JEE Main 2019 Paper 1 (B.E./B.Tech) Jan 11 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 11 January 2019, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is : [4 marks]

(A) $\frac{3}{4}$

(B) $\frac{5}{4}$

(C) $\frac{7}{8}$

(D) $\frac{9}{8}$

Correct Answer: (D) $\frac{9}{8}$

Solution:

Concept:

- For curves written as $y = f(x)$ and $y = g(x)$, the enclosed area is the integral of the upper ordinate minus the lower ordinate between their intersection points.
- Intersection points determine the limits; checking the sign of the difference on the interval prevents a signed integral from being mistaken for the geometric area.

Step 1: Find the intersection limits Write both curves in terms of y:

$$y = \frac{x^2}{4}, \qquad y = \frac{x+2}{4}.$$

Equating these ordinates gives

$$x^2 = x + 2 \quad \Longrightarrow \quad (x-2)(x+1) = 0.$$

Thus the horizontal interval of the bounded region is $[-1, 2]$. The corresponding points are $(-1, 1/4)$ and $(2, 1)$; both satisfy the line and the parabola.

Step 2: Integrate the difference of the ordinates Between the roots, $x + 2 - x^2 = (2 - x)(x + 1) \geq 0$, so the straight line is above the parabola throughout the interval. Hence

$$\begin{aligned} \mathcal{A} &= \frac{1}{4}\int_{-1}^{2} (x + 2 - x^2)\, dx \\ &= \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2} \\ &= \frac{1}{4}\left(\frac{10}{3} + \frac{7}{6}\right) = \frac{9}{8}. \end{aligned}$$

The positive value follows from the established ordering of the curves. The other proposed areas, $3/4, 5/4, 7/8$, do not equal this definite integral.

Step 3: Conclusion The enclosed area is $\boxed{9/8}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Express the line and parabola as ordinates first, then factor their difference to identify both limits and the upper curve in one calculation.

2. The straight line x + 2y = 1 meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is : [4 marks]

(A) $4\sqrt{5}$

(B) $\frac{\sqrt{5}}{2}$

(C) $2\sqrt{5}$

(D) $\frac{\sqrt{5}}{4}$

Correct Answer: (B) $\frac{\sqrt{5}}{2}$

Solution:

Concept:

- A circle through the origin has equation $x^2 + y^2 + Dx + Ey = 0$. Substituting its other two known points determines D and E.
- The tangent at the origin is $Dx+Ey = 0$. The perpendicular distance of (x_0, y_0) from $ax+by+c = 0$ is $|ax_0 + by_0 + c|/\sqrt{a^2 + b^2}$.

Step 1: Determine the circle and its tangent Setting $y = 0$ in the given line gives $A = (1, 0)$; setting $x = 0$ gives $B = (0, 1/2)$. Since the origin also lies on the circle, its constant term is zero. Substitution of the intercepts yields

$$1 + D = 0, \qquad \frac{1}{4} + \frac{E}{2} = 0.$$

Thus $D = -1$, $E = -1/2$, and the circle is

$$x^2 + y^2 - x - \frac{y}{2} = 0.$$

At the origin the tangent is $-x - y/2 = 0$, or $2x + y = 0$. As a geometric check, its normal is parallel to the radius joining the origin to the centre $(1/2, 1/4)$.

Step 2: Add the two perpendicular distances Apply the point-to-line distance formula separately to A and B, using the same tangent:

$$d_A = \frac{|2(1) + 0|}{\sqrt{2^2 + 1^2}} = \frac{2}{\sqrt{5}}, \qquad d_B = \frac{|2(0) + 1/2|}{\sqrt{5}} = \frac{1}{2\sqrt{5}}.$$

Their sum is therefore

$$d_A + d_B = \frac{5}{2\sqrt{5}} = \frac{\sqrt{5}}{2}.$$

One must add the distances, rather than take the distance of the midpoint or add their squares. The alternatives $4\sqrt{5}$, $2\sqrt{5}$ and $\sqrt{5}/4$ do not match the normalized distance sum.

Step 3: Conclusion The sum is $\boxed{\sqrt{5}/2}$ units. Hence, the correct option is (B).

Quick Tip to Solve: For a circle through the origin and the two axis intercepts, find only the linear coefficients: they immediately give the tangent at the origin.

3. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$.Then the common ratio of this series is : [4 marks]

(A) $\frac{4}{9}$

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{2}{9}$

Correct Answer: (C) $\frac{2}{3}$

Solution:

Concept:

- An infinite geometric progression with first term a and common ratio r, where $|r| < 1$, has sum $a/(1-r)$. Positive terms require $a > 0$ and $0 < r < 1$ here.
- Cubing each term produces another geometric progression, whose first term is a^3 and whose common ratio is r^3. Its sum is therefore $a^3/(1-r^3)$.

Step 1: Eliminate the first term The first given sum gives

$$\frac{a}{1-r} = 3 \quad \Longrightarrow \quad a = 3(1-r).$$

The second condition applies to the cubes of the individual terms, not to the cube of their total:

$$\frac{a^3}{1-r^3} = \frac{27}{19}.$$

Substitute the expression for a, then use $1 - r^3 = (1-r)(1+r+r^2)$. As $r < 1$, division by $1-r$ is valid:

$$\frac{27(1-r)^2}{1+r+r^2} = \frac{27}{19}.$$

Step 2: Solve and enforce convergence Cross-multiplying and collecting terms gives

$$19(1-2r+r^2) = 1+r+r^2,$$

$$18r^2 - 39r + 18 = 0 \quad \Longrightarrow \quad (3r-2)(2r-3) = 0.$$

The algebraic candidates are $r = 2/3$ and $r = 3/2$. The latter makes a positive infinite geometric series diverge, so it is inadmissible. For $r = 2/3$, the first term is $a = 1$. Directly,

$$\frac{1}{1-2/3} = 3, \qquad \frac{1}{1-8/27} = \frac{27}{19},$$

which verifies both data. The other listed ratios $4/9$, $1/3$ and $2/9$ fail the derived quadratic.

Step 3: Conclusion The common ratio is $\boxed{2/3}$. Hence, the correct option is (C).

Quick Tip to Solve: When both a geometric series and its termwise cubes are summed, eliminate the first term and factor $1 - r^3$ before solving for the ratio.

4. If $\int \frac{\sqrt{1-x^2}}{x^4}\,dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration, then $(A(x))^m$ equals : [4 marks]

(A) $\frac{1}{27x^6}$

(B) $\frac{-1}{27x^9}$

(C) $\frac{1}{9x^4}$

(D) $\frac{1}{3x^3}$

Correct Answer: (B) $\frac{-1}{27x^9}$

Solution:

Concept:

- For $\sqrt{1-x^2}$, use $x = \sin\theta$ on an interval with $-\pi/2 < \theta < \pi/2$, so $\cos\theta > 0$. The integrand is real for $|x| \le 1$ and requires $x \neq 0$.
- The identities $\csc^2\theta = 1 + \cot^2\theta$ and $d(\cot\theta) = -\csc^2\theta\,d\theta$ reduce the transformed expression to a polynomial integral.

Step 1: Transform and evaluate the integral For an interior point of either interval $(-1, 0)$ or $(0, 1)$, put $x = \sin\theta$. Then $dx = \cos\theta\,d\theta$ and $\sqrt{1-x^2} = \cos\theta$, giving

$$I = \int \frac{\cos^2\theta}{\sin^4\theta}\,d\theta = \int \cot^2\theta\csc^2\theta\,d\theta$$
$$= -\frac{\cot^3\theta}{3} + C = -\frac{(1-x^2)^{3/2}}{3x^3} + C.$$

No absolute-value sign is needed in the substitution because the chosen range makes the cosine positive; the denominator keeps the sign of x^3.

Step 2: Read the requested coefficient and exponent In the displayed factored form, the power of $\sqrt{1-x^2}$ is $m = 3$ and its coefficient is $A(x) = -1/(3x^3)$. Thus

$$(A(x))^m = \left(-\frac{1}{3x^3}\right)^3 = -\frac{1}{27x^9}.$$

A direct derivative checks the antiderivative:

$$\frac{d}{dx}\left[-\frac{(1-x^2)^{3/2}}{3x^3}\right] = \frac{\sqrt{1-x^2}}{x^2} + \frac{(1-x^2)^{3/2}}{x^4} = \frac{\sqrt{1-x^2}}{x^4}.$$

The positive alternatives miss the sign; $1/(9x^4)$ also has the wrong denominator power. None equals the cube just found.

Step 3: Conclusion Using $m = 3$ and $A(x) = -1/(3x^3)$, the requested expression is $\boxed{-1/(27x^9)}$. Hence, the correct option is (B).

Quick Tip to Solve: After integration, keep the factor $(\sqrt{1-x^2})^3$ intact and cube only its coefficient when evaluating $(A(x))^m$.

5. Two integers are selected at random from the set {1, 2, ..., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is : [4 marks]

(A) $\frac{2}{5}$

(B) $\frac{1}{2}$

(C) $\frac{7}{10}$

(D) $\frac{3}{5}$

Correct Answer: (A) $\frac{2}{5}$

Solution:

Concept:

- Conditional probability restricts the sample space: $P(E \mid S) = P(E \cap S)/P(S)$. Here E is the event that both selected integers are even, and S is that their sum is even.
- A sum of two integers is even precisely when both are even or both are odd. Selecting two integers from a set means choosing distinct elements without replacement; unordered pairs are equally likely.

Step 1: Count the pairs compatible with the condition The set contains five even integers, $2, 4, 6, 8, 10$, and six odd integers, $1, 3, 5, 7, 9, 11$. Among the $\binom{11}{2} = 55$ equally likely unordered pairs, the pairs with an even sum are of two disjoint types:

$$N(\text{both even}) = \binom{5}{2} = 10, \qquad N(\text{both odd}) = \binom{6}{2} = 15.$$

Hence the condition leaves $10 + 15 = 25$ allowable pairs. Mixed-parity pairs have odd sum and must be excluded before computing the desired probability.

Step 2: Form the conditional ratio Every both-even pair already has even sum, so $E \cap S = E$. Consequently,

$$P(E \mid S) = \frac{10/55}{25/55} = \frac{10}{25} = \frac{2}{5}.$$

The probability is not $1/2$: the two parity types are not equally numerous because there are more odd integers. The value $3/5$ is instead the conditional probability of a both-odd pair. The remaining choice $7/10$ also conflicts with the direct count. Counting ordered draws would double both the favourable and conditioned counts, giving the same ratio and providing a useful check on the method.

Step 3: Conclusion The required conditional probability is $\boxed{2/5}$. Hence, the correct option is (A).

Quick Tip to Solve: For an even-sum condition, count even-even and odd-odd pairs separately; their unequal counts prevent the tempting but incorrect answer $1/2$.

6. A square is inscribed in the circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the coordinate axes. Then the distance of the vertex of this square which is nearest to the origin is : [4 marks]

(A) $\sqrt{137}$

(B) 6

(C) $\sqrt{41}$

(D) 13

Correct Answer: (C) $\sqrt{41}$

Solution:

Concept:

- Completing squares in the circle equation gives its centre and radius. An inscribed square has that same centre, because its diagonals are diameters of the circle.
- For an axis-parallel square with half-side h, the displacement of a vertex from the centre is $(\pm h, \pm h)$; therefore $2h^2 = R^2$. Distance from the origin is found from $\sqrt{x^2 + y^2}$.

Step 1: Find the centre and the half-side Complete the squares carefully, retaining the constant terms added on each side:

$$x^2 - 6x + y^2 + 8y = 103,$$

$$(x - 3)^2 + (y + 4)^2 = 103 + 9 + 16 = 128.$$

Thus the centre is $(3, -4)$ and the radius is $\sqrt{128} = 8\sqrt{2}$. With its sides parallel to the axes, each vertex differs from the centre by equal horizontal and vertical magnitudes h. The radius relation yields

$$h^2 + h^2 = 128 \quad \Longrightarrow \quad h = 8.$$

This is the half-side, whereas the square's full side is 16.

Step 2: Compare the four vertex distances The vertices are

$$(11, 4), \quad (11, -12), \quad (-5, 4), \quad (-5, -12).$$

Compute squared distances first, which preserves the ordering without introducing four square roots:

$$11^2 + 4^2 = 137, \qquad 11^2 + (-12)^2 = 265,$$
$$(-5)^2 + 4^2 = 41, \qquad (-5)^2 + (-12)^2 = 169.$$

The least is 41, attained at $(-5, 4)$. Hence the nearest distance is $\sqrt{41}$. The options $\sqrt{137}$ and $13 = \sqrt{169}$ are distances of other vertices, and 6 is not the distance of any vertex. The answer concerns a vertex, not the nearest point of the circle.

Step 3: Conclusion The nearest vertex is $(-5, 4)$, at distance $\boxed{\sqrt{41}}$ units from the origin. Hence, the correct option is (C).

Quick Tip to Solve: For an axis-parallel inscribed square, use centre coordinates plus or minus $R/\sqrt{2}$; then compare squared distances to avoid unnecessary radicals.

7. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is [4 marks]

(A) -81

(B) -300

(C) 100

(D) 144

Correct Answer: (B) -300

Solution:

Concept:

- If the roots of $ax^2 + bx + c = 0$ are α and β, Vieta's relations give $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$.
- Writing the related real roots as t and t^3 converts their product into t^4. Both signs of a real fourth root must be retained before comparing the listed coefficient values.

Step 1: Use the product to find the possible roots Let the real roots be t and t^3. The product formula gives

$$t^4 = \frac{256}{81} = \left(\frac{4}{3}\right)^4.$$

Therefore $t = 4/3$ or $t = -4/3$. For the positive value, the two roots are $4/3$ and $64/27$; for the negative value they are $-4/3$ and $-64/27$. Their product is positive in both cases, as required by the positive constant term divided by the leading coefficient. There are no other real possibilities, since the positive number has exactly two real fourth roots.

Step 2: Use the sum to determine the coefficient For positive roots,

$$t + t^3 = \frac{4}{3} + \frac{64}{27} = \frac{100}{27}.$$

But the sum is $-k/81$, so

$$-\frac{k}{81} = \frac{100}{27} \quad \Longrightarrow \quad k = -300.$$

The negative pair similarly gives $k = 300$. The question asks for a value, and only -300 occurs among the choices. To check, the corresponding polynomial factors as

$$81x^2 - 300x + 256 = (3x - 4)(27x - 64),$$

whose roots satisfy $(4/3)^3 = 64/27$. The remaining listed numbers $-81, 100, 144$ are neither of the two allowed values ± 300.

Step 3: Conclusion A possible value is $\boxed{k = -300}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the product first when one root is a power of the other: it determines the root magnitude directly, and the sum then fixes the coefficient sign.

8. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set

$$S = \{x \in R : x^2 + 30 \leq 11x\}$$

is : [4 marks]

(A) -222

(B) -122

(C) 122

(D) 222

Correct Answer: (C) 122

Solution:

Concept:

- A continuous function on a closed bounded interval attains its maximum. Check stationary points inside the interval and its endpoints.
- The derivative determines monotonicity: if $f\prime(x) > 0$ throughout an interval, its largest value occurs at the right endpoint.

Step 1: Find the allowed interval Rearrange the condition defining the set and factor its quadratic:

$$x^2 - 11x + 30 \leq 0 \quad \Longleftrightarrow \quad (x-5)(x-6) \leq 0.$$

The two factors have opposite signs or one is zero precisely between their roots. Thus $S = [5, 6]$, including both endpoints. The stationary points of the cubic on the whole real line cannot be used unless they belong to this set.

Step 2: Determine the sign of the derivative Differentiate the polynomial term by term:

$$f\prime(x) = 9x^2 - 36x + 27 = 9(x-1)(x-3).$$

For every $x \in [5, 6]$, both $x - 1$ and $x - 3$ are positive, so $f\prime(x) > 0$. Hence the function is strictly increasing on the entire allowed interval; its stationary points 1 and 3 lie outside it.

Step 3: Evaluate the right endpoint Substitute $x = 6$, where the maximum is attained:

$$f(6) = 3(216) - 18(36) + 27(6) - 40 = 648 - 648 + 162 - 40 = 122.$$

As an endpoint check, $f(5) = 375 - 450 + 135 - 40 = 20 < 122$. The positive value 122 rules out both negative options, and direct substitution rules out 222.

Step 4: Conclusion The maximum value on S is $\boxed{122}$, attained at $x = 6$. Hence, the correct option is (C).

Quick Tip to Solve: Solve the constraint first: stationary points outside the permitted interval do not compete for its maximum.

9. Let A = $\begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then $|p|$ is : [4 marks]

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{1}{\sqrt{5}}$

(C) $\frac{1}{\sqrt{6}}$

(D) $\frac{1}{\sqrt{3}}$

Correct Answer: (A) $\frac{1}{\sqrt{2}}$

Solution:

Concept:

- The entry (i, j) of AA^T is the dot product of row i with row j. Thus $AA^T = I_3$ means that the rows have unit length and are mutually perpendicular.
- The diagonal entries give sums of squares, while off-diagonal entries give zero dot products. These equations determine the magnitudes without requiring signs.

Step 1: Use perpendicularity of the first two rows The first two rows are $(0, 2q, r)$ and $(p, q, -r)$. Their dot product must vanish:

$$0 \cdot p + (2q)q + r(-r) = 2q^2 - r^2 = 0.$$

Consequently $r^2 = 2q^2$. The dot product of the first and third rows gives $-2q^2 + r^2 = 0$, the same condition, so it introduces no conflicting requirement.

Step 2: Use the length of the first row Its squared norm is the first diagonal entry of AA^T, so

$$4q^2 + r^2 = 1.$$

Substitute $r^2 = 2q^2$ to obtain $6q^2 = 1$, hence $q^2 = 1/6$ and $r^2 = 1/3$. The factor $2q$ must be squared as $4q^2$; treating it as $2q^2$ would give an incorrect value.

Step 3: Find and check the magnitude of p The second row also has squared norm one:

$$p^2 + q^2 + r^2 = 1 \quad \Longrightarrow \quad p^2 = 1 - \frac{1}{6} - \frac{1}{3} = \frac{1}{2}.$$

The remaining row dot product is $p^2 - q^2 - r^2 = 1/2 - 1/6 - 1/3 = 0$, as required. The other offered magnitudes square to $1/5, 1/6, 1/3$, none of which satisfies this equation.

Step 4: Conclusion Therefore $\boxed{|p| = 1/\sqrt{2}}$. The sign of p is immaterial because its absolute value is requested. Hence, the correct option is (A).

Quick Tip to Solve: For an orthogonal matrix, use one convenient row dot product and one row norm before multiplying the whole matrix.

10. Let f : R → R be defined by f(x) = $\frac{x}{1+x^2}, x \in R$. Then the range of f is : [4 marks]

(A) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(B) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $(-1, 1) - \{0\}$

(D) R − [−1, 1]

Correct Answer: (A) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Solution:

Concept:

- The range consists of all output values actually attained by the function, not merely its declared codomain $\mathbb{R}$.
- The inequality $(|x| - 1)^2 \geq 0$ gives $1 + x^2 \geq 2|x|$, which bounds the absolute value of the given rational function.

Step 1: Bound every output Since $1 + x^2 > 0$ for all real x, division preserves inequalities. Thus

$$\left|\frac{x}{1+x^2}\right| \leq \frac{1}{2}, \qquad -\frac{1}{2} \leq f(x) \leq \frac{1}{2}.$$

Equality is possible: $f(1) = 1/2$ and $f(-1) = -1/2$. The endpoints therefore belong to the range; they cannot be removed by using open brackets.

Step 2: Show that all intermediate values occur The rational function is continuous on $[-1, 1]$ because its denominator never vanishes. As its endpoint values are $-1/2$ and $1/2$, the intermediate value theorem guarantees every value between them. An algebraic check is also possible: $y = x/(1 + x^2)$ gives

$$yx^2 - x + y = 0.$$

For $y \neq 0$, real solutions exist exactly when $1 - 4y^2 \geq 0$. The excluded algebraic case $y = 0$ is attained at $x = 0$.

Step 3: Compare the proposed sets The complement in option (B) omits the attainable central interval and includes outputs whose magnitude is too large. Option (C) omits 0, although $f(0) = 0$, and includes values such as $3/4$, which violate the bound. Option (D) likewise consists of magnitudes greater than one and so cannot contain any output of this function.

Step 4: Conclusion The exact range is $\boxed{[-1/2, 1/2]}$. Hence, the correct option is (A).

Quick Tip to Solve: For a rational-function range, use a universal bound and then prove attainment; a bound alone does not show every intermediate value occurs.

11. If the system of linear equations

2x + 2y + 3z = a

3x – y + 5z = b

x – 3y + 2z = c

where a, b, c are non zero real numbers, has more one solution, then : [4 marks]

(A) b – c – a = 0

(B) a + b + c = 0

(C) b – c + a = 0

(D) b + c – a = 0

Correct Answer: (A) b – c – a = 0

Solution:

Concept:

- A linear system with more than one solution must be consistent, and its coefficient rows must be linearly dependent.
- Any exact linear dependence between the left sides must also hold between their right sides. Otherwise row reduction produces an impossible equation.

Step 1: Identify the row dependence Subtract the third equation from the second. The left side simplifies as

$$(3x - y + 5z) - (x - 3y + 2z) = 2x + 2y + 3z.$$

This is exactly the left side of the first equation. Therefore the coefficient rows satisfy $R_2 - R_3 - R_1 = 0$. Applying the same operation to the augmented column must give zero for a consistent system.

Step 2: Impose the compatibility condition The right side obtained by that subtraction is $b - c$, whereas the first equation prescribes a. Thus

$$b - c = a \quad \Longleftrightarrow \quad b - c - a = 0.$$

If this equality fails, the two equations demand different values for the same expression, giving no solution rather than more than one solution. This argument establishes necessity directly without expanding a determinant.

Step 3: Verify that the system then has a free variable The first two coefficient rows are independent, since their first two columns have minor

$$\begin{vmatrix} 2 & 2 \\ 3 & -1 \end{vmatrix} = -2 - 6 = -8 \neq 0.$$

The coefficient rank is consequently two, and under the compatibility condition the augmented rank is also two. With three unknowns there is one free parameter, so infinitely many solutions are possible. Under $b = a + c$, option (B) would require $2b = 0$, option (C) would require $2a = 0$, and option (D) would require $2c = 0$. Each contradicts the stated nonzero constants.

Step 4: Conclusion The required relation is $\boxed{b - c - a = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: Look for a simple sum or difference of coefficient rows before computing determinants; apply the identical operation to the constants.

12. If y(x) is the solution of the differential equation $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right) y = e^{-2x},\ x > 0,$ where $y(1) = \frac{1}{2}e^{-2}$, then [4 marks]

(A) $y(\log_e 2) = \log_e 4$

(B) y(x) is decreasing in (0, 1)

(C) $y(\log_e 2) = \frac{\log_e 2}{4}$

(D) y(x) is decreasing in $\left(\frac{1}{2}, 1\right)$

Correct Answer: (D) y(x) is decreasing in $\left(\frac{1}{2}, 1\right)$

Solution:

Concept:

- For $y\prime + P(x)y = Q(x)$, an integrating factor is $e^{\int P(x)\,dx}$. Multiplying by it turns the left side into a product derivative.
- Monotonicity follows from the sign of $y\prime$. A function decreasing on one subinterval need not decrease on a larger interval containing a turning point.

Step 1: Apply the integrating factor Here $P(x) = 2 + 1/x$ on $x > 0$. Its integrating factor is

$$e^{2x+\ln x} = xe^{2x}.$$

Multiplication gives $\frac{d}{dx}(xe^{2x}y) = x$. Integrating both sides yields

$$xe^{2x}y = \frac{x^2}{2} + C, \qquad y = e^{-2x}\left(\frac{x}{2} + \frac{C}{x}\right).$$

Step 2: Use the prescribed initial value At $x = 1$, the given condition gives $e^2 y(1) = 1/2$, so $1/2 = 1/2 + C$ and $C = 0$. Hence

$$y(x) = \frac{x}{2}e^{-2x}.$$

Substituting this expression back into the differential equation gives e^{-2x}, confirming that the coefficient and exponential have been handled correctly.

Step 3: Check the value and monotonicity options Differentiate the obtained expression:

$$y\prime(x) = \frac{1}{2}e^{-2x}(1 - 2x).$$

It is positive on $(0, 1/2)$ and negative on $(1/2, 1)$. Thus option (B) is false and option (D) is true. Also

$$y(\ln 2) = \frac{\ln 2}{2}e^{-2\ln 2} = \frac{\ln 2}{8}.$$

This is neither $\ln 4$ nor $(\ln 2)/4$, excluding options (A) and (C). In particular the exponential contributes a factor $1/4$, which must still be multiplied by the prefactor $x/2$.

Step 4: Conclusion The solution is decreasing on $\boxed{(1/2, 1)}$. Hence, the correct option is (D).

Quick Tip to Solve: After solving a linear differential equation, test all listed assertions with the explicit solution; distinguish the factors from the integrating factor and the final exponential.

13. Let $f(x) = \begin{cases} -1 & -2 \le x < 0 \\ x^2 - 1, & 0 \le x \le 2 \end{cases}$ and

$$g(x) = |f(x)| + f(|x|).$$

Then, in the interval (–2, 2), g is : [4 marks]

(A) non continuous

(B) differentiable at all points

(C) not differentiable at two points

(D) not differentiable at one point

Correct Answer: (D) not differentiable at one point

Solution:

Concept:

- To differentiate an expression containing absolute values and piecewise functions, first remove them separately on intervals where their signs and formulas are fixed.
- At a joining point, differentiability requires continuity and equality of the left and right derivatives. A change of formula alone does not imply a corner.

Step 1: Find the formula on the negative interval For $-2 < x < 0$, $f(x) = -1$, while $|x| = -x > 0$ uses the quadratic branch. Therefore

$$|f(x)| = 1, \qquad f(|x|) = x^2 - 1, \qquad g(x) = x^2.$$

There is no switch at $x = -1$: $f(x)$ remains the constant -1 on the whole negative interval. The first absolute value is applied to the output $f(x)$.

