JEE Main 2019 Paper 1 (B.E./B.Tech) Jan 9 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 9 January 2019, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$, If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then arg z is equal to : [4 marks]

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) 0

Correct Answer: (A) $\frac{\pi}{4}$

Solution:

Concept:

- A root of $x^2 + x + 1 = 0$ is a non-real cube root of unity. The identity $(x-1)(x^2+x+1) = x^3 - 1$ relates the quadratic to cube roots.
- For $z = a + ib$, its argument is the angle of the vector (a, b), with the quadrant fixed by the signs of its real and imaginary parts.

Step 1: Reduce the powers of the root Since $z_0^2 + z_0 + 1 = 0$, multiplication by $z_0 - 1$ gives $z_0^3 = 1$. Neither root is zero, and neither is the real cube root 1. Both roots nevertheless have the same third power, so the choice of root makes no difference here. The exponents 81 and 93 are both multiples of 3. Hence

$$z_0^{81} = (z_0^3)^{27} = 1, \qquad z_0^{93} = (z_0^3)^{31} = 1.$$

Step 2: Simplify the complex number and locate its quadrant Insert these values in the given expression without changing the factors of i:

$$z = 3 + 6i(1) - 3i(1) = 3 + 3i.$$

Thus $\mathrm{Re}\, z = 3 > 0$ and $\mathrm{Im}\, z = 3 > 0$, placing the number in the first quadrant. Its modulus is $3\sqrt{2}$, and its direction satisfies

$$\tan\theta = \frac{3}{3} = 1, \qquad 0 < \theta < \frac{\pi}{2}.$$

The first-quadrant angle with this tangent is $\pi/4$. Angles $\pi/6$ and $\pi/3$ would require unequal real and imaginary parts; zero would require a vanishing imaginary part. This also directly checks the other listed choices.

Step 3: Conclusion Therefore $\arg z = \pi/4$ (principal argument). Hence, the correct option is (A).

Quick Tip to Solve: When a quadratic root is a cube root of unity, reduce exponents modulo 3 before doing any complex arithmetic; then check the quadrant.

2. If $\int_0^{\frac{\pi}{3}} \frac{\tan\theta}{\sqrt{2k\sec\theta}}\,d\theta = 1 - \frac{1}{\sqrt{2}}, (k>0)$, **then value of k is :** [4 marks]

(A) 4

(B) $\frac{1}{2}$

(C) 1

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- Use a substitution whose differential contains the numerator. Since $d(\sec\theta) = \sec\theta\tan\theta\,d\theta$, choosing $u = \sec\theta$ turns this integral into a power integral.
- On $[0, \pi/3]$, $\sec\theta > 0$; with $k > 0$, all square roots are real and the positive square root must be used.

Step 1: Transform the variable and limits Write the given integral as I. Put $u = \sec\theta$, so $du = u\tan\theta\,d\theta$, or $\tan\theta\,d\theta = du/u$. The limits change from $\theta = 0, \pi/3$ to $u = 1, 2$, respectively. Consequently

$$I = \frac{1}{\sqrt{2k}}\int_1^2 u^{-3/2}\,du.$$

The extra factor $1/u$ from the differential is essential: simply integrating $u^{-1/2}$ would give a different result.

Step 2: Evaluate and equate the positive factors The power rule gives an antiderivative $-2u^{-1/2}$, so

$$I = \frac{2}{\sqrt{2k}}\left(1 - \frac{1}{\sqrt{2}}\right) = \sqrt{\frac{2}{k}}\left(1 - \frac{1}{\sqrt{2}}\right).$$

The specified value is $1 - 1/\sqrt{2}$, which is strictly positive. Dividing by that common nonzero factor gives

$$\sqrt{\frac{2}{k}} = 1 \quad \Longrightarrow \quad k = 2.$$

The stipulated positivity excludes negative square-root choices. As a check, for $k = 2$ the prefactor is exactly 1. The remaining options $4, 1/2, 1$ produce prefactors $1/\sqrt{2}, 2, \sqrt{2}$, respectively, and therefore cannot give the required integral.

Step 3: Conclusion The required parameter is $k = 2$. Hence, the correct option is (D).

Quick Tip to Solve: With a square root of secant in the denominator, use secant as the variable and retain the additional secant factor in its differential.

3. The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1, 3, 7, 9 (repitition of digits allowed) is equal to : [4 marks]

(A) 374

(B) 372

(C) 375

(D) 250

Correct Answer: (A) 374

Solution:

Concept:

- The multiplication principle counts independent digit choices. Repetition being allowed means a used digit remains available at each subsequent position.
- A natural number has a nonzero leading digit. Count separately by number of digits so that each positive integer is included exactly once.

Step 1: Count numbers with fewer than four digits The allowed digits are $0, 1, 3, 7, 9$. A one-digit natural number can be 1, 3, 7 or 9, giving 4 choices; zero is not included as a positive natural number. For a two-digit number, the leading digit has 4 choices and the units digit has all 5 choices, giving $4 \cdot 5 = 20$. Similarly, a three-digit number has $4 \cdot 5 \cdot 5 = 100$ choices. Every such number is below 7,000. Thus the subtotal is

$$4 + 20 + 100 = 124.$$

Step 2: Apply the upper bound to four-digit numbers In a four-digit number below 7,000 the thousands digit can only be 1 or 3. A leading 0 does not form a four-digit number, while a leading 7 gives at least 7,000 and a leading 9 exceeds the bound. Once the first digit is chosen, each of the last three positions has all five allowed choices. Therefore this case contributes

$$2 \cdot 5^3 = 250.$$

No number with five or more digits can lie below 7,000. Adding the disjoint cases gives

$$N = 124 + 250 = 374.$$

The option 250 counts only four-digit numbers and omits all shorter ones. Counting 0 would incorrectly raise the answer to 375. Neither 372 nor 375 equals the positive-integer count with the strict upper bound.

Step 3: Conclusion Exactly 374 natural numbers satisfy the conditions. Hence, the correct option is (A).

Quick Tip to Solve: For a strict digit bound, count shorter lengths first, then restrict only the leading digits of the longest possible length.

4. If x = $\sin^{-1}(\sin 10)$ and y = $\cos^{-1}(\cos 10)$, then y − x is equal to : [4 marks]

(A) 0

(B) 10

(C) 7π

(D) π

Correct Answer: (D) π

Solution:

Concept:

- The principal range of $\sin^{-1}$ is $[-\pi/2, \pi/2]$, while that of $\cos^{-1}$ is $[0, \pi]$. An inverse function returns the representative in its principal range.
- Trigonometric inputs without a degree symbol are in radians. Use periodicity and reflection to place the angle 10 in the appropriate inverse-function range.

Step 1: Find the principal value of the inverse sine The angle 10 satisfies $3\pi < 10 < 7\pi/2$. Subtracting 3π puts $10 - 3\pi$ between 0 and $\pi/2$. Because

$$\sin 10 = -\sin(10 - 3\pi) = \sin(3\pi - 10),$$

and $-\pi/2 < 3\pi - 10 < 0$, the angle $3\pi - 10$ belongs to the principal range of inverse sine. Thus

$$x = \sin^{-1}(\sin 10) = 3\pi - 10.$$

One cannot cancel sine and inverse sine at 10 directly, because 10 lies outside their principal interval.

Step 2: Find the principal inverse cosine and subtract The angle $10 - 2\pi$ is between π and 2π, so reflect it about 2π. The angle $4\pi - 10$ lies strictly between 0 and π, and

$$\cos(4\pi - 10) = \cos 10.$$

It is therefore the correct principal value:

$$y = 4\pi - 10.$$

Subtracting the two expressions gives

$$y - x = (4\pi - 10) - (3\pi - 10) = \pi.$$

Numerically, x is about -0.575 and y about 2.566, consistent with the ranges and the difference π. The zero choice would require equal principal angles, while 10 and 7π also disagree with the explicit subtraction.

Step 3: Conclusion Therefore $y - x = \pi$. Hence, the correct option is (D).

Quick Tip to Solve: Before cancelling an inverse trigonometric function, move the angle into that inverse function's principal range using periodicity and reflection.

5. If the lines x = ay + b, z = cy + d and x = a'z + b', y = c'z + d' are perpendicular, then : [4 marks]

(A) ab' + bc' + 1 = 0

(B) cc' + a + a' = 0

(C) bb' + cc' + 1 = 0

(D) aa' + c + c' = 0

Correct Answer: (D) aa' + c + c' = 0

Solution:

Concept:

- A parametric line $\mathbf{r} = \mathbf{r}_0 + \lambda\mathbf{d}$ has direction vector $\mathbf{d}$; constant offsets determine its position, not its direction.
- Perpendicular directions have zero dot product. For vectors (l, m, n) and $(l\prime, m\prime, n\prime)$, the condition is $ll\prime + mm\prime + nn\prime = 0$.

Step 1: Parameterise each line using its free coordinate For the first line, use $y = s$. Then its coordinates are

$$(x, y, z) = (as + b, s, cs + d) = (b, 0, d) + s(a, 1, c).$$

Its direction vector is therefore $\mathbf{d}_1 = (a, 1, c)$. For the second line use $z = u$, giving

$$(x, y, z) = (a\prime u + b\prime, c\prime u + d\prime, u) = (b\prime, d\prime, 0) + u(a\prime, c\prime, 1).$$

Thus $\mathbf{d}_2 = (a\prime, c\prime, 1)$. Taking the components in the same order x, y, z avoids mixing coefficients from different coordinates.

Step 2: Impose the perpendicularity condition Each direction vector is nonzero, since one of its components is 1. Hence the cosine formula for the angle between directions has a nonzero denominator, and a right angle requires its numerator to vanish:

$$\mathbf{d}_1 \cdot \mathbf{d}_2 = aa\prime + 1 \cdot c\prime + c \cdot 1 = 0.$$

Equivalently, $aa\prime + c + c\prime = 0$. The constants $b, d, b\prime, d\prime$ disappear because translating a line does not change its direction. Options involving b or $b\prime$ cannot be general angle conditions. The remaining alternative $cc\prime + a + a\prime = 0$ pairs the wrong coordinate components and does not equal this dot product.

Step 3: Conclusion The required relation is $aa\prime + c + c\prime = 0$. Hence, the correct option is (D).

Quick Tip to Solve: When two coordinates are written in terms of the third, use that third coordinate as the parameter and read direction components in x, y, z order.

6. A data consists of n observations : $x_1, x_2, \ldots\ldots, x_n$.

If $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ **and**

$$\sum_{i=1}^{n}(x_i-1)^2 = 5n,$$

then the standard deviation of this data is : [4 marks]

(A) 2

(B) $\sqrt{5}$

(C) 5

(D) $\sqrt{7}$

Correct Answer: (B) $\sqrt{5}$

Solution:

Concept:

- For n observations, the mean is $\bar{x} = (\sum x_i)/n$ and the population variance is $\sigma^2 = (\sum x_i^2)/n - \bar{x}^2$. Standard deviation is the nonnegative square root of variance.
- Adding and subtracting the sums of $(x_i+1)^2$ and $(x_i-1)^2$ isolates the second and first moments, respectively.

Step 1: Extract the first and second moment sums Let $S_1 = \sum_{i=1}^{n} x_i$ and $S_2 = \sum_{i=1}^{n} x_i^2$. Expanding the two given expressions gives

$$S_2 + 2S_1 + n = 9n, \qquad S_2 - 2S_1 + n = 5n.$$

Subtract the second equation from the first:

$$4S_1 = 4n \quad \Longrightarrow \quad S_1 = n.$$

Now add them instead:

$$2S_2 + 2n = 14n \quad \Longrightarrow \quad S_2 = 6n.$$

Thus $\bar{x} = S_1/n = 1$ and the mean of the squared observations is $S_2/n = 6$.

Step 2: Calculate the variance and take its square root Use the central-moment identity, rather than treating the raw second moment as the variance:

$$\sigma^2 = \frac{S_2}{n} - \left(\frac{S_1}{n}\right)^2 = 6 - 1^2 = 5.$$

Standard deviation is therefore $\sigma = \sqrt{5}$. This result also checks the original shifted sums: the means of $(x_i+1)^2$ and $(x_i-1)^2$ become $6+2+1=9$ and $6-2+1=5$. The option 5 is the variance rather than its square root. Neither 2 nor $\sqrt{7}$ follows from the centered calculation. The formula uses division by n, appropriate for the standard deviation of the stated complete data, rather than an unbiased sample-variance estimate.

Step 3: Conclusion The standard deviation is $\sqrt{5}$. Hence, the correct option is (B).

Quick Tip to Solve: For symmetric shifts by +1 and -1, subtract to obtain the mean and add to obtain the mean square; only then calculate variance.

7. **If** $A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}cost \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$

then A is : [4 marks]

(A) invertible for all t∈**R**.

(B) invertible only if t = π

(C) not invertible for any t∈**R**

(D) invertible only if t = $\frac{\pi}{2}$.

Correct Answer: (A) invertible for all t∈**R**.

Solution:

Concept:

- A square matrix is invertible exactly when its determinant is nonzero. Factoring a scalar from each column extracts the product of those scalars from the determinant.
- Subtracting one row from another leaves a determinant unchanged. The identity $\sin^2 t + \cos^2 t = 1$ can remove all trigonometric dependence after expansion.

Step 1: Factor the exponentials and simplify the rows Put $c = \cos t$, $s = \sin t$. Factor e^t from column 1 and e^{-t} from each of columns 2 and 3. Their product is e^{-t}, so

$$\det A = e^{-t}\begin{vmatrix} 1 & c & s \\ 1 & -c-s & c-s \\ 1 & 2s & -2c \end{vmatrix}.$$

Apply $R_2 \leftarrow R_2 - R_1$ and $R_3 \leftarrow R_3 - R_1$. The first column becomes $(1,0,0)^T$. Expanding along it reduces the determinant to

$$\det A = e^{-t}\begin{vmatrix} -2c-s & c-2s \\ 2s-c & -2c-s \end{vmatrix}.$$

Step 2: Evaluate the reduced determinant for every real parameter The lower-left entry is the negative of the upper-right entry. Hence the determinant is a sum of two squares:

$$\det A = e^{-t}\big((2c+s)^2 + (c-2s)^2\big).$$

Expanding makes the mixed terms cancel:

$$(2c+s)^2 + (c-2s)^2 = 5c^2 + 5s^2 = 5.$$

Therefore $\det A = 5e^{-t}$. The exponential is strictly positive for every finite real t, so this determinant never vanishes. In particular $t = 0$ already gives determinant 5, ruling out choices that allow invertibility only at π or $\pi/2$. The assertion of noninvertibility at every real value is also contradicted by the same determinant calculation.

Step 3: Conclusion The matrix is invertible for all $t \in \mathbb{R}$, because $\det A = 5e^{-t} > 0$. Hence, the correct option is (A).

Quick Tip to Solve: Extract column factors first; subtract the first row from the others and look for a sum of squares before expanding a trigonometric determinant.

8. Let f : [0,1] $\to$ R be such that f(xy) = f(x).f(y), for all x, y $\in$ [0, 1], and f(0) $\neq$ 0. If y = y(x) satiesfies the differential equation, $\frac{dy}{dx}$ = f(x) with y(0) = 1, then y$\left(\frac{1}{4}\right)$ + y$\left(\frac{3}{4}\right)$ is equal to : [4 marks]

(A) 3

(B) 4

(C) 2

(D) 5

Correct Answer: (A) 3

Solution:

Concept:

- A multiplicative identity that holds for every pair in the domain may be evaluated at zero. The assumption $f(0) \neq 0$ then permits division by $f(0)$.
- After determining $f(x)$, solve $dy/dx = f(x)$ by integration. The initial value fixes the integration constant uniquely.

Step 1: Use the zero input in the functional equation For any $x \in [0, 1]$, choose $y = 0$. Because $x \cdot 0 = 0$, the given equation becomes

$$f(0) = f(x)f(0).$$

Division by the nonzero number $f(0)$ gives

$$f(x) = 1 \quad \text{for every } x \in [0, 1].$$

This argument needs no continuity assumption and rules out all nonconstant possibilities. In particular, setting both inputs to zero also confirms $f(0) = f(0)^2$, so the allowed nonzero value is indeed one.

Step 2: Integrate and impose the initial value The differential equation therefore reduces to

$$\frac{dy}{dx} = 1.$$

Its general solution is $y = x + C$. At $x = 0$, the condition $y(0) = 1$ gives $C = 1$, hence

$$y(x) = x + 1.$$

Evaluate the two required values separately:

$$y\left(\frac{1}{4}\right) = \frac{5}{4}, \qquad y\left(\frac{3}{4}\right) = \frac{7}{4}.$$

Adding gives $12/4 = 3$. The other proposed totals, 4, 2 and 5, cannot satisfy these explicit evaluations; the initial constant contributes twice to the sum, while the two arguments add to one.

Step 3: Conclusion $y(1/4) + y(3/4) = 3$. Hence, the correct option is (A).

Quick Tip to Solve: For a multiplicative function with a nonzero value at zero, put one argument equal to zero before trying logarithms or differentiation.

9. Let the equations of two sides of a triangle be 3x − 2y + 6 = 0 and 4x + 5y − 20 = 0. If the orthocentre of this triangle is at (1, 1), then the equation of its third side is : [4 marks]

(A) 122y − 26x − 1675 = 0

(B) 122y + 26x + 1675 = 0

(C) 26x + 61y + 1675 = 0

(D) 26x − 122y − 1675 = 0

Correct Answer: (D) 26x − 122y − 1675 = 0

Solution:

Concept:

- The orthocentre is the common point of the three altitudes. An altitude through a vertex is perpendicular to the opposite side.
- For finite nonzero slopes, perpendicular lines have slope product -1. Intersect each known side with the altitude through the orthocentre to obtain its other vertex.

Step 1: Find the vertex on the first side Call the common vertex of the given sides A, and let B lie on $3x - 2y + 6 = 0$, while C lies on $4x + 5y - 20 = 0$. The slope of AC is $-4/5$, so the altitude BH, through $H = (1, 1)$, has slope $5/4$. Therefore

$$y - 1 = \frac{5}{4}(x - 1), \qquad 5x - 4y - 1 = 0.$$

On the first side $y = (3x + 6)/2$. Substitution yields $5x - 2(3x + 6) - 1 = 0$, giving

$$B = \left(-13, -\frac{33}{2}\right).$$

Step 2: Find the second vertex and join the vertices The slope of AB is $3/2$, so CH has slope $-2/3$:

$$y - 1 = -\frac{2}{3}(x - 1), \qquad 2x + 3y - 5 = 0.$$

Solving this with $4x + 5y - 20 = 0$ gives

$$C = \left(\frac{35}{2}, -10\right).$$

Thus

$$m_{BC} = \frac{-10 + 33/2}{35/2 + 13} = \frac{13}{61}.$$

The required line is

$$y + \frac{33}{2} = \frac{13}{61}(x + 13),$$

or $26x - 122y - 1675 = 0$. Both vertices satisfy this equation. Option A has the wrong constant after matching the signs of the variable terms; B has the wrong slope; C also has the wrong slope. Only D passes both intersection checks.

Step 3: Conclusion The third side is $26x - 122y - 1675 = 0$. Hence, the correct option is (D).

Quick Tip to Solve: Use the orthocentre to construct the two altitudes opposite the known sides, rather than first finding their common vertex.

10. Let A = {x ∈ R : x is not a positive integer}.
Define a function f : A → R as $f(x) = \frac{2x}{x-1}$,
then f is : [4 marks]

(A) not injective

(B) neither injective nor surjective

(C) surjective but not injective

(D) injective but not surjective

Correct Answer: (D) injective but not surjective

Solution:

Concept:

- A function is injective if equal output values imply equal inputs. It is surjective onto $\mathbf{R}$ only if every real number has an admissible preimage.
- The domain excludes every positive integer, in particular $x = 1$; cross multiplication by the denominators is valid for all allowed inputs.

Step 1: Prove that the function is injective Let $a, b \in A$, and suppose $f(a) = f(b)$. Since neither input equals one,

$$\frac{2a}{a-1} = \frac{2b}{b-1} \quad \Longrightarrow \quad a(b-1) = b(a-1).$$

Expanding both sides and cancelling ab gives $-a = -b$, hence $a = b$. Consequently distinct allowed inputs cannot have the same image, so the function is injective. Removing additional positive integers from the domain cannot destroy this property: the equality argument holds on the whole set where the denominator is nonzero.

Step 2: Test whether every real output is attained The real number 2 is not an output. Indeed,

$$\frac{2x}{x-1} = 2 \quad \Longrightarrow \quad 2x = 2x - 2,$$

which is impossible. Equivalently, solving $y = 2x/(x-1)$ for an input gives

$$x = \frac{y}{y-2} \quad (y \neq 2),$$

and no such formula exists for $y = 2$. There can also be further missing outputs because the domain excludes positive integers, but one missing value already disproves surjectivity onto $\mathbf{R}$. Thus A, B and C fail because each asserts noninjectivity; C also incorrectly asserts surjectivity. D states the two properties established directly.

Step 3: Conclusion The function is injective but not surjective onto $\mathbf{R}$. Hence, the correct option is (D).

Quick Tip to Solve: To disprove surjectivity of a rational function, first test its horizontal-asymptote value as a possible output.

11. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 sq. units, then the number of elements in the set S is : [4 marks]

(A) 9

(B) 18

(C) 36

(D) 32

Correct Answer: (C) 36

Solution:

Concept:

- A nondegenerate triangle with vertices $(0,0),(a,0),(0,b)$ has area $|ab|/2$. Its nonzero axis intercepts a,b must be integers here.
- For a positive integer $N=\prod p_i^{e_i}$, its number of positive divisors is $\prod(e_i+1)$. Each positive divisor fixes an ordered pair of positive factors.

Step 1: Translate the area condition into an integer product The two non-origin vertices cannot lie on the same coordinate axis, because that would make the area zero. Thus write them uniquely as $(a,0)$ and $(0,b)$, with $a,b\neq 0$. The area condition gives

$$\frac{|ab|}{2}=50 \quad\Longrightarrow\quad |ab|=100.$$

Set $u=|a|$ and $v=|b|$. Then $uv=100$, so choosing a positive divisor u of 100 determines $v=100/u$. Because the axes are distinguished, (u,v) and (v,u) generally give different triangles.

Step 2: Count factor choices and signs without duplication The factorisation $100=2^2\cdot 5^2$ gives

$$(2+1)(2+1)=9$$

positive choices of u. They are $1,2,4,5,10,20,25,50,100$, with the corresponding v determined uniquely. For each pair there are four independent sign choices,

$$(a,b)=(u,v),(u,-v),(-u,v),(-u,-v).$$

They place the triangle in four different quadrants and are all distinct. No division by two is needed: swapping the unordered names of the vertices does not swap their fixed x-axis and y-axis positions. Hence the total is $9\cdot 4=36$. The answer 9 ignores all sign choices, 18 misses half of them, and 32 uses the wrong divisor count.

Step 3: Conclusion There are 36 triangles in S. Hence, the correct option is (C).

Quick Tip to Solve: For integer intercept triangles, count positive divisors of twice the area, then multiply by four for the independent signs.

12. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1, 5], then m lies in the interval : [4 marks]

(A) $(-5, -4)$

(B) (4, 5)

(C) (5, 6)

(D) (3, 4)

Correct Answer: The exact interval is (4,5], which none of the printed options gives.

Solution:

Concept:

- If the roots are r, s, Vieta's relations give $r + s = m$ and $rs = 4$. The interval $[1, 5]$ includes its endpoints.
- Real distinct positive roots with product four have sum strictly greater than four. The endpoint restrictions must also be checked explicitly rather than replaced by strict inequalities.

Step 1: Determine the necessary lower bound Both roots lie in $[1, 5]$, so they are positive. Their product is four and they are distinct. By the arithmetic-geometric mean inequality,

$$m = r + s > 2\sqrt{rs} = 4.$$

Equality would give the repeated root $r = s = 2$, which is excluded. Order the roots as $r < s$. Since $rs = 4$, the smaller root is below two and the larger is above two. The condition $r \geq 1$ implies $s = 4/r \leq 4$, so the larger root automatically satisfies the upper limit five.

Step 2: Find and verify the upper endpoint For $1 \leq r < 2$, the sum is

$$m = r + \frac{4}{r}.$$

Its derivative $1 - 4/r^2$ is negative on $[1, 2)$, so this expression decreases from 5 at $r = 1$ towards 4 as $r \to 2^-$. Therefore the exact range is

$$4 < m \leq 5.$$

Conversely, for every m in this range, the roots $(m \pm \sqrt{m^2 - 16})/2$ are distinct, positive and lie between 1 and 4. Crucially, $m = 5$ yields $(x - 1)(x - 4) = 0$; both 1 and 4 belong to the printed closed interval. Thus the printed option B, $(4, 5)$, omits a valid endpoint. A and D contradict the lower bound, and C exceeds the upper bound.

Step 3: Conclusion The exact answer is $m \in (4, 5]$. None of the printed options is exactly correct: option (B) gives only $(4, 5)$ and omits the valid value $m = 5$.

Quick Tip to Solve: For root-location intervals, substitute each proposed endpoint into the original polynomial; a closed input interval can produce a closed parameter endpoint.

13. The area of the region

A = {(x, y) : 0 ≤ y ≤x |x| + 1 and −1 ≤ x ≤1} in sq. units, is : [4 marks]

(A) $\frac{2}{3}$

(B) 2

(C) $\frac{4}{3}$

(D) $\frac{1}{3}$

Correct Answer: (B) 2

Solution:

Concept:

- The area between $y = 0$ and a nonnegative upper function $y = g(x)$ is $\int g(x)\,dx$ over the specified x-interval.
- The function $x|x|$ equals $-x^2$ for $x < 0$ and x^2 for $x \geq 0$. It is odd, so its integral over a symmetric interval is zero.

