JEE Main 2019 Paper 1 (B.E./B.Tech) Apr 10 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 10 April 2019, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. The number of real roots of the equation

$5 + |2^x - 1| = 2^x(2^x - 2)$ is [4 marks]

(A) 2

(B) 1

(C) 3

(D) 4

Correct Answer: (B) 1

Solution:

Concept:

- Set $t = 2^x$. For real x, this substitution gives $t > 0$, and every positive value of t corresponds to exactly one real x.
- The absolute value $|t-1|$ must be resolved separately on $t \geq 1$ and $0 < t < 1$; a quadratic root is admissible only if it lies in its case interval.

Step 1: Solve the case at or above one For $t \geq 1$, use $|t-1| = t-1$. The equation becomes

$$5 + t - 1 = t(t-2).$$

Rearranging and factoring gives

$$t^2 - 3t - 4 = 0, \qquad (t-4)(t+1) = 0.$$

The roots are 4 and -1. Only $t = 4$ satisfies both positivity and this case condition. Thus $2^x = 4$, giving $x = 2$.

Step 2: Check the interval below one For $0 < t < 1$, use $|t-1| = 1-t$. We obtain

$$6 - t = t^2 - 2t, \qquad t^2 - t - 6 = 0.$$

Hence $(t-3)(t+2) = 0$, giving $t = 3$ or $t = -2$. Neither lies in $(0,1)$, so this case contributes no root. The endpoint $t = 1$ was included in the first case and does not satisfy the equation. Counting all four algebraic roots without checking their intervals would incorrectly suggest option (D). Options (A) and (C) also count rejected roots.

Step 3: Conclusion There is exactly one real root, $x = 2$. Substitution gives $5 + |4-1| = 8 = 4(4-2)$. Hence, the correct option is (B).

Quick Tip to Solve: After replacing an exponential by t, write $t > 0$ immediately, and retain each quadratic root only in the absolute-value interval that produced it.

2. The sum of the real roots of the equation

$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$, **is equal to :** [4 marks]

(A) - 4

(B) 0

(C) 1

(D) 6

Correct Answer: (B) 0

Solution:

Concept:

- Expand the determinant along its first row using the cofactor signs $+, -, +$. A scalar factor does not affect the roots when it is nonzero.
- The sum of all roots of a cubic with no quadratic term is zero, but the question asks for real roots, so first check that every root of this determinant polynomial is real.

Step 1: Expand the determinant carefully Writing the determinant as $D(x)$, expansion along the first row gives

$$\begin{aligned} D(x) = x\{-3x(x+2) - 2x(x-3)\} \\ + 6\{2(x+2) + 3(x-3)\} \\ - \{4x - 9x\}. \end{aligned}$$

The first minor simplifies to $-5x^2$. The second minor is $5x - 5$, while the last minor is $-5x$. In particular, the last contribution is positive because it is multiplied by the first-row entry -1.

Step 2: Factor and identify the real roots Therefore

$$\begin{aligned} D(x) &= -5x^3 + 30x - 30 + 5x \\ &= -5(x^3 - 7x + 6) \\ &= -5(x-1)(x-2)(x+3). \end{aligned}$$

The determinant vanishes precisely at $x = 1, 2, -3$. All three are real and distinct, so no complex roots need to be removed from the root sum. Their sum is

$$1 + 2 - 3 = 0.$$

The alternatives $-4, 1, 6$ cannot be the sum of this complete root list; 1 and 6 may arise from confusing an individual root or a coefficient with the requested sum.

Step 3: Conclusion The sum of the real roots is 0. Hence, the correct option is (B).

Quick Tip to Solve: Before applying the cubic root-sum formula to a question about real roots, factor the polynomial enough to verify that its remaining roots are real.

3. Let a_1, a_2, a_3,.....be an A.P. with $a_6 = 2$. Then the common difference of this A.P., which maximises the product $a_1a_4a_5$, is : [4 marks]

(A) $\frac{3}{2}$

(B) $\frac{6}{5}$

(C) $\frac{8}{5}$

(D) $\frac{2}{3}$

Correct Answer: (C) $\frac{8}{5}$

Solution:

Concept:

- If an A.P. has common difference d and $a_6 = 2$, then $a_n = 2 + (n-6)d$. This expresses every requested term in one variable.
- A stationary point of a differentiable function satisfies $P\prime(d) = 0$. The second derivative distinguishes a local maximum from a local minimum; a global maximum also requires checking unbounded behaviour.

Step 1: Express the product in terms of the common difference Using the sixth term as the reference gives

$$a_1 = 2 - 5d, \quad a_4 = 2 - 2d, \quad a_5 = 2 - d.$$

Thus the product is

$$\begin{aligned} P(d) &= (2-5d)(2-2d)(2-d) \\ &= 8 - 32d + 34d^2 - 10d^3. \end{aligned}$$

There is no restriction on d in the printed question. In particular, negative common differences define valid real arithmetic progressions with the stated sixth term.

Step 2: Classify the stationary values Differentiate to obtain

$$P\prime(d) = -32 + 68d - 30d^2 = -2(3d-2)(5d-8).$$

The stationary differences are $2/3$ and $8/5$. Since $P\prime\prime(d) = 68 - 60d$, it is positive at $2/3$ and negative at $8/5$. Thus option (D) gives a local minimum, while option (C) gives a local maximum. Neither $3/2$ nor $6/5$ is stationary, so options (A) and (B) do not give a local extremum.

Step 3: Check whether the stated maximum exists globally The leading term is $-10d^3$. Consequently, as $d \to -\infty$, we have $P(d) \to +\infty$. For example, all three factors $2-5d, 2-2d, 2-d$ grow without bound for sufficiently large negative d. Therefore the unrestricted product has no global maximum. The stationary-point calculation identifies the intended local maximum, but cannot establish a maximum over all real common differences.

Step 4: Conclusion As printed, the product has no global maximum. If "maximises" is interpreted as giving the local maximum, $d = 8/5$. Under that interpretation, the correct option is (C).

Quick Tip to Solve: For a cubic objective, classify the stationary points and also inspect the two ends of the real line before claiming that a local maximum is a global maximum.

4. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is : [4 marks]

(A) 6

(B) 5

(C) 8

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- For independent tosses of a fair coin, any specified sequence of n outcomes has probability $(1/2)^n$.
- The complement of getting at least one head is getting no heads, namely the single sequence of all tails. Thus $P(\text{at least one head}) = 1 - 2^{-n}$.

Step 1: Translate the probability condition Let n be the required number of tosses. It is a positive integer. Instead of adding the probabilities of one head, two heads and so on, subtract the probability of all tails from one:

$$P(\text{at least one head}) = 1 - \frac{1}{2^n}.$$

The requirement is strictly more than $99\% = 0.99$, so

$$1 - \frac{1}{2^n} > 0.99 \quad \Longleftrightarrow \quad \frac{1}{2^n} < 0.01.$$

All quantities are positive; taking reciprocals reverses the inequality and gives $2^n > 100$.

Step 2: Find and verify the smallest integer The neighbouring powers of two are $2^6 = 64$ and $2^7 = 128$. Hence $n = 6$ does not satisfy the inequality, whereas $n = 7$ does. Direct checks give

$$P_6 = \frac{63}{64} = 0.984375 < 0.99,$$

$$P_7 = \frac{127}{128} = 0.9921875 > 0.99.$$

The probability increases with n, because 2^{-n} decreases. Therefore no smaller integer works. Option (B), five tosses, fails even more strongly than six; option (C), eight tosses, satisfies the probability condition but is not minimal. Option (A) is the last integer before the threshold.

Step 3: Conclusion The minimum number of tosses is 7. Hence, the correct option is (D).

Quick Tip to Solve: For "at least one" in repeated independent trials, calculate the probability of none and subtract it from one; then check both integers around the threshold.

5. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is : [4 marks]

(A) 180

(B) 210

(C) 170

(D) 190

Correct Answer: (C) 170

Solution:

Concept:

- A beam is determined by an unordered pair of distinct pillars. There are $\binom{n}{2} = n(n-1)/2$ unordered pairs among n pillars.
- The adjacent pairs are exactly the sides of the circular polygon, including the pair joining the last pillar back to the first; an n-pillar circle has n such pairs.

Step 1: Count all possible pairs and remove adjacent pairs If every pair of distinct pillar tops were connected, the number of beams would be

$$\binom{20}{2} = \frac{20 \cdot 19}{2} = 190.$$

The problem allows beams only between non-adjacent pillars. There are 20 adjacent pairs around the stadium, not 19, because the boundary is circular. Removing these pairs gives

$$190 - 20 = 170.$$

The fact that the pillars have equal height changes the spatial position of the beams but does not change which pairs of tops can be connected.

Step 2: Check the count by using one pillar From any one pillar, we must exclude that pillar itself and its two neighbours. Thus it connects to $20 - 3 = 17$ non-adjacent pillars. Counting over all pillars gives $20 \cdot 17 = 340$ incidences. Each physical beam is counted once from each of its two ends, so

$$\frac{340}{2} = 170.$$

This agrees with the subtraction method. Option (D), 190, includes all forbidden adjacent beams. Option (B), 210, exceeds even the number of all distinct pairs. Option (A), 180, would exclude only ten adjacent pairs and therefore retains forbidden beams.

Step 3: Conclusion The total number of beams is 170. Hence, the correct option is (C).

Quick Tip to Solve: For a circular arrangement, the number of adjacent pairs equals the number of points; subtract all those sides from the unordered-pair count.

6. Let λ be a real number for which the system of linear equations x + y + z = 6, 4x + λy – λz = λ – 2, 3x + 2y – 4z = – 5 has infinitely many solutions. Then λ is a root of the quadratic equation: [4 marks]

(A) $\lambda^2 + \lambda - 6 = 0$

(B) $\lambda^2 - \lambda - 6 = 0$

(C) $\lambda^2 - 3\lambda - 4 = 0$

(D) $\lambda^2 + 3\lambda - 4 = 0$

Correct Answer: (B) $\lambda^2 - \lambda - 6 = 0$

Solution:

Concept:

- Infinitely many solutions require dependent equations with compatible constants. A zero coefficient determinant alone is not enough: the augmented equations must also be consistent.
- Eliminating one variable reduces the test to two linear equations in two unknowns. Those two equations must be proportional, including their right-hand sides, to leave one free parameter.

Step 1: Eliminate the same variable from both remaining equations From the first equation, $y = 6 - x - z$. Substitution in the second gives

$$4x + \lambda(6 - x - z) - \lambda z = \lambda - 2,$$

$$(4 - \lambda)x - 2\lambda z = -5\lambda - 2.$$

Substitution in the third gives

$$3x + 2(6 - x - z) - 4z = -5,$$

$$x - 6z = -17.$$

The latter equation always gives a genuine relation between x and z. To have infinitely many solutions, the preceding relation must repeat it rather than impose a second independent condition.

Step 2: Impose and check proportionality If the multiplier is $k = 4 - \lambda$, matching the coefficient of z requires

$$-2\lambda = -6(4 - \lambda),$$

$$8\lambda = 24, \qquad \lambda = 3.$$

At this value the second reduced equation is exactly $x - 6z = -17$, including the constant, so the system is consistent and has infinitely many solutions. Explicitly, put $z = t$; then $x = 6t - 17$ and $y = 23 - 7t$, with arbitrary real t.

Step 3: Test the proposed quadratic equations At $\lambda = 3$, the left sides of options (A), (B), (C), (D) are $6, 0, -4, 14$, respectively. Thus only $\lambda^2 - \lambda - 6 = 0$ has the required parameter as a root. Its other root need not make the original system indeterminate.

Step 4: Conclusion The required parameter is $\lambda = 3$, a root of $\lambda^2 - \lambda - 6 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: After making coefficient rows proportional, always check the constants too; otherwise a determinant-zero parameter can produce no solution instead of infinitely many.

7. If $\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=5$, then a + b is equal to : [4 marks]

(A) 1

(B) - 4

(C) - 7

(D) 5

Correct Answer: (C) - 7

Solution:

Concept:

- If a polynomial numerator divided by $x-1$ has a finite limit at $x=1$, the numerator must vanish there. This provides the first relation between its coefficients.
- After factoring out the vanishing factor, the remaining polynomial is continuous. Its value at the limiting point gives the stated finite limit without any division by zero.

Step 1: Use finiteness to relate the coefficients The numerator is $f(x)=x^2-ax+b$, a continuous polynomial. If $f(1)\neq 0$, the denominator tends to zero while the numerator tends to a nonzero constant, so the quotient cannot have the given finite two-sided limit. Therefore

$$1-a+b=0, \qquad b=a-1.$$

This relation is necessary before trying to substitute $x=1$. Direct substitution into the original fraction would otherwise leave an undefined expression.

Step 2: Factor the numerator and evaluate the limit Using $b=a-1$, factor the numerator as

$$x^2-ax+a-1=(x-1)(x+1-a).$$

For $x\neq 1$ the common factor cancels, giving

$$\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=\lim_{x\to 1}(x+1-a)=2-a.$$

The limit is 5, so $2-a=5$, whence $a=-3$ and $b=-4$. The requested sum, rather than either individual coefficient, is

$$a+b=-3-4=-7.$$

As a check, the original numerator is then $x^2+3x-4=(x-1)(x+4)$, whose quotient tends to 5. Option (B) is only b, option (D) repeats the limit, and option (A) is neither the sum nor consistent with both coefficient conditions.

Step 3: Conclusion The value of $a+b$ is -7. Hence, the correct option is (C).

Quick Tip to Solve: For a finite limit with denominator $x-c$, first set the numerator to zero at c, then cancel the factor and use the remaining limit to determine the coefficients.

8. If $\int x^5 e^{-x^2} dx = g(x)\, e^{-x^2} + c$, where c is a constant of integration, then $g(-1)$ is equal to : [4 marks]

(A) 1

(B) - 1

(C) $-\frac{5}{2}$

(D) $-\frac{1}{2}$

Correct Answer: (C) $-\frac{5}{2}$

Solution:

Concept:

- For an integral containing an odd power of x and e^{-x^2}, put $u = x^2$, so that $du = 2x\, dx$.
- Repeated integration by parts evaluates a polynomial times an exponential: $\int u^2 e^{-u}\, du = -(u^2 + 2u + 2)e^{-u} + C$. The constant belongs outside the antiderivative.

Step 1: Transform the integral Write $x^5 dx = x^4(x\, dx)$. Under the substitution $u = x^2$, the fourth power becomes u^2. Thus the entire integral, including its differential, becomes

$$\int x^5 e^{-x^2}\, dx = \frac{1}{2}\int u^2 e^{-u}\, du.$$

The factor one-half is essential; omitting it doubles the answer.

Step 2: Integrate the polynomial factor Integrating by parts once reduces the power from two to one, and a second application removes it:

$$\begin{aligned}\int u^2 e^{-u}\, du &= -u^2 e^{-u} + 2\int u e^{-u}\, du \\ &= -u^2 e^{-u} - 2ue^{-u} - 2e^{-u} + C.\end{aligned}$$

Substituting back gives $g(x) = -\frac{1}{2}(x^4 + 2x^2 + 2)$, with the arbitrary additive constant already represented by c.

Step 3: Evaluate and check the coefficient At $x = -1$, both even powers equal one, so

$$g(-1) = -\frac{1}{2}(1 + 2 + 2) = -\frac{5}{2}.$$

As a check, differentiating $-\frac{1}{2}(x^4 + 2x^2 + 2)e^{-x^2}$ gives $x^5 e^{-x^2}$: the lower powers cancel. The positive value 1 and the values -1, $-1/2$ do not equal the coefficient obtained after integration.

Step 4: Conclusion $g(-1) = -\frac{5}{2}$. Hence, the correct option is (C).

Quick Tip to Solve: With an odd power multiplying e^{-x^2}, reserve one factor $x\, dx$ for the substitution before integrating.

9. The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, – 4) and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is : [4 marks]

(A) 6

(B) 7

(C) $2\sqrt{13}$

(D) $4\sqrt{3}$

Correct Answer: (B) 7

Solution:

Concept:

- If A is on a line of direction $\mathbf{d}$, the distance of P from it is $h = |\overrightarrow{AP} \times \mathbf{d}|/|\mathbf{d}|$.
- The cross product gives the parallelogram area. Dividing by its base length gives the perpendicular height; the direction vector need not be a unit vector.

Step 1: Form the displacement vector The given point is $P = (-1, 2, 6)$, while a point on the line is $A = (2, 3, -4)$. Therefore

$$\mathbf{u} = \overrightarrow{AP} = (-3, -1, 10), \qquad \mathbf{d} = (6, 3, -4).$$

In particular, the last component is $6 - (-4) = 10$. Reversing all components of $\mathbf{u}$ would give the same distance, but changing only one sign would not.

Step 2: Compute the perpendicular area Expanding the cross product component by component gives

$$\mathbf{u} \times \mathbf{d} = \big(4 - 30,\ 60 - 12,\ -9 + 6\big) = (-26, 48, -3).$$

Consequently its squared magnitude is

$$|\mathbf{u} \times \mathbf{d}|^2 = 26^2 + 48^2 + 3^2 = 676 + 2304 + 9 = 2989.$$

The line direction has squared magnitude $36 + 9 + 16 = 61$.

Step 3: Divide by the direction length The required squared distance is

$$h^2 = \frac{2989}{61} = 49, \qquad h = 7.$$

The positive square root is taken because distance is nonnegative. As a second check, the projection formula yields $|\mathbf{u}|^2 - (\mathbf{u} \cdot \mathbf{d})^2/|\mathbf{d}|^2 = 110 - 61 = 49$. Options 6, $2\sqrt{13}$, and $4\sqrt{3}$ have squares 36, 52, and 48 respectively, so none is the perpendicular distance.

Step 4: Conclusion The perpendicular distance is 7 units. Hence, the correct option is (B).

Quick Tip to Solve: Compare squared distances first: $|\mathbf{u}|^2 - (\mathbf{u} \cdot \mathbf{d})^2/|\mathbf{d}|^2$ often avoids a lengthy cross-product square root.

10. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3 /min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is : [4 marks]

(A) $\frac{5}{6\pi}$
(B) $\frac{1}{9\pi}$
(C) $\frac{1}{36\pi}$
(D) $\frac{1}{18\pi}$

Correct Answer: (D) $\frac{1}{18\pi}$

Solution:

Concept:

- The volume of ice is the outer sphere volume minus the fixed iron sphere volume: $V = \frac{4\pi}{3}(R^3 - 10^3)$.
- If the ice thickness is h, the outer radius is $R = 10 + h$, so $dR/dt = dh/dt$. A melting rate is a negative volume derivative.

Step 1: Identify the changing radius Only the ice melts; the radius of the iron ball remains $10\,\text{cm}$. When the ice thickness is $5\,\text{cm}$, the outer radius is

$$R = 10 + 5 = 15\,\text{cm}.$$

The surface losing ice is the outer spherical surface, so its area must be evaluated using 15 rather than either the thickness 5 or the iron radius 10.

Step 2: Differentiate the shell volume With time t measured in minutes, differentiating the volume expression gives

$$\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}.$$

The constant iron volume differentiates to zero. Since the volume decreases by $50\,\text{cm}^3$ every minute, substitute $dV/dt = -50$ and $R = 15$:

$$-50 = 4\pi(15)^2 \frac{dh}{dt} = 900\pi \frac{dh}{dt}.$$

Step 3: Report the rate of decrease Solving for the signed thickness derivative gives

$$\frac{dh}{dt} = -\frac{50}{900\pi} = -\frac{1}{18\pi}\,\text{cm min}^{-1}.$$

The question asks how fast the thickness decreases, which is the positive magnitude of this derivative. The units follow from volume rate divided by area. The other listed values, $5/(6\pi)$, $1/(9\pi)$, and $1/(36\pi)$, do not satisfy the differentiated shell-volume equation at the stated outer radius.

Step 4: Conclusion The thickness decreases at $\frac{1}{18\pi}\,\text{cm min}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: For a coating on a solid sphere, add the thickness to the fixed core radius before using $dV/dt = 4\pi R^2\, dR/dt$.

11. If z and w are two complex numbers such that |zw| = 1 and arg(z) – arg(w) = $\frac{\pi}{2}$, then : [4 marks]

(A) $z\overline{w} = \frac{1-i}{\sqrt{2}}$

(B) $\overline{z}w = i$

(C) $z\overline{w} = \frac{-1+i}{\sqrt{2}}$

(D) $\overline{z}w = -i$

Correct Answer: (D) $\overline{z}w = -i$

Solution:

Concept:

- Write $z = re^{i\theta}$ and $w = se^{i\phi}$, where $r, s > 0$. Since $|zw| = 1$, their moduli satisfy $rs = 1$.
- Conjugation reverses an argument: $\overline{z} = re^{-i\theta}$. Therefore $\overline{z}w$ has argument $\phi - \theta$ and modulus rs.

Step 1: Use the given argument difference The condition $\arg z - \arg w = \pi/2$ means $\theta - \phi = \pi/2$. Multiplying z by the conjugate of w subtracts these arguments rather than adding them:

$$z\overline{w} = rse^{i(\theta-\phi)} = e^{i\pi/2} = i.$$

This calculation uses both conditions: the argument determines the direction, and $rs = 1$ fixes the magnitude.

Step 2: Conjugate to obtain the listed product Take the complex conjugate of the preceding equality. Conjugation distributes over multiplication, while $\overline{i} = -i$, so

$$\overline{z\overline{w}} = \overline{z}w = -i.$$

Equivalently, evaluating the polar expression directly gives $\overline{z}w = rse^{i(\phi-\theta)} = e^{-i\pi/2}$, the same result.

Step 3: Exclude the other arguments The two quantities $(1 - i)/\sqrt{2}$ and $(-1 + i)/\sqrt{2}$ lie along directions $-\pi/4$ and $3\pi/4$, respectively. Neither is $z\overline{w} = i$, which lies on the positive imaginary axis. Also $\overline{z}w = i$ would reverse the required argument difference. Thus the modulus condition alone is insufficient to choose an option; the order of conjugation determines the sign.

Step 4: Conclusion $\overline{z}w = -i$. Hence, the correct option is (D).

Quick Tip to Solve: For $z\overline{w}$, subtract the argument of w from that of z; conjugating the other factor reverses this difference.

12. Let f(x) = $\log_e$(sin x), (0 < x < π) and g(x) = $\sin^{-1}$ (e^{-x}), (x $\geq$ 0). If α is a positive real number such that a = (fog)'(α) and b = (fog)(α), then : [4 marks]

(A) $a\alpha^2 + b\alpha - a = -2\alpha^2$

(B) $a\alpha^2 + b\alpha + a = 0$

(C) $a\alpha^2 - b\alpha - a = 0$

(D) $a\alpha^2 - b\alpha - a = 1$

Correct Answer: (D) $a\alpha^2 - b\alpha - a = 1$

Solution:

Concept:

- Simplify a composition before differentiating: $\sin(\sin^{-1} u) = u$ for $-1 \leq u \leq 1$.
- For positive x, $0 < e^{-x} < 1$, so $g(x) = \sin^{-1}(e^{-x})$ lies in the domain of f. Also $\log_e(e^{-x}) = -x$.

Step 1: Simplify the composite function For $x > 0$, substitute $g(x)$ into f:

$$(f \circ g)(x) = \log_e \left[\sin\left(\sin^{-1}(e^{-x})\right)\right]$$
$$= \log_e(e^{-x}) = -x.$$

The inverse sine is the principal real inverse, whose output lies in $(0, \pi/2)$ here. Hence the logarithm has a positive argument, and no additional absolute value or branch adjustment is needed.

Step 2: Evaluate the function and its derivative Differentiation of the simplified expression now gives

$$(f \circ g)\prime(x) = -1.$$

At the specified positive number α, the two constants are therefore

$$a = -1, \qquad b = -\alpha.$$

The derivative is constant, whereas the function value depends on α; interchanging these two roles would produce an incorrect option.

Step 3: Test the proposed identities Substitution yields

$$a\alpha^2 - b\alpha - a = -\alpha^2 + \alpha^2 + 1 = 1.$$

Thus option (D) is an identity for every allowed α. The same left side cannot be zero as option (C) asserts. For (A), the left side equals $-2\alpha^2 + 1$, differing from its proposed right side by 1. For (B), it is $-2\alpha^2 - 1$, strictly negative for positive α, so it cannot be zero.

Step 4: Conclusion $a\alpha^2 - b\alpha - a = 1$. Hence, the correct option is (D).

Quick Tip to Solve: Cancel an inverse trigonometric function inside a composition before applying the chain rule; here the entire composite becomes the straight line $-x$.

13. If both the mean and the standard deviation of 50 observations x_1, x_2,..., x_{50} are equal to 16, then the mean of $(x_1 - 4)^2$, $(x_2 - 4)^2$,....., $(x_{50} - 4)^2$ is : [4 marks]

(A) 400

(B) 480

(C) 380

(D) 525

Correct Answer: (A) 400

Solution:

Concept:

- For equally weighted observations, variance is $\sigma^2 = \overline{x^2} - \bar{x}^2$, where the bar denotes the arithmetic mean.
- The average squared displacement from any number c is $\overline{(x-c)^2} = \sigma^2 + (\bar{x} - c)^2$. Both the spread and the displacement of the mean contribute.

Step 1: Recover the mean square The problem gives the mean $\bar{x} = 16$ and the standard deviation $\sigma = 16$. The variance is the square of the standard deviation, not the standard deviation itself:

$$\sigma^2 = 16^2 = 256.$$

Using the variance identity gives

$$\overline{x^2} = \sigma^2 + \bar{x}^2 = 256 + 256 = 512.$$

The number of observations, 50, is already incorporated in the meaning of each average.

Step 2: Average the expanded expression Expand each transformed observation before taking its average:

$$(x_i - 4)^2 = x_i^2 - 8x_i + 16.$$

Averaging is linear, so

$$\frac{1}{50}\sum_{i=1}^{50}(x_i - 4)^2 = \overline{x^2} - 8\bar{x} + 16.$$

Substituting the two known averages gives $512 - 8(16) + 16 = 512 - 128 + 16 = 400$. The constant term remains 16 after averaging, since it occurs in every observation.

