JEE Main 2019 Paper 1 (B.E./B.Tech) Jan 9 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 9 January 2019, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. Let $a_1, a_2, ..., a_{30}$ be an A.P.,
$S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$.
If a_5 = 27 and S - 2T = 75, then a_{10} is equal to : [4 marks]

(A) 47

(B) 42

(C) 52

(D) 57

Correct Answer: (C) 52

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, the term at position r is $a_r = a + (r-1)d$. In particular, the difference between an even-positioned term and its preceding odd-positioned term is always d.
- The sum S contains all thirty terms, whereas T contains only the fifteen odd-positioned terms. Splitting the full sum into its odd and even positions avoids calculating either sum separately.

Step 1: Separate the odd and even positions Write $E = a_2 + a_4 + \cdots + a_{30}$. Since $S = E + T$, the stated difference is

$$S - 2T = E - T = \sum_{i=1}^{15}(a_{2i} - a_{2i-1}).$$

There are fifteen consecutive odd-even pairs in this expression. Each difference is d, with the same sign because the even term follows the odd term. Thus $S - 2T = 15d$, rather than $30d$.

Step 2: Use the given term to find the required term The numerical condition gives

$$15d = 75 \quad \Longrightarrow \quad d = 5.$$

From position 5 to position 10 there are five increments, not ten. Hence

$$a_{10} = a_5 + (10-5)d = 27 + 5(5) = 52.$$

As a check, the first term would be $a = 27 - 4(5) = 7$, and then $a_{10} = 7 + 9(5) = 52$ again. The other proposed values, 47, 42 and 57, are respectively four, three and six increments above 27; their positions are therefore 9, 8 and 11, rather than 10.

Step 3: Conclusion The common difference is 5 and the required term is $a_{10} = 52$. Hence, the correct option is (C).

Quick Tip to Solve: For a full sum minus twice the odd-position sum, pair each even term with the immediately preceding odd term; every pair contributes one common difference.

2. If a, b, c be three distinct real numbers in G.P. and a + b + c = xb , then x cannot be [4 marks]

(A) 2

(B) -3

(C) 4

(D) -2

Correct Answer: (A) 2

Solution:

Concept:

- Three distinct real terms in geometric progression can be written as a, ar, ar^2, where both a and r are nonzero. A zero first term or zero ratio would repeat a zero term, violating distinctness.
- After dividing the sum by the middle term, the parameter is $x = 1 + r + 1/r$. For real nonzero r, the quantity $r + 1/r$ lies in $(-\infty, -2] \cup [2, \infty)$.

Step 1: Express the parameter through the common ratio Substituting the progression into $a+b+c = xb$ gives

$$a + ar + ar^2 = xar \quad \Longrightarrow \quad r^2 + (1 - x)r + 1 = 0.$$

Division by ar is legitimate because distinctness rules out $a = 0$ and $r = 0$. The common ratio must be a real root of this quadratic, whose discriminant is

$$\Delta = (1 - x)^2 - 4.$$

In fact the boundary ratios $r = 1$ and $r = -1$ also fail distinctness: the first repeats all three terms, and the second repeats the first as the third.

Step 2: Test the four listed parameter values For $x = 2$, the quadratic becomes $r^2 - r + 1 = 0$, with discriminant -3. It has no real ratio, so this value is impossible even before the distinctness condition is applied. For comparison, $x = -3$ gives $r = -2 \pm \sqrt{3}$, $x = 4$ gives $r = (3 \pm \sqrt{5})/2$, and $x = -2$ gives $r = (-3 \pm \sqrt{5})/2$. Each of those choices has a real, nonzero ratio different from both 1 and -1. Taking the first term as 1 then produces three distinct real terms in each case. Thus the remaining three values are actually attainable, not merely values with a nonnegative discriminant.

Step 3: Conclusion The value $x = 2$ cannot arise from three distinct real terms in geometric progression. Hence, the correct option is (A).

Quick Tip to Solve: Divide a three-term geometric-progression sum by its middle term, then check the real-ratio discriminant and exclude ratios 1 and -1 for distinct terms.

3. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression

$3(\cos\theta - \sin\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

equals : [4 marks]

(A) 13 – 4 $\cos^2\theta$ + 6$\sin^2\theta\cos^2\theta$

(B) 13 – 4 $\cos^6\theta$

(C) 13 – 4 $\cos^2\theta$ + 6$\cos^2\theta$

(D) 13 – 4 $\cos^4\theta$ + 2$\sin^2\theta\cos^2\theta$

Correct Answer: (B) 13 – 4 $\cos^6\theta$

Solution:

Concept:

- Use $\sin^2\theta + \cos^2\theta = 1$ to simplify the squares of $\sin\theta \pm \cos\theta$. Their squares are $1 \pm 2\sin\theta\cos\theta$, so expansion makes the terms linear in the product cancel.
- After that cancellation, writing $t = \cos^2\theta$ changes $\sin^2\theta$ to $1 - t$. The remaining sixth power becomes the cubic $(1-t)^3$, allowing all lower powers to cancel exactly.

Step 1: Expand the two compound-angle factors Put $s = \sin\theta$ and $c = \cos\theta$, and denote the required expression by F. Since $s^2 + c^2 = 1$,

$$\begin{aligned} F &= 3(1-2sc)^2 + 6(1+2sc) + 4s^6 \\ &= 3 - 12sc + 12s^2c^2 + 6 + 12sc + 4s^6 \\ &= 9 + 12s^2c^2 + 4s^6. \end{aligned}$$

The opposite signs of the two linear product terms are essential; discarding the cross term before squaring would give an incorrect result.

Step 2: Rewrite the result using a single squared function Set $t = c^2$, so $s^2 = 1 - t$. Then

$$\begin{aligned} F &= 9 + 12t(1-t) + 4(1-t)^3 \\ &= 9 + 12t - 12t^2 + 4 - 12t + 12t^2 - 4t^3 \\ &= 13 - 4t^3 = 13 - 4\cos^6\theta. \end{aligned}$$

This is an identity, so it holds throughout the stated open interval. The other expressions retain second or fourth powers and do not reproduce this polynomial. For example, at $\theta = \pi/3$, which lies in the interval, the original expression and option (B) equal $207/16$, whereas options (A), (C) and (D) equal $105/8$, $27/2$ and $105/8$, respectively.

Step 3: Conclusion The expression simplifies to $13 - 4\cos^6\theta$. Hence, the correct option is (B).

Quick Tip to Solve: When both sine-plus-cosine and sine-minus-cosine occur, expand in their product first; switch to $\cos^2\theta$ only after the linear product terms cancel.

4. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to : [4 marks]

(A) -256

(B) 512

(C) -512

(D) 256

Correct Answer: (A) -256

Solution:

Concept:

- The quadratic formula gives the conjugate roots $-1+i$ and $-1-i$. Their power sum is real because the imaginary parts of conjugate powers cancel.
- Rather than expanding fifteenth powers directly, find a small power that is real. For these roots the fourth power is -4, so the fifteenth power can be evaluated as the product of the twelfth and third powers.

Step 1: Find the roots and their convenient low powers Solving the given equation yields

$$\alpha = -1 + i, \qquad \beta = -1 - i.$$

Their squares and fourth powers are

$$\alpha^2 = -2i, \quad \beta^2 = 2i, \qquad \alpha^4 = \beta^4 = -4.$$

Consequently $\alpha^{12} = \beta^{12} = (-4)^3 = -64$. The sign here is negative: although the exponent 12 is even, the roots are complex, so a positivity rule for even powers of real numbers does not apply.

Step 2: Reduce the fifteenth-power sum The sum and product of the roots are $\alpha + \beta = -2$ and $\alpha\beta = 2$. Using the identity for a sum of cubes gives

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = -8 + 12 = 4.$$

Now factor the common twelfth power:

$$\alpha^{15} + \beta^{15} = -64(\alpha^3 + \beta^3) = -64(4) = -256.$$

For a direct consistency check, $\alpha^3 = 2 + 2i$ and $\beta^3 = 2 - 2i$, whose sum is indeed 4. The answer must therefore be negative with magnitude 256. This excludes both positive options immediately, and the remaining negative option -512 has twice the required magnitude.

Step 3: Conclusion The required power sum is $\alpha^{15} + \beta^{15} = -256$. Hence, the correct option is (A).

Quick Tip to Solve: For roots involving $\pm i$, compute their square and fourth power first; a real low power often reduces a large exponent to a small one.

5. For $x \in R - \{0, 1\}$, Let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then J(x) is equal to : [4 marks]

(A) $f_1(x)$

(B) $\frac{1}{x} f_3(x)$

(C) $f_2(x)$

(D) $f_3(x)$

Correct Answer: (D) $f_3(x)$

Solution:

Concept:

- Function composition is evaluated from right to left. The relation $(f_2 \circ J \circ f_1)(x) = f_3(x)$ therefore means $f_2(J(f_1(x))) = f_3(x)$, rather than a product of functions.
- The reciprocal map $x \mapsto 1/x$ maps $\mathbb{R} \setminus \{0, 1\}$ onto itself. Thus replacing $1/x$ by a new variable is reversible on the stated domain and determines J throughout that domain.

Step 1: Undo the outer function Since $f_1(x) = 1/x$ and $f_2(y) = 1 - y$, the composition equation becomes

$$1 - J(1/x) = \frac{1}{1-x}.$$

Isolate the unknown middle function:

$$J(1/x) = 1 - \frac{1}{1-x} = \frac{(1-x)-1}{1-x} = \frac{-x}{1-x} = \frac{x}{x-1}.$$

The restriction $x \neq 1$ ensures that these fractions are defined. Notice that this equation gives the value at $1/x$, not yet the value at x; failing to replace the argument would select the wrong expression.

Step 2: Replace the reciprocal argument and verify Let $t = 1/x$, so $x = 1/t$. Substitution gives

$$J(t) = \frac{1/t}{1/t - 1} = \frac{1}{1-t} = f_3(t).$$

Verify in the original order:

$$1 - J(1/x) = 1 - \frac{1}{1 - 1/x} = 1 - \frac{x}{x-1} = \frac{1}{1-x}.$$

This equals the required right-hand side for every admissible x. The other candidates are different functions: at $t = 2$, they give $f_1(2) = 1/2$, $f_3(2)/2 = -1/2$ and $f_2(2) = -1$, with the last coinciding only at that point. At $t = 3$, $f_2(3) = -2$ differs from $J(3) = -1/2$, ruling it out as an identity.

Step 3: Conclusion On the given domain, $J(x) = 1/(1-x) = f_3(x)$. Hence, the correct option is (D).

Quick Tip to Solve: After undoing the outer function, rename the complete inner argument; an equation for J(1/x) is not yet a formula for J(x).

6. If the fractional part of the number $\left\{\frac{2^{403}}{15}\right\}$ is $\frac{k}{15}$, then k is equal to : [4 marks]

(A) 8

(B) 14

(C) 6

(D) 1

Correct Answer: (A) 8

Solution:

Concept:

- For a positive integer N, division by 15 gives $N = 15q + r$, with $0 \leq r < 15$. The fractional part of $N/15$ is exactly $r/15$, so the problem asks for a remainder, not a large power.
- The powers of 2 repeat modulo 15 because $2^4 = 16 \equiv 1 \pmod{15}$. Reducing the exponent modulo 4 determines the remainder without approximating the quotient.

Step 1: Reduce the exponent using the repeating remainder Divide the exponent by the cycle length:

$$403 = 4(100) + 3.$$

Thus

$$2^{403} = (2^4)^{100}2^3 = 16^{100} \cdot 8.$$

Since each factor 16 is congruent to 1 modulo 15, this gives $2^{403} \equiv 8 \pmod{15}$. The complete cycle of positive remainders is $2, 4, 8, 1$, so exponent 403, which is three beyond a multiple of four, belongs to the remainder-8 position.

Step 2: Convert the remainder to the requested fractional part There is therefore an integer q such that

$$2^{403} = 15q + 8, \qquad \frac{2^{403}}{15} = q + \frac{8}{15}.$$

The fraction $8/15$ lies strictly between 0 and 1, so it is already the fractional part; no further integer must be removed. Comparing with the given form $k/15$ yields $k = 8$. Among the alternatives, 1 is a possible remainder for exponents divisible by 4, but 403 is not such an exponent. Remainders 14 and 6 never occur in this four-term cycle. This also checks that the chosen value comes from the exact remainder rather than an estimate of a very large real number.

Step 3: Conclusion The fractional part is $8/15$, so $k = 8$. Hence, the correct option is (A).

Quick Tip to Solve: For the fractional part of an integer power divided by a small integer, compute the numerator modulo the denominator before doing any division.

7. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral $\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}dx$ is equal to :
(where c is a constant of integration) [4 marks]

(A) $\log_e \left|\frac{1}{2}\sec^2\left(x^2-1\right)\right| + c$
(B) $\frac{1}{2}\log_e \left|\sec\left(x^2-1\right)\right| + c$
(C) $\frac{1}{2}\log_e \left|\sec^2\left(\frac{x^2-1}{2}\right)\right| + c$
(D) $\log_e \left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$

Correct Answer: Options (C) and (D) are identical, and the principal radical requires an interval-dependent sign; the supplied key selects (D).

Solution:

Concept:

- Put $u = x^2 - 1$, so $du = 2x\,dx$. The identity $\sin 2u = 2\sin u\cos u$ reduces the ratio under the radical to $(1-\cos u)/(1+\cos u) = \tan^2(u/2)$ wherever the original fraction is defined.
- The principal real square root satisfies $\sqrt{z^2} = |z|$, not always z. Consequently the sign of $\tan(u/2)$ must be fixed separately on each connected interval of the integrand domain.

Step 1: Simplify the literal radical and integrate on an interval Let $v = (x^2-1)/2$; then $dv = x\,dx$. Cancelling the nonzero factor $2\sin u$ in the original ratio gives

$$I = \int x\,|\tan v|\,dx = \int |\tan v|\,dv.$$

On any interval where $\tan v > 0$, this is $\log|\sec v| + c$. Where $\tan v < 0$, it is $-\log|\sec v| + c$. In a single local formula, with σ the constant sign of $\tan v$ on the interval,

$$I = \sigma \log|\sec v| + c.$$

Differentiating gives $\sigma x \tan v = x|\tan v|$, exactly the original integrand after the identity. Points at which its original denominator vanishes must remain excluded; cancelling a factor does not define those points.

Step 2: Compare the printed alternatives and the key The last two printed answers are identical functions wherever defined:

$$\tfrac{1}{2}\log|\sec^2 v| = \log|\sec v|.$$

Thus (C) and (D) both give the positive-sign antiderivative on intervals with $\tan v > 0$. The supplied key selects (D), but the printed expression (C) is mathematically equivalent. Neither gives the negative-sign branch needed when $\tan v < 0$, a possibility not removed by the stated exclusion. For example, $x^2 - 1 = 3\pi/2$ is allowed and gives $\tan v = -1$. Options (A) and (B) instead involve $x^2 - 1$, rather than its half-angle, and their derivatives do not match the radical in general. A unique globally valid printed option therefore cannot be asserted.

Step 3: Conclusion The supplied key selects (D); (C) is identical to (D). They apply on intervals where $\tan((x^2-1)/2) > 0$; on negative-sign intervals the antiderivative is their negative. The printed question is ambiguous, so no unique option is selected.

Quick Tip to Solve: Retain the absolute value when taking the square root of a squared tangent. Compare logarithmic options using $\log(z^2) = 2\log|z|$ before selecting one.

8. The value of $\int_0^{\pi} |\cos x|^3 \, dx$ is : [4 marks]

(A) $\frac{4}{3}$

(B) $-\frac{4}{3}$

(C) 0

(D) $\frac{2}{3}$

Correct Answer: (A) $\frac{4}{3}$

Solution:

Concept:

- An absolute value removes the sign: $|\cos x|^3 \geq 0$. Thus the negative part of the cosine curve contributes positive area rather than cancelling the first half.
- The change $x \mapsto \pi - x$ leaves $|\cos x|$ unchanged. On $[0, \pi/2]$, $\cos x \geq 0$, and the substitution $u = \sin x$ evaluates the remaining cubic cosine integral.

Step 1: Use symmetry about the midpoint Let I denote the required integral. For $0 \leq x \leq \pi/2$, the cosine is nonnegative, whereas on the second half it is nonpositive. Since $\cos(\pi - x) = -\cos x$, the absolute values on the two halves agree. Therefore

$$I = 2\int_0^{\pi/2} \cos^3 x \, dx.$$

Without the absolute value, the integral of $\cos^3 x$ over $[0, \pi]$ would be zero. That cancellation is inapplicable to the printed integrand.

Step 2: Evaluate the integral on the first half Write $\cos^3 x = \cos x(1 - \sin^2 x)$. Set $u = \sin x$, so $du = \cos x \, dx$, and the bounds become 0 and 1:

$$\begin{aligned} I &= 2\int_0^1 (1 - u^2) \, du \\ &= 2\left[u - \frac{u^3}{3}\right]_0^1 = 2\left(1 - \frac{1}{3}\right) = \frac{4}{3}. \end{aligned}$$

The value $2/3$ counts only the first half. The zero and negative choices contradict the positive integrand on an interval of nonzero length.

Step 3: Conclusion The integral equals $\boxed{4/3}$. Hence, the correct option is (A).

Quick Tip to Solve: For powers of $|\cos x|$ over $[0, \pi]$, double the integral over $[0, \pi/2]$ before using a sine substitution.

9. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is : [4 marks]

(A) 500

(B) 350

(C) 200

(D) 300

Correct Answer: (D) 300

Solution:

Concept:

- A team is an unordered selection, so use combinations: $\binom{n}{r} = n!/[r!(n-r)!]$. Choosing girls and boys are independent decisions, and their counts multiply.
- The restriction forbids teams containing both A and B. It permits teams containing neither, as well as teams containing exactly one of them. Subtract the forbidden teams from all possible teams.

Step 1: Count all teams and the forbidden ones Choose the two girls from five in $\binom{5}{2} = 10$ ways. Ignoring the restriction temporarily, choose three boys from seven in $\binom{7}{3} = 35$ ways. This gives

$$N_{\text{all}} = \binom{5}{2}\binom{7}{3} = 10 \times 35 = 350.$$

For a forbidden team, A and B are already fixed. Its third boy can be any one of the other five boys. The choice of girls remains unrestricted, so

$$N_{\text{forbidden}} = \binom{5}{2}\binom{5}{1} = 10 \times 5 = 50.$$

There is no ordering factor: exchanging positions of members does not create a new team.

Step 2: Subtract and verify by separate cases The required number is

$$N = 350 - 50 = 300.$$

As an independent check, a valid selection of boys has either neither special boy, in $\binom{5}{3} = 10$ ways, or exactly one, in $2\binom{5}{2} = 20$ ways. Thus there are $10 + 20 = 30$ valid boy selections and $10 \times 30 = 300$ teams. The value 350 ignores the restriction, while 200 counts only the teams with exactly one special boy. The value 500 even exceeds the unrestricted count, so it cannot be correct.

Step 3: Conclusion There are $\boxed{300}$ permissible teams. Hence, the correct option is (D).

Quick Tip to Solve: When two named people cannot serve together, subtract selections containing both; do not accidentally require exactly one of them.

10.

$$\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$$

[4 marks]

(A) exists and equals $\frac{1}{2\sqrt{2}}$

(B) exists and equals $\frac{1}{4\sqrt{2}}$

(C) exists and equals $\frac{1}{2\sqrt{2}(1+\sqrt{2})}$

(D) does not exists

Correct Answer: (B) exists and equals $\frac{1}{4\sqrt{2}}$

Solution:

Concept:

- A radical difference can be rationalised using $(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})=a-b$. Applying this identity twice removes the two nested differences.
- The expression is evaluated for $y\neq 0$ near zero. Cancelling y^4 there is legitimate, and the resulting denominator is continuous and nonzero at zero.

Step 1: Rationalise the outer radical Direct substitution gives the indeterminate form $0/0$, which does not imply that the limit fails to exist. Denote the given quotient by $F(y)$. Multiply numerator and denominator by the conjugate of its numerator:

$$F(y)=\frac{\sqrt{1+y^4}-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}.$$

The outer rationalisation leaves another radical difference, so one more conjugate is needed. The plus sign in each conjugate is essential; replacing it with a minus sign would not use the difference-of-squares identity.

Step 2: Rationalise again and take the limit Since $(\sqrt{1+y^4}-1)(\sqrt{1+y^4}+1)=y^4$, for $y\neq 0$ we obtain

$$F(y)=\frac{1}{(\sqrt{1+y^4}+1)\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}.$$

Both factors tend to positive values as $y\to 0$, from either side. Their limits are 2 and $2\sqrt{2}$, respectively. Hence

$$\lim_{y\to 0}F(y)=\frac{1}{2(2\sqrt{2})}=\frac{1}{4\sqrt{2}}.$$

The first choice misses one factor of 2, and the third has an extra factor incompatible with the two conjugates. A finite two-sided limit has been obtained, excluding the last choice.

Step 3: Conclusion The limit exists and equals $\boxed{1/(4\sqrt{2})}$. Hence, the correct option is (B).

Quick Tip to Solve: For nested square-root limits with a small power in the denominator, rationalise from the outside inward until that power cancels.

11. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is : [4 marks]

(A) 16

(B) 22

(C) 20

(D) 18

Correct Answer: (C) 20

Solution:

Concept:

- For n observations with mean $\bar{h}$, the population variance is $\sigma^2 = \frac{1}{n}\sum h_i^2 - \bar{h}^2$. The divisor is the number of students, not $n-1$.
- Adding a new observation changes both the sum of heights and the mean. Recompute the six-student mean before finding deviations; the old mean cannot be retained.

Step 1: Recover the totals and the new mean For the original five students,

$$\sum_{i=1}^{5} h_i = 5(150) = 750 \text{ cm}.$$

The given variance implies

$$\sum_{i=1}^{5} h_i^2 = 5(18 + 150^2) = 112590 \text{ cm}^2.$$

After the student of height 156 cm joins, the sum becomes $750 + 156 = 906$ cm, so the new mean is $906/6 = 151$ cm. The new sum of squares is

$$112590 + 156^2 = 112590 + 24336 = 136926 \text{ cm}^2.$$

Step 2: Calculate the new variance and check it Apply the variance formula to all six students:

$$\sigma^2_{\text{new}} = \frac{136926}{6} - 151^2$$
$$= 22821 - 22801 = 20 \text{ cm}^2.$$

A shorter verification uses deviations. The original squared deviations from 150 total $5 \times 18 = 90$. Shifting their reference mean to 151 adds $5 \times 1^2 = 5$, since their original deviations sum to zero. The added student's squared deviation is $(156 - 151)^2 = 25$. Thus the total is $90 + 5 + 25 = 120$, and dividing by 6 again gives 20. Neither the old variance 18 nor the other two choices includes the correct update of the mean and total spread.

Step 3: Conclusion The variance is $\boxed{20 \text{ cm}^2}$. Hence, the correct option is (C).

Quick Tip to Solve: Update the mean as well as the squared-deviation total when adding a data point; here the mean moves from 150 to 151 cm.

12. If $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}$ (x $>\frac{3}{4}$), then x is equal to : [4 marks]

(A) $\frac{\sqrt{145}}{10}$

(B) $\frac{\sqrt{145}}{11}$

(C) $\frac{\sqrt{145}}{12}$

(D) $\frac{\sqrt{146}}{12}$

Correct Answer: (C) $\frac{\sqrt{145}}{12}$

Solution:

Concept:

- If two principal inverse-cosine values sum to $\pi/2$, they are complementary angles. For acute angles, the cosine of one equals the sine of the other.
- The condition $x > 3/4$ makes both $2/(3x)$ and $3/(4x)$ positive and less than 1. Their inverse cosines are therefore acute, so the positive square root in the sine formula is appropriate.

Step 1: Convert the angle equation into an algebraic equation Put

$$\alpha=\cos^{-1}\left(\frac{2}{3x}\right), \qquad \beta=\cos^{-1}\left(\frac{3}{4x}\right).$$

The question gives $\alpha+\beta=\pi/2$, so $\cos\alpha=\sin\beta$. Since β is acute,

$$\frac{2}{3x}=\sqrt{1-\frac{9}{16x^2}}.$$

Both sides are nonnegative. Squaring this equality therefore preserves it and gives

$$\frac{4}{9x^2}+\frac{9}{16x^2}=1.$$

Step 2: Solve and verify the permitted root Multiply by $144x^2$:

$$64+81=144x^2, \qquad x^2=\frac{145}{144}.$$

The given positivity condition selects

$$x=\frac{\sqrt{145}}{12}.$$

This exceeds $3/4$, as $145 > 81$. At this value, the two cosines are $8/\sqrt{145}$ and $9/\sqrt{145}$. Their squares sum to 1, and both angles are acute; hence the cosine of one is the sine of the other and the original angles do sum to $\pi/2$. Denominators 10 and 11 do not satisfy $144x^2 = 145$, while replacing 145 by 146 changes the required sum of squares.

Step 3: Conclusion The required value is $\boxed{\sqrt{145}/12}$. Hence, the correct option is (C).

Quick Tip to Solve: For acute complementary inverse-cosine angles, square their cosine arguments and add; the result must be 1.

13. The area (in sq. units) bounded by the parabolae $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is : [4 marks]

(A) $\frac{56}{3}$

(B) $\frac{32}{3}$

(C) $\frac{8}{3}$

(D) $\frac{14}{3}$

Correct Answer: (C) $\frac{8}{3}$

Solution:

Concept:

- The tangent slope to $y = f(x)$ at $x = a$ is $f\prime(a)$. Its equation through $(a, f(a))$ is $y - f(a) = f\prime(a)(x - a)$.
- The area between two curves is the integral of upper minus lower over the interval enclosed by the boundaries. The y-axis supplies the endpoint $x = 0$; tangency supplies the endpoint $x = 2$.

