JEE Main 2019 Paper 1 (B.E./B.Tech) Apr 12 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 12 April 2019, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point : [4 marks]

(A) (1, 5)

(B) (2, 3)

(C) (3, 5)

(D) (3, 10)

Correct Answer: (D) (3, 10)

Solution:

Concept:

- A circle tangent to the x-axis at $(3, 0)$ has its centre on the perpendicular through the point of contact, so its centre is $(3, k)$ and its radius is $|k|$.
- A chord at distance d from the centre has length $2\sqrt{r^2 - d^2}$. Here the y-axis is at distance 3 from the centre.

Step 1: Determine the centre and radius The y-axis intercept is a chord of length 8. Its half-length is 4. Applying Pythagoras to the radius, perpendicular distance and half-chord gives

$$k^2 = 3^2 + 4^2 = 25.$$

Thus $k = 5$ or $k = -5$, and the two possible circles are

$$(x - 3)^2 + (y - 5)^2 = 25, \qquad (x - 3)^2 + (y + 5)^2 = 25.$$

Step 2: Test the listed points All four listed points have positive y-coordinate, so only the circle centred at $(3, 5)$ can contain one of them. The squared distances of the first three points from this centre are respectively

$$4, \quad 5, \quad 0,$$

none of which equals 25. For $(3, 10)$, the squared distance is $0^2 + 5^2 = 25$. The circle centred below the x-axis has no positive-y points, so it creates no further listed solution.

Step 3: Conclusion The circle with centre $(3, 5)$ passes through $(3, 10)$. Hence, the correct option is (D).

Quick Tip to Solve: For a tangent circle, locate its centre directly above or below the contact point, then use half the intercepted chord in a right triangle.

2. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs. 12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is : [4 marks]

(A) $\frac{1}{4}$ loss

(B) $\frac{1}{2}$ gain

(C) $\frac{1}{2}$ loss

(D) 2 gain

Correct Answer: (C) $\frac{1}{2}$ loss

Solution:

Concept:

- Two fair dice give 36 equally likely ordered outcomes. The expected monetary change is the sum of each net payoff multiplied by its probability.
- The doublet and sum-9 events are disjoint: a doublet has an even total. Use negative payoffs for losses.

Step 1: Count the outcomes for each payoff The six doublets are $(1,1),(2,2),\ldots,(6,6)$, each paying Rs. 15. The outcomes with total 9 are $(3,6),(4,5),(5,4),(6,3)$, giving four outcomes paying Rs. 12. Since these events do not overlap, the remaining

$$36-6-4=26$$

outcomes each incur a loss of Rs. 6. These three counts exhaust the sample space.

Step 2: Calculate the weighted average Let X be the gain in rupees on one throw. Then

$$\begin{aligned} E(X) &= 15\left(\frac{6}{36}\right)+12\left(\frac{4}{36}\right)-6\left(\frac{26}{36}\right) \\ &= \frac{90+48-156}{36}=\frac{-18}{36}=-\frac{1}{2}. \end{aligned}$$

The negative sign means an expected loss, rather than a gain. Thus neither positive-gain option is possible, and the loss magnitude is one-half rather than one-quarter.

Step 3: Conclusion The expected result is a loss of Rs. $\frac{1}{2}$ per throw. Hence, the correct option is (C).

Quick Tip to Solve: List disjoint payoff categories before averaging; for dice, keep the ordered outcomes equally likely and put a minus sign on every loss.

3. Let $a \in \left(0, \frac{\pi}{2}\right)$ be fixed. If the integral $\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx$ = A(x) cos 2α + B(x) sin 2α + C, where C is a constant of integration, then the functions A(x) and B(x) are respectively : [4 marks]

(A) $x - \alpha$ and $\log_e |\cos(x - \alpha)|$

(B) $x + \alpha$ and $\log_e |\sin(x - \alpha)|$

(C) $x + \alpha$ and $\log_e |\sin(x + \alpha)|$

(D) $x - \alpha$ and $\log_e |\sin(x - \alpha)|$

Correct Answer: (D) $x - \alpha$ and $\log_e |\sin(x - \alpha)|$

Solution:

Concept:

- Convert the sum and difference of tangents to sines using a common denominator. On any interval where the original integrand is defined, this gives $\sin(x + \alpha)/\sin(x - \alpha)$.
- The identities $\sin(u+2\alpha) = \sin u \cos 2\alpha + \cos u \sin 2\alpha$ and $\int \cot u \, du = \log|\sin u|$ reduce the integral to elementary terms.

Step 1: Rewrite the integrand Put $u = x - \alpha$, so $du = dx$ because α is fixed. Then

$$\frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} = \frac{\sin(x + \alpha)}{\sin(x - \alpha)} = \cos 2\alpha + \sin 2\alpha \cot u.$$

This identity is applied only where the original quotient exists; in particular its denominator must be nonzero.

Step 2: Integrate and compare the choices Integrating term by term yields

$$I = (x - \alpha)\cos 2\alpha + \log|\sin(x - \alpha)| \sin 2\alpha + C.$$

This is the representative pair in (D). Differentiation returns $\cos 2\alpha + \sin 2\alpha \cot(x - \alpha)$, verifying the result. Option (A) instead differentiates to a term containing $-\tan(x - \alpha)$, and (C) uses the incorrect shifted argument $x + \alpha$. There is a literal ambiguity: (B) differs from (D) only by the constant $2\alpha \cos 2\alpha$, which is absorbed into the arbitrary integration constant. Consequently (B) also defines a valid antiderivative.

Step 3: Conclusion The supplied key selects (D): $A(x) = x - \alpha$, $B(x) = \log|\sin(x - \alpha)|$. With unrestricted C, (B) is also mathematically valid because its difference from (D) is constant; the printed question does not uniquely distinguish these two pairs.

Quick Tip to Solve: For indefinite integrals, compare options by differentiating: two formulas differing only by a parameter-dependent constant represent the same family.

4. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right).\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to : [4 marks]

(A) 36

(B) - 108

(C) - 36

(D) - 72

Correct Answer: (C) - 36

Solution:

Concept:

- The general term of $(a+b)^6$ is $\binom{6}{r}a^{6-r}b^r$, for integer $0 \le r \le 6$. Track its power of x before evaluating its coefficient.
- Multiplication by a prefactor changes exponents. A constant in the final product can arise both from an inner constant term and from an inner x^{-8} term.

Step 1: Find the relevant inner exponents Write the term with r factors of $-3/x^2$ as

$$T_{r+1}=\binom{6}{r}2^{6-r}(-3)^r x^{12-4r}.$$

When multiplied by $1/60$, this term is constant if $12-4r=0$, giving $r=3$. When multiplied by $-x^8/81$, it is constant if $20-4r=0$, giving $r=5$. Both indices are in the permitted range, so both contributions must be retained.

Step 2: Evaluate and add both contributions The first contribution is

$$\frac{1}{60}\binom{6}{3}2^3(-3)^3=\frac{20\cdot 8\cdot(-27)}{60}=-72.$$

The second is

$$-\frac{1}{81}\binom{6}{5}2^1(-3)^5=-\frac{6\cdot 2\cdot(-243)}{81}=36.$$

Therefore the required constant is $-72+36=-36$. The numbers 36 and -72 represent only one contribution each; -108 incorrectly combines their signs.

Step 3: Conclusion The term independent of x is -36. Hence, the correct option is (C).

Quick Tip to Solve: With a two-term prefactor, solve the zero-exponent condition separately for each prefactor term before adding coefficients.

5. Let z ∈ C with Im(z) = 10 and it satisfies $\frac{2z-n}{2z+n}$ = 2i - 1 for some natural number n. Then : [4 marks]

(A) n = 20 and Re(z) = –10

(B) n = 40 and Re(z) = 10

(C) n = 40 and Re(z) = –10

(D) n = 20 and Re(z) = 10

Correct Answer: (C) n = 40 and Re(z) = –10

Solution:

Concept:

- A complex equation can be solved algebraically by cross-multiplication, provided the original denominator is nonzero. Rationalise a complex denominator using its conjugate.
- If $z = a + ib$, then $\mathrm{Re}\, z = a$ and $\mathrm{Im}\, z = b$. The given imaginary part determines the real parameter n.

Step 1: Express z in terms of n The relation gives

$$2z - n = (-1 + 2i)(2z + n).$$

Collecting terms containing z yields

$$(4 - 4i)z = 2in, \qquad z = \frac{in}{2(1 - i)}.$$

Multiply numerator and denominator by $1 + i$:

$$z = \frac{in(1 + i)}{4} = \frac{n}{4}(-1 + i).$$

The negative real part appears because $i(1 + i) = i - 1$, not $1 + i$.

Step 2: Use the imaginary part and check From the expression above, $\mathrm{Im}\, z = n/4 = 10$, hence $n = 40$. Its real part is $-n/4 = -10$, so $z = -10 + 10i$. The original denominator is $2z + n = 20 + 20i \neq 0$, and

$$\frac{2z - n}{2z + n} = \frac{-60 + 20i}{20 + 20i} = \frac{-3 + i}{1 + i} = -1 + 2i.$$

Thus the substitution satisfies the full equation. Choices with n equal to 20 have the wrong imaginary part; the remaining positive-real-part choice has the wrong sign.

Step 3: Conclusion We obtain $n = 40$ and $\mathrm{Re}(z) = -10$. Hence, the correct option is (C).

Quick Tip to Solve: Isolate z before separating real and imaginary parts; rationalising once gives both components and makes the sign check immediate.

6. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to : [4 marks]

(A) 24

(B) 25

(C) 27

(D) 28

Correct Answer: (B) 25

Solution:

Concept:

- An unordered team is counted by combinations. A three-person team containing both sexes consists of either two boys and one girl, or one boy and two girls.
- These two cases are disjoint, so add their counts. For n girls, $\binom{n}{2} = n(n-1)/2$.

Step 1: Count the two admissible team types For two boys and one girl, select the boys in $\binom{5}{2} = 10$ ways and the girl in n ways, giving $10n$ teams. For one boy and two girls, select the boy in 5 ways and the girls in $\binom{n}{2}$ ways, giving $5n(n-1)/2$ teams. No all-boy or all-girl team is included, and every permitted team belongs to exactly one case.

Step 2: Solve the resulting equation The given count requires

$$10n + \frac{5n(n-1)}{2} = 1750.$$

Multiply by $2/5$ and simplify:

$$4n + n(n-1) = 700, \qquad n^2 + 3n - 700 = 0.$$

Factoring gives

$$(n+28)(n-25) = 0.$$

The negative root is impossible for a number of girls, so $n = 25$. A direct check is $10(25) + 5\binom{25}{2} = 250 + 1500 = 1750$. None of 24, 27 or 28 satisfies the quadratic, so the other options are excluded.

Step 3: Conclusion The group contains 25 girls. Hence, the correct option is (B).

Quick Tip to Solve: For a three-person mixed team, count the disjoint patterns 2+1 and 1+2; avoid permutations because team order does not matter.

7. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to : [4 marks]

(A) $\alpha\gamma$

(B) 0

(C) $\beta\gamma$

(D) $\alpha\beta$

Correct Answer: (C) $\beta\gamma$

Solution:

Concept:

- Three consecutive terms of a geometric progression satisfy $\beta^2 = \alpha\gamma$. Write them as $\alpha, \alpha r, \alpha r^2$ to reveal a perfect square.
- A common root must satisfy both equations. Substitute the repeated root of the first quadratic into the second to obtain a relation among the progression terms.

Step 1: Identify the common root Because the progression is non-constant, $\alpha \neq 0$. Writing $\beta = \alpha r$ and $\gamma = \alpha r^2$, the first equation becomes

$$\alpha x^2 + 2\alpha r x + \alpha r^2 = \alpha(x + r)^2 = 0.$$

Its only root is $x = -r = -\beta/\alpha$. The common root therefore has to be this value, even though the second quadratic has two distinct roots.

Step 2: Use the second quadratic and simplify Substituting $x = -r$ into $x^2 + x - 1 = 0$ gives

$$r^2 - r - 1 = 0, \qquad r^2 = r + 1.$$

Hence $\gamma = \alpha r^2 = \alpha + \beta$. Now

$$\begin{aligned} \alpha(\beta + \gamma) &= \alpha\beta + \alpha\gamma \\ &= \alpha\beta + \beta^2 \\ &= \beta(\alpha + \beta) = \beta\gamma. \end{aligned}$$

The first equality uses distribution, and the second uses the GP identity. Since r is neither 0 nor 1, the alternative expressions 0, $\alpha\gamma$ and $\alpha\beta$ do not equal the required value.

Step 3: Conclusion The expression $\alpha(\beta + \gamma)$ equals $\beta\gamma$. Hence, the correct option is (C).

Quick Tip to Solve: When the coefficients are consecutive GP terms with a doubled middle coefficient, factor the quadratic as a square before eliminating the common root.

8. The general solution of the differential equation $(y^2 - x^3)dx - xydy = 0$ $(x \neq 0)$ is : (where c is a constant of integration) [4 marks]

(A) $y^2 + 2x^3 + cx^2 = 0$

(B) $y^2 + 2x^2 + cx^3 = 0$

(C) $y^2 - 2x + cx^3 = 0$

(D) $y^2 - 2x^3 + cx^2 = 0$

Correct Answer: (A) $y^2 + 2x^3 + cx^2 = 0$

Solution:

Concept:

- An equation containing $y\,dy$ and y^2 can often be converted into a linear equation by setting $v = y^2$, so that $dv/dx = 2y\,dy/dx$.
- For a linear equation $v' + P(x)v = Q(x)$, an integrating factor is $e^{\int P(x)\,dx}$. Work on an interval where the stated condition $x \neq 0$ holds.

Step 1: Convert to a linear equation From the given differential equation,

$$xy\frac{dy}{dx} = y^2 - x^3.$$

Put $v = y^2$. Multiplication by $2/x$ gives

$$\frac{dv}{dx} - \frac{2}{x}v = -2x^2.$$

This substitution avoids dividing by y, so it does not discard a point at which a solution has $y = 0$. The restriction needed for this step is precisely the printed $x \neq 0$.

Step 2: Apply the integrating factor The integrating factor is $e^{-2\log|x|} = x^{-2}$. Hence

$$\frac{d}{dx}\left(\frac{v}{x^2}\right) = -2, \qquad \frac{v}{x^2} = -2x + C.$$

Returning to $v = y^2$, and absorbing a sign into the arbitrary constant, gives

$$y^2 + 2x^3 + cx^2 = 0.$$

Step 3: Check the proposed family Differentiating this family gives $2yy' + 6x^2 + 2cx = 0$. Using $c = -(y^2 + 2x^3)/x^2$ recovers $xyy' = y^2 - x^3$. Option (D) reverses the required sign of the cubic term. Options (B) and (C) place the arbitrary constant with x^3 instead of the required homogeneous term x^2, and therefore do not describe the derived general family.

Step 4: Conclusion The general solution is $y^2 + 2x^3 + cx^2 = 0$, on intervals with $x \neq 0$. Hence, the correct option is (A).

Quick Tip to Solve: When $y\,dy$ appears together with y^2, try $v = y^2$ before looking for a separable form.

9. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line x + y = 0. Then an equation of the line L is : [4 marks]

(A) $x + \sqrt{3}y = 8$

(B) $\sqrt{3}x + y = 8$

(C) $(\sqrt{3} + 1)x + (\sqrt{3} - 1)y = 8\sqrt{2}$

(D) $(\sqrt{3} - 1)x + (\sqrt{3} + 1)y = 8\sqrt{2}$

Correct Answer: Both (C) and (D) are valid; the supplied key selects (D).

Solution:

Concept:

- The normal form of a line at distance p from the origin is $x\cos\phi + y\sin\phi = p$. Positive intercepts require both normal coefficients to be positive.
- The angle between two unoriented lines is their acute angle. For direction vectors u, v, its cosine is $|u \cdot v|/(|u||v|)$.

Step 1: Find the possible normal directions Write L as $x\cos\phi + y\sin\phi = 4$, with $0 < \phi < \pi/2$. A direction vector of $x + y = 0$ is $(1, -1)$. The required angle gives

$$\frac{|\cos\phi - \sin\phi|}{\sqrt{2}} = \frac{1}{2}.$$

Thus $|\cos\phi - \sin\phi| = 1/\sqrt{2}$, yielding $\phi = \pi/12$ or $5\pi/12$. Both directions lie in the first quadrant, so both yield positive intercepts.

Step 2: Write both admissible lines For $\phi = \pi/12$, use $\cos 15^\circ = (\sqrt{6} + \sqrt{2})/4$ and $\sin 15^\circ = (\sqrt{6} - \sqrt{2})/4$. Multiplication by $2\sqrt{2}$ gives

$$(\sqrt{3} + 1)x + (\sqrt{3} - 1)y = 8\sqrt{2}.$$

For $\phi = 5\pi/12$, the two coefficients interchange:

$$(\sqrt{3} - 1)x + (\sqrt{3} + 1)y = 8\sqrt{2}.$$

These are options (C) and (D), respectively. Their coefficient norms are $2\sqrt{2}$, so each is indeed at distance 4.

Step 3: Check the remaining choices and the key Options (A) and (B) have first-quadrant normal angles 60° and 30°; their acute angles with the line of direction 135° are 75°, rather than 60°. The supplied key selects (D), which satisfies all conditions, but (C) also does. No orientation convention is specified in the wording that eliminates (C).

Step 4: Conclusion Both (C) and (D) satisfy the printed geometric conditions. The supplied answer key selects (D); under the usual acute-angle interpretation the question has two valid options and is ambiguous.

Quick Tip to Solve: For an angle between lines, keep the absolute value in the dot-product formula; dropping it can discard a valid normal direction.

10. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true ? [4 marks]

(A) If (A – B) $\subseteq$ C, then A $\subseteq$ C

(B) B $\cap$ C $\neq \phi$

(C) (C $\cup$ A) $\cap$ (C $\cup$ B) = C

(D) If (A – C) $\subseteq$ B, then A $\subseteq$ B

Correct Answer: (D) If (A – C) $\subseteq$ B, then A $\subseteq$ B

Solution:

Concept:

- The hypothesis says $A \cap B$ is nonempty and every member of that intersection belongs to C. A universal claim is disproved by one example satisfying its premises.
- Use $A = (A \setminus B) \cup (A \cap B)$ and the distributive identity $(C \cup A) \cap (C \cup B) = C \cup (A \cap B)$.

Step 1: Verify the first three statements For (A), both pieces of the decomposition $A = (A \setminus B) \cup (A \cap B)$ are subsets of C: the first by its additional premise, and the second by the given condition. Thus $A \subseteq C$. For (B), take any $z \in A \cap B$, which exists because the intersection is nonempty. Then $z \in B$ and $z \in C$, so $B \cap C \neq \varnothing$. For (C), distributivity gives

$$(C \cup A) \cap (C \cup B) = C \cup (A \cap B) = C,$$

since the extra intersection is already contained in C. Thus all three statements are necessarily true.

Step 2: Test the final implication with a counterexample Choose

$$A = \{1, 2\}, \qquad B = \{1\}, \qquad C = \{1, 2\}.$$

Then $A \cap B = \{1\} \neq \varnothing$ and $A \cap B \subseteq C$, so the original hypothesis holds. Also $A \setminus C = \varnothing \subseteq B$, so the additional premise in (D) holds. Nevertheless $2 \in A$ and $2 \notin B$, so $A \not\subseteq B$. The premise constrains elements outside C, but it places no such requirement on elements of $A \cap C$.

Step 3: Conclusion The final implication need not hold, as the counterexample proves. Hence, the correct option is (D).

Quick Tip to Solve: To disprove an implication about sets, make its premise easy to satisfy, for example by choosing an empty set difference.

11. Let f(x) = 5 – |x – 2| and g(x) = |x + 1|, x ∈ R. If f(x) attains maximum value at α and g(x) attains minimum value at β, then $\lim_{x\to-\alpha\beta}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to : [4 marks]

(A) $\frac{1}{2}$
(B) $-\frac{1}{2}$
(C) $\frac{3}{2}$
(D) $-\frac{3}{2}$

Correct Answer: (A) $\frac{1}{2}$

Solution:

Concept:

- The absolute value $|u|$ is nonnegative and attains its minimum value zero exactly when $u = 0$. Therefore a constant minus an absolute value is maximised at that same zero.
- For a rational expression giving $0/0$, factor the numerator and denominator. A common factor can be cancelled for nearby points other than the limiting point.

Step 1: Locate the two extremising points For $f(x) = 5 - |x - 2|$, the subtracted quantity is smallest at $x = 2$, where it is zero. Thus the maximum is 5, attained at $\alpha = 2$. For $g(x) = |x + 1|$, the minimum is zero and occurs at $\beta = -1$. Notice that α and β are the locations of the extrema, rather than their values. Hence

$$-\alpha\beta = -(2)(-1) = 2.$$

The limit must therefore be evaluated as $x \to 2$.

Step 2: Factor before taking the limit The quadratic factors are

$$x^2 - 5x + 6 = (x - 2)(x - 3), \qquad x^2 - 6x + 8 = (x - 2)(x - 4).$$

For $x \neq 2$ sufficiently near 2, cancellation gives

$$\frac{(x - 1)(x^2 - 5x + 6)}{x^2 - 6x + 8} = \frac{(x - 1)(x - 3)}{x - 4}.$$

The reduced denominator is nonzero at 2, so direct substitution is now valid:

$$\lim_{x\to2}\frac{(x - 1)(x - 3)}{x - 4} = \frac{(2 - 1)(2 - 3)}{2 - 4} = \frac{1}{2}.$$

The two negative factors in the last fraction cancel. This rules out the negative choices, and the numerator has magnitude one rather than three, excluding (C).

Step 3: Conclusion The required limit equals $\frac{1}{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the input values where the absolute-value terms vanish first; then factor the limit and cancel only in a punctured neighbourhood.

12. A value of α such that

$$\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e\left(\frac{9}{8}\right)$$

is: [4 marks]

(A) 2

(B) -2

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

Correct Answer: (B) -2

Solution:

Concept:

- The identity $1/[u(u+1)] = 1/u - 1/(u+1)$ converts this rational integrand into two logarithmic derivatives.
- A definite integral is evaluated from an antiderivative only when its interval contains no pole. The possible poles here are $x = -\alpha$ and $x = -\alpha - 1$, so candidate values must also be checked for admissibility.

Step 1: Integrate by partial fractions For an interval avoiding both poles,

$$\begin{aligned} I &= \left[\log\left|\frac{x+\alpha}{x+\alpha+1}\right|\right]_{\alpha}^{\alpha+1} \\ &= \log\left|\frac{2\alpha+1}{2\alpha+2}\right| - \log\left|\frac{2\alpha}{2\alpha+1}\right| \\ &= \log\left|\frac{(2\alpha+1)^2}{4\alpha(\alpha+1)}\right|. \end{aligned}$$

The numerator is a square. For either $\alpha > 0$ or $\alpha < -1$, the interval is free of poles and the denominator in this ratio is positive.

Step 2: Solve the resulting equation and check the options On these admissible ranges the required equation is

$$\frac{(2\alpha+1)^2}{4\alpha(\alpha+1)} = \frac{9}{8}.$$

Cross multiplication yields

$$8(4\alpha^2+4\alpha+1) = 36\alpha(\alpha+1), \quad \alpha^2+\alpha-2 = 0.$$

Thus $\alpha = 1$ or -2. Only -2 appears among the options. It gives integration limits $-2, -1$, while the poles are $2, 1$, both outside that interval, and substitution gives the ratio $9/8$ exactly. Option (A), $\alpha = 2$, gives $25/24$; option (C), $\alpha = 1/2$, gives $4/3$, so neither matches. For (D), $\alpha = -1/2$, the pole $x = 1/2$ is an endpoint, so the integral is divergent and cannot equal the stated finite logarithm.

Step 3: Conclusion The listed admissible value is $\alpha = -2$. Hence, the correct option is (B). The supplied key differs, but substituting its value $\alpha = 2$ gives $\log(25/24)$, rather than the required $\log(9/8)$, so it cannot satisfy the printed integral.

Quick Tip to Solve: After using logarithms to evaluate a definite rational integral, check that no denominator zero lies in its integration interval before accepting an algebraic root.

13. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points? [4 marks]

(A) $(2, \sqrt{2})$
(B) $(2, 2\sqrt{2})$
(C) $(\sqrt{2}, 2)$
(D) $(1, 2\sqrt{2})$

Correct Answer: (C) $(\sqrt{2}, 2)$

Solution:

Concept:

- For an ellipse centred at the origin with its major axis on the y-axis, the equation is $x^2/b^2+y^2/a^2 = 1$, where $a > b > 0$.
- The focal distance satisfies $c^2 = a^2 - b^2$. The full minor-axis length is $2b$, so the printed length must be halved before using the formula.

Step 1: Determine the semiaxes and equation The foci $(0, 2)$ and $(0, -2)$ have midpoint $(0, 0)$, so this is the centre. They lie on the y-axis, which is therefore the major axis, and their distance from the centre is $c = 2$. The minor axis has full length 4, hence $b = 2$, giving

$$a^2 = b^2 + c^2 = 4 + 4 = 8.$$

Consequently the ellipse is

$$\frac{x^2}{4} + \frac{y^2}{8} = 1.$$

In particular, the denominator under y^2 must be the larger one, since the foci are vertical.

