JEE Main 2019 Paper 1 (B.E./B.Tech) Jan 12 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 12 January 2019, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is : [4 marks]

(A) 12

(B) 9

(C) 7

(D) 11

Correct Answer: (A) 12

Solution:

Concept:

- Each game involves an unordered pair of participants. If there are m men, the number of distinct male pairs is $\binom{m}{2} = m(m-1)/2$.
- Every pair plays twice, so the game count is twice the pair count. A male-female pair is selected in $m \times 2$ ways; games between the two women do not enter the stated comparison.

Step 1: Count the two categories The men play two games for every pair of men, giving

$$G_{MM} = 2\binom{m}{2} = m(m-1).$$

There are two women and each plays twice against every man, so

$$G_{MW} = 2(m \times 2) = 4m.$$

These counts do not include the games between the women themselves, since the question compares only the two categories specified.

Step 2: Use the difference and solve The first category exceeds the second by 84, hence

$$m(m-1) - 4m = 84.$$

Rearranging and factoring gives

$$m^2 - 5m - 84 = 0, \qquad (m-12)(m+7) = 0.$$

Thus $m = 12$ or $m = -7$. A participant count is a nonnegative integer, so the negative root is inadmissible.

Step 3: Check the offered values For 12 men the two counts are $12 \times 11 = 132$ and $4 \times 12 = 48$, with difference 84 as required. The other offered values 9, 7 and 11 give differences $m(m-5)$ equal to 36, 14 and 66 respectively, so none satisfies the comparison.

Step 4: Conclusion There are $\boxed{12}$ men. Hence, the correct option is (A).

Quick Tip to Solve: Count pairs first and multiply by the games per pair; keep male-male and male-female categories separate.

2. If a curve passes through the point (1, –2) and has slope of the tangent at any point (x, y) on it as $\frac{x^2-2y}{x}$, then the curve also passes through the point : [4 marks]

(A) (–1, 2)

(B) $(-\sqrt{2}, 1)$

(C) $(\sqrt{3}, 0)$

(D) (3, 0)

Correct Answer: (C) $(\sqrt{3}, 0)$

Solution:

Concept:

- The slope gives $dy/dx = (x^2 - 2y)/x$, a linear differential equation on an interval with $x \neq 0$. Its standard form is $y\prime + (2/x)y = x$.
- For $y\prime + P(x)y = Q(x)$, multiplication by the integrating factor $e^{\int P(x)\,dx}$ converts the left side into a product derivative. Here the integrating factor can be taken as x^2.

Step 1: Integrate the equation Multiplying the equation by x^2 gives

$$x^2\frac{dy}{dx} + 2xy = x^3, \qquad \frac{d}{dx}(x^2y) = x^3.$$

Therefore

$$x^2y = \frac{x^4}{4} + C.$$

Since the given point has positive abscissa, the solution through it is considered on $x > 0$.

Step 2: Determine the integration constant Substitute the given point $(1, -2)$:

$$-2 = \frac{1}{4} + C, \qquad C = -\frac{9}{4}.$$

The required curve thus has equation

$$4x^2y = x^4 - 9, \qquad y = \frac{x^2}{4} - \frac{9}{4x^2}.$$

This also immediately reproduces the given ordinate at $x = 1$.

Step 3: Test the points For $x = \sqrt{3}$, $x^4 = 9$, so $4x^2y = 0$ and $y = 0$; option (C) is on the curve. At $x = 3$, the ordinate is 2, not 0. Even if the same algebraic equation is tested for negative abscissae, it gives $y = -2$ at $x = -1$ and $y = -5/8$ at $x = -\sqrt{2}$, excluding (A) and (B).

Step 4: Conclusion The curve passes through $\boxed{(\sqrt{3}, 0)}$. Hence, the correct option is (C).

Quick Tip to Solve: After finding the constant, test the zero-ordinate options by setting the numerator of the curve equation to zero.

3. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^{x}\right\} \log_e x \, dx$ is equal to : [4 marks]

(A) $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$

(B) $\frac{3}{2} - e - \frac{1}{2e^2}$

(C) $\frac{1}{2} - e - \frac{1}{e^2}$

(D) $\frac{3}{2} - \frac{1}{e} - \frac{1}{2x^2}$

Correct Answer: (B) $\frac{3}{2} - e - \frac{1}{2e^2}$

Solution:

Concept:

- For a positive variable base, differentiate through logarithms: if $t = (x/e)^x = e^{x\ln x - x}$, then $dt = t \ln x \, dx$. The factor $\ln x$ in the integrand suggests this substitution.
- The reciprocal expression is $(e/x)^x = 1/t$, whereas $(x/e)^{2x} = t^2$. A definite integral requires transforming both endpoints along with the differential.

Step 1: Transform the integral Write the integral as

$$I = \int_1^e \left(t^2 - \frac{1}{t}\right) \ln x \, dx.$$

Since $\ln x \, dx = dt/t$, this becomes

$$I = \int_{1/e}^{1} \left(t - \frac{1}{t^2}\right) dt.$$

At $x = 1$, $t = 1/e$; at $x = e$, $t = 1$. The substitution is increasing on the interior of this interval, so the order of limits remains unchanged.

Step 2: Evaluate at the transformed limits An antiderivative of $t - t^{-2}$ is $t^2/2 + t^{-1}$; the reciprocal term has a positive sign because its derivative is $-t^{-2}$. Hence

$$\begin{aligned} I &= \left[\frac{t^2}{2} + \frac{1}{t}\right]_{1/e}^{1} \\ &= \left(\frac{1}{2} + 1\right) - \left(\frac{1}{2e^2} + e\right) \\ &= \frac{3}{2} - e - \frac{1}{2e^2}. \end{aligned}$$

Step 3: Check the option and sign Throughout $1 < x < e$, one has $0 < t < 1$, so $t^2 - 1/t < 0$ and $\ln x > 0$. Thus the integral must be negative, consistently with the evaluated result. Option (B) has exactly the constant term, reciprocal-square term and $-e$ term obtained above. Options (A) and (C) have different coefficients; (D) even retains an x, which cannot occur in the value of this definite integral.

Step 4: Conclusion $\boxed{I = \frac{3}{2} - e - \frac{1}{2e^2}}$. Hence, the correct option is (B).

Quick Tip to Solve: For expressions involving a base raised to its own variable, try logarithmic differentiation before integration by parts.

4. If A = $\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$;

then for all $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$**, det (A) lies in the interval :** [4 marks]

(A) $\left(\frac{3}{2}, 3\right]$

(B) $\left(0, \frac{3}{2}\right]$

(C) $\left[\frac{5}{2}, 4\right)$

(D) $\left(1, \frac{5}{2}\right]$

Correct Answer: (A) $\left(\frac{3}{2}, 3\right]$

Solution:

Concept:

- Put $s = \sin\theta$ to simplify the determinant before considering the angle range. Cofactor expansion along the first row uses the alternating signs $+, -, +$.
- The interval $3\pi/4 < \theta < 5\pi/4$ contains π, where sine vanishes. Its endpoints are excluded, so the extreme endpoint values of $\sin^2\theta$ are not attained.

Step 1: Expand the determinant Using the first row,

$$\begin{aligned} \det A &= 1(1+s^2) - s(-s+s) + 1(s^2+1) \\ &= 2 + 2s^2. \end{aligned}$$

In particular, the middle cofactor contributes zero: deleting row 1 and column 2 leaves determinant $(-s)(1) - s(-1) = 0$. This check prevents a spurious linear sine term.

Step 2: Find the exact range On the stated interval, sine runs between $-1/\sqrt{2}$ and $1/\sqrt{2}$, without attaining either endpoint, and takes zero at $\theta = \pi$. Consequently

$$0 \le \sin^2\theta < \frac{1}{2}.$$

Substitution into the determinant expression yields

$$\boxed{2 \le \det A < 3}.$$

The lower endpoint 2 is attained, whereas 3 is only approached as the angle tends to an excluded endpoint.

Step 3: Choose the containing interval The question asks for an interval in which the determinant lies for every allowed angle. The entire range $[2, 3)$ is contained in $(3/2, 3]$, option (A). Option (B) excludes all these values. Option (C) excludes the value 2, while option (D) excludes values greater than $5/2$; such values occur when $\sin^2\theta > 1/4$. Thus only (A) works, even though it is slightly wider than the exact range.

Step 4: Conclusion The exact range is $[2, 3)$, contained in $\boxed{(3/2, 3]}$. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish the exact range from an offered interval containing it; test the interior angle $\theta = \pi$ to check endpoint claims.

5. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then n can be : [4 marks]

(A) 11

(B) 12

(C) 9

(D) 14

Correct Answer: (D) 14

Solution:

Concept:

- Three numbers a, b, c form an arithmetic progression exactly when $2b = a + c$. Here the common factors in adjacent binomial coefficients make their ratios easier to use than their full factorial forms.
- For integer $n \geq 6$, $\binom{n}{4} > 0$, and the adjacent ratios are $\binom{n}{5}/\binom{n}{4} = (n-4)/5$ and $\binom{n}{6}/\binom{n}{4} = (n-4)(n-5)/30$.

Step 1: Apply the arithmetic progression condition The condition is

$$2\binom{n}{5} = \binom{n}{4} + \binom{n}{6}.$$

Divide by the nonzero coefficient $\binom{n}{4}$, giving

$$\frac{2(n-4)}{5} = 1 + \frac{(n-4)(n-5)}{30}.$$

This equation preserves the order of the three coefficients: the middle coefficient, not the largest index, must be the average of its neighbours.

Step 2: Solve for the possible integers Multiplying by 30 produces

$$12(n-4) = 30 + (n-4)(n-5).$$

On expanding and collecting terms,

$$n^2 - 21n + 98 = 0, \qquad (n-7)(n-14) = 0.$$

Thus $n = 7$ and $n = 14$ are possible in general. Both are at least 6, so neither is excluded by the domain of the binomial coefficients.

Step 3: Identify the offered value Only 14 occurs among the four listed choices. As a direct check, for $n = 14$, the coefficients are $1001, 2002, 3003$, whose successive differences are both 1001. The other offered integers 11, 12 and 9 are not roots of the required quadratic, so their coefficients cannot meet the arithmetic progression condition. The additional valid value 7 is not offered and does not create a second correct option.

Step 4: Conclusion Among the choices, $\boxed{n = 14}$ is possible. Hence, the correct option is (D).

Quick Tip to Solve: Divide an arithmetic progression equation for neighbouring binomial coefficients by the first coefficient to cancel the factorials.

6. The number of integral values of m for which the quadratic expression, $(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, $x \in R$, is always positive, is : [4 marks]

(A) 7

(B) 8

(C) 3

(D) 6

Correct Answer: (A) 7

Solution:

Concept:

- A real quadratic $ax^2 + bx + c$ is strictly positive for every real x if and only if $a > 0$ and its discriminant $b^2 - 4ac < 0$. Zero discriminant is excluded because the expression would vanish at its vertex.
- Here $a = 1 + 2m$, $b = -2(1 + 3m)$, and $c = 4(1 + m)$. The parameter must satisfy both the leading-coefficient condition and the discriminant condition before integer values are counted.

Step 1: Impose the leading-coefficient condition For the expression to remain positive as $|x|$ becomes large, its quadratic coefficient must be positive:

$$1 + 2m > 0 \quad \Longrightarrow \quad m > -\frac{1}{2}.$$

If the leading coefficient vanished at $m = -1/2$, the remaining expression would have a nonzero linear term and could not stay positive for all real x.

Step 2: Simplify the discriminant inequality The discriminant is

$$\begin{aligned} \Delta &= 4(1 + 3m)^2 - 16(1 + 2m)(1 + m) \\ &= 4\left[m^2 - 6m - 3\right]. \end{aligned}$$

Therefore

$$m^2 - 6m - 3 < 0 \quad \Longleftrightarrow \quad 3 - 2\sqrt{3} < m < 3 + 2\sqrt{3}.$$

The two endpoints are approximately -0.464 and 6.464. This interval already satisfies $m > -1/2$, so its intersection with the first condition is unchanged.

Step 3: Count the integral parameters The integers strictly between those endpoints are

$$m = 0, 1, 2, 3, 4, 5, 6.$$

There are seven. For a boundary check, $m = 0$ gives discriminant -12, and $m = 6$ also gives -12, with positive leading coefficients in both cases. The next integer 7 has positive discriminant and fails. Thus 8 overcounts, while 3 and 6 omit admissible parameter values.

Step 4: Conclusion The number of integral values is $\boxed{7}$. Hence, the correct option is (A).

Quick Tip to Solve: For strict positivity on the whole real line, impose both an upward opening parabola and a strictly negative discriminant before counting integers.

7. If a straight line passing through the point P(–3, 4) is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is : [4 marks]

(A) x – y + 7 = 0

(B) 4x – 3y + 24 = 0

(C) 4x + 3y = 0

(D) 3x – 4y + 25 = 0

Correct Answer: (B) 4x – 3y + 24 = 0

Solution:

Concept:

- If a line cuts the axes at $A(a,0)$ and $B(0,b)$, the midpoint of the intercepted segment is $(a/2, b/2)$. The signs of the intercepts must be retained.
- The intercept form of a line with nonzero intercepts is $x/a + y/b = 1$. A line merely passing through the given point need not bisect the segment there; the midpoint condition supplies the additional restriction.

Step 1: Recover both intercepts from the midpoint The given point $P(-3,4)$ is the midpoint of the intercepted portion. Equating midpoint coordinates gives

$$\frac{a}{2} = -3, \qquad \frac{b}{2} = 4,$$

hence

$$a = -6, \qquad b = 8.$$

Therefore the line crosses the negative x-axis at $(-6,0)$ and the positive y-axis at $(0,8)$. Their midpoint is indeed $(-3,4)$.

Step 2: Write and simplify the line equation Substitute these signed intercepts in the intercept form:

$$\frac{x}{-6} + \frac{y}{8} = 1.$$

Multiplying by 24 gives

$$-4x + 3y = 24,$$

or equivalently

$$4x - 3y + 24 = 0.$$

At $(-3,4)$, the left side is $-12 - 12 + 24 = 0$, providing an independent incidence check.

Step 3: Exclude the other equations For (A), the intercepts are -7 and 7, with midpoint $(-7/2, 7/2)$, not P. Option (C) passes through the origin and has no nonzero axis-intercept segment with the required midpoint. For (D), the intercepts are $-25/3$ and $25/4$, so the midpoint is $(-25/6, 25/8)$, again different. Thus passing through P, which several choices do, is insufficient; (B) alone has the required intercept midpoint.

Step 4: Conclusion The line is $\boxed{4x - 3y + 24 = 0}$. Hence, the correct option is (B).

Quick Tip to Solve: If the midpoint of an axis-intercept segment is given, double its coordinates to get the signed intercepts immediately.

8. Let S and S' be the foci of an ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right angled triangle with right angle at B and area ($\Delta S'BS$) = 8 sq. units, then the length of a latus rectum of the ellipse is : [4 marks]

(A) 2

(B) $4\sqrt{2}$

(C) 4

(D) $2\sqrt{2}$

Correct Answer: (C) 4

Solution:

Concept:

- For an ellipse with semi-major axis a, semi-minor axis b, and focal distance c, the relation is $a^2 = b^2 + c^2$. Its full latus rectum has length $2b^2/a$.
- Coordinates can be chosen as $S = (c, 0)$, $S\prime = (-c, 0)$, and $B = (0, b)$. Perpendicular vectors have zero scalar product; the triangle area is half its base times its perpendicular height.

Step 1: Use the right angle at the minor-axis endpoint The vectors from B to the foci are $(c, -b)$ and $(-c, -b)$. Since the angle between them is a right angle, their scalar product vanishes:

$$(c, -b) \cdot (-c, -b) = -c^2 + b^2 = 0.$$

Thus $b = c$, since both lengths are positive. The same argument works at the other endpoint of the minor axis; the orientation does not affect the lengths.

Step 2: Find the axes from the stated area The focal segment has length $2c$, and its perpendicular distance from B is b. Hence

$$\text{Area} = \tfrac{1}{2}(2c)b = bc = 8.$$

Combining this with $b = c$ gives $b^2 = c^2 = 8$. Consequently

$$a^2 = b^2 + c^2 = 16, \qquad a = 4.$$

These are semi-axis lengths; using $2a$ in the latus-rectum formula would introduce an erroneous factor of two.

Step 3: Evaluate the full latus rectum Substitute the semi-axis values into the standard full-chord formula:

$$L = \frac{2b^2}{a} = \frac{2 \cdot 8}{4} = 4.$$

The value 2 is only the semi-latus rectum. The values $4\sqrt{2}$ and $2\sqrt{2}$ do not equal the required chord length after the area and right-angle constraints are imposed.

Step 4: Conclusion The length of a latus rectum is 4 units. Hence, the correct option is (C).

Quick Tip to Solve: At a minor-axis endpoint, a right angle between the focal segments immediately gives $b = c$; use the triangle area before calculating a.

9. The set of all values of λ for which the system of linear equations

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$

has a non-trivial solutions : [4 marks]

(A) is an empty set

(B) contains more than two elements

(C) is a singleton

(D) contains exactly two elements

Correct Answer: (C) is a singleton

Solution:

Concept:

- A homogeneous system $M\mathbf{x} = \mathbf{0}$ has a nonzero solution exactly when its coefficient determinant is zero. Here the allowed values of λ are the eigenvalues of the original coefficient matrix.
- One counts distinct parameter values, not the multiplicity with which a root occurs in the determinant polynomial. A repeated root can therefore define a singleton set.

Step 1: Bring every term to the left-hand side The system becomes

$$\begin{aligned}(1-\lambda)x - 2y - 2z &= 0,\\ x + (2-\lambda)y + z &= 0,\\ -x - y - \lambda z &= 0.\end{aligned}$$

Thus

$$M = \begin{pmatrix} 1-\lambda & -2 & -2 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & -\lambda \end{pmatrix}.$$

The determinant condition is necessary because an invertible matrix would force $x = y = z = 0$, contrary to the required non-trivial solution.

Step 2: Expand and simplify the determinant Expanding along the first row, with the alternating cofactor signs, gives

$$\begin{aligned}\det M &= (1-\lambda)(\lambda^2 - 2\lambda + 1)\\ &\quad + 2(1-\lambda) - 2(1-\lambda)\\ &= (1-\lambda)(\lambda-1)^2\\ &= (1-\lambda)^3.\end{aligned}$$

Hence the determinant vanishes only at $\lambda = 1$. Its triple algebraic multiplicity does not give three different real values.

Step 3: Check that the surviving value really works At $\lambda = 1$, the first equation gives $y + z = 0$, and the second then gives $x = 0$. The third equation is consistent with these conditions. Hence every vector $(x, y, z) = (0, t, -t)$, for any nonzero real t, is a non-trivial solution. For example, $(0, 1, -1)$

yields 0, 1, and -1 on the three left sides, equal to λx, λy, and λz, respectively. The set is therefore neither empty nor a set of two or more elements.

Step 4: Conclusion The complete parameter set is $\{1\}$, a singleton. Hence, the correct option is (C).

Quick Tip to Solve: After factoring the determinant, count distinct roots. Confirm one nonzero vector at a repeated root to avoid confusing multiplicity with set size.

10. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the students selected has opted neither for NCC nor for NSS is : [4 marks]

(A) $\frac{1}{3}$
(B) $\frac{1}{6}$
(C) $\frac{2}{3}$
(D) $\frac{5}{6}$

Correct Answer: (B) $\frac{1}{6}$

Solution:

Concept:

- For finite sets A and B, inclusion-exclusion gives $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. The intersection is subtracted once because it was counted twice.
- The event of selecting neither activity is the complement of the union. With uniform random selection, its probability is the favourable count divided by the total class strength.

Step 1: Count the students in at least one activity Let A denote the NCC students and B the NSS students. The data give $n(A) = 40$, $n(B) = 30$, and $n(A \cap B) = 20$. Therefore

$$n(A \cup B) = 40 + 30 - 20 = 50.$$

Adding 40 and 30 without subtracting the common group would produce 70, exceeding the class strength, precisely because the same 20 students would be counted twice.

Step 2: Take the complement within the class There are 60 students altogether. Those outside the union have opted for neither NCC nor NSS:

$$n((A \cup B)^c) = 60 - 50 = 10.$$

As a separate check, NCC only contains $40 - 20 = 20$ students, NSS only contains $30 - 20 = 10$, both contains 20, and neither contains 10. These disjoint counts total $20 + 10 + 20 + 10 = 60$, as required.

Step 3: Divide by the total number of equally likely selections Every one of the 60 students is equally likely to be selected. Consequently

$$P(\text{neither}) = \frac{10}{60} = \frac{1}{6}.$$

The value $5/6$ is the probability of at least one activity, so it is the complementary event. The value $1/3$ corresponds to 20 students and $2/3$ to 40 students; neither is the count outside both groups.

Step 4: Conclusion The required probability is $\boxed{1/6}$. Hence, the correct option is (B).

Quick Tip to Solve: For a neither-of-two event, first find the union using inclusion-exclusion and then subtract it from the whole class.

11. Let z_1 and z_2 be two complex numbers satisfying $|z_1| = 9$ and $|z_2 - 3 - 4i| = 4$. Then the minimum value of $|z_1 - z_2|$ is : [4 marks]

(A) 0

(B) 1

(C) 2

(D) $\sqrt{2}$

Correct Answer: (A) 0

Solution:

Concept:

- The locus $|z - z_0| = r$ is a circle of radius r centred at the complex point z_0. The modulus of the difference of two complex numbers is their Euclidean distance.
- Distance is always nonnegative. To establish that the minimum is zero, it is enough to exhibit one point belonging to both permitted loci.

Step 1: Identify the two circles The condition $|z_1| = 9$ places z_1 on the circle of radius 9 centred at the origin. The condition $|z_2 - (3 + 4i)| = 4$ places z_2 on a circle of radius 4 with centre $3 + 4i$. The distance between the centres is

$$|3 + 4i| = \sqrt{3^2 + 4^2} = 5.$$

Since $9 - 4 = 5$, the smaller circle is internally tangent to the larger circle.

Step 2: Construct their common point explicitly The unit complex number in the direction of the smaller centre is $(3 + 4i)/5$. The point at radius 9 in this direction is

$$w = \frac{9}{5}(3 + 4i) = \frac{27}{5} + \frac{36}{5}i.$$

Its modulus is 9. Furthermore

$$w - (3 + 4i) = \frac{4}{5}(3 + 4i), \qquad |w - (3 + 4i)| = 4.$$

Thus w lies on both circles, and taking $z_1 = z_2 = w$ meets both constraints simultaneously.

Step 3: Compare the attainable distance with the lower bound The exhibited pair gives $|z_1 - z_2| = |w - w| = 0$. No pair can give a negative modulus, so this attained value is the global minimum. In particular, 1, 2 and $\sqrt{2}$ are all positive and cannot be minimal when the two allowed points may coincide. Subtracting the radii without accounting for internal tangency would obscure this common

point.

Step 4: Conclusion The minimum value of $|z_1 - z_2|$ is 0. Hence, the correct option is (A).

Quick Tip to Solve: Before using a circle-distance formula, compare the centre distance with both the sum and the absolute difference of the radii.

12. If the function f given by $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation, $\frac{f(x)-14}{(x-1)^2} = 0\,(x \neq 1)$ is : [4 marks]

(A) -7

(B) 5

(C) 7

(D) 6

Correct Answer: (C) 7

Solution:

Concept:

- For a differentiable function that increases immediately before an interior point and decreases immediately after it, that point is a local maximum and its derivative is zero.
- A rational expression equals zero only where its numerator vanishes and its denominator is nonzero. A factor cancelled algebraically does not restore an excluded point to the domain.

Step 1: Use the change of monotonicity to determine the parameter The polynomial is differentiable everywhere. The given increase on $(0, 1]$ and decrease on $[1, 5)$ make $x = 1$ a local maximum. Differentiate:

$$f\prime(x) = 3x^2 - 6(a-2)x + 3a.$$

Thus

$$f\prime(1) = 3 - 6a + 12 + 3a = 15 - 3a = 0,$$

giving $a = 5$. This uses the sign change at the shared endpoint, rather than treating the endpoints of the two intervals separately.

Step 2: Check the derivative signs and simplify the numerator For $a = 5$,

$$f(x) = x^3 - 9x^2 + 15x + 7, \qquad f\prime(x) = 3(x-1)(x-5).$$

Both factors are negative on $(0, 1)$, so the derivative is positive there. On $(1, 5)$ the factors have opposite signs, so the derivative is negative. The determined parameter therefore satisfies the full given condition. Also

$$f(x) - 14 = x^3 - 9x^2 + 15x - 7 = (x-1)^2(x-7).$$

Step 3: Solve on the stated domain For $x \neq 1$, the equation reduces to

$$\frac{(x-1)^2(x-7)}{(x-1)^2} = x - 7 = 0.$$

Therefore $x = 7$. The value $x = 1$ remains excluded, even though it is a double root of the numerator. The alternatives -7, 5 and 6 give reduced values -14, -2 and -1, respectively, rather than zero. The root need not lie inside the intervals used to determine a.

Step 4: Conclusion A root of the equation is $\boxed{7}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the turning point first to find the parameter, then factor the numerator while retaining the original denominator restrictions.

13. The mean and the variance of five observations are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4 ; then the absolute value of the difference of the other two observations, is : [4 marks]

(A) 1

(B) 7

(C) 3

(D) 5

Correct Answer: (B) 7

Solution:

Concept:

- For n observations of mean $\bar{x}$, their sum is $n\bar{x}$, and the population variance is $\sigma^2 = \frac{1}{n}\sum x_i^2 - \bar{x}^2$.
- If the two unknown observations are u, v, their absolute difference can be obtained directly from $(u-v)^2 = 2(u^2+v^2) - (u+v)^2$, avoiding unnecessary quadratic roots.

Step 1: Find the sum of the missing observations The five observations have mean 4, so their total is

$$\sum x_i = 5 \cdot 4 = 20.$$

The three known observations sum to $3+4+4 = 11$. Writing the remaining two as u and v, we obtain

$$u + v = 20 - 11 = 9.$$

The mean fixes the sum but not the individual values; the variance supplies the additional information needed.

Step 2: Use the variance to find their squared sum Rearrange the variance identity before substituting:

$$\sum x_i^2 = 5(\sigma^2 + \bar{x}^2) = 5(5.20 + 16) = 106.$$

The known squares total $3^2 + 4^2 + 4^2 = 41$. Hence

$$u^2 + v^2 = 106 - 41 = 65.$$

Here 5.20 is the variance itself, not the standard deviation; squaring it again would give an incorrect total.