Step 3: Check through the shifted mean Subtracting 4 from every observation changes the mean to $16 - 4 = 12$ but leaves the variance at 256. The mean square of the shifted observations is consequently

$$256 + 12^2 = 256 + 144 = 400,$$

confirming the direct expansion. The other proposed values 480, 380, and 525 disagree with both computations. In particular, averaging a square is not the same as merely squaring the mean; the variance must be retained.

Step 4: Conclusion The required mean is 400. Hence, the correct option is (A).

Quick Tip to Solve: Use mean of $(x - c)^2 = \sigma^2 + (\bar{x} - c)^2$ to handle a shifted square without computing individual observations.

14. Let a, b and c be in G.P. with common ratio r, where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If $3a$, 7b and 15c are the first three terms of an A.P., then the 4th term of this A.P. is : [4 marks]

(A) a

(B) $\frac{7}{3}a$

(C) $5a$

(D) $\frac{2}{3}a$

Correct Answer: (A) a

Solution:

Concept:

- If a, b, c are consecutive terms of a G.P. of ratio r, then $b = ar$ and $c = ar^2$.
- Three consecutive A.P. terms satisfy twice the middle term equals the sum of the outer terms. A restriction on the ratio must be checked after solving the resulting equation.

Step 1: Apply the arithmetic progression condition The three A.P. terms are $3a$, $7ar$, and $15ar^2$. Equality of their successive differences gives

$$7ar - 3a = 15ar^2 - 7ar.$$

Because $a \neq 0$, division by a is permitted. Rearranging produces

$$15r^2 - 14r + 3 = 0.$$

This equation uses the A.P. property of the scaled terms, while the replacements for b and c use the original G.P. property.

Step 2: Enforce the permitted ratio Factor the quadratic:

$$(5r - 3)(3r - 1) = 0.$$

Thus $r = 3/5$ or $r = 1/3$. The first root exceeds $1/2$, violating the given restriction $0 < r \leq 1/2$. The admissible ratio is therefore $r = 1/3$, giving the first three A.P. terms

$$3a, \quad \frac{7a}{3}, \quad \frac{5a}{3}.$$

Step 3: Extend the arithmetic progression The common difference is

$$d = \frac{7a}{3} - 3a = -\frac{2a}{3}.$$

Hence the fourth term is

$$T_4 = \frac{5a}{3} + d = \frac{5a}{3} - \frac{2a}{3} = a.$$

This reasoning is valid for either sign of nonzero a. Option $7a/3$ is the second term rather than the fourth, and $5a$ and $2a/3$ do not continue the required common difference. Keeping the rejected ratio would give an unrelated progression that fails the interval condition.

Step 4: Conclusion The fourth term of the A.P. is a. Hence, the correct option is (A).

Quick Tip to Solve: When scaled G.P. terms form an A.P., write them as multiples of a, ar, ar^2, solve for r, and filter the roots before extending the A.P.

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15. The integral $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \, \csc^{4/3} x \, dx$ is equal to: [4 marks]

(A) $3^{\frac{5}{3}} - 3^{\frac{1}{3}}$

(B) $3^{\frac{5}{6}} - 3^{\frac{2}{3}}$

(C) $3^{\frac{4}{3}} - 3^{\frac{1}{3}}$

(D) $3^{\frac{7}{6}} - 3^{\frac{5}{6}}$

Correct Answer: (D) $3^{\frac{7}{6}} - 3^{\frac{5}{6}}$

Solution:

Concept:

- The identity $1 + \tan^2 x = \sec^2 x$ suggests using $t = \tan x$, with $dt = \sec^2 x \, dx$.
- On the interval $[\pi/6, \pi/3]$, both sine and cosine are positive, so fractional powers can be combined without sign or branch ambiguity.

Step 1: Rewrite the integrand in terms of tangent Let $t = \tan x$. Then $\sin x = t/\sqrt{1+t^2}$, $\cos x = 1/\sqrt{1+t^2}$, and $dx = dt/(1+t^2)$. Thus

$$\sec^{2/3} x \, \csc^{4/3} x = \frac{(1+t^2)^{1/3}(1+t^2)^{2/3}}{t^{4/3}} = \frac{1+t^2}{t^{4/3}}.$$

The factor $1 + t^2$ cancels against the denominator in dx. The lower and upper limits become $1/\sqrt{3}$ and $\sqrt{3}$, respectively.

Step 2: Integrate and simplify the endpoint powers The required integral is therefore

$$I = \int_{1/\sqrt{3}}^{\sqrt{3}} t^{-4/3} \, dt = \left[-3t^{-1/3}\right]_{1/\sqrt{3}}^{\sqrt{3}}$$
$$= 3\left(3^{1/6} - 3^{-1/6}\right) = 3^{7/6} - 3^{5/6}.$$

The sign is positive, as it must be because the original integrand is positive throughout the interval. Options (A) and (C) contain different powers, while (B) also fails the endpoint evaluation. Only (D) agrees with the transformed integral.

Step 3: Conclusion The integral equals $\boxed{3^{7/6} - 3^{5/6}}$. Hence, the correct option is (D).

Quick Tip to Solve: When the combined powers of sine and cosine in a denominator add to two, try tangent: its differential often cancels the entire remaining factor.

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16. The area (in sq.units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is : [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) $\frac{3}{2} - \frac{1}{\log_e 2}$

(D) $\log_e 2 + \frac{3}{2}$

Correct Answer: (C) $\frac{3}{2} - \frac{1}{\log_e 2}$

Solution:

Concept:

- In the first quadrant $x \geq 0$, so $|x + 1| = x + 1$. The area between two graphs is the integral of the upper ordinate minus the lower ordinate between their intersections.
- The exponential antiderivative is $\int 2^x \, dx = 2^x / \ln 2$. Here $\ln 2$ means the logarithm of 2 to base e.

Step 1: Find the bounded interval and identify the upper curve The equations meet at $x = 0$ and $x = 1$, since $2^0 = 1$ and $2^1 = 2$. The function 2^x is strictly convex, so its graph lies below the straight chord joining these two points. That chord is $y = x + 1$. Hence, on $0 < x < 1$, the line is above the exponential. Strict convexity also shows that there are no additional intersections: a strictly convex function minus a line cannot have three distinct zeros. The bounded region in the first quadrant is consequently the lens between these two intersection points.

Step 2: Integrate the difference of the ordinates The required area is

$$\begin{aligned} A &= \int_0^1 (x + 1 - 2^x) \, dx \\ &= \left[\frac{x^2}{2} + x - \frac{2^x}{\ln 2} \right]_0^1 \\ &= \frac{1}{2} + 1 - \frac{2 - 1}{\ln 2} = \frac{3}{2} - \frac{1}{\ln 2}. \end{aligned}$$

This is about 0.0573 square units, a small positive area consistent with the two curves being close together. Options (A) and (B) omit the exponential contribution, while adding the logarithmic term as in (D) cannot represent the small gap. The expression in (C) is the matching logarithmic form.

Step 3: Conclusion The bounded area is $\boxed{\frac{3}{2} - \frac{1}{\ln 2}}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: For a convex graph and the chord joining two intersection points, integrate chord minus curve; checking the sign prevents reversing the area.

17. Let y = y(x) be the solution of the differential equation,

$\frac{dy}{dx} + y\tan x = 2x + x^2\tan x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, **such that y(0) = 1. Then :** [4 marks]

(A) $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$

(B) $y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$

(C) $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$

(D) $y'\left(\frac{\pi}{4}\right) + y'\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

Correct Answer: (B) $y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$

Solution:

Concept:

- A visible particular solution can simplify a linear equation. Here $y_p = x^2$ gives $y_p\prime = 2x$, reproducing the entire right side.
- The difference of the complete solution and a particular solution satisfies the corresponding homogeneous equation. Its initial value fixes the remaining constant.

Step 1: Subtract the particular solution and use the initial value Put $z = y - x^2$. Substitution into the given equation gives

$$z' + z\tan x = 0.$$

Since $\cos x > 0$ on the stated interval, its solutions have the form $z = C\cos x$. This also follows by multiplying by the integrating factor $\sec x$, which makes $(z\sec x)' = 0$. The condition $y(0) = 1$ gives $z(0) = 1$, and hence $C = 1$. Therefore

$$y(x) = x^2 + \cos x, \qquad y'(x) = 2x - \sin x.$$

Direct substitution verifies both the original differential equation and the initial value.

Step 2: Evaluate the four proposed relations The solution y is even and its derivative is odd. Therefore the difference of the two stated y-values is zero, ruling out (A), and the sum of the two derivative values is zero, ruling out (D). At the positive endpoint,

$$y'(\pi/4) = \frac{\pi}{2} - \frac{\sqrt{2}}{2}.$$

Thus

$$y'(\pi/4) - y'(-\pi/4) = \pi - \sqrt{2}.$$

For completeness, the sum of the function values is

$$y(\pi/4) + y(-\pi/4) = \frac{\pi^2}{8} + \sqrt{2},$$

which is different from the expression in (C). All four alternatives are now tested using the same unique initial-value solution.

Step 3: Conclusion $\boxed{y\prime(\pi/4) - y\prime(-\pi/4) = \pi - \sqrt{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: If the right side resembles the left-side operator applied to a simple polynomial, subtract that polynomial before finding an integrating factor.

18. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$,where –1 ≤ x ≤ 1, – 2 ≤ y ≤ 2, x ≤ $\frac{y}{2}$, then for all x, y, $4x^2 - 4xy\cos\alpha + y^2$ is equal to : [4 marks]

(A) $4\sin^2\alpha$

(B) $2\sin^2\alpha$

(C) $4\sin^2\alpha - 2x^2y^2$

(D) $4\cos^2\alpha + 2x^2y^2$

Correct Answer: (A) $4\sin^2\alpha$

Solution:

Concept:

- Let $A = \cos^{-1}x$ and $B = \cos^{-1}(y/2)$. Their principal values lie in $[0, \pi]$, and the given difference is $A - B = \alpha$.
- The cosine addition formula relates $x = \cos(B + \alpha)$ to $y/2 = \cos B$. Squaring a derived equality yields an identity satisfied by every allowed pair.

Step 1: Express the cosine difference in useful form Since $A = B + \alpha$, the addition formula gives

$$x = \cos B\cos\alpha - \sin B\sin\alpha.$$

Using $\cos B = y/2$ and moving the first term,

$$x - \frac{y}{2}\cos\alpha = -\sin B\sin\alpha.$$

The restrictions ensure that both inverse cosines are defined; $x \le y/2$ additionally implies $A \ge B$. No division by $\sin\alpha$ is needed, so the derivation remains valid even when $\alpha = 0$.

Step 2: Square and collect the required expression Squaring the last equality and using $\sin^2 B = 1 - y^2/4$, we obtain

$$x^2 - xy\cos\alpha + \frac{y^2}{4}\cos^2\alpha = \left(1 - \frac{y^2}{4}\right)\sin^2\alpha.$$

Move the y^2-term on the right to the left. The identity $\sin^2\alpha + \cos^2\alpha = 1$ then gives

$$x^2 - xy\cos\alpha + \frac{y^2}{4} = \sin^2\alpha.$$

Multiplying by four gives exactly the expression asked for. This is an identity, rather than an equation whose extraneous roots need to be selected. For example $x = 0, y = 2$ gives $\alpha = \pi/2$ and value 4, immediately excluding (B) and (D); a nonzero product xy excludes the extra term in (C).

Step 3: Conclusion For every allowed pair, $\boxed{4x^2 - 4xy\cos\alpha + y^2 = 4\sin^2\alpha}$. Hence, the correct option is (A).

Quick Tip to Solve: Replace inverse cosines by angles, apply the addition formula, and square only after isolating the sine term; avoid dividing by a factor that may vanish.

19. Lines are drawn parallel to the line 4x – 3y + 2 = 0, at a distance $\frac{3}{5}$ from the origin. Then which one of the following points lies on any of these lines ? [4 marks]

(A) $\left(\frac{1}{4}, -\frac{1}{3}\right)$
(B) $\left(-\frac{1}{4}, \frac{2}{3}\right)$
(C) $\left(-\frac{1}{4}, -\frac{2}{3}\right)$
(D) $\left(\frac{1}{4}, \frac{1}{3}\right)$

Correct Answer: (B) $\left(-\frac{1}{4}, \frac{2}{3}\right)$

Solution:

Concept:

- All lines parallel to $4x - 3y + 2 = 0$ have equations $4x - 3y + c = 0$, with the same normal vector $(4, -3)$.
- The perpendicular distance of (x_0, y_0) from $ax + by + c = 0$ is $|ax_0 + by_0 + c|/\sqrt{a^2 + b^2}$. An absolute value generally produces two parallel lines.

Step 1: Determine the two lines at the specified distance For the origin, the distance from $4x-3y+c = 0$ is

$$d = \frac{|c|}{\sqrt{4^2 + (-3)^2}} = \frac{|c|}{5}.$$

Setting this equal to the given distance $3/5$ gives $|c| = 3$, so $c = 3$ or $c = -3$. The required lines are therefore

$$4x - 3y + 3 = 0 \quad \text{and} \quad 4x - 3y - 3 = 0.$$

Equivalently, a point belongs to either of these lines precisely when $4x - 3y$ equals -3 or 3. The original constant 2 determines a reference line's position; it need not be retained when moving to another parallel line.

Step 2: Test each proposed point Evaluate the shared linear expression rather than substituting into both line equations separately:

Option	$4x - 3y$
A	$1 + 1 = 2$
B	$-1 - 2 = -3$
C	$-1 + 2 = 1$
D	$1 - 1 = 0$

Only option (B) gives one of the permitted values. In fact, its coordinates satisfy $4x - 3y + 3 = 0$ exactly. The other three points lie on different members of the same parallel family and therefore do not satisfy the distance condition.

Step 3: Conclusion The point is $\boxed{(-1/4,\ 2/3)}$, lying on $4x - 3y + 3 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: For a fixed parallel family, calculate the required constant using the distance formula, then test just the common expression $ax + by$ for each point.

20. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is : [4 marks]

(A) $\left(\frac{5}{3}, 0\right)$

(B) (5, 0)

(C) (- 5, 0)

(D) $\left(-\frac{5}{3}, 0\right)$

Correct Answer: (C) (- 5, 0)

Solution:

Concept:

- For $x^2/a^2 - y^2/b^2 = 1$, the transverse axis is the x-axis. The focal distance is $c = \sqrt{a^2 + b^2}$, and the eccentricity is $e = c/a$.
- The foci are $(\pm c, 0)$, while the directrices are $x = \pm a/e = \pm a^2/c$. The corresponding focus and directrix have the same sign of their x-coordinate.

Step 1: Put the hyperbola into standard form Divide the given equation by 144:

$$\frac{x^2}{9} - \frac{y^2}{16} = 1.$$

Consequently $a^2 = 9$, $b^2 = 16$, and $a = 3$. The focal distance and eccentricity are

$$c = \sqrt{9 + 16} = 5, \qquad e = \frac{5}{3}.$$

The plus sign in $c^2 = a^2 + b^2$ is essential for a hyperbola; the ellipse relation would give the wrong geometry. Thus the two possible foci are $(5, 0)$ and $(-5, 0)$.

Step 2: Match the negative directrix to its focus The given directrix is

$$5x + 9 = 0 \quad \Longrightarrow \quad x = -\frac{9}{5}.$$

The calculated directrix magnitude is $a/e = 3/(5/3) = 9/5$, confirming the match. Since this is the negative directrix, its corresponding focus is the negative focus $(-5, 0)$. One may check the pairing at the vertex $(-3, 0)$: its distance from this focus is 2, while its perpendicular distance from the directrix is $6/5$. Their ratio is $2/(6/5) = 5/3 = e$, as required. Option (B) chooses the opposite focus, and (A) and (D) confuse a focal coordinate with the eccentricity rather than the distance c.

Step 3: Conclusion The focus corresponding to $5x + 9 = 0$ is $\boxed{(-5, 0)}$. Hence, the correct option is (C).

Quick Tip to Solve: Calculate the magnitude of the focal coordinate first, then use the sign of the directrix to choose the corresponding focus.

21. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is : [4 marks]

(A) 23

(B) 58

(C) 38

(D) 35

Correct Answer: (C) 38

Solution:

Concept:

- Choosing r factors of x^{-3} and $n-r$ factors of x^2 gives the general term $\binom{n}{r}x^{2n-5r}$.
- For fixed n, binomial coefficients increase strictly up to the middle and then decrease symmetrically. Hence $\binom{n}{r}=\binom{n}{23}$ requires $r=23$ or $r=n-23$, when both indices are valid.

Step 1: Find the index of the term containing the first power The general term is

$$T_{r+1}=\binom{n}{r}(x^2)^{n-r}(x^{-3})^r=\binom{n}{r}x^{2n-5r}, \qquad 0\le r\le n.$$

For the coefficient of x, the exponent must be one, so

$$2n-5r=1.$$

Since distinct values of r give distinct exponents, this term is unique. Its coefficient is $\binom{n}{r}$, which must match the stated coefficient. The symmetry $\binom{n}{r}=\binom{n}{n-r}$ means that simply setting $r=23$ may miss a smaller answer.

Step 2: Use both possible coefficient indices and choose the smaller value First, if $r=23$, then $2n-115=1$, giving $n=58$. Second, if $r=n-23$, then

$$2n-5(n-23)=1 \quad\Longrightarrow\quad 3n=114 \quad\Longrightarrow\quad n=38.$$

For $n=38$, the index is $r=15$, and indeed $2(38)-5(15)=1$ while $\binom{38}{15}=\binom{38}{23}$. Thus this smaller candidate works. Option (B) is the valid but larger candidate; (A) and (D) do not give the required coefficient. Strict increase of binomial coefficients up to the middle ensures that the two symmetry-related indices exhaust the possibilities, so no smaller value is omitted.

Step 3: Conclusion The smallest natural number is $\boxed{38}$. Hence, the correct option is (C).

Quick Tip to Solve: When a binomial coefficient is specified, check both its displayed lower index and its complementary index; the complementary choice may give the smaller exponent.

22. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is : [4 marks]

(A) $\frac{14}{3}$
(B) $\frac{16}{3}$
(C) $\frac{68}{15}$
(D) $\frac{34}{15}$

Correct Answer: (C) $\frac{68}{15}$

Solution:

Concept:

- The derivative of an implicit curve gives the tangent slope; the normal slope is its negative reciprocal when both slopes are finite.
- For a triangle whose base lies on the x-axis, its area is half the base length times the absolute y-coordinate of the opposite vertex.

Step 1: Find the tangent and normal slopes Differentiate the ellipse implicitly, retaining the coefficients of both squared terms:

$$6x + 10y\frac{dy}{dx} = 0, \qquad \frac{dy}{dx} = -\frac{3x}{5y}.$$

At $P(2, 2)$, the tangent slope is $-3/5$, so the normal slope is $5/3$. The point is on the ellipse because $3(2)^2 + 5(2)^2 = 32$.

Step 2: Locate the two intercepts The tangent through P has equation $y - 2 = -\frac{3}{5}(x - 2)$. Setting $y = 0$ gives

$$x_Q = 2 + \frac{10}{3} = \frac{16}{3}.$$

The normal is $y - 2 = \frac{5}{3}(x - 2)$, giving

$$x_R = 2 - \frac{6}{5} = \frac{4}{5}.$$

Both intercepts lie to the right of the origin, so their separation is a difference, not a sum.

Step 3: Use the x-axis segment as the base The base and altitude are therefore

$$QR = \frac{16}{3} - \frac{4}{5} = \frac{68}{15}, \qquad h = 2.$$

Hence

$$\text{Area} = \frac{1}{2} \cdot \frac{68}{15} \cdot 2 = \frac{68}{15}.$$

Options (A) and (B) do not use the separation of the two intercepts; option (D) is half the required area, omitting the altitude factor of 2.

Step 4: Conclusion The area of triangle PQR is $\frac{68}{15}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: When tangent and normal meet the x-axis, calculate their intercept difference first; the altitude is simply the y-coordinate of P.

23. The negation of the Boolean expression $\sim s \vee (\sim r \wedge s)$ is equivalent to : [4 marks]

(A) $\sim s \wedge \sim r$

(B) r

(C) $s \vee r$

(D) $s \wedge r$

Correct Answer: (D) $s \wedge r$

Solution:

Concept:

- De Morgan laws interchange conjunction and disjunction under negation: $\neg(p \vee q) = (\neg p) \wedge (\neg q)$.
- Double negation cancels, and the absorption law is $p \wedge (p \vee q) = p$. These identities apply to truth values, independently of the meaning of the statements.

Step 1: Negate the complete expression Let $E = (\neg s) \vee ((\neg r) \wedge s)$. The required negation applies to the entire disjunction, including both its terms. By De Morgan laws,

$$\neg E = s \wedge \neg((\neg r) \wedge s).$$

In particular, merely changing the first negation or changing one connecting symbol would not negate the whole expression.

Step 2: Simplify the inner negation Apply De Morgan laws once more to the conjunction:

$$\neg((\neg r) \wedge s) = r \vee (\neg s).$$

Substitute and distribute to obtain

$$\begin{aligned} \neg E &= s \wedge (r \vee \neg s) \\ &= (s \wedge r) \vee (s \wedge \neg s) \\ &= (s \wedge r) \vee \text{False} = s \wedge r. \end{aligned}$$

Thus both s and r must be true for the negation to be true.

Step 3: Check the alternatives by truth values When s is false, the original expression is true because its first term is true, so its negation must be false. This rules out (A) by taking both r and s false, and rules out (B) and (C) by taking r true and s false. When both r and s are true the original expression is false, confirming the conjunction in (D).

Step 4: Conclusion The negation is $s \wedge r$. Hence, the correct option is (D).

Quick Tip to Solve: Put brackets around the whole expression before negating it; switch each connective only when a negation crosses that bracket.

24. The sum

$1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+.....+\frac{1^3+2^3+3^3+...+15^3}{1+2+3+...+15}-\frac{1}{2}(1+2+3+...+15)$ **is equal to :** [4 marks]

(A) 620

(B) 1240

(C) 1860

(D) 660

Correct Answer: (A) 620

Solution:

Concept:

- The identity $1^3+2^3+\cdots+n^3=[n(n+1)/2]^2$ connects the sum of cubes to the square of the corresponding arithmetic sum.
- The standard sums are $\sum_{n=1}^{N} n=N(N+1)/2$ and $\sum_{n=1}^{N} n^2=N(N+1)(2N+1)/6$.

Step 1: Reduce the general fraction For each upper limit n, the denominator is $1+2+\cdots+n=n(n+1)/2$, which is nonzero. Thus

$$\frac{1^3+2^3+\cdots+n^3}{1+2+\cdots+n}=\frac{[n(n+1)/2]^2}{n(n+1)/2}=\frac{n(n+1)}{2}.$$

The first isolated term 1 is exactly this formula for $n=1$; it is included rather than added a second time.

Step 2: Combine the sums before evaluating Writing the requested value as S gives

$$\begin{aligned} S&=\sum_{n=1}^{15}\frac{n(n+1)}{2}-\frac{1}{2}\sum_{n=1}^{15} n \\ &=\frac{1}{2}\sum_{n=1}^{15}(n^2+n)-\frac{1}{2}\sum_{n=1}^{15} n \\ &=\frac{1}{2}\sum_{n=1}^{15} n^2. \end{aligned}$$

The linear terms cancel exactly. The subtraction at the end is made once, after the full sequence of fractions has been summed.

Step 3: Evaluate and compare the choices Now

$$\sum_{n=1}^{15} n^2=\frac{15\cdot 16\cdot 31}{6}=1240, \qquad S=\frac{1240}{2}=620.$$

Option (B) is the sum of squares before halving. Option (D), 660, and option (C), 1860, do not satisfy the reduced expression. As a direct check, the sum of the fractions is $(1240+120)/2=680$, and the final subtraction is 60.

Step 4: Conclusion The sum is 620. Hence, the correct option is (A).

Quick Tip to Solve: Use the sum-of-cubes identity inside each fraction, then combine the final subtraction symbolically before inserting numbers.

25. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is : [4 marks]

(A) $x = \sqrt{1+2y}, y \geq 0$

(B) $y = \sqrt{1+2x}, x \geq 0$

(C) $y = \sqrt{1+4x}, x \geq 0$

(D) $x = \sqrt{1+4y}, y \geq 0$

Correct Answer: (B) $y = \sqrt{1+2x}, x \geq 0$

Solution:

Concept:

- A circle tangent to the y-axis has radius equal to the perpendicular distance of its centre from that axis.
- Externally tangent circles have distance between their centres equal to the sum of their radii. The given unit circle has centre O(0,0) and radius 1.

Step 1: Express the unknown radius Let the variable centre be $C(h,k)$ in the first quadrant, so $h > 0$ and $k > 0$. If its radius is r, tangency to the y-axis requires

$$r = |h| = h.$$

This is the horizontal distance to the y-axis; using k here would incorrectly impose tangency to the x-axis.

Step 2: Apply external tangency to the unit circle The distance from the origin to C is $\sqrt{h^2+k^2}$. Since the circles touch externally, this distance is $r + 1 = h + 1$. Therefore

$$\sqrt{h^2+k^2} = h + 1.$$

Both sides are positive, so squaring introduces no sign ambiguity:

$$h^2 + k^2 = h^2 + 2h + 1, \qquad k^2 = 1 + 2h.$$

Because the centre is above the x-axis, only the positive square root is relevant.

Step 3: Identify the locus and reject the other forms Replacing the centre coordinates by x and y gives

$$y = \sqrt{1+2x}.$$

This is the upper branch of the parabola $y^2 = 1 + 2x$, restricted to centres with positive x. Option (A) interchanges the roles of the coordinates and would correspond to the other axis. Options (C) and (D) have an incorrect coefficient 4 rather than 2, inconsistent with squaring the sum of radii. The nonnegative range printed in (B) describes the same listed branch, including its boundary limit.

Step 4: Conclusion The required listed locus is $y = \sqrt{1+2x}$, as in option (B); actual nondegenerate first-quadrant circles have $x > 0$. Hence, the correct option is (B).

Quick Tip to Solve: For tangency to the y-axis, write radius = x-coordinate first; external tangency then gives the locus without finding the contact point.

