JEE Main 2019 Paper 1 (B.E./B.Tech) Apr 9 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 9 April 2019, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. If the sum and product of the first three term in an A.P. are 33 and 1155, respectively, then a value of its 11^{th} term is :- [4 marks]

(A) –25

(B) –36

(C) 25

(D) –35

Correct Answer: (A) –25

Solution:

Concept:

- Write three consecutive terms of an arithmetic progression symmetrically as $a-d, a, a+d$, where d is the common difference. Their sum is $3a$.
- The product of the outer terms is $(a-d)(a+d) = a^2 - d^2$. The eleventh term is the first term plus ten common differences.

Step 1: Use the sum to find the middle term The given three terms have sum 33. The symmetric form cancels the common difference and therefore determines the middle term directly:

$$(a-d) + a + (a+d) = 33 \quad \Longrightarrow \quad a = 11.$$

Thus the first term is $11-d$; it is not 11, since 11 is the second term.

Step 2: Use the product to find both differences The product condition now gives

$$(11-d)11(11+d) = 1155.$$

Divide by 11 and use the difference of squares:

$$121 - d^2 = 105, \qquad d^2 = 16, \qquad d = 4 \text{ or } -4.$$

Both signs are allowed because the question does not state that the progression is increasing.

Step 3: Compute the eleventh term and compare Counting from the actual first term gives

$$T_{11} = (11-d) + 10d = 11 + 9d.$$

For $d = 4$, this is 47; for $d = -4$, it is -25. The decreasing progression begins $15, 11, 7$, whose sum and product are indeed 33 and 1155. Among the displayed alternatives, only -25 occurs in this pair of possible values; -36, 25 and -35 do not.

Step 4: Conclusion A possible eleventh term is $\boxed{-25}$. Hence, the correct option is (A).

Quick Tip to Solve: For three A.P. terms with a known sum, use the middle term as the variable; retain both signs when taking the square root of d^2.

2. If $\cos x\frac{dy}{dx} - y\sin x = 6x$, $(0 < x < \frac{\pi}{2})$ and $y\left(\frac{\pi}{3}\right) = 0$ then $y\left(\frac{\pi}{6}\right)$ is equal to :- [4 marks]

(A) $-\frac{\pi^2}{2}$

(B) $-\frac{\pi^2}{4\sqrt{3}}$

(C) $\frac{\pi^2}{2\sqrt{3}}$

(D) $-\frac{\pi^2}{2\sqrt{3}}$

Correct Answer: (D) $-\frac{\pi^2}{2\sqrt{3}}$

Solution:

Concept:

- The product rule gives $\frac{d}{dx}(y\cos x) = \cos x\frac{dy}{dx} - y\sin x$. Recognising the full derivative avoids unnecessary division.
- An initial condition determines the integration constant. On $0 < x < \pi/2$, $\cos x > 0$, so dividing by $\cos x$ is valid.

Step 1: Recognise and integrate the derivative The left side of the given differential equation is precisely the derivative of $y\cos x$. Therefore

$$\frac{d}{dx}(y\cos x) = 6x.$$

Integrating with respect to x yields

$$y\cos x = 3x^2 + C.$$

The constant must be retained because different values of it give different solutions satisfying the differential equation.

Step 2: Apply the condition at the given point Substitute $x = \pi/3$ and $y(\pi/3) = 0$. The product on the left is zero, so

$$0 = 3\left(\frac{\pi}{3}\right)^2 + C = \frac{\pi^2}{3} + C.$$

Hence $C = -\pi^2/3$, and the required solution on the stated interval satisfies

$$y\cos x = 3x^2 - \frac{\pi^2}{3}.$$

This condition selects one member of the integrated family.

Step 3: Evaluate at the requested point At $x = \pi/6$, the numerator is

$$3\frac{\pi^2}{36} - \frac{\pi^2}{3} = -\frac{\pi^2}{4}.$$

Since $\cos(\pi/6) = \sqrt{3}/2$,

$$y\left(\frac{\pi}{6}\right) = \frac{-\pi^2/4}{\sqrt{3}/2} = -\frac{\pi^2}{2\sqrt{3}}.$$

This excludes (C) by its sign, (B) by its extra factor of two in the denominator, and (A) by its missing

factor of $\sqrt{3}$.

Step 4: Conclusion $y\left(\frac{\pi}{6}\right) = \boxed{-\frac{\pi^2}{2\sqrt{3}}}$. Hence, the correct option is (D).

Quick Tip to Solve: Before finding an integrating factor, check whether two adjacent terms already form a product derivative such as $(y\cos x)'$.

3. If f : R → R is a differentiable function and f(2) = 6,

then $\lim\limits_{x\to 2} \frac{\int_6^{f(x)} 2tdt}{(x-2)}$ **is :-** [4 marks]

(A) 2f'(2)

(B) 24f'(2)

(C) 0

(D) 12f'(2)

Correct Answer: (D) 12f'(2)

Solution:

Concept:

- The definite integral of $2t$ from 6 to $f(x)$ is $f(x)^2 - 36$; the upper limit is a function of x.
- Differentiability at 2 implies continuity there and $\lim_{x\to 2}[f(x) - f(2)]/(x-2) = f'(2)$.

Step 1: Evaluate and factor the numerator First perform the elementary integration with respect to the dummy variable t:

$$\int_6^{f(x)} 2t\,dt = [t^2]_6^{f(x)} = f(x)^2 - 36.$$

Because $f(2) = 6$, this numerator vanishes at $x = 2$. Factor it rather than substituting zero for the entire quotient:

$$f(x)^2 - 36 = (f(x) - 6)(f(x) + 6).$$

Step 2: Separate the derivative quotient For $x \neq 2$, the expression whose limit is sought becomes

$$\frac{f(x) - f(2)}{x - 2}\,[f(x) + 6].$$

The first factor tends to $f'(2)$ by the definition of derivative. For the second factor, differentiability guarantees $f(x) \to f(2) = 6$, so it tends to 12. No continuity of the derivative itself is needed for this argument.

Step 3: Take the limit and check the coefficients The limit of the product is consequently

$$f'(2)(6 + 6) = 12f'(2).$$

The coefficient 2 alone would ignore the upper limit value 6; the coefficient 24 would introduce an extra factor of two after the integral has already been evaluated. Although the numerator tends to

zero, its ratio to $x-2$ need not tend to zero, so option (C) does not follow. The derivative may itself be zero for a particular function, but the general expression is the one just obtained.

Step 4: Conclusion The required limit is $\boxed{12f'(2)}$. Hence, the correct option is (D).

Quick Tip to Solve: When a variable-limit integral appears over $x-a$, evaluate it and factor out $f(x)-f(a)$ to expose the derivative definition.

4. The value of sin 10° sin30° sin50° sin70° is :- [4 marks]

(A) $\frac{1}{36}$
(B) $\frac{1}{16}$
(C) $\frac{1}{32}$
(D) $\frac{1}{18}$

Correct Answer: (B) $\frac{1}{16}$

Solution:

Concept:

- The identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ relates three angles separated by 60°.
- Complementary-angle identities give $\sin(60^\circ-\theta)\sin(60^\circ+\theta) = 3/4 - \sin^2\theta$. Angles are measured in degrees here.

Step 1: Combine the factors at 50 and 70 degrees Use the product-to-sum identity with the two factors symmetric about 60°:

$$\sin 50^\circ \sin 70^\circ = \frac{\cos 20^\circ - \cos 120^\circ}{2} = \frac{\cos 20^\circ + 1/2}{2}.$$

Since $\cos 20^\circ = 1 - 2\sin^2 10^\circ$, this becomes

$$\sin 50^\circ \sin 70^\circ = \frac{3}{4} - \sin^2 10^\circ.$$

Step 2: Apply the triple-angle identity Multiply the last result by $\sin 10^\circ$. With $s = \sin 10^\circ$, we obtain

$$\sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{3s - 4s^3}{4} = \frac{\sin 30^\circ}{4} = \frac{1}{8}.$$

This is an exact calculation; no decimal approximation to any of the three sines is needed.

Step 3: Include the remaining factor The requested expression also contains the separate factor $\sin 30^\circ = 1/2$. Hence

$$\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}\cdot\frac{1}{2} = \frac{1}{16}.$$

The four factors are all positive, and the exact product has now been reduced to a single fraction. The other proposed fractions, $1/36$, $1/32$, and $1/18$, differ from this value; in particular, $1/32$ inserts an extra factor of one half.

Step 4: Conclusion The exact value is $\boxed{\frac{1}{16}}$. Hence, the correct option is (B).

Quick Tip to Solve: Look for factors at $\theta, 60^\circ - \theta, 60^\circ + \theta$; their product is $\sin 3\theta/4$.

5. The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34, x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to [4 marks]

(A) $\frac{7}{2}$
(B) $\frac{8}{3}$
(C) $\frac{9}{4}$
(D) $\frac{7}{3}$

Correct Answer: (D) $\frac{7}{3}$

Solution:

Concept:

- For ten ordered observations, the median is the average of the fifth and sixth observations, not either observation individually.
- The arithmetic mean equals the sum of all observations divided by their number; thus a mean of 42 for ten numbers gives total 420.

Step 1: Use the middle pair to find x The observations are already stated to be in increasing order. Their fifth and sixth entries are 34 and x. The median condition therefore gives

$$\frac{34 + x}{2} = 35 \quad \Longrightarrow \quad 34 + x = 70 \quad \Longrightarrow \quad x = 36.$$

This is consistent with its ordered position, since $34 < 36 < 42$. Taking $x = 35$ would incorrectly identify the median with one central observation.

Step 2: Use the total to find y Add the eight known observations, leaving out x and y:

$$10 + 22 + 26 + 29 + 34 + 42 + 67 + 70 = 300.$$

The mean of all ten numbers is 42, so

$$\frac{300 + x + y}{10} = 42.$$

After substituting $x = 36$, this becomes

$$336 + y = 420, \qquad y = 84.$$

The last entry is indeed greater than 70, so the increasing-order condition is preserved.

Step 3: Form and verify the ratio The requested ratio is

$$\frac{y}{x} = \frac{84}{36} = \frac{7}{3}.$$

As a check, the completed data have sum 420 and central-pair average $(34 + 36)/2 = 35$, satisfying both conditions independently. The other ratios $7/2$, $8/3$, and $9/4$ would give different values of the last observation with $x = 36$, and therefore would not reproduce the required mean of 42.

Step 4: Conclusion The ratio is $\boxed{\frac{7}{3}}$. Hence, the correct option is (D).

Quick Tip to Solve: For an even number of ordered observations, find the central unknown from the median first, then use the mean to determine the remaining end value.

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6. Two newspapers A and B are published in a city. It is known that 25% of the city populations reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisement is :- [4 marks]

(A) 13.5

(B) 13

(C) 12.8

(D) 13.9

Correct Answer: (D) 13.9

Solution:

Concept:

- Partition readers into the disjoint groups A only, B only, and both newspapers. A reader in the overlap must not be counted twice.
- To convert a percentage within a group into a percentage of the whole population, multiply the group percentage by the given fraction of interested readers.

Step 1: Find the three disjoint reader groups Take a total population of 100 people; the result then reads directly as a percentage. There are 25 readers of A, 20 readers of B, and 8 readers of both. Subtract the overlap from each separate readership:

$$A \text{ only} = 25 - 8 = 17, \qquad B \text{ only} = 20 - 8 = 12.$$

The group reading both contains 8 people. These groups together contain $17 + 12 + 8 = 37$ readers.

Step 2: Calculate advertisement readers in each group Apply each rate only to the group named with

it. The contributions per 100 city residents are

$$\begin{aligned} A \text{ only}: &\quad 0.30(17) = 5.1, \\ B \text{ only}: &\quad 0.40(12) = 4.8, \\ \text{Both}: &\quad 0.50(8) = 4.0. \end{aligned}$$

The 30 percent rate does not apply to all 25 A readers, and the 40 percent rate does not apply to all 20 B readers: doing that would incorrectly reuse the overlap.

Step 3: Add the contributions and compare options The three groups have no members in common, so their contributions can be added:

$$5.1 + 4.8 + 4.0 = 13.9.$$

Thus 13.9 out of every 100 residents look into the advertisements under the stated readership categories. The answer is a percentage of the city population, so we do not divide this total by 37, which would instead give a conditional percentage among newspaper readers. The alternatives 13.5, 13 and 12.8 do not equal the sum of the three correctly weighted contributions.

Step 4: Conclusion The required percentage is $\boxed{13.9\%}$. Hence, the correct option is (D).

Quick Tip to Solve: Whenever different rates apply to overlapping sets, subtract the intersection first and apply each rate to its own disjoint region.

7. If m is chosen in the quadratic equation

$(m^2 + 1)\,x^2 - 3x + (m^2 + 1)^2 = 0$

such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is :- [4 marks]

(A) $4\sqrt{3}$

(B) $8\sqrt{3}$

(C) $8\sqrt{5}$

(D) $10\sqrt{5}$

Correct Answer: (C) $8\sqrt{5}$

Solution:

Concept:

- For $ax^2 + bx + c = 0$, the sum of the roots is $-b/a$, and their product is c/a. Here m is a real parameter.
- For roots α, β, $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)$, and $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$.

Step 1: Maximise the sum of the roots For the given quadratic, Vieta's relations give

$$S = \alpha + \beta = \frac{3}{m^2 + 1}, \qquad P = \alpha\beta = m^2 + 1.$$

Since $m^2 \geq 0$, the denominator in S is at least one. The sum is greatest when $m = 0$, and its greatest value is 3. At this value the quadratic is $x^2 - 3x + 1 = 0$, with positive discriminant 5, so the selected roots are real.

Step 2: Determine the difference of roots and the remaining factor For the selected quadratic, $S = 3$ and $P = 1$. We can avoid explicitly writing the individual roots:

$$|\alpha - \beta| = \sqrt{S^2 - 4P} = \sqrt{9 - 4} = \sqrt{5}.$$

Also,

$$\alpha^2 + \alpha\beta + \beta^2 = (\alpha + \beta)^2 - \alpha\beta = S^2 - P = 8.$$

Notice that the second factor is 8, not 10; the product of the roots must be subtracted from the square of the sum.

Step 3: Use the difference-of-cubes factorisation Take absolute values in the factorisation:

$$|\alpha^3 - \beta^3| = |\alpha - \beta|\,|\alpha^2 + \alpha\beta + \beta^2| = 8\sqrt{5}.$$

The alternatives involving $\sqrt{3}$ have the wrong discriminant factor, and $10\sqrt{5}$ has the wrong symmetric quadratic factor. Thus both factors in the selected answer have been determined from the maximising value of the parameter rather than from an arbitrary quadratic in the family.

Step 4: Conclusion The absolute difference is $\boxed{8\sqrt{5}}$. Hence, the correct option is (C).

Quick Tip to Solve: Maximise the Vieta sum before finding the roots; then compute a difference of cubes using only the sum and product of those roots.

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8. If $f(x) = [x] - \left[\frac{x}{4}\right]$,x ∈ 4 , where [x] denotes the greatest integer function, then [4 marks]

(A) Both $\lim_{x \to 4-} f(x)$ and $\lim_{x \to 4+} f(x)$ exist but are not equal

(B) f is continuous at x = 4

(C) $\lim_{x \to 4+} f(x)$ exists but $\lim_{x \to 4-} f(x)$ does not exist

(D) $\lim_{x \to 4-} f(x)$ exists but $\lim_{x \to 4+} f(x)$ does not exist

Correct Answer: (B) f is continuous at x = 4

Solution:

Concept:

- The greatest integer $\lfloor t \rfloor$ is the unique integer n such that $n \leq t < n + 1$. Its values must be determined separately on each side of an integer.
- A function is continuous at $x = 4$ when its left-hand limit, its right-hand limit and its value at 4 are all equal. Jumps in two floor terms can cancel in their difference.

Step 1: Evaluate the expression just below 4 Choose $3 < x < 4$, sufficiently close to 4. Then $\lfloor x \rfloor = 3$,

while $3/4 < x/4 < 1$, so $\lfloor x/4 \rfloor = 0$. Thus throughout this left-hand neighbourhood,

$$f(x) = 3 - 0 = 3, \qquad \lim_{x \to 4^-} f(x) = 3.$$

The existence of the individual floor jumps does not make this one-sided limit fail.

Step 2: Evaluate the right-hand limit and the point value For $4 < x < 5$, we have $\lfloor x \rfloor = 4$ and $\lfloor x/4 \rfloor = 1$. Consequently,

$$\lim_{x \to 4^+} f(x) = 4 - 1 = 3.$$

At the point itself, both floor functions take their new integer values:

$$f(4) = \lfloor 4 \rfloor - \lfloor 1 \rfloor = 4 - 1 = 3.$$

Both one-sided limits therefore exist and are equal, which excludes (A), (C) and (D). In particular, it would be incorrect to decide continuity by considering only the first floor term: the second term jumps by exactly the same amount.

Step 3: Conclusion Both one-sided limits equal $f(4) = 3$, so f is continuous at $x = 4$. Hence, the correct option is (B).

Quick Tip to Solve: Near a common jump point, calculate each floor term on both sides before subtracting; equal jumps can cancel completely.

9. The area (in sq. units) of the region

A = {(x, y) : $\frac{y^2}{2} \le x \le y + 4$} is :- [4 marks]

(A) 30

(B) 18

(C) $\frac{53}{3}$

(D) 16

Correct Answer: (B) 18

Solution:

Concept:

- When a region is specified by $x_{\text{left}}(y) \le x \le x_{\text{right}}(y)$, its area is $\int (x_{\text{right}} - x_{\text{left}})\, dy$. Horizontal strips avoid solving the parabola for two separate branches.
- The intersection values of y supply the integration limits. The integrand must be nonnegative on the interval representing the bounded region.

Step 1: Find the intersection ordinates The two boundaries are $x = y^2/2$ and $x = y + 4$. At an intersection,

$$\frac{y^2}{2} = y + 4 \quad \Longrightarrow \quad y^2 - 2y - 8 = (y - 4)(y + 2) = 0.$$

Thus the region extends from $y = -2$ to $y = 4$. Between these values, $y + 4 - y^2/2 \ge 0$; the straight line is the right boundary and the parabola is the left boundary.

Step 2: Integrate the horizontal width Each thin horizontal strip has width $y+4-y^2/2$ and thickness dy. Therefore,

$$\begin{aligned}\text{Area} &= \int_{-2}^{4}\left(y+4-\frac{y^2}{2}\right)dy \\ &= \left[\frac{y^2}{2}+4y-\frac{y^3}{6}\right]_{-2}^{4} \\ &= \left(8+16-\frac{64}{6}\right)-\left(2-8+\frac{8}{6}\right) \\ &= \frac{40}{3}-\left(-\frac{14}{3}\right)=18.\end{aligned}$$

Both intersections are required: integrating only above the horizontal axis would omit part of the region. This exact evaluation rules out 30, $53/3$ and 16; no absolute-value correction is needed because the chosen width is already nonnegative.

Step 3: Conclusion The required area is 18 square units. Hence, the correct option is (B).

Quick Tip to Solve: For inequalities that bound x in terms of y, integrate right boundary minus left boundary with respect to y.

10. $\int e^{\sec x}\,(\sec x\tan x f(x)+\sec x\tan x+sex^2x)dx$

= e^{secx}f(x) + C then a possible choice of f(x) is :- [4 marks]

(A) x sec x + tan x + 1/2

(B) sec x + xtan x - 1/2

(C) sec x - tan x - 1/2

(D) sec x + tan x + 1/2

Correct Answer: The last printed term sex^2x is undefined; option (D) follows only if it means $\sec^2 x$.

Solution:

Concept:

- The product and chain rules give $\frac{d}{dx}(e^{\sec x}f(x))=e^{\sec x}(\sec x\tan x\,f(x)+f\prime(x))$, wherever $\cos x\neq 0$ and f is differentiable.
- An indefinite-integral identity can be tested by differentiating its proposed right-hand side. Every term in the given integrand must have a defined meaning before a unique choice can be established.

Step 1: Compare the derivatives literally Differentiating the asserted identity and cancelling the nonzero factor $e^{\sec x}$ and the common term $\sec x\tan x\,f(x)$ would require

$$f\prime(x)=\sec x\tan x+T(x),$$

where $T(x)$ denotes exactly the last printed term. That term is printed as sex^2x; neither a function named se nor constants s,e representing that expression are defined. It therefore cannot be silently identified with $\sec^2 x$. The literal expression does not specify a unique derivative equation.

Step 2: Check the conventional reading against every choice If the last term is read as $\sec^2 x$, the

derivative equation becomes $f' = \sec x \tan x + \sec^2 x$. Direct differentiation of the four choices gives

$$f_A' = \sec x + x \sec x \tan x + \sec^2 x,$$
$$f_B' = \sec x \tan x + \tan x + x \sec^2 x,$$
$$f_C' = \sec x \tan x - \sec^2 x,$$
$$f_D' = \sec x \tan x + \sec^2 x.$$

Thus only (D) meets that explicitly stated reading. For $f_D = \sec x + \tan x + 1/2$, substitution into the product-rule formula exactly reproduces an integrand whose last term is $\sec^2 x$, on each interval avoiding the zeros of $\cos x$. The other three derivatives have an extra term or the wrong sign. This calculation verifies the conditional result without replacing the printed term in the question.

Step 3: Conclusion The literal printed term is undefined, so the question as written has no uniquely justified option. If that term denotes $\sec^2 x$, the result is $f(x) = \sec x + \tan x + 1/2$, namely option (D); this conditional reading explains the keyed choice.

Quick Tip to Solve: Differentiate the proposed antiderivative first: after cancelling the common exponential product term, only the derivative of f remains to be checked.

11. The total number of matrices

$$A = \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$$

(x, y $\in$ R,x $\neq$ y) for which $A^TA = 3I_3$ is :- [4 marks]

(A) 3

(B) 4

(C) 2

(D) 6

Correct Answer: (B) 4

Solution:

Concept:

- The (i, j) entry of A^TA is the dot product of columns i and j of A. Thus $A^TA = 3I_3$ requires mutually orthogonal columns, each of squared length 3.
- Check both the diagonal and off-diagonal entries. After solving the squared-length conditions, count the signed choices and apply the extra restriction $x \neq y$.

Step 1: Calculate the column dot products The columns are $c_1 = (0, 2x, 2x)^T$, $c_2 = (2y, y, -y)^T$ and $c_3 = (1, -1, 1)^T$. Their cross products satisfy

$$c_1 \cdot c_2 = 2xy - 2xy = 0, \qquad c_1 \cdot c_3 = -2x + 2x = 0,$$

$$c_2 \cdot c_3 = 2y - y - y = 0.$$

They are orthogonal for every real x, y. Their squared lengths give

$$A^T A = \text{diag}(8x^2, 6y^2, 3).$$

Thus there are no additional cross-term equations to impose.

Step 2: Solve the length conditions and count Equating the diagonal entries to 3 yields

$$8x^2 = 3, \qquad 6y^2 = 3,$$

$$x = \pm\sqrt{\frac{3}{8}}, \qquad y = \pm\frac{1}{\sqrt{2}}.$$

Each choice for x can be combined with either choice for y. Since $x^2 = 3/8 \neq 1/2 = y^2$, none of these four pairs has $x = y$; the stated restriction removes no solution. Different pairs produce different matrices, because the matrix entries display both $2x$ and y. Therefore the number is $2 \times 2 = 4$, excluding the proposed counts 3, 2 and 6.

Step 3: Conclusion There are exactly 4 matrices satisfying all the conditions. Hence, the correct option is (B).

Quick Tip to Solve: Interpret a product A-transpose A as a table of column dot products; this often avoids a full symbolic matrix multiplication.

12. A rectangle is inscribed in a circle with a diameter lying along the line 3y = x + 7. If the two adjacent vertices of the rectangle are (–8, 5) and (6, 5), then the area of the rectangle (in sq. units) is : [4 marks]

(A) 72

(B) 84

(C) 56

(D) 98

Correct Answer: (B) 84

Solution:

Concept:

- The centre of a circle lies on the perpendicular bisector of every chord. A line containing a diameter passes through the centre, so its intersection with that perpendicular bisector locates the centre.
- For a rectangle inscribed in a circle, the circle centre is the intersection of the diagonals and therefore the rectangle centre. The distance from this centre to a side equals half the perpendicular side length.

