JEE Main 2019 Paper 1 (B.E./B.Tech) Apr 12 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 12 April 2019, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. If $\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x+\cos ecx} dx$ = m(π + n), then m.n is equal to [4 marks]

(A) - 1

(B) 1

(C) $-\frac{1}{2}$

(D) $\frac{1}{2}$

Correct Answer: (A) - 1

Solution:

Concept:

- Multiply numerator and denominator by $\sin x$ to express cotangent and cosecant in terms of cosine.
- The half-angle identity $1 + \cos x = 2\cos^2(x/2)$ converts the remaining integral into a standard secant-squared integral.

Step 1: Simplify the integrand Let the given integral be I. For $0 < x < \pi/2$, multiplying by $\sin x$ gives

$$\frac{\cot x}{\cot x + \csc x} = \frac{\cos x}{1 + \cos x} = 1 - \frac{1}{1 + \cos x}.$$

Although cotangent and cosecant are individually undefined at zero, their ratio has the finite limiting value $1/2$. Thus the endpoint causes no divergence, and the simplified continuous expression can be integrated on the full interval.

Step 2: Integrate and identify the constants Use the half-angle formula before integrating:

$$I = \int_0^{\pi/2} \left(1 - \frac{1}{2}\sec^2\frac{x}{2}\right) dx = \left[x - \tan\frac{x}{2}\right]_0^{\pi/2}.$$

At the upper limit, $\tan(\pi/4) = 1$; at the lower limit both terms vanish. Consequently,

$$I = \frac{\pi}{2} - 1 = \frac{1}{2}(\pi - 2).$$

Comparing coefficients in the stated form $m(\pi + n)$, we get $m = 1/2$ and $n = -2$. Their product is negative: the positive choices cannot match the constant term, and $-1/2$ would omit the factor 2 in n.

Step 3: Conclusion Therefore $mn = (1/2)(-2) = -1$. Hence, the correct option is (A).

Quick Tip to Solve: For a ratio containing cotangent and cosecant, multiply through by sine first; the half-angle identity then removes the denominator.

2. Consider the differential equation, $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$, If value of y is 1 when x = 1, then the value of x for which y = 2, is : [4 marks]

(A) $\frac{3}{2} - \frac{1}{\sqrt{e}}$

(B) $\frac{1}{2} + \frac{1}{\sqrt{e}}$

(C) $\frac{5}{2} + \frac{1}{\sqrt{e}}$

(D) $\frac{3}{2} - \sqrt{e}$

Correct Answer: (A) $\frac{3}{2} - \frac{1}{\sqrt{e}}$

Solution:

Concept:

- Treat x as a function of y. A linear equation $dx/dy + P(y)x = Q(y)$ has integrating factor $e^{\int P(y)dy}$.
- The initial condition determines the integration constant; it must be applied before substituting the requested value of y.

Step 1: Put the equation in linear form Divide the printed equation by y^2dy, valid near the initial value $y = 1$:

$$\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}.$$

The coefficient of x depends only on y, so this is linear in x, even though it is not linear when read as an equation for $y(x)$. Its integrating factor is

$$\text{IF} = \exp\left(\int y^{-2}dy\right) = e^{-1/y}.$$

Step 2: Integrate the product derivative Multiplication by the integrating factor gives

$$\frac{d}{dy}(xe^{-1/y}) = y^{-3}e^{-1/y}.$$

For the right-hand integral use $t = 1/y$; then $y^{-3}dy = -t\,dt$. Therefore

$$xe^{-1/y} = \int -te^{-t}dt = (t+1)e^{-t} + C.$$

Returning to y and dividing by the positive exponential yields $x = 1 + 1/y + Ce^{1/y}$. Differentiating this formula also verifies the linear equation.

Step 3: Apply the initial condition and evaluate At $(x, y) = (1, 1)$, the formula gives $1 = 2 + Ce$, hence $C = -e^{-1}$. The required solution is

$$x = 1 + \frac{1}{y} - e^{1/y-1}.$$

For $y = 2$, this becomes $x = 3/2 - e^{-1/2}$. The exponential has a negative half exponent, excluding option (D). Options (B) and (C) also have the wrong sign or constant term.

Step 4: Conclusion The required value is $x = \frac{3}{2} - \frac{1}{\sqrt{e}}$. Hence, the correct option is (A).

Quick Tip to Solve: When a differential equation is linear in x rather than y, use y as the independent variable and form its integrating factor accordingly.

3. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is : [4 marks]

(A) $\frac{13}{5}$

(B) $\frac{60}{13}$

(C) $\frac{120}{13}$

(D) $\frac{13}{2}$

Correct Answer: (C) $\frac{120}{13}$

Solution:

Concept:

- The angle between two intersecting circles is the angle between their tangents; their corresponding radii have the same right-angle condition.
- The line joining the centres bisects the common chord at right angles. The chord length is twice the altitude from an intersection point to that line.

Step 1: Find the distance between the centres Let the centres be O_1, O_2, and let P, Q be the common points. The tangents at P are perpendicular. Since each radius is perpendicular to its own tangent, $O_1P \perp O_2P$ as well. Thus the triangle O_1PO_2 is right angled at P, with perpendicular sides 5 cm and 12 cm. Pythagoras gives

$$O_1O_2 = \sqrt{5^2 + 12^2} = \sqrt{169} = 13 \text{ cm}.$$

The centre distance is the hypotenuse, not either radius.

Step 2: Use the area to obtain half the chord Let H be the intersection of PQ with O_1O_2. The common chord is the radical axis, so $PH \perp O_1O_2$ and $PH = HQ$. Compute the same triangle area using its two perpendicular radii, then its hypotenuse as base:

$$\frac{1}{2}(5)(12) = \frac{1}{2}(13)(PH).$$

Hence $PH = 60/13$ cm. The requested chord is $PQ = 2PH = 120/13$ cm. Option (B) is only the half-chord; (A) and (D) do not satisfy this area relation. The result is also less than the smaller diameter 10 cm, as a chord of the smaller circle must be.

Step 3: Conclusion The common chord has length $\frac{120}{13}$ cm. Hence, the correct option is (C).

Quick Tip to Solve: For orthogonal circles of radii r and R, use the right triangle of centres: the common chord is twice its altitude to the hypotenuse.

4. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is : [4 marks]

(A) $\frac{1}{10}$
(B) $\frac{3}{10}$
(C) $\frac{3}{20}$
(D) $\frac{1}{5}$

Correct Answer: (A) $\frac{1}{10}$

Solution:

Concept:

- For equally likely choices, probability equals the number of favourable unordered selections divided by all unordered selections.
- Equal chords of a circle subtend equal minor central angles. An inscribed equilateral triangle has its vertices equally spaced around the circle.

Step 1: Count all possible vertex selections Label the hexagon vertices A, B, C, D, E, F in cyclic order. Choosing three distinct vertices without regard to order gives

$$N = \binom{6}{3} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20.$$

All such three-element subsets are equally likely. We do not count permutations because choosing the same vertices in a different order produces the same triangle.

Step 2: Identify every equilateral triangle The six vertices are separated by central angles of 60°. For an equilateral triangle on the same circumcircle, the successive arcs between its three vertices must each subtend 120°. Each arc therefore contains exactly two hexagon edges. Starting from any vertex, we must choose every second vertex. This gives precisely the two distinct sets

$$\{A, C, E\}, \qquad \{B, D, F\}.$$

Choosing a different starting vertex within either set repeats that same triangle, so there are two favourable outcomes, not six. Adjacent vertices would create a 60° arc and hence unequal sides. Thus no further three-vertex sets qualify.

Step 3: Form the probability Dividing favourable choices by all choices gives

$$P = \frac{2}{20} = \frac{1}{10}.$$

The alternative fractions would require six, three or four favourable triangles respectively, none of which agrees with the complete cyclic-spacing count above.

Step 4: Conclusion The probability is $\frac{1}{10}$. Hence, the correct option is (A).

Quick Tip to Solve: For an equilateral triangle inscribed in a regular hexagon, take alternate vertices and count each resulting three-vertex set only once.

5. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to : [4 marks]

(A) - 320

(B) - 380

(C) - 460

(D) - 210

Correct Answer: (A) - 320

Solution:

Concept:

- The sum of the first n terms of an arithmetic progression with first term a and common difference d is $S_n = \frac{n}{2}[2a + (n-1)d]$.
- Two specified partial sums give two linear equations for a and d; use those same parameters for the requested sum.

Step 1: Translate both given sums into equations The condition $S_4 = 16$ gives

$$2(2a + 3d) = 16 \quad \Longrightarrow \quad 2a + 3d = 8.$$

Similarly, the negative value $S_6 = -48$ gives

$$3(2a + 5d) = -48 \quad \Longrightarrow \quad 2a + 5d = -16.$$

The prefactors 2 and 3 come from $n/2$, so they must be divided out before subtracting the equations. In particular the second right-hand side remains negative.

Step 2: Solve for the progression Subtract the first reduced equation from the second:

$$2d = -16 - 8 = -24 \quad \Longrightarrow \quad d = -12.$$

Substitution into $2a + 3d = 8$ gives $2a - 36 = 8$, hence $a = 22$. As a check, the fourth and sixth partial sums are $2(44 - 36) = 16$ and $3(44 - 60) = -48$, agreeing with both data. The decreasing progression can have a positive early partial sum and a negative later one.

Step 3: Calculate the tenth partial sum Use the sum formula with $n = 10$:

$$S_{10} = 5[2(22) + 9(-12)] = 5(44 - 108) = 5(-64) = -320.$$

The ninth multiple of the common difference is required inside the bracket; using ten multiples would be an off-by-one error. The other offered totals fail the parameters determined uniquely by both given sums.

Step 4: Conclusion Therefore $S_{10} = -320$. Hence, the correct option is (A).

Quick Tip to Solve: Divide each known AP sum by n/2 first; subtract the resulting equations to isolate the common difference with minimal algebra.