26. The angles A, B and C of a triangle ABC are in A.P. and a : b = 1 : $\sqrt{3}$. If c = 4 cm, then the area (in sq. cm) of this triangle is : [4 marks]

(A) $2\sqrt{3}$

(B) $4\sqrt{3}$

(C) $\frac{4}{\sqrt{3}}$

(D) $\frac{2}{\sqrt{3}}$

Correct Answer: (A) $2\sqrt{3}$

Solution:

Concept:

- Three angles in arithmetic progression satisfy $2B = A + C$. Together with $A + B + C = 180°$, this fixes the middle angle.
- The sine rule gives $a/b = \sin A/\sin B$. The area can then be found from the two perpendicular sides of a right triangle.

Step 1: Find the middle angle Since the triangle angles are in A.P., $A + C = 2B$. Adding B and using the angle sum gives

$$3B = 180°, \qquad B = 60°.$$

This statement concerns the angles, not the side lengths: the side lengths must be related through the sine rule.

Step 2: Determine the remaining angles and sides Using the specified side ratio,

$$\frac{1}{\sqrt{3}} = \frac{a}{b} = \frac{\sin A}{\sin 60°} = \frac{2\sin A}{\sqrt{3}},$$

so $\sin A = 1/2$. The formal possibilities are $A = 30°$ and $150°$, but the latter cannot coexist with $B = 60°$ in a triangle. Hence

$$A = 30°, \qquad C = 90°.$$

The side c opposite C is the hypotenuse. Since $c = 4\,\text{cm}$, the sine rule gives $a = 4\sin 30° = 2\,\text{cm}$ and $b = 4\sin 60° = 2\sqrt{3}\,\text{cm}$.

Step 3: Calculate the area and compare The sides a and b enclose the right angle C. Consequently

$$\Delta = \frac{1}{2}ab = \frac{1}{2}(2)(2\sqrt{3}) = 2\sqrt{3}\,\text{cm}^2.$$

Option (B) omits the factor of one-half. The values in (C) and (D) do not use the side ratio and the hypotenuse together; direct substitution of the legs into the area formula excludes them.

Step 4: Conclusion The area is $2\sqrt{3}\,\text{cm}^2$. Hence, the correct option is (A).

Quick Tip to Solve: For triangle angles in A.P., the middle angle is always 60 degrees; use the sine rule before applying any side-length pattern.

27. A perpendicular is drawn from a point on the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$ to the plane x + y + z = 3 such that the foot of the perpendicular Q also lies on the plane x – y + z = 3. Then the co-ordinates of Q are : [4 marks]

(A) (4, 0, – 1)

(B) (2, 0, 1)

(C) (1, 0, 2)

(D) (– 1, 0, 4)

Correct Answer: (B) (2, 0, 1)

Solution:

Concept:

- A line in symmetric form is parameterised by setting all three ratios equal to the same real number.
- The perpendicular to $x + y + z = 3$ has direction equal to its normal vector $(1, 1, 1)$. The foot must also satisfy both given plane equations.

Step 1: Parameterise the source point and perpendicular Let P be the point on the given line and set its common ratio equal to t. Then

$$P = (1 + 2t, -1 - t, t).$$

A point on the perpendicular through P is obtained by subtracting a multiple of the plane normal. Write the foot as

$$Q = (1 + 2t - \lambda, -1 - t - \lambda, t - \lambda).$$

The same displacement must occur in all three coordinates; choosing an arbitrary point in the intersection of the planes is insufficient.

Step 2: Use the two plane conditions Subtracting the equations $x + y + z = 3$ and $x - y + z = 3$ shows that Q has $2y = 0$, hence $y_Q = 0$. Thus

$$-1 - t - \lambda = 0, \qquad \lambda = -1 - t.$$

The sum of the coordinates of P is $2t$, so the first plane requires

$$2t - 3\lambda = 3.$$

Substitution yields

$$2t + 3 + 3t = 3, \qquad t = 0, \qquad \lambda = -1.$$

Step 3: Find and verify the foot It follows that

$$P = (1, -1, 0), \qquad Q = (2, 0, 1).$$

Q satisfies both plane equations, since $2 + 0 + 1 = 3$ and $2 - 0 + 1 = 3$. Also $Q - P = (1, 1, 1)$, proving perpendicularity. All listed options have y-coordinate zero and x+z equal to 3, so membership in the two planes alone does not distinguish them. Only (B) also arises from the stated line by a displacement parallel to the normal.

Step 4: Conclusion The foot is $Q = (2, 0, 1)$. Hence, the correct option is (B).

Quick Tip to Solve: Intersect the plane conditions first to fix y = 0, then enforce equal normal displacements from the parameterised line point.

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28. If the tangent to the curve $y = \frac{x}{x^2-3}$, $x \in \rho, \left(x \neq \pm\sqrt{3}\right)$, at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line 2x + 6y – 11 = 0, then : [4 marks]

(A) $|\ 6\alpha + 2\beta\ | = 9$

(B) $|\ 2\alpha + 6\beta\ | = 11$

(C) $|\ 2\alpha + 6\beta\ | = 19$

(D) $|\ 6\alpha + 2\beta\ | = 19$

Correct Answer: (D) $|\ 6\alpha + 2\beta\ | = 19$

Solution:

Concept:

- Parallel lines have equal slopes. The line $2x + 6y - 11 = 0$ has slope $-1/3$.
- The quotient rule gives the derivative of $x/(x^2 - 3)$; excluded domain points and the excluded origin must be checked after solving the slope equation.

Step 1: Differentiate and impose the tangent slope At a permitted value of x,

$$y' = \frac{(x^2 - 3) - 2x^2}{(x^2 - 3)^2} = -\frac{x^2 + 3}{(x^2 - 3)^2}.$$

Therefore the tangent at (α, β) is parallel to the given line precisely when

$$-\frac{\alpha^2 + 3}{(\alpha^2 - 3)^2} = -\frac{1}{3}.$$

The denominator is nonzero because the curve excludes $x = \pm\sqrt{3}$.

Step 2: Solve for the admissible contact points Cross-multiplication and expansion give

$$\begin{aligned} 3(\alpha^2 + 3) &= (\alpha^2 - 3)^2, \\ 3\alpha^2 + 9 &= \alpha^4 - 6\alpha^2 + 9, \\ \alpha^2(\alpha^2 - 9) &= 0. \end{aligned}$$

The root $\alpha = 0$ gives $\beta = 0$, explicitly excluded in the question. Hence $\alpha = 3$ or -3. Substitution into the original curve gives

$$(\alpha, \beta) = (3, 1/2) \quad \text{or} \quad (-3, -1/2).$$

Neither point is excluded by the domain.

Step 3: Test the listed absolute-value relations At these paired points,

$$6\alpha + 2\beta = \pm(18 + 1) = \pm 19,$$

whereas

$$2\alpha + 6\beta = \pm(6+3) = \pm 9.$$

Thus the first expression has absolute value 19 and the second has absolute value 9. This excludes (A), which assigns 9 to the first expression, and both (B) and (C), which assign 11 or 19 to the second expression.

Step 4: Conclusion The valid relation is $|6\alpha + 2\beta| = 19$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the origin root until you check the exclusions; then evaluate both proposed linear combinations at one of the symmetric points.

29. If the plane 2x – y + 2z + 3 = 0 has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes 4x – 2y + 4z + λ = 0 and 2x – y + 2z + μ = 0, respectively, then the maximum value of $\lambda + \mu$ is equal to : [4 marks]

(A) 13

(B) 9

(C) 5

(D) 15

Correct Answer: (A) 13

Solution:

Concept:

- Parallel planes must first be written with identical coefficients of x, y, z. Their distance is the absolute difference of their constant terms divided by the length of their common normal.
- For the normal $(2, -1, 2)$, its length is $\sqrt{2^2 + (-1)^2 + 2^2} = 3$. An absolute-value distance condition gives two possible positions of each parallel plane.

Step 1: Normalize the first comparison plane Divide its entire equation by 2, including its constant term. The two equations to compare are therefore $2x - y + 2z + 3 = 0$ and $2x - y + 2z + \lambda/2 = 0$. Scaling a plane equation changes its coefficients but does not change its geometric location.

Step 2: Use the first prescribed distance Apply the parallel-plane distance formula with the common normal of length 3:

$$\frac{|\lambda/2 - 3|}{3} = \frac{1}{3} \quad \Longrightarrow \quad |\lambda/2 - 3| = 1.$$

Thus $\lambda/2 = 4$ or 2, so $\lambda = 8$ or 4. In particular, the largest allowed value of λ is 8.

Step 3: Use the second distance independently The second plane already has the required normal coefficients. Hence

$$\frac{|\mu - 3|}{3} = \frac{2}{3} \quad \Longrightarrow \quad |\mu - 3| = 2.$$

Its two possible constants are $\mu = 5$ and $\mu = 1$. There is no additional relation between the two comparison planes, so either value can accompany either allowed value of λ.

Step 4: Maximize the sum and compare the options Choose the larger constant in each independent pair. This gives $\lambda + \mu = 8 + 5 = 13$. The possible sums are 13, 9, 9 and 5; although 9 and 5 also occur

for other positions, neither is the maximum. A sum of 15 is impossible because it exceeds the sum of the individual upper bounds.

Step 5: Conclusion The maximum value is $\boxed{13}$. Hence, the correct option is (A).

Quick Tip to Solve: Normalize the normal vectors before comparing constants; otherwise a plane equation multiplied by 2 gives a spurious distance.

30. If the line ax + y = c, touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then |c| is equal to : [4 marks]

(A) 2

(B) $\sqrt{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{1}{2}$

Correct Answer: (B) $\sqrt{2}$

Solution:

Concept:

- A line tangent to the unit circle has distance 1 from the origin. For $ax + y - c = 0$, this condition is $|c|/\sqrt{a^2 + 1} = 1$.
- A line is tangent to a parabola when substitution into its equation produces a quadratic with a repeated root. Its discriminant must therefore vanish.

Step 1: Apply tangency to the circle The centre of $x^2 + y^2 = 1$ is the origin and the radius is 1. The perpendicular-distance condition gives

$$\frac{|c|}{\sqrt{a^2 + 1}} = 1 \quad \Longrightarrow \quad c^2 = a^2 + 1.$$

We keep the square because the question asks for $|c|$, and either sign of the intercept can occur.

Step 2: Apply tangency to the parabola From the line, $y = c - ax$. Substitute this into $y^2 = 4\sqrt{2}\,x$:

$$a^2x^2 - (2ac + 4\sqrt{2})x + c^2 = 0.$$

If $a = 0$, a horizontal line intersects the parabola in one finite point but is not its tangent there; equivalently, the tangency condition below cannot hold. For a genuine nonvertical tangent here, impose a zero discriminant.

Step 3: Combine the two tangency conditions The discriminant simplifies as follows:

$$(2ac + 4\sqrt{2})^2 - 4a^2c^2 = 0 \quad \Longrightarrow \quad 16\sqrt{2}\,ac + 32 = 0.$$

Consequently $ac = -\sqrt{2}$, and squaring yields $a^2c^2 = 2$. With $c^2 = a^2 + 1$, let $u = a^2 \geq 0$. Then

$$u(u + 1) = 2, \qquad (u - 1)(u + 2) = 0.$$

Only $u = 1$ is admissible, since a real squared coefficient cannot be negative.

Step 4: Select the magnitude of the intercept It follows that $c^2 = 1 + 1 = 2$, hence $|c| = \sqrt{2}$. The candidate magnitudes 2, $1/\sqrt{2}$ and $1/2$ do not satisfy both equations $c^2 = a^2 + 1$ and $a^2c^2 = 2$, so none can describe the stated common tangent.

Step 5: Conclusion $\boxed{|c| = \sqrt{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a common tangent, use centre-to-line distance for the circle and a zero discriminant for the parabola, then eliminate the slope.

Physics (120 marks)

31. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet?

[Given ; Mass of planet = 8 $\times$ 10^{22} kg, Radius of planet = 2 $\times$ 10^6 m, Gravitational constant G = 6.67 $\times$ 10^{-11} Nm^2 /kg^2] [4 marks]

(A) 13

(B) 9

(C) 17

(D) 11

Correct Answer: (D) 11

Solution:

Concept:

- For a circular orbit, gravity supplies the centripetal force: $GMm/r^2 = mv^2/r$. Thus the satellite mass cancels and $v = \sqrt{GM/r}$.
- The period is $T = 2\pi r/v = 2\pi\sqrt{r^3/(GM)}$, where r is measured from the planet's centre. The count of complete revolutions is the integer part of elapsed time divided by the period.

Step 1: Convert the height and find the orbital radius The orbital height is measured above the surface, whereas the gravitational force uses the distance from the centre. Convert $20\,\text{km} = 2.0 \times 10^4\,\text{m}$ and add it to the planetary radius:

$$r = 2.0 \times 10^6 + 2.0 \times 10^4 = 2.02 \times 10^6\,\text{m}.$$

Using just the height as the orbital radius would place the orbit inside the planet and is physically incorrect.

Step 2: Evaluate the gravitational parameter With the supplied values,

$$GM = (6.67 \times 10^{-11})(8 \times 10^{22}) = 5.336 \times 10^{12}\,\text{m}^3\,\text{s}^{-2}.$$

There is no atmospheric drag or propulsion in the stated model, so this gravitational parameter and

the orbital radius completely determine the circular orbital period.

Step 3: Calculate the time for one revolution Substitute into the period formula:

$$T = 2\pi\sqrt{\frac{(2.02\times10^6)^3}{5.336\times10^{12}}} \simeq 7809\,\text{s}.$$

This is about 2.17 hours per orbit. The dimensions inside the square root are s^2, confirming that the result is a time.

Step 4: Count only the completed orbits Convert the observation interval to seconds: $24\times3600 = 86400\,\text{s}$. The revolution count is

$$\frac{86400}{7809} \simeq 11.064.$$

Thus 11 full revolutions have finished and only a small fraction of the next has occurred. Counts 9, 13 and 17 do not agree with the period; rounding up to include an unfinished orbit would also violate the word complete.

Step 5: Conclusion The spaceship makes $\boxed{11}$ complete revolutions in 24 hours. Hence, the correct option is (D).

Quick Tip to Solve: Always add the height to the planet radius, and take the integer part when the question asks for complete revolutions.

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32. One mole of ideal gas passes through a process where pressure and volume obey the relation $P = P_0\left[1-\frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$. Here P_0 and V_0 are constants. Calculate the change in the temperature of the gas if its volume changes form V_0 to $2V_0$ [4 marks]

(A) $\frac{3}{4}\frac{P_0V_0}{R}$
(B) $\frac{1}{2}\frac{P_0V_0}{R}$
(C) $\frac{5}{4}\frac{P_0V_0}{R}$
(D) $\frac{1}{4}\frac{P_0V_0}{R}$

Correct Answer: (C) $\frac{5}{4}\frac{P_0V_0}{R}$

Solution:

Concept:

- For one mole of an ideal gas, the equation of state is $PV = RT$, so $T = PV/R$. Temperature is determined by the pressure and volume at a state.
- The given relation specifies how the pressure changes with volume. The temperature change is an endpoint difference, $\Delta T = (P_fV_f - P_iV_i)/R$; no work integral is required.

Step 1: Express temperature as a function of volume Multiply the prescribed pressure by V and divide by R:

$$T(V) = \frac{P_0}{R}\left(V - \frac{V_0^2}{2V}\right).$$

The second term decreases in magnitude as the volume increases. In particular, the pressure is not

constant during this process, so replacing it everywhere by P_0 would give an incorrect temperature change.

Step 2: Evaluate the initial state At $V_i = V_0$, the ratio V_0/V_i is 1. Therefore

$$P_i = P_0\left(1 - \frac{1}{2}\right) = \frac{P_0}{2}, \qquad T_i = \frac{P_i V_0}{R} = \frac{P_0 V_0}{2R}.$$

This is the initial temperature, not the requested change in temperature.

Step 3: Evaluate the final state At $V_f = 2V_0$, the squared ratio is $1/4$. Hence

$$P_f = P_0\left(1 - \frac{1}{2}\cdot\frac{1}{4}\right) = \frac{7P_0}{8},$$

$$T_f = \frac{P_f(2V_0)}{R} = \frac{7P_0V_0}{4R}.$$

Both volume and pressure have increased, so the final temperature must be higher, consistent with this expression.

Step 4: Subtract the endpoint temperatures The requested temperature change is

$$\Delta T = T_f - T_i = \left(\frac{7}{4} - \frac{1}{2}\right)\frac{P_0V_0}{R} = \frac{5}{4}\frac{P_0V_0}{R}.$$

The coefficients $3/4$, $1/2$ and $1/4$ fail this endpoint subtraction. The factor P_0V_0/R has temperature units because pressure times volume has energy units.

Step 5: Conclusion The temperature increases by $\boxed{\frac{5P_0V_0}{4R}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a temperature change of an ideal gas, compute the two endpoint products PV first; integrating P with respect to V instead gives work.

33. A bullet of mass 20 g has an initial speed of 1 ms^{-1} , just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5 $\times$ 10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to : [4 marks]

(A) 0.3 ms^{-1}

(B) 0.1 ms^{-1}

(C) 0.7 ms^{-1}

(D) 0.4 ms^{-1}

Correct Answer: (C) 0.7 ms^{-1}

Solution:

Concept:

- The work-energy theorem states that the change in kinetic energy equals the net work: $K_f - K_i = W$. A resistive force opposite the displacement does negative work.

- For a mean resisting force F over thickness d, the energy loss is Fd. Thus $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - Fd$, provided the remaining kinetic energy is positive.

Step 1: Convert the data into SI units The bullet mass is $m = 20\,\mathrm{g} = 0.020\,\mathrm{kg}$, the wall thickness is $d = 20\,\mathrm{cm} = 0.20\,\mathrm{m}$, and its incoming speed is $u = 1\,\mathrm{m\,s^{-1}}$. The mean resisting force is already given in newtons: $F = 2.5 \times 10^{-2}\,\mathrm{N}$. Keeping mass in grams would make the energy calculation inconsistent.

Step 2: Find the incoming energy and the energy lost The initial kinetic energy is

$$K_i = \frac{1}{2}(0.020)(1)^2 = 0.010\,\mathrm{J}.$$

The work of the wall on the bullet is

$$W = -Fd = -(2.5 \times 10^{-2})(0.20) = -0.0050\,\mathrm{J}.$$

The minus sign is essential: the wall removes mechanical energy instead of accelerating the bullet along its motion.

Step 3: Find the outgoing speed The remaining kinetic energy is positive:

$$K_f = 0.010 - 0.0050 = 0.0050\,\mathrm{J}.$$

The bullet therefore emerges from the wall. Solve for its speed:

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(0.0050)}{0.020}} = \sqrt{0.5} \simeq 0.707\,\mathrm{m\,s^{-1}}.$$

Step 4: Check the result against the choices The wall removes one half of the incoming kinetic energy, so the speed becomes $1/\sqrt{2}$ times the original speed, rather than half the original speed. The choices 0.3, 0.1 and $0.4\,\mathrm{m\,s^{-1}}$ would require greater energy losses than the supplied force can produce over this thickness. The closest listed speed is $0.7\,\mathrm{m\,s^{-1}}$.

Step 5: Conclusion The emergence speed is approximately $\boxed{0.7\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the mean resistance to find the energy loss Fd, then take a square root to recover speed; energy and speed do not fall in the same ratio.

34. In free space, a particle A of charge 1μC is held fixed at a point P. Another particle B of the same charge and mass 4μg is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is : $\left[Take\ \frac{1}{4\pi\in_0} = 9\times10^9 Nm^2C^{-2}\right]$ [4 marks]

(A) 1.0 m/s

(B) 6.32 $\times$ 10^4 m/s

(C) 2.0 $\times$ 10^3 m/s

(D) 1.5 $\times$ 10^2 m/s

Correct Answer: (B) 6.32 $\times$ 10^4 m/s

Solution:

Concept:

- The potential energy of two point charges separated by r is $U = kq_Aq_B/r$. Like charges repel, so their potential energy falls as their separation increases.
- Particle A is held fixed and does not move. Its support therefore does no work, and the decrease in electrostatic potential energy becomes kinetic energy of particle B: $\frac{1}{2}mv^2 = kq^2(1/r_i - 1/r_f)$.

Step 1: Convert charges, mass and distances Each charge is $q = 1\,\mu\text{C} = 10^{-6}\,\text{C}$. The moving mass is $m = 4\,\mu\text{g} = 4\times10^{-9}\,\text{kg}$, since one microgram is 10^{-9} kilograms. The two distances are $r_i = 10^{-3}\,\text{m}$ and $r_f = 9\times10^{-3}\,\text{m}$. The particle is released from rest.

Step 2: Evaluate the released potential energy The common charge factor is

$$kq^2 = (9\times10^9)(10^{-6})^2 = 9\times10^{-3}\,\text{J m}.$$

Consequently the initial and final potential energies are

$$U_i = \frac{9\times10^{-3}}{10^{-3}} = 9\,\text{J}, \qquad U_f = \frac{9\times10^{-3}}{9\times10^{-3}} = 1\,\text{J}.$$

The decrease is $8\,\text{J}$; the final potential energy is not zero at the finite separation.

Step 3: Convert this energy to the speed of B Energy conservation gives

$$\frac{1}{2}(4\times10^{-9})v^2 = 9-1 = 8.$$

It follows that

$$v^2 = 4\times10^9\,\text{m}^2\,\text{s}^{-2}, \qquad v = \sqrt{4\times10^9} \simeq 6.32\times10^4\,\text{m s}^{-1}.$$

Its direction is radially away from P because the interaction is repulsive.

Step 4: Check the scale and the alternatives The very small mass explains why a few joules produce a large speed. Inserting the speeds 1.0, 2.0×10^3 or $1.5\times10^2\,\text{m s}^{-1}$ into $mv^2/2$ gives much less than $8\,\text{J}$; they cannot conserve the energy released. Only the second option has the correct magnitude.

Step 5: Conclusion Particle B moves away from P at $\boxed{6.32\times10^4\,\text{m s}^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert micrograms to kilograms carefully: one microgram is $10^{-9}\,\text{kg}$. Retain the final potential energy when the final separation is finite.

35. A 2 mW laser operates at wavelength of 500 nm. The number of photons that will be emitted per second is : [Given Planck's constant h = 6.6 $\times$ 10^{-34} Js, speed of light c = 3.0 $\times$ 10^{8} m/s] [4 marks]

(A) 5×10^{15}

(B) 1.5×10^{16}

(C) 1×10^{16}

(D) 2×10^{16}

Correct Answer: (A) 5×10^{15}

Solution:

Concept:

- A photon of wavelength λ carries energy $E_\gamma = h\nu = hc/\lambda$. The wavelength must be expressed in metres when SI values of h and c are used.
- Optical power is energy emitted per unit time. For photons of the same wavelength, the number emission rate is $\dot{N} = P/E_\gamma = P\lambda/(hc)$.

Step 1: Convert the laser power and wavelength The power is $P = 2\,\mathrm{mW} = 2.0 \times 10^{-3}\,\mathrm{J\,s^{-1}}$. The wavelength is $\lambda = 500\,\mathrm{nm} = 5.0 \times 10^{-7}\,\mathrm{m}$. In one second, the laser emits $2.0 \times 10^{-3}\,\mathrm{J}$ of light energy. Power is not itself the energy of a single photon.

Step 2: Calculate the energy of one photon Use the constants supplied in the question:

$$E_\gamma = \frac{(6.6 \times 10^{-34})(3.0 \times 10^{8})}{5.0 \times 10^{-7}} = 3.96 \times 10^{-19}\,\mathrm{J}.$$

This is a very small energy per photon, so a milliwatt beam can contain a very large number of photons each second. The unit is joules because h has units $\mathrm{J\,s}$ and c/λ has units $\mathrm{s^{-1}}$.

Step 3: Divide the energy per second by the photon energy The required photon emission rate is

$$\dot{N} = \frac{2.0 \times 10^{-3}}{3.96 \times 10^{-19}} = \frac{2.0}{3.96} \times 10^{16} \simeq 5.05 \times 10^{15}\,\mathrm{s^{-1}}.$$

To the accuracy of the listed choices, this is 5×10^{15} photons per second. No extra factor for the wave period or wavelength is needed after the photon energy has been calculated.

Step 4: Compare the alternatives by reconstructing the power Multiplying the selected rate by $3.96 \times 10^{-19}\,\mathrm{J}$ gives approximately $2 \times 10^{-3}\,\mathrm{W}$, as required. Rates 1×10^{16}, 1.5×10^{16} and $2 \times 10^{16}\,\mathrm{s^{-1}}$ would instead imply approximately 4, 6 and $8\,\mathrm{mW}$, respectively, so those options overestimate the given output.

Step 5: Conclusion The emission rate is approximately $\boxed{5 \times 10^{15}\,\mathrm{s^{-1}}}$. Hence, the correct option is (A).

Quick Tip to Solve: First find hc/lambda, the energy of one photon, and then divide power by that energy; milliwatts must be converted to watts.

36. The time dependence of the position of a particle of mass m = 2 is given by $\overrightarrow{r}(t) = 2t\widehat{i} - 3t^2\widehat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is [4 marks]

(A) 36 $\widehat{k}$

(B) - 48 $\widehat{k}$

(C) $-34\left(\widehat{k} - \widehat{i}\right)$

(D) $48\left(\widehat{i} + \widehat{j}\right)$

Correct Answer: (B) - 48 $\widehat{k}$

Solution:

Concept:

- The angular momentum about a fixed origin is $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$. Its direction follows the right-hand rule for the cross product.
- Velocity is the time derivative of position. For motion in the xy-plane, $L_z = m(xv_y - yv_x)$, and the other two components vanish.

Step 1: Differentiate the position vector The given position is $\vec{r} = 2t\hat{i} - 3t^2\hat{j}$, so differentiation must be applied separately to both components. The constant mass is $m = 2$. Thus

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} - 6t\hat{j}.$$

At $t = 2$, the position and velocity are

$$\vec{r} = 4\hat{i} - 12\hat{j}, \qquad \vec{v} = 2\hat{i} - 12\hat{j}.$$

The velocity component in the y-direction is negative; retaining this sign is essential in calculating angular momentum.

