JEE Main 2019 Paper 1 (B.E./B.Tech) Jan 11 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 11 January 2019, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. Let S = {1, 2, ..., 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randonly chosen subset of S is "nice" is : [4 marks]

(A) $\frac{5}{2^{20}}$

(B) $\frac{7}{2^{20}}$

(C) $\frac{4}{2^{20}}$

(D) $\frac{6}{2^{20}}$

Correct Answer: (A) $\frac{5}{2^{20}}$

Solution:

Concept:

- Every element can independently be included or excluded, giving 2^{20} equally likely subsets of S. Thus the required probability is the number of favourable subsets divided by 2^{20}.
- The complement map $B \mapsto S \setminus B$ is a bijection. Since the sum of all elements of S is $20 \cdot 21/2 = 210$, a subset of sum 203 corresponds to a complement of sum 7.

Step 1: Replace the large sum by its complement Let $C = S \setminus B$. Then

$$\sum_{c \in C} c = 210 - 203 = 7.$$

Every member of C must belong to $\{1, 2, \ldots, 7\}$, and no value may repeat because C is a set. Counting ordered compositions or partitions with repeated terms would therefore overcount.

Step 2: List all distinct-element subsets with sum seven The complete list is

$$\{7\}, \quad \{1, 6\}, \quad \{2, 5\}, \quad \{3, 4\}, \quad \{1, 2, 4\}.$$

With one element there is one possibility. With two elements the smaller member is 1, 2 or 3, giving three possibilities. With three elements the minimum is $1 + 2 + 3 = 6$, and only replacing 3 by 4 reaches 7. Four distinct positive elements have sum at least 10, so no larger subset is possible.

Step 3: Divide by the full sample space There are exactly five complements and therefore five nice subsets. Their probability is

$$P = \frac{5}{2^{20}}.$$

The numerators 7, 4 and 6 in the other options do not equal the exhaustive distinct-subset count; including repeated elements would describe a different sample space.

Step 4: Conclusion The probability is $5/2^{20}$. Hence, the correct option is (A).

Quick Tip to Solve: When a subset sum is close to the total sum, count complements with the small leftover sum; remember that set elements cannot repeat.

2. Let K be the set of all real values of x where the function f(x) = sin |x| – |x| + 2(x – π) cos |x| is not differentiable. Then the set K is equal to : [4 marks]

(A) {0, π}

(B) ϕ (an empty set)

(C) { r }

(D) {0}

Correct Answer: (B) ϕ (an empty set)

Solution:

Concept:

- The absolute-value function changes its formula only at zero. All constituent functions are differentiable for $x \neq 0$, so only the one-sided derivatives at zero need special attention.
- The identity $\cos|x| = \cos x$ holds because cosine is even. A cusp in individual terms can cancel in their sum, so the complete function must be tested.

Step 1: Check smoothness away from zero Write

$$f(x) = \sin|x| - |x| + 2(x-\pi)\cos x.$$

On each open half-line, $|x|$ equals either x or $-x$, so the expression is a combination of smooth elementary functions. In particular, $x = \pi$ is not exceptional: the factor $x - \pi$ is a polynomial factor, not an absolute value or a denominator.

Step 2: Compute the right-hand derivative For $x > 0$,

$$f(x) = \sin x - x + 2(x-\pi)\cos x.$$

Differentiating gives

$$f'(x) = 3\cos x - 1 - 2(x-\pi)\sin x.$$

Its limit as $x \to 0^+$ is $3 - 1 = 2$. The function itself is continuous at zero, with $f(0) = -2\pi$.

Step 3: Compare the left-hand derivative For $x < 0$,

$$f(x) = -\sin x + x + 2(x-\pi)\cos x,$$

and hence

$$f'(x) = 1 + \cos x - 2(x-\pi)\sin x.$$

The limit as $x \to 0^-$ is also 2. The continuous pieces therefore join with the same derivative. Neither zero nor any other real point belongs to the nondifferentiability set; options containing zero or π are excluded.

Step 4: Conclusion The function is differentiable on all of $\mathbb{R}$, so $K = \varnothing$. Hence, the correct option is (B).

Quick Tip to Solve: Do not declare a cusp just because an absolute value appears; compare derivatives of the entire expression, since cusp terms may cancel.

3. All x satisfying the inequality $(\cot^{-1} x)^2 - 7(\cot^{-1} x) + 10 > 0$, lie in the interval : [4 marks]

(A) $(\cot 2, \infty)$

(B) $(-\infty, \cot 5) \cup (\cot 2, \infty)$

(C) $(\cot 5, \cot 4)$

(D) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$

Correct Answer: (A) $(\cot 2, \infty)$

Solution:

Concept:

- For the principal inverse cotangent, $t = \cot^{-1} x$ lies in $(0, \pi)$. This range restriction must be imposed before converting an inequality back to x.
- The polynomial $t^2 - 7t + 10$ factors as $(t-2)(t-5)$. Also, $\cot t$ is strictly decreasing on $(0, \pi)$, since its derivative is $-\csc^2 t$.

Step 1: Solve the quadratic inequality in the angle Set $t = \cot^{-1} x$. The inequality becomes

$$(t-2)(t-5) > 0,$$

giving $t < 2$ or $t > 5$. The inequality is strict, so the roots 2 and 5 themselves are excluded. These are real radian values, not angles measured in degrees.

Step 2: Intersect with the principal-value range Since $0 < t < \pi < 5$, the branch $t > 5$ is impossible. The only admissible angles are therefore

$$0 < t < 2.$$

Keeping the discarded branch would introduce extraneous real values of x. Periodicity of cotangent does not enlarge the principal range of its inverse.

Step 3: Use monotonicity to recover the real interval Cotangent decreases continuously from $+\infty$ at 0^+ to $\cot 2$ at 2. Thus

$$x = \cot t > \cot 2.$$

Conversely, each real $x > \cot 2$ has its principal inverse cotangent between zero and 2, so this condition is sufficient as well as necessary. The other displayed unions or intervals introduce unnecessary ranges or omit valid values; the exact solution set is the single interval in (A).

Step 4: Conclusion The solution set is $(\cot 2, \infty)$. Hence, the correct option is (A).

Quick Tip to Solve: Solve in the inverse-trigonometric angle first, intersect with its principal range, and only then use the monotonicity of the original function.

4. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the othertwo vertices are the points of intersection of the parabola and y-axis, is 250 sq. units, then a value of 'a' is : [4 marks]

(A) $5\sqrt{5}$

(B) $(10)^{2/3}$

(C) $5\left(2^{1/3}\right)$

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- The equation $y^2 = -4(x - a^2)$ has vertex $(a^2, 0)$ and a horizontal axis. Its intersections with the y-axis follow by setting $x = 0$.
- A triangle has area $\frac{1}{2} \times$ base $\times$ perpendicular height. Absolute values are needed for geometric lengths even if the parameter a is negative.

Step 1: Locate the three vertices The vertex is $V = (a^2, 0)$. On the y-axis, the parabola equation gives

$$y^2 = 4a^2, \qquad y = \pm 2|a|.$$

Thus the other vertices may be taken as $P = (0, 2|a|)$ and $Q = (0, -2|a|)$. These are distinct here because the specified area is nonzero.

Step 2: Express the base and height in terms of the parameter The segment PQ lies on the y-axis and has length $4|a|$. The perpendicular distance of V from that axis is a^2, not $|a|$. Therefore

$$\text{Area} = \frac{1}{2}(4|a|)a^2 = 2|a|^3.$$

The parabola opens to the left, but its direction of opening does not alter these positive distances.

Step 3: Solve the area condition and compare the options Using the given area,

$$2|a|^3 = 250, \qquad |a|^3 = 125, \qquad |a| = 5.$$

Both $a = 5$ and $a = -5$ produce the same parabola because its equation contains a^2. The question asks for a value, and 5 is the listed one. The other positive choices have cubes different from 125 and hence cannot give the stated area.

Step 4: Conclusion A permissible value is $a = 5$. Hence, the correct option is (D).

Quick Tip to Solve: Use the two axis-intercepts as the base; the vertex-to-axis distance gives the height directly without a determinant.

5. Let a function f : (0, ∞) $\to$ (0, ∞) be defined by f(x) = $\left|1-\frac{1}{x}\right|$. Then f is : [4 marks]

(A) not injective but it is surjective

(B) neiter injective nor surjective

(C) injective only

(D) both injective as well as surjective

Correct Answer: No option is formally valid: the printed rule gives $f(1) = 0$, outside its stated codomain. The positive-target preimage test would give (A), whereas the supplied key lists (B).

Solution:

Concept:

- Injectivity requires distinct inputs to have distinct outputs. Surjectivity onto the positive reals means that each target $y > 0$ is attained by some positive input.
- For $0 < x < 1$, the formula is $f(x) = 1/x - 1$; for $x \geq 1$, it is $f(x) = 1 - 1/x$. These branches allow direct tests of both properties.

Step 1: Exhibit repeated output values Take the distinct positive inputs $x = 2$ and $x = 2/3$. Then

$$\left|1-\frac{1}{2}\right| = \frac{1}{2}, \qquad \left|1-\frac{1}{2/3}\right| = \frac{1}{2}.$$

Consequently the displayed rule is not injective. This explicit pair excludes both options that assert injectivity; monotonicity on one branch would not prove injectivity on the whole domain.

Step 2: Find a preimage of every positive target For an arbitrary $y > 0$, choose

$$x = \frac{1}{1+y}.$$

This lies strictly between zero and one, and substitution yields

$$\left|1-\frac{1}{x}\right| = |1-(1+y)| = y.$$

Thus every strictly positive target has a preimage, which is the surjectivity test intended by the options. In particular, large targets are attained near zero, so the rule is not restricted to outputs less than one.

Step 3: Identify the printed codomain defect At $x = 1$, however, the formula gives zero, whereas the printed codomain $(0, \infty)$ excludes zero. Strictly, this declaration does not define a function into that codomain. This defect does not change the two computations above: the displayed rule has range $[0, \infty)$, is not injective, and attains every positive target. Option (A) records the positive-target classification if the declaration is repaired; a formally valid declaration would use codomain $[0, \infty)$ or exclude the input 1.

Step 4: Conclusion The printed declaration is not a function into $(0, \infty)$, because $f(1) = 0$. Hence no option is formally correct as written. The supplied answer key lists (B), but every positive target has the preimage $1/(1 + y)$, so its non-surjectivity claim is not justified. Under a valid codomain $[0, \infty)$, the classification would be (A): surjective and not injective.

Quick Tip to Solve: Prove noninjectivity with two inputs sharing one output, and prove surjectivity by constructing an input for an arbitrary target; also check that every output lies in the stated codomain.

6. The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is : [4 marks]

(A) $\frac{8}{3}$

(B) $\frac{14}{3}$

(C) $\frac{187}{24}$

(D) $\frac{37}{24}$

Correct Answer: (D) $\frac{37}{24}$

Solution:

Concept:

- For $y = x^2 + 1$, the derivative $dy/dx = 2x$ gives the slope of the tangent. Area is computed by integrating the upper curve minus the appropriate lower boundary.
- In the first quadrant the tangent contributes to the boundary only after crossing the positive x-axis. Before that crossing the lower boundary is the axis itself.

Step 1: Find the tangent and its intercept The tangent slope at $(2, 5)$ is $2(2) = 4$. Its equation is

$$y - 5 = 4(x - 2), \qquad y = 4x - 3.$$

Setting $y = 0$ gives the x-intercept $x = 3/4$. The parabola meets the positive y-axis at $(0, 1)$, completing the boundary with the coordinate axes.

Step 2: Split the first-quadrant region at the intercept For $0 \le x \le 3/4$, the tangent is below the x-axis, so the region lies between $y = 0$ and $y = x^2 + 1$. For $3/4 \le x \le 2$, it lies between the tangent and parabola. Their difference is

$$x^2 + 1 - (4x - 3) = (x - 2)^2 \ge 0.$$

Thus no sign reversal or absolute-value integration is required on either part.

Step 3: Evaluate the area Equivalently subtract the area below the positive part of the tangent from the area below the parabola:

$$A = \int_0^2 (x^2 + 1)\, dx - \int_{3/4}^2 (4x - 3)\, dx.$$

The first integral is $8/3 + 2 = 14/3$. The second is a triangle of base $2 - 3/4 = 5/4$ and height 5, so it equals $25/8$. Therefore

$$A = \frac{14}{3} - \frac{25}{8} = \frac{112 - 75}{24} = \frac{37}{24}.$$

Integrating the difference from zero would include a portion below the axis; taking only $14/3$ would fail to subtract the tangent triangle.

Step 4: Conclusion The required area is $37/24$ square units. Hence, the correct option is (D).

Quick Tip to Solve: For a first-quadrant area involving a tangent, locate its axis-intercept before choosing the lower boundary of integration.

7. Let α and β be the roots of the quadratic equation $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$ $(0 < \theta < 45^\circ)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to : [4 marks]

(A) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$
(B) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$
(C) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$
(D) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

Correct Answer: (B) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

Solution:

Concept:

- Factoring the quadratic identifies its roots without the quadratic formula. A geometric series $\sum_{n=0}^{\infty} r^n$ equals $1/(1-r)$ only for $|r| < 1$.
- The given range $0 < \theta < 45^\circ$ ensures that both $\sin\theta$ and $\cos\theta$ are positive and strictly less than one. It also determines the order of the two roots.

Step 1: Factor the quadratic and identify the ordered roots The polynomial factors exactly as

$$x^2\sin\theta - x(1+\sin\theta\cos\theta) + \cos\theta = (x - \cos\theta)(x\sin\theta - 1).$$

Hence the roots are $\cos\theta$ and $1/\sin\theta$. Since $\cos\theta < 1 < 1/\sin\theta$, their required ordering is

$$\alpha = \cos\theta, \qquad \beta = \frac{1}{\sin\theta}.$$

Swapping the roots would create a divergent first geometric series.

Step 2: Rewrite and check both geometric ratios The summand is

$$\alpha^n + \frac{(-1)^n}{\beta^n} = (\cos\theta)^n + (-\sin\theta)^n.$$

Both ratios have absolute value less than one in the prescribed open interval. Therefore each series converges absolutely, and their sums may be added. The lower limit $n = 0$ is essential: each component starts with 1.

Step 3: Sum the two series and distinguish the signs Applying the geometric-series formula separately gives

$$\sum_{n=0}^{\infty}(\cos\theta)^n = \frac{1}{1-\cos\theta}, \qquad \sum_{n=0}^{\infty}(-\sin\theta)^n = \frac{1}{1+\sin\theta}.$$

Both contributions are added, despite the alternating signs inside the second series. Thus the alternatives with a minus between the fractions or with the denominators $1+\cos\theta$ and $1-\sin\theta$ use incorrect ratios or signs.

Step 4: Conclusion The sum is $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$. Hence, the correct option is (B).

Quick Tip to Solve: Factor the quadratic first, use the angle range to order its roots, and check the modulus of each geometric ratio before summing.

8. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it? [4 marks]

(A) $\left(4\sqrt{2}, 2\sqrt{3}\right)$
(B) $\left(4\sqrt{3}, 2\sqrt{3}\right)$
(C) $\left(4\sqrt{3}, 2\sqrt{2}\right)$
(D) $\left(4\sqrt{2}, 2\sqrt{2}\right)$

Correct Answer: (C) $\left(4\sqrt{3}, 2\sqrt{2}\right)$

Solution:

Concept:

- For the ellipse $x^2/a^2 + y^2/b^2 = 1$, with major axis along the x-axis, $a > b > 0$, the focal distance is $2c$, where $c^2 = a^2 - b^2$.
- The minor axis has length $2b$, and the latus rectum has length $2b^2/a$. A point lies on the ellipse exactly when its coordinates satisfy its equation.

Step 1: Translate the focal condition The distance between the foci equals the length of the minor axis, so $2c = 2b$, or $c = b$. Substitute this in the focal relation to obtain

$$a^2 = b^2 + c^2 = 2b^2.$$

This relation fixes the shape of the ellipse, but its size still comes from the given latus rectum.

Step 2: Find the semiaxes Use the latus-rectum length, keeping in mind that the stated length is the whole latus rectum:

$$\frac{2b^2}{a} = 8.$$

Since $b^2 = a^2/2$, this becomes $a = 8$. Thus $a^2 = 64$ and $b^2 = 32$, giving

$$\frac{x^2}{64} + \frac{y^2}{32} = 1.$$

Step 3: Test the printed points For option (C), $x^2 = 48$ and $y^2 = 8$, so the left side is $48/64 + 8/32 = 3/4 + 1/4 = 1$. For (A), (B) and (D), the respective values are $7/8$, $9/8$ and $3/4$; none equals 1. Thus being inside or outside the ellipse is insufficient: the required point must lie on its boundary.

Step 4: Conclusion The point $\left(4\sqrt{3}, 2\sqrt{2}\right)$ lies on the ellipse. Hence, the correct option is (C).

Quick Tip to Solve: First use the focal condition to relate the semiaxes, then use the whole latus-rectum length to determine their scale.

9. Let $\sqrt{3}\hat{i}+\hat{j}, \hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is : [4 marks]

(A) 4

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Concept:

- For two vectors of equal magnitude, their sum points along the internal angle bisector. The vectors here have magnitudes $\sqrt{3+1}=2$.
- The perpendicular distance of (x_0, y_0) from $ax+by+c=0$ is $|ax_0+by_0+c|/\sqrt{a^2+b^2}$.

Step 1: Identify the acute-angle bisector The directions of OA and OB are $(\sqrt{3},1)$ and $(1,\sqrt{3})$. Their dot product is $2\sqrt{3}>0$, so their smaller angle is acute. As both vectors have length 2, their sum $(1+\sqrt{3},1+\sqrt{3})$ points along the internal bisector. Its equation through the origin is therefore

$$y=x.$$

The other bisector, $y=-x$, belongs to the supplementary angle and is not the requested line.

Step 2: Apply the distance condition The coordinates of C are $(\beta,1-\beta)$. Its distance from $x-y=0$ is

$$\frac{|\beta-(1-\beta)|}{\sqrt{2}}=\frac{|2\beta-1|}{\sqrt{2}}.$$

Equating this to the given distance $3/\sqrt{2}$ gives $|2\beta-1|=3$. The absolute value must be retained because C may lie on either side of the bisector.

Step 3: Find both values and compare the choices Solve the two linear equations:

$$2\beta-1=3 \quad \text{or} \quad 2\beta-1=-3.$$

They give $\beta=2$ or $\beta=-1$. Both produce the required perpendicular distance, so the sum is $2+(-1)=1$. The numbers 4, 2 and 3 are not the sum of the complete pair of solutions; choosing only one signed distance would miss a valid value.

Step 4: Conclusion The sum of all possible values of β is 1. Hence, the correct option is (B).

Quick Tip to Solve: When the two position vectors have equal lengths, add them to find the internal bisector; then retain both signs in a distance equation.

10. The number of functions f from {1, 2, 3, ..., 20} onto {1, 2, 3, ..., 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is : [4 marks]

(A) $6^5 \times (15)!$

(B) $5^6 \times 15$

(C) $(15)! \times 6!$

(D) $5! \times 6!$

Correct Answer: (C) $(15)! \times 6!$

Solution:

Concept:

- An onto function between two finite sets of the same size is bijective. Thus every output must occur exactly once in the required function.
- Assigning distinct images to r specified inputs from s available outputs can be done in ${}^sP_r = s!/(s-r)!$ ways; remaining distinct outputs can then be permuted.

Step 1: Count the restricted inputs and allowed images The inputs divisible by 4 are $4, 8, 12, 16, 20$, so there are five restricted inputs. Their images must belong to the six-element set $\{3, 6, 9, 12, 15, 18\}$. Because the function is bijective, these five inputs must receive five distinct members of that set. No two of them can share an image.

Step 2: Assign images to these five inputs Choose their images in the fixed input order $4, 8, 12, 16, 20$. There are respectively $6, 5, 4, 3, 2$ choices. Hence the number of assignments is

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = {}^6P_5 = 6!.$$

The one unused multiple of 3 remains available to the unrestricted inputs; the condition does not prohibit that.

Step 3: Complete the bijection After the restricted images have been fixed, 15 inputs and 15 outputs remain. Any bijection between them is allowed, giving $15!$ completions for each earlier assignment. The multiplication principle gives

$$N = 6!\,15!.$$

Option (A) incorrectly allows repeated images for the restricted inputs. Option (D) uses $5!$ instead of arranging all 15 remaining outputs, and option (B) likewise does not count bijections of those remaining inputs.

Step 4: Conclusion The number of functions is $15! \times 6!$. Hence, the correct option is (C).

Quick Tip to Solve: For an onto map between equally sized finite sets, replace function counting by permutation counting before imposing the restrictions.

11. If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is : [4 marks]

(A) 5x + 3y – 11 = 0

(B) 5x – 3y + 1 = 0

(C) 3x – 5y + 7 = 0

(D) 3x + 5y – 13 = 0

Correct Answer: (B) 5x – 3y + 1 = 0

Solution:

Concept:

- The diagonals of a parallelogram bisect each other. In the printed cyclic order ABDC, A and D are opposite vertices, while B and C are opposite vertices.
- A line through (x_1, y_1) with direction (u, v) satisfies $v(x - x_1) - u(y - y_1) = 0$.

Step 1: Respect the printed vertex order The parallelogram is named ABDC, so its sides run A to B to D to C to A. Thus AD is a diagonal, and BC is the other diagonal. Treating the name as ABCD would identify the opposite vertices incorrectly. Since the midpoint of AD equals that of BC,

$$A + D = B + C.$$

Step 2: Find the fourth vertex and diagonal direction Use the given coordinates to calculate

$$D = B + C - A = (3 + 2 - 1,\ 4 + 5 - 2) = (4, 7).$$

The direction from $A(1, 2)$ to $D(4, 7)$ is therefore $(3, 5)$. The slope is $5/3$, and the point-slope equation of AD is

$$y - 2 = \frac{5}{3}(x - 1).$$

Step 3: Simplify and check the choices Multiplying by 3 and collecting terms gives

$$3y - 6 = 5x - 5 \quad \Longrightarrow \quad 5x - 3y + 1 = 0.$$

Both A and D satisfy this equation. Options (A) and (D) also pass through A, so testing A alone does not determine the diagonal; their slopes have the wrong sign. Option (C) passes through A but has slope $3/5$, the reciprocal of the required slope. Checking D distinguishes all four alternatives.

Step 4: Conclusion The equation of diagonal AD is $5x - 3y + 1 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: Read a parallelogram name in cyclic order: its first and third vertices are opposite, even when the lettering is unusual.

12. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is : [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{m+n}{6mn}$

(D) 1

Correct Answer: (B) $\frac{1}{4}$

Solution:

Concept:

- For every positive real number u, $(u-1)^2 \geq 0$ implies $1+u^2 \geq 2u$, with equality exactly at $u=1$.
- The variables x and y are independent and positive. Therefore the two positive factors in the given expression can attain their individual upper bounds simultaneously.

Step 1: Separate the two positive factors Put $u = x^m$ and $v = y^n$. Both are positive because x and y are positive and m and n are positive integers. The expression becomes

$$E = \frac{u}{1+u^2}\,\frac{v}{1+v^2}.$$

This separates the dependence on the two original variables and avoids differentiating their powers.

Step 2: Bound each factor From $1+u^2 \geq 2u$, division by the positive quantity $2(1+u^2)$ gives $u/(1+u^2) \leq 1/2$. Likewise $v/(1+v^2) \leq 1/2$. Multiplying these nonnegative inequalities yields

$$E \leq \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}.$$

Step 3: Verify that the bound is attained Equality requires $u = v = 1$. Choose $x = y = 1$; then $x^m = y^n = 1$ for every allowed m and n, so the expression actually equals $1/4$. Thus this is a maximum, not merely an unattained upper bound. Options (A) and (D) exceed the bound. The expression in (C) varies with m and n and is not the general maximum; for example, at $m = n = 1$ it is $1/3$, which exceeds the proven bound.

Step 4: Conclusion The maximum value is $\frac{1}{4}$, attained at $x = y = 1$. Hence, the correct option is (B).

Quick Tip to Solve: For a positive variable, recognize $u/(1+u^2) \leq 1/2$; check equality in each factor before calling the product bound a maximum.

13. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x(\tan^5 x+\cot^5 x)}$ equals : [4 marks]

(A) $\frac{\pi}{40}$

(B) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

(C) $\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(D) $\frac{1}{5}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

Correct Answer: (C) $\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

Solution:

Concept:

- The substitution $t=\tan x$ gives $dt=(1+t^2)\,dx$ and $\sin 2x=2t/(1+t^2)$, so $dx/\sin 2x=dt/(2t)$.
- The derivative $d(\tan^{-1} z)/dz=1/(1+z^2)$ suggests a further power substitution when the numerator is the derivative of that power.

Step 1: Convert the trigonometric integrand On $[\pi/6,\pi/4]$, $t=\tan x$ is positive and increases from $1/\sqrt{3}$ to 1. Also $\tan^5 x+\cot^5 x=t^5+t^{-5}=(t^{10}+1)/t^5$. Hence

$$I=\frac{1}{2}\int_{1/\sqrt{3}}^{1}\frac{t^4}{1+t^{10}}\,dt.$$

The factor $1/2$ comes from $\sin 2x$ and must not be dropped.