Step 3: Calculate the absolute difference and verify it The squared difference is

$$(u - v)^2 = 2(65) - 9^2 = 130 - 81 = 49.$$

Consequently $|u - v| = 7$. Indeed the sum and difference give the missing values 8 and 1 in either order. The deviations from the mean are then $-1, 0, 0, 4, -3$, whose squared sum is $1 + 16 + 9 = 26$; dividing by 5 recovers 5.20. The other proposed differences 1, 3 and 5 have squares 1, 9 and 25, inconsistent with the required squared difference 49.

Step 4: Conclusion The absolute value of the difference is $\boxed{7}$. Hence, the correct option is (B).

Quick Tip to Solve: With two missing observations, use the mean for their sum and the variance for their squared sum; then compute the difference directly.

14. Let f be a differentiable function such that f(1) = 2 and f '(x) = f(x) for all x $\in$ R R. If h(x) = f(f(x)), then h'(1) is equal to : [4 marks]

(A) 4e

(B) $2e^2$

(C) $4e^2$

(D) 2e

Correct Answer: (A) 4e

Solution:

Concept:

- The differential equation $f\prime(x) = f(x)$ has solution $f(x) = Ce^x$. The constant is fixed by one given function value.
- For the composition $h(x) = f(f(x))$, the chain rule gives $h\prime(x) = f\prime(f(x))f\prime(x)$. Both the outer derivative at the inner value and the inner derivative are necessary.

Step 1: Determine the function from its initial value Solving $f\prime = f$ gives $f(x) = Ce^x$. One may obtain this without dividing by the unknown function: the derivative of $e^{-x}f(x)$ is $e^{-x}(f\prime - f) = 0$, so that product is constant. Since $f(1) = 2$,

$$Ce = 2, \qquad C = \frac{2}{e}, \qquad f(x) = 2e^{x-1}.$$

This satisfies both the differential equation and the specified value.

Step 2: Apply the chain rule at the required point Differentiate the composition with its inner

argument intact:

$$h\prime(x) = f\prime(f(x))f\prime(x).$$

Since $f\prime(t) = f(t)$ for every real argument t, and $f(1) = 2$, this becomes

$$h\prime(1) = f\prime(2)f\prime(1) = f(2)f(1).$$

The outer derivative is evaluated at 2, not at 1. This distinction is essential in a nested function.

Step 3: Evaluate both factors and compare the options The explicit function gives

$$f(2) = 2e^{2-1} = 2e, \qquad f(1) = 2.$$

Therefore

$$h\prime(1) = (2e)(2) = 4e.$$

The alternative $2e$ omits the inner derivative factor. The alternatives $2e^2$ and $4e^2$ use an incorrect exponential factor: the initial condition changes Ce^2 into $2e$, not $2e^2$. As a direct check, differentiating $h(x) = 2\exp(2e^{x-1} - 1)$ at 1 also gives $4e$.

Step 4: Conclusion The required derivative is $\boxed{4e}$. Hence, the correct option is (A).

Quick Tip to Solve: For $f(f(x))$, write the two chain-rule factors before substituting the initial value; the outer factor uses the argument $f(1) = 2$.

15. Let Z be the set of integers.

If $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and

$B = \{x \in Z : -3 < 2x - 1 < 9\}$,

then the number of subsets of the set $A \times B$, is [4 marks]

(A) 2^{12}

(B) 2^{18}

(C) 2^{10}

(D) 2^{15}

Correct Answer: (D) 2^{15}

Solution:

Concept:

- For a real exponent, $2^t = 1$ holds exactly when $t = 0$. Thus the exponential condition becomes a polynomial equation.
- A finite set with n distinct elements has 2^n subsets, including the empty set and the entire set. Cartesian-product cardinalities multiply: $|A \times B| = |A||B|$.

Step 1: Find the integers in A The base is 2, so its exponent must vanish. Factor the quadratic to obtain

$$(x+2)(x^2 - 5x + 6) = (x+2)(x-2)(x-3) = 0.$$

All three roots are integers and are distinct. Therefore

$$A = \{-2, 2, 3\}, \qquad |A| = 3.$$

The exponent is a product: setting only the quadratic equal to zero would incorrectly lose the root -2.

Step 2: Count the product and its subsets Solve both strict inequalities defining B:

$$-3 < 2x - 1 < 9 \implies -2 < 2x < 10 \implies -1 < x < 5.$$

Consequently $B = \{0, 1, 2, 3, 4\}$, so $|B| = 5$. The endpoints -1 and 5 are excluded because the inequalities are strict. Each of the three elements of A pairs with each of the five elements of B, giving

$$|A \times B| = 3 \cdot 5 = 15.$$

A subset independently includes or excludes each of these 15 ordered pairs, so there are 2^{15} subsets. The exponents 12, 18 and 10 in the other options would require incorrect product cardinalities; in particular, 10 arises from omitting one root of the exponent equation.

Step 3: Conclusion The number of subsets is $\boxed{2^{15}}$. Hence, the correct option is (D).

Quick Tip to Solve: First count the roots and the allowed integers separately; multiply those counts before raising 2 to the result.

16. The total number of irrational terms in the binomial expansion of $(7^{1/5} - 3^{1/10})^{60}$ is : [4 marks]

(A) 54

(B) 55

(C) 49

(D) 48

Correct Answer: (A) 54

Solution:

Concept:

- The expansion of $(a + b)^n$ contains $n + 1$ indexed terms, with general term $T_{r+1} = \binom{n}{r} a^{n-r} b^r$, where $0 \le r \le n$.
- In a product of rational powers of distinct primes, rationality requires each prime exponent to be an integer. Prime-factor uniqueness prevents a fractional power of 7 from cancelling a fractional power of 3.

Step 1: Express the general term For the given difference, use the negative sign as part of the second base:

$$T_{r+1} = (-1)^r \binom{60}{r} 7^{(60-r)/5} 3^{r/10}, \qquad r = 0, 1, \ldots, 60.$$

The sign and binomial coefficient are nonzero integers, so neither affects whether the term is rational. Only the two prime exponents need to be tested. There are 61 total terms, not 60, because the index

includes both endpoints.

Step 2: Count rational terms and subtract Rationality demands simultaneously

$$\frac{60-r}{5} \in \mathbb{Z}, \qquad \frac{r}{10} \in \mathbb{Z}.$$

The second condition says that r is a multiple of 10; every such r automatically satisfies the first condition as well. The allowed indices are

$$r = 0, 10, 20, 30, 40, 50, 60,$$

which give seven rational terms. Thus the irrational terms number

$$61 - 7 = 54.$$

Counting only one endpoint would produce 55 incorrectly. Requiring only divisibility by 5 would wrongly count terms in which the power of 3 is still a half-integer; the alternatives 49 and 48 do not meet both prime-exponent conditions.

Step 3: Conclusion There are $\boxed{54}$ irrational terms. Hence, the correct option is (A).

Quick Tip to Solve: For distinct prime radicals, impose integrality on each prime exponent and remember that the index runs from 0 through n.

17. The integral $\int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}\,dx$ is equal to : (where C is a constant of integration) [4 marks]

(A) $\frac{x^{12}}{6(2x^4+3x^2+1)^3} + C$

(B) $\frac{x^4}{6(2x^4+3x^2+1)^3} + C$

(C) $\frac{x^{12}}{(2x^4+3x^2+1)^3} + C$

(D) $\frac{x^4}{(2x^4+3x^2+1)^3} + C$

Correct Answer: (A) $\frac{x^{12}}{6(2x^4+3x^2+1)^3} + C$

Solution:

Concept:

- When the proposed answers have a common polynomial denominator, reverse differentiation can identify the antiderivative exactly without a long partial-fraction calculation.
- For $D = 2x^4 + 3x^2 + 1$, use the product and chain rules on $x^{12}D^{-3}$. An antiderivative is verified when its derivative equals the entire printed integrand.

Step 1: Differentiate the first proposed expression Write $D = 2x^4 + 3x^2 + 1$, so $D' = 8x^3 + 6x$. For

option (A), let $F = x^{12}/(6D^3)$. Then

$$F' = \frac{12x^{11}D - 3x^{12}D'}{6D^4} = \frac{x^{11}(12D - 3xD')}{6D^4}.$$

The cancellation of the highest powers in this numerator is the reason the proposed form is useful.

Step 2: Simplify and exclude the remaining answers Substitution gives

$$12D - 3xD' = 24x^4 + 36x^2 + 12 - 24x^4 - 18x^2 = 18x^2 + 12.$$

Therefore

$$F' = \frac{x^{11}(18x^2 + 12)}{6D^4} = \frac{3x^{13} + 2x^{11}}{D^4},$$

exactly the required integrand. Option (C) is six times F and hence has six times the required derivative. Options (B) and (D) have numerator x^4, whose differentiation starts with a nonzero x^3 term near zero, whereas the required integrand starts with $2x^{11}$. They therefore cannot differ from F merely by a constant. Finally, $D = (x^2 + 1)(2x^2 + 1) > 0$ for real x, so this verification applies on the whole real line.

Step 3: Conclusion The integral is $\boxed{\frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C}$. Hence, the correct option is (A).

Quick Tip to Solve: In an integration MCQ, differentiate the most plausible answer; collect the numerator before expanding the denominator.

18. In a game, a man wins Rs. 100 if he gets 5 or 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is : [4 marks]

(A) $\frac{400}{3}$ loss

(B) 0

(C) $\frac{400}{9}$ loss

(D) $\frac{400}{3}$ gain

Correct Answer: (B) 0

Solution:

Concept:

- Expectation is the probability-weighted sum of the net gain over mutually exclusive terminal outcomes. Net gain includes losses on throws made before the winning throw.
- Here a success has probability $p = 2/6 = 1/3$ and a failure has probability $q = 4/6 = 2/3$. The game stops at the first success or after the third throw.

Step 1: List the terminal outcomes A success on the first throw ends the game with net gain Rs 100 and probability $1/3$. A failure followed by a success gives net gain $-50+100=50$ and probability $(2/3)(1/3)=2/9$. Two failures followed by a success give net gain $-50-50+100=0$ and probability $(2/3)^2(1/3)=4/27$. Three failures give net gain Rs -150 and probability $(2/3)^3=8/27$. These four events exhaust the possibilities: their probabilities sum to

$$\frac{9}{27}+\frac{6}{27}+\frac{4}{27}+\frac{8}{27}=1.$$

Step 2: Calculate and check the expectation Let X be the total monetary gain, with losses represented negatively. Then

$$E(X)=100\left(\frac{1}{3}\right)+50\left(\frac{2}{9}\right)+0\left(\frac{4}{27}\right)-150\left(\frac{8}{27}\right)$$
$$=\frac{900+300-1200}{27}=0.$$

As a check, each throw that is actually reached has conditional expected gain $100/3-50(2/3)=0$. The bounded stopping rule changes how many throws occur, but not that zero expected contribution. The two loss options and the positive-gain option therefore all fail the direct weighted-sum calculation; counting only the final throw would omit the previous losses.

Step 3: Conclusion The expected net gain is $\boxed{\text{Rs } 0}$, so there is neither expected gain nor expected loss. Hence, the correct option is (B).

Quick Tip to Solve: Build terminal sequences first and include every earlier loss in each sequence before calculating its net payoff.

19. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to : [4 marks]

(A) $-\sqrt{2}$

(B) 0

(C) -1

(D) $\sqrt{2}$

Correct Answer: (A) $-\sqrt{2}$

Solution:

Concept:

- A sum of squares of real quantities is zero only when each square is zero. This gives exact equality conditions instead of a numerical estimate.
- The identity $\cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\sin\alpha\sin\beta$ reduces the required expression to a product. The interval restrictions determine the signs.

Step 1: Convert the equation to a sum of squares Set $u=\sin\alpha$ and $v=\sqrt{2}\cos\beta$. The given equation becomes

$$u^4+v^4+2=4uv.$$

Rearrange it using an exact identity:

$$\begin{aligned} 0 &= u^4 + v^4 + 2 - 4uv \\ &= (u^2 - v^2)^2 + 2(uv - 1)^2. \end{aligned}$$

Both terms on the last line are nonnegative real squares. They must therefore vanish separately, yielding $u^2 = v^2$ and $uv = 1$.

Step 2: Apply the interval restrictions and evaluate Since $\alpha \in [0, \pi]$, we have $u = \sin\alpha \geq 0$. The equation $uv = 1$ forces u and v to be positive. Together with $u^2 = v^2$, it gives $u = v = 1$. Thus

$$\sin\alpha = 1, \quad \alpha = \frac{\pi}{2}, \qquad \cos\beta = \frac{1}{\sqrt{2}}, \quad \beta = \frac{\pi}{4}.$$

The beta value is unique in its specified interval, and hence $\sin\beta = 1/\sqrt{2}$. Now

$$\cos(\alpha + \beta) - \cos(\alpha - \beta) = -2(1)\left(\frac{1}{\sqrt{2}}\right) = -\sqrt{2}.$$

The positive option has the wrong sign because both sine factors are positive. Zero would require one sine to vanish, and -1 has the wrong magnitude; neither agrees with the equality conditions.

Step 3: Conclusion The required value is $\boxed{-\sqrt{2}}$. Hence, the correct option is (A).

Quick Tip to Solve: When quartic terms balance a product, try a sum-of-squares identity and retain all equality conditions, including their signs.

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20. $\lim\limits_{x\to 1^-} \frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ **is equal to :** [4 marks]

(A) $\sqrt{\frac{2}{\pi}}$

(B) $\frac{1}{\sqrt{2\pi}}$

(C) $\sqrt{\frac{\pi}{2}}$

(D) $\sqrt{\pi}$

Correct Answer: (A) $\sqrt{\frac{2}{\pi}}$

Solution:

Concept:

- Rationalising a difference of square roots replaces it by the difference of their radicands divided by the sum of the roots.
- For $x \to 1^-$, let $t = \arccos x$, so $t \to 0^+$, $\arcsin x = \pi/2 - t$, and $1 - \cos t = 2\sin^2(t/2)$. Use $\sin u/u \to 1$.

Step 1: Rationalise the numerator Denote the required limit by L. Multiplying numerator and

denominator by the conjugate gives

$$\frac{\sqrt{\pi}-\sqrt{2\arcsin x}}{\sqrt{1-x}}=\frac{\pi-2\arcsin x}{\sqrt{1-x}\left(\sqrt{\pi}+\sqrt{2\arcsin x}\right)}.$$

The sum of square roots tends to $2\sqrt{\pi}$. The remaining zero-over-zero ratio requires care because the derivative of arcsine is unbounded at x=1; replacing it by a finite derivative would be invalid.

Step 2: Use the small positive angle Set $x=\cos t$ with $t\to 0^{+}$. Then the numerator above is $2t$, and

$$\sqrt{1-x}=\sqrt{1-\cos t}=\sqrt{2}\sin(t/2).$$

The positive sign is valid because t approaches zero through positive values. Therefore

$$\frac{\pi-2\arcsin x}{\sqrt{1-x}}=\frac{2t}{\sqrt{2}\sin(t/2)}\longrightarrow 2\sqrt{2}.$$

Combining the two factors yields

$$L=\frac{2\sqrt{2}}{2\sqrt{\pi}}=\sqrt{\frac{2}{\pi}}.$$

Option (B) is half this value, resulting from losing the factor 2 in $\pi-2\arcsin x$. Options (C) and (D) have the wrong dependence on pi after the conjugate denominator is evaluated.

Step 3: Conclusion The limit is $\boxed{\sqrt{2/\pi}}$. Hence, the correct option is (A).

Quick Tip to Solve: Near an endpoint of arcsine, substitute x=cos t and use the half-angle identity to handle the square-root behaviour exactly.

21. If a circle of radius R passes through the origin O and intersects the coordinates axes at A and B, then the locus of the foot of perpendicular from O on AB is : [4 marks]

(A) $(x^2 + y^2)^2 = 4R^2x^2y^2$

(B) $(x^2 + y^2)(x + y) = R^2xy$

(C) $(x^2 + y^2)^2 = 4Rx^2y^2$

(D) $(x^2 + y^2)^3 = 4R^2x^2y^2$

Correct Answer: (D) $(x^2 + y^2)^3 = 4R^2x^2y^2$

Solution:

Concept:

- A circle passing through the origin can be written $X^2+Y^2-aX-bY=0$. Its centre is $(a/2, b/2)$, so its radius satisfies $a^2+b^2=4R^2$.
- The intercepts are $A=(a,0)$, $B=(0,b)$. The line joining them is $bX+aY=ab$, and a perpendicular from the origin has direction equal to the normal vector (b,a).

Step 1: Relate the circle to its intercept line Use capital coordinates for a general point of the circle

and lowercase (x, y) for the moving perpendicular foot. The radius condition is

$$a^2 + b^2 = 4R^2.$$

For nonzero intercepts, AB has equation $X/a + Y/b = 1$. Its normal is proportional to $(1/a, 1/b)$. Since the foot lies on a normal through the origin, there is a real scalar k such that

$$x = \frac{k}{a}, \qquad y = \frac{k}{b}.$$

Substitution in the intercept-line equation gives $k(1/a^2 + 1/b^2) = 1$.

Step 2: Eliminate both intercept parameters Let $s = x^2 + y^2$. From the preceding relations,

$$s = k^2\left(\frac{1}{a^2} + \frac{1}{b^2}\right) = k.$$

Consequently $a = s/x$ and $b = s/y$. Substitute in the radius condition:

$$\frac{s^2}{x^2} + \frac{s^2}{y^2} = 4R^2 \quad \Longrightarrow \quad s^3 = 4R^2x^2y^2.$$

This is the algebraic locus, with the origin included as the limiting position when an intercept degenerates. Every nonzero point satisfying it reconstructs intercepts with $a^2 + b^2 = 4R^2$, so the elimination is reversible there. Option (A) has the wrong exponent of s; (B) is not the elimination result; (C) also replaces the required squared radius by R. Dimensional consistency independently excludes those alternatives.

Step 3: Conclusion The locus is $\boxed{(x^2 + y^2)^3 = 4R^2x^2y^2}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the circle intercepts as parameters, then exploit the normal direction of AB to eliminate them in terms of the foot coordinates.

22. The tangent to the curve y = x^2 – 5x + 5, parallel to the line 2y = 4x + 1, also passes through the point : [4 marks]

(A) $\{\frac{1}{4}, \frac{7}{2}\}$
(B) $\left(-\frac{1}{8}, 7\right)$
(C) $\left(\frac{7}{2}, \frac{1}{4}\right)$
(D) $\left(\frac{1}{8}, -7\right)$

Correct Answer: (D) $\left(\frac{1}{8}, -7\right)$

Solution:

Concept:

- The derivative gives the tangent slope: for $y = f(x)$, the slope at $x = a$ is $f\prime(a)$. Parallel lines have equal slopes.
- The tangent at $(a, f(a))$ has equation $y - f(a) = f\prime(a)(x - a)$. A proposed point belongs to it only

if its coordinates satisfy this equation.

Step 1: Identify the required slope The reference line is $2y = 4x + 1$, or $y = 2x + \frac{1}{2}$, so its slope is 2. Differentiate the curve and equate its slope to 2:

$$\frac{dy}{dx} = 2x - 5, \qquad 2a - 5 = 2 \quad \Rightarrow \quad a = \frac{7}{2}.$$

The constant intercept of the reference line is irrelevant to parallelism; it is the slope that must agree.

Step 2: Find the tangent equation At the point of contact,

$$f\left(\frac{7}{2}\right) = \frac{49}{4} - \frac{35}{2} + 5 = -\frac{1}{4}.$$

Thus the tangent is

$$y + \frac{1}{4} = 2\left(x - \frac{7}{2}\right), \qquad y = 2x - \frac{29}{4}.$$

Notice that the contact point is $(7/2, -1/4)$, so the positive ordinate in option (C) cannot describe it.

Step 3: Test the listed points For option (D), substituting $x = 1/8$ gives

$$y = \frac{1}{4} - \frac{29}{4} = -7,$$

exactly the listed ordinate. At $x = 1/4$, the tangent instead gives $-27/4$, excluding (A). At $x = -1/8$, it gives $-15/2$, excluding (B). At $x = 7/2$, it gives $-1/4$, excluding (C).

Step 4: Conclusion The tangent passes through $\boxed{(1/8, -7)}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the tangent equation before testing options; a point with the correct contact abscissa can still have the wrong ordinate.

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23. Let S be the set of all real values of λ such that a plane passing through the points ($-\lambda^2$, 1, 1), (1, $-\lambda^2$, 1) and (1, 1, $-\lambda^2$) also passes through the point (–1, –1, 1). Then S is equal to : [4 marks]

(A) $\{1, -1\}$

(B) $\{3, -3\}$

(C) $\{\sqrt{3}\}$

(D) $\{\sqrt{3}, -\sqrt{3}\}$

Correct Answer: (D) $\{\sqrt{3}, -\sqrt{3}\}$

Solution:

Concept:

- Three non-collinear points determine a unique plane. A plane has equation $Ax + By + Cz = D$, with a nonzero normal vector (A, B, C).
- Here the three points are obtained by permuting the coordinates. Their coordinate sums are equal, suggesting a plane with normal vector $(1, 1, 1)$.

Step 1: Find the plane through the three points Write $t = \lambda^2$, so $t \geq 0$. Each point $(-t, 1, 1)$, $(1, -t, 1)$, and $(1, 1, -t)$ satisfies

$$x + y + z = 2 - t.$$

To verify uniqueness, subtract the first point from the other two. The resulting vectors are $(1+t, -1-t, 0)$ and $(1+t, 0, -1-t)$. Since $1 + t > 0$, these vectors are not parallel, so the three points are non-collinear. Therefore this is their plane, rather than merely one possible equation containing them.

Step 2: Impose the fourth-point condition The additional point is $(-1, -1, 1)$; its coordinate sum is -1. Substitute it into the plane equation:

$$-1 = 2 - \lambda^2 \quad \Rightarrow \quad \lambda^2 = 3.$$

Both real square roots must be retained, because the original coordinates depend only on the square of λ:

$$\lambda = \sqrt{3} \quad \text{or} \quad \lambda = -\sqrt{3}.$$

For either value the plane is $x + y + z = -1$, and all four listed points lie on it.

Step 3: Compare the sets offered Option (A) would give $\lambda^2 = 1$, so its plane would have coordinate sum 1, not -1. Option (B) gives $\lambda^2 = 9$, also failing the fourth-point test. Option (C) includes only the positive root and omits another valid real value. Option (D) contains exactly both roots, with no extra value.

Step 4: Conclusion $\boxed{S = \{\sqrt{3}, -\sqrt{3}\}}$. Hence, the correct option is (D).

Quick Tip to Solve: For points formed by permuting one exceptional coordinate, check their common coordinate sum before expanding a plane determinant.

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24. $\lim\limits_{x\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots..+\frac{1}{5n}\right)$ **is equal to :** [4 marks]

(A) $\tan^{-1}(2)$

(B) $\tan^{-1}(3)$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

Correct Answer: (A) $\tan^{-1}(2)$

Solution:

Concept:

- A sum of the form $\frac{1}{n}\sum_{r=1}^{mn} f(r/n)$ tends to $\int_0^m f(t)\,dt$ as the positive integer n increases without bound, provided f is continuous.
- The antiderivative of $1/(1+t^2)$ is $\tan^{-1} t$. Determine the upper endpoint from the final summand, rather than assuming that there are only n terms.

Step 1: Identify the endpoint and limiting parameter The displayed expression prints $x \to \infty$, although the summands and number of terms depend on n. The sequence limit intended by this

notation is $n \to \infty$. The final term determines the last integer index r:

$$\frac{n}{n^2+r^2} = \frac{1}{5n} \quad \Rightarrow \quad r^2 = 4n^2 \quad \Rightarrow \quad r = 2n.$$

Thus the terms run from $r = 1$ to $r = 2n$, with no change to their printed values.

Step 2: Convert the sum to an integral Divide the numerator and denominator of a general term by n^2:

$$\frac{n}{n^2+r^2} = \frac{1}{n}\frac{1}{1+(r/n)^2}.$$

The width of each interval is $1/n$, the right endpoints are r/n, and the last endpoint is 2. Since $1/(1+t^2)$ is continuous throughout $[0, 2]$, the Riemann-sum limit is

$$\int_0^2 \frac{dt}{1+t^2} = \left[\tan^{-1} t\right]_0^2 = \tan^{-1} 2.$$

Step 3: Check the answer choices Option (B) would require the final index to be $3n$, whose final term would be $1/(10n)$. Option (C) corresponds to an upper endpoint of 1, which would end with $1/(2n)$. Option (D) corresponds to integration to infinity. None matches the explicitly printed final term $1/(5n)$.

Step 4: Conclusion Interpreting the printed limiting variable as the sequence parameter n, the value is $\boxed{\tan^{-1} 2}$. Hence, the correct option is (A).

Quick Tip to Solve: The last summand fixes the integration limit: solve for its index before writing the Riemann integral.

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25. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$, then $|\alpha - \beta|$ is equal to : [4 marks]

(A) 90°

(B) 30°

(C) 45°

(D) 60°

Correct Answer: (B) 30°

Solution:

Concept:

- The vector triple-product identity is $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$. The order and the minus sign are essential.
- For unit vectors, $\vec{a} \cdot \vec{b} = \cos\alpha$ and $\vec{a} \cdot \vec{c} = \cos\beta$. Two non-parallel nonzero vectors are linearly independent, so their coefficients in a zero linear combination must vanish.

Step 1: Expand and rearrange the vector equation Apply the triple-product identity to the given

equation. Because all three vectors have unit magnitude, it becomes

$$\vec{b}\cos\beta - \vec{c}\cos\alpha = \frac{1}{2}\vec{b}.$$

Bring both terms involving $\vec{b}$ to the same side:

$$\left(\cos\beta - \frac{1}{2}\right)\vec{b} - \cos\alpha\,\vec{c} = \vec{0}.$$

This is a linear combination of the two non-parallel vectors $\vec{b}$ and $\vec{c}$, so neither coefficient can remain nonzero.

Step 2: Find both angles Equating the independent coefficients to zero gives

$$\cos\beta - \frac{1}{2} = 0, \qquad \cos\alpha = 0.$$

Angles between vectors are chosen in $[0, \pi]$, so these equations have the unique angle values

$$\beta = 60^\circ, \qquad \alpha = 90^\circ.$$

There is no additional negative or reflex-angle branch when defining the angle between two vectors.

