JEE Main 2019 Paper 1 (B.E./B.Tech) Apr 10 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 10 April 2019, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. If a directrix of a hyperbola centred at the origin and passing through the point (4, $-2\sqrt{3}$) is $5x = 4\sqrt{5}$ and its eccentricity is e, then : [4 marks]

(A) $4e^4 - 24e^2 + 27 = 0$

(B) $4e^4 - 24e^2 + 35 = 0$

(C) $4e^4 - 12e^2 - 27 = 0$

(D) $4e^4 + 8e^2 - 35 = 0$

Correct Answer: (B) $4e^4 - 24e^2 + 35 = 0$

Solution:

Concept:

- A hyperbola centred at the origin with a vertical directrix has standard form $x^2/a^2 - y^2/b^2 = 1$. Its transverse axis is the x-axis, and its eccentricity satisfies $e^2 = 1 + b^2/a^2 > 1$.
- The directrices are $x = \pm a/e$. Thus the stated directrix determines a in terms of e; substituting the given point then eliminates the remaining axis parameter.

Step 1: Use the directrix The equation $5x = 4\sqrt{5}$ means $x = 4/\sqrt{5}$. Comparing with $x = a/e$, and taking the positive semi-axis length, gives

$$a = \frac{4e}{\sqrt{5}}, \qquad a^2 = \frac{16e^2}{5}, \qquad b^2 = \frac{16e^2(e^2-1)}{5}.$$

The relation for b^2 follows directly from $b^2 = a^2(e^2 - 1)$; we must retain this factor rather than replace b by a.

Step 2: Substitute the point and simplify The coordinates give $x^2 = 16$ and $y^2 = 12$, so

$$\frac{16}{16e^2/5} - \frac{12}{16e^2(e^2-1)/5} = 1.$$

Multiply by $4e^2(e^2 - 1)$, which is nonzero because $e > 1$:

$$20(e^2 - 1) - 15 = 4e^2(e^2 - 1).$$

Consequently $4e^4 - 24e^2 + 35 = 0$. For an independent check, writing $u = e^2$ factors this as $(2u - 5)(2u - 7) = 0$. Both possible values exceed one and are consistent with a hyperbola; the problem asks for the relation, not a unique eccentricity. Options A, C and D have different coefficients and do not give the equation obtained by substituting the point.

Step 3: Conclusion The eccentricity satisfies $4e^4 - 24e^2 + 35 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: Convert a directrix equation to $x = a/e$ first; use $b^2 = a^2(e^2 - 1)$ before substituting a point, so only e^2 remains.

2. If $\lim_{x\to1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, then k is : [4 marks]

(A) $\frac{3}{2}$

(B) $\frac{8}{3}$

(C) $\frac{4}{3}$

(D) $\frac{3}{8}$

Correct Answer: (B) $\frac{8}{3}$

Solution:

Concept:

- A difference of powers can be factored before evaluating a limit. The limit $\lim_{x\to a}(x^n-a^n)/(x-a)=na^{n-1}$ follows from that factorisation.
- When cancelling $x-k$, the simplified expression agrees with the original for points near the limit, although the original fraction is undefined at the limiting point itself.

Step 1: Evaluate the left-hand limit Factor $x^4-1=(x-1)(x^3+x^2+x+1)$. For $x\neq1$, cancel the common factor and then substitute:

$$\lim_{x\to1}\frac{x^4-1}{x-1}=1+1+1+1=4.$$

Directly substituting into the original fraction would give $0/0$, which is an indeterminate form rather than the answer.

Step 2: Evaluate the right-hand limit and equate For $k\neq0$, use $x^3-k^3=(x-k)(x^2+kx+k^2)$ and $x^2-k^2=(x-k)(x+k)$. Therefore

$$\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}=\frac{3k^2}{2k}=\frac{3k}{2}.$$

Equating the two limits yields $4=3k/2$, hence $k=8/3$. The exceptional case $k=0$ gives the fraction x for nonzero x, whose limit is zero, so it cannot solve the equation. Checking the remaining listed values, $k=3/2,4/3,3/8$ would make the right-hand limit $9/4,2,9/16$, respectively. None equals four, while $8/3$ does.

Step 3: Conclusion $k=8/3$. Hence, the correct option is (B).

Quick Tip to Solve: Cancel the factor $x-k$, then substitute $x=k$. Keep a separate check for $k=0$ before dividing by k.

3. If α and β are the roots of the quadratic equation,
$x^2 + x \sin\theta - 2\sin\theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then
$\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12}).(\alpha-\beta)^{24}}$ is equal to : [4 marks]

(A) $\frac{2^{12}}{(\sin\theta-8)^6}$

(B) $\frac{2^6}{(\sin\theta+4)^{12}}$

(C) $\frac{2^{12}}{(\sin\theta+8)^{12}}$

(D) $\frac{2^{12}}{(\sin\theta-4)^{12}}$

Correct Answer: (C) $\frac{2^{12}}{(\sin\theta+8)^{12}}$

Solution:

Concept:

- For $x^2 + sx - 2s = 0$, the sum and product of roots are $\alpha + \beta = -s$ and $\alpha\beta = -2s$, where $s = \sin\theta$.
- The identity $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ gives the squared root difference without solving explicitly for either root.

Step 1: Cancel the reciprocal-power sum Here $0 < s < 1$, so the product $-2s$ is nonzero and the roots are real and distinct, since their discriminant is $s^2 + 8s > 0$. Their even powers have a strictly positive sum. Thus

$$\alpha^{-12} + \beta^{-12} = \frac{\alpha^{12} + \beta^{12}}{(\alpha\beta)^{12}}.$$

The numerator sum cancels from the requested ratio, leaving

$$\frac{(\alpha\beta)^{12}}{(\alpha - \beta)^{24}}.$$

This cancellation avoids computing twelfth powers of the individual roots.

Step 2: Express the ratio through the coefficients Using the sum and product,

$$(\alpha - \beta)^2 = s^2 + 8s = s(s + 8).$$

Because the exponent is even, no sign choice for $\alpha - \beta$ is needed. The ratio becomes

$$\frac{(-2s)^{12}}{\{s(s + 8)\}^{12}} = \frac{2^{12}}{(s + 8)^{12}}.$$

The factor s^{12} cancels legally since $s > 0$. Restoring $s = \sin\theta$ gives option C. Option A uses both the wrong sign and exponent in the denominator; B uses the wrong coefficient and shift; D incorrectly replaces the discriminant factor $s + 8$ by $s - 4$. The quantity derived remains positive throughout the stated interval, as an even-power ratio must.

Step 3: Conclusion The expression equals $\frac{2^{12}}{(\sin\theta + 8)^{12}}$. Hence, the correct option is (C).

Quick Tip to Solve: For reciprocal-power sums, rewrite the negative powers over $(\alpha\beta)^n$ before using the sum and product of roots.

4. If the length of the perpendicular from the point (β, 0, β) ($\beta \neq 0$) to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to : [4 marks]

(A) 2

(B) 1

(C) -2

(D) -1

Correct Answer: (D) -1

Solution:

Concept:

- A symmetric line with a zero denominator means that coordinate is fixed. Here $y = 1$, and a parametric point on the line is $Q(t) = (t, 1, -1 - t)$.
- The foot of a perpendicular minimises the squared distance to the line. Equivalently, its displacement from the given point is orthogonal to the line direction $(1, 0, -1)$.

Step 1: Locate the foot of the perpendicular Let $P = (\beta, 0, \beta)$. The displacement from $Q(t)$ to P is $(\beta - t, -1, \beta + 1 + t)$. Orthogonality to $(1, 0, -1)$ requires

$$(\beta - t) - (\beta + 1 + t) = 0,$$

so $t = -1/2$. The foot is consequently $Q = (-1/2, 1, -1/2)$. Notice that it is independent of β: the two equal coordinates of P cancel when taking the dot product with this line direction.

Step 2: Use the given perpendicular length The squared distance from P to this foot is

$$PQ^2 = (\beta + 1/2)^2 + 1 + (\beta + 1/2)^2 = 2\beta^2 + 2\beta + \frac{3}{2}.$$

The given distance is $\sqrt{3/2}$, so equating squares gives $2\beta^2 + 2\beta = 0$, or $\beta(\beta + 1) = 0$. The question excludes $\beta = 0$, leaving $\beta = -1$. A direct check gives $P - Q = (-1/2, -1, -1/2)$, whose squared magnitude is $3/2$ and whose dot product with the direction is zero. The listed values $2, 1, -2$ produce squared distances $27/2, 11/2, 11/2$, respectively, rather than $3/2$.

Step 3: Conclusion The allowed value is $\beta = -1$. Hence, the correct option is (D).

Quick Tip to Solve: Replace a zero denominator by a fixed coordinate. Parametrise the remaining coordinates and impose a zero dot product to find the perpendicular foot.

5. If a_1, a_2, a_3, ... a_n are in A.P. and $a_1 + a_4 + a_7 + ... + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to : [4 marks]

(A) 38

(B) 98

(C) 76

(D) 64

Correct Answer: (C) 76

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, $a_r = a + (r-1)d$. Equidistant terms have a constant sum: $a_r + a_{17-r} = a_1 + a_{16}$.
- Any evenly spaced selection of terms is itself an arithmetic progression. Its sum equals the number of selected terms times the average of its first and last terms.

Step 1: Find the common endpoint sum The selected indices $1, 4, 7, 10, 13, 16$ give six terms, not sixteen. Pairing terms from opposite ends,

$$a_1 + a_{16} = a_4 + a_{13} = a_7 + a_{10}.$$

Thus the stated sum is $3(a_1 + a_{16}) = 114$, which gives $a_1 + a_{16} = 38$. In terms of a and d, this same calculation is $6a + 45d = 114$, or $2a + 15d = 38$. It does not determine a and d separately, and no separate determination is necessary.

Step 2: Evaluate the second selection The required indices $1, 6, 11, 16$ form a four-term arithmetic progression. Both endpoint pairs have the sum just found:

$$a_1 + a_{16} = 38, \qquad a_6 + a_{11} = 2a + 15d = 38.$$

Therefore

$$a_1 + a_6 + a_{11} + a_{16} = 2 \times 38 = 76.$$

Equivalently, both selections have the same first and last terms and hence the same average, 19. The sums are in the ratio of their term counts, $4 : 6$, giving $(4/6)114 = 76$. Option A, 38, is just one pair rather than the sum of four terms; 98 and 64 do not preserve this ratio and cannot follow from the stated arithmetic progression.

Step 3: Conclusion The required sum is 76. Hence, the correct option is (C).

Quick Tip to Solve: When two evenly spaced selections share their first and last terms, their averages agree; compare their numbers of terms to compare the sums.

6. If for some $x \in \mathbb{R}$, the frequency distribution of the marks obtained by 20 students in a test is:

Marks	2	3	5	7
Frequency	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is [4 marks]

(A) 3.0

(B) 2.8

(C) 2.5

(D) 3.2

Correct Answer: (B) 2.8

Solution:

Concept:

- The total frequency equals the number of observations. Each frequency must be nonnegative, so an algebraic root that creates negative frequencies must be rejected.
- The arithmetic mean of a discrete frequency distribution is $\bar{m} = \sum f_i m_i / \sum f_i$, where m_i denotes the observed marks and f_i their frequencies.

Step 1: Use the total number of students Adding the four given frequencies and setting their sum equal to twenty gives

$$(x+1)^2 + (2x-5) + (x^2-3x) + x = 20.$$

On collecting terms this is $2x^2+2x-4=20$, or $x^2+x-12=0$. Factorisation yields $(x+4)(x-3)=0$, so $x=3$ or $x=-4$. The latter gives the frequency $x=-4$ for the last group and is inadmissible. With $x=3$, the frequencies corresponding to marks $2,3,5,7$ are $16,1,0,3$, which are nonnegative and add to twenty. A zero frequency is valid: it means no student scored five.

Step 2: Compute the weighted mean Multiply each score by its frequency before adding:

$$\sum f_i m_i = 2(16) + 3(1) + 5(0) + 7(3) = 32 + 3 + 0 + 21 = 56.$$

Hence $\bar{m} = 56/20 = 2.8$. The zero-frequency score contributes nothing to the numerator. Taking an unweighted average of the four scores would ignore how many students obtained each score and is incorrect. As a reverse check, the proposed means $3.0, 2.5, 3.2$ would require weighted totals $60, 50, 64$, respectively; none agrees with the verified total of 56. Only option B gives that total.

Step 3: Conclusion The mean of the marks is 2.8. Hence, the correct option is (B).

Quick Tip to Solve: First solve the total-frequency equation and reject negative frequencies; then average scores with their frequencies as weights, including zero-frequency classes correctly.

7. If $a > 0$ and $z = \dfrac{(1+i)^2}{a-i}$ has magnitude $\sqrt{\dfrac{2}{5}}$, then z is equal to: [4 marks]

(A) $-\frac{1}{5} + \frac{3}{5}i$

(B) $-\frac{1}{5} - \frac{3}{5}i$

(C) $\frac{1}{5} - \frac{3}{5}i$

(D) $-\frac{3}{5} - \frac{1}{5}i$

Correct Answer: (A) $-\frac{1}{5} + \frac{3}{5}i$

Solution:

Concept:

- For complex numbers, $|z_1/z_2| = |z_1|/|z_2|$, provided the denominator is nonzero. Here $|(1+i)^2| = |1+i|^2 = 2$.
- Multiplying numerator and denominator by the conjugate of the denominator gives a real denominator. The result can then be compared directly with the four printed complex numbers.

Step 1: Determine the positive real parameter The magnitude condition becomes

$$\frac{2}{\sqrt{a^2+1}} = \sqrt{\frac{2}{5}}.$$

Both sides are positive, so squaring gives $4/(a^2+1) = 2/5$. Thus $20 = 2(a^2+1)$, yielding $a^2 = 9$. The question states $a > 0$, which selects $a = 3$. The negative root is not permitted and would change the sign of the imaginary part of z. This sign restriction must be applied before selecting an option.

Step 2: Rationalise and compare the options Since $(1+i)^2 = 2i$, substitute $a = 3$ and multiply numerator and denominator by $3+i$:

$$z = \frac{2i}{3-i} = \frac{2i(3+i)}{(3-i)(3+i)} = \frac{-2+6i}{10} = -\frac{1}{5} + \frac{3}{5}i.$$

The requested number is therefore the value in option A. Its magnitude is $\sqrt{1/25 + 9/25} = \sqrt{2/5}$, consistent with the given condition. Option B is its conjugate and would give $a = -3$ if substituted into the original equation, contradicting $a > 0$. Option C also has the wrong real part, and D interchanges the real and imaginary magnitudes. These alternatives are excluded by rationalising the original expression, even though a magnitude check by itself need not distinguish every option. In particular, reversing the imaginary sign after the calculation would answer a different question.

Step 3: Conclusion The printed question asks for z, giving $z = -\dfrac{1}{5} + \dfrac{3}{5}i$, so the correct option is (A). The supplied key selects the second listed option, which is the conjugate instead; direct substitution into $z = 2i/(3-i)$ proves (A) for the printed demand.

Quick Tip to Solve: Use the modulus to find the positive parameter, rationalise the original expression, and check whether the question asks for the number itself or its conjugate.

8. If $f(x) = \begin{cases} \frac{\sin(p+1)x+\sin x}{x} & , x < 0 \\ q & , x = 0 \\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}} & , x > 0 \end{cases}$

is continuous at x = 0, then the ordered pair (p, q) is equal to [4 marks]

(A) $\left(-\frac{3}{2}, -\frac{1}{2}\right)$

(B) $\left(-\frac{1}{2}, \frac{3}{2}\right)$

(C) $\left(-\frac{3}{2}, \frac{1}{2}\right)$

(D) $\left(\frac{5}{2}, \frac{1}{2}\right)$

Correct Answer: (C) $\left(-\frac{3}{2}, \frac{1}{2}\right)$

Solution:

Concept:

- Continuity at zero requires the left-hand limit, the right-hand limit and the assigned value to be equal: $\lim_{x\to 0^-} f(x) = f(0) = \lim_{x\to 0^+} f(x)$. Here the assigned value is q.
- Use $\lim_{u\to 0} \sin u/u = 1$ on the negative branch. For the positive branch, factor out $\sqrt{x}$ and rationalise the resulting difference; this avoids substituting zero into an indeterminate quotient.

Step 1: Find the left-hand limit Separate the two sine terms and apply the standard sine limit. The parameter is a fixed real constant when the limit is taken, so

$$L = \lim_{x\to 0^-} \left(\frac{\sin((p+1)x)}{x} + \frac{\sin x}{x} \right) = (p+1) + 1 = p + 2.$$

This identity also holds when $p + 1 = 0$, since its sine term then vanishes identically.

Step 2: Simplify the right-hand limit For $x > 0$, the numerator is $\sqrt{x}(\sqrt{1+x} - 1)$, while $x^{3/2} = x\sqrt{x}$. Cancelling the positive factor and multiplying by the conjugate gives

$$R = \lim_{x\to 0^+} \frac{\sqrt{1+x} - 1}{x} = \lim_{x\to 0^+} \frac{1}{\sqrt{1+x} + 1} = \frac{1}{2}.$$

Thus continuity fixes $q = 1/2$ and $p + 2 = 1/2$, giving $p = -3/2$. Option A has the wrong value of q, option B has both limits mismatched, and option D has the wrong p.

Step 3: Conclusion The ordered pair is $\boxed{(p, q) = (-3/2, 1/2)}$. Hence, the correct option is (C).

Quick Tip to Solve: Obtain q from the branch with no unknown parameter first; then equate the other limit to it to find p.

9. The value of $\int_0^{2\pi} [\sin 2x\,(1+\cos 3x)]dx$**,**

where [t] denotes the greatest integer function is : [4 marks]

(A) 2π

(B) π

(C) -2π

(D) -π

Correct Answer: (D) -π

Solution:

Concept:

- For every noninteger real number t, the greatest integers satisfy $\lfloor t\rfloor + \lfloor -t\rfloor = -1$. For integer t, the sum is zero.
- A change of variable $x \mapsto 2\pi - x$ pairs points in the same integration interval. Changing an integrand at finitely many points does not change its definite integral.

Step 1: Pair the trigonometric arguments Write $F(x) = \sin 2x(1+\cos 3x)$ and $I = \int_0^{2\pi} \lfloor F(x)\rfloor\, dx$. Reflection about the midpoint changes the sine sign but preserves the cosine:

$$\sin(4\pi - 2x) = -\sin 2x, \qquad \cos(6\pi - 3x) = \cos 3x.$$

Consequently $F(2\pi - x) = -F(x)$, and substitution yields $I = \int_0^{2\pi} \lfloor -F(x)\rfloor\, dx$.

Step 2: Add the reflected integrals Since $|F(x)| \le 2$, only finitely many integer levels are possible. For each level, the nonconstant trigonometric polynomial $F(x) - k$ has finitely many zeros on this interval. Thus the integer-valued exceptions have no effect on integration. Everywhere else the two floors add to -1, so

$$2I = \int_0^{2\pi} \left(\lfloor F(x)\rfloor + \lfloor -F(x)\rfloor\right) dx = \int_0^{2\pi} (-1)\, dx = -2\pi.$$

Hence $I = -\pi$. The positive options have the wrong sign; -2π is the sum $2I$, not the integral itself.

Step 3: Conclusion The required integral is $\boxed{-\pi}$. Hence, the correct option is (D).

Quick Tip to Solve: For a floor-function integral, first test whether a reflection changes the inner expression to its negative; the paired floors then simplify immediately.

10. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and

$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$;

then for all $\theta \in \left(0, \frac{\pi}{2}\right)$ **:** [4 marks]

(A) $\Delta_1 - \Delta_2$ = x (cos 2θ – cos 4θ)

(B) $\Delta_1 + \Delta_2$ = - $2x^3$

(C) $\Delta_1 + \Delta_2$ = – 2(x^3 + x –1)

(D) $\Delta_1 - \Delta_2$ = - $2x^3$

Correct Answer: (B) $\Delta_1 + \Delta_2$ = - $2x^3$

Solution:

Concept:

- Expand a third-order determinant along its first row, using the cofactor signs $+, -, +$. A minus sign in a matrix entry must be retained inside the corresponding minor.
- The identity $\sin^2\alpha + \cos^2\alpha = 1$ can eliminate angle dependence after expansion. Evaluate the determinant for a general angle α once, then substitute the two given angles.

Step 1: Expand for a general angle Put $s = \sin\alpha$, $c = \cos\alpha$. The common determinant has first row (x, s, c). Its expansion is

$$\begin{aligned} D(\alpha) &= x(-x^2 - 1) - s(-sx - c) + c(-s + xc) \\ &= -x^3 - x + xs^2 + sc - sc + xc^2. \end{aligned}$$

The mixed terms cancel because they have opposite signs. In particular, the second cofactor contributes $+sc$, and the third contributes $-sc$; overlooking this cancellation introduces an incorrect angular term.

Step 2: Compare the two determinants Using $s^2 + c^2 = 1$ in the remaining terms gives

$$D(\alpha) = -x^3 - x + x(s^2 + c^2) = -x^3.$$

This expression is independent of the angle, so $\Delta_1 = \Delta_2 = -x^3$. Therefore their sum is $-2x^3$, while their difference is zero. Option D cannot give that difference because $x \neq 0$. Option A also fails as an identity: at $\theta = \pi/4$, its proposed right side is x, which is nonzero. Option C adds an unwarranted term $-2x + 2$, so it is not valid for general nonzero x.

Step 3: Conclusion For every allowed angle, $\boxed{\Delta_1 + \Delta_2 = -2x^3}$. Hence, the correct option is (B).

Quick Tip to Solve: When two determinants differ only in their angle, expand one with a general angle and simplify before calculating a sum or difference.

11. Let f : R $\to$ R be differentiable at c $\in$ R and f(c) = 0. If g(x) = |f(x)| , then at x = c, g is : [4 marks]

(A) differentiable if f '(c) = 0

(B) differentiable if f '(c) $\neq$ 0

(C) not differentiable

(D) not differentiable if f '(c) = 0

Correct Answer: (A) differentiable if f '(c) = 0

Solution:

Concept:

- If $f(c) = 0$, the difference quotient of $g(x) = |f(x)|$ at c is $|f(c+h)|/h$, since $g(c) = 0$. The sign of the denominator matters.
- Differentiability of f at c means $f(c+h)/h \to f\prime(c)$. Taking absolute values preserves a finite limit, but the sign of h produces opposite one-sided derivatives for g.

Step 1: Calculate the right derivative For $h > 0$, dividing the absolute value by h is the same as taking the absolute value of the quotient. Thus

$$g\prime_+(c) = \lim_{h\to 0^+} \frac{|f(c+h)|}{h} = \lim_{h\to 0^+} \left|\frac{f(c+h) - f(c)}{h}\right| = |f\prime(c)|.$$

This calculation uses the given differentiability of f, rather than assuming that f keeps one sign near its zero.

Step 2: Calculate the left derivative and compare For $h < 0$, the negative denominator introduces a minus sign outside the absolute value:

$$g\prime_-(c) = -\lim_{h\to 0^-} \left|\frac{f(c+h) - f(c)}{h}\right| = -|f\prime(c)|.$$

The two derivatives agree exactly when $|f\prime(c)| = 0$. In that case both are zero and g is differentiable. If $f\prime(c) \neq 0$, they are opposite nonzero numbers, so g has a corner. Hence B reverses the condition; C wrongly denies the zero-derivative case; and D also reverses what happens when $f\prime(c) = 0$.

Step 3: Conclusion The absolute-value function is differentiable at the given zero exactly when the derivative of the original function vanishes there; its own derivative is then zero. Hence, the correct option is (A).

Quick Tip to Solve: At a zero inside an absolute value, compare the two difference quotients; a zero slope removes the corner, whereas a nonzero slope creates one.

12. The region represented by| x – y | $\leq$ 2 and | x + y| $\leq$ 2 is bounded by a : [4 marks]

(A) rhombus of area $8\sqrt{2}$ sq. units

(B) square of side length $2\sqrt{2}$ units

(C) square of area 16 sq. units

(D) rhombus of side length 2 units

Correct Answer: (B) square of side length $2\sqrt{2}$ units

Solution:

Concept:

- The inequality $|x - y| \leq 2$ is the strip between $x - y = 2$ and $x - y = -2$. Similarly, $|x + y| \leq 2$ is the strip between $x + y = 2$ and $x + y = -2$.
- Intersect boundary lines to obtain the vertices. The distance formula gives side lengths, while slopes or direction vectors determine whether adjacent sides meet at right angles.

Step 1: Find the vertices of the intersection Let $u = x - y$ and $v = x + y$. The four extreme pairs (u, v) are $(2, 2), (2, -2), (-2, -2), (-2, 2)$. Recover the original coordinates using

$$x = \frac{u + v}{2}, \qquad y = \frac{v - u}{2}.$$

These produce the consecutive vertices $(2, 0), (0, -2), (-2, 0), (0, 2)$. The origin satisfies both inequalities, confirming that the required region is their common interior, including its boundary.

Step 2: Identify the shape and its size The lines $x - y = \pm 2$ have slope 1, and the lines $x + y = \pm 2$ have slope -1. Their slope product is -1, so adjacent sides are perpendicular. The distance between consecutive vertices is

$$a = \sqrt{(2 - 0)^2 + (0 + 2)^2} = \sqrt{8} = 2\sqrt{2}.$$

All four sides have this length, giving a square of area $a^2 = 8$ square units. Thus A has the wrong area, C has the wrong area, and D has the wrong side length. Although a square is also a rhombus, its stated dimensions must still be correct.

Step 3: Conclusion The region is bounded by a $\boxed{\text{square of side } 2\sqrt{2} \text{ units}}$. Hence, the correct option is (B).

Quick Tip to Solve: For simultaneous bounds on $x - y$ and $x + y$, use their endpoint values to find four vertices; remember that the axes of the resulting square are rotated.

13. Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall\, x \in R$. Then the set of all $x \in R$, where the function $h(x) = (fog)(x)$ is increasing, is : [4 marks]

(A) $[0, \infty)$

(B) $\left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$

(C) $\left[-\frac{1}{2}, 0\right] \cup [1, \infty)$

(D) $\left[0, \frac{1}{2}\right] \cup [1, \infty)$

Correct Answer: (D) $\left[0, \frac{1}{2}\right] \cup [1, \infty)$

Solution:

Concept:

- The chain rule gives $(f \circ g)\prime(x) = f\prime(g(x))g\prime(x)$. Both factors must be included when checking where the composite increases.
- For real t, $e^t - 1$ is positive, zero or negative exactly when t is positive, zero or negative. A derivative sign chart determines increasing intervals.

