$ is $\binom{6}{3}a^3b^3$. The term index is one more than the exponent of the second summand.
- Set $t=\log_8 x$, so $x=8^t$ because $x>0$. This turns powers whose exponents contain a logarithm into a quadratic equation in $t$.

**Step 1:** Write the fourth term accurately Here $a=2/x$ and $b=x^{\log_8 x}$. Consequently,

$$T_4=\binom{6}{3}\left(\frac{2}{x}\right)^3\left(x^{\log_8 x}\right)^3=20\cdot 8\,x^{3\log_8 x-3}.$$

Equating this to $20\cdot 8^7$ and dividing by $20\cdot 8$ yields

$$x^{3\log_8 x-3}=8^6.$$

The coefficient 8 must be removed before comparing exponents.

**Step 2:** Solve the resulting quadratic in the logarithm Put $t=\log_8 x$, hence $x=8^t$. The equation becomes

$$8^{3t^2-3t}=8^6.$$

Since the positive base 8 is different from 1, its exponential is one-to-one. Thus

$$3t^2-3t=6,\qquad (t-2)(t+1)=0.$$

Both roots are permitted: $t=2$ or $t=-1$. They give $x=8^2$ or $x=8^{-1}$, respectively.

**Step 3:** Select the value offered in the question The question asks for a value of $x$, rather than the complete solution set. Only $8^2$ occurs among the alternatives. For the listed logarithms $-2,2,3,1$, the exponent $3t^2-3t$ is respectively $18,6,18,0$; only 6 satisfies the equation. The valid but unlisted value $8^{-1}$ must not be confused with option (A), which is $8^{-2}$.

**Step 4:** Conclusion The positive solutions are $8^2$ and $8^{-1}$; the listed valid value is $8^2$. Hence, the correct option is (B).

**Quick Tip to Solve:** Convert the variable to a power of the logarithm base; then compare exponents only after accounting for the coefficient.

**6. All the points in the set**

$S = \left\{ \frac{\alpha+i}{\alpha-i} : \alpha \in R \right\} (i = \sqrt{-1})$ **lie on a :** [4 marks]

(A) straight line whose slope is –1

(B) straight line whose slope is 1.

(C) circle whose radius is 1.

(D) circle whose radius is $\sqrt{2}$ .

**Correct Answer:** (C) circle whose radius is 1.

**Solution:**

**Concept:**

- The modulus of a quotient is the quotient of the moduli: $|z_1/z_2| = |z_1|/|z_2|$ when $z_2 \neq 0$.
- For real $\alpha$, the complex numbers $\alpha + i$ and $\alpha - i$ are conjugates and therefore have equal modulus. The equation $|z| = 1$ describes the unit circle in the Argand plane.

**Step 1:** Check the domain and compute the modulus Write $z = (\alpha + i)/(\alpha - i)$. Since $\alpha$ is real, $|\alpha - i| = \sqrt{\alpha^2 + 1} > 0$; the denominator never vanishes. Moreover,

$$|z| = \frac{\sqrt{\alpha^2+1}}{\sqrt{\alpha^2+1}} = 1.$$

Thus every point represented by the set is exactly one unit from the origin, independently of the parameter.

**Step 2:** Confirm the Cartesian equation Multiplication by the conjugate of the denominator gives

$$z = \frac{(\alpha+i)^2}{\alpha^2+1} = \frac{\alpha^2-1}{\alpha^2+1} + i\frac{2\alpha}{\alpha^2+1}.$$

If its real and imaginary parts are $u, v$, then

$$u^2 + v^2 = \frac{(\alpha^2-1)^2 + 4\alpha^2}{(\alpha^2+1)^2} = 1.$$

This verifies the same circle directly, without needing a sketch.

**Step 3:** Interpret the locus and eliminate the alternatives The finite real parameter does not produce the point $z = 1$, since the numerator and denominator would then be equal, forcing $i = -i$. This omission does not affect the statement that all points lie on the unit circle. Taking $\alpha = 0, 1, -1$ gives $-1, i, -i$, three non-collinear points, so neither straight-line option works. A circle of radius $\sqrt{2}$ is inconsistent with the modulus 1.

**Step 4:** Conclusion All points lie on the circle centred at the origin with radius 1. Hence, the correct option is (C).

**Quick Tip to Solve:** For a quotient of conjugates, take the modulus first; it often gives the locus without eliminating the parameter.

**7. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx$ is equal to (Hence C is a constant of integration)** [4 marks]

(A) -3/4 $\tan^{-4/3} x + C$

(B) $3\tan^{-1/3} x + C$

(C) $-3\cot^{-1/3} x + C$

(D) $-3\tan^{-1/3} x + C$

**Correct Answer:** (D) $-3\tan^{-1/3} x + C$

**Solution:**

**Concept:**

- The integrand is $\sec^{2/3} x \, \csc^{4/3} x$. Fractional powers are interpreted with real cube roots on any interval where $\sin x$ and $\cos x$ are nonzero.
- Use $d(\cot x) = -\csc^2 x \, dx$. Extract a factor $\csc^2 x$ and express the remaining factor in powers of $\cot x$.

**Step 1:** Rewrite the integrand to expose a standard differential On the stated domain, real cube roots allow us to write

$$\begin{aligned} \sec^{2/3} x \, \csc^{4/3} x &= \csc^2 x \left( \frac{\sin x}{\cos x} \right)^{2/3} \\ &= \csc^2 x \, (\cot x)^{-2/3}. \end{aligned}$$

The product is positive wherever it is defined, because its fractional-power denominators have even numerators. This also avoids choosing an unnecessary restriction to the first quadrant.

**Step 2:** Substitute and apply the power rule Set $u = \cot x$, so $du = -\csc^2 x \, dx$. Then

$$I = -\int u^{-2/3} \, du = -\frac{u^{1/3}}{1/3} + C = -3u^{1/3} + C.$$

Restoring the trigonometric function,

$$I = -3(\cot x)^{1/3} + C = -3(\tan x)^{-1/3} + C.$$

The reciprocal identity is valid on each connected interval of the original domain.

**Step 3:** Differentiate and compare the printed expressions Differentiating the last expression gives

$$\frac{d}{dx}\left[-3(\tan x)^{-1/3}\right] = (\tan x)^{-4/3} \sec^2 x = \sec^{2/3} x \, \csc^{4/3} x.$$

Option (B) has the opposite sign, option (C) uses the reciprocal power of cotangent, and option (A) has the wrong power of tangent; their derivatives do not reproduce the integrand.

**Step 4:** Conclusion An antiderivative is $-3\tan^{-1/3} x + C$, with the exponent indicating a power, not an inverse trigonometric function. Hence, the correct option is (D).

**Quick Tip to Solve:** For fractional powers of sine and cosine, extract a squared cosecant or secant factor and use cotangent or tangent.

**8.** If $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\cdots\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$,

**then the inverse of** $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ **is** [4 marks]

(A) $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$

(B) $\begin{bmatrix}1 & 0\\13 & 1\end{bmatrix}$

(C) $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

(D) $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$

**Correct Answer:** (C) $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

**Solution:**

**Concept:**

- For the upper triangular matrix $U(t)=\begin{pmatrix}1 & t\\0 & 1\end{pmatrix}$, multiplication adds the upper-right entries: $U(a)U(b)=U(a+b)$.
- The inverse of a nonsingular matrix is the matrix whose product with it is the identity. Here $\det U(t)=1$, so the inverse always exists.

**Step 1:** Combine the matrices by direct multiplication Multiply two consecutive factors before handling the full product. Their diagonal entries remain 1, the lower-left entry remains 0, and the upper-right entry is the sum of the two parameters:

$$\begin{pmatrix}1 & a\\0 & 1\end{pmatrix}\begin{pmatrix}1 & b\\0 & 1\end{pmatrix}=\begin{pmatrix}1 & a+b\\0 & 1\end{pmatrix}.$$

Repeating this rule converts the given product into $U(1+2+\cdots+(n-1))$.

**Step 2:** Determine the natural number Compare the upper-right entry with 78 and use the sum of the first $n-1$ natural numbers:

$$\frac{n(n-1)}{2}=78,\qquad n^2-n-156=0.$$

Factoring gives $(n-13)(n+12)=0$. The number of factors must be a nonnegative integer, so the relevant solution is $n=13$; the negative root cannot describe this product.

**Step 3:** Invert and check the requested matrix The question asks for the inverse of $U(n)$, not of the

product $U(78)$. Since $U(13)U(-13) = U(0) = I$, the inverse is

$$\begin{pmatrix} 1 & -13 \\ 0 & 1 \end{pmatrix}.$$

Options (A) and (B) change the lower-left entry rather than cancel the upper-right entry; option (D) uses 12 instead of 13 and leaves an upper-right entry 1 when multiplied by $U(13)$.

**Step 4:** Conclusion The required inverse is $\begin{pmatrix} 1 & -13 \\ 0 & 1 \end{pmatrix}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For matrices with diagonal entries 1 and only an upper-right parameter, add parameters in products and negate the parameter to take the inverse.

**9. Slope of a line passing through P(2, 3) and intersecting the line, x + y = 7 at a distance of 4 units from P, is :** [4 marks]

(A) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$

(B) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

(C) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$

(D) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$

**Correct Answer:** (D) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$

**Solution:**

**Concept:**

- If the displacement from $P$ to the intersection point is $(u, v)$, then its length is $\sqrt{u^2+v^2}$ and the joining line has slope $v/u$.
- A point on $x + y = 7$ has coordinates whose sum is 7. Using displacement coordinates avoids solving for a line intercept.

**Step 1:** Express the intersection point using displacements Let $Q = (2 + u, 3 + v)$ be the intersection with the given line. Since $Q$ is on $x + y = 7$,

$$2 + u + 3 + v = 7, \qquad u + v = 2.$$

The distance condition is $PQ = 4$, so $u^2 + v^2 = 16$. These equations account for both possible intersection points at the specified distance.

**Step 2:** Find both possible displacements Put $v = 2 - u$ into the distance equation:

$$u^2 + (2 - u)^2 = 16.$$

This becomes $u^2 - 2u - 6 = 0$, hence

$$u = 1 \pm \sqrt{7}, \qquad v = 1 \mp \sqrt{7}.$$

Both values of $u$ are nonzero, so the slopes exist. Also $uv = -6$, giving another check on the signs.

**Step 3:** Compare the possible slopes with the options The two possible slopes are

$$m_1 = \frac{1-\sqrt{7}}{1+\sqrt{7}}, \qquad m_2 = \frac{1+\sqrt{7}}{1-\sqrt{7}}.$$

Both are negative: one displacement component is positive and the other negative. Therefore the positive options (A) and (B) fail. Option (C) involves $\sqrt{5}$ and does not satisfy the distance condition. Of the two valid slopes, only $m_1$ appears in the list, as (D). The wording asks for a slope, so identifying this listed possibility is sufficient.

**Step 4:** Conclusion A listed possible slope is $\frac{1-\sqrt{7}}{1+\sqrt{7}}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For a fixed distance from a point to a point on a line, use displacement variables: the line gives a linear relation and the distance gives a circle equation.

**10. The solution of the differential equation**

**$x\frac{dy}{dx} + 2y = x^2$ ($x \neq 0$) with $y(1) = 1$, is :** [4 marks]

(A) $y = \frac{4}{5}x^3 + \frac{1}{5x^2}$

(B) $y = \frac{3}{4}x^2 + \frac{1}{4x^2}$

(C) $y = \frac{x^2}{4} + \frac{3}{4x^2}$

(D) $y = \frac{x^3}{5} + \frac{1}{5x^2}$

**Correct Answer:** (C) $y = \frac{x^2}{4} + \frac{3}{4x^2}$

**Solution:**

**Concept:**

- A first-order linear equation $y' + P(x)y = Q(x)$ has integrating factor $e^{\int P(x)\,dx}$. Multiplying by it makes the left side an exact product derivative.
- The initial condition determines the integration constant on the interval containing the initial point. Here $x \neq 0$, and $x = 1$ lies in the positive interval.

**Step 1:** Put the equation in linear form Divide the equation by $x$, which is allowed because $x \neq 0$:

$$\frac{dy}{dx} + \frac{2}{x}y = x.$$

The integrating factor is

$$e^{\int 2/x\,dx} = e^{2\log|x|} = x^2.$$

The right-hand side after division is $x$, not $x^2$; keeping this distinction prevents an erroneous cubic term.

**Step 2:** Integrate the product derivative Multiply the linear equation by $x^2$:

$$x^2\frac{dy}{dx} + 2xy = x^3.$$

By the product rule, this is $\frac{d}{dx}(x^2y) = x^3$. Therefore

$$x^2y = \frac{x^4}{4} + C, \qquad y = \frac{x^2}{4} + \frac{C}{x^2}.$$

This form is valid on any interval avoiding the singular point 0.

**Step 3:** Apply the initial value and verify At $x = 1$, the condition $y = 1$ gives $1 = 1/4 + C$, so $C = 3/4$. Thus

$$y = \frac{x^2}{4} + \frac{3}{4x^2}.$$

Its derivative is $x/2 - 3/(2x^3)$. Substitution yields

$$xy\prime + 2y = \frac{x^2}{2} - \frac{3}{2x^2} + \frac{x^2}{2} + \frac{3}{2x^2} = x^2.$$

Option (B) satisfies the initial value but gives $3x^2$ on substitution. The cubic expressions in (A) and (D) do not produce the required forcing term.

**Step 4:** Conclusion The initial-value solution is $y = \frac{x^2}{4} + \frac{3}{4x^2}$ on the interval $x > 0$. Hence, the correct option is (C).

**Quick Tip to Solve:** After finding an integrating-factor solution, check both the differential equation and the initial value; satisfying the initial value alone does not select the correct option.

**11. Let $\sum_{k=1}^{10} f(a+k) = 16\left(2^{10}-1\right)$ where the function f satisfies f(x + y) = f(x)f(y) for all natural numbers x, y and f(1) = 2. then the natural number 'a' is** [4 marks]

(A) 2

(B) 16

(C) 4

(D) 3

**Correct Answer:** (D) 3

**Solution:**

**Concept:**

- The identity $f(x+y) = f(x)f(y)$, restricted to natural numbers, converts repeated addition of 1 into repeated multiplication by $f(1)$.
- A geometric sum with first term $t$, common ratio $r \neq 1$, and $N$ terms equals $t(r^N - 1)/(r-1)$.

**Step 1:** Determine the function at natural numbers Use the functional relation with $y = 1$:

$$f(m+1) = f(m)f(1) = 2f(m).$$

Starting from $f(1) = 2$, this gives $f(2) = 4$, $f(3) = 8$, and, by induction, $f(m) = 2^m$ for every positive integer $m$. No extension of the function to real arguments is required, because $a + k$ is a natural number.

**Step 2:** Evaluate the ten-term sum The terms are $2^{a+1}, 2^{a+2}, \ldots, 2^{a+10}$, a geometric progression of ratio 2. Hence

$$\sum_{k=1}^{10} f(a+k) = 2^{a+1}\frac{2^{10}-1}{2-1} = 2^{a+1}(2^{10}-1).$$

The exponent in the first term is $a + 1$, since the summation starts at $k = 1$, rather than at 0.

**Step 3:** Compare coefficients and test the choices Equate this expression to the given sum:

$$2^{a+1}(2^{10}-1) = 16(2^{10}-1).$$

Since $2^{10} - 1 \neq 0$, cancellation is valid and gives $2^{a+1} = 16 = 2^4$. Thus $a + 1 = 4$ and $a = 3$. For the other listed values $a = 2, 16, 4$, the coefficient would be $8, 2^{17}, 32$, respectively; none equals 16. Substituting $a = 3$ makes the first term 16, exactly the needed geometric-sum coefficient.

**Step 4:** Conclusion The natural number is $a = 3$. Hence, the correct option is (D).

**Quick Tip to Solve:** When a natural-number function turns sums into products, first use its value at 1 to find the sequence, then identify the first term of the required geometric sum.

**12. Let $\alpha$ and $\beta$ be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in $\mathbb{R}$,**

$$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$$

**is equal to** [4 marks]

(A) $y(y^2 - 1)$

(B) $y(y^2 - 3)$

(C) $y^3$

(D) $y^3 - 1$

**Correct Answer:** (C) $y^3$

**Solution:**

**Concept:**

- For roots of $x^2 + x + 1 = 0$, Vieta's relations give $\alpha + \beta = -1$ and $\alpha\beta = 1$. Each root also satisfies its own quadratic equation.

- The determinant of a symmetric matrix $\begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix}$ is $abc + 2fgh - af^2 - bg^2 - ch^2$.

**Step 1:** Collect the root identities Since $(x-1)(x^2+x+1) = x^3 - 1$, each root obeys $\alpha^3 = \beta^3 = 1$. Neither root is 1. Also

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 1 - 2 = -1.$$

These symmetric identities allow us to evaluate the determinant without substituting explicit complex radicals.

**Step 2:** Expand the determinant in symmetric form Using the stated determinant formula with diagonal entries $y+1, y+\beta, y+\alpha$, we obtain

$$\begin{aligned} D = {} & (y+1)(y+\beta)(y+\alpha) + 2\alpha\beta \\ & - (y+1) - (y+\beta)\beta^2 - (y+\alpha)\alpha^2. \end{aligned}$$

The positive triple-product term is $2\alpha\beta$, because the three off-diagonal entries are $\alpha, \beta, 1$. Keeping the negative square terms paired with their opposite diagonal entries avoids sign errors.

**Step 3:** Simplify all powers of the roots First, $(y+\beta)(y+\alpha) = y^2 - y + 1$, so $(y+1)(y^2-y+1) = y^3+1$. The remaining terms give

$$\begin{aligned} D &= y^3 + 1 + 2 - (y+1) - y(\alpha^2+\beta^2) - (\alpha^3+\beta^3) \\ &= y^3 + 1 + 2 - y - 1 + y - 2 = y^3. \end{aligned}$$

Thus no linear term or constant survives. Options (A) and (B) have incorrect linear terms, and (D) has an incorrect constant. The simplification holds for every real $y$, in particular for the stated nonzero values.

**Step 4:** Conclusion The determinant equals $y^3$. Hence, the correct option is (C).

**Quick Tip to Solve:** For determinants involving roots of a quadratic, use the sum, product and the polynomial relation to reduce powers before substituting explicit roots.

**13. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k $>$ 0, then k is equal to** [4 marks]

(A) $2\sqrt{6}$

(B) $\sqrt{6}$

(C) $2\sqrt{\frac{5}{6}}$

(D) $2\sqrt{\frac{10}{3}}$

**Correct Answer:** (A) $2\sqrt{6}$

**Solution:**

**Concept:**

- For $N$ observations, the variance is $\sigma^2 = \frac{\sum x_i^2}{N} - \bar{x}^2$, where $\bar{x} = \frac{\sum x_i}{N}$.
- The standard deviation is the nonnegative square root of the variance. These four listed numbers are the full data set, so the divisor is 4, not the sample divisor 3.

**Step 1:** Find the mean and the mean of squares The observations are $-1, 0, 1, k$. Their sum is $k$, because the first and third observations cancel. Their squared sum is $1 + 0 + 1 + k^2 = k^2 + 2$. Thus

$$\bar{x} = \frac{k}{4}, \qquad \frac{\sum x_i^2}{4} = \frac{k^2 + 2}{4}.$$

The squared mean is $k^2/16$, which must be subtracted from the mean of squares.

**Step 2:** Use the specified standard deviation As the standard deviation is $\sqrt{5}$, the variance is 5. Therefore

$$\frac{k^2 + 2}{4} - \frac{k^2}{16} = 5.$$

Multiplying by 16 gives $4k^2 + 8 - k^2 = 80$, hence $3k^2 = 72$ and $k^2 = 24$. The algebra produces two signs for $k$, but the question states $k > 0$. Consequently $k = \sqrt{24} = 2\sqrt{6}$.

**Step 3:** Verify the value and eliminate alternatives For $k^2 = 24$, the variance is $26/4 - 24/16 = 6.5 - 1.5 = 5$, as required. Option (B), with $k^2 = 6$, gives $13/8$; option (C), with $k^2 = 10/3$, gives $9/8$; option (D), with $k^2 = 40/3$, gives 3. None has variance 5. Squaring the given standard deviation at the outset is essential: setting the variance equal to $\sqrt{5}$ would solve a different condition.

**Step 4:** Conclusion The positive value is $k = 2\sqrt{6}$. Hence, the correct option is (A).

**Quick Tip to Solve:** When one observation is unknown, compute variance as mean of squares minus square of mean; it is shorter than expanding four deviations separately.

**14. If f(x) is a non-zero polynomial of degree four, having local extreme points at x = –1, 0, 1; then the set**

**S = {x ∈ R : f(x) = f(0)}**

**Contains exactly :** [4 marks]

(A) four rational numbers.

(B) four irrational numbers.

(C) two irrational and one rational number.

(D) two irrational and two rational numbes.

**Correct Answer:** (C) two irrational and one rational number.

**Solution:**

**Concept:**

- At an interior local extremum of a differentiable polynomial, the derivative vanishes. A quartic polynomial has a cubic derivative.
- A set lists distinct solutions: a repeated algebraic root contributes only one element. Rationality is decided after factoring the equation $f(x) - f(0) = 0$.

**Step 1:** Reconstruct the derivative from the three extrema The derivative vanishes at $-1, 0, 1$. Since it is a cubic and the polynomial has degree exactly four, its leading coefficient is nonzero. Therefore, for some real constant $c \neq 0$,

$$f\prime(x) = c(x+1)x(x-1) = c(x^3 - x).$$

The three distinct roots already account for every root of the cubic derivative; no additional linear factor is possible.

**Step 2:** Integrate and remove the additive constant Integrating gives

$$f(x) = c\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + C.$$

At $x = 0$, this yields $f(0) = C$. Thus

$$f(x) - f(0) = \frac{c}{4}x^2(x^2 - 2).$$

The additive constant cancels, so the requested set is independent of the vertical position of the quartic, as well as of its nonzero scale.

**Step 3:** Count distinct rational and irrational solutions Since $c \neq 0$, equality $f(x) = f(0)$ is equivalent to $x^2(x^2 - 2) = 0$. Hence

$$S = \{-\sqrt{2}, 0, \sqrt{2}\}.$$

The number 0 is rational, while both $\sqrt{2}$ and $-\sqrt{2}$ are irrational. Although 0 is a double root of the polynomial equation, it is a single member of the set. The alternatives listing four numbers confuse degree or multiplicity with distinct solutions, and the all-irrational alternative also overlooks 0.

**Step 4:** Conclusion The set contains exactly two irrational numbers and one rational number. Hence,

the correct option is (C).

**Quick Tip to Solve:** If a quartic has three specified stationary points, factor its cubic derivative first; then subtract the value at the comparison point before counting distinct roots.

**15. The area (in sq. units) of the region**
**A = {(x, y) : $x^2 \le y \le x + 2$} is** [4 marks]

(A) $\frac{31}{6}$
(B) $\frac{10}{3}$
(C) $\frac{13}{6}$
(D) $\frac{9}{2}$

**Correct Answer:** (D) $\frac{9}{2}$

**Solution:**

**Concept:**

- The area enclosed by two graphs is the integral of the upper ordinate minus the lower ordinate between their intersection points.
- For the region $x^2 \le y \le x + 2$, the line is above the parabola precisely where $x^2 - x - 2 \le 0$.

**Step 1:** Find the intersection limits Set the boundary ordinates equal and factor the resulting quadratic:

$$x^2 = x + 2 \quad \Longrightarrow \quad (x - 2)(x + 1) = 0.$$

Thus the curves meet at $x = -1$ and $x = 2$. Between these roots, $x + 2 - x^2$ is nonnegative, so no absolute-value split is required.

**Step 2:** Integrate the vertical width The horizontal range includes both the negative and positive parts of the enclosed region. Hence

$$\begin{aligned} A &= \int_{-1}^{2} (x + 2 - x^2)\, dx \\ &= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2} \\ &= \frac{10}{3} - \left(-\frac{7}{6}\right) = \frac{27}{6} = \frac{9}{2}. \end{aligned}$$

The value $10/3$ counts only the contribution from $0$ to $2$, omitting the positive area left of the y-axis. The other fractions also differ from the full boundary integral.

**Step 3:** Conclusion The enclosed area is $\boxed{9/2}$ square units. Hence, the correct option is (D).

**Quick Tip to Solve:** Always solve the intersection equation before integrating; the region here extends left of the y-axis to $x = -1$.

**16. Let p, q $\in$ R. If 2 - $\sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then :** [4 marks]

(A) $p^2 - 4q - 12 = 0$

(B) $q^2 - 4p - 16 = 0$

(C) $q^2 + 4p + 14 = 0$

(D) $p^2 - 4q + 12 = 0$

**Correct Answer:** No option is forced for arbitrary real p,q; the usual conjugate-root choice requires rational coefficients.

**Solution:**

**Concept:**

- A real polynomial can have an arbitrary real irrational root without also having its algebraic conjugate as a root.
- The conjugate-root rule for $2 \pm \sqrt{3}$ requires rational coefficients. Merely assuming $p, q \in \mathbb{R}$ does not give that rule.

