JEE Main 2019 Paper 1 (B.E./B.Tech) Apr 8 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 8 April 2019, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ **is equal to**
(where c is a constant of integration) [4 marks]

(A) 2x + sinx + 2sin2x + c

(B) x + 2sinx + sin2x + c

(C) x + 2sinx + 2sin2x + c

(D) 2x + sinx + sin2x + c

Correct Answer: (B) x + 2sinx + sin2x + c

Solution:

Concept:

- The sine quotient can be expanded into a finite cosine sum. This avoids treating the numerator and denominator as unrelated functions.
- Use $2\sin a\cos b = \sin(a+b) + \sin(a-b)$, then integrate $\cos(kx)$ as $\sin(kx)/k$.

Step 1: Expand the sine quotient Apply the product-to-sum identity twice:

$$2\sin(x/2)\cos x = \sin(3x/2) - \sin(x/2),$$
$$2\sin(x/2)\cos 2x = \sin(5x/2) - \sin(3x/2).$$

Adding these relations gives

$$\sin(5x/2) = \sin(x/2)(1 + 2\cos x + 2\cos 2x).$$

Thus the integrand is $1 + 2\cos x + 2\cos 2x$ wherever its printed denominator is nonzero. This is an identity on each interval of the original integrand's domain.

Step 2: Integrate and test the options Integrate the three terms separately:

$$\int \frac{\sin(5x/2)}{\sin(x/2)}\,dx = x + 2\sin x + \sin 2x + c.$$

Differentiating the result gives $1 + 2\cos x + 2\cos 2x$, exactly the expanded integrand. Option (A) differentiates to $2 + \cos x + 4\cos 2x$; option (C) gives $1 + 2\cos x + 4\cos 2x$; and option (D) gives $2 + \cos x + 2\cos 2x$. Each has at least one incorrect coefficient, so none can differ from the required primitive by merely a constant.

Step 3: Conclusion The integral is $\boxed{x + 2\sin x + \sin 2x + c}$, on each interval where the original quotient is defined. Hence, the correct option is (B).

Quick Tip to Solve: For a quotient $\sin((2m+1)t)/\sin t$, first look for a finite cosine sum; remember the factor $1/k$ when integrating $\cos kx$.

2. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is : [4 marks]

(A) 40

(B) 48

(C) 49

(D) 45

Correct Answer: (B) 48

Solution:

Concept:

- For n observations with mean $\bar{x}$, $\sum x_i = n\bar{x}$. Population variance is $\sigma^2 = (\sum x_i^2)/n - \bar{x}^2$.
- For two unknown observations u, v, the identity $(u+v)^2 = u^2 + v^2 + 2uv$ determines their product from their sum and sum of squares.

Step 1: Recover the sum of the missing observations There are seven observations with mean 8, so their total is $7 \cdot 8 = 56$. The five given observations sum to

$$2 + 4 + 10 + 12 + 14 = 42.$$

If the other two are u and v, then

$$u + v = 56 - 42 = 14.$$

Neither unknown needs to be found separately to obtain the requested product.

Step 2: Use the variance to find their squared sum With variance 16, the second-moment relation gives

$$\frac{\sum x_i^2}{7} = 16 + 8^2 = 80, \qquad \sum x_i^2 = 560.$$

The known squared observations contribute

$$2^2 + 4^2 + 10^2 + 12^2 + 14^2 = 4 + 16 + 100 + 144 + 196 = 460.$$

Hence $u^2 + v^2 = 560 - 460 = 100$. The divisor is 7, as for the variance of the given observations; replacing it by 6 would use a different statistic.

Step 3: Evaluate and verify the product The square-of-a-sum identity now gives

$$2uv = (u+v)^2 - (u^2 + v^2) = 14^2 - 100 = 96,$$

so $uv = 48$. In fact u, v are the roots of $t^2 - 14t + 48 = 0$, namely 6 and 8. Appending these restores sum 56 and squared sum 560, and therefore variance $560/7 - 64 = 16$. The other offered products, 40, 49 and 45, fail this required identity.

Step 4: Conclusion The product of the remaining observations is 48. Hence, the correct option is (B).

Quick Tip to Solve: Find the total sum and total sum of squares from mean and variance; the product follows without solving for both missing observations.

3. The greatest value of c $\in$ R for which the system of linear equations

x – cy – cz = 0

cx – y + cz = 0

cx + cy – z = 0

has a non-trivial solution, is : [4 marks]

(A) -1

(B) 0

(C) 1/2

(D) 2

Correct Answer: (C) 1/2

Solution:

Concept:

- A homogeneous system has a non-trivial solution precisely when its coefficient matrix is singular: its determinant must vanish.
- The parameter must satisfy the determinant equation; after factoring, compare all real roots rather than selecting just any admissible value.

Step 1: Form and expand the determinant The coefficient matrix, retaining every printed sign, is

$$M = \begin{pmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{pmatrix}.$$

If $\det M \neq 0$, multiplication by the inverse matrix gives only $x = y = z = 0$. A nonzero solution therefore requires $\det M = 0$. Expanding along the first row gives

$$\begin{aligned} \det M &= (1 - c^2) + c(-c - c^2) - c(c^2 + c) \\ &= 1 - 3c^2 - 2c^3. \end{aligned}$$

The signs in the cofactor expansion are essential because the first row already contains negative entries.

Step 2: Factor the condition and compare the roots The determinant factors as

$$1 - 3c^2 - 2c^3 = (1 - 2c)(c + 1)^2.$$

Consequently the only admissible real values are $c = -1$ and $c = 1/2$; the larger is $1/2$. Sufficiency can be checked explicitly: at $c = 1/2$, the vector $(x, y, z) = (1, 1, 1)$ satisfies all three equations and is non-trivial. Option (A), -1, is admissible but is not the greatest value. At $c = 0$ the determinant is 1, and at $c = 2$ it is -27, so options (B) and (D) give only the trivial solution.

Step 3: Conclusion The greatest possible value is $\boxed{c = 1/2}$. Hence, the correct option is (C).

Quick Tip to Solve: For a homogeneous parameter system, set the determinant to zero first; repeated roots are still single candidate values when asking for the greatest parameter.

4. The sum of all natural numbers 'n' such that 100 < n < 200 and H.C.F. (91, n) > 1 is : [4 marks]

(A) 3221

(B) 3121

(C) 3203

(D) 3303

Correct Answer: (B) 3121

Solution:

Concept:

- Factor $91 = 7 \times 13$. The condition $\text{HCF}(91, n) > 1$ means that n is divisible by 7 or by 13, including multiples of both.
- Sum each arithmetic progression using $S = N(a + \ell)/2$, and subtract the overlap once by inclusion-exclusion. The inequalities exclude 100 and 200.

Step 1: Sum the multiples of each prime The multiples of 7 strictly between 100 and 200 are

$$105, 112, \ldots, 196.$$

Their count is $(196 - 105)/7 + 1 = 14$, so their sum is

$$S_7 = \frac{14(105 + 196)}{2} = 2107.$$

Likewise the multiples of 13 in the required interval are

$$104, 117, \ldots, 195.$$

There are $(195 - 104)/13 + 1 = 8$ such terms, giving

$$S_{13} = \frac{8(104 + 195)}{2} = 1196.$$

These are exactly the numbers with at least one prime factor in common with 91.

Step 2: Subtract the common multiple once Numbers included in both progressions are multiples of $\text{lcm}(7, 13) = 91$. The sole such number in the interval is 182; 91 is too small and 273 is too large. It was counted twice, so

$$S = S_7 + S_{13} - 182 = 2107 + 1196 - 182 = 3121.$$

The count check is $14 + 8 - 1 = 21$ distinct integers. The other offered sums, 3221, 3203 and 3303, do not equal this inclusion-exclusion total; adding the progressions without the subtraction would incorrectly give 3303, option (D). The required sum counts every eligible integer once.

Step 3: Conclusion The required sum is $\boxed{3121}$. Hence, the correct option is (B).

Quick Tip to Solve: When a greatest-common-divisor condition says greater than 1, take the union of prime-multiple sets and subtract their overlap; it does not require divisibility by the whole number.

5. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is : [4 marks]

(A) 210

(B) 160

(C) 320

(D) 105

Correct Answer: (A) 210

Solution:

Concept:

- The circle $x^2 + y^2 = 16$ has centre $(0, 0)$ and radius $r = 4$. The distance of $x + y = n$ from the centre is $d = n/\sqrt{2}$.
- A chord at distance d from the centre has squared length $L^2 = 4(r^2 - d^2)$, obtained from a right triangle formed by a radius and half the chord.

Step 1: Find which lines intercept the circle A real chord requires $d < r$; equality would be a tangent of zero chord length. Thus

$$\frac{n}{\sqrt{2}} < 4 \quad \Longrightarrow \quad n < 4\sqrt{2}.$$

Since n is a positive natural number, the allowed values are $1, 2, 3, 4, 5$. The next line, $x + y = 6$, is already farther from the origin than the radius. There is no integer tangent value, so using a non-strict intersection inequality produces the same list here. The value $n = 0$ is excluded by the stated natural-number indexing.

Step 2: Sum the squared chord lengths For each allowed value,

$$L_n^2 = 4\left(16 - \frac{n^2}{2}\right) = 64 - 2n^2.$$

Therefore

$$\sum_{n=1}^{5} L_n^2 = 5(64) - 2(1^2 + 2^2 + 3^2 + 4^2 + 5^2)$$

$$= 320 - 2(55) = 210.$$

The individual squared lengths are 62, 56, 46, 32 and 14, whose sum independently checks the result. Option (C), 320, ignores the distances from the centre, while option (D), 105, omits the factor of two in the final expression. Option (B), 160, also disagrees with the five explicit squared lengths.

Step 3: Conclusion The sum of the squared chord lengths is $\boxed{210}$. Hence, the correct option is (A).

Quick Tip to Solve: Square the chord formula before summing: $L^2 = 4r^2 - 4d^2$. This avoids unnecessary square roots and makes the sum a sum of consecutive squares.

6. A point on the straight line, 3x + 5y = 15 which is equidistant from the coordinate axes will lie only in : [4 marks]

(A) 1^{st} and 2^{nd} qudratants

(B) 4^{th} qudratant

(C) 1^{st} and 2^{nd} and 4^{th} qudratants

(D) 1^{st} qudratant

Correct Answer: (A) 1^{st} and 2^{nd} qudratants

Solution:

Concept:

- The perpendicular distances of (x, y) from the coordinate axes are $|y|$ and $|x|$. Equidistance therefore means $|x| = |y|$.
- The locus is the pair of angle bisectors $y = x$ and $y = -x$. Intersect both with the given line before deciding the quadrants.

Step 1: Intersect the line with the first bisector On $y = x$, substitute into $3x + 5y = 15$:

$$3x + 5x = 15 \quad \Longrightarrow \quad x = y = \frac{15}{8}.$$

Both coordinates are positive, so this intersection is in the first quadrant. It satisfies the original line since $3(15/8) + 5(15/8) = 15$, and its two distances from the axes are both $15/8$. It is important to retain the second angle bisector as well: equality of distances does not require that the coordinates themselves have the same sign.

Step 2: Intersect with the second bisector and classify On $y = -x$, substitution gives

$$3x - 5x = 15 \quad \Longrightarrow \quad x = -\frac{15}{2}, \qquad y = \frac{15}{2}.$$

This point is in the second quadrant, and both distances from the axes equal $15/2$. There are no more possibilities because $|x| = |y|$ implies exactly $y = x$ or $y = -x$. The origin is not on the line, so no point on an axis is an additional case. Option (B) wrongly gives only the fourth quadrant, and option (C) wrongly includes it. Option (D) omits the valid second-quadrant point. Thus the full set of possible quadrants is precisely the first and second.

Step 3: Conclusion The point can lie only in the $\boxed{\text{first and second quadrants}}$. Hence, the correct option is (A).

Quick Tip to Solve: Equal distances from the two axes means $|x| = |y|$, so always test both $y = x$ and $y = -x$; testing just one loses a possible quadrant.

7. If cos(α + β) = 3/5 ,sin (α - β) = 5/13 and 0 $< \alpha, \beta < \frac{\pi}{4}$, then tan(2$\alpha$) is equal to : [4 marks]

(A) 21/16

(B) 63/52

(C) 33/52

(D) 63/16

Correct Answer: (D) 63/16

Solution:

Concept:

- Put $u = \alpha + \beta$ and $v = \alpha - \beta$, so $2\alpha = u + v$. Determine the signs of sine and cosine from the given angle ranges.
- Use $\tan(u+v) = (\tan u + \tan v)/(1 - \tan u \tan v)$. Both sum and difference angles here have known signs.

Step 1: Determine the tangents with their correct signs Since $0 < \alpha, \beta < \pi/4$, their sum obeys $0 < u < \pi/2$. From $\cos u = 3/5$, its positive sine is $4/5$, hence

$$\tan u = \frac{4}{3}.$$

Also $-\pi/4 < v < \pi/4$. As $\sin v = 5/13 > 0$, this difference angle is positive and has positive cosine. Therefore

$$\cos v = \sqrt{1 - 25/169} = \frac{12}{13}, \qquad \tan v = \frac{5}{12}.$$

The range information prevents choosing negative square roots, which would change the resulting tangent.

Step 2: Apply the tangent addition formula Now $u + v = (\alpha + \beta) + (\alpha - \beta) = 2\alpha$. Consequently

$$\begin{aligned} \tan 2\alpha &= \frac{4/3 + 5/12}{1 - (4/3)(5/12)} \\ &= \frac{21/12}{1 - 5/9} = \frac{7/4}{4/9} = \frac{63}{16}. \end{aligned}$$

A direct sine-cosine check gives

$$\sin 2\alpha = \frac{63}{65}, \qquad \cos 2\alpha = \frac{16}{65},$$

whose ratio is again $63/16$, and their squares sum to 1. The alternatives $21/16$, $63/52$, and $33/52$ fail this independently determined sine-to-cosine ratio. In particular the denominator must use the subtraction in $\cos(u + v)$, not the addition used for $\cos(u - v)$.

Step 3: Conclusion $\boxed{\tan(2\alpha) = 63/16}$. Hence, the correct option is (D).

Quick Tip to Solve: When data involve $\alpha + \beta$ and $\alpha - \beta$, add those two angles to reach 2α, checking quadrants before taking square roots.

8. Let y = y(x) be the solution of the differential equation,

$$(x^2+1)^2\frac{dy}{dx}+2x(x^2+1)y=1$$

such that y(0) = 0. If $\sqrt{a}y(1)=\frac{\pi}{32}$, then the value of 'a' is : [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{1}{16}$

(C) 1

(D) $\frac{1}{4}$

Correct Answer: (B) $\frac{1}{16}$

Solution:

Concept:

- A first-order linear differential equation becomes a product derivative after multiplication by its integrating factor. Here the coefficient of y, after division by $(1+x^2)^2$, is $2x/(1+x^2)$.
- The integrating factor is $\exp\left(\int 2x/(1+x^2)\,dx\right)=1+x^2$, and $\int dx/(1+x^2)=\tan^{-1}x$.

Step 1: Reduce the equation to a product derivative Dividing by $(1+x^2)^2$, which is positive for every real x, gives

$$y'+\frac{2x}{1+x^2}y=\frac{1}{(1+x^2)^2}.$$

Multiplication by the integrating factor produces

$$(1+x^2)y'+2xy=\frac{1}{1+x^2},\qquad \frac{d}{dx}\left[(1+x^2)y\right]=\frac{1}{1+x^2}.$$

This product rule is useful because it combines the two terms involving the unknown function into one derivative.

Step 2: Use the initial condition and evaluate the solution Integrating both sides yields

$$(1+x^2)y=\tan^{-1}x+C.$$

At $x=0$, the left side is zero because $y(0)=0$, and $\tan^{-1}0=0$; hence $C=0$. Therefore

$$y(x)=\frac{\tan^{-1}x}{1+x^2},\qquad y(1)=\frac{\pi/4}{2}=\frac{\pi}{8}.$$

The initial condition is essential: an arbitrary integration constant would change the required value.

Step 3: Determine the parameter and compare the choices The stated relation is $\sqrt{a}\,y(1)=\pi/32$. Substitution gives

$$\sqrt{a}\,\frac{\pi}{8}=\frac{\pi}{32}\quad\Longrightarrow\quad\sqrt{a}=\frac{1}{4}\quad\Longrightarrow\quad a=\frac{1}{16}.$$

The value $1/4$ is the square root of the required parameter, not the parameter itself. The choices $1/2$ and 1 also fail the given relation when their positive square roots are multiplied by $\pi/8$.

Step 4: Conclusion The parameter is $\boxed{a=1/16}$. Hence, the correct option is (B).

Quick Tip to Solve: Look for the product derivative of $(1+x^2)y$, then remember to square the value obtained for $\sqrt{a}$.

9. The sum of the solutions of the equation

$$\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0$$

(x > 0) is equal to: [4 marks]

(A) 9

(B) 12

(C) 4

(D) 10

Correct Answer: (D) 10

Solution:

Concept:

- For a square-root equation with $x>0$, the substitution $t=\sqrt{x}$ gives $t>0$. Each root must satisfy the interval used to remove the absolute value.
- The sign change of $|t-2|$ occurs at $t=2$. Solve separately on $0<t<2$ and $t\geq 2$, then convert the valid roots back using $x=t^2$.

Step 1: Transform the equation Put $t=\sqrt{x}$. The original expression becomes

$$|t-2|+t(t-4)+2=0, \qquad t>0.$$

We cannot remove the absolute value with the same sign for every positive t. Also, the question asks for the sum of the original x-solutions, so the final sum must be formed after squaring the admissible t-values.

Step 2: Solve on both sides of the sign change For $t\geq 2$, use $|t-2|=t-2$. Then

$$t-2+t^2-4t+2=t(t-3)=0.$$

The only root in this interval is $t=3$; $t=0$ is excluded both by the interval and by $x>0$. For $0<t<2$, use $|t-2|=2-t$. Thus

$$2-t+t^2-4t+2=t^2-5t+4=(t-1)(t-4)=0.$$

Here only $t=1$ lies in the interval. The algebraic root $t=4$ is invalid because this formula for the absolute value applies only below 2.

Step 3: Check the roots and form the requested sum The valid square roots $t=1,3$ give $x=1,9$. Direct substitution confirms them: at $x=1$, $|1-2|+1(1-4)+2=1-3+2=0$; at $x=9$, $|3-2|+3(3-4)+2=1-3+2=0$. At the boundary $t=2$, the expression equals -2, so no boundary solution was missed. Consequently

$$\sum x=1+9=10.$$

The option 4 is the sum of the square roots rather than the required sum of x; 9 omits one valid root, and 12 is neither the checked sum nor a valid alternative.

Step 4: Conclusion The two solutions are $x=1$ and $x=9$, with sum $\boxed{10}$. Hence, the correct option

is (D).

Quick Tip to Solve: After splitting an absolute-value equation into cases, reject roots outside their case and return from $\sqrt{x}$ to x before summing.

10. If $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$, $|x| < 1$ then $f\left(\frac{2x}{1+x^2}\right)$ is equal to [4 marks]

(A) 2f(x^2)

(B) 2f(x)

(C) (f(x))^2

(D) -2f(x)

Correct Answer: (B) 2f(x)

Solution:

Concept:

- For $f(x) = \ln((1-x)/(1+x))$, composition means substituting the full new argument into both occurrences of x.
- The identity $\ln(u^2) = 2\ln u$ is valid for $u > 0$. The given interval $-1 < x < 1$ ensures that $(1-x)/(1+x) > 0$.

Step 1: Check that the composed argument is allowed Set $u = 2x/(1+x^2)$. Since $|x| < 1$,

$$1 - |u| = \frac{1 + x^2 - 2|x|}{1 + x^2} = \frac{(1-|x|)^2}{1+x^2} > 0.$$

Thus $|u| < 1$, so the composition lies inside the stated domain of f. This also ensures that the logarithms used below are real.

Step 2: Substitute and simplify the logarithmic ratio Applying the definition of f gives

$$f(u) = \ln\left(\frac{1 - 2x/(1+x^2)}{1 + 2x/(1+x^2)}\right).$$

Multiply the numerator and denominator of the inner fraction by $1 + x^2$. The resulting quadratic expressions are perfect squares:

$$\frac{1 + x^2 - 2x}{1 + x^2 + 2x} = \frac{(1-x)^2}{(1+x)^2} = \left(\frac{1-x}{1+x}\right)^2.$$

Both $1 - x$ and $1 + x$ are positive on the given interval, so no sign ambiguity arises when applying the logarithm rule.

Step 3: Apply the logarithm law and identify the option Therefore

$$f\left(\frac{2x}{1+x^2}\right) = 2\ln\left(\frac{1-x}{1+x}\right) = 2f(x).$$

The square acts on the logarithm argument, not on the logarithm value, so $(f(x))^2$ is incorrect.

Also, $f(x^2)$ is an even function, whereas this result is odd; hence $2f(x^2)$ cannot be the identity. For $0 < x < 1$, both $f(x)$ and the composition are negative, ruling out $-2f(x)$. These checks distinguish the expressions even though they all happen to vanish at $x = 0$.

Step 4: Conclusion The composition is $\boxed{2f(x)}$ throughout $|x| < 1$. Hence, the correct option is (B).

Quick Tip to Solve: In a logarithmic composition containing $2x/(1 + x^2)$, combine each $1 \pm u$ first; the perfect squares reveal the factor 2.

11. Let f : [0, 2] $\to$ R be a twice differentiable function such that f"(x) $>$ 0, for all x $\in$ (0, 2). If ϕ(x) = f(x) + f(2 – x), then ϕ is : [4 marks]

(A) decreasing on (0, 2)

(B) decreasing on (0, 1) and increasing on (1, 2)

(C) increasing on (0, 2)

(D) increasing on (0, 1) and decreasing on (1, 2)

Correct Answer: (B) decreasing on (0, 1) and increasing on (1, 2)

Solution:

Concept:

- The condition $f''(x) > 0$ on an interval makes f' strictly increasing there. Monotonicity of a function is determined by the sign of its first derivative.
- For $\phi(x) = f(x) + f(2 - x)$, the chain rule gives $\phi'(x) = f'(x) - f'(2 - x)$. The minus sign comes from differentiating $2 - x$.

Step 1: Differentiate the symmetric sum For every $0 < x < 2$, both x and $2 - x$ belong to the interval where the hypothesis applies. Differentiating gives

$$\phi'(x) = f'(x) - f'(2 - x).$$

To establish the comparison needed in this expression, take any $u < v$ in $(0, 2)$. The mean value theorem applied to f' gives $f'(v) - f'(u) = f''(c)(v - u) > 0$ for some $c \in (u, v)$. Thus the ordering of the arguments determines the ordering of the derivative values.

Step 2: Compare the two arguments on each subinterval If $0 < x < 1$, then $x < 2 - x$. Since f' is strictly increasing,

$$f'(x) < f'(2 - x), \qquad \phi'(x) < 0.$$

Hence ϕ is decreasing on $(0, 1)$. If $1 < x < 2$, the inequality of the arguments reverses:

$$x > 2 - x, \qquad f'(x) > f'(2 - x), \qquad \phi'(x) > 0.$$

Hence ϕ is increasing on $(1, 2)$. At $x = 1$, the two derivative terms coincide and $\phi'(1) = 0$.

Step 3: Use symmetry to check the behaviour The identity $\phi(2 - x) = \phi(x)$ shows symmetry about $x = 1$, consistent with a minimum there. Therefore a function strictly decreasing or strictly increasing

on the entire interval cannot describe this sum. The option with increasing behaviour before 1 and decreasing behaviour after 1 would instead give a maximum, contradicting the signs of $\phi\prime$ just obtained. The decreasing-then-increasing choice is the only one consistent with strict convexity of f.

Step 4: Conclusion ϕ is decreasing on $(0, 1)$ and increasing on $(1, 2)$. Hence, the correct option is (B).

Quick Tip to Solve: For a symmetric sum $f(x) + f(a - x)$, compare the arguments in its derivative; they exchange order at $x = a/2$.

12. Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of $\triangle$AOP is 4, is : [4 marks]

(A) $9x^2 + 8y^2 - 8y = 16$

(B) $8x^2 - 9y^2 + 9y = 18$

(C) $8x^2 + 9y^2 - 9y = 18$

(D) $9x^2 - 8y^2 + 8y = 16$

Correct Answer: (A) $9x^2 + 8y^2 - 8y = 16$

Solution:

Concept:

- An ellipse is the locus whose sum of distances from two fixed foci is constant and greater than the distance between the foci. The constant sum is $2a$.
- For an ellipse with semi-major axis a and focal distance c from its centre, the semi-minor axis satisfies $b^2 = a^2 - c^2$. Its major axis lies along the line joining the foci.

Step 1: Convert the perimeter into a distance sum The fixed side has length $OA = 1$. Thus the perimeter condition $OP + AP + OA = 4$ gives

$$OP + AP = 3.$$

Since $3 > OA$, this is an ellipse with foci $O = (0, 0)$ and $A = (0, 1)$. Its centre is their midpoint, $(0, 1/2)$, and its major axis is vertical, along the y-axis. These observations determine both the shift of the equation and which denominator is the larger one.

Step 2: Find the semi-axes and write the standard equation The focal separation is $2c = 1$, so $c = 1/2$. The constant distance sum is $2a = 3$, giving $a = 3/2$. Therefore

$$b^2 = a^2 - c^2 = \frac{9}{4} - \frac{1}{4} = 2.$$

With a vertical major axis, the locus equation is

$$\frac{x^2}{2} + \frac{(y - 1/2)^2}{9/4} = 1.$$

The larger denominator belongs to the vertical coordinate because the foci lie on a vertical line; interchanging the denominators would produce the wrong orientation.

Step 3: Expand and compare the proposed equations Multiply the standard equation by 18:

$$9x^2 + 8(y - 1/2)^2 = 18.$$

Expanding and simplifying gives

$$9x^2 + 8y^2 - 8y + 2 = 18, \qquad 9x^2 + 8y^2 - 8y = 16.$$

The two options with negative squared-coordinate coefficients are hyperbolas, so they cannot represent a fixed sum of distances. The remaining ellipse option with coefficients 8 and 9 has its major axis horizontal after completing the square, inconsistent with the vertical foci. Only the derived equation has the correct centre, axes and distance sum.

Step 4: Conclusion The locus is $\boxed{9x^2 + 8y^2 - 8y = 16}$. Hence, the correct option is (A).

Quick Tip to Solve: Subtract the fixed side from the perimeter first. The remaining constant distance sum gives $2a$, while half the fixed side gives c.

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13. $\lim\limits_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ **equals:** [4 marks]

(A) $\sqrt{2}$

(B) $2\sqrt{2}$

(C) 4

(D) $4\sqrt{2}$

Correct Answer: (D) $4\sqrt{2}$

Solution:

Concept:

- When a denominator contains a difference of square roots, multiply by its conjugate. Here this changes the difference into $1 - \cos x$.
- The identity $\sin^2 x = (1 - \cos x)(1 + \cos x)$ cancels the small factor. A limit permits cancellation for nonzero x sufficiently close to the limiting point.