Step 1: Differentiate the composite Since $f(t) = e^t - t$ and $g(x) = x^2 - x$, the composite is $h(x) = e^{x^2-x} - (x^2 - x)$. Applying the chain rule yields

$$h\prime(x) = (e^{x^2-x} - 1)(2x - 1).$$

The first factor vanishes when $x^2 - x = 0$, namely $x = 0, 1$. The second vanishes at $x = 1/2$. These three points divide the real line into intervals on which the signs remain fixed.

Step 2: Build the sign chart The quadratic $x(x - 1)$ is positive outside $[0, 1]$ and negative inside it. Consequently the derivative signs are

x	$(-\infty, 0)$	$(0, 1/2)$	$(1/2, 1)$	$(1, \infty)$
$e^{x^2-x} - 1$	$+$	$-$	$-$	$+$
$2x - 1$	$-$	$-$	$+$	$+$
$h\prime(x)$	$-$	$+$	$-$	$+$

Thus the increasing intervals, with their endpoints included as in the options, are $[0, 1/2]$ and $[1, \infty)$. A wrongly includes the decreasing interval $(1/2, 1)$; B also includes it and a negative interval; C wrongly includes $(-1/2, 0)$, where the derivative is negative.

Step 3: Conclusion The stated set is $\boxed{[0, 1/2] \cup [1, \infty)}$. Hence, the correct option is (D).

Quick Tip to Solve: For an exponential composite, replace the sign of $e^{g(x)} - 1$ by the sign of $g(x)$, then combine it with the sign of $g\prime(x)$.

14. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated is : [4 marks]

(A) 36

(B) 60

(C) 72

(D) 48

Correct Answer: (B) 60

Solution:

Concept:

- A decimal integer is divisible by eleven when the difference between the sums of its digits in alternating positions is a multiple of eleven. The six available digits sum to 24.
- Choose which three digits occupy positions $1, 3, 5$, then arrange both groups. A six-digit number cannot have zero in its first position, so count and subtract such arrangements separately.

Step 1: Determine the two alternating groups Let S be the sum of the digits in positions $1, 3, 5$. The other three sum to $24 - S$, so divisibility requires $2S - 24$ to be a multiple of eleven. The smallest possible triple sum is $0 + 1 + 2 = 3$, and the largest is $5 + 7 + 9 = 21$. Thus $-18 \leq 2S - 24 \leq 18$. This expression is even, so the only possible multiple of eleven in that range is zero. Therefore $S = 12$. The triples with this sum are precisely $\{0, 5, 7\}$ and $\{1, 2, 9\}$, which are complementary.

Step 2: Count arrangements and remove leading zero Either triple may occupy the odd positions. For each choice, its three distinct digits can be arranged in $3!$ ways, and the remaining triple can be placed in the even positions in $3!$ ways. The initial count is therefore

$$2(3!)(3!) = 72.$$

A leading zero is possible only when $\{0, 5, 7\}$ occupies the odd positions. Fixing zero first leaves $2!$ odd-position arrangements and $3!$ even-position arrangements, so 12 must be excluded. The required count is $72 - 12 = 60$. Option C includes leading-zero strings; A counts only one group assignment before exclusions; D subtracts the invalid count twice.

Step 3: Conclusion There are $\boxed{60}$ such six-digit numbers. Hence, the correct option is (B).

Quick Tip to Solve: Apply the divisibility test to choose alternating-position digit groups first; only afterward permute them and subtract arrangements beginning with zero.

15. Let f(x) = x^2 , x $\in$ R. For any A $\subseteq$ R, define g (A) = { x $\in$ R : f(x) $\in$ A}. If S = [0,4], then which one of the following statements is not true ? [4 marks]

(A) $g(f(S)) \neq S$

(B) $f(g(S)) = S$

(C) $f(g(S)) \neq f(S)$

(D) $g(f(S)) = g(S)$

Correct Answer: (D) $g(f(S)) = g(S)$

Solution:

Concept:

- The image $f(A)$ contains the squares of elements of A. Here $g(A) = f^{-1}(A)$ is an inverse image of a set, not an inverse function.
- For $a \geq 0$, the inequality $0 \leq x^2 \leq a$ is equivalent to $-\sqrt{a} \leq x \leq \sqrt{a}$. Both signs must be retained.

Step 1: Compute the image and inverse image of S First use the nonnegative interval $S = [0, 4]$. Squaring is increasing on this interval, so its image is $f(S) = [0, 16]$. For the inverse image solve the inequality defining membership, rather than just squaring the endpoints:

$$g(S) = \{x : 0 \leq x^2 \leq 4\} = [-2, 2].$$

The negative half of this interval is essential because negative inputs can also have squares in S.

Step 2: Apply the operations a second time Now square every element of $[-2, 2]$, and take the inverse image of $[0, 16]$:

$$f(g(S)) = [0, 4] = S, \qquad g(f(S)) = [-4, 4].$$

Therefore option (A) is true because $[-4, 4] \neq [0, 4]$; option (B) is true because $f(g(S)) = S$. Option (C) is also true because $[0, 4] \neq [0, 16] = f(S)$. However, $[-4, 4] \neq [-2, 2]$, so the equality asserted in option (D) is false. For example, 3 belongs to $g(f(S))$ but not to $g(S)$.

Step 3: Conclusion The statement that is not true is $g(f(S)) = g(S)$. Hence, the correct option is (D).

Quick Tip to Solve: Write all four sets explicitly before comparing them. An inverse image under squaring includes negative as well as positive square roots.

16. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$, ($\lambda, \mu \in$ R), has infinitely many solutions, then the value of $\lambda + \mu$ is : [4 marks]

(A) 10

(B) 9

(C) 7

(D) 12

Correct Answer: (A) 10

Solution:

Concept:

- Infinitely many solutions require a dependent equation and consistency: one free variable must remain, with no contradictory constant equation.
- Elementary row subtraction preserves the solution set and exposes the conditions on the parameters. A zero determinant alone does not establish consistency.

Step 1: Eliminate x using the first equation Subtract the first equation from each of the other two. This gives

$$y + z = 1, \qquad 2y + (\lambda - 1)z = \mu - 5.$$

The first equation of the original system still reads $x + y + z = 5$. Combining it with $y + z = 1$ fixes $x = 4$. Thus only the relation between y and z can provide a free parameter; it is not enough merely for the third equation to have a special coefficient.

Step 2: Require dependence and consistency together Insert $y = 1 - z$ into the second reduced equation:

$$2(1 - z) + (\lambda - 1)z = \mu - 5,$$

hence

$$(\lambda - 3)z = \mu - 7.$$

If $\lambda \neq 3$, this fixes z, giving a unique solution. If $\lambda = 3$ but $\mu \neq 7$, the equation is contradictory. Infinitely many solutions occur exactly when $\lambda = 3$ and $\mu = 7$. Indeed, $(x, y, z) = (4, 1 - t, t)$ then satisfies all three equations for every real t. Therefore the required sum is $3 + 7 = 10$, excluding 9, 7 and 12.

Step 3: Conclusion $\lambda + \mu = 10$. Hence, the correct option is (A).

Quick Tip to Solve: After elimination, a parameter equation must reduce to 0=0 for infinitely many solutions; 0 equal to a nonzero constant means inconsistency.

17. Let $A(3,0,-1)$, $B(2,10,6)$ and $C(1,2,1)$ be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio $2:1$, then $\cos(\angle GOA)$, O being the origin, is equal to: [4 marks]

(A) $\frac{1}{\sqrt{15}}$
(B) $\frac{1}{6\sqrt{10}}$
(C) $\frac{1}{\sqrt{30}}$
(D) $\frac{1}{2\sqrt{15}}$

Correct Answer: (A) $\frac{1}{\sqrt{15}}$

Solution:

Concept:

- The midpoint formula gives $M = (A + C)/2$. For $BG : GM = 2 : 1$, internal division gives $G = (B + 2M)/3$.
- The angle at the origin is the angle between the position vectors: $\cos\angle GOA = (\vec{OG} \cdot \vec{OA})/(|\vec{OG}|\,|\vec{OA}|)$.

Step 1: Find the midpoint and the point G Average the corresponding coordinates of A and C:

$$M = \left(\frac{3+1}{2}, \frac{0+2}{2}, \frac{-1+1}{2}\right) = (2,1,0).$$

Since G is two-thirds of the way from B to M, the weight of M is two:

$$G = \frac{B+2M}{3} = \left(\frac{2+4}{3}, \frac{10+2}{3}, \frac{6+0}{3}\right) = (2,4,2).$$

Reversing these weights would locate a different point on the median and would change the angle.

Step 2: Evaluate the scalar product and magnitudes The position vectors are $\vec{OG} = (2,4,2)$ and $\vec{OA} = (3,0,-1)$. Their product is

$$\vec{OG} \cdot \vec{OA} = 2(3) + 4(0) + 2(-1) = 4.$$

Their lengths are $\sqrt{2^2+4^2+2^2} = 2\sqrt{6}$ and $\sqrt{3^2+0^2+(-1)^2} = \sqrt{10}$. Therefore

$$\cos\angle GOA = \frac{4}{2\sqrt{6}\sqrt{10}} = \frac{4}{4\sqrt{15}} = \frac{1}{\sqrt{15}}.$$

The other three denominators do not equal $\sqrt{15}$; in particular the factor 4 in the dot product must cancel when the radical is simplified.

Step 3: Conclusion $\cos(\angle GOA) = 1/\sqrt{15}$. Hence, the correct option is (A).

Quick Tip to Solve: An angle whose middle letter is O uses position vectors from the origin. Check the ratio weights before applying the dot-product formula.

18. **If** $\int \frac{dx}{(x^2-2x+10)^2} = A\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{f(x)}{x^2-2x+10}\right) + C$

where C is a constant of integration then : [4 marks]

(A) $A = \frac{1}{54}$ and $f(x) = 9(x-1)^2$

(B) $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

(C) $A = \frac{1}{81}$ and $f(x) = 3(x-1)$

(D) $A = \frac{1}{27}$ and $f(x) = 9(x-1)^2$

Correct Answer: (B) $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

Solution:

Concept:

- Complete the square: $x^2 - 2x + 10 = (x-1)^2 + 9$. A substitution $u = x - 1 = 3\tan\theta$ reduces the squared quadratic denominator.
- The identities $1 + \tan^2\theta = \sec^2\theta$ and $\cos^2\theta = (1 + \cos 2\theta)/2$ give an elementary antiderivative.

Step 1: Reduce the integral to a trigonometric form Put $u = x - 1$ and then $u = 3\tan\theta$. We have $du = 3\sec^2\theta\, d\theta$ and $u^2 + 9 = 9\sec^2\theta$, so

$$I = \int \frac{du}{(u^2+9)^2} = \frac{1}{27}\int \cos^2\theta\, d\theta.$$

Using the double-angle identity and integrating both terms gives

$$I = \frac{1}{54}(\theta + \sin\theta\cos\theta) + C.$$

The coefficient is determined by the factor 3 from the differential and 81 from the squared denominator.

Step 2: Return to x and compare the coefficients Here $\theta = \tan^{-1}(u/3)$, and

$$\sin\theta\cos\theta = \frac{\tan\theta}{1+\tan^2\theta} = \frac{3u}{u^2+9}.$$

Consequently

$$I = \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{3(x-1)}{x^2-2x+10}\right] + C.$$

Thus the required numerator is linear in $x - 1$, ruling out (A) and (D), and its common coefficient is $1/54$, ruling out (C). As a check, differentiating the bracket with respect to u gives $54/(u^2+9)^2$; multiplication by $1/54$ returns the original integrand.

Step 3: Conclusion $A = 1/54$ and $f(x) = 3(x-1)$. Hence, the correct option is (B).

Quick Tip to Solve: For a squared quadratic denominator, complete the square first. Keep the substitution scale in both dx and the denominator before integrating.

19. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is : [4 marks]

(A) $\frac{1}{10}$
(B) $\frac{1}{17}$
(C) $\frac{1}{11}$
(D) $\frac{1}{12}$

Correct Answer: (C) $\frac{1}{11}$

Solution:

Concept:

- Treat the four children as four distinct positions. Independent births with equal boy/girl likelihood give $2^4 = 16$ equally likely outcomes.
- For events E and F, conditional probability is $P(E \mid F) = P(E \cap F)/P(F)$, provided $P(F) > 0$.

Step 1: Count the outcomes allowed by the condition Let F be the event that at least two of the four children are girls. The possible numbers of girls are 2, 3 and 4. Their counts are

$$\binom{4}{2} = 6, \qquad \binom{4}{3} = 4, \qquad \binom{4}{4} = 1.$$

Therefore F contains $6 + 4 + 1 = 11$ equally likely outcomes and $P(F) = 11/16$. The division into two families only labels the four positions; the condition is about the total number of girls, not a separate requirement in each family.

Step 2: Restrict the sample space and form the ratio Let E be the event that all four children are girls. There is precisely one such outcome, GGGG, so $P(E) = 1/16$. Since four girls certainly include at least two girls, $E \subseteq F$ and $E \cap F = E$. Thus

$$P(E \mid F) = \frac{1/16}{11/16} = \frac{1}{11}.$$

The denominator is 11, not 10 or 12, because the all-girl outcome itself must remain in the conditioning event along with the six two-girl and four three-girl outcomes. A denominator of 17 is impossible here: the entire unconditioned space has only 16 equally likely outcomes.

Step 3: Conclusion The conditional probability is $1/11$. Hence, the correct option is (C).

Quick Tip to Solve: For an at-least condition, count each allowed number separately and include the extreme case. Then divide favourable outcomes by the conditioned total.

20. If y = y(x) is the solution of the differential equation

$\frac{dy}{dx} = (\tan x - y)\sec^2 x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

such that y (0) = 0, then $y\left(-\frac{\pi}{4}\right)$ is equal to : [4 marks]

(A) $\frac{1}{2} - e$

(B) $e - 2$

(C) $2 + \frac{1}{e}$

(D) $\frac{1}{e} - 2$

Correct Answer: (B) $e - 2$

Solution:

Concept:

- The substitution $t = \tan x$ has $dt/dx = \sec^2 x$ on the given interval and reduces the differential equation to $dy/dt + y = t$.
- A linear equation $dy/dt + y = t$ has integrating factor e^t. The initial condition determines the arbitrary constant.

Step 1: Solve the transformed differential equation Set $t = \tan x$. Dividing the original equation by $dt/dx = \sec^2 x$, which is positive on the stated interval, gives

$$\frac{dy}{dt} = t - y, \qquad \frac{dy}{dt} + y = t.$$

Multiply by e^t to obtain

$$\frac{d}{dt}(ye^t) = te^t.$$

Integration by parts yields $\int te^t\,dt = e^t(t-1) + C$. Therefore $ye^t = e^t(t-1) + C$, or $y = t - 1 + Ce^{-t}$. This includes every solution on the transformed real t-axis.

Step 2: Use the initial value and evaluate the target At $x = 0$, $t = 0$ and $y = 0$, so $0 = -1 + C$ and $C = 1$. Hence

$$y(x) = \tan x - 1 + e^{-\tan x}.$$

At $x = -\pi/4$, the tangent is -1, so

$$y(-\pi/4) = -1 - 1 + e = e - 2.$$

Notice that the exponent becomes positive: $-\tan(-\pi/4) = 1$. This excludes options involving $1/e$. The remaining option $1/2 - e$ also differs in sign and value. As a direct check, $dy/dt = 1 - e^{-t} = t - y$, so the expression satisfies the equation and its initial value.

Step 3: Conclusion $y(-\pi/4) = e - 2$. Hence, the correct option is (B).

Quick Tip to Solve: When both tan x and sec-squared x occur in a linear equation, substitute t=tan x first; apply the initial condition before evaluating negative angles.

21. All the pairs (x, y) that satisfy the inequality

$$2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \leq 1$$

also satisfy the equation [4 marks]

(A) sin x = |sin y|

(B) sin x = 2sin y

(C) 2 sin x = sin y

(D) 2 |sin x | = 3 sin y

Correct Answer: (A) sin x = |sin y|

Solution:

Concept:

- Because 2^u is strictly increasing, $2^u \leq 1$ is equivalent to $u \leq 0$. Rewrite powers of 4 as powers of 2.
- For real angles, $-1 \leq \sin x \leq 1$ and $0 \leq \sin^2 y \leq 1$. Equality at both extreme bounds will determine the admissible pairs.

Step 1: Express the condition as an inequality of exponents Using $4^{\sin^2 y} = 2^{2\sin^2 y}$, the given inequality becomes

$$2^{\sqrt{\sin^2 x - 2\sin x + 5} - 2\sin^2 y} \leq 1.$$

Monotonicity therefore gives

$$\sqrt{(\sin x - 1)^2 + 4} \leq 2\sin^2 y.$$

The left side is at least 2 because a square is nonnegative. The right side is at most 2 because $\sin^2 y \leq 1$. Thus the left side cannot be strictly below the right side; the only possibility is that both equal their common bound 2.

Step 2: Determine the equality cases and test the options Equality on the left requires $(\sin x - 1)^2 = 0$, giving $\sin x = 1$. Equality on the right requires $\sin^2 y = 1$, giving $\sin y = 1$ or -1. Both signs are permitted, and substitution makes the original expression exactly 1. Consequently every admissible pair satisfies

$$\sin x = 1 = |\sin y|.$$

Option (B) would require $1 = \pm 2$, and (C) would require $2 = \pm 1$, so neither holds. Option (D) would require $2 = \pm 3$, which is also impossible. The absolute value in (A) correctly accommodates both allowed signs of $\sin y$.

Step 3: Conclusion All the admissible pairs satisfy $\sin x = |\sin y|$. Hence, the correct option is (A).

Quick Tip to Solve: When one side is always at least a constant and the other is always at most it, an inequality between them forces equality at both bounds.

22. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a,b) is equal to : [4 marks]

(A) (28, 861)

(B) (28, 315)

(C) (–21, 714)

(D) (–54, 315)

Correct Answer: (B) (28, 315)

Solution:

Concept:

- The coefficient of x^r in $(1-3x)^{15}$ is $\binom{15}{r}(-3)^r$. Only powers through x^3 are needed.
- In a product of polynomials, collect every pair of terms whose degrees add to the required degree. Both prescribed coefficients must vanish for the same a, b.

Step 1: Expand the relevant terms Using the binomial theorem, the first four terms are

$$(1-3x)^{15} = 1 - 45x + 945x^2 - 12285x^3 + \cdots .$$

Here $\binom{15}{2}9 = 945$, while $\binom{15}{3}(-27) = -12285$. Terms of degree four or higher cannot contribute to the coefficients under consideration because the other factor has no negative powers.

Step 2: Equate the two coefficients to zero Multiplication by $1 + ax + bx^2$ gives the coefficient equations

$$945 - 45a + b = 0, \qquad -12285 + 945a - 45b = 0.$$

The first equation yields $b = 45a - 945$. Substitute this into the second equation:

$$-12285 + 945a - 45(45a - 945) = 0,$$

$$30240 - 1080a = 0 \quad \Longrightarrow \quad a = 28.$$

Consequently, $b = 45(28) - 945 = 315$.

Step 3: Check the ordered pair For $(a, b) = (28, 315)$, the two coefficients are

$$945 - 1260 + 315 = 0,$$

$$-12285 + 26460 - 14175 = 0.$$

Thus both conditions hold simultaneously. Option (A) has the correct first component but its second component fails the quadratic coefficient equation. Options (C) and (D) have different first components and cannot satisfy the unique pair of linear equations.

Step 4: Conclusion The required ordered pair is $(a, b) = (28, 315)$. Hence, the correct option is (B).

Quick Tip to Solve: For vanishing adjacent coefficients, expand only to the higher required degree; retaining the sign of each binomial term prevents most errors.

23. If Q(0, –1, –3) is the image of the point P in the plane 3x – y + 4z = 2 and R is the point (3, –1, –2), then the area (in sq. units) of $\triangle PQR$ is : [4 marks]

(A) $\frac{\sqrt{65}}{2}$

(B) $2\sqrt{13}$

(C) $\frac{\sqrt{91}}{2}$

(D) $\frac{\sqrt{91}}{4}$

Correct Answer: (C) $\frac{\sqrt{91}}{2}$

Solution:

Concept:

- Reflection in a plane changes a point only along the normal. For $Ax + By + Cz = D$, the image of $\mathbf{r}$ is $\mathbf{r} - 2\frac{Ax+By+Cz-D}{A^2+B^2+C^2}(A, B, C)$.
- The area of a triangle formed by two vectors $\mathbf{u}, \mathbf{v}$ from the same vertex is $\frac{1}{2}|\mathbf{u} \times \mathbf{v}|$.

Step 1: Recover the reflected point Reflection is reversible, so reflect $Q = (0, -1, -3)$ in the plane to obtain P. The normal is $\mathbf{n} = (3, -1, 4)$, with squared length 26. At Q, the plane expression is

$$3(0) - (-1) + 4(-3) - 2 = -13.$$

Hence

$$P = (0, -1, -3) - 2\frac{-13}{26}(3, -1, 4) = (3, -2, 1).$$

As a check, the midpoint of PQ is $(3/2, -3/2, -1)$, and substituting it in the plane gives 2, as required.

Step 2: Form two sides from the same vertex Take Q as the common starting point. Then

$$\overrightarrow{QP} = (3, -1, 4), \qquad \overrightarrow{QR} = (3, 0, 1).$$

Their cross product is

$$\overrightarrow{QP} \times \overrightarrow{QR} = (-1, 9, 3).$$

Its magnitude is $\sqrt{1 + 81 + 9} = \sqrt{91}$. This is the area of the parallelogram generated by the two sides, so the triangle has half this area.

Step 3: Compare with the choices The triangle area is therefore

$$[PQR] = \frac{1}{2}\sqrt{91}.$$

The factor of one-half is essential: using the cross-product magnitude directly would give a parallelogram area. Option (D) halves this result once too often, while (A) and (B) have squared radical magnitudes inconsistent with the independently computed cross product.

Step 4: Conclusion The area of $\triangle PQR$ is $\frac{\sqrt{91}}{2}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Reflect the known image back to recover the original point, then use two side vectors with the same starting vertex for the cross product.

24. The sum

$\frac{3\times1^3}{1^3} + \frac{5\times(1^3+2^3)}{1^2+2^2} + \frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2} + \ldots\ldots$ **upto 10 terms is:** [4 marks]

(A) 600

(B) 660

(C) 680

(D) 620

Correct Answer: (B) 660

Solution:

Concept:

- The standard identities are $\sum_{k=1}^{n} k^3 = [n(n+1)/2]^2$ and $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$.
- The multiplier of the nth fraction is $2n+1$. Simplify a general term before adding the ten terms.

Step 1: Express the general term The first three multipliers are $3, 5, 7$, so the nth term is

$$T_n = (2n+1)\frac{1^3 + 2^3 + \cdots + n^3}{1^2 + 2^2 + \cdots + n^2}.$$

For $n = 1$, the printed denominator is 1^3, whose value is also $1^2 = 1$, so this expression agrees with the first term as well. There are ten terms, meaning n runs from 1 through 10, including both endpoints.

Step 2: Simplify using sums of powers Insert the two standard sum formulae:

$$T_n = (2n+1)\frac{n^2(n+1)^2/4}{n(n+1)(2n+1)/6}.$$

Cancel $2n+1$, one factor n, and one factor $n+1$. This leaves

$$T_n = \frac{3}{2}n(n+1) = \frac{3}{2}(n^2+n).$$

A quick check at $n = 2$ gives $T_2 = 5(1+8)/(1+4) = 9$, agreeing with $3(2)(3)/2 = 9$.

Step 3: Add the ten simplified terms Thus the required sum is

$$S_{10} = \frac{3}{2}\left(\sum_{n=1}^{10} n^2 + \sum_{n=1}^{10} n\right).$$

Using $\sum n^2 = 10(11)(21)/6 = 385$ and $\sum n = 10(11)/2 = 55$,

$$S_{10} = \frac{3}{2}(385 + 55) = \frac{3}{2}(440) = 660.$$

This exact total agrees only with (B). The values 600, 680 and 620 do not follow from summing all ten terms of the simplified expression; no approximation or rounding is involved.

Step 4: Conclusion The sum up to ten terms is 660. Hence, the correct option is (B).

Quick Tip to Solve: When a ratio contains sums of cubes and squares, use their closed forms first: the odd multiplier often cancels immediately.

25. Which one of the following Boolean expressions is a tautology? [4 marks]

(A) (p ∨ q) ∧ (∼ p ∨ ∼ q)

(B) (p ∨ q) ∨ (p ∨ ∼ q)

(C) (p ∧ q) ∨ (p ∧ ∼ q)

(D) (p ∨ q) ∧ (p ∨ ∼ q)

Correct Answer: (B) (p ∨ q) ∨ (p ∨ ∼ q)

Solution:

Concept:

- A tautology is true for every assignment of truth values to its component statements. The law of excluded middle states that $q \vee \neg q$ is always true.
- Use associativity of disjunction and the distributive laws to simplify compound expressions; a single false assignment rules out a proposed tautology.

Step 1: Simplify the expression in option B Let T denote a statement that is always true. Associativity and idempotence of disjunction give

$$(p \vee q) \vee (p \vee \neg q) = p \vee (q \vee \neg q).$$

Since exactly one of q and $\neg q$ is true for either truth value of q, their disjunction is T. Consequently,

$$p \vee (q \vee \neg q) = p \vee T = T.$$

The result is independent of whether p itself is true or false. This establishes the required universal truth, rather than checking only a favourable assignment.

Step 2: Rule out the other expressions Option (A) is $(p \vee q) \wedge (\neg p \vee \neg q)$. When $p = q = T$, its second bracket is false, so the whole expression is false.

By distribution, option (C) reduces to

$$(p \wedge q) \vee (p \wedge \neg q) = p \wedge (q \vee \neg q) = p.$$

Similarly, option (D) reduces to

$$(p \vee q) \wedge (p \vee \neg q) = p \vee (q \wedge \neg q) = p.$$

Both (C) and (D) are false when p is false, regardless of q. Thus neither is a tautology. In contrast, option (B) explicitly contains a disjunction of a statement and its negation under an outer disjunction, which forces truth in every row of a truth table.

