JEE Main 2019 Paper 1 (B.E./B.Tech) Jan 12 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 12 January 2019, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is : [4 marks]

(A) 36

(B) 28

(C) 32

(D) 24

Correct Answer: (B) 28

Solution:

Concept:

- For three consecutive terms of a geometric progression, the square of the middle term equals the product of the other two. If the terms are a, b, c, then $ac = b^2$ and $abc = b^3$.
- Three numbers form an arithmetic progression when twice the middle number equals the sum of the end numbers. Apply this condition to the changed terms, while retaining the original G.P. condition.

Step 1: Find the middle term from the product The original product is positive and equal to 512. Since the product of three consecutive G.P. terms is the cube of the middle term,

$$b^3 = 512 = 8^3 \quad \Rightarrow \quad b = 8.$$

Thus the middle term of the original progression is fixed even though the common ratio is not yet known. There is no need to find that ratio to calculate the requested sum.

Step 2: Apply the arithmetic-progression condition Only the first and second terms receive an addition of 4. The new ordered terms are therefore $a + 4, b + 4, c$, not $a + 4, b, c + 4$. Their A.P. condition gives

$$2(b + 4) = (a + 4) + c.$$

Hence

$$a + c = 2b + 4 = 2(8) + 4 = 20.$$

Adding the unchanged middle term to this end-term sum now gives

$$a + b + c = 20 + 8 = 28.$$

Step 3: Check consistency and eliminate the alternatives The G.P. condition requires $ac = 64$. Together with $a + c = 20$, this gives end terms 4 and 16. The possible original progressions are $4, 8, 16$ and $16, 8, 4$. They change to $8, 12, 16$ and $20, 12, 4$, respectively, both valid A.P.s. Both original sums are 28; the alternatives 36, 32 and 24 therefore fail the derived sum condition.

Step 4: Conclusion The sum of the original three terms is $\boxed{28}$. Hence, the correct option is (B).

Quick Tip to Solve: For three G.P. terms, obtain the middle term directly from the cube root of their product; use the A.P. condition to find the sum without solving for the common ratio.

2. Let y = y(x) be the solution of the differential equation, $x\frac{dy}{dx}$ + y = x $\log_e$ x, (x > 1). If 2y(2) = $\log_e$ 4 − 1, then y(e) is equal to : [4 marks]

(A) $-\frac{e}{2}$
(B) $-\frac{e^2}{2}$
(C) $\frac{e^2}{4}$
(D) $\frac{e}{4}$

Correct Answer: (D) $\frac{e}{4}$

Solution:

Concept:

- The product rule gives $\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$. Recognising this derivative avoids introducing a separate integrating factor.
- Integration by parts gives $\int x \ln x \, dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$. The domain $x > 1$ makes the logarithm and all subsequent divisions valid.

Step 1: Integrate the product derivative Rewrite the given equation as

$$\frac{d}{dx}(xy) = x \ln x.$$

For integration by parts, take $u = \ln x$ and $dv = x\,dx$, so $du = dx/x$ and $v = x^2/2$. Thus

$$xy = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C.$$

Dividing by x yields

$$y(x) = \frac{x}{2}\ln x - \frac{x}{4} + \frac{C}{x}.$$

Step 2: Determine the integration constant The initial condition supplies the value of the product xy at $x = 2$, not the value of y alone. Substitution in the integrated equation gives

$$2y(2) = 2\ln 2 - 1 + C.$$

Since $\ln 4 = 2\ln 2$, the stated condition is $2y(2) = 2\ln 2 - 1$. Comparing these two expressions proves $C = 0$.

Step 3: Evaluate the solution at the required point Using $\ln e = 1$ in the particular solution,

$$y(e) = \frac{e}{2} - \frac{e}{4} = \frac{e}{4}.$$

This value is positive and proportional to e, eliminating the negative options (A) and (B) and the quadratic-in-e value in (C). As a check, differentiating $y = (x/2)\ln x - x/4$ gives $y' = (1/2)\ln x + 1/4$, and substitution recovers $xy' + y = x \ln x$.

Step 4: Conclusion The required value is $\boxed{e/4}$. Hence, the correct option is (D).

Quick Tip to Solve: When the left side is $xy' + y$, integrate $(xy)'$ directly and apply any condition on xy before dividing by x.

3. Let S = {1, 2, 3, ... , 100}. The number of non-empty subsets A of S such that the product of elements in A is even is : [4 marks]

(A) $2^{50} - 1$

(B) $2^{50}\,(2^{50} - 1)$

(C) $2^{100} - 1$

(D) $2^{50} + 1$

Correct Answer: (B) $2^{50}\,(2^{50} - 1)$

Solution:

Concept:

- A product of positive integers is even exactly when at least one of its factors is even. Thus a qualifying subset must contain a nonempty selection of the even elements of S.
- A set of n distinct elements has 2^n subsets, because every element independently has two choices: inclusion or exclusion. Exactly one of these subsets is empty.

Step 1: Separate the even and odd elements The numbers 1 through 100 include 50 even numbers, $2, 4, \ldots, 100$, and 50 odd numbers, $1, 3, \ldots, 99$. Any subset containing only odd numbers has an odd product and cannot be counted. Conversely, the presence of even one even element makes the entire product even, regardless of which odd numbers accompany it.

Step 2: Count the independent selections The even elements can be selected in 2^{50} ways, but the empty selection must be excluded because it provides no even factor. This leaves $2^{50} - 1$ choices. There is no restriction on the odd elements: any of their 2^{50} subsets, including the empty one, may accompany that selection. By the multiplication principle,

$$N = (2^{50} - 1)\,2^{50} = 2^{100} - 2^{50}.$$

Every subset counted here is automatically nonempty because its even-element selection is nonempty.

Step 3: Check by complement and compare the options There are $2^{100} - 1$ nonempty subsets altogether. Of these, $2^{50} - 1$ contain only odd elements. Subtracting gives

$$(2^{100} - 1) - (2^{50} - 1) = 2^{100} - 2^{50},$$

agreeing with the direct count. Option (A) counts only the nonempty even-element selections and omits arbitrary odd companions. Option (C) includes odd-only subsets. Option (D) does not represent the required independent choices.

Step 4: Conclusion The required count is $\boxed{2^{50}(2^{50} - 1)}$. Hence, the correct option is (B).

Quick Tip to Solve: To make a subset product even, choose a nonempty subset of the even numbers and any subset of the odd numbers; the resulting subset is already nonempty.

4. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to : [4 marks]

(A) $\frac{200}{6^5}$

(B) $\frac{225}{6^5}$

(C) $\frac{150}{6^5}$

(D) $\frac{175}{6^5}$

Correct Answer: (D) $\frac{175}{6^5}$

Solution:

Concept:

- For a fair die, each specified ordered sequence of five outcomes has probability 6^{-5}. Counting successful sequences therefore gives the required probability directly.
- Ending on the fifth throw means the first consecutive pair of fours must occur on throws 4 and 5. A pair on any earlier adjacent throws would have ended the experiment sooner.

Step 1: Fix the last three throws Throws 4 and 5 must both be four. Throw 3 must be different from four; otherwise throws 3 and 4 would already have formed the required pair, ending the experiment on throw 4. Hence throw 3 has five possibilities, while throws 4 and 5 each have only the single prescribed possibility.

Step 2: Exclude an earlier ending in the first two throws The first two throws have $6^2 = 36$ ordered outcomes. Only the outcome $(4, 4)$ is prohibited, because it would end the experiment on throw 2. Therefore 35 ordered choices remain. Since throw 3 is already known to be non-four, throws 2 and 3 cannot produce an additional forbidden pair, whatever the allowed second throw is. The number of successful five-throw sequences is consequently

$$35 \times 5 \times 1 \times 1 = 175.$$

Step 3: Form the probability and compare the alternatives All 6^5 possible sequences of five independent throws are equally likely. Thus

$$P(T = 5) = \frac{175}{6^5}.$$

Equivalently, independence gives

$$\left(1 - \frac{1}{36}\right)\frac{5}{6}\left(\frac{1}{6}\right)^2 = \frac{35}{36}\frac{5}{6}\frac{1}{36} = \frac{175}{6^5}.$$

The numerators 200, 225 and 150 do not count the required 35 allowed initial pairs followed by a non-four third throw, so options (A), (B) and (C) are excluded.

Step 4: Conclusion The probability that the experiment ends on the fifth throw is $\boxed{175/6^5}$. Hence, the correct option is (D).

Quick Tip to Solve: For the first consecutive pair to appear at the end, fix the final pair, forbid a matching outcome immediately before it, and then exclude all earlier pairs.

5. A ratio of the 5th term from the beginning to the 5th term from the end in the binomial expansion of $\left(2^{1/3}+\frac{1}{2(3)^{1/3}}\right)^{10}$ is : [4 marks]

(A) $1 : 2(6)^{1/3}$

(B) $1 : 4(6)^{1/3}$

(C) $2(36)^{1/3} : 1$

(D) $4(36)^{1/3} : 1$

Correct Answer: (D) $4(36)^{1/3} : 1$

Solution:

Concept:

- In $(a+b)^n$, the $(r+1)$th term from the beginning is $T_{r+1}=\binom{n}{r}a^{n-r}b^r$. There are $n+1$ terms in total.
- The fifth term from the end of an eleven-term expansion is the seventh term from the beginning. Symmetric coefficients satisfy $\binom{10}{4}=\binom{10}{6}$, so they cancel in the ratio.

Step 1: Identify both terms correctly Put $a=2^{1/3}$ and $b=1/(2\,3^{1/3})$. The exponent is 10, so the expansion has 11 terms. The fifth term from the beginning is T_5, corresponding to $r=4$. Counting five places backward from T_{11} gives T_7, corresponding to $r=6$. Therefore

$$T_5=\binom{10}{4}a^6b^4, \qquad T_7=\binom{10}{6}a^4b^6.$$

Step 2: Cancel common factors and simplify Taking the ratio in the order asked and cancelling equal binomial coefficients gives

$$\frac{T_5}{T_7}=\frac{a^2}{b^2}=\left(\frac{a}{b}\right)^2.$$

Now

$$\frac{a}{b}=2^{1/3}\cdot 2\,3^{1/3}=2\,6^{1/3}.$$

Hence

$$\frac{T_5}{T_7}=4\,6^{2/3}=4\sqrt[3]{36}.$$

Both terms are positive, so this quotient directly represents their ratio.

Step 3: Check the ratio direction and reject other choices The required ratio is thus $4\sqrt[3]{36} : 1$, with the first term larger than the second because $a>b>0$. Options (A) and (B) are less than one when written as quotients, so they reverse or otherwise miscompute the ratio. Option (C) has only half the required coefficient. The factor 4 must be retained because the factor 2 in a/b is squared, as well as the cube-root factor.

Step 4: Conclusion The ratio is $\boxed{4\sqrt[3]{36} : 1}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert a term counted from the end into its position from the beginning first; then exploit equal symmetric binomial coefficients before substituting the bases.

6. Let f and g be continuous functions on [0, a] such that f(x) = f(a – x) and g(x) + g(a – x) = 4, then $\int_0^a$ f(x) g(x) dx is equal to : [4 marks]

(A) $4\int_0^a f(x)dx$

(B) $-3\int_0^a f(x)dx$

(C) $\int_0^a f(x)dx$

(D) $2\int_0^a f(x)dx$

Correct Answer: (D) $2\int_0^a f(x)dx$

Solution:

Concept:

- Reflection of the integration variable gives $\int_0^a h(x)\,dx = \int_0^a h(a-x)\,dx$. The limits reverse together with the sign of the differential.
- The condition $f(a-x) = f(x)$ makes f symmetric about the midpoint. Pairing the two reflected forms then uses the given sum $g(x) + g(a-x) = 4$.

Step 1: Reflect the original integral Denote the required integral by

$$I = \int_0^a f(x)g(x)\,dx.$$

Set $x = a - t$, so $dx = -dt$, with $x = 0$ corresponding to $t = a$ and $x = a$ to $t = 0$. Reversing the limits gives

$$I = \int_0^a f(a-t)g(a-t)\,dt.$$

Since the integration variable is a dummy variable, rename t as x, and use the stated symmetry to obtain

$$I = \int_0^a f(x)g(a-x)\,dx.$$

Step 2: Add the two equal expressions Add the original and reflected representations of the same number I. By linearity,

$$2I = \int_0^a f(x)\{g(x) + g(a-x)\}\,dx.$$

The expression in braces is identically 4 on the entire interval, hence

$$2I = 4\int_0^a f(x)\,dx.$$

Dividing by 2 yields the requested relation

$$I = 2\int_0^a f(x)\,dx.$$

Continuity ensures that all integrals used in this argument exist.

Step 3: Check the coefficient against the choices The coefficient is 2 because adding the reflected expression doubles the original integral. Keeping the coefficient 4 would omit this final division. As a simple independent check, take $f(x) = 1$ and $g(x) = 2$, which satisfy both hypotheses. Then the original integral is $2a$, whereas the coefficients 4, -3 and 1 in options (A), (B) and (C) give $4a, -3a, a$, respectively, for a positive interval length.

Step 4: Conclusion The integral equals $\boxed{2\int_0^a f(x)\,dx}$. Hence, the correct option is (D).

Quick Tip to Solve: When a definite integral contains functions evaluated at complementary points, write its reflected form and add; remember that the left side becomes twice the original integral.

7. If $\frac{z-\alpha}{z+\alpha}$ $(\alpha \in R)$ is a purely imaginary number and | z | = 2, then a value of α is : [4 marks]

(A) $\frac{1}{2}$

(B) $\sqrt{2}$

(C) 2

(D) 1

Correct Answer: (C) 2

Solution:

Concept:

- A complex number is purely imaginary only if its real part is zero. For a ratio, multiply numerator and denominator by the conjugate of the denominator to isolate that real part.
- For $z = x + iy$, its modulus satisfies $|z|^2 = x^2 + y^2$. Here $|z| = 2$, so $x^2 + y^2 = 4$, while α is real.

Step 1: Rationalise the complex denominator Write $z = x + iy$. For the given quotient to be defined, $z + \alpha \neq 0$. Multiply by the conjugate $x + \alpha - iy$:

$$\frac{z-\alpha}{z+\alpha} = \frac{(x-\alpha+iy)(x+\alpha-iy)}{(x+\alpha)^2+y^2}.$$

Expanding the numerator, its real part is $x^2 - \alpha^2 + y^2$ and its imaginary part is $2\alpha y$. Hence

$$\frac{z-\alpha}{z+\alpha} = \frac{x^2+y^2-\alpha^2+2i\alpha y}{(x+\alpha)^2+y^2}.$$

Step 2: Impose the purely imaginary condition The denominator is a strictly positive real number whenever the quotient is defined. Therefore its real part is zero precisely when

$$x^2 + y^2 - \alpha^2 = 0.$$

Using $x^2 + y^2 = 4$,

$$4 - \alpha^2 = 0 \quad \Rightarrow \quad \alpha = \pm 2.$$

The question asks for a value, so the positive possibility 2 is sufficient; it does not require both signs to be listed.

Step 3: Confirm that the listed value is attainable To verify existence without a zero denominator, take $z = 2i$ and $\alpha = 2$. Then

$$\frac{2i-2}{2i+2} = \frac{i-1}{i+1} = i,$$

which is nonzero and purely imaginary, while $|z| = 2$. The other proposed values $1/2, \sqrt{2}, 1$ have squares $1/4, 2, 1$, so none makes the real numerator $4 - \alpha^2$ vanish. This directly excludes (A), (B) and (D).

Step 4: Conclusion One possible value is $\boxed{\alpha = 2}$. Hence, the correct option is (C).

Quick Tip to Solve: For a real parameter in $(z-\alpha)/(z+\alpha)$, rationalising shows that the real numerator is $|z|^2 - \alpha^2$; impose its vanishing before choosing a listed sign.

8. Let S be the set of all points in (–π, π) at which the function, f(x) = min{sin x, cos x} is not differentiable. Then S is a subset of which of the following ? [4 marks]

(A) $\left\{-\frac{\pi}{2}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right\}$
(B) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$
(C) $\left\{-\frac{\pi}{4}, 0, \frac{\pi}{4}\right\}$
(D) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$
Correct Answer: (D) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$

Solution:

Concept:

- The minimum of two differentiable functions agrees locally with the smaller function whenever their values are unequal. A possible corner can therefore occur only where the two functions meet.
- At a crossing, compare the one-sided derivatives of the selected branches. Equal function values alone do not prove nondifferentiability; unequal branch slopes establish a corner.

Step 1: Find all possible switching points The two branches are equal precisely when

$$\sin x = \cos x \quad \Longleftrightarrow \quad \sqrt{2}\sin(x - \pi/4) = 0.$$

Thus $x = \pi/4 + k\pi$, where k is an integer. In the open interval $(-\pi, \pi)$, the only possibilities are $-3\pi/4$ and $\pi/4$. Every other point has a neighbourhood in which one branch remains strictly smaller, so the minimum is differentiable there.

Step 2: Check the slopes at each crossing The derivatives of the sine and cosine branches are $\cos x$ and $-\sin x$. At $\pi/4$, these are $1/\sqrt{2}$ and $-1/\sqrt{2}$; at $-3\pi/4$, they are $-1/\sqrt{2}$ and $1/\sqrt{2}$. The sign of $\sin x - \cos x$ changes at both simple crossings, so the minimum changes branches and its left and right derivatives differ. Hence

$$S = \{-3\pi/4, \pi/4\}.$$

Option (A) and option (C) omit $-3\pi/4$, while option (B) omits $\pi/4$. Only option (D) contains both required points.

Step 3: Conclusion $S = \{-3\pi/4, \pi/4\}$ is a subset of the set in (D). Hence, the correct option is (D).

Quick Tip to Solve: For a minimum or maximum of smooth branches, first solve their equality, then compare slopes only at those crossing points.

9. Let P(4, –4) and Q(9, 6) be two points on the parabola, $y^2 = 4x$ and let x be any point on the arc POQ of this parabola, where O is the vertex of this parabola, such that the area of ΔPXQ is maximum. Then this maximum area (in sq. units) is : [4 marks]

(A) $\frac{625}{4}$
(B) $\frac{125}{4}$
(C) $\frac{75}{2}$
(D) $\frac{125}{2}$

Correct Answer: (B) $\frac{125}{4}$

Solution:

Concept:

- Parametrize the parabola $y^2 = 4x$ by $X = (t^2, 2t)$. The arc through the vertex between the two specified points corresponds to a closed interval of the parameter.
- The area of a triangle is half the absolute value of the determinant of two side vectors. After substitution it can be maximized as a quadratic in the parameter.

Step 1: Express the area using the parameter The points $P = (4, -4)$ and $Q = (9, 6)$ correspond to $t = -2$ and $t = 3$. Thus the given arc has $-2 \le t \le 3$. Using $Q - P = (5, 10)$ and $X - P = (t^2 - 4, 2t + 4)$, the area is

$$A(t) = \frac{1}{2}\left|5(2t + 4) - 10(t^2 - 4)\right| = 5\left|6 + t - t^2\right|.$$

Now $6 + t - t^2 = (t + 2)(3 - t) \ge 0$ throughout this arc. The absolute value therefore introduces no additional branch of the area formula.

Step 2: Maximize the quadratic and check the choices Complete the square:

$$A(t) = 30 + 5t - 5t^2 = \frac{125}{4} - 5\left(t - \frac{1}{2}\right)^2.$$

The squared term is nonnegative, so the area is at most $125/4$. Equality occurs at $t = 1/2$, which lies in the permitted interval, with $X = (1/4, 1)$. At both endpoints the triangle collapses and the area is zero, consistent with this maximum being attained in the interior. The values $625/4$, $75/2$, and $125/2$ in (A), (C), and (D) all exceed the proven upper bound and cannot be the maximum.

Step 3: Conclusion The maximum area is $\frac{125}{4}$ square units, attained at $X = (1/4, 1)$. Hence, the correct option is (B).

Quick Tip to Solve: For a fixed chord of a parabola, substitute $(t^2, 2t)$ into the triangle determinant; completing the resulting square gives the maximum directly.

10. Consider three boxes, each containing, 10 balls labelled 1, 2, ... , 10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1, 2, 3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is : [4 marks]

(A) 164

(B) 240

(C) 82

(D) 120

Correct Answer: (D) 120

Solution:

Concept:

- A strict inequality $n_1 < n_2 < n_3$ requires three distinct labels. Choosing a set of three labels determines a unique increasing order.
- The boxes are distinct, but the increasing condition fixes which selected label comes from which box: the smallest from the first, the middle from the second, and the largest from the third.

Step 1: Establish a one-to-one correspondence Choose any three labels $a < b < c$ from $\{1, 2, \ldots, 10\}$. There is exactly one allowed draw using these labels: choose label a from box 1, label b from box 2, and label c from box 3. Conversely, every admissible draw yields exactly one such set $\{a, b, c\}$. Therefore no valid choice is lost and no choice is counted twice when we count three-element subsets of the ten labels. Equal labels are excluded by the strict inequalities, even though equal labels can occur in unrestricted draws.

Step 2: Evaluate the combination and verify the count The desired number is

$$\binom{10}{3} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120.$$

An independent check starts with the $10 \cdot 9 \cdot 8 = 720$ ordered triples of distinct labels. Their six possible relative orders are equally represented; exactly one order is strictly increasing, giving $720/6 = 120$. We do not multiply the combination by 3!, because that would include forbidden decreasing or mixed orders. Thus 164 and 82 in (A) and (C) have no place in this complete count, and 240 in (B) counts twice as many choices as permitted.

Step 3: Conclusion There are 120 admissible choices of the balls. Hence, the correct option is (D).

Quick Tip to Solve: When successive selections must be strictly increasing and each box offers the same labels, count subsets rather than unrestricted ordered triples.

11. The integral $\int \cos(\log_e x)\, dx$ is equal to : (where C is a constant of integration) [4 marks]

(A) $\frac{x}{2}[\sin(\log_e x) - \cos(\log_e x)] + C$

(B) $x[\cos(\log_e x) + \sin(\log_e x)] + C$

(C) $\frac{x}{2}[\cos(\log_e x) + \sin(\log_e x)] + C$

(D) $x[\cos(\log_e x) - \sin(\log_e x)] + C$

Correct Answer: (C) $\frac{x}{2}[\cos(\log_e x) + \sin(\log_e x)] + C$

Solution:

Concept:

- Use integration by parts, $\int u\, dv = uv - \int v\, du$. For $u = \cos(\ln x)$, the derivative brings a factor $1/x$, which cancels the factor x obtained by integrating dx.
- The natural logarithm requires $x > 0$. A second integration by parts produces the original integral, allowing an algebraic equation for it.

Step 1: Introduce the linked sine integral Let $I = \int \cos(\ln x)\, dx$. Taking $u = \cos(\ln x)$ and $dv = dx$, we have $du = -\sin(\ln x)\, dx/x$ and $v = x$. Hence

$$I = x\cos(\ln x) + \int \sin(\ln x)\, dx.$$

Write the remaining integral as J. Applying parts again, with $u = \sin(\ln x)$, gives

$$J = x\sin(\ln x) - \int \cos(\ln x)\, dx = x\sin(\ln x) - I.$$

Step 2: Solve for the integral and differentiate to check Substitution yields $I = x\cos(\ln x) + x\sin(\ln x) - I$, so

$$I = \frac{x}{2}\big[\cos(\ln x) + \sin(\ln x)\big] + C.$$

For verification, differentiating the displayed nonconstant part gives

$$\frac{1}{2}(\cos(\ln x) + \sin(\ln x)) + \frac{1}{2}(-\sin(\ln x) + \cos(\ln x)) = \cos(\ln x).$$

Option (A) differentiates to $\sin(\ln x)$, option (B) to $2\cos(\ln x)$, and option (D) to $-2\sin(\ln x)$. None has the required derivative. The arbitrary constant absorbs the constants introduced in the intermediate integrations.

Step 3: Conclusion For $x > 0$, the integral is $\frac{x}{2}[\cos(\ln x) + \sin(\ln x)] + C$. Hence, the correct option is (C).

Quick Tip to Solve: After two integrations by parts, bring the repeated original integral to the left. Differentiate the final option to catch a missing factor of two.

12. An ordered pair (α, β) for which the system of linear equations

$(1 + \alpha)\,x + \beta y + z = 2$

$\alpha x + (1 + \beta)y + z = 3$

$\alpha x + \beta y + 2z = 2$

has a unique solution, is : [4 marks]

(A) (–3, 1)

(B) (1, –3)

(C) (–4, 2)

(D) (2, 4)

Correct Answer: (D) (2, 4)

Solution:

Concept:

- A square linear system has a unique solution exactly when its coefficient matrix is nonsingular, meaning its determinant is nonzero. The right-hand constants do not change this criterion.
- Replacing one row by itself minus another row preserves the determinant. Here subtracting the third row from the first two creates two simple rows.

Step 1: Simplify the coefficient determinant The coefficient matrix is

$$A = \begin{pmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{pmatrix}.$$

Apply $R_1 \leftarrow R_1 - R_3$ and $R_2 \leftarrow R_2 - R_3$. This gives

$$\det A = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix} = (2+\beta) + \alpha = \alpha + \beta + 2.$$

Consequently, uniqueness requires $\alpha + \beta + 2 \neq 0$. A zero determinant would rule out uniqueness regardless of whether the resulting equations were inconsistent or dependent.

Step 2: Test every proposed pair For (A), $(\alpha, \beta) = (-3, 1)$ gives $-3 + 1 + 2 = 0$. For (B), $(1, -3)$ gives $1 - 3 + 2 = 0$. For (C), $(-4, 2)$ also gives zero. For (D), $(2, 4)$ gives $2 + 4 + 2 = 8 \neq 0$, so only this pair has an invertible coefficient matrix. As a direct check for this pair, subtraction of the third equation from the first two gives $x - z = 0$ and $y - z = 1$. Substituting $x = z$, $y = z + 1$ in the third equation gives $8z + 4 = 2$, uniquely fixing $z = -1/4$, $x = -1/4$, and $y = 3/4$.

Step 3: Conclusion The ordered pair is $(\alpha, \beta) = (2, 4)$, whose coefficient determinant is 8. Hence, the correct option is (D).

Quick Tip to Solve: For uniqueness questions, evaluate the coefficient determinant before solving the system; row subtraction can expose a simple condition on the parameters.

13. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices such that $Q - P^5 = I_3$.

Then $\frac{q_{21}+q_{31}}{q_{32}}$ is equal to : [4 marks]

(A) 15

(B) 9

(C) 135

(D) 10

Correct Answer: (D) 10

Solution:

Concept:

- Write a unit lower triangular matrix as $P = I + N$, where N is strictly lower triangular. For a 3×3 matrix, $N^3 = 0$, so its binomial expansion terminates after N^2.
- The identity matrix commutes with every matrix. Adding I changes only diagonal entries, so the required off-diagonal entries of $Q = P^5 + I$ equal those of P^5.

Step 1: Compute the nilpotent part and its square Here

$$N = P - I = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}, \qquad N^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{pmatrix}.$$

The only nonzero product contributing to $(N^2)_{31}$ is $N_{32}N_{21} = 3 \cdot 3 = 9$. Multiplication once more gives the zero matrix, so $N^3 = 0$. Therefore every term involving N^3 or a higher power vanishes in the binomial expansion.

Step 2: Extract the required entries and form the ratio Using the binomial coefficients,

$$P^5 = (I + N)^5 = I + 5N + \binom{5}{2} N^2 = I + 5N + 10N^2.$$

It follows that $(P^5)_{21} = 15$, $(P^5)_{32} = 15$, and $(P^5)_{31} = 5 \cdot 9 + 10 \cdot 9 = 135$. Since $Q - P^5 = I$, all these off-diagonal entries remain unchanged in Q. Thus

$$\frac{q_{21} + q_{31}}{q_{32}} = \frac{15 + 135}{15} = 10.$$

Option (A) gives the individual entry q_{21}, not the ratio. Option (C) gives q_{31}, also not the ratio. Option (B), 9, would result from omitting q_{21} from the numerator. The complete numerator instead gives 10.