Step 1: Locate the circle centre Let the adjacent vertices be $P = (-8, 5)$ and $Q = (6, 5)$. Their

common side is horizontal, its length is

$$PQ = 6 - (-8) = 14,$$

and its midpoint is $M = (-1, 5)$. Thus its perpendicular bisector is $x = -1$. The given diameter line $3y = x + 7$ must contain the circle centre O. Substitution gives

$$3y = -1 + 7 = 6 \quad \Longrightarrow \quad O = (-1, 2).$$

The diameter direction need not coincide with a rectangle diagonal; only the fact that it passes through the circle centre is used.

Step 2: Calculate the second side and the area The perpendicular distance from $O = (-1, 2)$ to the side $y = 5$ is $5 - 2 = 3$. Since O is the rectangle centre, the opposite side is the same distance below it and hence lies on $y = -1$. The other side length is therefore

$$5 - (-1) = 6.$$

Consequently,

$$\text{Area} = 14 \times 6 = 84 \text{ square units.}$$

Using 3 instead of 6 would take only half the rectangle height. The dimensions determined from the centre exclude 72, 56 and 98 without needing to calculate the circle radius.

Step 3: Conclusion The rectangle has side lengths 14 and 6, giving area 84 square units. Hence, the correct option is (B).

Quick Tip to Solve: Locate the circle centre using a chord perpendicular bisector, then double its distance from that chord to obtain the rectangle height.

13. The domain of the definition of the function

$f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ **is** [4 marks]

(A) (-1, 0) $\cup$ (1, 2) $\cup$ (2, ∞)

(B) (-2, -1) $\cup$ (-1,0) $\cup$ (2, ∞)

(C) (1, 2) $\cup$ (2, ∞)

(D) (-1, 0) $\cup$ (1,2) $\cup$ (3, ∞)

Correct Answer: (A) (-1, 0) $\cup$ (1, 2) $\cup$ (2, ∞)

Solution:

Concept:

- A real logarithm $\log_{10} u$ is defined only for $u > 0$. Zero must be excluded along with negative arguments.
- A rational term is defined where its denominator is nonzero. The domain of a sum is the intersection of the domains of all its terms, not their union.

Step 1: Solve the logarithmic inequality The logarithm requires

$$x^3 - x = x(x-1)(x+1) > 0.$$

Its three simple roots are $-1, 0, 1$. Checking signs on the four resulting intervals gives a negative product on $(-\infty, -1)$, a positive product on $(-1, 0)$, a negative product on $(0, 1)$, and a positive product on $(1, \infty)$. Thus the logarithmic term has domain

$$(-1, 0) \cup (1, \infty).$$

All three roots are excluded because the logarithm of zero is undefined.

Step 2: Apply the denominator restriction The rational term additionally requires

$$4 - x^2 \neq 0 \quad \Longrightarrow \quad x \neq -2,\ 2.$$

The value -2 is already outside the logarithmic domain, but 2 lies within $(1, \infty)$, so it must be removed. The final intersection is

$$(-1, 0) \cup (1, 2) \cup (2, \infty).$$

Option (B) wrongly includes $(-2, -1)$, where the logarithm argument is negative. Option (C) loses the valid interval $(-1, 0)$. Option (D) wrongly omits valid values between 2 and 3; for example, $x = 5/2$ satisfies both restrictions. No further upper or lower bound is introduced by the base 10.

Step 3: Conclusion The domain is $\boxed{(-1, 0) \cup (1, 2) \cup (2, \infty)}$. Hence, the correct option is (A).

Quick Tip to Solve: First solve the strict sign inequality for the logarithm argument; then remove any remaining zeros of the rational denominator.

---

14. If the function $f(x) = \begin{cases} a|\pi - x| + 1, x \leq 5 \\ b|x - \pi| + 3, x > 5 \end{cases}$ **is continuous at x = 5, then the value of a – b is :-** [4 marks]

(A) $\frac{2}{\pi - 5}$
(B) $\frac{2}{5 - \pi}$
(C) $\frac{-2}{\pi + 5}$
(D) $\frac{2}{\pi + 5}$

Correct Answer: (B) $\frac{2}{5 - \pi}$

Solution:

Concept:

- Continuity at the joining point of a piecewise function requires the two limiting expressions to agree with the value supplied by the branch containing equality.
- An absolute value $|t|$ equals t when $t \geq 0$ and $-t$ otherwise. At $x = 5$, both $|\pi - x|$ and $|x - \pi|$ have value $5 - \pi > 0$.

Step 1: Evaluate both sides at the joining point The branch for $x \leq 5$ gives

$$f(5) = a|\pi - 5| + 1 = a(5 - \pi) + 1.$$

This also equals the left-hand limit because the expression $a|\pi - x| + 1$ is continuous as a function of x. The branch for $x > 5$ gives the right-hand limit

$$\lim_{x \to 5^+} f(x) = b|5 - \pi| + 3 = b(5 - \pi) + 3.$$

Although the right-hand branch is not used to define the value at 5, it is used for the limit from larger values of x.

Step 2: Impose continuity and isolate the difference Equating the two expressions yields

$$a(5 - \pi) + 1 = b(5 - \pi) + 3,$$

$$(a - b)(5 - \pi) = 2.$$

As $5 - \pi \neq 0$, division gives

$$a - b = \frac{2}{5 - \pi}.$$

No separate values of a and b are needed: continuity fixes their difference. Option (A) has the opposite sign because $\pi - 5 < 0$. Options (C) and (D) use the incorrect magnitude $\pi + 5$, which is not the distance between π and 5 on the real line.

Step 3: Conclusion The required value is $\boxed{a - b = 2/(5 - \pi)}$. Hence, the correct option is (B).

Quick Tip to Solve: At the joining point, simplify every absolute value numerically before equating the branch limits; this prevents a sign reversal.

---

15. If a unit vector $\vec{a}$ makes angles $\pi/3$ with $\hat{i}$, $\pi/4$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of θ is :- [4 marks]

(A) $\frac{5\pi}{6}$

(B) $\frac{5\pi}{12}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{4}$

Correct Answer: (C) $\frac{2\pi}{3}$

Solution:

Concept:

- A unit vector has components equal to its direction cosines. If its angles with the positive coordinate axes are α, β, θ, then $\cos^2\alpha + \cos^2\beta + \cos^2\theta = 1$.
- The sum-of-squares condition fixes the magnitude of the third component, but its sign can be positive or negative. The specified interval for the angle must therefore be applied after solving for

the cosine.

Step 1: Substitute the two known direction cosines The first two components of the unit vector are $\cos(\pi/3) = 1/2$ and $\cos(\pi/4) = 1/\sqrt{2}$. Using the unit-length condition gives

$$\frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1, \qquad \cos^2\theta = \frac{1}{4}.$$

These are components along mutually perpendicular axes; the angles themselves are not added.

Step 2: Find the admissible angles and check the choices Taking both signs, $\cos\theta = \pm 1/2$. In the interval $(0, \pi)$, the corresponding angles are

$$\theta = \frac{\pi}{3} \quad \text{or} \quad \theta = \frac{2\pi}{3}.$$

Only the second is listed. Option (A) has cosine squared $3/4$, and option (D) has cosine squared $1/2$, so neither satisfies the unit-length equation. Option (B), with angle $5\pi/12$, also does not have cosine squared $1/4$.

Step 3: Conclusion A listed possible value is $\boxed{\theta = 2\pi/3}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the squared missing direction cosine first, then retain both signs before matching the angles to the options.

16. The vertices B and C of a ΔABC lie on the line,
$\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that BC = 5 units.
Then the area (in sq. units) of this triangle, given that the point A(1, –1, 2), is : [4 marks]

(A) 6
(B) $5\sqrt{17}$
(C) $\sqrt{34}$
(D) $2\sqrt{34}$

Correct Answer: (C) $\sqrt{34}$

Solution:

Concept:

- The area of a triangle is $\frac{1}{2} \times$ base $\times$ perpendicular height. Here the base length is fixed, so the exact positions of B and C along their line are unnecessary.
- For a line through P with direction vector $\mathbf{d}$, the distance of A from the line is $h = |\overrightarrow{PA} \times \mathbf{d}|/|\mathbf{d}|$. The zero direction component means that y is constant on the line.

Step 1: Identify a point and the direction of the base line Write the line parametrically as $x = -2 + 3t,\ y = 1,\ z = 4t$. Thus take $P = (-2, 1, 0)$ and $\mathbf{d} = (3, 0, 4)$, for which $|\mathbf{d}| = 5$. The vector from P to the given vertex is

$$\overrightarrow{PA} = (1 + 2, -1 - 1, 2 - 0) = (3, -2, 2).$$

The perpendicular distance to the infinite line is the required altitude even if the foot lies outside

segment BC.

Step 2: Calculate the height and then the area The cross product is

$$\overrightarrow{PA} \times \mathbf{d} = (-8, -6, 6).$$

Its magnitude is $\sqrt{64 + 36 + 36} = \sqrt{136} = 2\sqrt{34}$. Consequently

$$h = \frac{2\sqrt{34}}{5}, \qquad \text{Area} = \frac{1}{2}(5)\frac{2\sqrt{34}}{5} = \sqrt{34}.$$

The factor of one-half is essential: omitting it would incorrectly give option (D). Neither 6 nor $5\sqrt{17}$ equals this base-height result, so (A) and (B) are excluded as well.

Step 3: Conclusion The triangle has area $\boxed{\sqrt{34} \text{ square units}}$. Hence, the correct option is (C).

Quick Tip to Solve: When a triangle has two vertices on a fixed line, use the third vertex's distance from that line as the height; do not try to locate both base vertices.

---

17. If the two lines x + (a – 1) y = 1 and 2x + a^2y = 1 (a∈R – {0, 1}) are perpendicular, then the distance of their point of intersection from the origin is : [4 marks]

(A) $\frac{2}{\sqrt{5}}$

(B) $\frac{\sqrt{2}}{5}$

(C) $\frac{2}{5}$

(D) $\sqrt{\frac{2}{5}}$

Correct Answer: (D) $\sqrt{\frac{2}{5}}$

Solution:

Concept:

- For lines $A_1x + B_1y = C_1$ and $A_2x + B_2y = C_2$, perpendicularity is equivalent to $A_1A_2 + B_1B_2 = 0$. This avoids unnecessary divisions when using slopes.
- The distance of an intersection point (x_0, y_0) from the origin is $\sqrt{x_0^2 + y_0^2}$. Determine the parameter from perpendicularity before solving for the intersection.

Step 1: Use perpendicularity to determine a The coefficient vectors of the two lines are $(1, a - 1)$ and $(2, a^2)$. Their scalar product must vanish:

$$2 + a^2(a - 1) = 0.$$

Factor the resulting cubic:

$$a^3 - a^2 + 2 = (a + 1)(a^2 - 2a + 2) = 0.$$

Since $a^2 - 2a + 2 = (a - 1)^2 + 1 > 0$ for real a, the only real choice is $a = -1$. It is allowed by the

given exclusion of 0 and 1.

Step 2: Solve the resulting lines and evaluate the distance The equations become $x - 2y = 1$ and $2x + y = 1$. Substitution of $y = 1 - 2x$ in the first gives $5x = 3$, so

$$(x, y) = \left(\frac{3}{5}, -\frac{1}{5}\right).$$

Therefore

$$d = \sqrt{\frac{9}{25} + \frac{1}{25}} = \frac{\sqrt{10}}{5} = \sqrt{\frac{2}{5}}.$$

The square of this distance is $2/5$. The other three listed values have squares $4/5, 2/25, 4/25$, respectively, and hence cannot be the required distance.

Step 3: Conclusion The distance is $\boxed{\sqrt{2/5}}$ units. Hence, the correct option is (D).

Quick Tip to Solve: Use the dot product of the line coefficient vectors for perpendicularity, and compare squared distances when the options contain radicals.

---

18. Let z $\in$ C be such that |z| < 1.

If $\omega = \frac{5+3z}{5(1-z)}$z, then : [4 marks]

(A) 4Im(ω) > 5

(B) 5Im(ω) < 1

(C) 5Re(ω) > 4

(D) 5Re(ω) > 1

Correct Answer: With the trailing z read as multiplication, none of the four options is universally valid.

Solution:

Concept:

- A claim made for every complex z with $|z| < 1$ can be disproved by one admissible counterexample. The point $z = 0$ is included in this open disc.
- Reading the printed trailing z literally as a factor gives $\omega = z(5 + 3z)/(5(1 - z))$. Since $|z| < 1$, its denominator is nonzero. Real and imaginary parts must be checked using this complete expression.

Step 1: Test the inequalities at the centre of the disc At $z = 0$, the expression including its trailing factor gives $\omega = 0$. Therefore $4\,\mathrm{Im}\,\omega = 0$, so option (A), which requires it to exceed 5, is false. Also $5\,\mathrm{Re}\,\omega = 0$, which disproves both the bound greater than 4 in (C) and the bound greater than 1 in (D). The strict inequality $|z| < 1$ does not exclude zero; hence this is a valid test of all three proposed universal statements.

Step 2: Test the remaining imaginary-part inequality Choose $z = i/2$, whose modulus is $1/2 < 1$. Rationalising the defining fraction first gives

$$\frac{5 + 3i/2}{5(1 - i/2)} = \frac{17}{25} + \frac{16}{25}i.$$

Including the trailing factor now yields

$$\omega = \frac{i}{2}\left(\frac{17}{25} + \frac{16}{25}i\right) = -\frac{8}{25} + \frac{17}{50}i.$$

Consequently $5\,\mathrm{Im}\,\omega = 17/10 > 1$, disproving option (B). Thus each proposed inequality has an explicit counterexample. If the trailing z were absent, the inverse transformation would instead give $|5\omega - 5| < |5\omega + 3|$, equivalent to $5\,\mathrm{Re}\,\omega > 1$; this explains the keyed choice but is a different expression.

Step 3: Conclusion For the expression including the printed trailing z, no valid option. The key gives (D), but $z = 0$ disproves (D) for the literal expression; (D) follows only for the fraction without that factor.

Quick Tip to Solve: Before applying a unit-disc transformation, substitute z=0 into the complete printed expression; it immediately tests any claimed positive lower bound on the real part.

19. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic meter per minute. The the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10m; is :- [4 marks]

(A) $\frac{1}{15\pi}$
(B) $\frac{1}{5\pi}$
(C) $\frac{1}{10\pi}$
(D) $\frac{2}{\pi}$

Correct Answer: (B) $\frac{1}{5\pi}$

Solution:

Concept:

- The semi-vertical angle is the angle between the cone axis and its sloping side. Similar cross-sections of the water therefore satisfy $r/h = \tan\alpha$, where r is the water-surface radius and h is its depth.
- Use $V = \pi r^2 h/3$ for the water volume and differentiate with respect to time only after expressing r in terms of h. The inflow rate is $dV/dt = 5\,\mathrm{m}^3\,\mathrm{min}^{-1}$.

Step 1: Express the water volume using its depth Here $\alpha = \tan^{-1}(1/2)$, so $r/h = 1/2$ and $r = h/2$. Therefore

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}.$$

The water surface expands as the depth increases; treating its radius as a constant would give an incorrect derivative. Both dimensions must be connected by the fixed tank angle.

Step 2: Differentiate and substitute the instantaneous depth Differentiating the volume gives

$$\frac{dV}{dt} = \frac{\pi h^2}{4}\frac{dh}{dt}.$$

At $h = 10\,\text{m}$ and the specified constant inflow,

$$5 = \frac{\pi(10)^2}{4}\frac{dh}{dt} = 25\pi\frac{dh}{dt}, \qquad \frac{dh}{dt} = \frac{1}{5\pi}.$$

The derivative is positive, as water is entering the tank. Options (A), (C), and (D) would give volume rates $5/3, 5/2, 50$ cubic metres per minute when multiplied by the instantaneous surface area 25π; none equals the required 5.

Step 3: Conclusion The water level is rising at $\boxed{1/(5\pi)\,\text{m}\,\text{min}^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: The instantaneous rise rate equals inflow divided by the water-surface area; first find the surface radius from the semi-vertical angle.

20. If the system of equations 2x + 3y – z = 0, x + ky – 2z = 0 and 2x – y + z = 0 has a non-trival solution (x, y, z), then $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k$ is equal to :- [4 marks]

(A) -4

(B) $\frac{3}{4}$

(C) $\frac{1}{2}$

(D) $-\frac{1}{4}$

Correct Answer: (C) $\frac{1}{2}$

Solution:

Concept:

- A homogeneous system always has the zero solution; a non-trivial solution requires a singular coefficient matrix. Here elimination directly gives both the coordinate ratios and the required parameter.
- The requested expression is unchanged when (x, y, z) is multiplied by a common nonzero constant. Ratios are therefore sufficient; there is no need to determine a unique solution vector.

Step 1: Use the first and third equations to obtain the ratios Add $2x + 3y - z = 0$ and $2x - y + z = 0$. This gives $4x + 2y = 0$, so $y = -2x$. Substitution into the third equation yields

$$2x + 2x + z = 0, \qquad z = -4x.$$

If x were zero, these relations would force y and z to vanish as well. Non-triviality therefore requires $x \neq 0$; consequently y and z are nonzero, and all ratios in the question are defined.

Step 2: Find k and substitute the ratios The second equation is $x + ky - 2z = 0$. Using the relations already found gives

$$x - 2kx + 8x = (9 - 2k)x = 0.$$

Since $x \neq 0$, it follows that $k = 9/2$. Now

$$\frac{x}{y} = -\frac{1}{2}, \qquad \frac{y}{z} = \frac{1}{2}, \qquad \frac{z}{x} = -4.$$

Thus

$$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k = -\frac{1}{2} + \frac{1}{2} - 4 + \frac{9}{2} = \frac{1}{2}.$$

The first two ratios cancel exactly. Option (A) is the partial sum before adding k; the other two proposed fractions do not equal the complete sum.

Step 3: Conclusion The required expression is $\boxed{1/2}$. Hence, the correct option is (C).

Quick Tip to Solve: In a homogeneous system, eliminate between equations without the parameter first; this often fixes the ratios before the singularity condition is needed.

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21. The value of the integral $\int_0^1 x\cot^{-1}(1-x^2+x^4)dx$ is :- [4 marks]

(A) $\frac{\pi}{2} - \frac{1}{2}\log_e 2$

(B) $\frac{\pi}{4} - \log_e 2$

(C) $\frac{\pi}{4} - \frac{1}{2}\log_e 2$

(D) $\frac{\pi}{2} - \log_e 2$

Correct Answer: (C) $\frac{\pi}{4} - \frac{1}{2}\log_e 2$

Solution:

Concept:

- For $0 \leq t \leq 1$, the addition formula gives $\tan(\tan^{-1} t + \tan^{-1}(1-t)) = 1/(1-t+t^2)$. The sum lies in $(0, \pi/2)$, so it equals $\cot^{-1}(1-t+t^2)$.
- The substitution $t = x^2$ uses the factor x in the integrand. Reflection $t \mapsto 1-t$ then makes the two inverse-tangent integrals equal.

Step 1: Reduce the quartic expression with a substitution Let $t = x^2$, giving $x\,dx = dt/2$, with limits 0 and 1 unchanged. The integral becomes

$$I = \frac{1}{2}\int_0^1 \cot^{-1}(1-t+t^2)\,dt.$$

For this interval, $1 - t + t^2 \geq 3/4 > 0$. The inverse cotangent is therefore on its positive acute branch, and the addition formula can be used without an extra multiple of π:

$$\cot^{-1}(1-t+t^2) = \tan^{-1} t + \tan^{-1}(1-t).$$

Step 2: Use symmetry and integrate by parts The substitution $u = 1-t$ shows that $\int_0^1 \tan^{-1}(1-t)dt =$

$\int_0^1 \tan^{-1} t\,dt$. Hence

$$I = \int_0^1 \tan^{-1} t\,dt.$$

Integration by parts gives

$$I = \left[t\tan^{-1} t - \frac{1}{2}\ln(1+t^2)\right]_0^1 = \frac{\pi}{4} - \frac{1}{2}\ln 2.$$

Option (B) subtracts twice the correct logarithmic contribution; (A) has twice the correct angular contribution, and (D) doubles the entire answer. The substitution factor of one-half and the two equal integrals explain why neither doubling is valid.

Step 3: Conclusion The integral is $\boxed{\pi/4 - (\ln 2)/2}$. Hence, the correct option is (C).

Quick Tip to Solve: For an integrand containing $x\,dx$ and $x^4 - x^2 + 1$, try $t = x^2$, then pair t with $1 - t$ in an inverse-tangent addition formula.

22. If the tangent to the parabola $y^2 = x$ at a point (α, β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then α is equal to : [4 marks]

(A) $\sqrt{2} + 1$

(B) $\sqrt{2} - 1$

(C) $2\sqrt{2} + 1$

(D) $2\sqrt{2} - 1$

Correct Answer: (A) $\sqrt{2} + 1$

Solution:

Concept:

- A tangent at a point on a parabola is obtained by differentiating its equation. Since (α, β) lies on $y^2 = x$, we have $\alpha = \beta^2 > 0$.
- A line $lx + my + c = 0$ is tangent to the ellipse $x^2/a^2 + y^2/b^2 = 1$ when $c^2 = a^2l^2 + b^2m^2$. Here the squared semiaxes are 1 and $1/2$.

Step 1: Write the parabola tangent Differentiating gives $2y\,dy/dx = 1$. At the specified point its slope is $1/(2\beta)$, which is defined because $\beta > 0$. The point-slope equation is therefore

$$2\beta(y - \beta) = x - \alpha.$$

Using $\alpha = \beta^2$, this reduces to

$$x - 2\beta y + \beta^2 = 0.$$

Its constant term and coefficient of y must both be retained when imposing the second tangency condition.

Step 2: Impose tangency to the ellipse Apply the ellipse condition with $l = 1$, $m = -2\beta$, and $c = \beta^2$:

$$\beta^4 = 1 + \frac{1}{2}(4\beta^2) = 1 + 2\beta^2.$$

Equivalently,

$$\alpha^2 - 2\alpha - 1 = 0, \qquad \alpha = 1 \pm \sqrt{2}.$$

The negative root $1 - \sqrt{2}$ cannot equal β^2, so the positive root is required. Substitution of $1 + \sqrt{2}$ gives $\alpha^2 = 3 + 2\sqrt{2} = 1 + 2\alpha$, checking tangency. The values in (B), (C) and (D) do not satisfy this positive-root equation.

Step 3: Conclusion The required abscissa is $\boxed{\alpha = 1 + \sqrt{2}}$. Hence, the correct option is (A).

Quick Tip to Solve: Express the parabola point using $\alpha = \beta^2$ before applying the ellipse tangent condition; the sign restriction then selects the root immediately.

23. The sum of the series 1 + 2 × 3 + 3 × 5 + 4 × 7 +.... upto 11th term is :- [4 marks]

(A) 945

(B) 916

(C) 915

(D) 946

Correct Answer: (D) 946

Solution:

Concept:

- The successive terms are products of the row number and the corresponding odd number: the rth term is $r(2r - 1) = 2r^2 - r$.
- Use $\sum_{r=1}^{n} r = n(n + 1)/2$ and $\sum_{r=1}^{n} r^2 = n(n + 1)(2n + 1)/6$. The upper limit is the number of terms, not the largest odd factor.