Step 2: Use the fifth power as a new variable Set $z=t^5$, so $dz=5t^4\,dt$. At the lower endpoint, $z=(1/\sqrt{3})^5=1/(9\sqrt{3})$; at the upper endpoint, z is 1. Therefore

$$I=\frac{1}{10}\int_{1/(9\sqrt{3})}^{1}\frac{dz}{1+z^2}.$$

All substitutions are monotone on this interval, so the order of the limits is unchanged.

Step 3: Evaluate and identify the option Integrating gives

$$I=\frac{1}{10}\left[\tan^{-1} z\right]_{1/(9\sqrt{3})}^{1}=\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\frac{1}{9\sqrt{3}}\right).$$

Option (A) keeps only the upper endpoint contribution and omits the lower one. Option (B) omits the upper endpoint and has the wrong prefactor. Option (D) uses the wrong lower power and a prefactor twice as large. The resulting positive value is smaller than $\pi/40$, as the subtraction also confirms.

Step 4: Conclusion The integral equals $\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\frac{1}{9\sqrt{3}}\right)$. Hence, the correct option is (C).

Quick Tip to Solve: After setting $t=\tan x$, combine reciprocal powers first; the factor $t^4\,dt$ then points directly to $z=t^5$.

14. A circle cuts a chord of length 4a on the x-axis and passes through a point on the y-axis, distant 2b from the origin. Then the locus of the centre of this circle, is : [4 marks]

(A) an ellipse

(B) a parabola

(C) a hyperbola

(D) a straight line

Correct Answer: (B) a parabola

Solution:

Concept:

- The perpendicular from a circle centre to a chord bisects the chord. A chord of length $4a$ has half-length $2a$.
- If the centre is (h, k), its distance to the x-axis is $|k|$. The radius R therefore satisfies $R^2 = k^2 + 4a^2$.

Step 1: Express the radius using the chord Let the variable centre be (h, k). The chord lies on the x-axis, so its midpoint is the perpendicular foot $(h, 0)$, and its endpoints have x-coordinates $h - 2a$ and $h + 2a$. A right triangle from the centre to either endpoint gives

$$R^2 = k^2 + (2a)^2 = k^2 + 4a^2.$$

This relation uses the prescribed chord length for every circle in the family.

Step 2: Use the fixed point on the y-axis Take the specified point to be $P(0, 2b)$. Since it lies on the circle,

$$R^2 = h^2 + (k - 2b)^2.$$

Equating the two radius expressions and expanding yields

$$h^2 + k^2 - 4bk + 4b^2 = k^2 + 4a^2,$$

and therefore

$$h^2 = 4b\left(k - b + \frac{a^2}{b}\right).$$

Here b is positive, as $2b$ is the stated nonzero distance.

Step 3: Recognize the locus The last equation has the form $h^2 = 4b(k - k_0)$, with $k_0 = b - a^2/b$, so it is a parabola with vertical axis. Choosing the fixed point on the negative y-axis instead reflects the equation and still gives a parabola. Only one squared centre coordinate survives after the radius is eliminated. Thus the locus is neither an ellipse nor a hyperbola, and the nonzero quadratic term prevents it from being a straight line.

Step 4: Conclusion The locus of the centre is a parabola. Hence, the correct option is (B).

Quick Tip to Solve: For a circle with a chord on a coordinate axis, express the radius using the half-chord before imposing passage through a fixed point.

15. If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is : [4 marks]

(A) 2 : 1

(B) 4 : 1

(C) 1 : 3

(D) 3 : 1

Correct Answer: (D) 3 : 1

Solution:

Concept:

- An arithmetic progression has general term $T_n = a + (n-1)d$, where a is its first term and d its common difference.
- Once one term is known, another term is found by counting the intervening common differences: $T_n = T_r + (n-r)d$.

Step 1: Use the zero term The nineteenth term is zero, so $a+18d = 0$, giving $a = -18d$. Consequently the progression can be described more conveniently relative to its zero term:

$$T_n = (n-19)d.$$

The non-zero qualification matters: if $d = 0$, the zero nineteenth term would force every term to be zero, contrary to the question. Thus $d \neq 0$, and cancellation of d in the desired ratio is legitimate.

Step 2: Evaluate both terms and compare the options The forty-ninth term lies thirty positions after the zero term, whereas the twenty-ninth term lies ten positions after it. Therefore

$$T_{49} = 30d, \qquad T_{29} = 10d.$$

Hence

$$T_{49} : T_{29} = 30d : 10d = 3 : 1.$$

This holds even for a negative common difference, because both terms change sign together. Options (A) and (B) give ratios 2 and 4, neither of which equals 30/10; option (C) reverses the required ratio. Counting the term numbers themselves as 49:29 would also ignore the specified zero term.

Step 3: Conclusion The required ratio is $3 : 1$. Hence, the correct option is (D).

Quick Tip to Solve: When an A.P. has a zero term, measure each requested index from that zero index before taking the ratio.

16. The solution of the differential equation,

$\frac{dy}{dx}$ **= (x – y)^2, when y(1) = 1, is :** [4 marks]

(A) $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x + y - 2$

(B) $\log_e\left|\frac{2-x}{2-y}\right| = x - y$

(C) $\log_e\left|\frac{2-y}{2-x}\right| = 2(y - 1)$

(D) $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x - 1)$

Correct Answer: (D) $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x - 1)$

Solution:

Concept:

- For an equation involving $x - y$, set $v = x - y$; then $dy/dx = 1 - dv/dx$.
- The separable integral is $\int dv/(1 - v^2) = \frac{1}{2}\log|(1 + v)/(1 - v)|$, valid away from $v = \pm 1$.

Step 1: Reduce to a separable equation Write $v = x - y$. The given equation becomes

$$1 - \frac{dv}{dx} = v^2, \qquad \frac{dv}{1 - v^2} = dx.$$

At the initial point $x = 1, y = 1$, we have $v = 0$; thus division by $1 - v^2$ is valid in a neighbourhood of the initial point. Using partial fractions and integrating gives

$$\frac{1}{2}\log\left|\frac{1 + v}{1 - v}\right| = x + C.$$

Substitution of $v = 0, x = 1$ yields $C = -1$.

Step 2: Restore the variables and check the equation Multiply by two and substitute $v = x - y$:

$$\log\left|\frac{1 + x - y}{1 - x + y}\right| = 2(x - 1).$$

Reversing the fraction changes the sign of its logarithm, so this is precisely option (D). Equivalently $v = \tanh(x-1)$, and $y = x - \tanh(x-1)$. Differentiating gives $y\prime = 1 - \operatorname{sech}^2(x-1) = \tanh^2(x-1) = (x - y)^2$, and $y(1) = 1$, confirming both requirements. The equilibrium values $v = \pm 1$ cannot satisfy the initial condition. Option (A) has the opposite logarithm sign and a different right-hand side; the expressions in (B) and (C) do not give the separated relation just derived.

Step 3: Conclusion

$$-\log\left|\frac{1 - x + y}{1 + x - y}\right| = 2(x - 1).$$

Hence, the correct option is (D).

Quick Tip to Solve: For a differential equation in x minus y, differentiate the substitution carefully: its derivative is one minus y prime.

17. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ **= (a + b + c) (x + a + b + c)^2, x $\neq$ 0, then x is equal to :** [4 marks]

(A) –2(a + b + c)

(B) 2(a + b + c)

(C) abc

(D) –(a + b + c)

Correct Answer: (A) –2(a + b + c)

Solution:

Concept:

- Put $s = a+b+c$. The diagonal entries become $2a-s, 2b-s, 2c-s$, so the matrix is a scalar matrix plus a rank-one matrix.
- For a three-dimensional matrix $\lambda I + uv^T$, its determinant is $\lambda^3 + \lambda^2 v^T u$; this polynomial identity also holds when $\lambda = 0$.

Step 1: Evaluate the determinant Let $u = (2a, 2b, 2c)^T$ and $v = (1, 1, 1)^T$. Each row of uv^T is constant, and subtracting sI gives exactly the printed matrix. Since $v^T u = 2s$, its determinant is

$$D = (-s)^3 + (-s)^2(2s) = s^3.$$

This identity can also be obtained by determinant multilinearity: terms taking two columns from the rank-one matrix vanish because those columns are equal, leaving the scalar determinant and three single-column contributions.

Step 2: Compare with the stated expression The equation is therefore

$$s^3 = s(x+s)^2.$$

In the nondegenerate case $s \neq 0$, division by s gives $(x+s)^2 = s^2$, hence

$$x(x+2s) = 0.$$

The given condition $x \neq 0$ selects $x = -2s = -2(a+b+c)$, which is option (A). Options (B), (C) and (D) are not the resulting nonzero root for general values of the parameters. There is a degeneracy in the literal wording: when $s = 0$, both sides of the original equation vanish for every nonzero x, so no unique value is determined in that special case. The listed answer is the usual nondegenerate result, with this restriction made explicit.

Step 3: Conclusion For $a+b+c \neq 0$, $x = -2(a+b+c)$. Hence, the correct option is (A). If $a+b+c = 0$, the printed equation does not determine a unique nonzero x.

Quick Tip to Solve: Before cancelling a parameter from a determinant equation, check its zero case separately; rank-one structure often makes the determinant immediate.

18. If $\int \frac{x+1}{\sqrt{2x-1}}\,dx$ = f(x) $\sqrt{2x-1}$ + C, where C is a constant of integration, then f(x) is equal to :

[4 marks]

(A) $\frac{2}{3}$ (x − 4)

(B) $\frac{1}{3}$ (x + 4)

(C) $\frac{1}{3}$ (x + 1)

(D) $\frac{2}{3}$ (x + 2)

Correct Answer: (B) $\frac{1}{3}$ (x + 4)

Solution:

Concept:

- For an integrand containing $\sqrt{2x-1}$, the substitution $t=\sqrt{2x-1}$ removes the radical.
- The real-valued integrand is defined for $x > 1/2$. An antiderivative can be checked by differentiation using the product rule.

Step 1: Transform and integrate Set $t=\sqrt{2x-1}$, so $t^2=2x-1$, $x=(t^2+1)/2$, and $dx=t\,dt$. Then

$$\int \frac{x+1}{\sqrt{2x-1}}\,dx=\frac{1}{2}\int (t^2+3)\,dt=\frac{t^3}{6}+\frac{3t}{2}+C.$$

The factor from dx cancels the radical in the denominator. Retaining that differential factor is essential; omitting it would produce the wrong coefficients.

Step 2: Factor the radical and verify Factor out t and return to x:

$$\frac{t^3}{6}+\frac{3t}{2}=t\frac{t^2+9}{6}=\sqrt{2x-1}\frac{x+4}{3}.$$

Thus the coefficient in the requested form is $f(x)=(x+4)/3$. As a check,

$$\frac{d}{dx}\left(\frac{x+4}{3}\sqrt{2x-1}\right)=\frac{\sqrt{2x-1}}{3}+\frac{x+4}{3\sqrt{2x-1}}=\frac{x+1}{\sqrt{2x-1}}.$$

The derivative agrees exactly with the printed integrand. Options (A), (C) and (D) have different coefficients or constant terms and do not pass this product-rule check. The arbitrary additive constant remains in C, rather than being inserted into the displayed linear factor.

Step 3: Conclusion $f(x)=\frac{x+4}{3}$. Hence, the correct option is (B).

Quick Tip to Solve: After integrating, factor out the radical specified in the question, then differentiate the proposed product to check the coefficients.

19. $\lim_{x\to 0} \frac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$ **is equal to :** [4 marks]

(A) 0

(B) 4

(C) 1

(D) 2

Correct Answer: (C) 1

Solution:

Concept:

- The standard small-angle limits in radians are $\sin t/t \to 1$ and $\cos t \to 1$ as $t \to 0$.
- A product or quotient of factors with finite limits can be evaluated factor by factor when its limiting denominator is nonzero.

Step 1: Evaluate the numerator For nonzero x sufficiently near zero, write the cotangent as cosine divided by sine:

$$x\cot(4x) = \frac{1}{4}\frac{4x}{\sin(4x)}\cos(4x).$$

Both dimensionless trigonometric factors tend to one, giving

$$\lim_{x\to 0} x\cot(4x) = \frac{1}{4}.$$

Direct substitution into cotangent is not valid because cotangent itself diverges; multiplication by x removes that divergence.

Step 2: Evaluate the denominator and form the ratio The squared sine and squared cotangent must be considered together:

$$\sin^2 x\cot^2(2x) = \left(\frac{\sin x}{x}\right)^2 (x\cot(2x))^2.$$

Here

$$x\cot(2x) = \frac{1}{2}\frac{2x}{\sin(2x)}\cos(2x) \longrightarrow \frac{1}{2}.$$

Thus the denominator tends to $1 \times (1/2)^2 = 1/4$, which is nonzero. The required limit is

$$(1/4)/(1/4) = 1.$$

Both one-sided limits agree, because all the standard factors used have the same limit from either side. Option (A) incorrectly treats the sine factor alone as decisive; (B) and (D) lose factors associated with the doubled or quadrupled angle.

Step 3: Conclusion The limit is 1. Hence, the correct option is (C).

Quick Tip to Solve: Combine a vanishing sine with a divergent cotangent before taking limits, and keep every angle multiplier inside the standard sine-over-angle ratio.

20. Let z be a complex number such that |z| + z = 3 + i (where i = $\sqrt{-1}$). Then |z| is equal to : [4 marks]

(A) $\frac{\sqrt{34}}{3}$

(B) $\frac{5}{3}$

(C) $\frac{5}{4}$

(D) $\frac{\sqrt{41}}{4}$

Correct Answer: (B) $\frac{5}{3}$

Solution:

Concept:

- For $z = u + iv$, its modulus $r = |z| = \sqrt{u^2 + v^2}$ is real and nonnegative.
- Equality of complex numbers requires equality of their real and imaginary parts separately.

Step 1: Compare real and imaginary parts Let $r = |z|$ and write $z = u + iv$, with real u, v. The condition becomes

$$r + u + iv = 3 + i.$$

Its imaginary part gives $v = 1$, and its real part gives $u = 3 - r$. Since the modulus has no imaginary part, it must not be combined with iv. Therefore

$$z = (3 - r) + i, \qquad r^2 = (3 - r)^2 + 1.$$

This expresses the modulus in terms of itself using its defining relation, with no approximation.

Step 2: Solve and check the modulus Expand and cancel the quadratic terms:

$$r^2 = 9 - 6r + r^2 + 1, \qquad 6r = 10, \qquad r = \frac{5}{3}.$$

The resulting complex number is $z = 4/3 + i$. Its modulus is indeed

$$\sqrt{(4/3)^2 + 1} = \sqrt{25/9} = \frac{5}{3},$$

and $|z| + z = 5/3 + 4/3 + i = 3 + i$, confirming the original condition. The positive root is required because a modulus cannot be negative. The other three listed values do not satisfy the necessary linear relation $6r = 10$, so they are rejected without guessing the real and imaginary parts. Merely taking the modulus of the right side would give $\sqrt{10}$, which is $||z| + z|$, not the requested $|z|$.

Step 3: Conclusion $|z| = \frac{5}{3}$. Hence, the correct option is (B).

Quick Tip to Solve: When a complex equation contains its own modulus, name that modulus r and compare real and imaginary parts before squaring.

21. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is : [4 marks]

(A) $\frac{13}{6}$

(B) 2

(C) $\frac{13}{12}$

(D) $\frac{13}{8}$

Correct Answer: (C) $\frac{13}{12}$

Solution:

Concept:

- For a standard hyperbola, the conjugate axis has length $2b$, the focal separation is $2c$, and $c^2 = a^2 + b^2$.
- The eccentricity is $e = c/a > 1$; a, b, and c are semi-axis and centre-to-focus distances, not the full given lengths.

Step 1: Convert the lengths into parameters The conjugate axis length is five, so

$$2b = 5, \qquad b = \frac{5}{2}.$$

The distance between the two foci is thirteen, hence

$$2c = 13, \qquad c = \frac{13}{2}.$$

Both quantities must be halved because the standard hyperbola parameters are measured from the centre. The orientation of the transverse axis does not affect these length relations or the eccentricity.

Step 2: Find the transverse semi-axis and eccentricity For a hyperbola the focal relation has a plus sign:

$$a^2 = c^2 - b^2 = \left(\frac{13}{2}\right)^2 - \left(\frac{5}{2}\right)^2 = \frac{169 - 25}{4} = 36.$$

Taking the positive length gives $a = 6$. Thus

$$e = \frac{c}{a} = \frac{13/2}{6} = \frac{13}{12}.$$

This exceeds one, as a hyperbola requires. Option (A) uses the full focal separation in the numerator while retaining the semi-transverse axis below it. Option (B) is not the ratio of the calculated lengths, and option (D) would require a different transverse semi-axis. Equivalently, using full lengths consistently gives $e = 13/12$, since the full transverse axis has length twelve; mixing full lengths and half-lengths creates the factor-of-two error.

Step 3: Conclusion The eccentricity is $\frac{13}{12}$. Hence, the correct option is (C).

Quick Tip to Solve: Halve both the conjugate-axis length and focal separation first, then apply c squared equals a squared plus b squared.

22. Let f(x) = $\frac{x}{\sqrt{a^2+x^2}} - \frac{d-x}{\sqrt{b^2+(d-x)^2}}$, x $\in$ R, where a, b and d are non-zero real constants. Then :

[4 marks]

(A) f is an increasing function of x

(B) f is neither increasing nor decreasing function of x

(C) f ' is not a continuous function of x

(D) f is a decreasing function of x

Correct Answer: (A) f is an increasing function of x

Solution:

Concept:

- A differentiable function is strictly increasing on an interval if its derivative is positive at every point of that interval.
- For a nonzero real constant c, the function $g_c(t) = t(c^2 + t^2)^{-1/2}$ has no zero denominator, since $c^2 + t^2 > 0$.

Step 1: Differentiate the basic expression Apply the product rule, followed by the chain rule, to obtain

$$g_c\prime(t) = (c^2 + t^2)^{-1/2} - t^2(c^2 + t^2)^{-3/2} = \frac{c^2}{(c^2 + t^2)^{3/2}}.$$

The numerator is strictly positive because the constant is nonzero; the denominator is also strictly positive. This sign does not depend on whether the constant itself is positive or negative.

Step 2: Apply the chain rule to both terms Write the given expression as $f(x) = g_a(x) - g_b(d - x)$. The inner derivative of $d - x$ is -1, which cancels the minus sign before the second term. Consequently,

$$f\prime(x) = \frac{a^2}{(a^2 + x^2)^{3/2}} + \frac{b^2}{\{b^2 + (d - x)^2\}^{3/2}} > 0.$$

Both summands are continuous for every real x, so the derivative is continuous too. Thus option (C) fails. Positivity throughout the real line rules out decreasing behaviour and also rules out the claim in option (B).

Step 3: Conclusion The derivative is positive for every real x, so f is strictly increasing on $\mathbb{R}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the second term as $-g_b(d - x)$: its two minus signs cancel on differentiation, leaving a sum of positive terms.

23. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual notation.
If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value : [4 marks]

(A) (19, 7, 25)

(B) (7, 19, 25)

(C) (5, 12, 13)

(D) (3, 4, 5)

Correct Answer: (B) (7, 19, 25)

Solution:

Concept:

- The cosine rule gives $\cos A = (b^2 + c^2 - a^2)/(2bc)$, with the analogous expressions for the other two angles.
- Only side ratios matter for triangle angles. A common positive scale factor therefore cancels from every cosine.

Step 1: Recover the side ratios Put $(b+c)/11 = (c+a)/12 = (a+b)/13 = k$. Since the side lengths are positive, $k > 0$. Adding the last two equalities and subtracting the first gives $2a = (12+13-11)k = 14k$. Similarly,

$$a = 7k, \qquad b = 6k, \qquad c = 5k.$$

These lengths satisfy the triangle inequalities. It is therefore enough to use the representative sides $7, 6, 5$; their common scale has no influence on the angles.

Step 2: Evaluate and compare the cosines The three cosine-rule calculations give

$$\cos A = \frac{36 + 25 - 49}{60} = \frac{1}{5},$$
$$\cos B = \frac{25 + 49 - 36}{70} = \frac{19}{35},$$
$$\cos C = \frac{49 + 36 - 25}{84} = \frac{5}{7}.$$

With a common denominator these are $7/35, 19/35, 25/35$. Hence their ordered ratio is $7 : 19 : 25$. For this triad each of the given quotients is $1/35$. Option (A) interchanges the first two entries and fails that equality. The ratios $5 : 12 : 13$ and $3 : 4 : 5$ also do not match these cosine values.

Step 3: Conclusion One possible ordered triad is $(\alpha, \beta, \gamma) = (7, 19, 25)$. Hence, the correct option is (B).

Quick Tip to Solve: First extract the three side lengths from the pairwise sums; the required ratio is the ratio of cosines, not the ratio of sides.

24. A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, then $\left(\frac{mean\ of\ X}{standard\ deviation\ of\ X}\right)$ is equal to : [4 marks]

(A) 4

(B) $3\sqrt{2}$

(C) $\frac{4\sqrt{3}}{3}$

(D) $4\sqrt{3}$

Correct Answer: (D) $4\sqrt{3}$

Solution:

Concept:

- Replacement keeps the probability of drawing a white ball fixed at $p = 30/40 = 3/4$ on every draw, and the independent count is binomial.
- For $X \sim \text{Bin}(n, p)$, the mean is np, the variance is $np(1-p)$, and the standard deviation is its positive square root.

Step 1: Identify the distribution and its mean There are 40 balls altogether, of which 30 are white. A white draw is a success, and the number of trials is $n = 16$. Because the selected ball is returned before the next trial, no draw alters the composition of the bag. Therefore

$$X \sim \text{Bin}\left(16, \frac{3}{4}\right), \qquad \mu = 16 \cdot \frac{3}{4} = 12.$$

The complementary probability of a red draw is $q = 1/4$. Without replacement a binomial model would not be justified, but replacement is explicit here.

Step 2: Find the standard deviation and form the ratio The variance and standard deviation are

$$\sigma^2 = 16 \cdot \frac{3}{4} \cdot \frac{1}{4} = 3, \qquad \sigma = \sqrt{3}.$$

Hence the requested ratio is

$$\frac{\mu}{\sigma} = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}.$$

Notice that dividing by the variance instead would give 4, the distractor in option (A). The question asks for the standard deviation, so that substitution is invalid. Neither $3\sqrt{2}$ nor $4\sqrt{3}/3$ equals the ratio obtained from these binomial parameters.

Step 3: Conclusion The mean divided by the standard deviation is $4\sqrt{3}$. Hence, the correct option is (D).

Quick Tip to Solve: Use $\mu/\sigma = \sqrt{np/q}$ for a binomial count; here it becomes $\sqrt{48} = 4\sqrt{3}$.

25. Let A and B be two invertible matrices of order 3 × 3. If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B^T)$ is equal to : [4 marks]

(A) $\frac{1}{4}$

(B) 16

(C) $\frac{1}{16}$

(D) 1

Correct Answer: (C) $\frac{1}{16}$

Solution:

Concept:

- Determinants multiply: $\det(PQ) = \det P \det Q$. Also $\det(P^T) = \det P$ and $\det(P^{-1}) = 1/\det P$ for an invertible matrix.
- Reducing matrix identities to scalar determinant equations avoids unnecessary matrix multiplication. The determinant of a product is unaffected by the order of its factors.

Step 1: Translate the two conditions Let $u = \det A$ and $v = \det B$. Invertibility gives $u, v \neq 0$, so the divisions below are legitimate. The first condition yields

$$\det(ABA^T) = uvu = u^2v = 8.$$

The second condition gives

$$\det(AB^{-1}) = \frac{u}{v} = 8, \qquad u = 8v.$$

Transposition does not introduce an extra power depending on the matrix order: it simply leaves the determinant unchanged.

Step 2: Solve and evaluate the required determinant Substitute $u = 8v$ into $u^2v = 8$:

$$64v^3 = 8, \qquad v^3 = \frac{1}{8}, \qquad v = \frac{1}{2}, \qquad u = 4.$$

These are real determinant values, so the real cube root is unique. Now apply the same product and inverse rules to the expression asked for:

$$\det(BA^{-1}B^T) = v\frac{1}{u}v = \frac{v^2}{u} = \frac{(1/2)^2}{4} = \frac{1}{16}.$$

The values $1/4$, 16, and 1 in the remaining options do not equal this product. In particular, inverting the target by mistake would produce the distractor 16.

Step 3: Conclusion The required determinant equals $1/16$. Hence, the correct option is (C).

Quick Tip to Solve: Replace each matrix by its determinant, a transpose by the same scalar, and an inverse by its reciprocal before doing any algebra.