Step 2: Evaluate the cross product Use the component formula, with the position measured from the requested origin:

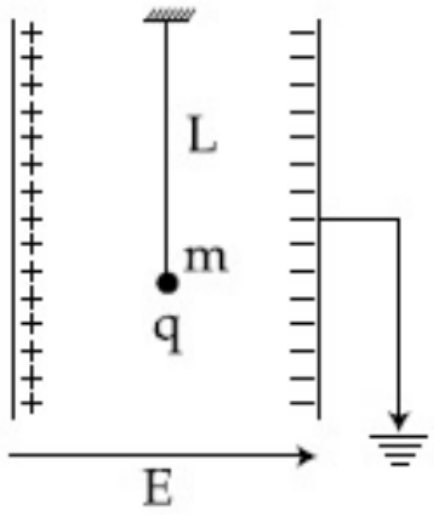
$$\begin{aligned} L_z &= m(xv_y - yv_x) \\ &= 2\{4(-12) - (-12)(2)\} \\ &= 2(-48 + 24) = -48. \end{aligned}$$

Both vectors have zero z-component, so their cross product is normal to the plane and cannot have an $\hat{i}$ or $\hat{j}$ component. This rules out options (C) and (D) directly. Option (A) has both the wrong magnitude and the wrong sign. The negative sign means that the angular momentum points along the negative z-axis, consistent with clockwise motion about the origin at this instant.

Step 3: Conclusion The angular momentum is $\vec{L} = -48\hat{k}$ (in SI angular-momentum units when the given data are in SI). Hence, the correct option is (B).

Quick Tip to Solve: For planar motion, compute $m(xv_y - yv_x)$ directly, but differentiate position before substituting the time.

37. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by : [4 marks]



(A) $2\pi\sqrt{\frac{L}{\sqrt{g^2-\frac{q^2E^2}{m^2}}}}$

(B) $2\pi\sqrt{\frac{L}{(g+\frac{qE}{m})}}$

(C) $2\pi\sqrt{\frac{L}{\sqrt{g^2+\frac{q^2E^2}{m^2}}}}$

(D) $2\pi\sqrt{\frac{L}{(g-\frac{qE}{m})}}$

Correct Answer: (C) $2\pi\sqrt{\frac{L}{\sqrt{g^2+\frac{q^2E^2}{m^2}}}}$

Solution:

Concept:

- A constant electric field exerts a force $q\vec{E}$ on the charged bob. Together with gravity, this gives a constant effective acceleration $\vec{g}_{\text{eff}} = \vec{g} + q\vec{E}/m$.
- For small oscillations about the shifted equilibrium position, a pendulum of length L has period $T = 2\pi\sqrt{L/g_{\text{eff}}}$.

Step 1: Combine the perpendicular accelerations The field in the figure is horizontal, from the positively charged plate toward the negatively charged plate. Gravity acts vertically downward. Therefore the electric acceleration of magnitude $|q|E/m$ is perpendicular to gravity, regardless of the sign of the charge. The magnitude of the resultant is

$$g_{\text{eff}} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}.$$

The string settles along this resultant rather than remaining vertical. Its equilibrium tilt satisfies

$\tan\theta_0 = qE/(mg)$ with a signed convention for the horizontal direction.

Step 2: Find the oscillation frequency about equilibrium Let ϕ be a small angular displacement from the new equilibrium direction. The restoring tangential force is $-mg_{\text{eff}}\sin\phi$, because both constant body forces can be represented by the single resultant. Hence

$$mL\ddot{\phi} = -mg_{\text{eff}}\sin\phi \simeq -mg_{\text{eff}}\phi.$$

Thus $\omega^2 = g_{\text{eff}}/L$. Options (B) and (D) add or subtract accelerations as though the electric field were vertical. Option (A) subtracts their squares, although they are perpendicular; it can even become imaginary for a strong field. The sum of squares in option (C) is the physically appropriate combination.

Step 3: Conclusion The small-oscillation period is

$$T = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + q^2E^2/m^2}}}.$$

Hence, the correct option is (C).

Quick Tip to Solve: First find the direction and magnitude of effective gravity; measure the small oscillation from the tilted equilibrium, not from the vertical.

38. A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. if this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop will be: [4 marks]

(A) $\frac{m}{\pi}$
(B) $\frac{3m}{\pi}$
(C) $\frac{2m}{\pi}$
(D) $\frac{4m}{\pi}$

Correct Answer: (D) $\frac{4m}{\pi}$

Solution:

Concept:

- The magnitude of the magnetic dipole moment of a single planar current loop is $\mu = IA$, where I is its current and A is its enclosed area.
- Reshaping the same wire conserves its total length. For unchanged current, the ratio of magnetic moments is therefore the ratio of the enclosed areas.

Step 1: Express the square in terms of its side Let the side of the original square be a. Its perimeter is $4a$ and its area is a^2. Since the question denotes the original magnetic dipole moment by m,

$$m = Ia^2.$$

Here m is a magnetic moment, not a mass. The given current remains unchanged when the wire is

bent, so there is no additional current factor to introduce. The geometry alone determines the change in moment.

Step 2: Conserve the perimeter and calculate the circular area Let the circle have radius r. Its circumference equals the original perimeter:

$$2\pi r = 4a \quad \Rightarrow \quad r = \frac{2a}{\pi}.$$

Its area and magnetic moment are consequently

$$A_c = \pi r^2 = \frac{4a^2}{\pi}, \qquad \mu_c = IA_c = \frac{4Ia^2}{\pi}.$$

Using $Ia^2 = m$ gives $\mu_c = 4m/\pi$. As a consistency check, a circle encloses more area than a square for a fixed perimeter, so the new moment must exceed m. The factors $1/\pi$, $2/\pi$, and $3/\pi$ in the other options are all less than unity, which rules them out even before comparing the exact result. The factor $4/\pi$ is about 1.27, as expected.

Step 3: Conclusion The magnitude of the circular loop's magnetic dipole moment is $4m/\pi$. Hence, the correct option is (D).

Quick Tip to Solve: When a current loop is reshaped, conserve wire length rather than area; then use magnetic moment proportional to enclosed area.

39. Space between two concentric conducting spheres of radii a and b (b > a) is filled with a medium of resistivity ρ. The resistance between the two spheres will be : [4 marks]

(A) $\frac{\rho}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$
(B) $\frac{\rho}{4\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$
(C) $\frac{\rho}{2\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$
(D) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$

Correct Answer: (D) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$

Solution:

Concept:

- For a small uniform conducting element, $dR = \rho\, dl/A$, where ρ is resistivity, dl is the path length and A is the area normal to current.
- By spherical symmetry, current flows radially between concentric spherical conductors. At radius r, it crosses the entire spherical area $4\pi r^2$.

Step 1: Write the resistance of a thin spherical shell Choose a thin shell of the resistive medium at radius r and thickness dr. Every radial current path crosses this shell, and symmetry distributes the current uniformly over its surface. The area available for conduction is therefore $4\pi r^2$, not the circular cross-section πr^2. Thus

$$dR = \frac{\rho\, dr}{4\pi r^2}.$$

The shells are crossed successively, so their resistances add in series. It would be incorrect to insert a single fixed area into $R = \rho l/A$, because the conduction area changes continuously with radius.

Step 2: Integrate from the inner sphere to the outer sphere The medium occupies $a \le r \le b$, so

$$\begin{aligned} R &= \frac{\rho}{4\pi}\int_a^b r^{-2}\,dr \\ &= \frac{\rho}{4\pi}\left[-\frac{1}{r}\right]_a^b \\ &= \frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right). \end{aligned}$$

Because $b > a$, this result is positive. It tends to zero as b approaches a, when the conducting gap disappears. The plus signs in (A) and (B) fail this limiting check. Option (C) has twice the resistance because it uses an incorrect geometrical factor. Dimensionally, resistivity multiplied by inverse length has the required unit of resistance.

Step 3: Conclusion The resistance is

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right).$$

Hence, the correct option is (D).

Quick Tip to Solve: For radial current, take the local area perpendicular to current and integrate thin shells in series; the full-sphere area is $4\pi r^2$.

40. A coil of self inductance 10 mH and resistance 0.1 Ω is connected through a switch to a battery of internal resistance 0.9 Ω. After the switch is closed, the time taken for the current to attain 80% of the saturation value is: [take ln 5 = 1.6] [4 marks]

(A) 0.324 s

(B) 0.002 s

(C) 0.103 s

(D) 0.016 s

Correct Answer: (D) 0.016 s

Solution:

Concept:

- In a series circuit containing inductance L and total resistance R, the current after switching on is $i(t) = I_\infty(1 - e^{-t/\tau})$, where $\tau = L/R$.
- The internal resistance of the battery is in series with the coil resistance. Both affect the time constant and must be included in R.

Step 1: Calculate the circuit time constant The coil inductance must first be converted into henry:

$$L = 10\,\text{mH} = 0.010\,\text{H}.$$

The total series resistance is the sum of the coil resistance and battery internal resistance:

$$R = 0.1 + 0.9 = 1.0\,\Omega.$$

Therefore

$$\tau = \frac{L}{R} = \frac{0.010}{1.0} = 0.010\,\text{s}.$$

The battery emf determines the saturation current I_∞, but it cancels when the current is specified as a fraction of that saturation value. No numerical emf is needed.

Step 2: Solve for eighty percent of the saturation current Set $i = 0.80I_\infty$ in the growth equation:

$$0.80 = 1 - e^{-t/\tau},$$
$$e^{-t/\tau} = 0.20 = \frac{1}{5},$$
$$t = \tau \ln 5.$$

Using the supplied approximation $\ln 5 = 1.6$,

$$t = (0.010)(1.6) = 0.016\,\text{s}.$$

This exceeds one time constant, as it should: at one time constant the current has reached only about 63 percent of its final value. Option (B) is too short even for this 63-percent level, and options (A) and (C) are far longer than the calculated growth time. Using only the coil resistance would incorrectly enlarge the time constant by a factor of ten.

Step 3: Conclusion The required time is $0.016\,\text{s}$. Hence, the correct option is (D).

Quick Tip to Solve: For a specified fraction f of the final RL current, use $t = -(L/R)\ln(1-f)$, including every series resistance.

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41. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1} . The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be : (Take g = 10 ms^{-2}) [4 marks]

(A) 5×10^{-4} m^2

(B) 2×10^{-5} m^2

(C) 5×10^{-5} m^2

(D) 1×10^{-5} m^2

Correct Answer: (C) 5×10^{-5} m^2

Solution:

Concept:

- For steady incompressible flow, conservation of volume gives $A_1v_1 = A_2v_2$. A faster portion of the same stream must have a smaller cross-sectional area.

- Bernoulli's equation at constant pressure gives $v_2^2 = v_1^2 + 2gh$ for a stream falling vertically through height h.

Step 1: Calculate the speed below the tap Take the tap as the initial point and the point $0.15\,\text{m}$ below it as the final point. Since the pressure is the same throughout the stream, its pressure terms cancel in Bernoulli's equation. The loss in gravitational potential energy per unit mass becomes kinetic energy:

$$\frac{v_2^2 - v_1^2}{2} = gh.$$

Using $v_1 = 1.0\,\text{m s}^{-1}$, $g = 10\,\text{m s}^{-2}$ and $h = 0.15\,\text{m}$,

$$v_2 = \sqrt{1.0^2 + 2(10)(0.15)} = \sqrt{4} = 2.0\,\text{m s}^{-1}.$$

The speed doubles, so the stream must become narrower rather than wider.

Step 2: Apply the continuity equation The same volume of water passes each cross-section per unit time. With the tap area $A_1 = 10^{-4}\,\text{m}^2$,

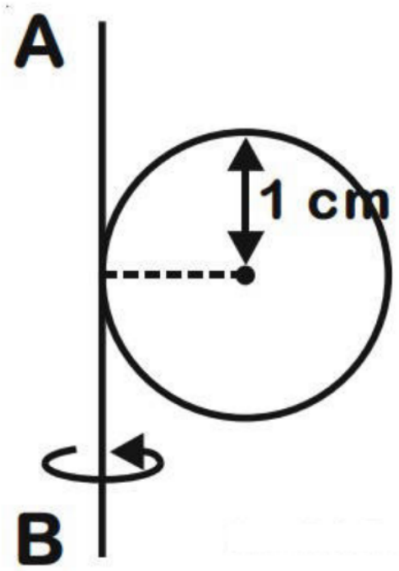
$$A_2 = \frac{A_1 v_1}{v_2} = \frac{(10^{-4})(1.0)}{2.0} = 5.0 \times 10^{-5}\,\text{m}^2.$$

Option (A) would make the falling stream wider despite its increased speed, contradicting continuity. Options (B) and (D) would require speeds of 5 and $10\,\text{m s}^{-1}$, respectively, from the same initial volume flow rate; neither is compatible with the available fall of $0.15\,\text{m}$. Both energy conservation and mass conservation are needed: the first determines speed, and the second determines area.

Step 3: Conclusion The cross-sectional area is $5 \times 10^{-5}\,\text{m}^2$. Hence, the correct option is (C).

Quick Tip to Solve: For a falling free jet, calculate its speed from the loss of height, then reduce its area in the inverse ratio of the speeds.

42. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5s, is close to : [4 marks]



(A) 7.9×10^{-6} Nm

(B) 4.0×10^{-6} Nm

(C) 2.0×10^{-5} Nm

(D) 1.6×10^{-5} Nm

Correct Answer: (C) 2.0×10^{-5} Nm

Solution:

Concept:

- For a thin uniform circular disc, the moment of inertia about a diameter in its plane is $I_d = MR^2/4$.
- The parallel-axis theorem gives $I = I_d + Md^2$ for a parallel displaced axis. A constant torque satisfies $\tau = I\alpha$, with $\alpha = (\omega - \omega_0)/t$.

Step 1: Identify the rotation axis and moment of inertia The diagram shows the stick AB along a tangent to the coin, lying in the plane of the coin. The tangent is parallel to a diameter and is displaced from the centre by one radius. The stick has negligible mass, so only the coin contributes. With $M = 5 \times 10^{-3}\,\text{kg}$ and $R = 10^{-2}\,\text{m}$,

$$\begin{aligned} I_{AB} &= \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2 \\ &= \frac{5}{4}(5 \times 10^{-3})(10^{-2})^2 \\ &= 6.25 \times 10^{-7}\,\text{kg}\,\text{m}^2. \end{aligned}$$

Using the perpendicular symmetry axis would give $MR^2/2$, but that axis is not parallel to AB and therefore cannot be used here.

Step 2: Calculate angular acceleration and torque The system starts from rest and reaches 25 rotations

per second after $5\,\mathrm{s}$. Convert rotations per second to radians per second before finding acceleration:

$$\omega = 2\pi(25) = 50\pi\,\mathrm{rad\,s^{-1}}, \qquad \alpha = \frac{50\pi}{5} = 10\pi\,\mathrm{rad\,s^{-2}}.$$

Hence

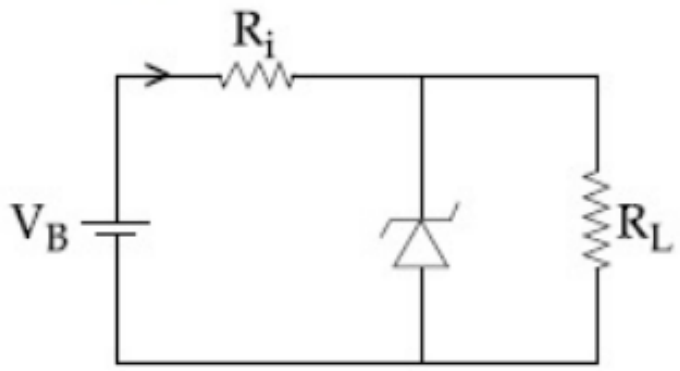
$$\tau = I_{AB}\alpha = (6.25 \times 10^{-7})(10\pi) \simeq 1.96 \times 10^{-5}\,\mathrm{N\,m}.$$

This is closest to option (C). Options (A) and (B) underestimate the rotational inertia by neglecting the displacement of the axis or choosing an inappropriate central axis. Option (D) is also smaller than the correctly calculated value. The units are torque units because $\mathrm{kg\,m^2}$ multiplied by angular acceleration gives $\mathrm{N\,m}$.

Step 3: Conclusion The required constant torque is approximately $2.0 \times 10^{-5}\,\mathrm{N\,m}$. Hence, the correct option is (C).

Quick Tip to Solve: Before using $\tau = I\alpha$, identify whether the drawn axis is a diameter, a normal axis or a tangent; then convert rotations to radians.

43. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6 V and the load resistance is, R_L = 4kΩ. The series resistance of the circuit is R_i = 1 kΩ. If the battery voltage V_B varies from 8 V to 16 V, what are the minimum and maximum values of the current through Zener diode? [4 marks]



(A) 0.5 mA; 8.5 mA

(B) 1.5 mA; 8.5 mA

(C) 1 mA; 8.5 mA

(D) 0.5 mA; 6 mA

Correct Answer: (A) 0.5 mA; 8.5 mA

Solution:

Concept:

- A reverse-biased Zener diode in breakdown maintains the load voltage at its breakdown voltage, here $V_Z = 6\,\mathrm{V}$. The load and Zener branches are in parallel.

- Kirchhoff's current law gives $I_i = I_L + I_Z$, where $I_i = (V_B - V_Z)/R_i$ and $I_L = V_Z/R_L$. The series resistor limits the total current.

Step 1: Find the constant load current The load resistance is $4\,\text{k}\Omega$, and the regulated output remains at $6\,\text{V}$. Thus the load current does not change as the battery varies within the stated range.

$$I_L = \frac{6}{4 \times 10^3}\,\text{A} = 1.5\,\text{mA}.$$

The current through the series resistor is larger than this, with the remaining current flowing through the Zener branch.

Step 2: Calculate the lower and upper Zener currents At the minimum battery voltage the series current is smallest:

$$I_{i,\min} = \frac{8-6}{1 \times 10^3} = 2\,\text{mA}, \qquad I_{Z,\min} = 2 - 1.5 = 0.5\,\text{mA}.$$

At the maximum battery voltage it is largest:

$$I_{i,\max} = \frac{16-6}{1 \times 10^3} = 10\,\text{mA}, \qquad I_{Z,\max} = 10 - 1.5 = 8.5\,\text{mA}.$$

Both Zener currents are positive, consistent with operation in breakdown. Option B mistakes the load current for the minimum Zener current; C has an incorrect minimum and D an incorrect maximum.

Step 3: Conclusion The minimum and maximum currents through the Zener diode are respectively $0.5\,\text{mA}$ and $8.5\,\text{mA}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the constant load current first, then subtract it from the series-resistor current at each input-voltage limit.

44. In a Young's double slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be : [4 marks]

(A) 25 : 9

(B) 4 : 1

(C) $\left(\sqrt{3}+1\right)^4 : 16$

(D) 9 : 1

Correct Answer: (D) 9 : 1

Solution:

Concept:

- For equally illuminated slits in the standard Young's experiment, the intensities supplied by the two slits are proportional to their widths. Thus $I_1/I_2 = 4$.
- Intensity is proportional to the square of wave amplitude. The interference extremes are $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$.

Step 1: Convert the intensity ratio into an amplitude ratio Let the intensity due to the narrower

slit alone be I_0. The wider slit supplies $4I_0$, so their amplitudes are proportional to $\sqrt{I_0}$ and $2\sqrt{I_0}$. Consequently the amplitude ratio is $2:1$, rather than $4:1$. Near the centre the diffraction envelope changes negligibly between the neighbouring interference maximum and minimum, so these two incident intensities can be used to calculate the fringe contrast.

Step 2: Calculate the two interference extremes At a maximum the amplitudes add, while at a minimum their difference remains because the sources are unequal. Therefore

$$I_{\max} = (2\sqrt{I_0} + \sqrt{I_0})^2 = 9I_0,$$

$$I_{\min} = (2\sqrt{I_0} - \sqrt{I_0})^2 = I_0.$$

Dividing eliminates the unknown illumination:

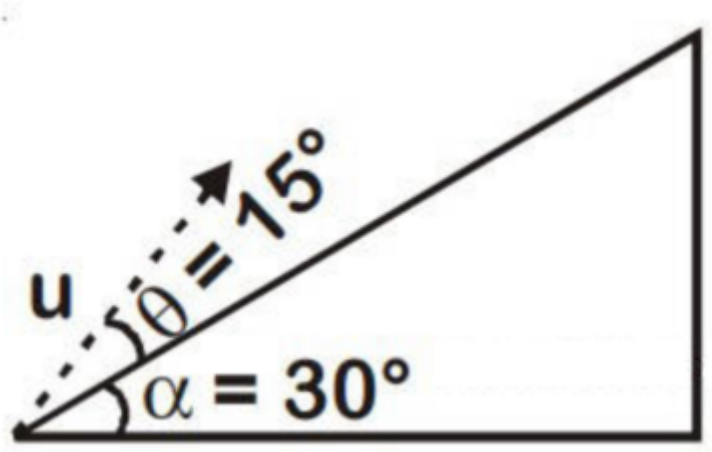
$$\frac{I_{\max}}{I_{\min}} = \frac{9I_0}{I_0} = 9.$$

The ratio $25:9$ would follow from incorrectly treating the width ratio as the amplitude ratio. The ratio $4:1$ is the individual slit-intensity ratio, not the fringe-extreme ratio; the expression in C also differs from the calculated value.

Step 3: Conclusion The intensity ratio of maxima to minima is $9:1$. Hence, the correct option is (D).

Quick Tip to Solve: Take the square root of the individual intensity ratio before adding and subtracting amplitudes; square only after interference.

45. A plane is inclined at an angle $\alpha = 30^\circ$ with respect to the horizontal. A particle is projected with a speed u = 2 ms^{-1} , from the base of the plane, making an angle $\theta = 15^\circ$ with respect to the plane as shown in the figure. the distance from the base, at which the particle hits the plane is close to : (Take g = 10 ms $^{-2}$) [4 marks]



(A) 14 cm

(B) 18 cm

(C) 20 cm

(D) 26 cm

Correct Answer: (C) 20 cm

Solution:

Concept:

- The launch angle relative to the horizontal is $\beta = \alpha + \theta = 45^\circ$. The incline is described by $y = x\tan\alpha$.
- Projectile coordinates are $x = u\cos\beta\, t$ and $y = u\sin\beta\, t - gt^2/2$. The required distance along the plane is $s = x/\cos\alpha$, not the horizontal displacement.

Step 1: Find the nonzero time at which the projectile meets the plane Substitute the projectile coordinates into the plane equation. Apart from the initial intersection at $t = 0$, we obtain

$$u\sin\beta - \frac{gt}{2} = u\cos\beta\tan\alpha.$$

Using $\sin\beta\cos\alpha - \cos\beta\sin\alpha = \sin(\beta - \alpha) = \sin\theta$, the flight time is

$$t = \frac{2u\sin\theta}{g\cos\alpha}.$$

This uses the angle above the plane correctly; using 15° as the angle above the horizontal would describe a different launch.

Step 2: Calculate the distance measured along the incline The horizontal distance is $u\cos\beta\, t$. Divide it by $\cos\alpha$ to obtain the length along the inclined surface:

$$s = \frac{2u^2\sin\theta\cos(\alpha + \theta)}{g\cos^2\alpha}.$$

Insert $u = 2\,\mathrm{m\,s^{-1}}$, $g = 10\,\mathrm{m\,s^{-2}}$, $\theta = 15^\circ$ and $\alpha = 30^\circ$:

$$s = \frac{8(0.258819)(0.707107)}{10(0.75)}\,\mathrm{m} = 0.1952\,\mathrm{m} = 19.52\,\mathrm{cm}.$$

This lies closest to 20 cm; 14 cm and 18 cm underestimate the distance, while 26 cm overestimates it.

Step 3: Conclusion The particle meets the plane about $19.5\,\mathrm{cm}$ from its base, closest to $20\,\mathrm{cm}$. Hence, the correct option is (C).

Quick Tip to Solve: Add the incline and relative projection angles to get the horizontal launch angle, and convert the horizontal range to distance along the incline.

46. A submarine experiences a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then $d_2 - d_1$ is approximately (density of water = 10^3 kg/m^3 and acceleration due to gravity = 10 ms^{-2}) : [4 marks]

(A) 600 m

(B) 400 m

(C) 300 m

(D) 500 m

Correct Answer: (C) 300 m

Solution:

Concept:

- For a stationary liquid of constant density, the pressure at depth d is $P = P_0 + \rho g d$, where P_0 is the pressure at its surface.
- Taking a difference at two depths cancels the same surface pressure: $P_2 - P_1 = \rho g(d_2 - d_1)$. Therefore the required depth difference does not require a numerical atmospheric pressure.

Step 1: Subtract the pressures at the two positions The submarine experiences a larger pressure at the second position, so the depth change must be positive. The pressure difference is

$$\Delta P = (8.08 - 5.05) \times 10^6\,\text{Pa} = 3.03 \times 10^6\,\text{Pa}.$$

We use this increment rather than either total pressure: the question asks how much deeper the submarine goes, not its depth below the sea surface. The result is the same whether the listed pressures include atmospheric pressure or are gauge pressures, provided the convention is common to both.

Step 2: Use the pressure gradient to find the depth change The given water density and gravitational acceleration give

$$\rho g = (10^3)(10) = 10^4\,\text{Pa}\,\text{m}^{-1}.$$

Hence

$$d_2 - d_1 = \frac{\Delta P}{\rho g} = \frac{3.03 \times 10^6}{10^4}\,\text{m} = 303\,\text{m}.$$

The pressure rises by 10^4 Pa per metre, so this result also directly checks the magnitude of the increment. Among the choices, 300 m is closest; 400 m, 500 m and 600 m would produce pressure increases of 4×10^6, 5×10^6 and 6×10^6 Pa, respectively, exceeding the given increment.

Step 3: Conclusion The additional depth is $303\,\text{m}$, approximately $300\,\text{m}$. Hence, the correct option is (C).

Quick Tip to Solve: For a change of depth, subtract the two pressures first; the surface-pressure contribution then cancels automatically.

47. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. the heat required to produce the same change in temperature, at a constant pressure is : [4 marks]

(A) $\frac{7}{5}Q$
(B) $\frac{3}{2}Q$
(C) $\frac{2}{3}Q$
(D) $\frac{5}{3}Q$

Correct Answer: (A) $\frac{7}{5}Q$

Solution:

Concept:

- A rigid diatomic molecule has five active degrees of freedom: three translational and two rotational. Its molar constant-volume heat capacity is $C_V = 5R/2$; vibrational modes are excluded by rigidity.
- For an ideal gas, Mayer's relation gives $C_P = C_V + R = 7R/2$. Heating at constant pressure requires extra heat because the expanding gas also does work.