Step 3: Conclusion The required ratio is 10. Hence, the correct option is (D).

Quick Tip to Solve: For a unit triangular matrix, subtract the identity and check nilpotency; a short binomial expansion is usually faster than repeated multiplication.

14. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m-4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is [4 marks]

(A) $-2 + \sqrt{2}$

(B) $4-3\sqrt{2}$

(C) $2 - \sqrt{3}$

(D) $4 - 2\sqrt{3}$

Correct Answer: (B) $4-3\sqrt{2}$

Solution:

Concept:

- For nonzero roots u, v, the sum of their ratio and reciprocal is $u/v + v/u = (u+v)^2/(uv) - 2$. Thus it can be written directly in terms of the sum and product of roots.
- For $ax^2 + bx + c = 0$, Vieta's relations give $u + v = -b/a$ and $uv = c/a$. A quadratic here requires $m \neq 0$; the nonzero constant term then ensures neither root is zero.

Step 1: Translate the ratio condition using Vieta relations The coefficients are $a = 3m^2$, $b = m(m-4)$, and $c = 2$. Hence

$$u + v = -\frac{m-4}{3m}, \qquad uv = \frac{2}{3m^2}.$$

The ratio may be u/v or v/u; the given symmetric expression is the same in either case. Therefore

$$\lambda + \frac{1}{\lambda} = \frac{(u+v)^2}{uv} - 2 = \frac{(m-4)^2}{6} - 2.$$

This calculation uses only the root relations and remains valid for complex conjugate roots.

Step 2: Solve the parameter equation and choose the smaller value Setting this expression equal to 1 yields

$$\frac{(m-4)^2}{6} - 2 = 1 \quad \Longrightarrow \quad (m-4)^2 = 18,$$

so $m = 4 + 3\sqrt{2}$ or $m = 4 - 3\sqrt{2}$. Both are nonzero, as required for the original quadratic. Their smaller value is $4 - 3\sqrt{2}$. The discriminant at these values is $m^2[(m-4)^2 - 24] = -6m^2 < 0$; the question does not require real roots, and conjugate complex roots satisfy the ratio condition. The other options fail $(m-4)^2 = 18$: their squares are respectively $38 - 12\sqrt{2}$, $7 + 4\sqrt{3}$, and 12 for (A), (C), and (D).

Step 3: Conclusion The least value is $m = 4 - 3\sqrt{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Replace a symmetric root-ratio expression by (sum of roots)^2/(product of roots) $- 2$, avoiding the quadratic formula.

15. The maximum value of 3cosθ + 5sin $\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is : [4 marks]

(A) $\sqrt{34}$

(B) $\sqrt{31}$

(C) $\sqrt{19}$

(D) $\frac{\sqrt{79}}{2}$

Correct Answer: (C) $\sqrt{19}$

Solution:

Concept:

- Expand a sine of a difference using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$.
- For real constants a, b, the expression $a\cos\theta + b\sin\theta$ has maximum $\sqrt{a^2 + b^2}$. This bound is attained when the unit vector $(\cos\theta, \sin\theta)$ points along (a, b).

Step 1: Expand the shifted sine Since $\cos(\pi/6) = \sqrt{3}/2$ and $\sin(\pi/6) = 1/2$, the given expression becomes

$$3\cos\theta + 5\sin(\theta - \pi/6) = 3\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta - \frac{5}{2}\cos\theta$$
$$= \frac{1}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta.$$

The two cosine terms must be combined before finding the amplitude. The original coefficients 3 and 5 multiply functions with different phases.

Step 2: Find the amplitude and verify attainment The required upper bound is

$$R = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1+75}{4}} = \sqrt{19}.$$

Equality is possible: choose $\cos\theta = 1/(2\sqrt{19})$ and $\sin\theta = 5\sqrt{3}/(2\sqrt{19})$. Their squares add to 1, so a real angle with these values exists. Thus the bound is a maximum, not merely an estimate. The other options have squared values 34, 31 and $79/4$; none equals the squared amplitude 19.

Step 3: Conclusion The maximum value is $\boxed{\sqrt{19}}$. Hence, the correct option is (C).

Quick Tip to Solve: Expand a phase shift first, collect the coefficients of sine and cosine, and only then take the square root of the sum of their squares.

16. Considering only the principal values of inverse functions, the set

$A = \{ x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4} \}$ [4 marks]

(A) contains two elements

(B) contains more than two elements

(C) is an empty set

(D) is a singleton

Correct Answer: (D) is a singleton

Solution:

Concept:

- The principal value of $\tan^{-1} t$ lies in $(-\pi/2, \pi/2)$. For $t \geq 0$, it lies in $[0, \pi/2)$.
- If $u + v = \pi/4$, then $\tan(u + v) = 1$. Tangent addition gives $\tan(u + v) = (\tan u + \tan v)/(1 - \tan u \tan v)$, but any roots must also satisfy the original principal-value equation.

Step 1: Obtain the algebraic candidates Write $u = \tan^{-1}(2x)$ and $v = \tan^{-1}(3x)$. An actual solution obeys $u + v = \pi/4$, so its tangent is defined and

$$\frac{2x + 3x}{1 - 6x^2} = 1.$$

Multiplying by the nonzero denominator and factoring gives

$$6x^2 + 5x - 1 = 0, \qquad (6x - 1)(x + 1) = 0.$$

The candidates are $x = 1/6$ and $x = -1$. The latter is immediately excluded by the required condition $x \geq 0$.

Step 2: Check the remaining principal values At $x = 1/6$, the two tangents are $1/3$ and $1/2$. Each corresponding principal angle lies strictly between 0 and $\pi/4$; their sum is therefore between 0 and $\pi/2$. Moreover,

$$\tan(u + v) = \frac{1/3 + 1/2}{1 - 1/6} = 1.$$

In that interval tangent takes the value 1 only at $\pi/4$. The candidate consequently satisfies the original equation. There are no further candidates, so the set is neither empty nor a set with two or more elements. This branch check prevents accepting angles whose tangents agree but whose principal sums differ by π.

Step 3: Conclusion $A = \{1/6\}$ contains exactly one element and is a singleton. Hence, the correct option is (D).

Quick Tip to Solve: After applying tangent addition, enforce the given sign restriction and verify the sum's interval to eliminate branch errors.

17. $\lim\limits_{x\to\pi/4} \frac{\cot^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ **is :** [4 marks]

(A) $8\sqrt{2}$

(B) 4

(C) $4\sqrt{2}$

(D) 8

Correct Answer: (D) 8

Solution:

Concept:

- Direct substitution gives the indeterminate form $0/0$. Factor the numerator before evaluating the limit.
- Use $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \cos 2x$ and $2\cos A \cos B = \cos(A+B) + \cos(A-B)$.

Step 1: Express the numerator with a common denominator In a neighbourhood of $x = \pi/4$, both $\sin x$ and $\cos x$ are nonzero. Hence

$$\begin{aligned}\cot^3 x - \tan x &= \frac{\cos^3 x}{\sin^3 x} - \frac{\sin x}{\cos x} \\ &= \frac{\cos^4 x - \sin^4 x}{\sin^3 x \cos x} = \frac{\cos 2x}{\sin^3 x \cos x}.\end{aligned}$$

Keeping the cube on the sine in this common denominator is essential; replacing it with a first power would change the answer.

Step 2: Cancel the vanishing factor and take the limit Product-to-sum gives

$$\cos 2x = 2\cos(x+\pi/4)\cos(x-\pi/4).$$

Away from the limiting point, cancel $\cos(x+\pi/4)$. The original quotient thus equals

$$\frac{2\cos(x-\pi/4)}{\sin^3 x \cos x}.$$

This simplified expression is continuous at $\pi/4$, where its denominator is $(1/\sqrt{2})^4 = 1/4$. Therefore

$$L = \frac{2\cos 0}{1/4} = 8.$$

Both sides approach the same value because the simplified expression is continuous throughout a neighbourhood of the point. The alternatives $8\sqrt{2}$, 4 and $4\sqrt{2}$ do not equal this exact value.

Step 3: Conclusion The limit is $\boxed{8}$. Hence, the correct option is (D).

Quick Tip to Solve: For a difference involving odd powers of tangent and cotangent, combine fractions first; a difference of fourth powers can reveal the factor that cancels.

18. For x $>$ 1, if $(2x)^{2y} = 4e^{2x-2y}$, then $(1 + \log_e 2x)^2 \frac{dy}{dx}$ is equal to : [4 marks]

(A) $\frac{x\log_e 2x - \log_e 2}{x}$

(B) $\log_e 2x$

(C) $x \log_e 2x$

(D) $\frac{x\log_e 2x + \log_e 2}{x}$

Correct Answer: (A) $\frac{x\log_e 2x - \log_e 2}{x}$

Solution:

Concept:

- Taking natural logarithms turns variable exponents into products: $\ln(a^b) = b\ln a$ for $a > 0$.
- Here $x > 1$, so $2x > 0$ and $1 + \ln(2x) > 0$. Thus the logarithms and the denominator obtained below are well defined. Also $d\ln(2x)/dx = 1/x$.

Step 1: Isolate the dependent variable Taking natural logarithms of $(2x)^{2y} = 4e^{2x-2y}$ gives

$$2y\ln(2x) = \ln 4 + 2x - 2y.$$

Divide by 2 and collect the terms containing y:

$$y\ln(2x) = \ln 2 + x - y, \qquad y[1 + \ln(2x)] = x + \ln 2.$$

Therefore $y = (x + \ln 2)/[1 + \ln(2x)]$. The constant term is $\ln 2$, because $\ln 4/2 = \ln 2$; it must not be differentiated as though it depended on x.

Step 2: Differentiate and multiply by the requested factor Apply the quotient rule, differentiating the numerator to 1 and the denominator to $1/x$:

$$\frac{dy}{dx} = \frac{1 + \ln(2x) - (x + \ln 2)/x}{[1 + \ln(2x)]^2}.$$

Multiplication by the stated squared factor leaves

$$[1 + \ln(2x)]^2 \frac{dy}{dx} = \ln(2x) - \frac{\ln 2}{x} = \frac{x\ln(2x) - \ln 2}{x}.$$

The subtraction arises from the second term of the quotient rule. Option (D) has the wrong sign for this correction, while (B) omits it and (C) has an extra factor of x. Since $\ln 2 \neq 0$, that correction cannot be discarded.

Step 3: Conclusion The required expression is $\boxed{[x\ln(2x) - \ln 2]/x}$. Hence, the correct option is (A).

Quick Tip to Solve: With a variable exponent, take logarithms and solve for the dependent variable before differentiating; the requested squared factor often cancels the quotient-rule denominator.

19. If the vertices of a hyperbola be at (–2, 0) and (2, 0) and one of its foci be at (–3, 0), then which one of the following points does not lie on this hyperbola? [4 marks]

(A) $(6, 5\sqrt{2})$

(B) $(2\sqrt{6}, 5)$

(C) $(-6, 2\sqrt{10})$

(D) $(4, \sqrt{15})$

Correct Answer: (A) $(6, 5\sqrt{2})$

Solution:

Concept:

- A hyperbola centred at the origin with transverse axis on the x-axis has equation $x^2/a^2 - y^2/b^2 = 1$, vertices $(\pm a, 0)$, and foci $(\pm c, 0)$.
- The focal relation for a hyperbola is $c^2 = a^2 + b^2$. A proposed point lies on the curve exactly when substitution makes the left-hand side equal to 1.

Step 1: Determine the equation from the vertices and focus The midpoint of the vertices $(-2, 0)$ and $(2, 0)$ is the origin, and their direction gives a horizontal transverse axis. Thus $a = 2$. The focus $(-3, 0)$ gives $c = 3$, so

$$b^2 = c^2 - a^2 = 9 - 4 = 5.$$

The equation to test is consequently

$$\frac{x^2}{4} - \frac{y^2}{5} = 1.$$

Notice the minus sign between the two squared terms. Using the ellipse relation would incorrectly give a negative value for b^2.

Step 2: Substitute all four proposed points Let $F(x, y) = x^2/4 - y^2/5$. Evaluate it without approximating the radicals:

$$F(6, 5\sqrt{2}) = 36/4 - 50/5 = 9 - 10 = -1,$$
$$F(2\sqrt{6}, 5) = 24/4 - 25/5 = 6 - 5 = 1,$$
$$F(-6, 2\sqrt{10}) = 36/4 - 40/5 = 9 - 8 = 1,$$
$$F(4, \sqrt{15}) = 16/4 - 15/5 = 4 - 3 = 1.$$

The sign of the negative x-coordinate in the third point disappears on squaring. Options (B), (C) and (D) satisfy the required equation exactly. Option (A) produces -1, which is different from the right-hand side 1, and therefore is the requested exception.

Step 3: Conclusion The point $\boxed{(6, 5\sqrt{2})}$ does not lie on the hyperbola. Hence, the correct option is (A).

Quick Tip to Solve: For a hyperbola question asking which point does not lie on the curve, derive its equation once and test squared coordinates; keep the required right-hand side in view.

20. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is : [4 marks]

(A) 31

(B) 50

(C) 51

(D) 30

Correct Answer: (A) 31

Solution:

Concept:

- For observations $x_1, \ldots, x_n$, the arithmetic mean is $\bar{x} = (\sum_{i=1}^{n} x_i)/n$.
- The signed deviation from an assumed mean a is $d_i = x_i - a$. Summing gives $\sum d_i = n(\bar{x} - a)$, or $\bar{x} = a + (\sum d_i)/n$.

Step 1: Convert the given deviations into a total There are 50 observations and the reference value is 30. A deviation from 30 means $x_i - 30$, so the given condition is

$$\sum_{i=1}^{50}(x_i - 30) = 50.$$

The number 30 is subtracted for each of the 50 observations, not just once. Expanding the sum therefore gives

$$\sum_{i=1}^{50} x_i - 50 \times 30 = 50, \qquad \sum_{i=1}^{50} x_i = 1500 + 50 = 1550.$$

These are signed deviations; their positive total indicates that the actual mean exceeds the reference value.

Step 2: Divide by the number of observations Use the total 1550 and the count 50:

$$\bar{x} = \frac{1550}{50} = 31.$$

Equivalently, the average deviation is $50/50 = 1$, so adding it to the assumed mean yields $30 + 1 = 31$. This also checks the condition: $50(31 - 30) = 50$. A mean of 30 would give zero total deviation, so option (D) is ruled out. Means of 50 and 51 would instead give deviation totals $50(50 - 30) = 1000$ and $50(51 - 30) = 1050$, respectively, ruling out (B) and (C). The value 50 supplied in the question is a total deviation and cannot itself be used as the mean.

Step 3: Conclusion The mean of the observations is $\boxed{31}$. Hence, the correct option is (A).

Quick Tip to Solve: Divide the total signed deviation by the number of observations, then add that average deviation to the given reference value.

21. If the straight line, 2x – 3y + 17 = 0 is perpendicular to the line passing through the points (7, 17) and (15, β), then β equals : [4 marks]

(A) $\frac{35}{3}$

(B) -5

(C) $-\frac{35}{3}$

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- A line $Ax + By + C = 0$, with $B \neq 0$, has slope $-A/B$. The constant term affects its position but not its direction.
- For two nonvertical perpendicular lines, their slopes satisfy $m_1 m_2 = -1$. The slope through two points is $(y_2 - y_1)/(x_2 - x_1)$ when their x-coordinates differ.

Step 1: Find the required perpendicular slope Rewrite the given equation in slope-intercept form:

$$2x - 3y + 17 = 0 \quad \Longrightarrow \quad y = \frac{2}{3}x + \frac{17}{3}.$$

Its slope is $m_1 = 2/3$, not $-2/3$. A line perpendicular to it therefore has slope

$$m_2 = -\frac{1}{m_1} = -\frac{3}{2}.$$

Since the first slope is positive, the perpendicular slope must be negative; this is a useful check on the signs.

Step 2: Apply the two-point slope formula The line through $(7, 17)$ and $(15, \beta)$ has a horizontal change of $15 - 7 = 8$. Thus

$$\frac{\beta - 17}{8} = -\frac{3}{2}, \qquad \beta - 17 = -12, \qquad \beta = 5.$$

Checking, the slope through $(7, 17)$ and $(15, 5)$ is $(5 - 17)/8 = -3/2$, whose product with $2/3$ is -1. The other proposed values $35/3$, -5, and $-35/3$ give slopes $-2/3$, $-11/4$, and $-43/12$, respectively; none is the required negative reciprocal. The given intercept $17/3$ does not impose a condition on either of the two points, because their line is a different, perpendicular line.

Step 3: Conclusion The value is $\boxed{\beta = 5}$. Hence, the correct option is (D).

Quick Tip to Solve: Read the slope as minus the coefficient of x divided by the coefficient of y, take its negative reciprocal, and then use the two-point formula.

22. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is [4 marks]

(A) $\frac{15}{4}$

(B) $\frac{15}{2}$

(C) $\frac{21}{2}$

(D) $\frac{17}{4}$

Correct Answer: (B) $\frac{15}{2}$

Solution:

Concept:

- The area between two curves over a fixed interval is the integral of upper ordinate minus lower ordinate. An area must be nonnegative, so the vertical order of the curves must be checked before integrating.
- For the parabola and line here, the vertical separation is $h(x) = (x^2 + 2) - (x + 1) = x^2 - x + 1$. The bounds $x = 0$ and $x = 3$ are the two vertical boundaries, not roots of the separation.

Step 1: Identify the upper curve Complete the square in the difference of the ordinates:

$$x^2 - x + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} > 0.$$

Thus the parabola remains above the straight line throughout the entire interval. In particular, there is no crossing at which the area integral needs to be split. Using line minus parabola would give a signed negative integral rather than the required area.

Step 2: Integrate the positive separation Integrate with respect to x between the given vertical lines:

$$\begin{aligned} A &= \int_0^3 \left[(x^2 + 2) - (x + 1)\right] dx \\ &= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^3 \\ &= 9 - \frac{9}{2} + 3 = \frac{15}{2}. \end{aligned}$$

The values $15/4$, $21/2$ and $17/4$ do not equal this definite integral. There is no factor of one-half outside the integral: each vertical strip already has its full height equal to the ordinate difference.

Step 3: Conclusion The required area is $\boxed{15/2}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: Complete the square in the ordinate difference first; if it stays positive, use a single upper-minus-lower integral over the full interval.

23. The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, y = 12 – x^2 such that the rectangle lies inside the parabola, is : [4 marks]

(A) 36

(B) $20\sqrt{2}$

(C) $18\sqrt{3}$

(D) 32

Correct Answer: (D) 32

Solution:

Concept:

- A rectangle with a horizontal base has its upper vertices at the same height. On the symmetric parabola $y = 12 - x^2$, these vertices therefore have abscissae $-a$ and a, where $a \geq 0$.
- The rectangle has width $2a$, height $12 - a^2$, and area $A(a) = 2a(12 - a^2)$. The feasible interval is $0 \leq a \leq \sqrt{12}$, since the height cannot be negative.

Step 1: Express the area using one variable Let the two upper corners be $(-a, 12-a^2)$ and $(a, 12-a^2)$. Their projections onto the x-axis give the lower corners. Every horizontal segment between the upper vertices is within the parabola, so this rectangle satisfies the stated containment condition. Hence

$$A(a) = (2a)(12 - a^2) = 24a - 2a^3.$$

The quantity a is the half-width; using it as the entire width would incorrectly halve the area.

Step 2: Locate and verify the maximum Differentiate the area and solve for an interior stationary point:

$$A\prime(a) = 24 - 6a^2 = 0 \quad \Longrightarrow \quad a = 2.$$

Also $A\prime\prime(a) = -12a$, which is negative at $a = 2$. At both endpoints of the feasible interval the area is zero, while this stationary point gives positive area. Thus it is the global maximum:

$$A_{\max} = 2(2)(12 - 4) = 4 \cdot 8 = 32.$$

Consequently 36, $20\sqrt{2}$, and $18\sqrt{3}$ cannot be the maximum: none is the area at the unique interior maximum.

Step 3: Conclusion The maximum area is $\boxed{32}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Use the positive upper-corner abscissa as the half-width: the width is twice that number, while the parabola supplies the height.

24. The Boolean expression $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to : [4 marks]

(A) $p \wedge q$

(B) $p \wedge (\sim q)$

(C) $p \vee (\sim q)$

(D) $(\sim p) \wedge (\sim q)$

Correct Answer: (D) $(\sim p) \wedge (\sim q)$

Solution:

Concept:

- The absorption identity is $(p \wedge q) \vee p = p$. Associativity allows this identity to be applied inside a larger disjunction without changing its truth value.
- Distribution gives $(a \vee b) \wedge c = (a \wedge c) \vee (b \wedge c)$. A statement and its negation cannot both hold: $p \wedge \sim p$ is false; repeated conjunctions obey $q \wedge q = q$.

Step 1: Simplify the first bracket Denote the given expression by E. In its first bracket the term $p \wedge q$ already implies p, so it adds nothing to the disjunction with p:

$$\begin{aligned}(p \wedge q) \vee (p \vee \sim q) &= ((p \wedge q) \vee p) \vee \sim q \\ &= p \vee \sim q.\end{aligned}$$

Therefore $E = (p \vee \sim q) \wedge (\sim p \wedge \sim q)$. This simplification preserves the final conjunction, which cannot be discarded.

Step 2: Distribute and remove the contradiction Distribute the simplified bracket over the final conjunction:

$$\begin{aligned}E &= (p \wedge \sim p \wedge \sim q) \\ &\quad \vee (\sim q \wedge \sim p \wedge \sim q) \\ &= \text{False} \vee (\sim p \wedge \sim q) \\ &= \sim p \wedge \sim q.\end{aligned}$$

A direct check is that E is true precisely when both p and q are false. Options (A) and (B) instead require p true. Option (C) is true also when p is true, so it is too broad. Only (D) has exactly the required truth assignments.

Step 3: Conclusion The expression is equivalent to $\boxed{(\sim p) \wedge (\sim q)}$. Hence, the correct option is (D).

Quick Tip to Solve: First apply absorption inside the disjunction; then use the final conjunction to eliminate any term containing both a proposition and its negation.

25. A tetrahedron has vertices P(1, 2, 1), Q(2, 1, 3), R(–1, 1, 2) and O(0, 0, 0). The angle between the faces OPQ and PQR is : [4 marks]

(A) $\cos^{-1}\left(\frac{17}{31}\right)$

(B) $\cos^{-1}\left(\frac{9}{35}\right)$

(C) $\cos^{-1}\left(\frac{19}{35}\right)$

(D) $\cos^{-1}\left(\frac{7}{31}\right)$

Correct Answer: (C) $\cos^{-1}\left(\frac{19}{35}\right)$

Solution:

Concept:

- A normal to a plane containing two nonparallel vectors is their cross product. For face OPQ, use $\overrightarrow{OP}$ and $\overrightarrow{OQ}$; for face PQR, use the edge vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.
- The acute angle θ between the faces satisfies $\cos\theta = |\mathbf{n}_1 \cdot \mathbf{n}_2|/(|\mathbf{n}_1||\mathbf{n}_2|)$. Reversing either cross product changes its sign but does not change this angle.

Step 1: Calculate the two normal vectors From the given vertices, the normal to the first face is

$$\mathbf{n}_1 = (1, 2, 1) \times (2, 1, 3) = (5, -1, -3).$$

For the other face, subtract the coordinates of P from those of Q and R:

$$\overrightarrow{PQ} = (1, -1, 2), \qquad \overrightarrow{PR} = (-2, -1, 1).$$

Their cross product is

$$\mathbf{n}_2 = (1, -1, 2) \times (-2, -1, 1) = (1, -5, -3).$$

Each normal is perpendicular to both edge vectors of its corresponding face, as can also be checked by dot products.

Step 2: Evaluate the normal-angle formula The scalar product and lengths are

$$\mathbf{n}_1 \cdot \mathbf{n}_2 = 5 + 5 + 9 = 19,$$

$$|\mathbf{n}_1| = \sqrt{25 + 1 + 9} = \sqrt{35}, \qquad |\mathbf{n}_2| = \sqrt{1 + 25 + 9} = \sqrt{35}.$$

Therefore

$$\cos\theta = \frac{19}{\sqrt{35}\sqrt{35}} = \frac{19}{35}.$$

The ratios $17/31$, $9/35$, and $7/31$ do not result from these normals; they would require a different dot product or different normal lengths. Using the angle between two edges instead would not measure the angle between the faces.

Step 3: Conclusion The required angle is $\boxed{\cos^{-1}(19/35)}$. Hence, the correct option is (C).

Quick Tip to Solve: For a face not passing through the origin, form its edge vectors by subtracting one common vertex before taking the cross product.

26. Let C_1 and C_2 be the centres of the circles $x^2 + y^2 - 2x - 2y - 2 = 0$ and $x^2 + y^2 - 6x - 6y + 14 = 0$ respectively. If P and Q are the points of intersection of these circles, then the area (in sq. units) of the quadrilateral PC_1QC_2 is : [4 marks]

(A) 4

(B) 6

(C) 9

(D) 8

Correct Answer: (A) 4

Solution:

Concept:

- Completing squares converts a circle equation into centre-radius form. The common chord of two intersecting circles lies on their radical axis, obtained by subtracting their equations.
- The line joining equal-radius circle centres perpendicularly bisects their common chord. A quadrilateral with perpendicular diagonals has area half the product of their lengths; here those diagonals are PQ and C_1C_2.

Step 1: Find the centres and common chord line The two equations become

$$(x-1)^2 + (y-1)^2 = 4, \qquad (x-3)^2 + (y-3)^2 = 4.$$

Thus $C_1 = (1,1)$, $C_2 = (3,3)$, and both radii equal 2. Subtracting the original equations gives

$$-4x - 4y + 16 = 0 \quad \Longrightarrow \quad x + y = 4.$$

The midpoint M of the centre segment is $(2,2)$, which lies on this common chord. Moreover $C_1C_2 = \sqrt{2^2+2^2} = 2\sqrt{2}$ and $C_1M = \sqrt{2}$. The perpendicularity follows because the centre segment has slope 1 and the chord has slope -1.

Step 2: Calculate the chord and quadrilateral area In the right triangle C_1MP, the hypotenuse is the radius 2. Hence

$$MP = \sqrt{2^2 - (\sqrt{2})^2} = \sqrt{2}, \qquad PQ = 2MP = 2\sqrt{2}.$$

The quadrilateral PC_1QC_2 has perpendicular diagonals of lengths PQ and C_1C_2, giving

$$A = \frac{1}{2}(2\sqrt{2})(2\sqrt{2}) = 4.$$

Equivalently it is the sum of the two triangles on base PQ, whose heights add to C_1C_2. Option (D), 8, omits the factor $1/2$, while 6 and 9 agree with neither this formula nor the computed lengths.

Step 3: Conclusion The quadrilateral has area $\boxed{4}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: For equal intersecting circles, locate the midpoint of the centres first; one right triangle then gives half the common chord.

27. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i}+\hat{j}+\hat{k}, \hat{i}+\mu\hat{j}+\hat{k}, \hat{i}+\hat{j}+\mu\hat{k}$ are co-planar, is : [4 marks]

(A) 2

(B) -1

(C) 0

(D) 1

Correct Answer: (B) -1

Solution:

Concept:

- Three vectors are coplanar exactly when their scalar triple product is zero. Equivalently the determinant of the matrix formed from their three component rows is zero.
- The word distinct means each admissible real value is included only once. A repeated root of the determinant equation still contributes only one value to the requested sum.

Step 1: Write and factor the coplanarity equation The three rows of the component matrix are $(\mu, 1, 1)$, $(1, \mu, 1)$, and $(1, 1, \mu)$. Therefore coplanarity requires

$$\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0.$$

Expanding along its first row gives

$$\begin{aligned} D &= \mu(\mu^2 - 1) - (\mu - 1) + (1 - \mu) \\ &= \mu^3 - 3\mu + 2 \\ &= (\mu - 1)^2(\mu + 2). \end{aligned}$$

For example, the factorisation can be checked by multiplying $(\mu^2 - 2\mu + 1)(\mu + 2)$, which gives exactly the expanded cubic.