26. Two lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates : [4 marks]

(A) (2, 4, 7)

(B) (2, − 4, −7)

(C) (2, − 4, 7)

(D) (− 2, 4, 7)

Correct Answer: (B) (2, − 4, −7)

Solution:

Concept:

- A symmetric line equation is converted to parametric form by equating its three ratios to one parameter. Distinct lines require independent parameters.
- Reflection in the xy-plane preserves the x- and y-coordinates and reverses the z-coordinate: $(x, y, z) \mapsto (x, y, -z)$.

Step 1: Find the intersection parameters Use parameter t for the first line and s for the second. Their coordinates are

$$(x, y, z) = (3 + t, -1 + 3t, 6 - t),$$
$$(x, y, z) = (-5 + 7s, 2 - 6s, 3 + 4s).$$

Equating the first two coordinates gives

$$t - 7s = -8, \qquad 3t + 6s = 3.$$

Divide the second equation by 3 to get $t + 2s = 1$, then subtract the first equation. This yields $9s = 9$, so $s = 1$ and $t = -1$.

Step 2: Check the third coordinate and reflect At $t = -1$, the first line gives $R = (2, -4, 7)$. At $s = 1$, the second line gives exactly the same point, including $z = 3 + 4 = 7$; this verifies a genuine intersection rather than only matching its projection. Reflection in the xy-plane therefore gives

$$R^* = (2, -4, -7).$$

Option (C) is the original point and does not perform the reflection. Option (A) incorrectly reverses the sign of y, while option (D) also changes the first coordinate. The required mirror changes only the coordinate perpendicular to its plane.

Step 3: Conclusion The reflection of R in the xy-plane is $(2, -4, -7)$. Hence, the correct option is (B).

Quick Tip to Solve: After solving two coordinate equations for an intersection, check the third one; for an xy-plane reflection, change only the sign of z.

27. Contrapositive of the statement " If two numbers are not equal, then their squares are not equal." is : [4 marks]

(A) If the squares of two numbers are equal, then the numbers are not equal

(B) If the squares of two numbers are equal, then the numbers are equal

(C) If the squares of two numbers are not equal, then the numbers are equal

(D) If the squares of two numbers are not equal, then the numbers are not equal

Correct Answer: (B) If the squares of two numbers are equal, then the numbers are equal

Solution:

Concept:

- The contrapositive of an implication $p \Rightarrow q$ is $\neg q \Rightarrow \neg p$: reverse the order and negate both component statements.
- Logical transformation depends on the form of the given statement. It does not require its assertion about all numbers to be mathematically true.

Step 1: Name and negate the two component statements For two numbers a and b, put

$$p: \ a \neq b, \qquad q: \ a^2 \neq b^2.$$

The printed statement has the form $p \Rightarrow q$. The negation of an inequality $a \neq b$ is the equality $a = b$, not another inequality. Thus

$$\neg p: \ a = b, \qquad \neg q: \ a^2 = b^2.$$

We must use these exact negations even though distinct numbers such as 1 and -1 show that the original universal assertion is false.

Step 2: Reverse the implication and select its wording The contrapositive starts with the negation of the original conclusion and ends with the negation of the original premise:

$$a^2 = b^2 \quad \Rightarrow \quad a = b.$$

In words: if the squares of two numbers are equal, then the numbers are equal. Option (A) is $\neg q \Rightarrow p$: it reverses the order and negates the original conclusion, but leaves the original premise unnegated. Option (C) is $q \Rightarrow \neg p$: it reverses the original order but negates only the original premise, leaving the original conclusion unnegated. Option (D) is the converse, $q \Rightarrow p$, which reverses without negation. None of those is the prescribed contrapositive. Its truth value remains the same as that of the original implication; truth and syntactic form are separate issues here.

Step 3: Conclusion The contrapositive is: "If the squares of two numbers are equal, then the numbers are equal. Hence, the correct option is (B).

Quick Tip to Solve: For a contrapositive, write the pattern $\neg q \Rightarrow \neg p$ first, then substitute the two negated statements.

28. Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1x + a_2x^2 + \ldots + a_{50}x^{50}$, for all $x \in R$; then $\frac{a_2}{a_0}$ is equal to [4 marks]

(A) 12.25

(B) 12.75

(C) 12.00

(D) 12.50

Correct Answer: (A) 12.25

Solution:

Concept:

- The binomial theorem gives the coefficient of x^r in $(x + c)^{50}$ as $\binom{50}{r}c^{50-r}$.
- Adding $(x + 10)^{50}$ and $(x - 10)^{50}$ cancels odd powers of x and doubles each even-power coefficient, since the exponent 50 is even.

Step 1: Extract the constant and quadratic coefficients The constant term is obtained by choosing no factors of x. The two expansions contribute 10^{50} and $(-10)^{50} = 10^{50}$, respectively. Therefore

$$a_0 = 2 \cdot 10^{50}.$$

For the quadratic term, choose x from exactly two of the fifty factors. Each expansion contributes $\binom{50}{2}10^{48}$, because the remaining exponent 48 is even. Hence

$$a_2 = 2\binom{50}{2}10^{48}.$$

Both terms have the same sign; the cancellation affecting odd powers does not apply to the quadratic coefficient.

Step 2: Cancel the common factors in the ratio Divide the two coefficients before evaluating the large powers:

$$\frac{a_2}{a_0} = \frac{2\binom{50}{2}10^{48}}{2 \cdot 10^{50}} = \frac{\binom{50}{2}}{100} = \frac{50 \cdot 49}{2 \cdot 100} = \frac{1225}{100} = 12.25.$$

The common factor of two cancels, while the powers of ten leave a factor 10^{-2}. The three other proposed values differ from the exact binomial coefficient calculation: using $50 \cdot 50/2$, for instance, would incorrectly replace 49 by 50 and give the distractor 12.50.

Step 3: Conclusion The coefficient ratio is $a_2/a_0 = 12.25$. Hence, the correct option is (A).

Quick Tip to Solve: For a low-degree coefficient, count how many x factors are chosen; divide the coefficients symbolically before evaluating large powers.

29. Let $S_n = 1 + q + q^2 + \ldots\ldots + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \ldots\ldots + \left(\frac{q+1}{2}\right)^n$ where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2 \cdot S_1 + \ldots. + {}^{101}C_{101} \cdot S_{100} = \alpha T_{100}$ then α is equal to [4 marks]

(A) 202

(B) 200

(C) 2^{100}

(D) 2^{99}

Correct Answer: (C) 2^{100}

Solution:

Concept:

- The finite geometric sum is $S_{k-1} = (q^k - 1)/(q - 1)$, since $q \neq 1$. Multiplication by $q - 1$ makes the weighted sum suitable for the binomial theorem.
- The binomial identity $\sum_{k=0}^{101} \binom{101}{k} x^k = (1 + x)^{101}$ evaluates both the power-weighted sum and the sum of coefficients.

Step 1: Express the given weighted sum Call the left-hand side L. The first term is $\binom{101}{1} S_0$, where $S_0 = 1$, so every term fits the same index pattern. Therefore

$$L = \sum_{k=1}^{101} \binom{101}{k} \frac{q^k - 1}{q - 1}.$$

Subtracting the coefficient sum from the power sum gives

$$L = \frac{[(1 + q)^{101} - 1] - (2^{101} - 1)}{q - 1} = \frac{(1 + q)^{101} - 2^{101}}{q - 1}.$$

The two constant terms cancel; retaining either one would introduce an incorrect extra term.

Step 2: Evaluate the second geometric sum Put $r = (q + 1)/2$. Since $q \neq 1$, we also have $r \neq 1$. Apply the finite-sum formula to its 101 terms:

$$T_{100} = \frac{r^{101} - 1}{r - 1} = \frac{(q + 1)^{101} - 2^{101}}{2^{100}(q - 1)}.$$

Comparing this expression with L immediately yields $L = 2^{100} T_{100}$. The factors 202 and 200 are not generated by the binomial expansion, and 2^{99} misses the factor of two from $r - 1 = (q - 1)/2$. For real $q \neq 1$, the numerator cannot vanish, because the odd power equation $(q + 1)^{101} = 2^{101}$ would force $q = 1$. Thus the comparison uniquely determines the coefficient.

Step 3: Conclusion $\alpha = 2^{100}$. Hence, the correct option is (C).

Quick Tip to Solve: For a binomially weighted geometric sum, multiply by the geometric denominator first; then apply the binomial theorem to the two resulting sums.

30. If the point (2, α, β) lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane 2x – 5y = 15, then $2\alpha - 3\beta$ is equal to [4 marks]

(A) 12

(B) 7

(C) 17

(D) 5

Correct Answer: (B) 7

Solution:

Concept:

- A plane normal $\mathbf{n} = (a, b, c)$ is perpendicular to every displacement vector joining two points of that plane.
- Perpendicular planes have perpendicular normals. The normal to $2x - 5y = 15$ is $(2, -5, 0)$; its constant term does not affect the orientation.

Step 1: Find the normal to the required plane The displacement between the given points is $(7, 0, 6) - (3, 4, 2) = (4, -4, 4)$, which is parallel to $(1, -1, 1)$. Hence the required normal satisfies

$$a - b + c = 0.$$

Perpendicularity to the given plane supplies the independent condition

$$2a - 5b = 0.$$

Choose $a = 5$ and $b = 2$; the first equation then gives $c = -3$. Thus a normal is $(5, 2, -3)$. Multiplying all three components by a common nonzero number would describe the same plane; there is no need to normalize the vector.

Step 2: Use a point and substitute the unknown coordinates The plane through $(3, 4, 2)$ with this normal has equation

$$5(x - 3) + 2(y - 4) - 3(z - 2) = 0,$$

or

$$5x + 2y - 3z = 17.$$

Check the other point: $5(7) + 2(0) - 3(6) = 17$, so both given points lie on it. Its normal also obeys $(5, 2, -3) \cdot (2, -5, 0) = 10 - 10 = 0$, confirming the required perpendicularity. Now insert $(2, \alpha, \beta)$:

$$10 + 2\alpha - 3\beta = 17 \quad \Rightarrow \quad 2\alpha - 3\beta = 7.$$

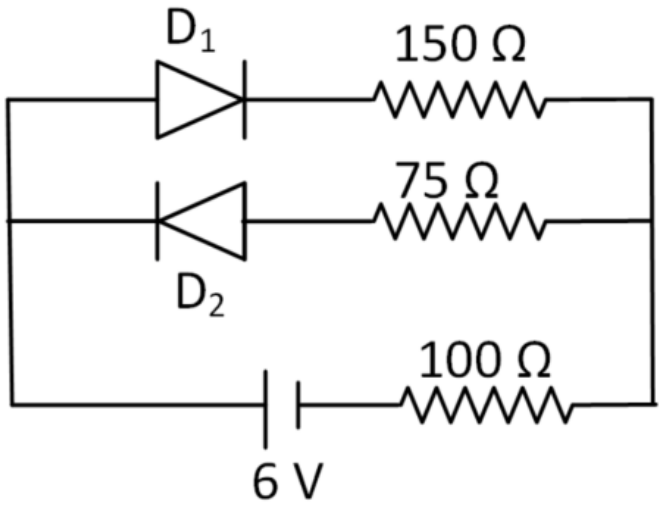
The option 17 is the plane constant before accounting for $x = 2$; it is not the requested combination. The values 12 and 5 do not satisfy this substitution.

Step 3: Conclusion $2\alpha - 3\beta = 7$. Hence, the correct option is (B).

Quick Tip to Solve: Find a plane normal from two dot-product conditions: one with its known in-plane direction and one with the other plane normal.

Physics (120 marks)

31. The circuit shown below contains two ideal diodes, each with a forward resistance of 50 Ω. If the battery voltage is 6 V, the current through the 100 Ω resistance (in Amperes) is : [4 marks]



(A) 0.027

(B) 0.030

(C) 0.036

(D) 0.020

Correct Answer: (D) 0.020

Solution:

Concept:

- A forward-biased diode conducts with the stated forward resistance, while an ideal reverse-biased diode is an open circuit.
- Once diode states are identified, apply Ohm's law to the closed loop, including the diode forward resistance as well as the external resistors.

Step 1: Identify the conducting branch The longer battery plate is on the left, so the left junction is at higher potential than the right junction. Diode D_1, in the top branch, has its anode on the left and cathode on the right; it is forward biased. Diode D_2 points the opposite way and is reverse biased. Consequently, no current flows through the middle branch containing $75\,\Omega$. This branch cannot be placed in parallel with the upper branch as though both diodes were conducting.

Step 2: Calculate the current in the closed loop The only conducting loop contains the battery, the $100\,\Omega$ resistor, the $150\,\Omega$ resistor, and the $50\,\Omega$ forward resistance of D_1. All carry the same current. The total loop resistance is

$$R_{\text{loop}} = 100 + 150 + 50 = 300\,\Omega.$$

Thus

$$I_{100} = \frac{6}{300} = 0.020\,\text{A}.$$

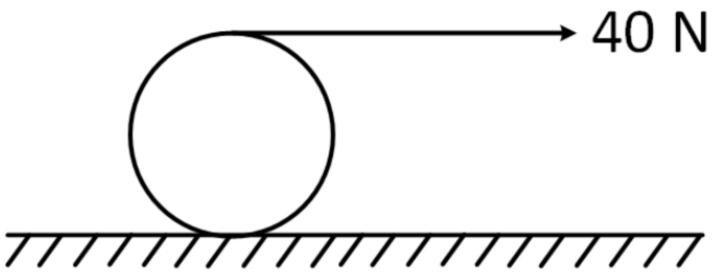
The voltage across the upper branch is $I(150+50)=4\,\text{V}$, positive on its left. This confirms that D_1 remains forward biased and D_2 remains reverse biased. The $100\,\Omega$ resistor drops the remaining $2\,\text{V}$, so Kirchhoff's voltage law is also satisfied. Each other proposed current would give a loop voltage different from $6\,\text{V}$ for this physically consistent diode state.

Step 3: Conclusion The current through the $100\,\Omega$ resistor is $0.020\,\text{A}$. Hence, the correct option is (D).

Quick Tip to Solve: Determine diode bias from the battery polarity before simplifying the resistor network; then verify the assumed bias using the calculated junction voltage.

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32. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) : [4 marks]



(A) 16 rad/s^2

(B) 20 rad/s^2

(C) 10 rad/s^2

(D) 12 rad/s^2

Correct Answer: (A) 16 rad/s^2

Solution:

Concept:

- For a thin hollow cylinder about its symmetry axis, $I=MR^2$. Rolling without slipping relates centre acceleration and angular acceleration by $a=\alpha R$.
- Translation obeys $\sum F_x=Ma$, and rotation about the centre obeys $\sum\tau=I\alpha$. The pull at the top contributes both a force and a torque.

Step 1: Write the coupled equations of motion Take rightward translation and clockwise rotation as positive. Let f be the signed frictional force, positive if it acts rightward. The horizontal force balance is

$$F+f=Ma.$$

The pull at the top gives clockwise torque FR, whereas a rightward force at the bottom gives counterclockwise torque fR. Hence

$$FR-fR=I\alpha.$$

Using $I = MR^2$ and $\alpha = a/R$, this reduces to $F - f = Ma$. These signs follow from the points where the forces act; treating both torques as clockwise would contradict rolling.

Step 2: Solve and substitute the given values Add the translational and rotational equations to eliminate friction:

$$2F = 2Ma \quad \Rightarrow \quad a = \frac{F}{M} = \frac{40}{5} = 8\,\mathrm{m\,s^{-2}}.$$

Therefore

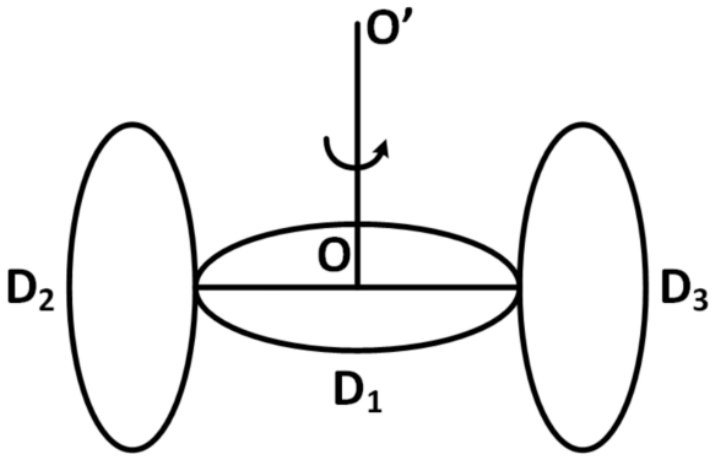
$$\alpha = \frac{a}{R} = \frac{8}{0.5} = 16\,\mathrm{rad\,s^{-2}}.$$

Subtracting the equations gives $f = 0$. This is consistent with a thin hollow cylinder pulled tangentially at its top: the applied force alone supplies the required translation and rotation in the rolling ratio. The angular acceleration is clockwise. The other values, 20, 10 and 12, fail either $a = \alpha R$ or the torque equation with $I = MR^2$; the solid-cylinder inertia $MR^2/2$ must not be used for this hollow cylinder.

Step 3: Conclusion The angular acceleration is $16\,\mathrm{rad\,s^{-2}}$, clockwise. Hence, the correct option is (A).

Quick Tip to Solve: For a tangential pull at the top, add the translation and rotation equations after using the rolling constraint; friction then cancels immediately.

33. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO' ,passing through the centre of D_1 as shown in the figure, will be : [4 marks]



(A) $3MR^2$

(B) MR^2

(C) $\frac{2}{3}$ MR^2

(D) $\frac{4}{5}$ MR^2

Correct Answer: (A) $3MR^2$

Solution:

Concept:

- A uniform disc has moment of inertia $MR^2/2$ about its central perpendicular axis and $MR^2/4$ about any central diameter.
- The parallel-axis theorem is $I = I_{\text{cm}} + Md^2$, where d is the perpendicular separation between the parallel axes. Moments of inertia add for rigidly attached bodies.

Step 1: Find the contribution of the central disc The figure shows D_1 horizontal, with OO' passing through its centre perpendicular to its plane. Its contribution is therefore

$$I_1 = \frac{1}{2}MR^2.$$

The two outer discs are vertical, attached at the opposite ends of a diameter of D_1. Their centres are each a horizontal distance R from OO'. The relevant central axis for either outer disc is a vertical diameter, parallel to OO', rather than the axis perpendicular to that outer disc.

Step 2: Apply the parallel-axis theorem to the outer discs For either outer disc, the moment about its own vertical diameter is $MR^2/4$. Shift this axis by R to the specified common axis:

$$I_2 = I_3 = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2.$$

Add the three contributions:

$$I_{\text{total}} = \frac{1}{2}MR^2 + 2\left(\frac{5}{4}MR^2\right) = 3MR^2.$$

The centre shifts of both outer discs are essential. Omitting them produces an unrealistically small result; indeed, their translation terms alone sum to $2MR^2$, already exceeding options (B), (C), and (D). This also gives a useful independent check on the option selection. The radius of the central disc locates the outer centres; their own radii determine their diameter-axis inertia.

Step 3: Conclusion The moment of inertia about OO' is $3MR^2$. Hence, the correct option is (A).

Quick Tip to Solve: For attached discs in different planes, identify whether each parallel central axis is a diameter or a perpendicular axis before adding the centre-shift term.

34. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation VT = K, where K is a constant. In this process the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (R is gas constant) : [4 marks]

(A) $\frac{1}{2}$ KRΔT

(B) $\frac{1}{2}$ RΔT

(C) $\frac{3}{2}$ RΔT

(D) $\frac{2K}{3}$ ΔT

Correct Answer: (B) $\frac{1}{2}$ RΔT

Solution:

Concept:

- For one mole of a monoatomic ideal gas, $\Delta U = C_V \Delta T = (3/2)R\Delta T$; internal energy depends only on temperature.
- With work W defined as work done by the gas, the first law is $Q = \Delta U + W$. Quasistatic boundary work is $W = \int P\,dV$.

Step 1: Use the path relation to evaluate the work The specified process obeys $VT = K$, so $V = K/T$ and

$$dV = -\frac{K}{T^2}\,dT.$$

For one mole, the ideal-gas equation gives

$$P = \frac{RT}{V} = \frac{RT^2}{K}.$$

Consequently, the differential work done by the gas is

$$P\,dV = \frac{RT^2}{K}\left(-\frac{K}{T^2}\,dT\right) = -R\,dT.$$

Integrating between the initial and final temperatures yields $W = -R\Delta T$. The negative sign is expected: increasing temperature along $VT = K$ reduces the volume, so the surroundings compress the gas.

Step 2: Combine work and internal-energy change The increase in internal energy is $(3/2)R\Delta T$. Substituting into the first law gives

$$Q = \frac{3}{2}R\Delta T - R\Delta T = \frac{1}{2}R\Delta T.$$

Since $\Delta T > 0$, the gas absorbs heat, while compression supplies the remaining energy needed for its temperature rise. The expression $(3/2)R\Delta T$ would be the heat only in an isochoric process with zero work, which is not this path. The constant K cancels from the work integrand, so options retaining a factor K cannot represent this process heat. The units of $R\Delta T$ are energy for the stated one mole.

Step 3: Conclusion The heat absorbed is $Q = \frac{1}{2}R\Delta T$. Hence, the correct option is (B).

Quick Tip to Solve: Do not substitute a familiar heat capacity until the path is identified; use its temperature-volume relation to calculate the work first.

35. Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T°C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is [4 marks]

(A) 200°C

(B) 270°C

(C) 230°C

(D) 250°C

Correct Answer: (C) 230°C

Solution:

Concept:

- For a rod with initial length L_0, linear expansion gives $L = L_0[1 + \alpha\Delta T]$, with α its coefficient of linear expansion.
- Identical initial lengths and equal final lengths imply equal fractional expansions: $\alpha_A\Delta T_A = \alpha_B\Delta T_B$. Temperature differences must be measured from the common initial temperature.

Step 1: Set up equality of the final lengths Both rods initially have the same length at $30°\text{C}$. Rod A is heated to $180°\text{C}$, so its temperature rise is

$$\Delta T_A = 180 - 30 = 150°\text{C}.$$

Rod B is heated to $T°\text{C}$, so its rise is $\Delta T_B = T - 30$. Equate the final lengths:

$$L_0[1 + \alpha_A(150)] = L_0[1 + \alpha_B(T - 30)].$$

Cancel the common initial length and then the unchanged part of each expression to obtain $150\alpha_A = (T - 30)\alpha_B$.

Step 2: Use the coefficient ratio and check the result The ratio is $\alpha_A/\alpha_B = 4/3$, giving

$$T - 30 = \frac{4}{3}(150) = 200.$$

Thus

$$T = 230°\text{C}.$$

Rod B has the smaller coefficient, so it must undergo a larger temperature rise to reach the same length: $200°\text{C}$ rather than $150°\text{C}$, as the result confirms. Option (A), $200°\text{C}$, is the rise of B and not its final temperature. The other proposed final temperatures give rises 240 and 220, whose products with α_B do not equal $150\alpha_A$. A Celsius temperature difference has the same numerical magnitude in kelvin, so no absolute-temperature conversion is needed in the expansion equation.

Step 3: Conclusion The final temperature of rod B is $230°\text{C}$. Hence, the correct option is (C).

Quick Tip to Solve: In equal-expansion questions, compare coefficient times temperature rise, then add the initial temperature to recover the requested final temperature.

36. A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice. What is the temperature of an object in °C, if this thermometer in the contact with the object reads $x_0/2$? [4 marks]

(A) 60

(B) 35

(C) 25

(D) 40

Correct Answer: (C) 25

Solution:

Concept:

- A linear thermometer has a reading of the form $x = a + bT$, where T is the Celsius temperature and a, b are calibration constants.
- The ice and steam points correspond to $0°\text{C}$ and $100°\text{C}$. Equal changes of reading represent equal changes of temperature.

Step 1: Locate the reading within the fixed-point interval The lower fixed-point reading is $x_i = x_0/3$ and the upper reading is $x_s = x_0$. The full reading interval is therefore

$$x_s - x_i = x_0 - \frac{x_0}{3} = \frac{2x_0}{3}.$$

For the object, $x = x_0/2$, so its reading lies above the ice point by

$$x - x_i = \frac{x_0}{2} - \frac{x_0}{3} = \frac{x_0}{6}.$$

The zero of the thermometer scale is not the Celsius zero; that offset must be removed before taking a ratio.

Step 2: Convert the fraction of the interval to Celsius Using the two-point calibration,

$$\frac{T}{100} = \frac{x - x_i}{x_s - x_i} = \frac{x_0/6}{2x_0/3} = \frac{1}{4}.$$

Hence

$$T = 25°\text{C}.$$

The result is one quarter of the way from freezing to boiling. The alternatives $35, 40, 60$ would require the reading to occupy those corresponding percentages of the fixed-point interval, whereas the given reading occupies only 25 percent. The unknown scale factor x_0 cancels as it should.

Step 3: Conclusion The temperature of the object is $25°\text{C}$. Hence, the correct option is (C).

Quick Tip to Solve: For a linear thermometer, subtract the ice-point reading first, then divide by the full ice-to-steam reading interval.