Step 3: Conclusion The Boolean expression $(p \vee q) \vee (p \vee \neg q)$ is a tautology. Hence, the correct option is (B).

Quick Tip to Solve: Look for a pair $q \vee \neg q$ under an OR operation; for the remaining choices, one false truth assignment is enough to reject tautology.

26. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\text{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is : [4 marks]

(A) $\frac{100}{3\sqrt{3}}$

(B) 25

(C) 20

(D) $10\sqrt{5}$

Correct Answer: (C) 20

Solution:

Concept:

- In an isosceles triangle, the median from the vertex to the base is perpendicular to the base. Thus if D is the midpoint of BC, then $AD \perp BD$.
- For a vertical tower of height h, an elevation angle satisfies $\tan\theta = h/(\text{horizontal distance})$. Reciprocal trigonometric ratios convert the given inverse functions.

Step 1: Relate the height to the two ground distances Let the tower top be T and its foot be D. Write $DT = h > 0$. At A, the elevation angle α has $\cot\alpha = 3\sqrt{2}$. Therefore

$$\frac{AD}{h} = 3\sqrt{2}, \qquad AD = 3\sqrt{2}\,h.$$

At B, the elevation angle β has $\csc\beta = 2\sqrt{2}$. The identity $\cot^2\beta = \csc^2\beta - 1$ gives $\cot^2\beta = 8 - 1 = 7$. As an elevation angle is acute,

$$BD = h\cot\beta = \sqrt{7}\,h.$$

Step 2: Apply the isosceles-triangle geometry Because $AB = AC$ and D is the midpoint of BC, $\triangle ADB$ is right-angled at D. Its hypotenuse is $AB = 100$ metres. Hence

$$AD^2 + BD^2 = AB^2.$$

Substituting the expressions in terms of h gives

$$(3\sqrt{2}h)^2 + (\sqrt{7}h)^2 = 10000,$$

$$18h^2 + 7h^2 = 10000 \quad \Longrightarrow \quad h^2 = 400.$$

Only the positive root describes a tower height, so $h = 20$ metres.

Step 3: Check the reciprocal ratios and options For $h = 20$, the two distances are $AD = 60\sqrt{2}$ and $BD = 20\sqrt{7}$. Their squared sum is $7200 + 2800 = 10000$, confirming the given side length. Also $BT = \sqrt{BD^2 + h^2} = 40\sqrt{2}$, so $BT/h = 2\sqrt{2}$, as the given cosecant requires. Options (A), (B) and (D) do not satisfy $25h^2 = 10000$.

Step 4: Conclusion The tower is $20\,\text{m}$ high. Hence, the correct option is (C).

Quick Tip to Solve: Convert a given cosecant elevation to cotangent using $\cot^2\theta = \csc^2\theta - 1$, then express both ground distances in terms of the height.

27. If the line x – 2y = 12 is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $\left(3, -\frac{9}{2}\right)$, then the length of the latus rectum of the ellipse is : [4 marks]

(A) 5

(B) 9

(C) $8\sqrt{3}$

(D) $12\sqrt{2}$

Correct Answer: (B) 9

Solution:

Concept:

- The tangent to $x^2/a^2 + y^2/b^2 = 1$ at (x_1, y_1) is $xx_1/a^2 + yy_1/b^2 = 1$. Compare equations after making their constant terms equal.
- The length of the latus rectum is twice the square of the semi-minor axis divided by the semi-major axis; identify the longer axis before applying the formula.

Step 1: Compare the tangent coefficients At the given point $(3, -9/2)$, the tangent formula gives

$$\frac{3x}{a^2} - \frac{9y}{2b^2} = 1.$$

The supplied tangent is $x - 2y = 12$. Dividing every term by 12 writes it in the same normalization:

$$\frac{x}{12} - \frac{y}{6} = 1.$$

The coefficients of x and y must therefore coincide. This comparison uses the complete tangent equation and does not equate coefficients of equations with different right-hand sides.

Step 2: Determine the two semi-axes Equating coefficients yields

$$\frac{3}{a^2} = \frac{1}{12} \implies a^2 = 36,$$

$$\frac{9}{2b^2} = \frac{1}{6} \implies b^2 = 27.$$

Thus $a = 6$ and $b = 3\sqrt{3}$, with $a > b$, so the major axis is the horizontal axis. The contact point indeed lies on the ellipse:

$$\frac{3^2}{36} + \frac{(-9/2)^2}{27} = \frac{1}{4} + \frac{3}{4} = 1.$$

Step 3: Find the full latus-rectum length For this horizontal ellipse, the full latus-rectum length is

$$\frac{2b^2}{a} = \frac{2(27)}{6} = 9.$$

The factor two includes both halves of the chord through a focus perpendicular to the major axis. Only option (B) equals this exact length; the other three values are therefore excluded.

Step 4: Conclusion The length of the latus rectum is 9 units. Hence, the correct option is (B).

Quick Tip to Solve: Normalize the given tangent to right-hand side 1 before comparing its coefficients with the tangent-at-a-point formula.

28.

$$\lim_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}}+\frac{(n+2)^{1/3}}{n^{4/3}}+\ldots\ldots+\frac{(2n)^{1/3}}{n^{4/3}}\right)$$

is equal to : [4 marks]

(A) $\frac{4}{3}(2)^{3/4}$

(B) $\frac{3}{4}(2)^{4/3}-\frac{3}{4}$

(C) $\frac{4}{3}(2)^{4/3}$

(D) $\frac{3}{4}(2)^{4/3}-\frac{4}{3}$

Correct Answer: (B) $\frac{3}{4}(2)^{4/3}-\frac{3}{4}$

Solution:

Concept:

- A sum of the form $\frac{1}{n}\sum_{k=1}^{n} f(k/n)$ converges to $\int_0^1 f(x)\,dx$ when f is continuous on the interval.
- Factor the power of n from each numerator before identifying the mesh width; the power rule gives $\int u^{1/3}du=\frac{3}{4}u^{4/3}+C$.

Step 1: Normalize a general summand The given expression contains exactly n terms. Label them by $k=1,2,\ldots,n$, so the general numerator is $(n+k)^{1/3}$. Then

$$\frac{(n+k)^{1/3}}{n^{4/3}}=\frac{n^{1/3}(1+k/n)^{1/3}}{n^{4/3}}=\frac{1}{n}(1+k/n)^{1/3}.$$

The factor $1/n$ is the width of each equal subinterval of $[0,1]$. The sample points k/n are their right endpoints; in particular the final sample gives the printed numerator $(2n)^{1/3}$.

Step 2: Convert to a definite integral The function $f(x)=(1+x)^{1/3}$ is continuous throughout $[0,1]$, so its right-endpoint Riemann sums converge. Therefore the limit is

$$\int_0^1 (1+x)^{1/3}\,dx.$$

Put $u=1+x$, with $du=dx$; the lower endpoint becomes 1 and the upper endpoint becomes 2. Thus

$$\int_1^2 u^{1/3}\,du=\left[\frac{3}{4}u^{4/3}\right]_1^2=\frac{3}{4}\,2^{4/3}-\frac{3}{4}.$$

Step 3: Check the endpoint subtraction The antiderivative coefficient is $1/(1+1/3)=3/4$, and evaluation at the lower endpoint contributes exactly $3/4$. This distinguishes option (B) from (D), which subtracts $4/3$. Options (A) and (C) use an incorrect coefficient, and (A) additionally uses the wrong exponent. As a magnitude check, the integrand ranges from 1 to $2^{1/3}$, so its integral over an interval of length 1 lies between those values, consistent with the result.

Step 4: Conclusion The limit is $\frac{3}{4}\,2^{4/3}-\frac{3}{4}$. Hence, the correct option is (B).

Quick Tip to Solve: Pull out the numerator factor $n^{1/3}$ first: the remaining $1/n$ exposes a Riemann sum and prevents an incorrect limiting power.

29. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q, for : [4 marks]

(A) exactly two values of K

(B) no value of K

(C) exactly one value of K

(D) infinitely many values of K

Correct Answer: (B) no value of K

Solution:

Concept:

- The common chord of two intersecting circles lies on their radical axis. Subtract equations only after making the coefficients of $x^2 + y^2$ equal.
- Two equations represent the same straight line only when all three coefficients, including the constant terms, are proportional.

Step 1: Eliminate the quadratic terms Write the equations as $S_1 = 0$ and $S_2 = 0$, with the second equation as printed, including its factor 2. At each common point both equations vanish, so $2S_1 - S_2 = 0$. This gives

$$(10K - 2K)x + (4 - 3)y + (2K + 1) = 0,$$

$$8Kx + y + 2K + 1 = 0.$$

Since the coefficient of y is nonzero, this is always a genuine straight line.

Step 2: Compare all coefficients The distinct intersection points P, Q determine a unique line. Therefore this radical axis would have to coincide with $4x + 5y - K = 0$. Comparing the y-coefficients fixes the multiplier at $1/5$, so

$$8K = \frac{4}{5} \quad \Longrightarrow \quad K = \frac{1}{10}.$$

The constants must also satisfy

$$2K + 1 = -\frac{K}{5} \quad \Longrightarrow \quad K = -\frac{5}{11}.$$

These conditions contradict each other. Comparing only the coefficients of x and y would miss this obstruction; two parallel distinct lines cannot pass through the same two distinct points. Options (A), (C) and (D) all require at least one admissible value, whereas the coefficient equations have no simultaneous solution.

Step 3: Conclusion No real value of K satisfies the necessary proportionality conditions. Hence, the correct option is (B).

Quick Tip to Solve: For a proposed common chord, compare the constant term as well as the slopes of the radical axis.

30. The line x = y touches a circle at the point (1,1). If the circle also passes through the point (1, –3), then its radius is : [4 marks]

(A) 3

(B) 2

(C) $2\sqrt{2}$

(D) $3\sqrt{2}$

Correct Answer: (C) $2\sqrt{2}$

Solution:

Concept:

- The radius to a point of tangency is perpendicular to the tangent. A line of slope 1 therefore has a normal of slope -1.
- All points on a circle have the same distance from its centre. Equate squared distances to avoid unnecessary square roots.

Step 1: Locate the centre on the normal Let the centre be $C(h, k)$. The tangent $x = y$ passes through $T(1, 1)$ and has slope 1. The radius CT lies on the normal through T, whose equation is

$$y - 1 = -(x - 1).$$

Thus the centre satisfies $h + k = 2$. This condition uses tangency, not merely the fact that T lies on the circle.

Step 2: Use the second point The other given point is $P(1, -3)$. Since $CT = CP$,

$$(h - 1)^2 + (k - 1)^2 = (h - 1)^2 + (k + 3)^2.$$

Cancelling the identical horizontal terms and expanding gives

$$k^2 - 2k + 1 = k^2 + 6k + 9, \qquad k = -1.$$

The normal condition then gives $h = 3$. Hence

$$r^2 = (3 - 1)^2 + (-1 - 1)^2 = 8, \qquad r = 2\sqrt{2}.$$

As a check, the distance from $(3, -1)$ to $x - y = 0$ is $4/\sqrt{2} = 2\sqrt{2}$, so that line really is tangent.

Step 3: Conclusion The radius is $2\sqrt{2}$ units. Hence, the correct option is (C). The other proposed radii have squares 9, 4 and 18, whereas equidistance and tangency require the squared radius to be exactly 8.

Quick Tip to Solve: The centre lies at the intersection of the tangent normal and the perpendicular bisector of any chord through the tangency point.

Physics (120 marks)

31. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of thier densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to : [4 marks]

(A) 2/5

(B) 2/3

(C) 3/5

(D) 4/5

Correct Answer: (A) 2/5

Solution:

Concept:

- The signed capillary height is $h = 2T\cos\theta/(\rho g r)$, where T is surface tension, ρ density and r tube radius.
- An obtuse contact angle gives depression because $\cos\theta < 0$. Equal rise and depression means equal magnitudes, not equal signed heights.

Step 1: Equate the magnitudes of the heights Use subscripts m and w for mercury and water. Since their displacements have equal magnitude,

$$\frac{2T_m|\cos 135^\circ|}{\rho_m g r_1} = \frac{2T_w\cos 0^\circ}{\rho_w g r_2}.$$

The negative cosine for mercury explains why it goes down; its absolute value enters the comparison of amounts. Gravity and the factor 2 cancel because both observations occur under the same conditions.

Step 2: Substitute the given ratios Rearrange before inserting numbers:

$$\frac{r_1}{r_2} = \frac{T_m}{T_w}\frac{\rho_w}{\rho_m}\frac{|\cos 135^\circ|}{\cos 0^\circ}.$$

Using $T_m/T_w = 7.5$, $\rho_m/\rho_w = 13.6$, $|\cos 135^\circ| = 1/\sqrt{2}$ and $\cos 0^\circ = 1$,

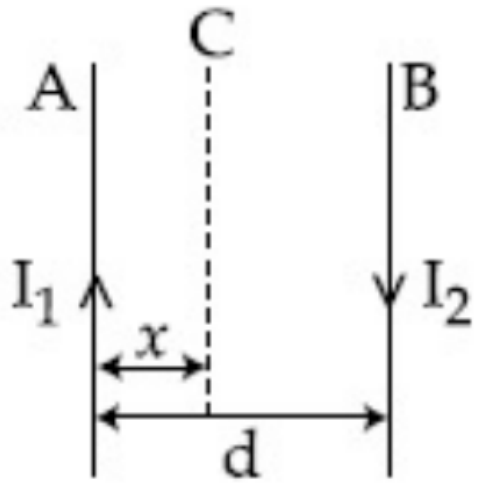
$$\frac{r_1}{r_2} = \frac{7.5}{13.6\sqrt{2}} \simeq 0.390.$$

This is close to $2/5 = 0.400$. Inverting the density ratio would predict an unrealistically large value and would reverse the role of the heavy liquid.

Step 3: Conclusion The radius ratio is approximately 0.39, closest to $2/5$. Hence, the correct option is (A). The values $2/3$, $3/5$ and $4/5$ are all substantially larger than the calculated ratio.

Quick Tip to Solve: For equal capillary displacements, tube radius is proportional to $T|\cos\theta|/\rho$; retain the contact-angle factor.

32. Two wires A & B are carrying currents I_1 & I_2 as shown in the figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are : [4 marks]



(A) $x = \left(\frac{I_1}{I_1+I_2}\right) d \, and \, x = \left(\frac{I_2}{I_1-I_2}\right) d$

(B) $x = \left(\frac{I_2}{I_1+I_2}\right) d \, and \, x = \left(\frac{I_2}{I_1-I_2}\right) d$

(C) $x = \left(\frac{I_1}{I_1-I_2}\right) d \, and \, x = \left(\frac{I_2}{I_1+I_2}\right) d$

(D) $x = \pm\frac{I_1 d}{I_1-I_2}$

Correct Answer: (D) $x = \pm\frac{I_1 d}{I_1-I_2}$

Solution:

Concept:

- The force per unit length between parallel wires a distance s apart has magnitude $\mu_0 I_a I_b/(2\pi s)$. Parallel currents attract and antiparallel currents repel.
- For a nonzero test current, zero resultant force is equivalent to zero magnetic field of the two fixed wires. Opposite fixed currents cannot cancel their fields between the wires.

Step 1: Determine the possible region Take A at signed coordinate $X = 0$ and B at $X = d$, with upward current positive. Their currents are $+I_1$ and $-I_2$. Between A and B their magnetic fields point in the same direction, so the position sketched between them cannot give equilibrium. Outside the pair, the fields oppose. The test current I cancels from the force-balance equation; reversing it reverses both forces and leaves the zero unchanged.

Step 2: Balance the fields with signed distances Along the line joining the wires, use the signed field equation

$$\frac{I_1}{X} - \frac{I_2}{X-d} = 0.$$

Consequently

$$I_1(X-d) = I_2 X, \qquad X = \frac{I_1 d}{I_1 - I_2}.$$

If $I_1 > I_2$, this is greater than d, outside B; if $I_1 < I_2$, it is negative, outside A. The positive distance from A is therefore $x = I_1 d/|I_1 - I_2|$. For equal currents there is no finite zero. Only one side is allowed for any fixed unequal pair; the two signs describe the alternative relative-current cases, not two simultaneous roots. Options (A)–(C) include a sum-denominator position between opposite currents, where the two fields add rather than cancel.

Step 3: Conclusion The equilibrium distance is $x = I_1 d/|I_1 - I_2|$, on the side of the weaker current. Hence, the correct option is (D), with its printed $\pm$ understood as the side convention.

Quick Tip to Solve: Locate the cancellation region before equating force magnitudes: opposite currents have their zero field outside the two wires.

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33. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is : [4 marks]

(A) $\frac{MR^2}{3}$
(B) $\frac{MR^2}{6}$
(C) $\frac{2MR^2}{3}$
(D) $\frac{MR^2}{2}$

Correct Answer: (C) $\frac{2MR^2}{3}$

Solution:

Concept:

- For an axis perpendicular to the disc through its centre, a concentric ring of radius r contributes $dI = r^2\, dm$.
- The surface density varies as $\sigma(r) = kr^2$, so a ring of width dr has mass $dm = \sigma(r)\, 2\pi r\, dr$. Rotational symmetry keeps the centre of mass at the geometric centre.

Step 1: Find the total mass The circumference of the ring is $2\pi r$, so its area is $dA = 2\pi r\, dr$. Multiplying by the local density, rather than treating the whole disc as uniform, gives

$$dm = 2\pi k r^3\, dr.$$

Integrating from the centre to the rim,

$$M = 2\pi k \int_0^R r^3\, dr = \frac{\pi k R^4}{2}.$$

Thus $k = 2M/(\pi R^4)$. The normalization is essential because the question asks for the answer in terms of total mass, not the density constant.

Step 2: Weight each ring by its squared radius Every particle in a thin ring is effectively the same

distance r from the required axis. Therefore

$$I = \int_0^R r^2\, dm = 2\pi k \int_0^R r^5\, dr = \frac{\pi k R^6}{3}.$$

Eliminating k using the mass integral gives

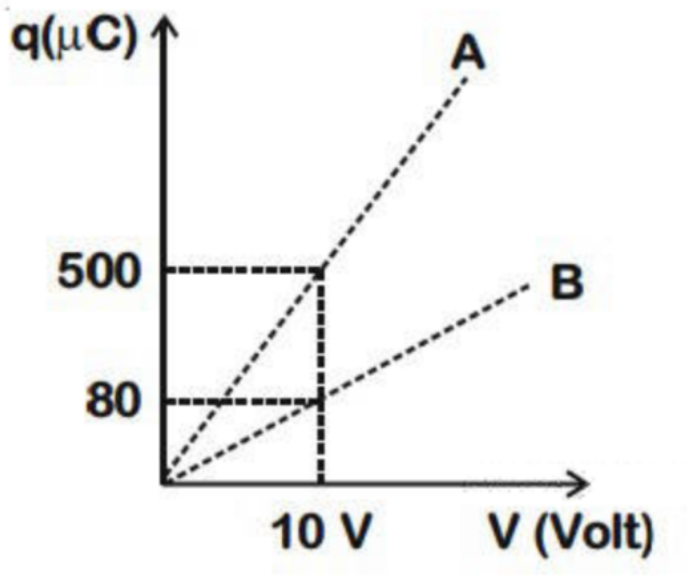
$$I = \frac{\pi R^6}{3}\frac{2M}{\pi R^4} = \frac{2}{3}MR^2.$$

The units are mass times length squared. Since the density increases towards the rim, the answer should exceed the uniform-disc value $MR^2/2$, while remaining below the thin-ring value MR^2. The uniform-disc result in (D) ignores the radial density variation; (A) and (B) are even smaller and contradict the greater concentration of mass near the rim.

Step 3: Conclusion The moment of inertia is $I = 2MR^2/3$. Hence, the correct option is (C).

Quick Tip to Solve: For a radial density, form I/M as a ratio of ring integrals; the unknown density constant cancels immediately.

34. Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are : [4 marks]



(A) 40 μF and 10 μF

(B) 60 μF and 40 μF

(C) 20 μF and 30 μF

(D) 50 μF and 30 μF

Correct Answer: (A) 40 μF and 10 μF

Solution:

Concept:

- The slope of a charge–voltage graph is capacitance: $C = q/V$. A charge in microcoulombs divided by volts gives capacitance in microfarads.
- For two positive capacitances, $C_p = C_1 + C_2$ and $C_s = C_1C_2/(C_1 + C_2)$. The parallel value exceeds the series value.

Step 1: Read the two slopes At the marked voltage $V = 10\,\text{V}$, the steeper line A gives $q = 500\,\mu\text{C}$, while B gives $q = 80\,\mu\text{C}$. Hence

$$C_A = \frac{500}{10} = 50\,\mu\text{F}, \qquad C_B = \frac{80}{10} = 8\,\mu\text{F}.$$

The steeper line must represent the parallel combination, so $C_p = 50\,\mu\text{F}$ and $C_s = 8\,\mu\text{F}$. Do not interpret the charge labels as capacitances without dividing by the voltage.

Step 2: Recover the individual capacitors Express both capacitances numerically in microfarads. Their sum and product obey

$$C_1 + C_2 = 50, \qquad C_1C_2 = C_sC_p = 8 \times 50 = 400.$$

Each capacitance is a root of

$$t^2 - 50t + 400 = 0 = (t - 40)(t - 10).$$

Thus the pair is 40 and 10. Substituting back checks both observations: the parallel capacitance is $40 + 10 = 50\,\mu\text{F}$, and the series capacitance is $40 \times 10/50 = 8\,\mu\text{F}$. The pair 20 and 30 has the right sum but the wrong product, so the sum alone cannot identify the answer. Options (B) and (D) already give incorrect parallel sums; (C) gives a series value of $12\,\mu\text{F}$, inconsistent with line B.

Step 3: Conclusion The individual capacitances are $40\,\mu\text{F}$ and $10\,\mu\text{F}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert the graph slopes into the sum and product of the capacitances, then solve the resulting quadratic.

35. A 25×10^{-3} m^3 volume cylinder is filled with 1 mol of O_2 gas at room temperature (300K). The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm, and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule ? [4 marks]

(A) $\sim 10^{13}$

(B) $\sim 10^{12}$

(C) $\sim 10^{11}$

(D) $\sim 10^{10}$

Correct Answer: No listed option; the printed data give approximately $1.8 \times 10^9\,\mathrm{s}^{-1}$ per molecule.

Solution:

Concept:

- For hard-sphere molecules of diameter d and number density n, the mean free path is $\lambda = 1/(\sqrt{2}\pi d^2 n)$.
- The collision frequency per molecule is approximately $z = \bar{v}/\lambda$. For a Maxwell distribution, $\bar{v} = \sqrt{8/(3\pi)}\, v_{\mathrm{rms}}$.

Step 1: Find the number density and mean free path The printed volume is $25 \times 10^{-3}\,\mathrm{m}^3 = 0.025\,\mathrm{m}^3$, and one mole contains $N_A = 6.022 \times 10^{23}$ molecules. Therefore

$$n = \frac{N_A}{V} = 2.409 \times 10^{25}\,\mathrm{m}^{-3}.$$

The molecular diameter is $d = 0.3\,\mathrm{nm} = 3.0 \times 10^{-10}\,\mathrm{m}$. Thus

$$\lambda = \frac{1}{\sqrt{2}\pi(3.0 \times 10^{-10})^2(2.409 \times 10^{25})} \simeq 1.04 \times 10^{-7}\,\mathrm{m}.$$

The diameter must be converted to metres before squaring; a nanometre conversion error changes the result dramatically.

Step 2: Calculate and compare the rate The given rms speed is $200\,\mathrm{m\,s}^{-1}$, so the mean speed is

$$\bar{v} = \sqrt{\frac{8}{3\pi}}\,(200) \simeq 184\,\mathrm{m\,s}^{-1}.$$

The collision frequency is consequently

$$z = \frac{184}{1.04 \times 10^{-7}} \simeq 1.8 \times 10^9\,\mathrm{s}^{-1}.$$

Using the rms speed itself as an estimate gives $1.9 \times 10^9\,\mathrm{s}^{-1}$, which has the same scale. Counting collisions for the entire gas, instead of per molecule, would answer a different question. No standard mean-speed versus rms-speed convention can introduce a factor of hundreds.

Step 3: Conclusion The printed data give approximately $1.8 \times 10^9\,\mathrm{s}^{-1}$ per molecule. None of the four printed options is consistent with this result. The supplied key selects (B), approximately $10^{12}\,\mathrm{s}^{-1}$, but $\sqrt{2}\pi d^2(N_A/V)\bar{v}$ directly evaluates to the value above; the printed volume, diameter and speed do not support the keyed value. Even (D), the closest listed rate, is several times larger, so no unique correct option is asserted.

Quick Tip to Solve: Check the power of ten in $d^2 N_A/V$ before comparing rates; the requested frequency is per molecule, not for all molecules.

36. The electric field of a plane electromagnetic wave is given by

$$\overrightarrow{E} = E_0\widehat{i}\cos(kz)cos(\omega t)$$

The corresponding magnetic field $\overrightarrow{B}$ is then given by [4 marks]

(A) $\overrightarrow{B} = \frac{E_0}{C}\widehat{j}\sin(kz)\sin(\omega t)$

(B) $\overrightarrow{B} = \frac{E_0}{C}\widehat{j}\sin(kz)\cos(\omega t)$

(C) $\overrightarrow{B} = \frac{E_0}{C}\widehat{j}\cos(kz)\sin(\omega t)$

(D) $\overrightarrow{B} = \frac{E_0}{C}\widehat{k}\sin(kz)\cos(\omega t)$

Correct Answer: (A) $\overrightarrow{B} = \frac{E_0}{C}\widehat{j}\sin(kz)\sin(\omega t)$

Solution:

Concept:

- Faraday's law in differential form is $\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$. A field along x varying with z gives the curl along y.
- For an electromagnetic wave in vacuum, $\omega = ck$; integrating with respect to time determines the oscillating magnetic field.

Step 1: Differentiate the electric field The field has only an x-component, $E_x = E_0\cos(kz)\cos(\omega t)$. Thus the relevant curl component is

$$(\nabla \times \mathbf{E})_y = \frac{\partial E_x}{\partial z} = -kE_0\sin(kz)\cos(\omega t).$$

Faraday's minus sign then gives $\partial B_y/\partial t = kE_0\sin(kz)\cos(\omega t)$.