Step 2: Select the distinct values and add them The zero determinant occurs for $\mu = 1$ or $\mu = -2$. Both are real. When $\mu = 1$, all three vectors coincide, which certainly makes them coplanar. When $\mu = -2$, their sum is the zero vector, providing a direct linear dependence check. Thus the distinct-value sum is

$$1 + (-2) = -1.$$

The double factor at 1 does not create a third distinct value. In particular, counting the root 1 twice would give 0, the distractor in option (C). The values 2 and 1 omit or mis-handle the negative root and do not give the requested sum.

Step 3: Conclusion The sum of the distinct real values of μ is $\boxed{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: After factorising the scalar triple product, list the different real roots before adding them; multiplicity is irrelevant when the question says distinct.

28. The perpendicular distance from the origin to the plane containing the two lines,
$\frac{x+2}{3} = \frac{y-2}{5} = \frac{z+5}{7}$ **and**
$\frac{x-1}{1} = \frac{y-4}{4} = \frac{z+4}{7}$, **is :** [4 marks]

(A) $6\sqrt{11}$

(B) $\frac{11}{\sqrt{6}}$

(C) 11

(D) $11\sqrt{6}$

Correct Answer: (B) $\frac{11}{\sqrt{6}}$

Solution:

Concept:

- A plane containing two nonparallel lines has a normal perpendicular to both line directions. Their direction-vector cross product therefore supplies the plane normal.
- The distance from $(0,0,0)$ to $ax+by+cz+d=0$ is $|d|/\sqrt{a^2+b^2+c^2}$. The normal and constant must be scaled together when simplifying the plane equation.

Step 1: Determine the plane normal The first line passes through $A=(-2,2,-5)$ with direction $\mathbf{u}=(3,5,7)$. The second passes through $B=(1,4,-4)$ with direction $\mathbf{v}=(1,4,7)$. Their cross product is

$$\mathbf{u}\times\mathbf{v}=(35-28,7-21,12-5)=(7,-14,7).$$

Thus use the proportional normal $\mathbf{n}=(1,-2,1)$. The displacement $B-A=(3,2,1)$ satisfies $\mathbf{n}\cdot(B-A)=3-4+1=0$, confirming that the two lines actually lie in one plane rather than being skew.

Step 2: Find the equation and apply the distance formula The plane through A with that normal is

$$(x+2)-2(y-2)+(z+5)=0,$$

so its equation simplifies to $x-2y+z+11=0$. The point B also satisfies this equation, and both direction vectors have zero dot product with its normal. Hence this is the required containing plane. Its perpendicular distance from the origin is

$$d=\frac{|11|}{\sqrt{1^2+(-2)^2+1^2}}=\frac{11}{\sqrt{6}}.$$

Option (C) uses the constant without normalising the normal, while (D) multiplies by the normal length instead of dividing. Option (A) likewise does not satisfy the point-to-plane distance formula.

Step 3: Conclusion The required perpendicular distance is $\boxed{11/\sqrt{6}}$ units. Hence, the correct option is (B).

Quick Tip to Solve: Use the cross product for the normal, verify the second line belongs to the plane, then divide the absolute constant by the normal length.

29. If a variable line, $3x + 4y - \lambda = 0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 2y + 78 = 0$ are on its opposite sides, then the set of all values of λ is the interval : [4 marks]

(A) (23, 31)

(B) (2, 17)

(C) [13, 23]

(D) [12, 21]

Correct Answer: (D) [12, 21]

Solution:

Concept:

- Complete squares to obtain a circle's centre and radius. A line leaves an entire circle on one side when its distance from the centre is at least the radius.
- For $3x + 4y - \lambda = 0$, the signed centre distance is $(3h + 4k - \lambda)/5$. Opposite signs place the centres on opposite sides; the radius conditions prevent the line cutting either circle.

Step 1: Find the centres and radii Completing squares gives

$$(x-1)^2 + (y-1)^2 = 1, \qquad (x-9)^2 + (y-1)^2 = 4.$$

Thus the first circle has centre $C_1 = (1, 1)$, radius $r_1 = 1$; the second has centre $C_2 = (9, 1)$, radius $r_2 = 2$. The different radii must both be included: separating just the centres would be insufficient.

Step 2: Place the line between the circles The quantities $3x + 4y$ at the two centres are 7 and 31. For separation, the first circle must lie in $3x + 4y \le \lambda$ and the second in $3x + 4y \ge \lambda$. Their distance conditions are therefore

$$\frac{\lambda - 7}{5} \ge 1, \qquad \frac{31 - \lambda}{5} \ge 2.$$

These inequalities give $\lambda \ge 12$ and $\lambda \le 21$. At either endpoint the line is tangent to one circle without entering it; the circles still lie on opposite sides.

Step 3: Check the offered intervals Combining the conditions yields $12 \le \lambda \le 21$. Option A lies beyond the upper admissible value. Option B includes lines that cut or pass to the wrong side of the first circle, and option C extends above 21, cutting the second circle. Only option D contains exactly the admissible values, with the tangency endpoints included.

Step 4: Conclusion The required interval is $[12, 21]$. Hence, the correct option is (D).

Quick Tip to Solve: For a separating line, compare its parameter with $ah + bk \pm r\sqrt{a^2 + b^2}$ for both circles; centre signs alone do not suffice.

30. Let $S_k = \frac{1+2+3+....+k}{k}$. If $S_1^2 + S_2^2 + ... + S_{10}^2 = \frac{5}{12}A$, then A is equal to : [4 marks]

(A) 283

(B) 156

(C) 301

(D) 303

Correct Answer: (D) 303

Solution:

Concept:

- The sum of the first k natural numbers is $k(k+1)/2$, so their arithmetic mean is $S_k = (k+1)/2$.
- The square-sum formula is $\sum_{j=1}^{N} j^2 = N(N+1)(2N+1)/6$. Keep the shifted summation limits when replacing $k+1$ by j.

Step 1: Simplify the term before summing By the arithmetic-series formula,

$$S_k = \frac{1+2+\cdots+k}{k} = \frac{k(k+1)}{2k} = \frac{k+1}{2}.$$

Consequently $S_k^2 = (k+1)^2/4$. In particular $S_1 = 1$ and $S_{10} = 11/2$: the sequence to be squared starts at 1, not at $1/2$. This checks the shift in the index.

Step 2: Evaluate the ten squared means As k runs from 1 to 10, $j = k+1$ runs from 2 to 11. Hence

$$\begin{aligned}\sum_{k=1}^{10} S_k^2 &= \frac{1}{4}\sum_{j=2}^{11} j^2 \\ &= \frac{1}{4}\left(\frac{11\cdot 12\cdot 23}{6} - 1\right) \\ &= \frac{1}{4}(506-1) = \frac{505}{4}.\end{aligned}$$

The subtraction of 1 removes the $j = 1$ term that is not in the required sum. We are summing squares of the means, rather than squaring the sum of those means.

Step 3: Compare the coefficient and eliminate alternatives The given relation is $505/4 = (5/12)A$. Multiplying both sides by $12/5$ gives

$$A = \frac{505}{4}\cdot\frac{12}{5} = 303.$$

This exact coefficient excludes 283, 156 and 301. For an additional direct check, $(5/12)\cdot 303 = 1515/12 = 505/4$, which agrees with the sum just calculated.

Step 4: Conclusion Thus $A = 303$. Hence, the correct option is (D).

Quick Tip to Solve: Simplify S_k to $(k+1)/2$ first; the square sum then runs from 2^2 to 11^2, so subtract 1^2.

Physics (120 marks)

31. A particle of mass m moves in a circular orbit in a central potential field U(r) = $\frac{1}{2}$ kr^2. If Bohr 's quantization conditions are applied, radii of possible orbitls and energy levels vary with quantum number n as : [4 marks]

(A) $r_n \propto \sqrt{n}$, $E_n \propto n$

(B) $r_n \propto \sqrt{n}$, $E_n \propto \frac{1}{n}$

(C) $r_n \propto n$, $E_n \propto n$

(D) $r_n \propto n^2$, $E_n \propto \frac{1}{n^2}$

Correct Answer: (A) $r_n \propto \sqrt{n}$, $E_n \propto n$

Solution:

Concept:

- For a central potential $U(r) = kr^2/2$, the inward force magnitude is $dU/dr = kr$. Circular motion requires $mv^2/r = kr$.
- Bohr quantization imposes angular momentum $mvr = n\hbar$, where $\hbar = h/(2\pi)$ and n is a positive integer. The total orbital energy is kinetic plus potential energy.

Step 1: Relate orbital speed to radius Balance the attractive central force and the required centripetal force:

$$\frac{mv^2}{r} = kr \quad \Longrightarrow \quad v = \sqrt{\frac{k}{m}}\, r.$$

Thus the angular speed $v/r = \sqrt{k/m}$ is independent of radius. This force is proportional to radius, so the Coulomb-orbit formulas for hydrogen do not apply to this potential.

Step 2: Apply angular momentum quantization Substitute this speed into $mvr = n\hbar$:

$$\sqrt{mk}\, r_n^2 = n\hbar, \qquad r_n = \left(\frac{n\hbar}{\sqrt{mk}}\right)^{1/2}.$$

The mass and spring-like constant are fixed, leaving $r_n \propto n^{1/2}$. The two factors of radius in angular momentum are essential: retaining only one would incorrectly suggest a linear dependence.

Step 3: Calculate the total energy and test the choices The circular-force relation gives $K = mv^2/2 = kr_n^2/2$, equal to the potential energy. Therefore

$$E_n = K + U = kr_n^2 = n\hbar\sqrt{\frac{k}{m}} \propto n.$$

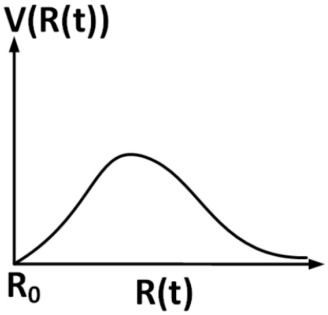
Option B has the correct radius scaling but the wrong inverse energy scaling. Option C incorrectly makes radius proportional to n. Option D gives the familiar Coulomb pattern, which is incompatible with the present r^2 potential. Both required dependences occur together only in A.

Step 4: Conclusion The allowed circular orbits obey $r_n \propto \sqrt{n}$ and $E_n \propto n$. Hence, the correct option is (A).

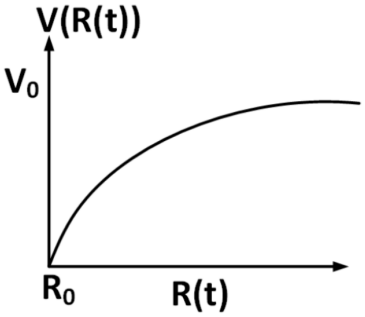
Quick Tip to Solve: Derive the force from the given potential before using quantization; Bohr's angular momentum rule alone does not imply hydrogen-like scaling.

32. **There is a uniform spherically symmetric surface charge density at a distance R_0 from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed V (R(t)) of the distribution as a function of its instantaneous radius R (t) is :** [4 marks]

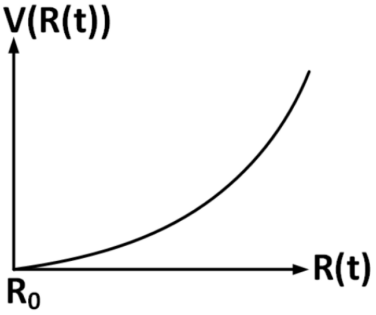
(A)



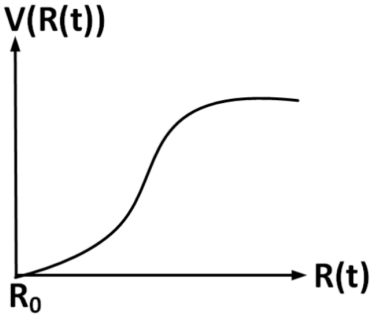
(B)



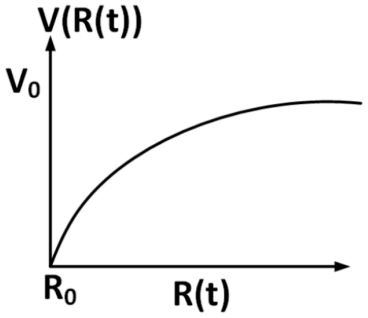
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- A uniformly charged spherical shell with fixed total charge Q and radius R stores electrostatic energy $U(R) = Q^2/(8\pi\varepsilon_0 R)$. Its decreasing field energy becomes radial kinetic energy.
- Spherical symmetry makes all surface elements move radially with the same speed V. If M is the shell's total mass, its kinetic energy is $MV^2/2$; total mechanical energy is conserved.

Step 1: Use the initially stationary state Initially $R = R_0$, $V = 0$, and all the energy is electrostatic. At a later radius $R \geq R_0$, conservation gives

$$\frac{1}{2}MV^2 + \frac{Q^2}{8\pi\varepsilon_0 R} = \frac{Q^2}{8\pi\varepsilon_0 R_0}.$$

The charge stays on the expanding spherical surface, so its total charge is constant even though its surface charge density decreases as the sphere grows.

Step 2: Determine the speed and limiting behaviour Solving for the speed gives

$$V(R) = V_0\sqrt{1 - \frac{R_0}{R}}, \qquad V_0 = \sqrt{\frac{Q^2}{4\pi\varepsilon_0 M R_0}}.$$

It starts at zero, increases for all $R > R_0$, and tends to the finite nonzero value V_0 as R tends to infinity. The repulsive force weakens, but never reverses: declining acceleration does not mean declining speed.

Step 3: Select the graph using its curvature For $R > R_0$, the slope is

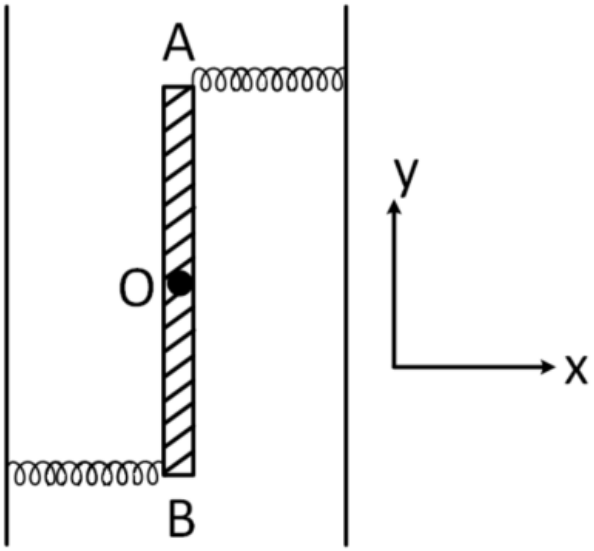
$$\frac{dV}{dR} = \frac{V_0 R_0}{2R^2\sqrt{1 - R_0/R}} > 0.$$

The slope decreases with radius and is very large just after R_0, so the curve rises steeply and then flattens. Graph A wrongly makes the speed decrease toward zero. Graph C has increasing slope and no finite saturation. Graph D begins with a shallow, upward-curving part, inconsistent with the initial steep slope. Graph B best matches all the required features.

Step 4: Conclusion The speed increases from zero with decreasing slope and approaches V_0. Hence, the correct option is (B).

Quick Tip to Solve: For electrostatic expansion, plot speed against radius using energy conservation; decreasing repulsion reduces the slope but cannot make the speed fall.

33. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length ℓ and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is : [4 marks]



(A) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

(B) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(D) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

Correct Answer: (B) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$

Solution:

Concept:

- A uniform rod of mass m and length ℓ has moment of inertia $I = m\ell^2/12$ about its centre perpendicular to the horizontal plane.
- For a small angular displacement θ, each end moves transversely by approximately $(\ell/2)\theta$. The two springs provide restoring torques, and $I\ddot{\theta} = -\kappa\theta$ gives angular frequency $\omega = \sqrt{\kappa/I}$.

Step 1: Find the small spring displacements Let the rod turn through a small angle θ from equilibrium. End A and end B move in opposite horizontal directions, each through magnitude

$$x = \frac{\ell}{2}\sin\theta \simeq \frac{\ell\theta}{2}.$$

One spring extends while the other compresses. Each incremental force has magnitude kx and opposes the end displacement. The pivot fixes the centre, so translational motion does not contribute to the oscillation.

Step 2: Add the restoring torques Each end is a distance $\ell/2$ from the pivot. To first order in θ, the two torques act in the same restoring sense:

$$\tau = -2\left(k\frac{\ell\theta}{2}\right)\frac{\ell}{2} = -\frac{k\ell^2}{2}\theta.$$

The effective torsional constant is therefore $\kappa = k\ell^2/2$. Opposite spring-force directions do not cancel the torques, because the position vectors of the two ends are also opposite.

Step 3: Obtain angular frequency and ordinary frequency With $I = m\ell^2/12$, the equation of motion becomes

$$\ddot{\theta} + \frac{k\ell^2/2}{m\ell^2/12}\theta = 0, \qquad \omega^2 = \frac{6k}{m}.$$

Thus $f = \omega/(2\pi)$. The factor 3 in A would result from including only one spring torque. The factors 1 and 2 in C and D do not account correctly for the rod's rotational inertia. The length cancels because torque constant and inertia both scale as ℓ^2.

Step 4: Conclusion The frequency is $f = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: Add the two restoring torques, then divide by $m\ell^2/12$; using a translational mass instead of the rod's moment of inertia loses the factor 6.

34. A proton and an α-particle (with their masses in the ratio of 1 : 4 and charges in the ratio of 1 : 2) are accelerated from rest through a potential difference V. If a uniform magnetic field (B) is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them will be ; [4 marks]

(A) $1 : \sqrt{3}$

(B) $1 : 3$

(C) $1 : \sqrt{2}$

(D) $1 : 2$

Correct Answer: (C) $1 : \sqrt{2}$

Solution:

Concept:

- For a nonrelativistic particle accelerated from rest through potential difference V, electric work gives kinetic energy $qV = mv^2/2$. The common voltage does not give equal speeds or equal energies when charges differ.
- In a uniform magnetic field perpendicular to the velocity, $qvB = mv^2/r$, so $r = mv/(qB) = p/(qB)$. Combining acceleration and bending makes $r \propto \sqrt{m/q}$ at fixed V, B.

Step 1: Express the radius in terms of mass and charge From the electric acceleration,

$$v = \sqrt{\frac{2qV}{m}}.$$

Substitute this into the magnetic radius rather than assuming either speed is shared:

$$r = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}.$$

The field changes direction of motion without changing kinetic energy, because its force is always perpendicular to velocity.

Step 2: Substitute the proton and alpha-particle ratios Write $m_p = m$, $q_p = e$, $m_\alpha = 4m$ and $q_\alpha = 2e$. Therefore

$$\frac{r_p}{r_\alpha} = \sqrt{\frac{m_p/q_p}{m_\alpha/q_\alpha}} = \sqrt{\frac{m/e}{4m/(2e)}} = \frac{1}{\sqrt{2}}.$$

Thus the alpha-particle path has the larger radius, by a factor $\sqrt{2}$. Both the common field magnitude and potential difference cancel in this ratio.

Step 3: Check why the other ratios fail Option D, $1 : 2$, could arise by treating the speeds as equal and using only m/q; they are not equal after a common accelerating voltage. Indeed $v_\alpha/v_p = 1/\sqrt{2}$. The factors $\sqrt{3}$ and 3 in A and B cannot follow from the specified mass-to-charge ratios of 1 and 2. Only C includes the square root that comes from the energy relation.

Step 4: Conclusion The required ratio is $r_p : r_\alpha = 1 : \sqrt{2}$. Hence, the correct option is (C).

Quick Tip to Solve: At a common accelerating voltage, magnetic radius scales as $\sqrt{m/q}$; the simpler m/q scaling applies only when the speeds are equal.

35. A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when : (i) they are moving in the same direction , and (ii) in the opposite direction is : [4 marks]

(A) $\frac{25}{11}$
(B) $\frac{3}{2}$
(C) $\frac{11}{5}$
(D) $\frac{5}{2}$

Correct Answer: (C) $\frac{11}{5}$

Solution:

Concept:

- Two trains completely cross when their relative displacement equals the sum of their lengths. Thus the crossing distance here is $60 + 120 = 180\,\text{m}$ in either direction arrangement.
- Relative speed is the speed difference for motion in the same direction and the speed sum for opposite directions. Time equals relative distance divided by relative speed; equal distances make the time ratio the inverse speed ratio.

Step 1: Find the required relative displacement For complete crossing, the passenger train's rear must

clear the freight train's farther end. The relative distance is therefore the combined length:

$$L = 60 + 120 = 180\,\text{m}.$$

This distance is the same in both cases, since changing the directions changes relative speed, not the trains' lengths.

Step 2: Calculate both relative speeds and times For the same direction,

$$u_{\text{same}} = 80 - 30 = 50\,\text{km h}^{-1} = \frac{125}{9}\,\text{m s}^{-1}, \qquad t_{\text{same}} = \frac{180}{125/9} = 12.96\,\text{s}.$$

For opposite directions,

$$u_{\text{opp}} = 80 + 30 = 110\,\text{km h}^{-1} = \frac{275}{9}\,\text{m s}^{-1}, \qquad t_{\text{opp}} = \frac{180}{275/9} = \frac{324}{55}\,\text{s}.$$

The smaller relative speed in the same-direction case gives the longer crossing time.

Step 3: Form the time ratio in the requested order The identical crossing distance and speed-conversion factor cancel:

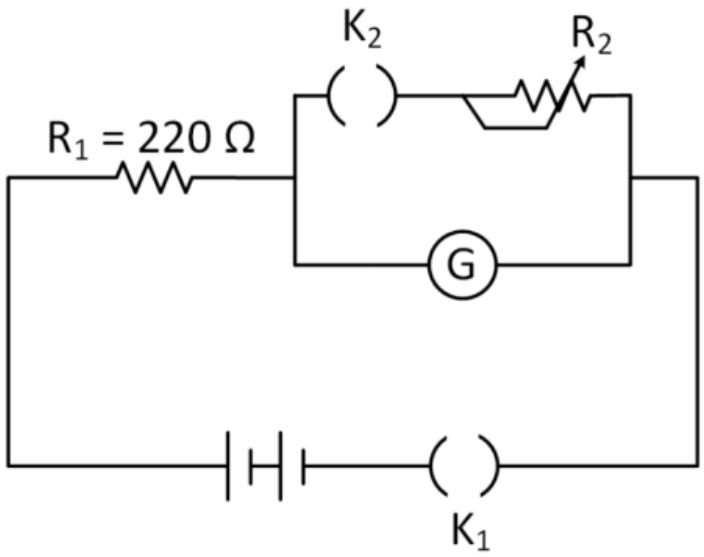
$$\frac{t_{\text{same}}}{t_{\text{opp}}} = \frac{180/u_{\text{same}}}{180/u_{\text{opp}}} = \frac{110}{50} = \frac{11}{5}.$$

Options A, B and D give different values and fail this ratio. Using either train's ground speed would ignore the motion of the other train.

Step 4: Conclusion The ratio of the times in cases (i) and (ii) is $11 : 5$, or $11/5$. Hence, the correct option is (C).

Quick Tip to Solve: When the same two trains cross in both cases, their combined length cancels: take $(v_1 + v_2)/(v_1 - v_2)$ for same-direction time over opposite-direction time.

36. The galvanometer deflection, when key K_1 is closed but K_2 is open, equals θ_0 (see figure). On closing K_2 also and adjusting R_2 to 5Ω, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. . The resistance of the galvanometer is, then, given by [Neglect the internal resistance of battery] : [4 marks]



(A) 5 Ω

(B) 25 Ω

(C) 12 Ω

(D) 22 Ω

Correct Answer: (D) 22 Ω

Solution:

Concept:

- Deflection is proportional to current through a moving-coil galvanometer: $\theta \propto I_g$. The specified change therefore gives $I_g/I_0 = 1/5$.
- A shunt carries current parallel to the galvanometer. Parallel branches have equal voltage, while the series resistance limits the total current. Let the galvanometer resistance be R_g and the battery emf be E.

Step 1: Write the current before connecting the shunt With K_1 closed and K_2 open, the shunt branch is disconnected. The same current passes through $R_1 = 220\,\Omega$ and the galvanometer. Neglecting battery internal resistance as directed, Ohm's law gives

$$I_0 = \frac{E}{220 + R_g}.$$

This produces the initial deflection θ_0.

Step 2: Find the galvanometer current after closing the second key The resistance $R_2 = 5\,\Omega$ now shunts the galvanometer. Their equivalent resistance is $5R_g/(R_g + 5)$. Current division sends the

fraction $5/(R_g+5)$ of the circuit current through the galvanometer, so

$$I_g = \frac{E}{220 + 5R_g/(R_g+5)} \frac{5}{R_g+5} = \frac{5E}{225R_g + 1100}.$$

The shunt changes the circuit current as well as dividing it; treating the original total current as constant would be incorrect.

Step 3: Use the deflection ratio and test the alternatives Cancel the emf in the ratio:

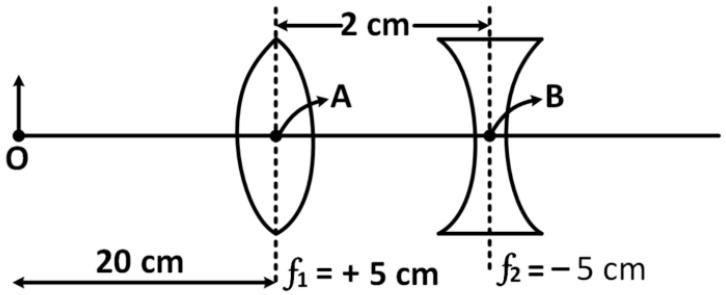
$$\frac{I_g}{I_0} = \frac{5(220+R_g)}{225R_g + 1100} = \frac{1}{5}.$$

Hence $5500+25R_g = 225R_g+1100$, giving $200R_g = 4400$ and $R_g = 22\,\Omega$. Substitution gives numerator 1210 and denominator 6050, exactly the required ratio $1/5$. The resistances 5, 25 and 12 ohms do not satisfy this ratio.

Step 4: Conclusion The galvanometer resistance is $\boxed{22\,\Omega}$. Hence, the correct option is (D).

Quick Tip to Solve: When adding a shunt, recompute the series-plus-parallel circuit before applying current division; the battery current generally changes.

37. What is the position and nature of image formed by lens combination shown in figure ? (f_1, f_2 are focal lengths) [4 marks]



(A) $\frac{20}{3}$ cm from point B at right, real

(B) 70 cm from point B at right ; real

(C) 40 cm from point B at right; real

(D) 70 cm from point B at left ; virtual

Correct Answer: (B) 70 cm from point B at right ; real

Solution:

Concept:

- Use the Cartesian lens formula $1/v - 1/u = 1/f$, with distances measured from each lens and positive in the direction of incident light.
- The image of the first lens serves as the object for the second. If the converging rays would meet to the right of the second lens, that lens has a virtual object and its object distance is positive.

Step 1: Locate the image formed by the converging lens The object lies 20 cm to the left of lens A, hence $u_1 = -20\,\text{cm}$, while $f_1 = +5\,\text{cm}$. Thus

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{5} - \frac{1}{20} = \frac{3}{20}, \qquad v_1 = \frac{20}{3}\,\text{cm}.$$

Without lens B, these rays would converge to a real image this far to the right of A.

Step 2: Measure the object distance from the second lens Lens B is only 2 cm to the right of A, so it intercepts the rays before they meet. Their intended meeting point is to the right of B by

$$u_2 = \frac{20}{3} - 2 = \frac{14}{3}\,\text{cm}.$$

This is a virtual object for B, so the positive sign is essential. Using a negative sign merely because it is called an object would give the wrong image location.

Step 3: Apply the second lens formula and identify the image For the concave lens, $f_2 = -5\,\text{cm}$. Therefore

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{5} + \frac{3}{14} = \frac{1}{70}, \qquad v_2 = 70\,\text{cm}.$$

The positive image distance means that the emerging rays actually converge to the right of B: the final image is real. Option A retains the first image distance and ignores B; option C does not satisfy the second lens equation; option D reverses the side and nature of the final image.