37. When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C, will be : [4 marks]

(A) 60°C

(B) 70°C

(C) 85°C

(D) 80°C

Correct Answer: (D) 80°C

Solution:

Concept:

- For an insulated mixture with no phase change, heat lost by the hotter liquid equals heat gained by the colder liquid: $m_A c_A(T_A - T) = m_B c_B(T - T_B)$.
- The heat capacities $C_A = m_A c_A$ and $C_B = m_B c_B$ determine the weighted mean temperature. They remain the same in both experiments.

Step 1: Find the ratio of the heat capacities In the first mixing experiment, liquid A cools from 100°C to 90°C, while liquid B warms from 75°C to 90°C. Conservation of energy gives

$$C_A(100 - 90) = C_B(90 - 75).$$

Thus

$$10C_A = 15C_B, \qquad \frac{C_A}{C_B} = \frac{3}{2}.$$

This ratio already includes the unequal masses; using just the mass ratio would incorrectly assume equal specific heats.

Step 2: Apply the heat balance to the second mixture Let the new equilibrium temperature be T. The same quantities of both liquids are used, but B now begins at 50°C. Therefore

$$C_A(100 - T) = C_B(T - 50).$$

Substitute the capacity ratio:

$$\frac{3}{2}(100 - T) = T - 50.$$

Simplifying gives

$$150 - 1.5T = T - 50, \qquad 2.5T = 200, \qquad T = 80^\circ\text{C}.$$

This lies between the two initial temperatures and closer to 100 because A has the larger heat capacity. None of 60, 70 or 85 satisfies the same energy-balance equation.

Step 3: Conclusion The second mixture reaches 80°C. Hence, the correct option is (D).

Quick Tip to Solve: Infer the ratio of total heat capacities from the first mixture; carry that ratio directly into the second heat-balance equation.

38. An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C.m. What is the potential energy of the electric dipole? [4 marks]

(A) - 7×10^{-27} J

(B) -9×10^{-20} J

(C) -10×10^{-29} J

(D) -20×10^{-18} J

Correct Answer: (A) - 7×10^{-27} J

Solution:

Concept:

- The interaction energy of a permanent dipole in a uniform electric field is $U = -\boldsymbol{p} \cdot \boldsymbol{E} = -pE\cos\theta$.
- The conventional zero of dipole energy is at $\theta = 90°$. A dipole making an acute angle with the field has negative potential energy.

Step 1: Substitute the dipole moment and field The data are $p = 10^{-29}\,\mathrm{C\,m}$, $E = 1000\,\mathrm{V/m} = 10^{3}\,\mathrm{V/m}$ and $\theta = 45°$. First calculate the energy scale:

$$pE = 10^{-29} \times 10^{3} = 10^{-26}\,\mathrm{J}.$$

The units are consistent because $\mathrm{C\,m} \times \mathrm{V/m} = \mathrm{C\,V} = \mathrm{J}$. The angle enters through the scalar product, not through the sine that appears in the torque formula.

Step 2: Evaluate the angular factor and compare options With $\cos 45° = 1/\sqrt{2}$,

$$U = -10^{-26}\frac{1}{\sqrt{2}} = -7.071 \times 10^{-27}\,\mathrm{J}.$$

Rounded to the precision of the options, this is -7×10^{-27} J. The negative sign is necessary for an acute alignment: rotating toward the parallel configuration lowers the energy. Option C has the wrong magnitude, while options B and D are many orders of magnitude larger than pE, even though $|\cos\theta| \le 1$ requires $|U| \le pE$.

Step 3: Conclusion The potential energy is approximately -7×10^{-27} J. Hence, the correct option is (A).

Quick Tip to Solve: Use cosine for dipole energy and sine for torque; first calculate the scale pE to catch exponent errors.

39. A 27 mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by :

[Given permittivity of space $\in_0$ = 9 $\times$ 10^{-12} SI units, Speed of light c = 3 $\times$ 10^8 m/s] [4 marks]

(A) 2 kV/m

(B) 1 kV/m

(C) 1.4 kV/m

(D) 0.7 kV/m

Correct Answer: (C) 1.4 kV/m

Solution:

Concept:

- The average intensity of a sinusoidal electromagnetic wave is $I = P/A = \frac{1}{2}\varepsilon_0 c E_0^2$, where E_0 is the maximum electric field.
- The factor $1/2$ comes from the time average of the squared sinusoidal field. The maximum field differs from the rms field by $\sqrt{2}$.

Step 1: Convert the power and beam area to SI units The beam power is

$$P = 27\,\text{mW} = 27 \times 10^{-3}\,\text{W}.$$

Since $1\,\text{mm}^2 = 10^{-6}\,\text{m}^2$, its area is

$$A = 10\,\text{mm}^2 = 10^{-5}\,\text{m}^2.$$

Thus the average intensity is

$$I = \frac{27 \times 10^{-3}}{10^{-5}} = 2.7 \times 10^3\,\text{W/m}^2.$$

The area conversion is squared; converting a millimetre as though it were an area would give a wrong power of ten.

Step 2: Find the peak electric field Rearrange the average-intensity expression using the supplied constants:

$$E_0^2 = \frac{2I}{\varepsilon_0 c} = \frac{2(2.7 \times 10^3)}{(9 \times 10^{-12})(3 \times 10^8)}.$$

The denominator equals 2.7×10^{-3}, hence

$$E_0^2 = 2.0 \times 10^6\,\text{V}^2/\text{m}^2, \qquad E_0 = \sqrt{2} \times 10^3\,\text{V/m}.$$

Therefore the maximum field is about $1.4\,\text{kV/m}$. The 1 kV/m value is the rms field; 2 and 0.7 kV/m do not satisfy the stated average intensity.

Step 3: Conclusion The maximum electric field is approximately $1.4\,\text{kV/m}$. Hence, the correct option is (C).

Quick Tip to Solve: When intensity is given and the maximum field is asked, retain the factor 2 in $E_0 = \sqrt{2I/(\varepsilon_0 c)}$.

40. The magnitude of torque on a particle of mass 1 kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians) : [4 marks]

(A) $\frac{\pi}{8}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{3}$

Correct Answer: (B) $\frac{\pi}{6}$

Solution:

Concept:

- The torque about an origin is $\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$, so its magnitude is $\tau = rF\sin\theta$.
- The angle θ is between the position vector from the origin and the force. The particle mass does not enter the instantaneous torque formula.

Step 1: Calculate the sine of the angle The position-vector magnitude is $r = 5\,\mathrm{m}$, the force is $F = 1\,\mathrm{N}$, and the torque magnitude is $2.5\,\mathrm{N\,m}$. Substitution gives

$$2.5 = 5(1)\sin\theta,$$

and therefore

$$\sin\theta = \frac{2.5}{5} = \frac{1}{2}.$$

Equivalently, the perpendicular distance from the origin to the line of action is $\tau/F = 2.5\,\mathrm{m}$, half the given radius.

Step 2: Identify the angle among the listed choices The acute angle with sine $1/2$ is

$$\theta = 30^\circ = \frac{\pi}{6}\,\mathrm{rad}.$$

The alternatives have sines $\sin(\pi/8) \approx 0.383$, $\sin(\pi/4) = 1/\sqrt{2}$, and $\sin(\pi/3) = \sqrt{3}/2$, none equal to one half. Because only the magnitude of torque is specified, the supplementary angle $5\pi/6$ would also yield the same magnitude in an unrestricted geometry; it is not among the choices. Thus the unique listed angle is $\pi/6$. Multiplying by mass would be inappropriate because the force is already supplied directly.

Step 3: Conclusion The listed angle consistent with the torque is $\pi/6$ radians. Hence, the correct option is (B).

Quick Tip to Solve: Form the dimensionless ratio $\tau/(rF)$ first; it is the sine of the angle, and its value must never exceed one.

41. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20 $\times$ 10^{-6} J/ T when a magnetic intensity of 60 $\times$ 10^{3} A/m is applied. Its magnetic susceptibility is [4 marks]

(A) 3.3×10^{-4}

(B) 2.3×10^{-2}

(C) 4.3×10^{-2}

(D) 3.3×10^{-2}

Correct Answer: (A) 3.3×10^{-4}

Solution:

Concept:

- Magnetisation M is the magnetic dipole moment per unit volume: $M = m_{\text{dip}}/V$, measured in A/m.
- For a linear paramagnetic material, $M = \chi H$, where H is the applied magnetic intensity and χ is the dimensionless magnetic susceptibility.

Step 1: Find the magnetisation of the cube The cube has side $1\,\text{cm} = 10^{-2}\,\text{m}$, so its volume is

$$V = (10^{-2})^3 = 10^{-6}\,\text{m}^3.$$

The magnetic moment is $20 \times 10^{-6}\,\text{J/T}$. Since $1\,\text{J/T} = 1\,\text{A m}^2$, the magnetisation is

$$M = \frac{20 \times 10^{-6}}{10^{-6}} = 20\,\text{A/m}.$$

The cubic conversion is essential: a centimetre cube is a millionth of a cubic metre.

Step 2: Divide magnetisation by the field intensity The applied intensity is $H = 60 \times 10^3\,\text{A/m}$. Therefore

$$\chi = \frac{M}{H} = \frac{20}{60 \times 10^3} = \frac{1}{3} \times 10^{-3} = 3.33 \times 10^{-4}.$$

Both numerator and denominator have the same units, so the result is dimensionless. It is positive and small, as expected for a paramagnet. The other three options are of order 10^{-2}, much larger than the ratio obtained from the supplied volume and moment. No factor of μ_0 is needed because the question supplies H, rather than magnetic induction B.

Step 3: Conclusion The magnetic susceptibility is approximately 3.3×10^{-4}. Hence, the correct option is (A).

Quick Tip to Solve: Convert the side to metres before cubing, then use $\chi = m_{\text{dip}}/(VH)$; do not insert μ_0 when the given field is H.

42. In a hydrogen like atom, when an electron jumps from the M-shell to the L-shell, the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be: [4 marks]

(A) $\frac{25}{16}\ \lambda$
(B) $\frac{27}{20}\ \lambda$
(C) $\frac{16}{25}\ \lambda$
(D) $\frac{20}{27}\ \lambda$

Correct Answer: (D) $\frac{20}{27}\ \lambda$

Solution:

Concept:

- For a hydrogen-like atom, $1/\lambda = RZ^2(1/n_f^2 - 1/n_i^2)$, with $n_i > n_f$ for emission.
- Wavelength is inversely proportional to the transition energy. In a ratio for the same atom, both the Rydberg constant and the factor Z^2 cancel.

Step 1: Write the wave number for the first transition The M, L and N shells correspond to principal quantum numbers $3, 2, 4$, respectively. For the given M-to-L transition,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{4} - \frac{1}{9}\right) = RZ^2\frac{5}{36}.$$

This is an emission transition because the electron moves to the lower-energy shell. The inverse wavelength, rather than the wavelength, is proportional to the energy difference.

Step 2: Compare with the N-to-L transition If the required wavelength is λ_2, then

$$\frac{1}{\lambda_2} = RZ^2\left(\frac{1}{4} - \frac{1}{16}\right) = RZ^2\frac{3}{16}.$$

Taking the ratio in the wavelength direction gives

$$\frac{\lambda_2}{\lambda} = \frac{5/36}{3/16} = \frac{5 \times 16}{36 \times 3} = \frac{20}{27}.$$

The second transition releases more energy, since the initial shell is farther above L, so its wavelength must be shorter; this immediately excludes the two ratios greater than one. The remaining $16/25$ does not equal the ratio of the two reciprocal-square energy differences.

Step 3: Conclusion The N-to-L transition has wavelength $\frac{20}{27}\lambda$. Hence, the correct option is (D).

Quick Tip to Solve: For spectral ratios, first calculate the two reciprocal-square differences, then invert their ratio when converting from wave number to wavelength.

43. A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30°C. What is the approximate percentage increment in the temperature of the water? [Specific Heat Capacities of water and metal are, respectively, 4200 Jkg^{-1} and 400 Jkg^{-1} K^{-1} [4 marks]

(A) 20%

(B) 25%

(C) 15%

(D) 30%

Correct Answer: (A) 20%

Solution:

Concept:

- In an insulated calorimetric mixture, heat lost by the hot metal equals heat gained by the water and the vessel. A heat capacity multiplies a temperature change directly, whereas a specific heat must first be multiplied by the mass.
- Let the final common temperature be T in degrees Celsius. The water and vessel together have heat capacity $0.5(4200) + 800 = 2900\,\mathrm{J\,K^{-1}}$; the metal has heat capacity $0.1(400) = 40\,\mathrm{J\,K^{-1}}$.

Step 1: Write the heat balance The ball cools from $500^\circ\mathrm{C}$ to T, while both the water and the vessel warm from $30^\circ\mathrm{C}$ to T. Thus the temperature changes must be kept distinct:

$$40(500 - T) = 2900(T - 30).$$

Including the vessel matters because its heat capacity is appreciable compared with that of the water. There is no phase change at the resulting temperature.

Step 2: Find the percentage rise Put $T = 30 + \Delta T$. Then

$$40(470 - \Delta T) = 2900\Delta T, \qquad \Delta T = \frac{18800}{2940} = 6.39\,\mathrm{K}.$$

The final temperature is approximately $36.39^\circ\mathrm{C}$. Using the initial Celsius temperature, as intended by the given alternatives, the percentage increment is

$$\frac{6.39}{30} \times 100\% = 21.3\%.$$

The closest offered approximation is 20%; the alternatives 15, 25 and 30 percent are farther from the heat-balance result.

Step 3: Conclusion The temperature rise is approximately $6.4^\circ\mathrm{C}$, or 21.3% of the initial Celsius reading, closest to 20%. Hence, the correct option is (A).

Quick Tip to Solve: Add the vessel heat capacity to the water heat capacity before applying heat balance; compare the resulting percentage with the approximate options.

44. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of simple pendulum on the Earth is 2 s. The period of oscillation of the same pendulum on the planet would be : [4 marks]

(A) $\frac{\sqrt{3}}{2}$ s

(B) $\frac{3}{2}$ s

(C) $\frac{2}{\sqrt{3}}$ s

(D) $2\sqrt{3}$ s

Correct Answer: (D) $2\sqrt{3}$ s

Solution:

Concept:

- The surface gravitational acceleration of a spherical planet is $g = GM/R^2$, where M is its mass and R its radius. A tripled diameter means a tripled radius, not an unchanged radius.
- For small oscillations, a simple pendulum of length L has period $T = 2\pi\sqrt{L/g}$. The same pendulum keeps the same length, so its period varies inversely as the square root of the local gravity.

Step 1: Compare the surface accelerations Write Earth parameters as M_E, R_E, g_E. The planet has $M_P = 3M_E$ and $R_P = 3R_E$. Therefore

$$g_P = \frac{G(3M_E)}{(3R_E)^2} = \frac{1}{3}\frac{GM_E}{R_E^2} = \frac{g_E}{3}.$$

The increase in radius reduces gravity by a factor of nine, outweighing the factor of three from the mass. It would be incorrect to use mass alone to estimate the acceleration.

Step 2: Scale the oscillation period Take the ratio of the two pendulum formulas, canceling the common length and 2π:

$$\frac{T_P}{T_E} = \sqrt{\frac{g_E}{g_P}} = \sqrt{3}.$$

Since $T_E = 2\,\text{s}$,

$$T_P = 2\sqrt{3}\,\text{s}.$$

Weaker gravity provides a weaker restoring acceleration and increases the period. This also rules out the first three alternatives, all of which are less than the original two-second period; a shorter period would require stronger gravity.

Step 3: Conclusion The period on the planet is $2\sqrt{3}\,\text{s}$. Hence, the correct option is (D).

Quick Tip to Solve: For a planet with both mass and radius multiplied by the same factor, gravity is divided by that factor and a fixed pendulum period grows by its square root.

45. A particle moves from the point $\left(2.0\hat{i}+4.0\hat{j}\right)$ m, at t = 0, with an initial velocity $\left(5.0\hat{i}+4.0\hat{j}\right)$ ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $\left(4.0\hat{i}+4.0\hat{j}\right)$ ms^{-2}. What is the distance of the particle from the origin at time 2 s? [4 marks]

(A) 15 m

(B) $20\sqrt{2}$ m

(C) $10\sqrt{2}$ m

(D) 5 m

Correct Answer: (B) $20\sqrt{2}$ m

Solution:

Concept:

- For constant acceleration, the position vector is $\boldsymbol{r}(t) = \boldsymbol{r}_0 + \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$. Apply this equation independently to the perpendicular Cartesian components.
- The distance from the origin is the magnitude of the final position, $r = \sqrt{x^2+y^2}$. It is distinct from the magnitude of displacement from the starting point and from the distance travelled along the path.

Step 1: Calculate the final coordinates At $t = 2\,\text{s}$, use the initial coordinates $x_0 = 2\,\text{m}$, $y_0 = 4\,\text{m}$, velocities $u_x = 5\,\text{m}\,\text{s}^{-1}$, $u_y = 4\,\text{m}\,\text{s}^{-1}$, and equal accelerations $a_x = a_y = 4\,\text{m}\,\text{s}^{-2}$. This gives

$$x = 2 + 5(2) + \frac{1}{2}(4)(2)^2 = 20\,\text{m},$$

$$y = 4 + 4(2) + \frac{1}{2}(4)(2)^2 = 20\,\text{m}.$$

The initial position must be included because the reference point requested is the origin.

Step 2: Take the position magnitude The final vector is $\boldsymbol{r} = 20\hat{\boldsymbol{i}} + 20\hat{\boldsymbol{j}}$ metres. Its components are perpendicular, so

$$|\boldsymbol{r}| = \sqrt{20^2+20^2} = \sqrt{800} = 20\sqrt{2}\,\text{m}.$$

Adding components would give a non-Euclidean distance, and omitting the initial coordinates would calculate a different vector. The values 15 m, $10\sqrt{2}$ m and 5 m do not equal the norm of the final position obtained from either component equation.

Step 3: Conclusion The particle is $20\sqrt{2}\,\text{m}$ from the origin after two seconds. Hence, the correct option is (B).

Quick Tip to Solve: First find the final x and y coordinates including the initial position, then take their root-sum-square; do not take the magnitude too early.

46. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young,s modulus will be: [4 marks]

(A) $V^{-2}A^2F^2$

(B) $V^{-4}A^{-2}F$

(C) $V^{-4}A^2F$

(D) $V^{-2}A^2F^{-2}$

Correct Answer: (C) $V^{-4}A^2F$

Solution:

Concept:

- Young's modulus is longitudinal stress divided by longitudinal strain. Strain is a ratio of lengths and is dimensionless, so the modulus has the dimensions of force per area, $[Y] = [F]L^{-2}$.
- The proposed fundamental quantities have dimensions $[V] = LT^{-1}$, $[A] = LT^{-2}$ and $[F] = MLT^{-2}$. A dimensional expression must reproduce the mass, length and time exponents simultaneously.

Step 1: Express a length using speed and acceleration The ratio of squared speed to acceleration has dimensions of length:

$$\left[\frac{V^2}{A}\right] = \frac{L^2T^{-2}}{LT^{-2}} = L.$$

Consequently an area has dimensions V^4A^{-2}, and its reciprocal has dimensions $V^{-4}A^2$. This argument concerns dimensions only and does not assert a particular motion equation or numerical coefficient.

Step 2: Build and verify the modulus dimensions Multiply the inverse-area dimensions by force:

$$[Y] = [V^{-4}A^2F].$$

As an independent check in the usual mass-length-time basis,

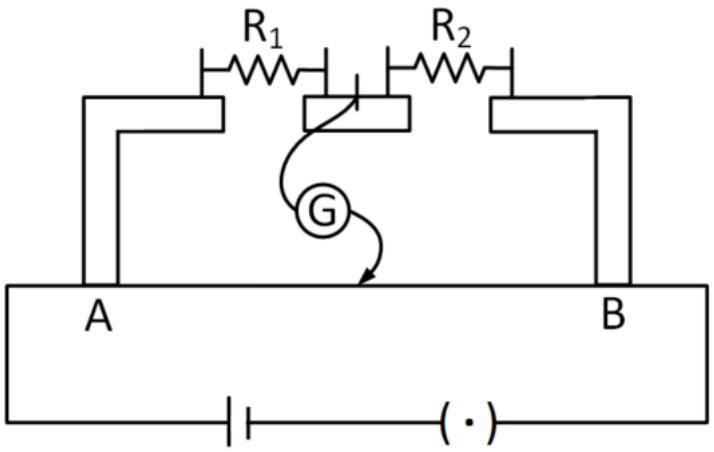
$$(LT^{-1})^{-4}(LT^{-2})^2(MLT^{-2}) = ML^{-1}T^{-2},$$

which is the dimension of pressure and hence of Young's modulus. Options with F^2 or F^{-2} have the wrong mass exponent. The alternative with A^{-2} also gives the wrong length and time exponents. Only the expression with $V^{-4}A^2F$ satisfies all three checks.

Step 3: Conclusion The required dimensional expression is $V^{-4}A^2F$. Hence, the correct option is (C).

Quick Tip to Solve: Reduce any elastic modulus to force per area, then use the dimensional length V squared divided by A to change the fundamental basis.

47. In the experimental set up of metre bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a 10 Ω resistor is connected in series with R_1, the null point shifts by 10 cm. The resistance that should be connected in parallel with (R_1 + 10) Ω such that the null point shifts back to its initial position is : [4 marks]



(A) 40 Ω

(B) 30 Ω

(C) 20 Ω

(D) 60 Ω

Correct Answer: (D) 60 Ω

Solution:

Concept:

- At the balance point of a metre bridge with a uniform one-metre wire, $R_1/R_2 = l/(100 - l)$, with l measured from the left end A in centimetres. No current flows through the galvanometer at balance.
- Adding resistance in series increases that arm resistance, so the left balance length increases. To restore the original ratio while keeping R_2 fixed, the modified left arm must again have its original effective resistance.

Step 1: Determine the two arm resistances The original balance at 40 cm gives

$$\frac{R_1}{R_2} = \frac{40}{60} = \frac{2}{3}.$$

After adding $10\,\Omega$ to the left arm, its ratio increases, so the shifted point is 50 cm rather than 30 cm. The new balance equation is

$$\frac{R_1 + 10}{R_2} = \frac{50}{50} = 1.$$

Therefore $R_2 = R_1 + 10$. Combining with $R_1 = 2R_2/3$ gives $R_2 = 30\,\Omega$ and $R_1 = 20\,\Omega$.

Step 2: Restore the original left-arm value The modified left resistance is now $R_1 + 10 = 30\,\Omega$. Let X be placed in parallel with it. For the original balance position to return, the equivalent left resistance

must be $20\,\Omega$:

$$\frac{30X}{30+X} = 20.$$

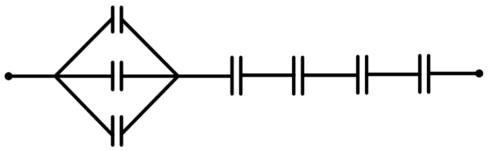
Thus $30X = 600 + 20X$, giving $X = 60\,\Omega$. For comparison, parallel values 40, 30 and 20 ohms would give effective resistances $120/7$, 15 and 12 ohms respectively, none of which restores the required 20-ohm arm.

Step 3: Conclusion A resistance of $60\,\Omega$ must be connected in parallel. Hence, the correct option is (D).

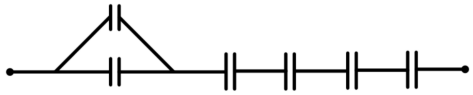
Quick Tip to Solve: Use the direction of the ratio change to decide whether the balance length increases or decreases before writing the second bridge equation.

48. Seven capacitors, each of capacitance 2 μF, are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right)\mu F$. Which of the combinations, shown in figures below, will achieve the desired value [4 marks]

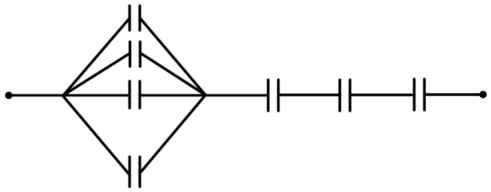
(A)



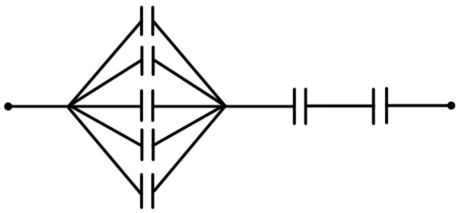
(B)



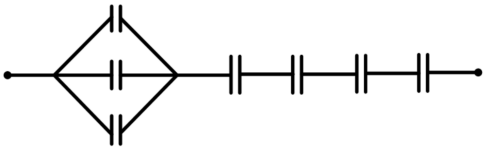
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- Capacitors connected between the same two nodes are in parallel and add directly: $C_p = \sum C_i$. Capacitors on a single series chain instead obey $1/C_s = \sum 1/C_i$.
- Every elementary capacitor has $C = 2\,\mu\text{F}$. In each diagram, first identify the group of branches between the two common junctions, reduce that group, and then combine it with the capacitors on the right-hand series chain.