**Step 1:** Write the actual coefficient constraint Let $r = 2 - \sqrt{3}$. Substitution gives

$$r^2 + pr + q = 0, \qquad q = -r^2 - pr.$$

This is a single linear relation between the two real parameters $p, q$. In particular, $p$ remains free; it cannot be fixed at $-4$ by equating rational and irrational parts when $p, q$ themselves may be irrational.

**Step 2:** Test the proposed universal relations Choose $p = 0$ and $q = -r^2 = 4\sqrt{3} - 7$. Then the quadratic is $x^2 - r^2 = 0$, so the specified root is indeed present. However the four option expressions become, respectively,

$$p^2 - 4q - 12 = 16 - 16\sqrt{3},$$
$$q^2 - 4p - 16 = 81 - 56\sqrt{3},$$
$$q^2 + 4p + 14 = 111 - 56\sqrt{3},$$
$$p^2 - 4q + 12 = 40 - 16\sqrt{3}.$$

All are nonzero, disproving every listed relation under the stated real-coefficient hypothesis.

**Step 3:** Separate the intended special case If the additional condition $p, q \in \mathbb{Q}$ were imposed, the other root would be $2 + \sqrt{3}$, giving $p = -4, q = 1$ and $p^2 - 4q - 12 = 0$. That familiar calculation explains the apparent intended choice, but its extra assumption is absent from the question.

**Step 4:** Conclusion As printed with real coefficients, no listed relation is necessary, so the question has no uniquely valid option. The question-sheet key selects (A); that relation follows with the additional rational-coefficient assumption, but the counterexample above proves it need not hold under the stated hypothesis.

**Quick Tip to Solve:** Check the coefficient field before using irrational-conjugate roots: real coefficients guarantee conjugate pairs only for nonreal complex roots.

**17. If the function f : R – {1, –1} → A defined by**

**f(x) = $\frac{x^2}{1-x^2}$ , is surjective, then A is equal to** [4 marks]

(A) R – (–1, 0)

(B) R – {–1}

(C) R – [–1, 0)

(D) [0, ∞)

**Correct Answer:** (C) R – [–1, 0)

**Solution:**

**Concept:**

- For a surjective function, the codomain equals the set of all values actually attained by the function.
- Put $t = x^2$. Then $t \geq 0$, $t \neq 1$, and $y = t/(1-t)$. Solving for $t$ gives the range restriction.

**Step 1:** Solve for the squared variable From $y = t/(1-t)$, rearrange to obtain

$$y = t(1+y), \qquad t = \frac{y}{1+y}.$$

The value $y = -1$ is impossible, since substitution in $y(1-t) = t$ would give $-1 = 0$. For every other candidate value, a real $x$ exists exactly when $t \geq 0$.

**Step 2:** Find and check the attainable intervals The sign condition $y/(1+y) \geq 0$ holds for $y < -1$ or $y \geq 0$. The resulting $t$ is never 1, since $y = 1+y$ is impossible. Thus every value in those two intervals has a permitted preimage $x = \pm\sqrt{y/(1+y)}$, with $x = 0$ when $y = 0$. Consequently

$$\text{range}(f) = (-\infty, -1) \cup [0, \infty) = \mathbb{R} \setminus [-1, 0).$$

The first option incorrectly includes $-1$, the second includes values between $-1$ and $0$, and the fourth omits all values below $-1$.

**Step 3:** Conclusion Surjectivity requires $A = \boxed{\mathbb{R} \setminus [-1, 0)}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For expressions involving $x^2$, solve for $x^2$ and impose nonnegativity; this also settles excluded endpoint values.

**18. If the function $f$ defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by**

$$f(x) = \begin{cases} \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1}, & x \neq \dfrac{\pi}{4}, \\ k, & x = \dfrac{\pi}{4} \end{cases}$$

**is continuous, then $k$ is equal to** [4 marks]

(A) 1

(B) $1 / \sqrt{2}$

(C) $\frac{1}{2}$

(D) 2

**Correct Answer:** (C) $\frac{1}{2}$

**Solution:**

**Concept:**

- Continuity at $x = \pi/4$ requires the assigned value $k$ to equal the two-sided limit of the surrounding expression.
- The numerator and denominator both vanish at $\pi/4$. Rationalisation and the double-angle identity expose a common factor that can be cancelled for nearby $x \neq \pi/4$.

**Step 1:** Remove the vanishing factor Write $\cot x - 1 = (\cos x - \sin x)/\sin x$. Rationalising the numerator gives

$$\sqrt{2}\cos x - 1 = \frac{2\cos^2 x - 1}{\sqrt{2}\cos x + 1} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\sqrt{2}\cos x + 1}.$$

The denominator $\sqrt{2}\cos x + 1$ is positive throughout the given interval, so this manipulation introduces no additional singularity.

**Step 2:** Evaluate the simplified limit For $x \neq \pi/4$ sufficiently near that point, cancellation yields

$$f(x) = \frac{\sin x(\cos x + \sin x)}{\sqrt{2}\cos x + 1}.$$

This expression is continuous at $\pi/4$, so

$$k = \frac{(1/\sqrt{2})(1/\sqrt{2} + 1/\sqrt{2})}{\sqrt{2}(1/\sqrt{2}) + 1} = \frac{1}{2}.$$

Direct substitution into the original quotient would give $0/0$, which is not a value. Neither $1$, $1/\sqrt{2}$ nor $2$ equals this unique limit.

**Step 3:** Conclusion For continuity, $\boxed{k = 1/2}$. Hence, the correct option is (C).

**Quick Tip to Solve:** When $\sqrt{2}\cos x - 1$ and $\cot x - 1$ vanish together, rationalise the first and rewrite cotangent to reveal $\cos x - \sin x$.

**19. Let the sum of the first n terms of a non-constant A.P., $a_1$, $a_2$, $a_3$, ..... be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, $a_{50}$) is equal to** [4 marks]

(A) (A, 50+45A)

(B) (50, 50+45A)

(C) (A, 50+46A)

(D) (50, 50+46A)

**Correct Answer:** (C) (A, 50+46A)

**Solution:**

**Concept:**

- The nth term is the difference of successive partial sums: $a_n = S_n - S_{n-1}$. A quadratic partial sum therefore gives a linear term formula.
- In an A.P., $a_{n+1} - a_n = d$, and the coefficient of $n$ in the linear expression for $a_n$ is the common difference.

**Step 1:** Recover the general term The given sum is $S_n = 50n + \frac{A}{2}n(n-7)$. Hence

$$\begin{aligned} a_n &= 50 + \frac{A}{2}\{n(n-7) - (n-1)(n-8)\} \\ &= 50 + \frac{A}{2}(2n-8) \\ &= 50 + A(n-4). \end{aligned}$$

This formula also works at $n = 1$, because $S_0 = 0$; thus the first term is $a_1 = 50 - 3A$, rather than 50.

**Step 2:** Read the difference and the fiftieth term Successive terms increase by

$$d = [50 + A(n-3)] - [50 + A(n-4)] = A.$$

At $n = 50$,

$$a_{50} = 50 + A(50-4) = 50 + 46A.$$

Equivalently, $a_{50} = a_1 + 49d = 50 - 3A + 49A$. The non-constant condition tells us $A \neq 0$; it does not turn the linear coefficient 50 in $S_n$ into the common difference. The expression $50 + 45A$ instead gives $a_{49}$, so it is one index too low.

**Step 3:** Conclusion The ordered pair is $\boxed{(d, a_{50}) = (A, 50 + 46A)}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Use $a_n = S_n - S_{n-1}$ rather than interpreting coefficients of $S_n$ directly as the first term and common difference.

**20. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then :** [4 marks]

(A) n = m – 8

(B) m = n = 78

(C) m + n = 68

(D) m = n = 68

**Correct Answer:** (B) m = n = 78

**Solution:**

**Concept:**

- A committee is an unordered selection, so choosing $r$ people from $N$ uses $\binom{N}{r}$, without an extra factorial.
- There are only five females and eight males available. These upper bounds force minimum numbers of the other sex in every eleven-person committee.

**Step 1:** Check whether the restrictions exclude anything Even a committee containing all five females must contain $11 - 5 = 6$ males. Thus every possible committee automatically satisfies the condition defining $m$. Likewise, with at most eight males available, every eleven-person committee contains at least $11 - 8 = 3$ females, so every committee satisfies the condition defining $n$. The two counts therefore refer to the same collection of committees.

**Step 2:** Count the committees and verify by cases There are thirteen people in all, and choosing eleven is equivalent to omitting two:

$$m = n = \binom{13}{11} = \binom{13}{2} = \frac{13 \cdot 12}{2} = 78.$$

A direct check separates the possible compositions into $(6, 5), (7, 4), (8, 3)$ males and females:

$$\binom{8}{6}\binom{5}{5} + \binom{8}{7}\binom{5}{4} + \binom{8}{8}\binom{5}{3} = 28 + 40 + 10 = 78.$$

Hence $m = n$, not $n = m - 8$; the common value is not $68$, and their sum is $156$, not $68$.

**Step 3:** Conclusion Both conditions are automatic, and $\boxed{m = n = 78}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Before counting restricted committees, use the available group sizes to check whether the restriction is already forced by the committee size.

**21. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x+\cos x}dx$ is** [4 marks]

(A) $\frac{\pi-2}{8}$

(B) $\frac{\pi-2}{4}$

(C) $\frac{\pi-1}{2}$

(D) $\frac{\pi-1}{4}$

**Correct Answer:** (D) $\frac{\pi-1}{4}$

**Solution:**

**Concept:**

- For an integrable function on $[0, a]$, reflection gives $\int_0^a g(x)\,dx = \int_0^a g(a-x)\,dx$.
- The sum-of-cubes factorisation $u^3+v^3 = (u+v)(u^2-uv+v^2)$ simplifies the sum of the original and reflected integrands.

**Step 1:** Reflect and add the integrals Let $I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x+\cos x}\,dx$. Under $x \mapsto \pi/2 - x$, sine and cosine interchange, giving

$$I = \int_0^{\pi/2} \frac{\cos^3 x}{\sin x + \cos x}\,dx.$$

The denominator is positive throughout the interval, so both integrals are well defined. Adding them,

$$2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}\,dx = \int_0^{\pi/2} (1 - \sin x \cos x)\,dx.$$

**Step 2:** Evaluate the elementary remainder Using $\int \sin x \cos x\,dx = \frac{1}{2}\sin^2 x$, obtain

$$2I = \left[x - \frac{1}{2}\sin^2 x\right]_0^{\pi/2} = \frac{\pi}{2} - \frac{1}{2}, \qquad I = \frac{\pi-1}{4}.$$

The factor 2 on the left must be retained when recovering the requested integral; omitting it would give the third option. The first two choices also have the wrong constant term, because the integral of $\sin x \cos x$ over this interval is $1/2$.

**Step 3:** Conclusion The integral equals $\boxed{(\pi-1)/4}$. Hence, the correct option is (D).

**Quick Tip to Solve:** When the denominator is $\sin x + \cos x$, try reflecting about $\pi/4$ and adding; a sum of cubes often cancels the denominator.

**22. Four persons can hit a target correctly with probabilities $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{8}$ respectively. if all hit at the target independently, then the probability that the target would be hit, is :** [4 marks]

(A) $\frac{25}{32}$
(B) $\frac{25}{192}$
(C) $\frac{1}{192}$
(D) $\frac{7}{32}$

**Correct Answer:** (A) $\frac{25}{32}$

**Solution:**

**Concept:**

- The target is hit if at least one person hits it. Its complementary event is that all four persons miss, which is easier to calculate than listing every possible combination of successful shots.
- For independent attempts, multiply the individual miss probabilities: $P(\text{all miss}) = \prod_{i=1}^{4}(1-p_i)$. Then subtract that product from one.

**Step 1:** Find the individual miss probabilities The given probabilities refer to hitting the target, so first complement each one. In the same order as the four persons, the miss probabilities are

$$1-\frac{1}{2}=\frac{1}{2},\quad 1-\frac{1}{3}=\frac{2}{3},\quad 1-\frac{1}{4}=\frac{3}{4},\quad 1-\frac{1}{8}=\frac{7}{8}.$$

These are independent miss events because the original attempts are independent. There is no need for a binomial formula: the four success probabilities are different.

**Step 2:** Calculate the probability of at least one hit Multiply the four miss probabilities, cancelling factors before multiplying the numerators:

$$P(\text{all miss}) = \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{7}{8}=\frac{7}{32}.$$

Therefore,

$$P(\text{target hit}) = 1-\frac{7}{32}=\frac{25}{32}.$$

Option (D) is the probability of the complementary event, so it cannot answer the question. Options (B) and (C) are also ruled out by a simple bound: the required probability must be at least $1/2$, the first person's success probability, whereas both those fractions are less than $1/2$.

**Step 3:** Conclusion The probability that the target is hit at least once is $\boxed{25/32}$. Hence, the correct option is (A).

**Quick Tip to Solve:** For an independent "at least one" event, multiply the failure probabilities and subtract from one; this avoids double-counting overlapping successes.

**23. If the line y = mx + $7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$ , then a value of m is :** [4 marks]

(A) $\frac{3}{\sqrt{5}}$

(B) $\frac{\sqrt{15}}{2}$

(C) $\frac{\sqrt{5}}{2}$

(D) $\frac{2}{\sqrt{5}}$

**Correct Answer:** (D) $\frac{2}{\sqrt{5}}$

**Solution:**

**Concept:**

- Implicit differentiation of $x^2/24 - y^2/18 = 1$ gives the tangent slope at a point on the hyperbola. A normal with finite nonzero slope has slope equal to the negative reciprocal of the tangent slope.
- The normal line must both pass through its point of contact and have the required slope. Using the line's intercept first determines the contact point's ordinate without solving a cubic intersection equation.

**Step 1:** Express the normal through its contact point Let the contact point be $(x_0, y_0)$. Differentiating the hyperbola gives

$$\frac{x}{12} - \frac{y}{9}\frac{dy}{dx} = 0, \qquad m_{\tan} = \frac{3x_0}{4y_0}.$$

Hence the normal slope is $m = -4y_0/(3x_0)$. The possible vertex normals have zero intercept and cannot be the line in the question. Substituting the general normal slope into its equation yields

$$y - y_0 = -\frac{4y_0}{3x_0}(x - x_0), \qquad y = mx + \frac{7y_0}{3}.$$

Comparison with the prescribed intercept $7\sqrt{3}$ gives $y_0 = 3\sqrt{3}$.

**Step 2:** Use the hyperbola to calculate the slope Substitute $y_0^2 = 27$ into the hyperbola:

$$\frac{x_0^2}{24} - \frac{27}{18} = 1 \quad \Longrightarrow \quad x_0^2 = 60.$$

Thus the two contact points have $x_0 = \pm 2\sqrt{15}$, and

$$m^2 = \frac{16y_0^2}{9x_0^2} = \frac{16 \cdot 27}{9 \cdot 60} = \frac{4}{5}.$$

The negative abscissa gives the positive slope $m = 2/\sqrt{5}$, while the positive abscissa gives its negative. Only the positive value is listed. The squared values in (A), (B), and (C) are $9/5$, $15/4$, and $5/4$, respectively; none equals $4/5$.

**Step 3:** Conclusion A permissible value is $\boxed{m = 2/\sqrt{5}}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For a normal to a central conic, write its point-slope equation and compare the intercept before calculating its slope; keep the negative reciprocal correct.

**24. If the line, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane, x + 2y + 3z = 15 at a point P, then the distance of P from the origin is :** [4 marks]

(A) $\frac{\sqrt{5}}{2}$

(B) $2\sqrt{5}$

(C) 9/2

(D) 7/2

**Correct Answer:** (C) 9/2

**Solution:**

**Concept:**

- The common value in a symmetric line equation is a single parameter. It determines all three coordinates of the same point, so the plane condition produces one linear equation in that parameter.
- The distance of $P = (x, y, z)$ from the origin is $OP = \sqrt{x^2 + y^2 + z^2}$. This is a point-to-point distance, not the perpendicular distance from the origin to the entire plane.

**Step 1:** Find the line parameter at the plane Set each ratio equal to $t$. The line is then

$$x = 1 + 2t, \qquad y = -1 + 3t, \qquad z = 2 + 4t.$$

The minus sign in the starting ordinate is essential because the given numerator is $y + 1$. At the intersection, substitution in $x + 2y + 3z = 15$ gives

$$(1 + 2t) + 2(-1 + 3t) + 3(2 + 4t) = 15,$$

$$5 + 20t = 15 \quad \Longrightarrow \quad t = \frac{1}{2}.$$

The nonzero coefficient 20 also confirms that there is a unique intersection, rather than a line lying in the plane.

**Step 2:** Calculate and check the distance The required point is

$$P = \left(2, \frac{1}{2}, 4\right).$$

As a check, $2 + 2(1/2) + 3(4) = 15$, so it does lie on the plane. Now apply the three-dimensional distance formula:

$$OP = \sqrt{2^2 + \left(\frac{1}{2}\right)^2 + 4^2} = \sqrt{4 + \frac{1}{4} + 16} = \sqrt{\frac{81}{4}} = \frac{9}{2}.$$

The other listed positive lengths cannot equal this exact result. In particular, $2\sqrt{5}$ has square 20, whereas the actual squared distance is $81/4$; the values are close but distinct. Neither $\sqrt{5}/2$ nor $7/2$ has the required square.

**Step 3:** Conclusion The distance of $P$ from the origin is $\boxed{9/2}$ units. Hence, the correct option is (C).

**Quick Tip to Solve:** After solving the common line parameter, check the resulting point in the plane before computing its distance; a sign error in the starting coordinates is then easy to catch.

**25. If the tangent to the curve, $y = x^3 + ax - b$ at the point (1, –5) is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on the curve ?** [4 marks]

(A) (2, –2)

(B) (2, –1)

(C) (–2, 2)

(D) (–2, 1)

**Correct Answer:** (A) (2, –2)

**Solution:**

**Concept:**

- The derivative of $y = x^3 + ax - b$ is $dy/dx = 3x^2 + a$, which is the tangent slope at the chosen abscissa.
- Two nonvertical perpendicular lines have slopes whose product is $-1$. The condition on slope determines $a$, while the fact that $(1, -5)$ lies on the curve determines $b$.

**Step 1:** Use perpendicularity to determine the coefficients The given line is $-x + y + 4 = 0$, or $y = x - 4$, so its slope is $1$. The tangent to the curve must therefore have slope $-1$. At $x = 1$,

$$3(1)^2 + a = -1 \quad \Longrightarrow \quad a = -4.$$

Next substitute the stated point into the original curve, not its derivative:

$$-5 = 1 - 4 - b = -3 - b \quad \Longrightarrow \quad b = 2.$$

Consequently the curve is $y = x^3 - 4x - 2$. These are two independent conditions: perpendicularity fixes the derivative at the point, and incidence fixes the constant term.

**Step 2:** Test the listed abscissas in the curve For either option with abscissa $2$, the curve gives

$$y(2) = 8 - 8 - 2 = -2.$$

Thus $(2, -2)$ lies on the curve, whereas $(2, -1)$ does not. For the remaining two options, both of which have abscissa $-2$,

$$y(-2) = -8 + 8 - 2 = -2.$$

Neither $(-2, 2)$ nor $(-2, 1)$ satisfies the equation. This checks every alternative without requiring a sketch. Finally, $y(1) = 1 - 4 - 2 = -5$ and $y'(1) = 3 - 4 = -1$, so the reconstructed curve satisfies both original conditions.

**Step 3:** Conclusion The point on the curve is $\boxed{(2, -2)}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Use a tangent condition on the derivative to find the coefficient of the linear term, then substitute the given point into the original curve to find the constant term.

**26. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points (1, f(1)) and (–1, f(–1)), then S is equal to :** [4 marks]

(A) $\left\{\frac{1}{3}, -1\right\}$

(B) $\left\{-\frac{1}{3}, 1\right\}$

(C) $\left\{-\frac{1}{3}, -1\right\}$

(D) $\left\{\frac{1}{3}, 1\right\}$

**Correct Answer:** (B) $\left\{-\frac{1}{3}, 1\right\}$

**Solution:**

**Concept:**

- The slope of the line joining two graph points is the secant slope $\big(f(b) - f(a)\big)/(b - a)$. A tangent parallel to that line has derivative equal to this slope.
- The tangent abscissa is a new variable: it need not equal either endpoint of the secant. Solving $f'(x) = m_{\text{sec}}$ finds every admissible tangent location.

**Step 1:** Calculate the slope of the specified chord For $f(x) = x^3 - x^2 - 2x$, first evaluate the endpoint ordinates carefully:

$$f(1) = 1 - 1 - 2 = -2, \qquad f(-1) = -1 - 1 + 2 = 0.$$

Therefore the line joining $(1, f(1))$ and $(-1, f(-1))$ has slope

$$m_{\text{sec}} = \frac{-2 - 0}{1 - (-1)} = -1.$$

Reversing both endpoint orders gives the same result. Notice that the negative input has square 1, so the middle term in $f(-1)$ remains negative.

**Step 2:** Equate the derivative to the chord slope Differentiating the polynomial gives $f'(x) = 3x^2 - 2x - 2$. The required parallelism condition is

$$3x^2 - 2x - 2 = -1 \quad \Longrightarrow \quad 3x^2 - 2x - 1 = 0.$$

Factor the quadratic:

$$(3x + 1)(x - 1) = 0 \quad \Longrightarrow \quad x = -\frac{1}{3} \text{ or } x = 1.$$

Both roots are valid because the polynomial and its derivative exist for all real $x$. Direct substitution gives derivative $-1$ at both roots. Option (A) has the two incorrect signs, (C) wrongly includes $-1$ instead of 1, and (D) wrongly includes $1/3$ instead of $-1/3$. In fact the derivative at $-1$ is 3 and at $1/3$ is $-7/3$, so neither extra value qualifies.

**Step 3:** Conclusion The complete set is $\boxed{S = \{-1/3, 1\}}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Find the secant slope from actual endpoint values before differentiating. Then factor the equation for the tangent slope and check both roots, including endpoint roots.

**27. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is :** [4 marks]

(A) $x^2 + y^2 - 4x^2y^2 = 0$

(B) $x^2 + y^2 - 2xy = 0$

(C) $x^2 + y^2 - 2x^2y^2 = 0$

(D) $x^2 + y^2 - 16x^2y^2 = 0$

**Correct Answer:** (A) $x^2 + y^2 - 4x^2y^2 = 0$

**Solution:**

**Concept:**

- A line with finite nonzero intercepts $a$ and $b$ has equation $x/a + y/b = 1$. Its intercept midpoint is $(a/2, b/2)$.
- A line is tangent to the unit circle centred at the origin precisely when its perpendicular distance from the origin is one. Eliminating the intercepts with the midpoint coordinates gives the locus equation.

**Step 1:** Write the tangent in terms of the midpoint Let $P = (a, 0)$, $Q = (0, b)$, and let their midpoint be $M = (h, k)$. Then $a = 2h$, $b = 2k$. The tangent has finite nonzero intercepts: a horizontal or vertical tangent would not meet both axes at distinct points. Its equation is

$$\frac{x}{2h} + \frac{y}{2k} = 1 \quad \Longrightarrow \quad kx + hy - 2hk = 0.$$

Here the lower-case $x, y$ in this line equation denote an arbitrary point of the tangent, while $h, k$ are fixed coordinates of its midpoint.

**Step 2:** Apply the tangency condition and eliminate parameters The distance from $(0, 0)$ to the tangent is

$$\frac{|2hk|}{\sqrt{h^2 + k^2}} = 1.$$

Squaring and rearranging gives

$$4h^2k^2 = h^2 + k^2, \qquad h^2 + k^2 - 4h^2k^2 = 0.$$

Renaming the variable midpoint coordinates as $x, y$ gives the expression in (A). The origin, although it satisfies this polynomial equation algebraically, is excluded by the finite nonzero intercept condition; it is not a midpoint of an admissible tangent. The coefficients 2 and 16 in (C) and (D) fail the distance-one condition. Option (B) reduces to $(x - y)^2 = 0$, an entire line rather than the required tangent-intercept locus.

**Step 3:** Conclusion The locus has equation $\boxed{x^2 + y^2 - 4x^2y^2 = 0}$, with the inadmissible origin excluded. Hence, the correct option is (A).

**Quick Tip to Solve:** If a midpoint of axis intercepts is $(h, k)$, replace the intercepts by $2h, 2k$ immediately and apply the centre-to-line distance condition for tangency.

**28. Let S = {$\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0$}. Then the sum of the elements of S is** [4 marks]

(A) $\pi$

(B) $2\pi$

(C) $\frac{13\pi}{6}$

(D) $\frac{5\pi}{3}$

**Correct Answer:** (B) $2\pi$

**Solution:**

**Concept:**

- Use $\cos^2\theta = 1 - \sin^2\theta$ to turn the equation into a quadratic in $s = \sin\theta$. Only quadratic roots in $[-1, 1]$ correspond to real angles.
- After solving for the sine, enumerate every angle in the entire specified interval $[-2\pi, 2\pi]$, including negative angles. A four-pi interval covers two full periods.

**Step 1:** Solve the quadratic for the sine The equation becomes

$$2(1 - \sin^2\theta) + 3\sin\theta = 0.$$

Writing $s = \sin\theta$, rearrange and factor:

$$2s^2 - 3s - 2 = 0, \qquad (2s + 1)(s - 2) = 0.$$

Thus $s = -1/2$ or $s = 2$. The latter is impossible because the sine of a real angle cannot exceed one. Therefore every admissible angle, and only such an angle, must satisfy $\sin\theta = -1/2$.