Step 2: Substitute each proposed point At $(\sqrt{2}, 2)$, the left side is

$$\frac{(\sqrt{2})^2}{4} + \frac{2^2}{8} = \frac{1}{2} + \frac{1}{2} = 1,$$

so option (C) lies on the ellipse. For (A), the left side at $(2, \sqrt{2})$ is $1 + 1/4 = 5/4$. For (B), at $(2, 2\sqrt{2})$, it is $1 + 1 = 2$. For (D), at $(1, 2\sqrt{2})$, it is $1/4 + 1 = 5/4$. Since none of these three values is one, those points are outside the ellipse rather than on it. This explicit substitution also avoids mistaking the focal distance for either semiaxis.

Step 3: Conclusion The point $(\sqrt{2}, 2)$ lies on the ellipse $x^2/4 + y^2/8 = 1$. Hence, the correct option is (C).

Quick Tip to Solve: Use the foci to identify the major-axis direction, then halve the minor-axis length and apply $a^2 = b^2 + c^2$.

14. A value of $\theta \in \left(0, \frac{\pi}{3}\right)$, for which

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0, \text{ is :}$$ [4 marks]

(A) $\frac{\pi}{18}$

(B) $\frac{\pi}{9}$

(C) $\frac{7\pi}{24}$

(D) $\frac{7\pi}{36}$

Correct Answer: (B) $\frac{\pi}{9}$

Solution:

Concept:

- Adding a multiple of one row or column to another does not change a determinant. A column sum can expose the identity $\cos^2\theta + \sin^2\theta = 1$.
- A trigonometric equation obtained from a determinant must still satisfy the stated interval. Checking the offered angles in the simplified expression is quicker than expanding the original determinant separately for each.

Step 1: Reduce the determinant with row operations Write $a = \cos^2\theta$, $b = \sin^2\theta$, and $d = 4\cos 6\theta$, so $a + b = 1$. The matrix is

$$\begin{pmatrix} 1+a & b & d \\ a & 1+b & d \\ a & b & 1+d \end{pmatrix}.$$

Apply $R_1 \leftarrow R_1 - R_3$ and $R_2 \leftarrow R_2 - R_3$. This gives

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ a & b & 1+d \end{vmatrix}.$$

Next use $C_3 \leftarrow C_3 + C_1 + C_2$. The last column becomes $(0, 0, 1 + d + a + b)^T$, so the determinant is

$$1 + d + a + b = 2 + 4\cos 6\theta.$$

Step 2: Solve and check the listed angles Vanishing of the determinant is equivalent to $\cos 6\theta = -1/2$. Because $0 < \theta < \pi/3$, the angle 6θ lies in $(0, 2\pi)$. Its possible values are $2\pi/3$ and $4\pi/3$, giving $\theta = \pi/9$ or $2\pi/9$. Of these, only $\pi/9$ is listed. For (A), $6\theta = \pi/3$ and the cosine is $1/2$. For (C), $6\theta = 7\pi/4$ and the cosine is $\sqrt{2}/2$. For (D), it is $7\pi/6$ and the cosine is $-\sqrt{3}/2$. None makes the determinant zero.

Step 3: Conclusion One required value is $\theta = \pi/9$, which belongs to the stated interval. Hence, the correct option is (B).

Quick Tip to Solve: When every row contains the same base vector plus a different diagonal one, subtract one row from the others before expanding the determinant.

15. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to : [4 marks]

(A) $4\sqrt{3}$

(B) $2\sqrt{6}$

(C) 48

(D) 24

Correct Answer: (D) 24

Solution:

Concept:

- Integrate the horizontal separation of the curves: area is $\int (x_{\text{right}} - x_{\text{left}})\, dy$. The parameter λ is positive, so division by it preserves inequalities.
- The intersections determine the integration limits. Solving the line for x makes both boundaries polynomials in y, avoiding square roots.

Step 1: Find both intersections From the line, $x = y/\lambda$. Substitute this into the parabola:

$$y^2 = 4\lambda \frac{y}{\lambda} = 4y \quad \Longrightarrow \quad y(y-4) = 0.$$

Thus the endpoints have ordinates 0 and 4, with corresponding abscissae 0 and $4/\lambda$. For $0 < y < 4$, we have $y > y^2/4$, so the line is the right boundary and the parabola is the left boundary.

Step 2: Integrate the width and impose the area The horizontal width is $y/\lambda - y^2/(4\lambda)$, giving

$$\begin{aligned} A &= \frac{1}{\lambda}\int_0^4 \left(y - \frac{y^2}{4}\right) dy \\ &= \frac{1}{\lambda}\left[\frac{y^2}{2} - \frac{y^3}{12}\right]_0^4 = \frac{1}{\lambda}\left(8 - \frac{16}{3}\right) = \frac{8}{3\lambda}. \end{aligned}$$

Equating this to $1/9$ gives $3\lambda = 72$, hence $\lambda = 24$. As a check, $\lambda = 48$ would give $1/18$, not $1/9$; the two radical options also fail $8/(3\lambda) = 1/9$.

Step 3: Conclusion The required positive value is $\boxed{\lambda = 24}$. Hence, the correct option is (D).

Quick Tip to Solve: When both curves are easy to express as x in terms of y, use horizontal strips; here the intersection ordinates are independent of λ.

16. $\lim\limits_{x\to 0} \frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ **is :** [4 marks]

(A) 6

(B) 1

(C) 3

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- For a difference of square roots, multiply numerator and denominator by the sum of the roots. This removes the indeterminate difference without approximating either radical separately.
- The standard limit $\sin x/x \to 1$ implies $(x^2 - \sin^2 x)/(x + 2\sin x) \to 0$. Both radicands tend to 1, so the radicals are real in a neighbourhood of zero.

Step 1: Rationalise the denominator Put $s = \sin x$. Multiplication by the conjugate gives

$$\frac{(x+2s)\left(\sqrt{1+x^2+2s}+\sqrt{1+s^2-x}\right)}{x^2-s^2+x+2s}.$$

The denominator follows by subtracting the second radicand from the first. In particular, the linear terms combine as $x + 2s$, exactly matching the original numerator. Keep the sign of $-x$ in the second radicand when performing this subtraction.

Step 2: Evaluate the remaining factors For sufficiently small nonzero x, divide the polynomial factor by $x + 2s$. The expression becomes

$$\frac{\sqrt{1+x^2+2s}+\sqrt{1+s^2-x}}{1+\dfrac{x^2-s^2}{x+2s}}.$$

Now

$$\frac{x^2-s^2}{x+2s} = x\frac{1-(\sin x/x)^2}{1+2\sin x/x} \longrightarrow 0.$$

The numerator tends to $1 + 1 = 2$, and the denominator tends to 1. Thus the limit is 2. The values 6, 1 and 3 do not retain both the matching linear factor and the conjugate sum, so none satisfies this evaluation.

Step 3: Conclusion The limit is $\boxed{2}$. Hence, the correct option is (D).

Quick Tip to Solve: After rationalising, look for the original numerator inside the new denominator; its leading term often cancels before applying $\sin x/x \to 1$.

17. The derivative of $\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x \in \left(0, \frac{\pi}{2}\right)\right)$ is : [4 marks]

(A) 1

(B) 2

(C) $\frac{2}{3}$

(D) $\frac{1}{2}$

Correct Answer: (B) 2

Solution:

Concept:

- The tangent subtraction identity gives $\tan(x-\pi/4) = (\sin x-\cos x)/(\sin x+\cos x)$. The denominator is positive on the specified interval.
- The principal range of $\tan^{-1}$ is $(-\pi/2, \pi/2)$. Differentiation with respect to $u = x/2$ uses $df/du = (df/dx)/(du/dx)$.

Step 1: Reduce the inverse tangent with its range Let $f(x)$ denote the given function. Since $0 < x < \pi/2$,

$$-\frac{\pi}{4} < x - \frac{\pi}{4} < \frac{\pi}{4}.$$

This entire interval lies inside the principal range of inverse tangent. Consequently,

$$f(x) = \tan^{-1}\left(\tan\left(x - \frac{\pi}{4}\right)\right) = x - \frac{\pi}{4}.$$

The interval check matters: inverse tangent of tangent cannot be replaced by its argument on every real interval. Here no branch adjustment is necessary, and the simplified function is differentiable throughout the given domain.

Step 2: Change the differentiation variable Write $u = x/2$, so $x = 2u$ and $f = 2u - \pi/4$. It follows directly that

$$\frac{df}{d(x/2)} = \frac{df}{du} = 2.$$

Equivalently, $df/dx = 1$ and $du/dx = 1/2$, so their quotient is 2. Option (A) is the derivative with respect to x, rather than the variable asked for. Option (D) multiplies by $1/2$ instead of dividing; option (C) has no factor arising from either the identity or the chain rule.

Step 3: Conclusion The derivative with respect to $x/2$ is $\boxed{2}$. Hence, the correct option is (B).

Quick Tip to Solve: First check the principal range before simplifying an inverse trigonometric composition; then explicitly name the new variable when differentiating with respect to $x/2$.

18. If a_1, a_2, a_3, ... are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is : [4 marks]

(A) 120

(B) 200

(C) 150

(D) 280

Correct Answer: (B) 200

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, the term formula is $a_n = a + (n-1)d$.
- The sum of the first n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$. For an odd number of terms, the average equals the middle term.

Step 1: Extract the combination fixed by the condition The given three terms are

$$a_1 = a, \qquad a_7 = a + 6d, \qquad a_{16} = a + 15d.$$

Adding them, as prescribed, gives

$$3a + 21d = 40 \quad \Longrightarrow \quad a + 7d = \frac{40}{3}.$$

This fixes a_8, the middle term among the first fifteen terms. It does not determine a and d separately, and separate values are unnecessary. Notice that the coefficient of d in the sixteenth term is fifteen; using sixteen would alter the combination and the answer.

Step 2: Use the middle term to obtain the sum Apply the sum formula without introducing an extra unknown:

$$S_{15} = \frac{15}{2}(2a + 14d)$$
$$= 15(a + 7d) = 15 \cdot \frac{40}{3} = 200.$$

A second check comes from pairing terms equidistant from the eighth term: $a_1 + a_{15} = a_2 + a_{14} = 2a_8$, and similarly for the other pairs. Seven such pairs plus a_8 total $15a_8$. Thus 120, 150 and 280 are excluded because each would give a middle term different from the fixed value $40/3$.

Step 3: Conclusion The sum of the first fifteen terms is $\boxed{200}$. Hence, the correct option is (B).

Quick Tip to Solve: In an A.P. sum problem, compare the given linear combination of a and d with the sum formula before trying to determine the two unknowns separately.

19. The equation of common tangent to the curves $y^2 = 16x$ and $xy = -4$, is : [4 marks]

(A) $x - y + 4 = 0$

(B) $x + y + 4 = 0$

(C) $x - 2y + 16 = 0$

(D) $2x - y + 2 = 0$

Correct Answer: (A) $x - y + 4 = 0$

Solution:

Concept:

- For $y^2 = 4ax$, a tangent with nonzero finite slope m has equation $y = mx + a/m$. Here $a = 4$.
- A line is tangent to a conic when substitution produces a quadratic with a repeated root. The repeated-root condition is discriminant zero.

Step 1: Write a general tangent to the parabola The parabola $y^2 = 16x$ has $a = 4$. Its finite-slope tangent can therefore be written as

$$y = mx + \frac{4}{m}, \qquad m \neq 0.$$

The excluded vertical tangent is $x = 0$; it cannot be a tangent to $xy = -4$, since that hyperbola has no point on $x = 0$. Thus the finite-slope family suffices for the common tangent. The intercept and slope must obey this relation; an arbitrary straight line need not touch the parabola.

Step 2: Impose tangency to the hyperbola Substitute the line into $xy = -4$:

$$mx^2 + \frac{4}{m}x + 4 = 0.$$

For a repeated intersection,

$$\left(\frac{4}{m}\right)^2 - 16m = 0 \quad \Longrightarrow \quad m^3 = 1.$$

The real slope is $m = 1$, so $y = x + 4$, or $x - y + 4 = 0$. Directly, this line gives $(y - 4)^2 = 0$ on the parabola and $(x + 2)^2 = 0$ on the hyperbola. It touches them at $(1, 4)$ and $(-2, 2)$, respectively. Option (B) has slope -1, which fails $m^3 = 1$. Options (C) and (D) have slopes $1/2$ and 2, which likewise fail the common-tangency condition.

Step 3: Conclusion The common tangent is $\boxed{x - y + 4 = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the tangent family of one conic first, then require a zero discriminant after substitution into the other; this leaves only one slope unknown.

20. If [x] denotes the greatest integer $\leq$ x, then the system of linear equations [sin θ]x + [–cosθ]y = 0, [cotθ]x + y = 0 [4 marks]

(A) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(B) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(C) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(D) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

Correct Answer: (B) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

Solution:

Concept:

- The greatest-integer function $[t]$ is the integer k satisfying $k \leq t < k+1$. In particular, a negative number strictly between -1 and 0 has floor -1, not zero.
- A homogeneous system always has the zero solution. With two unknowns, a nonzero determinant gives a unique solution; a singular coefficient matrix gives infinitely many solutions.

Step 1: Evaluate the floors in the second quadrant For $\pi/2 < \theta < 2\pi/3$, the ranges are

$$0 < \sin\theta < 1, \qquad 0 < -\cos\theta < \frac{1}{2}, \qquad -1 < \cot\theta < 0.$$

Therefore $[\sin\theta] = 0$, $[-\cos\theta] = 0$, and $[\cot\theta] = -1$. The equations reduce to

$$0 = 0, \qquad -x + y = 0.$$

Every pair $(x, y) = (t, t)$, with real t, is a solution. Hence there are infinitely many solutions in the first stated interval. The endpoints are excluded, so no boundary value of a trigonometric function changes these floors.

Step 2: Evaluate the floors in the third quadrant For $\pi < \theta < 7\pi/6$, one has $-1/2 < \sin\theta < 0$ and $\sqrt{3}/2 < -\cos\theta < 1$. Hence their floors are -1 and 0, respectively. Although $k = [\cot\theta]$ can vary, the system is always

$$-x = 0, \qquad kx + y = 0.$$

The first equation forces $x = 0$, and the second then forces $y = 0$. Equivalently, the determinant of $\begin{pmatrix} -1 & 0 \\ k & 1 \end{pmatrix}$ is -1. Thus this interval has a unique solution. Option (A) reverses the two conclusions; (C) wrongly makes the second interval singular; (D) wrongly makes the first interval nonsingular.

Step 3: Conclusion There are infinitely many solutions for $\theta \in (\pi/2, 2\pi/3)$ and a unique solution for $\theta \in (\pi, 7\pi/6)$. Hence, the correct option is (B).

Quick Tip to Solve: For floor-function coefficient systems, determine signs and integer bounds first. A variable floor coefficient need not be evaluated exactly when another equation already forces its unknown to zero.

21. The Boolean expression ~(p ⇒ (~q)) is equivalent to : [4 marks]

(A) p ∧ q

(B) q ⇒ ~p

(C) p ∨ q

(D) (~p) ⇒ q

Correct Answer: (A) p ∧ q

Solution:

Concept:

- An implication satisfies $p \Rightarrow r \equiv (\neg p) \vee r$. It is false only when its antecedent is true and its consequent is false.
- De Morgan's law gives $\neg(A \vee B) \equiv (\neg A) \wedge (\neg B)$, and double negation gives $\neg\neg p \equiv p$.

Step 1: Remove the implication and apply negation In the given implication the consequent is $\neg q$, so

$$p \Rightarrow (\neg q) \equiv (\neg p) \vee (\neg q).$$

Negating this complete statement and applying De Morgan's law yields

$$\begin{aligned} \neg\big(p \Rightarrow (\neg q)\big) &\equiv \neg\big((\neg p) \vee (\neg q)\big) \\ &\equiv (\neg\neg p) \wedge (\neg\neg q) \equiv p \wedge q. \end{aligned}$$

The outer negation changes the disjunction into a conjunction; it must act on both components, rather than on just the consequent.

Step 2: Check the truth condition and reject the alternatives The original implication is false precisely when p is true and $\neg q$ is false, that is, when both p and q are true. Its negation is therefore true only for that same pair, exactly as $p \wedge q$ is. Option (B), $q \Rightarrow \neg p$, is false when both are true, so it has the opposite value in that decisive case. Option (C), $p \vee q$, is also true when exactly one statement is true, so it is too broad. Option (D), $(\neg p) \Rightarrow q$, also has extra true cases: taking p true and q false makes this implication true while the required expression is false.

Step 3: Conclusion The expression is equivalent to $\boxed{p \wedge q}$. Hence, the correct option is (A).

Quick Tip to Solve: To negate an implication quickly, use $\neg(A \Rightarrow B) \equiv A \wedge \neg B$, then simplify any double negation in the consequent.

22. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is : [4 marks]

(A) $15\left(1+\sqrt{3}\right)$
(B) $15\left(3-\sqrt{3}\right)$
(C) $15\left(3+\sqrt{3}\right)$
(D) $15\left(5-\sqrt{3}\right)$

Correct Answer: (C) $15\left(3+\sqrt{3}\right)$

Solution:

Concept:

- For an angle of elevation θ, $\tan\theta$ equals the vertical rise to the object divided by the horizontal distance to it.
- Points A and B lie on the same vertical line, so their horizontal distances from the tower are equal. Raising the observation point by $30\,\mathrm{m}$ reduces the vertical rise by that amount.

Step 1: Express the tower height using A Let the foot and top of the tower be F and T. Put $AF = d$ metres and $FT = h$ metres. The tower is vertical and the ground horizontal, so the triangle at A is right-angled. Using the elevation from A gives

$$\tan 45^\circ = \frac{h}{d} = 1, \qquad h = d.$$

Thus the height and the required horizontal distance have the same numerical value.

Step 2: Use the elevation from B The horizontal through B meets the tower 30 metres above F. Hence the vertical rise from B to T is $h - 30$, while the horizontal distance is still d. Therefore

$$\tan 30^\circ = \frac{h-30}{d} = \frac{1}{\sqrt{3}}.$$

Substituting $h = d$, and multiplying through by $\sqrt{3}$, gives $\sqrt{3}(d-30) = d$.

Step 3: Solve and compare the options Rearranging and rationalising the denominator,

$$d = \frac{30\sqrt{3}}{\sqrt{3}-1} = \frac{30\sqrt{3}(\sqrt{3}+1)}{2} = 15(3+\sqrt{3}).$$

This is greater than 30, as required for the top to be above B. Options (A), (B) and (D) give different values and do not satisfy $d - 30 = d/\sqrt{3}$; the plus sign in the rationalised numerator is essential.

Step 4: Conclusion The distance of the foot of the tower from A is $15(3+\sqrt{3})\,\mathrm{m}$. Hence, the correct option is (C).

Quick Tip to Solve: When two observation points are vertically aligned, keep the horizontal distance unchanged and subtract their height difference from the second vertical rise.

23. Let $\alpha \in$ R and the three vectors
$\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$
and $\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$.
Then the set S = {α : $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar} : [4 marks]

(A) contains exactly two numbers only one of which is positive

(B) is singleton

(C) contains exactly two positive numbers

(D) is empty

Correct Answer: (D) is empty

Solution:

Concept:

- Three vectors in three dimensions are coplanar exactly when their scalar triple product, or the determinant of their component rows, is zero.
- Subtracting one row from another leaves a determinant unchanged. A square of a real number is nonnegative, which can settle whether the resulting equation has a real root.

Step 1: Form the coplanarity determinant Using the coefficients of $\hat{i}, \hat{j}, \hat{k}$ in the given order, the necessary and sufficient condition is

$$D = \begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -\alpha \\ \alpha & -2 & 3 \end{vmatrix} = 0.$$

We must find real values of α; complex roots would not belong to the set S specified in the question.

Step 2: Simplify by a row operation Replace the third row by the third row minus the first row. Its entries become $(0, -3, 0)$. Expanding along that row gives

$$D = \begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -\alpha \\ 0 & -3 & 0 \end{vmatrix} = 3 \begin{vmatrix} \alpha & 3 \\ 2 & -\alpha \end{vmatrix} = 3(-\alpha^2 - 6).$$

The cofactor sign in position $(3, 2)$ is negative; together with the entry -3, it produces the factor $+3$.

Step 3: Test the real solution set Thus coplanarity would require $\alpha^2 + 6 = 0$. For every real α, however, $\alpha^2 + 6 \geq 6 > 0$, so this cannot hold. In particular $D \leq -18$ and never vanishes. There are neither two real roots nor a single real root, so options (A), (B) and (C) are all ruled out.

Step 4: Conclusion No real value of α makes the three vectors coplanar, so $S = \varnothing$. Hence, the correct option is (D).

Quick Tip to Solve: Look for rows with two equal components before expanding a determinant: subtraction here leaves a single nonzero entry and avoids a long cubic expansion.

24. The tangents to the curve $y = (x - 2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point : [4 marks]

(A) $\left(\frac{5}{2}, -1\right)$

(B) $\left(-\frac{5}{2}, -1\right)$

(C) $\left(\frac{5}{2}, 1\right)$

(D) $\left(-\frac{5}{2}, 1\right)$

Correct Answer: (A) $\left(\frac{5}{2}, -1\right)$

Solution:

Concept:

- The points where two curves meet satisfy both equations simultaneously. Here the line gives $y = x - 3$, which can be substituted in the parabola equation.
- The tangent to $y = f(x)$ at (x_0, y_0) has equation $y - y_0 = f\prime(x_0)(x - x_0)$. The derivative supplies the tangent slope, not the slope of the given intersecting line.

Step 1: Find the points of contact Equating the two expressions for y yields

$$(x - 2)^2 - 1 = x - 3 \quad \Longrightarrow \quad x^2 - 5x + 6 = 0.$$

Factoring gives $(x - 2)(x - 3) = 0$, so $x = 2$ or $x = 3$. Using $y = x - 3$, the two contact points are $P = (2, -1)$ and $Q = (3, 0)$. Each lies on both the parabola and the specified line.

Step 2: Write both tangent equations Differentiate the parabola:

$$\frac{dy}{dx} = 2(x - 2).$$

At P the slope is zero, so its tangent is the horizontal line $y = -1$. At Q the slope is 2, giving

$$y - 0 = 2(x - 3), \qquad y = 2x - 6.$$

The original line joins P and Q; it is a secant and must not be used as either tangent.

Step 3: Intersect the tangents Substitute $y = -1$ into the second tangent equation: $-1 = 2x - 6$, whence $2x = 5$ and $x = 5/2$. Thus the intersection is $(5/2, -1)$. It satisfies both tangent equations. Options (B) and (D) have the wrong sign of x, while option (C) has y equal to 1 and therefore cannot lie on the horizontal tangent.

Step 4: Conclusion The tangents intersect at $\left(\frac{5}{2}, -1\right)$. Hence, the correct option is (A).

Quick Tip to Solve: If one point of contact is the vertex of a vertical parabola, its tangent is horizontal; use that fixed y-coordinate to intersect the other tangent quickly.

25. A plane which bisects the angle between the two given planes 2x – y + 2z – 4 = 0 and x + 2y + 2z – 2 = 0, passes through the point : [4 marks]

(A) (1, –4, 1)

(B) (1, 4, –1)

(C) (2, 4, 1)

(D) (2, –4, 1)

Correct Answer: (D) (2, –4, 1)

Solution:

Concept:

- A point on a bisector plane is at equal perpendicular distances from the two original planes. For $ax + by + cz + d = 0$, that distance is $|ax + by + cz + d|/\sqrt{a^2 + b^2 + c^2}$.
- There are two angle-bisector planes, obtained by equating the normalised plane expressions with both possible signs. The normal-vector lengths must be included before cancelling denominators.

Step 1: Normalise the plane expressions Write $f_1 = 2x - y + 2z - 4$ and $f_2 = x + 2y + 2z - 2$. Both normals have length $\sqrt{4 + 1 + 4} = \sqrt{1 + 4 + 4} = 3$. The bisector condition is therefore

$$\frac{|f_1|}{3} = \frac{|f_2|}{3}, \qquad f_1 = f_2 \text{ or } f_1 = -f_2.$$

Equal unnormalised expressions are valid here specifically because the two normal lengths agree.

Step 2: Obtain the two bisector planes Taking the difference and then the sum gives

$$f_1 - f_2 = x - 3y - 2 = 0, \qquad f_1 + f_2 = 3x + y + 4z - 6 = 0.$$

A proposed point need lie on either one of these planes, since the question asks for a plane bisecting the angle rather than prescribing a particular bisector.

Step 3: Check the listed points Evaluate the left sides of the two bisector equations in order. For (A), $(1, -4, 1)$, the results are $11, -3$; for (B), $(1, 4, -1)$, they are $-13, -3$; for (C), $(2, 4, 1)$, they are $-12, 8$. None is zero. For (D), $(2, -4, 1)$, they are $12, 0$. Thus only (D) lies on a bisector, specifically the second one. Directly, the original expressions there are 6 and -6, confirming equal distances of 2.

Step 4: Conclusion The bisector plane $3x + y + 4z - 6 = 0$ passes through $(2, -4, 1)$. Hence, the correct option is (D).

Quick Tip to Solve: For equal-length plane normals, test both the sum and the difference of the plane expressions; testing only their difference misses one entire angle bisector.