Step 1: Reduce the first configuration In option (A), the three left-hand capacitors connect between the same two junctions, so they give $C_p = 3C = 6\,\mu\text{F}$. Four further capacitors, each of value C, lie in series with this parallel group. Hence

$$\frac{1}{C_{\text{eq}}} = \frac{1}{3C} + \frac{4}{C} = \frac{13}{3C}, \qquad C_{\text{eq}} = \frac{3C}{13} = \frac{6}{13}\,\mu\text{F}.$$

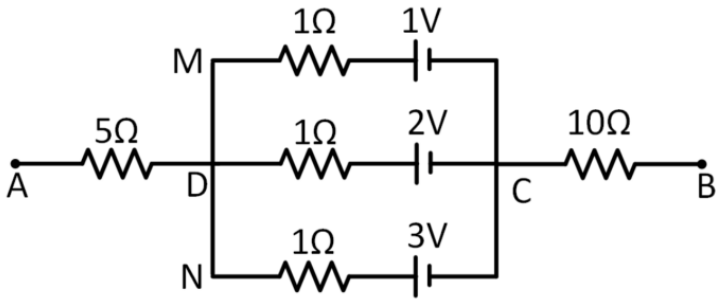
The arrangement also contains precisely seven elementary capacitors: three in the parallel group and four in the chain.

Step 2: Check the competing diagrams Option (B) shows two parallel capacitors followed by four series capacitors, giving $C_{\text{eq}} = C/(4 + 1/2) = 4/9\,\mu\text{F}$. In (C), four capacitors form the parallel group and three are in series afterwards, giving $C_{\text{eq}} = C/(3 + 1/4) = 8/13\,\mu\text{F}$. In (D), five are parallel and two follow in series, giving $C_{\text{eq}} = C/(2 + 1/5) = 10/11\,\mu\text{F}$. None of these equals $6/13\,\mu\text{F}$. Junction connectivity, rather than the visual slopes of the wires, determines which capacitors are parallel.

Step 3: Conclusion Three parallel capacitors in series with four individual capacitors give the required $6/13\,\mu\text{F}$. Hence, the correct option is (A).

Quick Tip to Solve: For n identical capacitors in parallel followed by m identical capacitors in series, use $C_{\text{eq}} = C/(m + 1/n)$, after identifying the common junctions.

49. In the circuit shown, the potential difference between A and B is : [4 marks]



(A) 6 V

(B) 3 V

(C) 2 V

(D) 1 V

Correct Answer: (C) 2 V

Solution:

Concept:

- An open terminal cannot draw steady current. Therefore the external $5\,\Omega$ and $10\,\Omega$ resistors carry no current and have zero voltage drop; $V_A = V_D$ and $V_B = V_C$.
- Cells with unequal emfs connected in parallel can have internal circulating currents even when the external terminals are open. Kirchhoff's current law at either common node determines their common terminal voltage.

Step 1: Write the branch currents Let $V = V_D - V_C$, positive from the left node relative to the right node. All three cells have their positive terminals on the left, and each has a series resistance of $1\,\Omega$. Taking branch current positive from D to C, the voltage relation is $V = ir + E$. Thus

$$i_1 = \frac{V-1}{1}, \qquad i_2 = \frac{V-2}{1}, \qquad i_3 = \frac{V-3}{1}.$$

Here emfs are in volts and resistances in ohms, giving currents in amperes. The external arms provide no additional current path.

Step 2: Apply current conservation and find the terminal voltage With no net external current, the algebraic sum of branch currents is zero:

$$(V-1) + (V-2) + (V-3) = 0, \qquad 3V = 6.$$

Hence $V = 2\,\mathrm{V}$. The branch currents are respectively $+1, 0, -1\,\mathrm{A}$, which confirms that their sum vanishes: the strongest cell drives the weakest one internally. Since there is no drop in either external resistor, $V_A - V_B = V_D - V_C = 2\,\mathrm{V}$. Adding the emfs would apply to series cells, not these parallel branches; choosing one extreme emf also ignores the current balance.

Step 3: Conclusion The potential difference between A and B is $2\,\mathrm{V}$, with A at higher potential. Hence, the correct option is (C).

Quick Tip to Solve: For open-circuit parallel cells, use the conductance-weighted mean of their emfs; equal branch resistances make it the simple arithmetic mean.

50. The region between y = 0 and y = d contains a magnetic field $\vec{B} = B\hat{z}$. A particle of mass m and charge q enters the region with a velocity $\vec{v} = v\hat{i}$. If d $= \frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is : [4 marks]

(A) $\frac{qvB}{m}\left(-\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)$

(B) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right)$

(C) $\frac{qvB}{m}\left(\frac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$

(D) $\frac{qvB}{m}\left(\frac{\hat{j}+\hat{i}}{\sqrt{2}}\right)$

Correct Answer: (A) $\frac{qvB}{m}\left(-\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)$

Solution:

Concept:

- The magnetic force is $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, perpendicular to velocity, so the speed remains v. The acceleration has magnitude qvB/m for the positive charge implicit in the given positive width.
- The orbit inside a uniform field is circular with radius $r = mv/(qB)$. The strip width measures displacement normal to the initial velocity, rather than the arc length traversed.

Step 1: Find the angle swept out in the field The initial force is along $-\hat{j}$, since $\hat{i} \times \hat{k} = -\hat{j}$. Thus the particle enters from the upper boundary and bends downward. If the tangent turns clockwise through θ, its downward displacement is $r(1 - \cos\theta)$. At the opposite boundary this equals the strip width. Consequently,

$$r(1 - \cos\theta) = d = \frac{r}{2} \quad \Longrightarrow \quad \cos\theta = \frac{1}{2}, \qquad \theta = 60^\circ.$$

The first encounter with the opposite boundary uses this angle, not a later point on the circle.

Step 2: Apply the vector force law at emergence The velocity just before leaving the field is therefore

$$\mathbf{v}_{\text{exit}} = v\left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right).$$

Taking its cross product with $B\hat{k}$ gives

$$\mathbf{a} = \frac{qvB}{m}\left(-\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right).$$

Both components are negative because the centre lies left and below the exit point. Option B incorrectly uses positive horizontal acceleration; C and D have equal component magnitudes, corresponding to the wrong deflection angle. The acceleration meant here is its value at the field-side of emergence; after leaving an ideal sharply bounded field it becomes zero.

Step 3: Conclusion The acceleration at emergence is $\frac{qvB}{m}(-\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j})$. Hence, the correct option is (A).

Quick Tip to Solve: Use $d = r(1 - \cos\theta)$ for the transverse displacement, then compute $\mathbf{v} \times \mathbf{B}$; the velocity direction is not the acceleration direction.

51. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to: ($\frac{hc}{e}$ = 1240 nm eV) [4 marks]

(A) 0.5 V

(B) 1.0 V

(C) 2.0 V

(D) 1.5 V

Correct Answer: (B) 1.0 V

Solution:

Concept:

- Einstein's photoelectric equation is $eV_s = hc/\lambda - \phi$, where V_s is stopping potential, e the elementary charge and ϕ the work function of the metal.
- For the same emitting metal, the work function is unchanged. Subtracting two measurements eliminates it, so the change in stopping potential depends only on the photon-energy change.

Step 1: Subtract the photoelectric equations Write the stopping potentials at the two wavelengths as

$$eV_{300} = \frac{hc}{300\,\text{nm}} - \phi, \qquad eV_{400} = \frac{hc}{400\,\text{nm}} - \phi.$$

The longer wavelength has lower photon energy, so it produces the smaller maximum electron kinetic energy and the smaller stopping potential. The requested decrease is the positive difference $V_{300} - V_{400}$, rather than its negative. Assuming photoemission at both wavelengths, subtraction gives

$$\Delta V_s = \frac{hc}{e}\left(\frac{1}{300\,\text{nm}} - \frac{1}{400\,\text{nm}}\right).$$

Step 2: Evaluate and compare with the options The supplied numerical conversion gives photon energies $1240/300 = 4.133\,\text{eV}$ and $1240/400 = 3.100\,\text{eV}$. An electron losing $1\,\text{eV}$ of maximum kinetic energy requires $1\,\text{V}$ less stopping potential. Therefore

$$\Delta V_s = 1240\frac{400 - 300}{300 \times 400}\,\text{V} = \frac{1240}{1200}\,\text{V} \simeq 1.033\,\text{V}.$$

This is closest to $1.0\,\text{V}$. The values 0.5, 1.5 and $2.0\,\text{V}$ do not match the reciprocal-wavelength difference. Neither the work function nor the intensity is needed to calculate this difference.

Step 3: Conclusion The stopping potential decreases by approximately $1.03\,\text{V}$, close to $1.0\,\text{V}$. Hence, the correct option is (B).

Quick Tip to Solve: For a change of wavelength on the same metal, subtract the two Einstein equations first so the unknown work function cancels.

52. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is: [4 marks]

(A) 60°

(B) 45°

(C) 90°

(D) 30°

Correct Answer: (A) 60°

Solution:

Concept:

- For a prism the internal refraction angles satisfy $r_1 + r_2 = A$, where A is the prism angle. An equilateral triangular prism has $A = 60^\circ$.
- At minimum deviation the ray path is symmetric: incidence equals emergence, and $r_1 = r_2 = A/2$. At the air-to-prism surface Snell's law is $\sin i = \mu \sin r_1$.

Step 1: Use symmetry of the minimum-deviation path The statement that the deviation is minimum fixes the internal angles and removes the need to search through arbitrary incident rays. For the equilateral prism,

$$r_1 = r_2 = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ.$$

These angles are measured from the normals to the refracting faces. They are not the external incidence and emergence angles. The equality $i = e$ holds only for this symmetric minimum-deviation condition; it does not mean $i = r_1$, because the ray bends on entering a denser medium.

Step 2: Apply Snell's law at the first face Using the refractive index $\mu = \sqrt{3}$ and air of index approximately one,

$$\sin i = \sqrt{3} \sin 30^\circ = \frac{\sqrt{3}}{2}.$$

The physical incidence angle lies between 0° and 90°, hence $i = 60^\circ$. As a consistency check, symmetry gives $e = 60^\circ$, so $\delta_{\min} = i + e - A = 60^\circ$. An incidence of 30° would incorrectly identify the internal angle with the external one; 45° would require a different refractive index, while 90° is grazing incidence and does not satisfy the calculated sine. Thus only the first listed angle fits both refraction and minimum deviation.

Step 3: Conclusion The angle of incidence is 60°. Hence, the correct option is (A).

Quick Tip to Solve: At minimum deviation set each internal angle equal to half the prism angle before applying Snell's law; distinguish internal refraction from external incidence.

53. In a double-slit experiment, green light (5303$\mathring{A}$) falls on a double slit having a separation of 19.44 μm and awidht of 4.05 μm. The number of bright fringes between the first and the second diffraction minima is : [4 marks]

(A) 04

(B) 05

(C) 10

(D) 09

Correct Answer: (B) 05

Solution:

Concept:

- For slit width a, single-slit diffraction minima obey $a\sin\theta = n\lambda$. The first and second positive-side minima have $n = 1$ and $n = 2$.
- For slit separation d, interference bright fringes occur at $d\sin\theta = m\lambda$, where m is an integer. Count the integer orders strictly inside the specified diffraction interval.

Step 1: Express the diffraction interval in interference orders The two diffraction minima bound the interval

$$\frac{\lambda}{a} < \sin\theta < \frac{2\lambda}{a}.$$

Substitution of $\sin\theta = m\lambda/d$ converts this to

$$\frac{d}{a} < m < \frac{2d}{a}.$$

The wavelength cancels because the same monochromatic light produces both interference and diffraction. Thus its numerical value is unnecessary for this count. We are considering the band between successive first and second minima on one side of the central maximum; the symmetric band on the other side is a separate interval.

Step 2: Count the integer bright-fringe orders Using the given slit dimensions in the same units,

$$\frac{d}{a} = \frac{19.44}{4.05} = 4.8, \qquad \frac{2d}{a} = 9.6.$$

Therefore the allowed orders are

$$m = 5,\ 6,\ 7,\ 8,\ 9.$$

There are five bright fringes between the minima. Neither boundary is an integer interference order, so there is no missing coincident order to remove here. Merely rounding d/a down to four would undercount: the actual count comes from the integers between 4.8 and 9.6. Ten counts both symmetric side bands, whereas nine does not represent the integers in the stated interval. The interference maxima are modulated by the diffraction envelope but the standard fringe-order counting uses these constructive-interference conditions.

Step 3: Conclusion There are 5 bright fringes between the first and second diffraction minima on one side. Hence, the correct option is (B).

Quick Tip to Solve: Convert the two diffraction minima into bounds on interference order: count integers in $d/a < m < 2d/a$, rather than rounding the interval width.

54. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt, is: [4 marks]

(A) 120 Ω

(B) 125 Ω

(C) 80 Ω

(D) 100 Ω

Correct Answer: (C) 80 Ω

Solution:

Concept:

- The figure of merit k of a galvanometer is the current needed per scale division. Thus full-scale current is $I_g = kn$, with n measured from the central zero to one end.
- A voltmeter is obtained by adding a high resistance R_s in series with the galvanometer resistance G. At full-scale reading, $V = I_g(G + R_s)$.

Step 1: Find the full-scale current The instrument has 30 divisions on each side of the zero. A deflection from zero to full scale in either direction is therefore 30 divisions, not 60. Using the specified figure of merit,

$$I_g = (0.005\,\text{A/division})(30\,\text{division}) = 0.15\,\text{A}.$$

The two sides permit opposite polarities of current; their division counts must not be added for a single full-scale voltage. Here the phrase figure of merit denotes a sensitivity parameter and is unrelated to a drawing.

Step 2: Calculate the required series resistance For a maximum voltmeter reading of $15\,\text{V}$, the total resistance of the meter arrangement must be

$$G + R_s = \frac{V}{I_g} = \frac{15}{0.15}\,\Omega = 100\,\Omega.$$

Since the existing galvanometer already contributes $20\,\Omega$, the added resistor is

$$R_s = 100 - 20 = 80\,\Omega.$$

Check directly: the voltage across the galvanometer at full scale is $0.15 \times 20 = 3\,\text{V}$, leaving $12\,\text{V}$ across the added $80\,\Omega$ resistor. These drops add to $15\,\text{V}$. Choosing $100\,\Omega$ mistakes total resistance for added resistance, while $120\,\Omega$ adds the galvanometer resistance instead of subtracting it. The $125\,\Omega$ option also fails the required full-scale current condition.

Step 3: Conclusion The required resistance to connect in series is $80\,\Omega$. Hence, the correct option is (C).

Quick Tip to Solve: Use divisions from the centre to one extreme, calculate V/I_g, and then subtract the galvanometer's own resistance to obtain the added series resistor.

55. A particle of mass m is moving in a straight line with momentum p. Starting at time t = 0, a force F = kt acts in the same direction on the moving particle during time interval T so that its momentum changes from p to 3p. Here k is a constant. The value of T is : [4 marks]

(A) $2\sqrt{\frac{k}{p}}$

(B) $2\sqrt{\frac{p}{k}}$

(C) $\sqrt{\frac{2p}{2}}$

(D) $\sqrt{\frac{2k}{p}}$

Correct Answer: (B) $2\sqrt{\frac{p}{k}}$

Solution:

Concept:

- Newton's second law in momentum form is $F = dp/dt$. A force acting over time produces impulse $\int F\,dt$, equal to final momentum minus initial momentum.
- For the time-dependent force $F(t) = kt$, the force-time graph is a straight line through the origin. Its impulse over duration T is the triangular area $kT^2/2$.

Step 1: Calculate the required impulse The force acts in the same direction as the initial motion, so the momentum increases without a reversal of sign. The given initial momentum is p, and the final momentum is $3p$. Hence the change is

$$\Delta p = 3p - p = 2p.$$

It is this difference, rather than the final momentum alone, that must be equated to the impulse. The particle mass is not needed because the question directly supplies the change in momentum rather than velocities.

Step 2: Integrate the force and solve for time Integrating the instantaneous force from the start to the final time gives

$$\int_0^T kt\,dt = \frac{1}{2}kT^2.$$

Therefore

$$\frac{1}{2}kT^2 = 2p \quad \Longrightarrow \quad T^2 = \frac{4p}{k}, \qquad T = 2\sqrt{\frac{p}{k}}.$$

We take the positive root because T is a duration. A dimensional check is decisive: since $k = F/t$, p/k has dimensions of time squared, whereas k/p has dimensions of inverse time squared. Thus the options involving $\sqrt{k/p}$ cannot represent time. The printed third option also lacks the required dependence on the force coefficient. Using the final force kT as if it acted throughout the interval would overestimate the impulse by a factor of two; the force rises from zero, so its average is $kT/2$.

Step 3: Conclusion The time interval is $T = 2\sqrt{p/k}$. Hence, the correct option is (B).

Quick Tip to Solve: For a force increasing linearly from zero, use the triangular force-time area and equate it to the momentum change, not the final momentum.

56. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self inductance of the coil: [4 marks]

(A) decreases by a factor of $9\sqrt{3}$

(B) increases by a factor of 27

(C) decreases by a factor of 9

(D) increases by a factor of 3

Correct Answer: (D) increases by a factor of 3

Solution:

Concept:

- A wire wound around a closed wooden frame forms a toroidal winding. For uniform turn density $n = N/\ell$, its inductance is $L = \mu n^2 A\ell = \mu N^2 A/\ell$, where A is the cross-sectional area of the frame and ℓ its magnetic path length.
- The triangular shape fixes the length of the closed magnetic path. The area relevant to flux per turn is the cross-section enclosed by each small winding, not the large planar area inside the triangle.

Step 1: Identify the quantities held fixed The wire is wound on the triangular frame, rather than being a collection of large triangular loops. Let each side of the frame have length a, so the path length is $\ell = 3a$. Increasing each side by a factor of three makes ℓ three times as large. The number of turns per unit length n is explicitly unchanged, so $N = n\ell$ also becomes three times as large. The question changes the side lengths of the frame, without changing the cross-section of the winding support; its cross-sectional area A remains the same.

Step 2: Calculate the inductance scaling For a current I, Ampere's law gives the characteristic field in the winding as $B = \mu nI$. Since the turn density and material are unchanged, this field is unchanged at the same current. The flux through each turn, $\Phi = BA$, therefore stays the same. There are now three times as many turns, so the total flux linkage and the self-inductance become three times as large:

$$\frac{L_{\text{new}}}{L_{\text{old}}} = \frac{\mu n^2 A(3\ell)}{\mu n^2 A\ell} = 3.$$

A factor of 27 would additionally scale the winding cross-sectional area by nine; that change is not specified. The large area enclosed by the triangular centreline is not the area of each turn. The options predicting a decrease are inconsistent with the increased number of identical flux-linking turns.

Step 3: Conclusion The self-inductance increases by a factor of 3. Hence, the correct option is (D).

Quick Tip to Solve: For a wire wound on a closed frame, use the winding cross-section in $L = \mu n^2 A\ell$; do not substitute the large area enclosed by the frame.

57. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then : [4 marks]

(A) $K_2 = \frac{K_1}{2}$

(B) $K_2 = 2K_1$

(C) $K_2 = K_1$

(D) $K_2 = \frac{K_1}{4}$

Correct Answer: (B) $K_2 = 2K_1$

Solution:

Concept:

- For a pendulum at small angular amplitude θ_0, the maximum kinetic energy equals its gravitational potential energy at the extreme position.
- The small-angle energy is $K_{\max} = mgL(1-\cos\theta_0) \simeq \frac{1}{2}mgL\theta_0^2$. Linear amplitude $A = L\theta_0$ and angular amplitude θ_0 are different quantities.

Step 1: Use the angular amplitude of the pendulum Taking the stated amplitude as the pendulum angular amplitude θ_0, the vertical rise of the bob at the extreme position is $h = L(1-\cos\theta_0)$. At the mean position this potential energy is entirely kinetic. Thus

$$K_1 = mgL(1-\cos\theta_0) \simeq \frac{1}{2}mgL\theta_0^2.$$

The mass, gravity and angular amplitude are unchanged in the second experiment. Consequently the energy is proportional to the pendulum length. This is the amplitude convention underlying the supplied answer key.

Step 2: Double the length and distinguish linear amplitude For $L_2 = 2L$ at the same angular amplitude,

$$K_2 = mg(2L)(1-\cos\theta_0) = 2K_1.$$

Although $\omega_2^2 = \omega_1^2/2$, the linear amplitude doubles: $A_2 = L_2\theta_0 = 2A_1$. Therefore $K_2/K_1 = (1/2)\times 2^2 = 2$, consistently. The wording does not explicitly specify angular versus linear amplitude. If instead the same linear displacement amplitude were held fixed, $K = mgA^2/(2L)$ would give option (A). Under the angular-amplitude interpretation used by the key, option (B) follows; options (C) and (D) follow neither of these fixed-amplitude conventions.

Step 3: Conclusion For the same angular amplitude, $K_2 = 2K_1$, matching the supplied key. Hence, the correct option is (B). The alternate linear-displacement convention is distinguished in the working.

Quick Tip to Solve: Check the amplitude convention: fixed angle gives $K \propto L$, whereas fixed linear displacement gives $K \propto 1/L$.

58. A particle of mass m and charge q is in an electric and magnetic field given by

$$\overrightarrow{E} = 2\hat{i} + 3\hat{j}; \quad \overrightarrow{B} = 4\hat{j} + 6\hat{k}.$$

The charged particle is shifted from he origin to the point P(x = 1; y = 1) along a straight path. The magnitude of the total work done is : [4 marks]

(A) (2.5) q

(B) (0.35) q

(C) (0.15) q

(D) 5 q

Correct Answer: (D) 5 q

Solution:

Concept:

- The Lorentz force is $\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$. Work is the line integral of force along displacement.
- A magnetic force does no work because $(\mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} = 0$. A uniform electric field does work $W_E = q\mathbf{E} \cdot \Delta\mathbf{r}$.

Step 1: Eliminate the magnetic contribution During a small time interval, $d\mathbf{r} = \mathbf{v}\, dt$. The magnetic work is therefore

$$dW_B = q(\mathbf{v} \times \mathbf{B}) \cdot \mathbf{v}\, dt = 0.$$

This vanishes at every point along the motion, regardless of the numerical components of the magnetic field. The magnetic field can bend the trajectory but cannot supply kinetic energy. Hence neither the particle mass nor the components $4\hat{j} + 6\hat{k}$ are required for the work calculation.

Step 2: Take the electric-field dot product The displacement from the origin to $P(1, 1)$ is $\Delta\mathbf{r} = \hat{i} + \hat{j}$, with zero displacement along the third direction. Since the electric field is constant,

$$W = W_E = q(2\hat{i} + 3\hat{j}) \cdot (\hat{i} + \hat{j}) = q(2 + 3) = 5q.$$

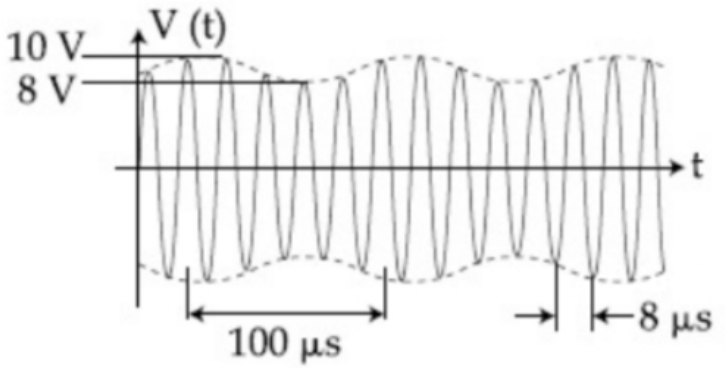
Thus the work magnitude is $5|q|$, or $5q$ using the positive charge magnitude in the choices. There is no factor of one half: that factor would arise for a field varying linearly with position, whereas this field is uniform. The other three coefficients fail the direct dot product.

Step 3: Conclusion The total work has magnitude $5q$ when q denotes the charge magnitude. Hence, the correct option is (D).

Quick Tip to Solve: For work in simultaneous electric and magnetic fields, discard the magnetic term first and dot the electric field with the displacement.

59. An amplitude modulated signal is plotted below :

Which one of the following best describes the above signal ? [4 marks]



(A) (1 + 9sin ($2\pi \times 10^4$ t)) sin($2.5\pi \times 10^5$t) V

(B) (9 + sin ($4\pi \times 10^4$ t)) sin($5\pi \times 10^5$t) V

(C) (9 + sin ($2\pi \times 10^4$ t)) sin($2.5\pi \times 10^5$t) V

(D) (9 + sin ($2.5\pi \times 10^4$ t)) sin($2\pi \times 10^4$t) V

Correct Answer: (C) (9 + sin ($2\pi \times 10^4$ t)) sin($2.5\pi \times 10^5$t) V

Solution:

Concept:

- An amplitude-modulated voltage can be written $V(t) = [A_c + A_m \sin(2\pi f_m t)] \sin(2\pi f_c t)$, with carrier frequency f_c and modulation frequency f_m.
- The envelope extremes give $V_{\max} = A_c + A_m$ and $V_{\min} = A_c - A_m$. The fast oscillations determine the carrier period.