Step 1: Check the vertical bounds and split at zero On $[-1, 0]$, the upper bound is $1 - x^2$, which is nonnegative. On $[0, 1]$, it is $1 + x^2$, also nonnegative. Thus every vertical section has the stated lower bound zero, and no extra change of sign for an area calculation is needed. The required area is

$$\mathcal{A} = \int_{-1}^{0} (1 - x^2)\,dx + \int_{0}^{1} (1 + x^2)\,dx.$$

Splitting at zero is essential because replacing $x|x|$ by x^2 throughout would change the negative half of the region.

Step 2: Evaluate both contributions and compare the options The left contribution is

$$\left[x - \frac{x^3}{3}\right]_{-1}^{0} = \frac{2}{3},$$

while the right contribution is

$$\left[x + \frac{x^3}{3}\right]_{0}^{1} = \frac{4}{3}.$$

Hence

$$\mathcal{A} = \frac{2}{3} + \frac{4}{3} = 2 \text{ square units.}$$

As an independent check, integrate $1 + x|x|$ on $[-1, 1]$. The constant contributes 2 and the odd term contributes zero, reproducing the answer. Options A and C are just the separate left and right contributions, not the entire region. Option D is neither their sum nor a valid signed-area correction because the upper curve never lies below the x-axis on this interval.

Step 3: Conclusion The area is 2 square units. Hence, the correct option is (B).

Quick Tip to Solve: On a symmetric interval, recognise that x times its absolute value is odd; check nonnegativity before identifying its integral with area.

14. A hyperbola has its centre at the origin, passes through the point (4, 2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is : [4 marks]

(A) $\frac{3}{2}$

(B) $\sqrt{3}$

(C) 2

(D) $\frac{2}{\sqrt{3}}$

Correct Answer: (D) $\frac{2}{\sqrt{3}}$

Solution:

Concept:

- A hyperbola centred at the origin with transverse axis along the x-axis has equation $x^2/a^2 - y^2/b^2 = 1$, where $a, b > 0$. Its transverse axis length is $2a$.
- For a hyperbola, $c^2 = a^2 + b^2$ and eccentricity $e = c/a = \sqrt{1 + b^2/a^2} > 1$. Here c is the distance from the centre to either focus.

Step 1: Use the transverse axis and the given point The stated transverse axis length is four, so

$$2a = 4 \quad \Longrightarrow \quad a = 2, \qquad a^2 = 4.$$

This is the full axis length, not the semiaxis. Since the transverse direction is horizontal, the positive term is the x-term. Substitute $(4, 2)$ into

$$\frac{x^2}{4} - \frac{y^2}{b^2} = 1.$$

We obtain

$$\frac{16}{4} - \frac{4}{b^2} = 1, \qquad \frac{4}{b^2} = 3, \qquad b^2 = \frac{4}{3}.$$

Both squared semiaxes are positive, as required for the standard equation.

Step 2: Calculate the focal distance and eccentricity Using the hyperbola relation rather than the ellipse relation gives

$$c^2 = 4 + \frac{4}{3} = \frac{16}{3}, \qquad c = \frac{4}{\sqrt{3}}.$$

Therefore

$$e = \frac{c}{a} = \frac{4/\sqrt{3}}{2} = \frac{2}{\sqrt{3}}.$$

This value exceeds one, which is necessary for a hyperbola. For a direct check, $e^2 - 1 = 4/3 - 1 = 1/3 = b^2/a^2$. The other choices, $3/2, \sqrt{3}, 2$, would imply ratios b^2/a^2 of $5/4, 2, 3$, respectively, whereas the point condition uniquely gives $1/3$. Thus none satisfies both the known axis and the given point.

Step 3: Conclusion The eccentricity is $e = 2/\sqrt{3}$. Hence, the correct option is (D).

Quick Tip to Solve: Halve the transverse axis length to get a, then substitute the point before using the hyperbola relation c squared equals a squared plus b squared.

15. For each x∈R, let [x] be the greatest integer less than or equal to x.

Then $\lim_{x\to 0^-} \frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to : [4 marks]

(A) $-\sin 1$

(B) 1

(C) $\sin 1$

(D) 0

Correct Answer: (A) $-\sin 1$

Solution:

Concept:

- The greatest integer function is constant between successive integers. In particular, $[x] = -1$ whenever $-1 < x < 0$. The approach from the left determines which constant to use.
- For negative x, the absolute value satisfies $|x| = -x$, so $x/|x| = -1$. Trigonometric arguments are measured in radians.

Step 1: Replace the floor and absolute value on the left neighbourhood Take $-1 < x < 0$, which includes every sufficiently small negative value used in the limit. Then $[x] = -1$, $|x| = -x$, and $\sin[x] = \sin(-1) = -\sin 1$. Therefore the expression becomes

$$\frac{x([x] + |x|)\sin[x]}{|x|} = (-1)(-1 - x)(-\sin 1) = -(1 + x)\sin 1.$$

These substitutions are exact throughout that interval; replacing the floor by zero would describe the right-hand side instead.

Step 2: Take the limit and compare the options The remaining expression is continuous at zero, so

$$\lim_{x\to 0^-} [-(1 + x)\sin 1] = -\sin 1.$$

The two negative signs from $x/|x|$ and the floor term first give a positive factor, but $\sin(-1)$ contributes the final negative sign. Thus positive $\sin 1$ has the wrong sign. The floor does not approach zero on this side, so neither zero nor one can result.

Step 3: Conclusion The required left-hand limit is $\boxed{-\sin 1}$. Hence, the correct option is (A).

Quick Tip to Solve: Before evaluating a floor-function limit at an integer, write its exact value on the specified side; here it is -1, not zero.

16. If x = 3 tan t and y = 3 sec t, then the value of $\frac{d^2y}{dx^2}$ at t = $\frac{\pi}{4}$, is : [4 marks]

(A) $\frac{1}{3\sqrt{2}}$

(B) $\frac{1}{6\sqrt{2}}$

(C) $\frac{3}{2\sqrt{2}}$

(D) $\frac{1}{6}$

Correct Answer: (B) $\frac{1}{6\sqrt{2}}$

Solution:

Concept:

- For a parametric curve with $dx/dt \neq 0$, $dy/dx = (dy/dt)/(dx/dt)$. A second derivative requires another application of the chain rule.
- The correct operator is $d/dx = (1/(dx/dt))d/dt$; differentiating dy/dx with respect to t alone is not enough.

Step 1: Calculate the first derivative Using the given parameter equations,

$$\frac{dx}{dt} = 3\sec^2 t, \qquad \frac{dy}{dt} = 3\sec t \tan t.$$

Consequently,

$$\frac{dy}{dx} = \frac{3\sec t \tan t}{3\sec^2 t} = \sin t.$$

At $t = \pi/4$, the denominator is nonzero, so this division and the next chain-rule step are valid locally.

Step 2: Differentiate again with respect to x Apply the parametric derivative formula to the first derivative:

$$\frac{d^2y}{dx^2} = \frac{d(\sin t)/dt}{dx/dt} = \frac{\cos t}{3\sec^2 t} = \frac{\cos^3 t}{3}.$$

Since $\cos(\pi/4) = 1/\sqrt{2}$, this gives

$$\left.\frac{d^2y}{dx^2}\right|_{t=\pi/4} = \frac{1}{3(\sqrt{2})^3} = \frac{1}{6\sqrt{2}}.$$

The scale factor three in the coordinates belongs in the denominator. Omitting the additional division by dx/dt, or confusing $\cos t$ with $\cos^3 t$, produces the wrong magnitude. The other three proposed values do not equal this result.

Step 3: Conclusion The second derivative is $\boxed{1/(6\sqrt{2})}$. Hence, the correct option is (B).

Quick Tip to Solve: For a second parametric derivative, divide by dx/dt twice: once to form dy/dx, and once after differentiating that ratio.

17. Let f be a differentiable function from R to R such that $|f(x) - f(y)| \leq 2|x-y|^{\frac{3}{2}}$, for all $x, y \in$ R.

If $f(0) = 1$

then $\int_0^1 f^2(x)\,dx$ is equal to : [4 marks]

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) 0

Correct Answer: (A) 1

Solution:

Concept:

- The derivative at x is the limit of $(f(x+h) - f(x))/h$ as $h \to 0$. Bounding its absolute value can determine the derivative without finding a formula first.
- A differentiable function whose derivative is zero everywhere on an interval is constant there, by the mean value theorem.

Step 1: Bound the difference quotient Set $y = x$ and replace the first argument by $x + h$ in the given inequality. For every nonzero h,

$$|f(x+h) - f(x)| \leq 2|h|^{3/2}.$$

Dividing by the positive quantity $|h|$ gives

$$\left|\frac{f(x+h) - f(x)}{h}\right| \leq 2|h|^{1/2}.$$

As $h \to 0$, the right side tends to zero. The squeeze theorem therefore yields $f\prime(x) = 0$ at every real x. The crucial point is that the exponent remaining after division is positive.

Step 2: Identify the function and integrate its square For any real $a < b$, differentiability supplies the hypotheses of the mean value theorem on $[a, b]$, so $f(b) - f(a) = f\prime(c)(b-a) = 0$ for some c between them. Hence f is constant throughout $\mathbb{R}$. The condition $f(0) = 1$ fixes that constant:

$$f(x) = 1, \qquad \int_0^1 f^2(x)\,dx = \int_0^1 1\,dx = 1.$$

The value cannot be zero because the function is identically one; the interval has length one, ruling out both two and one-half.

Step 3: Conclusion The integral equals $\boxed{1}$. Hence, the correct option is (A).

Quick Tip to Solve: In an estimate $|f(x) - f(y)| \leq C|x-y|^p$ with $p > 1$, divide by $|x-y|$ to see that every derivative must vanish.

18. If $f(x) = \int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}\,dx,\ (x \geq 0)$,
$f(0) = 0$, **then the value of** $f(1)$ **is :** [4 marks]

(A) $-\frac{1}{2}$
(B) $-\frac{1}{4}$
(C) $\frac{1}{2}$
(D) $\frac{1}{4}$

Correct Answer: (D) $\frac{1}{4}$

Solution:

Concept:

- For differentiable functions u, v with $v \neq 0$, the quotient rule is $(u/v)\prime = (u\prime v - uv\prime)/v^2$. A squared denominator in an integrand suggests checking this form.
- An antiderivative is determined up to a constant. The condition $f(0) = 0$ fixes that constant before evaluating $f(1)$.

Step 1: Find a quotient with the required derivative Put $D = x^2 + 1 + 2x^7$, so $D\prime = 2x + 14x^6$. Since the numerator contains x^6 and x^8, test x^7/D:

$$\begin{aligned}\frac{d}{dx}\left(\frac{x^7}{D}\right) &= \frac{7x^6(x^2+1+2x^7) - x^7(2x+14x^6)}{D^2} \\ &= \frac{7x^8 + 7x^6 + 14x^{13} - 2x^8 - 14x^{13}}{D^2} \\ &= \frac{5x^8 + 7x^6}{(x^2+1+2x^7)^2}.\end{aligned}$$

The highest-power terms cancel exactly. This verifies the antiderivative against the entire integrand, including its positive sign.

Step 2: Apply the initial value and evaluate The denominator is at least one for $x \geq 0$, so there is no singularity on the required interval. It follows that

$$f(x) = \frac{x^7}{x^2+1+2x^7} + C.$$

At zero the fraction vanishes, and $f(0) = 0$ gives $C = 0$. Therefore

$$f(1) = \frac{1}{1+1+2} = \frac{1}{4}.$$

Both negative options conflict with the nonnegative integrand and the initial value. The positive one-half also has the wrong denominator: all three denominator contributions at one must be included.

Step 3: Conclusion Thus $\boxed{f(1) = 1/4}$. Hence, the correct option is (D).

Quick Tip to Solve: For a squared polynomial denominator, test a simple quotient and look for cancellation of its highest-power terms before attempting expansion or substitution.

19. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is : [4 marks]

(A) 3

(B) 2

(C) 4

(D) 5

Correct Answer: (A) 3

Solution:

Concept:

- For a quadratic with integer coefficients, rational roots require its discriminant to be a nonnegative perfect square. Indeed, a rational square root of an integer must itself be an integer.
- Here the discriminant is $\Delta = 121 - 24\alpha$. The parameter α is restricted to positive integers, which makes a finite complete check possible.

Step 1: Bound the possible values of alpha The quadratic formula gives

$$x = \frac{11 \pm \sqrt{121 - 24\alpha}}{12}.$$

Rational roots must be real, so $121 - 24\alpha \geq 0$. Thus $\alpha \leq 121/24$, and positivity and integrality restrict the candidates to

$$\alpha \in \{1, 2, 3, 4, 5\}.$$

No larger positive integer can work: it would give a negative discriminant, hence nonreal rather than rational roots.

Step 2: Check which discriminants are squares Evaluate the five candidates without omitting any:

α	1	2	3	4	5
$121 - 24\alpha$	97	73	49	25	1

The first two entries lie strictly between consecutive integer squares and are not squares. The remaining values are $7^2, 5^2, 1^2$. Consequently precisely $\alpha = 3, 4, 5$ produce rational roots. As a direct check their roots are respectively $(3/2, 1/3)$, $(4/3, 1/2)$, and $(1, 5/6)$; each pair has product $\alpha/6$. There are three valid parameters, not two, four or five.

Step 3: Conclusion The number of possible positive integral values is $\boxed{3}$. Hence, the correct option is (A).

Quick Tip to Solve: First use the nonnegative-discriminant condition to bound an integer parameter, then test perfect squares only within that finite range.

20. Let $\vec{a} = \hat{i}+\hat{j}+\sqrt{2}\hat{k}$, $\vec{b} = b_1\hat{i}+b_2\hat{j}+\sqrt{2}\hat{k}$, $\vec{c} = 5\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$.
If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $\left|\vec{b}\right|$ is equal to : [4 marks]

(A) $\sqrt{32}$
(B) 6
(C) $\sqrt{22}$
(D) 4

Correct Answer: (B) 6

Solution:

Concept:

- The vector projection of $\vec{b}$ on a nonzero vector $\vec{a}$ is $(\vec{b}\cdot\vec{a})/(\vec{a}\cdot\vec{a})\,\vec{a}$. Equality to $\vec{a}$ forces this scalar coefficient to be one.
- Perpendicular vectors have zero dot product. The magnitude of a vector with components (u, v, w) is $\sqrt{u^2+v^2+w^2}$.

Step 1: Use the projection condition The given vector satisfies

$$\vec{a}\cdot\vec{a} = 1+1+2 = 4, \qquad \vec{b}\cdot\vec{a} = b_1+b_2+2.$$

Since $\vec{a}\neq\vec{0}$, its equality to the projection gives

$$\frac{b_1+b_2+2}{4} = 1 \quad\Longrightarrow\quad b_1+b_2 = 2.$$

This is a vector projection condition; using the scalar projection formula without its unit direction would impose a different equation.

Step 2: Use perpendicularity and calculate the magnitude Now $\vec{a}+\vec{b} = (1+b_1, 1+b_2, 2\sqrt{2})$. Its dot product with $\vec{c} = (5, 1, \sqrt{2})$ must vanish:

$$5(1+b_1)+(1+b_2)+(2\sqrt{2})(\sqrt{2}) = 0,$$

so $5b_1+b_2 = -10$. Subtracting $b_1+b_2 = 2$ yields $4b_1 = -12$, hence $b_1 = -3$ and $b_2 = 5$. Therefore

$$|\vec{b}| = \sqrt{(-3)^2+5^2+(\sqrt{2})^2} = \sqrt{9+25+2} = 6.$$

As a check, the projection dot product is $-3+5+2 = 4$, and the perpendicularity dot product is $-10+6+4 = 0$. The squared magnitude is 36, ruling out the alternatives whose squares are 32, 22 and 16.

Step 3: Conclusion The required magnitude is $\boxed{6}$. Hence, the correct option is (B).

Quick Tip to Solve: Translate vector projection into one dot-product equation and perpendicularity into another; solve those scalar equations before computing a length.

21. Let A(4, − 4) and B(9, 6) be points on the parabola, $y^2 = 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of $\triangle ACB$ is maximum. Then, the area (in sq. units) of $\triangle ACB$, is : [4 marks]

(A) $31\frac{1}{4}$

(B) $30\frac{1}{2}$

(C) 32

(D) $31\frac{3}{4}$

Correct Answer: (A) $31\frac{1}{4}$

Solution:

Concept:

- Points on $y^2 = 4x$ can be written as $(t^2, 2t)$. The arc from $A = (4, -4)$ to $B = (9, 6)$ through the origin corresponds to $-2 \le t \le 3$.
- The area of a triangle is half the absolute value of the two-dimensional determinant of two side vectors. With a fixed base, this reduces the area to a function of the moving parameter.

Step 1: Express the area in terms of t Write $C = (t^2, 2t)$. The two vectors drawn from A are

$$\overrightarrow{AB} = (5, 10), \qquad \overrightarrow{AC} = (t^2 - 4, 2t + 4).$$

Their determinant gives

$$\begin{aligned} \mathcal{A}(t) &= \frac{1}{2}|5(2t + 4) - 10(t^2 - 4)| \\ &= 5|-t^2 + t + 6|. \end{aligned}$$

On the specified arc, $-t^2 + t + 6 = (3 - t)(t + 2) \ge 0$, so the absolute value may be removed throughout the interval.

Step 2: Maximise the resulting quadratic Complete the square to locate the global maximum directly:

$$\mathcal{A}(t) = -5t^2 + 5t + 30 = \frac{125}{4} - 5\left(t - \frac{1}{2}\right)^2.$$

The square is nonnegative, and $t = 1/2$ belongs to $[-2, 3]$, so the upper bound $125/4$ is attained at $C = (1/4, 1)$. At the endpoints $C = A$ or $C = B$, the area is zero, consistent with this downward-opening quadratic. The maximum is 31.25, excluding 30.5, 32 and 31.75.

Step 3: Conclusion The maximum area is $\boxed{125/4 = 31\frac{1}{4}}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: Parametrise the parabola before expanding a triangle determinant; with a fixed chord, the area often becomes a simple quadratic whose vertex gives the maximum.

22. Let a, b and c be the 7^{th}, 11^{th} and 13^{th} terms respectively of a non-constant A.P. If these are also three consecutive terms of a G.P., then $\frac{a}{c}$ equal to : [4 marks]

(A) 2

(B) $\frac{1}{2}$

(C) $\frac{7}{13}$

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- In an arithmetic progression, the term at position n is $u + (n-1)d$, where u is the first term and d the common difference. Non-constant means $d \neq 0$.
- Three consecutive geometric-progression terms a, b, c satisfy $b^2 = ac$. The unequal gaps in the A.P. positions must be retained when expressing these terms.

Step 1: Express all three terms using the first selected term The seventh term is $a = u + 6d$. From the seventh to the eleventh position there are four steps, while from the seventh to the thirteenth there are six. Therefore

$$b = a + 4d, \qquad c = a + 6d.$$

The terms themselves need not be positive: an arithmetic progression may have negative terms and a negative common difference. No assumption about positivity is needed in using the G.P. condition.

Step 2: Apply the geometric-progression condition Substitute the expressions for b and c into $b^2 = ac$:

$$\begin{aligned}(a + 4d)^2 &= a(a + 6d),\\ a^2 + 8ad + 16d^2 &= a^2 + 6ad,\\ 2d(a + 8d) &= 0.\end{aligned}$$

Since the A.P. is non-constant, $d \neq 0$, so $a = -8d$. Consequently $b = -4d$ and $c = -2d$, giving a G.P. with common ratio $1/2$.

Step 3: Calculate the requested ratio and check the options We need the ratio of the first selected term to the third, not the common ratio between consecutive G.P. terms:

$$\frac{a}{c} = \frac{-8d}{-2d} = 4.$$

Thus $1/2$ is the G.P. common ratio and 2 is a/b; neither answers the question. The ratio $7/13$ incorrectly treats term positions as term values.

Step 4: Conclusion The required value is $a/c = 4$. Hence, the correct option is (D).

Quick Tip to Solve: Write selected A.P. terms relative to one of them; use the actual position gaps before imposing the G.P. relation.

23. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is : [4 marks]

(A) $\frac{21}{49}$

(B) $\frac{27}{49}$

(C) $\frac{26}{49}$

(D) $\frac{32}{49}$

Correct Answer: (D) $\frac{32}{49}$

Solution:

Concept:

- The law of total probability gives $P(R_2) = P(R_1)P(R_2 \mid R_1) + P(G_1)P(R_2 \mid G_1)$, where subscripts denote draw numbers.
- The drawn ball is removed before a ball of the opposite colour is added. Thus the urn again contains seven balls before the second draw, but its composition depends on the first colour.

Step 1: Find the probabilities of the first draw Initially the urn holds five red and two green balls, seven in total. All balls are equally likely to be drawn, so

$$P(R_1) = \frac{5}{7}, \qquad P(G_1) = \frac{2}{7}.$$

These events are mutually exclusive and exhaustive: every possible first draw is exactly one of red or green. It is therefore appropriate to condition the second draw on these two cases.

Step 2: Update the urn separately in each case If a red ball is drawn, removing it leaves four red and two green balls. Adding one green ball then gives four red and three green balls, so $P(R_2 \mid R_1) = 4/7$. If a green ball is drawn, removing it leaves five red and one green ball. Adding one red ball gives six red and one green ball, so $P(R_2 \mid G_1) = 6/7$. In each case the total is seven; retaining the original ball would incorrectly make the total eight.

Step 3: Combine the two disjoint paths A red second draw can occur through a red first draw followed by red, or through a green first draw followed by red. Hence

$$\begin{aligned} P(R_2) &= \frac{5}{7} \cdot \frac{4}{7} + \frac{2}{7} \cdot \frac{6}{7} \\ &= \frac{20 + 12}{49} = \frac{32}{49}. \end{aligned}$$

The other numerators $21, 27, 26$ do not equal the sum of these weighted paths. As a check, the green-second probability is $(5/7)(3/7) + (2/7)(1/7) = 17/49$, and the two final probabilities sum to one.

Step 4: Conclusion The probability that the second ball is red is $32/49$. Hence, the correct option is (D).

Quick Tip to Solve: For an urn whose contents change after a draw, write the new red and green counts in each branch before calculating conditional probabilities.

24. If the system of linear equations

$x - 4y + 7z = g$

$3y - 5z = h$

$-2x + 5y - 9z = k$

is consistent, then : [4 marks]

(A) $g + 2h + k = 0$

(B) $g + h + 2k = 0$

(C) $2g + h + k = 0$

(D) $g + h + k = 0$

Correct Answer: (C) $2g + h + k = 0$

Solution:

Concept:

- A consistent linear system must preserve every linear dependence among its left-hand sides in the corresponding right-hand sides.
- If one equation is a linear combination of the others, that equation is redundant only when the same combination of constants agrees. Otherwise it is a contradiction.

Step 1: Find the dependence between the coefficient rows Denote the three equations by E_1, E_2, E_3 in their printed order. Twice the left-hand side of E_1, plus the left-hand sides of E_2 and E_3, is

$$\begin{aligned} &2(x - 4y + 7z) + (3y - 5z) + (-2x + 5y - 9z) \\ &= (2 - 2)x + (-8 + 3 + 5)y + (14 - 5 - 9)z = 0. \end{aligned}$$

The coefficient rows therefore have the dependence $2E_1 + E_2 + E_3$ on the left. The factor two belongs to the first row, not to the second or third.

Step 2: Impose the same relation on the constants Applying this row combination to the right-hand sides gives the necessary equation

$$0 = 2g + h + k.$$

If this is violated, adding the equations produces a false statement of the form zero equals a nonzero number. No triple (x, y, z) can then solve the system. Thus consistency forces precisely the relation shown in option (C).

Step 3: Check that the condition is sufficient Let $z = t$ be arbitrary. From the second equation,

$$y = \frac{h + 5t}{3}, \qquad x = g + 4y - 7t.$$

These choices satisfy the first two equations for every g, h. Substitution in the third left-hand side gives

$$-2x + 5y - 9t = -2g - 3y + 5t = -2g - h.$$

It equals k exactly when $2g + h + k = 0$. The other options place the coefficients differently or omit the required factor two and are not the consistency relation.

Step 4: Conclusion The system is consistent when $2g + h + k = 0$. Hence, the correct option is (C).

Quick Tip to Solve: Look for a simple linear combination of coefficient rows before expanding a determinant; apply exactly the same weights to the constants.

25. The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is : [4 marks]

(A) x − 2y + z = 0

(B) 3x + 2y − 3z = 0

(C) x + 2y − 2z = 0

(D) 5x + 2y − 4z = 0

Correct Answer: (A) x − 2y + z = 0

Solution:

Concept:

- A plane containing a line through the origin also passes through the origin. If the line has direction $\boldsymbol{d}$, the plane normal $\boldsymbol{n}$ satisfies $\boldsymbol{n} \cdot \boldsymbol{d} = 0$.
- The normal to a plane containing two intersecting lines is their direction-vector cross product. Two planes are perpendicular when their normals have zero dot product.

Step 1: Find the normal of the given plane The two lines defining the given plane have direction vectors

$$\boldsymbol{u} = (3, 4, 2), \qquad \boldsymbol{v} = (4, 2, 3).$$

They meet at the origin and are not parallel, so they determine a plane. Its normal is

$$\boldsymbol{m} = \boldsymbol{u} \times \boldsymbol{v} = (12 - 4,\ 8 - 9,\ 6 - 16) = (8, -1, -10).$$

Its equation can therefore be written $8x - y - 10z = 0$. A normal is defined only up to a nonzero scalar, which does not affect the perpendicularity test.

