JEE Main 2019 Paper 1 (B.E./B.Tech) Jan 10 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 10 January 2019, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. If the system of equations

x + y + z = 5

x + 2y + 3z = 9

x + 3y + az = β

has infinitely many solutions, then $\beta - \alpha$ equals - [4 marks]

(A) 8

(B) 21

(C) 18

(D) 5

Correct Answer: (A) 8

Solution:

Concept:

- Infinitely many solutions require the equations to be consistent and the coefficient rank to be smaller than the number of unknowns. A dependent equation must have the same linear dependence on both sides.
- Elementary elimination subtracts equations without changing their solution set. Here the first two equations are independent, so a third dependent equation leaves one free parameter.

Step 1: Eliminate the common variable Subtract the first equation from the second and the second from the third. This gives

$$y + 2z = 4, \qquad y + (\alpha - 3)z = \beta - 9.$$

Subtract these two reduced equations in turn:

$$(\alpha - 5)z = \beta - 13.$$

If the coefficient on the left were nonzero, this equation would fix z; the preceding equation would then fix y, and the first original equation would fix x. That would be a unique solution, contrary to the requirement.

Step 2: Enforce dependence and verify the free parameter The last equation must therefore be the identity $0 = 0$, rather than a nonzero constant contradiction. Consequently,

$$\alpha - 5 = 0, \qquad \beta - 13 = 0.$$

Thus $\alpha = 5$ and $\beta = 13$. For any real parameter t, take $z = t$. Then

$$y = 4 - 2t, \qquad x = 1 + t.$$

Substitution gives $x + y + z = 5$, $x + 2y + 3z = 9$, and $x + 3y + 5z = 13$, independently of t. This explicitly verifies infinitely many solutions, not merely a zero determinant.

Step 3: Conclusion Therefore $\beta - \alpha = 13 - 5 = 8$. The other values 21, 18 and 5 do not equal this

difference. Hence, the correct option is (A).

Quick Tip to Solve: After eliminating a common variable, require the last reduced equation to be an identity: zero coefficient and zero constant are both necessary.

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2. Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k}$, and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is : [4 marks]

(A) (1, 5, 1)

(B) (1, 3, 1)

(C) $\left(-\frac{1}{2}, 4, 0\right)$

(D) $\left(\frac{1}{2}, 4, -2\right)$

Correct Answer: (C) $\left(-\frac{1}{2}, 4, 0\right)$

Solution:

Concept:

- Equal vectors have equal coefficients of each unit vector. The condition $\vec{b} = 2\vec{a}$ therefore gives three component equations, of which the middle one restricts the parameters.
- Two nonzero vectors are perpendicular exactly when their scalar product is zero: $\vec{a} \cdot \vec{c} = a_x c_x + a_y c_y + a_z c_z = 0$. Both conditions must hold for a proposed triple.

Step 1: Use the equality of vectors Doubling $\vec{a}$ gives components $(4, 2\lambda_1, 6)$. Comparing with $\vec{b} = (4, 3 - \lambda_2, 6)$, the first and third coordinates already agree. The remaining restriction is

$$2\lambda_1 + \lambda_2 = 3.$$

For option (A), the left side is $2 + 5 = 7$; for option (B), it is $2 + 3 = 5$; for option (D), it is $1 + 4 = 5$. Each fails the required vector equality, regardless of perpendicularity. Option (C) gives $-1 + 4 = 3$, so only this candidate remains.

Step 2: Check perpendicularity independently For the given general vectors,

$$\vec{a} \cdot \vec{c} = 2(3) + 6\lambda_1 + 3(\lambda_3 - 1) = 3 + 6\lambda_1 + 3\lambda_3.$$

Dividing the zero-dot-product equation by 3 gives

$$1 + 2\lambda_1 + \lambda_3 = 0.$$

With option (C), $\lambda_1 = -1/2$ and $\lambda_3 = 0$, so this expression is $1 - 1 + 0 = 0$. More explicitly, the vectors are $\vec{a} = (2, -1/2, 3)$ and $\vec{c} = (3, 6, -1)$, whose dot product is $6 - 3 - 3 = 0$. Also $\vec{b} = (4, -1, 6) = 2\vec{a}$, verifying both demands.

Step 3: Conclusion The possible triple is $\left(-\frac{1}{2}, 4, 0\right)$, which satisfies both vector equality and perpendicularity. Hence, the correct option is (C).

Quick Tip to Solve: Filter proposed triples using the simplest component equation first, then verify the dot product only for the survivors.

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3. **Let** $f(x) = \begin{cases} \max\{|x|, x^2\} & |x| \le 2 \\ 8 - 2|x| & 2 < |x| \le 4 \end{cases}$

Let S be the set of points in the interval (– 4, 4) at which f is not differentiable. Then S [4 marks]

(A) equals $\{-2, -1, 1, 2\}$

(B) equals $\{-2, -1, 0, 1, 2\}$

(C) equals $\{-2, 2\}$

(D) is an empty set

Correct Answer: (B) equals $\{-2, -1, 0, 1, 2\}$

Solution:

Concept:

- For $0 \le |x| \le 1$, the maximum of $|x|$ and x^2 is $|x|$; for $1 \le |x| \le 2$, it is x^2. Splitting the maximum exposes every potential corner.
- Each individual formula is differentiable on its open interval. At a joining point, differentiability requires continuity and equality of the left and right derivatives. The endpoints $-4, 4$ are excluded.

Step 1: Identify all transition points The formula can be described on successive intervals as follows:

$$f(x) = \begin{cases} 8 + 2x, & -4 < x < -2, \\ x^2, & -2 \le x \le -1, \\ -x, & -1 \le x \le 0, \\ x, & 0 \le x \le 1, \\ x^2, & 1 \le x \le 2, \\ 8 - 2x, & 2 < x < 4. \end{cases}$$

The overlapping endpoint descriptions agree: the values at $-2, 2$ are 4, at $-1, 1$ are 1, and at 0 are 0. Thus these joins are continuous. Only the five interior joins $-2, -1, 0, 1, 2$ need further checking; no other point changes its local formula.

Step 2: Compare the one-sided derivatives At $x = -2$, the left derivative is 2 and the right derivative is $2(-2) = -4$. At $x = -1$, the derivatives are -2 and -1. At $x = 0$, the derivatives are -1 and 1, giving the familiar absolute-value corner. At $x = 1$, the derivatives are 1 and 2. At $x = 2$, they are 4 and -2. Each pair is unequal, so differentiability fails at every listed point. On each remaining open interval the function is linear or quadratic and hence differentiable. This also proves that the list contains all failures, not just some examples. Option (A) omits the corner at 0, option (C) omits the transitions at ± 1 and 0, and option (D) misses all five.

Step 3: Conclusion Therefore $S = \{-2, -1, 0, 1, 2\}$. Hence, the correct option is (B).

Quick Tip to Solve: For a maximum involving absolute value, first solve where the competing formulas agree, then include the absolute-value corner at zero.

4. For each t ∈ R , let [t] be the greatest integer less than or equal to t

Then $\lim_{x\to 1^+} \frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|.[1-x]}$ [4 marks]

(A) equals -1

(B) equals 1

(C) equals 0

(D) does not exist

Correct Answer: (C) equals 0

Solution:

Concept:

- The greatest-integer function must be evaluated on the side of approach before taking a limit. If $0 < h < 1$, then $[-h] = -1$, even though $-h$ approaches zero.
- The elementary trigonometric limit is $\lim_{h\to 0} \sin h/h = 1$. Absolute values simplify once the signs of their arguments are known.

Step 1: Resolve the floor and absolute values Write $x = 1 + h$ with $h \to 0^+$, so $0 < h < 1$ close enough to the limit. Then

$$|x| = 1 + h, \quad |1 - x| = h, \quad [1 - x] = [-h] = -1.$$

In particular, the greatest integer is not zero: the numbers $-h$ lie just below zero. The sine factor involving it is constant throughout this right-hand neighbourhood:

$$\sin\left(\frac{\pi}{2}[1 - x]\right) = \sin(-\pi/2) = -1.$$

The remaining first factor is $1 - (1 + h) + \sin h = \sin h - h$.

Step 2: Simplify before applying the standard limit Substitution transforms the complete quotient into

$$\frac{(\sin h - h)(-1)}{h(-1)} = \frac{\sin h - h}{h} = \frac{\sin h}{h} - 1.$$

The two negative factors cancel; retaining only one of them would produce an incorrect sign during simplification. Hence

$$\lim_{h\to 0^+} \left(\frac{\sin h}{h} - 1\right) = 1 - 1 = 0.$$

Although both the original numerator and denominator tend to zero, this by itself does not mean the limit fails to exist. The simplified quotient has the finite limit above. Neither -1 nor 1 accounts for the cancellation of the two leading terms in $\sin h - h$.

Step 3: Conclusion The required right-hand limit exists and equals 0. Hence, the correct option is (C).

Quick Tip to Solve: Put $x = 1 + h$ with $h > 0$ first. Floor functions are determined by the approach interval, not by substituting the limiting endpoint.

5. If $\frac{dy}{dx} + \frac{3}{\cos^2 x}y = \frac{1}{\cos^2 x}$, $x \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$ and $y\left(\frac{\pi}{4}\right) = \frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals - [4 marks]

(A) $\frac{1}{3} + e^6$

(B) $\frac{1}{3}$

(C) $\frac{1}{3} + e^3$

(D) $-\frac{4}{3}$

Correct Answer: (A) $\frac{1}{3} + e^6$

Solution:

Concept:

- A linear equation $y\prime + P(x)y = Q(x)$ has integrating factor $e^{\int P(x)\,dx}$. Here $P(x) = 3\sec^2 x$, and $\int \sec^2 x\,dx = \tan x$.
- The constant particular solution is $y = 1/3$. Subtracting it reduces this equation to a homogeneous equation; the initial value then determines the remaining constant uniquely.

Step 1: Find the general solution The integrating factor is

$$I(x) = e^{3\tan x}.$$

Multiplying the differential equation throughout by it gives

$$\frac{d}{dx}\left(ye^{3\tan x}\right) = \sec^2 x\, e^{3\tan x}.$$

Since the derivative of $3\tan x$ is $3\sec^2 x$, integration yields

$$ye^{3\tan x} = \frac{1}{3}e^{3\tan x} + C, \qquad y = \frac{1}{3} + Ce^{-3\tan x}.$$

The given interval lies inside $(-\pi/2, \pi/2)$, so $\cos x$ is nonzero everywhere under consideration. Thus the equation and integrating factor are valid at both required points.

Step 2: Apply the initial value and evaluate the second point At $x = \pi/4$, $\tan x = 1$, and the specified value gives

$$\frac{4}{3} = \frac{1}{3} + Ce^{-3} \quad \Longrightarrow \quad Ce^{-3} = 1 \quad \Longrightarrow \quad C = e^3.$$

At $x = -\pi/4$, the tangent is -1, so

$$y(-\pi/4) = \frac{1}{3} + e^3 e^{-3(-1)} = \frac{1}{3} + e^6.$$

The exponent is 6 because the initial point contributes a factor e^3 and the final point contributes another e^3. Option (B) would discard the nonzero homogeneous term and violate the initial value; option (C) omits one factor; option (D) does not satisfy the evaluated solution.

Step 3: Conclusion Therefore $y(-\pi/4) = \frac{1}{3} + e^6$. Hence, the correct option is (A).

Quick Tip to Solve: For proportional forcing and coefficient terms, subtract the constant particular solution first; then track the change in $\tan x$ between the two points.

6. If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, $(k > 0)$, is 1 square unit. Then k is -

[4 marks]

(A) $\sqrt{3}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{2}{\sqrt{3}}$

(D) $\frac{1}{\sqrt{3}}$

Correct Answer: (D) $\frac{1}{\sqrt{3}}$

Solution:

Concept:

- The enclosed area between two graphs is the integral of upper ordinate minus lower ordinate between their intersection abscissae. Here $k > 0$ makes the bounded region lie in the first quadrant.
- From $x = ky^2$ and the nonnegative ordinates of the common region, the upper branch is $y = \sqrt{x/k}$; the other boundary is $y = kx^2$.

Step 1: Determine the intersections and boundary order Substitute $y = kx^2$ into $x = ky^2$:

$$x = k(kx^2)^2 = k^3x^4, \qquad x(k^3x^3 - 1) = 0.$$

Thus the intersections are $(0, 0)$ and $(1/k, 1/k)$. Both equations force nonnegative coordinates on their common bounded region. For $0 < x < 1/k$, writing $u = kx$ gives $0 < u < 1$, and $\sqrt{u} > u^2$. Therefore $\sqrt{x/k}$ is the upper ordinate and kx^2 is the lower one. The negative branch of the sideways parabola does not form another bounded region with the upward parabola.

Step 2: Integrate the difference and use the stated area The area is consequently

$$\begin{aligned} A &= \int_0^{1/k} \left(\sqrt{\frac{x}{k}} - kx^2\right) dx \\ &= \left[\frac{2x^{3/2}}{3\sqrt{k}} - \frac{kx^3}{3}\right]_0^{1/k} \\ &= \frac{2}{3k^2} - \frac{1}{3k^2} = \frac{1}{3k^2}. \end{aligned}$$

Set $A = 1$. Then $3k^2 = 1$, and positivity selects $k = 1/\sqrt{3}$. As a check on the alternatives, $k = \sqrt{3}, \sqrt{3}/2, 2/\sqrt{3}$ would give areas $1/9, 4/9, 1/4$, respectively, none of which is the required unit area.

Step 3: Conclusion The positive value is $k = \frac{1}{\sqrt{3}}$. Hence, the correct option is (D).

Quick Tip to Solve: Find both intersections before integrating, and use the positive branch of the sideways parabola because the other curve has nonnegative ordinates.

7. A point P moves on the line 2x – 3y + 4 = 0. If Q(1, 4) and R (3, – 2) are fixed points, then the locus of the centroid of $\triangle$PQR is a line : [4 marks]

(A) parallel to y-axis

(B) with slope $\frac{2}{3}$

(C) parallel to x-axis

(D) with slope $\frac{3}{2}$

Correct Answer: (B) with slope $\frac{2}{3}$

Solution:

Concept:

- The centroid of a triangle is the average of its three vertices: if the coordinates are (x_i, y_i), its coordinates are $((x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3)$.
- To obtain a locus, express the moving point in terms of the centroid coordinates and substitute into the moving point's constraint. A line $aX + bY + c = 0$, with $b \neq 0$, has slope $-a/b$.

Step 1: Express the moving point through the centroid Let the moving point be $P = (x, y)$, and write the centroid as $G = (X, Y)$. With the fixed points $Q = (1, 4)$ and $R = (3, -2)$, the centroid formula gives

$$X = \frac{x + 1 + 3}{3} = \frac{x + 4}{3}, \qquad Y = \frac{y + 4 - 2}{3} = \frac{y + 2}{3}.$$

Solve these two relations for the original moving coordinates:

$$x = 3X - 4, \qquad y = 3Y - 2.$$

This change is invertible, so substituting it into the line equation describes every possible centroid rather than only a necessary subset of its positions.

Step 2: Substitute into the line and identify its slope The point P satisfies $2x - 3y + 4 = 0$. Therefore

$$2(3X - 4) - 3(3Y - 2) + 4 = 0,$$

which simplifies to

$$6X - 9Y + 2 = 0, \qquad Y = \frac{2}{3}X + \frac{2}{9}.$$

The locus is a straight line of slope $2/3$. Its slope is finite and nonzero, excluding a line parallel to either coordinate axis. The value $3/2$ is the reciprocal of the required slope and would result from interchanging the roles of the two coordinates. Geometrically, averaging with two fixed vertices applies the same positive scale factor $1/3$ to both coordinates of P, together with a translation, so it also preserves the direction of the original line.

Step 3: Conclusion The centroid lies on $6X - 9Y + 2 = 0$, a line with slope $\frac{2}{3}$. Hence, the correct option is (B).

Quick Tip to Solve: A centroid with two fixed vertices is a translated one-third-scale image of the moving point; its line locus must be parallel to the moving point's line.

8. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4|z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then : [4 marks]

(A) $Im(z) = 0$

(B) $|z| = \sqrt{\frac{17}{2}}$

(C) $|z| = \frac{1}{2}\sqrt{9 + 16\cos^2\theta}$

(D) Re(z) = 0

Correct Answer: (C) $|z| = \frac{1}{2}\sqrt{9 + 16\cos^2\theta}$

Solution:

Concept:

- The modulus of a quotient is the quotient of the moduli. Write a non-zero complex number of modulus 2 as $2(\cos\theta + i\sin\theta)$, where θ is its argument.
- For $z = u + iv$, its modulus satisfies $|z|^2 = u^2 + v^2$. The reciprocal of $re^{i\theta}$ has modulus $1/r$ and argument $-\theta$.

Step 1: Introduce the ratio Put $w = 3z_1/(2z_2)$. The given condition is $3|z_1| = 4|z_2|$, so

$$|w| = \frac{3|z_1|}{2|z_2|} = 2.$$

Hence, for some real angle $\theta = \arg w$, we have $w = 2(\cos\theta + i\sin\theta)$. The other term in the question is exactly $1/w$; it is not the complex conjugate of w, because its modulus is $1/2$.

Step 2: Separate the real and imaginary parts Taking the reciprocal changes the sign of the argument:

$$\frac{1}{w} = \frac{1}{2}(\cos\theta - i\sin\theta).$$

Therefore

$$z = w + \frac{1}{w} = \frac{5}{2}\cos\theta + \frac{3}{2}i\sin\theta.$$

Both the real and imaginary parts generally vary with the argument. In particular, at $\theta = 0$ the real part is non-zero, and at $\theta = \pi/2$ the imaginary part is non-zero.

Step 3: Calculate and compare the modulus Using the sum of squares of these two components gives

$$\begin{aligned} |z|^2 &= \frac{25}{4}\cos^2\theta + \frac{9}{4}\sin^2\theta \\ &= \frac{9}{4} + 4\cos^2\theta. \end{aligned}$$

Taking the non-negative square root produces the expression in option (C). Option (B) cannot be a fixed value: for instance $\theta = 0$ gives $|z| = 5/2$, whereas $\theta = \pi/2$ gives $|z| = 3/2$. Options (A) and (D) fail by the component checks above.

Step 4: Conclusion With $\theta = \arg(3z_1/(2z_2))$, $|z| = \frac{1}{2}\sqrt{9 + 16\cos^2\theta}$. Hence, the correct option is (C).

Quick Tip to Solve: For an expression $w + 1/w$, write $w = re^{i\theta}$ first; its components are $(r + r^{-1})\cos\theta$ and $(r - r^{-1})\sin\theta$.

9. If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to : [4 marks]

(A) $\frac{7}{4}$

(B) $\frac{5}{4}$

(C) $\frac{3}{4}$

(D) $\frac{3}{2}$

Correct Answer: (A) $\frac{7}{4}$

Solution:

Concept:

- Three positive lengths form a non-degenerate triangle exactly when the sum of every two is greater than the third. Equality gives a straight line, not a triangle.
- A common positive scale factor does not change the triangle inequalities. Thus divide the lengths $5, 5r, 5r^2$ by 5 and test $1, r, r^2$, with $r > 0$.

Step 1: Write the necessary inequalities The three conditions for the scaled lengths are

$$1 + r > r^2, \qquad 1 + r^2 > r, \qquad r + r^2 > 1.$$

The middle condition is always true because

$$r^2 - r + 1 = \left(r - \frac{1}{2}\right)^2 + \frac{3}{4} > 0.$$

The first and third conditions, however, restrict the common ratio. We must keep both, since the largest side changes when r passes through 1.

Step 2: Find the allowed interval From $r^2 - r - 1 < 0$ and $r > 0$, we get $r < (1 + \sqrt{5})/2$. From $r^2 + r - 1 > 0$, we get $r > (\sqrt{5} - 1)/2$. Thus

$$\frac{\sqrt{5} - 1}{2} < r < \frac{1 + \sqrt{5}}{2}.$$

Numerically this is approximately $0.618 < r < 1.618$, so $7/4 = 1.75$ lies outside the interval.

Step 3: Check the listed ratios directly At $r = 7/4$, the two shorter scaled sides have sum $1 + 7/4 = 44/16$, less than the longest side $r^2 = 49/16$. Thus option (A) is impossible. The others satisfy all three strict inequalities: for $r = 5/4$, the largest side $25/16$ is less than $36/16$; for $r = 3/4$, the largest side 1 is less than $3/4 + 9/16 = 21/16$; for $r = 3/2$, the largest side $9/4$ is less than $1 + 3/2 = 10/4$.

Step 4: Conclusion The ratio $r = 7/4$ violates the triangle inequality and cannot occur. Hence, the correct option is (A).

Quick Tip to Solve: When a triangle has sides in geometric progression, divide by the first side and compare the largest of $1, r, r^2$ with the sum of the other two.

10. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1, 2, 3, ... , 9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is : [4 marks]

(A) $\frac{19}{36}$
(B) $\frac{15}{72}$
(C) $\frac{13}{36}$
(D) $\frac{19}{72}$

Correct Answer: (D) $\frac{19}{72}$

Solution:

Concept:

- Condition on the first random experiment. For an event E, the total-probability formula is $P(E) = P(H)P(E \mid H) + P(T)P(E \mid T)$.
- Two fair dice have 36 equally likely ordered outcomes. A uniformly selected card from nine distinct numbered cards gives probability $1/9$ to each number.

Step 1: Count successful dice outcomes Let E be the event that the noted number is 7 or 8. If the coin shows a head, the noted number is the sum of two dice. Sum 7 occurs for six ordered pairs:

$$(1,6), (2,5), (3,4), (4,3), (5,2), (6,1).$$

Sum 8 occurs for five ordered pairs:

$$(2,6), (3,5), (4,4), (5,3), (6,2).$$

The sets are disjoint. There are 36 equally likely ordered outcomes in total, so

$$P(E \mid H) = \frac{6+5}{36} = \frac{11}{36}.$$

Step 2: Count successful card outcomes If the coin shows a tail, only one card is selected. The successful cards are the card numbered 7 and the card numbered 8. Each appears once in a pack of nine cards, giving

$$P(E \mid T) = \frac{2}{9}.$$

The card and dice branches have different sample spaces, so their outcome counts cannot simply be combined into one equally likely list.

Step 3: Weight by the coin probabilities The coin is unbiased, hence $P(H) = P(T) = 1/2$. The head and tail branches are mutually exclusive and cover every possibility. Therefore

$$\begin{aligned} P(E) &= \frac{1}{2} \cdot \frac{11}{36} + \frac{1}{2} \cdot \frac{2}{9} \\ &= \frac{11}{72} + \frac{8}{72} = \frac{19}{72}. \end{aligned}$$

Option (A) is twice this value and omits the coin weighting. Options (B) and (C) disagree with the exact count of eleven successful ordered dice outcomes and two successful cards.

Step 4: Conclusion The required probability is $19/72$. Hence, the correct option is (D).

Quick Tip to Solve: For a random choice between two experiments, compute the event probability within each branch first, then weight each by the probability of choosing that branch.

11. If the line 3x + 4y – 24 = 0 intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is : [4 marks]

(A) (3, 4)

(B) (2, 2)

(C) (4, 4)

(D) (4, 3)

Correct Answer: (B) (2, 2)

Solution:

Concept:

- The incentre is the intersection of the internal angle bisectors and is at equal perpendicular distance from all three sides of a triangle.
- For a right triangle with perpendicular sides a, b and hypotenuse c, the inradius is $r = (a+b-c)/2$. This also follows from area $\Delta = rs$, where s is the semiperimeter.

Step 1: Find the intercepts and side lengths Put $y = 0$ in $3x + 4y - 24 = 0$. This gives $x = 8$, so $A = (8, 0)$. Put $x = 0$, giving $y = 6$, so $B = (0, 6)$. The triangle lies in the first quadrant and is right-angled at $O = (0, 0)$. Its side lengths are

$$OA = 8, \qquad OB = 6, \qquad AB = \sqrt{8^2 + 6^2} = 10.$$

Step 2: Determine the inradius The area and semiperimeter are

$$\Delta = \frac{1}{2}(8)(6) = 24, \qquad s = \frac{8+6+10}{2} = 12.$$

Using $\Delta = rs$, we obtain $r = 24/12 = 2$. Equivalently, the right-triangle formula gives $r = (8 + 6 - 10)/2 = 2$. This is the distance from the incentre to either coordinate axis.

Step 3: Locate and verify the incentre A point inside the first quadrant at distance 2 from both coordinate axes has coordinates $(2, 2)$. Its distance from the third side is

$$\frac{|3(2) + 4(2) - 24|}{\sqrt{3^2 + 4^2}} = \frac{10}{5} = 2,$$

confirming equal distances to all sides. Options (A) and (D) have unequal positive coordinates, so their distances to the axes differ. Although option (C) has equal coordinates, its distance to the line is $|12 + 16 - 24|/5 = 4/5$, not 4; it is not an incentre.

Step 4: Conclusion The incentre of triangle OAB is $(2, 2)$. Hence, the correct option is (B).

Quick Tip to Solve: For a right triangle on the positive coordinate axes, the incentre is (r, r); find r from area divided by semiperimeter.

12. If the third term in the binomial expansion of $\left(1 + x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is - [4 marks]

(A) $2\sqrt{2}$

(B) $4\sqrt{2}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

Correct Answer: (D) $\frac{1}{4}$

Solution:

Concept:

- The general term of $(1 + u)^5$ is $T_{k+1} = \binom{5}{k} u^k$. The third term has $k = 2$, not $k = 3$.
- The logarithm $\log_2 x$ requires $x > 0$. Setting $t = \log_2 x$ gives $x = 2^t$, which turns $x^{\log_2 x}$ into 2^{t^2}.

Step 1: Identify the third term Here the second term inside the binomial is $u = x^{\log_2 x}$. Consequently

$$T_3 = \binom{5}{2} \left(x^{\log_2 x}\right)^2 = 10x^{2\log_2 x}.$$

The given equality is $T_3 = 2560$, so dividing by 10 gives

$$x^{2\log_2 x} = 256 = 2^8.$$

All powers used are defined over the real numbers because $x > 0$.

Step 2: Solve using a logarithmic substitution Write $t = \log_2 x$, so $x = 2^t$. Substitution into the equation gives

$$\left(2^t\right)^{2t} = 2^8 \quad \Longrightarrow \quad 2t^2 = 8.$$

The exponential function with base 2 is one-to-one, so $t^2 = 4$, yielding $t = 2$ or $t = -2$. Hence

$$x = 2^2 = 4 \quad \text{or} \quad x = 2^{-2} = \frac{1}{4}.$$

Both satisfy the logarithm domain condition.

Step 3: Compare the proposed values Of these two possible values, only $1/4$ appears in the options. To verify, $\log_2(1/4) = -2$, and then $(1/4)^{-2} = 16$. The third term is $10(16)^2 = 2560$, exactly as required. For options (A), (B) and (C), the corresponding logarithms are $3/2$, $5/2$ and -3; their squares are $9/4$, $25/4$ and 9, respectively, rather than 4. Thus none satisfies the derived equation.

Step 4: Conclusion A possible value is $x = 1/4$; the other real solution is $x = 4$, which is not listed. Hence, the correct option is (D).