26. A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (2, 3). Then the centroid of this triangle is : [4 marks]

(A) $\left(\frac{1}{3}, 2\right)$
(B) $\left(\frac{1}{3}, \frac{5}{3}\right)$
(C) $\left(1, \frac{7}{3}\right)$
(D) $\left(\frac{1}{3}, 1\right)$

Correct Answer: (A) $\left(\frac{1}{3}, 2\right)$

Solution:

Concept:

- If M is the midpoint of a side joining vertices A and B, then $M = (A+B)/2$, so the unknown vertex is $B = 2M - A$. Apply this coordinatewise.
- The centroid is the arithmetic mean of the three vertex position vectors: $G = (A + B + C)/3$. Midpoints are not vertices, so averaging A and the two given midpoints would give a different point.

Step 1: Recover the second vertex Let the known vertex be $A = (1, 2)$, and let $M = (-1, 1)$ be the midpoint of AB. Then

$$B = 2M - A = \big(2(-1) - 1,\ 2(1) - 2\big) = (-3, 0).$$

As a check, averaging $(1, 2)$ and $(-3, 0)$ gives $(-1, 1)$, exactly the first stated midpoint.

Step 2: Recover the third vertex Let $N = (2, 3)$ be the midpoint of AC. The same midpoint formula gives

$$C = 2N - A = \big(2(2) - 1,\ 2(3) - 2\big) = (3, 4).$$

The midpoint of A and this C is indeed $(2, 3)$. Interchanging the names B and C would leave the centroid unchanged, so no choice of side names affects the result.

Step 3: Average the three vertices Now use the actual vertices:

$$G = \left(\frac{1 - 3 + 3}{3}, \frac{2 + 0 + 4}{3}\right) = \left(\frac{1}{3}, 2\right).$$

The y-coordinate is exactly 2, ruling out (B), (C) and (D). One can also combine the two midpoint equations directly: $A + B + C = 2M + 2N - A = (1, 6)$; dividing by 3 gives the same centroid and independently checks the coordinate arithmetic.

Step 4: Conclusion The centroid is $\left(\frac{1}{3}, 2\right)$. Hence, the correct option is (A).

Quick Tip to Solve: With one vertex A and its two adjacent-side midpoints M and N, use $G = (2M + 2N - A)/3$ to find the centroid without naming the other vertices.

27. If $^{20}C_1 + (2^2)\ ^{20}C_2 + (3^2)\ ^{20}C_3 + ... + (20^2)\ ^{20}C_{20} = A(2^{\beta})$, then the ordered pair (A, β) is equal to : [4 marks]

(A) (420, 19)

(B) (420, 18)

(C) (380, 18)

(D) (380, 19)

Correct Answer: (B) (420, 18)

Solution:

Concept:

- The binomial theorem gives $(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$. Differentiation produces weighted sums involving k and $k(k-1)$.
- Use $k^2 = k(k-1) + k$. The first term is obtained from the second derivative of the binomial expansion, while the second comes from its first derivative.

Step 1: Find the two standard weighted sums Differentiate $(1+x)^{20}$, and set $x = 1$:

$$\sum_{k=1}^{20} k\binom{20}{k} = 20 \cdot 2^{19}.$$

Differentiate a second time, then set $x = 1$, to obtain

$$\sum_{k=2}^{20} k(k-1)\binom{20}{k} = 20 \cdot 19 \cdot 2^{18}.$$

The missing k=1 term in the second sum is zero, so the two sums can be added over the same range.

Step 2: Combine them to get the square weight The printed first term is $1^2\binom{20}{1}$, so the entire left side is exactly the sum with weight k^2. Therefore

$$\begin{aligned}\sum_{k=1}^{20} k^2\binom{20}{k} &= 20 \cdot 19 \cdot 2^{18} + 20 \cdot 2^{19} \\ &= (380 + 40)2^{18} = 420 \cdot 2^{18}.\end{aligned}$$

Keeping the common power 2^{18} makes the contribution of the first derivative visible.

Step 3: Identify the offered ordered pair The expression just found matches $A2^{\beta}$ with the offered values $A = 420$ and $\beta = 18$. Option (A) is twice the computed sum. Option (C) omits the extra $20 \cdot 2^{19}$ term and therefore counts $k(k-1)$ instead of k^2. Option (D) also has the wrong coefficient and power. Only the pair in (B) gives the required value.

Step 4: Conclusion The sum is $420 \cdot 2^{18}$, so the matching ordered pair is $(A, \beta) = (420, 18)$. Hence, the correct option is (B).

Quick Tip to Solve: For sums weighted by a square, first rewrite k^2 as $k(k-1) + k$; this prevents confusing the square weight with the second-derivative weight.

28. For an initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is : [4 marks]

(A) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$

(B) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$

(C) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$

(D) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

Correct Answer: (D) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

Solution:

Concept:

- For fifty independent problems with a constant probability of solving each one, the number X of unsolved problems has the binomial distribution with $n = 50$ and failure probability $q = 1/5$.
- The phrase "unable to solve less than two problems" means $X < 2$, hence X is 0 or 1. The binomial formula is $P(X = r) = \binom{50}{r} q^r (1-q)^{50-r}$.

Step 1: Translate the event into failure counts The probability of solving one problem is $p = 4/5$, and that of being unable to solve it is $q = 1 - p = 1/5$. The required event consists of solving all fifty problems or solving exactly forty-nine of them. These two cases are disjoint, so their probabilities add:

$$P(X < 2) = P(X = 0) + P(X = 1).$$

The count is of unsolved problems, rather than solved problems.

Step 2: Compute both probabilities All fifty solved has probability p^{50}. Exactly one unsolved problem can occupy any of fifty positions, giving

$$P(X = 0) = \left(\frac{4}{5}\right)^{50}, \qquad P(X = 1) = 50\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^{49}.$$

The factor 50 counts the possible positions of the sole failure; it must not be dropped.

Step 3: Factor and compare the options Take out the common factor $(4/5)^{49}$:

$$P(X < 2) = \left(\frac{4}{5}\right)^{49}\left(\frac{4}{5} + 10\right) = \frac{54}{5}\left(\frac{4}{5}\right)^{49}.$$

This matches (D). Options (A) and (C) use powers of $1/5$ appropriate to many failures rather than at most one. Written with power 48, our coefficient would be $216/25$, so (B), whose coefficient is $316/25$, is also different.

Step 4: Conclusion The required probability is $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$. Hence, the correct option is (D).

Quick Tip to Solve: Define the random variable as the number of failures when the wording says "unable to solve"; then "less than two" immediately becomes the two cases 0 and 1.

29. Let S be the set of all $\alpha \in \mathbb{R}$ such that the equation $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to: [4 marks]

(A) $[2, 6]$

(B) $[3, 7]$

(C) $[1, 4]$

(D) $\mathbb{R}$

Correct Answer: (A) $[2, 6]$

Solution:

Concept:

- A real solution exists exactly when the equation can hold for some $t = \sin x$ in the closed interval $[-1, 1]$. Every value in that interval is attained by the sine function.
- Use $\cos 2x = 1 - 2\sin^2 x$ to express the equation in one variable. Factoring before dividing keeps the parameter dependence simple and allows the denominator to be checked.

Step 1: Replace the trigonometric expressions Put $t = \sin x$. The given equation becomes $1 - 2t^2 + \alpha t = 2\alpha - 7$. Collecting the parameter on the right gives

$$8 - 2t^2 = \alpha(2 - t).$$

Since $-1 \le t \le 1$, the factor $2 - t$ lies between 1 and 3 and is never zero; division by it therefore loses no solution.

Step 2: Factor and find the complete range Factor the numerator as $8 - 2t^2 = 2(2 - t)(2 + t)$. Consequently

$$\alpha = \frac{2(2 - t)(2 + t)}{2 - t} = 4 + 2t.$$

This increasing linear expression ranges from 2 at $t = -1$ to 6 at $t = 1$. Conversely, any $\alpha \in [2, 6]$ gives $t = (\alpha - 4)/2 \in [-1, 1]$, so a real angle exists. This proves sufficiency as well as necessity. The endpoint values are included, ruling out the shifted or shortened intervals in (B) and (C); parameters outside this interval rule out (D).

Step 3: Conclusion The required parameter set is $S = [2, 6]$. Hence, the correct option is (A).

Quick Tip to Solve: When a parameter multiplies sine, solve for the parameter after substituting $t = \sin x$; factor before finding its range.

30. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines $\vec{r} = \left(\hat{i}+\hat{j}\right) + \lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)$ and $\vec{r} = \left(\hat{i}+\hat{j}\right) + \mu\left(-\hat{i}+\hat{j}-2\hat{k}\right)$ is : [4 marks]

(A) $\frac{1}{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) 3

(D) $\sqrt{3}$

Correct Answer: (D) $\sqrt{3}$

Solution:

Concept:

- Two intersecting nonparallel lines determine a plane. Their direction vectors lie in that plane, so their cross product is perpendicular to it.
- For a plane $ax + by + cz + d = 0$, the perpendicular distance from (x_0, y_0, z_0) is $|ax_0 + by_0 + cz_0 + d|/\sqrt{a^2 + b^2 + c^2}$. The denominator normalizes the plane normal.

Step 1: Find a normal to the required plane Both lines pass through $A = (1, 1, 0)$, obtained by setting the respective parameters to zero. Their direction vectors are $\mathbf{u} = (1, 2, -1)$ and $\mathbf{v} = (-1, 1, -2)$. Their cross product is

$$\mathbf{u} \times \mathbf{v} = (-3, 3, 3).$$

It is nonzero, confirming that the plane is uniquely determined. An equivalent, simpler normal is $\mathbf{n} = (-1, 1, 1)$; scaling a normal does not change its plane.

Step 2: Write the plane and calculate the distance The plane through A with this normal satisfies

$$-(x - 1) + (y - 1) + z = 0, \qquad -x + y + z = 0.$$

For the given point $P = (2, 1, 4)$, the signed numerator is $-2 + 1 + 4 = 3$. Therefore

$$d = \frac{|-2 + 1 + 4|}{\sqrt{(-1)^2 + 1^2 + 1^2}} = \frac{3}{\sqrt{3}} = \sqrt{3}.$$

The value 3 in option (C) is only the unnormalized plane expression. Options (A) and (B) have the wrong numerator; neither is the distance to this plane.

Step 3: Conclusion The perpendicular length is $\sqrt{3}$ units. Hence, the correct option is (D).

Quick Tip to Solve: Use the two line directions to obtain the plane normal, then simplify that normal before applying the point-to-plane distance formula.

Physics (120 marks)

31. A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60° to the pass axis of P_3. When a beam of unpolarized light of intensity I0 is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio ($\frac{I_0}{I}$) equals (nearly) : [4 marks]

(A) 10.67

(B) 5.33

(C) 16.00

(D) 1.80

Correct Answer: (A) 10.67

Solution:

Concept:

- An ideal first polarizer transmits half the intensity of incident unpolarized light: $I_1 = I_0/2$. The transmitted beam is linearly polarized along its pass axis.
- Malus law for an already polarized beam is $I_{\text{out}} = I_{\text{in}} \cos^2 \phi$, where ϕ is the angle between the incident polarization direction and the next pass axis. Apply it at each later polarizer.

Step 1: Determine the successive axis angles The axes of P_1 and P_3 are crossed, so they differ by $90°$. The middle axis makes $60°$ with P_3; its effective acute angle with P_1 is therefore $30°$. The first transmission is $I_1 = I_0/2$, and the second transmission is

$$I_2 = \frac{I_0}{2} \cos^2 30° = \frac{3I_0}{8}.$$

The middle polarizer changes the polarization direction, so the crossed end polarizers do not force zero final intensity.

Step 2: Apply Malus law at the last polarizer The polarization incident on P_3 is along the axis of P_2. The final angle is $60°$, giving

$$I = I_2 \cos^2 60° = \frac{3I_0}{8}\frac{1}{4} = \frac{3I_0}{32}.$$

Thus

$$\frac{I_0}{I} = \frac{32}{3} = 10.666\ldots \simeq 10.67.$$

The value 5.33 would arise if the initial halving of unpolarized light were omitted. The values 16.00 and 1.80 do not equal the product of the two required squared cosine factors with the initial factor one-half.

Step 3: Conclusion The requested ratio is $I_0/I \simeq 10.67$. Hence, the correct option is (A).

Quick Tip to Solve: Include the one-half factor only for the first polarizer; use squared cosines of successive axis differences for the remaining polarizers.

32. One kg of water, at 20°C, heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to : [Specific heat of water = 4200 J/(kg °C), Latent heat of water = 2260 kJ/kg] [4 marks]

(A) 10 minutes

(B) 22 minutes

(C) 3 minutes

(D) 16 minutes

Correct Answer: (B) 22 minutes

Solution:

Concept:

- To evaporate all the water, energy must first raise it from its initial temperature to its boiling point and then provide latent heat at constant temperature: $Q = mc\Delta T + mL$.
- For a resistive element supplied by an alternating voltage, the average power is $P = V_{\text{rms}}^2/R$. With negligible heat loss, all this electrical energy heats or vaporizes the water, so $t = Q/P$.

Step 1: Calculate the heating and vaporization energies Take the boiling point as 100°C. The mass is $m = 1\,\text{kg}$, so the rise before boiling is $100 - 20 = 80^\circ\text{C}$. The sensible heat is

$$Q_1 = 1 \times 4200 \times 80 = 336000\,\text{J}.$$

Convert the latent heat to SI units: $L = 2260\,\text{kJ}\,\text{kg}^{-1} = 2260000\,\text{J}\,\text{kg}^{-1}$. Complete evaporation then requires $Q_2 = mL = 2260000\,\text{J}$.

Step 2: Divide the total energy by the electrical power The total required heat is

$$Q = 336000 + 2260000 = 2596000\,\text{J}.$$

Using the given mean resistance and rms voltage gives

$$P = \frac{200^2}{20} = 2000\,\text{W}, \qquad t = \frac{2596000}{2000} = 1298\,\text{s}.$$

In minutes, $t = 1298/60 \simeq 21.63\,\text{min}$, close to 22 minutes. Heating only to the boiling point would take 2.8 minutes, explaining why 3 minutes is insufficient. The other choices, 10 and 16 minutes, also supply less than the total sensible-plus-latent heat.

Step 3: Conclusion The time is approximately 21.6 minutes, closest to 22 minutes. Hence, the correct option is (B).

Quick Tip to Solve: For complete evaporation, add $mc\Delta T$ and mL before dividing by power; convert kJ to J before calculating seconds.

33. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constant, k_1 and k_2 will be : [4 marks]

(A) $\frac{1}{n^2}$

(B) $\frac{1}{n}$

(C) n^2

(D) n

Correct Answer: (B) $\frac{1}{n}$

Solution:

Concept:

- For pieces cut from the same uniform spring, the force constant is inversely proportional to the unstretched length: $k_i\ell_i = k\ell$. Shortening a spring reduces the number of active coils and makes it stiffer.
- Under the same tensile force, each uniform segment has the same extension per unit length. Hooke law $F = k_i x_i$ therefore relates the ratios of segment lengths and stiffnesses without requiring the absolute force.

Step 1: Relate the extensions of the two pieces Imagine applying the same force F separately to each spring piece, within its elastic limit. Uniform construction implies $x_1/x_2 = \ell_1/\ell_2$. Because $\ell_1 = n\ell_2$, the longer piece stretches n times as much: $x_1 = nx_2$. By Hooke law, however, $x_1 = F/k_1$ and $x_2 = F/k_2$, so

$$\frac{x_1}{x_2} = \frac{k_2}{k_1} = n.$$

Step 2: Calculate and check the stiffness ratio Inverting this relation yields

$$\frac{k_1}{k_2} = \frac{1}{n}.$$

Equivalently, since $\ell = \ell_1 + \ell_2 = (n+1)\ell_2$, one obtains $k_1 = k(n+1)/n$ and $k_2 = k(n+1)$; their ratio again equals $1/n$. For $n > 1$, piece 1 is longer and must be less stiff, ruling out the increasing ratios n and n^2. The dependence on length is linear in the inverse, so $1/n^2$ is also incorrect. If $n = 1$, equal lengths correctly give equal force constants.

Step 3: Conclusion The ratio of force constants is $k_1/k_2 = 1/n$. Hence, the correct option is (B).

Quick Tip to Solve: For a uniform spring cut into pieces, remember $k_i\ell_i =$ constant: the shorter piece has the larger force constant.

34. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process ? [4 marks]

(A) 35 J

(B) 30 J

(C) 25 J

(D) 40 J

Correct Answer: (A) 35 J

Solution:

Concept:

- A rigid diatomic ideal-gas molecule has five active degrees of freedom: three translational and two rotational. The vibrational modes are excluded by the rigid-molecule description, giving $C_V = 5R/2$.
- For an ideal gas, $C_P = C_V + R = 7R/2$. At constant pressure, $W = p\Delta V = \nu R\Delta T$, while the absorbed heat is $Q = \nu C_P \Delta T$; here ν is the number of moles.

Step 1: Use the given work to find the temperature-change factor For the constant-pressure expansion, the ideal-gas relation gives

$$W = \nu R\Delta T = 10\,\text{J}.$$

Expansion means positive work by the gas and positive temperature change. The internal-energy rise follows from the five active degrees of freedom:

$$\Delta U = \nu C_V \Delta T = \frac{5}{2}\nu R\Delta T = 25\,\text{J}.$$

No value of ν or ΔT individually is needed because their product is already fixed by the work.

Step 2: Add internal-energy increase and expansion work Using the first law with work done by the gas positive,

$$Q = \Delta U + W = 25 + 10 = 35\,\text{J}.$$

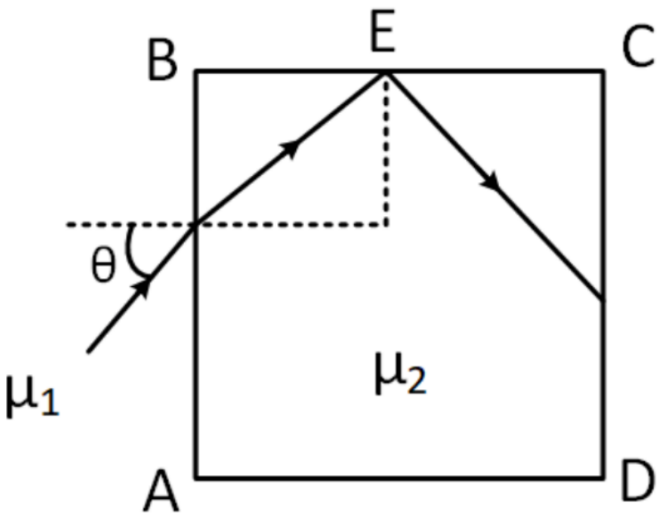
The same result follows directly from $Q/W = C_P/R = 7/2$. The option 25 J accounts only for internal energy and omits the energy spent on expansion. A monatomic-gas heat capacity would also give a different result and is inappropriate for a rigid diatomic molecule. Neither 30 J nor 40 J satisfies the first law with $C_V = 5R/2$ and the specified work.

Step 3: Conclusion The heat energy absorbed by the gas is $35\,\text{J}$. Hence, the correct option is (A).

Quick Tip to Solve: At constant pressure, obtain $\nu R\Delta T$ directly from the work; for a rigid diatomic gas, multiply it by $7/2$ to obtain heat.

35. A transparent cube of side, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1(\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle θ(shown in the figure). Total internal reflection takes place at point E on the face BC.

Then θ must satisfy : [4 marks]



(A) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$

(B) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$

(C) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$

(D) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$

Correct Answer: (B) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$

Solution:

Concept:

- Snell law at the liquid-to-cube entrance is $\mu_1 \sin\theta = \mu_2 \sin r$, where r is measured from the normal to face AB inside the cube.
- For passage from the cube into the less refractive liquid, the critical angle satisfies $\sin i_c = \mu_1/\mu_2$. Total internal reflection requires incidence strictly greater than i_c, not equality.

Step 1: Relate the two internal angles Faces AB and BC are perpendicular, so their inward normals are perpendicular as well. A ray making angle r with the normal to AB therefore has incidence $i = 90^\circ - r$ on BC. The total-internal-reflection condition is

$$\sin i > \frac{\mu_1}{\mu_2}, \qquad \cos r > \frac{\mu_1}{\mu_2}.$$

These angles are acute, so squaring preserves the inequality.

Step 2: Eliminate the refraction angle From Snell law, $\sin r = (\mu_1/\mu_2)\sin\theta$. Substitution into

$1 - \sin^2 r > \mu_1^2/\mu_2^2$ gives

$$\sin^2\theta < \frac{\mu_2^2}{\mu_1^2} - 1.$$

When the inverse-sine threshold is real, the equivalent angular condition is

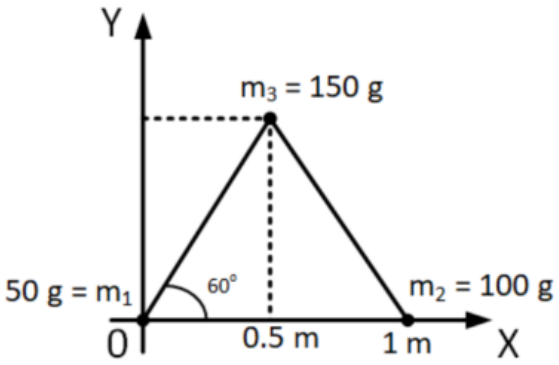
$$\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}.$$

The inequality points downward, excluding (A); (C) and (D) use the critical angle without accounting for the perpendicular faces and entrance refraction. More generally, the sine-squared inequality is the valid criterion: if its right side exceeds 1, every physically entering ray meets it.

Step 3: Conclusion The angular condition, in the domain of the displayed inverse sine, is $\theta < \sin^{-1}\sqrt{\mu_2^2/\mu_1^2 - 1}$. Hence, the correct option is (B).

Quick Tip to Solve: For perpendicular refracting faces, first use $i + r = 90°$; apply the critical-angle test at the reflecting face, then Snell law at entry.

36. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be : [4 marks]



(A) $\left(\frac{\sqrt{3}}{4}m, \frac{5}{12}m\right)$

(B) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{8}m\right)$

(C) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$

(D) $\left(\frac{\sqrt{3}}{8}m, \frac{7}{12}m\right)$

Correct Answer: (C) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$

Solution:

Concept:

- The centre of mass is the mass-weighted average of the particle positions: $x_{\text{cm}} = \sum m_i x_i / \sum m_i$, and $y_{\text{cm}} = \sum m_i y_i / \sum m_i$.

- An equilateral triangle of side a has height $\sqrt{3}a/2$. Its upper vertex lies directly above the midpoint of its horizontal base.

Step 1: Assign the vertex coordinates Use the origin and axes shown in the figure. The 50 g mass is at $(0, 0)$, the 100 g mass at $(1, 0)\,\mathrm{m}$, and the 150 g mass at $(1/2, \sqrt{3}/2)\,\mathrm{m}$. The total mass is $50 + 100 + 150 = 300\,\mathrm{g}$. It is unnecessary to convert the masses to kilograms, since the same mass unit occurs in each numerator and denominator.

Step 2: Calculate the two weighted averages For the horizontal coordinate, both the right-hand base mass and the top mass contribute. For the vertical coordinate, only the top mass contributes because the other two lie on the x-axis. Thus

$$x_{\mathrm{cm}} = \frac{50(0) + 100(1) + 150(1/2)}{300} = \frac{175}{300} = \frac{7}{12}\,\mathrm{m},$$

$$y_{\mathrm{cm}} = \frac{50(0) + 100(0) + 150(\sqrt{3}/2)}{300} = \frac{\sqrt{3}}{4}\,\mathrm{m}.$$

The vertical coordinate is half the triangle height because the top particle carries half the total mass. Option B gives only one quarter of the height; A and D interchange or change the coordinate values.

Step 3: Conclusion The centre of mass is $\left(\frac{7}{12}\,\mathrm{m}, \frac{\sqrt{3}}{4}\,\mathrm{m}\right)$. Hence, the correct option is (C).

Quick Tip to Solve: Put the base on the x-axis first. Then only the upper mass enters the numerator of the vertical centre-of-mass coordinate.

37. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound ? [Given reference intensity of sound as 10^{-12} W/m^2] [4 marks]

(A) 20 cm

(B) 10 cm

(C) 40 cm

(D) 30 cm

Correct Answer: (C) 40 cm

Solution:

Concept:

- The sound intensity level is $\beta = 10\log_{10}(I/I_0)$, where I_0 is the reference intensity. The logarithm acts on an intensity ratio, not on the power alone.
- For a small source emitting uniformly in all directions, its power P crosses a sphere of area $4\pi r^2$, so $I = P/(4\pi r^2)$.

Step 1: Convert the decibel level into intensity The stated level is $120\,\mathrm{dB}$ and the reference intensity is $I_0 = 10^{-12}\,\mathrm{W\,m^{-2}}$. Therefore

$$120 = 10\log_{10}\frac{I}{10^{-12}}, \qquad \frac{I}{10^{-12}} = 10^{12}.$$

Hence $I = 1\,\mathrm{W\,m^{-2}}$. The factor 10 applies because intensity is proportional to the square of the wave amplitude; the factor 20 belongs to amplitude ratios.