Step 2: Impose both conditions on the required normal The line that must lie in the required plane has direction $\boldsymbol{d} = (2, 3, 4)$. The required normal must be perpendicular both to $\boldsymbol{d}$, for line containment, and to $\boldsymbol{m}$, for perpendicular planes. Hence it is parallel to

$$\boldsymbol{d} \times \boldsymbol{m} = (-26, 52, -26) = -26(1, -2, 1).$$

Choosing the simpler normal $(1, -2, 1)$, and using the fact that the plane passes through the origin, gives $x - 2y + z = 0$.

Step 3: Verify the proposed plane Along the required line, $(x, y, z) = (2t, 3t, 4t)$, and $x - 2y + z = (2 - 6 + 4)t = 0$. Also

$$(1, -2, 1) \cdot (8, -1, -10) = 8 + 2 - 10 = 0.$$

The other listed normals also pass the line-containment test, but their dot products with $\boldsymbol{m}$ are $52, 26, 78$, respectively, and so those planes are not perpendicular to the specified plane.

Step 4: Conclusion The required plane is $x - 2y + z = 0$. Hence, the correct option is (A).

Quick Tip to Solve: For a plane containing a line and perpendicular to another plane, make its normal perpendicular to both the line direction and the other plane normal.

26. If the circles

$x^2 + y^2 - 16x - 20y + 164 = r^2$

and $(x - 4)^2 + (y - 7)^2 = 36$

intersect at two distinct points, then : [4 marks]

(A) $r > 11$

(B) $0 < r < 1$

(C) $r = 11$

(D) $1 < r < 11$

Correct Answer: (D) $1 < r < 11$

Solution:

Concept:

- The equation $(x-h)^2+(y-k)^2 = R^2$ represents a circle with centre (h, k) and radius R. Completing squares identifies these quantities.
- Two circles of radii R_1, R_2, with centres separated by d, intersect at two distinct points exactly when $|R_1 - R_2| < d < R_1 + R_2$. Equalities give tangency.

Step 1: Identify the centres and radii Complete squares in the first equation:

$$x^2 - 16x + y^2 - 20y + 164 = r^2.$$

Completing the squares gives

$$(x - 8)^2 + (y - 10)^2 = r^2.$$

Thus its centre is $C_1 = (8, 10)$ and its radius is r, taking r as the positive radius as in the options. The second circle is already in standard form: its centre is $C_2 = (4, 7)$ and its radius is 6. The constant $164 = 64 + 100$ cancels exactly in completing the squares.

Step 2: Apply the strict intersection inequalities The distance between the centres is

$$d = \sqrt{(8 - 4)^2 + (10 - 7)^2} = \sqrt{16 + 9} = 5.$$

Two distinct intersections therefore require

$$|r - 6| < 5 < r + 6.$$

The first inequality gives $-5 < r - 6 < 5$, or $1 < r < 11$. For every r in this interval the second inequality is automatically satisfied, since $r + 6 > 7 > 5$.

Step 3: Exclude tangency and non-intersection At $r = 11$, the radius difference is five, so the circles touch internally at one point, rather than meeting twice. At $r = 1$, the same internal tangency occurs. For $0 < r < 1$, or for $r > 11$, the radius difference exceeds the centre distance, placing one circle entirely inside the other without intersection. Thus the two endpoints must both be excluded, and only the open interval in option (D) meets the demand.

Step 4: Conclusion The circles intersect at two distinct points for $1 < r < 11$. Hence, the correct option is (D).

Quick Tip to Solve: Use both the sum and the absolute difference of the radii; strict inequalities distinguish two intersections from tangency.

27. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is : [4 marks]

(A) 14

(B) 15

(C) 10

(D) 12

Correct Answer: (B) 15

Solution:

Concept:

- The identity $(1-t^6)/(1-t) = 1+t+t^2+t^3+t^4+t^5$ converts the expression into a polynomial before cubing.
- In the binomial expansion, $(1-t)^{-3} = \sum_{n\geq 0}\binom{n+2}{2}t^n$. Terms starting at degree six cannot affect a coefficient of degree four.

Step 1: Separate the low-degree contribution Write the given expression as

$$\left(\frac{1-t^6}{1-t}\right)^3 = (1-t^6)^3(1-t)^{-3}.$$

The numerator expands to $1-3t^6+3t^{12}-t^{18}$. Every nonconstant term has degree at least six, while the power series for the second factor contains only nonnegative powers. Therefore none of those numerator terms can contribute to t^4. We only need the coefficient of t^4 in $(1-t)^{-3}$.

Step 2: Extract the coefficient The negative binomial theorem gives

$$(1-t)^{-3} = \sum_{n=0}^{\infty}\frac{(n+1)(n+2)}{2}t^n.$$

Putting $n=4$, the desired coefficient is

$$\frac{(4+1)(4+2)}{2} = \frac{5\cdot 6}{2} = 15.$$

Using a power series here is simply a coefficient-extraction device; the original expression equals a polynomial, and only its terms through degree four are being used.

Step 3: Confirm by counting polynomial products In $(1+t+t^2+t^3+t^4+t^5)^3$, a contribution to t^4 chooses exponents i,j,k with $i+j+k=4$. Their upper bound of five never restricts a sum of four. For $i=0,1,2,3,4$, the numbers of choices for (j,k) are respectively $5,4,3,2,1$, totaling 15. This directly confirms the binomial result and excludes the other proposed coefficients $14, 10, 12$.

Step 4: Conclusion The coefficient of t^4 is 15. Hence, the correct option is (B).

Quick Tip to Solve: When extracting a low-degree coefficient, discard numerator terms whose degrees already exceed the target degree.

28. If $0 \leq x < \frac{\pi}{2}$, then the number of values of x for which sin x − sin 2x + sin 3x = 0, is : [4 marks]

(A) 3

(B) 1

(C) 4

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- The sum-to-product identity $\sin A + \sin B = 2\sin((A+B)/2)\cos((A-B)/2)$ groups the first and third terms effectively.
- A product vanishes when at least one factor vanishes. Solve each factor separately within the stated interval, retaining allowed endpoints and excluding forbidden ones.

Step 1: Factor the trigonometric expression Group $\sin x$ and $\sin 3x$. The sum-to-product formula gives

$$\sin x + \sin 3x = 2\sin 2x \cos x.$$

Hence the printed equation becomes

$$2\sin 2x\cos x - \sin 2x = 0 \quad \Longleftrightarrow \quad \sin 2x(2\cos x - 1) = 0.$$

This is an exact factorisation, so it does not add or remove any solutions. In particular, dividing immediately by $\sin 2x$ would lose any solutions where that factor is zero.

Step 2: Solve the two factors in the allowed interval Since $0 \leq x < \pi/2$, the doubled angle satisfies $0 \leq 2x < \pi$. In this interval, $\sin 2x = 0$ only at $2x = 0$, giving $x = 0$; the other endpoint $2x = \pi$ is excluded. The second factor gives

$$2\cos x - 1 = 0 \quad \Longrightarrow \quad \cos x = \frac{1}{2}.$$

Cosine decreases from one to zero on this interval, so this equation has exactly one solution, $x = \pi/3$.

Step 3: Count distinct roots and check them The two roots 0 and $\pi/3$ are distinct and both lie in the required interval. At zero all three sine terms vanish. At $\pi/3$, $\sin x = \sin 2x = \sqrt{3}/2$ and $\sin 3x = 0$, again making the expression zero. There are no other roots because both factors have been completely solved on the interval. Counting one misses the allowed endpoint zero, while three or four would introduce angles outside the interval or duplicate a root.

Step 4: Conclusion There are exactly two values, $x = 0$ and $x = \pi/3$. Hence, the correct option is (D).

Quick Tip to Solve: Factor first and solve every factor; check whether each endpoint belongs to the interval before counting roots.

29. The logical statement

[$\sim$ ($\sim$ p $\vee$ q) $\vee$ (p $\wedge$ r)] $\wedge$ ($\sim$ q $\wedge$ r) is equivalent to : [4 marks]

(A) ($\sim$ p $\wedge$ $\sim$ q) $\wedge$ r

(B) $\sim$ p $\vee$ r

(C) (p $\wedge$ r) $\wedge$ $\sim$ q

(D) (p $\wedge$ $\sim$ q) $\vee$ r

Correct Answer: (C) (p $\wedge$ r) $\wedge$ $\sim$ q

Solution:

Concept:

- De Morgan's law interchanges conjunction and disjunction under negation: $\neg(A \vee B) = \neg A \wedge \neg B$. A double negation restores the original statement.
- Distributivity gives $(A \wedge B) \vee (A \wedge C) = A \wedge (B \vee C)$. The absorption identity $A \wedge (A \vee B) = A$ removes a redundant condition.

Step 1: Remove the inner negation Let the given statement be S. Negating $\neg p \vee q$ changes both truth values and changes the connective to conjunction:

$$\neg(\neg p \vee q) = p \wedge \neg q.$$

Therefore

$$S = [(p \wedge \neg q) \vee (p \wedge r)] \wedge (\neg q \wedge r).$$

The first term involves p, rather than $\neg p$, because the two negations on p cancel.

Step 2: Factor and absorb the repeated conditions Factor p out of the bracket and rearrange the conjunctions:

$$S = p \wedge (\neg q \vee r) \wedge \neg q \wedge r.$$

The last two factors already require both $\neg q$ and r to hold. Consequently their disjunction is automatically true, so it imposes no additional restriction. Hence

$$S = p \wedge \neg q \wedge r = (p \wedge r) \wedge \neg q.$$

Option (A) wrongly requires p to be false. Options (B) and (D) are too weak: for example, they can be true merely because r is true, even when p is false. The original conjunction cannot be true in that case.

Step 3: Conclusion The equivalent statement is $(p \wedge r) \wedge \neg q$. Hence, the correct option is (C).

Quick Tip to Solve: Use the conditions outside a bracket first: if they already force a disjunction inside it to be true, that bracket factor can be absorbed.

30. The sum of the following series

$$1+6+\frac{9\left(1^2+2^2+3^2\right)}{7}+\frac{12\left(1^2+2^2+3^2+4^2\right)}{9}$$

$+\frac{15\left(1^2+2^2+\ldots+5^2\right)}{11}+\ldots..$ **up to 15 terms, is :** [4 marks]

(A) 7520

(B) 7510

(C) 7830

(D) 7820

Correct Answer: (D) 7820

Solution:

Concept:

- The sum of the first n squares is $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$. This formula simplifies the partial sum that occurs inside each term.
- For N positive integers, $\sum n^2 = N(N+1)(2N+1)/6$ and $\sum n^3 = [N(N+1)/2]^2$. A polynomial general term can be summed using these identities.

Step 1: Identify and simplify the general term The coefficient of the sum of squares is $3n$, and the denominator is $2n+1$. Thus the term indexed by n is

$$T_n = \frac{3n}{2n+1}\left(1^2+2^2+\cdots+n^2\right).$$

This also reproduces the first two printed terms: $T_1 = 1$ and $T_2 = 6$. Substituting the square-sum formula gives

$$T_n = \frac{3n}{2n+1}\frac{n(n+1)(2n+1)}{6} = \frac{n^2(n+1)}{2}.$$

The denominator cancels exactly; no approximation to the series is required.

Step 2: Sum all fifteen terms Write $n^2(n+1) = n^3 + n^2$, so the desired sum is

$$S_{15} = \frac{1}{2}\left(\sum_{n=1}^{15} n^3 + \sum_{n=1}^{15} n^2\right).$$

The two standard sums are

$$\sum_{n=1}^{15} n^3 = \left(\frac{15\cdot 16}{2}\right)^2 = 14400,$$

$$\sum_{n=1}^{15} n^2 = \frac{15\cdot 16\cdot 31}{6} = 1240.$$

Consequently $S_{15} = (14400+1240)/2 = 7820$. The alternatives 7520, 7510 and 7830 fail this exact evaluation. In particular the first printed term corresponds to $n = 1$, so there are precisely fifteen indices, not fourteen or sixteen.

Step 3: Conclusion The sum of the first fifteen terms is 7820. Hence, the correct option is (D).

Quick Tip to Solve: Before summing a series of partial square sums, insert the square-sum formula into its general term; the factor $2n+1$ often cancels.

Physics (120 marks)

31. Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from the second mirror (M_2) parallel to the first mirror (M_1). The angle between the two mirrors will be : [4 marks]

(A) 45°

(B) 60°

(C) 75°

(D) 90°

Correct Answer: (B) 60°

Solution:

Concept:

- At each plane mirror, the incident and reflected rays make equal angles with the normal. Equivalently, the incident and reflected lines make equal acute angles with the mirror surface.
- When a ray travels from one inclined mirror to the other, the two reflection points and the intersection of the mirrors form a triangle. Its interior angles sum to $180°$.

Step 1: Relate the first reflection to the mirror angle Let the angle between the mirror surfaces be α, and let O be their intersection. Write P and Q for the reflection points on M_1 and M_2, respectively. The incident ray is parallel to M_2, so the acute angle it makes with M_1 is α. The law of reflection makes the angle between the segment PQ and the surface of M_1 also equal to α. Thus the interior angle at P of the ray triangle OPQ is α.

Step 2: Use the final direction and close the triangle The final reflected ray is parallel to M_1. It therefore makes an angle α with M_2. Reflection at Q requires the incoming segment PQ to make the same angle with M_2, giving the interior angle at Q as α. The interior angle at O is the angle between the mirrors, also α. Hence

$$\alpha + \alpha + \alpha = 180°, \qquad \alpha = 60°.$$

The proposed angles $45°$, $75°$, and $90°$ give triangle-angle sums of $135°$, $225°$, and $270°$, respectively, and cannot meet both stated parallelism conditions. The normals need not be confused with the mirror surfaces: the corresponding incidence angles are $90° - \alpha = 30°$.

Step 3: Conclusion The angle between the two plane mirrors is $60°$. Hence, the correct option is (B).

Quick Tip to Solve: Join the two reflection points to the mirrors' intersection. The two parallel-ray conditions make all three interior angles of this triangle equal.

32. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength λ = 500 nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30^{o} \leq \theta \leq 30^{o}$ is : [4 marks]

(A) 640

(B) 320

(C) 321

(D) 641

Correct Answer: (D) 641

Solution:

Concept:

- For slit separation d, the path difference in a direction θ is $d\sin\theta$. A bright fringe occurs when $d\sin\theta = m\lambda$, where m is any integer.
- Both positive and negative orders must be counted in a symmetric angular interval. If the limiting angle itself satisfies the bright-fringe condition and the inequality is inclusive, that endpoint is included.

Step 1: Find the largest permitted order Convert the given separation and wavelength to metres:

$$d = 0.320 \times 10^{-3}\,\text{m}, \qquad \lambda = 500 \times 10^{-9}\,\text{m}.$$

Within $-30^{\circ} \leq \theta \leq 30^{\circ}$, the sine ranges from $-1/2$ to $1/2$. Thus the interference order obeys

$$-\frac{d}{2\lambda} \leq m \leq \frac{d}{2\lambda}.$$

The upper magnitude is

$$\frac{0.320 \times 10^{-3}}{2(500 \times 10^{-9})} = 320.$$

The exact sine relation is needed here: the angle is too large to justify replacing sine by the angle in radians.

Step 2: Count the integer orders including the centre Every integer from -320 through $+320$ corresponds to one bright direction. There are 320 orders on the negative side, 320 on the positive side, and the central order $m = 0$:

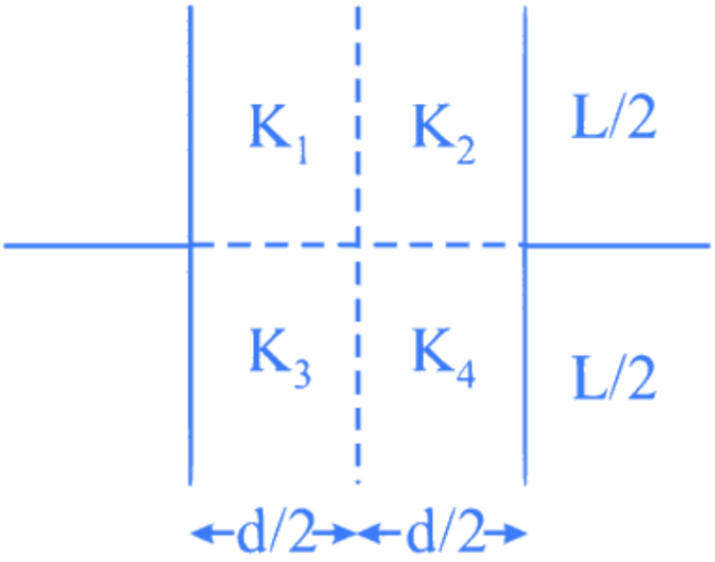
$$N = 320 + 1 + 320 = 641.$$

The extreme orders occur exactly at the stated boundary angles and must be counted because the inequalities include equality. Option (A), 640, omits the central bright fringe. Option (B), 320, counts only the noncentral fringes on one side. Option (C), 321, counts the centre plus one side, but leaves out the opposite side. None of those is the total for the full angular interval.

Step 3: Conclusion The total number of bright fringes is 641. Hence, the correct option is (D).

Quick Tip to Solve: Find the maximum integer fringe order from $d\sin\theta/\lambda$, then use $2m_{\max} + 1$ for an inclusive symmetric interval.

33. A parallel plate capacitor with square plates is filled with four dielecytrics of dielectrics constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be : [4 marks]



(A) $K = \frac{K_1K_2}{K_1+K_2} + \frac{K_3K_4}{K_3+K_4}$

(B) $K = \frac{(K_1+K_2)(K_3+K_4)}{2(K_1+K_2+K_3+K_4)}$

(C) $K = \frac{(K_1+K_2)(K_3+K_4)}{K_1+K_2+K_3+K_4}$

(D) $K = \frac{(K_1+K_4)(K_2+K_3)}{2(K_1+K_2+K_3+K_4)}$

Correct Answer: (A) $K = \frac{K_1K_2}{K_1+K_2} + \frac{K_3K_4}{K_3+K_4}$

Solution:

Concept:

- For plate area A, separation d, and dielectric constant K, the capacitance is $C = \varepsilon_0 KA/d$. Layers encountered successively across the separation combine in series.
- Side-by-side regions spanning the same plate-to-plate potential difference combine in parallel. The effective constant is defined by $C_{\text{eq}} = \varepsilon_0 KA/d$ for the entire original plate area.

Step 1: Combine the upper two dielectric regions Let each square plate have side L, so $A = L^2$. The upper half has area $A/2$. Within that half, K_1 and K_2 each occupy half the plate separation, $d/2$. Their individual capacitances are therefore

$$C_1 = \frac{\varepsilon_0 K_1 (A/2)}{d/2} = \frac{\varepsilon_0 A K_1}{d}, \qquad C_2 = \frac{\varepsilon_0 A K_2}{d}.$$

They lie successively along the electric-field direction, so

$$C_{\text{upper}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_0 A}{d} \frac{K_1 K_2}{K_1 + K_2}.$$

Step 2: Combine the lower half and add the parallel branches The lower half similarly has K_3 and

K_4 in series, giving

$$C_{\text{lower}} = \frac{\varepsilon_0 A}{d}\frac{K_3 K_4}{K_3 + K_4}.$$

Both halves connect the same two plates, so their equivalent capacitances add:

$$C_{\text{eq}} = \frac{\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2} + \frac{K_3 K_4}{K_3 + K_4}\right).$$

Comparison with $\varepsilon_0 K A/d$ identifies the required K. The other options group dielectrics contrary to the depicted field direction. A useful check is to put all four constants equal to k: the result must reduce to k, which immediately rules out the extra factor of one-half in options (B) and (D).

Step 3: Conclusion

$$K = \frac{K_1 K_2}{K_1 + K_2} + \frac{K_3 K_4}{K_3 + K_4}.$$

Hence, the correct option is (A).

Quick Tip to Solve: First trace the field from one plate to the other: dielectrics encountered in succession are series layers; separate strips between the plates are parallel branches.

34. The position co-ordinates of a particle moving in a 3-D coordinate system is given by

x = a cosωt

y = a sinωt and

z = aωt

The speed of the particle is : [4 marks]

(A) $\sqrt{2}\,a\omega$

(B) $a\omega$

(C) $\sqrt{3}\,a\omega$

(D) 2aω

Correct Answer: (A) $\sqrt{2}\,a\omega$

Solution:

Concept:

- The Cartesian components of velocity are the time derivatives of position: $v_x = dx/dt$, $v_y = dy/dt$, and $v_z = dz/dt$. Speed is the magnitude of this velocity vector.
- Perpendicular velocity components combine as $v^2 = v_x^2 + v_y^2 + v_z^2$. The identity $\sin^2 u + \cos^2 u = 1$ makes the transverse speed of circular motion constant.

Step 1: Differentiate each coordinate with respect to time The constants a and ω give the particle's coordinates as

$$x = a\cos(\omega t), \qquad y = a\sin(\omega t), \qquad z = a\omega t.$$

Applying the chain rule to the first two coordinates gives

$$v_x = -a\omega \sin(\omega t), \qquad v_y = a\omega \cos(\omega t).$$

The third coordinate is linear in time, so its derivative is $v_z = a\omega$. Thus the particle has circular motion in its projection on the xy-plane together with uniform motion along z; the three-dimensional path is a helix.

Step 2: Calculate the magnitude of the velocity Square and add the three mutually perpendicular components:

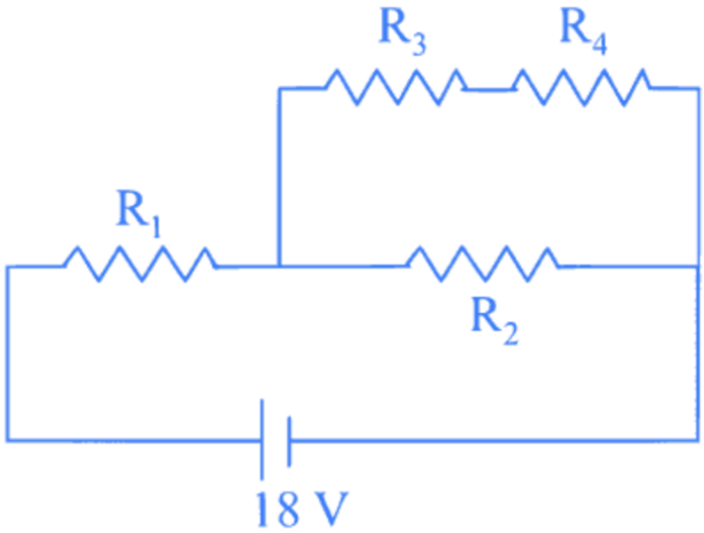
$$\begin{aligned} v^2 &= a^2\omega^2 \sin^2(\omega t) + a^2\omega^2 \cos^2(\omega t) + a^2\omega^2 \\ &= a^2\omega^2(1+1) = 2a^2\omega^2. \end{aligned}$$

For the stated amplitude and angular frequency, $v = \sqrt{2}\, a\omega$, constant in time. Option (B) accounts for only the circular projection or only the axial component. Option (C) treats all three Cartesian components as having constant magnitude $a\omega$, although the first two vary with sine and cosine. Option (D) adds the two perpendicular speeds arithmetically rather than in quadrature. Speed is nonnegative; if signed parameters were allowed, its general expression would be $\sqrt{2}\,|a\omega|$.

Step 3: Conclusion The particle has constant speed $\sqrt{2}\, a\omega$. Hence, the correct option is (A).

Quick Tip to Solve: For a helix, combine the circular transverse speed and the uniform axial speed in quadrature after differentiating the coordinates.

35. In the given circuit the the internal resistance of the 18 V cell is negligible. If R_1 = 400 Ω, R_3 = 100 Ω and R_4 = 500 Ω and the reading of an ideal voltmeter across R_4 is 5V, then the value of R_2 will be : [4 marks]



(A) 300 Ω

(B) 450 Ω

(C) 550 Ω

(D) 230 Ω

Correct Answer: (A) 300 Ω

Solution:

Concept:

- An ideal voltmeter has infinite resistance and draws no current. Ohm's law $V = IR$ therefore gives the current through the resistor across which it is connected without loading the circuit.
- Series resistors carry the same current, parallel branches have equal potential differences, and Kirchhoff's junction rule equates the incoming current to the sum of the outgoing branch currents.

Step 1: Use the voltmeter reading to find the parallel-branch voltage The upper branch contains R_3 and R_4 in series. Since the voltmeter reads $5\,\text{V}$ across $R_4 = 500\,\Omega$, their common current is

$$I_{34} = \frac{5}{500} = 0.010\,\text{A}.$$

The drop across $R_3 = 100\,\Omega$ is consequently

$$V_3 = I_{34}R_3 = 0.010(100) = 1\,\text{V}.$$

Thus the total voltage across the upper branch is $V_{34} = 1 + 5 = 6\,\text{V}$. Resistor R_2 joins the same two junctions, so it also has $6\,\text{V}$ across it.

Step 2: Find the remaining branch current and resistance The ideal source supplies $18\,\text{V}$, leaving a drop of $18 - 6 = 12\,\text{V}$ across $R_1 = 400\,\Omega$. The current entering the junction is

$$I_1 = \frac{12}{400} = 0.030\,\text{A}.$$

Subtract the upper-branch current to obtain the current through R_2:

$$I_2 = I_1 - I_{34} = 0.030 - 0.010 = 0.020\,\text{A}.$$

It follows that

$$R_2 = \frac{6}{0.020} = 300\,\Omega.$$

As a check, $300\,\Omega$ in parallel with $600\,\Omega$ gives $200\,\Omega$; with R_1, the total is $600\,\Omega$, giving the required $0.030\,\text{A}$. The other resistance choices cannot satisfy both this source current and the measured $5\,\text{V}$ drop.