Step 2: Find the formulas on the positive interval For $0 \le x \le 1$, $x^2 - 1 \le 0$, so

$$g(x) = (1 - x^2) + (x^2 - 1) = 0.$$

For $1 < x < 2$, the quadratic is positive and both terms equal it. Thus $g(x) = 2x^2 - 2$ there. The complete interior description is

$$g(x) = \begin{cases} x^2, & -2 < x < 0, \\ 0, & 0 \le x \le 1, \\ 2x^2 - 2, & 1 < x < 2. \end{cases}$$

Step 3: Check the joining points At $x = 0$, both limiting values are zero, and the left derivative $2x \to 0$ equals the right derivative zero. Thus g is differentiable there. At $x = 1$, both sides have value zero, so it is continuous; however, the left derivative is zero and the right derivative is $4x \to 4$. These unequal derivatives give a corner. Every other interior point lies in a polynomial piece. The function is therefore continuous throughout, excluding (A), but not everywhere differentiable, excluding (B). Only one point fails differentiability, excluding (C).

Step 4: Conclusion The only point of nondifferentiability is $x = 1$. Hence, the correct option is (D).

Quick Tip to Solve: Simplify the entire expression before counting corners: cancellation can remove an apparent absolute-value corner, as it does at zero here.

14. Let [x] denote the greatest integer less than or equal to x. Then $\lim_{x\to 0} \frac{\tan\left(\pi\sin^2 x\right)+(|x|-\sin(x[x]))^2}{x^2}$

[4 marks]

(A) equals $\pi + 1$

(B) equals 0

(C) does not exist

(D) equals π

Correct Answer: (C) does not exist

Solution:

Concept:

- Near zero, the greatest-integer function has different fixed values: $[x] = 0$ to the right and $[x] = -1$ to the left.
- Use $\sin x/x \to 1$ and $\tan u/u \to 1$. A two-sided limit exists only when the two finite one-sided limits agree.

Step 1: Evaluate the common tangent contribution On either side of zero the tangent term can be factored into standard limits:

$$\frac{\tan(\pi \sin^2 x)}{x^2} = \frac{\tan(\pi \sin^2 x)}{\pi \sin^2 x}\, \pi \left(\frac{\sin x}{x}\right)^2 \longrightarrow \pi.$$

For sufficiently small nonzero x, this expression is defined because the tangent argument is close to zero rather than a pole.

Step 2: Compute the right-hand limit If $0 < x < 1$, then $[x] = 0$ and $|x| = x$. Consequently $\sin(x[x]) = 0$, so

$$\frac{(|x| - \sin(x[x]))^2}{x^2} = \frac{x^2}{x^2} = 1.$$

Adding the common tangent contribution, the right-hand limit is $\pi + 1$. Evaluating only this side would incorrectly suggest option (A).

Step 3: Compute the left-hand limit and compare For $-1 < x < 0$, use $[x] = -1$ and $|x| = -x$. Now $\sin(x[x]) = \sin(-x) = -\sin x$, giving

$$\frac{(|x| - \sin(x[x]))^2}{x^2} = \frac{(\sin x - x)^2}{x^2} = \left(\frac{\sin x}{x} - 1\right)^2 \longrightarrow 0.$$

Thus the left-hand limit is π, different from the right-hand value $\pi + 1$. Option (D) describes only the left-hand value; option (B) is inconsistent with either side. The mismatch proves nonexistence without assigning a value at zero.

Step 4: Conclusion Since $\pi \neq \pi + 1$, the two-sided limit does not exist. Hence, the correct option is (C).

Quick Tip to Solve: When a greatest-integer term occurs at an integer boundary, freeze its value on each side before simplifying any absolute values.

15. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is : [4 marks]

(A) 0

(B) 8

(C) 6

(D) 4

Correct Answer: (A) 0

Solution:

Concept:

- An expansion of $(a+b)^n$, with even n, has $n+1$ terms and a unique middle term numbered $n/2+1$. Here $n=8$, so the fifth term is required.
- The general term is $T_{r+1}=\binom{n}{r}a^{n-r}b^r$. Because the original expression contains $3/x$, every admissible value must satisfy $x\neq 0$.

Step 1: Identify and simplify the fifth term There are nine terms, so put $r=4$, rather than $r=5$, in the general-term formula. With $a=x^3/3$ and $b=3/x$,

$$T_5=\binom{8}{4}\left(\frac{x^3}{3}\right)^4\left(\frac{3}{x}\right)^4=70\frac{x^{12}}{81}\frac{81}{x^4}=70x^8.$$

The factors of 3 cancel, while the powers of x subtract. In particular, the exponent is eight, so both positive and negative real roots must be considered.

Step 2: Find and add all real roots Equating this middle term to the stated value gives

$$70x^8=5670 \quad\Longrightarrow\quad x^8=81=3^4.$$

For a real x, this is equivalent to $|x|=81^{1/8}=\sqrt{3}$. Thus the two real solutions are $x=\sqrt{3}$ and $x=-\sqrt{3}$, both allowed because neither is zero. Their sum is zero. The other options, 8, 6 and 4, are positive and therefore cannot equal the sum of this opposite pair; taking only the positive root would omit a valid solution.

Step 3: Conclusion The real values are $\sqrt{3}$ and $-\sqrt{3}$, whose sum is $\boxed{0}$. Hence, the correct option is (A).

Quick Tip to Solve: For an even power equation with a positive right-hand side, retain both real signs before adding the roots.

16. The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}\,dx$ (where [x] denotes the greatest integer less than or equal to x) is [4 marks]

(A) 0

(B) 4

(C) 4− sin 4

(D) sin 4

Correct Answer: (A) 0

Solution:

Concept:

- The greatest-integer function satisfies $\lfloor t \rfloor = -1$ for $-1 < t < 0$, and $\lfloor t \rfloor = 0$ for $0 \le t < 1$. The floor must be evaluated before simplifying the denominator.
- Here $2 < \pi$, so x/π remains strictly between -1 and 1 throughout the interval. Also $\sin^2 x$ is even: $\sin^2(-x) = \sin^2 x$.

Step 1: Split at the jump of the floor function On $[-2, 0)$, the quantity x/π is negative but greater than -1, so $\lfloor x/\pi \rfloor = -1$. The denominator is consequently $-1 + 1/2 = -1/2$. On $[0, 2]$, the floor equals zero, so the denominator is $1/2$. Thus

$$I = -2\int_{-2}^{0} \sin^2 x\,dx + 2\int_{0}^{2} \sin^2 x\,dx.$$

There is no zero denominator on either interval. The single point $x = 0$ does not affect the definite integral, and the numerator there is itself zero.

Step 2: Use evenness to cancel the two contributions Substituting $u = -x$ in the first integral shows

$$\int_{-2}^{0} \sin^2 x\,dx = \int_{0}^{2} \sin^2 u\,du.$$

The two identical positive integrals therefore occur with opposite coefficients, giving $I = 0$. For an explicit check, the half-angle identity gives $\int_0^2 \sin^2 x\,dx = 1 - \sin 4/4$; multiplying this by -2 and $+2$ still gives zero. The options 4, $4 - \sin 4$, and $\sin 4$ fail because they do not account for this sign reversal across zero.

Step 3: Conclusion The negative and positive halves cancel exactly, so the integral is $\boxed{0}$. Hence, the correct option is (A).

Quick Tip to Solve: With a floor function on a symmetric interval, first make a small interval table; do not assume the entire integrand is even.

17. Let $a_1, a_2, \ldots\ldots, a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals [4 marks]

(A) 5^3

(B) $2(5^2)$

(C) $4(5^2)$

(D) 5^4

Correct Answer: (D) 5^4

Solution:

Concept:

- If a geometric progression has first term a and common ratio r, then $a_n = ar^{n-1}$. Dividing two terms cancels the common first term.
- The ratio $a_m/a_n = r^{m-n}$, whenever the denominator is nonzero. The exponent depends on the difference of the indices, not on their ratio or sum.

Step 1: Determine the information about the common ratio The given quotient $a_3/a_1 = 25$ ensures that $a_1 \neq 0$. Writing the third term in terms of the first gives

$$\frac{a_3}{a_1} = \frac{ar^2}{a} = r^2 = 25.$$

For a real progression this permits both $r = 5$ and $r = -5$. It is unnecessary to choose between these possibilities because the required quotient will involve an even power. Also $r \neq 0$, so $a_5 = ar^4 \neq 0$ and the requested quotient is defined.

Step 2: Evaluate the requested quotient and compare the options The ninth and fifth terms are $a_9 = ar^8$ and $a_5 = ar^4$. Hence

$$\frac{a_9}{a_5} = \frac{ar^8}{ar^4} = r^4 = (r^2)^2 = 25^2 = 625 = 5^4.$$

Both possible signs of r give this same positive result. Option (A), $5^3 = 125$, uses the wrong exponent. Options (B) and (C) are respectively 50 and 100; multiplying r^2 by the number of steps is incorrect, since equal steps in a geometric progression multiply successive ratios rather than add them.

Step 3: Conclusion The ratio is $\boxed{625 = 5^4}$. Hence, the correct option is (D).

Quick Tip to Solve: For ratios in a G.P., subtract the term indices first; then express the resulting power in terms of the given power of the common ratio.

18. If $x\log_e(\log_e x) - x^2 + y^2 = 4 (y > 0)$, then $\frac{dy}{dx}$ at x = e is equal to : [4 marks]

(A) $\frac{(1+2e)}{2\sqrt{4+e^2}}$

(B) $\frac{(1+2e)}{\sqrt{4+e^2}}$

(C) $\frac{(2e-1)}{2\sqrt{4+e^2}}$

(D) $\frac{e}{\sqrt{4+e^2}}$

Correct Answer: (C) $\frac{(2e-1)}{2\sqrt{4+e^2}}$

Solution:

Concept:

- Implicit differentiation treats y as a function of x, so $d(y^2)/dx = 2y\,dy/dx$. The product rule is needed for $x\ln(\ln x)$.
- The chain rule gives $d[\ln(\ln x)]/dx = 1/(x\ln x)$ for $x > 1$. At $x = e$, $\ln e = 1$ and $\ln 1 = 0$.

Step 1: Find the positive value of y at the specified point Substitute $x = e$ into the original relation before evaluating the derivative:

$$e\ln(\ln e) - e^2 + y^2 = 4 \quad \Longrightarrow \quad y^2 = 4 + e^2.$$

The condition $y > 0$ selects $y = \sqrt{4+e^2}$. The negative square root is not permitted. Since this value is nonzero, division by $2y$ in the implicit derivative is valid near the point.

Step 2: Differentiate and substitute the point Applying both the product and chain rules gives

$$\ln(\ln x) + \frac{1}{\ln x} - 2x + 2y\frac{dy}{dx} = 0.$$

Therefore, at $x = e$,

$$\left.\frac{dy}{dx}\right|_{x=e} = \frac{2e - 0 - 1}{2\sqrt{4+e^2}} = \frac{2e-1}{2\sqrt{4+e^2}}.$$

The -1 in the numerator comes from moving the derivative contribution $1/\ln e$ to the other side. Options (A) and (B) have the wrong sign for this contribution, while (D) omits it altogether. Option (B) also misses the factor 2 arising from differentiating y^2.

Step 3: Conclusion The derivative is $\boxed{\frac{2e-1}{2\sqrt{4+e^2}}}$. Hence, the correct option is (C).

Quick Tip to Solve: For nested logarithms, write the product-rule and chain-rule terms separately, and use the original equation to determine the correct sign of y.

19. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to [4 marks]

(A) $\frac{1}{4}$
(B) $\frac{5}{12}$
(C) $\frac{-1}{12}$
(D) $\frac{1}{12}$

Correct Answer: (D) $\frac{1}{12}$

Solution:

Concept:

- The basic identity $\sin^2 x + \cos^2 x = 1$ holds for every real x. Setting $u = \sin^2 x$ and $v = \cos^2 x$ turns the higher even powers into ordinary polynomials.
- For $u + v = 1$, the identities $u^2 + v^2 = 1 - 2uv$ and $u^3 + v^3 = 1 - 3uv$ follow by expanding the square and cube of $u + v$.

Step 1: Express the two functions using a common product Put $p = \sin^2 x \cos^2 x = uv$. From the definition of f_k,

$$f_4(x) = \frac{\sin^4 x + \cos^4 x}{4} = \frac{1 - 2p}{4},$$

while

$$f_6(x) = \frac{\sin^6 x + \cos^6 x}{6} = \frac{1 - 3p}{6}.$$

In the cubic identity, the mixed terms combine to $3uv(u + v) = 3uv$, which is why no additional trigonometric factor is left.

Step 2: Subtract and check that the answer is constant Using the common denominator 12,

$$f_4(x) - f_6(x) = \frac{3(1 - 2p) - 2(1 - 3p)}{12} = \frac{3 - 6p - 2 + 6p}{12} = \frac{1}{12}.$$

The terms containing p cancel, proving the result for every real x, rather than just for a special angle. As a quick option check, at $x = 0$ the difference is $1/4 - 1/6 = 1/12$. This rules out $1/4$, $5/12$, and $-1/12$; in particular, the negative option would correspond to reversing the requested subtraction.

Step 3: Conclusion For all real x, $f_4(x) - f_6(x) = \boxed{\frac{1}{12}}$. Hence, the correct option is (D).

Quick Tip to Solve: When even sine and cosine powers appear together, set u equal to sine squared and v equal to cosine squared, so their sum becomes 1.

20. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2}$ – d each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2}$+ d each. If the variance of this outcome data is $\frac{4}{3}$ then |d| equals : [4 marks]

(A) $\frac{2}{3}$

(B) $\frac{\sqrt{5}}{2}$

(C) $\sqrt{2}$

(D) 2

Correct Answer: (C) $\sqrt{2}$

Solution:

Concept:

- The mean of observations x_i with frequencies f_i is $\bar{x} = \sum f_i x_i / \sum f_i$. Equal frequencies at equal distances on opposite sides of a centre give that centre as the mean.
- The variance of all the observations is $\sigma^2 = \sum f_i (x_i - \bar{x})^2 / \sum f_i$. Use the population denominator 30 here, because the question gives the variance of the complete outcome data.

Step 1: Find the mean using the three equally frequent outcomes Each of the values $1/2 - d$, $1/2$, and $1/2 + d$ appears ten times, so

$$\bar{x} = \frac{10(1/2 - d) + 10(1/2) + 10(1/2 + d)}{30} = \frac{1}{2}.$$

The terms involving d cancel exactly. Consequently, the deviations from the mean are $-d$, 0, and d, with frequency ten each. This symmetry holds regardless of the sign of d, so the data determine its magnitude rather than its sign.

Step 2: Use the given variance to find the magnitude Squaring the deviations removes their signs:

$$\sigma^2 = \frac{10d^2 + 10(0)^2 + 10d^2}{30} = \frac{2d^2}{3}.$$

Equating this to the stated $4/3$ gives $2d^2 = 4$, hence $d^2 = 2$ and $|d| = \sqrt{2}$. The other choices fail on substitution: $|d| = 2/3$ gives variance $8/27$, $|d| = \sqrt{5}/2$ gives $5/6$, and $|d| = 2$ gives $8/3$, none equal to $4/3$. The square root is taken only after solving for the squared deviation.

Step 3: Conclusion The required magnitude is $\boxed{|d| = \sqrt{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: For three equally frequent outcomes at centre minus d, centre, and centre plus d, the variance is two-thirds of d squared.

21. Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is : [4 marks]

(A) $-10\hat{i} - 5\hat{j}$

(B) $-10\hat{i} + 5\hat{j}$

(C) $-14\hat{i} + 5\hat{j}$

(D) $-14\hat{i} - 5\hat{j}$

Correct Answer: (B) $-10\hat{i} + 5\hat{j}$

Solution:

Concept:

- Three vectors are coplanar exactly when their scalar triple product is zero. Row operations that subtract multiples of another row preserve the determinant representing that product.
- The cross product $\vec{a} \times \vec{c}$ is perpendicular to both vectors. The stated nonzero condition must be used to discard parameter values for which the two vectors are parallel.

Step 1: Apply coplanarity to determine the possible parameter values With the components given in the question,

$$\begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix} = 0.$$

Subtract the first row from the second and twice the first row from the third. The determinant becomes triangular:

$$\begin{vmatrix} 1 & 2 & 4 \\ 0 & \lambda - 2 & 0 \\ 0 & 0 & \lambda^2 - 9 \end{vmatrix} = (\lambda - 2)(\lambda^2 - 9) = 0.$$

Thus $\lambda = 2, 3, -3$. For either $\lambda = 3$ or $\lambda = -3$, $\lambda^2 - 1 = 8$, so $\vec{c} = (2, 4, 8) = 2\vec{a}$, giving a zero cross product. The required nonzero cross product therefore forces $\lambda = 2$.

Step 2: Compute the cross product for the admissible value For $\lambda = 2$, $\vec{c} = (2, 4, 3)$. Hence

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = (6 - 16)\hat{i} - (3 - 8)\hat{j} + (4 - 4)\hat{k} = -10\hat{i} + 5\hat{j}.$$

This is nonzero. Its dot product with $\vec{a}$ is $-10 + 10 = 0$, and with $\vec{c}$ is $-20 + 20 = 0$, checking both signs. The other three listed vectors fail at least one of these perpendicularity checks.

Step 3: Conclusion The required vector is $\boxed{-10\hat{i} + 5\hat{j}}$. Hence, the correct option is (B).

Quick Tip to Solve: After solving a coplanarity determinant, test any nonzero-cross-product condition before substituting the parameter into the final determinant.

22. **Let** $\left(-2-\frac{1}{3}i\right)^3=\frac{x+iy}{27}\left(i=\sqrt{-1}\right)$, **where x and y are real numbers, then y − x equals :** [4 marks]

(A) − 85

(B) 85

(C) − 91

(D) 91

Correct Answer: (D) 91

Solution:

Concept:

- For a complex number, expand using ordinary algebra and then use $i^2=-1$ and $i^3=-i$. Real and imaginary parts of equal complex numbers must agree separately.
- The denominator $27=3^3$ suggests writing $-2-i/3=(-6-i)/3$. This removes fractions before cubing; the final numerator is exactly $x+iy$.

Step 1: Remove the denominator before expansion The given equality becomes

$$\frac{(-6-i)^3}{27}=\frac{x+iy}{27},\qquad x+iy=(-6-i)^3.$$

The cube applies to the entire numerator, including its negative real part. Keeping the common denominator outside avoids distributing a factor of three incorrectly.

Step 2: Expand the cube and collect both parts First square the numerator, then multiply once more:

$$\begin{aligned}(-6-i)^2&=36+12i+i^2=35+12i,\\(35+12i)(-6-i)&=-210-35i-72i-12i^2\\&=-198-107i.\end{aligned}$$

The last product contributes $+12$ to the real part because $i^2=-1$; it does not contribute to the imaginary coefficient.

Step 3: Identify the two real numbers and subtract Comparing coefficients gives $x=-198$ and $y=-107$. Consequently,

$$y-x=-107-(-198)=91.$$

The value -91 in (C) reverses the requested subtraction. Neither -85 nor 85 in (A) and (B) agrees with the correctly expanded numerator; changing the sign of the i^2 contribution produces an incorrect real part.

Step 4: Conclusion $y-x=91$. Hence, the correct option is (D).

Quick Tip to Solve: For a fractional complex number, clear its denominator before raising it to a power, then compare real and imaginary coefficients only at the end.

23. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is : [4 marks]

(A) $x + y + 1 = 0$

(B) $4x + 2y + 1 = 0$

(C) $x - 2y + 4 = 0$

(D) $x + 2y + 4 = 0$

Correct Answer: (D) $x + 2y + 4 = 0$

Solution:

Concept:

- A nonvertical tangent of slope $m \neq 0$ to $y^2 = 4ax$ has equation $y = mx + a/m$. Here $a = 1$, so its intercept must be $1/m$.
- A line is tangent to a conic when substituting the line leaves a repeated intersection root. For a quadratic this is the condition that its discriminant is zero.

Step 1: Write a tangent to the parabola Let the common tangent be $y = mx + b$. The parabola tangent condition gives $b = 1/m$, hence

$$y = mx + \frac{1}{m}.$$

The vertex tangent $x = 0$ is vertical, but it never intersects $xy = 2$, so it cannot be the required common tangent.

Step 2: Impose tangency to the hyperbola Substitute the same line into $xy = 2$:

$$mx^2 + \frac{x}{m} - 2 = 0.$$

For a repeated real root,

$$\frac{1}{m^2} + 8m = 0 \quad \Longrightarrow \quad 8m^3 + 1 = 0.$$

There is one real solution, $m = -1/2$. Thus $b = 1/m = -2$, and the line is $y = -x/2 - 2$.

Step 3: Verify the line and exclude the other choices Rearranging gives $x + 2y + 4 = 0$. Substitution in the parabola yields $(y + 2)^2 = 0$, while substitution in the hyperbola yields $(x + 2)^2 = 0$, confirming tangency to both. In (A), slope -1 fails $8m^3 + 1 = 0$. In (B), slope -2 and intercept $-1/2$ satisfy only the parabola condition, not the hyperbola condition. In (C), slope $1/2$ again fails the hyperbola discriminant condition.

Step 4: Conclusion The common tangent is $x + 2y + 4 = 0$. Hence, the correct option is (D).

Quick Tip to Solve: Use the tangent form of one conic first, then demand a zero discriminant after substitution into the second; this reduces the unknowns to one slope.

24. The value of r for which $^{20}C_r\ ^{20}C_0 + {}^{20}C_{r-1}\ ^{20}C_1 + {}^{20}C_{r-2}\ ^{20}C_2 + \ldots\ldots + {}^{20}C_0\ ^{20}C_r$ is maximum, is [4 marks]

(A) 20

(B) 15

(C) 10

(D) 11

Correct Answer: (A) 20

Solution:

Concept:

- In a product of two polynomials, the coefficient of t^r is the sum of products of coefficients whose exponents add to r.
- The binomial theorem gives $(1+t)^{20}(1+t)^{20} = (1+t)^{40}$. This coefficient comparison is Vandermonde's identity.

Step 1: Interpret the displayed sum as a coefficient Write the sum as

$$S_r = \sum_{k=0}^{r} \binom{20}{r-k}\binom{20}{k}.$$

Every term selects t^{r-k} from the first factor $(1+t)^{20}$ and t^k from the second. Thus S_r is precisely the coefficient of t^r in their product. Terms outside the binomial coefficient range are zero.

Step 2: Combine the two factors Because the bases agree,

$$(1+t)^{20}(1+t)^{20} = (1+t)^{40}.$$

Comparing the same coefficient on both sides gives

$$S_r = \binom{40}{r}.$$

We must maximize a coefficient of degree forty, rather than maximize a single factor from either original degree-twenty expansion.

Step 3: Locate the maximum and check the options For consecutive coefficients,

$$\frac{\binom{40}{r+1}}{\binom{40}{r}} = \frac{40-r}{r+1}.$$

This ratio exceeds one up to the increase from $r = 19$ to 20, and is below one for the increase from 20 to 21. Therefore the unique maximum is at $r = 20$. The offered values $10, 11, 15$ all lie on the increasing side of the sequence, so their coefficients are strictly smaller than the central coefficient.

Step 4: Conclusion The sum is largest for $r = 20$. Hence, the correct option is (A).

Quick Tip to Solve: When a sum contains complementary binomial subscripts, view it as a coefficient in a product; the central coefficient of the combined power gives the maximum.

25. The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane 2x + 3y − z = 5, contains which one of the following points ? [4 marks]

(A) (− 2, 2, 2)

(B) (2, 2, 0)

(C) (2, 0, − 2)

(D) (0, − 2, 2)

Correct Answer: (C) (2, 0, − 2)

Solution:

Concept:

- The line has a point $A = (3, -2, 1)$ and direction $\mathbf{d} = (2, -1, 3)$. The plane onto which it is projected has normal $\mathbf{n} = (2, 3, -1)$.
- Orthogonal projection moves a point along the normal to the projection plane. A plane containing both the original line and its projected line therefore contains the directions $\mathbf{d}$ and $\mathbf{n}$.

Step 1: Find a normal to the required plane The two spanning directions are not parallel. Their cross product gives a normal to the plane containing the line and its normal projection:

$$\begin{aligned}\mathbf{d} \times \mathbf{n} &= ((-1)(-1) - 3(3),\ 3(2) - 2(-1),\ 2(3) - (-1)(2)) \\ &= (-8, 8, 8).\end{aligned}$$

We can divide by -8, so a simpler normal is $(1, -1, -1)$.

Step 2: Use a known point of the original line The required plane passes through A, so its equation is

$$(x - 3) - (y + 2) - (z - 1) = 0,$$

which simplifies to $x - y - z = 4$. Its normal is perpendicular to $\mathbf{n}$, since $2 - 3 + 1 = 0$. Therefore moving any point of the original line along $\mathbf{n}$, including to its perpendicular projection, keeps it in this plane.

Step 3: Test each offered point Evaluate the left side $x - y - z$ at the four points:

Option	A	B	C	D
$x - y - z$	-6	0	4	0

Only $(2, 0, -2)$ gives the required value 4. The other three points fail the plane equation, even though they might lie on other planes related to the data.

Step 4: Conclusion The plane $x - y - z = 4$ contains $(2, 0, -2)$. Hence, the correct option is (C).

Quick Tip to Solve: The plane joining a line to its orthogonal projection is spanned by the line direction and the normal of the projection plane; cross these to find its normal.

26. If tangents are drawn to the ellipse x2 + $2y^2$ = 2 at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve : [4 marks]

(A) $\frac{x^2}{2} + \frac{y^2}{4} = 1$

(B) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

(C) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$

(D) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

Correct Answer: The printed $x2$ contradicts the stated ellipse; (B) follows only if it denotes x^2.

Solution:

Concept:

- The printed expression is $x2 + 2y^2 = 2$, with the first 2 on the baseline. Reading $x2$ literally as $2x$ gives a parabola, whereas the wording calls the curve an ellipse with four vertices.
- For a tangent meeting the axes at $(a, 0)$ and $(0, b)$, its intercepted segment has midpoint $(a/2, b/2)$. The curve equation must be unambiguous before deriving its tangent and midpoint locus.

Step 1: Check the literal printed equation On the literal reading $2x + 2y^2 = 2$, the curve is $x + y^2 = 1$, which has one vertex rather than the stated four. At its point $(1 - v^2, v)$, its tangent is

$$x + 2vy = 1 + v^2.$$

For $v \neq 0$, the midpoint (h, k) of its intercepts satisfies

$$h = \frac{1 + v^2}{2}, \qquad k = \frac{1 + v^2}{4v}.$$

Eliminating v gives $h^2 = 4k^2(2h - 1)$, which is none of the offered curves. Thus the literal equation does not support a listed answer.