Quick Tip to Solve: When the variable occurs both as a base and inside a logarithmic exponent, set $t = \log_2 x$ before taking powers; keep both signs when solving t^2.

13. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is - [4 marks]

(A) 1356

(B) 1256

(C) 1365

(D) 1465

Correct Answer: (A) 1356

Solution:

Concept:

- Positive integers with a fixed remainder upon division by 7 form an arithmetic progression of common difference 7.
- The sum of an arithmetic progression is $S_n = n(a+l)/2$, where a and l are its first and last terms. Count only integers from 10 through 99.

Step 1: List the terms with remainder 2 The integers congruent to 2 modulo 7 are $7k+2$. The first two-digit member is 16, and the last is 93, since the next member would be 100. Thus the required progression is

$$16, 23, 30, \ldots, 93.$$

Its number of terms is

$$n_2 = \frac{93-16}{7} + 1 = 12.$$

The addition of 1 counts both endpoints; merely dividing the difference by 7 would count the gaps.

Step 2: List the terms with remainder 5 The integers congruent to 5 modulo 7 are $7k+5$. Their two-digit members run from 12 to 96, with the next term 103 outside the permitted range. Thus

$$12, 19, 26, \ldots, 96, \qquad n_5 = \frac{96-12}{7} + 1 = 13.$$

No integer can have both remainder 2 and remainder 5 on division by 7, so these two lists do not overlap.

Step 3: Add the two progression sums Using the first-and-last-term formula separately gives

$$S_2 = \frac{12}{2}(16+93) = 6(109) = 654,$$

$$S_5 = \frac{13}{2}(12+96) = \frac{13}{2}(108) = 702.$$

Their total is $654 + 702 = 1356$. This accounts for all 25 eligible two-digit positive integers exactly once. As a direct check, the expression is a sum of integers and both individual sums are even, so the odd totals in options (C) and (D) are impossible. Option (B) is 100 smaller than the exact sum and does not equal either arithmetic-progression calculation.

Step 4: Conclusion The required sum is 1356. Hence, the correct option is (A).

Quick Tip to Solve: For remainder conditions, split into separate arithmetic progressions and find the first and last terms inside the specified digit range before applying the sum formula.

14. **Let** $I = \int_a^b (x^4 - 2x^2)dx$. **If I is minimum then the ordered pair (a, b) is -** [4 marks]

(A) $(\sqrt{2}, -\sqrt{2})$

(B) $(0, \sqrt{2})$

(C) $(-\sqrt{2}, \sqrt{2})$

(D) $(-\sqrt{2}, 0)$

Correct Answer: (C) $(-\sqrt{2}, \sqrt{2})$

Solution:

Concept:

- The integrand factors as $f(x) = x^4 - 2x^2 = x^2(x^2 - 2)$. It is non-positive on $[-\sqrt{2}, \sqrt{2}]$ and positive outside this interval.
- For the usual increasing limits $a \leq b$, a minimum signed integral includes the entire negative part and avoids positive contributions. Reversing the limits reverses the sign of an integral.

Step 1: Find the sign-changing points Solving $x^2(x^2 - 2) = 0$ gives the zeros $-\sqrt{2}, 0, \sqrt{2}$. The double zero at 0 does not change the sign: $x^2 \geq 0$, while $x^2 - 2 < 0$ between the two outer zeros. Thus the integrand is strictly negative on both open intervals $(-\sqrt{2}, 0)$ and $(0, \sqrt{2})$. It is positive for $|x| > \sqrt{2}$.

Step 2: Choose the interval giving the negative area Taking $a = -\sqrt{2}$ and $b = \sqrt{2}$ includes both negative lobes. Any shorter increasing interval omits some negative contribution, or includes positive contribution outside these endpoints, and cannot improve this value. The isolated zero at the origin does not justify stopping there, because the integrand is negative again immediately to its right. An antiderivative is

$$F(x) = \frac{x^5}{5} - \frac{2x^3}{3}.$$

Step 3: Compare all four ordered pairs At the positive endpoint,

$$F(\sqrt{2}) = \frac{4\sqrt{2}}{5} - \frac{4\sqrt{2}}{3} = -\frac{8\sqrt{2}}{15}.$$

Since F is odd, option (C) gives

$$I = F(\sqrt{2}) - F(-\sqrt{2}) = -\frac{16\sqrt{2}}{15}.$$

Options (B) and (D) each give $-8\sqrt{2}/15$, since each includes only one negative lobe. Option (A) reverses the full interval and gives $+16\sqrt{2}/15$. Therefore (C) is the smallest among the listed ordered pairs; it also gives the global minimum when the limits are taken in their usual order $a \leq b$.

Step 4: Conclusion The minimum among the given choices occurs at $(a, b) = (-\sqrt{2}, \sqrt{2})$, giving $I = -16\sqrt{2}/15$. Hence, the correct option is (C).

Quick Tip to Solve: To minimise an integral with selectable increasing limits, use a sign chart of the integrand and include the full negative region, rather than stopping at every zero.

15. Let d $\in$ R, and

$$A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix},$$

$\theta \in [0, 2\pi]$ **If the minimum value of det(A) is 8, then a value of d is -** [4 marks]

(A) -7

(B) $2\left(\sqrt{2}+2\right)$

(C) -5

(D) $2\left(\sqrt{2}+1\right)$

Correct Answer: (C) -5

Solution:

Concept:

- Row operations of the form $R_i \mapsto R_i + kR_j$ leave a determinant unchanged. A carefully chosen operation can expose a constant row and simplify the expansion.
- For $s = \sin\theta$ with $0 \le \theta \le 2\pi$, we have $-1 \le s \le 1$, and hence $0 \le s^2 \le 1$. Minimise the determinant over this entire attainable interval.

Step 1: Simplify the determinant Put $s = \sin\theta$. In the given matrix apply $R_3 \mapsto R_3 - 2R_2$, followed by $R_3 \mapsto R_3 + R_1$. The third row becomes $(1, 0, 0)$. Expanding along this row gives

$$\det A = \begin{vmatrix} d+4 & s-2 \\ s+2 & d \end{vmatrix} = d(d+4) - (s-2)(s+2).$$

Therefore

$$\det A = (d+2)^2 - s^2.$$

Step 2: Use the minimum condition For fixed d, the determinant is smallest when s^2 is largest. The value $s^2 = 1$ is attained at $\theta = \pi/2$ and $3\pi/2$, so this is an actual minimum, not just a bound. Thus

$$(d+2)^2 - 1 = 8, \qquad (d+2)^2 = 9.$$

Taking both signs gives $d + 2 = \pm 3$, and hence $d = 1$ or $d = -5$. Of the listed values, only -5 occurs. For example, $d = -7$ gives minimum $25 - 1 = 24$, while each listed positive surd exceeds 4 and gives a minimum greater than 35; none equals 8.

Step 3: Conclusion The admissible listed value is $d = -5$. Hence, the correct option is (C).

Quick Tip to Solve: When a determinant simplifies to a constant minus $\sin^2\theta$, its minimum occurs at $\sin^2\theta = 1$; retain both signs when solving the resulting square.

16. The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve y = $\sqrt{x}$, (x > 0), is - [4 marks]

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{5}{4}$

(C) $\frac{3}{2}$

(D) $\frac{\sqrt{5}}{2}$

Correct Answer: (D) $\frac{\sqrt{5}}{2}$

Solution:

Concept:

- The distance between (x, y) and (a, b) is $D = \sqrt{(x-a)^2 + (y-b)^2}$. Because the square-root function is increasing on nonnegative numbers, minimising D^2 gives the same location as minimising D.
- A point on the given curve has coordinates $(x, \sqrt{x})$ with $x > 0$. Substituting this relation reduces the distance problem to a quadratic in one real variable.

Step 1: Express the squared distance Let $P = (3/2, 0)$ be the fixed point and $Q = (x, \sqrt{x})$ be any point of the curve. Then

$$D^2 = \left(x - \frac{3}{2}\right)^2 + (\sqrt{x})^2 = x^2 - 2x + \frac{9}{4}.$$

Complete the square rather than differentiating a radical:

$$D^2 = (x-1)^2 + \frac{5}{4}.$$

The square is nonnegative for every allowed value of x, so $D^2 \geq 5/4$. This bound applies to every point of the curve, rather than just a stationary point.

Step 2: Check attainment and take the square root Equality requires $x = 1$, which satisfies the strict domain restriction $x > 0$. At this point $Q = (1, 1)$, and direct substitution gives

$$PQ = \sqrt{\left(1 - \frac{3}{2}\right)^2 + 1^2} = \frac{\sqrt{5}}{2}.$$

This establishes the global minimum. The number $5/4$ is the minimum squared distance and must not be reported as the distance itself. The option $\sqrt{3}/2$ would give squared distance $3/4$, contradicting the proven lower bound. The value $3/2$ is larger than the attained minimum and is associated with the limiting endpoint $(0, 0)$, which is excluded anyway.

Step 3: Conclusion The shortest distance is $\sqrt{5}/2$. Hence, the correct option is (D).

Quick Tip to Solve: For distance to $y = \sqrt{x}$, work with the squared distance: the term $y^2 = x$ turns the expression into a quadratic that can be completed to a square.

17. Let f : R $\to$ R be a function such that f(x) = x^3 + x^2f'(1) + xf"(2) + f"'(3), x $\in$ R. Then f(2) equals - [4 marks]

(A) 30

(B) -2

(C) -4

(D) 8

Correct Answer: (B) -2

Solution:

Concept:

- The quantities $f\prime(1)$, $f\prime\prime(2)$ and $f\prime\prime\prime(3)$ are constants with respect to x. Introduce separate constants before differentiating the given polynomial identity.
- An identity valid for every real x can be differentiated and evaluated at the specified points. The resulting linear equations determine its coefficients consistently.

Step 1: Introduce and determine the derivative constants Write $a = f\prime(1)$, $b = f\prime\prime(2)$ and $c = f\prime\prime\prime(3)$. The identity becomes

$$f(x) = x^3 + ax^2 + bx + c.$$

Differentiating, keeping a, b, c constant, gives

$$f\prime(x) = 3x^2 + 2ax + b, \qquad f\prime\prime(x) = 6x + 2a.$$

At $x = 1$, the first equation yields $a = 3 + 2a + b$, so $a + b = -3$. At $x = 2$, the second equation gives $b = 12 + 2a$. Combining these conditions gives $3a = -15$, hence $a = -5$ and $b = 2$.

Step 2: Find the constant term and evaluate The third derivative of $x^3 + ax^2 + bx + c$ is identically 6. Therefore $c = f\prime\prime\prime(3) = 6$, and the uniquely determined polynomial is

$$f(x) = x^3 - 5x^2 + 2x + 6.$$

Substitute $x = 2$:

$$f(2) = 8 - 20 + 4 + 6 = -2.$$

As a consistency check, this polynomial gives $f\prime(1) = 3 - 10 + 2 = -5$ and $f\prime\prime(2) = 12 - 10 = 2$, exactly the constants used in the identity. The alternatives 30, -4 and 8 do not equal the sum of these four polynomial terms; 8 alone retains only the cubic term and omits the derivative-dependent contributions.

Step 3: Conclusion We obtain $f(2) = -2$. Hence, the correct option is (B).

Quick Tip to Solve: Treat derivatives evaluated at fixed points as unknown coefficients, then differentiate the polynomial and impose each coefficient definition at its own point.

18. Consider the quadratic equation $(c-5)x^2 - 2cx + (c-4) = 0$, $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is - [4 marks]

(A) 12

(B) 18

(C) 10

(D) 11

Correct Answer: (D) 11

Solution:

Concept:

- For a real quadratic $F(x) = a(x - r_1)(x - r_2)$ with distinct roots $r_1 < r_2$, its sign is opposite to a between the roots and the same as a outside them.
- To put one root in $(0, 2)$ and the other in $(2, 3)$, the values at 0 and 3 must have the sign of the leading coefficient, while the value at 2 has the opposite sign. Strict intervals require strict inequalities.

Step 1: Evaluate the boundary signs Let

$$F(x) = (c-5)x^2 - 2cx + c - 4.$$

The leading coefficient is nonzero because $c \neq 5$. Direct evaluation gives

$$F(0) = c - 4, \qquad F(2) = c - 24, \qquad F(3) = 4c - 49.$$

If $c > 5$, the parabola opens upward. The required signs are $F(0) > 0$, $F(2) < 0$ and $F(3) > 0$. Thus

$$c > 4, \qquad c < 24, \qquad c > 49/4.$$

Together with $c > 5$, these reduce to $49/4 < c < 24$. The sign changes guarantee a root in each open interval by continuity, and a quadratic has no additional roots.

Step 2: Exclude the downward case and count integers If $c < 5$, the parabola opens downward, so the required signs would instead be $F(0) < 0$, $F(2) > 0$ and $F(3) < 0$. But $F(2) > 0$ requires $c > 24$, contradicting $c < 5$. Hence no value comes from this case. In the upward case the admissible integers are

$$13, 14, 15, \ldots, 23.$$

Their number is $23 - 13 + 1 = 11$. The strict upper endpoint excludes $c = 24$, which would place a root exactly at 2; it cannot be counted to obtain 12. The alternatives 18 and 10 also do not equal the number of integers in the derived interval.

Step 3: Conclusion The set is $S = \{13, 14, \ldots, 23\}$, with 11 elements. Hence, the correct option is (D).

Quick Tip to Solve: Use signs at the three interval boundaries instead of solving the quadratic. Check both possible signs of the leading coefficient and keep endpoint inequalities strict.

19. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is [4 marks]

(A) 42

(B) 102

(C) 1

(D) 38

Correct Answer: (D) 38

Solution:

Concept:

- Let M, P, C be the students choosing Mathematics, Physics and Chemistry. Their union contains students choosing at least one course, and its complement in the class contains students choosing none.
- For three finite sets, $|M \cup P \cup C| = |M| + |P| + |C| - |M \cap P| - |M \cap C| - |P \cap C| + |M \cap P \cap C|$. Common multiples are counted using least common multiples of the relevant divisors.

Step 1: Count the individual and common multiples Among the integers from 1 through 140, the number divisible by a positive integer k is $\lfloor 140/k \rfloor$. Therefore

$$|M| = 70, \qquad |P| = 46, \qquad |C| = 28.$$

A student in both Mathematics and Physics has a number divisible by 6, so there are 23 such students. Similarly, the pairwise intersection counts are

$$|M \cap P| = 23, \quad |M \cap C| = 14, \quad |P \cap C| = 9.$$

A number divisible by 2, 3 and 5 must be divisible by 30. Thus the triple intersection contains $\lfloor 140/30 \rfloor = 4$ students.

Step 2: Find the union and take its complement Apply inclusion-exclusion:

$$|M \cup P \cup C| = 70 + 46 + 28 - 23 - 14 - 9 + 4 = 102.$$

Add back the triple intersection to avoid removing those students too many times. Consequently the number choosing no course is

$$140 - 102 = 38.$$

The value 102 in another option counts at least one course, which is the complementary group and not the requested group. The value 42 results if the triple-intersection correction is omitted when finding the union, while 1 does not agree with the complete count.

Step 3: Conclusion There are 38 students who did not opt for any of the three courses. Hence, the correct option is (D).

Quick Tip to Solve: Translate simultaneous course choices into divisibility by least common multiples. Compute the union by inclusion-exclusion before subtracting from the class total.

20. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is - [4 marks]

(A) 6 : 7

(B) 10 : 3

(C) 4 : 9

(D) 5 : 8

Correct Answer: (C) 4 : 9

Solution:

Concept:

- For five observations with mean $\bar{x}$, their sum is $5\bar{x}$. The population variance used here is $\sigma^2 = \frac{1}{5}\sum x_i^2 - \bar{x}^2$, with division by five rather than four.
- If two unknown observations are u, v, their sum and sum of squares determine their product through $(u+v)^2 = u^2 + v^2 + 2uv$. They can then be recovered as roots of a quadratic.

Step 1: Obtain the sum and sum of squares The mean is 5, so the sum of all five observations is $5 \times 5 = 25$. The known observations have sum $1 + 3 + 8 = 12$, giving

$$u + v = 25 - 12 = 13.$$

Using the given variance,

$$9.20 = \frac{1^2 + 3^2 + 8^2 + u^2 + v^2}{5} - 25.$$

Thus the total sum of squares is $5(9.20 + 25) = 171$. The known squares sum to $1 + 9 + 64 = 74$, and consequently

$$u^2 + v^2 = 171 - 74 = 97.$$

Step 2: Find the two observations and compare ratios Now

$$2uv = (u+v)^2 - (u^2 + v^2) = 169 - 97 = 72,$$

so $uv = 36$. Each unknown observation therefore satisfies

$$t^2 - 13t + 36 = 0, \qquad (t-4)(t-9) = 0.$$

The observations are 4 and 9, in either order, and an available ratio is $4 : 9$. All listed ratio pairs sum to 13, so the mean alone cannot distinguish them. Their square sums do: $6^2 + 7^2 = 85$, $10^2 + 3^2 = 109$, and $5^2 + 8^2 = 89$; none equals 97. Only $4^2 + 9^2 = 97$ reproduces the specified variance. Substituting the recovered observations into the original variance formula gives $171/5 - 25 = 9.20$, completing the check.

Step 3: Conclusion The unknown observations are 4 and 9; hence their ratio can be $4 : 9$. Hence, the correct option is (C).

Quick Tip to Solve: Convert mean and variance into the sum and sum of squares first. Then recover the product using the square of the sum, rather than guessing the observations.

21. Let n $\geq$ 2 be a natural number and $0 < \theta < \frac{\pi}{2}$. Then $\int \frac{(\sin^n\theta - \sin\theta)^{1/n}\cos\theta}{\sin^{n+1}\theta}\,d\theta$ is equal to - (where C is a constant of integration) [4 marks]

(A) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

(B) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}} + C$

(C) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

(D) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

Correct Answer: (C) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

Solution:

Concept:

- Factor $\sin^n\theta$ out of the radical and use $u = 1 - \sin^{-(n-1)}\theta$; its derivative supplies the remaining trigonometric factor.
- The power rule gives $\int u^{1/n}\,du = \frac{n}{n+1}u^{(n+1)/n} + C$ on a consistent domain and branch. Here $0 < \sin\theta < 1$, so the inner quantity u is negative; real radicals exist for odd n, while even n needs a consistent complex-root interpretation.

Step 1: Rewrite the integrand and substitute Write $s = \sin\theta > 0$ and factor $s^n - s = s^n(1 - s^{1-n})$. On a consistent root branch, the integrand becomes

$$\frac{(s^n - s)^{1/n}\cos\theta}{s^{n+1}} = \left(1 - s^{1-n}\right)^{1/n}\frac{\cos\theta}{s^n}.$$

Choose $u = 1 - s^{1-n}$. Differentiating gives

$$du = (n-1)\frac{\cos\theta}{s^n}\,d\theta.$$

Step 2: Integrate and verify the coefficient The integral is therefore

$$\frac{1}{n-1}\int u^{1/n}\,du = \frac{n}{(n-1)(n+1)}u^{(n+1)/n} + C.$$

Substituting back gives

$$\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{(n+1)/n} + C.$$

Differentiation multiplies the coefficient by $(n+1)/n$ and then by $(n-1)\cos\theta/\sin^n\theta$; the factors cancel to 1, recovering the integrand. The plus sign inside option (A), the inner exponent $n+1$ in (B), and the denominator n^2+1 in (D) all fail this derivative check.

Step 3: Conclusion The integral is the expression in option (C), on the domain and branch described above. Hence, the correct option is (C).

Quick Tip to Solve: After taking out $\sin^n\theta$ from the radical, look for $1 - \csc^{n-1}\theta$; its derivative supplies exactly the leftover factor. Check the radical domain as well.

22. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is - [4 marks]

(A) $\frac{5\pi}{4}$

(B) $\frac{\pi}{2}$

(C) π

(D) $\frac{3\pi}{8}$

Correct Answer: (B) $\frac{\pi}{2}$

Solution:

Concept:

- Use $\sin^2 u + \cos^2 u = 1$ to express the equation in one squared trigonometric function. A square is nonnegative and vanishes only when its base is zero.
- The interval for θ must be doubled when setting $u = 2\theta$: here $0 < u < \pi$. All solutions in that interval must be retained before summing.

Step 1: Reduce the equation to a perfect square Set $u = 2\theta$ and $v = \cos^2 u$. Then $\sin^2 u = 1 - v$, so the given equation becomes

$$1 - v + v^2 = \frac{3}{4} \quad \Longrightarrow \quad v^2 - v + \frac{1}{4} = 0 \quad \Longrightarrow \quad \left(v - \frac{1}{2}\right)^2 = 0.$$

Thus the only possible value of the squared cosine is $v = 1/2$. It is admissible because $0 \le \cos^2 u \le 1$. The repeated algebraic root does not mean that there is only one angle: both signs of the cosine remain possible.

Step 2: Find every angle in the stated interval We require $\cos^2 u = 1/2$, or $\cos u = \pm 1/\sqrt{2}$. In $0 < u < \pi$, the positive value occurs at $u = \pi/4$ and the negative value at $u = 3\pi/4$. Therefore

$$\theta = \frac{\pi}{8}, \ \frac{3\pi}{8}.$$

Both lie strictly between the stated endpoints. At either angle, $\sin^2(2\theta) = 1/2$ and $\cos^4(2\theta) = 1/4$, checking the equation directly. Their sum is

$$\frac{\pi}{8} + \frac{3\pi}{8} = \frac{\pi}{2}.$$

The remaining proposed sums $5\pi/4$, π, and $3\pi/8$ differ from this total; the last is only one of the two admissible angles.

Step 3: Conclusion The sum of all admissible values is $\boxed{\pi/2}$. Hence, the correct option is (B).

Quick Tip to Solve: After reducing to a squared cosine, include both signs of the cosine and apply the interval to the doubled angle.

23. If a circle C passing through the point (4, 0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, – 1), then the radius of C is : [4 marks]

(A) 5

(B) $2\sqrt{5}$

(C) 4

(D) $\sqrt{37}$

Correct Answer: (A) 5

Solution:

Concept:

- Completing squares identifies the centre and radius of a circle. At a point of external contact, the centres and the contact point are collinear, with the contact point between the centres.
- Every point of a circle is at the same distance from its centre. Thus the known contact point and the additional point give an equation for the unknown centre.

Step 1: Parametrise the second centre on the line of centres The given circle is

$$(x+2)^2 + (y-3)^2 = 25,$$

so its centre is $O = (-2, 3)$ and its radius is 5. The contact point is $T = (1, -1)$, and $\overrightarrow{OT} = (3, -4)$, whose length is 5. External tangency places the unknown centre S beyond T on this ray. Write

$$S = T + t(3, -4) = (1 + 3t, -1 - 4t), \qquad t > 0.$$

Its radius is the distance $ST = 5t$. The positivity restriction explicitly chooses external rather than internal contact.

Step 2: Use the additional point and check external tangency Since $P = (4, 0)$ lies on the required circle, $SP^2 = ST^2$. Substitution gives

$$(3t - 3)^2 + (-1 - 4t)^2 = 25t^2.$$

Expanding yields $25t^2 - 10t + 10 = 25t^2$, hence $t = 1$. Thus $S = (4, -5)$ and the radius is 5. For an independent geometric check, $SP = 5$, $ST = 5$, and

$$OS = \sqrt{6^2 + (-8)^2} = 10 = 5 + 5.$$

The distance between centres is exactly the sum of the radii, confirming external tangency. The contact point is also the midpoint of O and S. The alternative radii $2\sqrt{5}$, 4, and $\sqrt{37}$ do not satisfy the derived equality $SP = ST$ on the required external ray.

Step 3: Conclusion The radius of C is $\boxed{5}$. Hence, the correct option is (A).

Quick Tip to Solve: At external contact, put the unknown centre on the ray from the first centre through the contact point, then equate two radii.

24. The equation of a tangent to the hyperbola $4x^2 - 5y^2 = 20$ parallel to the line $x - y = 2$ is :

[4 marks]

(A) $x - y + 9 = 0$

(B) $x - y - 3 = 0$

(C) $x - y + 1 = 0$

(D) $x - y + 7 = 0$

Correct Answer: (C) $x - y + 1 = 0$

Solution:

Concept:

- Parallel lines have the same slope. A line is tangent to a conic when substituting its equation gives a repeated root, equivalently a zero quadratic discriminant.
- For $x^2/a^2 - y^2/b^2 = 1$, a tangent $y = mx + k$ satisfies $k^2 = a^2m^2 - b^2$, provided the right-hand side is nonnegative.

Step 1: Write the family of parallel lines The given line is $y = x - 2$, with slope 1. Therefore every line parallel to it has the form $y = x + k$, where k is its intercept. Substituting in the hyperbola gives

$$4x^2 - 5(x + k)^2 = 20.$$

After expansion and multiplication by -1, this is

$$x^2 + 10kx + 5k^2 + 20 = 0.$$

Its roots describe the intersection abscissae. For tangency the two intersections coincide, so this quadratic must have a repeated real root.

Step 2: Apply the tangency condition and compare the options Set its discriminant equal to zero:

$$(10k)^2 - 4(5k^2 + 20) = 80(k^2 - 1) = 0.$$

Hence $k = 1$ or $k = -1$, giving the tangents $x-y+1 = 0$ and $x-y-1 = 0$. Only the first appears among the options. For $k = 1$, the intersection quadratic becomes $(x + 5)^2 = 0$; the contact point is $(-5, -4)$, which satisfies $4(25) - 5(16) = 20$. Differentiating the hyperbola also gives $dy/dx = 4x/(5y) = 1$ there, confirming the required slope. Options (A), (B), and (D) have intercepts $9, -3, 7$, respectively, whose squares are not 1; each therefore intersects in two distinct points rather than touching.

Step 3: Conclusion A listed tangent is $\boxed{x - y + 1 = 0}$. Hence, the correct option is (C).

Quick Tip to Solve: For a specified tangent slope, substitute the line with unknown intercept and set the resulting quadratic discriminant to zero.

25. Consider the statement : "P(n) : $n^2 - n + 41$ is prime". Then which one of the following is true ? [4 marks]

(A) P(5) is false but P(3) is true

(B) Both P(3) and P(5) are true

(C) P(3) is false but P(5) is true

(D) Both P(3) and P(5) are false

Correct Answer: (B) Both P(3) and P(5) are true

Solution:

Concept:

- A predicate $P(n)$ becomes a statement with a definite truth value after a particular integer is substituted for n. Each requested value must be tested separately.
- An integer greater than one is prime if it has no prime divisor at most its square root. A composite integer must have such a divisor, so this is a complete test.

Step 1: Evaluate and test the statement at three At $n = 3$, the integer in the predicate is

$$3^2 - 3 + 41 = 9 - 3 + 41 = 47.$$

Since $6^2 < 47 < 7^2$, any prime divisor capable of proving compositeness would be among $2, 3, 5$. The integer 47 is odd, its digit sum is 11 rather than a multiple of 3, and its last digit is neither 0 nor 5. None of these primes divides it. Therefore 47 is prime and $P(3)$ is true.

