JEE Main 2019 Paper 1 (B.E./B.Tech) Jan 10 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 10 January 2019, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. Two vertices of a triangle are (0, 2) and (4, 3). If its orthocenter is at the origin, then its third vertex lies in which quadrant : [4 marks]

(A) third

(B) fourth

(C) second

(D) first

Correct Answer: (C) second

Solution:

Concept:

- The orthocentre is the common intersection of the three altitudes. An altitude through a vertex is perpendicular to the opposite side.
- For two perpendicular nonvertical lines the product of their slopes is -1. A vertical altitude instead makes its opposite side horizontal.

Step 1: Use the altitude through the first vertex Let $A = (0, 2)$, $B = (4, 3)$, $H = (0, 0)$, and let the third vertex be $C = (x, y)$. The line AH is the vertical line $x = 0$. Since it is an altitude, the opposite side BC must be horizontal. As B has ordinate 3, this gives $y = 3$. This step uses the given orthocentre rather than treating the origin as the centroid or circumcentre.

Step 2: Use the altitude through the second vertex The slope of BH is $3/4$. Therefore the side AC, perpendicular to it, has slope $-4/3$. Using $A = (0, 2)$ and $C = (x, 3)$, we obtain

$$\frac{3-2}{x-0} = -\frac{4}{3} \quad \Longrightarrow \quad x = -\frac{3}{4}.$$

Thus $C = (-3/4, 3)$. As a check, the slope of AB is $1/4$, while the slope of CH is -4, so the third altitude also passes through the origin.

Step 3: Identify the signs of the coordinates The abscissa of C is negative and its ordinate is positive. This is precisely the second quadrant. The third quadrant would require both coordinates negative, the fourth would require a positive abscissa and negative ordinate, and the first would require both positive; none matches the calculated vertex.

Step 4: Conclusion The third vertex is $(-3/4, 3)$, which lies in the second quadrant. Hence, the correct option is (C).

Quick Tip to Solve: When the orthocentre and one vertex lie on a vertical line, make the opposite side horizontal before using any slope formula.

2. If $\sum_{r=0}^{25}\left\{{}^{50}C_r.{}^{50-r}C_{25-r}\right\} = K\left({}^{50}C_{25}\right)$, **then K is equal to :** [4 marks]

(A) 2^{24}

(B) $2^{25}-1$

(C) 2^{25}

(D) $(25)^2$

Correct Answer: (C) 2^{25}

Solution:

Concept:

- Write combinations in factorial form to expose a factor independent of the summation index.
- The binomial theorem gives $\sum_{r=0}^{m}\binom{m}{r} = 2^m$; both endpoint terms must be included.

Step 1: Simplify a general term For each integer r from 0 to 25, the factorial definition yields

$$\binom{50}{r}\binom{50-r}{25-r} = \frac{50!}{r!(50-r)!}\frac{(50-r)!}{(25-r)!25!}.$$

Cancel the common factorial and regroup to get

$$\binom{50}{r}\binom{50-r}{25-r} = \frac{50!}{25!25!}\frac{25!}{r!(25-r)!} = \binom{50}{25}\binom{25}{r}.$$

All factorials here have nonnegative integer arguments over the given summation range.

Step 2: Sum using the binomial theorem The factor $\binom{50}{25}$ does not depend on r, so

$$\sum_{r=0}^{25}\binom{50}{r}\binom{50-r}{25-r} = \binom{50}{25}\sum_{r=0}^{25}\binom{25}{r} = \binom{50}{25}(1+1)^{25}.$$

Comparing this exact expression with the stated right-hand side $K\binom{50}{25}$, and cancelling the nonzero combination, determines $K = 2^{25}$.

Step 3: Check the offered alternatives The exponent is 25 because the remaining sum is the complete binomial sum of order 25. The value 2^{24} would halve that sum, and $2^{25}-1$ would incorrectly remove an endpoint term even though both endpoints are present. The value 25^2 is not the binomial sum and does not satisfy the derived identity.

Step 4: Conclusion $K = 2^{25}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $\binom{n}{r}\binom{n-r}{m-r} = \binom{n}{m}\binom{m}{r}$ to turn this product sum into a complete binomial expansion.

3. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ **is :** [4 marks]

(A) $\frac{22}{23}$

(B) $\frac{23}{22}$

(C) $\frac{21}{19}$

(D) $\frac{19}{21}$

Correct Answer: (C) $\frac{21}{19}$

Solution:

Concept:

- For positive arguments, $\cot^{-1}u=\tan^{-1}(1/u)$, taking the principal acute angle.
- The tangent subtraction identity can express an inverse tangent as a difference of consecutive inverse tangents, allowing a finite sum to telescope.

Step 1: Evaluate the inner sum Since $\sum_{p=1}^{n}p=n(n+1)/2$, the argument of the inverse cotangent is

$$1+\sum_{p=1}^{n}2p=n^2+n+1.$$

It is positive for every index in the problem. Consequently the term is the acute angle

$$\tan^{-1}\left(\frac{1}{n^2+n+1}\right).$$

Keeping the initial 1 is essential for the telescoping identity.

Step 2: Express each term as a difference For $n\geq 1$, let $a_n=\tan^{-1}(1/n)$. The difference a_n-a_{n+1} is positive and less than $\pi/2$. Its tangent is

$$\frac{1/n-1/(n+1)}{1+1/[n(n+1)]}=\frac{1}{n^2+n+1}.$$

The range just checked makes the inverse-tangent equality exact, without an extra multiple of π. Thus the required angle sum telescopes to $a_1-a_{20}=\pi/4-\tan^{-1}(1/20)$.

Step 3: Take the cotangent of the total Applying the tangent subtraction formula once more gives

$$\tan\left(\frac{\pi}{4}-\tan^{-1}\frac{1}{20}\right)=\frac{1-1/20}{1+1/20}=\frac{19}{21}.$$

The question asks for the cotangent, so the answer is the reciprocal, $21/19$. The value $19/21$ is the tangent rather than the cotangent. Ratios involving 22 and 23 do not arise because the last surviving inverse tangent has denominator 20.

Step 4: Conclusion The requested value is $\frac{21}{19}$. Hence, the correct option is (C).

Quick Tip to Solve: When an inverse-trigonometric argument contains n^2+n+1, test consecutive angles $\tan^{-1}(1/n)-\tan^{-1}(1/(n+1))$.

4. If $\overrightarrow{\alpha} = (\lambda - 2)\,\overrightarrow{a} + \overrightarrow{b}$ and $\overrightarrow{\beta} = (4\lambda - 2)\,\overrightarrow{a} + 3\,\overrightarrow{b}$ be two given vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ are non-collinear. The value of λ for which vectors $\overrightarrow{\alpha}$ and $\overrightarrow{\beta}$ are collinear, is - [4 marks]

(A) 4

(B) 3

(C) -3

(D) -4

Correct Answer: (D) -4

Solution:

Concept:

- Two nonzero vectors are collinear precisely when one is a scalar multiple of the other.
- Non-collinear vectors $\vec{a}, \vec{b}$ are linearly independent, so equal linear combinations have equal coefficients of each vector.

Step 1: Introduce the proportionality factor The given vectors are $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$. Since the coefficient of $\vec{b}$ in $\vec{\alpha}$ is 1, independence ensures that $\vec{\alpha} \neq \vec{0}$. If the vectors are collinear, write $\vec{\beta} = t\vec{\alpha}$ for a scalar t. Comparing the coefficients of $\vec{b}$ immediately gives $t = 3$.

Step 2: Compare the remaining coefficients The coefficient of $\vec{a}$ must obey the same proportionality factor. Therefore

$$4\lambda - 2 = 3(\lambda - 2) = 3\lambda - 6.$$

Subtracting 3λ and then adding 2 yields $\lambda = -4$. The non-collinearity of the original vectors is what permits this separate comparison of coefficients; comparing only the lengths of the new vectors would not establish collinearity.

Step 3: Verify the result and exclude the other values At $\lambda = -4$, we have $\vec{\alpha} = -6\vec{a} + \vec{b}$ and $\vec{\beta} = -18\vec{a} + 3\vec{b} = 3\vec{\alpha}$, as required. More generally the coefficient mismatch is $(4\lambda - 2) - 3(\lambda - 2) = \lambda + 4$. This is 8, 7 and 1 for the alternatives 4, 3 and -3, respectively, so none of those alternatives makes the two vectors collinear.

Step 4: Conclusion $\lambda = -4$, for which $\vec{\beta} = 3\vec{\alpha}$. Hence, the correct option is (D).

Quick Tip to Solve: For vectors expressed in two independent directions, first compare the coefficient that is constant to find the proportionality factor.

5. Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If R(z) and 1(z) respectively denote the real and imaginary parts of z, then : [4 marks]

(A) R(z) = − 3

(B) R(z) < 0 and I(z) > 0

(C) I(z) = 0

(D) R(z) > 0 and I(z) > 0

Correct Answer: (C) I(z) = 0

Solution:

Concept:

- Complex conjugation commutes with positive integer powers: $\overline{w^n} = (\overline{w})^n$. Hence the sum of conjugate powers is real.
- De Moivre theorem states $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$ for integer n.

Step 1: Identify the conjugate pair Let $w = \sqrt{3}/2 + i/2$. Its conjugate is $\overline{w} = \sqrt{3}/2 - i/2$, so the given expression is $z = w^5 + \overline{w}^5 = w^5 + \overline{w^5}$. The imaginary parts cancel exactly. This already proves $I(z) = 0$; to distinguish the other alternatives completely, we can also calculate its real part.

Step 2: Evaluate the fifth powers The modulus of w is $\sqrt{3/4 + 1/4} = 1$, and its argument is $\pi/6$. Thus

$$w = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}, \qquad \overline{w} = \cos\frac{\pi}{6} - i\sin\frac{\pi}{6}.$$

Raising both to the fifth power and adding gives

$$z = 2\cos\frac{5\pi}{6} = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}.$$

No imaginary term survives, consistently with the conjugate argument.

Step 3: Compare all statements The exact values are $R(z) = -\sqrt{3}$ and $I(z) = 0$. The first option incorrectly gives the real part as -3. The second has the correct negative sign for the real part but wrongly requires a positive imaginary part. The fourth wrongly makes both parts positive. Only the statement that the imaginary part vanishes agrees with the calculation.

Step 4: Conclusion $z = -\sqrt{3}$, so $I(z) = 0$. Hence, the correct option is (C).

Quick Tip to Solve: Before expanding conjugate powers, use their conjugacy: their sum has zero imaginary part, regardless of the integer exponent.

6. Let A = $\begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where b > 0.

Then the minimum value of $\frac{\det(A)}{b}$ is - [4 marks]

(A) $\sqrt{3}$

(B) $-2\sqrt{3}$

(C) $-\sqrt{3}$

(D) $2\sqrt{3}$

Correct Answer: (D) $2\sqrt{3}$

Solution:

Concept:

- A determinant can be evaluated by cofactor expansion along any row; the signs across the first row are $+, -, +$.
- For positive u, v, the arithmetic-geometric mean inequality gives $u + v \geq 2\sqrt{uv}$, with equality precisely when $u = v$.

Step 1: Expand the determinant Expanding the given matrix along the first row yields

$$\det A = 2\{2(b^2+1) - b^2\} - b(2b - b) + \{b^2 - (b^2+1)\}.$$

The three minors simplify respectively to $b^2 + 2$, b, and -1. Consequently

$$\det A = 2(b^2+2) - b^2 - 1 = b^2 + 3.$$

The middle cofactor carries a minus sign, so its contribution is $-b^2$, not $+b^2$.

Step 2: Minimise the required quotient Since $b > 0$, division by b is allowed and both terms in

$$\frac{\det A}{b} = b + \frac{3}{b}$$

are positive. The arithmetic-geometric mean inequality gives

$$b + \frac{3}{b} \geq 2\sqrt{b \cdot \frac{3}{b}} = 2\sqrt{3}.$$

Equality requires $b = 3/b$, equivalently $b^2 = 3$. The permitted solution $b = \sqrt{3}$ attains the bound, so it is an actual minimum rather than merely an infimum.

Step 3: Check the offered values The negative alternatives cannot be minima because the quotient is positive for every allowed b. The positive alternative $\sqrt{3}$ is below the proven lower bound $2\sqrt{3}$, so it is also impossible. Substituting $b = \sqrt{3}$ directly gives $\sqrt{3} + 3/\sqrt{3} = 2\sqrt{3}$, confirming the remaining alternative.

Step 4: Conclusion The minimum is $2\sqrt{3}$, attained at $b = \sqrt{3}$. Hence, the correct option is (D).

Quick Tip to Solve: After evaluating the determinant, look for a positive expression $b + c/b$; its minimum is $2\sqrt{c}$ at $b = \sqrt{c}$.

7. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is - [4 marks]

(A) 1

(B) 2

(C) $\frac{15}{8}$

(D) $\frac{4}{9}$

Correct Answer: (B) 2

Solution:

Concept:

- For a monic quadratic $x^2 + px + q = 0$ with roots α, β, Vieta relations give $\alpha + \beta = -p$ and $\alpha\beta = q$.
- The identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ avoids solving individually for the roots.

Step 1: Put the equation in standard form Move the right-hand side to the left:

$$x^2 + (3 - \lambda)x + (2 - \lambda) = 0.$$

Hence the sum and product of the roots are $\alpha + \beta = \lambda - 3$ and $\alpha\beta = 2 - \lambda$. It is essential to include the term $-\lambda$ in the constant coefficient; leaving the constant as 2 changes the optimisation problem.

Step 2: Write and minimise the sum of squares Using Vieta relations,

$$\begin{aligned} \alpha^2 + \beta^2 &= (\lambda - 3)^2 - 2(2 - \lambda) \\ &= \lambda^2 - 4\lambda + 5 \\ &= (\lambda - 2)^2 + 1. \end{aligned}$$

For real λ, the square is nonnegative, and it vanishes exactly at $\lambda = 2$. Therefore the least value of the sum of squares is 1, attained for that parameter. The question asks for the parameter, rather than this minimum value.

Step 3: Verify the roots and the alternatives At $\lambda = 2$, the equation becomes $x^2 + x = 0$, with real roots 0 and -1; their squared sum is indeed 1. Thus the minimiser is admissible even if real roots are required. The alternative $\lambda = 1$ gives a squared sum 2, while $15/8$ and $4/9$ both leave a strictly positive term $(\lambda - 2)^2$. Each therefore gives a larger sum than the value at $\lambda = 2$.

Step 4: Conclusion The required parameter is $\lambda = 2$; the corresponding least sum of squared roots is 1. Hence, the correct option is (B).

Quick Tip to Solve: Optimise the roots through their sum and product; complete the square in the parameter instead of using the quadratic formula.

8. Let f be a differentiable function such that f '(x) = 7 - $\frac{3}{4}\frac{f(x)}{x}$, (x > 0) and f(1) $\neq$ 4. Then $\lim_{x\to 0'}$ xf$\left(\frac{1}{x}\right)$: [4 marks]

(A) does not exist

(B) exists and equals $\frac{4}{7}$

(C) exists and equals 4

(D) exists and equals 0

Correct Answer: (C) exists and equals 4

Solution:

Concept:

- A first-order linear equation $f\prime(x) + P(x)f(x) = Q(x)$ is solved by multiplying by the integrating factor $e^{\int P(x)\,dx}$. Here $x > 0$, so fractional powers are real.
- The expression $xf(1/x)$ must be simplified after finding the complete family of solutions. A condition on $f(1)$ fixes whether the arbitrary constant can vanish, but need not change the limit.

Step 1: Find the integrating factor Rewrite the given relation as

$$f\prime(x) + \frac{3}{4x}f(x) = 7.$$

Its integrating factor is $x^{3/4}$. Multiplication converts the entire left side into a product derivative:

$$\frac{d}{dx}\{x^{3/4}f(x)\} = 7x^{3/4}.$$

This step uses the product rule, with derivative of the factor equal to $(3/4)x^{-1/4}$.

Step 2: Integrate and apply the stated condition Integration gives

$$x^{3/4}f(x) = 4x^{7/4} + C, \qquad f(x) = 4x + Cx^{-3/4}.$$

At $x = 1$, this gives $f(1) = 4 + C$. Thus the printed condition $f(1) \neq 4$ means $C \neq 0$; it does not remove the leading term $4x$.

Step 3: Substitute the reciprocal argument For positive x, substitute $1/x$ into the complete solution before multiplying:

$$xf(1/x) = x\{4/x + Cx^{3/4}\} = 4 + Cx^{7/4}.$$

As x approaches zero through positive values, the power term tends to zero for every finite constant C. Therefore the limit exists. The values zero and $4/7$ do not result from this expression, and the nonzero integration constant does not cause divergence.

Step 4: Conclusion The required limit exists and equals 4. Hence, the correct option is (C).

Quick Tip to Solve: For a reciprocal argument, first write the full solution and then multiply by the outside factor; the resulting power immediately decides the limit.

9. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0.$

has a non-trival solution, is - [4 marks]

(A) two

(B) one

(C) four

(D) three

Correct Answer: (A) two

Solution:

Concept:

- A homogeneous system has a nontrivial solution precisely when its coefficient matrix is singular. Equivalently, eliminate variables and require the remaining relation to allow a nonzero free parameter.
- For $0 < \theta < \pi$, $s = \sin\theta$ lies in $(0, 1]$. Use $\sin 3\theta = 3s - 4s^3$ and $\cos 2\theta = 1 - 2s^2$, then count angles rather than merely roots in s.

Step 1: Reduce the first two equations Adding the first two equations gives $7y + 14z = 0$, hence $y = -2z$. Substitution into the first gives $x = -z$. Consequently every possible solution is

$$(x, y, z) = (-z, -2z, z).$$

If $z = 0$, all three unknowns vanish. A nontrivial solution therefore requires a nonzero z.

Step 2: Find the trigonometric condition Insert these expressions into the third equation and divide by the nonzero parameter:

$$-\sin 3\theta - 2\cos 2\theta + 2 = 0.$$

Writing $s = \sin\theta$, the condition becomes

$$3s - 4s^3 + 2(1 - 2s^2) = 2,$$

$$s(4s^2 + 4s - 3) = 0.$$

The endpoint root $s = 0$ is excluded because the interval is open.

Step 3: Count the permitted angles The quadratic factors as $(2s - 1)(2s + 3) = 0$. Its roots are $1/2$ and $-3/2$, of which only $1/2$ is an attainable positive sine. Thus

$$\theta = \frac{\pi}{6}, \quad \frac{5\pi}{6}.$$

Each angle permits any nonzero value of z, so both genuinely produce nontrivial solutions. The negative root supplies no angle, while zero would supply only excluded endpoints. Counts of one, three or four therefore do not fit the specified interval.

Step 4: Conclusion There are exactly two values of θ. Hence, the correct option is (A).

Quick Tip to Solve: Eliminate two variables before expanding a determinant, and remember that one admissible positive sine usually gives two angles in $(0, \pi)$.

10. If $\int x^5.e^{-4x^3}\,dx = \frac{1}{48}e^{-4x^3}\,f(x) + C$, where C is a constant of inegration, then f(x) is equal to - [4 marks]

(A) $-2x^3 - 1$

(B) $-2x^3 + 1$

(C) $4x^3 + 1$

(D) $-4x^3 - 1$

Correct Answer: (D) $-4x^3 - 1$

Solution:

Concept:

- When an exponential contains x^3, set $u = x^3$; the factor $x^2\,dx$ becomes $du/3$. The remaining power x^3 then becomes u.
- The integral of a polynomial times an exponential follows from integration by parts. Differentiate the final antiderivative as an independent check of its sign and numerical coefficient.

Step 1: Transform the integral Set $u = x^3$, so $du = 3x^2\,dx$ and $x^5\,dx = u\,du/3$. Hence

$$I = \int x^5 e^{-4x^3}\,dx = \frac{1}{3}\int ue^{-4u}\,du.$$

The factor $1/3$ is essential; overlooking it changes the coefficient used to identify $f(x)$.

Step 2: Integrate by parts Take the algebraic factor as the first function. Since an antiderivative of e^{-4u} is $-e^{-4u}/4$,

$$\begin{aligned} I &= \frac{1}{3}\left(-\frac{u}{4}e^{-4u} + \frac{1}{4}\int e^{-4u}\,du\right) \\ &= -\frac{u}{12}e^{-4u} - \frac{1}{48}e^{-4u} + C \\ &= \frac{1}{48}e^{-4x^3}(-4x^3 - 1) + C. \end{aligned}$$

Both terms have negative signs because the exponential has a negative coefficient in its exponent.

Step 3: Verify the polynomial For $f(x) = -4x^3 - 1$, differentiating the proposed antiderivative gives

$$\frac{e^{-4x^3}}{48}\{-12x^2 - 12x^2(-4x^3 - 1)\} = x^5 e^{-4x^3}.$$

The x^2 terms cancel exactly. The alternatives with $-2x^3$ have the wrong leading coefficient. The alternative $4x^3 + 1$ is the negative of the required polynomial and would differentiate to the negative integrand.

Step 4: Conclusion We obtain $f(x) = -4x^3 - 1$. Hence, the correct option is (D).

Quick Tip to Solve: After substitution, factor out the exact prefactor printed in the question; a quick product-rule differentiation checks both polynomial terms.

11. If the area of an equilateral triangle inscribed in the circle $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq units then c is equal to : [4 marks]

(A) 20

(B) 25

(C) -25

(D) 13

Correct Answer: (B) 25

Solution:

Concept:

- An equilateral triangle of side a has area $\sqrt{3}a^2/4$. Its circumradius is $R = a/\sqrt{3}$, since the circumcentre lies two-thirds of the way along an altitude from a vertex.
- Completing squares in $x^2 + y^2 + 2gx + 2fy + c = 0$ gives centre $(-g, -f)$ and radius squared $g^2 + f^2 - c$. An inscribed triangle has this circle as its circumcircle.

Step 1: Recover the side from the area Let the side of the equilateral triangle be $a > 0$. Equating its area to the given value gives

$$\frac{\sqrt{3}}{4}a^2 = 27\sqrt{3}, \qquad a^2 = 108.$$

Thus $a = 6\sqrt{3}$. The positive root is used because a side length cannot be negative.

Step 2: Find the radius of the circle The altitude of an equilateral triangle is $h = \sqrt{3}a/2$. Its circumcentre and centroid coincide, so the radius is $2h/3 = a/\sqrt{3}$. Therefore

$$R^2 = \frac{a^2}{3} = 36.$$

One must use the circumradius here, rather than the inradius: the vertices lie on the circle, whereas the sides would be tangent to an incircle.

Step 3: Determine and check the constant Complete the squares in the circle equation:

$$(x+5)^2 + (y+6)^2 = 25 + 36 - c = 61 - c.$$

Matching this with $R^2 = 36$ gives $61 - c = 36$, and hence $c = 25$. For comparison, the other choices $20, -25, 13$ give radius squared $41, 86, 48$, respectively. None equals 36, so none can support an equilateral triangle of the stated area.

Step 4: Conclusion The required constant is $c = 25$. Hence, the correct option is (B).

Quick Tip to Solve: For an equilateral triangle inscribed in a circle, combine the area formula directly with $a^2 = 3R^2$, then compare radius squared after completing squares.

12. Let f : (−1, 1) → R be a function defined by f(x) = max $\left\{-|x|, -\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly - [4 marks]

(A) one element

(B) three elements

(C) five elements

(D) two elements

Correct Answer: (B) three elements

Solution:

Concept:

- The maximum of two smooth expressions follows whichever has the greater value on each interval. Potential corners occur at switching points and at any nonsmooth point of an active expression.
- Here $-|x|$ has a corner at zero, while $-\sqrt{1-x^2}$ is smooth throughout $(-1, 1)$. Endpoints ± 1 are outside the stated domain and are not counted.

Step 1: Locate where the branches switch Equality of the two candidates means $|x| = \sqrt{1-x^2}$. Both sides are nonnegative, so squaring is valid and gives

$$x^2 = 1 - x^2, \qquad x = \pm\frac{1}{\sqrt{2}}.$$

For $|x| < 1/\sqrt{2}$, the smaller positive magnitude is $|x|$, so its negative is the larger candidate. Outside this central interval the square-root branch is larger.

Step 2: Examine zero and the positive switch Thus the active middle branch is $-|x|$. At zero its left derivative is 1, whereas its right derivative is -1, so differentiability fails. At $x = 1/\sqrt{2}$, the derivative from the central interval is -1. The outer branch has derivative

$$\frac{d}{dx}(-\sqrt{1-x^2}) = \frac{x}{\sqrt{1-x^2}},$$

which equals 1 at that switch. These unequal derivatives produce another corner.

Step 3: Examine the negative switch and completeness At $x = -1/\sqrt{2}$, the outer derivative is -1, while the derivative of the central branch x is 1. This is a third corner. Away from these three points each active formula is differentiable, so no further failures occur. Counting only switching points would give two and miss the absolute-value corner; counting only zero would miss both switches. There are no additional interior singularities to give five.

Step 4: Conclusion $K = \{-1/\sqrt{2}, 0, 1/\sqrt{2}\}$, which has three elements. Hence, the correct option is (B).

Quick Tip to Solve: For a maximum, inspect both branch crossings and corners of the active branches; an inactive branch singularity does not create a corner of the maximum.