Step 1: Rationalize the denominator At $x = 0$, both the numerator and denominator vanish, so direct substitution gives the indeterminate form $0/0$. Multiply numerator and denominator by $\sqrt{2} + \sqrt{1 + \cos x}$:

$$\frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}} = \frac{\sin^2 x\left(\sqrt{2} + \sqrt{1 + \cos x}\right)}{2 - (1 + \cos x)}.$$

The denominator is now $1 - \cos x$. The conjugate is nonzero near zero, so this multiplication preserves the expression wherever the original quotient is defined.

Step 2: Cancel the trigonometric factor Using $\sin^2 x = (1 - \cos x)(1 + \cos x)$, we obtain, for sufficiently small nonzero x,

$$\frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}} = (1 + \cos x)\left(\sqrt{2} + \sqrt{1 + \cos x}\right).$$

Cancelling $1-\cos x$ does not claim that the original expression is defined at zero. It produces an equal expression on a punctured neighbourhood, which is all that is needed to calculate the limit.

Step 3: Evaluate the continuous expression and check its scale Now $\cos x \to 1$, and the square root is continuous at the positive argument 2. Hence

$$L = 2\left(\sqrt{2}+\sqrt{2}\right) = 4\sqrt{2}.$$

As a scale check, $\sin^2 x \sim x^2$ and $1-\cos x \sim x^2/2$; rationalization therefore contributes a factor 2 from their ratio and a further $2\sqrt{2}$ from the conjugate. Their product confirms $4\sqrt{2}$, rather than $\sqrt{2}$, $2\sqrt{2}$, or 4. No sign difference arises from approaching zero on the two sides.

Step 4: Conclusion The limit equals $\boxed{4\sqrt{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: Rationalize before using small-angle limits: the factor $1-\cos x$ cancels exactly with a factor of $\sin^2 x$.

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14. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is : [4 marks]

(A) 2

(B) 5

(C) 4

(D) 3

Correct Answer: (C) 4

Solution:

Concept:

- The quadratic formula gives the two complex roots of $x^2-2x+2=0$. Their ratio can be simplified by multiplying numerator and denominator by a complex conjugate.
- For the imaginary unit i, $i^2=-1$, and its positive integer powers repeat every four steps. The least positive exponent returning a nonzero complex number to 1 is its multiplicative order.

Step 1: Find the roots of the quadratic The discriminant of the equation is

$$(-2)^2 - 4(1)(2) = -4.$$

Therefore the roots are

$$x = \frac{2 \pm \sqrt{-4}}{2} = 1 \pm i.$$

We may label them $\alpha = 1+i$ and $\beta = 1-i$. Both are nonzero, so the ratio in the question is well defined. Interchanging the root labels replaces the ratio by its reciprocal; we will check that the required least positive exponent is unchanged.

Step 2: Simplify the ratio and inspect its powers With the chosen labels,

$$\frac{\alpha}{\beta} = \frac{1+i}{1-i} = \frac{(1+i)^2}{(1-i)(1+i)} = \frac{2i}{2} = i.$$

Its successive positive powers are

$$i^1 = i, \qquad i^2 = -1, \qquad i^3 = -i, \qquad i^4 = 1.$$

None of the first three powers equals 1, so the first return occurs at exponent 4. Equivalently, the ratio has argument $\pi/2$, and multiplying that argument by 4 gives the first positive full turn.

Step 3: Check the root ordering and the alternatives If the labels are reversed, $\alpha/\beta = -i$, whose powers are $-i, -1, i, 1$; the first return is again at 4. Thus the answer does not depend on naming the roots. Exponents 2 and 3 produce -1 and a purely imaginary number, respectively. Exponent 5 repeats the first power and is also not 1. The customary least exponent in this roots-of-unity question is a positive integer; exponent zero is the trivial identity for any nonzero ratio and is not among the proposed choices.

Step 4: Conclusion The least positive integer is $\boxed{n = 4}$. Hence, the correct option is (C).

Quick Tip to Solve: Compute the root ratio before raising it to powers; recognizing i or $-i$ immediately gives a cycle of length four.

15. The length of the perpendicular from the point (2, –1, 4) on the straight line, $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is : [4 marks]

(A) less than 2

(B) greater than 4

(C) greater than 2 but less than 3

(D) greater than 3 but less than 4

Correct Answer: (D) greater than 3 but less than 4

Solution:

Concept:

- For a line through a point with position vector $\mathbf{a}$ and direction vector $\mathbf{u}$, the perpendicular distance from $\mathbf{p}$ satisfies $d^2 = |\mathbf{p}-\mathbf{a}|^2 - \dfrac{[(\mathbf{p}-\mathbf{a})\cdot\mathbf{u}]^2}{|\mathbf{u}|^2}$.
- The scalar projection removes the component parallel to the line. The remaining component is perpendicular, so its length is the shortest distance.

Step 1: Read a point and direction from the line The symmetric equation gives a point $Q = (-3, 2, 0)$ on the line and direction $\mathbf{u} = (10, -7, 1)$. For $P = (2, -1, 4)$, the displacement from this point of the line is

$$\overrightarrow{QP} = (5, -3, 4).$$

Thus $|\overrightarrow{QP}|^2 = 25 + 9 + 16 = 50$ and $|\mathbf{u}|^2 = 100 + 49 + 1 = 150$.

Step 2: Subtract the parallel component The dot product is

$$\overrightarrow{QP} \cdot \mathbf{u} = 5(10) + (-3)(-7) + 4(1) = 75.$$

Therefore the squared perpendicular distance is

$$d^2 = 50 - \frac{75^2}{150} = 50 - \frac{75}{2} = \frac{25}{2}.$$

The distance is positive, so $d = 5/\sqrt{2}$.

Step 3: Compare with the offered intervals Since $9 < 25/2 < 16$, taking positive square roots gives $3 < d < 4$. This excludes the intervals below 2 and between 2 and 3, and also excludes a value greater than 4. As a check, the perpendicular foot is $Q + \frac{1}{2}\mathbf{u} = (2, -\frac{3}{2}, \frac{1}{2})$; its displacement to P has squared length $0^2 + (\frac{1}{2})^2 + (\frac{7}{2})^2 = 25/2$.

Step 4: Conclusion The perpendicular distance is $5/\sqrt{2}$ units, which is greater than 3 but less than 4. Hence, the correct option is (D).

Quick Tip to Solve: Compare squared distances with 9 and 16 to identify this interval without approximating the square root.

16. Let A and B be two non-null events such that A $\subset$ B . Then, which of the following statements is always correct? [4 marks]

(A) P(A|B) = 1

(B) P(A|B) = P(B) – P(A)

(C) P(A|B) $\leq$ P(A)

(D) P(A|B) $\geq$ P(A)

Correct Answer: (D) P(A|B) $\geq$ P(A)

Solution:

Concept:

- Conditional probability is $P(A \mid B) = P(A \cap B)/P(B)$, with $P(B) > 0$. For $A \subset B$, the intersection is exactly A.
- Every probability is at most 1. Dividing a nonnegative number by a positive number no greater than 1 cannot reduce it.

Step 1: Use the containment of events Whenever A occurs, B also occurs, so $A \cap B = A$. It follows that

$$P(A \mid B) = \frac{P(A)}{P(B)}.$$

Notice that containment would make $P(B \mid A) = 1$, but it does not generally make $P(A \mid B) = 1$: observing the larger event need not force the smaller one to occur.

Step 2: Compare the conditional and original probabilities Because $0 < P(B) \le 1$,

$$P(A \mid B) - P(A) = \frac{P(A)[1 - P(B)]}{P(B)} \ge 0.$$

Both factors in the numerator are nonnegative and the denominator is positive. Thus $P(A \mid B) \ge P(A)$, including the equality case when $P(B) = 1$. The result needs no independence assumption.

Step 3: Test the alternatives For an explicit example, let $P(A) = 1/4$ and $P(B) = 1/2$, with $A \subset B$. Then $P(A \mid B) = 1/2$. It is neither 1 nor $P(B) - P(A) = 1/4$, so (A) and (B) fail. It exceeds $P(A) = 1/4$, so (C) is not always correct either. The inequality in (D) follows for every admissible pair, rather than only this example.

Step 4: Conclusion For all the stated events, the conditional probability of the smaller event is at least its original probability. Hence, the correct option is (D).

Quick Tip to Solve: For nested events first replace the intersection by the smaller event, then compare the denominator with 1.

17. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is : [4 marks]

(A) 175

(B) 162

(C) 160

(D) 180

Correct Answer: (D) 180

Solution:

Concept:

- If n objects have repeated multiplicities $r_1, r_2, \ldots$, their distinct arrangements number $n!/(r_1!r_2!\cdots)$. Identical copies are not distinguishable.
- The nine digit positions have four even places, numbered 2, 4, 6 and 8. Only the three odd digits, 1, 1 and 3, are restricted to these places.

Step 1: Arrange the odd digits in permitted places Choose three of the four even places for the odd digits. In those chosen places, the multiset $1, 1, 3$ has $3!/2!$ distinct orders, since exchanging the two 1s creates no new number. Consequently the placements of odd digits number

$$\binom{4}{3}\frac{3!}{2!} = 4 \cdot 3 = 12.$$

Equivalently choose the place of 3 in four ways and the two places of 1 in $\binom{3}{2}$ ways.

Step 2: Fill every remaining place After placing the odd digits, six positions remain: the five odd-numbered places and the unused even-numbered place. All of them receive even digits. The remaining

multiset is $2, 2, 2, 2, 4, 4$, whose arrangements number

$$\frac{6!}{4!2!} = \frac{720}{24 \cdot 2} = 15.$$

There is no leading-zero restriction because none of the available digits is zero.

Step 3: Multiply the independent counts For each of the 12 arrangements of the odd digits, all 15 fillings of the remaining positions are valid and distinct. Every valid nine-digit number determines exactly one such pair of placements, so nothing is missed or counted twice. Thus

$$N = 12 \cdot 15 = 180.$$

The values 175, 162 and 160 do not equal this complete count. Requiring every even place to contain an odd digit would be an extra condition, and would be impossible with only three odd digits.

Step 4: Conclusion There are 180 such numbers. Hence, the correct option is (D).

Quick Tip to Solve: Place the restricted digits first; divide by factorials for identical digits both before and after filling the unrestricted positions.

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18. Let $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, **($\alpha \in$ R)**

such that $A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ **then a value of** α **is** [4 marks]

(A) 0

(B) $\frac{\pi}{16}$

(C) $\frac{\pi}{32}$

(D) $\frac{\pi}{64}$

Correct Answer: (D) $\frac{\pi}{64}$

Solution:

Concept:

- The matrix $R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ represents a plane rotation. Angle-addition identities give $R(\theta)R(\phi) = R(\theta + \phi)$.
- Therefore $R(\alpha)^n = R(n\alpha)$. Equality of these matrices means equality of both sine and cosine, hence equality of their angles modulo 2π.

Step 1: Recognise the angle of the given power The right-hand matrix is

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = R(\pi/2),$$

since $\cos(\pi/2) = 0$ and $\sin(\pi/2) = 1$. The signs of the off-diagonal entries distinguish this positive

quarter-turn from a negative quarter-turn.

Step 2: Apply the rotation multiplication rule For the stated matrix $A = R(\alpha)$, repeated multiplication gives $A^{32} = R(32\alpha)$. Thus

$$32\alpha = \frac{\pi}{2} + 2k\pi, \qquad k \in \mathbb{Z}.$$

Dividing the whole equation by 32 yields

$$\alpha = \frac{\pi}{64} + \frac{k\pi}{16}.$$

The question asks for a value, so choosing $k = 0$ is sufficient.

Step 3: Check each proposed value At $\alpha = \pi/64$, the power rotates through $32\alpha = \pi/2$, exactly as required. At $\alpha = 0$, the power is the identity matrix. At $\alpha = \pi/16$, the angle becomes 2π, again giving the identity. At $\alpha = \pi/32$, it becomes π, giving minus the identity. Neither identity nor minus the identity has the nonzero off-diagonal entries printed in the question.

Step 4: Conclusion One admissible value is $\alpha = \pi/64$. Hence, the correct option is (D).

Quick Tip to Solve: For a rotation matrix power, multiply the angle by the exponent; check the sine sign as well as the cosine.

19. The area (in sq. units) of the region

$A = \{(x, y) \in R \times R \mid 0 \le x \le 3, 0 \le y \le 4, y \le x^2 + 3x\}$ is : [4 marks]

(A) $\frac{59}{6}$

(B) $\frac{26}{3}$

(C) 8

(D) $\frac{53}{6}$

Correct Answer: (A) $\frac{59}{6}$

Solution:

Concept:

- For a region between $y = 0$ and a nonnegative upper boundary $y = g(x)$, its area is $\int g(x)\,dx$. Here both $y \le 4$ and $y \le x^2 + 3x$ must hold.
- The actual upper boundary is the smaller of 4 and $x^2 + 3x$. The integral must be split where these two bounds cross inside $0 \le x \le 3$.

Step 1: Locate the change in the upper boundary Solve $x^2 + 3x = 4$, giving

$$(x + 4)(x - 1) = 0.$$

Only $x = 1$ lies in the stated interval. On $[0, 1]$, the parabola rises from 0 to 4 and lies below the line $y = 4$. On $[1, 3]$, the horizontal line is lower. The root -4 is outside the region and supplies no further split.

Step 2: Integrate the permitted vertical heights The lower boundary is always $y = 0$, so

$$\begin{aligned}\text{Area} &= \int_0^1 (x^2 + 3x)\,dx + \int_1^3 4\,dx \\ &= \left[\frac{x^3}{3} + \frac{3x^2}{2}\right]_0^1 + 4(3-1) \\ &= \frac{1}{3} + \frac{3}{2} + 8 = \frac{59}{6}.\end{aligned}$$

Both pieces have positive height; no signed-area cancellation is involved.

Step 3: Check by subtracting the excluded cap The containing rectangle has area $3 \cdot 4 = 12$. Its excluded portion lies above the parabola only for $0 \le x \le 1$, and has area

$$\int_0^1 (4 - x^2 - 3x)\,dx = 4 - \frac{1}{3} - \frac{3}{2} = \frac{13}{6}.$$

Subtracting gives $12 - 13/6 = 59/6$, agreeing with the direct calculation. The other three fractions or integers offered do not match this value; using the parabola over the entire interval would ignore the cap at $y = 4$.

Step 4: Conclusion The required area is $59/6$ square units. Hence, the correct option is (A).

Quick Tip to Solve: When two inequalities bound y from above, integrate their minimum and split at their intersection inside the x interval.

20. If $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$, $\beta = \tan^{-1}\left(\frac{1}{3}\right)$ where $0 < \alpha, \beta < \frac{\pi}{2}$, then α - β is equal to : [4 marks]

(A) $\tan^{-1}\left(\frac{9}{14}\right)$

(B) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

(C) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

(D) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

Correct Answer: (B) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

Solution:

Concept:

- For an acute angle, a given cosine fixes a positive sine. Here $\cos\alpha = 3/5$, so $\sin\alpha = 4/5$.
- Use $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$, and check the range before taking an inverse function.

Step 1: Find the sine and cosine values Since $\tan\beta = 1/3$ and β is acute, a right triangle with opposite side 1 and adjacent side 3 has hypotenuse $\sqrt{10}$. Hence

$$\sin\beta = \frac{1}{\sqrt{10}}, \qquad \cos\beta = \frac{3}{\sqrt{10}}, \qquad \tan\alpha = \frac{4}{3}.$$

The tangent is increasing on the acute interval, and $4/3 > 1/3$; therefore $\alpha > \beta$. In particular

$0 < \alpha - \beta < \pi/2$, which removes any inverse-sine ambiguity.

Step 2: Evaluate the difference and eliminate alternatives Substitute into the sine subtraction formula:

$$\sin(\alpha - \beta) = \frac{4}{5}\frac{3}{\sqrt{10}} - \frac{3}{5}\frac{1}{\sqrt{10}} = \frac{9}{5\sqrt{10}}.$$

Thus the acute difference equals the inverse sine in option (B). For a direct check of the tangent alternatives,

$$\tan(\alpha - \beta) = \frac{4/3 - 1/3}{1 + (4/3)(1/3)} = \frac{9}{13}.$$

Neither $9/14$ nor $9/(5\sqrt{10})$ equals $9/13$, excluding (A) and (D). Also $\cos(\alpha - \beta) = 13/(5\sqrt{10})$, not $9/(5\sqrt{10})$, excluding (C).

Step 3: Conclusion $\boxed{\alpha - \beta = \sin^{-1}\left(9/(5\sqrt{10})\right)}$. Hence, the correct option is (B).

Quick Tip to Solve: Compute a sine or tangent of the difference first, then use the acute-angle range to select the correct inverse trigonometric expression.

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21. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in R$, then : [4 marks]

(A) $S_1 = \{-1\}$; $S_2 = \{0, 2\}$

(B) $S_1 = \{-2\}$; $S_2 = \{0, 1\}$

(C) $S_1 = \{-2, 0\}$; $S_2 = \{1\}$

(D) $S_1 = \{-2, 1\}$; $S_2 = \{0\}$

Correct Answer: (D) $S_1 = \{-2, 1\}$; $S_2 = \{0\}$

Solution:

Concept:

- For a differentiable function on the real line, every local extremum must occur at a stationary point where $f\prime(x) = 0$. A derivative sign change determines its type.
- At a stationary point, a positive second derivative proves a strict local minimum and a negative second derivative proves a strict local maximum. The sets here list the x-coordinates of these points.

Step 1: Find every stationary point Differentiate the polynomial and factor the result:

$$\begin{aligned} f\prime(x) &= 36x^3 + 36x^2 - 72x \\ &= 36x(x + 2)(x - 1). \end{aligned}$$

Hence the stationary values of x are $-2, 0, 1$. A polynomial is differentiable everywhere, so there are no additional candidates from a derivative that fails to exist, and the domain has no finite endpoints.

Step 2: Classify the stationary points The second derivative is

$$f''(x) = 108x^2 + 72x - 72.$$

Its values are $f''(-2) = 216 > 0$, $f''(0) = -72 < 0$, and $f''(1) = 108 > 0$. Thus -2 and 1 are local minimum positions, whereas 0 is a local maximum position. None has zero second derivative, so the test is conclusive at all three.

Step 3: Confirm the result and compare the sets The derivative signs on the four successive intervals are negative, positive, negative and positive. Thus it changes from negative to positive at -2 and 1, confirming minima, and from positive to negative at 0, confirming a maximum. Therefore $S_1 = \{-2, 1\}$ and $S_2 = \{0\}$. The first option even includes nonstationary positions; the second and third assign at least one stationary point the wrong type. Only the fourth lists all three with their correct classifications.

Step 4: Conclusion The sets are $S_1 = \{-2, 1\}$ and $S_2 = \{0\}$. Hence, the correct option is (D).

Quick Tip to Solve: Factor the first derivative before classifying extrema; check its sign on each interval to avoid swapping the maxima and minima sets.

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22. If $f(x) = \frac{2-x\cos x}{2+x\cos x}$ and g(x) = $\log_e$x, (x > 0) then the value of integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g(f(x))\,dx$ is [4 marks]

(A) $\log_e 3$

(B) $\log_e 2$

(C) $\log_e 1$

(D) $\log_e e$

Correct Answer: (C) $\log_e 1$

Solution:

Concept:

- On symmetric limits, a continuous odd function has integral zero: $\int_{-a}^{a} h(x)\,dx = 0$ if $h(-x) = -h(x)$.
- The identity $\ln(1/u) = -\ln u$, valid for $u > 0$, converts a reciprocal relation into odd symmetry.

Step 1: Check that the logarithm is defined For $|x| \le \pi/4$, $|x\cos x| \le |x| \le \pi/4 < 2$. Thus both $2 - x\cos x$ and $2 + x\cos x$ are positive, so $f(x) > 0$ throughout the interval. Consequently $h(x) = g(f(x)) = \ln f(x)$ is continuous and its integral exists.

Step 2: Use reflection about the origin Since cosine is even, changing x to $-x$ changes the sign of $x\cos x$. Therefore

$$f(-x) = \frac{2 + x\cos x}{2 - x\cos x} = \frac{1}{f(x)}, \qquad h(-x) = \ln\frac{1}{f(x)} = -h(x).$$

The integrand is odd. In particular, its positive and negative contributions cancel pairwise, even though neither logarithmic term is itself an odd function.

Step 3: Evaluate and compare the options The symmetric limits now give

$$\int_{-\pi/4}^{\pi/4} g(f(x))\,dx = 0 = \ln 1.$$

The other proposed values are $\ln 3 > 0$, $\ln 2 > 0$, and $\ln e = 1$; none equals the cancelling integral.

Step 4: Conclusion The integral is $0 = \log_e 1$. Hence, the correct option is (C).

Quick Tip to Solve: For a logarithm on symmetric limits, check whether the inner function satisfies $f(-x)f(x) = 1$ before attempting integration.

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23. If $2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2$,
x $\in \left(0, \frac{\pi}{2}\right)$ **then** $\frac{dy}{dx}$ **is equal to:** [4 marks]

(A) $2x - \frac{\pi}{3}$
(B) $\frac{\pi}{6} - x$
(C) $\frac{\pi}{3} - x$
(D) $x - \frac{\pi}{6}$

Correct Answer: The keyed choice is (D), but the standard principal inverse-cotangent branch gives a different derivative for $x > \pi/6$, and the expression is undefined at $x = \pi/6$.

Solution:

Concept:

- The addition formula gives $\tan(x + \pi/3) = (\sqrt{3}\cos x + \sin x)/(\cos x - \sqrt{3}\sin x)$.
- For the standard principal inverse cotangent, $0 < \cot^{-1} t < \pi$. Reducing an angle modulo π must respect this range before differentiating.

Step 1: Rewrite the argument as a cotangent Put $u = \pi/6 - x$. The given quotient is

$$\tan(x + \pi/3) = \cot(\pi/6 - x) = \cot u.$$

Because the printed interval is $0 < x < \pi/2$, we have $-\pi/3 < u < \pi/6$. Also, at $x = \pi/6$ the original denominator is zero, so the stated function is not defined there. This point must be excluded from any derivative formula.

Step 2: Apply the principal branch separately For $0 < x < \pi/6$, the angle u is positive and lies in the principal range. For $\pi/6 < x < \pi/2$, it is negative, so add π. Thus

$$\cot^{-1}(\cot u) = \begin{cases} \pi/6 - x, & 0 < x < \pi/6, \\ 7\pi/6 - x, & \pi/6 < x < \pi/2. \end{cases}$$

Writing the applicable angle as v, the equation is $2y = v^2$. Since v has derivative -1 on each interval,

y has derivative $-v$.

Step 3: Differentiate and interpret the listed answer It follows that

$$\frac{dy}{dx} = \begin{cases} x - \pi/6, & 0 < x < \pi/6, \\ x - 7\pi/6, & \pi/6 < x < \pi/2. \end{cases}$$

The keyed expression $x - \pi/6$, option (D), follows by retaining the first branch; it also follows from a signed inverse-cotangent convention. On the standard principal branch, however, it does not hold over the whole printed interval. Options (A), (B), and (C) already fail on the first interval by their slope or constant term.

Step 4: Conclusion Option (D), $x - \pi/6$, is the keyed answer and is valid on $0 < x < \pi/6$. With the standard principal $\cot^{-1}$ range $(0, \pi)$, the full answer is the piecewise derivative above; none of the four options is valid throughout the printed interval.

Quick Tip to Solve: Always reduce the angle into the chosen inverse-trigonometric principal range; differentiating an unreduced angle can miss an entire branch.

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24. The magnitude of the projection of the vector $\hat{2i} + \hat{3j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + \hat{2j} + \hat{3k}$, is : [4 marks]

(A) $\frac{\sqrt{3}}{2}$

(B) $\sqrt{6}$

(C) $\sqrt{\frac{3}{2}}$

(D) $3\sqrt{6}$

Correct Answer: (C) $\sqrt{\frac{3}{2}}$

Solution:

Concept:

- The cross product of two nonparallel vectors in a plane is normal to that plane.
- The magnitude of the scalar projection of $\mathbf{v}$ onto a nonzero vector $\mathbf{n}$ is $|\mathbf{v} \cdot \mathbf{n}|/|\mathbf{n}|$.

Step 1: Construct a normal to the plane Let $\mathbf{a} = (1, 1, 1)$, $\mathbf{b} = (1, 2, 3)$, and $\mathbf{v} = (2, 3, 1)$. A normal is

$$\mathbf{n} = \mathbf{a} \times \mathbf{b} = (3 - 2,\ 1 - 3,\ 2 - 1) = (1, -2, 1).$$

Indeed, $\mathbf{n} \cdot \mathbf{a} = 1 - 2 + 1 = 0$ and $\mathbf{n} \cdot \mathbf{b} = 1 - 4 + 3 = 0$, confirming that the direction is perpendicular to both spanning vectors.

Step 2: Normalize and calculate the projection The normal has length $\sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}$, whereas

$$\mathbf{v} \cdot \mathbf{n} = 2 - 6 + 1 = -3.$$

The signed projection onto this chosen normal is negative; the question asks for its magnitude. Hence

$$p = \frac{|-3|}{\sqrt{6}} = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}.$$

Reversing the normal changes the sign before taking the magnitude, but cannot change this result.

Step 3: Distinguish the other values The dot product alone is not a projection onto a unit direction, so division by $\sqrt{6}$ is essential. The square of the result is $3/2$, while the squares of the other options are $3/4$, 6, and 54. This directly distinguishes the required magnitude from every alternative.

Step 4: Conclusion The magnitude is $\sqrt{3/2}$. Hence, the correct option is (C).

Quick Tip to Solve: Take a cross product for the normal, then divide the absolute dot product by the normal length; never assume a cross product is a unit vector.

25. The shortest distance between the line y = x and the curve $y^2 = x - 2$ is : [4 marks]

(A) $\frac{7}{4\sqrt{2}}$
(B) $\frac{7}{8}$
(C) $\frac{11}{4\sqrt{2}}$
(D) 2

Correct Answer: (A) $\frac{7}{4\sqrt{2}}$

Solution:

Concept:

- The perpendicular distance of (x_0, y_0) from $x - y = 0$ is $|x_0 - y_0|/\sqrt{2}$.
- Every point on $y^2 = x - 2$ can be parametrized as $(t^2 + 2, t)$, where t is real. Completing a square finds the global minimum.

Step 1: Express the distance using one parameter For the point $P = (t^2 + 2, t)$ on the parabola, its closest point on the line is its perpendicular projection. Thus the least distance from this point to the line is

$$d(t) = \frac{|t^2 - t + 2|}{\sqrt{2}}.$$

Minimizing this point-to-line distance over all real t therefore gives the distance between the entire curve and the line; no separate search along the line is required.

Step 2: Complete the square and locate the minimum Write

$$t^2 - t + 2 = \left(t - \frac{1}{2}\right)^2 + \frac{7}{4}.$$

This expression is always positive, so the absolute value can be removed. The square is nonnegative

for every real parameter and vanishes at $t = 1/2$. Consequently

$$d_{\min} = \frac{7}{4\sqrt{2}}, \qquad P_{\min} = \left(\frac{9}{4}, \frac{1}{2}\right).$$

This is a global minimum, rather than merely a stationary value.

Step 3: Check the nearest line point and the choices The perpendicular foot on $y = x$ is $H = (11/8, 11/8)$. The difference vector is $(7/8, -7/8)$, whose length is $7\sqrt{2}/8 = 7/(4\sqrt{2})$. The value $7/8$ is only either coordinate difference, while $11/(4\sqrt{2})$ and 2 exceed the attained minimum.