Step 2: Evaluate and test the statement at five At $n = 5$, direct substitution gives

$$5^2 - 5 + 41 = 25 - 5 + 41 = 61.$$

Now $7^2 < 61 < 8^2$, so test the primes $2, 3, 5, 7$. The integer is odd, its digit sum 7 is not divisible by 3, and its last digit excludes divisibility by 5. Finally $61 = 7 \cdot 8 + 5$, so it is not divisible by 7. This exhausts the possible small prime divisors, proving that 61 is prime and $P(5)$ is true. Option (A) incorrectly denies $P(5)$, option (C) incorrectly denies $P(3)$, and option (D) denies both. No assertion that the polynomial is prime for all integers is needed: these two evaluations settle the question.

Step 3: Conclusion Both $P(3)$ and $P(5)$ are true. Hence, the correct option is (B).

Quick Tip to Solve: Test only the specified inputs, and certify each prime by checking prime divisors up to its square root.

26. Let A be a point on the line $\vec{r} = (1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$ and B(3, 2, 6) be a point in the space. Then the value of μ for which the vector $\overrightarrow{AB}$ is parallel to the plane x − 4y + 3z = 1 is -

[4 marks]

(A) $\frac{1}{8}$
(B) $\frac{1}{2}$
(C) $\frac{1}{4}$
(D) $-\frac{1}{4}$

Correct Answer: (C) $\frac{1}{4}$

Solution:

Concept:

- A direction vector is parallel to a plane precisely when it is perpendicular to the plane normal. The coefficients of x, y, z provide that normal.
- The vector from a point A to a point B is $\overrightarrow{AB} = B - A$. Coordinates of a point on a parametrised line must first be expressed using its parameter.

Step 1: Form the displacement and identify the normal The line gives

$$A = (1-3\mu, \mu-1, 2+5\mu), \qquad B = (3, 2, 6).$$

Subtracting each coordinate of A from the corresponding coordinate of B gives

$$\overrightarrow{AB} = (2+3\mu, 3-\mu, 4-5\mu).$$

The plane $x - 4y + 3z = 1$ has normal $\mathbf{n} = (1, -4, 3)$. Its constant term determines position, not orientation, so it does not enter the dot product for the parallelism test.

Step 2: Impose zero normal component and verify For $\overrightarrow{AB}$ to lie in a direction parallel to the plane,

$$\begin{aligned}\overrightarrow{AB} \cdot \mathbf{n} &= (2+3\mu) - 4(3-\mu) + 3(4-5\mu) \\ &= 2 - 8\mu = 0.\end{aligned}$$

Therefore $\mu = 1/4$. At this value $A = (1/4, -3/4, 13/4)$, and the displacement is $(11/4, 11/4, 11/4)$, a nonzero multiple of $(1, 1, 1)$. Its dot product with the normal is proportional to $1 - 4 + 3 = 0$, confirming the result. Substituting the other offered parameter values $1/8, 1/2, -1/4$ into $2 - 8\mu$ gives $1, -2, 4$, respectively, all nonzero. Those displacement vectors have a normal component and fail the condition.

Step 3: Conclusion The required parameter is $\boxed{\mu = 1/4}$. Hence, the correct option is (C).

Quick Tip to Solve: For a vector parallel to a plane, take its dot product with the plane normal; do not substitute its components into the plane equation.

27. Consider a triangular plot ABC with sides AB = 7m, BC = 5m and CA = 6m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is - [4 marks]

(A) $2\sqrt{21}$

(B) $\frac{3}{2}\sqrt{21}$

(C) $7\sqrt{3}$

(D) $\frac{2}{3}\sqrt{21}$

Correct Answer: (D) $\frac{2}{3}\sqrt{21}$

Solution:

Concept:

- Apollonius' theorem gives the median from B to the midpoint D of AC: $BD^2 = (2AB^2 + 2BC^2 - AC^2)/4$.
- A vertical post is perpendicular to the horizontal plot. In the right triangle formed by its top, its foot and the observer, tangent of the angle of elevation is height divided by horizontal distance.

Step 1: Calculate the observer-to-post distance The lamp-post stands at the midpoint D of the side AC, so $AD = DC = 3\,\text{m}$. The horizontal distance from the observer B to the post is the median BD, rather than either given side AB or BC. By Apollonius' theorem,

$$BD^2 = \frac{2(7)^2 + 2(5)^2 - (6)^2}{4} = \frac{98 + 50 - 36}{4} = 28\,\text{m}^2.$$

Hence $BD = 2\sqrt{7}\,\text{m}$. Taking the positive square root is required because this is a physical distance.

Step 2: Use the angle subtended by the vertical post Let L be the top of the lamp-post and $DL = h$. Since BD lies in the horizontal plot and DL is vertical, triangle BDL is right angled at D. The post subtends $\angle DBL = 30°$ at B. Therefore

$$\tan 30° = \frac{h}{BD} \quad \Longrightarrow \quad h = 2\sqrt{7} \cdot \frac{1}{\sqrt{3}} = \frac{2\sqrt{21}}{3}\,\text{m}.$$

As a check, this height is less than the horizontal distance because $30° < 45°$. Options (A), (B), and (C) would all make $h/BD > 1$, corresponding to an elevation exceeding $45°$; they cannot represent the stated geometry. The last option alone gives the tangent ratio $1/\sqrt{3}$.

Step 3: Conclusion The lamp-post height is $\boxed{\frac{2}{3}\sqrt{21}\,\text{m}}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the median to the midpoint first; that median, not a side of the triangular plot, is the base of the elevation triangle.

28. If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal, then which on of the following is a valid choice for the ordered triad (a, b, c)? [4 marks]

(A) (1, 1, 3)

(B) (1, 1, 0)

(C) $\left(\frac{1}{2}, 2, 0\right)$

(D) $\left(\frac{1}{2}, 2, 3\right)$

Correct Answer: All four triads share the axial normal y=0. With the additional non-axial restriction, only option (A) works.

Solution:

Concept:

- The normal at the vertex of a horizontal parabola is its axis. Both given parabolas have the same axis, namely $y = 0$, regardless of the horizontal shift c.
- For a non-axial normal of slope m to $y^2 = 4px$, its equation is $y = mx - 2pm - pm^3$. A shift of the vertex replaces x by the coordinate measured from that vertex.

Step 1: Check the axial normal under the literal condition At the vertex $(c, 0)$ of $y^2 = 4b(x - c)$, the tangent is vertical and the normal is $y = 0$. At the vertex $(0, 0)$ of $y^2 = 8ax$, the tangent is also vertical and its normal is again $y = 0$. Every listed choice has nonzero positive a, b, so neither parabola is degenerate. Consequently all four listed triads have this common normal. The wording does not exclude the axis, and therefore does not specify a unique option.

Step 2: Examine the intended non-axial condition separately For a nonzero slope m, the normal equations are

$$y = mx - mc - 2bm - bm^3,$$
$$y = mx - 4am - 2am^3.$$

Equality of their intercepts, after dividing by $m \neq 0$, gives

$$c = (2a - b)(2 + m^2).$$

Option (A), with $(a, b, c) = (1, 1, 3)$, gives $3 = 2 + m^2$, so $m = \pm 1$. For example $y = x - 6$ is a normal to both parabolas in (A). Option (B) would require $m^2 = -2$; option (C) would also require $m^2 = -2$; and option (D) would require $m^2 = -5$. Thus (A) is the unique listed triad admitting a non-axial common normal. This additional restriction, however, is not present in the question as written.

Step 3: Conclusion As written, all four options admit the common normal $y = 0$, so there is no unique correct option. If a non-axial common normal is intended, the valid triad is $(1, 1, 3)$, option (A).

Quick Tip to Solve: Before dividing a common-normal equation by its slope, check the zero-slope case; it may be a normal to both curves.

29. The plane passing through the point (4, –1, 2) and parallel to the lines $\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$ and $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$ also passes through the point - [4 marks]

(A) (1, 1, – 1)

(B) (1, 1, 1)

(C) (– 1, – 1, –1)

(D) (– 1, – 1, 1)

Correct Answer: (B) (1, 1, 1)

Solution:

Concept:

- A plane parallel to two nonparallel lines has a normal perpendicular to both direction vectors. The denominators in the symmetric equations give those vectors, not the coordinates of a point.
- For a normal vector $\mathbf{n} = (a, b, c)$, the plane through (x_0, y_0, z_0) is $a(x-x_0)+b(y-y_0)+c(z-z_0) = 0$. A candidate point belongs to it exactly when substitution satisfies this equation.

Step 1: Read the direction vectors The first line has direction $\mathbf{u} = (3, -1, 2)$, while the second has direction $\mathbf{v} = (1, 2, 3)$. Their offsets, such as $x + 2$ and $y - 3$, locate the lines but do not affect which planes are parallel to them.

Step 2: Find a normal and the plane equation Take the cross product to obtain a common perpendicular:

$$\mathbf{u} \times \mathbf{v} = (-7, -7, 7).$$

We may divide by -7 and use the simpler normal $(1, 1, -1)$. Through the given point $(4, -1, 2)$, the plane is therefore

$$(x - 4) + (y + 1) - (z - 2) = 0, \qquad x + y - z = 1.$$

As a check, the normal has dot products $3 - 1 - 2 = 0$ and $1 + 2 - 3 = 0$ with the two directions.

Step 3: Test every candidate point For option (A), $x + y - z = 1 + 1 - (-1) = 3$, so it fails. For (B), $1 + 1 - 1 = 1$, so it lies on the plane. For (C), $-1 - 1 - (-1) = -1$, and for (D), $-1 - 1 - 1 = -3$. Neither of these equals the required constant. This also shows the listed choices contain just one valid point.

Step 4: Conclusion The required plane is $x + y - z = 1$, and it passes through $(1, 1, 1)$. Hence, the correct option is (B).

Quick Tip to Solve: Use only the denominators to read direction ratios; after finding a plane, substitution is the quickest way to test its proposed points.

30. If $\sum_{i=1}^{20}\left(\frac{^{20}C_{i-1}}{^{20}C_i+^{20}C_{i-1}}\right)^3=\frac{k}{21}$ **then k is equal to** [4 marks]

(A) 100

(B) 200

(C) 50

(D) 400

Correct Answer: (A) 100

Solution:

Concept:

- Pascal's identity gives $\binom{20}{i}+\binom{20}{i-1}=\binom{21}{i}$. Ratios of adjacent binomial coefficients simplify efficiently by cancelling factorials.
- The cube-sum identity is $\sum_{i=1}^{n} i^3=[n(n+1)/2]^2$. Here the exponent applies to each individual ratio inside the sum.

Step 1: Simplify one ratio For each integer $1\le i\le 20$, Pascal's identity changes the denominator to $\binom{21}{i}$. Thus

$$\frac{\binom{20}{i-1}}{\binom{20}{i}+\binom{20}{i-1}}=\frac{20!}{(i-1)!(21-i)!}\frac{i!(21-i)!}{21!}=\frac{i}{21}.$$

The factorials are defined over the full stated range, including the two endpoints, so the same reduction applies to every term.

Step 2: Evaluate the sum of cubes Cube the simplified ratio before summing. The left side of the given equation becomes

$$\sum_{i=1}^{20}\left(\frac{i}{21}\right)^3=\frac{1}{21^3}\left(\frac{20\cdot 21}{2}\right)^2=\frac{210^2}{21^3}=\frac{100}{21}.$$

In the last step, $210=10\cdot 21$, so its square cancels two powers of 21. Keeping this cancellation symbolic avoids an unnecessary large decimal calculation.

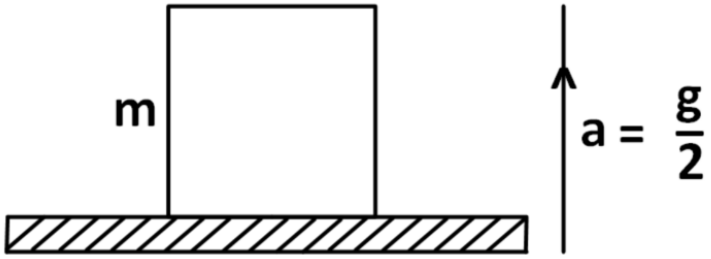
Step 3: Compare with the stated form The question defines the same sum to be $k/21$. Multiplication by 21 therefore gives $k=100$. Option (B) doubles this value, option (C) halves it, and option (D) is four times it; none equals the exactly evaluated sum when divided by 21. In particular, there is no extra factor from the number of terms after using the cube-sum formula.

Step 4: Conclusion The sum equals $100/21$, so $k=100$. Hence, the correct option is (A).

Quick Tip to Solve: Before summing ratios of neighbouring binomial coefficients, apply Pascal's identity and cancel factorials; this often leaves a simple power sum.

Physics (120 marks)

31. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in figure. Work done by normal reaction on block in time is - [4 marks]



(A) $\frac{mg^2t^2}{8}$

(B) $\frac{3mg^2t^2}{8}$

(C) $-\frac{mg^2t^2}{8}$

(D) 0

Correct Answer: (B) $\frac{3mg^2t^2}{8}$

Solution:

Concept:

- The block moves with the accelerating platform while contact is maintained. Newton's second law must include both the upward normal force N and the downward weight mg.
- Work by a constant force is $W = Fs\cos\theta$, where s is the displacement in the chosen ground frame. For motion from rest with constant acceleration, $s = at^2/2$.

Step 1: Find the normal reaction Choose upward as positive. The acceleration of the block is $a = g/2$, so its force equation is

$$N - mg = m\frac{g}{2}.$$

Consequently,

$$N = \frac{3}{2}mg.$$

This positive reaction confirms that the platform can maintain contact. Using $N = mg$ would incorrectly treat the vertically accelerating block as being in equilibrium.

Step 2: Find the displacement and normal work The platform and block start from rest and have the same upward displacement in time t:

$$s = \frac{1}{2}\left(\frac{g}{2}\right)t^2 = \frac{gt^2}{4}.$$

The normal force points along this displacement, so its work is positive:

$$W_N = Ns = \frac{3mg}{2}\frac{gt^2}{4} = \frac{3mg^2t^2}{8}.$$

The contact force does work because its point of application moves vertically in the ground frame.

Step 3: Distinguish normal work from net work The speed at time t is $gt/2$, giving $\Delta K = mg^2t^2/8$. Gravity does work $-mgs = -mg^2t^2/4$; adding this to W_N reproduces that kinetic-energy increase. Thus option (A) is the net work, not the work of the normal alone. Option (C) has the wrong sign, and option (D) incorrectly assumes a normal force always does zero work.

Step 4: Conclusion The work done by the normal reaction is $3mg^2t^2/8$. Hence, the correct option is (B).

Quick Tip to Solve: A normal force can do work when the supporting surface moves; first find its magnitude from acceleration, then use the block's displacement in the ground frame.

32. A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is - [4 marks]

(A) 12 Ω

(B) 2 Ω

(C) 4 Ω

(D) 8 Ω

Correct Answer: (C) 4 Ω

Solution:

Concept:

- For a uniform wire of constant cross-section and material, resistance is proportional to length: $R = \rho L/A$. Bending it changes its shape but not the resistance of each chosen length.
- Two resistive paths are in parallel if their ends meet at the same pair of terminals. Resistances add along a series path, while $R_{\text{eq}} = R_1R_2/(R_1 + R_2)$ for two parallel paths.

Step 1: Assign resistance to each side The total wire resistance is $18\,\Omega$. The three sides of an equilateral triangle have equal lengths, and uniformity gives each one resistance

$$r = \frac{18}{3} = 6\,\Omega.$$

Choose any two vertices as terminals, say A and B; symmetry ensures that every choice of vertex pair gives the same result.

Step 2: Identify the two conducting paths One current path runs directly along side AB, with resistance $6\,\Omega$. The other runs through the third vertex C, along AC and then CB. Those two sides are in series within that path, giving $6 + 6 = 12\,\Omega$. Both paths begin at A and end at B, so they have the same potential difference and must be combined in parallel.

Step 3: Calculate and check the equivalent resistance The equivalent resistance is

$$R_{AB} = \frac{6 \times 12}{6 + 12}\,\Omega = \frac{72}{18}\,\Omega = 4\,\Omega.$$

This is smaller than the smaller path resistance $6\,\Omega$, as parallel combinations require. Option (A), $12\,\Omega$, counts only the two-side route. Option (B), $2\,\Omega$, would treat all three sides as parallel even

though they do not share the same terminal pair. Option (D), $8\,\Omega$, exceeds the direct $6\,\Omega$ path and is impossible.

Step 4: Conclusion The resistance between any two vertices is $4\,\Omega$. Hence, the correct option is (C).

Quick Tip to Solve: For a closed uniform wire, split the loop into the two arcs between the measurement terminals and combine those arc resistances in parallel.

33. An insulating thin rod of length l has a linear charge density $\rho(x) = \rho_0 \frac{x}{l}$ on it. The rod is rotated about an axis passing through the origin (x = 0) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is - [4 marks]

(A) $\frac{\pi}{3} n\rho l^3$

(B) $\frac{\pi}{4} n\rho l^3$

(C) $n\rho l^3$

(D) $\pi n\rho l^3$

Correct Answer: (B) $\frac{\pi}{4} n\rho l^3$

Solution:

Concept:

- A charge dq completing n revolutions each second produces a time-averaged current $dI = n\,dq$. Its circular trajectory of radius x has area πx^2.
- The magnetic moment of a current loop is current times its area. For a distributed rotating charge, add the moments of the infinitesimal circular paths; the charge density must be included inside this integral.

Step 1: Write the charge on a short element Take the rod along $0 \le x \le l$, with the rotation axis at its end $x = 0$. The given linear density is $\rho(x) = \rho_0 x/l$. A length dx therefore carries

$$dq = \rho(x)\,dx = \frac{\rho_0 x}{l}\,dx.$$

Elements farther from the axis carry more charge per unit length, as well as tracing larger circles.

Step 2: Add the elemental magnetic moments The element at distance x circulates n times per second. Its average current is $dI = n\,dq$ and its moment is

$$d\mu = (n\,dq)(\pi x^2) = \frac{\pi n\rho_0}{l} x^3\,dx.$$

All the moments point along the same rotation axis for positive charge and the same sense of rotation. Hence their magnitudes add:

$$\mu = \frac{\pi n\rho_0}{l}\int_0^l x^3\,dx = \frac{\pi n\rho_0}{l}\frac{l^4}{4} = \frac{\pi}{4} n\rho_0 l^3.$$

Step 3: Check the coefficient and units The options use ρ for the density scale denoted by ρ_0 in the question. The coefficient $\pi/4$ identifies option (B). A coefficient $\pi/3$, as in (A), would arise for a

uniform linear density, which this rod does not have. Options (C) and (D) omit the correct integral factor. The units are $(\mathrm{s}^{-1})(\mathrm{C\,m}^{-1})\mathrm{m}^3 = \mathrm{A\,m}^2$, the units of magnetic moment.

Step 4: Conclusion The time-averaged magnetic moment has magnitude $\mu = \pi n \rho_0 l^3/4$, along the rotation axis according to the current sense. Hence, the correct option is (B).

Quick Tip to Solve: For rotating line charge, integrate $\pi n x^2 \rho(x)\, dx$; using the total charge at one representative radius generally gives a wrong coefficient.

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34. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are - [4 marks]

(A) 400 nm, 500 nm

(B) 625 nm, 500 nm

(C) 380 nm, 500 nm

(D) 380 nm, 525 nm

Correct Answer: (B) 625 nm, 500 nm

Solution:

Concept:

- A bright fringe in Young's double-slit experiment satisfies $d \sin\theta = m\lambda$, with integer order m. At a small angle in radians, the usual paraxial approximation is $\sin\theta \simeq \theta$.
- Two wavelengths give bright fringes at the same angle only when the common path difference is an integer multiple of each wavelength. The visible-range restriction bounds the allowed integer orders.

Step 1: Calculate the common path difference The slit separation is $d = 0.1\,\mathrm{mm} = 10^{-4}\,\mathrm{m}$, and the observation angle is $1/40$ radian. Using the small-angle approximation,

$$\Delta = d\theta = 10^{-4}\frac{1}{40}\,\mathrm{m} = 2.5 \times 10^{-6}\,\mathrm{m} = 2500\,\mathrm{nm}.$$

Both bright fringes must therefore obey $m\lambda = 2500\,\mathrm{nm}$, though their integer orders may be different.

Step 2: Use the visible wavelength interval The stated range gives

$$380 \le \frac{2500}{m} \le 740, \qquad \frac{2500}{740} \le m \le \frac{2500}{380}.$$

Thus the integer orders allowed are $4, 5, 6$. They correspond approximately to $625\,\mathrm{nm}$, $500\,\mathrm{nm}$, and $416.7\,\mathrm{nm}$. The choices need not list every possible visible wavelength; they ask for the pair among the supplied alternatives.

Step 3: Check the listed pairs For option (B), $2500/625 = 4$ and $2500/500 = 5$, both integers. Option (A) fails because $2500/400 = 6.25$. Options (C) and (D) contain $380\,\mathrm{nm}$, for which $2500/380$ is not an integer; in (D), $2500/525$ is also nonintegral. The angular approximation changes the path difference

only at relative order θ^2, so the wavelengths are quoted to the precision intended by these options.

Step 4: Conclusion The listed wavelengths are $\lambda_1 = 625\,\text{nm}$ and $\lambda_2 = 500\,\text{nm}$, corresponding to orders 4 and 5. Hence, the correct option is (B).

Quick Tip to Solve: At a fixed bright-fringe angle, compute the common path difference once and test whether dividing it by each proposed wavelength gives an integer.

35. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is - [4 marks]

(A) $\frac{F}{2mR}$
(B) $\frac{2F}{3mR}$
(C) $\frac{3F}{2mR}$
(D) $\frac{F}{3mR}$

Correct Answer: (B) $\frac{2F}{3mR}$

Solution:

Concept:

- A homogeneous solid cylinder has moment of inertia $I = MR^2/2$ about its symmetry axis. Rolling without slipping connects the magnitudes of translational and angular acceleration by $a = \alpha R$.
- A horizontal pull through the roller's axle accelerates its centre, while static friction supplies the torque needed for rolling. Both the translation and the rotation must satisfy Newton's laws simultaneously.

Step 1: Set up the force and torque equations Let the pull F act to the right through the axle of the cylindrical roller. Let f be the leftward static friction at the ground contact. The equation for translation is

$$F - f = Ma.$$

The horizontal pull and weight have no moment about the centre; friction produces the clockwise torque that makes the roller rotate in the direction needed for rightward rolling.

Step 2: Use the rolling constraint With clockwise angular acceleration written as a positive magnitude, the torque equation is

$$fR = I\alpha = \frac{1}{2}MR^2\alpha.$$

Since the motion is rolling without slipping, $a = \alpha R$. It follows that $f = Ma/2$. Substitution in the translational equation gives

$$F - \frac{1}{2}Ma = Ma, \qquad a = \frac{2F}{3M}.$$

The two motions share the applied force through the required friction; one cannot simply set $a = F/M$.

Step 3: Obtain the angular acceleration and compare Dividing the centre acceleration by the radius gives

$$\alpha = \frac{a}{R} = \frac{2F}{3MR}.$$

The options write the cylinder mass as m rather than the M used in the statement; the matching expression is option (B). Option (A) has an incorrect inertia coefficient, option (C) reverses the factor $2/3$, and option (D) is too small by a factor of two. The result has units s^{-2}, as an angular acceleration should.

Step 4: Conclusion The angular acceleration is $2F/(3MR)$, clockwise for a rightward pull. Hence, the correct option is (B).

Quick Tip to Solve: For a force through the axle, use $a = F/(M + I/R^2)$ together with $\alpha = a/R$; a solid cylinder contributes the extra effective mass $M/2$.

36. In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12} m, the minimum electron energy required is close to - [4 marks]

(A) 25 keV

(B) 500 keV

(C) 100 keV

(D) 1 keV

Correct Answer: (A) 25 keV

Solution:

Concept:

- The resolving length is of the order of the electron wavelength, so the threshold momentum is obtained from the de Broglie relation $p = h/\lambda$, where h is Planck's constant.
- For a first estimate, the electron kinetic energy is $K = p^2/(2m_e) = h^2/(2m_e\lambda^2)$. The relativistic expression is $K = \sqrt{p^2c^2 + m_e^2c^4} - m_ec^2$.

Step 1: Find the momentum needed to resolve the width Take the largest usable wavelength, $\lambda = 7.5 \times 10^{-12}\,\mathrm{m}$, because a shorter wavelength would require more momentum and energy. Using $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$, obtain

$$p = \frac{6.626 \times 10^{-34}}{7.5 \times 10^{-12}} = 8.83 \times 10^{-23}\,\mathrm{kg\,m\,s^{-1}}.$$

Step 2: Calculate and compare the electron energies With $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$, the nonrelativistic estimate gives

$$K = \frac{(8.83 \times 10^{-23})^2}{2(9.11 \times 10^{-31})} = 4.28 \times 10^{-15}\,\mathrm{J} \simeq 26.7\,\mathrm{keV}.$$

Here $1\,\mathrm{keV} = 1.602 \times 10^{-16}\,\mathrm{J}$. A relativistic check uses $pc \simeq 165.5\,\mathrm{keV}$ and $m_ec^2 = 511\,\mathrm{keV}$, giving $K \simeq \sqrt{511^2 + 165.5^2} - 511 \simeq 26.1\,\mathrm{keV}$. Thus the closest listed energy is 25 keV. The 1 keV choice has a wavelength too large to provide the stated resolution; 100 keV and 500 keV exceed the energy scale needed and are not the minimum threshold.

Step 3: Conclusion The minimum energy is close to $25\,\mathrm{keV}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert the resolving width into momentum first. Since $K \propto \lambda^{-2}$ at low energies, even a small wavelength error strongly changes the energy.

37. A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as - [4 marks]

(A) $3\mu_2 - 2\mu_1 = 1$

(B) $\mu_1 + \mu_2 = 3$

(C) $2\mu_1 - \mu_2 = 1$

(D) $2\mu_2 - \mu_1 = 1$

Correct Answer: (C) $2\mu_1 - \mu_2 = 1$

Solution:

Concept:

- For a thin plano-convex lens in air, the magnitude of its focal length is $f_1 = R/(\mu_1 - 1)$, since one surface is plane and the other has radius of curvature R.
- A plano-concave lens has negative signed power. Its focal-length magnitude is $f_2 = R/(\mu_2 - 1)$. The stated positive comparison $f_1 = 2f_2$ compares focal-length magnitudes.

Step 1: Write the focal-length magnitudes The lens maker formula is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$. A plane surface contributes zero reciprocal curvature. Consequently, the convex lens has power $(\mu_1 - 1)/R$, whereas the concave lens has signed power $-(\mu_2 - 1)/R$. Comparing their positive focal-length magnitudes therefore gives

$$\frac{f_1}{f_2} = \frac{R/(\mu_1 - 1)}{R/(\mu_2 - 1)} = \frac{\mu_2 - 1}{\mu_1 - 1}.$$

The common radius cancels; contact does not change either lens's own focal length.