13. A helicopter is flying along the curve given by $y - x^{3/2} = 7$, $(x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is - [4 marks]

(A) $\frac{1}{6}\sqrt{\frac{7}{3}}$

(B) $\frac{\sqrt{5}}{6}$

(C) $\frac{1}{2}$

(D) $\frac{1}{3}\sqrt{\frac{7}{3}}$

Correct Answer: (A) $\frac{1}{6}\sqrt{\frac{7}{3}}$

Solution:

Concept:

- A point on the helicopter path can be written as $(x, 7 + x^{3/2})$, with $x \geq 0$. The distance formula measures its separation from the fixed point $(1/2, 7)$.
- Minimise the square of the distance instead of its square root: the square-root function is increasing for nonnegative arguments. Check the boundary as well as stationary points on a restricted domain.

Step 1: Form the squared distance Subtract the coordinates of the soldier from those of a general point on the curve. The vertical separation is $x^{3/2}$, giving

$$D^2 = \left(x - \frac{1}{2}\right)^2 + x^3 =: F(x), \qquad x \geq 0.$$

This is the straight-line distance, so both horizontal and vertical contributions must be retained.

Step 2: Find the global minimum Differentiate the squared distance:

$$F\prime(x) = 2x - 1 + 3x^2 = (3x - 1)(x + 1).$$

The roots are $1/3$ and -1, but the latter lies outside the path domain. Also $F\prime\prime(x) = 2 + 6x > 0$ for $x \geq 0$. Hence $x = 1/3$ is the unique minimum; the function decreases to it and increases thereafter. The boundary gives $F(0) = 1/4$, confirming that the endpoint is farther away.

Step 3: Evaluate the nearest distance At $x = 1/3$,

$$F(1/3) = \left(-\frac{1}{6}\right)^2 + \frac{1}{27} = \frac{7}{108}.$$

Taking the positive square root gives

$$D_{\min} = \sqrt{\frac{7}{108}} = \frac{1}{6}\sqrt{\frac{7}{3}}.$$

The option $1/2$ is the distance at the endpoint, not the minimum. The other proposed radicals have squares $5/36$ and $7/27$, both larger than $7/108$, so they do not equal the stationary minimum.

Step 4: Conclusion The nearest distance is $\frac{1}{6}\sqrt{7/3}$ units. Hence, the correct option is (A).

Quick Tip to Solve: Minimise the squared distance and factor its derivative; use the domain to discard extraneous stationary points before comparing with the endpoint.

14. If $\int_0^x f(t)\,dt = x^2 + \int_x^1 t^2 f(t)\,dt$ then $f'\left(\frac{1}{2}\right)$ is - [4 marks]

(A) $\frac{18}{25}$

(B) $\frac{6}{25}$

(C) $\frac{24}{25}$

(D) $\frac{4}{5}$

Correct Answer: (C) $\frac{24}{25}$

Solution:

Concept:

- The fundamental theorem of calculus differentiates an integral with variable upper limit as the integrand at that limit. A variable lower limit introduces a minus sign.
- After obtaining $f(x)$ from the first derivative of the identity, use the quotient rule to find $f\prime(x)$. The requested derivative is different from the value of the function itself.

Step 1: Differentiate the given identity Differentiate both sides with respect to x. On the right, the lower limit of the integral is x, while its upper limit is the constant 1. Therefore

$$f(x) = 2x - x^2 f(x).$$

The minus sign comes from the moving lower limit: increasing that limit removes a short piece of the area. Moving terms to the left gives

$$(1 + x^2) f(x) = 2x.$$

Step 2: Find the derivative of the function Since $1 + x^2$ never vanishes for real x, division yields

$$f(x) = \frac{2x}{1 + x^2}.$$

Apply the quotient rule carefully:

$$f\prime(x) = \frac{2(1 + x^2) - 2x(2x)}{(1 + x^2)^2} = \frac{2(1 - x^2)}{(1 + x^2)^2}.$$

Both numerator terms must be included; differentiating only $2x$ would miss the variation of the denominator.

Step 3: Substitute the requested point At $x = 1/2$, the numerator is $2(1 - 1/4) = 3/2$, and the denominator is $(1 + 1/4)^2 = 25/16$. Thus

$$f\prime(1/2) = \frac{3/2}{25/16} = \frac{24}{25}.$$

The value $4/5$ is instead $f(1/2)$, so it answers the wrong quantity. The fractions $18/25$ and $6/25$ also fail the direct quotient-rule substitution. Any differentiable function satisfying the displayed identity must satisfy this derivative relation.

Step 4: Conclusion The requested derivative is $f\prime(1/2) = 24/25$. Hence, the correct option is (C).

Quick Tip to Solve: For variable-limit integrals, write the lower-limit minus sign first; after solving for the function, check whether the question asks for it or its derivative.

15. Let N be the set of natural numbers and two functions f and g be defined as f, g : N $\to$ N such that

f(n) = $\begin{cases} \frac{n+1}{2}; & if\ n\ is\ odd \\ \frac{n}{2}; & if\ n\ is\ even \end{cases}$ **;**

and g(n) = n $-(-1)^n$.

Then fog is - [4 marks]

(A) neither one-one nor onto

(B) onto but not one-one

(C) both one-one and onto

(D) one-one but not onto

Correct Answer: (B) onto but not one-one

Solution:

Concept:

- For functions on the natural numbers, $(f \circ g)(n) = f(g(n))$: apply g before applying f.
- A function is one-one if distinct inputs have distinct images. It is onto if every element of its stated codomain has a preimage.

Step 1: Evaluate the composition on consecutive inputs Write an odd input as $n = 2m - 1$, with $m \in \mathbb{N}$. Since $(-1)^n = -1$, the inner function gives an even output. Hence

$$g(2m-1) = 2m, \qquad f(g(2m-1)) = f(2m) = m.$$

For the even input $n = 2m$, the inner function gives an odd output:

$$g(2m) = 2m - 1, \qquad f(g(2m)) = f(2m-1) = m.$$

Thus each adjacent pair $2m-1, 2m$ has the same image, even though g interchanges the two members of the pair.

Step 2: Test injectivity and surjectivity separately The distinct inputs 1 and 2 both have image 1, so the composition is not one-one. More generally, the calculation gives two preimages for each positive integer m. Conversely, given any target $m \in \mathbb{N}$, choose $n = 2m$; then $(f \circ g)(n) = m$. This proves that the image is all of the codomain, so the composition is onto. Option (A) incorrectly denies surjectivity, while options (C) and (D) incorrectly assert injectivity. The two properties must be tested against the specified codomain, rather than inferred from the fact that g itself swaps inputs.

Step 3: Conclusion The composition maps both $2m - 1$ and $2m$ to m. It is onto but not one-one. Hence, the correct option is (B).

Quick Tip to Solve: Pair the inputs as $(1, 2), (3, 4), \ldots$; if each pair maps to its pair number, surjectivity is immediate and injectivity fails.

16. Let S = $\left\{(x,y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r}\right\}; r \neq \pm 1$. Then S represents : [4 marks]

(A) an ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, where r > 1

(B) an ellipse whose eccentricity is $\frac{2}{\sqrt{r+1}}$, where 0 < r < 1

(C) an ellipse whose eccentricity is $\frac{2}{\sqrt{r-1}}$, where 0 < r < 1

(D) an ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, where r > 1

Correct Answer: The displayed set definition contains no equation or inequality, so it does not determine a locus.

Solution:

Concept:

- A locus requires a condition on (x, y), such as an equality or inequality. An expression alone does not specify which points belong to a set.
- For the ellipse $x^2/b^2 + y^2/a^2 = 1$, with $a > b > 0$, its eccentricity is $e = \sqrt{1 - b^2/a^2}$.

Step 1: Check the displayed definition literally The displayed set contains the expression $y^2/(1+r) - x^2/(1-r)$ after the colon, followed by the closing brace. There is no equality to a number, inequality or other condition on the coordinates. The restriction $r \neq \pm 1$ prevents zero denominators, but does not supply the missing condition on x, y. Therefore the displayed expression does not define an ellipse, or indeed a determinate locus, as written. A unique eccentricity cannot be deduced from that incomplete definition.

Step 2: Distinguish the conditional ellipse calculation If the additional condition were

$$\frac{y^2}{1+r} - \frac{x^2}{1-r} = 1,$$

then for $r > 1$ it would become

$$\frac{x^2}{r-1} + \frac{y^2}{r+1} = 1.$$

Under that extra condition the squared semiaxes would be $a^2 = r + 1$ and $b^2 = r - 1$, and thus

$$e^2 = 1 - \frac{r-1}{r+1} = \frac{2}{r+1}.$$

That conditional result agrees with option (D), whereas (A) misses a factor $\sqrt{2}$. For $0 < r < 1$, the same added equation has opposite signs and defines a hyperbola, so (B) and (C) would not describe an ellipse. This calculation identifies the conventional ellipse result without treating the unprinted equation as given.

Step 3: Conclusion The question as displayed is incomplete and has no uniquely defensible option. The supplied key selects (D), $e = \sqrt{2/(r+1)}$ for $r > 1$; that result follows only if the additional equation with right-hand side 1 is supplied. The literal displayed definition does not state it.

Quick Tip to Solve: Before classifying a conic, check that a complete equation is stated; signs and denominators alone do not define its locus.

17. Let $a_1, a_2, a_3, \ldots\ldots a_{10}$ be in G.P. with $a_i > 0$ for i = 1, 2, ..., 10 and S be the set of pairs (r, k), r, k $\in$ N (the set of natural numbers) for which

$$\begin{vmatrix} \log_e a_1{}^r a_2{}^k & \log_e a_2{}^r a_3{}^k & \log_e a_3{}^r a_4{}^k \\ \log_e a_4{}^r a_5{}^k & \log_e a_5{}^r a_6{}^k & \log_e a_6{}^r a_7{}^k \\ \log_e a_7{}^r a_8{}^k & \log_e a_8{}^r a_9{}^k & \log_e a_9{}^r a_{10}{}^k \end{vmatrix} = \mathbf{0}.$$

Then the number of elements in S, is - [4 marks]

(A) 10

(B) 4

(C) 2

(D) infinitely many

Correct Answer: (D) infinitely many

Solution:

Concept:

- If $a_j = Aq^{j-1} > 0$ form a geometric progression, then their logarithms form an arithmetic progression: $\log a_j = \log A + (j-1)\log q$.
- A determinant is zero when its columns are linearly dependent. In particular, $C_1 - 2C_2 + C_3 = 0$ makes a three-column determinant vanish.

Step 1: Express every logarithm as a linear sequence Put $L = \log_e A$ and $d = \log_e q$. Positivity of all the terms gives $A > 0$, $q > 0$, so these logarithms are defined. For any natural numbers r, k, an entry beginning with a_j is

$$\begin{aligned} b_j &= \log_e(a_j^r a_{j+1}^k) \\ &= r\log_e a_j + k\log_e a_{j+1}. \end{aligned}$$

Substitution gives

$$b_j = (r+k)L + \big((r+k)(j-1) + k\big)d.$$

This is affine in j. Therefore the successive differences satisfy $b_{j+1} - b_j = (r+k)d$, independently of j. This includes the constant-progression case $q = 1$, where $d = 0$.

Step 2: Use column dependence to count the pairs The displayed determinant has rows (b_1, b_2, b_3), (b_4, b_5, b_6) and (b_7, b_8, b_9). Each row consists of three consecutive terms of the same arithmetic progression. Its first entry minus twice its second plus its third is therefore zero. Consequently

$$C_1 - 2C_2 + C_3 = 0,$$

so the determinant vanishes for every $(r, k) \in \mathbb{N}^2$, without an additional constraint on either exponent. The set contains, for example, every pair $(r, 1)$ as r ranges through the natural numbers. Hence none of the finite counts 10, 4 or 2 can be correct.

Step 3: Conclusion Every natural-number pair (r, k) satisfies the determinant condition, so $S = \mathbb{N}^2$ has infinitely many elements. Hence, the correct option is (D).

Quick Tip to Solve: For determinants made from logarithms of a geometric progression, first turn products into sums and look for equal column differences.

18. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations x_1, x_2, ..., x_5 and –50 is equal to [4 marks]

(A) 582.5

(B) 507.5

(C) 586.5

(D) 509.5

Correct Answer: (B) 507.5

Solution:

Concept:

- For n observations, $\sigma^2 = \frac{1}{n}\sum x_i^2 - \bar{x}^2$, where $\bar{x}$ is their mean. The variance is the square of the standard deviation.
- When an observation is appended, update both the sum and the sum of squares before computing the new mean and variance.

Step 1: Recover the original sums The five observations have mean 10, so their sum is

$$\sum_{i=1}^{5} x_i = 5(10) = 50.$$

Their standard deviation is 3, and hence their variance is $3^2 = 9$, not 3. Rearranging the population-variance formula gives

$$\sum_{i=1}^{5} x_i^2 = 5(9 + 10^2) = 5(109) = 545.$$

Thus the two pieces of information needed for the enlarged data set are fixed even though the individual observations are unknown. There is no need to construct a particular set of five numbers.

Step 2: Append the sixth value and calculate the variance The sixth observation is -50. The new sum is $50 - 50 = 0$, so the mean of all six observations is zero. Its contribution to the sum of squares is positive:

$$(-50)^2 = 2500.$$

Therefore

$$\sum_{i=1}^{6} x_i^2 = 545 + 2500 = 3045,$$

and the required variance is

$$\sigma_{\text{new}}^2 = \frac{3045}{6} - 0^2 = 507.5.$$

The original mean 10 cannot be retained after adding the large negative observation. The value 509.5, and the larger alternatives 582.5 and 586.5, do not equal the second moment about the new mean. Dividing by 5 would give a sample-variance expression, whereas this question uses the variance of the given observations.

Step 3: Conclusion The variance of the six observations is 507.5. Hence, the correct option is (B).

Quick Tip to Solve: An appended value equal to the negative of the old sum makes the new mean zero, so the new variance is simply the mean of the squares.

19. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is - [4 marks]

(A) $8\sqrt{2}$

(B) $6\sqrt{3}$

(C) $3\sqrt{2}$

(D) $2\sqrt{11}$

Correct Answer: (B) $6\sqrt{3}$

Solution:

Concept:

- The endpoints of a chord are the common solutions of the line equation and the parabola equation.
- If a line has slope m, the distance between two points on it is $|\Delta x|\sqrt{1+m^2}$.

Step 1: Find the intersection coordinates The chord line is $x - \sqrt{2}\,y + 4\sqrt{2} = 0$. Solving for y gives

$$y = \frac{x}{\sqrt{2}} + 4.$$

Insert this in the parabola $x^2 = 4y$:

$$x^2 = 2\sqrt{2}\,x + 16, \qquad x^2 - 2\sqrt{2}\,x - 16 = 0.$$

The quadratic factors as

$$(x - 4\sqrt{2})(x + 2\sqrt{2}) = 0.$$

Hence the two endpoint abscissae are $4\sqrt{2}$ and $-2\sqrt{2}$. Substitution into the line gives the corresponding ordinates 8 and 2. Both points satisfy $x^2 = 4y$, which independently checks the intersection calculation.

Step 2: Apply the distance formula and compare the options The endpoints are therefore $P = (4\sqrt{2}, 8)$ and $Q = (-2\sqrt{2}, 2)$. Their coordinate differences are $6\sqrt{2}$ and 6. The chord length is

$$PQ = \sqrt{(6\sqrt{2})^2 + 6^2} = \sqrt{72 + 36} = 6\sqrt{3}.$$

Equivalently, the line slope is $1/\sqrt{2}$, and $|\Delta x|\sqrt{1+m^2} = 6\sqrt{2}\sqrt{3/2} = 6\sqrt{3}$. A coordinate difference alone is not a chord length on this oblique line. Squaring the candidate lengths gives 128, 108, 18 and 44 respectively; the computed squared distance is 108, so only the second candidate matches.

Step 3: Conclusion The length of the chord is $6\sqrt{3}$ units. Hence, the correct option is (B).

Quick Tip to Solve: After solving the intersection quadratic, use the line slope to turn the root separation into the chord length.

20. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+[\sin x]+4}$, where [t] denotes the greatest integer less than or equal to t, is

[4 marks]

(A) $\frac{1}{12}(7\pi - 5)$
(B) $\frac{1}{12}(7\pi + 5)$
(C) $\frac{3}{10}(4\pi - 3)$
(D) $\frac{3}{20}(4\pi - 3)$

Correct Answer: (D) $\frac{3}{20}(4\pi - 3)$

Solution:

Concept:

- The greatest integer function $[t] = \lfloor t \rfloor$ is constant between consecutive integers. Negative values must be rounded downward, rather than toward zero.
- A bounded function may change at finitely many individual points without changing its definite integral. Integrate the constant values on the open subintervals.

Step 1: Locate all changes in the denominator On $(-\pi/2, 0)$, the sine is negative and $[\sin x] = -1$, except that its endpoint value is also -1. On $(0, \pi/2)$, the sine lies strictly between 0 and 1, so $[\sin x] = 0$. The factor $[x]$ changes at $-1, 0, 1$. Thus the denominator $[x] + [\sin x] + 4$ has values 1, 2, 4 and 5 on

$$(-\pi/2, -1), \quad (-1, 0), \quad (0, 1), \quad (1, \pi/2),$$

respectively. Its values at the breakpoints, including $x = \pi/2$, cannot affect the integral. In particular, the negative-sine floor is not zero.

Step 2: Integrate the four constants The interval lengths are $\pi/2 - 1, 1, 1, \pi/2 - 1$. Therefore

$$I = \left(\frac{\pi}{2} - 1\right) + \frac{1}{2} + \frac{1}{4} + \frac{1}{5}\left(\frac{\pi}{2} - 1\right).$$

Collect the coefficients of π and the constants separately:

$$I = \frac{3\pi}{5} - 1 + \frac{3}{4} - \frac{1}{5} = \frac{3\pi}{5} - \frac{9}{20}.$$

Thus $I = 3(4\pi - 3)/20$. Option (C) is twice this value, while (A) and (B) have the wrong coefficient of π. The integrand is not even, because the greatest integer functions take different values on the positive and negative halves, so doubling one half would not be valid.

Step 3: Conclusion The integral equals $\frac{3}{20}(4\pi - 3)$. Hence, the correct option is (D).

Quick Tip to Solve: For floor-function integrals, make a breakpoint table first; negative arguments are the most common source of an incorrect denominator.

21. The value of $\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}.....\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$ **is -** [4 marks]

(A) $\frac{1}{256}$
(B) $\frac{1}{2}$
(C) $\frac{1}{1024}$
(D) $\frac{1}{512}$

Correct Answer: (D) $\frac{1}{512}$

Solution:

Concept:

- The identity $2\sin t\cos t = \sin(2t)$ combines a sine with a cosine of the same angle.
- Repeated doubling telescopes a product of cosines whose angles are successive halves of one another. Each combination contributes one factor of $1/2$.

Step 1: Begin with the smallest angle Let $\theta = \pi/2^{10}$. The product in the question is

$$P = \sin\theta\cos\theta\cos(2\theta)\cdots\cos(2^8\theta).$$

The largest cosine angle is $2^8\theta = \pi/4$, as required by the first printed factor. Combining the sine and first cosine gives

$$\sin\theta\cos\theta = \frac{1}{2}\sin(2\theta).$$

Combining the resulting sine with $\cos(2\theta)$ gives $\frac{1}{4}\sin(4\theta)$. The same operation continues because each remaining cosine has precisely the new sine angle.

Step 2: Count the doublings and evaluate the last sine There are nine cosine factors, indexed by the denominator exponents $2, 3, \ldots, 10$. After using the double-angle formula nine times, the product is

$$P = \frac{1}{2^9}\sin(2^9\theta) = \frac{1}{512}\sin\frac{\pi}{2} = \frac{1}{512}.$$

All factors in the original product are positive, consistent with this result. Option (A), $1/256$, would use only eight halving factors; option (C), $1/1024$, would use ten. Option (B), $1/2$, would stop after one use and omit the rest of the cosines. Counting the factors before evaluating avoids these errors and makes explicit why the final angle is $\pi/2$, rather than π.

Step 3: Conclusion The value of the given product is $\frac{1}{512}$. Hence, the correct option is (D).

Quick Tip to Solve: Count the cosine factors before telescoping: exponents 2 through 10 give nine factors, hence a denominator 2^9.

22. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is - [4 marks]

(A) 4

(B) $2\sqrt{2}$

(C) 3

(D) $\sqrt{5}$

Correct Answer: (A) 4

Solution:

Concept:

- For a binomial raised to a positive integer power, the general term is $T_{r+1}=\binom{n}{r}a^{n-r}b^r$, where $0\le r\le n$.
- The exponent of x must include the outside factor x^2. Terms contribute to the requested coefficient only when their total exponent is exactly 2.

Step 1: Write the general term Taking $a=\sqrt{x}=x^{1/2}$, $b=\lambda x^{-2}$, and including the prefactor gives

$$x^2T_{r+1}=\binom{10}{r}\lambda^r x^{2+(10-r)/2-2r}=\binom{10}{r}\lambda^r x^{7-5r/2}.$$

Here r counts the number of selections of the second term. The negative exponent in that term must be multiplied by r; treating it as a constant would select the wrong binomial coefficient.

Step 2: Select the required power and coefficient For the power x^2, equate the exponent to 2:

$$7-\frac{5r}{2}=2 \quad\Longrightarrow\quad r=2.$$

This is an allowed integer index, so exactly this one term contributes. Its coefficient is

$$\binom{10}{2}\lambda^2=45\lambda^2=720.$$

Thus $\lambda^2=16$, and the positive root is $\lambda=4$. The negative root is excluded by the stated condition.

Step 3: Check the alternatives The alternatives $2\sqrt{2}$, 3, and $\sqrt{5}$ have squares 8, 9, and 5, respectively. They give coefficients 360, 405, and 225, so none equals 720. For $\lambda=4$, the coefficient is $45\times16=720$, as required.

Step 4: Conclusion The positive value is $\boxed{\lambda=4}$. Hence, the correct option is (A).

Quick Tip to Solve: Include every outside power of x before matching the exponent in the binomial general term.

23. Two sides of a parallelogram are along the lines, x + y = 3 & x – y + 3 = 0. If its diagonals intersect at (2, 4), then one of its vertex is : [4 marks]

(A) (2, 1)

(B) (2, 6)

(C) (3, 5)

(D) (3, 6)

Correct Answer: (D) (3, 6)

Solution:

Concept:

- The diagonals of a parallelogram bisect each other. If their intersection is O, opposite vertices A, C satisfy $C = 2O - A$.
- Intersecting lines containing two adjacent sides meet at a vertex; the opposite sides are parallel to the corresponding given lines.

Step 1: Find a vertex and its opposite The given side lines are $x + y = 3$ and $x - y = -3$. Adding them gives $2x = 0$, so their intersection is $A = (0, 3)$. Since $O = (2, 4)$ is the midpoint of the diagonal through A,

$$C = (2 \times 2 - 0,\ 2 \times 4 - 3) = (4, 5).$$

This uses the diagonal property, rather than assuming the intersection of diagonals lies on a side.

Step 2: Construct the remaining two vertices The line through C parallel to $x + y = 3$ is $x + y = 9$. The line through C parallel to $x - y = -3$ is $x - y = -1$. Intersect each of these with the other original side:

$$x + y = 3, \quad x - y = -1 \implies (x, y) = (1, 2),$$
$$x + y = 9, \quad x - y = -3 \implies (x, y) = (3, 6).$$

The four vertices therefore are $(0, 3), (1, 2), (4, 5), (3, 6)$, in cyclic order.

Step 3: Verify the listed point Of the supplied choices, only $(3, 6)$ occurs in this vertex list. As a check, the midpoint of the other diagonal is $((1 + 3)/2, (2 + 6)/2) = (2, 4)$, agreeing with the given intersection. The points $(2, 1), (2, 6), (3, 5)$ are absent from the complete list and hence cannot be vertices of this parallelogram.

Step 4: Conclusion One vertex is $\boxed{(3, 6)}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the intersection of the two side lines first, then reflect that vertex across the diagonal midpoint.

24. The curve amongst the family of curves represented by the differential equation, $(x^2 - y^2)dx + 2xy\,dy = 0$ which passes through (1, 1) is : [4 marks]

(A) a circle with centre on the y-axis

(B) an ellipse with major axis along the y-axis

(C) a circle with centre on the x-axis

(D) a hyperbola with transverse axis along the x-axis

Correct Answer: (C) a circle with centre on the x-axis

Solution:

Concept:

- When a differential equation contains y^2 and $2y\,dy/dx$, the substitution $z = y^2$ converts it into a linear equation.
- A linear equation $z\prime + P(x)z = Q(x)$ has integrating factor $\exp(\int P(x)\,dx)$. Completing squares identifies the resulting conic.

Step 1: Reduce the equation to linear form Near the given point $(1, 1)$, x is nonzero. Divide the equation by dx and use $z = y^2$, so $z\prime = 2y\,dy/dx$:

$$x^2 - y^2 + 2xy\frac{dy}{dx} = 0 \quad \Longrightarrow \quad xz\prime - z = -x^2.$$

Dividing by x gives $z\prime - z/x = -x$. On an interval containing x=1 the integrating factor can be chosen as $1/x$.

Step 2: Integrate and impose the point Multiplication by $1/x$ makes the left-hand side an exact derivative:

$$\frac{d}{dx}\left(\frac{z}{x}\right) = -1.$$

Integration gives $z/x = -x + C$, or $x^2 + y^2 = Cx$. Substitution of $(1, 1)$ yields $1 + 1 = C$, so the required member is $x^2 + y^2 = 2x$. No arbitrary constant remains once the specified point is used.