Step 2: Find the spherical spreading distance Substitute the audio output $P = 2\,\mathrm{W}$ into the intensity formula:

$$r = \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{2}{4\pi(1)}}\,\mathrm{m} = 0.399\,\mathrm{m} \simeq 40\,\mathrm{cm}.$$

The positive square root is the physical distance. For comparison, the intensities at 20 cm, 10 cm and 30 cm are about 3.98, 15.9 and $1.77\,\mathrm{W\,m^{-2}}$, respectively. Each exceeds the required intensity and hence gives a level above 120 dB; those distances do not satisfy the stated level.

Step 3: Conclusion The required distance is approximately $40\,\mathrm{cm}$. Hence, the correct option is (C).

Quick Tip to Solve: First convert dB to intensity, then apply inverse-square spreading. Here 120 dB with the stated reference immediately gives one watt per square metre.

38. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, l_1 = 30 cm and l_2 = 70 cm. Then, v is equal to - [4 marks]

(A) 338 ms^{-1}

(B) 384 ms^{-1}

(C) 379 ms^{-1}

(D) 332 ms^{-1}

Correct Answer: (B) 384 ms^{-1}

Solution:

Concept:

- For a tube closed at one end, resonance occurs at odd quarter-wavelengths: $l_n + e = (2n-1)\lambda/4$, where e is the end correction.
- Two successive resonant air-column lengths differ by $\lambda/2$. Subtraction eliminates the same end correction at the open end.

Step 1: Subtract the successive resonant lengths The first observed resonance need not be identified with the fundamental. Since the two lengths are explicitly successive resonances, their difference is half a wavelength in either case. Thus

$$l_2 - l_1 = 70 - 30 = 40\,\mathrm{cm} = 0.40\,\mathrm{m},$$

$$\frac{\lambda}{2} = 0.40\,\mathrm{m}, \qquad \lambda = 0.80\,\mathrm{m}.$$

Using $4l_1$ alone would neglect end correction and is unnecessary when a second resonance is given.

Step 2: Use the frequency to obtain the wave speed The tuning fork fixes the frequency of the sound

in the tube at $f = 480\,\text{Hz}$. For any progressive sound wave, $v = f\lambda$. Hence

$$v = 480 \times 0.80 = 384\,\text{m s}^{-1}.$$

Equivalently, the directly usable relation is $v = 2f(l_2 - l_1)$. This result follows from the measured resonance separation, rather than a memorised room-temperature sound speed. The other offered values, 338, 379 and 332 m s^{-1}, would require different separations and do not reproduce the stated 40 cm difference at 480 Hz.

Step 3: Conclusion The measured speed of sound is $384\,\text{m s}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: For successive resonances in the same resonance tube, use twice the length difference as the wavelength; this automatically removes the end correction.

39. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths, $\frac{\lambda_1}{\lambda_2}$, of the photons emitted in this process is : [4 marks]

(A) $\frac{22}{5}$
(B) $\frac{7}{5}$
(C) $\frac{9}{7}$
(D) $\frac{20}{7}$

Correct Answer: (D) $\frac{20}{7}$

Solution:

Concept:

- The ground state has $n = 1$; consequently the first, second and third excited states have $n = 2, 3, 4$, respectively.
- The hydrogen emission wavenumber obeys $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$. A larger photon energy corresponds to a shorter wavelength.

Step 1: Identify the two transitions The first jump is from the third excited state to the second excited state, so it is $4 \to 3$, not $3 \to 2$. The subsequent jump to the first excited state is $3 \to 2$. The emitted wavelengths λ_1 and λ_2 must be associated with these transitions in the order given in the question. Both differences in the Rydberg formula are positive because the final level is lower.

Step 2: Calculate and divide the wavelengths For the two transitions,

$$\frac{1}{\lambda_1} = R_H\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7R_H}{144},$$

$$\frac{1}{\lambda_2} = R_H\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R_H}{36}.$$

Since wavelength is the reciprocal of wavenumber, the required ratio is

$$\frac{\lambda_1}{\lambda_2} = \frac{5/36}{7/144} = \frac{20}{7}.$$

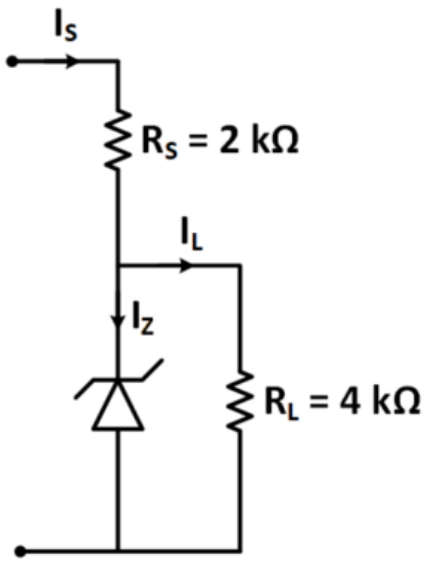
The common Rydberg constant cancels. The first transition releases less energy, so its wavelength must be longer, consistent with the ratio exceeding unity. Options A, B and C do not equal this exact quotient; in particular one must invert the wavenumber ratio when forming the wavelength ratio.

Step 3: Conclusion The wavelength ratio is $\lambda_1/\lambda_2 = 20/7$. Hence, the correct option is (D).

Quick Tip to Solve: Write the principal quantum numbers beside the words excited state before calculating. Then remember that wavelength ratios are inverse photon-energy ratios.

40. Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6V. If the unregulated input voltage varies between 10 V to 16 V, then what is maximum Zener current?

[4 marks]



(A) 3.5 mA

(B) 1.5 mA

(C) 2.5 mA

(D) 7.5 mA

Correct Answer: (A) 3.5 mA

Solution:

Concept:

- A reverse-biased Zener diode in breakdown holds the parallel load voltage at approximately its breakdown voltage V_Z. Here the regulated voltage is $6\,\mathrm{V}$.
- The series-resistor current divides at the output node: $I_S = I_L + I_Z$, with $I_S = (V_{\text{in}} - V_Z)/R_S$ and

$I_L = V_Z/R_L$.

Step 1: Calculate the fixed load current The diagram gives the series resistor $R_S = 2\,\text{k}\Omega$ and load resistor $R_L = 4\,\text{k}\Omega$. Since the load and Zener are connected across the same two nodes,

$$I_L = \frac{6}{4 \times 10^3}\,\text{A} = 1.5\,\text{mA}.$$

This is the load current and must not be mistaken for the Zener current. The Zener takes the excess current after the load requirement is met.

Step 2: Use the maximum input voltage The regulated voltage and load current remain fixed while the input rises, so the largest Zener current occurs at $V_{\text{in}} = 16\,\text{V}$. Thus

$$I_{S,\text{max}} = \frac{16 - 6}{2 \times 10^3}\,\text{A} = 5.0\,\text{mA},$$

$$I_{Z,\text{max}} = I_{S,\text{max}} - I_L = 5.0 - 1.5 = 3.5\,\text{mA}.$$

At the lower input of 10 V, the series current is 2 mA and the Zener still carries 0.5 mA, consistent with the ideal regulating model. Option B is merely the load current; C is not the current difference at either limiting voltage, and D exceeds the entire maximum series current, so is impossible.

Step 3: Conclusion The maximum Zener current is $3.5\,\text{mA}$. Hence, the correct option is (A).

Quick Tip to Solve: At maximum input, calculate the current through the series resistor and subtract the load current. The current through the Zener is the remainder.

41. The ratio of the weights of a body on the Earth's surface to that on the surface of a planets is 9 : 4. The mass of the planet is $\frac{1}{9}$ th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet ? (Take the planets to have the same mass density) [4 marks]

(A) $\frac{R}{9}$
(B) $\frac{R}{2}$
(C) $\frac{R}{3}$
(D) $\frac{R}{4}$

Correct Answer: (B) $\frac{R}{2}$

Solution:

Concept:

- The weight of a fixed test body at a spherical planet surface is $W = GMm/r^2$, where M and r are the planet mass and radius.
- For the same test body, the constants G and m cancel in a ratio. A separate equal-density assumption would instead require $M \propto r^3$.

Step 1: Use the given weight and mass ratios Let the Earth have mass M and radius R, and let the

other planet have radius r and mass $M/9$. The stated weight ratio gives

$$\frac{W_E}{W_p} = \frac{GMm/R^2}{G(M/9)m/r^2} = \frac{9r^2}{R^2} = \frac{9}{4}.$$

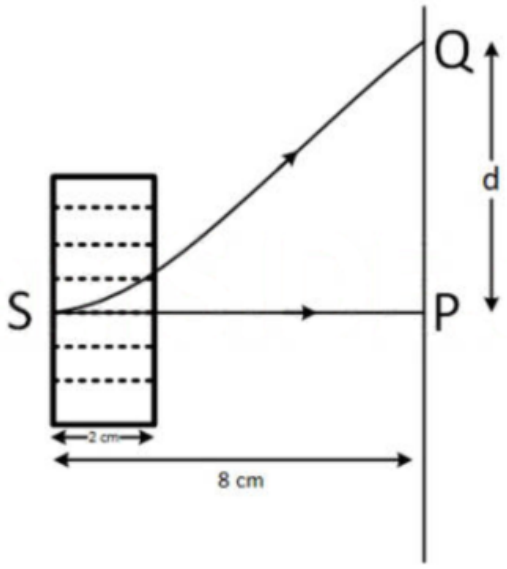
Therefore $r^2/R^2 = 1/4$. A radius is positive, so $r = R/2$. This uses both quantitative ratios exactly as printed.

Step 2: Check the options and the density qualification For the offered radii $R/9, R/2, R/3, R/4$, the weight ratio predicted with mass $M/9$ is respectively $1/9, 9/4, 1, 9/16$. Only $R/2$ reproduces $9/4$. However, if the parenthetical equal-density statement is read as requiring the Earth and this planet to have identical mean density, it contradicts these data: $M_p/M = (r/R)^3$ would give $r = R/\sqrt[3]{9}$, which is not $R/2$. Indeed the quantitative ratios imply $\rho_p/\rho_E = (1/9)/(1/8) = 8/9$. Thus all printed statements cannot hold simultaneously; the multiple-choice value follows from the explicitly specified mass and weight ratios.

Step 3: Conclusion The two specified quantitative ratios give radius $R/2$. Hence, the correct option is (B). The equal-density qualification, if applied between the Earth and the planet, is incompatible with those ratios and cannot also be satisfied.

Quick Tip to Solve: Form the ratio of surface weights before using any density relation. Check extra conditions separately: equal density requires a cubic mass-radius relation.

42. An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = (1.5 \times 10^{-3}T)\,\hat{k}$ at S (See figure). The field extends between x = 0 and x = 2 cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is : (electron's charge = 1.6 $\times$ 10^{-19} C, mass of electron = 9.1 $\times$ 10^{-31} kg) [4 marks]



(A) 2.25 cm

(B) 12.87 cm

(C) 1.22 cm

(D) 11.65 cm

Correct Answer: (B) 12.87 cm

Solution:

Concept:

- A magnetic force does no work, so the electron keeps its speed $v = \sqrt{2K/m}$. In the field it follows a circle of radius $r = mv/(eB) = \sqrt{2mK}/(eB)$.
- After leaving the finite field strip the electron travels along the tangent. The screen deflection is the curved-path rise plus the straight-path rise.

Step 1: Find the orbit radius and the exit angle Convert the kinetic energy to joules: $K = 100(1.6 \times 10^{-19}) = 1.6 \times 10^{-17}$ J. Consequently

$$r = \frac{\sqrt{2(9.1 \times 10^{-31})(1.6 \times 10^{-17})}}{(1.6 \times 10^{-19})(1.5 \times 10^{-3})} \simeq 0.02249\,\text{m} = 2.249\,\text{cm}.$$

For an electron initially moving along positive x in a positive z magnetic field, the force is along positive y, as in the figure. If θ is the exit tangent angle above x, the horizontal field width gives $r \sin\theta = 2\,\text{cm}$. Hence $\sin\theta \simeq 0.8895$, $\cos\theta \simeq 0.45694$, and $\tan\theta \simeq 1.94666$.

Step 2: Add the two contributions to the deflection The rise inside the field equals $r(1 - \cos\theta) \simeq 1.22\,\text{cm}$. The screen is 8 cm from S and the field ends 2 cm from S, leaving 6 cm of horizontal field-free travel. Thus

$$d = r(1 - \cos\theta) + (8 - 2)\tan\theta \simeq 1.221 + 11.680 = 12.901\,\text{cm}.$$

This is closest to the listed value 12.87 cm; early rounding of the orbit radius changes the result slightly. Option A gives the orbit radius, C gives only the curved-path rise, and D gives only the subsequent straight-path rise. Each omits a necessary part of the trajectory.

Step 3: Conclusion The supplied constants give total screen displacement about $12.90\,\text{cm}$, nearest to the listed $12.87\,\text{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: For a finite magnetic strip, use its horizontal width to find the exit tangent angle. Add the circular rise to the tangent drift across the remaining screen distance.

43. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct ? [4 marks]

(A) $R^2 = h_1 h_2$

(B) $R^2 = 16\, h_1 h_2$

(C) $R^2 = 4\, h_1 h_2$

(D) $R^2 = 2 h_1 h_2$

Correct Answer: (B) $R^2 = 16\, h_1 h_2$

Solution:

Concept:

- For a projectile returning to its launch level, the horizontal range is $R = u^2 \sin 2\theta/g$, where u is the launch speed and g is gravitational acceleration.

- The maximum height is $h = u^2 \sin^2\theta/(2g)$. Equal ranges at the same speed correspond to complementary distinct projection angles.

Step 1: Relate the two launch angles Let the launch angles above the horizontal be θ_1 and θ_2. Equal ranges give $\sin 2\theta_1 = \sin 2\theta_2$. Since the maximum heights differ, the angles cannot be identical. For ordinary forward projectiles, the second possibility is $2\theta_2 = 180^\circ - 2\theta_1$, hence $\theta_2 = 90^\circ - \theta_1$. One particle takes the steeper path and the other takes the shallower path.

Step 2: Multiply the two maximum heights Use the complementary-angle identity to express both heights in terms of one angle:

$$h_1 = \frac{u^2 \sin^2\theta_1}{2g}, \qquad h_2 = \frac{u^2 \cos^2\theta_1}{2g}.$$

Therefore

$$h_1 h_2 = \frac{u^4 \sin^2\theta_1 \cos^2\theta_1}{4g^2}.$$

Meanwhile, squaring the common range gives

$$R^2 = \frac{u^4 (2\sin\theta_1 \cos\theta_1)^2}{g^2} = \frac{4u^4 \sin^2\theta_1 \cos^2\theta_1}{g^2}.$$

Comparing these expressions gives a factor of 16. The factors 1, 4 and 2 in the other options miss the combined factors from the height formula and the double-angle identity.

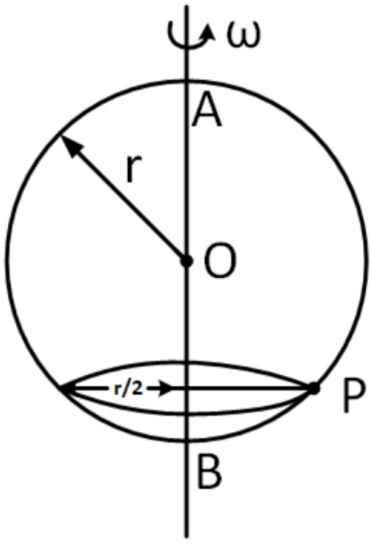
Step 3: Conclusion

$$\boxed{R^2 = 16 h_1 h_2}.$$

Hence, the correct option is (B).

Quick Tip to Solve: For two different trajectories with equal speed and range, replace the second angle by the complement of the first before multiplying their heights.

44. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to - [4 marks]



(A) $\frac{\sqrt{3}g}{2r}$

(B) $\frac{2g}{(r\sqrt{3})}$

(C) $\frac{(g\sqrt{3})}{r}$

(D) $\frac{2g}{r}$

Correct Answer: (B) $\frac{2g}{(r\sqrt{3})}$

Solution:

Concept:

- In the frame rotating with the hoop, a bead at rest has gravity mg downward and centrifugal force $m\omega^2\rho$ horizontally away from the vertical axis, where ρ is its distance from that axis.
- The smooth wire provides only a normal reaction. Equilibrium therefore requires the net force component along the tangent to the circle to vanish.

Step 1: Read the geometry at the bead The wire length $2\pi r$ gives a ring of radius r. Let θ be the angle between OP and the downward vertical. The horizontal radius of the bead's circular motion about AB is $\rho = r\sin\theta$. The figure labels this horizontal distance as $r/2$; the left and right halves of the horizontal orbit have the same radius. Thus

$$\sin\theta = \frac{1}{2}, \qquad \cos\theta = \frac{\sqrt{3}}{2}.$$

The bead is on the lower half of the hoop, so the positive value of the cosine applies.

Step 2: Balance the tangential force components Along the tangent, gravity contributes $mg\sin\theta$ toward the lowest point. Centrifugal force contributes $m\omega^2 r\sin\theta\cos\theta$ in the opposite direction.

Equating their magnitudes gives

$$mg\sin\theta = m\omega^2 r\sin\theta\cos\theta.$$

Since the shown position is away from the axis, $\sin\theta \neq 0$, and hence

$$\omega^2 = \frac{g}{r\cos\theta} = \frac{2g}{r\sqrt{3}}.$$

Mass cancels, as expected for this equilibrium. The alternatives with factors $\sqrt{3}/2$, $\sqrt{3}$ or 2 do not satisfy this tangential balance at the marked horizontal radius.

Step 3: Conclusion

$$\boxed{\omega^2 = \frac{2g}{r\sqrt{3}}}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Use the distance from the rotation axis in the centrifugal force, and resolve along the hoop tangent so that the unknown normal reaction disappears.

45. The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to : [4 marks]

(A) $n_0\alpha^{-3/4}$

(B) $n_0\alpha^{-3}$

(C) $n_0\alpha^{1/4}$

(D) $\sqrt{n_0}\alpha^{1/2}$

Correct Answer: (A) $n_0\alpha^{-3/4}$

Solution:

Concept:

- Number density n has dimensions L^{-3}. The exponential argument must be dimensionless, so αr^4 implies $[\alpha] = L^{-4}$.
- The total number of molecules is the volume integral of number density. Spherical symmetry gives the volume element $dV = 4\pi r^2\,dr$.

Step 1: Identify the characteristic length scale For a gas distributed according to $n(r) = n_0 e^{-\alpha r^4}$, take $\alpha > 0$ so that the density decreases and the total over all space is finite. Since $[\alpha] = L^{-4}$, the characteristic radial scale is $\ell = \alpha^{-1/4}$. Density is proportional to n_0 at every point, so the total number must be proportional to n_0, not its square root. A volume based on this length scale is proportional to $\ell^3 = \alpha^{-3/4}$.

Step 2: Confirm the dependence by rescaling the integral The total is

$$N = 4\pi n_0 \int_0^\infty r^2 e^{-\alpha r^4}\,dr.$$

Put $x = \alpha^{1/4} r$. Then $r^2 dr = \alpha^{-3/4} x^2 dx$, and

$$N = 4\pi n_0 \alpha^{-3/4} \int_0^\infty x^2 e^{-x^4}\, dx.$$

The remaining convergent integral is a pure numerical constant independent of n_0 and α; no evaluation of it is needed. Options (B) and (C) have the wrong length dependence, while (D) also violates the linear dependence on the density normalization. Increasing positive α makes the gas more concentrated near the origin and reduces the total at fixed n_0, consistent with the negative exponent.

Step 3: Conclusion

$$\boxed{N \propto n_0 \alpha^{-3/4}}.$$

Hence, the correct option is (A).

Quick Tip to Solve: Extract the length scale from the exponential first: an exponent proportional to r^4 gives $\ell = \alpha^{-1/4}$, and total number scales as density times ℓ^3.

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46. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current Ig flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to $I_0 (I_0 > I_g)$ by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to V ($V = GI_0$) by connecting a series resistance R_V to it. Then, [4 marks]

(A) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

(B) $R_A R_V = G^2 \left(\frac{I_g}{I_0 - I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

(C) $R_A R_V = G^2 \left(\frac{I_0 - I_g}{I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

(D) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

Correct Answer: (D) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

Solution:

Concept:

- A shunt resistor is connected in parallel with a galvanometer to make an ammeter; equal voltage drops determine how the total current divides.
- A voltmeter uses a series resistor so that its full-scale voltage produces only the galvanometer full-scale current I_g.

Step 1: Calculate the ammeter shunt At the ammeter full-scale reading, the total current is I_0, but only I_g may pass through resistance G. The remaining current $I_0 - I_g$ passes through the shunt R_A. The two parallel branches have equal potential differences:

$$I_g G = (I_0 - I_g) R_A.$$

Therefore

$$R_A = \frac{GI_g}{I_0 - I_g}.$$

The condition $I_0 > I_g$ ensures that this required resistance is positive. A shunt provides a bypass; putting this resistor in series instead would not extend the current range.

Step 2: Calculate the series resistor and compare At the voltmeter full-scale value $V = GI_0$, the current through both series elements is I_g. Thus

$$GI_0 = I_g(G + R_V), \qquad R_V = G\frac{I_0 - I_g}{I_g}.$$

Multiplying and dividing the two resistance formulas gives

$$R_A R_V = G^2, \qquad \frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2.$$

Both relations must hold simultaneously. Option (A) loses one power in the ratio. Options (B) and (C) attach a spurious current factor to the product, while (B) also inverts the ratio. The reciprocal current factors in the correct resistances cancel exactly in their product.

Step 3: Conclusion

$$\boxed{R_A R_V = G^2, \quad \frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2}.$$

Hence, the correct option is (D).

Quick Tip to Solve: Derive the parallel shunt from equal voltages and the series resistor from the full-scale voltage; then form the product and ratio rather than memorising them.

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47. A plane electromagnetic wave having a frequency v = 23.9 GHz propagates along the positive z-direction in free space. The peak value of the Electric Field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave ? [4 marks]

(A) $\vec{B} = 2 \times 10^{-7} \sin(1.5 \times 10^2 \text{ x} + 0.5 \times 10^{11}\text{t})\ \hat{j}$

(B) $\vec{B} = 60 \sin(0.5 \times 10^3\text{x} + 0.5 \times 10^{11}\text{t})\ \hat{k}$

(C) $\vec{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 \text{ z} + 1.5 \times 10^{11}\text{t})\ \hat{i}$

(D) $\vec{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 \text{ z} - 1.5 \times 10^{11}\text{t})\ \hat{i}$

Correct Answer: (D) $\vec{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 \text{ z} - 1.5 \times 10^{11}\text{t})\ \hat{i}$

Solution:

Concept:

- For an electromagnetic wave in free space, $E_0 = cB_0$, $\omega = 2\pi\nu$, and $k = \omega/c$, where $c = 3.0 \times 10^8\,\text{m s}^{-1}$.
- A sinusoidal phase $kz - \omega t$ travels along positive z. Both electric and magnetic fields are transverse to the propagation direction.

Step 1: Find the magnetic amplitude and phase constants The peak magnetic field is

$$B_0 = \frac{60}{3.0 \times 10^8} = 2.0 \times 10^{-7}\,\text{T}.$$

Converting the given frequency gives $\nu = 23.9 \times 10^9$ Hz, so

$$\omega = 2\pi(23.9 \times 10^9) \simeq 1.50 \times 10^{11}\,\text{rad}\,\text{s}^{-1}.$$

The wave number is

$$k = \frac{1.50 \times 10^{11}}{3.0 \times 10^8} = 0.50 \times 10^3\,\text{rad}\,\text{m}^{-1}.$$

These values fix the amplitude and the two coefficients in an acceptable wave expression.

Step 2: Check direction and reject the other expressions For propagation in positive z, a fixed phase requires $kz - \omega t =$ constant; increasing time then increases z. A magnetic field along $\hat{\mathbf{i}}$ is transverse and is allowed: the electric field can point along $-\hat{\mathbf{j}}$, making $\mathbf{E} \times \mathbf{B}$ point along positive z. Option (A) depends on x and has incorrect phase constants. Option (B) has the wrong amplitude, spatial dependence and magnetic direction. Option (C) has the correct numerical coefficients but the plus sign gives propagation along negative z. Only (D) satisfies every condition together.

Step 3: Conclusion The acceptable component is

$$\boxed{\mathbf{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t)\,\hat{\mathbf{i}}\,\text{T}}.$$

Hence, the correct option is (D).

Quick Tip to Solve: Check a proposed electromagnetic wave in three passes: amplitude E_0/c, phase coefficients $2\pi\nu$ and ω/c, then the sign that sets its travel direction.

48. Consider an electron in a hydrogen atom revolving in its second excited state (having radius 4.65 $\mathring{A}$). The de-Broglie wavelength of this electron is : [4 marks]

(A) 6.6 $\mathring{A}$
(B) 3.5 $\mathring{A}$
(C) 9.7 $\mathring{A}$
(D) 12.9 $\mathring{A}$

Correct Answer: (C) 9.7 $\mathring{A}$

Solution:

Concept:

- The ground state has principal quantum number $n = 1$, so the first and second excited states have $n = 2$ and $n = 3$, respectively.
- Bohr angular-momentum quantisation $mvr = nh/(2\pi)$ combined with the de Broglie relation $\lambda = h/(mv)$ gives $2\pi r = n\lambda$.