Step 1: Read the voltage envelope The displayed upper envelope varies between $10\,\text{V}$ and $8\,\text{V}$. Adding and subtracting these readings gives

$$A_c = \frac{10+8}{2} = 9\,\text{V}, \qquad A_m = \frac{10-8}{2} = 1\,\text{V}.$$

The envelope therefore has the form $9+\sin(2\pi f_m t)$, and its modulation index is $A_m/A_c = 1/9$. Option (A) interchanges the carrier and modulating amplitudes and would give an overmodulated waveform, inconsistent with the positive envelope drawn.

Step 2: Read both time scales and assemble the signal The slow envelope repeats after $100\,\mu$s, while a single fast carrier cycle lasts $8\,\mu$s. Hence

$$f_m = \frac{1}{100 \times 10^{-6}} = 10^4\,\text{Hz}, \qquad f_c = \frac{1}{8 \times 10^{-6}} = 1.25 \times 10^5\,\text{Hz}.$$

Their angular frequencies are $2\pi \times 10^4$ and $2.5\pi \times 10^5$, respectively. Option (B) doubles both frequencies and halves both measured periods. Option (D) gives a carrier slower than the stated modulation and does not reproduce the fast $8\,\mu$s oscillations.

Step 3: Conclusion The matching signal is $V(t) = [9 + \sin(2\pi \times 10^4 t)] \sin(2.5\pi \times 10^5 t)\,\text{V}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the envelope extremes to determine amplitudes, then read the slow envelope period and the fast carrier period separately.

60. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by : [4 marks]

(A) 1 rad/s

(B) 10^{-3} rad/s

(C) 10^{-1} rad/s

(D) 10^{-5} rad/s

Correct Answer: (B) 10^{-3} rad/s

Solution:

Concept:

- The small-angle frequency obeys $\omega^2 = g_{\text{eff}}/L$. Vertical support acceleration changes the effective gravity in the support frame.
- For a support oscillation of amplitude a and angular frequency Ω, its acceleration amplitude is $a\Omega^2$. A small gravity change gives $\delta\omega/\omega \simeq \delta g/(2g)$.

Step 1: Find the size of the gravity perturbation The initial values $L = 1\,\text{m}$ and $\omega_0 = 10\,\text{rad}\,\text{s}^{-1}$ imply

$$g = \omega_0^2 L = 100\,\text{m}\,\text{s}^{-2}.$$

For the moving support, $a = 10^{-2}\,\text{m}$ and $\Omega = 1\,\text{rad}\,\text{s}^{-1}$. Its acceleration amplitude is

$$\delta g_{\max} = a\Omega^2 = 10^{-2}\,\text{m}\,\text{s}^{-2}.$$

The sign varies through the support cycle, so the pendulum frequency is weakly modulated rather than shifted by one fixed amount. The ratio $\Omega/\omega_0 = 0.1$ also permits the slow-support estimate.

Step 2: Estimate the fractional frequency change Expand the square root to first order, since $\delta g/g = 10^{-4}$ is small:

$$\omega \simeq \omega_0\left(1 + \frac{\delta g}{2g}\right), \qquad \frac{|\delta\omega|_{\max}}{\omega_0} \simeq \frac{10^{-2}}{2 \times 100} = 5 \times 10^{-5}.$$

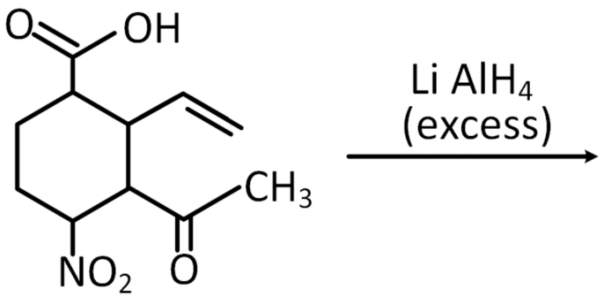
The corresponding absolute change is $|\delta\omega|_{\max} = 5 \times 10^{-4}\,\text{rad}\,\text{s}^{-1}$. The question calls the change relative but lists angular-frequency units. Interpreting the options in their printed units, the best order estimate for the absolute change is $10^{-3}\,\text{rad}\,\text{s}^{-1}$; values 1 and 10^{-1} are far too large, and 10^{-5} is too small for that absolute change.

Step 3: Conclusion The question is ambiguous as worded: the literal relative change is 5×10^{-5}, dimensionless, so none of the frequency-valued choices is dimensionally valid for that demand. If the choices intend the absolute frequency change, $5 \times 10^{-4}\,\text{rad}\,\text{s}^{-1}$, the supplied key selects (B) as its order estimate. Hence, the correct option is (B) only under that absolute-change interpretation.

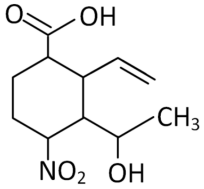
Quick Tip to Solve: Keep fractional and absolute changes distinct: first find $\delta\omega/\omega_0$, then multiply by ω_0 when the choices have frequency units.

Chemistry (120 marks)

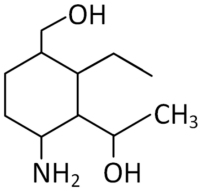
61. The major product obtained in the following reaction is : [4 marks]



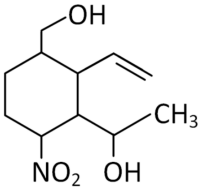
(A) Acid and nitro groups unchanged; acetyl reduced to $-CH(OH)CH_3$; vinyl retained.



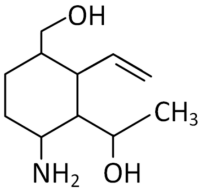
(B) Acid reduced to $-CH_2OH$, nitro to $-NH_2$, acetyl to $-CH(OH)CH_3$, and vinyl hydrogenated to ethyl.



(C) Acid reduced to $-CH_2OH$, acetyl to $-CH(OH)CH_3$; nitro and vinyl retained.

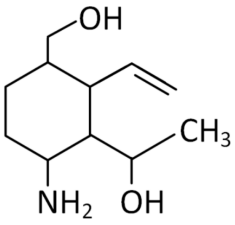


(D) Acid reduced to $-CH_2OH$, nitro to $-NH_2$, acetyl to $-CH(OH)CH_3$; vinyl retained.



Correct Answer: (D)

Acid reduced to $-CH_2OH$, nitro to $-NH_2$, acetyl to $-CH(OH)CH_3$; vinyl retained.



Solution:

Concept:

- Excess $LiAlH_4$ reduces carboxylic acids to primary alcohols, ketones to secondary alcohols and nitro groups to primary amino groups after work-up.
- An isolated carbon-carbon double bond ordinarily remains unchanged under $LiAlH_4$ reduction. Track every functional group independently.

Step 1: Identify all reducible groups The cyclohexane framework carries four substituents: a carboxylic acid $-COOH$, a vinyl group $-CH{=}CH_2$, an acetyl group $-COCH_3$ and a nitro group $-NO_2$. With excess reagent, the acid is not left unchanged merely because it initially consumes hydride through an acid-base reaction. Further hydride reduces its carbonyl carbon, so the attached carboxyl group becomes $-CH_2OH$. The ketone substituent separately becomes $-CH(OH)CH_3$. Neither conversion breaks the bond connecting that substituent to the ring.

Step 2: Complete the reduction and compare the options The nitro substituent is reduced to $-NH_2$, while the isolated vinyl group remains $-CH{=}CH_2$. Thus the required product has hydroxymethyl, vinyl, 1-hydroxyethyl and amino substituents at the same four ring positions as their precursors. Option (A) reduces only the ketone and incorrectly retains both the acid and nitro group. Option (B) also hydrogenates the isolated double bond to an ethyl group, which is not the usual action of this reagent. Option (C) correctly changes the acid and ketone but leaves the nitro group unreduced. Only option (D) simultaneously gives both alcohols and the amine while retaining the carbon-carbon double bond.

Step 3: Conclusion The product in (D) contains $-CH_2OH$, $-CH(OH)CH_3$ and $-NH_2$, with the vinyl double bond unchanged. Hence, the correct option is (D).

Quick Tip to Solve: With excess $LiAlH_4$, check every polar reducible group, but leave an isolated $C{=}C$ bond unchanged.

62. Given the equilibrium constant:

K_C of the reaction :

$Cu(s) + 2Ag^+(aq) \rightarrow Cu^{2+}(aq) + 2Ag(s)$ is

10×10^{15}, calculate the E^0_{cell} of this reaciton at 298 K

[2.303 $\frac{RT}{F}$ at 298 K = 0.059V] [4 marks]

(A) 0.4736 mV

(B) 0.04736 V

(C) 0.4736 V

(D) 0.04736 mV

Correct Answer: (C) 0.4736 V

Solution:

Concept:

- Combine $\Delta G^\circ = -nFE^\circ$ with $\Delta G^\circ = -RT\ln K$. The result is

$$E^\circ = \frac{2.303RT}{nF}\log_{10} K.$$

- The electron number n is determined by the balanced redox reaction, rather than by the sum of all stoichiometric coefficients.

Step 1: Determine the transferred electron number The oxidation half-reaction is $\mathrm{Cu} \longrightarrow \mathrm{Cu}^{2+} + 2\,\mathrm{e}^-$, and the reduction half-reaction is $2\,\mathrm{Ag}^+ + 2\,\mathrm{e}^- \longrightarrow 2\,\mathrm{Ag}$. Adding them cancels two electrons, so $n = 2$. Metallic copper and silver have unit activity in the equilibrium expression. The given equilibrium constant is

$$K = 10 \times 10^{15} = 10^{16}, \qquad \log_{10} K = 16.$$

In particular, the leading factor of ten must not be lost: using 15 for the logarithm would correspond to a different constant.

Step 2: Substitute and match the voltage scale At $298\,\mathrm{K}$, inserting the stated rounded factor gives

$$E^\circ = \frac{0.059}{2} \times 16 = 0.472\,\mathrm{V}.$$

The tabulated choice $0.4736\,\mathrm{V}$ results from the customary more precise factor $0.0592\,\mathrm{V}$: $(0.0592/2) \times 16 = 0.4736\,\mathrm{V}$. This is the intended matching option at about $0.47\,\mathrm{V}$. The small discrepancy is due to the rounded factor printed in the question, not a different electron count. Option (B) is smaller by a factor of ten; options (A) and (D) use millivolts and are smaller still. A large positive K must give a positive standard cell potential.

Step 3: Conclusion The standard potential is approximately $0.47\,\mathrm{V}$, matching $0.4736\,\mathrm{V}$. Hence, the correct option is (C).

Quick Tip to Solve: First convert 10×10^{15} to 10^{16}, then divide the logarithmic factor by the two transferred electrons; retain volt units.

63. The coordination number of Th in $K_4[Th(C_2O_4)_4(OH_2)_2]$ is :
($C_2O_4^{2-}$ = Oxalato) [4 marks]

(A) 14

(B) 10

(C) 8

(D) 6

Correct Answer: (B) 10

Solution:

Concept:

- The coordination number counts donor atoms directly bonded to the central metal, rather than counting ligand molecules or all atoms in the formula.
- Oxalato, ${C_2O_4}^{2-}$, is bidentate and supplies two oxygen donors; each coordinated water molecule is monodentate and supplies one oxygen donor.

Step 1: Count the donor atoms inside the coordination sphere The species inside square brackets is $[Th(C_2O_4)_4(OH_2)_2]^{4-}$. There are four oxalato ligands. Each chelates thorium through two oxygen atoms, so together they provide

$$N_{\text{oxalato}} = 4 \times 2 = 8.$$

Although each oxalate contains four oxygen atoms, its usual bidentate coordination uses two donor atoms on that particular metal centre. Counting every oxygen atom in the molecular formula would therefore overcount the coordination number.

Step 2: Include water and exclude the counterions The two OH_2 groups represent two water molecules bonded through oxygen. They are neutral, monodentate ligands and add

$$N_{\text{water}} = 2 \times 1 = 2.$$

Thus the thorium coordination number is

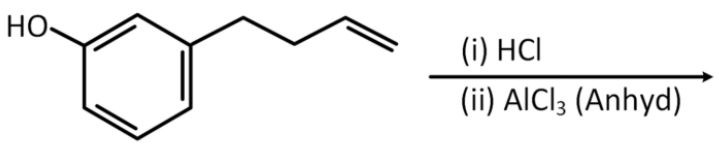
$$\text{CN(Th)} = 8 + 2 = 10.$$

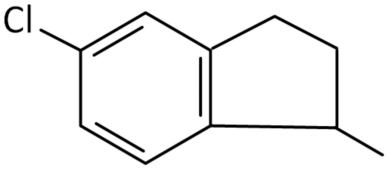
The four potassium ions lie outside the brackets and balance the anionic complex; they are not donor atoms attached to thorium. Option (C), 8, omits the two coordinated waters. Option (D), 6, counts ligand molecules without their denticity. Option (A), 14, incorrectly adds the four external counterions to the actual ten donor atoms. None of these alternatives follows the definition of coordination number.

Step 3: Conclusion Thorium is directly coordinated to ten oxygen donor atoms. Hence, the correct option is (B).

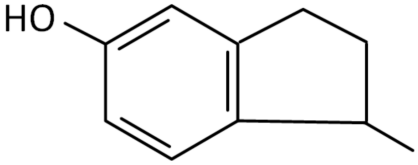
Quick Tip to Solve: Multiply each ligand count by its denticity, add the coordinated waters, and ignore all ions outside the square brackets.

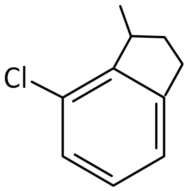
64. The major product of the following reaction is : [4 marks]



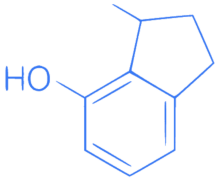
(A) Chloro-substituted methylindane shown 

(B) Hydroxy-substituted methylindane shown, cyclized para to hydroxyl

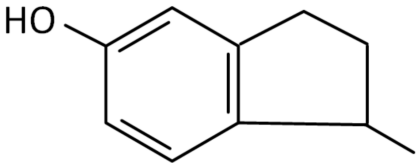


(C) Chloro-substituted alternative methylindane shown 

(D) Hydroxy-substituted alternative methylindane shown, cyclized ortho to hydroxyl



Correct Answer: (B)
Hydroxy-substituted methylindane shown, cyclized para to hydroxyl



Solution:

Concept:

- Addition of HCl to an unsymmetrical alkene follows Markovnikov orientation: the more stable carbocation gives the major alkyl chloride.
- Anhydrous $AlCl_3$ activates an alkyl chloride for intramolecular electrophilic aromatic substitution. A phenolic OH group directs substitution towards its ortho and para positions.

Step 1: Apply Markovnikov addition The side chain is

$$\mathrm{Ar-CH_2-CH_2-CH=CH_2}.$$

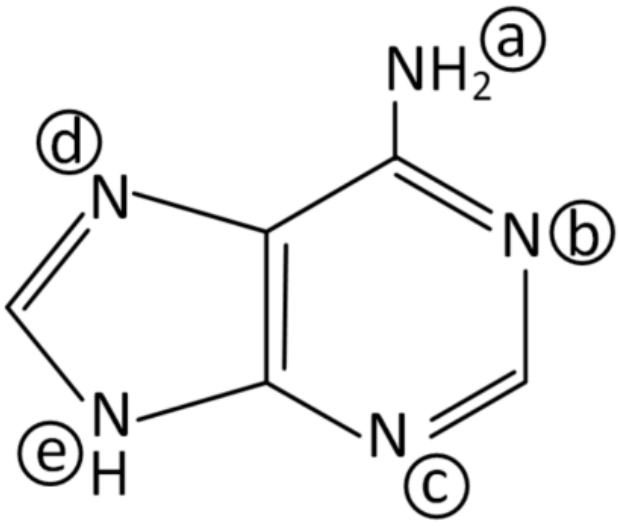
Protonation at the terminal carbon produces the secondary carbocation at the adjacent carbon. Chloride capture therefore gives $\mathrm{Ar-CH_2-CH_2-CH(Cl)-CH_3}$. Reversing this orientation would require an unstable primary carbocation and does not give the major product.

Step 2: Close the five-membered ring at the preferred aromatic site Activation of the secondary chloride by $\mathrm{AlCl_3}$ allows the aromatic ring to attack intramolecularly. Attack at the carbon adjacent to the side-chain attachment closes a five-membered ring. Of the two such positions, the site para to the phenolic group is less crowded than the site lying between the hydroxyl group and the side chain. Deprotonation restores aromaticity, giving the hydroxy-substituted methylindane shown in (B). The phenolic carbon–oxygen bond is retained; these conditions do not replace aromatic OH by chlorine. Thus (A) and (C) have the wrong substituent, while (D) represents the less favourable ortho cyclization.

Step 3: Conclusion The major product is the methylindane retaining the phenolic OH group, with cyclization para to it, as drawn in (B). Hence, the correct option is (B).

Quick Tip to Solve: First mark the carbon bearing chlorine after Markovnikov addition; count the ring formed by connecting that carbon to each neighbouring aromatic site.

65. In the following compound,
the favourable site/s for protonation is/are : [4 marks]



(A) (b), (c) and (d)
(B) (a) and (e)
(C) (a) and (d)
(D) (a)
Correct Answer: (A) (b), (c) and (d)

Solution:

Concept:

- Protonation is favoured at a nitrogen whose lone pair remains available for bonding to H^+.
- A pyridine-like nitrogen has its lone pair outside the aromatic pi system; a pyrrole-like nitrogen contributes its lone pair to aromaticity. An exocyclic amino lone pair can be delocalized into the ring.

Step 1: Identify the available ring-nitrogen lone pairs The nitrogens labelled (b) and (c) in the six-membered ring each have a pyridine-like bonding pattern. Their lone pairs are not the electrons that form the aromatic pi bonds. Nitrogen (d) in the five-membered ring is also imine-like: it has a double bond to carbon and a lone pair available outside the aromatic pi system. Protonation at these sites therefore does not require removal of an aromatic lone pair.

Step 2: Exclude the nitrogens with delocalized lone pairs The nitrogen at (e) already bears hydrogen and is pyrrole-like. Its lone pair participates in aromatic delocalization, so binding another proton at that lone pair sacrifices important aromatic stabilization. At (a), the exocyclic amino lone pair is conjugated with the electron-deficient fused ring. Its resonance donation makes it less available than the pyridine-like ring lone pairs. Consequently, the favourable set in the alternatives is (b), (c) and (d), rather than sets containing (a) or (e). This comparison concerns the favourable sites represented by the options, rather than an assertion that all nitrogens have equal basicity.

Step 3: Conclusion The favourable protonation sites are the three ring nitrogens labelled b, c and d. Hence, the correct option is (A).

Quick Tip to Solve: For heterocyclic nitrogen, draw the lone pair before judging basicity: a lone pair used to maintain aromaticity is much less available for protonation.

66. **A compound 'X' on treatment with Br_2/NaOH, provided C_3H_9N, which gives positive carbylamine test. Compound 'X' is :** [4 marks]

(A) $CH_3CH_2CH_2CONH_2$

(B) $CH_3CON(CH_3)_2$

(C) $CH_3CH_2COCH_2NH$

(D) $CH_3COCH_2NHCH_3$

Correct Answer: (A) $CH_3CH_2CH_2CONH_2$

Solution:

Concept:

- Hofmann bromamide degradation converts a primary amide $RCONH_2$ into a primary amine RNH_2, with loss of the carbonyl carbon.
- The carbylamine test is positive for primary amines: reaction with chloroform and alcoholic alkali forms an isocyanide.

Step 1: Infer the required amide from the product formula The product C_3H_9N gives the carbylamine test, so it is a primary amine rather than a secondary or tertiary amine. Under Br_2/NaOH, its precursor

must therefore have the primary-amide group $-CONH_2$. Because the degradation removes one carbon atom, the precursor must contain four carbons. In option (A), the group attached to the carbonyl is propyl, exactly the group required in propylamine.

Step 2: Apply the degradation and eliminate the alternatives The balanced net reaction is

$$CH_3CH_2CH_2CONH_2 + Br_2 + 4\,NaOH \longrightarrow CH_3CH_2CH_2NH_2 + Na_2CO_3 + 2\,NaBr + 2\,H_2O.$$

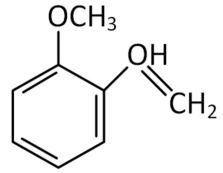
The organic product is propan-1-amine, which has molecular formula C_3H_9N and a primary amino group. Option (B) is an N,N-disubstituted amide, lacking the primary-amide hydrogens needed for the usual Hofmann rearrangement. Options (C) and (D), as printed, do not contain the required $-CONH_2$ group directly attached to the carbonyl carbon; they cannot undergo this conversion to the specified primary amine by the stated reaction. The carbon count and the functional-group requirement independently select (A).

Step 3: Conclusion Compound X is butanamide, $CH_3CH_2CH_2CONH_2$. Its Hofmann degradation yields C_3H_9N, a primary amine. Hence, the correct option is (A).

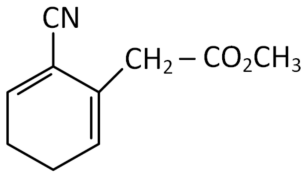
Quick Tip to Solve: Work backwards in Hofmann degradation: replace the product amino group by $-CONH_2$, adding one carbon to reconstruct the precursor.

67. Which of the following compounds reacts with ethylmagnesium bromide and also decolourizes bromine water solution: [4 marks]

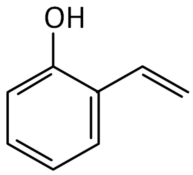
(A) The methoxy-substituted benzene with the oxygen-containing unsaturated substituent shown



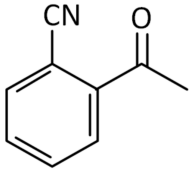
(B) The nonaromatic cyclohexadiene bearing cyano and methyl acetate side-chain groups shown



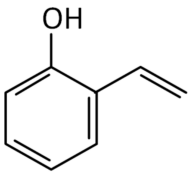
(C) 2-Vinylphenol



(D) 2-Acetylbenzonitrile



Correct Answer: (C)

2-Vinylphenol 

Solution:

Concept:

- An organomagnesium reagent reacts with an acidic hydroxyl proton by proton transfer and with electrophilic carbonyl or nitrile groups by nucleophilic addition.
- Bromine water is decolourized by reactive nonaromatic carbon–carbon double bonds and by strongly activated phenolic aromatic rings.

Step 1: Test the phenolic compound in option (C) Option (C) has both a phenolic hydroxyl group and a vinyl substituent. The hydroxyl proton reacts immediately with ethylmagnesium bromide, forming ethane and a magnesium phenoxide:

$$\mathrm{ArOH} + \mathrm{C_2H_5MgBr} \longrightarrow \mathrm{ArOMgBr} + \mathrm{C_2H_6}.$$

The compound also decolourizes bromine water: its side-chain double bond can add bromine, and its phenolic ring is activated towards bromination. Thus this compound certainly satisfies both requirements; an aromatic ring alone would not suffice for the ordinary bromine-water test.

Step 2: Compare the remaining structures and the accepted alternatives Option (B) is a nonaromatic cyclohexadiene bearing a nitrile group and an ester-containing side chain. Its carbon–carbon double bonds consume bromine, while its nitrile and ester functions react with a Grignard reagent by nucleophilic addition. It therefore also satisfies the two tests. Option (D) has a ketone and a nitrile that react with the Grignard reagent, but its benzene ring does not undergo the ordinary bromine-water decolourization test. Option (A) is printed with an unusual oxygen-containing unsaturated substituent; it is not needed to identify either of the two accepted, chemically clear alternatives. Distinguish an alkene double bond from the delocalized bonds of an unactivated benzene ring when applying the second test.

Step 3: Conclusion Option (C) reacts with ethylmagnesium bromide and decolourizes bromine water. Hence, the correct option is (C). The official final key accepts both the second and third options; the second option also satisfies the two tests through its nitrile/ester groups and nonaromatic double bonds.

Quick Tip to Solve: Apply the two tests independently: find an acidic proton or electrophilic group for the Grignard reagent, then check for alkene unsaturation or an activated phenolic ring.

68. Which of the following compounds will produce a precipitate with $AgNO_3$? [4 marks]

(A) 3-Bromopyridine 

(B) 7-Bromocyclohepta-1,3,5-triene 

(C) Bromocyclohexane 

(D) Bromobenzene 

Correct Answer: (B) 7-Bromocyclohepta-1,3,5-triene 

Solution:

Concept:

- Silver nitrate promotes halide ionization by removing released bromide as insoluble AgBr. The ease of reaction depends strongly on the stability of the carbocation formed.
- The cycloheptatrienyl (tropylium) cation has a planar cyclic conjugated system with six pi electrons and is aromatic by the $4n + 2$ rule.

Step 1: Examine ionization of the seven-membered-ring bromide In option (B), bromine is attached to the saturated carbon of cyclohepta-1,3,5-triene. Loss of bromide makes that carbon planar and gives a continuous ring of seven p orbitals. The three double bonds supply six pi electrons, so the resulting tropylium cation is aromatic and strongly stabilized. Bromide is trapped by silver ion:

$$Ag^{+} + Br^{-} \longrightarrow AgBr\downarrow.$$

This favourable ionization accounts for the characteristic precipitate expected for (B).