Step 2: Apply the given ratio and rearrange Insert $f_1/f_2 = 2$:

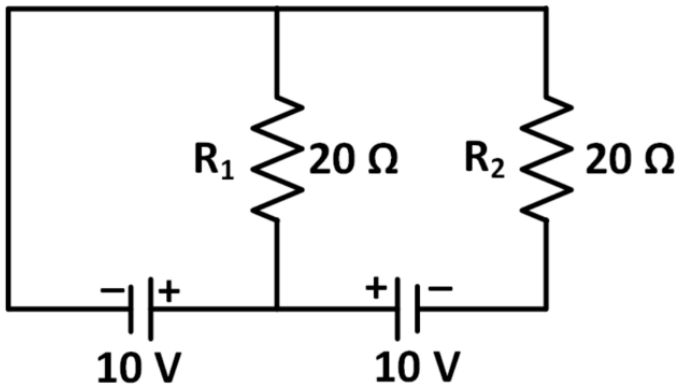
$$\mu_2 - 1 = 2(\mu_1 - 1) \quad \Longrightarrow \quad 2\mu_1 - \mu_2 = 1.$$

The larger focal-length magnitude belongs to the smaller refractive-index contrast, so $\mu_2 - 1$ must be twice $\mu_1 - 1$, as this relation states. Option (D) reverses that ratio. Option (B) constrains the sum of indices rather than the ratio of their contrasts. Option (A) also gives an unrelated contrast ratio. Signed focal lengths would have opposite signs; the positive ratio in the question must therefore be read as the usual comparison of their magnitudes.

Step 3: Conclusion The required relation is $2\mu_1 - \mu_2 = 1$. Hence, the correct option is (C).

Quick Tip to Solve: For equal curvatures, compare $\mu - 1$, not μ. Focal-length magnitudes are inversely proportional to this refractive-index contrast.

38. In the given circuit the cells have zero internal resistance. The currents (in Amperes) passing through resistance R_1 and R_2 respectively, are - [4 marks]



(A) 0.5, 0

(B) 0, 1

(C) 1, 2

(D) 2, 2

Correct Answer: (A) 0.5, 0

Solution:

Concept:

- An ideal cell fixes the potential difference between its terminals independently of the current. Its positive terminal is at the higher potential by the marked emf.
- Once the potentials at the two ends of a resistor are known, Ohm's law gives its current: $I = \Delta V/R$. A resistor with equal potentials at its ends carries zero current.

Step 1: Assign potentials using the left cell Call the common upper wire node A, the junction between the two cells B, and the bottom of R_2 node C. The entire left wire belongs to node A, because ideal wire has no potential drop. Set $V_A = 0$. The left 10 V cell has its positive terminal at B, so

$$V_B - V_A = 10\,\text{V}, \qquad V_B = 10\,\text{V}.$$

Resistor $R_1 = 20\,\Omega$ lies directly between B and A. Thus

$$I_1 = \frac{V_B - V_A}{20} = \frac{10}{20} = 0.5\,\text{A}.$$

Its conventional current flows upward from B through R_1 towards the upper wire.

Step 2: Use the right cell to find the second current The right cell also has its positive terminal at B, but its negative terminal is at C. Therefore

$$V_B - V_C = 10\,\text{V} \quad \Longrightarrow \quad V_C = 0 = V_A.$$

The two ends of R_2 are equipotential, giving

$$I_2 = \frac{V_A - V_C}{20} = 0\,\text{A}.$$

The equal, opposing cell emfs cancel around the outer loop. Options (B), (C) and (D) all assign a nonzero current to R_2, contrary to its zero voltage, and also fail the 10 V drop across R_1.

Step 3: Conclusion The currents through R_1 and R_2, respectively, are $0.5\,\text{A}$ and $0\,\text{A}$. Hence, the correct option is (A).

Quick Tip to Solve: Mark node potentials before combining resistors. Two resistors drawn side by side need not have the same pair of terminal potentials.

39. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to - [4 marks]

(A) 16.6 cm

(B) 10.0 cm

(C) 20.0 cm

(D) 33.3 cm

Correct Answer: (C) 20.0 cm

Solution:

Concept:

- The speed of transverse waves on a string is $v = \sqrt{T/\mu}$, where T is the tension and $\mu = M/L$ is mass per unit length.
- In a standing wave, adjacent nodes are separated by half a wavelength: $d = \lambda/2 = v/(2f)$. A string fixed at both ends has an integer number of these half-wavelength segments.

Step 1: Find the linear density and wave speed Convert the mass to SI units before using the tension formula:

$$M = 5\,\text{g} = 5 \times 10^{-3}\,\text{kg}, \qquad \mu = \frac{5 \times 10^{-3}}{1} = 5 \times 10^{-3}\,\text{kg m}^{-1}.$$

The wave speed is therefore

$$v = \sqrt{\frac{8.0}{5 \times 10^{-3}}} = \sqrt{1600} = 40\,\text{m s}^{-1}.$$

This is the propagation speed of either travelling component; the nodes themselves remain stationary.

Step 2: Find the wavelength and halve it The external vibrator fixes the frequency at $100\,\text{Hz}$, hence

$$\lambda = \frac{v}{f} = \frac{40}{100} = 0.40\,\text{m}, \qquad d = \frac{\lambda}{2} = 0.20\,\text{m} = 20.0\,\text{cm}.$$

As a boundary-condition check, $L/d = 1/0.20 = 5$, an integer, so five half-waves fit between the fixed

ends. This is the fifth harmonic because the fundamental frequency is $v/(2L) = 20\,\text{Hz}$. The other separations 16.6 cm, 10.0 cm and 33.3 cm correspond to different half-wavelengths and therefore different frequencies for the same tension and mass density.

Step 3: Conclusion The separation between successive nodes is $20.0\,\text{cm}$. Hence, the correct option is (C).

Quick Tip to Solve: First calculate wave speed from tension and linear density. Then use $v/(2f)$ directly for adjacent-node spacing; v/f is a full wavelength.

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40. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is - [4 marks]

(A) mv^2

(B) $\frac{1}{2}\,mv^2$

(C) $\frac{3}{2}\,mv^2$

(D) $2\,mv^2$

Correct Answer: (A) mv^2

Solution:

Concept:

- For a circular satellite orbit of radius r, gravitational force supplies centripetal force, giving $v^2 = GM/r$, where M is Earth's mass.
- For just escape, the object reaches infinity with zero speed. With gravitational potential energy defined as zero at infinity, its total mechanical energy immediately after ejection must be zero.

Step 1: Express the orbital potential energy using the given speed The original circular motion satisfies

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad \Longrightarrow \quad \frac{GM}{r} = v^2.$$

The ejected object is at the same distance r at the instant of release. Its gravitational potential energy at that instant is consequently

$$U = -\frac{GMm}{r} = -mv^2.$$

The symbol v here remains the satellite's original circular speed; it is not the object's speed after the ejection impulse.

Step 2: Apply the threshold escape condition Let u be the object's speed immediately after ejection in the Earth-centred frame. For just escape, conservation of energy gives

$$\frac{1}{2}mu^2 - \frac{GMm}{r} = 0 \quad \Longrightarrow \quad K = \frac{1}{2}mu^2 = \frac{GMm}{r} = mv^2.$$

Equivalently, the local escape speed is $u = \sqrt{2}v$. The choice $\frac{1}{2}mv^2$ is only its original circular-orbit kinetic energy and is insufficient for escape. The choices $\frac{3}{2}mv^2$ and $2mv^2$ exceed the threshold, leaving

positive kinetic energy at infinity. The question asks for the total kinetic energy after ejection, rather than just the additional energy imparted.

Step 3: Conclusion At the instant it is ejected for just escape, the object has kinetic energy mv^2. Hence, the correct option is (A).

Quick Tip to Solve: At a given orbital radius, escape speed is $\sqrt{2}$ times circular speed, so escape kinetic energy is twice the circular-orbit kinetic energy.

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41. If the magnetic field of a plane electromagnetic wave is given by (the speed of light = 3×10^8 B = 100×10^{-6} sin $\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right]$ then the maximum electric field associated with it is - [4 marks]

(A) 4.5×10^4 N/C

(B) 4×10^4 N/C

(C) 6×10^4 N/C

(D) 3×10^4 N/C

Correct Answer: (D) 3×10^4 N/C

Solution:

Concept:

- For a plane electromagnetic wave in vacuum, the electric and magnetic fields are in phase and their magnitudes satisfy $E = cB$ at each point and instant.
- The amplitude is the coefficient multiplying the sine, not its angular frequency. Accordingly, the peak fields obey $E_0 = cB_0$; rms fields would each be smaller by $\sqrt{2}$.

Step 1: Read the magnetic-field amplitude The field is sinusoidal with coefficient 100×10^{-6}. Its maximum magnetic field is therefore

$$B_0 = 100 \times 10^{-6}\,\mathrm{T} = 1.0 \times 10^{-4}\,\mathrm{T}.$$

The factor $2\pi \times 2 \times 10^{15}$ belongs to the phase and determines the oscillation frequency. It does not multiply the magnetic-field amplitude and is not required when calculating the ratio of the peak electric and magnetic fields. The sine can attain unity, so the given coefficient already represents a peak value.

Step 2: Multiply the amplitude by the wave speed Use the supplied speed of light, $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$:

$$E_0 = cB_0 = (3 \times 10^8)(1.0 \times 10^{-4}) = 3 \times 10^4\,\mathrm{N\,C^{-1}}.$$

The dimensions are consistent because $\mathrm{T\,m\,s^{-1}} = \mathrm{N\,C^{-1}}$. No factor of $\sqrt{2}$, two, or the frequency should be introduced for this peak-to-peak relation. The alternatives 4.5×10^4, 4×10^4 and $6 \times 10^4\,\mathrm{N/C}$ would all require a magnetic amplitude larger than the stated $10^{-4}\,\mathrm{T}$ at the given speed.

Step 3: Conclusion The maximum electric field is $3 \times 10^4\,\mathrm{N/C}$. Hence, the correct option is (D).

Quick Tip to Solve: For an electromagnetic-wave amplitude question, read the coefficient of the sine and use $E_0 = cB_0$. Leave the phase and frequency out of the amplitude calculation.

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42. A 2 W carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is - [4 marks]

(A) 0.4 mA

(B) 20 mA

(C) 63 mA

(D) 100 mA

Correct Answer: (B) 20 mA

Solution:

Concept:

- In a four-band resistor code, the first two colours give significant digits, the third is the power-of-ten multiplier, and the fourth specifies tolerance.
- For a resistor carrying steady current, the dissipated power is $P = I^2R$. A power rating sets the maximum allowed dissipation, so $I_{\max} = \sqrt{P_{\max}/R}$.

Step 1: Decode the resistance value Green represents the digit 5, black represents 0, and red gives the multiplier 10^2. Thus the nominal resistance is

$$R = 50 \times 10^2\,\Omega = 5000\,\Omega = 5.0\,\text{k}\Omega.$$

The final brown band means a tolerance of $\pm 1\%$, rather than an extra significant digit or multiplier. The usual colour-code calculation uses the nominal resistance. Even taking the upper tolerance value, $5050\,\Omega$, would change the current by less than one percent and would not change the closest choice.

Step 2: Use the power limit to calculate the current At the limiting dissipation $P_{\max} = 2\,\text{W}$,

$$I_{\max} = \sqrt{\frac{2}{5000}} = \sqrt{4 \times 10^{-4}} = 2.0 \times 10^{-2}\,\text{A} = 20\,\text{mA}.$$

Check by substitution: $(0.020)^2 \times 5000 = 2\,\text{W}$. A current of 0.4 mA would dissipate only $8 \times 10^{-4}\,\text{W}$, so it is not the maximum. At 63 mA the dissipation is about 19.8 W, and at 100 mA it is 50 W, both far above the rating. The units must be converted to amperes before squaring the current.

Step 3: Conclusion The maximum permissible current is approximately $20\,\text{mA}$. Hence, the correct option is (B).

Quick Tip to Solve: Decode the resistance before applying the wattage rating. Use $I = \sqrt{P/R}$, and convert amperes to milliamperes only after taking the square root.

43. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is - [4 marks]

(A) 1 : 16

(B) 1 : 8

(C) 1 : 2

(D) 1 : 4

Correct Answer: (A) 1 : 16

Solution:

Concept:

- For a projectile returning to the launch level, the horizontal range is $s = u^2 \sin(2\theta)/g$, where u is launch speed, θ is elevation and g is gravitational acceleration.
- The greatest range is obtained at $\theta = 45^\circ$. Allowing every azimuth gives a circular region on horizontal ground, whose radius is the greatest horizontal range.

Step 1: Find the largest reachable radius For a fixed speed, the sine factor varies from zero to one. Consequently the greatest distance from the firing point at which a bullet can land is

$$s_{\max} = \frac{u^2}{g}.$$

Rotation of the firing direction about the vertical sweeps this limiting range into a circle. Every smaller radial distance is also attainable by choosing a suitable elevation, so the covered region is a disc, rather than just its boundary. Both guns experience the same gravitational acceleration.

Step 2: Compare the areas using the speed dependence The disc area is

$$A = \pi s_{\max}^2 = \frac{\pi u^4}{g^2}.$$

The speed units cancel when taking the ratio, so conversion from kilometres per second is unnecessary:

$$\frac{A_A}{A_B} = \left(\frac{u_A}{u_B}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}.$$

A ratio of $1 : 2$ compares speeds only. The ratio $1 : 4$ compares maximum ranges, whereas $1 : 8$ would incorrectly use a cubic speed dependence. Area introduces a second square after the range has already depended quadratically on speed.

Step 3: Conclusion The maximum ground areas have ratio $A_A : A_B = 1 : 16$. Hence, the correct option is (A).

Quick Tip to Solve: First identify the radius of the reachable region: range scales as u^2, so the circular area scales as u^4.

44. A charge Q is distributed over three concentric spherical shells of radii a, b, c (a < b < c) such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where r < a, would be - [4 marks]

(A) $\frac{Q\left(a^2+b^2+c^2\right)}{4\pi\varepsilon_0(a^3+b^3+c^3)}$

(B) $\frac{Q}{4\pi\varepsilon_0(a+b+c)}$

(C) $\frac{Q}{12\pi\varepsilon_0}\frac{ab+bc+ca}{abc}$

(D) $\frac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$

Correct Answer: (D) $\frac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$

Solution:

Concept:

- An isolated uniformly charged spherical shell of radius s and charge q produces constant potential $q/(4\pi\varepsilon_0 s)$ at every interior point, with zero potential at infinity.
- Equal surface charge density σ means shell charge is proportional to surface area: $q = 4\pi s^2\sigma$. Potentials add algebraically by superposition.

Step 1: Express each shell charge in terms of the total Write the common surface charge density as σ. The three charges are

$$q_a = 4\pi a^2\sigma, \qquad q_b = 4\pi b^2\sigma, \qquad q_c = 4\pi c^2\sigma.$$

Adding these gives

$$Q = 4\pi\sigma(a^2 + b^2 + c^2).$$

Thus $\sigma = Q/[4\pi(a^2+b^2+c^2)]$. The larger shell carries a larger share of the total charge; equal density does not mean that each shell carries $Q/3$.

Step 2: Add the constant interior potentials Since $r < a < b < c$, the observation point is inside all three shells. Therefore

$$\begin{aligned} V &= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_a}{a} + \frac{q_b}{b} + \frac{q_c}{c}\right) \\ &= \frac{\sigma}{\varepsilon_0}(a+b+c) \\ &= \frac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}. \end{aligned}$$

There is no dependence on r, consistent with the zero electric field in this inner region. Option C would follow from equal charges on the shells, which is not the given condition. Options A and B use different radius weightings and do not follow from the required area-proportional charge distribution.

Step 3: Conclusion The potential is $V = Q(a+b+c)/[4\pi\varepsilon_0(a^2+b^2+c^2)]$, independent of r for $r < a$. Hence, the correct option is (D).

Quick Tip to Solve: For equal surface densities use charges proportional to squared radii, then divide each charge by its own radius before adding potentials.

45. To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is - [4 marks]

(A) μFR/2

(B) μFR/3

(C) μFR/6

(D) $\frac{2}{3}\mu$FR

Correct Answer: (D) $\frac{2}{3}\mu$FR

Solution:

Concept:

- Uniform normal loading gives pressure $p = F/(\pi R^2)$. A small contact area dA has normal force $p\,dA$ and sliding friction magnitude $\mu p\,dA$.
- The friction is tangential to the circular motion. Its torque magnitude about the rotation axis is $d\tau = r\,\mu p\,dA$, with r the local lever arm.

Step 1: Divide the mop into concentric rings Take a narrow annulus of radius r and radial width dr. Its area and normal force are

$$dA = 2\pi r\,dr, \qquad dN = \frac{F}{\pi R^2}\,2\pi r\,dr = \frac{2F}{R^2}r\,dr.$$

All elements on this annulus have the same lever arm r. Although their friction forces have different horizontal directions, their torques about the vertical axis have the same resisting sense. Thus their torque magnitudes add, rather than cancelling.

Step 2: Integrate the resisting torque and balance it For the annulus,

$$d\tau = r\mu\,dN = \frac{2\mu F}{R^2}r^2\,dr.$$

Summing from the centre to the rim gives

$$\tau_f = \frac{2\mu F}{R^2}\int_0^R r^2\,dr = \frac{2\mu F}{R^2}\frac{R^3}{3} = \frac{2}{3}\mu F R.$$

The constant angular speed means zero angular acceleration, so the driving torque equals this resisting torque in magnitude and acts opposite to it. Using $R/2$ as the effective lever arm would ignore the greater contact area of outer rings. The area-weighted mean lever arm is actually $2R/3$; this excludes the factors $1/2$, $1/3$ and $1/6$ in the other options.

Step 3: Conclusion The machine must apply torque of magnitude $\tau = \frac{2}{3}\mu F R$ in the direction of rotation. Hence, the correct option is (D).

Quick Tip to Solve: For uniform pressure on a disc, weight each radius by annular area $2\pi r\,dr$; the effective friction lever arm is $2R/3$.

46. A solid metal cube of edge length 2 cm is moving in a positive y-direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is - [4 marks]

(A) 2mV

(B) 12 mV

(C) 6 mV

(D) 1 mV

Correct Answer: (B) 12 mV

Solution:

Concept:

- A charge moving with a conductor experiences magnetic force $q\mathbf{v}\times\mathbf{B}$. Charge separation establishes an electric field until $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$ inside the moving metal.
- If $\mathbf{v}$ and $\mathbf{B}$ are perpendicular and the separation is along their cross product, the potential-difference magnitude is $|\Delta V| = vB\ell$.

Step 1: Identify the charge-separation direction The cube moves along $+y$, and the magnetic field points along $+z$. Therefore

$$\mathbf{v} \times \mathbf{B} = vB\,\hat{\mathbf{x}}.$$

Positive charges are pushed towards the face at larger x, while electrons are displaced towards the face at smaller x. These are precisely the two faces perpendicular to the x-axis. At equilibrium the internal electric field points towards $-x$, with magnitude $E = vB$. The higher-potential face is the one at larger x.

Step 2: Calculate the voltage across the cube edge The separation of the two faces is one edge length,

$$\ell = 2\,\text{cm} = 2 \times 10^{-2}\,\text{m}.$$

Thus

$$\begin{aligned} |\Delta V| &= vB\ell \\ &= 6 \times 0.1 \times 2 \times 10^{-2}\,\text{V} \\ &= 1.2 \times 10^{-2}\,\text{V} = 12\,\text{mV}. \end{aligned}$$

Only the edge component along the charge-separation direction enters; neither a face diagonal nor the face area is involved. The options 2, 6 and $1\,\text{mV}$ do not equal the product of the stated speed, magnetic field and correctly converted edge. The result has voltage units because $\text{T}\,\text{m}^2\,\text{s}^{-1} = \text{V}$.

Step 3: Conclusion The potential difference is $12\,\text{mV}$, with the larger-x face at higher potential. Hence, the correct option is (B).

Quick Tip to Solve: Use $\mathbf{v} \times \mathbf{B}$ to identify the pair of faces first, then insert their separation in $vB\ell$.

47. Water flows into a large tank with flat bottom at the rate of $10^{-4}\,\mathrm{m^3\,s^{-1}}$. Water is also leaking out of a hole of area $1\,\mathrm{cm^2}$ at its bottom. If the height of the water in the tank remains steady, then this height is - [4 marks]

(A) 2.9 cm

(B) 5.1 cm

(C) 4 cm

(D) 1.7 cm

Correct Answer: (B) 5.1 cm

Solution:

Concept:

- A steady water height requires equal volumetric inflow and outflow: $Q_{\text{in}} = A_h v$, where A_h is the hole area and v is the efflux speed.
- For a large tank open to the atmosphere, Bernoulli's equation gives Torricelli's law $v = \sqrt{2gh}$. The free-surface speed is negligible compared with the speed through the small bottom hole.

Step 1: Use continuity to find the outflow speed The inflow is $Q_{\text{in}} = 10^{-4}\,\mathrm{m^3\,s^{-1}}$. Convert the hole area before dividing:

$$A_h = 1\,\mathrm{cm^2} = 10^{-4}\,\mathrm{m^2}.$$

Since the depth remains constant, water cannot accumulate in the tank. Consequently

$$v = \frac{Q_{\text{in}}}{A_h} = \frac{10^{-4}}{10^{-4}} = 1\,\mathrm{m\,s^{-1}}.$$

The equal powers of ten cancel. Converting a square centimetre as though it were a centimetre would give the wrong speed and a much larger error in the inferred height.

Step 2: Relate the exit speed to the water depth Both the water surface and the emerging stream are at atmospheric pressure. Bernoulli's equation between them therefore converts gravitational head into kinetic energy:

$$gh = \frac{v^2}{2}.$$

Using $g = 9.8\,\mathrm{m\,s^{-2}}$,

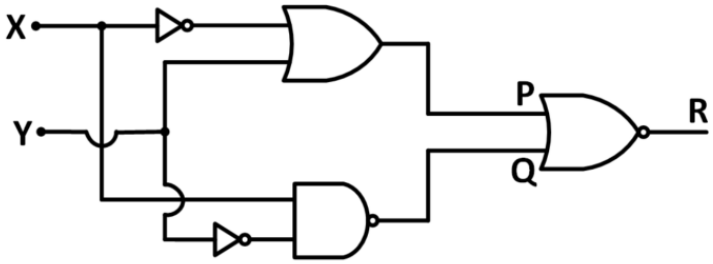
$$h = \frac{1^2}{2 \times 9.8}\,\mathrm{m} = 0.05102\,\mathrm{m} \simeq 5.1\,\mathrm{cm}.$$

The alternatives 2.9, 4 and $1.7\,\mathrm{cm}$ imply smaller efflux speeds than $1\,\mathrm{m\,s^{-1}}$, so their outflows cannot balance the stated inflow. No tank base area is needed because the question specifies steady depth, rather than the rate at which depth changes.

Step 3: Conclusion The steady water height is approximately $5.1\,\mathrm{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: At steady depth first find $v = Q/A_h$, then calculate $h = v^2/(2g)$; keep the area conversion in square metres.

48. To get output 1 at R, for the given logic gate circuit the input values must be [4 marks]



(A) x = 0, y = 0

(B) x = 1, y = 0

(C) x = 0, y = 1

(D) x = 1, y = 1

Correct Answer: (B) x = 1, y = 0

Solution:

Concept:

- An OR gate gives $a + b$, an AND gate gives ab, and a small output bubble denotes logical negation. The symbol $\overline{a}$ means NOT a.
- A NOR gate outputs one only when both its inputs are zero. Wire crossings drawn with a bridge are not junctions; only a joined connection transfers the input to that branch.

Step 1: Write the outputs of the upper and lower branches The upper branch inverts x and ORs it with y, giving

$$P = \overline{x} + y.$$

The lower branch inverts y and combines it with the unmodified x through a NAND gate, giving

$$Q = \overline{x\overline{y}}.$$

The wire bridges in the drawing prevent the crossed x and y conductors from shorting together. By De Morgan's law, $Q = \overline{x} + y$, so the two branches actually supply the same Boolean value.

Step 2: Impose the required final output and test the choices The final gate is NOR, so

$$R = \overline{P + Q} = \overline{\overline{x} + y} = x\overline{y}.$$

Thus $R = 1$ requires both $x = 1$ and $\overline{y} = 1$, or $y = 0$. Directly checking the four possible pairs gives

x	y	P	Q	R
0	0	1	1	0
1	0	0	0	1
0	1	1	1	0
1	1	1	1	0

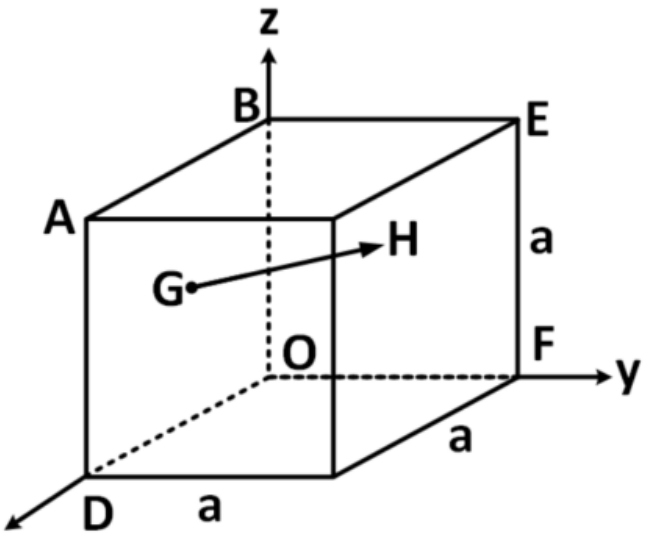
Only the second row satisfies the demand. For option A the inverted x already makes $P = 1$. For

options C and D the input $y = 1$ makes $P = 1$. In each of those cases the final NOR gate must output zero, irrespective of its other input.

Step 3: Conclusion The required inputs are $x = 1$ and $y = 0$. Hence, the correct option is (B).

Quick Tip to Solve: Work backwards from a final NOR output of one: force each incoming branch to zero before solving for the original inputs.

49. In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be - [4 marks]



(A) $\frac{1}{2}a\left(\hat{k}-\hat{i}\right)$

(B) $\frac{1}{2}a\left(\hat{j}-\hat{i}\right)$

(C) $\frac{1}{2}a\left(\hat{j}-\hat{k}\right)$

(D) $\frac{1}{2}a\left(\hat{i}-\hat{k}\right)$

Correct Answer: (B) $\frac{1}{2}a\left(\hat{j}-\hat{i}\right)$

Solution:

Concept:

- The position vector of a rectangle's centre is the average of its corner position vectors. A displacement from an initial point to a final point is final position minus initial position.
- Use the three cube edges through O as mutually perpendicular coordinate axes: OD is along $+x$, OF along $+y$, and OB along $+z$, each of length a.

Step 1: Find the coordinates of both face centres Set $O = (0, 0, 0)$. The face $ABOD$ lies in the plane

$y = 0$, with its centre halfway along the x and z edges. Its centre G therefore has position

$$\mathbf{r}_G = \frac{a}{2}\hat{\mathbf{i}} + \frac{a}{2}\hat{\mathbf{k}}.$$

The face $BEFO$ lies in the plane $x = 0$. Its centre H is halfway along the y and z directions, so

$$\mathbf{r}_H = \frac{a}{2}\hat{\mathbf{j}} + \frac{a}{2}\hat{\mathbf{k}}.$$

Both centres are at the same height $z = a/2$, which immediately implies zero vertical component of the displacement.