Step 1: Choose the orbit quantum number The wording specifies the second excited state, which is two levels above the ground state. Therefore use $n = 3$, rather than 2. The supplied orbital radius is $r = 4.65\,\text{Å}$. It is sufficient to use this radius directly; neither the electron mass nor its speed needs to be calculated. The standing-wave condition says that an integral number of electron wavelengths fits around the circumference of the allowed orbit.

Step 2: Apply the standing-wave condition Substituting $n = 3$ into $2\pi r = n\lambda$ gives

$$\lambda = \frac{2\pi r}{3} = \frac{2\pi \times 4.65}{3}\,\text{Å} \simeq 9.74\,\text{Å}.$$

Keeping the radius in angstroms also gives the wavelength in angstroms, so no intermediate conversion to metres is required. The listed value 9.7 agrees with this result to the precision of the options. Values 6.6, 3.5 and 12.9 do not satisfy the required three-wavelength circumference. For instance, the circumference is approximately $29.22\,\text{Å}$, and dividing it by 9.7 gives approximately 3, whereas dividing by the other proposed wavelengths does not give the specified principal quantum number. The result is a wavelength associated with electron momentum, not the wavelength of light emitted in a transition.

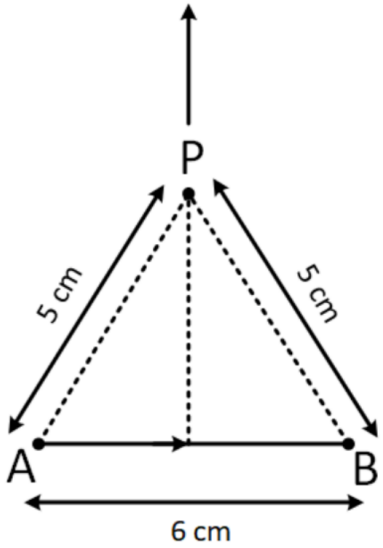
Step 3: Conclusion

$$\boxed{\lambda \simeq 9.7\,\text{Å}}.$$

Hence, the correct option is (C).

Quick Tip to Solve: Translate excited-state wording into n first, then use $\lambda = 2\pi r/n$; the second excited state means $n = 3$, not 2.

49. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5A. (See figure) ($\mu_0 = 4\pi \times 10^{-7}$ N-A^{-2}) [4 marks]



(A) 1.5×10^{-5} T

(B) 3.0×10^{-5} T

(C) 2.0×10^{-5} T

(D) 2.5×10^{-5} T

Correct Answer: (A) 1.5×10^{-5} T

Solution:

Concept:

- For a straight current-carrying segment, $B = \mu_0 I(\sin\theta_1 + \sin\theta_2)/(4\pi a)$, where a is the perpendicular distance from the field point to the wire.
- The endpoint angles are measured from the perpendicular to the wire. The field direction follows the right-hand rule around the current.

Step 1: Find the perpendicular distance and endpoint angles The figure gives $AP = BP = 5\,\text{cm}$ and $AB = 6\,\text{cm}$. Let M be the perpendicular foot from P onto AB. Symmetry gives $AM = MB = 3\,\text{cm}$. Pythagoras in either right triangle gives

$$PM = \sqrt{5^2 - 3^2}\,\text{cm} = 4\,\text{cm} = 0.04\,\text{m}.$$

Both endpoint angles are equal. Measured from PM, their sine is opposite side divided by hypotenuse, so $\sin\theta_1 = \sin\theta_2 = 3/5$. Using 5 cm as the perpendicular distance would confuse the sloping side with the altitude.

Step 2: Substitute in the finite-wire expression With $I = 5\,\text{A}$ and $\mu_0/(4\pi) = 10^{-7}\,\text{N A}^{-2}$,

$$B = \frac{10^{-7} \times 5}{0.04}\left(\frac{3}{5} + \frac{3}{5}\right) = 1.5 \times 10^{-5}\,\text{T}.$$

The current arrow along AB points from A toward B; at the point above the wire the field is out of the figure by the right-hand rule. The requested options compare magnitudes. Option (B) is twice the calculated value, while (C) and (D) also fail the finite-length angular factor. In particular, the infinite-wire formula would replace the sine sum by 2, producing $2.5 \times 10^{-5}\,\mathrm{T}$; it cannot be used for this short segment.

Step 3: Conclusion

$$\boxed{B = 1.5 \times 10^{-5}\,\mathrm{T}}.$$

Hence, the correct option is (A).

Quick Tip to Solve: Use the 3-4-5 triangle to get the perpendicular distance and the endpoint sine immediately; a finite segment needs the endpoint-angle factor.

50. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity . The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio (v_1/v_2) equals : [4 marks]

(A) $\frac{1}{9}$

(B) $\frac{1}{27}$

(C) 27

(D) 9

Correct Answer: (D) 9

Solution:

Concept:

- For a small sphere falling steadily through a viscous liquid, its effective weight equals the Stokes drag: $\frac{4}{3}\pi r^3(\rho_s - \rho_f)g = 6\pi\eta r v_t$. Here ρ_s and ρ_f are the sphere and fluid densities.
- Consequently, $v_t = \frac{2(\rho_s - \rho_f)gr^2}{9\eta}$. With the same material and fluid, terminal speed is proportional to the square of the sphere radius, not its volume.

Step 1: Find the radius of each fragment The sphere is divided into 27 identical solid spheres without changing its density. Conservation of total volume gives

$$\frac{4}{3}\pi R^3 = 27\left(\frac{4}{3}\pi r^3\right), \qquad r^3 = \frac{R^3}{27}, \qquad r = \frac{R}{3}.$$

Thus the radius decreases by a factor of three, although the mass of one fragment decreases by a factor of 27.

Step 2: Compare the two terminal speeds Apply the same terminal-speed formula to the original sphere and to one small sphere. Gravity, density difference and viscosity cancel in the ratio:

$$\frac{v_1}{v_2} = \frac{R^2}{r^2} = \frac{R^2}{(R/3)^2} = 9.$$

The smaller spheres settle more slowly. The reciprocal value $1/9$ would describe v_2/v_1, while 27 and $1/27$ confuse the volume ratio with the terminal-speed ratio. We compare each separate fragment with the original sphere; their speeds are not added.

Step 3: Conclusion The required ratio is $\boxed{v_1/v_2 = 9}$. Hence, the correct option is (D).

Quick Tip to Solve: For identical material in the same liquid, first obtain the radius ratio from volume conservation, then square it to obtain the terminal-speed ratio.

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51. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equal to : [4 marks]

(A) $\frac{3F}{(\pi r^2 YT)}$

(B) $\frac{6F}{(\pi r^2 YT)}$

(C) $\frac{F}{(3\pi r^2 YT)}$

(D) $9F\left(\pi r^2 YT\right)$

Correct Answer: (A) $\frac{3F}{(\pi r^2 YT)}$

Solution:

Concept:

- Linear expansion on a temperature rise T produces thermal strain αT, where α is the coefficient of linear expansion.
- A longitudinal compressive force F acting on area A produces strain of magnitude $F/(AY)$. For a nearly isotropic solid at small strain, the volume expansion coefficient is $\gamma \simeq 3\alpha$.

Step 1: Balance thermal expansion and compression The cross-sectional area of the cylindrical rod is $A = \pi r^2$. Heating alone would increase its length by αLT, while the imposed compressive force shortens it by $FL/(AY)$. Since the actual final length is unchanged, these two changes have equal magnitudes and opposite signs:

$$\Delta L = \alpha LT - \frac{FL}{\pi r^2 Y} = 0.$$

The rod length L cancels because both changes are proportional to it; the force is the net axial force, so it is not doubled.

Step 2: Convert the linear coefficient into a volume coefficient Dividing the balance equation by LT, and using the small-expansion relation between volume and linear expansion, gives

$$\alpha = \frac{F}{\pi r^2 YT}, \qquad \gamma \simeq 3\alpha = \frac{3F}{\pi r^2 YT}.$$

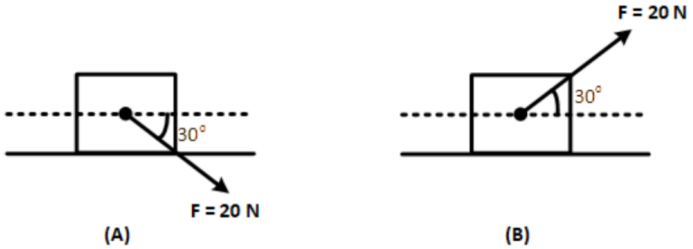
The factor of three comes from expansion in three mutually perpendicular directions, not from changing the longitudinal elastic strain. The factor six in (B) overcounts this conversion; (C) divides by three instead of multiplying. As printed, (D) multiplies by $\pi r^2 YT$, so it does not even have the

inverse-temperature dimensions required of an expansion coefficient.

Step 3: Conclusion The coefficient of volume expansion is approximately $\boxed{\gamma = 3F/(\pi r^2 YT)}$. Hence, the correct option is (A).

Quick Tip to Solve: Zero net extension means thermal strain equals compressive strain. Solve for the linear expansion coefficient first, then multiply by three for volume expansion.

52. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force F = 20 N, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is μ = 0.2. The difference between the accelerations of the blocks, in case (B) and case (A) will be : (g = 10 ms^{-2}) [4 marks]



(A) 3.2 ms^{-2}

(B) 0.8 ms^{-2}

(C) 0 ms^{-2}

(D) 0.4 ms^{-2}

Correct Answer: (B) 0.8 ms^{-2}

Solution:

Concept:

- The kinetic friction magnitude is $f = \mu N$, where N is the normal reaction. A downward force component increases N, whereas an upward component decreases it.
- The horizontal acceleration follows $ma = F\cos\theta - \mu N$. Both cases have the same horizontal applied force, so their acceleration difference depends only on their different friction forces.

Step 1: Find the two normal reactions The blocks have no vertical acceleration. Here $mg = 5 \times 10 = 50\,\text{N}$ and the magnitude of the vertical applied-force component is

$$F\sin 30^\circ = 20 \times \frac{1}{2} = 10\,\text{N}.$$

In case (A), pushing has a downward component, giving $N_A = 50 + 10 = 60\,\text{N}$. In case (B), pulling has an upward component, giving $N_B = 50 - 10 = 40\,\text{N}$. Therefore $f_A = 0.2 \times 60 = 12\,\text{N}$ and $f_B = 0.2 \times 40 = 8\,\text{N}$.

Step 2: Subtract the horizontal accelerations The horizontal applied force is $20\cos 30^\circ = 10\sqrt{3}\,\text{N}$,

exceeding either friction force, so both blocks accelerate to the right. Their accelerations are

$$a_A = \frac{10\sqrt{3} - 12}{5}, \qquad a_B = \frac{10\sqrt{3} - 8}{5}.$$

On subtraction the common applied-force term cancels:

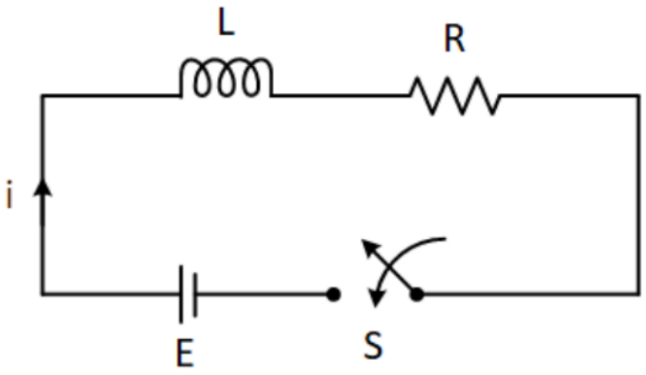
$$a_B - a_A = \frac{12 - 8}{5} = 0.8\,\mathrm{m\,s^{-2}}.$$

The difference cannot be zero because the normal reactions differ. The value 0.4 counts only one change of $\mu F \sin\theta$, whereas changing from push to pull gives twice that change; 3.2 does not follow from the force difference.

Step 3: Conclusion The acceleration of the pulled block exceeds that of the pushed block by $\boxed{0.8\,\mathrm{m\,s^{-2}}}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal oblique forces, subtract the acceleration equations directly: $a_{\text{pull}} - a_{\text{push}} = 2\mu F \sin\theta/m$.

53. Consider the LR circuit shown in the figure. If the switch S is closed at t = 0 then the amount of charge that passes through the battery between t = 0 and t = $\frac{L}{R}$ is : [4 marks]



(A) $\frac{2.7EL}{R^2}$
(B) $\frac{EL}{2.7R^2}$
(C) $\frac{7.3EL}{R^2}$
(D) $\frac{EL}{7.3R^2}$

Correct Answer: (B) $\frac{EL}{2.7R^2}$

Solution:

Concept:

- After an initially unenergised series LR circuit is connected to a battery of emf E, its current is $i(t) = \frac{E}{R}(1 - e^{-Rt/L})$. The time constant is $\tau_L = L/R$.
- Charge passing through the battery is the time integral of current, $q = \int i(t)\,dt$. The initial current

is zero; using the final steady current throughout the interval would overestimate the charge.

Step 1: Integrate the growing current The figure shows a single series loop containing the battery, resistance and inductor. Thus the battery current is the same growing current as in the inductor. Between closing the switch and one time constant,

$$q = \frac{E}{R}\int_0^{L/R}\left(1 - e^{-Rt/L}\right)dt.$$

An antiderivative of the integrand is $t + (L/R)e^{-Rt/L}$; its derivative is $1 - e^{-Rt/L}$, which checks the sign of the exponential contribution.

Step 2: Evaluate the limits and compare options Substituting the upper and lower limits gives

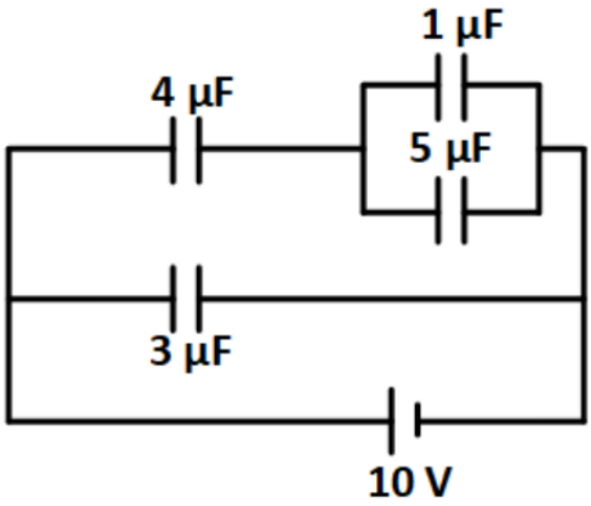
$$\begin{aligned} q &= \frac{E}{R}\left[\frac{L}{R} + \frac{L}{R}e^{-1} - \frac{L}{R}\right] \\ &= \frac{EL}{eR^2} \simeq \frac{EL}{2.7R^2}. \end{aligned}$$

The average current over this interval is therefore $E/(eR)$, below the steady value E/R, as required. Options (A) and (C) give charges greater than $(E/R)(L/R)$, even though the current never reaches E/R during this interval. Option (D) gives the coefficient $1/7.3$, inconsistent with the exact exponential integral; the correct coefficient is $1/e$.

Step 3: Conclusion The charge is $\boxed{q = EL/(eR^2) \simeq EL/(2.7R^2)}$. Hence, the correct option is (B).

Quick Tip to Solve: To find charge in a transient circuit, integrate the current rather than multiplying its final value by the elapsed time. Keep the lower-limit exponential term.

54. In the given circuit, the charge on 4 μF capacitor will be : [4 marks]



(A) 5.4 μC

(B) 9.6 μC

(C) 13.4 μC

(D) 24 μC

Correct Answer: (D) 24 μC

Solution:

Concept:

- Capacitances in parallel add because they have the same potential difference: $C_p = C_1 + C_2$. For a series pair, $C_s = C_a C_b/(C_a + C_b)$, and the two series elements carry equal charge magnitudes.
- A branch connected directly across an ideal battery has the battery voltage. A separate parallel capacitor changes the total charge supplied by the battery but does not change the voltage across the other branch.

Step 1: Reduce the upper capacitor branch The $1\,\mu$F and $5\,\mu$F capacitors are connected between the same two nodes in the small rectangular part of the upper branch. They are therefore parallel, with equivalent capacitance

$$C_p = 1 + 5 = 6\,\mu\text{F}.$$

This equivalent capacitor is in series with the $4\,\mu$F capacitor, so the equivalent capacitance of the whole upper branch is

$$C_{\text{upper}} = \frac{4 \times 6}{4 + 6} = 2.4\,\mu\text{F}.$$

Step 2: Find the series charge using the battery voltage The upper branch is connected between the same battery terminals as the separate $3\,\mu$F capacitor. Hence its potential difference is $10\,\text{V}$, giving

$$Q_{\text{upper}} = C_{\text{upper}} V = 2.4 \times 10 = 24\,\mu\text{C}.$$

Because the $4\,\mu\text{F}$ capacitor is in series with the parallel pair, its charge has this same magnitude. As a check, its voltage is $24/4 = 6\,\text{V}$, and the parallel pair has voltage $24/6 = 4\,\text{V}$; these add to $10\,\text{V}$. The other listed charges fail the series-charge condition and this voltage sum. The $3\,\mu\text{F}$ branch independently stores $30\,\mu\text{C}$ and is not included in the series reduction.

Step 3: Conclusion The charge on the $4\,\mu\text{F}$ capacitor is $\boxed{24\,\mu\text{C}}$. Hence, the correct option is (D).

Quick Tip to Solve: Mark the circuit nodes first: the 1 and 5 microfarad capacitors are parallel, their equivalent is in series with 4 microfarads, and that branch gets the full battery voltage.

55. A particle is moving with speed v = b$\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time t = τ(assume that the particle is at origin t = 0) [4 marks]

(A) $\frac{b^2\tau}{\sqrt{2}}$
(B) $b^2\tau$
(C) $\frac{b^2\tau}{2}$
(D) $\frac{b^2\tau}{4}$

Correct Answer: (C) $\frac{b^2\tau}{2}$

Solution:

Concept:

- For motion along the positive x-axis, speed equals dx/dt. The given relation $v = b\sqrt{x}$ can therefore be integrated by separating the variables.
- The acceleration can also be found from $a = v\,dv/dx$. This gives a useful independent check on the factor multiplying time in the final speed.

Step 1: Integrate the position equation Using the moving trajectory starting at the origin at $t = 0$, write

$$\frac{dx}{dt} = b\sqrt{x}, \qquad \frac{dx}{\sqrt{x}} = b\,dt.$$

Integration from the initial position and time to x, t gives

$$\int_0^x x^{-1/2}\,dx = \int_0^t b\,dt, \qquad 2\sqrt{x} = bt.$$

The integral at the lower endpoint is finite. The initial condition removes the integration constant, so $x = b^2t^2/4$ on the trajectory that begins moving at the stated initial time.

Step 2: Evaluate the speed and check the factor Substitute $\sqrt{x} = bt/2$ into the given speed law:

$$v(t) = b\sqrt{x} = \frac{b^2t}{2}, \qquad v(\tau) = \frac{b^2\tau}{2}.$$

Alternatively, differentiating the speed with respect to position gives

$$a = v\frac{dv}{dx} = b\sqrt{x}\frac{b}{2\sqrt{x}} = \frac{b^2}{2} \quad (x > 0).$$

Starting with zero speed, this constant acceleration yields the same result $v = at$. The factor two arises from integrating $x^{-1/2}$; omitting it gives (B). The factors $\sqrt{2}$ in (A) and four in (D) are inconsistent with either the integrated position relation or the acceleration check. The dimensions also agree: since $[b] = \mathrm{m}^{1/2}\mathrm{s}^{-1}$, $b^2\tau$ has the dimensions of speed.

Step 3: Conclusion The speed at $t = \tau$ is $\boxed{b^2\tau/2}$. Hence, the correct option is (C).

Quick Tip to Solve: When speed is given as a function of position, use $dx/dt = v(x)$ or $a = v\,dv/dx$; substituting time directly for position is invalid.

56. Let a total charge 2Q be distributed in a sphere of radius R, with the charge density given by ρ(r) = kr, where r is the distance from the centre. Two charges A and B, of –Q each, are placed on diametrically opposite points, at equal distance, a from the centre. If A and B do not experience any force, then : [4 marks]

(A) $a = 8^{-1/4}R$

(B) $a = 2^{-1/4}R$

(C) $a = \frac{3R}{2^{1/4}}$

(D) $a = \frac{R}{\sqrt{3}}$

Correct Answer: (A) $a = 8^{-1/4}R$

Solution:

Concept:

- For a spherically symmetric charge distribution, Gauss law gives the electric field at radius a from the charge enclosed inside that radius: $E(a) = KQ_{\text{enc}}(a)/a^2$, where $K = 1/(4\pi\varepsilon_0)$.
- The negative point charges attract toward the positively charged sphere centre, while they repel each other. Their separation is $2a$, and the forces on either charge cancel only when these two magnitudes are equal.

Step 1: Find the enclosed charge For density $\rho(r) = kr$, the charge of a thin spherical shell is $dq = kr\,4\pi r^2 dr$. Thus

$$Q_{\text{enc}}(a) = 4\pi k \int_0^a r^3 dr = \pi k a^4.$$

The total sphere charge is $2Q = \pi k R^4$. Eliminating the density constant yields

$$Q_{\text{enc}}(a) = 2Q\frac{a^4}{R^4}.$$

Consequently, for an interior point the field magnitude is $E(a) = 2KQa^2/R^4$; exterior spherical shells give no net field there.

Step 2: Balance the inward attraction and outward repulsion The distributed positive charge exerts an inward force of magnitude $QE(a) = 2KQ^2a^2/R^4$ on each negative charge. The other negative

charge is $2a$ away and repels it outward with magnitude $KQ^2/(4a^2)$. Equilibrium therefore requires

$$\frac{2KQ^2a^2}{R^4} = \frac{KQ^2}{4a^2}, \qquad 8a^4 = R^4.$$

Hence $a = R/8^{1/4}$, which is less than R, verifying use of the interior-field formula. Option (B) misses the separation factor in the repulsive force; (D) does not satisfy the fourth-power balance. Option (C) lies outside the sphere; outside, the inward force would be $2KQ^2/a^2$, eight times the repulsion, so it cannot give equilibrium.

Step 3: Conclusion The required distance is $\boxed{a = 8^{-1/4}R}$. Hence, the correct option is (A).

Quick Tip to Solve: For a radial density proportional to r, the enclosed charge scales as r^4. Also use separation $2a$, rather than a, for the two point charges.

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57. A Carnot engine has an efficiency of $\frac{1}{6}$. When the temperature of the sink is reduced by 62°C, its efficiency is doubled. The temperatures of the source and the sink are, respectively [4 marks]

(A) 99°C, 37°C

(B) 124°C, 62°C

(C) 37°C, 99°C

(D) 62°C, 124°C

Correct Answer: (A) 99°C, 37°C

Solution:

Concept:

- A Carnot engine working between source temperature T_h and sink temperature T_c has efficiency $\eta = 1 - T_c/T_h$, with both temperatures in kelvin.
- Temperature differences have the same numerical value in kelvin and degrees Celsius; lowering the sink alone increases the efficiency by $\Delta T_c/T_h$.

Step 1: Translate the initial efficiency The initial efficiency is $1/6$, so the fraction of input heat rejected is $5/6$. Consequently,

$$1 - \frac{T_c}{T_h} = \frac{1}{6} \quad \Rightarrow \quad T_c = \frac{5}{6}T_h.$$

This relation uses absolute temperatures: substituting Celsius temperatures directly in the ratio would be incorrect.

Step 2: Use the increased efficiency The new sink temperature is $T_c - 62$ K and the unchanged source remains at T_h. Doubling the efficiency gives

$$1 - \frac{T_c - 62}{T_h} = \frac{1}{3}.$$

Subtract the initial efficiency equation to eliminate the original sink temperature:

$$\frac{62}{T_h} = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}, \qquad T_h = 372\,\text{K}.$$

Step 3: Convert both temperatures and test the alternatives The original sink is $T_c = (5/6)(372) = 310\,\text{K}$. Using $T(^\circ\text{C}) = T(\text{K}) - 273$, the original temperatures are 99°C and 37°C. After cooling, the sink is $248\,\text{K}$, giving $1 - 248/372 = 1/3$, as required. Option (B) gives initial efficiency $62/397$, not $1/6$; (C) and (D) put the sink above the source and cannot operate as heat engines.

Step 4: Conclusion The source and original sink temperatures are 99°C and 37°C, respectively. Hence, the correct option is (A).

Quick Tip to Solve: When only the sink is cooled, use $\Delta\eta = \Delta T_c/T_h$ first; it isolates the source temperature without solving simultaneous equations.

58. In an amplitude modulator circuit, the carrier wave is given by, C(t) = 4 sin(20000 πt) while modulating signal is given by, m(t) = 2 sin (2000 πt). The values of modulation index and lower side band frequency are : [4 marks]

(A) 0.3 and 9 kHz

(B) 0.5 and 10 kHz

(C) 0.4 and 10 kHz

(D) 0.5 and 9 kHz

Correct Answer: (D) 0.5 and 9 kHz

Solution:

Concept:

- The modulation index is $\mu = A_m/A_c$, the ratio of modulating-signal amplitude to unmodulated-carrier amplitude.
- For a sinusoid $A\sin(\omega t)$, frequency is $f = \omega/(2\pi)$. Amplitude modulation produces sidebands at $f_c - f_m$ and $f_c + f_m$.