Step 4: Conclusion The shortest distance is $7/(4\sqrt{2})$. Hence, the correct option is (A).

Quick Tip to Solve: Parametrize the parabola by its free coordinate and minimize the point-to-line distance; completing a square proves global minimality immediately.

26. The sum of the co-efficients of all even degree terms in x in the expansion of $\left(x + \sqrt{x^3 - 1}\right)^6 + \left(x - \sqrt{x^3 - 1}\right)^6$, (x > 1) is equal to: [4 marks]

(A) 32

(B) 26

(C) 29

(D) 24

Correct Answer: (D) 24

Solution:

Concept:

- Adding $(a + b)^6$ and $(a - b)^6$ cancels terms containing odd powers of b. When $b^2 = x^3 - 1$, the result is a polynomial in x.
- For any polynomial $P(x)$, the sum of its even-degree coefficients is $[P(1) + P(-1)]/2$.

Step 1: Eliminate the radical algebraically Let $b = \sqrt{x^3 - 1}$. Pairing the two binomial expansions gives

$$P(x) = 2\left[x^6 + 15x^4(x^3 - 1) + 15x^2(x^3 - 1)^2 + (x^3 - 1)^3\right].$$

All radicals have disappeared, so the right-hand side is a polynomial identity for the expression on its stated real domain $x > 1$. Coefficient sums concern this polynomial and can be evaluated at 1 and -1 without inserting either value into a real square root.

Step 2: Apply the even-coefficient filter At $x = 1$, the factor $x^3 - 1$ is zero, so $P(1) = 2$. At $x = -1$, the even powers x^2, x^4, x^6 all equal one, while $x^3 - 1 = -2$. Hence

$$P(-1) = 2\left[1 - 30 + 60 - 8\right] = 46.$$

It follows that the sum of the even-degree coefficients is

$$\frac{P(1) + P(-1)}{2} = \frac{2 + 46}{2} = 24.$$

Step 3: Check why the other sums do not apply Adding $P(1)$ and $P(-1)$ cancels odd-degree coefficients and doubles even-degree coefficients, so the final division by two is necessary. The obtained value 24 differs from 32, 26, and 29; these choices cannot be sums of the even-degree coefficients of the polynomial above.

Step 4: Conclusion The sum is 24. Hence, the correct option is (D).

Quick Tip to Solve: Cancel paired radicals before using $P(1)$ and $P(-1)$; the coefficient filter applies to the resulting polynomial even outside the original radical domain.

27. The sum of the series

$2.^{20}C_0 + 5.^{20}C_1 + 8.^{20}C_2 + 11.^{20}C_3 + \ldots + 62.^{20}C_{20}$ is equal to : [4 marks]

(A) 2^{25}

(B) 2^{24}

(C) 2^{26}

(D) 2^{23}

Correct Answer: (A) 2^{25}

Solution:

Concept:

- The binomial theorem gives $\sum_{r=0}^{n} \binom{n}{r} = 2^n$.
- Differentiating $(1+x)^n$ and then setting $x = 1$ gives $\sum_{r=0}^{n} r\binom{n}{r} = n2^{n-1}$.

Step 1: Identify the weight of the general term The multipliers $2, 5, 8, 11, \ldots, 62$ form an arithmetic progression with first term 2 and common difference 3. Because the first term multiplies $\binom{20}{0}$, the coefficient paired with $\binom{20}{r}$ is $3r + 2$, rather than $3r - 1$. For $r = 20$, this gives 62, agreeing with the final displayed term.

Step 2: Separate the two standard sums Writing the required series as S, linearity gives

$$S = \sum_{r=0}^{20} (3r+2)\binom{20}{r} = 3\sum_{r=0}^{20} r\binom{20}{r} + 2\sum_{r=0}^{20} \binom{20}{r}.$$

For completeness, differentiating $(1+x)^{20} = \sum_{r=0}^{20} \binom{20}{r} x^r$ at $x = 1$ gives the weighted sum $20 \cdot 2^{19}$. The unweighted sum is 2^{20}.

Step 3: Simplify the power of two Substituting these values,

$$\begin{aligned} S &= 3(20 \cdot 2^{19}) + 2(2^{20}) \\ &= 60 \cdot 2^{19} + 4 \cdot 2^{19} \\ &= 64 \cdot 2^{19} = 2^{25}. \end{aligned}$$

Thus 2^{24} is half the sum, 2^{26} is twice it, and 2^{23} is one quarter of it. None of those alternatives equals the series.

Step 4: Conclusion The series sums to 2^{25}. Hence, the correct option is (A).

Quick Tip to Solve: For an arithmetic multiplier of $\binom{n}{r}$, write it as $ar + b$, then use only the two sums $\sum r\binom{n}{r}$ and $\sum \binom{n}{r}$.

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28. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is : [4 marks]

(A) If you are not a citizen of India, then you are not born in India.

(B) If you are born in India, then you are not a citizen of India.

(C) If you are a citizen of India, then you are born in India.

(D) If you are not born in India, then you are not a citizen of India

Correct Answer: (A) If you are not a citizen of India, then you are not born in India.

Solution:

Concept:

- For statements p and q, the contrapositive of $p \Rightarrow q$ is $\neg q \Rightarrow \neg p$. It reverses the implication and negates both component statements.
- An implication and its contrapositive are logically equivalent. The converse $q \Rightarrow p$ and inverse $\neg p \Rightarrow \neg q$ are different constructions.

Step 1: Name the hypothesis and the conclusion Let p mean "you are born in India" and q mean "you are a citizen of India". The given conditional statement has the form $p \Rightarrow q$. This question asks only for a logical transformation of that statement; deciding whether a real citizenship rule has exceptions is unnecessary for forming its contrapositive.

Step 2: Reverse and negate both statements The negation of q is "you are not a citizen of India", while the negation of p is "you are not born in India". Therefore $\neg q \Rightarrow \neg p$ reads: "If you are not a citizen of India, then you are not born in India." This is exactly the wording of option (A). The transformation uses negation of the entire conclusion as the new hypothesis.

Step 3: Distinguish the distractors Option (B) has the form $p \Rightarrow \neg q$, so it merely negates the conclusion and does not reverse the implication. Option (C) is $q \Rightarrow p$, the converse. Option (D) is $\neg p \Rightarrow \neg q$, the inverse. Only reversing the order and negating both sides gives the contrapositive required here.

Step 4: Conclusion The contrapositive is "If you are not a citizen of India, then you are not born in India." Hence, the correct option is (A).

Quick Tip to Solve: For a contrapositive, perform both operations: swap hypothesis and conclusion, then negate each one. Doing only one operation gives a distractor.

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29. The equation of a plane containing the line of intersection of the planes 2x – y – 4 = 0 and y + 2z – 4 = 0 and passing through the point (1, 1, 0) is : [4 marks]

(A) x – 3y – 2z = –2

(B) 2x – z = 2

(C) x – y – z = 0

(D) x + 3y + z = 4

Correct Answer: (C) x – y – z = 0

Solution:

Concept:

- Every plane through the intersection of two distinct planes $P = 0$ and $Q = 0$ can be represented by $P + \lambda Q = 0$, together with the limiting member $Q = 0$.
- The extra point determines the parameter. Multiplying an entire plane equation by a nonzero constant does not change its plane.

Step 1: Form the family containing the intersection Put $P = 2x - y - 4$ and $Q = y + 2z - 4$. Every point on the given intersection has both $P = 0$ and $Q = 0$, so it satisfies

$$(2x - y - 4) + \lambda(y + 2z - 4) = 0.$$

The required point does not lie on $Q = 0$, because $1 + 2(0) - 4 = -3$. Thus the desired plane is a finite member of this family.

Step 2: Use the point and simplify Substitute $(x, y, z) = (1, 1, 0)$ into the family:

$$(2 - 1 - 4) + \lambda(1 + 0 - 4) = 0 \quad \Rightarrow \quad -3 - 3\lambda = 0 \quad \Rightarrow \quad \lambda = -1.$$

Therefore the equation becomes

$$2x - y - 4 - (y + 2z - 4) = 0 \quad \Rightarrow \quad 2x - 2y - 2z = 0.$$

Dividing by 2 gives $x - y - z = 0$.

Step 3: Check the offered planes The result contains $(1, 1, 0)$. It also contains the entire intersection since it is $(P - Q)/2 = 0$. To exclude the other choices, take $(3, 2, 1)$, which satisfies both original planes. Option (A) has left side $3 - 6 - 2 = -5$, rather than -2; option (B) gives $6 - 1 = 5$, rather than 2; option (D) gives $3 + 6 + 1 = 10$, rather than 4. Hence those planes do not contain the intersection line.

Step 4: Conclusion The required plane is $\boxed{x - y - z = 0}$. Hence, the correct option is (C).

Quick Tip to Solve: For a plane through an intersection line, use a linear combination of the two plane equations; one point then fixes its single parameter.

30. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, then a^2 is equal to : [4 marks]

(A) $\frac{2}{17}$
(B) $\frac{64}{17}$
(C) $\frac{128}{17}$
(D) $\frac{4}{17}$

Correct Answer: (A) $\frac{2}{17}$

Solution:

Concept:

- For the ellipse $4x^2 + y^2 = 8$, implicit differentiation gives the tangent slope $dy/dx = -4x/y$ wherever $y \neq 0$.
- Two finite slopes m_1, m_2 represent perpendicular lines when $m_1 m_2 = -1$. The point of tangency must also lie on the ellipse.

Step 1: Find the slope of the first tangent Differentiate the ellipse equation with respect to x:

$$8x + 2y\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{4x}{y}.$$

At $(1, 2)$, this is $m_1 = -4/2 = -2$. Thus the tangent perpendicular to it has slope $m_2 = 1/2$, since $(-2)(1/2) = -1$.

Step 2: Determine the relation between the second coordinates At the second point (a, b), the slope condition is

$$-\frac{4a}{b} = \frac{1}{2} \quad \Rightarrow \quad b = -8a.$$

Neither a vertical nor a horizontal tangent can be perpendicular to a tangent of slope -2, so using the finite-slope formula here loses no valid case. Notice that the slope changes sign; keeping the same slope would describe parallel tangents instead.

Step 3: Use the ellipse equation and compare choices Since (a, b) is a point on the ellipse,

$$4a^2 + b^2 = 8 \quad \Rightarrow \quad 4a^2 + 64a^2 = 8 \quad \Rightarrow \quad a^2 = \frac{8}{68} = \frac{2}{17}.$$

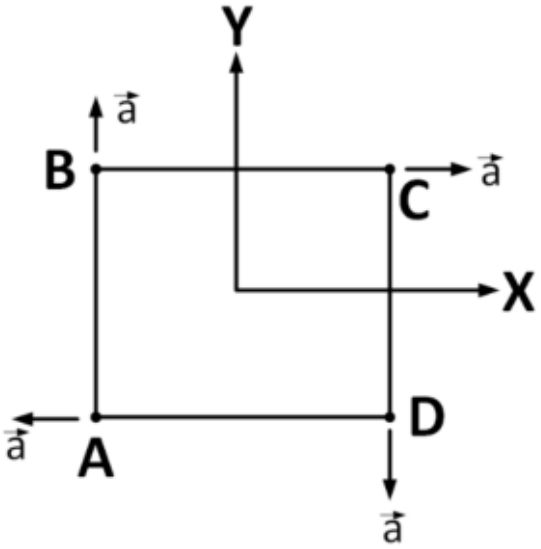
Both opposite points corresponding to the two signs of a produce this same square. The values $64/17$ and $128/17$ even exceed the largest possible value $a^2 = 2$ on this ellipse. The remaining choice $4/17$ would give $4a^2 + b^2 = 68(4/17) = 16$ under the required relation, rather than 8.

Step 4: Conclusion Therefore $\boxed{a^2 = 2/17}$. Hence, the correct option is (A).

Quick Tip to Solve: Apply the perpendicular-slope condition before substituting the second point into the ellipse; this reduces the problem to one squared coordinate.

Physics (120 marks)

31. Four particles A, B, C and D with masses m_A = m, m_B = 2m, m_C = 3m and m_D = 4m are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is : [4 marks]



(A) $\frac{a}{5}\left(\hat{i}+\hat{j}\right)$

(B) $\frac{a}{5}\left(\hat{i}-\hat{j}\right)$

(C) $a\left(\hat{i}+\hat{j}\right)$

(D) Zero

Correct Answer: (B) $\frac{a}{5}\left(\hat{i}-\hat{j}\right)$

Solution:

Concept:

- For particles of fixed masses, the acceleration of the centre of mass is the mass-weighted vector average: $\boldsymbol{a}_{\mathrm{CM}} = \sum_i m_i \boldsymbol{a}_i / \sum_i m_i$.
- Equal acceleration magnitudes need not cancel: their directions and the masses multiplying them must both be included. Take rightward as $+\hat{\boldsymbol{i}}$ and upward as $+\hat{\boldsymbol{j}}$.

Step 1: Read the four vector directions The diagram gives leftward acceleration for A, upward acceleration for B, rightward acceleration for C and downward acceleration for D. Their vectors are therefore

$$\boldsymbol{a}_A = -a\hat{\boldsymbol{i}}, \quad \boldsymbol{a}_B = a\hat{\boldsymbol{j}},$$
$$\boldsymbol{a}_C = a\hat{\boldsymbol{i}}, \quad \boldsymbol{a}_D = -a\hat{\boldsymbol{j}}.$$

The square's side length is unnecessary because the required quantity depends on these accelerations, rather than the particles' positions.

Step 2: Take the mass-weighted sum Using the masses $m, 2m, 3m, 4m$, the total mass is $10m$. The numerator of the centre-of-mass acceleration is

$$\sum m_i \boldsymbol{a}_i = -ma\hat{\boldsymbol{i}} + 2ma\hat{\boldsymbol{j}} + 3ma\hat{\boldsymbol{i}} - 4ma\hat{\boldsymbol{j}}$$
$$= 2ma\hat{\boldsymbol{i}} - 2ma\hat{\boldsymbol{j}}.$$

After dividing by the total mass,

$$\boldsymbol{a}_{\text{CM}} = \frac{a}{5}(\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}}).$$

Step 3: Check the direction and alternatives The heavier rightward particle C wins over A, so the horizontal component must be positive. The heavier downward particle D wins over B, so the vertical component must be negative. Options (A) and (C) have a positive vertical component and are excluded. Option (D) would require cancellation of each mass-weighted pair, which does not occur. The common scale m cancels, as it must for a mass-weighted average with fixed mass ratios.

Step 4: Conclusion The acceleration is $\boxed{\boldsymbol{a}_{\text{CM}} = \frac{a}{5}(\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}})}$. Hence, the correct option is (B).

Quick Tip to Solve: First combine the opposite horizontal pair and the opposite vertical pair with their masses; divide each resulting component by the same total mass.

32. A thermally insulated vessel contains 150g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to : (Latent heat of vaporization of water = 2.10 $\times$ 10^6 J kg^{-1} and Latent heat of Fusion of water = 3.36 $\times$ 10^5 J kg^{-1}) [4 marks]

(A) 35 g

(B) 130 g

(C) 20 g

(D) 150 g

Correct Answer: (C) 20 g

Solution:

Concept:

- With no heat supplied from outside and no temperature change, the latent heat consumed by evaporation is supplied by the latent heat released during freezing.
- For mass m evaporated from an initial mass M, the remaining frozen mass is $M - m$, so $mL_v = (M - m)L_f$. Both masses must use the same units.

Step 1: Identify the two complementary masses The initial water mass is $M = 150\,\text{g} = 0.150\,\text{kg}$. Let m kilograms evaporate. Since the rest becomes ice at the same $0°\text{C}$, the frozen mass is $0.150 - m$ kilograms. No sensible-heat term involving specific heat is needed: the stated process has no temperature change. Evaporation requires energy, whereas freezing releases energy.

Step 2: Balance the latent heats For the insulated process, equate the heat required and released:

$$m(2.10 \times 10^6) = (0.150 - m)(3.36 \times 10^5).$$

Rearrangement gives

$$m(2.10 \times 10^6 + 3.36 \times 10^5) = 0.150(3.36 \times 10^5),$$
$$m = \frac{50400}{2436000}\,\text{kg} = 0.02069\,\text{kg}.$$

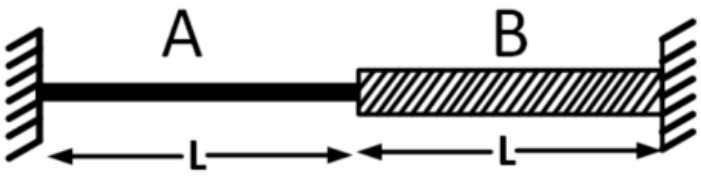
Thus about $20.69\,\text{g}$ evaporates, leaving about $129.31\,\text{g}$ of ice.

Step 3: Select the closest mass The closest offered value to $20.69\,\text{g}$ is $20\,\text{g}$. The value $130\,\text{g}$ describes approximately the frozen remainder rather than the evaporated portion. Neither $35\,\text{g}$ nor $150\,\text{g}$ satisfies the heat balance. In particular, evaporation of all the water would leave no freezing water to supply the latent heat. The small evaporated fraction is physically reasonable because L_v is much larger than L_f.

Step 4: Conclusion The evaporated mass is approximately $20.7\,\text{g}$, closest to $\boxed{20\,\text{g}}$. Hence, the correct option is (C).

Quick Tip to Solve: For evaporation powered by simultaneous freezing at the same temperature, use the evaporated fraction $m/M = L_f/(L_v + L_f)$, not L_f/L_v.

33. A wire of length 2L, is made by joining two wires A and B of same length but different radii r and 2r and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio p : q is : [4 marks]



(A) 3 : 5

(B) 4 : 9

(C) 1 : 2

(D) 1 : 4

Correct Answer: (C) 1 : 2

Solution:

Concept:

- For a stretched wire under tension T, wave speed is $v = \sqrt{T/\mu}$, where μ is mass per unit length. The connected wires carry the same tension.

- For a segment of length L with nodes at both ends and n antinodes, $L = n\lambda/2$, so its frequency is $f = nv/(2L)$.

Step 1: Compare the linear mass densities and speeds The wires have the same material density, but their radii are r and $2r$. Thus

$$\mu_A = \rho\pi r^2, \qquad \mu_B = \rho\pi(2r)^2 = 4\mu_A.$$

Since the tension is common,

$$\frac{v_A}{v_B} = \sqrt{\frac{\mu_B}{\mu_A}} = 2.$$

The thinner wire therefore transmits waves twice as fast; doubling the radius quadruples the mass per unit length, rather than merely doubling it.

Step 2: Use the node at the junction Each outer end is fixed, and the junction is given to be a node. Hence each length-L segment has nodes at both its ends. If A contains p antinodes and B contains q antinodes, then

$$f = \frac{pv_A}{2L} = \frac{qv_B}{2L}.$$

The wires oscillate together at the same frequency. Cancelling the equal lengths gives

$$\frac{p}{q} = \frac{v_B}{v_A} = \frac{1}{2}.$$

Step 3: Check the other ratios The slower, thicker segment has the shorter wavelength at the common frequency and therefore contains twice as many half-waves in the same length. This confirms $q = 2p$, rather than the reverse. Ratios $3 : 5$ and $4 : 9$ do not satisfy $pv_A = qv_B$ with $v_A = 2v_B$. The ratio $1 : 4$ confuses the factor of four in linear density with the factor of two in wave speed. The ratio specifies mode counts; it does not require that the individual counts be exactly 1 and 2.

Step 4: Conclusion The antinode ratio is $\boxed{p : q = 1 : 2}$. Hence, the correct option is (C).

Quick Tip to Solve: At a fixed common frequency, the equal-length wire with the lower wave speed has more antinodes: the antinode count is inversely proportional to speed.

34. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is : [4 marks]

(A) $\frac{1}{2}$
(B) $\frac{3}{2}$
(C) $\frac{8}{5}$
(D) $\frac{3}{5}$

Correct Answer: (C) $\frac{8}{5}$

Solution:

Concept:

- A thin annular element of radius r and width dr has area $2\pi r\,dr$; its mass must include the given radial density $\rho(r) = \rho_0 r$.
- The perpendicular central-axis moment is $I_C = \int r^2\,dm$. The parallel-axis theorem gives $I_{\text{edge}} = I_C + MR^2$ for a parallel axis through the edge.

Step 1: Express the mass of an annulus Use concentric annuli, since all points on one annulus have the same distance from the central axis. The varying density yields

$$dm = (\rho_0 r)(2\pi r\,dr) = 2\pi\rho_0 r^2\,dr.$$

Thus the total mass is

$$M = 2\pi\rho_0 \int_0^R r^2\,dr = \frac{2\pi\rho_0 R^3}{3}.$$

If the stated density is interpreted as volume density for a plate of uniform thickness, the same thickness factor multiplies both integrals and cancels from their ratio.

Step 2: Calculate the central moment and shift the axis The moment about the perpendicular axis through the centre is

$$I_C = 2\pi\rho_0 \int_0^R r^4\,dr = \frac{2\pi\rho_0 R^5}{5} = \frac{3}{5}MR^2.$$

Radial symmetry puts the centre of mass at the geometric centre. The requested axis through the edge is parallel to this central axis and a distance R away. Therefore

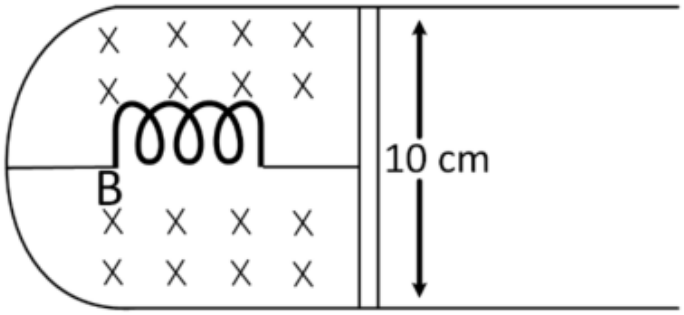
$$I_{\text{edge}} = I_C + MR^2 = \left(\frac{3}{5} + 1\right) MR^2 = \frac{8}{5}MR^2.$$

Step 3: Identify the coefficient and reject the distractors Comparison with $I = aMR^2$ gives $a = 8/5$. The value $3/5$ is the coefficient for the central axis before the required shift. The values $1/2$ and $3/2$ are familiar central and edge-axis coefficients of a uniform disc, respectively, but this disc's density grows outward. Its relatively greater mass near the rim increases the central coefficient from $1/2$ to $3/5$, consistently increasing the shifted coefficient as well.

Step 4: Conclusion The required coefficient is $\boxed{a = 8/5}$. Hence, the correct option is (C).

Quick Tip to Solve: For a radially varying plate, determine $I_C/(MR^2)$ by dividing the two annular integrals, then add 1 for the parallel axis at the edge.

35. A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1} (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillation it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10W and air drag negligible, N will be close to : [4 marks]



(A) 50000

(B) 1000

(C) 5000

(D) 10000

Correct Answer: (C) 5000

Solution:

Concept:

- A conducting strip of length ℓ moving at speed v across field B has motional emf $\mathcal{E} = B\ell v$. Its induced current produces a force opposing the motion.
- For circuit resistance R_e, the retarding force is $-cv$, where $c = B^2\ell^2/R_e$. In an underdamped oscillator the amplitude envelope is $A(t) = A_0 e^{-ct/(2m)}$.

Step 1: Find the magnetic damping coefficient Here $\ell = 0.10\,\text{m}$, $B = 0.10\,\text{T}$, $m = 0.050\,\text{kg}$, $k = 0.50\,\text{N m}^{-1}$, and the strip resistance is $R_e = 10\,\Omega$. The U-shaped wire has negligible resistance. From $i = B\ell v/R_e$, the magnetic force magnitude is $Bi\ell = B^2\ell^2 v/R_e$, so

$$c = \frac{(0.10)^2(0.10)^2}{10} = 1.0 \times 10^{-5}\,\text{kg s}^{-1}.$$

Step 2: Calculate the amplitude decay time and period The equation of motion is $m\ddot{x} + c\dot{x} + kx = 0$. The amplitude falls by a factor of e when $ct/(2m) = 1$, giving

$$t_e = \frac{2m}{c} = \frac{0.100}{10^{-5}} = 10^4\,\text{s}.$$

The damped angular frequency is

$$\omega_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} = \sqrt{10 - 10^{-8}}\,\text{s}^{-1} \simeq \sqrt{10}\,\text{s}^{-1}.$$

Damping is extremely weak here, so $T_d = 2\pi/\omega_d \simeq 1.987\,\text{s}$.

Step 3: Count the oscillations and compare the choices The number of cycles in the amplitude decay time is

$$N = \frac{t_e}{T_d} \simeq \frac{10^4\sqrt{10}}{2\pi} \simeq 5.03 \times 10^3.$$

Thus 5000 is the closest offered number. The values 1000, 10000 and 50000 differ substantially from the calculated cycle count. The factor of 2 in $t_e = 2m/c$ is essential: energy decays twice as rapidly in its exponent because it is proportional to amplitude squared. The question asks for the amplitude envelope, not an energy decay time.

Step 4: Conclusion The strip performs approximately 5.03×10^3 oscillations, closest to $\boxed{5000}$. Hence, the correct option is (C).

Quick Tip to Solve: For velocity-proportional magnetic damping, first obtain $c = B^2\ell^2/R_e$; divide the amplitude time $2m/c$ by the mechanical period.

36. Two particles move at right angle to each other. Their de-Broglie wavelengths are λ_1 and λ_2 respectively. The particles suffer perfectly inelastic collision. The de-Broglie wavelength λ_2 of the final particle, is given by : [4 marks]

(A) $\lambda = \frac{\lambda_1+\lambda_2}{2}$

(B) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$

(C) $\lambda = \sqrt{\lambda_1\lambda_2}$

(D) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

Correct Answer: (B) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$

Solution:

Concept:

- The de Broglie relation is $p = h/\lambda$, where p is the magnitude of momentum and h is Planck's constant. Wavelength is therefore inversely proportional to momentum, rather than to speed alone.
- Total vector momentum is conserved during the collision. A perfectly inelastic collision joins the particles, but it does not imply conservation of kinetic energy. Perpendicular incoming momenta combine by the Pythagorean rule.

Step 1: Express the two initial momenta Let the final composite particle have wavelength λ. The two initial momentum magnitudes are

$$p_1 = \frac{h}{\lambda_1}, \qquad p_2 = \frac{h}{\lambda_2}.$$

The stated right angle between velocities is also the angle between momenta, since multiplying a velocity by its positive mass does not change its direction. Choose perpendicular coordinate axes along these momenta.

Step 2: Conserve momentum and use the right angle The final momentum is $\boldsymbol{p} = \boldsymbol{p}_1 + \boldsymbol{p}_2$. Its

magnitude obeys

$$p^2 = p_1^2 + p_2^2.$$

There is no cross term because the scalar product of perpendicular vectors is zero. Substituting the three de Broglie expressions gives

$$\frac{h^2}{\lambda^2} = \frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}.$$

Cancel the common nonzero factor h^2. No mass or collision energy calculation is needed.