Step 1: Find the tangent and the enclosed interval For the given parabola,

$$y = x^2 - 1, \qquad \frac{dy}{dx} = 2x.$$

At $(2, 3)$, the slope is 4, so the tangent is

$$y - 3 = 4(x - 2), \qquad y = 4x - 5.$$

To check where the line meets the curve, solve $x^2 - 1 = 4x - 5$. This becomes $(x - 2)^2 = 0$, confirming the double intersection at the point of tangency. Together with the y-axis, the bounded region therefore extends from $x = 0$ to $x = 2$.

Step 2: Integrate the nonnegative vertical separation Subtracting the line ordinate from the parabola ordinate gives

$$(x^2 - 1) - (4x - 5) = x^2 - 4x + 4 = (x - 2)^2 \geq 0.$$

Thus the parabola is the upper curve throughout the interval. The required area is

$$\begin{aligned} A &= \int_0^2 (x^2 - 4x + 4)\, dx \\ &= \left[\frac{x^3}{3} - 2x^2 + 4x\right]_0^2 \\ &= \frac{8}{3} - 8 + 8 = \frac{8}{3}. \end{aligned}$$

The line crosses the x-axis inside this interval, but that does not require splitting the integral: the area is between the two curves, not between either curve and the x-axis. The remaining numerical choices do not equal the integral of their vertical separation.

Step 3: Conclusion The bounded area is $\boxed{8/3}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Subtract a parabola's tangent from the parabola first: the difference often becomes a perfect square, making the area integral immediate.

14. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to : [4 marks]

(A) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(B) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(D) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

Correct Answer: (C) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Solution:

Concept:

- The matrix $R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ represents a plane rotation. The angle-addition identities give $R(\alpha)R(\beta) = R(\alpha+\beta)$.
- Rotation matrices are invertible, with $R(\theta)^{-1} = R(-\theta) = R(\theta)^{\mathsf{T}}$. Consequently $R(\theta)^n = R(n\theta)$ for every integer n, including negative powers.

Step 1: Convert the negative matrix power to an angle Here $A = R(\theta)$, so the inverse-power rule gives

$$A^{-50} = R(-50\theta).$$

For $\theta = \pi/12$, the resulting angle is

$$-50\theta = -\frac{50\pi}{12} = -\frac{25\pi}{6}.$$

Sine and cosine repeat with period 2π. Add $4\pi = 24\pi/6$ to this angle to obtain $-\pi/6$, which has the same rotation matrix. The negative exponent reverses the angle; treating it as a positive power would interchange the signs of the off-diagonal entries.

Step 2: Evaluate the sine and cosine and compare options Using $\cos(-\pi/6) = \sqrt{3}/2$ and $\sin(-\pi/6) = -1/2$, we get

$$A^{-50} = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}.$$

The two diagonal entries are equal, and the off-diagonal entries have opposite signs, as required for a rotation. Its determinant is $3/4 + 1/4 = 1$, providing a check. Option (A) is rotation by $+\pi/6$, while option (D) is rotation by $-\pi/3$. Option (B) has unequal diagonal entries and is not the required rotation matrix. Only the third displayed matrix has both the correct angle and the correct sign pattern.

Step 3: Conclusion Thus $A^{-50} = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$. Hence, the correct option is (C).

Quick Tip to Solve: For a rotation matrix, multiply the angle by the exponent, reduce modulo 2π, and only then evaluate sine and cosine.

15. Let f : R → R be a function defined as

$$f(x) = \begin{cases} 5 & ; \quad x \leq 1 \\ a + bx & ; \quad 1 < x < 3 \\ b + 5x & ; \quad 3 \leq x < 5 \\ 30 & ; \quad x \geq 5 \end{cases}$$

Then, f is [4 marks]

(A) continuous if a = 0 and b = 5

(B) continuous if a = –5 and b = 10

(C) continuous if a = 5 and b = 5

(D) not continuous for any values of a and b

Correct Answer: (D) not continuous for any values of a and b

Solution:

Concept:

- A polynomial is continuous on its interval of definition. For a piecewise function, only the joining points need separate checks.
- Continuity at a joining point requires the left-hand limit, the right-hand limit and the assigned function value to agree.

Step 1: Write the matching conditions The only possible discontinuities occur at $x = 1, 3, 5$. At $x = 1$, the function value and left-hand limit are both 5, while the right-hand limit is $a + b$. Hence

$$a + b = 5.$$

At $x = 3$, the middle expression approaches $a + 3b$, whereas the expression valid at 3 gives $b + 15$. Thus

$$a + 3b = b + 15, \qquad a + 2b = 15.$$

These are necessary conditions; checking only one junction would not establish continuity on the whole real line.

Step 2: Solve and verify all three junctions Subtracting the first equation from the second gives $b = 10$, and then $a = -5$. However, the last junction supplies a further condition. At $x = 5$, continuity demands

$$b + 25 = 30, \qquad b = 5.$$

This contradicts $b = 10$. Equivalently, choosing $b = 5$ at the last junction makes $a = 0$ at the first junction, but then the limits at 3 are 15 and 20. Therefore none of the parameter pairs in (A), (B) or (C) satisfies every required junction. The pair in (B) satisfies the first two junctions but fails the last.

Step 3: Conclusion No values of a and b make f continuous on $\mathbb{R}$. Hence, the correct option is (D).

Quick Tip to Solve: For a piecewise function with three junctions, write all three matching conditions before choosing a parameter pair.

16. The maximum volume (in cu.m) of the right circular cone having slant height 3 m is : [4 marks]

(A) $2\sqrt{3}\pi$

(B) $3\sqrt{3}\pi$

(C) 6π

(D) $\frac{4}{3}\pi$

Correct Answer: (A) $2\sqrt{3}\pi$

Solution:

Concept:

- For a right circular cone, the radius r, height h and slant height l satisfy $r^2 + h^2 = l^2$.
- The volume is $V = \frac{1}{3}\pi r^2 h$. With fixed slant height, write the volume in terms of one variable and find its maximum on the allowed interval.

Step 1: Express the volume using the height The slant height is $3\,\mathrm{m}$, so $r^2 + h^2 = 9$. For a nondegenerate cone, $0 < h < 3$, and

$$r^2 = 9 - h^2.$$

Substituting this into the volume formula gives

$$V(h) = \frac{\pi}{3}(9h - h^3).$$

The radius and height cannot be maximised independently, because increasing either changes the other under the fixed-slant-height constraint.

Step 2: Find and classify the stationary point Differentiate with respect to h:

$$V\prime(h) = \pi(3 - h^2).$$

The positive stationary height is $h = \sqrt{3}\,\mathrm{m}$. Moreover,

$$V\prime\prime(h) = -2\pi h < 0 \quad (0 < h < 3),$$

so this stationary point is a maximum. The limiting endpoint volumes are zero, confirming that it is the global maximum. The corresponding radius obeys $r^2 = 9 - 3 = 6$, and hence

$$V_{\max} = \frac{1}{3}\pi \cdot 6 \cdot \sqrt{3} = 2\sqrt{3}\pi\,\mathrm{m}^3.$$

Options (B) and (C) exceed the maximum allowed by this constraint, while (D) is not its stationary maximum.

Step 3: Conclusion The greatest volume is $2\sqrt{3}\pi\ \mathrm{m}^3$. Hence, the correct option is (A).

Quick Tip to Solve: With slant height fixed at l, use the height as the variable; the maximum occurs at $h = l/\sqrt{3}$.

17. If y = y(x) is the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$, satisfying y(1) = 1, then $y(\frac{1}{2})$ is equal to : [4 marks]

(A) $\frac{7}{64}$

(B) $\frac{49}{16}$

(C) $\frac{1}{4}$

(D) $\frac{13}{16}$

Correct Answer: (B) $\frac{49}{16}$

Solution:

Concept:

- A linear differential equation $y' + P(x)y = Q(x)$ is solved by the integrating factor $e^{\int P(x)\,dx}$.
- The initial condition determines the integration constant; it must be used before evaluating the solution at a new point.

Step 1: Find the integrating factor On the interval $x > 0$, which contains both the initial point 1 and the required point $1/2$, divide the equation by x:

$$\frac{dy}{dx} + \frac{2}{x}y = x.$$

Here $P(x) = 2/x$, so the integrating factor is

$$e^{\int (2/x)\,dx} = x^2.$$

Multiplication by x^2 turns the left side into an exact product derivative:

$$x^2\frac{dy}{dx} + 2xy = x^3, \qquad \frac{d}{dx}(x^2y) = x^3.$$

Step 2: Apply the initial condition and evaluate Integrating gives

$$x^2y = \frac{x^4}{4} + C, \qquad y = \frac{x^2}{4} + \frac{C}{x^2}.$$

At $x = 1$, $y = 1$, so $1 = 1/4 + C$, giving $C = 3/4$. Consequently,

$$y\left(\frac{1}{2}\right) = \frac{1}{16} + \frac{3/4}{1/4} = \frac{1}{16} + 3 = \frac{49}{16}.$$

As a check, differentiating $y = x^2/4 + 3/(4x^2)$ and computing $xy\prime + 2y$ returns x^2; the same expression also gives $y(1) = 1$. Thus the other three proposed values cannot satisfy the uniquely determined initial-value solution.

Step 3: Conclusion $y(1/2) = 49/16$. Hence, the correct option is (B).

Quick Tip to Solve: After multiplying by the integrating factor x^2, integrate the product x^2y, rather than integrating y directly.

---

18. Let

A = $\left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3+2i\sin\theta}{1-2i\sin\theta} is\, purely\, imaginary\right\}$

. Then the sum of the elements in A is : [4 marks]

(A) $\frac{5\pi}{6}$

(B) π

(C) $\frac{3\pi}{4}$

(D) $\frac{2\pi}{3}$

Correct Answer: (D) $\frac{2\pi}{3}$

Solution:

Concept:

- A complex number is purely imaginary when its real part is zero. Rationalising the denominator isolates its real and imaginary parts.
- Trigonometric solutions must be restricted to the interval stated in the question; excluding an allowed negative angle changes the requested sum.

Step 1: Set the real part equal to zero Write $s = \sin\theta$, which is real. The denominator $1 - 2is$ cannot vanish, since its real part is 1. Multiplying numerator and denominator by its conjugate gives

$$\frac{3+2is}{1-2is} = \frac{(3+2is)(1+2is)}{1+4s^2} = \frac{3-4s^2+8is}{1+4s^2}.$$

Since $1 + 4s^2 > 0$, the real part vanishes precisely when

$$3 - 4s^2 = 0, \qquad \sin\theta = \pm\frac{\sqrt{3}}{2}.$$

The imaginary part is nonzero at these values, so they do give purely imaginary numbers.

Step 2: List every allowed angle and add The interval is $(-\pi/2, \pi)$. Within it, positive sine gives $\theta = \pi/3$ and $\theta = 2\pi/3$, while negative sine gives $\theta = -\pi/3$. The other familiar negative-sine angle $-2\pi/3$ lies outside the interval, and no other representatives fit. Thus

$$A = \left\{-\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}\right\}, \qquad \sum_{\theta\in A}\theta = \frac{2\pi}{3}.$$

In particular, summing only the two positive angles would give π, the distractor in (B). Options (A)

and (C) also disagree with the complete list of allowed roots.

Step 3: Conclusion The sum of the elements of A is $2\pi/3$. Hence, the correct option is (D).

Quick Tip to Solve: Rationalise first, then draw the permitted angular interval mentally before listing the sine solutions.

19. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it? [4 marks]

(A) (5, $2\sqrt{6}$)

(B) (6, $4\sqrt{2}$)

(C) (8, 6)

(D) (4, -4)

Correct Answer: (C) (8, 6)

Solution:

Concept:

- A right-opening parabola with vertex $(h, 0)$ and focus $(h+p, 0)$ has equation $y^2 = 4p(x-h)$, where $p > 0$.
- A point lies on a curve exactly when its coordinates satisfy the curve equation. To find a point that does not lie on it, test all the given points.

Step 1: Determine the vertex and focal parameter Both specified points lie on the positive x-axis. Their distances from the origin therefore give the vertex $V = (2, 0)$ and focus $F = (4, 0)$. The focus is to the right of the vertex, and its distance from the vertex is $p = 4 - 2 = 2$. Thus the required equation is

$$y^2 = 4(2)(x-2) = 8(x-2).$$

The shift by 2 is essential: using $y^2 = 8x$ would incorrectly place the vertex at the origin.

Step 2: Substitute the four candidate points For (A), $(2\sqrt{6})^2 = 24$ and $8(5-2) = 24$, so this point lies on the parabola. For (B), $(4\sqrt{2})^2 = 32$ and $8(6-2) = 32$, so it also lies on it. For (C), however,

$$y^2 = 6^2 = 36, \qquad 8(x-2) = 8(8-2) = 48.$$

The two sides differ, so $(8, 6)$ is not on the parabola. Finally, for (D), $(-4)^2 = 16 = 8(4-2)$, so the negative ordinate causes no difficulty. The curve is symmetric about its axis, and both signs of an ordinate are possible. Exactly one of the four points fails the equation.

Step 3: Conclusion The point $(8, 6)$ does not lie on the parabola. Hence, the correct option is (C).

Quick Tip to Solve: Use the vertex-to-focus distance as p, then compare y^2 with $4p(x-h)$ for each option.

20. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, the $|\tan\theta|$ is equal to : [4 marks]

(A) $\frac{8}{15}$
(B) $\frac{4}{9}$
(C) $\frac{7}{17}$
(D) $\frac{8}{17}$

Correct Answer: (A) $\frac{8}{15}$

Solution:

Concept:

- The angle between two smooth curves at an intersection is the angle between their tangents there.
- For finite tangent slopes m_1 and m_2, the acute angle satisfies $\tan\theta = |(m_2 - m_1)/(1 + m_1 m_2)|$, provided the denominator is nonzero.

Step 1: Find the intersections and tangent slopes At a common point, the two expressions for y are equal:

$$10 - x^2 = 2 + x^2, \qquad 2x^2 = 8, \qquad x = \pm 2.$$

Both points have $y = 6$. Differentiate the equations separately:

$$m_1 = \frac{d}{dx}(10 - x^2) = -2x, \qquad m_2 = \frac{d}{dx}(2 + x^2) = 2x.$$

At $x = 2$, the slopes are -4 and 4. At $x = -2$, these slopes exchange signs, leaving the acute angle unchanged.

Step 2: Apply the tangent-angle formula Using either intersection, the product of the slopes is -16, while the magnitude of their difference is 8. Hence

$$|\tan\theta| = \left|\frac{4 - (-4)}{1 + (-4)(4)}\right| = \left|\frac{8}{-15}\right| = \frac{8}{15}.$$

The denominator in this formula is $1 + m_1 m_2$, not $1 + |m_1 m_2|$; the latter mistake would give $8/17$, option (D). The absolute value is taken after forming the ratio so that the answer corresponds to the acute angle. Options (B) and (C) do not follow from the tangent slopes at either intersection.

Step 3: Conclusion $|\tan\theta| = 8/15$. Hence, the correct option is (A).

Quick Tip to Solve: Find the intersection before differentiating numerically, and preserve the sign of the product in $1 + m_1 m_2$.

21. The system of linear equations

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$ then [4 marks]

(A) has infinitely many solutions for $a = 4$

(B) has a unique solution for $|a| = \sqrt{3}$

(C) is inconsistent when $|a| = \sqrt{3}$

(D) is inconsistent when $a = 4$

Correct Answer: (C) is inconsistent when $|a| = \sqrt{3}$

Solution:

Concept:

- Elementary row operations preserve the solutions of a linear system. A row of the form $0 = c$, with $c \neq 0$, proves inconsistency.
- A nonzero coefficient for each successive variable after elimination gives a unique solution; a zero determinant by itself does not distinguish inconsistency from infinitely many solutions.

Step 1: Eliminate repeated terms Subtract twice the first equation from the second equation:

$$(2x + 3y + 2z) - 2(x + y + z) = 5 - 4, \qquad y = 1.$$

Next subtract the second equation from the third:

$$(a^2 - 3)z = a - 4.$$

The first equation now gives $x + z = 1$. Thus the whole system is equivalent to

$$y = 1, \qquad x + z = 1, \qquad (a^2 - 3)z = a - 4.$$

This form makes the exceptional parameter values explicit without expanding a determinant.

Step 2: Classify the exceptional and regular values When $|a| = \sqrt{3}$, the coefficient $a^2 - 3$ vanishes, but $a - 4$ is nonzero for both $a = \sqrt{3}$ and $a = -\sqrt{3}$. The last equation becomes $0 = a - 4$, a contradiction. Therefore the system is inconsistent for both these values, rather than having a unique solution. For $a^2 \neq 3$,

$$z = \frac{a - 4}{a^2 - 3}, \qquad y = 1, \qquad x = 1 - z,$$

which is a unique solution. In particular, at $a = 4$, the final equation is $13z = 0$; hence $(x, y, z) = (1, 1, 0)$. This rules out both the infinite-solution claim in (A) and the inconsistency claim in (D).

Step 3: Conclusion The system is inconsistent when $|a| = \sqrt{3}$. Hence, the correct option is (C).

Quick Tip to Solve: When two equations differ only in one coefficient and the constant, subtract them first to expose the consistency condition.

22. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval : [4 marks]

(A) $(3, \infty)$

(B) $\left(\frac{3}{2}, 2\right]$

(C) $\left(1, \frac{3}{2}\right]$

(D) $(2, 3]$

Correct Answer: (A) $(3, \infty)$

Solution:

Concept:

- For the hyperbola $x^2/a^2 - y^2/b^2 = 1$, the eccentricity is $e = \sqrt{1 + b^2/a^2}$ and the length of a latus rectum is $L = 2b^2/a$. The positive square root must be used for each semi-axis.
- Here $a = \cos\theta > 0$ and $b = \sin\theta > 0$, since the angle lies strictly in the first quadrant. The identity $a^2 + b^2 = 1$ relates the two parameters.

Step 1: Translate the eccentricity condition Using the given denominators, simplify the eccentricity before imposing its bound:

$$e^2 = 1 + \frac{\sin^2\theta}{\cos^2\theta} = \sec^2\theta.$$

Hence $e = \sec\theta$. The hypothesis $e > 2$ is equivalent to $0 < \cos\theta < 1/2$; equality is excluded because the eccentricity inequality is strict.

Step 2: Determine the full range of the latus rectum Put $u = \cos\theta$. Then $0 < u < 1/2$ and

$$L = \frac{2(1 - u^2)}{u} = \frac{2}{u} - 2u.$$

This continuous function decreases strictly, because $dL/du = -2/u^2 - 2 < 0$. As $u \to (1/2)^-$, $L \to 4 - 1 = 3$, while as $u \to 0^+$, $L \to \infty$. Thus every value strictly greater than 3 occurs, but 3 does not. Options (B), (C) and (D) all contain only values at most 3 and so cannot describe the required interval.

Step 3: Conclusion The length of the latus rectum lies in $\boxed{(3, \infty)}$. Hence, the correct option is (A).

Quick Tip to Solve: For this hyperbola, simplify the eccentricity with $\sin^2\theta + \cos^2\theta = 1$ first; then express the latus rectum using only $\cos\theta$.

23. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$ is :

[4 marks]

(A) $2\sqrt{3}y = 12x + 1$

(B) $\sqrt{3}y = x + 3$

(C) $2\sqrt{3}y = -x - 12$

(D) $\sqrt{3}y = 3x + 1$

Correct Answer: (B) $\sqrt{3}y = x + 3$

Solution:

Concept:

- The circle $x^2 + y^2 - 6x = 0$ is $(x-3)^2 + y^2 = 9$, so its centre is $(3, 0)$ and radius is 3. A line is tangent when its perpendicular distance from the centre equals the radius.
- A nonvertical tangent to $y^2 = 4ax$ has equation $y = mx + a/m$, where $m \neq 0$. In this question $a = 1$, so the intercept must be $1/m$.

Step 1: Impose tangency to both curves Write the parabola tangent as $mx - y + 1/m = 0$. Its distance from the circle centre is

$$\frac{|3m + 1/m|}{\sqrt{m^2 + 1}}.$$

Equating this to the radius and squaring gives

$$(3m + 1/m)^2 = 9(m^2 + 1).$$

After expanding and cancelling $9m^2$, we obtain $6 + 1/m^2 = 9$, hence $m^2 = 1/3$. Squaring introduces no unwanted sign condition here, since the original equation uses an absolute value and both sides are nonnegative.

Step 2: Select the printed common tangent For $m = 1/\sqrt{3}$, the intercept is $\sqrt{3}$, and the line becomes

$$y = \frac{x}{\sqrt{3}} + \sqrt{3}, \qquad \sqrt{3}\,y = x + 3.$$

The negative slope gives another common tangent, but it is not among the listed equations. In (A), $m = 2\sqrt{3}$ and the intercept is $1/(2\sqrt{3})$; although it touches the parabola, its slope fails the circle condition. In (C), the slope and intercept have product 1, but $m^2 = 1/12$, which also fails that condition. In (D), $m^2 = 3$, again different from $1/3$.

Step 3: Conclusion A common tangent is $\boxed{\sqrt{3}\,y = x + 3}$. Hence, the correct option is (B).

Quick Tip to Solve: Test parabola tangency through the product of slope and intercept, then use the centre-to-line distance to test circle tangency.

24. The plane through the intersection of the planes x + y + z = 1 and 2x + 3y – z + 4 = 0 and parallel to y-axis also passes through the point : [4 marks]

(A) $(-3, 0, -1)$

(B) $(3, 2, 1)$

(C) $(3, 3, -1)$

(D) $(-3, 1, 1)$

Correct Answer: (B) $(3, 2, 1)$

Solution:

Concept:

- A plane containing the intersection of $P_1 = 0$ and $P_2 = 0$ can be obtained from a nontrivial linear combination $\alpha P_1 + \beta P_2 = 0$. Every common point satisfies every such combination.
- The y-axis has direction vector $(0, 1, 0)$. A plane is parallel to that direction when its normal is perpendicular to it, so the coefficient of y in the plane equation must vanish.

Step 1: Construct the required member of the family Use $P_1 = x + y + z - 1$ and $P_2 = 2x + 3y - z + 4$. For the combination $P_2 + \lambda P_1 = 0$, the equation is

$$(2 + \lambda)x + (3 + \lambda)y + (-1 + \lambda)z + 4 - \lambda = 0.$$

Its normal has y-component $3 + \lambda$. Therefore parallelism to the y-axis requires $\lambda = -3$. The resulting equation is $-x - 4z + 7 = 0$, or equivalently $x + 4z = 7$. Notice that the y-coordinate is unrestricted, as it should be for a plane parallel to that axis.

Step 2: Check each candidate point Substitute the x- and z-coordinates into the left side $x + 4z$. For (A), $(-3, 0, -1)$ gives $-3 - 4 = -7$, not 7. For (B), $(3, 2, 1)$ gives $3 + 4 = 7$, so it lies on the plane. For (C), $(3, 3, -1)$ gives $3 - 4 = -1$, and for (D), $(-3, 1, 1)$ gives $-3 + 4 = 1$. Thus exactly one listed point satisfies the required plane. Changing y alone cannot correct a wrong value of $x + 4z$, since y does not appear in the final equation.

Step 3: Conclusion The plane is $x + 4z = 7$, and it passes through $\boxed{(3, 2, 1)}$. Hence, the correct option is (B).

Quick Tip to Solve: For a plane parallel to a coordinate axis, eliminate that coordinate from the plane family before testing the options.

25. Three circles of radii a, b, c (a < b < c) touch each other externally. If they have x-axis as a common tangent, then : [4 marks]

(A) a, b, c are in A.P.

(B) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P

(C) $\frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}}$

(D) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

Correct Answer: (C) $\frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}}$

Solution:

Concept:

- A circle of radius r tangent to the x-axis has its centre a perpendicular distance r from that axis. For externally touching circles of radii r and s, the centre distance is $r + s$.
- If two such circles lie on the same side of the axis, their vertical centre separation is $|r - s|$, so their horizontal separation is $\sqrt{(r+s)^2 - (r-s)^2} = 2\sqrt{rs}$.

Step 1: Relate the horizontal centre distances The three externally touching circles with a common tangent lie on the same side of that tangent: circles on opposite sides can touch only at their common tangency point on the axis, and a third positive-radius circle could not touch both without overlap. Their three horizontal centre distances are therefore

$$2\sqrt{ab}, \qquad 2\sqrt{ac}, \qquad 2\sqrt{bc}.$$

Since $a < b < c$, the largest is $2\sqrt{bc}$. On a line, the largest pairwise distance is the sum of the other two, so the centre of the smallest circle lies horizontally between the other centres.

Step 2: Obtain the reciprocal-square-root relation Consequently,

$$2\sqrt{bc} = 2\sqrt{ab} + 2\sqrt{ac}.$$

Divide by the strictly positive quantity $2\sqrt{abc}$ to obtain

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{c}} + \frac{1}{\sqrt{b}}.$$

This is option (C). Option (D) is impossible because $1/\sqrt{b} < 1/\sqrt{a}$, whereas its right side is greater than $1/\sqrt{a}$. The relation does not require radii or their square roots to form an arithmetic progression. For example, $a = 4/9$, $b = 1$, $c = 4$ satisfies the derived identity, but neither those radii nor their square roots form an arithmetic progression, excluding (A) and (B).

Step 3: Conclusion The radii satisfy $\boxed{\frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}}}$. Hence, the correct option is (C).

Quick Tip to Solve: For two circles touching both each other and a straight line, use the horizontal centre separation $2\sqrt{rs}$, not the sum of the radii.

26. The equation of the line passing through (–4, 3, 1), parallel to the plane x + 2y – z – 5 = 0 and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is : [4 marks]

(A) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(B) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

(C) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

(D) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Correct Answer: (A) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

Solution:

Concept:

- A direction vector $\boldsymbol{d}$ is parallel to the plane $x + 2y - z - 5 = 0$ when it is perpendicular to the normal $\boldsymbol{n} = (1, 2, -1)$, so $\boldsymbol{d} \cdot \boldsymbol{n} = 0$.
- Use the common value of the symmetric coordinates as a parameter to express an arbitrary point on the given line. The vector joining the fixed point to this variable point must satisfy the parallelism condition.