Step 1: Read the two amplitudes The coefficient of the carrier sine is $A_c = 4$, and that of the modulating sine is $A_m = 2$. Their frequencies do not enter the amplitude ratio. Thus

$$\mu = \frac{A_m}{A_c} = \frac{2}{4} = 0.5.$$

This is a dimensionless modulation depth of 50 percent and is below unity, so there is no overmodulation.

Step 2: Convert angular frequencies to ordinary frequencies Compare each sine argument with $2\pi ft$, keeping the factor 2π. The carrier and modulating frequencies are

$$f_c = \frac{20000\pi}{2\pi} = 10000\,\text{Hz} = 10\,\text{kHz}, \qquad f_m = \frac{2000\pi}{2\pi} = 1000\,\text{Hz} = 1\,\text{kHz}.$$

The given coefficients of t are angular frequencies, not frequencies in hertz.

Step 3: Choose the lower sideband The product of the carrier and modulation sinusoids produces sum and difference frequencies. The lower one is

$$f_{\text{LSB}} = f_c - f_m = 10 - 1 = 9\,\text{kHz}.$$

The upper sideband would be $11\,\text{kHz}$. Option (A) has the correct lower frequency but the wrong amplitude ratio. Option (B) uses the carrier itself as the lower sideband, and (C) has both an incorrect ratio and that same frequency error.

Step 4: Conclusion The modulation index is 0.5 and the lower sideband frequency is $9\,\text{kHz}$. Hence, the correct option is (D).

Quick Tip to Solve: Read amplitudes from the sine coefficients and frequencies from the arguments; the lower sideband is the carrier frequency minus the modulating frequency.

59. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equals : [4 marks]

(A) 10.0 m/s

(B) 5.5 m/s

(C) 15.0 m/s

(D) 2.5 m/s

Correct Answer: (D) 2.5 m/s

Solution:

Concept:

- For a stationary sound source of frequency f and sound speed v, a listener moving towards it at speed u hears $f(v+u)/v$; moving away gives $f(v-u)/v$.
- The beat frequency is the absolute difference between the two frequencies heard simultaneously. The sources remain at rest, so the Doppler denominators are both v.

Step 1: Write the frequencies heard from each source The listener travels away from S_1 and towards S_2. Therefore the first sound is shifted down and the second up:

$$f_1 = 660\frac{330-u}{330}, \qquad f_2 = 660\frac{330+u}{330}.$$

The waves arriving from opposite directions are both affected by the same listener speed, with opposite signs.

Step 2: Use the observed beat rate Subtract the lower frequency from the higher and equate the result to ten beats per second:

$$f_b = f_2 - f_1 = \frac{660}{330}\big[(330+u) - (330-u)\big] = 4u.$$

Hence

$$10 = 4u \quad \Rightarrow \quad u = 2.5\,\mathrm{m\,s^{-1}}.$$

Accounting for only one Doppler shift would miss the factor of two in the frequency difference.

Step 3: Check the individual frequencies and options At this speed the listener hears $f_1 = 655\,\mathrm{Hz}$ and $f_2 = 665\,\mathrm{Hz}$, whose difference is exactly $10\,\mathrm{Hz}$. The proposed speeds 10.0, 5.5 and $15.0\,\mathrm{m\,s^{-1}}$ instead predict beat rates 40, 22 and $60\,\mathrm{Hz}$, respectively, so none fits the observation. The obtained speed is much less than the sound speed, consistent with the stated motion.

Step 4: Conclusion The speed of the listener is $u = 2.5\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: Between two identical stationary sources, opposite observer Doppler shifts add in the beat difference: $f_b = 2fu/v$.

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60. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively, If initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be : [4 marks]

(A) 9 : 8

(B) 1 : 8

(C) 8 : 1

(D) 3 : 8

Correct Answer: (A) 9 : 8

Solution:

Concept:

- After time t, a radioactive sample with half-life $T_{1/2}$ contains $N = N_0\,2^{-t/T_{1/2}}$ undecayed nuclei.
- The number that have decayed is the initial population minus the survivors, $D = N_0 - N$. Equal initial populations cancel in the requested ratio.

Step 1: Count the half-lives and survivors Let both samples initially contain N_0 nuclei. Sixty minutes represents six half-lives for A and three for B:

$$\frac{t}{T_A} = \frac{60}{10} = 6, \qquad \frac{t}{T_B} = \frac{60}{20} = 3.$$

The numbers still present are therefore

$$N_A = \frac{N_0}{64}, \qquad N_B = \frac{N_0}{8}.$$

A decays more rapidly because its half-life is smaller.

Step 2: Find the decayed populations The question asks for nuclei that have decayed, so subtract the survivors from the corresponding initial population:

$$D_A = N_0 - \frac{N_0}{64} = \frac{63N_0}{64}, \qquad D_B = N_0 - \frac{N_0}{8} = \frac{7N_0}{8}.$$

The initial equality permits a direct ratio without knowing the sample size.

Step 3: Simplify the ratio and eliminate distractors Dividing gives

$$\frac{D_A}{D_B} = \frac{63/64}{7/8} = \frac{63}{56} = \frac{9}{8}.$$

Option (B), $1 : 8$, is the ratio of the surviving populations, not the decayed ones; (C) reverses that surviving ratio. Option (D) also fails the explicit population subtraction. Since A has the shorter half-life, its decayed population should exceed B's, as $9 : 8$ does.

Step 4: Conclusion The ratio of the decayed numbers of nuclei is $D_A : D_B = 9 : 8$. Hence, the correct option is (A).

Quick Tip to Solve: Underline whether a decay problem asks for survivors or decays; for decays, take $1 - 2^{-n}$ after n half-lives before forming the ratio.

Chemistry (120 marks)

61. Among the following, the energy of 2s orbital is lowest in : [4 marks]

(A) Li

(B) H

(C) Na

(D) K

Correct Answer: (D) K

Solution:

Concept:

- Orbital energies are measured relative to a free electron at rest. A lower orbital energy is more negative and means that the electron is more strongly bound.
- For the same inner orbital across different atoms, greater effective nuclear attraction generally lowers its energy. Outer-shell electrons shield an inner electron much less effectively than electrons in inner shells.

Step 1: Identify the orbital being compared The question compares the energy of the same $2s$ orbital in each atom; it does not compare the energies of their outermost occupied orbitals. Hydrogen has nuclear charge $Z = 1$, lithium has $Z = 3$, sodium has $Z = 11$, and potassium has $Z = 19$. In H the $2s$ orbital is an excited-state orbital; in Li it is the valence orbital; in Na and K it is a core orbital.

Step 2: Compare the attraction felt by a 2s electron As nuclear charge grows, the inner $2s$ electron is held more tightly. Added electrons in higher shells do not cancel the increased attraction at this inner orbital because those electrons are mainly outside it. A simple effective-charge picture has E_{2s} becoming more negative as the effective nuclear charge increases. Potassium therefore binds its $2s$ electron more strongly than sodium, lithium or hydrogen.

Step 3: Distinguish the core-orbital trend from valence trends The relevant qualitative ordering, from

most negative to least negative energy, is

$$E_{2s}(\mathrm{K}) < E_{2s}(\mathrm{Na}) < E_{2s}(\mathrm{Li}) < E_{2s}(\mathrm{H}).$$

Thus Li and H are too weakly binding to give the minimum, while Na has a less strongly bound $2s$ core than K. The familiar decrease of first ionisation enthalpy down the alkali-metal group concerns successive $2s$, $3s$, and $4s$ valence orbitals; it cannot be applied to a fixed $2s$ orbital.

Step 4: Conclusion The energy of the $2s$ orbital is lowest in potassium, K. Hence, the correct option is (D).

Quick Tip to Solve: For orbital-energy comparisons, first check whether the principal quantum number stays fixed; a fixed core orbital follows nuclear attraction rather than the valence ionisation trend.

62. In comparison to boron, berylium has : [4 marks]

(A) lesser nuclear charge and greater first ionisation enthalpy

(B) greater nuclear charge and greater first ionisation enthalpy

(C) greater nuclear charge and lesser first ionisation enthalpy

(D) lesser nuclear charge and lesser first ionisation ethalpy

Correct Answer: (A) lesser nuclear charge and greater first ionisation enthalpy

Solution:

Concept:

- Nuclear charge is $+Ze$, where Z is the atomic number. Beryllium has $Z = 4$ and boron has $Z = 5$.
- First ionisation enthalpy is the energy needed to remove one electron from an isolated gaseous atom. Besides nuclear charge, subshell energy, penetration and electronic configuration affect it.

Step 1: Compare nuclear charges directly Beryllium has four protons, whereas boron has five. Its nuclear charge is therefore $+4e$, less than boron's $+5e$. This immediately eliminates options (B) and (C), which describe beryllium as having the greater nuclear charge. Nuclear charge should not be confused with the effective nuclear charge after shielding.

Step 2: Write the electronic configurations The ground-state configurations are

$$\mathrm{Be} : 1s^2\,2s^2, \qquad \mathrm{B} : 1s^2\,2s^2\,2p^1.$$

Ionising Be removes an electron from its filled $2s$ subshell. Ionising B removes the single $2p$ electron while leaving the filled $2s^2$ subshell intact. For a given shell, $2s$ penetrates the inner region better and is lower in energy than $2p$.

Step 3: Explain the ionisation exception The boron $2p$ electron is less penetrating and more shielded than a beryllium $2s$ electron. It is consequently easier to remove despite boron's additional proton. Hence

$$\Delta_i H_1(\mathrm{Be}) > \Delta_i H_1(\mathrm{B}).$$

This is the well-known departure from the general increase of first ionisation enthalpy across a period.

Option (D) gets the nuclear-charge comparison right but reverses the ionisation-enthalpy comparison; only (A) has both properties correct.

Step 4: Conclusion Compared with boron, beryllium has lesser nuclear charge and greater first ionisation enthalpy. Hence, the correct option is (A).

Quick Tip to Solve: At the group 2 to group 13 boundary, check whether removal switches from a filled s subshell to a higher-energy p electron before applying the general periodic trend.

63. A solution is prepared by dissolving 0.6 g of urea (molar mass = 60 g mol^{-1}) and 1.8 g of glucose (molar mass = 180 g mol^{-1}) in 100 mL of water at 27°C. The osmotic pressure of the solution is : (R = 0.08206 L atm K^{-1} mol^{-1}) [4 marks]

(A) 8.2 atm

(B) 2.46 atm

(C) 4.92 atm

(D) 1.64 atm

Correct Answer: (C) 4.92 atm

Solution:

Concept:

- For a dilute solution, osmotic pressure is $\Pi = CRT = nRT/V$, where n is the total amount of dissolved solute particles and V is the solution volume.
- Urea and glucose are non-electrolytes and do not dissociate in water, so each has van't Hoff factor $i = 1$. Their particle amounts add for osmotic pressure.

Step 1: Calculate the amount of each solute Divide each given mass by its molar mass:

$$n_{\text{urea}} = \frac{0.6}{60} = 0.010\,\text{mol}, \qquad n_{\text{glucose}} = \frac{1.8}{180} = 0.010\,\text{mol}.$$

Although the masses differ, the two substances supply equal numbers of molecules. With no dissociation, the total solute particle amount is $0.020\,\text{mol}$.

Step 2: Convert volume and temperature consistently For the dilute solution, use the stated $100\,\text{mL}$ aqueous volume as $0.100\,\text{L}$, neglecting the small solute-volume change as in the ideal dilute-solution treatment. The absolute temperature is $27 + 273 = 300\,\text{K}$. Thus

$$C = \frac{0.020}{0.100} = 0.200\,\text{mol}\,\text{L}^{-1}.$$

The litre volume is required because the supplied gas constant uses litres and atmospheres.

Step 3: Evaluate the total osmotic pressure Substitute in the osmotic-pressure equation:

$$\Pi = (0.200)(0.08206)(300) = 4.9236\,\text{atm} \simeq 4.92\,\text{atm}.$$

Each solute contributes approximately $2.46\,\text{atm}$, so option (B) counts only one solute. Option (A)

does not match the total particle concentration, and option (D) is also inconsistent with CRT. No extra dissociation multiplier is justified for either molecular solute.

Step 4: Conclusion The osmotic pressure of the solution is approximately $4.92\,\text{atm}$. Hence, the correct option is (C).

Quick Tip to Solve: For mixed non-electrolytes, add moles rather than masses, then apply $\Pi = (n_1 + n_2)RT/V$ using kelvin and volume units consistent with R.

64. The decreasing order of electrical conductivity of the following aqueous solutions is :

0.1 M Formic acid (A),

0.1 M Acetic acid (B),

0.1 M Benzoic acid (C) [4 marks]

(A) A > B > C

(B) C > B > A

(C) A > C > B

(D) C > A > B

Correct Answer: (C) A > C > B

Solution:

Concept:

- Electrical conductivity of an aqueous acid depends on the concentration and mobility of the ions it produces. Hydrogen ions have particularly high ionic mobility.
- For a weak monobasic acid of concentration c, $K_a = c\alpha^2/(1-\alpha)$. With small dissociation, $\alpha \simeq \sqrt{K_a/c}$, where α is the degree of ionisation.

Step 1: Compare the acid strengths The three solutions have the same analytical concentration, $0.1\,\text{mol}\,\text{L}^{-1}$, and all three acids are weak and monobasic. Their approximate dissociation constants are

$$K_a(\text{formic}) = 1.8 \times 10^{-4}, \quad K_a(\text{benzoic}) = 6.3 \times 10^{-5}, \quad K_a(\text{acetic}) = 1.8 \times 10^{-5}.$$

Thus formic acid ionises most, benzoic acid next, and acetic acid least. The electron-donating methyl group makes acetic acid weaker than formic acid.

Step 2: Relate ionisation to conductivity For each acid, ionisation supplies one hydrogen ion and one carboxylate ion per molecule dissociated. The approximate hydrogen-ion concentrations are $\sqrt{K_a c}$: 4.2×10^{-3}, 2.5×10^{-3}, and $1.3 \times 10^{-3}\,\text{mol}\,\text{L}^{-1}$ for formic, benzoic and acetic acids respectively. Hence the concentration of conducting ions follows the same sequence. Differences in carboxylate mobility do not overturn this order; the highly mobile hydrogen ion gives the major contribution. Option A incorrectly places acetic acid above benzoic acid, while B and D incorrectly place benzoic acid above formic acid.

Step 3: Conclusion The decreasing conductivity is formic acid > benzoic acid > acetic acid, or $A > C > B$. Hence, the correct option is (C).

Quick Tip to Solve: For equal concentrations of weak monobasic acids, compare K_a, then use $[\text{H}^+] \simeq \sqrt{K_a c}$ to rank their conductivity.

65. Which one of the following is likely to give a precipitate with $AgNO_3$ solution ? [4 marks]

(A) $CHCl_3$

(B) $(CH_3)_3CCl$

(C) CCl_4

(D) $CH_2=CH–Cl$

Correct Answer: (B) $(CH_3)_3CCl$

Solution:

Concept:

- A silver nitrate test detects chloride ions released by an organic chloride: $Ag^+ + Cl^- \longrightarrow AgCl(s)$. A covalently bonded chlorine atom must first be liberated as chloride.
- Tertiary alkyl chlorides readily undergo ionisation because the resulting tertiary carbocation is stabilised by the electron-releasing alkyl groups and hyperconjugation.

Step 1: Identify the readily ionising chloride In tert-butyl chloride, $(CH_3)_3CCl$, the carbon bonded to chlorine is tertiary. Cleavage of its carbon–chlorine bond produces the relatively stable tert-butyl carbocation and chloride ion:

$$(CH_3)_3CCl \longrightarrow (CH_3)_3C^+ + Cl^-.$$

The released chloride is trapped by silver ion as a white precipitate of silver chloride. Removal of chloride also favours further ionisation, making the test particularly convenient for a tertiary alkyl chloride.

Step 2: Exclude the other bonding environments Chloroform, $CHCl_3$, and carbon tetrachloride, CCl_4, do not supply free chloride ions to an ordinary silver nitrate solution simply because they contain several chlorine atoms. Their carbon–chlorine bonds remain covalent under the test conditions. Vinyl chloride, $CH_2=CHCl$, has chlorine directly bonded to an unsaturated carbon. Conjugation gives its carbon–chlorine bond partial double-bond character, and ionisation would require an unstable vinyl carbocation. It therefore resists the substitution that produces chloride. Thus A, C and D cannot be chosen by merely counting chlorine atoms; the ease of releasing chloride is decisive.

Step 3: Conclusion $(CH_3)_3CCl$ gives the silver chloride precipitate. Hence, the correct option is (B).

Quick Tip to Solve: In the silver nitrate test, ask whether the carbon–halogen bond can ionise. A tertiary alkyl chloride reacts readily; a vinyl chloride does not.

66. Benzene diazonium chloride on reaction with aniline in the presence of dilute hydrochloric acid gives : [4 marks]

(A) $C_6H_5-N=N-C_6H_4-NH_2$ (para-aminoazobenzene)

(B) $C_6H_5-N=N-C_6H_4-NH_2$ (ortho-aminoazobenzene)

(C) $C_6H_5-N=N-NH-C_6H_5$

(D) $C_6H_5-C_6H_4-NH_2$ (para-aminobiphenyl)

Correct Answer: (A) $C_6H_5-N=N-C_6H_4-NH_2$ (para-aminoazobenzene)

Solution:

Concept:

- Benzene diazonium ion, $C_6H_5N_2^+$, acts as an electrophile in coupling with an activated aromatic ring. The diazonium nitrogen atoms are retained as the azo linkage $-N=N-$.
- The amino group activates the aniline ring and directs electrophilic substitution to its ortho and para positions. When the para position is free, para coupling is favoured over sterically more crowded ortho coupling.

Step 1: Locate the favoured coupling position Aniline has an amino group attached to benzene. Donation of its nitrogen lone pair increases electron density particularly at the ortho and para ring carbons. In the coupling reaction in dilute acid, the diazonium electrophile attaches predominantly to the available para carbon of the aniline ring. The para site is less crowded than the ortho sites. Loss of a proton from the attacked ring restores aromaticity; it does not remove the amino substituent.

Step 2: Identify the product and exclude the alternatives The resulting azo dye is para-aminoazobenzene, also called aniline yellow. Its connectivity is

$$C_6H_5-N=N-C_6H_4-NH_2 \quad (\text{para amino group}).$$

This is the structure in option A: the amino group and azo substituent are opposite each other on the second ring. Option B shows the ortho isomer rather than the favoured para product. Option C links the diazonium group to the amino nitrogen, producing diazoaminobenzene, rather than the ring-coupled azo dye formed under the stated acidic coupling conditions. Option D is a substituted biphenyl with a direct carbon–carbon bond between the rings and lacks the required azo linkage. No such loss of both nitrogen atoms occurs in azo coupling.

Step 3: Conclusion The product is para-aminoazobenzene, $C_6H_5-N=N-C_6H_4-NH_2$. Hence, the correct option is (A).

Quick Tip to Solve: For azo coupling with aniline, retain $-N=N-$, keep $-NH_2$ on the ring, and choose its free para position before an ortho position.

67. The compound used in the treatment of lead poisoning is : [4 marks]

(A) desferrioxime B

(B) Cis-platin

(C) D-penicillamine

(D) EDTA

Correct Answer: (D) EDTA

Solution:

Concept:

- A chelating ligand coordinates to a metal ion through more than one donor atom, producing a stable ring-containing complex. Stable complex formation can bind a toxic metal ion.
- EDTA is ethylenediaminetetraacetate. Its two nitrogen donors and four carboxylate oxygen donors make it a hexadentate ligand capable of tightly binding many metal ions.

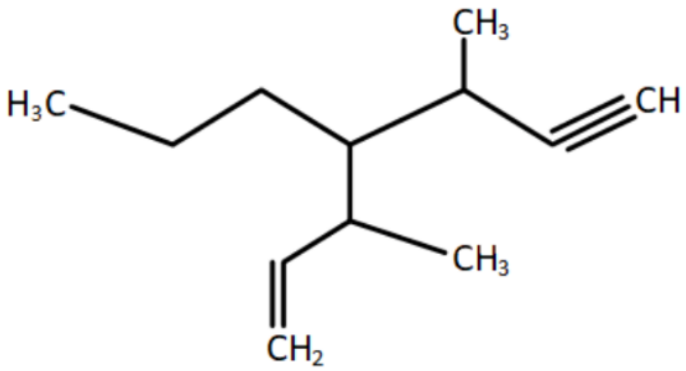
Step 1: Identify the chelator used for lead Lead poisoning involves toxic lead ions, Pb^{2+}. In the coordination-chemistry application asked here, EDTA binds lead into a stable chelate. The medicinal preparation is commonly calcium disodium EDTA, in which lead displaces calcium because the lead–EDTA complex is sufficiently stable. The bound lead can then be eliminated from the body. The key chemical idea is sequestration of the metal ion, rather than conversion of lead into an insoluble solid within tissues.

Step 2: Match the remaining compounds to their applications Desferrioxime B, also referred to as desferrioxamine B, is associated with chelation of excess iron, so option A is not the standard lead-poisoning match. Cis-platin is a platinum coordination compound used as an anticancer agent; it is not the chelator selected for lead removal, excluding B. D-penicillamine is especially associated with copper removal in copper accumulation disorders, so C is not the intended match either. These compounds have different coordination chemistry and applications despite all appearing in discussions of medicinal coordination compounds. EDTA is the direct textbook match for the stated toxic metal.

Step 3: Conclusion The compound is EDTA, used as its appropriate medicinal chelate preparation for lead removal. Hence, the correct option is (D).

Quick Tip to Solve: Keep the standard matches distinct: EDTA–lead, desferrioxamine–iron, penicillamine–copper, and cis-platin–anticancer action.

68. The IUPAC name for the following compound is : [4 marks]



(A) 3, 5-dimethyl-4-propylhept-1-en-6-yne

(B) 3-methyl-4-(3-methylprop-1-enyl)-1-heptyne

(C) 3-methyl-4-(1-methylprop-2-enyl)-1-heptene

(D) 3,5-dimethyl-4-propylhept-6-en-1-yne

Correct Answer: (A) 3, 5-dimethyl-4-propylhept-1-en-6-yne

Solution:

Concept:

- For this acyclic unsaturated hydrocarbon, select a parent chain that contains both the double bond and the triple bond. The multiple bonds must be represented in the parent suffix.
- Number the selected chain to give the multiple bonds the lowest locants. If the locant set ties, the double bond receives the lower locant; substituents are then located on that numbering.

Step 1: Trace the chain containing both multiple bonds Start at the terminal CH_2 of the double bond and follow the chain through the central junction to the terminal CH of the triple bond. This path contains seven carbons. Its condensed form, including the side groups, is

$$CH_2{=}CH{-}CH(CH_3){-}CH(CH_2CH_2CH_3){-}CH(CH_3){-}C{\equiv}CH.$$

The seven-carbon parent is therefore a heptenyne. The leftward three-carbon arm in the printed structure is a propyl substituent at the central carbon; it is not used as the end of the parent chain because that choice would omit one of the two multiple bonds.

Step 2: Number and assemble the name Numbering from the alkene end puts the double bond at 1 and the triple bond at 6. Reversing the direction puts the triple bond at 1 and the double bond at 6. Both give the same set $\{1, 6\}$, so the double bond receives locant 1. The methyl groups are at carbons 3 and 5, and the propyl group is at carbon 4. In alphabetical order, ignoring the multiplying prefix di-, methyl precedes propyl. The name is thus 3,5-dimethyl-4-propylhept-1-en-6-yne. Options B and C choose an unsuitable parent that does not include both multiple bonds, while D reverses the tie-breaking priority and gives the double bond locant 6.

Step 3: Conclusion The IUPAC name is 3,5-dimethyl-4-propylhept-1-en-6-yne. Hence, the correct option is (A).

Quick Tip to Solve: For an enyne, include both multiple bonds in the parent. If numbering ties at $\{1, 6\}$, give the double bond locant 1.

69. The INCORRECT match in the following is : [4 marks]

(A) $\Delta G^o = 0$, K = 1

(B) $\Delta G^o < 0$, K < 1

(C) $\Delta G^o > 0$, K < 1

(D) $\Delta G^o < 0$, K > 1

Correct Answer: (B) $\Delta G^o < 0$, K < 1

Solution:

Concept:

- The standard Gibbs energy change and the thermodynamic equilibrium constant are related by $\Delta G^\circ = -RT \ln K$, where $R > 0$ and absolute temperature $T > 0$.
- The sign of $\ln K$ is positive for $K > 1$, zero for $K = 1$, and negative for $0 < K < 1$. The leading minus sign therefore reverses this sign in ΔG°.

Step 1: Derive the possible sign combinations If $K = 1$, then $\ln K = 0$, so $\Delta G^\circ = 0$. If $K > 1$, the logarithm is positive and $-RT \ln K$ is negative, giving $\Delta G^\circ < 0$. If $K < 1$, the logarithm is negative and $-RT \ln K$ is positive, giving $\Delta G^\circ > 0$. Here the equilibrium constant is positive, so the last statement always means $0 < K < 1$. These are properties of the standard-state energy change; the actual Gibbs energy change is zero at equilibrium regardless of the value of K.