Step 3: Check the proposed wavelength rules The final momentum exceeds either incoming magnitude, so the final wavelength must be smaller than either initial wavelength. The arithmetic mean in (A), geometric mean in (C), and harmonic mean in (D) each lie between the two initial wavelengths and therefore cannot describe this perpendicular momentum sum. For example, equal initial wavelengths give $\lambda = \lambda_1/\sqrt{2}$, whereas those three means would all give λ_1.

Step 4: Conclusion The required relation is

$$\boxed{\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}}.$$

Hence, the correct option is (B).

Quick Tip to Solve: For perpendicular momenta, add the squares of reciprocal de Broglie wavelengths. Do not average the wavelengths or conserve kinetic energy in a sticking collision.

37. The bob of a simple pendulum has mass 2g and a charge of 5.0 μC. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is : (take g = 10 m/s^2) [4 marks]

(A) $\tan^{-1}(5.0)$

(B) $\tan^{-1}(2.0)$

(C) $\tan^{-1}(0.5)$

(D) $\tan^{-1}(0.2)$

Correct Answer: (C) $\tan^{-1}(0.5)$

Solution:

Concept:

- A charge q in a uniform electric field E experiences an electric force of magnitude qE. Here that force is horizontal, while the weight mg is vertically downward.
- At equilibrium, the string tension balances both perpendicular forces. Resolving tension at an angle θ to the vertical gives $T\sin\theta = qE$ and $T\cos\theta = mg$.

Step 1: Convert the mass and charge to SI units The mass must be converted from grams before using the gravitational acceleration. The given quantities are

$$m = 2 \times 10^{-3}\,\text{kg}, \qquad q = 5.0 \times 10^{-6}\,\text{C}.$$

Also $E = 2000\,\mathrm{V\,m^{-1}} = 2000\,\mathrm{N\,C^{-1}}$ and $g = 10\,\mathrm{m\,s^{-2}}$. The electric force points horizontally along the field because the bob carries a positive charge.

Step 2: Calculate the two forces Their magnitudes are

$$F_e = qE = (5.0 \times 10^{-6})(2000) = 0.010\,\mathrm{N},$$

$$W = mg = (2 \times 10^{-3})(10) = 0.020\,\mathrm{N}.$$

The weight is twice the electric force. These forces are balanced by different components of the same tension, rather than being set equal directly.

Step 3: Eliminate the unknown tension Divide the horizontal equilibrium equation by the vertical one:

$$\tan\theta = \frac{T\sin\theta}{T\cos\theta} = \frac{qE}{mg} = \frac{0.010}{0.020} = 0.5.$$

Thus the angle is approximately 26.6°, measured from the vertical towards the field. The reciprocal ratio 2.0 in (B) would give the complementary angle measured from the horizontal. The ratios 5.0 and 0.2 in (A) and (D) do not follow from the SI force values. Since the electric force is less than the weight, an angle below 45° also checks the result.

Step 4: Conclusion The equilibrium angle with the vertical is

$$\boxed{\theta = \tan^{-1}(0.5)}.$$

Hence, the correct option is (C).

Quick Tip to Solve: When the angle is measured from the vertical, use horizontal force divided by weight for its tangent. Convert both grams and microcoulombs before substituting.

38. If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of : [4 marks]

(A) 10^8 N/m^2

(B) 10^{16} N/m^2

(C) 10^4 N/m^2

(D) 2 N/m^2

Correct Answer: (D) 2 N/m^2

Solution:

Concept:

- For a normal elastic collision with a stationary wall, the component of molecular velocity perpendicular to the wall reverses. The magnitude of the momentum transferred to the wall by one molecule is $2mv$.
- Force is the rate of momentum transfer, and pressure is force divided by the collision area: $P = F/A$.

The stated number of collisions per second is already a rate, so it must not be multiplied by a further speed factor.

Step 1: Find the impulse from one collision Take the incident direction towards the surface as positive. Before collision the molecular momentum is $+mv$; after an elastic normal reflection it is $-mv$. Therefore the molecule changes momentum by $-2mv$, and the wall receives an equal and opposite impulse of magnitude

$$J = 2mv = 2(10^{-26})(10^{4}) = 2 \times 10^{-22}\,\mathrm{kg\,m\,s^{-1}}.$$

The factor of two is essential because the molecule reverses its direction instead of merely stopping.

Step 2: Use the collision rate to obtain force The number arriving per second is $\dot{N} = 10^{22}\,\mathrm{s^{-1}}$. Consequently the total momentum delivered each second is

$$F = \dot{N}J = (10^{22})(2 \times 10^{-22}) = 2\,\mathrm{N}.$$

The powers of ten cancel. All the collisions are perpendicular according to the question, so no directional average or factor of one-third is appropriate here.

Step 3: Divide by the area and compare the choices The exposed area is $A = 1\,\mathrm{m^2}$, giving

$$P = \frac{F}{A} = \frac{2}{1} = 2\,\mathrm{N\,m^{-2}}.$$

This is of order unity and matches (D). Values 10^{8}, 10^{16}, and $10^{4}\,\mathrm{N\,m^{-2}}$ in (A), (B), and (C) are incompatible with the calculated impulse rate. In particular, using an extra factor of speed would incorrectly count the collision rate twice: the question supplies the number striking the wall each second, rather than a molecular number density.

Step 4: Conclusion The pressure exerted on the surface is

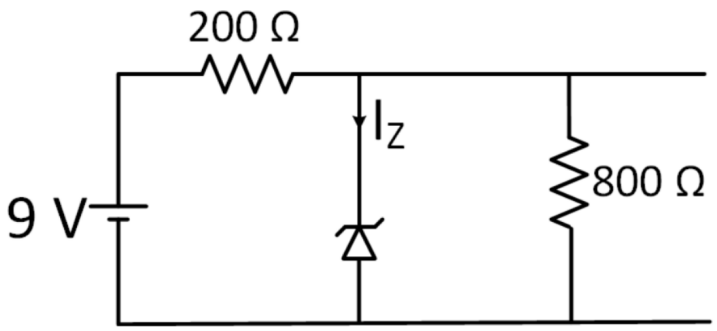
$$\boxed{P = 2\,\mathrm{N\,m^{-2}} = 2\,\mathrm{Pa}}.$$

Hence, the correct option is (D).

Quick Tip to Solve: If a problem supplies the actual collision rate, multiply it by the impulse $2mv$, then divide by area. Use $\frac{1}{3}nmv^2$ only for an isotropic gas described by number density.

39. The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.

The current IZ through the Zener is : [4 marks]



(A) 7 mA

(B) 17 mA

(C) 15mA

(D) 10 mA

Correct Answer: (D) 10 mA

Solution:

Concept:

- A reverse-biased Zener diode in breakdown holds its terminal voltage approximately at V_Z. Because the load resistor is parallel to the diode, it has the same voltage across it.
- The series resistor carries the total supply current, which splits between the load and the Zener. Kirchhoff's current law gives $I_S = I_L + I_Z$, so the diode current is found by subtraction.

Step 1: Check that breakdown is possible The circuit contains a $9\,\mathrm{V}$ source, a $200\,\Omega$ series resistor, and an $800\,\Omega$ load parallel to the Zener. Without the Zener conducting, the divider would place

$$V_L = 9\frac{800}{200+800} = 7.2\,\mathrm{V}$$

across the load. This exceeds the specified breakdown voltage $5.6\,\mathrm{V}$, so the reverse-biased diode does enter breakdown and clamps the output at that value.

Step 2: Calculate the supply current through the series resistor The potential drop across the $200\,\Omega$ resistor is the difference between the source and regulated output voltages:

$$V_R = 9 - 5.6 = 3.4\,\mathrm{V}.$$

Hence its current is

$$I_S = \frac{3.4}{200} = 0.017\,\mathrm{A} = 17\,\mathrm{mA}.$$

This is the total current before the junction. It is not entirely the Zener current, because the load also draws current.

Step 3: Subtract the current required by the load The parallel $800\,\Omega$ load draws

$$I_L = \frac{5.6}{800} = 0.007\,\text{A} = 7\,\text{mA}.$$

Applying current conservation at the top junction gives

$$I_Z = I_S - I_L = 17 - 7 = 10\,\text{mA}.$$

The positive result is consistent with the breakdown assumption. Option (A) is only the load current, while (B) is the total series-resistor current. The value $15\,\text{mA}$ in (C) satisfies neither the voltage drops nor the junction equation for these resistor values.

Step 4: Conclusion The current through the Zener in the indicated direction is

$$\boxed{I_Z = 10\,\text{mA}}.$$

Hence, the correct option is (D).

Quick Tip to Solve: For a Zener regulator, calculate the current through the series resistor first and subtract the parallel load current. Verify the result is positive so that breakdown can be sustained.

40. An alternating voltage v(t) = 220 sin 100 πt volt is applied to a purely resistance load of 50Ω . The time taken for the current to rise from half of the peak value to the peak value is : [4 marks]

(A) 5 ms

(B) 2.2 ms

(C) 3.3 ms

(D) 7.2 ms

Correct Answer: (C) 3.3 ms

Solution:

Concept:

- For a purely resistive load, voltage and current are in phase: $i(t) = v(t)/R = I_0 \sin \omega t$. Here $I_0 = 220/50\,\text{A}$ and $\omega = 100\pi\,\text{rad}\,\text{s}^{-1}$.
- The required time is the difference between two phase instants on the rising part of the sinusoid. Half the peak current means half the sine amplitude, not half the time required to reach the peak.

Step 1: Write the current as a sinusoidal function Ohm's law at every instant gives

$$i(t) = \frac{220}{50}\sin(100\pi t) = 4.4\sin(100\pi t)\,\text{A}.$$

Thus the peak current is $I_0 = 4.4\,\text{A}$, and its half-peak value is $2.2\,\text{A}$. Dividing the desired current values by I_0 eliminates the amplitude; the timing depends on the frequency alone.

Step 2: Find the two phases on the rising branch When the current first rises to half its positive peak,

$$\sin(100\pi t_1) = \frac{1}{2}, \qquad 100\pi t_1 = \frac{\pi}{6}.$$

At the positive peak,

$$\sin(100\pi t_2) = 1, \qquad 100\pi t_2 = \frac{\pi}{2}.$$

The phase $5\pi/6$ also has sine one-half, but it belongs to the falling branch and cannot be used for a current rising towards the peak.

Step 3: Convert the phase difference to a time interval The phase change is $\Delta\phi = \pi/2 - \pi/6 = \pi/3$. Therefore

$$\Delta t = \frac{\Delta\phi}{\omega} = \frac{\pi/3}{100\pi} = \frac{1}{300}\,\text{s} = 3.33\,\text{ms}.$$

The $5\,\text{ms}$ in (A) is the entire zero-to-peak quarter-period, not the interval beginning at half amplitude. The alternatives $2.2\,\text{ms}$ and $7.2\,\text{ms}$ in (B) and (D) do not correspond to the required $\pi/3$ phase advance at this angular frequency.

Step 4: Conclusion The time needed for the current to rise from half its peak to its peak is

$$\boxed{\Delta t \approx 3.3\,\text{ms}}.$$

Hence, the correct option is (C).

Quick Tip to Solve: Convert each specified fraction of the peak current into a sine phase, choose the rising branch, and divide the phase difference by angular frequency. Half amplitude is reached at $30°$, not $45°$.

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41. In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be : [4 marks]

(A) 2

(B) 4

(C) 18

(D) 9

Correct Answer: (B) 4

Solution:

Concept:

- For coherent waves with amplitudes a_1 and a_2, constructive interference gives resultant amplitude $a_1 + a_2$, while destructive interference gives magnitude $|a_2 - a_1|$.
- Intensity is proportional to the square of the resultant amplitude. Thus the fringe-intensity ratio is $I_{\max}/I_{\min} = (a_1 + a_2)^2/(a_2 - a_1)^2$, with the common proportionality factor cancelling.

Step 1: Express both amplitudes using one scale The given amplitude ratio is $a_1/a_2 = 1/3$. Write

$$a_1 = a, \qquad a_2 = 3a,$$

where a is a positive amplitude scale. This substitution preserves the ratio without requiring either amplitude to be known individually. The two waves are coherent, so a stable relative phase allows both maxima and minima to be formed.

Step 2: Calculate the two resultant intensities At a bright fringe the waves are in phase. Their amplitudes add, giving $a + 3a = 4a$. At a dark fringe their phase difference is π, so the resultant amplitude has magnitude $3a - a = 2a$. If K is the common intensity constant, then

$$I_{\max} = K(4a)^2 = 16Ka^2,$$

$$I_{\min} = K(2a)^2 = 4Ka^2.$$

The minimum is nonzero because the amplitudes are unequal; complete destructive cancellation would require equal amplitudes.

Step 3: Take the ratio and reject amplitude-only comparisons Dividing the expressions cancels both K and a^2:

$$\frac{I_{\max}}{I_{\min}} = \frac{16Ka^2}{4Ka^2} = 4.$$

Option (A), 2, is the ratio of the two resultant amplitudes rather than their intensities. Option (D), 9, is the ratio of the separate incident intensities, a_2^2/a_1^2, which is a different quantity. Option (C), 18, agrees with neither the constructive nor the destructive superposition result. Squaring the amplitude ratio only after forming the sum and difference avoids these errors.

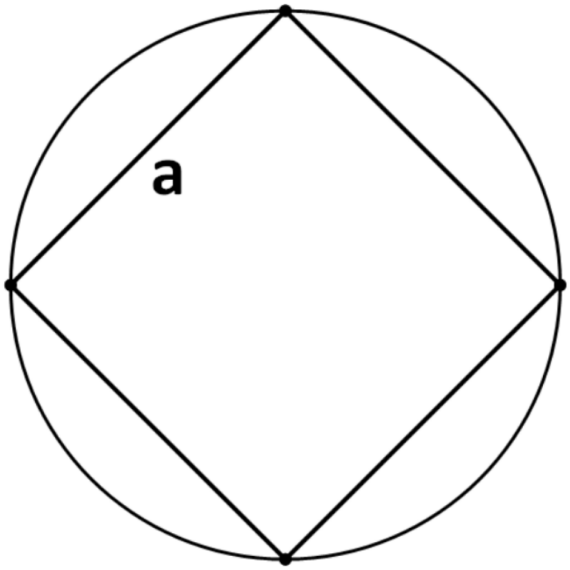
Step 4: Conclusion The maximum-to-minimum fringe intensity ratio is

$$\boxed{I_{\max} : I_{\min} = 4 : 1}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Use $(a_1 + a_2)^2/(a_1 - a_2)^2$ for fringe contrast. When amplitudes are in ratio $1 : 3$, first form resultant amplitudes $4 : 2$, then square the ratio.

42. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of other's gravitational field in a circular orbit circumscribing the square? [4 marks]



(A) $1.21\sqrt{\frac{GM}{a}}$

(B) $1.16\sqrt{\frac{GM}{a}}$

(C) $1.41\sqrt{\frac{GM}{a}}$

(D) $1.35\sqrt{\frac{GM}{a}}$

Correct Answer: (B) $1.16\sqrt{\frac{GM}{a}}$

Solution:

Concept:

- Each mass moves on the circumcircle of the square, whose radius is half its diagonal: $r = a/\sqrt{2}$. The required centripetal force on one mass is Mv^2/r.
- Gravitational forces add as vectors. The forces from the two adjacent corners have equal magnitudes and perpendicular directions, so their resultant lies along the diagonal towards the square's centre. The force from the opposite corner has the same inward direction.

Step 1: Resolve the gravitational attraction of the neighbours Consider one corner mass. Each adjacent mass is at distance a and exerts force

$$F_s = \frac{GM^2}{a^2}.$$

The two forces make an angle of $90°$. Their sum therefore has magnitude

$$F_{\text{adj}} = \sqrt{F_s^2 + F_s^2} = \sqrt{2}\frac{GM^2}{a^2},$$

directed along the inward diagonal. Equivalently, each contributes $F_s \cos 45^\circ$ towards the centre and their transverse components cancel.

Step 2: Add the force of the opposite mass The diagonal distance to the opposite corner is $a\sqrt{2}$, so that mass contributes

$$F_d = \frac{GM^2}{(a\sqrt{2})^2} = \frac{GM^2}{2a^2}.$$

This is already along the inward diagonal. Hence the net force on the chosen mass is

$$F_{\text{net}} = \frac{GM^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right).$$

By symmetry the same radial force acts on each corner particle, allowing the square configuration to rotate uniformly.

Step 3: Equate the inward force to the centripetal force Using the circumradius $r = a/\sqrt{2}$,

$$\frac{Mv^2}{a/\sqrt{2}} = \frac{GM^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right).$$

Cancel M and rearrange:

$$v^2 = \frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right), \qquad v = 1.1634\ldots\sqrt{\frac{GM}{a}}.$$

Thus the coefficient rounds to 1.16. The alternatives 1.21, 1.41, and 1.35 in (A), (C), and (D) do not equal the square root of the combined force coefficient after the correct radius is included. Adding the neighbour forces as scalars or using a as the orbit radius would give an incorrect coefficient.

Step 4: Conclusion Each particle must have tangential speed

$$\boxed{v \approx 1.16\sqrt{\frac{GM}{a}}}.$$

Hence, the correct option is (B).

Quick Tip to Solve: At a square corner, combine the two equal side forces with a $\sqrt{2}$ factor, then add the diagonal force. Use the circumradius $a/\sqrt{2}$, not the side length, for circular motion.

43. A circular coil having N turns and radius r carries a current I. It is held in the XZ plane in a magnetic field $B\hat{i}$. The torque on the coil due to the magnetic field is : [4 marks]

(A) $\frac{Br^2I}{\pi N}$

(B) $B\pi r^2IN$

(C) Zero

(D) $\frac{B\pi r^2I}{N}$

Correct Answer: (B) $B\pi r^2IN$

Solution:

Concept:

- The magnetic dipole moment of a planar coil with N turns, current I, and area A is $\boldsymbol{\mu} = NIA\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is normal to its plane.
- The torque is $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$, with magnitude $\tau = NIAB\sin\alpha$. Here α is the angle between the normal and the field, not the angle between the plane and the field.

Step 1: Identify the area vector The coil lies in the XZ plane, so its area vector points along either $+\hat{\mathbf{j}}$ or $-\hat{\mathbf{j}}$, depending on the current direction. The field is $\mathbf{B} = B\hat{\mathbf{i}}$, which lies in the plane of the coil. Therefore the area vector and field are perpendicular: $\alpha = 90^\circ$. The torque is consequently maximal, rather than zero.

Step 2: Insert the circular area For a circle of radius r, $A = \pi r^2$. Substitution gives

$$\tau = NI(\pi r^2)B\sin 90^\circ = B\pi r^2IN.$$

Each turn contributes the same torque, so the torques add in proportion to N. Without the current direction, the sign of the torque vector cannot be fixed, but its magnitude is fixed. Options (A) and (D) incorrectly make the torque decrease with the number of turns; (A) also has the wrong area factor. Option (C) would require the area vector to be parallel to the field, which is not this geometry.

Step 3: Conclusion The torque magnitude is $\boxed{B\pi r^2IN}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the angle between the magnetic field and the normal to the coil. A field lying in the coil plane produces maximum torque.

44. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that g = 3.1 p ms^{-2}, what will be the tensile stress that would be developed in the wire ? [4 marks]

(A) 3.1×10^6 Nm^{-2}

(B) 6.2×10^6 Nm^{-2}

(C) 4.8×10^6 Nm^{-2}

(D) 5.2×10^6 Nm^{-2}

Correct Answer: (A) 3.1×10^6 Nm^{-2}

Solution:

Concept:

- Tensile stress is the axial tensile force divided by the original cross-sectional area: $\sigma = F/A$. For a hanging mass at rest, $F = mg$.
- The cross-section of a wire of radius r is $A = \pi r^2$. Stress has SI unit $\mathrm{N\,m^{-2}} = \mathrm{Pa}$; neither the wire length nor Young's modulus is needed to find it.

Step 1: Calculate the area and retain the printed factor The mass is $m = 4\,\mathrm{kg}$ and the radius is $r = 2.0 \times 10^{-3}\,\mathrm{m}$. Therefore

$$A = \pi(2.0 \times 10^{-3})^2 = 4\pi \times 10^{-6}\,\mathrm{m}^2.$$

The question writes the gravitational acceleration as $g = 3.1p\,\mathrm{m\,s^{-2}}$, without defining p. Keeping that factor explicitly, the tensile force is $F = 4(3.1p) = 12.4p\,\mathrm{N}$. The mass itself is not the force; multiplying by gravitational acceleration is essential.

Step 2: Evaluate the stress and the numerical interpretation Divide the tensile force by the area:

$$\sigma = \frac{12.4p}{4\pi \times 10^{-6}} = \frac{3.1p}{\pi} \times 10^6\,\mathrm{Pa}.$$

If the printed factor p denotes π, this becomes $3.1 \times 10^6\,\mathrm{Pa}$, corresponding to option (A) and the supplied answer key. This reading also gives $g = 3.1\pi \simeq 9.74\,\mathrm{m\,s^{-2}}$. The other three choices do not follow from that acceleration: they would require different values of the otherwise undefined factor. If p is read literally as an unspecified parameter, however, no single numerical stress can be selected. The parameter-dependent expression above is then the exact answer; assigning it a value without explaining the factor would conceal the ambiguity.

Step 3: Conclusion Literally, $\boxed{\sigma = (3.1p/\pi) \times 10^6\,\mathrm{Pa}}$. With the conventional reading $p = \pi$, $\sigma = 3.1 \times 10^6\,\mathrm{Pa}$, and the correct option is (A), as in the supplied key; an undefined literal p leaves the numerical answer ambiguous.

Quick Tip to Solve: Convert the radius to metres before squaring it. In $mg/(\pi r^2)$, a factor π in the given acceleration cancels exactly.

45. Radiation coming from transitions n = 2 to n = 1 of hydrogen atoms fall on He^+ ions in n = 1 and n = 2 states. The possible transition of helium ions as they absorb energy from the radiation is : [4 marks]

(A) n = 1 → n = 4

(B) n = 2 → n = 5

(C) n = 2 → n = 4

(D) n = 2 → n = 3

Correct Answer: (C) n = 2 → n = 4

Solution:

Concept:

- The bound-state energies of a hydrogen-like ion are $E_n = -13.6Z^2/n^2\,\text{eV}$, where Z is the nuclear charge and n the principal quantum number.
- Absorption requires the photon energy to equal an allowed upward energy difference exactly. Having a photon energy larger than a particular level separation does not permit partial absorption of that photon.

Step 1: Find the incident photon energy For hydrogen, $Z = 1$. The photon emitted in the transition $2 \to 1$ has energy

$$E_\gamma = 13.6\left(1 - \frac{1}{4}\right) = 10.2\,\text{eV}.$$

For He^+, $Z = 2$, so $E_n = -54.4/n^2\,\text{eV}$. An ion initially in its ground state needs at least $54.4(1-1/4) = 40.8\,\text{eV}$ to reach any higher bound level; hence these photons cannot excite the $n = 1$ ions.

Step 2: Check the excited-state transitions For an ion starting from $n = 2$, the required final level satisfies

$$54.4\left(\frac{1}{4} - \frac{1}{n_f^2}\right) = 10.2.$$

Thus $13.6 - 54.4/n_f^2 = 10.2$, giving $n_f^2 = 16$ and $n_f = 4$. Directly, $\Delta E_{2\to4} = 54.4(1/4 - 1/16) = 10.2\,\text{eV}$. To test the distractors, $1 \to 4$ requires $51.0\,\text{eV}$, $2 \to 5$ requires $11.424\,\text{eV}$, and $2 \to 3$ requires approximately $7.56\,\text{eV}$. None equals the incident photon energy; also, the photon energy is below the $13.6\,\text{eV}$ ionisation threshold from $n = 2$.

Step 3: Conclusion The allowed absorption is $\boxed{n = 2 \to n = 4}$ in He^+. Hence, the correct option is (C).

Quick Tip to Solve: First calculate the photon energy from hydrogen, then retain the $Z^2 = 4$ factor when testing helium-ion energy gaps.

46. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in : [4 marks]

(A) 2.2 hrs

(B) 4.2 hrs

(C) 2.6 hrs

(D) 3.2 hrs

Correct Answer: (C) 2.6 hrs

Solution:

Concept:

- Use the relative position $\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$ and relative velocity $\mathbf{u} = \mathbf{v}_B - \mathbf{v}_A$. For constant velocities, $\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{u}t$.
- The separation is smallest when its squared magnitude is stationary: $d(\mathbf{r}\cdot\mathbf{r})/dt = 2\mathbf{r}\cdot\mathbf{u} = 0$. Equivalently, the separation vector is then perpendicular to the relative velocity.

Step 1: Form the relative-motion equation Take east as the positive x-direction and north as the positive y-direction. Initially,

$$\mathbf{r}_0 = 80\hat{\mathbf{i}} + 150\hat{\mathbf{j}}\ \text{km}.$$

Ship B moves west, so $\mathbf{v}_B = -10\hat{\mathbf{i}}\,\text{km h}^{-1}$. Subtracting the velocity of A gives

$$\mathbf{u} = (-10-30)\hat{\mathbf{i}} + (0-50)\hat{\mathbf{j}} = -40\hat{\mathbf{i}} - 50\hat{\mathbf{j}}\ \text{km h}^{-1}.$$

Hence the two coordinate separations after t hours are $80 - 40t$ and $150 - 50t$.

Step 2: Minimise the squared distance We minimise $s^2 = (80-40t)^2 + (150-50t)^2$, which has the same minimum as s. Differentiating,

$$-80(80-40t) - 100(150-50t) = 0.$$

Therefore $8200t = 21400$, and

$$t = \frac{107}{41}\,\text{h} \simeq 2.61\,\text{h}.$$

The second derivative is $8200 > 0$, confirming a minimum at a positive time. The nearest listed time is $2.6\,\text{h}$; the other choices do not satisfy $\mathbf{r}(t)\cdot\mathbf{u} = 0$. Equalising just one coordinate would give 2 or 3 hours and would not minimise the full two-dimensional separation.

Step 3: Conclusion The ships are nearest after approximately $\boxed{2.6\,\text{h}}$. Hence, the correct option is (C).

Quick Tip to Solve: For closest approach with constant velocities, use $t = -\mathbf{r}_0\cdot\mathbf{u}/|\mathbf{u}|^2$; subtract both velocity components consistently.

47. Water from a pipe is coming at a rate of 100 litres per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of : (density of water = 1000 kg/m^3, coefficient of viscosity of water = 1mPas) [4 marks]

(A) 10^6

(B) 10^4

(C) 10^3

(D) 10^2

Correct Answer: (B) 10^4

Solution:

Concept:

- The volume flow rate is $Q = Av = \pi r^2 v$, where v is the mean flow speed and r the pipe radius.
- The Reynolds number for flow through a circular pipe is $\mathrm{Re} = \rho v D/\eta$, using the diameter $D = 2r$, mass density ρ, and dynamic viscosity η. It is dimensionless.