Step 1: Locate the intersection point Let the common parameter on the given line be t. Its points have coordinates

$$Q = (-1 - 3t,\ 3 + 2t,\ 2 - t).$$

The required line passes through $P = (-4, 3, 1)$, so a possible direction is

$$\overrightarrow{PQ} = (3 - 3t,\ 2t,\ 1 - t).$$

Perpendicularity to the plane normal requires

$$(3 - 3t) + 2(2t) - (1 - t) = 0,$$

which simplifies to $2 + 2t = 0$. Thus $t = -1$, and the intersection point is $Q = (2, 1, 3)$.

Step 2: Write and verify the resulting line We have $\overrightarrow{PQ} = (6, -2, 2) = 2(3, -1, 1)$. The required symmetric equation is therefore

$$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}.$$

It passes through P, and parameter 2 on this line gives Q, establishing intersection with the prescribed line. Its direction also satisfies $3 + 2(-1) - 1 = 0$, confirming parallelism to the plane. Option (B) has a direction parallel to the plane but does not pass through the required point Q. Option (C) has normal dot product $-1 + 2 - 1 = 0$, but its direction is not proportional to $(6, -2, 2)$. Option (D) fails even to contain P.

Step 3: Conclusion The required line is $\boxed{\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Parameterise the line to be intersected, then dot its joining vector with the plane normal; this determines the intersection point directly.

27. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{c} = 4$, then $|\vec{c}|^2$ is equal to : [4 marks]

(A) 8

(B) $\frac{19}{2}$

(C) 9

(D) $\frac{17}{2}$

Correct Answer: (B) $\frac{19}{2}$

Solution:

Concept:

- For any vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, Lagrange's identity gives $|\boldsymbol{u} \times \boldsymbol{v}|^2 + (\boldsymbol{u} \cdot \boldsymbol{v})^2 = |\boldsymbol{u}|^2|\boldsymbol{v}|^2$. It combines the perpendicular and parallel contributions.
- The vector equation $\boldsymbol{a} \times \boldsymbol{c} + \boldsymbol{b} = \boldsymbol{0}$ implies $\boldsymbol{a} \times \boldsymbol{c} = -\boldsymbol{b}$. Taking squared magnitudes removes the minus sign but retains the exact cross-product magnitude.

Step 1: Evaluate the known scalar quantities From $\boldsymbol{a} = (1, -1, 0)$ and $\boldsymbol{b} = (1, 1, 1)$,

$$|\boldsymbol{a}|^2 = 1^2 + (-1)^2 = 2, \qquad |\boldsymbol{b}|^2 = 1^2 + 1^2 + 1^2 = 3.$$

Therefore $|\boldsymbol{a} \times \boldsymbol{c}|^2 = 3$, while the given dot product supplies $(\boldsymbol{a} \cdot \boldsymbol{c})^2 = 4^2 = 16$. These are squared quantities; replacing 16 by 4 would not be consistent with the identity.

Step 2: Apply the identity and check the vector Lagrange's identity now becomes

$$3 + 16 = 2|\boldsymbol{c}|^2, \qquad |\boldsymbol{c}|^2 = \frac{19}{2}.$$

An independent component check is useful. If $\boldsymbol{c} = (u, v, w)$, then $\boldsymbol{a} \times \boldsymbol{c} = (-w, -w, u + v) = (-1, -1, -1)$, giving $w = 1$ and $u + v = -1$. The dot product gives $u - v = 4$. Hence $u = 3/2$, $v = -5/2$, and $|\boldsymbol{c}|^2 = 9/4 + 25/4 + 1 = 19/2$, confirming the result. The alternatives 8, 9 and $17/2$ fail $2|\boldsymbol{c}|^2 = 19$.

Step 3: Conclusion The required squared magnitude is $\boxed{|\boldsymbol{c}|^2 = \frac{19}{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: When both a dot product and a cross-product magnitude are known, combine their squares with Lagrange's identity to find the unknown magnitude without solving components.

28. If the Boolean expression
$(p \oplus q) \wedge (\sim p \odot q)$ is equivalent
to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the
ordered pair $(\oplus, \odot)$ is : [4 marks]

(A) $(\vee, \wedge)$
(B) $(\vee, \vee)$
(C) $(\wedge, \vee)$
(D) $(\wedge, \wedge)$

Correct Answer: (C) $(\wedge, \vee)$

Solution:

Concept:

- The symbols $\oplus$ and $\odot$ here stand for unknown choices from AND $(\wedge)$ and OR $(\vee)$; they do not mean exclusive OR or a new logical operation.
- Two Boolean expressions are equivalent when they have the same truth value for every assignment of p and q. The identities $p \wedge \neg p = \mathrm{F}$ and $\mathrm{F} \vee X = X$ simplify expressions containing a statement and its negation.

Step 1: Test the pair that reproduces the required expression Choose $\oplus = \wedge$ and $\odot = \vee$. The expression becomes

$$(p \wedge q) \wedge (\neg p \vee q).$$

By distributivity, this equals

$$(p \wedge q \wedge \neg p) \vee (p \wedge q \wedge q) = \mathrm{F} \vee (p \wedge q) = p \wedge q.$$

Thus this pair gives the required equivalence for every truth assignment, including both the true case $p = q = \mathrm{T}$ and the cases when at least one statement is false.

Step 2: Exclude the other operator pairs For (A), the expression is $(p \vee q) \wedge (\neg p \wedge q) = \neg p \wedge q$; taking $p = \mathrm{T}, q = \mathrm{T}$ makes it false while $p \wedge q$ is true. For (B), the distributive identity gives $(p \vee q) \wedge (\neg p \vee q) = q$, which is true for $p = \mathrm{F}, q = \mathrm{T}$ even though $p \wedge q$ is false. For (D), the expression contains $p \wedge \neg p$ and is always false, failing when both statements are true. These checks leave exactly the pair in (C).

Step 3: Conclusion The ordered pair is $\boxed{(\oplus, \odot) = (\wedge, \vee)}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $p = q = \mathrm{T}$ to reject operator pairs that force $p \wedge \neg p$, then test $p = \mathrm{F}, q = \mathrm{T}$ to distinguish the survivors.

29. Consider the set of all lines px + qy + r = 0 such that 3p + 2q + 4r = 0. Which one of the following statements is true? [4 marks]

(A) The lines are not concurrent

(B) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

(C) The lines are all parallel

(D) Each line passes through the origin

Correct Answer: (B) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

Solution:

Concept:

- A family of lines is concurrent if every member passes through the same fixed point. For a point (x_0, y_0), this requires $px_0 + qy_0 + r = 0$ for every allowed triple of coefficients.
- The relation between the coefficients can be divided by a nonzero constant to put it into the same form as the line equation. A valid line has p, q not both zero.

Step 1: Rewrite the coefficient condition The condition involves the coefficients, rather than the variable coordinates of a general point. Divide it by 4 to obtain

$$3p + 2q + 4r = 0 \quad \Longrightarrow \quad p\left(\frac{3}{4}\right) + q\left(\frac{1}{2}\right) + r = 0.$$

This is exactly the expression obtained by substituting a particular point into the equation of every line in the family.

Step 2: Identify and verify the common point Substituting $x = 3/4$ and $y = 1/2$ into $px + qy + r$ gives zero because of the given coefficient relation. Consequently, every allowed line contains

$$P = \left(\frac{3}{4}, \frac{1}{2}\right).$$

The statement is independent of the slope of an individual line and includes vertical lines.

Step 3: Eliminate the other descriptions For example, the choices $(p, q, r) = (4, 0, -3)$ and $(0, 2, -1)$ satisfy the condition and give $x = 3/4$ and $y = 1/2$. These are distinct intersecting lines, so the family is neither nonconcurrent nor entirely parallel. Neither of these lines passes through the origin, which also disproves the last statement.

Step 4: Conclusion The lines are concurrent at $\left(\frac{3}{4}, \frac{1}{2}\right)$. Hence, the correct option is (B).

Quick Tip to Solve: Normalize the coefficient of r to 1; the resulting coefficients of p and q directly give the common point.

30. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P (X = 2) equals : [4 marks]

(A) $\frac{25}{169}$
(B) $\frac{49}{169}$
(C) $\frac{24}{169}$
(D) $\frac{52}{169}$

Correct Answer: (A) $\frac{25}{169}$

Solution:

Concept:

- An ordinary deck has 4 aces and 48 non-aces. Hence the probability of an ace on one draw is $p = 4/52 = 1/13$, while that of a non-ace is $q = 12/13$.
- Replacement restores the full deck before the second draw. The draws are independent, so X has a binomial distribution with two trials: $P(X = k) = \binom{2}{k} p^k q^{2-k}$.

Step 1: Find the probability of exactly one ace Exactly one ace can occur in two mutually exclusive orders: ace followed by non-ace, or non-ace followed by ace. Each order has probability $(1/13)(12/13)$. Thus

$$P(X = 1) = 2\left(\frac{1}{13}\right)\left(\frac{12}{13}\right) = \frac{24}{169}.$$

The factor of 2 is essential because the question counts aces, without prescribing the order.

Step 2: Add the probability of two aces Two aces means that both independent draws produce an ace. Therefore

$$P(X = 2) = \left(\frac{1}{13}\right)^2 = \frac{1}{169}.$$

The two events $X = 1$ and $X = 2$ are disjoint, so their probabilities add:

$$P(X = 1) + P(X = 2) = \frac{24 + 1}{169} = \frac{25}{169}.$$

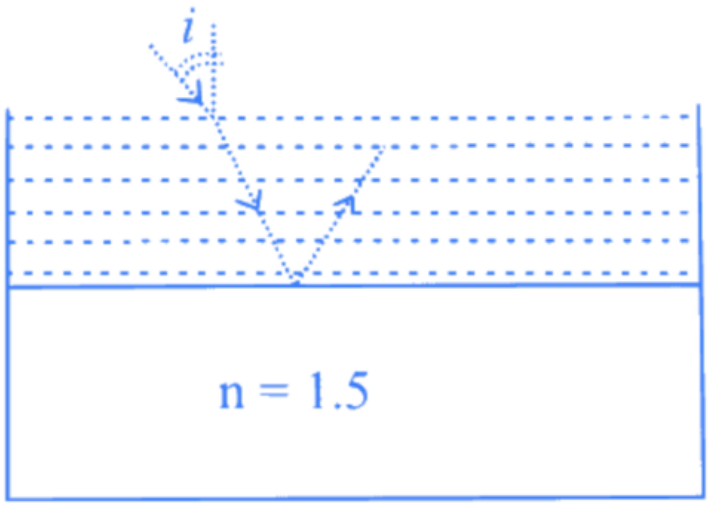
Step 3: Check using the complementary event The requested sum is the probability of at least one ace. Its complement is drawing no ace at all, whose probability is $(12/13)^2 = 144/169$. Hence $1 - 144/169 = 25/169$, confirming the result. The option $24/169$ omits the two-ace case; neither $49/169$ nor $52/169$ equals the complementary probability.

Step 4: Conclusion $P(X = 1) + P(X = 2) = \frac{25}{169}$. Hence, the correct option is (A).

Quick Tip to Solve: For at least one success in independent trials, calculate one minus the probability of no success; replacement keeps the success probability fixed.

Physics (120 marks)

31. Consider a tank made of glass(refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is : [4 marks]



(A) $\sqrt{\frac{5}{3}}$

(B) $\frac{3}{\sqrt{5}}$

(C) $\frac{5}{\sqrt{3}}$

(D) $\frac{4}{3}$

Correct Answer: (B) $\frac{3}{\sqrt{5}}$

Solution:

Concept:

- At the air-liquid surface, Snell's law gives $\sin i = \mu \sin r$, where r is the angle inside the liquid. The parallel bottom interface has the same angle of incidence r.
- Complete polarization of reflected light requires incidence at Brewster's angle r_B, with $\tan r_B = n_g/\mu$. Here the glass refractive index is $n_g = 3/2$.

Step 1: Find the accessible range inside the liquid Because $\sin i \leq 1$, the refracted ray satisfies

$$\sin r \leq \frac{1}{\mu}.$$

The limiting internal angle is $r_{\max} = \sin^{-1}(1/\mu)$, approached as the external incidence becomes grazing. For every beam actually entering through the top surface, the internal angle is below this grazing limit.

Step 2: Make the Brewster angle inaccessible No entering ray can reach Brewster incidence at the

bottom if $r_B \geq r_{\max}$. Express the sine of the Brewster angle in terms of its tangent:

$$\sin r_B = \frac{3/2}{\sqrt{\mu^2 + 9/4}}.$$

At the minimum permitted refractive index, the Brewster angle equals the limiting internal angle. Thus

$$\frac{3/2}{\sqrt{\mu^2 + 9/4}} = \frac{1}{\mu}.$$

Step 3: Solve and compare the choices All quantities are positive, so squaring and simplifying is legitimate:

$$\frac{9}{4}\mu^2 = \mu^2 + \frac{9}{4} \quad \Longrightarrow \quad \mu^2 = \frac{9}{5}.$$

Therefore the threshold is $\mu = 3/\sqrt{5}$. The smaller choices $\sqrt{5/3}$ and $4/3$ allow Brewster incidence for a finite external angle. The larger choice $5/\sqrt{3}$ also prevents it, but is not the minimum. At threshold, equality would require an exactly grazing external ray, which does not enter the liquid.

Step 4: Conclusion The minimum refractive index is $\mu_{\min} = \frac{3}{\sqrt{5}}$. Hence, the correct option is (B).

Quick Tip to Solve: Compare the Brewster angle at the bottom with the largest angle that refraction at the top surface can produce; the threshold occurs at grazing incidence.

32. A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The corecivity of the bar magnet is : [4 marks]

(A) 285 A/m

(B) 2600 A/m

(C) 520 A/m

(D) 1200 A/m

Correct Answer: (B) 2600 A/m

Solution:

Concept:

- Coercivity is the magnitude of the reverse magnetizing field intensity H_c required to reduce a magnet's residual magnetization to zero.
- For the ideal solenoid model, the magnetizing field intensity is $H = nI = NI/\ell$, where N is the number of turns, I the current and ℓ the solenoid length. Its SI unit is $\mathrm{A\,m^{-1}}$.

Step 1: Identify the field that demagnetizes the bar The solenoid supplies the reverse field required to demagnetize the inserted bar. Hence its magnetizing field intensity has magnitude equal to the coercivity, $H_c = NI/\ell$. The relevant quantity is H, rather than magnetic flux density B; no factor of μ_0 is needed in a result expressed in amperes per metre.

Step 2: Substitute the solenoid data The given length is already in metres: $\ell = 0.2\,\mathrm{m}$, with $N = 100$

and $I = 5.2\,\text{A}$. The number of turns per unit length is

$$n = \frac{100}{0.2} = 500\,\text{m}^{-1}.$$

Consequently,

$$H_c = nI = 500 \times 5.2 = 2600\,\text{A m}^{-1}.$$

Step 3: Check the units and reject the alternatives The product of turn density m^{-1} and current A has precisely the required unit A m^{-1}. The value 520 is only NI; it omits division by the solenoid length and has not yet become field intensity. Neither 285 nor $1200\,\text{A m}^{-1}$ follows from the stated turns, current and length. Coercivity is quoted as a positive magnitude even though the field opposes the original magnetization.

Step 4: Conclusion The bar magnet's coercivity is $2600\,\text{A m}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: When coercivity is requested in A/m, use the reverse magnetizing field NI/ℓ, rather than the flux density $\mu_0 NI/\ell$.

33. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔTK. Young's modulus, Y, for this metal is : [4 marks]

(A) $\frac{F}{A\alpha\Delta T}$
(B) $\frac{F}{A\alpha(\Delta T-273)}$
(C) $\frac{F}{2A\alpha\Delta T}$
(D) $\frac{2F}{A\alpha\Delta T}$

Correct Answer: (A) $\frac{F}{A\alpha\Delta T}$

Solution:

Concept:

- A rod free to expand would acquire thermal strain $\epsilon_{\text{th}} = \alpha\Delta T$, corresponding to an increase in length $\Delta L_{\text{th}} = \alpha L\Delta T$.
- Young's modulus relates axial stress and elastic strain: $Y = (F/A)/(\Delta L_{\text{el}}/L)$. A temperature difference has the same numerical value in kelvin and degrees Celsius.

Step 1: Balance thermal expansion with elastic compression The final length is unchanged because the compressive elastic strain exactly cancels the positive thermal strain. Taking magnitudes,

$$\Delta L_{\text{el}} = \Delta L_{\text{th}} = \alpha L\Delta T.$$

This is a balance of two changes in length, so the original length L will cancel from the modulus calculation.

Step 2: Use the internal compressive stress The two end forces have equal magnitude F and opposite

directions. Each cross-section transmits an axial force F, giving stress F/A, not $2F/A$. Therefore

$$Y = \frac{F/A}{\alpha L \Delta T/L} = \frac{F}{A\alpha\Delta T}.$$

Equivalently, the restraining force is $F = YA\alpha\Delta T$, which gives the same expression on rearrangement.

Step 3: Check temperature differences and other factors The stated ΔT is a temperature rise, not an absolute temperature. A rise of one kelvin equals a rise of one degree Celsius, so no subtraction of 273 is appropriate. This rules out the expression containing $\Delta T - 273$. The options with an extra factor of 2 confuse the pair of balancing end forces with the single internal force through any section. Finally, $\alpha\Delta T$ is dimensionless, leaving the correct pressure unit for Y.

Step 4: Conclusion Young's modulus is $Y = \frac{F}{A\alpha\Delta T}$. Hence, the correct option is (A).

Quick Tip to Solve: For a fully restrained rod, equate the elastic strain $F/(AY)$ to $\alpha\Delta T$; opposing end forces do not double the internal stress.

34. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is : [4 marks]

(A) 2.0 %

(B) 2.5 %

(C) 1.0 %

(D) 0.5 %

Correct Answer: (C) 1.0 %

Solution:

Concept:

- A uniform wire has resistance $R = \rho L/A$, where ρ is resistivity, L length and A cross-sectional area. The usual small-stretch model keeps resistivity unchanged.
- If volume $V = AL$ is constant, then $A = V/L$ and $R = \rho L^2/V$. Resistance is therefore proportional to the square of length, rather than to length alone.

Step 1: Relate the new dimensions to the original ones The fractional increase in length is $0.5/100 = 0.005$, so

$$L' = 1.005L.$$

The unchanged volume requires $A'L' = AL$, hence $A' = A/1.005$. Stretching simultaneously increases the conducting path and decreases its area; both effects increase the resistance.

Step 2: Calculate the resistance ratio Use the ratio form to avoid needing the original dimensions:

$$\frac{R'}{R} = \frac{L'}{L}\frac{A}{A'} = (1.005)^2 = 1.010025.$$

Thus the exact geometrical percentage increase under this model is

$$\frac{R' - R}{R} \times 100 = 1.0025\%.$$

At the precision of the supplied choices, this is 1.0%.

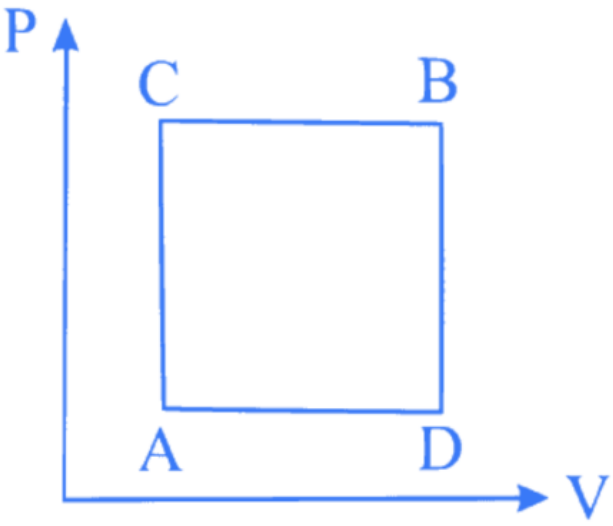
Step 3: Verify with the small-change approximation For constant volume and resistivity, logarithmic differentiation gives $\Delta R/R \simeq 2\Delta L/L$. Therefore the first-order increase is $2(0.5\%) = 1.0\%$, agreeing with the ratio calculation. The 0.5% option ignores the reduction in cross-sectional area. The values 2.0% and 2.5% do not follow from either the exact ratio or its first-order approximation.

Step 4: Conclusion The resistance increases by approximately 1.0% (the exact model value is 1.0025%). Hence, the correct option is (C).

Quick Tip to Solve: At fixed volume, substitute $A = V/L$ before comparing resistances: $R \propto L^2$, so a small percentage increase in length approximately doubles in resistance.

35. A gas can be taken from A to B via two different processes ACB and ADB.

When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is : [4 marks]



(A) 40 J

(B) 80 J

(C) 100 J

(D) 20 J

Correct Answer: (A) 40 J

Solution:

Concept:

- With heat entering the gas positive and work done by the gas positive, the first law is $\Delta U = Q - W$, or equivalently $Q = \Delta U + W$.
- Internal energy is a state function: the change between the same initial state A and final state B is identical for every path. Heat and work individually depend on the process.

Step 1: Find the internal-energy change along ACB Along the given path ACB, the heat absorbed is $Q_{ACB} = 60\,\text{J}$ and the work done by the gas is $W_{ACB} = 30\,\text{J}$. Hence

$$U_B - U_A = Q_{ACB} - W_{ACB} = 60 - 30 = 30\,\text{J}.$$

Part of the input heat appears as work, while the remaining energy increases the internal energy of the gas.

Step 2: Apply the same change along ADB The endpoints A and B are unchanged, so the path ADB also has $\Delta U = 30\,\text{J}$. Its stated work is only $W_{ADB} = 10\,\text{J}$. Rearranging the first law gives

$$Q_{ADB} = \Delta U + W_{ADB} = 30 + 10 = 40\,\text{J}.$$

This is positive, so it is heat flowing into the system, as requested.

Step 3: Check the result against the two paths The diagram is consistent with the smaller work on the lower-pressure route ADB. For fixed ΔU, reducing the work done by the gas by $20\,\text{J}$ reduces the heat input by the same amount: $60 - 20 = 40\,\text{J}$. The $20\,\text{J}$ option is the difference of path works rather than the required heat. Neither 80 nor $100\,\text{J}$ satisfies $Q - W = 30\,\text{J}$ for the ADB work of $10\,\text{J}$.

Step 4: Conclusion The heat flowing into the gas along ADB is $40\,\text{J}$. Hence, the correct option is (A).

Quick Tip to Solve: First obtain ΔU from the fully specified path, then reuse it for the other path; only the endpoints determine the internal-energy change.

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36. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\overrightarrow{E} = 6.3\hat{j}\,V/m$. The corresponding magnetic field $\overrightarrow{B}$, at that point will be : [4 marks]

(A) $18.9 \times 10^{-8}\,\hat{k}$T

(B) $2.1 \times 10^{-8}\,\hat{k}$T

(C) $6.3 \times 10^{-8}\,\hat{k}$T

(D) $18.9 \times 10^{8}\,\hat{k}$T

Correct Answer: (B) $2.1 \times 10^{-8}\,\hat{k}$T

Solution:

Concept:

- In a plane electromagnetic wave in free space, the instantaneous magnitudes satisfy $E = cB$, where $c = 3.0 \times 10^8\,\text{m}\,\text{s}^{-1}$. Electric and magnetic fields oscillate in phase.

- The direction of propagation is the direction of $\vec{E} \times \vec{B}$. Thus both fields are transverse to the propagation direction and perpendicular to one another.

Step 1: Calculate the magnetic-field magnitude The specified electric field is $\vec{E} = 6.3\hat{j}\,\mathrm{V\,m^{-1}}$, so its magnitude at the given point and instant is 6.3. Use the free-space field relation directly:

$$B = \frac{E}{c} = \frac{6.3}{3.0 \times 10^8} = 2.1 \times 10^{-8}\,\mathrm{T}.$$

The frequency is not needed: it determines the wavelength and time variation, but does not change the ratio of the simultaneous field amplitudes in free space.

Step 2: Determine the vector direction and compare the options The wave travels along $+\hat{i}$ and its electric field is along $+\hat{j}$. Since $\hat{j} \times \hat{k} = \hat{i}$, the magnetic field must point along $+\hat{k}$. A negative $\hat{k}$ direction would describe propagation along negative x instead. The magnitudes in (A) and (C) are respectively nine and three times the required value; (D) also has a positive rather than negative power of ten. Only (B) gives both the required magnitude and direction.

Step 3: Conclusion

$$\boxed{\vec{B} = 2.1 \times 10^{-8}\hat{k}\,\mathrm{T}.}$$

Hence, the correct option is (B).

Quick Tip to Solve: Use $B = E/c$ for the magnitude, then check the direction with $\vec{E} \times \vec{B}$; the frequency is irrelevant to this field ratio.

37. A block of mass m, lying on a smooth horizontal surface, is attached to a sring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is : [4 marks]



(A) $\frac{2F}{\sqrt{mk}}$
(B) $\frac{F}{\pi\sqrt{mk}}$
(C) $\frac{\pi F}{\sqrt{mk}}$
(D) $\frac{F}{\sqrt{mk}}$

Correct Answer: (D) $\frac{F}{\sqrt{mk}}$

Solution:

Concept:

- For a displacement x from the initial, unstretched equilibrium position, a constant pulling force does work Fx, while the spring stores elastic energy $kx^2/2$. The smooth surface eliminates friction.
- The speed is largest where the kinetic energy is largest. Equivalently, the net force $F - kx$ vanishes at the displaced equilibrium position $x = F/k$.

Step 1: Express the kinetic energy in terms of displacement The block begins from rest with zero spring extension. The work supplied by the applied force is shared between kinetic energy and elastic potential energy. Therefore

$$Fx = \frac{1}{2}mv^2 + \frac{1}{2}kx^2, \qquad \frac{1}{2}mv^2 = Fx - \frac{1}{2}kx^2.$$

The constant force continues acting as the block moves; consequently the spring energy alone is not conserved. This point distinguishes the problem from releasing an initially stretched spring without an external pull.