Step 2: Compare the other carbon–bromine bonds In (A) bromine is attached directly to an aromatic heterocyclic carbon, and in (D) it is attached directly to benzene. These aryl carbon–bromine bonds resist ordinary ionization: the corresponding aryl cations are unstable, and the bonds have partial double-bond character. Option (C) is cyclohexyl bromide; loss of bromide would give an ordinary secondary cyclohexyl cation with no aromatic stabilization. It is much less readily ionized than the cycloheptatrienyl derivative. Under the qualitative silver nitrate test intended here, the readily formed aromatic cation makes (B) the diagnostic choice. The comparison is based on facile precipitation, rather than claiming that secondary alkyl halides can never react under prolonged or more forcing solvolysis conditions.

Step 3: Conclusion The cycloheptatrienyl bromide in (B) readily yields the aromatic tropylium cation and an AgBr precipitate. Hence, the correct option is (B).

Quick Tip to Solve: When a cyclic halide ionizes, count pi electrons in the resulting cation; a six-pi-electron tropylium ring is exceptionally stabilized.

69. The number of bridging CO ligand(s) and Co-Co bond(s) in $Co_2(CO)_8$, respectively are : [4 marks]

(A) 2 and 0

(B) 0 and 2

(C) 4 and 0

(D) 2 and 1

Correct Answer: (D) 2 and 1

Solution:

Concept:

- The familiar bridged structure of $Co_2(CO)_8$ contains six terminal carbonyl ligands and two carbonyl ligands that bridge the two cobalt atoms.
- In neutral electron counting, cobalt contributes nine valence electrons, a terminal carbonyl donates two electrons to one metal, and a bridging carbonyl contributes one electron to each metal.

Step 1: Count the carbonyl ligands in the bridged structure Of the eight carbonyl ligands, three terminal ligands attach to each cobalt atom, giving six terminal ligands in total. The other two carbonyl groups each connect both cobalt atoms and are therefore bridging ligands. A bridging ligand is counted once in the molecular formula even though it binds both metals. Thus the number of bridging carbonyls is two, not four.

Step 2: Check the metal–metal bond by electron counting For either cobalt atom, the neutral electron count before the metal–metal bond is

$$9 + 3(2) + 2(1) = 17.$$

A single cobalt–cobalt bond contributes one electron to the count of each cobalt, bringing each metal to eighteen electrons. Therefore the usual bridged structure has one Co – Co bond. Option (A) omits this bond and leaves the metals at seventeen electrons in this counting scheme. Option (B) assigns the wrong carbonyl bridging pattern and two metal–metal bonds, and (C) incorrectly doubles the number of bridges. Although other structural forms of this carbonyl can occur, the standard bridged structure tested by the listed pairs is uniquely described by two bridges and one metal–metal bond.

Step 3: Conclusion The numbers of bridging carbonyl ligands and cobalt–cobalt bonds are 2 and 1, respectively. Hence, the correct option is (D).

Quick Tip to Solve: For bridged $Co_2(CO)_8$, remember six terminal plus two bridging carbonyls, then verify the single metal–metal bond with the eighteen-electron count.

70.

$$\underline{A} \xrightarrow{4KOH,O_2} \underset{(Green)}{2\underline{B}} + 2H_2O$$

$$3\underline{B} \xrightarrow{4HCl} \underset{(Purple)}{2\underline{C}} + MnO_2 + 2H_2O$$

$$2\underline{C} \xrightarrow{H_2O.KI} \underset{(Purple)}{2\underline{A}} + 2KOH + \underline{D}$$

In the above sequence of reactions, $\underline{A}$ and $\underline{D}$, respectively, are : [4 marks]

(A) Kl and $KMnO_4$

(B) Kl and K_2MnO_4

(C) KlO_3 and MnO_2

(D) MnO_2 and KlO_3

Correct Answer: (D) MnO_2 and KlO_3

Solution:

Concept:

- Manganese dioxide is oxidized by oxygen in hot concentrated alkali to green potassium manganate, K_2MnO_4.
- Manganate disproportionates in acid to purple permanganate and manganese dioxide. In neutral or mildly alkaline medium, permanganate can oxidize iodide to iodate while being reduced to manganese dioxide.

Step 1: Identify the green and purple manganese compounds The first conversion is the alkaline oxidation

$$2\,MnO_2 + 4\,KOH + O_2 \longrightarrow 2\,K_2MnO_4 + 2\,H_2O.$$

Hence A is MnO_2 and green B is K_2MnO_4. Acidification gives

$$3\,K_2MnO_4 + 4\,HCl \longrightarrow 2\,KMnO_4 + MnO_2 + 4\,KCl + 2\,H_2O.$$

Thus purple C is $KMnO_4$. Manganese changes from oxidation state +6 in manganate into +7 and +4 in the two disproportionation products.

Step 2: Use the final oxidation of iodide to identify D The final net reaction is

$$2\,KMnO_4 + KI + H_2O \longrightarrow 2\,MnO_2 + 2\,KOH + KIO_3.$$

Two manganese atoms each gain three electrons on changing from +7 to +4; iodide loses six electrons in changing from -1 to +5 in iodate. Both the atom balance and electron balance therefore identify D as KIO_3. The printed purple annotation beside regenerated A is inconsistent with the brown manganese dioxide formed in this reaction; the identities follow unambiguously from the reaction sequence. The other alternatives reverse the roles of the manganese and iodine compounds or assign iodide as the starting manganese compound.

Step 3: Conclusion $A = MnO_2$ and $D = KIO_3$. Hence, the correct option is (D).

Quick Tip to Solve: Use the colours to identify green manganate and purple permanganate first; then balance manganese and iodine oxidation-state changes in the last reaction.

71. The standard reaction Gibbs energy for a chemical reaction at an absolute temperature T is given by $\Delta_r G^o = A - BT$

Where A and B are non-zero constants. Which of the following is TRUE about this reaction? [4 marks]

(A) Exothermic if B < 0

(B) Endothermic if A > 0

(C) Exothermic if A > 0 and B < 0

(D) Endothermic if A < 0 and B > 0

Correct Answer: (B) Endothermic if A > 0

Solution:

Concept:

- The standard Gibbs energy is related to enthalpy and entropy by $\Delta_r G^\circ = \Delta_r H^\circ - T\Delta_r S^\circ$, where T is the absolute temperature.
- An endothermic reaction has $\Delta_r H^\circ > 0$; an exothermic reaction has $\Delta_r H^\circ < 0$. The sign of entropy alone does not decide the heat absorbed or released.

Step 1: Compare the temperature dependence The given expression is $\Delta_r G^\circ = A - BT$. Comparing it with the thermodynamic identity gives

$$\Delta_r H^\circ = A, \qquad \Delta_r S^\circ = B.$$

Equivalently, since A and B are constants, $-\partial\Delta_r G^\circ/\partial T = B$, and adding $T\Delta_r S^\circ$ to the Gibbs energy leaves A.

Step 2: Use the enthalpy sign to test the options If $A > 0$, the standard reaction enthalpy is positive and the reaction is endothermic, irrespective of the sign of B. Thus option (B) is true. Option (A) uses the sign of entropy to infer heat flow, which is invalid. Option (C) calls a positive-enthalpy reaction exothermic and is false. Option (D) calls a negative-enthalpy reaction endothermic and is also false. Spontaneity at a particular temperature is a different question and would require the sign of the entire expression $A - BT$.

Step 3: Conclusion The reaction is endothermic if $A > 0$, because $A = \Delta_r H^\circ$. Hence, the correct option is (B).

Quick Tip to Solve: In a linear expression $\Delta G = A - BT$, identify the intercept with enthalpy and the negative slope with entropy before judging heat flow.

72. The reaction $2X \rightarrow B$ is a zeroth order reaction. If the initial concentration of X is 0.2 M, the half-life is 6 h. When the initial concentration of X is 0.5 M, the time required to reach its final concentration of 0.2 M will be: [4 marks]

(A) 18.0 h

(B) 9.0 h

(C) 7.2 h

(D) 12.0 h

Correct Answer: (A) 18.0 h

Solution:

Concept:

- For a zero-order disappearance of a reactant, its concentration decreases linearly: $[X]_t = [X]_0 - k_X t$, where k_X is the constant rate of disappearance of X.
- The zero-order half-life is $t_{1/2} = [X]_0/(2k_X)$. It depends on the initial concentration, unlike a first-order half-life.

Step 1: Find the constant concentration decrease In the first experiment, the concentration falls from $0.2\,\mathrm{M}$ to $0.1\,\mathrm{M}$ in $6\,\mathrm{h}$. Hence

$$k_X = \frac{0.2 - 0.1}{6} = \frac{1}{60}\,\mathrm{M\,h^{-1}}.$$

This definition uses the disappearance of X directly. If reaction rate is instead defined with the stoichiometric factor $1/2$, its rate constant is half as large, but the concentration law and final time remain unchanged.

Step 2: Apply the same rate to the second experiment The required concentration loss is now $0.5 - 0.2 = 0.3\,\mathrm{M}$, three times the loss in the first experiment. Because the reaction is zero order, the rate does not change with concentration. Thus

$$t = \frac{0.5 - 0.2}{1/60} = 18.0\,\mathrm{h}.$$

The other proposed times would remove only 0.15, 0.12, or $0.20\,\mathrm{M}$, respectively, and would not reach the specified final concentration. Using the old half-life unchanged would incorrectly treat the reaction as first order.

Step 3: Conclusion The concentration reaches $0.2\,\mathrm{M}$ after $18.0\,\mathrm{h}$. Hence, the correct option is (A).

Quick Tip to Solve: For zero order, compare concentration losses: three times the loss takes three times the time, since the disappearance rate is constant.

73. The reaction, MgO(s) + C(s) $\rightarrow$ Mg(s) + CO(g), for which $\Delta_r H^o$ + 491.1 kJ mol^{-1} and $\Delta_r S^o$ = 198.0 JK^{-1} mol^{-1}, is not feasible at 298 K. Temperature above which reaciton will be feasible is : [4 marks]

(A) 2480.3 K

(B) 2040.5 K

(C) 2380.5 K

(D) 1890.0 K

Correct Answer: (A) 2480.3 K

Solution:

Concept:

- A reaction is thermodynamically feasible in the standard-state sense when $\Delta_r G^\circ = \Delta_r H^\circ - T\Delta_r S^\circ < 0$.
- When both enthalpy and entropy changes are positive, heating favours feasibility. The crossover temperature is $T_0 = \Delta_r H^\circ / \Delta_r S^\circ$, with consistent energy units.

Step 1: Convert the enthalpy to the entropy energy unit For $\mathrm{MgO(s) + C(s) \longrightarrow Mg(s) + CO(g)}$, the supplied positive enthalpy change is $491.1\,\mathrm{kJ\,mol^{-1}}$, while entropy is in joules. Therefore

$$\Delta_r H^\circ = 491100\,\mathrm{J\,mol^{-1}}, \qquad \Delta_r S^\circ = 198.0\,\mathrm{J\,K^{-1}\,mol^{-1}}.$$

The production of gas is consistent with the positive entropy change. At low temperature the positive enthalpy term dominates.

Step 2: Find the crossover and interpret the inequality At the boundary, $\Delta_r G^\circ = 0$, so

$$T_0 = \frac{491100}{198.0} = 2480.303\ldots\ \mathrm{K}.$$

Since $\Delta_r S^\circ > 0$, the Gibbs energy becomes negative for $T > T_0$, not for temperatures below it. At $298\,\mathrm{K}$, $\Delta_r G^\circ = 491.1 - 298(0.198) = 432.096\,\mathrm{kJ\,mol^{-1}} > 0$, confirming the stated lack of feasibility. Each of the other offered temperatures is below the crossover and still gives positive standard Gibbs energy.

Step 3: Conclusion The reaction becomes feasible above approximately $2480.3\,\mathrm{K}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert kJ to J before dividing ΔH by ΔS; a positive entropy change makes the high-temperature side favourable.

74. K_2HgI_4 is 40% ionised in aqueous solution. The value of its van't Hoff factor (i) is: [4 marks]

(A) 1.6

(B) 2.2

(C) 2.0

(D) 1.8

Correct Answer: (D) 1.8

Solution:

Concept:

- The van't Hoff factor i is the ratio of the actual number of solute particles to the number of undissociated formula units initially dissolved.
- For a fraction α dissociating into ν ions per formula unit, particle counting gives $i = (1-\alpha) + \nu\alpha = 1 + (\nu - 1)\alpha$.

Step 1: Count the ions from one formula unit Potassium tetraiodomercurate(II) contains a complex anion. Its ionisation in water is

$$\mathrm{K_2[HgI_4]} \longrightarrow 2\,\mathrm{K^+} + \mathrm{[HgI_4]^{2-}}.$$

Thus each dissociated formula unit produces three independently moving ions: two potassium ions and one complex ion. The four iodide ligands remain within the complex anion; they are not counted as four additional free iodide ions.

Step 2: Combine the dissociated and undissociated particles The degree of ionisation is $\alpha = 40/100 = 0.40$. Starting with one mole of formula units leaves 0.60 mol undissociated and produces 0.80 mol potassium ions plus 0.40 mol complex ions. Hence

$$i = 0.60 + 0.80 + 0.40 = 1.80.$$

Equivalently, $i = 1 + (3-1)(0.40) = 1.8$. The value must lie between 1 for no ionisation and 3 for complete ionisation. Option 1.6 would use the wrong particle increase, while 2.0 and 2.2 correspond to larger degrees of ionisation than the stated 40 percent.

Step 3: Conclusion The van't Hoff factor is $i = 1.8$. Hence, the correct option is (D).

Quick Tip to Solve: Keep a complex ion intact when counting dissociation particles; $\mathrm{K_2[HgI_4]}$ yields three ions, not separate mercury and iodide ions.

75. For the equilibrium,

$2H_2O \rightleftharpoons H_3O^+ + OH^-$, the value of ΔG^o at 298 K is approximately : [4 marks]

(A) $-$ 80 kJ mol^{-1}

(B) 100 kJ mol^{-1}

(C) $-$ 100 kJ mol^{-1}

(D) 80 kJ mol^{-1}

Correct Answer: (D) 80 kJ mol^{-1}

Solution:

Concept:

- The standard reaction Gibbs energy is $\Delta_r G^\circ = -RT \ln K$, where K is the dimensionless equilibrium constant and R is the gas constant.
- For the autoionisation $2\,H_2O \rightleftharpoons H_3O^+ + OH^-$ at $298\,K$, the activity of liquid water is unity and $K = K_w \simeq 10^{-14}$.

Step 1: Insert the ionisation constant The very small equilibrium constant shows that ion formation is strongly unfavourable relative to liquid water under standard-state conditions. Therefore the standard Gibbs energy should be positive. Using $R = 8.314\,J\,K^{-1}\,mol^{-1}$,

$$\Delta_r G^\circ = -8.314(298)\ln(10^{-14}).$$

Since $\ln(10^{-14}) = -14\ln 10$, the two negative signs cancel.

Step 2: Evaluate and compare the proposed magnitudes Converting the natural logarithm to a common logarithm gives

$$\Delta_r G^\circ = 2.303(8.314)(298)(14) \simeq 7.99 \times 10^4\,J\,mol^{-1} \simeq 80\,kJ\,mol^{-1}.$$

The two negative options have the wrong sign for $K \ll 1$, and the positive 100 option is not the calculated magnitude. Although the actual Gibbs energy is zero at equilibrium, the question asks for $\Delta_r G^\circ$, not $\Delta_r G$. At equilibrium the nonstandard activity term $RT \ln Q$ cancels this positive standard value.

Step 3: Conclusion The standard Gibbs energy change is approximately $+80\,kJ\,mol^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: For water ionisation at 298 K, use $pK_w = 14$: $\Delta G^\circ = 2.303RTpK_w$, which is positive.

76. The correct match between Item I and Item II is:

Item I		Item II	
(A)	Ester test	(P)	Tyr
(B)	Carbylamine test	(Q)	Asp
(C)	Phthalein dye test	(R)	Ser
		(S)	Lys

[4 marks]

(A) $(A) \rightarrow (Q);\ (B) \rightarrow (S);\ (C) \rightarrow (P)$

(B) $(A) \rightarrow (R);\ (B) \rightarrow (S);\ (C) \rightarrow (Q)$

(C) $(A) \rightarrow (R);\ (B) \rightarrow (Q);\ (C) \rightarrow (P)$

(D) $(A) \rightarrow (Q);\ (B) \rightarrow (S);\ (C) \rightarrow (R)$

Correct Answer: (A) $(A) \rightarrow (Q);\ (B) \rightarrow (S);\ (C) \rightarrow (P)$

Solution:

Concept:

- Amino acids have a common amino-acid backbone, but their side chains give characteristic functional-group reactions. Asp, Lys, Tyr and Ser denote aspartic acid, lysine, tyrosine and serine.
- An additional carboxyl group is associated with ester formation, a primary amino side chain gives the carbylamine test, and a phenolic side chain gives the phthalein dye test.

Step 1: Identify the ester and carbylamine matches Aspartic acid (Asp, Q) has the side chain $-CH_2COOH$. Its additional carboxyl group undergoes esterification with an alcohol in acidic conditions, giving the intended ester-test match $(A) \rightarrow (Q)$. Lysine (Lys, S) has a side chain $-(CH_2)_4NH_2$, providing a primary amino group. With chloroform and alcoholic alkali, a primary amine gives an isocyanide in the carbylamine test. Thus $(B) \rightarrow (S)$. These matches refer to the characteristic side-chain groups rather than merely the common amino-acid backbone.

Step 2: Identify the phenolic test and eliminate alternatives Tyrosine (Tyr, P) contains a phenolic $-C_6H_4OH$ group. Phenols react with phthalic anhydride in concentrated sulfuric acid and give a coloured phthalein dye in alkaline medium. Therefore $(C) \rightarrow (P)$. Serine has an aliphatic hydroxyl side chain, not a phenolic ring, so it cannot replace tyrosine for this test. Options (B) and (D) assign the phthalein test to Asp or Ser, respectively, and option (C) assigns the primary-amine test to Asp instead of Lys.

Step 3: Conclusion The ester, carbylamine and phthalein dye tests match Asp, Lys and Tyr, respectively. Hence, the correct option is (A).

Quick Tip to Solve: For amino-acid matching tests, inspect side chains first: Asp has extra carboxyl, Lys extra amino, Tyr phenolic hydroxyl and Ser alcoholic hydroxyl.

77. The correct option with respect to the Pauling electronegativity values of the element is : [4 marks]

(A) Ga $<$ Ge

(B) Si $<$ Al

(C) Te $>$ Se

(D) P $>$ S

Correct Answer: (A) Ga $<$ Ge

Solution:

Concept:

- Electronegativity measures an atom's tendency to attract a shared pair of electrons. Pauling values generally increase from left to right across a period.
- Down a group, increased size and shielding usually lower electronegativity. Numerical values can be used to check close comparisons and avoid reversing a trend.

Step 1: Check the gallium and germanium comparison Gallium and germanium are successive elements in the same period, with germanium to the right of gallium. Their approximate Pauling electronegativities are

$$\chi(\mathrm{Ga}) = 1.81, \qquad \chi(\mathrm{Ge}) = 2.01.$$

Thus $\chi(\mathrm{Ga}) < \chi(\mathrm{Ge})$, so the inequality in option (A) is correct. The greater effective attraction across the period makes germanium attract bonding electrons more strongly.

Step 2: Test each remaining inequality Silicon lies to the right of aluminium and has $\chi(\mathrm{Si}) \simeq 1.90$, compared with $\chi(\mathrm{Al}) \simeq 1.61$; therefore the proposed $\mathrm{Si} < \mathrm{Al}$ is reversed. Tellurium lies below selenium, with values about 2.10 and 2.55, respectively, so $\mathrm{Te} > \mathrm{Se}$ is also reversed. Finally, sulfur lies to the right of phosphorus: $\chi(\mathrm{P}) \simeq 2.19$ and $\chi(\mathrm{S}) \simeq 2.58$. Hence $\mathrm{P} > \mathrm{S}$ is false as well. All three distractors contradict the corresponding period or group trend.

Step 3: Conclusion The true Pauling electronegativity comparison is $\mathrm{Ga} < \mathrm{Ge}$. Hence, the correct option is (A).

Quick Tip to Solve: Check the pair positions before recalling numbers: moving right usually raises electronegativity, while moving down a group usually lowers it.

78. The de Broglie wavelength (λ) associated with a photoelectron varies with the frequency (v) of the incident radiation as, [v_0 is threshold frequency] : [4 marks]

(A) $\lambda \propto \frac{1}{(v-v_0)^{\frac{3}{2}}}$

(B) $\lambda \propto \frac{1}{(v-v_0)^{\frac{1}{4}}}$

(C) $\lambda \propto \frac{1}{(v-v_0)}$

(D) $\lambda \propto \frac{1}{(v-v_0)^{\frac{1}{2}}}$

Correct Answer: (D) $\lambda \propto \frac{1}{(v-v_0)^{\frac{1}{2}}}$

Solution:

Concept:

- Einstein's photoelectric equation gives the maximum electron kinetic energy: $K = h\nu - \phi = h(\nu - \nu_0)$, where h is Planck's constant and $\phi = h\nu_0$ is the work function.
- The de Broglie relation is $\lambda = h/p$. For a nonrelativistic electron of mass m_e, $K = p^2/(2m_e)$, so wavelength varies as the inverse square root of kinetic energy.

Step 1: Express the electron momentum in terms of frequency Only the photon energy above the work function becomes electron kinetic energy. Thus the threshold frequency must be subtracted before finding the momentum. Combining the energy and momentum equations gives

$$\frac{p^2}{2m_e} = h(\nu - \nu_0), \qquad p = \sqrt{2m_e h(\nu - \nu_0)}.$$

This expression applies to emitted electrons when $\nu > \nu_0$.

Step 2: Obtain the wavelength and compare the powers Substitute this momentum into the matter-wave relation:

$$\lambda = \frac{h}{\sqrt{2m_e h(\nu - \nu_0)}} = \sqrt{\frac{h}{2m_e}}(\nu - \nu_0)^{-1/2}.$$

The factor outside the frequency term is constant. The powers $-3/2$, $-1/4$, and -1 in the other options therefore do not follow from the kinetic-energy relation. In particular, momentum is proportional to the square root of energy, rather than to energy itself.

Step 3: Conclusion $\lambda \propto (\nu - \nu_0)^{-1/2}$. Hence, the correct option is (D).

Quick Tip to Solve: Combine $K = h(\nu - \nu_0)$ with $\lambda = h/\sqrt{2m_e K}$; take the square root only after subtracting the threshold frequency.

79. 25 ml of the given HCl solution requires 30 mL of 0.1 M sodium carbonate solution. What is the volume of this HCl solution required to titrate 30 mL of 0.2 M aqueous NaOH solutions ? [4 marks]

(A) 50 mL

(B) 12.5 mL

(C) 25 mL

(D) 75 mL

Correct Answer: (C) 25 mL

Solution:

Concept:

- Molarity times volume in litres gives moles: $n = MV$. Neutralisation must be compared using the balanced reaction, because carbonate consumes two moles of acid per mole.
- Sodium hydroxide reacts with hydrochloric acid in a one-to-one molar ratio, whereas sodium carbonate reacts in a one-to-two ratio.

Step 1: Standardise the hydrochloric acid solution The complete carbonate neutralisation is

$$\mathrm{Na_2CO_3} + 2\,\mathrm{HCl} \longrightarrow 2\,\mathrm{NaCl} + \mathrm{H_2O} + \mathrm{CO_2}.$$

The carbonate sample contains $0.1 \times 0.030 = 0.003$ mol. Therefore, the 25 mL acid sample contains $2 \times 0.003 = 0.006$ mol of HCl. Its molarity is

$$M_{\mathrm{HCl}} = \frac{0.006}{0.025} = 0.24\,\mathrm{mol\,L^{-1}}.$$

The factor of two is essential: using equal moles of carbonate and acid would underestimate the acid concentration.

Step 2: Calculate the acid volume for sodium hydroxide For the second titration, $\mathrm{HCl} + \mathrm{NaOH} \longrightarrow \mathrm{NaCl} + \mathrm{H_2O}$. The sodium hydroxide contains

$$n_{\mathrm{NaOH}} = 0.2 \times 0.030 = 0.006\,\mathrm{mol}.$$

It needs the same number of moles of acid, and hence

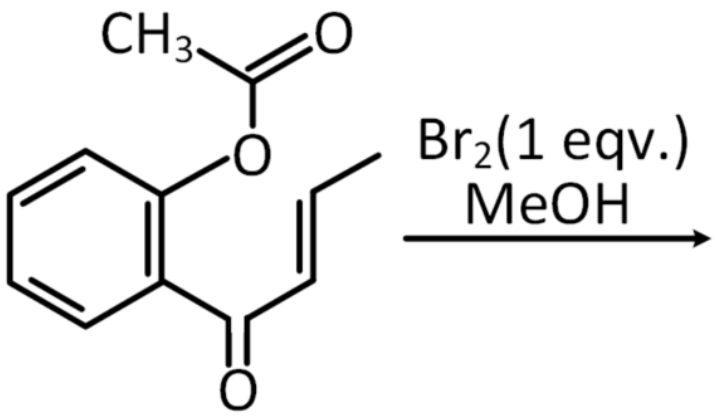
$$V_{\mathrm{HCl}} = \frac{0.006}{0.24} = 0.025\,\mathrm{L} = 25\,\mathrm{mL}.$$

The 50 mL choice results from omitting the carbonate factor of two; 12.5 mL and 75 mL also fail the one-to-one mole requirement for the second titration.