Step 1: Convert the data and obtain the speed Convert litres to cubic metres and minutes to seconds before evaluating the speed:

$$Q = \frac{100 \times 10^{-3}}{60} = \frac{1}{600}\,\mathrm{m^3\,s^{-1}}.$$

The radius is $r = 0.05\,\mathrm{m}$, so $D = 0.10\,\mathrm{m}$ and $A = \pi(0.05)^2 = 7.854 \times 10^{-3}\,\mathrm{m^2}$. Therefore

$$v = \frac{Q}{A} \simeq 0.212\,\mathrm{m\,s^{-1}}.$$

The viscosity $1\,\mathrm{mPa\,s}$ is $10^{-3}\,\mathrm{Pa\,s}$; the prefix milli must be included.

Step 2: Evaluate the dimensionless flow parameter Substitute the density, speed, diameter, and viscosity:

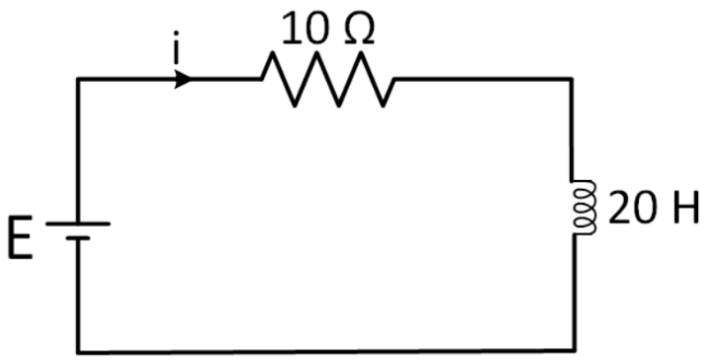
$$\mathrm{Re} = \frac{1000(0.212)(0.10)}{10^{-3}} \simeq 2.12 \times 10^4.$$

One may also combine the two formulas first: $\mathrm{Re} = 2\rho Q/(\pi r \eta)$, which gives the same result. Thus the flow parameter has order 10^4. The 10^6 option is two powers too large, while 10^3 and 10^2 are too small. Confusing litres with cubic metres, or retaining minutes when the viscosity is in SI units, would produce an incorrect order of magnitude. The value also exceeds the usual laminar-flow range in a pipe.

Step 3: Conclusion The Reynolds number is about 2.1×10^4, hence of order $\boxed{10^4}$. Hence, the correct option is (B).

Quick Tip to Solve: For a given volume flow rate, combine the formulas to get $\mathrm{Re} = 2\rho Q/(\pi r \eta)$, then convert every quantity to SI units.

48. A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor is : [4 marks]



(A) $\frac{2}{\ln 2}$

(B) $\ln 2$

(C) $2\ln 2$

(D) $\frac{1}{2}\ln 2$

Correct Answer: (C) $2\ln 2$

Solution:

Concept:

- For a series RL circuit switched onto a constant emf E, the current grows as $i = (E/R)(1 - e^{-t/\tau})$, where $\tau = L/R$ is the time constant.
- The resistor dissipates power $P_R = i^2R$, whereas the inductor stores energy $U_L = Li^2/2$ at a rate $P_L = dU_L/dt = Li\,di/dt$. These are instantaneous rates, not accumulated energies.

Step 1: Express the required power balance The circuit contains $L = 20\,\text{H}$ and $R = 10\,\Omega$, so

$$\tau = \frac{L}{R} = 2\,\text{s}.$$

At the nonzero-current instant of interest, equate the two powers:

$$i^2R = Li\frac{di}{dt} \quad \Rightarrow \quad iR = L\frac{di}{dt}.$$

Kirchhoff's voltage law gives $E = iR + L\,di/dt$. Thus equality of the powers means the applied emf is equally divided between resistor and inductor: $iR = E/2$, or $i = E/(2R)$.

Step 2: Find when the current reaches half its final value Insert $i = E/(2R)$ into the current-growth expression:

$$\frac{E}{2R} = \frac{E}{R}(1 - e^{-t/\tau}) \quad \Rightarrow \quad e^{-t/\tau} = \frac{1}{2}.$$

Taking logarithms gives

$$t = \tau \ln 2 = 2\ln 2\,\text{s} \simeq 1.386\,\text{s}.$$

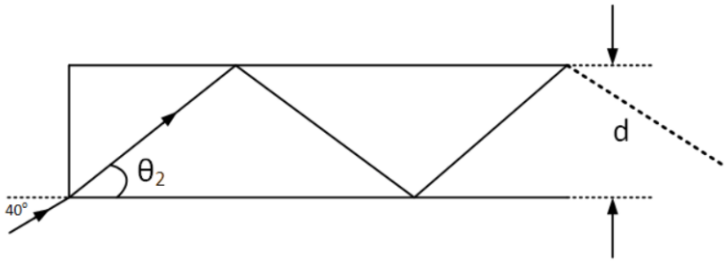
The answer must be proportional to L/R, which rules out expressions using its reciprocal or the

wrong factor. At the exact initial instant both powers vanish trivially; the listed choices seek the later crossing where both powers are positive. The emf cancels, so its numerical value is unnecessary. Options (A), (B), and (D) do not give the half-current condition with the specified time constant.

Step 3: Conclusion The nontrivial equality of the two rates occurs at $\boxed{2\ln 2\,\mathrm{s}}$. Hence, the correct option is (C).

Quick Tip to Solve: In current growth, equal resistor and inductor powers mean half the final current, reached after $(L/R)\ln 2$. At the crossing, verify $i = E/(2R)$; do not equate accumulated energies.

49. In figure, the optical fiber is l = 2m long and has a diameter of d = 20 μm. If a ray of light is incident on one end of the fiber at angle θ_1 = 40°, the number of reflection it makes before emerging from the other end is close to: (refractive index of fibre is 1.31 and sin 40° = 0.64) [4 marks]



(A) 57000

(B) 55000

(C) 66000

(D) 45000

Correct Answer: (B) 55000

Solution:

Concept:

- At the plane entrance face, Snell's law relates the angle to the fibre axis inside the material to the external angle: $\sin\theta_1 = n\sin\theta_2$, taking the surrounding air index as one.
- For successive reflections at opposite sides of a fibre of diameter d, the axial distance between reflections is $\Delta x = d/\tan\theta_2$. Hence the number in length l is approximately $N = l\tan\theta_2/d$.

Step 1: Find the internal ray direction The entrance-face normal is along the fibre axis, so the angle θ_2 in the figure is the refracted angle. With the supplied sine value,

$$\sin\theta_2 = \frac{0.64}{1.31} \simeq 0.48855.$$

Consequently,

$$\cos\theta_2 = \sqrt{1-(0.48855)^2} \simeq 0.87254, \qquad \tan\theta_2 \simeq 0.5599.$$

The side-wall incidence angle is the complementary angle; its sine is $\cos\theta_2$, which exceeds $1/1.31$. Thus total internal reflection at the fibre wall is consistent with this path.

Step 2: Count the axial reflection intervals Use $l = 2\,\text{m}$ and $d = 20\,\mu\text{m} = 2.0 \times 10^{-5}\,\text{m}$. The distance travelled along the axis between consecutive wall reflections is

$$\Delta x = \frac{2.0 \times 10^{-5}}{0.5599}\,\text{m}.$$

Hence

$$N \simeq \frac{l \tan\theta_2}{d} = \frac{2(0.5599)}{2.0 \times 10^{-5}} \simeq 5.60 \times 10^4.$$

The first and last partial traversals can change this count by about one, negligible at this scale. Using the supplied values more closely gives approximately 55990; among the options, 55000 is nearer than 57000, while 66000 and 45000 are much farther away. Using the external angle directly would wrongly ignore refraction at the entrance.

Step 3: Conclusion The calculation gives approximately 5.60×10^4 reflections. The closest listed value is $\boxed{55000}$, so the printed-data calculation gives option (B), rather than the supplied key's value of 57000.

Quick Tip to Solve: Use Snell's law first, then count one reflection per axial advance $d/\tan\theta_2$. Compare the final number with all listed choices.

50. Voltage rating of a parallel plate capacitor is 500V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is 10^{-4} m^2. What is the dielectric constant is the capacitance is 15 pF? (given $\varepsilon_0 = 8.86 \times 10^{-12}$ C^2/Nm^2) [4 marks]

(A) 8.5

(B) 4.5

(C) 3.8

(D) 6.2

Correct Answer: (A) 8.5

Solution:

Concept:

- For a uniform field between parallel plates, the potential difference is $V = Ed$, where d is the plate separation.
- A dielectric completely filling the gap gives $C = K\varepsilon_0 A/d$. Here K is dimensionless and A is the overlapping plate area.

Step 1: Find the separation from the voltage rating The rated voltage corresponds to the largest permitted field in the dielectric. Using the limiting field, rather than the field in an empty capacitor, gives the plate separation directly:

$$d = \frac{V_{\max}}{E_{\max}} = \frac{500}{10^6} = 5.0 \times 10^{-4}\,\text{m}.$$

Thus the gap is $0.50\,\text{mm}$. The dielectric constant does not multiply this voltage-field relation; its effect enters the capacitance formula.

Step 2: Substitute the measured capacitance Convert the capacitance before substituting: $15\,\text{pF} = 15 \times 10^{-12}\,\text{F}$. Rearranging the parallel-plate formula gives

$$K = \frac{Cd}{\varepsilon_0 A} = \frac{(15 \times 10^{-12})(5.0 \times 10^{-4})}{(8.86 \times 10^{-12})(10^{-4})} = \frac{75}{8.86} \simeq 8.47.$$

All lengths and areas have been used in SI units, so no additional millimetre conversion is needed.

Step 3: Compare the alternatives The computed value rounds to 8.5. The other proposed constants, 4.5, 3.8 and 6.2, would give capacitances of approximately 7.97, 6.73 and $10.99\,\text{pF}$, respectively, with the same gap. None equals the specified $15\,\text{pF}$.

Step 4: Conclusion The dielectric constant is $K \simeq 8.5$. Hence, the correct option is (A).

Quick Tip to Solve: Combine the two relations as $K = CV_{\max}/(\varepsilon_0 A E_{\max})$, but convert pF to farads before evaluating.

51. An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be : [4 marks]

(A) 20 cm from the convergent mirror, same size as the object

(B) 40 cm from the convergent mirror, same size as the object

(C) 40 cm from the convergent lens, same size as the object

(D) 20 cm from the convergent mirror, twice the size of the object

Correct Answer: (C) 40 cm from the convergent lens, same size as the object

Solution:

Concept:

- An object at twice the focal length of a converging lens forms a real image at twice the focal length on the opposite side, with equal size.
- An object at the centre of curvature of a concave mirror is imaged at that same position with magnification -1. The reflected light then passes through the lens again.

Step 1: Locate the first image formed by the lens Take the lens at $x = 0$, the original upright object at $x = -40\,\text{cm}$, and the mirror at $x = +60\,\text{cm}$. On the first passage, $u_1 = -40\,\text{cm}$ and $f = +20\,\text{cm}$. The lens formula gives

$$\frac{1}{v_1} - \frac{1}{-40} = \frac{1}{20}, \qquad v_1 = 40\,\text{cm}.$$

Its magnification is $m_1 = v_1/u_1 = -1$.

Step 2: Reflect the light at the mirror This first image lies $60 - 40 = 20\,\text{cm}$ in front of the converging mirror. Since the mirror focal length is $10\,\text{cm}$, that point is its centre of curvature. The mirror forms an equal-sized inverted image back at $x = +40\,\text{cm}$; its magnification is $m_2 = -1$. After reflection the light travels towards the lens.

Step 3: Include the second passage through the lens The mirror image is now a real object $40\,\text{cm}$ to

the right of the lens for the returning rays. That is again twice the lens focal length, so the final image is $40\,\text{cm}$ to the left of the lens, at $x = -40\,\text{cm}$. The final lens magnification is $m_3 = -1$, giving

$$m = m_1 m_2 m_3 = (-1)^3 = -1.$$

Its size therefore equals the original object size, and it is inverted. The final image is $100\,\text{cm}$ from the mirror, so statements placing it 20 or $40\,\text{cm}$ from that mirror are incorrect; the doubled-size statement is also incorrect.

Step 4: Conclusion The final image is $40\,\text{cm}$ from the converging lens on the original object side, with the same size as the object. Hence, the correct option is (C).

Quick Tip to Solve: In a lens-mirror combination, trace the return passage through the lens as a third imaging event; the mirror image alone is not the final image.

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52. A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is E = 6 V m^{-1} along y-direction. Its corresponding magnetic field component, B would be : [4 marks]

(A) 2×10^{-8} T along y-direction

(B) 6×10^{-8} T along z-direction

(C) 2×10^{-8} T along z-direction

(D) 6×10^{-8} T along x-direction

Correct Answer: (C) 2×10^{-8} T along z-direction

Solution:

Concept:

- For a plane electromagnetic wave in vacuum, the instantaneous magnitudes satisfy $E = cB$, with $c = 3.0 \times 10^8\,\text{m s}^{-1}$.
- The two fields are perpendicular to each other and to the propagation direction. The energy flux points along $\mathbf{E} \times \mathbf{B}$.

Step 1: Calculate the magnetic-field magnitude The given $6\,\text{V m}^{-1}$ is the electric field at the specified event, so the corresponding magnetic field at that same event is obtained without an averaging or root-mean-square factor:

$$B = \frac{E}{c} = \frac{6}{3.0 \times 10^8} = 2.0 \times 10^{-8}\,\text{T}.$$

A value of $6 \times 10^{-8}\,\text{T}$ would instead imply $E = 18\,\text{V m}^{-1}$, which is not the field stated.

Step 2: Use the propagation direction to fix the axis The electric field points along $+y$, and propagation is along $+x$. The magnetic field must therefore be transverse to both these directions. Since

$$\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}},$$

the required magnetic field points along $+z$. Choosing $-z$ would reverse the direction of wave travel.

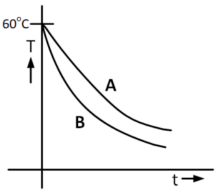
Step 3: Reject the other field vectors Option (A) has the correct magnitude but places the magnetic field along y, parallel to the electric field, violating transversality and giving zero cross product. Option (B) has the correct axis but a magnetic magnitude three times too large. Option (D) is both too large and longitudinal, along the propagation axis. Only the vector in (C) obeys both the vacuum magnitude relation and the right-hand direction rule.

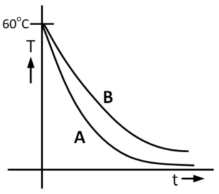
Step 4: Conclusion The magnetic field is $2 \times 10^{-8}\,\mathrm{T}$ along the z-direction. Hence, the correct option is (C).

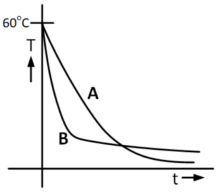
Quick Tip to Solve: Check an electromagnetic-wave option twice: first use $B = E/c$, then verify that $\mathbf{E} \times \mathbf{B}$ points along propagation.

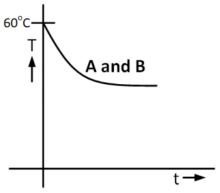
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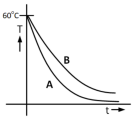
53. Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg/m^3 and specific heat of 2000 J kg^{-1} K^{-1} while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1} K^{-1}. Which of the following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same) [4 marks]

(A)


(B)


(C)


(D)


Correct Answer: (B)


Solution:

Concept:

- The heat capacity of a liquid of volume V, density ρ and specific heat c is $mc = \rho V c$. Equal volumes need not have equal heat capacities.
- For identical heat-loss conditions, the magnitude of the cooling rate is the heat-loss power divided by heat capacity: $|dT/dt| = P(T)/(\rho V c)$.

Step 1: Compare the heat capacities of the two liquids Both beakers hold the same volume V. The relevant heat capacities are therefore

$$C_A = (8 \times 10^2)(2000)V = 1.6 \times 10^6 V,$$

$$C_B = (10^3)(4000)V = 4.0 \times 10^6 V.$$

Thus $C_B/C_A = 2.5$. Beaker B requires two and a half times as much heat removal for the same temperature decrease.

Step 2: Relate heat loss to the temperature slope The beakers have the same geometry and emissivity. At a given common temperature their heat-loss powers are equal. Energy balance then gives

$$\frac{|dT_A/dt|}{|dT_B/dt|} = \frac{C_B}{C_A} = 2.5.$$

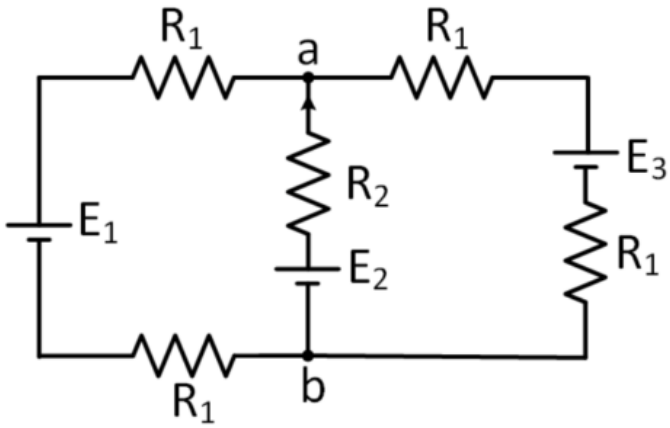
In particular, at the initial temperature 60°C, A has the more negative slope and starts cooling faster. As both approach room temperature their heat loss decreases, so the curves flatten.

Step 3: Identify the physically consistent pair of curves For a common ambient temperature T_s, Newton cooling has the schematic form $T - T_s = (60 - T_s)e^{-kt}$, with k inversely proportional to heat capacity. Consequently $k_A > k_B$, and A lies below B for every later time while both ultimately approach the same ambient temperature. This is the ordering shown in (B). Graph (A) reverses the ordering, graph (C) makes the curves cross, and graph (D) makes them identical despite unequal heat capacities. None of those follows from the stated identical cooling conditions.

Step 4: Conclusion Liquid A cools faster, so its temperature curve lies below that of liquid B after their common start at 60°C. Hence, the correct option is (B).

Quick Tip to Solve: For equal liquid volumes, compare ρc, not density or specific heat separately; the smaller product gives the steeper initial cooling curve.

54. For the circuit shown, with $R_1 = 1.0$W, $R_2 = 2.0$ W, $E_1 = 2$ V and $E_2 = E_3 = 4$ V, the potential difference between the points 'a' and 'b' is approximately (in V): [4 marks]



(A) 3.3

(B) 2.7

(C) 2.3

(D) 3.7

Correct Answer: (A) 3.3

Solution:

Concept:

- Between two junctions, each series branch has the same terminal voltage $V = V_a - V_b$. Resistances in each individual branch must first be added.
- For a branch whose cell has its positive terminal towards a, the current taken from a towards b is $I = (V - E)/R$. Kirchhoff's junction rule requires the algebraic sum of those currents to vanish.

Step 1: Reduce the three branches between the junctions The left route from a to b contains two resistors R_1 in series with E_1, so its resistance is $2R_1 = 2\,\Omega$ and emf is $2\,\mathrm{V}$. The middle route contains $R_2 = 2\,\Omega$ and $E_2 = 4\,\mathrm{V}$. The right route contains two R_1 resistors and $E_3 = 4\,\mathrm{V}$, again giving $2\,\Omega$. The long plate of every cell is towards the upper junction, so all three emf signs are the same.

Step 2: Apply the junction rule at a Define each branch current positive from a to b, even though some actual currents may be negative. With no external connection drawing current,

$$\frac{V-2}{2} + \frac{V-4}{2} + \frac{V-4}{2} = 0.$$

Multiplying by 2 gives $3V - 10 = 0$, hence

$$V_a - V_b = \frac{10}{3}\,\mathrm{V} \simeq 3.33\,\mathrm{V}.$$

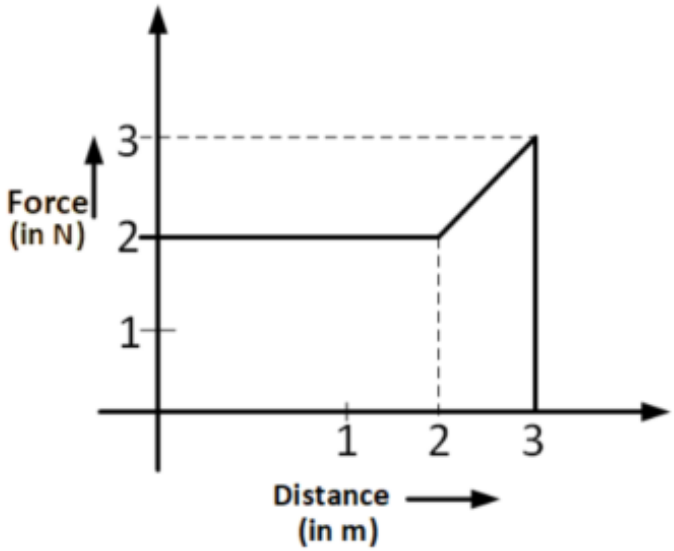
Because all branch resistances are equal, the voltage is also the arithmetic mean of the three emfs.

Step 3: Check the current balance and the options At this voltage, the left-branch current is $2/3\,\mathrm{A}$ downwards, while the middle and right currents are each $-1/3\,\mathrm{A}$, meaning upwards. Their sum is zero, confirming both the sign convention and the voltage. The approximate value is $3.3\,\mathrm{V}$; 2.7, 2.3 and $3.7\,\mathrm{V}$ do not satisfy the junction equation.

Step 4: Conclusion The potential difference is $V_a - V_b = 10/3\,\mathrm{V} \approx 3.3\,\mathrm{V}$, with a at the higher potential. Hence, the correct option is (A).

Quick Tip to Solve: For parallel cell branches use $V = (\sum E_i/R_i)/(\sum 1/R_i)$, after reducing every branch and checking each cell polarity.

55. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3m is : [4 marks]



(A) 6.5 J

(B) 2.5 J

(C) 5 J

(D) 4 J

Correct Answer: (A) 6.5 J

Solution:

Concept:

- The net work done by a position-dependent force in one dimension is $W = \int F\,dx$, equal to the signed area under its force-distance graph.
- The work-energy theorem gives $K_f - K_i = W$. Here the particle starts from rest, so $K_i = 0$ and the accumulated work is its final kinetic energy.

Step 1: Read the two force intervals from the graph From $x = 0$ to $x = 2\,\text{m}$, the force is constant at $2\,\text{N}$. From $x = 2$ to $x = 3\,\text{m}$, it increases linearly from $2\,\text{N}$ to $3\,\text{N}$. The entire graph is above the distance axis, so both intervals contribute positive work; no area needs to be subtracted.

Step 2: Add the rectangular and trapezoidal areas The first interval gives a rectangle:

$$W_1 = (2\,\text{N})(2\,\text{m}) = 4\,\text{J}.$$

In the second interval the average force is $(2+3)/2 = 2.5\,\text{N}$, since the force varies linearly. Thus

$$W_2 = \frac{2+3}{2}(3-2) = 2.5\,\text{J}.$$

Equivalently, take a $2\,\text{N}$ by $1\,\text{m}$ rectangle plus a triangle of height $1\,\text{N}$ and base $1\,\text{m}$.

Step 3: Convert the total work to kinetic energy The total accumulated work is

$$W = W_1 + W_2 = 4 + 2.5 = 6.5\,\text{J}.$$

As the initial kinetic energy is zero, the final kinetic energy is this same value. The $2.5\,\text{J}$ alternative counts only the last interval, and $4\,\text{J}$ counts only the first. The $5\,\text{J}$ alternative does not equal the full area. Multiplying only the final force by total distance would also be invalid because the force is not constant.

Step 4: Conclusion After travelling $3\,\text{m}$, the particle has kinetic energy $6.5\,\text{J}$. Hence, the correct option is (A).

Quick Tip to Solve: Split a piecewise-linear force-distance graph at every change of slope, add the signed areas, and only then apply $\Delta K = W$.

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56. In SI units, the dimensions of $\sqrt{\varepsilon_0/\mu_0}$ is : [4 marks]

(A) $A^{-1}\,TML^3$

(B) $A^2T^3M^{-1}L^{-2}$

(C) $AT^{-3}ML^{3/2}$

(D) $AT^2M^{-1}L^{-1}$

Correct Answer: (B) $A^2T^3M^{-1}L^{-2}$

Solution:

Concept:

- Coulomb's law gives $[\varepsilon_0] = [Q]^2/([F][r]^2)$, where $[Q] = \text{AT}$ and $[F] = \text{MLT}^{-2}$.
- The magnetic force per unit length between parallel currents gives $[\mu_0] = \text{MLT}^{-2}\text{A}^{-2}$. A square root halves every dimensional exponent.

Step 1: Find the dimensions of permittivity Using $F = Q_1Q_2/(4\pi\varepsilon_0 r^2)$, with the numerical factor dimensionless, gives

$$[\varepsilon_0] = \frac{\text{A}^2\text{T}^2}{(\text{MLT}^{-2})\text{L}^2} = \text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2.$$

The time exponent is four because charge itself contains one power of time.

Step 2: Divide by permeability and take the root For two parallel currents, $F/\ell = \mu_0 I_1 I_2/(2\pi r)$, so $[\mu_0] = [F]/[I]^2 = \text{MLT}^{-2}\text{A}^{-2}$. Hence

$$\left[\frac{\varepsilon_0}{\mu_0}\right] = \text{M}^{-2}\text{L}^{-4}\text{T}^6\text{A}^4,$$

and therefore

$$\left[\sqrt{\frac{\varepsilon_0}{\mu_0}}\right] = \text{M}^{-1}\text{L}^{-2}\text{T}^3\text{A}^2.$$

This has the same dimensions as electrical conductance, or inverse resistance.

Step 3: Compare every proposed dimension Reordering factors, the result is $\text{A}^2\text{T}^3\text{M}^{-1}\text{L}^{-2}$, exactly

as in (B). Options (A), (C) and (D) have current powers -1, 1 and 1, respectively, instead of 2, and their remaining powers also disagree. A useful independent check is the vacuum wave impedance $Z_0 = \sqrt{\mu_0/\varepsilon_0}$; the requested square root is its reciprocal, so it must have conductance dimensions.

Step 4: Conclusion The dimensions are $\mathrm{A}^2\mathrm{T}^3\mathrm{M}^{-1}\mathrm{L}^{-2}$. Hence, the correct option is (B).

Quick Tip to Solve: If you remember that $\sqrt{\mu_0/\varepsilon_0}$ is an impedance, invert the dimensions of resistance to check the requested ratio quickly.

57. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to: [4 marks]

(A) 10^4 Nm^{-2}

(B) 10^6 Nm^{-2}

(C) 10^8 Nm^{-2}

(D) 10^3 Nm^{-2}

Correct Answer: (B) 10^6 Nm^{-2}

Solution:

Concept:

- For a uniform stretched cord, Young's modulus is $Y = FL/(Ax)$, where L is the original length, A its cross-sectional area and x the extension.
- The elastic energy stored under the linear stress-strain approximation is $U = \frac{1}{2}Fx = YAx^2/(2L)$. With negligible cord mass and losses, this becomes the stone's kinetic energy.