Step 2: Locate and evaluate the maximum The kinetic-energy expression is a concave quadratic in x. Differentiating with respect to x gives $F - kx = 0$, hence $x = F/k$. Since its second derivative is $-k < 0$, this is a maximum. Substitution gives

$$\frac{1}{2}mv_{\max}^2 = F\frac{F}{k} - \frac{k}{2}\left(\frac{F}{k}\right)^2 = \frac{F^2}{2k}.$$

Thus $v_{\max}^2 = F^2/(mk)$. Alternatively, the new oscillation centre is at F/k, and the amplitude about that centre is also F/k. With $\omega = \sqrt{k/m}$, the result follows from $v_{\max} = \omega A$. The factor 2 in (A) confuses the maximum extension $2F/k$ with the amplitude; the factors of π in (B) and (C) do not enter a maximum instantaneous speed.

Step 3: Conclusion

$$\boxed{v_{\max} = \frac{F}{\sqrt{mk}}.}$$

Hence, the correct option is (D).

Quick Tip to Solve: Measure the oscillation amplitude from the new equilibrium F/k, not from the original position or from the full extension $2F/k$.

38. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant.

The general equation for its path is : [4 marks]

(A) y = x^2 + constant

(B) y^2 = x + constant

(C) y^2 = x^2 + constant

(D) xy = constant

Correct Answer: (C) y^2 = x^2 + constant

Solution:

Concept:

- For planar motion, $v_x = dx/dt$ and $v_y = dy/dt$. Wherever $v_x \neq 0$, the tangent slope of the path is $dy/dx = v_y/v_x$.
- Eliminating time from the two component equations gives the trajectory. The integration constant is fixed by the initial position; it need not be zero.

Step 1: Obtain the differential equation of the path Compare the given vector $\vec{v} = K(y\hat{i} + x\hat{j})$ with $v_x\hat{i} + v_y\hat{j}$. The x-component is proportional to y, while the y-component is proportional to x:

$$\frac{dx}{dt} = Ky, \qquad \frac{dy}{dt} = Kx.$$

Dividing these equations eliminates both the time and the constant K. The correct slope is therefore $dy/dx = x/y$, not y/x; interchanging the components would lead to a different curve.

Step 2: Integrate and check the invariant Rearrange as $y\,dy = x\,dx$, and integrate both sides:

$$\frac{y^2}{2} = \frac{x^2}{2} + C_1, \qquad y^2 = x^2 + C,$$

where $C = 2C_1$. This is generally a rectangular hyperbola; the special case $C = 0$ contains the lines $y = \pm x$. One can verify the result without division, which also handles a vertical tangent:

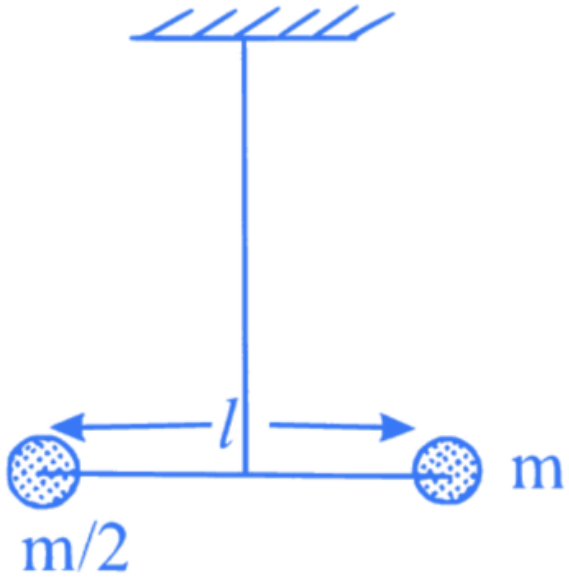
$$\frac{d}{dt}(y^2 - x^2) = 2y(Kx) - 2x(Ky) = 0.$$

Thus the difference of the squared coordinates is constant throughout the motion. Options (A) and (B) are parabolas and do not satisfy this invariant. For option (D), $d(xy)/dt = K(x^2 + y^2)$, which is not zero in general, so xy is not the required constant.

Step 3: Conclusion The general trajectory is $\boxed{y^2 = x^2 + \text{constant}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a velocity field whose components contain the opposite coordinates, test derivatives of $x^2 - y^2$ and xy to identify the conserved path relation quickly.

39. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k, at the centre of mass of the rod-mass system(see figure). Because of torsional constant k, the restoring torque is τ = kθ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be : [4 marks]



(A) $\frac{3k{\theta_0}^2}{l}$

(B) $\frac{2k{\theta_0}^2}{l}$

(C) $\frac{k{\theta_0}^2}{l}$

(D) $\frac{k{\theta_0}^2}{2l}$

Correct Answer: (C) $\frac{k{\theta_0}^2}{l}$

Solution:

Concept:

- The suspension passes through the centre of mass. If the distances of masses m and $m/2$ from the wire are r_1 and r_2, then $mr_1 = (m/2)r_2$ and $r_1 + r_2 = l$.
- Torsional potential energy is $k\theta^2/2$. On passing through the untwisted mean position it becomes rotational kinetic energy $I\omega^2/2$, where I is the moment of inertia about the suspension wire.
- At the mean position the torsional torque vanishes, but the masses still need radial centripetal force. The rod tension supplies $T = mr_1\omega^2 = (m/2)r_2\omega^2$.

Step 1: Find the radii and rotational inertia The centre of mass lies closer to the heavier mass. The two centre-of-mass equations give $r_1 = l/3$ for mass m and $r_2 = 2l/3$ for mass $m/2$. The massless rod contributes no inertia, so

$$I = m\left(\frac{l}{3}\right)^2 + \frac{m}{2}\left(\frac{2l}{3}\right)^2 = \frac{ml^2}{3}.$$

Using $l/2$ for both radii would incorrectly treat the unequal masses as equal.

Step 2: Use energy conservation and radial dynamics The system is released from rest at angular

displacement θ_0. At the mean position its torsional energy is zero and its speed is greatest:

$$\frac{1}{2}k\theta_0^2 = \frac{1}{2}I\omega^2, \qquad \omega^2 = \frac{3k\theta_0^2}{ml^2}.$$

For the heavier mass, the rod tension is the inward radial force. Thus

$$T = m\omega^2\frac{l}{3} = m\frac{3k\theta_0^2}{ml^2}\frac{l}{3} = \frac{k\theta_0^2}{l}.$$

The lighter mass gives exactly the same tension, since $(m/2)(2l/3) = ml/3$. The other options have extra factors 3, 2, or 1/2; none follows from both the correct centre of mass and energy balance. The tension remains nonzero even though the instantaneous restoring torque is zero.

Step 3: Conclusion

$$\boxed{T = \frac{k\theta_0^2}{l}.}$$

Hence, the correct option is (C).

Quick Tip to Solve: First locate the centre of mass; then convert torsional energy to angular speed and use either end mass to calculate the same radial tension.

40. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its center. Then value of h is : [4 marks]

(A) $\frac{R}{\sqrt{5}}$

(B) $\frac{R}{\sqrt{2}}$

(C) R

(D) $R\sqrt{2}$

Correct Answer: (B) $\frac{R}{\sqrt{2}}$

Solution:

Concept:

- For a uniformly charged ring of radius R and total charge Q, symmetry cancels the transverse components of the field at an axial point. Its magnitude at distance $h \geq 0$ is $E(h) = |Q|h/[4\pi\varepsilon_0(R^2 + h^2)^{3/2}]$.
- The largest field is found by differentiating with respect to axial distance. A stationary point is a maximum only if the field increases before it and decreases after it.

Step 1: Differentiate the axial field Write $A = |Q|/(4\pi\varepsilon_0)$, a positive constant that does not affect the location of the maximum. Then

$$E = Ah(R^2 + h^2)^{-3/2}.$$

Applying the product and chain rules gives

$$\frac{dE}{dh} = A\left[(R^2 + h^2)^{-3/2} - 3h^2(R^2 + h^2)^{-5/2}\right].$$

Combining the terms produces

$$\frac{dE}{dh} = A\frac{R^2 - 2h^2}{(R^2 + h^2)^{5/2}}.$$

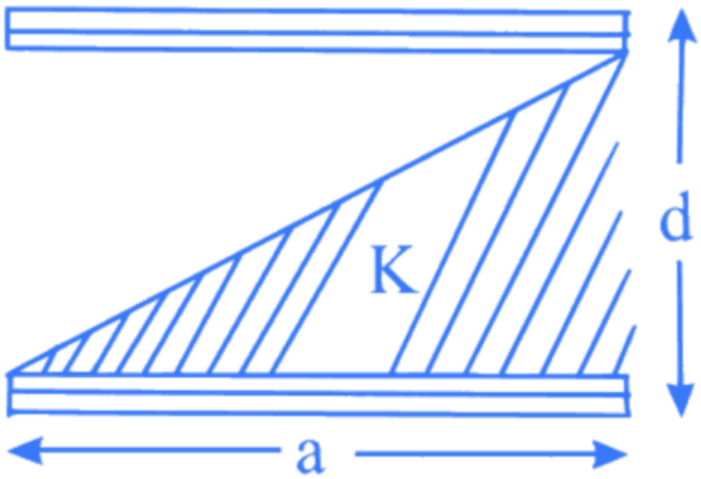
The power $3/2$ in the field formula includes the projection of each elemental field onto the axis; omitting this projection changes the problem.

Step 2: Identify the maximum and eliminate the other distances The denominator is positive, so the stationary condition is $R^2 - 2h^2 = 0$. Since h is a distance, choose its nonnegative root: $h = R/\sqrt{2}$. For smaller positive h, the numerator is positive and the field rises; for larger h, it is negative and the field falls. Also $E(0) = 0$ and $E \to 0$ as $h \to \infty$, confirming this is the global maximum on either side of the ring. At $R/\sqrt{5}$, option (A), the derivative is still positive; at R and $R\sqrt{2}$, options (C) and (D), it is already negative. None of those distances is a stationary point.

Step 3: Conclusion The largest magnitude occurs at $\boxed{h = R/\sqrt{2}}$ from the centre. Hence, the correct option is (B).

Quick Tip to Solve: For the ring field $h/(R^2 + h^2)^{3/2}$, differentiation reduces to the numerator $R^2 - 2h^2$; its zero fixes the maximum independently of charge.

41. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d (d $<<$ a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is : [4 marks]



(A) $\frac{K\epsilon_0 a^2}{2d(K+1)}$

(B) $\frac{K\epsilon_0 a^2}{d(K-1)} \ln K$

(C) $\frac{K\epsilon_0 a^2}{d} \ln K$

(D) $\frac{1}{2}\frac{K\epsilon_0 a^2}{d}$

Correct Answer: (B) $\frac{K\epsilon_0 a^2}{d(K-1)} \ln K$

Solution:

Concept:

- For dielectric layers stacked along the electric field, capacitances combine in series. A strip of area dA containing air thickness $d - t$ and dielectric thickness t has $dC = \varepsilon_0 dA/[d - t + t/K]$.
- Neighbouring vertical strips lie between the same two conducting plates and share the same voltage, so their capacitances add in parallel. The triangular wedge makes the dielectric thickness vary linearly across the plate.

Step 1: Express the capacitance of a narrow strip Let x measure horizontal distance from the thin end of the wedge, where its thickness is zero. From the straight sloping boundary, the dielectric thickness is $t = (d/a)x$, for $0 \le x \le a$. A strip of width dx extends through the full plate width a, so its area is $a\,dx$. Within that strip, the air and dielectric sections are in series along the field. Consequently

$$dC = \frac{\varepsilon_0 a\,dx}{d - (d/a)x + (d/(aK))x} = \frac{\varepsilon_0 a}{d}\frac{dx}{1 - \frac{K-1}{K}\frac{x}{a}}.$$

This is not a simple half-area parallel combination because the wedge changes thickness, rather than merely covering half the plate area.

Step 2: Integrate the strips and check the limiting case Adding all strip capacitances in parallel yields

$$C = \frac{\varepsilon_0 a}{d}\int_0^a \frac{dx}{1 - \frac{K-1}{K}\frac{x}{a}} = \frac{\varepsilon_0 a}{d}\frac{aK}{K-1}\ln K.$$

Hence $C = K\varepsilon_0 a^2 \ln K/[d(K-1)]$. As $K \to 1$, the factor $K \ln K/(K-1) \to 1$, restoring the empty-capacitor value $\varepsilon_0 a^2/d$. This check rules out (A), which gives only a quarter of that value at K=1, (C), which tends to zero, and (D), which tends to half of it. The expression in (B) alone has both the required logarithm and the correct empty-space limit.

Step 3: Conclusion

$$\boxed{C = \frac{K\varepsilon_0 a^2}{d(K-1)}\ln K.}$$

Hence, the correct option is (B).

Quick Tip to Solve: For a wedge, divide the plate into strips where the dielectric thickness is approximately constant: air and dielectric are in series within each strip, and the strips themselves combine in parallel.

42. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm^2, is v. If the electron density in copper is 9 $\times$ 10^{28}/m^3 the value of v. in mm/s is close to (Take charge of electron to be = 1.6 $\times$ 10^{-19}C) [4 marks]

(A) 0.02

(B) 3

(C) 2

(D) 0.2

Correct Answer: (A) 0.02

Solution:

Concept:

- The magnitude of electric current in a uniform wire is $I = neAv_d$, where n is the number of conduction electrons per unit volume, e is the positive elementary-charge magnitude, A is cross-sectional area, and v_d is drift speed.
- All quantities must first be expressed in consistent SI units. In particular, $1\,\text{mm}^2 = 10^{-6}\,\text{m}^2$, while $1\,\text{m s}^{-1} = 10^3\,\text{mm s}^{-1}$.

Step 1: Convert the area and substitute the data The cross section is $A = 5\times10^{-6}\,\text{m}^2$, not $5\times10^{-3}\,\text{m}^2$, because both dimensions of the area convert from millimetres. The electron density is $n = 9\times10^{28}\,\text{m}^{-3}$, and $e = 1.6\times10^{-19}\,\text{C}$. Therefore

$$neA = (9\times10^{28})(1.6\times10^{-19})(5\times10^{-6}) = 7.2\times10^{4}\,\text{C m}^{-1}.$$

Use the given current magnitude $I = 1.5\,\text{A}$. The electrons drift opposite to conventional current, but the question asks for speed, so the required value is positive.

Step 2: Calculate the speed and express it in millimetres per second Rearrange the current relation before evaluating:

$$v_d = \frac{I}{neA} = \frac{1.5}{7.2\times10^{4}} = 2.0833\times10^{-5}\,\text{m s}^{-1}.$$

Converting the final length unit gives

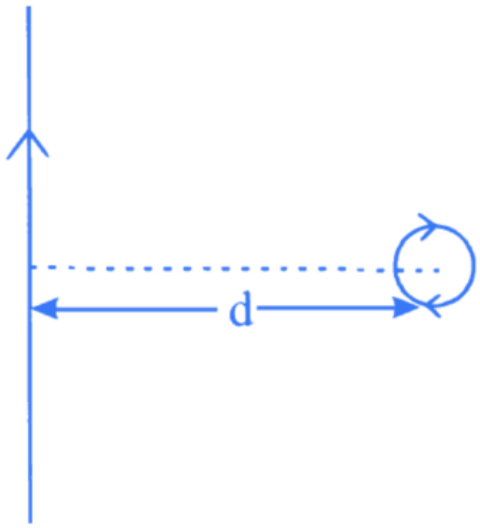
$$v_d = (2.0833\times10^{-5})\times10^{3} = 0.020833\,\text{mm s}^{-1}.$$

This is close to 0.02, option (A). Option (D), 0.2, is ten times too large, while 2 and 3 in (C) and (B) exceed the computed speed by approximately two orders of magnitude. Keeping area and final speed conversions separate avoids cancelling an incorrect power of ten accidentally.

Step 3: Conclusion The drift speed is approximately $\boxed{0.02\,\text{mm s}^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert mm^2 to m^2 using 10^{-6}, calculate $I/(neA)$ in SI, and multiply the resulting speed by 10^3 only at the end.

43. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is d (d $>>$ a). If the loop a applies a force F on the wire then : [4 marks]



(A) $F = 0$

(B) $F \propto \left(\frac{a}{d}\right)$

(C) $F \propto \left(\frac{a^2}{d^3}\right)$

(D) $F \propto \left(\frac{a}{d}\right)^2$

Correct Answer: (D) $F \propto \left(\frac{a}{d}\right)^2$

Solution:

Concept:

- A small circular current loop behaves as a magnetic dipole with moment $m = I_2\pi a^2$, directed normal to its plane by the right-hand rule.
- The field of a long straight wire is $B(r) = \mu_0 I_1/(2\pi r)$. A net force on a dipole requires a field gradient: $F_r = m\, dB/dr$ for a fixed parallel orientation.

Step 1: Identify the nonuniform field The loop lies to one side of the wire. The magnetic field is normal to the paper, and its magnitude is greater on the nearer side of the loop. Thus forces on opposite current elements do not cancel exactly. Since $d \gg a$, we may use the field and its gradient at the loop centre. The loop current shown is clockwise, so its moment is into the paper; the upward wire current also gives a field into the paper on its right.

Step 2: Calculate the leading force Use the dipole energy $U = -mB$. The radial force is the negative derivative of this energy, giving an attraction toward the stronger field. Its magnitude is

$$F = m\left|\frac{dB}{dd}\right| = I_2\pi a^2 \frac{\mu_0 I_1}{2\pi d^2} = \frac{\mu_0 I_1 I_2}{2}\frac{a^2}{d^2}.$$

The force exerted by the loop on the wire has the same magnitude and the opposite direction. For

fixed currents the remaining dependence is therefore $(a/d)^2$.

Step 3: Compare the proposed dependences Zero force would require a uniform field over the loop, whereas the wire field varies with distance. A dependence linear in a misses the loop area; the a^2/d^3 option contains an extra inverse power of distance. Differentiating the wire field $1/d$ gives $1/d^2$, not $1/d^3$.

Step 4: Conclusion $F \propto a^2/d^2 = (a/d)^2$. Hence, the correct option is (D).

Quick Tip to Solve: For a small current loop near a wire, multiply its magnetic moment by the gradient of the wire field; the area gives a^2 and the gradient gives d^{-2}.

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44. A heavy ball of mass M is suspendeed from the ceiling of a car by a light string of mass m (m $<<$ M). When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to : [4 marks]

(A) $\frac{g}{30}$
(B) $\frac{g}{5}$
(C) $\frac{g}{10}$
(D) $\frac{g}{20}$

Correct Answer: (B) $\frac{g}{5}$

Solution:

Concept:

- The transverse wave speed on a stretched string is $v = \sqrt{T/\mu}$, where T is its tension and μ is its mass per unit length.
- In a horizontally accelerating car, a suspended mass is in equilibrium relative to the car under gravity and the horizontal pseudo force. The effective gravitational acceleration has magnitude $\sqrt{g^2 + a^2}$.

Step 1: Obtain the two tensions Because $m \ll M$, the tension is nearly constant along the light string and is set by the suspended ball. At rest, $T_0 = Mg$. In the accelerating car the string tilts; resolving tension gives $T\cos\theta = Mg$ and $T\sin\theta = Ma$. Squaring and adding removes the unknown angle:

$$T = M\sqrt{g^2 + a^2}.$$

The horizontal and vertical forces are perpendicular, so adding their magnitudes as $M(g + a)$ would be incorrect.

Step 2: Use the fourth power of the speed ratio The same string is used in both situations, so its linear mass density cancels. Consequently,

$$\left(\frac{v}{v_0}\right)^2 = \frac{T}{T_0} = \sqrt{1 + \frac{a^2}{g^2}}, \qquad \frac{a}{g} = \sqrt{\left(\frac{60.5}{60}\right)^4 - 1}.$$

Numerically, $(60.5/60)^4 \simeq 1.03375$, so $a/g \simeq 0.1837$. For a quick approximation, write $60.5/60 = 1 + 1/120$; its fourth power is approximately $1 + 1/30$, yielding nearly the same estimate.

Step 3: Select the closest value The proposed ratios are approximately $0.0333, 0.20, 0.10, 0.05$. The calculated value 0.184 is closest to 0.20. The small fractional speed change should not be equated directly to a/g, since the acceleration changes the resultant tension quadratically and the wave speed introduces another square root.

Step 4: Conclusion The acceleration is approximately $0.184g$, closest to $g/5$. Hence, the correct option is (B).

Quick Tip to Solve: For a pendulum string in an accelerating car, square the wave-speed ratio to obtain the tension ratio, then square once more before extracting a/g.

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45. A conducting circular loop made of a thin wire, has area 3.5 $\times$ 10^{-3} m^2 and resistance 10 Ω. It is placed perpendicular to a time dependent magnetic field B(t) = (0.4T)sin(50πt). The field is uniform in space. Then the net charge flowing through the loop during t = 0 s and t = 10 ms is close to : [4 marks]

(A) 0.14 mC

(B) 0.7 mC

(C) 0.21 mC

(D) 0.6 mC

Correct Answer: (A) 0.14 mC

Solution:

Concept:

- Faraday's law gives $\mathcal{E} = -d\Phi/dt$, and Ohm's law gives $I = \mathcal{E}/R$ for the resistive loop.
- When the magnetic field is perpendicular to the loop, its flux is $\Phi = AB$. Integrating current over time gives the signed charge $q = -\Delta\Phi/R$.

Step 1: Find the endpoint magnetic fields Convert the final time before evaluating the trigonometric argument: $10\,\text{ms} = 0.010\,\text{s}$. The initial and final fields are

$$B(0) = 0, \qquad B(0.010) = 0.4\sin(50\pi \times 0.010) = 0.4\sin(\pi/2) = 0.4\,\text{T}.$$

The interval is the first quarter of the sine cycle. The field increases throughout it, so the induced current has one direction; the net charge magnitude also equals the total charge magnitude transported during this interval.

Step 2: Integrate the induced current Choose either orientation for the area normal. The sign of charge then follows from Lenz's law, but the options specify positive magnitudes. Integrating Faraday's law avoids treating a changing current as constant:

$$q = \int_0^{0.010} I\,dt = -\frac{A}{R}\int_{B(0)}^{B(0.010)} dB, \qquad |q| = \frac{A|\Delta B|}{R}.$$

Substitute the printed area and resistance:

$$|q| = \frac{(3.5 \times 10^{-3})(0.4)}{10} = 1.4 \times 10^{-4}\,\text{C} = 0.14\,\text{mC}.$$

Step 3: Check the interpretation and options The initial emf is largest, but it decreases to zero by the final time; multiplying the initial current by the duration would overestimate the charge. The endpoint flux change already includes that variation exactly. The values $0.7, 0.21, 0.6\,\text{mC}$ do not equal $A\Delta B/R$ for the given interval and resistance.

Step 4: Conclusion The net charge flowing through the loop has magnitude $0.14\,\text{mC}$. Hence, the correct option is (A).

Quick Tip to Solve: For charge induced in a resistive loop, use $|q| = |\Delta\Phi|/R$; the endpoint fluxes eliminate the need to integrate the sinusoidal emf explicitly.

46. Three charges + Q, q, + Q are placed respectively, at distance, 0, d/2 and d from the origin, on the x-axis. If the net force experienced by + Q, placed at x = 0, is zero, then value of q is : [4 marks]

(A) $-\frac{Q}{4}$
(B) $+\frac{Q}{2}$
(C) $+\frac{Q}{4}$
(D) $-\frac{Q}{2}$

Correct Answer: (A) $-\frac{Q}{4}$

Solution:

Concept:

- Coulomb's law gives the magnitude of the force between charges separated by r as $k|q_1 q_2|/r^2$. The force direction depends on whether the charges attract or repel.
- For equilibrium of the charge at the origin, the vector sum of the two forces on that particular charge must vanish. The nearer charge is at distance $d/2$, not d.

Step 1: Determine the necessary sign of the middle charge The positive charge at $x = d$ repels the positive charge at the origin toward negative x. To cancel this force, the middle charge must pull the origin charge toward positive x. Since the middle charge lies to the right, it must be negative. Either positive proposed value would repel the origin charge in the same direction as the distant positive charge and cannot give equilibrium.

Step 2: Equate the opposing force magnitudes The distant positive charge exerts the magnitude

$$F_{\text{far}} = \frac{kQ^2}{d^2}.$$

The charge q, only half as far away, exerts

$$F_{\text{middle}} = \frac{kQ|q|}{(d/2)^2} = \frac{4kQ|q|}{d^2}.$$

Equating these magnitudes cancels the common factor kQ/d^2, leaving $4|q| = Q$. Thus $|q| = Q/4$; combine this with the negative sign already established.

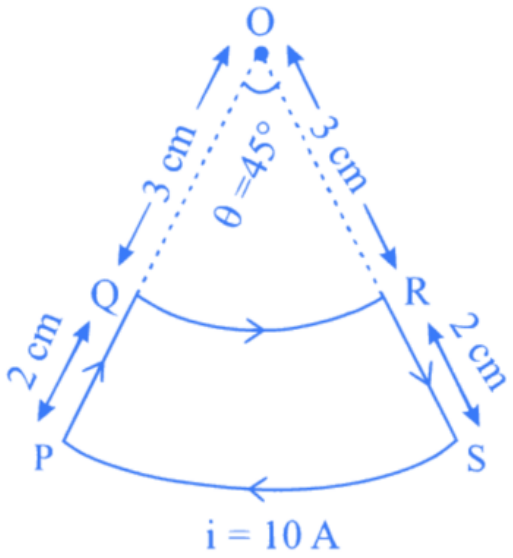
Step 3: Verify the balance and reject the other magnitude For $q = -Q/4$, the middle charge attracts the origin charge to the right with exactly kQ^2/d^2, cancelling the leftward force. The alternative $-Q/2$ would give twice that attractive force and leave a rightward resultant. The inverse-square distance dependence is essential: using a factor of two instead of four for the halved separation produces the wrong magnitude. No condition of zero force on the middle charge is needed to answer the question, which explicitly asks about the charge at $x = 0$.

Step 4: Conclusion The required middle charge is $q = -Q/4$. Hence, the correct option is (A).

Quick Tip to Solve: Fix the sign from force directions first, then use the inverse-square law: a charge at half the distance needs one quarter the magnitude to produce the same force.