Step 1: Identify the general term For $r = 1, 2, 3, 4$, the expression $r(2r - 1)$ gives $1, 6, 15, 28$, agreeing with $1, 2 \times 3, 3 \times 5, 4 \times 7$. Thus the initial isolated 1 is already included as the first term; no extra term should be added. The eleventh term is 11×21, and the requested sum is

$$S_{11} = \sum_{r=1}^{11}(2r^2 - r) = 2\sum_{r=1}^{11} r^2 - \sum_{r=1}^{11} r.$$

Step 2: Evaluate the two standard sums Substitute $n = 11$ into the standard formulae:

$$\sum_{r=1}^{11} r = \frac{11 \cdot 12}{2} = 66, \qquad \sum_{r=1}^{11} r^2 = \frac{11 \cdot 12 \cdot 23}{6} = 506.$$

Hence

$$S_{11} = 2(506) - 66 = 1012 - 66 = 946.$$

As a direct arithmetic check, the eleven terms are $1, 6, 15, 28, 45, 66, 91, 120, 153, 190, 231$; their sum is also 946. Option (A) differs by one, while (B) and (C) give 916 and 915, respectively, neither of which equals the evaluated sum.

Step 3: Conclusion The sum up to the eleventh term is $\boxed{946}$. Hence, the correct option is (D).

Quick Tip to Solve: Write a product of a counting number and an odd number as $r(2r-1)$; it separates directly into the standard sums of squares and integers.

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24. If p $\Rightarrow$ (q $\vee$ r) is false, then the truth values of p, q, r are respectively :- [4 marks]

(A) F, F, F

(B) T, F, F

(C) F, T, T

(D) T, T, F

Correct Answer: (B) T, F, F

Solution:

Concept:

- An implication $p \Rightarrow s$ is false precisely when its antecedent p is true and its consequent s is false.
- The disjunction $q \vee r$ is false only when both q and r are false. If either constituent is true, their disjunction is true.

Step 1: Use the condition for a false implication Here the consequent is the complete compound statement $q \vee r$. Since the implication is given to be false, we must have

$$p = T, \qquad q \vee r = F.$$

A false antecedent cannot produce a false implication: both $F \Rightarrow F$ and $F \Rightarrow T$ are true. Consequently, options (A) and (C), each of which assigns F to p, are excluded before considering the individual values of q and r.

Step 2: Resolve the disjunction and check the choices To make $q \vee r$ false we need

$$q = F, \qquad r = F.$$

Thus the required ordered triple is (T, F, F). Substitution gives

$$T \Rightarrow (F \vee F) = T \Rightarrow F = F,$$

exactly as required. For option (D), (T, T, F), the consequent is $T \vee F = T$, so the implication becomes $T \Rightarrow T = T$. It therefore fails the given condition. Option (B) is the only assignment that makes the antecedent true while keeping the entire consequent false.

Step 3: Conclusion The truth values of p, q, r, respectively, are $\boxed{T, F, F}$. Hence, the correct option is (B).

Quick Tip to Solve: For a false implication, first force the left statement to be true and the whole right statement to be false; only then resolve any connective inside the right statement.

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25. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are addded to the total number of balls used in forming the equilaterial triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is :- [4 marks]

(A) 262

(B) 190

(C) 157

(D) 225

Correct Answer: (B) 190

Solution:

Concept:

- If an equilateral triangular array has n balls on each side, its rows contain $1, 2, \ldots, n$ balls. The total is the triangular number $n(n+1)/2$.
- A complete square array with m balls on each side contains m^2 balls. Here the square has $n-2$ balls on each side, and its total includes the additional 99 balls.

Step 1: Translate the two arrangements into an equation Let the triangle have n rows. Its original number of balls is

$$N = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

After adding 99 balls, every ball is used in a square array with $n-2$ rows and $n-2$ columns. Therefore

$$\frac{n(n+1)}{2} + 99 = (n-2)^2.$$

The square-side count is two fewer than the triangular-side count; it is not the original total minus two.

Step 2: Solve and verify the admissible count Multiplying by two and simplifying gives

$$n^2 + n + 198 = 2n^2 - 8n + 8,$$

$$n^2 - 9n - 190 = 0 = (n-19)(n+10).$$

The negative solution $n = -10$ is impossible for a number of rows, so $n = 19$. The required original count is

$$N = \frac{19 \cdot 20}{2} = 190.$$

Indeed, $190 + 99 = 289 = 17^2$, and $17 = 19 - 2$, verifying both the addition and the side condition. The other proposed totals, $262, 157, 225$, are unequal to the unique positive solution for the original triangular array.

Step 3: Conclusion The equilateral triangular arrangement originally used $\boxed{190}$ balls. Hence, the correct option is (B).

Quick Tip to Solve: Introduce the number of balls along a side before writing totals: the triangle gives $n(n+1)/2$, while the enlarged square gives $(n-2)^2$.

26. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is :- [4 marks]

(A) $4\pi\left(3+\sqrt{2}\right)$

(B) $8\pi\left(2-\sqrt{2}\right)$

(C) $8\pi\left(3-2\sqrt{2}\right)$

(D) $4\pi\left(2-\sqrt{2}\right)$

Correct Answer: (C) $8\pi\left(3-2\sqrt{2}\right)$

Solution:

Concept:

- At a common point of tangency, a circle and a parabola have the same tangent. The circle centre therefore lies on the normal to the parabola at that point.
- A circle with centre (h,k) tangent to the x-axis has radius $|k|$. Its passage through $(1,2)$ imposes $(h-1)^2+(k-2)^2=k^2$.

Step 1: Find the locus of the circle centre Differentiating $y^2=4x$ gives $2y\,dy/dx=4$. At $(1,2)$, the tangent slope is 1, so the normal slope is -1. The centre (h,k) must lie on

$$k-2=-(h-1), \qquad h+k=3.$$

Thus $h=3-k$. Because the circle touches the x-axis, its squared radius is k^2, whether the centre is initially taken above or below the axis.

Step 2: Determine both radii and choose the smaller area The distance from the centre to $(1,2)$ equals its radius:

$$k^2=(h-1)^2+(k-2)^2.$$

Substitute $h=3-k$:

$$k^2=(2-k)^2+(k-2)^2=2(k-2)^2,$$

$$k^2-8k+8=0, \qquad k=4\pm 2\sqrt{2}.$$

Both values are positive, so they are the two radii. The smaller radius is $4-2\sqrt{2}$; hence

$$\pi r^2=\pi(4-2\sqrt{2})^2=\pi(24-16\sqrt{2})=8\pi(3-2\sqrt{2}).$$

This expression is option (C). Options (A), (B) and (D) are different positive values and do not equal the square of the smaller radius times π.

Step 3: Conclusion The smaller circle has area $\boxed{8\pi(3-2\sqrt{2})}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: For a circle tangent to a curve at a given point and to an axis, combine the curve normal with the distance-to-axis formula for the radius.

27. If some three consecutive in the binomial expansion of $(x + 1)^n$ is powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficient is :- [4 marks]

(A) 625

(B) 227

(C) 964

(D) 232

Correct Answer: (D) 232

Solution:

Concept:

- The coefficients in $(1+x)^n$ are $\binom{n}{r}$. Consecutive coefficients satisfy $\binom{n}{r+1}/\binom{n}{r} = (n-r)/(r+1)$.
- The given ratio determines two successive coefficient ratios. Solving them together fixes the integer indices and hence the actual coefficients, rather than merely their proportional values.

Step 1: Set up the two consecutive-ratio equations Let the three coefficients be $\binom{n}{r}, \binom{n}{r+1}, \binom{n}{r+2}$, in increasing powers of x. Their ratio $2 : 15 : 70$ gives

$$\frac{n-r}{r+1} = \frac{15}{2}, \qquad \frac{n-r-1}{r+2} = \frac{70}{15} = \frac{14}{3}.$$

Cross-multiplication yields

$$2n = 17r + 15, \qquad 3n = 17r + 31.$$

Both equations use the same initial index r; increasing the lower index in the second ratio also decreases its numerator by one.

Step 2: Find the coefficients and their average Subtract the two linear equations to obtain $n = 16$. Substitution into the first gives $32 = 17r + 15$, so $r = 1$. The coefficients are therefore

$$\binom{16}{1} = 16, \qquad \binom{16}{2} = 120, \qquad \binom{16}{3} = 560.$$

Their ratio is $16 : 120 : 560 = 2 : 15 : 70$, as required. The arithmetic average is

$$\frac{16 + 120 + 560}{3} = \frac{696}{3} = 232.$$

The values $625, 227, 964$ in the other options are not this average. Averaging the bare ratio entries would be incorrect because those entries are only proportional to the coefficients.

Step 3: Conclusion The average of the three coefficients is $\boxed{232}$. Hence, the correct option is (D).

Quick Tip to Solve: Use ratios of adjacent binomial coefficients twice; the two linear equations usually determine both the expansion power and the starting index.

28. The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point : [4 marks]

(A) (6, –2)

(B) (4, –2)

(C) (–4, 6)

(D) (–6, 4)

Correct Answer: (A) (6, –2)

Solution:

Concept:

- The circle $x^2 + y^2 = 4$ has centre $(0, 0)$ and radius 2. Completing squares in the second equation gives $(x + 3)^2 + (y + 4)^2 = 49$, so its centre is $(-3, -4)$ and radius 7.
- If the distance between two circle centres equals the difference of their radii, the circles touch internally and have a single common tangent at their contact point. That tangent is perpendicular to the line of centres.

Step 1: Locate the point of internal contact The distance between the centres is

$$\sqrt{(-3)^2 + (-4)^2} = 5 = 7 - 2.$$

Thus the two circles touch internally. The unit vector from the larger centre $(-3, -4)$ towards the origin is $(3/5, 4/5)$. The contact point lies a further distance 2 beyond the origin in that direction, giving

$$P = \left(\frac{6}{5}, \frac{8}{5}\right).$$

Its distances from the two centres are 2 and 7, respectively, confirming the location.

Step 2: Write the tangent and test each proposed point The tangent to $x^2 + y^2 = 4$ at P is

$$\frac{6}{5}x + \frac{8}{5}y = 4, \qquad 3x + 4y = 10.$$

Its distance from the larger centre is

$$\frac{|3(-3) + 4(-4) - 10|}{\sqrt{3^2 + 4^2}} = \frac{35}{5} = 7,$$

confirming that it is also tangent to the larger circle. Substituting the options into $3x + 4y$ gives 10 for $(6, -2)$, 4 for $(4, -2)$, 12 for $(-4, 6)$, and -2 for $(-6, 4)$. Only the first point lies on the common tangent.

Step 3: Conclusion The common tangent is $3x + 4y = 10$, which passes through $\boxed{(6, -2)}$. Hence, the correct option is (A).

Quick Tip to Solve: Check the centre distance before writing general tangent equations; internal contact gives the contact point and its perpendicular tangent directly.

29. Two poles standing on a horizontal ground are of heights 5m and 10 m respectively. The line joining their tops makes an angle of 15° with ground. Then the distance (in m) between the poles, is :- [4 marks]

(A) $5\left(2+\sqrt{3}\right)$
(B) $\frac{5}{2}\left(2+\sqrt{3}\right)$
(C) $10\left(\sqrt{3}-1\right)$
(D) $5\left(\sqrt{3}+1\right)$

Correct Answer: (A) $5\left(2+\sqrt{3}\right)$

Solution:

Concept:

- The horizontal separation of two vertical poles is the horizontal side of the right triangle formed by their tops. Its vertical side is the difference of their heights, not their sum.
- For an inclination angle α, $\tan\alpha = h/d$, where h is the vertical rise and d is the horizontal distance. The tangent subtraction identity evaluates $\tan 15^\circ$ exactly.

Step 1: Identify the right triangle Let the distance between the feet of the poles be d. Since both feet are on horizontal ground, a horizontal line from the top of the shorter pole meets the taller pole at the same height. The remaining vertical rise is

$$h = 10 - 5 = 5\,\text{m}.$$

The line joining the tops is the hypotenuse, and its angle with this horizontal line is 15°. Therefore $\tan 15^\circ = 5/d$.

Step 2: Evaluate the distance Use the difference of 45° and 30°:

$$\tan 15^\circ = \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}} = 2 - \sqrt{3}.$$

Hence

$$d = \frac{5}{2-\sqrt{3}} = \frac{5(2+\sqrt{3})}{4-3} = 5(2+\sqrt{3})\,\text{m}.$$

Option (B) gives only half the required horizontal distance. Options (C) and (D) also fail the condition $d\tan 15^\circ = 5$, so neither represents the separation of the poles.

Step 3: Conclusion The distance between the poles is $5(2+\sqrt{3})\,\text{m}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the difference of pole heights as the vertical side; rationalize $1/(2-\sqrt{3})$ to obtain $2+\sqrt{3}$.

30. Let P be the plane, which contains the line of intersection of the planes, x + y + z – 6 = 0 and 2x + 3y + z + 5 = 0 and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to : [4 marks]

(A) $205\sqrt{5}$

(B) $63\sqrt{5}$

(C) $11/\sqrt{5}$

(D) $17/\sqrt{5}$

Correct Answer: (C) $11/\sqrt{5}$

Solution:

Concept:

- Every plane through the intersection of $F_1 = 0$ and $F_2 = 0$ can be written $F_1 + \lambda F_2 = 0$, including the relevant member here.
- Perpendicular planes have orthogonal normal vectors. The normal of the xy-plane is $(0, 0, 1)$, so a plane perpendicular to it has zero coefficient of z.
- The distance of (x_0, y_0, z_0) from $ax + by + cz + d = 0$ is $|ax_0 + by_0 + cz_0 + d|/\sqrt{a^2 + b^2 + c^2}$.

Step 1: Choose the vertical member of the family The family containing the given intersection is

$$(x + y + z - 6) + \lambda(2x + 3y + z + 5) = 0.$$

Its normal is $(1 + 2\lambda, 1 + 3\lambda, 1 + \lambda)$. Orthogonality to the normal of the xy-plane gives

$$1 + \lambda = 0, \qquad \lambda = -1.$$

Subtracting the second plane from the first gives $-x - 2y - 11 = 0$, or equivalently $P : x + 2y + 11 = 0$.

Step 2: Apply the distance formula For the point $(0, 0, 256)$, substitution gives

$$D = \frac{|0 + 2(0) + 0(256) + 11|}{\sqrt{1^2 + 2^2 + 0^2}} = \frac{11}{\sqrt{5}}.$$

The large z-coordinate makes no contribution because this plane is parallel to the z-direction. Thus the two large expressions in (A) and (B) are excluded; (D) has the wrong constant term, since the subtraction produces 11, not 17.

Step 3: Conclusion The required distance is $11/\sqrt{5}$. Hence, the correct option is (C).

Quick Tip to Solve: For a plane perpendicular to the xy-plane, eliminate the z-term in the plane family before using the distance formula.

Physics (120 marks)

31. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is :- [4 marks]

(A) V

(B) $\frac{V}{K+n}$

(C) $\frac{(n+1)V}{K+n}$

(D) $\frac{nV}{K+n}$

Correct Answer: (C) $\frac{(n+1)V}{K+n}$

Solution:

Concept:

- Parallel capacitors share the same potential difference, and their equivalent capacitance is the sum of their capacitances.
- After the battery is removed, the connected capacitor combination is isolated, so its total free charge is conserved even though charge can redistribute between its capacitors.
- Completely filling an air capacitor with dielectric constant K multiplies its capacitance by K. The final voltage is $V_f = Q_{\text{total}}/C_{\text{final}}$.

Step 1: Calculate the charge before inserting the dielectric Initially both capacitors have voltage V. Their separate charges are CV and nCV, so the total charge available on the connected positive plates is

$$Q_i = CV + nCV = (n+1)CV.$$

The battery is disconnected before the dielectric is introduced. It therefore supplies no additional charge during this change; keeping the voltage fixed at V would apply only if the battery remained connected.

Step 2: Find the final capacitance and voltage Only the first capacitor receives the dielectric, so its capacitance becomes KC, while the second remains nC. Thus

$$C_f = KC + nC = (K+n)C.$$

Using conserved charge,

$$(K+n)CV_f = (n+1)CV, \qquad V_f = \frac{(n+1)V}{K+n}.$$

Option (A) neglects the voltage reduction when $K > 1$. Options (B) and (D) omit part of the initial charge and fail the check $K = 1$, for which no physical change occurs and the voltage must remain V.

Step 3: Conclusion The final common potential difference is $\frac{(n+1)V}{K+n}$. Hence, the correct option is (C).

Quick Tip to Solve: With a disconnected battery conserve total charge; with a connected battery keep voltage fixed. Check dielectric answers by setting $K = 1$.

32. Two materials having coefficients of thermal conductivity '3K' and 'K' and thickness 'd' and '3d', respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is :- [4 marks]



(A) $\frac{\theta_1}{10} + \frac{9\theta_2}{10}$
(B) $\frac{\theta_2+\theta_1}{2}$
(C) $\frac{\theta_1}{6} + \frac{5\theta_2}{6}$
(D) $\frac{\theta_1}{3} + \frac{2\theta_2}{3}$

Correct Answer: (A) $\frac{\theta_1}{10} + \frac{9\theta_2}{10}$

Solution:

Concept:

- For a slab of thickness ℓ, conductivity k, and area A, thermal resistance is $R = \ell/(kA)$. The heat current is the temperature difference divided by this resistance.
- At steady state the heat current through both joined slabs is equal. Temperature drops across slabs in series are proportional to their thermal resistances.

Step 1: Compare the two thermal resistances Let T be the interface temperature and A the common cross-sectional area. The hot-side slab has thickness d and conductivity $3K$; the cold-side slab has thickness $3d$ and conductivity K. Therefore

$$R_h = \frac{d}{3KA}, \qquad R_c = \frac{3d}{KA} = 9R_h.$$

The cold-side slab offers nine times as much resistance to heat flow, so nine times the temperature drop occurs across it. Equal thickness or equal conductivity cannot be assumed here.

Step 2: Use equal heat currents to determine the interface temperature Equating heat currents gives

$$\frac{3KA(\theta_2 - T)}{d} = \frac{KA(T - \theta_1)}{3d}.$$

Canceling the common factors,

$$9(\theta_2 - T) = T - \theta_1, \qquad 10T = 9\theta_2 + \theta_1.$$

Thus T is close to the hot boundary temperature, as expected from the small resistance on that side. Option (B) would require equal resistances; (C) and (D) imply resistance ratios of five and two, rather than the actual ratio nine.

Step 3: Conclusion The interface temperature is $T = \frac{\theta_1}{10} + \frac{9\theta_2}{10}$. Hence, the correct option is (A).

Quick Tip to Solve: Compute thickness divided by conductivity for each slab; the larger thermal resistance takes the larger temperature drop.

33. In a conductor, if the number of conduction electrons per unit volume is 8.5×10^{28} m^{-3} and mean free time is 25fs (femto second), it's approximate resistivity is :-

($m_e = 9.1 \times 10^{-31}$ kg) [4 marks]

(A) 10^{-8} Ωm

(B) 10^{-7} Ωm

(C) 10^{-5} Ωm

(D) 10^{-6} Ωm

Correct Answer: (A) 10^{-8} Ωm

Solution:

Concept:

- In the relaxation-time model, conductivity is $\sigma = ne^2\tau/m_e$, where n is electron number density, e the magnitude of electronic charge, and τ the mean free time.
- Resistivity is the reciprocal of conductivity: $\rho = m_e/(ne^2\tau)$. A femtosecond is $10^{-15}\,\mathrm{s}$; this time must be converted to SI units.

Step 1: Convert the data and select the relation The given values are

$$n = 8.5 \times 10^{28}\,\mathrm{m}^{-3}, \quad \tau = 25 \times 10^{-15}\,\mathrm{s}, \quad m_e = 9.1 \times 10^{-31}\,\mathrm{kg}.$$

Take $e = 1.6 \times 10^{-19}\,\mathrm{C}$. In an electric field, the drift speed magnitude is $eE\tau/m_e$. Substituting it into $J = nev_d$ gives $J = (ne^2\tau/m_e)E$, justifying the resistivity formula used.

Step 2: Substitute and identify the order of magnitude The denominator is

$$ne^2\tau = (8.5 \times 10^{28})(2.56 \times 10^{-38})(25 \times 10^{-15}) = 5.44 \times 10^{-23}.$$

Therefore

$$\rho = \frac{9.1 \times 10^{-31}}{5.44 \times 10^{-23}} \simeq 1.67 \times 10^{-8}\,\Omega\,\mathrm{m}.$$

The options give approximate powers of ten, so the matching scale is $10^{-8}\,\Omega\,\mathrm{m}$. The other choices are respectively about six, six hundred, and sixty times this result, and do not match the estimate.

Step 3: Conclusion The resistivity is approximately $1.7 \times 10^{-8}\,\Omega\,\mathrm{m}$, corresponding to $10^{-8}\,\Omega\,\mathrm{m}$. Hence, the correct option is (A).

Quick Tip to Solve: Keep the coefficient and power of ten separate in $ne^2\tau$, and remember that the electronic charge is squared.

34. A particle of mass 'm' is moving with speed '2v' and collides with a mass '2m' moving with speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the origianl direction. The speed of each of the moving particle will be :- [4 marks]

(A) 2 $\sqrt{2}$v

(B) v / (2 $\sqrt{2}$)

(C) v / $\sqrt{2}$

(D) $\sqrt{2}$v

Correct Answer: (A) 2 $\sqrt{2}$v

Solution:

Concept:

- For the collision and splitting considered together, total linear momentum is conserved in the absence of an external impulse. Kinetic energy need not be conserved in a fragmentation event.
- Momentum is a vector. The two final particles move on opposite sides of the original direction; their transverse components cancel, while their forward components add.

Step 1: Write the initial momentum and use transverse symmetry Choose the original direction of motion as the x-axis. Both initial velocities point along it, so

$$p_{i,x} = m(2v) + (2m)v = 4mv, \qquad p_{i,y} = 0.$$

Afterward the original particle of mass m is at rest. Let the speeds of the two fragments be u_1, u_2. Since their directions make opposite 45° angles with the x-axis, transverse conservation gives $mu_1 \sin 45^\circ = mu_2 \sin 45^\circ$, hence both speeds equal u.

Step 2: Conserve the forward momentum The final forward momentum is the sum of the two projected fragment momenta:

$$p_{f,x} = 2mu \cos 45^\circ = \sqrt{2}\, mu.$$

Equating initial and final values,

$$4mv = \sqrt{2}\, mu, \qquad u = 2\sqrt{2}\, v.$$

The alternatives in (B), (C), and (D) would give total forward momenta $mv/2$, mv, and $2mv$, respectively, all smaller than the required $4mv$. The increase in final kinetic energy can come from internal energy released in splitting; energy conservation for translation alone must not replace momentum conservation.

Step 3: Conclusion Each moving fragment has speed $2\sqrt{2}\, v$. Hence, the correct option is (A).

Quick Tip to Solve: First cancel transverse momentum to establish equal fragment speeds, then add their forward momentum components rather than their speed magnitudes.

35. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system , when the beads reach the opposite ends of the rod, will be :- [4 marks]

(A) $\frac{M\omega_0}{M+3m}$
(B) $\frac{M\omega_0}{M+m}$
(C) $\frac{M\omega_0}{M+6m}$
(D) $\frac{M\omega_0}{M+2m}$

Correct Answer: (C) $\frac{M\omega_0}{M+6m}$

Solution:

Concept:

- With no external torque about the fixed central axis, the angular momentum of the rod and both beads is conserved. Internal forces between rod and beads can redistribute angular momentum.
- A thin rod has central moment of inertia $ML^2/12$. A negligible-size bead at distance r contributes mr^2. Radial bead velocity contributes no angular momentum about the central axis.

Step 1: Calculate the initial and final moments of inertia Initially both beads are at the center, so their contributions to moment of inertia are zero. Thus

$$I_i = \frac{ML^2}{12}.$$

When the beads reach the opposite ends, each is a distance $L/2$ from the axis. Both must be included:

$$I_f = \frac{ML^2}{12} + 2m\left(\frac{L}{2}\right)^2 = \frac{L^2}{12}(M+6m).$$

Their radial sliding motion does not change this expression for the angular momentum associated with the angular speed of the rod.