Step 1: Convert the dimensions and calculate the area The original length is $L = 0.42\,\mathrm{m}$, extension $x = 0.20\,\mathrm{m}$, stone mass $m = 0.02\,\mathrm{kg}$, and speed $v = 20\,\mathrm{m\,s^{-1}}$. The cord diameter is $6\,\mathrm{mm}$, so use a radius of $3\,\mathrm{mm}$, not the diameter, in the area formula. Thus

$$A = \pi(3 \times 10^{-3})^2 = 9\pi \times 10^{-6}\,\mathrm{m}^2.$$

Step 2: Equate elastic energy to the kinetic energy The stone acquires

$$K = \frac{1}{2}(0.02)(20)^2 = 4\,\mathrm{J}.$$

Equating this with the stored energy gives

$$\frac{YAx^2}{2L} = \frac{1}{2}mv^2 \quad \Rightarrow \quad Y = \frac{mv^2L}{Ax^2}.$$

Substitution yields

$$Y = \frac{(0.02)(20)^2(0.42)}{(9\pi \times 10^{-6})(0.20)^2} \simeq 2.97 \times 10^6\,\mathrm{N\,m^{-2}}.$$

Step 3: Compare the listed orders of magnitude The result is of order $10^6\,\mathrm{N\,m^{-2}}$, closest to the value in option (B). Values 10^4 and 10^3 are far too small to store the required energy at this extension, while 10^8 is far too large. The factor of one-half in elastic energy cancels the one-half in kinetic energy.

Step 4: Conclusion The Young's modulus is approximately $3.0 \times 10^6\,\mathrm{N\,m^{-2}}$, closest to $10^6\,\mathrm{N\,m^{-2}}$. Hence, the correct option is (B).

Quick Tip to Solve: Equate $YAx^2/(2L)$ to $mv^2/2$; convert the diameter to a radius before calculating the cord area.

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58. A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of –4 Q, the new potential difference between the same two surfaces is : [4 marks]

(A) V

(B) 2V

(C) –2V

(D) 4V

Correct Answer: (A) V

Solution:

Concept:

- Inside the metal of an electrostatic conductor the electric field is zero. The charge on a cavity's inner surface is fixed by the charge enclosed in that cavity.
- The potential difference between two surfaces is the line integral of the electric field between them. An added charge on the outside of a spherical conducting shell contributes the same potential to every point inside that shell.

Step 1: Locate the induced and added charges The solid sphere retains its charge Q. To make the field zero in the shell metal, its inner surface carries $-Q$. Initially the shell is uncharged, so its outer surface carries $+Q$. After the shell receives $-4Q$, the inner-surface charge remains $-Q$, while the outer-surface charge becomes $-3Q$. The added charge has changed only the outer-surface charge.

Step 2: Evaluate the field contribution to the potential difference Let the solid sphere radius be a and the shell inner radius be b. In the spherical gap, Gauss's law gives

$$E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}.$$

In the shell metal the field remains zero. Consequently the potential difference from the sphere

surface to the shell's outer surface is

$$V_{\text{sphere}} - V_{\text{shell}} = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}\, dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right).$$

The added outer charge does not appear in this expression.

Step 3: Compare before and after charging the shell Both surface potentials change by the same additive amount due to the extra outer-surface charge. Their difference therefore stays unchanged. Options (B) and (D) incorrectly multiply the original difference; option (C) incorrectly reverses its sign as well as changing its magnitude. A negative net shell charge does not reverse the field due to the central charge within the gap.

Step 4: Conclusion The new potential difference between the same surfaces remains V. Hence, the correct option is (A).

Quick Tip to Solve: To find a potential difference across a cavity, integrate only the field between the surfaces; an external shell charge gives a common potential shift.

59. A 200 Ω resistor has a certain color code. If one replaces the red color by green in the code, the new resistance will be : [4 marks]

(A) 500 Ω

(B) 100 Ω

(C) 200 Ω

(D) 300 Ω

Correct Answer: (A) 500 Ω

Solution:

Concept:

- In the usual four-band resistor code, the first two bands give the significant digits and the third band is a power-of-ten multiplier: $R = (10a + b)10^c\,\Omega$.
- The colour-digit sequence begins black 0, brown 1, red 2, orange 3, yellow 4, green 5. A tolerance band does not set the nominal resistance.

Step 1: Express the given resistance using two significant digits Write

$$200\,\Omega = 20 \times 10^1\,\Omega.$$

Thus the significant-digit bands are red for 2 and black for 0, while the multiplier band is brown for 10^1. This identifies where the stated red colour occurs in the resistance code: it is the first significant-digit band, not the multiplier.

Step 2: Replace the red band by green Green denotes the digit 5. Changing only the red band therefore changes the first digit from 2 to 5, retaining the black second digit and brown multiplier. The new resistance is

$$R_{\text{new}} = 50 \times 10^1\,\Omega = 500\,\Omega.$$

Equivalently, the nominal value increases by the factor $5/2$: $200(5/2) = 500\,\Omega$.

Step 3: Eliminate the alternative values A resistance of $100\,\Omega$ would require brown as the first band, while $300\,\Omega$ would require orange. The value $200\,\Omega$ would mean leaving the first digit red. None describes a red-to-green substitution. Replacing a colour by its digit must be done in the correct band position; one must not interpret every coloured band as a multiplier.

Step 4: Conclusion The replacement changes the code from red-black-brown to green-black-brown, giving $500\,\Omega$. Hence, the correct option is (A).

Quick Tip to Solve: First rewrite the resistance as two significant digits times a power of ten; then change only the digit or multiplier represented by the specified band.

60. The wavelength of the carrier waves in a modern optical fiber communication network is close to : [4 marks]

(A) 1500 nm

(B) 2400 nm

(C) 600 nm

(D) 900 nm

Correct Answer: (A) 1500 nm

Solution:

Concept:

- An optical fibre guides a light carrier by total internal reflection, but its usable transmission distance also depends on attenuation and dispersion in the fibre material.
- Silica fibres have useful near-infrared transmission windows around 850, 1300 and $1550\,\text{nm}$. Long-distance modern links commonly use the low-loss window near $1.55\,\mu\text{m}$.

Step 1: Identify the relevant optical communication window The question asks for a carrier wavelength in a modern optical-fibre network, so the appropriate scale is the near infrared. The widely used long-haul window is approximately

$$\lambda \simeq 1.55\,\mu\text{m} = 1550\,\text{nm}.$$

Silica attenuation is especially small in this region, permitting long repeater or amplifier spacings. This is why a communication carrier need not lie in the visible range.

Step 2: Select the nearest listed wavelength The option $1500\,\text{nm}$ is only $50\,\text{nm}$ away from $1550\,\text{nm}$, and represents the same conventional telecom window. The other listed values are much farther away. A $600\,\text{nm}$ wave is visible light, $900\,\text{nm}$ is nearer the shorter-wavelength window, and $2400\,\text{nm}$ is beyond the usual silica telecom windows and suffers stronger infrared absorption.

Step 3: Check the scale through frequency For an additional scale check,

$$f = \frac{c}{\lambda} \simeq \frac{3 \times 10^8}{1.5 \times 10^{-6}} = 2 \times 10^{14}\,\text{Hz}.$$

This is an optical carrier frequency, much larger than radio carrier frequencies. The information signal modulates this light carrier; it does not determine the carrier wavelength directly.

Step 4: Conclusion The nearest wavelength is $1500\,\mathrm{nm}$, corresponding to the standard near-infrared telecom band. Hence, the correct option is (A).

Quick Tip to Solve: Remember the silica-fibre telecom windows as roughly 0.85, 1.30 and $1.55\,\mu\mathrm{m}$; a long-haul modern network points to the last one.

Chemistry (120 marks)

61. Which of the following amines can be prepared by Gabriel phthalimide reaction ? [4 marks]

(A) neo-pentylamine

(B) n-butylamine

(C) t-butylamine

(D) triethylamine

Correct Answer: (B) n-butylamine

Solution:

Concept:

- Gabriel synthesis alkylates the phthalimide anion with a suitable alkyl halide, followed by hydrolysis or hydrazinolysis to release a primary amine, $\mathrm{RNH_2}$.
- The alkylation proceeds through an $S_\mathrm{N}2$ displacement. An unhindered primary alkyl halide is suitable; severe steric hindrance suppresses the required backside attack.

Step 1: Identify a suitable alkyl group To prepare n-butylamine, potassium phthalimide can attack a primary straight-chain n-butyl halide such as $\mathrm{CH_3CH_2CH_2CH_2Br}$. The carbon bearing bromine is a terminal $\mathrm{CH_2}$ group, so it is accessible to the nucleophile. Alkylation attaches the n-butyl group to the imide nitrogen, and cleavage of the imide then gives $\mathrm{CH_3CH_2CH_2CH_2NH_2}$.

Step 2: Reject the sterically blocked substrates For t-butylamine the corresponding t-butyl halide has a tertiary reaction centre. Backside attack needed for $S_\mathrm{N}2$ is blocked, and elimination is favoured. Neopentyl halide, $\mathrm{(CH_3)_3CCH_2X}$, is formally primary but has a bulky quaternary carbon immediately beside the reacting carbon. This pronounced neopentyl steric effect makes the Gabriel alkylation unsuitable under its usual conditions.

Step 3: Reject the tertiary amine and choose the product Triethylamine is $\mathrm{N(CH_2CH_3)_3}$, a tertiary amine rather than a primary amine. Gabriel synthesis releases a nitrogen bearing one alkyl substituent and two hydrogens, so it is not the method for making triethylamine. Of the listed choices, n-butylamine satisfies both the primary-amine product requirement and the unhindered-substrate requirement.

Step 4: Conclusion n-Butylamine can be prepared by Gabriel phthalimide synthesis using a primary n-butyl halide. Hence, the correct option is (B).

Quick Tip to Solve: Check two conditions for Gabriel synthesis: the product must be a primary amine, and its alkyl halide must permit a reasonably unhindered $S_\mathrm{N}2$ attack.

62. The correct order of the spin-only magnetic moment of metal ions in the following low-spin complexes, $[V(CN)_6]^{4-}$,$[Fe(CN)_6]^{4-}$, $[Ru(NH_3)_6]^{3+}$, and $[Cr(NH_3)_6]^{2+}$, is : [4 marks]

(A) $V^{2+} > Ru^{3+} > Cr^{2+} > Fe^{2+}$

(B) $V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$

(C) $Cr^{2+} > V^{2+} > Ru^{3+} > Fe^{2+}$

(D) $Cr^{2+} > Ru^{3+} > Fe^{2+} > V^{2+}$

Correct Answer: (B) $V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired d electrons and μ_B is the Bohr magneton.
- For the stated low-spin octahedral complexes, electrons pair in the lower-energy t_{2g} set before occupying e_g, subject to the three orbitals available within t_{2g}. Determine oxidation states before counting d electrons.

Step 1: Determine the metal oxidation states and d counts Each cyanide ligand has charge -1, so $[V(CN)_6]^{4-}$ and $[Fe(CN)_6]^{4-}$ contain V^{2+} and Fe^{2+}, respectively. Ammonia is neutral, so the other ions are Ru^{3+} and Cr^{2+}. Their configurations are therefore d^3, d^6, d^5 and d^4, respectively; ionisation removes the outer s electrons before the d electrons.

Step 2: Count the unpaired electrons in the low-spin states For V^{2+}, t_{2g}^3 has three unpaired electrons. For Cr^{2+}, low-spin t_{2g}^4 has two. For Ru^{3+}, low-spin t_{2g}^5 has one. Finally Fe^{2+}, with t_{2g}^6, has none. Thus the unpaired-electron counts in decreasing order are $3 > 2 > 1 > 0$.

Step 3: Convert the counts into the moment order The corresponding spin-only moments are

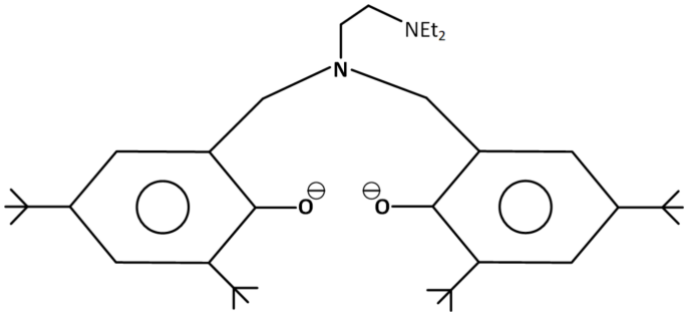
$$\sqrt{15} > \sqrt{8} > \sqrt{3} > 0 \qquad \text{in } \mu_B.$$

Since $\sqrt{n(n+2)}$ increases with n, the required order follows directly. Option (A) interchanges chromium and ruthenium, while (C) incorrectly places chromium above vanadium. Option (D) even places diamagnetic iron above vanadium, contradicting its zero unpaired-electron count.

Step 4: Conclusion The order is $V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$. Hence, the correct option is (B).

Quick Tip to Solve: For low-spin octahedral d^3, d^4, d^5, d^6, the unpaired-electron counts are $3, 2, 1, 0$; order the moments from these counts rather than from atomic number.

63. The following ligand is : [4 marks]



(A) hexadentate

(B) bidentate

(C) tetradentate

(D) tridentate

Correct Answer: (C) tetradentate

Solution:

Concept:

- Denticity is the number of donor atoms belonging to one ligand that can coordinate to the same metal centre. Count donor atoms with available lone pairs, not the number of rings or side chains.
- Phenoxide oxygen is an anionic donor and a neutral tertiary amine nitrogen is a lone-pair donor. The carbon framework linking these atoms controls the chelate arrangement but does not add ordinary donor sites.

Step 1: Locate the two oxygen donors The structure contains two negatively charged oxygen atoms, one attached to each aromatic ring. Each is a phenoxide oxygen and can donate an electron pair to a metal centre. These two oxygen atoms therefore contribute two coordination sites. The aromatic rings and tert-butyl substituents do not provide additional conventional donor atoms in the ligand shown.

Step 2: Locate both nitrogen donors The central nitrogen is bonded to two benzyl arms and to the upward carbon chain. As a neutral tertiary amine, it retains one lone pair available for coordination. At the end of the upward chain, the group labelled NEt_2 contains a second tertiary amine nitrogen. Its two ethyl substituents and its bond to the chain do not remove its lone pair. This terminal nitrogen supplies one further donor site.

Step 3: Add the donor sites and classify the ligand There are therefore two oxygen donors and two nitrogen donors:

$$\text{denticity} = 2\,(\text{O}) + 2\,(\text{N}) = 4.$$

The flexible linked framework allows this N_2O_2 donor set to bind one metal through four sites. Bidentate would omit both nitrogens; tridentate would miss one donor, usually the terminal NEt_2 nitrogen. Hexadentate would require two extra donor atoms that are absent from the structure.

Step 4: Conclusion The ligand has an N_2O_2 donor set and is tetradentate. Hence, the correct option is (C).

Quick Tip to Solve: Trace every heteroatom in the structure, including an amine hidden in an abbreviated NEt_2 group; each available nitrogen or oxygen donor counts once.

64. An organic compound neither reacts with neutral ferric chloride solution nor with fehling solution. it however, reacts with Grignard reagent and gives positive iodoform test. The compound is : [4 marks]

(A) o-$HOC_6H_4COCH_3$

(B) o-$CH_3OC_6H_4CHO$

(C) o-$CH_3OC_6H_4COC_2H_5$

(D) o-$(CH_3CH(OH))C_6H_4COC_2H_5$

Correct Answer: (D) o-$(CH_3CH(OH))C_6H_4COC_2H_5$

Solution:

Concept:

- Neutral ferric chloride gives a characteristic coloured complex with a phenolic hydroxyl group. An alcoholic hydroxyl group does not give this phenol test.
- An iodoform test is positive for a methyl ketone, CH_3CO-R, or an oxidisable secondary alcohol containing $CH_3CH(OH)^-$. Carbonyl compounds also react with a Grignard reagent.

Step 1: Apply the negative tests Option (A) contains a phenolic $-OH$ group directly attached to the aromatic ring, so its neutral ferric chloride test is positive and it is excluded. Options (B), (C) and (D) have no phenolic hydroxyl group. The aromatic aldehyde in (B) may fail Fehling's test, while the ordinary ketone groups in (C) and (D) also give negative Fehling results. Thus a negative Fehling test by itself does not identify the answer.

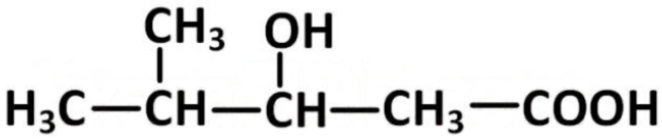
Step 2: Use the iodoform test to distinguish the remaining structures In (B), the $-CHO$ group is attached to an aromatic ring and is not ethanal; the methoxy group does not create a methyl ketone or a methyl secondary alcohol. Hence (B) does not give iodoform. In (C), the carbonyl is bonded to ethyl, $-COCH_2CH_3$, rather than directly to methyl, so it too fails this test. Option (D) has a $-CH(OH)CH_3$ side chain. Oxidation during the iodoform test converts it into a methyl ketone, which then yields yellow CHI_3.

Step 3: Check the Grignard reaction Option (D) also contains a ketone group that can undergo carbonyl addition with a Grignard reagent. Its alcoholic proton initially consumes reagent by an acid-base reaction; with sufficient reagent the carbonyl reacts as well. Thus the presence of this alcoholic hydroxyl is consistent with the stated reaction and does not imply a positive phenol test.

Step 4: Conclusion The compound with the ortho $-CH(OH)CH_3$ and $-COC_2H_5$ side chains satisfies all the tests. Hence, the correct option is (D).

Quick Tip to Solve: For an iodoform test, inspect both $-COCH_3$ and $-CH(OH)CH_3$; a positive result need not mean the starting compound is a methyl ketone.

65. The IUPAC name of the following compound is : [4 marks]



(A) 3-Hydroxy-4-methylpentanoic acid

(B) 4,4-Dimethyl-3-hydroxybutanoic acid

(C) 2-Methyl-3-hydroxypentan-5-oic acid

(D) 4-Methyl-3-hydroxypentanoic acid

Correct Answer: (A) 3-Hydroxy-4-methylpentanoic acid

Solution:

Concept:

- A carboxylic acid is the principal functional group here. The parent chain must contain its carbon, and numbering starts with the carboxyl carbon as carbon 1.
- An alcohol group is named by the prefix hydroxy when a higher-priority carboxylic acid is present. Different substituent prefixes are cited alphabetically, so hydroxy precedes methyl.

Step 1: Trace and number the carbon skeleton Follow the continuous chain from the $-\mathrm{COOH}$ carbon through the next three carbons to either terminal methyl group at the branched end. This gives five carbon atoms, so the parent is pentanoic acid. The hydroxyl is on carbon 3 and the remaining methyl branch is on carbon 4. The connectivity is therefore that of $\mathrm{HOOC}-\mathrm{CH_2}-\mathrm{CH(OH)}-\mathrm{CH(CH_3)}-\mathrm{CH_3}$. The internal carbon printed as $\mathrm{CH_3}$ between two carbon atoms in the figure has a valence typo: with two single carbon-carbon bonds it must be $\mathrm{CH_2}$. The name is determined by the displayed carbon skeleton.

Step 2: Assemble the name and eliminate the distractors The prefixes are 3-hydroxy and 4-methyl. Citing them alphabetically gives 3-hydroxy-4-methylpentanoic acid. Option (B) chooses a four-carbon parent even though a five-carbon chain including the acid carbon is available. Option (C) numbers from the wrong end: the acid carbon must receive locant 1 rather than 5. Option (D) has the correct substituent positions but reverses the required alphabetical order of hydroxy and methyl. Parent-chain length and functional-group priority must be fixed before alphabetising the prefixes.

Step 3: Conclusion The displayed carbon skeleton is named 3-hydroxy-4-methylpentanoic acid, with the internal hydrogen-count typo understood as explained above. Hence, the correct option is (A).

Quick Tip to Solve: Start numbering at the $-\mathrm{COOH}$ carbon, then locate the branches; alphabetise hydroxy and methyl only after their locants are fixed.

66. For silver, C_p(J K^{-1} mol^{-1}) = 23 +0.01 T. If the temperature (T) of 3 moles of silver is raised from 300 K to 1000 K at 1 atm pressure, the value of ΔH will be close to : [4 marks]

(A) 62 KJ

(B) 16 KJ

(C) 13 KJ

(D) 21 KJ

Correct Answer: (A) 62 KJ

Solution:

Concept:

- At constant pressure, the enthalpy increase for n moles is $\Delta H = n\int_{T_1}^{T_2} C_{p,m}(T)\,dT$, where the stated heat capacity is molar.
- When $C_{p,m} = a + bT$, integration gives $\Delta H = n[a(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2)]$. Temperatures must be in kelvin and the resulting energy is in joules.

Step 1: Integrate the molar heat capacity The given heat capacity is $C_{p,m} = 23 + 0.01T$ in $\mathrm{J\,K^{-1}\,mol^{-1}}$. It changes with temperature, so using only its initial value would neglect part of the heat absorbed. For one mole,

$$\begin{aligned}\Delta H_m &= \int_{300}^{1000} (23 + 0.01T)\,dT \\ &= 23(1000 - 300) + 0.005(1000^2 - 300^2) \\ &= 16100 + 4550 = 20650\,\mathrm{J\,mol^{-1}}.\end{aligned}$$

The factor 0.005 comes from integrating $0.01T$, rather than simply multiplying it by the temperature interval.

Step 2: Scale to three moles and compare the choices Heat capacity is an extensive quantity after multiplication by the number of moles. Therefore

$$\Delta H = 3(20650) = 61950\,\mathrm{J} = 61.95\,\mathrm{kJ}.$$

The enthalpy rises, so the sign is positive. The value $21\,\mathrm{kJ}$ in (D) is approximately the one-mole result and omits the factor 3. The values $16\,\mathrm{kJ}$ and $13\,\mathrm{kJ}$ in (B) and (C) do not include the complete integrated heat-capacity contribution for the three-mole sample. The nearest listed value is $62\,\mathrm{kJ}$.

Step 3: Conclusion The enthalpy change is $\Delta H = +61.95\,\mathrm{kJ}$, approximately $+62\,\mathrm{kJ}$. Hence, the correct option is (A).

Quick Tip to Solve: For a heat capacity linear in temperature, its average equals the value at the midpoint temperature; here $C_{p,m}(650) = 29.5$ gives $3(29.5)(700) = 61950\,\mathrm{J}$.

67. The size of the iso-electronic species Cl^-, Ar and Ca^{2+} is affected by : [4 marks]

(A) electron-electron interaction in the outer orbitals

(B) Principal quantum number of valence shell

(C) nuclear charge

(D) azimuthal quantum number of valence shell

Correct Answer: (C) nuclear charge

Solution:

Concept:

- Isoelectronic species contain the same total number of electrons and have the same ground-state electronic configuration. The comparison therefore keeps the occupied shells unchanged.
- For an isoelectronic series, increasing nuclear charge increases attraction for the same electron cloud. The ionic or atomic radius consequently decreases as atomic number increases.

Step 1: Compare the electron counts and occupied shells Chlorine has atomic number 17, so Cl^- has $17 + 1 = 18$ electrons. Argon has 18 electrons, and calcium has atomic number 20, so Ca^{2+} has $20 - 2 = 18$ electrons. All three have the closed-shell configuration

$$1s^2\,2s^2\,2p^6\,3s^2\,3p^6.$$

Their outermost occupied shell has principal quantum number $n = 3$, and the valence subshell is $3p$, with azimuthal quantum number $l = 1$. Thus neither (B) nor (D) supplies a changing quantity that explains the size differences.

Step 2: Identify the changing attractive force The nuclear charges are $+17e$, $+18e$ and $+20e$, respectively. With the same number of electrons and comparable shielding arrangement, the greater positive nuclear charge draws the electron cloud inward more strongly. Hence the radius order is

$$r(Cl^-) > r(Ar) > r(Ca^{2+}).$$

Electron-electron interactions exist in all three species, but the electron count and orbital occupancy are identical; they are not the primary distinguishing factor in this isoelectronic comparison. Consequently option (A) does not explain the systematic trend, whereas option (C) does.

Step 3: Conclusion The differing sizes are governed chiefly by the differing nuclear charge. Hence, the correct option is (C).

Quick Tip to Solve: In an isoelectronic series, first verify the equal electron count, then arrange the species by atomic number: larger nuclear charge means smaller radius.

68. If solublity product of $Zr_3(PO_4)_4$ is denoted by K_{sp} and its molar solubility is denoted by S, then which of the following relation between S and K_{sp} is correct ? [4 marks]

(A) $S = \left(\frac{K_{sp}}{929}\right)^{1/9}$

(B) $S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$

(C) $S = \left(\frac{K_{sp}}{144}\right)^{1/6}$

(D) $S = \left(\frac{K_{sp}}{216}\right)^{1/7}$

Correct Answer: (B) $S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$

Solution:

Concept:

- If a salt has molar solubility S, each ion concentration is its stoichiometric coefficient in the dissolution equation multiplied by S, in the absence of a common ion.
- The solubility product multiplies the dissolved-ion concentrations, each raised to its stoichiometric coefficient. The pure solid does not appear in the equilibrium expression.

Step 1: Write the balanced dissolution equilibrium Charge balance requires zirconium(IV) ions and phosphate ions. The dissolution is

$$\mathrm{Zr_3(PO_4)_4(s) \rightleftharpoons 3\,Zr^{4+}(aq) + 4\,{PO_4}^{3-}(aq).}$$

If S moles of salt dissolve per litre, every mole produces three moles of zirconium ions and four moles of phosphate ions. Hence

$$[\mathrm{Zr^{4+}}] = 3S, \qquad [{\mathrm{PO_4}}^{3-}] = 4S.$$

Using S for each individual ion concentration would ignore the stoichiometry and give an incorrect coefficient.

Step 2: Form the product and isolate the solubility Substitute these concentrations into the solubility-product expression:

$$\begin{aligned} K_{sp} &= [\mathrm{Zr^{4+}}]^3[{\mathrm{PO_4}}^{3-}]^4 \\ &= (3S)^3(4S)^4 \\ &= 3^3\,4^4\,S^{3+4} \\ &= 27(256)S^7 = 6912S^7. \end{aligned}$$

Thus the exponent must be 7 because a formula unit produces a total of seven ions. Options (A) and (C) have the wrong root powers. Option (D) uses the correct seventh root but the wrong numerical coefficient, 216 instead of $3^3 \times 4^4 = 6912$. Only (B) includes both the correct coefficient and exponent.

Step 3: Conclusion The relation is

$$\boxed{S = \left(\frac{K_{sp}}{6912}\right)^{1/7}}.$$

Hence, the correct option is (B).

Quick Tip to Solve: For A_aB_b, use $K_{sp} = a^a b^b S^{a+b}$; both the powers of the coefficients and the total ion count matter.

69. Maltose on treatment with dilute HCl gives : [4 marks]

(A) D-Galactose

(B) D-Glucose and D-Fructose

(C) D-Glucose

(D) D-Fructose

Correct Answer: (C) D-Glucose

Solution:

Concept:

- Maltose is a disaccharide formed from two D-glucose units joined through an $\alpha(1 \rightarrow 4)$ glycosidic linkage.
- Dilute aqueous acid catalyses hydrolysis of a glycosidic bond: water supplies the elements needed to separate the two monosaccharide residues without changing their carbon skeletons.

Step 1: Identify the monosaccharides in maltose The first glucose unit is linked through its anomeric carbon, carbon 1, to the oxygen on carbon 4 of the second glucose unit. Both residues are D-glucose; maltose contains neither a fructose residue nor a galactose residue. This composition distinguishes maltose from sucrose, which contains glucose and fructose, and from lactose, which contains glucose and galactose. The identity of the constituent units, rather than the common molecular formula of these disaccharides, determines the hydrolysis products.