6. The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is equal to : [4 marks]

(A) $\pi - \sin^{-1}\left(\frac{63}{65}\right)$

(B) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$

(C) $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$

(D) $\pi - \cos^{-1}\left(\frac{33}{65}\right)$

Correct Answer: (B) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$

Solution:

Concept:

- For $\alpha = \sin^{-1}(12/13)$ and $\beta = \sin^{-1}(3/5)$, both angles are acute, so their cosines are positive.
- The sine subtraction formula is $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.
- The cosine subtraction formula is $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

Step 1: Compute the complementary sides Principal inverse sine takes values in $[-\pi/2, \pi/2]$. The positive inputs here give acute angles. Thus

$$\cos\alpha = \sqrt{1-(12/13)^2} = \frac{5}{13}, \qquad \cos\beta = \sqrt{1-(3/5)^2} = \frac{4}{5}.$$

Since $12/13 > 3/5$ and inverse sine increases, $\alpha > \beta$. Their difference $\delta = \alpha - \beta$ therefore lies in $(0, \pi/2)$; this range prevents choosing a supplementary angle.

Step 2: Find the cosine of the difference The cosine subtraction identity gives

$$\cos\delta = \frac{5}{13}\frac{4}{5} + \frac{12}{13}\frac{3}{5} = \frac{20+36}{65} = \frac{56}{65}.$$

Because δ is acute,

$$\delta = \cos^{-1}\frac{56}{65} = \frac{\pi}{2} - \sin^{-1}\frac{56}{65}.$$

For an independent check, the sine subtraction identity gives $\sin\delta = (48-15)/65 = 33/65$, consistent with $33^2 + 56^2 = 65^2$.

Step 3: Check the remaining expressions Option (A) is obtuse, whereas δ is acute. Option (C) equals $\sin^{-1}(9/65)$, whose sine is not $33/65$. Option (D) has cosine $-33/65$ and is also obtuse. Thus only the complement expression with $56/65$ has both the correct value and principal range.

Step 4: Conclusion The value is $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$. Hence, the correct option is (B).

Quick Tip to Solve: After applying a sine or cosine difference formula, establish the angle range before converting back to an inverse trigonometric expression.

7. If m is the minimum value of k for which the function f(x) = x$\sqrt{kx - x^2}$ is increasing in the interval [0,3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M) is equal to : [4 marks]

(A) $(5, 3\sqrt{6})$

(B) $(4, 3\sqrt{3})$

(C) $(4, 3\sqrt{2})$

(D) $(3, 3\sqrt{3})$

Correct Answer: (B) $(4, 3\sqrt{3})$

Solution:

Concept:

- A real-valued square root requires $kx - x^2 \geq 0$. On positive x this first requires $k \geq x$.
- A differentiable function increasing on an interval has nonnegative derivative there; a positive derivative in the interior also proves strict increase.

Step 1: Check the domain and differentiate For $f(x) = x\sqrt{kx - x^2}$ to be real throughout $[0, 3]$, we need $k \geq 3$. For $0 < x < 3$, where the square root is positive, the product and chain rules give

$$f'(x) = \sqrt{kx - x^2} + \frac{x(k - 2x)}{2\sqrt{kx - x^2}} = \frac{x(3k - 4x)}{2\sqrt{kx - x^2}}.$$

Its denominator and the factor x are positive in the interior, so only $3k - 4x$ controls its sign.

Step 2: Determine the smallest admissible k Nonnegative derivative for every x arbitrarily close to 3 requires $3k - 12 \geq 0$, hence $k \geq 4$. If $3 \leq k < 4$, then $3k/4 < 3$, and there are interior points $x > 3k/4$ where $f'(x) < 0$; those choices cannot make the function increasing on the full interval. For $k = 4$, $3k - 4x = 12 - 4x > 0$ on $(0, 3)$. Continuity at both endpoints then gives increase on the closed interval, so $m = 4$.

Step 3: Find the maximum at the endpoint With $k = m = 4$, increase implies the maximum occurs at $x = 3$:

$$M = f(3) = 3\sqrt{4 \cdot 3 - 3^2} = 3\sqrt{3}.$$

Option (A) uses a possible but nonminimal k; option (C) gives the wrong endpoint value. Option (D) uses k=3, which makes the derivative negative near the upper end. Both components of the ordered pair are therefore fixed uniquely.

Step 4: Conclusion The ordered pair is $(m, M) = (4, 3\sqrt{3})$. Hence, the correct option is (B).

Quick Tip to Solve: For a square-root product, check the real domain first, then simplify the derivative to a sign factor and test its worst endpoint.

8. The integral $\int \frac{2x^3-1}{x^4+x}dx$ is equal to :
(Here C is a constant of integration) [4 marks]

(A) $\log_e \frac{|x^3+1|}{x^2} + C$

(B) $\frac{1}{2}\log_e \frac{|x^3+1|}{x^2} + C$

(C) $\log_e \left|\frac{x^3+1}{x}\right| + C$

(D) $\frac{1}{2}\log_e \frac{(x^3+1)^2}{|x^3|} + C$

Correct Answer: (C) $\log_e \left|\frac{x^3+1}{x}\right| + C$

Solution:

Concept:

- For a differentiable nonzero function $f(x)$, the derivative of $\ln|f(x)|$ is $f\prime(x)/f(x)$. This identifies integrals containing a polynomial divided by a related polynomial.
- Factor the denominator before separating the fraction: $x^4 + x = x(x^3 + 1)$. The original integrand is defined for real $x \neq 0, -1$; an antiderivative is interpreted on any interval avoiding these points.

Step 1: Separate the rational expression Seek constants multiplying the logarithmic derivatives of $x^3 + 1$ and x. We have

$$\frac{2x^3 - 1}{x(x^3 + 1)} = \frac{3x^2}{x^3 + 1} - \frac{1}{x},$$

because the numerator on the right, after taking the common denominator, is $3x^3 - (x^3 + 1) = 2x^3 - 1$. This decomposition avoids a needless factorisation of the cubic into linear and quadratic factors.

Step 2: Integrate and combine the logarithms Integrating each logarithmic derivative gives

$$\begin{aligned} I &= \ln|x^3 + 1| - \ln|x| + C \\ &= \ln\left|\frac{x^3 + 1}{x}\right| + C. \end{aligned}$$

The absolute value handles either sign of the quotient on each permitted interval. A constant can differ between disconnected intervals, as usual for an indefinite integral.

Step 3: Check the competing options by differentiation Option (A) differentiates to $3x^2/(x^3+1) - 2/x$, so its coefficient of $1/x$ is wrong. Option (B) gives half that expression. Option (D), written as $\ln|x^3 + 1| - \frac{3}{2}\ln|x| + C$, also has the wrong coefficient. Differentiating option (C) returns exactly the original numerator over $x^4 + x$, which confirms the answer without relying on visual similarity of the logarithms.

Step 4: Conclusion The integral is $\ln\left|\frac{x^3 + 1}{x}\right| + C$, for $x \neq 0, -1$. Hence, the correct option is (C).

Quick Tip to Solve: For logarithmic answer choices, differentiate them: the coefficient of $1/x$ quickly distinguishes a denominator x from x^2.

9. If A is a symmetric matrix and B is a skew-symmetric matrix such that A + B = $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to : [4 marks]

(A) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$

(B) $\begin{bmatrix} -4 & -2 \\ -1 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$

(D) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

Correct Answer: (D) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

Solution:

Concept:

- A symmetric matrix satisfies $A^T = A$, whereas a skew-symmetric matrix satisfies $B^T = -B$. In particular, every diagonal entry of a real skew-symmetric matrix is zero.
- If $M = A + B$, transposing gives $M^T = A - B$. Consequently the decomposition is unique: $A = (M + M^T)/2$ and $B = (M - M^T)/2$. Matrix multiplication must retain the requested order AB.

Step 1: Find the symmetric part Here

$$M = \begin{pmatrix} 2 & 3 \\ 5 & -1 \end{pmatrix}, \qquad M^T = \begin{pmatrix} 2 & 5 \\ 3 & -1 \end{pmatrix}.$$

Adding these matrices and dividing every entry by two gives

$$A = \frac{1}{2}\begin{pmatrix} 4 & 8 \\ 8 & -2 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 4 & -1 \end{pmatrix}.$$

The equal off-diagonal entries provide a direct check of symmetry.

Step 2: Find the skew-symmetric part Subtract the transpose instead:

$$B = \frac{1}{2}\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

Its diagonal entries vanish and its off-diagonal entries are negatives of each other. Adding this result to A restores both original entries 3 and 5, checking the sign of the skew-symmetric part.

Step 3: Multiply in the specified order Use each row of A against each column of B:

$$\begin{aligned} AB &= \begin{pmatrix} 2 & 4 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 2(0)+4(1) & 2(-1)+4(0) \\ 4(0)-1(1) & 4(-1)-1(0) \end{pmatrix} \\ &= \begin{pmatrix} 4 & -2 \\ -1 & -4 \end{pmatrix}. \end{aligned}$$

Option (A) has the wrong sign in its lower-left entry, while (B) and (C) already disagree with the upper-left value 4. Reversing the factors would generally give a different matrix, so no commutativity assumption is used.

Step 4: Conclusion $AB = \begin{pmatrix} 4 & -2 \\ -1 & -4 \end{pmatrix}$. Hence, the correct option is (D).

Quick Tip to Solve: Take half the sum and half the difference with the transpose before multiplying; then check one diagonal and one off-diagonal product entry for signs.

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10. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is : [4 marks]

(A) $4\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

(B) $4\left(-2\hat{i} - 2\hat{j} + \hat{k}\right)$

(C) $4\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$

(D) $4\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$

Correct Answer: (A) $4\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

Solution:

Concept:

- The cross product of two nonparallel vectors is perpendicular to each of them. All vectors perpendicular to both are scalar multiples of their cross product.
- To impose a specified magnitude, first divide a direction vector by its own magnitude and then multiply by the required length. Both opposite directions are allowed when the problem asks for one such vector.