Step 2: Conserve angular momentum and compare the choices Writing ω for the final angular speed gives

$$I_i\omega_0 = I_f\omega, \qquad \frac{ML^2}{12}\omega_0 = \frac{L^2}{12}(M+6m)\omega.$$

After canceling $L^2/12$,

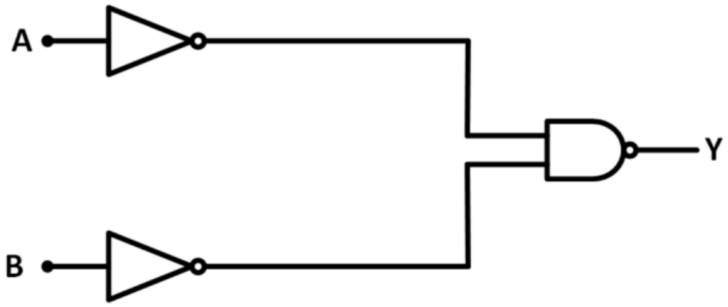
$$\omega = \frac{M\omega_0}{M+6m}.$$

The angular speed decreases because the moment of inertia increases. Option (A) counts only one end bead; options (B) and (D) also underestimate the beads' combined contribution. Conservation of rotational kinetic energy is not the applicable condition, because the beads also acquire radial motion.

Step 3: Conclusion The final angular speed is $\frac{M\omega_0}{M+6m}$. Hence, the correct option is (C).

Quick Tip to Solve: At the rod ends each bead contributes $mL^2/4$; include both before using $I_i\omega_i = I_f\omega_f$.

36. The logic gate equivalent to the given logic circuit is :- [4 marks]



(A) NAND

(B) AND

(C) NOR

(D) OR

Correct Answer: (D) OR

Solution:

Concept:

- A NOT gate complements its input: an input A becomes $\overline{A}$. A NAND gate complements the product of its two inputs.
- De Morgan's identity is $\overline{XY} = \overline{X} + \overline{Y}$, where the plus sign denotes Boolean OR, not ordinary arithmetic addition.

Step 1: Read the two input stages The triangular symbol on each input line has a small circle at its output, so each is an inverter. Thus the upper wire carries $\overline{A}$, while the lower carries $\overline{B}$. The final symbol has the shape of an AND gate and an output circle, so it is a NAND gate. It must therefore invert the product of these already inverted inputs, rather than simply multiply the original inputs.

Step 2: Simplify the output and check the alternatives Applying the NAND operation and then De Morgan's law gives

$$Y = \overline{\overline{A}\,\overline{B}} = A + B.$$

For input pairs $(A, B) = (0,0), (0,1), (1,0), (1,1)$, the output sequence is $0, 1, 1, 1$. This is the OR truth table. NAND instead gives $1, 1, 1, 0$, AND gives $0, 0, 0, 1$, and NOR gives $1, 0, 0, 0$. Thus none of the other three named gates reproduces the complete circuit.

Step 3: Conclusion The equivalent gate is OR, with output $Y = A + B$. Hence, the correct option is (D).

Quick Tip to Solve: Write the signal after each inversion before applying De Morgan's law; the two input bubbles and the output bubble together convert NAND to OR.

37. A metal wire of resistance 3 Ω is elongated to make a uniform wire of double its previous length. This new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60° at the centre, the equivalent resistance between these two points will be :- [4 marks]

(A) 5 /2 Ω

(B) 12/5 Ω

(C) 7/2 Ω

(D) 5 / 3 Ω

Correct Answer: (D) 5 / 3 Ω

Solution:

Concept:

- For a uniform wire, $R = \rho L/A$, where ρ is resistivity. Stretching conserves the volume AL of the metal.
- The two arcs of a circular wire connect the same pair of terminals, so their resistances are in parallel: $R_{\text{eq}} = R_1 R_2/(R_1 + R_2)$.

Step 1: Find the resistance after stretching Let the original length and cross-sectional area be L and A. The new length is $2L$, so volume conservation gives the new area $A/2$. Assuming the resistivity is unchanged, the new total resistance is

$$R_{\text{wire}} = \rho\frac{2L}{A/2} = 4\frac{\rho L}{A} = 4(3) = 12\,\Omega.$$

Bending the wire into a circle changes its shape, but does not change this total resistance. Doubling only the length while forgetting the reduced area would give an incorrect result.

Step 2: Divide the circle into parallel arcs The terminals subtend $60°$ at the centre. Hence the shorter and longer arcs have fractions $1/6$ and $5/6$ of the circumference. Their resistances are

$$R_1 = \frac{12}{6} = 2\,\Omega, \qquad R_2 = \frac{5(12)}{6} = 10\,\Omega.$$

Current can travel along either arc, so

$$R_{\text{eq}} = \frac{2 \times 10}{2 + 10} = \frac{5}{3}\,\Omega.$$

This is less than the smaller branch resistance, as a parallel combination must be. The values $5/2$, $12/5$, and $7/2\,\Omega$ do not satisfy the parallel-resistance calculation.

Step 3: Conclusion The equivalent resistance is $\boxed{5/3\,\Omega}$. Hence, the correct option is (D).

Quick Tip to Solve: When a stretched wire forms a ring, first use volume conservation to find its new total resistance, then split that total in the ratio of the two central angles.

38. Two coils 'P' and 'Q' are separated by some distance. When a current of 3 A flows through coil 'P', a magnetic flux of 10^{-3} Wb passes through 'Q'. No current is passed through 'Q'. When no current passes through 'P' and a current of 2 A passes through 'Q', the flux through 'P' is :- [4 marks]

(A) 3.67×10^{-4} Wb

(B) 3.67×10^{-3} Wb

(C) 6.67×10^{-4} Wb

(D) 6.67×10^{-3} Wb

Correct Answer: (C) 6.67×10^{-4} Wb

Solution:

Concept:

- Magnetic flux coupling between two fixed coils is linear in the current producing it, provided the magnetic response remains linear.
- The mutual inductance is reciprocal: $M_{PQ} = M_{QP} = M$. The coupled flux in this question is therefore related to the source current by $\Phi = MI$.

Step 1: Determine the mutual coupling from the first experiment With current only in coil P, the stated flux through Q is $10^{-3}\,\mathrm{Wb}$ for a current of $3\,\mathrm{A}$. The coupling coefficient inferred from these data is

$$M = \frac{\Phi_Q}{I_P} = \frac{10^{-3}}{3}\,\mathrm{H}.$$

The coils are not moved between the experiments. Reciprocity means that interchanging which coil carries the current does not require a new, unrelated coupling constant. The second coil is unenergised in the first experiment, so its own current makes no contribution to the stated flux.

Step 2: Use reciprocity with the second current When P carries no current and Q carries $2\,\mathrm{A}$, the flux coupled through P is

$$\Phi_P = MI_Q = \frac{10^{-3}}{3} \times 2 = 6.666\ldots \times 10^{-4}\,\mathrm{Wb}.$$

Rounding to three significant figures gives $6.67 \times 10^{-4}\,\mathrm{Wb}$. The current is reduced from 3 A to 2 A, so the flux must be two-thirds of its earlier value, not larger than it. This eliminates the options with exponent -3. The other -4 option has the wrong numerical coefficient and fails the same current ratio.

Step 3: Conclusion The flux is $\boxed{6.67 \times 10^{-4}\,\mathrm{Wb}}$. Hence, the correct option is (C).

Quick Tip to Solve: For unchanged reciprocal coil coupling, compare the two source currents directly: the second coupled flux is the first flux multiplied by $2/3$.

39. A wooden block floating in a bucket of water has 4/5 of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is :- [4 marks]

(A) 0.8

(B) 0.7

(C) 0.6

(D) 0.5

Correct Answer: (C) 0.6

Solution:

Concept:

- Archimedes' principle gives an upward force equal to the weight of displaced fluid: $F_b = \rho_{\text{fluid}} g V_{\text{displaced}}$.
- For a freely floating body in equilibrium, its weight equals the total buoyant force. Contributions from water and oil must be added separately.

Step 1: Infer the density of the wooden block Let the block volume be V, its density be ρ_b, and water density be ρ_w. Initially only water supplies buoyancy, and the submerged volume is $4V/5$. Balancing forces gives

$$\rho_b V g = \rho_w \frac{4V}{5} g, \qquad \rho_b = \frac{4}{5}\rho_w.$$

The remaining fifth of the block is initially above water. Its full volume still contributes to its weight; only the submerged fraction contributes to the water buoyancy.

Step 2: Balance the two displaced fluids After oil is added, half the block volume displaces water and half displaces oil. With oil density ρ_o, equilibrium requires

$$\rho_b V g = \rho_w \frac{V}{2} g + \rho_o \frac{V}{2} g.$$

Substituting the density already found and cancelling Vg yields

$$\frac{4}{5}\rho_w = \frac{1}{2}(\rho_w + \rho_o), \qquad \frac{\rho_o}{\rho_w} = \frac{8}{5} - 1 = \frac{3}{5} = 0.6.$$

The other proposed ratios 0.8, 0.7, and 0.5 would give average displaced-fluid densities 0.9, 0.85, and 0.75 times water density, rather than the required 0.8. Oil is less dense than water, consistent with its upper layer.

Step 3: Conclusion The density of oil relative to water is $\boxed{0.6}$. Hence, the correct option is (C).

Quick Tip to Solve: Equal block volumes in two fluids make the block density the arithmetic mean of the two fluid densities; use the original floating fraction to obtain the block density first.

40. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular accleration of 20 rad/s^2 must be applied about the axis for a duration of :- [4 marks]

(A) 2.5 s

(B) 3 s

(C) 5s

(D) 2 s

Correct Answer: (D) 2 s

Solution:

Concept:

- For rotation about a fixed axis, kinetic energy is $K = \frac{1}{2}I\omega^2$, where I is the moment of inertia and ω the angular speed.
- Under constant angular acceleration, $\omega = \omega_0 + \alpha t$. The stated initial rest condition gives $\omega_0 = 0$.

Step 1: Calculate the required final angular speed The moment of inertia about the specified axis is $I = 1.5\,\text{kg}\,\text{m}^2$, and the target rotational energy is $1200\,\text{J}$. Substituting into the energy formula gives

$$1200 = \frac{1}{2}(1.5)\omega^2, \qquad \omega^2 = \frac{2400}{1.5} = 1600.$$

The positive angular speed is therefore $\omega = 40\,\text{rad}\,\text{s}^{-1}$. The square root is essential: energy is proportional to the square of angular speed, rather than directly proportional to it.

Step 2: Find the duration and reject the other times Starting from rest with $\alpha = 20\,\text{rad}\,\text{s}^{-2}$, use

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{40}{20} = 2\,\text{s}.$$

A direct check combines the two equations:

$$K(t) = \frac{1}{2}(1.5)(20t)^2 = 300t^2\,\text{J}.$$

At $t = 2\,\text{s}$, this is $1200\,\text{J}$. The alternative durations 2.5 s, 3 s, and 5 s would instead produce 1875 J, 2700 J, and 7500 J. They all overshoot the target, so none is compatible with the specified constant acceleration.

Step 3: Conclusion The angular acceleration must act for $\boxed{2\,\text{s}}$. Hence, the correct option is (D).

Quick Tip to Solve: Combine the two rotation formulas into $K = I\alpha^2t^2/2$ when the body starts from rest; then solve for time with a square root.

41. A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T [4 marks]

(A) T^2/R^3 is a constant

(B) TR is a constant

(C) T/R^2 is a constant

(D) T/R is a constant

Correct Answer: (D) T/R is a constant

Solution:

Concept:

- For a spherically symmetric mass distribution, only the mass enclosed within radius R contributes to the gravitational field there; exterior spherical shells give zero net field.
- A thin spherical shell has mass $dM = 4\pi r^2 \rho(r)\, dr$. Circular motion requires $v^2/R = GM(R)/R^2$, and the period is $T = 2\pi R/v$.

Step 1: Integrate the enclosed mass Insert the given radial density $\rho(r) = K/r^2$ into the shell-mass expression. The factor r^2 in the volume element cancels the inverse-square density:

$$dM = 4\pi r^2 \frac{K}{r^2}\, dr = 4\pi K\, dr.$$

Consequently, the mass inside an orbit of radius R is

$$M(R) = \int_0^R 4\pi K\, dr = 4\pi K R.$$

Although the density diverges at the centre, its integral over this volume is finite. This enclosed mass increases linearly with radius and cannot be treated as the same fixed central mass for all orbits.

Step 2: Relate the orbital speed and period to radius The circular-orbit condition gives

$$\frac{v^2}{R} = \frac{G(4\pi K R)}{R^2}, \qquad v^2 = 4\pi G K.$$

Thus the orbital speed is independent of radius, while

$$T = \frac{2\pi R}{\sqrt{4\pi G K}}, \qquad \frac{T}{R} = \frac{2\pi}{\sqrt{4\pi G K}}.$$

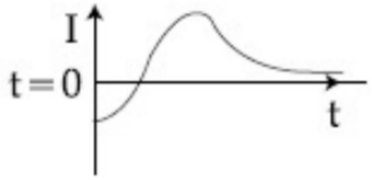
For fixed G and K, this ratio is constant. Kepler's T^2/R^3 law applies to a fixed enclosed central mass and is inapplicable here. Since $T \propto R$, the products or ratios TR and T/R^2 vary as R^2 and $1/R$, respectively.

Step 3: Conclusion The invariant is $\boxed{T/R}$. Hence, the correct option is (D).

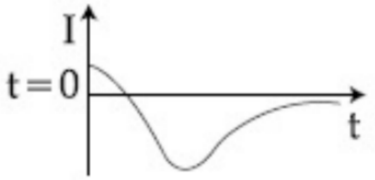
Quick Tip to Solve: For a density proportional to r^{-2}, remember that spherical shell volume contributes r^2: the enclosed mass grows as R, making circular speed constant.

42. A very long solenoid of radius R is carrying current I(t) = kte^{-at}(k $>$ 0), as a function of time (t $\geq$ 0). counter clockwise current is taken to be positive. A circular conducting coil of radius 2R is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by :- [4 marks]

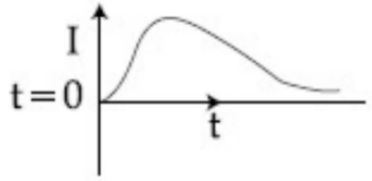
(A)



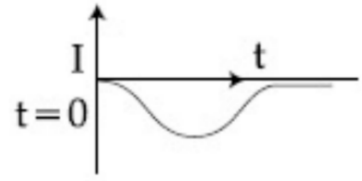
(B)



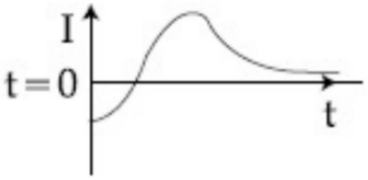
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- The field inside a long solenoid is proportional to its current. Flux through a larger concentric loop is proportional to $I(t)$, with the solenoid area πR^2 setting the effective flux area.
- Faraday's law is $\mathcal{E} = -d\Phi/dt$. With a fixed positive coil resistance, the sign and time dependence of induced current follow $-dI/dt$.

Step 1: Differentiate the solenoid current Let the positive constant relating the outer coil's flux to

the solenoid current be C. The outer radius $2R$ does not replace the flux area by four times its value, because the ideal long solenoid field outside its own radius is negligible. For the stated current,

$$I(t) = kte^{-at}, \qquad \frac{dI}{dt} = ke^{-at}(1 - at).$$

If the outer coil resistance is $\mathcal{R}$, its induced current has the form

$$i(t) = -\frac{Ck}{\mathcal{R}}e^{-at}(1 - at).$$

The minus sign implements Lenz's law with counterclockwise current defined as positive.

Step 2: Identify the initial sign, reversal, and late-time limit At $t = 0$, the solenoid current itself is zero but its derivative is $k > 0$; hence $i(0) = -Ck/\mathcal{R} < 0$. For the decaying exponential shown in the question, $a > 0$. At $t = 1/a$, the derivative vanishes and the induced current crosses zero. It is positive for $t > 1/a$, reaches a positive maximum at $t = 2/a$, then approaches zero from above as $t \to \infty$. Only graph (A) starts with a finite negative current, changes sign, and decays on the positive side. Graph (B) has the reversed signs, while (C) and (D) incorrectly begin at zero and omit the required sign reversal.

Step 3: Conclusion The induced current follows a negative initial value, a zero crossing, and a positive decaying tail, as in graph (A). Hence, the correct option is (A).

Quick Tip to Solve: A zero solenoid current at the initial instant does not imply zero induced current: inspect the initial slope dI/dt, then locate where that slope changes sign.

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43. The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$

What is the magnitude of the acceleration at t = 1 ? [4 marks]

(A) 50

(B) 25

(C) 40

(D) 100

Correct Answer: (A) 50

Solution:

Concept:

- Instantaneous velocity is the time derivative of the position vector: $\vec{v} = d\vec{r}/dt$. Acceleration is its second derivative, $\vec{a} = d^2\vec{r}/dt^2$. Differentiate the two Cartesian components independently because the unit vectors are constant.
- For perpendicular Cartesian components, the magnitude is $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$. The sign of a component specifies direction and must be retained during differentiation, even though it is squared in the magnitude.

Step 1: Differentiate the given coordinates The constant term in the vertical coordinate has zero

derivative. Differentiating each time-dependent term once gives the velocity, and differentiating once more gives the acceleration:

$$\vec{v}(t) = 30t\,\hat{i} - 40t\,\hat{j}, \qquad \vec{a}(t) = 30\,\hat{i} - 40\,\hat{j}.$$

Thus the acceleration is constant; evaluating at $t = 1$ does not introduce any additional factor of time.

Step 2: Combine the perpendicular components The horizontal and vertical acceleration components are 30 and -40, respectively. Their magnitudes cannot simply be added because they act at right angles. Therefore

$$|\vec{a}| = \sqrt{30^2 + (-40)^2} = \sqrt{900 + 1600} = \sqrt{2500} = 50.$$

The value 25 is the norm of the quadratic coefficients before applying the second-derivative factor of two; 40 includes only the vertical acceleration, and 100 introduces an extra factor of two. None gives the required vector magnitude.

Step 3: Conclusion The acceleration magnitude at $t = 1$ is 50 (in the acceleration units corresponding to the stated position and time). Hence, the correct option is (A).

Quick Tip to Solve: Differentiate each coordinate twice before taking a magnitude; the constant 4 affects the starting position but contributes nothing to acceleration.

44. A massless spring (k = 800 N/m), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely ? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K) [4 marks]

(A) 10^{-3} K

(B) 10^{-1} K

(C) 10^{-5}K

(D) 10^{-4} K

Correct Answer: (C) 10^{-5}K

Solution:

Concept:

- The mechanical energy initially stored in a stretched spring is $E = \frac{1}{2}kA^2$, where k is its force constant and A is the displacement from equilibrium. When the oscillations die out, this energy becomes heat.
- The water and immersed mass reach a common temperature rise ΔT. Their heat capacities add: $Q = (m_w c_w + m_s c_s)\Delta T$. The container and spring have negligible heat capacity as specified.

Step 1: Calculate the energy that is dissipated Convert the extension to metres and the attached mass to kilograms: $A = 0.020\,\text{m}$ and $m_s = 0.500\,\text{kg}$. The spring is massless, so its initial elastic energy

supplies the entire oscillatory energy:

$$E = \frac{1}{2}(800)(0.020)^2 = 400(0.0004) = 0.160\,\text{J}.$$

Damping changes this mechanical energy into thermal energy; it does not continually supply new energy to the system.

Step 2: Include both bodies in the heat balance The heat capacity of the water is $1 \times 4184 = 4184\,\text{J K}^{-1}$, while that of the attached mass is $0.500 \times 400 = 200\,\text{J K}^{-1}$. Hence

$$\Delta T = \frac{0.160}{4184 + 200} = \frac{0.160}{4384} = 3.65 \times 10^{-5}\,\text{K}.$$

The rise has the $10^{-5}\,\text{K}$ scale among the listed powers. The options 10^{-3} and $10^{-1}\,\text{K}$ require far more heat, whereas $10^{-4}\,\text{K}$ is above the calculated rise.

Step 3: Conclusion The common temperature rise is approximately $3.65 \times 10^{-5}\,\text{K}$, corresponding to the listed $10^{-5}\,\text{K}$ scale. Hence, the correct option is (C).

Quick Tip to Solve: Use the spring amplitude in metres and add the heat capacity of the immersed solid to that of the water before dividing the dissipated energy.

45. A wedge of mass M = 4m lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by :- [4 marks]

(A) $\frac{v^2}{g}$

(B) $\frac{2v^2}{7g}$

(C) $\frac{v^2}{2g}$

(D) $\frac{2v^2}{5g}$

Correct Answer: (D) $\frac{2v^2}{5g}$

Solution:

Concept:

- With no external horizontal force on the particle–wedge system, total horizontal momentum is conserved. The normal forces between the two bodies are internal; they can transfer momentum from one body to the other.
- All surfaces are smooth, so total mechanical energy is conserved. At the greatest height on the wedge, the particle momentarily has zero velocity relative to the wedge and both bodies have the same horizontal speed.

Step 1: Find the common speed at the turning point Initially only the particle moves, so the horizontal momentum is mv. At the highest point, its relative motion up the wedge momentarily stops. Writing

the common horizontal speed as V, momentum conservation gives

$$mv = (M+m)V = (4m+m)V = 5mV, \qquad V = \frac{v}{5}.$$

The system still moves at this instant. Setting its final kinetic energy to zero would incorrectly treat the wedge as fixed.

Step 2: Apply conservation of mechanical energy Take the particle's initial height as the zero of gravitational potential energy. The wedge does not rise, and the particle gains potential energy mgh. Thus

$$\frac{1}{2}mv^2 = \frac{1}{2}(5m)\left(\frac{v}{5}\right)^2 + mgh.$$

Rearranging,

$$mgh = \frac{1}{2}mv^2 - \frac{1}{10}mv^2 = \frac{2}{5}mv^2, \qquad h = \frac{2v^2}{5g}.$$

The value $v^2/(2g)$ would apply to an immovable wedge. The remaining values do not satisfy both the momentum and energy equations for $M = 4m$.

Step 3: Conclusion The maximum height is $\boxed{h = 2v^2/(5g)}$. Hence, the correct option is (D).

Quick Tip to Solve: At maximum ascent the relative speed is zero, not the laboratory speed. First find the shared speed from horizontal momentum, then use energy conservation.

46. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600nm. coming from a distant object, the limit of resolution of the telescope is close to :- [4 marks]

(A) 3.0×10^{-7} rad

(B) 4.5×10^{-7} rad

(C) 1.5×10^{-7} rad

(D) 2.0×10^{-7} rad

Correct Answer: (A) 3.0×10^{-7} rad

Solution:

Concept:

- For a telescope with a circular objective, the Rayleigh angular limit of resolution is $\theta_{\min} = 1.22\lambda/D$, in radians. Here λ is the light wavelength and D is the objective diameter.
- The angular limit is the smallest separation the telescope can resolve, not its reciprocal resolving power. Diffraction improves when the aperture grows and worsens when the wavelength grows.

Step 1: Express both lengths in the same unit The objective diameter is $250\,\text{cm} = 2.50\,\text{m}$, while the wavelength is $600\,\text{nm} = 600 \times 10^{-9}\,\text{m} = 6.00 \times 10^{-7}\,\text{m}$. The formula uses the full aperture diameter rather than its radius. This distinction matters because using half the given length would double the calculated angle.

Step 2: Evaluate the angular diffraction limit Substitute the two SI lengths directly in the Rayleigh

expression:

$$\theta_{\min} = \frac{1.22(6.00 \times 10^{-7})}{2.50} = \frac{7.32 \times 10^{-7}}{2.50} = 2.928 \times 10^{-7}\,\text{rad}.$$

Rounding to the precision of the alternatives gives 3.0×10^{-7} rad. The values 4.5×10^{-7}, 1.5×10^{-7}, and 2.0×10^{-7} rad are inconsistent with the circular-aperture factor and the supplied diameter.

Step 3: Conclusion The limit of resolution is approximately 2.93×10^{-7} rad, closest to 3.0×10^{-7} rad. Hence, the correct option is (A).