Step 4: Conclusion The final image is real, $\boxed{70\,\text{cm}}$ to the right of B. Hence, the correct option is (B).

Quick Tip to Solve: For separated lenses, subtract the separation from the first image distance and decide whether the second object is real or virtual before assigning its sign.

38. A straight rod of length L extends from x = a to x = L + a. The gravitational force it exerts on a point mass 'm' at x = 0, if the mass per unit length of the rod is A + Bx^2 , is given by : [4 marks]

(A) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) - BL\right]$

(B) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

(C) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) + BL\right]$

(D) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) - BL\right]$

Correct Answer: (B) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

Solution:

Concept:

- Newton's law for a small mass element is $dF = Gm\,dm/x^2$, where x is its distance from the point mass at the origin.
- For linear density $\lambda(x) = A + Bx^2$, a segment of length dx has mass $dm = (A + Bx^2)\,dx$. All elements lie on the same side of the point mass, so their attractive forces add along the rod.

Step 1: Express the force of a rod element Take a short element at coordinate x, between the stated endpoints a and $a + L$. The rod has negligible transverse size, so its distance from the mass is simply x. Substituting its mass into the inverse-square law gives

$$dF = \frac{Gm(A + Bx^2)}{x^2}\,dx = Gm\left(\frac{A}{x^2} + B\right)dx.$$

The x^2 in the density cancels the inverse-square factor in the second contribution.

Step 2: Integrate over the full rod Integrate the force components from the near end to the far end, keeping both endpoints:

$$F = Gm\int_a^{a+L}\left(\frac{A}{x^2} + B\right)dx = Gm\left[-\frac{A}{x} + Bx\right]_a^{a+L}.$$

Evaluating the upper limit minus the lower limit yields

$$F = Gm\left[A\left(\frac{1}{a} - \frac{1}{a + L}\right) + BL\right].$$

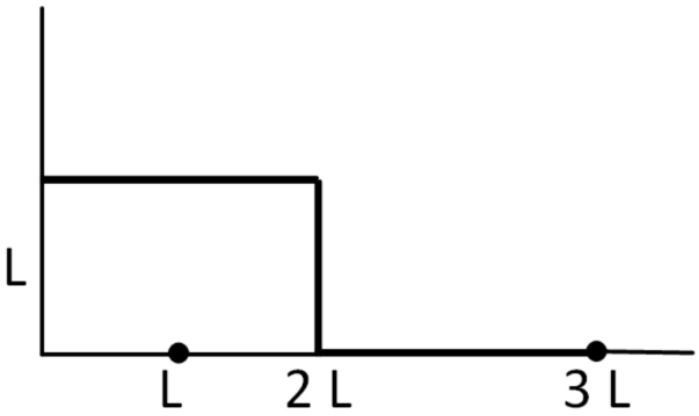
For a rod to the right of the origin, the force is directed along positive x, towards the rod.

Step 3: Check signs and exclude the other expressions The uniform-density contribution is positive because $1/a > 1/(a + L)$ for $a > 0$. For positive B, the density increment adds mass and therefore adds $GmBL$, rather than subtracting it. Option A has the wrong sign on this increment; C reverses the uniform contribution; D reverses both. As L tends to zero, both terms vanish, as a rod of vanishing length must produce zero force.

Step 4: Conclusion The force magnitude is $\boxed{Gm[A(1/a - 1/(a + L)) + BL]}$, directed towards the rod. Hence, the correct option is (B).

Quick Tip to Solve: Insert the varying density into the force integrand before integrating; here its quadratic term cancels the inverse-square distance factor.

39. The position vector of the centre of mass $\vec{r}_{cm}$ of an asymmetric uniform bar of negligible area of crosssection as shown in figure is : [4 marks]



(A) $\vec{r}_{cm} = \frac{11}{8}L\,\hat{x} + \frac{8}{8}L\hat{y}$
(B) $\vec{r}_{cm} = \frac{5}{8}L\,\hat{x} + \frac{13}{8}L\hat{y}$
(C) $\vec{r}_{cm} = \frac{13}{8}L\,\hat{x} + \frac{5}{8}L\hat{y}$
(D) $\vec{r}_{cm} = \frac{3}{8}L\,\hat{x} + \frac{11}{8}L\hat{y}$

Correct Answer: (C) $\vec{r}_{cm} = \frac{13}{8}L\,\hat{x} + \frac{5}{8}L\hat{y}$

Solution:

Concept:

- For a uniform thin bar, mass is proportional to segment length. Its centre of mass follows $x_{\text{cm}} = \sum l_i x_i / \sum l_i$, and similarly for y.
- Each straight segment has its own centre at its midpoint. The coordinate axes establish positions; only the thick bent bar contributes mass.

Step 1: Identify the three material segments The bar consists of the upper horizontal segment from $(0, L)$ to $(2L, L)$, the vertical segment from $(2L, L)$ to $(2L, 0)$, and the lower horizontal segment from $(2L, 0)$ to $(3L, 0)$. Their lengths are respectively $2L, L, L$, so the total length is $4L$. Their midpoints are respectively

$$(L, L), \qquad (2L, L/2), \qquad (5L/2, 0).$$

There is no material bar along the coordinate axes on the left or below the upper segment.

Step 2: Compute the horizontal centre coordinate Let the common linear density be λ. It cancels between numerator and denominator, leaving the length-weighted average:

$$x_{\text{cm}} = \frac{(2L)(L) + (L)(2L) + (L)(5L/2)}{4L} = \frac{(13/2)L^2}{4L} = \frac{13L}{8}.$$

A simple average of the three midpoint positions would be incorrect because the upper segment has twice the mass of either other segment.

Step 3: Compute the vertical centre coordinate and compare Use the same weights for the heights:

$$y_{\text{cm}} = \frac{(2L)(L) + (L)(L/2) + (L)(0)}{4L} = \frac{(5/2)L^2}{4L} = \frac{5L}{8}.$$

Both coordinates lie within the bounding rectangle, as expected. Option B swaps the two components and even puts the height above the bar's maximum height L; D likewise has a height above L. Option A places the centre at height L, impossible when some bar mass lies strictly below that height.

Step 4: Conclusion The position vector is $\boxed{\vec{r}_{\text{cm}} = (13L/8)\hat{x} + (5L/8)\hat{y}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a uniform bent wire, list each straight segment's length and midpoint; use lengths as mass weights and omit the coordinate axes.

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40. A travelling harmonic wave is represented by the equation y(x,t) = 10^{-3}sin (50t + 2x), where, x and y are in mater and t is in seconds. Which of the following is a correct statement about the wave ? [4 marks]

(A) The wave is propagating along the positive x-axis with speed 100 ms^{-1}

(B) The wave is propagating along the positive x-axis with speed 25 ms^{-1}

(C) The wave is propagating along the negative x-axis with speed 25 ms^{-1}

(D) The wave is propagating along the negative x-axis with speed 100 ms^{-1}

Correct Answer: (C) The wave is propagating along the negative x-axis with speed 25 ms^{-1}

Solution:

Concept:

- A travelling wave has constant phase along a moving crest. For $y = A\sin(\omega t + kx)$, the plus sign gives propagation towards decreasing x.
- The phase speed has magnitude $v = \omega/k$. Angular frequency and wave number must be read from the coefficients of time and position, not from the amplitude.

Step 1: Read the wave parameters Compare the given expression with the general harmonic form:

$$y(x,t) = 10^{-3}\sin(50t + 2x).$$

The amplitude is $A = 10^{-3}\,\text{m}$, angular frequency is $\omega = 50\,\text{rad}\,\text{s}^{-1}$, and wave number is $k = 2\,\text{rad}\,\text{m}^{-1}$. The amplitude sets the maximum transverse displacement and has no role in determining the propagation speed.

Step 2: Follow a point of fixed phase A crest is identified by a fixed value of the phase. Thus along its path,

$$50t + 2x = \text{constant}.$$

Differentiating with respect to time gives

$$50 + 2\frac{dx}{dt} = 0 \quad \Longrightarrow \quad \frac{dx}{dt} = -25\,\mathrm{m\,s^{-1}}.$$

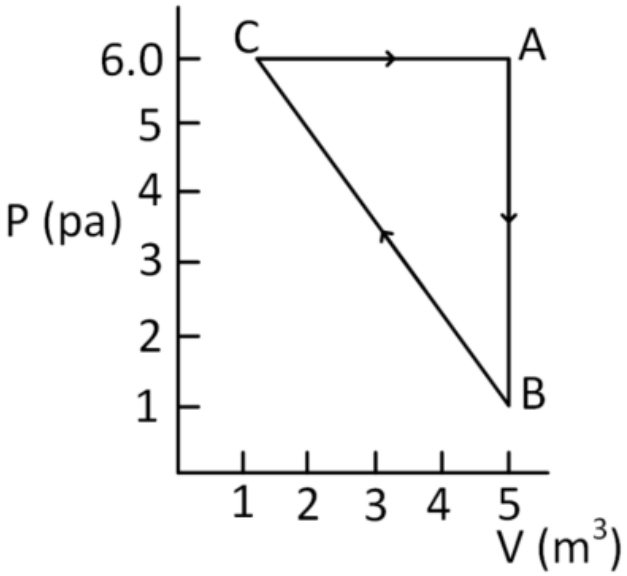
The negative sign means the wave pattern moves along the negative x-axis; the speed, being a magnitude, is positive.

Step 3: Check the magnitude and reject the alternatives The direct speed formula gives $v = \omega/k = 50/2 = 25\,\mathrm{m\,s^{-1}}$, confirming the phase calculation. Equivalently, $f = 50/(2\pi)$ and $\lambda = 2\pi/2$, so $f\lambda = 25\,\mathrm{m\,s^{-1}}$. Options A and B assign the wrong direction for a plus sign between the time and position terms. Option D has the right direction but uses 100 rather than the ratio 25 for the speed. Multiplying the coefficients does not give a wave speed.

Step 4: Conclusion The wave propagates along the negative x-axis with speed $\boxed{25\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: To determine direction without memorising signs, set the phase equal to a constant and solve for how the crest coordinate changes with time.

41. For the given cyclic process CAB as shown for a gas, the work done is : [4 marks]



(A) 1 J

(B) 10 J

(C) 5 J

(D) 30 J

Correct Answer: (B) 10 J

Solution:

Concept:

- Work done by a gas is $W = \int P\,dV$. Expansion contributes positive work, compression negative work, and a constant-volume path contributes zero.
- The net work in a complete pressure-volume cycle is the signed area enclosed. A clockwise cycle gives positive work by the gas.

Step 1: Read the vertices and direction of the cycle The diagram gives $C = (1\,\mathrm{m}^3, 6\,\mathrm{Pa})$, $A = (5\,\mathrm{m}^3, 6\,\mathrm{Pa})$, and $B = (5\,\mathrm{m}^3, 1\,\mathrm{Pa})$. The arrows trace $C \to A \to B \to C$, a clockwise cycle. The gas expands at the higher pressure along CA and returns by compression at lower pressures along BC; therefore the net work is positive.

Step 2: Calculate the work on each leg On CA the pressure remains 6 Pa while volume increases by $4\,\mathrm{m}^3$, giving

$$W_{CA} = 6(5-1) = 24\,\mathrm{J}.$$

On AB the volume is constant, so $W_{AB} = 0$. On BC pressure varies linearly from 1 Pa to 6 Pa as the volume decreases from 5 to $1\,\mathrm{m}^3$. Its average pressure is $(1+6)/2 = 3.5\,\mathrm{Pa}$, hence

$$W_{BC} = 3.5(1-5) = -14\,\mathrm{J}.$$

Step 3: Add the contributions and verify the enclosed area The full cycle gives

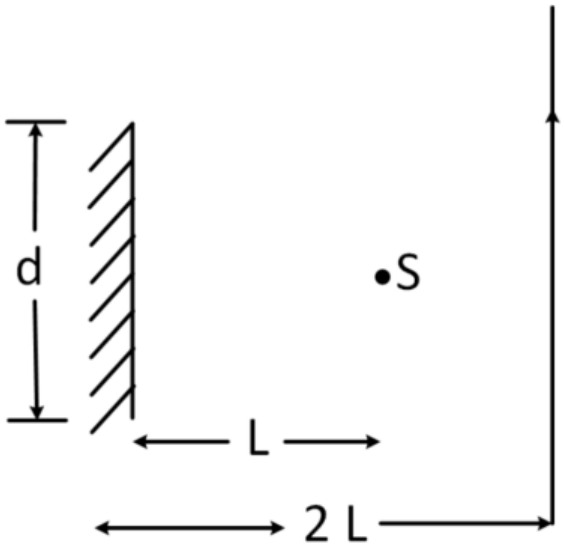
$$W = 24 + 0 - 14 = 10\,\mathrm{J}.$$

The triangle has horizontal base $5 - 1 = 4\,\mathrm{m}^3$ and vertical height $6 - 1 = 5\,\mathrm{Pa}$. Thus its area is $\frac{1}{2}(4)(5) = 10\,\mathrm{Pa\,m}^3 = 10\,\mathrm{J}$, in agreement. Neither 1 J nor 5 J equals this area. The 30 J alternative uses an origin-based rectangle rather than the actual enclosed triangular region.

Step 4: Conclusion The net work done by the gas in the specified cycle is $\boxed{10\,\mathrm{J}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a triangular pressure-volume cycle, use half the product of the pressure range and volume range; use the arrows to determine the sign.

42. A point source of light, S is placed at distance L in front of the centre of plane mirror of width d which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance 2L as shown below. The distance over which the man can see the image of the light source in the mirror is [4 marks]



(A) d

(B) d/2

(C) 3d

(D) 2d

Correct Answer: (C) 3d

Solution:

Concept:

- A plane mirror forms a virtual image as far behind the mirror as the object is in front. The observer sees the image only when the line from the eye to that image intersects the reflecting segment.
- The two extreme lines pass through the mirror edges. Similar triangles determine the accessible span on an observer line parallel to the mirror.

Step 1: Replace the source by its virtual image Place the mirror on $x = 0$, with its centre at $y = 0$, and its edges at $y = \pm d/2$. The source is at $(L, 0)$, so its virtual image is $S\prime = (-L, 0)$. The man moves along $x = 2L$. This replacement is valid because reflected rays entering his eye appear to have come in straight lines from the virtual image; it incorporates the law of reflection.

Step 2: Find where a limiting ray reaches the observer line The distance from the virtual image to the mirror plane is L, whereas its distance to the observer line is $L + 2L = 3L$. A straight ray from $S\prime$ through the upper edge has vertical displacement $d/2$ at distance L. By similar triangles, its displacement at distance $3L$ is

$$y_{\max} = \frac{3L}{L}\frac{d}{2} = \frac{3d}{2}.$$

The corresponding lower-edge ray reaches $y_{\min} = -3d/2$.

Step 3: Determine the entire visible interval The image is visible at every position between these two limiting rays, because each eye-to-image line then meets a point within the mirror. Outside that interval it misses the mirror. The length of the interval is

$$D = y_{\max} - y_{\min} = \frac{3d}{2} - \left(-\frac{3d}{2}\right) = 3d.$$

Options d and d/2 fail to include the geometric magnification of the mirror aperture at the more distant observer line. Option 2d uses only the observer-to-mirror distance; the correct triangle starts at the virtual image, so the total distance is 3L.

Step 4: Conclusion The man can see the image over a distance $\boxed{3d}$. Hence, the correct option is (C).

Quick Tip to Solve: Measure both similar-triangle distances from the virtual image behind the mirror; their ratio magnifies the finite mirror width into the viewing interval.

43. An ideal gas occupies a volume of 2m^3 at a pressure of 3 $\times$ 10^6 Pa. The energy of the gas is : [4 marks]

(A) 6 $\times$ 10^4 J

(B) 9$\times$ 10^6 J

(C) 3 $\times$ 10^2 J

(D) 10^8 J

Correct Answer: (B) 9$\times$ 10^6 J

Solution:

Concept:

- The random translational kinetic energy of an ideal gas is related to its absolute temperature by $K = \frac{3}{2}nRT$, where n is the amount of gas and R is the gas constant.
- The ideal-gas equation is $PV = nRT$. Combining these equations eliminates the unspecified amount and temperature and gives $K = \frac{3}{2}PV$.

Step 1: Identify the energy relation Here the energy is evaluated using the kinetic-theory relation for random translational motion. It follows equally from $PV = \frac{1}{3}Nm\langle c^2\rangle$ and $K = \frac{1}{2}Nm\langle c^2\rangle$, with N molecules of mass m. Thus no individual molecular mass, temperature or amount of gas is needed. Rotational and vibrational contributions would require additional information about the molecules.

Step 2: Calculate the pressure-volume product The given pressure is $P = 3 \times 10^6\,\mathrm{Pa}$ and the volume is $V = 2\,\mathrm{m}^3$. Therefore

$$PV = (3 \times 10^6)(2) = 6 \times 10^6\,\mathrm{J}.$$

The product has energy units because $\mathrm{Pa\,m^3} = \mathrm{N\,m} = \mathrm{J}$.

Step 3: Evaluate the kinetic energy Multiply the pressure-volume product by the kinetic-theory factor:

$$K = \frac{3}{2}(6 \times 10^6) = 9 \times 10^6\,\mathrm{J}.$$

The factor $3/2$ counts the three independent translational velocity components, each contributing one half of nRT.

Step 4: Compare the choices Only option (B) has this magnitude. Options (A) and (C) are much smaller and have incorrect powers of ten, while option (D) is larger than the value obtained from $3PV/2$. Merely using PV as the energy would also omit the required kinetic-theory factor.

Step 5: Conclusion The gas has translational kinetic energy 9×10^6 J. Hence, the correct option is (B).

Quick Tip to Solve: For a kinetic-theory energy question with only pressure and volume supplied, compute PV in joules first, then use $K = 3PV/2$.

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44. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5 μm diameter of a wire is : [4 marks]

(A) 500

(B) 100

(C) 200

(D) 50

Correct Answer: (C) 200

Solution:

Concept:

- A screw gauge converts one full rotation into an axial advance equal to its pitch p. With N equal circular-scale divisions, its least count is $\mathrm{LC} = p/N$.
- To resolve a diameter d, the least count must be no greater than d. The minimum division count is obtained at the limiting equality $p/N = d$.

Step 1: Express the required resolution in millimetres The wire diameter is $5\,\mu\mathrm{m}$. Since $1\,\mathrm{mm} = 1000\,\mu\mathrm{m}$, the required resolution in main-scale units is

$$d = \frac{5}{1000}\,\mathrm{mm} = 0.005\,\mathrm{mm}.$$

The conversion is essential: using 5 directly with a pitch expressed in millimetres would mix units.

Step 2: Use the screw advance per complete turn The stated main-scale step of $1\,\mathrm{mm}$ is used as the pitch for this gauge. Thus one complete revolution corresponds to an axial advance of $p = 1\,\mathrm{mm}$. Dividing that advance into N equal circular divisions gives $\mathrm{LC} = 1/N\,\mathrm{mm}$.

Step 3: Find the smallest acceptable division count Require the least count to resolve the given diameter:

$$\frac{1}{N} \le 0.005 \quad \Longrightarrow \quad N \ge \frac{1}{0.005} = 200.$$

The limiting count 200 therefore gives a least count exactly equal to $5\,\mu\mathrm{m}$. Increasing the number further improves resolution but does not give the minimum requested.

Step 4: Check all the proposed scales For 50 divisions the least count is $20\,\mu\mathrm{m}$, and for 100 it is

$10\,\mu\text{m}$; neither resolves the required diameter. A 500-division scale gives $2\,\mu\text{m}$, which is sufficient but unnecessarily fine. The 200-division scale is the first acceptable choice.

Step 5: Conclusion The minimum number of circular-scale divisions is 200. Hence, the correct option is (C).

Quick Tip to Solve: Convert the wire diameter to the pitch unit before dividing. A finer scale can work, but a question asking for the minimum requires the smallest sufficient division count.

45. A simple pendulum, made of a string of length ℓ and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then M is given by : [4 marks]

(A) $\frac{m}{2}\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$

(B) $\frac{m}{2}\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

(C) $m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$

(D) $m\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

Correct Answer: (C) $m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$

Solution:

Concept:

- For a pendulum released from a small angular amplitude θ, conservation of mechanical energy gives a lowest-point speed $v=\sqrt{g\ell}\,\theta$, with the angle in radians.
- In a one-dimensional elastic collision with a stationary block, momentum is conserved and the relative speed of separation equals the relative speed of approach. A rebound velocity must be assigned a negative sign.

Step 1: Relate both speeds to the angular amplitudes Let the bob approach the block with positive speed u and rebound with speed magnitude w. Its gravitational potential energy at the turning point is $mg\ell(1-\cos\theta)$. For the small angles specified, $1-\cos\theta \simeq \theta^2/2$, so

$$u=\sqrt{g\ell}\,\theta_0, \qquad w=\sqrt{g\ell}\,\theta_1.$$

Consequently $w/u=\theta_1/\theta_0$; the actual final bob velocity is $-w$.

Step 2: Apply the elastic-collision condition Let V be the block speed immediately after impact. Because the collision is elastic, the separating relative speed equals u:

$$V-(-w)=u, \qquad V=u-w.$$

The string is vertical at impact and provides no horizontal impulse, so horizontal momentum can be conserved during the short collision.

Step 3: Use momentum and eliminate the velocities Conservation of horizontal momentum gives

$$mu = -mw + MV.$$

Substitute $V = u - w$, then rearrange:

$$M(u - w) = m(u + w), \qquad M = m\frac{u + w}{u - w}.$$

Insert the small-angle speeds and cancel the common factor $\sqrt{g\ell}$.

Step 4: Check the mass ratio and alternatives The result is $M = m(\theta_0 + \theta_1)/(\theta_0 - \theta_1)$. A real rebound requires $M > m$, consistent with this ratio for $0 < \theta_1 < \theta_0$. Option (D) reverses the ratio and predicts a lighter block; options (A) and (B) introduce an unjustified factor $1/2$.

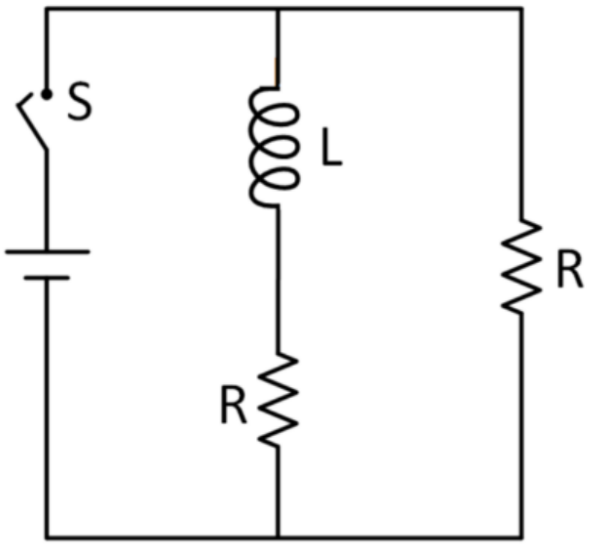
Step 5: Conclusion

$$M = m\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right).$$

Hence, the correct option is (C).

Quick Tip to Solve: For a rebounding bob use final velocity $-w$, not $+w$. At small amplitudes the bottom speed is proportional to the turning angle, so g and ℓ cancel.

46. In the figure shown, a circuit contains two identical resistors with resistance R = 5Ω and an inductance with L = 2mH. An ideal battery of 15 V is connected in the circuit. What will be the current through the battery long after the switch is closed ? [4 marks]



(A) 6 A

(B) 7.5 A

(C) 3 A

(D) 5.5 A

Correct Answer: (A) 6 A

Solution:

Concept:

- The potential difference across an ideal inductor is $V_L = L\,dI/dt$. In a steady direct-current state its current is constant, so $V_L = 0$.
- An ideal inductor therefore acts as a conducting connection after the transient has died away. Parallel resistors have the same potential difference, and their branch currents add at the battery.

Step 1: Identify the long-time circuit The requested current is long after closing the switch, rather than immediately after closing it. Hence the current in the branch containing $L = 2\,\mathrm{mH}$ has stopped changing. The inductor no longer contributes an inductive voltage; it can be replaced by a wire for this steady-state calculation. Its numerical inductance affects the transient duration, not the final current.

Step 2: Identify the two remaining branches The central branch in the figure contains the inductor in series with one $5\,\Omega$ resistor. Replacing only the inductor by a wire leaves that resistor connected between the upper and lower rails. The right-hand branch contains the other $5\,\Omega$ resistor between the same rails. Thus both resistors remain, connected in parallel across the $15\,\mathrm{V}$ battery.

Step 3: Calculate the equivalent resistance and current For two equal parallel resistors,

$$\frac{1}{R_{\mathrm{eq}}} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}, \qquad R_{\mathrm{eq}} = 2.5\,\Omega.$$

Ohm's law for the battery then gives

$$I = \frac{15}{2.5} = 6\,\mathrm{A}.$$

Equivalently, each resistor carries $15/5 = 3\,\mathrm{A}$, and the battery supplies their sum.

Step 4: Reject the alternatives Option (C), $3\,\mathrm{A}$, is just one branch current and would also describe the instant when the inductor branch has not yet developed current. It is not the total long-time current. The values $7.5\,\mathrm{A}$ and $5.5\,\mathrm{A}$ do not satisfy Ohm's law for two parallel $5\,\Omega$ resistors.

Step 5: Conclusion The steady current through the battery is $6\,\mathrm{A}$. Hence, the correct option is (A).

Quick Tip to Solve: At long times replace an ideal inductor by a wire, but retain every resistor in its branch. Add the parallel branch currents to obtain the current through the battery.

47. A light wave is incident normally on a glass slab of refractive index 1.5. If 4 % of light gets reflected and the amplitude of the electric field of the incident light is 30 V/m, then the amplitude of the electric field for the wave propagating in the glass medium will be : [4 marks]

(A) 6 V/m

(B) 10 V/m

(C) 30 V/m

(D) 24 V/m

Correct Answer: (D) 24 V/m

Solution:

Concept:

- For a sinusoidal electromagnetic wave of electric-field amplitude E_0 in a nonmagnetic dielectric, its mean intensity is $I = \frac{1}{2} n \varepsilon_0 c E_0^2$, where n is refractive index.
- The reflected fraction is an intensity fraction. With no absorption at the interface, $I_t/I_i = 1 - R$. The refractive-index factor must be retained when comparing fields in different media.

Step 1: Find the transmitted intensity fraction The reflected light carries 4% of the incident intensity. Thus the wave entering the glass carries the remaining fraction

$$\frac{I_t}{I_i} = 1 - 0.04 = 0.96.$$

This is a ratio of energy fluxes, not directly a ratio of electric-field amplitudes, so multiplying 30 by 0.96 would be incorrect.

Step 2: Relate intensity to electric-field amplitude Take the incident medium as air, with $n_i \simeq 1$, and the glass index as $n_t = 1.5$. On forming the ratio, the common factors $\varepsilon_0 c/2$ cancel:

$$\frac{I_t}{I_i} = \frac{n_t E_t^2}{n_i E_i^2} = 1.5 \frac{E_t^2}{30^2}.$$

The factor 1.5 expresses the different electromagnetic impedance inside the glass.

Step 3: Calculate the field transmitted into the slab Substitute the transmitted fraction and solve for the positive amplitude:

$$E_t = 30 \sqrt{\frac{0.96}{1.5}} = 30\sqrt{0.64} = 24\,\mathrm{V/m}.$$

As an independent normal-incidence check, the amplitude transmission coefficient is $2n_i/(n_i + n_t) = 2/2.5 = 0.8$, giving the same field 0.8×30.

Step 4: Compare the choices The $6\,\mathrm{V/m}$ value in option (A) equals the magnitude of the reflected field, since $\sqrt{0.04} \times 30 = 6$; it is not the transmitted field. Option (C) leaves the incident amplitude unchanged, and $10\,\mathrm{V/m}$ has no agreement with the intensity relation. Only $24\,\mathrm{V/m}$ gives the required transmitted energy flux.

Step 5: Conclusion The electric-field amplitude inside the glass is $24\,\mathrm{V/m}$. Hence, the correct option is (D).