Step 1: Form the two given combinations Write vectors as ordered component triples. The sum and difference are

$$\mathbf{u} = \mathbf{a} + \mathbf{b} = (4, 4, 0),$$
$$\mathbf{v} = \mathbf{a} - \mathbf{b} = (2, 0, 4).$$

A desired vector must have zero dot product with each of these two combinations, rather than merely

have the specified magnitude. These vectors are nonparallel, so their common perpendicular direction is well-defined.

Step 2: Calculate a common perpendicular direction Expanding the cross product gives

$$\begin{aligned}\mathbf{u}\times\mathbf{v} &= (4\cdot 4-0\cdot 0,\ 0\cdot 2-4\cdot 4,\ 4\cdot 0-4\cdot 2)\\ &= (16,-16,-8)=8(2,-2,-1).\end{aligned}$$

Thus $(2,-2,-1)$ is a convenient smaller direction vector. Its length is $\sqrt{4+4+1}=3$. Multiplication by 4 therefore gives a perpendicular vector of length 12.

Step 3: Verify magnitude and eliminate the other directions The vector $\mathbf{w}=(8,-8,-4)$ has

$$|\mathbf{w}|=\sqrt{64+64+16}=12,\quad \mathbf{w}\cdot\mathbf{u}=32-32=0,\quad \mathbf{w}\cdot\mathbf{v}=16-16=0.$$

All four offered vectors have magnitude 12, so that test alone cannot select the answer. For the direction in (B), its dot product with $(4,4,0)$ is nonzero; the same is true of the directions in (C) and (D). Only (A) has opposite first two components and the required relation between its first and third components.

Step 4: Conclusion One required vector is $4(2\hat{i}-2\hat{j}-\hat{k})$. Hence, the correct option is (A).

Quick Tip to Solve: For vectors perpendicular to $(4,4,0)$ and $(2,0,4)$, use $x+y=0$ and $x+2z=0$; these relations check the options immediately.

11. If the data x_1, x_2,....., x_{10} is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000 ; then the standard deviation of this data is : [4 marks]

(A) $\sqrt{2}$

(B) 2

(C) $2\sqrt{2}$

(D) 4

Correct Answer: Formal keyed result: 2, option (B); the printed summaries are inconsistent for real data.

Solution:

Concept:

- The overall mean of groups is found from the total of their observations, not from the unweighted average of the group means. A group with m observations and mean μ has sum $m\mu$.
- For all N observations, the variance is $\sigma^2=\frac{1}{N}\sum x_i^2-\bar{x}^2$, and the standard deviation is its nonnegative square root. This is the population standard deviation of the given data.

Step 1: Recover the sum and overall mean The first four observations have sum $4\times 11=44$. The

remaining six observations have sum $6 \times 16 = 96$. Therefore,

$$\sum_{i=1}^{10} x_i = 44 + 96 = 140, \qquad \bar{x} = \frac{140}{10} = 14.$$

The different group sizes are essential: averaging 11 and 16 directly would incorrectly give 13.5. We do not need the individual observations because their total and sum of squares determine the variance.

Step 2: Subtract the square of the mean The given sum of squares is 2,000, so the mean square is $2000/10 = 200$. Hence

$$\sigma^2 = \frac{2000}{10} - 14^2 = 200 - 196 = 4.$$

Equivalently, the total squared deviation from 14 is

$$\sum (x_i - 14)^2 = 2000 - 28(140) + 10(196) = 40.$$

Dividing this by 10 again gives the same variance 4. This second calculation verifies the subtraction and denominator.

Step 3: Take the square root and check the alternatives Standard deviation is measured in the same units as the observations, whereas variance is measured in their squared units. Therefore,

$$\sigma = \sqrt{4} = 2.$$

Option (D), 4, is the variance rather than the standard deviation. Options (A) and (C) would correspond to variances 2 and 8, neither of which equals the computed value. The denominator here is 10; using 9 would instead be an unbiased sample-variance estimate, which is not what this data question asks for.

Step 4: Check consistency of the supplied summaries There is a separate defect in the stated data: for real observations, each group satisfies $\sum x_i^2 \geq m\mu^2$. The two groups therefore require $\sum x_i^2 \geq 4(11)^2 + 6(16)^2 = 2020$, which exceeds the printed 2000. No real data set satisfies every stated summary. The formal variance calculation above explains the keyed choice (B); it must be distinguished from the existence of such observations.

Step 5: Conclusion Using the supplied totals formally gives $\sigma = 2$, the keyed option (B). However, the group means require a sum of squares at least 2020, so the printed data are internally inconsistent and no real data set satisfies them all.

Quick Tip to Solve: Compute $\sum x_i$ from group size times group mean, then use mean square minus square of mean; take the square root only at the end.

12. If $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all values of α for which det(A) + 1 = 0, is : [4 marks]

(A) 2

(B) - 1

(C) 0

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- If $B = A^{-1}$, then $\det(B) = 1/\det(A)$. Existence of the inverse requires both determinants to be nonzero.
- For a quadratic equation $a\alpha^2 + b\alpha + c = 0$, the sum of its roots is $-b/a$. Expanding a determinant along a row with a zero reduces the arithmetic.

Step 1: Translate the condition into a determinant of B The given condition is $\det(A) + 1 = 0$, so $\det(A) = -1$. Taking the reciprocal gives

$$\det(B) = \frac{1}{-1} = -1.$$

Thus we should evaluate the determinant of the displayed matrix directly. There is no need to find all nine entries of A, and the negative sign remains negative under reciprocation.

Step 2: Expand and simplify the determinant Expanding along the first row with alternating signs gives

$$\begin{aligned} \det(B) &= 5\{2(-1) - 1(3)\} - 2\alpha\{0(-1) - 1(\alpha)\} \\ &\quad + 1\{0(3) - 2\alpha\} \\ &= -25 + 2\alpha^2 - 2\alpha. \end{aligned}$$

In particular, the middle cofactor contributes $+2\alpha^2$, since its minor is $-\alpha$. Tracking this pair of negative signs prevents changing the root sum.

Step 3: Solve for the admissible values and add them Equating to -1 yields

$$2\alpha^2 - 2\alpha - 25 = -1, \qquad \alpha^2 - \alpha - 12 = 0.$$

Factoring gives $(\alpha - 4)(\alpha + 3) = 0$, so the possible values are 4 and -3. Both make $\det(B) = -1 \neq 0$, so both indeed define an inverse matrix and neither must be discarded. Their sum is $4 - 3 = 1$. The value -1 offered in (B) is the determinant of A, not the requested sum; 0 and 2 also disagree with the root sum. Vieta's formula independently gives the sum $-(-1)/1 = 1$.

Step 4: Conclusion The admissible values of α are 4 and -3, whose sum is 1. Hence, the correct option is (D).

Quick Tip to Solve: When an inverse matrix is supplied, convert a determinant condition by reciprocation; then use the quadratic root sum instead of computing the full inverse.

13. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is : [4 marks]

(A) 2^{20} - 1

(B) 2^{20}

(C) 2^{20} + 1

(D) 2^{21}

Correct Answer: (B) 2^{20}

Solution:

Concept:

- Identical objects contribute only their selected number: choosing k of the ten identical objects creates one type of choice, rather than $\binom{10}{k}$ distinguishable choices.
- Choosing r of 21 distinct objects can be done in $\binom{21}{r}$ ways. The binomial theorem gives $\sum_{r=0}^{21}\binom{21}{r} = 2^{21}$, and symmetry pairs terms r and $21-r$.

Step 1: Classify the choices by the number of distinct objects Let r be the number chosen from the 21 distinct objects. The remaining $10-r$ objects must be from the identical group. Every integer r from 0 through 10 is possible, since there are exactly ten identical objects available. For a fixed r, choose a subset of the distinct group in $\binom{21}{r}$ ways, and then include the required identical objects in one way. Thus

$$N = \sum_{r=0}^{10}\binom{21}{r}.$$

This includes the endpoint $r = 0$, when all ten selected objects are identical.

Step 2: Use binomial symmetry to evaluate the sum Since 21 is odd, no term is left unpaired at the middle. For each $r = 0, 1, \ldots, 10$, the coefficient $\binom{21}{r}$ equals $\binom{21}{21-r}$, whose index lies between 11 and 21. Hence

$$\sum_{r=0}^{10}\binom{21}{r} = \sum_{r=11}^{21}\binom{21}{r}.$$

Adding the two equal sums gives 2^{21}, so each sum is 2^{20}.

Step 3: Check why no correction term is needed Unlike a binomial sum with an even upper index, there is no central coefficient that needs to be halved separately. The all-identical choice and the choice of ten distinct objects were already counted, so adding or subtracting one would double-count or omit a valid selection. This rules out (A) and (C). Option (D) counts all subsets of the distinct group, including subsets with more than ten objects; such subsets cannot occur in a selection of ten total objects.

Step 4: Conclusion The number of selections is 2^{20}. Hence, the correct option is (B).

Quick Tip to Solve: Use the number selected from the distinct group as the summation index. For an odd upper index, binomial symmetry splits the total exactly into two equal halves.

14. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r + \lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ is equal to : [4 marks]

(A) $\frac{7}{116}$
(B) $\frac{29}{348}$
(C) $\frac{1}{12}$
(D) $\frac{21}{346}$

Correct Answer: Both (B) and (C) are equal: $29/348 = 1/12$; the printed choices do not give a unique answer.

Solution:

Concept:

- For a geometric series starting at its first power, $\sum_{r=1}^{\infty} t^r = t/(1-t)$, provided $|t| < 1$. The first term is t, not 1.
- For roots α, β of $ax^2 + bx + c = 0$, Vieta's relations give $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$. These symmetric quantities often avoid substituting radical roots.

Step 1: Establish convergence of both series The quadratic formula applied to $375x^2 - 25x - 2 = 0$ gives

$$\alpha, \beta = \frac{25 \pm \sqrt{625 + 3000}}{750} = \frac{5 \pm \sqrt{145}}{150}.$$

Since $12 < \sqrt{145} < 13$, the positive root lies between $17/150$ and $18/150$, and the negative root lies between $-8/150$ and $-7/150$. Both therefore have absolute value less than 1, which justifies applying the infinite geometric-series formula to each limit.