Quick Tip to Solve: For a circular telescope objective use 1.22 times wavelength divided by diameter; convert centimetres and nanometres before substituting.

47. A He^+ ion is in its first excited state. Its ionization energy is :- [4 marks]

(A) 13.60 eV

(B) 6.04 eV

(C) 48.36 eV

(D) 54.40 eV

Correct Answer: (A) 13.60 eV

Solution:

Concept:

- A singly ionized helium atom, He^+, has one electron and nuclear charge $Z = 2$. Its Bohr energy levels are $E_n = -13.6Z^2/n^2\,\text{eV}$, with zero energy assigned to a free electron at infinity.
- The first excited state has principal quantum number $n = 2$; the ground state has $n = 1$. Ionization energy from a given state is the positive energy needed to raise its electron from that level to $n \to \infty$.

Step 1: Identify the occupied energy level For the first excited state, insert $Z = 2$ and $n = 2$ in the hydrogen-like energy expression:

$$E_2 = -13.6\frac{2^2}{2^2}\,\text{eV} = -13.6\,\text{eV}.$$

Although the ion has a larger nuclear charge than hydrogen, the factor $Z^2 = 4$ is exactly cancelled here by $n^2 = 4$. The energy is negative because the electron remains bound to the nucleus.

Step 2: Find the energy required to reach infinity The continuum threshold is $E_\infty = 0$, so the energy that must be supplied is

$$E_{\text{ion}} = E_\infty - E_2 = 0 - (-13.6) = 13.60\,\text{eV}.$$

The value $54.40\,\text{eV}$ is the ionization energy from the ground state, which is not occupied here. The value $48.36\,\text{eV}$ describes a different level-energy difference, while $6.04\,\text{eV}$ corresponds approximately to binding in the third level; neither ionizes the stated first excited state.

Step 3: Conclusion The ionization energy of He^+ in its first excited state is $\boxed{13.60\,\text{eV}}$. Hence, the correct option is (A).

Quick Tip to Solve: First translate "first excited" into n = 2. Ionization asks for the magnitude of that level energy, not the energy gap from the ground state.

48. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a maganetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in Tesla) is approximately :- [4 marks]

(A) 10^{-4}

(B) 10^{-2}

(C) 10^{-1}

(D) 10^{-3}

Correct Answer: (D) 10^{-3}

Solution:

Concept:

- A current-carrying coil has magnetic moment $\mu = NIA$, where N is its number of turns, I its current and A its area. When the field is parallel to the coil plane, the moment is perpendicular to the field and the magnetic torque is $NIAB$.
- The torsion band exerts restoring torque $\kappa\theta$. At equilibrium these torques balance, so $B = \kappa\theta/(NIA)$. The angle must be in radians because the torsion constant is specified per radian.

Step 1: Convert the area, current and angle The numerical data in SI units are

$$N = 175, \quad A = 1\,\text{cm}^2 = 10^{-4}\,\text{m}^2, \quad I = 10^{-3}\,\text{A}, \quad \theta = \frac{\pi}{180} = 0.01745\,\text{rad}.$$

The restoring torque is therefore

$$\kappa\theta = (10^{-6})(0.01745) = 1.745 \times 10^{-8}\,\text{N m}.$$

Using one as the angle would treat a degree as a radian and overestimate the field greatly.

Step 2: Balance the magnetic and restoring torques The coefficient multiplying the field is

$$NIA = 175(10^{-3})(10^{-4}) = 1.75 \times 10^{-5}\,\text{A m}^2.$$

Therefore

$$B = \frac{1.745 \times 10^{-8}}{1.75 \times 10^{-5}} = 9.97 \times 10^{-4}\,\text{T}.$$

This is approximately $10^{-3}\,\text{T}$. The alternatives 10^{-4}, 10^{-2}, and $10^{-1}\,\text{T}$ differ by factors of ten or more and do not satisfy the measured torque balance.

Step 3: Conclusion The magnetic field is approximately $\boxed{1.0 \times 10^{-3}\,\text{T}}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert the deflection from degrees to radians and remember that one square centimetre is ten to the minus four square metres before applying torque balance.

49. The specific heats, C_P and C_V of a gas of diatomic molecules, A, are given (in units of J mol^{-1} K^{-1}) by 29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. If they are treated as ideal gases, then :- [4 marks]

(A) A is rigid but B has a vibrational mode

(B) A has a vibrational mode but B has none

(C) A has one vibrational mode and B has two

(D) Both A and B have a vibrational mode each

Correct Answer: (B) A has a vibrational mode but B has none

Solution:

Concept:

- For an ideal gas, Mayer's relation gives $C_P - C_V = R$, and the equipartition expression is $C_V = fR/2$, where f is the number of active quadratic degrees of freedom.
- A rigid diatomic molecule has three translational and two rotational degrees of freedom, giving $f = 5$ and $\gamma = C_P/C_V = 7/5 = 1.4$. An active vibration adds kinetic and potential energy terms, increasing heat capacity and reducing γ toward $9/7$.

Step 1: Compare the heat-capacity ratios Use the ratios to identify the molecular behavior indicated by the supplied values:

$$\gamma_A = \frac{29}{22} \simeq 1.318, \qquad \gamma_B = \frac{30}{21} \simeq 1.429.$$

Gas B is close to the rigid-diatomic ratio 1.4. Gas A has a clearly smaller ratio, closer to the vibrational value $9/7 \simeq 1.286$, indicating a contribution from vibrational energy storage.

Step 2: Identify which gas has a vibrational contribution The same comparison can be expressed using $f = 2/(\gamma - 1) = 2C_V/(C_P - C_V)$:

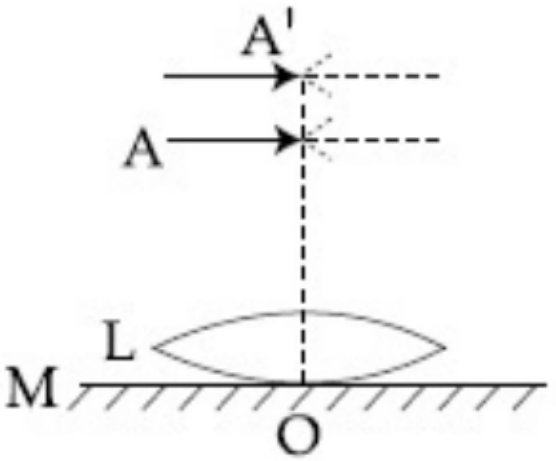
$$f_A = \frac{44}{7} \simeq 6.29, \qquad f_B = \frac{42}{9} \simeq 4.67.$$

These supplied approximate capacities indicate additional vibrational contribution for A and the rigid value near five for B. They are not exact ideal-gas heat capacities: the differences 7 and 9 do not both equal the universal gas constant. Thus the options describe the intended approximate molecular comparison. A cannot be identified as rigid while B vibrates; nor is there evidence that both gases have active vibration, or that B has two vibrational modes.

Step 3: Conclusion The ratios indicate that A has a vibrational contribution, whereas B behaves approximately as a rigid diatomic gas with no active vibration. Hence, the correct option is (B).

Quick Tip to Solve: Compare the ratio Cp/Cv with 1.4 for a rigid diatomic gas: additional vibrational energy storage lowers this ratio toward 9/7.

50. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index $\mu 1$ is put between the lens and the mirror, The pin has to be moved to A', such that OA' = 27 cm, to get its inverted real image at A' itself. The value of μ_1 will be :- [4 marks]



(A) $\sqrt{3}$

(B) 3 / 2

(C) $\sqrt{2}$

(D) 4 / 3

Correct Answer: (D) 4 / 3

Solution:

Concept:

- For a pin at the focal point of a lens above a plane mirror, emerging rays are parallel, return along their paths after reflection, and form an inverted image at the pin. The pin distance therefore gives the focal length.
- For the symmetric biconvex glass lens shown, the lens-maker formula is $1/f_g = (n_g - 1)(2/R)$. The liquid trapped below it makes a plano-concave lens with power $P_\ell = -(\mu_1 - 1)/R$. Thin lenses in contact have additive powers.

Step 1: Find the glass lens curvature The initial coincidence position gives $f_g = 18\,\text{cm}$. Using the two equal curvature magnitudes of the biconvex lens and $n_g = 1.5$,

$$\frac{1}{18} = (1.5 - 1)\frac{2}{R} = \frac{1}{R}, \qquad R = 18\,\text{cm}.$$

The plane mirror supplies the return path; it contributes no converging power of its own.

Step 2: Calculate the liquid power and index With liquid, coincidence occurs at $27\,\text{cm}$, so the combined focal length is $27\,\text{cm}$. The liquid decreases the positive power, consistently with its plano-concave shape:

$$\frac{1}{27} = \frac{1}{18} - \frac{\mu_1 - 1}{18}.$$

Multiplying by 18 gives $2/3 = 2 - \mu_1$, hence $\mu_1 = 4/3$. Options (A), (B) and (C) give different liquid

powers and do not reproduce the observed combined focal length.

Step 3: Conclusion The liquid refractive index is $\boxed{\mu_1 = 4/3}$. Hence, the correct option is (D).

Quick Tip to Solve: In the lens-on-mirror coincidence method, use the pin distance as the focal length, then subtract powers to isolate the liquid lens.

51. 50 W/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1m^2 surface area will be close to (c = 3 $\times$ 108 m/s) :- [4 marks]

(A) 20 $\times$ 10^{-8} N

(B) 35 $\times$ 10^{-8} N

(C) 10 $\times$ 10–8 N

(D) 15 $\times$ 10^{-8} N

Correct Answer: (A) 20 $\times$ 10^{-8} N

Solution:

Concept:

- Radiation of energy E carries momentum E/c. Absorption transfers that momentum, while normal reflection reverses it and transfers $2E/c$. Force is momentum transferred per second.
- For intensity I, illuminated area A, and reflected fraction r with the remainder absorbed, $F = [(1-r)+2r]IA/c = (1+r)IA/c$. The units $\mathrm{W\,m^{-2}}$ identify the given quantity as intensity.

Step 1: Separate absorbed and reflected powers The incident power on the stated area is $P = IA = 50 \times 1 = 50\,\mathrm{W}$. Of this, 25% is reflected and 75% is absorbed. Thus

$$P_{\mathrm{ref}} = 12.5\,\mathrm{W}, \qquad P_{\mathrm{abs}} = 37.5\,\mathrm{W}.$$

The two contributions push in the same direction, away from the incident beam, so their magnitudes must be added rather than subtracted.

Step 2: Add the momentum-transfer rates The printed speed appears as $c = 3 \times 108\,\mathrm{m/s}$, with no superscript on the 8. Here it is interpreted as the conventional speed of light, $3 \times 10^8\,\mathrm{m\,s^{-1}}$, consistent with the question naming c and with all four answer scales. Reading 108 literally would give $c = 324\,\mathrm{m/s}$ and $F = 62.5/324 \simeq 0.193\,\mathrm{N}$, matching none of the options. With the conventional speed-of-light interpretation,

$$F = \frac{P_{\mathrm{abs}} + 2P_{\mathrm{ref}}}{c} = \frac{37.5 + 25}{3 \times 10^8} = 2.083 \times 10^{-7}\,\mathrm{N}.$$

Equivalently this is $20.83 \times 10^{-8}\,\mathrm{N}$, closest to $20 \times 10^{-8}\,\mathrm{N}$. The values in (B), (C) and (D) do not follow from the required single momentum transfer for absorption and double transfer for reflection.

Step 3: Conclusion The force is approximately $\boxed{2.08 \times 10^{-7}\,\mathrm{N}}$, closest to $20 \times 10^{-8}\,\mathrm{N}$. Hence, the correct option is (A).

Quick Tip to Solve: For an opaque surface with reflectivity r, multiply IA/c by 1+r; do not double the absorbed contribution.

52. The area of a square is 5.29 cm^2. The area of 7 such squares taking into account the significant figures is :- [4 marks]

(A) 37.0 cm^2

(B) 37 cm^2

(C) 37.030 cm^2

(D) 37.03 cm^2

Correct Answer: (A) 37.0 cm^2

Solution:

Concept:

- In multiplication, the final measured result retains the number of significant figures of the least precise measured factor. Exact counting numbers introduce no measurement uncertainty.
- The measured area $5.29\,\mathrm{cm}^2$ has three significant figures. The number of squares, 7, is an exact count, so the total area must also have three significant figures.

Step 1: Multiply by the exact count Each square has area $5.29\,\mathrm{cm}^2$, and there are exactly seven equal squares. Additive area gives

$$A_{\text{total}} = 7(5.29) = 37.03\,\mathrm{cm}^2.$$

Keep this unrounded value until the final reporting stage. The factor seven does not restrict the answer to one significant figure because it counts objects rather than reporting a measured length or area.

Step 2: Round and distinguish the options The first three significant digits of 37.03 are 3, 7 and the zero in the tenths place. The next digit is 3, so the tenths digit is unchanged:

$$37.03\,\mathrm{cm}^2 \longrightarrow 37.0\,\mathrm{cm}^2.$$

The terminal zero is essential: it records precision to one decimal place and makes three significant figures. Option (B), 37, has only two significant figures. Options (C) and (D) have five and four respectively, claiming more precision than the measured input supports.

Step 3: Conclusion The appropriately reported total area is $\boxed{37.0\,\mathrm{cm}^2}$. Hence, the correct option is (A).

Quick Tip to Solve: Keep exact counts separate from measured data when deciding significant figures; a trailing decimal zero can carry necessary precision.

53. A particle 'P' is formed due to a completely inelastic collision of particles 'x' and 'y' having de-Broglie wavelengths 'λ_x' and 'λ_y' respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of 'P' is :- [4 marks]

(A) $\lambda_x - \lambda_y$
(B) $\frac{\lambda_x \lambda_y}{|\lambda_x - \lambda_y|}$
(C) $\lambda_x + \lambda_y$
(D) $\frac{\lambda_x \lambda_y}{\lambda_x + \lambda_y}$

Correct Answer: (B) $\frac{\lambda_x \lambda_y}{|\lambda_x - \lambda_y|}$

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/|p|$, where h is Planck's constant and $|p|$ is momentum magnitude. A wavelength is positive even when momentum has a negative component.
- Total momentum is conserved in a collision with no external impulse. In a completely inelastic collision the bodies stick together; kinetic energy need not be conserved, but their final momentum is still the vector sum of initial momenta.

Step 1: Assign opposite momentum directions Take the initial direction of particle x as positive. Its momentum component is $p_x = h/\lambda_x$, while that of y is $p_y = -h/\lambda_y$, because the particles approach in opposite directions. Therefore the momentum of their combined particle P is

$$p_P = h\left(\frac{1}{\lambda_x} - \frac{1}{\lambda_y}\right).$$

The mass of the combined particle is unnecessary since its wavelength depends directly on its momentum.

Step 2: Convert the resultant momentum to wavelength Taking the magnitude before applying the wave relation gives

$$\lambda_P = \frac{h}{|p_P|} = \frac{1}{|1/\lambda_x - 1/\lambda_y|} = \frac{\lambda_x \lambda_y}{|\lambda_x - \lambda_y|}.$$

For equal incident wavelengths, momenta cancel and the final wavelength diverges, as this expression predicts. Options (A) and (C) incorrectly add or subtract wavelengths directly. Option (D) uses a sum of momentum magnitudes, appropriate for parallel, same-direction momenta rather than the specified opposite directions.

Step 3: Conclusion $\boxed{\lambda_P = \lambda_x \lambda_y / |\lambda_x - \lambda_y|}$. Hence, the correct option is (B).

Quick Tip to Solve: Add signed momenta first and take the magnitude only at the end; de Broglie wavelengths themselves do not add in collisions.

54. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and x_2 ($x_1 > x_2$) from the lens. The ratio of x_1 and x_2 is :- [4 marks]

(A) 3 : 1

(B) 2 : 1

(C) 5 : 3

(D) 4 : 3

Correct Answer: (A) 3 : 1

Solution:

Concept:

- The thin-lens relation with Cartesian signs is $1/f = 1/v - 1/u$, and transverse magnification is $m = v/u$. A real object has $u < 0$, and a convex lens has $f > 0$.
- Two images of the same magnification magnitude can have opposite orientations: an inverted real image has $m = -2$, while an erect virtual image has $m = +2$. Their different signs are essential to finding two object positions.

Step 1: Find the position for the real image For the inverted image put $m = -2$, so $v = -2u$. Substitution into the lens equation gives

$$\frac{1}{20} = \frac{1}{-2u} - \frac{1}{u} = -\frac{3}{2u}, \qquad u = -30\,\text{cm}.$$

The positive object distance is therefore $30\,\text{cm}$. This lies between f and 2f, as required for an enlarged real image of a convex lens.

Step 2: Find the virtual-image position and compare For the erect image $m = +2$, giving $v = 2u$. Thus

$$\frac{1}{20} = \frac{1}{2u} - \frac{1}{u} = -\frac{1}{2u}, \qquad u = -10\,\text{cm}.$$

This object is inside the focal length, so an enlarged virtual image is consistent. Since $x_1 > x_2$, take $x_1 = 30\,\text{cm}$ and $x_2 = 10\,\text{cm}$. Their ratio is $30 : 10 = 3 : 1$; none of the ratios in (B), (C) or (D) satisfies these two lens-equation solutions.

Step 3: Conclusion The ratio is $\boxed{x_1 : x_2 = 3 : 1}$. Hence, the correct option is (A).

Quick Tip to Solve: When a convex lens gives the same enlargement at two object positions, solve separately with positive and negative magnifications.

55. The position of a particle as a function of time t, is given by

$x(t) = at + bt^2 - ct^3$

where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be : [4 marks]

(A) $a + \frac{b^2}{c}$

(B) $a + \frac{b^2}{4c}$

(C) $a + \frac{b^2}{3c}$

(D) $a + \frac{b^2}{2c}$

Correct Answer: (C) $a + \frac{b^2}{3c}$

Solution:

Concept:

- Instantaneous velocity is the derivative of position, $v = dx/dt$, and acceleration is $a_{\text{acc}} = dv/dt = d^2x/dt^2$. Differentiate each power of time using $d(t^n)/dt = nt^{n-1}$.
- The coefficient a in the given position expression is a constant, distinct from the particle's acceleration. To find velocity when acceleration vanishes, first find the corresponding time and then substitute it into the velocity expression.

Step 1: Differentiate and obtain the time For $x(t) = at + bt^2 - ct^3$, the derivatives are

$$v(t) = a + 2bt - 3ct^2, \qquad a_{\text{acc}}(t) = 2b - 6ct.$$

Set the acceleration to zero:

$$2b - 6ct = 0, \qquad t_0 = \frac{b}{3c}.$$

This uses the nonzero c implicit in the listed expressions. One must not set the velocity itself to zero: zero acceleration means stationary velocity, not stationary position.

Step 2: Evaluate the velocity without dropping terms Substitute the obtained time into both time-dependent velocity terms:

$$v(t_0) = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2$$
$$= a + \frac{2b^2}{3c} - \frac{b^2}{3c} = a + \frac{b^2}{3c}.$$

The factor 3 from differentiating the cubic term remains essential. The terms b^2/c, $b^2/(4c)$, and $b^2/(2c)$ in the other options cannot result from these derivatives and substitution. Dimensionally b^2/c has velocity units, consistent with adding it to a.

Step 3: Conclusion At zero acceleration, $\boxed{v = a + b^2/(3c)}$. Hence, the correct option is (C).

Quick Tip to Solve: For a cubic position law, acceleration is linear in time: solve that linear equation before substituting into the quadratic velocity.

56. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0 – 0.5 A ? [4 marks]

(A) 0.02 ohm

(B) 0.2 ohm

(C) 0.002 ohm

(D) 0.5 ohm

Correct Answer: (B) 0.2 ohm

Solution:

Concept:

- An ammeter of larger current range is made by connecting a small shunt resistance S in parallel with the galvanometer. The shunt carries the excess current and protects the sensitive coil.
- At full-scale reading, the galvanometer carries its allowed current I_g, the shunt carries $I - I_g$, and equal parallel voltage drops imply $I_g G = (I - I_g)S$. Thus $S = I_g G/(I - I_g)$.

Step 1: Determine the coil voltage and shunt current The specified galvanometer resistance is $G = 50\,\Omega$, its maximum current is $I_g = 0.002\,\text{A}$, and the desired full-scale current is $I = 0.5\,\text{A}$. The permissible coil voltage is

$$V_g = I_g G = 0.002 \times 50 = 0.100\,\text{V}.$$

At the same operating point the shunt must carry $I_s = 0.5 - 0.002 = 0.498\,\text{A}$, rather than the entire ammeter current.

Step 2: Calculate and check the parallel resistance The shunt has the same terminal voltage as the coil, so

$$S = \frac{0.100}{0.498} = 0.2008\,\Omega \simeq 0.2\,\Omega.$$

This is much smaller than $50\,\Omega$, as expected when almost all the current must bypass the coil. Values 0.02 and 0.002 ohm would divert too much current for the stated full-scale coil current, while 0.5 ohm would allow too much coil current at a total of 0.5 A. A series resistance would limit current rather than extend the ammeter range, so the connection must be parallel.

Step 3: Conclusion Connect a $\boxed{0.2\,\Omega}$ shunt in parallel with the galvanometer. Hence, the correct option is (B).

Quick Tip to Solve: Use the excess current I minus Ig in the shunt denominator, and check that a range-extending ammeter shunt is much smaller than the coil resistance.

57. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is :- [4 marks]

(A) 180m/s, 80 Hz

(B) 180m/s, 120 Hz

(C) 320m/s, 120 Hz

(D) 320m/s, 80 Hz

Correct Answer: (D) 320m/s, 80 Hz

Solution:

Concept:

- For a string fixed at both ends, each end is a displacement node. Its allowed wavelengths satisfy $L = n\lambda_n/2$, where $n = 1, 2, 3, \ldots$ is the harmonic number.
- The harmonic frequencies are $f_n = nv/(2L) = nf_1$, where v is wave speed and f_1 is the fundamental frequency. The driving frequency equals the frequency of the excited mode.

Step 1: Identify the third-harmonic frequency The vibrator drives the string at $240\,\text{Hz}$, and the question identifies the mode as the third harmonic. Thus $f_3 = 240\,\text{Hz}$, rather than the fundamental. Since three half-wavelengths fit along the string,

$$\lambda_3 = \frac{2L}{3} = \frac{2(2.0)}{3} = \frac{4}{3}\,\text{m}.$$

Step 2: Calculate the speed and fundamental Use the wave relation with the wavelength of the actual excited mode:

$$v = f_3\lambda_3 = 240\left(\frac{4}{3}\right) = 320\,\text{m}\,\text{s}^{-1}.$$

The fundamental corresponds to one half-wavelength along the same string, so

$$f_1 = \frac{f_3}{3} = 80\,\text{Hz}.$$

As an independent check, $v/(2L) = 320/4 = 80\,\text{Hz}$. Options (A) and (B) give the wrong speed, while (B) and (C) incorrectly use $120\,\text{Hz}$ for the fundamental.

Step 3: Conclusion The wave speed is $320\,\text{m}\,\text{s}^{-1}$ and the fundamental frequency is $80\,\text{Hz}$. Hence, the correct option is (D).

Quick Tip to Solve: For the nth harmonic, divide the driving frequency by n before using $v = 2Lf_1$; the driving frequency need not be the fundamental.

58. Four point charges –q, +q, +q and –q are placed on y-axis at y = –2d, y = –d, y = +d and y = +2d, respectively. The magnitude of the electric field E at a point on the x-axis at x = D, with D >> d, will behave as :- [4 marks]

(A) $E \propto \frac{1}{D^3}$

(B) $E \propto \frac{1}{D}$

(C) $E \propto \frac{1}{D^4}$

(D) $E \propto \frac{1}{D^2}$

Correct Answer: (C) $E \propto \frac{1}{D^4}$

Solution:

Concept:

- By superposition, the electric field is the vector sum of the fields of all four point charges. For charges at $y = \pm a$, their transverse components cancel at a point on the x-axis.
- The x-component from two equal charges Q at $y = \pm a$ is $2kQD/(D^2 + a^2)^{3/2}$, where $k = 1/(4\pi\varepsilon_0)$. For $D \gg d$, a binomial expansion identifies the leading nonzero term.