Step 3: Take the absolute difference Therefore

$$|\alpha - \beta| = |90^\circ - 60^\circ| = 30^\circ.$$

Option (A) gives α alone rather than the difference, and option (D) gives β alone. Option (C) is inconsistent with the independently forced cosine values. Substituting $\cos\beta = 1/2$ and $\cos\alpha = 0$ into the expanded equation also recovers exactly $\vec{b}/2$.

Step 4: Conclusion $\boxed{|\alpha - \beta| = 30^\circ}$. Hence, the correct option is (B).

Quick Tip to Solve: After expanding a vector triple product, use non-parallelism to compare coefficients; taking magnitudes too early loses information.

26. If an angle between the line, $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$ and the plane, $x - 2y - kz = 3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is : [4 marks]

(A) $\sqrt{\frac{3}{5}}$

(B) $-\frac{5}{2}$

(C) $-\frac{3}{2}$

(D) $\sqrt{\frac{5}{3}}$

Correct Answer: (D) $\sqrt{\frac{5}{3}}$

Solution:

Concept:

- For a line direction $\vec{d}$, a plane normal $\vec{n}$, and the acute line–plane angle θ, $\sin\theta = |\vec{d}\cdot\vec{n}|/(|\vec{d}|\,|\vec{n}|)$.
- The line–plane angle is complementary to the angle between the line and the plane normal. If $\cos\theta$ is supplied, first obtain $\sin\theta$, using its nonnegative value for an acute angle.

Step 1: Extract the direction and normal vectors The denominators in the symmetric line equation give

$$\vec{d} = (2, 1, -2), \qquad |\vec{d}| = \sqrt{4+1+4} = 3.$$

The plane equation $x - 2y - kz = 3$ gives

$$\vec{n} = (1, -2, -k), \qquad |\vec{n}| = \sqrt{5+k^2}.$$

Their scalar product is $2 - 2 + 2k = 2k$. The line point and the constant on the right side of the plane do not affect this angle.

Step 2: Use the given angle correctly The stated angle has $\cos\theta = 2\sqrt{2}/3$. Consequently,

$$\sin\theta = \sqrt{1 - \frac{8}{9}} = \frac{1}{3}.$$

Apply the line–plane formula:

$$\frac{2|k|}{3\sqrt{5+k^2}} = \frac{1}{3} \quad \Rightarrow \quad 2|k| = \sqrt{5+k^2}.$$

Both sides are nonnegative, so squaring introduces no sign inconsistency:

$$4k^2 = 5 + k^2 \quad \Rightarrow \quad k^2 = \frac{5}{3}.$$

Step 3: Select the listed permitted value The two mathematical values are $k = \pm\sqrt{5/3}$, but only the positive value appears among the options. Option (A) has square $3/5$, while options (B) and (C) have squares $25/4$ and $9/4$; none satisfies $3k^2 = 5$. The question asks for a value, so one valid listed root is sufficient.

Step 4: Conclusion A listed value is $\boxed{k = \sqrt{5/3}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a line and a plane use sine of their angle. Cosine applies directly to the line and the plane normal.

27. If the sum of the first 15 terms of the series $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + ...$ is equal to 225 k, then k is equal to : [4 marks]

(A) 9

(B) 108

(C) 27

(D) 54

Correct Answer: (C) 27

Solution:

Concept:

- The quantities being cubed form an arithmetic progression: $3/4, 6/4, 9/4, 12/4, \ldots$. Thus the r-th term of the given series is $(3r/4)^3$.
- The sum of the first n positive integer cubes is $\sum_{r=1}^{n} r^3 = [n(n+1)/2]^2$. A constant factor can be taken outside a finite sum.

Step 1: Express every term using an integer index Convert the mixed numbers before recognizing the pattern: $1\frac{1}{2} = 6/4$, $2\frac{1}{4} = 9/4$, and $3\frac{3}{4} = 15/4$. The bases increase by $3/4$ each time, so the first 15 terms give

$$S_{15} = \sum_{r=1}^{15} \left(\frac{3r}{4}\right)^3 = \frac{27}{64} \sum_{r=1}^{15} r^3.$$

It is the bases that form an arithmetic progression; the cubes themselves do not have a constant difference.

Step 2: Evaluate the cube sum Use the exact cube-sum identity:

$$\sum_{r=1}^{15} r^3 = \left(\frac{15 \cdot 16}{2}\right)^2 = 120^2 = 14400.$$

Therefore

$$S_{15} = \frac{27}{64} \cdot 14400 = 27 \cdot 225 = 6075.$$

The cancellation $14400/64 = 225$ makes comparison with the given expression immediate and avoids unnecessary decimal arithmetic.

Step 3: Compare with the prescribed multiple The question states $S_{15} = 225k$. Hence

$$225k = 225 \cdot 27 \quad \Rightarrow \quad k = 27.$$

Option (A), 9, loses a factor of 3 when cubing the scale factor. Options (B) and (D) would give sums four times and twice the computed sum, respectively. Directly multiplying $225 \cdot 27$ confirms the required total, so no further scaling is needed.

Step 4: Conclusion $\boxed{k = 27}$. Hence, the correct option is (C).

Quick Tip to Solve: Pull the cube of the common scale factor outside the sum, then use the square of the triangular-number formula.

28. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30º and the angle of depression of reflection of the cloud in the lake from P be 60º, then the height of the cloud (in meters) from the surface of the lake is : [4 marks]

(A) 45

(B) 42

(C) 50

(D) 60

Correct Answer: (C) 50

Solution:

Concept:

- The tangent of an angle of elevation or depression equals vertical separation divided by horizontal distance. The horizontal distance to a cloud and to its lake reflection is the same.
- A reflection in a horizontal water surface lies as far below the surface as the object lies above it. If the cloud height is h, its vertical separations from a point 25 m above the lake are $h - 25$ and $h + 25$.

Step 1: Write the elevation equation Let h metres be the cloud height above the surface, and let $d > 0$ metres be its horizontal distance from the vertical through P. Since the cloud is seen at an elevation of 30°, it lies above P, so $h > 25$. The first right triangle gives

$$\frac{h-25}{d} = \tan 30^\circ = \frac{1}{\sqrt{3}}, \qquad d = \sqrt{3}(h-25).$$

The height used here is relative to the observer, not to the lake.

Step 2: Use the reflected image The reflected image lies h metres below the water. Its vertical distance below P is therefore $h + 25$, and the depression angle is 60°:

$$\frac{h+25}{d} = \tan 60^\circ = \sqrt{3}.$$

Substitute the expression for the same horizontal distance:

$$h + 25 = 3(h-25) \quad \Rightarrow \quad 100 = 2h \quad \Rightarrow \quad h = 50.$$

Step 3: Check the height and reject the alternatives At $h = 50$, the horizontal distance is $25\sqrt{3}$. Then the elevation ratio is $25/(25\sqrt{3}) = 1/\sqrt{3}$ and the depression ratio is $75/(25\sqrt{3}) = \sqrt{3}$, as required. Options (A), (B), and (D) do not satisfy the necessary equation $h + 25 = 3(h-25)$, so they cannot reproduce both angles simultaneously.

Step 4: Conclusion The cloud is $\boxed{50\ \text{m}}$ above the lake surface. Hence, the correct option is (C).

Quick Tip to Solve: Use h minus observer height for the cloud and h plus observer height for its reflection; both triangles share one horizontal distance.

29. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive directions of x-axis, is : [4 marks]

(A) $x = y\cot\theta - 2\tan\theta$

(B) $y = x\tan\theta + 2\cot\theta$

(C) $x = y\cot\theta + 2\tan\theta$

(D) $y = x\tan\theta - 2\cot\theta$

Correct Answer: (C) $x = y\cot\theta + 2\tan\theta$

Solution:

Concept:

- The parabola $x^2 = 4ay$ has its axis along the positive y-axis. Here $4a = 8$, so $a = 2$.
- A nonvertical line inclined at angle θ to the positive x-axis has slope $m = \tan\theta$. Tangency means that substitution into the parabola gives a repeated intersection root.

Step 1: Write a line with the required slope Let the tangent be $y = mx + c$. Substitute this line into $x^2 = 8y$, keeping the sign of the intercept explicit. We obtain

$$x^2 - 8mx - 8c = 0.$$

This quadratic describes the abscissae of intersections. For a tangent the two intersections coincide, so its discriminant must vanish.

Step 2: Apply the tangency condition The repeated-root condition gives

$$(-8m)^2 - 4(1)(-8c) = 64m^2 + 32c = 0.$$

Thus $c = -2m^2$, and the tangent is

$$y = mx - 2m^2.$$

For nonzero $m = \tan\theta$, divide through by m and rearrange:

$$x = y\cot\theta + 2\tan\theta.$$

Option A has the wrong sign for the final term. Options B and D have intercepts proportional to $\cot\theta$, whereas tangency requires an intercept $-2\tan^2\theta$. The vertex tangent at zero slope is obtained as a limit of the slope form, $y = 0$; the cotangent form presumes the cotangent is defined.

Step 3: Conclusion The required tangent is $\boxed{x = y\cot\theta + 2\tan\theta}$. Hence, the correct option is (C).

Quick Tip to Solve: For $x^2 = 4ay$, use $y = mx - am^2$; the familiar $y = mx + a/m$ belongs to the differently oriented parabola $y^2 = 4ax$.

30. The expression $\sim (\sim p \rightarrow q)$ is logically equivalent to : [4 marks]

(A) $p \wedge q$

(B) $\sim p \wedge \sim q$

(C) $p \wedge \sim q$

(D) $\sim p \wedge q$

Correct Answer: (B) $\sim p \wedge \sim q$

Solution:

Concept:

- An implication $A \rightarrow B$ is false only when A is true and B is false. Therefore $\neg(A \rightarrow B) \equiv A \wedge \neg B$.
- De Morgan's law is $\neg(A \vee B) \equiv (\neg A) \wedge (\neg B)$, and double negation gives $\neg\neg p \equiv p$.

Step 1: Remove the implication The antecedent of the implication is $\neg p$, not p. Apply $A \rightarrow B \equiv \neg A \vee B$ with $A = \neg p$ and $B = q$:

$$(\neg p \rightarrow q) \equiv \neg(\neg p) \vee q \equiv p \vee q.$$

The outer negation in the question still acts on this whole expression. It must not be applied to only one of its components.

Step 2: Negate the disjunction and test the alternatives De Morgan's law now yields

$$\neg(\neg p \rightarrow q) \equiv \neg(p \vee q) \equiv (\neg p) \wedge (\neg q).$$

As a direct check, the original implication is false precisely when $\neg p$ is true and q is false, which requires both p and q false. Its negation is then true. Option A is true when both propositions are true; option C is true for p true and q false; option D is true for p false and q true. Each of those assignments makes the required expression false, ruling them out. Only option B is true exactly on the necessary false-false assignment.

Step 3: Conclusion The equivalent expression is $\boxed{\neg p \wedge \neg q}$. Hence, the correct option is (B).

Quick Tip to Solve: Negate an implication by keeping its entire antecedent and negating its consequent: $\neg(A \rightarrow B) = A \wedge \neg B$.

Physics (120 marks)

31. A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is ℓ_1, and that below the piston is ℓ_2, such that $\ell_1 > \ell_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass, m, will be given by :

(R is universal gas constant and g is the acceleration due to gravity) [4 marks]

(A) $\frac{nRT}{g}\left[\frac{\ell_1-\ell_2}{\ell_1\ell_2}\right]$

(B) $\frac{RT}{g}\left[\frac{2\ell_1+\ell_2}{\ell_1\ell_2}\right]$

(C) $\frac{nRT}{g}\left[\frac{1}{\ell_2}+\frac{1}{\ell_1}\right]$

(D) $\frac{RT}{ng}\left[\frac{\ell_1-3\ell_2}{\ell_1\ell_2}\right]$

Correct Answer: (A) $\frac{nRT}{g}\left[\frac{\ell_1-\ell_2}{\ell_1\ell_2}\right]$

Solution:

Concept:

- For n moles of an ideal gas, $PV = nRT$. With cylinder cross-sectional area A, a gas column of length ℓ has volume $A\ell$.
- The stationary frictionless piston has zero net vertical force. Gas below pushes upward, while gas above and the piston's weight act downward.

Step 1: Calculate the two pressures Let P_1 denote pressure in the upper part and P_2 that in the lower part. The two amounts of gas and their temperatures are equal, but their volumes differ:

$$P_1 = \frac{nRT}{A\ell_1}, \qquad P_2 = \frac{nRT}{A\ell_2}.$$

Since $\ell_1 > \ell_2$, the lower gas has the smaller volume and the larger pressure. This pressure difference can support a positive piston mass.

Step 2: Balance forces and simplify The upward force is P_2A; the total downward force is $P_1A + mg$. Equilibrium gives

$$P_2A = P_1A + mg.$$

Substituting the ideal-gas pressures cancels the unknown area:

$$mg = nRT\left(\frac{1}{\ell_2} - \frac{1}{\ell_1}\right).$$

Consequently,

$$m = \frac{nRT}{g}\frac{\ell_1 - \ell_2}{\ell_1\ell_2}.$$

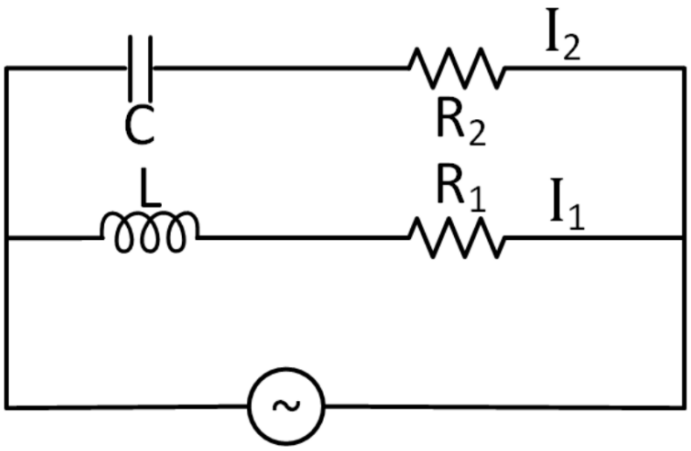
The difference, rather than the sum in option C, is required because the pressure forces oppose each other. Options B and D also have the wrong dependence on the number of moles. The result tends to

zero when the lengths become equal, as required for an unsupported weightless piston.

Step 3: Conclusion $\boxed{m = \frac{nRT}{g}\frac{\ell_1 - \ell_2}{\ell_1 \ell_2}}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the piston force balance before using the gas law; the common cross-sectional area cancels immediately.

32. In the above circuit, C = $\frac{\sqrt{3}}{2}\mu$F, R_2 = 20 Ω, L = $\frac{\sqrt{3}}{10}$ H and R_1 = 10 Ω. Current in L-R_1 path is I_1 and in C-R_2 path it is I_2 . The voltage of A.C. source is given by, V = 200 $\sqrt{2}$ sin (100 t) volts . The phase difference between I_1 and I_2 is : [4 marks]



(A) 150°

(B) 90°

(C) 30°

(D) 0°

Correct Answer: (A) 150°

Solution:

Concept:

- The two series branches are connected across the same AC source. Its angular frequency is $\omega = 100\,\mathrm{rad\,s^{-1}}$, read from $\sin(100t)$.
- In a series RL branch the current lags the applied voltage by $\tan^{-1}(\omega L/R)$. In a series RC branch it leads by $\tan^{-1}[1/(\omega CR)]$.

Step 1: Find the lag in the inductive branch The inductor reactance is

$$X_L = \omega L = 100\frac{\sqrt{3}}{10} = 10\sqrt{3}\,\Omega.$$

Thus

$$\phi_1 = \tan^{-1}\left(\frac{X_L}{R_1}\right) = \tan^{-1}(\sqrt{3}) = 60^\circ.$$

Taking the source voltage as zero phase, the phase of I_1 is -60°. The source amplitude determines current magnitudes, but does not enter these phase ratios.

Step 2: Find the capacitive lead and compare phases Using the stated capacitance $C = (\sqrt{3}/2)\,\mu\text{F}$, the capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{20000}{\sqrt{3}}\,\Omega.$$

Hence the current lead is

$$\phi_2 = \tan^{-1}\left(\frac{X_C}{20}\right) = \tan^{-1}\left(\frac{1000}{\sqrt{3}}\right) \approx 89.90^\circ.$$

The two currents lie on opposite sides of the common voltage phasor, so their separation is the sum:

$$\Delta\phi = 60^\circ + 89.90^\circ = 149.90^\circ \approx 150^\circ.$$

The stated microfarad value makes the result slightly below 150°; option A is the rounded listed value. Subtracting the two phase magnitudes would incorrectly suggest about 30°. The 90° and zero alternatives likewise do not describe their relative phases.

Step 3: Conclusion The phase difference is approximately $\boxed{150^\circ}$ (149.90° using the stated capacitance). This differs from the supplied key value of 30°: a 30° difference would subtract the lead and lag, whereas the branch currents are on opposite sides of the voltage phase. Hence, the correct option is (A).

Quick Tip to Solve: For parallel RL and RC branches, use the common voltage as phase reference; add the inductive lag and capacitive lead.

33. A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be : [4 marks]

(A) 2.0

(B) 1.2

(C) 0.1

(D) 0.4

Correct Answer: (A) 2.0

Solution:

Concept:

- For liquid in steady rotation with angular speed ω, radial pressure balance gives $\partial p/\partial r = \rho\omega^2 r$. Vertical hydrostatic balance gives $\partial p/\partial z = -\rho g$.

- The pressure along the free surface is atmospheric and constant. Its height therefore obeys $g\,dz = \omega^2 r\,dr$, where r is distance from the axis.

Step 1: Obtain the centre-to-wall height difference Integrate the free-surface relation from the rotation axis to the wall of radius R:

$$g\,[z(R) - z(0)] = \int_0^R \omega^2 r\,dr = \frac{\omega^2 R^2}{2}.$$

Consequently,

$$\Delta h = \frac{\omega^2 R^2}{2g}.$$

This is the height difference requested, rather than the rise of the edge relative to the initial level. Conservation of volume fixes the absolute height of the paraboloid but is unnecessary for its centre-to-wall difference.

Step 2: Convert the frequency and substitute The frequency is $f = 2\,\mathrm{s}^{-1}$, so $\omega = 2\pi f = 4\pi\,\mathrm{rad\,s}^{-1}$. The radius is $R = 0.05\,\mathrm{m}$. Therefore

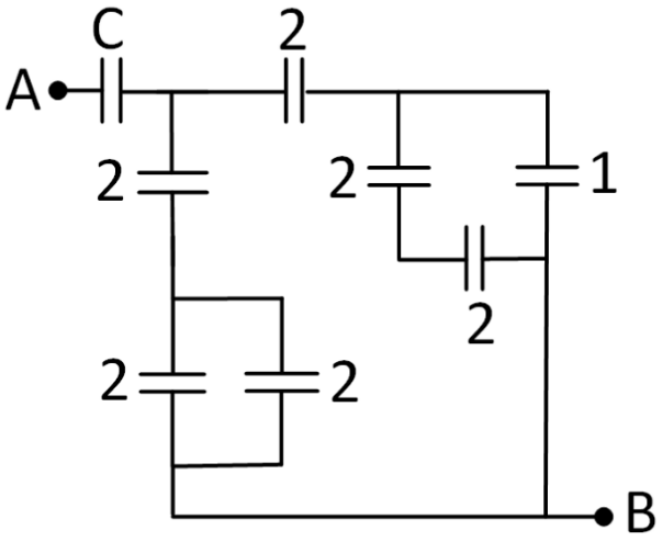
$$\Delta h = \frac{(4\pi)^2 (0.05)^2}{2(9.8)}\,\mathrm{m} \approx 0.0201\,\mathrm{m} = 2.01\,\mathrm{cm}.$$

Rounding to the precision of the options gives $2.0\,\mathrm{cm}$. Options B, C and D are inconsistent with this value; treating rotations per second directly as angular speed or confusing radius and diameter loses essential conversion factors. The density cancels from both equilibrium equations.

Step 3: Conclusion The side is approximately $\boxed{2.0\,\mathrm{cm}}$ above the centre. Hence, the correct option is (A).

Quick Tip to Solve: Use $\omega = 2\pi f$ before applying $\Delta h = \omega^2 R^2/(2g)$; keep the radius and the final height in consistent units.

34. In the circuit shown, find C if the effective capacitance of the whole circuit is to be 0.5 μF. All values in the circuit are in μF. [4 marks]



(A) $\frac{6}{5}\mu F$

(B) $\frac{7}{11}\mu F$

(C) 4μF

(D) $\frac{7}{10}\mu F$

Correct Answer: (B) $\frac{7}{11}\mu F$

Solution:

Concept:

- Capacitors connected across the same two nodes are in parallel: $C_p = C_1 + C_2$. Two capacitors in series have $C_s = C_1C_2/(C_1 + C_2)$.
- Trace ideal wires to identify common-potential nodes before reducing a network. Here the unknown C is in series with the entire fixed network between its right-hand junction and terminal B.

Step 1: Reduce the fixed left and right branches All capacitances in the following reductions are in μF. In the left downward branch the two bottom capacitors of value 2 share their upper and lower nodes, so they are parallel and give $2 + 2 = 4$. This is in series with the upper capacitor of value 2:

$$C_{\text{left}} = \frac{2 \times 4}{2 + 4} = \frac{4}{3}.$$

On the right, the vertical 2 and horizontal 2 form a series pair of value 1. That pair is parallel with the rightmost capacitor of value 1, giving 2. It is then in series with the top capacitor of value 2:

$$C_{\text{right}} = \frac{2 \times 2}{2 + 2} = 1.$$

Step 2: Combine the branches and solve for C The two reduced branches share the junction after C

and terminal B, hence

$$C_{\text{fixed}} = \frac{4}{3} + 1 = \frac{7}{3}.$$

Its series combination with C must equal $1/2$:

$$\frac{1}{1/2} = \frac{1}{C} + \frac{1}{7/3}.$$

Thus

$$\frac{1}{C} = 2 - \frac{3}{7} = \frac{11}{7}, \qquad C = \frac{7}{11}\,\mu\text{F}.$$

A direct substitution gives $(7/11)(7/3)/[(7/11) + (7/3)] = 1/2$. The other listed values do not satisfy this series equation, ruling out A, C and D.

Step 3: Conclusion The required capacitance is $\boxed{C = \frac{7}{11}\,\mu\text{F}}$. Hence, the correct option is (B).

Quick Tip to Solve: Mark the junction immediately after the unknown capacitor, then reduce each route from that junction to B separately.

35. A parallel plate capacitor with plates of area 1 m^2 each, are at a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge on each plate is :
(Take $\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{C^2}{N-m^2}$) [4 marks]

(A) 9.85×10^{-10} C

(B) 8.85×10^{-10} C

(C) 6.85×10^{-10} C

(D) 7.85×10^{-10} C

Correct Answer: (B) 8.85×10^{-10} C

Solution:

Concept:

- For two large oppositely charged parallel plates, neglecting edge effects, the field between them is $E = \sigma/\varepsilon_0$, where $\sigma = Q/A$ is the magnitude of surface charge density.
- The capacitance and potential difference provide an independent route: $C = \varepsilon_0 A/d$, $V = Ed$, and $Q = CV$. The separation cancels when the field is specified.

Step 1: Express charge using the specified field The fields of the two oppositely charged plates add in the gap. Each isolated sheet would contribute $\sigma/(2\varepsilon_0)$, so the total gap field is σ/ε_0, not half of this. Therefore

$$Q = \sigma A = \varepsilon_0 E A.$$

Use $A = 1\,\text{m}^2$, the area of one plate, rather than adding the areas of the two plates. The magnitude sought is the charge on each plate; their signs are opposite.

Step 2: Substitute and verify through capacitance Insert the supplied values:

$$Q = (8.85 \times 10^{-12})(100)(1)\,\mathrm{C} = 8.85 \times 10^{-10}\,\mathrm{C}.$$

The units reduce to coulombs because ε_0 has units $\mathrm{C}^2/(\mathrm{N\,m}^2)$ and E has units $\mathrm{N/C}$. As a cross-check,

$$C = \frac{8.85 \times 10^{-12} \times 1}{0.1} = 8.85 \times 10^{-11}\,\mathrm{F}, \qquad V = 100(0.1) = 10\,\mathrm{V}.$$

Their product gives the same charge. Options A, C and D have different numerical coefficients and do not follow from the stated permittivity, field and area.

Step 3: Conclusion The charge magnitude on each plate is $\boxed{8.85 \times 10^{-10}\,\mathrm{C}}$. Hence, the correct option is (B).

Quick Tip to Solve: When the field between capacitor plates is already given, use $Q = \varepsilon_0 EA$; plate separation is unnecessary.

36. A plano-convex lens (focal length f_2, refractive index μ_2, radius of curvature R) fits exactly into a plano-concave lens (focal length f_1, refractive index μ_1, radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be : [4 marks]

(A) $f_1 + f_2$

(B) $f_1 - f_2$

(C) $\frac{R}{\mu_2 - \mu_1}$

(D) $\frac{2f_1 f_2}{f_1 + f_2}$

Correct Answer: (C) $\frac{R}{\mu_2 - \mu_1}$

Solution:

Concept:

- The optical power of a thin lens in air is $P = 1/f = (\mu - 1)(1/R_1 - 1/R_2)$, with signed radii.
- Two thin lenses in contact have additive powers: $1/F = 1/f_1 + 1/f_2$. A plane refracting surface has zero paraxial power.

Step 1: Assign the powers of the fitted lenses Take light from left to right and use the positive magnitude R of the common curved surface. The plano-convex lens has positive power and the plano-concave lens negative power. Therefore, irrespective of which outer plane face light enters first, their powers in air are

$$\frac{1}{f_2} = \frac{\mu_2 - 1}{R}, \qquad \frac{1}{f_1} = -\frac{\mu_1 - 1}{R}.$$

The signed focal length of the concave lens must not be treated as a positive quantity.