Step 2: Express the sum through the root sum and product Let the required value be S. Then

$$\begin{aligned} S &= \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} \\ &= \frac{\alpha + \beta - 2\alpha\beta}{1 - (\alpha+\beta) + \alpha\beta}. \end{aligned}$$

Vieta's relations for the printed quadratic give $\alpha + \beta = 25/375 = 1/15$ and $\alpha\beta = -2/375$. The product is negative because the roots have opposite signs.

Step 3: Substitute and simplify the rational value Substitution produces

$$S = \frac{\frac{1}{15} + \frac{4}{375}}{1 - \frac{1}{15} - \frac{2}{375}} = \frac{29/375}{348/375} = \frac{29}{348}.$$

The numerator includes $-2\alpha\beta$, while the denominator includes only $+\alpha\beta$; using the same coefficient in both would be incorrect. As a check, the result is about 0.08333, consistent with one positive and one smaller negative geometric sum. Notice that $348 = 12 \times 29$, so $29/348 = 1/12$. Thus both (B) and (C) show the same correct value. Options (A) and (D) are different. The sheet key selects (B), but the printed options also make (C) correct; no unique selection can be justified.

Step 4: Conclusion The value is $\frac{29}{348} = \frac{1}{12}$. Both options (B) and (C) are correct as printed; the question-sheet key lists (B), but the duplicate value prevents a unique answer.

Quick Tip to Solve: Check both root magnitudes first, then combine $\alpha/(1-\alpha) + \beta/(1-\beta)$ using Vieta's root sum and product.

15. The equation $|z - i| = |z - 1|$, $i = \sqrt{-1}$, represents : [4 marks]

(A) a circle of radius 1

(B) the line through the origin with slope – 1

(C) a circle of radius $\frac{1}{2}$

(D) the line through the origin with slope 1

Correct Answer: (D) the line through the origin with slope 1

Solution:

Concept:

- The modulus $|z - a|$ is the Euclidean distance from the point representing z to the fixed point representing a in the Argand plane.
- A point equidistant from two distinct fixed points lies on their perpendicular bisector. Writing $z = x + iy$, with x, y real, converts this distance condition into a Cartesian equation.

Step 1: Write the two distances The points representing i and 1 have coordinates $(0, 1)$ and $(1, 0)$. Hence the given condition is

$$\sqrt{x^2 + (y-1)^2} = \sqrt{(x-1)^2 + y^2}.$$

Both distances are nonnegative, so squaring preserves equivalence; there is no extra branch or sign choice.

Step 2: Simplify the Cartesian equation Expand both sides and cancel the identical quadratic terms:

$$x^2 + y^2 - 2y + 1 = x^2 - 2x + 1 + y^2.$$

Thus $-2y = -2x$, giving $y = x$. This is a straight line through the origin, with slope 1. Its direction is perpendicular to the segment joining the two fixed points, whose slope is -1, as the locus interpretation predicts.

Step 3: Check the alternatives The origin has equal distances 1 from both fixed points and therefore belongs to the locus. The point $(2, 2)$ also satisfies the distance equation, so the locus is unbounded and cannot be either listed circle. The line of slope -1 does not satisfy the equation except at its intersection with $y = x$.

Step 4: Conclusion The equation represents the line $y = x$, through the origin with slope 1. Hence, the correct option is (D).

Quick Tip to Solve: For an equality of two complex moduli, identify the two fixed points first; the locus is their perpendicular bisector.

16. The coefficient of x^{18} in the product $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$ is : [4 marks]

(A) 126

(B) - 84

(C) - 126

(D) 84

Correct Answer: (D) 84

Solution:

Concept:

- The coefficient of a power can be found after algebraically simplifying the product; expansion of all three factors separately is unnecessary.
- Use $(1-x)(1+x+x^2) = 1-x^3$, followed by the binomial theorem $(1-u)^9 = \sum_{k=0}^{9}(-1)^k\binom{9}{k}u^k$.

Step 1: Combine the factors Pair nine of the ten factors $(1-x)$ with $(1+x+x^2)^9$. The remaining factor combines with $(1+x)$:

$$\begin{aligned}(1+x)(1-x)^{10}(1+x+x^2)^9 &= (1+x)(1-x)(1-x^3)^9\\ &= (1-x^2)(1-x^3)^9.\end{aligned}$$

This identity preserves every coefficient while making the possible exponents easy to track.

Step 2: Identify every contribution to the target power In $(1-x^3)^9$, all powers have exponent divisible by 3. The constant term of $(1-x^2)$ contributes the coefficient of x^{18}. The term $-x^2$ would require an x^{16} term from $(1-x^3)^9$, but no such term exists because 16 is not divisible by 3. Thus there is exactly one contribution.

Step 3: Evaluate the binomial coefficient The required exponent is obtained when $3k = 18$, so $k = 6$. Its coefficient is

$$(-1)^6\binom{9}{6} = \binom{9}{3} = \frac{9\cdot 8\cdot 7}{3\cdot 2\cdot 1} = 84.$$

The sign is positive because six negative terms are selected. The negative options have the wrong sign; 126 is not the binomial coefficient for this exponent.

Step 4: Conclusion The coefficient of x^{18} is 84. Hence, the correct option is (D).

Quick Tip to Solve: Before expanding a product containing $1+x+x^2$, look for a factor $1-x$; it converts the expression to powers of x^3.

17. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at x = 0 is equal to : [4 marks]

(A) $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$
(B) $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$
(C) $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$
(D) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$

Correct Answer: (B) $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$

Solution:

Concept:

- When y is defined implicitly as a function of x, differentiate every occurrence of y using the chain rule.
- For the product xy, the derivative is $y + xy\prime$. Differentiating a second time requires both the product rule and the derivative of e^y.

Step 1: Find the point and first derivative At $x = 0$, the equation gives $e^y = e$, hence $y = 1$. Differentiating the original relation gives

$$e^y y\prime + y + xy\prime = 0, \qquad (e^y + x)y\prime = -y.$$

At $(0, 1)$, therefore, $e\,y\prime + 1 = 0$, and $y\prime(0) = -1/e$. The denominator $e^y + x$ equals $e \neq 0$ here, so the local differentiation is valid.

Step 2: Differentiate once more Differentiate the first derivative equation before substituting the point:

$$e^y(y\prime)^2 + e^y y\prime\prime + 2y\prime + xy\prime\prime = 0.$$

The two copies of $y\prime$ come from differentiating y and from differentiating $xy\prime$. Substitute $x = 0$, $y = 1$, and $y\prime = -1/e$:

$$\frac{1}{e} + e\,y\prime\prime - \frac{2}{e} = 0.$$

Thus $e\,y\prime\prime = 1/e$, so $y\prime\prime(0) = 1/e^2$.

Step 3: Compare the signs with the options The first derivative is negative and the second derivative positive. This immediately rules out the two options with positive first derivative and the option with negative second derivative. Keeping the square $(y\prime)^2$ positive is essential in the second derivative equation.

Step 4: Conclusion The ordered pair is $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$. Hence, the correct option is (B).

Quick Tip to Solve: Find the point first, but substitute it only after differentiating; retain both product-rule terms in the second derivative.

18. For x $\in$ (0, 3/2), let f(x) = $\sqrt{x}$, g(x) = tan x and h(x) = $\frac{1-x^2}{1+x^2}$. If ϕ (x) = ((hof)og)(x), then $\phi\left(\frac{\pi}{3}\right)$ is equal to : [4 marks]

(A) $\tan\frac{7\pi}{12}$

(B) $\tan\frac{11\pi}{12}$

(C) $\tan\frac{\pi}{12}$

(D) $\tan\frac{5\pi}{12}$

Correct Answer: (B) $\tan\frac{11\pi}{12}$

Solution:

Concept:

- Composition acts from right to left: $((h\circ f)\circ g)(x) = h(f(g(x)))$. The square root is applied after the tangent.
- The identities $h(\sqrt{u}) = (1-u)/(1+u)$ for $u \geq 0$ and $\tan(A-B) = (\tan A - \tan B)/(1+\tan A\tan B)$ simplify the value.

Step 1: Evaluate the functions in their prescribed order For $x = \pi/3$, the tangent is positive:

$$g(\pi/3) = \tan(\pi/3) = \sqrt{3}.$$

The next function gives $f(g(\pi/3)) = \sqrt{\sqrt{3}} = 3^{1/4}$. In the rational function h, this number is squared, so its square is $\sqrt{3}$, not 3. Consequently

$$\phi(\pi/3) = \frac{1-\sqrt{3}}{1+\sqrt{3}}.$$

The denominator is positive and nonzero.

Step 2: Express the result as a tangent Use the tangent subtraction formula with $A = \pi/4$ and $B = \pi/3$:

$$\frac{1-\sqrt{3}}{1+\sqrt{3}} = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{12}\right).$$

Tangent has period π, so this equals $\tan(11\pi/12)$. Rationalizing also gives the value $\sqrt{3}-2$, which is negative and has magnitude less than 1.

Step 3: Eliminate the other listed angles The angles $\pi/12$ and $5\pi/12$ are in the first quadrant and have positive tangents. The angle $7\pi/12$ is in the second quadrant but its tangent has magnitude greater than 1. Only $11\pi/12$ gives the small negative value found above.

Step 4: Conclusion $\phi(\pi/3) = \sqrt{3}-2 = \tan(11\pi/12)$. Hence, the correct option is (B).

Quick Tip to Solve: Write a composition explicitly from right to left, and cancel a square root with the square inside h before calculating radicals.

19. If the area (in sq. units) of the region $\{(x, y) : y^2 \leq 4x, x + y \leq 1, x \geq 0, y \geq 0\}$ is a $\sqrt{2} + b$, then $a - b$ is equal to : [4 marks]

(A) $\frac{8}{3}$

(B) $-\frac{2}{3}$

(C) 6

(D) $\frac{10}{3}$

Correct Answer: (C) 6

Solution:

Concept:

- For horizontal strips, the area is $\int (x_{\text{right}} - x_{\text{left}})\, dy$. The inequalities determine both boundaries and the interval of integration.
- The parabola $y^2 = 4x$ gives $x = y^2/4$, while the line $x + y = 1$ gives $x = 1 - y$. Only their first-quadrant intersection is relevant.