Step 3: Conclusion The resistance is $R_2 = 300\,\Omega$. Hence, the correct option is (A).

Quick Tip to Solve: Start with the measured resistor voltage, propagate through its series branch, and only then use the source voltage and junction rule to find the unknown branch.

36. The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line.

The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is : [4 marks]

(A) 5.755 mm

(B) 5.950 mm

(C) 5.725 mm

(D) 5.740 mm

Correct Answer: (C) 5.725 mm

Solution:

Concept:

- The least count is the axial distance moved in one circular-scale division: $\mathrm{LC} = p/N$, where p is the pitch and N is the number of divisions. The observed thickness is the main-scale reading plus the circular-scale reading multiplied by this least count.
- A positive zero error is subtracted from an observed reading. When the circular-scale zero lies below the reference line with the jaws closed, the instrument already reads a small positive thickness.

Step 1: Calculate the least count The screw advances $0.5\,\mathrm{mm}$ in one complete revolution, and this revolution is divided into 100 equal circular-scale divisions. Hence each division represents

$$\mathrm{LC} = \frac{0.5}{100} = 0.005\,\mathrm{mm}.$$

This conversion must be made before adding the two scale readings; the number 48 is a count of divisions, not a length in millimetres.

Step 2: Apply the zero correction The zero lies three divisions below the reference line when the jaws touch. Thus the positive zero error is

$$e = 3(0.005) = 0.015\,\mathrm{mm}.$$

The uncorrected reading for the sheet is

$$t_{\mathrm{obs}} = 5.5 + 48(0.005) = 5.740\,\mathrm{mm}.$$

Subtracting the error gives

$$t = t_{\mathrm{obs}} - e = 5.740 - 0.015 = 5.725\,\mathrm{mm}.$$

Option (D) is just the reading before zero correction; (A) adds the positive error instead of removing it. Option (B) is inconsistent with the least count and the stated circular-scale count.

Step 3: Conclusion The corrected thickness is $\boxed{5.725\,\mathrm{mm}}$. Hence, the correct option is (C).

Quick Tip to Solve: First convert divisions into length, then use true reading = observed reading minus zero error. A zero below the reference line gives a positive error in the usual screw-gauge scale convention.

37. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the radio of the magnetic field at the central of the loop (B_L) to that at the center of the coil (B_C), i.e. $\frac{B_L}{B_C}$ will be : [4 marks]

(A) N

(B) $\frac{1}{N}$

(C) N^2

(D) $\frac{1}{N^2}$

Correct Answer: (D) $\frac{1}{N^2}$

Solution:

Concept:

- For a thin circular coil of radius r, N turns and current I, the central field is $B = \mu_0 NI/(2r)$; the fields of the turns add in the same direction.
- Equal wire lengths do not imply equal loop radii. A single loop satisfies $L = 2\pi R$, whereas an N-turn coil made from the same length satisfies $L = N(2\pi r)$.

Step 1: Relate the radii using the wire length Let R be the radius of the one-turn loop and r the radius of each turn of the coil. Neglecting the short connecting leads, all of the wire forms the circular turns. The two length relations give

$$R = \frac{L}{2\pi}, \qquad r = \frac{L}{2\pi N} = \frac{R}{N}.$$

Therefore the coil has a smaller radius by the factor N. This change in radius contributes an additional enhancement of its field, besides the direct contribution from having more turns.

Step 2: Form the magnetic-field ratio With the same current through both wires, their fields are

$$B_L = \frac{\mu_0 I}{2R}, \qquad B_C = \frac{\mu_0 NI}{2r}.$$

Dividing cancels the permeability and the common current:

$$\frac{B_L}{B_C} = \frac{r}{NR} = \frac{1}{N^2}.$$

The reciprocal ratio B_C/B_L would be N^2, so option (C) reverses what was asked. Option (B) misses the decrease in the radius, and option (A) has both the wrong dependence and the wrong sense for $N > 1$. For a check, when $N = 1$, both arrangements are the same and this result becomes unity.

Step 3: Conclusion $\boxed{B_L/B_C = 1/N^2}$. Hence, the correct option is (D).

Quick Tip to Solve: For a fixed total wire length and current, the central field of an N-turn circular coil scales as N squared: one factor comes from the turns and the other from the smaller radius.

38. The magnetic field associated with a light wave is given, at the origin, by B = B_0 [sin(3.14 $\times$ 10^7)ct + sin(6.28 $\times$ 10^7)ct]. If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photo electrons ?

(Take c = 3 $\times$ 10^8 ms^{-1}, h = 6.6 $\times$ 10^{-34}J-s) [4 marks]

(A) 6.82 eV

(B) 12.5 eV

(C) 8.52 eV

(D) 7.72 eV

Correct Answer: (D) 7.72 eV

Solution:

Concept:

- For a sinusoidal wave term $\sin(kct)$, the angular frequency is $\omega = kc$ and the ordinary frequency is $\nu = \omega/(2\pi)$. Photon energy is $h\nu$, rather than $h\omega$.
- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ϕ is the work function. In a mixture of frequencies, the largest photon frequency produces the largest kinetic energy; the two photon energies are not added.

Step 1: Select the higher-frequency component The two coefficients multiplying ct are 3.14×10^7 and $6.28 \times 10^7\,\mathrm{m}^{-1}$. The latter is larger and therefore sets the highest photoelectron energy. With $c = 3 \times 10^8\,\mathrm{m\,s}^{-1}$,

$$\nu_{\max} = \frac{(6.28 \times 10^7)(3 \times 10^8)}{2\pi} \simeq 3.0 \times 10^{15}\,\mathrm{Hz}.$$

The lower-frequency component has about half this frequency, so it cannot determine the maximum kinetic energy.

Step 2: Subtract the work function and compare options Using the stated rounded Planck constant and $1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$, the larger photon energy is

$$E_{\max} = 6.6 \times 10^{-34}(3.0 \times 10^{15}) = 1.98 \times 10^{-18}\,\mathrm{J} = 12.375\,\mathrm{eV}.$$

Thus

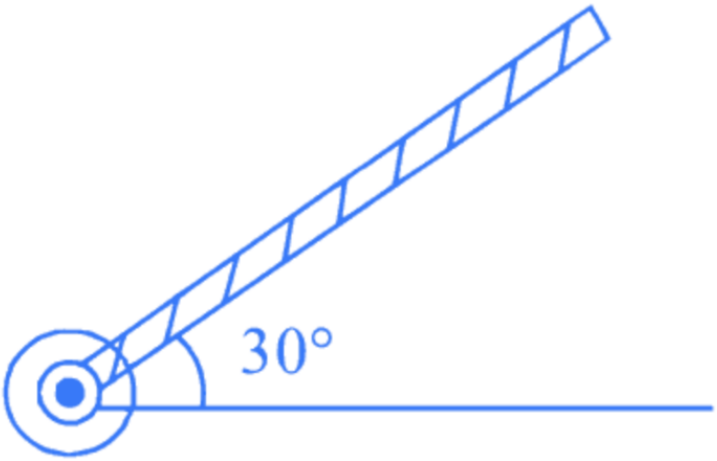
$$K_{\max} = 12.375 - 4.7 = 7.675\,\mathrm{eV} \simeq 7.7\,\mathrm{eV}.$$

The listed value closest to this is $7.72\,\mathrm{eV}$; the small difference reflects the rounding of the supplied constants. The lower-frequency component gives only about $1.49\,\mathrm{eV}$. Option (B) is approximately the high-frequency photon energy before subtracting the work function, while (A) and (C) do not agree with either component's maximum kinetic energy.

Step 3: Conclusion The maximum kinetic energy is about $7.7\,\mathrm{eV}$, matching the listed $7.72\,\mathrm{eV}$. Hence, the correct option is (D).

Quick Tip to Solve: In a superposition of two colours, use the higher frequency for maximum photoelectron energy. Convert angular frequency to ordinary frequency before multiplying by Planck's constant.

39. A rod of length 50 cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be (g = 10 ms^{-2}) [4 marks]



(A) $\sqrt{\frac{30}{7}}$

(B) $\sqrt{30}$

(C) $\frac{\sqrt{20}}{3}$

(D) $\frac{\sqrt{30}}{2}$

Correct Answer: (B) $\sqrt{30}$

Solution:

Concept:

- A uniform rod of length ℓ pivoted at one end has moment of inertia $I = m\ell^2/3$. Its centre of mass is at distance $\ell/2$ from the pivot.
- For an ideal fixed pivot, the pivot force does no work. The loss of gravitational potential energy therefore equals the rotational kinetic energy $I\omega^2/2$.

Step 1: Find the fall of the centre of mass Take the horizontal position of the rod as the reference level for gravitational potential energy. Initially the rod makes 30° above this horizontal. Its centre of mass is halfway along its length, not at its free end. Therefore it falls through

$$\Delta h = \frac{\ell}{2}\sin 30^\circ = \frac{0.50}{2} \times \frac{1}{2} = 0.125\,\text{m}.$$

The released rod begins from rest, so the initial kinetic energy is zero. The decrease $mg\Delta h$ is converted entirely into rotation about the end pivot.

Step 2: Equate potential-energy loss to kinetic energy Conservation of mechanical energy gives

$$mg\frac{\ell}{2}\sin 30^\circ = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2.$$

Cancel the mass and simplify before substituting:

$$\omega^2 = \frac{3g\sin 30^\circ}{\ell} = \frac{3(10)(1/2)}{0.50} = 30\,\mathrm{s}^{-2}.$$

Thus $\omega = \sqrt{30}\,\mathrm{rad\,s^{-1}}$. The other proposed values do not satisfy the energy balance with the rod's end-axis inertia. In particular, using the free end's fall or treating the entire mass as a point at that end would use the wrong mass distribution. The gravitational torque varies during the fall, so a constant-angular-acceleration shortcut is inappropriate.

Step 3: Conclusion The angular speed is $\boxed{\sqrt{30}\,\mathrm{rad\,s^{-1}}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a rod falling about an end, use the centre of mass for the potential-energy change and the end-axis moment of inertia for rotational kinetic energy.

40. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of radio m/M is close to : [4 marks]

(A) 0.77

(B) 0.57

(C) 0.37

(D) 0.17

Correct Answer: (C) 0.37

Solution:

Concept:

- The frequency of small torsional oscillations is $f = (1/2\pi)\sqrt{\kappa/I}$, where κ is the wire's torsional constant and I is the moment of inertia about the suspension axis.
- For a uniform rod of length $2L$ about its middle, $I_0 = M(2L)^2/12 = ML^2/3$. A point mass at perpendicular distance a adds ma^2 to the moment of inertia.

Step 1: Calculate the added rotational inertia The suspension wire and hence its torsional constant are unchanged when the two small masses are attached. Each mass lies at $L/2$ from the central axis. The two additions are equal and positive, giving

$$\Delta I = 2m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{2}.$$

The final moment of inertia is therefore

$$I_1 = \frac{ML^2}{3} + \frac{mL^2}{2}.$$

Although the masses are on opposite sides, their contributions to inertia add: they do not cancel as vector torques might.

Step 2: Use the fractional decrease in frequency A decrease of 20 percent means $f_1 = 0.80 f_0$. Since the frequency is inversely proportional to the square root of inertia,

$$\left(\frac{f_1}{f_0}\right)^2 = \frac{I_0}{I_1} = 0.64, \qquad \frac{I_1}{I_0} = \frac{25}{16}.$$

Substituting the inertia expressions and cancelling L^2 gives

$$1 + \frac{3m}{2M} = \frac{25}{16}, \qquad \frac{m}{M} = \frac{2}{3}\left(\frac{9}{16}\right) = \frac{3}{8} = 0.375.$$

The closest listed value is 0.37. Values 0.77, 0.57 and 0.17 lead to different frequency reductions. Using 20 percent directly as the inertia increase would overlook both the inverse dependence and its square root.

Step 3: Conclusion $\boxed{m/M = 0.375 \simeq 0.37}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert a percentage frequency change into an inertia ratio by squaring and inverting: I(final)/I(initial) = [f(initial)/f(final)] squared.

---

41. Two point charges $q_1(\sqrt{10}\mu C)$ and $q_2(-25\ \mu C)$ are placed on the x-axis at x = 1 m and x = 4 m respectively. The electric field (in V/m) at a point y = 3 m on y-axis is,
[take $\frac{1}{4\pi\epsilon_0}$ = 9 × 10^9 Nm^2C^{-2}] [4 marks]

(A) $\left(63\hat{i} - 27\hat{j}\right) \times 10^2$

(B) $\left(-63\hat{i} + 27\hat{j}\right) \times 10^2$

(C) $\left(81\hat{i} - 81\hat{j}\right) \times 10^2$

(D) $\left(-81\hat{i} + 81\hat{j}\right) \times 10^2$

Correct Answer: (A) $\left(63\hat{i} - 27\hat{j}\right) \times 10^2$

Solution:

Concept:

- The electric field of a point charge at position $\mathbf{r}_q$ is $\mathbf{E} = kq(\mathbf{r} - \mathbf{r}_q)/|\mathbf{r} - \mathbf{r}_q|^3$, where $k = 1/(4\pi\varepsilon_0)$. The charge sign determines the field direction.
- Electric fields add as vectors. Compute each displacement from its source charge to the field point, and add Cartesian components rather than the field magnitudes.

Step 1: Find the field of the positive charge The observation point is $P = (0, 3)\,\mathrm{m}$. From $q_1 = \sqrt{10}\,\mu\mathrm{C}$ at $(1, 0)$, the displacement is $-\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ metres and its magnitude is $\sqrt{10}\,\mathrm{m}$. Thus

$$\mathbf{E}_1 = \frac{9 \times 10^9 \sqrt{10} \times 10^{-6}}{(\sqrt{10})^3}(-\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) = (-900\hat{\mathbf{i}} + 2700\hat{\mathbf{j}})\,\mathrm{V\,m^{-1}}.$$

Its negative x-component and positive y-component agree with the direction away from this positive

charge.

Step 2: Add the field of the negative charge From $q_2 = -25\,\mu\text{C}$ at $(4, 0)$, the displacement to P is $-4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$, of magnitude $5\,\text{m}$. Keeping the charge sign gives

$$\mathbf{E}_2 = \frac{9 \times 10^9(-25 \times 10^{-6})}{5^3}(-4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) = (7200\hat{\mathbf{i}} - 5400\hat{\mathbf{j}})\,\text{V m}^{-1}.$$

Therefore

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (6300\hat{\mathbf{i}} - 2700\hat{\mathbf{j}})\,\text{V m}^{-1}.$$

The net field points right and downward, excluding (B) and (D). Option (C) has the same signs but incorrect component magnitudes; it does not use the distinct source-to-point distances correctly.

Step 3: Conclusion $\boxed{\mathbf{E} = (63\hat{\mathbf{i}} - 27\hat{\mathbf{j}}) \times 10^2\,\text{V m}^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use kq times the displacement vector divided by distance cubed. This automatically includes both the inverse-square magnitude and the direction, provided the charge sign is retained.

---

42. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about : [Take R = 8.3 J/K mole] [4 marks]

(A) 0.9 kJ

(B) 6 kJ

(C) 10 kJ

(D) 14 kJ

Correct Answer: (C) 10 kJ

Solution:

Concept:

- For a gas of fixed molecular mass, $v_{\text{rms}} = \sqrt{3RT/\mathcal{M}}$, where T is absolute temperature. Doubling the rms speed therefore requires four times the initial kelvin temperature.
- In a rigid vessel there is no expansion work. For an ideal diatomic gas with translational and rotational modes active, $Q = \Delta U = nC_V\Delta T$, with $C_V = 5R/2$.

Step 1: Determine the required rise in absolute temperature The initial temperature is $T_1 = 27+273 = 300\,\text{K}$. The ratio of speeds is two, so

$$\frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^2 = 4, \qquad T_2 = 1200\,\text{K}.$$

The temperature rise to use in the heat equation is $\Delta T = 1200 - 300 = 900\,\text{K}$. Doubling the Celsius temperature or merely doubling the absolute temperature would not meet the specified change in rms speed.

Step 2: Calculate the increase in internal energy Nitrogen is diatomic, with molar mass $28\,\text{g mol}^{-1}$.

Hence the amount of gas is

$$n = \frac{15}{28}\,\text{mol}.$$

Using the standard ideal-diatomic approximation and the given gas constant,

$$Q = n\frac{5R}{2}\Delta T = \frac{15}{28} \times \frac{5}{2} \times 8.3 \times 900 \simeq 1.00 \times 10^4\,\text{J} = 10.0\,\text{kJ}.$$

The vessel fixes the volume, so $C_P = 7R/2$ must not be used; that would incorrectly include energy for expansion work and give approximately 14 kJ, option (D). The smaller values (A) and (B) do not agree with the required temperature rise and the diatomic constant-volume heat capacity. As usual in this kinetic-theory model, vibrational excitation is neglected.

Step 3: Conclusion The required heat is approximately $\boxed{10\,\text{kJ}}$. Hence, the correct option is (C).

Quick Tip to Solve: First square the rms-speed ratio to obtain the kelvin-temperature ratio. In a rigid vessel, use constant-volume molar heat capacity and the temperature difference, not the final temperature.

---

43. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \frac{A}{r^2}e^{-\frac{2r}{s}}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is :

[4 marks]

(A) a log $\left(1 - \frac{Q}{2\pi aA}\right)$

(B) $\frac{a}{2}$ log $\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

(C) a log $\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

(D) $\frac{a}{2}$ log $\left(1 - \frac{Q}{2\pi aA}\right)$

Correct Answer: The density uses s but all options use a; option (B) is obtained only if these denote the same decay length.

Solution:

Concept:

- A thin spherical shell of radius r and thickness dr has volume $dV = 4\pi r^2 dr$. The enclosed charge is obtained from $Q = \int_0^R \rho(r)\,dV$.
- The exponential decay length in the displayed density is denoted by s, whereas the stated constants and every option use a. Without identifying these two symbols, the literal expression and the choices are inconsistent.

Step 1: Integrate the displayed charge density The inverse-square factor cancels the spherical volume factor. Although the density diverges at the centre, its integrated charge is finite because the charge of a thin shell remains finite. Retaining the symbol actually displayed in the exponent gives

$$dQ = 4\pi A e^{-2r/s}\,dr.$$

For a positive decay length s, integrate from the centre to the surface:

$$Q = 4\pi A \int_0^R e^{-2r/s}\, dr = 2\pi A s \left(1 - e^{-2R/s}\right).$$

Step 2: Invert the exponential and compare the options Rearrangement gives

$$e^{-2R/s} = 1 - \frac{Q}{2\pi A s}, \qquad R = \frac{s}{2} \ln\left(\frac{1}{1 - Q/(2\pi A s)}\right).$$

For $A, s > 0$, this requires $0 \le Q < 2\pi A s$, and the radius is nonnegative. If the decay length is written as a, this becomes option (B). Options (A) and (D) have the wrong sign for positive charge and radius; option (C) is twice the required value. However, identifying s with a is necessary for that comparison: the printed statement supplies no separate relation between them.

Step 3: Conclusion The literal density gives $R = \frac{s}{2} \ln\left[\frac{1}{1 - Q/(2\pi A s)}\right]$. The key selects (B), which follows when the decay-length symbol s is identified with a. With the two symbols treated as distinct, no printed option is uniquely justified.

Quick Tip to Solve: For radial densities, multiply by $4\pi r^2 dr$ before integrating; then check that the logarithm produces a positive radius and a dimensionless argument.

44. A power transmission line feeds input power at 2300 V to a srep down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be : [4 marks]

(A) 50 A

(B) 45 A

(C) 35 A

(D) 25 A

Correct Answer: (B) 45 A

Solution:

Concept:

- Transformer efficiency is the ratio of useful secondary power to input primary power: $\eta = P_s/P_p$. The given operating powers are $P_p = V_p I_p$ and $P_s = V_s I_s$.
- A step-down transformer reduces voltage and increases the available current. Losses reduce the output current below its ideal value when the input voltage and current are fixed.

Step 1: Calculate the input and useful output power The primary voltage is $2300\,\mathrm{V}$ and the primary current is $5\,\mathrm{A}$. Thus

$$P_p = 2300 \times 5 = 11500\,\mathrm{W}.$$

The efficiency is $90\% = 0.90$, so only this fraction of the input power is delivered at the secondary:

$$P_s = 0.90 \times 11500 = 10350\,\text{W}.$$

The remaining $1150\,\text{W}$ accounts for transformer losses; efficiency must multiply the input power, rather than divide it.

Step 2: Use the secondary voltage to find current The output voltage is $230\,\text{V}$. Consequently,

$$I_s = \frac{P_s}{V_s} = \frac{10350}{230} = 45\,\text{A}.$$

For comparison, a lossless transformer would give $I_s = (2300/230) \times 5 = 50\,\text{A}$, so option (A) neglects the stated losses. Options (C) and (D) would imply efficiencies of 70% and 50%, respectively, inconsistent with the question. The primary turn count is not required because both operating voltages are supplied; using it does not replace the efficiency condition.

Step 3: Conclusion The output current is $45\,\text{A}$. Hence, the correct option is (B).

Quick Tip to Solve: When both transformer voltages are known, use $I_s = \eta(V_p/V_s)I_p$; the actual current is the efficiency times its lossless value.

45. Expression for time in terms of G(universal gravitional constant), h (Planck constant) and c (speed of light) is proportional to : [4 marks]

(A) $\sqrt{\frac{hc^5}{G}}$

(B) $\sqrt{\frac{c^3}{Gh}}$

(C) $\sqrt{\frac{Gh}{c^5}}$

(D) $\sqrt{\frac{Gh}{c^3}}$

Correct Answer: (C) $\sqrt{\frac{Gh}{c^5}}$

Solution:

Concept:

- The dimensional formulae are $[G] = M^{-1}L^3T^{-2}$, $[h] = ML^2T^{-1}$ and $[c] = LT^{-1}$. A time expression must have overall dimensions $M^0L^0T^1$.
- In a product $G^\alpha h^\beta c^\gamma$, add the powers of each fundamental dimension separately. Dimensional analysis determines the exponents, but cannot determine a dimensionless numerical factor.

Step 1: Write equations for the three exponents Let the required time be proportional to $G^\alpha h^\beta c^\gamma$. Substituting dimensions gives

$$M^{-\alpha+\beta}L^{3\alpha+2\beta+\gamma}T^{-2\alpha-\beta-\gamma}.$$

Equating the mass, length and time powers with those of time yields

$$-\alpha + \beta = 0, \qquad 3\alpha + 2\beta + \gamma = 0,$$

$$-2\alpha - \beta - \gamma = 1.$$

The first equation gives $\beta = \alpha$, and the second therefore gives $\gamma = -5\alpha$.

Step 2: Solve and check the resulting dimensions The time equation becomes $-2\alpha - \alpha + 5\alpha = 1$, hence $\alpha = \beta = 1/2$ and $\gamma = -5/2$. Therefore

$$t \propto G^{1/2} h^{1/2} c^{-5/2} = \sqrt{\frac{Gh}{c^5}}.$$

Indeed, $[Gh] = L^5T^{-3}$ and $[c^5] = L^5T^{-5}$, so their ratio has dimension T^2. Its square root has dimension time. Option (D) instead has dimension length, option (B) inverse length, and option (A) retains a mass dimension, so none of those can represent time.

Step 3: Conclusion The required combination is $\sqrt{Gh/c^5}$. Hence, the correct option is (C).

Quick Tip to Solve: First cancel mass by multiplying G and h; then divide by the power of c that removes length before taking the square root.

46. The top of a water tank is open to air and its water level is mainted. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the center of the opening from the level of water in the tank is close to : [4 marks]

(A) 6.0 m

(B) 4.8 m

(C) 9.6 m

(D) 2.9 m

Correct Answer: (B) 4.8 m

Solution:

Concept:

- For an open tank with a maintained water surface and a small outlet, Bernoulli's equation gives the efflux speed $v = \sqrt{2gh}$, where h is the vertical depth of the outlet centre.
- The volumetric flow rate is $\dot{V} = Av$, with outlet area $A = \pi r^2$. Convert the supplied volume per minute to volume per second before finding speed.

Step 1: Convert flow rate and calculate outlet speed The opening radius is $2\,\text{cm} = 0.020\,\text{m}$; this is a radius, not a diameter. Its area and the flow rate are

$$A = \pi(0.020)^2 = 1.2566 \times 10^{-3}\,\text{m}^2,$$

$$\dot{V} = \frac{0.74}{60} = 1.2333 \times 10^{-2}\,\text{m}^3\,\text{s}^{-1}.$$

The mean efflux speed is therefore

$$v = \frac{\dot{V}}{A} = \frac{1.2333 \times 10^{-2}}{1.2566 \times 10^{-3}} \simeq 9.81\,\text{m}\,\text{s}^{-1}.$$

Step 2: Apply Bernoulli equation between surface and opening Both the water surface and the jet are exposed to atmospheric pressure, so the pressure terms cancel. The surface speed is negligible compared with the outlet speed in the tank approximation. The fall in gravitational potential energy per unit mass supplies $v^2/2$, giving

$$h = \frac{v^2}{2g} \simeq \frac{(9.81)^2}{2(9.8)} \simeq 4.91\,\text{m}.$$

Using $g = 10\,\text{m}\,\text{s}^{-2}$ gives $4.81\,\text{m}$. Either convention selects $4.8\,\text{m}$ as the closest listed depth. Values 6.0, 9.6 and $2.9\,\text{m}$ do not reproduce the given discharge through this area.

Step 3: Conclusion The depth is close to $4.8\,\text{m}$. Hence, the correct option is (B).