Step 1: Express the given heat at constant volume For n moles of gas undergoing the stated temperature rise, the heat supplied at fixed volume is

$$Q = nC_V\Delta T = n\frac{5R}{2}\Delta T.$$

No boundary work is done at constant volume. Thus all this heat increases the gas internal energy. The same temperature change for the same amount of gas produces the same internal-energy increase in the constant-pressure process as well, but it is not the whole heat input in that process.

Step 2: Compare with the constant-pressure heat Let the required heat at constant pressure be Q_P. Then

$$Q_P = nC_P\Delta T = n\frac{7R}{2}\Delta T.$$

Divide by the expression for the given heat:

$$\frac{Q_P}{Q} = \frac{C_P}{C_V} = \frac{7R/2}{5R/2} = \frac{7}{5}, \qquad Q_P = \frac{7}{5}Q.$$

Equivalently, the added expansion work is $nR\Delta T = (2/5)Q$, giving a total $Q + (2/5)Q$. The choices $3Q/2$ and $5Q/3$ do not correspond to five degrees of freedom; $2Q/3$ is smaller than the original heat and cannot include the positive expansion work.

Step 3: Conclusion The heat required at constant pressure is $\frac{7}{5}Q$. Hence, the correct option is (A).

Quick Tip to Solve: For the same gas and temperature rise, divide the heats to get C_P/C_V; a rigid diatomic molecule gives $7/5$.

48. In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units? [4 marks]

(A) $[M^{-3}L^{-2}T^8A^4]$

(B) $[M^{-2}L^{-2}T^6A^3]$

(C) $[M^{-1}L^{-2}T^4A^2]$

(D) $[M^{-2}L^0\ T^{-4}A^{-2}]$

Correct Answer: (A) $[M^{-3}L^{-2}T^8A^4]$

Solution:

Concept:

- The pure numerical constant 5 is dimensionless. Thus $X = 5YZ^2$ implies $[Y] = [X]/[Z]^2$. Here X is capacitance and Z is magnetic field.
- Charge has dimensions $[IT] = [AT]$, voltage has dimensions $[ML^2T^{-3}A^{-1}]$, and magnetic field follows from $F = qvB$. The symbol A denotes the SI current dimension.

Step 1: Derive the dimensions of capacitance and magnetic field Capacitance is charge divided by voltage, so

$$[X] = \frac{[AT]}{[ML^2T^{-3}A^{-1}]} = [M^{-1}L^{-2}T^4A^2].$$

From the magnetic force on a moving charge, $B = F/(qv)$, we find

$$[Z] = \frac{[MLT^{-2}]}{[AT][LT^{-1}]} = [MT^{-2}A^{-1}].$$

Notice that magnetic field contains no net power of length, because the length factors from force and speed cancel.

Step 2: Divide by the square of the magnetic-field dimensions Square the entire dimension expression for Z, including the current and time exponents:

$$[Z]^2 = [M^2T^{-4}A^{-2}].$$

Consequently

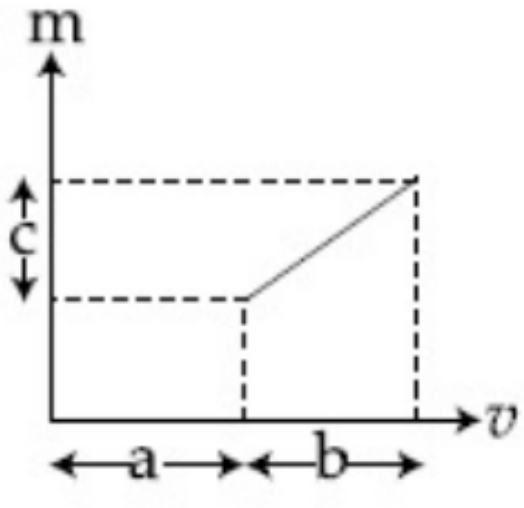
$$[Y] = \frac{[M^{-1}L^{-2}T^4A^2]}{[M^2T^{-4}A^{-2}]} = [M^{-3}L^{-2}T^8A^4].$$

Multiplying this result back by $[Z]^2$ recovers $[M^{-1}L^{-2}T^4A^2]$, the dimensions of capacitance, as required. Option B effectively divides by only one magnetic-field factor; C is capacitance itself, without dividing; D has incompatible length, time and current exponents.

Step 3: Conclusion The dimensions of Y are $[M^{-3}L^{-2}T^8A^4]$. Hence, the correct option is (A).

Quick Tip to Solve: Derive capacitance as charge per volt, then subtract twice every magnetic-field exponent; do not divide by only one field factor.

49. The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focal length of the lens used? [4 marks]



(A) $\frac{b^2}{ac}$
(B) $\frac{b^2c}{a}$
(C) $\frac{a}{c}$
(D) $\frac{b}{c}$

Correct Answer: (D) $\frac{b}{c}$

Solution:

Concept:

- With Cartesian signs, the thin-lens equation is $1/f = 1/v - 1/u$ and signed magnification is $m_s = v/u$. Eliminating u gives $m_s = 1 - v/f$.
- The slope magnitude of a magnification versus image-distance line is therefore $1/|f|$. If positive magnification denotes image size for a real image, $m = |m_s| = v/f - 1$, with positive slope $1/f$.

Step 1: Read changes from the two marked graph points The left marked point is at image distance $v_1 = a$, while the right point is at $v_2 = a + b$. Thus the horizontal separation of the points is $\Delta v = b$, not a. The vertical double arrow marks their magnification difference as $\Delta m = c$. Magnification is dimensionless, whereas both a and b represent image distances. We need the slope of the line rather than the coordinates of one point.

Step 2: Relate the graph slope to the focal length Eliminating the object distance from the lens and magnification equations makes magnification linear in image distance. Using the positive size magnification shown by the rising graph,

$$m = \frac{v}{f} - 1, \qquad \frac{\Delta m}{\Delta v} = \frac{1}{f}.$$

The marked separations give

$$\frac{1}{f} = \frac{c}{b}, \qquad f = \frac{b}{c}.$$

The signed-magnification convention yields the same focal-length magnitude from the absolute slope. Option C incorrectly uses the horizontal offset a in place of the run b; A and B introduce an unnecessary dependence on that offset. Translating the plotted segment along its line changes a but cannot change the lens focal length.

Step 3: Conclusion The focal length of the lens, as obtained from the reciprocal graph slope, is $\frac{b}{c}$. Hence, the correct option is (D).

Quick Tip to Solve: On a magnification versus image-distance graph, use the difference in coordinates: focal-length magnitude is horizontal run divided by vertical rise.

---

50. In an experiment, brass and steel wires of length 1 m each with areas of cross section 1mm^2 are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is, [Given, the Young's Modulus for steel and brass are, respectively, 120 $\times$ 10^9 N/m^2 and 60 $\times$ 10^9 N/m^2] [4 marks]

(A) 8.0×10^6 N/m^2

(B) 1.2×10^6 N/m^2

(C) 0.2×10^6 N/m^2

(D) 1.8×10^6 N/m^2

Correct Answer: (A) 8.0×10^6 N/m^2

Solution:

Concept:

- Young's modulus relates axial stress to strain: $Y = \sigma/(\Delta L/L)$, hence $\Delta L = \sigma L/Y$. Stress is force per unit cross-sectional area.
- Two wires in series carry the same tension. Their equal cross-sectional areas therefore make their stresses equal, while their extensions add to give the extension of the combination.

Step 1: Identify the common stress Let the common tensile force be F and each area be $A = 1\,\text{mm}^2$. Both wires have stress $\sigma = F/A$. The steel and brass wires each have original length $L = 1\,\text{m}$, but their extensions differ because brass has the smaller Young's modulus. No average of the two moduli should be inserted directly into the extension of one wire.

Step 2: Add the two extensions For the series connection the given total extension is $0.2\,\text{mm} = 2.0 \times 10^{-4}\,\text{m}$. Thus

$$\Delta L_{\text{total}} = \sigma\left(\frac{L}{Y_s} + \frac{L}{Y_b}\right).$$

$$2.0 \times 10^{-4} = \sigma\left(\frac{1}{120 \times 10^9} + \frac{1}{60 \times 10^9}\right) = \sigma\frac{1}{40 \times 10^9}.$$

The bracket has units of metres per pascal; multiplying it by stress correctly gives a length.

Step 3: Calculate and check the result Solving gives

$$\sigma = (2.0 \times 10^{-4})(40 \times 10^{9}) = 8.0 \times 10^{6}\,\mathrm{N\,m^{-2}}.$$

At this stress the steel extends $0.0667\,\mathrm{mm}$ and the brass extends $0.1333\,\mathrm{mm}$, whose sum is $0.2000\,\mathrm{mm}$. The other listed stresses, 1.2, 0.2, and 1.8 million pascals, give smaller total extensions and cannot satisfy the prescribed elongation.

Step 4: Conclusion The required stress is $8.0 \times 10^{6}\,\mathrm{N\,m^{-2}}$. Hence, the correct option is (A).

Quick Tip to Solve: For equal-area wires in series, add the compliances L/Y, not the Young's moduli; the softer wire supplies more extension.

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51. In $Li^{+\,+}$, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. When the ion gets deexcited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ?
(Given : H = 6.63 $\times$ 10^{-34} Js; c = 3 $\times$ 10^{8} ms $^{-1}$) [4 marks]

(A) 10.8 nm

(B) 12.3 nm

(C) 9.4 nm

(D) 11.4 nm

Correct Answer: (A) 10.8 nm

Solution:

Concept:

- A one-electron ion has Bohr energies $E_n = -13.6Z^2/n^2\,\mathrm{eV}$, where Z is the nuclear charge and n is the principal quantum number.
- When all downward transitions among levels from 1 through n occur, the number of distinct spectral lines is $N = n(n-1)/2$. An absorbed photon supplies the energy difference hc/λ.

Step 1: Find the excited principal quantum number The total number of lines includes direct emission to the ground state and transitions through intermediate levels. Therefore

$$\frac{n(n-1)}{2} = 6, \qquad n^2 - n - 12 = 0.$$

$$(n-4)(n+3) = 0.$$

The physical positive solution is $n = 4$. The electron was initially in $n = 1$, so the incident radiation excites the $1 \to 4$ transition, not one of the subsequent smaller-energy emissions.

Step 2: Compute the excitation energy For Li^{2+}, only one electron remains and the nuclear charge is $Z = 3$. Hence

$$\Delta E = 13.6(3)^2\left(1 - \frac{1}{4^2}\right)\,\mathrm{eV} = 114.75\,\mathrm{eV}.$$

Using $1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$, the excitation energy is $1.836 \times 10^{-17}\,\mathrm{J}$. The factor Z^2 is essential:

lithium's hydrogen-like energy spacings are nine times the hydrogen spacings.

Step 3: Convert photon energy to wavelength Substitute the supplied Planck constant and speed of light:

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34})(3.0 \times 10^{8})}{1.836 \times 10^{-17}}\,\text{m}.$$

$$\lambda = 1.083 \times 10^{-8}\,\text{m} = 10.83\,\text{nm}.$$

Rounding to the precision of the alternatives gives $10.8\,\text{nm}$. The wavelengths 12.3, 9.4, and $11.4\,\text{nm}$ would supply different photon energies and do not match the required $1 \to 4$ excitation.

Step 4: Conclusion The incident wavelength is approximately $10.8\,\text{nm}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the total line count to determine the highest level first; only then use the ground-to-highest energy difference for the absorbed photon.

52. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by : [4 marks]

(A) 65

(B) 140

(C) 185

(D) 285

Correct Answer: (B) 140

Solution:

Concept:

- For a uniform disc about its symmetry axis, $I = \frac{1}{2}mr^2$. For a uniform solid sphere about a diameter, $I = \frac{2}{5}mr^2$. The radii must belong to the new shapes.
- Reshaping the same material conserves density and mass. Thus a solid sphere's radius scales as the cube root of its mass when compared with another sphere of the same density.

Step 1: Determine the mass and radius of the new sphere The disc receives $7M/8$, leaving

$$m_2 = M - \frac{7M}{8} = \frac{M}{8}.$$

Writing the common material density as ρ, the new sphere satisfies

$$\frac{(4/3)\pi r_2^3}{(4/3)\pi R^3} = \frac{m_2}{M} = \frac{1}{8}.$$

Consequently $r_2 = R/2$. Its radius is not $R/8$: the volume, rather than radius, is proportional to mass. No density factor is needed for the disc because its final radius is already specified.

Step 2: Evaluate both axial moments of inertia The disc has radius $2R$, so

$$I_1 = \frac{1}{2}\left(\frac{7M}{8}\right)(2R)^2 = \frac{7}{4}MR^2.$$

For the smaller solid sphere about a diameter,

$$I_2 = \frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R}{2}\right)^2 = \frac{1}{80}MR^2.$$

Both results have the correct dimensions of mass times length squared. The disc has much greater rotational inertia because most of the mass is spread to a larger radius.

Step 3: Take the dimensionless ratio Divide the two expressions without substituting any arbitrary values for M or R:

$$\frac{I_1}{I_2} = \frac{(7/4)MR^2}{(1/80)MR^2} = \frac{7}{4} \times 80 = 140.$$

The common mass and radius scales cancel completely. The other numerical choices, 65, 185 and 285, do not follow from the disc and solid-sphere factors together with the conserved-density radius $R/2$. In particular, retaining the original sphere radius would underestimate this ratio by a factor of four.

Step 4: Conclusion The ratio is $I_1/I_2 = 140$. Hence, the correct option is (B).

Quick Tip to Solve: Before applying an inertia formula to a recast sphere, find its new radius from $r^3 \propto m$; then square that radius in I.

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53. Light is incident normally on a completely absorbing surface with an energy flux of 25 W cm^{-2}. If the surface has an area of 25 cm^2, the momentum transferred to the surface in 40 min time duration will be : [4 marks]

(A) 6.3×10^{-4} Ns

(B) 5.0×10^{-3} Ns

(C) 1.4×10^{-6} Ns

(D) 3.5×10^{-6} Ns

Correct Answer: (B) 5.0×10^{-3} Ns

Solution:

Concept:

- Electromagnetic radiation of energy E carries momentum $p = E/c$. At normal incidence, complete absorption transfers this incident momentum to the surface.
- Energy flux is power per unit area: $P = IA$. Over a time t, the total incident energy is $E = IAt$, so the transferred momentum is IAt/c.

Step 1: Find the incident power The energy flux is $I = 25\,\mathrm{W\,cm^{-2}}$, while the illuminated area is

$A = 25\,\text{cm}^2$. Since these area units already match, they cancel directly:

$$P = IA = (25)(25)\,\text{W} = 625\,\text{W}.$$

Equivalently, SI conversion would give $I = 2.5 \times 10^5\,\text{W m}^{-2}$ and $A = 2.5 \times 10^{-3}\,\text{m}^2$, again producing $625\,\text{W}$. Converting only one of these two area units would introduce a large error.

Step 2: Calculate the absorbed energy The illumination lasts for $40\,\text{min} = 2400\,\text{s}$. Thus

$$E = Pt = (625)(2400) = 1.50 \times 10^6\,\text{J}.$$

The surface is completely absorbing, so the outgoing light carries away none of this normally directed momentum. The momentum transfer is therefore E/c, without the factor of two that would apply to perfect reflection.

Step 3: Evaluate the momentum transfer Taking $c = 3.0 \times 10^8\,\text{m s}^{-1}$,

$$\Delta p = \frac{1.50 \times 10^6}{3.0 \times 10^8} = 5.0 \times 10^{-3}\,\text{kg m s}^{-1}.$$

Since $1\,\text{kg m s}^{-1} = 1\,\text{N s}$, this equals $5.0 \times 10^{-3}\,\text{N s}$. A check using radiation force gives $F = P/c = 2.083 \times 10^{-6}\,\text{N}$, and Ft gives the same impulse. The other alternatives do not equal the required energy divided by the speed of light.

Step 4: Conclusion The momentum transferred is $5.0 \times 10^{-3}\,\text{N s}$. Hence, the correct option is (B).

Quick Tip to Solve: For absorption use $\Delta p = IAt/c$; reserve the factor of two for normal reflection, and convert the exposure time to seconds.

54. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? [Take, density of water = 10^3 kg/m^3] [4 marks]

(A) 30.1 kg

(B) 87.5 kg

(C) 65.4 kg

(D) 46.3 kg

Correct Answer: (B) 87.5 kg

Solution:

Concept:

- Archimedes' principle gives the buoyant force as $F_b = \rho_w g V_{\text{submerged}}$. A freely floating body is in equilibrium when this equals its total weight.
- The largest additional load is reached when the block is just fully submerged. The extra buoyancy comes from immersing the initially unsubmerged part of the volume.

Step 1: Calculate the block volume and its own mass For side $a = 0.5\,\text{m}$,

$$V = a^3 = (0.5)^3 = 0.125\,\text{m}^3.$$

Initially only $0.30V$ is submerged. Equilibrium requires $m_b g = \rho_w g(0.30V)$. Cancelling g gives

$$m_b = 1000 \times 0.30 \times 0.125 = 37.5\,\text{kg}.$$

This is the mass of the block itself, and must be included in the total supported mass before determining how much more can be added.

Step 2: Find the limiting supported mass When the top surface is just at water level, the displaced volume is the full block volume. Its maximum buoyant force is $\rho_w g V$, equivalent to supporting

$$m_{\text{total,max}} = \rho_w V = 1000 \times 0.125 = 125\,\text{kg}.$$

Subtract the block's own mass:

$$m_{\text{added,max}} = 125 - 37.5 = 87.5\,\text{kg}.$$

This also follows directly from the remaining 70 percent of volume: $m_{\text{added,max}} = 0.70\rho_w V$.

Step 3: Interpret the requested weight and compare the choices The alternatives express the load in kilograms, so they refer to the load's mass (or its kilogram-weight equivalent), rather than an SI force. The corresponding actual weight is $87.5g\,\text{N}$, about $858\,\text{N}$ if $g = 9.8\,\text{m s}^{-2}$. Loads of 30.1, 65.4, and $46.3\,\text{kg}$ would leave some reserve buoyancy and therefore are not the maximum. Strictly below full submersion, the added mass must be just less than the limiting value; the conventional maximum quoted is this limiting load.

Step 4: Conclusion The limiting additional load corresponds to a mass of $87.5\,\text{kg}$, or weight $87.5g\,\text{N}$. Hence, the correct option is (B).

Quick Tip to Solve: For a floating block initially immersed by fraction f, the extra load capacity is $(1 - f)\rho_w V$, expressed as a mass.

55. The magnitude of the magnetic field at the centre of an equilateral triangular loop of side 1 m which is carrying a current of 10 A is :

[Take $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}] [4 marks]

(A) 3 μT

(B) 18 μT

(C) 9 μT

(D) 1 μT

Correct Answer: (B) 18 μT

Solution:

Concept:

- The field of a finite straight current-carrying wire at perpendicular distance d is $B = \mu_0 I(\sin\theta_1 + \sin\theta_2)/(4\pi d)$, with angles measured from the perpendicular.
- At the centre of an equilateral triangular loop, the three sides give equal fields normal to the loop and in the same direction. Their magnitudes therefore add.

Step 1: Find the distance and endpoint angles for one side Let the triangle side be $a = 1\,\text{m}$. The centre is one-third of the altitude above a side, hence its perpendicular distance is

$$d = \frac{1}{3}\left(\frac{\sqrt{3}a}{2}\right) = \frac{a}{2\sqrt{3}}.$$

The perpendicular bisects the side, so the endpoint angle satisfies

$$\tan\theta = \frac{a/2}{a/(2\sqrt{3})} = \sqrt{3}.$$

Thus $\theta_1 = \theta_2 = 60^\circ$, measured from the perpendicular to that side, and $\sin 60^\circ = \sqrt{3}/2$.

Step 2: Calculate the field of one side Using the finite-wire expression,

$$B_{\text{side}} = \frac{\mu_0 I}{4\pi d}(2\sin 60^\circ) = \frac{\mu_0 I\sqrt{3}}{4\pi}\frac{2\sqrt{3}}{a} = \frac{3\mu_0 I}{2\pi a}.$$

The field cannot be calculated by treating a side as infinitely long, because its endpoints subtend finite angles at the centre. Each side carries the same current and is at the same distance from the centre.

Step 3: Add all three fields and substitute By the right-hand rule all three contributions point along the same normal, so

$$B = 3B_{\text{side}} = \frac{9\mu_0 I}{2\pi a}.$$

$$B = \frac{9(4\pi \times 10^{-7})(10)}{2\pi(1)} = 18 \times 10^{-6}\,\text{T} = 18\,\mu\text{T}.$$

Each side contributes $6\,\mu$T. The alternatives 3, 9, and $1\,\mu$T neither equal this sum nor account correctly for all finite sides. In particular, the total field is not a vector cancellation: circulation around the triangle gives a common normal direction at its centre.

Step 4: Conclusion The field magnitude at the centre is $18\,\mu$T. Hence, the correct option is (B).

Quick Tip to Solve: For a regular polygon, calculate one finite-side field using the inradius, then multiply by the number of sides; their central fields reinforce.

56. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit? [4 marks]

(A) 1.16 mm

(B) 1.36 mm

(C) 1.00 mm

(D) 0.90 mm

Correct Answer: (A) 1.16 mm

Solution:

Concept:

- Axial stress in a rod is $\sigma = F/A$, where F is the tensile load and A is its cross-sectional area. For a circular rod, $A = \pi d^2/4$.
- The elastic limit is the largest permissible stress for elastic behaviour. To minimise the diameter under a fixed load, set the stress equal to this limiting value.

Step 1: Translate the elastic-limit condition The applied load is $F = 400\,\mathrm{N}$, while the permitted stress is

$$\sigma_{\lim} = 379\,\mathrm{MPa} = 379 \times 10^6\,\mathrm{N\,m^{-2}}.$$

A safe rod must satisfy $F/A \le \sigma_{\lim}$, or $A \ge F/\sigma_{\lim}$. Therefore the limiting minimum area is

$$A_{\min} = \frac{400}{379 \times 10^6} = 1.0554 \times 10^{-6}\,\mathrm{m^2}.$$

This is $1.0554\,\mathrm{mm^2}$, since one square millimetre is $10^{-6}\,\mathrm{m^2}$.

Step 2: Convert the area into a diameter Using the circular cross section,

$$d_{\min} = \sqrt{\frac{4A_{\min}}{\pi}} = \sqrt{\frac{4(400)}{\pi(379 \times 10^6)}}\,\mathrm{m}.$$

$$d_{\min} = 1.159 \times 10^{-3}\,\mathrm{m} \simeq 1.16\,\mathrm{mm}.$$

The square root is essential because stress depends on diameter squared. The calculated value is a diameter, not the radius; using $A = \pi d^2$ would incorrectly reduce the result by a factor of two.

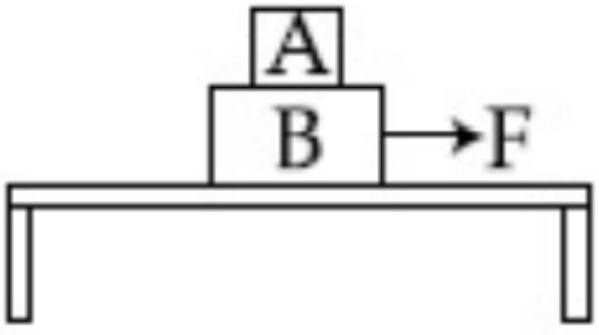
Step 3: Check the remaining choices At $d = 1.00\,\mathrm{mm}$, the area is $0.7854\,\mathrm{mm^2}$, giving stress about $509\,\mathrm{MPa}$, above the elastic limit. A $0.90\,\mathrm{mm}$ rod has still less area and is also unsafe. A $1.36\,\mathrm{mm}$ rod gives stress about $275\,\mathrm{MPa}$, so it is safe but is not the minimum. The $1.16\,\mathrm{mm}$ choice is the limiting diameter rounded to the given precision. No Young's modulus or rod length is needed because the question restricts stress rather than extension.

Step 4: Conclusion The minimum diameter is approximately $1.16\,\mathrm{mm}$. Hence, the correct option is (A).

Quick Tip to Solve: Combine $F/A \le \sigma_{\lim}$ with $A = \pi d^2/4$: minimum diameter varies as the square root of the load.

57. Two blocks A and B of masses m_A = 1 kg and m_B = 3 kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is :

[Take g = 10 m/s^2] [4 marks]



(A) 8 N

(B) 16 N

(C) 40 N

(D) 12 N

Correct Answer: (B) 16 N

Solution:

Concept:

- Static friction accelerates the upper block. Its maximum value is $f_{AB,\max} = \mu_{AB} m_A g$, so the common acceleration cannot exceed $\mu_{AB} g$.
- The normal reaction of the table supports both masses. For the combined system, friction between the two blocks is internal, but table friction opposes the applied force.

Step 1: Limit the acceleration of A Block A has no other horizontal force: the friction exerted by B must provide $m_A a$. Therefore

$$m_A a \le 0.2 m_A g, \qquad a_{\max} = 0.2(10) = 2\,\mathrm{m\,s^{-2}}.$$

At this limit, the inter-block friction is only $1(2) = 2\,\mathrm{N}$. It is not the friction exerted by the table.

Step 2: Apply Newton's law to both blocks The table supports a mass of $1+3 = 4\,\mathrm{kg}$, hence its normal force is $40\,\mathrm{N}$. Its opposing friction is

$$f_T = 0.2(40) = 8\,\mathrm{N}.$$

Treating A and B together cancels the equal and opposite contact friction forces. At the limiting common acceleration,

$$F_{\max} - 8 = (1+3)(2), \qquad F_{\max} = 16\,\mathrm{N}.$$

The value $8\,\mathrm{N}$ merely balances table friction; $12\,\mathrm{N}$ produces a smaller allowable acceleration. A force

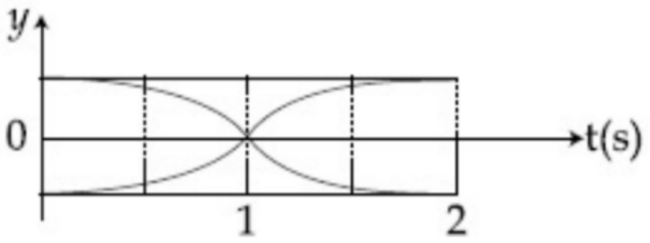
of $40\,\mathrm{N}$ would require $a = 8\,\mathrm{m\,s^{-2}}$ and make A slip.