Step 1: Find the bounds of the region The condition $y^2 \leq 4x$ places each horizontal strip to the right of $x = y^2/4$. The condition $x + y \leq 1$ places it to the left of $x = 1 - y$. At the intersection,

$$\frac{y^2}{4} = 1 - y, \qquad y^2 + 4y - 4 = 0.$$

The nonnegative root is $r = 2\sqrt{2} - 2$; the other root is negative and excluded. Thus $0 \leq y \leq r$.

Step 2: Integrate the horizontal width The width is nonnegative on this interval, and the area is

$$A = \int_0^r \left(1 - y - \frac{y^2}{4}\right) dy = r - \frac{r^2}{2} - \frac{r^3}{12}.$$

Because $r^2 = 4 - 4r$, multiplying by r also gives $r^3 = 20r - 16$. Substitute these expressions to simplify without expanding large radicals:

$$A = r - 2 + 2r - \frac{5}{3}r + \frac{4}{3} = \frac{4}{3}r - \frac{2}{3}.$$

Step 3: Read off the coefficients Using $r = 2\sqrt{2} - 2$, we obtain

$$A = \frac{8}{3}\sqrt{2} - \frac{10}{3}.$$

Therefore $a = 8/3$ and $b = -10/3$. In particular, the constant coefficient is negative; the area itself remains positive. Hence $a - b = 8/3 + 10/3 = 6$. The other listed values result from confusing an individual coefficient with their requested difference.

Step 4: Conclusion $a - b = 6$. Hence, the correct option is (C).

Quick Tip to Solve: When a parabola is given as $y^2 = 4x$, horizontal strips often avoid splitting the area into separate integrals.

20. Let f : R $\to$ R be a continuously differentiable function such that f(2) = 6 and f'(2) = $\frac{1}{48}$. If $\int_6^{f(x)} 4t^3 dt$ = (x - 2)g(x), then $\lim_{x\to 2} g(x)$ is equal to : [4 marks]

(A) 18

(B) 36

(C) 12

(D) 24

Correct Answer: (A) 18

Solution:

Concept:

- The fundamental theorem of calculus and the chain rule give $\frac{d}{dx}\int_a^{f(x)} F(t)\,dt = F(f(x))f\prime(x)$ for continuous F.
- If $H(2) = 0$, then $\lim_{x\to 2} H(x)/(x-2) = H\prime(2)$, directly by the definition of derivative. No derivative of g is required.

Step 1: Recognize the derivative quotient Define $H(x) = \int_6^{f(x)} 4t^3\,dt$. Since $f(2) = 6$, the lower and upper limits coincide at $x = 2$, giving $H(2) = 0$. For $x \neq 2$, the supplied equation implies

$$g(x) = \frac{H(x)}{x-2} = \frac{H(x) - H(2)}{x-2}.$$

Thus the requested limit is $H\prime(2)$. The value assigned to g at the single point 2 has no effect on this limit.

Step 2: Apply the chain rule and substitute The integrand $4t^3$ is continuous and f is continuously differentiable, so

$$H\prime(x) = 4[f(x)]^3 f\prime(x).$$

At the specified point,

$$H\prime(2) = 4 \cdot 6^3 \cdot \frac{1}{48} = \frac{864}{48} = 18.$$

Both the moving upper limit and its derivative must appear; omitting the factor $f\prime(2)$ would give an entirely different value.

Step 3: Verify by evaluating the integral Alternatively, $H(x) = [f(x)]^4 - 6^4$. Differentiating this expression gives exactly $4[f(x)]^3 f\prime(x)$, confirming the same calculation. None of the values 36, 12, or 24 equals the resulting derivative; no extra factor is introduced by the zero lower-to-upper interval at $x = 2$.

Step 4: Conclusion $\lim_{x\to 2} g(x) = 18$. Hence, the correct option is (A).

Quick Tip to Solve: A vanishing integral divided by $x - a$ is often a derivative quotient; apply the fundamental theorem before substituting the data.

21. The equation y = sinx sin (x + 2) – $\sin^2$ (x + 1) represents a straight line lying in : [4 marks]

(A) first, second and fourth quadrants

(B) first, third and fourth quadrants

(C) second and third quadrants only

(D) third and fourth quadrants only

Correct Answer: (D) third and fourth quadrants only

Solution:

Concept:

- The product-to-sum formula is $\sin A \sin B = [\cos(A - B) - \cos(A + B)]/2$.
- The square identity $\sin^2 u = (1 - \cos 2u)/2$ allows the variable-dependent cosine terms to cancel. A negative constant ordinate describes a horizontal line below the horizontal axis.

Step 1: Expand both trigonometric terms Set $A = x$ and $B = x+2$. Since cosine is even, $\cos(A-B) = \cos(-2) = \cos 2$, and

$$\sin x \sin(x + 2) = \frac{\cos 2 - \cos(2x + 2)}{2}.$$

Also,

$$\sin^2(x + 1) = \frac{1 - \cos(2x + 2)}{2}.$$

The angles here are in radians; the constant 1 is not an arbitrary multiple of π.

Step 2: Cancel the variable-dependent terms Subtract the two expressions exactly as the question specifies:

$$y = \frac{\cos 2 - \cos(2x + 2)}{2} - \frac{1 - \cos(2x + 2)}{2}$$
$$= \frac{\cos 2 - 1}{2} = -\sin^2 1.$$

The terms involving $2x + 2$ cancel for every real x. Since $\sin 1 \neq 0$, the resulting ordinate is strictly negative and independent of x.

Step 3: Locate the line in the coordinate plane For negative x, both coordinates are negative, so the line lies in the third quadrant. For positive x, the ordinate remains negative, so it lies in the fourth quadrant. It meets the negative vertical axis at $x = 0$; axis points are not assigned to a quadrant. It has no point in either upper quadrant, eliminating the other listed choices.

Step 4: Conclusion The line is $y = -\sin^2 1 < 0$, lying in the third and fourth quadrants only. Hence, the correct option is (D).

Quick Tip to Solve: When a product has angles equally spaced around the angle in a squared sine, use product-to-sum to expose cancellation.

22. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/sec, then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is : [4 marks]

(A) $\frac{25}{3}$

(B) 25

(C) $25\sqrt{3}$

(D) $\frac{25}{\sqrt{3}}$

Correct Answer: (D) $\frac{25}{\sqrt{3}}$

Solution:

Concept:

- The ladder, wall and ground form a right triangle. If the horizontal distance is x and the top height is y, the fixed ladder length gives $x^2 + y^2 = L^2$.
- Differentiate with respect to time, treating both distances as changing quantities. The downward motion makes dy/dt negative, whereas motion away from the wall makes dx/dt positive.

Step 1: Use one unit throughout Take all lengths in centimetres. The ladder has length $L = 200\,\text{cm}$, and the specified instant has $y = 100\,\text{cm}$. The top slides down at $25\,\text{cm}\,\text{s}^{-1}$, so $dy/dt = -25\,\text{cm}\,\text{s}^{-1}$. The horizontal separation at this instant follows from the triangle:

$$x = \sqrt{200^2 - 100^2} = 100\sqrt{3}\,\text{cm}.$$

The positive root is required because a distance cannot be negative.

Step 2: Differentiate the constraint The ladder length does not change as its ends move, so

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0, \qquad \frac{dx}{dt} = -\frac{y}{x}\frac{dy}{dt}.$$

Substitute the instantaneous distances and the signed vertical velocity:

$$\frac{dx}{dt} = -\frac{100}{100\sqrt{3}}(-25) = \frac{25}{\sqrt{3}}\,\text{cm}\,\text{s}^{-1}.$$

The result is positive, confirming outward motion. Equal end speeds would require equal horizontal and vertical distances; that does not hold here. Multiplying by $\sqrt{3}$ reverses the ratio, while dividing by 3 incorrectly uses a squared-distance ratio.

Step 3: Conclusion The bottom slides away from the wall at $25/\sqrt{3}\,\text{cm}\,\text{s}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the negative sign for the descending end; use the instantaneous ratio y/x, not its square.

23. If the volume of parallelopiped formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k}$, $\hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then λ is equal to : [4 marks]

(A) $-\frac{1}{\sqrt{3}}$

(B) $\sqrt{3}$

(C) $-\sqrt{3}$

(D) $\frac{1}{\sqrt{3}}$

Correct Answer: No offered value gives the global minimum of the absolute scalar triple product; option (D) is only a local minimum.

Solution:

Concept:

- The geometric volume of a parallelepiped is the absolute value of its scalar triple product: $V = |\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|$. A signed determinant alone is not a volume.
- A stationary local minimum need not be a global minimum. Both the zeroes of the determinant and its stationary values must be considered when the parameter has no stated restriction.

Step 1: Compute the actual volume In the order printed, the three vectors have components $(1,\lambda,1)$, $(0,1,\lambda)$, and $(\lambda,0,1)$. Expanding the determinant gives

$$f(\lambda)=\begin{vmatrix}1 & \lambda & 1\\ 0 & 1 & \lambda\\ \lambda & 0 & 1\end{vmatrix}=1+\lambda^3-\lambda.$$

Thus $V=|\lambda^3-\lambda+1|$. In particular, $f(-2)=-5$ and $f(-1)=1$. Continuity therefore guarantees a real zero between -2 and -1, at which the volume is zero, the smallest possible nonnegative value.

Step 2: Distinguish the offered stationary value Differentiation gives $f\prime(\lambda)=3\lambda^2-1$ and $f\prime\prime(\lambda)=6\lambda$. At $\lambda=1/\sqrt{3}$, $f=1-2/(3\sqrt{3})>0$ and $f\prime\prime>0$, so this is a local minimum of the volume. The negative reciprocal root is a local maximum; the values $\pm\sqrt{3}$ are not stationary. None of these four offered parameters makes the determinant zero. If a nondegenerate solid is implicitly required, zero is excluded but can be approached arbitrarily closely, so there is no attained global minimum in that domain either.