Step 3: Identify the curve Complete the square:

$$(x - 1)^2 + y^2 = 1.$$

This is a circle of radius 1 centred at $(1, 0)$, which lies on the x-axis. The equal positive coefficients of the squared terms rule out a hyperbola and an ellipse with a distinct major axis. Its centre is not on the y-axis. Differentiating $x^2 + y^2 = 2x$ also gives the original differential relation after using the curve equation.

Step 4: Conclusion The curve is a circle with centre on the x-axis. Hence, the correct option is (C).

Quick Tip to Solve: Use z=y squared when the combination 2y dy/dx occurs; it often turns a nonlinear equation into a linear one.

25. The plane which bisects the line segment joining the points (–3, –3, 4) and (3, 7, 6) at right angles, passes through which one of the following points ? [4 marks]

(A) (2, 1, 3)

(B) (4, − 1, 2)

(C) (4, 1, − 2)

(D) (− 2, 3, 5)

Correct Answer: (C) (4, 1, − 2)

Solution:

Concept:

- The perpendicular bisector plane of a segment passes through its midpoint and has a normal vector parallel to the segment.
- If $\boldsymbol{n} = (a, b, c)$ is normal and $M = (x_0, y_0, z_0)$ lies on the plane, its equation is $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.

Step 1: Find the midpoint and the normal Let the endpoints be $A = (-3, -3, 4)$ and $B = (3, 7, 6)$. Their midpoint and difference vector are

$$M = \left(\frac{-3+3}{2}, \frac{-3+7}{2}, \frac{4+6}{2}\right) = (0, 2, 5),$$

$$\overrightarrow{AB} = (6, 10, 2) = 2(3, 5, 1).$$

Thus $(3, 5, 1)$ is a convenient normal to the required plane. Halving the normal simplifies the coefficients without changing the plane.

Step 2: Write the plane equation The plane through M with this normal has equation

$$3(x - 0) + 5(y - 2) + (z - 5) = 0,$$

which simplifies to $3x + 5y + z = 15$. This construction enforces both conditions: M bisects the segment, while a normal parallel to AB makes the plane perpendicular to the segment. A plane parallel to AB would not satisfy the right-angle condition.

Step 3: Test each offered point Evaluate $3x + 5y + z$ for the four choices in their printed order. The results are

$$\begin{aligned}(2, 1, 3):&\ 6 + 5 + 3 = 14,\\ (4, -1, 2):&\ 12 - 5 + 2 = 9,\\ (4, 1, -2):&\ 12 + 5 - 2 = 15,\\ (-2, 3, 5):&\ -6 + 15 + 5 = 14.\end{aligned}$$

Exactly the third point satisfies the plane equation; the other three fail the necessary equality.

Step 4: Conclusion The plane passes through $\boxed{(4, 1, -2)}$. Hence, the correct option is (C).

Quick Tip to Solve: For a perpendicular bisector plane, use the segment direction as the normal and its midpoint as a point on the plane.

26. The tangent to the curve, $y = xe^{x^2}$ passing through the point (1, e) also passes through the point [4 marks]

(A) $\left(\frac{4}{3}, 2e\right)$

(B) (3, 6e)

(C) (2, 3e)

(D) $\left(\frac{5}{3}, 2e\right)$

Correct Answer: (A) $\left(\frac{4}{3}, 2e\right)$

Solution:

Concept:

- The tangent slope at a point of a differentiable curve $y = f(x)$ is $f\prime(x)$ evaluated at that point.
- For $y = xe^{x^2}$, the product rule differentiates x and the exponential separately, while the chain rule contributes the factor $2x$.

Step 1: Find the tangent slope at the given point The point $(1, e)$ lies on the curve since $1 \cdot e^{1^2} = e$. Differentiate the function:

$$\frac{dy}{dx} = e^{x^2} + xe^{x^2}(2x) = e^{x^2}(1 + 2x^2).$$

Therefore its slope at $x = 1$ is $m = e(1 + 2) = 3e$. Both derivative terms are needed; omitting the derivative of the exponential would give an incorrect slope.

Step 2: Form the tangent equation Using the point-slope form at $(1, e)$,

$$y - e = 3e(x - 1) \quad \Longrightarrow \quad y = e(3x - 2).$$

A point belongs to this line only if both its coordinates satisfy this relation. The question concerns the tangent through the displayed curve point, so its slope is fixed by the derivative there.

Step 3: Check the alternatives For $x = 4/3$, the line gives $y = e(4 - 2) = 2e$, agreeing with option A. At $x = 3$, it instead gives $7e$, not the $6e$ in B. At $x = 2$, it gives $4e$, not the $3e$ in C. At $x = 5/3$, it gives $3e$, not the $2e$ in D. Thus only A satisfies the tangent equation. Notice that checking the alternatives on the original curve would test a different condition and would not locate points of the tangent.

Step 4: Conclusion The tangent also passes through $\boxed{(4/3, 2e)}$. Hence, the correct option is (A).

Quick Tip to Solve: Differentiate first and form the tangent equation; substitute each candidate into that line rather than the curve.

27. If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target atleast once is greater than $\frac{5}{6}$ is : [4 marks]

(A) 4

(B) 6

(C) 5

(D) 3

Correct Answer: (C) 5

Solution:

Concept:

- For n independent trials with hit probability p, the probability of no hit is $(1-p)^n$.
- The event of at least one hit is the complement of no hits, so its probability is $1-(1-p)^n$. The requested number of shots must be an integer.

Step 1: Express the required probability The probability of missing on each shot is $1-1/3=2/3$. Independence allows the miss probabilities to be multiplied, giving

$$P(\text{no hits in } n \text{ shots}) = \left(\frac{2}{3}\right)^n.$$

Consequently,

$$P(\text{at least one hit}) = 1-\left(\frac{2}{3}\right)^n.$$

This complement includes one hit, two hits, and every higher hit count without counting each case separately.

Step 2: Translate the strict condition The condition in the question becomes

$$1-\left(\frac{2}{3}\right)^n > \frac{5}{6} \quad \Longleftrightarrow \quad \left(\frac{2}{3}\right)^n < \frac{1}{6}.$$

Since $0<2/3<1$, this power decreases as n increases. It is therefore sufficient to locate two successive integer shot counts where the inequality switches from false to true. The strict inequality must be retained throughout.

Step 3: Check the boundary integers For four shots, $(2/3)^4 = 16/81 > 1/6$, because $96>81$. For five shots, $(2/3)^5 = 32/243 < 1/6$, because $192<243$. Thus four shots fail but five succeed, with hit probability $1-32/243 = 211/243 > 5/6$. Three shots also fail by monotonicity. Six shots succeed but are not the minimum, so the alternatives 3, 4, and 6 are excluded for the stated demand.

Step 4: Conclusion The minimum number of shots is $\boxed{5}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the complement of all misses for at least one success, then check the two integers around the threshold.

28. Consider the following three statements :

P : 5 is a prime number

Q : 7 is a factor of 192

R : L.C.M. of 5 and 7 is 35

Then the truth value of which one of the following statements is true ? [4 marks]

(A) $(P \wedge Q) \vee (\sim R)$

(B) $P \vee (\sim Q \wedge R)$

(C) $(\sim P) \wedge (\sim Q \wedge R)$

(D) $(\sim P) \vee (Q \wedge R)$

Correct Answer: (B) $P \vee (\sim Q \wedge R)$

Solution:

Concept:

- Negation reverses a truth value. A conjunction $A \wedge B$ is true only when both components are true.
- A disjunction $A \vee B$ is true when at least one component is true. Evaluate grouped operations before combining their results.

Step 1: Determine the three basic truth values The statement P is true: 5 has exactly the positive factors 1 and 5. The statement Q is false: $192 = 7 \times 27 + 3$, so 7 does not divide 192. The statement R is true: 5 and 7 are coprime and their least common multiple is $5 \times 7 = 35$. Hence $P = T, Q = F, R = T$, and the negations are $\sim P = F, \sim Q = T, \sim R = F$.

Step 2: Evaluate the first two choices For A, $(P \wedge Q) \vee (\sim R) = (T \wedge F) \vee F = F \vee F = F$. For B, the parenthesised conjunction is $(\sim Q) \wedge R = T \wedge T = T$, so

$$P \vee ((\sim Q) \wedge R) = T \vee T = T.$$

In particular, the true statement P already makes this disjunction true, and direct calculation verifies the other component as well.

Step 3: Exclude the remaining choices For C, $(\sim P) \wedge ((\sim Q) \wedge R) = F \wedge (T \wedge T) = F$. For D, $(\sim P) \vee (Q \wedge R) = F \vee (F \wedge T) = F$. Thus the four alternatives have truth values F, T, F, F, in order. The negation symbol must apply to the statement immediately following it; negating an entire conjunction in place of its individual component would change the logical expression.

Step 4: Conclusion The true statement is $P \vee ((\sim Q) \wedge R)$. Hence, the correct option is (B).

Quick Tip to Solve: Write the truth values of P, Q, and R first; then evaluate the brackets before the outer connective.

29. On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, x + y + z = 2 ? [4 marks]

(A) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$
(B) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$
(C) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$
(D) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

Correct Answer: (C) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

Solution:

Concept:

- Write a symmetric line using a common parameter. Its three coordinates must also satisfy the plane equation at the intersection.
- A point lies on a candidate symmetric line only when all three displayed coordinate ratios have the same value; satisfying just one or two ratios is insufficient.

Step 1: Find the intersection point Put each ratio in the given line equal to t. Then

$$x = 4 + 2t, \qquad y = 5 + 2t, \qquad z = 3 + t.$$

Substitute all three coordinates in the plane, rather than equating the parameter to the plane constant:

$$(4 + 2t) + (5 + 2t) + (3 + t) = 2.$$

Thus $12+5t = 2$, so $t = -2$. The intersection point is therefore $P = (0, 1, 1)$. As a check, its coordinate sum is $0 + 1 + 1 = 2$, and its three original line ratios are all -2.

Step 2: Test the proposed lines For option (C), insert $P = (0, 1, 1)$:

$$\frac{0-1}{1} = -1, \qquad \frac{1-3}{2} = -1, \qquad \frac{1+4}{-5} = -1.$$

All three ratios agree, proving that this line contains the intersection. For completeness, the ratios in (A) are $(-4, -4, 4)$, which fail the third coordinate. In (B) they are $(-1, -1, 4/3)$, again unequal. In (D) they are $(1, 1, -1)$, so that line also misses the point. The negative sign in the last denominator of (C) is essential to the agreement.

Step 3: Conclusion The point is $(0, 1, 1)$, and it lies on $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the intersection once, then compare the three ratios in each option; do not solve four new line-plane problems.

30. With the usual notation, in ΔABC, if $\angle A + \angle B$ = 120°, a = $\sqrt{3}+1$, b = $\sqrt{3}-1$ then the ratio $\angle A : \angle B$, is : [4 marks]

(A) 9 : 7

(B) 7 : 1

(C) 5 : 3

(D) 3 : 1

Correct Answer: (B) 7 : 1

Solution:

Concept:

- The sine rule gives $a/b = \sin A/\sin B$. The sum-to-product identities convert a ratio of side differences and sums into a half-angle relation.
- In a triangle, angles lie between 0° and 180°. Since $a > b$, the opposite angles obey $A > B$.

Step 1: Form a half-angle equation The two given sides give

$$\frac{a-b}{a+b} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

Using the sine rule and then the sum-to-product formulae,

$$\frac{a-b}{a+b} = \frac{\sin A - \sin B}{\sin A + \sin B} = \frac{\tan((A-B)/2)}{\tan((A+B)/2)}.$$

Here $(A+B)/2 = 60^\circ$, whose tangent is $\sqrt{3}$. Consequently,

$$\frac{1}{\sqrt{3}} = \frac{\tan((A-B)/2)}{\sqrt{3}}.$$

The side difference must be divided by the side sum; using the side ratio directly would not produce this half-angle identity.

Step 2: Determine the two angles and compare the ratios It follows that $\tan((A-B)/2) = 1$. Because $0 < A - B < 120^\circ$, the relevant half-angle is 45°, not another periodic value. Thus $A - B = 90^\circ$. Combining this with $A + B = 120^\circ$,

$$A = 105^\circ, \qquad B = 15^\circ, \qquad A : B = 7 : 1.$$

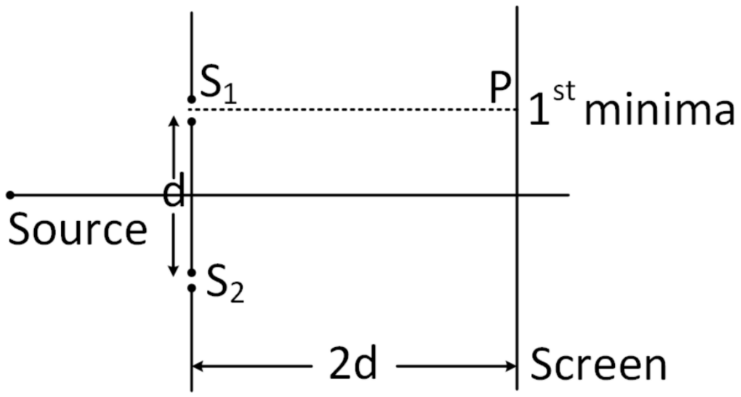
As a direct check, $\sin 105^\circ/\sin 15^\circ = 2+\sqrt{3}$, which equals $(\sqrt{3}+1)/(\sqrt{3}-1)$. Options (A), (C) and (D), when combined with the same angle sum, yield different angle differences and fail the half-angle equation. The larger side correctly faces the obtuse angle.

Step 3: Conclusion $\angle A = 105^\circ$ and $\angle B = 15^\circ$, giving $\angle A : \angle B = 7 : 1$. Hence, the correct option is (B).

Quick Tip to Solve: For two sides and the sum of their opposite angles, use the ratio (side difference)/(side sum) to determine the half-difference of the angles.

Physics (120 marks)

31. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1) ? [4 marks]



(A) $\frac{\lambda}{2(5-\sqrt{2})}$

(B) $\frac{\lambda}{2(\sqrt{5}-2)}$

(C) $\frac{\lambda}{(5-\sqrt{2})}$

(D) $\frac{\lambda}{(\sqrt{5}-2)}$

Correct Answer: (B) $\frac{\lambda}{2(\sqrt{5}-2)}$

Solution:

Concept:

- For coherent slits illuminated in phase, the first destructive-interference condition is a path difference $\lambda/2$.
- Use actual geometric path lengths when the screen distance is only twice the slit separation; the usual small-angle far-screen formula is not justified.

Step 1: Calculate the two distances to the specified point The source shown lies on the perpendicular bisector of the slits, so its distances to S_1 and S_2 are equal. It introduces no initial path difference between them. The point P is directly in front of S_1, and the screen is a horizontal distance $2d$ from the slit plane. Therefore,

$$S_1P = 2d.$$

The ray from S_2 to P has horizontal projection $2d$ and vertical projection d. By Pythagoras,

$$S_2P = \sqrt{(2d)^2 + d^2} = \sqrt{5}\,d.$$

The distances are of comparable scale, so replacing the oblique path by a paraxial approximation would change the answer.

Step 2: Apply the first-minimum condition The total path difference is the difference of the two

slit-to-screen paths because the source-to-slit lengths cancel:

$$\Delta = S_2P - S_1P = (\sqrt{5} - 2)d.$$

For the first minimum, $\Delta = \lambda/2$. Solving for the required slit separation gives

$$d = \frac{\lambda}{2(\sqrt{5} - 2)}.$$

Option (D) doubles this separation and gives a path difference λ, corresponding to constructive rather than first destructive interference. Options (A) and (C) contain $5 - \sqrt{2}$, which is not the difference obtained from the right triangle in the figure. Thus both their geometric factor and the physical path difference are incorrect.

Step 3: Conclusion The required separation is $d = \lambda/[2(\sqrt{5} - 2)]$. Hence, the correct option is (B).

Quick Tip to Solve: When the screen distance is a small multiple of slit spacing, compute both ray lengths exactly before imposing the interference condition.

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32. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separateds two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus -

[4 marks]

(A) 2 cm

(B) 3.1 cm

(C) 4.0 cm

(D) 1 cm

Correct Answer: (B) 3.1 cm

Solution:

Concept:

- For a spherical refracting surface, $n_2/v - n_1/u = (n_2 - n_1)/R$, with distances measured from the surface pole using the Cartesian sign convention.
- A parallel incident beam corresponds to an object at infinity. The image distance is therefore the image-side focal distance, which includes the factor n_2.

Step 1: Choose the signs and simplify the surface equation Take light to travel from air into the eye, from left to right. The incident medium has $n_1 = 1$, and the refracted medium has $n_2 = 1.34$. For the convex corneal surface, its centre of curvature lies on the refracted side, so $R = +7.8\,\text{mm}$. A parallel incident beam has $u = -\infty$, making $n_1/u = 0$. The refraction equation consequently becomes

$$\frac{1.34}{v} = \frac{1.34 - 1}{7.8\,\text{mm}}.$$

This is a single refracting interface; applying the thin-lens formula would incorrectly introduce a second surface.

Step 2: Evaluate the image-side focal distance Rearrange the preceding equation to obtain

$$v = \frac{n_2 R}{n_2 - n_1} = \frac{1.34 \times 7.8}{0.34}\,\text{mm} = 30.74\,\text{mm}.$$

Converting millimetres to centimetres gives $v = 3.074\,\text{cm}$, which rounds to $3.1\,\text{cm}$ at the precision of the options. Its positive sign means that the focus is inside the second medium, beyond the refracting surface. The alternatives 2, 4.0 and $1\,\text{cm}$ do not satisfy the spherical-refraction equation. In particular, omitting the factor 1.34 would calculate a different focal-distance expression and cannot describe the refracted beam here.

Step 3: Conclusion The parallel beam focuses approximately $3.1\,\text{cm}$ beyond the refracting surface, inside the eye. Hence, the correct option is (B).

Quick Tip to Solve: For a single refracting surface, remember that the image-side focal distance is n2 R/(n2-n1); it is not simply R/(n2-n1).

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33. A particle which is experiencing a force, given by $\vec{F} = 3\hat{i} - 12\hat{j}$, undergoes a displacement of $\vec{d} = 4\,\vec{i}$ particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ? [4 marks]

(A) 9 J

(B) 10 J

(C) 12 J

(D) 15 J

Correct Answer: (D) 15 J

Solution:

Concept:

- The work-energy theorem states that net work equals the change in kinetic energy: $W = K_f - K_i$.
- For a constant force, work is the dot product $\vec{F} \cdot \vec{d}$. A force component perpendicular to the displacement does no work.

Step 1: Evaluate the work from the vector components Resolve the vectors along the same Cartesian axes:

$$\vec{F} = (3\hat{i} - 12\hat{j})\,\text{N}, \qquad \vec{d} = 4\hat{i}\,\text{m}.$$

Their scalar product is

$$W = (3\hat{i} - 12\hat{j}) \cdot (4\hat{i}) = 3 \times 4 - 12 \times 0 = 12\,\text{J}.$$

Here $\hat{i} \cdot \hat{i} = 1$ and $\hat{j} \cdot \hat{i} = 0$. Although the vertical force has a large magnitude, there is no vertical displacement in the specified change, so it contributes no work. Multiplying the magnitudes of the two full vectors would omit their angle and give the wrong result.

Step 2: Add the work to the initial kinetic energy The particle begins with $K_i = 3\,\text{J}$. Since the

calculated work is positive, kinetic energy increases:

$$K_f = K_i + W = 3 + 12 = 15\,\text{J}.$$

The initial kinetic energy cannot be ignored: $12\,\text{J}$, option (C), is only the change in kinetic energy, not the final value. Option (A), $9\,\text{J}$, would incorrectly subtract the initial energy from the work. Option (B), $10\,\text{J}$, is inconsistent with the scalar product and the energy balance. No particle mass or travel time is needed because the work-energy theorem directly connects the supplied force and displacement with the energy change.

Step 3: Conclusion The final kinetic energy is $15\,\text{J}$. Hence, the correct option is (D).

Quick Tip to Solve: First compute the scalar product for the work, then add it to the initial kinetic energy; work itself is only the energy change.

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34. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $\left(\vec{A}+\vec{B}\right)$ is 'n' times the magnitude of $\left(\vec{A}-\vec{B}\right)$. The angle between $\vec{A}$ and $\vec{B}$ is - [4 marks]

(A) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$

(B) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

(C) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

(D) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$

Correct Answer: (C) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

Solution:

Concept:

- The dot-product identities give $|\vec{A}\pm\vec{B}|^2 = A^2 + B^2 \pm 2AB\cos\theta$, where θ is the angle between the vectors.
- For equal nonzero magnitudes, their common magnitude cancels from a ratio. Square a ratio of magnitudes before using the dot-product expressions.

Step 1: Express the squared magnitude ratio Let $|\vec{A}| = |\vec{B}| = a$, and let the angle between them be θ, with $0 \le \theta \le \pi$. The sum and difference obey

$$|\vec{A}+\vec{B}|^2 = 2a^2(1+\cos\theta),$$

$$|\vec{A}-\vec{B}|^2 = 2a^2(1-\cos\theta).$$

The statement gives a ratio of magnitudes equal to n, so the ratio of their squares is n^2, not n. Cancelling $2a^2$,

$$n^2 = \frac{1+\cos\theta}{1-\cos\theta}.$$

For a finite specified ratio the denominator is nonzero; the parallel case may be understood as its

limiting case.

Step 2: Solve for the angle and reject the alternatives Cross-multiplication gives

$$n^2 - n^2\cos\theta = 1 + \cos\theta,$$

so

$$\cos\theta = \frac{n^2-1}{n^2+1}, \qquad \theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right).$$

As a useful check, $n = 1$ means equal lengths for the sum and difference, which requires $\theta = 90^\circ$. Options (A) and (B), using inverse sine, instead give zero in this test. Option (D) happens to pass that one test but incorrectly uses n instead of n^2; for example $n = 2$ requires $\cos\theta = 3/5$, whereas (D) gives $1/3$. Thus the squared ratio is essential to obtaining the correct general result.

Step 3: Conclusion The angle is $\cos^{-1}[(n^2-1)/(n^2+1)]$. Hence, the correct option is (C).

Quick Tip to Solve: When a problem gives a ratio of vector magnitudes, square that ratio before substituting dot-product formulae; check n=1 for perpendicular vectors.

35. Consider the nuclear fission

$Ne^{20} \rightarrow 2He^4 + C^{12}$

Given that the binding energy/ nucleon of Ne^{20}, He^4 and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement - [4 marks]

(A) 8.3 MeV energy will be released

(B) energy of 11.9 MeV has to be supplied

(C) energy of 12.4 MeV will be supplied

(D) energy of 3.6 MeV will be released

Correct Answer: None of the printed options: 9.72 MeV must be supplied.

Solution:

Concept:

- Total nuclear binding energy equals the binding energy per nucleon multiplied by the mass number. The reaction energy is $Q = B_{\text{products}} - B_{\text{reactants}}$.
- Positive Q is released energy; negative Q means energy must be supplied. Account for both helium nuclei in the stated reaction.

Step 1: Calculate the binding energies from the printed values For the reactant nucleus, the printed binding energy per nucleon is $8.03\,\text{MeV}$ and its mass number is 20. Therefore,

$$B(^{20}\text{Ne}) = 20(8.03) = 160.60\,\text{MeV}.$$

Each helium nucleus contains four nucleons, and there are two helium nuclei among the products.

Their combined binding energy is

$$2B(^4\text{He}) = 2(4)(7.07) = 56.56\,\text{MeV}.$$

For the carbon product, the stated value is $7.86\,\text{MeV}$ per nucleon, giving

$$B(^{12}\text{C}) = 12(7.86) = 94.32\,\text{MeV}.$$

Thus the products have total binding energy $56.56 + 94.32 = 150.88\,\text{MeV}$. The mass numbers add to 20 on each side, as required.

Step 2: Determine the energy balance and compare every option The reaction energy computed from those values is

$$Q = 150.88 - 160.60 = -9.72\,\text{MeV}.$$

The products are less tightly bound, so $9.72\,\text{MeV}$ must be supplied. Options (A) and (D) both claim release and have the wrong sign. Options (B) and (C) correctly describe supply but give 11.9 and $12.4\,\text{MeV}$, neither of which equals the result. The discrepancy is much larger than rounding of the displayed two-decimal binding energies. In particular, selecting a supply option solely from the sign would not validate its numerical value.

Step 3: Conclusion With the printed binding energies, $9.72\,\text{MeV}$ must be supplied; none of the printed options is correct. The answer key lists (B), but its $11.9\,\text{MeV}$ contradicts $2(4)(7.07) + 12(7.86) - 20(8.03) = -9.72\,\text{MeV}$.

Quick Tip to Solve: Multiply each binding energy per nucleon by its mass number and reaction coefficient, then subtract reactant binding energy from product binding energy.

36. Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, –2) and (0, –2). The work required to put a fifth charge Q at the origin of the coordinate system will be - [4 marks]

(A) $\frac{Q_2}{4\pi\varepsilon_0}$

(B) $\frac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$

(C) $\frac{Q_2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$

(D) $\frac{Q_2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

Correct Answer: (D) $\frac{Q_2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

Solution:

Concept:

- The electric potential of a point charge is $V = kQ/r$, where $k = 1/(4\pi\varepsilon_0)$ and the zero of potential is at infinity.
- Potentials add algebraically because potential is a scalar. The external work in bringing an additional charge Q slowly from infinity is $W = QV$; the original four charges stay fixed.