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47. A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to : [4 marks]



(A) 1.0×10^{-7} T

(B) 1.5×10^{-7} T

(C) 1.5×10^{-5} T

(D) 1.0×10^{-5} T

Correct Answer: (D) 1.0×10^{-5} T

Solution:

Concept:

- A circular arc of radius r, current I, and central angle θ in radians produces a field $B = \mu_0 I\theta/(4\pi r)$ at its centre.
- For a radial straight conductor, every current element is parallel or antiparallel to the vector from

that element to the centre, so its Biot–Savart contribution there is zero.

Step 1: Read the two radii and the angular span The figure gives $OQ = OR = 3\,\text{cm}$ and radial segments $QP = RS = 2\,\text{cm}$. The inner radius is therefore $r_1 = 0.03\,\text{m}$, and the outer radius is $r_2 = 0.05\,\text{m}$, not $0.02\,\text{m}$. Both arcs subtend $45^\circ = \pi/4$ at O. The current is $10\,\text{A}$. The straight radial connectors have zero field at O because $d\boldsymbol{\ell} \times \hat{\boldsymbol{r}} = 0$.

Step 2: Subtract the fields of the two arcs The current traverses the inner and outer arcs in opposite senses. Their magnetic fields at the centre therefore oppose, with the smaller-radius inner arc giving the larger magnitude. Thus

$$B = \frac{\mu_0 I\theta}{4\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

Substituting SI units gives

$$B = 10^{-7} \times 10 \times \frac{\pi}{4}\left(\frac{1}{0.03} - \frac{1}{0.05}\right) = 1.047 \times 10^{-5}\,\text{T}.$$

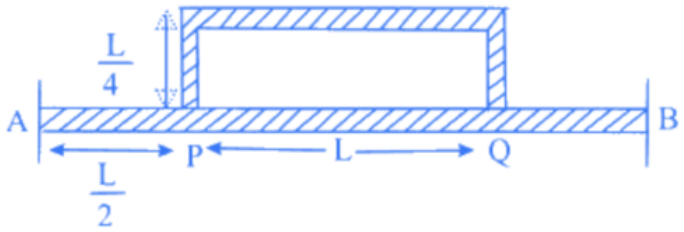
For the arrows shown the inner arc circulates anticlockwise about O, so the resultant field is out of the paper.

Step 3: Compare the numerical choices The result rounds to $1.0 \times 10^{-5}\,\text{T}$. The $10^{-7}\,\text{T}$ choices are too small by about two orders of magnitude, as happens if centimetres are not converted to metres. The $1.5 \times 10^{-5}\,\text{T}$ choice does not follow from the difference of reciprocal radii. Adding the arc fields would ignore their opposite current senses.

Step 4: Conclusion The magnetic field magnitude is approximately $1.0 \times 10^{-5}\,\text{T}$. Hence, the correct option is (D).

Quick Tip to Solve: For an annular-sector wire, radial sides give no field at the centre; subtract the two arc fields and remember that the outer radius includes the radial connector length.

48. Temperature difference of 120°C is maintained between ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (see figure). In steady state, temperature difference between P and Q will be close to : [4 marks]



(A) 45°C

(B) 75°C

(C) 60°C

(D) 35°C

Correct Answer: (A) 45°C

Solution:

Concept:

- The thermal resistance of a uniform rod is $R_{\text{th}} = \ell/(kA)$, where ℓ is its length, k its thermal conductivity and A its cross-sectional area.
- At steady state, sections in series carry the same total heat current. Conducting paths joining the same two junctions are in parallel and share their temperature difference.

Step 1: Replace the rods by thermal resistances Use the common rod material and cross-section and define $R = L/(kA)$. The diagram shows $AP = L/2$, $PQ = L$ along the straight rod, and hence $QB = L/2$, since $AB = 2L$. Their resistances are $R/2, R, R/2$, respectively. The bent connecting rod runs from P to Q with total length $3L/2$, so its resistance is $3R/2$. Its bend changes its path length but does not make it a series addition to the straight PQ segment.

Step 2: Combine the two paths between the junctions The straight and bent rods connect the same junction temperatures T_P, T_Q. Their parallel equivalent is

$$R_{PQ} = \frac{R(3R/2)}{R + 3R/2} = \frac{3}{5}R.$$

The complete equivalent resistance between the fixed-temperature ends is then

$$R_{AB} = \frac{R}{2} + \frac{3}{5}R + \frac{R}{2} = \frac{8}{5}R.$$

The total heat current enters through AP, splits between the two P-to-Q paths, recombines at Q, and leaves through QB.

Step 3: Use the thermal voltage divider The fraction of the total end-to-end temperature difference

across the equivalent PQ branch is its fraction of the series resistance:

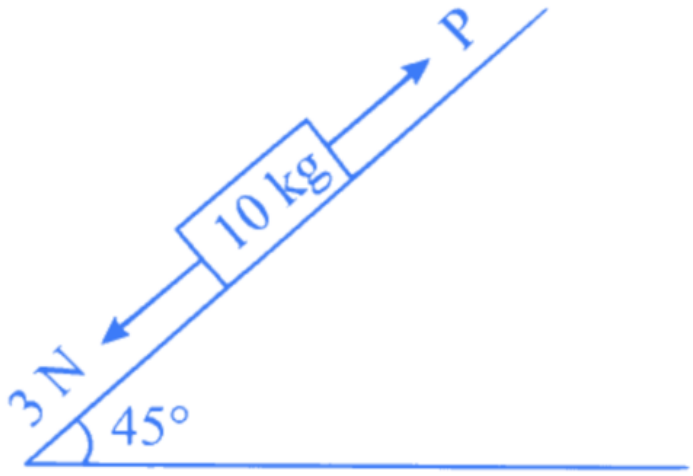
$$|T_P - T_Q| = 120\frac{(3/5)R}{(8/5)R} = 45\,^\circ\text{C}.$$

The value $60\,^\circ\text{C}$ would apply to the central length L of the original uniform rod without the added parallel path. The bypass reduces the central equivalent resistance and therefore reduces this temperature drop; $75\,^\circ\text{C}$ is in the wrong direction, and $35\,^\circ\text{C}$ does not satisfy the resistance ratio.

Step 4: Conclusion The steady temperature difference between P and Q is $45\,^\circ\text{C}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert the bent rod to a resistance using its full path length, combine it in parallel with the straight segment between the same junctions, then use a series temperature divider.

49. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block does not move downward? (take g = 10 ms^{-2}) [4 marks]



(A) 32 N

(B) 18 N

(C) 23 N

(D) 25 N

Correct Answer: (A) 32 N

Solution:

Concept:

- For a block on an incline of angle θ, gravity has components $mg\sin\theta$ down the plane and $mg\cos\theta$ into it.
- Static friction adjusts up to $f_{s,\max} = \mu_s N$. At the minimum uphill applied force preventing

downward motion, friction acts uphill at its limiting magnitude.

Step 1: Find the normal reaction and limiting friction The figure shows both P and the additional $3\,\mathrm{N}$ force parallel to the plane; neither changes the normal reaction. With $m = 10\,\mathrm{kg}$, $g = 10\,\mathrm{m\,s^{-2}}$, and $\theta = 45^\circ$,

$$N = mg\cos 45^\circ = 50\sqrt{2}\,\mathrm{N}.$$

Hence the largest uphill friction available is

$$f_{s,\max} = 0.6(50\sqrt{2}) = 30\sqrt{2}\,\mathrm{N}.$$

The $3\,\mathrm{N}$ arrow points downhill, so it must be added to the downhill gravitational component.

Step 2: Balance forces at impending downward motion Take uphill as positive. At the threshold of slipping down, the maximum static friction assists the uphill force P, giving

$$P + f_{s,\max} - mg\sin 45^\circ - 3 = 0.$$

Therefore the exact lower threshold is

$$P_{\min} = 50\sqrt{2} + 3 - 30\sqrt{2} = 20\sqrt{2} + 3 \simeq 31.28\,\mathrm{N}.$$

A force smaller than this would require static friction larger than its maximum and the block would slide downward. The friction direction is set by the impending downward tendency, rather than being chosen to oppose the applied uphill force.

Step 3: Choose the smallest listed force that works The choices $18, 23, 25\,\mathrm{N}$ all lie below the required threshold of approximately $31.3\,\mathrm{N}$. At $P = 32\,\mathrm{N}$, the required uphill friction is $50\sqrt{2}+3-32 \simeq 41.71\,\mathrm{N}$, below the available $30\sqrt{2} \simeq 42.43\,\mathrm{N}$. Thus the listed $32\,\mathrm{N}$ force can hold the block, consistent with the minimum threshold rounded to the available options.

Step 4: Conclusion The threshold force is $20\sqrt{2} + 3 \simeq 31.3\,\mathrm{N}$; the appropriate listed value is $32\,\mathrm{N}$. Hence, the correct option is (A).

Quick Tip to Solve: At the minimum uphill force, friction is uphill and limiting. Include every downhill applied force before subtracting $\mu_s mg\cos\theta$.

50. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M, Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of M/m ? [4 marks]



(A) 5

(B) 2

(C) 4

(D) 3

Correct Answer: (C) 4

Solution:

Concept:

- On the smooth horizontal surface, there is no external horizontal impulse, so the total horizontal momentum is conserved through both collisions.
- In a perfectly inelastic collision the bodies stick together. Kinetic energy is not conserved, but the final common speed follows from momentum conservation.

Step 1: Find the speed after the first collision Initially only A moves, so the total momentum is mv. After A and B stick, the moving mass is $2m$. Calling its speed u, momentum conservation gives

$$mv = 2mu \quad \Rightarrow \quad u = \frac{v}{2}.$$

The stationary block C has not yet acquired momentum. We must include the loss in this first collision as well as in the second collision.

Step 2: Find the final kinetic-energy fraction After the combined block sticks to C, all three blocks move at speed V:

$$(2m)\frac{v}{2} = (2m + M)V, \qquad V = \frac{mv}{2m + M}.$$

The initial and final energies are therefore

$$K_i = \frac{1}{2}mv^2, \qquad K_f = \frac{1}{2}(2m + M)\left(\frac{mv}{2m + M}\right)^2.$$

Dividing cancels the unknown initial speed:

$$\frac{K_f}{K_i} = \frac{m}{2m + M}.$$

Step 3: Use the stated total energy loss The loss is $5K_i/6$, so the retained fraction is $1/6$. Hence

$$\frac{m}{2m+M} = \frac{1}{6} \quad \Rightarrow \quad 6m = 2m + M \quad \Rightarrow \quad M = 4m.$$

The alternative ratios 5, 2 and 3 would retain fractions $1/7$, $1/4$ and $1/5$, respectively, rather than $1/6$.

Step 4: Conclusion

$$\boxed{\frac{M}{m} = 4}$$

Hence, the correct option is (C).

Quick Tip to Solve: For successive sticking collisions with stationary targets, conserve the momentum of the entire system and compare only its initial and final kinetic energies.

51. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is : [4 marks]

(A) $\frac{L}{m}$
(B) $\frac{4L}{m}$
(C) $\frac{L}{2m}$
(D) $\frac{2L}{m}$

Correct Answer: (C) $\frac{L}{2m}$

Solution:

Concept:

- Areal velocity is the area swept out by the radius vector per unit time. For a short displacement, the swept area is half the magnitude of the cross product of radius and displacement.
- The angular momentum about the Sun is $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$. Its magnitude is $L = mrv_\perp$, where $v_\perp$ is the transverse speed.

Step 1: Express the area swept in a short time Let the circular orbit have radius r, and let the planet turn through the small angle $d\theta$ in time dt. The area of this thin sector is

$$dA = \frac{1}{2}r^2 d\theta.$$

Dividing by the elapsed time gives

$$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}rv,$$

because the speed in a circular orbit is $v = r\,d\theta/dt$. The factor one-half comes from the sector area and must be retained.

Step 2: Relate the result to the given angular momentum The velocity is perpendicular to the orbital radius, so the angular-momentum magnitude is

$$L = mrv.$$

Consequently $rv = L/m$, and substitution into the area formula gives

$$\frac{dA}{dt} = \frac{1}{2}\frac{L}{m}.$$

This depends only on the specified angular momentum per unit mass; neither the orbital radius nor the speed needs to be separately calculated. A central gravitational force supplies no torque about the Sun, consistently making this areal velocity constant.

Step 3: Check the numerical factor and dimensions The units of L/m are $\mathrm{m^2\,s^{-1}}$, the correct units for area per time. All options share these dimensions, so the sector factor decides the answer. The alternatives L/m, $4L/m$ and $2L/m$ omit or incorrectly replace the required factor $1/2$.

Step 4: Conclusion

$$\boxed{\frac{dA}{dt} = \frac{L}{2m}}$$

Hence, the correct option is (C).

Quick Tip to Solve: Remember $2m\,dA/dt = L$: the factor two is geometrical, coming from the area of the thin triangle or sector swept by the radius vector.

52. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is 10^{19} $\mathrm{m^{-3}}$ and their mobility is 1.6 $\mathrm{m^2/(V.s)}$ then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to : [4 marks]

(A) $2\,\Omega$m

(B) $4\,\Omega$m

(C) 0.4 Ωm

(D) 0.2 Ωm

Correct Answer: (C) 0.4 Ωm

Solution:

Concept:

- Electron mobility is $\mu_e = v_d/E$, where v_d is the magnitude of the drift velocity and E is the electric-field magnitude.
- For an n-type semiconductor with negligible hole conduction, the conductivity is $\sigma = ne\mu_e$ and the resistivity is $\rho = 1/\sigma$. Here n is electron number density and e is the positive elementary-charge magnitude.

Step 1: Obtain conductivity from the drift-current relation The current density carried by electrons has magnitude

$$J = nev_d.$$

Using the given definition $v_d = \mu_e E$, we obtain

$$J = ne\mu_e E.$$

Comparison with the microscopic form of Ohm's law, $J = \sigma E$, identifies $\sigma = ne\mu_e$. Although electrons drift opposite to the electric field, conventional current is along the field, so the conductivity is positive.

Step 2: Substitute the carrier density and mobility With all quantities already in SI units, use

$$n = 10^{19}\,\mathrm{m}^{-3}, \qquad e = 1.6 \times 10^{-19}\,\mathrm{C}, \qquad \mu_e = 1.6\,\mathrm{m}^2\,\mathrm{V}^{-1}\,\mathrm{s}^{-1}.$$

The conductivity is

$$\sigma = (10^{19})(1.6 \times 10^{-19})(1.6) = 2.56\,\mathrm{S\,m}^{-1}.$$

No hole term is added, as the question explicitly directs us to ignore it.

Step 3: Take the reciprocal and select the closest value The required property is resistivity, rather than conductivity:

$$\rho = \frac{1}{2.56} = 0.390625\,\Omega\,\mathrm{m} \simeq 0.4\,\Omega\,\mathrm{m}.$$

The values 2, 4 and $0.2\,\Omega\,\mathrm{m}$ are not close to this reciprocal. Multiplying by the charge density without taking the final reciprocal would give the wrong physical quantity.

Step 4: Conclusion

$$\boxed{\rho \simeq 0.4\,\Omega\,\mathrm{m}}$$

Hence, the correct option is (C).

Quick Tip to Solve: First calculate $ne\mu_e$ as a conductivity; then invert it. Keeping that intermediate unit prevents confusing conductivity with resistivity.

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53. Surface of certain metal is first illuminated with light of wavelength λ_1 = 350 nm and then, by light of wavelength λ_2 = 540 nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to :

(Energy of photon n = $\frac{1240}{\lambda(in\,mm)}$eV) [4 marks]

(A) 1.8

(B) 2.5

(C) 5.6

(D) 1.4

Correct Answer: (A) 1.8

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = E_\gamma - \phi$, where ϕ is the work function and $E_\gamma = hc/\lambda$ is photon energy.

- The maximum electron kinetic energy is proportional to the square of its maximum speed: $K_{\max} = m_e v_{\max}^2/2$. A speed ratio of two therefore means an energy ratio of four.

Step 1: Find the two photon energies The shorter wavelength gives the larger photon energy, so the 350 nm light produces the faster electrons. Using $hc = 1240\,\text{eV nm}$, the photon energies are

$$E_1 = \frac{1240}{350} = 3.542857\,\text{eV}, \qquad E_2 = \frac{1240}{540} = 2.296296\,\text{eV}.$$

The wavelengths in these substitutions are in nanometres, consistently with the unit of hc. Equivalently one may use hc in joule metres and convert the resulting energies to electronvolts.

Step 2: Convert the speed ratio into an energy equation Since $v_1 = 2v_2$, the kinetic energies satisfy $K_1 = 4K_2$. Apply the same work function to both illuminations:

$$E_1 - \phi = 4(E_2 - \phi).$$

Collecting the work-function terms gives

$$3\phi = 4E_2 - E_1.$$

Using a factor of two directly between kinetic energies would be incorrect because kinetic energy depends on speed squared.

Step 3: Evaluate and check the emission conditions Substitution yields

$$\phi = \frac{4(2.296296) - 3.542857}{3} = 1.880776\,\text{eV}.$$

This is closest to $1.8\,\text{eV}$. It is less than both photon energies, so both illuminations can eject electrons. Indeed the resulting kinetic energies are about 1.6621 and $0.4155\,\text{eV}$, in the required ratio four. Work functions 2.5 or 5.6 eV would forbid emission at 540 nm; 1.4 eV fails the speed ratio.

Step 4: Conclusion The work function is approximately $1.88\,\text{eV}$, closest to $1.8\,\text{eV}$. Hence, the correct option is (A).

Quick Tip to Solve: Square the maximum-speed ratio before applying the two photoelectric equations; the common work function then follows from one linear equation.

54. A mixture of 2 moles of helium gas (atomic mass = 4 u), and 1 mole of argon gas (atomic mass = 40 u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{V_{rms}\,(helium)}{V_{rms}\,(\arg on)}\right]$, is close to : [4 marks]

(A) 3.16

(B) 0.32

(C) 0.45

(D) 2.24

Correct Answer: (A) 3.16

Solution:

Concept:

- For a gas at temperature T, the root-mean-square molecular speed is $v_{\text{rms}} = \sqrt{3RT/M}$, where M is its molar mass in kg mol^{-1}.
- The gases in the mixture are at the same temperature. Their rms speeds depend on their molecular masses, not on their mole numbers or individual contributions to the pressure.

Step 1: Write separate rms-speed expressions Let M_{He} and M_{Ar} denote the molar masses of helium and argon. Both are monatomic gases, so the given atomic masses directly give the corresponding molar masses:

$$M_{\text{He}} = 4 \times 10^{-3}\,\text{kg mol}^{-1}, \qquad M_{\text{Ar}} = 40 \times 10^{-3}\,\text{kg mol}^{-1}.$$

At the common temperature 300 K their individual rms speeds are $\sqrt{3RT/M_{\text{He}}}$ and $\sqrt{3RT/M_{\text{Ar}}}$.

Step 2: Take the ratio before substituting constants Dividing the two expressions cancels the temperature, the gas constant and the common factor three:

$$\frac{v_{\text{rms,He}}}{v_{\text{rms,Ar}}} = \sqrt{\frac{M_{\text{Ar}}}{M_{\text{He}}}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.1623.$$

The lighter helium atoms move faster on average, as required for the same mean translational kinetic energy per atom.

Step 3: Explain why the mixture composition does not enter Two moles of helium means twice as many helium atoms as argon atoms, but it does not double the rms speed of an individual helium atom. The rms speed is the square root of the mean squared speed within that species. Therefore no factor involving the 2:1 mole ratio belongs in the expression. The option 0.32 reverses the requested speed ratio; 0.45 and 2.24 incorrectly introduce a composition factor.

Step 4: Conclusion

$$\boxed{\frac{v_{\text{rms,He}}}{v_{\text{rms,Ar}}} \simeq 3.16}$$

Hence, the correct option is (A).

Quick Tip to Solve: At a common temperature, put the heavier molar mass in the numerator when finding the lighter gas's speed divided by the heavier gas's speed.

55. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then d is : [4 marks]

(A) 1.1 cm away from the lens

(B) 0

(C) 0.55 cm towards the lens

(D) 0.55 cm away from the lens

Correct Answer: (D) 0.55 cm away from the lens

Solution:

Concept:

- For a thin convex lens using positive object and real-image distances, $1/f = 1/u + 1/v$. Initially equal object and image distances of 10 cm imply $f = 5\,\text{cm}$.
- A plane-parallel slab of thickness t and refractive index μ reduces the apparent axial distance to an object behind it by $t(1 - 1/\mu)$, for paraxial rays emerging into air.

Step 1: Find the focal length from the original image The original source and screen distances from the lens are both 10 cm. Thus

$$\frac{1}{f} = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}, \qquad f = 5\,\text{cm}.$$

The original arrangement places the source and its image at twice the focal length on opposite sides of the lens. Inserting a plane slab does not change the lens power; it changes the effective object distance seen by the lens.

Step 2: Calculate the new apparent object distance The glass slab is on the source side. Its 1.5 cm thickness has apparent optical depth $t/\mu = 1.5/1.5 = 1.0\,\text{cm}$. The remaining air path is $10 - 1.5 = 8.5\,\text{cm}$. Therefore

$$u\prime = 8.5 + 1.0 = 9.5\,\text{cm}.$$

Equivalently the source appears shifted toward the lens by

$$\Delta u = t\left(1 - \frac{1}{\mu}\right) = 1.5\left(1 - \frac{1}{1.5}\right) = 0.5\,\text{cm}.$$

Step 3: Refocus the screen and determine the direction Apply the lens formula to the apparent object:

$$\frac{1}{v\prime} = \frac{1}{5} - \frac{1}{9.5} = \frac{9}{95}, \qquad v\prime = \frac{95}{9} = 10.5556\,\text{cm}.$$

The new image is farther from the lens, so the screen must move away by

$$d = v\prime - 10 = \frac{5}{9}\,\text{cm} \simeq 0.55\,\text{cm}.$$

The object remains outside the focal point, so the image stays real. A shift toward the lens has the wrong direction; zero ignores the slab, while 1.1 cm overestimates the image displacement.

Step 4: Conclusion The screen is shifted approximately $\boxed{0.55\,\text{cm}}$ away from the lens. Hence, the correct option is (D).

Quick Tip to Solve: On the object side a glass slab makes the source appear closer to the lens; calculate that apparent object distance first, then reapply the lens formula.

56. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio : [4 marks]

(A) 16 : 9

(B) 25 : 9

(C) 4 : 1

(D) 5 : 3

Correct Answer: (B) 25 : 9

Solution:

Concept:

- For coherent waves, the maximum and minimum resultant intensities are $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$.
- Intensity is proportional to the square of wave amplitude. We first find the amplitude ratio from the fringe contrast, then square it to obtain the individual intensity ratio.

Step 1: Introduce the larger-to-smaller amplitude ratio Let the individual amplitudes be a_1 and a_2, with $a_1 > a_2 > 0$, and let $r = a_1/a_2$. At constructive interference the amplitudes add, while at destructive interference they subtract. Hence the given ratio is

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{r+1}{r-1}\right)^2 = 16.$$

The different source intensities ensure that the minimum intensity is not zero.

Step 2: Solve for the amplitude ratio Because $r > 1$, the ratio inside the square is positive, so taking its positive square root gives

$$\frac{r+1}{r-1} = 4.$$

Rearrange to obtain

$$r + 1 = 4r - 4, \qquad 3r = 5, \qquad r = \frac{5}{3}.$$

This is the amplitude ratio only. It is not yet the requested ratio of the wave intensities.

Step 3: Square and verify the intensity ratio The individual intensities satisfy

$$\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}.$$

As a check, amplitudes proportional to 5 and 3 produce extrema proportional to $8^2 = 64$ and $2^2 = 4$,

whose ratio is exactly 16. Option 5:3 confuses amplitude with intensity. The ratios 16:9 and 4:1 correspond to amplitude ratios 4:3 and 2:1, giving maximum-to-minimum ratios 49 and 9, respectively.

Step 4: Conclusion

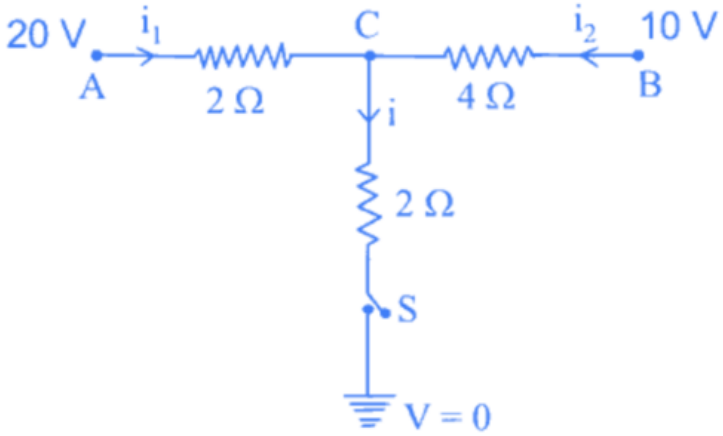
$$\boxed{I_1 : I_2 = 25 : 9}$$

Hence, the correct option is (B).

Quick Tip to Solve: Take the square root of the extrema ratio to find the sum-to-difference amplitude ratio; after solving for the amplitudes, square once more for source intensities.

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57. When the switch S, in the circuit shown, is closed, then the value of current i will be : [4 marks]



(A) 3A

(B) 5A

(C) 4A

(D) 2A

Correct Answer: (B) 5A

Solution:

Concept:

- For a resistor connecting potentials V_1 and V_2, the current from the first terminal to the second is $(V_1 - V_2)/R$. A negative value means the actual current is opposite to that reference direction.
- At a junction, charge conservation requires the sum of incoming currents to equal the sum of outgoing currents. Closing the switch connects the lower end of the vertical resistor to ground, whose potential is zero.