Step 3: Conclusion The maximum applied force without sliding of A relative to B is $\boxed{16\,\mathrm{N}}$. Hence, the correct option is (B).

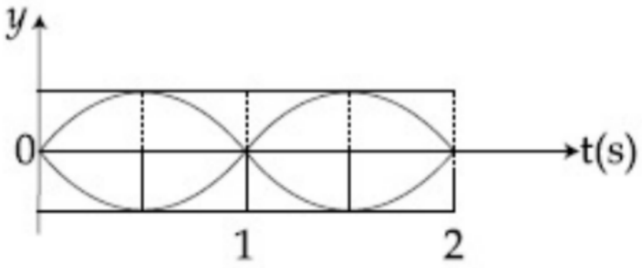
Quick Tip to Solve: First limit the acceleration using the upper contact, then add table friction to the force needed to accelerate the total mass.

58. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz, is : [4 marks]

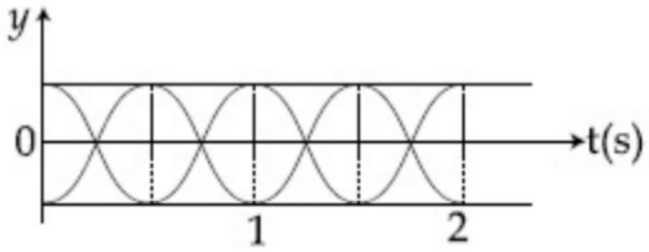
(A)



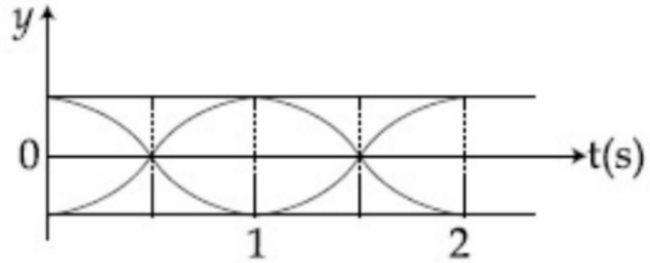
(B)



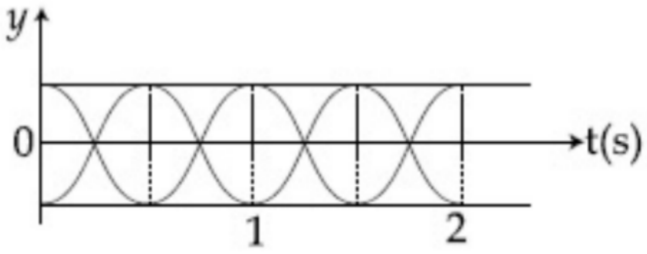
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- Two waves of nearby frequencies produce beats. Their beat frequency is $f_b = |f_2 - f_1|$, while the rapid oscillations have mean frequency $(f_1 + f_2)/2$.
- For equal amplitudes and a common initial phase, the sum-to-product identity gives $y = 2a\cos[\pi(f_2 - f_1)t]\sin[\pi(f_1 + f_2)t]$. The two envelope curves describe the schematic pattern.

Step 1: Calculate the envelope Here $f_1 = 9\,\text{Hz}$ and $f_2 = 11\,\text{Hz}$. Thus

$$y = 2a\cos(2\pi t)\sin(20\pi t).$$

The fast oscillation is at $10\,\text{Hz}$. The signed envelope factor is $2a\cos(2\pi t)$; its absolute magnitude sets the instantaneous amplitude. The positive and negative envelopes meet whenever this cosine vanishes.

Step 2: Count the beats in the displayed interval The loudness maxima repeat at the beat frequency

$$f_b = 11 - 9 = 2\,\text{Hz}, \qquad T_b = \frac{1}{2}\,\text{s}.$$

Over two seconds there must be four complete beat intervals, with maxima at $t = 0, 0.5, 1, 1.5, 2\,\text{s}$, and minima halfway between them. Option (C) shows precisely this repeated narrowing and widening. Option (A) shows only one broad narrowing over the interval; (B) and (D) have envelope maxima too widely separated. These schematics represent the envelope, so one must count amplitude repetitions rather than treat each curve as a rapid wave oscillation.

Step 3: Conclusion The pattern has two beats per second and four beat intervals in two seconds, matching figure (C). Hence, the correct option is (C).

Quick Tip to Solve: Count successive envelope maxima: their spacing is the reciprocal of the frequency difference, not the reciprocal of the mean frequency.

59. Two radioactive substances A and B have decay constants 5λ and λ respectively. At t = 0, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\frac{1}{e}\right)^2$ will be : [4 marks]

(A) $\frac{2}{\lambda}$
(B) $\frac{1}{4\lambda}$
(C) $\frac{1}{2\lambda}$
(D) $\frac{1}{\lambda}$

Correct Answer: (C) $\frac{1}{2\lambda}$

Solution:

Concept:

- Radioactive populations decay exponentially: $N(t) = N_0 e^{-kt}$, where k is the decay constant and t is elapsed time.
- For equal initial populations, their ratio depends only on the difference of decay constants. The faster-decaying population belongs in the numerator of a ratio below unity.

Step 1: Write the two populations Let the common initial number be N_0. Since A has decay constant 5λ and B has decay constant λ,

$$N_A = N_0 e^{-5\lambda t}, \qquad N_B = N_0 e^{-\lambda t}.$$

Both populations decrease, but A decreases more rapidly. Consequently N_A/N_B starts at one and falls below one, as required by the given target ratio. The reverse ratio would grow and cannot equal e^{-2} at positive time.

Step 2: Solve the ratio equation Dividing eliminates the unknown initial population:

$$\frac{N_A}{N_B} = e^{-(5\lambda-\lambda)t} = e^{-4\lambda t}.$$

The stated ratio is $(1/e)^2 = e^{-2}$. Since the exponential function is one-to-one,

$$-4\lambda t = -2, \qquad t = \frac{1}{2\lambda}.$$

As a check, this time gives $N_A/N_0 = e^{-5/2}$ and $N_B/N_0 = e^{-1/2}$, whose ratio is exactly e^{-2}. The times $2/\lambda$, $1/(4\lambda)$ and $1/\lambda$ instead give ratios e^{-8}, e^{-1} and e^{-4}, so the other choices fail.

Step 3: Conclusion The required time is $\boxed{t = 1/(2\lambda)}$, expressed in the time unit reciprocal to that of λ. Hence, the correct option is (C).

Quick Tip to Solve: For initially equal radioactive samples, subtract the decay constants before taking the logarithm of the desired population ratio.

60. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him? (Take velocity of sound in air is 350 m/s) [4 marks]

(A) 750 Hz

(B) 857 Hz

(C) 807 Hz

(D) 1143 Hz

Correct Answer: (A) 750 Hz

Solution:

Concept:

- For a stationary observer and a moving source in still air, the observed frequency is $f_{\text{app}} = fv/(v \mp v_s)$: use minus for approach and plus for recession.
- The actual source frequency f stays unchanged when the source crosses the observer. The measured approach frequency is not the actual source frequency.

Step 1: Recover the emitted frequency The speed of sound is $v = 350\,\text{m}\,\text{s}^{-1}$, and the source speed is $v_s = 50\,\text{m}\,\text{s}^{-1}$. During approach the wavefront spacing decreases, so

$$1000 = f\frac{350}{350-50} = f\frac{350}{300}.$$

Hence the actual emitted frequency is

$$f = 1000\frac{300}{350} = \frac{6000}{7}\,\text{Hz} \approx 857.14\,\text{Hz}.$$

This intermediate result explains the tempting $857\,\text{Hz}$ choice: it is the source frequency, not the recession frequency.

Step 2: Use the recession denominator After crossing the observer, the source moves away. The wavefront spacing increases, giving

$$f_{\text{away}} = f\frac{350}{350+50} = 1000\frac{350-50}{350+50} = 1000\frac{300}{400} = 750\,\text{Hz}.$$

The common factor 350 cancels when the two apparent frequencies are compared. The answer must be less than the actual frequency and less than the approach frequency. The value $1143\,\text{Hz}$ has the wrong trend, and $807\,\text{Hz}$ satisfies neither Doppler relation with the given speeds.

Step 3: Conclusion The apparent frequency after the source passes and recedes is $\boxed{750\,\text{Hz}}$. Hence, the correct option is (A).

Quick Tip to Solve: Directly compare the two readings using $f_{\text{away}}/f_{\text{toward}} = (v - v_s)/(v + v_s)$.

Chemistry (120 marks)

61. The major product Y in the following reaction is:

$$C_6H_5COCH_3 \xrightarrow{NaOCl} X \xrightarrow[\text{(ii) aniline}]{\text{(i) } SOCl_2} Y$$

[4 marks]

(A) $C_6H_5CONHC_6H_5$

(B) $C_6H_5COCH{=}NC_6H_5$

(C) p-$H_2NC_6H_4COC_6H_5$

(D) o-$H_2NC_6H_4COC_6H_5$

Correct Answer: (A) $C_6H_5CONHC_6H_5$

Solution:

Concept:

- A methyl ketone undergoes the haloform reaction with hypochlorite: its $-COCH_3$ group becomes a carboxylate group, while the methyl carbon leaves as haloform.
- Thionyl chloride converts a carboxylic acid to its acyl chloride. Aniline then undergoes nucleophilic acyl substitution at that carbonyl carbon to form an amide.

Step 1: Identify the benzoyl intermediate The starting ketone is acetophenone, $C_6H_5COCH_3$. Sodium hypochlorite chlorinates its methyl group and causes haloform cleavage. The carbonyl carbon remains attached to the phenyl group, producing benzoate; in the subsequent acid/acyl-chloride sequence this gives the benzoyl derivative:

$$C_6H_5COCH_3 \xrightarrow{NaOCl} C_6H_5COO^- \xrightarrow{\text{acidification}} C_6H_5COOH.$$

The methyl carbon is removed as $CHCl_3$. Thus the product cannot retain an extra carbon between the benzoyl group and the incoming nitrogen.

Step 2: Acylate aniline at nitrogen Thionyl chloride gives benzoyl chloride, which reacts with aniline:

$$C_6H_5COOH + SOCl_2 \longrightarrow C_6H_5COCl + SO_2 + HCl,$$

$$C_6H_5COCl + H_2NC_6H_5 \longrightarrow C_6H_5CONHC_6H_5 + HCl.$$

The lone pair on aniline nitrogen attacks the acyl carbon, and chloride is displaced. The product is benzanilide (N-phenylbenzamide), with a $-CO-NH-$ linkage, as in (A). Option (B) contains a carbon-nitrogen double bond and an extra carbon; neither is formed by acyl substitution. Options (C) and (D) would require acylation of the aromatic ring at para or ortho positions, whereas the given sequence acylates the amino nitrogen.

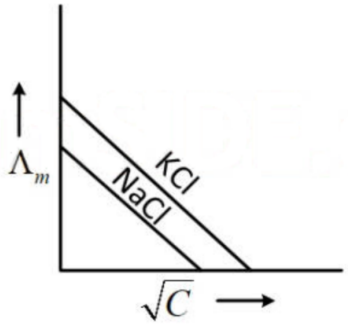
Step 3: Conclusion The major product is benzanilide, $\boxed{C_6H_5CONHC_6H_5}$, corresponding to (A). Hence, the correct option is (A).

Quick Tip to Solve: Trace the carbonyl carbon through the sequence: haloform cleavage removes the methyl carbon, then aniline replaces chloride on the benzoyl group.

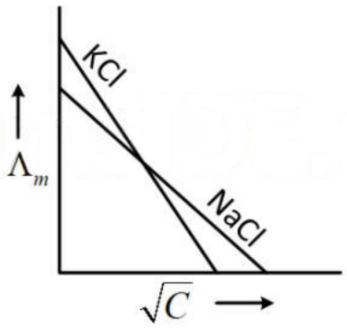
62. **Which one of the following graphs between molar conductivity (Λ_m) versus $\sqrt{C}$ is correct ?**

[4 marks]

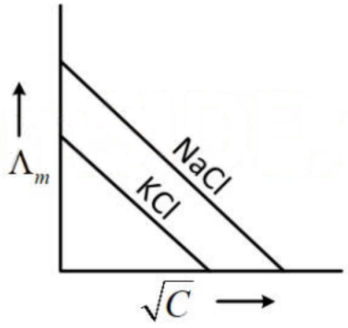
(A)



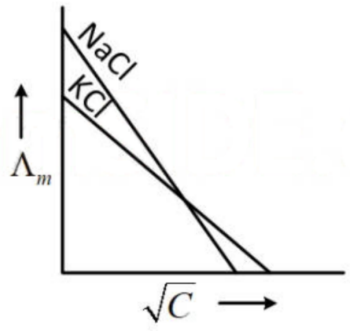
(B)



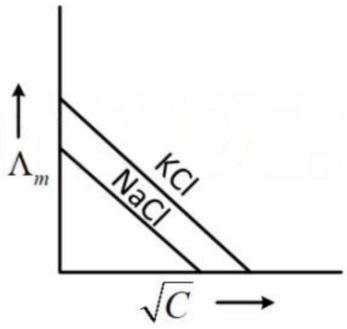
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- For dilute strong electrolytes, molar conductivity falls approximately linearly with $\sqrt{C}$: $\Lambda_m = \Lambda_m^\circ - K\sqrt{C}$. The intercept is the limiting molar conductivity.
- Both salts contain the same chloride ion. The hydrated sodium ion has lower mobility than the potassium ion, so $\Lambda_m^\circ(\mathrm{KCl}) > \Lambda_m^\circ(\mathrm{NaCl})$.

Step 1: Compare the intercepts At infinite dilution, independent ionic contributions add:

$$\Lambda_m^\circ(\mathrm{KCl}) = \lambda^\circ_{\mathrm{K}^+} + \lambda^\circ_{\mathrm{Cl}^-},$$

$$\Lambda_m^\circ(\mathrm{NaCl}) = \lambda^\circ_{\mathrm{Na}^+} + \lambda^\circ_{\mathrm{Cl}^-}.$$

The chloride contribution is common. Although the bare sodium ion is smaller, it is more strongly hydrated and therefore moves less readily through water. The potassium salt consequently has the larger conductivity intercept, ruling out graphs (C) and (D), which reverse the ordering.

Step 2: Match the concentration trend As concentration rises, interionic interactions reduce ionic mobility, so both lines slope downward. For these two 1:1 strong electrolytes the schematic comparison uses similar negative slopes, with the KCl line above the NaCl line in the dilute concentration region. Figure (A) shows precisely this ordering and trend. Figure (B) incorrectly makes the curves cross and predicts that increasing concentration reverses their relative conductivities within the displayed comparison. One should use the dilute-solution straight-line relation only within its range; the schematic extrapolation to zero conductivity is not a claim that this law holds at arbitrary high concentration.

Step 3: Conclusion The correct schematic graph is (A): both molar conductivities decrease with $\sqrt{C}$, with KCl above NaCl. Hence, the correct option is (A).

Quick Tip to Solve: For salts with a common anion, compare cation mobility at infinite dilution first; hydration makes sodium less mobile than potassium.

63. 1 g of a non-volatile non-electrolyte solute is dissolved in 100 g of two different solvents A and B whose ebullioscopic constants are in the ratio of 1 : 5. The ratio of the elevation in their boiling points, $\frac{\Delta T_b(A)}{\Delta T_b(B)}$, is : [4 marks]

(A) 5 : 1

(B) 1 : 0.2

(C) 10 : 1

(D) 1 : 5

Correct Answer: (D) 1 : 5

Solution:

Concept:

- A dilute solution of a non-volatile solute has boiling-point elevation $\Delta T_b = iK_b m$, where m is molality and K_b is the solvent's ebullioscopic constant.
- The solute is a non-electrolyte, so $i = 1$. Equal masses of the same solute in equal masses of solvent give equal molalities, even when the solvents differ.

Step 1: Compare the two molalities Let the solute molar mass be $M\,\mathrm{g\,mol^{-1}}$. Each solution contains $1/M$ mole of solute in $100\,\mathrm{g} = 0.100\,\mathrm{kg}$ of solvent. Thus

$$m_A = m_B = \frac{1/M}{0.100} = \frac{10}{M}\,\mathrm{mol\,kg^{-1}}.$$

The molar mass is not needed numerically because the solute is the same in both cases. Molality uses the mass of solvent, not the total mass of solution or its volume. Hence any difference in solvent density is irrelevant here.

Step 2: Take the elevation ratio Applying the elevation law to A and B separately gives

$$\Delta T_b(A) = K_b(A)\frac{10}{M}, \qquad \Delta T_b(B) = K_b(B)\frac{10}{M}.$$

Dividing cancels the common molality:

$$\frac{\Delta T_b(A)}{\Delta T_b(B)} = \frac{K_b(A)}{K_b(B)} = \frac{1}{5}.$$

The solution in the solvent with larger K_b has the larger elevation. The ratios $5 : 1$ and $1 : 0.2$ are both the reciprocal, namely five, while $10 : 1$ also contradicts the equal-molality cancellation. No extra factor arises from the common $1\,\mathrm{g}$ solute mass.

Step 3: Conclusion The boiling-point elevations are in the ratio $\boxed{1 : 5}$. Hence, the correct option is (D).

Quick Tip to Solve: Before calculating either elevation, cancel equal molalities: for the same solute mass and solvent mass, the elevation ratio equals the ratio of K_b.

64. For the reaction of H_2 with I_2, the rate constant is 2.5×10^{-4} dm^3 $mol^{-1}s^{-1}$ at 327°C and 1.0 dm^3 mol^{-1} at 527°C. The activation energy for the reaction, in kJ $mole^{-1}$ is : (R = 8.314 JK^{-1} mol^{-1}) [4 marks]

(A) 59

(B) 166

(C) 72

(D) 150

Correct Answer: (B) 166

Solution:

Concept:

- The Arrhenius equation is $k = Ae^{-E_a/(RT)}$, where E_a is the activation energy and T is absolute temperature.
- For the same reaction at two temperatures, $\ln(k_2/k_1) = (E_a/R)(1/T_1 - 1/T_2)$. The ratio uses rate constants expressed in the same units.

Step 1: Convert the temperatures The temperatures in the exponential must be in kelvin. Thus $T_1 = 327 + 273 = 600\,\text{K}$ and $T_2 = 527 + 273 = 800\,\text{K}$. The rate constant rises on heating, so the activation energy obtained from their ratio must be positive.

Step 2: Form the rate-constant ratio The given constants give

$$\frac{k_2}{k_1} = \frac{1.0}{2.5 \times 10^{-4}} = 4000, \qquad \ln 4000 = 8.294.$$

The factor 10^{-4} is essential; dropping it would produce an entirely different temperature dependence.

Step 3: Calculate and compare Substituting into the two-temperature equation,

$$E_a = \frac{8.314 \ln 4000}{1/600 - 1/800} = 8.314(8.294)(2400)\,\text{J mol}^{-1}.$$

Hence $E_a \approx 1.655 \times 10^5\,\text{J mol}^{-1} = 165.5\,\text{kJ mol}^{-1}$. This rounds to 166, whereas 59, 72 and 150 do not reproduce the observed factor of 4000 at these temperatures.

Step 4: Conclusion The activation energy is approximately $166\,\text{kJ mol}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert both Celsius temperatures to kelvin before subtracting their reciprocals; then check that an increasing rate constant gives positive activation energy.

65. The crystal field stabilization energy (CFSE) of $[Fe(H_2O)_6]Cl_2$ and $K_2[NiCl_4]$ respectively, are :

[4 marks]

(A) – 0.4 Δ_0 and – 0.8 Δ_t

(B) – 0.6 Δ_0 and – 0.8 Δ_t

(C) – 2.4 Δ_0 and – 1.2 Δ_t

(D) – 0.4 Δ_0 and – 1.2 Δ_t

Correct Answer: (A) – 0.4 Δ_0 and – 0.8 Δ_t

Solution:

Concept:

- Crystal field stabilization is the sum of the energies of the occupied split d orbitals, measured relative to the unsplit d-orbital barycentre.
- In an octahedral field, t_{2g} and e_g have energies $-0.4\Delta_o$ and $+0.6\Delta_o$; in a tetrahedral field, e and t_2 have energies $-0.6\Delta_t$ and $+0.4\Delta_t$.

Step 1: Find the metal-ion configurations In $[Fe(H_2O)_6]Cl_2$, the complex cation has charge $+2$, and neutral water leaves iron in oxidation state $+2$. Thus Fe^{2+} is d^6. In $K_2[NiCl_4]$, the complex has charge -2; four chloride ligands imply Ni^{2+}, a d^8 ion.

Step 2: Calculate the octahedral stabilization Water is a weak-field ligand for Fe^{2+}, so the six-coordinate ion is high spin with occupancy $t_{2g}^4 e_g^2$. Consequently,

$$\mathrm{CFSE_{Fe}} = 4(-0.4\Delta_o) + 2(0.6\Delta_o) = -0.4\Delta_o.$$

The value $-2.4\Delta_o$ would belong to low-spin d^6, which is not the water complex here.

Step 3: Calculate the tetrahedral stabilization The chloride complex of Ni^{2+} is tetrahedral. Its eight electrons occupy $e^4 t_2^4$, giving

$$\mathrm{CFSE_{Ni}} = 4(-0.6\Delta_t) + 4(0.4\Delta_t) = -0.8\Delta_t.$$

Thus options containing $-1.2\Delta_t$ are excluded. The remaining alternative with $-0.6\Delta_o$ also has the wrong octahedral electron count.

Step 4: Conclusion The CFSE values are $-0.4\Delta_o$ and $-0.8\Delta_t$, respectively. Hence, the correct option is (A).

Quick Tip to Solve: Find oxidation state and d count first, then use the correct lower-orbital label: octahedral t_{2g}, but tetrahedral e.

66. For the reaction,

$2SO_2(g) + O_2(g) = 2SO_3(g)$, $\Delta H = -57.2$ kJ mol^{-1} and $K_C = 1.7 \times 10^{16}$

Which of the following statement is incorrect ? [4 marks]

(A) The equilibrium will shift in forward direction as the pressure increase.

(B) The addition of inert gas at constant volume will be not affect the equilibrium constant.

(C) The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.

(D) The equilibrium constant decreases as the temperature increase.

Correct Answer: (C) The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.

Solution:

Concept:

- The equilibrium constant measures the equilibrium composition, whereas a catalyst controls the rate of reaching that composition.
- Le Chatelier's principle predicts the response to pressure and temperature. At fixed temperature the equilibrium constant is unchanged by a catalyst or an inert gas.

Step 1: Check the pressure statement The forward reaction $2\,SO_2(g) + O_2(g) \rightleftharpoons 2\,SO_3(g)$ reduces the gaseous amount from three stoichiometric moles to two. Increasing pressure by compression therefore favours the products, the side with fewer gas molecules. Statement (A) is correct.

Step 2: Check the inert-gas and temperature statements At constant volume and temperature, adding an inert gas does not change any reacting gas's partial pressure $p_i = n_i RT/V$. It therefore does not alter the equilibrium composition or K_C. Statement (B) is correct. Since the reaction is exothermic, increasing temperature favours the reverse, endothermic direction and decreases K_C. Equivalently, $d\ln K/dT = \Delta H/(RT^2) < 0$. Thus statement (D) is also correct.

Step 3: Distinguish extent from speed The large value $K_C = 1.7 \times 10^{16}$ means that products are strongly favoured at equilibrium. It says nothing about the activation barrier or the time required to attain equilibrium. A favourable reaction can still proceed slowly and need a catalyst for a useful rate. A catalyst lowers the kinetic barrier for both directions without changing K_C. The claim that no catalyst is required merely because K_C is large is therefore unjustified.

Step 4: Conclusion Statement (C) is incorrect: a large equilibrium constant does not imply a sufficiently rapid reaction. Hence, the correct option is (C).

Quick Tip to Solve: When an option infers reaction speed from the size of K, separate thermodynamics (extent) from kinetics (rate) before judging it.

67. The minimum amount of $O_2(g)$ consumed per gram of reactant is for the reaction :
(Given atomic mass : Fe = 56, O = 16, Mg = 24, P = 31, C = 12, H = 1) [4 marks]

(A) $4Fe(s) + 3O_2(g) \rightarrow 2Fe_2O_3(s)$

(B) $P_4(s) + 5O_2(g) \rightarrow P_4O_{10}(s)$

(C) $C_3H_8(g) + 5O_2(g) \rightarrow 3CO_2(g) + 4H_2O(l)$

(D) $2Mg(s) + O_2(g) \rightarrow 2MgO(s)$

Correct Answer: (A) $4Fe(s) + 3O_2(g) \rightarrow 2Fe_2O_3(s)$

Solution:

Concept:

- For a balanced equation, the oxygen consumed per gram of the other reactant is the oxygen mass divided by that reactant's stoichiometric mass.
- Compare all reactions on the same one-gram basis. Oxygen coefficients alone are insufficient because the reacting substances have different molar masses.

Step 1: Calculate the iron and phosphorus ratios For iron, four moles weigh $4(56) = 224\,\text{g}$, while three moles of oxygen weigh $3(32) = 96\,\text{g}$. Hence

$$r_{\text{Fe}} = \frac{96}{224} = \frac{3}{7} \approx 0.429\,\text{g O}_2 \text{ per g Fe}.$$

For P_4, the reactant mass is $4(31) = 124\,\text{g}$, and five moles of oxygen weigh $160\,\text{g}$. Its ratio is $160/124 \approx 1.290$.

Step 2: Calculate the propane and magnesium ratios The molar mass of C_3H_8 is $3(12) + 8(1) = 44\,\text{g mol}^{-1}$. Therefore its oxygen requirement is

$$r_{\text{propane}} = \frac{5(32)}{44} = \frac{40}{11} \approx 3.636.$$

For magnesium, two moles weigh $2(24) = 48\,\text{g}$, requiring $32\,\text{g}$ of oxygen. Thus $r_{\text{Mg}} = 32/48 \approx 0.667$. Each ratio has the same units, grams of oxygen per gram of the named reactant.