Step 3: Conclusion As printed without a restriction on λ, there is no correct option for the global minimum. Option (D), $1/\sqrt{3}$, gives only the local minimum associated with the positive stationary point.

Quick Tip to Solve: For a volume, minimize the absolute determinant and check its zeroes before using a second-derivative test.

24. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4,4) then PQ is equal to : [4 marks]

(A) $\frac{\sqrt{61}}{2}$

(B) $\frac{\sqrt{221}}{2}$

(C) $\frac{\sqrt{157}}{2}$

(D) $\frac{5\sqrt{5}}{2}$

Correct Answer: (D) $\frac{5\sqrt{5}}{2}$

Solution:

Concept:

- For an implicitly defined curve, the derivative gives the tangent slope and the normal slope is its negative reciprocal when both exist.
- The tangent to $3x^2 + 4y^2 = 12$ at (x_1, y_1) is $3xx_1 + 4yy_1 = 12$. Use the external point condition to choose between the two possible contact points.

Step 1: Find the possible contact points Differentiate the ellipse equation:

$$6x + 8y\frac{dy}{dx} = 0, \qquad m_{\mathrm{t}} = -\frac{3x}{4y}.$$

Therefore $m_{\mathrm{n}} = 4y/(3x)$. The line $2x + y = 4$ has slope -2, so parallelism of the normal requires

$$\frac{4y_1}{3x_1} = -2, \qquad y_1 = -\frac{3}{2}x_1.$$

Substitution into the ellipse yields $3x_1^2 + 9x_1^2 = 12$, hence $x_1 = \pm 1$. The possible points are $(1, -3/2)$ and $(-1, 3/2)$; neither has a vertical or horizontal normal.

Step 2: Use the tangent and compute the distance Since the tangent passes through $Q(4, 4)$, its equation requires

$$12x_1 + 16y_1 = 12.$$

Replacing y_1 by $-3x_1/2$ gives $-12x_1 = 12$. Thus $P = (-1, 3/2)$. The other candidate gives -12 on the left and fails the tangent condition. Now apply the distance formula:

$$PQ^2 = (4 + 1)^2 + \left(4 - \frac{3}{2}\right)^2 = 25 + \frac{25}{4} = \frac{125}{4}.$$

The horizontal separation is 5 units and the vertical separation is $5/2$ units; both are required in the Euclidean distance. The competing squared distances $61/4, 221/4, 157/4$ do not equal this value.

Step 3: Conclusion $PQ = \sqrt{125/4} = 5\sqrt{5}/2$. Hence, the correct option is (D).

Quick Tip to Solve: First obtain both antipodal contact points from the normal direction, then use the tangent-through-point condition to select one.

25. If the truth value of the statement p $\rightarrow$ (~q $\vee$ r) is false (F), then the truth values of the statements p, q, r are respectively : [4 marks]

(A) T, F, T

(B) F, T, T

(C) T, T, F

(D) T, F, F

Correct Answer: (C) T, T, F

Solution:

Concept:

- An implication $p \rightarrow s$ is false only in the case where its antecedent p is true and its consequent s is false.
- A disjunction $a \vee b$ is false precisely when both components are false. Negation reverses the truth value: $\neg q$ is false exactly when q is true.

Step 1: Apply the condition for a false implication Treat the complete parenthesis $(\neg q \vee r)$ as the consequent, rather than considering either term separately. The given false value forces

$$p = \mathrm{T}, \qquad (\neg q \vee r) = \mathrm{F}.$$

A false antecedent cannot work: an implication with antecedent false is true regardless of its consequent. Therefore the choice beginning with $p = \mathrm{F}$ is immediately excluded. We must now find the values of q and r which make the parenthesis false.

Step 2: Resolve the disjunction and verify each remaining case Neither component of the disjunction may be true. Hence

$$\neg q = \mathrm{F}, \qquad r = \mathrm{F}, \qquad q = \mathrm{T}.$$

This gives the ordered triple $(\mathrm{T}, \mathrm{T}, \mathrm{F})$. Substituting it back produces $\mathrm{T} \rightarrow (\mathrm{F} \vee \mathrm{F}) = \mathrm{T} \rightarrow \mathrm{F} = \mathrm{F}$, as required. In the first offered triple, both $\neg q$ and r are true, making the implication true. In the last triple, q is false, so $\neg q$ is true even though r is false; that implication is also true. Thus the condition identifies exactly one listed triple.

Step 3: Conclusion The truth values of p, q, r, respectively, are T, T, F. Hence, the correct option is (C).

Quick Tip to Solve: Work inward from the false implication: first force its consequent false, then force every member of its disjunction false.

26. For $x \in \mathbb{R}$, let [x] denote the greatest integer $\leq$ x, then the sum of the series $\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + \left[-\frac{1}{3} - \frac{2}{100}\right] + ... + \left[-\frac{1}{3} - \frac{99}{100}\right]$ is : [4 marks]

(A) - 153

(B) - 135

(C) - 133

(D) - 131

Correct Answer: (C) - 133

Solution:

Concept:

- The greatest integer function $\lfloor u \rfloor$ is the largest integer not exceeding u. For negative fractions it rounds downward, not toward zero.
- Instead of summing one hundred expressions individually, divide the index range where the argument crosses an integer. Count the terms in each constant-value block.

Step 1: Locate the change in floor value Write the sum as

$$S = \sum_{k=0}^{99} \left\lfloor -\frac{1}{3} - \frac{k}{100} \right\rfloor .$$

The first argument is $-1/3$, while the last is $-397/300$. Every argument therefore lies strictly between -2 and 0, so its floor is either -1 or -2. To have floor -1, the argument must be at least -1:

$$-\frac{1}{3} - \frac{k}{100} \geq -1 \quad \Longleftrightarrow \quad k \leq \frac{200}{3}.$$

Since k is an integer, the last such index is 66.

Step 2: Count the terms and form the sum The indices $0, 1, \ldots, 66$ provide $66 - 0 + 1 = 67$ terms, each equal to -1. The remaining indices $67, 68, \ldots, 99$ provide $99 - 67 + 1 = 33$ terms, each equal to -2. A direct boundary check confirms the switch: the arguments at indices 66 and 67 are $-298/300$ and $-301/300$, respectively. Thus

$$S = 67(-1) + 33(-2) = -67 - 66 = -133.$$

The counts add to 100, matching the number of terms from index zero through 99. The other proposed totals require incorrect counts or rounding negative values toward zero.

Step 3: Conclusion The series sums to -133. Hence, the correct option is (C).

Quick Tip to Solve: For floors of negative arithmetic progressions, solve the integer-boundary inequality first and include both endpoints when counting indices.

27. The number of solutions of the equation

$1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is : [4 marks]

(A) 5

(B) 3

(C) 7

(D) 4

Correct Answer: (A) 5

Solution:

Concept:

- For every real angle, $\sin^4 x \geq 0$ and $\cos^2(3x) \leq 1$. An equality between these two sides is strongly constrained by these bounds.
- The real zeroes of $\sin x$ are $x = n\pi$ for integers n. A candidate obtained from a bound must still be checked in the original equation.

Step 1: Force equality in both bounds The equation is

$$1 + \sin^4 x = \cos^2(3x).$$

Its left side is at least 1, while its right side is at most 1. Consequently, the only possible common value is exactly 1. Thus $\sin^4 x = 0$ and $\cos^2(3x) = 1$. The first condition implies $x = n\pi$, $n \in \mathbb{Z}$. For every such candidate, $\cos(3x) = \cos(3n\pi) = (-1)^{3n}$, whose square is 1. Therefore every candidate satisfies the second condition and the original equation; no other real values can satisfy the bounding argument.

Step 2: Count the permitted integer multiples Apply the closed interval restriction to $x = n\pi$:

$$-\frac{5\pi}{2} \leq n\pi \leq \frac{5\pi}{2} \quad \Longleftrightarrow \quad -\frac{5}{2} \leq n \leq \frac{5}{2}.$$

The integers in this interval are $-2, -1, 0, 1, 2$. Hence the solutions are $-2\pi, -\pi, 0, \pi, 2\pi$. The interval endpoints $\pm 5\pi/2$ are not additional solutions, because their sine has magnitude 1 and makes the left side 2. Counting only the central multiples misses the outer two, whereas counting all zeroes of $\sin 3x$ admits values that fail $\sin x = 0$.

Step 3: Conclusion There are exactly 5 solutions in the specified interval. Hence, the correct option is (A).

Quick Tip to Solve: Before expanding multiple-angle expressions, compare the numerical ranges of the two sides; equality at a common bound often fixes the angles.

28. Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio : [4 marks]

(A) 14 : 13

(B) 13 : 11

(C) 5 : 4

(D) 2 : 1

Correct Answer: (C) 5 : 4

Solution:

Concept:

- The finite-slope tangent to $y^2 = 4ax$ has equation $y = mx + a/m$, where $m \neq 0$. Here $a = 3$.
- A line $y = mx + c$ is tangent to $x^2/a_h^2 - y^2/b_h^2 = 1$ when $c^2 = a_h^2 m^2 - b_h^2$. The focal distance is $\sqrt{a_h^2 + b_h^2}$.

Step 1: Determine both common tangents Rewrite the hyperbola as $x^2 - y^2/8 = 1$, so $a_h^2 = 1$, $b_h^2 = 8$. For a common tangent the parabola gives $c = 3/m$, while the hyperbola gives $c^2 = m^2 - 8$. Equating these expressions yields

$$\frac{9}{m^2} = m^2 - 8, \qquad m^4 - 8m^2 - 9 = 0.$$

Factoring gives $(m^2 - 9)(m^2 + 1) = 0$, so the real slopes are $m = 3$ and $m = -3$. The corresponding common tangents are $y = 3x + 1$ and $y = -3x - 1$. The vertical tangent to the parabola, $x = 0$, is not a tangent to this hyperbola and adds no common line.