**Step 2:** List and add all angles in the interval The two solution families are

$$\theta = -\frac{\pi}{6} + 2n\pi \quad \text{or} \quad \theta = -\frac{5\pi}{6} + 2n\pi, \qquad n \in \mathbb{Z}.$$

Within the closed interval specified in the question, these give exactly

$$S = \left\{-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}.$$

Neither endpoint belongs to the set, since its sine is zero. Adding the four values gives

$$\sum_{\theta \in S} \theta = \frac{(-5 - 1 + 7 + 11)\pi}{6} = 2\pi.$$

Hence (B) is the unique listed result. Options (A), (C), and (D) do not equal this full sum; taking only positive solutions or omitting one negative solution would not answer the stated interval problem.

**Step 3:** Conclusion The sum of the elements of $S$ is $\boxed{2\pi}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Write both sine solution families, then enumerate the integers that place them inside the given interval; do not discard negative angles or assume there is only one period.

**29. For any two statements p and q, the negation of the expression**

**$p \vee (\sim p \wedge q)$ is :** [4 marks]

(A) $p \leftrightarrow q$

(B) $\sim p \wedge \sim q$

(C) $p \wedge q$

(D) $\sim p \vee \sim q$

**Correct Answer:** (B) $\sim p \wedge \sim q$

**Solution:**

**Concept:**

- The connective $\vee$ denotes inclusive OR, while $\wedge$ denotes AND. Negation reverses the truth value of an entire statement.
- Distributivity gives $p \vee (r \wedge q) = (p \vee r) \wedge (p \vee q)$. De Morgan's law changes the negation of an OR into the AND of the negations.

**Step 1:** Simplify the statement before negating Set $E = p \vee (\neg p \wedge q)$. Applying distributivity and using the fact that $p \vee \neg p$ is always true,

$$E = (p \vee \neg p) \wedge (p \vee q) = p \vee q.$$

Thus the original statement holds whenever at least one of $p, q$ is true; the nested expression introduces no additional condition.

**Step 2:** Apply De Morgan's law and test the alternatives Negating the simplified expression gives

$$\neg E = \neg(p \vee q) = \neg p \wedge \neg q.$$

This is true only when both statements are false. The biconditional in (A) is also true when both are true, so it is not the required negation. Option (C) is true when both are true, precisely the opposite case. Option (D) is already true when just one statement is false, so it is too broad.

**Step 3:** Conclusion The negation is $\neg p \wedge \neg q$. Hence, the correct option is (B).

**Quick Tip to Solve:** Simplify nested logical connectives before negating; then check the all-false and all-true rows to catch incorrect alternatives.

**30. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane y – z + 5 = 0, also passes through the point** [4 marks]

(A) $\left(\sqrt{2}, 1, 4\right)$

(B) $\left(-\sqrt{2}, 1, 4\right)$

(C) $\left(-\sqrt{2}, -1, -4\right)$

(D) $\left(\sqrt{2}, -1, 4\right)$

**Correct Answer:** The keyed answer is (A); the literal conditions also admit (B), on the other admissible plane.

**Solution:**

**Concept:**

- A plane $ax + by + cz + d = 0$ has normal vector $(a, b, c)$. The acute angle between planes is the acute angle between their normal vectors.
- The two given points impose $d = b$ and $c = -b$. For nonzero $b$, dividing by $b$ gives the family $kx + y - z + 1 = 0$.

**Step 1:** Impose the angle condition The normal of $y - z + 5 = 0$ is $(0, 1, -1)$, and that of the required plane is $(k, 1, -1)$. Hence

$$\frac{|2|}{\sqrt{k^2 + 2}\sqrt{2}} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, \qquad k^2 + 2 = 4.$$

Thus $k = \pm\sqrt{2}$. The omitted case $b = 0$ gives $x = 0$, which is perpendicular to the reference plane and fails the angle condition.

**Step 2:** Test the listed points on both possible planes For $k = \sqrt{2}$, substituting point (A) gives $2 + 1 - 4 + 1 = 0$. Therefore (A), the keyed answer, satisfies the requirements. However, the second allowed sign must also be retained: with $k = -\sqrt{2}$, point (B) gives exactly the same sum, $2 + 1 - 4 + 1 = 0$. Both planes contain the two prescribed points. For (C), the two possible residuals are 2 and 6; for (D), they are $-2$ and $-6$. Neither of those points qualifies.

**Step 3:** Conclusion The keyed answer is (A), corresponding to $\sqrt{2}x + y - z + 1 = 0$. As printed, the question does not select a unique plane: option (B) also satisfies all conditions on $-\sqrt{2}x + y - z + 1 = 0$. Thus the literal question admits both (A) and (B).

**Quick Tip to Solve:** An acute angle condition usually produces two possible signs for a plane normal. Test both before deciding that an option is unique.

**31. For a given gas at 1 atm pressure, rms speed of the molecule is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be :** [4 marks]

(A) 100 m/s

(B) 100 $\sqrt{5}$ m/s

(C) 80 $\sqrt{5}$ m/s

(D) 80 m/s

**Correct Answer:** (B) 100 $\sqrt{5}$ m/s

**Solution:**

**Concept:**

- The root-mean-square molecular speed of a gas is $v_{\text{rms}} = \sqrt{3RT/M}$, where $T$ is absolute temperature, $R$ the gas constant and $M$ the molar mass.
- For the same gas, $M$ is fixed, so $v_{\text{rms}}$ is proportional to $\sqrt{T}$. Pressure alone does not set the speed; its change can be accommodated by a change in density.

**Step 1:** Express both temperatures on the absolute scale Temperature ratios in the kinetic-theory relation must use kelvin. Using the conventional conversion for the given data,

$$T_1 = 127 + 273 = 400\,\text{K}, \qquad T_2 = 227 + 273 = 500\,\text{K}.$$

The temperature rises by a factor $500/400 = 5/4$. The doubling of pressure is not an additional factor in the molecular-speed ratio.

**Step 2:** Scale the initial rms speed Dividing the speed formula at the two temperatures cancels $3R/M$, giving

$$\frac{v_2}{200} = \sqrt{\frac{500}{400}} = \frac{\sqrt{5}}{2}, \qquad v_2 = 100\sqrt{5}\,\text{m s}^{-1}.$$

This is approximately $223.6\,\text{m s}^{-1}$, greater than the original speed as required for heating. Options (A), (C) and (D) give 100, about 178.9, and $80\,\text{m s}^{-1}$, respectively; all incorrectly predict a decrease despite the rise in temperature.

**Step 3:** Conclusion The new rms speed is $100\sqrt{5}\,\text{m s}^{-1}$. Hence, the correct option is (B).

**Quick Tip to Solve:** For the same gas, use the square root of the kelvin-temperature ratio; do not multiply by a separate pressure ratio.

**32. Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2nd Balmer line (n = 4 to n = 2) will be :** [4 marks]

(A) 642.7 nm

(B) 488.9 nm

(C) 889.2 nm

(D) 388.9 nm

**Correct Answer:** (B) 488.9 nm

**Solution:**

**Concept:**

- Balmer lines are hydrogen transitions ending at $n = 2$. The Rydberg relation is $\lambda^{-1} = R(1/2^2 - 1/n^2)$, with the upper principal quantum number $n > 2$.
- Taking a ratio of two lines eliminates the Rydberg constant. Wavelength is inversely proportional to the energy gap, not directly proportional to the upper quantum number.

**Step 1:** Write the reciprocal wavelengths of the two lines For the first line, the transition starts at $n = 3$; for the second, it starts at $n = 4$. Thus

$$\frac{1}{\lambda_1} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}, \qquad \frac{1}{\lambda_2} = R\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R}{16}.$$

The second gap is larger, so its wavelength must be shorter than the stipulated $660\,\text{nm}$.

**Step 2:** Eliminate the constant and calculate the wavelength Inverting the ratio correctly gives

$$\frac{\lambda_2}{\lambda_1} = \frac{5/36}{3/16} = \frac{20}{27}, \qquad \lambda_2 = 660\frac{20}{27} = 488.888\ldots\ \text{nm}.$$

Therefore the listed value is $488.9\,\text{nm}$. Option (C) exceeds the first-line wavelength and fails even the energy-order check. Options (A) and (D), though shorter than $660\,\text{nm}$, do not have the required ratio $20/27$ and therefore cannot describe the $4 \to 2$ transition.

**Step 3:** Conclusion The second Balmer line has wavelength $488.9\,\text{nm}$. Hence, the correct option is (B).

**Quick Tip to Solve:** For two hydrogen lines, divide their Rydberg expressions first; this avoids inserting an unnecessary value of the Rydberg constant.

**33. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its (1/n)$^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be :** [4 marks]

(A) $\frac{2MgL}{n^2}$

(B) nMgL

(C) $\frac{MgL}{2n^2}$

(D) $\frac{MgL}{n^2}$

**Correct Answer:** (C) $\frac{MgL}{2n^2}$

**Solution:**

**Concept:**

- For a slow lift, the external work against gravity equals the increase in gravitational potential energy. A distributed mass can be treated through its centre of mass for this calculation.
- A uniform cable has constant mass per unit length $M/L$. Its hanging part of length $L/n$ therefore has mass $M/n$, rather than the mass of the entire cable.

**Step 1:** Identify the mass and its initial centre of mass Choose the horizontal surface as zero height. The hanging segment has length $\ell = L/n$, with its centre at depth $\ell/2 = L/(2n)$. The segment initially resting on the surface does not change height. Hence only the hanging mass contributes to the change in gravitational energy:

$$m = \frac{M}{L}\frac{L}{n} = \frac{M}{n}, \qquad \Delta h = \frac{L}{2n}.$$

**Step 2:** Calculate the work of lifting the segment Raising the hanging part completely onto the surface gives

$$W = mg\Delta h = \frac{M}{n}g\frac{L}{2n} = \frac{MgL}{2n^2}.$$

Equivalently, an element at depth $x$ rises by $x$, so $W = (Mg/L)\int_0^{L/n} x\,dx$, giving the same result. Option (D) omits the one-half from the centre-of-mass height; (A) is four times the required value. Option (B) incorrectly grows with $n$, although a larger $n$ leaves less cable hanging and must require less work. No extra kinetic-energy change is needed for a slow lift.

**Step 3:** Conclusion The required work against gravity is $MgL/(2n^2)$. Hence, the correct option is (C).

**Quick Tip to Solve:** For a uniform hanging cable, use the hanging mass times the rise of its midpoint; both mass and length contribute a factor of $1/n$.

**34. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is Y = 0.3 sin(0.157x) cos(200pt). The length of the string is : (All quantities are in SI units.)** [4 marks]

(A) 60 m

(B) 20 m

(C) 80 m

(D) 40 m

**Correct Answer:** (C) 80 m

**Solution:**

**Concept:**

- A standing wave has spatial factor $\sin(kx)$, where the wave number is $k = 2\pi/\lambda$. The coefficient multiplying $x$ must be read independently of the time factor.
- A string fixed at both ends has a node at each end. In its $m$th harmonic, $L = m\lambda/2$, or equivalently $kL = m\pi$.

**Step 1:** Extract the wave number from the spatial dependence Comparing the printed equation with $Y = A\sin(kx)\cos(\omega t)$, its spatial factor gives

$$k = 0.157\,\text{rad}\,\text{m}^{-1}, \qquad \lambda = \frac{2\pi}{0.157} \simeq 40\,\text{m}.$$

The displacement amplitude $0.3$ and the coefficient in the time factor do not determine the node separation. Consecutive nodes are separated by $\lambda/2 \simeq 20\,\text{m}$.

**Step 2:** Fit four half-wavelengths between the clamps The fourth harmonic consists of four equal loops between the fixed ends. Therefore

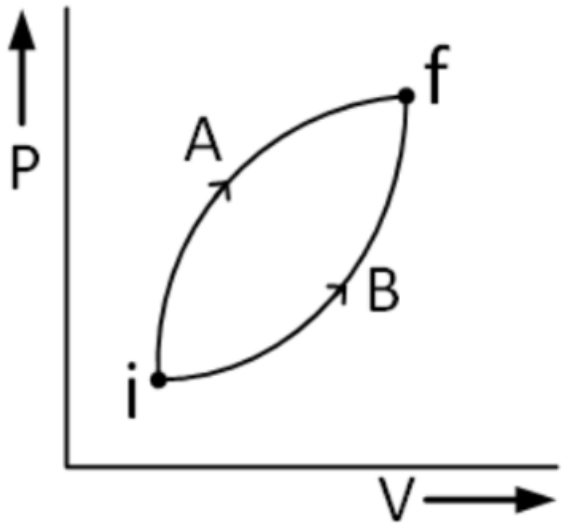
$$L = 4\frac{\lambda}{2} = 2\lambda = \frac{4\pi}{0.157} \simeq 80\,\text{m}.$$

The small rounding reflects the given three-digit wave number. Option (B), $20\,\text{m}$, fits only one half-wave and is the fundamental. Option (D), $40\,\text{m}$, fits two half-waves, while option (A), $60\,\text{m}$, fits three. Only $80\,\text{m}$ gives the specified fourth harmonic with a node at each clamp.

**Step 3:** Conclusion The string length is approximately $80\,\text{m}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Read $k$ from the coefficient of $x$, then use $kL = m\pi$; the harmonic number counts half-wavelengths, not whole wavelengths.

**35. Following figure shows two processes A and B for a gas. If $\Delta Q_A$ and $\Delta Q_B$ are the amount of heat absorbed by the system in two cases, and $\Delta U_A$ and $\Delta U_B$ are changes in internal energies, respectively, then :** [4 marks]




(A) $\Delta Q_A > \Delta Q_B$ ; $\Delta U_A > \Delta U_B$

(B) $\Delta Q_A < \Delta Q_B$ ; $\Delta U_A < \Delta U_B$

(C) $\Delta Q_A > \Delta Q_B$ ; $\Delta U_A = \Delta U_B$

(D) $\Delta Q_A = \Delta Q_B$ ; $\Delta U_A = \Delta U_B$

**Correct Answer:** (C) $\Delta Q_A > \Delta Q_B$ ; $\Delta U_A = \Delta U_B$

**Solution:**

**Concept:**

- Internal energy is a state function. Its change depends only on the initial and final equilibrium states, while heat and work generally depend on the path.
- With work done by the gas taken positive, the first law is $Q = \Delta U + W$. On a pressure-volume graph, $W = \int P\,dV$, the signed area under the path.

**Step 1:** Compare the changes in internal energy Both arrows in the figure lead from the same initial point $i$ to the same final point $f$. The gas therefore begins and ends in the same respective states in processes A and B. Since internal energy has a definite value at each state,

$$\Delta U_A = U_f - U_i = \Delta U_B.$$

The different intermediate pressures cannot change this endpoint difference. This immediately rules out (A) and (B), which claim unequal internal-energy changes.

**Step 2:** Compare the work and hence the heat absorbed Volume increases along both paths. Between the common endpoints, curve A is above curve B, so the pressure during A is greater at the same volume. Thus

$$W_A - W_B = \int_{V_i}^{V_f} (P_A - P_B)\,dV > 0.$$

Subtracting the two first-law equations then gives

$$Q_A - Q_B = (\Delta U_A - \Delta U_B) + (W_A - W_B) > 0.$$

The upper-path expansion requires more heat because more energy leaves as mechanical work, even though the stored-energy increase is identical. Option (D) incorrectly equates the heats and ignores the nonzero area enclosed between the paths.

**Step 3:** Conclusion The heat absorbed obeys $\Delta Q_A > \Delta Q_B$, while $\Delta U_A = \Delta U_B$. Hence, the correct option is (C).

**Quick Tip to Solve:** For paths with common endpoints on a pressure-volume graph, equate $\Delta U$ first; compare the areas under the paths to compare heat.

**36. The pressure wave, P = 0.01 sin [1000t – 3x] $Nm^{-2}$, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day, when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336 $ms^{-1}$ . Approximate value of T is** [4 marks]

(A) 12°C

(B) 15°C

(C) 4°C

(D) 11°C

**Correct Answer:** (C) 4°C

**Solution:**

**Concept:**

- A progressive wave of the form $P = P_0 \sin(\omega t - kx)$ has phase speed $v = \omega/k$, where $\omega$ is angular frequency and $k$ is wave number.
- For the same gas, the speed of sound is proportional to the square root of absolute temperature: $v^2 \propto \Theta$. Celsius temperatures must first be converted to kelvin.

**Step 1:** Read the initial speed from the wave equation The coefficient of time is $\omega = 1000\,\mathrm{rad\,s^{-1}}$, and that of position is $k = 3\,\mathrm{rad\,m^{-1}}$. Thus, on the day at $0^\circ\mathrm{C}$, or approximately $273\,\mathrm{K}$,

$$v_0 = \frac{\omega}{k} = \frac{1000}{3}\,\mathrm{m\,s^{-1}}.$$

The pressure amplitude $0.01\,\mathrm{N\,m^{-2}}$ does not enter this speed calculation. The frequency is the same on the two days, but the wavelength changes when the air temperature changes.

**Step 2:** Compare the absolute temperatures Write the second absolute temperature as $273 + T$, with $T$ measured in degrees Celsius. For unchanged gas composition,

$$\frac{273 + T}{273} = \left(\frac{336}{1000/3}\right)^2 = (1.008)^2 = 1.016064.$$

Therefore,

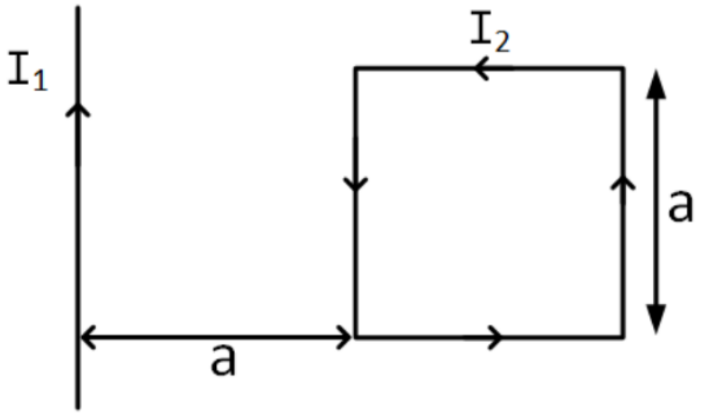
$$T = 273(0.016064) = 4.385472^\circ\text{C} \approx 4^\circ\text{C}.$$

This is only a small rise above freezing, consistent with the small increase in speed. The alternatives $11^\circ$C, $12^\circ$C and $15^\circ$C would each predict a noticeably larger speed increase than the stated value.

**Step 3:** Conclusion The approximate temperature is $T \approx 4^\circ$C. Hence, the correct option is (C).

**Quick Tip to Solve:** Find $\omega/k$ before using a temperature formula; the wave equation supplies the reference speed more accurately than a memorised rounded value.

**37. A rigid square loop of side 'a' and carrying current $I_2$ is lying on a horizontal surface near a long current $I_1$ carrying wire in the same plane as shown in figure. The net force on the loop due to wire will be :** [4 marks]




(A) Repulsive and equal to $\mu_0 I_1 I_2/4\pi$

(B) Attractive and equal to $\mu_0 I_1 I_2/3\pi$

(C) Repulsive and equal to $\mu_0 I_1 I_2/2\pi$

(D) Zero

**Correct Answer:** (A) Repulsive and equal to $\mu_0 I_1 I_2/4\pi$

**Solution:**

**Concept:**

- A long straight wire carrying current $I_1$ produces $B(r) = \mu_0 I_1/(2\pi r)$ at distance $r$. A perpendicular straight segment of length $a$, carrying current $I_2$, experiences force $I_2 a B$.
- Parallel currents attract and antiparallel currents repel. Forces must be added with their directions because the field strength varies across the square.

**Step 1:** Identify the forces on the four sides The diagram puts the nearer vertical side at distance $a$, and the farther one at distance $2a$, from the straight wire. The near-side current is downward whereas $I_1$ is upward, so that side is repelled. The far-side current is upward, so it is attracted. Forces on the horizontal top and bottom sides are opposite and cancel: corresponding small elements have equal current magnitude and are at the same distance from the wire. Thus only the unequal forces on the

vertical sides contribute to the resultant.

**Step 2:** Subtract the attractive force from the repulsive force Both vertical sides have length $a$. Their force magnitudes are

$$F_{\text{near}} = I_2 a \frac{\mu_0 I_1}{2\pi a} = \frac{\mu_0 I_1 I_2}{2\pi},$$

$$F_{\text{far}} = I_2 a \frac{\mu_0 I_1}{2\pi(2a)} = \frac{\mu_0 I_1 I_2}{4\pi}.$$

The stronger near-side force points away from the wire, hence

$$F_{\text{net}} = F_{\text{near}} - F_{\text{far}} = \frac{\mu_0 I_1 I_2}{4\pi}.$$

Option (C) gives only the near-side force and forgets the far-side attraction. Option (D) wrongly treats the field as uniform, while (B) has both an incorrect direction and magnitude.

**Step 3:** Conclusion The net force is repulsive, with magnitude $\mu_0 I_1 I_2/(4\pi)$. Hence, the correct option is (A).

**Quick Tip to Solve:** For a loop beside a straight wire, first cancel the transverse-side forces; then compare the two parallel sides using their different distances.

**38. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is :** [4 marks]

(A) 0.24 m

(B) 0.32 m

(C) 1.60 m

(D) 0.16 m

**Correct Answer:** (B) 0.32 m

**Solution:**

**Concept:**

- In the Cartesian sign convention, a concave mirror has negative focal length, and a real object placed in front of it has negative object distance.
- The mirror equation is $1/f = 1/v + 1/u$, and transverse magnification is $m = -v/u$. An upright image has positive magnification and is virtual for this concave mirror.

**Step 1:** Use the upright magnification to relate the distances Take $f = -0.40\,\text{m}$. Since the required image is upright and enlarged five times, $m = +5$, not $-5$. Consequently,

$$5 = -\frac{v}{u} \quad \Rightarrow \quad v = -5u.$$

The face is the real object, so $u < 0$, and this relation gives $v > 0$: the image lies behind the mirror. This is the expected configuration for a concave shaving mirror, with the object between the pole and focus.

**Step 2:** Substitute into the mirror equation and check Eliminate $v$ using the magnification relation:

$$\frac{1}{-0.40} = \frac{1}{-5u} + \frac{1}{u} = \frac{4}{5u}.$$

Thus,

$$u = \frac{4}{5}f = -0.32\,\text{m}, \qquad v = +1.60\,\text{m}.$$

The requested holding distance is the positive separation $|u| = 0.32\,\text{m}$. It is smaller than the focal length $0.40\,\text{m}$, as an upright enlarged image requires. Option (C), $1.60\,\text{m}$, is the image distance behind the mirror, not the mirror-to-face distance. For the other proposed object distances, $0.24\,\text{m}$ and $0.16\,\text{m}$, the magnifications would be $2.5$ and approximately $1.67$, respectively, rather than $5$.

**Step 3:** Conclusion Hold the mirror $0.32\,\text{m}$ from the face. Hence, the correct option is (B).

**Quick Tip to Solve:** Attach the positive sign to an upright magnification first. After solving with signed distances, report the physical object separation as a positive length.

**39. A rectangular coil (Dimension 5 cm $\times$ 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction is kept centered at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is :** [4 marks]

(A) 0.42 Nm

(B) 0.55 Nm

(C) 0.38 Nm

(D) 0.27 Nm

**Correct Answer:** (D) 0.27 Nm

**Solution:**

**Concept:**

- The magnetic moment of a plane coil is $\mu = NIA$, where $N$ is the number of turns, $I$ the current and $A$ the area of one turn.
- The torque magnitude is $\tau = \mu B \sin\theta$, where $\theta$ is the angle between the normal to the coil and the magnetic field, rather than the angle between the plane and the field.

**Step 1:** Determine the area and final angle Convert both dimensions to SI units before calculating the area:

$$A = (5 \times 10^{-2})(2.5 \times 10^{-2}) = 1.25 \times 10^{-3}\,\text{m}^2.$$

Initially the coil is in the $X$-$Z$ plane, so its normal is along the $Y$-axis, perpendicular to the field along the $X$-axis. Rotation through $45°$ about the $Z$-axis rotates this normal in the $X$-$Y$ plane. Its angle to the field becomes either $45°$ or $135°$, depending on the sense chosen; both give the same torque magnitude. The stated clockwise current fixes the moment direction but does not alter that magnitude.

**Step 2:** Calculate the torque magnitude For $N = 100$ and $I = 3\,\text{A}$,

$$\mu = NIA = 100(3)(1.25 \times 10^{-3}) = 0.375\,\text{A m}^2.$$

With $B = 1\,\text{T}$, the final torque is

$$\tau = 0.375(1)\sin 45^\circ = \frac{0.375}{\sqrt{2}} \approx 0.265\,\text{N m}.$$

Rounded to two decimal places this is $0.27\,\text{N m}$. The maximum torque is only $\mu B = 0.375\,\text{N m}$, ruling out $0.42$ and $0.55\,\text{N m}$. The value $0.38\,\text{N m}$ approximately describes the initial maximum torque, before tilting, rather than the final value.