Step 1: Assign the junction potential Let the potential of junction C be V_C, measured relative to ground. The two fixed terminal potentials are $20\,\text{V}$ at A and $10\,\text{V}$ at B. With S closed, the downward current in the vertical $2\,\Omega$ branch is $i = V_C/2$. The incoming branch currents, with the arrows shown, are

$$i_1 = \frac{20 - V_C}{2}, \qquad i_2 = \frac{10 - V_C}{4}.$$

Step 2: Apply the junction rule and solve Using $i_1 + i_2 = i$, we obtain

$$\frac{20 - V_C}{2} + \frac{10 - V_C}{4} = \frac{V_C}{2}.$$

Multiplying by four gives

$$40 - 2V_C + 10 - V_C = 2V_C, \qquad 5V_C = 50.$$

Thus $V_C = 10\,\text{V}$. Therefore $i_1 = 5\,\text{A}$, $i_2 = 0$, and the downward current is

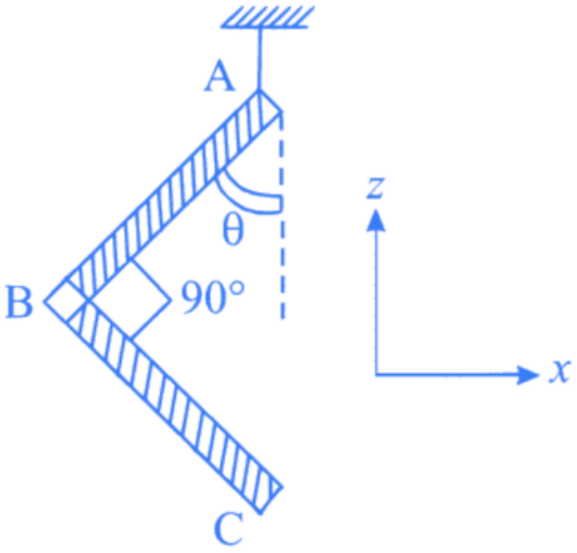
$$i = \frac{10}{2} = 5\,\text{A}.$$

The zero current in the right branch is consistent: both its ends are at $10\,\text{V}$. Values 3 A, 4 A and 2 A do not satisfy the node equation.

Step 3: Conclusion The current through the closed switch branch is $i = 5\,\text{A}$, downward as indicated. Hence, the correct option is (B).

Quick Tip to Solve: Use one unknown junction potential; do not combine the three resistors as a simple series circuit because their outer terminals have different fixed potentials.

58. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, thrown : [4 marks]



(A) $\tan\theta = \frac{1}{2\sqrt{3}}$
(B) $\tan\theta = \frac{1}{2}$
(C) $\tan\theta = \frac{2}{\sqrt{3}}$
(D) $\tan\theta = \frac{1}{3}$
Correct Answer: (D) $\tan\theta = \frac{1}{3}$

Solution:

Concept:

- A freely suspended rigid body rests with its centre of mass vertically below the suspension point. Otherwise its weight produces a nonzero turning moment about that point.
- For equal-length thin rods of the same uniform linear mass density, the masses are equal and each rod's centre of mass lies at its midpoint. The combined centre is the mean of the two midpoint position vectors.

Step 1: Find the centre of mass in rod-based coordinates Let $AB = BC = L$, and give each rod mass m. Choose A as origin, with one coordinate direction along AB toward B and a perpendicular direction parallel to BC toward C. The midpoint of AB then has coordinates $(L/2, 0)$; the midpoint of BC has coordinates $(L, L/2)$. Equal masses give

$$u_G = \frac{m(L/2) + mL}{2m} = \frac{3L}{4}, \qquad v_G = \frac{m(0) + m(L/2)}{2m} = \frac{L}{4}.$$

These coordinates locate the centre of mass G inside the angle between the two directions.

Step 2: Align the centre of mass with the vertical The string acts at A, so its force has no moment about A. In equilibrium the weight through G must also have no moment, which requires AG to be along the downward vertical. The angle between AB and AG is therefore the given θ. From the two perpendicular components of AG,

$$\tan\theta = \frac{v_G}{u_G} = \frac{L/4}{3L/4} = \frac{1}{3}.$$

This orientation puts G below A and is stable. The alternatives $1/(2\sqrt{3})$, $1/2$ and $2/\sqrt{3}$ would leave the centre of mass displaced sideways from A, producing a gravitational torque.

Step 3: Conclusion The equilibrium angle satisfies $\tan\theta = 1/3$. Hence, the correct option is (D).

Quick Tip to Solve: For a suspended compound rod, locate the weighted average of the rod midpoints first; the line from suspension point to this centre becomes vertical.

59. A sample of radioactive material A, that has an activity of 10 mCi(1 Ci = 3.7 $\times$ 10^{10} decays/s), has twice the number of nuclei as another sample of a different radioactive materail B which has an activity of 20 mCi. The correct choices for half-lives of A and B would then be respectively : [4 marks]

(A) 5 days and 10 days

(B) 10 days and 40 days

(C) 20 days and 5 days

(D) 20 days and 10 days

Correct Answer: (C) 20 days and 5 days

Solution:

Concept:

- The activity of a radioactive sample is $R = \lambda N$, where N is its number of undecayed nuclei and λ is the decay constant. More nuclei need not mean a greater activity if the decay constants differ.
- The half-life is $T_{1/2} = \ln 2/\lambda$. Consequently, at the stated instant, $R = (\ln 2)N/T_{1/2}$; activity is proportional to the number of nuclei and inversely proportional to half-life.

Step 1: Compare the decay constants Denote the samples' activities by R_A and R_B. The given values yield

$$\frac{R_A}{R_B} = \frac{10\,\text{mCi}}{20\,\text{mCi}} = \frac{1}{2}, \qquad \frac{N_A}{N_B} = 2.$$

Using $R = \lambda N$,

$$\frac{1}{2} = \frac{\lambda_A N_A}{\lambda_B N_B} = 2\frac{\lambda_A}{\lambda_B}.$$

Hence $\lambda_A/\lambda_B = 1/4$. Sample A has a smaller decay constant even though it contains twice as many nuclei.

Step 2: Convert to the half-life ratio and test the pairs Because half-life is inversely proportional to decay constant,

$$\frac{T_A}{T_B} = \frac{\lambda_B}{\lambda_A} = 4.$$

Among the given pairs, 20 days and 5 days have ratio four. The pairs in (A), (B) and (D) have respective ratios $1/2$, $1/4$ and 2, so none matches. As a direct check of (C), the activity ratio predicted is

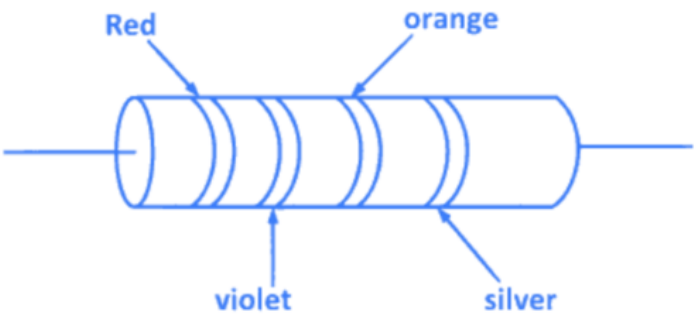
$$(N_A/N_B)(T_B/T_A) = 2(5/20) = \frac{1}{2},$$

exactly the measured ratio. The conversion from curie to decays per second cancels, since both activities use the same unit.

Step 3: Conclusion The compatible half-lives are $T_A = 20$ days and $T_B = 5$ days. Hence, the correct option is (C).

Quick Tip to Solve: Use the ratio $R_A/R_B = (N_A/N_B)(T_B/T_A)$; the half-life ratio reverses the decay-constant ratio, and the curie conversion is unnecessary.

60. A resistance is shown in the figure. Its value and tolerance are given respectively by : [4 marks]



(A) 270 Ω, 10 %

(B) 27 kΩ, 10 %

(C) 27 kΩ, 20 %

(D) 270 Ω, 5 %

Correct Answer: (B) 27 kΩ, 10 %

Solution:

Concept:

- In the four-band resistor code, the first two bands give the significant digits, the third gives a power-of-ten multiplier, and the fourth gives the tolerance.
- The digit codes needed here are red $= 2$, violet $= 7$, and orange $= 3$. As a multiplier, orange means 10^3; silver in the tolerance position means $\pm 10\%$.

Step 1: Read the significant digits and multiplier The figure identifies the bands, in reading order, as red, violet, orange and silver. Silver is the tolerance band and is read last; it is not a significant digit. Red and violet form the two-digit number 27. The orange multiplier then gives

$$R = 27 \times 10^3\,\Omega = 27\,\text{k}\Omega.$$

The third-band code is an exponent, so it does not mean multiply by three or append a single zero. This is why the $270\,\Omega$ alternatives have the wrong nominal value.

Step 2: Apply the tolerance band and compare the options The silver final band specifies a tolerance of $\pm 10\%$, not 5%, which would correspond to gold. The allowed departure from the nominal resistance is

$$\Delta R = \frac{10}{100} \times 27\,\text{k}\Omega = 2.7\,\text{k}\Omega.$$

Thus the resistance can lie between $24.3\,\text{k}\Omega$ and $29.7\,\text{k}\Omega$. Option (B) gives both the correct nominal value and the correct tolerance. Option (C) has the right nominal resistance but uses 20%, the tolerance associated with the absence of a tolerance band. Options (A) and (D) give the wrong resistance, and (D) also has the wrong tolerance.

Step 3: Conclusion The resistance and tolerance are $27\,\text{k}\Omega$ and $\pm 10\%$, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Locate the gold or silver tolerance band first and read from the other end; then use two digits followed by the multiplier, rather than treating all bands as digits.

Chemistry (120 marks)

61. In general, the properties that decrease and increase down a group in the periodic table, respectively, are :

[4 marks]

(A) Atomic Radius and Electronegativity

(B) Electron Gain Enthalpy and Electronegativity.

(C) Electronegativity and Atomic Radius.

(D) Electronegativity and Electron Gain Enthalpy.

Correct Answer: (C) Electronegativity and Atomic Radius.

Solution:

Concept:

- Atomic radius measures atomic size. Down a group, each successive element has its valence electrons in a new principal shell, so increased shell number and shielding generally increase the radius.
- Electronegativity is an atom's tendency to attract a shared electron pair in a chemical bond. Greater distance of the bonding electrons from the nucleus and stronger shielding generally reduce that attraction down a group.

Step 1: Determine which property decreases On moving down a group, the nuclear charge increases, but inner electron shells also increase and shield the outer electrons. The valence shell is farther from the nucleus. For a shared bonding pair this increased distance and shielding generally outweigh the rise in nuclear charge, so electronegativity decreases. The word "in general" asks for the overall periodic trend, rather than exceptional behaviour of particular elements. Thus the first property must be electronegativity, narrowing the choice to (C) or (D).

Step 2: Identify the increasing property and eliminate the alternatives The addition of occupied principal shells makes atomic size increase down the group. The unambiguous standard pair is therefore decreasing electronegativity and increasing atomic radius, as in (C). Option (A) reverses both trends, and (B) incorrectly puts electronegativity in the increasing position. Option (D) needs a convention check: electron gain enthalpy is a signed enthalpy change. It generally becomes less negative down a group, so its algebraic value generally increases, even though the magnitude of the energy released on adding an electron tends to decrease. These are different statements. Electron gain enthalpy also has notable exceptions, such as the more negative value for chlorine than fluorine. Under the signed enthalpy convention, (D) can also describe a general down-group trend. Thus (C) is the intended/keyed answer, but the wording does not cleanly exclude (D); do not interpret an increasing signed enthalpy as an increasing attraction for electrons.

Step 3: Conclusion The intended/keyed answer is (C): electronegativity decreases and atomic radius increases. The wording is ambiguous because (D) can also hold when electron gain enthalpy means its signed value, generally becoming less negative down a group.

Quick Tip to Solve: For size and electronegativity, use the added-shell trend. For electron gain enthalpy, distinguish a less negative signed value from a smaller magnitude of energy released.

62. Which one of the following statements regarding Henry's law is not correct ? [4 marks]

(A) Higher the value of K_H at a given pressure, higher is the solubility of the gas in the liquids

(B) Different gases have different K_H (Henry's law constant) values at the same temperature.

(C) The partial pressure of the gas in vapour phase is proportional to the mole fraction of the gas in the solution.

(D) The value of K_H increases with increase of temperature and K_H is function of the nature of the gas

Correct Answer: (A) Higher the value of K_H at a given pressure, higher is the solubility of the gas in the liquids

Solution:

Concept:

- For a gas dissolved in a liquid at fixed temperature, Henry's law in the stated convention is $p = K_H x$, where p is the gas's partial pressure and x is its mole fraction in solution.
- The Henry constant depends on the gas, solvent and temperature. In this pressure-over-mole-fraction convention, a larger K_H means a smaller solubility at a fixed gas partial pressure.

Step 1: Test the claimed relation between constant and solubility Rearrange the law to isolate the dissolved mole fraction:

$$x = \frac{p}{K_H}.$$

With pressure held fixed, doubling K_H halves x. The dissolved amount therefore decreases as this Henry constant increases. Statement (A) says the opposite: it predicts greater solubility for a greater constant. This statement is incorrect under the convention $p = K_H x$ used in the question.

Step 2: Check the remaining statements Different gases interact differently with a given solvent, so their Henry constants can differ even at the same temperature; (B) is correct. At fixed temperature and for a fixed gas-solvent pair, K_H is constant, and $p = K_H x$ explicitly says that gas partial pressure is proportional to dissolved mole fraction; (C) is correct. For the usual gases considered in this law, dissolution is exothermic, so heating reduces gas solubility. In the convention above this corresponds to an increase of K_H with temperature, and its dependence on the nature of the gas is also correct; thus (D) states the standard trend. Since the question asks for the statement that is not correct, select (A).

Step 3: Conclusion A higher K_H at fixed pressure gives lower gas solubility, so statement (A) is not correct. Hence, the correct option is (A).

Quick Tip to Solve: Write $x = p/K_H$ before judging a Henry-law option: the inverse relation prevents confusing this constant with the reciprocal solubility convention.

63. The highest value of the calculated spin only magnetic moment (in BM) among all the transition metal complexes is : [4 marks]

(A) 5.92

(B) 6.93

(C) 3.87

(D) 4.90

Correct Answer: (A) 5.92

Solution:

Concept:

- The calculated spin-only magnetic moment is $\mu_{\text{so}} = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons and BM denotes the Bohr magneton.
- The five orbitals of a transition-metal d subshell can contain at most five unpaired electrons. A high-spin d^5 configuration attains this maximum with one electron in each orbital.

Step 1: Find the maximum number of unpaired electrons A d subshell contains five orbitals. Hund's rule fills them singly before pairing, provided the ligand field permits a high-spin arrangement. Consequently, the maximum unpaired-electron count is $n = 5$, as in high-spin d^5. Adding a sixth d electron forces pairing in an already occupied orbital, so d^6 does not give six unpaired electrons. Ligand-induced pairing can reduce the moment but cannot raise this five-electron maximum for the transition-metal d configuration.

Step 2: Calculate the spin-only maximum and compare the values Substitute $n = 5$ into the spin-only expression:

$$\mu_{\text{max}} = \sqrt{5(5+2)}\,\text{BM} = \sqrt{35}\,\text{BM} \simeq 5.92\,\text{BM}.$$

The value $6.93\,\text{BM} = \sqrt{48}\,\text{BM}$ would require $n = 6$, which is not possible in the five d orbitals. The values $3.87\,\text{BM}$ and $4.90\,\text{BM}$ correspond to $n = 3$ and $n = 4$, respectively, and are achievable but smaller. This question specifically asks for the calculated spin-only value; an orbital contribution to an observed moment is not included in this formula.

Step 3: Conclusion The highest calculated spin-only moment is $\sqrt{35}\,\text{BM} \approx 5.92\,\text{BM}$. Hence, the correct option is (A).

Quick Tip to Solve: The largest spin-only moment of a transition-metal d configuration comes from high-spin d^5; never assign six unpaired electrons to five d orbitals.

64. For emission line of atomic hydrogen from $n_i = 8$ to $n_f = n$, the plot of wave number $(\overline{v})$ against $\left(\frac{1}{n^2}\right)$ will be (The Rydberg constant, R_H is in wave number unit) [4 marks]

(A) Linear with intercept $-R_H$

(B) Non linear

(C) Linear with slope R_H

(D) Linear with slope $-R_H$

Correct Answer: (C) Linear with slope R_H

Solution:

Concept:

- For a hydrogen emission transition from a higher level n_i to a lower level n_f, the emitted wave number is $\overline{\nu} = R_H(1/n_f^2 - 1/n_i^2)$. Wave number is the reciprocal of wavelength, and R_H has the same unit.
- A graph of $y = mx + c$ is linear, with slope m and vertical intercept c. Here the horizontal variable is $x = 1/n^2$, not n itself.

Step 1: Insert the fixed initial level The initial principal quantum number is fixed at eight and the final level is n. For emission $n < 8$, so the Rydberg expression gives a positive wave number:

$$\overline{\nu} = R_H\left(\frac{1}{n^2} - \frac{1}{8^2}\right) = R_H\frac{1}{n^2} - \frac{R_H}{64}.$$

The second term is a constant because the initial level does not change.

Step 2: Read the slope and intercept Putting $x = 1/n^2$ and $y = \overline{\nu}$ gives

$$y = R_H x - \frac{R_H}{64}.$$

Thus the allowed transitions give points on a straight line whose positive slope is R_H. The extrapolated vertical intercept is $-R_H/64$, rather than $-R_H$; the physical emission points occupy only the allowed discrete final levels. Option (A) has the wrong intercept, option (B) contradicts the linear form, and option (D) has the wrong slope sign.

Step 3: Conclusion The plot is linear with slope R_H and extrapolated intercept $-R_H/64$. Hence, the correct option is (C).

Quick Tip to Solve: Identify the plotted horizontal variable first: a dependence on $1/n^2$ becomes linear when that entire quantity is on the x-axis.

65. The correct match between Item-I and Item-II is:

Item-I (drug)		Item-II (test)	
A	Chloroxylenol	P	Carbylamine test
B	Norethindrone	Q	Sodium hydrogen carbonet test
C	Sulphapyridine	R	Ferric chloride test
D	Penicillin	S	Bayer's test

[4 marks]

(A) A → R; B → P; C → S; D → Q

(B) A → Q; B → S; C → P; D → R

(C) A → R; B → S; C → P; D → Q

(D) A → Q; B → P; C → S; D → R

Correct Answer: (C) A → R; B → S; C → P; D → Q

Solution:

Concept:

- A qualitative organic test detects a functional group in the drug molecule. A phenolic group responds to ferric chloride; a free primary amino group responds to the carbylamine test.
- Unsaturation decolourises cold dilute alkaline permanganate in Baeyer's test, whereas a carboxylic acid releases carbon dioxide with sodium hydrogen carbonate.

Step 1: Match the phenolic and unsaturated drugs Chloroxylenol contains a phenolic $-\mathrm{OH}$ group, which forms a coloured complex with ferric chloride. Therefore A matches R. Norethindrone contains carbon–carbon multiple bonding and responds to the permanganate test for unsaturation; therefore B matches S. This immediately eliminates the options assigning A to Q or B to P.

Step 2: Match the primary amine and carboxylic acid Sulphapyridine has a free aromatic primary amino group. With chloroform and alcoholic potassium hydroxide it gives the characteristic carbylamine reaction, so C matches P. The sulphonamide nitrogen is not the functional group responsible for this positive test. Penicillin contains a carboxylic acid group, which reacts with bicarbonate to release gas:

$$\mathrm{RCOOH} + \mathrm{NaHCO_3} \longrightarrow \mathrm{RCOONa} + \mathrm{H_2O} + \mathrm{CO_2}\uparrow.$$

Thus D matches Q. Reading all four matches together selects one complete option; identifying only one positive test would not be enough if several options shared that entry.

Step 3: Conclusion The correct matching is A → R, B → S, C → P, D → Q. Hence, the correct option is (C).

Quick Tip to Solve: In a drug-test matching question, locate the actual functional groups: phenol, free primary amine, unsaturation and carboxylic acid give four distinct anchors.

66. The major product of following reaction is : [4 marks]

$$R - C \equiv N \xrightarrow[(2)\, H_2O]{(1)\, AlH\,(i\text{–}Bu)_2}$$

(A) RCOOH

(B) $RCONH_2$

(C) RCHO

(D) RCH_2NH_2

Correct Answer: (C) RCHO

Solution:

Concept:

- Diisobutylaluminium hydride, $(\text{i-Bu})_2\text{AlH}$ or DIBAL-H, is a bulky reducing agent used for controlled partial reduction of a nitrile to an aldehyde after hydrolysis.
- The nitrile carbon remains in the carbon skeleton. Hydride addition produces an aluminium-bound imine intermediate, and the subsequent aqueous work-up converts its carbon–nitrogen bond into a carbonyl group.

Step 1: Identify the reagent and its selective action The starting group is $\text{R}-\text{C}\equiv\text{N}$, and the first reagent written over the arrow is AlH(i-Bu)_2. This is DIBAL-H. Under the usual controlled conditions for this transformation, hydride adds to the electrophilic nitrile carbon and produces an imine-type aluminium complex. The reduction is deliberately stopped before the carbon receives enough hydrogen to give a primary amine.

Step 2: Apply the aqueous work-up and compare the products The second step is treatment with water. Hydrolysis of the reduced imine intermediate gives the corresponding aldehyde, retaining the nitrile carbon as the aldehyde carbon:

$$\text{R}-\text{C}\equiv\text{N} \xrightarrow[\text{then } \text{H}_2\text{O}]{\text{DIBAL}-\text{H}} \text{R}-\text{CHO}.$$

An ordinary nitrile hydrolysis could produce an amide or carboxylic acid, but those would disregard the reducing reagent present in the first step. A strong exhaustive hydride reduction, such as with excess lithium aluminium hydride, would instead give RCH_2NH_2. That is not the selective DIBAL-H transformation shown here.

Step 3: Conclusion The major product after partial reduction and aqueous work-up is RCHO, an aldehyde. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish DIBAL-H from exhaustive hydride reduction: nitrile plus DIBAL-H followed by water gives an aldehyde, while stronger complete reduction gives an amine.

67. The increasing order of pka of the following amino acids in aqueous solution is :

Gly, Asp, Lys, Arg [4 marks]

(A) Asp $<$ Gly $<$ Arg $<$ Lys

(B) Gly $<$ Asp $<$ Arg $<$ Lys

(C) Asp $<$ Gly $<$ Lys $<$ Arg

(D) Arg $<$ Lys $<$ Gly $<$ Asp

Correct Answer: (C) Asp $<$ Gly $<$ Lys $<$ Arg

Solution:

Concept:

- Amino acids have several ionisable groups, so a literal pK_a value belongs to a specified dissociation step. The isoelectric pH, pI, is a different quantity: the pH at which the average net charge is zero.
- Acidic side chains lower the isoelectric pH, whereas basic side chains raise it. Aspartic acid has an additional carboxyl group; lysine and arginine have additional basic groups.

Step 1: Compare the acidic, neutral and basic side chains Aspartic acid (Asp) has a side-chain carboxyl group and is the acidic member of this set. Glycine (Gly) has hydrogen as its side chain and is the neutral reference. Lysine (Lys) contains an additional amino group, while arginine (Arg) contains a more strongly basic guanidino group. The guanidinium ion is stabilised by delocalisation, so arginine retains its positive side-chain charge up to a higher pH than lysine.

Step 2: Read the intended order while distinguishing the notation The acidic-to-basic order represented among the choices is

$$\text{Asp} < \text{Gly} < \text{Lys} < \text{Arg}.$$

For example, their approximate isoelectric pH values are $2.8, 6.0, 9.7, 10.8$, respectively. This is the order in option (C) and the accepted choice for the question. Strictly, the printed notation pK_a is underspecified: these polyprotic molecules do not have one common, uniquely defined pK_a, and the displayed order is the conventional order of pI, not a universal ordering of every dissociation constant. Options (A), (B) and (D) also reverse the acidic-neutral or lysine-arginine comparison.

Step 3: Conclusion The literal question is ambiguous because it does not identify which pK_a is to be compared. The accepted option is (C) under the intended isoelectric-pH interpretation: Asp $<$ Gly $<$ Lys $<$ Arg. This is a pI order and must not be treated as a unique order of unspecified dissociation constants.

Quick Tip to Solve: For amino acids, always distinguish a particular pK_a from pI; acidic and basic side chains predict the isoelectric-pH trend directly.

68. Which amongst the following is the strongest acid ? [4 marks]

(A) $CHBr_3$

(B) CHI_3

(C) $CH(CN)_3$

(D) $CHCl_3$

Correct Answer: (C) $CH(CN)_3$

Solution:

Concept:

- A carbon acid loses a proton to form a carbanion. Its acidity increases when the resulting negative charge is stabilised by electron-withdrawing groups and resonance.
- A cyano group, $-C{\equiv}N$, withdraws electron density by induction and can delocalise a neighbouring carbanion charge into its conjugated carbon–nitrogen system.

Step 1: Form the relevant conjugate base All four molecules contain a hydrogen bonded to carbon, so compare the species obtained by removing that proton rather than looking for an oxygen–hydrogen bond. For tricyanomethane the deprotonation is

$$HC(CN)_3 \rightleftharpoons H^+ + [C(CN)_3]^-.$$

The resulting tricyanomethanide anion is stabilised by all three cyano substituents. Its central carbon lone pair can conjugate with the adjacent nitrile groups, spreading negative charge onto their nitrogen atoms through resonance contributors.

Step 2: Compare cyano and halo substituents The three cyano groups combine a strong inductive withdrawal with extensive resonance stabilisation of the conjugate base. Haloforms $CHBr_3$, CHI_3 and $CHCl_3$ do have acidic carbon-bound hydrogen, and halogens influence their carbanion stability. However, none provides the same threefold nitrile conjugation available in $[C(CN)_3]^-$. Hence ranking only the electronegativities of chlorine, bromine and iodine would miss the decisive comparison. The cyano-substituted compound is appreciably more acidic than each of the three haloforms, ruling out options (A), (B) and (D).

Step 3: Conclusion The strongest acid among the given compounds is $CH(CN)_3$. Hence, the correct option is (C).