Quick Tip to Solve: Compute speed from volume flow divided by outlet area first; then use $h = v^2/(2g)$, keeping minutes and centimetres out of the SI calculation.

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47. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of 24V/50 Hz. The energy dissipated in the circuit in 60 s is : [4 marks]

(A) 5.65×10^2J

(B) 2.26×10^3J

(C) 5.17×10^2 J

(D) 3.39×10^3 J

Correct Answer: (C) 5.17×10^2 J

Solution:

Concept:

- For a series RLC circuit, $X_L = \omega L$, $X_C = 1/(\omega C)$ and $Z^2 = R^2 + (X_L - X_C)^2$, with $\omega = 2\pi f$.
- The stated AC voltage is its rms value. The average power is dissipated in the resistor: $P = I_{\text{rms}}^2 R = V_{\text{rms}}^2 R/Z^2$. The energy dissipated over time t is Pt.

Step 1: Find the reactances and impedance Convert the component values to SI units: $L = 0.020\,\text{H}$, $C = 120 \times 10^{-6}\,\text{F}$, and $R = 60\,\Omega$. At $f = 50\,\text{Hz}$,

$$\omega = 100\pi\,\text{rad}\,\text{s}^{-1}, \qquad X_L = 100\pi(0.020) \simeq 6.28\,\Omega,$$

$$X_C = \frac{1}{100\pi(120 \times 10^{-6})} \simeq 26.53\,\Omega.$$

Thus the net reactance is capacitive, $X_L - X_C \simeq -20.24\,\Omega$, and

$$Z^2 = 60^2 + (-20.24)^2 \simeq 4009.8\,\Omega^2.$$

Step 2: Calculate real power and energy dissipated The rms current is $I_{\text{rms}} = 24/\sqrt{4009.8} \simeq 0.379\,\text{A}$. Hence

$$P = \frac{24^2 \times 60}{4009.8} \simeq 8.62\,\text{W},$$

$$E = Pt = 8.62 \times 60 \simeq 517\,\text{J}.$$

The inductor and capacitor store and return energy during a cycle; their reactances reduce current but do not themselves contribute average Joule loss. Treating the circuit as a pure resistor overestimates power. Option (A) is not the value obtained from the actual impedance, and options (B) and (D) are much larger than the loss allowed by this current.

Step 3: Conclusion The dissipated energy is 5.17×10^2 J. Hence, the correct option is (C).

Quick Tip to Solve: Use rms voltage directly in $E = V^2Rt/[R^2 + (\omega L - 1/(\omega C))^2]$; do not insert an extra factor of one-half.

48. A force acts on a 2 kg object so that its position is given as a function of time as x = $3t^2$ + 5. What is the work done by this force in first 5 seconds ? [4 marks]

(A) 850 J

(B) 950 J

(C) 875 J

(D) 900 J

Correct Answer: (D) 900 J

Solution:

Concept:

- For one-dimensional motion, velocity is $v = dx/dt$, acceleration is $a = d^2x/dt^2$, and the resultant force is $F = ma$.
- The work-energy theorem states that the work of the resultant force equals $\Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$. A constant force also gives $W = F\Delta x$.

Step 1: Differentiate the position and calculate the force The position is $x(t) = 3t^2 + 5$, with the quantities interpreted in SI units. Differentiating with respect to time gives

$$v(t) = 6t\,\text{m s}^{-1}, \qquad a(t) = 6\,\text{m s}^{-2}.$$

For the $2\,\text{kg}$ object, the force producing this acceleration is constant:

$$F = ma = 2 \times 6 = 12\,\text{N}.$$

The additive 5 in the position specifies the initial coordinate; it does not affect velocity, acceleration or the work over a specified displacement.

Step 2: Find the displacement and verify by kinetic energy At the start $x(0) = 5\,\text{m}$, while after five seconds $x(5) = 3(25) + 5 = 80\,\text{m}$. Therefore

$$\Delta x = 80 - 5 = 75\,\text{m}, \qquad W = 12 \times 75 = 900\,\text{J}.$$

As an independent check, $v_i = 0$ and $v_f = 6(5) = 30\,\mathrm{m\,s^{-1}}$, so

$$\Delta K = \frac{1}{2}(2)(30^2 - 0^2) = 900\,\mathrm{J}.$$

The work is positive because force and displacement are both in the positive x direction. Using the final coordinate instead of displacement would incorrectly include the arbitrary initial position. Options (A), (B) and (C) disagree with both consistent calculations.

Step 3: Conclusion The work done during the first $5\,\mathrm{s}$ is $900\,\mathrm{J}$. Hence, the correct option is (D).

Quick Tip to Solve: For $x = At^2 + B$, differentiate before substituting time; the coordinate offset B cancels from the displacement and cannot affect work.

---

49. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point. the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is (g = 10 $\mathrm{ms^{-2}}$ [4 marks]

(A) 200 N

(B) 140 N

(C) 70 N

(D) 100 N

Correct Answer: (D) 100 N

Solution:

Concept:

- A mass at rest has zero resultant force. The vertical rope segment below the applied horizontal force therefore has tension $T_0 = mg$.
- At the point where the horizontal force is applied, resolve the upper rope tension into vertical and horizontal components. The upper rope makes 45° with the vertical roof direction.

Step 1: Find the downward load at the force application point The suspended mass is $10\,\mathrm{kg}$ and the given gravitational acceleration is $10\,\mathrm{m\,s^{-2}}$. Thus

$$T_0 = mg = 10 \times 10 = 100\,\mathrm{N}.$$

The mass is in equilibrium, so the lower segment transmits this $100\,\mathrm{N}$ vertical load to the point where the horizontal force acts. It is the upper segment that inclines; the free suspended mass still requires a vertical tension to balance its weight.

Step 2: Resolve the upper tension into its two components Let the upper-segment tension be T, and let F be the applied horizontal force. Vertical equilibrium of the application point requires

$$T\cos 45^\circ = 100\,\mathrm{N}.$$

Horizontal equilibrium then gives

$$F = T\sin 45^\circ = 100\tan 45^\circ = 100\,\mathrm{N}.$$

The upper tension itself is $100\sqrt{2}\,\mathrm{N} \simeq 141\,\mathrm{N}$, which explains why $140\,\mathrm{N}$ is a tempting but incorrect choice: it is not the horizontal force. The $200\,\mathrm{N}$ and $70\,\mathrm{N}$ options do not balance the horizontal and vertical components at this angle. Equal inclination components give the immediate check $F = mg$.

Step 3: Conclusion The horizontal force has magnitude $100\,\mathrm{N}$. Hence, the correct option is (D).

Quick Tip to Solve: For a rope deflected through angle θ from vertical, write $F = mg\tan\theta$; distinguish the applied horizontal force from the larger upper-rope tension.

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50. A particle having the same charge as of electron moves in a ciurcular path of radius 0.5 cm under the influence of a magnetic field 0f 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Given charge of electron = $1.6 \times 10{-}^{19}$C) [4 marks]

(A) 9.1×10^{-31} kg

(B) 1.6×10^{-27} kg

(C) 1.6×10^{-19} kg

(D) 2.0×10^{-24} kg

Correct Answer: (D) 2.0×10^{-24} kg

Solution:

Concept:

- The magnetic force on a particle moving perpendicular to a uniform field is $F_B = |q|vB$; it supplies the centripetal force mv^2/r in circular motion.
- For an undeflected beam in crossed fields, electric and magnetic forces balance in magnitude: $|q|E = |q|vB$. Thus $v = E/B$, independent of the charge sign.

Step 1: Obtain the speed from the straight path The straight trajectory means that the electric force cancels the transverse magnetic force. Using the magnitudes of the fields avoids a sign error for the negatively charged particle. Hence

$$v = \frac{E}{B} = \frac{100}{0.5} = 200\,\mathrm{m\,s^{-1}}.$$

The charge being equal to that of an electron fixes its charge magnitude, not its mass; the electron mass must not be inserted without calculating it.

Step 2: Use the circular motion to find the mass Before the electric field balances the magnetic force, the magnetic force bends the same-speed particle into the stated circle. Convert the radius to SI units:

$r = 0.5\,\text{cm} = 5.0 \times 10^{-3}\,\text{m}$. Then

$$\begin{aligned} |q|vB &= \frac{mv^2}{r}, \\ m &= \frac{|q|Br}{v} = \frac{|q|B^2 r}{E}, \\ m &= \frac{(1.6 \times 10^{-19})(0.5)(5.0 \times 10^{-3})}{200} \\ &= 2.0 \times 10^{-24}\,\text{kg}. \end{aligned}$$

The other three proposed masses do not give the observed radius at the speed selected by the crossed fields.

Step 3: Conclusion The particle has mass $2.0 \times 10^{-24}\,\text{kg}$. Hence, the correct option is (D).

Quick Tip to Solve: First find $v = E/B$ from the undeflected path, then use $m = |q|Br/v$; sharing an electron charge does not imply sharing its mass.

51. The energy required to take a satellite to a height 'h' above Earth surface (radius of Earth = 6.4 $\times$ 10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is [4 marks]

(A) 1.6×10^3 km

(B) 3.2×10^3 km

(C) 6.4×10^3 km

(D) 1.28×10^4 km

Correct Answer: (B) 3.2×10^3 km

Solution:

Concept:

- The gravitational potential energy of a satellite of mass m at distance r from the centre of Earth is $U(r) = -GMm/r$, taking zero at infinity.
- In a circular orbit of radius r, gravity supplies centripetal force, giving $v^2 = GM/r$ and orbital kinetic energy $K = GMm/(2r)$.

Step 1: Calculate the energy needed to gain height Let Earth have radius R and mass M. Moving the satellite from the surface to height h changes its distance from the centre from R to $R + h$. The lifting energy in the question is the increase in gravitational potential energy, not the kinetic energy needed for the orbit. It is positive because the final potential energy is less negative:

$$\begin{aligned} E_1 &= U(R+h) - U(R) \\ &= GMm\left(\frac{1}{R} - \frac{1}{R+h}\right) \\ &= \frac{GMmh}{R(R+h)}. \end{aligned}$$

Keeping the full inverse-distance expression is essential because the required height need not be small compared with Earth radius.

Step 2: Equate this energy to the orbital kinetic energy The circular orbit has radius $R+h$, so

$$E_2 = \frac{1}{2}mv^2 = \frac{GMm}{2(R+h)}.$$

Equating the two specified energies cancels the satellite mass and the common orbital denominator:

$$\begin{aligned}\frac{GMmh}{R(R+h)} &= \frac{GMm}{2(R+h)},\\ \frac{h}{R} &= \frac{1}{2},\\ h = \frac{R}{2} &= \frac{6.4\times10^3}{2}\\ &= 3.2\times10^3\,\text{km}.\end{aligned}$$

In fact $E_1/E_2 = 2h/R$; heights $R/4$, R, and $2R$ give ratios $1/2$, 2, and 4, respectively, so the remaining options fail.

Step 3: Conclusion The required height above Earth surface is 3.2×10^3 km. Hence, the correct option is (B).

Quick Tip to Solve: Keep the lifting energy and orbital kinetic energy separate. Their ratio simplifies immediately to $E_1/E_2 = 2h/R$.

52. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about x = 0. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be : [4 marks]

(A) $\frac{A}{2}$
(B) $\frac{A}{2\sqrt{2}}$
(C) $\frac{A}{\sqrt{2}}$
(D) A

Correct Answer: (C) $\frac{A}{\sqrt{2}}$

Solution:

Concept:

- For SHM about $x=0$, the restoring force is $-kx$, where $k=m\omega^2$; potential energy measured from equilibrium is $U=\frac{1}{2}kx^2$.
- The conserved mechanical energy is $E=\frac{1}{2}kA^2$, so kinetic energy is $K=\frac{1}{2}k(A^2-x^2)$. Both depend on displacement magnitude rather than its sign.

Step 1: Write the equality of the two energies At a turning point $x=\pm A$, the speed and kinetic energy vanish, while all the energy is potential. At equilibrium the potential energy is zero and all the

energy is kinetic. The equality must therefore occur at an intermediate displacement. Substituting the SHM energy expressions gives

$$\frac{1}{2}kx^2 = \frac{1}{2}k(A^2 - x^2),$$
$$x^2 = A^2 - x^2,$$
$$2x^2 = A^2.$$

The spring constant or its effective SHM equivalent cancels, which explains why the answer depends only on the amplitude, not on the mass or frequency.

Step 2: Find the displacement and compare the options Taking the square root gives the two symmetric positions

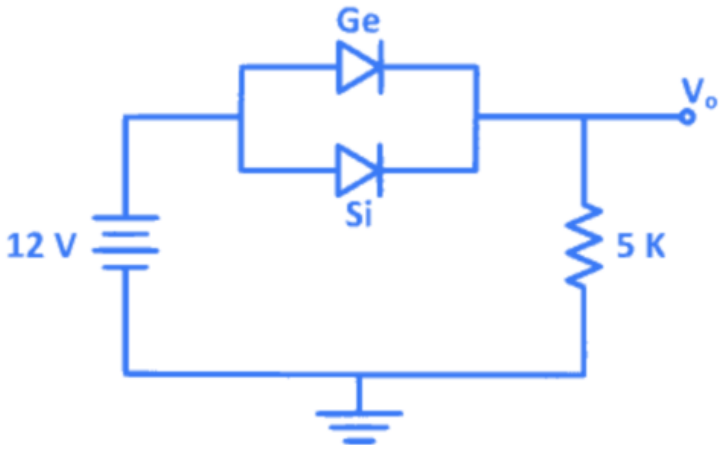
$$x = \pm\frac{A}{\sqrt{2}}, \qquad |x| = \frac{A}{\sqrt{2}}.$$

The options list the positive displacement magnitude. A direct energy-fraction check confirms it: $U/E = x^2/A^2 = 1/2$, leaving the same fraction for kinetic energy. At $x = A/2$, potential energy is only $E/4$, while at $x = A/(2\sqrt{2})$ it is $E/8$; neither equals the remaining kinetic energy. At $x = A$, the kinetic energy is zero. Thus only the magnitude in option (C) satisfies the energy equality.

Step 3: Conclusion The equal-energy positions are $x = \pm A/\sqrt{2}$; the positive magnitude listed is $A/\sqrt{2}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the energy fraction $U/E = (x/A)^2$. Equal kinetic and potential energies require each fraction to be one half, not a displacement of half the amplitude.

53. Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of V_0 changes by : (assume that the Ge diode has large breakdown voltage) [4 marks]



(A) 0.8 V

(B) 0.6 V

(C) 0.2 V

(D) 0.4 V

Correct Answer: (D) 0.4 V

Solution:

Concept:

- In the constant-threshold model, a conducting germanium diode drops $0.3\,\text{V}$, while a conducting silicon diode drops $0.7\,\text{V}$. A reverse-biased diode below breakdown is treated as open.
- Parallel diodes share the same terminal voltage. The branch with the smaller forward threshold conducts first and can prevent the other branch from reaching its threshold.

Step 1: Find the original output voltage Initially both diodes are directed from the positive $12\,\text{V}$ supply node toward the output node. As the supply drives the load, the germanium diode reaches conduction at a drop of only $0.3\,\text{V}$. It fixes the voltage across the parallel diode pair at that value. The silicon diode therefore has only $0.3\,\text{V}$ across it, below its $0.7\,\text{V}$ threshold, and remains off. Relative to the grounded lower rail,

$$V_{0,\text{initial}} = 12 - 0.3 = 11.7\,\text{V}.$$

The load current is $11.7/(5 \times 10^3) = 2.34\,\text{mA}$, confirming the direction of conduction assumed.

Step 2: Reverse the germanium branch and recompute the output After reversal the germanium diode is reverse biased and does not break down, as explicitly stated. The silicon branch is now the conducting path from the supply to the resistor. Its forward drop becomes $0.7\,\text{V}$, giving

$$V_{0,\text{final}} = 12 - 0.7 = 11.3\,\text{V}.$$

The signed change and its magnitude are

$$\Delta V_0 = 11.3 - 11.7 = -0.4\,\text{V}, \qquad |\Delta V_0| = 0.4\,\text{V}.$$

The voltage drops are not added, because the diodes are parallel rather than in series. Neither the sum nor twice either threshold represents the change, ruling out the other listed values.

Step 3: Conclusion The output voltage decreases from $11.7\,\text{V}$ to $11.3\,\text{V}$, a change of magnitude $0.4\,\text{V}$. Hence, the correct option is (D).

Quick Tip to Solve: For parallel forward diodes, start with the lower threshold. Reversing that diode transfers conduction to the other branch, so the output change equals the threshold difference.

54. The energy associated with electric field is (U_E) and with magnetic field is (U_B) for an electromagnetic wave in free space. Then :

[4 marks]

(A) $U_E = \frac{U_B}{2}$

(B) $U_E > U_B$

(C) $U_E < U_B$

(D) $U_E = U_B$

Correct Answer: (D) $U_E = U_B$

Solution:

Concept:

- In free space, the instantaneous electric and magnetic energy densities are $u_E = \frac{1}{2}\varepsilon_0 E^2$ and $u_B = B^2/(2\mu_0)$.
- For a travelling electromagnetic wave in vacuum, electric and magnetic fields are in phase and satisfy $E = cB$, with $c^2 = 1/(\mu_0\varepsilon_0)$.

Step 1: Express the electric energy density in terms of the magnetic field The field amplitudes have different SI units, so comparing the numerical sizes of E and B alone cannot compare their energies. Insert the wave relation into the electric energy density instead:

$$\begin{aligned} u_E &= \frac{1}{2}\varepsilon_0 (cB)^2 \\ &= \frac{1}{2}\varepsilon_0 c^2 B^2 \\ &= \frac{1}{2}\varepsilon_0 \frac{1}{\mu_0\varepsilon_0} B^2 \\ &= \frac{B^2}{2\mu_0} = u_B. \end{aligned}$$

The constants exactly cancel to give equality. Because the fields are in phase, this comparison applies at every position and time for the travelling wave, including the instants at which both fields vanish.

Step 2: Convert density equality to energy equality The energies must refer to the same region of the wave. Integrating equal energy densities over that common volume preserves the equality:

$$U_E = \int u_E \, dV = \int u_B \, dV = U_B.$$

Time averaging also preserves it, so the result does not depend on whether the question refers to instantaneous or cycle-averaged energies. The total electromagnetic energy is shared equally between the two fields: each contributes half the total, rather than the electric contribution being half the magnetic contribution. Option (A) confuses those two statements, and the strict inequalities in (B) and (C) contradict the derived equality.

Step 3: Conclusion The electric and magnetic fields carry equal energies in the free-space electromagnetic wave: $U_E = U_B$. Hence, the correct option is (D).

Quick Tip to Solve: Combine $E = cB$ with $\varepsilon_0 c^2 = 1/\mu_0$. Each field supplies half the total wave energy; neither supplies half the other field energy.

55. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to : [4 marks]

(A) $\frac{2a_1a_2}{a_1+a_2}t$

(B) $\sqrt{2a_1a_2}t$

(C) $\sqrt{a_1a_2}t$

(D) $\frac{a_1+a_2}{2}t$

Correct Answer: (C) $\sqrt{a_1a_2}t$

Solution:

Concept:

- A car starting from rest with constant acceleration a covers distance $s = \frac{1}{2}aT^2$ in time T, reaching speed $V = aT = \sqrt{2as}$.
- The finish-line speed difference must compare the cars at their respective arrival times. Their elapsed times differ by t, but their travelled race distance is the same.

Step 1: Write the finish times and speeds Let the common race distance be s, and write T_A and T_B for the two travel times. Since A finishes earlier, $T_B - T_A = t$, with $a_1 > a_2$. The equations for motion from rest give

$$T_A = \sqrt{\frac{2s}{a_1}}, \quad T_B = \sqrt{\frac{2s}{a_2}},$$
$$V_A = \sqrt{2sa_1}, \quad V_B = \sqrt{2sa_2}.$$

These expressions retain the same unknown distance for both cars. It would be incorrect to subtract a_1T and a_2T at one common time because the stated finish speeds occur at different times.

Step 2: Eliminate the race distance Use $v = V_A - V_B$ and form the ratio to the measured time difference:

$$t = \sqrt{2s}\left(\frac{1}{\sqrt{a_2}} - \frac{1}{\sqrt{a_1}}\right),$$
$$v = \sqrt{2s}(\sqrt{a_1} - \sqrt{a_2}),$$
$$\frac{v}{t} = \frac{\sqrt{a_1} - \sqrt{a_2}}{1/\sqrt{a_2} - 1/\sqrt{a_1}}$$
$$= \sqrt{a_1a_2}.$$

The common difference of square roots and the factor $\sqrt{2s}$ cancel exactly. Thus the factor multiplying t is the geometric mean of the accelerations. The arithmetic mean, harmonic mean, and extra factor $\sqrt{2}$ in the remaining choices do not follow from the common-distance equations.

Step 3: Conclusion The finish-line speed advantage is $v = \sqrt{a_1a_2}\,t$. Hence, the correct option is (C).

Quick Tip to Solve: Write both finish times and speeds in terms of the same distance, then divide their differences to eliminate that distance cleanly.

56. A carbon resistance has a following colour code. What is the value of the resistance ? [4 marks]



(A) 530 kΩ ± 5%

(B) 5.3 MΩ ± 5%

(C) 6.4 MΩ ± 5%

(D) 64 kΩ ± 10%

Correct Answer: (A) 530 kΩ ± 5%

Solution:

Concept:

- A four-band resistor uses the first two bands as significant digits, the third band as the power-of-ten multiplier, and the fourth band as the tolerance.
- The relevant colours here are green $= 5$, orange $= 3$, yellow multiplier $= 10^4$, and gold tolerance $= \pm 5\%$. Read the tolerance band last.

Step 1: Decode the significant digits and multiplier The diagram labels the bands G, O, Y, Golden, in that order. Thus the significant-digit colours are green and orange, giving the two-digit number 53, not 5.3 and not 64. Yellow is the third band, so it acts as a multiplier rather than a third significant digit:

$$R_{\text{nominal}} = 53 \times 10^4\,\Omega = 530000\,\Omega.$$

Dividing by 10^3 converts ohms to kilo-ohms, while dividing by 10^6 converts them to mega-ohms:

$$R_{\text{nominal}} = 530\,\text{k}\Omega = 0.53\,\text{M}\Omega.$$

Keeping the two-digit number intact until after applying the multiplier prevents a factor-of-ten error.

Step 2: Apply the gold tolerance and select the matching value The separated final golden band specifies a tolerance of $\pm 5\%$. It does not supply another significant digit or multiply the already decoded value. Therefore the absolute allowed deviation is

$$\Delta R = \frac{5}{100}(530\,\text{k}\Omega) = 26.5\,\text{k}\Omega.$$

The permitted range is consequently $503.5\,\text{k}\Omega$ to $556.5\,\text{k}\Omega$. Option (B) has a nominal value ten times too large, and option (C) uses the wrong leading digits as well as the wrong scale. Option (D) has both incorrect digits and a tolerance of 10%, which would require a silver final band rather than gold.

Step 3: Conclusion The resistance is $530\,\text{k}\Omega \pm 5\%$. Hence, the correct option is (A).

Quick Tip to Solve: Identify gold as the last, tolerance band first. Then read two digits, multiply by the third-band power of ten, and convert ohms to the requested prefix.

57. A musician using an open flute of length 50 cm producess second harmonic sound waves. A person runs towards the musician from another end of hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to : [4 marks]

(A) 666 Hz

(B) 753 Hz

(C) 500 Hz

(D) 333 Hz

Correct Answer: (A) 666 Hz

Solution:

Concept:

- An open pipe supports every harmonic: $f_n = nv/(2L)$, where L is its length and v is the sound speed.
- For a stationary source and an observer approaching at speed u_o, the Doppler frequency is $f_{\text{obs}} = f(v + u_o)/v$. The observer meets wavefronts more frequently.

Step 1: Find the frequency produced by the flute The flute is open at both ends, so its fundamental wavelength is twice its length. Use $L = 50\,\text{cm} = 0.50\,\text{m}$. The question specifies the second harmonic, not the fundamental:

$$f_1 = \frac{330}{2(0.50)} = 330\,\text{Hz}, \qquad f_2 = 2f_1 = 660\,\text{Hz}.$$

The source frequency therefore is 660 Hz. The speed of the listener does not alter the frequency with which the flute itself vibrates.

Step 2: Apply the approaching-observer Doppler shift Convert the running speed to the same units as the wave speed:

$$u_o = 10\frac{1000}{3600} = \frac{25}{9}\,\text{m s}^{-1}.$$

Since the source is at rest, only the numerator changes:

$$f_{\text{obs}} = 660\frac{330 + 25/9}{330} = 665.56\,\text{Hz}.$$

The positive correction is small because the listener runs much more slowly than sound. Thus 666 Hz is the nearest listed value. The value 333 Hz would follow from mistakenly using the fundamental; 500 Hz is below the emitted second harmonic, contrary to an approaching observer, and 753 Hz is far too large for this speed.

Step 3: Conclusion The running person hears approximately $666\,\text{Hz}$. Hence, the correct option is (A).

Quick Tip to Solve: For an open pipe first choose the harmonic, then apply Doppler. A slow approaching observer should hear only a small increase over the emitted frequency.

58. At a given instant, say t = 0, two radioactive substance A and B have equal activities. the ratio $\frac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t}. If the half-life of A is $\ln^2$, the half-life of B is : [4 marks]

(A) 4ln2

(B) $\frac{\ln 2}{2}$

(C) $\frac{\ln 2}{4}$

(D) 2ln2

Correct Answer: (C) $\frac{\ln 2}{4}$

Solution:

Concept:

- Radioactive activity obeys $R(t) = R(0)e^{-\lambda t}$. The decay constant λ is positive and the half-life is $T_{1/2} = \ln 2/\lambda$.
- The ratio of two exponentially decaying activities depends on the difference of their decay constants, rather than their sum.