Step 2: Add the powers and compare the alternatives The plane faces contribute no curvature, while the two touching curved faces form a single refracting boundary between the two materials. Adding

the individual thin-lens powers gives

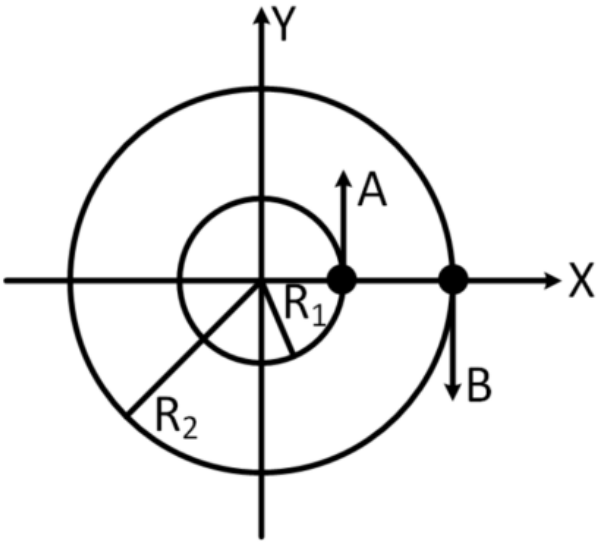
$$\frac{1}{F} = \frac{\mu_2 - 1}{R} - \frac{\mu_1 - 1}{R} = \frac{\mu_2 - \mu_1}{R}.$$

Thus the dependence is on the difference of refractive indices. Adding or subtracting focal lengths, as in (A) and (B), does not add optical powers. The usual contact-lens expression is $f_1 f_2/(f_1 + f_2)$, so the extra factor two in (D) is incorrect.

Step 3: Conclusion The focal length is $F = R/(\mu_2 - \mu_1)$. Hence, the correct option is (C).

Quick Tip to Solve: Check the equal-index limit: if $\mu_1 = \mu_2$, the assembly is a parallel-sided slab and must have zero power.

37. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure. :
The relative velocity $\vec{V}_A - \vec{V}_B$ at t = $\frac{\pi}{2\omega}$ is given by : [4 marks]



(A) $-\omega\,(R_1 + R_2)\,\hat{i}$
(B) $\omega\,(R_2 - R_1)\,\hat{i}$
(C) $\omega\,(R_1 + R_2)\,\hat{i}$
(D) $\omega\,(R_1 - R_1)\,\hat{i}$
Correct Answer: (B) $\omega\,(R_2 - R_1)\,\hat{i}$

Solution:

Concept:

- Uniform circular motion has tangential speed $v = \omega R$; its direction changes with the angular position.
- Relative velocity of A with respect to B is the vector difference $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$, not the difference

of speed magnitudes alone.

Step 1: Locate both particles after a quarter revolution The figure places both particles initially on the positive x-axis. The upward arrow at A shows anticlockwise motion on radius R_1; the downward arrow at B shows clockwise motion on radius R_2. In the given time their angular displacement magnitudes are

$$\omega t = \omega \frac{\pi}{2\omega} = \frac{\pi}{2}.$$

Thus A reaches the top of its circle, $(0, R_1)$, whereas B reaches the bottom of its circle, $(0, -R_2)$. Their rotations are opposite even though their angular speed magnitudes agree.

Step 2: Subtract the tangential velocity vectors At the upper point an anticlockwise-moving particle travels left. At the lower point a clockwise-moving particle also travels left. Consequently,

$$\vec{v}_A = -\omega R_1 \hat{i}, \qquad \vec{v}_B = -\omega R_2 \hat{i}.$$

Taking the specified order of subtraction,

$$\vec{v}_A - \vec{v}_B = (-\omega R_1 + \omega R_2)\hat{i} = \omega(R_2 - R_1)\hat{i}.$$

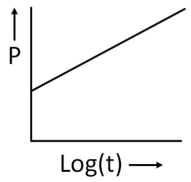
The sum-of-radii options (A) and (C) would require opposite tangential velocities at this instant; they are parallel instead. The printed expression in (D) is zero and cannot describe unequal radii.

Step 3: Conclusion The relative velocity is $\omega(R_2 - R_1)\hat{i}$. Hence, the correct option is (B).

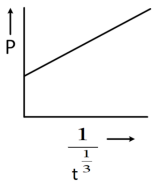
Quick Tip to Solve: First mark the new positions and tangents, then subtract vectors; opposite rotation senses do not imply opposite velocities at every instant.

38. A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by : [4 marks]

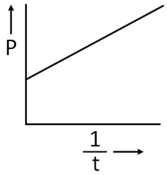
(A) P versus $\log t$



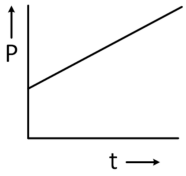
(B) P versus $1/t^{1/3}$



(C) P versus $1/t$

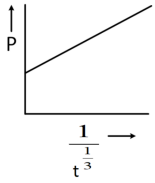


(D) P versus t



Correct Answer: (B)

P versus $1/t^{1/3}$



Solution:

Concept:

- A soap bubble has two liquid surfaces, so its excess pressure is $P - P_0 = 4S/r$, where S is surface tension and P_0 is external pressure.
- For a spherical bubble, $V = 4\pi r^3/3$. A constant rate of volume increase makes radius proportional to the cube root of elapsed inflation time.

Step 1: Express the bubble radius in terms of time Let the pump supply volume at the constant rate k. Taking the time origin at the start of bubble growth,

$$V = kt, \qquad \frac{4\pi r^3}{3} = kt.$$

Hence

$$r = \left(\frac{3k}{4\pi}\right)^{1/3} t^{1/3}.$$

The radius does not grow linearly with time: increasing the radius requires progressively more volume because volume is proportional to its cube. This geometric relation sets the exponent in the pressure graph.

Step 2: Identify the straight-line pressure graph Substitute the radius into the pressure relation:

$$P = P_0 + 4S\left(\frac{4\pi}{3k}\right)^{1/3} \frac{1}{t^{1/3}}.$$

This has the form $P = P_0 + CX$, with $X = 1/t^{1/3}$ and $C > 0$. The graph is therefore a straight rising line against $1/t^{1/3}$, with positive intercept P_0, exactly as in (B). Against actual time the pressure decreases towards the external pressure, ruling out the rising time graph (D). The logarithmic variable in (A) and inverse-time variable in (C) do not linearise the cube-root dependence.

Step 3: Conclusion The correct plot is the straight line of P against $1/t^{1/3}$. Hence, the correct option is (B).

Quick Tip to Solve: Combine $V \propto t$, $r \propto V^{1/3}$ and $P - P_0 \propto 1/r$ before choosing the horizontal-axis variable.

39. A simple harmonic motion is represented by :

$y = 5 (\sin 3\pi t + \sqrt{3} \cos 3\pi t)$ cm

The amplitude and time period of the motion are : [4 marks]

(A) 10 cm, $\frac{3}{2}$ s

(B) 5 cm, $\frac{2}{3}$ s

(C) 5 cm, $\frac{3}{2}$ s

(D) 10 cm, $\frac{2}{3}$ s

Correct Answer: (D) 10 cm, $\frac{2}{3}$ s

Solution:

Concept:

- For equal-frequency perpendicular phase components, $y = a\sin\Omega t + b\cos\Omega t = A\sin(\Omega t + \phi)$, where $A = \sqrt{a^2 + b^2}$.
- The angular frequency Ω, in radians per second, determines the period through $T = 2\pi/\Omega$; changing the phase does not change the period.

Step 1: Combine the sine and cosine contributions The expression contains a sine coefficient $a = 5\,\text{cm}$ and a cosine coefficient $b = 5\sqrt{3}\,\text{cm}$. Both terms have the same angular frequency and describe one harmonic motion. Its amplitude is

$$A = \sqrt{5^2 + (5\sqrt{3})^2} = \sqrt{25 + 75} = 10\,\text{cm}.$$

Equivalently, the displacement can be written as $y = 10\sin(3\pi t + \pi/3)\,\text{cm}$, since $10\cos(\pi/3) = 5$ and $10\sin(\pi/3) = 5\sqrt{3}$.

Step 2: Read the angular frequency and calculate the period The common argument of the trigonometric functions is $3\pi t$, so $\Omega = 3\pi\,\text{rad}\,\text{s}^{-1}$. Thus

$$T = \frac{2\pi}{3\pi} = \frac{2}{3}\,\text{s}.$$

The factor 5 outside the bracket is not the final amplitude, because both phase components contribute. This eliminates (B) and (C). Option (A) has the correct amplitude but uses the reciprocal period $3/2\,\text{s}$. Only (D) contains both the correct amplitude and the correct time period.

Step 3: Conclusion The amplitude is $10\,\text{cm}$ and the time period is $2/3\,\text{s}$. Hence, the correct option is (D).

Quick Tip to Solve: For $a\sin\Omega t + b\cos\Omega t$, use the root-sum-square of coefficients for amplitude and read the period from Ω separately.

40. The mean intensity of radiation on the surface of the Sun is about 10^8 W/m^2 . The rms value of the corresponding magnetic field is closet to : [4 marks]

(A) 10^2 T

(B) 10^{-2} T

(C) 10^{-4} T

(D) 1 T

Correct Answer: (C) 10^{-4} T

Solution:

Concept:

- The mean energy flux of an electromagnetic wave is $I = cB_{\text{rms}}^2/\mu_0$, where c is the speed of light.
- The rms field already includes the averaging over an oscillation. The factor $1/2$ is needed with the peak field, not again with the rms field.

Step 1: Relate the radiation intensity to the magnetic field For a plane electromagnetic wave the instantaneous energy flux is EB/μ_0. Using $E = cB$ and averaging gives

$$I = \frac{c}{\mu_0}\langle B^2 \rangle = \frac{cB_{\text{rms}}^2}{\mu_0}.$$

Therefore the required field is

$$B_{\text{rms}} = \sqrt{\frac{\mu_0 I}{c}}.$$

The supplied intensity is a mean intensity, so this formula directly returns the rms magnetic field rather than its maximum value.

Step 2: Substitute the intensity and compare orders of magnitude With $I = 10^8\,\text{W m}^{-2}$, $\mu_0 = 4\pi \times 10^{-7}\,\text{H m}^{-1}$, and $c = 3 \times 10^8\,\text{m s}^{-1}$,

$$B_{\text{rms}}^2 = \frac{4\pi \times 10^{-7} \times 10^8}{3 \times 10^8} \simeq 4.19 \times 10^{-7}\,\text{T}^2,$$

$$B_{\text{rms}} \simeq 6.47 \times 10^{-4}\,\text{T}.$$

The difference from 10^{-4} T is about 5.47×10^{-4} T, whereas that from 10^{-2} T is about 9.35×10^{-3} T. Thus (C) is the closest value; (B) is much larger, and the fields of 1 T and 100 T in (D) and (A) are still further away.

Step 3: Conclusion The calculated rms field is 6.47×10^{-4} T; its closest listed value is 10^{-4} T. Hence, the correct option is (C).

Quick Tip to Solve: Calculate the squared field first and take its square root carefully; $\sqrt{10^{-7}}$ is $10^{-3.5}$, not 10^{-7}.

41. A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of $5.0\,\mathrm{m\,s^{-1}}$, at right angles to the horizontal component of the earth's magnetic field of $0.3\times10^{-4}\,\mathrm{Wb\,m^{-2}}$. The value of the induced emf in wire is : [4 marks]

(A) $0.3\times10^{-3}\,\mathrm{V}$

(B) $2.5\times10^{-3}\,\mathrm{V}$

(C) $1.5\times10^{-3}\,\mathrm{V}$

(D) $1.1\times10^{-3}\,\mathrm{V}$

Correct Answer: (D) $1.1\times10^{-3}\,\mathrm{V}$

Solution:

Concept:

- The motional emf between the ends of a straight moving wire is $\mathcal{E}=(\vec{v}\times\vec{B})\cdot\vec{l}$, where $\vec{l}$ joins its ends.
- Only the wire component along $\vec{v}\times\vec{B}$ contributes. Here the horizontal terrestrial field points north and the falling velocity points vertically down.

Step 1: Find the effective length cutting the field Take east, north and up as positive coordinate directions. The horizontal field is northward, while the wire falls downward. Hence $\vec{v}\times\vec{B}$ is eastward. A wire directed North East to South West makes 45° with this east-west direction, so its effective length is

$$l_\perp = l\cos 45^\circ = \frac{10}{\sqrt{2}}\,\mathrm{m}.$$

Although the velocity is perpendicular to the magnetic field, the full wire length is not aligned with the magnetic force on its charges. This additional projection must be included.

Step 2: Calculate the emf magnitude Use the field $B_H = 0.3\times10^{-4}\,\mathrm{T}$, speed $v = 5.0\,\mathrm{m\,s^{-1}}$ and projected length:

$$|\mathcal{E}| = B_H v l\cos 45^\circ$$

$$= \frac{(0.3\times10^{-4})(5.0)(10)}{\sqrt{2}}\,\mathrm{V} \simeq 1.06\times10^{-3}\,\mathrm{V}.$$

To the precision of the choices this is $1.1\times10^{-3}\,\mathrm{V}$. Option (C), $1.5\times10^{-3}\,\mathrm{V}$, omits the direction projection. Options (A) and (B) do not follow from the given length, speed, field and 45° orientation.

Step 3: Conclusion The induced emf magnitude is approximately $1.1\times10^{-3}\,\mathrm{V}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the scalar triple product for a moving wire: perpendicular velocity and field alone do not guarantee that the emf is Blv.

42. Two satellites, A and B, have masses m and 2m respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius 2R around the earth. The ratio of their kinetic energies, T_A/T_B, is ; [4 marks]

(A) 2

(B) $\frac{1}{2}$

(C) $\sqrt{\frac{1}{2}}$

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- In a circular orbit, gravity supplies the centripetal force: $GMm/r^2 = mv^2/r$, where M is the mass of Earth.
- The resulting orbital kinetic energy is $K = GMm/(2r)$, so for the same central body it scales as satellite mass divided by orbital radius.

Step 1: Derive the circular-orbit energy relation For a satellite of mass m_s in an orbit of radius r, balance the gravitational and centripetal forces:

$$\frac{GMm_s}{r^2} = \frac{m_s v^2}{r} \quad \Rightarrow \quad v^2 = \frac{GM}{r}.$$

Substituting into the kinetic-energy expression gives

$$K = \frac{1}{2}m_s v^2 = \frac{GMm_s}{2r}.$$

Orbital speed is independent of the satellite mass, but kinetic energy is not: the satellite mass still multiplies $v^2/2$.

Step 2: Apply the mass and radius changes together For satellite A, $m_A = m$ and $r_A = R$, whereas satellite B has $m_B = 2m$ and $r_B = 2R$. Therefore,

$$T_A = \frac{GMm}{2R}, \qquad T_B = \frac{GM(2m)}{2(2R)} = \frac{GMm}{2R}.$$

Their energies are equal because doubling the mass exactly compensates the inverse dependence on orbit radius. Thus

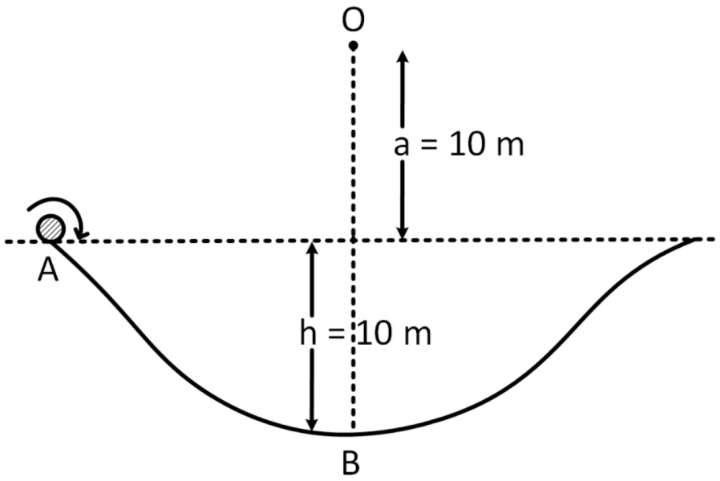
$$\frac{T_A}{T_B} = \frac{m/R}{(2m)/(2R)} = 1.$$

The factor $1/2$ is v_B^2/v_A^2, whereas the question asks for T_A/T_B, which also includes the masses. A square-root factor would arise for speed, while 2 would ignore the larger mass of B. These checks exclude (A), (B) and (C).

Step 3: Conclusion The kinetic-energy ratio is $T_A/T_B = 1$. Hence, the correct option is (D).

Quick Tip to Solve: For circular-orbit energy ratios around the same planet, compare m/r; do not confuse the energy ratio with the speed ratio.

43. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. when the particle reaches point b, its angular momentum about O will be : (Take g = 10 m/s^2) [4 marks]



(A) 6 kg-m^2/s

(B) 8 kg-m^2/s

(C) 2 kg-m^2/s

(D) 3 kg-m^2/s

Correct Answer: (A) 6 kg-m^2/s

Solution:

Concept:

- On a frictionless fixed track the normal reaction does no work, so mechanical energy is conserved: $\frac{1}{2}mv_B^2 = \frac{1}{2}mu^2 + mgh$.
- Angular momentum about a point has magnitude $L = mvb$, where b is the perpendicular distance from that point to the instantaneous line of motion.

Step 1: Read the geometry and convert the mass The figure gives $h = 10\,\text{m}$ and $a = 10\,\text{m}$. Thus $OB = a + h = 20\,\text{m}$. Convert the mass before substitution: $m = 20\,\text{g} = 0.020\,\text{kg}$. At the lowest point B the tangent to the smooth curve is horizontal, while OB is vertical. The perpendicular distance to the momentum line at B is therefore the full 20 m, rather than only the height lost from A.

Step 2: Find the speed at the bottom Only the 10 m fall from A to B changes gravitational potential energy. With $u = 5\,\text{m s}^{-1}$ and $g = 10\,\text{m s}^{-2}$, energy conservation gives

$$v_B^2 = 5^2 + 2(10)(10) = 225, \qquad v_B = 15\,\text{m s}^{-1}.$$

The distance a above A affects the moment arm about O, but does not contribute to this energy change.

Step 3: Calculate the angular momentum about O Since the radius OB and velocity are perpendicular

at B,

$$L_O = mv_B\, OB = (0.020)(15)(20) = 6\,\mathrm{kg\,m^2\,s^{-1}}.$$

The 3 value would use only a 10 m lever arm and misses the other 10 m. Values 2 and 8 also disagree with the actual speed and perpendicular distance. Angular momentum about O is not assumed conserved along the track: the normal reaction can exert a torque about O.

Step 4: Conclusion The magnitude of angular momentum at B is $6\,\mathrm{kg\,m^2\,s^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use energy to get the bottom speed, then measure the perpendicular distance from O to the velocity line; the energy height and moment arm need not be equal.

44. When a certain photosensitive surface is illuminated with monochromatic light of frequency v, the stopping potential for the current is $-V_0/2$. When the surface is illuminated by monochromatic light of frequency v/2, the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is : [4 marks]

(A) $2v$

(B) $\frac{4}{3}v$

(C) $\frac{3v}{2}$

(D) $\frac{5v}{3}$

Correct Answer: (C) $\frac{3v}{2}$

Solution:

Concept:

- The algebraic photoelectric relation is $eV_s = h\nu - \phi = h(\nu - \nu_0)$, where $e > 0$, ϕ is the work function and $\nu_0 = \phi/h$.
- The two signed values printed in this question can be used to find the intercept of the linear relation. A physical emitted electron must still have nonnegative maximum kinetic energy.

Step 1: Keep the signs given in the question Write the two relations using the printed signed potentials. For frequencies ν and $\nu/2$, respectively, these are

$$h\nu - \phi = -\frac{eV_0}{2}, \qquad \frac{h\nu}{2} - \phi = -eV_0.$$

Here V_0 is the magnitude used in the printed expressions. Replacing both minus signs by plus signs would describe a different pair of data and would not reproduce any of the listed threshold values.

Step 2: Eliminate the work function Subtract the second equation from the first, so that the same work function cancels:

$$\frac{h\nu}{2} = \frac{eV_0}{2}, \qquad eV_0 = h\nu.$$

Insert this into the first equation to obtain

$$\phi = h\nu + \frac{eV_0}{2} = \frac{3}{2}h\nu.$$

Consequently the zero of the extrapolated linear relation occurs at $\nu_0 = \phi/h = 3\nu/2$.

Step 3: Interpret the result carefully The formal threshold selected by the supplied signed data is therefore $3\nu/2$, rather than 2ν, $4\nu/3$ or $5\nu/3$; those values do not satisfy both equations. There is a physical inconsistency in the literal emission wording: both stated illuminating frequencies are below this threshold, so they cannot produce ordinary single-photon photoemission. A negative right-hand side is not a negative electron kinetic energy. The option follows the intended signed linear extrapolation, with this physical limitation made explicit.

Step 4: Conclusion The signed equations give the formal threshold $\nu_0 = 3\nu/2$. Hence, the correct option is (C). Literally, actual emission at the two stated subthreshold frequencies is not possible.

Quick Tip to Solve: Subtract the two photoelectric equations before finding the work function, and check afterwards that an actual emission frequency exceeds the threshold.

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45. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to [4 marks]

(A) 0.5×10^{-8} s

(B) 4×10^{-8} s

(C) 3×10^{-6} s

(D) 2×10^{-7} s

Correct Answer: (B) 4×10^{-8} s

Solution:

Concept:

- The mean free path is $\lambda = 1/(\sqrt{2}\, n\sigma)$, where n is number density and σ is the collision cross-section.
- The mean collision time is $\tau = \lambda/\bar{v}$. For the same gas, $n = P/(k_B T)$ and $\bar{v} \propto \sqrt{T}$, so $\tau \propto \sqrt{T}/P$.

Step 1: Find the combined pressure and temperature dependence Increasing pressure makes molecules more closely spaced and reduces the free path; increasing temperature at a specified pressure lowers number density and increases that path. Explicitly, $\lambda \propto T/P$. The speed also changes and must not be held constant: $\bar{v} \propto \sqrt{T}$. Dividing free path by speed therefore leaves $\tau \propto (T/P)/\sqrt{T} = \sqrt{T}/P$. The molecular mass and cross-section cancel in the ratio for this unchanged gas.

Step 2: Substitute the two states The pressure changes from 2 atm to 4 atm, while temperature changes from 300 K to 500 K. Thus

$$\frac{\tau_2}{\tau_1} = \frac{P_1}{P_2}\sqrt{\frac{T_2}{T_1}} = \frac{1}{2}\sqrt{\frac{500}{300}} \simeq 0.6455.$$

Multiplying the initial time gives

$$\tau_2 = (6 \times 10^{-8})(0.6455)\,\text{s} \simeq 3.87 \times 10^{-8}\,\text{s}.$$

No conversion of atmospheres is needed because only their ratio enters.

Step 3: Select the nearest magnitude The result rounds to 4×10^{-8} s, which is option B. It is smaller than the initial time because the pressure increase dominates. Option A is almost eight times smaller than the calculation, while C and D are larger than the original time and cannot represent the calculated ratio. Using only the inverse pressure dependence would give 3×10^{-8} s, missing the speed and density temperature factors.

Step 4: Conclusion The new mean time is approximately 3.87×10^{-8} s, close to 4×10^{-8} s. Hence, the correct option is (B).

Quick Tip to Solve: For the same ideal gas use the ratio $\tau_2/\tau_1 = (P_1/P_2)\sqrt{T_2/T_1}$; do not confuse collision time with mean free path.

46. A load of mass M kg is suspended from a steel wire of length 2m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8.

The new value of increase in length of the steel wire is: [4 marks]

(A) 5.0 mm

(B) zero

(C) 3.0 mm

(D) 4.0 mm

Correct Answer: (C) 3.0 mm

Solution:

Concept:

- For a wire obeying Hooke's law, $\Delta L = FL/(AY)$, where F is its tension, L its length, A its cross-sectional area and Y Young's modulus.
- A fully immersed body experiences buoyancy $F_b = \rho_l V g$. Its weight is $W = \rho_s V g$, so $F_b/W = \rho_l/\rho_s$.

Step 1: Find the fraction of weight supported by the liquid The liquid and load have relative densities 2 and 8 respectively, measured with respect to the same reference density. Their actual density ratio is therefore $\rho_l/\rho_s = 2/8 = 1/4$. Full immersion displaces the entire volume of the load, so the upward buoyant force is exactly one quarter of its weight. The remaining downward apparent weight must be carried by the wire.

Step 2: Calculate the new wire tension Originally the wire supports $F_1 = Mg$, giving extension $\Delta L_1 = 4.0\,\text{mm}$. After immersion, equilibrium of the load gives

$$F_2 = Mg - F_b = Mg\left(1 - \frac{2}{8}\right) = \frac{3}{4}Mg.$$

The mass M need not be determined. The wire remains the same, so its length, area and Young's modulus cancel when the two extensions are divided.

Step 3: Scale the extension by the tension ratio Using Hooke's law for the two loads,

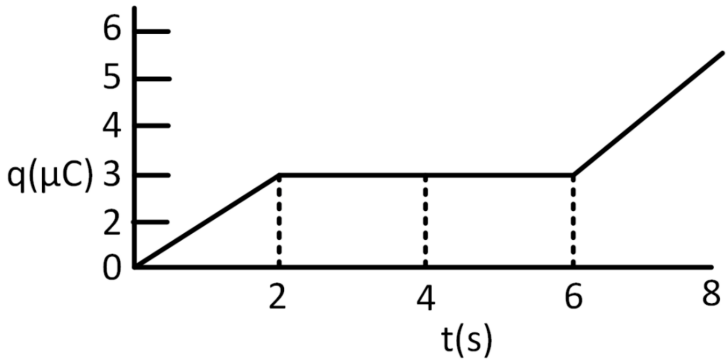
$$\frac{\Delta L_2}{\Delta L_1} = \frac{F_2}{F_1} = \frac{3}{4}, \qquad \Delta L_2 = 4.0\left(\frac{3}{4}\right) = 3.0\,\text{mm}.$$

Zero extension would require the liquid density to equal the load density, which it does not. An unchanged 4.0 mm ignores buoyancy, while 5.0 mm would require an increased tension even though the liquid supports part of the load. The quoted wire dimensions are unnecessary for this ratio calculation.

Step 4: Conclusion The new increase in wire length is $3.0\,\text{mm}$. Hence, the correct option is (C).

Quick Tip to Solve: When the same wire supports an immersed load, multiply its original extension by $1 - \rho_l/\rho_s$; all wire constants cancel.

47. The charge on a capacitor plate in a circuit, as a function of time, is shown in the figure :
When is the value of current at t = 4 s ? [4 marks]



(A) zero

(B) 1.5 μA

(C) 2 μA

(D) 3 μA

Correct Answer: (A) zero

Solution:

Concept:

- Instantaneous current is the rate of charge flow: $i(t) = dq/dt$. On a charge-time graph it equals the tangent slope, not the height of the graph.
- A horizontal segment has constant charge and therefore zero current, even if the stored charge is nonzero. One μC per second equals one μA.