Step 2: Test every printed match Option A pairs zero standard Gibbs energy change with $K = 1$, which agrees with the logarithmic equation. Option C pairs a positive standard Gibbs energy change with $K < 1$, also correctly. Option D pairs a negative standard Gibbs energy change with $K > 1$, again correctly. Option B instead pairs $\Delta G^\circ < 0$ with $K < 1$. Substituting a value below one makes $\ln K$ negative and forces ΔG° to be positive, contradicting B. Since the question asks for the incorrect match, select this contradiction rather than one of the valid combinations.

Step 3: Conclusion The incorrect match is $\Delta G^\circ < 0,\ K < 1$. Hence, the correct option is (B).

Quick Tip to Solve: Write $\Delta G^\circ = -RT \ln K$ before comparing signs: a larger-than-one equilibrium constant means a negative standard Gibbs energy change.

70. 25 g of an unknown hydrocarbon upon burning produces 88 g of CO_2 and 9 g of H_2O. This unknown hydrocarbon contains : [4 marks]

(A) 18 g of carbon and 7 g of hydrogen

(B) 20 g of carbon and 5 g of hydrogen

(C) 22 g of carbon and 3 g of hydrogen

(D) 24 g of carbon and 1 g of hydrogen

Correct Answer: (D) 24 g of carbon and 1 g of hydrogen

Solution:

Concept:

- Complete combustion converts every carbon atom in a hydrocarbon into CO_2 and every hydrogen atom into H_2O. The oxygen in the products is supplied by the oxidant.
- Using molar masses $44\,\text{g mol}^{-1}$ for CO_2 and $18\,\text{g mol}^{-1}$ for H_2O, the carbon mass fraction in carbon dioxide is $12/44$, and the hydrogen mass fraction in water is $2/18$.

Step 1: Calculate the carbon in the sample The $88\,\text{g}$ of carbon dioxide represents

$$n(CO_2) = \frac{88}{44} = 2\,\text{mol}.$$

Each mole of carbon dioxide contains one mole of carbon atoms. Therefore the original hydrocarbon contained $2\,\text{mol}$ of carbon atoms, with mass

$$m(C) = 2 \times 12 = 24\,\text{g}.$$

Only the carbon portion of the carbon dioxide is counted; including its oxygen would incorrectly assign oxygen from the air to the hydrocarbon.

Step 2: Calculate the hydrogen and check mass conservation The $9\,\text{g}$ of water corresponds to

$$n(H_2O) = \frac{9}{18} = 0.5\,\text{mol}.$$

There are two hydrogen atoms in each water molecule, so this is $1\,\text{mol}$ of hydrogen atoms, or $1\,\text{g}$ of hydrogen. The total carbon and hydrogen mass is $24 + 1 = 25\,\text{g}$, exactly the given sample mass. Option A would have too little carbon and too much hydrogen, as would B and C; in particular none of their carbon masses could produce the measured $88\,\text{g}$ of carbon dioxide. Both independent product measurements select D.

Step 3: Conclusion The hydrocarbon contains $24\,\text{g}$ of carbon and $1\,\text{g}$ of hydrogen. Hence, the correct option is (D).

Quick Tip to Solve: In combustion analysis, multiply the carbon dioxide mass by $12/44$ and the water mass by $2/18$, then check that the two elemental masses sum to the sample mass.

71. In which one of the following equilibria, $K_p \neq K_C$? [4 marks]

(A) $2NO(g) \rightleftharpoons N_2(g) + O_2(g)$

(B) $2C(s) + O_2(g) \rightleftharpoons 2CO(g)$

(C) $2HI(g) \rightleftharpoons H_2(g) + I_2(g)$

(D) $NO_2(g) + SO_2(g) \rightleftharpoons NO(g) + SO_3(g)$

Correct Answer: (B) $2C(s) + O_2(g) \rightleftharpoons 2CO(g)$

Solution:

Concept:

- For an ideal gaseous equilibrium, $K_p = K_c(RT)^{\Delta n_g}$, where Δn_g is the sum of gaseous product coefficients minus that of gaseous reactants.
- Pure solids have unit activity. They do not enter the gaseous mole count, even when their stoichiometric coefficients are large.

Step 1: Count gas molecules in each equation For (A), $2\mathrm{NO}\,(\mathrm{g}) \rightleftharpoons \mathrm{N_2(g)} + \mathrm{O_2(g)}$, the change is $1+1-2=0$. For (C), $2\mathrm{HI}\,(\mathrm{g}) \rightleftharpoons \mathrm{H_2(g)}+\mathrm{I_2(g)}$, it is also zero. For (D), there are two gaseous molecules on each side, so again $\Delta n_g = 0$. Thus each of these three equilibria has $K_p = K_c$ in the usual textbook relation.

Step 2: Exclude solid carbon from the second equilibrium In (B), $2\,\mathrm{C\,(s)} + \mathrm{O_2(g)} \rightleftharpoons 2\,\mathrm{CO\,(g)}$, only oxygen and carbon monoxide are counted. Consequently,

$$\Delta n_g = 2 - 1 = 1, \qquad K_p = K_c RT.$$

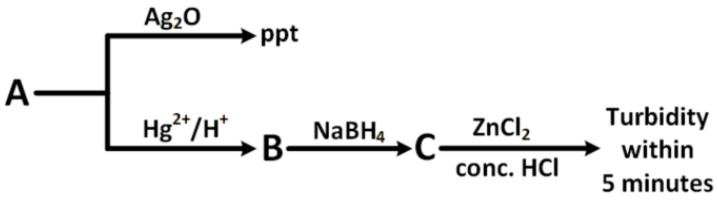
The two carbon atoms on the reactant side are in a solid phase; including them would incorrectly give a negative mole change. Directly, $K_p = p_{\mathrm{CO}}^2/p_{\mathrm{O_2}}$ and $K_c = [\mathrm{CO}]^2/[\mathrm{O_2}]$. Substituting $p_i = [i]RT$ leaves exactly one factor of RT. Therefore (B) is the equilibrium for which the pressure and concentration constants do not have the same form or numerical value under the conventional units.

Step 3: Conclusion For $2\,\mathrm{C\,(s)} + \mathrm{O_2(g)} \rightleftharpoons 2\,\mathrm{CO\,(g)}$, $\Delta n_g = 1$, giving $K_p = K_c RT \neq K_c$. Hence, the correct option is (B).

Quick Tip to Solve: Cross out pure solids first, then subtract gaseous coefficients; equal gas counts make the RT factor disappear.

72. Consider the following reactions :

'A' is : [4 marks]



(A) $CH \equiv CH$

(B) $CH_3 - C \equiv C - CH_3$

(C) $CH_3 - C \equiv CH$

(D) $CH_2 = CH_2$

Correct Answer: (C) $CH_3 - C \equiv CH$

Solution:

Concept:

- A terminal alkyne contains an acidic $-\mathrm{C}{\equiv}\mathrm{C}{-}\mathrm{H}$ hydrogen and gives an insoluble silver acetylide with the silver-ion reagent used in the terminal-alkyne test.
- Mercury(II)-catalysed acidic hydration of an alkyne produces an enol which tautomerises to a carbonyl compound. $\mathrm{NaBH_4}$ then reduces that carbonyl group to an alcohol.
- In the Lucas test, $\mathrm{ZnCl_2}$ and concentrated HCl convert a secondary alcohol to an insoluble chloride, producing turbidity within a few minutes.

Step 1: Apply the precipitate test The upper branch shows a precipitate with the silver reagent. This requires a terminal triple bond, so ethyne (A) and propyne (C) remain candidates. But-2-yne (B) has no terminal acetylenic hydrogen and cannot give the indicated acetylide precipitate. Ethene (D) is an alkene, so it also fails this test. The lower reaction sequence must now distinguish the two terminal alkynes.

Step 2: Track the carbonyl compound and the alcohol For propyne the sequence is

$$\mathrm{CH_3C{\equiv}CH} \xrightarrow{\mathrm{Hg^{2+}/H^+}} \mathrm{CH_3COCH_3} \xrightarrow{\mathrm{NaBH_4}} \mathrm{CH_3CH(OH)CH_3}.$$

Hydration gives propanone through its enol, and reduction gives propan-2-ol, a secondary alcohol. This alcohol gives the stated turbidity within five minutes with Lucas reagent. In contrast, ethyne hydrates to ethanal and reduction yields ethanol, a primary alcohol which does not give this prompt room-temperature turbidity. Thus ethyne satisfies the first branch but fails the second, whereas propyne satisfies both independent observations.

Step 3: Conclusion A is propyne, $\mathrm{CH_3C{\equiv}CH}$. Its hydration and reduction give secondary propan-2-ol. Hence, the correct option is (C).

Quick Tip to Solve: Use both branches of an identification scheme: the silver precipitate selects terminal alkynes, and the Lucas test identifies the class of alcohol formed.

73. Heating of 2-chloro-1-phenylbutane with EtOK/EtOH gives X as the major product. Reaction of X with $Hg(OAc)_2/H_2O$ followed by $NaBH_4$ gives Y as the major product. Y is : [4 marks]

(A) $PhCH_2CH(OH)CH_2CH_3$

(B) $PhCH_2CH_2CH(OH)CH_3$

(C) $PhCH(OH)CH_2CH_2CH_3$

(D) $PhCH_2CH_2CH_2CH_3$

Correct Answer: (C) $PhCH(OH)CH_2CH_2CH_3$

Solution:

Concept:

- Heating an alkyl chloride with ethanolic potassium ethoxide favours beta-elimination. An alkene conjugated with a phenyl ring is stabilised by delocalisation.
- Oxymercuration-demercuration adds water across an alkene with Markovnikov orientation and without the skeletal rearrangements associated with a freely formed carbocation.

Step 1: Find the favoured alkene Write the starting compound as $PhCH_2CH(Cl)CH_2CH_3$. Ethoxide can remove a beta-hydrogen from either neighbouring carbon while chloride leaves. Removal from the benzylic CH_2 group gives $X = PhCH{=}CHCH_2CH_3$. Removal on the other side gives $PhCH_2CH{=}CHCH_3$. Although both are disubstituted alkenes, the first is directly conjugated with the aromatic ring and is therefore the major elimination product. The phenyl group must be directly attached to a double-bond carbon for this stabilisation; an intervening saturated carbon would interrupt conjugation.

Step 2: Place the hydroxyl group at the benzylic carbon Treatment of the conjugated alkene with $Hg(OAc)_2/H_2O$, followed by $NaBH_4$, hydrates the double bond. Water attacks with the orientation that develops and stabilises positive character at the benzylic carbon. After demercuration,

$$PhCH{=}CHCH_2CH_3 \longrightarrow PhCH(OH)CH_2CH_2CH_3.$$

This is 1-phenylbutan-1-ol, the printed structure (C). Structure (A) instead puts OH one carbon away from the phenyl group, (B) puts it still farther along the chain, and (D) lacks oxygen entirely. None matches hydration of the major conjugated alkene with the required orientation.

Step 3: Conclusion $Y = PhCH(OH)CH_2CH_2CH_3$, 1-phenylbutan-1-ol. Hence, the correct option is (C).

Quick Tip to Solve: First choose the elimination product using conjugation, then apply hydration orientation to that alkene; reversing these decisions can place OH on the wrong carbon.

74. The pair that has similar atomic radii is : [4 marks]

(A) Mo and W

(B) Mn and Fe

(C) Ti and Hf

(D) Sc and Ni

Correct Answer: (A) Mo and W

Solution:

Concept:

- Atomic radii generally increase down a group because an extra principal shell is added, but the intervening lanthanoids cause an additional contraction before the 5d series.
- The 4f electrons shield nuclear charge poorly. Across the lanthanoids the increasing effective nuclear attraction contracts the electron cloud, making corresponding 4d and 5d elements similar in size.

Step 1: Identify a corresponding 4d and 5d pair Molybdenum, Mo, is a 4d element and tungsten, W, is the corresponding 5d element in group 6. Before tungsten, the lanthanoid series has been filled. Its contraction largely compensates for the size increase that would normally accompany the additional electron shell. Consequently Mo and W have very similar atomic radii, commonly listed at approximately $139\,\mathrm{pm}$ in the standard transition-metal radius comparison. This is the characteristic consequence tested by the question.

Step 2: Compare the remaining pairs Mn and Fe are neighbouring 3d elements; any closeness in their radii comes from the small changes across a transition row, rather than the matched 4d/5d contraction relationship. Their standard tabulated atomic radii are not equal. Ti and Hf are group-4 elements separated by two periods: the nearly equal-radius partner of Hf is Zr, not Ti. Sc and Ni are widely separated within the 3d series, over which the radius decreases substantially. Thus the other listed pairs do not display the distinctive equality caused by lanthanoid contraction. Selecting the same group alone is insufficient: the relevant comparison is specifically between the second and third transition series.

Step 3: Conclusion Mo and W have similar atomic radii because of lanthanoid contraction. Hence, the correct option is (A).

Quick Tip to Solve: For almost equal transition-metal sizes, look for a corresponding 4d/5d pair, such as Zr/Hf or Mo/W, after the lanthanoids have intervened.

75. An 'Assertion' and a 'Reason' are given below. Choose the correct answer from the following options :

Assertion (A) : Vinyl halides do not undergo nucleophilic substitution easily.

Reason (R) : Even though the intermediate carbocation is stabilized by loosely held p-electrons, the cleavage is difficult because of strong bonding. [4 marks]

(A) Both (A) and (R) are correct statements but (R) is not the correct explanation of (A)

(B) Both (A) and (R) are correct statements and (R) is the correct explanation of (A)

(C) (A) is correct statement but (R) is a wrong statement

(D) Both (A) and (R) are wrong statements.

Correct Answer: (C) (A) is correct statement but (R) is a wrong statement

Solution:

Concept:

- A vinyl halide has the halogen bonded directly to an sp^2-hybridised alkene carbon, rather than to an allylic saturated carbon.
- Overlap of a halogen lone pair with the double-bond system gives the carbon-halogen bond partial double-bond character. This bond is shorter and stronger than a normal alkyl carbon-halogen bond.
- A vinylic carbocation is unstable; the stabilisation familiar for allylic or benzylic carbocations must not be assigned to a positive charge directly on a vinyl carbon.

Step 1: Evaluate the assertion The assertion is correct. Ordinary vinyl halides resist both usual nucleophilic substitution routes. Ionisation for an S_N1 reaction would break the strong carbon-halogen bond and produce a highly unstable vinylic carbocation. The normal backside displacement geometry for an S_N2 reaction is also unfavourable at the double-bond carbon. Therefore the statement that vinyl halides do not undergo nucleophilic substitution easily agrees with their bonding and reaction behaviour.

Step 2: Evaluate the reason independently The reason correctly refers to difficult cleavage of a strong bond, but it additionally claims that the intermediate carbocation is stabilised by loosely held p-electrons. That claim is false for a vinyl halide. The carbocation produced by direct halide loss is vinylic, not an allylic carbocation whose empty p orbital can delocalise positive charge through an adjacent double bond. A reason must be correct as a whole; one true clause cannot rescue its false stabilisation claim.

Step 3: Match the two truth values to the options The truth values are assertion true and reason false. Thus (A) and (B), both of which call the reason correct, are excluded. Option (D) is excluded because the assertion is true. Only (C) expresses the independently established truth values.

Step 4: Conclusion The assertion is correct, while the reason is wrong because it falsely assigns stabilisation to the vinylic carbocation. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish vinyl, $CH_2{=}CH{-}X$, from allyl, $CH_2{=}CH{-}CH_2{-}X$; the extra saturated carbon changes carbocation stabilisation and substitution behaviour.

76. What will be the major product when m-cresol is reacted with propargyl bromide ($HC{\equiv}C{-}CH_2Br$) in presence of K_2CO_3 in acetone? [4 marks]

(A) $3\text{-}CH_3{-}C_6H_4{-}O{-}CH_2{-}C{\equiv}CH$ (the propargyl ether of m-cresol)

(B) 2-(Prop-2-yn-1-yl)-3-methylphenol

(C) 2-(Prop-2-yn-1-yl)-5-methylphenol

(D) 4-(Prop-2-yn-1-yl)-3-methylphenol

Correct Answer: (A) $3\text{-}CH_3{-}C_6H_4{-}O{-}CH_2{-}C{\equiv}CH$ (the propargyl ether of m-cresol)

Solution:

Concept:

- Phenols are acidic enough to form a useful concentration of phenoxide in the presence of potassium carbonate. The phenoxide oxygen is an effective nucleophile.
- A primary propargyl bromide undergoes nucleophilic displacement at its CH_2Br carbon. Acetone is a polar aprotic solvent that favours this substitution.
- Williamson ether synthesis joins the phenolic oxygen to the alkyl group; the carbon skeleton of the aromatic ring remains unchanged.

Step 1: Generate the nucleophile from m-cresol The starting phenol is m-cresol, $3\text{-}CH_3{-}C_6H_4{-}OH$. Potassium carbonate removes the phenolic hydrogen sufficiently for the resulting cresoxide ion to react with the alkyl bromide. The methyl substituent stays meta to the oxygen-bearing ring carbon. The nucleophilic atom in the cresoxide ion is oxygen, so the reaction forms an oxygen-carbon bond rather than replacing a hydrogen on the aromatic ring.

Step 2: Displace bromide from the propargyl group The primary propargylic carbon bears bromide in $HC{\equiv}C{-}CH_2Br$. Cresoxide attacks that carbon and bromide leaves in an S_N2 step:

$$3\text{-}CH_3{-}C_6H_4{-}O^- + BrCH_2C{\equiv}CH \longrightarrow 3\text{-}CH_3{-}C_6H_4{-}OCH_2C{\equiv}CH + Br^-.$$

The terminal triple bond remains intact. The product is the propargyl ether of m-cresol, matching structure (A). Options (B), (C) and (D) retain a free phenolic OH and place the propargyl substituent directly on a ring carbon at different positions. Those are carbon-alkylated phenols, whereas the stated carbonate/acetone conditions give ordinary oxygen alkylation. No heating or rearrangement conditions are stated that would justify replacing this ether product with a ring-alkylated isomer.

Step 3: Conclusion The major product is $3\text{-}CH_3{-}C_6H_4{-}OCH_2C{\equiv}CH$, the propargyl ether of m-cresol. Hence, the correct option is (A).

Quick Tip to Solve: With phenol, carbonate and a primary alkyl bromide in acetone, connect the alkyl group to oxygen first; do not move it onto the aromatic ring.

77. The coordination numbers of Co and Al in $[Co(Cl)(en)_2]Cl$ and $K_3[Al(C_2O_4)_3]$, respectively, are : (en = ethane-1, 2-diamine) [4 marks]

(A) 3 and 3

(B) 6 and 6

(C) 5 and 3

(D) 5 and 6

Correct Answer: (D) 5 and 6

Solution:

Concept:

- The coordination number counts donor atoms directly bonded to the central metal. It is not necessarily the number of ligand molecules or the oxidation state.
- Ethane-1,2-diamine, en, is bidentate and donates through two nitrogen atoms. Oxalate, $C_2O_4{}^{2-}$, is also bidentate and normally donates through two oxygen atoms.
- Only species inside the coordination brackets contribute to the metal coordination number. Ions outside the brackets balance charge.

Step 1: Count donors attached to cobalt The cobalt complex is $[Co(Cl)(en)_2]Cl$. Inside the brackets, the single chlorido ligand supplies one donor atom, and each of the two en ligands supplies two nitrogen donors. Therefore

$$CN(Co) = 1 + 2(2) = 5.$$

The chloride written outside the square brackets is a counter-ion, rather than a ligand bonded directly to Co. Counting it would incorrectly produce six. Merely counting the three ligand units inside the brackets would also be wrong, because two of those units each attach at two sites.

Step 2: Count donors attached to aluminium In $K_3[Al(C_2O_4)_3]$, all three oxalate ions are in the coordination sphere. Each attaches through two oxygen donors, giving

$$CN(Al) = 3(2) = 6.$$

The three potassium ions are counter-ions and do not coordinate to the aluminium centre in this formula. Thus the answer is ordered as cobalt five, aluminium six. Option (A) confuses the count of ligand units with donor atoms in both complexes, (B) overcounts cobalt, and (C) treats oxalate as monodentate. The requested values describe connectivity and require no inference about the metals having the same coordination number.

Step 3: Conclusion The coordination numbers of Co and Al are 5 and 6, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Write each ligand count multiplied by its denticity, add only inside the brackets, and ignore the counter-ions outside them.

78. The molar solubility of $Cd(OH)_2$ is 1.84×10^{-5} M in water. The expected solubility of $Cd(OH)_2$ in a buffer solution of pH = 12 is : [4 marks]

(A) 2.49×10^{-10} M

(B) 1.84×10^{-9} M

(C) 6.23×10^{-11} M

(D) $\frac{2.49}{1.84} \times 10^{-9} M$

Correct Answer: (A) 2.49×10^{-10} M

Solution:

Concept:

- The solubility product is the equilibrium constant for dissolution of a sparingly soluble salt. For $\mathrm{Cd(OH)_2(s)} \rightleftharpoons \mathrm{Cd^{2+}(aq)} + 2\,\mathrm{OH^-(aq)}$, $K_{\mathrm{sp}} = [\mathrm{Cd^{2+}}][\mathrm{OH^-}]^2$.
- In pure water a molar solubility s supplies s molar cadmium ions and $2s$ molar hydroxide ions; a buffer instead maintains its own hydroxide concentration. Use $\mathrm{pH} + \mathrm{pOH} = 14$ at the usual temperature.

Step 1: Calculate the solubility product Take $s = 1.84 \times 10^{-5}\,\mathrm{mol\,L^{-1}}$. The hydroxide produced by the salt greatly exceeds that from water, so the stoichiometric concentrations can be used. Thus

$$K_{\mathrm{sp}} = s(2s)^2 = 4s^3 = 4(1.84 \times 10^{-5})^3 = 2.4918 \times 10^{-14}.$$

The factor four is essential: each dissolved formula unit produces two hydroxide ions, whose concentration is squared.

Step 2: Apply the buffer concentration At $\mathrm{pH} = 12$,

$$\mathrm{pOH} = 2, \qquad [\mathrm{OH^-}] = 10^{-2}\,\mathrm{mol\,L^{-1}}.$$

Let the new solubility be s_b. Since the buffer fixes the hydroxide concentration,

$$s_b = \frac{K_{\mathrm{sp}}}{[\mathrm{OH^-}]^2} = \frac{2.4918 \times 10^{-14}}{10^{-4}} = 2.4918 \times 10^{-10}\,\mathrm{mol\,L^{-1}}.$$

The very small added hydroxide does not appreciably change the buffer. Options B and D are too large; C misses the stoichiometric factor four.

Step 3: Conclusion The expected solubility is 2.49×10^{-10} M. Hence, the correct option is (A).

Quick Tip to Solve: First obtain K_{sp} from pure-water solubility, then replace $2s$ by the hydroxide concentration fixed by the buffer.

79. NO_2 required for a reaction is produced by the decomposition of N_2O_5 in CCl_4 as per the equation, $2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$.

The initial concentration of N_2O_5 is 3.00 mol L^{-1} and it is 2.75 mol L^{-1} after 30 minutes. The rate of formation of NO_2 is : [4 marks]

(A) 2.083×10^{-3} mol L^{-1} min^{-1}

(B) 8.333×10^{-3} mol L^{-1} min^{-1}

(C) 4.167×10^{-3} mol L^{-1} min^{-1}

(D) 1.667×10^{-2} mol L^{-1} min^{-1}

Correct Answer: (D) 1.667×10^{-2} mol L^{-1} min^{-1}

Solution:

Concept:

- For a reaction $aA \rightarrow bB$, the positive disappearance rate of A and formation rate of B obey $-\frac{1}{a}\frac{\Delta[A]}{\Delta t} = \frac{1}{b}\frac{\Delta[B]}{\Delta t}$.
- The concentration change over a finite time interval gives an average rate. Stoichiometric coefficients determine the relative concentration changes, independently of the rate law.

Step 1: Find the average disappearance rate The concentration of N_2O_5 decreases from 3.00 to $2.75\,\text{mol}\,\text{L}^{-1}$ in $30\,\text{min}$. Therefore

$$-\frac{\Delta[N_2O_5]}{\Delta t} = \frac{3.00 - 2.75}{30} = 8.333 \times 10^{-3}\,\text{mol}\,\text{L}^{-1}\,\text{min}^{-1}.$$

The negative concentration change is converted to a positive disappearance rate. No first-order rate constant or logarithm is needed because the requested rate is obtained directly from the measured interval.

Step 2: Convert to the formation rate of nitrogen dioxide The balanced reaction is $2\,N_2O_5 \longrightarrow 4\,NO_2 + O_2$. Two moles of reactant yield four moles of nitrogen dioxide, so

$$\frac{\Delta[NO_2]}{\Delta t} = \frac{4}{2}\left(-\frac{\Delta[N_2O_5]}{\Delta t}\right) = 2(8.333 \times 10^{-3}) = 1.667 \times 10^{-2}\,\text{mol}\,\text{L}^{-1}\,\text{min}^{-1}.$$

Option B is only the disappearance rate of the reactant. Option C is the coefficient-normalized reaction rate; option A is smaller still. Neither is the requested product formation rate.

Step 3: Conclusion The average rate of formation of NO_2 is $1.667 \times 10^{-2}\,\text{mol}\,\text{L}^{-1}\,\text{min}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: Calculate the reactant disappearance rate first, then multiply by the product-to-reactant stoichiometric coefficient ratio, here $4/2$.