Step 1: Extract the difference of decay constants Write the separate activity laws using the common initial activity R_0:

$$R_A = R_0 e^{-\lambda_A t}, \qquad R_B = R_0 e^{-\lambda_B t}.$$

On dividing, the initial activities cancel, giving

$$\frac{R_B}{R_A} = e^{-(\lambda_B - \lambda_A)t} = e^{-3t}.$$

Equality for all times requires $\lambda_B - \lambda_A = 3$. Thus B loses activity faster than A. The coefficient 3 is a difference of decay constants in the reciprocal time unit implicit in the question; it is not the decay constant of B alone.

Step 2: Use the half-life of A to find that of B The logarithmic half-life value stated for A is $\ln 2$, with the 2 being the argument of the natural logarithm. Accordingly,

$$\lambda_A = \frac{\ln 2}{\ln 2} = 1, \qquad \lambda_B = 1 + 3 = 4.$$

Substituting in the half-life relation yields

$$T_{1/2,B} = \frac{\ln 2}{4}.$$

It is one quarter of the half-life of A, consistent with B decaying faster. The alternatives $4\ln 2$ and $2\ln 2$ would make B slower than A, while $(\ln 2)/2$ would give a decay-constant difference of only 1, contradicting the ratio supplied.

Step 3: Conclusion The half-life of B is $\boxed{(\ln 2)/4}$ in the same time unit as that of A. Hence, the correct option is (C).

Quick Tip to Solve: Take the ratio of activities before substituting half-lives: the exponent immediately gives the difference of decay constants, including its sign.

59. In a communication system operating at wavelength 800 nm, only one percent of source frequency is available as signal bandwith. The number of channels accomodated for transmitting TV signals of band width 6 MHz are (Take velocity of light c = 3 $\times$ 10^8m/s, h = 6.6 $\times$ 10^{-34} J-s) [4 marks]

(A) 3.75 $\times$ 10^6

(B) 3.86 $\times$ 10^6

(C) 6.25 $\times$ 10^5

(D) 4.87 $\times$ 10^5

Correct Answer: (C) 6.25 $\times$ 10^5

Solution:

Concept:

- The source frequency and wavelength are related by $\nu = c/\lambda$. The usable signal bandwidth here is one percent of this source frequency.
- If adjacent channels each require bandwidth b, their ideal number is $N = B_{\text{available}}/b$, neglecting guard bands because none are specified.

Step 1: Calculate the available frequency interval Express the wavelength in metres before using the wave equation:

$$\lambda = 800\,\text{nm} = 8.00 \times 10^{-7}\,\text{m}.$$

The carrier frequency is therefore

$$\nu = \frac{3 \times 10^8}{8.00 \times 10^{-7}} = 3.75 \times 10^{14}\,\text{Hz}.$$

Only one percent can be used to accommodate signals, so the available bandwidth is

$$B_{\text{available}} = 0.01\nu = 3.75 \times 10^{12}\,\text{Hz}.$$

This is a frequency interval, even though it is expressed using the same unit as the carrier frequency.

Step 2: Divide by the bandwidth of one TV channel Each television signal occupies $6\,\text{MHz} = 6 \times 10^6$ Hz. Thus

$$N = \frac{3.75 \times 10^{12}}{6 \times 10^6} = 0.625 \times 10^6 = 6.25 \times 10^5.$$

The ratio is dimensionless, as a count of channels must be. Planck's constant is not needed: no photon energy is involved in allocating a frequency band. Options (A) and (B) overestimate this quotient by several times; option (D) also fails the direct division. Using the whole carrier frequency instead of its one-percent interval would give a count one hundred times too large.

Step 3: Conclusion The system can accommodate 6.25×10^5 channels under the stated bandwidth allocation. Hence, the correct option is (C).

Quick Tip to Solve: Keep three quantities separate: carrier frequency, the usable fraction of that frequency, and bandwidth per channel. Divide only after converting MHz to Hz.

60. Two Carnot engines A and B are operated in series. The first one, A, receives heat at T_1 (= 600 K) and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and, in tum, rejects to a heat reservoir at T_3 (=400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal : [4 marks]

(A) 600 K

(B) 400 K

(C) 300 K

(D) 500 K

Correct Answer: (D) 500 K

Solution:

Concept:

- A reversible Carnot engine has $Q_{\text{cold}}/Q_{\text{hot}} = T_{\text{cold}}/T_{\text{hot}}$, with absolute temperatures in kelvin.
- The work delivered in one cycle is the heat received minus the heat rejected. For engines in series, the heat rejected by A is the heat received by B.

Step 1: Express both work outputs using the same heat input Let Q_1 be the heat absorbed by engine A, Q_2 the heat it rejects at the intermediate temperature, and Q_3 the heat finally rejected by B. For engine A,

$$Q_2 = Q_1\frac{T_2}{T_1}, \qquad W_A = Q_1 - Q_2 = \frac{Q_1}{T_1}(T_1 - T_2).$$

For engine B,

$$Q_3 = Q_2\frac{T_3}{T_2} = Q_1\frac{T_3}{T_1}, \qquad W_B = Q_2 - Q_3 = \frac{Q_1}{T_1}(T_2 - T_3).$$

Both expressions have the same positive factor Q_1/T_1.

Step 2: Equate the outputs and check the result The stated equality of work gives

$$T_1 - T_2 = T_2 - T_3, \qquad 2T_2 = T_1 + T_3.$$

Hence

$$T_2 = \frac{600 + 400}{2} = 500\,\text{K}.$$

For a check, A rejects $5Q_1/6$, and the combined pair rejects $2Q_1/3$. Each engine then delivers $Q_1/6$. At 600 K the first engine would have zero output, and at 400 K the second would have zero output; neither could satisfy equal positive outputs. A temperature of 300 K is below the final cold reservoir and is incompatible with the specified ordering. Equal work does not mean equal efficiencies because the two engines absorb different quantities of heat.

Step 3: Conclusion The intermediate reservoir must be at $\boxed{500\,\text{K}}$. Hence, the correct option is (D).

Quick Tip to Solve: For reversible engines in series with equal work, use the arithmetic mean of the end temperatures. Equal efficiencies instead would lead to a different condition.

Chemistry (120 marks)

61. If the standard electrode potential for a cell is 2 V at 300 K, the equilibrium constant (K) for the reaction

$Zn(s) + Cu^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Cu(s)$

at 300 K is approximately,

($R = 8\ JK^{-1}mol^{-1}$, $F = 96000\ C\ mol^{-1}$) [4 marks]

(A) e^{-80}

(B) e^{-160}

(C) e^{320}

(D) e^{160}

Correct Answer: (D) e^{160}

Solution:

Concept:

- The standard Gibbs energy of a cell reaction is $\Delta G^\circ = -nFE^\circ$, where n is the number of electrons transferred per balanced reaction.
- Equilibrium thermodynamics gives $\Delta G^\circ = -RT\ln K$. Combining the equations yields $\ln K = nFE^\circ/(RT)$.

Step 1: Find the electron transfer for the balanced reaction Zinc is oxidised from oxidation state zero to +2, while copper(II) is reduced to elemental copper:

$$\mathrm{Zn} \longrightarrow \mathrm{Zn^{2+}} + 2\,\mathrm{e^-}, \qquad \mathrm{Cu^{2+}} + 2\,\mathrm{e^-} \longrightarrow \mathrm{Cu}.$$

Adding these half-reactions gives exactly the cell reaction printed in the question, with no extra multiplication. Therefore $n = 2$. This number counts transferred electrons, not the number of species in the overall equation. Since the standard cell potential is positive, the reaction as written has negative standard Gibbs energy and an equilibrium constant greater than one.

Step 2: Calculate the natural logarithm of the equilibrium constant Use the supplied approximate constants and the temperature 300 K:

$$\ln K = \frac{2 \times 96000 \times 2}{8 \times 300} = \frac{384000}{2400} = 160.$$

Consequently

$$K = e^{160}.$$

The exponent is dimensionless: both nFE° and RT have energy-per-mole units. Options (A) and (B) have negative exponents and incorrectly predict $K < 1$. Option (C) doubles the correct exponent, effectively counting twice the actual electron transfer for the reaction as written. No conversion to a base-ten logarithm is needed because the choices already use powers of e.

Step 3: Conclusion The equilibrium constant is approximately $\boxed{e^{160}}$. Hence, the correct option is (D).

Quick Tip to Solve: Write the electron-balanced half-reactions first. A positive standard cell potential must give a positive natural logarithm of the equilibrium constant.

62. Which of the following combination of statements is true regarding the interpretation of the atomic orbitals ?

(a) An electron in an orbital of high angular momentum stays away from the nucleus than an electron in the orbital of lower angular momentum.

(b) For a given value of the principal quantum number, the size of the orbit is inversely proportional to the azimuthal quantum number.

(c) According to wave mechanics, the ground state angular momentum is equal to $\frac{h}{2\pi}$.

(d) The plot of ψ vs r for various azimuthal quantum numbers, shows peak shifting towards higher r value. [4 marks]

(A) (a), (b)

(B) (a), (d)

(C) (b), (c)

(D) (a), (c)

Correct Answer: (B) (a), (d)

Solution:

Concept:

- The orbital angular momentum magnitude in wave mechanics is $L = \sqrt{\ell(\ell+1)}\,\hbar$, with $\hbar = h/(2\pi)$. An s orbital has $\ell = 0$.
- Penetration describes how much of an orbital's electron distribution reaches the nuclear region. The usual penetration order within a shell is s greater than p greater than d greater than f.

Step 1: Assess the radial-distribution statements Statement (a) expresses the usual penetration trend: increasing orbital angular momentum suppresses the wavefunction close to the nucleus. Higher-angular-momentum orbitals thus have less access to the nuclear region than the penetrating lower-angular-momentum orbitals. In that qualitative sense (a) is true. The radial-wavefunction comparison in (d) refers to the outward displacement of the peaks as the azimuthal quantum number increases: s character can penetrate the nuclear region, whereas higher-angular-momentum functions vanish there. This is the radial-penetration interpretation intended by (d). It identifies (d) as true; it should not be replaced by a claim that every radius measure obeys one universal monotonic formula.

Step 2: Reject the inverse-size and Bohr-model claims Statement (b) claims that size is inversely proportional to ℓ. There is no such orbital-size law; in particular s orbitals with $\ell = 0$ have finite size, so the proposed inverse proportionality would be undefined. Thus (b) is false. For the ground-state s electron, wave mechanics gives

$$\ell = 0 \quad \Longrightarrow \quad L = \sqrt{0(0+1)}\,\hbar = 0.$$

Therefore (c) is false: the value $\hbar$ is associated with the old Bohr quantisation, not the orbital angular momentum of the quantum-mechanical ground state. Options (A) and (C) include false statement (b), and option (D) includes false statement (c).

Step 3: Conclusion The first and fourth statements form the intended correct combination. Hence, the correct option is (B).

Quick Tip to Solve: Test the ground state with $\ell = 0$: it immediately rules out $L = \hbar$. Do not confuse orbital penetration with an inverse-proportionality law for orbital size.

63. A solution containing 62 g ethylene glycol in 250 g water is cooled to − 10°C. If K_f for water is 1.86 K kg mol^{-1} , the amount of water (in g) separated as ice is : [4 marks]

(A) 48

(B) 32

(C) 64

(D) 16

Correct Answer: (C) 64

Solution:

Concept:

- For a non-electrolyte, freezing-point depression is $\Delta T_f = K_f m$, where m is moles of solute per kilogram of liquid solvent.
- As pure ice separates, ethylene glycol remains in the liquid phase. Its amount stays fixed, while the mass of liquid water decreases and its molality rises.

Step 1: Find the amount of solute and the final molality Ethylene glycol, $C_2H_6O_2$, has molar mass $2(12) + 6(1) + 2(16) = 62\,\text{g mol}^{-1}$. Thus the initial 62 g represents

$$n_{\text{glycol}} = \frac{62}{62} = 1.00\,\text{mol}.$$

It is a non-electrolyte, so its van't Hoff factor is one. At equilibrium with ice at $-10\,^\circ\text{C}$, the freezing-point depression is $10\,\text{K}$, giving

$$m_{\text{final}} = \frac{10}{1.86} = 5.376\,\text{mol kg}^{-1}.$$

This molality uses water remaining liquid, not the original total mass of water.

Step 2: Compute the remaining liquid water and subtract Let w be the mass of liquid water in kilograms at the final temperature. With all one mole of glycol still dissolved,

$$m_{\text{final}} = \frac{1}{w}, \qquad w = \frac{1}{10/1.86} = 0.186\,\text{kg} = 186\,\text{g}.$$

The ice separated is therefore

$$m_{\text{ice}} = 250 - 186 = 64\,\text{g}.$$

Initially the solution has molality $1/0.250 = 4$, corresponding to a depression of only 7.44 K. Hence some water must indeed freeze before the remaining solution can coexist with ice at the lower temperature. The other listed ice masses leave liquid-water masses inconsistent with the required depression of 10 K.

Step 3: Conclusion The mass of water separated as ice is $\boxed{64\,\text{g}}$. Hence, the correct option is (C).

Quick Tip to Solve: In partial freezing, first find the solvent mass still liquid from ΔT_f. Subtract that mass from the initial solvent mass only at the end.

64. The transition element that has lowest enthalpy of atomisation, is : [4 marks]

(A) Fe

(B) Cu

(C) V

(D) Zn

Correct Answer: (B) Cu

Solution:

Concept:

- Enthalpy of atomisation is the energy needed to form isolated gaseous atoms from one mole of the element in its standard state. Stronger metallic bonding means a larger value.
- A transition element has an incompletely filled d subshell in its atom or in a common oxidation state. A d-block element with only filled d subshells does not automatically qualify.

Step 1: Apply the transition-element restriction Although zinc is placed in the d block, its atom has configuration $[\mathrm{Ar}]3d^{10}4s^2$, and its common ion Zn^{2+} has $3d^{10}$. Neither has an incomplete d subshell. Thus Zn must be excluded from the comparison requested here. Its very low atomisation enthalpy cannot make it the answer to a question specifically asking for a transition element.

Step 2: Compare the remaining metals Vanadium and iron have several unpaired d electrons and relatively strong metallic bonding. Copper has the configuration $[\mathrm{Ar}]3d^{10}4s^1$; the filled d subshell makes its metallic cohesion weaker than that of V and Fe. Approximate atomisation enthalpies illustrate the comparison: Cu is about 337, Fe about 416, and V about $515\,\mathrm{kJ\,mol^{-1}}$. Copper still qualifies as a transition element because Cu^{2+} has a $3d^9$ configuration. Consequently Fe and V are rejected by the energy comparison, whereas Zn is rejected by the definition.

Step 3: Conclusion Among the transition elements listed, copper has the lowest enthalpy of atomisation. Hence, the correct option is (B).

Quick Tip to Solve: Check the word transition before ranking d-block metals: Zn has a low atomisation enthalpy but does not meet the incomplete-d-subshell definition.

65. The pH of rain water, is approximately : [4 marks]

(A) 5.6

(B) 7.5

(C) 7.0

(D) 6.5

Correct Answer: (A) 5.6

Solution:

Concept:

- Unpolluted rainwater is slightly acidic because atmospheric CO_2 dissolves in water and establishes carbonic-acid equilibria.

- The pH scale is defined by $\mathrm{pH} = -\log_{10}[\mathrm{H}^+]$, using the usual dilute-solution concentration approximation.

Step 1: Account for dissolved carbon dioxide Rain forms in contact with the atmosphere, so it is not equivalent to freshly prepared carbon-dioxide-free pure water. Dissolved carbon dioxide produces a weakly acidic solution through the equilibria

$$\mathrm{CO_2} + \mathrm{H_2O} \rightleftharpoons \mathrm{H_2CO_3}$$

$$\mathrm{H_2CO_3} \rightleftharpoons \mathrm{H^+} + \mathrm{HCO_3^-}.$$

Only a small fraction ionises, but the hydrogen-ion concentration becomes greater than that of neutral water. Therefore the expected pH is below 7.

Step 2: Select the approximate natural value The conventional approximate pH of ordinary, unpolluted rainwater is 5.6. This corresponds to

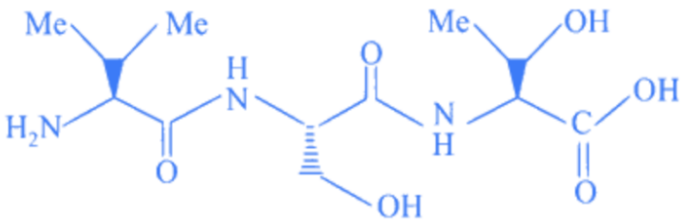
$$[\mathrm{H^+}] \simeq 10^{-5.6} = 2.5 \times 10^{-6}\,\mathrm{mol\,L^{-1}},$$

which is consistent with a weakly acidic solution. A value of 7.0 would describe neutral water at ordinary laboratory temperature without the atmospheric carbon-dioxide effect. The value 7.5 is alkaline and has the wrong direction of change. The value 6.5 is weakly acidic but is not the standard approximate value asked for. More strongly acidic polluted rain can have a still lower pH; the question gives no pollution-specific condition.

Step 3: Conclusion The approximate pH of ordinary rainwater is 5.6. Hence, the correct option is (A).

Quick Tip to Solve: For natural rainwater remember dissolved carbon dioxide: neutral pure water gives 7, whereas ordinary rainwater is approximately 5.6.

66. The correct sequence of amino acids present in the tripeptide given below is : [4 marks]



(A) Val - Ser - Thr

(B) Thr - Ser - Val

(C) Leu - Ser - Thr

(D) Thr - Ser - Leu

Correct Answer: (A) Val - Ser - Thr

Solution:

Concept:

- A peptide sequence is written from the free amino terminus (N terminus) to the free carboxyl terminus (C terminus), irrespective of how the structure is oriented on the page.
- Identify each amino-acid residue from the substituent R on its alpha carbon in the repeating backbone $-NH-CH(R)-CO-$.

Step 1: Locate the two ends of the peptide The left-hand residue in the displayed structure carries the free NH_2 group, while the right-hand residue carries the free COOH group. The sequence must therefore be read from left to right. The two intervening $-CO-NH-$ groups are peptide bonds: they join three amino-acid residues without changing the identity of their side chains. Reading from the carboxyl end would incorrectly reverse the sequence.

Step 2: Identify the side chains in order The first alpha carbon bears $-CH(CH_3)_2$, the isopropyl side chain of valine (Val). The middle alpha carbon bears $-CH_2OH$, identifying serine (Ser). The last alpha carbon bears $-CH(OH)CH_3$, identifying threonine (Thr). Thus the N-to-C order is

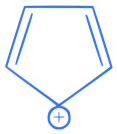
$$\text{Val} \longrightarrow \text{Ser} \longrightarrow \text{Thr}.$$

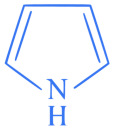
Leucine would require $-CH_2CH(CH_3)_2$, with an additional methylene between the alpha carbon and the branch point; that is absent from the first residue. Hence the leucine-containing alternatives fail. The Thr–Ser–Val alternative contains the same three residues but reads them in the wrong direction.

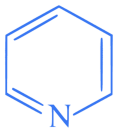
Step 3: Conclusion The tripeptide sequence is Val–Ser–Thr. Hence, the correct option is (A).

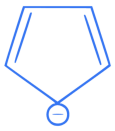
Quick Tip to Solve: Find the free amino group first, then read the side chains toward the free carboxyl group; do not identify a peptide by reading its drawing backward.

67. Which of the following compounds is not aromatic ? [4 marks]

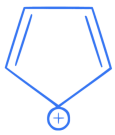
(A) Cyclopentadienyl cation 

(B) Pyrrole 

(C) Pyridine 

(D) Cyclopentadienyl anion 

Correct Answer: (A)

Cyclopentadienyl cation 

Solution:

Concept:

- Aromaticity requires a cyclic, planar, continuously conjugated system with $4n + 2$ pi electrons. A corresponding planar conjugated $4n$-electron system is antiaromatic.
- A carbocation contributes an empty p orbital and no electron pair; a conjugated carbanion or a pyrrole-type nitrogen can contribute a lone pair to the ring.

Step 1: Count electrons in the cyclopentadienyl cation Option (A) has two carbon–carbon double bonds in a five-membered ring and a positive charge at the remaining carbon. The double bonds supply four pi electrons altogether. The positively charged carbon has an empty p orbital, so it does not add two more electrons. Therefore its conjugated ring has

$$N_\pi = 2(2) = 4,$$

which does not satisfy the aromatic $4n + 2$ rule. In its planar conjugated description it is antiaromatic, and in particular it is not aromatic.

Step 2: Check the other three rings Pyrrole in (B) has four electrons from its two double bonds plus two from the nitrogen lone pair, giving six aromatic pi electrons. Pyridine in (C) has six pi electrons from its three ring double bonds; its nitrogen lone pair lies outside this pi system and is not counted. The cyclopentadienyl anion in (D) has four electrons from the double bonds plus two from the negatively charged carbon, again giving six. Thus each of (B), (C), and (D) satisfies the six-electron aromatic count, unlike the cation. The sign of the charge is decisive when comparing the two five-membered carbon rings.

Step 3: Conclusion The cyclopentadienyl cation in option (A) is not aromatic. Hence, the correct option is (A).

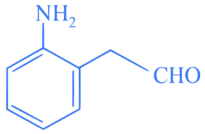
Quick Tip to Solve: For the cyclopentadienyl pair, count four electrons for the cation and six for the anion; the negative charge adds the lone pair needed for aromaticity.

68. The tests performed on compound X and their inferences are:

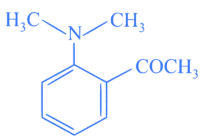
	Test	Inference
(a)	2,4-DNP test	Coloured precipitate
(b)	Iodoform test	Yellow precipitate
(c)	Azo-dye test	No dye formation

Compound X is: [4 marks]

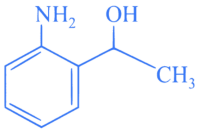
(A) o-$H_2NC_6H_4CH_2CHO$

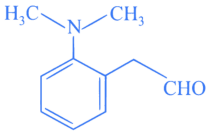


(B) o-$(CH_3)_2NC_6H_4COCH_3$

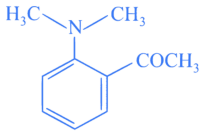


(C) o-$H_2NC_6H_4CH(OH)CH_3$



(D) *o*-$(CH_3)_2NC_6H_4CH_2CHO$ 

Correct Answer: (B)

o-$(CH_3)_2NC_6H_4COCH_3$ 

Solution:

Concept:

- 2,4-Dinitrophenylhydrazine gives a coloured hydrazone precipitate with aldehydes and ketones, revealing a carbonyl group.
- The iodoform test identifies a methyl ketone $RCOCH_3$, or a compound oxidisable to one; a primary aromatic amine can be diazotised and coupled to give an azo dye.

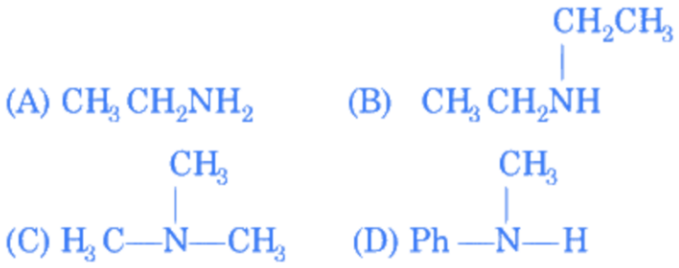
Step 1: Combine the positive carbonyl and iodoform tests The 2,4-DNP result requires an aldehyde or ketone, so the secondary alcohol in (C) is eliminated despite its ability to give iodoform after oxidation. Among the remaining structures, (B) contains $ArCOCH_3$, a methyl ketone. Its carbonyl forms the coloured hydrazone, and the methyl group adjacent to the carbonyl undergoes the haloform reaction. In contrast, (A) and (D) contain the side chain $ArCH_2CHO$; these aldehydes are not ethanal and have no methyl-ketone unit, so they do not give the required iodoform result.

Step 2: Use the negative azo-dye test Option (B) has a dimethylamino group, $-N(CH_3)_2$, bonded to the benzene ring. It is a tertiary aromatic amine, with no primary $-NH_2$ group to convert into an arenediazonium salt in the usual functional-group test. Hence it gives no azo dye by that diazotisation-and-coupling procedure. The primary aromatic amino groups in (A) and (C) would instead permit dye formation. Option (D) also lacks a primary amino group but has already failed the iodoform requirement. Applying all three observations together therefore selects only (B), rather than choosing a compound from a single positive test.

Step 3: Conclusion Compound X is the ortho-dimethylamino-substituted acetophenone shown in (B). Hence, the correct option is (B).

Quick Tip to Solve: Intersect the tests: DNP plus iodoform points to a methyl ketone, and the negative azo-dye test then excludes a primary aromatic amino group.

69. The increasing basicity order of the following compounds is : [4 marks]



(A) A < B < D < C

(B) D < C < A < B

(C) D < C < B < A

(D) A < B < C < D

Correct Answer: (B) D < C < A < B

Solution:

Concept:

- Amine basicity depends on the availability of the nitrogen lone pair and on the stabilisation of the protonated amine. Resonance, inductive donation and hydration all matter.
- For a base B, $B + H_2O \rightleftharpoons BH^+ + OH^-$. A smaller pK_b corresponds to a stronger base in water.