Step 1: Locate the required time on the graph The graph rises from the origin to $q = 3\,\mu\text{C}$ at $t = 2\,\text{s}$, then remains horizontal until $t = 6\,\text{s}$. The requested instant $t = 4\,\text{s}$ lies strictly inside this horizontal

portion, rather than at either corner. Hence a derivative exists there and can be read directly from that segment. The later rising portion is irrelevant to the current at 4 s.

Step 2: Differentiate the constant-charge interval Throughout $2 < t < 6\,\text{s}$, the plate charge is $q(t) = 3\,\mu\text{C}$, independent of time. Therefore

$$i(4\,\text{s}) = \left.\frac{dq}{dt}\right|_{t=4\,\text{s}} = 0.$$

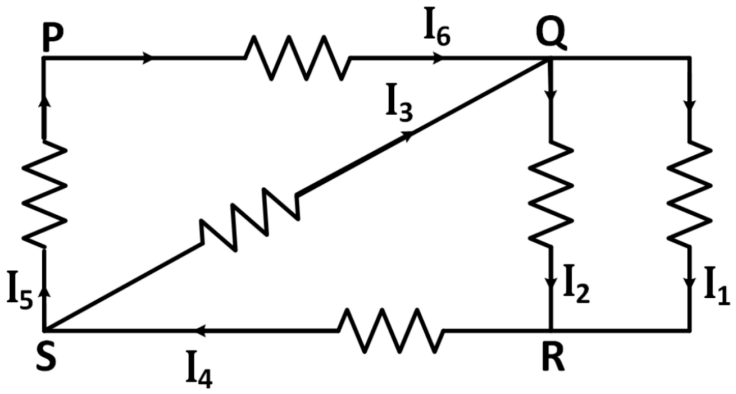
Equivalently, the slope of any two points within this straight segment is $(3-3)/(6-2) = 0\,\mu\text{C}\,\text{s}^{-1}$. The presence of stored charge does not itself imply that charge continues to flow.

Step 3: Distinguish current from other graph quantities The value $1.5\,\mu\text{A}$ is the slope of the initial rise, $3/2$, and belongs to the interval before 2 s. It cannot be carried over to the flat part. Values $2\,\mu\text{A}$ and $3\,\mu\text{A}$ are likewise not the slope at 4 s; in particular the graph height 3 is a charge in microcoulombs, not a current. Taking total charge divided by elapsed time would give an average from the origin, not the instantaneous derivative requested.

Step 4: Conclusion The current at $t = 4\,\text{s}$ is zero. Hence, the correct option is (A).

Quick Tip to Solve: Read the slope of the segment containing the specified time. A charged capacitor can have zero current whenever its charge is constant.

48. In the given circuit diagram, the currents, I_1 = – 0.3 A, I_4 = 0.8 A and I_5 = 0.4 A, are flowing as shown. The currents I_2, I_3 and I_6, respectively, are : [4 marks]



(A) 1.1 A, – 0.4 A, 0.4 A

(B) – 0.4 A, 0.4 A, 1.1 A

(C) 0.4 A, 1.1 A, 0.4 A

(D) 1.1 A, 0.4 A, 0.4 A

Correct Answer: (D) 1.1 A, 0.4 A, 0.4 A

Solution:

Concept:

- Kirchhoff's current law follows from charge conservation: the algebraic sum of currents entering a junction equals the sum leaving it.
- Treat every arrow as a reference direction. A negative given current means actual flow opposite that arrow; keep its sign when substituting into the junction equations.

Step 1: Use the unbranched path through P The branch from S to P carries $I_5 = 0.4\,\text{A}$ upward. At P this branch connects only to the upper resistor leading to Q. There is no additional branch at P, so all this current becomes I_6:

$$I_6 = I_5 = 0.4\,\text{A}.$$

This immediately excludes the option assigning 1.1 A to I_6. The current arrows themselves need not be redrawn because signed values refer to those already shown.

Step 2: Apply charge conservation at S The bottom arrow I_4 enters S from R. The arrows I_5 and I_3 both leave S, through the upper-left and diagonal branches respectively. Consequently

$$I_4 = I_5 + I_3, \qquad I_3 = 0.8 - 0.4 = 0.4\,\text{A}.$$

A negative diagonal value would violate this balance with the printed reference directions.

Step 3: Find the current in the central vertical branch At R, the reference arrows for I_1 and I_2 enter the junction, and I_4 leaves it. Hence

$$I_1 + I_2 = I_4, \qquad I_2 = 0.8 - (-0.3) = 1.1\,\text{A}.$$

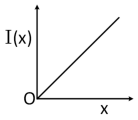
The negative I_1 must be retained: its actual flow leaves R. As an independent check at Q, $I_3 + I_6 = 0.4 + 0.4 = 0.8\,\text{A}$, while $I_2 + I_1 = 1.1 - 0.3 = 0.8\,\text{A}$. Thus both independent junction balances agree, excluding the other permutations and the wrong negative sign.

Step 4: Conclusion In the requested order, $(I_2, I_3, I_6) = (1.1, 0.4, 0.4)\,\text{A}$. Hence, the correct option is (D).

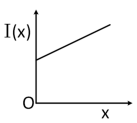
Quick Tip to Solve: Write the junction equation using the printed arrows first, then insert signed currents; subtracting a negative given current increases the unknown here.

49. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is 'I(x)'. Which one of the graphs represents the variation of I(x) with x correctly ? [4 marks]

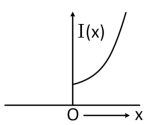
(A) Straight line through the origin.



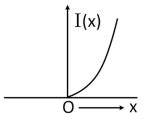
(B) Straight line with a positive I-axis intercept.



(C) Upward-curving parabola with a positive I-axis intercept.

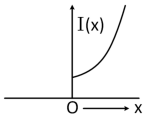


(D) Upward-curving parabola through the origin.



Correct Answer: (C)

Upward-curving parabola with a positive I-axis intercept.



Solution:

Concept:

- The moment of inertia of a uniform solid sphere of mass M and radius R about any diameter is $I_{\text{cm}} = 2MR^2/5$.
- The parallel-axis theorem gives $I(x) = I_{\text{cm}} + Mx^2$ for an axis parallel to that diameter and displaced by perpendicular distance x.

Step 1: Identify the value at zero displacement A diameter passes through the centre of mass, so it is the appropriate reference axis for the parallel-axis theorem. At $x = 0$ the displaced axis coincides with that diameter and

$$I(0) = \frac{2}{5}MR^2 > 0.$$

The graph cannot pass through the origin. This immediately rules out options A and D, because both of those show zero inertia for the central axis. A sphere with nonzero mass and radius retains rotational inertia about its diameter.

Step 2: Determine the variation away from the centre For a nonzero displacement the additional inertia is Mx^2, so

$$I(x) = \frac{2}{5}MR^2 + Mx^2, \qquad \frac{dI}{dx} = 2Mx, \qquad \frac{d^2I}{dx^2} = 2M > 0.$$

Thus the graph on the shown nonnegative-x range starts at a positive intercept, has zero initial slope and bends upward with increasing slope. The quantity x is the axis separation, not a distance of an individual mass element from the centre.

Step 3: Compare all four plots Option B has a positive intercept but is a straight line, representing a linear dependence on x; the theorem requires a quadratic dependence, so B is excluded. Option C has both essential features: a positive value at zero displacement and upward curvature. Option D has the correct type of curvature but its zero intercept omits the central moment of inertia. Option A misses both the intercept and the quadratic dependence. The correct shape follows directly without needing numerical values of M or R.

Step 4: Conclusion The required graph is the upward-curving parabola with nonzero intercept $2MR^2/5$. Hence, the correct option is (C).

Quick Tip to Solve: For a parallel-axis graph, test the intercept at zero displacement first, then check whether the added term depends on x or on x^2.

50. Let ℓ, r, C and V represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{\ell}{rCV}$ in SI units will be : [4 marks]

(A) [A^{-1}]

(B) [LTA]

(C) [LA^{-2}]

(D) [LT^2]

Correct Answer: (A) [A^{-1}]

Solution:

Concept:

- Inductance relates the induced emf to the rate of change of current: $V = \ell\, dI/dt$. Thus $[\ell] = [V]TA^{-1}$, where A is the dimension of electric current.
- Resistance obeys Ohm's law, $r = V/I$, and capacitance is charge per potential difference, $C = Q/V$. Charge has dimension AT.

Step 1: Express each circuit quantity using voltage, current and time Keeping voltage as a common dimensional symbol avoids unnecessarily expanding it into mass and length. The relevant dimensions are

$$[r] = [V]A^{-1}, \qquad [C] = AT[V]^{-1}.$$

Their product therefore satisfies

$$[rC] = ([V]A^{-1})(AT[V]^{-1}) = T.$$

This is also consistent with the familiar resistance-capacitance time constant.

Step 2: Cancel the dimensions in the required ratio Substitute the inductance dimension and the product just calculated:

$$\left[\frac{\ell}{rCV}\right] = \frac{[V]TA^{-1}}{T[V]} = A^{-1}.$$

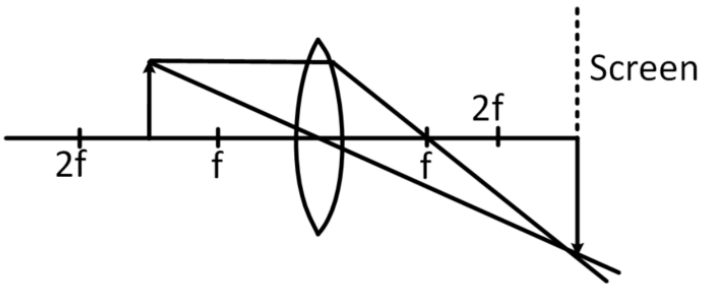
Both voltage and time cancel completely, so no length dimension remains. Every one of options (B), (C) and (D) contains a factor of length and consequently cannot describe this ratio. In particular, confusing inductance with a length because its symbol resembles a lower-case letter would give a wrong result.

Step 3: Conclusion The required SI dimension is $[A^{-1}]$, the inverse of electric current. Hence, the correct option is (A).

Quick Tip to Solve: Use rC as a time constant and $\ell/V = T/I$; this reduces the dimensional ratio before expanding any base units.

51. Formation of real image using a biconvex lens is shown below :

If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen ? [4 marks]



(A) Image disappears

(B) Magnified image

(C) Erect real image

(D) No change

Correct Answer: (A) Image disappears

Solution:

Concept:

- For a thin lens in a surrounding medium, $1/f = (n_{\text{lens}}/n_{\text{medium}} - 1)(1/R_1 - 1/R_2)$. Its power depends on the relative refractive index.
- A screen displays a sharp real image only when it is at the image distance selected by the lens formula for the fixed object distance.

Step 1: Determine how immersion changes the lens power In air the glass biconvex lens converges light strongly enough to form the real inverted image at the screen shown. Immersion in water increases the surrounding refractive index. Thus the relative index of glass with respect to its surroundings decreases, and so does the positive refracting power of the lens. Its focal length increases. The physical curvatures do not change, but keeping the lens shape fixed does not keep its focal length fixed.

Step 2: Check the unchanged screen position Write the lens relation using positive object and real-image distances as $1/f = 1/u + 1/v$. Originally the fixed values of u and v obey this relation. After immersion, $1/f$ changes, while the two fixed positions leave $1/u + 1/v$ unchanged. They therefore cannot still satisfy the focusing condition. Rays no longer converge sharply on the original screen. A new real-image position, if available, would require moving the screen; that is expressly forbidden. The magnified-image option does not give the observation at this screen, an erect real image is not produced by this single lens for a real object, and no change contradicts the change in lens power.

Step 3: Conclusion The sharp image disappears from the fixed screen when the setup is immersed in water. Hence, the correct option is (A).

Quick Tip to Solve: When a lens is immersed, first change its relative refractive index; then test focusing at the fixed screen rather than merely discussing magnification.

52. A galvanometer, whose resistance is 50 ohm, has 25 divisions in it. When a current of 4×10^{-4} A passes through it, its needle (pointer) deflects by one division. To use this galvanometer as a voltmeter of range 2.5 V, it should be connected to a resistance of : [4 marks]

(A) 200 ohm

(B) 250 ohm

(C) 6200 ohm

(D) 6250 ohm

Correct Answer: (A) 200 ohm

Solution:

Concept:

- For a moving-coil galvanometer, current is proportional to deflection. The full-scale current is the current per division multiplied by the total divisions.
- A voltmeter is formed by adding a series resistance R_s. Its range satisfies $V = I_g(G + R_s)$, where G is the galvanometer resistance.

Step 1: Calculate the full-scale current The given $4 \times 10^{-4}\,\text{A}$ is the current for only one division, not for the full scale. For 25 divisions,

$$I_g = 25(4 \times 10^{-4}) = 1.0 \times 10^{-2}\,\text{A}.$$

This is the maximum permitted current through the galvanometer when the converted instrument reads its full range of $2.5\,\text{V}$. Using the one-division current here would incorrectly make the range calculation twenty-five times too large.

Step 2: Find the added series resistance The complete instrument must have total resistance

$$R_{\text{total}} = \frac{2.5}{1.0 \times 10^{-2}} = 250\,\Omega.$$

The coil already contributes $G = 50\,\Omega$, so only the difference must be added:

$$R_s = 250 - 50 = 200\,\Omega.$$

This resistance is connected in series, so the same full-scale current flows through it and the coil. Option (B) is the total resistance rather than the added one. Options (C) and (D) result from treating a one-division current as full-scale current, with or without subtracting the coil resistance.

Step 3: Conclusion Connect a $200\,\Omega$ resistance in series with the galvanometer. Hence, the correct option is (A).

Quick Tip to Solve: Find the full-scale current from all divisions before calculating V/I_g, and subtract the coil resistance to obtain the external multiplier.

53. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to : [4 marks]

(A) 335 ms^{-1}

(B) 328 ms^{-1}

(C) 341 ms^{-1}

(D) 322 ms^{-1}

Correct Answer: (B) 328 ms^{-1}

Solution:

Concept:

- For the first resonance of an air column closed by water, its effective length is one quarter of the wavelength: $L_{\text{eff}} = v/(4f)$.
- The unknown distance from the reference mark to the effective open end is the same in both measurements. Subtracting the two resonance equations eliminates it.

Step 1: Set up the two effective-length equations Let x, in metres, include both the offset of the reference mark and the open-end correction. For the first fork and then the second fork, the first-resonance condition gives

$$0.11 + x = \frac{v}{4(512)}, \qquad 0.27 + x = \frac{v}{4(256)}.$$

The jagged end prevents a reliable direct length measurement, but it does not require assigning a numerical value to x. The same reference and tube end are used each time.

Step 2: Eliminate the unknown correction and find the speed Subtract the first equation from the second:

$$0.27 - 0.11 = \frac{v}{4}\left(\frac{1}{256} - \frac{1}{512}\right).$$

Since $1/256 - 1/512 = 1/512$, this reduces to

$$0.16 = \frac{v}{2048}, \qquad v = 327.68\,\text{m}\,\text{s}^{-1}.$$

The nearest listed value is $328\,\text{m}\,\text{s}^{-1}$. The alternatives 335, 341 and 322 do not follow from the measured length difference. Notice that these are first resonances for different frequencies, so the rule for successive resonances of one unchanged fork is not applicable.

Step 3: Conclusion The measured speed is about $328\,\text{m}\,\text{s}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: For an uncertain tube end, subtract first-resonance equations at two frequencies to remove the common offset and end correction.

54. In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to : [4 marks]

(A) 2020 nm

(B) 250 nm

(C) 1700 nm

(D) 220 nm

Correct Answer: (B) 250 nm

Solution:

Concept:

- In an inelastic collision, an electron transfers the decrease of its kinetic energy to the atom as excitation energy.
- A photon obeys $E = hc/\lambda$. The shortest wavelength corresponds to the largest photon energy available in de-excitation.

Step 1: Find the excitation energy transferred to mercury The electron enters with $5.6\,\text{eV}$ and emerges with $0.7\,\text{eV}$. The energy lost by it is therefore

$$\Delta E = 5.6 - 0.7 = 4.9\,\text{eV}.$$

This is the energy supplied to the mercury atom in the excitation process. Neither the incident energy alone nor the residual electron energy is the energy of the emitted photon. The residual energy is still carried away by the electron.

Step 2: Convert the maximum photon energy to wavelength The largest possible emitted photon energy in the return from this excitation is $4.9\,\text{eV}$. A direct transition gives the minimum wavelength; any cascade splits the available energy among photons and gives longer wavelengths. Using $hc \simeq 1240\,\text{eV nm}$,

$$\lambda_{\text{min}} = \frac{1240}{4.9}\,\text{nm} \simeq 253\,\text{nm}.$$

This is closest to 250 nm. The 220 nm value would require about $5.64\,\text{eV}$, exceeding the transferred excitation energy. The 1700 nm and 2020 nm values correspond to much smaller photon energies and do not describe the minimum wavelength.

Step 3: Conclusion The minimum wavelength is approximately $253\,\text{nm}$, close to $250\,\text{nm}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the electron's loss of energy, rather than its incident or final energy, in $\lambda = hc/\Delta E$.

55. A paramagnetic material has 10^{28} atoms/m^3. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4}. Its susceptibility at 300 K is : [4 marks]

(A) 3.726×10^{-4}

(B) 2.672×10^{-4}

(C) 3.672×10^{-4}

(D) 3.267×10^{-4}

Correct Answer: (D) 3.267×10^{-4}

Solution:

Concept:

- For a paramagnetic material obeying Curie law, $\chi = C/T$, where χ is susceptibility, T is absolute temperature and C is the Curie constant.
- For the same sample, the atomic number density and microscopic magnetic properties remain unchanged. Hence χT is constant between the two temperatures.

Step 1: Relate the susceptibilities at the two temperatures Denote the initial and final values by subscripts 1 and 2. From Curie law,

$$\chi_1 T_1 = \chi_2 T_2, \qquad \chi_2 = \chi_1 \frac{T_1}{T_2}.$$

Here $T_1 = 350\,\mathrm{K}$, $T_2 = 300\,\mathrm{K}$ and $\chi_1 = 2.8 \times 10^{-4}$. The given $10^{28}\,\mathrm{atoms\,m^{-3}}$ is the same at both temperatures and is already contained in the material constant, so it is not an additional multiplication factor.

Step 2: Substitute and compare with the options Cooling makes thermal agitation weaker and allows a greater induced alignment of the atomic magnetic moments. We therefore expect a positive susceptibility slightly larger than its initial value. Numerically,

$$\chi_2 = 2.8 \times 10^{-4} \frac{350}{300} = 3.2666\ldots \times 10^{-4} \simeq 3.267 \times 10^{-4}.$$

Susceptibility is dimensionless. Option (B) incorrectly decreases it on cooling. Options (A) and (C) increase it by the wrong factor; only (D) has the required ratio $350/300 = 7/6$.

Step 3: Conclusion The susceptibility at $300\,\mathrm{K}$ is 3.267×10^{-4}. Hence, the correct option is (D).

Quick Tip to Solve: For one paramagnetic sample use $\chi_2/\chi_1 = T_1/T_2$; cooling must increase the susceptibility under Curie law.

56. An alpha-particle of mass m suffers 1-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is [4 marks]

(A) 2m

(B) 4m

(C) 1.5m

(D) 3.5m

Correct Answer: (B) 4m

Solution:

Concept:

- In a one-dimensional elastic collision, both linear momentum and kinetic energy are conserved for the pair of colliding bodies.
- For an incident mass m with speed u and a stationary target mass M, the incident particle's final velocity is $v = (m - M)u/(m + M)$.

Step 1: Translate the energy loss into a signed speed ratio The alpha-particle retains $100\% - 64\% = 36\%$ of its own initial kinetic energy. Its mass remains unchanged, so

$$\frac{\frac{1}{2}mv^2}{\frac{1}{2}mu^2} = 0.36, \qquad \left|\frac{v}{u}\right| = 0.6.$$

The question also says that it scatters directly backwards. Taking its incident direction as positive therefore fixes the sign:

$$v/u = -0.6.$$

The collision is elastic for the complete two-body system even though the alpha-particle alone loses kinetic energy to the nucleus.

Step 2: Use the elastic-collision relation to solve for the target mass Substitute the signed velocity ratio into the standard collision equation:

$$\frac{m - M}{m + M} = -\frac{3}{5}.$$

Cross-multiplication yields

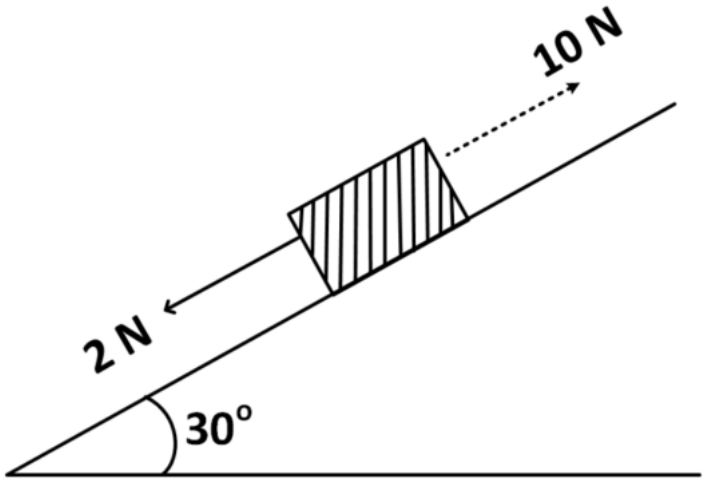
$$5m - 5M = -3m - 3M, \qquad 8m = 2M, \qquad M = 4m.$$

This is heavier than the alpha-particle, as required for backward scattering. For comparison, $M = 2m$, $1.5m$ and $3.5m$ give speed ratios of magnitudes $1/3$, $1/5$ and $5/9$, respectively; none leaves exactly 36 percent of the initial alpha-particle energy.

Step 3: Conclusion The stationary nucleus has mass $4m$. Hence, the correct option is (B).

Quick Tip to Solve: Convert the retained energy fraction to a speed ratio by taking its square root, then apply a minus sign when the projectile rebounds.

57. A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is : [Take g = 10 m/s^2] [4 marks]



(A) $\frac{\sqrt{3}}{4}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{2}{3}$

Correct Answer: (C) $\frac{\sqrt{3}}{2}$

Solution:

Concept:

- Static friction adjusts to prevent relative motion, up to its limiting magnitude $f_{\max} = \mu_s N$, where N is the normal reaction.
- For a block on an incline of angle θ, gravity has components $mg\sin\theta$ along the plane and $mg\cos\theta$ perpendicular to it. Applied forces parallel to the plane do not change N.

Step 1: Write the two limiting equilibrium conditions Let $P = mg\sin 30°$ and $f = \mu_s mg\cos 30°$. With the largest downward applied force, the impending motion is down the slope, so friction acts upwards. Equilibrium gives

$$f = P + 2.$$

With the largest upward force, the impending motion is upwards, so friction acts downwards. Therefore

$$10 = P + f.$$

The friction directions are different in the two experiments, although their limiting magnitudes are equal.

Step 2: Determine gravity and limiting friction Adding and subtracting these equations gives

$$2f = 12, \qquad f = 6\,\text{N},$$

$$2P = 8, \qquad P = 4\,\text{N}.$$

Since $\sin 30^\circ = 1/2$, the weight is $mg = 8\,\text{N}$. The normal reaction is thus

$$N = 8\cos 30^\circ = 4\sqrt{3}\,\text{N}.$$

Step 3: Calculate the coefficient and compare the choices The ratio of limiting friction to the normal reaction is

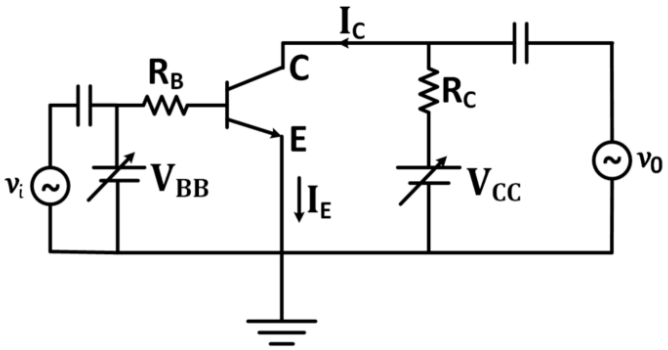
$$\mu_s = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}.$$

Options (A), (B) and (D) give different limiting-friction values when multiplied by $4\sqrt{3}\,\text{N}$, and cannot satisfy both threshold forces simultaneously. A coefficient of friction is dimensionless; neither threshold force itself is its value.

Step 4: Conclusion The coefficient of static friction is $\boxed{\mu_s = \sqrt{3}/2}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the sum of the two opposite limiting forces to find twice the limiting friction, and their difference to find twice the downslope weight component.

58. In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, V_{CC} = 5V, β_{dc} = 200, R_B = 100 kΩ, R_C = 1 kΩ and V_{BE} = 1.0 V. The minimum base current and the input voltage at which the transistor will go to saturation, will be respectively : [4 marks]



(A) 20 μA and 2.8 V

(B) 25 μA and 2.8 V

(C) 20 μA and 3.5 V

(D) 25 μA and 3.5 V

Correct Answer: (D) 25 μA and 3.5 V

Solution:

Concept:

- In the active region of a common-emitter transistor, $I_C = \beta_{\text{dc}} I_B$. The collector circuit limits the maximum available collector current.
- At the onset of ideal saturation take $V_{CE} \simeq 0$, so $I_{C,\text{sat}} = V_{CC}/R_C$. The base loop obeys $V_{BB} = V_{BE} + I_B R_B$.

Step 1: Find the collector-current limit The collector supply is $5\,\text{V}$ and the collector resistance is $1\,\text{k}\Omega$. At saturation essentially the entire collector supply falls across this resistor in the ideal model used for the options:

$$I_{C,\text{sat}} = \frac{5}{1000} = 5.0 \times 10^{-3}\,\text{A} = 5.0\,\text{mA}.$$

Increasing base drive beyond the onset of saturation cannot continue increasing the collector current according to the active-region gain law.