Step 2: Separate the ellipse reading from the literal one If the baseline $x2$ instead denotes the squared term x^2, the ellipse is $x^2 + 2y^2 = 2$. At contact (u, v), its tangent is $ux + 2vy = 2$. Its intercept midpoint then has

$$h = \frac{1}{u}, \qquad k = \frac{1}{2v}.$$

Substituting $u = 1/h$ and $v = 1/(2k)$ into $u^2 + 2v^2 = 2$ gives

$$\frac{1}{2h^2} + \frac{1}{4k^2} = 1.$$

This conditional ellipse result is (B). Option (C) swaps the unequal coefficients, while (A) and (D) incorrectly use squares instead of reciprocals. This calculation does not remove the inconsistency in the literal printing.

Step 3: Conclusion As printed, the equation and the description are inconsistent, so no unique option follows literally. Under the ellipse interpretation $x2 = x^2$, the midpoint locus is $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$, option (B).

Quick Tip to Solve: Check that each exponent agrees with the named conic before applying its tangent formula; for an ellipse, eliminate the contact coordinates using the tangent midpoint.

27. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is : [4 marks]

(A) $\frac{y}{\sqrt{3}}$
(B) $\frac{c}{3}$
(C) $\frac{c}{\sqrt{3}}$
(D) $\frac{3}{2}$y

Correct Answer: (C) $\frac{c}{\sqrt{3}}$

Solution:

Concept:

- For triangle sides a, b, c, with angle C opposite c, the cosine rule gives $c^2 = a^2 + b^2 - 2ab\cos C$.
- The extended sine rule relates the circumradius R to a side and its opposite angle: $c = 2R\sin C$. The side sum and product determine the included angle here.

Step 1: Rewrite the given relation in side lengths Let the two specified sides be a, b. Then $x = a + b$ and $y = ab$, with $a, b > 0$. The condition gives

$$c^2 = x^2 - y = (a+b)^2 - ab = a^2 + ab + b^2.$$

Keeping the product term is essential: expanding the square contributes $2ab$, of which only one ab is subtracted.

Step 2: Determine the angle opposite the third side Compare this with the cosine rule:

$$a^2 + b^2 - 2ab\cos C = a^2 + b^2 + ab.$$

Since $ab > 0$, division gives $\cos C = -1/2$. A triangle angle lies between 0 and π, so $C = 120^\circ$ and

$$\sin C = \frac{\sqrt{3}}{2}.$$

The positive sine is required; the circumradius is a positive length.

Step 3: Apply the sine rule and check dimensions Using the side opposite this angle,

$$R = \frac{c}{2\sin 120^\circ} = \frac{c}{\sqrt{3}}.$$

Option (B), $c/3$, has the correct dimension but the wrong factor in the extended sine rule. Options (A) and (D) are proportional to $y = ab$, which has the dimension of length squared, so neither can be a general expression for a radius. Only (C) matches both the geometry and dimensions.

Step 4: Conclusion The circumradius is $R = c/\sqrt{3}$. Hence, the correct option is (C).

Quick Tip to Solve: When both the sum and product of two sides are given, expand the sum squared and compare with the cosine rule; then use the extended sine rule for the radius.

28. If q is false and p $\wedge$ q $\leftrightarrow$ r is true, then which one of the following statements is a tautology ?

[4 marks]

(A) P $\wedge$ r

(B) (p $\vee$ r) $\rightarrow$ (p $\wedge$ r)

(C) p $\vee$ r

(D) (p $\wedge$ r) $\rightarrow$ (p $\vee$ r)

Correct Answer: (D) (p $\wedge$ r) $\rightarrow$ (p $\vee$ r)

Solution:

Concept:

- A biconditional $s \leftrightarrow r$ is true exactly when s and r have equal truth values. A conjunction is false whenever either component is false.
- An implication $A \rightarrow B$ is false only for a true antecedent and a false consequent. A tautology is true for every allowed assignment of its variables.

Step 1: Use the given truth values As q is false, $p \wedge q$ is false regardless of p. For $(p \wedge q) \leftrightarrow r$ to be true, r must also be false. The premises do not fix p, so both its truth values must be considered; selecting one value of p would not prove a tautology.

Step 2: Check the four statements under the premises With $r = F$, option (A) is $p \wedge F = F$. Option (C) is $p \vee F = p$, hence is false when $p = F$. Option (B) reduces to $p \rightarrow F$, which is false when $p = T$. Option (D) reduces to $F \rightarrow p$, which is true for either value of p. Therefore only (D) is always true under the supplied information.

Step 3: Verify the tautology without restricting the variables In fact, (D) holds for every p, r, even before using the premises. If $p \wedge r$ is true, then both statements are true, so $p \vee r$ is true as well. If $p \wedge r$ is false, the implication is automatically true. Equivalently, the implication cannot have a true antecedent and a false consequent. This verifies its status as a tautology rather than a statement true only in one selected row of a truth table.

Step 4: Conclusion $(p \wedge r) \rightarrow (p \vee r)$ is a tautology. Hence, the correct option is (D).

Quick Tip to Solve: To test an implication for tautology, look for a true antecedent with a false consequent; if conjunction implies disjunction, that counterexample is impossible.

29. The direction ratios of normal to the plane through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane y − z + 5 = 0 are : [4 marks]

(A) 2, -1, 1

(B) $2\sqrt{3}, 1, -1$

(C) $\sqrt{2}, 1, -1$

(D) $\sqrt{2}, -\sqrt{2}$

Correct Answer: (C) $\sqrt{2}, 1, -1$

Solution:

Concept:

- A normal vector to a plane is perpendicular to every displacement vector joining two points of the plane. Let its direction ratios be (a, b, c).
- The acute angle between two planes is the acute angle between their normals: $\cos\theta = |\mathbf{n} \cdot \mathbf{m}|/(|\mathbf{n}||\mathbf{m}|)$. Direction ratios can be multiplied by any nonzero common factor.

Step 1: Use the two given points The displacement from $(0, -1, 0)$ to $(0, 0, 1)$ is $(0, 1, 1)$. Perpendicularity to the required normal therefore gives

$$(a, b, c) \cdot (0, 1, 1) = b + c = 0.$$

Thus $c = -b$ and the normal has the form $(a, b, -b)$. This condition uses the fact that the whole line through the given points lies in the plane.

Step 2: Apply the angle condition The normal to $y - z + 5 = 0$ is $(0, 1, -1)$. With $\theta = \pi/4$,

$$\frac{|2b|}{\sqrt{a^2 + 2b^2}\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

Squaring and simplifying yields $4b^2 = a^2 + 2b^2$, hence $a^2 = 2b^2$. We must have $b \neq 0$, since otherwise this relation would give the zero normal. Taking $b = 1$ and the positive choice gives $(\sqrt{2}, 1, -1)$. The negative choice for a also defines a permissible plane, but is not listed.

Step 3: Check the listed ratios For option (C), $b + c = 0$ and $a^2 = 2b^2$, so both independent conditions hold. Option (A) satisfies the first condition but gives $a^2 = 4$, not $2b^2 = 2$. Option (B) similarly gives $a^2 = 12$, so its angle is different. Option (D), as printed, supplies only two entries and does not specify the three direction ratios of a normal in space.

Step 4: Conclusion The listed admissible direction ratios are $\boxed{\sqrt{2}, 1, -1}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the line joining the given points first: it reduces the unknown normal to $(a, b, -b)$, so the angle equation contains only one essential ratio.

30. Two circles with equal radii are intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is : [4 marks]

(A) $2\sqrt{2}$

(B) $\sqrt{2}$

(C) 2

(D) 1

Correct Answer: (C) 2

Solution:

Concept:

- The centre of any circle through $(0, 1)$ and $(0, -1)$ lies on the perpendicular bisector of this chord, namely the x-axis.
- A tangent at a point of a circle is perpendicular to the radius at that point. Equal radii constrain the distances of both centres from either common point.

Step 1: Locate the two centres Write the centres as $C_1 = (a, 0)$ and $C_2 = (b, 0)$. The squared radii measured from the common point $P = (0, 1)$ are $a^2 + 1$ and $b^2 + 1$. As the radii are equal,

$$a^2 + 1 = b^2 + 1 \quad \Rightarrow \quad b = \pm a.$$

The centres are distinct because the circles intersect in two points rather than coincide. Therefore $b = -a$ and $a \neq 0$.

Step 2: Use the tangent through the other centre The line PC_2 is tangent to the circle with centre C_1, so $C_1P \perp PC_2$. Using displacement vectors,

$$\overrightarrow{C_1P} = (-a, 1), \qquad \overrightarrow{PC_2} = (-a, -1).$$

Their dot product must vanish:

$$(-a)(-a) + (1)(-1) = a^2 - 1 = 0.$$

Thus $|a| = 1$. The centres lie at $(1, 0)$ and $(-1, 0)$, with their names possibly interchanged.

Step 3: Calculate and compare the distances The distance is

$$C_1C_2 = |a - (-a)| = 2|a| = 2.$$

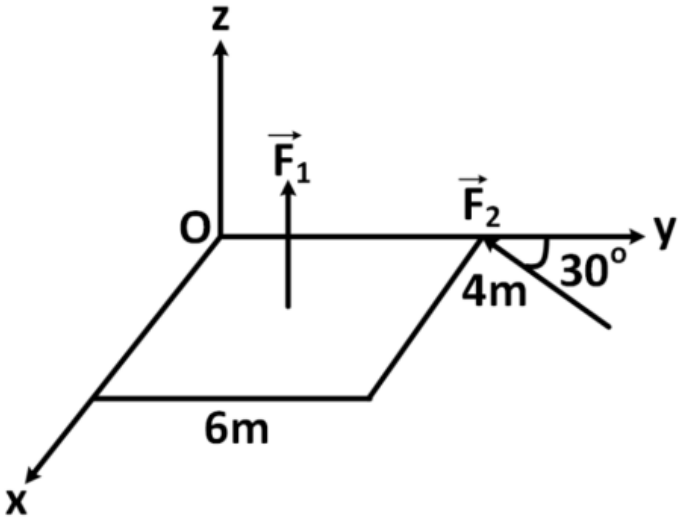
As a check, each circle has radius $\sqrt{1^2 + 1^2} = \sqrt{2}$, while the line from one centre to the other has length 2. The two radii joining the centres to P are perpendicular. Consequently $\sqrt{2}$ is the radius, not the required separation; $2\sqrt{2}$ would be the diameter, and 1 only the distance from either centre to the common chord. These account for the incorrect alternatives.

Step 4: Conclusion The distance between the centres is $\boxed{2}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal circles, the centres are mirror images across the common chord. Combine that symmetry with radius–tangent perpendicularity before writing circle equations.

Physics (120 marks)

31. A slab is subjected to two forces $\vec{F_1}$ and $\vec{F_2}$ of same magnitude F as shown in the figure. Force $\vec{F_2}$ is in XY-plane while force $\vec{F_1}$ acts along z = axis at the point $\left(2\vec{i}+3\vec{j}\right)$ .. The moment of these forces about point O will be : [4 marks]



(A) $\left(3\hat{i}-2\hat{j}-3\hat{k}\right)F$

(B) $\left(3\hat{i}+2\hat{j}-3\hat{k}\right)F$

(C) $\left(3\hat{i}+2\hat{j}+3\hat{k}\right)F$

(D) $\left(3\hat{i}-2\hat{j}+3\hat{k}\right)F$

Correct Answer: (D) $\left(3\hat{i}-2\hat{j}+3\hat{k}\right)F$

Solution:

Concept:

- The moment of a force about O is $\boldsymbol{\tau}=\mathbf{r}\times\mathbf{F}$, where $\mathbf{r}$ points from O to a point on the force line of action.
- Moments add vectorially. Use the right-handed products $\hat{\imath}\times\hat{\jmath}=\hat{k}$, $\hat{\jmath}\times\hat{k}=\hat{\imath}$, and $\hat{\imath}\times\hat{k}=-\hat{\jmath}$.

Step 1: Find the moment of the vertical force The first force is $\mathbf{F}_1=F\hat{k}$ and acts at $\mathbf{r}_1=2\hat{\imath}+3\hat{\jmath}$. Its moment is

$$\boldsymbol{\tau}_1=(2\hat{\imath}+3\hat{\jmath})\times F\hat{k}=F(3\hat{\imath}-2\hat{\jmath}).$$

The signs follow from the order $\mathbf{r}\times\mathbf{F}$; reversing that order would reverse the torque. The vertical force supplies no $\hat{k}$ component of moment.

Step 2: Resolve the force in the plane The second force acts on the positive y-axis at a distance $6\,\mathrm{m}$ from O, so $\mathbf{r}_2=6\hat{\jmath}$. Its arrow points towards the slab. The component along x is therefore negative, with magnitude $F\sin 30^\circ=F/2$. Its component parallel to y gives zero moment about O, because its

line of action passes along that axis. Hence

$$\boldsymbol{\tau}_2 = 6\hat{\jmath} \times \left(-\frac{F}{2}\hat{\imath}\right) = 3F\hat{k}.$$

The indicated $4\,\mathrm{m}$ side is not the lever arm of this force about O.

Step 3: Add and check the components The resultant moment is

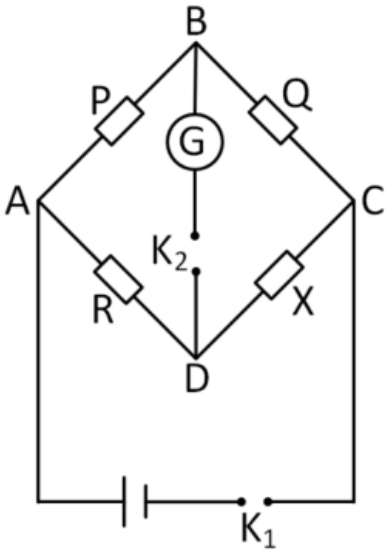
$$\boldsymbol{\tau} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 = F(3\hat{\imath} - 2\hat{\jmath} + 3\hat{k}).$$

Options (B) and (C) have the wrong sign for the $\hat{\jmath}$ component generated by $\mathbf{F}_1$. Option (A) gives a negative $\hat{k}$ component, contrary to the direction of the second force in the figure. Only option (D) has all three signs and magnitudes correct. With the distances expressed in metres, the moment is in newton metres when F is in newtons.

Step 4: Conclusion The moment about O is $\boxed{(3\hat{\imath} - 2\hat{\jmath} + 3\hat{k})F}$. Hence, the correct option is (D).

Quick Tip to Solve: For a force applied on the y-axis, discard its y-component immediately; only its transverse components can contribute to the moment about the origin.

32. In a Wheatstone bridge(see fig.), Resistances P and Q are approximately equal. When R = 400 Ω, the bridge is balanced. On interchanging P and Q, the value of R, for balance, is 405 Ω. The value of X is close to : [4 marks]



(A) 402.5 ohm

(B) 401.5 ohm

(C) 403.5 ohm

(D) 404.5 ohm

Correct Answer: (A) 402.5 ohm

Solution:

Concept:

- A balanced Wheatstone bridge carries no galvanometer current, so the corresponding resistance ratios satisfy $P/Q = R/X$.
- Interchanging the two ratio-arm resistances inverts the ratio. Multiplying the two balance equations removes the unknown ratio without assuming $P = Q$ exactly.

Step 1: Write the first balance condition In the diamond shown, P and R are connected to the left junction, while Q and X are connected to the right junction. Equality of the potentials of the galvanometer junctions gives

$$\frac{P}{Q} = \frac{400}{X}.$$

The numerical resistance values in these equations are in ohms. The statement that P and Q are approximately equal suggests X will be close to $400\,\Omega$, but does not permit replacing their ratio by exactly one.

Step 2: Use the interchanged arrangement After interchanging only P and Q, balance requires $R = 405\,\Omega$, so

$$\frac{Q}{P} = \frac{405}{X}.$$

Multiplying by the first equation eliminates P and Q:

$$1 = \frac{400 \times 405}{X^2}, \qquad X = \sqrt{162000}\,\Omega.$$

The positive square root is selected because X is an ordinary resistance.

Step 3: Evaluate the geometric mean Numerically,

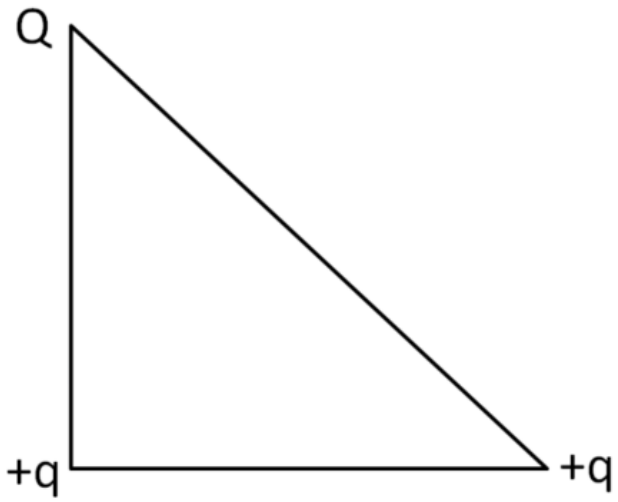
$$X = 402.492\ldots\,\Omega \simeq 402.5\,\Omega.$$

The arithmetic mean $(400 + 405)/2 = 402.5$ is an excellent approximation here because the two balance values are close. More precisely, $X^2 = 402.5^2 - 2.5^2$, so the geometric mean is only about $0.008\,\Omega$ below that arithmetic mean. The other proposed values, 401.5, 403.5 and $404.5\,\Omega$, differ by roughly one ohm or more and do not satisfy the product relation. No separate measurement of P or Q is needed.

Step 4: Conclusion The unknown resistance is approximately $\boxed{402.5\,\Omega}$. Hence, the correct option is (A).

Quick Tip to Solve: When ratio arms are swapped, multiply the two balance equations: the unchanged resistance is the geometric mean of the two balancing resistances.

33. Three charges Q, + q and + q are placed at the vertices of a right-angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is : [4 marks]



(A) $\frac{-q}{1+\sqrt{2}}$
(B) + q
(C) − 2q
(D) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$

Correct Answer: (D) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$

Solution:

Concept:

- The electrostatic energy of point charges is the sum over distinct pairs: $U = \sum_{i<j} kq_iq_j/r_{ij}$, where $k = 1/(4\pi\varepsilon_0)$. Each pair is counted once.
- For a right isosceles triangle with equal legs a, the hypotenuse has length $\sqrt{2}a$. Like-charge pairs contribute positive energy and unlike-charge pairs negative energy.

Step 1: Identify all three pair separations Let each equal perpendicular side be a. The two positive charges at the lower vertices are separated by a. The charge Q is at the upper vertex, so its distance from the lower-left $+q$ is a, and its distance from the lower-right $+q$ is $\sqrt{2}a$. The position of Q matters: it is not at the right-angle vertex. Thus its two interaction terms have different denominators.

Step 2: Impose zero total energy Adding the three pair energies gives

$$U = \frac{kq^2}{a} + \frac{kQq}{a} + \frac{kQq}{\sqrt{2}a}.$$

For $U = 0$, divide by the common nonzero factor kq/a:

$$q + Q\left(1 + \frac{1}{\sqrt{2}}\right) = 0.$$

Solving for the unknown charge,

$$Q = -\frac{q}{1 + 1/\sqrt{2}} = -\frac{\sqrt{2}q}{\sqrt{2}+1}.$$

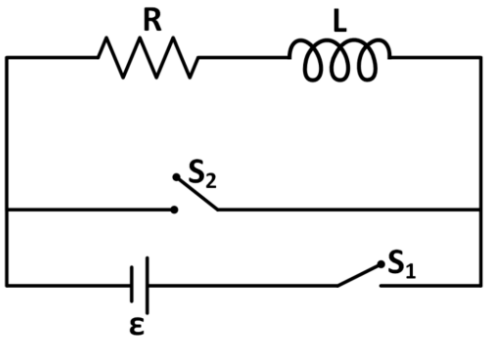
The negative sign is physically necessary to cancel the positive energy of the $+q, +q$ pair.

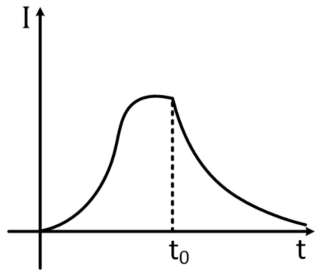
Step 3: Test the other charge values With this value of Q, the two attractive interaction energies sum to exactly $-kq^2/a$. Option (B), $Q = +q$, makes all three terms positive and cannot give zero energy. Option (C), $Q = -2q$, makes the attractive terms too large in magnitude. Option (A) lacks the factor $\sqrt{2}$ in the numerator and leaves a positive residual energy. Substituting its value into the reduced equation gives $q - q/\sqrt{2} \neq 0$. Only the charge in option (D) cancels the repulsive term exactly.

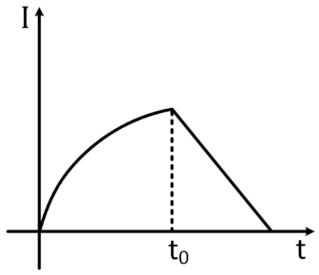
Step 4: Conclusion The required charge is $\boxed{Q = -\frac{\sqrt{2}q}{\sqrt{2}+1}}$. Hence, the correct option is (D).

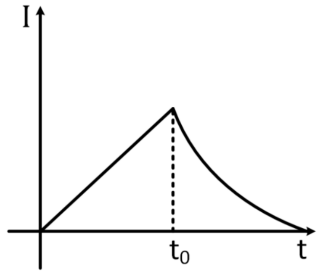
Quick Tip to Solve: List the three pair distances before writing energy. The unknown charge lies at an acute vertex here, so its two distances are a and $\sqrt{2}a$, not two equal legs.

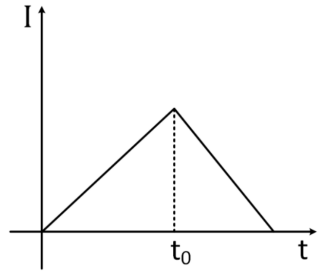
34. In the circuit shown, switch S_1 is closed at time $t = 0$ and switch S_2 is kept open. At some later time t_0, switch S_1 is opened and S_2 is closed. The behaviour of the current I as a function of time t is given by: [4 marks]



(A)


(B)


(C)


(D)


Correct Answer: No listed graph shows both the exact exponential rise and exponential decay required by this RL circuit.

Solution:

Concept:

- An ideal inductor opposes changes in current: $V_L = L\,dI/dt$. In a series RL circuit the time constant is $\tau = L/R$.
- The current through an ideal inductor is continuous at switching when no impulsive voltage is applied. Removing the source while completing the resistor–inductor loop produces exponential decay, not an abrupt fall.

Step 1: Find the current while the battery is connected For $0 \le t < t_0$, switch S_1 connects the battery and S_2 is open. Kirchhoff's voltage law gives

$$L\frac{dI}{dt} + RI = \varepsilon, \qquad I(0) = 0.$$

Therefore

$$I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right).$$

This curve rises from zero with the largest slope initially; its slope then decreases. It is concave down throughout the growth interval, since $d^2I/dt^2 < 0$. It is neither a straight line nor an S-shaped growth curve.

Step 2: Find the current after switching At $t = t_0$, opening S_1 removes the battery and closing S_2 leaves a closed loop containing R and L. Write $I_0 = (\varepsilon/R)(1 - e^{-t_0/\tau})$. Continuity requires $I(t_0^+) = I_0$. The source-free loop obeys

$$L\frac{dI}{dt} + RI = 0, \qquad I(t) = I_0 e^{-(t-t_0)/\tau} \quad (t \ge t_0).$$

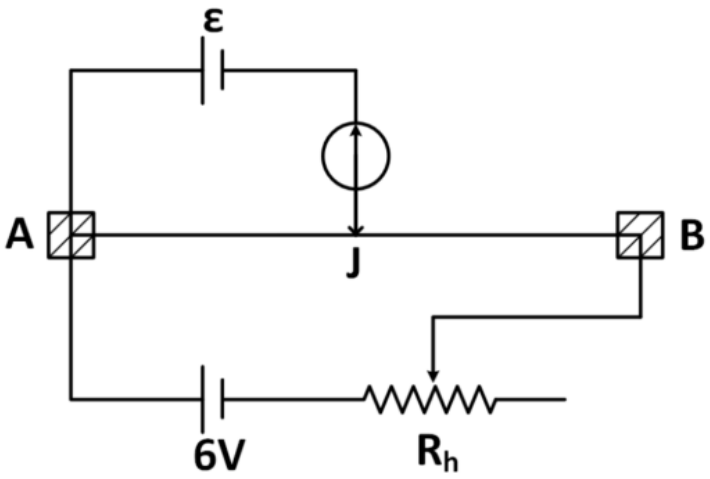
The decay is concave up and approaches zero asymptotically; it does not reach zero after a finite straight-line ramp.

Step 3: Compare the printed graph shapes The physically required graph has exponential, concave-down growth followed continuously by exponential, concave-up decay. Graph (A) has an initially increasing growth slope, unlike $1 - e^{-t/\tau}$. Graph (B) has the correct qualitative rising curvature but a straight falling segment reaching zero at finite time. Graph (C) has a straight rising segment, and graph (D) has straight segments on both sides. None of the four printed graphs depicts both parts of the exact RL response. A straight segment can be a local short-time approximation, but the question gives no such approximation or restriction on t_0.

Step 4: Conclusion The current grows as $\varepsilon(1 - e^{-tR/L})/R$ and then decays as $I_0 e^{-(t-t_0)R/L}$. No printed option has both required curvatures, so there is no unique correct option as drawn.

Quick Tip to Solve: Check both curvatures: RL growth is concave down, RL decay is concave up, and the two branches join at the same nonzero current.

35. The resistance of the meter bridge AB in given figure is 4 Ω. With a cell of emf ε = 0.5 V and rheostat resistance R_h = 2 Ω the null point is obtained at some point J. When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$ the same null point J is found for R_h = 6 Ω. The emf ε_2 is, : [4 marks]



(A) 0.3 V

(B) 0.6 V

(C) 0.5 V

(D) 0.4 V

Correct Answer: (A) 0.3 V

Solution:

Concept:

- At a potentiometer null point, no current flows through the test cell or galvanometer; the test emf equals the potential drop along the balancing length of the uniform wire.
- For a fixed balancing length, the balancing emf is proportional to the current through the wire. The driving current is determined by the total resistance of the primary circuit.