Step 1: Find the four distances from the origin The charges at $(0, 2)$ and $(0, -2)$ are each a distance 2 from the origin. Those at $(4, 2)$ and $(4, -2)$ are each a distance $\sqrt{4^2 + 2^2} = 2\sqrt{5}$. Thus there are two equal contributions from the nearer pair and two from the farther pair. Opposite vertical positions do not cancel potentials; they would cancel particular components of the electric field.

Step 2: Add the potentials and multiply by the incoming charge At the origin the potential created by the existing charges is

$$V = kQ\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2\sqrt{5}} + \frac{1}{2\sqrt{5}}\right) = kQ\left(1 + \frac{1}{\sqrt{5}}\right).$$

Therefore the increase in interaction energy is

$$W = QV = \frac{Q^2}{4\pi\varepsilon_0}\left(1 + \frac{1}{\sqrt{5}}\right).$$

There is no factor of one half: only the interactions involving the newly added fifth charge are being added. Option A omits the farther charges, B uses the wrong distances, and C uses $\sqrt{3}$ instead of the actual $\sqrt{5}$.

Step 3: Conclusion The required work is $\frac{Q^2}{4\pi\varepsilon_0}(1 + 1/\sqrt{5})$, which is positive for adding an equal charge. Hence, the correct option is (D).

Quick Tip to Solve: For adding one charge to an existing fixed arrangement, find the existing potential at its destination and use $W = qV$.

37. A particle starts from the origin at time t = 0 and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5s ? [4 marks]



(A) 3 m

(B) 9 m

(C) 10 m

(D) 6 m

Correct Answer: (B) 9 m

Solution:

Concept:

- Displacement equals the signed area under a velocity-time graph: $x(t) - x(0) = \int_0^t v\,dt$. The initial position here is $x(0) = 0$.
- For straight segments and constant velocities, this integral is the sum of triangle and rectangle areas. The height represents velocity and the horizontal width represents elapsed time.

Step 1: Read the graph only up to the required instant Between 0 and 2 seconds, the velocity rises linearly from zero to $2\,\mathrm{m\,s^{-1}}$. Between 2 and 4 seconds it remains at $2\,\mathrm{m\,s^{-1}}$. At 4 seconds it changes to $3\,\mathrm{m\,s^{-1}}$, and this is the value throughout the interval from 4 to 5 seconds. A vertical jump in velocity has no horizontal width and contributes no separate area.

Step 2: Add the three positive areas The triangular area for the first interval is

$$s_1 = \tfrac{1}{2}(2)(2) = 2\,\mathrm{m}.$$

The next rectangle contributes

$$s_2 = (4-2)(2) = 4\,\mathrm{m},$$

and the final one-second rectangle contributes

$$s_3 = (5-4)(3) = 3\,\mathrm{m}.$$

Hence

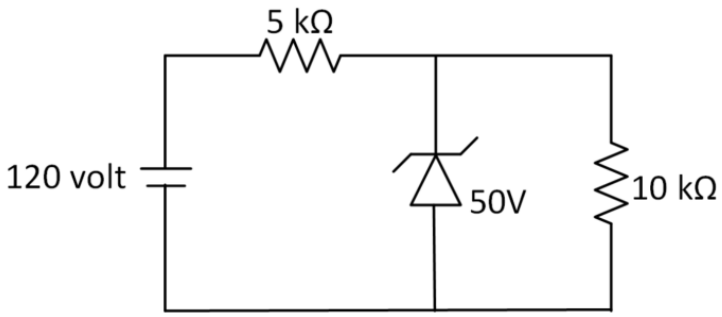
$$x(5) = 0 + s_1 + s_2 + s_3 = 9\,\mathrm{m}.$$

All these areas are positive, consistent with motion along the positive axis. The value 3 m counts only the last interval, 6 m is the position at 4 seconds, and 10 m does not equal the graph area up to 5 seconds.

Step 3: Conclusion At $t = 5\,\mathrm{s}$, the particle is at $x = 9\,\mathrm{m}$. Hence, the correct option is (B).

Quick Tip to Solve: Mark the requested time on the graph before calculating areas, so later segments do not enter the displacement.

38. For the circuit shown below, the current through the Zener diode is - [4 marks]



(A) 5 mA

(B) zero

(C) 14 mA

(D) 9 mA

Correct Answer: (D) 9 mA

Solution:

Concept:

- A reverse-biased ideal Zener diode in breakdown maintains its rated voltage across the parallel load. Here that regulated voltage is $V_Z = 50\,\text{V}$.
- Kirchhoff's current law at the output node gives $I_s = I_L + I_Z$, where I_s is the series-resistor current and I_L is the load current.

Step 1: Check whether the Zener can enter breakdown If the Zener branch were open, the $5\,\text{k}\Omega$ series resistor and $10\,\text{k}\Omega$ load would form a voltage divider. Its output would be

$$V_{\text{open}} = 120\frac{10}{5+10} = 80\,\text{V}.$$

Since this exceeds the 50 V breakdown rating and the diode has the reverse polarity shown, it conducts and clamps the load to 50 V. The available supply therefore supports the assumed breakdown condition.

Step 2: Calculate the branch currents separately The voltage across the series resistor is $120 - 50 = 70\,\text{V}$, so

$$I_s = \frac{70}{5000} = 0.014\,\text{A} = 14\,\text{mA}.$$

The load shares the Zener voltage and draws

$$I_L = \frac{50}{10000} = 0.005\,\text{A} = 5\,\text{mA}.$$

Subtracting at the junction gives

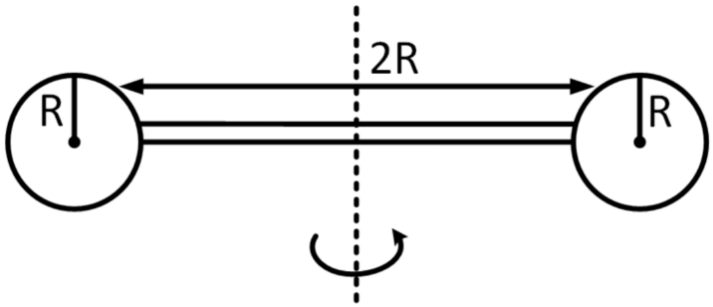
$$I_Z = 14 - 5 = 9\,\text{mA}.$$

This is positive, confirming breakdown. The 14 mA option is the total series current, the 5 mA option is the load current, and zero would contradict the 80 V open-branch output.

Step 3: Conclusion The current through the Zener diode is $9\,\mathrm{mA}$. Hence, the correct option is (D).

Quick Tip to Solve: In a Zener regulator, the series current splits between the load and Zener; always subtract the load current.

39. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is : [4 marks]



(A) $\frac{17}{15}$ MR^2
(B) $\frac{137}{15}$ MR^2
(C) $\frac{209}{15}$ MR^2
(D) $\frac{152}{15}$ MR^2

Correct Answer: (B) $\frac{137}{15}$ MR^2

Solution:

Concept:

- A uniform solid sphere has moment of inertia $I_{\text{cm}} = 2MR^2/5$ about a diameter. About a parallel axis a distance d away, $I = I_{\text{cm}} + Md^2$.
- A uniform thin rod of mass M and length L has moment of inertia $ML^2/12$ about its perpendicular central axis. The moments of all parts about the common axis add.

Step 1: Locate the centres of the spheres correctly The rod length is $2R$, and each spherical ball is attached at an end of the rod. The distance from the rod midpoint to its end is R; the ball centre lies another radius R beyond that end. Each sphere centre is therefore at $d = 2R$ from the indicated rotation axis. Using only R would locate the centre at the contact point and omit the ball radius.

Step 2: Add the two shifted spheres and the rod For each ball, the parallel-axis theorem gives

$$I_{\text{ball}} = \frac{2}{5}MR^2 + M(2R)^2 = \frac{22}{5}MR^2.$$

For the rod,

$$I_{\text{rod}} = \frac{M(2R)^2}{12} = \frac{1}{3}MR^2.$$

Hence the full system has

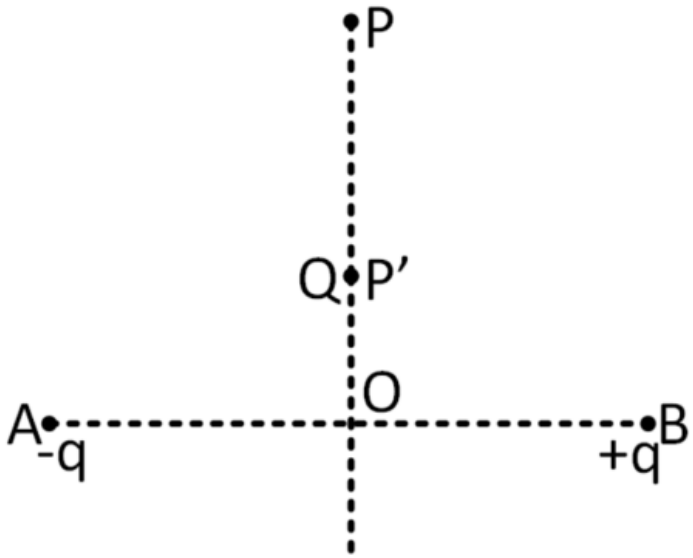
$$I = 2\left(\frac{22}{5}MR^2\right) + \frac{1}{3}MR^2 = \frac{132+5}{15}MR^2 = \frac{137}{15}MR^2.$$

The sphere masses are each M, not a combined mass M. The alternative coefficients do not equal this sum; in particular the much smaller coefficient in A misses the large translation terms.

Step 3: Conclusion The moment of inertia is $\frac{137}{15}MR^2$. Hence, the correct option is (B).

Quick Tip to Solve: Before applying the parallel-axis theorem, measure the axis-to-centre distance, including the radius beyond each rod end.

40. Charges –q and +q located at A and B, respectively, constitude an electric dipole. Distance AB = 2a, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where OP = y and y $>>$ 2a. The charge Q experiences an electrostatic force F. If Q is now moved along the equatorial line to P' such that OP' = $\left(\frac{y}{3}\right)$, the force on Q will be close to - $\left(\frac{y}{3} >> 2a\right)$ [4 marks]



(A) 9F

(B) 3F

(C) F/3

(D) 27F

Correct Answer: (D) 27F

Solution:

Concept:

- For a dipole of moment magnitude $p = 2aq$, the electric field at a distant equatorial point is approximately $E = kp/r^3$, directed opposite to the dipole moment.
- The electric force on a fixed test charge has magnitude $|Q|E$. Moving that charge along the same

distant equatorial line changes the magnitude by the inverse cube of its distance.

Step 1: Identify the appropriate distance dependence On the equatorial line, the components of the two charge fields perpendicular to the dipole cancel, while those parallel to the dipole add opposite to its moment. The exact magnitude is

$$E(r) = \frac{k(2aq)}{(r^2 + a^2)^{3/2}}.$$

Because the stated distances are both much greater than the separation, $r^2 + a^2 \simeq r^2$, giving $E(r) \simeq kp/r^3$. This is a dipole field, so the inverse-square law for a single point charge cannot be applied to the final resultant.

Step 2: Take the ratio at the two positions The first distance is y, and the new distance is $y/3$. Neither the source dipole nor the charge Q changes, so

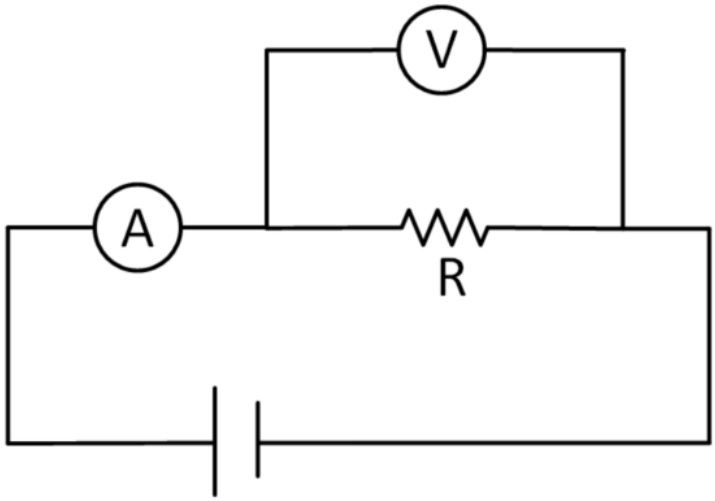
$$\frac{F\prime}{F} \simeq \frac{kp|Q|/(y/3)^3}{kp|Q|/y^3} = \left(\frac{y}{y/3}\right)^3 = 27.$$

The stated condition $y/3 \gg 2a$ ensures that the dipole approximation remains valid even after the move. Thus 9F incorrectly assumes inverse-square dependence, 3F assumes inverse-linear dependence, and F/3 predicts weakening on approaching the dipole.

Step 3: Conclusion The new force is approximately $27F$, with the same direction as before along the equatorial line. Hence, the correct option is (D).

Quick Tip to Solve: For a distant dipole, reducing the distance by a factor of three multiplies the field and force by 3^3.

41. The actual value of resistance R, shown in the figure is 30Ω. This is measured in an experiment as shown using the standard formula R = $\frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is - [4 marks]



(A) $570\,\Omega$

(B) $600\,\Omega$

(C) $350\,\Omega$

(D) $35\,\Omega$

Correct Answer: (A) $570\,\Omega$

Solution:

Concept:

- A real voltmeter has a finite resistance R_V and draws current. In this connection it is parallel to the resistor, while the ammeter reads their total current.
- The quotient V/I therefore measures the parallel combination $R_m = RR_V/(R + R_V)$, which is smaller than the actual resistor resistance.

Step 1: Translate the stated percentage error The true resistance is $R = 30\,\Omega$, whereas the measured resistance is 5 percent less. Therefore

$$R_m = (1 - 0.05)30 = 28.5\,\Omega.$$

If the current through the resistor alone had been measured, the quotient would give exactly 30 ohms. Here some of the ammeter current bypasses the resistor through the voltmeter, increasing the denominator and reducing the measured value.

Step 2: Solve for the voltmeter resistance The branch currents obey $I = V/R + V/R_V$, so

$$\frac{1}{28.5} = \frac{1}{30} + \frac{1}{R_V}.$$

Consequently

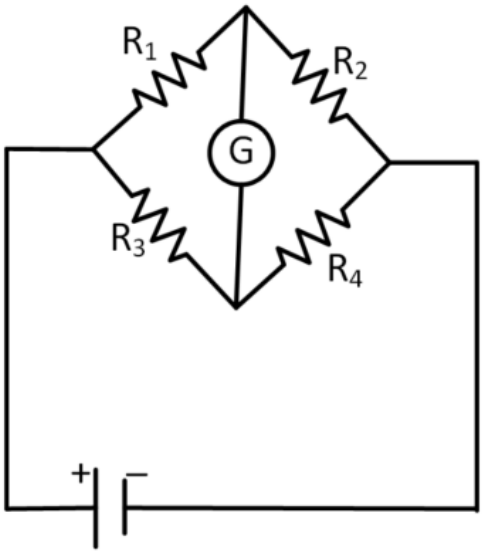
$$R_V = \frac{30(28.5)}{30 - 28.5} = \frac{855}{1.5} = 570\,\Omega.$$

Checking, $30 \times 570/(30 + 570) = 28.5\,\Omega$, as required. A 600-ohm voltmeter would give about 28.57 ohms and only an approximate 5 percent error. The smaller proposed resistances would draw much larger currents and produce much larger errors, so they cannot fit the exact statement.

Step 3: Conclusion The internal resistance of the voltmeter is $570\,\Omega$. Hence, the correct option is (A).

Quick Tip to Solve: First identify whether the ammeter includes the voltmeter current; if it does, V/I is a parallel resistance.

42. The Wheatstone bridge shown in figure, here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80Ω and 40Ω, respectively. Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3, would be - [4 marks]



(A) Brown, Blue, Brown

(B) Grey, Black, Brown

(C) Red, Green, Brown

(D) Brown, Blue, Black

Correct Answer: (A) Brown, Blue, Brown

Solution:

Concept:

- In a balanced Wheatstone bridge, no current flows through the galvanometer and the divider ratios satisfy $R_1/R_2 = R_3/R_4$ for the arrangement shown.
- In a three-band resistance code, the first two colours give the two significant digits and the third gives the power-of-ten multiplier: orange 3, red 2, brown 1, blue 6 and black 0.

Step 1: Decode the known resistor and apply balance The colour sequence orange, red, brown gives the digits 3 and 2 with multiplier 10^1. Therefore

$$R_1 = 32 \times 10 = 320\,\Omega.$$

At balance the upper and lower central nodes have equal potentials, so the ratios of the two left-to-right branch resistances are equal. Substituting the specified values gives

$$\frac{320}{80} = \frac{R_3}{40}.$$

Thus

$$R_3 = \frac{320 \times 40}{80} = 160\,\Omega.$$

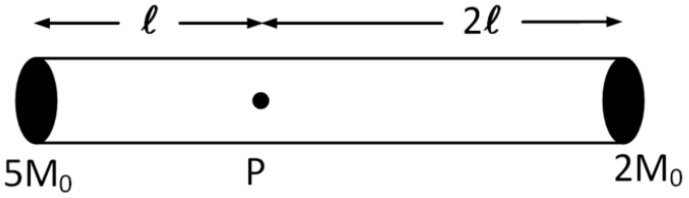
The order of the bridge arms matters: swapping a ratio would yield an incorrect resistance.

Step 2: Encode the resulting resistance and reject the alternatives Write the required resistance as $160 = 16 \times 10^1$. The first digit 1 is brown, the second digit 6 is blue, and the multiplier 10^1 is brown. Thus its code is brown, blue, brown. Grey, black, brown represents $80 \times 10 = 800\,\Omega$; red, green, brown represents $25 \times 10 = 250\,\Omega$; and brown, blue, black represents $16 \times 1 = 16\,\Omega$. None of those alternatives gives the 160-ohm arm required for balance.

Step 3: Conclusion The required colour code is Brown, Blue, Brown. Hence, the correct option is (A).

Quick Tip to Solve: Decode the known resistor numerically, solve the bridge ratio, then convert the final value back into colour bands.

43. A rigid massless rod of length 3l has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be - [4 marks]



(A) $\frac{g}{13l}$
(B) $\frac{g}{2l}$
(C) $\frac{g}{3l}$
(D) $\frac{7g}{3l}$

Correct Answer: (A) $\frac{g}{13l}$

Solution:

Concept:

- For rotation about a fixed pivot, the net external torque and angular acceleration obey $\tau_P = I_P\alpha$. The pivot force has no moment about the pivot.
- For a point mass at perpendicular distance r, its moment of inertia is mr^2, whereas its gravitational torque in the horizontal position has magnitude mgr.

Step 1: Find the opposing gravitational torques The figure puts the mass $5M_0$ a distance l to the left of P and the mass $2M_0$ a distance $2l$ to its right. Their weights produce opposite rotations. Taking anticlockwise as positive gives

$$\tau_P = (5M_0g)l - (2M_0g)(2l) = M_0gl.$$

Thus the heavier left end initially moves down. Adding the two torque magnitudes would incorrectly treat opposing effects as acting in the same direction.

Step 2: Calculate the inertia and divide The massless rod contributes no moment of inertia. Both masses contribute positively, irrespective of the signs of their position coordinates:

$$I_P = 5M_0l^2 + 2M_0(2l)^2 = 13M_0l^2.$$

Consequently, immediately after release,

$$\alpha = \frac{\tau_P}{I_P} = \frac{M_0gl}{13M_0l^2} = \frac{g}{13l}.$$

The initial angular velocity is zero but the unbalanced gravitational torque makes the angular acceleration nonzero. Options (B), (C), and (D) do not use both the opposing torque signs and the squared lever arms in the inertia.

Step 3: Conclusion The instantaneous angular acceleration is $g/(13l)$, anticlockwise as drawn. Hence, the correct option is (A).

Quick Tip to Solve: Subtract opposing weight torques, but always add the positive terms mr^2 when finding the pivot inertia.

44. The electric field of a plane polarized electromagnetic wave in free space at time t = 0 is given by an expression $\vec{E}(x, y) = 10\hat{j}\cos[(6x + 8z)]$. The magnetic field $\vec{B}$(x,z, t) is given by – (c is the velocity of light) [4 marks]

(A) $\frac{1}{c}\left(6\hat{k} + 8\hat{i}\right)\cos[(6x + 8z - 10ct)]$

(B) $\frac{1}{c}\left(6\hat{k} - 8\hat{i}\right)\cos[(6x + 8z - 10ct)]$

(C) $\frac{1}{c}\left(6\hat{k} + 8\hat{i}\right)\cos[(6x - 8z + 10ct)]$

(D) $\frac{1}{c}\left(6\hat{k} - 8\hat{i}\right)\cos[(6x - 8z + 10ct)]$

Correct Answer: (B) $\frac{1}{c}\left(6\hat{k} - 8\hat{i}\right)\cos[(6x + 8z - 10ct)]$

Solution:

Concept:

- A plane electromagnetic wave in vacuum has $\omega = c|\mathbf{k}|$ and magnetic amplitude $B_0 = E_0/c$. Its fields and propagation direction are mutually perpendicular.
- For propagation along the unit vector $\hat{\mathbf{n}}$, the fields obey $\mathbf{B} = (\hat{\mathbf{n}} \times \mathbf{E})/c$; the electric and magnetic fields have the same phase.

Step 1: Read the wave vector and frequency The spatial phase at zero time is $6x + 8z$, so for the forward travelling wave represented in the options,

$$\mathbf{k} = 6\hat{i} + 8\hat{k}, \qquad |\mathbf{k}| = \sqrt{6^2 + 8^2} = 10.$$

Hence $\omega = 10c$, the full phase is $6x + 8z - 10ct$, and the unit propagation vector is $(6\hat{i} + 8\hat{k})/10$. The field remains polarized along the y direction, with amplitude 10.

Step 2: Use the vector relation for the magnetic field Using $\hat{i} \times \hat{j} = \hat{k}$ and $\hat{k} \times \hat{j} = -\hat{i}$, we obtain

$$\mathbf{B} = \frac{1}{c}(6\hat{k} - 8\hat{i})\cos(6x + 8z - 10ct).$$

Its amplitude is $\sqrt{36 + 64}/c = 10/c$, as required. Its direction is perpendicular to $\mathbf{k}$, since $6(-8) + 8(6) = 0$. Option (A) fails that perpendicularity check. Options (C) and (D) change the spatial phase at zero time to $6x - 8z$, which does not match the given electric field.

Step 3: Conclusion The required magnetic field is $\mathbf{B} = c^{-1}(6\hat{k} - 8\hat{i})\cos(6x + 8z - 10ct)$. Hence, the correct option is (B).

Quick Tip to Solve: Find the unit propagation vector first, then use $\mathbf{B} = \hat{\mathbf{n}} \times \mathbf{E}/c$; a dot product with $\mathbf{k}$ checks the signs.

45. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures ?

[4 marks]

(A) 4264.4 ± 81.0 cm^3

(B) 4264 ± 81 cm^3

(C) 4300 ± 80 cm^3

(D) 4260 ± 80 cm^3

Correct Answer: (D) 4260 ± 80 cm^3

Solution:

Concept:

- The volume of a cylinder in terms of its diameter and height is $V = \pi d^2 h/4$. The factor $\pi/4$ is exact for this error calculation.
- For a product of powers, the maximum fractional uncertainties add with their powers: $\Delta V/V = 2\Delta d/d + \Delta h/h$. Round the uncertainty before choosing the precision of the reported value.

Step 1: Calculate the central volume Use the diameter directly, rather than treating it as the radius. With the lengths in centimetres,

$$V = \frac{\pi}{4}(12.6)^2(34.2) = 4264.44\ldots \text{ cm}^3.$$

These extra digits are retained only during the calculation. They do not mean that the metre scale determines the volume to a fraction of a cubic centimetre.

Step 2: Propagate the uncertainties and round consistently The diameter occurs squared, so its fractional error must be counted twice:

$$\frac{\Delta V}{V} = 2\frac{0.1}{12.6} + \frac{0.1}{34.2} = 0.018797\ldots.$$

$$\Delta V = 4264.44\ldots(0.018797\ldots) = 80.16\ldots \text{ cm}^3.$$

Expressing the uncertainty to one significant figure gives $80\,\text{cm}^3$, at the tens place. Round the central volume to that same place, giving $4260\,\text{cm}^3$. Option (A) and option (B) retain unjustified digits in both quantities. Option (C) rounds the central value to hundreds, although the quoted uncertainty resolves tens; it unnecessarily loses precision.

Step 3: Conclusion The appropriately reported volume is $(4260 \pm 80)\,\text{cm}^3$. Hence, the correct option is (D).

Quick Tip to Solve: Keep guard digits while calculating; then round the uncertainty and the central value to the same decimal place.

46. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is - (Take Gravitational constant; G = 6.67×10^{-11} Nm^2 kg^{-2}) [4 marks]

(A) 2.4×10^4 m/s

(B) 1.4×10^5 m/s

(C) 3.8×10^4 m/s

(D) 2.8×10^5 m/s

Correct Answer: (D) 2.8×10^5 m/s

Solution:

Concept:

- Gravitational potential is a scalar and adds for different masses: $\Phi = -\sum GM_i/r_i$, with zero potential at infinity.
- The minimum escape speed satisfies $\frac{1}{2}v_{\text{esc}}^2 + \Phi = 0$. A zero resultant force at one point does not imply zero potential or zero escape energy.