**Step 3:** Conclusion The torque is approximately $0.27\,\text{N m}$. Hence, the correct option is (D).

**Quick Tip to Solve:** Use the coil normal when finding the angle in $NIAB\sin\theta$, and convert each side from centimetres to metres before multiplying.

**40. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness 2a and mass 2M. The gravitational field at distance '3a' from the centre will be :** [4 marks]

(A) $\frac{GM}{3a^2}$
(B) $\frac{2GM}{9a^2}$
(C) $\frac{GM}{9a^2}$
(D) $\frac{2GM}{3a^2}$

**Correct Answer:** (A) $\frac{GM}{3a^2}$

**Solution:**

**Concept:**

- Outside a spherically symmetric mass distribution, its gravitational field is the same as if its entire mass were concentrated at its centre.
- Gravitational fields superpose. The field at radius $r$ is radially inward, with magnitude $g = GM_{\text{enc}}/r^2$, where $M_{\text{enc}}$ is the mass enclosed within that radius.

**Step 1:** Locate the observation point relative to both bodies The inner solid sphere has radius $a$ and mass $M$. The concentric surrounding shell has thickness $2a$, extending from radius $a$ to radius $a + 2a = 3a$. Thus the point at radius $3a$ is at the outer surface of the entire spherical distribution. All of the solid sphere and all of the shell mass are enclosed. The shell has a finite volume density, so the field is continuous at its outer boundary and may be evaluated using the exterior expression.

**Step 2:** Add the enclosed masses and apply the field formula The shell theorem allows each complete concentric distribution to be replaced, for this external or surface point, by its central point-mass equivalent. Hence

$$M_{\text{enc}} = M + 2M = 3M.$$

Using the actual distance $r = 3a$,

$$g = \frac{G(3M)}{(3a)^2} = \frac{3GM}{9a^2} = \frac{GM}{3a^2}.$$

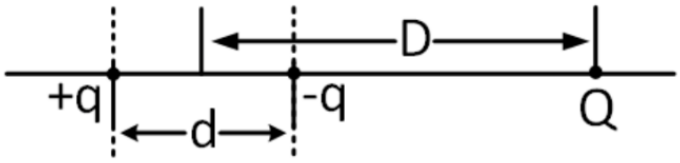
The direction is towards the common centre. Option (B) accounts only for the shell mass, and option (C) accounts only for the solid sphere; neither includes the whole system. Option (D) has twice the required magnitude. The shell thickness determines the observation radius here; it is not the distance to insert in the denominator independently of the inner radius.

**Step 3:** Conclusion The gravitational field has magnitude $GM/(3a^2)$, directed towards the centre. Hence, the correct option is (A).

**Quick Tip to Solve:** Add the shell thickness to its inner radius to find the outer radius. At that surface, use all enclosed mass and the square of the full radius.

**41. A system of three charges are placed as shown in the figure :**

**If D $>>$ d, the potential energy of the system is best given by :** [4 marks]




(A) $\frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{d} + \frac{qQd}{D^2}\right]$

(B) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{2D^2}\right]$

(C) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right]$

(D) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} + \frac{2qQd}{D^2}\right]$

**Correct Answer:** (C) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right]$

**Solution:**

**Concept:**

- The electrostatic potential energy of three point charges is the sum over their three distinct pairs: $U = \sum_{i<j} kq_iq_j/r_{ij}$, where $k = 1/(4\pi\varepsilon_0)$.
- For $d \ll D$, keep the leading dipole contribution in $d/D$. The opposite-charge pair has negative internal energy, which must be included along with its interaction with $Q$.

**Step 1:** Write all three pair energies The figure shows $+q$ on the left and $-q$ on the right, separated by $d$; $Q$ is farther to the right, at distance $D$ from their midpoint. Therefore the distances from $Q$ to

$+q$ and $-q$ are $D + d/2$ and $D - d/2$, respectively. Adding each pair exactly once gives

$$U = k\left[-\frac{q^2}{d} + \frac{qQ}{D + d/2} - \frac{qQ}{D - d/2}\right].$$

The first term is the mutual energy of $+q$ and $-q$, not a self-energy. The nearer negative charge makes the net interaction with a positive $Q$ negative.

**Step 2:** Simplify and retain the leading distant-field term Combine the last two fractions before approximating:

$$\frac{1}{D + d/2} - \frac{1}{D - d/2} = \frac{-d}{D^2 - d^2/4}.$$

Because $D \gg d$, the small correction $d^2/4$ can be neglected compared with $D^2$. Thus

$$U \simeq \frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right].$$

Option (A) has the wrong sign for the internal pair energy. Option (B) loses a factor of two when combining the two distant pair terms. Option (D) gives the wrong sign and coefficient for the dipole interaction. All three pair contributions are already present, so no extra factor of one-half belongs in the sum.

**Step 3:** Conclusion The leading potential energy is $U \simeq \frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right]$. Hence, the correct option is (C).

**Quick Tip to Solve:** List distinct charge pairs first, then expand the distant terms together. This preserves the internal dipole energy and prevents a spurious factor of one-half.

**42. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '2r' is :** [4 marks]

(A) M

(B) 4M

(C) M/2

(D) 2M

**Correct Answer:** (D) 2M

**Solution:**

**Concept:**

- Capillary rise in a circular tube obeys $h = 2\gamma\cos\theta/(\rho g r)$, where $\gamma$ is surface tension, $\theta$ the contact angle and $\rho$ the liquid density.
- The mass of the raised cylindrical liquid column is $M = \rho\pi r^2 h$. Both the cross-sectional area and the height change when the tube radius changes.

**Step 1:** Find how the height changes with radius For water in tubes of the same material under the same conditions, the surface tension, contact angle, density and gravitational acceleration are

unchanged. The capillary-rise formula therefore gives $h \propto 1/r$. When the radius is doubled to $r_2 = 2r$,

$$h_2 = \frac{h}{2}.$$

The wider tube raises water to a smaller height. That observation alone does not determine the mass: the wider tube simultaneously contains a larger cross-sectional area of water.

**Step 2:** Combine the changed area and height The original mass is $M = \rho\pi r^2 h$. For the wider tube,

$$M_2 = \rho\pi(2r)^2\frac{h}{2} = 2\rho\pi r^2 h = 2M.$$

Equivalently, the upward surface-tension force is $2\pi r\gamma\cos\theta$, which balances the raised column weight $Mg$; hence $M \propto r$, confirming the result directly. Option (B), $4M$, accounts for the increased area but ignores the reduced height. Option (C), $M/2$, follows only the height change and ignores the area. Option (A), $M$, incorrectly treats those changes as cancelling, whereas an area factor of four and height factor of one-half give a net factor of two.

**Step 3:** Conclusion The mass of water raised in the tube of radius $2r$ is $2M$. Hence, the correct option is (D).

**Quick Tip to Solve:** Capillary height scales as $1/r$, but the raised mass scales as $r^2 h$, so it scales as $r$. Distinguish mass from height before taking ratios.

**43. A moving coil galvanometer has resistance 50Ω and it indicates full deflection at 4mA current. A voltmeter is made using this galvanometer and a 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to :** [4 marks]

(A) 15 V

(B) 10 V

(C) 40 V

(D) 20 V

**Correct Answer:** (D) 20 V

**Solution:**

**Concept:**

- A voltmeter is made by connecting a large resistance in series with the galvanometer. The same current flows through both resistances.
- At the maximum measurable voltage the galvanometer current equals its full-scale value: $V_{\max} = I_g(R_s + R_g)$. Here $R_s$ is the added resistance and $R_g$ is the coil resistance.

**Step 1:** Convert the supplied values The full-scale current is $I_g = 4\,\text{mA} = 4 \times 10^{-3}\,\text{A}$. The series resistor is $R_s = 5\,\text{k}\Omega = 5000\,\Omega$, while the galvanometer contributes $R_g = 50\,\Omega$. Hence the total resistance across which the measured voltage acts is $5050\,\Omega$.

**Step 2:** Calculate the full-scale voltage Apply Ohm's law to the complete series combination:

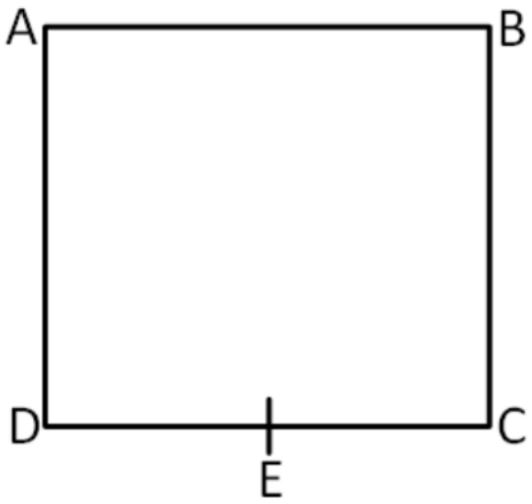
$$V_{\max} = (4 \times 10^{-3})(5000 + 50) = 20.2\,\text{V}.$$

The question asks for the closest value, so round to $20\,\text{V}$. The other choices, 10, 15 and 40 V, are not close to 20.2 V; 40 V would require nearly twice the full-scale current.

**Step 3:** Conclusion The maximum voltage is approximately $20.2\,\text{V}$, closest to $20\,\text{V}$. Hence, the correct option is (D).

**Quick Tip to Solve:** Add the coil resistance to the series resistance before multiplying by the full-scale current.

**44. A wire of resistance R is bent to form a square ABCD as shown in the figure. The effective resistance between E and C is : (E is mid-point of arm CD)** [4 marks]




(A) $\frac{7}{64}R$

(B) $\frac{1}{16}R$

(C) R

(D) $\frac{3}{4}R$

**Correct Answer:** (A) $\frac{7}{64}R$

**Solution:**

**Concept:**

- For a uniform wire, resistance is proportional to length. Each side of this square has resistance $R/4$; half a side has resistance $R/8$.
- The two paths connecting terminals E and C are in parallel because they have the same endpoints. Resistances along one path add in series.

**Step 1:** Identify the two conducting paths The short path is the segment EC. Since E is the midpoint of CD, its resistance is $R_1 = R/8$. The other path goes from E through D, A and B to C. Its resistance includes ED and three complete sides:

$$R_2 = \frac{R}{8} + 3\frac{R}{4} = \frac{7R}{8}.$$

These are separate branches; one must not add their resistances as if current had to traverse the full loop.

**Step 2:** Combine the branches in parallel The equivalent resistance is

$$R_{EC} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(R/8)(7R/8)}{R} = \frac{7R}{64}.$$

This is less than the shorter branch resistance $R/8$, as required for parallel paths. The values $R$ and $3R/4$ exceed that branch; $R/16$ would correspond to two equal $R/8$ branches, which this geometry does not provide.

**Step 3:** Conclusion The effective resistance is $7R/64$. Hence, the correct option is (A).

**Quick Tip to Solve:** Cut a closed uniform loop mentally at the two terminals: its two arcs become parallel resistors.

**45. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\overline{v}$ , m is its mass and $k_B$ is Boltzmann constant, then its temperature will be :** [4 marks]

(A) $\frac{m\overline{v}^2}{5k_B}$

(B) $\frac{m\overline{v}^2}{6k_B}$

(C) $\frac{m\overline{v}^2}{7k_B}$

(D) $\frac{m\overline{v}^2}{3k_B}$

**Correct Answer:** (D) $\frac{m\overline{v}^2}{3k_B}$

**Solution:**

**Concept:**

- The molecular rms speed is defined by $\overline{v}^2 = \langle v_x^2 + v_y^2 + v_z^2 \rangle$. It describes the translational motion of the molecular centre of mass.
- Equipartition assigns $k_B T/2$ to each quadratic energy term. Translation has three independent velocity components, giving mean translational energy $3k_B T/2$.

**Step 1:** Relate the rms speed to temperature The mean translational kinetic energy of one molecule is

$$\frac{1}{2} m \overline{v}^2 = \frac{3}{2} k_B T.$$

Solving for the temperature therefore gives

$$T = \frac{m\overline{v}^2}{3k_B}.$$

The mass here is that of one molecule, so Boltzmann's constant, rather than the gas constant, is appropriate.

**Step 2:** Distinguish speed from total molecular energy Rotation and vibration contribute additional

internal energy but do not add components to the translational rms speed. A fully excited diatomic molecule has seven quadratic terms in total: three translational, two rotational, and two vibrational. Using 7 in the denominator would wrongly equate translational kinetic energy to total molecular energy. The denominators 5 and 6 similarly do not describe the three centre-of-mass velocity components.

**Step 3:** Conclusion The literal rms-speed relation gives $T = m\overline{v}^2/(3k_B)$. Hence, the correct option is (D). Although the supplied key selects the denominator 7, that counts total molecular energy; the printed question specifies rms velocity, for which only the three translational components enter.

**Quick Tip to Solve:** For molecular rms speed always use three translational degrees of freedom, regardless of internal rotation or vibration.

**46. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane. : (i) a ring of radius R, (ii) a solid cylinder of radius R/2 and (iii) a solid sphere of radius R/4 . If in each case, the speed of the centre of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is :** [4 marks]

(A) 20 : 15 : 14

(B) 4 : 3 : 2

(C) 2 : 3 : 4

(D) 10 : 15 : 7

**Correct Answer:** (A) 20 : 15 : 14

**Solution:**

**Concept:**

- For rolling without slipping, $\omega = v/r$ and total kinetic energy is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.
- On climbing the incline, static friction does no work and the initial kinetic energy becomes $mgh$. Writing $I = kmr^2$ gives $h = (1+k)v^2/(2g)$.

**Step 1:** Write the energy balance for each shape At the maximum height, both centre-of-mass speed and angular speed vanish because the rolling constraint holds. Thus

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(kmr^2)\frac{v^2}{r^2}, \qquad h = \frac{(1+k)v^2}{2g}.$$

The factors of mass and radius cancel, so the different stated radii do not change the height ratio.

**Step 2:** Insert the moments of inertia For a ring $k = 1$, for a solid cylinder $k = 1/2$, and for a solid sphere $k = 2/5$. Since the initial centre-of-mass speeds are equal,

$$h_{\text{ring}} : h_{\text{cylinder}} : h_{\text{sphere}} = 2 : \frac{3}{2} : \frac{7}{5} = 20 : 15 : 14.$$

The ring rises highest because it has the largest rotational energy per unit mass at the given translational speed. The remaining choices fail these inertia factors; in particular, a ratio placing the sphere highest reverses the energy ordering.

**Step 3:** Conclusion The ratio of maximum heights is $20 : 15 : 14$. Hence, the correct option is (A).

**Quick Tip to Solve:** With equal rolling speeds, compare $1 + I/(mr^2)$; the specified radii cancel automatically.

**47. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is :** [4 marks]

(A) 0.01 kg/m$^3$

(B) 0.10 kg/m$^3$

(C) 0.31 kg/m$^3$

(D) 0.07 kg/m$^3$

**Correct Answer:** No printed option is correct: fractional error is 0.31 (dimensionless), whereas absolute density error is $3100\,\mathrm{kg\,m^{-3}}$.

**Solution:**

**Concept:**

- The density of a cube is $\rho = m/a^3$, where $a$ is its edge length. The first-order maximum fractional uncertainty adds the magnitudes of the relative contributions.
- For independent specified error limits, the standard maximum-error rule is $\Delta\rho/\rho = \Delta m/m + 3\Delta a/a$. This fractional error is dimensionless; the absolute density error has units of $\mathrm{kg\,m^{-3}}$.

**Step 1:** Calculate the fractional error Using the central values and their error limits gives

$$\frac{\Delta m}{m} = \frac{0.10}{10.00} = 0.01, \qquad \frac{\Delta a}{a} = \frac{0.01}{0.10} = 0.10.$$

Hence

$$\frac{\Delta\rho}{\rho} = 0.01 + 3(0.10) = 0.31.$$

The edge length contribution must be multiplied by three because volume depends on its cube.

**Step 2:** Convert to absolute uncertainty and inspect the units The nominal density is

$$\rho = \frac{10.00}{(0.10)^3} = 10^4\,\mathrm{kg\,m^{-3}},$$

so the first-order absolute uncertainty is

$$\Delta\rho = 0.31(10^4) = 3100\,\mathrm{kg\,m^{-3}}.$$

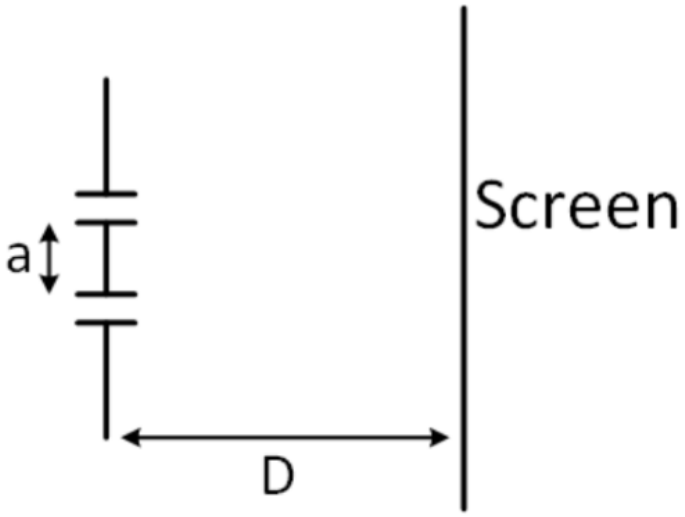
None of the printed dimensional options equals this. The number 0.31 in the third choice matches the fractional uncertainty only; attaching density units to that number is incorrect. The numbers 0.01 and 0.10 are only individual fractional inputs, and 0.07 is neither the combined fractional nor absolute uncertainty.

**Step 3:** Conclusion The fractional uncertainty is $0.31$ (31%), and the first-order absolute density uncertainty is $3100\,\mathrm{kg\,m^{-3}}$. No printed option is dimensionally correct. The supplied key selects (C),

whose number matches the fractional error, but its printed unit $\mathrm{kg\,m^{-3}}$ makes it unsuitable as the absolute error.

**Quick Tip to Solve:** After adding relative errors, multiply by the measured quantity only if an absolute error with units is required.

**48. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index $\mu$ is put in front of one of the slits, the central maximum gest shifted by a distance equal to n fringe widths. If the wavelength of light used is $\lambda$, t will be :** [4 marks]




(A) $\frac{D\lambda}{a(\mu-1)}$
(B) $\frac{2nD\lambda}{a(\mu-1)}$
(C) $\frac{2D\lambda}{a(\mu-1)}$
(D) $\frac{n\lambda}{(\mu-1)}$

**Correct Answer:** (D) $\frac{n\lambda}{(\mu-1)}$

**Solution:**

**Concept:**

- A thin sheet replacing an equal thickness of air introduces extra optical path $\Delta = (\mu - 1)t$, where $\mu$ is its refractive index.
- In Young's experiment the fringe width is $\beta = D\lambda/a$. A fixed extra path difference shifts the whole pattern by $x = D\Delta/a$.

**Step 1:** Express the central fringe shift The central maximum occurs where the geometrical path difference compensates the sheet's extra optical path. Taking the magnitude of its displacement,

$$x = \frac{D}{a}(\mu - 1)t.$$

The sign determines which side the central maximum moves towards; only the displacement magnitude is required to obtain the thickness.

**Step 2:** Use the stated number of fringe widths The observed shift is $n\beta$, so

$$\frac{D}{a}(\mu-1)t = n\frac{D\lambda}{a}.$$

Cancel the common factors $D/a$ and solve:

$$t = \frac{n\lambda}{\mu-1}.$$

The answer cannot depend on screen distance or slit separation when the shift is specified in fringe widths: increasing either geometrical factor affects both shift and fringe width equally. The first three choices retain such an unwanted factor, and the extra factors of two have no origin in the single sheet's optical path.

**Step 3:** Conclusion The sheet thickness is $t = n\lambda/(\mu-1)$. Hence, the correct option is (D).

**Quick Tip to Solve:** A shift of one fringe requires one wavelength of added optical path: set $(\mu-1)t = n\lambda$.

**49. The electric field of a light wave is given as**

$$\vec{E} = 10^{-3}\cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14}t\right)\hat{x}\,\frac{\mathrm{N}}{\mathrm{C}}.$$

**This light falls on a metal plate of work function $2\,\mathrm{eV}$. The stopping potential of the photoelectrons is: Given, $E$ (in eV) $= 12375/\lambda$ ($\lambda$ in Å).** [4 marks]

(A) 2.48 V

(B) 0.48 V

(C) 0.72 V

(D) 2.0 V

**Correct Answer:** (B) 0.48 V

**Solution:**

**Concept:**

- Einstein's photoelectric equation is $eV_s = E_\gamma - \phi$, where $\phi$ is the work function and $V_s$ is the stopping potential.
- In a travelling wave, the coefficient of position is $2\pi/\lambda$. Thus its wavelength can be read directly from the spatial term; the electric-field amplitude controls intensity, not photon energy.

**Step 1:** Read the wavelength and photon energy The spatial phase in the given wave is $2\pi x/(5\times10^{-7})$, so

$$\lambda = 5\times10^{-7}\,\mathrm{m} = 5000\,\text{Å}.$$

Using the supplied conversion,

$$E_\gamma = \frac{12375}{5000}\,\text{eV} = 2.475\,\text{eV}.$$

This also agrees with the optical frequency $6 \times 10^{14}\,\text{Hz}$ to the precision of the constants.

**Step 2:** Subtract the work function The maximum kinetic energy is

$$K_{\max} = 2.475 - 2 = 0.475\,\text{eV}.$$

A stopping potential of one volt removes one electronvolt of kinetic energy from an electron; hence $V_s = 0.475\,\text{V}$, about 0.48 V. The choice 2.48 V uses the photon energy without subtracting the work function; 2.0 V uses the work function itself. The value 0.72 V is inconsistent with the given wavelength and conversion constant.

**Step 3:** Conclusion The stopping potential is approximately $0.48\,\text{V}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Read wavelength from the spatial phase, convert metres to angstroms, then subtract the work function in electronvolts.

**50. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of $\theta$, where $\theta$ is the angle by which it has rotated, is given as k$\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is :** [4 marks]

(A) $\frac{k}{4I}\theta$
(B) $\frac{k}{I}\theta$
(C) $\frac{k}{2I}\theta$
(D) $\frac{2k}{I}\theta$

**Correct Answer:** (D) $\frac{2k}{I}\theta$

**Solution:**

**Concept:**

- For rotation about a fixed axis, the work done by torque equals the change in rotational kinetic energy: $dK = \tau\, d\theta$. This relation applies to an angle-dependent torque as well.
- The rotational equation of motion is $\tau = I\alpha$, where $I$ is the constant moment of inertia and $\alpha$ is angular acceleration.

**Step 1:** Differentiate the given energy with respect to angle The specified energy is $K = k\theta^2$. The torque is the rate at which rotational work is done per unit angular displacement, not the energy divided by the angle. Therefore,

$$\tau(\theta) = \frac{dK}{d\theta} = \frac{d}{d\theta}(k\theta^2) = 2k\theta.$$

The derivative supplies the factor of two. No assumption of constant torque is appropriate, since the given energy is quadratic in angle.

**Step 2:** Apply the rotational equation of motion Dividing the torque by the disc's moment of inertia

gives

$$\alpha(\theta) = \frac{\tau(\theta)}{I} = \frac{2k}{I}\theta.$$

Equivalently, differentiating $\frac{1}{2}I\omega^2 = k\theta^2$ with respect to time gives $I\omega\alpha = 2k\theta\omega$, yielding the same result wherever the disc is moving. The positive result is consistent with increasing kinetic energy as the positive rotation increases.

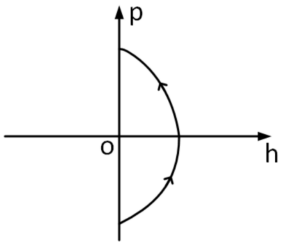
**Step 3:** Compare the four coefficients The required coefficient is $2k/I$. The expressions in (A), (B) and (C) respectively give $k/(4I)$, $k/I$ and $k/(2I)$; none equals the derivative of the given energy divided by $I$.

**Step 4:** Conclusion The angular acceleration is $\boxed{\alpha = 2k\theta/I}$. Hence, the correct option is (D).

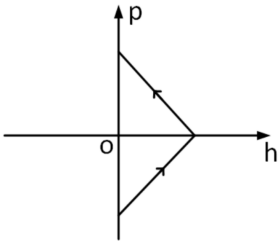
**Quick Tip to Solve:** When kinetic energy is given as a function of angular displacement, first use $\tau = dK/d\theta$, then divide by $I$; do not use $K/\theta$ for variable torque.

**51. A ball is thrown vertically up (taken as +z-axis) from the ground. The correct momentum-height (p-h) diagram is:** [4 marks]

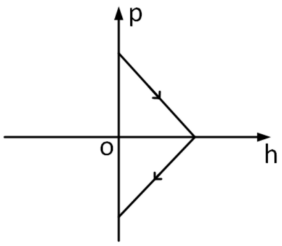
(A)



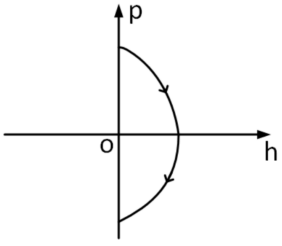

(B)



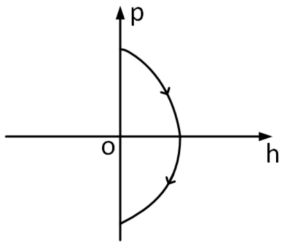

(C)




(D)



**Correct Answer:** (D)




**Solution:**

**Concept:**

- With upward positive, the momentum is $p = mv$, so it is positive during ascent and negative during descent. Gravity always changes momentum at the rate $dp/dt = -mg$.
- Conservation of mechanical energy gives $p^2/(2m) + mgh = \frac{1}{2}mu^2$, where $u$ is the launch speed and $h$ is height above the ground.