Step 1: Calculate the first driving current The lower circuit contains the $6\,\mathrm{V}$ supply, the full wire AB of resistance $4\,\Omega$, and the rheostat. The upper $0.5\,\mathrm{V}$ cell is the test cell, not the source driving the wire. With $R_h = 2\,\Omega$, the current in the wire is

$$I_1 = \frac{6}{4+2} = 1\,\mathrm{A}.$$

At balance, $\varepsilon_1 = I_1 R_{AJ}$, where R_{AJ} is the resistance of the wire segment from A to the unchanged null point J. Thus $R_{AJ} = 0.5\,\Omega$.

Step 2: Use the same balancing length After replacement of the test cell, the primary supply remains $6\,\mathrm{V}$, while the rheostat is set to $6\,\Omega$. The new current is

$$I_2 = \frac{6}{4+6} = 0.6\,\mathrm{A}.$$

Because J is unchanged, R_{AJ} is unchanged. The second emf is therefore

$$\varepsilon_2 = I_2 R_{AJ} = 0.6 \times 0.5 = 0.3\,\text{V}.$$

Equivalently, $\varepsilon_2/\varepsilon_1 = I_2/I_1 = (4+2)/(4+6) = 3/5$.

Step 3: Check the direction of the change Increasing the rheostat resistance reduces the current and hence the potential gradient along the wire. For the same null length, the balancing emf must consequently be smaller than $0.5\,\text{V}$, excluding $0.6\,\text{V}$ and $0.5\,\text{V}$. The value $0.4\,\text{V}$ also fails the required ratio $3/5$. Using the rheostat resistance alone would ignore the substantial $4\,\Omega$ wire resistance; both are in series in the driving circuit. The correct balance uses the whole primary-circuit resistance.

Step 4: Conclusion The new cell emf is $\boxed{0.3\,\text{V}}$. Hence, the correct option is (A).

Quick Tip to Solve: At the same null length, compare primary currents. Include the entire potentiometer wire resistance along with the rheostat when forming the current ratio.

---

36. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle? [4 marks]

(A) zero

(B) 10 m/s

(C) $10\sqrt{2}m/s$

(D) $10\sqrt{3}m/s$

Correct Answer: (B) 10 m/s

Solution:

Concept:

- Speed is the magnitude of velocity. Constant speed does not imply constant velocity, because the tangent direction changes during circular motion.
- The angle between the initial and final tangential velocity vectors equals the angle swept by the radius. For equal magnitudes v, the change has magnitude $2v\sin(\theta/2)$.

Step 1: Identify the two velocity vectors At both positions, the velocity is tangent to the circle and has magnitude $v = 10\,\text{m}\,\text{s}^{-1}$. Since each tangent is perpendicular to its radius, turning the radius through 60° also turns the velocity through 60°. We must subtract these vectors; subtracting their scalar magnitudes would only give the change in speed.

Step 2: Subtract the velocities as vectors Use $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$ and take its squared magnitude. The scalar product includes the cosine of the angle between the two velocities:

$$|\Delta\vec{v}|^2 = v_f^2 + v_i^2 - 2v_f v_i \cos 60^\circ = 100 + 100 - 200\left(\frac{1}{2}\right) = 100.$$

Taking the nonnegative square root gives

$$|\Delta\vec{v}| = 10\,\mathrm{m\,s^{-1}}.$$

Equivalently, $2(10)\sin 30^\circ = 10\,\mathrm{m\,s^{-1}}$.

Step 3: Check the other choices Zero would require the initial and final velocity vectors to point in the same direction, which they do not. The values $10\sqrt{2}$ and $10\sqrt{3}\,\mathrm{m\,s^{-1}}$ would correspond to turns of 90° and 120°, respectively. Neither is the stated angular displacement.

Step 4: Conclusion The magnitude of the change in velocity is $\boxed{10\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a constant-speed turn, use the chord of the velocity circle: $|\Delta\vec{v}| = 2v\sin(\theta/2)$, never just final speed minus initial speed.

37. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is TV^x = constant, then x is : [4 marks]

(A) $\frac{5}{3}$
(B) $\frac{2}{5}$
(C) $\frac{3}{5}$
(D) $\frac{2}{3}$

Correct Answer: (B) $\frac{2}{5}$

Solution:

Concept:

- A rigid diatomic molecule at room temperature has five active degrees of freedom: three translational and two rotational. Its vibrational modes are neglected.
- For an ideal gas, $C_V = fR/2$, $C_P = C_V + R$, and $\gamma = C_P/C_V$. The adiabatic temperature-volume relation is $TV^{\gamma-1} = \text{constant}$.

Step 1: Find the heat-capacity ratio Here $f = 5$, so the molar heat capacities are

$$C_V = \frac{5}{2}R, \qquad C_P = \frac{7}{2}R.$$

Therefore

$$\gamma = \frac{C_P}{C_V} = \frac{7R/2}{5R/2} = \frac{7}{5}.$$

The word rigid describes the molecular model; it does not say that the gas is enclosed in a fixed-volume container. The volume may change in the stated process.

Step 2: Convert the adiabatic equation For the usual quasistatic adiabatic process, $PV^\gamma = \text{constant}$. Substitute the ideal-gas equation $P = nRT/V$ with a fixed number n of moles:

$$\frac{nRT}{V}V^\gamma = \text{constant}.$$

The factor nR is constant and may be absorbed into the right-hand side, leaving

$$TV^{\gamma-1} = \text{constant}.$$

Comparing with $TV^x = \text{constant}$, we obtain

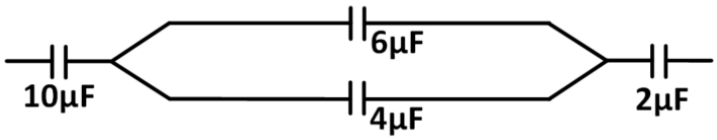
$$x = \gamma - 1 = \frac{7}{5} - 1 = \frac{2}{5}.$$

Step 3: Distinguish the competing exponents The ratio $5/3$ is the heat-capacity ratio of a monatomic gas, not the required exponent here. The exponent $2/3$ is $\gamma - 1$ for a monatomic gas. The value $3/5$ is not the diatomic adiabatic exponent in this temperature-volume equation. Thus only $2/5$ fits the molecular degrees of freedom and the correct relation.

Step 4: Conclusion For the rigid diatomic ideal gas, $\boxed{x = 2/5}$. Hence, the correct option is (B).

Quick Tip to Solve: For an ideal gas with f active degrees of freedom, the exponent in $TV^x = \text{constant}$ is $x = 2/f$. A rigid diatomic gas has $f = 5$.

38. In the figure shown below, the charge on the left plate of the 10μF capacitor is –30μC. The charge on the right plate of the 6 μF capacitor is : [4 marks]



(A) + 12 μC

(B) + 18 μC

(C) − 18 μC

(D) − 12 μC

Correct Answer: (B) + 18 μC

Solution:

Concept:

- Capacitors in parallel have the same potential difference, so their charge magnitudes divide in the ratio of their capacitances: $Q_6/Q_4 = 6/4$.
- The opposite plates of an ideal capacitor carry equal and opposite charges. Charge conservation on an initially neutral isolated connecting conductor determines the signs of the charges on the adjoining plates.

Step 1: Identify the charge entering the parallel block The $6\,\mu$F and $4\,\mu$F capacitors are connected

between the same two junctions. Their parallel equivalent is

$$C_p = 6 + 4 = 10\,\mu\text{F}.$$

The left plate of the leftmost $10\,\mu\text{F}$ capacitor has charge $-30\,\mu\text{C}$, so its right plate has $+30\,\mu\text{C}$. This right plate and the left plates of the two parallel capacitors form one isolated junction. Consequently their left-plate charges sum to $-30\,\mu\text{C}$.

Step 2: Divide the charge between the parallel capacitors Let $V_L - V_R$ be the potential difference from the left junction to the right junction of the parallel pair. The total charge on its left plates is

$$C_p(V_L - V_R) = -30\,\mu\text{C}.$$

Hence

$$V_L - V_R = \frac{-30\,\mu\text{C}}{10\,\mu\text{F}} = -3\,\text{V}.$$

The left plate charge of the $6\,\mu\text{F}$ capacitor is therefore

$$Q_{6,L} = 6\,\mu\text{F}\,(-3\,\text{V}) = -18\,\mu\text{C}.$$

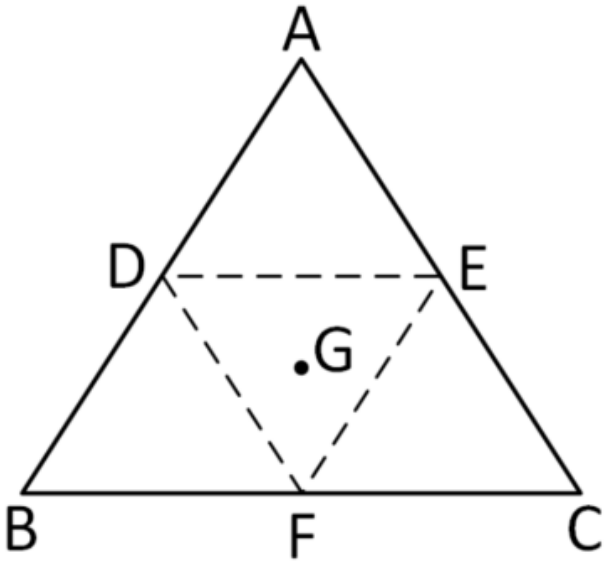
Its right plate has the opposite charge, $+18\,\mu\text{C}$.

Step 3: Check the sign and the other branch The $4\,\mu\text{F}$ capacitor carries a charge magnitude $4 \times 3 = 12\,\mu\text{C}$, which explains the magnitude appearing in two distractors. Those choices refer to the other capacitor. The choice $-18\,\mu\text{C}$ is the charge on the left plate of the $6\,\mu\text{F}$ capacitor; the question explicitly asks for its right plate.

Step 4: Conclusion The right plate of the $6\,\mu\text{F}$ capacitor carries $\boxed{+18\,\mu\text{C}}$. Hence, the correct option is (B).

Quick Tip to Solve: First use junction charge conservation to fix the sign. Then split the charge of a parallel group in proportion to its capacitances.

39. An equilateral triangle ABC is cut from a thin solid sheet of wood. (see figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. then : [4 marks]



(A) $I = \frac{I_0}{4}$

(B) $I = \frac{15}{16}I_0$

(C) $I = \frac{9}{16}I_0$

(D) $I = \frac{3}{4}I_0$

Correct Answer: (B) $I = \frac{15}{16}I_0$

Solution:

Concept:

- For geometrically similar uniform laminae of the same thickness and density, mass scales as the square of the linear scale factor.
- The moment of inertia about corresponding perpendicular centroidal axes scales as mass times length squared. It therefore scales as the fourth power of the linear scale factor.

Step 1: Compare the two triangles The line joining the midpoints of two sides of a triangle is parallel to the third side and has half its length. Therefore triangle DEF is equilateral and similar to ABC, with linear scale factor

$$k = \frac{DE}{BC} = \frac{1}{2}.$$

Their centroids coincide at G: the central inverted triangle is symmetric about the same three median lines. Thus the stated axis is the centroidal perpendicular axis for both triangles.

Step 2: Calculate the inertia removed Let the original triangle have mass M and side a. Since both

triangles are parts of the same uniform sheet, the smaller one has mass

$$M_s = k^2 M = \frac{M}{4}.$$

Write the centroidal moment of inertia as $I_0 = cMa^2$, where c is a shape-dependent numerical constant. The same constant applies to the smaller similar triangle, so

$$I_s = c\left(\frac{M}{4}\right)\left(\frac{a}{2}\right)^2 = \frac{1}{16}cMa^2 = \frac{I_0}{16}.$$

No parallel-axis addition is required because the centroids coincide.

Step 3: Subtract the removed contribution Moment of inertia is additive over pieces of a lamina about a fixed axis. Removing the central triangle therefore gives

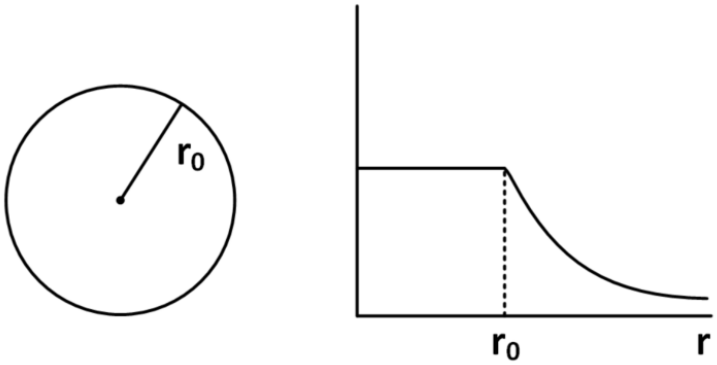
$$I = I_0 - I_s = I_0 - \frac{I_0}{16} = \frac{15I_0}{16}.$$

The fraction $1/4$ concerns the removed mass, rather than its inertia. The result $3I_0/4$ would incorrectly assume equal average squared distances for the removed and original material; the central piece lies much closer to the axis. The fraction $9/16$ has no such scaling basis.

Step 4: Conclusion The remaining lamina has moment of inertia $\boxed{I = \frac{15}{16}I_0}$. Hence, the correct option is (B).

Quick Tip to Solve: For a similar piece cut from a uniform sheet about the same centroid, multiply the area factor k^2 by the squared-distance factor k^2: its inertia factor is k^4.

40. The given graph shows variation (with distance r form centre) of : [4 marks]



(A) Electric field of a uniformly charged sphere

(B) Electric field of a uniformly charged spherical shell

(C) Potential of a uniformly charged sphere

(D) Potential of a uniformly charged spherical shell

Correct Answer: (D) Potential of a uniformly charged spherical shell

Solution:

Concept:

- The potential inside a uniformly charged spherical shell is constant and equals the surface potential, taking zero potential at infinity.
- Outside any spherically symmetric charge distribution of total charge Q, the potential is $V = kQ/r$, where $k = 1/(4\pi\varepsilon_0)$. The electric field and the potential have different radial profiles.

Step 1: Read the two portions of the graph The plotted quantity has a nonzero constant value from the centre to $r = r_0$. Beyond r_0, it decreases continuously toward zero. A spherical shell of radius r_0 has zero electric field inside it, so there is no change of potential between any two interior points. Its potential is the nonzero surface value throughout that entire region.

Step 2: Write the matching potential function For a shell with positive total charge Q, the radial potential is

$$V(r) = \begin{cases} kQ/r_0, & 0 \le r \le r_0, \\ kQ/r, & r \ge r_0. \end{cases}$$

The inside branch is horizontal, as in the figure. At $r = r_0$, both expressions give kQ/r_0, so the graph joins continuously. The exterior branch decreases inversely with distance and approaches zero at infinity, agreeing with the curved tail shown.

Step 3: Reject the other distributions The electric field of a uniformly charged solid sphere grows linearly from zero inside, so option (A) cannot give a nonzero horizontal interior segment. The field of a shell is zero inside and jumps to a nonzero value just outside its surface, unlike the continuous graph; this rules out (B). For a uniformly charged solid sphere,

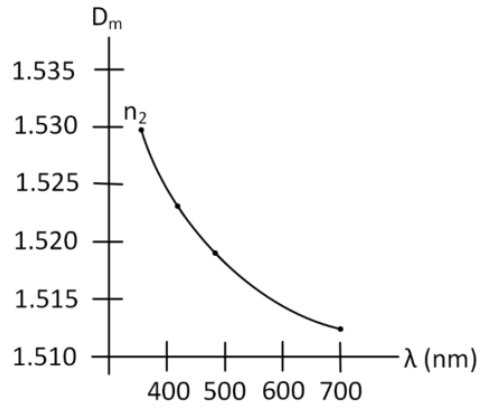
$$V(r) = \frac{kQ}{2r_0}\left(3 - \frac{r^2}{r_0^2}\right) \quad (r \le r_0),$$

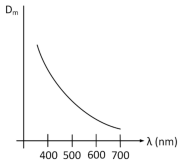
which decreases quadratically rather than staying constant. Thus (C) also fails. Only the shell potential has both features together.

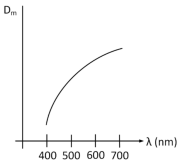
Step 4: Conclusion The graph represents the potential of a uniformly charged spherical shell. Hence, the correct option is (D).

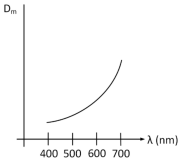
Quick Tip to Solve: A nonzero flat interior followed by a $1/r$ exterior tail identifies shell potential. A flat zero interior followed by a jump identifies shell electric field.

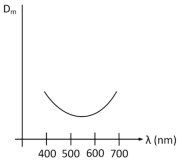
41. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation? [4 marks]



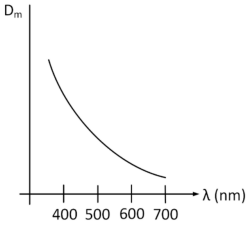
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- For a thin prism of fixed apex angle A, its minimum deviation is approximately $D_m = (n-1)A$, where n is its refractive index relative to air.
- Because A is a positive constant, subtracting one from n and multiplying by A preserves the increasing or decreasing character and the curvature of the given refractive-index curve.

Step 1: Relate deviation to refractive index The exact minimum-deviation relation is

$$n = \frac{\sin[(A + D_m)/2]}{\sin(A/2)}.$$

For a thin prism, the involved angles are small, so replacing each sine by its argument gives

$$n \simeq \frac{A + D_m}{A}, \qquad D_m \simeq (n-1)A.$$

We therefore compare the shapes as functions of wavelength at the same fixed prism angle; there is no need to know its numerical value.

Step 2: Transfer the wavelength dependence The supplied curve decreases as wavelength increases from the violet region toward the red region. It decreases more steeply at the short-wavelength end and becomes flatter toward $700\,\text{nm}$. Multiplication by $A > 0$ and subtraction of the constant A leave this shape unchanged. In differential form,

$$\frac{dD_m}{d\lambda} = A\frac{dn}{d\lambda} < 0, \qquad \frac{d^2D_m}{d\lambda^2} = A\frac{d^2n}{d\lambda^2} > 0.$$

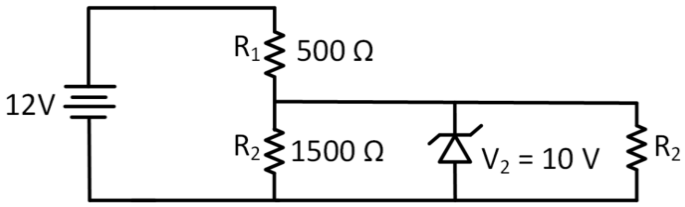
Thus the deviation curve must be decreasing and concave upward over the shown range.

Step 3: Compare all four graphical options Option (A) falls monotonically with wavelength and gradually flattens, matching both observations. Options (B) and (C) rise as wavelength increases, which contradicts the decreasing refractive index. Option (D) first falls and then rises, introducing a minimum that is absent from the input curve. Therefore neither an increasing graph nor a U-shaped graph can describe the thin-prism deviation in this range.

Step 4: Conclusion The minimum-deviation curve decreases and flattens as wavelength increases, as shown in graph (A). Hence, the correct option is (A).

Quick Tip to Solve: For the same thin prism, D_m is a positive linear function of n; transfer the shape of the refractive-index graph directly to the deviation graph.

42. In the given circuit the current through Zener Diode is close to: [4 marks]



(A) 0.0 mA

(B) 6.7 mA

(C) 4.0 mA

(D) 6.0 mA

Correct Answer: (A) 0.0 mA

Solution:

Concept:

- A reverse-biased Zener diode conducts appreciable breakdown current only when the reverse voltage reaches its Zener voltage V_Z. Below breakdown it is approximately an open circuit.
- Always check the open-circuit divider voltage before imposing voltage regulation. A passive load connected across the lower divider resistor can only reduce that voltage.

Step 1: Find the largest possible reverse voltage The series resistor is $R_1 = 500\,\Omega$, the lower divider resistor is $R_2 = 1500\,\Omega$, and the supply is $12\,\text{V}$. Temporarily ignore both the Zener and the extra parallel load branch. This gives the greatest possible output voltage across the lower resistor:

$$V_{\text{out,max}} = 12\frac{1500}{500 + 1500} = 9\,\text{V}.$$

The additional resistor drawn across that lower branch can only lower its equivalent resistance and hence lower the divider output; it cannot raise it above $9\,\text{V}$.

Step 2: Test the breakdown assumption The diode is reverse biased, and its marked breakdown voltage is $10\,\text{V}$. Since even the upper bound on its reverse voltage is

$$V_{\text{out}} \leq 9\,\text{V} < 10\,\text{V},$$

it cannot enter breakdown. It therefore carries only a negligible reverse leakage current in the idealized circuit. In particular, forcing $V_{\text{out}} = 10\,\text{V}$ would give a supply current $(12 - 10)/500 = 4\,\text{mA}$, while the $1500\,\Omega$ resistor alone would demand $10/1500 \simeq 6.7\,\text{mA}$. That contradiction independently rules out regulation.

Step 3: Separate resistor currents from diode current The numbers 4.0 and $6.7\,\text{mA}$ arise from incorrectly assuming an established $10\,\text{V}$ output and calculating currents in the series or lower resistor. They are not possible breakdown currents here. A nonzero diode current of $6.0\,\text{mA}$ likewise requires breakdown, which the supply and resistor network cannot support. The closest offered current is zero.

Step 4: Conclusion The Zener diode stays below breakdown, so $\boxed{I_Z \simeq 0.0\,\text{mA}}$. Hence, the correct option is (A).

Quick Tip to Solve: Check the divider output with the Zener open first. If that voltage is below V_Z, the diode cannot clamp the output and its breakdown current is zero.

43. The force of interaction between two atoms is given by F = $\alpha\beta\exp\left(-\frac{x^2}{\alpha kt}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is : [4 marks]

(A) $M^2L^2T^{-2}$

(B) M^2LT^{-4}

(C) MLT^{-4}

(D) $M^0L^2LT^{-4}$

Correct Answer: (B) M^2LT^{-4}

Solution:

Concept:

- The argument of an exponential must be dimensionless. Thus $x^2/(\alpha kT)$ has no dimensions.
- Boltzmann constant times absolute temperature is an energy: $[kT] = ML^2T^{-2}$. The symbol T in a dimensional formula denotes time, whereas temperature occurs only in the product kT.
- The exponential factor is dimensionless, so the prefactor $\alpha\beta$ must have the dimensions of force, MLT^{-2}.

Step 1: Determine the dimensions of alpha Since x is a distance, $[x^2] = L^2$. Equating numerator and denominator dimensions in the exponential gives

$$[\alpha][kT] = L^2.$$

Therefore

$$[\alpha] = \frac{L^2}{ML^2T^{-2}} = M^{-1}T^2.$$

No independent temperature dimension remains because kT, taken as a product, already has energy dimensions.

Step 2: Use the force prefactor to determine beta The dimensionless exponential cannot supply any dimensions to the force. Consequently,

$$[F] = [\alpha][\beta], \qquad [\beta] = \frac{MLT^{-2}}{M^{-1}T^2} = M^2LT^{-4}.$$

Option (A) has an extra length power and the wrong time power. Option (C) has only one mass power, and option (D) has no mass power. Only option (B) has the required combination. Substituting back gives $[\alpha\beta] = M^{-1}T^2\, M^2LT^{-4} = MLT^{-2}$, confirming the result.

Step 3: Conclusion The required dimensional formula is $\boxed{[\beta] = M^2LT^{-4}}$. Hence, the correct option is (B).

Quick Tip to Solve: For an exponential force law, make its argument dimensionless first; then use the prefactor to recover the dimensions of force.

44. There are two long co-axial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2, respectively. The ratio of mutual inductance to the self - inductance of the inner-coil is : [4 marks]

(A) $\frac{n_2}{n_1}.\frac{{r_2}^2}{{r_1}^2}$

(B) $\frac{n_2}{n_1}$

(C) $\frac{n_1}{n_2}$

(D) $\frac{n_2}{n_1}.\frac{r_1}{r_2}$

Correct Answer: (B) $\frac{n_2}{n_1}$

Solution:

Concept:

- For a long air-core solenoid, the field inside is $B = \mu_0 nI$, where n is its turn density and I is the current.
- Self-inductance is flux linkage per current, $L = N\Phi/I$. Mutual inductance is the flux linkage in one coil per current in the other.
- The common flux is restricted to the cross-section $A_1 = \pi r_1^2$ of the smaller inner solenoid; the larger outer area is not the shared area.

Step 1: Calculate the inner solenoid self-inductance The inner coil has $N_1 = n_1 l$ turns. Its own field is $B_1 = \mu_0 n_1 I_1$, and each turn encloses flux $\Phi_1 = B_1 A_1$. Hence

$$L_1 = \frac{N_1 \Phi_1}{I_1} = \mu_0 n_1^2 l \pi r_1^2.$$

The result contains the square of its own turn density because that density determines both the field and the number of linked turns.

Step 2: Calculate the mutual inductance and form the ratio Pass a current I_2 through the outer solenoid. Its uniform interior field is $B_2 = \mu_0 n_2 I_2$, which threads the entire inner area. The flux linkage of the inner coil is $n_1 l B_2 A_1$, so

$$M = \frac{n_1 l B_2 A_1}{I_2} = \mu_0 n_1 n_2 l \pi r_1^2.$$

Dividing by the inner self-inductance gives

$$\frac{M}{L_1} = \frac{\mu_0 n_1 n_2 l \pi r_1^2}{\mu_0 n_1^2 l \pi r_1^2} = \frac{n_2}{n_1}.$$

Both geometric factors cancel. Options (A) and (D) retain an unjustified radius factor, while (C) reverses the turn-density ratio.

Step 3: Conclusion The ratio is $\boxed{M/L_1 = n_2/n_1}$. Hence, the correct option is (B).

Quick Tip to Solve: For coaxial long solenoids, use the smaller cross-section in mutual flux; compare with the specified coil self-inductance only after writing both expressions.

45. An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be : [4 marks]

(A) 1.16×10^{-3} m/s towards the lens

(B) 2.26×10^{-3} m/s away from the lens

(C) 3.22×10^{-3} m/s towards the lens

(D) 0.92×10^{-3} m/s away from the lens

Correct Answer: (A) 1.16×10^{-3} m/s towards the lens

Solution:

Concept:

- For a real object a positive distance s from a convex lens and its real image at distance v, the thin-lens equation is $1/f = 1/s + 1/v$.
- The image distance depends on the object distance: $v = fs/(s-f)$. Differentiate this relation to find the instantaneous image velocity.
- Take increasing image distance as motion away from the lens. A negative dv/dt therefore means motion towards it.