Step 2: Find the intersection and focal ratio At their intersection, $3x + 1 = -3x - 1$, which gives

$$P = (-1/3, 0).$$

The hyperbola has focal distance $\sqrt{1 + 8} = 3$, hence $S = (3, 0)$ and $S\prime = (-3, 0)$. The point P lies between these foci and closer to the negative focus. The ordered distances are

$$SP = 3 + \frac{1}{3} = \frac{10}{3}, \qquad PS\prime = 3 - \frac{1}{3} = \frac{8}{3}.$$

Their ratio is $10 : 8 = 5 : 4$. Reversing the focus order would reverse the ratio, but the question fixes S on the positive axis. None of the other listed ratios equals these distances.

Step 3: Conclusion P divides $SS\prime$ internally in the ratio $SP : PS\prime = 5 : 4$. Hence, the correct option is (C).

Quick Tip to Solve: Use the parabola tangent intercept a/m in the hyperbola tangency condition, then retain the stated ordering of the foci.

29. If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the plane 2x + 3y – z + 13 = 0 at a point P and the plane 3x + y + 4z = 16 at a point Q, then PQ is equal to : [4 marks]

(A) $2\sqrt{7}$

(B) 14

(C) $2\sqrt{14}$

(D) $\sqrt{14}$

Correct Answer: (C) $2\sqrt{14}$

Solution:

Concept:

- A point on a line in symmetric form is obtained by equating its three ratios to a common real parameter t. Its direction vector is $\boldsymbol{d} = (3, 2, -1)$.
- The intersection parameter is found by substituting the parametric coordinates in the plane equation. Two points with parameters t_1, t_2 have separation $|t_2 - t_1|\,|\boldsymbol{d}|$.

Step 1: Parametrise the line Equating the ratios to t gives

$$x = 2 + 3t, \qquad y = -1 + 2t, \qquad z = 1 - t.$$

These expressions retain the negative direction ratio in the third coordinate; replacing it by a positive ratio would describe a different line. A unit increment in t therefore produces the displacement $(3, 2, -1)$.

Step 2: Locate both intersections For the first plane, substitute the three coordinates:

$$2(2 + 3t) + 3(-1 + 2t) - (1 - t) + 13 = 13 + 13t = 0.$$

Thus $t_P = -1$ and $P = (-1, -3, 2)$. For the second plane,

$$3(2 + 3t) + (-1 + 2t) + 4(1 - t) = 9 + 7t = 16.$$

Thus $t_Q = 1$, giving $Q = (5, 1, 0)$. Both parameters are unique since the coefficient of t is nonzero in each plane equation.

Step 3: Evaluate the distance The displacement from P to Q is $(6, 4, -2)$, so

$$PQ = \sqrt{6^2 + 4^2 + (-2)^2} = \sqrt{56} = 2\sqrt{14}.$$

Equivalently, the parameter difference is 2 and the direction-vector magnitude is $\sqrt{14}$. Options (A), (B) and (D) do not equal this distance; in particular, (D) omits the parameter difference.

Step 4: Conclusion The required separation is $PQ = 2\sqrt{14}$. Hence, the correct option is (C).

Quick Tip to Solve: After finding the two intersection parameters, multiply their absolute difference by the magnitude of the direction-ratio vector; this avoids a second coordinate-distance calculation.

30. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(X \leq 2) = \frac{k}{2^{16}}$, then k is equal to : [4 marks]

(A) 17

(B) 1

(C) 137

(D) 121

Correct Answer: (C) 137

Solution:

Concept:

- For $X \sim \text{Bin}(n, p)$, the mean is np and the variance is $np(1-p)$, with n a positive integer and $0 \leq p \leq 1$.
- The binomial probability is $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$. A cumulative probability includes every allowed integer outcome in its stated range.

Step 1: Recover the binomial parameters The supplied mean and variance imply

$$np = 8, \qquad np(1-p) = 4.$$

Dividing the variance by the nonzero mean gives $1-p = 4/8 = 1/2$. Hence $p = 1/2$ and $n = 8/(1/2) = 16$. These values satisfy both conditions: the mean is $16/2 = 8$, while the variance is $16/4 = 4$.

Step 2: Sum the required outcomes Since a binomial count cannot be negative, $X \leq 2$ consists precisely of the mutually exclusive events $X = 0, 1, 2$. Here the success and failure probabilities are equal, so each term has the same denominator:

$$P(X = r) = \binom{16}{r} \left(\frac{1}{2}\right)^{16}.$$

Consequently,

$$P(X \leq 2) = \frac{\binom{16}{0} + \binom{16}{1} + \binom{16}{2}}{2^{16}} = \frac{1 + 16 + 120}{2^{16}} = \frac{137}{2^{16}}.$$

The numerator 121 would omit the contribution from exactly one success, 17 omits exactly two successes, and 1 counts only zero successes. Thus none of those alternatives represents the full cumulative probability.

Step 3: Conclusion Comparing with $P(X \leq 2) = k/2^{16}$ yields $k = 137$. Hence, the correct option is (C).

Quick Tip to Solve: For a binomial variable, variance divided by mean immediately gives the failure probability. With equal success and failure probabilities, sum the binomial coefficients over the desired outcomes.

Physics (120 marks)

31. An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n, corresponding to its initial excited state is (for photon of wavelength λ, energy $E = \frac{1240\,eV}{\lambda(in\,nm)}$) : [4 marks]

(A) n = 4

(B) n = 7

(C) n = 5

(D) n = 6

Correct Answer: (C) n = 5

Solution:

Concept:

- A hydrogen-like ion has energy levels $E_n = -13.6Z^2/n^2\,\text{eV}$. For He^+, $Z = 2$, so its ground-state energy is $-54.4\,\text{eV}$.
- The energies of successively emitted photons add to the net decrease in atomic energy. With the given units, a photon has energy $1240/\lambda\,\text{eV}$ when λ is in nanometres.

Step 1: Add the two photon energies Both photons are emitted, so both energies are positive contributions to the ion's energy loss. Using the supplied conversion,

$$E_{\gamma1} = \frac{1240}{108.5} \simeq 11.4286\,\text{eV}, \qquad E_{\gamma2} = \frac{1240}{30.4} \simeq 40.7895\,\text{eV}.$$

Their sum is approximately $52.2181\,\text{eV}$. The wavelengths themselves must not be added; energy is proportional to reciprocal wavelength.

Step 2: Find the initial level For a transition from initial level n to the ground state, the net energy release is

$$54.4\left(1 - \frac{1}{n^2}\right) = 52.2181.$$

Therefore,

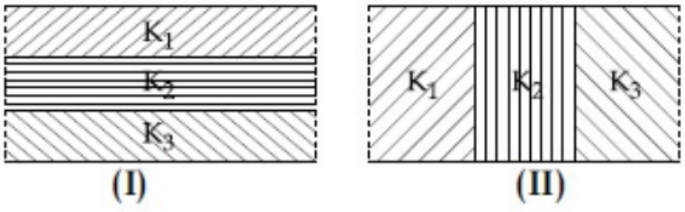
$$\frac{1}{n^2} = 1 - \frac{52.2181}{54.4} \simeq 0.04011, \qquad n \simeq 4.993.$$

The small departure from an integer is due to the wavelengths being rounded. Thus the level is $n = 5$. For comparison, the net releases for $n = 4, 6, 7$ are approximately $51.0, 52.889, 53.290\,\text{eV}$, respectively, whereas $n = 5$ gives $52.224\,\text{eV}$. The latter agrees with the supplied photon energies to their stated precision.

Step 3: Conclusion The initial excited level has principal quantum number $n = 5$. Hence, the correct option is (C).

Quick Tip to Solve: For a cascade ending in a specified level, add the photon energies first. The intermediate level cancels from the total energy difference, so it need not be determined.

32. Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in fig.I, and the second one is filled as shown in fig II. If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)): [4 marks]



(A) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$

(B) $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(C) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$

(D) $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

Correct Answer: (D) $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

Solution:

Concept:

- The original vacuum capacitance is $C = \varepsilon_0 A/d$. Dielectric layers stacked along the electric field behave as capacitors in series; regions side by side across the plate area behave in parallel.
- At the same applied voltage, stored energy is $E = \frac{1}{2}C_{\text{eq}}V^2$. Thus the energy ratio equals the ratio of the equivalent capacitances, rather than their inverse.

Step 1: Find the capacitance of arrangement I In figure I the three slabs divide the plate separation into three equal thicknesses $d/3$, each covering area A. Their individual capacitances are $3CK_1, 3CK_2, 3CK_3$. The charge is common to successive layers, so they are in series:

$$\frac{1}{C_I} = \frac{1}{3C}\left(\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}\right).$$

Writing $S = K_1K_2 + K_2K_3 + K_3K_1$ and $P = K_1K_2K_3$, this gives $C_I = 3CP/S$.

Step 2: Find the capacitance of arrangement II In figure II the three regions span the full separation d, but each has plate area $A/3$. They experience the same potential difference and act in parallel. Their capacitances are $CK_1/3, CK_2/3, CK_3/3$. Therefore,

$$C_{II} = \frac{C}{3}(K_1 + K_2 + K_3).$$

This distinction between division of thickness and division of area supplies the two factors of three.

Step 3: Compare the stored energies The common factor $V^2/2$ cancels, leaving

$$\frac{E_1}{E_2} = \frac{C_I}{C_{II}} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_1K_2+K_2K_3+K_3K_1)}.$$

For equal dielectric constants both arrangements must coincide and this expression becomes one. Options (A) and (B) fail that check, giving nine and one ninth. Option (C) reverses the required capacitance ratio for unequal dielectrics.

Step 4: Conclusion The energy ratio is $9K_1K_2K_3/[(K_1+K_2+K_3)(K_1K_2+K_2K_3+K_3K_1)]$. Hence, the correct option is (D).

Quick Tip to Solve: Identify whether the dielectric divides the plate gap or the plate area before applying series or parallel rules. At fixed voltage, energy follows capacitance directly.