Quick Tip to Solve: When light crosses into a dielectric, use $I \propto nE_0^2$, rather than $I \propto E_0^2$ alone. The latter comparison works only within the same medium.

48. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, speed of the plane is : [4 marks]

(A) $\frac{\sqrt{3}}{2}$v

(B) $\frac{2v}{\sqrt{3}}$

(C) v

(D) $\frac{v}{2}$

Correct Answer: (D) $\frac{v}{2}$

Solution:

Concept:

- The sound heard at an instant was emitted at an earlier position of the plane. If its path length to the observer is r, the travel time is $t = r/v$, where v is the speed of sound.
- During that same time the plane moves horizontally from the emission point to its present position above the observer. If the horizontal displacement is x, its speed is $u = x/t$.

Step 1: Distinguish the emission and observation positions Let the observer be at O, the emission point of the sound be P, and the plane's current position vertically above the observer be Q. The plane travels horizontally at constant altitude from P to Q while the sound travels diagonally from P to O. The apparent sound direction is therefore along OP, not along the present vertical line OQ.

Step 2: Use the direction of the arriving sound The line from the observer to the earlier position makes $60°$ with the ground. Let $OP = r$ and let the horizontal distance $PQ = x$. The right triangle gives

$$x = r\cos 60° = \frac{r}{2}.$$

The angle is measured from the ground, so the horizontal projection uses cosine. Using sine here would interchange horizontal and vertical components.

Step 3: Compare the two distances covered in the same time The sound takes

$$t = \frac{r}{v}$$

to reach the observer. In precisely this interval the plane covers $x = r/2$, so

$$u = \frac{x}{t} = \frac{r/2}{r/v} = \frac{v}{2}.$$

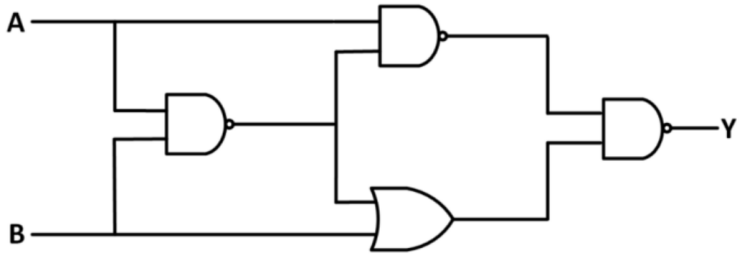
The unknown altitude and the path length cancel, which explains why neither is needed in the question.

Step 4: Check the competing ratios Option (A) uses $\sin 60°$, the vertical projection rather than the horizontal flight distance. Option (B) exceeds the speed of sound and does not satisfy the specified right-triangle timing. Option (C) would require equal sound and plane path lengths, whereas the horizontal path here is only half the sound path.

Step 5: Conclusion The speed of the plane is $v/2$. Hence, the correct option is (D).

Quick Tip to Solve: For a delayed sound direction, mark the source's earlier position first. If the angle is measured from the horizontal, equal travel times give $u = v\cos\theta$.

49. The output of the given logic circuit is : [4 marks]



(A) $\overline{A}B$

(B) $AB + \overline{AB}$

(C) $A\overline{B} + \overline{A}B$

(D) $A\overline{B}$

Correct Answer: (D) $A\overline{B}$

Solution:

Concept:

- A NAND gate gives the complement of the product of its inputs, while an OR gate gives their Boolean sum. A small bubble at a gate's output denotes inversion.
- Boolean simplification uses De Morgan's law, $\overline{AB} = \overline{A} + \overline{B}$, and the identities $A + \overline{A} = 1$, $A \cdot 1 = A$ and $AA = A$.

Step 1: Name the output of the first gate The leftmost gate takes A and B, and its output has an inversion bubble. It is a NAND gate, so write its output as

$$X = \overline{AB} = \overline{A} + \overline{B}.$$

This intermediate signal is fed to both the upper NAND gate and the lower OR gate. Naming it avoids losing track of the shared wire.

Step 2: Simplify the upper and lower branches The upper gate has inputs A and X, hence

$$U = \overline{AX} = \overline{A(\overline{A} + \overline{B})} = \overline{A}\overline{B} = \overline{A} + B.$$

The lower gate is OR with inputs B and X:

$$V = B + X = B + \overline{A} + \overline{B} = 1.$$

Its output is always high because it includes both B and its complement.

Step 3: Apply the final NAND operation The last gate also has an output bubble, so

$$Y = \overline{UV} = \overline{U \cdot 1} = \overline{U} = A\overline{B}.$$

Thus the circuit gives a high output only when input $A = 1$ and input $B = 0$. For input pairs $00, 01, 10, 11$, its output sequence is $0, 0, 1, 0$.

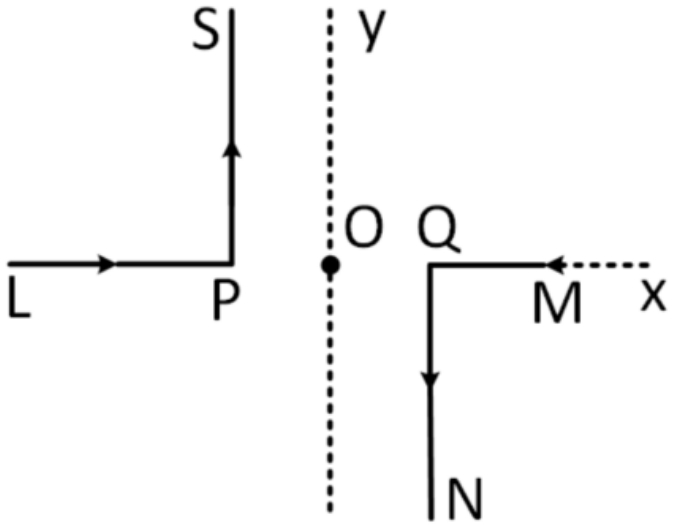
Step 4: Check the option expressions Option (A), $\overline{A}B$, selects the opposite unequal input pair. Option

(B), $AB + \overline{AB}$, is identically 1, so cannot match the three low outputs. Option (C) is exclusive OR, high for both unequal input pairs, whereas this circuit is low for $A = 0, B = 1$. The remaining expression is exactly option (D).

Step 5: Conclusion The output is $Y = A\overline{B}$. Hence, the correct option is (D).

Quick Tip to Solve: Label the first shared NAND output before simplifying either branch. Spotting $B + \overline{B} = 1$ makes the lower branch constant and turns the final NAND into an inverter.

50. As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the x-axis, while segments PS and QN are parallel to the y-axis. If OP = OQ = 4cm, and the magnitude of the magnetic field at O is 10^{-4} T, and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be ($\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}) : [4 marks]



(A) 40 A, perpendicular into the page

(B) 40 A, perpendicular out of the page

(C) 20 A, perpendicular into the page

(D) 40 A, perpendicular out of the page

Correct Answer: (C) 20 A, perpendicular into the page

Solution:

Concept:

- For a straight current element, the Biot–Savart law gives $d\vec{B} = \mu_0 I\, d\vec{\ell} \times \hat{r}/(4\pi r^2)$. A collinear point receives no field from that element.
- A semi-infinite straight wire produces $B = \mu_0 I/(4\pi d)$ at perpendicular distance d from its end. Fields add as vectors; their direction follows the right-hand rule.

Step 1: Identify the contributing segments The point O lies on the straight extensions of LP and QM.

For every element of these horizontal segments, the displacement to O is parallel or antiparallel to the current element, so the cross product vanishes. Only the vertical semi-infinite segments PS and QN contribute. Their perpendicular distances from O are both $d = 4\,\text{cm} = 0.04\,\text{m}$.

Step 2: Add the two magnetic fields On the left, current flows upward along PS and O lies to its right, so its field points into the page. On the right, current flows downward along QN and O lies to its left, again giving a field into the page. The two equal fields reinforce:

$$B = 2\left(\frac{\mu_0 I}{4\pi d}\right) = \frac{\mu_0 I}{2\pi d}.$$

Each vertical segment is semi-infinite, so using the infinite-wire field for each would introduce an erroneous factor of two.

Step 3: Calculate the current and test the choices Substitute the given field and permeability:

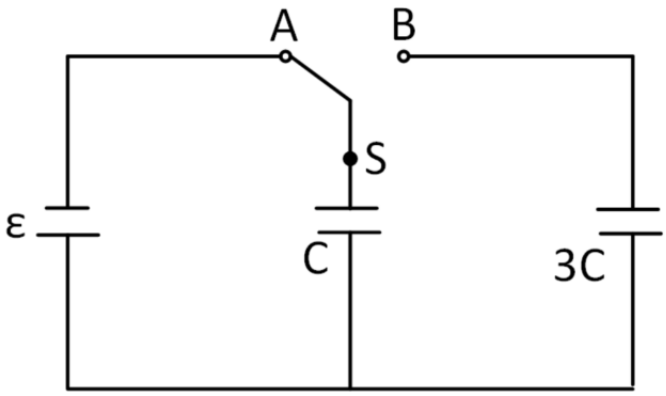
$$I = \frac{2\pi d B}{\mu_0} = \frac{2\pi(0.04)(10^{-4})}{4\pi \times 10^{-7}} = 20\,\text{A}.$$

A current of $40\,\text{A}$, as in A, B and D, would instead produce $2 \times 10^{-4}\,\text{T}$. B and D also reverse the direction. C has both the required magnitude and direction.

Step 4: Conclusion The current in each wire is $20\,\text{A}$, and the magnetic field at O is perpendicular into the page. Hence, the correct option is (C).

Quick Tip to Solve: First discard collinear segments: their Biot–Savart cross product is zero. Then check the direction of each remaining segment before adding magnitudes.

51. In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is : [4 marks]



(A) $\frac{1}{8}\frac{Q^2}{C}$

(B) $\frac{5}{8}\frac{Q^2}{C}$

(C) $\frac{3}{4}\frac{Q^2}{C}$

(D) $\frac{3}{8}\frac{Q^2}{C}$

Correct Answer: (D) $\frac{3}{8}\frac{Q^2}{C}$

Solution:

Concept:

- The electrostatic energy stored in a capacitor is $U = Q^2/(2C) = CV^2/2$, where Q is its charge and V its potential difference.
- After the battery is disconnected, connecting capacitors in parallel conserves their total charge. They attain a common voltage; the decrease in electrostatic energy is dissipated in the circuit.

Step 1: Find the energy before switching With S at A, the capacitor C is charged by the battery while the upper plate of the capacitor $3C$ is isolated. Thus the initial charge Q resides on C and $3C$ is uncharged. The initial voltage and stored energy are

$$V_i = \frac{Q}{C}, \qquad U_i = \frac{Q^2}{2C}.$$

The common lower connection alone does not charge the isolated capacitor.

Step 2: Find the common final voltage Moving S to B disconnects the charging battery from the upper capacitor plates and joins C and $3C$ in parallel. Their equivalent capacitance is

$$C_f = C + 3C = 4C.$$

Conservation of the charge on the connected upper plates gives

$$V_f = \frac{Q}{4C}, \qquad Q_C = \frac{Q}{4}, \qquad Q_{3C} = \frac{3Q}{4}.$$

The charges add to Q, but the voltage falls because a larger capacitance now shares that charge.

Step 3: Subtract the final stored energy The final electrostatic energy is

$$U_f = \frac{1}{2}(4C)\left(\frac{Q}{4C}\right)^2 = \frac{Q^2}{8C}.$$

Therefore the energy dissipated during redistribution is

$$U_{\text{loss}} = U_i - U_f = \frac{Q^2}{2C} - \frac{Q^2}{8C} = \frac{3Q^2}{8C}.$$

A is the final stored energy rather than the loss. B exceeds the initial energy and is impossible for this isolated redistribution. C is also larger than the available initial energy.

Step 4: Conclusion The dissipated energy is $\frac{3Q^2}{8C}$. Hence, the correct option is (D).

Quick Tip to Solve: In charge-sharing problems conserve charge first, then calculate the two stored energies. Electrical energy itself is not conserved during the transient.

52. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is : [4 marks]

(A) 16 cm

(B) 12 cm

(C) 14 cm

(D) 18 cm

Correct Answer: (A) 16 cm

Solution:

Concept:

- The axial moment of inertia of a uniform thick hollow cylinder of mass M is $I = \frac{1}{2}M(r_i^2 + r_o^2)$, where r_i and r_o are its inner and outer radii.
- A thin cylindrical shell has its mass at a single radius r, so its axial moment of inertia is $I_{\text{thin}} = Mr^2$. Equal mass allows M to cancel when the moments are equated.

Step 1: Write the moment of the thick cylinder The given inner radius is $10\,\text{cm}$ and the outer radius is $20\,\text{cm}$. For rotation about the common longitudinal axis,

$$I = \frac{M}{2}\left[(10)^2 + (20)^2\right] = 250M\,\text{cm}^2.$$

The length $30\,\text{cm}$ does not enter this axial formula: changing an element's axial coordinate does not change its distance from the rotation axis.

Step 2: Equate the thin-shell moment to the given moment Let the radius of the thin cylindrical shell of the same mass be r. Its thickness is negligible, so every mass element has perpendicular distance r from the axis. Hence

$$Mr^2 = 250M\,\text{cm}^2, \qquad r^2 = 250\,\text{cm}^2.$$

Taking the positive square root,

$$r = 5\sqrt{10}\,\text{cm} \simeq 15.81\,\text{cm}.$$

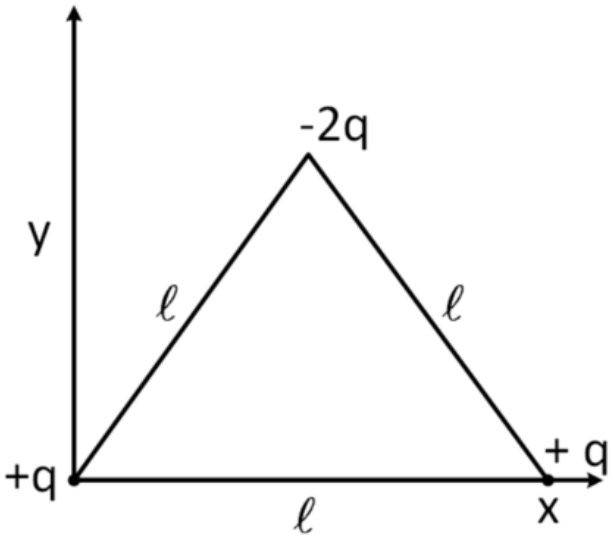
This is the radius of gyration of the original hollow cylinder about its axis.

Step 3: Choose the nearest listed radius The listed radii are approximate integer values. The calculated $15.81\,\text{cm}$ is closest to $16\,\text{cm}$. The alternatives 12, 14, and $18\,\text{cm}$ would give axial moments $144M$, $196M$, and $324M\,\text{cm}^2$, respectively, instead of $250M\,\text{cm}^2$. Do not use the solid-cylinder formula for the thin shell; a thin shell has Mr^2, not $Mr^2/2$.

Step 4: Conclusion The required thin-cylinder radius is approximately $15.81\,\text{cm}$, corresponding to $16\,\text{cm}$. Hence, the correct option is (A).

Quick Tip to Solve: For a thick annular cylinder, average the squared inner and outer radii and then take the square root; averaging the radii themselves gives a different answer.

53. Determine the electric dipole moment of the system of the three charges, placed on the vertices of an equilateral triangle, as shown in the figure : [4 marks]



(A) $2q\ell\hat{j}$

(B) $(q\ell)\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

(C) $\sqrt{3}\,q\ell\frac{\hat{j}-\hat{i}}{\sqrt{2}}$

(D) $-\sqrt{3}\,q\ell\hat{j}$

Correct Answer: (D) $-\sqrt{3}\,q\ell\hat{j}$

Solution:

Concept:

- The electric dipole moment of point charges is $\vec{p}=\sum_i q_i\vec{r}_i$. For a neutral system, $\sum_i q_i=0$, this vector is independent of the choice of origin.
- An equilateral triangle of side ℓ has altitude $\sqrt{3}\ell/2$. Its apex lies horizontally halfway between the two base vertices.

Step 1: Assign coordinates from the diagram Use the left base vertex as the origin, with the base along the positive x-axis and y upward. The three charges and their position vectors are

$$+q:(0,0),$$
$$+q:(\ell,0),$$
$$-2q:(\ell/2,\sqrt{3}\ell/2).$$

The two base charges are equal and positive; the apex charge is negative with twice their magnitude. The total charge is therefore zero.

Step 2: Sum the charge-weighted position vectors The charge at the origin contributes zero. Adding

the other two contributions gives

$$\begin{aligned}\vec{p} &= q(\ell\hat{i}) - 2q\left(\frac{\ell}{2}\hat{i} + \frac{\sqrt{3}\ell}{2}\hat{j}\right)\\ &= q\ell\hat{i} - q\ell\hat{i} - \sqrt{3}q\ell\hat{j}\\ &= -\sqrt{3}q\ell\hat{j}.\end{aligned}$$

The horizontal components cancel exactly; the remaining vertical component is directed downward.

Step 3: Check the direction and reject the other vectors Dipole moment points from negative charge toward positive charge. Here the positive charges have their charge centre at the midpoint of the base, below the negative apex, confirming a negative y direction. A points upward and also has the wrong magnitude. B and C have nonzero x-components, contradicting the left-right symmetry of the distribution. D has zero x-component, negative y-component, and magnitude $\sqrt{3}q\ell$.

Step 4: Conclusion The electric dipole moment is $\vec{p} = -\sqrt{3}q\ell\hat{j}$, with units of charge times length. Hence, the correct option is (D).

Quick Tip to Solve: For equal base charges in an equilateral triangle, symmetry immediately cancels horizontal dipole components. Use the altitude, rather than the side length, for the vertical separation.

54. A particle A of mass 'm' and charge 'q' is accelerated by a potential difference of 50 V. Another particle B of mass ' 4 m' and charge 'q' is accelerated by a potential difference of 2500 V. The ratio of de-Broglie wavelengths $\frac{\lambda_A}{\lambda_B}$ is close to : [4 marks]

(A) 4.47

(B) 10.00

(C) 14.14

(D) 0.07

Correct Answer: (C) 14.14

Solution:

Concept:

- A nonrelativistic particle accelerated from rest through a potential difference V acquires kinetic energy $qV = p^2/(2m)$, so its momentum is $p = \sqrt{2mqV}$.
- The de Broglie relation is $\lambda = h/p = h/\sqrt{2mqV}$. For equal charge magnitudes, the ratio depends on the products of mass and accelerating voltage.

Step 1: Express the two wavelengths For A, $m_A = m$ and $V_A = 50\,\text{V}$; for B, $m_B = 4m$ and $V_B = 2500\,\text{V}$. The common charge is q. Thus

$$\lambda_A = \frac{h}{\sqrt{2mq(50)}}, \qquad \lambda_B = \frac{h}{\sqrt{2(4m)q(2500)}}.$$

The larger mass and larger acceleration voltage both increase the momentum of B and therefore

decrease its wavelength.

Step 2: Form the ratio before substituting constants Dividing the expressions eliminates Planck's constant, the common mass scale and the charge:

$$\frac{\lambda_A}{\lambda_B} = \sqrt{\frac{m_B V_B}{m_A V_A}} = \sqrt{\frac{4 \times 2500}{50}}.$$

The quantity inside the square root is dimensionless because it is a ratio of two mass-voltage products. Simplify it to obtain

$$\frac{\lambda_A}{\lambda_B} = \sqrt{200} = 10\sqrt{2} \simeq 14.14.$$

Step 3: Test the scale and the alternatives B has $4 \times 50 = 200$ times the mass-voltage product of A, hence $\sqrt{200}$ times the momentum. Its wavelength must be smaller, so the requested ratio exceeds one. D is approximately the reciprocal and reverses the requested ratio. A and B do not equal $\sqrt{200}$; they result from incorrect handling of the voltage or mass factor. C is the required numerical ratio.

Step 4: Conclusion The ratio is $\lambda_A/\lambda_B = 10\sqrt{2} \simeq 14.14$. Hence, the correct option is (C).

Quick Tip to Solve: When comparing accelerated particles, write $\lambda \propto (mqV)^{-1/2}$ first. Check that the heavier, more energetic particle has the shorter wavelength.

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55. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth, collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same, just before the collision. The subsequent motion of the combined body will be : [4 marks]

(A) in the same circular orbit of radius R

(B) such that it escapes to infinity

(C) in a circular orbit of a different radius

(D) in an elliptical orbit

Correct Answer: (D) in an elliptical orbit

Solution:

Concept:

- During a brief completely inelastic collision, the bodies stick together and linear momentum is conserved. Kinetic energy need not be conserved.
- For a circular orbit of radius R, $v_c^2 = GM_E/R$, where M_E is the earth's mass. The specific orbital energy is $\varepsilon = v^2/2 - GM_E/r$; a negative value describes a bound Kepler orbit.

Step 1: Conserve the two perpendicular momentum components The satellite moves tangentially with speed v_c, whereas the falling meteorite moves radially inward with the same speed. Choose outward radial and forward tangential unit vectors $\hat{r}$ and $\hat{t}$. The total initial momentum is

$$\vec{P} = Mv_c\hat{t} - Mv_c\hat{r}.$$

After sticking, the mass is $2M$, so

$$\vec{v}_f = \frac{v_c}{2}(\hat{t} - \hat{r}), \qquad v_f^2 = \frac{v_c^2}{2}.$$

A nonzero inward radial velocity remains after impact.

Step 2: Determine whether the new orbit is bound At the collision point the distance is still R. The specific energy of the combined body is

$$\varepsilon = \frac{v_f^2}{2} - \frac{GM_E}{R} = \frac{v_c^2}{4} - v_c^2 = -\frac{3GM_E}{4R} < 0.$$

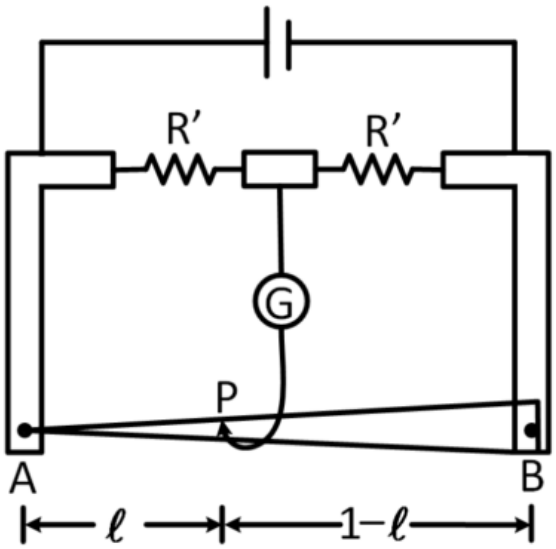
The combined body cannot escape to infinity because its orbital energy is negative. Its nonzero tangential component also gives nonzero angular momentum, so its trajectory is not a purely radial fall.

Step 3: Distinguish the ellipse from a circle A circular orbit requires zero radial velocity and the circular speed appropriate to its radius. Here $v_r = -v_c/2 \neq 0$, while the position at impact is R; the body cannot instantly move onto a circle of another radius. Thus A and C fail the circular-orbit conditions, and B fails the energy test. The bound noncircular Kepler trajectory is an ellipse. For a further check, $\varepsilon = -GM_E/(2a)$ gives semimajor axis $a = 2R/3$.

Step 4: Conclusion The combined body follows an elliptical orbit under the earth's gravitational field. Hence, the correct option is (D).

Quick Tip to Solve: After an orbital collision, test the radial velocity and the sign of orbital energy separately: nonzero radial motion excludes a circle, and negative energy excludes escape.

56. In a meter bridge, the wire of length 1 m has a non-uniform cross-section such that, the variation $\frac{dR}{d\ell}$ of its resistance R with length ℓ is $\frac{dR}{d\ell} \propto \frac{1}{\sqrt{\ell}}$. Two equal resistances are connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P. What is the length AP ? [4 marks]



(A) 0.3 m

(B) 0.25 m

(C) 0.35 m

(D) 0.2 m

Correct Answer: (B) 0.25 m

Solution:

Concept:

- At the balance point of a meter bridge, the ratio of the resistances in its two wire segments equals the ratio of the two gap resistances. Equal gap resistances therefore require equal wire-segment resistances.
- For a nonuniform wire, resistance must be integrated from its resistance per unit length. Here $dR/dx = k/\sqrt{x}$, where x is measured from A and $k > 0$ is a constant.

Step 1: Integrate the resistance from A to the jockey Let the full wire length be $L = 1\,\mathrm{m}$, and let AP be x. Use a dummy variable s for the integration:

$$R_{AP} = \int_0^x \frac{k}{\sqrt{s}}\, ds = 2k\sqrt{x}.$$

Although the resistance density grows without bound near A, its integral is finite: the lower endpoint contributes $2k\sqrt{0} = 0$. The resistance is not proportional to length in this wire.

Step 2: Write the resistance of the remaining segment The same coordinate is used along the entire

wire, so

$$R_{PB} = \int_x^L \frac{k}{\sqrt{s}}\, ds = 2k(\sqrt{L} - \sqrt{x}).$$

It would be incorrect to write $R_{PB} = 2k\sqrt{L - x}$, because that would reset the given resistance-density law at P rather than continue measuring from A.

Step 3: Apply the balance condition and compare the choices The two upper gap resistances are equal, hence zero galvanometer current requires

$$\frac{R_{AP}}{R_{PB}} = 1, \qquad 2k\sqrt{x} = 2k(\sqrt{L} - \sqrt{x}).$$

Thus

$$2\sqrt{x} = \sqrt{L}, \qquad x = \frac{L}{4} = 0.25\,\text{m}.$$

This places the jockey nearer A, where resistance per unit length is greater. A, C and D do not divide the total resistance equally; for example the fraction accumulated up to x is $\sqrt{x/L}$, which equals one-half only at $x/L = 1/4$.

Step 4: Conclusion The balance length is $AP = 0.25\,\text{m}$. Hence, the correct option is (B).

Quick Tip to Solve: In a nonuniform bridge wire, balance divides resistance, not length. Integrate the given resistance density using one fixed origin for both wire segments.

57. Two electric bulbs, rated at (25 W, 220 V) and (100 W, 220 V), are connected in series across a 220 V voltage source. If the 25 W and 100 W bulbs draw powers P_1 and P_2 respectively, then : [4 marks]

(A) P_1 = 4W, P_2 = 16 W

(B) P_1 = 16W, P_2 = 4 W

(C) P_1 = 9W, P_2 = 16 W

(D) P_1 = 16W, P_2 = 9 W

Correct Answer: (B) P_1 = 16W, P_2 = 4 W

Solution:

Concept:

- A bulb rating gives the power at its rated voltage. Its resistance is obtained from $P = V^2/R$, treating the filament resistance as fixed for this circuit calculation.
- In a series connection both bulbs carry the same current I, and their actual powers are I^2R_1 and I^2R_2. The rated powers cannot be used as the actual powers at the reduced individual voltages.

Step 1: Find the filament resistances Use the common rated voltage $220\,\text{V}$ separately for the two ratings:

$$R_1 = \frac{220^2}{25} = 1936\,\Omega, \qquad R_2 = \frac{220^2}{100} = 484\,\Omega.$$

Thus the lower-rated bulb has four times the resistance of the other bulb. This is why it dissipates more power when both carry the same current.

Step 2: Calculate the common current The series resistances add, so the full source voltage is applied across $R_1 + R_2$, rather than across either bulb individually:

$$R_{\text{total}} = 1936 + 484 = 2420\,\Omega, \qquad I = \frac{220}{2420} = \frac{1}{11}\,\text{A}.$$

The individual voltage drops are $176\,\text{V}$ and $44\,\text{V}$, which sum to $220\,\text{V}$.

Step 3: Evaluate and compare the powers Substitute this same current into both power expressions:

$$P_1 = \frac{1936}{121} = 16\,\text{W}, \qquad P_2 = \frac{484}{121} = 4\,\text{W}.$$

Their ratio is $4 : 1$. Option (A) reverses this ratio; (C) and (D) have an incorrect power for at least one bulb. The sum $20\,\text{W}$ also agrees with the source power $VI = 220/11$.