**Step 1:** Obtain the shape of the curve Multiplying the energy equation by $2m$ gives

$$p^2 = m^2u^2 - 2m^2gh.$$

Thus

$$h = \frac{u^2}{2g} - \frac{p^2}{2m^2g}.$$

With $h$ on the horizontal axis and $p$ on the vertical axis, this is a parabola opening towards decreasing height. At the greatest height $H = u^2/(2g)$, momentum is zero. At ground level the two momenta are $+mu$ and $-mu$, corresponding to launch and return.

**Step 2:** Determine the arrows for the actual flight The ball starts at the positive intercept $(h, p) = (0, mu)$. While it rises, $h$ increases and $p$ decreases, so the upper branch must point downwards and to the right towards $(H, 0)$. After the top, the ball falls: height decreases while momentum becomes increasingly negative. The lower branch therefore points downwards and to the left towards $(0, -mu)$. Momentum passes continuously through zero, rather than reversing discontinuously.

**Step 3:** Reject the incompatible plots Options (B) and (C) have straight sides; energy requires a quadratic relation between momentum and height. Option (A) has the curved shape but its arrows run in the reverse time direction. Option (D) combines the parabolic curve with the correct arrows on both branches.

**Step 4:** Conclusion The correct momentum-height plot is the left-opening parabola traversed from positive to negative momentum, as in option (D). Hence, the correct option is (D).

**Quick Tip to Solve:** For a momentum-height plot, find its shape from energy, then use $dp/dt = -mg$ to fix the arrows; the shape alone cannot distinguish reversed trajectories.

**52. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is 1/16 th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :** [4 marks]

(A) $2T\sqrt{\frac{1}{10}}$

(B) $4T\sqrt{\frac{1}{14}}$

(C) $4T\sqrt{\frac{1}{15}}$

(D) $2T\sqrt{\frac{1}{14}}$

**Correct Answer:** (C) $4T\sqrt{\frac{1}{15}}$

**Solution:**

**Concept:**

- A completely submerged bob experiences buoyancy $F_b = \rho_\ell V g$, opposing its weight $\rho_b V g$, where $\rho_\ell$ and $\rho_b$ are liquid and bob densities.
- For small oscillations the inertial mass remains $m = \rho_b V$, while buoyancy reduces the restoring force. With effective gravity $g_{\text{eff}}$, the period is $2\pi\sqrt{l/g_{\text{eff}}}$.

**Step 1:** Find the reduced restoring acceleration The density ratio is $\rho_\ell/\rho_b = 1/16$. The net downward force relevant to restoring motion is

$$mg - F_b = \rho_b V g\left(1 - \frac{\rho_\ell}{\rho_b}\right) = mg\frac{15}{16}.$$

For an angular displacement $\theta$, the restoring torque is $-(mg - F_b)l\sin\theta$. Using $\sin\theta \simeq \theta$ and moment of inertia $ml^2$ gives

$$\ddot{\theta} + \frac{15g}{16l}\theta = 0.$$

The liquid is non-viscous, so no damping correction is needed in the stated model.

**Step 2:** Compare the liquid and air periods In air, $T = 2\pi\sqrt{l/g}$. In the liquid,

$$T_\ell = 2\pi\sqrt{\frac{l}{(15/16)g}} = \frac{4}{\sqrt{15}}T.$$

The increase is small: $4/\sqrt{15} \simeq 1.033$. This is reasonable because buoyancy removes only one sixteenth of the bob's effective weight, while its inertia is unchanged.

**Step 3:** Check the alternatives Option (C) is exactly $4T\sqrt{1/15}$. Options (A) and (D) predict a period shorter than $T$, although the restoring force has decreased. Option (B) uses $14$ instead of the required density difference factor $15$.

**Step 4:** Conclusion The period is $\boxed{T_\ell = 4T\sqrt{1/15}}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Buoyancy changes the restoring weight, not the bob's inertia: replace $g$ by $g(1 - \rho_\ell/\rho_b)$ and take the inverse square root for the period.

**53. The magnetic field of a plane electromagnetic wave is given by:**

$$\overline{B} = B_0\hat{i}\cos(kz - \omega t) + B_i\hat{j}\cos(kz + \omega t).$$

$B_0 = 3 \times 10^{-5}\,\mathrm{T}$ **and** $B_1 = 2 \times 10^{-6}\,\mathrm{T}$**. The rms value of the force experienced by a stationary charge** $Q = 10^{-4}\,\mathrm{C}$ **at** $z = 0$ **is closest to:** [4 marks]

(A) 0.6 N

(B) 0.9 N

(C) $3 \times 10^{-2}$ N

(D) 0.1 N

**Correct Answer:** (A) 0.6 N

**Solution:**

**Concept:**

- For each plane electromagnetic wave in vacuum, $\mathbf{E} = c\,\mathbf{B} \times \hat{n}$, with propagation direction $\hat{n}$. The two phases $kz - \omega t$ and $kz + \omega t$ propagate along $+z$ and $-z$, respectively.
- A stationary charge feels electric force $\mathbf{F} = Q\mathbf{E}$; its magnetic Lorentz force $Q\mathbf{v} \times \mathbf{B}$ vanishes. A sinusoidal field of amplitude $E_a$ has rms magnitude $E_a/\sqrt{2}$.

**Step 1:** Find the two electric-field directions The first magnetic field is along $\hat{i}$ and travels along $\hat{k}$, so its electric field is along $\hat{i} \times \hat{k} = -\hat{j}$. The second is along $\hat{j}$ and travels along $-\hat{k}$, so its electric field is along $-\hat{i}$. Denote its given amplitude by $B_1 = 2 \times 10^{-6}\,\mathrm{T}$. At $z = 0$, both cosine factors equal $\cos\omega t$, hence

$$\mathbf{E}(0, t) = -c(B_1\hat{i} + B_0\hat{j})\cos\omega t.$$

The components are perpendicular, so their amplitudes combine in quadrature, not by direct addition or subtraction.

**Step 2:** Compute the rms force Using $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$ gives

$$F_{\mathrm{rms}} = \frac{Qc}{\sqrt{2}}\sqrt{B_0^2 + B_1^2}.$$

Substitution yields

$$F_{\mathrm{rms}} = \frac{10^{-4}(3 \times 10^8)}{\sqrt{2}}\sqrt{(3 \times 10^{-5})^2 + (2 \times 10^{-6})^2} \simeq 0.638\,\mathrm{N}.$$

The field exists at the charge even though the charge is stationary; only the magnetic contribution to its force is absent.

**Step 3:** Choose the nearest numerical option The value $0.638\,\mathrm{N}$ is closest to $0.6\,\mathrm{N}$. The $0.9\,\mathrm{N}$ option is close to the peak force rather than its rms value; $0.03\,\mathrm{N}$ and $0.1\,\mathrm{N}$ are much too small for the dominant $B_0$ contribution.

**Step 4:** Conclusion The rms force is approximately $\boxed{0.64\,\mathrm{N}}$, closest to $0.6\,\mathrm{N}$. Hence, the correct option is (A).

**Quick Tip to Solve:** For counter-propagating waves, find each electric-field direction separately. At a stationary charge add the electric vectors, then take the rms; the magnetic force is zero.

**54. A capacitor with capacitance 5$\mu$F is charged to 5$\mu$C. If the plates are pulled apart to reduce the capacitance to 2$\mu$F, how much work is done ?** [4 marks]

(A) $2.16 \times 10^{-6}$ J

(B) $2.55 \times 10^{-6}$ J

(C) $3.75 \times 10^{-6}$ J

(D) $6.25 \times 10^{-6}$ J

**Correct Answer:** (C) $3.75 \times 10^{-6}$ J

**Solution:**

**Concept:**

- For an isolated charged capacitor the charge $Q$ stays constant when its plates are moved. Its electrostatic energy is $U = Q^2/(2C)$.
- In slow separation with no losses, work done by the external agent equals the increase in stored field energy: $W = U_f - U_i$. Pulling attracting plates apart requires positive work.

**Step 1:** Identify the quantity that remains constant The capacitor is charged to $Q = 5\,\mu\text{C}$, and no battery connection is specified. Thus its fixed charge is $5 \times 10^{-6}\,\text{C}$ as capacitance decreases from $C_i = 5 \times 10^{-6}\,\text{F}$ to $C_f = 2 \times 10^{-6}\,\text{F}$. The voltage changes: initially it is $Q/C_i = 1\,\text{V}$, and finally it is $Q/C_f = 2.5\,\text{V}$. Therefore a constant-voltage energy expression must not be used with one unchanged voltage.

**Step 2:** Calculate the initial and final stored energies The two field energies are

$$U_i = \frac{(5 \times 10^{-6})^2}{2(5 \times 10^{-6})} = 2.50 \times 10^{-6}\,\text{J},$$

$$U_f = \frac{(5 \times 10^{-6})^2}{2(2 \times 10^{-6})} = 6.25 \times 10^{-6}\,\text{J}.$$

As expected, reducing capacitance at fixed charge increases the energy. The required external work is consequently

$$W = U_f - U_i = (6.25 - 2.50) \times 10^{-6} = 3.75 \times 10^{-6}\,\text{J}.$$

**Step 3:** Distinguish work from final energy Option (D) gives the final stored energy only, without subtracting the initial energy. The values in (A) and (B) are not the difference of the two constant-charge energies. Option (C) equals the positive increase in energy.

**Step 4:** Conclusion The work done in separating the plates is $\boxed{3.75 \times 10^{-6}\,\text{J}}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Before calculating capacitor work, check whether charge or voltage is fixed. For an isolated capacitor use $Q^2/(2C)$ and subtract the initial energy from the final energy.

**55. A body of mass 2 kg makes an eleastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body ?** [4 marks]

(A) 1.2 kg

(B) 1.0 kg

(C) 1.8 kg

(D) 1.5 kg

**Correct Answer:** (A) 1.2 kg

**Solution:**

**Concept:**

- In an elastic collision both linear momentum and kinetic energy are conserved. In one dimension, the relative speed of separation equals the relative speed of approach.
- For a stationary target, the first body obeys $v_1/u = (m_1 - m_2)/(m_1 + m_2)$, with velocities signed along the initial direction.

**Step 1:** Translate the direction and speed correctly The first body has $m_1 = 2\,\text{kg}$ and continues in its original direction. Its final velocity is therefore $v_1 = +u/4$, not $-u/4$. If the second body's final velocity is $v_2$, the elastic relative-velocity relation gives

$$v_2 - v_1 = u, \qquad v_2 = \frac{5u}{4}.$$

The second body can move faster than the first body's original speed because it is lighter; this does not by itself violate energy conservation.

**Step 2:** Use momentum to determine the unknown mass Momentum conservation now reads

$$2u = 2\left(\frac{u}{4}\right) + m_2\left(\frac{5u}{4}\right).$$

Cancelling the nonzero launch speed and multiplying by four gives $8 = 2 + 5m_2$, when masses are measured in kilograms. Thus

$$m_2 = \frac{6}{5}\,\text{kg} = 1.2\,\text{kg}.$$

As a check, the final kinetic energy is

$$\frac{1}{2}(2)\frac{u^2}{16} + \frac{1}{2}(1.2)\frac{25u^2}{16} = u^2,$$

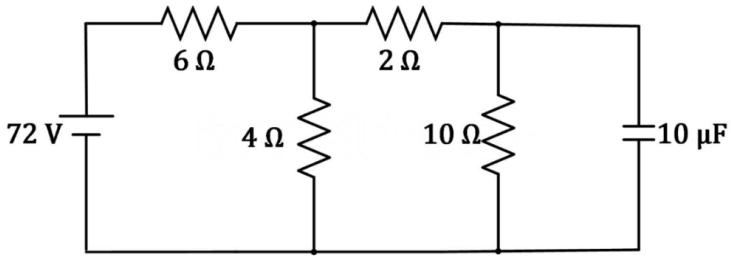
identical to the initial $\frac{1}{2}(2)u^2$.

**Step 3:** Check the proposed masses Using $(2 - m_2)/(2 + m_2)$, a mass of $1.0\,\text{kg}$ gives $v_1/u = 1/3$, $1.8\,\text{kg}$ gives $1/19$, and $1.5\,\text{kg}$ gives $1/7$. Only $1.2\,\text{kg}$ gives the required positive ratio $1/4$.

**Step 4:** Conclusion The second body has mass $\boxed{1.2\,\text{kg}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Keep the sign of the first body's final velocity: continued forward motion means $(m_1 - m_2)/(m_1 + m_2) > 0$, so the target must be lighter than the projectile.

**56. Determine the charge on the capacitor in the following circuit :** [4 marks]




(A) 200$\mu$C

(B) 2$\mu$C

(C) 10$\mu$C

(D) 60$\mu$C

**Correct Answer:** (A) 200$\mu$C

**Solution:**

**Concept:**

- In a steady direct-current circuit an ideal capacitor draws no current, but it can retain a nonzero potential difference and charge. Its charge is $Q = CV$.
- The capacitor is parallel to the $10\,\Omega$ resistor, so its final voltage equals the drop across that resistor. Kirchhoff's laws determine this voltage from the conducting resistor network.

**Step 1:** Reduce the resistor network at steady state Open the capacitor branch for the steady-current calculation. The $2\,\Omega$ and $10\,\Omega$ resistors remain in series, making $12\,\Omega$. That branch is parallel to the $4\,\Omega$ resistor, hence

$$R_p = \frac{4 \times 12}{4 + 12} = 3\,\Omega.$$

The $6\,\Omega$ resistor is in series with this parallel combination. Thus $R_{\text{total}} = 6 + 3 = 9\,\Omega$, and the source current is $72/9 = 8\,\text{A}$.

**Step 2:** Find the voltage across the capacitor The voltage across the parallel network is $8 \times 3 = 24\,\text{V}$. The current through its $12\,\Omega$ branch is therefore $24/12 = 2\,\text{A}$. Consequently the voltage across the $10\,\Omega$ resistor, and across the capacitor connected to the same two nodes, is

$$V_C = (2\,\text{A})(10\,\Omega) = 20\,\text{V}.$$

The remaining $4\,\text{V}$ drops across the $2\,\Omega$ resistor, as required.

**Step 3:** Calculate the stored charge and compare With $C = 10\,\mu\text{F}$,

$$Q = CV_C = (10 \times 10^{-6})(20) = 2.0 \times 10^{-4}\,\text{C} = 200\,\mu\text{C}.$$

The options 2, 10 and $60\,\mu$C would imply capacitor voltages of only 0.2, 1 and $6\,\text{V}$, respectively, inconsistent with the resistor-network voltage. Zero capacitor current does not mean zero capacitor charge.

**Step 4:** Conclusion The magnitude of charge on the capacitor is $\boxed{200\,\mu\text{C}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** At DC steady state remove the capacitor branch only when finding currents. Then calculate the voltage between its original terminals and restore its charge through $Q = CV$.

**57. The stream of a river is flowing with a speed of 2km/h. A swimmer can swim at a speed of 4km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight ?** [4 marks]

(A) 150°

(B) 120°

(C) 60°

(D) 90°

**Correct Answer:** (B) 120°

**Solution:**

**Concept:**

- The velocity of the swimmer relative to the bank is the vector sum of the swimming velocity relative to water and the water velocity relative to the bank.
- To cross straight across a river, the resultant component parallel to the river flow must vanish. Measure the swimming angle $\theta$ from the downstream direction.

**Step 1:** Resolve the velocities along the stream Choose the positive horizontal direction along the flowing water. The river contributes $+2\,\mathrm{km\,h^{-1}}$, while the swimmer contributes $4\cos\theta\,\mathrm{km\,h^{-1}}$. These are velocities in different reference frames that must be added; the swimmer must therefore aim partly upstream rather than directly perpendicular to the current.

**Step 2:** Cancel the downstream drift For a straight crossing the horizontal displacement must remain zero, so its horizontal velocity is zero throughout. Thus

$$2 + 4\cos\theta = 0 \quad \Rightarrow \quad \cos\theta = -\frac{1}{2}.$$

The swimming direction lies between $90°$ and $180°$ from downstream; hence $\theta = 120°$. The perpendicular speed remains $4\sin 120° = 2\sqrt{3}\,\mathrm{km\,h^{-1}}$, so crossing is possible.

**Step 3:** Check the other proposed directions At $90°$, the swimmer has no upstream component, and the current causes drift. At $60°$, the horizontal swimming component is downstream, making the drift worse. At $150°$, the upstream component is $2\sqrt{3}$, larger than the current, so the swimmer drifts upstream. Only $120°$ cancels the current exactly.

**Step 4:** Conclusion The swimmer should head at $120°$ to the direction of river flow, equivalently $30°$ upstream of the straight-across direction. Hence, the correct option is (B).

**Quick Tip to Solve:** For a straight crossing, cancel the current first: $u\cos\theta = -v$, when the angle is measured from downstream. Do not confuse this angle with the angle from the bank normal.

**58. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to :** [4 marks]

(A) $1/L^2$

(B) $1/L$

(C) L

(D) $L^2$

**Correct Answer:** (B) $1/L$

**Solution:**

**Concept:**

- For a long solenoid of length $L$, area $S$, total turns $N$, and fixed magnetic permeability $\mu$, the internal field is $B = \mu NI/L$.
- Self-inductance is flux linkage per unit current: $\mathcal{L} = N\Phi/I$. Here $\mathcal{L}$ denotes inductance, so it is distinguished from the given length $L$.

**Step 1:** Express the linked magnetic flux Each turn encloses the fixed cross-sectional area $S$. For the usual long-solenoid approximation the field is uniform over this area, and the flux through one turn is

$$\Phi = BS = \frac{\mu NIS}{L}.$$

There are $N$ turns linking approximately the same flux, so the total linkage is $N\Phi$, rather than just the single-turn flux.

**Step 2:** Find the length dependence Substitute the flux in the definition of self-inductance:

$$\mathcal{L} = \frac{N\Phi}{I} = \frac{\mu N^2 S}{L}.$$

The current cancels because inductance is a property of the winding geometry and magnetic medium in this linear model. The question keeps both the number of turns and the area fixed; changing their separation changes only the denominator $L$. Therefore $\mathcal{L} \propto L^{-1}$.

**Step 3:** Distinguish fixed turns from fixed turn density If the length doubles at fixed $N$, the turn density $N/L$ halves, the field for a given current halves, and so does the flux linkage. This rejects the positive powers $L$ and $L^2$, and the inverse-square option would predict a factor of four rather than two. The familiar form $\mu n^2 SL$ does not imply direct proportionality here, because $n = N/L$ is changing.

**Step 4:** Conclusion With total turns, area, and magnetic medium unchanged, the solenoid inductance varies as $1/L$. Hence, the correct option is (B).

**Quick Tip to Solve:** Before using $\mathcal{L} = \mu n^2 SL$, check whether $n$ or total turns $N$ is fixed. Here substitute $n = N/L$ to reveal the inverse-length dependence.

**59. A signal Acoswt is transmitted using $v_0 \sin \omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is** [4 marks]

(A) $v_0 \sin \omega_0 t + A\cos\omega t$

(B) $(v_0 + A)\cos\omega t \sin\omega_0 t$

(C) $v_0 \sin[\omega_0 (1+ 0.01A\sin\omega t)t]$

(D) $v_0 \sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$

**Correct Answer:** (D) $v_0 \sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$

**Solution:**

**Concept:**

- In amplitude modulation, the message changes the carrier amplitude while the carrier angular frequency and phase remain fixed.
- For message $A\cos\omega t$ and carrier $v_0 \sin\omega_0 t$, the AM waveform is $[v_0 + A\cos\omega t]\sin\omega_0 t$. The product-to-sum identity gives its sidebands.

**Step 1:** Write the varying carrier amplitude The unmodulated carrier has amplitude $v_0$. Adding the instantaneous message to this amplitude gives

$$v(t) = [v_0 + A\cos\omega t]\sin\omega_0 t.$$

This must be distinguished from merely adding the message voltage to the carrier voltage: the message multiplies the carrier sine because it controls the envelope of the high-frequency oscillation.

**Step 2:** Expand the modulation product Separate the carrier term and apply $\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$:

$$\begin{aligned} v(t) &= v_0 \sin\omega_0 t + A\cos\omega t \sin\omega_0 t \\ &= v_0 \sin\omega_0 t + \frac{A}{2}\sin[(\omega_0 - \omega)t] \\ &\quad + \frac{A}{2}\sin[(\omega_0 + \omega)t]. \end{aligned}$$

The result contains the original carrier and sidebands at the difference and sum frequencies, each with amplitude $A/2$.

**Step 3:** Compare the alternative waveforms Option (A) adds a separate low-frequency signal without producing these two sidebands. Option (B) modulates the entire amplitude as $(v_0 + A)\cos\omega t$, so it lacks the required constant carrier-amplitude term and gives a suppressed-carrier product. Option (C) changes the argument of the carrier sine; that is angle modulation rather than the stated amplitude modulation. Option (D) is precisely the expansion above.

**Step 4:** Conclusion The AM signal is the carrier plus equal-amplitude lower and upper sidebands, as displayed in option (D). Hence, the correct option is (D).

**Quick Tip to Solve:** Form [carrier amplitude + message] × carrier oscillation, then use product-to-sum. AM changes the envelope, not the sine argument.

**60. An NPN transistor is used in common emitter configuration as an amplifier with 1 kΩ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15$\mu$A change in the base current of the amplifier. The input resistance and voltage gain are :** [4 marks]

(A) 0.67 kW, 200

(B) 0.33 kW, 1.5

(C) 0.67 kW, 300

(D) 0.33 kW, 300

**Correct Answer:** (C) 0.67 kW, 300

**Solution:**

**Concept:**

- The small-signal input resistance is $r_{\text{in}} = \Delta V_{BE}/\Delta I_B$; the output voltage magnitude across a resistive load is $|\Delta V_o| = R_L \Delta I_C$.
- The magnitude of voltage gain is $|A_v| = |\Delta V_o|/\Delta V_{BE}$. A common-emitter stage inverts the voltage signal, so its signed voltage gain is negative.

**Step 1:** Calculate the input resistance Use the base-current change for the input resistance, since the applied input is across the base-emitter junction. Convert the data before division:

$$r_{\text{in}} = \frac{10 \times 10^{-3}}{15 \times 10^{-6}} = \frac{2000}{3}\,\Omega \simeq 0.67\,\text{k}\Omega.$$

Using the collector current in this ratio would instead give the wrong input resistance. The input current is much smaller than the collector-current change because the transistor provides current amplification.

**Step 2:** Calculate the voltage-gain magnitude The changing collector current produces a load-voltage change of magnitude

$$|\Delta V_o| = (3 \times 10^{-3})(1 \times 10^{3}) = 3\,\text{V}.$$

Divide this by the input signal voltage:

$$|A_v| = \frac{3}{10 \times 10^{-3}} = 300.$$

Thus a $10\,\text{mV}$ input produces a $3\,\text{V}$ output change in the ideal small-signal calculation. The minus sign of a common-emitter voltage gain describes inversion, not a negative amplification magnitude.

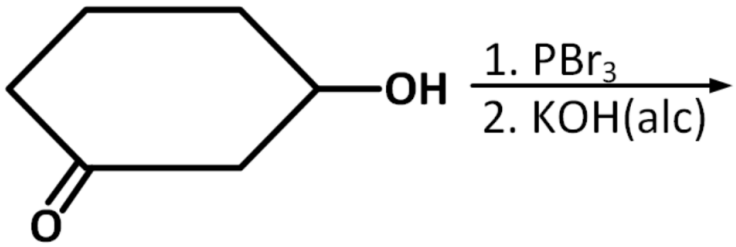
**Step 3:** Check the numerical alternatives and units The current gain is $\beta = \Delta I_C/\Delta I_B = 200$; this explains the tempting value in option (A), but it is not the voltage gain. The $0.33$ values in (B) and (D) do not follow from the input data. Option (C) supplies the numerical pair $0.67, 300$. Its printed unit kW is dimensionally inappropriate for a resistance; the calculated resistance unit is $\text{k}\Omega$.

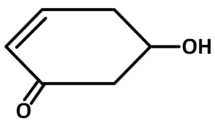
**Step 4:** Conclusion The input resistance is approximately $0.67\,\text{k}\Omega$, and the voltage-gain magnitude is $300$ (signed gain $-300$). The numerical pair is option (C), with its printed kW understood as the resistance-unit error described above. Hence, the correct option is (C).

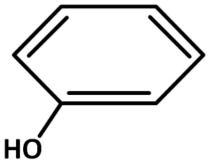
**Quick Tip to Solve:** Use base current to calculate input resistance, and collector current times load resistance to calculate output voltage. Current gain and voltage gain need not be equal.