Quick Tip to Solve: For carbon acids, first draw the conjugate base mentally; resonance into several cyano groups usually outweighs a simple halogen electronegativity argument.

69. The correct decreasing order for acid strength is : [4 marks]

(A) $NO_2CH_2COOH > FCH_2COOH > CNCH_2COOH > ClCH_2COOH$

(B) $FCH_2COOH > NCCH_2COOH > NO_2CH_2COOH > ClCH_2COOH$

(C) $CNCH_2COOH > O_2NCH_2COOH > FCH_2COOH > ClCH_2COOH$

(D) $NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$

Correct Answer: (D) $NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$

Solution:

Concept:

- A substituted acetic acid is stronger when its substituent stabilises the carboxylate ion. Electron-withdrawing groups on the adjacent carbon exert a strong negative inductive effect.
- For these alpha-substituted acetic acids in water, the relevant decreasing acid-strength order is nitroacetic, cyanoacetic, fluoroacetic, then chloroacetic acid. Their approximate pK_a values are $1.7, 2.5, 2.6, 2.9$, respectively.

Step 1: Compare the groups at the same distance Each structure has the form $X{-}CH_2{-}COOH$, so the substituent is one carbon away from the carboxyl carbon in every case. Distance is therefore controlled, and the difference is the electron-withdrawing effect of X. Withdrawal reduces the electron density of the carboxylate group and stabilises the conjugate base relative to the undissociated acid. Greater stabilisation means a larger acid dissociation constant and a smaller pK_a.

Step 2: Arrange all four acids and test the options The nitro substituent produces the strongest acid in the list. The cyano-substituted acid is next, followed closely by fluoroacetic acid. Fluorine withdraws more strongly than chlorine at the same position, so fluoroacetic acid is stronger than chloroacetic acid. Thus

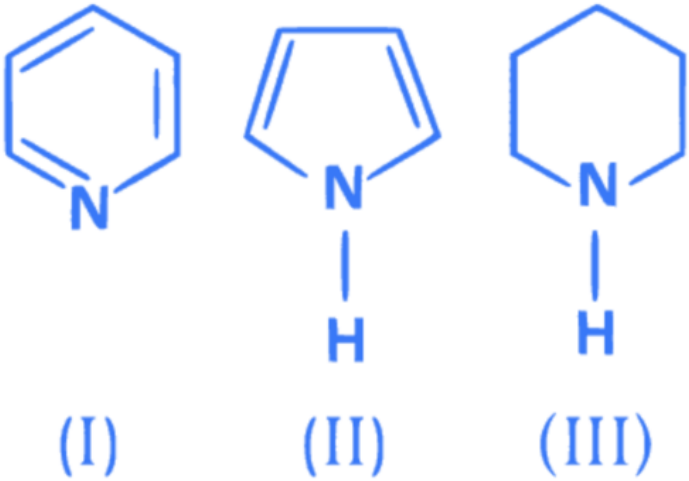
$$NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH.$$

Here $NCCH_2COOH$ in the option denotes the cyano-substituted acid, with $-C{\equiv}N$ attached to the methylene carbon. Option (A) incorrectly exchanges the cyano and fluoro entries; (B) places fluorine first, and (C) places cyano ahead of nitro.

Step 3: Conclusion The decreasing acid-strength order is nitroacetic $>$ cyanoacetic $>$ fluoroacetic $>$ chloroacetic acid. Hence, the correct option is (D).

Quick Tip to Solve: When substituents are equally far from $-COOH$, compare their effects on the carboxylate; remember that smaller pK_a means stronger acid.

70. Arrange the following amines in the decreasing order of basicity : [4 marks]



(A) I > II > III

(B) III > I > II

(C) III > II > I

(D) I > III > II

Correct Answer: (B) III > I > II

Solution:

Concept:

- A nitrogen base accepts a proton using its lone pair. Basicity depends on how available that lone pair is and whether protonation sacrifices aromatic stabilisation.
- The structures are pyridine (I), pyrrole (II) and piperidine (III). Pyridine has an available sp^2 lone pair outside the aromatic sextet; pyrrole uses its lone pair in aromaticity; piperidine has a localised sp^3 amine lone pair.

Step 1: Identify why pyrrole is the weakest base In pyrrole, two double bonds supply four pi electrons and the nitrogen lone pair supplies two more, completing an aromatic six-electron ring. Accepting a proton at nitrogen would make this lone pair unavailable to that aromatic sextet. Consequently nitrogen protonation loses important aromatic stabilisation, and the lone pair is poorly available for basic action. Compound II must therefore lie last, eliminating the options that place it above pyridine or piperidine.

Step 2: Compare piperidine with pyridine Pyridine nitrogen has its lone pair in an sp^2 orbital, separate from the ring pi system, so protonation can occur without destroying the aromatic sextet. It is therefore more basic than pyrrole. Piperidine is a saturated secondary amine: its nitrogen lone pair is localised in an sp^3 orbital, and adjacent alkyl groups donate electron density. Relative to the higher-s-character pyridine orbital, this lone pair is held less tightly and accepts a proton more readily. Hence piperidine is the strongest base. The complete decreasing order is

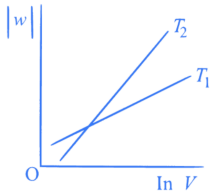
$$\text{III} > \text{I} > \text{II}.$$

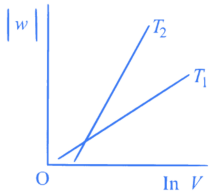
Only option (B) places both the saturated amine first and the aromatic-lone-pair nitrogen last.

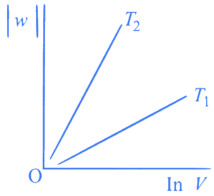
Step 3: Conclusion The decreasing basicity order is piperidine > pyridine > pyrrole, that is III > I > II. Hence, the correct option is (B).

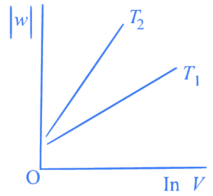
Quick Tip to Solve: For a nitrogen heterocycle, check first whether its lone pair completes an aromatic sextet; such a lone pair is much less available to accept a proton.

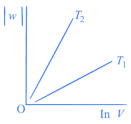
71. Consider the reversible isothermal expansion of an ideal gas in a closed system at two different temperatures T_1 and T_2 ($T_1 < T_2$). The correct graphical depiction of the dependence of work done (w) on the final volume (V) is: [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- For a reversible expansion of an ideal gas, the external pressure differs infinitesimally from the gas pressure, so integration uses $P = nRT/V$. The temperature is constant along each isothermal path.
- With the chemistry sign convention, work on the gas is $w = -nRT\ln(V_f/V_i)$. The graphs show the positive magnitude $|w| = nRT\ln(V_f/V_i)$, where n is the amount of gas and R is the gas constant.

Step 1: Express the work in terms of the plotted variable Let the same gas start at the same initial volume V_i in the two experiments. Integrating the reversible pressure-volume work gives

$$w = -\int_{V_i}^{V_f} \frac{nRT}{V}\,dV = -nRT\ln\frac{V_f}{V_i}.$$

For an expansion, $V_f > V_i$, so $w < 0$, while its magnitude is positive. Expanding the logarithm gives

$$|w| = nRT\ln V_f - nRT\ln V_i.$$

Thus a plot against $\ln V_f$, as shown on the horizontal axes, must be a straight line.

Step 2: Compare the slopes and the common zero The slope of each line is nRT, which is larger at T_2

because $T_2 > T_1$. Both lines give zero work when $V_f = V_i$, so their extrapolated horizontal intercept must be the same, namely $\ln V_i$. Option (C) depicts the two straight rays with a common zero and the steeper ray labelled T_2. The intersections at nonzero work in (A) and (B) cannot represent a common initial volume. The unequal intercepts depicted in (D) also fail that condition. The higher-temperature line must lie above the lower-temperature line throughout the expansion region.

Step 3: Conclusion The required graph has a common zero-work point and a larger slope at T_2. Hence, the correct option is (C).

Quick Tip to Solve: For isothermal work, first read the horizontal axis: the dependence is logarithmic in V_f, but linear in $\ln V_f$. Then compare both the slopes and the zero-work intercept.

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72. Two complexes $[Cr(H_2O)_6]Cl_3$ (A) and $[Cr(NH_3)_6]Cl_3$ (B) are violet and yellow coloured, respectively. The incorrect statement regarding them is :

[4 marks]

(A) Δ_0 values of (A) and (B) are calculated from the energies of violet and yellow light, respectively

(B) both are paramagnetic with three unpaired electrons

(C) both absorb energies corresponding to their complementry colors.

(D) Δ_0 value for (A) is less than that of (B).

Correct Answer: (A) Δ_0 values of (A) and (B) are calculated from the energies of violet and yellow light, respectively

Solution:

Concept:

- In an octahedral complex, the crystal-field splitting Δ_o is the energy needed to promote an electron between the t_{2g} and e_g sets. The relevant photon is absorbed, with $\Delta_o = h\nu = hc/\lambda$.
- The observed colour is complementary to the light absorbed. Both complexes contain Cr^{3+}, a $3d^3$ ion, and NH_3 is a stronger-field ligand than H_2O.

Step 1: Determine the electronic configurations Each complex cation has charge $+3$, because its three counterions are Cl^- and its six ligands are neutral. Chromium therefore has oxidation state $+3$ in both. Removing the $4s$ electron and two $3d$ electrons from chromium gives $3d^3$. In an octahedral field the three electrons occupy separate t_{2g} orbitals:

$$Cr^{3+}: \quad t_{2g}^3 e_g^0.$$

There are three unpaired electrons in both complexes, so both are paramagnetic. Consequently statement (B) is correct; changing from water to ammonia does not change this unpaired-electron count for a d^3 ion.

Step 2: Identify which colour determines the splitting The violet and yellow colours specified are the colours that reach the observer, rather than the photons removed in the electronic transition. Thus the absorption energies corresponding to the complementary colours must be used to determine Δ_o. Statement (C) correctly expresses this distinction, whereas statement (A) incorrectly uses the

observed violet and yellow light themselves. Finally, ammonia produces a stronger crystal field than water in the spectrochemical series, so

$$\Delta_o\left([\mathrm{Cr}(\mathrm{H_2O})_6]^{3+}\right) < \Delta_o\left([\mathrm{Cr}(\mathrm{NH_3})_6]^{3+}\right).$$

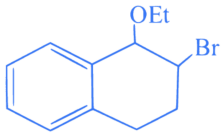
Statement (D) is therefore correct as well. Only (A) confuses the observed colour with the absorbed colour.

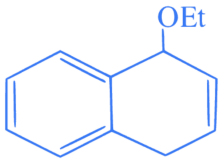
Step 3: Conclusion The splitting must be calculated from absorbed complementary light, so statement (A) is incorrect. Hence, the correct option is (A).

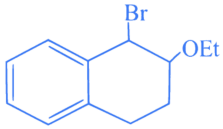
Quick Tip to Solve: When a coordination compound is described as coloured, distinguish the observed colour from the absorbed complementary colour before using hc/λ.

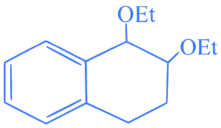
73. The major product of the following reaction is: [4 marks]

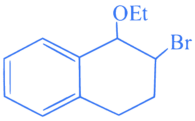


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- An alkene reacts with electrophilic bromine to form a bridged bromonium intermediate. A nucleophilic solvent can open that intermediate, giving addition of a halogen and a solvent-derived group across the double bond.
- For an unsymmetrical bromonium intermediate, the carbon that best stabilizes positive character is preferentially attacked by the solvent. A benzylic carbon is stabilized by the adjacent aromatic ring.

Step 1: Locate the reactive double bond The starting material has a nonaromatic double bond in the six-membered ring fused to benzene. One end of this double bond is adjacent to the aromatic ring and is therefore benzylic in the addition intermediate. The aromatic benzene ring is retained:

electrophilic addition occurs at the more readily reacting nonaromatic alkene rather than destroying aromatic stabilization. Bromine initially interacts with this alkene to give a bromonium intermediate, so the original alkene bond is consumed in the principal addition product.

Step 2: Open the intermediate with ethanol Ethanol attacks the carbon of the intermediate with greater benzylic positive character. Its oxygen therefore bonds to the alkene carbon next to the aromatic ring; loss of the proton from the attacking ethanol gives an ethoxy substituent, $-OCH_2CH_3$. Bromine remains attached to the adjacent carbon, farther from the benzene ring. The resulting connectivity is precisely that shown in option (A): benzylic ethoxy and adjacent bromo groups on the saturated ring. Option (C) reverses this preferred orientation. Option (B) retains a shifted nonaromatic double bond and omits bromine, while option (D) replaces both new groups by ethoxy groups. Neither represents the ordinary bromine/ethanol addition product. Ring opening occurs by nucleophilic attack opposite the bridge, consistent with anti addition where stereochemistry is specified.

Step 3: Conclusion The major product has $-OEt$ at the benzylic carbon and $-Br$ at the adjacent carbon, as drawn in (A). Hence, the correct option is (A).

Quick Tip to Solve: For bromination in an alcohol solvent, add Br and OR across the alkene; put OR on the carbon with better stabilized positive character, here the benzylic carbon.

74. A solution of sodium sulphate contains 92 g of Na^+ ions per kilogram of water. The molality of Na^+ ions in that solution in mol kg^{-1} is : [4 marks]

(A) 12

(B) 4

(C) 8

(D) 16

Correct Answer: (B) 4

Solution:

Concept:

- Molality is the amount of the specified solute species per kilogram of solvent: $m = n_{\text{solute}}/m_{\text{solvent}}$, with the solvent mass expressed in kilograms.
- The question gives the mass of Na^+ itself. Its molar mass is approximately $23\,\text{g mol}^{-1}$; the negligible electron mass does not affect this conversion.

Step 1: Convert the sodium-ion mass into moles Use the mass given for the ions whose molality is asked for, rather than treating it as the mass of sodium sulphate. For sodium ions,

$$n(Na^+) = \frac{\text{mass of } Na^+}{\text{molar mass of } Na^+} = \frac{92\,\text{g}}{23\,\text{g mol}^{-1}} = 4\,\text{mol}.$$

Thus the solution contains four moles of sodium ions for every kilogram of water. The solvent mass is already in the required unit, so no density or total solution volume is needed.

Step 2: Calculate the requested ionic molality The molality of sodium ions is directly

$$m(\mathrm{Na^+}) = \frac{4\,\mathrm{mol}}{1\,\mathrm{kg}} = 4\,\mathrm{mol\,kg^{-1}}.$$

Although each formula unit of $\mathrm{Na_2SO_4}$ produces two sodium ions, that factor must not be applied again: the given 92 g already refers to sodium ions. As a check, four moles of sodium ions correspond to two moles of sodium sulphate in the same kilogram of water; dissociation of those two moles returns the same four moles of sodium ions. The value 8 would double-count the sodium coefficient, and 12 or 16 also cannot follow from the given ionic mass. This illustrates why the species named in the numerator of a concentration expression must be matched to the mass supplied.

Step 3: Conclusion The molality of $\mathrm{Na^+}$ ions is $\boxed{4\,\mathrm{mol\,kg^{-1}}}$. Hence, the correct option is (B).

Quick Tip to Solve: If the given mass already belongs to an ion, divide by that ion's molar mass directly; do not multiply again by its coefficient in the salt formula.

75. The anodic half-cell of lead-acid battery is recharged using electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process is : (Molar mass of $PbSO_4$ = 303 g mol^{-1}) [4 marks]

(A) 22.8

(B) 15.2

(C) 7.6

(D) 11.4

Correct Answer: (C) 7.6

Solution:

Concept:

- One faraday of charge corresponds to one mole of electrons. A charge of $0.05\,F$ therefore supplies 0.05 mole of electrons to the electrode process.
- In recharging the lead electrode of a lead-acid battery, $\mathrm{PbSO_4}$ is reduced to Pb. Lead changes from oxidation state $+2$ to zero, so each mole of lead sulphate requires two moles of electrons.

Step 1: Write the charging half-reaction During discharge the lead electrode is oxidized and becomes coated with lead sulphate. Recharging reverses that process at the same electrode:

$$\mathrm{PbSO_4(s) + 2\,e^- \longrightarrow Pb\,(s) + {SO_4}^{2-}(aq)}.$$

The charge-to-substance relation follows from the two electrons in this half-reaction. The phrase anodic half-cell identifies the electrode by its discharge role; it does not make the number of electrons one during charging. At the positive plate the reverse conversion of lead sulphate to lead dioxide also involves two electrons, so the electron requirement for the stated amount is unambiguous.

Step 2: Use the charge to find the mass converted The supplied electron amount and the reacting

lead-sulphate amount are related by

$$n(e^-) = 0.05\,\text{mol}, \qquad n(\text{PbSO}_4) = \frac{0.05}{2} = 0.025\,\text{mol}.$$

Multiplying by the specified molar mass gives

$$m = 0.025\,\text{mol} \times 303\,\text{g mol}^{-1} = 7.575\,\text{g} \simeq 7.6\,\text{g}.$$

This agrees with option (C). The value 15.2 g would incorrectly use only one electron per formula unit; the larger values 22.8 g and 11.4 g likewise do not satisfy the two-electron stoichiometry for the supplied charge. Equivalently, the electrochemical equivalent is $303/2$ grams per faraday.

Step 3: Conclusion The amount of PbSO_4 electrolyzed is $\boxed{7.6\,\text{g}}$ to the precision of the options. Hence, the correct option is (C).

Quick Tip to Solve: Balance the electrode half-reaction first: for lead sulphate at the lead electrode, two electrons mean that its equivalent mass is half its molar mass.

76. 20 mL of 0.1 M H_2SO_4 solution is added to 30 mL of of 0.2 M NH_4OH solution. The pH of the resultant mixture is : [pk_b of NH_4OH = 4.7]. [4 marks]

(A) 5.2

(B) 9.0

(C) 5.0

(D) 9.4

Correct Answer: (B) 9.0

Solution:

Concept:

- Strong-acid equivalents first neutralize the weak base. If weak base remains together with its conjugate acid, the mixture is a basic buffer and $\text{pOH} = pK_b + \log([\text{salt}]/[\text{base}])$.
- Sulphuric acid supplies two acid equivalents per mole in this neutralization. For the ammonium hydroxide buffer, the conjugate pair is aqueous ammonia/weak base and $NH_4{}^+$.

Step 1: Complete the neutralization before using equilibrium The initial amount of weak base is

$$n_{\text{base}} = 0.030 \times 0.2 = 0.006\,\text{mol}.$$

The sulphuric acid amount is $0.020 \times 0.1 = 0.002\,\text{mol}$, which supplies $0.004\,\text{mol}$ of acid equivalents. Therefore 0.004 mol of weak base is converted to ammonium ions, leaving

$$n_{\text{base,left}} = 0.006 - 0.004 = 0.002\,\text{mol}, \quad n_{NH_4{}^+} = 0.004\,\text{mol}.$$

The remaining weak base and the ammonium salt share the same final volume, 50 mL, so their concentration ratio equals their mole ratio.

Step 2: Evaluate the buffer pH Using the weak-base buffer equation,

$$\mathrm{pOH} = 4.7 + \log\frac{0.004}{0.002} = 4.7 + \log 2 \simeq 4.7 + 0.301 = 5.001.$$

At the standard temperature assumed in this calculation, $\mathrm{pH} + \mathrm{pOH} = 14$, giving

$$\mathrm{pH} = 14 - 5.001 = 8.999 \simeq 9.0.$$

The result is basic, as expected for excess weak base with its ammonium salt. Option (C), 5.0, is approximately the pOH rather than the pH; option (A) has the same acidic-versus-basic error. Option (D), 9.4, fails to use the actual salt-to-base ratio of two. Treating sulphuric acid as monoprotic would also give the wrong neutralization amounts.

Step 3: Conclusion The resultant mixture has $\boxed{\mathrm{pH} = 9.0}$. Hence, the correct option is (B).

Quick Tip to Solve: Count acid equivalents before applying a buffer formula; here each mole of sulphuric acid neutralizes two moles of the weak base.

77. The following results were obtained during kinetic studies of the reaction:

$$2\,\mathrm{A} + \mathrm{B} \longrightarrow \text{Products}$$

Experiment	$[A]$ $(\mathrm{mol\,L^{-1}})$	$[B]$ $(\mathrm{mol\,L^{-1}})$	Initial rate of reaction $(\mathrm{mol\,L^{-1}\,min^{-1}})$
I	**0.10**	**0.20**	6.93×10^{-3}
II	**0.10**	**0.25**	6.93×10^{-3}
III	**0.20**	**0.30**	1.386×10^{-2}

The time (in minutes) required to consume half of A is: [4 marks]

(A) 5

(B) 10

(C) 1

(D) 100

Correct Answer: (A) 5

Solution:

Concept:

- The initial-rate method gives the powers in $r = k_r[A]^x[B]^y$ by comparing concentrations and measured rates. Reaction coefficients do not determine these experimental orders.
- For the printed reaction $2\,\mathrm{A} + \mathrm{B} \longrightarrow \text{Products}$, the formal reaction rate is $r = -\frac{1}{2}d[A]/dt$. Consequently a first-order law $r = k_r[A]$ gives $-d[A]/dt = 2k_r[A]$, and the disappearance constant for A is $2k_r$.

Step 1: Determine the experimental reaction-rate law In experiments I and II, A remains at $0.10\,\mathrm{mol\,L^{-1}}$, while B changes from 0.20 to $0.25\,\mathrm{mol\,L^{-1}}$; the rate is unchanged. Therefore the

order in B is zero. Comparing I with III then shows that doubling A doubles the rate, so the order in A is one. Thus

$$r = k_r[A], \qquad k_r = \frac{6.93 \times 10^{-3}}{0.10} = 0.0693\,\text{min}^{-1}.$$

Experiment III gives the same quotient, $1.386 \times 10^{-2}/0.20$, confirming this law.

Step 2: Apply the coefficient of A to its disappearance The column is labelled initial rate of reaction. For the reaction as printed, two moles of A disappear per unit reaction extent, so

$$-\frac{d[A]}{dt} = 2r = 2k_r[A] = 0.1386[A].$$

Integrating this first-order disappearance equation gives

$$\ln \frac{[A]_0}{[A]_t} = 0.1386t.$$

At half consumption, $[A]_t = [A]_0/2$, hence

$$t_{1/2} = \frac{\ln 2}{0.1386} = 5.00\,\text{min}.$$

The supplied answer key selects 10 min, which results if the table is instead treated as the disappearance rate $-d[A]/dt$ of A. Under that convention its constant is 0.0693 and the half-life is 10 min. Under the formal meaning of the printed label, however, the factor of two is required and 5 min follows. The alternatives 1 min and 100 min follow from neither interpretation.

Step 3: Conclusion Using the formal reaction-rate definition for the printed $2\,\text{A}^{+}\text{B}$ equation, the half-consumption time is $\boxed{5\,\text{min}}$. Hence, the correct option is (A). The supplied final key gives 10 min, which assumes the tabulated rate is the disappearance rate of A rather than the stoichiometrically normalized reaction rate.

Quick Tip to Solve: Before using a half-life formula, distinguish the normalized reaction rate from a species disappearance rate. A coefficient of two multiplies the latter by two.

78. According to molecular orbital theory, which of the following is true with respect to Li_2^+ and Li_2^- ? [4 marks]

(A) Li_2^+ is unstable and Li_2^- is stable

(B) Li_2^+ is stable and Li_2^- unstable

(C) Both are stable

(D) Both are unstable

Correct Answer: (C) Both are stable

Solution:

Concept:

- Molecular orbital bond order is $\text{B.O.} = (N_b - N_a)/2$, where N_b and N_a are the numbers of electrons in bonding and antibonding orbitals.
- A positive bond order predicts a bonded species. The filled inner $1s$ bonding and antibonding orbitals cancel in the bond-order calculation.

Step 1: Write the configuration of the cation Each lithium atom contributes one valence electron from its $2s$ orbital. Neutral Li_2 therefore has two valence electrons in the bonding σ_{2s} orbital. Removing one electron gives the cation configuration $(\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^1$. Its valence contribution is thus

$$\text{B.O.}(Li_2^+) = \frac{1-0}{2} = \frac{1}{2}.$$

Because this is positive, the cation is stable with respect to formation of a bond according to the molecular orbital criterion.

Step 2: Write the configuration of the anion Adding one electron to neutral Li_2 places the additional electron in the next available orbital, the antibonding σ_{2s}^*. The valence configuration is $(\sigma_{2s})^2(\sigma_{2s}^*)^1$, so

$$\text{B.O.}(Li_2^-) = \frac{2-1}{2} = \frac{1}{2}.$$

An antibonding electron weakens the bond but does not necessarily eliminate it. Here its contribution only reduces the neutral bond order from one to one-half. Both ions therefore retain positive bond order. Options A and B wrongly reject one ion, while D wrongly rejects both.

Step 3: Conclusion Both Li_2^+ and Li_2^- have bond order $1/2$, so both are stable according to molecular orbital theory. Hence, the correct option is (C).

Quick Tip to Solve: When comparing molecular ions, remove an electron from the highest occupied orbital or add one to the lowest vacant orbital before calculating bond order.

79. Aluminium is usually found in +3 oxidation state. In contrast, thallium exists in + 1 and + 3 oxidation states. This is due to : [4 marks]

(A) inert pair effect

(B) diagonal relationship

(C) lattice structure

(D) lanthanoid contraction

Correct Answer: (A) inert pair effect

Solution:

Concept:

- Group 13 elements have valence configuration ns^2np^1, allowing oxidation states $+1$ and $+3$.
- The inert pair effect is the increasing reluctance of the outer ns^2 electron pair to participate in bonding down a p-block group.

Step 1: Compare the available valence electrons Aluminium has the outer configuration $3s^2 3p^1$. Its three valence electrons commonly participate in bonding, producing the familiar $+3$ oxidation state. Thallium has outer configuration $6s^2 6p^1$. Loss or bonding participation of only the $6p$ electron leaves the $6s^2$ pair intact and gives the $+1$ state. If the $6s^2$ pair also participates, the oxidation state becomes $+3$. Thus the two states correspond directly to whether the outer s pair remains unshared.