Step 2: Subtract the initial position from the final position The requested direction is from $ABOD$ to $BEFO$, that is, from G to H. Hence

$$\begin{aligned}\overrightarrow{GH} &= \mathbf{r}_H - \mathbf{r}_G \\ &= \frac{a}{2}\hat{\mathbf{j}} + \frac{a}{2}\hat{\mathbf{k}} - \frac{a}{2}\hat{\mathbf{i}} - \frac{a}{2}\hat{\mathbf{k}} \\ &= \frac{a}{2}(\hat{\mathbf{j}} - \hat{\mathbf{i}}).\end{aligned}$$

The negative x component reflects motion towards the plane $x = 0$; the positive y component reflects motion away from $y = 0$. Options A, C and D all contain a nonzero z component, contrary to the equal heights of the two centres. This also checks the orientation independently of the algebra.

Step 3: Conclusion The vector is $\overrightarrow{GH} = \frac{a}{2}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$. Hence, the correct option is (B).

Quick Tip to Solve: Locate the two face centres as coordinate triples, then subtract final minus initial; their common height cancels the vertical component.

50. A heat source at T = 10^3 K is connected to another heat reservoir at T = 10^2 K by a copper slab which is 1 mthick. Given that the thermal conductivity of copper is 0.1 $WK^{-1}m^{-1}$, the energy flux through it in the steady state is - [4 marks]

(A) 200 Wm^{-2}

(B) 65 Wm^{-2}

(C) 120 Wm^{-2}

(D) 90 Wm^{-2}

Correct Answer: (D) 90 Wm^{-2}

Solution:

Concept:

- Fourier's law gives the rate of heat conduction through a uniform slab: $\dot{Q} = kA(T_h - T_c)/L$, where k is thermal conductivity, A is cross-sectional area and L is thickness.
- Energy flux means heat transferred per unit time per unit area, $j = \dot{Q}/A$. Its SI unit is $\mathrm{W\,m^{-2}}$, distinct from the unit watt for total heat current.
- In steady state the temperature gradient in a homogeneous slab is constant. Heat flows from the

hotter reservoir to the colder reservoir, and the flux magnitude uses their temperature difference.

Step 1: Identify the temperature drop and slab thickness The reservoir temperatures are $T_h = 10^3 = 1000\,\mathrm{K}$ and $T_c = 10^2 = 100\,\mathrm{K}$. Thus the driving temperature difference is

$$\Delta T = T_h - T_c = 1000 - 100 = 900\,\mathrm{K}.$$

The thickness along the heat-flow direction is $L = 1\,\mathrm{m}$. Use the supplied conductivity $k = 0.1\,\mathrm{W\,m^{-1}K^{-1}}$; a tabulated value for copper must not replace the value specified in this question.

Step 2: Calculate the steady energy flux Divide Fourier's heat-current equation by the slab area, so that its unknown area cancels:

$$j = \frac{k\Delta T}{L} = \frac{0.1 \times 900}{1} = 90\,\mathrm{W\,m^{-2}}.$$

The dimensional check is $(\mathrm{W\,m^{-1}K^{-1}})(\mathrm{K})/(\mathrm{m}) = \mathrm{W\,m^{-2}}$. This is the amount crossing each square metre per second.

Step 3: Compare all the numerical alternatives The calculated flux is 90, rather than 200, 65 or $120\,\mathrm{W\,m^{-2}}$. Those three numbers fail direct substitution into $jL = k\Delta T$. A negative sign would only indicate direction relative to a chosen coordinate; the requested flux magnitude is positive.

Step 4: Conclusion The energy flux is $\boxed{90\,\mathrm{W\,m^{-2}}}$, directed from the $1000\,\mathrm{K}$ source towards the $100\,\mathrm{K}$ reservoir. Hence, the correct option is (D).

Quick Tip to Solve: For a slab-flux question, use $k\Delta T/L$ directly: area cancels, and the temperature difference must include both reservoir temperatures.

51. The density of a material in SI units is 128 kg m^{-3} . In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is - [4 marks]

(A) 40

(B) 640

(C) 16

(D) 410

Correct Answer: (A) 40

Solution:

Concept:

- Density is mass per unit volume: $\rho = M/V$. Its dimensions are $[ML^{-3}]$, so both the new mass unit and the cube of the new length unit matter.
- A physical density remains unchanged when its numerical value changes with the unit system. If one new density unit is u_ρ in SI units, the new numerical value is $\rho_{\mathrm{SI}}/u_\rho$.
- The new volume unit is the cube of the new length unit, not merely its numerical length. Convert centimetres to metres and grams to kilograms before combining them.

Step 1: Express each new fundamental unit in SI units Let the new mass unit be M_0 and the new length unit be L_0. The definitions give

$$M_0 = 50\,\text{g} = 0.050\,\text{kg}, \qquad L_0 = 25\,\text{cm} = 0.25\,\text{m}.$$

One new volume unit is therefore

$$L_0^3 = (0.25)^3\,\text{m}^3 = 0.015625\,\text{m}^3.$$

Cubing the length is essential because density refers to a three-dimensional volume.

Step 2: Find the size of the new density unit The density unit built from these fundamental units is

$$u_\rho = \frac{M_0}{L_0^3} = \frac{0.050}{0.015625}\,\text{kg m}^{-3} = 3.2\,\text{kg m}^{-3}.$$

Thus each unit of numerical density in the new system represents $3.2\,\text{kg m}^{-3}$ in SI units.

Step 3: Convert the stated density and verify the option The given density is $128\,\text{kg m}^{-3}$, so its numerical value in the new units is

$$n = \frac{128}{3.2} = 40.$$

As a reverse check, $40 \times 0.050/(0.25)^3 = 128$. The alternatives 640, 16 and 410 do not reconstruct the given SI density, so none of them can be the required numerical value.

Step 4: Conclusion The density has numerical value $\boxed{40}$ in the specified new unit system. Hence, the correct option is (A).

Quick Tip to Solve: Build one new density unit as mass unit divided by length unit cubed; divide the SI density by that unit size.

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52. A magnet of total magnetic moment $10^{-2}\,\hat{i}$ A-m^2 is placed in a time varying magnetic field, B$\hat{i}$ (cos ωt) where B = 1 Tesla and ω = 0.125 rad/s. The work done for reversing the direction of the magnetic moment at t = 1 second, is - [4 marks]

(A) 0.014 J

(B) 0.028 J

(C) 0.01 J

(D) 0.007 J

Correct Answer: None of the printed options equals $2(10^{-2})(1)\cos(0.125) = 0.01984\,\text{J}$.

Solution:

Concept:

- A magnetic dipole of moment $\boldsymbol{m}$ in an external field has potential energy $U = -\boldsymbol{m}\cdot\boldsymbol{B} = -mB\cos\theta$, where θ is their relative angle.
- The external work needed to reverse a dipole quasistatically at a specified instant equals its

potential-energy increase from parallel to antiparallel: $W = 2mB(t)$.

- The printed field is $\boldsymbol{B}(t) = B_0\cos(\omega t)\,\hat{\boldsymbol{i}}$. The argument of the cosine is in radians because ω is given in radians per second.

Step 1: Evaluate the field at the specified instant The printed quantities are $B_0 = 1\,\mathrm{T}$, $\omega = 0.125\,\mathrm{rad\,s^{-1}}$, $m = 10^{-2}\,\mathrm{A\,m^2}$ and $t = 1\,\mathrm{s}$. Consequently,

$$B(1) = 1\cos(0.125 \times 1)\,\mathrm{T} = 0.9921977\,\mathrm{T}.$$

This is positive, so the initially stated moment along $+\hat{\boldsymbol{i}}$ is parallel to the instantaneous field.

Step 2: Calculate the energy change on reversal Before reversal $\theta_i = 0$ and afterwards $\theta_f = \pi$. At the specified instant,

$$U_i = -mB(1), \qquad U_f = +mB(1).$$

Hence the required work is

$$W = U_f - U_i = 2(10^{-2})(1)\cos(0.125) = 0.01984395\,\mathrm{J}.$$

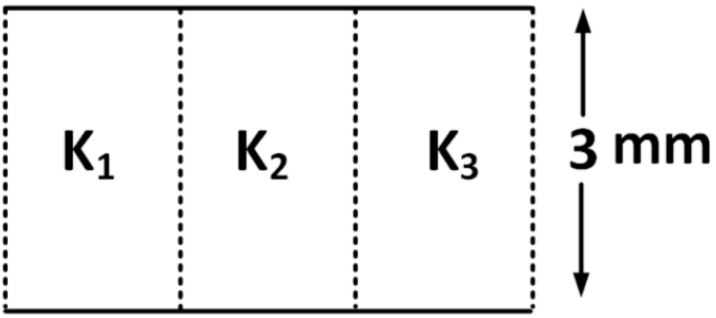
The factor of two is essential: reversal changes energy from a negative value to the corresponding positive value, rather than merely raising it to zero.

Step 3: Test the printed options against the calculation The printed options are 0.014, 0.028, 0.01 and $0.007\,\mathrm{J}$; none equals approximately $0.01984\,\mathrm{J}$. In particular, $0.014\,\mathrm{J}$ would be close to $2mB_0\cos(\pi/4)$, but $\pi/4$ is not the printed angular phase. Substituting that phase would change the question. The other alternatives likewise fail the stated energy difference, so no unique listed option can be justified.

Step 4: Conclusion For the printed $\omega = 0.125\,\mathrm{rad\,s^{-1}}$, the reversal work is $\boxed{0.01984\,\mathrm{J}}$, and none of the listed options is correct. The answer key selects (A), but the explicit calculation $2mB_0\cos(\omega t) = 0.02\cos(0.125)$ proves that (A) does not follow from the printed data.

Quick Tip to Solve: For reversal, first evaluate the field at the stated time and then use $\Delta U = 2mB(t)$; keep a radian phase in radian mode.

53. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$. The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be - [4 marks]



(A) 12

(B) 36

(C) 14

(D) 4

Correct Answer: (A) 12

Solution:

Concept:

- A parallel-plate capacitor completely filled with dielectric constant K has capacitance $C = \varepsilon_0 K A/d$, where A is plate area and d is separation.
- Dielectric regions placed side by side between the same two plates share the same potential difference. Their individual capacitances therefore add in parallel.
- The diagram shows three equal-area regions, each spanning the full $3\,\mathrm{mm}$ plate separation. This geometry determines the electrical combination; three layers stacked along the field would instead be in series.

Step 1: Read the arrangement from the figure The plate area is $A = 6\,\mathrm{cm}^2$. Each dielectric occupies one third of that area, $A_i = A/3 = 2\,\mathrm{cm}^2$, and extends from the upper plate to the lower plate. The three capacitances are therefore

$$C_1 = \frac{\varepsilon_0 K_1 A}{3d}, \quad C_2 = \frac{\varepsilon_0 K_2 A}{3d}, \quad C_3 = \frac{\varepsilon_0 K_3 A}{3d}.$$

All three experience the same plate-to-plate voltage.

Step 2: Add the parallel contributions For the side-by-side arrangement, total charge is the sum of charges in the three regions. Dividing by their common voltage gives

$$C = C_1 + C_2 + C_3 = \frac{\varepsilon_0 A}{d}\,\frac{K_1 + K_2 + K_3}{3}.$$

A single dielectric filling the whole capacitor has $C_{\mathrm{eq}} = \varepsilon_0 K_{\mathrm{eq}} A/d$. Equating these expressions cancels both geometry factors.

Step 3: Evaluate the effective dielectric constant Using $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$,

$$K_{\text{eq}} = \frac{10 + 12 + 14}{3} = 12.$$

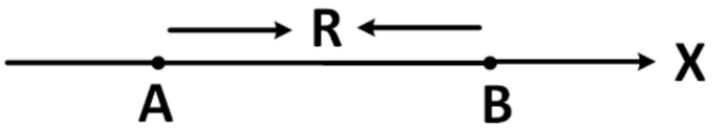
The value 36 is only the sum before the equal-area factor is included. The value 14 is the largest constituent constant, not their equal-area average; 4 is below every constituent constant and cannot describe this parallel combination.

Step 4: Conclusion The required material has dielectric constant $\boxed{12}$. Hence, the correct option is (A).

Quick Tip to Solve: Inspect which way the dielectrics are arranged: side-by-side equal areas give the arithmetic mean, while layers along the field give a harmonic mean.

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54. Two electric dipoles, A, B with respective dipole moments $\vec{d}_A = -4qai$ and $\vec{d}_B = -2qai$ are placed on the x-axis with a separation R, as shown in the figure. The distance from A at which both of them produce the same potential is - [4 marks]



(A) $\frac{\sqrt{2}R}{\sqrt{2}+1}$

(B) $\frac{R}{\sqrt{2}+1}$

(C) $\frac{\sqrt{2}R}{\sqrt{2}-1}$

(D) $\frac{R}{\sqrt{2}-1}$

Correct Answer: (C) $\frac{\sqrt{2}R}{\sqrt{2}-1}$

Solution:

Concept:

- The dipole approximation gives $V = k\boldsymbol{p} \cdot \hat{\boldsymbol{r}}/r^2$, with $k = 1/(4\pi\varepsilon_0)$. Potential has a sign determined by the observation direction relative to the dipole moment.
- Both dipole moments point along the negative x direction. At a point to the right of both dipoles their potentials are both negative; between the dipoles their potentials have opposite signs.
- The magnitude of A's moment is twice B's. Equal axial potential magnitudes therefore require the observation point to be $\sqrt{2}$ times farther from A than from B.

Step 1: Choose coordinates and identify the possible region Place A at $x = 0$ and B at $x = R$. For $0 < x < R$, A produces negative potential and B produces positive potential, so they cannot be equal. For $x < 0$, both potentials are positive, but A is both stronger and closer to the point than B; A's potential is therefore always greater. The finite axial equal-potential point must lie to the right of B, where $x > R$.

Step 2: Equate the signed axial potentials In this region the distances from A and B are respectively x and $x - R$. Their potentials are

$$V_A = -\frac{4kqa}{x^2}, \qquad V_B = -\frac{2kqa}{(x-R)^2}.$$

Setting $V_A = V_B$ and cancelling the common negative factor gives

$$\frac{4}{x^2} = \frac{2}{(x-R)^2}, \qquad x^2 = 2(x-R)^2.$$

Because both distances are positive, take $x = \sqrt{2}(x - R)$.

Step 3: Solve for the distance from A and reject other regions Rearranging the last relation,

$$(\sqrt{2}-1)x = \sqrt{2}R, \qquad x = \frac{\sqrt{2}R}{\sqrt{2}-1}.$$

This exceeds R, consistent with the chosen region. Options (A) and (B) lie between the dipoles where the potentials have opposite signs. Option (D) lies beyond B but fails the required ratio $x/(x - R) = \sqrt{2}$, whereas option (C) satisfies it.

Step 4: Conclusion The finite point on the x-axis is at distance $\boxed{\frac{\sqrt{2}R}{\sqrt{2}-1}}$ from A, beyond B. Hence, the correct option is (C).

Quick Tip to Solve: For dipole potentials, check their signs in each region before equating magnitudes; the square-root distance ratio alone can give an inadmissible root.

55. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is - (g = 10 ms^{-2}) [4 marks]

(A) 30 m

(B) 40 m

(C) 20 m

(D) 10 m

Correct Answer: (B) 40 m

Solution:

Concept:

- Before collision both bodies undergo vertical motion with acceleration $-g$. Their equal acceleration makes their relative acceleration zero, so their initial relative speed determines the meeting time.
- The bullet embeds in the wood, making the collision perfectly inelastic. Vertical momentum is conserved over the short impact, but kinetic energy is not.

- After collision the joined mass rises under gravity until its speed becomes zero. The additional rise is $v^2/(2g)$, measured from the collision point rather than from the building top.

Step 1: Find when and where the bodies meet Take upward as positive and ground level as $y = 0$. The bullet and wood positions at time t are

$$y_b = 100t - 5t^2, \qquad y_w = 100 - 5t^2.$$

Equating them cancels the gravity terms: $100t = 100$, giving $t = 1\,\text{s}$. The common collision height is

$$y_c = 100(1) - 5(1)^2 = 95\,\text{m}.$$

The wood has already fallen 5 m below the top when impact occurs.

Step 2: Conserve momentum at impact Immediately before impact, the bullet velocity is $v_b = 100 - 10(1) = 90\,\text{m s}^{-1}$, while the wood velocity is $v_w = -10\,\text{m s}^{-1}$. For combined velocity V,

$$(0.02)(90) + (0.03)(-10) = (0.02 + 0.03)V.$$

Thus $1.8 - 0.3 = 0.05V$, so $V = 30\,\text{m s}^{-1}$ upward. The negative wood momentum must be included; it opposes the bullet's momentum.

Step 3: Find the highest point relative to the building top The further rise after impact is

$$h = \frac{V^2}{2g} = \frac{30^2}{2(10)} = 45\,\text{m}.$$

Hence the maximum height above ground is $95 + 45 = 140\,\text{m}$, and the height above the 100 m building top is $140 - 100 = 40\,\text{m}$. Option 30 confuses the post-impact speed with a height, while 20 and 10 m fail the energy calculation for the combined mass.

Step 4: Conclusion The joined system reaches $\boxed{40\,\text{m}}$ above the top of the building. Hence, the correct option is (B).

Quick Tip to Solve: Keep the reference levels separate: find collision height, add the post-impact rise, then subtract the building height.

56. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio f_1/f_2 is - [4 marks]

(A) 19/18

(B) 20/19

(C) 21/20

(D) 18/17

Correct Answer: (A) 19/18

Solution:

Concept:

- For a source approaching a stationary observer through still air, the observed frequency is $f_{\text{obs}} = fv/(v - u_s)$, where f is emitted frequency, v is sound speed and u_s is source speed.
- Source motion compresses the wavelength ahead of it while sound speed in the medium remains v. A faster approaching source produces a greater observed frequency.
- The same whistle is used in both situations, so its emitted frequency and the common sound-speed numerator cancel when the two observed frequencies are divided.

Step 1: Write the frequency for each source speed For $v = 340\,\text{m}\,\text{s}^{-1}$ and the first train speed $u_1 = 34\,\text{m}\,\text{s}^{-1}$,

$$f_1 = f\frac{340}{340 - 34} = f\frac{340}{306}.$$

For the reduced train speed $u_2 = 17\,\text{m}\,\text{s}^{-1}$,

$$f_2 = f\frac{340}{340 - 17} = f\frac{340}{323}.$$

Both denominators are smaller than 340, as required for an approaching source.

Step 2: Take and simplify the frequency ratio Dividing the two expressions eliminates the unknown emitted frequency:

$$\frac{f_1}{f_2} = \frac{340/306}{340/323} = \frac{323}{306}.$$

Both numbers are divisible by 17, giving

$$\frac{f_1}{f_2} = \frac{19 \times 17}{18 \times 17} = \frac{19}{18}.$$

The ratio is dimensionless; no additional factor of train speed is introduced.

Step 3: Check the direction and compare every option The first source speed is larger, so $f_1 > f_2$; the computed ratio exceeds one, as expected. Its precise value is approximately 1.0556. The alternatives $20/19$, $21/20$ and $18/17$ have different values and do not equal the exact denominator ratio $(340 - 17)/(340 - 34)$. Applying the moving-observer formula would incorrectly put the speed correction in the numerator instead of accounting for source wavelength compression.

Step 4: Conclusion The required ratio is $\boxed{f_1/f_2 = 19/18}$. Hence, the correct option is (A).

Quick Tip to Solve: For two speeds of the same approaching source, write the ratio directly as $(v - u_2)/(v - u_1)$, retaining the source-speed correction in the denominator.

57. Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At t = 0 it was 1600 counts per second and t = 8 seconds it was 100 counts per second. The count rate observed, as counts per second, at t = 6 seconds is close to - [4 marks]

(A) 200

(B) 150

(C) 400

(D) 360

Correct Answer: (A) 200

Solution:

Concept:

- The activity and the observed count rate of a radioactive source are proportional to the number of undecayed nuclei when the counter geometry remains fixed. They obey the exponential decay law $R(t) = R_0 e^{-\lambda t}$.
- The half-life $T_{1/2}$ is the time in which the count rate falls to half its earlier value: $R(t) = R_0(1/2)^{t/T_{1/2}}$. The count rate is not a linearly decreasing function of time.

Step 1: Determine the number of half-lives in eight seconds The measured rates are $R_0 = 1600\,\mathrm{s}^{-1}$ and $R(8) = 100\,\mathrm{s}^{-1}$. Their ratio is

$$\frac{R(8)}{R_0} = \frac{100}{1600} = \frac{1}{16} = \left(\frac{1}{2}\right)^4.$$

Thus four half-lives have elapsed in eight seconds, so $T_{1/2} = 8/4 = 2\,\mathrm{s}$. Equivalently, $e^{-8\lambda} = 1/16$ gives $\lambda = (\ln 2)/2\,\mathrm{s}^{-1}$.

Step 2: Calculate the count rate after six seconds Six seconds corresponds to three half-lives. Therefore

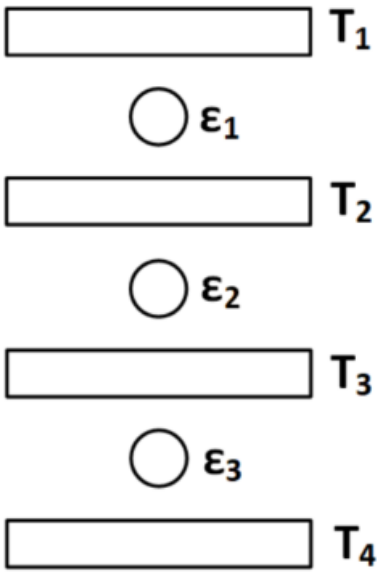
$$R(6) = 1600\left(\frac{1}{2}\right)^3 = \frac{1600}{8} = 200\,\mathrm{s}^{-1}.$$

The sequence at two-second intervals is $1600, 800, 400, 200, 100$, which also checks the supplied endpoint. Option (C), 400, belongs to four seconds rather than six. The values 150 and 360 do not follow the measured exponential decay constant, so neither (B) nor (D) is consistent with both observations.

Step 3: Conclusion The count rate at $t = 6\,\mathrm{s}$ is 200 counts per second. Hence, the correct option is (A).

Quick Tip to Solve: Express the ratio of two measured count rates as a power of two before using logarithms; here a factor of 16 immediately reveals four half-lives.

58. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if - [4 marks]



(A) $T_2 = (T_1^3T_4)^{1/4}$; $T_3 = (T_1T_4^3)^{1/4}$

(B) $T_2 = (T_1T_4)^{1/2}$; $T_3 = (T_1^2T_4)^{1/3}$

(C) $T_2 = (T_1T_4^2)^{1/3}$; $T_3 = (T_1^2T_4)^{1/3}$

(D) $T_2 = (T_1^2T_4)^{1/3}$; $T_3 = (T_1T_4^2)^{1/3}$

Correct Answer: (D) $T_2 = (T_1^2T_4)^{1/3}$; $T_3 = (T_1T_4^2)^{1/3}$

Solution:

Concept:

- A reversible Carnot engine operating between absolute temperatures T_h and T_c has efficiency $\eta = 1 - T_c/T_h$. Temperatures in this formula must be measured in kelvin.
- Equal efficiencies for engines connected through intermediate reservoirs require equal ratios of successive temperatures, not equal temperature differences. The four reservoir temperatures therefore form a geometric progression.

Step 1: Equate the efficiencies of the three engines The first engine operates from T_1 to T_2, the second from T_2 to T_3, and the third from T_3 to T_4. Hence

$$\eta_1 = 1 - \frac{T_2}{T_1}, \quad \eta_2 = 1 - \frac{T_3}{T_2}, \quad \eta_3 = 1 - \frac{T_4}{T_3}.$$

Writing their common cold-to-hot ratio as r, equality gives

$$\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = r, \qquad 0 < r < 1.$$

Multiplication cancels the intermediate temperatures and yields $r^3 = T_4/T_1$.

Step 2: Find the intermediate reservoir temperatures Since $r = (T_4/T_1)^{1/3}$,

$$T_2 = T_1 r = (T_1^2 T_4)^{1/3}, \qquad T_3 = T_1 r^2 = (T_1 T_4^2)^{1/3}.$$

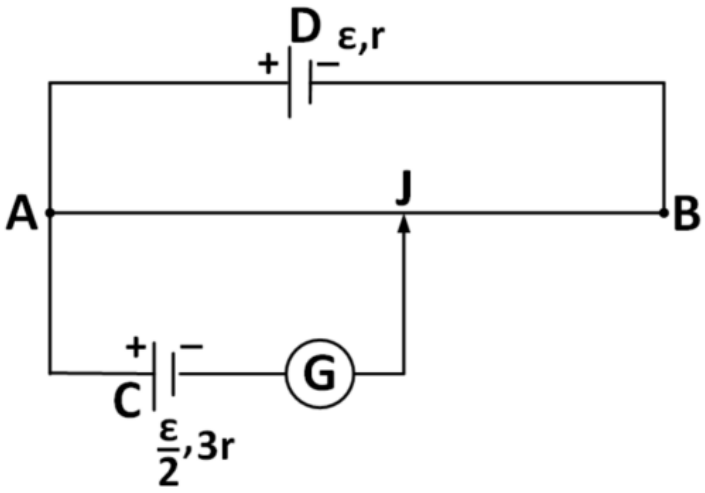
These expressions give the stated order $T_1 > T_2 > T_3 > T_4$. Option (A) uses quarter powers and leaves the middle engine with a different ratio. Option (B) puts the first intermediate reservoir at a geometric mean inconsistent with three equal stages. Option (C) interchanges the correct intermediate temperatures and violates $T_2 > T_3$. Only (D) gives the same ratio at all three stages.

Step 3: Conclusion The required temperatures are $T_2 = (T_1^2 T_4)^{1/3}$ and $T_3 = (T_1 T_4^2)^{1/3}$. Hence, the correct option is (D).

Quick Tip to Solve: For equal-efficiency Carnot engines in series, write a geometric progression of absolute temperatures; the number of engines sets the root in the common ratio.

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59. A potentiometer wire AB having length L and resistance 12 r is joined to a cell D of emf ε and internal resistance r. A cell C having emf $\varepsilon/2$ and internal resistance 3r is connected. The length AJ at which the galvanometer as shown in figure shows no deflection is – [4 marks]



(A) $\frac{11}{12}L$
(B) $\frac{13}{24}L$
(C) $\frac{5}{12}L$
(D) $\frac{11}{24}L$

Correct Answer: (B) $\frac{13}{24}L$

Solution:

Concept:

- A uniform potentiometer wire has a constant potential gradient $k = V_{AB}/L$, where V_{AB} is the terminal potential difference across its full length.
- At the null point no current flows through the galvanometer or test cell. Thus the test cell balances

its emf, and its internal resistance produces no voltage drop; the driver cell still carries the main-circuit current.

Step 1: Calculate the potential gradient along the wire The driver cell D supplies the wire resistance $12r$ in series with its own internal resistance r. Thus

$$I = \frac{\varepsilon}{12r + r} = \frac{\varepsilon}{13r}.$$

Only the potential difference across the wire contributes to its gradient:

$$V_{AB} = I(12r) = \frac{12\varepsilon}{13}, \qquad k = \frac{12\varepsilon}{13L}.$$

Using ε/L would incorrectly neglect the driver cell's internal voltage drop.