Step 3: Choose the minimum The comparison is

$$0.429\ (\text{Fe}) < 0.667\ (\text{Mg}) < 1.290\ (P_4) < 3.636\ (C_3H_8).$$

Iron has the smallest oxygen requirement on the required mass basis. The magnesium reaction uses fewer moles of oxygen as written, but it also represents much less reactant mass; that is why comparing coefficients would give the wrong answer.

Step 4: Conclusion The minimum oxygen consumption per gram of reactant occurs for $4\,\text{Fe} + 3\,O_2 \longrightarrow 2\,Fe_2O_3$. Hence, the correct option is (A).

Quick Tip to Solve: For every candidate, divide $32\times$ the oxygen coefficient by the coefficient-weighted molar mass of the other reactant.

68. The increasing order of nucleophilicity of the following nucleophiles is :

(a) $CH_3CO_2^-$ (b) H_2O

(c) $CH_3SO_3^-$ (d) $\bar{O}H$ [4 marks]

(A) (b) < (c) < (a) < (d)

(B) (b) < (c) < (d) < (a)

(C) (a) < (d) < (c) < (b)

(D) (d) < (a) < (c) < (b)

Correct Answer: (A) (b) < (c) < (a) < (d)

Solution:

Concept:

- A nucleophile donates an electron pair to an electrophile. For the oxygen nucleophiles listed here, charge and resonance delocalization determine the qualitative order.
- An electron pair held in a strongly resonance-stabilized anion is less available for bond formation than a localized pair on hydroxide.

Step 1: Identify the four species The labels denote (a) acetate, $CH_3CO_2^-$; (b) water, H_2O; (c) methanesulfonate, $CH_3SO_3^-$; and (d) hydroxide, OH^-. All attack through oxygen, so differences in electronic stabilization can be compared without changing the donor element. Neutral water is the weakest nucleophile in the listed qualitative series.

Step 2: Compare the anionic nucleophiles In methanesulfonate the negative charge is extensively delocalized over the sulfonate oxygen atoms. This strong stabilization makes it a weak electron-pair donor. Acetate also has a resonance-delocalized charge, but it is less stabilized than sulfonate and is the stronger nucleophile of the two. Hydroxide has no such resonance delocalization: its negative charge is localized on oxygen, making its electron pair most available among these species.

Step 3: Arrange and eliminate alternatives Putting the neutral species first and then ordering the anions by increasing availability gives

$$H_2O < CH_3SO_3^- < CH_3CO_2^- < OH^-.$$

In labels this is $(b) < (c) < (a) < (d)$. Option (B) incorrectly puts resonance-stabilized acetate above hydroxide. Options (C) and (D) place water at the strongest end and reverse the principal charge/resonance trend. Only (A) gives the required increasing sequence.

Step 4: Conclusion The increasing nucleophilicity order is $H_2O < CH_3SO_3^- < CH_3CO_2^- < OH^-$. Hence, the correct option is (A).

Quick Tip to Solve: For this oxygen-donor series, locate the neutral species first, then compare how strongly resonance stabilizes each anion; localized hydroxide is the strongest.

69. The difference between ΔH and ΔU (ΔH – ΔU), when the combustion of one mole of heptane(l) is carried out at a temperature T, is equal to : [4 marks]

(A) – 4 RT

(B) 3 RT

(C) – 3 RT

(D) 4 RT

Correct Answer: (A) – 4 RT

Solution:

Concept:

- For a reaction involving ideal gases, $\Delta H - \Delta U = \Delta n_g RT$, where Δn_g counts gaseous product moles minus gaseous reactant moles.
- In the usual complete combustion equation of a liquid hydrocarbon, carbon forms $\mathrm{CO_2(g)}$ and hydrogen forms $\mathrm{H_2O\,(l)}$. Liquid reactants and products are excluded from Δn_g.

Step 1: Balance complete combustion Heptane is $\mathrm{C_7H_{16}}$. Seven carbon atoms produce seven carbon dioxide molecules, and sixteen hydrogen atoms produce eight water molecules. The products contain $14 + 8 = 22$ oxygen atoms, requiring eleven oxygen molecules:

$$\mathrm{C_7H_{16}(l) + 11\,O_2(g) \longrightarrow 7\,CO_2(g) + 8\,H_2O\,(l).}$$

The equation is written for exactly one mole of heptane, as requested.

Step 2: Count only gaseous coefficients The gaseous reactants consist of eleven moles of oxygen. The gaseous products consist of seven moles of carbon dioxide. Heptane and water are liquids in this combustion equation and do not enter the ideal-gas mole difference. Therefore

$$\Delta n_g = 7 - 11 = -4.$$

Counting liquid water as a gas would change the physical reaction and give an inappropriate positive result.

Step 3: Apply the enthalpy relation From $H = U + PV$, the ideal-gas contribution changes by $\Delta n_g RT$ at temperature T. Substitution gives

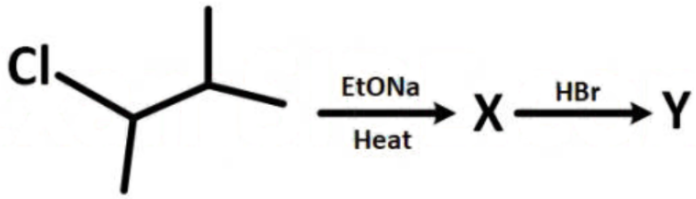
$$\Delta H - \Delta U = (-4)RT.$$

The negative sign agrees with the decrease in gaseous moles. The options $3RT$ and $-3RT$ have an incorrect gas-mole difference; $4RT$ has the reversed sign and does not describe combustion to liquid water.

Step 4: Conclusion For combustion of one mole of liquid heptane, $\Delta H - \Delta U = -4RT$. Hence, the correct option is (A).

Quick Tip to Solve: Write the balanced combustion equation with physical states before using $\Delta n_g RT$; liquid water contributes no gaseous moles.

70. The major product 'Y' in the following reactions is : [4 marks]



(A) $BrCH_2CH_2CH(CH_3)CH_3$

(B) $CH_3CH(OH)CH(CH_3)CH_3$

(C) $CH_3CH_2C(Br)(CH_3)CH_3$

(D) $CH_3CH(Br)CH(CH_3)CH_3$

Correct Answer: (C) $CH_3CH_2C(Br)(CH_3)CH_3$

Solution:

Concept:

- Hot sodium ethoxide promotes beta-elimination from a haloalkane. With a small alkoxide base, the more substituted alkene is the major Saytzeff product.
- Addition of HBr to an unsymmetrical alkene without peroxide follows the ionic Markovnikov pathway through the more stable carbocation.

Step 1: Identify the elimination product The starting structure is $CH_3CH(Cl)CH(CH_3)CH_3$, namely 2-chloro-3-methylbutane. Ethoxide removes a beta hydrogen while chloride leaves. Elimination towards the adjacent methine carbon gives

$$CH_3CH(Cl)CH(CH_3)CH_3 \longrightarrow CH_3CH{=}C(CH_3)CH_3.$$

This internal alkene is 2-methylbut-2-ene, X. It is more substituted than the competing terminal alkene, so it predominates under the stated hot ethoxide conditions.

Step 2: Add hydrogen bromide For $CH_3CH{=}C(CH_3)CH_3$, protonation at the double-bond carbon bearing hydrogen places the positive charge on the carbon bonded to three carbon groups. This gives a tertiary carbocation rather than the competing secondary one. Bromide attacks that tertiary centre to form

$$CH_3CH_2C(Br)(CH_3)CH_3.$$

No peroxide is specified, so the radical anti-Markovnikov addition rule is not applicable.

Step 3: Match the product to the options The product is 2-bromo-2-methylbutane, which has bromine on the tertiary carbon. This is structure (C). Structure (A) has bromine at a terminal carbon and does not follow the ionic addition pathway. Structure (B) is an alcohol, although the second reagent is hydrogen bromide. Structure (D) places bromine on a secondary carbon; it corresponds to the less favourable addition orientation rather than the major product.

Step 4: Conclusion The major product Y is 2-bromo-2-methylbutane, $CH_3CH_2C(Br)(CH_3)CH_3$. Hence, the correct option is (C).

Quick Tip to Solve: Apply the two selectivity rules in order: hot ethoxide gives the more substituted alkene, then ordinary HBr puts bromine at its more substituted carbon.

71. Which of these factors does not govern the stability of a conformation in acyclic compounds?

[4 marks]

(A) Angle strain

(B) Torsional strain

(C) Electrostatic forces of interaction

(D) Steric interactions

Correct Answer: (A) Angle strain

Solution:

Concept:

- Conformers interconvert by rotation about a single bond, without breaking bonds or changing the connectivity of the molecule.
- Torsional strain comes from eclipsing bonds; steric interactions come from nonbonded groups approaching too closely. Electrostatic attraction or repulsion can also change the relative energies.

Step 1: Identify what changes during rotation For an acyclic molecule, rotation about a carbon–carbon single bond primarily changes the dihedral angles between bonds on neighbouring atoms. For example, staggered and eclipsed ethane differ in the relative orientation of their carbon–hydrogen bonds. Their carbon bond angles remain essentially tetrahedral. Thus, the conformational energy comparison does not require compression or expansion of a ring bond angle.

Step 2: Compare the listed factors Torsional strain is greater in eclipsed arrangements and smaller in staggered ones, so option (B) does govern stability. In substituted chains, the approach of bulky groups raises steric repulsion; an anti arrangement may therefore be more stable than a gauche arrangement, confirming (D). If polar bonds or charged groups occur, changing their separation and orientation changes electrostatic interactions, confirming (C). Angle strain instead concerns departure of bond angles from their preferred values and is characteristic of constrained cyclic structures. It is not the factor governing the ordinary rotational conformers of an acyclic compound.

Step 3: Conclusion Angle strain is the factor that does not govern the conformational stability of acyclic compounds in this comparison. Hence, the correct option is (A).

Quick Tip to Solve: For an open-chain Newman projection, compare eclipsing, crowding and polar interactions; reserve angle-strain arguments for bond-angle distortion, especially in rings.

72. The ratio of the shortest wavelength of two spectral series of hydrogen spectrum is found to be about 9. The spectral series are : [4 marks]

(A) Paschen and Pfund

(B) Balmer and Brackett

(C) Lyman and Paschen

(D) Brackett and Pfund

Correct Answer: (C) Lyman and Paschen

Solution:

Concept:

- The Rydberg formula is $1/\lambda = R_{\mathrm{H}}(1/n_1^2 - 1/n_2^2)$, where n_1 is fixed for a series and $n_2 > n_1$.
- The shortest wavelength is the series limit, obtained as $n_2 \to \infty$. Therefore $\lambda_{\min} = n_1^2/R_{\mathrm{H}}$.

Step 1: Find the limiting wavelengths As the initial level rises, the transition energy to a fixed final level increases. The largest photon energy, and hence the smallest wavelength, occurs when the initial level approaches infinity. Substituting this limit gives

$$\frac{1}{\lambda_{\min}} = \frac{R_{\mathrm{H}}}{n_1^2}.$$

The final levels for Lyman, Balmer, Paschen, Brackett and Pfund are respectively $1, 2, 3, 4, 5$, so their limiting wavelengths are proportional to $1, 4, 9, 16, 25$.

Step 2: Test each pair Take the larger limiting wavelength divided by the smaller, since the stated ratio is about 9. For Paschen and Pfund the ratio is $25/9$, for Balmer and Brackett it is $16/4 = 4$, and for Brackett and Pfund it is $25/16$. None is 9. For Lyman and Paschen,

$$\frac{\lambda_{\min,\mathrm{Paschen}}}{\lambda_{\min,\mathrm{Lyman}}} = \frac{3^2}{1^2} = 9.$$

The constant cancels, so no numerical value of the Rydberg constant is needed. Reversing this last ratio gives $1/9$, representing the same pair of series.

Step 3: Conclusion The two series are Lyman and Paschen, whose shortest wavelengths are in the ratio $1 : 9$. Hence, the correct option is (C).

Quick Tip to Solve: For a shortest-wavelength question, use the square of the final quantum number: series-limit wavelengths follow $1, 4, 9, 16, 25, \ldots$.

73. The highest possible oxidation states of uranium and plutonium, respectively are : [4 marks]

(A) 4 and 6

(B) 6 and 4

(C) 7 and 6

(D) 6 and 7

Correct Answer: (D) 6 and 7

Solution:

Concept:

- Actinoids exhibit variable oxidation states because their $5f$, $6d$ and $7s$ electrons have comparable energies and can participate in bonding.
- The highest oxidation state observed for an element must be distinguished from its most stable or most common oxidation state.

Step 1: Identify the uranium limit Uranium has the ground-state configuration $[\mathrm{Rn}]5f^3 6d^1 7s^2$. Participation of its six outer electrons gives the highest usual oxidation state $+6$. A familiar example is the uranyl ion $UO_2{}^{2+}$: assigning oxygen its usual oxidation number -2,

$$x + 2(-2) = +2 \quad \Rightarrow \quad x = +6.$$

Uranium also has lower oxidation states, including $+4$, but the question asks for the highest possible state rather than simply a common one.

Step 2: Identify the plutonium limit and eliminate the pairs Plutonium is an early actinoid with a wider oxidation-state range than a single common ionic charge might suggest. Its highest state in the standard actinoid oxidation-state table is $+7$, observed under strongly oxidising conditions. The increasing nuclear charge across the series progressively stabilises the $5f$ electrons, so one cannot infer the maximum merely by counting every electron outside radon. The required ordered pair is therefore $(+6, +7)$. Option (A) incorrectly restricts uranium to $+4$; option (B) restricts plutonium to $+4$; option (C) assigns the higher value to the wrong element. Only (D) gives both maxima in the requested uranium–plutonium order.

Step 3: Conclusion The highest oxidation states of uranium and plutonium are respectively $+6$ and $+7$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the highest state separate from the most common state: uranium reaches $+6$, while plutonium can reach $+7$.

74. In chromatography, which of the following statements is incorrect for R_f ? [4 marks]

(A) R_f value depends on the type of chromatography.

(B) Higher R_f value means higher adsorption.

(C) R_f value is dependent on the mobile phase.

(D) The value of R_f can not be more than one.

Correct Answer: (B) Higher R_f value means higher adsorption.

Solution:

Concept:

- The retention factor is $R_f = d_s/d_f$, where d_s is the distance travelled by the solute spot from the origin and d_f is the distance travelled by the solvent front.
- In adsorption chromatography, stronger adsorption to the stationary phase reduces migration, whereas a greater preference for the mobile phase increases migration.

Step 1: Relate migration to adsorption Both distances are measured from the same starting line along the direction of solvent movement. A strongly adsorbed compound spends more time attached to the stationary phase and less time moving with the solvent. Its spot therefore travels a shorter distance. With the solvent-front distance fixed for that run, a smaller numerator gives a smaller R_f. Conversely, a larger R_f means that the solute has migrated farther and is relatively less strongly retained. Thus the statement linking a higher R_f to higher adsorption reverses the physical relationship.

Step 2: Check the other three statements Option (A) is acceptable: the stationary phase and the mechanism used in a chromatographic method affect how a solute partitions or adsorbs. Option (C) is also correct because mobile-phase composition and solvent strength affect solute migration. Thus R_f is meaningful only when the experimental conditions are specified. Finally, a solute spot cannot outrun the solvent front in the ordinary definition, so

$$0 \le d_s \le d_f \quad \Rightarrow \quad 0 \le R_f \le 1.$$

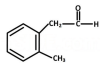
Hence option (D) is correct. The question asks for the incorrect statement, so the reversed adsorption claim must be selected.

Step 3: Conclusion "Higher R_f value means higher adsorption" is incorrect: stronger adsorption gives a lower R_f. Hence, the correct option is (B).

Quick Tip to Solve: Remember the moving spot: stronger sticking to the stationary phase means less travel and therefore a smaller R_f.

75. Compound A($C_9H_{10}O$) shows positive iodoform test. Oxidation of A with $KMnO_4$/KOH given acid B($C_8H_6O_4$). Anhydride of B is used for the preparation of phenolphthalein. Compound A is: [4 marks]

(A) p-Methylacetophenone 

(B) o-Methylphenylacetaldehyde 

(C) m-Methylacetophenone 

(D) o-Methylacetophenone 

Correct Answer: (D) o-Methylacetophenone 

Solution:

Concept:

- A methyl ketone, $RCOCH_3$, gives the iodoform test. An ordinary aldehyde other than ethanal does not satisfy this test.
- Phthalic anhydride is used with phenol to prepare phenolphthalein. Its precursor is phthalic acid, the ortho isomer $C_6H_4(COOH)_2$.

Step 1: Identify the oxidation product The acid B has formula $C_8H_6O_4$ and forms the anhydride used in phenolphthalein preparation. This identifies it as benzene-1,2-dicarboxylic acid, or phthalic acid. Its adjacent carboxyl groups permit formation of a five-membered cyclic anhydride on dehydration. The adjacency is decisive: the meta and para dicarboxylic acids do not give this same phthalic anhydride. Therefore the two oxidisable substituents on the benzene ring of A must be ortho to one another.

Step 2: Apply both tests to the options Option (D) is ortho-methylacetophenone, o-$CH_3C_6H_4COCH_3$. Its $-COCH_3$ group gives the positive iodoform test. Strong alkaline permanganate oxidation followed by acidification converts its two side chains into adjacent carboxyl groups, giving phthalic acid. Its formula is also correct: the disubstituted ring contributes C_6H_4, the methyl group CH_3, and the acetyl group $COCH_3$, totalling $C_9H_{10}O$. Options (A) and (C) contain methyl ketones but have para and meta substitution, respectively, giving the wrong dicarboxylic acid. Option (B) is an aryl-substituted ethanal with a $-CH_2CHO$ side chain, not the required methyl ketone, so it fails the iodoform criterion.

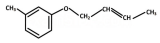
Step 3: Conclusion Compound A is ortho-methylacetophenone (2-methylacetophenone), o-$CH_3C_6H_4COCH_3$. Hence, the correct option is (D).

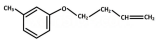
Quick Tip to Solve: Use the anhydride clue to fix the ortho relationship first, then use iodoform to choose the methyl ketone rather than the aldehyde.

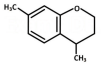
76. The major product obtained in the given reaction is:

$$\text{m-}CH_3C_6H_4OCH_2CH_2CH(Cl)CH_3 \xrightarrow{AlCl_3} \text{Product}$$

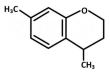
[4 marks]

(A) $\text{m-}CH_3C_6H_4OCH_2CH{=}CHCH_3$ 

(B) $\text{m-}CH_3C_6H_4OCH_2CH_2CH{=}CH_2$ 

(C) 4,7-Dimethylchroman 

(D) 4,5-Dimethylchroman 

Correct Answer: (C) 4,7-Dimethylchroman 

Solution:

Concept:

- Anhydrous $AlCl_3$ activates an alkyl chloride to generate an electrophilic carbon centre, allowing Friedel–Crafts alkylation of an activated aromatic ring.
- Intramolecular attack can close a favourable six-membered ring. An aryloxy group directs attack to its ortho and para positions; steric crowding distinguishes the available ortho sites.

Step 1: Locate the electrophile and possible ring closures The side chain has the connectivity

$$Ar{-}O{-}CH_2{-}CH_2{-}CH(Cl){-}CH_3.$$

Activation of its carbon–chlorine bond by $AlCl_3$ creates an electrophilic centre at the carbon bearing chlorine. An ortho carbon of the same aromatic ring can attack this centre. The new ring contains the ether oxygen, two methylene carbons, the electrophilic carbon and two adjacent aromatic carbons: six atoms altogether. Loss of a proton restores aromaticity after formation of the new carbon–carbon bond.

Step 2: Choose the less crowded aromatic position The methyl group of the starting aromatic ring is meta to the aryloxy substituent. One possible ortho attack site lies between the oxygen-bearing carbon and the methyl-bearing carbon and is crowded by both substituents. The other ortho site is farther from the aromatic methyl group and is para to it. Ring closure at this less hindered, activated site gives the fused cyclic ether drawn in (C), 4,7-dimethylchroman. Closure at the crowded ortho site would give the regioisomer drawn in (D), which is disfavoured. Options (A) and (B) are elimination products: they retain an open chain and do not represent the favoured intramolecular electrophilic aromatic substitution under the stated Lewis-acid conditions.

Step 3: Conclusion The major product is the six-membered fused cyclic ether shown in option (C), formed by attack at the less hindered ortho position. Hence, the correct option is (C).

Quick Tip to Solve: Count the ring formed by intramolecular attack, then compare the two ortho sites: avoid closure at the site trapped between the aryloxy and methyl substituents.

77. The pH of a 0.02 M NH_4Cl solution will be :
[given K_b (NH_4OH) = 10^{-5} and log 2 = 0.301] [4 marks]

(A) 2.56

(B) 5.35

(C) 4.35

(D) 4.65

Correct Answer: (B) 5.35

Solution:

Concept:

- NH_4Cl is a salt of a weak base and a strong acid. Its NH_4^+ ion is a weak acid, with $K_a = K_w/K_b$; Cl^- does not appreciably hydrolyse.
- For a dilute weak acid of concentration C, $[H^+] \simeq \sqrt{K_a C}$ when its ionisation is small, and $pH = -\log_{10}[H^+]$.

Step 1: Calculate the acid strength The hydrolysis is

$$NH_4^+ + H_2O \rightleftharpoons NH_3 + H_3O^+.$$

Using $K_w = 10^{-14}$ at the usual temperature and the given base constant,

$$K_a = \frac{10^{-14}}{10^{-5}} = 10^{-9}.$$

Let the hydronium concentration produced be x. Neglecting the much smaller contribution from pure water gives $K_a = x^2/(0.02 - x)$. Since K_a is small, initially approximate $0.02 - x$ by 0.02.

Step 2: Find the concentration and take the logarithm The acid approximation gives

$$x = \sqrt{10^{-9} \times 0.02} = \sqrt{2 \times 10^{-11}} = 4.47 \times 10^{-6}\ \text{mol}\,\text{L}^{-1}.$$

Its fractional ionisation is $x/0.02 \simeq 2.24 \times 10^{-4}$, verifying that the neglected change in concentration is negligible. Using the supplied logarithm directly,

$$pH = \frac{1}{2}\big(9 - \log(0.02)\big) = \frac{1}{2}(9 + 1.699) = 5.3495.$$

Thus the rounded value is 5.35. The values 2.56, 4.35 and 4.65 do not follow from the given concentration and equilibrium constant; treating the salt as a fully dissociated strong acid would also be incorrect because the acidic species is weak ammonium.

Step 3: Conclusion The pH of the solution is approximately 5.35. Hence, the correct option is (B).

Quick Tip to Solve: For a weak-base/strong-acid salt, first replace K_b by $K_a = K_w/K_b$, then use the weak-acid square-root approximation.

78. The correct order of the first ionization enthalpies is : [4 marks]

(A) Ti $<$ Mn $<$ Zn $<$ Ni

(B) Zn $<$ Ni $<$ Mn $<$ Ti

(C) Mn $<$ Ti $<$ Zn $<$ Ni

(D) Ti $<$ Mn $<$ Ni $<$ Zn

Correct Answer: (D) Ti $<$ Mn $<$ Ni $<$ Zn

Solution:

Concept:

- First ionization enthalpy is the energy required to remove one electron from an isolated gaseous atom: $\mathrm{M(g)} \longrightarrow \mathrm{M^+(g) + e^-}$.
- Across the first transition series the effective nuclear attraction generally increases, but electronic configurations cause irregularities; a simple perfectly monotonic rule is insufficient.

Step 1: Compare the relevant measured trends Titanium has the lowest first ionization enthalpy of these four elements, about $659\,\mathrm{kJ\,mol^{-1}}$. Manganese is higher at about $717\,\mathrm{kJ\,mol^{-1}}$, and nickel is higher again at about $737\,\mathrm{kJ\,mol^{-1}}$. Thus their increasing sequence begins $\mathrm{Ti} < \mathrm{Mn} < \mathrm{Ni}$. These approximate values are useful for checking the close Mn–Ni comparison; the stable half-filled configuration of manganese does not reverse this particular order.

Step 2: Place zinc and test the alternatives Zinc has configuration $[\mathrm{Ar}]3d^{10}4s^2$ and a substantially higher first ionization enthalpy, about $906\,\mathrm{kJ\,mol^{-1}}$. Its filled subshell configuration and stronger nuclear attraction make removal of an electron comparatively difficult. Therefore zinc belongs at the high-energy end, after nickel. Option (A) wrongly puts zinc below nickel. Option (B) reverses the overall trend and makes titanium the highest. Option (C) wrongly places manganese below titanium and zinc below nickel. Only (D) has all four comparisons correct.

Step 3: Conclusion The increasing order is $\mathrm{Ti} < \mathrm{Mn} < \mathrm{Ni} < \mathrm{Zn}$. Hence, the correct option is (D).

Quick Tip to Solve: For these four atoms, remember the approximate sequence 659, 717, 737 and 906; do not use half-filled stability alone to interchange Mn and Ni.

79. The INCORRECT statement is : [4 marks]

(A) the color of $[CoCl(NH_3)_5]^{2+}$ is violet as it absorbs the yellow light.

(B) the gemstone, ruby, has Cr^{3+} ions occupying the octahedral sites of beryl.

(C) the spin-only magnetic moment of $[Ni(NH_3)_4(H_2O)_2]^{2+}$ is 2.83 BM.

(D) the spin-only magnetic moments of $[Fe(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$ are nearly similar.

Correct Answer: (B) the gemstone, ruby, has Cr^{3+} ions occupying the octahedral sites of beryl.

Solution:

Concept:

- A complex appears in the colour complementary to the light it absorbs. The host crystal as well as the dopant ion determines the identity of a gemstone.

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mathrm{BM}$, where n is the number of unpaired electrons.

Step 1: Check the gemstone and colour statements Ruby consists of corundum, Al_2O_3, in which a small proportion of Al^{3+} sites is occupied by Cr^{3+}. These sites are octahedral, but the host is not beryl. Chromium-doped beryl is associated with emerald. Thus statement (B) has the wrong host crystal and is incorrect. In (A), absorption in the yellow region leaves the complementary violet colour, so the stated colour relationship is consistent.