Step 2: Calculate the minimum base drive The smallest base current that would demand this collector current in the active region is

$$I_{B,\text{min}} = \frac{I_{C,\text{sat}}}{\beta_{\text{dc}}} = \frac{5.0 \times 10^{-3}}{200} = 25 \times 10^{-6}\,\text{A} = 25\,\mu\text{A}.$$

The gain is a current ratio, and the factor of 10^{-3} in the collector current must be retained.

Step 3: Determine the input supply and reject the alternatives Including the given base-emitter drop, the required base supply is

$$V_{BB} = 1.0 + (25 \times 10^{-6})(100 \times 10^{3}) = 1.0 + 2.5 = 3.5\,\text{V}.$$

This lies within the allowed range of the supply. A base current of $20\,\mu\text{A}$ yields only $4\,\text{mA}$, below the collector limit. Also, $2.8\,\text{V}$ gives $(2.8 - 1.0)/100000 = 18\,\mu\text{A}$, which is insufficient. Thus neither alternative current nor the lower voltage satisfies both circuits.

Step 4: Conclusion The minimum base current is $\boxed{25\,\mu\text{A}}$ and the corresponding input voltage is $\boxed{3.5\,\text{V}}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the collector circuit limit first, divide by the current gain, and then add the base-emitter drop to the base-resistor voltage.

59. In a radioactive decay chain, the initial nucleus is $^{232}_{90}$Th. At the end there are 6 α-particles and 4 β-particles which are emitted. If the end nucleus is $^{A}_{Z}$X, A and Z are given by : [4 marks]

(A) A = 208; Z = 80

(B) A = 208; Z = 82

(C) A = 200; Z = 81

(D) A = 202; Z = 80

Correct Answer: (B) A = 208; Z = 82

Solution:

Concept:

- An alpha particle is a helium nucleus $^{4}_{2}\mathrm{He}$. Each alpha emission lowers the parent mass number by four and atomic number by two.
- In beta-minus decay a neutron changes into a proton, electron and antineutrino. The daughter mass number remains unchanged and its atomic number increases by one.

Step 1: Account for all six alpha emissions The initial nucleus has mass number 232 and atomic number 90. Six alpha particles carry away a total mass number $6 \times 4 = 24$ and a total nuclear charge number $6 \times 2 = 12$. Immediately after allowing for these emissions, the daughter numbers are

$$A_1 = 232 - 24 = 208, \qquad Z_1 = 90 - 12 = 78.$$

The mass number counts nucleons, rather than the actual mass measured in kilograms.

Step 2: Include the four beta emissions The beta particles in this decay chain are electrons, so each is a beta-minus emission. Its emitted charge is negative; conservation of charge therefore requires the daughter nuclear charge to rise by one. Four such emissions give

$$A = 208, \qquad Z = 78 + 4 = 82.$$

Beta-minus decay interconverts nucleons inside the nucleus and does not remove a nucleon, so it cannot reduce A. The order of the individual alpha and beta emissions does not affect these total changes.

Step 3: Check the numbers against conservation laws The combined nuclear bookkeeping can be written as

$$^{232}_{90}\mathrm{Th} \longrightarrow {}^{208}_{82}\mathrm{X} + 6\,{}^{4}_{2}\mathrm{He} + 4\,{}^{0}_{-1}\mathrm{e} + \text{antineutrinos}.$$

The right-hand mass numbers sum to $208 + 24 = 232$, and charge numbers sum to $82 + 12 - 4 = 90$. Option (A) has the correct mass number but the wrong nuclear charge. Options (C) and (D) already fail nucleon-number conservation. The daughter with atomic number 82 is lead.

Step 4: Conclusion The end nucleus has $\boxed{A = 208,\ Z = 82}$, corresponding to $^{208}_{82}\mathrm{Pb}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep separate tallies: alpha changes both numbers, while beta-minus changes only the atomic number, increasing it by one.

60. To double the covering range of a TV transmittion tower, its height should be multiplied by :

[4 marks]

(A) 4

(B) 2

(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{2}}$

Correct Answer: (A) 4

Solution:

Concept:

- Television signals in the relevant high-frequency band propagate primarily by line of sight. The curvature of Earth limits the horizon distance from a tower.
- For tower height $h \ll R$, the horizon range is $d \simeq \sqrt{2Rh}$, where R is the radius of Earth. Thus linear range is proportional to the square root of height.

Step 1: Express the tower range in terms of its height A line from the tower top to the radio horizon is tangent to Earth at the horizon point. The radius to that point is perpendicular to this line. The right triangle therefore gives

$$(R+h)^2 = R^2 + d^2,$$

and hence

$$d^2 = 2Rh + h^2 \simeq 2Rh.$$

The h^2 term is negligible because a transmission tower is very short compared with Earth's radius. This is the usual terrestrial communication range model.

Step 2: Apply the required doubling of distance Let the new height be h_2, and let the original height be h_1. Since the Earth radius remains the same, dividing the two range formulas yields

$$\frac{d_2}{d_1} = \sqrt{\frac{h_2}{h_1}}.$$

The condition is a doubling of linear covering range, so

$$2 = \sqrt{\frac{h_2}{h_1}} \quad \Rightarrow \quad \frac{h_2}{h_1} = 4.$$

Squaring the distance ratio, rather than multiplying height by the distance ratio directly, is essential.

Step 3: Compare the proposed multipliers Multiplying the height by 2 would multiply the range only by $\sqrt{2}$. Multiplying it by $\sqrt{2}$ would multiply the range by $2^{1/4}$, also less than two. Multiplying the height by $1/\sqrt{2}$ would reduce the range. Only a factor of four gives the required factor of two in distance. The requested quantity is the covering range, not the area enclosed by the horizon circle; confusing distance with area would lead to a different scaling.

Step 4: Conclusion The tower height must be multiplied by $\boxed{4}$. Hence, the correct option is (A).

Quick Tip to Solve: For horizon-limited transmission, square the desired factor in linear range to obtain the required factor in tower height.

Chemistry (120 marks)

61. The element that does NOT show catenation is : [4 marks]

(A) Ge

(B) Si

(C) Sn

(D) Pb

Correct Answer: (D) Pb

Solution:

Concept:

- Catenation is the ability of an element to form covalent bonds with atoms of the same element, producing chains, rings or extended frameworks.
- In Group 14, effective self-linking depends strongly on the strength of the element-element bond. Increasing atomic size down the group weakens overlap and generally reduces catenation.

Step 1: Identify the property being compared The question asks specifically about catenation, so the relevant bond is a bond between two atoms of the same element. It is not enough that an element forms compounds with other elements, exhibits metallic bonding in the bulk solid, or has four valence electrons. Stable molecular self-linked units require sufficiently strong directional covalent element-element bonds. Thus membership of Group 14 alone does not guarantee appreciable catenation.

Step 2: Compare the listed Group 14 elements Silicon forms silicon-silicon bonds and shows catenation, although to a smaller extent than carbon. Germanium can also form germanium-germanium linked compounds. Tin shows still weaker but limited self-linking. Lead lies below tin, has larger atoms and poor effective overlap for the corresponding covalent self-bonds. Its lead-lead bond is too weak to support the usual catenation chemistry considered in the Group 14 trend. The standard qualitative trend is

$$\mathrm{C} \gg \mathrm{Si} > \mathrm{Ge} \approx \mathrm{Sn},$$

with lead treated as showing no catenation in this classification.

Step 3: Select the exception Options (A), (B) and (C) are eliminated because germanium, silicon and tin do show catenation to differing extents. Option (D), lead, is the requested exception. This answer concerns the textbook catenation tendency rather than an assertion that no lead-lead interaction can ever occur under any specialised condition. The trend reflects covalent self-bond stability, not merely the ability to form a crystalline metallic lattice.

Step 4: Conclusion The element that does not show catenation in the Group 14 classification is $\boxed{\mathrm{Pb}}$. Hence, the correct option is (D).

Quick Tip to Solve: For Group 14 catenation, remember the sharp decrease from carbon and the absence of ordinary catenation for lead; metallic bonding does not count as a chain of covalent self-bonds.

62. Molecules of benzoic acid (C_6H_5COOH) dimerise in benzene. 'w' g of the acid dissolved in 30 g of benzene shows a depression in freezing point equal to 2K. If the percentage association of the acid to form dimmer in the solution is 80, then w is – (Its given that K_f = 5 K kg mol^{-1}, Molar mass of benzoic acid = 122 g mol^{-1}) [4 marks]

(A) 1.5 g

(B) 1.8 g

(C) 1.0 g

(D) 2.4 g

Correct Answer: (D) 2.4 g

Solution:

Concept:

- Freezing-point depression is $\Delta T_f = iK_f m$, where m is the molality calculated from the formula units initially dissolved and i accounts for association.
- For dimerisation of a fraction α of the solute molecules, two monomers become one dimer. The van't Hoff factor is $i = 1 - \alpha/2$.

Step 1: Count particles after association Begin with one mole of benzoic acid formula units. If 80% associate, 0.80 mole of monomers forms 0.40 mole of dimers, leaving 0.20 mole of free monomers. The total number of independently moving solute particles is therefore $0.20 + 0.40 = 0.60$ mole per original mole. Thus

$$i = 1 - \frac{0.80}{2} = 0.60.$$

Association decreases the particle count, so using a factor greater than one would be physically inconsistent.

Step 2: Write the molality with the solvent in kilograms The number of moles of acid initially dissolved is $w/122$, with w in grams. The mass of benzene is $30\,\text{g} = 0.030\,\text{kg}$. Consequently

$$m = \frac{w/122}{0.030}.$$

The denominator must be the solvent mass, not the total solution mass. Substituting the measured freezing-point depression gives

$$2 = 0.60 \times 5 \times \frac{w}{122 \times 0.030}.$$

Step 3: Solve and match the approximate option Rearranging the equation gives

$$w = \frac{2 \times 122 \times 0.030}{0.60 \times 5} = \frac{7.32}{3} = 2.44\,\text{g}.$$

To the precision of the given choices, this is $2.4\,\text{g}$. As a check, without association the inferred mass would be $1.464\,\text{g}$, approximately option (A); that choice ignores the decrease in particle count. Options (B) and (C) likewise do not satisfy the measured depression when $i = 0.60$ is included. More acid is required because association reduces the number of particles per mole dissolved.

Step 4: Conclusion The required acid mass is $\boxed{w = 2.44\,\text{g} \simeq 2.4\,\text{g}}$. Hence, the correct option is (D).

Quick Tip to Solve: For dimers use $i = 1 - \alpha/2$, and convert the solvent mass into kilograms before substituting in the freezing-point formula.

63. The correct order of atomic radii is : [4 marks]

(A) Ho > N > Eu > Ce

(B) Ce > Eu > Ho > N

(C) N > Ce > Eu > Ho

(D) Eu > Ce > Ho > N

Correct Answer: (D) Eu > Ce > Ho > N

Solution:

Concept:

- The lanthanoid contraction is the general decrease in size across the series as nuclear charge rises and added $4f$ electrons incompletely screen that charge.
- Atomic radii of the lanthanoid metals have notable anomalies: europium has an unusually large radius associated with its stable half-filled $4f^7$ configuration and predominantly divalent metallic state. A smooth contraction alone is insufficient here.

Step 1: Place nitrogen last Nitrogen is a compact second-period element. Its occupied electron shells extend only to $n = 2$, whereas cerium, europium and holmium are sixth-period lanthanoids. Even with the lanthanoid contraction, these metal atoms are substantially larger than nitrogen. Nitrogen must therefore be the smallest member of this comparison. Options (A) and (C), which place nitrogen above one or more lanthanoids, cannot be correct.

Step 2: Include the europium anomaly For most lanthanoids, increasing effective nuclear attraction produces a gradual decrease in metallic atomic radius. Cerium occurs before holmium, so its radius is larger than that of holmium. Europium is a conspicuous exception to the smooth trend: the stable half-filled $4f^7$ arrangement favours a divalent metallic state, reducing the electrons participating in metallic bonding relative to the predominantly trivalent neighbours. The weaker metallic bonding gives europium an unusually large atomic radius. It is large enough to exceed cerium, rather than lying between cerium and holmium.

Step 3: Combine the comparisons The metal-radius comparison is therefore

$$r(\mathrm{Eu}) > r(\mathrm{Ce}) > r(\mathrm{Ho}).$$

Appending the much smaller nitrogen atom gives

$$\mathrm{Eu} > \mathrm{Ce} > \mathrm{Ho} > \mathrm{N}.$$

Option (B) is tempting because it follows a perfectly smooth contraction, but it misses the europium anomaly. Option (D) includes that exception and also places nitrogen correctly. This comparison is of atomic radii of the elements; it should not be replaced by radii of hypothetical ions all forced to have the same charge.

Step 4: Conclusion The correct decreasing order is $\boxed{\mathrm{Eu} > \mathrm{Ce} > \mathrm{Ho} > \mathrm{N}}$. Hence, the correct option is (D).

Quick Tip to Solve: Apply lanthanoid contraction with its atomic-radius exceptions: europium and ytterbium are unusually large because of their divalent metallic states.

64. If the de Broglie wavelength of the electron in n^{th} Bohr orbit in a hydrogenic atom is equal to 1.5 πa_0 (a_0 is Bohr radius), then the value of n/z is - [4 marks]

(A) 0.75

(B) 0.40

(C) 1.50

(D) 1.0

Correct Answer: (A) 0.75

Solution:

Concept:

- A standing electron wave in the nth allowed Bohr orbit satisfies $2\pi r_n = n\lambda$, where r_n is the orbit radius and λ is the de Broglie wavelength.
- For a hydrogenic, one-electron species with nuclear charge Z, the Bohr radius formula is $r_n = a_0 n^2/Z$. The symbol z in the question denotes this nuclear charge.

Step 1: Combine the orbit and wavelength relations The circumference contains an integer number of wavelengths. Hence the wavelength is the circumference divided by the orbit number, not simply the circumference. Substituting the hydrogenic radius gives

$$\lambda = \frac{2\pi r_n}{n} = \frac{2\pi}{n}\frac{a_0 n^2}{Z} = 2\pi a_0 \frac{n}{Z}.$$

One power of n cancels, so the wavelength is proportional to the ratio requested in the question.

Step 2: Insert the given wavelength and compare options Using $\lambda = 1.5\pi a_0$, divide both sides by $2\pi a_0$:

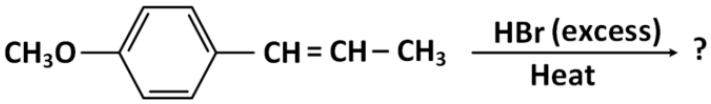
$$\frac{n}{Z} = \frac{1.5\pi a_0}{2\pi a_0} = \frac{1.5}{2} = 0.75.$$

Both the length unit and the factor of pi cancel, as a ratio of two pure numbers requires. Options (B), (C) and (D) would instead predict wavelengths $0.80\pi a_0$, $3.00\pi a_0$ and $2.00\pi a_0$, respectively, none equal to the stated wavelength.

Step 3: Conclusion The required ratio is $\boxed{n/z = 0.75}$. Hence, the correct option is (A).

Quick Tip to Solve: First use the standing-wave condition to obtain $\lambda = 2\pi a_0(n/Z)$; this avoids confusing orbit radius with wavelength.

65. The major product in the following conversion is – [4 marks]



(A) p-HO$-$C$_6$H$_4-$CH$_2-$CHBr$-$CH$_3$

(B) p-CH$_3$O$-$C$_6$H$_4-$CH$_2-$CHBr$-$CH$_3$

(C) p-CH$_3$O$-$C$_6$H$_4-$CHBr$-$CH$_2-$CH$_3$

(D) p-HO$-$C$_6$H$_4-$CHBr$-$CH$_2-$CH$_3$

Correct Answer: (D) p-HO$-$C$_6$H$_4-$CHBr$-$CH$_2-$CH$_3$

Solution:

Concept:

- Electrophilic addition of HBr to an unsymmetrical alkene follows the pathway producing the more stable carbocation when no peroxide is specified.
- Hot excess HBr cleaves an aryl methyl ether at the methyl–oxygen bond. The aryl–oxygen bond remains, yielding a phenol and CH$_3$Br.

Step 1: Locate bromine in the side chain The starting side chain is Ar$-$CH$=$CH$-$CH$_3$. Proton addition at the carbon farther from the ring puts the positive charge at the benzylic carbon. This carbocation is stabilised by resonance with the aromatic ring, unlike the alternative ordinary secondary carbocation. Bromide therefore attacks the benzylic centre to give Ar$-$CHBr$-$CH$_2-$CH$_3$. There is no peroxide in the reagents, so a radical anti-Markovnikov pathway is not appropriate.

Step 2: Apply the hot excess acid to the methoxy group The oxygen of Ar$-$O$-$CH$_3$ is protonated. Bromide then displaces the protonated aryloxy group by substitution at the unhindered methyl carbon:

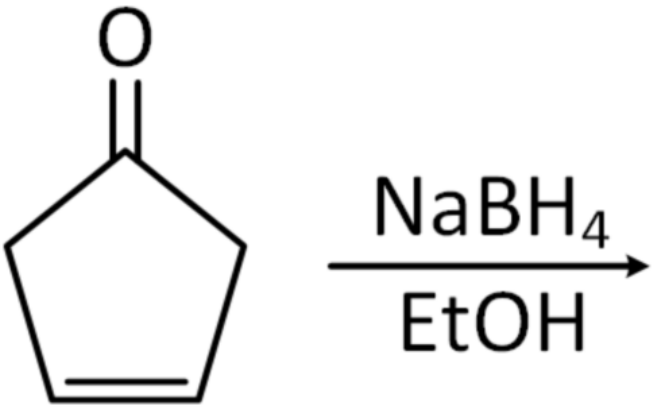
$$\text{Ar}-\text{O}-\text{CH}_3 + \text{HBr} \longrightarrow \text{Ar}-\text{OH} + \text{CH}_3\text{Br}.$$

Substitution at an aromatic sp2 carbon does not occur by this mechanism. Thus the ring retains oxygen as an OH substituent. The product is p-HO$-$C$_6$H$_4-$CHBr$-$CH$_2-$CH$_3$. Option (A) puts bromine at the wrong side-chain carbon. Option (B) has that same regioselectivity error and retains methoxy; option (C) has the correct side chain but neglects ether cleavage under hot excess acid.

Step 3: Conclusion The major product is p-HO$-$C$_6$H$_4-$CHBr$-$CH$_2-$CH$_3$. Hence, the correct option is (D).

Quick Tip to Solve: With hot excess hydrogen halide, check both the alkene and the ether: benzylic addition determines Br position, while methyl ether cleavage gives phenol.

66. The major product of the following reaction is – [4 marks]



(A) Cyclopent-3-en-1-ol

(B) 3-Ethoxycyclopentan-1-ol

(C) Cyclopentanone

(D) Cyclopentanol

Correct Answer: (A) Cyclopent-3-en-1-ol

Solution:

Concept:

- $NaBH_4$ transfers hydride to the electrophilic carbonyl carbon of an aldehyde or ketone, converting the carbonyl group into an alcohol after protonation.
- An isolated carbon–carbon double bond is normally unchanged by $NaBH_4$ in ethanol. Reduction of a ketone gives a secondary alcohol.

Step 1: Identify the two functional groups The five-membered starting ring contains a ketone at carbon 1 and an isolated double bond between carbons 3 and 4. The alkene is not the carbonyl bond: the ring skeleton must be preserved while deciding which bond the reagent reduces. Sodium borohydride is a mild carbonyl reducing reagent rather than a reagent for hydrogenation of an ordinary alkene. Thus a product formed only by saturating the carbon–carbon double bond is inconsistent with these conditions.

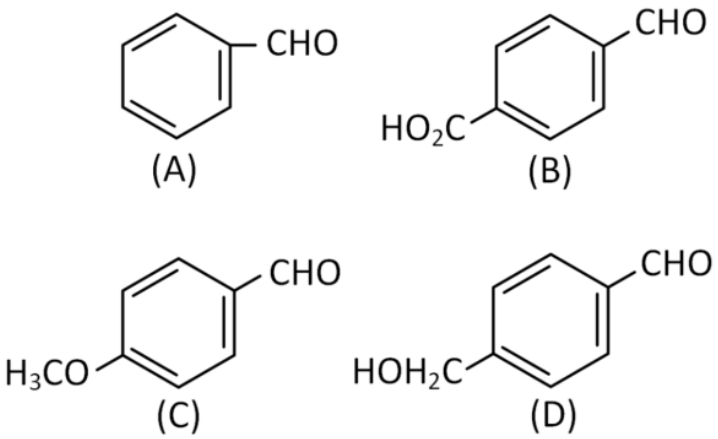
Step 2: Reduce the carbonyl and protonate the intermediate Hydride from borohydride attacks the carbonyl carbon and the carbonyl pi electrons move to oxygen, forming an alkoxide. Ethanol supplies a proton to this oxygen to furnish the alcohol. The net change is $C{=}O \longrightarrow CH{-}OH$, while the remote double bond stays in its original position. The product is cyclopent-3-en-1-ol. Option (B) additionally introduces an ethoxy group and saturates the double bond, neither of which follows from simple hydride reduction. Option (C) reduces the alkene while leaving the ketone, the reverse of the reagent selectivity. Option (D) requires reduction of both bonds, so it also predicts an extra reaction absent here.

Step 3: Conclusion The product is cyclopent-3-en-1-ol: the ketone becomes a secondary alcohol and the isolated alkene remains. Hence, the correct option is (A).

Quick Tip to Solve: For $NaBH_4$ in ethanol, change the carbonyl to an alcohol first and leave an isolated alkene untouched.

67. The aldehydes which will not form Grignard product with one equivalent Grignard reagents are -

[4 marks]



(A) B and D

(B) C and D

(C) B, C and D

(D) B and C

Correct Answer: (A) B and D

Solution:

Concept:

- A Grignard reagent RMgX is both a carbon nucleophile and a strong base. A sufficiently acidic O–H proton consumes it by acid–base reaction.
- One equivalent used in deprotonating a carboxylic acid or alcohol is no longer available to add to the aldehyde. An ether has no such O–H proton.

Step 1: Test the four aldehydes for acidic protons Compound A is benzaldehyde and contains no O–H group. Compound B has a carboxylic acid substituent, $-\mathrm{CO_2H}$, whose proton reacts rapidly with the Grignard reagent. Compound C has a methoxy substituent, $-\mathrm{OCH_3}$, which is an ether and has no acidic hydrogen on oxygen. Compound D has a hydroxymethyl substituent, $-\mathrm{CH_2OH}$, so its alcohol proton also consumes the organomagnesium reagent.

Step 2: Track the single equivalent of reagent For either an alcohol or carboxylic acid, the important competing reaction has the form

$$\mathrm{R{-}MgX + H{-}OY \longrightarrow R{-}H + Y{-}O{-}MgX.}$$

This acid–base reaction consumes the one equivalent of reagent before normal nucleophilic addition to the aldehyde can give the expected alcohol after workup. Therefore B and D do not give the usual Grignard addition product with only one equivalent. A and C can undergo carbonyl addition because neither has an O–H proton. The presence of oxygen alone is not sufficient to quench the reagent: this is why including C in options (B), (C) or (D) is incorrect. Additional reagent beyond that consumed by acidic protons would be a different experimental condition.

Step 3: Conclusion The aldehydes that consume the single equivalent without giving normal aldehyde addition are B and D. Hence, the correct option is (A).

Quick Tip to Solve: Before adding a Grignard reagent to a carbonyl, count acidic O–H groups; each consumes an equivalent, whereas an ether oxygen does not.

68. The correct statement (s) among I to III with respect to potassium ions that are abundant within the cell fluid is /are

I. They activate many enzymes

II. They participate in the oxidation of glucose to produce ATP

III. Along with sodium ions they are responsible for the transmission of nerve signals [4 marks]

(A) III only

(B) I and II only

(C) I and III only

(D) I, II and III only

Correct Answer: (D) I, II and III only

Solution:

Concept:

- K^+ is the principal intracellular alkali-metal cation. It acts as an enzyme activator and supports important cellular metabolic processes.
- Complementary sodium and potassium concentration gradients across a membrane provide the electrochemical basis for nerve signal transmission.

Step 1: Assess enzyme activation and glucose metabolism Statement I is correct: potassium ions activate many enzymes, helping maintain the conditions required for their catalytic activity. Statement II is also correct in the biological role described here: potassium participates in the cellular processes by which glucose is oxidised to produce ATP. This does not mean that potassium itself supplies the energy or changes oxidation state; glucose oxidation provides the chemical energy, and potassium supports the enzyme-controlled metabolic processes. Its role as an ion is compatible with remaining in the +1 oxidation state.

Step 2: Assess nerve transmission and combine the statements Statement III is correct. Sodium is concentrated mainly outside cells and potassium mainly inside; their unequal distribution and regulated movement across the membrane establish membrane potentials and enable propagation of nerve signals. Sodium and potassium therefore act together in this physiological function. All three statements are true, so an option omitting any one of them is incomplete. Option (A) omits the two metabolic roles, option (B) omits nerve transmission, and option (C) omits the role in glucose metabolism. None of these restricted selections includes the full set of correct statements.

Step 3: Conclusion Statements I, II and III are all correct. Hence, the correct option is (D).

Quick Tip to Solve: Remember the intracellular potassium triad: enzyme activation, glucose metabolism linked to ATP production, and nerve signalling with sodium.

69. Given

(i) C (graphite) + $O_2(g) \rightarrow CO_2(g)$; $\Delta rH^{\ominus}$ = x kJ mol^{-1}

(ii) C(graphite) + $\frac{1}{2}O_2(g) \rightarrow CO(g)$; $\Delta rH^{\ominus}$ = y kJ mol^{-1}

(iii) $CO(g) + \frac{1}{2} O_2(g) \rightarrow CO_2(g)$; $\Delta rH^{\ominus}$ = z kJ mol^{-1}

Based on the above thermochemical equations, find out which one of the following algebraic relationships is correct?

[4 marks]

(A) z = x + y

(B) x = y + z

(C) x = y – z

(D) y = 2z – x

Correct Answer: (B) x = y + z

Solution:

Concept:

- Enthalpy is a state function. Hess's law therefore states that reaction enthalpies add when their balanced thermochemical equations are added.
- An intermediate produced in one step and consumed in the next cancels from the net reaction. Reversing a reaction reverses its enthalpy sign; no reversal is needed here.