Step 1: Separate the aromatic amine from the aliphatic amines The structures are A = ethylamine, B = diethylamine, C = trimethylamine, and D = N-methylaniline. In D the nitrogen lone pair is delocalised into the phenyl ring. Protonation removes that resonance participation, so the lone pair is much less readily available than in the three aliphatic amines. D is therefore the weakest base. Any proposed order putting D above an aliphatic amine, as in (A) or (D), fails this resonance comparison.

Step 2: Compare inductive donation and hydration Among the aliphatic amines in water, electron-donating alkyl groups increase electron density at nitrogen, while solvation of the conjugate acid stabilises protonation. Diethylamine combines strong ethyl-group donation with effective hydration of its secondary ammonium ion and is the strongest here. Trimethylamine has three methyl groups, but its conjugate acid is less effectively hydrated than those of the less hindered amines; ethylamine is consequently stronger than trimethylamine in water. Representative pK_b values, about 4.2 for C, 3.3 for A and roughly 2.9 for B, reflect the required comparison. Thus

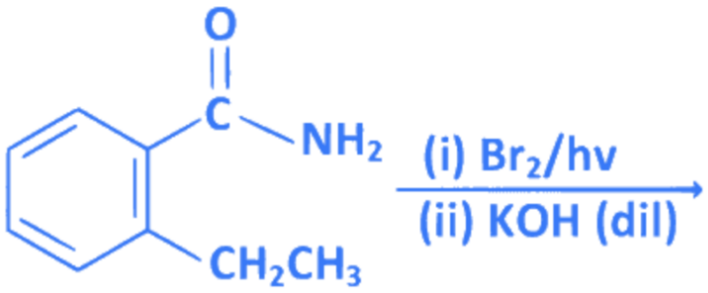
$$D < C < A < B.$$

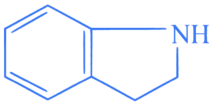
Option (C) incorrectly reverses the relative positions of ethylamine and diethylamine.

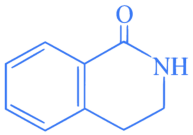
Step 3: Conclusion The increasing basicity order is $D < C < A < B$. Hence, the correct option is (B).

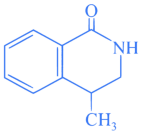
Quick Tip to Solve: In aqueous amine basicity, first demote aniline-type nitrogen by resonance, then compare alkyl donation together with hydration instead of simply counting alkyl groups.

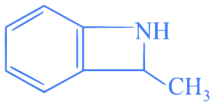
70. The major product of the following reaction is : [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: None of the printed options is the five-membered lactam expected from benzylic bromination and ring closure.

Solution:

Concept:

- Bromine under light preferentially replaces a benzylic hydrogen through a resonance-stabilised benzylic radical. Thus an ethyl substituent becomes $-CHBrCH_3$, not $-CH_2CH_2Br$.
- An ortho amide can close onto the adjacent benzylic carbon by intramolecular nitrogen alkylation. Track every carbon and the carbonyl when comparing the resulting lactam with structural options.

Step 1: Identify the benzylic bromination site The printed starting compound is ortho-ethylbenzamide, o-$C_2H_5C_6H_4CONH_2$. Photochemical bromination occurs at the carbon directly attached to the benzene ring, giving o-$CH_3CHBrC_6H_4CONH_2$. It does not create a new carbon atom and it does not preferentially brominate the terminal methyl group. The amide carbonyl is still present after this radical substitution.

Step 2: Trace the ring formed in base Intramolecular displacement of bromide by the amide nitrogen gives a five-membered lactam. Around its newly closed ring the atom sequence is: aromatic carbon bearing the carbonyl, carbonyl carbon, nitrogen, benzylic carbon bearing CH_3, and the adjacent aromatic carbon. The product is therefore 3-methylisoindolin-1-one. This atom-by-atom trace

preserves the amide carbonyl and all nine starting carbons; it introduces no additional methylene group.

Step 3: Compare the literal product with the printed drawings None of the four drawings represents that five-membered methyl-substituted lactam. Option (A) is a nitrogen heterocycle without the carbonyl. Option (B) is a six-membered lactam with two methylene groups; it would require terminal rather than benzylic bromination. Option (C) is a six-membered lactam with an extra methyl substituent and therefore has an extra carbon relative to the substrate. Option (D) has a four-membered nitrogen ring and lacks the carbonyl. Simply invoking a Hofmann rearrangement also does not justify these options: the stated first step is photochemical bromination, and a rearrangement would remove the amide carbon rather than produce the required five-membered lactam. Thus the literal printed alternatives do not provide a defensible unique selection.

Step 4: Conclusion The expected cyclisation product is 3-methylisoindolin-1-one: its new ring has five atoms and the molecule retains all nine starting carbons and its carbonyl. A and D lack that carbonyl, B has the wrong six-membered ring, and C has an extra carbon. Thus no printed option represents the product; a unique option cannot be selected from the question as printed.

Quick Tip to Solve: Trace the ring atoms and count carbons before selecting a cyclic product: benzylic bromination of an ethyl group leaves its terminal methyl substituent intact.

71. The complex that has highest crystal field splitting energy (Δ), is : [4 marks]

(A) $[Co(NH_3)_5(H_2O)]Cl_3$

(B) $K_2[CoCl_4]$

(C) $[Co(NH_3)_5Cl]Cl_2$

(D) $K_3[Co(CN)_6]$

Correct Answer: (D) $K_3[Co(CN)_6]$

Solution:

Concept:

- Crystal field splitting Δ is the energy separation between the two sets of metal d orbitals produced by the surrounding ligands. Strong-field ligands produce a larger splitting than weak-field ligands.
- For the same metal, splitting increases with metal oxidation state and depends on geometry. The relevant spectrochemical order is $Cl^- < H_2O < NH_3 < CN^-$; tetrahedral splitting is smaller than comparable octahedral splitting.

Step 1: Identify the metal ions and geometries In $[Co(NH_3)_5(H_2O)]Cl_3$ and $[Co(NH_3)_5Cl]Cl_2$, cobalt has oxidation state $+3$, and six donor atoms give octahedral coordination. In $K_3[Co(CN)_6]$, six cyanide ions and a complex charge of -3 likewise give cobalt(III), with octahedral geometry. However, $K_2[CoCl_4]$ contains cobalt(II) in a weak-field tetrahedral environment, so it cannot give the largest splitting.

Step 2: Compare the ligands at the same oxidation state The three cobalt(III) complexes can be compared through ligand field strength. Cyanide is substantially stronger-field than ammonia, water

or chloride. All six sites in $[Co(CN)_6]^{3-}$ are occupied by cyanide, whereas the ammine complexes contain the weaker ammonia ligands plus water or chloride. Thus the cyanide complex produces the strongest ligand field. Replacing water by chloride lowers rather than raises splitting; neither ammine option exceeds the cyanide complex.

Step 3: Conclusion The highest crystal field splitting energy belongs to $K_3[Co(CN)_6]$. Hence, the correct option is (D).

Quick Tip to Solve: First compare oxidation state and geometry, then use ligand strength; six cyanide ligands make the cobalt(III) complex the clear choice here.

72. When the first electron gain enthalpy $(\Delta_{eg}H)$ of oxygen is $-$ 141 kJ/mol, its second electron gain enthalpy is : [4 marks]

(A) a more negative value than the first

(B) almost the same as that of the first

(C) negative, but less negative than the first

(D) a positive value

Correct Answer: (D) a positive value

Solution:

Concept:

- Electron gain enthalpy is the enthalpy change when an electron is added to an isolated gaseous species. A negative value means energy is released; a positive value means energy must be supplied.
- The first electron is added to a neutral atom, whereas the second is added to an already negatively charged ion. Electron–electron repulsion makes formation of a gaseous dianion endothermic.

Step 1: Distinguish the two electron-addition processes The given value refers specifically to the first process:

$$O(g) + e^- \longrightarrow O^-(g), \qquad \Delta_{eg}H_1 = -141\,kJ\,mol^{-1}.$$

The electron is attracted by the neutral oxygen atom, and the net energy change for this step is negative. The second electron, however, is not being added to neutral oxygen, so the same value and sign cannot simply be carried over.

Step 2: Account for the repulsion in the second addition The second process is

$$O^-(g) + e^- \longrightarrow O^{2-}(g).$$

Both the incoming electron and the ion have negative charge. Work must therefore be done against their electrostatic repulsion, and the second electron gain enthalpy is positive. The stability of oxide ions in ionic crystals does not reverse this gaseous sign: lattice formation supplies a separate, strongly exothermic contribution. Options (A), (B) and (C) all retain a negative second enthalpy and so describe the wrong sign.

Step 3: Conclusion The second electron gain enthalpy of oxygen is a positive value. Hence, the

correct option is (D).

Quick Tip to Solve: Always write the charge of the species accepting the electron: addition to a gaseous negative ion requires energy, even when the final ion is stable in a crystal.

73. For the reaction, 2A + B $\rightarrow$ products, when the concentrations of A and B both were doubled, the rate of the reaction increased from 0.3 mol $L^{-1}s^{-1}$ to 2.4 mol $L^{-1}s^{-1}$. When the concentration of A alone is doubled, the rate increased from 0.3 mol $L^{-1}s^{-1}$ to 0.6 mol $L^{-1}s^{-1}$. [4 marks]

(A) Total order of the reaction is 4

(B) Order of the reaction with respect to B is 2

(C) Order of the reaction with respect to B is 1

(D) Order of the reaction with respect to A is 2

Correct Answer: (B) Order of the reaction with respect to B is 2

Solution:

Concept:

- Write the rate law as $r = k[\mathrm{A}]^m[\mathrm{B}]^n$, where m and n are experimentally determined orders. The coefficients in a balanced overall reaction do not, by themselves, determine these exponents.
- At fixed temperature the rate constant cancels in a rate ratio. Doubling one concentration multiplies the rate by 2 raised to that reactant's order.

Step 1: Find the order with respect to A When only $[\mathrm{A}]$ is doubled, the concentration of B stays fixed. Divide the new rate by the original rate:

$$\frac{r_2}{r_1} = 2^m = \frac{0.6}{0.3} = 2.$$

Consequently, $m = 1$. This directly excludes the statement that the order with respect to A is two; the stoichiometric coefficient 2 in the reaction must not be mistaken for its kinetic order.

Step 2: Use the simultaneous doubling experiment When both concentrations are doubled,

$$2^{m+n} = \frac{2.4}{0.3} = 8 = 2^3.$$

Thus $m + n = 3$, and with $m = 1$, $n = 2$. The rate law is therefore $r = k[\mathrm{A}][\mathrm{B}]^2$. As a check, doubling A contributes a factor of 2 and doubling B contributes a factor of 4, giving the observed factor of 8. The total order is three, not four, and the order in B is two, not one.

Step 3: Conclusion The order of reaction with respect to B is 2. Hence, the correct option is (B).

Quick Tip to Solve: Use the single-reactant doubling first; divide the combined rate factor by that factor to isolate the effect of the other reactant.

74. The major product obtained in the following reaction is : [4 marks]



(A) $C_6H_5C(CH_3)=C(CH_3)NHCOCH_3$

(B) o-$CH_3COC_6H_4C(CH_3)(OH)CH(NH_2)CH_3$

(C) $C_6H_5C(CH_3)(OCOCH_3)CH(NH_2)CH_3$

(D) $C_6H_5C(CH_3)(OH)CH(NHCOCH_3)CH_3$

Correct Answer: (D) $C_6H_5C(CH_3)(OH)CH(NHCOCH_3)CH_3$

Solution:

Concept:

- Write $Ph = C_6H_5$ for the phenyl group; this abbreviation keeps the carbon skeleton clear.
- Acetic anhydride is an acylating reagent: a nucleophile attacks an anhydride carbonyl, and acetate leaves. A primary amine gives an N-acetyl derivative, while an alcohol can give an acetate ester.
- In an amino alcohol, the amino nitrogen is more nucleophilic than the neutral alcohol oxygen. One equivalent of acetic anhydride under mild conditions therefore favours selective N-acetylation; pyridine accepts the acid formed.

Step 1: Identify the competing functional groups The substrate is $PhC(CH_3)(OH)CH(NH_2)CH_3$. It contains both an amino group and an alcoholic hydroxyl group. The reagent is one equivalent of acetic anhydride in pyridine at room temperature. These conditions favour attack by the more nucleophilic amine on an anhydride carbonyl, rather than substitution on the aromatic ring or dehydration of the alcohol.

Step 2: Follow the acetyl group onto nitrogen Nucleophilic acyl substitution replaces one N–H bond by an acetyl group:

$$RNH_2 + (CH_3CO)_2O \longrightarrow RNHCOCH_3 + CH_3COOH.$$

Here R denotes the unchanged carbon skeleton of the amino alcohol. The product has the following condensed structure:

$$PhC(CH_3)(OH)CH(NHCOCH_3)CH_3.$$

In this product, the hydroxyl remains free and the amino group becomes an amide. Option (C) instead acetylates oxygen while leaving the more reactive amine untouched. Option (B) requires aromatic ring acylation, which is not caused by these conditions, and option (A) additionally dehydrates the alcohol without an appropriate dehydration treatment.

Step 3: Conclusion The major product is the N-acetyl amino alcohol with the original hydroxyl group retained, $PhC(CH_3)(OH)CH(NHCOCH_3)CH_3$. Hence, the correct option is (D).

Quick Tip to Solve: With one equivalent of an acylating reagent and both amino and hydroxyl groups present, check the more nucleophilic nitrogen before selecting an O-acetyl product.

75. For the following reaction, in the mass of water produced from 445 g of $C_{57}H_{110}O_6$ is :

$2C_{57}H_{110}O_6(s) + 163\ O_2(g) \rightarrow 114\ CO_2(g) + 110\ H_2O(l)$ [4 marks]

(A) 490 g

(B) 445 g

(C) 495 g

(D) 890 g

Correct Answer: (C) 495 g

Solution:

Concept:

- A balanced equation gives mole ratios rather than mass ratios. Convert the stated mass of reactant into moles, apply the coefficient ratio, and finally convert the product moles into mass.
- Take atomic molar masses $C = 12$, $H = 1$ and $O = 16$, in $g\,mol^{-1}$. The equation shows that two moles of $C_{57}H_{110}O_6$ form 110 moles of water.

Step 1: Calculate the amount of the organic reactant The molar mass is

$$M(C_{57}H_{110}O_6) = 57(12) + 110(1) + 6(16) = 890\,g\,mol^{-1}.$$

Therefore the given 445 g corresponds to

$$n(C_{57}H_{110}O_6) = \frac{445}{890} = 0.500\,mol.$$

The calculation uses the amount of the organic reactant in the supplied equation, with oxygen available for its combustion.

Step 2: Apply the water ratio and convert to mass The balanced coefficients give

$$n(H_2O) = 0.500 \times \frac{110}{2} = 27.5\,mol.$$

Since $M(H_2O) = 18\,g\,mol^{-1}$,

$$m(H_2O) = 27.5(18) = 495\,g.$$

The mass need not equal the initial organic reactant mass, since oxygen from the gas also enters the products. Thus 445 g is not justified by conservation of mass for water alone. The 490 g and 890 g alternatives do not follow the mole ratio and molar-mass conversion.

Step 3: Conclusion The mass of water produced is $495\,g$. Hence, the correct option is (C).

Quick Tip to Solve: Reduce the coefficient ratio first: each mole of this reactant gives 55 moles of water, so the half-mole sample gives 27.5 moles directly.

76. The products formed in the reaction of cumene with O_2 followed by treatment with dil. HCl are : [4 marks]

(A) Benzene and acetone: C_6H_6 and CH_3COCH_3.

(B) Acetophenone and methanol: $C_6H_5COCH_3$ and CH_3OH.

(C) Phenol and acetone: C_6H_5OH and CH_3COCH_3.

(D) Phenol and propan-2-ol: C_6H_5OH and $CH_3CH(OH)CH_3$.

Correct Answer: (C) Phenol and acetone: C_6H_5OH and CH_3COCH_3.

Solution:

Concept:

- Cumene is isopropylbenzene, $C_6H_5CH(CH_3)_2$. Oxidation by molecular oxygen gives cumene hydroperoxide at its benzylic tertiary carbon.
- Acid-catalysed cleavage of cumene hydroperoxide, the Hock rearrangement, produces phenol and acetone. The aromatic group migrates to oxygen during cleavage, creating the phenolic C–O bond.

Step 1: Form the hydroperoxide intermediate Oxygen reacts at the benzylic C–H position of cumene to give $C_6H_5C(CH_3)_2OOH$. This first stage is an oxidation, not simple substitution of the aromatic ring. The intermediate retains both methyl groups on the original side-chain carbon, which is useful for tracking the three-carbon fragment into the final ketone.

Step 2: Cleave the hydroperoxide in dilute acid Treatment with dilute hydrochloric acid promotes rearrangement and cleavage. The phenyl group becomes bonded to oxygen and yields phenol; the remaining central carbon and its two methyl groups yield acetone:

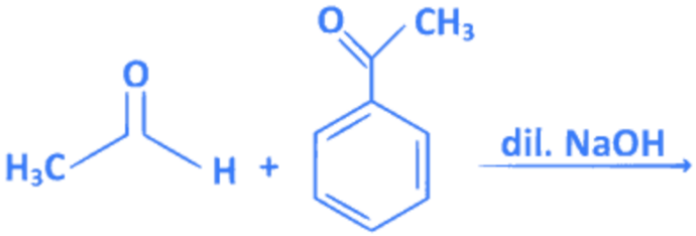
$$C_6H_5C(CH_3)_2OOH \xrightarrow{H^+} C_6H_5OH + CH_3COCH_3.$$

Option (A) lacks the hydroxyl group on the benzene ring. Option (B) predicts acetophenone and methanol, which do not represent this rearrangement. Option (D) has propan-2-ol instead of acetone and would require a reduction of the carbonyl product. No reducing reagent is supplied in the sequence.

Step 3: Conclusion The products are phenol, C_6H_5OH, and acetone, CH_3COCH_3. Hence, the correct option is (C).

Quick Tip to Solve: Remember the paired products of the cumene process together: the aromatic fragment gives phenol and the isopropyl fragment gives acetone.

77. The major product in the following reaction is : [4 marks]



(A) $CH_3CH(OH)CH_2CHO$

(B) $OHCCH_2C(OH)(CH_3)C_6H_5$

(C) $CH_3CH(OH)CH_2COC_6H_5$

(D) $C_6H_5COCH_2C(OH)(CH_3)C_6H_5$

Correct Answer: (C) $CH_3CH(OH)CH_2COC_6H_5$

Solution:

Concept:

- Write $Ph = C_6H_5$ for the phenyl group; this abbreviation keeps the carbon skeleton clear.
- Dilute aqueous base generates an enolate from a carbonyl compound containing alpha hydrogen. The enolate carbon attacks another carbonyl carbon; protonation then gives a beta-hydroxy carbonyl compound.
- In a crossed aldol reaction of an aromatic methyl ketone and an aldehyde, the aldehyde is the more electrophilic carbonyl partner. Acetophenone enolate attacking acetaldehyde gives the crossed beta-hydroxy ketone.

Step 1: Generate the nucleophile and identify the electrophile The reactants are acetaldehyde, CH_3CHO, and acetophenone, $PhCOCH_3$. Hydroxide removes an alpha hydrogen from acetophenone to give its resonance-stabilised enolate. The enolate carbon is nucleophilic. Acetaldehyde has a less hindered and more electrophilic carbonyl than the ketone, so it is the preferred acceptor in the crossed addition represented by the options.

Step 2: Connect the enolate carbon to the aldehyde carbonyl The acetophenone alpha carbon bonds to the carbonyl carbon of acetaldehyde. After protonation, the latter carbon bears OH and its original methyl group:

$$PhCOCH_2^- + CH_3CHO \longrightarrow PhCOCH_2CH(O^-)CH_3$$

$$PhCOCH_2CH(O^-)CH_3 \xrightarrow{H_2O} PhCOCH_2CH(OH)CH_3.$$

This is the beta-hydroxy ketone in option (C). The dilute-base conditions specify aldol addition; heating for dehydration is not shown. Option (A) is the acetaldehyde self-aldol product, option (B) uses the less electrophilic ketone as acceptor and retains an aldehyde group, and option (D) is the acetophenone self-aldol product.

Step 3: Conclusion The major product is $PhCOCH_2CH(OH)CH_3$, equivalently $CH_3CH(OH)CH_2COPh$. Hence, the correct option is (C).

Quick Tip to Solve: Trace the donor carbonyl unchanged and turn the acceptor carbonyl into OH; this distinguishes the two possible crossed aldol connectivities immediately.

78. Homoleptic octahedral complexes of a metal ion 'M^{3+}' with three monodenate ligands L_1, L_2 and L_3 adsorb wavelenths in the region of green, blue and red respectively. The increasing order of the ligands strength is : [4 marks]

(A) $L_3 < L_1 < L_2$

(B) $L_3 < L_2 < L_1$

(C) $L_1 < L_2 < L_3$

(D) $L_2 < L_1 < L_3$

Correct Answer: (A) $L_3 < L_1 < L_2$

Solution:

Concept:

- In an octahedral complex, the ligand field separates the five metal d orbitals into the lower t_{2g} and upper e_g sets. Their energy gap is Δ_o.
- For an electronic transition between these sets, the absorbed photon satisfies $\Delta_o = h\nu = hc/\lambda$, where λ is its wavelength. For the same metal ion and geometry, a stronger ligand gives a larger splitting.

Step 1: Order the absorbed wavelengths The question gives the colours of the radiation absorbed, rather than the colours transmitted or observed. In the visible spectrum, red has a longer wavelength than green, and green has a longer wavelength than blue. Therefore, for the complexes containing the respective ligands,

$$\lambda_3 > \lambda_1 > \lambda_2.$$

No complementary-colour conversion is needed because the absorption colours are already specified.

Step 2: Convert wavelength order into energy order Photon energy varies inversely with wavelength. Since h and c are positive constants common to all three comparisons, reversing the wavelength order gives

$$\Delta_o(L_3) < \Delta_o(L_1) < \Delta_o(L_2).$$

Thus the red-absorbing complex has the smallest ligand-field gap and the blue-absorbing complex has the largest. Options (B), (C) and (D) each violate at least one of these pairwise comparisons; treating a longer wavelength as a larger energy would incorrectly reverse the order.

Step 3: Conclusion The increasing order of ligand strength is $L_3 < L_1 < L_2$. Hence, the correct option is (A).

Quick Tip to Solve: When absorption colours are given, order their wavelengths first and reverse that order for ligand-field strength; use complementary colours only for observed colours.

79. Consider the following reversible chemical reactions :

The relation between K_1 and K_2 is : [4 marks]

$$A_2(g) + B_2(g) \underset{}{\overset{K_1}{\rightleftharpoons}} 2AB(g)..... (1)$$

$$6AB(g) \underset{}{\overset{K_2}{\rightleftharpoons}} 3A_2(g) + 3B_2(g)..... (2)$$

(A) $K_1K_2 = \frac{1}{3}$

(B) $K_2 = {K_1}^3$

(C) $K_2 = {K_1}^{-3}$

(D) $K_1K_2 = 3$

Correct Answer: (C) $K_2 = {K_1}^{-3}$

Solution:

Concept:

- For a reaction at a fixed temperature, the equilibrium constant is the product of equilibrium activities of products divided by those of reactants, each raised to its stoichiometric coefficient.
- Reversing a reaction replaces K by $1/K$. Multiplying every stoichiometric coefficient by n raises its equilibrium constant to the power n; the constant is not multiplied by n.

Step 1: Write the first equilibrium expression For the first reaction, $\mathrm{A_2(g) + B_2(g) \rightleftharpoons 2\,AB\,(g)}$, the constant is

$$K_1 = \frac{a_{\mathrm{AB}}^2}{a_{\mathrm{A_2}} a_{\mathrm{B_2}}}.$$

The same algebra can be written using equilibrium concentrations or partial pressures consistently. Both reactions have zero change in total gaseous stoichiometric amount, so the requested relationship has no additional pressure-concentration conversion factor.

Step 2: Reverse and triple the reaction The second printed equation is $\mathrm{6\,AB\,(g) \rightleftharpoons 3\,A_2(g) + 3\,B_2(g)}$. It is precisely three times the reverse of reaction (1). Consequently,

$$K_2 = \frac{a_{\mathrm{A_2}}^3 a_{\mathrm{B_2}}^3}{a_{\mathrm{AB}}^6} = \left(\frac{1}{K_1}\right)^3 = K_1^{-3}.$$

Option (B) includes the tripling but misses the reversal. Options (A) and (D) incorrectly treat the coefficient three as an ordinary multiplier instead of an exponent; no universal product K_1K_2 equal to either $1/3$ or 3 follows.

Step 3: Conclusion The required relation is $K_2 = K_1^{-3}$. Hence, the correct option is (C).

Quick Tip to Solve: Compare equations before writing expressions: reverse means reciprocal, and scaling all coefficients means taking a power.

80. The correct match between Item I and Item II is:

Item I		Item II	
(A)	Benzaldehyde	(P)	Mobile phase
(B)	Alumina	(Q)	Adsorbent
(C)	Acetonitrile	(R)	Adsorbate

[4 marks]

(A) $(A) \rightarrow (Q);\ (B) \rightarrow (P);\ (C) \rightarrow (R)$

(B) $(A) \rightarrow (R);\ (B) \rightarrow (Q);\ (C) \rightarrow (P)$

(C) $(A) \rightarrow (Q);\ (B) \rightarrow (R);\ (C) \rightarrow (P)$

(D) $(A) \rightarrow (P);\ (B) \rightarrow (R);\ (C) \rightarrow (Q)$

Correct Answer: (B) $(A) \rightarrow (R);\ (B) \rightarrow (Q);\ (C) \rightarrow (P)$

Solution:

Concept:

- Adsorption chromatography separates substances through their different affinities for a stationary solid adsorbent and a moving liquid mobile phase.
- The adsorbent is the solid surface that holds molecules. The adsorbate is the substance retained on that surface; the mobile phase is the solvent that moves past the surface.