Step 1: Find the potential at the common centre The stars have equal masses, so O is halfway between them. The distance from O to each star is therefore $r = 10^{11}$ m, rather than the full separation. The potential per unit meteorite mass is

$$\Phi_O = -\frac{GM}{r} - \frac{GM}{r} = -\frac{2GM}{r}.$$

The gravitational forces cancel at O, but both negative potential contributions must still be included.

Step 2: Apply the threshold escape condition For a meteorite of mass m, energy conservation at the escape threshold gives

$$\frac{1}{2}mv^2 - \frac{2GMm}{r} = 0, \qquad v = \sqrt{\frac{4GM}{r}}.$$

Substitute the specified quantities:

$$v^2 = \frac{4(6.67 \times 10^{-11})(3 \times 10^{31})}{10^{11}} = 8.004 \times 10^{10}\,\text{m}^2\text{s}^{-2}.$$

$$v = 2.829\ldots \times 10^5\,\text{m}\,\text{s}^{-1}.$$

Along the perpendicular symmetry axis, both stars remain at the same distance from the meteorite, so their rotation does not change this potential-energy calculation. Options (A) and (C) are far too small; (B) also fails the threshold energy equation.

Step 3: Conclusion The minimum speed is approximately $2.8 \times 10^5\,\text{m}\,\text{s}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: At the midpoint of two equal masses, add the potentials even though the gravitational forces cancel.

47. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω is close to – (density of water = 10^3 kg/m^3). [4 marks]

(A) 2.50 rad s^{-1}

(B) 3.75 rad s^{-1}

(C) 5.00 rad s^{-1}

(D) 7.90 rad s^{-1}

Correct Answer: (D) 7.90 rad s^{-1}

Solution:

Concept:

- A floating body at equilibrium displaces its own weight of water. A small extra downward displacement x of a cylindrical body increases its displaced volume by Ax, where A is its cross-sectional area.
- The extra upward buoyant force is $\rho g A x$. Thus the restoring-force constant is $k_{\text{eff}} = \rho g A$, and $\omega = \sqrt{k_{\text{eff}}/m}$.

Step 1: Find the moving mass and restoring force The bottle has negligible mass; its contained water supplies the inertia. Convert the volume before using SI units:

$$V = 310\,\text{ml} = 310 \times 10^{-6}\,\text{m}^3, \qquad m = \rho V = 0.310\,\text{kg}.$$

At equilibrium the weight and buoyancy balance. If the bottle is pushed down through a small distance x, its extra immersed volume is $\pi r^2 x$. The excess upward force is therefore $\rho g \pi r^2 x$, directed opposite to the displacement.

Step 2: Identify the angular frequency Taking downward displacement as positive, the equation of motion is

$$m\ddot{x} = -\rho g \pi r^2 x, \qquad \omega^2 = \frac{\rho g \pi r^2}{m}.$$

With $r = 0.025\,\text{m}$,

$$\omega^2 = \frac{1000(9.8)\pi(0.025)^2}{0.310} \simeq 62.1\,\text{s}^{-2}, \qquad \omega \simeq 7.88\,\text{rad}\,\text{s}^{-1}.$$

The density cancels if one writes $m = \rho V$. Options (A), (B), and (C) yield substantially smaller restoring-force-to-mass ratios than the one fixed by the given radius and water volume.

Step 3: Conclusion The angular frequency is close to $7.90\,\text{rad}\,\text{s}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: For vertical oscillations of a floating cylinder, use the extra displaced volume Ax, not the total equilibrium displacement.

48. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then - [4 marks]

(A) $T_h = 1.5\ T_c$

(B) $T_h = T_c$

(C) $T_h = 2T_c$

(D) $T_h = 0.5\ T_c$

Correct Answer: (B) $T_h = T_c$

Solution:

Concept:

- A magnetic dipole displaced through a small angle θ in a uniform field experiences restoring torque $-\mu B \sin\theta \simeq -\mu B\theta$.
- The equation $I\ddot{\theta} = -\mu B\theta$ gives $T = 2\pi\sqrt{I/(\mu B)}$. Both rotational inertia and magnetic moment enter the period; neither can be compared alone.

Step 1: Compare the inertia and magnetic moment factors For the standard hoop and solid circular-cylinder model, with the same mass M and radius R, the corresponding moments of inertia have ratio

$$\frac{I_h}{I_c} = \frac{MR^2}{\frac{1}{2}MR^2} = 2.$$

The same ratio is obtained with the diametral moments of a thin hoop and solid disc, $MR^2/2$ and $MR^2/4$, for small tilts of their magnetic axes. The given magnetic moments satisfy $\mu_h = 2\mu_c$, so the stronger restoring torque of the hoop must be compared with its larger inertia.

Step 2: Form the ratio of the oscillation periods The field strength is the same for the two bodies and cancels from the ratio:

$$\frac{T_h}{T_c} = \sqrt{\frac{I_h}{I_c}\frac{\mu_c}{\mu_h}} = \sqrt{2 \times \frac{1}{2}} = 1.$$

Equivalently, both bodies have the same ratio I/μ, so they have the same angular frequency. Comparing the magnetic moments alone would incorrectly predict a shorter period for the hoop, while comparing only the inertias would incorrectly predict a longer one. Options (A), (C), and (D) are not consistent with the simultaneous factor-of-two changes in inertia and restoring torque.

Step 3: Conclusion The two small-oscillation periods are equal: $T_h = T_c$. Hence, the correct option is (B).

Quick Tip to Solve: For magnetic oscillations compare I/μ; an equal increase in inertia and magnetic moment leaves the period unchanged.

49. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2. The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photoelectrons per second and their maximum energy, respectively, will be [4 marks]

(A) 10^{14} and 10 eV

(B) 10^{12} and 5 eV

(C) 10^{11} and 5 eV

(D) 10^{10} and 5 eV

Correct Answer: (C) 10^{11} and 5 eV

Solution:

Concept:

- The incident optical power is intensity times illuminated area: $P = IA$. The photon arrival rate is P/E_γ, and the electron emission rate is this rate multiplied by the quantum efficiency.
- Einstein's photoelectric equation gives $K_{\max} = E_\gamma - \phi$. Intensity controls the number of photons, whereas photon energy and work function determine the maximum electron energy.

Step 1: Calculate the emitted-electron rate Convert milliwatts to watts and use the stated illuminated area:

$$P = (16 \times 10^{-3})(10^{-4}) = 1.6 \times 10^{-6}\,\text{W}.$$

Each photon carries

$$E_\gamma = 10\,\text{eV} = 10(1.6 \times 10^{-19}) = 1.6 \times 10^{-18}\,\text{J}.$$

Thus the incident photon rate is $P/E_\gamma = 10^{12}\,\text{s}^{-1}$. Only 10 percent of these photons produce photoelectrons, so the emission rate is $0.10 \times 10^{12} = 10^{11}\,\text{s}^{-1}$. The conversion efficiency changes the count; it does not reduce each successful photon's energy.

Step 2: Find the maximum kinetic energy and compare The work function is the minimum energy used to release an electron. The remainder is the greatest possible kinetic energy:

$$K_{\max} = 10\,\text{eV} - 5\,\text{eV} = 5\,\text{eV}.$$

Option (A) has both an excessive count and the full photon energy rather than the kinetic energy. Option (B) neglects the 10 percent efficiency. Option (D) includes an extra, unjustified factor of one tenth in the emission rate. The calculated pair agrees with option (C).

Step 3: Conclusion The emission rate is 10^{11} photoelectrons per second and the maximum kinetic energy is $5\,\text{eV}$. Hence, the correct option is (C).

Quick Tip to Solve: Apply quantum efficiency only to the photon count, then subtract the work function separately to find the electron energy.

50. Two forces P and Q, of magnitude 2F and 3F, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is - [4 marks]

(A) 90°

(B) 60°

(C) 30°

(D) 120°

Correct Answer: (D) 120°

Solution:

Concept:

- The squared magnitude of the resultant of two forces is $R^2 = P^2 + Q^2 + 2PQ\cos\theta$, where θ is the angle between them.
- Doubling one force changes its squared contribution by a factor of four and its cross term by a factor of two; it does not generally double the resultant.

Step 1: Write the original resultant For $P = 2F$ and $Q = 3F$, apply the parallelogram law. Retain the common force scale so that it can cancel later:

$$R^2 = (2F)^2 + (3F)^2 + 2(2F)(3F)\cos\theta = F^2(13 + 12\cos\theta).$$

The angle is between the two original vectors, so it is the same angle after changing the magnitude of Q.

Step 2: Apply the doubled-resultant condition Only Q doubles, becoming $6F$, while P remains $2F$. The new resultant has magnitude $2R$. Therefore

$$(2R)^2 = (2F)^2 + (6F)^2 + 2(2F)(6F)\cos\theta.$$

$$4(13 + 12\cos\theta) = 40 + 24\cos\theta.$$

$$24\cos\theta = -12, \qquad \cos\theta = -\frac{1}{2}.$$

For the angle between two vectors in the range zero to 180 degrees, this gives 120 degrees.

Step 3: Check the options At 120 degrees, the original squared resultant is $7F^2$ and the new one is $28F^2$, confirming a factor of two in magnitude. A right angle gives squared magnitudes 13 and 40 rather than a ratio of four. The acute angles 60 and 30 degrees have positive cosine, whereas the condition demands negative cosine; neither can satisfy the equation.

Step 4: Conclusion The required angle is $\boxed{\theta = 120^\circ}$. Hence, the correct option is (D).

Quick Tip to Solve: When a resultant doubles, multiply its squared magnitude by four before comparing the two vector-addition equations.

51. The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is - [4 marks]

(A) 740 J

(B) 637.5 J

(C) 540 J

(D) 437.5 J

Correct Answer: (D) 437.5 J

Solution:

Concept:

- The magnitude of self-induced emf is $|\mathcal{E}| = L\,|dI/dt|$, where L is the self-inductance.
- The magnetic energy stored at current I is $U = \frac{1}{2}LI^2$. The energy change depends on the difference of squared currents, not the square of their difference.

Step 1: Determine the self-inductance The current increases uniformly from 10 A to 25 A in one second, so the current-change rate is

$$\frac{dI}{dt} = \frac{25-10}{1} = 15\,\mathrm{A\,s^{-1}}.$$

Using the given induced-emf magnitude of 25 V gives

$$L = \frac{25}{15} = \frac{5}{3}\,\mathrm{H}.$$

The induced emf opposes the increase in current, but this sign does not make L negative; inductance and stored magnetic energy are positive.

Step 2: Calculate the increase in stored energy Use the same inductance at both current values:

$$\Delta U = U_f - U_i = \frac{1}{2}L(I_f^2 - I_i^2).$$

$$\Delta U = \frac{1}{2} \times \frac{5}{3} \times (25^2 - 10^2) = \frac{5}{6} \times 525 = 437.5\,\mathrm{J}.$$

Equivalently, the initial energy is $83.33\,\mathrm{J}$ and the final energy is $520.83\,\mathrm{J}$, whose difference is 437.5 J. The positive sign agrees with the increase of current.

Step 3: Check the offered values The average current during the uniform rise is $(10+25)/2 = 17.5\,\mathrm{A}$. Thus the energy supplied to the magnetic field is also $|\mathcal{E}|I_{\mathrm{av}}\Delta t = 25 \times 17.5 \times 1 = 437.5\,\mathrm{J}$. This independent calculation rejects 740 J, 637.5 J and 540 J; none is the difference of the initial and final magnetic energies.

Step 4: Conclusion The energy stored in the inductance increases by $\boxed{437.5\,\mathrm{J}}$. Hence, the correct option is (D).

Quick Tip to Solve: First infer L from the emf and current slope, then use the difference of squared currents in the magnetic-energy formula.

52. Two kg of a monoatomic gas is at a pressure of 4×10^4 N/m^2. The density of the gas is 8 kg/m^3. What is the order of energy of the gas due to its thermal motion ? [4 marks]

(A) 10^4 J

(B) 10^3 J

(C) 10^5 J

(D) 10^6 J

Correct Answer: (A) 10^4 J

Solution:

Concept:

- For an ideal monoatomic gas, only the three translational degrees of freedom contribute: $U = \frac{3}{2}nRT$.
- Using the ideal-gas law $PV = nRT$, write $U = \frac{3}{2}PV$; the volume follows from total mass and density as $V = m/\rho$.

Step 1: Obtain the volume from the density The mass is $m = 2\,\text{kg}$ and the density is $\rho = 8\,\text{kg}\,\text{m}^{-3}$. Therefore

$$V = \frac{m}{\rho} = \frac{2}{8} = 0.25\,\text{m}^3.$$

Density is mass per unit volume, so multiplying mass by density would not give a volume and would have the wrong dimensions. No molar mass or number of particles is needed because pressure times volume already determines nRT.

Step 2: Calculate the total thermal energy The kinetic energy associated with random thermal motion is the internal energy of this monoatomic gas. Hence

$$U = \frac{3}{2}PV = \frac{3}{2}(4 \times 10^4)(0.25) = 1.5 \times 10^4\,\text{J}.$$

The units are consistent: $\text{N}\,\text{m}^{-2} \times \text{m}^3 = \text{N}\,\text{m} = \text{J}$. This is a total energy for the given two kilograms, rather than an energy per kilogram or per molecule.

Step 3: Identify the order of magnitude The numerical coefficient 1.5 is of order unity, so the characteristic power of ten is 10^4 J. In fact, the result is 1.5 times this value. It is fifteen times 10^3 J, while 10^5 J and 10^6 J are approximately 6.7 and 67 times larger than the calculated energy. Those scales do not describe the requested order.

Step 4: Conclusion The thermal energy is 1.5×10^4 J, of order $\boxed{10^4\,\text{J}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a monoatomic ideal gas with known mass and density, combine the formulas directly as U = 3Pm/(2 density).

53. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is - [4 marks]

(A) $\frac{4\pi}{3}$

(B) $\frac{3}{8}\pi$

(C) $\frac{7}{3}\pi$

(D) $\frac{8\pi}{3}$

Correct Answer: (D) $\frac{8\pi}{3}$

Solution:

Concept:

- In SHM, the speed at displacement x is $v = \omega\sqrt{A^2 - x^2}$, with amplitude A and angular frequency ω.
- The acceleration magnitude is $|a| = \omega^2|x|$, and the period is $T = 2\pi/\omega$. Numerical equality of speed and acceleration must be applied using the stated SI units.

Step 1: Convert the lengths and evaluate the speed factor The amplitude is $A = 0.05\,\mathrm{m}$ and the displacement magnitude is $|x| = 0.04\,\mathrm{m}$. Thus

$$\sqrt{A^2 - x^2} = \sqrt{0.05^2 - 0.04^2} = \sqrt{0.0009} = 0.03\,\mathrm{m}.$$

Consequently the numerical speed in SI units is 0.03ω, and the numerical acceleration magnitude is $0.04\omega^2$. The acceleration points towards the mean position, but its direction is irrelevant to the equality of magnitudes.

Step 2: Apply the numerical equality and find the period For the nonzero oscillation frequency, the stated condition gives

$$0.03\omega = 0.04\omega^2, \qquad \omega = \frac{0.03}{0.04} = \frac{3}{4}\,\mathrm{s}^{-1}.$$

Therefore

$$T = \frac{2\pi}{3/4} = \frac{8\pi}{3}\,\mathrm{s}.$$

This does not assert that velocity and acceleration are dimensionally identical: their numerical magnitudes in the specified SI units are equal.

Step 3: Verify the result and reject other periods With this angular frequency, the speed is $0.0225\,\mathrm{m\,s^{-1}}$, and the acceleration magnitude is $0.0225\,\mathrm{m\,s^{-2}}$, as required. The equality fixes a unique positive angular frequency. The periods $4\pi/3$, $3\pi/8$ and $7\pi/3$ seconds give different angular frequencies, so none reproduces that equality.

Step 4: Conclusion The periodic time is $\boxed{8\pi/3\,\mathrm{s}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the 3-4-5 triangle in the SHM speed formula, then equate the numerical SI values before converting angular frequency to period.

54. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is - [4 marks]

(A) 11×10^{-5} W

(B) 11×10^{-3} W

(C) 11×10^{5} W

(D) 11×10^{-4} W

Correct Answer: (A) 11×10^{-5} W

Solution:

Concept:

- For an ohmic resistor, electrical power is $P = I^2R = V^2/R$. Choose the expression matching the known current or voltage.
- The same resistor has the same resistance in both situations. An ideal voltage supply fixes the voltage across it, while its current follows from Ohm's law.

Step 1: Determine the unknown resistance The initial current is $I_1 = 2\,\text{mA} = 2 \times 10^{-3}\,\text{A}$, while the initial power is $P_1 = 4.4\,\text{W}$. Thus

$$R = \frac{P_1}{I_1^2} = \frac{4.4}{(2 \times 10^{-3})^2} = \frac{4.4}{4 \times 10^{-6}} = 1.1 \times 10^{6}\,\Omega.$$

Squaring the milliampere conversion is essential: its factor becomes 10^{-6}, not 10^{-3}. The large resistance is consistent with substantial power at such a small current.

Step 2: Apply the new voltage supply The supply now maintains $V_2 = 11\,\text{V}$ across this same resistance. The new power is

$$P_2 = \frac{V_2^2}{R} = \frac{121}{1.1 \times 10^{6}} = 1.1 \times 10^{-4}\,\text{W}.$$

To compare directly with the printed alternatives, rewrite this as

$$P_2 = 11 \times 10^{-5}\,\text{W}.$$

There is no reason to retain the original 2 mA current when the new voltage is imposed.

Step 3: Check the scale and the options The new current is $I_2 = 11/(1.1 \times 10^{6}) = 10^{-5}\,\text{A}$. Its product with 11 V gives the same $1.1 \times 10^{-4}\,\text{W}$. The other choices 11×10^{-3}, 11×10^{5} and $11 \times 10^{-4}\,\text{W}$ exceed this result by factors of 100, 10^{10} and 10 respectively, and are inconsistent with this megaohm resistance.

Step 4: Conclusion The dissipated power is $\boxed{11 \times 10^{-5}\,\text{W}}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the resistance from the first experiment; changing to a voltage supply requires V squared over R, rather than reusing the old current.

55. Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by gas is close to – (Gas constant R = 8.31 J/mol.K) [4 marks]

(A) 581 J

(B) 73 J

(C) 146 J

(D) 291 J

Correct Answer: (D) 291 J

Solution:

Concept:

- At constant pressure, work done by a gas is $W = P(V_f - V_i)$. Heating an ideal gas at fixed pressure causes expansion and positive work.
- From $PV = nRT$, the isobaric work is $W = nR\Delta T$. A Celsius temperature difference equals the same difference in kelvins.

Step 1: Identify the temperature change The amount of gas is $n = 0.5\,\text{mol}$, and its temperature rises from 20°C to 90°C. Therefore

$$\Delta T = 90 - 20 = 70\,\text{K}.$$

One could equivalently subtract the absolute temperatures 363.15 K and 293.15 K. Converting either Celsius temperature alone into a temperature change would be incorrect. The pressure is constant at 1 atm throughout.

Step 2: Calculate the expansion work Use the ideal-gas equation at the two end states with the same pressure:

$$P\Delta V = nR(T_f - T_i).$$

Since the pressure is constant, this is also the boundary work. Substitution gives

$$W = 0.5 \times 8.31 \times 70 = 290.85\,\text{J}.$$

Thus the work done by the gas is positive and close to 291 J. The pressure need not be converted to pascals because it has been eliminated algebraically using the gas law.

Step 3: Distinguish work from heat and compare choices The monoatomic nature sets the heat capacities, but does not change the isobaric work formula. Heat supplied would be $nC_P\Delta T$, while the internal-energy rise would be $nC_V\Delta T$; neither is the work asked here. The 581 J choice would correspond approximately to using one mole instead of half a mole. The 146 J and 73 J choices underestimate the expansion work by factors of about two and four, respectively.

Step 4: Conclusion The work done by the gas is $\boxed{291\,\text{J}}$, to the nearest offered value. Hence, the correct option is (D).

Quick Tip to Solve: For constant-pressure heating of an ideal gas, use nR times the temperature rise; the pressure and atomicity do not enter the final work formula.

56. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is : [4 marks]

(A) 508 pJ

(B) 692 pJ

(C) 560 pJ

(D) 600 pJ

Correct Answer: (A) 508 pJ

Solution:

Concept:

- Once the charging battery is disconnected, charge Q remains constant on the isolated capacitor. Filling its gap with dielectric increases its capacitance to $C_f = KC_0$.
- At fixed charge, stored electrostatic energy is $U = Q^2/(2C)$. The work done by the capacitor on an entering dielectric equals the decrease in its stored energy.

Step 1: Find the initial charge and energy The initial capacitance is $C_0 = 12\,\text{pF} = 12 \times 10^{-12}\,\text{F}$, charged to $V_0 = 10\,\text{V}$. Hence

$$Q = C_0 V_0 = 120\,\text{pC},$$

$$U_i = \frac{1}{2} C_0 V_0^2 = \frac{1}{2}(12 \times 10^{-12})(10)^2 = 600\,\text{pJ}.$$

Disconnecting the battery prevents any subsequent supply or removal of charge. It does not keep the potential difference at 10 V during insertion.

Step 2: Evaluate the final energy at fixed charge The dielectric constant is $K = 6.5$, so the filled capacitance becomes $C_f = 6.5C_0$. Keeping the same Q gives

$$U_f = \frac{Q^2}{2KC_0} = \frac{U_i}{K} = \frac{600}{6.5}\,\text{pJ} \approx 92.31\,\text{pJ}.$$

The potential correspondingly falls to V_0/K. Using $\frac{1}{2}C_f V_0^2$ instead would incorrectly describe a capacitor still held at the original voltage by its battery.

Step 3: Calculate the work and check the alternatives The attractive electric force transfers the lost field energy to the slab:

$$W = U_i - U_f = 600\left(1 - \frac{1}{6.5}\right)\,\text{pJ} \approx 507.69\,\text{pJ}.$$

The work is positive and less than the initial 600 pJ. Thus 692 pJ is impossible because it exceeds all the initially stored energy. The value 600 pJ would require zero final field energy, and 560 pJ does not give the required final energy of 92.31 pJ.

Step 4: Conclusion The work done by the capacitor on the slab is approximately $\boxed{508\,\text{pJ}}$. Hence, the correct option is (A).

Quick Tip to Solve: For dielectric insertion, decide first whether charge or voltage stays fixed; a disconnected capacitor releases the fraction (1 - 1/K) of its initial energy.

57. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C. (Specific heat of brass is 394 J kg^{-1} K^{-1}) [4 marks]

(A) 458 J kg^{-1} K^{-1}

(B) 1232 J kg^{-1} K^{-1}

(C) 654 J kg^{-1} K^{-1}

(D) 916 J kg^{-1} K^{-1}

Correct Answer: (D) 916 J kg^{-1} K^{-1}

Solution:

Concept:

- For an insulated calorimeter, the heat lost by the hot metal equals the heat gained by the water and the brass vessel. Use $Q = mc\Delta T$ for each body.
- The water and brass start at the same temperature and reach the same final temperature. Their heat capacities therefore add; the metal has a different temperature fall.

Step 1: Identify the temperature changes The unknown metal cools from $100°\text{C}$ to $21.5°\text{C}$, a fall of $78.5\,\text{K}$. The water and calorimeter warm from $8.4°\text{C}$ to $21.5°\text{C}$, a rise of $13.1\,\text{K}$. Convert the masses to kilograms: metal 0.192, water 0.240, brass 0.128.

Step 2: Balance the heat transfers Using the usual specific heat of water, $c_w = 4200\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$, and writing the unknown specific heat as c, conservation of energy gives

$$0.192c(78.5) = [0.240(4200) + 0.128(394)](13.1).$$

The water heat capacity is $1008\,\text{J}\,\text{K}^{-1}$, and the brass contributes $50.432\,\text{J}\,\text{K}^{-1}$; neither contribution should be omitted.

Step 3: Evaluate and check the alternatives Thus

$$c = \frac{1058.432 \times 13.1}{0.192 \times 78.5} \simeq 920\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}.$$

With the more precise water value $4180\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$, this becomes approximately $916\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$, the listed value. The choices 458, 654 and 1232 fail the same heat-balance equation. The small difference between 916 and 920 is due to the convention for water specific heat, not a different physical process.

Step 4: Conclusion The specific heat is approximately $9.16 \times 10^2\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: Add the heat capacities of the water and vessel first; both warm through the same temperature interval.

58. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license what broadcast frequency will you allot ? [4 marks]

(A) 2900 kHz

(B) 2750 kHz

(C) 2250 kHz

(D) 2000 kz

Correct Answer: (D) 2000 kz

Solution:

Concept:

- An amplitude-modulated carrier of frequency f_c with maximum modulation frequency f_m occupies the interval $f_c - f_m$ to $f_c + f_m$. Its bandwidth is $2f_m$.
- Two stations using the same modulation bandwidth require carrier separation of at least $2f_m$, so that their sideband intervals do not overlap.

Step 1: Find the present carrier The modulation frequency is $250\,\text{kHz}$, stated to be ten percent of the carrier frequency. Therefore

$$250 = 0.10 f_c, \qquad f_c = 2500\,\text{kHz}.$$

Do not confuse the modulation frequency with the carrier itself: the radiated spectrum is centred on 2500 kHz.

Step 2: Locate the occupied channel The lower and upper sideband edges of this station are

$$2500 - 250 = 2250\,\text{kHz}, \qquad 2500 + 250 = 2750\,\text{kHz}.$$

Thus its occupied channel is 500 kHz wide. Assigning a new carrier at either existing edge does not by itself avoid overlap, since the new carrier also has sidebands extending on both sides.