Step 1: Combine the two symmetric pairs The inner pair carries $+q$ at $y = \pm d$; the outer pair carries $-q$ at $y = \pm 2d$. Both pairs have zero net y-component. Their x-components oppose each other, giving

$$E_x = 2kqD\left[\frac{1}{(D^2 + d^2)^{3/2}} - \frac{1}{(D^2 + 4d^2)^{3/2}}\right].$$

The bracket is positive for $D > 0$, so this is also the magnitude.

Step 2: Expand for a distant observation point Factor out D^3 from each denominator and use $(1 + u)^{-3/2} = 1 - 3u/2 + O(u^2)$:

$$E = \frac{2kq}{D^2}\left[\left(1 - \frac{3d^2}{2D^2}\right) - \left(1 - \frac{6d^2}{D^2}\right) + O\left(\frac{d^4}{D^4}\right)\right].$$

Therefore

$$E = \frac{9kqd^2}{D^4} + O(D^{-6}).$$

The D^{-2} term cancels because total charge is zero; the dipole moment also vanishes by symmetry, ruling out D^{-3}. A D^{-1} field is not the leading field of these localized charges.

Step 3: Conclusion The leading distant field behaves as $E \propto D^{-4}$. Hence, the correct option is (C).

Quick Tip to Solve: Check both total charge and dipole moment before selecting a distance power: if both vanish, expand the exact symmetric-pair field to the next order.

59. The physical sizes of the transmitter and receiver antenna in a communication system are :-

[4 marks]

(A) proportional to carrier frequency

(B) inversely proportional to carrier frequency

(C) inversely proportional to modulation frequency

(D) independent of both carrier and modulation frequency

Correct Answer: (B) inversely proportional to carrier frequency

Solution:

Concept:

- Efficient radiation and reception require an antenna whose characteristic dimension is comparable to a suitable fraction of the electromagnetic wavelength, commonly $\lambda/4$ for a monopole or $\lambda/2$ for a dipole.
- The carrier wavelength is $\lambda_c = c/f_c$, where c is electromagnetic wave speed and f_c is the carrier frequency. Thus an antenna designed for the carrier has characteristic length proportional to $1/f_c$.

Step 1: Relate the antenna to the radiated wave The information signal modulates a high-frequency carrier. The transmitting antenna radiates this modulated carrier, and the receiving antenna is designed to respond efficiently to the corresponding radio-frequency wave. It is therefore the carrier wavelength that sets their relevant physical dimensions. For example, a quarter-wave antenna has

$$\ell = \frac{\lambda_c}{4}.$$

The exact numerical fraction depends on antenna design, but the frequency scaling is the same.

Step 2: Insert the wavelength-frequency relation Substituting $\lambda_c = c/f_c$,

$$\ell = \frac{c}{4f_c} \quad \Longrightarrow \quad \ell \propto \frac{1}{f_c}.$$

Doubling the carrier frequency halves the required characteristic length for the same antenna type. Option (A) reverses this dependence. Option (C) uses the much lower information or modulation frequency, which does not set the radio antenna size. Option (D) ignores the wavelength requirement and is also incorrect.

Step 3: Conclusion The physical sizes of the transmitting and receiving antennas are inversely proportional to the carrier frequency. Hence, the correct option is (B).

Quick Tip to Solve: Use the wavelength of the wave actually radiated: antenna length is a fraction of c/f_c, so a higher carrier frequency allows a smaller antenna.

60. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B ? (speed of sound in air = 340 ms^{-1}) :- [4 marks]

(A) 2300 Hz

(B) 2060 Hz

(C) 2250 Hz

(D) 2150 Hz

Correct Answer: (C) 2250 Hz

Solution:

Concept:

- For sound, source and observer speeds must be measured relative to the medium. Their effects enter separately; the relative speed of the cars cannot simply replace both speeds in the Doppler formula.
- When source and observer recede from one another, the observed frequency is $f_o = f_s(v - v_o)/(v + v_s)$. Here v is sound speed, v_o the observer speed away from the source and v_s the source speed away from the observer.

Step 1: Choose the signs for recession Car B is the source, and the observer is in car A. Each moves at $20\,\mathrm{m\,s^{-1}}$ relative to the ground, which is the reference for the stationary air in this problem. Because both cars move away from one another, the observer encounters fewer wavefronts per second and the source increases the wavefront spacing. Both effects lower the detected frequency. Hence

$$2000 = f_s\frac{340 - 20}{340 + 20} = f_s\frac{320}{360}.$$

Step 2: Recover the source frequency and check it Rearrange the equation rather than using the observed frequency as the natural frequency:

$$f_s = 2000\frac{360}{320} = 2000\frac{9}{8} = 2250\,\mathrm{Hz}.$$

The answer exceeds $2000\,\mathrm{Hz}$, as it must for recession. Substituting it back gives $2250(8/9) = 2000\,\mathrm{Hz}$. The alternatives 2300, 2060 and $2150\,\mathrm{Hz}$ do not satisfy the exact ratio $f_o/f_s = 8/9$.

Step 3: Conclusion The natural frequency of the source in car B is $2250\,\mathrm{Hz}$. Hence, the correct option is (C).

Quick Tip to Solve: For recession, use a minus sign in the observer factor and a plus sign in the source factor; then check that the emitted frequency is larger than the heard frequency.

Chemistry (120 marks)

61. The correct statements among I to III are :

(I) Valence bond theory cannot explain the color exhibited by transition metal complexes.

(II) Valence bond theory can predict quantitatively the magnetic properties of transtition metal complexes.

(III) Valence bond theory cannot distinguish ligands as weak and strong field ones. [4 marks]

(A) (II) and (III) only

(B) (I) and (II) only

(C) (I), (II) and (III)

(D) (I) and (III) only

Correct Answer: (D) (I) and (III) only

Solution:

Concept:

- Valence bond theory describes coordination bonding by overlap of ligand donor orbitals with vacant metal hybrid orbitals. With an assumed electron arrangement it can describe geometry and qualitatively classify a complex as paramagnetic or diamagnetic.
- This hybrid-orbital picture does not provide the ligand-field energy splitting and transition energies needed to explain optical spectra, quantitative magnetic behaviour, or the relative field strengths of ligands.

Step 1: Assess the colour statement Statement (I) is correct. The colour of a transition-metal complex depends on absorption of particular visible-light energies associated with electronic transitions. To predict which light is absorbed, a theory must describe the relevant energy levels and their separation. Valence bond theory supplies a bonding and hybridisation picture, but it does not explain those optical transition energies and therefore cannot explain the colours of complexes.

Step 2: Assess magnetism and ligand strength Statement (II) is incorrect. VBT may indicate the presence or absence of unpaired electrons after the electron configuration and pairing are specified, but its treatment of magnetic properties is qualitative, not the quantitative prediction claimed in the statement. Do not confuse a qualitative paramagnetic/diamagnetic classification with a complete quantitative theory. Statement (III) is correct: VBT does not provide a basis for ranking ligands as weak-field or strong-field, nor explain why one ligand promotes pairing more strongly than another. Those distinctions require an energy-level treatment such as crystal field theory. Thus every option containing (II), namely (A), (B) and (C), must be rejected.

Step 3: Conclusion Only statements (I) and (III) are correct. Hence, the correct option is (D).

Quick Tip to Solve: When testing VBT statements, distinguish qualitative geometry and unpaired-electron descriptions from quantitative spectra, magnetic behaviour and ligand-field strength.

62. A solution of $Ni(NO_3)_2$ is electrolysed between platinum electrodes using 0.1 Faraday electricity. How many mole of Ni will be deposited at the cathode? [4 marks]

(A) 0.10

(B) 0.15

(C) 0.20

(D) 0.05

Correct Answer: (D) 0.05

Solution:

Concept:

- One Faraday of electric charge corresponds to one mole of electrons. Thus a charge of $0.1F$ supplies $0.1\,\text{mol}$ of electrons to the electrode reaction.
- For electrodeposition, balance the cathode half-reaction first. If depositing one metal atom requires z electrons, the deposited amount is $n(\text{metal}) = n(e^-)/z = Q/(zF)$.

Step 1: Identify the nickel reduction reaction In $Ni(NO_3)_2$, each nitrate ion has charge -1, so nickel is present as Ni^{2+}. At the cathode nickel ions gain electrons and form nickel metal:

$$Ni^{2+} + 2\,e^- \longrightarrow Ni\,(s).$$

Consequently two moles of electrons are required to deposit one mole of nickel. The platinum electrode is inert; its presence does not alter this electron-to-nickel stoichiometric ratio.

Step 2: Convert charge into deposited moles The supplied charge is $Q = 0.1F$, so

$$n(e^-) = \frac{Q}{F} = 0.1\,\text{mol}.$$

Applying the two-electron requirement,

$$n(\text{Ni}) = \frac{0.1}{2} = 0.05\,\text{mol}.$$

Option (A), 0.10, would incorrectly assume a one-electron nickel ion. Option (C), 0.20, multiplies by two instead of dividing, while option (B), 0.15, has no basis in the balanced half-reaction. As a check, $0.05\,\text{mol}$ of nickel consumes $2(0.05) = 0.1\,\text{mol}$ electrons.

Step 3: Conclusion The amount of nickel deposited at the cathode is $0.05\,\text{mol}$. Hence, the correct option is (D).

Quick Tip to Solve: Divide the number of Faradays by the metal-ion charge to obtain deposited moles; the oxidation state comes from the electrolyte formula, not the electrode material.

63. The peptide that gives positive ceric ammonium nitrate and carbylamine tests is : [4 marks]

(A) Ser-Lys

(B) Asp-Gln

(C) Gln-Asp

(D) Lys-Asp

Correct Answer: (A) Ser-Lys

Solution:

Concept:

- Ceric ammonium nitrate gives the characteristic positive alcohol test with a free alcoholic hydroxyl group. The serine side chain is $-CH_2OH$, so a peptide retaining that side chain can respond to this test.
- The carbylamine test is given by a free primary amino group with chloroform and alcoholic potassium hydroxide. Peptide formation converts the linking amino and carboxyl groups into an amide bond, while other suitable side-chain groups can remain free.

Step 1: Find a peptide with an alcoholic hydroxyl group Among the amino acids listed, serine (Ser) has the side chain $-CH_2OH$. Lysine (Lys) has $-(CH_2)_4NH_2$, aspartic acid (Asp) has $-CH_2COOH$, and glutamine (Gln) has $-CH_2CH_2CONH_2$. The latter three do not have a free alcoholic hydroxyl group. A carboxylic-acid $-OH$ is not an alcoholic hydroxyl, and an amide group is not an alcohol. Thus only Ser–Lys has the functionality needed for the positive ceric ammonium nitrate alcohol test.

Step 2: Confirm a primary amino group survives In Ser–Lys, the peptide linkage uses the serine carboxyl group and the lysine alpha-amino group. Serine retains its N-terminal primary amino group, and lysine also retains its terminal side-chain $-NH_2$ group. These are not part of the peptide amide linkage and can give the carbylamine reaction. Therefore Ser–Lys meets both tests simultaneously. Asp–Gln, Gln–Asp and Lys–Asp may retain a free amino group, but all lack the alcoholic side chain required for the other test, so options (B), (C) and (D) fail the combined condition.

Step 3: Conclusion Ser–Lys contains both a free alcoholic hydroxyl group and free primary amino groups. Hence, the correct option is (A).

Quick Tip to Solve: Test the two requirements separately: locate serine for an alcoholic hydroxyl, then check that peptide bonding has left a free primary amino group.

64. The correct statements among I to III regarding group 13 element oxides are,

(I) Boron trioxide is acidic.

(II) Oxides of aluminium and gallium are amphoteric.

(III) Oxides of indium and thalliumare basic. [4 marks]

(A) (II) and (III) only

(B) (I), (II) and (III)

(C) (I) and (III) only

(D) (I) and (II) only

Correct Answer: (B) (I), (II) and (III)

Solution:

Concept:

- The acidic or basic character of an oxide follows the nonmetallic or metallic character of its element. Nonmetal oxides are generally acidic, whereas metallic oxides become increasingly basic.
- Down group 13, metallic character increases. Thus the oxides change from acidic through amphoteric to predominantly basic; an amphoteric oxide reacts with both acids and bases.

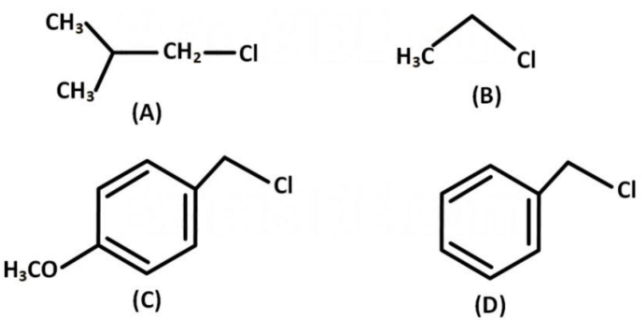
Step 1: Check boron and the intermediate elements Boron is the nonmetal at the top of the group. Its oxide, B_2O_3, is acidic and reacts with bases to form borates; statement I is therefore true. Aluminium and gallium occupy the intermediate region of the trend. Their oxides, Al_2O_3 and Ga_2O_3, can react with both acids and alkalis, so statement II is also true. For example, $Al_2O_3 + 6\,HCl \longrightarrow 2\,AlCl_3 + 3\,H_2O$ shows the basic side of amphoterism.

Step 2: Check the heavier elements and combine the results Indium and thallium show stronger metallic character than aluminium and gallium. Their oxides are consequently classified as basic in the group trend, establishing statement III. This does not mean that every oxide has exactly the same quantitative basic strength: the question asks for the qualitative classification. Since each of I, II and III is correct, any option containing only two statements leaves out a true statement. In particular, options A, C and D each omit one independently valid classification.

Step 3: Conclusion All three statements, (I), (II) and (III), are correct. Hence, the correct option is (B).

Quick Tip to Solve: Move down group 13 in the order acidic, amphoteric, basic, and locate each named element before judging the statements.

65. Increasing order of reactivity of the following compounds for S_N1 substitution is: [4 marks]



(A) (A) < (B) < (D) < (C)

(B) (B) < (C) < (D) < (A)

(C) (B) < (A) < (D) < (C)

(D) (B) < (C) < (A) < (D)

Correct Answer: (C) (B) < (A) < (D) < (C)

Solution:

Concept:

- An S_N1 substitution begins with slow ionisation of the carbon-halogen bond. The rate increases when the developing carbocation is better stabilised.
- Compare resonance stabilisation first, then electron donation by alkyl groups. A para methoxy group can donate its lone pair through the aromatic ring and stabilise a benzylic cation.

Step 1: Rank the two aliphatic chlorides Compound B is ethyl chloride, whereas A is isobutyl chloride. Cleavage at the carbon bearing chlorine initially gives primary carbocation character in each case. The more electron-releasing isopropyl group attached to the ionising carbon in A stabilises the developing positive charge more effectively than the methyl group in B. Thus B ionises less readily than A. The comparison concerns the initial ionisation step; a possible rearrangement after ionisation does not change which bond must first break.

Step 2: Compare the benzylic compounds and form the sequence Compound D is benzyl chloride. Its benzylic positive charge can be delocalised into the benzene ring, so it is much better stabilised than the nonbenzylic primary cations from A and B. Compound C is para-methoxybenzyl chloride. In addition to ordinary benzylic delocalisation, the para methoxy group supplies electron density by resonance and further lowers the energy of the developing cation. Hence C reacts faster than D. Combining both comparisons gives

$$\mathrm{B} < \mathrm{A} < \mathrm{D} < \mathrm{C}.$$

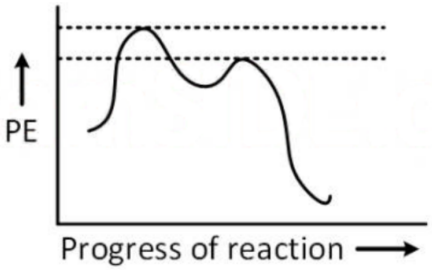
Option A reverses the two aliphatic chlorides, while options B and D incorrectly put the resonance-enhanced methoxybenzyl substrate below less stabilised substrates.

Step 3: Conclusion The increasing order is B < A < D < C. Hence, the correct option is (C).

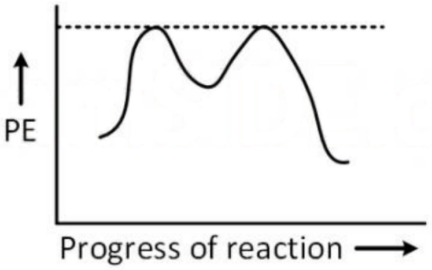
Quick Tip to Solve: For SN1 orders, identify the carbon that loses the leaving group and then compare the resonance stabilisation of its developing cation.

66. **Which of the following potential energy (PE) diagrams represents the S_N1 reaction?** [4 marks]

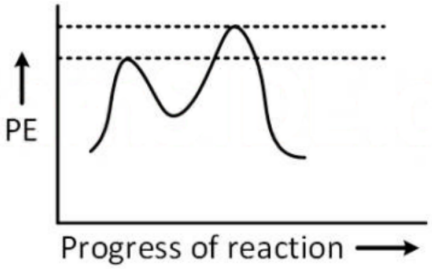
(A)



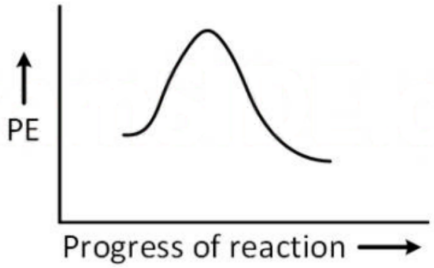
(B)



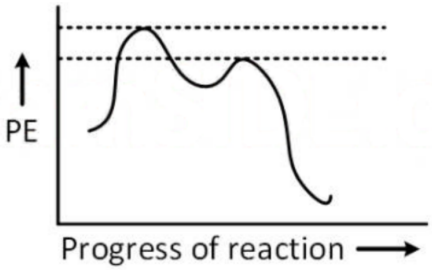
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- An S_N1 reaction proceeds through a carbocation intermediate: slow carbon-halogen bond ionisation followed by faster nucleophilic attack.
- A transition state is a maximum on a potential-energy profile, whereas an intermediate is a local minimum between successive maxima. Activation energies are measured from the starting minimum of each individual step.

Step 1: Identify the required two-step profile The first elementary step produces a carbocation and the leaving-group anion. The second consumes that carbocation as the nucleophile forms the new bond. Therefore the reaction coordinate must contain two peaks with a valley between them. The valley represents a chemical intermediate, not a transition state. A single smooth peak, as drawn in option D, describes a one-step process and cannot represent the stated stepwise mechanism.

Step 2: Locate the slow ionisation step Ionisation is the difficult, rate-determining step in the standard SN1 mechanism. It therefore has a large first barrier, while capture of the carbocation has a smaller barrier measured upward from the intermediate. Option A shows the usual energy diagram: the first transition state lies above the second, with a carbocation valley separating them. Option B places both peaks at the same potential energy rather than displaying the characteristic higher ionisation transition state; option C instead places the second transition state higher. The conventional SN1 profile among these sketches is consequently A. The depth of the final product minimum indicates the overall energy change, and should not be confused with the number of elementary steps.

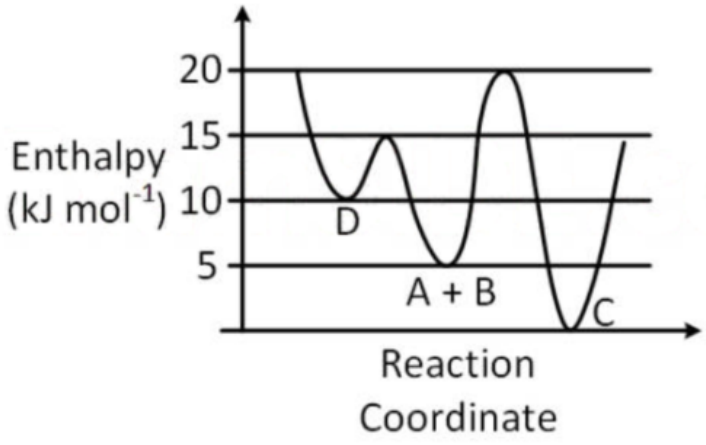
Step 3: Conclusion The two-step profile with the higher first transition state is diagram (A). Hence, the correct option is (A).

Quick Tip to Solve: Count peaks to count elementary steps, then identify the carbocation valley and the large initial ionisation barrier.

67. Consider the given plot of enthalpy of the following reaction between A and B.

A+ B → C + D

Identify the incorrect statement. [4 marks]



(A) Formation of A and B from C has highest enthalpy of activation.

(B) D is kinetically stable product.

(C) C is the thermodynamically stable product

(D) Activation enthalpy to form C is 5kJ mol^{-1} less than that to form D.

Correct Answer: (D) Activation enthalpy to form C is 5kJ mol^{-1} less than that to form D.

Solution:

Concept:

- Activation enthalpy is the enthalpy of the relevant transition state minus that of the species from which that step starts: $\Delta H^{\ddagger} = H_{\text{TS}} - H_{\text{initial}}$.
- The kinetic product is reached through the lower barrier; the thermodynamically more stable product has the lower final enthalpy. These two criteria need not select the same product.

Step 1: Read the forward barriers from the plot The reactants A+B lie at $5\,\text{kJ}\,\text{mol}^{-1}$. The peak toward D is at 15, whereas the peak toward C is at 20 on the same enthalpy scale. Thus

$$\Delta H_D^{\ddagger} = 15 - 5 = 10\,\text{kJ}\,\text{mol}^{-1}, \qquad \Delta H_C^{\ddagger} = 20 - 5 = 15\,\text{kJ}\,\text{mol}^{-1}.$$

Formation of D has the smaller barrier, so D is the kinetically favoured product, as intended by statement B. Formation of C requires a barrier 5 units greater, rather than 5 units less, than formation of D.

Step 2: Check the other stability statements The product minimum C is at zero while D is at $10\,\text{kJ}\,\text{mol}^{-1}$, so C is the lower-enthalpy, thermodynamically stable product. Statement C is correct. The reverse barrier from C to A+B is $20 - 0 = 20\,\text{kJ}\,\text{mol}^{-1}$, exceeding the forward barriers 10 and 15 and the reverse D barrier $15 - 10 = 5\,\text{kJ}\,\text{mol}^{-1}$. Hence statement A also agrees with the plotted pathway. The question asks for the incorrect statement, so the direction of the comparison in D is decisive.

Step 3: Conclusion The activation enthalpy for C is $5\,\text{kJ}\,\text{mol}^{-1}$ more than that for D. Statement (D)

incorrectly says less. Hence, the correct option is (D).

Quick Tip to Solve: Always subtract the reactant minimum from the peak; compare product minima separately when deciding thermodynamic stability.

68. Among the following species, the diamagnetic molecule is [4 marks]

(A) CO

(B) B_2

(C) O_2

(D) NO

Correct Answer: (A) CO

Solution:

Concept:

- A molecule is diamagnetic when all its electrons are paired. One or more unpaired electrons makes it paramagnetic.
- For diatomic molecules, fill molecular orbitals in increasing energy and apply Hund's rule to degenerate orbitals: electrons occupy them singly before pairing.