Step 2: Integrate and compare the options Integrating the oscillating field with respect to time gives

$$B_y = \frac{kE_0}{\omega}\sin(kz)\sin(\omega t) = \frac{E_0}{c}\sin(kz)\sin(\omega t).$$

There is no additional static magnetic field in the specified wave. Options with $\cos(\omega t)$ have the wrong time derivative, and the option with $\cos(kz)$ has the wrong spatial dependence. A z-directed magnetic field would also violate the transverse wave geometry.

Step 3: Conclusion

$$\mathbf{B} = \frac{E_0}{c}\widehat{\mathbf{j}}\sin(kz)\sin(\omega t).$$

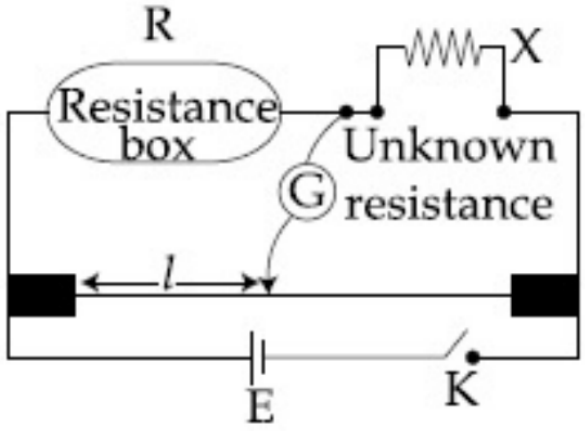
Hence, the correct option is (A).

Quick Tip to Solve: For a product of separate space and time factors, use Maxwell's curl law rather than applying the travelling-wave relation point by point.

37. In a meter bridge experiment, the circuit diagram and the corresponding observation table are shown in figure.

SI. No.	R(Ω)	l(cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the readings is inconsistent? [4 marks]



(A) 4

(B) 3

(C) 2

(D) 1

Correct Answer: (A) 4

Solution:

Concept:

- At the null point of a metre bridge, the ratio of the gap resistances equals the ratio of the corresponding wire lengths: $R/X = l/(100 - l)$.
- The unknown resistance X must be approximately the same in all trials; small differences arise from experimental error.

Step 1: Calculate the unknown resistance in each trial The resistance box is in the left gap and l is measured from the left end, so

$$X = R\frac{100 - l}{l}.$$

For the four readings this gives

$$X_1 = 1000\frac{40}{60} = 666.7\,\Omega,$$
$$X_2 = 100\frac{87}{13} = 669.2\,\Omega,$$
$$X_3 = 10\frac{98.5}{1.5} = 656.7\,\Omega,$$
$$X_4 = 1\frac{99}{1} = 99\,\Omega.$$

Step 2: Identify the inconsistent reading The first three estimates cluster around $6.6 \times 10^2\,\Omega$; their small spread is compatible with the limited precision of the recorded balance lengths. The fourth estimate is only $99\,\Omega$, far outside that cluster. For $R = 1\,\Omega$ and $X \simeq 667\,\Omega$, the expected balance length is $100/(668) \simeq 0.15\,\text{cm}$, rather than $1.0\,\text{cm}$. Therefore readings 1, 2 and 3 do not provide the outlier.

Step 3: Conclusion Reading 4 is inconsistent with the common unknown resistance. Hence, the correct option is (A).

Quick Tip to Solve: Calculate the same unknown resistance from every row; do not judge consistency merely by whether the balance length increases with resistance.

---

38. Two coaxial discs, having moments of inertia I_1 and $I_1/2$, are rotating with respective angular velocities ω_1 and $\omega_1/2$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then (E_f - E_i) is: [4 marks]

(A) $\frac{I_1\omega_1^2}{24}$

(B) $\frac{I_1\omega_1^2}{12}$

(C) $\frac{3}{8}I_1\omega_1^2$

(D) $\frac{I_1\omega_1^2}{6}$

Correct Answer: All printed options are positive, whereas the requested final minus initial energy is negative.

Solution:

Concept:

- With no external torque about the common axis, total angular momentum is conserved when the two discs come into contact.
- Friction makes their angular velocities equal and converts some rotational kinetic energy into heat; final kinetic energy must be lower.

Step 1: Find the common angular velocity The initial angular momentum is

$$L_i = I_1\omega_1 + \frac{I_1}{2}\frac{\omega_1}{2} = \frac{5}{4}I_1\omega_1.$$

The combined moment of inertia is $3I_1/2$, so conservation gives

$$\Omega = \frac{L_i}{3I_1/2} = \frac{5}{6}\omega_1.$$

This lies between the two initial angular velocities, as required by frictional equalisation.

Step 2: Subtract the initial energy from the final energy Using $E = I\omega^2/2$,

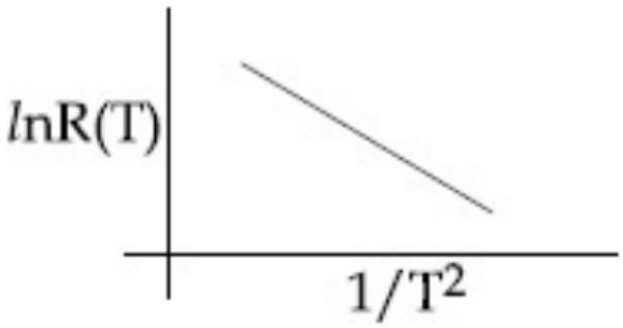
$$E_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}\frac{I_1}{2}\frac{\omega_1^2}{4} = \frac{9}{16}I_1\omega_1^2,$$
$$E_f = \frac{1}{2}\frac{3I_1}{2}\frac{25\omega_1^2}{36} = \frac{25}{48}I_1\omega_1^2,$$
$$E_f - E_i = \left(\frac{25}{48} - \frac{27}{48}\right)I_1\omega_1^2 = -\frac{I_1\omega_1^2}{24}.$$

All four printed choices are positive, so none equals the requested signed difference. Their magnitudes other than the first also disagree with the direct energy calculation.

Step 3: Conclusion The literal quantity asked is $E_f - E_i = -I_1\omega_1^2/24$, so no printed option is correct. The question-sheet key selects (A), which is the positive energy loss $E_i - E_f$, but has the opposite sign to the printed demand.

Quick Tip to Solve: Calculate the signed quantity exactly as written. Frictional energy loss is positive, while final minus initial kinetic energy is negative.

39. In an experiment, the resistance of a material is plotted as a function of temperature (in some range). As shown in the figure, it is a straight line. One may conclude that : [4 marks]



(A) $R(T) = R_0 e^{-T^2/T_0^2}$
(B) $R(T) = \frac{R_0}{T^2}$
(C) $R(T) = R_0 e^{T^2/T_0^2}$
(D) $R(T) = R_0 e^{-T_0^2/T^2}$

Correct Answer: (D) $R(T) = R_0 e^{-T_0^2/T^2}$

Solution:

Concept:

- The plotted variables are $y = \ln R(T)$ and $x = 1/T^2$, rather than R and T themselves.
- A straight line of negative slope satisfies $y = a - bx$, with $b > 0$. Exponentiating converts a linear logarithmic relation into an exponential law.

Step 1: Translate the axes into an equation The graph falls as $1/T^2$ increases, so the coefficient of $1/T^2$ in $\ln R$ is negative. Write

$$\ln R(T) = \ln R_0 - T_0^2 \left(\frac{1}{T^2}\right).$$

Here $\ln R_0$ is the intercept and $-T_0^2$ is the slope. The ratio T_0^2/T^2 is dimensionless, as an exponential argument must be.

Step 2: Exponentiate and reject the other dependences Taking exponentials gives

$$R(T) = R_0 \exp\left(-\frac{T_0^2}{T^2}\right).$$

The laws with exponents $\pm T^2/T_0^2$ give $\ln R = \ln R_0 \pm 1/(T_0^2 x)$, which is not linear in $x = 1/T^2$. The inverse-square resistance law instead gives $\ln R = \ln R_0 + \ln x$, also a curved graph. Only the negative inverse-square exponent supplies a constant negative slope on these axes.

Step 3: Conclusion $R(T) = R_0 e^{-T_0^2/T^2}$. Hence, the correct option is (D).

Quick Tip to Solve: Read the axis labels before selecting a functional form: a straight line of logarithm versus inverse-square temperature implies an exponential in that inverse square.

---

40. A proton, an electron, and a Helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicualr to the plane. Let r_p, r_e and r_{He} be their respective radii, then [4 marks]

(A) $r_e < r_p < r_{He}$

(B) $r_e < r_p = r_{He}$

(C) $r_e > r_p > r_{He}$

(D) $r_e > r_p = r_{He}$

Correct Answer: (B) $r_e < r_p = r_{He}$

Solution:

Concept:

- The magnetic Lorentz force provides centripetal force: $|q|vB = mv^2/r$, hence $r = p/(|q|B)$.
- For the same nonrelativistic kinetic energy K, momentum is $p = \sqrt{2mK}$, so the orbit radius is proportional to $\sqrt{m}/|q|$.

Step 1: Compare the electron and proton Both have the same charge magnitude e, and their kinetic

energies and field strengths are equal. Therefore

$$\frac{r_e}{r_p} = \sqrt{\frac{m_e}{m_p}} \ll 1.$$

The electron has the smaller radius. Its negative charge reverses the direction of circulation, but does not change the positive magnitude of the radius. Thus both choices that put $r_e > r_p$ are excluded.

Step 2: Compare the helium nucleus and proton A helium nucleus has approximately four proton masses and charge $+2e$. Consequently,

$$\frac{r_{\mathrm{He}}}{r_p} = \sqrt{\frac{4m_p}{m_p}}\,\frac{e}{2e} = 1.$$

The increase by a factor of two in momentum is exactly offset by the doubled charge. The choice claiming $r_p < r_{\mathrm{He}}$ neglects this charge factor. The comparison concerns radius magnitudes, and the common energy means kinetic energy in this magnetic-orbit problem.

Step 3: Conclusion The radii satisfy $r_e < r_p = r_{\mathrm{He}}$. Hence, the correct option is (B).

Quick Tip to Solve: At equal kinetic energy compare $\sqrt{m}/|q|$; at equal speed the appropriate comparison would instead be $m/|q|$.

41. The value of acceleration due to gravity at Earth's surface is $9.8\,\mathrm{m\,s^{-2}}$. The altitude above its surface at which the acceleration due to gravity decreases to $4.9\,\mathrm{m\,s^{-2}}$, is close to: (Radius of earth $= 6.4 \times 10^6$ m). [4 marks]

(A) 1.6×10^6 m

(B) 9.0×10^6 m

(C) 6.4×10^6 m

(D) 2.6×10^6 m

Correct Answer: (D) 2.6×10^6 m

Solution:

Concept:

- Outside a spherical Earth, gravitational acceleration is $g(r) = GM/r^2$, where r is measured from the centre.
- At altitude h, the centre distance is $R + h$, giving $g_h/g_0 = [R/(R+h)]^2$.

Step 1: Use the given ratio of accelerations The acceleration changes from 9.8 to $4.9\,\mathrm{m\,s^{-2}}$, so it is halved:

$$\frac{1}{2} = \left(\frac{R}{R+h}\right)^2.$$

Taking the positive square root, since both distances are positive, gives

$$R + h = \sqrt{2}\,R, \qquad h = (\sqrt{2} - 1)R.$$

The requested altitude is the excess over R, not the total centre distance.

Step 2: Substitute the radius and compare the choices With $R = 6.4 \times 10^6\,\text{m}$,

$$h \simeq 0.4142(6.4 \times 10^6) = 2.65 \times 10^6\,\text{m}.$$

This is closest to $2.6 \times 10^6\,\text{m}$. The $9.0 \times 10^6\,\text{m}$ choice approximately represents the centre distance $R + h$. At $h = R$, gravity would fall to one quarter, not one half, ruling out $6.4 \times 10^6\,\text{m}$. The remaining smaller altitude cannot provide the required inverse-square reduction. The small-altitude linear approximation is unsuitable because h/R is about 0.41.

Step 3: Conclusion The required altitude is approximately $2.65 \times 10^6\,\text{m}$, closest to $2.6 \times 10^6\,\text{m}$. Hence, the correct option is (D).

Quick Tip to Solve: When gravity is reduced by a large fraction, use the exact inverse-square relation and subtract Earth's radius only after finding the centre distance.

42. A particle of mass m is moving along a trajectory given by

$x = x_0 + a\cos\omega_1 t$

$y = y_0 + b\sin\omega_2 t$

The torque, acting on the particle about the origin, at t = 0 is : [4 marks]

(A) Zero

(B) $+my_0a\,\omega_1^2\widehat{k}$

(C) $-m\left(x_0b\omega_2^2 - y_0a\omega_1^2\right)\widehat{k}$

(D) $m(-x_0b + y_0a)\omega_1^2\widehat{k}$

Correct Answer: (B) $+my_0a\,\omega_1^2\widehat{k}$

Solution:

Concept:

- Torque about the origin is $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = m(\mathbf{r} \times \mathbf{a})$. In a plane its z-component is $m(x\ddot{y} - y\ddot{x})$.
- The acceleration follows by differentiating each coordinate twice; the sine and cosine factors have different values at $t = 0$.

Step 1: Find the position and acceleration at the specified instant The given coordinates imply

$$\ddot{x} = -a\omega_1^2\cos(\omega_1 t), \quad \ddot{y} = -b\omega_2^2\sin(\omega_2 t).$$

At $t = 0$, use $\cos 0 = 1$ and $\sin 0 = 0$:

$$\mathbf{r}(0) = (x_0 + a)\widehat{\mathbf{i}} + y_0\widehat{\mathbf{j}}, \qquad \mathbf{F}(0) = -ma\omega_1^2\widehat{\mathbf{i}}.$$

In particular, the y-component of force vanishes, even though the y-component of velocity need not vanish.

Step 2: Take the cross product Using $\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$,

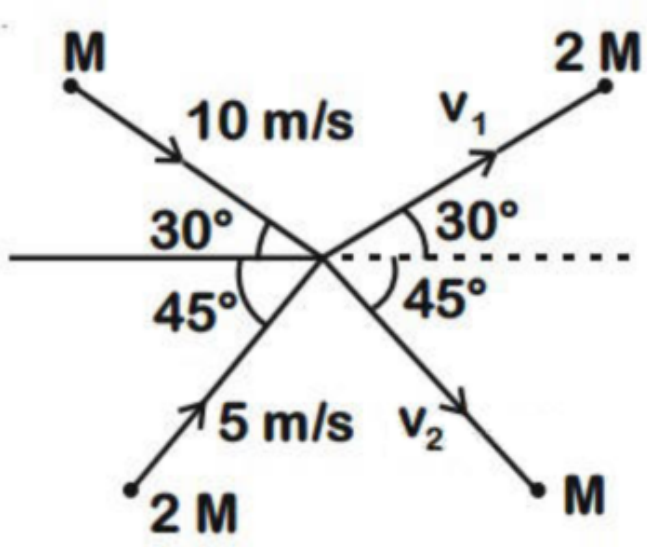
$$\boldsymbol{\tau}(0) = \left[(x_0 + a)\hat{\mathbf{i}} + y_0\hat{\mathbf{j}}\right] \times \left[-ma\omega_1^2\hat{\mathbf{i}}\right] = my_0a\omega_1^2\hat{\mathbf{k}}.$$

The x-coordinate cannot contribute because the force points along x. Terms involving $x_0b\omega_2^2$ incorrectly retain a nonzero sine at zero time; replacing ω_2 with ω_1 does not fix that error. Torque is not generally zero because the line of force is displaced from the origin by y_0.

Step 3: Conclusion $\boldsymbol{\tau}(0) = +my_0a\omega_1^2\hat{\mathbf{k}}$. Hence, the correct option is (B).

Quick Tip to Solve: Evaluate the acceleration at the requested instant before expanding the torque; zero sine terms often eliminate most of the calculation.

43. Two particles, of masses M and 2M, moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds u_1 and u_2, respectively. The values of u_1 and u_2 are nearly : [4 marks]



(A) 6.5 m/s and 3.2 m/s

(B) 6.5 m/s and 6.3 m/s

(C) 3.2 m/s and 6.3 m/s

(D) 3.2 m/s and 12.6 m/s

Correct Answer: (B) 6.5 m/s and 6.3 m/s

Solution:

Concept:

- In the absence of an external impulse, both Cartesian components of total momentum are conserved. Resolve velocities using the angles measured from the horizontal in the diagram.
- An elastic collision must also conserve kinetic energy, $K = \frac{1}{2}mv^2$. This provides a separate consistency check on the given directions and speeds.

Step 1: Identify the incoming momentum Take rightward and upward as positive. The incoming particle of mass M has components $(10\cos 30^\circ, -10\sin 30^\circ)$; that of mass $2M$ has components $(5\cos 45^\circ, 5\sin 45^\circ)$. Consequently,

$$p_x/M = 5\sqrt{3} + 5\sqrt{2}, \qquad p_y/M = -5 + 5\sqrt{2}.$$

Use the indicated upper and lower outgoing rays to identify the two requested speeds, in that order.

Step 2: Resolve the outgoing momentum The diagram puts mass $2M$ on the upper ray at 30°, with speed u_1, and mass M on the lower ray at 45°, with speed u_2. The momentum equations are

$$\sqrt{3}u_1 + u_2/\sqrt{2} = 5\sqrt{3} + 5\sqrt{2},$$

$$u_1 - u_2/\sqrt{2} = -5 + 5\sqrt{2}.$$

Adding eliminates the lower-ray contribution, giving

$$u_1 = \frac{5\sqrt{3} + 10\sqrt{2} - 5}{\sqrt{3} + 1} \simeq 6.52\,\mathrm{m\,s^{-1}}.$$

Then $u_2 = \sqrt{2}(u_1 + 5 - 5\sqrt{2}) \simeq 6.29\,\mathrm{m\,s^{-1}}$.

Step 3: Check the elastic-collision statement The momentum-based pair is the one listed in the second choice. The alternatives with a speed near $3.2\,\mathrm{m\,s^{-1}}$, or with $12.6\,\mathrm{m\,s^{-1}}$, fail these simultaneous component equations. However,

$$K_i = \tfrac{1}{2}M(10)^2 + \tfrac{1}{2}(2M)(5)^2 = 75M,$$

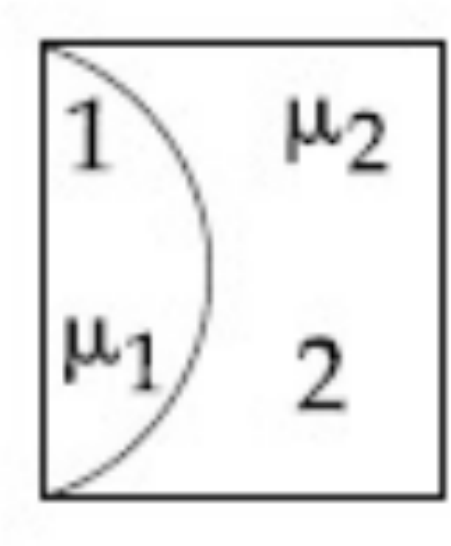
$$K_f = \tfrac{1}{2}(2M)(6.52)^2 + \tfrac{1}{2}M(6.29)^2 \simeq 62.2M.$$

Thus the printed directions are incompatible with an elastic collision. They support the intended momentum answer, but no listed pair satisfies all the printed physical conditions.

Step 4: Conclusion The intended answer from the indicated rays is $u_1 \simeq 6.5\,\mathrm{m\,s^{-1}}$, $u_2 \simeq 6.3\,\mathrm{m\,s^{-1}}$, corresponding to option (B). Strictly, the additional word "elastically" makes the printed data inconsistent, since kinetic energy is not conserved.

Quick Tip to Solve: After resolving momentum in two directions, check kinetic energy separately whenever a collision is called elastic.

44. One plano-convex and one plano-concave lens of same radius of curvature 'R' but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is : [4 marks]



(A) $\frac{2R}{\mu_1-\mu_2}$
(B) $\frac{R}{2-(\mu_1-\mu_2)}$
(C) $\frac{R}{\mu_1-\mu_2}$
(D) $\frac{R}{2(\mu_1-\mu_2)}$

Correct Answer: (C) $\frac{R}{\mu_1-\mu_2}$

Solution:

Concept:

- For a thin lens in air, the lens-maker formula is $P = 1/f = (\mu - 1)(1/R_1 - 1/R_2)$, with Cartesian signs for the surface radii.
- Powers of thin lenses in contact add: $P = P_1 + P_2$. A plane surface has zero curvature, so its reciprocal radius is zero.

Step 1: Assign the powers of the component lenses The figure shows a plano-convex piece of index μ_1 fitting into a plano-concave piece of index μ_2. Both curved surfaces have the same radius magnitude R, and the outer faces are plane. Consider each piece first as a thin lens surrounded by air. For the convex piece, the left surface is plane and the right curved surface has radius $-R$, so

$$P_1 = (\mu_1 - 1)\left(0 - \frac{1}{-R}\right) = \frac{\mu_1 - 1}{R}.$$

The positive sign expresses its converging power when $\mu_1 > 1$.

Step 2: Add the positive and negative powers The concave piece has its curved first surface of radius

$-R$ and its second surface plane. Its power is therefore

$$P_2 = (\mu_2 - 1)\left(\frac{1}{-R} - 0\right) = -\frac{\mu_2 - 1}{R}.$$

In the thin-contact approximation the intermediate air interfaces cancel in the sum, leaving the power of the actual combination:

$$\frac{1}{f} = P_1 + P_2 = \frac{\mu_1 - 1 - (\mu_2 - 1)}{R} = \frac{\mu_1 - \mu_2}{R}.$$

It follows that $f = R/(\mu_1 - \mu_2)$.

Step 3: Test the limiting case and the alternatives When the two indices are equal, the curved internal boundary produces no refraction. The combination then becomes a plane-parallel slab and has zero power, so its focal length must tend to infinity. The expression containing $2 - (\mu_1 - \mu_2)$ fails this test. The expressions with an extra factor of two in the numerator or denominator have the wrong power: only one curved interface contributes, not two independently exposed curved surfaces. If $\mu_1 > \mu_2$, the result is positive; reversing that inequality makes the assembly diverging.

Step 4: Conclusion The focal length is $f = \dfrac{R}{\mu_1 - \mu_2}$. Hence, the correct option is (C).

Quick Tip to Solve: For complementary lens pieces with plane outer faces, the net power depends on the difference of their refractive indices.

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45. A moving coil galvanometer allows a full scale current of 10^{-4} A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0-5 V. Therefore the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0.10 mA is : [4 marks]

(A) 200 Ω

(B) 500 Ω

(C) 100 Ω

(D) None of the options are correct

Correct Answer: (D) None of the options are correct

Solution:

Concept:

- A galvanometer of resistance G and full-scale current I_g, in series with resistance R_s, measures full-scale voltage $V = I_g(G + R_s)$.
- For a shunted ammeter, equal voltages across the parallel branches give $I_g G = (I - I_g)S$. A passive resistance cannot be negative.

Step 1: Use the voltmeter data exactly as printed The given full-scale current is $I_g = 10^{-4}\,\text{A}$, the series resistance is $R_s = 2 \times 10^6\,\Omega$, and the voltmeter range is $5\,\text{V}$. Thus the total resistance needed

for this voltage and current would be

$$G + R_s = \frac{V}{I_g} = \frac{5}{10^{-4}} = 5 \times 10^4\,\Omega.$$

Subtracting the stated series resistance gives

$$G = 5 \times 10^4 - 2 \times 10^6 = -1.95 \times 10^6\,\Omega.$$

A conventional moving-coil galvanometer cannot have this negative resistance. The supplied voltmeter data are already inconsistent.

Step 2: Examine the ammeter range independently The desired upper limit is printed as $0.10\,\mathrm{mA}$, namely

$$I = 0.10 \times 10^{-3}\,\mathrm{A} = 10^{-4}\,\mathrm{A} = I_g.$$

At full scale, the entire current is therefore already equal to the galvanometer current. There is no excess current to divert through a finite shunt. Formally,

$$S = \frac{I_g G}{I - I_g}$$

has zero denominator. For a normal positive galvanometer resistance, this limit means an open shunt, not any of the finite values offered.

Step 3: Compare the proposed resistances A finite shunt of $200\,\Omega$, $500\,\Omega$, or $100\,\Omega$ would carry additional positive current and increase the total full-scale range above I_g. None can produce the printed range, even if the inconsistent voltmeter information is set aside. In addition, substituting the voltmeter data literally cannot produce a physically acceptable galvanometer resistance. It would be unjustified to change either the megohm value or the decimal range to obtain one of the numerical choices.

Step 4: Conclusion No stated finite shunt resistance satisfies the printed data. Hence, the correct option is (D), "None of the options are correct."

Quick Tip to Solve: Before calculating a shunt, check that the target ammeter current exceeds the galvanometer full-scale current and that its inferred resistance is positive.

46. In a photoelectric effect experiment the threshold wavelength of the light is 380 nm. If the wavelentgh of incident light is 260 nm, the maximum kinetic energy of emitted electrons will be:
Given E (in eV) = 1237/λ (in nm) [4 marks]

(A) 4.5 eV

(B) 15.1 eV

(C) 3.0 eV

(D) 1.5 eV

Correct Answer: (D) 1.5 eV

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = E_\gamma - \phi$, where ϕ is the work function and E_γ is the incident photon energy.
- Threshold light ejects electrons with zero maximum kinetic energy, so $\phi = hc/\lambda_0$. Use the supplied conversion $E(\text{eV}) = 1237/\lambda(\text{nm})$.

Step 1: Calculate the work function The threshold wavelength is $\lambda_0 = 380\,\text{nm}$. Threshold photons provide just enough energy for an electron to leave the metal. Therefore

$$\phi = \frac{1237}{380}\,\text{eV} \simeq 3.2553\,\text{eV}.$$

This energy is consumed in overcoming the surface binding, and must be subtracted from the incident energy. The threshold wavelength itself is not the wavelength of the incoming photon in this experiment.

Step 2: Calculate the incident energy and the excess For the incident wavelength $\lambda = 260\,\text{nm}$,

$$E_\gamma = \frac{1237}{260}\,\text{eV} \simeq 4.7577\,\text{eV}.$$

Since this wavelength is shorter than the threshold wavelength, the photon energy exceeds the work function and emission is possible. The maximum electron energy is

$$\begin{aligned} K_{\max} &= 1237\left(\frac{1}{260} - \frac{1}{380}\right)\text{eV} \\ &= \frac{1237 \times 120}{260 \times 380}\,\text{eV} \\ &\simeq 1.5024\,\text{eV}. \end{aligned}$$

The maximum refers to electrons that reach the surface without losing some of this available excess energy in collisions.