Step 1: Express the image distance and differentiate The object is well outside the focal point, so the image is real and lies on the opposite side of the lens. With f constant,

$$v = \frac{fs}{s-f}, \qquad \frac{dv}{ds} = \frac{f(s-f) - fs}{(s-f)^2} = -\frac{f^2}{(s-f)^2}.$$

The negative derivative already shows that the image approaches the lens as the object moves farther away. This agrees with the image tending to the focal plane when the object becomes very distant.

Step 2: Substitute the speed and compare the options The given motion has $ds/dt = +5\,\mathrm{m\,s^{-1}}$, with $s = 20\,\mathrm{m}$ and $f = 0.3\,\mathrm{m}$. By the chain rule,

$$\frac{dv}{dt} = -\frac{(0.3)^2}{(20-0.3)^2}(5) = -\frac{0.45}{388.09}\,\mathrm{m\,s^{-1}}.$$

Thus

$$\frac{dv}{dt} \simeq -1.16 \times 10^{-3}\,\mathrm{m\,s^{-1}}.$$

The magnitude matches (A), and its direction also matches. Option (C) gives the wrong magnitude even though its direction is correct. Options (B) and (D) have both an incorrect magnitude and motion away from the lens.

Step 3: Conclusion The image moves at $\boxed{1.16 \times 10^{-3}\,\mathrm{m\,s^{-1}}}$ towards the lens. Hence, the correct option is (A).

Quick Tip to Solve: Image speed is the object speed multiplied by the square of the magnification; use the derivative sign separately to determine its direction.

46. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 $\mathring{A}$. The radius of the atom in the excited state, in terms of Bohr radius a_0 will be : (hc = 12500 eV $\mathring{A}$) [4 marks]

(A) $4a_0$

(B) $9a_0$

(C) $25a_0$

(D) $16a_0$

Correct Answer: (D) $16a_0$

Solution:

Concept:

- Absorbed photon energy is $E_\gamma = hc/\lambda$. For hydrogen, the Bohr levels are $E_n = -13.6/n^2\,\text{eV}$.
- Excitation from the ground state requires $E_\gamma = 13.6(1 - 1/n^2)\,\text{eV}$, with integer principal quantum number n.
- The radius of the hydrogen orbit is $r_n = n^2 a_0$; energy and radius must be connected through the same final value of n.

Step 1: Calculate the photon energy and excited level Use the value of hc stated in the question, keeping wavelength in angstroms so the length units cancel:

$$E_\gamma = \frac{12500}{980}\,\text{eV} \simeq 12.755\,\text{eV}.$$

The excited energy is therefore approximately $-13.6 + 12.755 = -0.845\,\text{eV}$. Hence

$$n^2 = \frac{13.6}{13.6 - 12500/980} \simeq 16.1, \qquad n \simeq 4.01.$$

With the rounded data supplied, this identifies the discrete level $n = 4$.

Step 2: Use the radius law and check the other levels The final radius is

$$r_4 = 4^2 a_0 = 16a_0.$$

As a check, the exact Bohr energy gap for $n = 4$ is $13.6(1 - 1/16) = 12.75\,\text{eV}$, consistent with the photon energy at the stated precision. The radii $4a_0$, $9a_0$, and $25a_0$ correspond to $n = 2, 3, 5$, requiring gaps 10.2, 12.09, and $13.056\,\text{eV}$, respectively. None matches the absorbed photon. The photon is below the ground-state ionisation threshold, so it excites rather than ionises the atom.

Step 3: Conclusion The atom is excited to $n = 4$, giving $\boxed{r = 16a_0}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the final principal quantum number from the energy gap before applying the radius law; hydrogen orbital radius scales as the square of that integer.

47. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be : [4 marks]

(A) $\frac{3}{4}\rho v^2$

(B) $\frac{1}{4}\rho v^2$

(C) $\frac{1}{2}\rho v^2$

(D) ρv^2

Correct Answer: (A) $\frac{3}{4}\rho v^2$

Solution:

Concept:

- The mass flow rate through hose cross-section $A = \pi a^2$ is $\dot{m} = \rho A v$. Force follows from the rate of change of horizontal momentum.
- A mass element stopped from speed v transfers momentum mv; one reflected at the same speed transfers $2mv$.
- Pressure on the impacted area is the force divided by A. Liquid passing through without a velocity change transfers no momentum.

Step 1: Separate the three contributions to momentum transfer Choose the incident direction as positive. Half the arriving mass flow continues at velocity $+v$, so this portion has zero change of momentum and exerts no force. One quarter stops, changing velocity from $+v$ to zero. Its contribution to the force magnitude on the mesh is

$$F_{\text{stop}} = \frac{1}{4}\dot{m}\, v.$$

The remaining quarter returns at velocity $-v$, so its momentum changes by $-2v$ per unit mass and the reaction on the mesh points forward.

Step 2: Add the forces and divide by the impact area The reflected part supplies

$$F_{\text{return}} = \frac{1}{4}\dot{m}(2v) = \frac{1}{2}\dot{m}\, v.$$

Thus the total forward force on the mesh is

$$F = \left(\frac{1}{4} + \frac{1}{2}\right)\dot{m}\, v = \frac{3}{4}\rho A v^2.$$

The resultant pressure is consequently

$$P = \frac{F}{A} = \frac{3}{4}\rho v^2.$$

The radius cancels because both force and impacted area scale as πa^2. Option (B) counts only the stopped liquid; (C) counts only the reflected contribution. Option (D) incorrectly treats the weighted momentum transfer as equal to that of stopping the entire stream.

Step 3: Conclusion The resultant pressure is $\boxed{P = \frac{3}{4}\rho v^2}$. Hence, the correct option is (A).

Quick Tip to Solve: Weight each stream fraction by its velocity change: use zero for transmission, one incident speed for stopping and twice that speed for reversal.

48. A satellite is revolving in a circular orbit at a height h form the earth surface, such that h $<<$ R where R is the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is : [4 marks]

(A) $\sqrt{gR}\left(\sqrt{2}-1\right)$

(B) $\sqrt{2gR}$

(C) $\sqrt{gR}$

(D) $\frac{\sqrt{gR}}{2}$

Correct Answer: (A) $\sqrt{gR}\left(\sqrt{2}-1\right)$

Solution:

Concept:

- At orbital radius $r = R+h$, circular speed is $v_c = \sqrt{GM/r}$, found by equating gravity to centripetal force.
- For just escaping to infinity with zero final speed, the total mechanical energy must be zero, giving $v_e = \sqrt{2GM/r}$.
- The smallest speed increase is obtained by accelerating along the existing velocity. Near the surface, $GM = gR^2$ and $h \ll R$ implies $r \simeq R$.

Step 1: Compare the initial orbital speed with the escape threshold The satellite is already moving; it does not require the full escape speed as an additional speed. At the same position,

$$v_c = \sqrt{\frac{GM}{R+h}}, \qquad v_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{2}\,v_c.$$

A forward tangential impulse increases its speed most efficiently. An impulse in any other direction would need a larger velocity change to reach the same final speed, by the vector triangle inequality.

Step 2: Calculate the minimum increase near the surface The required increase is the difference of the two speeds,

$$\Delta v_{\min} = v_e - v_c = (\sqrt{2}-1)\sqrt{\frac{GM}{R+h}}.$$

Since $h \ll R$, replace $R+h$ by R to leading order and use $GM = gR^2$:

$$\Delta v_{\min} \simeq (\sqrt{2}-1)\sqrt{gR}.$$

Option (B) is the full near-surface escape speed, not its increase. Option (C) is the initial circular speed. Option (D) uses a factor $1/2$ instead of the required $\sqrt{2}-1 \simeq 0.414$. The atmospheric assumption allows mechanical-energy conservation during the subsequent escape.

Step 3: Conclusion The minimum speed increase is $\boxed{\sqrt{gR}(\sqrt{2}-1)}$, to leading order in h/R. Hence, the correct option is (A).

Quick Tip to Solve: For a circular-orbit escape impulse, subtract the existing circular speed from escape speed at the same radius before making a small-height approximation.

49. Ice at –20°C is added to 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to (Specific heat of water = 4.2J/g/°C Specific heat of Ice = 2.1J/g/°C Heat of fusion of water at 0°C= 334J/g) [4 marks]

(A) 100 g

(B) 60 g

(C) 50 g

(D) 40 g

Correct Answer: (D) 40 g

Solution:

Concept:

- Heat lost by the initially warm water equals heat gained by the ice when no heat escapes to the surroundings.
- All added ice must warm from $-20^\circ\mathrm{C}$ to $0^\circ\mathrm{C}$, but only the mass that melts absorbs latent heat.
- If the added ice mass is m grams and $20\,\mathrm{g}$ remains, the melted mass is $(m-20)\,\mathrm{g}$. The final ice-water mixture is at its melting temperature.

Step 1: Write the sensible-heat and latent-heat terms The $50\,\mathrm{g}$ of water cools through $40^\circ\mathrm{C}$, releasing

$$Q_{\text{water}} = 50 \times 4.2 \times 40 = 8400\,\mathrm{J}.$$

Warming all m grams of ice through $20^\circ\mathrm{C}$ consumes

$$Q_{\text{warm}} = m \times 2.1 \times 20 = 42m\,\mathrm{J}.$$

Melting only $m-20$ grams consumes

$$Q_{\text{melt}} = 334(m-20)\,\mathrm{J}.$$

There is no subsequent temperature rise of the melted water because the final mixture remains at zero degrees.

Step 2: Solve the energy balance and verify the mass Equate the heat gained and lost:

$$8400 = 42m + 334(m-20) = 376m - 6680.$$

Therefore

$$m = \frac{8400+6680}{376} = \frac{15080}{376} \simeq 40.1\,\mathrm{g}.$$

About $20.1\,\mathrm{g}$ melts and $20\,\mathrm{g}$ remains, as specified. The warming term is about $1684\,\mathrm{J}$, and the melting term about $6716\,\mathrm{J}$, adding to $8400\,\mathrm{J}$. The nearest offered mass is $40\,\mathrm{g}$; the alternatives 50, 60, and $100\,\mathrm{g}$ would require more heat than the warm water supplies under the stated final condition.

Step 3: Conclusion The mass of ice added was approximately $\boxed{40\,\mathrm{g}}$. Hence, the correct option is (D).

Quick Tip to Solve: When some ice remains, warm the entire initial ice mass, but charge latent heat only to the part that actually melts.

50. If the de Broglie wavelength of an electron is equal to the 10^{-3} times the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to : (Speed of light = 3×10^8 m/s, Planck's constant = 6.63×10^{-34} J.s, Mass of electron = 9.1×10^{-31} kg) [4 marks]

(A) 1.7×10^6 m/s

(B) 1.45×10^6 m/s

(C) 1.1×10^6 m/s

(D) 1.8×10^6 m/s

Correct Answer: (B) 1.45×10^6 m/s

Solution:

Concept:

- The photon wavelength is related to its frequency by $\lambda_\gamma = c/\nu$. The frequency determines its wavelength before the given wavelength ratio is applied.
- For a nonrelativistic electron, $\lambda_e = h/(m_e v)$, where h is Planck's constant, m_e is the electron mass and v is its speed.

Step 1: Calculate the photon and electron wavelengths Use the given frequency and the speed of light:

$$\lambda_\gamma = \frac{3 \times 10^8}{6 \times 10^{14}} = 5.0 \times 10^{-7}\,\text{m}.$$

The electron wavelength is one thousandth of this wavelength, rather than one thousand times it:

$$\lambda_e = 10^{-3}\lambda_\gamma = 5.0 \times 10^{-10}\,\text{m}.$$

Step 2: Find the electron speed and compare the options Rearrange the matter-wave relation and substitute in SI units:

$$v = \frac{h}{m_e \lambda_e} = \frac{6.63 \times 10^{-34}}{(9.1 \times 10^{-31})(5.0 \times 10^{-10})} = 1.457 \times 10^6\,\text{m s}^{-1}.$$

This is only about $0.0049c$, so the nonrelativistic momentum used here is consistent. The value rounds to the listed $1.45 \times 10^6\,\text{m s}^{-1}$. The other three speeds do not reproduce the specified electron wavelength when substituted into $h/(m_e v)$.

Step 3: Conclusion The electron speed is approximately $1.45 \times 10^6\,\text{m s}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: First find the photon wavelength, multiply it by the stated ratio, and only then use the electron mass in the de Broglie relation.

51. A body of mass 1 kg falls freely from a height of 100 m, on a platform mass 3 kg which is mounted on a spring having spring constant k = 1.25 $\times$ 10^6 N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that g = 10 ms^{-2} , the value of x will be close to : [4 marks]

(A) 8 cm

(B) 4 cm

(C) 40 cm

(D) 80 cm

Correct Answer: No listed option matches the approximately 2.0 cm compression required by the printed data.

Solution:

Concept:

- The falling body acquires speed $u = \sqrt{2gh}$. During a brief perfectly inelastic impact, the body and platform have a common speed fixed by conservation of vertical momentum.
- Mechanical energy is conserved during the subsequent spring compression, but not during the sticking collision. The initial platform compresses the spring by $x_0 = Mg/k$.

Step 1: Find the common speed just after impact Let the falling mass be $m = 1\,\mathrm{kg}$ and the platform mass be $M = 3\,\mathrm{kg}$. The speed immediately before impact is

$$u = \sqrt{2(10)(100)} = \sqrt{2000}\,\mathrm{m\,s^{-1}}.$$

Neglecting the finite external impulse during the very short impact gives

$$mu = (m + M)V, \qquad V = \frac{\sqrt{2000}}{4}.$$

The combined kinetic energy is consequently

$$K = \frac{1}{2}(4)V^2 = 250\,\mathrm{J}.$$

It is not the original $1000\,\mathrm{J}$; most of that energy is lost during sticking.

Step 2: Use energy after the collision and compare the listed values The platform initially has $x_0 = 30/(1.25 \times 10^6) = 2.4 \times 10^{-5}\,\mathrm{m}$ compression. If y is the additional downward displacement, energy balance up to the turning point is

$$250 + 40y = \frac{1}{2}k\left[(x_0 + y)^2 - x_0^2\right].$$

Since $kx_0 = 30\,\mathrm{N}$, this becomes

$$625000y^2 - 10y - 250 = 0, \qquad y \simeq 0.020008\,\mathrm{m}.$$

The total maximum compression is about $2.003\,\mathrm{cm}$. No option equals this physical value. The smallest listed value, $4\,\mathrm{cm}$, is about twice this result, and 8, 40 and $80\,\mathrm{cm}$ are still farther away. Thus none is

a defensible approximation to the calculated compression. Treating all the falling energy as spring energy would instead give $4\,\text{cm}$, but would incorrectly ignore the inelastic collision.

Step 3: Conclusion The maximum compression required by the literal data is approximately $2.0\,\text{cm}$, so no listed option matches. The supplied key selects (B), $4\,\text{cm}$, but that value follows from incorrectly conserving mechanical energy through the sticking collision; it is not the physical result.

Quick Tip to Solve: For a body that sticks to a platform, use momentum at impact, then energy for the spring motion. Do not conserve mechanical energy across the collision.

---

52. Two equal resistances when connected in series to a battery, consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be : [4 marks]

(A) 240 W

(B) 60 W

(C) 30 W

(D) 120 W

Correct Answer: (A) 240 W

Solution:

Concept:

- For an ideal battery maintaining terminal voltage V, the total power delivered to a resistive combination is $P = V^2/R_{\text{eq}}$.
- Two equal resistances R have equivalent resistance $2R$ in series and $R/2$ in parallel. The total load power is inversely proportional to this equivalent resistance.

Step 1: Express the power in the series circuit Let either resistance be R. Their series equivalent is

$$R_s = R + R = 2R.$$

Thus the stated $60\,\text{W}$ is the total power in both resistances:

$$P_s = \frac{V^2}{2R} = 60\,\text{W}, \qquad \frac{V^2}{R} = 120\,\text{W}.$$

Each series resistor therefore dissipates $30\,\text{W}$. The battery voltage is shared equally in that connection, so each resistor initially has only $V/2$ across it.

Step 2: Calculate the power in the parallel circuit In parallel each resistor is directly across the same battery. Their equivalent resistance follows from

$$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}, \qquad R_p = \frac{R}{2}.$$

Consequently,

$$P_p = \frac{V^2}{R/2} = \frac{2V^2}{R} = 2(120) = 240\,\text{W}.$$

Equivalently $P_p/P_s = R_s/R_p = 4$. Each resistor now consumes $120\,\text{W}$, giving the same total. A total of $120\,\text{W}$ counts only one parallel branch; $60\,\text{W}$ would require unchanged equivalent resistance, while $30\,\text{W}$ is the earlier power in one series resistor.

Step 3: Conclusion The electric power consumed by the parallel combination is $240\,\text{W}$. Hence, the correct option is (A).

Quick Tip to Solve: At fixed battery voltage, compare equivalent resistances: changing two equal resistors from series to parallel divides resistance by four and multiplies total power by four.

53. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by: [4 marks]

(A) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$

(B) $(\sqrt{n}, \sqrt{n})$

(C) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$

(D) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$

Correct Answer: (C) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$

Solution:

Concept:

- In the usual nonmagnetic transparent-medium model, the wave speed is $v = c/n$, the permeability is μ_0, and the electric and magnetic amplitudes obey $E = vB$.
- The time-averaged intensity is $I = E_0 B_0/(2\mu_0)$. A lossless entry with unchanged intensity therefore keeps the product of the electric and magnetic field amplitudes unchanged.

Step 1: Relate the two field ratios using wave speed Use subscripts 1 and 2 for the wave before and after entry, respectively. Initially $E_1/B_1 = c$, whereas inside the medium

$$\frac{E_2}{B_2} = \frac{c}{n}.$$

Writing $a = E_1/E_2$ and $b = B_1/B_2$, division of these two relations gives

$$\frac{a}{b} = n.$$

Thus the electric and magnetic fields cannot have identical before-to-after ratios for a general n. This already rules out options (A) and (B).

Step 2: Use the conserved intensity to determine each ratio Since the stated intensity is $50\,\text{W}\,\text{m}^{-2}$ on

both sides in the lossless model,

$$\frac{E_1 B_1}{2\mu_0} = \frac{E_2 B_2}{2\mu_0}, \qquad ab = 1.$$

Combining $a/b = n$ with $ab = 1$, and taking positive magnitudes, gives

$$a^2 = n, \qquad a = \sqrt{n}, \qquad b = \frac{1}{\sqrt{n}}.$$

The numerical intensity cancels because only ratios are required. The electric amplitude decreases on entry, while the magnetic amplitude increases. Option (D) reverses these changes and would imply a wave speed nc, rather than c/n.

Step 3: Conclusion The required before-to-after ratios are $\left(\sqrt{n},\ 1/\sqrt{n}\right)$ for the electric and magnetic fields, respectively. Hence, the correct option is (C).

Quick Tip to Solve: For lossless propagation in a nonmagnetic dielectric, combine constant field product from intensity with the changed field quotient from wave speed.

54. Equation of travelling wave on a stretched string of linear density 5 g/m is y = 0.03 sin(450 t – 9x) where distance and time are measured in SI units. The tension in the string is : [4 marks]

(A) 10 N

(B) 7.5 N

(C) 5 N

(D) 12.5 N

Correct Answer: (D) 12.5 N

Solution:

Concept:

- A progressive sinusoidal wave has the form $y = A\sin(\omega t - k_w x)$. Its speed is $v = \omega/k_w$; the subscript distinguishes the wave number from a spring constant.
- For transverse waves on a stretched string of linear mass density μ, $v = \sqrt{T/\mu}$, where T is the string tension. Hence $T = \mu v^2$.

Step 1: Read the angular frequency and wave number Compare the given equation with the standard travelling-wave form:

$$y = 0.03\sin(450t - 9x).$$

The coefficient of time is $\omega = 450\,\mathrm{rad\,s^{-1}}$, and the magnitude of the coefficient of position is $k_w = 9\,\mathrm{rad\,m^{-1}}$. Therefore

$$v = \frac{\omega}{k_w} = \frac{450}{9} = 50\,\mathrm{m\,s^{-1}}.$$

The minus sign specifies travel in the positive x direction; it does not make the speed or tension negative.

Step 2: Convert the density and calculate the tension The linear density must be converted from

grams per metre to kilograms per metre:

$$\mu = 5\,\mathrm{g\,m^{-1}} = 5 \times 10^{-3}\,\mathrm{kg\,m^{-1}}.$$

Apply the speed-tension relation:

$$T = \mu v^2 = (5 \times 10^{-3})(50)^2 = 0.005(2500) = 12.5\,\mathrm{N}.$$

The amplitude $0.03\,\mathrm{m}$ does not enter this relation: in the small-amplitude string model, speed depends on tension and mass per unit length, not amplitude. None of 10, 7.5 or $5\,\mathrm{N}$ gives the required $50\,\mathrm{m\,s^{-1}}$ when substituted in $\sqrt{T/\mu}$.

Step 3: Conclusion The tension in the stretched string is $12.5\,\mathrm{N}$. Hence, the correct option is (D).

Quick Tip to Solve: Read wave speed as the time coefficient divided by the position coefficient, then convert linear density into SI units before squaring the speed.

55. A body is projected at t = 0 with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. neglecting air resistance and taking acceleration due to gravity g = 10 ms^{-2}, the value of R is : [4 marks]

(A) 2.8 m

(B) 5.1 m

(C) 2.5 m

(D) 10.3 m

Correct Answer: (A) 2.8 m

Solution:

Concept:

- The local radius of curvature satisfies $a_\perp = v^2/R$, where $a_\perp$ is the component of acceleration perpendicular to the instantaneous velocity.
- For a projectile with horizontal speed v_x and downward acceleration g, $a_\perp = gv_x/v$. Thus $R = v^3/(gv_x)$, using the speed at the specified instant.

Step 1: Find the instantaneous velocity components Resolve the launch velocity and evolve the vertical component for one second:

$$v_x = 10\cos 60^\circ = 5\,\mathrm{m\,s^{-1}},$$

$$v_y = 10\sin 60^\circ - 10(1) = 5\sqrt{3} - 10 \simeq -1.340\,\mathrm{m\,s^{-1}}.$$

The negative sign shows the projectile is descending. Its speed at this instant is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{25 + (5\sqrt{3} - 10)^2} \simeq 5.176\,\mathrm{m\,s^{-1}}.$$

Step 2: Use the normal acceleration to calculate the radius Gravity is vertical, so its component

normal to the velocity has magnitude gv_x/v. Equating this component to v^2/R yields

$$R = \frac{v^3}{gv_x} = \frac{(5.176)^3}{10(5)} \simeq 2.77\,\text{m}.$$

This rounds to $2.8\,\text{m}$. The value $2.5\,\text{m}$ applies at the highest point, where the velocity is exactly horizontal; that instant is $\sqrt{3}/2\,\text{s}$, not $1\,\text{s}$. The values 5.1 and $10.3\,\text{m}$ likewise do not satisfy $a_\perp = v^2/R$ for the computed velocity. Using the full g as normal acceleration away from the top would omit its tangential component.

Step 3: Conclusion The instantaneous radius of curvature is approximately $2.8\,\text{m}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the instantaneous speed in the curvature formula. Gravity is fully normal to the trajectory only at the highest point.

56. In an experiment, electrons are accelerated, from rest, by applying a voltage of 500 V. Calculate the radius of the path if a magnetic field 100 mT is then applied. [Charge of the electron = 1.6 $\times$ 10^{-19} C Mass of the electron = 9.1 $\times$ 10^{-31} kg] [4 marks]

(A) 7.5 $\times$ 10^{-4} m

(B) 7.5 $\times$ 10^{-3} m

(C) 7.5 m

(D) 7.5 $\times$ 10^{-2} m

Correct Answer: (A) 7.5 $\times$ 10^{-4} m

Solution:

Concept:

- Acceleration through a potential difference V gives an electron kinetic energy $eV = \frac{1}{2}m_e v^2$, where e is the positive magnitude of its charge.
- For the circular path intended here, velocity is perpendicular to the magnetic field. The magnetic force supplies centripetal acceleration: $evB = m_e v^2/r$, giving $r = m_e v/(eB)$.

Step 1: Calculate the speed acquired through the voltage The energy supplied by the accelerating voltage is

$$eV = (1.6 \times 10^{-19})(500) = 8.0 \times 10^{-17}\,\text{J}.$$

Therefore the electron speed is

$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2(1.6 \times 10^{-19})(500)}{9.1 \times 10^{-31}}} \simeq 1.326 \times 10^7\,\text{m s}^{-1}.$$

This is about 4.4% of the speed of light, so a nonrelativistic calculation gives the required option accurately.

Step 2: Find the radius and check its order of magnitude Convert the magnetic field before substitut-

ing:

$$B = 100\,\mathrm{mT} = 0.100\,\mathrm{T}.$$

The radius is then

$$r = \frac{(9.1 \times 10^{-31})(1.326 \times 10^{7})}{(1.6 \times 10^{-19})(0.100)} = 7.54 \times 10^{-4}\,\mathrm{m}.$$

This is about $0.75\,\mathrm{mm}$, consistent with option (A). The negative charge changes the direction of bending but not this positive radius. The radii in (B) and (D) are larger by factors of ten and one hundred; option (C) is larger by ten thousand. They do not satisfy the magnetic-force balance for the given field and accelerating energy.

Step 3: Conclusion The path radius is approximately $7.5 \times 10^{-4}\,\mathrm{m}$. Hence, the correct option is (A).

Quick Tip to Solve: Combine the two relations as $r = \sqrt{2m_eV/e}/B$, and convert millitesla to tesla before doing the numerical calculation.

57. In a Young's double slit experiment, the path difference, at a certain point on the screen, between two interfering waves is $\frac{1}{8}$ th of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to : [4 marks]

(A) 0.94

(B) 0.85

(C) 0.74

(D) 0.80

Correct Answer: (B) 0.85

Solution:

Concept:

- For coherent waves of equal individual intensity I_0, the resultant intensity is $I = 2I_0(1 + \cos\phi) = 4I_0\cos^2(\phi/2)$, where ϕ is their phase difference.
- The path difference δ and phase difference are related by $\phi = 2\pi\delta/\lambda$. At a bright-fringe centre the waves are in phase, giving $I_{\max} = 4I_0$.