Step 2: Apply the null condition for cell C Let $AJ = \ell$. At zero galvanometer deflection the current through cell C is zero, so there is no drop across its internal resistance $3r$. The balance equation is therefore

$$k\ell = \frac{\varepsilon}{2}.$$

Substituting the gradient gives

$$\ell = \frac{\varepsilon}{2}\frac{13L}{12\varepsilon} = \frac{13L}{24}.$$

This lies inside the wire, as a balance requires. For the other proposed lengths, the wire drops would be $11\varepsilon/13$, $5\varepsilon/13$, and $11\varepsilon/26$ for (A), (C), and (D), respectively. None equals $\varepsilon/2$, so they fail the null condition.

Step 3: Conclusion The balance point is $AJ = 13L/24$. Hence, the correct option is (B).

Quick Tip to Solve: Include the driver cell's internal resistance when finding the wire gradient, but omit the test cell's internal resistance at a true potentiometer null.

60. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower is LOS (Line of Sight) mode ? (Given : radius of earth = 6.4 $\times$ 10^6 m). [4 marks]

(A) 40 km

(B) 65 km

(C) 48 km

(D) 80 km

Correct Answer: (B) 65 km

Solution:

Concept:

- The limiting line of sight from an antenna is tangent to Earth. For antenna height $h \ll R$, its distance to the horizon is $d \simeq \sqrt{2Rh}$, where R is Earth's radius.
- Both antennas extend the visible separation. The maximum transmitter-to-receiver distance is the

sum of their individual horizon distances: $d_{\max} = \sqrt{2Rh_t} + \sqrt{2Rh_r}$.

Step 1: Compute the horizon distances of the two antennas The right triangle formed by Earth's centre, a tangent point and an antenna gives $d^2 = (R+h)^2 - R^2 = 2Rh + h^2$. With heights only tens or hundreds of metres, the h^2 correction is negligible compared with $2Rh$. For the tower,

$$d_t = \sqrt{2(6.4 \times 10^6)(140)} = \sqrt{1.792 \times 10^9}\,\text{m} \simeq 42.33\,\text{km}.$$

For the receiving antenna,

$$d_r = \sqrt{2(6.4 \times 10^6)(40)} = \sqrt{5.12 \times 10^8}\,\text{m} \simeq 22.63\,\text{km}.$$

Step 2: Add the two visible ranges and compare the options At the greatest separation the radio path is tangent to Earth between the antennas, and the two horizon distances add:

$$d_{\max} = 42.33 + 22.63 = 64.96\,\text{km} \simeq 65\,\text{km}.$$

A value near 40 km includes roughly the tower horizon alone and misses the receiver's contribution. A value of 48 km is too small for the sum, while 80 km is larger than the geometric line-of-sight limit. There is no need to introduce ionospheric reflection because the question specifically asks for LOS propagation.

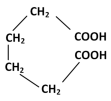
Step 3: Conclusion The maximum line-of-sight broadcast distance is approximately $65\,\text{km}$. Hence, the correct option is (B).

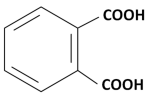
Quick Tip to Solve: Add the two square-root horizon distances separately; putting the sum of the heights under a single square root does not give the LOS separation.

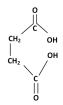
Chemistry (120 marks)

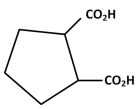
61. Which dicarboxylic acid in presence of a dehydrating agent is least reactive to give an anhydride?

[4 marks]

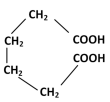
(A) Adipic acid: $HOOC-(CH_2)_4-COOH$ 

(B) Phthalic acid (benzene-1,2-dicarboxylic acid) 

(C) Succinic acid: $HOOC-CH_2-CH_2-COOH$ 

(D) Cyclopentane-1,2-dicarboxylic acid 

Correct Answer: (A)

Adipic acid: $HOOC-(CH_2)_4-COOH$ 

Solution:

Concept:

- Dehydration of a dicarboxylic acid removes water between its two carboxyl groups and can produce a cyclic acid anhydride by intramolecular acyl substitution.
- Five- and six-membered anhydride rings form most readily because the reacting groups can approach with favourable geometry. A seven-membered ring is less favourable owing to conformational and ring-closure constraints.

Step 1: Identify the ring size obtained from each acid Option (A) is adipic acid, $HOOC-(CH_2)_4-COOH$. Its chain has four methylene carbons between the two carboxyl carbons. Closing it into an anhydride includes these four atoms, both carbonyl carbons and the bridging oxygen: a seven-membered ring. Option (B) is phthalic acid, with adjacent carboxyl groups on a benzene ring; their proximity permits a fused five-membered anhydride ring. Option (C) is succinic acid, $HOOC-CH_2-CH_2-COOH$, whose dehydration also closes a five-membered ring.

Step 2: Compare the ease of intramolecular dehydration In option (D), the two carboxyl groups occupy neighbouring carbons of a cyclopentane skeleton. The anhydride closure uses those two neighbouring skeleton carbons, two carbonyl carbons and one oxygen, again making a five-membered anhydride ring. Thus (B), (C) and (D) have the favourable local five-membered closure arrangement. Adipic acid in (A) instead requires a less favourable seven-membered closure, so it is the least reactive of the displayed acids for this purpose. The comparison concerns ease of cyclic anhydride formation, not simply the number of carboxyl groups, which is two in every option.

Step 3: Conclusion Adipic acid, shown in option (A), is least reactive towards cyclic anhydride formation because its intramolecular dehydration requires a seven-membered ring. Hence, the correct option is (A).

Quick Tip to Solve: Count the atoms of the proposed anhydride ring, including both carbonyl carbons and the bridging oxygen; do not count only the carbons between the acid groups.

62. Liquids A and B form an ideal solution in the entire composition range. At 350 K, the vaapor pressures of pure A and pure B are 7×10^3 Pa and 12×10^3 Pa, respectively . The composition of the vapor in equilibriumwith a solution containing 40 mole percent of A at this temperature is : [4 marks]

(A) $x_A = 0.76$; $x_B = 0.24$

(B) $x_A = 0.28$; $x_B = 0.72$

(C) $x_A = 0.4$; $x_B = 0.6$

(D) $x_A = 0.37$; $x_B = 0.63$

Correct Answer: (B) $x_A = 0.28$; $x_B = 0.72$

Solution:

Concept:

- For an ideal liquid solution, Raoult's law gives the partial vapour pressures $p_A = x_A p_A^0$ and $p_B = x_B p_B^0$, where x_A, x_B denote liquid-phase mole fractions.

- Dalton's law gives vapour-phase mole fractions $y_A = p_A/(p_A+p_B)$ and $y_B = p_B/(p_A+p_B)$. Vapour composition generally differs from liquid composition because the pure-liquid vapour pressures differ.

Step 1: Find the partial and total vapour pressures The liquid contains 40 mole percent A, so $x_A = 0.40$ and $x_B = 0.60$. At the specified temperature,

$$p_A = 0.40(7 \times 10^3) = 2.8 \times 10^3 \text{ Pa},$$

$$p_B = 0.60(12 \times 10^3) = 7.2 \times 10^3 \text{ Pa}.$$

The total pressure is

$$P = p_A + p_B = 10.0 \times 10^3 \text{ Pa}.$$

Both the mole fraction and the pure-component vapour pressure are needed; the pure-component pressure ratio alone does not determine this mixture's vapour composition.

Step 2: Convert partial pressures to vapour mole fractions Using Dalton's law,

$$y_A = \frac{2.8 \times 10^3}{10.0 \times 10^3} = 0.28, \qquad y_B = \frac{7.2 \times 10^3}{10.0 \times 10^3} = 0.72.$$

Their sum is one. B has the greater pure vapour pressure and is enriched from 0.60 in the liquid to 0.72 in the vapour, a useful physical check. The symbols x_A, x_B in the printed options label the requested vapour composition. Option (C) merely repeats the liquid fractions. Option (A) predicts enrichment of the less volatile A, and option (D) does not equal the calculated partial-pressure fractions.

Step 3: Conclusion The vapour contains 0.28 mole fraction A and 0.72 mole fraction B, or 28 mole percent A and 72 mole percent B. Hence, the correct option is (B).

Quick Tip to Solve: Use Raoult's law first for partial pressures, then divide each by their sum; copying the liquid mole fractions misses the vapour enrichment of the more volatile liquid.

63. Wilkinson catalyst is : (Et = C_2H_5) [4 marks]

(A) $[(Ph_3P)_3\ RhCl]$

(B) $[(Et_3P)_3RhCl]$

(C) $[(Et_3P)_3IrCl]$

(D) $[(Ph_3P)_3IrCl]$

Correct Answer: (A) $[(Ph_3P)_3\ RhCl]$

Solution:

Concept:

- Wilkinson's catalyst is chloridotris(triphenylphosphine)rhodium(I), conventionally written $[RhCl(PPh_3)_3]$, where Ph denotes a phenyl group C_6H_5.
- Triphenylphosphine is a neutral phosphine ligand, while chloride has charge -1. The neutral complex therefore contains rhodium in oxidation state $+1$, a d^8 metal centre.

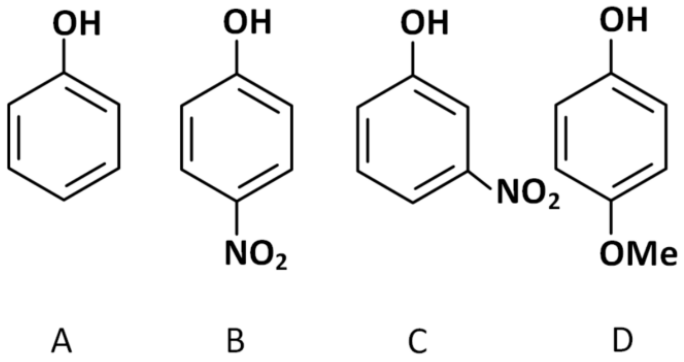
Step 1: Identify the required metal and phosphine ligand The characteristic components are one rhodium atom, one coordinated chloride and three triphenylphosphine ligands. Written with the ligands first, this is $[(Ph_3P)_3RhCl]$, the formula in option (A). The notation Ph_3P denotes the same triphenylphosphine ligand as PPh_3; rearranging the formula does not change the compound. The catalyst is commonly used for homogeneous hydrogenation of carbon-carbon multiple bonds, so both its ligand identity and its metal are significant.

Step 2: Check the alternative formulae In (B), the rhodium and chloride are retained, but triphenylphosphine has been replaced by triethylphosphine PEt_3, where Et is C_2H_5. This is a different complex. Option (C) changes both the metal to iridium and the ligand to triethylphosphine. Option (D) has triphenylphosphine but still contains iridium rather than rhodium. Neither is Wilkinson's named catalyst. As a consistency check for (A), three neutral phosphines and one chloride require $Rh(I)$; the d^8 Rh(I) centre forms the familiar approximately square-planar complex. A similar-looking formula with a different metal is not sufficient.

Step 3: Conclusion Wilkinson's catalyst is $[(Ph_3P)_3RhCl]$, chloridotris(triphenylphosphine)rhodium(I). Hence, the correct option is (A).

Quick Tip to Solve: Remember the combination Rh, chloride and three triphenylphosphines; replacing Ph by Et or Rh by Ir gives a different complex.

64. The increasing order of the pKa values of the following compounds is : [4 marks]



(A) B < C < D < A

(B) B < C < A < D

(C) D < A < C < B

(D) C < B < A < D

Correct Answer: (B) B < C < A < D

Solution:

Concept:

- Acid strength increases when the conjugate phenoxide ion is stabilised. Since $pK_a = -\log_{10} K_a$, a stronger acid has a smaller pK_a.
- An electron-withdrawing group stabilises the negative charge of phenoxide; a resonance electron-

donating group tends to destabilise it. Position controls whether resonance operates.

Step 1: Identify the four phenols A is phenol, B is para-nitrophenol, C is meta-nitrophenol and D is para-methoxyphenol. All four lose the proton of the hydroxyl group to give a phenoxide ion. Thus the comparison concerns substituent effects on the same oxygen-bearing aromatic conjugate base, rather than breaking a carbon–hydrogen bond.

Step 2: Compare the nitro-substituted compounds The nitro group withdraws electrons by induction. At the para position it also withdraws by resonance, allowing the negative charge of the phenoxide ion to be delocalised towards the nitro group. B therefore has the most stabilised conjugate base and the lowest $\mathrm{p}K_a$. In C the nitro group is meta to hydroxyl; its stabilising inductive effect remains, but direct resonance withdrawal from the phenoxide charge is absent. Consequently $\mathrm{p}K_a(B) < \mathrm{p}K_a(C) < \mathrm{p}K_a(A)$.

Step 3: Place the methoxy derivative and eliminate alternatives At the para position the methoxy group donates electron density through resonance. Its net effect here makes loss of the phenolic proton less favourable than in phenol, so D has the largest $\mathrm{p}K_a$. Option A incorrectly puts D before A. Option C reverses the broad electron-withdrawing versus electron-donating trend. Option D incorrectly makes meta-nitrophenol more acidic than para-nitrophenol.

Step 4: Conclusion The increasing order is $B < C < A < D$. Hence, the correct option is (B).

Quick Tip to Solve: Rank conjugate-base stability first, then reverse that ranking for $\mathrm{p}K_a$; check whether the substituent can act by resonance at its position.

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65. A mixture of 100 m mol of $Ca(OH)_2$ and 2 g of sodium sulphate was dissolved in water and the volume was made up to 100 mL. The mass of calcium sulphate formed and the concentration of OH^- in resulting solution, respectively, are : (Molar mass of Ca $(OH)_2$, Na_2SO_4 and $CaSO_4$ are 74, 143 and 136 g mol^{-1} , respectively; K_{sp} of $Ca(OH)_2$ is 5.5 $\times$ 10^{-6}) [4 marks]

(A) 13.6g, 0.28 mol L^{-1}

(B) 13.6g, 0.14 mol L^{-1}

(C) 1.9g, 0.28 mol L^{-1}

(D) 1.9g, 0.14 mol L^{-1}

Correct Answer: (C) 1.9g, 0.28 mol L^{-1}

Solution:

Concept:

- The precipitation reaction is $\mathrm{Ca(OH)_2 + Na_2SO_4 \longrightarrow CaSO_4(s) + 2\,NaOH}$. One mole of sulphate precipitates one mole of calcium.
- Excess solid calcium hydroxide can supply calcium as precipitation removes it. At equilibrium, $K_{sp} = [\mathrm{Ca^{2+}}][\mathrm{OH^-}]^2$, with concentrations in $\mathrm{mol\,L^{-1}}$.

Step 1: Find the limiting amount of sulphate Use the molar mass stated in the question, $143\,\mathrm{g\,mol^{-1}}$,

rather than replacing it with a tabulated value. The sulphate amount is

$$n(\mathrm{Na_2SO_4}) = \frac{2}{143} = 0.013986\,\mathrm{mol}.$$

There are $0.100\,\mathrm{mol}$ of calcium hydroxide, far more than this amount. Sulphate therefore limits the precipitate; it is incorrect to precipitate the whole $0.100\,\mathrm{mol}$ of calcium.

Step 2: Calculate the calcium sulphate mass To the precision of the options, the sulphate is removed as calcium sulphate. Hence

$$m(\mathrm{CaSO_4}) = \frac{2}{143} \times 136 = 1.902\,\mathrm{g} \simeq 1.9\,\mathrm{g}.$$

The alternatives containing $13.6\,\mathrm{g}$ would require $0.100\,\mathrm{mol}$ of sulphate, which the mixture does not contain.

Step 3: Find hydroxide and check saturation Each mole of precipitated calcium leaves two moles of hydroxide in the solution. In $0.100\,\mathrm{L}$,

$$[\mathrm{OH^-}] \simeq \frac{2(2/143)}{0.100} = 0.27972 \simeq 0.28\,\mathrm{mol\,L^{-1}}.$$

The given solubility product gives residual dissolved calcium of approximately

$$[\mathrm{Ca^{2+}}] \simeq \frac{5.5 \times 10^{-6}}{(0.28)^2} = 7.0 \times 10^{-5}\,\mathrm{mol\,L^{-1}}.$$

Its additional hydroxide contribution is only about $1.4 \times 10^{-4}\,\mathrm{mol\,L^{-1}}$, negligible at the precision requested. The value 0.14 misses the two hydroxide ions supplied per calcium hydroxide unit.

Step 4: Conclusion The mass formed is approximately $1.9\,\mathrm{g}$, and $[\mathrm{OH^-}] = 0.28\,\mathrm{mol\,L^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use sulphate to count precipitated calcium, then double its mole amount for hydroxide; the excess sparingly soluble hydroxide is not all dissolved.

66. The type of hybridisation and number of lone pair (s) of electrons of Xe in $XeOF_4$, respectively, are: [4 marks]

(A) sp^3d and 2

(B) sp^3d^2 and 2

(C) sp^3d and 1

(D) sp^3d^2 and 1

Correct Answer: (D) sp^3d^2 and 1

Solution:

Concept:

- In VSEPR counting, each bonded atom gives one electron domain at the central atom; a multiple bond still counts as one domain.
- Six electron domains correspond to octahedral electron-pair arrangement and the conventional sp^3d^2 hybridisation description. A lone pair is a nonbonding pair on the central atom.

Step 1: Count the valence electrons and bonds Xenon belongs to group 18 and has eight valence electrons. In $XeOF_4$, four fluorine atoms form four single bonds to xenon and oxygen forms a double bond. The molecule is neutral. The usual Lewis structure has an $Xe{=}O$ bond, four $Xe{-}F$ bonds and one lone pair on xenon; terminal atoms complete their octets.

Step 2: Verify the lone pair by electron accounting The total valence-electron count is

$$8 + 6 + 4(7) = 42.$$

Four single bonds and one double bond use 12 electrons. The four fluorine atoms need 24 nonbonding electrons, and oxygen needs four more. Thus

$$42 - 12 - 24 - 4 = 2$$

electrons remain on xenon: exactly one lone pair. A proposed second xenon lone pair would not satisfy this electron count.

Step 3: Count domains and match the option Xenon is bonded to five atoms, so there are five bonding domains, including the double bond to oxygen as only one domain. Adding its one lone pair gives steric number $5 + 1 = 6$. The electron-pair arrangement is octahedral, while the molecular shape is square pyramidal. The conventional hybridisation is therefore sp^3d^2, not sp^3d. Options A and C use only five domains; option B supplies an extra lone pair that the Lewis count does not allow.

Step 4: Conclusion Xenon has sp^3d^2 hybridisation and one lone pair in $XeOF_4$. Hence, the correct option is (D).

Quick Tip to Solve: For oxyhalides, count the double bond as one VSEPR domain but use both shared pairs when doing the total electron bookkeeping.

67. Consider the following reduction processes :

$Zn^{2+} + 2e^- \rightarrow Zn(s)$; $E^o = -0.76$ V

$Ca^{2+} + 2e^- \rightarrow Ca(s)$; $E^o = -2.87$ V

$Mg^{2+} + 2e^- \rightarrow Mg(s)$; $E^o = -2.36$ V

$Ni^2 + 2e^- \rightarrow Ni(s)$; $E^o = -0.25$

The reducing power of the metals increases in the order : [4 marks]

(A) Ca < Mg < Zn < Ni

(B) Ni < Zn < Mg < Ca

(C) Zn < Mg < Ni < Ca

(D) Ca < Zn < Mg < Ni

Correct Answer: (B) Ni < Zn < Mg < Ca

Solution:

Concept:

- A reducing metal donates electrons and itself undergoes oxidation: $\mathrm{M(s)} \longrightarrow \mathrm{M^{2+}} + 2\,\mathrm{e^-}$.
- The listed values are reduction potentials. Reversing a half-reaction reverses the sign, so $E^\circ_{\text{ox}} = -E^\circ_{\text{red}}$; a more negative reduction potential indicates a stronger reducing metal.

Step 1: Identify the direction relevant to a reducing metal The printed half-reactions describe metal ions gaining electrons to become metals. Reducing power, however, asks how readily the metal gives electrons away. Thus we must compare the reverse direction. A metal whose ion is difficult to reduce is correspondingly more ready to oxidise, and so is the stronger reducing agent.

Step 2: Reverse the signs and compare the values The oxidation potentials corresponding to the stated reductions are

$$E^\circ_{\text{ox}}(\mathrm{Ni}) = +0.25\,\mathrm{V},$$
$$E^\circ_{\text{ox}}(\mathrm{Zn}) = +0.76\,\mathrm{V},$$
$$E^\circ_{\text{ox}}(\mathrm{Mg}) = +2.36\,\mathrm{V},$$
$$E^\circ_{\text{ox}}(\mathrm{Ca}) = +2.87\,\mathrm{V}.$$

Their increasing order is nickel, zinc, magnesium, calcium. For example, calcium can reduce nickel ions because the corresponding cell potential is $-0.25 - (-2.87) = +2.62\,\mathrm{V}$, consistent with calcium being the stronger electron donor.

Step 3: Check all listed orders Option A is the reverse order and would describe increasing ease of reduction of the ions, not increasing reducing power of the metals. Option C incorrectly puts nickel above zinc and magnesium, despite its least negative reduction potential. Option D incorrectly makes calcium the weakest and nickel the strongest. Only option B keeps every neighbouring comparison consistent with the given potentials.

Step 4: Conclusion The reducing power increases as $\mathrm{Ni} < \mathrm{Zn} < \mathrm{Mg} < \mathrm{Ca}$. Hence, the correct option is (B).

Quick Tip to Solve: Underline whether the question asks about the metal or its ion: the metal is a stronger reducing agent when its listed reduction potential is more negative.

68. The electronegativity of aluminium is similar to : [4 marks]

(A) Beryllium

(B) Carbon

(C) Boron

(D) Lithium

Correct Answer: (A) Beryllium

Solution:

Concept:

- Electronegativity measures an atom's tendency to attract a shared electron pair in a chemical bond.
- Across a period electronegativity generally increases; down a group it generally decreases. These opposing changes can produce similar values for diagonally related elements.

Step 1: Locate aluminium and the candidate elements Aluminium is in period 3, group 13. Beryllium is in period 2, group 2, immediately above and to its left in the main-group diagonal comparison. Aluminium and beryllium form a familiar diagonal pair. Their comparable charge density and electronegativity help explain related chemical behaviour, including amphoteric oxides and appreciable covalent character of some compounds.

Step 2: Compare the electronegativity values On the commonly used Pauling scale, representative values are

$$\chi(\mathrm{Al}) \simeq 1.61, \qquad \chi(\mathrm{Be}) \simeq 1.57.$$

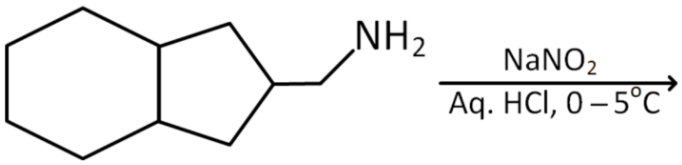
The difference is only about 0.04. By comparison, lithium is near 0.98, boron near 2.04, and carbon near 2.55. Thus beryllium is clearly closest. The exact last decimal may differ among tables; the close aluminium–beryllium similarity and separation from the other choices remain the same.

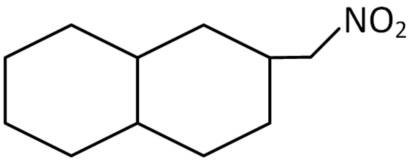
Step 3: Eliminate the tempting group-based choice Boron shares aluminium's group, but the smaller second-period boron atom attracts bonding electrons considerably more strongly. Carbon is still more electronegative, so option B is also unsuitable. Lithium is an electropositive alkali metal and lies well below aluminium in electronegativity. Therefore options B, C and D do not match the required similarity. The useful comparison is the diagonal one, rather than merely selecting the element from the same group.

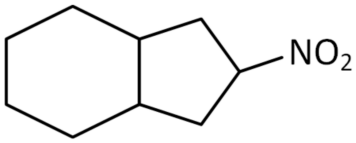
Step 4: Conclusion Aluminium has electronegativity similar to beryllium. Hence, the correct option is (A).

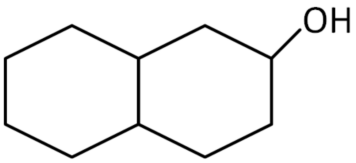
Quick Tip to Solve: Remember the diagonal pairs Li–Mg and Be–Al; the opposing period and group trends can make a diagonal neighbour closer than a group neighbour.

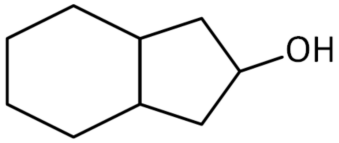
69. The major product formed in the reaction given below will be : [4 marks]



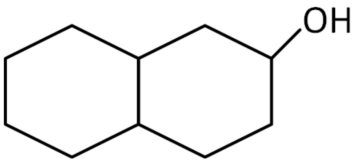
(A) 

(B) 

(C) 

(D) 

Correct Answer: (C)



Solution:

Concept:

- $NaNO_2$ with aqueous HCl generates nitrous acid, which converts a primary aliphatic amine into an unstable aliphatic diazonium intermediate.
- Loss of N_2 generates carbocation character. A neighbouring carbon–carbon bond can migrate to expand a five-membered ring, after which water captures the rearranged cation.

Step 1: Identify the group that reacts The side chain is an aminomethyl group attached to the five-membered ring of a fused six–five bicyclic system. The amine is primary and aliphatic, so it does not give a persistent aromatic diazonium salt. Nitrosation followed by diazonium formation makes the terminal nitrogen group an excellent leaving group. Nitrogen gas is released, creating electron deficiency at the carbon that originally carried NH_2.

Step 2: Follow the neighbouring bond migration The initially generated primary cation is next to a

ring carbon. Migration of an adjacent carbon–carbon bond of the five-membered ring towards this electron-deficient side-chain carbon incorporates that side-chain carbon into the ring. The migrating bond carries its electron pair, so the positive charge moves to the original ring attachment carbon. This is a ring expansion from five to six members, preserving the total number of carbon atoms. The fused six-membered ring on the other side remains intact.

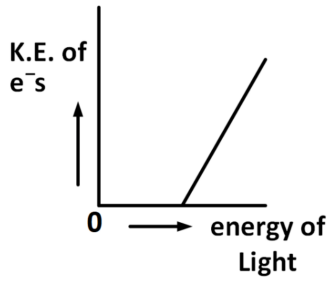
Step 3: Capture the cation and distinguish the products Water attacks the rearranged cation and deprotonation gives an alcohol. The product therefore has two fused six-membered rings and the hydroxyl group at the position shown in C. Options A and B are nitro compounds; nitrous-acid deamination is not replacement of the amine by NO_2. Option D is an alcohol with the original five-membered ring and fails to include the aminomethyl carbon in the expanded ring. It does not represent the favoured rearranged skeleton.

Step 4: Conclusion The major product is the ring-expanded fused six–six bicyclic alcohol shown in option C. Hence, the correct option is (C).

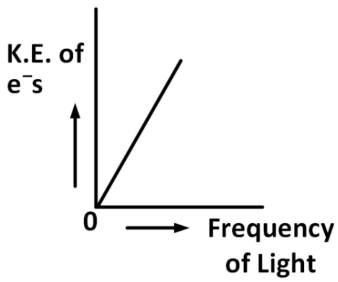
Quick Tip to Solve: For a primary aminomethyl group next to a small ring, follow loss of nitrogen by a neighbouring C–C bond shift; count carbons before choosing the alcohol.