Step 3: Reject the other energies The value $4.5\,\text{eV}$ is close to the total incident photon energy and neglects the substantial work function. The value $15.1\,\text{eV}$ exceeds the photon energy, violating energy conservation. The value $3.0\,\text{eV}$ is about twice the calculated excess and does not follow from the given wavelengths. Rounding the correctly subtracted energy to the precision of the alternatives gives $1.5\,\text{eV}$, rather than rounding each wavelength-dependent contribution prematurely.

Step 4: Conclusion The maximum kinetic energy is approximately $1.5\,\text{eV}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the difference of reciprocal wavelengths directly to avoid confusing the photon energy with the emitted electron energy.

47. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma u^2$ (where m is mass of the ball, u is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is : [4 marks]

(A) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(B) $\frac{1}{\sqrt{\gamma g}}ln\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$

(C) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(D) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

Correct Answer: (A) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

Solution:

Concept:

- During ascent, both gravity and drag act downward. With upward velocity $u > 0$, Newton's law gives $du/dt = -g - \gamma u^2$.
- Separation of variables converts the rise time into an integral over speed. The identity $\int dz/(1+z^2) = \tan^{-1} z + C$ evaluates it.

Step 1: Write the equation of motion with signs Take upward as positive. The weight is $-mg$, and the drag opposing upward motion is $-m\gamma u^2$. Hence

$$m\frac{du}{dt} = -mg - m\gamma u^2, \qquad dt = -\frac{du}{g+\gamma u^2}.$$

The mass cancels because both forces are proportional to it. This equation applies over the ascent, from initial velocity V_0 to the zero velocity at the highest point; no descent equation is required.

Step 2: Integrate between the physical limits Let T be the time to the zenith. Reverse the velocity limits to remove the negative sign:

$$T = \int_0^{V_0} \frac{du}{g+\gamma u^2}.$$

Use $z = u\sqrt{\gamma/g}$, so $du = \sqrt{g/\gamma}\,dz$. Then

$$T = \frac{1}{\sqrt{\gamma g}} \int_0^{V_0\sqrt{\gamma/g}} \frac{dz}{1+z^2}.$$

Thus

$$T = \frac{1}{\sqrt{\gamma g}} \tan^{-1}\left(V_0\sqrt{\frac{\gamma}{g}}\right).$$

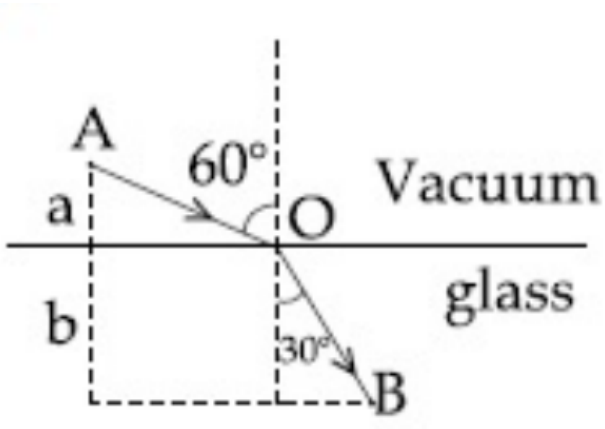
The inverse tangent is taken on its usual nonnegative principal branch for nonnegative initial speed.

Step 3: Check the limit and distinguish the forms For weak drag, $\tan^{-1} z \simeq z$, and this expression tends to V_0/g, the usual vacuum rise time. For positive drag, $\tan^{-1} z < z$, so the rise time is shorter, as expected from the additional downward force. A logarithm corresponds to a different denominator structure, while an inverse sine arises from a square-root quadratic denominator. Neither integrates $1/(g+\gamma u^2)$. The expression with 2γ would describe twice the specified drag coefficient and is therefore also excluded. Dimensional consistency follows from $[\gamma] = \text{length}^{-1}$.

Step 4: Conclusion The rise time is $T = \frac{1}{\sqrt{\gamma g}} \tan^{-1}\left(V_0\sqrt{\frac{\gamma}{g}}\right)$. Hence, the correct option is (A).

Quick Tip to Solve: During upward motion, add the magnitudes of gravity and drag in the speed-decay equation; the quadratic denominator produces an inverse tangent.

48. A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure. The optical path length of light ray from A to B is: [4 marks]



(A) 2a + 2b/$\sqrt{3}$

(B) 2a + 2b/3

(C) 2a + 2b

(D) 2$\sqrt{3}$/a + 2b

Correct Answer: (C) 2a + 2b

Solution:

Concept:

- Optical path length is $\mathcal{L} = \int n\, ds$, the refractive index times the geometrical distance in each uniform medium.
- Snell's law at a vacuum-glass interface gives $n = \sin i / \sin r$. Angles of incidence and refraction are measured from the normal.

Step 1: Find the refractive index The incident ray makes $60°$ with the normal, while the refracted ray

makes 30°. The refractive index of vacuum is one, so

$$n = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}.$$

The glass part of the path must therefore be weighted by $\sqrt{3}$; merely adding the geometrical lengths would not give the optical path.

Step 2: Find the actual distances along the rays The diagram labels the perpendicular distance of A above the interface as a, and that of B below the interface as b. These are normal projections of the two sloping ray segments. Hence

$$AO\cos 60^\circ = a, \qquad OB\cos 30^\circ = b.$$

The path lengths are consequently

$$AO = \frac{a}{\cos 60^\circ} = 2a, \qquad OB = \frac{b}{\cos 30^\circ} = \frac{2b}{\sqrt{3}}.$$

Using the angles to the interface instead of the normal would interchange sine and cosine and give incorrect projections.

Step 3: Weight each segment by its medium The total optical path from A to B is

$$\begin{aligned}\mathcal{L} &= 1 \times AO + n \times OB \\ &= 2a + \sqrt{3}\left(\frac{2b}{\sqrt{3}}\right) \\ &= 2a + 2b.\end{aligned}$$

The alternative $2a + 2b/\sqrt{3}$ is the unweighted geometrical path and misses the slower propagation through glass. The expression $2a + 2b/3$ applies the refractive factor in the wrong direction. The remaining expression has a term proportional to $1/a$, inconsistent with the dimensions of an optical length, and also disagrees with the vacuum ray projection. Both distances contribute positively, with the same length units as a and b.

Step 4: Conclusion The optical path length is $\boxed{2a + 2b}$. Hence, the correct option is (C).

Quick Tip to Solve: First convert normal depths into ray lengths with cosines; only then multiply each ray length by the refractive index of its medium.

49. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10A, then the input voltage and current in the primary coil are : [4 marks]

(A) 220 V and 20 A

(B) 220 V and 10A

(C) 440 V and 5A

(D) 440 V and 20 A

Correct Answer: (C) 440 V and 5A

Solution:

Concept:

- For an ideal transformer, $V_p/V_s = N_p/N_s$, where N_p, N_s are the primary and secondary turn counts.
- Neglecting losses, input and output powers are equal: $V_pI_p = V_sI_s$. Thus the current ratio is inverse to the turns ratio.

Step 1: Recover the secondary voltage from the output The output power is $P_s = 2.2\,\text{kW} = 2200\,\text{W}$, and the secondary current is $I_s = 10\,\text{A}$. Under the ideal-transformer conditions implicit in the question,

$$V_s = \frac{P_s}{I_s} = \frac{2200}{10} = 220\,\text{V}.$$

This is the secondary voltage, not the requested primary voltage. Treating it immediately as the input voltage would ignore the change in turn count between the coils.

Step 2: Apply the turns ratio to the voltage The primary contains 300 turns and the secondary 150. Therefore

$$\frac{V_p}{V_s} = \frac{300}{150} = 2, \qquad V_p = 2(220) = 440\,\text{V}.$$

There are fewer turns on the secondary, so this is a step-down transformer: the secondary voltage must be smaller than the primary voltage. The proposed input values of $220\,\text{V}$ fail this turns-ratio condition.

Step 3: Use power conservation to determine the current With no specified losses or efficiency correction, use equal input and output powers:

$$I_p = \frac{P_s}{V_p} = \frac{2200}{440} = 5\,\text{A}.$$

The same result follows from $I_p/I_s = N_s/N_p = 1/2$. This inverse relation is essential: increasing primary voltage relative to secondary voltage decreases primary current for the same transmitted power. The pair $440\,\text{V}, 20\,\text{A}$ would imply $8800\,\text{W}$ input, four times the stated output, and is incompatible with the ideal model. The $220\,\text{V}$ pairs also fail the voltage transformation even if one were to check their powers separately.

Step 4: Conclusion The primary voltage is $440\,\text{V}$ and the primary current is $5\,\text{A}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the secondary voltage from output power first; then use turns for voltage and conserved power for the primary current.

50. A uniformly charged ring of radius 3a and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed u at z = 4a. The minimum value of u such that it crosses the origin is : [4 marks]

(A) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(B) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(C) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(D) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

Correct Answer: (A) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

Solution:

Concept:

- The potential on the axis of a uniformly charged ring of radius R, at axial distance z, is $V(z) = kq/\sqrt{R^2+z^2}$, where $k = 1/(4\pi\varepsilon_0)$.
- The potential energy of a second charge q is $U = qV$. Conservation of mechanical energy gives the initial kinetic energy needed to overcome the increase in potential energy.

Step 1: Find the initial and central potentials Here the radius is $R = 3a$ and the charge starts at $z = 4a$. Every element of the ring is then at distance $\sqrt{9a^2+16a^2} = 5a$ from the moving charge. Hence

$$V_i = \frac{kq}{5a}, \qquad V_O = \frac{kq}{3a}.$$

The potential rises as the charge approaches the origin, because its distance from every ring element decreases. Thus the origin is the highest potential-energy point on the axial path.

Step 2: Apply energy conservation at the threshold For the limiting initial speed, the kinetic energy at the origin tends to zero. If m is the mass of the moving point charge,

$$\frac{1}{2}mu_{\min}^2 = U_O - U_i = kq^2\left(\frac{1}{3a} - \frac{1}{5a}\right) = \frac{2kq^2}{15a}.$$

Therefore

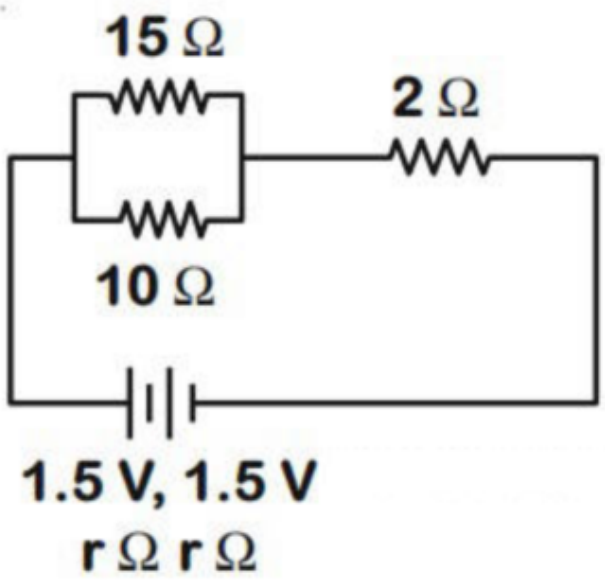
$$u_{\min} = \sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}.$$

A speed just above this threshold takes the charge through the origin with nonzero speed. The other options use an incorrect energy barrier: the required coefficient is $1/3 - 1/5 = 2/15$, rather than $1/15$, $1/5$ or $4/15$.

Step 3: Conclusion The threshold speed is $\sqrt{2/m}\left[(2/15)q^2/(4\pi\varepsilon_0 a)\right]^{1/2}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the difference of the ring potentials, not the potential at the centre alone; the starting distance from a ring element is the 3-4-5 triangle distance $5a$.

51. In the given circuit, an ideal voltmeter connected across the 10Ω resistance reads 2V. The internal resistance r, of each cell is: [4 marks]



(A) 1.5 Ω

(B) 1 Ω

(C) 0.5 Ω

(D) 0 Ω

Correct Answer: (C) 0.5 Ω

Solution:

Concept:

- An ideal voltmeter draws no current, so its reading is the potential difference across the parallel $10\,\Omega$ and $15\,\Omega$ branches.
- For cells connected in series aiding, the emfs and internal resistances add: $\mathcal{E} = 3\,\text{V}$ and $r_{\text{total}} = 2r$. Kirchhoff's loop rule includes both external and internal voltage drops.

Step 1: Find the current delivered by the cells The two parallel resistors have the same potential difference $2\,\text{V}$. Their separate currents are

$$I_{10} = \frac{2}{10} = \frac{1}{5}\,\text{A}, \qquad I_{15} = \frac{2}{15}\,\text{A}.$$

Consequently the current in the series $2\,\Omega$ resistor and in both cells is

$$I = I_{10} + I_{15} = \frac{1}{3}\,\text{A}.$$

Equivalently, the parallel resistance is $10 \times 15/(10+15) = 6\,\Omega$, so the total external resistance is $8\,\Omega$.

Step 2: Account for the internal voltage drop The voltage across the external circuit is the parallel-

branch voltage plus the series-resistor voltage:

$$V_{\text{ext}} = 2 + \frac{1}{3}(2) = \frac{8}{3}\,\text{V}.$$

The cells supply $1.5 + 1.5 = 3\,\text{V}$. Their combined internal drop must therefore be $3 - 8/3 = 1/3\,\text{V}$. Using $I(2r) = 1/3$,

$$\frac{1}{3}(2r) = \frac{1}{3}, \qquad r = 0.5\,\Omega.$$

The result is for each cell, not for the pair. The choices 1.5, 1 and $0\,\Omega$ would give total currents $3/11$, $3/10$ and $3/8\,\text{A}$, respectively, and hence parallel voltages different from $2\,\text{V}$.

Step 3: Conclusion Each cell has internal resistance $r = 0.5\,\Omega$. Hence, the correct option is (C).

Quick Tip to Solve: First obtain the total current by adding the two parallel branch currents; divide the combined internal resistance by two only at the end.

52. n moles of an ideal gas with constant volume heat capcity C_V undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is : [4 marks]

(A) $\frac{nR}{C_V - nR}$
(B) $\frac{4nR}{C_V - nR}$
(C) $\frac{4nR}{C_V + nR}$
(D) $\frac{nR}{C_V + nR}$

Correct Answer: (D) $\frac{nR}{C_V + nR}$

Solution:

Concept:

- For an ideal gas undergoing an isobaric process, $W = P\Delta V = nR\Delta T$, where W is work done by the gas.
- The first law gives $Q = \Delta U + W$. Here C_V denotes the constant-volume heat capacity of the given gas sample, so $\Delta U = C_V \Delta T$; its constant-pressure heat capacity is $C_V + nR$.

Step 1: Express the work in terms of temperature change At constant pressure, subtract the ideal-gas equation for the initial state from that for the final state:

$$P(V_f - V_i) = nR(T_f - T_i).$$

Writing $\Delta T = T_f - T_i$, the work in the expansion is

$$W = P\Delta V = nR\Delta T.$$

Expansion means $\Delta V > 0$, and hence $\Delta T > 0$, so the gas does positive work. There is no extra factor of four in this expression; the number of moles is already included as n.

Step 2: Find the heat supplied and divide The temperature rise increases the internal energy by

$C_V\Delta T$. The heat supplied must provide this increase as well as the expansion work:

$$Q = C_V\Delta T + nR\Delta T = (C_V + nR)\Delta T.$$

Thus the common temperature interval cancels:

$$\frac{W}{Q} = \frac{nR\Delta T}{(C_V + nR)\Delta T} = \frac{nR}{C_V + nR}.$$

The ratio is positive and less than one because part of the heat raises the internal energy. Options (A) and (B) wrongly subtract the work contribution from the heat capacity. Option (C) uses the correct denominator but introduces an unsupported factor of four in the numerator. If a molar heat capacity were used instead, one would write $C_V = nc_{V,m}$ and obtain the equivalent ratio $R/(c_{V,m} + R)$.

Step 3: Conclusion The ratio of work done to heat supplied is $nR/(C_V + nR)$. Hence, the correct option is (D).

Quick Tip to Solve: At constant pressure, add the internal-energy rise and expansion work before taking W/Q; check whether the heat capacity supplied is molar or for the whole sample.

53. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is : [Given that R = 8.31 J mol^{-1} K^{-1}] [4 marks]

(A) 374 J

(B) 700 J

(C) 748 J

(D) 350 J

Correct Answer: (C) 748 J

Solution:

Concept:

- A rigid cylinder has constant volume, so its boundary work is $W = P\Delta V = 0$. The supplied heat equals the internal-energy increase.
- Helium is a monatomic ideal gas: its molar constant-volume heat capacity is $c_{V,m} = 3R/2$. At STP, one mole occupies $22.4\,\mathrm{L}$.

Step 1: Calculate the amount of helium The volume at STP is $67.2\,\mathrm{L}$. Using the STP molar volume gives

$$n = \frac{67.2}{22.4} = 3\,\mathrm{mol}.$$

The temperature is raised by $20°\mathrm{C}$, which is an interval of $20\,\mathrm{K}$. Celsius and kelvin temperature differences have the same numerical value; adding 273 would convert an absolute temperature and would be inappropriate here. The number of moles remains constant because the cylinder is closed.

Step 2: Use the constant-volume heat capacity Since the cylinder has fixed capacity, the gas cannot

expand. Therefore all the supplied heat increases its translational internal energy:

$$Q = \Delta U = nc_{V,m}\Delta T = n\frac{3}{2}R\Delta T.$$

Substitute the given gas constant and the calculated amount:

$$Q = 3 \times \frac{3}{2} \times 8.31 \times 20 = 747.9\,\text{J} \simeq 748\,\text{J}.$$

Using $5R/2$ would describe constant-pressure heating of a monatomic gas, which would include expansion work and does not apply to a rigid cylinder. The $374\,\text{J}$ choice is about half of the required heat, while 700 and $350\,\text{J}$ do not follow from the stated values. Keep the final rounding until after the product has been evaluated.

Step 3: Conclusion The heat needed is $747.9\,\text{J}$, or approximately $748\,\text{J}$. Hence, the correct option is (C).

Quick Tip to Solve: For a rigid container, immediately use $Q = nc_V\Delta T$; for helium select $c_V = 3R/2$, and convert the STP volume to moles first.

54. A current of 5 A passes through a copper conductor (resistivity = 1.7×10^{-8} Ωm) of radius of cross-section 5 mm. Find the mobility of the charges if their drift velocity is 1.1×10^{-3} m/s. [4 marks]

(A) 1.3 m^2/Vs

(B) 1.0 m^2/Vs

(C) 1.8 m^2/Vs

(D) 1.5 m^2/Vs

Correct Answer: (B) 1.0 m^2/Vs

Solution:

Concept:

- The current density in a uniform conductor is $J = I/A$, where its cross-sectional area is $A = \pi r^2$.
- The microscopic form of Ohm's law gives the electric-field magnitude $E = \rho J$. Charge-carrier mobility is the magnitude ratio $\mu = v_d/E = v_d A/(\rho I)$.

Step 1: Determine the electric field in the conductor The given radius is $5\,\text{mm} = 5 \times 10^{-3}\,\text{m}$. Thus

$$A = \pi(5 \times 10^{-3})^2 = 7.854 \times 10^{-5}\,\text{m}^2.$$

The current density is

$$J = \frac{5}{7.854 \times 10^{-5}} = 6.366 \times 10^4\,\text{A m}^{-2}.$$

Multiplying by resistivity, rather than resistance, gives the field:

$$E = (1.7 \times 10^{-8})(6.366 \times 10^4) = 1.082 \times 10^{-3}\,\text{V m}^{-1}.$$

A wire length is not needed because current density and resistivity already specify the local field.

Step 2: Divide the drift speed by the field magnitude The given drift speed is $v_d = 1.1 \times 10^{-3}\,\mathrm{m\,s^{-1}}$. Hence

$$\mu = \frac{1.1 \times 10^{-3}}{1.082 \times 10^{-3}} = 1.016\,\mathrm{m^2\,V^{-1}\,s^{-1}}.$$

To the precision of the choices this is $1.0\,\mathrm{m^2\,V^{-1}\,s^{-1}}$. Mobility is conventionally a positive magnitude, although electrons drift opposite to the electric field. The alternatives 1.3, 1.8 and 1.5 are not consistent with the computed field and speed: multiplying any of them by $1.082 \times 10^{-3}\,\mathrm{V/m}$ would predict a drift speed larger than the specified value.

Step 3: Conclusion The mobility is approximately $1.0\,\mathrm{m^2\,V^{-1}\,s^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Combine the definitions into $\mu = v_d \pi r^2/(\rho I)$; convert millimetres to metres before squaring the radius.

55. An n-p-n transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100Ω and the output load resistance is 10 kΩ. The common emitter current gain β is : [4 marks]

(A) 10^4

(B) 10^2

(C) 6×10^2

(D) 60

Correct Answer: (B) 10^2

Solution:

Concept:

- Power gain in decibels is $G_{\mathrm{dB}} = 10 \log_{10}(P_{\mathrm{out}}/P_{\mathrm{in}})$. It must first be converted to an ordinary power ratio.
- For current gain $\beta = I_{\mathrm{out}}/I_{\mathrm{in}}$, the signal-power ratio is $P_{\mathrm{out}}/P_{\mathrm{in}} = \beta^2 R_L/R_{\mathrm{in}}$, since power dissipated in a resistance is I^2R.

Step 1: Convert the specified gain and resistances The gain is $60\,\mathrm{dB}$, so

$$60 = 10 \log_{10} G_P, \qquad G_P = 10^6.$$

The output load is $10\,\mathrm{k\Omega} = 10^4\,\Omega$, while the input resistance is $100\,\Omega = 10^2\,\Omega$. Therefore

$$\frac{R_L}{R_{\mathrm{in}}} = \frac{10^4}{10^2} = 100.$$

The decibel number 60 is logarithmic; it is neither the linear power gain nor the transistor current gain.

Step 2: Relate the power gain to the current gain Let the input and output signal-current magnitudes

be I_b and I_c, respectively. Then

$$G_P = \frac{I_c^2 R_L}{I_b^2 R_{\text{in}}} = \beta^2 \frac{R_L}{R_{\text{in}}}.$$

Substitution gives

$$10^6 = 100\beta^2, \qquad \beta^2 = 10^4, \qquad \beta = 10^2.$$

The positive root is taken because β is a gain magnitude. A value 10^4 confuses β^2 with β; 6×10^2 and 60 do not give the required power ratio when inserted in $100\beta^2$. As a check, the voltage-gain magnitude is $\beta R_L/R_{\text{in}} = 10^4$, and multiplying it by the current gain 10^2 reproduces the power gain 10^6.

Step 3: Conclusion The common emitter current gain is $\beta = 100 = 10^2$. Hence, the correct option is (B).

Quick Tip to Solve: Convert decibels to a power ratio with $10\log_{10}$, then use $G_P = \beta^2 R_L/R_{\text{in}}$; remember to take the square root.

56. The displacement of a damped harmonic oscillator is given by

$x(t) = e^{-0.1t} \cos(10\pi t + f)$.

Here t is in seconds. The time taken for its amplitude of vibration to drop to half of its initial value is close to : [4 marks]

(A) 27 s

(B) 13 s

(C) 7 s

(D) 4 s

Correct Answer: (C) 7 s

Solution:

Concept:

- For $x(t) = A_0 e^{-\gamma t}\cos(\omega t + \phi)$, the amplitude envelope is $A(t) = A_0 e^{-\gamma t}$, not the instantaneous displacement.
- The half-amplitude time satisfies $A(t)/A_0 = 1/2$. Taking natural logarithms gives $t_{1/2} = \ln 2/\gamma$, independent of angular frequency and initial phase.

Step 1: Identify the slowly decaying amplitude The given displacement is an exponential factor multiplying a cosine. Therefore

$$\frac{A(t)}{A(0)} = e^{-0.1t}, \qquad \gamma = 0.1\,\text{s}^{-1}.$$

The factor $\cos(10\pi t + f)$ specifies where the oscillator is within an individual cycle. It does not alter the amplitude envelope. An instantaneous displacement can vanish even when the oscillator still has a large amplitude, so setting the entire expression for $x(t)$ equal to one-half would answer a different question.

Step 2: Solve the half-amplitude condition Set the envelope ratio equal to one-half:

$$e^{-0.1t} = \frac{1}{2}.$$

Taking the natural logarithm of both sides gives

$$-0.1t = -\ln 2, \qquad t = \frac{0.6931}{0.1} = 6.931\,\text{s}.$$

This is closest to $7\,\text{s}$. The given oscillation angular frequency $10\pi\,\text{rad/s}$ corresponds to a period $0.2\,\text{s}$, but the number of cycles is irrelevant to the exponential half-life. At 4, 13, and $27\,\text{s}$, the amplitude ratios are approximately 0.670, 0.273, and 0.067, respectively; none equals 0.5. Do not use the decay rate of energy, which is twice the amplitude decay rate.

Step 3: Conclusion The amplitude halves after approximately $6.93\,\text{s}$, closest to $7\,\text{s}$. Hence, the correct option is (C).

Quick Tip to Solve: Read the exponential envelope directly: when amplitude decays as $e^{-\gamma t}$, its half-life is $\ln 2/\gamma$, regardless of the cosine factor.

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57. Two radioactive materials A and B have decay constants 10λ and λ, respectively. It initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be 1/e after a time : [4 marks]

(A) $1/9\lambda$

(B) $11/10\lambda$

(C) $1/10\lambda$

(D) $1/11\lambda$

Correct Answer: (A) $1/9\lambda$

Solution:

Concept:

- Radioactive nuclei obey $N(t) = N_0 e^{-kt}$, where k is the decay constant and t is the elapsed time.
- For two samples with equal initial populations, dividing their decay laws cancels N_0; the ratio is governed by the difference of their decay constants.

Step 1: Write the two decay laws Let the common initial population be N_0. The given constants are $k_A = 10\lambda$ and $k_B = \lambda$, so

$$N_A = N_0 e^{-10\lambda t}, \qquad N_B = N_0 e^{-\lambda t}.$$

Both populations decrease. Sample A decreases more rapidly because its decay constant is ten times that of B; therefore the ratio must decrease from its initial value of one.