Step 1: Add the two reactions through carbon monoxide Reaction (ii) forms carbon monoxide from graphite, and reaction (iii) consumes exactly one mole of this carbon monoxide to form carbon dioxide. Add them as printed:

$$\mathrm{C(graphite)} + \frac{1}{2}\,\mathrm{O_2(g)} \longrightarrow \mathrm{CO\,(g)},$$
$$\mathrm{CO\,(g)} + \frac{1}{2}\,\mathrm{O_2(g)} \longrightarrow \mathrm{CO_2(g)}.$$

The carbon monoxide appears once on each side, so it cancels. The two half moles of oxygen combine to one mole.

Step 2: Compare the net equation and its enthalpy The resulting equation is

$$\mathrm{C(graphite)} + \mathrm{O_2(g)} \longrightarrow \mathrm{CO_2(g)},$$

which is exactly reaction (i). Since equations (ii) and (iii) were used in their original direction with coefficients unchanged, their enthalpies add without sign or numerical factors. Therefore

$$x = y + z.$$

Equivalently, $z = x - y$. Option (A) incorrectly adds the complete combustion enthalpy and the first-stage enthalpy. Option (C) subtracts the second-stage enthalpy despite using that step forward. Option (D) introduces a factor of two unsupported by the balanced equations. The argument uses the stated reactions and does not require numerical enthalpy values.

Step 3: Conclusion The correct algebraic relationship is $\boxed{x = y + z}$. Hence, the correct option is (B).

Quick Tip to Solve: Cancel the intermediate CO by adding the printed equations; add their enthalpies with exactly the same coefficients and directions.

70. 8 g of NaOH is dissolved in 18g of H_2O. Mole fraction of NaOH in solution and molality (in mol kg^{-1}) of the solution respectively are - [4 marks]

(A) 0.2, 11.11

(B) 0.167, 22.20

(C) 0.167, 11.11

(D) 0.2, 22.20

Correct Answer: (C) 0.167, 11.11

Solution:

Concept:

- The analytical mole fraction of solute is $x_s = n_s/(n_s + n_w)$, with n_s and n_w the amounts of solute and solvent in moles.
- Molality is $m = n_s/(\text{mass of solvent in kg})$. Its denominator is the solvent mass, not the mass or volume of the whole solution.

Step 1: Convert solute and solvent masses to moles The molar mass of sodium hydroxide is $23 + 16 + 1 = 40\,\text{g mol}^{-1}$. Thus

$$n_{\text{NaOH}} = \frac{8}{40} = 0.200\,\text{mol}.$$

The molar mass of water is $18\,\text{g mol}^{-1}$, giving

$$n_{\text{H}_2\text{O}} = \frac{18}{18} = 1.00\,\text{mol}.$$

The question uses the conventional composition in terms of NaOH formula units and water, rather than separately counting the ions formed by dissociation.

Step 2: Calculate the two concentration measures The total analytical amount is $0.200 + 1.00 = 1.200\,\text{mol}$. Hence

$$x_{\text{NaOH}} = \frac{0.200}{1.200} = \frac{1}{6} \approx 0.167.$$

Convert the given solvent mass independently: $18\,\text{g} = 0.018\,\text{kg}$. Therefore

$$m = \frac{0.200}{0.018} = 11.11\,\text{mol kg}^{-1}.$$

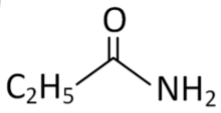
The mole fraction is dimensionless, while molality has the stated unit. Options (A) and (D) incorrectly use 0.200 as the mole fraction, failing to divide by total moles. Options (B) and (D) give twice the correct molality; counting ions does not double the analytical moles of NaOH used in this definition.

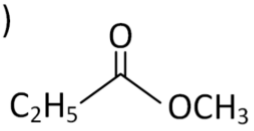
Step 3: Conclusion The mole fraction and molality are $\boxed{0.167,\ 11.11\,\text{mol kg}^{-1}}$, respectively. Hence,

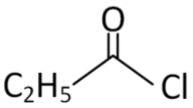
the correct option is (C).

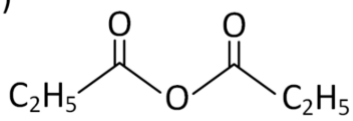
Quick Tip to Solve: Use total moles for mole fraction but kilograms of water alone for molality; keep these two denominators separate.

71. The increasing order of the reactivity of the following with $LiAlH_4$ is [4 marks]

(A) 

(B) 

(C) 

(D) 

(A) A < B < C < D

(B) B < A < C < D

(C) B < A < D < C

(D) A < B < D < C

Correct Answer: (D) A < B < D < C

Solution:

Concept:

- Lithium aluminium hydride, $LiAlH_4$, supplies hydride to the electrophilic carbonyl carbon. The relative ease of this attack depends on the group bonded to that carbonyl.
- For the derivatives shown, electron donation into the carbonyl and the ability of the attached group to leave give the increasing reactivity order: amide, ester, anhydride, acyl chloride.

Step 1: Identify the four functional groups A is propanamide, $C_2H_5CONH_2$; B is methyl propanoate, $C_2H_5COOCH_3$; C is propanoyl chloride, C_2H_5COCl; and D is propanoic anhydride, $(C_2H_5CO)_2O$. Their alkyl groups are comparable, so the functional group attached to the acyl carbon decides the order rather than the length of the hydrocarbon chain.

Step 2: Compare carbonyl activation and leaving groups The nitrogen lone pair in the amide donates strongly into the carbonyl, reducing the positive character of the carbonyl carbon and making A least reactive. An ester also has resonance donation from oxygen, but it is weaker than the amide donation, so B is more reactive than A. The anhydride can expel a resonance-stabilised carboxylate ion; it is more reactive than an ester. The acyl chloride has the most reactive carbonyl of these derivatives, with chloride as a good leaving group. Thus the sequence is

$$A < B < D < C.$$

Step 3: Check the alternative sequences Option (A) incorrectly puts the acyl chloride below the anhydride. Options (B) and (C) incorrectly put the ester below the amide. Only option (D) simultaneously places the amide first and the acyl chloride last.

Step 4: Conclusion The increasing order is A < B < D < C. Hence, the correct option is (D).

Quick Tip to Solve: Identify the acyl derivative first: amide < ester < anhydride < acyl chloride. Compare functional groups before considering the alkyl chains.

72. If K_{sp} of Ag_2CO_3 is 8×10^{-12}, the molar solubility of Ag_2CO_3 in $0.1\,M$ $AgNO_3$ is – [4 marks]

(A) $8 \times 10^{-12}\,M$

(B) $8 \times 10^{-10}\,M$

(C) $8 \times 10^{-13}\,M$

(D) $8 \times 10^{-11}\,M$

Correct Answer: (B) $8 \times 10^{-10}\,M$

Solution:

Concept:

- For $Ag_2CO_3(s) \rightleftharpoons 2\,Ag^+(aq) + {CO_3}^{2-}(aq)$, the solubility product is $K_{sp} = [Ag^+]^2[{CO_3}^{2-}]$. The exponent two belongs to the silver-ion concentration.
- A soluble silver salt supplies a common ion and suppresses dissolution. If the molar solubility is s, carbonate concentration is s, while the total silver concentration is $0.1 + 2s$.

Step 1: Set up the equilibrium expression Let s moles of silver carbonate dissolve per litre in the silver nitrate solution. Since silver nitrate dissociates essentially completely, the initial silver concentration is $0.1\,mol\,L^{-1}$. The dissolved carbonate contributes one carbonate and two silver ions per formula unit. Therefore

$$8 \times 10^{-12} = (0.1 + 2s)^2 s.$$

We must use the silver already present, rather than the pure-water expression $4s^3$.

Step 2: Apply and verify the common-ion approximation The very small solubility product suggests $2s \ll 0.1$, so $[Ag^+] \simeq 0.1\,M$. Substitution gives

$$s = \frac{8 \times 10^{-12}}{(0.1)^2} = \frac{8 \times 10^{-12}}{10^{-2}} = 8 \times 10^{-10}\,M.$$

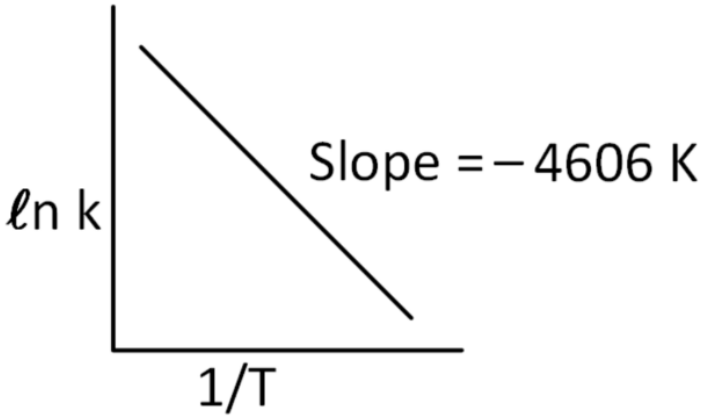
The additional silver is $2s = 1.6 \times 10^{-9}\,M$, negligible beside $0.1\,M$; hence the approximation is self-consistent.

Step 3: Eliminate the other powers of ten Option (A) forgets the division by the square of the common-ion concentration. Options (C) and (D) give smaller solubilities and do not satisfy $(0.1)^2 s = 8 \times 10^{-12}$. In particular, division by 10^{-2} increases the numerical solubility by a factor of one hundred relative to K_{sp}.

Step 4: Conclusion The molar solubility is $8 \times 10^{-10}\,mol\,L^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the dissolution stoichiometry before using the common-ion concentration: for silver carbonate, square the silver concentration, not the solubility.

73. For a reaction consider the plot of $\ln k$ versus $1/T$ given in the figure. If the rate constant of this reaction at $400\,\mathrm{K}$ is $10^{-5}\,\mathrm{s}^{-1}$, then the rate constant at $500\,\mathrm{K}$ is – [4 marks]



(A) $10^{-4}\,\mathrm{s}^{-1}$

(B) $4 \times 10^{-4}\,\mathrm{s}^{-1}$

(C) $10^{-6}\,\mathrm{s}^{-1}$

(D) $2 \times 10^{-4}\,\mathrm{s}^{-1}$

Correct Answer: (A) $10^{-4}\,\mathrm{s}^{-1}$

Solution:

Concept:

- The Arrhenius equation is $\ln k = \ln A - E_a/(RT)$. A plot of natural logarithm $\ln k$ against $1/T$ has slope $-E_a/R$, measured in kelvin.
- For two points on the same straight line, $\ln(k_2/k_1) = m(1/T_2 - 1/T_1)$, where m is the printed slope. This form avoids separately calculating the pre-exponential factor.

Step 1: Read the slope and temperature change The graph gives $m = -4606\,\mathrm{K}$. Take $T_1 = 400\,\mathrm{K}$, $k_1 = 10^{-5}\,\mathrm{s}^{-1}$, and $T_2 = 500\,\mathrm{K}$. The inverse-temperature difference is negative:

$$\frac{1}{500} - \frac{1}{400} = \frac{4-5}{2000} = -\frac{1}{2000}\,\mathrm{K}^{-1}.$$

Thus its product with the negative slope is positive, as required for an increasing rate constant.

Step 2: Calculate the ratio of the rate constants Using the difference form of the straight-line equation,

$$\ln \frac{k_2}{10^{-5}} = (-4606)\left(-\frac{1}{2000}\right) = 2.303 \simeq \ln 10.$$

Exponentiating therefore gives

$$\frac{k_2}{10^{-5}} \simeq 10, \qquad k_2 \simeq 10^{-4}\,\mathrm{s}^{-1}.$$

The rounded slope makes this a tenfold increase, rather than an exactly defined transcendental numerical value.

Step 3: Check the proposed alternatives Option (C) predicts a decrease on heating and has the wrong sign of change. Options (B) and (D) imply ratios of forty and twenty respectively, whose natural logarithms are not 2.303. Option (A) alone gives the required ratio of ten; no extra factor of 2.303 should be inserted because the graph already uses natural logarithms.

Step 4: Conclusion At $500\,\mathrm{K}$, $k = 10^{-4}\,\mathrm{s}^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the graph slope directly in the difference equation. A change of 2.303 in $\ln k$ means a factor of ten in k.

---

74. The upper stratosphere consisting of the ozone layer protects us from sun's radiation that falls in the wavelength region of - [4 marks]

(A) 200-315 nm

(B) 400-550 nm

(C) 0.8-1.5 nm

(D) 600-750 nm

Correct Answer: (A) 200-315 nm

Solution:

Concept:

- Stratospheric ozone absorbs harmful ultraviolet radiation through photochemical absorption and dissociation. This protective screening chiefly concerns the short ultraviolet wavelengths reaching the ozone layer.
- The ultraviolet region lies below visible light in wavelength. Visible wavelengths are approximately 400–$750\,\mathrm{nm}$, whereas nanometre-scale wavelengths near one nanometre belong to the X-ray region.

Step 1: Identify the radiation screened by ozone The ozone layer acts as a filter for harmful solar ultraviolet radiation. An absorbed photon can dissociate ozone,

$$\mathrm{O_3} + h\nu \longrightarrow \mathrm{O_2} + \mathrm{O}.$$

Subsequent oxygen–ozone photochemical cycling dissipates absorbed energy in the stratosphere. The question asks for the relevant wavelength interval, so we compare the spectral location of each option instead of treating all solar radiation alike.

Step 2: Choose the ultraviolet interval The interval 200–$315\,\mathrm{nm}$ is in the ultraviolet region and includes wavelengths strongly screened by ozone. It is shorter than the visible region and corresponds to photons energetic enough to produce the photochemical effects relevant to ozone shielding. The atmospheric ozone layer prevents much of this biologically harmful ultraviolet radiation from reaching the surface.

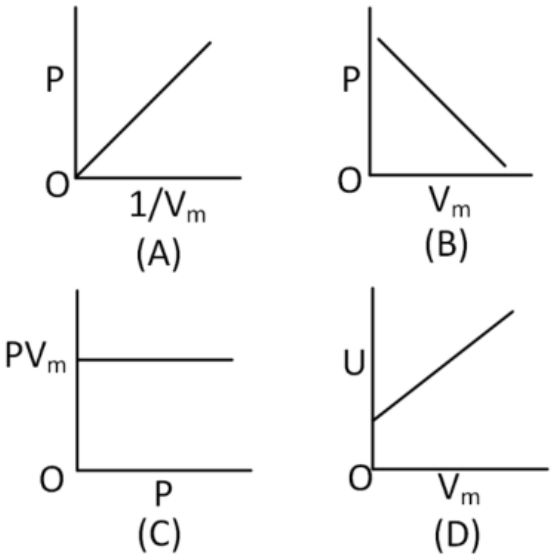
Step 3: Reject the visible and X-ray intervals The intervals 400–$550\,\mathrm{nm}$ and 600–$750\,\mathrm{nm}$ are visible light, which must substantially reach the ground and is not the radiation singled out by ozone protection. The interval 0.8–$1.5\,\mathrm{nm}$ is far shorter, in the X-ray region. It therefore does not describe the standard ultraviolet screening range of the ozone layer. Of the four choices, only the first has the

appropriate ultraviolet wavelengths.

Step 4: Conclusion The ozone layer protects us from radiation in the 200–$315\,\text{nm}$ region. Hence, the correct option is (A).

Quick Tip to Solve: Locate each wavelength on the spectrum: hundreds of nanometres below visible light indicate ultraviolet, whereas wavelengths near one nanometre indicate X-rays.

75. The combination of plots which does not represent isothermal expansion of an ideal gas is – [4 marks]



(A) A and D

(B) B and D

(C) B and C

(D) A and C

Correct Answer: (B) B and D

Solution:

Concept:

- For one mole of an ideal gas, $PV_m = RT$, where V_m is molar volume. During an isothermal change, T and therefore RT remain constant.
- The internal energy U of a fixed amount of ideal gas depends only on temperature. It remains constant during an isothermal expansion, even though pressure and volume change.

Step 1: Test graphs A and B using the gas equation Rearrange the equation as

$$P = RT\left(\frac{1}{V_m}\right).$$

At fixed temperature, a plot of P against $1/V_m$ is a straight line through the origin with positive slope

RT. Graph A has this form and is valid. In contrast, a plot of P against V_m must be a rectangular hyperbola, because their product is constant. Graph B is a straight decreasing line and is therefore invalid; a negative slope alone is insufficient.

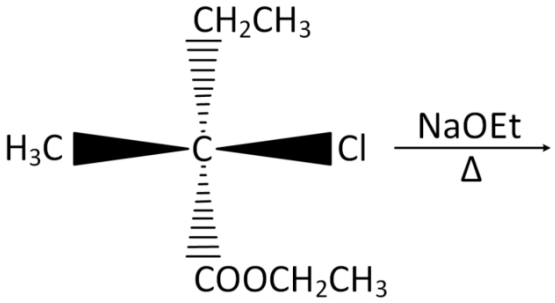
Step 2: Test graphs C and D separately The quantity $PV_m = RT$ has the same value at every pressure along the isotherm. Hence a plot of PV_m against P must be horizontal, just as graph C shows. For an ideal gas, $\Delta U = 0$ when $\Delta T = 0$. Consequently U should also be horizontal when plotted against V_m, not increase linearly as in graph D. Thus D is invalid.

Step 3: Select the pair of invalid plots The plots that do not represent the process are B and D. Option (A) incorrectly rejects valid graph A, option (C) incorrectly rejects valid graph C, and option (D) contains both valid graphs. The question asks for the invalid pair, so choosing A and C would reverse the requested classification.

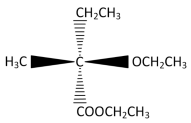
Step 4: Conclusion Graphs B and D do not represent isothermal expansion of an ideal gas. Hence, the correct option is (B).

Quick Tip to Solve: Check the axes before judging a graph: P is linear in $1/V_m$, while PV_m and U are constant on an ideal-gas isotherm.

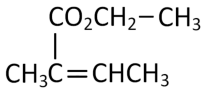
76. The major product of the following reaction is – [4 marks]



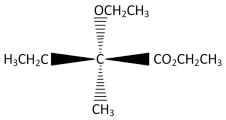
(A) $CH_3C(OC_2H_5)(C_2H_5)COOC_2H_5$, with the stereochemical arrangement shown.



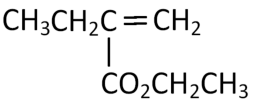
(B) $CH_3C(COOC_2H_5)=CHCH_3$.



(C) $CH_3C(OC_2H_5)(C_2H_5)COOC_2H_5$, with the alternative stereochemical arrangement shown.

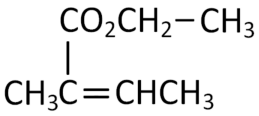


(D) $CH_3CH_2C(COOC_2H_5)=CH_2$.



Correct Answer: (B)

$CH_3C(COOC_2H_5)=CHCH_3$.



Solution:

Concept:

- Sodium ethoxide under heating promotes beta-elimination of hydrogen chloride from an alkyl chloride. An E2 elimination removes a beta-hydrogen and the leaving chloride while forming a carbon–carbon double bond.
- With a small base such as ethoxide, the more substituted alkene is generally favoured. Here both possible alkenes are conjugated with the ester carbonyl, so their relative substitution helps select the major product.

Step 1: Locate the available beta-hydrogens The carbon carrying chlorine is bonded to methyl, ethyl and ethoxycarbonyl groups. It has no hydrogen of its own. A beta-hydrogen may be removed from the methyl group or from the methylene carbon of the ethyl group; the ester carbonyl carbon has no hydrogen available for this elimination. Removing a hydrogen from the ethyl side gives $CH_3CH{=}C(CH_3)COOC_2H_5$, whereas removal from the methyl side gives $CH_2{=}C(CH_2CH_3)COOC_2H_5$.

Step 2: Apply the reaction conditions and substitution rule The reagent $NaOC_2H_5$ is a strong, relatively unhindered base, and heat favours elimination. Rotation around the relevant single bond permits an anti arrangement for E2 elimination. The internal alkene $CH_3CH{=}C(CH_3)COOC_2H_5$ has three carbon substituents on the double bond; the terminal alkene has two. The internal, more substituted alkene is therefore the major product under these conditions.

Step 3: Compare all four products Option (B) depicts this internal alkene, ethyl 2-methylbut-2-enoate. Option (D) is the less substituted terminal alkene and is a minor elimination possibility. Options (A) and (C) instead replace chlorine by ethoxy and retain a saturated carbon centre; these are substitution products, which do not give the favoured heated-base elimination product. The stereochemical wedges of those substitution alternatives do not change this distinction.

Step 4: Conclusion The major product is $CH_3CH{=}C(CH_3)COOC_2H_5$, ethyl 2-methylbut-2-enoate, shown in (B). Hence, the correct option is (B).

Quick Tip to Solve: With ethoxide and heat, identify both beta positions and compare the resulting alkenes; the more substituted internal alkene normally dominates over the terminal alkene.

77. The element that shows greater ability to form pπ-pπ multiple bonds, is : [4 marks]

(A) Si

(B) Ge

(C) C

(D) Sn

Correct Answer: (C) C

Solution:

Concept:

- A $p\pi$–$p\pi$ bond forms by sideways overlap of parallel p orbitals on adjacent atoms. Efficient sideways overlap requires orbitals of suitable size and sufficiently short interatomic separation.
- Down Group 14, atomic size and bond length increase. The compact second-shell $2p$ orbitals of carbon form strong pi bonds more readily than the larger orbitals of silicon, germanium and tin.

Step 1: Relate multiple bonds to orbital overlap A double bond has a sigma bond and one pi bond, while a triple bond has a sigma bond and two pi bonds. The pi contribution comes from sideways p-orbital overlap, not simply from the ability to make four sigma bonds. The question compares the tendency to form such multiple bonds among members of the carbon family, so the effectiveness of this sideways overlap is the deciding feature.

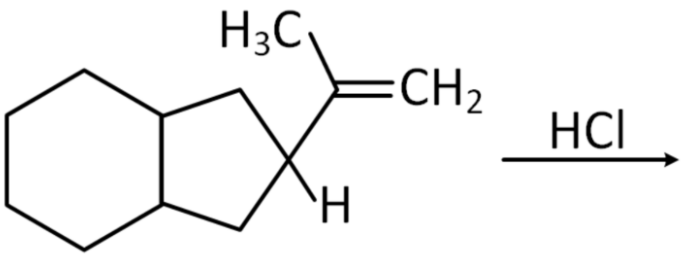
Step 2: Compare carbon with its heavier congeners Carbon is the smallest of the listed atoms. Adjacent carbon atoms can approach closely enough for strong $2p$–$2p$ sideways overlap, as illustrated by the stable double bonds in alkenes and triple bonds in alkynes. Silicon uses larger $3p$ orbitals, and germanium and tin use still larger valence p orbitals. Their longer interatomic distances make ordinary pi overlap less effective, so their tendency toward strong multiple bonds is lower.

Step 3: Eliminate the heavier Group 14 elements Silicon, germanium and tin can form multiple bonds in specially stabilised compounds, but that does not give them a greater general ability than carbon. Their larger size works against the strong pi overlap asked about here. Thus (A), (B) and (D) do not fit the comparison. Carbon, listed as option (C), has the greatest tendency to form $p\pi$–$p\pi$ multiple bonds among these elements.

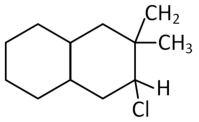
Step 4: Conclusion Carbon has the greatest ability to form $p\pi$–$p\pi$ multiple bonds among the listed elements. Hence, the correct option is (C).

Quick Tip to Solve: For ordinary pi bonding within a group, check atomic size and orbital overlap: compact carbon $2p$ orbitals are especially effective at sideways overlap.

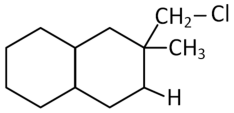
78. The major product of the following reaction is – [4 marks]



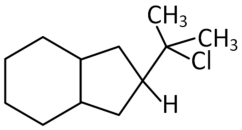
(A) Ring-expanded fused six-membered rings, with chlorine on the adjacent ring carbon (structure shown).



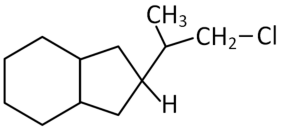
(B) Ring-expanded fused six-membered rings with a chloromethyl substituent (structure shown).



(C) Unchanged fused-ring skeleton bearing $-C(Cl)(CH_3)_2$ (structure shown).

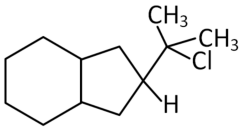


(D) Unchanged fused-ring skeleton bearing $-CH(CH_3)CH_2Cl$ (structure shown).



Correct Answer: (C)

Unchanged fused-ring skeleton bearing $-C(Cl)(CH_3)_2$ (structure shown).



Solution:

Concept:

- Hydrogen chloride adds to an alkene by protonation of the double bond followed by nucleophilic capture of the carbocation by Cl^-.
- Markovnikov orientation places the positive charge at the more substituted carbon. Rearrangement is favoured only when it gives a sufficiently more stable intermediate.

Step 1: Locate the favoured carbocation The side chain is of the form $R-C(CH_3)=CH_2$, where R is the fused-ring carbon bearing hydrogen. Protonation at the terminal methylene converts that

group into methyl and places the positive charge on the internal carbon. Thus the intermediate is $R-C^{+}(CH_3)_2$, a tertiary carbocation. Protonation in the opposite direction would generate a primary terminal carbocation and is disfavoured.

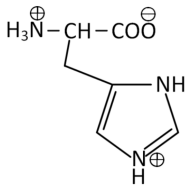
Step 2: Capture the ion without an unfavourable rearrangement Chloride attacks the tertiary centre to form $R-C(Cl)(CH_3)_2$. The fused five-membered ring remains intact. Expanding it by a carbon-carbon shift would move the charge away from this already tertiary centre and does not provide the necessary stability advantage. Therefore the two ring-expanded structures A and B are not the major product. Structure D places chlorine at the terminal carbon, the opposite regioselectivity from ordinary ionic HCl addition. No peroxide-induced reversal applies to HCl.

Step 3: Conclusion The major product retains the fused-ring skeleton and has $-C(Cl)(CH_3)_2$ as the side chain. Hence, the correct option is (C).

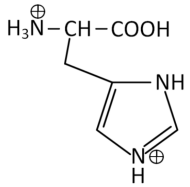
Quick Tip to Solve: Draw the carbocation before considering ring expansion: a pre-existing tertiary ion need not rearrange merely because a five-membered ring is nearby.

79. The correct structure of histidine in a strongly acidic solution (pH = 2) is - [4 marks]

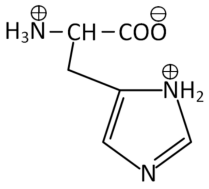
(A) NH_3^{+}, COO^{-}, and protonation at the pyridine-like imidazole nitrogen (structure shown).