70. Which of the graphs shown below does not represent the relationship between incident light and the electron ejected from metal surface ? [4 marks]

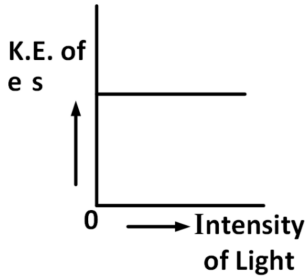
(A)



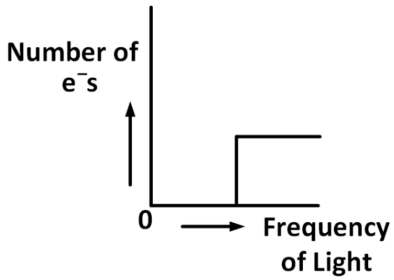
(B)



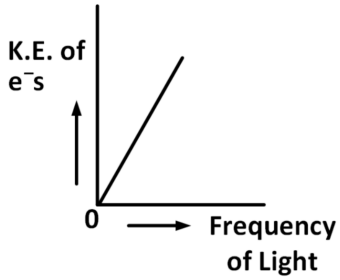
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- Einstein's equation is $K_{\max} = h\nu - \phi = h(\nu - \nu_0)$, where ϕ is the metal's positive work function and $\nu_0 = \phi/h$ is its threshold frequency.
- At fixed frequency above threshold, increasing intensity increases the rate of photon arrival and the number of emitted electrons, but not the maximum kinetic energy of an electron.

Step 1: Check the energy graph If the horizontal variable is photon energy $E = h\nu$, Einstein's equation becomes $K_{\max} = E - \phi$. Thus the graph is a straight line with a positive energy-axis intercept $E = \phi$. Below that threshold there is no photoemission. The rising line in A begins at a nonzero incident photon energy and is consistent with this relationship.

Step 2: Check the frequency intercept For kinetic energy against frequency, the straight line must start at the positive threshold frequency ν_0, not at the origin. Its slope is h; extending the line backwards gives vertical intercept $-\phi$. Graph B instead passes through the origin, implying $\phi = 0$ and emission at arbitrarily small positive frequency. That is not the relationship for a metal surface with a finite work function.

Step 3: Check the remaining two plots Graph C keeps electron kinetic energy constant as intensity changes at fixed frequency, which is correct: extra photons produce more electrons rather than more energetic individual electrons. Graph D represents the idealised threshold dependence of the electron count when photon flux is fixed: no electrons below threshold and emission above it. The number emitted is controlled by incident photon flux; increasing frequency above threshold principally changes their energy. Hence A, C and D express the usual qualitative photoelectric relationships, while B omits the essential threshold intercept.

Step 4: Conclusion The kinetic-energy versus frequency line cannot pass through the origin as shown in B. Hence, the correct option is (B).

Quick Tip to Solve: Whenever kinetic energy is plotted against frequency, locate the threshold intercept first; a positive work function forbids an origin-passing line.

71. The total number of isomers for a square planar complex $[M(F)(Cl)(SCN)(NO_2)]$ is [4 marks]

(A) 16

(B) 8

(C) 12

(D) 4

Correct Answer: (C) 12

Solution:

Concept:

- A square-planar complex with four different monodentate ligands has three geometrical arrangements: fix one ligand, then choose which of the other three lies trans to it. Rotations and reflections within the plane do not create further arrangements.
- An ambidentate ligand can attach through either of two donor atoms, producing linkage isomers. Here thiocyanate can be S-bound or N-bound, and nitrite can be N-bound or O-bound.

Step 1: Count the linkage combinations The ligands F^- and Cl^- each have one possible donor atom. The SCN^- ligand has the thiocyanato-S and isothiocyanato-N linkages, while $NO_2{}^-$ gives nitro-N and nitrito-O linkages. Their choices are independent, so the number of linkage combinations is

$$2 \times 2 = 4.$$

The four combinations are (S,N), (S,O), (N,N) and (N,O), where the first entry identifies the donor of thiocyanate and the second that of nitrite.

Step 2: Include the square-planar arrangements For each fixed linkage combination, place fluoride at one vertex of the square. Chloride, the thiocyanate ligand or the nitrite ligand can be opposite fluoride, giving exactly three geometrical isomers. The two remaining ligands occupy the adjacent positions; interchanging their displayed positions gives an equivalent planar arrangement, not a new optical isomer. Hence

$$N_{\text{total}} = 4 \times 3 = 12.$$

The value 4 counts linkage choices alone, whereas 8 and 16 do not include the correct factor of three for four different ligands.

Step 3: Conclusion The complex has 12 isomers after both linkage and geometrical isomerism are included. Hence, the correct option is (C).

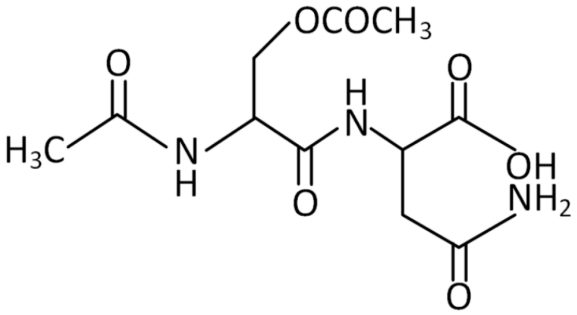
Quick Tip to Solve: For square-planar $MABCD$, multiply the three trans-pair arrangements by the independent linkage choices of each ambidentate ligand.

72. The correct structure of product 'P' in the following reactions is :

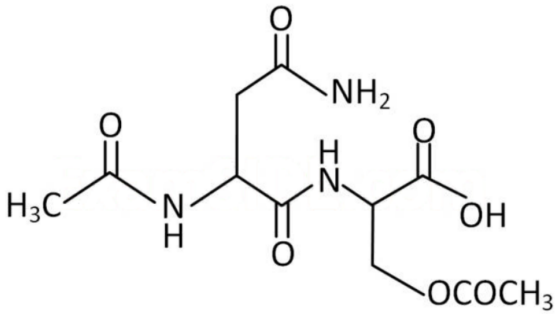
$$\text{Asn} - \text{Ser} + (CH_3CO)_2O \text{ (excess)} \xrightarrow{NEt_3} P$$

[4 marks]

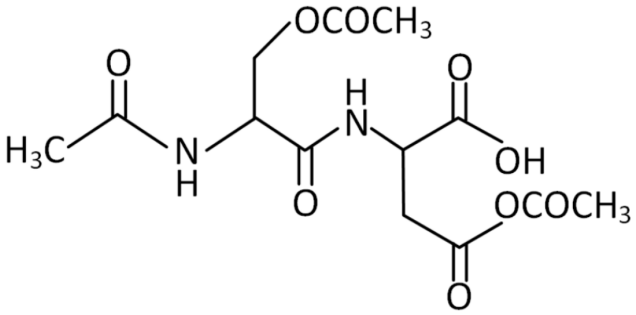
(A) $\mathrm{Ac-Ser(OAc)-Asn}$



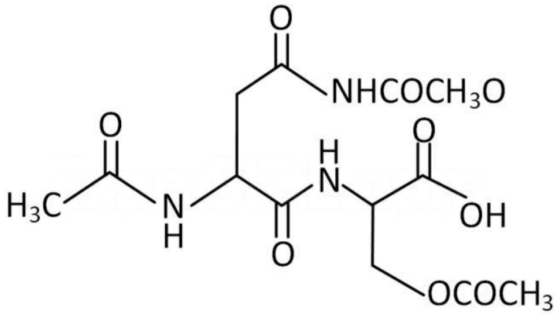
(B) $\mathrm{Ac-Asn-Ser(OAc)}$



(C) $\mathrm{Ac-Ser(OAc)-Asp}$ with side-chain $-\mathrm{C(=O)OCOCH_3}$

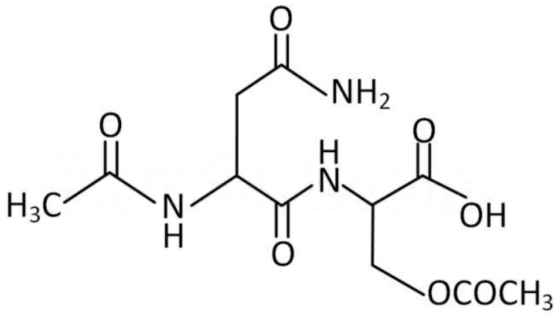


(D) $\mathrm{Ac-Asn-Ser(OAc)}$ with the side-chain label $\mathrm{NHCOCH_3O}$ as shown



Correct Answer: (B)

$\mathrm{Ac-Asn-Ser(OAc)}$



Solution:

Concept:

- In a peptide name the amino-terminal residue is written first. Thus Asn-Ser has the backbone $\mathrm{H_2N-CH(CH_2CONH_2)-CO-NH-CH(CH_2OH)-COOH}$, with asparagine on the N-terminal side and serine on the C-terminal side.
- Acetic anhydride acetylates a free amino group to an amide and a free alcohol to an acetate ester. Triethylamine acts as a base; resonance makes existing amide nitrogens much less nucleophilic than a free amine.

Step 1: Locate the reactive groups The free terminal $-\mathrm{NH_2}$ group belongs to asparagine, whereas the serine side chain contains $-\mathrm{CH_2OH}$. Excess acetic anhydride supplies enough acetylating reagent for both sites. The peptide $-\mathrm{CONH-}$ linkage remains intact, and the asparagine side-chain $-\mathrm{CONH_2}$ is an amide rather than a second free amino group. Acetylation therefore changes the terminal group to $\mathrm{CH_3CONH^-}$ and the serine side chain to $-\mathrm{CH_2OCOCH_3}$.

Step 2: Preserve the residue order and identify the structure The resulting product has the condensed structure

$$\mathrm{CH_3CONH-CH(CH_2CONH_2)-CO-NH-CH(CH_2OCOCH_3)-COOH.}$$

It is N-acetyl-Asn-Ser with an O-acetylated serine side chain, exactly the structure in option (B). Option (A) reverses the residue order to Ser-Asn, which acetylation cannot do. Option (C) also changes the asparagine side-chain amide into an oxygen-containing acyl derivative. Option (D) modifies the side-chain amide instead of leaving $-\mathrm{CONH_2}$ intact; its printed extra side-chain label does not match the selective product.

Step 3: Conclusion The free N-terminal amino group and serine hydroxyl group are acetylated without reversing the peptide sequence. Hence, the correct option is (B).

Quick Tip to Solve: Write the peptide from its N terminus first; then acetylate free $-\mathrm{NH_2}$ and $-\mathrm{OH}$ groups while preserving the amide bonds.

73. The effect of lanthanoid contraction in the lanthanoid series of elements by and large means :

[4 marks]

(A) increase in atomic radii and decrease in ionic radii

(B) increase in both atomic and ionic radii

(C) decrease in both atomic and ionic radii

(D) decrease in atomic radii and increase in ionic radii

Correct Answer: (C) decrease in both atomic and ionic radii

Solution:

Concept:

- Lanthanoid contraction is the gradual decrease in the sizes of atoms and corresponding ions across the lanthanoid series as atomic number increases. The term describes the overall trend, rather than claiming perfect regularity for every neutral atom.
- The added electrons enter the inner $4f$ subshell. These electrons shield the increasing nuclear charge poorly, so the outer electrons experience a progressively stronger effective nuclear attraction.

Step 1: Follow the increase in effective nuclear charge On moving from one lanthanoid to the next, the nucleus gains one proton and the differentiating electron enters the $4f$ subshell. The extra electron does not fully screen the added positive charge from the remaining electrons. The effective nuclear charge therefore increases across the series, pulling the electron cloud closer to the nucleus. This is the physical origin of the contraction; simply adding electrons does not imply a larger radius when nuclear attraction increases more strongly.

Step 2: Apply the trend to both atoms and ions The stronger attraction produces an overall decrease in atomic radii. It also decreases the radii of the corresponding ions, especially the commonly compared trivalent lanthanoid ions Ln^{3+}, whose charge is kept the same throughout the comparison. Thus both kinds of radius contract. Option (A) has the wrong trend for atoms, option (B) reverses both trends, and option (D) incorrectly makes the ionic radii increase. Small local irregularities do not change the broad meaning requested by the phrase "by and large".

Step 3: Conclusion Lanthanoid contraction means a decrease in both atomic and ionic radii across the series. Hence, the correct option is (C).

Quick Tip to Solve: For a size trend across the lanthanoids, combine increasing nuclear charge with poor $4f$-electron shielding; compare ions of the same charge.

74. A process has $\Delta H = 200$ J mol^{-1} and $\Delta S = 40$ JK^{-1} mol^{-1}. Out of the values given below, choose the minimum temperature above which the process will be spontaneous : [4 marks]

(A) 4 K

(B) 20 K

(C) 5 K

(D) 12 K

Correct Answer: (C) 5 K

Solution:

Concept:

- At constant temperature and pressure, a process is spontaneous in the stated direction when its Gibbs energy change is negative: $\Delta G = \Delta H - T\Delta S < 0$. Here T is the absolute temperature in kelvin.
- For positive ΔH and positive ΔS, enthalpy opposes the process but the entropy contribution favours it. The threshold is $T = \Delta H/\Delta S$; above that temperature the entropy term dominates.

Step 1: Insert the supplied enthalpy and entropy changes Both quantities are already expressed using joules, so no conversion from kilojoules is needed. Using $\Delta H = 200\,\mathrm{J\,mol^{-1}}$ and $\Delta S = 40\,\mathrm{J\,K^{-1}\,mol^{-1}}$,

$$\Delta G = (200 - 40T)\,\mathrm{J\,mol^{-1}}.$$

The ratio has units of kelvin because the molar energy units cancel. Mixing joules and kilojoules here would shift the threshold by a factor of one thousand.

Step 2: Solve the strict spontaneity inequality For spontaneity,

$$200 - 40T < 0 \quad \Longrightarrow \quad 40T > 200 \quad \Longrightarrow \quad T > 5\,\mathrm{K}.$$

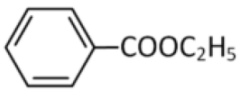
At exactly $5\,\mathrm{K}$, $\Delta G = 0$, so this is the boundary, not a temperature at which the forward direction is strictly spontaneous. At $4\,\mathrm{K}$, $\Delta G = 40\,\mathrm{J\,mol^{-1}} > 0$. The process is spontaneous at 12 K and 20 K, but neither is the minimum threshold above which it becomes spontaneous.

Step 3: Conclusion The threshold temperature is $5\,\mathrm{K}$, and the process is spontaneous for $T > 5\,\mathrm{K}$. Hence, the correct option is (C).

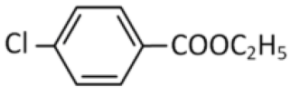
Quick Tip to Solve: For $\Delta H > 0$ and $\Delta S > 0$, calculate $\Delta H/\Delta S$ in consistent units and retain the strict condition $T > \Delta H/\Delta S$.

75. The decreasing order of ease of alkaline hydrolysis for the following esters in [4 marks]

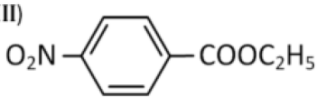
(I)



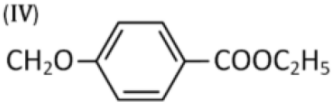
(II)



(III)



(IV)



(A) III > II > IV > I

(B) IV > II > III > I

(C) III > II > I > IV

(D) II > III > I > IV

Correct Answer: (C) III > II > I > IV

Solution:

Concept:

- Alkaline hydrolysis of an ester begins with nucleophilic attack by OH^- at the ester carbonyl carbon, followed by formation and breakdown of a tetrahedral intermediate. A more electrophilic carbonyl generally undergoes attack more readily.
- For these para-substituted ethyl benzoates, the alkoxy leaving group is the same. The distinguishing factor is the electronic effect transmitted by the aromatic substituent to the acyl group.

Step 1: Compare the withdrawing substituents Compound I is unsubstituted ethyl benzoate. Compound II has a para-chloro group, whose net electron-withdrawing inductive effect increases the susceptibility of the acyl group to hydroxide attack relative to I. Compound III has a para-nitro group, a much stronger electron-withdrawing substituent through both induction and resonance. It lowers electron density in the conjugated aromatic acyl system more strongly, so III hydrolyses more readily than II. The first part of the order is therefore III > II > I.

Step 2: Place the electron-donating ester last Compound IV has a para-methoxy-type oxygen donor. Donation of an oxygen lone pair through the aromatic ring increases electron density in the conjugated acyl system and reduces the electrophilicity of the ester carbonyl. It is therefore less reactive towards hydroxide attack than unsubstituted I. Combining the comparisons gives

$$\text{III} > \text{II} > \text{I} > \text{IV}.$$

Option (A) wrongly puts IV ahead of I; (B) makes the donating derivative the most reactive; and (D)

incorrectly places chloro ahead of the much more strongly withdrawing nitro group.

Step 3: Conclusion The decreasing order of ease of alkaline hydrolysis is III $>$ II $>$ I $>$ IV. Hence, the correct option is (C).

Quick Tip to Solve: When the ester leaving groups match, rank para substituents by their effect on carbonyl electrophilicity: strong withdrawal accelerates attack and resonance donation retards it.

76. Two pi and half sigma bonds are present in : [4 marks]

(A) O_2

(B) N_2^+

(C) O_2^+

(D) N_2

Correct Answer: (B) N_2^+

Solution:

Concept:

- In molecular orbital theory, the bond order contributed by a set of orbitals is $(N_b - N_a)/2$, where N_b and N_a count bonding and antibonding electrons in that set. Sigma and pi contributions must be counted separately.
- For N_2, the valence configuration is $(\sigma 2s)^2(\sigma^* 2s)^2(\pi 2p_x)^2(\pi 2p_y)^2(\sigma 2p_z)^2$. The highest occupied orbital is the bonding $\sigma 2p_z$ orbital.

Step 1: Remove one electron from nitrogen Formation of N_2^+ removes one electron from the bonding $\sigma 2p_z$ orbital. The occupied sigma orbitals then contain three bonding electrons and two antibonding electrons, giving

$$\mathrm{BO}_\sigma = \frac{3-2}{2} = \frac{1}{2}.$$

The four electrons in the two bonding pi orbitals remain unchanged, with no occupied pi antibonding orbitals, so

$$\mathrm{BO}_\pi = \frac{4-0}{2} = 2.$$

Thus the ion has two pi bonds and a net half sigma bond, for a total bond order of 2.5.

Step 2: Distinguish the other molecules Neutral N_2 has a full sigma bond and two pi bonds, so its sigma contribution is not one half. In O_2, the sigma contribution is one but the occupied pi antibonding orbitals reduce the pi contribution to one. Removing one pi antibonding electron gives O_2^+, whose sigma contribution is still one and pi contribution is 1.5. Although O_2^+ and N_2^+ both have total bond order 2.5, only the nitrogen cation has the required split into sigma and pi components.

Step 3: Conclusion N_2^+ contains a net half sigma bond and two pi bonds. Hence, the correct option is (B).

Quick Tip to Solve: A total bond order of 2.5 is not sufficient here: identify whether ionisation removes a sigma bonding electron or a pi antibonding electron.

77. The values of K_p/K_c for the following reactions at 300 K are, respectively : (At 300 K, RT = 24.62 dm^3 atm mol^{-1})

$N_2(g) + O_2(g) \rightleftharpoons 2\,NO(g)$

$N_2O_4(g) \rightleftharpoons 2\,NO(g)$

$N_2(g) + 3H_2(g) \rightleftharpoons 2\,NH_3(g)$ [4 marks]

(A) 24.62 dm^3 atm mol^{-1}, 606.0 dm^6 atm^2 mol^{-2} 1.65×10^{-3} dm^{-6} atm^{-2} mol^2

(B) 1,4.1 $\times 10^{-2}$ dm^{-3} atm^{-1} mol, 606 dm^6 atm^2 mol^{-2}

(C) 1,24.62 dm^3 atm mol^{-1} , 1.65×10^{-3} dm^{-6} atm $^{-2}$ mol^2

(D) 1, 24.62 dm^3 atm mol^{-1}, 606.0 dm^6 atm^2 mol^{-2}

Correct Answer: (C) 1,24.62 dm^3 atm mol^{-1} , 1.65×10^{-3} dm^{-6} atm $^{-2}$ mol^2

Solution:

Concept:

- For ideal gaseous equilibria written with the conventional pressure and concentration units, $K_p = K_c(RT)^{\Delta n_g}$, where Δn_g is the sum of gaseous product coefficients minus the sum of gaseous reactant coefficients.
- Use the stoichiometric coefficients of each printed equation, not the number of different chemical species. With $RT = 24.62\,\mathrm{dm^3\,atm\,mol^{-1}}$, a negative exponent requires both the reciprocal numerical value and reciprocal units.

Step 1: Count the gaseous mole changes For $\mathrm{N_2(g) + O_2(g) \rightleftharpoons 2\,NO\,(g)}$, $\Delta n_g = 2 - (1+1) = 0$. For the second printed equation, one gaseous reactant unit gives two gaseous product units, so $\Delta n_g = 2 - 1 = 1$. The displayed $\mathrm{N_2O_4 \rightleftharpoons 2\,NO}$ equation omits oxygen and is not atom-balanced as written; the usual dissociation product is $\mathrm{NO_2}$, but this chemical-label defect does not change the intended coefficient difference or the requested power of RT. For $\mathrm{N_2(g) + 3\,H_2(g) \rightleftharpoons 2\,NH_3(g)}$, $\Delta n_g = 2 - (1+3) = -2$.

Step 2: Evaluate the three ratios in their stated order The ratios are

$$\left(\frac{K_p}{K_c}\right)_1 = (24.62)^0 = 1,$$
$$\left(\frac{K_p}{K_c}\right)_2 = 24.62\,\mathrm{dm^3\,atm\,mol^{-1}},$$
$$\left(\frac{K_p}{K_c}\right)_3 = \frac{1}{(24.62)^2} \simeq 1.65 \times 10^{-3}\,\mathrm{dm^{-6}\,atm^{-2}\,mol^2}.$$

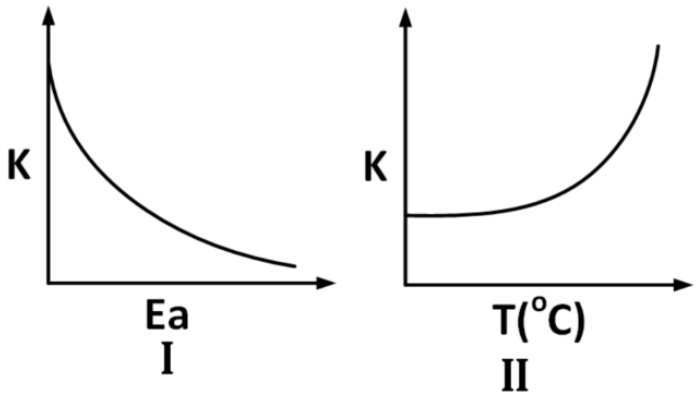
Option (A) incorrectly assigns a non-unit first ratio. Option (B) inverts the second ratio and squares rather than inverts the third; option (D) also uses $(RT)^2$ instead of $(RT)^{-2}$ for ammonia formation.

Step 3: Conclusion The three ratios are 1, $24.62\,\mathrm{dm^3\,atm\,mol^{-1}}$, and $1.65 \times 10^{-3}\,\mathrm{dm^{-6}\,atm^{-2}\,mol^2}$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Write Δn_g beside each reaction first; zero gives 1, positive one gives RT, and negative two gives its reciprocal square.

78. Consider the given plots for a reaction obeying Arrhenius equation (0°C < T < 300°C) : (K and Ea are rate constant and activation energy, respectively)

Choose the correct option : [4 marks]



(A) I is right but II is wrong

(B) Both I and II are correct

(C) Both I and II are wrong

(D) I is wrong but II is right

Correct Answer: (B) Both I and II are correct

Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where A is the frequency factor, E_a is the activation energy, R is the gas constant and T is absolute temperature.
- At fixed temperature a larger activation barrier lowers the rate constant. For a positive activation energy, increasing temperature increases the fraction of molecules that can cross that barrier.

Step 1: Check plot I at fixed temperature Hold A and T fixed while comparing activation energies. The exponential has a negative exponent, so increasing E_a makes its value smaller:

$$\left(\frac{\partial k}{\partial E_a}\right)_T = -\frac{k}{RT} < 0.$$

Thus a decreasing curve of k against E_a, as in plot I, is correct. It is not a straight-line decrease because the dependence is exponential.

Step 2: Check plot II using absolute temperature In the equation use $T = t + 273.15$, where t is the Celsius temperature shown on the axis. For the usual positive activation barrier,

$$\frac{dk}{dT} = \frac{kE_a}{RT^2} > 0.$$

The rate constant therefore rises with temperature. A curved, increasingly steep rise over the stated ordinary-temperature interval is the standard Arrhenius behaviour represented in plot II; the Celsius zero does not mean zero absolute temperature.

Step 3: Compare the offered statements Plot I correctly represents suppression of the rate constant by a higher barrier, and plot II correctly represents its increase on heating. Option A incorrectly rejects II; option D incorrectly rejects I; option C rejects both valid trends. The graph labels refer to the rate constant, not to an equilibrium constant.

Step 4: Conclusion Both I and II show the correct Arrhenius trends. Hence, the correct option is (B).

Quick Tip to Solve: Use the sign of the exponent: a larger barrier decreases k, while a higher absolute temperature makes that exponent less negative.

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79. If dichloromethane (DCM) and water (H_2O) are used for differential extraction, which one of the following statements is correct ? [4 marks]

(A) DCM and H_2O would stay as upper and lower layer respectively in the separating funnel (S.F.)

(B) DCM and H_2O will make turbid/colloidal mixture

(C) DCM and H_2O would stay as lower and upper layer respectively in the separating funnel(S.F).

(D) DCM and H_2O will be miscible clearly

Correct Answer: (C) DCM and H_2O would stay as lower and upper layer respectively in the separating funnel(S.F).

Solution:

Concept:

- Differential extraction uses two liquids that form separate phases, so that a solute can distribute between them according to its relative solubilities.
- The denser immiscible liquid forms the lower layer in a separating funnel; the less dense one forms the upper layer. An organic solvent need not be the upper layer.

Step 1: Identify the two phases Dichloromethane, CH_2Cl_2, and water have only limited mutual solubility and form two distinct bulk layers under the usual extraction conditions. They do not mix to give a single clear homogeneous phase. This phase separation is the reason DCM is suitable as an extracting solvent for many organic solutes.

Step 2: Compare their densities At ordinary temperature the density of DCM is about $1.33\,\mathrm{g\,cm^{-3}}$, whereas water has density approximately $1.00\,\mathrm{g\,cm^{-3}}$. Therefore

$$\rho_{\mathrm{DCM}} > \rho_{\mathrm{water}}.$$

Gravity places the denser DCM beneath the water. After shaking, the funnel is allowed to stand until the interface is clear, and the lower DCM layer can be drained through the stopcock.