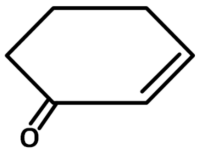
## Chemistry (120 marks)

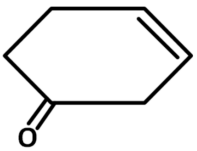
**61.** **The major product of the following reaction is :** [4 marks]

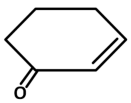



(A) 


(B) 


(C) 


(D) 


**Correct Answer:** (C) 


**Solution:**

**Concept:**

- $PBr_3$ converts an aliphatic alcohol into the corresponding alkyl bromide, replacing the hydroxyl group without reducing a ketone group.
- Alcoholic KOH favours elimination of hydrogen bromide. An alkene conjugated with a carbonyl group is more stable than an otherwise comparable isolated alkene.

**Step 1:** Identify the position of the alcohol Number the carbonyl carbon of the six-membered ring as C1. The lower right carbon is C2 and the carbon bearing the hydroxyl group is C3. Thus, the starting compound is 3-hydroxycyclohexan-1-one. The ketone and the alcohol are separate functional groups: the oxygen attached to C1 is not the hydroxyl oxygen that is replaced in the first reaction.

**Step 2:** Replace the hydroxyl group by bromine Treatment with $PBr_3$ replaces the hydroxyl group at C3 by bromine, giving 3-bromocyclohexan-1-one. The six-carbon ring remains intact and the carbonyl group at C1 remains a ketone. This step creates the leaving group needed for the subsequent dehydrohalogenation; it neither aromatises the ring nor introduces an extra hydroxyl group.

**Step 3:** Choose the conjugated elimination product Alcoholic KOH removes a hydrogen from a carbon adjacent to C3 while bromide leaves C3. Removal from C2 produces a C2=C3 bond, giving the conjugated sequence $O{=}C{-}C{=}C$. Removal from C4 would instead give a C3=C4 bond separated from the carbonyl by a saturated carbon. The conjugated enone is favoured because its pi electrons are delocalised across the carbonyl and the adjacent double bond. In the drawings, option (C) has

the double bond on the lower right edge adjacent to the carbonyl. Option (D) has an isolated double bond, option (A) incorrectly retains the alcohol and adds unsaturation elsewhere, and option (B) would require further unsaturation to form phenol, which this sequence does not provide.

**Step 4:** Conclusion The major product is cyclohex-2-en-1-one, the conjugated enone shown in option (C). Hence, the correct option is (C).

**Quick Tip to Solve:** Number the carbonyl carbon first, then eliminate towards it when that creates an alpha,beta-unsaturated ketone; distinguish conjugated and isolated ring double bonds carefully.

**62. The element having greatest difference between its first and second ionization energies, is :**

[4 marks]

(A) Ba

(B) Ca

(C) Sc

(D) K

**Correct Answer:** (D) K

**Solution:**

**Concept:**

- The first ionization energy removes an electron from a neutral gaseous atom, whereas the second removes an electron from its singly charged gaseous ion.
- A particularly large increase occurs when the first removal gives a noble-gas configuration, so the second removal must take an electron from a closed inner shell.

**Step 1:** Write the relevant outer configurations The outer configurations of the candidates are

$$\mathrm{K} : [\mathrm{Ar}]\, 4s^1, \qquad \mathrm{Ca} : [\mathrm{Ar}]\, 4s^2,$$
$$\mathrm{Sc} : [\mathrm{Ar}]\, 3d^1 4s^2, \quad \mathrm{Ba} : [\mathrm{Xe}]\, 6s^2.$$

Potassium alone has just one electron outside a noble-gas core. The two alkaline-earth metals have two outer s electrons, while scandium has three electrons beyond its argon core.

**Step 2:** Identify the electron removed in each step The first ionization of potassium removes its single $4s$ electron:

$$\mathrm{K} \longrightarrow \mathrm{K}^+ + \mathrm{e}^-, \qquad \mathrm{K}^+ : [\mathrm{Ar}].$$

The second ionization must disturb this stable closed shell, removing a $3p$ electron that is closer to the nucleus and more strongly held than the original $4s$ electron. The first ionization is comparatively easy, while the second is exceptionally costly, so their difference is especially large.

**Step 3:** Rule out the other elements For calcium the first two electrons are both outer $4s$ electrons; its dramatic closed-shell jump occurs between the second and third ionizations. Barium behaves similarly with its two $6s$ electrons. Scandium also loses its outer s electrons in the first two stages rather than removing an electron from an argon core at the second stage. Increased attraction after

the first removal raises their second energies, but none has potassium's immediate transition from a lone valence electron to a closed inner shell.

**Step 4:** Conclusion Potassium has the greatest difference between the first and second ionization energies because $K^+$ has the closed-shell argon configuration. Hence, the correct option is (D).

**Quick Tip to Solve:** Locate the first noble-gas ion. An element with one valence electron shows the large ionization-energy jump immediately after the first electron is removed.

---

**63. Liquid 'M' and liquid 'N' form an ideal solution. The vapour pressures of pure liquids 'M' and 'N' are 450 and 700 mmHg, respectively, at the same temperature. Then correct statement is:**

**($x_M$ = Mole fraction of 'M' in solution ;**

**$x_N$ = Mole fraction of 'N' in solution ;**

**$y_M$ = Mole fraction of 'M' in vapour phase ;**

**$y_N$ = Mole fraction of 'N' in vapour phase)** [4 marks]

(A) $\frac{x_M}{x_N} < \frac{y_N}{y_N}$

(B) $(x_M - y_M) < (x_N - y_N)$

(C) $\frac{x_M}{x_N} = \frac{y_N}{y_N}$

(D) $\frac{x_M}{x_N} > \frac{y_M}{y_N}$

**Correct Answer:** (D) $\frac{x_M}{x_N} > \frac{y_M}{y_N}$

**Solution:**

**Concept:**

- For an ideal binary solution, Raoult's law gives $p_M = x_M p_M^\circ$ and $p_N = x_N p_N^\circ$. Dalton's law gives $y_i = p_i/(p_M + p_N)$.
- The component with larger pure vapour pressure is more volatile and is enriched in the equilibrium vapour. For a mixture containing both liquids, $0 < x_M, x_N < 1$.

**Step 1:** Write the two partial pressures At the specified temperature the pure vapour pressures are $p_M^\circ = 450\,\text{mmHg}$ and $p_N^\circ = 700\,\text{mmHg}$. Therefore

$$p_M = 450x_M, \qquad p_N = 700x_N.$$

The common pressure unit cancels from composition ratios. Since the solution is ideal, no activity coefficient or extra interaction correction is introduced into these expressions.

**Step 2:** Compare the vapour and liquid ratios Dividing the two vapour mole fractions eliminates the total pressure:

$$\frac{y_M}{y_N} = \frac{p_M}{p_N} = \frac{450x_M}{700x_N} = \frac{9}{14}\frac{x_M}{x_N}.$$

Both liquid mole fractions are positive for the binary mixture. Since $9/14 < 1$, the vapour ratio is smaller than the liquid ratio:

$$\frac{x_M}{x_N} > \frac{y_M}{y_N}.$$

This agrees with physical intuition: the more volatile liquid $N$ contributes disproportionately to the vapour.

**Step 3:** Check the remaining printed statements The total pressure lies strictly between $450$ and $700\,\text{mmHg}$, so $y_M = 450x_M/P < x_M$ and $y_N = 700x_N/P > x_N$. Consequently $x_M - y_M > 0$, while $x_N - y_N < 0$, contradicting (B). In printed (A) and (C), $y_N/y_N = 1$; whether $x_M/x_N$ is below or equal to one depends on the unspecified liquid composition. Those statements are not general consequences of the given pressures, whereas (D) holds throughout the mixture range.

**Step 4:** Conclusion The less volatile component has the smaller relative abundance in the vapour, so $x_M/x_N > y_M/y_N$. Hence, the correct option is (D).

**Quick Tip to Solve:** For an ideal binary mixture, write $y_M/y_N = (x_M/x_N)(p_M^\circ/p_N^\circ)$. This ratio avoids calculating total pressure or either vapour mole fraction separately.

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**64. The degenerate orbitals of $[Cr(H_2O)_6]^{3+}$ are :** [4 marks]

(A) $d_{x^2}$ and $d_{xz}$

(B) $d_{xz}$ and $d_{yz}$

(C) $d_{yz}$ and $d_{z^2}$

(D) $d_{x^2-y^2}$ and $d_{xy}$

**Correct Answer:** (B) $d_{xz}$ and $d_{yz}$

**Solution:**

**Concept:**

- The six water ligands in $[Cr(H_2O)_6]^{3+}$ surround chromium octahedrally. Ligand repulsions split the five metal d orbitals into two sets of different energy.
- The lower-energy $t_{2g}$ set contains $d_{xy}, d_{xz}, d_{yz}$; the upper-energy $e_g$ set contains $d_{x^2-y^2}, d_{z^2}$. Orbitals within one set are degenerate.

**Step 1:** Identify the geometry and metal configuration Water is a neutral ligand, so chromium has oxidation state $+3$. Chromium has atomic number 24, and $Cr^{3+}$ has configuration $3d^3$. The coordination number is six, giving the usual octahedral arrangement. Although electron occupancy helps describe the complex, degeneracy here follows from octahedral symmetry rather than from the number of electrons.

**Step 2:** Separate the two orbital sets The $d_{x^2-y^2}$ and $d_{z^2}$ orbitals point along the coordinate axes, towards the ligands, and suffer greater repulsion. The $d_{xy}, d_{xz}, d_{yz}$ orbitals point between these axes and experience equal, smaller repulsion. Their crystal-field energies relative to the barycentre are $-0.4\Delta_o$ for each $t_{2g}$ orbital and $+0.6\Delta_o$ for each $e_g$ orbital.

**Step 3:** Compare the printed pairs Both $d_{xz}$ and $d_{yz}$ in option (B) belong to $t_{2g}$, so their energies are equal. The pairs in (C) and (D) combine one orbital from each set and are split by $\Delta_o$. Option (A), as printed with $d_{x^2}$, does not give the required valid same-set pair; interpreting its axial orbital as the usual $d_{x^2-y^2}$ would also mix the two sets.

**Step 4:** Conclusion The degenerate pair is $d_{xz}$ and $d_{yz}$, both members of the octahedral $t_{2g}$ set. Hence,

the correct option is (B).

**Quick Tip to Solve:** For an octahedral complex, first list $xy, xz, yz$ together and $x^2 - y^2, z^2$ together; a pair is degenerate only within one list.

**65. The major product of the following reaction is :**

$$CH_3CH{=}CHCO_2CH_3 \xrightarrow{LiAlH_4}$$

[4 marks]

(A) $CH_3CH_2CH_2CHO$

(B) $CH_3CH_2CH_2CO_2CH_3$

(C) $CH_3CH{=}CHCH_2OH$

(D) $CH_3CH_2CH_2CH_2OH$

**Correct Answer:** (C) $CH_3CH{=}CHCH_2OH$

**Solution:**

**Concept:**

- Lithium aluminium hydride, $LiAlH_4$, reduces an ester group $RCOOR'$ to a primary alcohol $RCH_2OH$, with the alkoxy part giving $R'OH$ after work-up.
- Under the usual ester-reduction conditions, the carbon-carbon double bond of an alpha,beta-unsaturated ester is retained; hydride reduction targets its polar carbonyl-containing functional group.

**Step 1:** Identify the reducible functional group The starting compound is methyl but-2-enoate. It contains an ester at one end and a carbon-carbon double bond in the four-carbon acyl chain. The ester carbonyl carbon belongs to this four-carbon chain, whereas the terminal methoxy carbon is attached through oxygen. Track these two fragments separately so that reduction does not accidentally add or remove a carbon from the main chain.

**Step 2:** Convert the ester end into a primary alcohol Hydride addition to the ester carbonyl is followed by loss of the methoxy fragment, giving an aldehyde intermediate. That intermediate is more readily reduced and undergoes a second hydride addition. Protonation during work-up then produces the primary alcohol. Therefore, the net conversion of the main organic chain is

$$CH_3CH{=}CHCO_2CH_3 \xrightarrow[H_2O]{LiAlH_4} CH_3CH{=}CHCH_2OH.$$

The methoxy fragment forms methanol after work-up; it is not appended to the four-carbon product. The carbon-carbon double bond remains between the same two carbon atoms.

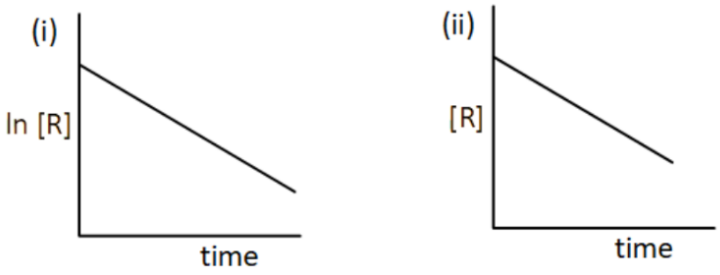
**Step 3:** Compare the four product choices Option (C) is but-2-en-1-ol, which has exactly the required primary alcohol and retained double bond. Option (A) is butanal: it both removes the original double bond and stops at an aldehyde, although an aldehyde does not survive excess $LiAlH_4$. Option (B) leaves the ester unreduced and changes only the double bond. Option (D) gives a saturated alcohol and would require reduction of the carbon-carbon double bond as well, which is not the selective

transformation specified here. Thus, only (C) matches both the functional-group change and the preserved carbon skeleton.

**Step 4:** Conclusion The major product is $CH_3CH{=}CHCH_2OH$, but-2-en-1-ol. Hence, the correct option is (C).

**Quick Tip to Solve:** For an unsaturated ester with LiAlH4, replace the ester end by CH2OH and retain the C=C bond; do not confuse carbonyl reduction with catalytic hydrogenation.

**66. The given plots represent the variation of the concentration of a reactant R with time for two different reactions (i) and (ii). The respective orders of the reactions are :** [4 marks]




(A) 0,1

(B) 1,0

(C) 0,2

(D) 1,1

**Correct Answer:** (B) 1,0

**Solution:**

**Concept:**

- A reaction order is identified by the concentration function that varies linearly with time. The vertical-axis label is essential; a decreasing line alone does not specify the order.
- For a first-order reaction, $\ln[R] = \ln[R]_0 - kt$. For a zero-order reaction, $[R] = [R]_0 - kt$, where $k$ is the appropriate rate constant.

**Step 1:** Read the vertical axis of plot (i) The first graph plots $\ln[R]$, rather than the concentration itself, against time. Integrating $-d[R]/dt = k[R]$ gives the first-order law

$$\ln[R] = \ln[R]_0 - kt.$$

This is of the straight-line form $y = c + mt$, with intercept $\ln[R]_0$ and negative slope $-k$. Therefore reaction (i) is first order.

**Step 2:** Read the vertical axis of plot (ii) The second graph plots the untransformed concentration $[R]$ against time. Its constant negative slope means the concentration decreases by the same amount in equal time intervals. Integrating $-d[R]/dt = k$ gives

$$[R] = [R]_0 - kt.$$

Thus the reaction rate is independent of concentration, and reaction (ii) is zero order. This linear law applies until the reactant is exhausted, not to negative concentrations.

**Step 3:** Match the ordered pair The requested order is (i), followed by (ii), giving $(1, 0)$. Option (A) reverses these assignments. Option (C) misidentifies both plots; a second-order reaction would instead give a straight line in $1/[R]$ versus time. Option (D) wrongly treats the concentration-time graph in (ii) as a first-order graph, which would show exponential rather than linear decay.

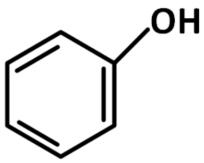
**Step 4:** Conclusion The respective orders are first order and zero order, $(1, 0)$. Hence, the correct option is (B).

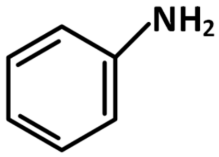
**Quick Tip to Solve:** Inspect the vertical-axis transformation before the slope: linear $\ln[R]$ means first order, while linear $[R]$ means zero order.

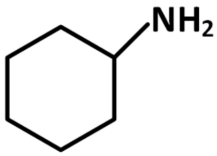
**67. The organic compound that gives following qualitative analysis is :**

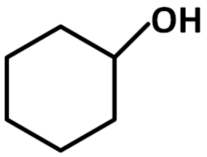
| | Test | Inference |
|---|---|---|
| (a) | Dil. HCl | Insoluble |
| (b) | NaOH solution | soluble |
| (c) | $Br_2$/water | Decolourization |

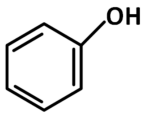
[4 marks]

(A) 


(B) 


(C) 


(D) 


**Correct Answer:** (A) 


**Solution:**

**Concept:**

- Phenols are weak acids: they form water-soluble phenoxide salts with NaOH, but do not form soluble salts with dilute HCl. Ordinary alcohols do not behave this way with aqueous hydroxide.
- An amino group is basic and forms an ammonium salt with dilute acid. An activated phenol ring rapidly consumes bromine water by electrophilic substitution.

**Step 1:** Use the acid and alkali solubility tests The compound is insoluble in dilute HCl, so it is not behaving as an amine that forms a soluble ammonium chloride. Both aniline in (B) and cyclohexanamine in (C) contain basic amino groups and would react with the acid. The positive solubility test with NaOH instead indicates an acidic hydrogen whose removal produces an ionic, water-soluble salt.

**Step 2:** Distinguish phenol from cyclohexanol Phenol, shown in (A), loses its hydroxyl proton to hydroxide because the phenoxide ion is resonance-stabilised.

$$C_6H_5OH + NaOH \longrightarrow C_6H_5ONa + H_2O$$

Cyclohexanol in (D) is an ordinary aliphatic alcohol; its alkoxide is not resonance-stabilised, and aqueous NaOH does not convert it into a soluble salt to the extent required by the test. Thus (D) does not explain the alkali solubility.

**Step 3:** Confirm the bromine-water result The hydroxyl group strongly activates the phenol ring at the ortho and para positions. Phenol consumes bromine water and forms 2,4,6-tribromophenol:

$$C_6H_5OH + 3\,Br_2 \longrightarrow C_6H_2Br_3OH + 3\,HBr.$$

This accounts for decolourization. Aniline can also consume bromine, so that test alone is insufficient; combining it with both solubility observations uniquely identifies phenol.

**Step 4:** Conclusion Phenol satisfies all three observations: insoluble in dilute acid, soluble in aqueous alkali, and decolourization of bromine water. Hence, the correct option is (A).

**Quick Tip to Solve:** Use the solubility tests together: acid solubility suggests an amine, whereas alkali solubility combined with bromine-water decolourization points to phenol.

**68. Among the following, the molecule expected to be stabilized by anion formation is :**

**$C_2$, $O_2$, NO, $F_2$** [4 marks]

(A) $C_2$

(B) NO

(C) $O_2$

(D) $F_2$

**Correct Answer:** (A) $C_2$

**Solution:**

**Concept:**

- The bond order is $\mathrm{BO} = (N_b - N_a)/2$, where $N_b$ and $N_a$ count electrons in bonding and antibonding molecular orbitals.
- Adding one electron to a bonding orbital increases the bond order by $1/2$; adding it to an antibonding orbital decreases it by $1/2$. Here stabilisation is judged by the resulting bond strength.

**Step 1:** Locate the accepting orbital in carbon In $C_2$, the occupied valence orbitals have configuration $(\sigma_{2s})^2(\sigma^*_{2s})^2(\pi_{2p_x})^2(\pi_{2p_y})^2$. The lowest vacant orbital is the bonding $\sigma_{2p_z}$ orbital. The neutral molecule has bond order $(6-2)/2 = 2$. Adding one electron to this bonding orbital gives $C_2^-$ with bond order $(7-2)/2 = 2.5$, strengthening the bond.

**Step 2:** Compare oxygen and nitric oxide For $O_2$, the added electron enters a $\pi^*$ antibonding orbital, so the bond order falls from 2 to 1.5 in $O_2^-$. In NO, the accepting orbital is also antibonding; the

bond order decreases from 2.5 to 2 in $NO^-$. Therefore neither option (B) nor option (C) gives the bond-order increase sought in the question.

**Step 3:** Check fluorine and select the molecule In $F_2$, the additional electron occupies the vacant antibonding $\sigma^*_{2p}$ orbital. The bond order decreases from 1 to 0.5, ruling out (D). The comparison concerns occupancy of the next molecular orbital and the corresponding bond strengthening, rather than assuming that every species capable of accepting an electron must become more strongly bonded.

**Step 4:** Conclusion Only $C_2$ gains an electron in a bonding orbital, increasing its bond order from 2 to 2.5. Hence, the correct option is (A).

**Quick Tip to Solve:** For anion-formation questions, identify the lowest vacant or partly filled molecular orbital before counting: bonding addition raises bond order, antibonding addition lowers it.

**69. Which of the following statements is not true about sucrose?** [4 marks]

(A) The glycosidic linkage is present between $C_1$ of $\alpha$-glucose and $C_1$ of $\beta$-fructose

(B) It is also named as invert sugar

(C) It is a non reducing sugar

(D) On hydrolysis, it produces glucose and fructose

**Correct Answer:** Both (A) and (B) are false: sucrose has a C1–C2 linkage, and invert sugar is its hydrolysis mixture.

**Solution:**

**Concept:**

- Sucrose consists of an $\alpha$-D-glucose unit and a $\beta$-D-fructose unit joined through their anomeric carbons: C1 of glucose and C2 of fructose.
- Both anomeric centres participate in the glycosidic bond, making sucrose non-reducing. Invert sugar is the glucose-fructose mixture formed by hydrolysis, rather than unhydrolysed sucrose.

**Step 1:** Check the linkage stated in option (A) The printed statement joins C1 of glucose to C1 of fructose. This is incorrect: glucose is an aldose with its anomeric centre at C1, whereas fructose is a ketose with its anomeric centre at C2. The sucrose linkage is therefore $\alpha(1 \rightarrow 2)\beta$, not a C1–C1 linkage. Thus (A) is a false statement and agrees with the supplied key choice.

**Step 2:** Check the term invert sugar in option (B) Sucrose itself is dextrorotatory. Its hydrolysis yields equimolar glucose and fructose, and the stronger levorotation of fructose makes the mixture levorotatory. This change in the sign of rotation is inversion. The resulting mixture is called invert sugar. Consequently the printed assertion that sucrose is also named invert sugar is false as well; it confuses the starting disaccharide with its hydrolysis products.

**Step 3:** Verify the remaining statements Option (C) is true because neither anomeric centre remains free to open into a reducing carbonyl group. Option (D) is also true:

$$C_{12}H_{22}O_{11} + H_2O \longrightarrow C_6H_{12}O_6(\text{glucose}) + C_6H_{12}O_6(\text{fructose}).$$

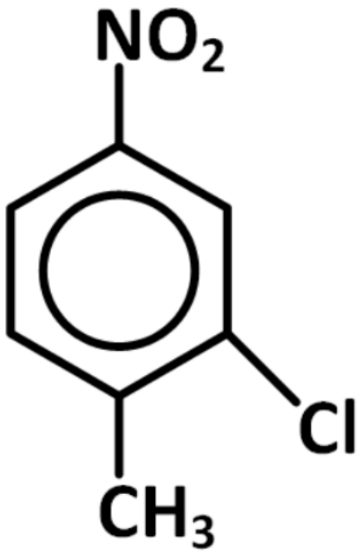
Therefore the ambiguity is not caused by (C) or (D), but by the two independently false assertions in

(A) and (B). A single-answer format cannot distinguish between those two as printed.

**Step 4:** Conclusion The supplied key selects (A), and its C1–C1 linkage is indeed wrong. However, (B) is also false because invert sugar is hydrolysed sucrose, not sucrose itself. Thus the printed question admits both (A) and (B) and has no unique correct option.

**Quick Tip to Solve:** Remember the two separate facts: sucrose links glucose C1 to fructose C2, and the name invert sugar refers to the glucose-fructose hydrolysate.

**70. The correct IUPAC name of the following compound is :** [4 marks]




(A) 2-methyl-5-nitro-1-chlorobenzene

(B) 2-chloro-1-methyl-4-nitrobenzene

(C) 3-chloro-4-methyl-1-nitrobenzene

(D) 5-chloro-4-methyl-1-nitrobenzene

**Correct Answer:** (B) 2-chloro-1-methyl-4-nitrobenzene

**Solution:**

**Concept:**

- For a benzene ring bearing only prefix substituents, choose the numbering that gives the lowest ordered set of locants. Compare the sets at their first point of difference.
- Write different substituent prefixes alphabetically in the final name: chloro precedes methyl, which precedes nitro. Alphabetical order does not override a strictly lower locant set.

**Step 1:** Read the substituent positions in the structure The ring bears a methyl group, an adjacent chloro group, and a nitro group opposite to the methyl group. Choose the methyl-bearing carbon as carbon 1 and number towards chlorine. The resulting assignments are methyl at 1, chloro at 2, and nitro at 4. This gives the increasing locant set $(1, 2, 4)$.

**Step 2:** Compare alternative numbering schemes If chlorine is carbon 1 and numbering proceeds

towards methyl, the locants are $(1, 2, 5)$. If nitro is carbon 1 and numbering proceeds towards chlorine, they are $(1, 3, 4)$; proceeding the other way gives $(1, 4, 5)$. Compare $(1, 2, 4)$ with these: it beats $(1, 2, 5)$ at the third entry, and beats the other two at the second entry. Consequently methyl must receive locant 1 in the preferred numbering.