Step 1: Convert the path difference to a phase difference The stated path difference is one-eighth of a wavelength, rather than one-eighth of a complete phase cycle measured in radians. Thus

$$\phi = \frac{2\pi}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4}.$$

The intensity formula contains the cosine of half this phase difference, so its argument is $\pi/8$.

Step 2: Form and evaluate the intensity ratio Dividing by the central bright-fringe intensity removes the unknown slit intensity:

$$\frac{I}{I_{\max}} = \cos^2\frac{\pi}{8} = \frac{1 + \cos(\pi/4)}{2} = \frac{2 + \sqrt{2}}{4} \simeq 0.8536.$$

The phase is small enough that the interference remains predominantly constructive, but the intensity is below its maximum. Of the four proposed numbers, 0.85 is nearest; 0.94 is too high, while 0.80 and 0.74 are too low. Using $\cos^2\phi$ instead would incorrectly omit the half-angle.

Step 3: Conclusion The intensity ratio is approximately 0.854, close to 0.85. Hence, the correct option is (B).

Quick Tip to Solve: First calculate the phase $2\pi\delta/\lambda$, then use half that phase in the normalized intensity $I/I_{\max} = \cos^2(\phi/2)$.

58. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. considering only translational and rotational modes, the total internal energy of the system is : [4 marks]

(A) 12 RT

(B) 20 RT

(C) 4 RT

(D) 15 RT

Correct Answer: (D) 15 RT

Solution:

Concept:

- For an ideal gas with f active quadratic degrees of freedom, equipartition gives the internal energy $U = (f/2)nRT$, where n is the number of moles.
- Oxygen is diatomic: with translations and rotations only, $f = 3 + 2 = 5$. Argon is monatomic and has only the three translational degrees of freedom, so $f = 3$.

Step 1: Calculate the contribution of oxygen The three moles of oxygen share the mixture temperature T. Vibrational modes are excluded by the question, so the energy per mole is $5RT/2$:

$$U_{\text{oxygen}} = 3\left(\frac{5}{2}RT\right) = \frac{15}{2}RT.$$

The rotational contribution is that of a linear diatomic molecule, with two active rotational axes; it is not three rotational degrees of freedom.

Step 2: Add the contribution of argon Argon atoms do not have an independent molecular rotational contribution in this model. Their five moles contribute

$$U_{\text{argon}} = 5\left(\frac{3}{2}RT\right) = \frac{15}{2}RT.$$

Internal energies are additive, even though the gases have different degrees of freedom:

$$U_{\text{total}} = \frac{15}{2}RT + \frac{15}{2}RT = 15RT.$$

The value $12RT$ would treat all eight moles as monatomic, while $20RT$ would treat all eight as diatomic. The value $4RT$ counts neither contribution correctly; the individual gases must be weighted

by their own mole numbers.

Step 3: Conclusion The mixture has total internal energy $15RT$, in joules when R is in $\mathrm{J\,mol^{-1}K^{-1}}$ and T is in kelvin. Hence, the correct option is (D).

Quick Tip to Solve: For a mixture, add $n_i f_i RT/2$ species by species; do not assign one common degree-of-freedom count to unlike gases.

59. A particle undergoing simple harmonic motion has time dependent displacement given by x(t) = Asin$\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at t = 210 s will be: [4 marks]

(A) $\frac{1}{9}$

(B) 3

(C) 2

(D) 1

Correct Answer: The literal ratio is $K/U = 1/3$; none of the four printed options matches it.

Solution:

Concept:

- For SHM $x = A\sin\omega t$, velocity is $v = A\omega\cos\omega t$. Thus $K = \frac{1}{2}m\omega^2A^2\cos^2\omega t$, where m is the particle mass.
- Taking the SHM potential energy zero at equilibrium, $U = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2\sin^2\omega t$. Hence $K/U = \cot^2\omega t$.

Step 1: Evaluate the displacement phase at the stated time The printed displacement is a sine, with angular frequency $\omega = \pi/90\ \mathrm{s^{-1}}$. At $t = 210$ s,

$$\omega t = \frac{\pi}{90}(210) = \frac{7\pi}{3} = 2\pi + \frac{\pi}{3}.$$

Consequently $\sin^2(\omega t) = 3/4$ and $\cos^2(\omega t) = 1/4$. These factors must be assigned to potential and kinetic energy, respectively.

Step 2: Compare the kinetic and potential energies Writing the conserved total energy as $E = \frac{1}{2}m\omega^2A^2$, the two energies are

$$U = \frac{3}{4}E, \qquad K = E - U = \frac{1}{4}E.$$

Therefore

$$\frac{K}{U} = \frac{E/4}{3E/4} = \frac{1}{3}.$$

None of $1/9, 3, 2, 1$ equals this ratio: $1/9$ would square the ratio again, 3 is the reciprocal U/K, and 2 and 1 do not match either energy fraction. The printed sine and time have been used without replacing them by an intended alternative.

Step 3: Conclusion For the question as printed, $K/U = 1/3$, so none of the listed options is correct. The supplied key selects (B), namely 3, but that is U/K, whereas the question explicitly asks for

kinetic to potential energy.

Quick Tip to Solve: For $x = A\sin\omega t$, potential energy follows $\sin^2\omega t$ and kinetic energy follows $\cos^2\omega t$; check which ratio is requested before dividing.

60. An amplitude modulated signal is given by V(t) = 10[1 + 0.3cos(2.2 × 10^4 t)] sin(5.5 × 10^5 t). Here t is in seconds. The sideband frequencies (in kHz) are, [Given π = 22/7] [4 marks]

(A) 892.5 and 857.5

(B) 89.25 and 85.75

(C) 1785 and 1715

(D) 178.5 and 171.5

Correct Answer: The printed frequencies imply sidebands 91.0 and 84.0 kHz; none of the four options matches them.

Solution:

Concept:

- An amplitude-modulated wave has the form $V = A_c[1 + \mu\cos(\omega_m t)]\sin(\omega_c t)$, where ω_c and ω_m are carrier and modulating angular frequencies.
- The product-to-sum identity produces sidebands at $\omega_c + \omega_m$ and $\omega_c - \omega_m$. Ordinary frequencies are $f = \omega/(2\pi)$, not ω itself.

Step 1: Identify the carrier and modulating frequencies Comparing the two trigonometric arguments with the standard form gives

$$\omega_c = 5.5 \times 10^5 \text{ rad s}^{-1}, \qquad \omega_m = 2.2 \times 10^4 \text{ rad s}^{-1}.$$

The factors 10 and 0.3 specify the carrier amplitude and modulation depth; they determine sideband amplitudes but do not set their frequencies.

Step 2: Convert the sum and difference to kilohertz Using the given $\pi = 22/7$,

$$f_c = \frac{5.5 \times 10^5 \times 7}{44} = 87500 \text{ Hz} = 87.5 \text{ kHz},$$

$$f_m = \frac{2.2 \times 10^4 \times 7}{44} = 3500 \text{ Hz} = 3.5 \text{ kHz}.$$

It follows that

$$f_{\text{upper}} = 87.5 + 3.5 = 91.0 \text{ kHz}, \qquad f_{\text{lower}} = 87.5 - 3.5 = 84.0 \text{ kHz}.$$

These values are not the numbers in any printed option; the pair 89.25 and 85.75 would instead require $f_m = 1.75$ kHz, half the value fixed by the printed angular frequency.

Step 3: Conclusion The printed signal gives sidebands 91.0 kHz and 84.0 kHz, so none of the listed options is correct. The supplied key selects (B), but its frequency separation corresponds to half the

printed modulation angular frequency.

Quick Tip to Solve: Read both angular frequencies directly from the sine and cosine arguments and divide each by 2π before adding or subtracting; the modulation index does not change the sideband locations.

Chemistry (120 marks)

61. Consider the reaction

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

The equilibrium constant of the above reaction is K_p. If pure ammonia is left to dissociate, the partial pressure of ammonia at equilibrium is given by (Assume that $P_{NH_3} << P_{total}$ at equilibrium) [4 marks]

(A) $\frac{3^{\frac{3}{2}} K_P^{\frac{1}{2}} P^2}{4}$

(B) $\frac{K_P^{\frac{1}{2}} P^2}{4}$

(C) $\frac{3^{\frac{3}{2}} K_P^{\frac{1}{2}} P^2}{16}$

(D) $\frac{K_P^{\frac{1}{2}} P^2}{16}$

Correct Answer: (C) $\frac{3^{\frac{3}{2}} K_P^{\frac{1}{2}} P^2}{16}$

Solution:

Concept:

- For $N_2(g) + 3\,H_2(g) \rightleftharpoons 2\,NH_3(g)$, the pressure equilibrium constant is $K_p = p_{NH_3}^2/(p_{N_2} p_{H_2}^3)$. The reaction is written in the ammonia-formation direction.
- Dissociating pure ammonia produces nitrogen and hydrogen in the mole ratio $1:3$. Their partial pressures therefore remain in that ratio. Let P denote the total equilibrium pressure.

Step 1: Express the product pressures using the approximation Write the ammonia partial pressure as p. The other two partial pressures sum to $P - p$ and satisfy $p_{H_2} = 3p_{N_2}$. Hence

$$p_{N_2} = \frac{P-p}{4}, \qquad p_{H_2} = \frac{3(P-p)}{4}.$$

The question specifies $p \ll P$, so these become $P/4$ and $3P/4$, respectively. Neglecting ammonia in the total pressure does not mean setting its equilibrium partial pressure to zero.

Step 2: Solve the formation equilibrium expression Substitute the approximate nitrogen and hydrogen pressures into the stated formation constant:

$$K_p = \frac{p^2}{(P/4)(3P/4)^3} = \frac{256p^2}{27P^4}.$$

Taking the physically positive root gives

$$p = \frac{\sqrt{27}\sqrt{K_p}P^2}{16} = \frac{3^{3/2}K_p^{1/2}P^2}{16}.$$

The factor 16 comes from the square root of 4^4, and $3^{3/2}$ comes from the three hydrogen factors. Option (A) has the wrong denominator; (B) and (D) also omit the stoichiometric factor from hydrogen.

Step 3: Conclusion The ammonia partial pressure is approximately $3^{3/2}K_p^{1/2}P^2/16$, with P the total equilibrium pressure. Hence, the correct option is (C).

Quick Tip to Solve: For dissociation of pure ammonia, assign one-quarter of the non-ammonia pressure to nitrogen and three-quarters to hydrogen, then use the equilibrium constant in the direction actually written.

62. The correct order of the atomic radii of C, Cs, Al, and S is : [4 marks]

(A) S < C < Al < Cs

(B) S < C < Cs < Al

(C) C < S < Al < Cs

(D) C < S < Cs < Al

Correct Answer: (C) C < S < Al < Cs

Solution:

Concept:

- Atomic size generally increases down a group because the valence shell moves to a higher principal quantum number and shielding increases.
- Across a period, increasing effective nuclear charge pulls electrons in the same shell closer to the nucleus, so the atomic radius generally decreases from left to right. Comparisons must use a consistent atomic-radius convention.

Step 1: Compare aluminium, sulfur and caesium Aluminium and sulfur lie in period 3. Aluminium lies to the left of sulfur, so the greater effective nuclear attraction in sulfur gives

$$r(\mathrm{S}) < r(\mathrm{Al}).$$

Caesium is far down group 1, in period 6, with a very diffuse outer electron in a high principal shell. It is consequently larger than all three other atoms here:

$$r(\mathrm{Al}) < r(\mathrm{Cs}).$$

Thus any option putting caesium below aluminium in the size order is eliminated.

Step 2: Place carbon relative to sulfur Carbon is a small period-2 atom with its valence electrons in the second shell; sulfur has its valence electrons in the third shell. The additional shell makes sulfur larger than carbon despite its more rightward position in the table. For scale, commonly used covalent radii are about $77\ \mathrm{pm}$ for carbon and $104\ \mathrm{pm}$ for sulfur. Combining the comparisons gives

$$r(\mathrm{C}) < r(\mathrm{S}) < r(\mathrm{Al}) < r(\mathrm{Cs}).$$

Options (A) and (B) incorrectly place sulfur below carbon. Options (B) and (D) incorrectly make aluminium larger than caesium; only (C) satisfies both size comparisons.

Step 3: Conclusion The increasing atomic-radius order is $\mathrm{C} < \mathrm{S} < \mathrm{Al} < \mathrm{Cs}$. Hence, the correct option is (C).

Quick Tip to Solve: First compare elements in the same period, then use shell number to place the period-2 carbon and the much larger period-6 caesium.

63. Two blocks of the same metal having same mass and at temperature T_1 and T_2, respectively, are brought in contact with each other and allowed to attain thermal equilibrium at constant pressure. The change in entropy, ΔS, for this process is : [4 marks]

(A) $2C_p \ln\left[\frac{(T_1+T_2)^{\frac{1}{2}}}{T_1T_2}\right]$

(B) $2C_p \ln\left[\frac{(T_1+T_2)}{2T_1T_2}\right]$

(C) $C_p \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$

(D) $2C_p \ln\left[\frac{(T_1+T_2)}{4T_1T_2}\right]$

Correct Answer: (C) $C_p \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$

Solution:

Concept:

- Equal masses of the same metal have equal heat capacities. Denote the constant-pressure heat capacity of each block by C_p, and assume no heat is lost to the surroundings.
- Entropy is a state function. For a block with constant heat capacity, its change between temperatures T_i and T_f is $\Delta S = C_p \ln(T_f/T_i)$, calculated along a reversible reference path. Temperatures are absolute.

Step 1: Find the common equilibrium temperature Energy conservation for the two equal-capacity blocks gives

$$C_p(T_f - T_1) + C_p(T_f - T_2) = 0, \qquad T_f = \frac{T_1 + T_2}{2}.$$

The hotter block loses exactly the energy gained by the cooler one. The final temperature is the arithmetic mean because the masses and heat capacities are equal, not because entropy is conserved.

Step 2: Add the entropy changes of both blocks The two state-function changes add even though the actual heat flow across a finite temperature difference is irreversible:

$$\begin{aligned}\Delta S &= C_p \ln\frac{T_f}{T_1} + C_p \ln\frac{T_f}{T_2} \\ &= C_p \ln\frac{T_f^2}{T_1T_2} \\ &= C_p \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right].\end{aligned}$$

The logarithm argument is dimensionless. The arithmetic-mean/geometric-mean inequality makes it at least one, so total entropy cannot decrease and is zero only for $T_1 = T_2$. Options (A), (B) and

(D) fail the temperature-ratio structure; (B) and (D), for example, do not even yield a dimensionless argument or the required zero at equal temperatures.

Step 3: Conclusion The entropy change is $\Delta S = C_p \ln[(T_1 + T_2)^2/(4T_1T_2)]$, with C_p the heat capacity of each block. Hence, the correct option is (C).

Quick Tip to Solve: For two equal heat capacities, use the arithmetic mean for the final temperature, then add the two logarithmic entropy changes; check that the result vanishes when the initial temperatures coincide.

64. The freezing point of a diluted milk sample is found to be –0.2°C, while it should have been –0.5°C for pure milk. How much water has been added to pure milk to make the diluted sample? [4 marks]

(A) 1 cup of water to 2 cups of pure milk

(B) 2 cups of water to 3 cups of pure milk

(C) 3 cups of water to 2 cups of pure milk

(D) 1 cup of water to 3 cups of pure milk

Correct Answer: (C) 3 cups of water to 2 cups of pure milk

Solution:

Concept:

- Freezing-point depression is a colligative property: $\Delta T_f = K_f m$, where m is solute molality and K_f belongs to the solvent.
- Adding water conserves the dissolved milk solids while lowering their concentration. In the dilution model used here, the depression ratio equals the original milk fraction of the final volume.

Step 1: Compare the two depressions The freezing point of water is $0\,^\circ\mathrm{C}$. Therefore the magnitudes of the depressions for pure milk and the diluted sample are $0.5\,^\circ\mathrm{C}$ and $0.2\,^\circ\mathrm{C}$, respectively. The negative freezing temperatures must not be added; their magnitudes measure concentration. Thus

$$\frac{\Delta T_{f,\text{dilute}}}{\Delta T_{f,\text{pure}}} = \frac{0.2}{0.5} = \frac{2}{5}.$$

Only two-fifths of the original concentration remains after dilution.

Step 2: Find the water-to-milk ratio Let M cups of pure milk be mixed with W cups of water. The solute amount is unchanged and the final volume is $M + W$, so

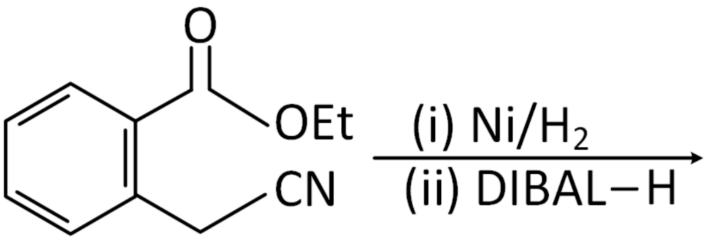
$$\frac{M}{M+W} = \frac{2}{5} \quad \Longrightarrow \quad 5M = 2M + 2W \quad \Longrightarrow \quad \frac{W}{M} = \frac{3}{2}.$$

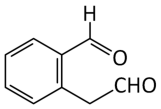
Hence choose two cups of milk and three cups of added water. As a check, this gives a milk fraction $2/(2+3) = 0.4$, and $0.5 \times 0.4 = 0.2$. Options (A), (B) and (D) instead give milk fractions $2/3$, $3/5$ and $3/4$; none gives the observed depression.

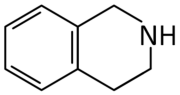
Step 3: Conclusion Three cups of water have been added to every two cups of pure milk. Hence, the correct option is (C).

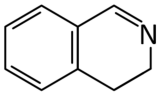
Quick Tip to Solve: Use the ratio of freezing-point depressions to obtain the milk fraction first; subtract that fraction from one to obtain the added-water fraction.

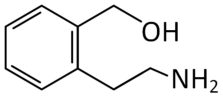
65. The major product of the following reaction is : [4 marks]



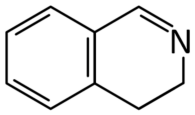
(A) 

(B) 

(C) 

(D) 

Correct Answer: (C)



Solution:

Concept:

- Catalytic hydrogenation converts a nitrile into a primary amine: $\mathrm{R{-}CN + 2\,H_2 \longrightarrow R{-}CH_2NH_2}$. The carbon of the nitrile remains in the chain.
- A suitably placed amino group attacks an ester intramolecularly to give a lactam. Controlled DIBAL-H reduction of a lactam gives the corresponding cyclic imine.

Step 1: Reduce the nitrile and close the ring The ortho side chain in the starting ester is $-\mathrm{CH_2{-}CN}$. Treatment with $\mathrm{H_2/Ni}$ converts it into $-\mathrm{CH_2{-}CH_2{-}NH_2}$, while retaining the ester carbonyl. The new primary amino group is positioned to attack the neighbouring ester carbonyl within the same molecule. Loss of ethanol closes a favourable six-membered ring, producing the benzene-fused cyclic amide, 3,4-dihydroisoquinolin-1(2H)-one. Count the ring atoms: two fused aromatic carbons, the carbonyl carbon, nitrogen, and the two methylene carbons. This rules out treating the nitrile and ester as unrelated groups.

Step 2: Apply the second reagent and identify the structure DIBAL-H partially reduces the lactam carbonyl. The carbonyl oxygen is removed in the reduction/work-up sequence and the corresponding $\mathrm{C{=}N}$ linkage is formed, giving 3,4-dihydroisoquinoline. Its imine bond is next to the benzene ring, and two methylene groups remain between nitrogen and the other fused ring carbon. This is precisely the cyclic imine in option (C). Option (A) would require leaving nitrogen out and generating aldehydes, contrary to the first hydrogenation. Option (B) is the further-reduced saturated cyclic amine, whereas the controlled reduction here stops at the imine. Option (D) is an open-chain amino alcohol and ignores favourable lactam formation before the second reduction.

Step 3: Conclusion The product is 3,4-dihydroisoquinoline, the benzene-fused six-membered cyclic imine shown in option (C). Hence, the correct option is (C).

Quick Tip to Solve: After reducing an ortho nitrile, count the ring size available for intramolecular aminolysis before applying the next reducing reagent.

66. The correct match between items I and II is :

Item-I (Mixture)	Item-II (Separation method)
(A) H_2O : Sugar	(P) Sublimation
(B) H_2O : Aniline	(Q) Recrystallization
(C) H_2O : Toluene	(R) Steam distillation
	(S) Differential extraction

[4 marks]

(A) $A \to (R);\ (B) \to (P);\ (C) \to (S)$

(B) $A \to (Q);\ (B) \to (R);\ (C) \to (S)$

(C) $A \to (Q);\ (B) \to (P);\ (C) \to (R)$

(D) $A \to (S);\ (B) \to (R);\ (C) \to (P)$

Correct Answer: (B) $A \to (Q);\ (B) \to (R);\ (C) \to (S)$

Solution:

Concept:

- Separation uses a difference in physical properties: solubility, volatility, or distribution between immiscible liquid phases.
- Recrystallization recovers a dissolved crystalline solid; steam distillation separates a steam-volatile organic compound; differential extraction separates components according to their preference for a solvent phase.

Step 1: Match the sugar and aniline mixtures Sugar is a nonvolatile solid dissolved in water. Concentration followed by cooling permits the sugar to crystallize, so the appropriate listed method is recrystallization: $A \to Q$. Sublimation is unsuitable because sugar does not pass cleanly from solid to vapour without decomposition. Aniline is an organic liquid that is only sparingly soluble in water and can be carried over with steam. Steam distillation allows it to distil below its normal boiling point because water and the organic component contribute to the total vapour pressure. Thus $B \to R$. Neither sugar nor aniline is isolated by the sublimation method (P).

Step 2: Match toluene and compare all combinations Toluene is essentially immiscible with water and preferentially occupies the organic phase. Separation according to its different solubility in the two phases corresponds to differential extraction among the listed choices, giving $C \to S$. Combining the assignments gives

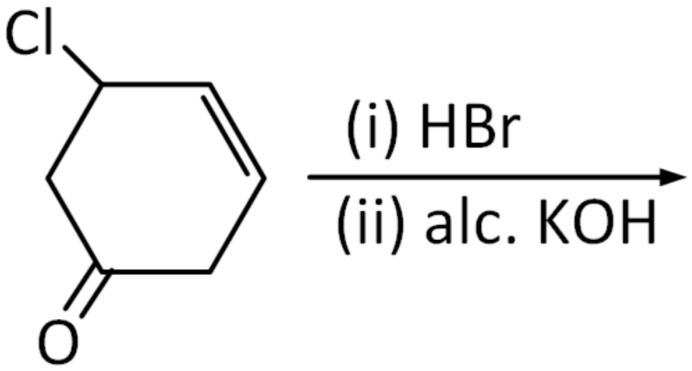
$$A \to Q, \qquad B \to R, \qquad C \to S.$$

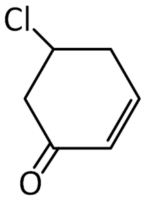
This complete combination appears in option (B). Option (A) incorrectly sends sugar to steam distillation and aniline to sublimation. Option (C) also assigns aniline to sublimation, while option (D) assigns toluene to sublimation and sugar to extraction. Each therefore fails at least one essential physical-property test.

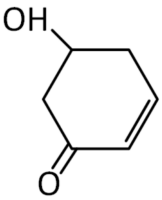
Step 3: Conclusion The correct matching is $A \to Q,\ B \to R,\ C \to S$. Hence, the correct option is (B).

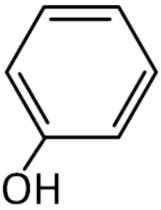
Quick Tip to Solve: Eliminate every combination assigning sublimation to an ordinary liquid or to sugar; then distinguish the crystalline solute from the steam-volatile organic liquid.

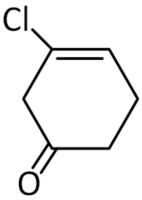
67. **The major product of the following reaction is :** [4 marks]



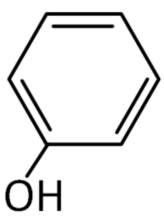
(A) 

(B) 

(C) 

(D) 

Correct Answer: (C)



Solution:

Concept:

- Addition of HBr to an alkene provides a bromo intermediate. Alcoholic KOH favours elimination of hydrogen halides.
- Formation of a conjugated dienone followed by keto-enol tautomerism can produce a phenol; the aromatic benzene ring strongly stabilizes that endpoint.

Step 1: Track the addition and the available eliminations Number the carbonyl carbon as C1 and proceed through the lower-right ring carbon to the alkene: the original double bond is C3–C4 and chlorine is on C5. Addition of HBr across C3–C4 gives a bromocyclohexanone intermediate. On treatment with alcoholic KOH, elimination can introduce unsaturation toward the carbonyl, forming C2=C3, while loss of hydrogen chloride introduces C4=C5. The elimination sequence therefore supplies a cyclohexa-2,4-dien-1-one framework rather than requiring substitution of chlorine by hydroxyl. Both halogens are lost as the unsaturated ring system develops.

Step 2: Use tautomerism to identify the stable endpoint The dienone has an enol tautomer in which the carbonyl oxygen becomes $-OH$, while the third ring double bond forms between C1 and C6. The resulting ring contains three alternating double bonds and six delocalized π electrons: it is phenol. Aromatic stabilization makes this enol the favoured final product even though simple nonaromatic carbonyl compounds usually favour their keto form. The phenol is shown in option (C). Option (A) retains chlorine and contains only one ring double bond; it stops before the aromatic endpoint. Option (B) depicts a hydroxycyclohexenone, confusing alcoholic-base elimination with aqueous substitution. Option (D) also retains chlorine and lacks the three alternating double bonds required for aromatic stabilization.

Step 3: Conclusion The major product is phenol, C_6H_5OH, obtained by elimination followed by aromatizing keto-enol tautomerism. Hence, the correct option is (C).

Quick Tip to Solve: When elimination can create a cyclic dienone, check whether its enol is aromatic; that stabilization can reverse the usual preference for the keto tautomer.

68. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H atom is suitable for this purpose?

[$R_H = 1 \times 10^5$ cm^{-1}, h = 6.6×10^{-34} Js, c = 3×10^8 ms^{-1}] [4 marks]

(A) Balmer, $\infty \to 2$

(B) Paschen, $5 \to 3$

(C) Paschen, $\infty \to 3$

(D) Lyman, $\infty \to 1$

Correct Answer: (C) Paschen, $\infty \to 3$

Solution:

Concept:

- For hydrogen emission, $\lambda^{-1} = R_H(1/n_f^2 - 1/n_i^2)$, where $n_i > n_f$. For a series limit, $n_i \to \infty$, so $\lambda = n_f^2/R_H$.
- The Paschen series ends at $n_f = 3$, Balmer at $n_f = 2$, and Lyman at $n_f = 1$. Use the supplied Rydberg constant consistently in centimetres or metres.