Step 3: Test the proposed new carriers For a new station with the same maximum modulation frequency, a 2000 kHz carrier occupies 1750–2250 kHz. It just meets the existing channel at its lower boundary without an overlapping frequency interval. Carriers at 2250 or 2750 kHz are only 250 kHz away and overlap extensively. A 2900 kHz carrier is 400 kHz away, still less than the required 500 kHz separation; its lower sideband extends to 2650 kHz and overlaps the old channel. Among the listed allocations, only 2000 kHz provides the required spacing.

Step 4: Conclusion Allocate a carrier frequency of $2000\,\text{kHz}$, treating the printed "kz" as the frequency unit kHz. Hence, the correct option is (D).

Quick Tip to Solve: Compare carrier separation with the full AM bandwidth, not with only one sideband width.

59. At some location on earth, the horizontal component of earth's magnetic field is 18 $\times$ 10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is - [4 marks]

(A) 3.6×10^{-5} N

(B) 1.8×10^{-5} N

(C) 1.3×10^{-5} N

(D) 6.5×10^{-5} N

Correct Answer: (D) 6.5×10^{-5} N

Solution:

Concept:

- The dip angle δ satisfies $\tan\delta = B_V/B_H$, where B_V and B_H are the vertical and horizontal components of the terrestrial field.
- A magnetic needle of pole strength p and pole separation L has magnetic moment $\mu = pL$. In a horizontal orientation its tipping torque is μB_V.

Step 1: Determine the vertical field The freely suspended needle aligns with the local magnetic field. Since it makes 45° with the horizontal,

$$B_V = B_H \tan 45^\circ = 18 \times 10^{-6}\,\text{T}.$$

The total field is larger by a factor of $\sqrt{2}$, but it is the vertical component alone that tips a horizontal needle.

Step 2: Balance torques about the suspension point The magnetic moment is

$$\mu = pL = (1.8)(0.12) = 0.216\,\text{A m}^2.$$

When the needle is horizontal, the magnetic torque has magnitude μB_V. A vertical force applied at one end has a perpendicular lever arm $L/2 = 0.06\,\text{m}$. Hence

$$F\frac{L}{2} = pLB_V, \qquad F = 2pB_V.$$

Choose the force direction so that its torque opposes the magnetic tipping torque; the supporting thread adjusts its tension to maintain translational equilibrium.

Step 3: Substitute and distinguish the choices Numerically,

$$F = 2(1.8)(18 \times 10^{-6}) = 64.8 \times 10^{-6}\,\text{N} = 6.48 \times 10^{-5}\,\text{N}.$$

This rounds to 6.5×10^{-5} N. The needle length cancels because both torques are proportional to it. A calculation using the full length as the applied-force lever arm would miss the factor two. The three smaller listed forces cannot balance the magnetic torque with the actual half-length lever arm.

Step 4: Conclusion The required vertical force has magnitude 6.5×10^{-5} N. Hence, the correct option is (D).

Quick Tip to Solve: For a needle suspended at its midpoint, an end force acts through half the needle length, giving $F = 2pB_V$.

60. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20,000 Hz) [4 marks]

(A) 4

(B) 7

(C) 6

(D) 5

Correct Answer: (C) 6

Solution:

Concept:

- A pipe closed at one end has a displacement node at the closed end and an antinode at the open end. Its allowed frequencies are odd multiples of its fundamental.
- An overtone is an allowed frequency above the fundamental. Counting all audible allowed modes includes one frequency that must subsequently be removed.

Step 1: Write the allowed spectrum For fundamental frequency $f_1 = 1.5\,\text{kHz}$, the allowed frequencies are

$$f_n = (2n-1)f_1, \qquad n = 1, 2, 3, \ldots.$$

Thus the successive modes have frequencies 1.5, 4.5, 7.5, 10.5, 13.5, 16.5, 19.5, 22.5 kHz and so on. Even harmonics such as 3.0 and 6.0 kHz are absent for an ideal closed pipe.

Step 2: Apply the hearing limit The upper audible frequency is $20000\,\text{Hz} = 20\,\text{kHz}$. Therefore

$$(2n-1)(1.5) \le 20, \qquad 2n-1 \le \frac{40}{3}.$$

The largest permitted odd multiplier is 13: it gives 19.5 kHz. The next odd multiplier, 15, gives 22.5 kHz and lies beyond the stated hearing limit. Consequently seven allowed modes, corresponding to multipliers 1 through 13 in odd steps, are audible.

Step 3: Count only the overtones The multiplier 1 represents the fundamental, not an overtone. Remove it from the seven audible modes, leaving six audible overtones: 4.5, 7.5, 10.5, 13.5, 16.5 and 19.5 kHz. The answer 7 incorrectly counts the fundamental. The answers 4 and 5 omit one or more permitted odd harmonics below 20 kHz; no such omission is justified by the given ideal-pipe assumptions.

Step 4: Conclusion The number of distinctly audible overtones is 6. Hence, the correct option is (C). This differs from the question-sheet key value 7, which counts seven modes including the fundamental; an overtone excludes the fundamental, as the explicit frequency list proves.

Quick Tip to Solve: List the odd harmonics below the hearing limit, then subtract the fundamental to obtain the overtone count.

Chemistry (120 marks)

61. Elevation in the boiling point for 1 molar solution of glucose is 2 K. The depression in the freezing point for 2 molal solution of glucose in the same solvent is 2 K. The relation between K_b and K_f is [4 marks]

(A) $K_b = K_f$

(B) $K_b = 0.5\ K_f$

(C) $K_b = 1.5\ K_f$

(D) $K_b = 2\ K_f$

Correct Answer: As printed, the ratio is underdetermined because molarity cannot be converted to molality without density. The conventional intended answer is (D), $K_b = 2K_f$, only if the first concentration is interpreted as 1 molal.

Solution:

Concept:

- For glucose, a non-electrolyte, $i = 1$. The colligative-property equations are $\Delta T_b = K_b m$ and $\Delta T_f = K_f m$, with m the molality in moles per kilogram of solvent.
- Molarity is moles per litre of solution; it is not molality. Conversion requires the solution density (and solute molar mass). A numerical concentration without matching units cannot be substituted into these equations.

Step 1: Use the freezing-point data The second solution is explicitly 2 molal and its freezing-point depression is 2 K. Thus

$$K_f = \frac{\Delta T_f}{m} = \frac{2}{2} = 1\,\mathrm{K\,kg\,mol^{-1}}.$$

This determination does not involve the molarity of the first solution and is fixed by the stated data.

Step 2: Keep the first concentration literal The first solution is stated to be 1 molar, not 1 molal. Let its unknown molality be m_1. Its boiling-point elevation gives

$$K_b m_1 = 2, \qquad K_b = \frac{2}{m_1}.$$

One litre contains one mole of glucose, approximately 180 g. If its density is ρ in $\mathrm{kg\,L^{-1}}$, the solvent mass in that litre is $\rho - 0.180$ kg, so

$$m_1 = \frac{1}{\rho - 0.180}.$$

No density is supplied, and the solvent is not identified sufficiently to infer it.

Step 3: Separate the literal result from the conventional interpretation Consequently the printed information does not determine a unique numerical ratio K_b/K_f. If "1 molar" were instead "1 molal", the usual intended calculation would give $K_b = 2$ and hence $K_b = 2K_f$, option (D). That calculation requires an extra concentration assumption which is not a consequence of the literal wording. None of options (A)–(D) follows uniquely from the stated molarity; selecting one without this qualification confuses the two definitions.

Step 4: Conclusion As printed, the ratio is underdetermined because molarity cannot be converted

to molality without density. The conventional intended answer is (D), $K_b = 2K_f$, only if the first concentration is interpreted as 1 molal.

Quick Tip to Solve: Check that both concentrations are molal before comparing boiling-point elevation with freezing-point depression.

62. What will be the major product in the following mononitration reaction? Benzanilide, $C_6H_5-NH-CO-C_6H_5$, is treated with HNO_3 and concentrated H_2SO_4. [4 marks]

(A) p-$NO_2-C_6H_4-NH-CO-C_6H_5$

(B) $C_6H_5-NH-CO-C_6H_4-NO_2$, with NO_2 para to $CO-NH-C_6H_5$.

(C) m-$NO_2-C_6H_4-NH-CO-C_6H_5$

(D) $C_6H_5-NH-CO-C_6H_4-NO_2$, with NO_2 ortho to $CO-NH-C_6H_5$.

Correct Answer: (A) p-$NO_2-C_6H_4-NH-CO-C_6H_5$

Solution:

Concept:

- Nitration with HNO_3/H_2SO_4 is electrophilic aromatic substitution through the nitronium ion, NO_2^+. The relative activation and directing effects of substituents decide which ring reacts preferentially.
- An acylamino substituent, $-NHCOPh$, donates by resonance to its attached ring and directs substitution ortho/para. A directly attached carbonyl substituent, $-CONHPh$, withdraws electrons and deactivates its ring.

Step 1: Distinguish the two aromatic rings Benzanilide contains two benzene rings joined through $-NH-CO-$. The ring attached directly to nitrogen sees an acylamino group. Although the nitrogen lone pair is also conjugated with the carbonyl, it can donate electron density into this ring and stabilise its ortho/para substitution intermediates. The other ring is attached directly to the carbonyl carbon and is deactivated by the electron-withdrawing carbonyl group.

Step 2: Choose the favoured site The more reactive nitrogen-side ring is nitrated preferentially. Within that ring, both ortho and para positions receive resonance activation, but para substitution avoids the steric crowding between the entering nitro group and the bulky benzamido substituent. The major mononitration product is therefore the para-nitro derivative on the nitrogen-side ring, p-$NO_2-C_6H_4-NH-CO-C_6H_5$.

Step 3: Reject the other structures Option (B) puts the nitro group para to a carbonyl substituent on the less reactive benzoyl ring; that is neither the preferred ring nor its usual directing position. Option (C) nitrates meta on the nitrogen-side ring, contrary to its ortho/para resonance direction. Option (D) places nitro ortho on the carbonyl-side ring, again choosing the deactivated ring and an unfavoured direction. Option (A) alone combines the activated ring with its less hindered para site.

Step 4: Conclusion The major product is para-nitrobenzanilide with nitration on the nitrogen-side phenyl ring. Hence, the correct option is (A).

Quick Tip to Solve: In an aromatic amide, first identify which ring is bonded to nitrogen and which is bonded to the carbonyl carbon; their directing effects differ.

63. An aromatic compound 'A' having molecular formula $C_7H_6O_2$ on treating with aqueous ammonia and heating forms compound 'B'. The compound 'B' on reaction with molecular bromine and potassium hydroxide provides compound 'C' having molecular formula C_6H_7N.. The structure of 'A' is [4 marks]

(A) 4-Hydroxybenzaldehyde, p-HO$-C_6H_4-$CHO.

(B) Benzoic acid, C_6H_5COOH.

(C) 2-Hydroxybenzaldehyde, o-HO$-C_6H_4-$CHO.

(D) 3-(Furan-2-yl)prop-2-enal (a furan ring bearing $-CH=CH-CHO$ at its 2-position).

Correct Answer: (B) Benzoic acid, C_6H_5COOH.

Solution:

Concept:

- A carboxylic acid reacts with ammonia to form an ammonium carboxylate, which loses water on heating to form the corresponding primary amide.
- Hofmann bromamide degradation converts $RCONH_2$ into RNH_2 using bromine and aqueous alkali. The carbonyl carbon is lost, so the amine contains one fewer carbon atom.

Step 1: Work backwards from the amine The product has molecular formula C_6H_7N, which is that of aniline, $C_6H_5NH_2$. In Hofmann degradation the group attached to the amide carbonyl migrates to nitrogen. Therefore aniline must come from benzamide, $C_6H_5CONH_2$, rather than from an aldehyde or a nitrogen-free ring transformation. The intermediate B consequently contains seven carbon atoms.

Step 2: Identify the acid and write the transformations The acid that yields benzamide with aqueous ammonia followed by heating is benzoic acid:

$$C_6H_5COOH + NH_3 \longrightarrow C_6H_5COONH_4$$

$$C_6H_5COONH_4 \xrightarrow{\Delta} C_6H_5CONH_2 + H_2O.$$

Its molecular formula is $C_7H_6O_2$, exactly that given for A. The subsequent degradation is

$$C_6H_5CONH_2 + Br_2 + 4\,KOH \longrightarrow C_6H_5NH_2 + 2\,KBr + K_2CO_3 + 2\,H_2O.$$

This equation also shows explicitly where the missing carbon goes.

Step 3: Check the four proposed starting compounds Option (B) is benzoic acid and gives both the required amide intermediate and final aniline. Options (A) and (C) are hydroxybenzaldehydes: although they share the formula $C_7H_6O_2$, they contain an aldehyde and phenolic group, not a carboxylic acid, so the stated simple amidation sequence does not apply. Option (D) has a furan ring and an unsaturated aldehyde side chain; it likewise cannot provide benzamide and then aniline through the two specified reactions.

Step 4: Conclusion Compound A is benzoic acid, C_6H_5COOH; B is benzamide and C is aniline. Hence, the correct option is (B).

Quick Tip to Solve: In a Hofmann degradation sequence, add one carbonyl carbon to the product amine to reconstruct the precursor amide and then its acid.

64. The difference in the number of unpaired electrons of a metal ion in its high spin and low-spin octahedral complexes is two. The metal ion is : [4 marks]

(A) Mn^{2+}

(B) Ni^{2+}

(C) Co^{2+}

(D) Fe^{2+}

Correct Answer: (C) Co^{2+}

Solution:

Concept:

- In an octahedral field the three lower-energy t_{2g} orbitals and two higher-energy e_g orbitals are filled according to the competition between splitting energy and pairing energy.
- A weak-field high-spin complex maximises unpaired electrons, whereas a strong-field low-spin complex pairs electrons in t_{2g} before occupying e_g.

Step 1: Find the d-electron counts Remove the outer 4s electrons first when forming the divalent ions. Thus Mn^{2+}, Ni^{2+}, Co^{2+} and Fe^{2+} have d^5, d^8, d^7, d^6 configurations respectively. Count singly occupied orbitals rather than just the number of electrons in the upper set.

Step 2: Compare the high-spin and low-spin occupancies For d^5, high spin is $t_{2g}^3 e_g^2$ with five unpaired electrons and low spin is t_{2g}^5 with one: the difference is four. For octahedral d^8, both cases have $t_{2g}^6 e_g^2$, with two unpaired electrons, so the difference is zero. For d^7, high spin is $t_{2g}^5 e_g^2$, with three unpaired electrons; low spin is $t_{2g}^6 e_g^1$, with one. Its difference is exactly two. Finally, d^6 changes from $t_{2g}^4 e_g^2$ with four unpaired electrons to t_{2g}^6 with none, a difference of four.

Step 3: Conclusion Only the d^7 ion Co^{2+} has the required difference, $3 - 1 = 2$. Hence, the correct option is (C).

Quick Tip to Solve: For a spin-state comparison, draw the five split d orbitals and count single occupancies in both arrangements; do not confuse the total d count with the unpaired count.

65. The ground state energy of hydrogen atom is – 13.6 eV. The energy of second excited state of He^+ ion in eV is : [4 marks]

(A) -6.04

(B) -54.4

(C) -27.2

(D) -3.4

Correct Answer: (A) -6.04

Solution:

Concept:

- A one-electron ion follows the hydrogen-like energy expression $E_n = -13.6Z^2/n^2$ eV, with zero energy assigned to the separated electron and nucleus.

- The ground state is $n = 1$, the first excited state is $n = 2$, and the second excited state is $n = 3$. For He^+, the nuclear charge is $Z = 2$.

Step 1: Identify the ion and quantum number Helium has atomic number two. Removing one electron gives the one-electron ion He^+, so the Bohr energy formula applies directly. The word second counts excitations above the ground state; it does not mean the second energy level. We therefore use $Z = 2$ and $n = 3$.

Step 2: Substitute and check the options The energy is

$$E_3 = -13.6\frac{2^2}{3^2} = -\frac{54.4}{9} = -6.044\ldots \text{ eV}.$$

Rounding to the precision of the options gives -6.04 eV. The value -54.4 eV is the ground-state energy of this ion, not an excited-state energy. The value -3.4 eV is the hydrogen $n = 2$ energy and omits the correct nuclear-charge scaling. The value -27.2 eV results from neither the required Z^2 factor nor the required $n^2 = 9$ denominator. The negative sign is necessary because the electron remains bound; excitation makes the energy less negative, rather than positive.

Step 3: Conclusion The energy of the second excited state of He^+ is $\boxed{-6.04 \text{ eV}}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the actual principal quantum number before substituting: the kth excited state has $n = k + 1$. Scale hydrogen energies by Z^2 only for one-electron species.

66. Which of the following tests cannot be used for identifying amino acids? [4 marks]

(A) Ninhydrin test

(B) Barfoed test

(C) Xanthoproteic test

(D) Biuret test

Correct Answer: (B) Barfoed test

Solution:

Concept:

- Ninhydrin detects free amino groups in amino acids. The xanthoproteic reaction detects nitration of suitable aromatic amino-acid side chains.
- Barfoed's test is a carbohydrate test: reducing monosaccharides reduce copper(II) in acidic medium to red copper(I) oxide. The biuret test instead requires peptide bonds.

Step 1: Identify the clearly unrelated test Barfoed's reagent contains copper(II) acetate in dilute acetic acid. Its characteristic precipitate is formed by reducing sugars under the test conditions. It does not identify the amino and carboxyl functional groups of an amino acid. Consequently, Barfoed's test is the intended exception among the listed biochemical tests.

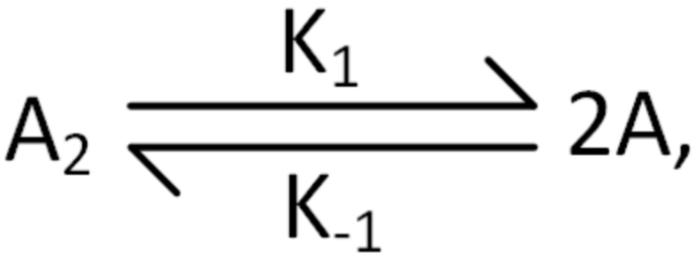
Step 2: Distinguish the scope of the remaining tests Ninhydrin is the general amino-acid test: most alpha-amino acids give the familiar purple colour, while some imino acids behave differently. The

xanthoproteic test can help identify aromatic amino acids such as tyrosine and tryptophan, but is not universal for every amino acid. The biuret test gives a violet complex with copper(II) in alkaline medium when enough peptide bonds are present, as in proteins and suitable peptides. A free amino acid has no peptide bonds and therefore does not give this test. Thus, if the wording is interpreted strictly as isolated free amino acids, biuret is also inapplicable; it is a protein/peptide test rather than a direct test for a free amino acid. This distinction must be retained rather than claiming that free amino acids give a positive biuret test.

Step 3: Conclusion The intended answer is Barfoed test. Hence, the correct option is (B). Strictly, the wording is non-unique for free amino acids because the biuret test also requires peptide bonds; it does not identify free amino acids.

Quick Tip to Solve: Associate each test with the group it actually detects: free amino groups for ninhydrin, aromatic rings for xanthoproteic, peptide bonds for biuret, and reducing sugars for Barfoed.

67. For an elementary chemical reaction,
the expression for $\frac{d[A]}{dt}$ is [4 marks]



(A) $2K_1[A_2] - K_{-1}[A]^2$
(B) $K_1[A_2] - K_{-1}[A]^2$
(C) $K_1[A_2] + K_{-1}[A]^2$
(D) $2K_1[A_2] - 2K_{-1}[A]^2$
Correct Answer: (D) $2K_1[A_2] - 2K_{-1}[A]^2$

Solution:

Concept:

- For an elementary step, the powers in its rate law follow its molecularity: $r_f = K_1[A_2]$ and $r_b = K_{-1}[A]^2$.
- The rate of concentration change includes the stoichiometric coefficient. Each forward event makes two A particles and each reverse event consumes two A particles.

Step 1: Write the two event rates The printed reaction is $A_2 \rightleftharpoons 2A$. The forward direction is unimolecular in the dimer, giving $r_f = K_1[A_2]$. The reverse direction involves two A particles, so $r_b = K_{-1}[A]^2$. These are rates of reaction events per unit volume; neither is yet the concentration rate for A itself.

Step 2: Apply stoichiometry and subtract consumption The forward contribution to $d[A]/dt$ is $+2r_f$;

the reverse contribution is $-2r_b$. Therefore

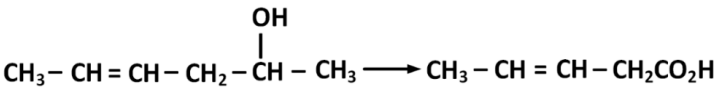
$$\frac{d[\mathrm{A}]}{dt} = 2K_1[\mathrm{A}_2] - 2K_{-1}[\mathrm{A}]^2.$$

The factor two must multiply both terms because two A particles are involved in either direction. Option (A) omits it only from the reverse term. Option (B) gives the net event rate rather than the rate of change of A concentration. Option (C) also adds the reverse rate, although the reverse process removes A. As an additional check, at equilibrium the expression vanishes when $K_1[\mathrm{A}_2] = K_{-1}[\mathrm{A}]^2$, which is exactly equality of the forward and reverse event rates.

Step 3: Conclusion The required expression is $\boxed{2K_1[\mathrm{A}_2] - 2K_{-1}[\mathrm{A}]^2}$. Hence, the correct option is (D).

Quick Tip to Solve: First write each elementary event rate, then multiply by the number of particles of the species formed or consumed; the stoichiometric factor belongs on both directions.

68. Which is the most suitable reagent for the following transformation ? [4 marks]



(A) alkaline $KMnO_4$

(B) Tollen's reagent

(C) I_2/NaOH

(D) CrO_2Cl_2/CS_2

Correct Answer: (C) I_2/NaOH

Solution:

Concept:

- A methyl carbinol, $R{-}CH(OH){-}CH_3$, is oxidised by iodine in alkali to a methyl ketone and then undergoes the haloform reaction.
- The haloform reaction cleaves $RCOCH_3$ to $RCOO^-$ and yellow CHI_3; acidification gives $RCOOH$.

Step 1: Identify the structural change The starting alcohol is $CH_3CH{=}CHCH_2CH(OH)CH_3$. Let $R = CH_3CH{=}CHCH_2$. Then the alcohol has precisely the $RCH(OH)CH_3$ group required for the iodoform reaction. The target acid $CH_3CH{=}CHCH_2CO_2H$ has one fewer carbon: the terminal methyl carbon is lost while the carbon bearing the hydroxyl group becomes the carboxyl carbon. The remote double bond is retained.

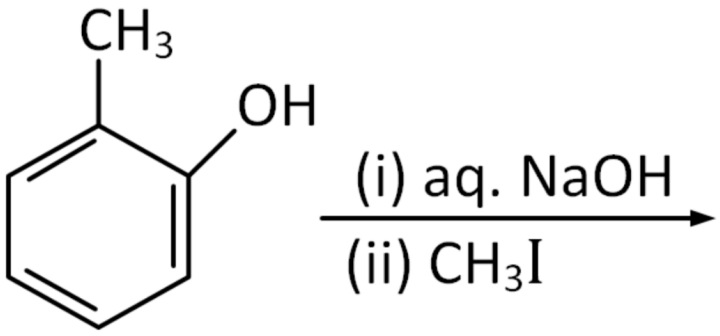
Step 2: Choose the selective reagent and reject alternatives With $I_2/NaOH$, oxidation first gives $RCOCH_3$. Repeated iodination at the methyl group followed by alkaline cleavage gives $RCOO^- + CHI_3$. The acid shown is the protonated product after acid work-up. Alkaline $KMnO_4$ is unsuitable because it can attack the carbon–carbon double bond and does not selectively perform this one-carbon-shortening haloform transformation. Tollens'reagent is mainly used to oxidise aldehydes; it does not give this conversion of a secondary methyl alcohol. CrO_2Cl_2/CS_2 is associated with Etard oxidation of

appropriate benzylic methyl groups, not the required methyl-carbinol cleavage. Only iodine in alkali directly explains the retained alkene and the missing terminal carbon together.

Step 3: Conclusion Use $\boxed{I_2/NaOH}$, followed by acidification of the carboxylate to give the acid shown. Hence, the correct option is (C).

Quick Tip to Solve: When a methyl secondary alcohol becomes an acid with one fewer carbon, look for the iodoform reaction: the removed methyl carbon leaves as CHI_3.

69. The major product of the following reaction [4 marks]



(A) 1-Methoxy-2-methylbenzene 

(B) 2,4-Dimethylphenol 

(C) 2,5-Dimethylphenol 

(D) 2,6-Dimethylphenol 

Correct Answer: (A) 1-Methoxy-2-methylbenzene 

Solution:

Concept:

- Phenols are weak acids. Aqueous $NaOH$ removes the phenolic proton to give a phenoxide ion, whose oxygen is nucleophilic.
- Williamson ether synthesis forms a carbon–oxygen bond when an alkoxide or phenoxide attacks an alkyl halide by the S_N2 mechanism. Methyl iodide is particularly suitable because its methyl carbon is unhindered and iodide is a good leaving group.

Step 1: Form the sodium phenoxide The starting compound is 2-methylphenol, also called o-cresol: its methyl group is adjacent to the hydroxyl group. The first reagent neutralises that hydroxyl group without changing the aromatic carbon skeleton.

$$2\text{-}CH_3C_6H_4OH + NaOH \longrightarrow 2\text{-}CH_3C_6H_4ONa + H_2O$$

The original ring methyl group therefore remains at the same position throughout the reaction.