**Step 3:** Arrange the prefixes and reject the other names Using the chosen numbering, the prefixes are 2-chloro, 1-methyl and 4-nitro. Put them in alphabetical order before benzene to obtain 2-chloro-1-methyl-4-nitrobenzene. Option (A) uses the higher set $(1, 2, 5)$ and does not put the different prefixes in the correct order. Options (C) and (D) use the higher sets $(1, 3, 4)$ and $(1, 4, 5)$, respectively. A ring having chlorine is not automatically numbered from chlorine when another start yields a lower set.

**Step 4:** Conclusion The correct IUPAC name is 2-chloro-1-methyl-4-nitrobenzene. Hence, the correct option is (B).

**Quick Tip to Solve:** First minimise the locant set for all substituents together; only then alphabetise the prefixes when writing the final benzene name.

**71. The correct order of the oxidation states of nitrogen in NO, $N_2O$, $NO_2$ and $N_2O_3$ is :** [4 marks]

(A) $NO_2 < NO < N_2O_3 < N_2O$

(B) $NO_2 < N_2O_3 < NO < N_2O$

(C) $N_2O < NO < N_2O_3 < NO_2$

(D) $N_2O < N_2O_3 < NO < NO_2$

**Correct Answer:** (C) $N_2O < NO < N_2O_3 < NO_2$

**Solution:**

**Concept:**

- In an ordinary oxide, oxygen has oxidation number $-2$. The algebraic sum of oxidation numbers in a neutral molecule is zero.
- For $N_aO_b$, let the average oxidation number of nitrogen be $x$. Charge balance gives $ax - 2b = 0$, or $x = 2b/a$. This is an average over nitrogen atoms, not a claim that inequivalent atoms must have identical individual values.

**Step 1:** Calculate each average oxidation number Apply charge balance separately, using the atom counts in each molecular formula:

$$\begin{aligned} NO: &\quad x - 2 = 0 &&\Rightarrow x = +2, \\ N_2O: &\quad 2x - 2 = 0 &&\Rightarrow x = +1, \\ NO_2: &\quad x - 4 = 0 &&\Rightarrow x = +4, \\ N_2O_3: &\quad 2x - 6 = 0 &&\Rightarrow x = +3. \end{aligned}$$

In particular, the two nitrogen atoms in nitrous oxide share a total oxidation number of $+2$, so the formula-based average is $+1$.

**Step 2:** Arrange the values and test the choices Increasing numerical order is $+1 < +2 < +3 < +4$.

Therefore the oxides must appear as

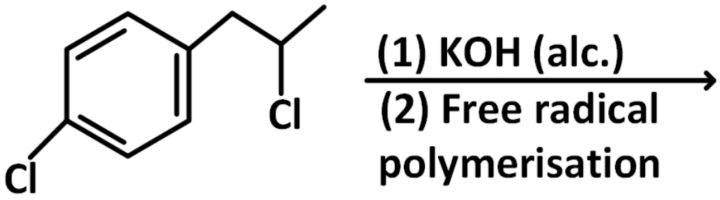
$$N_2O < NO < N_2O_3 < NO_2.$$

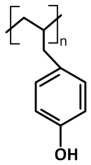
Option (A) wrongly places the highest value, that of nitrogen dioxide, first and the lowest last. Option (B) essentially reverses the increasing trend. Option (D) correctly starts and ends the sequence but reverses the middle two oxides: $+3$ cannot come before $+2$ in increasing order. Only option (C) preserves all four comparisons.

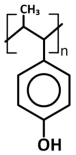
**Step 3:** Conclusion The increasing order is $N_2O < NO < N_2O_3 < NO_2$, with average nitrogen oxidation states $+1, +2, +3, +4$, respectively. Hence, the correct option is (C).

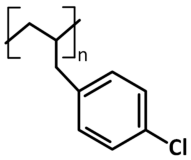
**Quick Tip to Solve:** For a neutral nitrogen oxide, divide twice the number of oxygen atoms by the number of nitrogen atoms; compare these average values before ordering the formulae.

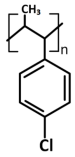
**72. The major product of the following reaction is :** [4 marks]



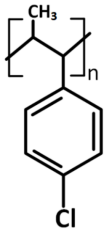

(A) 


(B) 


(C) 


(D) 


**Correct Answer:** (D)




**Solution:**

**Concept:**

- Alcoholic KOH promotes beta-elimination from an alkyl chloride. A conjugated alkene is favoured when the eliminated hydrogen can be taken from a benzylic carbon.

- Free-radical addition polymerisation consumes the alkene $C{=}C$ bond while retaining substituents on its two carbon atoms. An unactivated aromatic $C{-}Cl$ bond survives these conditions.

**Step 1:** Identify the major alkene after elimination Write $Ar = p\text{-}ClC_6H_4$. The side chain shown is $Ar{-}CH_2{-}CH(Cl){-}CH_3$. Removal of chloride and a hydrogen from the adjacent benzylic carbon gives

$$Ar{-}CH_2{-}CH(Cl){-}CH_3 \xrightarrow{\text{alc}\cdot\ KOH} Ar{-}CH{=}CH{-}CH_3.$$

This internal alkene is conjugated with the aromatic ring and is also more substituted than $Ar{-}CH_2{-}CH{=}CH_2$, the competing terminal alkene. It is therefore the major monomer. The chlorine directly bonded to the benzene ring is not the chloride removed in this step.

**Step 2:** Convert the alkene into its polymer repeat The double bond opens during addition polymerisation; the former double-bond carbons become successive atoms of the saturated backbone:

$$n\,Ar{-}CH{=}CH{-}CH_3 \longrightarrow [-CH(Ar) - CH(CH_3)-]_n\,.$$

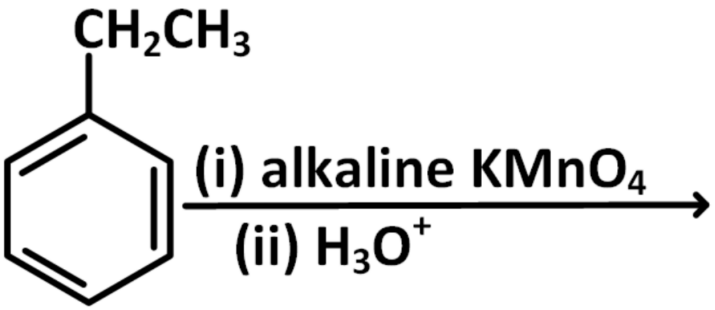
One backbone carbon thus bears a directly attached para-chlorophenyl group and its neighbour bears a methyl group. Reversing the direction in which this repeat is written describes the same polymer.

**Step 3:** Compare the four structural options Options (A) and (B) replace the aromatic chlorine with an $OH$ group, although no reaction here makes a phenol. Option (C) retains chlorine but corresponds to polymerisation of the less favoured terminal alkene: its ring is attached through a $CH_2$ spacer. Option (D) retains the ring chlorine, attaches the ring directly to a backbone carbon, and places methyl on the neighbouring backbone carbon, exactly as required.

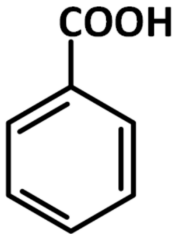
**Step 4:** Conclusion The major product has repeat unit $[-CH(p\text{-}ClC_6H_4) - CH(CH_3)-]_n$, the structure in (D). Hence, the correct option is (D).

**Quick Tip to Solve:** Choose the conjugated elimination product first, then trace its two alkene carbons into the polymer backbone without moving their substituents.

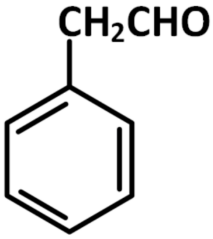
**73.** **The major product of the following reaction is :** [4 marks]



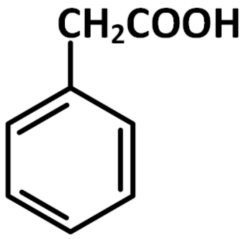

(A)



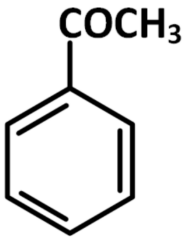

(B)



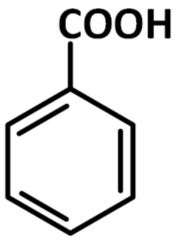

(C)




(D)




**Correct Answer:** (A)



**Solution:**

**Concept:**

- An alkylbenzene with at least one benzylic hydrogen undergoes strong side-chain oxidation with alkaline $KMnO_4$. The carbon attached to the ring becomes the carboxyl carbon.
- The alkaline reaction gives a benzoate salt; acidic work-up with $H_3O^+$ protonates it to the carboxylic acid. The aromatic ring remains intact in this standard transformation.

**Step 1:** Check the substrate and oxidation site The structure is ethylbenzene, $C_6H_5-CH_2-CH_3$. Its benzylic carbon is the $CH_2$ carbon directly bonded to the benzene ring, and it has two hydrogens. Thus the requirement for side-chain oxidation is satisfied. Under strong permanganate oxidation, the reaction does not simply convert the terminal methyl group into an aldehyde or preserve every side-chain carbon in the main aromatic product.

**Step 2:** Apply oxidation and the acid work-up The benzylic side chain is converted to a benzoate group in the alkaline medium. The sequence can be represented as

$$C_6H_5CH_2CH_3 \xrightarrow{\text{alkaline } KMnO_4} C_6H_5COO^- \xrightarrow{H_3O^+} C_6H_5COOH.$$

The original benzylic carbon supplies the carboxyl carbon attached directly to the ring. The extra terminal carbon is oxidatively removed from that side chain; it does not remain as a methylene spacer between the ring and the acid group.

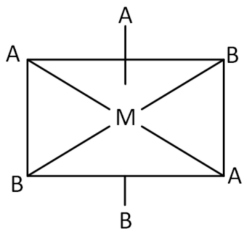
**Step 3:** Exclude the other oxygen-containing products Option (A) is benzoic acid and matches complete benzylic oxidation. Option (B), $C_6H_5CH_2CHO$, is an aldehyde retaining both side-chain carbons and is not the final product of these strongly oxidising conditions. Option (C), $C_6H_5CH_2COOH$, incorrectly retains a methylene group next to the ring. Option (D), acetophenone, is a ketone and represents incomplete side-chain oxidation rather than the carboxylic acid formed here.

**Step 4:** Conclusion The major product after the acid work-up is benzoic acid, $C_6H_5COOH$, as drawn in option (A). Hence, the correct option is (A).

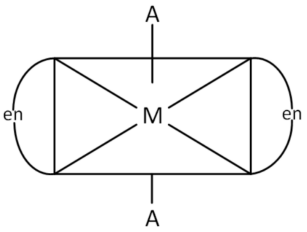
**Quick Tip to Solve:** With strong permanganate, first look for a benzylic hydrogen: an oxidisable alkyl side chain on benzene generally ends as a directly attached carboxyl group.

**74. The one that will show optical activity is :**
**(en = ethane-1,2-diamine)** [4 marks]

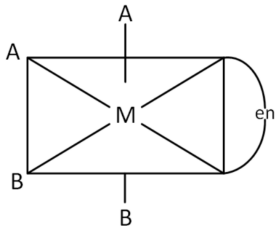
(A)



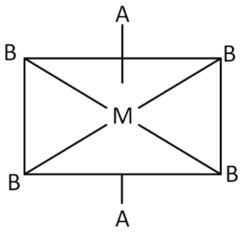
(B)



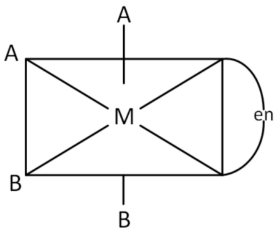

(C)




(D)




**Correct Answer:** (C)




**Solution:**

**Concept:**

- A complex is optically active as an individual enantiomer when its mirror image is not superimposable. A plane of symmetry or centre of inversion makes a structure achiral.
- The ligand en is bidentate: its two nitrogen donors occupy adjacent octahedral positions. When testing symmetry, preserve each depicted chelate connection, not merely the identities of the donor atoms.

**Step 1:** Exclude the symmetric monodentate arrangements In option (A), the three A ligands and three B ligands form the meridional $MA_3B_3$ arrangement. A plane containing the axial sites and the appropriate equatorial diagonal interchanges the equivalent equatorial A sites and equivalent equatorial B sites. The structure is achiral. In option (D), the two A ligands are trans, while the four B ligands occupy the equatorial positions. This $\text{trans-}MA_2B_4$ arrangement also has mirror symmetry and is achiral.

**Step 2:** Test the two chelated arrangements Option (B) has two identical en chelates and two trans A ligands. Reflection in the equatorial plane preserves the two A sites as a pair and maps each chelate onto itself, so this trans arrangement is achiral. Option (C), however, has one en chelate on one octahedral edge, two cis A ligands and two cis B ligands. The A and B assignments on the remaining four sites remove the mirror symmetry that an unlabelled drawing might appear to suggest.

**Step 3:** Check why the third structure is chiral Take the en donors as the adjacent $+x$ and $+y$ sites. The other sites can be labelled A at $-x, +z$ and B at $-y, -z$, consistently with the drawing. Any symmetry must either fix both en donor sites or exchange them. Fixing the two sites fixes the x and y axes; the only remaining nonidentity possibility is reversal of z, which interchanges the axial A and B ligands and is forbidden. Exchanging the en sites would map the negative x site onto the negative y site, again interchanging an A ligand with a B ligand. Thus only the identity preserves all ligand assignments and the chelate connection; in particular, no reflection, inversion or other improper symmetry is possible. Its mirror form is therefore a distinct enantiomer. This distinguishes (C) from the symmetric arrangements (A), (B) and (D).

**Step 4:** Conclusion The octahedral chelated arrangement in (C) is chiral and can show optical activity as an individual enantiomer. Hence, the correct option is (C).

**Quick Tip to Solve:** For optical isomerism, keep each en bridge intact while testing mirror symmetry; counting ligand types alone cannot establish chirality.

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**75. For a reaction,**

**$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$ ;**

**identify dihydrogen ($H_2$) as a limiting reagent in the following reaction mixtures.** [4 marks]

(A) 56g of $N_2$ + 10g of $H_2$

(B) 14g of $N_2$ + 4g of $H_2$

(C) 28g of $N_2$ + 6g of $H_2$

(D) 35g of $N_2$ + 8g of $H_2$

**Correct Answer:** (A) 56g of $N_2$ + 10g of $H_2$

**Solution:**

**Concept:**

- The balanced equation is $\mathrm{N_2 + 3\,H_2 \longrightarrow 2\,NH_3}$: one mole of nitrogen requires three moles of hydrogen. A limiting reagent is exhausted first and fixes the maximum reaction extent.
- Compare $n(\mathrm{N_2})/1$ with $n(\mathrm{H_2})/3$, using molar masses $28$ and $2\,\mathrm{g\,mol^{-1}}$. Equivalently, each $28\,\mathrm{g}$ of nitrogen requires $6\,\mathrm{g}$ of hydrogen.

**Step 1:** Test the first two mixtures For option (A),

$$n(\mathrm{N_2}) = \frac{56}{28} = 2\,\mathrm{mol}, \qquad n(\mathrm{H_2}) = \frac{10}{2} = 5\,\mathrm{mol}.$$

The nitrogen would require $6\,\mathrm{mol}$ of hydrogen, but only $5\,\mathrm{mol}$ is available. Hence hydrogen is limiting. For option (B), $14\,\mathrm{g}$ of nitrogen is $0.5\,\mathrm{mol}$, requiring $1.5\,\mathrm{mol}$, or $3\,\mathrm{g}$, of hydrogen. The supplied $4\,\mathrm{g}$ is

excess, so nitrogen is limiting in (B).

**Step 2:** Test the remaining mixtures For option (C), $28\,\mathrm{g}$ of nitrogen is exactly $1\,\mathrm{mol}$ and $6\,\mathrm{g}$ of hydrogen is exactly $3\,\mathrm{mol}$. The ratio is stoichiometric, so neither reagent is present in excess; it is not a mixture with hydrogen deficient relative to nitrogen. For option (D),

$$n(\mathrm{N_2}) = \frac{35}{28} = 1.25\,\mathrm{mol}.$$

It requires $3.75\,\mathrm{mol}$, or $7.5\,\mathrm{g}$, of hydrogen. Since $8\,\mathrm{g}$ is provided, hydrogen is again in excess and nitrogen is limiting.

**Step 3:** Identify the uniquely hydrogen-deficient choice Only the first mixture supplies less hydrogen than the balanced equation demands for the given nitrogen. The absolute mass of hydrogen alone is not a useful criterion: the required amount depends on the nitrogen present. Comparing stoichiometric mole amounts separates the hydrogen-deficient first mixture from the nitrogen-deficient second and fourth mixtures and the exactly balanced third mixture.

**Step 4:** Conclusion Dihydrogen is the limiting reagent in the mixture containing $56\,\mathrm{g}$ of $\mathrm{N_2}$ and $10\,\mathrm{g}$ of $\mathrm{H_2}$. Hence, the correct option is (A).

**Quick Tip to Solve:** For this reaction, multiply the nitrogen mass by $6/28$; hydrogen is limiting precisely when the supplied hydrogen mass is smaller than that requirement.

---

**76. The standard Gibbs energy for the given cell reaction in kJ mol$^{-1}$ at 298 K is :**

**Zn(s) + Cu$^{2+}$ (aq) $\rightarrow$ Zn$^{2+}$ (aq) + Cu (s),**

**E$^\circ$ = 2 V at 298 K**

**(Faraday's constant, F = 96000 C mol$^{-1}$)** [4 marks]

(A) 384

(B) –192

(C) –384

(D) 192

**Correct Answer:** (C) –384

**Solution:**

**Concept:**

- The standard Gibbs energy change of a cell reaction is $\Delta_r G^\circ = -nFE^\circ_{\text{cell}}$, where $n$ is the number of electrons transferred in the balanced reaction as written.
- Use the cell potential and Faraday constant stated in the question. Since $1\,\mathrm{C\,V} = 1\,\mathrm{J}$, the direct calculation gives joules per mole of reaction; divide by $1000$ for kilojoules.

**Step 1:** Find the electron-transfer number The oxidation and reduction half-reactions are

$$\mathrm{Zn\,(s)} \longrightarrow \mathrm{Zn^{2+}(aq)} + 2\,\mathrm{e^-},$$

$$Cu^{2+}(aq) + 2\,e^- \longrightarrow Cu\,(s).$$

Adding them gives the stated overall reaction with two electrons transferred per reaction event. Thus $n = 2$, not 1. The cancellation of electrons in the overall equation does not mean the electron-transfer number is zero.

**Step 2:** Substitute the specified data and convert units At the stated temperature, insert $E^\circ = 2\,V$ and $F = 96000\,C\,mol^{-1}$:

$$\begin{aligned}\Delta_r G^\circ &= -(2)(96000)(2)\,J\,mol^{-1}\\ &= -384000\,J\,mol^{-1}\\ &= -384\,kJ\,mol^{-1}.\end{aligned}$$

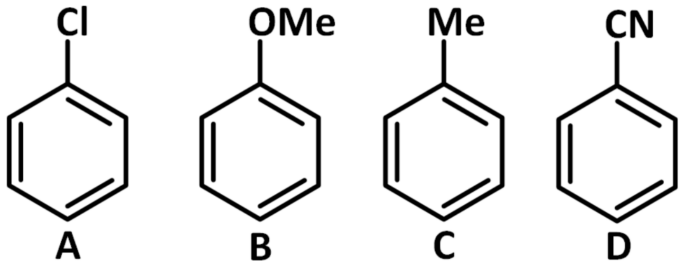
The temperature is already the temperature associated with the supplied standard potential, so it is not multiplied into this expression. A separate factor of $T$ would belong to another thermodynamic relation.

**Step 3:** Check the sign and reject the distractors A positive standard cell potential implies a negative standard Gibbs energy for the forward reaction, in agreement with spontaneous galvanic operation. Therefore the positive values in (A) and (D) have the wrong sign. The value $-192\,kJ\,mol^{-1}$ in (B) would follow from incorrectly taking only one electron. The two-electron balanced reaction gives both the sign and magnitude in (C).

**Step 4:** Conclusion The standard Gibbs energy change is $\boxed{-384\,kJ\,mol^{-1}}$ for the reaction as written. Hence, the correct option is (C).

**Quick Tip to Solve:** Balance the half-reactions to determine $n$ before using $-nFE^\circ$, and check that a positive cell voltage produces a negative Gibbs energy.

**77. The increasing order of reactivity of the following compounds towards aromatic electrophilic substitution reaction is :** [4 marks]




(A) A < B < C < D

(B) D < A < C < B

(C) D < B < A < C

(D) B < C < A < D

**Correct Answer:** (B) D < A < C < B

**Solution:**

**Concept:**

- An electron-donating substituent activates a benzene ring towards electrophilic substitution by stabilising the positively charged sigma complex; electron-withdrawing groups deactivate it.
- Resonance donation from methoxy is strongly activating. Methyl activates more weakly by hyperconjugation and its positive inductive effect. Cyano withdraws through both resonance and induction, whereas chlorine is overall deactivating despite donating a lone pair by resonance.

**Step 1:** Identify the four substituents The labelled structures are A: chlorobenzene ($-\mathrm{Cl}$); B: anisole ($-\mathrm{OCH_3}$); C: toluene ($-\mathrm{CH_3}$); and D: benzonitrile ($-\mathrm{CN}$). Their relative reactivity is decided by how these groups affect the ring and the sigma complex, not by the alphabetical labels in the figure or by the presence of a heteroatom alone.

**Step 2:** Rank the deactivators and activators The cyano group is a strong electron-withdrawing group: its negative resonance and negative inductive effects greatly reduce ring nucleophilicity, so D is least reactive. Chlorine also deactivates through its strong negative inductive effect, but its lone-pair resonance donation partly offsets this withdrawal, making A more reactive than D. The methyl group donates electron density and stabilises the sigma complex by hyperconjugation, so C is more reactive than chlorobenzene. Methoxy provides strong direct lone-pair resonance donation and activates the ring more strongly than methyl, putting B last in increasing order.

**Step 3:** Write the order and compare all choices Combining the comparisons gives

$$\text{benzonitrile} < \text{chlorobenzene} < \text{toluene} < \text{anisole},$$

that is, $D < A < C < B$. Option (A) incorrectly makes strongly deactivated benzonitrile the most reactive. Option (C) places anisole below chlorobenzene and toluene, contrary to its strong resonance activation. Option (D) reverses the principal activation trend by placing anisole first and benzonitrile last. Only option (B) agrees with all four substituent effects.

**Step 4:** Conclusion The increasing reactivity order is $\boxed{D < A < C < B}$, corresponding to benzonitrile, chlorobenzene, toluene and anisole. Hence, the correct option is (B).

**Quick Tip to Solve:** Distinguish directing effect from activation: chlorine directs ortho/para but still slows substitution, while methoxy both directs ortho/para and strongly activates the ring.

**78. Among the following, the set of parameters that represents path function, is :**

**(A) q + w**

**(B) q**

**(C) w**

**(D) H–TS** [4 marks]

(A) (B) and (C)

(B) (A) and (D)

(C) (B), (C) and (D)

(D) (A), (B) and (C)

**Correct Answer:** (A) (B) and (C)

**Solution:**

**Concept:**

- A state function depends only on the equilibrium state. Its change between fixed initial and final states is independent of the process followed.
- Heat $q$ and work $w$ describe energy transferred during a process. With work done on the system taken positive, the first law gives $\Delta U = q + w$.

**Step 1:** Identify the process-dependent quantities Heat is energy transferred because of a temperature difference, while work is transferred through a mechanical or other generalized displacement. Neither is energy stored as a property of the system. The amount of each depends on the process. For example, connecting the same initial and final states by a different expansion route changes the pressure integral for work. The heat exchanged correspondingly changes so that the required internal-energy change remains the same. Thus parameters (B), $q$, and (C), $w$, are path functions.

**Step 2:** Test both combinations of state variables The sum in (A) is not a path function: $q + w = \Delta U$. Although its two terms can vary separately, their sum is fixed by the two end states. Also, $H$, $T$, and $S$ are state variables, so $H - TS = G$, the Gibbs energy, is a state function. Therefore (D) is excluded. Option (B) includes two state-function expressions; option (C) incorrectly includes (D); option (D) incorrectly includes (A). Only the first combination contains the two path functions and no state function.

**Step 3:** Conclusion The required parameters are heat $q$ and work $w$. Hence, the correct option is (A).

**Quick Tip to Solve:** Check combinations before classifying them: heat and work depend on the path, but their sum $q + w = \Delta U$ does not.

**79. For any given series of spectral lines of atomic hydrogen, let $\Delta\overline{\nu} = \Delta\overline{\nu}_{\max} - \Delta\overline{\nu}_{\min}$ be the difference in maximum and minimum frequencies in $\mathrm{cm}^{-1}$. The ratio Lyman Balmer $\frac{\Delta\overline{\nu}_{\text{Lyman}}}{\Delta\overline{\nu}_{\text{Balmer}}}$ is :** [4 marks]

(A) 9 : 4

(B) 4 : 1

(C) 27 : 5

(D) 5 : 4

**Correct Answer:** (A) 9 : 4

**Solution:**

**Concept:**

- The hydrogen-series wavenumber is $\overline{\nu} = R_{\mathrm{H}}(1/n^2 - 1/m^2)$, where $n$ is the fixed final level and $m > n$ is the initial level.
- The smallest wavenumber in a series corresponds to $m = n + 1$; the series limit, its largest wavenumber, corresponds to $m \to \infty$. Wavenumber and frequency differ by the common factor $c$.