Step 3: Conclusion The required hydrochloric acid volume is $25\,\mathrm{mL}$. Hence, the correct option is (C).

Quick Tip to Solve: Compare acid equivalents: 30 mL of 0.1 M carbonate and 30 mL of 0.2 M hydroxide each consume 0.006 mol of HCl.

80. The major product obtained in the following conversion is [4 marks]



(A) $\mathrm{ArCOCH(Br)CH(OCH_3)CH_3}$, where Ar is 2-acetoxyphenyl.

(B) The starting enone with Br ortho to the acetoxy group on the aromatic ring.

(C) The starting enone with Br para to the acetoxy group on the aromatic ring.

(D) $\mathrm{ArCOCH(OCH_3)CH(Br)CH_3}$, where Ar is 2-acetoxyphenyl.

Correct Answer: (A) $\mathrm{ArCOCH(Br)CH(OCH_3)CH_3}$, where Ar is 2-acetoxyphenyl.

Solution:

Concept:

- The substrate contains an α,β-unsaturated ketone. Number the side-chain carbon adjacent to the carbonyl as α, and the carbon bearing the terminal methyl group as β.
- With bromine in methanol, addition across the alkene can incorporate bromine and methoxy groups. The solvent is a nucleophile and is not simply an inert medium.

Step 1: Identify the reactive double bond and solvent participation Write the reacting side chain as $\mathrm{ArCOCH{=}CHCH_3}$, with Ar the printed 2-acetoxyphenyl group. The alkene reacts with electrophilic bromine to generate a bromonium-like intermediate. Methanol opens this intermediate, and subsequent loss of a proton gives a neutral methoxy group. The ketone carbonyl and the aromatic acetoxy substituent remain intact during this addition. Aromatic substitution products would leave the side-chain double bond unchanged and do not represent the favoured conversion under these conditions.

Step 2: Place bromine and methoxy on the correct carbons The carbonyl substituent withdraws electron density from the adjacent alkene carbon. Opening places the methanol-derived methoxy group on the β carbon, while bromine remains at the α carbon. Thus the side chain becomes

$$\mathrm{ArCOCH(Br)CH(OCH_3)CH_3}.$$

This is the structure drawn in option (A). Option (D) reverses the positions of bromine and methoxy. Options (B) and (C) instead brominate different aromatic-ring positions and retain the original enone double bond. The use of methanol specifically supports the alkene addition product rather than those ring-substitution choices.

Step 3: Conclusion The major product is the α-bromo-β-methoxy ketone shown in (A). Hence, the

correct option is (A).

Quick Tip to Solve: For bromine in an alcohol solvent, check whether the alkene product contains one Br and one alkoxy group; do not automatically choose aromatic bromination.

81. The reaction that does NOT define calcination is : [4 marks]

(A) $2Cu_2S + 3O_2 \xrightarrow{\Delta} 2Cu_2O + 2SO_2$

(B) $CaCO_3 . MgCO_3 \xrightarrow{\Delta} CaO + MgO + 2CO_2$

(C) $ZnCO_3 \xrightarrow{\Delta} ZnO + CO_2$

(D) $Fe_2O_3 . XH_2O \xrightarrow{\Delta} Fe_2O_3 + XH_2O$

Correct Answer: (A) $2Cu_2S + 3O_2 \xrightarrow{\Delta} 2Cu_2O + 2SO_2$

Solution:

Concept:

- Calcination is strong heating of an ore in the absence or limited supply of air. It commonly removes water of hydration or carbon dioxide, converting hydrated oxides or carbonates into oxides.
- Roasting heats a sulphide ore in an adequate supply of oxygen. Oxygen is a reactant, and sulphur is removed as sulphur dioxide.

Step 1: Identify the reaction that consumes oxygen In option (A), the sulphide reacts directly with oxygen:

$$2\,Cu_2S + 3\,O_2 \xrightarrow{\Delta} 2\,Cu_2O + 2\,SO_2.$$

Sulphur changes from sulphide in the ore to sulphur dioxide. This is oxidative heating, so it is an example of roasting. The fact that an oxide is produced does not by itself make the process calcination; both roasting and calcination can produce oxides, and the atmosphere and starting compound distinguish them.

Step 2: Check why the remaining reactions are calcination Option (B) decomposes the double carbonate into CaO, MgO, and CO_2. Option (C) similarly decomposes zinc carbonate:

$$ZnCO_3 \xrightarrow{\Delta} ZnO + CO_2.$$

Neither reaction requires oxygen as a reactant. Option (D) expels water from hydrated iron(III) oxide, leaving the anhydrous oxide. These are the characteristic carbonate-decomposition and dehydration changes of calcination. Thus (B), (C), and (D) all define calcination, while (A) requires an oxygen supply and belongs to roasting.

Step 3: Conclusion The oxidation of Cu_2S to Cu_2O and SO_2 is roasting, not calcination. Hence, the correct option is (A).

Quick Tip to Solve: Look for oxygen on the reactant side: sulphide plus oxygen giving sulphur dioxide identifies roasting; loss of water or carbon dioxide identifies calcination.

82. Taj Mahal is being slowly disfigured and discoloured. This is primarily due to : [4 marks]

(A) acid rain

(B) soil pollution

(C) global warming

(D) water pollution

Correct Answer: (A) acid rain

Solution:

Concept:

- Acid rain contains acids produced from atmospheric sulphur oxides and nitrogen oxides. These pollutants dissolve and oxidise in atmospheric moisture, increasing the acidity of precipitation.
- The marble of the Taj Mahal contains calcium carbonate, $CaCO_3$. Carbonates react with acids, so acidic deposition attacks the surface and damages its appearance.

Step 1: Connect air pollutants to acidic deposition Industrial emissions and fuel combustion release SO_2 and nitrogen oxides. Their atmospheric reactions lead mainly to sulphuric and nitric acids, which are carried to the monument in rain or deposited on damp surfaces. This establishes a direct chemical route from polluted air to deterioration of the marble. The phrase disfigured and discoloured therefore points to acid rain and associated atmospheric deposition, rather than to a general temperature trend.

Step 2: Show how the acid damages marble and reject the alternatives A representative attack by sulphuric acid is

$$CaCO_3 + H_2SO_4 \longrightarrow CaSO_4 + CO_2 + H_2O.$$

Reaction converts the original carbonate surface into calcium sulphate and releases carbon dioxide. Repeated attack erodes details, alters the surface, and contributes to disfigurement. Soil pollution does not provide this direct reaction at the exposed marble surface. Global warming describes a climate effect rather than the primary chemical cause here, and water pollution concerns contaminated water bodies rather than acidic atmospheric deposition. Hence only acid rain matches the mechanism asked about.

Step 3: Conclusion Acid rain is the primary cause among the listed choices. Hence, the correct option is (A).

Quick Tip to Solve: When a marble monument is damaged, link its $CaCO_3$ to acid attack; the carbonate reaction distinguishes acid rain from unrelated pollution categories.

83. The homopolymer formed from 4-hydroxy-butanoic acid is : [4 marks]

(A) $[-CO(CH_2)_2COO-]_n$

(B) $[-OCO(CH_2)_3O-]_n$

(C) $[-CO(CH_2)_3O-]_n$

(D) $[-CO(CH_2)_2CO-]_n$

Correct Answer: (C) $[-CO(CH_2)_3O-]_n$

Solution:

Concept:

- 4-Hydroxybutanoic acid is $HOCH_2CH_2CH_2COOH$, containing both an alcohol group and a carboxylic acid group in each monomer molecule.
- A hydroxy acid undergoes self-condensation: an alcohol group of one molecule reacts with the carboxyl group of another, forming an ester linkage and eliminating water. Only one monomer type is involved, so the product is a homopolymer.

Step 1: Locate the groups that form each chain linkage The monomer has three methylene groups between its hydroxyl oxygen and carboxyl carbon: $HO-(CH_2)_3-COOH$. During condensation, the hydroxyl end of one unit joins the acid end of the next. The chain therefore contains recurring ester groups, $-CO-O-$, while all three methylene groups of each monomer remain in the backbone. The acid carbonyl is retained; no reaction introduces a second carbonyl into each monomer-derived unit.

Step 2: Write the repeat unit and compare the choices The repeat unit can be written in either direction as

$$[-O(CH_2)_3CO-]_n \equiv [-CO(CH_2)_3O-]_n .$$

This is exactly the connectivity in option (C). Joining the right-hand oxygen of one repeat to the left-hand carbonyl carbon of the next creates the ester linkage. Option (A) has two carbonyl groups and only two methylenes; option (B) contains an extra backbone oxygen; option (D) has two carbonyls but lacks the required ester oxygen. Each therefore fails the atom and connectivity check for the given hydroxy-acid monomer.

Step 3: Conclusion The homopolymer has repeat unit $[-CO(CH_2)_3O-]_n$, a polyester. Hence, the correct option is (C).

Quick Tip to Solve: Count the three methylene groups before joining the ends; a hydroxy acid preserves one carbonyl and one ester oxygen per monomer-derived repeat.

84. The relative stability of +1 oxidation state of group 13 elements follows the order : [4 marks]

(A) Tl < In < Ga < Al

(B) Al < Ga < TI < In

(C) AI < Ga < In < TI

(D) Ga < AI < In < TI

Correct Answer: (C) AI < Ga < In < TI

Solution:

Concept:

- Group 13 elements have outer electronic configuration ns^2np^1. The +1 oxidation state results when the np electron participates while the ns^2 pair remains unshared.
- The inert-pair effect becomes stronger down the group. Consequently the lower oxidation state, +1, becomes progressively more stable relative to +3 from aluminium to thallium.

Step 1: Explain why the lower oxidation state is stabilised In the +3 state all three valence electrons are involved, whereas in the +1 state only the outer p electron is involved and the s pair is retained. In the heavier group members, poor shielding by inner d and f electrons and the resulting strong effective attraction for the s electrons make this pair less available for bonding. This stabilises the +1 state down the group. Thus aluminium strongly favours +3, while thallium shows a particularly stable +1 state.

Step 2: Arrange the elements and eliminate inconsistent orders The order of the listed elements down group 13 is aluminium, gallium, indium, thallium. Applying the increasing inert-pair effect gives

$$\mathrm{Al} < \mathrm{Ga} < \mathrm{In} < \mathrm{Tl} \qquad (+1\text{-state stability}).$$

Option (A) reverses the trend and would make aluminium(I) the most stable. Option (B) incorrectly puts indium(I) above thallium(I), despite the stronger inert-pair effect in thallium. Option (D) incorrectly puts gallium(I) below aluminium(I). Option (C) alone follows the complete down-group trend and the enhanced stability of thallium(I).

Step 3: Conclusion The relative stability is $\mathrm{Al} < \mathrm{Ga} < \mathrm{In} < \mathrm{Tl}$. Hence, the correct option is (C).

Quick Tip to Solve: For group 13, increasing inert-pair effect stabilises +1 down the group; distinguish this from the opposite trend in the relative stability of +3.

85. The radius of the largest sphere which fits properly at the centre of the edge of a body centred cubic unit cell is : (Edge length is represented by 'a') [4 marks]

(A) 0.134 a

(B) 0.067 a

(C) 0.047 a

(D) 0.027 a

Correct Answer: (B) 0.067 a

Solution:

Concept:

- In a body-centred cubic (bcc) unit cell, atoms touch along the body diagonal, so the host-atom radius R obeys $4R = \sqrt{3}a$, where a is the edge length.
- The largest sphere at a specified empty position is limited by the nearest host atoms. Its radius r equals the shortest centre-to-centre distance minus R.

Step 1: Calculate the host-atom radius The body diagonal passes through one corner centre, the body-centred atom and the opposite corner centre. The total touching length is four host radii. Therefore,

$$R = \frac{\sqrt{3}}{4}a \simeq 0.4330a.$$

The edge-centre position is not a regular occupied lattice position of bcc; the question asks about the space available between the existing atoms.

Step 2: Find the limiting neighbours of the edge centre Take the edge centre as $(a/2, 0, 0)$. Its distance from either endpoint $(0, 0, 0)$ or $(a, 0, 0)$ is $a/2$. Its distance from the body centre $(a/2, a/2, a/2)$ is $a/\sqrt{2}$, which is larger. Thus the corner atoms at the ends of the edge provide the tighter restriction, and tangency requires

$$R + r = \frac{a}{2}.$$

Step 3: Subtract the occupied radius and compare Substitution gives

$$r = \frac{a}{2} - \frac{\sqrt{3}a}{4} = \frac{2 - \sqrt{3}}{4}a \simeq 0.06699a.$$

The proposed $0.134a$ is twice this radius and corresponds to the available diameter. Values $0.047a$ and $0.027a$ are smaller spheres that could fit, but neither is the largest possible sphere.

Step 4: Conclusion The largest sphere has radius $r \simeq 0.067a$. Hence, the correct option is (B).

Quick Tip to Solve: At an edge centre first check the two endpoint atoms; use $a/2 - R$, and do not confuse the free diameter with the free radius.

86. Match the following items in column I with the corresponding items in column II.

Item I		Item II	
(i)	$Na_2CO_3.10H_2O$	(A)	Portland cement ingredient
(ii)	$Mg(HCO_3)_2$	(B)	Castner-Kellner process
(iii)	NaOH	(C)	Solvay process
(iv)	$Ca_3Al_2O_6$	(D)	Temporary hardness

[4 marks]

(A) (i)→(D); (ii)→(A); (iii)→(B); (iv)→(C)

(B) (i)→(B); (ii)→(C); (iii)→(A); (iv)→(D)

(C) (i)→(C); (ii)→(B); (iii)→(D); (iv)→(A)

(D) (i)→(C); (ii)→(D); (iii)→(B); (iv)→(A)

Correct Answer: (D) (i)→(C); (ii)→(D); (iii)→(B); (iv)→(A)

Solution:

Concept:

- Match a compound with its characteristic preparation, use or effect. Washing soda is $Na_2CO_3 \cdot 10\,H_2O$, whereas caustic soda is $NaOH$; their industrial processes are different.
- Temporary hardness is caused by dissolved calcium and magnesium hydrogencarbonates. Tricalcium aluminate, $Ca_3Al_2O_6$, is a constituent of Portland cement.

Step 1: Match washing soda and magnesium hydrogencarbonate The Solvay process produces sodium carbonate through precipitation of sodium hydrogencarbonate followed by heating. Crystallisation with water gives washing soda, $Na_2CO_3 \cdot 10\,H_2O$. Thus (i) matches (C). Dissolved $Mg(HCO_3)_2$ contributes to temporary hardness, which can be removed by boiling; it is not a cement ingredient or the product of either sodium-alkali process. Hence (ii) matches (D).

Step 2: Match caustic soda and the cement compound In the Castner–Kellner process, electrolysis of brine using a mercury cathode forms sodium amalgam; its reaction with water gives sodium hydroxide. Thus $NaOH$, item (iii), matches (B). Tricalcium aluminate is one of the major clinker compounds in Portland cement, so item (iv), $Ca_3Al_2O_6$, matches (A).

Step 3: Select the complete consistent mapping Collecting the four independent identifications gives

$$(\mathrm{i}) \to \mathrm{C}, \quad (\mathrm{ii}) \to \mathrm{D}, \quad (\mathrm{iii}) \to \mathrm{B}, \quad (\mathrm{iv}) \to \mathrm{A}.$$

Option (A) incorrectly associates washing soda with hardness and magnesium hydrogencarbonate with cement. Option (B) assigns caustic soda to cement, while option (C) interchanges the hardness and caustic-soda matches. Only the fourth option satisfies all four identifications.

Step 4: Conclusion The four compounds match Solvay process, temporary hardness, Castner–Kellner process and Portland cement, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Anchor the matching with $NaOH$: Castner–Kellner makes caustic soda, while Solvay makes sodium carbonate.

87. The higher concentration of which gas in air can cause stiffness of flower buds? [4 marks]

(A) CO_2

(B) SO_2

(C) NO_2

(D) CO

Correct Answer: (B) SO_2

Solution:

Concept:

- Air pollutants produce characteristic effects on plants. The effect depends on the chemical nature and concentration of the gas, not simply on whether the gas occurs in air.
- High concentrations of sulphur dioxide, SO_2, injure vegetation and can cause stiffness of flower buds, so the symptom identifies the sulphur oxide among these choices.

Step 1: Identify the pollutant associated with the symptom Sulphur dioxide is released particularly by combustion of sulphur-containing fossil fuels and by some metallurgical operations. It enters plant tissues and harms sensitive growing organs. A high concentration can cause flower buds to become stiff and eventually fall from the plant. The wording asks for this characteristic injury rather than a general greenhouse effect or a gas toxic to human blood. Thus the relevant gas is SO_2.

Step 2: Distinguish the remaining gases Carbon dioxide, CO_2, is a reactant in photosynthesis and its increased atmospheric concentration is chiefly associated with the greenhouse effect; it is not the characteristic answer for stiffness of flower buds. Nitrogen dioxide, NO_2, is an oxidising air pollutant involved in photochemical smog and plant damage, but it does not give the specific standard association asked here. Carbon monoxide, CO, is chiefly discussed as a poisonous gas because it binds strongly to haemoglobin and reduces oxygen transport in humans.

Step 3: Use the specific effect to select the option Although several listed gases can be undesirable at sufficiently high concentrations, the named flower-bud symptom specifically points to sulphur dioxide. Selecting a gas merely because it is a pollutant would not distinguish SO_2, NO_2 and CO. Match the precise biological effect to the chemical species: here that eliminates (A), (C) and (D).

Step 4: Conclusion A higher concentration of SO_2 can cause stiffness of flower buds. Hence, the correct option is (B).

Quick Tip to Solve: For pollutant-effect questions, match the precise symptom: sulphur dioxide is associated with stiff flower buds and damage to vegetation.

88. The correct match between Item I and Item II is :

Item I		Item II	
(A)	Allosteric effect	(P)	Molecule binding to the active site of enzyme
(B)	Competitive inhibitor	(Q)	Molecule crucial for communication in the body
(C)	Receptor	(R)	Molecule binding to a site other than the active site of enzyme
(D)	Poison	(S)	Molecule binding to the enzyme covalently

[4 marks]

(A) (A)→(P); (B)→(R); (C)→(Q); (D)→(S)

(B) (A)→(P); (B)→(R); (C)→(S); (D)→(Q)

(C) (A)→(R); (B)→(P); (C)→(S); (D)→(Q)

(D) (A)→(R); (B)→(P); (C)→(Q); (D)→(S)

Correct Answer: (D) (A)→(R); (B)→(P); (C)→(Q); (D)→(S)

Solution:

Concept:

- An enzyme has an active site where its substrate binds. A competitive inhibitor binds at this site; an allosteric molecule binds at another site and alters the enzyme's activity.
- Receptors are proteins involved in chemical communication in the body. In the given matching, a poison is associated with covalent binding that can irreversibly inactivate an enzyme.

Step 1: Match the two enzyme-binding mechanisms Allosteric action is exerted through a site different from the active site. Binding there can change the shape or activity of the active site, even though the allosteric molecule does not compete for the substrate's exact position. Hence (A) matches (R). A competitive inhibitor resembles the substrate sufficiently to occupy the active site and competes with substrate binding, so (B) matches (P).

Step 2: Match the communication and poison entries Receptors receive chemical messengers and help transmit their signals, making them crucial for communication in the body. Thus receptor, entry (C), matches description (Q). Covalent attachment of a poison to an enzyme can prevent its normal action irreversibly, so entry (D) matches description (S). This association concerns the mechanism specified in the table; it does not claim that every possible poison acts by the same biochemical mechanism.

Step 3: Compare all four options The full correspondence is

$$\mathrm{A} \to \mathrm{R}, \quad \mathrm{B} \to \mathrm{P}, \quad \mathrm{C} \to \mathrm{Q}, \quad \mathrm{D} \to \mathrm{S}.$$

The first two options put allosteric binding at the active site and therefore fail. The third correctly places allosteric and competitive binding but wrongly interchanges receptor communication and covalent poison binding. The fourth gives each of the four associations correctly.

Step 4: Conclusion The correct match is A–R, B–P, C–Q, D–S. Hence, the correct option is (D).

Quick Tip to Solve: First separate active-site competition from allosteric binding elsewhere; then pair receptors with communication and covalent binding with enzyme poisoning.

89. Among the colloids cheese (C), milk (M), and smoke (S), the correct combination of the dispersed phase and dispersion medium, respectively is : [4 marks]

(A) C : liquid in solid ; M : liquid in liquid ; S : solid in gas

(B) C : liquid in solid ; M : liquid in solid ; S : solid in gas

(C) C : solid in liquid ; M : liquid in liquid ; S : gas in solid

(D) C : solid in liquid ; M : solid in liquid ; S : solid in gas

Correct Answer: (A) C : liquid in solid ; M : liquid in liquid ; S : solid in gas

Solution:

Concept:

- A colloid consists of a dispersed phase distributed through a continuous dispersion medium. Name the dispersed phase first and the medium second.
- A gel is liquid dispersed in solid, an emulsion is liquid dispersed in liquid, and a solid aerosol is solid dispersed in gas. The overall appearance alone does not identify both phases.

Step 1: Classify cheese Cheese is conventionally classified as a gel. Liquid is trapped within a continuous solid network, so its dispersed phase is liquid and its dispersion medium is solid. Therefore C corresponds to liquid in solid. It is not classified as solid in liquid merely because the finished food appears predominantly solid; the classification concerns the dispersed component and the supporting medium.

Step 2: Classify milk and smoke Milk is an emulsion: droplets of liquid fat form the dispersed phase in an aqueous liquid dispersion medium. Thus M is liquid in liquid. Smoke contains fine solid particles dispersed in air. Its dispersed phase is solid and its medium is gas, making it a solid aerosol. It is not gas in solid, which would describe a solid foam rather than a suspension of particles in air.

Step 3: Compare the triples in the options The required sequence is

C: liquid in solid; M: liquid in liquid; S: solid in gas.

Option (B) changes milk to liquid in solid and so mistakes an emulsion for a gel. Option (C) reverses the phases of cheese and smoke. Option (D) classifies both cheese and milk as solid in liquid, inconsistent with their standard gel and emulsion classifications. The first option alone preserves the correct phase order for all three systems.

Step 4: Conclusion Cheese is liquid in solid, milk is liquid in liquid, and smoke is solid in gas. Hence, the correct option is (A).

Quick Tip to Solve: Always read a colloid description as dispersed phase in dispersion medium: cheese is a gel, milk an emulsion and smoke a solid aerosol.

90. The hydride that is NOT electron deficient : [4 marks]

(A) B_2H_6

(B) AlH_3

(C) GaH_3

(D) SiH_4

Correct Answer: (D) SiH_4

Solution:

Concept:

- An electron-deficient hydride lacks enough valence electrons to describe every bond using ordinary two-centre, two-electron bonds with complete octets on the central atoms.
- Group 14 hydrides such as SiH_4 are electron-precise: four valence electrons from the central atom combine with four hydrogen electrons to form four ordinary covalent bonds.

Step 1: Count the electrons in silane Silicon has four valence electrons and each of four hydrogen atoms contributes one. The total number of valence electrons in SiH_4 is

$$4 + 4(1) = 8.$$

These eight electrons form four Si–H sigma bonds. Each hydrogen has a duet and silicon is surrounded by four shared pairs, giving its octet. Therefore silane has exactly the electrons required for its normal covalent structure and is not electron deficient.

Step 2: Check the group 13 hydrides Boron, aluminium and gallium have three valence electrons. A simple EH_3 unit for such an element has only six electrons for three E–H bonds and leaves the central atom short of an octet. Their hydrides consequently display electron deficiency and may associate through bridging interactions. Diborane, B_2H_6, is the familiar example: its two B–H–B bridges involve three-centre, two-electron bonding rather than an independent conventional bond for every atom pair.

Step 3: Apply the negative wording of the question The question asks for the hydride that is *not* electron deficient. Thus selecting a hydride simply because it has bridging bonds would reverse the demand. B_2H_6, AlH_3 and GaH_3 belong to the electron-deficient group 13 hydrides, whereas SiH_4 is the electron-precise group 14 hydride. This eliminates the first three choices.

Step 4: Conclusion SiH_4 is electron-precise and is not electron deficient. Hence, the correct option is (D).

Quick Tip to Solve: For main-group hydrides, count the central atom's valence electrons: group 13 trihydrides are deficient, while group 14 tetrahydrides are electron-precise.