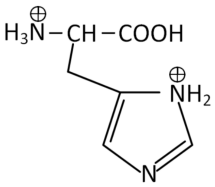
(B) NH_3^{+}, COOH, and protonation at the pyridine-like imidazole nitrogen (structure shown).



(C) NH_3^{+}, COO^{-}, and protonation at the pyrrole-like imidazole nitrogen (structure shown).

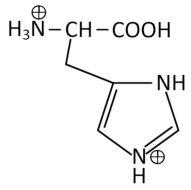


(D) NH_3^{+}, COOH, and protonation at the pyrrole-like imidazole nitrogen (structure shown).



Correct Answer: (B)

NH_3^+, COOH, and protonation at the pyridine-like imidazole nitrogen (structure shown).



Solution:

Concept:

- Histidine has an alpha-amino group, a carboxyl group and an imidazole side chain. An acidic medium favours protonation of basic sites.
- The pyridine-like imidazole nitrogen has a lone pair available for protonation; the pyrrole-like nitrogen contributes its lone pair to aromaticity.

Step 1: Protonate the amino and carboxyl groups In the strongly acidic representation requested, the amino group is $-NH_3^+$ and the carboxyl group is represented as $-COOH$. It is not the neutral amino acid or the usual zwitterionic form. Options A and C retain $-COO-$, so they do not represent the fully protonated acidic form sought here. Acid-base populations are equilibria, but the choice concerns the appropriate limiting acidic structure rather than a mixture.

Step 2: Identify the correct imidazole nitrogen In the original imidazole ring, one nitrogen already bears hydrogen and has its lone pair participating in the aromatic sextet. The other nitrogen is pyridine-like: its lone pair is outside that sextet and accepts the added proton. Protonation there produces an imidazolium ring without disrupting aromaticity. Option B shows this nitrogen bearing the additional H and positive charge, together with $-NH_3^+$ and $-COOH$. Option D instead adds a proton to the already hydrogen-bearing, pyrrole-like nitrogen and is not the favoured conjugate acid.

Step 3: Conclusion The required acidic structure has $-NH_3^+$, $-COOH$, and an imidazolium side chain protonated at the pyridine-like nitrogen. Hence, the correct option is (B).

Quick Tip to Solve: For a heteroaromatic nitrogen, ask whether its lone pair is part of aromaticity before assigning the site of protonation.

80. The magnetic moment of an octahedral homoleptic Mn(II) complex is 5.9 BM. The suitable ligand for this complex is - [4 marks]

(A) CN^-

(B) ethylenediamine

(C) NCS^-

(D) CO

Correct Answer: (C) NCS^-

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons.
- Mn^{2+} is a $3d^5$ ion. In an octahedral field, a weak ligand field favours the high-spin arrangement, while sufficiently strong splitting favours pairing.

Step 1: Determine the spin state from the measured moment For five unpaired electrons, the predicted moment is

$$\mu = \sqrt{5(5+2)} = \sqrt{35} = 5.92\,\text{BM}.$$

This agrees with the given 5.9 BM. The complex must therefore retain the high-spin d^5 configuration $t_{2g}^3 e_g^2$: all five d orbitals are singly occupied. A low-spin d^5 arrangement, t_{2g}^5, would have only one unpaired electron and a much smaller moment, about 1.73 BM.

Step 2: Choose the ligand consistent with high spin The ligand must give relatively small octahedral splitting so that promotion to the upper e_g orbitals costs less energy than pairing. Among the choices, NCS^- is the suitable weaker-field ligand. Cyanide and carbon monoxide are strong-field ligands, so A and D do not match the indicated high-spin state. Ethylenediamine produces a stronger field than thiocyanate and is not the intended weak-field choice in this comparison. Homoleptic means that only one kind of ligand is present; it does not itself determine the number of unpaired electrons.

Step 3: Conclusion The observed moment corresponds to five unpaired electrons in high-spin octahedral Mn^{2+}, for which NCS^- is suitable. Hence, the correct option is (C).

Quick Tip to Solve: Recognise 5.9 BM as the fingerprint of five unpaired electrons; then choose the ligand that preserves high spin.

81. Λ_m° **for** NaCl, HCl **and** NaA **are 126.4, 425.9 and 100.5** $\mathrm{S\,cm^2\,mol^{-1}}$**, respectively. If the conductivity of 0.001 M** HA **is** $5 \times 10^{-5}\,\mathrm{S\,cm^{-1}}$**, degree of dissociation of** HA **is -** [4 marks]

(A) 0.50

(B) 0.125

(C) 0.25

(D) 0.75

Correct Answer: (B) 0.125

Solution:

Concept:

- Kohlrausch's law adds limiting ionic conductivities: $\Lambda_m^\circ(HA) = \Lambda_m^\circ(HCl) + \Lambda_m^\circ(NaA) - \Lambda_m^\circ(NaCl)$.
- For a weak electrolyte, $\alpha = \Lambda_m/\Lambda_m^\circ$. When concentration c is in mol per litre and conductivity in S per cm, $\Lambda_m = 1000\kappa/c$.

Step 1: Calculate the limiting molar conductivity of HA Combine the strong-electrolyte data so that the sodium and chloride contributions cancel. The hydrogen and A-ion contributions remain:

$$\Lambda_m^\circ(HA) = 425.9 + 100.5 - 126.4 = 400.0\,\mathrm{S\,cm^2\,mol^{-1}}.$$

This is the conductivity per mole if HA were fully dissociated at infinite dilution; it is not the measured molar conductivity at the given concentration.

Step 2: Find the actual molar conductivity and compare Using $c = 0.001\,\mathrm{mol\,L^{-1}}$ and $\kappa = 5 \times 10^{-5}\,\mathrm{S\,cm^{-1}}$,

$$\Lambda_m = \frac{1000(5 \times 10^{-5})}{0.001} = 50\,\mathrm{S\,cm^2\,mol^{-1}}.$$

Hence

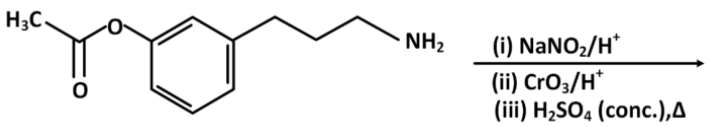
$$\alpha = \frac{50}{400} = 0.125.$$

The factor 1000 converts litres into cubic centimetres. Omitting it would give a degree of dissociation smaller by a thousand. The values 0.50, 0.25 and 0.75 do not equal the measured-to-limiting conductivity ratio, so A, C and D are excluded.

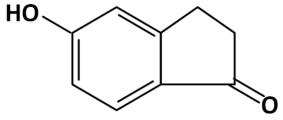
Step 3: Conclusion The degree of dissociation of HA is $\alpha = 0.125$, or 12.5 percent. Hence, the correct option is (B).

Quick Tip to Solve: Cancel the spectator-ion conductivities first; convert conductivity into molar conductivity before taking the dissociation ratio.

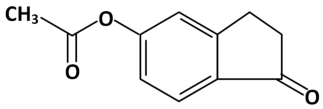
82. The major product of the following reaction is – [4 marks]



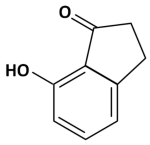
(A) 5-Hydroxyindan-1-one (structure shown).



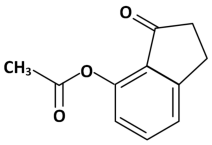
(B) 5-Acetoxyindan-1-one (structure shown).



(C) 7-Hydroxyindan-1-one (structure shown).

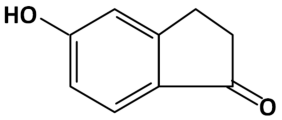


(D) 7-Acetoxyindan-1-one (structure shown).



Correct Answer: (A)

5-Hydroxyindan-1-one (structure shown).



Solution:

Concept:

- A primary aliphatic amine reacts with nitrous acid to give an unstable aliphatic diazonium intermediate, which loses nitrogen and gives the corresponding alcohol.
- A primary alcohol is oxidised to a carboxylic acid by acidic CrO_3. An arylpropanoic acid can undergo acid-promoted intramolecular acylation to form an indanone.

Step 1: Convert the terminal amino group into a carboxyl group The side chain attached to the aromatic ring is $-CH_2CH_2CH_2NH_2$. Treatment with $NaNO_2/H^+$ generates nitrous acid and replaces the terminal amino group by hydroxyl, giving $Ar-CH_2CH_2CH_2OH$ with evolution of N_2. Oxidation with CrO_3/H^+ converts that primary alcohol into $Ar-CH_2CH_2COOH$. The carboxyl carbon remains in the molecule and becomes the ketone carbonyl in the ring-forming step.

Step 2: Cyclise at the favoured aromatic position Hot concentrated sulphuric acid activates the carboxylic acid for intramolecular electrophilic acylation. Closure onto the aromatic carbon adjacent to the side-chain attachment forms a five-membered ring. Of the two available positions, attack para to the oxygen substituent is less hindered than attack ortho to it, giving the 5-substituted indan-1-one skeleton. The acidic reaction conditions also remove the acetate protecting group, exposing the

phenolic hydroxyl group. Thus A has both the appropriate regiochemistry and the final hydroxyl group. B retains acetate, whereas C and D correspond to closure at the more hindered position.

Step 3: Conclusion The major product is 5-hydroxyindan-1-one, the fused ketone shown in option A. Hence, the correct option is (A).

Quick Tip to Solve: Track the terminal carbon through alcohol to acid to cyclic ketone, and use the oxygen substituent to distinguish the two possible ring-closure positions.

83. The major product of the following reaction is:

$$CH_3CH_2CHBrCH_2Br \xrightarrow[\text{(ii) } NaNH_2 \text{ in liquid } NH_3]{\text{(i) alcoholic KOH}} \textbf{Product}$$

[4 marks]

(A) $CH_3CH_2C{\equiv}CH$

(B) $CH_3CH{=}C{=}CH_2$

(C) $CH_3CH_2CH(NH_2)CH_2NH_2$

(D) $CH_3CH{=}CHCH_2NH_2$

Correct Answer: (A) $CH_3CH_2C{\equiv}CH$

Solution:

Concept:

- Vicinal dihalides can lose two molecules of hydrogen halide to form an alkyne. The carbon skeleton is preserved during dehydrohalogenation.
- Alcoholic KOH effects the first elimination. A much stronger base, $NaNH_2$ in liquid ammonia, removes hydrogen halide from the vinyl halide formed.

Step 1: Recognise the adjacent brominated carbons The starting compound is $CH_3CH_2CHBrCH_2Br$, a four-carbon vicinal dibromide. The two bromine atoms are attached to adjacent carbons at the end of the chain. Elimination of one molecule of HBr with alcoholic KOH introduces a carbon-carbon double bond and leaves a vinyl bromide. This first step increases the degree of unsaturation without replacing bromine by an amino group.

Step 2: Carry out the second elimination with the stronger base The vinyl halide is harder to dehydrohalogenate because its carbon-bromine bond involves an sp2 carbon, so sodium amide in liquid ammonia is used. A second HBr is eliminated, producing

$$CH_3CH_2CHBrCH_2Br \xrightarrow{-2\,HBr} CH_3CH_2C{\equiv}CH.$$

The final neutral hydrocarbon is but-1-yne. Option B is an allene and does not represent the standard double elimination between the original vicinal halogen-bearing carbons. Options C and D contain amino groups and correspond to substitution rather than the intended successive eliminations. If excess amide transiently forms an acetylide, protonation during ordinary isolation restores the terminal alkyne.

Step 3: Conclusion The major hydrocarbon product is but-1-yne, $CH_3CH_2C{\equiv}CH$. Hence, the correct option is (A).

Quick Tip to Solve: A vicinal dibromide followed by alcoholic base and sodium amide signals two eliminations: retain the carbon chain and replace the adjacent C–C bond by a triple bond.

84. An open vessel at 27°C is heated until two fifth of the air (assumed as an ideal gas) it has escaped from the vessel assuming that the volume of the vessel remains constant, the temperature at which the vessel has been heated is - [4 marks]

(A) 750 K

(B) 500°C

(C) 750°C

(D) 500 K

Correct Answer: (D) 500 K

Solution:

Concept:

- The ideal-gas equation is $PV = nRT$. For an open vessel, pressure stays equal to atmospheric pressure; the vessel volume is fixed.
- With P and V constant, nT is constant. Temperatures in gas-law ratios must be absolute temperatures in kelvin.

Step 1: Determine the fraction of gas left The vessel initially contains n_1 moles of air at $27^\circ\mathrm{C}$. Convert this to

$$T_1 = 27 + 273 = 300\,\mathrm{K}.$$

Two fifths of the original air escapes, so the remaining amount is

$$n_2 = \left(1 - \frac{2}{5}\right) n_1 = \frac{3}{5} n_1.$$

It would be incorrect to use two fifths as the final amount: that is the portion lost, not the portion retained.

Step 2: Apply the gas law at constant pressure and volume Because the vessel is open, it cannot sustain a pressure rise above its surroundings in the final equilibrium state. Applying the ideal-gas equation before and after heating gives

$$n_1 R T_1 = PV = n_2 R T_2.$$

Hence

$$T_2 = \frac{n_1}{n_2} T_1 = \frac{5}{3}(300) = 500\,\mathrm{K}.$$

This corresponds to about $227^\circ\mathrm{C}$, not $500^\circ\mathrm{C}$. Option A results from confusing the escaped fraction with the retained fraction, while B and C mix the temperature scales or use the wrong fraction. The

decrease in moles is consistent with the increase in temperature at fixed PV.

Step 3: Conclusion The vessel is heated to $500\,\mathrm{K}$. Hence, the correct option is (D).

Quick Tip to Solve: For an open rigid vessel, keep pressure and volume fixed and use nT constant; always calculate the fraction remaining before substituting.

85. The pair that does NOT require calcination is - [4 marks]

(A) Fe_2O_3 and $CaCO_3{\cdot}MgCO_3$

(B) $ZnCO_3$ and CaO

(C) ZnO and $Fe_2O_3{\cdot}xH_2O$

(D) ZnO and MgO

Correct Answer: (D) ZnO and MgO

Solution:

Concept:

- Calcination is strong heating in the absence or limited supply of air, used to remove water and volatile matter or to decompose a carbonate ore into its oxide.
- An anhydrous metal oxide is already in the form produced by calcination. Do not confuse this preliminary conversion with the subsequent reduction of an oxide to metal.

Step 1: Identify the compounds that need conversion A carbonate loses carbon dioxide on heating. Thus zinc carbonate requires calcination:

$$\mathrm{ZnCO_3} \xrightarrow{\Delta} \mathrm{ZnO} + \mathrm{CO_2}.$$

The double carbonate $\mathrm{CaCO_3 \cdot MgCO_3}$ also contains carbonate groups and decomposes on heating. The fact that the other member of a pair is an oxide does not make the entire pair free from calcination.

Step 2: Check hydrated and anhydrous oxides The hydrated iron oxide $\mathrm{Fe_2O_3}.x\mathrm{H_2O}$ contains combined water, which is removed on heating:

$$\mathrm{Fe_2O_3}.x\mathrm{H_2O} \xrightarrow{\Delta} \mathrm{Fe_2O_3} + x\mathrm{H_2O}.$$

In contrast, ZnO and MgO contain neither carbonate nor water of hydration. Both are already anhydrous oxides.

Step 3: Eliminate each unsuitable pair Option (A) fails because its double carbonate requires decomposition. Option (B) fails because $\mathrm{ZnCO_3}$ must lose $\mathrm{CO_2}$. Option (C) fails because the hydrated iron oxide must lose water. Only (D) contains two compounds for which this preliminary thermal conversion is unnecessary.

Step 4: Conclusion The pair is ZnO and MgO. Hence, the correct option is (D).

Quick Tip to Solve: For a pair question, test both members: carbonate or hydrated oxide in either position means calcination is needed.

86. The compound that is NOT a common component of photochemical smog is : [4 marks]

(A) O_3

(B) $CH_3C(=O)OONO_2$

(C) $CH_2=CHCHO$

(D) CF_2Cl_2

Correct Answer: (D) CF_2Cl_2

Solution:

Concept:

- Photochemical smog forms when sunlight drives reactions involving nitrogen oxides and volatile organic compounds. Its characteristic pollutants are strong oxidants and irritating organic products.
- Ozone, peroxyacetyl nitrate (PAN), and aldehydes such as acrolein are common components. A chlorofluorocarbon is primarily associated with stratospheric ozone depletion.

Step 1: Identify the common oxidants Option (A) is ozone, O_3. Nitrogen dioxide absorbs sunlight and supplies oxygen atoms, which combine with molecular oxygen to form ozone:

$$NO_2 \xrightarrow{h\nu} NO + O, \qquad O + O_2 \longrightarrow O_3.$$

Consequently ozone is an important secondary pollutant in photochemical smog and cannot be the requested exception.

Step 2: Recognise the organic smog products The structure in (B), $CH_3C(=O)OONO_2$, is peroxyacetyl nitrate, commonly abbreviated PAN. It is formed through oxidation of organic compounds in the presence of nitrogen oxides and causes eye irritation and plant damage. Option (C), $CH_2=CHCHO$, is acrolein, an unsaturated aldehyde and another irritating component of photochemical smog. These two options therefore also fail the word NOT.

Step 3: Distinguish the chlorofluorocarbon Option (D), CF_2Cl_2, is dichlorodifluoromethane, a chlorofluorocarbon. Its environmental significance is release of chlorine radicals under ultraviolet radiation in the stratosphere, leading to ozone depletion. That process is distinct from formation of the common ground-level photochemical-smog mixture.

Step 4: Conclusion CF_2Cl_2 is not a common component of photochemical smog. Hence, the correct option is (D).

Quick Tip to Solve: Separate ground-level smog products (ozone, PAN and aldehydes) from CFCs associated with stratospheric ozone destruction.

87. The volume strength of 1 M H_2O_2 is : (Molar mass of H_2O_2 = $34\,g\,mol^{-1}$) [4 marks]

(A) 5.6

(B) 22.4

(C) 11.35

(D) 16.8

Correct Answer: (C) 11.35

Solution:

Concept:

- Volume strength is the volume of oxygen, measured at the reference standard conditions, liberated by complete decomposition of one volume of hydrogen peroxide solution.
- The balanced decomposition is $2\,H_2O_2 \longrightarrow 2\,H_2O + O_2$. Hence one mole of hydrogen peroxide yields half a mole of oxygen; molarity refers to moles per litre of solution.

Step 1: Convert molarity into oxygen amount Take $1\,L$ of the given $1\,mol\,L^{-1}$ solution. It contains $1\,mol$ of H_2O_2, equivalent to $34\,g$. From the balanced equation,

$$n(O_2) = \frac{1}{2}n(H_2O_2) = 0.5\,mol.$$

The solution volume is not the oxygen volume: the stoichiometric factor must be applied first.

Step 2: Calculate the volume at standard pressure Using standard temperature $273.15\,K$ and standard pressure $1\,bar$, the ideal-gas molar volume is approximately $22.7\,L\,mol^{-1}$. Therefore

$$V(O_2) = 0.5 \times 22.7 = 11.35\,L.$$

Each litre of solution releases 11.35 litres of oxygen at these conditions, so its volume strength is 11.35.

Step 3: Compare the reference convention and options The familiar value $22.4\,L\,mol^{-1}$ uses $1\,atm$, giving 11.2-volume strength instead. The printed value 11.35 corresponds to the $1\,bar$ convention. Option (B) ignores the factor of one-half; (A) is half the already halved older-convention value, and (D) does not follow the decomposition stoichiometry for a 1 M solution.

Step 4: Conclusion At standard temperature and $1\,bar$, the volume strength is 11.35 (11.35 volumes of oxygen per volume of solution). Hence, the correct option is (C).

Quick Tip to Solve: Always state the gas reference conditions: 1 bar gives about 22.7 litres per mole, whereas 1 atm gives about 22.4.

88. Among the following the false statement is - [4 marks]

(A) Lyophilic sol can be coagulated by adding an electrote

(B) Tyndall effect can be used to distinguish between a colloidal solution and a true solution

(C) It is possible to cause artificial rain by throwing electrified sand carrying charge opposite to the one on clouds from an aeroplane

(D) Latex is a colloidal solution of rubber particles which are positively charged

Correct Answer: (D) Latex is a colloidal solution of rubber particles which are positively charged

Solution:

Concept:

- Colloidal particles scatter light (the Tyndall effect) and commonly carry an electric charge. Neutralisation of that charge can permit particles to aggregate or coagulate.
- Lyophilic sols are stabilised by solvation as well as charge, so their coagulation generally needs a much larger electrolyte concentration than that of lyophobic sols.

Step 1: Check coagulation and light scattering Option (A) says a lyophilic sol can be coagulated by adding an electrolyte. This is possible with sufficient electrolyte, which reduces solvation and allows aggregation; the statement does not say that a small amount is enough. Option (B) is also true: colloidal particles scatter a beam strongly enough to make its path visible, whereas particles in a true solution are too small to show the usual observable Tyndall cone.

Step 2: Check the rain-production statement Clouds contain dispersed charged water droplets. Introducing particles with an opposite charge can help neutralise the droplets and promote their coalescence. The resulting larger drops can fall as rain. This is the colloid-coagulation principle behind the statement in (C), so it is not the false option sought here.

Step 3: Determine the sign of latex particles Latex is a colloidal dispersion of rubber particles in water, but the rubber particles carry a negative charge. The sign in option (D) is therefore reversed. Its identification of latex as a colloid does not make the whole sentence true: the claim that the rubber particles are positively charged is the decisive error. Oppositely charged ions can destabilise latex and cause coagulation.

Step 4: Conclusion The false statement is (D): latex particles are negatively charged, not positively charged. Hence, the correct option is (D).

Quick Tip to Solve: In a false-statement question, check qualifiers and charge signs carefully; lyophilic stability means difficult coagulation, not impossible coagulation.

89. Chlorine on reaction with hot and concentrated sodium hydroxide give : [4 marks]

(A) $C\ell O_3^-$ and $C\ell O_2^-$

(B) $C\ell^-$ and $C\ell O^-$

(C) $C\ell^-$ and $C\ell O_2^-$

(D) $C\ell^-$ and $C\ell O_3^-$

Correct Answer: (D) $C\ell^-$ and $C\ell O_3^-$

Solution:

Concept:

- Chlorine undergoes disproportionation in alkali: atoms of the same element in oxidation state zero are simultaneously reduced to chloride and oxidised to an oxyanion.
- The temperature and concentration of the alkali determine the oxidised product: cold dilute alkali gives hypochlorite; hot concentrated alkali gives chlorate.

Step 1: Write the reaction for the stated conditions The question specifies hot and concentrated sodium hydroxide, so chlorine forms sodium chloride and sodium chlorate, together with water:

$$3\,Cl_2 + 6\,NaOH \xrightarrow{\text{hot}} 5\,NaCl + NaClO_3 + 3\,H_2O.$$

The equation contains six chlorine atoms and six sodium atoms on each side. The six oxygen atoms on the left appear as three in chlorate and three in water on the right.

Step 2: Check the oxidation-state balance In Cl_2, chlorine has oxidation state zero. Five chlorine atoms become Cl^-, each gaining one electron. The remaining atom becomes chlorine in $ClO_3{}^-$, with oxidation state $+5$, losing five electrons. Thus electron gain and loss balance, confirming the 5:1 ratio of chloride to chlorate in the equation.

Step 3: Reject products for other conditions Option (B), chloride and hypochlorite, belongs to cold dilute alkali:

$$Cl_2 + 2\,NaOH \longrightarrow NaCl + NaClO + H_2O.$$

It is inappropriate for the stated hot concentrated solution. Options (A) and (C) contain chlorite, $ClO_2{}^-$, which is not the product of the standard reaction here. Option (A) also omits chloride, the reduced species needed in this disproportionation.

Step 4: Conclusion The ions formed are Cl^- and $ClO_3{}^-$. Hence, the correct option is (D).

Quick Tip to Solve: Read hot/concentrated before choosing the oxyanion: chlorate accompanies chloride; cold/dilute conditions instead give hypochlorite.

90. The two monomers for the synthesis of Nylon 6, 6 are [4 marks]

(A) $HOOC(CH_2)_6COOH$, $H_2N(CH_2)_4NH_2$

(B) $HOOC(CH_2)_4COOH$, $H_2N(CH_2)_6NH_2$

(C) $HOOC(CH_2)_4COOH$, $H_2N(CH_2)_4NH_2$

(D) $HOOC(CH_2)_6COOH$, $H_2N(CH_2)_6NH_2$

Correct Answer: (B) $HOOC(CH_2)_4COOH$, $H_2N(CH_2)_6NH_2$

Solution:

Concept:

- Nylon 6,6 is a condensation polyamide formed from a diamine and a dicarboxylic acid. Each amine group reacts with a carboxyl group to make an amide linkage and eliminate water.
- The two sixes count the carbon atoms in the diamine and the dicarboxylic acid respectively. The carboxyl carbon atoms are included in the acid carbon count.

Step 1: Identify the six-carbon diamine The diamine is hexamethylenediamine, $H_2N(CH_2)_6NH_2$, also called hexane-1,6-diamine. Its six methylene groups supply six carbon atoms; the terminal amino groups supply no additional carbon. A diamine with only four methylene groups would have four carbons and would not supply the required diamine component.

Step 2: Count the acid carbons correctly The acid is adipic acid, $HOOC(CH_2)_4COOH$, also called hexanedioic acid. Its four methylene carbons plus the two carboxyl carbons give six in all. In contrast, $HOOC(CH_2)_6COOH$ has eight carbon atoms, because the two terminal carboxyl carbon atoms must also be counted.

Step 3: Match the pair to the polymer repeat The repeating chain contains

$$[-NH-(CH_2)_6-NH-CO-(CH_2)_4-CO-]_n .$$

Option (B) supplies both correct monomers. Option (A) pairs an eight-carbon acid with a four-carbon diamine. Option (C) uses the correct acid but the wrong four-carbon diamine. Option (D) uses the correct diamine but an eight-carbon acid. None of these three pairs forms Nylon 6,6.

Step 4: Conclusion The monomers are adipic acid, $HOOC(CH_2)_4COOH$, and hexamethylenediamine, $H_2N(CH_2)_6NH_2$. Hence, the correct option is (B).

Quick Tip to Solve: For the acid, include both COOH carbon atoms: four internal methylene groups make a six-carbon dicarboxylic acid.