Step 2: Apply the required population ratio Divide the expressions before taking logarithms:

$$\frac{N_A}{N_B} = e^{-(10\lambda-\lambda)t} = e^{-9\lambda t}.$$

The specified ratio is $1/e = e^{-1}$. Equating the real exponents gives

$$-9\lambda t = -1, \qquad t = \frac{1}{9\lambda}.$$

The factor 9λ is a difference, rather than a sum: B is also decaying during the same interval. Using $1/(10\lambda)$ would incorrectly treat B's population as fixed. The denominators 11λ and the factor $11/10$ likewise do not satisfy the ratio equation. The result has units of time because a decay constant has units of inverse time.

Step 3: Conclusion The required elapsed time is $\boxed{t = 1/(9\lambda)}$. Hence, the correct option is (A).

Quick Tip to Solve: For a ratio of two exponentially decaying populations, subtract their decay constants before solving for time.

58. Given below in the left column are different modes of communication using the kinds of waves given in the right column.

A	**Optical fibre communication**	**P**	**Ultrasound**
B	**Radar**	**Q**	**Infrared light**
C	**Sonar**	**R**	**Microwaves**
D	**Mobile phones**	**S**	**Radio waves**

[4 marks]

(A) A-Q, B-S, C-R, D-P

(B) A-Q, B-S, C-P, D-R

(C) A-S, B-Q, C-R, D-P

(D) A-R, B-P, C-S, D-Q

Correct Answer: (B) A-Q, B-S, C-P, D-R

Solution:

Concept:

- Communication systems use waves suited to their medium and purpose: optical fibres guide light, radar uses electromagnetic waves, and sonar uses sound.
- Ultrasound is a mechanical wave requiring a medium; radio and infrared waves are electromagnetic. Microwaves form a high-frequency region of the radio spectrum.

Step 1: Match the optical and acoustic systems Optical fibre communication employs light guided through a transparent fibre by total internal reflection. The listed optical choice is infrared light, so A matches Q. This does not match ultrasound or the ordinary radio-wave choices. Sonar sends sound pulses through water and detects their echoes. Ultrasound is suitable because its short wavelength provides useful directionality and resolution. Thus C matches P. This immediately excludes options (A) and (C), both of which assign microwaves to sonar, and excludes (D), which assigns radio waves to sonar.

Step 2: Complete the electromagnetic matches Radar detects objects by transmitting electromagnetic

signals and receiving reflected signals. Radio waves are the listed match S for B. Mobile-phone communication employs microwave-frequency radio signals, giving D-R in the classification used by the question. The complete correspondence is therefore

$$\mathrm{A}-\mathrm{Q}, \quad \mathrm{B}-\mathrm{S}, \quad \mathrm{C}-\mathrm{P}, \quad \mathrm{D}-\mathrm{R}.$$

Although practical radar also commonly operates in the microwave band, microwaves are themselves radio waves; the matching task uses the separate listed labels to describe radar and mobile phones. Only option (B) simultaneously identifies the optical-fibre and sonar waves correctly and supplies the displayed remaining matches. Option (D) is also untenable because radar cannot work through ultrasound as its electromagnetic probing signal.

Step 3: Conclusion The matching is A-Q, B-S, C-P, D-R. Hence, the correct option is (B).

Quick Tip to Solve: Match the unmistakable pairs first: optical fibre with infrared light and sonar with ultrasound; then check the remaining pairs.

59. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are, in $\mathrm{m\,s^{-1}}$, (Given speed of sound = 300 m/s) [4 marks]

(A) 12, 18

(B) 16, 14

(C) 12, 16

(D) 8, 18

Correct Answer: (A) 12, 18

Solution:

Concept:

- For a stationary source and a moving observer, $f_{\text{obs}} = f(v \pm u)/v$, where v is sound speed and u is observer speed relative to the medium.
- The plus sign applies when the observer approaches the source; the minus sign applies when the observer recedes. Observer motion changes the rate of encountering wavefronts.

Step 1: Find the speed of the receding observer The emitted frequency is $f = 500\,\mathrm{Hz}$. Since $480\,\mathrm{Hz}$ is smaller, the first observer moves away from the source. With $v = 300\,\mathrm{m\,s^{-1}}$, write

$$480 = 500\left(1 - \frac{u_1}{300}\right).$$

Consequently,

$$u_1 = 300\left(1 - \frac{480}{500}\right) = 300 \times 0.04 = 12\,\mathrm{m\,s^{-1}}.$$

The frequency decrease of twenty hertz is four percent of the original frequency, so the speed is four percent of the sound speed.

Step 2: Find the speed of the approaching observer The second detected frequency is greater than the emitted frequency, so this observer approaches the source:

$$530 = 500\left(1 + \frac{u_2}{300}\right).$$

It follows that

$$u_2 = 300\left(\frac{530}{500} - 1\right) = 300 \times 0.06 = 18\,\mathrm{m\,s^{-1}}.$$

Both speeds are positive magnitudes; the directions are accounted for by the signs in the Doppler equations. Substitution of 12 and 18 reproduces 480 and 530 Hz respectively. Option (B) does not reproduce either shift, option (C) gives the wrong second shift, and option (D) gives the wrong first shift. A denominator containing a source speed is unnecessary because the source is stationary.

Step 3: Conclusion The respective speeds are $\boxed{12\,\mathrm{m\,s^{-1}},\ 18\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (A).

Quick Tip to Solve: With a stationary source, use $u = v|f_{\mathrm{obs}} - f|/f$; decide approach or recession from whether the frequency rises or falls.

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60. A message signal of frequency 100 MHz and peak voltage 100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are : [4 marks]

(A) 0.25; 1×10^8 Hz

(B) 4; 2×10^8 Hz

(C) 4; 1×10^8 Hz

(D) 0.25; 2×10^8 Hz

Correct Answer: (D) 0.25; 2×10^8 Hz

Solution:

Concept:

- For amplitude modulation, the modulation index is $\mu = A_m/A_c$, where A_m and A_c are the message and carrier peak voltages.
- The lower and upper sideband frequencies are $f_c - f_m$ and $f_c + f_m$. Their separation is $2f_m$, independent of carrier frequency.

Step 1: Calculate the modulation index Use the voltage amplitudes, rather than the two frequencies, in the modulation-index formula. Here the message peak voltage is $100\,\mathrm{V}$ and the carrier peak voltage is $400\,\mathrm{V}$. Therefore

$$\mu = \frac{100}{400} = 0.25.$$

The index is dimensionless because the voltage units cancel. It is below one, so the carrier amplitude remains sufficient for ordinary amplitude modulation without overmodulation. Taking the reciprocal would give four, which is the incorrect index appearing in options (B) and (C).

Step 2: Find the sideband separation Convert the message frequency before calculating the separation:

$$f_m = 100\,\text{MHz} = 10^8\,\text{Hz}, \qquad f_c = 300\,\text{GHz} = 3 \times 10^{11}\,\text{Hz}.$$

Thus the upper and lower sidebands lie at $3 \times 10^{11} + 10^8$ and $3 \times 10^{11} - 10^8$ hertz. Subtracting gives

$$f_{\text{USB}} - f_{\text{LSB}} = (f_c + f_m) - (f_c - f_m) = 2f_m = 2 \times 10^8\,\text{Hz}.$$

The carrier cancels because the sidebands are symmetrically located about it. Option (A) gives only the spacing from the carrier to one sideband, rather than the spacing between both sidebands. Only option (D) has both the correct index and the correct separation.

Step 3: Conclusion The modulation index is $\boxed{0.25}$, and the sideband separation is $\boxed{2 \times 10^8\,\text{Hz}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use amplitude ratio for modulation index and twice the message frequency for the separation of the two sidebands.

Chemistry (120 marks)

61. At room temperature, a dilute solution of urea is prepared by dissolving 0.60 of urea in 360 g of water. If the vapour pressure of pure water at this temperature is 35 mm Hg, lowering of vapour pressure will be. (molar mass of urea = 60 g mol^{-1}) [4 marks]

(A) 0.031 mmHg

(B) 0.017 mmHg

(C) 0.028 mmHg

(D) 0.027 mmHg

Correct Answer: (B) 0.017 mmHg

Solution:

Concept:

- For an ideal dilute solution containing a nonvolatile solute, the vapour-pressure law gives $p = x_w p^0$, so $\Delta p = p^0 x_s$.
- Urea is a nonelectrolyte, so one mole contributes one mole of solute particles. The mole fraction uses moles of solute divided by total moles.

Step 1: Calculate the component amounts Take the stated urea quantity as $0.60\,\text{g}$, consistent with the given molar mass $60\,\text{g mol}^{-1}$. The corresponding amount of urea is

$$n_s = \frac{0.60}{60} = 0.010\,\text{mol}.$$

Using $18\,\text{g mol}^{-1}$ for water gives

$$n_w = \frac{360}{18} = 20\,\text{mol}.$$

The solute amount is very small relative to the solvent amount, as required for the dilute-solution approximation. Urea does not dissociate in water, so no ionic multiplicity factor is introduced.

Step 2: Apply the relative lowering relation The exact solute mole fraction for these amounts is

$$x_s = \frac{0.010}{20 + 0.010} = 4.9975 \times 10^{-4}.$$

Multiply it by the pure-water vapour pressure:

$$\Delta p = 35 \times 4.9975 \times 10^{-4} = 0.01749\,\text{mmHg}.$$

Equivalently, the dilute approximation $x_s \simeq n_s/n_w = 0.0005$ gives $0.0175\,\text{mmHg}$. The listed value $0.017\,\text{mmHg}$ is the closest choice. The other listed values, 0.031, 0.028 and 0.027 mmHg, are substantially larger and do not follow from the mole ratio. This is the lowering, not the solution's total vapour pressure, which remains close to 35 mmHg.

Step 3: Conclusion The vapour-pressure lowering is approximately $0.0175\,\text{mmHg}$, corresponding to the listed $0.017\,\text{mmHg}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert both masses into moles before using relative lowering; for a very dilute solution, $\Delta p \simeq p^0 n_s/n_w$.

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62. The isoelectronic set of ions is : [4 marks]

(A) F^-, Li^+, Na^+ and Mg^{2+}

(B) Li^+, Na^+, O^{2-} and F^-

(C) N^{3-}, O^{2-}, F^- and Na^+

(D) N^{3-}, Li^+, Mg^{2+} and O^{2-}

Correct Answer: (C) N^{3-}, O^{2-}, F^- and Na^+

Solution:

Concept:

- Isoelectronic species have the same total number of electrons, even when their nuclear charges and ionic charges differ.
- For atomic number Z, a cation of charge $+r$ has $Z - r$ electrons and an anion of charge $-r$ has $Z + r$ electrons.

Step 1: Count electrons in the proposed common set The atomic numbers needed are N: 7, O: 8, F: 9 and Na: 11. Account for the ionic charges explicitly:

$$N^{3-} : 7 + 3 = 10, \quad O^{2-} : 8 + 2 = 10,$$
$$F^- : 9 + 1 = 10, \quad Na^+ : 11 - 1 = 10.$$

Each ion in option (C) therefore has ten electrons. All four have the closed-shell electron arrangement $1s^2\,2s^2\,2p^6$, the same electron configuration as neon. Their different proton numbers do not prevent

them from being isoelectronic.

Step 2: Check why the remaining sets fail Lithium has atomic number three, so Li^+ has only $3-1=2$ electrons and the helium-like configuration $1s^2$. Magnesium has atomic number twelve, so Mg^{2+} has $12-2=10$ electrons. Option (A) mixes the two-electron lithium ion with ten-electron fluoride, sodium and magnesium ions. Option (B) also includes lithium alongside three ten-electron species. Option (D) again combines two-electron lithium with ten-electron nitrogen, oxygen and magnesium ions. Thus the presence of lithium rules out all three distractors. Similar charge signs or membership in nearby periods alone is insufficient; equality of electron count is the defining test.

Step 3: Conclusion The isoelectronic set is $\boxed{N^{3-},\ O^{2-},\ F^-,\ Na^+}$, each with ten electrons. Hence, the correct option is (C).

Quick Tip to Solve: Count electrons as atomic number minus signed ionic charge; here Li^+ immediately eliminates every incorrect set.

63. The principle of column chromatography is : [4 marks]

(A) Gravitational force.

(B) Capillary action.

(C) Differential adsorption of the substances on the solid phase.

(D) Differential absorption of the substances on the solid phase.

Correct Answer: (C) Differential adsorption of the substances on the solid phase.

Solution:

Concept:

- Adsorption is accumulation on a surface; absorption is entry into the bulk of a material. These two processes are different.
- In adsorption column chromatography, a solid adsorbent such as silica or alumina forms the stationary phase, while an eluting solvent forms the mobile phase.

Step 1: Identify the separation mechanism A mixture is placed on a packed column of solid adsorbent. As the solvent travels through the column, each constituent repeatedly distributes between the solvent and the adsorbent surface. Constituents do not all attach to that surface equally strongly. A constituent adsorbed more strongly spends a larger fraction of the time retained on the solid, whereas a weakly adsorbed constituent travels more readily with the mobile solvent. This difference in retention causes initially mixed substances to move at different rates and emerge in separate fractions.

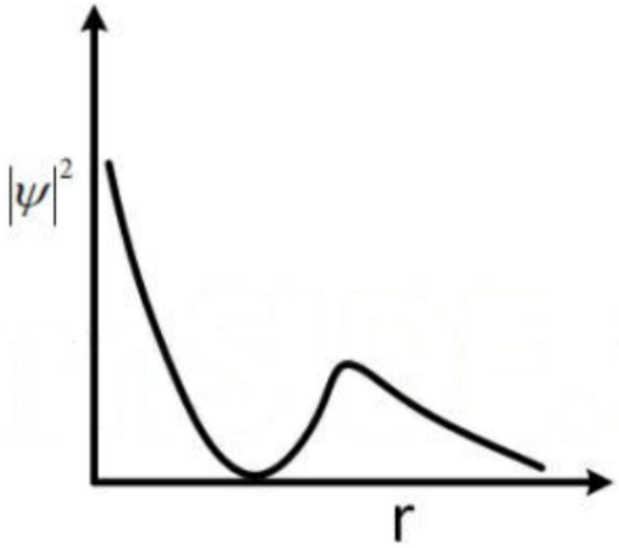
Step 2: Distinguish the principle from the distractors Gravity can help solvent pass down a simple column, but gravitational force does not explain why different dissolved substances separate. Thus option (A) names a possible driving force for flow, not the separation principle. Capillary action concerns liquid movement through narrow spaces and is especially associated with the rise of solvent in paper methods; it does not identify the differential retention used here, eliminating (B). Option (D) says differential absorption, which would mean uptake into the solid's bulk. The relevant interaction is retention on the adsorbent surface, so adsorption is the correct term. Option (C) states that distinction

precisely: differential adsorption on the solid stationary phase produces different migration rates and hence separation.

Step 3: Conclusion The principle is differential adsorption of the substances on the solid phase. Hence, the correct option is (C).

Quick Tip to Solve: In a packed adsorption column, stronger surface adsorption means slower elution; keep adsorption distinct from bulk absorption.

64. The graph between $|\psi|^2$ and r (radial distance) is shown below. This represents : [4 marks]



(A) 3s orbital

(B) 2s orbital

(C) 2p orbital

(D) 1s orbital

Correct Answer: (B) 2s orbital

Solution:

Concept:

- The probability density at a point is $|\psi|^2$. A radial node is a finite positive radius where the wavefunction and its probability density vanish.
- The number of radial nodes is $n - l - 1$, where n is the principal quantum number and l is the orbital angular momentum quantum number. For an s orbital, $l = 0$.

Step 1: Read the origin and the node The plotted density is nonzero near the nucleus, falls to zero once at a finite radius, rises to a second maximum, and finally approaches zero at large distance. The finite-radius zero is a radial node. The asymptotic approach to zero at infinity is not another node. A nonzero nuclear density identifies an s orbital among the choices; a p wavefunction vanishes at the nucleus.

Step 2: Determine the principal quantum number For an s orbital the node count is $n-1$. The graph shows one radial node, so

$$n-1=1 \quad \Longrightarrow \quad n=2.$$

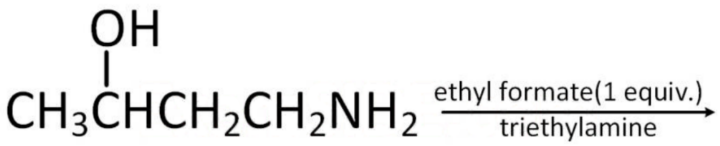
It therefore represents a 2s orbital. The two lobes in a radial-density graph are regions on either side of a radial node, not two angular lobes of a p orbital.

Step 3: Exclude the other orbitals A 3s orbital has $3-0-1=2$ radial nodes, whereas only one is shown. A 1s orbital has no radial node and its density decreases continuously. A 2p orbital has no radial node and zero density at the origin. Notice that the vertical axis is $|\psi|^2$, not the radial probability $4\pi r^2|\psi|^2$; the extra factor in the latter would force the value at the origin to zero even for an s orbital.

Step 4: Conclusion The finite density at the nucleus and the single radial node identify the $2s$ orbital. Hence, the correct option is (B).

Quick Tip to Solve: Check the vertical axis first. For $|\psi|^2$, nonzero nuclear density indicates s character; then count finite positive radial zeros.

65. The major product of the following reaction is : [4 marks]



(A) $CH_3CH(OH)CH{=}CH_2$

(B) $CH_3CH(OH)CH_2CH_2NHCHO$

(C) $CH_3CH(OCHO)CH_2CH_2NH_2$

(D) $CH_3CH{=}CHCH_2NH_2$

Correct Answer: (B) $CH_3CH(OH)CH_2CH_2NHCHO$

Solution:

Concept:

- A primary amine attacks the carbonyl carbon of an ester to give an amide by nucleophilic acyl substitution. Ethyl formate supplies the formyl group HCO^-.
- An unprotonated amino group is more nucleophilic than an alcohol group. With one equivalent of an acylating reagent, selective N-acylation of an amino alcohol is favoured.

Step 1: Identify the competing reactive groups The starting compound is $CH_3CH(OH)CH_2CH_2NH_2$, containing both a secondary alcohol and a primary amine. The nitrogen lone pair is more readily available for attack than an oxygen lone pair. Triethylamine provides basic conditions rather than a strongly acidic medium that would convert the amino group into an unreactive ammonium ion.

Step 2: Follow substitution at the ester carbonyl The amino nitrogen attacks ethyl formate, $HCOOC_2H_5$, forming a tetrahedral intermediate. Proton transfers and elimination of the ethoxy

group give the formamide. The overall change is

$$RNH_2 + HCOOC_2H_5 \longrightarrow RNHCHO + C_2H_5OH.$$

Here R is $CH_2CH_2CH(OH)CH_3$; the alcohol remains free. The new linkage is nitrogen to the formyl carbonyl carbon, not oxygen to that carbon.

Step 3: Compare the four proposed products The N-formylated amino alcohol is

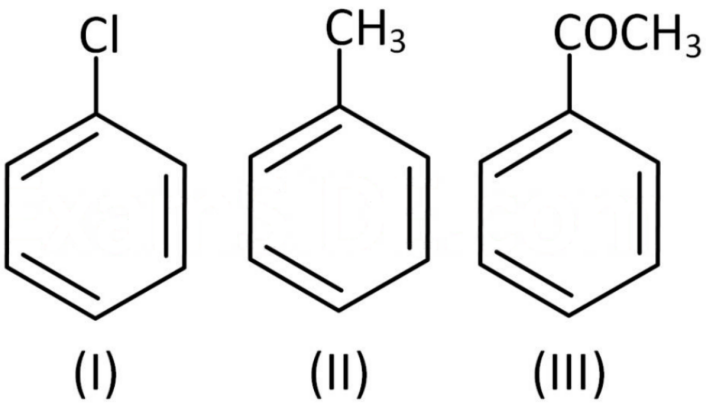
$$CH_3CH(OH)CH_2CH_2NHCHO.$$

This is the second structure. The O-formate in (C) would require the less nucleophilic alcohol to be acylated preferentially. Products (A) and (D) introduce carbon-carbon double bonds by eliminating an amino or hydroxyl group; ethyl formate and triethylamine are not the reagents for those transformations. One equivalent also does not imply simultaneous protection of both heteroatoms.

Step 4: Conclusion The major product is the N-formyl derivative $CH_3CH(OH)CH_2CH_2NHCHO$, with the hydroxyl group unchanged. Hence, the correct option is (B).

Quick Tip to Solve: For an amino alcohol with one equivalent of ethyl formate, compare nucleophilicity: the free amino group is formylated before the alcohol.

66. The increasing order of the reactivity of the following compounds towards electrophilic aromatic substitution reactions is : [4 marks]



(A) III < II < I

(B) III < I < II

(C) II < I < III

(D) I < III < II

Correct Answer: (B) III < I < II

Solution:

Concept:

- Electrophilic aromatic substitution is faster when a substituent increases ring electron density and stabilises the positively charged sigma-complex intermediate.
- A methyl group activates a benzene ring through electron donation, while a directly attached carbonyl group withdraws by resonance. Halogens deactivate overall through their inductive effect despite donating by resonance.

Step 1: Identify the three substituents Compound I is chlorobenzene, compound II is toluene, and compound III is acetophenone. The acetyl substituent is attached through its carbonyl carbon, $-COCH_3$. This attachment matters: it withdraws electron density from the ring rather than donating as an alkoxy substituent would.

Step 2: Compare donation and withdrawal The methyl group in II donates electron density by the positive inductive effect and hyperconjugation. It stabilises the sigma complex, so toluene reacts most readily. Chlorine in I withdraws strongly by induction, making chlorobenzene less reactive than benzene. Its lone-pair resonance donation explains its ortho/para orientation but does not make it an activating substituent. The acetyl group in III withdraws by both induction and resonance, strongly reducing the availability of ring electrons to an electrophile.

Step 3: Arrange the increasing reactivity Acetophenone is more strongly deactivated than chlorobenzene, and toluene is activated. Thus

$$\text{III} < \text{I} < \text{II}.$$

Option (A) incorrectly puts chlorobenzene above toluene. Option (C) incorrectly puts the activated methyl compound below both deactivated compounds and makes the carbonyl compound the most reactive. Option (D) incorrectly makes acetophenone more reactive than chlorobenzene. Only (B) combines both required comparisons.

Step 4: Conclusion The increasing order is acetophenone $<$ chlorobenzene $<$ toluene, or III $<$ I $<$ II. Hence, the correct option is (B).

Quick Tip to Solve: Keep directing influence separate from activation: chlorine directs ortho/para but still slows electrophilic substitution overall.

67. The species that can have a trans-isomer is :

(en = ehane-1, 2-diamine, ox = oxalate) [4 marks]

(A) $[Cr(en)^2(ox)]^+$

(B) $[Pt(en)Cl_2]$

(C) $[Pt(en)_2Cl_2]^{2+}$

(D) $[Zn(en)Cl_2]$

Correct Answer: (C) $[Pt(en)_2Cl_2]^{2+}$

Solution:

Concept:

- In a trans arrangement two specified ligands occupy opposite coordination positions, making an angle of 180° at the metal.
- Ethylenediamine (en) and oxalate (ox) are bidentate chelating ligands. Each normally occupies adjacent positions; the bite of these chelates does not span opposite sites.

Step 1: Count coordination sites for each complex For $[Pt(en)_2Cl_2]^{2+}$, two en ligands supply four donor atoms and two chlorido ligands supply two more. Its coordination number is six and its geometry is octahedral. Platinum is in the +4 oxidation state because en is neutral and each chlorido ligand has charge -1.

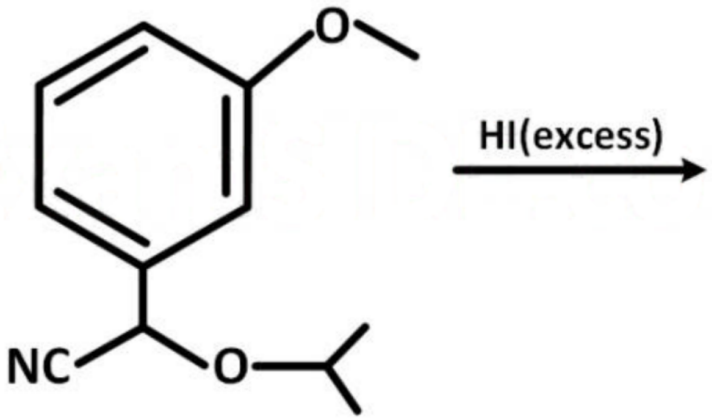
Step 2: Place the two chlorido ligands In an octahedron the two chlorido ligands may be adjacent, giving the cis isomer, or opposite, giving the trans isomer. In the opposite arrangement the remaining four positions can still be occupied by two en ligands, each joining a pair of adjacent sites. Thus a valid trans configuration exists without forcing either en ligand to stretch between opposite vertices.

Step 3: Reject the other coordination geometries The chromium option contains only bidentate chelating ligands (the printed superscript on en is read in the usual bis(en) composition); such a tris-chelate arrangement does not give a cis/trans pair of monodentate chlorido ligands. $[Pt(en)Cl_2]$ is a square-planar platinum(II) complex: en must occupy adjacent positions, leaving the two chlorido positions adjacent as well. They cannot be trans. $[Zn(en)Cl_2]$ has tetrahedral coordination; all pairs of positions have the tetrahedral angle, so a trans isomer is impossible. This leaves only the six-coordinate platinum(IV) complex.

Step 4: Conclusion $[Pt(en)_2Cl_2]^{2+}$ can place its two chlorido ligands opposite each other in an octahedron. Hence, the correct option is (C).

Quick Tip to Solve: Count donor atoms before testing cis/trans isomerism. A chelating en ligand occupies adjacent sites, while octahedral remaining ligands may be opposite.

68. The major product of the following reaction is : [4 marks]



(A) $(3\text{-HO}-C_6H_4)CH(OH)CN$

(B) $(3\text{-I}-C_6H_4)CH(OH)CN$

(C) $(3\text{-HO}-C_6H_4)CH(I)CN$

(D) $(3\text{-I}-C_6H_4)CH(I)CN$

Correct Answer: (C) $(3\text{-HO}-C_6H_4)CH(I)CN$

Solution:

Concept:

- Hydrogen iodide protonates an ether oxygen and supplies iodide for carbon-oxygen bond cleavage. An aryl-oxygen bond resists substitution because its carbon is sp^2-hybridised and the bond has partial double-bond character.
- Excess HI cleaves alkyl ethers and can convert the resulting benzylic alcohol into an iodide; phenolic hydroxyl is retained.