Step 1: Identify the solid stationary phase Alumina, Al_2O_3, is a standard polar solid used as an adsorbent in adsorption chromatography. Its surface can hold organic molecules through intermolecular interactions. Thus item (B), alumina, matches (Q), adsorbent. This single identification eliminates options (A), (C) and (D), which classify alumina as either the moving phase or the adsorbed solute.

Step 2: Assign the solute and solvent Benzaldehyde is the organic compound being separated and becomes adsorbed on the alumina surface. It is therefore the adsorbate: (A) matches (R). Acetonitrile is a liquid solvent that carries dissolved solute through the stationary bed, so it is the mobile phase: (C) matches (P). The fact that both benzaldehyde and acetonitrile are organic substances does not make their roles identical; their functions in the separation distinguish the solute from the eluting solvent. Combining all three assignments gives

$$(A) \rightarrow (R), \qquad (B) \rightarrow (Q), \qquad (C) \rightarrow (P).$$

This checks the complete match rather than relying only on a single correct pair.

Step 3: Conclusion Benzaldehyde is the adsorbate, alumina the adsorbent and acetonitrile the mobile phase. Hence, the correct option is (B).

Quick Tip to Solve: Remember the roles by physical function: the solid adsorbs, the solute is adsorbed, and the solvent moves.

81. The entropy change associated with the conversion of 1 kg of ice at 273 K to water vapours at 383 K is :

(Specific heat of water liquid and water vapour are 4.2 kJ K^{-1} kg^{-1} and 2.0 kJ K^{-1} kg^{-1}; heat of liquid fusion and vapourisation of water are 334 kJ^{-1} and 2491 kJ kg^{-1}, respectively). (log 273 = 2.436, log 373 = 2.572, log 383 = 2.583) [4 marks]

(A) 7.90 kJ kg^{-1} K^{-1}

(B) 2.64 kJ kg^{-1} K^{-1}

(C) 8.49 kJ kg^{-1} K^{-1}

(D) 9.26 kJ kg^{-1} K^{-1}

Correct Answer: (D) 9.26 kJ kg^{-1} K^{-1}

Solution:

Concept:

- Entropy is a state function, so the total change can be calculated along a reversible path connecting the initial ice at 273 K and final water vapour at 383 K.
- At a reversible phase transition, $\Delta S = mL/T$. For heating a phase with constant specific heat, $\Delta S = mc_p \ln(T_2/T_1)$. Use absolute temperatures and $\ln x = 2.303 \log_{10} x$.

Step 1: Include melting and heating the liquid For $m = 1\,\text{kg}$, first melt the ice at 273 K, then heat the resulting water from 273 K to 373 K. The two contributions are

$$\Delta S_1 = \frac{334}{273} = 1.223\,\text{kJ K}^{-1},$$

$$\Delta S_2 = 4.2(2.303)(2.572 - 2.436) = 1.315\,\text{kJ K}^{-1}.$$

The logarithm is required because the temperature changes continuously during heating; dividing the heat for this stage by either endpoint temperature would be incorrect.

Step 2: Include vaporisation and superheating Vaporise the water reversibly at 373 K and then heat the vapour from 373 K to 383 K. This gives

$$\Delta S_3 = \frac{2491}{373} = 6.678\,\text{kJ K}^{-1},$$

$$\Delta S_4 = 2.0(2.303)(2.583 - 2.572) = 0.0507\,\text{kJ K}^{-1}.$$

Add all four positive contributions:

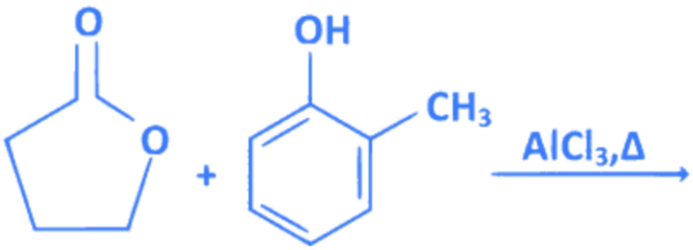
$$\Delta S = 1.223 + 1.315 + 6.678 + 0.0507 \simeq 9.26\,\text{kJ K}^{-1}.$$

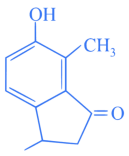
Since the sample mass is 1 kg, its numerical specific entropy change is also 9.26 in the printed specific units. The other options omit or miscalculate at least one contribution; in particular, vaporisation is the largest contribution and must not be left out.

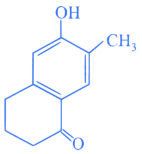
Step 3: Conclusion The entropy increases by approximately $9.26\,\text{kJ K}^{-1}$ for the 1 kg sample, or $9.26\,\text{kJ kg}^{-1}\,\text{K}^{-1}$ per unit mass. Hence, the correct option is (D).

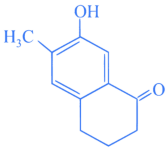
Quick Tip to Solve: Draw a four-stage thermal path mentally: melt, heat liquid, vaporise, heat vapour. Use latent heat divided by transition temperature only on the two phase-change stages.

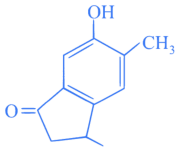
82. The major product of the following reaction is : [4 marks]



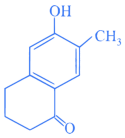
(A) 6-Hydroxy-3,7-dimethylindan-1-one 

(B) 6-Hydroxy-7-methyl-1-tetralone 

(C) 7-Hydroxy-6-methyl-1-tetralone 

(D) 6-Hydroxy-3,5-dimethylindan-1-one 

Correct Answer: (B)

6-Hydroxy-7-methyl-1-tetralone 

Solution:

Concept:

- Anhydrous $AlCl_3$ activates an ester or lactone carbonyl towards electrophilic aromatic acylation. The phenolic hydroxyl group directs substitution predominantly to an available ortho or para position.
- The five-membered lactone is gamma-butyrolactone. Its carbonyl carbon and three methylene carbons provide a four-carbon chain; subsequent intramolecular cyclisation can incorporate two adjacent aromatic carbons into a six-membered ring.

Step 1: Place the acyl group on the activated aromatic ring The aromatic reactant is ortho-cresol: its methyl group occupies one of the two positions ortho to OH. Activation of the lactone by $AlCl_3$ allows ring-opening acylation, introducing the gamma-hydroxybutanoyl chain, $-COCH_2CH_2CH_2OH$, at the less hindered para position relative to the phenolic hydroxyl group. Thus the carbonyl carbon is attached directly to the original para carbon of the aromatic ring. This fixes which end of the eventual

fused ring carries the ketone.

Step 2: Close the six-membered ring and compare structures Under the heated Lewis-acid conditions, activation of the terminal hydroxyl group permits intramolecular electrophilic substitution at an adjacent aromatic carbon. The ring formed contains the carbonyl carbon, three methylene-derived carbons and two aromatic fusion carbons: six ring atoms in total. Cyclisation occurs on the side farther from the original methyl substituent. The resulting tetralone has the phenolic hydroxyl at position 6 and the methyl at position 7, as in structure (B). Options (A) and (D) instead show a five-membered ketone ring and an extra methyl branch, which do not follow from this unbranched four-carbon acyl chain. Option (C) reverses the position of the carbonyl relative to the phenolic group and would not result from para acylation.

Step 3: Conclusion The major product is 6-hydroxy-7-methyl-1-tetralone, the fused six-membered cyclic ketone shown in (B). Hence, the correct option is (B).

Quick Tip to Solve: For lactone annulation, track the carbonyl end first and count the chain carbons before choosing between a five- and a six-membered fused ring.

83. In which of the following process, the bond order has increased and paramagnetic character has charged to diamagnetic ? [4 marks]

(A) $NO \rightarrow NO^+$

(B) $N_2 \rightarrow {N_2}^+$

(C) $O_2 \rightarrow {O_2}^+$

(D) $O_2 \rightarrow {O_2}^{2-}$

Correct Answer: (A) $NO \rightarrow NO^+$

Solution:

Concept:

- Molecular orbital bond order is $\mathrm{BO} = (N_b - N_a)/2$, where N_b and N_a are the numbers of electrons in bonding and antibonding orbitals.
- A species is paramagnetic when it contains an unpaired electron and diamagnetic when all its electrons are paired. Removing an antibonding electron raises bond order by one half; removing a bonding electron lowers it.

Step 1: Check the nitric oxide ionisation Neutral NO has eleven valence electrons and one unpaired electron in an antibonding π^* orbital. It is paramagnetic and has bond order 2.5. Ionisation removes this antibonding electron:

$$\mathrm{NO} \longrightarrow \mathrm{NO^+} + \mathrm{e^-}, \qquad \mathrm{BO} : 2.5 \longrightarrow 3.$$

The resulting NO^+ has ten valence electrons, is isoelectronic with N_2, and has all electrons paired. Hence both requested changes occur together.

Step 2: Eliminate the other ionisations and reduction In $N_2 \longrightarrow {N_2}^+$, an electron is removed from a bonding orbital. Bond order falls from 3 to 2.5 and the product is paramagnetic, so this fails both

requirements. In $O_2 \longrightarrow {O_2}^+$, removing one of the two unpaired antibonding electrons raises bond order from 2 to 2.5, but one unpaired electron remains; the cation is still paramagnetic. Finally, adding two electrons to O_2 produces ${O_2}^{2-}$, pairing the electrons in the antibonding orbitals. It becomes diamagnetic, but its bond order decreases from 2 to 1. Thus neither increasing bond order alone nor becoming diamagnetic alone is enough.

Step 3: Conclusion Only $NO \longrightarrow NO^+$ increases bond order while changing a paramagnetic species into a diamagnetic one. Hence, the correct option is (A).

Quick Tip to Solve: For each option, check both tests separately: whether the changed electron is bonding or antibonding, and whether any unpaired electron remains.

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84. At 100°C, copper (Cu) has FCC unit cell structure with cell edge length of x $\overset{o}{A}$. What is the approximate density of Cu (in g cm^{-3}) at this temperature?

[Atomic Mass of Cu = 63.55 u] [4 marks]

(A) $\frac{205}{x^3}$

(B) $\frac{105}{x^3}$

(C) $\frac{211}{x^3}$

(D) $\frac{422}{x^3}$

Correct Answer: (D) $\frac{422}{x^3}$

Solution:

Concept:

- The density of a crystal is the mass contained in one unit cell divided by its volume: $\rho = ZM/(N_A a^3)$, where Z is the atoms per cell, M the molar mass and a the cell edge.
- An FCC unit cell contains $8(1/8)+6(1/2) = 4$ atoms. To obtain density in $\mathrm{g\,cm^{-3}}$, use M in $\mathrm{g\,mol^{-1}}$ and convert 1 angstrom $= 10^{-8}$ cm.

Step 1: Find the unit-cell mass and volume Copper has four atoms per FCC unit cell. With the given atomic mass, its molar mass is $63.55\,\mathrm{g\,mol^{-1}}$, so

$$m_{\text{cell}} = \frac{4 \times 63.55}{6.022 \times 10^{23}}\ \mathrm{g}.$$

At the specified temperature the edge length is already supplied as x angstrom. Thus no thermal-expansion correction is needed. Converting and then cubing gives

$$a = x \times 10^{-8}\ \mathrm{cm}, \qquad V_{\text{cell}} = x^3 \times 10^{-24}\ \mathrm{cm^3}.$$

Step 2: Divide the cell mass by its volume Substitute these quantities without rounding prematurely:

$$\rho = \frac{4(63.55)}{(6.022 \times 10^{23})(x^3 \times 10^{-24})} = \frac{254.20}{0.6022x^3}\ \mathrm{g\,cm^{-3}}.$$

Therefore,

$$\rho \simeq \frac{422.1}{x^3}\,\mathrm{g\,cm^{-3}}.$$

The coefficient near 105 would correspond to counting only one atom per cell, while the coefficient near 211 counts only two. These are not the FCC atom count. The coefficient 205 likewise fails the given molar-mass and unit-cell calculation. The inverse cube dependence is expected because changing the edge changes the volume cubically while the mass per unit cell remains fixed.

Step 3: Conclusion The approximate density is $\rho = 422/x^3\,\mathrm{g\,cm^{-3}}$. Hence, the correct option is (D).

Quick Tip to Solve: Write the FCC atom count and cube the length-conversion factor explicitly; most density errors come from using two atoms or forgetting that the volume conversion is cubed.

85. The metal that forms nitride by reacting directly with N_2 of air, is : [4 marks]

(A) K

(B) Li

(C) Rb

(D) Cs

Correct Answer: (B) Li

Solution:

Concept:

- Lithium is the exceptional alkali metal that combines directly with atmospheric nitrogen to form the stable ionic nitride Li_3N.
- The small size of Li^+ gives lithium nitride a large lattice enthalpy, helping compensate for the energy required to break the strong nitrogen triple bond.

Step 1: Identify the exceptional alkali metal All four choices are Group 1 metals, so their usual oxidation state is $+1$. However, similar valence configurations do not imply identical reactions with nitrogen. Lithium is the smallest member and its nitride is sufficiently stable to form by direct combination with N_2. This is a characteristic anomalous behaviour of lithium among the alkali metals.

Step 2: Write the direct combination reaction The nitride ion is N^{3-}, so three lithium ions are needed for electrical neutrality. The formula is therefore Li_3N, and the balanced reaction is

$$6\,Li + N_2 \longrightarrow 2\,Li_3N.$$

Here lithium is oxidised from zero to $+1$, while nitrogen is reduced from zero to -3. The lattice stability of the product accounts for the feasibility of this direct reaction.

Step 3: Eliminate the remaining metals Potassium, rubidium and caesium have much larger cations. They do not show this characteristic direct formation of a stable simple nitride on reaction with nitrogen in air. Their high reactivity towards oxygen or water is not a reason to choose them for the specific nitrogen reaction asked here.

Step 4: Conclusion The metal is lithium, which forms Li_3N directly from nitrogen. Hence, the correct

option is (B).

Quick Tip to Solve: For direct nitride formation in Group 1, remember lithium as the exception; balance the product using Li^+ and N^{3-}.

86. The temporary hardness of water is due to : [4 marks]

(A) Na_2SO_4

(B) NaCl

(C) $Ca(HCO_3)_2$

(D) $CaCl_2$

Correct Answer: (C) $Ca(HCO_3)_2$

Solution:

Concept:

- Hardness arises mainly from dissolved calcium and magnesium ions, which form insoluble salts with soap and reduce its lathering ability.
- Temporary hardness is caused by calcium and magnesium hydrogencarbonates. It can be removed by boiling, unlike the usual chloride and sulphate hardness.

Step 1: Recognise the hydrogencarbonate salt Among the choices, $Ca(HCO_3)_2$ contains both a hardness-producing cation, Ca^{2+}, and the hydrogencarbonate anion. Dissolution supplies calcium ions, so soap is consumed in precipitating calcium salts before a lasting lather forms. This composition identifies the salt associated with temporary hardness.

Step 2: Explain why the hardness is temporary On boiling, calcium hydrogencarbonate decomposes and insoluble calcium carbonate is removed as a precipitate:

$$Ca(HCO_3)_2 \xrightarrow{\text{heat}} CaCO_3\downarrow + CO_2\uparrow + H_2O.$$

Since dissolved calcium is removed from the water, the hardness decreases. The word temporary refers to this removal by boiling; it does not mean that the water becomes soft merely by standing for a short time.

Step 3: Compare the other choices Na_2SO_4 and $NaCl$ supply sodium ions rather than the calcium or magnesium ions responsible for ordinary hardness. They therefore do not cause the specified temporary hardness. $CaCl_2$ does supply calcium ions, but its hardness is permanent: ordinary boiling does not precipitate the calcium through hydrogencarbonate decomposition.

Step 4: Conclusion Temporary hardness is due to calcium hydrogencarbonate, $Ca(HCO_3)_2$. Hence, the correct option is (C).

Quick Tip to Solve: Check both ions: calcium or magnesium makes water hard, while hydrogencarbonate makes that hardness removable by boiling.

87. For coagulation of arsenious sulphate sol, which one of the following salt solution will be most effective ? [4 marks]

(A) $BaCl_2$

(B) $AlCl_3$

(C) NaCl

(D) Na_3PO_4

Correct Answer: (B) $AlCl_3$

Solution:

Concept:

- The arsenious sol in this coagulation question is a negatively charged sol. Its coagulating ions are therefore the positive ions supplied by an electrolyte.
- By the Hardy–Schulze rule, a counter-ion of greater charge has greater coagulating power. For a negative sol, compare cation valencies rather than anion valencies.

Step 1: Identify the ions that neutralise the sol charge Coagulation occurs when the charge stabilising the colloidal particles is sufficiently neutralised, allowing particles to aggregate and settle. Since the arsenious sol is negatively charged, cations are attracted to its surface and act as the effective counter-ions. Comparing the total number of ions in a salt formula would miss the decisive role of counter-ion charge.

Step 2: List the relevant cations The salts supply Ba^{2+} from $BaCl_2$, Al^{3+} from $AlCl_3$, and Na^+ from both $NaCl$ and Na_3PO_4. Their positive charges are respectively 2, 3, 1 and 1. Thus the aluminium ion has the highest counter-ion valency in this list.

Step 3: Apply the coagulating-power order The Hardy–Schulze comparison gives

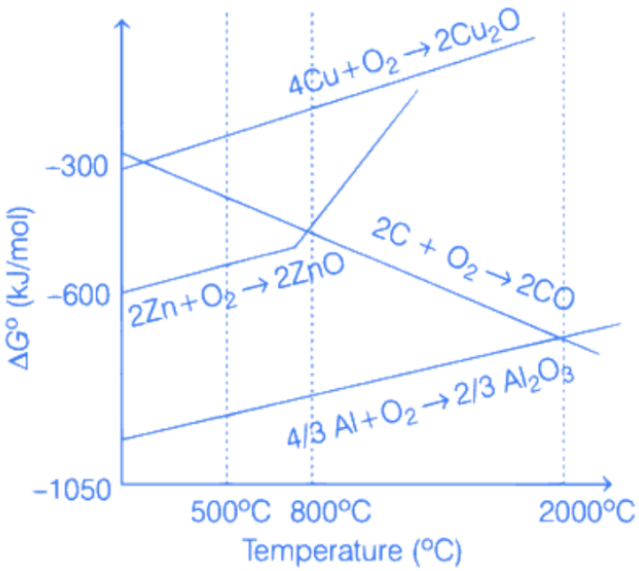
$$Al^{3+} > Ba^{2+} > Na^+$$

for coagulating power towards this negative sol. Consequently aluminium chloride is most effective. The $PO_4{}^{3-}$ ion in sodium phosphate is highly charged but has the same sign as the sol; its trivalent charge does not make sodium phosphate the correct choice. Barium chloride is stronger than a sodium salt but weaker than aluminium chloride under this rule.

Step 4: Conclusion $AlCl_3$, through its trivalent Al^{3+} ion, is the most effective coagulant. Hence, the correct option is (B).

Quick Tip to Solve: First determine the sol charge, then compare only the oppositely charged ions. A trivalent co-ion is not a trivalent counter-ion.

88. The correct statement regarding the given Ellingham diagram is : [4 marks]



(A) At 1400°C, Al can be used for the extraction of Zn from ZnO

(B) At 500°C, coke can be used for the extraction of Zn from ZnO

(C) Coke cannot be used for the extraction of Cu from Cu_2O.

(D) At 800°C, Cu can be used for the extraction of Zn from ZnO.

Correct Answer: (A) At 1400°C, Al can be used for the extraction of Zn from ZnO

Solution:

Concept:

- An Ellingham diagram plots the standard Gibbs energy change for oxide formation against temperature. A lower oxidation line indicates a more favourable oxidation at that temperature.
- A reducing agent can remove oxygen from an oxide when its own oxidation line lies below that of the metal being extracted, giving a negative standard Gibbs energy for the combined reduction.

Step 1: Compare aluminium and zinc at the stated temperature At $1400^\circ C$, the aluminium-to-alumina line lies below the zinc-to-zinc-oxide line. Aluminium therefore has a greater affinity for oxygen at this temperature. Adding aluminium oxidation to the reverse of zinc oxidation gives a favourable overall reduction. In integer coefficients this is

$$2\,Al + 3\,ZnO \longrightarrow Al_2O_3 + 3\,Zn.$$

Thus the statement about aluminium extracting zinc at this temperature agrees with the diagram.

Step 2: Check coke against the zinc oxide line At $500^\circ C$, the carbon-to-carbon-monoxide line is above the zinc oxide formation line. Carbon oxidation then does not supply a sufficiently favourable Gibbs energy change to drive zinc oxide reduction under the standard comparison. The carbon and zinc lines cross near $800^\circ C$; the claim for $500^\circ C$ is therefore false.

Step 3: Test the copper statements The carbon oxidation line lies below the copper oxide formation line over the relevant region shown, so coke can reduce Cu_2O; the assertion that it cannot is false. At

$800^\circ C$, the copper oxide line is above the zinc oxide line, meaning copper has less oxygen affinity than zinc. Copper cannot serve as the indicated reducing agent for ZnO. The relative vertical positions, rather than line slopes alone, decide each comparison.

Step 4: Conclusion At $1400^\circ C$, aluminium can extract zinc from ZnO. Hence, the correct option is (A).

Quick Tip to Solve: At the specified temperature, draw a mental vertical line: the reducing agent must have its oxidation line below the oxide being reduced.

89. Which of the following conditions in drinking water causes methemoglobinemia? [4 marks]

(A) $>$ 50 ppm of lead

(B) $>$ 50 ppm of chloride

(C) $>$ 50 ppm of nitrate

(D) $>$ 100 ppm of sulphate

Correct Answer: (C) $>$ 50 ppm of nitrate

Solution:

Concept:

- Excess nitrate in drinking water is associated with methemoglobinemia, especially in infants. The characteristic examination threshold in the options is nitrate above 50 ppm.
- Nitrate can be reduced to nitrite in the body. Nitrite oxidises the iron of haemoglobin from Fe^{2+} to Fe^{3+}, producing methemoglobin with impaired oxygen-carrying capacity.

Step 1: Connect the contaminant with the named disorder The disorder specifically concerns conversion of haemoglobin into methemoglobin. Nitrate contamination is the relevant water-pollution cause because nitrate can generate nitrite in the digestive system. It is the subsequent oxidation of haemoglobin, rather than a simple change in the taste or salinity of the water, that links this ion to the disorder.

Step 2: Explain the oxidation-state change Normal haemoglobin contains iron in the ferrous state, Fe^{2+}. Oxidation changes it to the ferric state, Fe^{3+}, forming methemoglobin. This form cannot perform the normal oxygen-binding function at the affected sites, so sufficient conversion reduces effective oxygen delivery. This is why nitrate-contaminated drinking water is associated with the condition often called blue-baby syndrome.

Step 3: Match the concentration and exclude distractors The option stating more than 50 ppm of nitrate matches the specified association. Lead is a toxic heavy-metal contaminant with other serious effects, but it is not the nitrate-related cause being tested here. Chloride mainly affects salinity and taste at elevated concentrations, and sulphate is associated with gastrointestinal or laxative effects at excessive concentrations. Neither is the characteristic answer to methemoglobinemia in this list.

Step 4: Conclusion The relevant condition is nitrate concentration greater than 50 ppm in drinking water. Hence, the correct option is (C).

Quick Tip to Solve: Associate nitrate with nitrite formation and oxidation of haemoglobin iron; this mechanism distinguishes methemoglobinemia from other water-pollution effects.

90. Good reducing nature of H_3PO_2 is attributed to the presence of : [4 marks]

(A) Two $P-OH$ bonds

(B) One $P-H$ bond

(C) Two $P-H$ bonds

(D) One $P-OH$ bond

Correct Answer: (C) Two $P-H$ bonds

Solution:

Concept:

- Hypophosphorous acid, H_3PO_2, has the structural arrangement $HOP(=O)H_2$: one hydroxyl group and two hydrogen atoms bonded directly to phosphorus.
- The reducing character of phosphorus oxoacids is associated with their $P-H$ bonds. Hydroxyl hydrogens control acidity and must not be confused with the directly bonded hydrogens.

Step 1: Count the two kinds of hydrogen The molecular formula alone gives three hydrogens but does not say how they are attached. In hypophosphorous acid, only one hydrogen is attached to oxygen as $P-OH$; the other two are attached directly to phosphorus. Thus there are two $P-H$ bonds and one $P-OH$ group. Counting all three hydrogens as acidic would give an incorrect picture of the structure.

Step 2: Relate the structure to oxidation The phosphorus oxidation state in H_3PO_2 is obtained from

$$x + 3(+1) + 2(-2) = 0, \qquad x = +1.$$

The acid can be oxidised to higher phosphorus oxidation states while reducing another species. For example, its oxidation to phosphoric acid can be represented by

$$H_3PO_2 + 2\,H_2O \longrightarrow H_3PO_4 + 4\,H^+ + 4\,e^-.$$

This balanced oxidation half-reaction shows the loss of electrons underlying its reducing action.

Step 3: Select the structural feature The two directly bonded hydrogens are the feature sought. The choice with one $P-H$ bond has the wrong count. The choice with two $P-OH$ bonds is structurally false, while one $P-OH$ bond is present but explains its monobasic nature rather than the strong reducing character asked about.

Step 4: Conclusion The reducing nature of H_3PO_2 is attributed to its two $P-H$ bonds. Hence, the correct option is (C).

Quick Tip to Solve: For phosphorus oxoacids, count $P-OH$ groups for basicity and $P-H$ bonds for reducing character; the two counts answer different questions.