80. Thermal decomposition of a Mn compound (X) at 513 K results in compound Y, MnO_2 and gaseous product. MnO_2 reacts with NaCl and concentrated H_2O_4 to give a pungent gas Z. X, Y and Z, respectively, are : [4 marks]

(A) $KMnO_4$, K_2MnO_4 and Cl_2

(B) K_2MnO_4, $KMnO_4$ and SO_2

(C) K_3MnO_4, K_2MnO_4 and Cl_2

(D) K_2MnO_4, $KMnO_4$ and Cl_2

Correct Answer: (A) $KMnO_4$, K_2MnO_4 and Cl_2

Solution:

Concept:

- On heating, potassium permanganate forms potassium manganate, manganese dioxide and oxygen. The manganese oxidation states change from $+7$ to $+6$ and $+4$.
- Manganese dioxide oxidizes chloride to chlorine in an acidic medium. Chlorine is a pungent, greenish-yellow gas; oxygen from the thermal step is colourless and odourless.

Step 1: Identify the thermal decomposition products The characteristic decomposition at the given temperature is

$$2\,KMnO_4 \xrightarrow{\Delta} K_2MnO_4 + MnO_2 + O_2.$$

This equation is balanced in potassium, manganese and oxygen: two manganese atoms appear as one manganate unit and one dioxide unit. Consequently $X = KMnO_4$, $Y = K_2MnO_4$, and the gaseous thermal product is O_2. Among the options only A has this ordered pair. Options B and D reverse the permanganate and manganate roles, while C specifies a manganese(V) salt rather than potassium permanganate.

Step 2: Identify the gas released from chloride The chloride oxidation can be written without spectator ions as

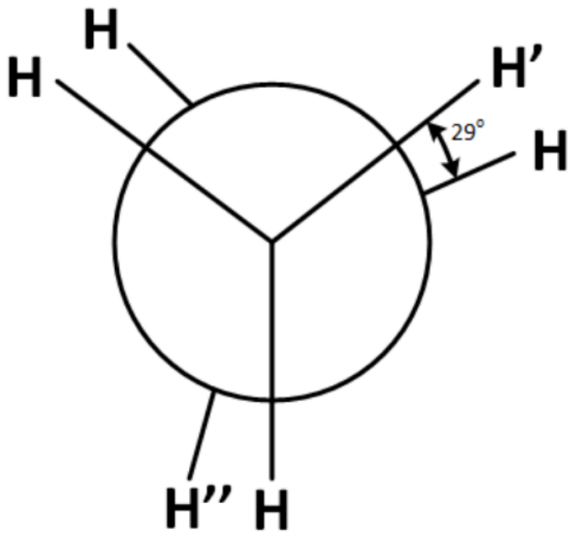
$$MnO_2 + 4\,H^+ + 2\,Cl^- \longrightarrow Mn^{2+} + Cl_2 + 2\,H_2O.$$

Manganese is reduced from $+4$ to $+2$, accepting the two electrons released as two chloride ions form one chlorine molecule. Thus the pungent gas Z is Cl_2, not SO_2. The printed acid formula H_2O_4 lacks sulfur; the familiar laboratory preparation uses concentrated sulfuric acid to acidify sodium chloride. The independently decisive thermal step and the chloride-oxidation identification both select the same option.

Step 3: Conclusion $X = KMnO_4$, $Y = K_2MnO_4$, and $Z = Cl_2$. Hence, the correct option is (A).

Quick Tip to Solve: Recognize the three-product decomposition of permanganate first; then distinguish odourless oxygen from pungent chlorine in the second reaction.

81. In the following skew conformation of ethane, H'–C–C–H" dihedral angle is : [4 marks]



(A) 120°

(B) 58°

(C) 151°

(D) 149°

Correct Answer: (D) 149°

Solution:

Concept:

- A dihedral angle $\mathrm{H-C-C-H}$ is the angle between the two planes containing the selected C–H bonds and the central C–C bond. In a Newman projection it is read between the corresponding radial bond directions.
- The three C–H bonds attached to either one carbon are separated by 120° when viewed along the carbon–carbon axis. Front-carbon bonds start at the centre; rear-carbon bonds start at the circle.

Step 1: Distinguish the front and rear bonds In the projection, the bond to $\mathrm{H'}$ starts at the centre and points to the upper right. The nearby unprimed rear hydrogen is attached at the circle, and its direction is separated from this front bond by the marked 29°. The selected $\mathrm{H''}$ belongs to the rear carbon and points downwards to the left. The angle asked is between the primed front bond and this double-primed rear bond, rather than between the nearby pair used to mark the skew rotation.

Step 2: Add the rear-carbon angular separation From the nearby rear C–H bond to the rear $\mathrm{C-H''}$ bond, the rotation is 120°. The marked 29° lies on the same rotational path from $\mathrm{H'}$. Therefore the smaller selected angle is

$$\theta = 29^\circ + 120^\circ = 149^\circ.$$

Option A overlooks the skew displacement and uses only the separation of bonds on a single carbon. Option B doubles the marked angle even though the selected hydrogens are not that pair. Option C would require the complementary staggered displacement $60^\circ - 29^\circ = 31^\circ$ to be added to 120°, which does not follow the labelled bonds.

Step 3: Conclusion The $H' - C - C - H''$ dihedral angle is $149°$. Hence, the correct option is (D).

Quick Tip to Solve: Trace the two labelled radial bonds, distinguishing the centre from the circle; add the marked skew rotation only along the path that actually reaches the selected rear bond.

82. Which of the given statements is INCORRECT about glycogen? [4 marks]

(A) It is a straight chain polymer similar to amylose

(B) It is present in animal cells

(C) It is present in some yeast and fungi

(D) Only α-linkages are present in the molecule

Correct Answer: (A) It is a straight chain polymer similar to amylose

Solution:

Concept:

- Glycogen is a storage polysaccharide made of α-D-glucose units. Its chains contain $\alpha(1 \rightarrow 4)$ glycosidic bonds and its branch points contain $\alpha(1 \rightarrow 6)$ bonds.
- Amylose is predominantly unbranched, whereas glycogen is highly branched. A negative question asks for the false statement, so every remaining statement must be checked for consistency.

Step 1: Compare the chain structures The glucose units in a glycogen chain are linked through carbon 1 of one unit and carbon 4 of the next. Additional chains branch through carbon 6, giving a compact, highly branched macromolecule with many non-reducing ends. Amylose, in contrast, is an essentially straight-chain polymer with $\alpha(1 \rightarrow 4)$ linkages. Therefore describing glycogen as a straight-chain polymer similar to amylose is incorrect. Glycogen is structurally closer to the branched component of starch, amylopectin, although it is even more extensively branched.

Step 2: Check the occurrence and linkage statements Option B is correct because glycogen stores glucose in animal cells, especially liver and muscle cells. Option C is also correct: glycogen occurs as a reserve carbohydrate in some yeast and fungi, so its occurrence is not restricted to animals. Option D is correct because both the main-chain $1 \rightarrow 4$ linkages and the branch-point $1 \rightarrow 6$ linkages have the α configuration. Branching does not require a β linkage; linkage position and anomeric configuration describe different features. Thus B, C and D are true statements and cannot answer the request for the incorrect one.

Step 3: Conclusion The incorrect statement is that glycogen is a straight-chain polymer similar to amylose. Hence, the correct option is (A).

Quick Tip to Solve: Keep branching separate from α/β configuration: glycogen branches through $\alpha(1 \rightarrow 6)$ bonds while retaining $\alpha(1 \rightarrow 4)$ main chains.

83. The temporary hardness of a water sample is due to compound X. Boiling this sample converts X to compound Y. X and Y, respectively, are : [4 marks]

(A) $Mg(HCO_3)_2$ and $MgCO_3$

(B) $Ca(HO_3)_2$ and CaO

(C) $Ca(HCO_3)_2$ and $Ca(OH)_2$

(D) $Mg(HCO_3)_2$ and $Mg(OH)_2$

Correct Answer: (D) $Mg(HCO_3)_2$ and $Mg(OH)_2$

Solution:

Concept:

- Temporary hardness results from dissolved hydrogen carbonates of calcium or magnesium. Boiling removes it by converting soluble salts into insoluble precipitates.
- Calcium hydrogen carbonate yields calcium carbonate, whereas prolonged boiling of magnesium hydrogen carbonate gives magnesium hydroxide. Escaping carbon dioxide helps drive the transformations.

Step 1: Identify the magnesium precipitate on boiling The relevant overall change is

$$\mathrm{Mg(HCO_3)_2(aq)} \xrightarrow{\text{boiling}} \mathrm{Mg(OH)_2(s) + 2\,CO_2(g)}.$$

It is balanced: each side contains one magnesium, two carbon, two hydrogen and six oxygen atoms. The precipitated hydroxide removes dissolved magnesium ions from the water. A carbonate intermediate can be considered, but its hydrolysis during boiling gives the hydroxide:

$$\mathrm{MgCO_3 + H_2O \longrightarrow Mg(OH)_2 + CO_2}.$$

Thus the final pair required is magnesium hydrogen carbonate followed by magnesium hydroxide.

Step 2: Eliminate the remaining proposed pairs Option A stops at magnesium carbonate rather than the hydroxide obtained on boiling in water. For calcium, the standard reaction is

$$\mathrm{Ca(HCO_3)_2} \xrightarrow{\text{boiling}} \mathrm{CaCO_3(s) + CO_2 + H_2O}.$$

This produces neither calcium oxide nor calcium hydroxide under the stated water-boiling conditions. Therefore B and C do not give the correct calcium transformation; B also does not display the usual hydrogen carbonate formula. Calcium carbonate requires much stronger heating in the absence of bulk water to form the oxide. Only D correctly combines the soluble hardness-causing salt with the insoluble final magnesium product.

Step 3: Conclusion $X = \mathrm{Mg(HCO_3)_2}$ and $Y = \mathrm{Mg(OH)_2}$. Hence, the correct option is (D).

Quick Tip to Solve: For boiling away temporary hardness, remember the different final precipitates: calcium gives carbonate, while magnesium gives hydroxide.

84. The correct statement is : [4 marks]

(A) leaching of bauxite using concentrated NaOH solution gives sodium aluminate and sodium silicate

(B) the Hall-Heroult process is used for the production of aluminium and iron

(C) pig iron is obtained from cast iron

(D) the blistered appearance of copper during the metallurgical process is due to the evolution of CO_2

Correct Answer: (A) leaching of bauxite using concentrated NaOH solution gives sodium aluminate and sodium silicate

Solution:

Concept:

- Bayer-process leaching exploits the amphoteric character of alumina: concentrated aqueous sodium hydroxide dissolves alumina as aluminate. Silica impurity can also dissolve to form sodium silicate.
- The Hall–Heroult process electrolytically reduces alumina dissolved in molten cryolite. Metallurgical process names, the sequence of iron products and the escaping gas in copper refining must be matched separately.

Step 1: Verify the products of bauxite leaching Concentrated NaOH reacts with the alumina component of bauxite and with silica impurity. The familiar formula equations are

$$\mathrm{Al_2O_3 + 2\,NaOH \longrightarrow 2\,NaAlO_2 + H_2O},$$

$$\mathrm{SiO_2 + 2\,NaOH \longrightarrow Na_2SiO_3 + H_2O}.$$

In aqueous solution the aluminate can be represented as hydrated hydroxoaluminate; the sodium aluminate description in the option is the conventional metallurgical form. Insoluble iron oxide remains in the residue. Therefore formation of both sodium aluminate and sodium silicate during alkaline leaching is correctly stated in A.

Step 2: Check why the other statements fail Option B incorrectly includes iron: Hall–Heroult electrolysis produces aluminium, whereas iron is commonly obtained by reduction of its oxides in a blast furnace. Option C reverses the sequence. Pig iron is the initial high-carbon blast-furnace product; cast iron is prepared by remelting pig iron with suitable materials, not the other way round. In copper metallurgy, the reaction

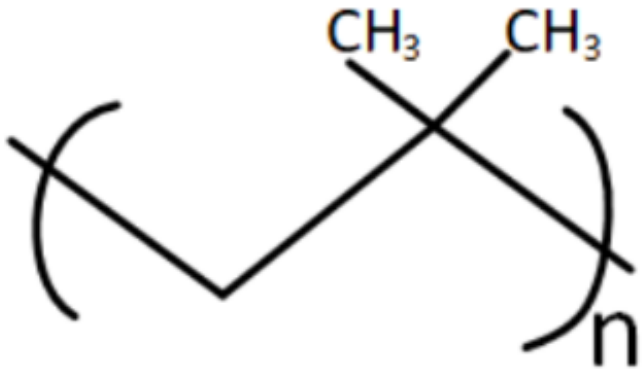
$$\mathrm{Cu_2S + 2\,Cu_2O \longrightarrow 6\,Cu + SO_2}$$

releases sulfur dioxide. Its escaping bubbles cause the blistered appearance, so option D incorrectly names carbon dioxide. Each of B, C and D therefore contains a specific factual error.

Step 3: Conclusion Alkaline leaching of bauxite can produce sodium aluminate and sodium silicate. Hence, the correct option is (A).

Quick Tip to Solve: Check each metallurgy option by pairing one process with its product or escaping gas: Hall–Heroult/aluminium, blast furnace/pig iron, blister copper/sulfur dioxide.

85. The correct name of the following polymer is : [4 marks]



(A) Polytert-butylene

(B) Polyisobutane

(C) Polyisobutylene

(D) Polyisoprene

Correct Answer: (C) Polyisobutylene

Solution:

Concept:

- An addition polymer is identified by its repeating unit. Replacing a backbone single bond between the two repeat-unit carbons by a double bond reconstructs the corresponding alkene monomer.
- Isobutylene is the common name of 2-methylpropene, $CH_2{=}C(CH_3)_2$. Addition across its double bond gives a saturated carbon backbone while retaining the two methyl substituents on the same carbon.

Step 1: Read the substituents in the repeating unit The displayed backbone contains two carbon atoms per repeating unit: one is a methylene group, and the other carries two methyl groups. Thus the repeat has the connectivity $-CH_2-C(CH_3)_2-$. Both methyl groups belong to the same backbone carbon; they are not part of the main chain. The brackets and the subscript n show that this two-carbon sequence repeats throughout the macromolecule.

Step 2: Reconstruct the monomer and compare the names Form a double bond between those two backbone carbons without moving either methyl group. The monomer is therefore $CH_2{=}C(CH_3)_2$, namely isobutylene. Its polymer is polyisobutylene. Polyisobutane incorrectly names a saturated alkane as the addition monomer, while polytert-butylene is not the name corresponding to the alkene obtained here. Polyisoprene comes from the conjugated diene isoprene and has a different repeat-unit carbon skeleton; it does not match the two methyl groups on this backbone carbon.

Step 3: Conclusion The given repeat unit is obtained by addition polymerisation of isobutylene (2-methylpropene), so the polymer is polyisobutylene. Hence, the correct option is (C).

Quick Tip to Solve: To name an addition polymer, recover the alkene from one repeat unit first; keep every side group attached to its original carbon.

86. The INCORRECT statement is : [4 marks]

(A) Lithium is least reactive with water among the alkali metals

(B) LiCl crystallises from aqueous solution as $LiCl.2H_2O$

(C) Lithium is the strongest reducing agent among the alkali metals

(D) $LiNO_3$ decomposes on heating to give $LiNO_2$ and O_2

Correct Answer: (D) $LiNO_3$ decomposes on heating to give $LiNO_2$ and O_2

Solution:

Concept:

- Lithium differs from the other alkali metals because its small cation has a high charge density. Several of its salts show behaviour closer to the corresponding magnesium salts.
- On heating, lithium nitrate forms lithium oxide, nitrogen dioxide and oxygen. Most other alkali-metal nitrates instead form the corresponding nitrite and oxygen.

Step 1: Check the three true statements Lithium reacts least vigorously with water among the alkali metals, so statement (A) is correct. This kinetic trend should not be confused with reducing strength in aqueous solution. The exceptionally large hydration enthalpy of Li^+ makes lithium the strongest aqueous reducing agent among these metals, so (C) is also correct. Lithium chloride can crystallise from aqueous solution as the hydrate $LiCl \cdot 2\,H_2O$; the small lithium ion is strongly hydrated, consistent with statement (B).

Step 2: Write the actual thermal decomposition The balanced equation for heating lithium nitrate is

$$4\,LiNO_3 \xrightarrow{\Delta} 2\,Li_2O + 4\,NO_2 + O_2.$$

The lithium, nitrogen and oxygen atom counts are respectively 4, 4 and 12 on both sides. Thus the solid product is lithium oxide, not lithium nitrite. The nitrite pathway quoted in (D) is typical of other alkali-metal nitrates, for example $2\,NaNO_3 \xrightarrow{\Delta} 2\,NaNO_2 + O_2$, and cannot be transferred to lithium nitrate. Since the question asks for the incorrect statement, select this exception rather than one of the three true observations.

Step 3: Conclusion Lithium nitrate decomposes to Li_2O, NO_2 and O_2, not to $LiNO_2$ and O_2. Hence, the correct option is (D).

Quick Tip to Solve: Remember lithium nitrate with magnesium nitrate: heating gives the oxide and nitrogen dioxide, rather than stopping at the nitrite.

87. The primary pollutant that leads to photochemical smog is : [4 marks]

(A) acrolein

(B) ozone

(C) sulphur dioxide

(D) nitrogen oxides

Correct Answer: (D) nitrogen oxides

Solution:

Concept:

- A primary pollutant is released directly into the atmosphere. A secondary pollutant is produced in air by subsequent chemical or photochemical reactions.
- Photochemical smog forms when nitrogen oxides and hydrocarbons react in sunlight. Nitrogen dioxide photolysis provides oxygen atoms that lead to ozone formation.

Step 1: Identify the directly emitted precursor High-temperature combustion in vehicle engines produces nitrogen oxides, collectively denoted NO_x. Nitrogen monoxide is oxidised to nitrogen dioxide in air. In sunlight, nitrogen dioxide undergoes photolysis and initiates the chemistry responsible for oxidising smog. The essential sequence may be represented as

$$NO_2 \xrightarrow{h\nu} NO + O, \qquad O + O_2 \longrightarrow O_3.$$

Hydrocarbons and their radicals sustain ozone accumulation by participating in additional reactions, so the smog is a photochemical mixture rather than a single emitted gas.

Step 2: Distinguish the remaining options from the precursor Ozone is an important oxidising component of photochemical smog, but it is formed in the atmosphere and is therefore a secondary pollutant, not the requested primary one. Acrolein is also associated with the irritating secondary products of hydrocarbon oxidation. Sulphur dioxide is a primary pollutant, but it is mainly associated with the reducing, sulphurous smog found in cool humid conditions; it is not the relevant precursor in this comparison. Nitrogen oxides satisfy both conditions: direct emission and a central role in producing photochemical smog.

Step 3: Conclusion The relevant primary pollutants are nitrogen oxides, which initiate the sunlight-driven formation of photochemical smog. Hence, the correct option is (D).

Quick Tip to Solve: When a smog question says primary, look for an emitted precursor such as nitrogen oxides; ozone is a secondary product.

88. The ratio of number of atoms present in a simple cubic, body centered cubic and face centered cubic structure are, respectively : [4 marks]

(A) 8 : 1 : 6

(B) 4 : 2 : 1

(C) 1 : 2 : 4

(D) 4 : 2 : 3

Correct Answer: (C) 1 : 2 : 4

Solution:

Concept:

- The effective number of atoms in a unit cell is obtained by counting each boundary atom only by the fraction that belongs to that cell.
- A corner atom is shared by eight unit cells and contributes $1/8$. A face-centred atom is shared by two cells and contributes $1/2$, whereas a body-centred atom belongs entirely to one cell.

Step 1: Count the simple cubic and body-centred cubic cells A simple cubic cell has atoms only at its eight corners. Therefore its effective population is

$$Z_{\text{sc}} = 8\left(\frac{1}{8}\right) = 1.$$

A body-centred cubic cell has the same eight corners together with one atom wholly inside its body. The interior atom is not shared with a neighbouring cell, so

$$Z_{\text{bcc}} = 8\left(\frac{1}{8}\right) + 1 = 2.$$

Counting the visible corners as eight complete atoms would overcount the contribution and give a misleading ratio.

Step 2: Count the face-centred cubic cell and form the ratio A face-centred cubic cell has eight corner atoms and one atom at the centre of each of its six faces. Consequently,

$$Z_{\text{fcc}} = 8\left(\frac{1}{8}\right) + 6\left(\frac{1}{2}\right) = 1 + 3 = 4.$$

The required sequence is explicitly simple cubic, body-centred cubic, then face-centred cubic. Keeping that order, the ratio is $1 : 2 : 4$. Option (B) reverses this sequence, and options (A) and (D) do not give the effective populations after boundary sharing. The ratio concerns atoms per unit cell, rather than coordination numbers or the numbers of atoms merely visible on the boundary.

Step 3: Conclusion The effective atom counts are 1, 2 and 4, respectively, giving $1 : 2 : 4$. Hence, the correct option is (C).

Quick Tip to Solve: Use the contributions corner $= 1/8$, face $= 1/2$, body $= 1$; multiply by the number of each site before comparing cells.

89. The C–C bond length is maximum in : [4 marks]

(A) C_{60}

(B) diamond

(C) graphite

(D) C_{70}

Correct Answer: (B) diamond

Solution:

Concept:

- Carbon-carbon bond length depends on bonding and hybridisation. A conventional sp^3-sp^3 single bond is longer than the bonds with appreciable sp^2 or multiple-bond character.
- Diamond contains a three-dimensional tetrahedral network of sp^3-hybridised carbon atoms. Graphite and the fullerenes have carbon frameworks based mainly on sp^2 bonding.

Step 1: Compare diamond with graphite In diamond, each carbon forms four sigma bonds to four tetrahedrally arranged neighbours. The nearest-neighbour carbon-carbon distance is approximately $154\,\text{pm}$, characteristic of a carbon-carbon single bond. Within a graphite layer, carbon is sp^2-hybridised and the delocalised bonding gives a shorter carbon-carbon distance, approximately $142\,\text{pm}$. The much larger separation between graphite layers is not a carbon-carbon bond length and must not be used in this comparison.

Step 2: Compare the fullerene bonds and select the maximum The curved cages of C_{60} and C_{70} contain inequivalent carbon-carbon bonds, so they should not be treated as having one perfectly uniform bond length. Nevertheless, their bonded carbon neighbours are linked through the largely sp^2 cage framework, with the usual fullerene bond distances shorter than the $154\,\text{pm}$ diamond distance. For example, C_{60} has bonds of roughly 140 and $145\,\text{pm}$. Thus neither fullerene in (A) or (D), nor graphite in (C), exceeds the single-bond distance in diamond. Compare bonded neighbours throughout, not cage diameters or lattice spacings.

Step 3: Conclusion Among the listed allotropes, diamond has the maximum carbon-carbon bond length, approximately $154\,\text{pm}$. Hence, the correct option is (B).

Quick Tip to Solve: For carbon allotropes, compare nearest bonded neighbours: diamond has sp^3 single bonds; graphite layer spacing is not a bond length.

90. Among the following, the INCORRECT statement about colloids is : [4 marks]

(A) The range of diameters of colloidal particles is between 1 and to 1000 nm

(B) they are larger than small molecules and have high molar mass

(C) They can scatter light

(D) The osmotic pressure of a colloidal solution is of higher order than the true solution at the same concentration.

Correct Answer: (D) The osmotic pressure of a colloidal solution is of higher order than the true solution at the same concentration.

Solution:

Concept:

- Colloidal particles have characteristic sizes approximately 1 to $1000\,\text{nm}$. Their relatively large dimensions let them scatter a beam of light, giving the Tyndall effect.
- Osmotic pressure is a colligative property. For a dilute dispersion, $\pi = cRT/M$, where c is solute mass per unit volume, M is molar mass and T is absolute temperature.

Step 1: Check the size and optical statements The conventional colloidal size range is 1–$1000\,\text{nm}$, so statement (A), despite its extra word, gives the correct range. Colloidal particles are larger than ordinary small dissolved molecules; macromolecular colloids or aggregates have large molar masses, agreeing with (B). Colloids can scatter visible light and show the Tyndall effect, so (C) is also correct. These observations concern particle size and optical behaviour, and neither implies that the number of independently dispersed particles is especially large.

Step 2: Use particle number to compare osmotic pressures At the same mass concentration and temperature, osmotic pressure varies inversely with molar mass:

$$\pi = \frac{cRT}{M}.$$

A colloidal particle contains many atoms or molecules and therefore has a very large effective molar mass. A fixed mass of colloidal solute supplies far fewer independently moving particles than the same mass of an ordinary low-molar-mass solute. Its osmotic pressure is correspondingly much smaller, not of a higher order. This small pressure is measurable and is useful for determining macromolecular molar masses. If number concentration were fixed instead, the ideal osmotic pressures would be equal; neither comparison supports the asserted larger pressure.

Step 3: Conclusion The claim of a higher osmotic pressure is incorrect: at the same mass concentration, a colloidal dispersion generally has a much smaller osmotic pressure than an ordinary true solution. Hence, the correct option is (D).

Quick Tip to Solve: For osmotic-pressure comparisons, ask how many particles a fixed solute mass provides: larger molar mass means fewer particles and lower pressure.