Step 1: Calculate the Paschen limit The desired wavelength is

$$900\,\text{nm} = 9 \times 10^{-5}\,\text{cm}.$$

For the Paschen series limit, substitute $n_f = 3$, $n_i = \infty$ and $R_H = 10^5\,\text{cm}^{-1}$:

$$\frac{1}{\lambda} = 10^5\left(\frac{1}{9} - 0\right) \quad \Longrightarrow \quad \lambda = \frac{9}{10^5}\,\text{cm} = 900\,\text{nm}.$$

Thus this transition gives the stated infrared wavelength directly. Planck's constant and the speed of light would be needed to convert wavelength into photon energy, but not to choose the spectral line.

Step 2: Check the competing transitions The Balmer limit gives $\lambda = 4/10^5\,\text{cm} = 400\,\text{nm}$, so option (A) is too short. For the finite Paschen transition $5 \to 3$,

$$\lambda = \frac{1}{10^5(1/9 - 1/25)} = \frac{225}{16 \times 10^5}\,\text{cm} = 1406.25\,\text{nm},$$

so option (B) is too long. The Lyman limit gives $1/10^5\,\text{cm} = 100\,\text{nm}$, ruling out option (D). Only the Paschen limit matches the requested value using the constants explicitly provided.

Step 3: Conclusion The suitable transition is Paschen, $\infty \to 3$, with wavelength $900\,\text{nm}$. Hence, the correct option is (C).

Quick Tip to Solve: For a hydrogen series limit, use $\lambda = n_f^2/R_H$; keep the length unit reciprocal to the unit in the supplied Rydberg constant.

69. The element that usually does NOT show variable oxidation states is : [4 marks]

(A) V

(B) Cu

(C) Sc

(D) ti

Correct Answer: (C) Sc

Solution:

Concept:

- Variable oxidation states arise when different numbers of outer ns and $(n-1)d$ electrons can participate in bonding.
- Scandium has configuration $[\mathrm{Ar}]\,3d^1 4s^2$. Loss of its three outer electrons produces the especially stable argon-core configuration of Sc^{3+}.

Step 1: Examine scandium Scandium normally uses the two $4s$ electrons and its single $3d$ electron to give the +3 oxidation state:

$$\mathrm{Sc} : [\mathrm{Ar}]\,3d^1 4s^2 \quad \longrightarrow \quad \mathrm{Sc}^{3+} : [\mathrm{Ar}].$$

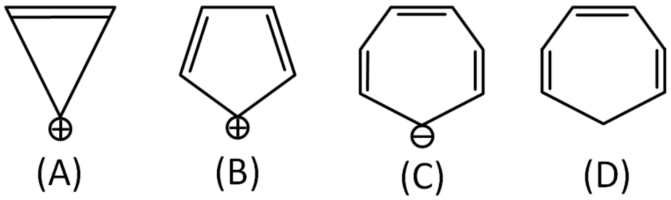
The resulting ion has no remaining d electrons. Removing an additional electron would disrupt the inner closed shell, while the lower oxidation states are not the usual stable states of ordinary scandium compounds. This explains why scandium usually has a single +3 oxidation state. The word "usually" concerns common chemical behaviour, rather than every unusual species that could be generated under exceptional conditions.

Step 2: Compare the other elements Vanadium has several outer d electrons and commonly shows +2, +3, +4 and +5 states. Copper commonly gives both +1 and +2, corresponding to Cu^{+} with $3d^{10}$ and Cu^{2+} with $3d^9$. Titanium, represented by the final option, commonly exhibits +2, +3 and +4. Thus each of the other listed elements shows variable oxidation states in its usual chemistry. Scandium is the one distinguished by its overwhelmingly predominant +3 state, so choosing copper merely because its neutral atom has a filled $3d$ shell would overlook the well-established +2 ion.

Step 3: Conclusion Scandium, Sc, usually shows only the +3 oxidation state. Hence, the correct option is (C).

Quick Tip to Solve: Count the outer electrons of scandium: removing $4s^2 3d^1$ reaches a noble-gas core, explaining its characteristic single +3 oxidation state.

70. Which compound (s) out of the following is/are not aromatic ? [4 marks]



(A) (A) and (C)

(B) (C) and (D)

(C) (B)

(D) (B), (C) and (D)

Correct Answer: (D) (B), (C) and (D)

Solution:

Concept:

- Aromaticity requires a cyclic, planar, continuously conjugated system with $(4n+2)\pi$ electrons. Counting electrons alone is insufficient if a ring atom breaks conjugation.
- A carbocation can supply an empty p orbital, whereas an anionic carbon can contribute a lone pair to the π system. A planar conjugated $4n\pi$ ring is antiaromatic rather than aromatic.

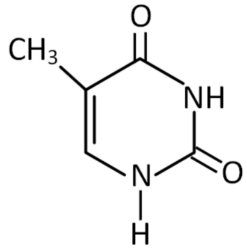
Step 1: Test the two positively charged rings Structure (A) is the cyclopropenyl cation. Its three carbon atoms have overlapping p orbitals, and its single double bond contributes two π electrons. The empty orbital at the positive carbon preserves cyclic conjugation. Since $2 = 4(0) + 2$, (A) is aromatic. Structure (B) is the cyclopentadienyl cation. Its two double bonds contribute four π electrons and the positive carbon contributes no additional electrons. The 4π count does not satisfy the aromatic rule; a planar conjugated form is antiaromatic. Therefore (B) must be included among the structures that are not aromatic.

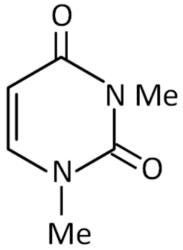
Step 2: Test the seven-membered rings and select the combination In (C), the cycloheptatrienyl anion has six electrons from three double bonds plus two from the negatively charged carbon's lone pair. Its total is eight, a $4n$ count. A fully planar conjugated form would be antiaromatic, and distortion can avoid that destabilization; either way, this is not an aromatic 6π tropylium cation. In (D), neutral cycloheptatriene contains an sp^3 methylene carbon that interrupts the cyclic chain of p orbitals. Its three double bonds do not by themselves make it aromatic. Consequently (B), (C) and (D) all fail aromaticity. Options (A) and (B) omit (B), while option (C) omits both nonaromatic seven-membered structures.

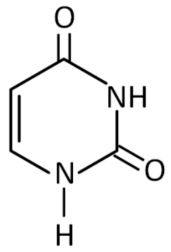
Step 3: Conclusion The cyclopentadienyl cation, cycloheptatrienyl anion and neutral cycloheptatriene are not aromatic. Hence, the correct option is (D).

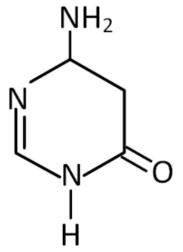
Quick Tip to Solve: Check uninterrupted conjugation first, then count π electrons; distinguish a seven-membered anion from the aromatic tropylium cation.

71. Among the following compounds, which one is found in RNA ? [4 marks]

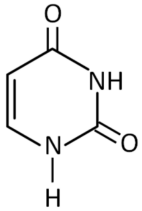
(A) 

(B) 

(C) 

(D) 

Correct Answer: (C)



Solution:

Concept:

- RNA contains ribose, phosphate and four principal nitrogenous bases: adenine, guanine, cytosine and uracil. DNA normally uses thymine in place of uracil.
- Uracil is a pyrimidine base with two ring nitrogen atoms and carbonyl groups at positions 2 and 4. Its carbon at position 5 has hydrogen, unlike the methyl-substituted carbon of thymine.

Step 1: Identify the unsubstituted dioxopyrimidine Inspect the ring substituents rather than just counting nitrogen atoms. The structure in option (C) has two carbonyl groups, two ring nitrogen atoms and no methyl substituent. This is uracil, the base used by RNA. Its free-base drawing has two N–H groups; incorporation into a nucleotide forms a glycosidic bond at one nitrogen without changing its identity.

Step 2: Distinguish the other structures Option (A) has a methyl group on the carbon between the carbonyl-bearing carbon and the carbon–carbon double bond: it is thymine, or 5-methyluracil. Option (B) instead has methyl groups on both nitrogen atoms, so it is not ordinary uracil. Option (D) has an amino group, only one carbonyl, and the different ring unsaturation shown; it does not have the uracil structure. Thus merely having a nitrogen heterocycle or an oxygen atom is not enough to select an RNA base.

Step 3: Conclusion The uracil structure is option (C), which is found in RNA. Hence, the correct option is (C).

Quick Tip to Solve: For an RNA-base structure, locate uracil by its two carbonyl groups and absence of the 5-methyl group that identifies thymine.

72. For the chemical reaction X $\rightleftharpoons$ Y, the standard reaction Gibbs energy depends on temperature T (in K) as $\Delta_r G^o$ (in kJ mol^{-1}) = 120 $-\frac{3}{8}$ T.

The major component of the reaction mixture at T is : [4 marks]

(A) Y if T = 300 K

(B) Y if T = 280 K

(C) X if T = 350 K

(D) X if T = 315 K

Correct Answer: (D) X if T = 315 K

Solution:

Concept:

- The standard reaction Gibbs energy determines the equilibrium constant through $\Delta_r G^\circ = -RT \ln K$, where K is the product-to-reactant activity ratio for X $\rightleftharpoons$ Y.
- When $\Delta_r G^\circ > 0$, $K < 1$, so X is favoured; when $\Delta_r G^\circ < 0$, $K > 1$, so Y is favoured. The crossover temperature follows by setting the given expression to zero.

Step 1: Find the crossover temperature Using the supplied dependence, set

$$120 - \frac{3}{8}T = 0 \quad \Longrightarrow \quad T = 120 \times \frac{8}{3} = 320\,\text{K}.$$

Below this temperature the expression is positive, while above it the expression is negative. This determines which side of the equilibrium is favoured; it does not mean that the less abundant component disappears entirely.

Step 2: Test all four temperatures At $300\,\text{K}$, $\Delta_r G^\circ = 7.5\,\text{kJ mol}^{-1} > 0$, so X rather than Y predominates, excluding (A). At $280\,\text{K}$, the value is $15\,\text{kJ mol}^{-1} > 0$, again excluding Y in (B). At $350\,\text{K}$, the value is $-11.25\,\text{kJ mol}^{-1} < 0$, so Y predominates and (C) is false. Finally,

$$\Delta_r G^\circ(315) = 120 - \frac{3}{8}(315) = 1.875\,\text{kJ mol}^{-1} > 0.$$

Thus $K < 1$ at 315 K and X is the major component.

Step 3: Conclusion At $315\,\text{K}$, X is the major equilibrium component. Hence, the correct option is (D).

Quick Tip to Solve: First solve $\Delta_r G^\circ = 0$; here the 320 K boundary lets you classify every option immediately.

73. A 10 mg effervescent tablet containing sodium bicarbonate and oxalic acid releases 0.25 ml of CO_2 at T = 298.15 K and p = 1 bar. If molar volume of CO_2 is 25.0 L under such condition, what is the percentage of sodium bicarbonate in each tablet ?

[Molar mass of $NaHCO_3$ = 84 g mol^{-1}] [4 marks]

(A) 33.6

(B) 0.84

(C) 8.4

(D) 16.8

Correct Answer: (C) 8.4

Solution:

Concept:

- Acid reacts with each mole of bicarbonate to release one mole of carbon dioxide: $HCO_3^- + H^+ \longrightarrow CO_2 + H_2O$. Therefore $n(NaHCO_3) = n(CO_2)$.
- At the specified temperature and pressure, the amount of gas is $n = V/V_m$. Calculate mass percentage as 100 times the bicarbonate mass divided by the tablet mass, using the same mass unit for both.

Step 1: Convert the gas volume into moles The molar volume is already specified for the experimental conditions, so no standard-condition molar volume should be substituted. Convert the measured volume to litres:

$$V(CO_2) = 0.25\,\text{mL} = 2.5 \times 10^{-4}\,\text{L}.$$

Hence

$$n(CO_2) = \frac{2.5 \times 10^{-4}}{25.0} = 1.0 \times 10^{-5}\,\text{mol}.$$

Oxalic acid supplies protons; its being diprotic does not change the one-to-one relation between bicarbonate consumed and carbon dioxide evolved.

Step 2: Find the bicarbonate mass and percentage The bicarbonate amount is also 1.0×10^{-5} mol, giving

$$m(NaHCO_3) = (1.0 \times 10^{-5})(84)\,\text{g} = 8.4 \times 10^{-4}\,\text{g} = 0.84\,\text{mg}.$$

The total tablet mass is 10 mg, not 1 mg. Therefore

$$\%NaHCO_3 = \frac{0.84}{10} \times 100 = 8.4\%.$$

Option (B) confuses the mass in milligrams with the percentage; (A) and (D) introduce incorrect stoichiometric factors rather than the required one-to-one bicarbonate-to-gas ratio.

Step 3: Conclusion The sodium bicarbonate content is 8.4% by mass. Hence, the correct option is (C).

Quick Tip to Solve: For acid plus bicarbonate, match bicarbonate moles directly to carbon dioxide moles before calculating the mass percentage.

74. An organic compound is estimated through Duma's method and was found to evolve 6 moles of CO_2. 4 moles of H_2O and 1 mole of nitrogen gas. The formula of the compound is : [4 marks]

(A) C_6H_8N

(B) $C_6H_8N_2$

(C) $C_{12}H_8N$

(D) $C_{12}H_8N_2$

Correct Answer: (B) $C_6H_8N_2$

Solution:

Concept:

- On complete combustion, every carbon atom in the compound gives one molecule of CO_2, whereas two hydrogen atoms give one molecule of H_2O.
- In the Dumas determination, nitrogen is collected as N_2, so one mole of nitrogen gas corresponds to two moles of nitrogen atoms. Compare atom ratios, not just gas-molecule ratios.

Step 1: Recover the amounts of constituent atoms The reported product quantities give

$$n(\mathrm{C}) = n(\mathrm{CO_2}) = 6\,\mathrm{mol},$$

$$n(\mathrm{H}) = 2n(\mathrm{H_2O}) = 2(4) = 8\,\mathrm{mol},$$

$$n(\mathrm{N}) = 2n(\mathrm{N_2}) = 2(1) = 2\,\mathrm{mol}.$$

The factors of two for both water and nitrogen gas are essential. The oxygen atoms in the combustion products are not evidence for oxygen in the original compound, since oxygen is supplied during combustion.

Step 2: Compare the formula ratios The elemental atom ratio is

$$\mathrm{C} : \mathrm{H} : \mathrm{N} = 6 : 8 : 2 = 3 : 4 : 1.$$

Thus the empirical formula is C_3H_4N. Among the proposed molecular formulae, $C_6H_8N_2$ has exactly twice this empirical composition. In contrast, C_6H_8N has too little nitrogen; $C_{12}H_8N$ and $C_{12}H_8N_2$ have too much carbon relative to hydrogen. Product amounts establish the composition ratio, so the selection is made by matching that ratio to the listed alternatives, without inferring a separate molar mass.

Step 3: Conclusion Only $C_6H_8N_2$ matches the observed carbon, hydrogen and nitrogen ratio. Hence, the correct option is (B).

Quick Tip to Solve: Convert each collected product to moles of atoms first: multiply water and nitrogen-gas amounts by two, but leave carbon dioxide unchanged.

75. For the cell $\mathrm{Zn(s)\,|\,Zn^{2+}(aq)\,||\,M^{x+}(aq)\,|\,M(s)}$, different half cells and their standard electrode potentials are given below:

Half cell	$\mathrm{Au^{3+}/Au}$	$\mathrm{Ag^{+}/Ag}$	$\mathrm{Fe^{3+}/Fe^{2+}}$	$\mathrm{Fe^{2+}/Fe}$
$E^{\circ}/(\mathrm{V})$	**1.40**	**0.80**	**0.77**	-0.44

If $E^{\circ}_{\mathrm{Zn^{2+}/Zn}} = -0.76\,\mathrm{V}$, which cathode will give maximum value of $E^{\circ}_{\mathrm{cell}}$ per electron transferred?

[4 marks]

(A) $\mathrm{Ag^{+}/Ag}$

(B) $\mathrm{Fe^{3+}/Fe^{2+}}$

(C) $\mathrm{Au^{3+}/Au}$

(D) $\mathrm{Fe^{2+}/Fe}$

Correct Answer: (A) $\mathrm{Ag^{+}/Ag}$

Solution:

Concept:

- For the zinc-anode cell, the standard cell potential is $E^{\circ}_{\mathrm{cell}} = E^{\circ}_{\mathrm{cathode}} - E^{\circ}_{\mathrm{Zn^{2+}/Zn}} = E^{\circ}_{\mathrm{cathode}} + 0.76\,\mathrm{V}$.
- The question asks for the comparison per electron transferred. For this requested comparison, tabulate $E^{\circ}_{\mathrm{cell}}/x$, where x is the number of electrons in the cathodic reduction as written. The reduction electron counts are 1, 1, 3 and 2 for the listed options.

Step 1: Calculate each cell potential For silver, iron(III)/iron(II), gold and iron(II)/iron respectively, subtraction of the same zinc reduction potential gives

$$E^{\circ}_{\mathrm{Ag^{+}/Ag}}(\mathrm{cell}) = 0.80 + 0.76 = 1.56\,\mathrm{V},$$
$$E^{\circ}_{\mathrm{Fe^{3+}/Fe^{2+}}}(\mathrm{cell}) = 0.77 + 0.76 = 1.53\,\mathrm{V},$$
$$E^{\circ}_{\mathrm{Au^{3+}/Au}}(\mathrm{cell}) = 1.40 + 0.76 = 2.16\,\mathrm{V},$$
$$E^{\circ}_{\mathrm{Fe^{2+}/Fe}}(\mathrm{cell}) = -0.44 + 0.76 = 0.32\,\mathrm{V}.$$

These potentials themselves are not multiplied when half-reactions are balanced: electrode potential is an intensive quantity.

Step 2: Apply the requested electron-count comparison The reductions are $\mathrm{Ag^{+} + e^{-} \longrightarrow Ag}$, $\mathrm{Fe^{3+} + e^{-} \longrightarrow Fe^{2+}}$, $\mathrm{Au^{3+} + 3\,e^{-} \longrightarrow Au}$ and $\mathrm{Fe^{2+} + 2\,e^{-} \longrightarrow Fe}$. Their requested ratios are consequently

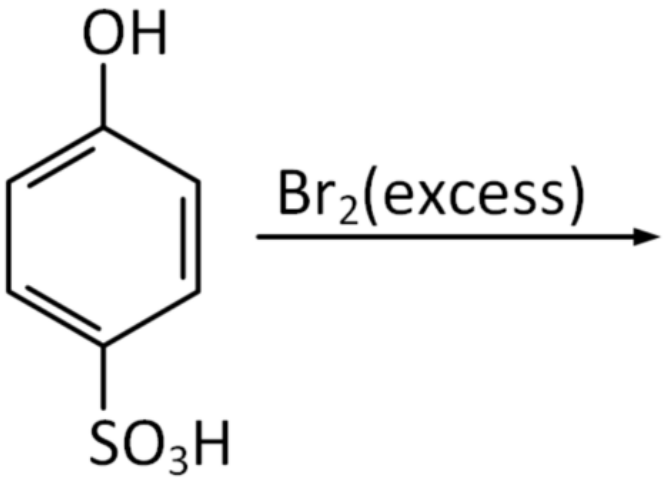
$$\frac{1.56}{1} = 1.56, \qquad \frac{1.53}{1} = 1.53, \qquad \frac{2.16}{3} = 0.72, \qquad \frac{0.32}{2} = 0.16.$$

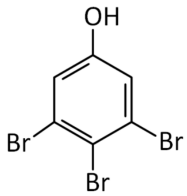
Silver gives the largest ratio. Gold has the highest unnormalised cell potential, but that is a different comparison from the one requested here; the two iron alternatives give smaller ratios.

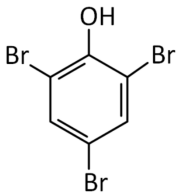
Step 3: Conclusion Using the requested per-electron comparison, the $\mathrm{Ag^{+}/Ag}$ cathode is selected. Hence, the correct option is (A).

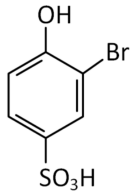
Quick Tip to Solve: Keep the two comparisons distinct: first calculate the cell potential, then use the cathodic electron count only for the ratio explicitly requested here.

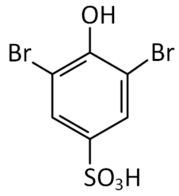
76. The major product of the following reaction is : [4 marks]

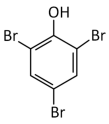


(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- The hydroxyl group strongly activates the aromatic ring by resonance and directs electrophilic substitution to its ortho and para positions.
- Excess bromine gives extensive substitution of activated phenols. A sulfonic acid group can be displaced during bromination at its ring position, allowing the fully ortho/para-brominated phenol to form.

Step 1: Locate the activated positions The starting compound is para-hydroxybenzenesulfonic acid: take the carbon bonded to hydroxyl as position 1 and the sulfonic acid substituent as position 4. The hydroxyl group activates positions 2, 4 and 6. Positions 2 and 6 initially carry hydrogen and can undergo bromination. The sulfonic acid group also directs ordinary substitution toward positions 2 and 6, which are meta to itself, so the first substitutions are consistent with both groups.

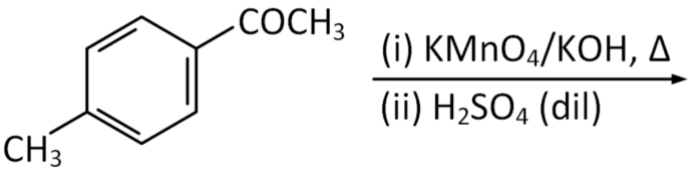
Step 2: Follow the effect of excess bromine With excess bromine, bromination does not stop at the monobromo product or at the dibromo compound retaining the sulfonic acid group. Brominative

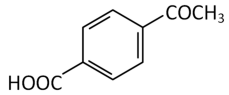
replacement of the para sulfonic acid substituent furnishes bromine at position 4 as well. The final phenol therefore carries bromine at positions 2, 4 and 6, while retaining its hydroxyl group. Option (B) shows precisely this 2,4,6-tribromophenol structure. Option (A) instead places bromines at positions 3,4,5, inconsistent with hydroxyl-directed attack. Option (C) is the incompletely brominated monosubstitution product, while (D) retains the sulfonic acid group after ortho dibromination and is not the final product with excess reagent.

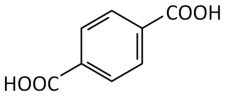
Step 3: Conclusion The major product is 2,4,6-tribromophenol, the structure shown in option (B). Hence, the correct option is (B).

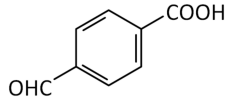
Quick Tip to Solve: For a phenol with a para sulfonic acid group, excess bromine can replace that group as well as brominate the two free ortho positions.

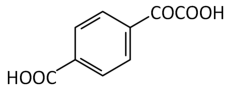
77. The major product of the following reaction is [4 marks]

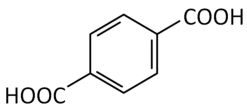


(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- Hot alkaline potassium permanganate is a strong oxidant. A benzylic methyl group with benzylic hydrogen is oxidised to a carboxylate group; acidic work-up then gives the carboxylic acid.
- Under vigorous permanganate oxidation, an aryl methyl ketone can undergo oxidative cleavage to give the aromatic carboxylate. The two substituent positions on the benzene ring are preserved.

Step 1: Identify both oxidisable side chains The initial compound has a methyl group and an acetyl group, $-COCH_3$, para to each other on a benzene ring. The methyl carbon is directly bonded to the ring and has benzylic hydrogens, so its oxidation gives $-COO-$. The reaction conditions are hot alkaline $KMnO_4$, not a mild oxidation that stops at an aldehyde. Therefore a proposed product retaining $-CHO$ at this position cannot be the major final product.

Step 2: Complete oxidation and acidify The acetyl substituent is also oxidatively cleaved under these vigorous conditions, leaving the ring bonded to a carboxylate carbon. After the dilute sulfuric acid

work-up, both carboxylates are protonated:

$$\text{p-CH}_3\text{–C}_6\text{H}_4\text{–COCH}_3 \xrightarrow{\text{hot alkaline KMnO}_4} \text{p-}^{-}\text{OOC–C}_6\text{H}_4\text{–COO}^{-} \xrightarrow{\text{H}^+} \text{p-HOOC–C}_6\text{H}_4\text{–COOH}.$$

The product is benzene-1,4-dicarboxylic acid, or terephthalic acid. Option (A) retains the acetyl group and is incompletely oxidised. Option (C) retains an aldehyde, which would be oxidised further. Option (D) retains an extra carbonyl carbon in a $-\text{COCOOH}$ side chain rather than the aryl carboxylic acid obtained by oxidative cleavage.

Step 3: Conclusion The major product is terephthalic acid, $\text{HOOC–C}_6\text{H}_4\text{–COOH}$ with the groups para, shown in option (B). Hence, the correct option is (B).

Quick Tip to Solve: Under hot alkaline permanganate, inspect every side chain for oxidation; the acid work-up protonates carboxylates but does not stop oxidation at an aldehyde.

---

78. If a reaction follows the Arrhenius equation, the plot ln k vs $\frac{1}{(RT)}$ gives straight line with a gradient (− y) unit.

The energy required to active the reactant is : [4 marks]

(A) y unit

(B) y/R unit

(C) yR unit

(D) −y unit

Correct Answer: (A) y unit

Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where A is the pre-exponential factor, E_a is activation energy, R is the gas constant and T is absolute temperature.
- Taking natural logarithms gives a linear relation. The slope depends on the exact reciprocal-temperature variable selected for the horizontal axis.

Step 1: Linearise the given equation Taking the natural logarithm on both sides gives

$$\ln k = \ln A - \frac{E_a}{RT}.$$

The question plots $\ln k$, not $\log_{10} k$, so no factor of 2.303 is introduced. The intercept is $\ln A$.