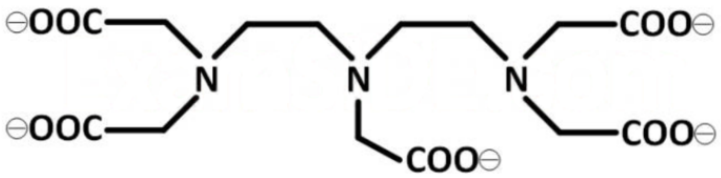
Step 1: Check carbon monoxide Carbon monoxide has $6 + 8 = 14$ electrons in total, or ten valence electrons. It is isoelectronic with N_2 and has a closed-shell ground state. In the valence-orbital description, the lower $\sigma(2s)$ and $\sigma^*(2s)$ orbitals are doubly occupied, the two bonding $\pi(2p)$ orbitals are each doubly occupied, and the remaining bonding $\sigma(2p)$ orbital contains a pair. No electron is left unpaired. Thus CO satisfies the condition for diamagnetism.

Step 2: Eliminate the remaining species For B_2, after filling the 2s bonding and antibonding orbitals, the two remaining valence electrons occupy the two degenerate bonding $\pi(2p)$ orbitals singly. It has two unpaired electrons. Oxygen, O_2, has two unpaired electrons in its degenerate antibonding $\pi^*(2p)$ orbitals. Finally NO has $7 + 8 = 15$ electrons, an odd electron count, so its ground state cannot have every electron paired; it contains one unpaired electron. Consequently options B, C and D are paramagnetic, leaving a unique diamagnetic choice.

Step 3: Conclusion Carbon monoxide, CO, has all electrons paired and is diamagnetic. Hence, the correct option is (A).

Quick Tip to Solve: Use electron counts first: CO is a closed-shell N2 analogue, whereas odd-electron NO must be paramagnetic.

69. The maximum possible denticities of a ligand given below towards a common transition and inner-transition metal ion, respectively, are : [4 marks]



(A) 8 and 6

(B) 8 and 8

(C) 6 and 8

(D) 6 and 6

Correct Answer: (C) 6 and 8

Solution:

Concept:

- Denticity is the number of donor atoms of one ligand that simultaneously bind to the same metal ion; it is not the total number of atoms carrying lone pairs.
- The actual maximum denticity depends on the available donor groups and the coordination capacity and size of the metal. Inner-transition ions commonly accommodate higher coordination numbers than common transition ions.

Step 1: Count the available donor groups The illustrated ligand has three amine nitrogen atoms, each with a lone pair available for donation, and five carboxylate arms. In the usual polyaminocarboxylate chelation pattern, each carboxylate arm contributes one oxygen donor to the same central metal ion. The complete donor set therefore supplies

$$3 \text{ nitrogen donors} + 5 \text{ oxygen donors} = 8 \text{ donor atoms.}$$

Counting both oxygens of each carboxylate as independent simultaneously binding chelate donors would overcount the ordinary coordination pattern intended here.

Step 2: Apply the metal-dependent coordination limit A common transition metal ion generally accommodates six donor atoms in its usual octahedral complex. The ligand can therefore bind hexadentately, with two of its potential donors left uncoordinated. The larger inner-transition metal ion can accept eight donors, allowing the same ligand to bind octadentately. Thus the required ordered pair, transition ion first and inner-transition ion second, is (6,8). Option A reverses these roles. Option B assigns eightfold coordination to the common transition ion, and option D fails to use the higher coordination capacity of the inner-transition ion.

Step 3: Conclusion The maximum possible denticities in the stated order are 6 and 8. Hence, the correct option is (C).

Quick Tip to Solve: Count the ligand donors, then check how many the named metal class can accommodate; potential donor count and actual denticity differ.

70. In the following reaction Rate of the reaction is the highest for : [4 marks]

$$\text{Carbonyl compound} + \text{MeOH} \underset{}{\overset{HCL}{\rightleftharpoons}} \text{acetal}$$

(A) Propanal as substrate and methanol in stoichiometric amount.

(B) Acetone as substrate and methanol in stoichiometric amount

(C) Propanal as substrate and methanol in excess

(D) Acetone as substrate and methanol in excess

Correct Answer: (C) Propanal as substrate and methanol in excess

Solution:

Concept:

- Acetal formation is acid-catalysed addition of an alcohol to a carbonyl compound followed by replacement of the carbonyl oxygen through the hemiacetal intermediate.
- Aldehydes are generally more reactive than ketones toward nucleophilic addition because they have less steric hindrance and fewer electron-donating alkyl groups at the carbonyl carbon.

Step 1: Compare propanal with acetone Propanal has one alkyl group and one hydrogen attached to its carbonyl carbon. Acetone has two alkyl groups. The second alkyl group in acetone both donates electron density, reducing the electrophilicity of the carbonyl carbon, and crowds the incoming methanol molecule. For comparable acid catalysis and alcohol conditions, nucleophilic addition therefore proceeds more readily with propanal. Options B and D use acetone, the less reactive carbonyl substrate, and cannot give the highest rate among the combinations listed.

Step 2: Compare the methanol conditions Methanol is a reacting nucleophile, rather than merely an inert additive. Using it in excess increases its availability for attack on the activated carbonyl and the subsequent intermediates. Excess alcohol also favours acetal formation in this reversible process, although the equilibrium displacement and the kinetic concentration effect are distinct ideas. Between the two propanal choices, option C combines the more reactive aldehyde with excess methanol; option A uses only the stoichiometric amount. Thus C incorporates both favourable factors. No claim that stoichiometric alcohol prevents reaction is needed: it simply does not provide the greatest alcohol availability among the listed choices.

Step 3: Conclusion The reaction is fastest for propanal as substrate with methanol in excess. Hence, the correct option is (C).

Quick Tip to Solve: For acetal formation, first favour the less hindered, more electrophilic carbonyl; then compare the amount of alcohol available.

71. The maximum number of possible oxidation states of actinoides are shown by : [4 marks]

(A) nobelium (No) and lawrencium (Lr)

(B) actinium (Ac) and thorium (Th)

(C) neptunium (Np) and plutonium (Pu)

(D) berkelium (Bk) and californium (Cf)

Correct Answer: (C) neptunium (Np) and plutonium (Pu)

Solution:

Concept:

- Actinoids exhibit variable oxidation states because the energies of their $5f$, $6d$ and $7s$ orbitals are comparable, allowing different numbers of electrons to participate in bonding.
- The accessible range is widest among the early-middle actinoids, notably neptunium and plutonium. A maximum oxidation state and the number of different oxidation states are distinct quantities.

Step 1: Identify the pair with the widest range Neptunium and plutonium show several oxidation states, commonly tabulated from $+3$ through $+7$. Their $5f$ electrons are sufficiently available for bonding that oxidation need not stop at the stable $+3$ state. Thus the pair in option (C) has the broad range sought in this question.

Step 2: Compare the remaining pairs Actinium is predominantly trivalent and thorium predominantly tetravalent, so the pair in (B) does not have a large range. Toward the end of the series, the $5f$ electrons are held more tightly and participate less readily in bonding. Berkelium and californium therefore have a narrower commonly encountered range than neptunium and plutonium. Nobelium strongly favours $+2$, while lawrencium is predominantly $+3$; this excludes (A). These trends also exclude (D), even though some later actinoids can have more than one oxidation state.

Step 3: Conclusion Neptunium (Np) and plutonium (Pu) exhibit the maximum variety of oxidation states among the listed pairs. Hence, the correct option is (C).

Quick Tip to Solve: For variable actinoid oxidation states, look near Np and Pu; do not confuse the largest oxidation number with the widest range of states.

72. Which one of the following about an electron occupying the 1s orbital in a hydrogen atom is incorrect ?

(The Bohr radius is represented by a_0) [4 marks]

(A) The probability density of finding the electron is maximum at the nucleus.

(B) The electron can be found at a distance $2a_0$ from the nucleus

(C) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus.

(D) The magnitude of potential energy is double that of its kinetic energy on an average.

Correct Answer: (C) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus.

Solution:

Concept:

- For hydrogen in the $1s$ state, $\psi_{1s} = (\pi a_0^3)^{-1/2} e^{-r/a_0}$. The probability density is $|\psi|^2$, while the radial probability includes the spherical-shell factor $4\pi r^2$.
- A stationary hydrogen state has a fixed energy eigenvalue: $E_1 = -13.6\,\text{eV}$. The Coulomb virial theorem gives $\langle V \rangle = -2\langle T \rangle$.

Step 1: Check the probability statements The density $|\psi_{1s}|^2 = (\pi a_0^3)^{-1} e^{-2r/a_0}$ decreases as r increases, so it is largest at the nucleus. Option (A) is correct. At $r = 2a_0$, this density is still positive, proportional to e^{-4}; the electron is not confined to a sphere of radius a_0. Option (B) is also correct. The radius a_0 maximises the radial probability distribution, not the probability density itself.

Step 2: Check energy and the averaged energies The electron occupying a definite $1s$ orbital has the same total energy eigenvalue irrespective of where a position measurement finds it. There is no special maximum of total energy at $r = a_0$. Thus option (C) incorrectly attaches a radial maximum to total energy. In contrast, the virial relation gives $|\langle V \rangle| = 2\langle T \rangle$, so the average-energy statement in (D) is correct. The question asks for the incorrect statement, not for the most probable radius.

Step 3: Conclusion The total energy is fixed in the hydrogen $1s$ state; it does not become maximum at a_0. Hence, the correct option is (C).

Quick Tip to Solve: Keep three ideas separate: density peaks at the nucleus, radial probability peaks at a_0, and the energy of a stationary orbital is fixed.

73. HF has highest boiling point among hydrogen halides, because it has : [4 marks]

(A) lowest dissociation enthalpy

(B) strongest hydrogen bonding

(C) lowest ionic character

(D) strongest van der Waals' interactions

Correct Answer: (B) strongest hydrogen bonding

Solution:

Concept:

- Boiling separates molecules from the liquid into the gas. Its temperature depends on intermolecular attractions, rather than directly on the energy required to break a covalent bond within each molecule.
- A hydrogen atom directly bonded to small, strongly electronegative fluorine can form strong intermolecular hydrogen bonds, represented as $F-H\cdots F$.

Step 1: Identify the exceptional intermolecular attraction In HF, fluorine strongly polarises the $H-F$ bond and its compact size permits strong hydrogen bonding between neighbouring molecules. Molecules associate through these attractions in the liquid. Extra energy is needed to disrupt this association during vaporisation. This raises the boiling point of HF above those of the other hydrogen halides, despite its comparatively low molecular mass.

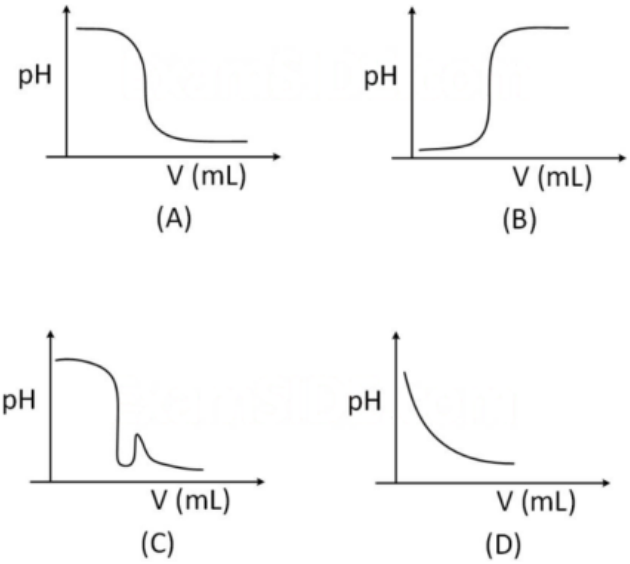
Step 2: Eliminate the alternative explanations Option (A) concerns dissociation enthalpy, which refers to breaking the intramolecular bond into atoms; it is not the cause of the boiling-point anomaly. In fact the $H-F$ bond is very strong. Option (C) is also inappropriate: the large electronegativity difference makes the bond strongly polar, rather than the least ionic in character. Dispersion forces become stronger for the larger, more polarisable halogen atoms, so they cannot make HF the highest-boiling member. Hence option (D) misses the specific exceptional attraction present in HF.

Step 3: Conclusion The unusually high boiling point of HF is caused by its strongest intermolecular hydrogen bonding. Hence, the correct option is (B).

Quick Tip to Solve: When a small molecule containing H bonded to F, O or N has an unexpectedly high boiling point, test for hydrogen bonding before comparing molecular masses.

74. In an acid-base titration, 0.1 M HCl solution was added to the NaOH solution of unknown strength. Which of the following correctly shows the change of pH of the titraction mixture in this experiment?

[4 marks]



(A) (C)

(B) (A)

(C) (B)

(D) (D)

Correct Answer: (B) (A)

Solution:

Concept:

- In this experiment the flask initially contains the strong base NaOH, and the added solution is the strong acid HCl. Neutralisation follows $\mathrm{H^+ + OH^- \longrightarrow H_2O}$.
- Before equivalence excess $\mathrm{OH^-}$ controls pH; after equivalence excess $\mathrm{H^+}$ controls it. A strong acid-strong base titration has a steep pH change near equivalence.

Step 1: Follow the pH as acid is added Initially the solution is alkaline, so the curve must start at high pH. Each addition of acid consumes hydroxide ions and the pH decreases. Near the equivalence volume, a very small further volume of acid changes the solution from a slight excess of base to a slight excess of acid. This produces a sharp downward section. Beyond equivalence the solution remains acidic and approaches the pH of the added acid as its volume becomes very large.

Step 2: Match the graph and then the option code The graph labelled (A) has a high initial pH, a steep descending middle region and a low final pH, exactly as required. Graph (B) rises and would correspond to adding base to acid. Graph (C) has a nonphysical upward excursion during simple neutralisation, while graph (D) lacks the characteristic sharp equivalence region. The printed answer choices list graph (C), graph (A), graph (B) and graph (D), in that order. Therefore the correct graph (A) is selected by the second answer choice, namely option (B).

Step 3: Conclusion The mixture follows the first graph, which is the second printed answer choice.

Hence, the correct option is (B).

Quick Tip to Solve: First identify what is in the flask and what is being added. Then distinguish a graph label from the answer-choice letter selecting that graph.

75. What would be the molality of 20% (mass/ mass) aqueous solution of KI? (molar mass of KI = 166 g mol^{-1}) [4 marks]

(A) 1.51

(B) 1.35

(C) 1.08

(D) 1.48

Correct Answer: (A) 1.51

Solution:

Concept:

- Molality is the number of moles of solute per kilogram of solvent: $m = n_{\text{solute}}/w_{\text{solvent}}(\text{kg})$. The denominator is solvent mass, not solution mass.
- A 20% mass-by-mass solution contains $20\,\text{g}$ of solute in $100\,\text{g}$ of total solution. The remaining mass is water.

Step 1: Choose a convenient solution mass Take $100\,\text{g}$ of the aqueous solution. It contains $20\,\text{g}$ of KI, so the mass of water is $100 - 20 = 80\,\text{g} = 0.080\,\text{kg}$. No density or volume is required, because both the concentration given and the concentration sought are based on masses. Using $0.100\,\text{kg}$ in the denominator would incorrectly count the solute as solvent.

Step 2: Convert solute mass to moles and calculate Using the supplied molar mass, the amount of KI is

$$n = \frac{20}{166} = 0.12048\,\text{mol}.$$

Therefore

$$m = \frac{0.12048}{0.080} = 1.5060\,\text{mol kg}^{-1} \simeq 1.51\,\text{mol kg}^{-1}.$$

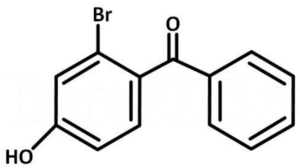
The other listed values, 1.35, 1.08 and 1.48, do not result from the given mass fraction and molar mass. In particular, 1.48 is not the rounding of 1.5060. Since the chosen total mass was arbitrary, choosing any other solution mass gives the same molality.

Step 3: Conclusion The molality of the solution is $1.51\,\text{mol kg}^{-1}$. Hence, the correct option is (A).

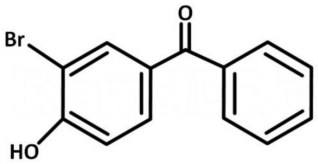
Quick Tip to Solve: For a mass percentage w and molar mass M, use $m = 1000w/[M(100 - w)]$; subtract the solute mass before forming the solvent denominator.

76. p-Hydroxybenzophenone upon reaction with bromine in carbon tetrachloride gives: [4 marks]

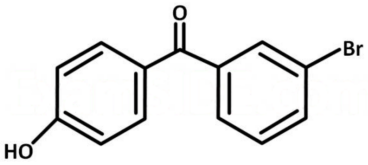
(A) 2-bromo-4-hydroxybenzophenone



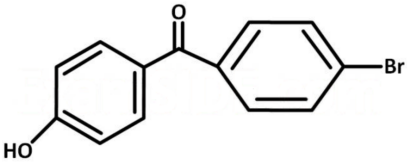
(B) 3-bromo-4-hydroxybenzophenone



(C) 3'-bromo-4-hydroxybenzophenone

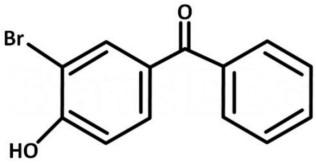


(D) 4'-bromo-4-hydroxybenzophenone



Correct Answer: (B)

3-bromo-4-hydroxybenzophenone



Solution:

Concept:

- The phenolic $-\mathrm{OH}$ group activates an aromatic ring by resonance donation and directs electrophilic substitution to its ortho and para positions.
- The benzoyl group $-\mathrm{COC_6H_5}$ withdraws electron density and is meta-directing. In the para-hydroxy substrate, its direction agrees with the ortho direction of the hydroxyl group.

Step 1: Locate the mutually favoured positions Number the benzophenone ring bearing hydroxyl from the carbon attached to the carbonyl group: the hydroxyl group is at position 4. Its para position is already occupied by the benzoyl linkage. Positions 3 and 5 are ortho to hydroxyl and meta to the carbonyl substituent. These two positions are equivalent by symmetry in the starting compound. Bromination at either one therefore gives the same constitutional product, 3-bromo-4-hydroxybenzophenone.

Step 2: Compare the four structures Bromine in the nonpolar solvent $\mathrm{CCl_4}$ produces the monobrominated phenolic-ring product represented by the second structure. The first structure places bromine meta to hydroxyl and ortho to the withdrawing benzoyl group, so it does not follow the dominant directing effects. The third and fourth structures brominate the other phenyl ring, which is deactivated by its directly attached carbonyl group and lacks the activating hydroxyl substituent. Thus neither is favoured over substitution on the hydroxyl-bearing ring. The reaction is aromatic electrophilic substitution; no carbonyl reduction or replacement of the phenolic hydroxyl group is involved.

Step 3: Conclusion The product is 3-bromo-4-hydroxybenzophenone, the second printed structure.

Hence, the correct option is (B).

Quick Tip to Solve: For a para-hydroxy aryl ketone, mark the positions ortho to OH and meta to the carbonyl group; agreement between the directors identifies the bromination site.

77. Hinsberg's reagent is : [4 marks]

(A) C_6H_5COCl

(B) $C_6H_5SO_2Cl$

(C) $SOCl_2$

(D) $(COCl)_2$

Correct Answer: (B) $C_6H_5SO_2Cl$

Solution:

Concept:

- Hinsberg's reagent is benzenesulphonyl chloride, $C_6H_5SO_2Cl$. It reacts with primary and secondary amines in aqueous alkali to form sulphonamides.
- The test distinguishes amines by sulphonamide formation and by whether an acidic nitrogen-bound hydrogen remains in the product, allowing dissolution in alkali.

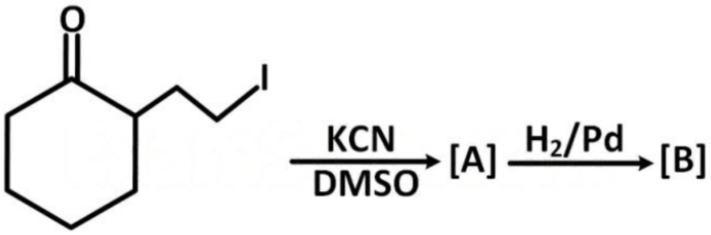
Step 1: Recall the reagent and the diagnostic reaction A primary amine reacts with benzenesulphonyl chloride to form a sulphonamide containing an $N-H$ bond. This hydrogen is acidic because the sulphonyl group withdraws electron density, so the product dissolves in alkali by salt formation. A secondary amine forms a sulphonamide without an $N-H$ bond and that product is insoluble in alkali. A tertiary amine does not form the corresponding sulphonamide under the test conditions. These contrasting behaviours are the basis of the Hinsberg separation.

Step 2: Identify the sulphonyl chloride among the choices The required reagent must have a phenyl group bonded to sulphur in a $-SO_2Cl$ group, giving $C_6H_5SO_2Cl$, option (B). Option (A), C_6H_5COCl, is benzoyl chloride, an acylating reagent rather than the Hinsberg reagent. Option (C), $SOCl_2$, is thionyl chloride, commonly used for converting alcohols to chlorides. Option (D), $(COCl)_2$, is oxalyl chloride. Although all four contain chlorine and can act as reagents, only benzenesulphonyl chloride supplies the sulphonyl group needed in this specific test.

Step 3: Conclusion Hinsberg's reagent is benzenesulphonyl chloride, $C_6H_5SO_2Cl$. Hence, the correct option is (B).

Quick Tip to Solve: Remember the S in Hinsberg with the sulphonyl group $-SO_2Cl$; benzoyl chloride contains $-COCl$ and belongs to acylation chemistry.

78. The major products A and B for the following reactions are, respectively: [4 marks]



(A) A: 1-cyano-2-(2-iodoethyl)cyclohexan-1-ol; B: 1-(aminomethyl)-2-(2-iodoethyl)cyclohexan-1-ol.

(B) A: 2-(2-cyanoethyl)cyclohexan-1-one; B: 2-(3-aminopropyl)cyclohexan-1-one.

(C) A: 2-(2-cyanoethyl)cyclohexan-1-one; B: 2-(3-aminopropyl)cyclohexan-1-ol.

(D) A: 1-cyano-2-(2-iodoethyl)cyclohexan-1-ol; B: 1-(aminomethyl)-2-ethylcyclohexan-1-ol.

Correct Answer: (B) A: 2-(2-cyanoethyl)cyclohexan-1-one; B: 2-(3-aminopropyl)cyclohexan-1-one.

Solution:

Concept:

- In a primary alkyl iodide, cyanide from ionic KCN attacks through carbon and replaces iodide by an $S_{\mathrm{N}}2$ reaction. DMSO is a polar aprotic solvent that favours this substitution.
- Catalytic hydrogenation converts a nitrile group into a primary amino group: $\mathrm{R{-}C{\equiv}N + 2\,H_2 \longrightarrow R{-}CH_2NH_2}$. The carbon of cyanide becomes part of the product chain.

Step 1: Identify the reactive group in the first step The starting material is cyclohexanone bearing a $-\mathrm{CH_2CH_2I}$ group on the carbon adjacent to the carbonyl. The terminal carbon carrying iodine is primary. Cyanide displaces iodide at that carbon, giving $-\mathrm{CH_2CH_2CN}$. The ring carbonyl remains a ketone; these substitution conditions do not give the cyanohydrin shown in options (A) and (D). Thus product A is 2-(2-cyanoethyl)cyclohexan-1-one.

Step 2: Follow the added carbon through hydrogenation The nitrile carbon is retained when $-\mathrm{C{\equiv}N}$ is reduced to $-\mathrm{CH_2NH_2}$. Hence the side chain becomes $-\mathrm{CH_2CH_2CH_2NH_2}$, while the cyclohexanone carbonyl is retained under the stated nitrile hydrogenation conditions. Product B is therefore 2-(3-aminopropyl)cyclohexan-1-one. Option (C) incorrectly also reduces the ketone to a cyclohexanol, while (A) and (D) begin with the wrong cyanohydrin intermediate.

Step 3: Conclusion A is 2-(2-cyanoethyl)cyclohexan-1-one and B is 2-(3-aminopropyl)cyclohexan-1-one. Hence, the correct option is (B).

Quick Tip to Solve: With KCN, first replace the leaving group by CN and count its carbon; reduction of CN adds a terminal CH2NH2 group without losing that carbon.