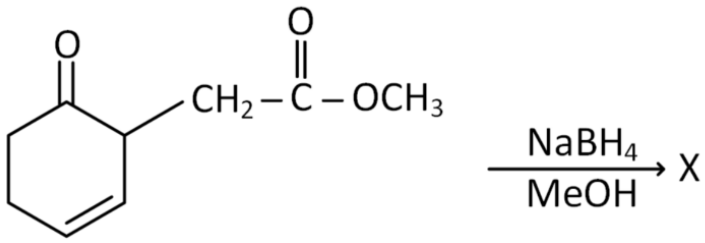
Step 3: Reject the other descriptions Option A reverses the actual density order and so assigns the layers incorrectly. Option B describes a persistent colloidal or turbid mixture rather than the normal settled extraction layers; temporary emulsification during shaking is not the final phase arrangement. Option D wrongly claims complete miscibility. Option C alone identifies both separate phases in their correct vertical positions.

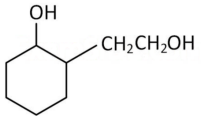
Step 4: Conclusion DCM forms the lower layer and water forms the upper layer in the separating

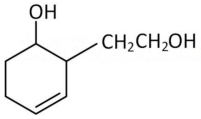
funnel. Hence, the correct option is (C).

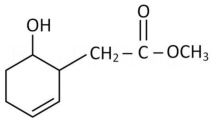
Quick Tip to Solve: Before draining an extraction solvent, compare densities: chlorinated solvents such as DCM are commonly heavier than water.

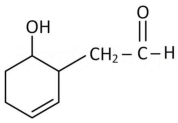
80. The major product 'X' formed in the following reaction is : [4 marks]



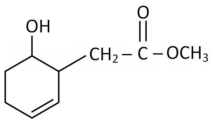
(A) Ring alcohol with saturated ring and $-CH_2CH_2OH$ side chain. 

(B) Ring alcohol retaining the ring double bond, with $-CH_2CH_2OH$ side chain. 

(C) Ring alcohol retaining the ring double bond and $-CH_2COOCH_3$ side chain. 

(D) Ring alcohol retaining the ring double bond, with $-CH_2CHO$ side chain. 

Correct Answer: (C)

Ring alcohol retaining the ring double bond and $-CH_2COOCH_3$ side chain. 

Solution:

Concept:

- Sodium borohydride, $NaBH_4$, is a mild hydride donor that normally reduces aldehydes and ketones to alcohols in methanol.
- Under these conditions an ordinary ester group and an isolated carbon-carbon double bond remain unchanged. Identify each functional group separately before choosing a product.

Step 1: List the reducible groups in the substrate The cyclic substrate contains a ketone carbonyl

within the six-membered ring, a carbon-carbon double bond elsewhere in that ring, and a methyl ester side chain $-CH_2COOCH_3$. The double bond is not the carbonyl being reduced. Methanol is the reaction medium and can protonate the alkoxide formed after hydride transfer.

Step 2: Reduce the ketone selectively Hydride attacks the electrophilic carbon atom of the ring ketone and converts its $C=O$ bond to an alkoxide. Protonation by methanol produces a secondary ring alcohol:

$$R_2C=O \xrightarrow{NaBH_4/MeOH} R_2CHOH.$$

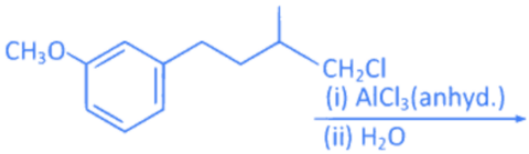
The ring carbonyl oxygen remains in the molecule as the hydroxyl oxygen. The ring skeleton and the position of the existing alkene are preserved.

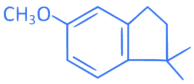
Step 3: Match the unchanged groups to the options The correct product must retain both $C=C$ and $-CH_2COOCH_3$, while replacing only the ketone by $-CHOH-$. Structure C has precisely these features. A incorrectly reduces the ester and also saturates the ring alkene. B incorrectly reduces the ester to $-CH_2CH_2OH$. D incorrectly converts the ester to an aldehyde, a selective transformation not caused by the stated reagent. No catalytic hydrogenation or stronger ester-reducing reagent is present.

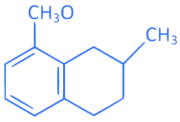
Step 4: Conclusion Only the ring ketone is reduced; the alkene and methyl ester remain intact, giving structure C. Hence, the correct option is (C).

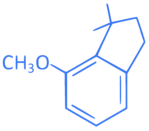
Quick Tip to Solve: For $NaBH_4$, circle aldehydes and ketones first; do not automatically reduce every multiple bond or every carbonyl group.

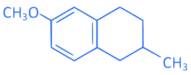
81. The major product of the following reaction is : [4 marks]



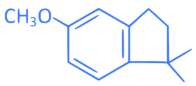
(A) Gem-dimethyl indane with ring closure para to methoxy, as drawn. 

(B) Methoxy-substituted methyl tetralin with the substitution pattern drawn.


(C) Gem-dimethyl indane with ring closure ortho to methoxy, as drawn. 

(D) Methoxy-substituted methyl tetralin with the substitution pattern drawn.


Correct Answer: (A)

Gem-dimethyl indane with ring closure para to methoxy, as drawn. 

Solution:

Concept:

- Anhydrous $AlCl_3$ activates an alkyl chloride for Friedel-Crafts alkylation. Intramolecular attack can produce a fused ring.
- Carbocation rearrangement favours a more stable centre, and the methoxy group activates aromatic ortho and para positions. A less crowded para position is favoured when available.

Step 1: Generate and rearrange the electrophile Write the side chain as

$$Ar-CH_2-CH_2-CH(CH_3)-CH_2Cl.$$

The methoxy group is meta to the chain attachment on the ring. Activation of the terminal chloride by $AlCl_3$ develops positive charge at the terminal carbon. A neighbouring hydride shifts from $CH(CH_3)$ to that carbon, replacing the unfavourable primary cationic centre by a tertiary one:

$$Ar-CH_2-CH_2-C^+(CH_3)_2.$$

The carbon that originally carried chlorine has become a methyl group.

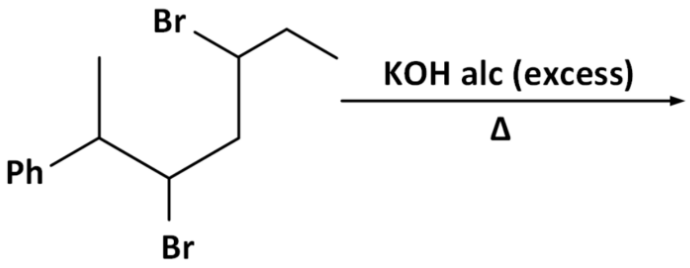
Step 2: Close the five-membered ring The aromatic ring attacks the rearranged tertiary centre at a position ortho to the side-chain attachment. This makes a five-membered ring fused to the benzene ring: the new ring contains two aromatic fusion atoms, the two methylene carbons and the tertiary centre. The latter consequently bears two methyl groups. Loss of an aromatic proton restores aromaticity, and aqueous work-up releases the product from its Lewis-acid complex.

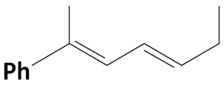
Step 3: Choose the favoured aromatic position The available ring positions for this closure are ortho and para to methoxy. Closure para to methoxy avoids the greater crowding beside the methoxy substituent and gives the gem-dimethyl indane drawn in A. C is the more hindered ortho cyclisation product. B and D have a six-membered saturated fused ring and a single methyl substituent; those do not represent the favoured rearranged five-membered-ring pathway.

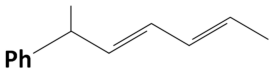
Step 4: Conclusion Hydride rearrangement followed by intramolecular para-directed cyclisation gives the gem-dimethyl indane in A. Hence, the correct option is (A).

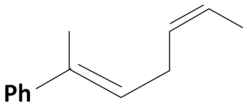
Quick Tip to Solve: Before counting a new Friedel-Crafts ring, check for a hydride shift: rearrangement changes both the ring size and methyl substitution.

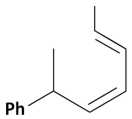
82. The major product of the following reaction is [4 marks]



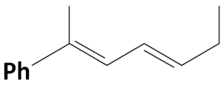
(A) $PhC(CH_3)=CH-CH=CH-CH_2CH_3$, with the geometry drawn. 

(B) $PhCH(CH_3)-CH=CH-CH=CH-CH_3$, with the geometry drawn. 

(C) $PhC(CH_3)=CH-CH_2-CH=CH-CH_3$, with the geometry drawn. 

(D) $PhCH(CH_3)-CH=CH-CH=CH-CH_3$, with the geometry drawn. 

Correct Answer: (A)

$PhC(CH_3)=CH-CH=CH-CH_2CH_3$, with the geometry drawn. 

Solution:

Concept:

- Hot alcoholic KOH favours beta elimination of hydrogen halide. Excess base permits both bromine substituents to be eliminated.
- The major alkene arrangement is stabilised by substitution and conjugation. A diene conjugated with a phenyl ring gains an extended delocalised system.

Step 1: Write the carbon skeleton without changing it The starting chain is

$$PhCH(CH_3)-CHBr-CH_2-CHBr-CH_2CH_3.$$

. Number its main chain from the carbon bonded to phenyl: bromine is present on C2 and C4. Each bromine can leave with a hydrogen from an adjacent carbon. The carbon skeleton stays unchanged; the reaction changes single bonds to double bonds rather than moving the phenyl or methyl groups.

Step 2: Select the stabilised pair of eliminations Remove H from C1 with Br from C2 to create C1=C2. Remove H from C3 with Br from C4 to create C3=C4. The product connectivity is

$$PhC(CH_3)=CH-CH=CH-CH_2CH_3.$$

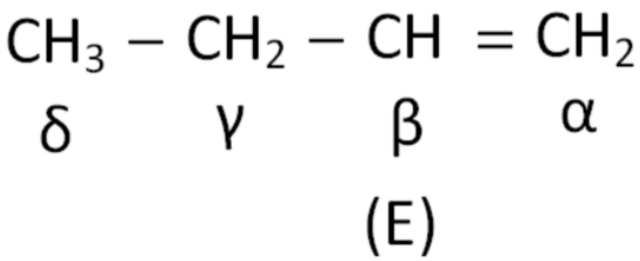
The two double bonds are separated by one single bond, so they form a conjugated diene. The first double bond is also directly attached to the phenyl ring, extending conjugation and giving the more substituted benzylic alkene.

Step 3: Compare the four drawings A shows this extended conjugated arrangement. In B and D the phenyl-bearing carbon remains saturated, so the diene is separated from the aromatic ring by an sp^3 carbon and loses that stabilisation, regardless of the drawn diene geometry. C retains an alkene next to phenyl but its two alkenes are separated by a saturated methylene group, making an isolated rather than a conjugated diene. Thus both conjugation requirements distinguish A from the alternatives.

Step 4: Conclusion The major product is $\mathrm{PhC(CH_3){=}CH{-}CH{=}CH{-}CH_2CH_3}$, represented by A. Hence, the correct option is (A).

Quick Tip to Solve: For two eliminations, compare the whole final pi system: a phenyl-conjugated diene can be favoured over two individually plausible isolated alkenes.

83. Which hydrogen in compound (E) is easily replaceable during bromination reaction in presence of light ? [4 marks]



(A) β – hydrogen

(B) α – hydrogen

(C) γ – hydrogen

(D) δ – hydrogen

Correct Answer: (C) γ – hydrogen

Solution:

Concept:

- Bromination in the presence of light proceeds through radical formation. The hydrogen most readily abstracted is the one whose removal gives the most stable carbon radical.
- An allylic radical is resonance-stabilised by an adjacent double bond. A vinylic radical, formed on a double-bond carbon, does not have that same allylic stabilisation.

Step 1: Locate each printed hydrogen label The structure is $\mathrm{CH_3{-}CH_2{-}CH{=}CH_2}$. The labels are assigned in the figure: the terminal alkene $\mathrm{CH_2}$ carries alpha hydrogens, the other alkene carbon carries the beta hydrogen, the saturated $\mathrm{CH_2}$ beside the double bond carries gamma hydrogens, and the terminal $\mathrm{CH_3}$ carries delta hydrogens. Use these printed labels rather than importing a different

numbering convention.

Step 2: Compare the radicals formed by abstraction Removing a gamma hydrogen gives an allylic radical, whose unpaired electron is delocalised over two carbon centres:

$$CH_3-CH^{\bullet}-CH=CH_2 \longleftrightarrow CH_3-CH=CH-CH_2^{\bullet}.$$

This resonance lowers the energy of the radical intermediate and makes the corresponding allylic carbon-hydrogen bond easier to break than a comparable non-allylic bond. A bromine radical can abstract this hydrogen, after which the allylic radical reacts with bromine.

Step 3: Reject vinylic and ordinary alkyl positions Alpha and beta hydrogens are attached directly to the alkene carbons. Their abstraction forms relatively unstable vinylic radicals and breaks stronger vinylic carbon-hydrogen bonds. A delta hydrogen gives an ordinary primary radical on the terminal methyl carbon, without adjacent pi-bond resonance. Thus A, B and D are less favourable than the allylic gamma site. Light is the cue to compare radical substitution sites rather than simply add bromine across the double bond.

Step 4: Conclusion The gamma hydrogen is most easily replaced because its abstraction forms a resonance-stabilised allylic radical. Hence, the correct option is (C).

Quick Tip to Solve: In light-induced bromination, locate the saturated carbon next to C=C; its allylic hydrogen is usually the preferred abstraction site.

84. The total number of isotopes of hydrogen and number of radioactive isotopes among them, respectively, are : [4 marks]

(A) 3 and 2

(B) 2 and 1

(C) 2 and 0

(D) 3 and 1

Correct Answer: (D) 3 and 1

Solution:

Concept:

- Isotopes of an element have the same atomic number but different mass numbers because their nuclei contain different numbers of neutrons.
- The three standard isotopes of hydrogen are protium, deuterium and tritium. Nuclear stability, rather than chemical valency, determines whether an isotope is radioactive.

Step 1: List the three standard isotopes Hydrogen has atomic number one, so every hydrogen nucleus contains one proton. Its familiar isotope set is

$$^{1}_{1}H \quad \text{(protium)}, \qquad ^{2}_{1}H \quad \text{(deuterium)}, \qquad ^{3}_{1}H \quad \text{(tritium)}.$$

Their neutron counts are obtained by subtracting the atomic number from the mass number: zero,

one and two, respectively. Hence the isotope count in the standard chemical classification is three, not two.

Step 2: Identify which nuclei are unstable Protium and deuterium are stable. Tritium is radioactive and undergoes beta-minus decay, converting a neutron to a proton:

$$^{3}_{1}\mathrm{H} \longrightarrow {}^{3}_{2}\mathrm{He} + e^{-} + \bar{\nu}_e.$$

Thus only one member of the three-isotope set is radioactive. The extra neutron in deuterium does not make it radioactive; heavy water therefore should not be confused with tritiated water.

Step 3: Check both numbers in every option The required ordered pair is total isotopes followed by radioactive isotopes. Option A correctly begins with three but incorrectly classifies two as radioactive. Options B and C omit one of the familiar isotopes and therefore have an incorrect total; C also incorrectly says none is radioactive. Only D supplies three total isotopes and one radioactive isotope in the requested order.

Step 4: Conclusion Hydrogen has three standard isotopes, of which only tritium is radioactive: the pair is $3, 1$. Hence, the correct option is (D).

Quick Tip to Solve: Remember the isotope names in mass order: protium and deuterium are stable; the third, tritium, is radioactive.

85. Hall- Heroult's process is given by : [4 marks]

(A) Cu^{2+} (aq) + H_2(g) → Cu(s) + $2H^+$ (aq)

(B) Cr_2O_3 + 2AI → Al_2O_3 + 2Cr

(C) $2Al_2O_3$ + 3C → 4Al + $3CO_2$

(D) ZnO + C $\overset{Coke,\,1673\,K}{\longrightarrow}$ Zn + CO

Correct Answer: (C) $2Al_2O_3$ + 3C → 4Al + $3CO_2$

Solution:

Concept:

- Hall-Heroult extraction produces aluminium by electrolysing alumina, Al_2O_3, dissolved in a molten fluoride electrolyte. Cryolite lowers the operating temperature and improves conductivity.
- Aluminium ions are reduced at the cathode. Oxygen formed from oxide ions reacts with the carbon anode, so carbon is consumed and appears in the overall equation.

Step 1: Identify the cathode and anode changes The metal extracted is aluminium, rather than copper, chromium or zinc. At the cathode aluminium ions gain electrons, while oxide ions lose electrons at the anode. The corresponding idealised half-reactions are

$$Al^{3+} + 3\,e^{-} \longrightarrow Al, \qquad 2\,O^{2-} \longrightarrow O_2 + 4\,e^{-}.$$

The oxygen does not simply accumulate because the anode is made of carbon. It attacks that anode, producing predominantly carbon dioxide in the standard overall description.

Step 2: Combine the reactions and eliminate the other processes Combining electrolysis with consumption of the carbon anode gives

$$2\,Al_2O_3 + 3\,C \longrightarrow 4\,Al + 3\,CO_2.$$

There are four aluminium atoms, six oxygen atoms and three carbon atoms on each side, so this is the balanced overall reaction. Option (A) describes reduction of copper ions by hydrogen. Option (B) is aluminothermic reduction of chromium(III) oxide: aluminium is the reducing agent there, not the extracted metal. Option (D) is the high-temperature carbon reduction of zinc oxide. None represents the electrolytic extraction of aluminium.

Step 3: Conclusion The Hall-Heroult process has the net reaction $2\,Al_2O_3 + 3\,C \longrightarrow 4\,Al + 3\,CO_2$. Hence, the correct option is (C).

Quick Tip to Solve: Identify aluminium as the product and carbon dioxide as the anode product; these immediately distinguish Hall-Heroult extraction from thermite and zinc reduction.

86. The metal used for marking X–ray tube window is : [4 marks]

(A) Ca

(B) Na

(C) Mg

(D) Be

Correct Answer: (D) Be

Solution:

Concept:

- An X-ray tube window must allow the useful X-rays to leave the evacuated tube with relatively little absorption, while maintaining a strong, sealed barrier.
- Beryllium has low atomic number $Z = 4$ and low density. A thin beryllium window therefore offers relatively low attenuation of X-rays and adequate mechanical rigidity.

Step 1: Relate the window material to its function The window is the exit opening for radiation generated inside the tube. It must not absorb most of this radiation before it reaches the object being examined. A thin, light material with low atomic number is consequently preferred. Beryllium meets this requirement and can be made into a thin, mechanically useful window. The question concerns the radiation-transmitting window, rather than the heavy metal target on which the electrons strike.

Step 2: Compare the listed metals The atomic numbers of the listed metals are

$$Z(\mathrm{Ca}) = 20, \quad Z(\mathrm{Na}) = 11, \quad Z(\mathrm{Mg}) = 12, \quad Z(\mathrm{Be}) = 4.$$

Of these, beryllium provides the characteristic combination of a very low atomic number, low density and useful stiffness for this application. Calcium and magnesium have larger atomic numbers and are not the standard X-ray window material. Sodium is soft and highly reactive, making it unsuitable as the required robust window. Thus simply choosing a light-looking metal is insufficient: the recognised

application and structural requirements both point to beryllium.

Step 3: Conclusion Beryllium, Be, is used for X-ray tube windows because thin sheets transmit X-rays relatively well while providing mechanical support. Hence, the correct option is (D).

Quick Tip to Solve: Associate beryllium with the X-ray exit window and tungsten with the target; the window transmits radiation whereas the target generates it.

87. Which of the following is not an example of heterogenous catalytic reaction? [4 marks]

(A) Ostwald's process

(B) Combustion of coal

(C) Hydrogenation of vegetable oils

(D) Haber's process

Correct Answer: (B) Combustion of coal

Solution:

Concept:

- In heterogeneous catalysis, the catalyst and the reacting substances are in different phases. Gas molecules commonly react on the surface of a solid catalyst.
- A substance consumed as a reactant is not a catalyst. A reaction between a solid and a gas does not become a heterogeneous catalytic reaction merely because two phases are present.

Step 1: Identify the genuinely catalysed industrial reactions In the Ostwald process, gaseous ammonia is oxidised over a solid platinum-rhodium catalyst. In the Haber process, gaseous nitrogen and hydrogen react over a solid iron catalyst. Hydrogenation of vegetable oils uses a solid catalyst such as nickel in contact with hydrogen gas and the oil. In each case the catalyst supplies a reaction surface and is in a phase different from at least the reacting fluid. Thus options (A), (C) and (D) are standard examples of heterogeneous catalytic reactions.

Step 2: Check whether coal acts as a catalyst Coal contains carbonaceous material which burns in oxygen. The simplified complete-combustion equation is

$$\mathrm{C(s) + O_2(g) \longrightarrow CO_2(g).}$$

Here carbon is consumed and becomes part of the gaseous product. Oxygen is also a reactant. No separate catalyst is specified in the option. Although solid coal and gaseous oxygen make this a heterogeneous reaction, that alone does not establish catalysis. The distinction is between different phases participating in a reaction and a separate catalyst accelerating that reaction. Combustion of coal therefore fails the catalytic requirement of the question.

Step 3: Conclusion Combustion of coal is not a heterogeneous catalytic reaction: coal is consumed as fuel rather than acting as a catalyst. Hence, the correct option is (B).

Quick Tip to Solve: For a catalysis question, identify the catalyst first. Different physical phases alone establish heterogeneity, but do not establish catalysis.

88. The chemical nature of hydrogen peroxide is : [4 marks]

(A) Oxidising agent in acidic medium, but not in basic medium

(B) Oxidising and reducing agent in both acidic and basic medium

(C) Reducing agent in basic medium, but not in acidic medium

(D) Oxidising and reducing agent in acidic medium, but not in basic medium.

Correct Answer: (B) Oxidising and reducing agent in both acidic and basic medium

Solution:

Concept:

- In H_2O_2, oxygen has oxidation number -1, intermediate between -2 in water or hydroxide and 0 in molecular oxygen.
- An oxidising agent accepts electrons and is reduced; a reducing agent donates electrons and is oxidised. Peroxide can undergo either change in acidic as well as alkaline solution.

Step 1: Exhibit both roles in acidic solution When hydrogen peroxide accepts electrons, its oxygen changes from -1 to -2. When it loses electrons, its oxygen changes from -1 to 0. The balanced acidic half-reactions are

$$H_2O_2 + 2\,H^+ + 2\,e^- \longrightarrow 2\,H_2O,$$

$$H_2O_2 \longrightarrow O_2 + 2\,H^+ + 2\,e^-.$$

The first shows peroxide acting as an oxidising agent; the second shows it acting as a reducing agent. Which role occurs depends on the other reactant, not merely on whether the solution is acidic.

Step 2: Exhibit both roles in basic solution Balance the corresponding half-reactions using water and hydroxide ions:

$$H_2O_2 + 2\,e^- \longrightarrow 2\,OH^-,$$

$$H_2O_2 + 2\,OH^- \longrightarrow O_2 + 2\,H_2O + 2\,e^-.$$

Both atoms and charges balance in each equation. The first again represents reduction of peroxide, while the second represents oxidation of peroxide. Thus alkalinity does not eliminate either possible redox role. Option (A) wrongly denies oxidising behaviour in base; option (C) wrongly restricts reducing behaviour to base; option (D) wrongly excludes both roles in base.

Step 3: Conclusion Hydrogen peroxide can act as both an oxidising agent and a reducing agent in both acidic and basic media. Hence, the correct option is (B).

Quick Tip to Solve: Remember oxygen at oxidation number -1 in peroxide: it can move down to -2 or up to 0, so check both possible redox roles before imposing a medium restriction.

89. Which premitive unit cell has unequal edge lengths (a $\neq$ b $\neq$ c) and all axial angles different from 90°? [4 marks]

(A) Hexagonal

(B) Tetragonal

(C) Triclinic

(D) Monoclinic

Correct Answer: (C) Triclinic

Solution:

Concept:

- A unit cell is described by edge lengths a, b, c and interaxial angles α, β, γ. Crystal systems are distinguished by the equalities and right-angle restrictions on these parameters.
- The triclinic system has unequal edge lengths and no required right angles: $a \neq b \neq c$ and $\alpha \neq \beta \neq \gamma \neq 90^\circ$ in the standard classification.

Step 1: Match the stated metric conditions The question specifies two independent features: all three edges have unequal lengths, and none of the three axial angles is a right angle. These are the characteristic metric conditions of the triclinic system. Its primitive lattice has lattice points at the corners of its unit cell, with no additional centring translation. The word primitive specifies this lattice-point arrangement; the edge and angle conditions identify the crystal system.

Step 2: Eliminate the other three crystal systems For a tetragonal cell, $a = b \neq c$ and $\alpha = \beta = \gamma = 90^\circ$, so both the equal-edge relation and the angles disagree. In the conventional hexagonal description, $a = b \neq c$, $\alpha = \beta = 90^\circ$ and $\gamma = 120^\circ$, so two right angles remain. A monoclinic cell has unequal edges, but two angles are right angles, usually written $\alpha = \gamma = 90^\circ$, with $\beta \neq 90^\circ$. Hence unequal edges alone would not distinguish monoclinic from triclinic. Requiring all angles to differ from a right angle removes monoclinic and leaves triclinic as the only match.

Step 3: Conclusion The primitive unit cell with unequal edge lengths and all axial angles different from 90° is triclinic. Hence, the correct option is (C).

Quick Tip to Solve: After checking unequal edges, count the right angles: monoclinic has two, orthorhombic has three, and triclinic has none in its standard metric description.

90. Water filled in two glasses A and B have BOD values of 10 and 20, respectively. The correct statement regarding them, is : [4 marks]

(A) B is more polluted than A.

(B) A is suitable for drinking, whereas B is not .

(C) Both A and B are suitable for dirnking

(D) A is more polluted than B.

Correct Answer: (A) B is more polluted than A.

Solution:

Concept:

- Biochemical oxygen demand (BOD) measures the oxygen required by microorganisms to decompose biodegradable organic matter in a water sample under specified test conditions.
- For samples tested under the same conditions, higher BOD indicates a greater load of biodegradable organic pollution. BOD is oxygen demand, not the concentration of oxygen already dissolved in the sample.

Step 1: Compare the two given oxygen demands The values for glasses A and B are 10 and 20, respectively. Their ratio is

$$\frac{\text{BOD}_B}{\text{BOD}_A} = \frac{20}{10} = 2.$$

Thus the sample in B imposes twice the biochemical oxygen demand of the sample in A under the same measurement conditions. Microbial decomposition of its biodegradable contaminants needs more oxygen. Greater consumption of oxygen can deplete the dissolved oxygen available to aquatic organisms, which is why a high BOD is an indicator of water pollution.

Step 2: Separate relative pollution from drinking-water safety Because $20 > 10$, B is more polluted with respect to biodegradable organic load. Option (D) reverses this comparison. Options (B) and (C) make claims about suitability for drinking that do not follow from a comparison of these two BOD values. A lower BOD than another sample does not by itself certify potable water: pathogens, toxic chemicals and other quality parameters must also be considered. Neither sample can be declared safe simply because its value is the smaller of the two. The supported conclusion is therefore the relative pollution statement in option (A), rather than either drinking-water claim.

Step 3: Conclusion Glass B has the higher BOD and hence the greater biodegradable organic pollution load. B is more polluted than A. Hence, the correct option is (A).

Quick Tip to Solve: Compare BOD directly for organic pollution: higher demand means more biodegradable contamination. Do not confuse a lower BOD with proof that a sample is safe to drink.