Step 2: Methylate the oxygen with methyl iodide The negatively charged oxygen attacks the methyl carbon of CH_3I, displacing iodide and forming an ether. The methyl group supplied by methyl iodide

is attached to oxygen, rather than introduced as a second substituent on the aromatic ring.

$$2\text{-}CH_3C_6H_4O^- + CH_3I \longrightarrow 2\text{-}CH_3C_6H_4OCH_3 + I^-$$

This is an S_N2 substitution at the methyl carbon; it does not require an aromatic substitution step or a methyl carbocation.

Step 3: Compare the four structures Option (A) alone replaces the phenolic hydrogen by a methyl group, giving adjacent CH_3 and OCH_3 groups. Options (B), (C) and (D) retain OH and place a second methyl group on the ring, at different positions. Those structures require carbon methylation of the aromatic ring and are not the Williamson ether product under the stated reagents.

Step 4: Conclusion The major product is 1-methoxy-2-methylbenzene (o-methylanisole), $2\text{-}CH_3C_6H_4OCH_3$. Hence, the correct option is (A). The reaction mechanism gives oxygen methylation, so none of the ring-methylated phenols can be selected in place of (A).

Quick Tip to Solve: With phenol followed by $NaOH$ and CH_3I, replace the hydroxyl hydrogen by CH_3; keep all existing ring substituents in place.

70. A reaction of cobalt (III) chloride and ethylenediamine in a 1 : 2 mole ratio generates two isomeric products A (violet coloured) and B (green coloured). A can show optical activity, but B is optically inactive. What type of isomers does A and B represcent? [4 marks]

(A) Ionisation isomers

(B) Linkage isomers

(C) Coordination isomers

(D) Geometrical isomers

Correct Answer: (D) Geometrical isomers

Solution:

Concept:

- Ethylenediamine, abbreviated en, is a neutral bidentate ligand. Two en ligands occupy four coordination sites around octahedral cobalt(III).
- For $[Co(en)_2Cl_2]^+$, the two coordinated chlorides can be adjacent (cis) or opposite (trans), giving geometrical isomers with the same connectivity.

Step 1: Identify the coordination sphere The 1:2 cobalt-to-en ratio supplies two bidentate en ligands per cobalt. They occupy four donor positions, leaving two positions for chloride in an octahedral complex. The products are conventionally written $[Co(en)_2Cl_2]Cl$, with two coordinated chlorides and one counter-ion. The two coloured forms differ in how the coordinated chlorides are arranged, not in the number or identity of ligands.

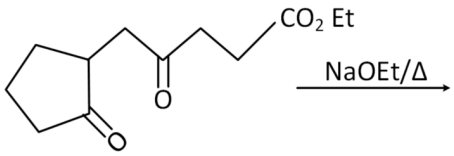
Step 2: Use the optical behaviour to assign cis and trans In the cis arrangement the chloride ligands occupy neighbouring positions. The two chelate rings can give non-superimposable mirror-image arrangements, so the cis form can show optical activity. This is the violet product A. In the trans arrangement the chlorides occupy opposite positions and the complex has the symmetry needed

for optical inactivity; it is the green product B. Thus A and B form a cis/trans pair. They are not ionisation isomers, since no ligand/counter-ion interchange is required. They are not linkage isomers, because en is not changing its donor atom and chloride has no alternative donor end. They are not coordination isomers, which require ligand redistribution between separate complex cations and complex anions. Optical isomerism is possible within the cis form, but the relationship between A and B themselves is geometrical.

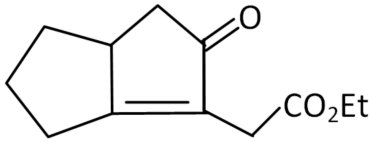
Step 3: Conclusion A is the optically active cis form and B the optically inactive trans form of $[Co(en)_2Cl_2]^+$. They are geometrical isomers. Hence, the correct option is (D).

Quick Tip to Solve: For an octahedral $[M(en)_2X_2]$ complex, check whether the two X ligands are adjacent or opposite before considering optical activity of the cis form.

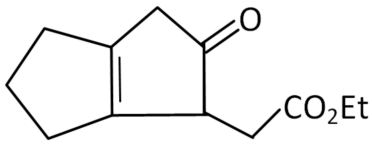
71. The major product obtained in the following reaction is : [4 marks]



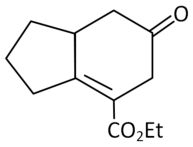
(A) Fused five-five bicyclic enone, double bond conjugated with the ketone; side chain CH_2CO_2Et.



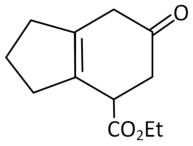
(B) Fused five-five bicyclic ketone with the double bond at the ring junction; side chain CH_2CO_2Et.



(C) Fused five-six bicyclic unsaturated keto ester, with CO_2Et attached to the alkene carbon, as shown.

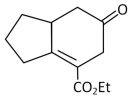


(D) Fused five-six bicyclic ketone with the double bond at the ring junction, as shown.



Correct Answer: (C)

Fused five-six bicyclic unsaturated keto ester, with CO_2Et attached to the alkene carbon, as shown.



Solution:

Concept:

- An enolate attacks a ketone intramolecularly to form a new carbon–carbon bond. Formation of a six-membered ring is particularly favourable.
- On heating, the cyclic hydroxy ester loses water to form an alkene conjugated with the ester carbonyl; conjugation stabilises the product.

Step 1: Identify the attacking carbon The starting material contains a cyclopentanone ring and a side-chain ketone followed by two methylene groups and an ester. Ethoxide removes a hydrogen from the terminal methylene directly adjacent to the ester group. The resulting ester enolate is the nucleophile. Its carbon can attack the carbonyl carbon of the original cyclopentanone ring, making a new carbon–carbon bond within the same molecule.

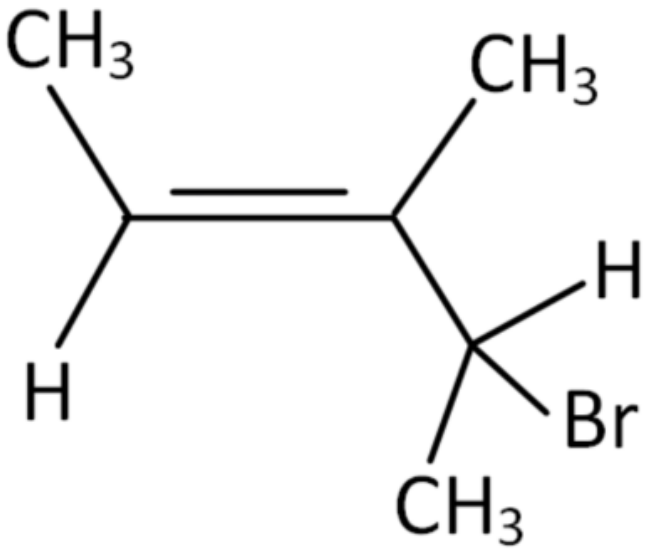
Step 2: Count the ring and eliminate water Follow the chain from the ester-activated methylene through the adjacent methylene, the side-chain ketone carbonyl carbon, the next methylene, the original substituted ring carbon, and the original ring carbonyl carbon. Closing these six atoms makes a six-membered ring fused to the original five-membered ring. This carbon is also alpha to the ester and is readily deprotonated. Proton transfer gives the aldol alcohol. Heating removes water between the attacking carbon and the former ring carbonyl carbon. The newly formed double bond is conjugated with the ester group; the side-chain ketone remains in the six-membered ring.

Step 3: Compare the alternatives Option C has the six-membered newly formed ring and the ester substituent directly on the alkene carbon. Options A and B contain a newly formed five-membered ring and do not follow this favourable closure. Option D has its double bond at the ring junction instead of at the ester-stabilised position. Thus it misses the conjugated product obtained by dehydration.

Step 4: Conclusion The major product is the fused five-six bicyclic unsaturated keto ester drawn in option C. Hence, the correct option is (C).

Quick Tip to Solve: For an intramolecular condensation, mark every enolisable carbon, count the ring formed by each possible attack, and then check conjugation in the dehydration product.

72. What is the IUPAC name of the following compound ? [4 marks]



(A) 4–Bromo-3-methylpent-2-ene

(B) 3–Bromo 1, 2-dimethylbut-1-ene

(C) 3–Bromo-3-methyl-1, 2-dimethylprop-1-ene

(D) 2-Bromo-3-methylpent-3-ene

Correct Answer: (A) 4–Bromo-3-methylpent-2-ene

Solution:

Concept:

- The parent chain is the longest carbon chain containing the carbon–carbon double bond.
- Number the parent chain so that the double bond receives the lowest possible locant; then name substituents alphabetically.

Step 1: Select and number the parent chain Trace the continuous chain from the methyl group attached to the left alkene carbon, through the two alkene carbons, through the carbon bearing bromine, and finally to its methyl group. This chain has five carbon atoms, so the parent is pentene. Numbering from the left methyl end places the double bond between C2 and C3. Numbering from the other end would place it between C3 and C4. The alkene must therefore be numbered as pent-2-ene.

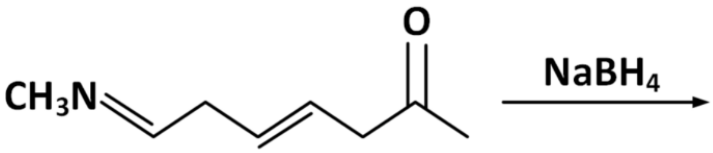
Step 2: Locate and arrange the substituents The methyl group projecting upwards from the right alkene carbon is attached to C3. The bromine atom is attached to C4. These give the prefixes 4-bromo and 3-methyl. Alphabetical ordering puts bromo before methyl; the numerical locants do not determine the alphabetical order. Combining the prefixes with the parent gives 4-bromo-3-methylpent-2-ene. This specifies the connectivity requested by the choices.

Step 3: Reject the shorter or wrongly numbered names The butene and propene alternatives choose shorter chains even though a five-carbon chain containing the double bond is available. They therefore violate parent-chain selection. The name 2-bromo-3-methylpent-3-ene chooses the correct chain length but numbers it from the wrong end: its double-bond locant is 3 rather than the available lower value 2. A lower bromo locant cannot override the multiple-bond numbering rule.

Step 4: Conclusion The IUPAC name represented by the choices is 4-bromo-3-methylpent-2-ene. Hence, the correct option is (A).

Quick Tip to Solve: Give the double bond its lowest locant before comparing the locants of bromo or methyl substituents.

73. The major product of the following reaction is : [4 marks]



(A) $CH_3NH(CH_2)_5COCH_3$

(B) $CH_3N{=}CH(CH_2)_4CH(OH)CH_3$

(C) $CH_3N{=}CHCH_2CH{=}CHCH_2CH(OH)CH_3$

(D) $CH_3NHCH_2CH_2CH{=}CHCH_2CH(OH)CH_3$

Correct Answer: (D) $CH_3NHCH_2CH_2CH{=}CHCH_2CH(OH)CH_3$

Solution:

Concept:

- $NaBH_4$ supplies hydride to polar multiple bonds such as a ketone carbonyl and an imine $C{=}N$ bond.
- An isolated carbon–carbon double bond is not normally hydrogenated by sodium borohydride. Protonation of the reduction intermediates gives an alcohol and an amine.

Step 1: Identify the reducible groups The molecule contains three distinct multiple bonds: a terminal N-methyl imine, an internal carbon–carbon alkene, and a methyl ketone. The carbonyl and imine bonds are polar: their carbon atoms can accept hydride. In contrast, the isolated alkene does not have the electrophilic character needed for ordinary borohydride addition. Therefore the alkene is retained while the two heteroatom-containing double bonds are reduced.

Step 2: Apply the reductions without changing the skeleton The ketone fragment $-COCH_3$ becomes $-CH(OH)CH_3$, a secondary alcohol. At the other end, $CH_3N{=}CH^-$ becomes $CH_3NHCH_2{}^-$, a secondary amine after protonation. Neither reduction breaks a carbon–carbon bond or moves the central alkene. The product therefore has the connectivity $CH_3NHCH_2CH_2CH{=}CHCH_2CH(OH)CH_3$. Both functional-group changes must be included in the same answer. The carbon skeleton is conserved throughout both hydride additions.

Step 3: Compare the four structures Option A retains the ketone and removes the alkene, so it does not follow borohydride selectivity. Option B retains the imine while reducing the alkene, again the wrong selectivity. Option C changes only the ketone and leaves the imine unchanged, so it is incomplete. Option D shows both the amine and alcohol while preserving the original carbon–carbon double bond, exactly as required.

Step 4: Conclusion The product is the unsaturated amino alcohol shown in option D. Hence, the

correct option is (D).

Quick Tip to Solve: For borohydride questions, inspect every polar double bond separately; preservation of an isolated alkene is a useful first filter.

74. In the reaction of oxalate with permanganate in acidic medium, the number of electrons involved in producing one molecule of CO_2 is : [4 marks]

(A) 10

(B) 2

(C) 1

(D) 5

Correct Answer: (C) 1

Solution:

Concept:

- For an ion, the sum of oxidation numbers equals its ionic charge. Oxygen normally has oxidation number -2.
- An increase in oxidation number represents loss of electrons. Count electrons per requested product molecule, rather than per full balanced equation.

Step 1: Find the oxidation state of carbon Let the oxidation state of either equivalent carbon atom in oxalate, $C_2O_4{}^{2-}$, be x. Then

$$2x + 4(-2) = -2, \qquad x = +3.$$

In carbon dioxide the oxidation state y obeys

$$y + 2(-2) = 0, \qquad y = +4.$$

Thus each carbon atom increases its oxidation state from $+3$ to $+4$. This one-unit increase means that each carbon loses one electron.

Step 2: Write the oxidation half-reaction The balanced oxalate oxidation is

$$C_2O_4{}^{2-} \longrightarrow 2\,CO_2 + 2\,e^-.$$

There are two carbon atoms and two electrons lost, but two molecules of carbon dioxide are also produced. Therefore the number of electrons per molecule of carbon dioxide is

$$\frac{2 \text{ electrons}}{2 \text{ molecules of } CO_2} = 1.$$

The acidic permanganate half-reaction consumes five electrons per permanganate ion, but that is a different counting unit.

Step 3: Separate the common distractors Two electrons is the loss per entire oxalate ion, not per carbon dioxide molecule. Five is associated with reduction of one permanganate ion to manganese(II)

in acid. Ten is the electron exchange in the usual combined equation involving two permanganate ions and five oxalate ions. None of these larger equation-based counts is the quantity explicitly requested here.

Step 4: Conclusion One electron is involved in producing each molecule of CO_2. Hence, the correct option is (C).

Quick Tip to Solve: First calculate the oxidation-number change per atom that enters the requested product, then scale only if the question asks for a full equation.

75. 5.1 g NH_4SH is introduced in 3.0 L evacuated flask at 327°C. 30% of the solid NH_4SH decomposed to NH_3 and H_2S as gases . The Kp of the reaction at 327°C is (R = 0.082 L atm mol^{-1} K^{-1}, Molar mass of S = 32 g mol^{-1} molar mass of N = 14 g mol^{-1}) [4 marks]

(A) 0.242×10^{-4} atm^2

(B) 1×10^{-4} atm^2

(C) 4.9×10^{-3} atm^2

(D) 0.242 atm^2

Correct Answer: (D) 0.242 atm^2

Solution:

Concept:

- The decomposition is $NH_4SH(s) \rightleftharpoons NH_3(g) + H_2S(g)$. A pure solid has unit activity and is omitted from K_p.
- Each gaseous partial pressure is obtained from $p_i = n_i RT/V$, using temperature in kelvin. Here $K_p = p_{NH_3} p_{H_2S}$.

Step 1: Find the amount decomposed The molar mass of ammonium hydrosulphide is

$$M = 14 + 5(1) + 32 = 51\,\text{g mol}^{-1}.$$

Thus the initially introduced amount is

$$n_0 = \frac{5.1}{51} = 0.100\,\text{mol}.$$

Only 30 percent decomposes, so the reaction extent is

$$\xi = 0.30(0.100) = 0.0300\,\text{mol}.$$

Each mole of solid produces one mole of each gas, giving 0.0300 mol of ammonia and 0.0300 mol of hydrogen sulphide.

Step 2: Calculate each partial pressure The absolute temperature is $327 + 273 = 600\,\text{K}$. The flask

volume is $3.0\,\mathrm{L}$. For either gas,

$$p_i = \frac{(0.0300)(0.082)(600)}{3.0} = 0.492\,\mathrm{atm}.$$

Their equal amounts in the same volume at the same temperature guarantee equal partial pressures. The total pressure would be twice this value, but the equilibrium expression requires the individual pressures.

Step 3: Evaluate the equilibrium constant The remaining solid does not appear in the expression. Therefore

$$K_p = (0.492)(0.492) = 0.242064\,\mathrm{atm}^2 \simeq 0.242\,\mathrm{atm}^2.$$

The much smaller values in A, B and C are inconsistent with partial pressures of nearly half an atmosphere. They can arise from incorrect powers of ten, use of Celsius in the gas law, or failure to convert the decomposed fraction into moles correctly.

Step 4: Conclusion $K_p = 0.242\,\mathrm{atm}^2$. Hence, the correct option is (D).

Quick Tip to Solve: In a solid-to-two-gases equilibrium, find moles of each gas first; do not substitute total pressure directly into each factor of the equilibrium expression.

76. The amount of sugar ($C_{12}H_{22}O_{11}$) required to prepare 2L of its 0.1 M aqueous solution is :

[4 marks]

(A) 17.1 g

(B) 34.2 g

(C) 68.4 g

(D) 136.8 g

Correct Answer: (C) 68.4 g

Solution:

Concept:

- Molarity is the number of moles of solute per litre of final solution: $c = n/V$.
- The required solute mass is $m = nM_{\mathrm{molar}}$; molarity refers to final solution volume, not to the starting volume of water.

Step 1: Calculate the required moles of sugar The final solution must have volume $2\,\mathrm{L}$ and concentration $0.1\,\mathrm{mol\,L^{-1}}$. Consequently,

$$n = cV = (0.1\,\mathrm{mol\,L^{-1}})(2\,\mathrm{L}) = 0.2\,\mathrm{mol}.$$

The litre units cancel. There is no need to convert this volume to millilitres because the concentration is already expressed per litre. Converting only one of these two quantities would introduce an erroneous factor of one thousand.

Step 2: Convert moles into mass Using atomic masses C = 12, H = 1 and O = 16, the molar mass of

$C_{12}H_{22}O_{11}$ is

$$M_{\text{molar}} = 12(12) + 22(1) + 11(16) = 342\,\text{g mol}^{-1}.$$

The mass needed is therefore

$$m = nM_{\text{molar}} = (0.2)(342) = 68.4\,\text{g}.$$

For preparation, dissolve this mass in some water and then make the final solution volume up to 2 L; adding it to 2 L of water would not guarantee the specified final volume.

Step 3: Check the alternatives against molarity The value 34.2 g corresponds to 0.1 mol and would make only a 0.05 M solution in 2 L. The value 17.1 g is 0.05 mol and gives 0.025 M. The value 136.8 g is 0.4 mol and gives 0.2 M. Only 68.4 g gives the requested 0.2 mol in the specified final volume, confirming the selection without relying solely on the first calculation.

Step 4: Conclusion The mass of sugar required is $68.4\,\text{g}$. Hence, the correct option is (C).

Quick Tip to Solve: Use mass = molarity times final volume in litres times molar mass; checking the units prevents the common millilitre conversion error.

77. The correct match between item 'I' and item 'II' is:

Item I (compound)		Item II (reagent)	
(A)	Lysine	(P)	1-naphthol
(B)	Furfural	(Q)	Ninhydrin
(C)	Benzyl alcohol	(R)	$KMnO_4$
(D)	Styrene	(S)	Ceric ammonium nitrate

[4 marks]

(A) $(A) \to (Q);\ (B) \to (R);\ (C) \to (S);\ (D) \to (P)$
(B) $(A) \to (Q);\ (B) \to (P);\ (C) \to (S);\ (D) \to (R)$
(C) $(A) \to (R);\ (B) \to (P);\ (C) \to (Q);\ (D) \to (S)$
(D) $(A) \to (Q);\ (B) \to (P);\ (C) \to (R);\ (D) \to (S)$
Correct Answer: (B) $(A) \to (Q);\ (B) \to (P);\ (C) \to (S);\ (D) \to (R)$

Solution:

Concept:

- Choose a qualitative reagent by the functional group responsible for its characteristic test; distinguish amino acids, alcohols and unsaturation.
- Ninhydrin detects amino acids, ceric ammonium nitrate detects alcohols, and dilute permanganate detects a carbon–carbon double bond.

Step 1: Match the amino acid and alcohol Lysine is an amino acid containing amino groups. It gives the characteristic ninhydrin colour test, so (A) matches (Q). Benzyl alcohol contains a hydroxyl group attached to a saturated carbon. It gives the ceric ammonium nitrate alcohol test, so (C) matches (S). These two secure matches already eliminate the choices assigning lysine to permanganate or benzyl

alcohol to ninhydrin.

Step 2: Match furfural and styrene Furfural is associated with the 1-naphthol colour reaction used in carbohydrate testing: furfural-type dehydration products condense with the naphthol reagent to produce the characteristic colour. Thus (B) matches (P). Styrene contains a carbon–carbon double bond in its vinyl group. Cold dilute $\mathrm{KMnO_4}$ is decolourised through oxidation of that unsaturation, giving (D) matches (R). This is the familiar Baeyer test for an alkene.

Step 3: Combine and compare all four assignments The complete sequence is

$$(A) \to (Q), \quad (B) \to (P), \quad (C) \to (S), \quad (D) \to (R).$$

Option A wrongly assigns styrene to 1-naphthol and furfural to permanganate instead of the diagnostic pairings. Option C assigns lysine to permanganate and benzyl alcohol to ninhydrin. Option D swaps the diagnostic alcohol and alkene reagents for benzyl alcohol and styrene. Option B alone preserves every characteristic match; a single correct pair would not be enough for a matching question.

Step 4: Conclusion The correct matching is A–Q, B–P, C–S and D–R. Hence, the correct option is (B).

Quick Tip to Solve: Fix the most specific tests first: amino acid–ninhydrin and alcohol–ceric ammonium nitrate make this matching problem almost immediate.

78. The 71st electron of an element X with an atomic number of 71 enters into the orbital : [4 marks]

(A) 4f

(B) 6s

(C) 6p

(D) 5d

Correct Answer: (D) 5d

Solution:

Concept:

- The atomic number is the number of electrons in a neutral atom. For $Z = 71$, the element is lutetium, whose ground-state configuration is $[\mathrm{Xe}]4f^{14}5d^{1}6s^{2}$.
- An orbital can hold two electrons, whereas a complete f subshell holds fourteen. The differentiating electron is identified from the actual ground-state configuration, rather than from the name of the series alone.

Step 1: Write the configuration through ytterbium The xenon core accounts for 54 electrons. The two electrons in $6s$ and fourteen in $4f$ then account for another sixteen. Thus the preceding element, ytterbium, has the closed-subshell configuration $[\mathrm{Xe}]4f^{14}6s^{2}$, containing 70 electrons. In particular, the $4f$ subshell is already filled; it cannot accept a fifteenth electron.

Step 2: Place the remaining electron and compare the choices Adding one electron gives lutetium:

$$54 + 14 + 2 + 1 = 71, \qquad \mathrm{Lu} : [\mathrm{Xe}]4f^{14}5d^{1}6s^{2}.$$

The remaining electron occupies $5d$. Option (A) incorrectly puts it into the already full $4f$ subshell. Option (B) ignores that $6s$ already contains its allowed pair, and option (C) starts the $6p$ subshell before the lower-energy $5d$ state required here. The word "71st" refers to the differentiating electron, not to the outermost shell number.

Step 3: Conclusion The 71st electron enters the $5d$ orbital. Hence, the correct option is (D).

Quick Tip to Solve: For the last lanthanoid, first count the closed $4f^{14}6s^2$ configuration: the next electron is $5d^1$.

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79. An ideal gas undergoes isothermal compression from 5m^3 to 1 m^3 against a constant external pressure of 4 Nm^{-2}. Heat released in this process is used to increase the temperature of 1 mole of Al. If molar heat capacity of Al is 24 J mol^{-1} K^{-1}, the temperature of Al increases by : [4 marks]

(A) $\frac{2}{3}K$

(B) $\frac{3}{2}K$

(C) 1 K

(D) 2 K

Correct Answer: (A) $\frac{2}{3}K$

Solution:

Concept:

- For an ideal gas, internal energy depends only on temperature. An isothermal change therefore has $\Delta U = 0$, and the chemical sign convention gives $\Delta U = q + w$.
- Against constant external pressure, $w = -P_{\text{ext}}(V_f - V_i)$. Heat transferred to a solid sample raises its temperature by $Q = nC_m\Delta T$, where C_m is its molar heat capacity.

Step 1: Find the work done on the gas The volume decreases from 5 to $1\,\text{m}^3$, so $\Delta V = -4\,\text{m}^3$. Use the constant external pressure, not the reversible logarithmic work formula:

$$w = -(4\,\text{N m}^{-2})(-4\,\text{m}^3) = 16\,\text{J}.$$

Work is positive because the surroundings compress the gas. Since its temperature stays fixed, this energy cannot remain as an increase in the ideal gas internal energy.