79. During compression of a spring the work done is 10kJ and 2kJ escaped to the surroundings as heat. The change in internal energy, ΔU(inkJ) is : [4 marks]

(A) - 12

(B) 8

(C) - 8

(D) 12

Correct Answer: (B) 8

Solution:

Concept:

- The first law in the chemistry sign convention is $\Delta U = q + w$, where heat entering the system and work done on the system are positive.
- The spring is the system. Compression transfers energy to the spring mechanically, whereas heat escaping to the surroundings removes energy from it.

Step 1: Assign the signs from the physical transfers The work done during compression is work done on the spring. Therefore $w = +10\,\text{kJ}$. The statement that $2\,\text{kJ}$ escapes as heat means that the system loses this heat, so $q = -2\,\text{kJ}$. These signs refer to the same boundary, namely the spring. A positive magnitude of work must not automatically be assigned a negative sign merely because the spring becomes shorter.

Step 2: Apply the energy balance and compare the choices Substitute the signed transfers into the first law:

$$\Delta U = (-2) + 10 = +8\,\text{kJ}.$$

The spring retains eight kilojoules of the ten kilojoules supplied by compression, after losing two kilojoules as heat. The value $+12\,\text{kJ}$ would count heat loss as heat input. The values $-8\,\text{kJ}$ and $-12\,\text{kJ}$ incorrectly make compression an energy loss by the spring. Thus only the positive value eight agrees with both the equation and the physical process.

Step 3: Conclusion The internal energy increases by $8\,\text{kJ}$. Hence, the correct option is (B).

Quick Tip to Solve: Draw energy arrows into and out of the system before substituting: compression adds energy and escaping heat subtracts it.

80. Molal depression constant for a solvent is 4.0 kg mol^{-1}. The depression in the freezing point of the solvent for 0.03 mol kg^{-1} solution of K_2SO_4 is :

(Assume complete dissociation of the electrolyte) [4 marks]

(A) 0.18 K

(B) 0.24 K

(C) 0.36 K

(D) 0.12 K

Correct Answer: (C) 0.36 K

Solution:

Concept:

- The freezing-point depression is $\Delta T_f = iK_f m$, where m is solute molality and i counts the effective dissolved particles per formula unit.
- Complete dissociation of K_2SO_4 produces two potassium ions and one sulfate ion, so its van't Hoff factor is three.

Step 1: Count the ions produced by dissociation Write the dissolution explicitly:

$$K_2SO_4 \longrightarrow 2\,K^+ + {SO_4}^{2-}.$$

One mole of potassium sulfate gives three moles of independently moving ions under the complete-dissociation assumption. Thus $i = 3$, not two: the sulfate ion remains one polyatomic ion, but there are two potassium ions. The supplied molality $0.03\,\text{mol kg}^{-1}$ refers to formula units of the salt before this particle-count correction.

Step 2: Substitute the molality and compare the depressions Using the stated molal depression constant numerically as $K_f = 4.0\,\text{K kg mol}^{-1}$,

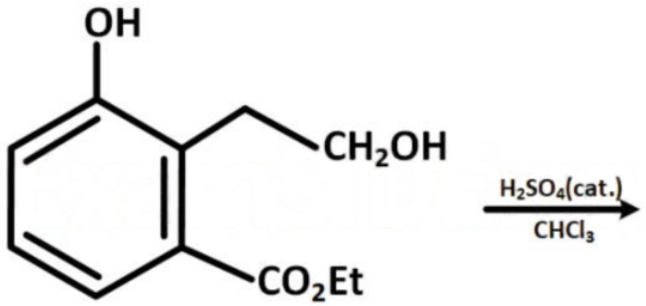
$$\Delta T_f = 3 \times 4.0 \times 0.03 = 0.36\,\text{K}.$$

The product $K_f m = 0.12\,\text{K}$ would describe a non-dissociating solute and explains the tempting option (D). Option (B), $0.24\,\text{K}$, corresponds to only two ions, while option (A), $0.18\,\text{K}$, corresponds to an incorrect factor of 1.5. None uses the three particles specified by complete dissociation.

Step 3: Conclusion The depression in freezing point is $0.36\,\text{K}$. Hence, the correct option is (C).

Quick Tip to Solve: For complete dissociation, count every ion including stoichiometric coefficients before multiplying Kf by the salt molality.

81. The major product of the following reaction is: [4 marks]



(A) The six-membered lactone formed between the terminal alcohol and the adjacent ester; the phenolic OH remains unchanged (5-hydroxy-3,4-dihydroisocoumarin).

(B) The side chain is dehydrated to $-CH=CH_2$; the phenolic OH and ethyl ester remain unchanged.

(C) A five-membered cyclic ether joins the phenolic oxygen to the end of the two-carbon side chain; the ester becomes COOH.

(D) A five-membered cyclic ether joins the phenolic oxygen to the end of the two-carbon side chain; the ethyl ester remains unchanged.

Correct Answer: (A) The six-membered lactone formed between the terminal alcohol and the adjacent ester; the phenolic OH remains unchanged (5-hydroxy-3,4-dihydroisocoumarin).

Solution:

Concept:

- An alcohol can attack an acid-activated ester carbonyl, replacing the original alkoxy group. Intramolecular ester exchange produces a cyclic ester, called a lactone.
- Five- and six-membered rings form readily. The side-chain alcohol and neighbouring ester in this substrate are placed to form a six-membered lactone.

Step 1: Locate the groups that close into a favourable ring The aromatic ring carries a phenolic hydroxyl, a neighbouring $-CH_2CH_2OH$ side chain and an ethyl ester next to that side chain. Catalytic sulfuric acid activates the ester carbonyl toward nucleophilic attack. The terminal alcoholic oxygen can reach that carbonyl intramolecularly. Counting the new ring gives the carbonyl carbon, its attached aromatic carbon, the adjacent aromatic carbon, two methylene carbons and the attacking oxygen: six ring atoms.

Step 2: Complete ester exchange and distinguish the products After alcohol attack and proton transfers, ethanol leaves from the activated ester. This yields the fused six-membered lactone shown in (A), with the original phenolic hydroxyl still present. Option (B) would require elimination of water to give a vinyl group instead of the favourable ring closure. Options (C) and (D) form a cyclic ether using the phenolic oxygen rather than the ester carbonyl; (C) additionally changes the ethyl ester into an acid. They therefore do not represent the direct intramolecular transesterification favoured by this arrangement and catalyst.

Step 3: Conclusion The major product is the six-membered lactone, 5-hydroxy-3,4-dihydroisocoumarin, retaining the phenolic hydroxyl. Hence, the correct option is (A).

Quick Tip to Solve: When an alcohol and ester are tethered in the same molecule, count the prospective ring atoms before choosing dehydration or ether formation.

82. 10 mL of 1mM surfactant solution forms a monolayer covering 0.24 cm^2 on a polar substrate. If the polar head is approximated as cube, what is its edge length? [4 marks]

(A) 1.0 pm

(B) 2.0 nm

(C) 0.1 nm

(D) 2.0 pm

Correct Answer: (D) 2.0 pm

Solution:

Concept:

- For a close-packed monolayer, total covered area equals the number of adsorbed molecules multiplied by the area occupied by one polar head.
- If the polar head is a cube of edge a, the contacting face has area a^2. Thus $a = \sqrt{A/N}$, where $N = nN_A$.

Step 1: Calculate the number of surfactant molecules Convert concentration and volume consistently: $1\,\text{mM} = 10^{-3}\,\text{mol}\,\text{L}^{-1}$ and $10\,\text{mL} = 10^{-2}\,\text{L}$. Therefore

$$n = CV = 10^{-3} \times 10^{-2} = 10^{-5}\,\text{mol}, \qquad N = 10^{-5} \times 6.022 \times 10^{23} = 6.022 \times 10^{18}.$$

Each molecule contributes one polar head to the monolayer. The given area must be distributed over this entire number, not merely over the numerical molarity.

Step 2: Find the face area and take its square root The covered area is $0.24\,\text{cm}^2 = 2.4 \times 10^{-5}\,\text{m}^2$. Hence

$$a^2 = \frac{2.4 \times 10^{-5}}{6.022 \times 10^{18}} = 3.985 \times 10^{-24}\,\text{m}^2, \qquad a = 1.996 \times 10^{-12}\,\text{m}.$$

Since $1\,\text{pm} = 10^{-12}\,\text{m}$, this is approximately $2.0\,\text{pm}$. Option (A) is half the calculated edge. Options (B) and (C) are nanometre values and give much larger areas per molecule than the stated coverage permits. The very small result follows directly from the supplied numerical data.

Step 3: Conclusion The edge length is approximately $2.0\,\text{pm}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert the total area into square metres before taking the square root; a factor of 10 in length becomes a factor of 100 in area.

83. At a given temperature T, gases Ne, Ar, Xe and Kr are found to deviate from ideal gas behaviour. Their equation of state is given as

$p = \frac{RT}{V-b}$ **at T.**

Here, b is the van der Waals constant. Which gas will exhibit steepest increase in the plot of Z (compression factor) vs p? [4 marks]

(A) Xe

(B) Ne

(C) Ar

(D) Kr

Correct Answer: (A) Xe

Solution:

Concept:

- For one mole, the compressibility factor is $Z = pV/(RT)$. The given equation includes the excluded-volume parameter b, but no attractive-pressure correction.
- At fixed temperature, the slope of the resulting straight line in a Z-versus-p plot depends directly on the molecular-size parameter b.

Step 1: Express molar volume in terms of pressure Begin with the equation supplied in the question rather than adding the omitted attraction term:

$$p(V-b) = RT, \qquad V = b + \frac{RT}{p}.$$

Substitute this expression into the definition of compressibility:

$$Z = \frac{p}{RT}\left(b + \frac{RT}{p}\right) = 1 + \frac{b}{RT}p.$$

Thus every gas has the same intercept, one, and a positive slope $b/(RT)$ at the common temperature. The slope is independent of pressure in this specified equation of state.

Step 2: Compare atomic sizes and eliminate the smaller gases The van der Waals parameter b measures excluded volume and increases with atomic size. Among these noble gases the size order is $\mathrm{Ne} < \mathrm{Ar} < \mathrm{Kr} < \mathrm{Xe}$. Xenon therefore has the largest value of b and hence the largest slope. Neon, argon and krypton give progressively larger slopes in that order, but each is less steep than xenon. Attractive interactions need not be compared: they are absent from the equation that this question explicitly asks us to use.

Step 3: Conclusion Xenon exhibits the steepest increase of Z with pressure. Hence, the correct option is (A).

Quick Tip to Solve: Rearrange the stated equation of state into Z = intercept + slope times pressure; then compare only the parameter appearing in the slope.

84. Assertion: For the extraction of iron, haematite ore is used.

Reason: Haematite is a carbonate ore of iron. [4 marks]

(A) Both the assertion and reason are correct and the reason is the correct explanation for the assertion.

(B) Only the reason is correct.

(C) Both the assertion and reason are correct, but the reason is not the correct explanation for the assertion.

(D) Only the assertion is correct.

Correct Answer: (D) Only the assertion is correct.

Solution:

Concept:

- Haematite is the oxide ore Fe_2O_3, and it is an important ore used for the extraction of iron in a blast furnace.
- A carbonate ore contains the carbonate ion CO_3^{2-}. Siderite, $FeCO_3$, is the carbonate ore of iron; haematite contains no carbonate group.

Step 1: Evaluate the assertion independently The assertion says that haematite ore is used for iron extraction. This is correct. In the blast furnace its iron(III) oxide is reduced, principally by carbon monoxide in the relevant temperature zones. An overall reduction equation is

$$Fe_2O_3 + 3\,CO \longrightarrow 2\,Fe + 3\,CO_2.$$

This equation is balanced in iron, carbon and oxygen, and shows why an oxide ore can serve as the starting material for metallic iron. The assertion does not require haematite to be a carbonate.

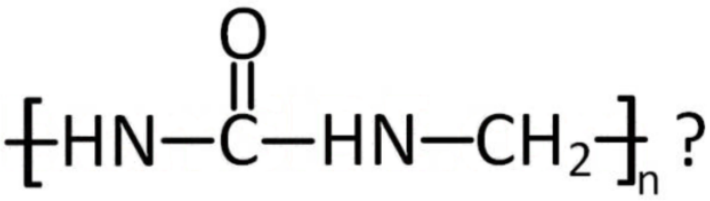
Step 2: Evaluate the reason independently The reason calls haematite a carbonate ore. This is false because its formula is Fe_2O_3, an oxide. The formula $FeCO_3$ belongs to siderite instead. Confusing these mineral names would reverse the evaluation, so the chemical formula is the decisive check.

Step 3: Select the matching logical combination Because the assertion is true and the reason is false, the question of whether the reason explains the assertion cannot rescue it. Options (A) and (C) both incorrectly declare the reason true. Option (B) incorrectly declares only the reason correct. Only option (D) states the independently verified combination: the assertion alone is correct.

Step 4: Conclusion The assertion is correct, while the reason is incorrect: haematite is Fe_2O_3, not a carbonate. Hence, the correct option is (D).

Quick Tip to Solve: Write the ore formula first: haematite is Fe2O3, magnetite is Fe3O4 and siderite is FeCO3; the formula settles the ore type.

85. Which of the following compounds is a constituent of the polymer [4 marks]



(A) N-Methyl urea

(B) Ammonia

(C) Formaldehyde

(D) Methylamine

Correct Answer: (C) Formaldehyde

Solution:

Concept:

- Condensation polymers are formed by joining suitable multifunctional monomers, generally with elimination of a small molecule such as water.
- The displayed repeat unit contains both the urea linkage, $-\mathrm{NH}-\mathrm{CO}-\mathrm{NH}-$, and a methylene group, $-\mathrm{CH_2}-$. These two fragments identify the monomers.

Step 1: Identify the urea fragment The carbonyl carbon in the repeat unit is bonded to two nitrogen atoms. This is the characteristic skeleton of urea, $\mathrm{NH_2CONH_2}$. During condensation, hydrogen atoms on the nitrogen atoms are replaced by bonds joining adjacent units. Thus the carbonyl group belongs to the urea component; it does not come from an aldehyde carbonyl retained unchanged.

Step 2: Trace the methylene bridge Formaldehyde, HCHO, reacts with urea to give methylol derivatives, which condense to form methylene bridges between nitrogen atoms. The relevant structural change can be represented locally as

$$-\mathrm{NH_2} + \mathrm{HCHO} + \mathrm{H_2N^-} \longrightarrow -\mathrm{NH}-\mathrm{CH_2}-\mathrm{NH}- + \mathrm{H_2O}.$$

Repetition produces the urea-formaldehyde polymer represented by the given unit.

Step 3: Eliminate the other choices N-Methyl urea contains an N-methyl substituent, rather than supplying the formaldehyde-derived linking group in the displayed polymer. Ammonia has no carbon and cannot provide the methylene carbon. Methylamine contains only one amino functionality and does not supply the shown urea-plus-methylene repeating skeleton. Formaldehyde is therefore the constituent among the listed compounds.

Step 4: Conclusion The polymer is a urea-formaldehyde condensation polymer, and formaldehyde supplies its methylene bridges. Hence, the correct option is (C).

Quick Tip to Solve: Split the repeat unit into a recognisable functional group and its bridge: $-\mathrm{NH}-\mathrm{CO}-\mathrm{NH}-$ suggests urea and $-\mathrm{CH_2}-$ suggests formaldehyde.

86. The layer of atmosphere between 10 km to 50 km above the sea level is called as : [4 marks]

(A) mesosphere

(B) thermosphere

(C) troposphere

(D) stratosphere

Correct Answer: (D) stratosphere

Solution:

Concept:

- The atmosphere is divided into layers according to its temperature variation with altitude. The usual sequence upward is troposphere, stratosphere, mesosphere and thermosphere.
- The stratosphere extends approximately from the tropopause, near 10–$12\,\mathrm{km}$, to about $50\,\mathrm{km}$ above sea level. Layer boundaries are approximate, not fixed worldwide.

Step 1: Match the given altitude interval The question specifies the region between $10\,\mathrm{km}$ and $50\,\mathrm{km}$. This interval starts near the upper boundary of the troposphere and ends at the stratopause. It is therefore the altitude band conventionally identified as the stratosphere. The use of an approximate lower boundary of 10 km is consistent with this classification.

Step 2: Use the characteristic feature as a check The stratosphere contains the ozone-rich region. Ozone absorbs ultraviolet radiation from the Sun, giving a temperature rise with increasing altitude through much of this layer. This distinguishes it from the lower troposphere, in which the average temperature generally decreases as altitude increases and most ordinary weather occurs.

Step 3: Exclude the remaining layers The troposphere is the lowest atmospheric layer and extends from the surface to roughly the stated lower boundary, so it cannot name the whole 10–50 km band. The mesosphere lies above the stratosphere, approximately from 50 km to 80–85 km. The thermosphere begins still higher. Thus neither the mesosphere nor the thermosphere fits both limits in the question.

Step 4: Conclusion The atmospheric layer occupying approximately 10–$50\,\mathrm{km}$ above sea level is the stratosphere. Hence, the correct option is (D).

Quick Tip to Solve: Use 50 km as the stratosphere–mesosphere boundary, and remember that the ozone layer belongs to the stratosphere.

87. The amorphous form of silica is : [4 marks]

(A) kieselguhr

(B) quartz

(C) tridymite

(D) cristobalite

Correct Answer: (A) kieselguhr

Solution:

Concept:

- Silica is silicon dioxide, SiO_2. Its forms may have the same chemical composition but different arrangements of the silicon–oxygen network.
- A crystalline solid has long-range periodic order, whereas an amorphous solid lacks that long-range periodic arrangement. Composition alone does not decide whether silica is crystalline.

Step 1: Identify the noncrystalline material Kieselguhr, also called diatomaceous earth, is a naturally occurring porous siliceous material associated with the remains of diatoms. In the standard classification used here, it is an amorphous form of silica. Its silicon–oxygen network does not have the extended regular crystal lattice characteristic of the crystalline silica polymorphs.

Step 2: Classify the competing forms Quartz, tridymite and cristobalite are crystalline modifications of SiO_2. They differ in their network arrangements and in the conditions under which the forms are stable, but each has crystalline order. Their different names therefore do not signify different chemical formulae or turn any of them into the amorphous choice.

Step 3: Apply the criterion in the question The question asks specifically for an amorphous form, not merely for a material containing silicon dioxide. Kieselguhr meets that criterion. Quartz is rejected because it is crystalline; tridymite and cristobalite are rejected for the same reason. No calculation of silicon oxidation state or oxygen content can distinguish these alternatives, since the essential distinction is structural order.

Step 4: Conclusion Kieselguhr is the amorphous form of silica among the listed substances. Hence, the correct option is (A).

Quick Tip to Solve: Keep the crystalline trio quartz, tridymite and cristobalite together; kieselguhr is the amorphous silica choice.

88. Noradrenaline is a/an [4 marks]

(A) Antacid

(B) Antihistamine

(C) Neurotransmitter

(D) Antidepressant

Correct Answer: (C) Neurotransmitter

Solution:

Concept:

- A neurotransmitter is a chemical messenger released by a neuron that transmits a signal to another cell by interacting with receptors.
- Noradrenaline, also called norepinephrine, is a catecholamine neurotransmitter involved in nervous-system signalling. Its biological role must be distinguished from the action of drugs that alter its concentration.

Step 1: Identify the substance by its function Noradrenaline participates in the transmission of nerve impulses across synapses. After release from a nerve ending, it interacts with receptors on the receiving cell and contributes to a physiological response. A substance performing this signalling role is classified as a neurotransmitter, which directly matches one of the options.

Step 2: Distinguish the messenger from an antidepressant Some antidepressant drugs affect the availability or action of neurotransmitters such as noradrenaline. For example, changing its removal or breakdown can influence signalling. That connection does not make noradrenaline itself an antidepressant in the classification asked here: the chemical messenger and a drug that modifies its action are different categories.

Step 3: Reject the unrelated therapeutic classes An antacid neutralises excess gastric acid; noradrenaline is not being classified by such acid-neutralising action. An antihistamine blocks particular effects of histamine, and histamine is a different signalling substance. Consequently antacid and antihistamine do not describe noradrenaline. Of the four terms, neurotransmitter is the direct and chemically appropriate identification.

Step 4: Conclusion Noradrenaline is a neurotransmitter. Hence, the correct option is (C).

Quick Tip to Solve: When a question connects depression with noradrenaline, separate the natural neurotransmitter from drugs that change its synaptic availability.

89. The one that is not a carbonate ore is : [4 marks]

(A) siderite

(B) bauxite

(C) malachite

(D) calamine

Correct Answer: (B) bauxite

Solution:

Concept:

- A carbonate ore contains the carbonate ion, CO_3^{2-}, as a constituent of its mineral composition. Writing the mineral formula is the most direct classification test.
- Bauxite is a hydrated aluminium oxide ore, commonly represented at this level as $Al_2O_3 \cdot xH_2O$; it is not an aluminium carbonate.

Step 1: Check siderite and calamine Siderite is iron(II) carbonate, $FeCO_3$, so its formula explicitly contains carbonate. Calamine in the standard school-level ore list is zinc carbonate, $ZnCO_3$, and also contains carbonate. Both substances therefore belong to the carbonate-ore category and cannot be the required exception.

Step 2: Check the basic carbonate Malachite is a basic copper carbonate, represented as $CuCO_3 \cdot Cu(OH)_2$ or equivalently $Cu_2CO_3(OH)_2$. The presence of hydroxide does not remove its carbonate group. It is still classified as a carbonate ore, although it is not a simple carbonate like siderite or calamine.

Step 3: Identify the exception Bauxite supplies aluminium and consists predominantly of hydrated aluminium oxide minerals. Its hydrated-oxide representation contains no CO_3^{2-} unit. Thus bauxite differs from the other three by the anionic component of the ore, not merely by the metal that it contains. This formula-based comparison selects exactly one non-carbonate alternative.

Step 4: Conclusion Bauxite is a hydrated oxide ore, whereas siderite, malachite and calamine contain carbonate. Hence, the correct option is (B).

Quick Tip to Solve: Write the ore formula before choosing: even a basic carbonate such as malachite still contains CO_3^{2-}.

90. The structures of beryllium chloride in the solid state and vapour, phase, respectively,are :

[4 marks]

(A) chain and chain

(B) dimeric and chain

(C) chain and dimeric

(D) dimeric and dimeric

Correct Answer: (C) chain and dimeric

Solution:

Concept:

- Beryllium chloride, $BeCl_2$, is largely covalent because the small Be^{2+} ion has high polarising power. A simple monomer leaves the beryllium centre electron deficient.
- Chlorine atoms can bridge beryllium centres by donating lone-pair electron density. Bridging allows association, with the extent of association depending on the physical state.

Step 1: Identify the solid-state arrangement In solid beryllium chloride, chloride bridges link beryllium centres into extended polymeric chains. Each beryllium is coordinated tetrahedrally by four chlorine atoms in the extended structure, with chlorides shared between neighbouring centres. Thus the first entry in the required pair must be chain, rather than dimeric.

Step 2: Identify the vapour species in the comparison In the vapour phase, beryllium chloride is associated as the dimer Be_2Cl_4, containing bridging chlorides between two beryllium atoms. This is a finite molecular unit rather than an indefinitely extended chain. At sufficiently high temperature the dimer can dissociate further to monomeric $BeCl_2$, but the chain-versus-dimer comparison offered in the question refers to the usual associated vapour form.

Step 3: Match the order of the two states The wording gives solid state first and vapour phase second, so the required ordered pair is chain and dimeric. Option (A) incorrectly retains an extended chain in the vapour, while (B) reverses the pair and also assigns a chain to the vapour. Option (D) incorrectly describes the solid as only discrete dimers. Only (C) places both descriptions in the stated order.

Step 4: Conclusion Beryllium chloride has a chain structure in the solid and a dimeric structure in the associated vapour. Hence, the correct option is (C).

Quick Tip to Solve: For electron-deficient $BeCl_2$, remember the association sequence: extended chains in the solid, dimers in the vapour, and monomers at high temperature.