Step 4: Conclusion The actual powers are $P_1 = 16\,\text{W}$ and $P_2 = 4\,\text{W}$. Hence, the correct option is (B).

Quick Tip to Solve: For bulbs with the same rated voltage, resistance is inversely proportional to rated power; in series, actual power is proportional to resistance.

58. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius 2R. The thermal conductivity of the material of the inner cylinder is K_1 and the of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is : [4 marks]

(A) $K_1 + K_2$

(B) $\frac{K_1+3K_2}{4}$

(C) $\frac{K_1+K_2}{2}$

(D) $\frac{2K_1+3K_2}{5}$

Correct Answer: (B) $\frac{K_1+3K_2}{4}$

Solution:

Concept:

- Fourier's law gives the heat current through a uniform conductor: $H = KA\Delta T/L$, where K is thermal conductivity, A the area normal to heat flow and L its length.
- For heat flowing along the cylinder, the core and shell are parallel thermal paths. Both have the same length and end-to-end temperature difference, and their heat currents add.

Step 1: Find the areas normal to heat flow The inner solid cylinder has radius R. The shell occupies the annulus between R and $2R$. Therefore,

$$A_1 = \pi R^2, \qquad A_2 = \pi(2R)^2 - \pi R^2 = 3\pi R^2.$$

The total area is $A = 4\pi R^2$. The shell therefore carries three times the area weighting of the core, rather than an equal weighting.

Step 2: Add the parallel heat currents Let ΔT be the temperature difference between the common

end faces and L their common length. With no heat loss, the total current is

$$H = \frac{K_1 \pi R^2 \Delta T}{L} + \frac{3K_2 \pi R^2 \Delta T}{L}.$$

For an equivalent single cylinder, write

$$H = \frac{K_{\text{eff}}(4\pi R^2)\Delta T}{L}.$$

Equating the two expressions cancels the common geometric and temperature factors.

Step 3: Obtain the effective conductivity The resulting equation is $4K_{\text{eff}} = K_1 + 3K_2$. Option (A) adds conductivities without normalising by total area. Option (C) wrongly assigns equal areas, and option (D) gives weights inconsistent with the annular area. A useful check is $K_1 = K_2 = K$, which must give $K_{\text{eff}} = K$; the area-weighted result does so.

Step 4: Conclusion

$$K_{\text{eff}} = \frac{K_1 + 3K_2}{4}.$$

Hence, the correct option is (B).

Quick Tip to Solve: First identify the heat-flow direction: along the axis means parallel paths, so use area-weighted conductivities, with shell area equal to outer area minus inner area.

59. A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index ? [4 marks]

(A) 0.5

(B) 0.6

(C) 0.4

(D) 0.3

Correct Answer: (B) 0.6

Solution:

Concept:

- For amplitude modulation, the envelope varies from $A_{\max} = A_c + A_m$ to $A_{\min} = A_c - A_m$, where A_c is carrier amplitude and A_m is the amplitude variation produced by the signal.
- The modulation index is the dimensionless ratio $m = A_m/A_c = (A_{\max} - A_{\min})/(A_{\max} + A_{\min})$. The extrema describe amplitudes, not average carrier power.

Step 1: Recover the carrier amplitude The stated envelope maximum and minimum are $160\,\text{V}$ and $40\,\text{V}$. Adding the envelope equations removes the modulating amplitude:

$$2A_c = A_{\max} + A_{\min} = 160 + 40 = 200\,\text{V}.$$

Thus $A_c = 100\,\text{V}$, agreeing with the carrier specified in the question. The two extrema are symmetric about this value.

Step 2: Find the amplitude variation Subtracting the envelope equations gives

$$2A_m = A_{\max} - A_{\min} = 160 - 40 = 120\,\text{V}, \qquad A_m = 60\,\text{V}.$$

Equivalently, the envelope rises by $60\,\text{V}$ above the carrier and falls by the same amount below it. Dividing the full peak-to-peak envelope change by the carrier would incorrectly introduce an extra factor of two.

Step 3: Calculate and check the index The ratio is

$$m = \frac{60}{100} = \frac{160 - 40}{160 + 40} = 0.6.$$

There is no unit because both amplitudes are voltages. Values 0.5, 0.4 and 0.3 would predict minima 50, 60 and $70\,\text{V}$, respectively, for a $100\,\text{V}$ carrier, so they do not reproduce the given envelope. Since $m < 1$, the envelope is not overmodulated.

Step 4: Conclusion The modulation index is $m = 0.6$, or 60%. Hence, the correct option is (B).

Quick Tip to Solve: Use the envelope ratio (maximum minus minimum)/(maximum plus minimum); the sum recovers twice the carrier amplitude.

60. An ideal battery of 4 V and resistance R are connected in series in the primary circuit of a potentiometer of length 1 m and esistance 5 Ω. The value of R, to give a potential difference of 5 mV across 10 cm of potentiometer wire, is : [4 marks]

(A) 480 Ω

(B) 495 Ω

(C) 490 Ω

(D) 395 Ω

Correct Answer: (D) 395 Ω

Solution:

Concept:

- A uniform potentiometer wire has constant resistance per unit length, so its potential drop is proportional to length when a steady current flows.
- The wire resistance and series resistance R carry the same primary current. Use the required wire drop to find that current, then apply Ohm's law to the complete primary circuit.

Step 1: Find the voltage across the full wire A length of $10\,\text{cm} = 0.10\,\text{m}$ must have a potential difference of $5\,\text{mV} = 0.005\,\text{V}$. The required gradient is

$$k = \frac{0.005}{0.10} = 0.05\,\text{V m}^{-1}.$$

The full one-metre wire must consequently have a drop $V_w = k(1) = 0.05\,\text{V}$. The remaining battery voltage is dropped across R.

Step 2: Determine the primary current The resistance of the entire potentiometer wire is $5\,\Omega$, hence

$$I = \frac{V_w}{5} = \frac{0.05}{5} = 0.010\,\text{A}.$$

The ideal battery has no internal resistance to include. Notice that applying the battery's $4\,\text{V}$ directly to the wire would give a much larger gradient than requested.

Step 3: Solve for the series resistance The total resistance necessary to draw this current from $4\,\text{V}$ is

$$R + 5 = \frac{4}{0.010} = 400\,\Omega, \qquad R = 395\,\Omega.$$

A direct check gives $I = 4/(395+5) = 0.01\,\text{A}$, and the resistance of ten centimetres is $0.5\,\Omega$; their product is $0.005\,\text{V}$. The other options give total resistances 485, 500 and $495\,\Omega$, so none supplies the required current and gradient.

Step 4: Conclusion The required series resistance is $395\,\Omega$. Hence, the correct option is (D).

Quick Tip to Solve: Scale the specified short-length drop to the whole potentiometer wire before finding current; subtract the wire resistance from the total circuit resistance.

Chemistry (120 marks)

61. Two solids dissociate as follows –

$$\text{A}\,(\text{s}) \rightleftharpoons \text{B}\,(\text{g}) + \text{C}\,(\text{g}); \quad K_{p1} = x\,\text{atm}^2$$

$$\text{D}\,(\text{s}) \rightleftharpoons \text{C}\,(\text{g}) + \text{E}\,(\text{g}); \quad K_{p2} = y\,\text{atm}^2$$

The total pressure when both the solids dissociated simultaneously is - [4 marks]

(A) (x + y) atm

(B) $(\sqrt{x+y})$ atm

(C) $2(\sqrt{x+y})$ atm

(D) $x^2 + y^2$ atm

Correct Answer: (C) $2(\sqrt{x+y})$ atm

Solution:

Concept:

- Pure solids have unit activity and do not appear in K_p. For the equilibria shown, $K_{p1} = p_B p_C = x\,\text{atm}^2$ and $K_{p2} = p_C p_E = y\,\text{atm}^2$.
- Both decompositions produce the common gas C. Starting from the solids, the number of moles of C formed equals the sum of the moles of B and E, so $p_C = p_B + p_E$ at the common temperature and volume.

Step 1: Express the partial pressures through reaction amounts Let the equilibrium partial pressures of B and E be $a\,\text{atm}$ and $b\,\text{atm}$. Each mole of B produced from $\text{A}\,(\text{s})$ is accompanied by one mole of

C; the same holds for E from D(s). Therefore the numerical partial pressure of C, in atmospheres, is $a + b$. The gases cannot be treated as two independent mixtures because they share C.

Step 2: Combine the equilibrium equations The equilibrium constants give

$$a(a + b) = x, \qquad b(a + b) = y.$$

Adding the equations gives

$$(a + b)^2 = x + y, \qquad a + b = \sqrt{x + y}.$$

Take the nonnegative root because partial pressures cannot be negative. This step automatically incorporates the suppression of each dissociation by the common product gas.

Step 3: Calculate the total gas pressure Dalton's law includes all three gases:

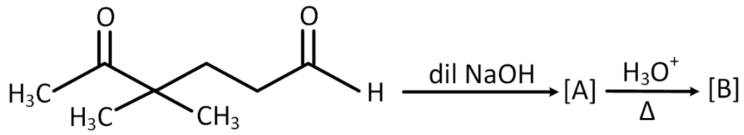
$$P_{\text{total}} = p_B + p_C + p_E = [a + (a + b) + b]\,\text{atm} = 2\sqrt{x + y}\,\text{atm}.$$

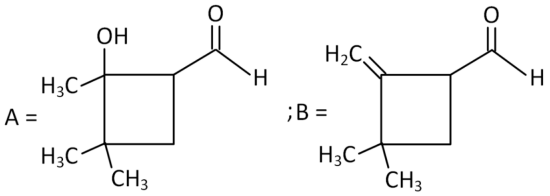
Option (B) counts only the common gas pressure and misses the equal combined pressure of the other gases. Option (A) adds equilibrium constants rather than pressures; option (D) also fails the square-root dependence. Pure solid pressures make no contribution to this gas pressure sum.

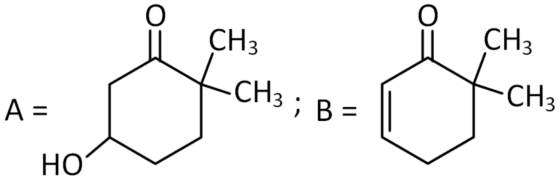
Step 4: Conclusion The total equilibrium pressure is $2\sqrt{x + y}\,\text{atm}$. Hence, the correct option is (C).

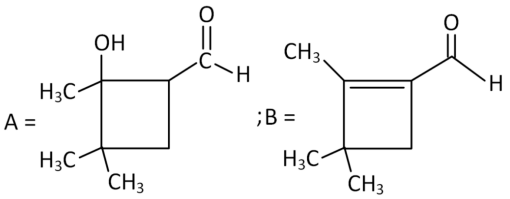
Quick Tip to Solve: For two solid decompositions sharing one gaseous product, write its pressure as the sum of the two unique-gas pressures before combining the equilibrium constants.

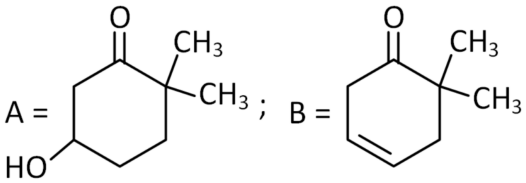
62. In the following reaction, products A and B are – [4 marks]



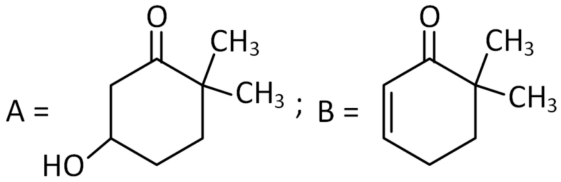
(A) 

(B) 

(C) 

(D) 

Correct Answer: (B)



Solution:

Concept:

- Dilute base forms an enolate at a carbon adjacent to a carbonyl group. An intramolecular aldol addition joins this nucleophilic carbon to the other carbonyl carbon, forming a beta-hydroxy carbonyl compound.
- Formation of a six-membered ring is favoured over a strained four-membered ring. Acid and heat

then remove water from the aldol product to form a conjugated alpha,beta-unsaturated carbonyl compound.

Step 1: Identify the favourable enolate and ring closure The starting chain can be read as $CH_3COC(CH_3)_2CH_2CH_2CHO$. The ketone-side methyl group has alpha hydrogens. Its enolate carbon attacks the terminal aldehyde carbon, and the six atoms along the intervening chain close into a six-membered ring. The ketone carbonyl remains in the product, while the aldehyde carbonyl oxygen becomes the hydroxyl oxygen after protonation. The carbon bearing the two methyl groups remains a ring carbon next to the ketone.

Step 2: Identify aldol product A Number the retained ketone carbonyl carbon as C1 and the gem-dimethyl carbon as C2. The attacked aldehyde carbon becomes C5, so the product is 5-hydroxy-2,2-dimethylcyclohexan-1-one. It is the six-membered beta-hydroxy ketone shown as A in options (B) and (D). The four-membered hydroxy aldehydes in options (A) and (C) require the less favourable ring closure and retain the wrong carbonyl for this pathway.

Step 3: Dehydrate to identify product B On treatment with H_3O^+ and heat, loss of the C5 hydroxyl group and a hydrogen from the adjacent C6 position creates the C5=C6 bond in this numbering. This alkene lies next to the C1 ketone and is conjugated with it. Renumbering for the alkene gives 6,6-dimethylcyclohex-2-en-1-one. Option (B) shows this conjugated enone; option (D) places the double bond one position farther away and gives a nonconjugated ketone.

Step 4: Conclusion A is the six-membered beta-hydroxy ketone and B is its conjugated cyclohexenone dehydration product, as drawn in option (B). Hence, the correct option is (B).

Quick Tip to Solve: Count ring atoms before choosing the aldol donor; then place the dehydration double bond next to the retained carbonyl to obtain the conjugated enone.

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63. Decomposition of X exhibits a rate constant of 0.05 μg/year. How many year are required for the decomposition of 5μg of X into 2.5 μg? [4 marks]

(A) 50

(B) 20

(C) 25

(D) 40

Correct Answer: (A) 50

Solution:

Concept:

- The units of a rate constant reveal reaction order. A constant given as mass per time, $\mu g\,year^{-1}$, represents a constant mass-disappearance rate for the fixed sample, corresponding to zero-order kinetics.
- For zero-order disappearance, the amount remaining follows $m_t = m_0 - kt$. Thus the time is the amount consumed divided by the constant disappearance rate, rather than a logarithm of an amount ratio.

Step 1: Infer the kinetic law from the units The value $k = 0.05\,\mu\text{g year}^{-1}$ means that $0.05\,\mu\text{g}$ of X decomposes in each year while the zero-order law applies. A first-order rate constant would instead have units of inverse time alone. Therefore the familiar first-order half-life expression $0.693/k$ does not apply to this value of k.

Step 2: Find the amount decomposed The initial amount is $5\,\mu\text{g}$, and the desired remaining amount is $2.5\,\mu\text{g}$. The mass that disappears is

$$\Delta m = m_0 - m_t = 5 - 2.5 = 2.5\,\mu\text{g}.$$

Although this is half the initial amount, its time must still be computed from the zero-order integrated law. The amount remaining and the amount consumed happen to be equal here, but their roles in the equation are different.

Step 3: Calculate and verify the time Rearrange the integrated rate law:

$$t = \frac{m_0 - m_t}{k} = \frac{2.5\,\mu\text{g}}{0.05\,\mu\text{g year}^{-1}} = 50\,\text{years}.$$

Substitution gives $m_t = 5 - 0.05(50) = 2.5\,\mu\text{g}$, as required. The other times 20, 25 and 40 years would leave 4, 3.75 and $3\,\mu\text{g}$, respectively, and therefore cannot match the requested final amount.

Step 4: Conclusion The required decomposition time is $50\,\text{years}$. Hence, the correct option is (A).

Quick Tip to Solve: Check the units of k first: an amount-per-time rate constant signals zero order, so divide the amount lost by k.

---

64. Freezing point of a 4% aqueous solution of X is equal to freezing point of 12% aqueous solution of Y. If molecular weight of X is A, then molecular weight of Y is - [4 marks]

(A) 4A

(B) 2A

(C) 3A

(D) A

Correct Answer: (C) 3A

Solution:

Concept:

- For a dilute solution of a non-electrolyte, freezing-point depression is $\Delta T_f = K_f m$, where m is solute molality and K_f depends only on the solvent.
- Equal freezing points in the same solvent imply equal particle molalities. In the dilute-solution convention used by these options, concentration divided by molar mass is compared.

Step 1: Equate the freezing-point depressions Let the molar mass of Y be M_Y. Both solutions are aqueous, so the solvent and its cryoscopic constant are the same. Taking X and Y as non-electrolytes, their equal depressions give $m_X = m_Y$. In the dilute approximation, equal volumes or approximately

equal masses of water allow the relative solute concentrations to be used directly:

$$\frac{4}{A} = \frac{12}{M_Y}.$$

The factor converting percentage to the common solvent basis cancels. Comparing percentages alone without dividing by molar mass would compare masses rather than numbers of particles.

Step 2: Solve for the molar-mass ratio Cross-multiplication gives

$$4M_Y = 12A, \qquad M_Y = 3A.$$

Thus three times as much Y by mass supplies the same number of solute particles because each mole of Y has three times the mass. The proposed values A, $2A$ and $4A$ would make the concentration-to-molar-mass ratios unequal, so they do not satisfy the intended comparison.

Step 3: Identify the concentration convention The percentage basis is not specified. If the percentages were interpreted strictly as mass percentages, the solvent masses in 100 g solution would instead be 96 g and 88 g; exact molality equality would give $M_Y = 36A/11$, which is absent from the options. The listed choice therefore uses the usual dilute-concentration approximation; $3A$ should not be described as the exact mass-percentage result.

Step 4: Conclusion Using the dilute-solution convention represented by the options, the molecular weight of Y is $3A$. Hence, the correct option is (C).

Quick Tip to Solve: For equal colligative effects, compare moles of solute per common solvent basis; check whether a percentage means mass percent before doing an exact calculation.

65. $Mn_2(CO)_{10}$ is an organometallic compound due to the presence of - [4 marks]

(A) C–O bond

(B) Mn – Mn bond

(C) Mn – O bond

(D) Mn – C bond

Correct Answer: (D) Mn – C bond

Solution:

Concept:

- An organometallic compound contains a direct bond between a metal atom and the carbon atom of a carbon-containing ligand or group.
- Metal carbonyls are included among organometallic compounds because carbon monoxide binds through carbon, producing a metal–carbon bond.

Step 1: Identify the donor atom in the carbonyl ligand In $\mathrm{Mn_2(CO)_{10}}$, the ligand CO is coordinated to manganese through its carbon atom. Carbon monoxide donates an electron pair from carbon to a suitable orbital on the metal, and metal-to-ligand back donation also stabilises the bond. The connectivity relevant to classification is therefore $\mathrm{Mn{-}C{-}O}$, not $\mathrm{Mn{-}O{-}C}$. The molecule has ten

carbonyl ligands and a manganese–manganese linkage, but only one of these kinds of linkage supplies the defining criterion.

Step 2: Apply the definition to the listed bonds The direct $\mathrm{Mn-C}$ bond joins a metal to ligand carbon and makes the compound organometallic. A $\mathrm{C-O}$ bond belongs to the ligand itself and occurs in ordinary nonmetal compounds, so it cannot establish the classification. The $\mathrm{Mn-Mn}$ bond joins two metals and likewise contains no carbon. A proposed $\mathrm{Mn-O}$ linkage would be a metal–oxygen bond, which does not meet the direct metal–carbon requirement and is not the carbonyl attachment here.

Step 3: Distinguish the defining bond from other stabilising bonds The question asks why this compound belongs to a class, rather than which bonds can occur somewhere in its structure. The presence of a metal–metal bond affects its structure and electron count, but does not replace the classification rule. Even a carbonyl complex without a metal–metal bond remains organometallic when the carbonyl carbon is directly bonded to the metal.

Step 4: Conclusion The defining linkage is the direct $\mathrm{Mn-C}$ bond. Hence, the correct option is (D).

Quick Tip to Solve: For organometallic classification, trace the atom directly attached to the metal: carbon qualifies, whereas oxygen or another metal does not.

66. The hardness of a water sample (in terms of equivalents of $CaCO_3$) containing 10^{-3} M $CaSO_4$ is (Molar mass of $CaSO_4$ = 136 g mol^{-1}) [4 marks]

(A) 90 ppm

(B) 100 ppm

(C) 50 ppm

(D) 10 ppm

Correct Answer: (B) 100 ppm

Solution:

Concept:

- Water hardness is expressed as the mass concentration of an equivalent amount of $\mathrm{CaCO_3}$, regardless of which calcium or magnesium salt causes it.
- For dilute aqueous samples, $1\,\mathrm{ppm}$ corresponds to approximately $1\,\mathrm{mg\,L^{-1}}$. Both $\mathrm{CaSO_4}$ and $\mathrm{CaCO_3}$ contain one divalent calcium ion per formula unit.

Step 1: Calculate the calcium concentration The sample contains $10^{-3}\,\mathrm{mol\,L^{-1}}$ of $\mathrm{CaSO_4}$. Each formula unit yields one $\mathrm{Ca^{2+}}$, so the calcium concentration is also $10^{-3}\,\mathrm{mol\,L^{-1}}$. One mole of this salt has the same hardness contribution as one mole of $\mathrm{CaCO_3}$, since both supply the same number of divalent calcium ions. Sulfate is the counter-ion; its different mass does not change the number of hardness-causing calcium ions.

Step 2: Convert to calcium-carbonate equivalents Using $M(\mathrm{CaCO_3}) = 100\,\mathrm{g\,mol^{-1}}$, the equivalent mass concentration is

$$10^{-3} \times 100 = 0.100\,\mathrm{g\,L^{-1}} = 100\,\mathrm{mg\,L^{-1}}.$$

Equivalently, the actual salt concentration is $10^{-3} \times 136 = 0.136\,\mathrm{g\,L^{-1}}$, and multiplying by the

equivalent-weight ratio $50/68$ again gives $0.100\,\mathrm{g\,L^{-1}}$ as $\mathrm{CaCO_3}$.

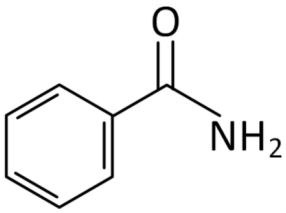
Step 3: Express the result in ppm and reject the alternatives The dilute-water conversion gives hardness $= 100\,\mathrm{ppm}$. The value 50 ppm would incorrectly include an extra division by two after the valency cancellation; both salts have the same valency factor. The values 10 ppm and 90 ppm do not follow from the stated concentration. Reporting 136 ppm would give the mass of the sulfate salt itself rather than the required calcium-carbonate equivalent.

Step 4: Conclusion The hardness is $100\,\mathrm{ppm}$ as $\mathrm{CaCO_3}$. Hence, the correct option is (B).

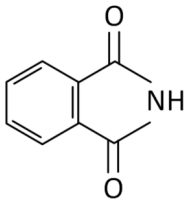
Quick Tip to Solve: For a calcium salt with one calcium per formula unit, multiply its molarity by 100,000 to obtain dilute-water hardness in ppm as calcium carbonate.

67. The increasing order of reactivity of the following compounds towards reaction with alkyl halides directly is [4 marks]

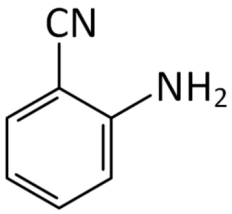
(A)



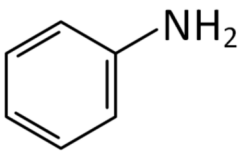
(B)



(C)



(D)



(A) A < B < C < D

(B) B < A < C < D

(C) B < A < D < C

(D) A < C < D < B

Correct Answer: (B) B < A < C < D

Solution:

Concept:

- Direct alkylation by an alkyl halide requires an available nitrogen lone pair to attack the electrophilic carbon and displace the halide.
- Resonance with a carbonyl strongly reduces nitrogen nucleophilicity; an electron-withdrawing cyano group also reduces the nucleophilicity of an aniline nitrogen.

Step 1: Identify the four nitrogen environments Compound A is benzamide, $C_6H_5CONH_2$, with the nitrogen lone pair conjugated with one carbonyl group. B is phthalimide: its nitrogen lies between two carbonyl groups, which delocalise its lone pair even more strongly. C is an ortho-cyano-substituted aniline, while D is aniline itself. In C and D, the amino nitrogen is attached directly to an aromatic ring; in C the additional CN group withdraws electron density.

Step 2: Rank lone-pair availability for direct attack The two carbonyl groups in B make its neutral nitrogen least available for attack on an alkyl halide. A, with one adjacent carbonyl, is somewhat more reactive but remains much less nucleophilic than an aromatic amine. Between C and D, the cyano substituent in C exerts electron-withdrawing effects and decreases electron density at nitrogen. Unsubstituted aniline D therefore reacts more readily than C. Combining these comparisons gives

$$B < A < C < D.$$

Step 3: Keep direct alkylation separate from the Gabriel reaction Phthalimide is useful in the Gabriel synthesis only after conversion into its anionic salt by base; that activated nucleophile is different from neutral B reacting directly. Its familiar synthetic use must not place B high in this direct-reactivity order. Option (A) reverses the order of the imide and amide, option (C) places the cyanoaniline above aniline, and option (D) incorrectly places neutral phthalimide at the most reactive end.

Step 4: Conclusion The increasing order for direct reaction with alkyl halides is $B < A < C < D$. Hence, the correct option is (B).

Quick Tip to Solve: Count the carbonyl groups conjugated with nitrogen before ranking alkylation rates, and do not confuse neutral phthalimide with its reactive potassium salt.

68. In the following reaction:

$$\textbf{Aldehyde} + \textbf{Alcohol} \xrightarrow{\text{HCl}} \textbf{Acetal}$$

Aldehyde	Alcohol
HCHO	t-BuOH
CH_3CHO	MeOH

The best combination is [4 marks]

(A) CH_3CHO and MeOH

(B) HCHO and MeOH

(C) CH_3CHO and tBuOH

(D) HCHO and tBuOH

Correct Answer: (B) HCHO and MeOH

Solution:

Concept:

- Acetal formation is acid-catalysed addition of alcohol to an aldehyde followed by replacement of the hemiacetal hydroxyl group by a second alkoxy group.
- A less hindered, more electrophilic aldehyde and a less hindered alcohol favour nucleophilic addition and formation of the acetal under acidic conditions.

Step 1: Compare the two aldehydes The carbonyl carbon of $HCHO$ is bonded to two hydrogen atoms, so it offers minimum steric hindrance to attack by an alcohol oxygen. In CH_3CHO, a methyl group both adds steric bulk and donates electron density, reducing carbonyl electrophilicity relative to formaldehyde. Hence formaldehyde is the more favourable aldehyde when selecting the combination that forms an acetal most readily. Protonation by the acid catalyst increases the electrophilicity of either carbonyl but does not reverse this comparison.

Step 2: Compare the two alcohols Methanol, CH_3OH, has a small methyl group and can approach the protonated carbonyl with little steric interference. tert-Butanol, $(CH_3)_3COH$, is much more hindered around oxygen and its bulky alkoxy group also crowds the acetal centre. Under acid, a tertiary alcohol can undergo competing reactions such as dehydration. Thus methanol is the better alcohol for this acetal-forming combination.

Step 3: Combine the favourable choices and check all options Combining the more accessible aldehyde with the less hindered alcohol selects $HCHO$ and CH_3OH. The overall equilibrium can be written

$$HCHO + 2\,CH_3OH \overset{H^+}{\rightleftharpoons} CH_2(OCH_3)_2 + H_2O.$$

Option (A) uses the less reactive aldehyde, (C) combines that aldehyde with bulky tert-butanol, and (D) retains the bulky alcohol. None improves on the simultaneous steric and electronic advantages of option (B).

Step 4: Conclusion The best combination is $HCHO$ and methanol, CH_3OH. Hence, the correct option is (B).