Step 2: Read the slope for the printed horizontal axis Set $x = 1/(RT)$. The equation becomes $\ln k = \ln A - E_a x$, whose gradient is $-E_a$. Comparing with the given gradient $-y$ gives

$$-E_a = -y \quad \Longrightarrow \quad E_a = y.$$

Multiplying or dividing by R would correspond to misreading the axis as $1/T$. The negative slope does not mean negative activation energy; the explicit minus sign already appears in the Arrhenius

equation.

Step 3: Conclusion The energy required to activate the reactants is $E_a = y$ unit. Hence, the correct option is (A).

Quick Tip to Solve: Before using an Arrhenius slope formula, check whether the horizontal axis is $1/T$ or $1/(RT)$.

79. Match the metals (column I) with the coordination compound(s)/enzyme(s) (column II).

Column I: Metals		Column II	
(A)	Co	(i)	Wilkinson catalyst
(B)	Zn	(ii)	Chlorophyl
(C)	Rh	(iii)	Vitamin B_{12}
(D)	Mg	(iv)	Carbonic anhydrase

[4 marks]

(A) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii)

(B) (A)-(iv); (B)-(iii); (C)-(i); (D)-(ii)

(C) (A)-(i); (B)-(ii); (C)-(iii); (D)-(iv)

(D) (A)-(ii); (B)-(i); (C)-(iv); (D)-(iii)

Correct Answer: (A) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii)

Solution:

Concept:

- Metal centres in coordination compounds have characteristic biological or catalytic roles. Identifying the actual central metal gives the matching directly.
- Vitamin B_{12} contains cobalt, carbonic anhydrase contains zinc, Wilkinson catalyst contains rhodium, and chlorophyll contains magnesium.

Step 1: Identify the two biological metal centres Vitamin B_{12}, also called cobalamin, contains a cobalt centre in its corrin coordination framework. Thus metal (A), Co, matches entry (iii). Carbonic anhydrase is a zinc-containing enzyme that catalyses the interconversion of carbon dioxide and bicarbonate in water. Therefore metal (B), Zn, matches entry (iv). These two assignments already exclude the options interchanging cobalt with zinc or assigning either to Wilkinson catalyst.

Step 2: Complete the catalyst and pigment matches Wilkinson catalyst is $RhCl[P(C_6H_5)_3]_3$, a rhodium coordination compound used in homogeneous hydrogenation. Thus metal (C), Rh, matches (i). Chlorophyll has magnesium at the centre of its coordinating ring system, so metal (D), Mg, matches (ii). The complete sequence is therefore (A)-(iii), (B)-(iv), (C)-(i), (D)-(ii). The other listed sequences place at least one metal in a compound in which it is not the central metal.

Step 3: Conclusion The required matching sequence is Co–vitamin B_{12}, Zn–carbonic anhydrase, Rh–Wilkinson catalyst, and Mg–chlorophyll. Hence, the correct option is (A).

Quick Tip to Solve: Remember the four metal anchors: cobalamin–Co, carbonic anhydrase–Zn, Wilkinson–Rh, chlorophyll–Mg.

80. An example of solid sol is : [4 marks]

(A) Butter

(B) Hair cream

(C) Gem stones

(D) Paint

Correct Answer: (C) Gem stones

Solution:

Concept:

- A colloid is classified using both the dispersed phase and the dispersion medium. A solid sol consists of a solid dispersed phase distributed through a solid medium.
- The word sol alone commonly denotes solid particles dispersed in a liquid; the qualifier solid specifies that the continuous medium is also solid.

Step 1: Identify a solid dispersed in a solid Gem stones provide the standard example of a solid sol: small amounts of solid material are dispersed in a solid host, often producing characteristic colour. Both the dispersed component and the continuous phase are solid. They therefore satisfy the two-phase description required by the term solid sol, rather than merely being a colloid that looks thick or solid.

Step 2: Compare the other examples by their phases Butter is a gel-type example, with liquid dispersed in a solid or semisolid continuous structure; it is not the solid-in-solid class. Hair cream is an emulsion-type colloidal preparation involving liquid phases, rather than a solid sol. Paint is the familiar example of solid pigment particles dispersed in a liquid medium, so it is a sol but not a solid sol. Thus only gem stones have the phase combination asked for among these options.

Step 3: Conclusion Gem stones are an example of a solid sol, with solid dispersed in solid. Hence, the correct option is (C).

Quick Tip to Solve: Classify a colloid by writing dispersed phase first and medium second; solid sol means solid/solid.

81. The chloride that CANNOT get hydrolysed is : [4 marks]

(A) $SnCl_4$

(B) $SiCl_4$

(C) $PbCl_4$

(D) CCl_4

Correct Answer: (D) CCl_4

Solution:

Concept:

- Hydrolysis replaces chloride by oxygen-containing groups through reaction with water. The tendency of group-14 tetrachlorides to hydrolyse depends on the accessibility and reactivity of the central atom.
- Carbon tetrachloride is kinetically resistant to hydrolysis under ordinary conditions, whereas the corresponding silicon, tin and lead tetrachlorides can undergo hydrolysis.

Step 1: Examine the carbon centre In CCl_4, carbon is a small second-period atom surrounded by four chlorine atoms in a tetrahedral arrangement. It already has a complete octet and cannot accommodate the expanded coordination characteristic of the water-assisted substitution pathways available to the heavier members. Nucleophilic attack by water at this carbon centre is therefore strongly hindered. Consequently carbon tetrachloride does not undergo ordinary hydrolysis simply on contact with water.

Step 2: Contrast the heavier tetrachlorides The larger and more accessible central atoms in $SiCl_4$, $SnCl_4$ and $PbCl_4$ allow water-assisted reaction and formation of strong bonds to oxygen. For example, the overall silicon reaction is

$$SiCl_4 + 2\,H_2O \longrightarrow SiO_2 + 4\,HCl.$$

The oxide product may be hydrated in practice. Tin and lead tetrachlorides also hydrolyse, so none of those three can be the chloride requested. The distinguishing property is resistance to hydrolysis, not whether the compound contains polar carbon–chlorine bonds.

Step 3: Conclusion CCl_4 is the chloride that cannot be hydrolysed under the ordinary conditions intended here. Hence, the correct option is (D).

Quick Tip to Solve: For group-14 tetrachlorides, remember the hydrolysis contrast: carbon tetrachloride resists water; silicon tetrachloride reacts readily.

82. NaH is an example of : [4 marks]

(A) metallic hydride

(B) saline hydride

(C) electron-rich hydrid

(D) molecular hydride

Correct Answer: (B) saline hydride

Solution:

Concept:

- Saline or ionic hydrides form when strongly electropositive metals combine with hydrogen. Hydrogen is then present as the hydride ion, H^-.
- Covalent molecular hydrides contain discrete molecules, while metallic hydrides typically involve hydrogen in a metal lattice without the simple alkali-metal ionic description.

Step 1: Determine the ionic constituents of sodium hydride Sodium has one valence electron and readily forms Na^+. Hydrogen accepts an electron to attain its filled $1s^2$ configuration as H^-. Thus sodium hydride is represented by

$$NaH \equiv Na^+ H^-.$$

Its formula denotes an ionic solid lattice of these ions, rather than a discrete covalent molecule. Such alkali-metal hydrides are described as saline hydrides because of their salt-like ionic character.

Step 2: Check the alternative classifications A metallic hydride is associated with hydrogen in a metallic host, typically a transition-metal system; sodium hydride is instead the standard ionic hydride. An electron-rich hydride is a covalent molecular hydride with lone pairs on its central atom, such as ammonia or water, so that description does not apply to NaH. Nor is it a molecular hydride, since it forms an extended ionic lattice. Its hydride ion also explains its reaction with water to release hydrogen: $NaH + H_2O \longrightarrow NaOH + H_2$.

Step 3: Conclusion NaH is a saline (ionic) hydride. Hence, the correct option is (B).

Quick Tip to Solve: An alkali metal plus hydrogen usually signals an ionic hydride containing H^-, not a molecular hydride.

83. The correct match between item (I) and item (II) is:

Item-I		Item-II	
(A)	Norethindrone	(P)	Anti-biotic
(B)	Ofloxacin	(Q)	Anti-fertility
(C)	Equanil	(R)	Hypertension
		(S)	Analgesics

[4 marks]

(A) $(A) \to (R);\ (B) \to (P);\ (C) \to (R)$
(B) $(A) \to (R);\ (B) \to (P);\ (C) \to (S)$
(C) $(A) \to (Q);\ (B) \to (P);\ (C) \to (R)$
(D) $(A) \to (Q);\ (B) \to (R);\ (C) \to (S)$

Correct Answer: (C) $(A) \to (Q);\ (B) \to (P);\ (C) \to (R)$

Solution:

Concept:

- Drugs are matched to their therapeutic classes by their characteristic use. Antibiotics act against bacteria, while antifertility drugs regulate reproduction.
- Norethindrone is a synthetic progestational antifertility agent; ofloxacin is an antibiotic. Equanil is a tranquillizer associated in the textbook classification with control of anxiety and hypertension.

Step 1: Assign norethindrone and ofloxacin Norethindrone is used as a component of oral contraceptive preparations and therefore belongs to the anti-fertility entry (Q). It is not an antibiotic or a drug identified here with hypertension. Hence item-I (A) must map to (Q). Ofloxacin is a fluoroquinolone antibacterial agent, so item-I (B) maps to the anti-biotic entry (P), rather than to hypertension or analgesics. These assignments establish the first two members of the matching sequence.

Step 2: Identify the listed use of Equanil Equanil is the trade name associated with meprobamate, a tranquillizer. In this textbook classification it is associated with control of depression/anxiety and hypertension; it is therefore paired with entry (R). It is not classified as an analgesic, whose primary purpose is relief of pain. The complete matching is consequently (A) to (Q), (B) to (P), and (C) to (R). The first two alternatives wrongly assign norethindrone to hypertension, and the last alternative wrongly assigns ofloxacin to hypertension and Equanil to analgesics.

Step 3: Conclusion The correct sequence is norethindrone–anti-fertility, ofloxacin–anti-biotic, and Equanil–hypertension. Hence, the correct option is (C).

Quick Tip to Solve: Use the most distinctive drug first: norethindrone identifies antifertility and ofloxacin identifies antibiotic; then check Equanil.

84. The concentration of dissolved oxygen (DO) in cold water can go upto : [4 marks]

(A) 14 ppm

(B) 16 ppm

(C) 8 ppm

(D) 10 ppm

Correct Answer: (D) 10 ppm

Solution:

Concept:

- Dissolved oxygen (DO) measures the oxygen present in water, normally expressed in ppm. For dilute aqueous solutions, a concentration in ppm is approximately the same numerical concentration in $\mathrm{mg\,L^{-1}}$.
- Gases generally become less soluble in water as temperature increases. Cold water can therefore retain more dissolved oxygen than warm water under comparable conditions.

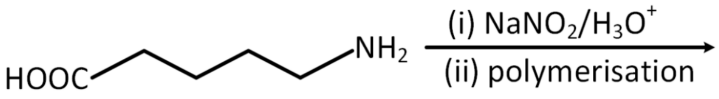
Step 1: Recall the cold-water value used in environmental chemistry In the textbook discussion of water pollution and aquatic life, the concentration of dissolved oxygen in cold water can reach about $10\,\mathrm{ppm}$. This is the characteristic value requested by the question. It describes dissolved oxygen content, not the oxygen percentage of air, and not the biochemical oxygen demand (BOD), which measures oxygen consumption during decomposition of organic matter.

Step 2: Separate this value from other oxygen measures A value such as $8\,\mathrm{ppm}$ may describe oxygen content at another temperature or condition, but it is not the quoted upper cold-water value in this question. The listed 14 and $16\,\mathrm{ppm}$ values likewise do not match that textbook statement. Actual oxygen saturation also depends on pressure, salinity and precise temperature; no such numerical conditions are supplied here for a solubility calculation. The intended selection is therefore the stated environmental-chemistry value, consistent with greater oxygen solubility in colder water.

Step 3: Conclusion The dissolved oxygen concentration in cold water can go up to approximately $10\,\mathrm{ppm}$ in the stated textbook context. Hence, the correct option is (D).

Quick Tip to Solve: Do not confuse DO with BOD: DO is oxygen already dissolved, and the quoted cold-water value here is about $10\,\mathrm{ppm}$.

85. The polymer obtained from the following reaction is : [4 marks]



(A) $[-C(=O)-(CH_2)_4-NH-]_n$

(B) $[-NH-C(=O)-(CH_2)_4-C(=O)-NH-]_n$

(C) $[-O-C(=O)-(CH_2)_4-O-]_n$

(D) $[-O-(CH_2)_4-C(=O)-]_n$

Correct Answer: (D) $[-O-(CH_2)_4-C(=O)-]_n$

Solution:

Concept:

- A primary aliphatic amine reacts with nitrous acid, generated by $NaNO_2$ in acid, to give the corresponding alcohol with evolution of N_2.
- A molecule containing one hydroxyl and one carboxyl group undergoes self-condensation to form a polyester. Each ester link joins the alcohol oxygen of one molecule to the carboxyl carbon of another.

Step 1: Identify the product of the first reaction The starting chain is $HOOC-(CH_2)_4-NH_2$, with four methylene groups between the functional groups. The amino group is replaced by hydroxyl, while the carboxyl group and carbon skeleton remain unchanged:

$$HOOC-(CH_2)_4-NH_2 + HNO_2 \longrightarrow HOOC-(CH_2)_4-OH + N_2 + H_2O.$$

Thus the monomer entering polymerisation is a hydroxy acid, not the original amino acid.

Step 2: Join the two complementary functional groups The hydroxyl end of one molecule reacts with the carboxyl end of the next, eliminating water. Repetition produces the chain segment

$$\cdots - O-(CH_2)_4-C(=O)-O-(CH_2)_4-C(=O)-\cdots .$$

The repeat unit is therefore $[-O-(CH_2)_4-C(=O)-]_n$. The terminal bonds on a repeat unit continue into neighbouring units; they are not additional atoms.

Step 3: Eliminate the other structures Options (A) and (B) retain nitrogen as amide groups, although the first reaction has expelled the amino nitrogen as N_2. Option (C) has an extra oxygen in the repeated chain compared with the hydroxy-acid condensation product. Option (D) alone preserves four methylene groups, one carbonyl group and one ester oxygen per monomer residue.

Step 4: Conclusion The product is the polyester $[-O-(CH_2)_4-C(=O)-]_n$. Hence, the correct option is (D).

Quick Tip to Solve: Apply the reagent before naming the polymer: nitrous acid removes the amino group, so the later polymer cannot be a polyamide.

86. Peroxyacetyl nitrate (PAN), an eye irritant is produced by : [4 marks]

(A) classical smog

(B) acid rain

(C) photochemical smog

(D) organic waste

Correct Answer: (C) photochemical smog

Solution:

Concept:

- Photochemical smog is an oxidising mixture formed when nitrogen oxides and volatile organic compounds react in sunlight. Its secondary pollutants include ozone, aldehydes and peroxyacyl nitrates.
- Peroxyacetyl nitrate (PAN), $CH_3C(O)OONO_2$, is a secondary pollutant: it is produced by atmospheric reactions rather than being the principal material emitted directly from a source.

Step 1: Identify the conditions that form PAN Sunlight initiates reactions involving NO_2 and oxygen, while oxidation of hydrocarbons and oxygenated organic compounds generates reactive organic radicals. Oxidation of acetaldehyde can form an acetyl radical, which adds oxygen to give the peroxyacetyl radical. It combines with NO_2 to form PAN. In a condensed radical notation, the decisive association is

$$CH_3C(O)OO^{\bullet} + NO_2^{\bullet} \rightleftharpoons CH_3C(O)OONO_2.$$

These reactions explain why PAN is linked to sunlight-driven urban pollution.

Step 2: Distinguish the other pollution categories Classical smog is the cool, humid, reducing smog associated mainly with smoke, sulphur dioxide and fog; PAN is characteristic of the oxidising photochemical variety instead. Acid rain results chiefly from atmospheric conversion of sulphur and nitrogen oxides into acids and is identified by acidic precipitation, not by formation of PAN. Organic waste is discarded biological material: it can release gases during decomposition, but it is not the atmospheric reaction mixture asked for here. PAN irritates eyes and can damage vegetation, consistent with its role among photochemical-smog oxidants.

Step 3: Conclusion PAN is produced in photochemical smog by sunlight-driven reactions of nitrogen oxides and organic pollutants. Hence, the correct option is (C).

Quick Tip to Solve: Associate PAN and ozone with sunny, oxidising photochemical smog; associate sulphur dioxide, smoke and fog with classical smog.

87. The amphoteric hydroxide is : [4 marks]

(A) $Ca(OH)_2$

(B) $Be(OH)_2$

(C) $Mg(OH)_2$

(D) $Sr(OH)_2$

Correct Answer: (B) $Be(OH)_2$

Solution:

Concept:

- An amphoteric hydroxide reacts with both acids and strong bases. Reaction with an acid alone proves basic behaviour, but does not establish amphoterism.
- The very small Be^{2+} ion has a high charge density and strong polarising power. Beryllium compounds therefore show greater covalent character than the corresponding compounds of the heavier Group 2 metals.

Step 1: Demonstrate both kinds of reaction Beryllium hydroxide consumes acid to produce a beryllium salt and water, showing its basic side:

$$Be(OH)_2 + 2\,HCl \longrightarrow BeCl_2 + 2\,H_2O.$$

It also dissolves in excess strong alkali through formation of the tetrahydroxoberyllate complex:

$$Be(OH)_2 + 2\,OH^- \longrightarrow [Be(OH)_4]^{2-}.$$

The second reaction is the distinguishing evidence. A substance that merely neutralises an acid would not necessarily react with alkali in this way.

Step 2: Compare the other Group 2 hydroxides $Mg(OH)_2$, $Ca(OH)_2$ and $Sr(OH)_2$ are classified as basic hydroxides in this comparison. They neutralise acids, but do not display the characteristic amphoteric dissolution of $Be(OH)_2$ in excess aqueous alkali. Down Group 2, the cations become larger, their polarising power decreases and the hydroxides become more ionic and strongly basic. Low water solubility must not be confused with amphoterism: $Mg(OH)_2$ is sparingly soluble, yet that fact does not make it an acid as well as a base. Thus (A), (C) and (D) fail the two-reaction criterion, whereas (B) satisfies it.

Step 3: Conclusion $Be(OH)_2$ reacts with both acids and strong alkalis and is amphoteric. Hence, the correct option is (B).

Quick Tip to Solve: Test amphoterism by asking for a reaction with excess alkali as well as with acid; in Group 2, beryllium is the exception.

88. The correct statements among (a) to (d) regarding H_2 as a fuel are :

(a) It produces less pollutants than petrol.

(b) A cylinder of compressed dihydrogen weighs $\sim$ 30 times more than a petrol tank producing the same amount of energy.

(c) Dihydrogen is stored in tanks of metal alloys like $NaNi_5$.

(d) On combustion, values of energy released per gram of liquid dihydrogen and LPG are 50 and 142 KJ, respectively. [4 marks]

(A) (a) and (c) only

(B) (b) and (d) only

(C) (a), (b) and (c) only

(D) (b), (c) and (d) only

Correct Answer: Literal wording has no matching option because of $NaNi_5$; the accepted answer is (C) with the alloy example corrected to $LaNi_5$.

Solution:

Concept:

- Dihydrogen has a high energy output per unit mass and its ideal combustion product is water: $2\,H_2 + O_2 \longrightarrow 2\,H_2O$. Fuel performance must distinguish gravimetric energy density from the mass and volume of the storage system.
- Metal-hydride storage uses reversible uptake of hydrogen by suitable alloys. The conventional textbook example is $LaNi_5$; the printed statement (c) gives $NaNi_5$.

Step 1: Evaluate pollution and compressed-gas storage Statement (a) is correct in the usual comparison with petrol: hydrogen contains no carbon, so its combustion does not directly produce carbon monoxide, carbon dioxide or unburnt hydrocarbon residues. Very hot air combustion can still form nitrogen oxides, so this is a relative pollution advantage. Statement (b) is also the quoted textbook storage comparison: for the same energy output, a compressed-hydrogen cylinder and its pressure-resistant container can weigh about thirty times as much as the petrol tank. This refers to the whole storage system, not to the mass of hydrogen fuel alone.

Step 2: Check alloy storage and the calorific values Statement (c) expresses the use of hydrogen-absorbing metal alloys. Its familiar chemical example is $LaNi_5$, which can reversibly absorb hydrogen; the printed $NaNi_5$ is a formula error in that example. In the intended textbook classification, (c) is accepted. Statement (d) reverses the approximate energy outputs: liquid hydrogen releases about $142\ \mathrm{kJ\,g^{-1}}$, whereas LPG releases about $50\ \mathrm{kJ\,g^{-1}}$. Therefore (d) is false. Option (A) omits the true storage-mass statement (b), while (B) and (D) include the false statement (d).

Step 3: Conclusion Literally, only statements (a) and (b) are valid, and their combination is absent from the options because statement (c) prints $NaNi_5$ instead of the conventional alloy $LaNi_5$. With that formula corrected, the accepted answer is (C): the first three statements only.

Quick Tip to Solve: Keep fuel mass separate from cylinder mass, and remember the approximate pair: hydrogen 142, LPG 50, in $\mathrm{kJ\,g^{-1}}$.

89. A solid having density of $9 \times 10^3\ \mathrm{kg\,m^{-3}}$ forms face centred cubic crystals of edge length $200\sqrt{2}\ \mathrm{pm}$. What is the molar mass of the solid?

[Avogadro constant $\cong 6 \times 10^{23}\ \mathrm{mol^{-1}}$, $\pi \cong 3$] [4 marks]

(A) $0.0305\ \mathrm{kg\,mol^{-1}}$

(B) $0.4320\ \mathrm{kg\,mol^{-1}}$

(C) $0.0216\ \mathrm{kg\,mol^{-1}}$

(D) $0.0432\ \mathrm{kg\,mol^{-1}}$

Correct Answer: (A) $0.0305\ \mathrm{kg\,mol^{-1}}$

Solution:

Concept:

- The density of a cubic crystal is $\rho = ZM/(N_A a^3)$, where Z is the number of particles per unit cell, M is molar mass, N_A is Avogadro constant and a is the unit-cell edge.
- An FCC unit cell contains $8(1/8) + 6(1/2) = 4$ particles. Density in $\mathrm{kg\,m^{-3}}$ and edge in metres give molar mass in $\mathrm{kg\,mol^{-1}}$.

Step 1: Convert the edge and compute the volume Use the stated edge directly; there is no need to calculate an atomic radius. Since $1\ \mathrm{pm} = 10^{-12}\ \mathrm{m}$,

$$a = 200\sqrt{2} \times 10^{-12} = 2\sqrt{2} \times 10^{-10}\ \mathrm{m}.$$

Cubing the entire expression gives

$$a^3 = (2\sqrt{2})^3 \times 10^{-30} = 16\sqrt{2} \times 10^{-30}\ \mathrm{m^3}.$$

The factor $\sqrt{2}$ must also be cubed; treating it as a simple factor of two would change the result.

Step 2: Use the four particles per FCC cell Rearrange the density equation and substitute the supplied approximate Avogadro constant:

$$\begin{aligned} M &= \frac{\rho N_A a^3}{Z} \\ &= \frac{(9 \times 10^3)(6 \times 10^{23})(16\sqrt{2} \times 10^{-30})}{4} \\ &= 0.0216\sqrt{2}\ \mathrm{kg\,mol^{-1}} \\ &\simeq 0.03055\ \mathrm{kg\,mol^{-1}}. \end{aligned}$$

This rounds to the value in (A). Option (C) loses the remaining $\sqrt{2}$ factor, while (B) and (D) do not equal the FCC density result. The stated value of π is not needed because the calculation uses the cubic cell volume, not a spherical atomic volume.

Step 3: Conclusion The molar mass is approximately $0.0305\ \mathrm{kg\,mol^{-1}}$, or $30.5\ \mathrm{g\,mol^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: For an FCC density problem insert $Z = 4$ and cube the full edge expression before substituting; keep metres and kilograms consistent.

90. Match the ores (column A) with the metals (column B):

(Column A) Ores		(Column B) Metals	
(I)	Siderite	(a)	Zinc
(II)	Kaolinite	(b)	Copper
(III)	Malachite	(c)	Iron
(IV)	Calamine	(d)	Aluminium

[4 marks]

(A) (I)-(c); (II)-(d); (III)-(b); (IV)-(a)

(B) (I)-(a); (II)-(b); (III)-(c); (IV)-(d)

(C) (I)-(b); (II)-(c); (III)-(d); (IV)-(a)

(D) (I)-(c); (II)-(d); (III)-(a); (IV)-(b)

Correct Answer: (A) (I)-(c); (II)-(d); (III)-(b); (IV)-(a)

Solution:

Concept:

- An ore-metal match follows the identity of the metallic element in the ore mineral. Similar-sounding mineral names do not imply the same metal.
- Carbonate and silicate formulae provide a useful check: siderite is an iron carbonate, malachite a basic copper carbonate, calamine a zinc carbonate, and kaolinite an aluminium silicate mineral.

Step 1: Match siderite and kaolinite Siderite has formula $FeCO_3$; its metallic component is iron. Therefore (I) matches (c). Kaolinite is a hydrated aluminium silicate, commonly written $Al_2Si_2O_5(OH)_4$, equivalently $Al_2O_3 \cdot 2\,SiO_2 \cdot 2\,H_2O$. Its metal is aluminium, so (II) matches (d). The silicon in this mineral belongs to the silicate framework and is not one of the four listed metals. These two matches already exclude (B) and (C), which assign zinc or copper to siderite and do not assign aluminium to kaolinite.

Step 2: Match malachite and calamine Malachite is basic copper carbonate, $CuCO_3 \cdot Cu(OH)_2$, also written $Cu_2CO_3(OH)_2$. It is therefore a copper mineral, giving (III)-(b). Calamine is represented in this syllabus as zinc carbonate, $ZnCO_3$, giving (IV)-(a). Both (A) and (D) have the first two matches right, but (D) swaps zinc and copper for the final two minerals. Checking the actual formulae prevents that tempting interchange. No redox calculation is needed: the question asks which metal each ore contains, rather than the processing route used to extract it.

Step 3: Conclusion The ores match as siderite–iron, kaolinite–aluminium, malachite–copper and calamine–zinc. Hence, the correct option is (A).

Quick Tip to Solve: Use the formula as the memory check: $FeCO_3$ is siderite, $ZnCO_3$ is calamine, and the green basic carbonate malachite contains copper.