Step 2: Hydrolyse the glycosidic linkage and compare the options In dilute hydrochloric acid the glycosidic linkage is cleaved by water. The overall stoichiometry is

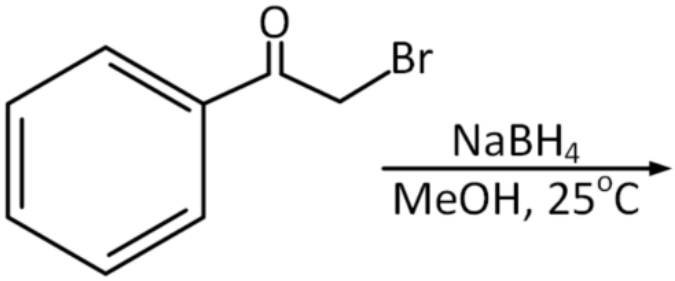
$$\mathrm{C_{12}H_{22}O_{11} + H_2O \xrightarrow{H^+} 2\,C_6H_{12}O_6.}$$

For maltose the two molecules on the right are both D-glucose. The acid acts as a catalyst and does not convert glucose into a different hexose under this hydrolysis treatment. Option (A) incorrectly introduces galactose. Option (B), glucose and fructose, is the characteristic hydrolysis pair for sucrose. Option (D) supplies only fructose, although no fructose unit is present in maltose. Therefore only option (C) names the monosaccharide actually obtained; the fact that two molecules form does not require two different product names.

Step 3: Conclusion Maltose yields two molecules of D-glucose on acid hydrolysis. Hence, the correct option is (C).

Quick Tip to Solve: Remember the constituent pairs: maltose gives glucose plus glucose, sucrose gives glucose plus fructose, and lactose gives glucose plus galactose.

70. The major product of the following reaction is : [4 marks]



(A) $C_6H_5CH(OH)CH_2Br$

(B) $C_6H_5CH(OH)CH_2OCH_3$

(C) $C_6H_5CH{=}CHOCH_3$

(D) 2-Phenyloxirane (styrene oxide; a three-membered epoxide ring)

Correct Answer: (A) $C_6H_5CH(OH)CH_2Br$

Solution:

Concept:

- Sodium borohydride is a mild hydride donor that reduces an aldehyde or ketone carbonyl to an alcohol. Methanol provides a proton to the alkoxide formed after hydride addition.
- For $C_6H_5COCH_2Br$, carbonyl reduction changes $C{=}O$ into $CH(OH)$ while retaining the aromatic ring and the carbon-bromine bond under the stated mild conditions.

Step 1: Reduce the ketone carbonyl The substrate is phenacyl bromide, $C_6H_5COCH_2Br$. A hydride from BH_4^- attacks the electrophilic carbonyl carbon and the carbonyl oxygen becomes an alkoxide. Its carbon skeleton remains intact:

$$C_6H_5COCH_2Br \xrightarrow{H^-} C_6H_5CH(O^-)CH_2Br.$$

The added hydrogen is attached to the former carbonyl carbon. This step reduces the ketone without replacing bromide by hydrogen or altering the benzene ring.

Step 2: Protonate the alkoxide and select the product Methanol protonates the alkoxide to form the secondary alcohol:

$$C_6H_5CH(O^-)CH_2Br + CH_3OH \longrightarrow C_6H_5CH(OH)CH_2Br + CH_3O^-.$$

This is 2-bromo-1-phenylethan-1-ol, shown in (A). Option (B) additionally replaces bromide by methoxy, which is not the specified reduction. Option (C), an enol ether, requires a different transformation rather than hydride addition and protonation. Option (D) is an epoxide; its formation from the halohydrin requires base-promoted intramolecular substitution, rather than the direct mild carbonyl reduction in methanol. No chiral reagent is present, so neither alcohol enantiomer is selected.

Step 3: Conclusion The carbonyl group is reduced to give $C_6H_5CH(OH)CH_2Br$. Hence, the correct option is (A).

Quick Tip to Solve: For mild $NaBH_4$ reduction, replace only $C{=}O$ by $CH(OH)$ first; do not introduce substitution, dehydration or cyclisation without the needed conditions.

71. The lathanide ion that would show colour is : [4 marks]

(A) Sm^{3+}

(B) Gd^{3+}

(C) Lu^{3+}

(D) La^{3+}

Correct Answer: (A) Sm^{3+}

Solution:

Concept:

- Colour in a lanthanoid ion is commonly associated with absorption of visible light in transitions between its partially occupied $4f$ energy levels. The position of the absorption bands matters as well as the number of unpaired electrons.
- The trivalent lanthanoid configurations are obtained after removing the two $6s$ electrons and one further valence electron. Empty and completely filled $4f$ subshells do not give the usual visible f-f absorption.

Step 1: Write the trivalent electronic configurations The relevant configurations, with the xenon core suppressed only after stating it, are

$$\mathrm{Sm}^{3+} : [\mathrm{Xe}]\, 4f^5,$$
$$\mathrm{Gd}^{3+} : [\mathrm{Xe}]\, 4f^7,$$
$$\mathrm{Lu}^{3+} : [\mathrm{Xe}]\, 4f^{14},$$
$$\mathrm{La}^{3+} : [\mathrm{Xe}]\, 4f^0.$$

Samarium therefore has an incompletely filled $4f$ shell with accessible transitions that can absorb in the visible region.

Step 2: Distinguish visible colour from paramagnetism Sm^{3+} is coloured, commonly pale yellow in its salts or aqueous solutions. Although Gd^{3+} has seven unpaired electrons and is strongly paramagnetic, its lowest suitable excited electronic levels lie too high for the characteristic visible absorption: it is ordinarily colourless. The half-filled configuration must not automatically be identified as visibly coloured merely because it contains unpaired electrons. Lu^{3+}, with $4f^{14}$, and La^{3+}, with $4f^0$, are also colourless. Thus options (B), (C) and (D) fail the specific visible-colour test.

Step 3: Conclusion The ion that shows colour is Sm^{3+}, with configuration $[\mathrm{Xe}]4f^5$. Hence, the correct option is (A).

Quick Tip to Solve: For lanthanoid colour questions, check visible absorption as well as occupancy: Gd^{3+} is a familiar colourless exception despite its seven unpaired electrons.

72. Given that $E^{\Theta}_{O_2/H_2O} = 1.23\,V$ **;**

$$E^{\Theta}_{S_2O_8^{2-}/SO_4^{2-}} = 2.05\,V$$

$$E^{\Theta}_{Br_2/Br^-} = 1.09\,V$$

$$E^{\Theta}_{Au^{3+}/Au} = 1.4\,V$$

The strongest oxidizing agent is : [4 marks]

(A) O_2

(B) Au^{3+}

(C) Br_2

(D) $S_2O_8^{2-}$

Correct Answer: (D) $S_2O_8^{2-}$

Solution:

Concept:

- An oxidizing agent accepts electrons and is itself reduced. A larger standard reduction potential, E°, indicates a greater thermodynamic tendency to accept electrons under standard conditions.
- Compare the listed reduction potentials directly. Electrode potentials are intensive quantities: multiplying a half-reaction to balance electrons does not multiply its E° value.

Step 1: Identify the electron acceptors in the couples The species written on the oxidized side of each listed couple are O_2, $S_2O_8{}^{2-}$, Br_2, and Au^{3+}. For example, peroxodisulfate accepts two electrons according to

$$S_2O_8{}^{2-} + 2\,e^- \longrightarrow 2\,SO_4{}^{2-}.$$

Its quoted standard reduction potential is $2.05\,V$. Oxygen, bromine and gold(III) also accept electrons in their respective reduction half-reactions, so the comparison must use the same reduction-potential convention for all four.

Step 2: Rank the standard reduction potentials The numerical ordering is

$$2.05 > 1.40 > 1.23 > 1.09 \quad (V).$$

Thus the oxidizing-strength order under the specified standard conditions is

$$S_2O_8{}^{2-} > Au^{3+} > O_2 > Br_2.$$

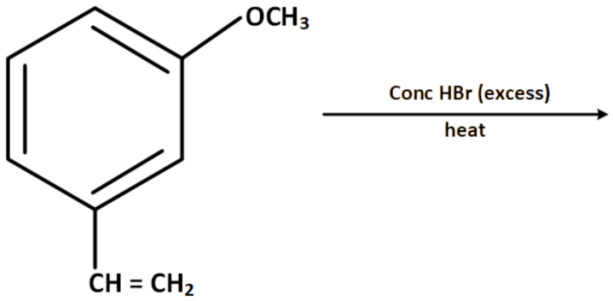
Option (A), oxygen, has a lower potential of $1.23\,V$; option (B), gold(III), has $1.40\,V$; option (C), bromine, has $1.09\,V$. None exceeds peroxodisulfate. The number of electrons in a half-reaction is not an alternative ranking criterion, and a more negative ionic charge does not by itself imply weaker oxidizing power.

Step 3: Conclusion $S_2O_8{}^{2-}$, having the largest standard reduction potential $2.05\,V$, is the strongest

oxidizing agent. Hence, the correct option is (D).

Quick Tip to Solve: For oxidizing strength, choose the oxidized member of the couple with the largest reduction potential; do not multiply potentials while balancing electrons.

73. The major product of the following reaction is : [4 marks]



(A) m-$BrC_6H_4CH(Br)CH_3$

(B) m-$HOC_6H_4CH(Br)CH_3$

(C) m-$BrC_6H_4CH_2CH_2Br$

(D) m-$HOC_6H_4CH_2CH_2Br$

Correct Answer: (B) m-$HOC_6H_4CH(Br)CH_3$

Solution:

Concept:

- Hot concentrated HBr cleaves an aryl methyl ether at its methyl–oxygen bond. Bromide attacks the methyl group of the protonated ether; it does not replace the oxygen directly on an aromatic sp^2 carbon.
- In the absence of peroxides, HBr adds to a styrene-type double bond in the Markovnikov direction through the more stable benzylic carbocation. Excess reagent allows both transformations.

Step 1: Convert the methoxy group into a phenolic group The starting compound contains an aryl $-OCH_3$ group and a vinyl $-CH{=}CH_2$ group in the meta relationship. Protonation of the ether oxygen is followed by bromide attack on the methyl carbon:

$$Ar{-}OCH_3 + HBr \longrightarrow Ar{-}OH + CH_3Br.$$

The aryl–oxygen bond remains intact because ordinary S_N2 displacement cannot take place at the aromatic carbon. Consequently the product must carry $-OH$, not an aryl $-Br$ group. This rules out options (A) and (C).

Step 2: Add HBr across the side-chain double bond Proton addition to the terminal carbon of $Ar{-}CH{=}CH_2$ gives the resonance-stabilized benzylic cation $Ar{-}CH^{+}{-}CH_3$. Bromide then attacks that carbon, producing $Ar{-}CH(Br){-}CH_3$. The alternative $Ar{-}CH_2{-}CH_2Br$ would arise from the anti-Markovnikov radical orientation and is not the outcome with the stated conditions; no peroxide initiation is specified. Combining both reactions gives m-$HOC_6H_4CH(Br)CH_3$. The two ring sub-

stituents retain their original meta positions; the ring itself undergoes neither rearrangement nor replacement of phenolic oxygen by bromine.

Step 3: Conclusion The major product is m-$HOC_6H_4CH(Br)CH_3$, the phenol with a benzylic bromide side chain. Hence, the correct option is (B).

Quick Tip to Solve: Treat the ether bond and alkene separately: aryl methyl ether gives phenol, while ordinary HBr puts bromine on the benzylic carbon of the vinyl group.

74. In order to oxidise a mixture of one mole of each of FeC_2O_4, $Fe_2(C_2O_4)_3$, $FeSO_4$ and $Fe_2(SO_4)_3$ in acidic medium, the number of moles of $KMnO_4$ required is : [4 marks]

(A) 1.5

(B) 3

(C) 2

(D) 1

Correct Answer: (C) 2

Solution:

Concept:

- In acidic medium, permanganate is reduced from manganese(VII) to manganese(II): $MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O$. One mole of $KMnO_4$ therefore accepts five moles of electrons.
- Iron(II) loses one electron on oxidation to iron(III). Each oxalate ion loses two electrons on conversion to carbon dioxide: $C_2O_4^{2-} \longrightarrow 2\,CO_2 + 2\,e^-$. Iron(III) and sulfate supply no electrons in these reactions.

Step 1: Count the reducing equivalents of each mole One mole of FeC_2O_4 contains one mole of Fe^{2+} and one mole of oxalate, so it loses $1 + 2 = 3$ moles of electrons. One mole of $Fe_2(C_2O_4)_3$ already contains ferric iron, but its three oxalate ions lose $3 \times 2 = 6$ moles of electrons. One mole of $FeSO_4$ contains one mole of ferrous iron and loses one mole of electrons. Finally, $Fe_2(SO_4)_3$ contains ferric iron and sulfate, so contributes zero reducing equivalents. Counting ferric iron again would overestimate the oxidant requirement.

Step 2: Balance the total electron loss against permanganate The total electron loss from the stated mixture is

$$n(e^-) = 3 + 6 + 1 + 0 = 10\,\text{mol}.$$

Equating this to the five-electron accepting capacity of each mole of permanganate gives

$$5n(KMnO_4) = 10, \qquad n(KMnO_4) = 2\,\text{mol}.$$

The proposed amounts 1.5, 3 and 1 mol would accept 7.5, 15 and 5 mol of electrons respectively, rather than the required 10. Thus options (A), (B) and (D) do not satisfy electron conservation.

Step 3: Conclusion The required amount of $KMnO_4$ is $2\,\text{mol}$. Hence, the correct option is (C).

Quick Tip to Solve: For mixtures of ferrous and oxalate salts, add one electron per Fe^{2+} and two per oxalate, then divide by five for acidic permanganate.

75. For the reaction 2A + B → C, the values of initial rate at diffrent reactant concentrations are given in the table below. The rate law for the reaction is :

[A] (mol L^{-1})	[B] (mol L^{-1})	Initial Rate (mol $L^{-1}s^{-1}$)
0.05	0.05	0.045
0.10	0.05	0.090
0.20	0.10	0.72

[4 marks]

(A) Rate = k[A][B]^2

(B) Rate = k[A]^2[B]^2

(C) Rate = k[A]^2[B]

(D) Rate = k[A][B]

Correct Answer: (A) Rate = k[A][B]^2

Solution:

Concept:

- Write the experimental rate law as $r = k[A]^m[B]^n$, where m and n are found from initial-rate comparisons. They need not equal the coefficients in the overall reaction.
- When concentrations are changed by known factors, divide the two rate equations to eliminate k. First isolate one concentration where possible, then use the remaining experiment to find the other order.

Step 1: Find the order with respect to A Compare the first and second experiments. The concentration of B remains $0.05\,\text{mol}\,\text{L}^{-1}$, whereas A doubles from 0.05 to $0.10\,\text{mol}\,\text{L}^{-1}$. The rate also doubles, from 0.045 to $0.090\,\text{mol}\,\text{L}^{-1}\,\text{s}^{-1}$. Hence

$$\frac{0.090}{0.045} = \left(\frac{0.10}{0.05}\right)^m, \qquad 2 = 2^m, \qquad m = 1.$$

This immediately excludes options (B) and (C), both of which predict a first-to-second rate ratio of four.

Step 2: Determine the B order and verify the law Between experiments two and three, both concentrations double, while the rate rises by a factor of eight:

$$\frac{0.72}{0.090} = 2^m 2^n = 2^{1+n}.$$

Therefore $8 = 2^{1+n}$, giving $n = 2$. The resulting law is $r = k[A][B]^2$. A numerical check from experiment one gives

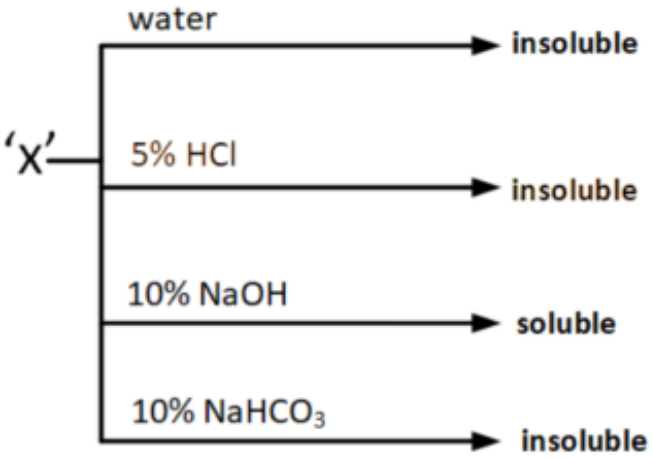
$$k = \frac{0.045}{(0.05)(0.05)^2} = 360\,\text{L}^2\,\text{mol}^{-2}\,\text{s}^{-1}.$$

It predicts experiment three as $360(0.20)(0.10)^2 = 0.72$, as required. Option (D) would predict only a fourfold rise when both concentrations double, whereas the observed rise is eightfold.

Step 3: Conclusion The reaction is first order in A and second order in B: $\boxed{\text{Rate} = k[A][B]^2}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the experiments with B unchanged first. After finding the A order, divide the eightfold rate change by A's twofold contribution to obtain B's fourfold contribution.

76. An organic compound 'X' showing the following solubility profile is : [4 marks]



(A) o-Toluidine

(B) Benzamide

(C) Oleic acid

(D) m-Cresol

Correct Answer: (D) m-Cresol

Solution:

Concept:

- An organic substance may become water-soluble when acid or base converts it into an ionic salt. Dilute hydrochloric acid tests basic compounds, while sodium hydroxide can dissolve acidic compounds by deprotonation.
- Phenols are acidic enough to form phenoxide ions with NaOH, but normally do not react with $\mathrm{NaHCO_3}$. Carboxylic acids are stronger acids and do react with bicarbonate, releasing $\mathrm{CO_2}$.

Step 1: Translate the four observations into functional-group tests The profile shows insolubility in water and in 5% hydrochloric acid, solubility in 10% sodium hydroxide, and insolubility in 10% sodium bicarbonate. Insolubility in hydrochloric acid excludes a readily protonated aromatic amine. Dissolution in hydroxide indicates an acidic group that forms a soluble salt. Failure to dissolve in bicarbonate distinguishes a weak phenolic acid from a carboxylic acid. These observations together are characteristic of a water-insoluble phenol.

Step 2: Test each named compound against the profile m-Cresol contains a phenolic $-\mathrm{OH}$ group and a methyl group on an aromatic ring. Its hydroxide reaction is

$$\mathrm{CH_3C_6H_4OH + NaOH \longrightarrow CH_3C_6H_4ONa + H_2O.}$$

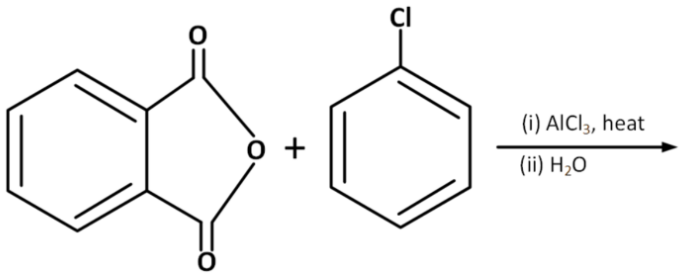
The resulting ionic cresolate is soluble. Bicarbonate is too weak a base to deprotonate cresol appreciably, so that test remains negative. In contrast, o-toluidine is an amine and forms a soluble ammonium salt with hydrochloric acid, contradicting the acid test. Benzamide is not a phenolic acid and does not show this simple hydroxide-to-phenoxide solubility response. Oleic acid has a carboxyl

group and reacts with bicarbonate, contradicting the final observation. Thus options (A), (B) and (C) each fail a decisive part of the profile.

Step 3: Conclusion The compound is m-cresol: insoluble in water, HCl and bicarbonate solution, but soluble in aqueous NaOH through phenoxide formation. Hence, the correct option is (D).

Quick Tip to Solve: NaOH-positive but bicarbonate-negative solubility points to a phenol; bicarbonate-positive solubility instead points to a carboxylic acid.

77. The major product of the following reaction is : [4 marks]



(A) 2-Chloroanthracene-9,10-dione (2-chloroanthraquinone)

(B) 2-(3-Chlorobenzoyl)benzoic acid

(C) 1-Chloroanthracene-9,10-dione (1-chloroanthraquinone)

(D) 2-(4-Chlorobenzoyl)benzoic acid

Correct Answer: (D) 2-(4-Chlorobenzoyl)benzoic acid

Solution:

Concept:

- Phthalic anhydride reacts with an aromatic substrate under $AlCl_3$-catalysed Friedel–Crafts acylation to give an ortho-carboxybenzophenone after aqueous work-up. Ring opening creates the acylating functionality and leaves a carboxyl group next to the new benzoyl linkage.
- Chlorine deactivates an aromatic ring inductively but directs electrophilic substitution to ortho and para positions through resonance donation. For a bulky acylating group, the less crowded para product is favoured.

Step 1: Identify the intermediate skeleton and the directing effect The reagents are phthalic anhydride and chlorobenzene with $AlCl_3$, followed by water. Lewis-acid activation of the anhydride enables electrophilic acylation of the chlorobenzene ring. Hydrolysis of the aluminium-containing intermediate gives a benzophenone bearing a carboxyl group ortho to its carbonyl on the original phthalic ring. Chlorine remains attached to the other ring; it is not displaced in this electrophilic substitution.

Step 2: Choose the para-acylated open-chain acid The bulky acyl group enters mainly para to chlorine rather than ortho. The product is therefore

$$\text{2-(4-chlorobenzoyl)benzoic acid},$$

which can be represented by the connectivity $o\text{-}HO_2C-C_6H_4-CO-C_6H_4-Cl$, with chlorine para

to the ketone attachment on the second ring. Option (B) places chlorine meta to that attachment and conflicts with the ortho/para directing effect. Options (A) and (C) depict already cyclized chloroanthraquinones, containing a second ketone in a new central ring. Obtaining that anthraquinone skeleton requires a subsequent cyclizing/dehydrating conversion of the benzoylbenzoic acid; the indicated aqueous work-up gives the acid and does not supply that separate conversion.

Step 3: Conclusion The major product is 2-(4-chlorobenzoyl)benzoic acid, with para substitution on the chlorobenzene-derived ring. Hence, the correct option is (D).

Quick Tip to Solve: With phthalic anhydride, identify the benzoylbenzoic-acid skeleton first; apply the ring substituent's directing effect before considering any later anthraquinone cyclization.

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78. In the following compounds, the decreasing order of basic strength will be : [4 marks]

(A) $(C_2H_5)_2NH > NH_3 > C_2H_5NH_2$

(B) $NH_3 > C_2H_5NH_2 > (C_2H_5)_2NH$

(C) $(C_2H_5)_2NH > C_2H_5NH_2 > NH_3$

(D) $C_2H_5NH_2 > NH_3 > (C_2H_5)_2NH$

Correct Answer: (C) $(C_2H_5)_2NH > C_2H_5NH_2 > NH_3$

Solution:

Concept:

- A base accepts a proton by donating its available lone pair. The more readily nitrogen donates its lone pair, the greater the basic strength of the amine.
- An ethyl group has a positive inductive effect: it releases electron density towards nitrogen. Alkylamines are therefore more basic than ammonia; for these compounds diethylamine is more basic than ethylamine.

Step 1: Identify the proton-accepting site Each compound contains nitrogen with a lone pair. Protonation gives an ammonium ion:

$$NH_3 + H^+ \longrightarrow NH_4^+, \qquad C_2H_5NH_2 + H^+ \longrightarrow C_2H_5NH_3^+.$$

Ethyl groups increase electron density at the atom that binds the proton, so replacing hydrogen by ethyl strengthens the base.

Step 2: Compare the number of electron-releasing groups Ammonia has no ethyl group, ethylamine has one and diethylamine has two. The cumulative inductive effect of two ethyl groups makes the nitrogen lone pair in diethylamine more available than in ethylamine. In aqueous solution the protonated primary and secondary amines are also effectively solvated; the secondary amine remains the stronger of these two.

Step 3: Match the complete order The order is

$$(C_2H_5)_2NH > C_2H_5NH_2 > NH_3.$$

Option (A) incorrectly puts ammonia above ethylamine. Option (B) reverses both comparisons, while (D) incorrectly makes the secondary amine the weakest base.

Step 4: Conclusion The decreasing basic-strength order is $(C_2H_5)_2NH > C_2H_5NH_2 > NH_3$. Hence, the correct option is (C).

Quick Tip to Solve: For ammonia, ethylamine and diethylamine, count the ethyl groups attached directly to nitrogen: their electron-releasing effect strengthens basicity in this sequence.

79. The vapour pressures of pure liquids A and B are 400 and 600 mmHg, respectively at 298 K on mixing the two liquids, the sum of their initial volume is equal ot the volume of the final mixture. The mole fraction of liquid B is 0.5 in the mixture, The vapour pressure of the final solution, the mole fractions of components A and B in vapour phase, respectively are : [4 marks]

(A) 500 mmHg. 0.5,0.5

(B) 500 mmHg, 0.4, 0.6

(C) 450 mmHg, 0.4,0.6

(D) 450 mmHg.0.5,0.5

Correct Answer: (B) 500 mmHg, 0.4, 0.6

Solution:

Concept:

- For an ideal binary solution, Raoult's law gives $p_A = x_A p_A^0$ and $p_B = x_B p_B^0$, where x_i is a liquid-phase mole fraction and p_i^0 is the pure-liquid vapour pressure.
- The total pressure is $P = p_A + p_B$. Dalton's law gives the vapour mole fractions $y_A = p_A/P$ and $y_B = p_B/P$; these generally differ from the liquid mole fractions.

Step 1: Find both liquid mole fractions The mixture has $x_B = 0.5$, so $x_A = 1 - 0.5 = 0.5$. The stated absence of a volume change is the intended ideal-solution condition in this question. Thus apply Raoult's law at the specified temperature of $298\,\text{K}$.

Step 2: Calculate partial and total pressures Using the pure-component pressures,

$$\begin{aligned} p_A &= 0.5(400) = 200\,\text{mmHg}, \\ p_B &= 0.5(600) = 300\,\text{mmHg}, \\ P &= 200 + 300 = 500\,\text{mmHg}. \end{aligned}$$

The component with the larger pure vapour pressure contributes more to the total pressure even though the liquid amounts are equimolar.

Step 3: Calculate the vapour composition Divide each partial pressure by the total:

$$y_A = \frac{200}{500} = 0.4, \qquad y_B = \frac{300}{500} = 0.6.$$

These fractions sum to one. Option (A) incorrectly copies the liquid composition into the vapour. Options (C) and (D) give $450\,\text{mmHg}$, inconsistent with the sum of the calculated partial pressures.

Step 4: Conclusion Thus $P = 500\,\text{mmHg}$, $y_A = 0.4$ and $y_B = 0.6$. Hence, the correct option is (B).

Quick Tip to Solve: Use liquid mole fractions in Raoult's law first, then divide the resulting partial pressures by their sum to obtain the vapour mole fractions.