Quick Tip to Solve: In acetal comparisons, first choose the less hindered carbonyl compound, then the smaller alcohol; acid catalysis does not remove steric crowding.

69. Among the following compounds most basic amino acid is - [4 marks]

(A) Lysine

(B) Asparagine

(C) Histidine

(D) Serine

Correct Answer: (A) Lysine

Solution:

Concept:

- An amino acid is especially basic when its side chain contains an additional group that readily accepts a proton, beyond the common alpha-amino group.
- An aliphatic amine lone pair is more available for protonation than an amide lone pair or the basic nitrogen in an imidazole ring.

Step 1: Identify the basic side chain of lysine Lysine has the side chain $-(CH_2)_4NH_2$. Its terminal amino group is a primary aliphatic amine and readily accepts a proton to form $-(CH_2)_4NH_3{}^+$. This extra proton-binding site makes lysine strongly basic compared with amino acids whose side chains are neutral. The carboxyl and alpha-amino groups occur in all four choices, so the distinguishing factor in this question is the side chain rather than the common amino-acid backbone.

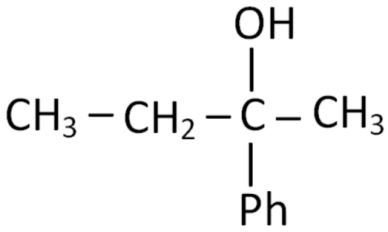
Step 2: Compare the other three side chains Asparagine has the side chain $-CH_2CONH_2$. Although it contains nitrogen, that nitrogen belongs to an amide, and resonance delocalises its lone pair into the carbonyl group; it is not an extra strongly basic amine. Serine contains $-CH_2OH$, an alcohol side chain, so it also lacks a strongly basic extra nitrogen. Histidine has an imidazole side chain that can accept a proton, making histidine basic, but this site is appreciably weaker than the terminal aliphatic amine of lysine.

Step 3: Choose the strongest base from the listed amino acids The conjugate-acid dissociation of the lysine side-chain ammonium group has a higher pK_a than that of protonated histidine imidazole. This is another way of saying that lysine binds a proton more strongly at its side chain. Thus histidine does not outrank lysine, while the amide of asparagine and hydroxyl group of serine cannot supply comparable basicity. The presence of nitrogen by itself is not enough: its bonding environment determines lone-pair availability.

Step 4: Conclusion Lysine is the most basic amino acid among the four choices. Hence, the correct option is (A).

Quick Tip to Solve: Look for an extra free aliphatic amino group when identifying a basic amino acid; an amide nitrogen does not behave like a free amine.

70. can not be prepared by - [4 marks]



(A) $CH_3CH_2COCH_3$ + PhMgX

(B) $PhCOCH_3$ + CH_3CH_2MgX

(C) $PhCOCH_2CH_3$ + CH_3MgX

(D) HCHO + $PhCH(CH_3)CH_2MgX$

Correct Answer: (D) HCHO + $PhCH(CH_3)CH_2MgX$

Solution:

Concept:

- A Grignard reagent adds its carbon group to a carbonyl carbon; hydrolysis of the magnesium alkoxide converts that carbonyl oxygen into an alcohol hydroxyl group.
- Ketones give tertiary alcohols, whereas formaldehyde gives primary alcohols. The carbon carrying the hydroxyl group retains the original carbonyl substituents.

Step 1: Read the substituents on the target alcohol The illustrated product has its hydroxyl-bearing carbon bonded to a methyl group, an ethyl group and a phenyl group. It is therefore the tertiary alcohol $CH_3CH_2C(OH)(CH_3)Ph$. To prepare it by a Grignard addition, a ketone can provide any two of those carbon groups, and the Grignard reagent must supply the remaining one. Hydrolysis is understood after the addition; the target connectivity is fixed at the former carbonyl carbon.

Step 2: Check the three ketone routes In option (A), butan-2-one supplies methyl and ethyl groups, while $PhMgX$ adds phenyl. In (B), acetophenone supplies phenyl and methyl, while CH_3CH_2MgX adds ethyl. In (C), propiophenone supplies phenyl and ethyl, while CH_3MgX adds methyl. Each route therefore gives the same three groups on the hydroxyl-bearing carbon, and all three can prepare the illustrated tertiary alcohol.

Step 3: Find the connectivity produced by formaldehyde Option (D) uses HCHO. Its carbonyl carbon initially has two hydrogen atoms, so Grignard addition followed by hydrolysis gives RCH_2OH, a primary alcohol. With the listed reagent, $R = PhCH(CH_3)CH_2$, the product is $PhCH(CH_3)CH_2CH_2OH$. Although its carbon skeleton contains the same kinds of groups, the hydroxyl group is now on a terminal carbon rather than on the carbon bonded to phenyl, methyl and ethyl. No rearrangement in ordinary Grignard addition moves the hydroxyl to the required tertiary centre.

Step 4: Conclusion The formaldehyde route in option (D) gives a primary alcohol and cannot prepare the illustrated tertiary alcohol. Hence, the correct option is (D).

Quick Tip to Solve: Trace the carbonyl carbon through a Grignard reaction: it becomes the hydroxyl-bearing carbon, so formaldehyde always gives a primary alcohol after hydrolysis.

71. The pair of metal ions that can give a spin only magnetic moment of 3.9 BM for the complex $[M(H_2O)_6]Cl_2$, is - [4 marks]

(A) V^{2+} and Fe^{2+}

(B) V^{2+} and Co^{2+}

(C) Co^{2+} and Fe^{2+}

(D) Cr^{2+} and Mn^{2+}

Correct Answer: (B) V^{2+} and Co^{2+}

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons; paired electrons make no net spin contribution.
- Water is a weak-field ligand. Its octahedral complexes with the divalent first-row ions considered here are high spin, so the electrons occupy the five d orbitals to maximise unpaired spins.

Step 1: Determine the oxidation state and required spin count The two chloride ions are outside the coordination sphere. Each carries charge -1, while the six water ligands are neutral, so the metal is in oxidation state $+2$. The required value is close to $\sqrt{15} = 3.87$, which rounds to $3.9\,\text{BM}$. Thus $n(n+2) = 15$ and the physically meaningful solution is $n = 3$. We need two divalent ions with three unpaired electrons, rather than ions with three total electrons.

Step 2: Count the unpaired d electrons in each candidate Vanadium has configuration $[\text{Ar}]3d^3 4s^2$; removing the two 4s electrons gives V^{2+}, a d^3 ion. Its octahedral arrangement is t_{2g}^3, giving three unpaired electrons. Cobalt gives $Co^{2+} : d^7$; with weak-field water the arrangement is $t_{2g}^5 e_g^2$, also with three unpaired electrons. In contrast, $Fe^{2+} : d^6$ has four unpaired electrons, $Cr^{2+} : d^4$ has four, and $Mn^{2+} : d^5$ has five. Their moments are $\sqrt{24}$ or $\sqrt{35}\,\text{BM}$, rather than $\sqrt{15}\,\text{BM}$. Hence every pair containing iron(II), chromium(II), or manganese(II) fails the stated spin-only condition.

Step 3: Conclusion Both V^{2+} and Co^{2+} have three unpaired electrons in these aqua complexes and give $\mu \simeq 3.9\,\text{BM}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert the magnetic moment to an unpaired-electron count first; then remove 4s electrons before counting the d electrons of a transition-metal ion.

72. The element with Z = 120 (not yet discovered) will be an/a - [4 marks]

(A) Alkaline earth metal

(B) Alkali metal

(C) Transition metal

(D) Inner transition metal

Correct Answer: (A) Alkaline earth metal

Solution:

Concept:

- The periodic classification follows the subshell receiving the differentiating electron. The s block contains groups 1 and 2, with outer configurations ns^1 and ns^2, respectively.
- An alkaline earth metal has a filled outer s subshell, ns^2, outside a closed-shell core; the predicted position is found by continuing the electronic filling sequence.

Step 1: Locate the element after the period-seven closed shell The noble-gas element at the end of the seventh period has atomic number 118. For the periodic-table prediction in this question, the next electron enters the new $8s$ subshell. Thus the element of atomic number 119 has the predicted outer configuration $8s^1$, while adding one more electron to reach $Z = 120$ produces $8s^2$. The two added electrons begin period 8; they do not begin a d or f subshell before the 8s orbital is filled.

Step 2: Identify the group and eliminate the other classes The configuration for the proposed element can therefore be written as $[\mathrm{Og}]8s^2$, with the bracket denoting the closed-shell $Z = 118$ core. Two electrons in the outermost s orbital place it in group 2, below the other alkaline earth metals. An alkali metal would instead have $8s^1$, corresponding to the immediately preceding atomic number. A transition metal requires the characteristic filling of a d subshell, whereas an inner transition metal involves an f subshell; neither description fits the differentiating 8s electron here. The large atomic number alone does not make an element an inner transition element.

Step 3: Conclusion The predicted $8s^2$ valence configuration places $Z = 120$ in group 2 as an alkaline earth metal. Hence, the correct option is (A).

Quick Tip to Solve: For a hypothetical element just after a noble gas, count electrons into the next s subshell: one gives group 1 and two give group 2.

73. What is the work function of the metal if the light of wavelength 4000$\mathring{A}$ generates photoelectrons of velocity 6×10^5 ms^{-1} from it ?

(Mass of electron = 9×10^{-31} kg;

Velocity of light = 3×10^8 ms^{-1}

Plank's constant = 6.626×10^{-34} Js;

Charge of electron = 1.6×10^{-19} JeV^{-1}) [4 marks]

(A) 4.0 eV

(B) 0.9 eV

(C) 2.1 eV

(D) 3.1 eV

Correct Answer: (C) 2.1 eV

Solution:

Concept:

- Einstein's photoelectric equation is $hc/\lambda = \phi + K_{\max}$, where ϕ is the metal's work function and $K_{\max} = mv^2/2$ is the emitted electron's kinetic energy.
- A wavelength in angstroms must be converted using $1\,\text{Å} = 10^{-10}\,\text{m}$. Divide an energy in joules by 1.6×10^{-19} to express it in electronvolts.

Step 1: Calculate the incident photon energy The wavelength is $4000\,\text{Å} = 4.000 \times 10^{-7}\,\text{m}$. Using the constants given in the question,

$$E_\gamma = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{4.000 \times 10^{-7}} = 4.9695 \times 10^{-19}\,\text{J}.$$

Hence $E_\gamma = 4.9695/1.6 = 3.10594\,\text{eV}$. This is the total energy available to release the electron and give it kinetic energy; it is not itself the work function.

Step 2: Subtract the photoelectron kinetic energy The speed is $6 \times 10^5\,\text{m s}^{-1}$, so

$$K_{\max} = \tfrac{1}{2}(9 \times 10^{-31})(6 \times 10^5)^2 = 1.62 \times 10^{-19}\,\text{J} = 1.0125\,\text{eV}.$$

Therefore

$$\phi = E_\gamma - K_{\max} = (4.9695 - 1.62) \times 10^{-19} = 3.3495 \times 10^{-19}\,\text{J} = 2.09344\,\text{eV}.$$

The result rounds to $2.1\,\text{eV}$. Option (D), $3.1\,\text{eV}$, would incorrectly equate the work function to the entire photon energy and leave no energy for the observed motion. Option (A) exceeds the photon energy and cannot permit emission; option (B) leaves too much kinetic energy for the stated speed.

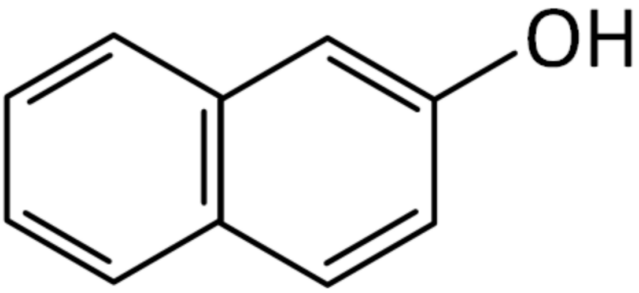
Step 3: Conclusion The work function is approximately $2.1\,\text{eV}$. Hence, the correct option is (C).

Quick Tip to Solve: Keep photon energy and kinetic energy in the same unit before subtracting; the work function must be positive and smaller than the incident photon energy.

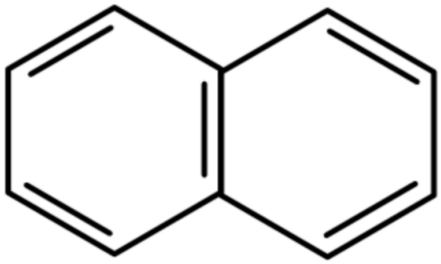
74. **Among the following four aromatic compounds, which one will have the lowest melting point ?**

[4 marks]

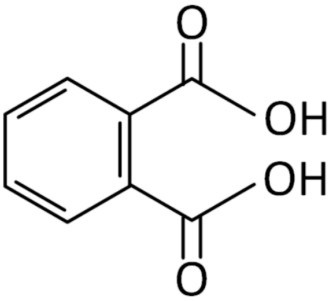
(A)



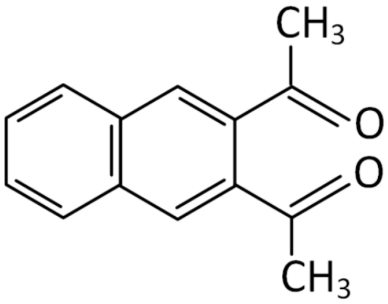
(B)



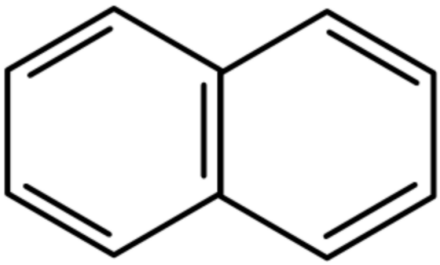
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- Melting a molecular crystal overcomes the intermolecular attractions stabilising its lattice. Hydrogen bonding and strong permanent-dipole interactions can increase lattice cohesion relative to a comparable nonpolar hydrocarbon.
- The structures must be read as whole molecules: the presence of an oxygen atom matters through its functional group and intermolecular interactions, while molecular shape and packing also influence melting.

Step 1: Identify the functional groups in the four structures Option (A) is a naphthol: the fused aromatic rings carry a phenolic $-\mathrm{OH}$ group. Option (B) is unsubstituted naphthalene, a fused-ring hydrocarbon with no oxygen-containing group. Option (C) is phthalic acid, with two adjacent $-\mathrm{COOH}$ groups on a benzene ring. Option (D) contains the fused naphthalene framework bearing two acetyl groups, each with a polar $\mathrm{C{=}O}$ bond. Identifying these groups is more useful than comparing only the number of aromatic rings.

Step 2: Compare the attractions holding the crystals together Naphthalene has no hydrogen-bond donor or acceptor and no strongly polar functional group, so its intermolecular attraction is predominantly dispersion. The phenolic molecules in (A) can associate through intermolecular hydrogen bonds. The carboxylic-acid groups in (C) permit particularly strong hydrogen-bonded association in the solid. The carbonyl-bearing compound in (D) has substantial permanent dipoles and a larger substituted framework, giving stronger cohesive interactions than unsubstituted naphthalene for this set. Thus the hydrocarbon in (B) is the most readily melted of the four. This comparison applies to these specific structures: melting points depend on packing as well as functional groups, so the statement is not a universal rule that every nonpolar molecule melts below every polar molecule.

Step 3: Conclusion Naphthalene, the unsubstituted fused-ring hydrocarbon shown in (B), has the lowest melting point among these four compounds. Hence, the correct option is (B).

Quick Tip to Solve: For this set, first separate the hydrocarbon from the phenol, dicarboxylic acid and diketone; then compare their intermolecular association rather than aromatic ring count alone.

75. 50 mL of 0.5 M oxalic acid is needed to neutralize 25 mL of sodium hydroxide solution. The amount of NaOH in 50 mL of the given sodium hydroxide solution is - [4 marks]

(A) 20 g

(B) 4 g

(C) 80 g

(D) 10 g

Correct Answer: (B) 4 g

Solution:

Concept:

- Oxalic acid, $H_2C_2O_4$, is diprotic: one mole is completely neutralised by two moles of $NaOH$. Molarity counts moles of acid molecules, not moles of replaceable protons.
- The amount of solute in a homogeneous solution scales directly with the solution volume. Once the moles of base in the titrated aliquot are known, the amount in 50 mL follows by doubling.

Step 1: Use the balanced neutralisation equation The complete reaction is

$$H_2C_2O_4 + 2\,NaOH \longrightarrow Na_2C_2O_4 + 2\,H_2O.$$

The acid supplied contains

$$n(H_2C_2O_4) = MV = 0.5 \times \frac{50}{1000} = 0.025\,\text{mol}.$$

Every acid molecule supplies two acidic protons at complete neutralisation, so the 25 mL base aliquot contains $2 \times 0.025 = 0.050\,\text{mol}$ of $NaOH$. A one-to-one acid-base mole ratio would miss the diprotic nature of oxalic acid.

Step 2: Scale to 50 mL and convert moles into mass The molarity of the base is

$$M_{\text{base}} = \frac{0.050}{0.025} = 2.0\,\text{mol}\,\text{L}^{-1}.$$

Consequently 50 mL contains

$$n(NaOH) = 2.0 \times 0.050 = 0.100\,\text{mol}.$$

Using $M_r(NaOH) = 23 + 16 + 1 = 40\,\text{g}\,\text{mol}^{-1}$, the required mass is

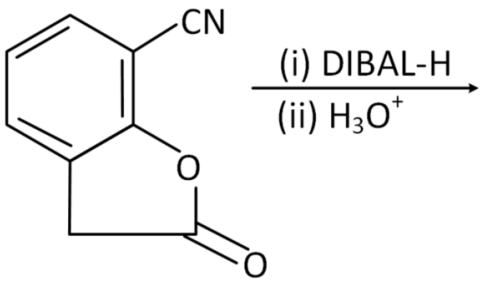
$$m = 0.100 \times 40 = 4.0\,\text{g}.$$

As a consistency check, the titrated 25 mL portion contains 2.0 g, so doubling its volume doubles its solute mass to 4.0 g. Options (A), (C), and (D) would correspond to much larger moles of base in the same volume and would require more than the stated 0.025 mol of diprotic acid for neutralisation.

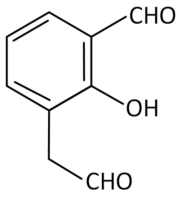
Step 3: Conclusion The given sodium hydroxide solution contains $4\,\text{g}$ of $NaOH$ in 50 mL. Hence, the correct option is (B).

Quick Tip to Solve: Write the acid-base mole ratio before applying the titration data; for oxalic acid, the factor of two comes from its two ionisable protons.

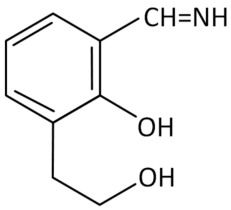
76. **The major product of the following reaction is –** [4 marks]



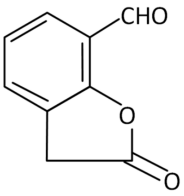
(A)



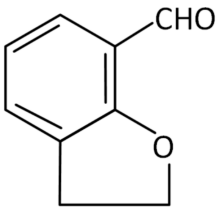
(B)



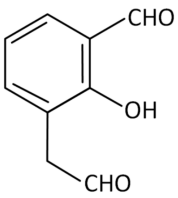
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- DIBAL-H is a controlled hydride reducing agent. A nitrile is partially reduced to an aluminium-bound imine intermediate, which gives an aldehyde after acidic hydrolysis.
- A lactone is a cyclic ester. Controlled reduction followed by hydrolysis can cleave its acyl-oxygen bond and open the ring, furnishing an aldehyde at the former ester carbonyl carbon and an alcohol or phenol at the oxygen-bearing fragment.

Step 1: Reduce the aromatic nitrile group The starting aromatic ring bears a $-\mathrm{CN}$ substituent in addition to the fused cyclic ester. Hydride attack on the nitrile carbon produces an imine-type aluminium intermediate. In the second step, $\mathrm{H_3O^+}$ hydrolyses that intermediate, converting the original nitrile carbon into a formyl carbon:

$$\mathrm{Ar{-}CN} \xrightarrow[\mathrm{H_3O^+}]{\mathrm{DIBAL{-}H}} \mathrm{Ar{-}CHO}.$$

The product must therefore carry $-\mathrm{CHO}$, rather than retain $-\mathrm{CH{=}NH}$. This immediately excludes (B), whose imine group is not the final product after the explicitly stated acidic work-up.

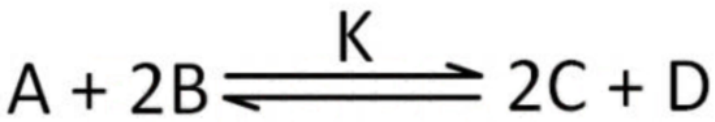
Step 2: Open the lactone and match the entire product The ester carbonyl in the fused five-membered ring is also reduced. Cleavage of its bond to the ring oxygen opens this cyclic ester; protonation gives a phenolic $-\mathrm{OH}$ on the aromatic carbon that originally carried the ester oxygen. The other end becomes the side chain $-\mathrm{CH_2{-}CHO}$, since the original carbonyl carbon is retained as an aldehyde and the adjoining methylene stays bonded to the ring. Together with nitrile reduction, the product has three consecutive aromatic substituents: $-\mathrm{CHO}$, $-\mathrm{OH}$, and $-\mathrm{CH_2CHO}$, exactly as in (A). Option (C) leaves the lactone unchanged. Option (D) removes its carbonyl while retaining a cyclic ether, rather than representing controlled ester reduction and ring opening. Neither matches the combined transformations under the stated sequence.

Step 3: Conclusion The product has the nitrile converted to an aldehyde and the lactone opened to a phenol with an aldehyde side chain, as shown in (A). Hence, the correct option is (A).

Quick Tip to Solve: For DIBAL-H followed by acidic work-up, inspect every reducible functional group; a cyclic ester can open, while a nitrile gives an aldehyde rather than a persistent imine.

77. **In a chemical reaction,**

the initial concentration of B was 1.5 times of the concentration of A, but the equilibrium concentrations of A and B were found to be equal. The equilibrium constant (K) for the aforesaid chemical reaction is - [4 marks]



(A) 16

(B) 1

(C) 1/4

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- For $\mathrm{A} + 2\,\mathrm{B} \rightleftharpoons 2\,\mathrm{C} + \mathrm{D}$, the concentration equilibrium constant is $K = [\mathrm{C}]^2[\mathrm{D}]/([\mathrm{A}][\mathrm{B}]^2)$; stoichiometric coefficients become powers in the expression.
- Use a single reaction-extent concentration x: the decreases in A and B are x and $2x$, and the increases in C and D are $2x$ and x. The starting reactant ratio fixes the extent when their equilibrium concentrations are equal.

Step 1: Set up the equilibrium concentrations Let the initial concentration of A be a; then that of B is $1.5a$. For the initially mixed reactants, take the initial product concentrations as zero. If x is the amount of A reacted per unit volume, the equilibrium concentrations are

$$[\mathrm{A}] = a - x, \quad [\mathrm{B}] = 1.5a - 2x,$$
$$[\mathrm{C}] = 2x, \qquad [\mathrm{D}] = x.$$

The stated equality of the reactant concentrations gives

$$a - x = 1.5a - 2x \quad \Rightarrow \quad x = 0.5a.$$

Thus A and B each have concentration $0.5a$, while C has concentration a and D has concentration $0.5a$. All concentrations are nonnegative, so this extent is physically consistent.

Step 2: Substitute into the equilibrium expression Insert each species with its correct stoichiometric exponent:

$$K = \frac{(a)^2(0.5a)}{(0.5a)(0.5a)^2} = \frac{0.5a^3}{0.125a^3} = 4.$$

The unknown initial concentration cancels because both numerator and denominator have total concentration power three. Equal equilibrium concentrations of A and B do not imply $K = 1$; the product concentrations and all the powers must be included. The value $1/4$ is the reciprocal for the reverse reaction, not the reaction printed. The value 16 would result from an incorrect concentration

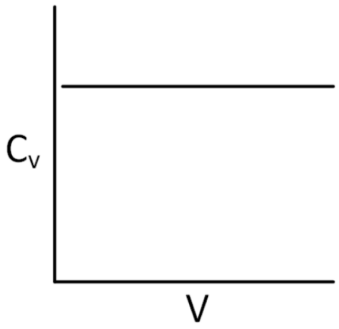
ratio or exponent treatment.

Step 3: Conclusion The equilibrium constant for the reaction in the forward direction as printed is $K = 4$. Hence, the correct option is (D).

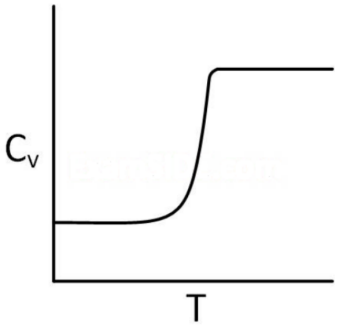
Quick Tip to Solve: Use an extent table before substituting into K; equality of two equilibrium concentrations is a constraint on the extent, not a statement that K equals one.

78. For diatomic ideal gas in a closed system, which of the following plots does not correctly describe the relation between various thermodynamic quantities? [4 marks]

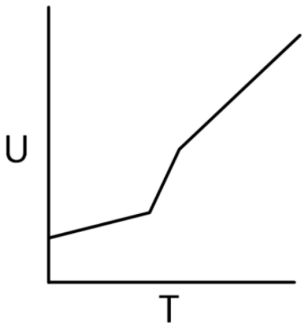
(A)



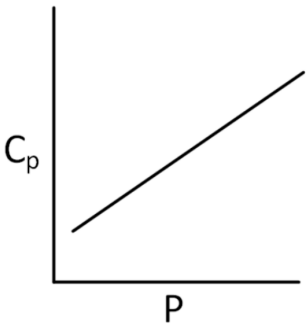
(B)



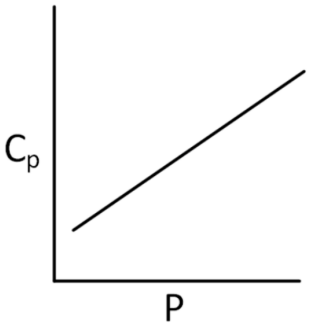
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- For an ideal gas, internal energy depends only on temperature: $U = U(T)$. Consequently, $C_V = (\partial U/\partial T)_V$ is independent of volume.
- The molar heat capacities satisfy Mayer's relation $C_{P,m} - C_{V,m} = R$. Translational, rotational and vibrational modes determine their temperature dependence, rather than pressure.

Step 1: Check the volume and temperature plots In (A), the horizontal line shows that changing volume does not change C_V, consistent with an ideal gas. In (B), the rise between approximately constant ranges represents the thermal activation of additional molecular modes. For a diatomic molecule, rotations contribute when excited; vibrations contribute at still higher temperatures. Thus heat capacity need not remain constant over an arbitrarily wide temperature interval. The familiar $C_{V,m} = 5R/2$ describes the range where rotations are active and vibrations are effectively frozen.

Step 2: Relate the energy slope to heat capacity The slope of an internal-energy versus temperature plot is the heat capacity:

$$\frac{dU}{dT} = C_V.$$

An increasing internal energy with changing slope, as represented schematically in (C), reflects the change in accessible energy modes. It is therefore not a claim that internal energy depends on volume or pressure.

Step 3: Identify the incorrect pressure dependence At a fixed temperature, both ideal-gas heat

capacities are independent of pressure. In particular,

$$C_P = C_V + nR$$

for the fixed amount n in the closed system. Hence a C_P-versus-P graph should be horizontal when other independent conditions are fixed. The rising straight line in (D) introduces a pressure dependence that an ideal gas does not possess. This distinguishes (D) from the volume independence and temperature effects in the other plots.

Step 4: Conclusion The increasing C_P-versus-pressure line is incorrect for an ideal gas. Hence, the correct option is (D).

Quick Tip to Solve: Check the horizontal axis first: ideal-gas heat capacities can vary with temperature as modes activate, but not with pressure or volume at fixed temperature.