Step 2: Identify the effect and reject the distractors Down the group, poor shielding by intervening d and f electrons and the increased stabilization of the outer s orbital make the s pair less available. This is called the inert pair effect, and it stabilizes the lower oxidation state particularly strongly for thallium. A diagonal relationship compares elements in diagonally adjacent positions, whereas aluminium and thallium are in the same group. Lattice structure describes solid-state arrangement and does not name this valence-electron trend. Lanthanoid contraction affects atomic sizes but is not the specific explanation for retaining the $6s^2$ pair in the $+1$ state.

Step 3: Conclusion The coexistence of $+1$ and $+3$ oxidation states in thallium, compared with the usual $+3$ state of aluminium, is explained by the inert pair effect. Hence, the correct option is (A).

Quick Tip to Solve: For a heavy p-block element, a stable oxidation state two below the group maximum is a direct clue that the outer s-electron pair stays inert.

80. The compounds A and B in the following reaction are, respectively : [4 marks]



(A) A = Benzyl alcohol, B = Benzyl cyanide

(B) A = Benzyl chloride, B = Benzyl cyanide

(C) A = Benzyl alcohol, B = Benzyl isocyanide

(D) A = Benzyl chloride, B = Benzyl isocyanide

Correct Answer: (D) A = Benzyl chloride, B = Benzyl isocyanide

Solution:

Concept:

- Benzene reacts with formaldehyde and hydrogen chloride by chloromethylation to introduce the $-CH_2Cl$ side chain.
- Cyanide is ambident: alkali-metal cyanides usually give nitriles through carbon, whereas predominantly covalent $AgCN$ gives isocyanides through nitrogen.

Step 1: Identify the chloromethylation product A The reagent combination $HCHO + HCl$ converts benzene into chloromethylbenzene, commonly called benzyl chloride. The group attached to the aromatic ring contains one carbon from formaldehyde, and chlorine remains bonded to that side-chain carbon:

$$C_6H_6 + HCHO + HCl \longrightarrow C_6H_5CH_2Cl + H_2O.$$

Thus $A = C_6H_5CH_2Cl$. Benzyl alcohol would contain an $-OH$ group instead of chlorine; it is not the chloromethylation product represented here. This eliminates options A and C before the second reagent is considered.

Step 2: Apply the nitrogen-end reaction of silver cyanide The cyanide group can form a bond through either carbon or nitrogen, so the identity of its metal salt matters. With silver cyanide, the carbon end is strongly associated with silver and the available nitrogen end forms the new bond to the benzyl carbon. Consequently,

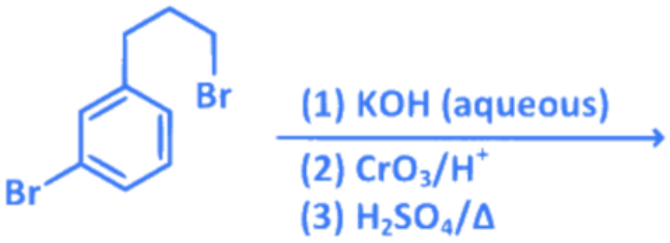
$$C_6H_5CH_2Cl + AgCN \longrightarrow C_6H_5CH_2NC + AgCl.$$

The product is benzyl isocyanide, not benzyl cyanide $C_6H_5CH_2CN$. Option B would correspond to carbon-end substitution, as normally favoured by ionic KCN, and therefore fails for the stated reagent.

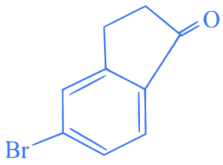
Step 3: Conclusion A is benzyl chloride and B is benzyl isocyanide, $C_6H_5CH_2NC$. Hence, the correct option is (D).

Quick Tip to Solve: For cyanide substitution, read the counter-ion: KCN points to $-CN$, while AgCN points to $-NC$.

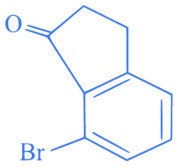
81. The major product of the following reaction is : [4 marks]



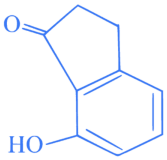
(A) 5-Bromoindan-1-one



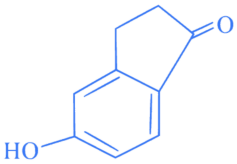
(B) 7-Bromoindan-1-one



(C) 7-Hydroxyindan-1-one

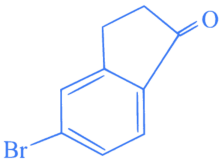


(D) 5-Hydroxyindan-1-one



Correct Answer: (A)

5-Bromoindan-1-one



Solution:

Concept:

- Aqueous KOH replaces a primary alkyl bromide by an alcohol, while an ordinary aryl $C-Br$ bond remains intact under these conditions.
- Acidified CrO_3 oxidizes a primary alcohol to a carboxylic acid. A suitably placed aromatic side-chain acid can cyclize by intramolecular acylation in hot concentrated H_2SO_4.

Step 1: Convert the terminal bromide into an acid The starting molecule has a bromine atom on the aromatic ring and a three-carbon side chain ending in bromine: $m\text{-}BrC_6H_4CH_2CH_2CH_2Br$. Aqueous potassium hydroxide substitutes only the primary side-chain bromine, producing

$\text{m-BrC}_6\text{H}_4\text{CH}_2\text{CH}_2\text{CH}_2\text{OH}$. The ring bromine is bonded directly to an sp^2 carbon and is not replaced by this ordinary aqueous-alkali treatment. Acidified chromium trioxide then oxidizes the terminal primary alcohol to the acid:

$$\text{m-BrC}_6\text{H}_4\text{CH}_2\text{CH}_2\text{CH}_2\text{OH} \xrightarrow{\text{CrO}_3/\text{H}^+} \text{m-BrC}_6\text{H}_4\text{CH}_2\text{CH}_2\text{COOH}.$$

This keeps the side-chain carbon skeleton and the aromatic bromine unchanged.

Step 2: Choose the favoured ring closure Heating the acid with sulfuric acid promotes intramolecular electrophilic acylation at an ortho position relative to the side chain, closing a five-membered ring and forming an indanone. Of the two available ortho positions, one lies ortho to ring bromine and the other lies para to bromine. Closure at the less crowded position para to bromine is favoured, giving 5-bromoindan-1-one, the structure in A. Option B is the less favoured regioisomer formed by attack adjacent to bromine. Options C and D incorrectly replace the aryl bromine with hydroxyl; none of the listed steps effects that replacement under the stated conditions.

Step 3: Conclusion The major product is 5-bromoindan-1-one, with the aromatic bromine retained and ring closure para to it. Hence, the correct option is (A).

Quick Tip to Solve: Track aryl and alkyl halogens separately: ordinary aqueous KOH substitutes the alkyl halogen, and the surviving ring substituent then controls cyclization orientation.

82. The one that is extensively used as a piezoelectric material is : [4 marks]

(A) tridymite

(B) amorphous silica

(C) quartz

(D) mica

Correct Answer: (C) quartz

Solution:

Concept:

- A piezoelectric crystal develops an electric polarization under mechanical stress; conversely, an applied electric field can produce a small deformation.
- The effect requires a suitable non-centrosymmetric crystal structure. Quartz is the standard widely used silica-based piezoelectric material.

Step 1: Relate the property to the material structure Quartz is crystalline silicon dioxide, SiO_2. In its suitable non-centrosymmetric crystal arrangement, compression or stretching displaces the centres of positive and negative charge relative to one another. This produces a potential difference across appropriate crystal faces. The reverse response allows an alternating voltage to drive regular mechanical vibrations. Because the crystal has stable resonant frequencies, the property is useful in electronic timing, frequency control, oscillators and pressure-sensitive devices.

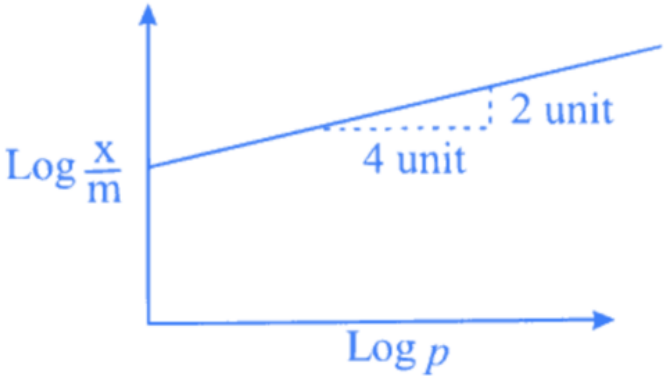
Step 2: Compare the four listed choices The material extensively used for this purpose is quartz. Amorphous silica lacks the ordered non-centrosymmetric crystal arrangement needed for the usual

bulk quartz piezoelectric response. Tridymite is a different structural form of silica, but it is not the standard extensively used piezoelectric material asked for here. Mica is familiar chiefly for its sheet structure and insulating uses, rather than as the conventional quartz-type piezoelectric oscillator material. The fact that several choices are silicate or silica-related materials does not mean that they share the same electrical response: arrangement of atoms is decisive.

Step 3: Conclusion Quartz is extensively used as a piezoelectric material. Hence, the correct option is (C).

Quick Tip to Solve: Associate quartz with crystal oscillators and pressure sensors; piezoelectricity depends on crystal symmetry, not merely on chemical composition.

83. Adsorption of a gas follows Freundlich adsorption isotherm. In the given plot, x is the mass of the gas adsorbed on mass m of the adsorbent at pressure P. $\frac{x}{m}$ is proportional to : [4 marks]



(A) P^2

(B) $P^{\frac{1}{4}}$

(C) $P^{\frac{1}{2}}$

(D) P

Correct Answer: (C) $P^{\frac{1}{2}}$

Solution:

Concept:

- The Freundlich isotherm is $x/m = kP^{1/n}$, where x/m is the amount adsorbed per unit mass and k, n are constants at fixed temperature.
- Taking logarithms gives $\log(x/m) = \log k + (1/n)\log P$. Therefore the slope of the plotted line equals the pressure exponent $1/n$.

Step 1: Extract the slope from the triangle The vertical axis is $\log(x/m)$, and the horizontal axis is $\log P$. The dotted triangle marked on the line gives a vertical rise of two units for a horizontal run of four units. Hence

$$\text{slope} = \frac{\Delta \log(x/m)}{\Delta \log P} = \frac{2}{4} = \frac{1}{2}.$$

The distances must be read in the order rise divided by run. Taking their reciprocal would give two, but that would be the slope of the inverse graph, not the graph printed in the question.

Step 2: Convert the logarithmic slope into the power law Compare the straight-line expression with $y = c + sx$: the slope is $s = 1/n$, whereas the intercept is $\log k$. Thus

$$\frac{1}{n} = \frac{1}{2}, \qquad \log(x/m) = \log k + \frac{1}{2}\log P.$$

Exponentiating gives $x/m = kP^{1/2}$, and at fixed temperature the multiplier k is constant. The requested proportionality is therefore to the square root of pressure. The other powers would imply slopes two, one-quarter and one, respectively, all inconsistent with the labelled rise and run.

Step 3: Conclusion $\frac{x}{m} \propto P^{1/2}$. Hence, the correct option is (C).

Quick Tip to Solve: On a log-log adsorption plot, the slope itself is the pressure exponent; calculate vertical change divided by horizontal change first.

84. The ore that contains both iron and copper is : [4 marks]

(A) Copper Pyrites

(B) Malachite

(C) Dolomite

(D) Azurite

Correct Answer: (A) Copper Pyrites

Solution:

Concept:

- An ore is a naturally occurring mineral from which a metal can be extracted economically; its chemical formula reveals which metals it contains.
- Copper pyrites, also called chalcopyrite, is the mixed copper-iron sulfide $\mathrm{CuFeS_2}$.

Step 1: Check the formula of copper pyrites Copper pyrites contains copper, iron and sulfur in the formula

$$\mathrm{CuFeS_2}.$$

Thus each formula unit includes both a copper atom and an iron atom, meeting both conditions in the question. Its common mineral name is chalcopyrite. The word pyrites can also occur in the name iron pyrites, $\mathrm{FeS_2}$, but copper pyrites is a different mineral and must be identified by its complete name and composition. Selecting a mineral merely because it contains copper would not be sufficient here.

Step 2: Eliminate the other minerals by composition Malachite is basic copper carbonate, $\mathrm{CuCO_3 \cdot Cu(OH)_2}$, also written $\mathrm{Cu_2CO_3(OH)_2}$; its formula contains copper but no iron. Azurite is another basic copper carbonate, $\mathrm{2\,CuCO_3 \cdot Cu(OH)_2}$, equivalently $\mathrm{Cu_3(CO_3)_2(OH)_2}$, and likewise contains no iron. Dolomite is $\mathrm{CaMg(CO_3)_2}$, a calcium-magnesium carbonate containing neither of the two requested metals. Consequently, malachite and azurite fail the iron requirement, while dolomite fails both the copper and iron requirements. Only copper pyrites has both metal symbols in

its formula.

Step 3: Conclusion The ore containing both iron and copper is copper pyrites, $CuFeS_2$. Hence, the correct option is (A).

Quick Tip to Solve: Use mineral formulae to check multi-metal questions: chalcopyrite is $CuFeS_2$, whereas malachite and azurite are copper carbonates without iron.

85. A water sample has ppm level concentration of the following metals : Fe = 0.2; Mn = 5.0; Cu = 3.0; Zn = 5.0. The metal that makes the water sample unsuitable for drinking is : [4 marks]

(A) Cu

(B) Mn

(C) Fe

(D) Zn

Correct Answer: (B) Mn

Solution:

Concept:

- Drinking-water quality is assessed by comparing each dissolved metal concentration with its own permissible limit. Equal concentrations of different metals do not imply equal safety.
- The textbook maximum prescribed concentrations relevant here are Fe: 0.2 ppm, Mn: 0.05 ppm, Cu: 3.0 ppm and Zn: 5.0 ppm. In a dilute aqueous sample, ppm is approximately milligrams per litre.

Step 1: Compare the reported concentrations with their limits The iron concentration is 0.2 ppm, which reaches but does not exceed the stated 0.2 ppm limit. Copper is 3.0 ppm, at its 3.0 ppm limit, and zinc is 5.0 ppm, at its 5.0 ppm limit. These three values therefore do not identify the contaminant that exceeds the prescribed concentration. The criterion is exceedance, rather than simply the presence of a dissolved metal.

Step 2: Identify the excessive metal Manganese is present at 5.0 ppm, whereas its prescribed maximum is only 0.05 ppm. The ratio is

$$\frac{5.0}{0.05} = 100.$$

Thus the manganese level is one hundred times the permissible concentration. Option (B) identifies this metal. Copper, iron and zinc, listed in options (A), (C) and (D), respectively, are at their stated limits rather than above them.

Step 3: Conclusion The water sample is unsuitable for drinking because its manganese concentration is excessive. Hence, the correct option is (B).

Quick Tip to Solve: Compare concentration divided by its own permissible limit; the largest concentration alone need not identify the unsafe metal.

86. Correct statements among a to d regarding silicones are :

(a) They are polymers with hydrophobic character.

(b) They are biocompatible.

(c) In general, they have high thermal stability and low dielectric strength.

(d) Usually, they are resistant to oxidation and used as greases. [4 marks]

(A) (a), (b), (c) and (d)

(B) (a), (b) and (c) only

(C) (a) and (b) only

(D) (a), (b) and (d) only

Correct Answer: (D) (a), (b) and (d) only

Solution:

Concept:

- Silicones are organosilicon polymers containing a repeating siloxane backbone, $-Si-O-Si-O-$, with organic groups attached to silicon.
- The strong siloxane framework gives thermal and oxidation stability, while the outward-facing organic groups produce water repellency. Silicones are also useful electrical insulators.

Step 1: Evaluate statements (a) and (b) Statement (a) is correct: silicones are polymers and their organic substituents give them a hydrophobic, or water-repelling, surface. This makes them useful for waterproof coatings. Statement (b) is also correct in the usual textbook description: suitable silicone materials are biocompatible and have applications in medical devices. The claim concerns their general material properties; it does not mean that every possible silicone formulation is interchangeable in medical use.

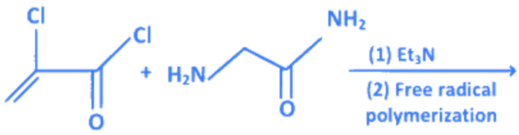
Step 2: Evaluate statements (c) and (d) Statement (c) combines one correct property with one incorrect property. Silicones generally have high thermal stability, but they have high dielectric strength, not low dielectric strength. Their insulating behaviour is an important reason for their electrical applications, so statement (c) as a whole is false. Statement (d) is correct: they usually resist oxidation, and silicone oils or greases function as lubricants over a useful temperature range.

Step 3: Select the matching combination The true statements are therefore (a), (b) and (d). Options (A) and (B) include the false statement (c). Option (C) omits the true statement (d). Only option (D) contains exactly the complete set of correct statements.

Step 4: Conclusion The hydrophobic, biocompatible and oxidation-resistant properties are correct; the claim of low dielectric strength is false. Hence, the correct option is (D).

Quick Tip to Solve: For a statement joined by "and", check both properties: thermal stability is high and dielectric strength is also high for silicones.

87. Major product of the following reaction is : [4 marks]



(A) $\left[-\mathrm{CH(COCl)-CH(NHCH_2CONH_2)^-}\right]_n$
(B) $\left[-\mathrm{CH(COCl)-CH(NHCOCH_2NH_2)^-}\right]_n$
(C) $\left[-\mathrm{CH_2-C(Cl)(CONHCOCH_2NH_2)^-}\right]_n$
(D) $\left[-\mathrm{CH_2-C(Cl)(CONHCH_2CONH_2)^-}\right]_n$

Correct Answer: (D) $\left[-\mathrm{CH_2-C(Cl)(CONHCH_2CONH_2)^-}\right]_n$

Solution:

Concept:

- An acid chloride reacts preferentially with a free amino group by nucleophilic acyl substitution. Triethylamine removes the hydrogen chloride formed during amide formation.
- Free-radical addition polymerisation consumes a carbon-carbon double bond and retains the substituents on its two carbon atoms. It does not transfer an amino substituent onto the carbon chain.

Step 1: Form the unsaturated amide The first reactant is $\mathrm{CH_2{=}C(Cl)COCl}$, and the second is glycinamide, $\mathrm{H_2NCH_2CONH_2}$. The free amino group is more nucleophilic than the amide nitrogen because the latter lone pair is delocalised towards its carbonyl group. Consequently, the acid-chloride chlorine is replaced by $-\mathrm{NHCH_2CONH_2}$:

$$\mathrm{CH_2{=}C(Cl)COCl + H_2NCH_2CONH_2 \longrightarrow CH_2{=}C(Cl)CONHCH_2CONH_2 + HCl.}$$

Triethylamine traps the hydrogen chloride; the chlorine bonded directly to the alkene carbon remains in the molecule.

Step 2: Polymerise the remaining double bond Radical addition joins the terminal $\mathrm{CH_2}$ carbon and the substituted alkene carbon into the backbone. The resulting repeat unit is

$$\left[-\mathrm{CH_2-C(Cl)(CONHCH_2CONH_2)^-}\right]_n.$$

Thus chlorine and the $-\mathrm{CONHCH_2CONH_2}$ side chain are attached to the same backbone carbon, exactly as in option (D). Options (A) and (B) incorrectly retain an acid-chloride group and attach nitrogen directly to the backbone. Option (C) places the side-chain atoms in the wrong order by acylating the original amide nitrogen rather than the free amino group.

Step 3: Conclusion The product contains the repeat unit $\left[-\mathrm{CH_2-C(Cl)(CONHCH_2CONH_2)^-}\right]_n$, with an amide side chain formed before polymerisation. Hence, the correct option is (D).

Quick Tip to Solve: Perform the acylation first, then replace only the alkene double bond by backbone single bonds; keep the side-chain atom order unchanged.

88. The isotopes of hydrogen are : [4 marks]

(A) Tritium and protium only

(B) Protium and Deuterium only

(C) Protium, deuterium and tritium

(D) Deuterium and tritium only

Correct Answer: (C) Protium, deuterium and tritium

Solution:

Concept:

- Isotopes of an element have the same atomic number, and therefore the same number of protons, but different mass numbers because their neutron counts differ.
- Hydrogen has atomic number 1. Its three standard named isotopes are protium ${}_{1}^{1}\mathrm{H}$, deuterium ${}_{1}^{2}\mathrm{H}$, and tritium ${}_{1}^{3}\mathrm{H}$.

Step 1: List the nuclear compositions Protium has one proton and no neutron, so its mass number is 1. Deuterium has one proton and one neutron, giving mass number 2. Tritium has one proton and two neutrons, giving mass number 3. In every case the proton count is exactly one, which establishes that all three are hydrogen atoms rather than different elements. A neutral atom of each isotope also has one electron.

Step 2: Distinguish stability from isotope identity Protium and deuterium are stable isotopes, whereas tritium is radioactive. Radioactivity does not prevent a nucleus from being an isotope of hydrogen: isotope identity is determined by atomic number and neutron count. Similarly, deuterium is not a different element merely because it is often denoted by D, and tritium is commonly denoted by T.

Step 3: Check the offered lists Option (C) includes all three named isotopes. Option (A) omits deuterium, option (B) omits tritium, and option (D) omits protium. The word "only" makes each of those incomplete two-isotope lists incorrect, even though the names they do contain individually refer to hydrogen isotopes.

Step 4: Conclusion The isotopes are protium, deuterium and tritium. Hence, the correct option is (C).

Quick Tip to Solve: Remember hydrogen isotope mass numbers 1, 2 and 3: their neutron counts are respectively 0, 1 and 2.

89. 0.5 moles of gas A and x moles of gas B exert a pressure of 200 Pa in a container of volume 10 m^3 at 1000 K. Given R is the gas constant in $JK^{-1}mol^{-1}$, x is : [4 marks]

(A) $\frac{2R}{4+R}$
(B) $\frac{2R}{4-R}$
(C) $\frac{4+R}{2R}$
(D) $\frac{4-R}{2R}$

Correct Answer: (D) $\frac{4-R}{2R}$

Solution:

Concept:

- For an ideal gas mixture at a common temperature, the total pressure obeys $PV = n_{\text{total}}RT$. Dalton's law gives the same result by adding the partial pressures.
- The total amount here is $n_{\text{total}} = 0.5 + x$. Using pressure in pascals, volume in cubic metres and temperature in kelvin is consistent with R expressed in $\text{J K}^{-1}\,\text{mol}^{-1}$.

Step 1: Substitute the total amount into the gas equation Use the pressure of the whole mixture, rather than assigning 200 Pa separately to each gas. Since both gases occupy the full 10 cubic metre container,

$$200 \times 10 = (0.5 + x)R \times 1000.$$

Dividing by 1000 gives

$$2 = (0.5 + x)R, \qquad 0.5 + x = \frac{2}{R}.$$

No molar-mass information is required, because the ideal gas equation depends on the number of moles.

Step 2: Isolate the unknown amount and compare the forms Subtract the given 0.5 moles of A and combine the fractions:

$$x = \frac{2}{R} - \frac{1}{2} = \frac{4 - R}{2R}.$$

This is option (D). Option (C) has the wrong sign for the subtraction. Options (A) and (B) are reciprocal-type expressions that do not satisfy the displayed gas equation.

Step 3: Check the physical consistency of the data With the physical SI value $R \approx 8.314$, the formal result is about -0.259 mol. A negative amount of gas is impossible: indeed, the given pressure and volume allow only about 0.241 mol in total, already less than the stated 0.5 mol of A. The algebraic expression selected by the printed symbolic options is still uniquely (D); the numerical data do not describe a physically realisable mixture.

Step 4: Conclusion Formal substitution gives $x = (4 - R)/(2R)$. Hence, the correct option is (D). With the SI value of R, the printed data yield a negative amount and therefore have no physical mixture interpretation.

Quick Tip to Solve: Add the mole amounts before applying the mixture gas law, then check that the inferred amount of each component is nonnegative.

90. The alkaline earth metal nitrate that does not crystallise with water molecules, is : [4 marks]

(A) $Mg(NO_3)_2$

(B) $Sr(NO_3)_2$

(C) $Ca(NO_3)_2$

(D) $Ba(NO_3)_2$

Correct Answer: (D) $Ba(NO_3)_2$

Solution:

Concept:

- Alkaline earth metal cations form nitrate salts of general formula $M(NO_3)_2$, because each metal ion has charge $+2$ and nitrate has charge -1.
- The tendency to bind water in a crystal depends on cation size, charge density and crystal structure. Hydration generally becomes weaker as the Group 2 cation becomes larger down the group.

Step 1: Recall the hydration trend for the listed salts Magnesium, calcium and strontium nitrates are commonly obtained as hydrated crystalline salts. Representative hydrates are $Mg(NO_3)_2 \cdot 6\,H_2O$, $Ca(NO_3)_2 \cdot 4\,H_2O$ and $Sr(NO_3)_2 \cdot 4\,H_2O$. Their water molecules are part of the crystalline material, rather than merely liquid water adhering to the surface. Thus the salts in options (A), (B) and (C) do not meet the condition in the question.

Step 2: Identify the nitrate that crystallises without water Barium is the largest cation among the four listed Group 2 metals. Its lower charge density produces weaker interaction with water than that of the smaller cations. Barium nitrate normally crystallises as the anhydrous salt $Ba(NO_3)_2$; its formula contains no water of crystallisation. This is the standard exception highlighted when comparing these alkaline earth metal nitrates.

Step 3: Match the exception to the options The question asks about water of crystallisation, not solubility in water or thermal decomposition of nitrate. A salt can dissolve in water and still crystallise without incorporating water into its lattice. Therefore the anhydrous barium salt, rather than one of the hydrated magnesium, calcium or strontium salts, is the required choice.

Step 4: Conclusion Barium nitrate, $Ba(NO_3)_2$, does not crystallise with water molecules. Hence, the correct option is (D).

Quick Tip to Solve: For these Group 2 nitrates, remember the anhydrous barium salt as the exception; do not confuse hydration with solubility.