Step 1: Cleave the aromatic methyl ether The aromatic methoxy group is $Ar-O-CH_3$. After oxygen protonation, iodide attacks the methyl carbon by an S_N2 process. The products are phenol and methyl iodide:

$$ArOCH_3 + HI \longrightarrow ArOH + CH_3I.$$

The alternative attack at an aromatic carbon is not an ordinary substitution pathway, so the ring carbon does not acquire iodine. This immediately rules out structures (B) and (D).

Step 2: Apply excess reagent to the side-chain ether The other ether is attached to the benzylic carbon bearing the cyano group: $Ar-CH(CN)-O-CH(CH_3)_2$. Protonation makes its oxygen a good leaving group. Cleavage and further reaction with excess hydrogen iodide replace the benzylic oxygen functionality by iodine. The net side-chain product is $Ar-CH(I)-CN$; the cyano group and the carbon skeleton remain in place. If cleavage first supplies a benzylic alcohol, the excess reagent converts that alcohol further to the iodide.

Step 3: Select the structure retaining phenolic oxygen The final aromatic substituents remain meta to one another. The product must have phenolic OH on the ring and I at the side-chain carbon, giving $(3\text{-HO}-C_6H_4)CH(I)CN$. Structure (A) stops at the benzylic alcohol and overlooks the stated excess of hydrogen iodide. The aryl iodides (B) and (D) incorrectly replace phenolic hydroxyl through cleavage

of an aryl-oxygen bond. Structure (C) alone has both correct changes.

Step 4: Conclusion The major product is the meta-hydroxy aryl compound with a benzylic iodide, $(3\text{-HO}-\text{C}_6\text{H}_4)\text{CH(I)CN}$. Hence, the correct option is (C).

Quick Tip to Solve: With excess HI, distinguish the two oxygens: aromatic methoxy becomes phenol, whereas a benzylic oxygen group can be replaced by iodine.

69. A process will be spontaneous at all temperatures if : [4 marks]

(A) $\Delta H < 0$ and $\Delta S > 0$

(B) $\Delta H < 0$ and $\Delta S < 0$

(C) $\Delta H > 0$ and $\Delta S < 0$

(D) $\Delta H > 0$ and $\Delta S > 0$

Correct Answer: (A) $\Delta H < 0$ and $\Delta S > 0$

Solution:

Concept:

- At constant temperature and pressure a process is thermodynamically spontaneous when $\Delta G < 0$, with $\Delta G = \Delta H - T\Delta S$. Temperature is measured on the absolute Kelvin scale.
- An exothermic enthalpy change favours spontaneity. A positive entropy change also favours spontaneity because it contributes the negative term $-T\Delta S$ at any positive temperature.

Step 1: Require both terms to favour the process If $\Delta H < 0$ and $\Delta S > 0$, then ΔH is negative and $-T\Delta S$ is negative for every $T > 0$. Their sum is therefore negative:

$$\Delta G = \underbrace{\Delta H}_{<0} + \underbrace{(-T\Delta S)}_{<0} < 0.$$

No competition between the enthalpic and entropic contributions remains.

Step 2: Check the remaining sign combinations For option (B), put $\Delta H = -h$ and $\Delta S = -s$, with $h, s > 0$. Then $\Delta G = -h + Ts$, so spontaneity holds only below h/s. For option (C), $\Delta H > 0$ and $\Delta S < 0$ make both contributions to ΔG positive, so the process is never spontaneous under these conditions. For option (D), write $\Delta H = h$ and $\Delta S = s$; then $\Delta G = h - Ts$, which becomes negative only above h/s.

Step 3: Interpret the temperature statement The phrase all temperatures requires a sign combination that cannot change the sign of Gibbs energy as the relative size of the two terms varies. Only negative enthalpy together with positive entropy meets that condition in the usual treatment where their signs persist over the range considered. This is a thermodynamic prediction of direction; it does not assert that the process has a fast reaction rate or a small activation barrier.

Step 4: Conclusion For spontaneity at all temperatures, $\Delta H < 0$ and $\Delta S > 0$. Hence, the correct option is (A).

Quick Tip to Solve: Use $\Delta G = \Delta H - T\Delta S$: negative enthalpy and positive entropy make both contributions negative at every positive temperature.

70. Amylopectin is compound of : [4 marks]

(A) α-D-glucose, C_1 – C_4 and C_1 – C_6 linkages

(B) β-D-glucose, C_1 – C_4 and C_2 – C_6 linkages

(C) β-D-glucose, C_1 – C_4 and C_1 – C_6 linkages

(D) α-D-glucose, C_1 – C_4 and C_2 – C_6 linkages

Correct Answer: (A) α-D-glucose, C_1 – C_4 and C_1 – C_6 linkages

Solution:

Concept:

- Amylopectin is the branched component of starch and is built from α-D-glucose units joined by glycosidic bonds.
- The main chains contain $\alpha(1 \rightarrow 4)$ linkages. Branch points use $\alpha(1 \rightarrow 6)$ linkages between glucose residues.

Step 1: Identify the monomer and its anomeric form The structural unit of starch is α-D-glucose. The alpha designation specifies the configuration at the anomeric carbon, carbon 1, involved in forming the glycosidic bond. Amylopectin is not a polymer of beta-D-glucose; that distinction is essential because a change in glycosidic configuration changes the shape and properties of the polysaccharide.

Step 2: Locate the main-chain and branch bonds Within a chain, carbon 1 of one glucose residue is connected through oxygen to carbon 4 of another. At a branch, carbon 1 of a glucose residue is connected through oxygen to carbon 6 of a residue in the existing chain. Thus the two types are

$$\alpha(1 \rightarrow 4) \quad \text{and} \quad \alpha(1 \rightarrow 6).$$

The carbon 6 group provides the extra attachment point that allows a branched structure rather than a single unbranched chain.

Step 3: Compare every option with this structure Option (A) combines alpha-D-glucose with the correct 1-to-4 and 1-to-6 links. Options (B) and (C) use beta-D-glucose and therefore fail the monomer configuration test. Option (B) also gives a 2-to-6 bond, which is not the starch branch linkage. Option (D) has the correct alpha monomer but the wrong branch carbon pair, 2-to-6. Both ordinary starch linkages use an anomeric carbon 1, not carbon 2. Amylose, the other starch component, mainly contains the 1-to-4 links and lacks amylopectin's branching.

Step 4: Conclusion Amylopectin contains α-D-glucose with $C_1 - C_4$ and $C_1 - C_6$ glycosidic linkages. Hence, the correct option is (A).

Quick Tip to Solve: For starch remember alpha glucose: 1-to-4 builds the chains and 1-to-6 creates the branch points of amylopectin.

71. Consider the following statements:

(a) The pH of a mixture containing 400 mL of 0.1 M H_2SO_4 and 400 mL of 0.1 M NaOH will be approximately 1.3.

(b) Ionic product of water is temperature dependent.

(c) A monobasic acid with $K_a = 10^{-5}$ has pH = 5. The degree of dissociation of this acid is 50%.

(d) The Le Chatelier's principle is not applicable to common-ion effect.

The correct statements are: [4 marks]

(A) (a) and (b)

(B) (a), (b) and (c)

(C) (a), (b) and (d)

(D) (b) and (c)

Correct Answer: (B) (a), (b) and (c)

Solution:

Concept:

- Neutralization removes equal mole equivalents of acid and base. The remaining hydrogen-ion concentration gives $\text{pH} = -\log_{10}[\text{H}^+]$.
- For $\text{HA} \rightleftharpoons \text{H}^+ + \text{A}^-$, $K_a = [\text{H}^+]\alpha/(1-\alpha)$, where α is the fraction dissociated. Equilibrium constants depend on temperature.

Step 1: Test statements (a) and (b) In the usual complete-neutralization approximation for this question, sulfuric acid supplies two acid equivalents per mole. Its amount is $0.400(0.1) = 0.040$ mol, while the base supplies 0.040 mol of hydroxide. Thus the excess acid equivalents are $0.080 - 0.040 = 0.040$ mol in 0.800 L.

$$[\text{H}^+] \simeq \frac{0.040}{0.800} = 0.050\,\text{mol}\,\text{L}^{-1}, \qquad \text{pH} \simeq 1.30.$$

Statement (a) uses this approximation and is correct. Since water ionization is an equilibrium whose position changes with temperature, its equilibrium constant K_w is temperature dependent. Statement (b) is correct as well.

Step 2: Test statements (c) and (d) The pH fixes $[\text{H}^+] = 10^{-5}\,\text{mol}\,\text{L}^{-1}$. Using the exact dissociation relation rather than the small-dissociation approximation gives

$$10^{-5} = 10^{-5}\frac{\alpha}{1-\alpha} \quad \Longrightarrow \quad \alpha = \frac{1}{2}.$$

Hence statement (c) is correct: this is 50 percent dissociation. Adding a common ion shifts the dissociation equilibrium towards the undissociated species, exactly as Le Chatelier's principle predicts. Statement (d) is false. Options A and D each omit a true statement, while C includes the false statement (d).

Step 3: Conclusion The first three statements are correct in the approximation used for the acid mixture; the common-ion statement is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: For a weak acid with a known pH, use $\alpha = K_a/(K_a + [\text{H}^+])$; do not assume small dissociation when $K_a = [\text{H}^+]$.

72. The oxoacid of sulphur that does not contain bond between sulphur atoms is [4 marks]

(A) $H_2S_2O_7$

(B) $H_2S_2O_3$

(C) $H_2S_4O_6$

(D) $H_2S_2O_4$

Correct Answer: (A) $H_2S_2O_7$

Solution:

Concept:

- The molecular formula alone does not establish a direct sulphur–sulphur bond. Inspect whether the sulphur centres are joined directly or through oxygen.
- Pyrosulphuric acid results from condensation of two sulphuric acid molecules with loss of water; its bridging linkage is $S-O-S$.

Step 1: Identify the oxygen-bridged acid Pyrosulphuric acid, $H_2S_2O_7$, has the connectivity

$$HO-S(=O)_2-O-S(=O)_2-OH.$$

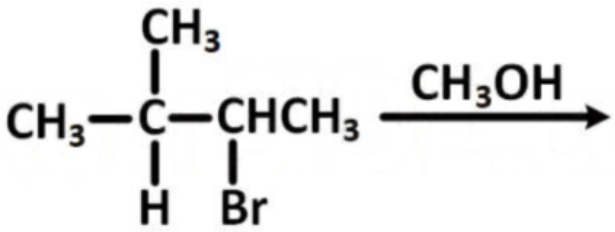
The central oxygen atom is bonded to both sulphur atoms. There are two terminal hydroxyl groups and four terminal doubly bonded oxygen atoms, making seven oxygens altogether. Thus the sulphur atoms are connected through an intervening oxygen, rather than by an S–S bond. This distinguishes an oxygen bridge from a direct bond even though the formula contains two sulphur atoms.

Step 2: Reject the other sulphur acids Thiosulphuric acid, $H_2S_2O_3$, has the connectivity $HO-S(=O)_2-SH$; replacing an oxygen of sulphuric acid by sulphur introduces a direct S–S bond. Tetrathionic acid, $H_2S_4O_6$, has the chain $HO-S(=O)_2-S-S-S(=O)_2-OH$, so it contains direct sulphur–sulphur links. Dithionous acid, $H_2S_2O_4$, has $HO-S(=O)-S(=O)-OH$, again containing an S–S bond. Thus B, C and D each possess the bond excluded by the question, whereas A does not.

Step 3: Conclusion Only pyrosulphuric acid, $H_2S_2O_7$, lacks a direct sulphur–sulphur bond. Hence, the correct option is (A).

Quick Tip to Solve: Remember pyrosulphuric acid as two sulphuric acid units joined by an oxygen bridge; thio and polythionic acids contain direct sulphur links.

73. The major product of the following reaction is : [4 marks]



(A) $(CH_3)_2CH-CH=CH_3$

(B) $(CH_3)_2C=CHCH_3$

(C) $(CH_3)_2C(OCH_3)CH_2CH_3$

(D) $(CH_3)_2CHCH(OCH_3)CH_3$

Correct Answer: (C) $(CH_3)_2C(OCH_3)CH_2CH_3$

Solution:

Concept:

- A secondary alkyl bromide in methanol can undergo solvolysis: ionization first forms a carbocation, followed by nucleophilic attack by methanol.
- A 1,2-hydride shift can convert a secondary carbocation into a more stable tertiary carbocation. Methanol capture followed by deprotonation gives an ether.

Step 1: Form and rearrange the carbocation The substrate is $CH_3CH(CH_3)CH(Br)CH_3$. Loss of bromide from its secondary carbon initially gives

$$CH_3CH(CH_3)CH^+CH_3.$$

The adjacent carbon bears a hydrogen and is bonded to three carbon groups. Migration of this hydrogen with its bonding electron pair to the positively charged carbon is a 1,2-hydride shift. The new cation is

$$CH_3C^+(CH_3)CH_2CH_3,$$

which is tertiary and more stable than the initial secondary cation. The rearrangement changes the position of positive charge without changing the carbon skeleton.

Step 2: Capture the rearranged cation Methanol supplies an oxygen lone pair to the tertiary centre. Its initial product is a protonated ether; removal of the oxygen-bound proton by solvent gives

$$CH_3C(CH_3)(OCH_3)CH_2CH_3.$$

This is 2-methoxy-2-methylbutane. Option D instead places methoxy at the original secondary centre and corresponds to unrearranged capture. Options A and B depict elimination products rather than the rearranged ether selected under the stated methanol solvolysis conditions. Therefore the tertiary-centre substitution product is the required major product.

Step 3: Conclusion The major product is $(CH_3)_2C(OCH_3)CH_2CH_3$, formed after a hydride shift. Hence, the correct option is (C).

Quick Tip to Solve: Before trapping a secondary carbocation, inspect its neighbouring carbon for a hydride shift that can produce a tertiary carbocation.

74. Three complexes,
$[CoCl(NH_3)_5]^{2+}$ (I),
$[Co(NH_3)_5H_2O]^{3+}$ (II) and
$[Co(NH_3)_6]^{3+}$ (III)
absorb light in the visible region. The correct order of the wavelength of light absorbed by them is : [4 marks]

(A) (III) > (I) > (II)
(B) (III) > (II) > (I)
(C) (I) > (II) > (III)
(D) (II) > (I) > (III)

Correct Answer: (C) (I) > (II) > (III)

Solution:

Concept:

- For otherwise comparable octahedral complexes, stronger-field ligands give a larger crystal-field splitting Δ_o.
- Absorption across the splitting satisfies $\Delta_o = h\nu = hc/\lambda$; thus the wavelength decreases as the energy gap increases.

Step 1: Compare the ligand field strengths The complexes contain cobalt in the same oxidation state, +3: chloride contributes -1 in I, while ammonia and water are neutral. Thus variation in the metal oxidation state is not responsible for their ordering. The relevant spectrochemical-series order is

$$Cl^- < H_2O < NH_3.$$

Five ammonia ligands are common to I and II, so replacing chloride by water increases the field strength. Replacing water by a sixth ammonia ligand increases it further. Therefore

$$\Delta_o(\mathrm{I}) < \Delta_o(\mathrm{II}) < \Delta_o(\mathrm{III}).$$

Step 2: Reverse the order for wavelength An absorbed photon promotes an electron between crystal-field levels. Its energy is inversely proportional to its wavelength, so the weakest field requires the smallest photon energy and the longest wavelength. Consequently,

$$\lambda(\mathrm{I}) > \lambda(\mathrm{II}) > \lambda(\mathrm{III}).$$

Option A incorrectly assigns the longest wavelength to III and reverses the I–II comparison. Option B also makes III longest, confusing increasing splitting with increasing wavelength. Option D correctly makes III shortest but reverses I and II. Only option C consistently follows both ligand strength and the inverse energy–wavelength relation.

Step 3: Conclusion The wavelengths of light absorbed are ordered (I) > (II) > (III). Hence, the correct option is (C).

Quick Tip to Solve: Write the splitting order first from the spectrochemical series, then reverse it once to obtain the wavelength order.

75. At 300 K and 1 atmospheric pressure, 10 mL of a hydrocarbon required 55 mL of O_2 for complete combustion, and 40 mL of CO_2 is formed. The formula of the hydrocarbon is : [4 marks]

(A) C_4H_7Cl

(B) C_4H_6

(C) C_4H_8

(D) C_4H_{10}

Correct Answer: (B) C_4H_6

Solution:

Concept:

- At the same temperature and pressure, volumes of gases are proportional to their mole amounts. Volume ratios therefore give the coefficients of a gas reaction.
- Complete combustion of C_xH_y follows $C_xH_y + (x^+y/_4)O_2 \longrightarrow xCO_2 + (y/_2)H_2O$.

Step 1: Find the carbon count from carbon dioxide Each molecule of a hydrocarbon containing x carbon atoms produces x molecules of carbon dioxide on complete combustion. Since the hydrocarbon and carbon dioxide volumes are measured at the same temperature and pressure,

$$x = \frac{V_{CO_2}}{V_{\text{hydrocarbon}}} = \frac{40}{10} = 4.$$

The carbon count is therefore four. This step does not require the molar gas volume at 300 K; only the ratio is used. Water's physical state does not affect this carbon balance.

Step 2: Use oxygen consumption to obtain hydrogen The oxygen required per molecule is $x + y/4$: the carbon atoms consume x molecules of oxygen, while hydrogen consumes $y/4$. The observed oxygen-to-hydrocarbon volume ratio is 55/10, giving

$$4 + \frac{y}{4} = 5.5 \quad \Longrightarrow \quad y = 6.$$

The resulting balanced equation is

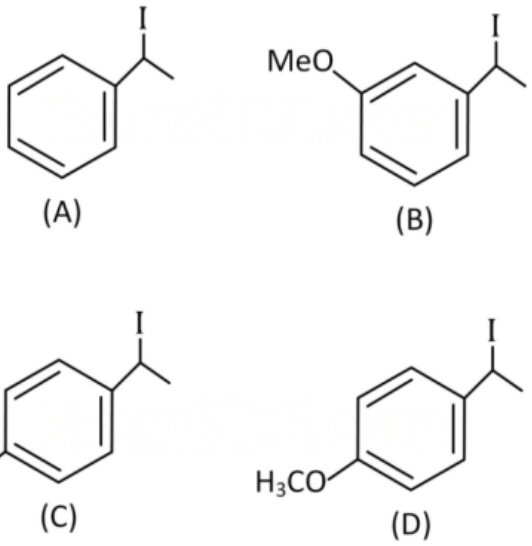
$$2\,C_4H_6 + 11\,O_2 \longrightarrow 8\,CO_2 + 6\,H_2O.$$

It predicts 55 mL oxygen and 40 mL carbon dioxide for 10 mL hydrocarbon. Option A contains chlorine and is not a hydrocarbon. Options C and D require oxygen-to-fuel ratios of 6 and 6.5 respectively, exceeding the observed 5.5.

Step 3: Conclusion The hydrocarbon has molecular formula C_4H_6. Hence, the correct option is (B).

Quick Tip to Solve: First obtain carbon atoms from the carbon dioxide volume ratio; then subtract that number from the oxygen ratio and multiply the remainder by four.

76. Increasing rate of S_N1 reaction in the following compounds is : [4 marks]



(A) $(B) < (A) < (D) < (C)$

(B) $(A) < (B) < (C) < (D)$

(C) $(A) < (B) < (D) < (C)$

(D) $(B) < (A) < (C) < (D)$

Correct Answer: (D) $(B) < (A) < (C) < (D)$

Solution:

Concept:

- The rate-determining step of an S_N1 reaction is ionization to a carbocation. With the same leaving group and comparable conditions, greater carbocation stabilization generally means a faster reaction.
- An ortho or para electron donor can stabilize a benzylic cation through resonance. A meta methoxy group lacks this direct resonance interaction and its inductive effect is electron withdrawing.

Step 1: Compare the parent and meta-substituted compounds All four substrates have the same secondary benzylic iodide centre, so they lose the same leaving group and form the same basic type of benzylic carbocation. Compound A has an unsubstituted phenyl ring. In B, methoxy is meta to the benzylic side chain. The positive charge of the benzyl cation is delocalized to the ortho and para ring positions, so meta methoxy cannot donate directly into those cationic resonance positions. Its oxygen exerts a negative inductive effect. Thus B forms the carbocation less readily than A:

$$(B) < (A).$$

Step 2: Compare the para donors and complete the order Compound C has a para methyl group, which supplies electron density by hyperconjugation and its positive inductive effect, stabilizing the benzylic cation relative to A. Compound D has para methoxy. Its lone-pair resonance donation is particularly effective because positive charge can appear at the para ring carbon in the benzylic cation.

This provides stronger stabilization than methyl. The increasing ionization rate is therefore

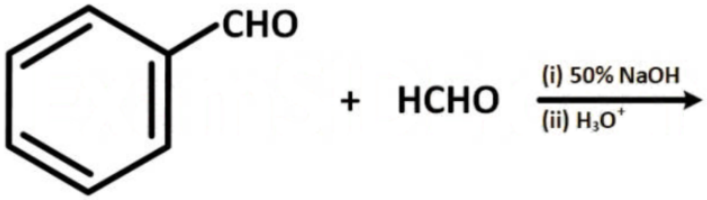
$$(B) < (A) < (C) < (D).$$

Option A reverses C and D, while options B and C place meta methoxy above the parent compound. Those orders neglect either the strong para resonance donation or the position dependence of methoxy's effects.

Step 3: Conclusion The increasing rate follows meta-methoxy substituted < unsubstituted < para-methyl substituted < para-methoxy substituted benzylic iodide. Hence, the correct option is (D).

Quick Tip to Solve: For benzylic substitution, locate the substituent before using its resonance effect: para methoxy strongly stabilizes the cation, whereas meta methoxy acts mainly inductively.

77. Major products of the following reaction are : [4 marks]



(A) $C_6H_5CH_2OH$ and C_6H_5COOH

(B) CH_3OH and C_6H_5COOH

(C) $HCOOH$ and $C_6H_5CH_2OH$

(D) CH_3OH and HCO_2H

Correct Answer: (C) $HCOOH$ and $C_6H_5CH_2OH$

Solution:

Concept:

- Aldehydes without an alpha hydrogen undergo Cannizzaro disproportionation in concentrated alkali: one aldehyde is oxidized to carboxylate and another is reduced to alcohol.
- In the crossed Cannizzaro reaction of formaldehyde with an aromatic aldehyde, formaldehyde is preferentially oxidized, while the aromatic aldehyde is reduced. Acid work-up converts carboxylate into the acid.

Step 1: Identify the roles of the two aldehydes The reactants are benzaldehyde, C_6H_5CHO, and formaldehyde, $HCHO$. Neither has an enolizable alpha hydrogen, so the concentrated sodium hydroxide conditions favour Cannizzaro chemistry rather than an aldol reaction. Hydroxide readily adds to the less hindered formaldehyde carbonyl. The resulting intermediate transfers hydride to benzaldehyde, oxidizing the formaldehyde-derived carbon and reducing the benzaldehyde carbonyl carbon. The carbon skeleton of benzaldehyde is retained in the alcohol.

Step 2: Apply the acid work-up and compare options The net alkaline reaction can be written

$$C_6H_5CHO + HCHO + NaOH \longrightarrow C_6H_5CH_2OH + HCOONa.$$

The second step supplies acid, which protonates sodium formate:

$$HCOONa + H_3O^+ \longrightarrow HCOOH + Na^+ + H_2O.$$

Thus the isolated products are benzyl alcohol and formic acid. Option A describes products of benzaldehyde self-disproportionation. Option B reverses the preferential donor and acceptor roles, while D omits any benzene-containing product and cannot conserve the carbon skeleton of benzaldehyde. Option C contains precisely the reduction and oxidation products of the crossed reaction.

Step 3: Conclusion The major products are $HCOOH$ and $C_6H_5CH_2OH$. Hence, the correct option is (C).

Quick Tip to Solve: In crossed Cannizzaro reactions with formaldehyde, remember that formaldehyde sacrifices itself to formate while the other aldehyde becomes an alcohol.

78. Consider the hydrated ions of Ti^{2+}, V^{2+}, Ti^{3+}, and Sc^{3+}. The correct order of their spin-only magnetic moments is : [4 marks]

(A) $Sc^{3+} < Ti^{3+} < V^{2+} < Ti^{2+}$

(B) $Sc^{3+} < Ti^{3+} < Ti^{2+} < V^{2+}$

(C) $Ti^{3+} < Ti^{2+} < Sc^{3+} < V^{2+}$

(D) $V^{2+} < Ti^{2+} < Ti^{3+} < Sc^{3+}$

Correct Answer: (B) $Sc^{3+} < Ti^{3+} < Ti^{2+} < V^{2+}$

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons. This expression increases with nonnegative n.
- For first-row transition-metal cations, remove the $4s$ electrons before removing $3d$ electrons. Water is a weak-field ligand; these d^1, d^2 and d^3 ions have one, two and three unpaired electrons respectively.

Step 1: Find the electronic configurations Neutral scandium, titanium and vanadium have configurations $[Ar]3d^14s^2$, $[Ar]3d^24s^2$ and $[Ar]3d^34s^2$, respectively. Removal of the outer electrons therefore gives $Sc^{3+} : 3d^0$, $Ti^{3+} : 3d^1$, $Ti^{2+} : 3d^2$ and $V^{2+} : 3d^3$. In their hydrated forms the first three d electrons occupy different lower-energy d orbitals according to Hund's rule, so their unpaired-electron counts are 0, 1, 2 and 3.

Step 2: Calculate and compare the moments Substitution into the spin-only expression gives

$$\mu(\mathrm{Sc}^{3+}) = 0,$$
$$\mu(\mathrm{Ti}^{3+}) = \sqrt{3}\,\mu_{\mathrm{B}},$$
$$\mu(\mathrm{Ti}^{2+}) = \sqrt{8}\,\mu_{\mathrm{B}},$$
$$\mu(\mathrm{V}^{2+}) = \sqrt{15}\,\mu_{\mathrm{B}}.$$

Thus the magnetic moment grows in exactly the order of the unpaired-electron counts. Option A reverses the last two ions; C wrongly places diamagnetic scandium above titanium; D reverses the trend entirely.