Step 2: Transfer the released heat to aluminium The first law gives $q = -w = -16\,\text{J}$ for the gas. Consequently the aluminium receives $Q = +16\,\text{J}$. For one mole,

$$\Delta T = \frac{Q}{nC_m} = \frac{16}{1 \times 24}\,\text{K} = \frac{2}{3}\,\text{K}.$$

The sign is positive for aluminium because it gains heat. The value $3/2\,\text{K}$ reverses the ratio of heat capacity and heat, whereas 1 and 2 K would require 24 and 48 J respectively, more than the released heat.

Step 3: Conclusion The temperature of aluminium rises by $\frac{2}{3}\,\text{K}$. Hence, the correct option is (A).

Quick Tip to Solve: For an isothermal ideal-gas compression, the heat released equals the work supplied; divide that energy by the sample heat capacity.

80. The pair that contains two P – H bonds in each of the oxoacids is [4 marks]

(A) H_3PO_2 and $H_4P_2O_5$

(B) $H_4P_2O_5$ and $H_4P_2O_6$

(C) $H_4P_2O_5$ and H_3PO_3

(D) H_3PO_3 and H_3PO_2

Correct Answer: (A) H_3PO_2 and $H_4P_2O_5$

Solution:

Concept:

- Count hydrogen atoms attached directly to phosphorus separately from those attached through oxygen. A P–H bond does not contribute an ionisable proton in these oxoacids; O–H hydrogens determine their basicity.
- The structural units are $H_2P(=O)(OH)$ for hypophosphorous acid and $HP(=O)(OH)_2$ for phosphorous acid. Condensation can join phosphorus centres through an oxygen bridge without removing their directly bonded hydrogens.

Step 1: Count the bonds in each relevant acid In H_3PO_2, two of the three hydrogen atoms are directly attached to phosphorus and the third forms the hydroxyl group. It therefore has two P–H bonds. In H_3PO_3, only one hydrogen is attached directly to phosphorus; its other two hydrogens belong to hydroxyl groups. It therefore has just one P–H bond, even though its molecular formula also contains three hydrogens.

Step 2: Check the condensed two-phosphorus acids Pyrophosphorous acid, $H_4P_2O_5$, has the skeleton $HO-P(=O)(H)-O-P(=O)(H)-OH$. Each phosphorus contributes one P–H bond, giving two in the molecule. In hypophosphoric acid, $H_4P_2O_6$, the phosphorus atoms are directly bonded and each carries hydroxyl groups; its hydrogens are on oxygen, not phosphorus. Thus (B) fails because of $H_4P_2O_6$. Options (C) and (D) fail because H_3PO_3 has only one P–H bond. Only (A) contains two qualifying acids.

Step 3: Conclusion Both H_3PO_2 and $H_4P_2O_5$ contain two P–H bonds per molecule. Hence, the correct option is (A).

Quick Tip to Solve: Count direct P–H bonds from the structure; the total number of H atoms in the molecular formula does not give the answer.

81. The process with negative entropy change is : [4 marks]

(A) Dissociation of $CaSO_4$ (s) to CaO(s) and SO_3(g)

(B) Dissolution of iodine in water

(C) Synthesis of ammonia from N_2 and H_2

(D) Sublimation of dry ice

Correct Answer: (C) Synthesis of ammonia from N_2 and H_2

Solution:

Concept:

- Entropy measures the number of accessible microscopic arrangements. Gas particles have much more freedom of motion than particles confined to a solid, so forming a gas usually increases entropy.
- For reactions involving gases, compare the total gaseous stoichiometric amounts. Converting several moles of gas into fewer moles of gas generally lowers the entropy of the reacting system.

Step 1: Examine ammonia synthesis The balanced reaction is

$$N_2(g) + 3\,H_2(g) \longrightarrow 2\,NH_3(g).$$

Four moles of gaseous reactants become two moles of gaseous product. This substantial reduction in independently moving gas particles reduces the number of available arrangements, so the entropy change for ammonia synthesis is negative. The comparison concerns the system entropy of the stated process, rather than the combined entropy of the system and surroundings.

Step 2: Eliminate the other three processes In option (A), the stated decomposition of a solid produces gaseous SO_3; the appearance of a gas increases disorder relative to the solid reactant. In (B), iodine molecules become dispersed through water instead of remaining together in the iodine crystal, giving the usual increase in mixing entropy. In (D), solid carbon dioxide becomes gaseous carbon dioxide: sublimation greatly increases translational freedom. None of these represents the clear contraction in gaseous particle number seen in ammonia synthesis. A spontaneous process need not have positive system entropy by itself; heat exchange can affect the surroundings.

Step 3: Conclusion Synthesis of ammonia from N_2 and H_2 has a negative entropy change. Hence, the correct option is (C).

Quick Tip to Solve: Balance a gaseous reaction before counting gas moles: ammonia synthesis changes four gas moles into two.

82. In the cell

Pt|(s)|H_2(g, 1 bar)|$HCl\,(aq)$|AgCl|(s)|Ag(s)|Pt(s)

the cell potential is 0.92 V when a 10^{-6} molal HCl solution is used. The standard electrode potential of (AgCl/ $AgCl^-$) electrode is :

{**Given,** $\frac{2.303RT}{F} = 0.06V$ **at** 298} [4 marks]

(A) 0.94 V

(B) 0.40 V

(C) 0.76 V

(D) 0.20 V

Correct Answer: (D) 0.20 V

Solution:

Concept:

- The hydrogen electrode is the left-hand anode and the silver-silver chloride electrode is the right-hand cathode. The standard hydrogen electrode potential is zero, so E°_{cell} equals the standard reduction potential of the silver-silver chloride electrode.
- The Nernst equation is $E = E^\circ - (0.06/n)\log_{10} Q$ at the stated temperature. Pure solids have unit activity, and the very dilute HCl supplies both H^+ and Cl^- at approximately 10^{-6} activity.

Step 1: Write the cell reaction and reaction quotient The half-reactions are $\mathrm{H_2} \longrightarrow 2\,\mathrm{H^+} + 2\,\mathrm{e^-}$ and $2\,\mathrm{AgCl} + 2\,\mathrm{e^-} \longrightarrow 2\,\mathrm{Ag} + 2\,\mathrm{Cl^-}$. Adding gives

$$\mathrm{H_2(g)} + 2\,\mathrm{AgCl(s)} \longrightarrow 2\,\mathrm{H^+(aq)} + 2\,\mathrm{Cl^-(aq)} + 2\,\mathrm{Ag\,(s)}.$$

Hence $n = 2$. At hydrogen pressure 1 bar,

$$Q = a^2_{\mathrm{H^+}} a^2_{\mathrm{Cl^-}} = (10^{-6})^4 = 10^{-24}.$$

Both ions must be included: the chloride concentration affects the cathode potential, and the hydrogen-ion concentration affects the anode potential.

Step 2: Extract the standard electrode potential Substitute the measured emf into the Nernst equation:

$$0.92 = E^\circ_{\text{cell}} - \frac{0.06}{2}\log_{10}(10^{-24}) = E^\circ_{\text{cell}} + 0.72.$$

Therefore $E^\circ_{\text{cell}} = 0.20\,\mathrm{V}$. Options (A), (B) and (C) do not reproduce 0.92 V after adding the concentration contribution of 0.72 V; they would give 1.66, 1.12 and 1.48 V respectively. The very dilute electrolyte makes the measured emf larger than the standard emf, so subtract the concentration contribution.

Step 3: Conclusion The standard reduction potential of the silver-silver chloride electrode is $0.20\,\mathrm{V}$. Hence, the correct option is (D).

Quick Tip to Solve: In this cell HCl contributes twice: include both hydrogen-ion and chloride-ion activities before applying the Nernst equation.

83. Sodium metal on dissolution in liquid ammonia gives a deep blue solution due to the formation of : [4 marks]

(A) ammoniated electrons

(B) sodamide

(C) sodium-ammonia complex

(D) sodium ion-ammonia complex

Correct Answer: (A) ammoniated electrons

Solution:

Concept:

- An alkali metal dissolving in liquid ammonia forms solvated metal cations and solvated electrons. Ammonia molecules surround and stabilise each species; the electrons remain mobile in the solution.
- The characteristic deep blue colour of a dilute alkali-metal ammonia solution comes from the absorption of visible light by the solvated, or ammoniated, electrons. The cation alone is not the colour-producing species.

Step 1: Identify the species formed on dissolution Sodium loses its valence electron on entering the liquid-ammonia medium. Both the sodium ion and the electron become surrounded by ammonia molecules, represented schematically as

$$\mathrm{Na} + (\mathrm{x}^{+}\mathrm{y})\mathrm{NH_3} \longrightarrow [\mathrm{Na}(\mathrm{NH_3})_x]^{+} + [\mathrm{e}(\mathrm{NH_3})_y]^{-}.$$

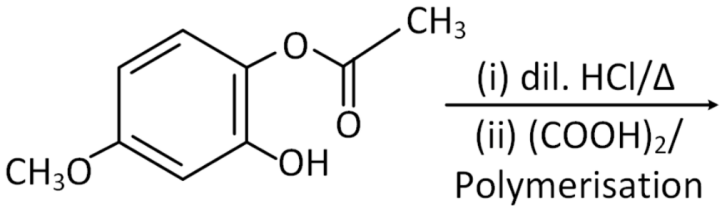
The symbols x and y represent solvation by ammonia; they need not have a fixed value for the present colour argument. The solution also conducts electricity because it contains these charged species.

Step 2: Assign the colour and reject the alternatives The solvated electron has accessible electronic energy states and absorbs light in the visible region, leaving the transmitted solution with its deep blue appearance. Thus (A) names the species responsible. Sodamide in (B) is a chemical product that can form on reaction of sodium with ammonia, but it is not the origin of the immediate blue colour. The sodium-ammonia description in (C) does not identify the solvated electron. The sodium ion-ammonia complex in (D) describes the solvated cation, which has the closed-shell sodium-ion configuration and does not account for this characteristic blue absorption. Both ions and electrons may be present, but the question asks which causes the colour.

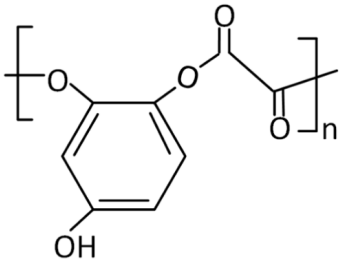
Step 3: Conclusion The deep blue colour is due to ammoniated electrons. Hence, the correct option is (A).

Quick Tip to Solve: Link the blue colour and reducing behaviour of dilute metal-ammonia solutions to their solvated electrons, rather than to the metal cations.

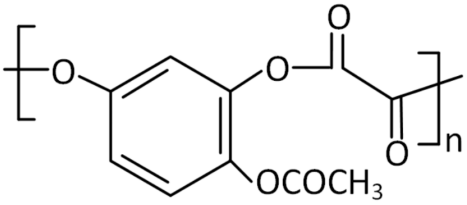
84. The major product of the following reaction is [4 marks]



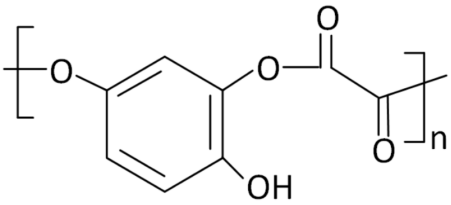
(A) Polyester with adjacent ring oxygen links and a free OH substituent.



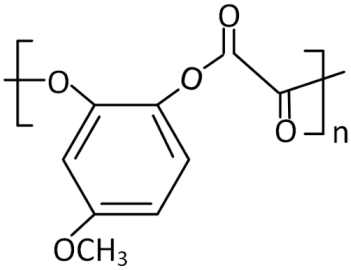
(B) Polyester with meta ring oxygen links and a retained $OCOCH_3$ substituent.



(C) Polyester with meta ring oxygen links and a free OH substituent.

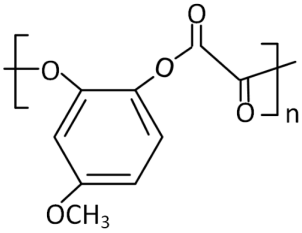


(D) Polyester with adjacent ring oxygen links and a retained OCH_3 substituent.



Correct Answer: (D)

Polyester with adjacent ring oxygen links and a retained OCH_3 substituent.



Solution:

Concept:

- Acidic hydrolysis of an aryl acetate converts the $Ar-OCOCH_3$ group into a phenolic $Ar-OH$ group. The aryl methyl ether $Ar-OCH_3$ remains intact under the stated dilute-HCl hydrolysis conditions.
- A molecule with two phenolic hydroxyl groups can condense with oxalic acid, $HOOC-COOH$, to form a polyester. Each new ester link uses a phenolic oxygen and a carboxyl group, with loss of water.

Step 1: Identify the dihydroxy intermediate The starting ring has an acetoxy group, a neighbouring hydroxyl group, and a methoxy substituent. Step (i), dilute HCl with heating, hydrolyses the acetate protecting group:

$$Ar-OCOCH_3 + H_2O \longrightarrow Ar-OH + CH_3COOH.$$

The ring consequently has two adjacent phenolic hydroxyl groups. Its methoxy substituent stays

at its original position. Cleaving an aryl methyl ether requires different, substantially stronger ether-cleavage conditions; it is not the transformation indicated by dilute hydrochloric acid here.

Step 2: Build the polyester and compare all structures In step (ii), the two adjacent phenolic oxygens enter the polymer backbone by esterification with oxalic acid. The linking portion is $-O-C(=O)-C(=O)-O-$, and the aromatic unit still bears OCH_3. Option (D) has both the correct adjacent ring attachment sites and the retained methoxy group. Option (A) instead replaces the methoxy substituent with a free hydroxyl group. Option (B) retains an acetate substituent and places the backbone oxygen connections at incorrect ring positions. Option (C) likewise changes the substituent to hydroxyl and uses the wrong ring connection pattern. Track each original ring substituent separately to avoid mistaking an acetate ester for a methyl ether.

Step 3: Conclusion The polyester retaining the methoxy substituent and adjacent phenolic oxygen links is formed. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish $Ar-OCOCH_3$ from $Ar-OCH_3$: dilute-acid hydrolysis removes the acetyl group, leaving the methyl ether intact.

85. Haemoglobin and gold sol are examples of : [4 marks]

(A) positively charged sols

(B) negatively charged sols

(C) Positively and negatively charged sols, respectively

(D) negatively and positively charged sols, respectively

Correct Answer: (C) Positively and negatively charged sols, respectively

Solution:

Concept:

- Colloidal particles usually carry the same sign of charge within a given sol. Their mutual electrostatic repulsion helps prevent aggregation.
- The charge belongs to the dispersed colloidal particle and its surface layer; the complete dispersion remains electrically neutral because counter-ions are also present.

Step 1: Identify the charge on haemoglobin Haemoglobin is a standard example of a positively charged colloid in the usual classification of sols. This classification refers to the conditions implicit in the question; it does not mean that a protein has the same net charge at every possible pH. Consequently, any answer that labels haemoglobin negative is inconsistent with the stated textbook classification.

Step 2: Identify gold sol and compare the ordered pair Gold sol is a negatively charged metallic sol. Thus the first substance in the question is positive and the second negative. Option (A) incorrectly assigns a positive charge to gold, while option (B) incorrectly assigns a negative charge to haemoglobin. Option (D) reverses both assignments. The word respectively fixes the order, so the pair cannot be interchanged.

Step 3: Conclusion Haemoglobin and gold sol are positively and negatively charged sols, respectively.

Hence, the correct option is (C).

Quick Tip to Solve: Read respectively as an ordered pair: write haemoglobin (+), then gold sol (-), before comparing the options.

86. The reaction that is NOT involved in the ozone layer depletion mechanism in the stratosphere is : [4 marks]

(A) $Cl\overset{\bullet}{O}\,(g) + O\,(g) \rightarrow \overset{\bullet}{C}l\,(g) + O_2(g)$

(B) $CF_2Cl_2\,(g) \xrightarrow{uv} \overset{\bullet}{C}l\,(g) + \overset{\bullet}{C}F_2Cl\,(g)$

(C) $CH_4 + 2O_3 \rightarrow 3CH_2 = O + 3H_2O$

(D) $HOCl\,(g) \xrightarrow{hv} \overset{\bullet}{O}H\,(g) + \overset{\bullet}{C}l\,(g)$

Correct Answer: (C) $CH_4 + 2O_3 \rightarrow 3CH_2 = O + 3H_2O$

Solution:

Concept:

- Ozone depletion in the stratosphere involves reactive radicals, especially chlorine radicals released by ultraviolet photolysis of chlorofluorocarbons.
- A chlorine radical can consume ozone and then be regenerated. Regeneration makes chlorine a catalyst rather than a reactant consumed once.

Step 1: Check the radical-producing and recycling reactions Photolysis of CF_2Cl_2 breaks a carbon–chlorine bond and releases a chlorine radical, as in option (B). Hypochlorous acid can also undergo photolysis to give hydroxyl and chlorine radicals, as in option (D). Option (A) regenerates the chlorine radical when chlorine monoxide reacts with atomic oxygen. Together with $Cl^{\bullet} + O_3 \rightarrow ClO^{\bullet} + O_2$, this recycling contributes to catalytic ozone destruction.

Step 2: Reject the methane reaction as printed Option (C) proposes $CH_4 + 2\,O_3 \longrightarrow 3\,CH_2O + 3\,H_2O$. It is not part of the stratospheric radical mechanism. In fact, the printed equation is not even atom-balanced: its left side has one carbon and four hydrogens, whereas its right side has three carbons and twelve hydrogens. It therefore cannot represent the claimed elementary depletion step. The other three options have the radical generation or regeneration roles identified above.

Step 3: Conclusion The methane–ozone equation in option (C) is not involved in the ozone-layer depletion mechanism. Hence, the correct option is (C).

Quick Tip to Solve: For a NOT question, first label each reaction as radical generation, radical recycling or neither; also check atom balance.

87. Among the following reactions of hydrogen with halogens, the one that requires a catalyst is : [4 marks]

(A) $H_2 + Br_2 \rightarrow 2HBr$

(B) $H_2 + Cl_2 \rightarrow 2HCl$

(C) $H_2 + F_2 \rightarrow 2HF$

(D) $H_2 + I_2 \rightarrow 2HI$

Correct Answer: (D) $H_2 + I_2 \rightarrow 2HI$

Solution:

Concept:

- The direct combination of hydrogen with halogens becomes progressively less facile down the group. The required initiation conditions differ for fluorine, chlorine, bromine and iodine.
- A catalyst provides a lower-activation-energy pathway and increases the rate of approach to equilibrium; it does not alter the equilibrium constant or the final equilibrium composition.

Step 1: Compare fluorine, chlorine and bromine Fluorine combines extremely vigorously with hydrogen, even under very mild conditions, so option (C) does not need the catalyst sought. Hydrogen and chlorine react on exposure to light, which initiates the radical chain, excluding option (B). Bromine reacts with hydrogen on heating; this is the standard condition distinguished from the catalytic iodine reaction, excluding option (A). Light or heat as an initiating condition must not be confused with the presence of a catalyst.

Step 2: Select the iodine reaction Hydrogen and iodine combine much less readily, and direct preparation of hydrogen iodide uses heating in the presence of a catalyst such as platinum. The reaction is reversible:

$$H_2 + I_2 \rightleftharpoons 2\,HI.$$

Platinum accelerates both directions and helps the system reach equilibrium. It does not remove the reversibility or force complete conversion. Among the four listed hydrogen-halogen combinations, this is the one identified with catalytic assistance.

Step 3: Conclusion The reaction $H_2 + I_2 \rightleftharpoons 2\,HI$ requires a catalyst in the standard comparison. Hence, the correct option is (D).

Quick Tip to Solve: Remember the progression: fluorine reacts vigorously, chlorine needs light, bromine needs heat, and iodine uses heat with a catalyst.

88. The number of 2-centre-2-electron and 3-centre -2-electron bonds in B_2H_6, respectively, are :

[4 marks]

(A) 2 and 2

(B) 2 and 1

(C) 2 and 4

(D) 4 and 2

Correct Answer: (D) 4 and 2

Solution:

Concept:

- Diborane, B_2H_6, is electron deficient: its total number of valence electrons is insufficient for all its connections to be described by ordinary two-centre bonds.
- Each terminal B–H bond is a two-centre two-electron bond. Each bridging B–H–B unit is one three-centre two-electron bond shared by two boron atoms and one hydrogen.

Step 1: Count the terminal bonds Diborane has four terminal hydrogen atoms, two attached terminally to each boron atom. Each such hydrogen forms a conventional B–H bond containing an electron pair shared between the two atoms. Therefore there are four 2-centre-2-electron bonds. The two bridging hydrogens are not included in this terminal count, because their bonding cannot be represented by independent ordinary B–H pairs.

Step 2: Count the bridges and verify the electrons The remaining two hydrogens bridge the boron atoms, one above and one below the framework. Each bridge is one 3-centre-2-electron bond, giving two such bonds. The count uses $4 \times 2 + 2 \times 2 = 12$ bonding electrons, exactly the total $2 \times 3 + 6 \times 1 = 12$ valence electrons of B_2H_6. Options (A) and (B) undercount the terminal bonds, and option (C) interchanges the terminal and bridge counts.

Step 3: Conclusion The numbers of 2-centre-2-electron and 3-centre-2-electron bonds are 4 and 2, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Count each B–H–B bridge as one multicentre bond, rather than two separate conventional B–H bonds.

89. The electrolytes usually used in the electroplating of gold and silver, respectively are : [4 marks]

(A) $[Au(CN)_2]^-$ and $[Ag(CN)_2]^-$

(B) $[Au(CN)_2]^-$ and $[Ag(Cl)_2]^-$

(C) $[Au(OH)_4]^-$ and $[Ag(OH)_2]^-$

(D) $[Au(NH_3)_2]^-$ and $[Ag(CN)_2]^-$

Correct Answer: (A) $[Au(CN)_2]^-$ and $[Ag(CN)_2]^-$

Solution:

Concept:

- Electroplating deposits a thin metal layer by reduction at the cathode. A suitable complex electrolyte controls the supply of reducible metal species and supports a smooth deposit.
- Cyanide forms stable, soluble complexes with gold(I) and silver(I). The commonly used complex anions are dicyanidoaurate(I) and dicyanidoargentate(I).

Step 1: Identify the gold electrolyte The conventional gold-plating electrolyte contains $[Au(CN)_2]^-$, usually supplied as its soluble potassium salt. Gold is in oxidation state $+1$ because two cyanido ligands each contribute -1 and the overall complex charge is -1. Reduction at the cathode deposits metallic gold, while complex formation keeps the concentration of freely available metal ions low enough for controlled deposition.

Step 2: Identify silver and eliminate the alternatives The analogous silver-plating electrolyte contains $[Ag(CN)_2]^-$, also commonly used as a soluble alkali-metal salt. Its cathodic reduction can be written

$$[Ag(CN)_2]^- + e^- \longrightarrow Ag + 2\,CN^-.$$

Option (A) supplies both familiar cyanide complexes in the requested gold-then-silver order. Option (B) replaces the silver cyanide complex by a chloride complex; option (C) gives hydroxo complexes; option (D) replaces the gold cyanide complex by an ammine. Those are not the usual pair asked here.

Step 3: Conclusion The electrolytes contain $[Au(CN)_2]^-$ and $[Ag(CN)_2]^-$, respectively. Hence, the correct option is (A).

Quick Tip to Solve: For conventional gold and silver plating, look for the same dicyanido ligand pattern around Au(I) and Ag(I).

90. A compound of formula A_2B_3 has the hcp lattice. Which atom forms the hcp lattice and what fraction of tetrahedral voids is occupied by the other atoms : [4 marks]

(A) hcp lattice - A, $\frac{1}{3}$ Tetrahedral voids-B

(B) hcp lattice - B, $\frac{1}{3}$ Tetrahedral voids - A

(C) hcp lattice - A, $\frac{2}{3}$ Tetrahedral voids - B

(D) hcp lattice - B, $\frac{2}{3}$ Tetrahedral voids - A

Correct Answer: (B) hcp lattice - B, $\frac{1}{3}$ Tetrahedral voids - A

Solution:

Concept:

- For N atoms in a close-packed lattice, there are 2N tetrahedral voids. This relation is the same for hcp and ccp packing.
- The formula fixes the ratio of occupied interstitial sites to host atoms. Count lattice atoms first, then divide the number of guest atoms by the available voids.

Step 1: Take B as the hcp host Write the composition as A_2B_3. If there are $3n$ B atoms forming the hcp lattice, they generate $2(3n) = 6n$ tetrahedral voids. The compound contains $2n$ A atoms. Therefore the required occupied fraction is

$$f = \frac{2n}{6n} = \frac{1}{3}.$$

This gives an hcp lattice of B with A occupying one third of its tetrahedral voids, precisely the combination in option (B).

Step 2: Check the competing assignments If A were the host, $2n$ A atoms would create $4n$ tetrahedral voids and the $3n$ B atoms would require a fraction $3/4$, not either of the A-host fractions listed in (A) or (C). If B is the host but the occupied fraction is $2/3$, as in (D), the number of A atoms becomes $(2/3)(6n) = 4n$, producing A_4B_3 rather than A_2B_3. Thus checking the stoichiometric ratio rules out every other listed pair.

Step 3: Conclusion B forms the hcp lattice, and A occupies one third of the tetrahedral voids. Hence, the correct option is (B).

Quick Tip to Solve: Use guest/host = 2 times the occupied tetrahedral fraction; here 2/3 = 2 times 1/3.