Step 2: Check both magnetic-moment statements In $[Ni(NH_3)_4(H_2O)_2]^{2+}$, nickel is Ni^{2+}, with a $3d^8$ configuration in an octahedral environment. It has two unpaired electrons, giving

$$\mu = \sqrt{2(2+2)} = \sqrt{8} \approx 2.83\,\mathrm{BM}.$$

Hence (C) is correct. Water is a weak-field ligand: octahedral Fe^{2+} is high-spin d^6, while Cr^{2+} is high-spin d^4. Both have four unpaired electrons and therefore the same spin-only value, $\sqrt{24} \approx 4.90\,\mathrm{BM}$. This confirms (D), leaving only (B) incorrect.

Step 3: Conclusion Ruby contains Cr^{3+} in corundum, not beryl. Hence, the correct option is (B).

Quick Tip to Solve: In gemstone questions check the host lattice separately from the dopant: ruby is chromium in corundum, whereas emerald is chromium-containing beryl.

80. The noble gas that does not occur in the atmosphere is : [4 marks]

(A) Ne

(B) He

(C) Kr

(D) Ra

Correct Answer: No printed option satisfies the statement: Ne, He and Kr occur in air, while Ra is not a noble gas. The supplied key selects D.

Solution:

Concept:

- Neon, helium and krypton are Group 18 elements and occur in the atmosphere in small amounts.
- Chemical symbols must be read literally: Ra is radium, a Group 2 metal; Rn is radon, a Group 18 gas. They are different elements.

Step 1: Eliminate the three atmospheric noble gases Neon is a minor constituent of air and can be obtained from liquefied air. Helium also occurs in the atmosphere, although only in a very small concentration and natural gas is its usual commercial source. Krypton is another trace constituent recovered during fractional processing of liquid air. Consequently none of (A), (B) and (C) answers the stated condition. A low atmospheric concentration does not mean that an element is entirely absent from the atmosphere.

Step 2: Examine the remaining printed symbol Option (D) is printed as Ra. Radium is an alkaline-

earth metal with atomic number 88, so it is not a noble gas and cannot literally satisfy the question. The supplied answer key selects (D), but that selection does not make radium a Group 18 element. One must not silently replace the printed symbol by Rn. Radon is radioactive and is conventionally distinguished from the stable noble gases in school discussions of atmospheric occurrence, but it can occur transiently in air through radioactive decay. This also cautions against treating an absolute claim of absence as a precise description.

Step 3: Conclusion The supplied key selects (D), but the printed Ra is radium, not a noble gas. Thus the literal question has no scientifically valid option; (D) is the keyed choice only.

Quick Tip to Solve: Check an element symbol before applying a periodic-group rule: Ra means radium, while Rn means radon. A one-letter difference changes the element completely.

81. Which of the following is not a correct method of the preparation of benzylamine from cyanobenzene ?

[4 marks]

(A) (i) $SnCl_2$ + HCl(gas) (ii) $NaBH_4$

(B) H_2/Ni

(C) (i) $LiAlH_4$ (ii) H_3O^+

(D) (i) HCl/H_2O (ii) $NaBH_4$

Correct Answer: (D) (i) HCl/H_2O (ii) $NaBH_4$

Solution:

Concept:

- Reduction of a nitrile $\mathrm{R{-}C{\equiv}N}$ to a primary amine retains the nitrile carbon: $\mathrm{R{-}C{\equiv}N} \longrightarrow \mathrm{R{-}CH_2NH_2}$.
- Hydrolysis instead converts a nitrile into a carboxylic acid, and $\mathrm{NaBH_4}$ does not normally reduce a carboxylic acid to a primary amine.

Step 1: Identify the two direct reduction methods Cyanobenzene is $\mathrm{C_6H_5CN}$, and benzylamine is $\mathrm{C_6H_5CH_2NH_2}$. Catalytic hydrogenation with $\mathrm{H_2/Ni}$ reduces the nitrile group to $\mathrm{-CH_2NH_2}$, so method (B) is suitable. Likewise $\mathrm{LiAlH_4}$, followed by aqueous acidic work-up, is a standard conversion of a nitrile into a primary amine. Therefore (C) also produces the required benzylamine without losing the nitrile carbon.

Step 2: Compare the two sequential reagent sets In method (A), $\mathrm{SnCl_2}$ and dry HCl partially reduce the nitrile to an imine salt. Reduction of that imine functionality by $\mathrm{NaBH_4}$ gives the amine; hydrolysis to an aldehyde is not the stated intervening step. Thus (A) is a possible route. By contrast, aqueous HCl in (D) hydrolyses cyanobenzene to benzoic acid through the amide stage:

$$\mathrm{C_6H_5CN} \xrightarrow{\mathrm{HCl/H_2O}} \mathrm{C_6H_5COOH}.$$

Subsequent $\mathrm{NaBH_4}$ does not restore a nitrogen atom or convert this acid into benzylamine. The nitrogen required in the product has already been removed from the organic molecule during

hydrolysis.

Step 3: Conclusion Aqueous acid hydrolysis followed by $NaBH_4$ does not convert cyanobenzene into benzylamine. Hence, the correct option is (D).

Quick Tip to Solve: Track the nitrile nitrogen: reduction retains it in the amine, whereas complete hydrolysis removes it from the organic product, so a later hydride cannot regenerate it.

82. Number of stereo centers present in linear and cyclic structures of glucose are respectively :

[4 marks]

(A) 4 & 4

(B) 4 & 5

(C) 5 & 4

(D) 5 & 5

Correct Answer: (B) 4 & 5

Solution:

Concept:

- A tetrahedral carbon is a stereocentre when it is attached to four different groups. A trigonal carbonyl carbon is not a tetrahedral stereocentre.
- Cyclization of glucose forms a hemiacetal and converts its aldehyde carbon into the anomeric carbon, creating one additional stereocentre.

Step 1: Count the centres in open-chain glucose The open-chain formula is $CHO-(CHOH)_4-CH_2OH$. Carbon atoms C2, C3, C4 and C5 each carry hydrogen, hydroxyl and two different carbon chains, so all four are stereocentres. The carbonyl carbon C1 is planar and does not meet the tetrahedral criterion. Carbon C6 is CH_2OH, containing two identical hydrogen atoms, so it is not a stereocentre either. The linear structure therefore contains exactly four stereocentres, not five.

Step 2: Count the extra centre produced by ring closure In cyclic glucose, the hydroxyl group at C5 adds intramolecularly to the aldehyde at C1. The carbonyl carbon becomes tetrahedral and now has four different substituents: hydrogen, hydroxyl, the ring oxygen and the C2 side of the ring. It is consequently a new stereocentre, called the anomeric centre. The existing centres C2–C5 remain, and C6 still has two identical hydrogen atoms. Thus the cyclic structure has five stereocentres. The alpha and beta forms differ in configuration at C1, but each form has the same count of five. This excludes (A), which misses the added centre, and (C) and (D), which overcount the linear form.

Step 3: Conclusion Linear and cyclic glucose contain four and five stereocentres, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Count the four CHOH carbons in open-chain glucose, then add the anomeric carbon upon cyclization; never count the terminal CH2OH carbon.

83. The correct option among the following is : [4 marks]

(A) Addition of alum to water makes it unfit for drinking.

(B) Colloidal medicines are more effective because they have small surface area.

(C) Colloidal paraticles in lyophobic sols can be precipitated by electrophoresis.

(D) Brownian motion in colloidal solution is faster if the viscosity of the solution is very high.

Correct Answer: (C) Colloidal paraticles in lyophobic sols can be precipitated by electrophoresis.

Solution:

Concept:

- Lyophobic colloidal particles are stabilized by electrical charge. Electrophoresis is their migration under an applied electric field toward the oppositely charged electrode.
- Large surface area favours the effectiveness of colloidal materials, while greater viscosity resists particle motion and slows Brownian diffusion.

Step 1: Test the electrophoresis statement A charged colloidal particle moves toward the oppositely charged electrode during electrophoresis. On reaching the electrode it can lose its charge, removing the electrostatic repulsion that kept the particles apart. The discharged particles can aggregate and precipitate. Hence the statement that particles in a lyophobic sol can be precipitated by electrophoresis, option (C), is correct. Migration itself and final coagulation are distinct stages of the same process; the charge neutralization explains why precipitation can occur.

Step 2: Explain why the remaining statements fail Alum is used in water treatment to help coagulate suspended impurities, allowing them to settle or be filtered. Its controlled use does not make drinking water unfit, so the general claim in (A) is false. Colloidal medicines have a large accessible surface area and can be readily assimilated; the phrase small surface area in (B) gives the opposite reason. Finally, increasing viscosity increases the resistance to particle motion. At fixed temperature and particle size, the diffusion coefficient varies as $D = k_{\mathrm{B}}T/(6\pi\eta r)$, so a larger viscosity η decreases diffusion rather than making Brownian movement faster. This rejects (D).

Step 3: Conclusion Lyophobic colloidal particles may discharge and precipitate during electrophoresis. Hence, the correct option is (C).

Quick Tip to Solve: For colloid statements check the direction of the effect: large surface area aids action, high viscosity slows motion, and loss of charge promotes coagulation.

84. The correct statements among (a) to (d) are :

(a) saline hydrides produce H_2 gas when reacted with H_2O.

(b) reaction of $LiAlH_4$ with BF_3 leads to B_2H_6.

(c) PH_3 and CH_4 are electron - rich and electron - precise hydrides, respectively.

(d) HF and CH_4 are called as molecular [4 marks]

(A) (a), (b), (c) and (d)

(B) (a), (c) and (d) only

(C) (c) and (d) only

(D) (a), (b) and (c) only

Correct Answer: (A) (a), (b), (c) and (d)

Solution:

Concept:

- Saline hydrides contain hydride ions, H^-, which react with water to form hydrogen gas. Covalent hydrides can be classified by their electron count.
- Electron-rich hydrides have lone pairs on the central atom, whereas electron-precise hydrides use the available valence electrons in ordinary covalent bonds.

Step 1: Check the two chemical reactions Statement (a) is correct: an ionic hydride such as sodium hydride reacts with water according to

$$\mathrm{NaH} + \mathrm{H_2O} \longrightarrow \mathrm{NaOH} + \mathrm{H_2}.$$

Hydride accepts a proton from water, producing hydrogen. Statement (b) is also correct: lithium aluminium hydride reduces boron trifluoride to diborane. A balanced representation is

$$3\,\mathrm{LiAlH_4} + 4\,\mathrm{BF_3} \longrightarrow 2\,\mathrm{B_2H_6} + 3\,\mathrm{LiF} + 3\,\mathrm{AlF_3}.$$

The boron and hydrogen atoms, as well as the metal fluorides, balance on the two sides, confirming that diborane is the product.

Step 2: Classify the molecular hydrides In PH_3, phosphorus forms three covalent bonds and retains a lone pair; it is therefore electron-rich. Carbon in CH_4 forms four bonds with no lone pair or electron deficiency, making methane electron-precise. Thus (c) is correct. Both HF and CH_4 are covalent compounds consisting of discrete molecules, so they belong to molecular hydrides, confirming the classification in (d). Hydrogen bonding between HF molecules does not change that molecular classification. All four statements are consequently accepted; (B) incorrectly omits (b), (C) omits both reactions, and (D) incorrectly omits the molecular-hydride classification.

Step 3: Conclusion All four statements are correct. Hence, the correct option is (A).

Quick Tip to Solve: Use three separate checks for hydrides: reaction of H- with water, lone pairs for electron-rich species, and ordinary covalent bonding for molecular hydrides.

85. Air pollution that occurs in sunlight is : [4 marks]

(A) oxidising smog

(B) reducing smog

(C) fog

(D) acid rain

Correct Answer: (A) oxidising smog

Solution:

Concept:

- Photochemical smog forms when sunlight initiates reactions of nitrogen oxides and hydrocarbons in polluted air. It is called oxidising smog because its characteristic products are oxidants.
- Photolysis of nitrogen dioxide produces oxygen atoms, which combine with molecular oxygen to produce ozone: $NO_2 \xrightarrow{h\nu} NO + O$, followed by $O + O_2 \longrightarrow O_3$.

Step 1: Identify the role of sunlight The wording singles out pollution that occurs in sunlight. This is the defining condition for photochemical smog, commonly associated with warm, dry and sunny conditions. Nitrogen oxides and unburnt hydrocarbons from vehicles act as precursors. Sunlight supplies the energy for the initiating photochemical reactions, rather than merely warming a pre-existing fog.

Step 2: Classify the resulting pollutants The products include ozone, nitrogen dioxide and peroxyacetyl nitrate (PAN). Their oxidising action gives this mixture the name oxidising smog. Ozone at ground level damages vegetation and irritates the respiratory tract. Thus the chemical character and the sunlight requirement both point to the same option.

Step 3: Eliminate the alternatives Reducing smog, the classical London type, is associated with smoke, sulphur dioxide and cool humid conditions; direct sunlight is not its defining requirement. Fog is a suspension of condensed water droplets rather than a sunlight-generated chemical pollutant. Acid rain results from acidic oxides and their atmospheric oxidation and dissolution; it is not the name of the characteristic sunlight-driven smog. These distinctions eliminate (B), (C) and (D).

Step 4: Conclusion The sunlight-dependent air pollution is photochemical, or oxidising, smog. Hence, the correct option is (A).

Quick Tip to Solve: Use the sunlight clue to identify photochemical smog, then recall its oxidants: ozone, nitrogen dioxide and PAN.

86. A hydrated solid X on heating initially gives a monohydrated compound Y. Y upon heating above 373 K leads to an anhydrous white powder Z. X and Z, respectively, are : [4 marks]

(A) Baking soda and dead burnt plaster.

(B) Washing soda and soda ash

(C) Washing soda and dead burnt plaster.

(D) Baking soda and soda as.

Correct Answer: (B) Washing soda and soda ash

Solution:

Concept:

- Washing soda is hydrated sodium carbonate, $Na_2CO_3 \cdot 10\,H_2O$. Soda ash is anhydrous sodium carbonate, Na_2CO_3.
- Heating a hydrated salt removes water of crystallisation stepwise. A monohydrate contains one water molecule per formula unit; an anhydrous salt contains none.

Step 1: Identify the initial hydrate The sequence requires an initial hydrated solid that can first lose some, but not all, of its water of crystallisation. Washing soda satisfies this condition: its decahydrate can lose nine water molecules to give sodium carbonate monohydrate. The sodium carbonate framework remains unchanged during this dehydration.

Step 2: Write the two dehydration stages The first stage is represented by

$$Na_2CO_3 \cdot 10\,H_2O \xrightarrow{\Delta} Na_2CO_3 \cdot H_2O + 9\,H_2O.$$

Thus $X = Na_2CO_3 \cdot 10\,H_2O$ and $Y = Na_2CO_3 \cdot H_2O$. On further heating above the stated temperature, the remaining water is expelled:

$$Na_2CO_3 \cdot H_2O \xrightarrow{\Delta} Na_2CO_3 + H_2O.$$

The white anhydrous powder Z is therefore soda ash.

Step 3: Reject compounds from different sequences Baking soda is sodium hydrogen carbonate, $NaHCO_3$, and is not the initial hydrated salt described here. Its thermal decomposition releases carbon dioxide as well as water. Dead burnt plaster is anhydrous calcium sulphate, $CaSO_4$, belonging to the gypsum dehydration sequence, not the sodium carbonate sequence. Hence options containing baking soda or dead burnt plaster do not fit both stages, even if one product sounds familiar.

Step 4: Conclusion X is washing soda and Z is soda ash. Hence, the correct option is (B).

Quick Tip to Solve: Follow the salt framework through dehydration: $Na_2CO_3 \cdot 10\,H_2O \longrightarrow Na_2CO_3 \cdot H_2O \longrightarrow Na_2CO_3$; only the water content changes.

87. The number of pentagons in C_{60} and trigons (triangles) in white phosphorus, respectively, are :

[4 marks]

(A) 12 and 3

(B) 20 and 3

(C) 20 and 4

(D) 12 and 4

Correct Answer: (D) 12 and 4

Solution:

Concept:

- Buckminsterfullerene, C_{60}, has a closed cage of 12 pentagons and 20 hexagons, with three carbon-carbon connections at every vertex.
- White phosphorus consists of discrete tetrahedral P_4 molecules. A tetrahedron has four triangular faces, six edges and four vertices.

Step 1: Count the pentagonal faces of the fullerene The football-like C_{60} cage contains pentagonal and hexagonal rings. It is important to distinguish their numbers: 12 rings are pentagons, whereas 20 are hexagons. One can verify the pentagon count from the cage topology. With 60 vertices each of degree three, the number of edges is $E = 3(60)/2 = 90$. Euler's formula gives $F = 2 - 60 + 90 = 32$.

Step 2: Verify the ring count algebraically Let p and h denote the numbers of pentagonal and hexagonal faces. Then

$$p + h = 32, \qquad 5p + 6h = 2E = 180.$$

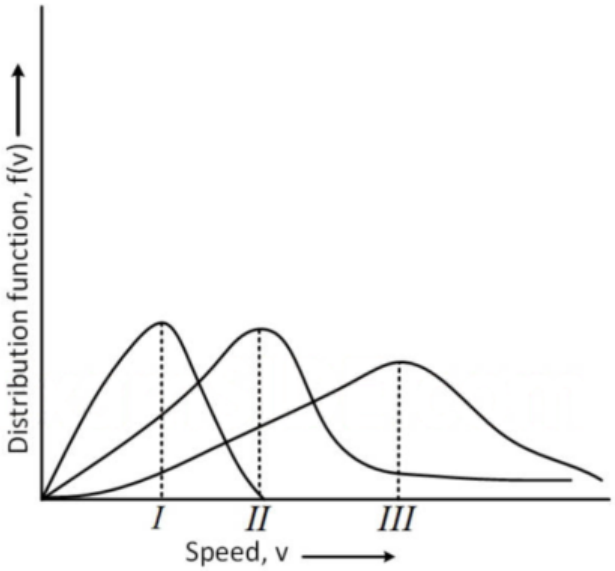
Each edge is counted twice in the second equation because it borders two faces. Subtracting five times the first equation from the second gives $h = 20$, and hence $p = 12$. This independently confirms which of the two familiar fullerene counts is requested.

Step 3: Count the triangular faces of phosphorus The four atoms of a white-phosphorus molecule occupy the corners of a tetrahedron. Every choice of three corners forms one face, giving $\binom{4}{3} = 4$ triangular faces. Three is the number of neighbours of each atom, not the number of faces. Therefore the pair is 12 and 4. Options (B) and (C) confuse hexagons with pentagons, while (A) gives the wrong tetrahedral face count.

Step 4: Conclusion C_{60} has 12 pentagons and tetrahedral white phosphorus has 4 triangular faces. Hence, the correct option is (D).

Quick Tip to Solve: Keep the paired structural facts separate: fullerene has 12 pentagons and 20 hexagons; a tetrahedron has four triangular faces.

88. Points I, II and III in the following plot respectively correspond to (V_{mp} : most probable velocity) [4 marks]



(A) V_{mp} of O_2 (400 K); V_{mp} of N_2 (300 K); V_{mp} of H_2 (300 K)

(B) V_{mp} of N_2 (300 K); V_{mp} of H_2 (300 K); V_{mp} of O_2 (400 K)

(C) V_{mp} of H_2 (300 K); V_{mp} of N_2 (300 K); V_{mp} of O_2 (400 K)

(D) V_{mp} of N_2 (300 K); V_{mp} of O_2 (400 K); V_{mp} of H_2 (300 K)

Correct Answer: (D) V_{mp} of N_2 (300 K); V_{mp} of O_2 (400 K); V_{mp} of H_2 (300 K)

Solution:

Concept:

- For an ideal gas, the most probable speed is $v_{\text{mp}} = \sqrt{2RT/M}$, where T is the absolute temperature and M is the molar mass in consistent units.
- The peak of a Maxwell speed-distribution curve occurs at its most probable speed. Consequently, the left-to-right order of the peaks is the increasing order of T/M.

Step 1: Compare temperature-to-molar-mass ratios Use molar masses $M(\text{N}_2) = 28$, $M(\text{O}_2) = 32$ and $M(\text{H}_2) = 2$, all in the same units. The common positive factor $2R$ need not be evaluated for ordering, since the square-root function is increasing. The relevant ratios are

$$\frac{300}{28} \approx 10.71, \qquad \frac{400}{32} = 12.50, \qquad \frac{300}{2} = 150.$$

Thus nitrogen at 300 K has the smallest most probable speed, oxygen at 400 K the next, and hydrogen at 300 K by far the largest.

Step 2: Match the ordered peaks to the graph In the plot, I is the leftmost peak, II is the middle peak, and III is the rightmost peak. Increasing horizontal position means increasing speed. Therefore I represents N_2 at 300 K, II represents O_2 at 400 K, and III represents H_2 at 300 K. The very small molar mass of hydrogen outweighs the fact that its temperature is lower than that of oxygen.

Step 3: Check the other proposed assignments Option (A) reverses the order of nitrogen and oxygen:

their temperatures differ, so ranking molar masses alone is insufficient. Option (B) puts hydrogen in the middle even though its T/M is much larger. Option (C) puts hydrogen first, contradicting its large most probable speed. Only (D) preserves the calculated increasing order of all three peaks.

Step 4: Conclusion I, II and III correspond respectively to v_{mp} of N_2 (300 K), O_2 (400 K) and H_2 (300 K). Hence, the correct option is (D).

Quick Tip to Solve: Rank Maxwell-distribution peaks by T/M, then take the same order for speed; do not compare molar masses alone when temperatures differ.

89. The correct match between Item-I and Item-II is :

	Item - I		Item - II
(a)	High density polythene	(I)	Peroxide catalyst
(b)	Polyacrylonitrile	(II)	Condensation at high temperature & pressure
(c)	Novolac	(III)	Ziegler-Natta Catalyst
(d)	Nylon 6	(IV)	Acid or base catalyst

[4 marks]

(A) (a) → (IV), (b) → (II), (c) → (I), (d) → (III)

(B) (a) → (III), (b) → (I), (c) → (IV), (d) → (II)

(C) (a) → (II), (b) → (IV), (c) → (I), (d) → (III)

(D) (a) → (III), (b) → (I), (c) → (II), (d) → (IV)

Correct Answer: (B) (a) → (III), (b) → (I), (c) → (IV), (d) → (II)

Solution:

Concept:

- Addition polymerisation joins unsaturated monomers without eliminating a small molecule. Coordination catalysts and radical initiators select different polymerisation pathways.
- Condensation and ring-opening polymerisations have characteristic reagents and conditions. Match each named polymer to its preparation rather than treating all polymerisations as identical.

Step 1: Match high density polythene and polyacrylonitrile High density polythene is prepared by coordination polymerisation of ethene using a Ziegler–Natta catalyst, typically a titanium compound with an organoaluminium compound. This gives the match (a) to (III). Polyacrylonitrile is formed by addition polymerisation of acrylonitrile, $CH_2{=}CHCN$, using a peroxide catalyst as the radical source. Thus (b) matches (I).

Step 2: Match the condensation and ring-opening products Novolac is a phenol-formaldehyde condensation polymer; acid or base catalysis is the listed preparation category for this resin system. Hence (c) matches (IV). Nylon 6 is obtained from caprolactam by polymerisation under high-temperature conditions with water initiating ring opening; the listed high-temperature and pressure condensation condition is its corresponding entry (II). Thus (d) matches (II).

Step 3: Compare the complete mapping with the options The four assignments together are

$$(a) \to (III), \quad (b) \to (I), \quad (c) \to (IV), \quad (d) \to (II).$$

Option (A) already mismatches high density polythene with acid/base catalysis, and (C) mismatches it with condensation. Option (D) gets the first two entries right but interchanges the conditions for Novolac and nylon 6. It is necessary to test the entire mapping rather than accept an option after checking only its first entry.

Step 4: Conclusion The correct sequence of entries is III, I, IV and II for the four polymers in the stated order. Hence, the correct option is (B).

Quick Tip to Solve: First anchor HDPE to Ziegler–Natta and PAN to peroxide; then distinguish phenol-formaldehyde resin from the caprolactam route to nylon 6.

90. The correct statement is : [4 marks]

(A) zincite is a carbonate ore.

(B) Sodium cyanide cannot be used in the metallurgy of silver.

(C) aniline is a froth stabilizer.

(D) zone refining process is used for the refining of titanium.

Correct Answer: (C) aniline is a froth stabilizer.

Solution:

Concept:

- In froth flotation, collectors promote attachment of sulphide ore particles to air bubbles, while froth stabilisers help maintain the froth. Aniline is a froth stabiliser.
- Ore composition and refining method are separate facts: zincite is ZnO, cyanide leaching is used for silver, and zone refining depends on different impurity solubilities in solid and molten phases.

Step 1: Check the ore and extraction statements Zincite is zinc oxide, ZnO, not a carbonate. The familiar carbonate ore of zinc is calamine, $\mathrm{ZnCO_3}$, so (A) is false. Sodium cyanide is used in the cyanide process for silver extraction because silver forms a soluble dicyanoargentate complex, $[\mathrm{Ag(CN)_2}]^-$. Silver can subsequently be recovered by displacement with zinc. Therefore the claim that sodium cyanide cannot be used is false, eliminating (B).

Step 2: Identify the flotation additive In froth flotation, air bubbles carry suitably wetted sulphide mineral particles into a froth that can be skimmed off. Froth must persist long enough for separation, and stabilising agents help prevent it from collapsing too quickly. Aniline and cresols are examples of froth stabilisers. This role differs from that of collectors, such as xanthates, which alter the surface affinity of the mineral particles. Statement (C) correctly names the function of aniline.

Step 3: Check the refining claim Zone refining is chiefly used to obtain very pure semiconductors such as silicon and germanium. Titanium is associated with the van Arkel iodide refining process, in which volatile titanium iodide is formed and then decomposed at a hot filament to deposit pure metal. Thus (D) assigns titanium the wrong standard refining process. After checking all four statements,

only the statement about aniline remains correct.

Step 4: Conclusion Aniline is a froth stabiliser used in froth flotation. Hence, the correct option is (C).

Quick Tip to Solve: Separate three lists in metallurgy: ore formulae, flotation-additive roles, and refining methods; aniline belongs with froth stabilisers.