**Step 1:** Find the width for a general final level For a fixed $n$, increasing $m$ decreases $1/m^2$ and hence increases the emitted wavenumber. Therefore the first line and series limit give respectively

$$\overline{\nu}_{\min} = R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right), \qquad \overline{\nu}_{\max} = \frac{R_{\mathrm{H}}}{n^2}.$$

Subtracting removes the common lower-level contribution:

$$\Delta\overline{\nu} = \overline{\nu}_{\max} - \overline{\nu}_{\min} = \frac{R_{\mathrm{H}}}{(n+1)^2}.$$

**Step 2:** Substitute the Lyman and Balmer final levels For Lyman lines $n = 1$, so the width is $R_{\mathrm{H}}/4$. For Balmer lines $n = 2$, the width is $R_{\mathrm{H}}/9$. Their ratio is consequently

$$\frac{\Delta\overline{\nu}_{\text{Lyman}}}{\Delta\overline{\nu}_{\text{Balmer}}} = \frac{R_{\mathrm{H}}/4}{R_{\mathrm{H}}/9} = \frac{9}{4}.$$

The ratio $4 : 1$ compares the series limits rather than the widths. The ratio $27 : 5$ compares their first lines rather than their widths. The value $5 : 4$ is neither the difference required nor its ratio. No conversion from wavenumber to frequency changes the result because the same factor multiplies both widths.

**Step 3:** Conclusion The spectral-width ratio is $9 : 4$, a dimensionless ratio. Hence, the correct option is (A).

**Quick Tip to Solve:** For a hydrogen series terminating at level $n$, remember that the width between its first line and limit is $R_{\mathrm{H}}/(n+1)^2$.

**80. Aniline dissolved in dilute HCl is reacted with sodium nitrite at 0°C. This solution was added dropwise to a solution containing equimolar mixture of aniline and phenol in dil. HCl. The structure of the major product is** [4 marks]

(A) $p$-HO$-$C$_6$H$_4-$N$=$N$-$C$_6$H$_5$

(B) C$_6$H$_5-$N$=$N$-$NH$-$C$_6$H$_5$

(C) $p$-H$_2$N$-$C$_6$H$_4-$N$=$N$-$C$_6$H$_5$

(D) C$_6$H$_5-$N$=$N$-$O$-$C$_6$H$_5$

**Correct Answer:** (C) $p$-H$_2$N$-$C$_6$H$_4-$N$=$N$-$C$_6$H$_5$

**Solution:**

**Concept:**

- Aniline undergoes diazotisation with nitrous acid generated from $NaNO_2$ and dilute HCl at 0–5 °C, forming benzenediazonium chloride.
- Azo coupling is electrophilic substitution on an activated aromatic ring. Aniline is coupled in a mildly acidic medium, whereas phenol couples most effectively in alkaline solution as phenoxide.

**Step 1:** Identify the electrophile and the suitable coupling component The first operation converts the amino group of aniline into the diazonium group:

$$C_6H_5NH_2 + NaNO_2 + 2\,HCl \longrightarrow C_6H_5N_2Cl + NaCl + 2\,H_2O.$$

Low temperature keeps the diazonium salt available for the following coupling reaction. Its terminal nitrogen acts as the electrophilic centre. In the mixture supplied, the acidic conditions are those used for coupling with aniline; they do not generate the strongly activated phenoxide ion needed for efficient phenol coupling. Although some aniline is protonated, its available free-base fraction provides the activated aromatic coupling partner.

**Step 2:** Choose the aromatic substitution site and compare structures The amino group directs electrophilic substitution to the ortho and para positions. The less hindered para site is favoured, so the new bond joins the terminal diazonium nitrogen to the carbon para to $NH_2$. The major azo product is $p$-aminoazobenzene, $p$-H$_2$N$-$C$_6$H$_4-$N$=$N$-$C$_6$H$_5$. Option (A), the hydroxyazo dye, is favoured when phenol is coupled in alkaline medium. Option (B) is the nitrogen-linked diazoamino compound, rather than the acid-promoted para aminoazo product. Option (D) has an oxygen bridge and is not the aromatic carbon-coupling product.

**Step 3:** Conclusion The major product is $p$-aminoazobenzene, the para-amino azo structure. Hence, the correct option is (C).

**Quick Tip to Solve:** For azo-dye questions, inspect the coupling medium first: aniline couples under mildly acidic conditions, while phenol is activated as phenoxide in alkaline solution.

**81. The major product of the following reaction is :** [4 marks]




(A) $CH_3CD_2CH(Cl)(I)$

(B) $CH_3CD(Cl)CHD(I)$

(C) $CH_3C(I)(Cl)CHD_2$

(D) $CH_3CD(I)CHD(Cl)$

**Correct Answer:** (C) $CH_3C(I)(Cl)CHD_2$

**Solution:**

**Concept:**

- Addition of a hydrogen halide to an unsymmetrical terminal alkyne normally follows Markovnikov orientation. Deuterium, D, follows the same regiochemical role as hydrogen.
- One equivalent adds across one of the two $\pi$ bonds to give a vinyl halide. A second addition to that alkene gives the saturated dihalide.

**Step 1:** Add one equivalent of deuterium chloride The starting compound is propyne, $CH_3-C\equiv CH$. The terminal carbon already has hydrogen; Markovnikov addition places deuterium on that carbon and chlorine on the internal carbon. With precisely one equivalent, the first product retains a double bond:

$$CH_3-C\equiv CH + DCl \longrightarrow CH_3-C(Cl)=CHD.$$

The original terminal hydrogen remains in the molecule. No exchange of the methyl hydrogens is implied by the stated reagents.

**Step 2:** Add deuterium iodide to the resulting alkene In the vinyl chloride intermediate, the second electrophilic addition places the incoming deuterium at the terminal carbon again. The developing carbocation at the chlorine-bearing carbon is stabilized by lone-pair donation from chlorine, and iodide bonds at this internal carbon. Thus

$$CH_3-C(Cl)=CHD + DI \longrightarrow CH_3-C(Cl)(I)-CHD_2.$$

Both halogens end up on the same internal carbon, while both newly added deuterium atoms are terminal. Option (A) puts both halogens on the terminal carbon, reversing the first addition. Options (B) and (D) separate the halogens onto different carbons, contrary to the second addition. Atom counting also confirms retention of the original terminal hydrogen.

**Step 3:** Conclusion The major product is $CH_3-C(I)(Cl)-CHD_2$. Hence, the correct option is (C).

**Quick Tip to Solve:** Follow isotopes through each addition separately: mark the original terminal H, then add one D to that same terminal carbon in each Markovnikov step.

**82. The osmotic pressure of a dilute solution of an ionic compound XY in water is four times that of a solution of 0.01 M $BaCl_2$ in water. Assuming complete dissociation of the given ionic compounds in water, the concentration of XY (in mol $L^{-1}$) in solution is :** [4 marks]

(A) $4 \times 10^{-4}$

(B) $6 \times 10^{-2}$

(C) $4 \times 10^{-2}$

(D) $16 \times 10^{-4}$

**Correct Answer:** (B) $6 \times 10^{-2}$

**Solution:**

**Concept:**

- The osmotic pressure of a dilute solution is $\pi = iCRT$, where $i$ is the van't Hoff factor, $C$ the formula-unit molarity, and $T$ the absolute temperature.
- With complete dissociation, $i$ equals the number of independently dissolved ions per formula unit. Osmotic pressures at the same temperature therefore compare through $iC$.

**Step 1:** Count ions from each formula unit The binary ionic compound dissociates as $XY \rightarrow X^+ + Y^-$, producing two ions per formula unit, so $i_{XY} = 2$. The charges need not be assumed to obtain that count: one X ion and one Y ion are released. Barium chloride dissociates as

$$BaCl_2 \longrightarrow Ba^{2+} + 2\,Cl^-,$$

giving $i_{BaCl_2} = 3$. Its ionic particle concentration is therefore $3(0.01) = 0.03\,\text{mol}\,L^{-1}$.

**Step 2:** Use the fourfold osmotic-pressure condition Let the concentration of XY be $C$. At a common temperature, the factors $R$ and $T$ cancel from the ratio:

$$\frac{\pi_{XY}}{\pi_{BaCl_2}} = \frac{2C}{3(0.01)} = 4.$$

Consequently,

$$2C = 0.12, \qquad C = 0.06 = 6 \times 10^{-2}\,\text{mol}\,L^{-1}.$$

Option (C), $0.04$, would follow from ignoring that the salts produce different numbers of ions. Options (A) and (D) give particle concentrations far smaller than $0.12\,\text{mol}\,L^{-1}$, so neither could yield four times the reference pressure. As a direct check, $2(0.06)/(3 \times 0.01) = 4$.

**Step 3:** Conclusion The concentration is $6 \times 10^{-2}\,\text{mol}\,L^{-1}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Compare total particle molarities, $iC$, rather than salt molarities; count ions before forming the pressure ratio.

**83. Excessive release of $CO_2$ into the atomosphere results in :** [4 marks]

(A) depletion of ozone

(B) polar vortex

(C) global warming

(D) formation of smog

**Correct Answer:** (C) global warming

**Solution:**

**Concept:**

- Carbon dioxide is a greenhouse gas: it absorbs part of the terrestrial infrared radiation emitted by the heated surface and atmosphere.
- Increasing greenhouse-gas concentration changes the outgoing radiative energy balance. The resulting long-term rise in global mean surface temperature is global warming.

**Step 1:** Connect carbon dioxide with retention of infrared radiation Incoming solar energy heats the Earth. The surface then radiates energy outward mainly at infrared wavelengths. $CO_2$ absorbs in infrared bands and exchanges that energy with the atmosphere; the atmosphere also emits radiation in all directions, including back towards the surface. Increasing the abundance of this gas reduces the rate at which energy escapes to space at a given initial temperature. The climate system consequently warms until a new radiative balance is reached. This enhancement of the greenhouse effect is the relevant consequence of excessive release of $CO_2$.

**Step 2:** Distinguish the other atmospheric phenomena Ozone depletion principally involves catalytic destruction of stratospheric ozone by radicals, especially chlorine or bromine released from ozone-depleting substances; it is not the characteristic direct effect of increased $CO_2$. A polar vortex is a large-scale circulation of cold polar air, not the phenomenon named by this greenhouse-gas increase. Smog involves pollutants such as nitrogen oxides, volatile organic compounds, particulates, or sulfur compounds under suitable atmospheric conditions. $CO_2$ is not the reactant that produces the characteristic photochemical smog. Therefore the other three choices do not describe the expected direct association.

**Step 3:** Conclusion Excessive release of $CO_2$ enhances the greenhouse effect and results in global warming. Hence, the correct option is (C).

**Quick Tip to Solve:** Keep the pollutant-effect pairs distinct: $CO_2$ points to greenhouse warming, while halogen radicals point to stratospheric ozone depletion.

**84. Magnisium powder burns in air to give :** [4 marks]

(A) MgO and $Mg_3N_2$

(B) MgO only

(C) $Mg(NO_3)_2$ and $Mg_3N_2$

(D) MgO and $Mg(NO_3)_2$

**Correct Answer:** (A) MgO and $Mg_3N_2$

**Solution:**

**Concept:**

- Air supplies both oxygen and nitrogen. Burning magnesium is sufficiently reactive to combine with either gas under the high-temperature conditions of combustion.
- Magnesium forms $Mg^{2+}$; oxygen forms $O^{2-}$ and nitrogen forms $N^{3-}$ in their binary salts. Charge balance therefore gives MgO and $Mg_3N_2$.

**Step 1:** Write the oxidation reaction Magnesium powder burns with a dazzling white flame. Oxygen in air oxidizes the metal to magnesium oxide:

$$2\,Mg + O_2 \longrightarrow 2\,MgO.$$

Each magnesium atom loses two electrons and each oxygen atom gains two, yielding the ionic oxide. This is a major product of combustion. However, the wording specifies air rather than pure oxygen, so the possible reaction with the nitrogen component must also be considered before selecting the oxide-only option.

**Step 2:** Include reaction with atmospheric nitrogen and eliminate nitrates At the temperature of burning magnesium, some metal combines directly with nitrogen to form magnesium nitride:

$$3\,Mg + N_2 \longrightarrow Mg_3N_2.$$

The formula follows from the charge balance $3(+2) + 2(-3) = 0$. Thus combustion in air gives oxide together with nitride. Option (B) neglects nitrogen and would describe the oxygen-only reaction instead. Options (C) and (D) wrongly introduce magnesium nitrate, $Mg(NO_3)_2$; direct burning in air does not convert atmospheric nitrogen into the nitrate salt. Moreover, magnesium nitrate is thermally unstable at high temperatures, so it is not the stated combustion product. The pair in option (A) accounts for both reactive gases present in air.

**Step 3:** Conclusion Magnesium powder burns in air to give MgO and $Mg_3N_2$. Hence, the correct option is (A).

**Quick Tip to Solve:** When a reactive metal burns in air, check nitrogen as well as oxygen; magnesium forms a nitride alongside its oxide.

**85. The number of water molecule(s) not coordinated to copper ion directly in $CuSO_4.5H_2O$, is :**

[4 marks]

(A) 2

(B) 4

(C) 3

(D) 1

**Correct Answer:** (D) 1

**Solution:**

**Concept:**

- Water of crystallisation includes both water directly bonded to the metal and water held in the crystal by hydrogen bonding. These two roles must be counted separately.
- In copper sulphate pentahydrate, four water molecules coordinate directly to copper. The remaining water molecule belongs to the hydrogen-bonded crystal lattice.

**Step 1:** Identify the directly coordinated water The hydrate formula is $\mathrm{CuSO_4 \cdot 5\,H_2O}$, so there are five water molecules per formula unit in total. In its crystal structure four of these water molecules provide oxygen donor atoms directly bonded to $\mathrm{Cu^{2+}}$. Sulphate oxygen atoms also participate in the copper coordination environment; therefore, a possible larger coordination number must not be confused with the number of coordinated water molecules. The question asks specifically for water molecules that lack a direct copper–oxygen bond.

**Step 2:** Subtract the coordinated water from the total The fifth water molecule is linked into the crystal through hydrogen bonds rather than coordinated directly to the copper ion. Hence,

$$N_{\text{water not directly coordinated}} = 5 - 4 = 1.$$

Counting four would give the coordinated water, which is the opposite of what is asked. Two and three would require only three and two directly coordinated waters, respectively, inconsistent with the pentahydrate structure.

**Step 3:** Conclusion Only $1$ water molecule per formula unit is not directly coordinated to copper in $\mathrm{CuSO_4 \cdot 5\,H_2O}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For copper sulphate pentahydrate, remember the split as four directly coordinated waters plus one hydrogen-bonded lattice water; do not equate total hydration with direct coordination.

**86. The aerosol is a kind of colloid in which :** [4 marks]

(A) solid is dispersed in gas

(B) gas is dispersed in solid

(C) gas is dispersed in liquid

(D) liquid is dispersed in water

**Correct Answer:** (A) solid is dispersed in gas

**Solution:**

**Concept:**

- A colloid is classified by both its dispersed phase (the distributed particles) and its dispersion medium (the continuous phase). Their order in the description matters.
- An aerosol has a gas as its dispersion medium. Its dispersed phase can be a solid, as in smoke, or a liquid, as in mist.

**Step 1:** Identify the continuous phase required for an aerosol In smoke, tiny solid particles are suspended throughout air. The solid particles form the dispersed phase, while air is the gaseous dispersion medium. This is a solid aerosol. Mist or fog is another aerosol, with liquid droplets dispersed in air. Although both types exist, the question requires selecting the valid aerosol description among the four given combinations; a choice need not list every possible type of aerosol.

**Step 2:** Classify each listed combination Option (A), solid dispersed in gas, is precisely the smoke-type aerosol. Gas dispersed in a solid, option (B), forms a solid foam, such as foam rubber. Gas dispersed in a liquid, option (C), forms a foam, such as soap lather. In option (D) the medium is water, a liquid, so it cannot be an aerosol: an immiscible liquid dispersed in water would be an emulsion, whereas a miscible liquid would make a solution. Only the first option has the required gaseous continuous phase.

**Step 3:** Conclusion A solid dispersed in a gas is a solid aerosol. Hence, the correct option is (A).

**Quick Tip to Solve:** Locate the dispersion medium first: aerosols always have a gaseous medium, whereas foams have gas as the dispersed phase.

**87. The ore that contains the metal in the form of fluoride is :** [4 marks]

(A) sphalerite

(B) magnetite

(C) malachite

(D) cryolite

**Correct Answer:** (D) cryolite

**Solution:**

**Concept:**

- The chemical formula of an ore identifies whether the metal occurs as a sulphide, oxide, carbonate or fluoride. Mineral names alone do not specify the anion.

- Cryolite is sodium hexafluoroaluminate, $Na_3AlF_6$. Its fluorine-containing anion distinguishes it from the other three minerals listed.

**Step 1:** Write the formulae of the candidate minerals Sphalerite is zinc blende, ZnS, so its metal is present as a sulphide. Magnetite has formula $Fe_3O_4$, a mixed oxide of iron, so it is an oxide mineral. Malachite is basic copper carbonate, $CuCO_3 \cdot Cu(OH)_2$, equivalently $Cu_2CO_3(OH)_2$. Its anions are carbonate and hydroxide; the formula contains no fluorine. Thus, the first three names correspond to familiar minerals, but none meets the fluoride condition.

**Step 2:** Identify the fluoride-containing mineral Cryolite has formula $Na_3AlF_6$, which can be described as $3\,NaF \cdot AlF_3$. It contains the hexafluoroaluminate ion $[AlF_6]^{3-}$, in which fluoride ligands surround aluminium. The presence of six fluorine atoms is a direct compositional test that does not apply to sphalerite, magnetite or malachite. Cryolite is also used in aluminium extraction to assist the electrolytic process, but its fluoride composition alone is sufficient to select the answer here.

**Step 3:** Conclusion The fluoride-containing mineral is cryolite, $Na_3AlF_6$. Hence, the correct option is (D).

**Quick Tip to Solve:** Match each mineral name to its formula: ZnS is a sulphide, Fe3O4 an oxide, malachite a basic carbonate, and Na3AlF6 the fluoride.

**88. $C_{60}$ an allotrope of carbon contains :** [4 marks]

(A) 12 hexagons and 20 pentagons.

(B) 20 hexagons and 12 pentagons.

(C) 16 hexagons and 16 pentagons

(D) 18 hexagons and 14 pentagons.

**Correct Answer:** (B) 20 hexagons and 12 pentagons.

**Solution:**

**Concept:**

- Buckminsterfullerene, $C_{60}$, has a closed cage with sixty carbon vertices, each bonded to three neighbouring carbon atoms.
- The cage is a truncated icosahedron consisting of pentagonal and hexagonal faces. Euler's relation for a closed polyhedron is $V - E + F = 2$.

**Step 1:** Find the number of edges and faces Each of the sixty carbon atoms contributes three bonds to the vertex count. Every bond belongs to two carbon atoms, so the number of edges is

$$E = \frac{3 \times 60}{2} = 90.$$

Using $V = 60$ in Euler's relation gives

$$F = 2 - V + E = 2 - 60 + 90 = 32.$$

Thus, if $p$ and $h$ denote the numbers of pentagons and hexagons, respectively, then $p + h = 32$.

Counting faces alone cannot distinguish the listed choices, because each choice proposes thirty-two faces.

**Step 2:** Count the sides shared by the faces Every edge is shared by two faces. The total face-side count is therefore

$$5p + 6h = 2E = 180.$$

Subtracting $5(p + h) = 160$ gives $h = 20$, and hence $p = 12$. Option (A) reverses these two counts. Options (C) and (D) also give thirty-two faces, but their face-side counts are $176$ and $178$, rather than the required $180$, so neither represents the sixty-vertex cage.

**Step 3:** Conclusion The $C_{60}$ cage contains $20$ hexagons and $12$ pentagons. Hence, the correct option is (B).

**Quick Tip to Solve:** Remember the football-like C60 cage as twelve pentagons surrounded by twenty hexagons; Euler's relation and edge counting provide a check if the counts are reversed.

**89. Match the catalysts (Column I) with products (Column II).**

| Column I | Column II |
|---|---|
| (A) $V_2O_5$ | (i) Polyethylene |
| (B) $TiCl_4/Al(Me)_3$ | (ii) ethanal |
| (C) $PdCl_2$ | (iii) $H_2SO_4$ |
| (D) Iron Oxide | (iv) $NH_3$ |

[4 marks]

(A) (A)-(iii); (B)-(i); (C)-(ii); (D)-(iv)

(B) (A)-(ii); (B)-(iii); (C)-(i); (D)-(iv)

(C) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii)

(D) (A)-(iv); (B)-(iii); (C)-(ii); (D)-(i)

**Correct Answer:** (A) (A)-(iii); (B)-(i); (C)-(ii); (D)-(iv)

**Solution:**

**Concept:**

- A catalyst is matched with the particular industrial reaction it accelerates; its chemical formula does not imply that it supplies atoms to the product.
- The standard associations are vanadium(V) oxide with the Contact process, Ziegler–Natta catalysts with alkene polymerisation, palladium chloride with Wacker oxidation, and an iron-based catalyst with ammonia synthesis.

**Step 1:** Match the first two catalysts $V_2O_5$ catalyses the oxidation of $SO_2$ to $SO_3$ in the Contact process. The latter is used to manufacture sulphuric acid, so (A) matches (iii), $H_2SO_4$; this immediately excludes options (B) and (D). The $TiCl_4/Al(Me)_3$ combination is a Ziegler–Natta-type catalyst used for addition polymerisation of ethene. Thus (B) matches (i), polyethylene, rather than ammonia. That excludes option (C).

**Step 2:** Complete and check the remaining matches $PdCl_2$ is used in the Wacker oxidation of ethene

to ethanal, $CH_3CHO$, giving (C)–(ii). An iron-based catalyst is employed in the Haber synthesis,

$$N_2 + 3\,H_2 \rightleftharpoons 2\,NH_3.$$

The iron oxide listed is associated with preparation of the active iron catalyst under the process conditions; it is therefore paired with ammonia, giving (D)–(iv). All four associations agree in the first option. The other choices incorrectly connect the polymerisation catalyst or palladium catalyst with unrelated products.

**Step 3:** Conclusion The catalysts match sulphuric acid, polyethylene, ethanal and ammonia, respectively, in the order listed. Hence, the correct option is (A).

**Quick Tip to Solve:** Use the distinctive pair TiCl4/alkylaluminium–polyethylene first, then check V2O5–sulphuric acid and PdCl2–ethanal to complete the matching.

**90. Consider the van der Waals constants, a and b, for the following gases.**

| Gas | Ar | Ne | Kr | Xe |
|---|---|---|---|---|
| a/ (atm $dm^6$ $mol^{-2}$) | 1.3 | 0.2 | 5.1 | 4.1 |
| b/ ($10^{-2}$ $dm^3$ $mol^{-1}$) | 3.2 | 1.7 | 1.0 | 5.0 |

**Which gas is expected to have the highest critical temperature?** [4 marks]

(A) Ne

(B) Kr

(C) Xe

(D) Ar

**Correct Answer:** (B) Kr

**Solution:**

**Concept:**

- For one mole of a van der Waals gas, $P = RT/(V_m - b) - a/V_m^2$, where $a$ measures attractive interactions and $b$ accounts for excluded molar volume.
- The critical temperature is $T_c = 8a/(27Rb)$. With common units and the same gas constant, the largest ratio $a/b$ gives the highest critical temperature.

**Step 1:** Use a consistent comparison of the tabulated constants The parameter $a$ alone is insufficient because $b$ also enters the critical-temperature expression. Every tabulated $b$ entry has the same multiplier $10^{-2}$. Consequently, for ranking the gases we may divide the displayed $a$ number by the displayed $b$ number; the common factor of 100 in the actual $a/b$ ratios cannot affect the ranking. The

four dimensionless numerical comparisons are

$$\begin{aligned} \text{Ar}: &\quad 1.3/3.2 = 0.40625, \\ \text{Ne}: &\quad 0.2/1.7 \simeq 0.11765, \\ \text{Kr}: &\quad 5.1/1.0 = 5.10, \\ \text{Xe}: &\quad 4.1/5.0 = 0.82. \end{aligned}$$

**Step 2:** Rank the critical temperatures from the ratios The resulting order is

$$T_c(\text{Kr}) > T_c(\text{Xe}) > T_c(\text{Ar}) > T_c(\text{Ne}).$$

Krypton has by far the largest $a/b$, because its listed attraction parameter is large while its listed excluded-volume parameter is small. Neon, xenon and argon therefore cannot be the maximum. Use the constants actually supplied in the table for this comparison; a remembered experimental trend for noble gases cannot replace the numerical data specified in the question.

**Step 3:** Conclusion Using the given constants, krypton has the largest $a/b$ and hence the highest critical temperature. Hence, the correct option is (B).

**Quick Tip to Solve:** For critical-temperature comparisons, rank a/b rather than a alone; a common power-of-ten factor in every b entry cancels from the ranking.