Step 3: Conclusion The order is $\mathrm{Sc}^{3+} < \mathrm{Ti}^{3+} < \mathrm{Ti}^{2+} < \mathrm{V}^{2+}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the d-electron counts first: d^0, d^1, d^2, d^3 immediately give increasing spin-only moments for these hydrated ions.

79. During the change of O_2 to O_2^- , the incoming electron goes to the orbital : [4 marks]

(A) $\pi 2p_y$

(B) $\sigma^* 2p_z$

(C) $\pi^* 2p_x$

(D) $\pi 2p_x$

Correct Answer: (C) $\pi^* 2p_x$

Solution:

Concept:

- Oxygen and superoxide are described using molecular orbitals. Added electrons enter the lowest available molecular orbital, subject to the Pauli principle and Hund's rule.
- For oxygen, the highest occupied orbitals are the degenerate antibonding $\pi^* 2p_x$ and $\pi^* 2p_y$ orbitals. The higher-energy $\sigma^* 2p_z$ orbital remains empty.

Step 1: Write the valence-shell occupancy Taking the internuclear axis as z, the valence configuration of O_2 is

$$(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2(\pi 2p_x)^2(\pi 2p_y)^2(\pi^* 2p_x)^1(\pi^* 2p_y)^1.$$

The two bonding pi orbitals are already full. Each of the two antibonding pi orbitals contains one electron, so either can accept a second electron of opposite spin. Their degeneracy means the labels x and y do not imply different energies.

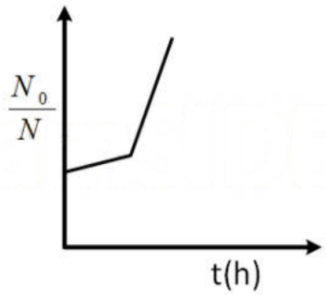
Step 2: Place the incoming electron and reject the alternatives Formation of O_2^- adds one electron without changing the nuclei. It pairs in a $\pi^* 2p$ orbital rather than entering the higher $\sigma^* 2p_z$ level. Among the printed choices, only $\pi^* 2p_x$ names this orbital. Options A and D are bonding pi orbitals, already occupied by two electrons each, while option B is energetically too high. As a consistency check, the extra antibonding electron lowers the bond order from $(8-4)/2 = 2$ to $(8-5)/2 = 1.5$, leaving one unpaired electron in superoxide.

Step 3: Conclusion The incoming electron occupies the antibonding π^*2p_x orbital among the listed choices. Hence, the correct option is (C).

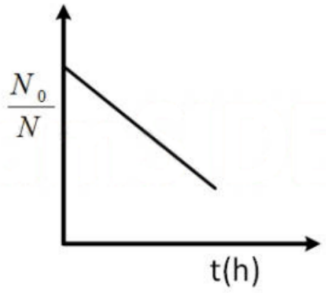
Quick Tip to Solve: For oxygen and its anions, add electrons to π^*2p before σ^*2p; each extra antibonding electron lowers the bond order by one-half.

80. A bacterial infection in an internal wound grows as $N'(t) = N_0 \exp(t)$, where the time t is in hours. A does of antibiotic, taken orally, needs 1 hour to reach the wound. Once it reaches there, the bacterial population goes down as $\frac{dN}{dt} = -5N^2$. What will be the plot of N_0/N vs. t after 1 hour? [4 marks]

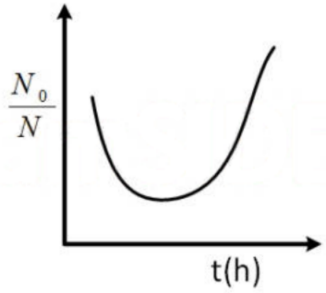
(A)



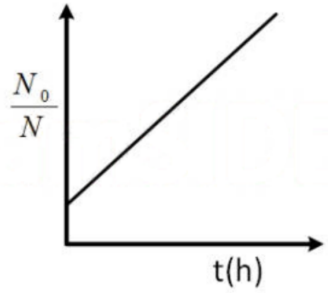
(B)



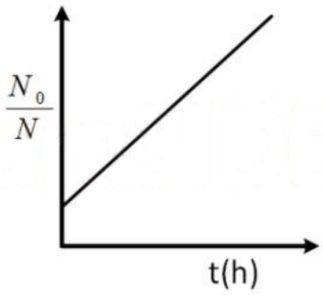
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- After the antibiotic reaches the wound, the stated equation is $dN/dt = -5N^2$. A negative rate makes the bacterial population decrease, but its reciprocal increases.
- The chain rule gives $d(1/N)/dt = -N^{-2}(dN/dt)$. Consequently a second-order decay law produces a straight-line plot of reciprocal population against time.

Step 1: Transform the decay equation For the interval after the first hour, differentiate the ordinate asked for in the question:

$$\frac{d}{dt}\left(\frac{N_0}{N}\right) = -\frac{N_0}{N^2}\frac{dN}{dt} = -\frac{N_0}{N^2}(-5N^2) = 5N_0.$$

Here N_0 is a fixed positive reference population. The slope is positive and independent of time. Neither the earlier growth law nor the precise population at the moment of arrival changes this constant slope.

Step 2: Integrate from the arrival time Let $N_1 = N(1) > 0$ denote the population when the drug arrives. Integrating only over the interval in which the decay equation applies gives

$$\frac{1}{N(t)} - \frac{1}{N_1} = 5(t-1), \qquad t \geq 1.$$

Therefore

$$\frac{N_0}{N(t)} = \frac{N_0}{N_1} + 5N_0(t-1).$$

This is an increasing straight line over the specified interval, with a positive value at its starting time. Option A has a change of slope, B has a negative slope, and C is curved; none can represent a function with the constant positive derivative just found. The fourth graph has the required linear rise.

Step 3: Conclusion After the antibiotic arrives, N_0/N increases linearly with time. Hence, the correct option is (D).

Quick Tip to Solve: For $dN/dt = -kN^2$, differentiate $1/N$ directly: its derivative is k. This identifies the graph without needing the earlier growth history.

81. Ethylamine ($C_2H_5NH_2$) can be obtained from N-ethylphatalimide on treatment with : [4 marks]

(A) CaH_2

(B) H_2O

(C) $NaBH_4$

(D) NH_2NH_2

Correct Answer: (D) NH_2NH_2

Solution:

Concept:

- Gabriel synthesis prepares a primary amine by alkylating phthalimide and then cleaving its imide protecting group. In N-ethylphthalimide, the ethyl group is already bonded to nitrogen.
- Hydrazine, NH_2NH_2, cleaves N-alkylphthalimides by hydrazinolysis. Formation of stable phthalhydrazide releases the corresponding primary amine.

Step 1: Identify the bond cleavage needed The required product is ethylamine, $C_2H_5NH_2$. The starting imide nitrogen is attached both to the ethyl group and to the two carbonyl-containing parts of the phthalimide ring. We must release that nitrogen as the primary amine while retaining its bond to the ethyl group. This is deprotection of an N-alkylphthalimide, rather than reduction of the ethyl group or construction of a new carbon skeleton.

Step 2: Select the reagent and compare the choices Hydrazine attacks the imide carbonyl groups and the resulting acyl substitution cleaves the imide linkage. The net transformation can be summarized as

$$\text{N-ethylphthalimide} + NH_2NH_2 \longrightarrow \text{phthalhydrazide} + C_2H_5NH_2.$$

Calcium hydride, option A, is not the reagent used to perform this selective cleavage. Water alone, option B, does not give the standard cleavage under the stated conditions; hydrolysis of an imide normally needs appropriate acidic or basic conditions and heating. Sodium borohydride, option C, is a mild reducing agent, not the standard Gabriel deprotection reagent. Hydrazine is the specific reagent supplied in option D.

Step 3: Conclusion Treat N-ethylphthalimide with hydrazine, NH_2NH_2, to liberate ethylamine. Hence, the correct option is (D).

Quick Tip to Solve: An N-alkylphthalimide already contains the desired alkyl–nitrogen bond; use hydrazinolysis to uncover the primary amine without changing the alkyl chain.

82. Consider the statements S1 and S2

S1 : Conductivity always increases with decrease in the concentration of electrolyte.

S2 : Molar conductivity always increases with decrease in the concentration of electrolyte.

The correct option among the following is : [4 marks]

(A) Both S1 and S2 are wrong

(B) S1 is correct and S2 is wrong

(C) Both S1 and S2 are correct

(D) S1 is wrong and S2 is correct

Correct Answer: (D) S1 is wrong and S2 is correct

Solution:

Concept:

- Conductivity, κ, measures the conducting ability of a solution of specified dimensions and depends on the concentration of mobile ions per unit volume.
- Molar conductivity is $\Lambda_m = 1000\kappa/c$ when c is in $\mathrm{mol\,L^{-1}}$ and κ in $\mathrm{S\,cm^{-1}}$. It measures conductance associated with one mole of electrolyte and has a different dilution trend.

Step 1: Evaluate statement S1 Dilution reduces the number of ions in a unit volume of solution. Although ionic mobility can improve as interionic interactions weaken, this does not cause conductivity always to increase. In the usual electrolyte solutions considered here, conductivity decreases on dilution because there are fewer charge carriers per unit volume. Hence the assertion in S1 that conductivity always increases as concentration decreases is false. Confusing conductivity with molar conductivity would reverse this conclusion.

Step 2: Evaluate statement S2 and combine the results For a strong electrolyte, dilution reduces interionic attractions and allows ions to migrate more freely, so molar conductivity increases toward its limiting value. For a weak electrolyte, dilution additionally increases the degree of dissociation, producing more ions from each mole of electrolyte; its molar conductivity also increases. Thus S2 expresses the standard dilution trend correctly. Options A and B incorrectly call S2 false, and C incorrectly calls S1 true. Only option D combines false S1 with true S2. The factor $1/c$ in the definition also shows why a falling conductivity can coexist with rising molar conductivity.

Step 3: Conclusion S1 is wrong and S2 is correct. Hence, the correct option is (D).

Quick Tip to Solve: Keep the reference amount in mind: conductivity concerns a unit volume, whereas molar conductivity concerns one mole, so dilution affects them differently.

83. A gas undergoes physical adsorption on a surface and follows the given Freundlich adsorption isotherm equation

$$\frac{x}{m} = kp^{0.5}$$

Adsorption of the gas increase with : [4 marks]

(A) Decrease in p and increase in T

(B) Increase in p and decrease in T

(C) Decrease in p and decrease in T

(D) Increase in p and increase in T

Correct Answer: (B) Increase in p and decrease in T

Solution:

Concept:

- The Freundlich equation here is $x/m = kp^{0.5}$, where x/m is the mass adsorbed per unit mass of adsorbent and k is constant at a specified temperature.
- Physical adsorption is generally exothermic. Lowering the temperature favours the adsorbed state, while increasing pressure favours transfer of gas molecules onto the surface.

Step 1: Determine the pressure dependence from the equation At fixed temperature and positive pressure,

$$\frac{d(x/m)}{dp} = \frac{k}{2\sqrt{p}} > 0.$$

Therefore adsorption increases with pressure. For example, multiplying pressure by four doubles x/m according to the stated isotherm. Decreasing pressure cannot increase adsorption in this model. This immediately excludes options A and C, both of which specify a decrease in pressure. The exponent 0.5 weakens the increase relative to direct proportionality but does not reverse it.

Step 2: Apply the temperature effect of physical adsorption Physical adsorption releases heat because attractions form between the gas and the surface. For the equilibrium between free gas molecules and adsorbed molecules, heat can be regarded as a product. Cooling favours adsorption; heating favours desorption, allowing molecules to escape from the surface. Thus between the remaining choices, the decrease in temperature in B is favourable, whereas the increase in temperature in D is unfavourable. The Freundlich equation is an isotherm, so its pressure dependence alone should not be used to infer temperature independence: its constant also depends on temperature.

Step 3: Conclusion Adsorption increases on increasing pressure and decreasing temperature. Hence, the correct option is (B).

Quick Tip to Solve: Use the exponent to settle the pressure effect, then remember that physical adsorption releases heat and is therefore favoured by cooling.

84. The regions of the atmosphere, where clouds form and where we live, respectively, are : [4 marks]

(A) Stratosphere and Stratosphere

(B) Stratosphere and Troposphere

(C) Troposphere and Stratosphere

(D) Troposphere and Troposphere

Correct Answer: (D) Troposphere and Troposphere

Solution:

Concept:

- The troposphere is the lowest atmospheric layer, extending upward from the Earth's surface. It contains most atmospheric water vapour and is the region of ordinary weather.
- The stratosphere lies above the troposphere and contains the ozone-rich region. It is not the atmospheric layer in which people ordinarily live or in which ordinary weather clouds form.

Step 1: Locate the cloud-forming region Clouds form when moist air cools sufficiently for water vapour to condense into small droplets or ice crystals. The troposphere contains most of the atmosphere's water vapour and undergoes the vertical mixing associated with weather. Consequently ordinary clouds, rain and storms belong to the troposphere. This fixes the first member of the requested pair as troposphere, eliminating A and B, which identify the cloud-forming layer as stratosphere. The question concerns the standard atmospheric classification, rather than rare specialised high-altitude clouds.

Step 2: Locate the region where we live Human settlements are on the Earth's surface and within its immediate atmospheric surroundings, all in the lowest atmospheric layer. Therefore we live in the troposphere as well. Option C correctly identifies the first layer but incorrectly places human life in the stratosphere. Option D names the troposphere for both entries and is the only pair meeting both requirements. The ozone layer is a useful identifier of the stratosphere, but its importance in absorbing ultraviolet light does not make it the layer containing our normal living environment.

Step 3: Conclusion Clouds form in the troposphere, and we live in the troposphere. Hence, the correct option is (D).

Quick Tip to Solve: Associate the troposphere with the surface, water vapour and weather; associate the stratosphere with the ozone layer above it.

85. The synonym for water gas when used in the production of methanol is : [4 marks]

(A) fuel gas

(B) laughing gas

(C) syn gas

(D) natural gas

Correct Answer: (C) syn gas

Solution:

Concept:

- Water gas is a mixture of carbon monoxide, CO, and hydrogen, H_2. The same components are feedstocks for several industrial syntheses.
- The name synthesis gas, shortened to syngas or syn gas, describes a carbon monoxide–hydrogen mixture used to prepare useful chemicals, including methanol.

Step 1: Identify the components of water gas Passing steam over red-hot carbon forms water gas. The carbon removes oxygen from steam, producing carbon monoxide and hydrogen together:

$$C + H_2O\,(g) \longrightarrow CO + H_2.$$

This preparation explains why the mixture contains both a carbon source and hydrogen. Water gas is a mixture, not the name of a single molecular substance.

Step 2: Connect the mixture to methanol synthesis Methanol manufacture uses carbon monoxide and hydrogen over a suitable catalyst under pressure. The net synthesis is

$$CO + 2\,H_2 \longrightarrow CH_3OH.$$

The hydrogen-to-carbon-monoxide ratio can be adjusted for this reaction. When water gas is used as a chemical-synthesis feedstock, its relevant name is synthesis gas, or syn gas.

Step 3: Reject the remaining names Fuel gas is a broad description based on combustibility and does not give the specific synthesis-feedstock name requested. Laughing gas is nitrous oxide, N_2O, which contains neither of the two required feedstock components. Natural gas consists mainly of methane, CH_4; although it can be converted into synthesis gas, it is not itself the carbon monoxide–hydrogen mixture.

Step 4: Conclusion The synonym is **syn gas**, short for synthesis gas. Hence, the correct option is (C).

Quick Tip to Solve: For methanol manufacture, look for the pair CO and H_2: their industrial feedstock is called synthesis gas.

86. The alloy used in the construction of aircrafts is : [4 marks]

(A) Mg – Zn

(B) Mg – Al

(C) Mg – Mn

(D) Mg – Sn

Correct Answer: (B) Mg – Al

Solution:

Concept:

- Aircraft construction favours materials with low density and a useful strength-to-weight ratio. Alloying can improve the mechanical performance of a light metal.
- Magnesium and aluminium are both light metals. Their alloy is the standard magnesium alloy associated with aircraft construction in this question.

Step 1: Identify the required material property An aircraft structure must carry loads while keeping its own mass low. A dense metal may be strong but adds an avoidable weight penalty. Magnesium has a low density, while aluminium is also a light metal and is widely used in structural alloys. A magnesium–aluminium alloy combines the light-metal character with better practical strength than pure magnesium.

Step 2: Select the magnesium–aluminium pair Among the listed compositions, the pair Mg–Al identifies the familiar lightweight alloy used in aircraft construction. This is commonly associated with magnalium in elementary descriptions. The choice concerns the principal alloying pair, rather than a pure metal or an exact percentage composition, which the question does not specify.

Step 3: Compare the alternatives Options (A), (C), and (D) give magnesium paired with zinc, manganese, and tin respectively. Those are different alloy compositions and do not name the standard magnesium–aluminium aircraft alloy sought here. Merely containing magnesium is therefore insufficient to select an option: the second named constituent must also agree. The aluminium-containing option is the unique intended match among these four choices.

Step 4: Conclusion The alloy is Mg–Al. Hence, the correct option is (B).

Quick Tip to Solve: Associate aircraft alloys with low density and good strength per unit mass; in this list the familiar pair is magnesium–aluminium.

87. Match the refining methods (Column I) with metals (Column II).

Column I (Refining Methods)	Column II (Metals)
(I) Liquation	(a) Zr
(II) Zone Refining	(b) Ni
(III) Mond Process	(c) Sn
(IV) Van Arkel Method	(d) Ga

[4 marks]

(A) (I)-(c) ; (II)-(a) ; (III)-(b) ; (IV)-(d)

(B) (I)-(c) ; (II)-(d) ; (III)-(b) ; (IV)-(a)

(C) (I)-(b) ; (II)-(d) ; (III)-(a) ; (IV)-(c)

(D) (I)-(b) ; (II)-(c) ; (III)-(d) ; (IV)-(a)

Correct Answer: (B) (I)-(c) ; (II)-(d) ; (III)-(b) ; (IV)-(a)

Solution:

Concept:

- A refining method is chosen by the physical or chemical difference between the desired metal and its impurities. Liquation uses melting behaviour; zone refining uses differential solubility.
- Mond refining relies on a volatile nickel carbonyl, whereas the Van Arkel method uses a volatile metal iodide that decomposes on a hot filament.

Step 1: Match liquation and zone refining In liquation, a low-melting metal is melted and drained away from higher-melting impurities on a sloping surface. Tin is the suitable match, so (I) corresponds to (c). Zone refining moves a narrow molten zone through the solid: impurities favour the melt and travel with it. This method supplies very pure semiconductor materials, including gallium; hence (II) corresponds to (d).

Step 2: Match the chemical transport methods In the Mond process, impure nickel reacts with carbon monoxide to form volatile nickel tetracarbonyl. On stronger heating, the carbonyl decomposes and deposits pure nickel:

$$\mathrm{Ni} + 4\,\mathrm{CO} \rightleftharpoons \mathrm{Ni(CO)_4}.$$

Thus (III) corresponds to (b). The Van Arkel method purifies zirconium through volatile zirconium tetraiodide, which decomposes on a hot filament:

$$\mathrm{ZrI_4} \longrightarrow \mathrm{Zr} + 2\,\mathrm{I_2}.$$

Thus (IV) corresponds to (a).

Step 3: Compare the complete sequences The resulting order is (c), (d), (b), (a). Option (A) incorrectly exchanges the zone-refining and Van Arkel matches. Options (C) and (D) begin by assigning liquation to nickel and also mismatch the chemical methods. Only option (B) preserves all four established method–metal associations.

Step 4: Conclusion Liquation matches tin, zone refining matches gallium, the Mond process matches nickel, and the Van Arkel method matches zirconium. Hence, the correct option is (B).

Quick Tip to Solve: Remember the chemical signatures: nickel forms the carbonyl in Mond refining; zirconium forms the iodide in the Van Arkel method.

88. Which of the following is a condensation polymer? [4 marks]

(A) Neoprene

(B) Buna-S

(C) Nylon 6, 6

(D) Teflon

Correct Answer: (C) Nylon 6, 6

Solution:

Concept:

- Condensation polymerisation joins multifunctional monomers through repeated reactions that eliminate small molecules, often water. The resulting linkages may be amide or ester groups.
- Addition polymerisation joins unsaturated monomers without eliminating a small molecule at each chain-forming step; several familiar synthetic rubbers and fluoropolymers form this way.

Step 1: Identify the monomers of nylon 6,6 Nylon 6,6 is produced from hexamethylenediamine and adipic acid. Each monomer has two reactive functional groups, enabling chain growth in both directions. An amino group of the diamine reacts with a carboxyl group of the dicarboxylic acid. This produces an amide linkage, $-CO-NH-$, with elimination of water.

Step 2: Recognise the condensation linkage The repeating structural portion of nylon 6,6 can be written

$$\left[-NH-(CH_2)_6-NH-CO-(CH_2)_4-CO-\right]_n.$$

Its amide links originate from reactions of the two different functional groups; they are not formed by simply opening a carbon–carbon double bond. Repeated elimination of water during the formation of these links identifies nylon 6,6 as a condensation polymer.

Step 3: Classify the other three polymers Neoprene is made by addition polymerisation of chloroprene. Buna-S is an addition copolymer of buta-1,3-diene and styrene. Teflon is formed by addition polymerisation of tetrafluoroethene, $CF_2=CF_2$. Their unsaturated monomers add into chains without the characteristic water-eliminating amide formation seen in nylon 6,6, so options (A), (B), and (D) do not fit.

Step 4: Conclusion **Nylon 6,6** is the condensation polymer. Hence, the correct option is (C).

Quick Tip to Solve: Look for two functional groups reacting with loss of a small molecule: diamine plus dicarboxylic acid gives a condensation polyamide.

89. Consider the following table:

Gas	$a/(\mathrm{k\,Pa\,dm^6\,mol^{-1}})$	$b/(\mathrm{dm^3\,mol^{-1}})$
A	**642.32**	**0.05196**
B	**155.21**	**0.04136**
C	**431.91**	**0.05196**
D	**155.21**	**0.4382**

a and b are vander Waals constants. The correct statement about the gases is: [4 marks]

(A) Gas C will occupy more volume than gas A; gas B will be more compressible than gas D

(B) Gas C will occupy lesser volume than gas A; gas B will be more compressible than gas D

(C) Gas C will occupy lesser volume than gas A; gas B will be lesser compressible than gas D

(D) Gas C will occupy more volume than gas A; gas B will be lesser compressible than gas D

Correct Answer: (A) Gas C will occupy more volume than gas A; gas B will be more compressible than gas D

Solution:

Concept:

- For one mole, the van der Waals equation is $\left(P + a/V_m^2\right)(V_m - b) = RT$. The constant a measures intermolecular attraction and b measures excluded molecular volume.
- At comparable temperature and pressure, stronger attractions favour contraction, whereas larger excluded volume opposes compression. Compare one parameter at a time when the other is equal.

Step 1: Compare gases A and C using their attraction constants The table gives identical excluded-volume constants, $b_A = b_C = 0.05196$, but different attraction constants:

$$a_A = 642.32 > 431.91 = a_C.$$

Gas A has the stronger intermolecular attractions. These attractions pull its molecules closer together and favour a smaller molar volume. Gas C, with weaker attractions and the same excluded-volume parameter, therefore occupies more volume than gas A under the intended comparison.

Step 2: Compare gases B and D using excluded volume For these two gases the attraction constants are equal: $a_B = a_D = 155.21$. However,

$$b_B = 0.04136 < 0.4382 = b_D.$$

The smaller excluded-volume constant of B means a smaller volume is unavailable to molecular motion. B can consequently be compressed more readily; the larger molecular-size correction for D makes reduction in volume more difficult.

Step 3: Combine the statements and eliminate alternatives The first comparison requires more volume for C than A, eliminating options (B) and (C). The second requires B to be more compressible than D, eliminating option (D). Both required conclusions appear together only in (A). The units displayed in the supplied table do not affect this ranking because each comparison uses numerical constants from the same column.

Step 4: Conclusion Gas C will occupy more volume than gas A, and gas B will be more compressible

than gas D. Hence, the correct option is (A).

Quick Tip to Solve: First find equal constants in the table: equal b isolates attraction, while equal a lets you compare the excluded-volume effect.

90. The correct order of catenation is : [4 marks]

(A) C $>$ Sn $>$ Si $\approx$ Ge

(B) Si $>$ Sn $>$ C $>$ Ge

(C) C $>$ Si $>$ Ge $\approx$ Sn

(D) Ge $>$ Sn $>$ Si $>$ C

Correct Answer: (C) C $>$ Si $>$ Ge $\approx$ Sn

Solution:

Concept:

- Catenation is an element's ability to form bonds with atoms of the same element, producing chains, rings, and networks. Strong element–element bonds support extensive catenation.
- Down Group 14, increasing atomic size generally weakens effective orbital overlap in like-atom covalent bonds. Carbon is therefore especially effective at forming stable chains.

Step 1: Place carbon first Carbon atoms are small and form strong carbon–carbon covalent bonds. Stable single and multiple bonds permit a very large range of chains and rings. This is the structural basis of the vast diversity of organic compounds. Carbon consequently has the greatest catenation tendency among the four elements listed; any ordering that places another element above carbon is rejected.

Step 2: Compare silicon with germanium and tin Silicon can catenate, but its larger orbitals overlap less effectively than those of carbon, and silicon–silicon bonds are weaker than carbon–carbon bonds. Germanium and tin have still larger atoms and lower catenation tendencies. The standard Group 14 comparison places silicon after carbon, with the much weaker catenation of germanium and tin treated as approximately comparable in the given choices.

Step 3: Choose the ordering that preserves both comparisons These considerations give

$$\mathrm{C} > \mathrm{Si} > \mathrm{Ge} \approx \mathrm{Sn}.$$

Option (A) incorrectly places tin above silicon and makes silicon approximately equal to germanium. Option (B) places silicon above carbon and tin above germanium. Option (D) puts carbon last, opposite to its strong self-bonding ability. Only option (C) agrees with the established qualitative trend.

Step 4: Conclusion The correct order is $\mathrm{C} > \mathrm{Si} > \mathrm{Ge} \approx \mathrm{Sn}$. Hence, the correct option is (C).

Quick Tip to Solve: Use carbon's exceptionally strong self-bond as the first anchor, then place silicon ahead of the weakly catenating heavier Group 14 elements.