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79. The correct order for acid strength of compounds :

CH ≡ CH, CH_3–C ≡ CH and CH_2 = CH_2 is as follows [4 marks]

(A) CH ≡ CH > CH_2 = CH_2 > CH_3 – C ≡ CH

(B) CH_3 – C ≡ CH > CH_2 = CH_2 > HC ≡ CH

(C) HC ≡ CH > CH_3 – C ≡ CH > CH_2 = CH_2

(D) CH_3 – CH ≡ CH > CH ≡ CH > CH_2 = CH_2

Correct Answer: (C) HC ≡ CH > CH_3 – C ≡ CH > CH_2 = CH_2

Solution:

Concept:

- Hydrocarbon acidity is compared through the stability of the carbanion remaining after loss of H^+. A more stable conjugate base corresponds to a stronger acid.
- An sp carbon has 50% s-character, whereas an sp^2 carbon has about 33%. Greater s-character stabilises a negative charge; an alkyl group has an electron-releasing inductive effect.

Step 1: Compare the hybridisation of the acidic carbons Removing a terminal hydrogen from acetylene or propyne leaves the negative charge on an sp-hybridised carbon. Removing hydrogen from ethene leaves it on an sp^2-hybridised carbon. The greater s-character in an sp orbital holds the electron pair nearer the nucleus and stabilises the acetylide ion more strongly. Therefore both terminal alkynes are more acidic than ethene. This immediately excludes (A) and (B), since each puts ethene above one terminal alkyne.

Step 2: Compare acetylene with propyne The methyl group in propyne pushes electron density towards the negatively charged carbon in its conjugate base. This $+I$ effect destabilises that base relative to the conjugate base of acetylene, which has hydrogen instead of methyl. Thus acetylene loses its terminal proton more readily than propyne:

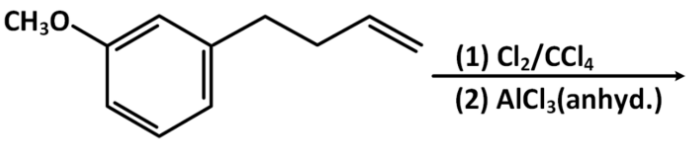
$$HC{\equiv}CH > CH_3{-}C{\equiv}CH > CH_2{=}CH_2$$

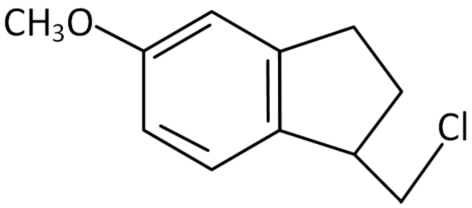
in acid strength. The reversed order of the two alkynes in (D) is therefore incorrect. The comparison is about the terminal acetylenic hydrogen, not about removing a hydrogen from the methyl group.

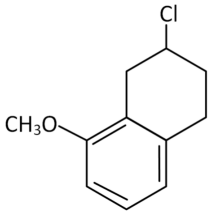
Step 3: Conclusion Acetylene is strongest, followed by propyne and then ethene. Hence, the correct option is (C).

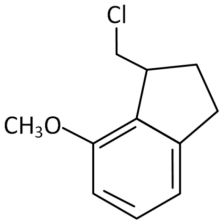
Quick Tip to Solve: For hydrocarbon acidity, compare s-character first; if hybridisation is the same, an electron-releasing alkyl group weakens the acid.

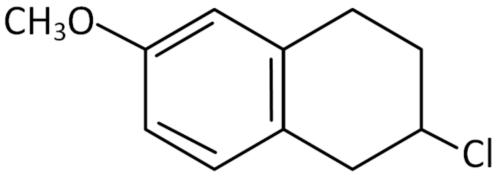
80. The major product of the following reaction is [4 marks]



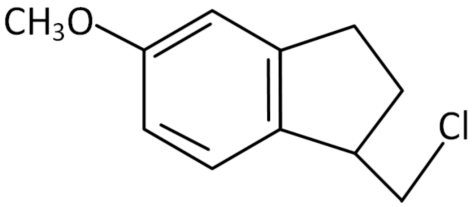
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- Chlorine in CCl_4 adds across the isolated alkene double bond to form a vicinal dichloride; these conditions do not initially chlorinate the aromatic ring.
- Anhydrous $AlCl_3$ activates an alkyl chloride towards intramolecular Friedel–Crafts alkylation. Ring size, carbocation stability and the directing effect of methoxy determine the major cyclisation.

Step 1: Add chlorine across the side-chain double bond Write the original side chain as $Ar-CH_2-CH_2-CH=CH_2$, with methoxy meta to the chain on the aromatic ring. Chlorine addition gives

$$Ar-CH_2-CH_2-CHCl-CH_2Cl.$$

The aromatic methoxy group remains present. Both carbon atoms of the former double bond now bear chlorine; a subsequent cyclisation removes one of those chlorides rather than removing both.

Step 2: Close the favoured ring at the secondary carbon Lewis-acid activation of the chloride on the secondary carbon favours attack by the nearby aromatic ring. Bond formation to this carbon closes a five-membered saturated ring fused to benzene, while the terminal CH_2Cl group remains attached to that ring. Closure involving the terminal primary carbon would instead give a six-membered ring and is less favourable under these conditions. This rules out the tetralin structures in (B) and (D).

Step 3: Use the methoxy group to select the ring position The two positions ortho to the side chain can close the five-membered ring. One is ortho to methoxy and the other is para to methoxy. The methoxy group activates both, but the para position is less crowded than the position between methoxy and the tether. Attack at that less hindered, activated site followed by loss of a proton restores aromaticity and gives (A). Structure (C) represents closure at the more crowded ortho-to-methoxy position and is not the major product.

Step 4: Conclusion The major product is the five-membered fused ring with a retained chloromethyl substituent formed by para-to-methoxy closure, shown in (A). Hence, the correct option is (A).

Quick Tip to Solve: After alkene chlorination, mark both chlorides. Activate the secondary one, count the new ring, then choose the less crowded methoxy-activated aromatic position.

81. The standard electrode potential E^o and its temperature coefficient $\left(\frac{dE^o}{dT}\right)$ for a cell are 2V and -5×10^{-4} VK^{-1} at 300 K respectively.

The cell reaction is

Zn(s) + Cu^{2+} (aq) $\longrightarrow$ Zn^{2+} (aq) + Cu(s)

The standard reaction enthalpy ($\Delta_r H^o$) at 300 K in kJ mol^{-1} is, [Use R = 8 JK^{-1} mol^{-1} and F = 96,000C mol^{-1}] [4 marks]

(A) $-$ 412.8

(B) $-$ 384.0

(C) 192.0

(D) 206.4

Correct Answer: (A) $-$ 412.8

Solution:

Concept:

- The standard Gibbs energy is $\Delta_r G^\circ = -nFE^\circ$, where n is the number of electrons transferred per written cell reaction and F is the Faraday constant.
- The relations $\Delta_r S^\circ = nF(dE^\circ/dT)$ and $\Delta_r H^\circ = \Delta_r G^\circ + T\Delta_r S^\circ$ connect the cell potential to reaction enthalpy.

Step 1: Find the electron transfer and Gibbs energy Zinc is oxidised according to $\mathrm{Zn} \longrightarrow \mathrm{Zn}^{2+} + 2\,\mathrm{e}^-$, while Cu^{2+} accepts two electrons. Thus $n = 2$ for the written reaction. Substituting the supplied potential and Faraday constant gives

$$\Delta_r G^\circ = -2(96000)(2) = -384000\,\mathrm{J\,mol^{-1}}.$$

This is a Gibbs energy, not yet the requested enthalpy. Selecting (B) at this stage would omit the temperature correction.

Step 2: Calculate the entropy and enthalpy correction The given temperature coefficient is negative, so the standard reaction entropy is also negative:

$$\Delta_r S^\circ = 2(96000)(-5 \times 10^{-4}) = -96\,\mathrm{J\,K^{-1}\,mol^{-1}}.$$

At $300\,\mathrm{K}$,

$$T\Delta_r S^\circ = 300(-96) = -28800\,\mathrm{J\,mol^{-1}}.$$

Adding this to the Gibbs energy yields

$$\Delta_r H^\circ = -384000 - 28800 = -412800\,\mathrm{J\,mol^{-1}}.$$

The gas constant is not needed because the potential and its temperature coefficient already supply the required information.

Step 3: Convert units and check the alternatives Dividing by 1000 gives $-412.8\,\mathrm{kJ\,mol^{-1}}$. The negative entropy contribution makes the enthalpy more negative than the Gibbs energy. Options (C) and (D) also have the wrong positive sign and correspond to neither the electron count nor the

correction for the written reaction.

Step 4: Conclusion $\Delta_r H^\circ = -412.8\,\text{kJ mol}^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $\Delta H^\circ = -nF[E^\circ - T(dE^\circ/dT)]$; a negative temperature coefficient makes the enthalpy more negative than $-nFE^\circ$.

82. The metal d-orbitals that are directly facing the ligands in $K_3[Co(CN)_6]$ are - [4 marks]

(A) $d_{x^2-y^2}$ and d_{z^2}

(B) d_{xy}, d_{xz} and d_{yz}

(C) d_{xz}, d_{yz} and d_{z^2}

(D) d_{xy} and $d_{x^2-y^2}$

Correct Answer: (A) $d_{x^2-y^2}$ and d_{z^2}

Solution:

Concept:

- In $K_3[Co(CN)_6]$, the complex anion has six monodentate cyanide ligands, so its coordination number is six and its geometry is octahedral.
- In an octahedron, ligands approach along the Cartesian axes. The e_g set, $d_{x^2-y^2}$ and d_{z^2}, points along those axes; the t_{2g} set lies between them.

Step 1: Identify the coordination environment The three potassium ions balance the charge of $[Co(CN)_6]^{3-}$. Each cyanide has charge -1, so cobalt has oxidation state $+3$. More directly relevant to the question, there are six donor atoms around cobalt. For this complex they occupy the six octahedral positions along $+x, -x, +y, -y, +z, -z$. The question asks which metal orbitals directly face these ligand positions, rather than which orbitals contain electrons.

Step 2: Locate the lobes of the five d orbitals The four lobes of $d_{x^2-y^2}$ point along the positive and negative x and y axes, directly towards four ligands. The two principal lobes of d_{z^2} point along the z axis towards the other two ligands. In contrast, d_{xy}, d_{xz} and d_{yz} have their lobes between the relevant axes. They therefore experience less direct ligand repulsion and form the lower-energy t_{2g} group in an octahedral field.

Step 3: Check each proposed group Option (A) contains precisely the two directly facing e_g orbitals. Option (B) lists the entire between-axis t_{2g} set. Option (C) combines two between-axis orbitals with one axis-directed orbital, while (D) similarly mixes one of each. Strong-field cyanide affects the electron pairing, but it does not rotate these orbital directions or change the identity of the axis-directed pair.

Step 4: Conclusion The orbitals directly facing the ligands are $d_{x^2-y^2}$ and d_{z^2}. Hence, the correct option is (A).

Quick Tip to Solve: For octahedral complexes, remember that the two e_g orbitals point at ligands, while all three t_{2g} orbitals point between them.

83. The volume of gas A is twice than that of gas B. The compressibility factor of gas A is thrice than that of gas B at same temperature. The pressures of the gases for equal number of moles are - [4 marks]

(A) $2P_A = 3P_B$

(B) $3P_A = 2P_B$

(C) $P_A = 3P_B$

(D) $P_A = 2P_B$

Correct Answer: (A) $2P_A = 3P_B$

Solution:

Concept:

- The compressibility factor is $Z = PV/(nRT)$. It measures a real gas relative to ideal-gas behaviour without requiring $Z = 1$.
- For equal amounts at the same temperature, nRT is common, and the pressure ratio is $P_A/P_B = (Z_A/Z_B)(V_B/V_A)$. Both the volume and compressibility ratios must be included.

Step 1: Write the real-gas relation for each sample Rearranging the definition gives $PV = ZnRT$. Hence

$$P_AV_A = Z_AnRT, \qquad P_BV_B = Z_BnRT.$$

The question specifies equal numbers of moles and a common temperature, so dividing these equations cancels n, R and T:

$$\frac{P_AV_A}{P_BV_B} = \frac{Z_A}{Z_B} = 3.$$

No assumption about either individual value of Z is necessary.

Step 2: Substitute the stated volume ratio Since $V_A = 2V_B$, the preceding equation becomes

$$\frac{P_A(2V_B)}{P_BV_B} = 3 \quad \Longrightarrow \quad 2\frac{P_A}{P_B} = 3.$$

Thus

$$\frac{P_A}{P_B} = \frac{3}{2}, \qquad 2P_A = 3P_B.$$

The larger compressibility factor increases the pressure ratio by three, while the doubled volume reduces that effect by two. This is why neither ratio can be used alone.

Step 3: Check the other pressure relations Option (B) gives the reciprocal ratio $2/3$, reversing the effect of Z_A/Z_B. Option (C) would follow from ignoring the doubled volume. Option (D) gives a factor of two and does not satisfy the compressibility-factor equation: it would imply $P_AV_A/(P_BV_B) = 4$, rather than the supplied value three. Only (A) obeys both given ratios simultaneously.

Step 4: Conclusion The pressure relation is $2P_A = 3P_B$, or $P_A = 1.5P_B$. Hence, the correct option is (A).

Quick Tip to Solve: At common n, T, take ratios of $PV = ZnRT$: pressure varies directly with Z and inversely with volume.

84. Given

Gas	H_2	CH_4	CO_2	SO_2
Critical Temperature / K	33	190	304	630

On the basis of data given above, predict which of the following gases shows least adsorption on a definite amount of charcoal? [4 marks]

(A) SO_2

(B) CH_4

(C) H_2

(D) CO_2

Correct Answer: (C) H_2

Solution:

Concept:

- Physical adsorption of gases on charcoal is favoured by stronger intermolecular attractions between the gas and the adsorbent surface.
- A higher critical temperature indicates a gas that is more readily liquefied and generally more strongly adsorbed under comparable conditions. The least critical temperature therefore identifies the least readily adsorbed gas here.

Step 1: Use critical temperature as the measure of attraction The gas molecules are retained on a charcoal surface by intermolecular forces in physical adsorption. A gas that condenses readily has stronger attractive interactions and tends to be adsorbed more readily. Critical temperature is the highest temperature at which a gas can be liquefied by pressure alone, so a very low value signifies weak attractions and difficult liquefaction. This qualitative comparison applies to the same amount of charcoal under comparable temperature and pressure conditions.

Step 2: Arrange the supplied gases by critical temperature The table gives

$$T_c(\mathrm{H_2}) = 33\,\mathrm{K}, \quad T_c(\mathrm{CH_4}) = 190\,\mathrm{K},$$
$$T_c(\mathrm{CO_2}) = 304\,\mathrm{K}, \quad T_c(\mathrm{SO_2}) = 630\,\mathrm{K}.$$

Consequently, their approximate tendency towards physical adsorption increases in the order

$$\mathrm{H_2} < \mathrm{CH_4} < \mathrm{CO_2} < \mathrm{SO_2}.$$

Hydrogen has the smallest critical temperature by a substantial margin and hence the weakest tendency to be retained by this criterion.

Step 3: Exclude the alternative gases Methane and carbon dioxide have higher critical temperatures than hydrogen, so neither represents the minimum adsorption. Sulfur dioxide has the largest value and is the most readily liquefiable of the four; selecting it would reverse the criterion. The question asks for the least adsorption, so the relevant end of the ordering is hydrogen, not sulfur dioxide.

Step 4: Conclusion H_2 shows the least adsorption on the given charcoal because its critical temperature is lowest. Hence, the correct option is (C).

Quick Tip to Solve: For comparable physical adsorption on charcoal, rank gases by their critical temperatures: the lowest T_c usually means the least adsorption.

85. In the Hall-Heroult process, aluminium is formed at the cathode. The cathode is made out of - [4 marks]

(A) Copper

(B) Platinum

(C) Carbon

(D) Pure aluminium

Correct Answer: (C) Carbon

Solution:

Concept:

- The Hall-Heroult process extracts aluminium by electrolysing alumina dissolved in a molten cryolite-containing bath. The bath conducts ions and operates at a much lower temperature than pure alumina would require.
- The cathode is the negative electrode, where aluminium-containing ions accept electrons. The carbon lining inside the steel tank provides the cathode surface; the steel shell supports this lining.

Step 1: Identify the electrode construction The electrolytic cell is a steel tank whose inner surface is lined with carbon. This conducting carbon lining is connected to the negative terminal and therefore acts as the cathode. Graphite or carbon blocks suspended in the molten bath act as the positive anodes. Thus both electrode systems contain carbon, but they have different shapes, positions and reactions.

Step 2: Relate the cathode to aluminium formation At the cathode, reduction produces liquid aluminium, represented by the formal half-reaction

$$\mathrm{Al}^{3+} + 3\,\mathrm{e}^{-} \longrightarrow \mathrm{Al}(\mathrm{l}).$$

The liquid metal collects at the bottom of the cell and is tapped off. Aluminium is therefore the product deposited at the cathode; its formation does not mean that the original cathode is made of pure aluminium.

Step 3: Eliminate the other materials Copper and platinum are not the materials used for the carbon-lined cathode of the industrial Hall-Heroult cell. Pure aluminium names the desired product rather than the lining of the cell. Carbon is both a conductor and the stated construction material, so it is the only matching choice.

Step 4: Conclusion The cathode is the carbon lining of the steel tank. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish the electrode material from the deposited metal: carbon is the cathode lining, while molten aluminium is collected as its product.

86. Poly β –hydroxybutyrate-co-β-hydroxyvalerate (PHBV) is a copolymer of [4 marks]

(A) 2-hydroxybutanoic acid and 3-hydroxypentanoic acid

(B) 3-hydroxybutanoic acid and 2-hydroxypentanoic acid

(C) 3-hydroxybutanoic acid and 3-hydroxypentanoic acid

(D) 3-hydroxybutanoic acid and 4-hydroxypentanoic acid

Correct Answer: (C) 3-hydroxybutanoic acid and 3-hydroxypentanoic acid

Solution:

Concept:

- PHBV is a biodegradable copolyester obtained from two hydroxycarboxylic acids: β-hydroxybutyric acid and β-hydroxyvaleric acid. Both supply hydroxyl and carboxyl groups for polyester linkages.
- In a carboxylic acid, the carboxyl carbon is carbon 1. The adjacent carbon is the α carbon (carbon 2), and the next is the β carbon (carbon 3).

Step 1: Translate the first monomer name Butyric acid has a four-carbon parent chain, so its systematic parent name is butanoic acid. A hydroxyl group on its β carbon occupies carbon 3, giving 3-hydroxybutanoic acid. Its condensed structure can be written as

$$\mathrm{HOOC{-}CH_2{-}CH(OH){-}CH_3}.$$

Numbering from the methyl end would give the wrong locant because the carboxyl group controls the numbering.

Step 2: Translate the second monomer name Valeric acid has a five-carbon parent chain and is systematically pentanoic acid. Again, the β-hydroxyl group is on carbon 3. The second monomer is consequently 3-hydroxypentanoic acid, with condensed structure

$$\mathrm{HOOC{-}CH_2{-}CH(OH){-}CH_2{-}CH_3}.$$

The difference between the two monomers is the alkyl portion attached to the hydroxyl-bearing carbon, not the hydroxyl locant.

Step 3: Compare all pairs Only option (C) places the hydroxyl group at carbon 3 in both acids. Option (A) puts the first hydroxyl at carbon 2; option (B) puts the second at carbon 2. Option (D) incorrectly puts the pentanoic-acid hydroxyl at carbon 4. None of those altered positions represents the two specified beta-hydroxy monomers.

Step 4: Conclusion PHBV is a copolymer of 3-hydroxybutanoic acid and 3-hydroxypentanoic acid. Hence, the correct option is (C).

Quick Tip to Solve: For hydroxyacids, count the carboxyl carbon as 1: alpha means 2 and beta means 3, irrespective of the length of the carbon chain.

87. Water samples with BOD values of 4 ppm and 18 ppm, respectively are : [4 marks]

(A) Clean and Highly polluted

(B) Highly polluted and Clean

(C) Highly polluted and Highly polluted

(D) Clean and Clean

Correct Answer: (A) Clean and Highly polluted

Solution:

Concept:

- Biochemical oxygen demand (BOD) measures the dissolved oxygen required by microorganisms to decompose biodegradable organic matter in a water sample under specified test conditions.
- A larger BOD indicates a greater biodegradable organic pollution load, because more oxygen is consumed during microbial oxidation. Clean water has a small BOD; highly polluted water has a much larger value.

Step 1: Interpret the lower BOD value The first sample has a BOD of 4 ppm. In the usual textbook classification, clean water has BOD below about 5 ppm. Thus this sample falls in the clean category. The value is the oxygen demand of the sample and must not be mistaken for its existing dissolved oxygen content: a low oxygen demand is favourable for water quality.

Step 2: Interpret the higher BOD value The second sample has a BOD of 18 ppm, considerably above the clean-water range. Water with BOD of about 17 ppm or more is described as highly polluted in this classification. A large amount of biodegradable matter leads microorganisms to consume substantial dissolved oxygen, which can deprive aquatic organisms of the oxygen needed for respiration.

Step 3: Preserve the order of the samples The question asks for the descriptions respectively, so the first classification must belong to 4 ppm and the second to 18 ppm. The required pair is therefore clean followed by highly polluted. Option (B) reverses this order. Option (C) wrongly treats 4 ppm as highly polluted, while option (D) wrongly treats 18 ppm as clean. The two samples cannot receive the same classification here.

Step 4: Conclusion The 4 ppm sample is clean and the 18 ppm sample is highly polluted, respectively. Hence, the correct option is (A).

Quick Tip to Solve: BOD is oxygen demand, not oxygen supply: increasing BOD generally means increasing biodegradable organic pollution and decreasing water quality.

88. A metal on combustion in excess air forms X. X upon hydrolysis with water yields H_2O_2 and O_2 along with another product. The metal is : [4 marks]

(A) Rb

(B) Mg

(C) Na

(D) Li

Correct Answer: (A) Rb

Solution:

Concept:

- On combustion in excess air, larger alkali metals such as potassium, rubidium and caesium preferentially form superoxides containing the O_2^- ion. Sodium mainly forms a peroxide, while lithium forms the normal oxide.
- A superoxide reacts with water to give the metal hydroxide, hydrogen peroxide and oxygen together. This combination of products distinguishes it from the simple hydrolysis of a normal oxide or a peroxide.

Step 1: Identify the oxygen-containing compound The simultaneous production of H_2O_2 and O_2 points to a superoxide. For rubidium the combustion product is rubidium superoxide, RbO_2. Its formation in excess oxygen is represented by

$$Rb + O_2 \longrightarrow RbO_2.$$

The large rubidium cation stabilises the relatively large superoxide anion.

Step 2: Verify the hydrolysis products The reaction with water is

$$2\,RbO_2 + 2\,H_2O \longrightarrow 2\,RbOH + H_2O_2 + O_2.$$

This equation gives precisely the two named products and the additional product, rubidium hydroxide. It is balanced: both sides contain two rubidium atoms, four hydrogen atoms and six oxygen atoms. The oxygen gas is generated directly in the stated superoxide-water reaction.

Step 3: Exclude the remaining metals Sodium predominantly forms Na_2O_2, whose hydrolysis gives hydroxide and hydrogen peroxide without oxygen as a product of that hydrolysis equation. Lithium forms Li_2O, and magnesium forms mainly MgO; their normal oxides do not give the specified peroxide-plus-oxygen combination with water. Thus the stated products identify rubidium among the choices.

Step 4: Conclusion The metal is rubidium, which forms RbO_2 in excess air. Hence, the correct option is (A).

Quick Tip to Solve: Use the products to identify the oxide: a superoxide gives hydroxide, hydrogen peroxide and oxygen together on reaction with water.

89. Iodine reacts with concentrated HNO_3 to yield Y along with other products. The oxidation state of iodine in Y, is : [4 marks]

(A) 3

(B) 1

(C) 7

(D) 5

Correct Answer: (D) 5

Solution:

Concept:

- Concentrated nitric acid is a strong oxidising agent. It oxidises iodine to iodic acid, HIO_3, while the nitrogen in nitric acid is reduced to nitrogen dioxide.
- For a neutral compound, the sum of oxidation states is zero. In iodic acid hydrogen contributes $+1$, and each oxygen contributes -2; the remaining contribution determines iodine's oxidation state.

Step 1: Identify the iodine-containing product Iodine reacts with concentrated nitric acid to form iodic acid rather than a simple iodide. A balanced equation is

$$I_2 + 10\,HNO_3 \longrightarrow 2\,HIO_3 + 10\,NO_2 + 4\,H_2O.$$

Thus the substance designated by Y is HIO_3. The equation preserves two iodine atoms, ten nitrogen atoms, ten hydrogen atoms and thirty oxygen atoms.

Step 2: Calculate the oxidation state Let the oxidation state of iodine in HIO_3 be x. Using the usual oxidation states of hydrogen and oxygen gives

$$(+1) + x + 3(-2) = 0.$$

Therefore

$$x = 6 - 1 = +5.$$

Iodine begins at zero in elemental I_2, so its change to $+5$ is consistent with oxidation by concentrated nitric acid.

Step 3: Check the numerical alternatives The values 1 and 3 do not balance the oxidation-state sum in iodic acid. The value 7 would correspond to a different iodine oxoacid, not the iodic acid formed in this reaction. In the balanced reaction, two iodine atoms each lose five electrons, while ten nitrogen atoms each gain one electron on changing from $+5$ to $+4$, which independently checks the redox balance.

Step 4: Conclusion The product is iodic acid, HIO_3, and iodine has oxidation state $+5$. Hence, the correct option is (D).

Quick Tip to Solve: First identify the actual oxoacid formed; then use hydrogen as $+1$ and oxygen as -2 rather than guessing the halogen's highest oxidation state.

90. The molecule that has minimum /no role in the formation of photochemical smog, is : [4 marks]

(A) $CH_2 = O$

(B) O_3

(C) N_2

(D) NO

Correct Answer: (C) N_2

Solution:

Concept:

- Photochemical smog forms when nitrogen oxides and hydrocarbons react in sunlight. Its characteristic secondary pollutants include ozone, aldehydes and peroxyacyl nitrates.
- The nitrogen molecule, N_2, has a strong triple bond and is relatively unreactive under ordinary atmospheric conditions. It is distinct from reactive nitrogen oxides such as NO and NO_2.

Step 1: Identify the reactive nitrogen oxide Nitric oxide, NO, participates in atmospheric reactions involving nitrogen dioxide and ozone. Sunlight can photolyse nitrogen dioxide, and oxygen atoms then combine with molecular oxygen to form ozone. A simplified part of this sequence is

$$NO_2 \xrightarrow{\text{sunlight}} NO + O, \qquad O + O_2 \longrightarrow O_3.$$

Hydrocarbon-derived radicals help sustain the reactions that lead to the accumulation of photochemical oxidants.

Step 2: Assess ozone and formaldehyde Ozone, O_3, is a major oxidant and characteristic component of photochemical smog. Formaldehyde, CH_2O, is an aldehyde associated with hydrocarbon oxidation and contributes to photochemical air-pollution chemistry. Thus neither ozone nor formaldehyde can be selected as having no relevant role. Nitric oxide is also involved, as shown by the nitrogen-oxide reaction sequence.

Step 3: Select the least involved molecule Molecular nitrogen, N_2, makes up much of the atmosphere but does not ordinarily enter the sunlight-driven smog-forming reactions because breaking its triple bond requires substantial energy. Although high-temperature combustion can first convert some atmospheric nitrogen into nitrogen oxides, nitrogen gas itself has minimum direct role among the listed molecules. This distinguishes option (C) from the three reactive pollutants or participants.

Step 4: Conclusion Molecular nitrogen, N_2, has minimum or no direct role in photochemical-smog formation under ordinary atmospheric conditions. Hence, the correct option is (C).

Quick Tip to Solve: Do not confuse atmospheric nitrogen with nitrogen oxides: the stable nitrogen molecule is largely inactive, while NO and NO2 participate in smog chemistry.