80. Which one of the following equations does not correctly represent the first law of thermodynamics for the given processes involving an ideal gas? (Assume non-expansion work is zero) [4 marks]

(A) Adiabatic process : $\Delta U = -w$

(B) Cyclic process : $q = -w$

(C) Isochoric process : $\Delta U = q$

(D) Isothermal process : $q = -w$

Correct Answer: (A) Adiabatic process : $\Delta U = -w$

Solution:

Concept:

- The chemistry sign convention takes heat supplied to the system as positive and work done on the system as positive. The first law is $\Delta U = q + w$, with expansion work $w = -\int P_{\text{ext}}\, dV$.
- For an ideal gas, internal energy depends only on temperature. Therefore an isothermal change has $\Delta U = 0$; a complete cycle also has zero internal-energy change because internal energy is a state function.

Step 1: Apply the adiabatic condition An adiabatic process exchanges no heat, so $q = 0$. Substituting into the first law gives

$$\Delta U = 0 + w = w.$$

Thus the stated equation $\Delta U = -w$ has the wrong sign under the work convention used by the other alternatives. For expansion, $w < 0$, so the correct equation also predicts the expected decrease in internal energy.

Step 2: Check the cyclic and isochoric cases In a cyclic process the initial and final states coincide, giving $\Delta U = 0$. Therefore $q + w = 0$, or $q = -w$, so (B) is correct. In an isochoric process, $dV = 0$. With non-expansion work excluded, $w = 0$, hence $\Delta U = q$, so (C) is correct.

Step 3: Check the isothermal case For an ideal gas at fixed temperature, $\Delta U = 0$. The first law then gives $q = -w$, exactly the equation in (D). The last three alternatives are therefore consistent, leaving only the adiabatic equation as incorrect.

Step 4: Conclusion For an adiabatic process the correct relation is $\Delta U = w$, rather than $\Delta U = -w$. Hence, the correct option is (A).

Quick Tip to Solve: Write $\Delta U = q + w$ once, then set the quantity fixed by the process to zero. This avoids mixing work done by the gas with work done on it.

81. The quantum number of four electrons are given below :

n = 4, l = 2, m_l =–2, m_s = –1/2

n = 3, l = 2, m_l = 1, m_s = +1/2

n = 4, l = 1, m_l = 0, m_s = +1/2

n = 3, l = 1, m_l = 1, m_s = –1/2

The correct order of their increasing enegies will be : [4 marks]

(A) IV < II < III < I

(B) I < III < II < IV

(C) IV < III < II < I

(D) I < II < III < IV

Correct Answer: (A) IV < II < III < I

Solution:

Concept:

- For the usual ordering of orbitals in a many-electron atom, compare $n + l$: the orbital with the smaller value has lower energy. If $n + l$ is equal, the orbital with smaller n has lower energy.
- The azimuthal quantum numbers $l = 0, 1, 2$ correspond to s, p and d orbitals. The values of m_l and m_s specify orbital orientation and spin; they do not change this ordering in the absence of an external field.

Step 1: Identify each orbital The four sets are listed in their stated order as I, II, III and IV. Their (n, l) values identify I as $4d$, II as $3d$, III as $4p$ and IV as $3p$. The different magnetic and spin quantum numbers do not require a different energy comparison here.

Step 2: Evaluate the energy-ordering sums Calculate the sum for each orbital:

$$\begin{aligned} \text{I}: &\quad n + l = 4 + 2 = 6, \\ \text{II}: &\quad n + l = 3 + 2 = 5, \\ \text{III}: &\quad n + l = 4 + 1 = 5, \\ \text{IV}: &\quad n + l = 3 + 1 = 4. \end{aligned}$$

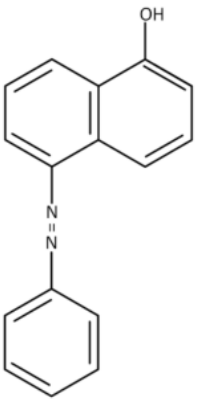
Thus IV must be the lowest and I the highest of these four. This immediately excludes (B) and (D).

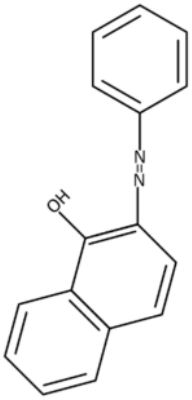
Step 3: Resolve the tie between the middle orbitals Both $3d$ and $4p$ have $n + l = 5$. Their principal quantum numbers are 3 and 4, respectively, so $3d$ lies below $4p$. Therefore II precedes III. Option (C) reverses precisely this tied pair, while (A) has every comparison in the correct order.

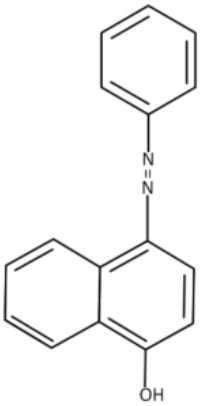
Step 4: Conclusion The increasing energy order is $3p < 3d < 4p < 4d$, that is, IV < II < III < I. Hence, the correct option is (A).

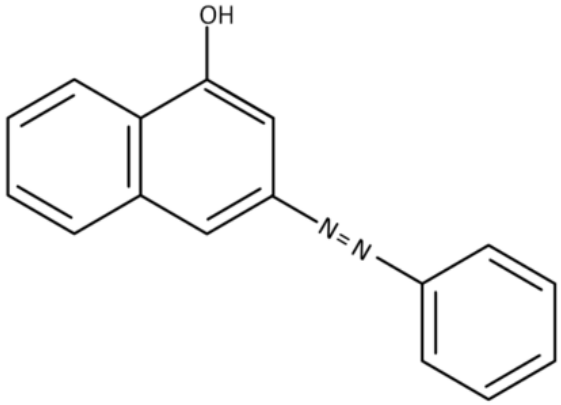
Quick Tip to Solve: List $n + l$ beside every quantum-number set before sorting; use n only to break a tie, and do not sort these orbitals by spin or magnetic quantum number.

82. Coupling of benzene diazonium chloride with 1 - naphthol in alkaline medium will give : [4 marks]

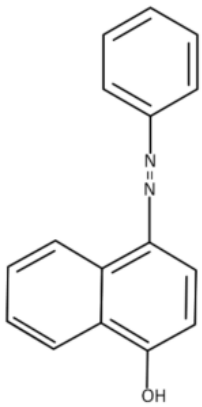
(A) 

(B) 

(C) 

(D) 

Correct Answer: (C)



Solution:

Concept:

- Aromatic diazonium ions undergo electrophilic aromatic substitution with activated aromatic rings. The new bond produces an azo linkage, $\mathrm{Ar{-}N{=}N{-}Ar'}$, joining the two aromatic systems.
- In alkaline medium 1-naphthol forms the resonance-stabilised naphthoxide ion. The oxygen substituent activates the same ring towards coupling, particularly its 4-position, para to the oxygen-bearing 1-position.

Step 1: Activate the naphthol ring The alkaline medium removes the phenolic proton:

$$\mathrm{C_{10}H_7OH + OH^- \rightleftharpoons C_{10}H_7O^- + H_2O.}$$

The negatively charged oxygen donates electron density into the aromatic system by resonance, making the ring sufficiently nucleophilic to react with the relatively weak diazonium electrophile. The carbon skeleton of naphthalene is retained.

Step 2: Locate the coupling position Number the carbon bearing the hydroxyl group as position 1. In 1-naphthol the favoured coupling site is position 4, the para position within that oxygen-bearing ring. Attack on the terminal nitrogen of the benzene diazonium ion forms a carbon–nitrogen bond, followed by loss of the ring proton and restoration of aromaticity. The diazonium nitrogen pair remains as the $\mathrm{-N{=}N-}$ bridge.

Step 3: Identify the matching structure The product is 4-(phenyldiazenyl)naphthalen-1-ol, conventionally called 4-phenylazo-1-naphthol. Structure (C) shows the phenylazo group opposite the hydroxyl group on the same naphthalene ring. Structure (A) places the linkage on the other fused ring, (B) at the adjacent position, and (D) at the meta position of the hydroxyl-bearing ring. Those connectivities do not represent the principal para-coupling product.

Step 4: Conclusion Azo coupling occurs at the 4-position of 1-naphthol, producing the para-linked phenylazo dye shown in (C). Hence, the correct option is (C).

Quick Tip to Solve: Before comparing rotated naphthalene drawings, fix the carbon bearing OH as position 1 and trace the same six-membered ring to its para carbon at position 4.

83. Which is wrong with respect to our responsiblity as a human being to protect our environment?

[4 marks]

(A) Restricting the use of vehicles

(B) Setting up compost tin in gardens.

(C) Avoiding the use of floodlighted facilities

(D) Using plastic bags.

Correct Answer: (D) Using plastic bags.

Solution:

Concept:

- Environmental protection includes reducing pollutant generation, conserving energy and managing waste at its source. A responsible action should reduce a persistent burden on air, water, soil or living organisms.
- Conventional plastic bags are persistent waste: they degrade very slowly, may fragment into microplastics, can block drains and can harm animals. Avoiding unnecessary single-use bags reduces this waste stream.

Step 1: Assess the transport and organic-waste actions Restricting unnecessary vehicle use reduces fuel consumption and emissions such as carbon monoxide, nitrogen oxides and particulate matter. Therefore (A) is a protective action. A compost container in a garden converts suitable biodegradable kitchen and garden waste into useful manure, reducing the amount of organic waste discarded in mixed rubbish. Thus the action described in (B) is also environmentally beneficial.

Step 2: Assess lighting and plastic-bag use Avoiding unnecessary floodlighted facilities saves electrical energy and reduces light pollution, so (C) represents a useful conservation measure. In contrast, using disposable plastic bags adds non-biodegradable litter and increases the demand for plastic production. Careless disposal may contaminate soil and water and obstruct drainage.

Step 3: Select the action that conflicts with the responsibility The question asks which behaviour is wrong, rather than which behaviour protects the environment. Only (D) increases a persistent waste problem. Reusable bags used repeatedly are a better practical choice, while vehicle restriction, composting and limiting unnecessary floodlighting all reduce an environmental burden.

Step 4: Conclusion Using plastic bags conflicts with the stated responsibility to protect the environment. Hence, the correct option is (D).

Quick Tip to Solve: For an environmental-responsibility question, compare the waste or energy consequence of every action, then check whether the question asks for the helpful action or the harmful one.

84. With respect to an ore, Ellingham diagram helps to predict the feasibility of its. [4 marks]

(A) Electrolysis

(B) Thermal reducation

(C) Vapour phase refining

(D) Zone refining

Correct Answer: (B) Thermal reducation

Solution:

Concept:

- An Ellingham diagram plots the standard Gibbs energy change ΔG° for oxide formation against temperature. It compares the thermodynamic stability of different metal oxides at a selected temperature.
- For a proposed reduction, combine oxide decomposition with oxidation of the reducing agent. The overall standard Gibbs energy must be negative for the reduction to be thermodynamically favourable under standard-state conditions.

Step 1: Interpret the relative positions of oxidation lines A lower oxidation line means a more negative Gibbs energy of oxide formation and hence a more stable oxide at that temperature. If oxidation of a reducing agent has a line below the formation line of the metal oxide, the reducing agent can take oxygen from that oxide with a negative overall Gibbs energy change.

Step 2: Apply the diagram to oxide reduction For a suitable reducing metal R, represent the oxygen transfer schematically as

$$\mathrm{MO} + \mathrm{R} \longrightarrow \mathrm{M} + \mathrm{RO}.$$

Its Gibbs energy change is the oxide-formation Gibbs energy for RO minus that for MO, using consistent stoichiometry. An analogous comparison with carbon or carbon monoxide oxidation lines predicts favourable temperature ranges for metallurgical reduction.

Step 3: Distinguish the other processes This is a prediction of thermal reduction, the process in (B). Electrolysis in (A) is driven by electrical energy supplied to an electrolytic cell. Vapour phase refining in (C) depends on forming and decomposing a volatile compound, while zone refining in (D) depends on different impurity solubilities in solid and molten phases. Their defining mechanisms are not what the oxide-stability lines directly describe.

Step 4: Conclusion The Ellingham diagram predicts the thermodynamic feasibility of thermal reduction of an oxide ore at a given temperature. Hence, the correct option is (B).

Quick Tip to Solve: To test a reducing agent, compare oxidation lines at the same temperature: a lower line for the agent means it has the stronger thermodynamic tendency to bind oxygen.

85. Diborane (B_2H_6) reacts with O_2 and H_2O independently, then products formed are : [4 marks]

(A) B_2O_3 and HBO_2

(B) B_2O_3 and H_3BO_3

(C) HBO_2 and H_3BO_3

(D) HBO_2 and HBO_3

Correct Answer: (B) B_2O_3 and H_3BO_3

Solution:

Concept:

- Diborane, B_2H_6, is a boron hydride. Its reaction with oxygen is combustion, whereas its reaction with water is hydrolysis; these are separate reactions.
- Combustion converts boron into boron(III) oxide, B_2O_3. Hydrolysis replaces boron–hydrogen bonds by boron–hydroxyl bonds and releases hydrogen.

Step 1: Identify the product formed with oxygen Diborane burns in oxygen to form boron(III) oxide and water. Balance the boron atoms first, then the hydrogen atoms, and finally the oxygen atoms:

$$B_2H_6 + 3\,O_2 \longrightarrow B_2O_3 + 3\,H_2O.$$

There are two boron atoms, six hydrogen atoms and six oxygen atoms on each side. Thus the boron-containing product of the first reaction is B_2O_3, not metaboric acid, HBO_2.

Step 2: Identify the product formed with water Hydrolysis gives orthoboric acid, commonly called boric acid, together with hydrogen gas:

$$B_2H_6 + 6\,H_2O \longrightarrow 2\,H_3BO_3 + 6\,H_2.$$

Both boron atoms occur in the two molecules of H_3BO_3, and the six water molecules supply the six oxygen atoms. The hydrogen count is eighteen on either side. This reaction does not require oxygen gas, because the question asks for independent reactions.

Step 3: Compare the listed pairs The required pair of boron-containing products, in the stated order of reagents, is therefore B_2O_3 and H_3BO_3. Option (A) incorrectly uses metaboric acid as the hydrolysis product. Options (C) and (D) incorrectly give HBO_2 for combustion; (D) also fails to give the correct boric-acid formula. Only option (B) satisfies both reactions.

Step 4: Conclusion The products are B_2O_3 with oxygen and H_3BO_3 with water. Hence, the correct option is (B).

Quick Tip to Solve: Write the combustion and hydrolysis equations separately: oxygen gives the oxide, whereas water gives boric acid and hydrogen.

86. The correct order of hydration enthapies of alkali metal ions is : [4 marks]

(A) $Li^+ > Na^+ > Cs^+ > Rb^+$

(B) $Na^+ > Li^+ > K^+ > Cs^+ > Rb^+$

(C) $Na^+ > Li^+ > K^+ > Rb^+ > Cs^+$

(D) $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$

Correct Answer: (D) $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$

Solution:

Concept:

- Hydration of a gaseous metal ion is exothermic because water molecules are attracted to the ion. The usual hydration-enthalpy trend compares the magnitudes of this released energy.
- For ions with the same charge, a smaller ionic radius gives higher charge density and stronger ion–dipole attraction. The magnitude of hydration enthalpy therefore decreases as ionic size increases.

Step 1: Arrange the alkali-metal ions by size Each listed alkali-metal ion has charge $+1$, so differences in charge do not determine the trend. Down the group, the number of occupied electron shells increases and the ionic radius increases:

$$r(Li^+) < r(Na^+) < r(K^+) < r(Rb^+) < r(Cs^+).$$

Lithium ion is the smallest and caesium ion the largest. The small lithium ion exerts the strongest electric field on nearby polar water molecules.

Step 2: Convert the size trend into the hydration trend The oxygen end of each water molecule points toward the positive ion. Stronger attraction stabilises the hydrated ion more and releases more energy. Hence the order of hydration-enthalpy magnitudes is

$$|\Delta H_{hyd}(Li^+)| > |\Delta H_{hyd}(Na^+)| > |\Delta H_{hyd}(K^+)| > |\Delta H_{hyd}(Rb^+)| > |\Delta H_{hyd}(Cs^+)|.$$

The actual signed enthalpies are negative: lithium has the most negative value. The option notation conventionally expresses the magnitude of hydration, rather than the algebraic order of negative numbers.

Step 3: Eliminate the alternative orders Option (A) places caesium above rubidium, although caesium is larger and less strongly hydrated; it also omits potassium. Options (B) and (C) place sodium above lithium, contrary to the much smaller radius of lithium ion. Option (B) additionally reverses rubidium and caesium. Option (D) alone follows the complete decrease in hydration magnitude down the group.

Step 4: Conclusion The hydration-enthalpy magnitudes follow $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$. Hence, the correct option is (D).

Quick Tip to Solve: For singly charged ions, rank hydration strength opposite to ionic radius; remember that stronger hydration means a more negative signed enthalpy.

87. 100 mL of a water sample contains 0.81 g of calcium bicrabonate and 0.73 g of magnesium bicarbonate. The hardness of this water sample expressed in terms of equivalents of $CaCO_3$ is : (Molar mass of calcium bicarbonate is 162 g mol^{-1} and magnesium bicarbonate is 146 g mol^{-1}) [4 marks]

(A) 1,000 ppm

(B) 100 ppm

(C) 10,000 ppm

(D) 5,000 ppm

Correct Answer: (C) 10,000 ppm

Solution:

Concept:

- Water hardness is reported as the concentration of an equivalent amount of $CaCO_3$, usually in $mg\,L^{-1}$, which is numerically equal to ppm for dilute water samples.
- Each mole of $Ca(HCO_3)_2$ or $Mg(HCO_3)_2$ provides one mole of a divalent metal ion. It is therefore equivalent to one mole, or $100\,g$, of $CaCO_3$.

Step 1: Convert the calcium salt to calcium-carbonate equivalent The sample volume is $100\,mL = 0.100\,L$. Using the supplied molar mass, the amount of calcium bicarbonate is

$$n_{Ca} = \frac{0.81}{162} = 0.0050\,mol.$$

Its calcium-carbonate equivalent is

$$m_{eq,Ca} = 0.0050 \times 100 = 0.50\,g.$$

Equivalently, the salt and calcium carbonate have equivalent masses of 81 and $50\,g\,equiv^{-1}$, respectively, giving the same conversion factor.

Step 2: Add the magnesium contribution The amount of magnesium bicarbonate is

$$n_{Mg} = \frac{0.73}{146} = 0.0050\,mol.$$

Its calcium-carbonate equivalent is also $0.0050 \times 100 = 0.50\,g$. Both salts cause hardness, so their contributions must be added rather than averaged:

$$m_{eq,total} = 0.50 + 0.50 = 1.00\,g = 1000\,mg.$$

Step 3: Express the result per litre and compare options The equivalent mass above belongs to only $0.100\,L$, not one litre. Therefore

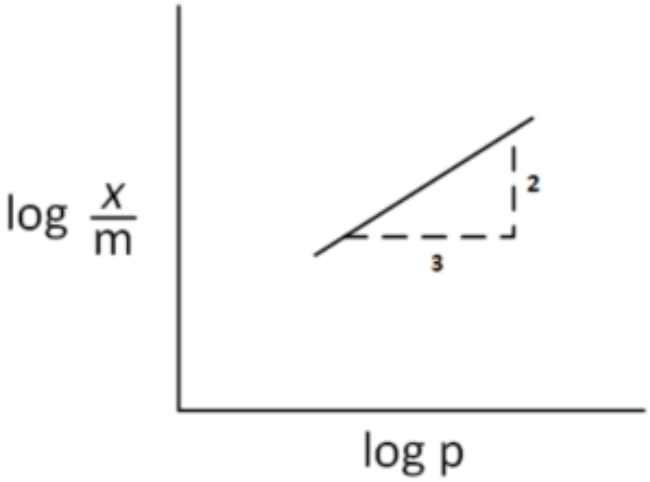
$$\text{Hardness} = \frac{1000\,mg}{0.100\,L} = 10000\,mg\,L^{-1} = 10000\,ppm.$$

Option (D), $5000\,ppm$, counts only one of the two equally contributing salts. Options (A) and (B) are too small because they do not correctly convert the sample quantity to its per-litre concentration.

Step 4: Conclusion The total hardness is $10{,}000\,ppm$ as $CaCO_3$. Hence, the correct option is (C).

Quick Tip to Solve: Convert each divalent bicarbonate to moles, multiply the total by 100 g of calcium carbonate per mole, and only then divide by the sample volume.

88. Adsorption of a gas follows freundlich adosorbed isotherm. x is the mass of the gas adsorbed on mass m of the adsorbent. The plot log $\frac{x}{m}$ versus log p is shown in the given graph. $\frac{x}{m}$ is proportional to : [4 marks]



(A) $p^{2/3}$

(B) p^3

(C) p^2

(D) $p^{3/2}$

Correct Answer: (A) $p^{2/3}$

Solution:

Concept:

- The Freundlich adsorption isotherm at fixed temperature is $x/m = kp^{1/n}$, where x is the adsorbed gas mass, m the adsorbent mass, and k, n constants.
- Taking logarithms gives $\log(x/m) = \log k + (1/n)\log p$. A plot of $\log(x/m)$ against $\log p$ therefore has slope $1/n$, the required pressure exponent.

Step 1: Read the slope from the graph The graph gives a vertical rise of 2 and a horizontal run of 3 along the straight line. Since the vertical coordinate is $\log(x/m)$ and the horizontal coordinate is $\log p$, its slope is

$$s = \frac{\Delta \log(x/m)}{\Delta \log p} = \frac{2}{3}.$$

These changes are measured along the logarithmic axes. The exponent is their ratio, not either of the individual numbers printed beside the triangle.

Step 2: Identify the pressure exponent Comparing the straight-line form of the isotherm with $y = c + sx$ gives

$$\frac{1}{n} = s = \frac{2}{3}, \qquad n = \frac{3}{2}.$$

Substituting the exponent into the original isotherm,

$$\frac{x}{m} = kp^{2/3}.$$

At fixed temperature k is constant, so it affects the vertical intercept but does not alter the requested proportionality.

Step 3: Check the distractors Option (D), $p^{3/2}$, confuses the parameter n with the actual exponent $1/n$; it also corresponds to taking run divided by rise. Options (B) and (C) use the horizontal or vertical increment alone as an exponent, instead of their ratio. A slope of $2/3$ selects only option (A). As a consistency check, this exponent is between zero and one, as in the usual Freundlich adsorption regime.

Step 4: Conclusion The graph implies $x/m \propto p^{2/3}$. Hence, the correct option is (A).

Quick Tip to Solve: In a log–log Freundlich plot, the pressure exponent equals rise divided by run; do not invert the slope to obtain the exponent.

89. Element 'B' forms ccp structure and 'A' occupies half of the octahedral voids, while oxygen atoms occupy all the tetrahedral voids, The structure of bimetallic oxide is : [4 marks]

(A) AB_2O_4

(B) A_2BO_4

(C) A_4B_2O

(D) A_2B_2O

Correct Answer: (A) AB_2O_4

Solution:

Concept:

- A cubic close-packed (ccp or fcc) unit cell contains four atoms of the element forming the lattice: $8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$.
- If a close-packed lattice contains N atoms, it has N octahedral voids and $2N$ tetrahedral voids. Multiply these counts by the stated occupied fractions before reducing the composition.

Step 1: Count the atoms of the lattice-forming element Element B forms the ccp lattice. Taking one conventional unit cell, the eight corner contributions total one atom and the six face contributions total three atoms:

$$N_B = 8\left(\frac{1}{8}\right) + 6\left(\frac{1}{2}\right) = 1 + 3 = 4.$$

The void counts must be based on these four B atoms. B is the host lattice here; neither A nor oxygen is to be counted as the close-packed host.

Step 2: Count A atoms and oxygen atoms in the voids For four host atoms, there are four octahedral voids. A occupies half of them, so

$$N_A = \frac{1}{2} \times 4 = 2.$$

The same unit cell has $2 \times 4 = 8$ tetrahedral voids. All are occupied by oxygen atoms, giving

$$N_O = 1 \times 8 = 8.$$

An occupied void contributes one whole guest atom to the unit-cell composition. The stated fractional occupancy is applied to the total void count only once.

Step 3: Reduce the atom ratio and compare the formulae The unit-cell composition is $A_2B_4O_8$. Dividing each subscript by the common factor two gives

$$N_A : N_B : N_O = 2 : 4 : 8 = 1 : 2 : 4.$$

Thus the empirical formula is AB_2O_4. Option (B) reverses the relative counts of A and B. Options (C) and (D) assign far too few oxygen atoms, although the eight tetrahedral voids are all occupied. Only option (A) preserves both independent occupancy conditions.

Step 4: Conclusion The bimetallic oxide has formula AB_2O_4. Hence, the correct option is (A).

Quick Tip to Solve: Start with N host atoms: octahedral voids are N and tetrahedral voids are 2N. Apply occupancies before simplifying the A:B:O ratio.

90. Assertion : Ozone is destroyed by CFCs in the upper stratosphere.
Reason : Ozone holes increase the amount of UV radiation the earth. [4 marks]

(A) Assertion and reason are both correct, and the reason is the correct explanation for the assertion.

(B) Assertion and reason are incorrect.

(C) Assertion and reason are correct but, the reason is not the explanation for the assertion

(D) Assertion is false, but the reason is correct.

Correct Answer: (C) Assertion and reason are correct but, the reason is not the explanation for the assertion

Solution:

Concept:

- Stratospheric ozone absorbs harmful ultraviolet radiation. Chlorofluorocarbons (CFCs) are sufficiently persistent to reach the stratosphere, where UV light can liberate chlorine radicals.
- In an Assertion–Reason question, first decide the truth of each statement separately. A true consequence of the assertion is not necessarily a causal explanation for it.

Step 1: Evaluate the assertion The assertion is correct in the standard description of stratospheric ozone depletion. Ultraviolet radiation breaks carbon–chlorine bonds in CFC molecules and produces chlorine radicals. These radicals react with ozone:

$$Cl^{\bullet} + O_3 \longrightarrow ClO^{\bullet} + O_2.$$

The chlorine-containing intermediate can then react with atomic oxygen:

$$ClO^{\bullet} + O \longrightarrow Cl^{\bullet} + O_2.$$

Chlorine is regenerated and can destroy further ozone molecules; it acts catalytically rather than being consumed after one reaction.

Step 2: Evaluate the reason The reason states that ozone holes increase the ultraviolet radiation reaching the Earth. This is also correct. A reduced stratospheric ozone column absorbs less of the incoming harmful UV radiation, especially UV-B, so more reaches the surface. Here an ozone hole refers to severe depletion of the ozone column, not a literal empty opening in the atmosphere.

Step 3: Test whether the reason explains the assertion The reason describes what happens after ozone has been depleted. It does not explain how CFCs cause that depletion. The required causal mechanism is the release of chlorine radicals and their catalytic reactions, established in the first step. Thus both statements are true but the reason does not explain the assertion. Option (A) incorrectly treats the effect as the mechanism; option (B) incorrectly rejects both statements; option (D) incorrectly rejects the assertion.

Step 4: Conclusion Both the assertion and reason are correct, but the reason is not the explanation of the assertion. Hence, the correct option is (C).

Quick Tip to Solve: For Assertion–Reason questions, ask whether the reason supplies the mechanism or cause; an environmental consequence alone does not explain the cause.