33. An electromagnetic wave is represented by the electric field $\vec{E} = E_0\hat{n}\sin[\omega t + (6y - 8z)]$. Taking unit vectors in x, y and z directions to be $\hat{i}, \hat{j}, \hat{k}$, the direction of propagation $\hat{s}$, is : [4 marks]

(A) $\hat{s} = \frac{3\hat{i}-4\hat{j}}{5}$

(B) $\hat{s} = \frac{-4\hat{k}+3\hat{j}}{5}$

(C) $\hat{s} = \left(\frac{-3\hat{j}+4\hat{k}}{5}\right)$

(D) $\hat{s} = \frac{4\hat{j}-3\hat{k}}{5}$

Correct Answer: (C) $\hat{s} = \left(\frac{-3\hat{j}+4\hat{k}}{5}\right)$

Solution:

Concept:

- A plane wave written as $\sin(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})$ propagates along $\boldsymbol{k}$. With a plus sign before the spatial phase, the propagation vector points opposite to its spatial coefficient vector.
- The propagation direction is a unit vector. Divide the wave vector by its magnitude; the polarization vector in the electric-field amplitude specifies field orientation, not the travel direction.

Step 1: Read the wave vector with the correct sign The given phase is $\omega t + 6y - 8z$. Rewrite it in the standard minus-spatial-phase form:

$$\omega t + 6y - 8z = \omega t - (-6y + 8z).$$

Hence the wave vector is $\boldsymbol{k} = -6\hat{j} + 8\hat{k}$. There is no x-dependent term, so the propagation direction has no $\hat{i}$ component. This immediately rules out option (A).

Step 2: Normalise and verify the direction The magnitude of this vector is

$$|\boldsymbol{k}| = \sqrt{(-6)^2 + 8^2} = 10.$$

Its unit vector is therefore

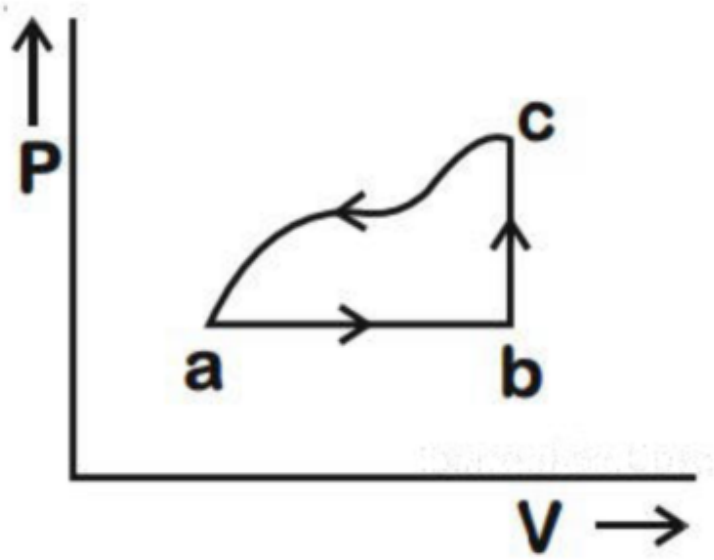
$$\hat{s} = \frac{-6\hat{j} + 8\hat{k}}{10} = \frac{-3\hat{j} + 4\hat{k}}{5}.$$

The sign can also be checked by following a constant-phase surface: $\omega\, dt + 6\, dy - 8\, dz = 0$. Motion along the displayed unit vector makes the spatial contribution negative, offsetting increasing time. Option (B) points in the opposite direction, while option (D) interchanges the relative magnitudes of the two spatial components. Neither reproduces the phase evolution of the given wave.

Step 3: Conclusion The wave propagates along $\hat{s} = (-3\hat{j} + 4\hat{k})/5$. Hence, the correct option is (C).

Quick Tip to Solve: First rewrite the phase as frequency times time minus wave vector dotted with position. Only then read and normalise the spatial coefficients; the original plus sign reverses the direction.

34. A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is –180 J. The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work done by the gas along the path abc is: [4 marks]



(A) 120 J

(B) 130 J

(C) 100 J

(D) 140 J

Correct Answer: (B) 130 J

Solution:

Concept:

- Internal energy is a state function. Its net change over a closed cycle is zero, irrespective of the heat exchanged and work done on the individual paths.
- With heat absorbed by the gas positive and work done by the gas positive, the first law reads

$Q = \Delta U + W$, or $W = Q - \Delta U$.

Step 1: Determine the internal-energy rise from a to c The path ca returns from state c to state a, and its given change is $\Delta U_{ca} = -180\,\text{J}$. Hence $U_a - U_c = -180\,\text{J}$. The forward route abc joins the same endpoints in the opposite sense, so

$$\Delta U_{abc} = U_c - U_a = +180\,\text{J}.$$

Equivalently, adding the changes around $abca$ gives $\Delta U_{abc} + \Delta U_{ca} = 0$. The change along abc is not zero merely because it is part of a cycle.

Step 2: Apply the first law to the complete forward route Both specified heat quantities are absorbed, and therefore add:

$$Q_{abc} = Q_{ab} + Q_{bc} = 250 + 60 = 310\,\text{J}.$$

The work done by the gas is consequently

$$W_{abc} = Q_{abc} - \Delta U_{abc} = 310 - 180 = 130\,\text{J}.$$

The vertical segment bc on the pressure-volume diagram is isochoric and does no work; the nonzero work is associated with expansion along ab. The numerical pressure and volume values are unnecessary because heat and the endpoint energy change already determine the work. The alternatives $120, 100, 140\,\text{J}$ do not satisfy $310 = 180 + W_{abc}$ with the given data.

Step 3: Conclusion The work done by the gas along abc is $130\,\text{J}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the return path to infer the opposite internal-energy change, then apply the first law to the entire requested route. Keep absorbed heat and work by the gas positive consistently.

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35. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his center of mass moves by a distance $\ell(\ell << \text{L})$, is close to; [4 marks]

(A) mgℓ(1 + θ_0^2)

(B) mgℓ

(C) mgℓ(1 + $\frac{\theta_0^2}{2}$)

(D) mgℓ(1 - θ_0^2)

Correct Answer: (A) mgℓ(1 + θ_0^2)

Solution:

Concept:

- At the lowest point, shortening the distance of the centre of mass from the pivot conserves angular momentum about the pivot during the rapid standing motion; the external forces exert no appreciable angular impulse there.
- The person's muscular work raises gravitational potential energy and increases kinetic energy. Retain terms to first order in ℓ/L; the option expressions also use the small-angle approximation

$2(1-\cos\theta_0) \simeq \theta_0^2$, with angle in radians.

Step 1: Find the speed before standing The centre of mass falls through height $L(1-\cos\theta_0)$ between an extreme position and the lowest point. Conservation of mechanical energy during this part of the swing gives

$$\frac{1}{2}Mv^2 = MgL(1-\cos\theta_0), \qquad v^2 = 2gL(1-\cos\theta_0).$$

For a small angular amplitude this becomes $v^2 \simeq gL\theta_0^2$.

Step 2: Calculate the kinetic-energy increase After standing, the pivot-to-centre-of-mass distance is $L-\ell$. Conservation of angular momentum gives

$$MLv = M(L-\ell)v\prime, \qquad v\prime = \frac{Lv}{L-\ell}.$$

Thus, using $\ell \ll L$,

$$\Delta K = \frac{1}{2}Mv^2\left[\left(1-\frac{\ell}{L}\right)^{-2}-1\right] \simeq \frac{Mv^2\ell}{L}.$$

The radial inward movement therefore increases kinetic energy rather than leaving the speed unchanged.

Step 3: Add the gravitational contribution The centre of mass rises by ℓ, so $\Delta U = Mg\ell$. The work done is

$$W \simeq Mg\ell + \frac{Mv^2\ell}{L} = Mg\ell[1+2(1-\cos\theta_0)] \simeq Mg\ell(1+\theta_0^2).$$

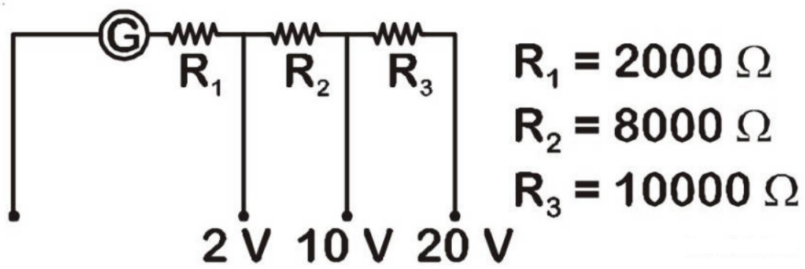
The options use m for the same person's mass denoted M in the stem. Option (B) omits the kinetic-energy increase, option (C) underestimates that increase by a factor of two, and option (D) gives it the wrong sign.

Step 4: Conclusion To the approximations represented by the options, $W \simeq Mg\ell(1+\theta_0^2)$, equivalently $mg\ell(1+\theta_0^2)$. Hence, the correct option is (A).

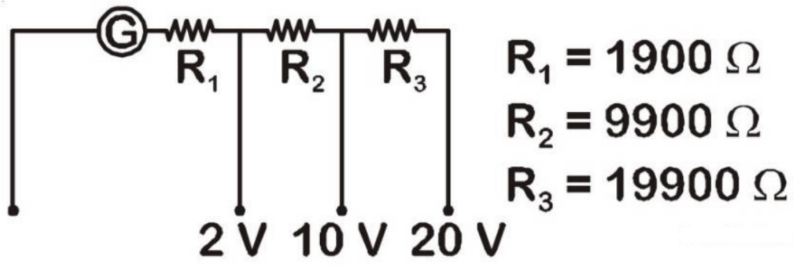
Quick Tip to Solve: When a swinging person moves inward at the bottom, conserve angular momentum first. Their work supplies both the rise in potential energy and the kinetic-energy increase.

36. A galvanometer of resistance 100 Ω has 50 divisions on its scale and has sensitivitv of 20 μA/division. It is to be converted to a voltmeter with three ranges of 0-2V, 0-10 V and 0-20 V. The appropriate circuit to do so is [4 marks]

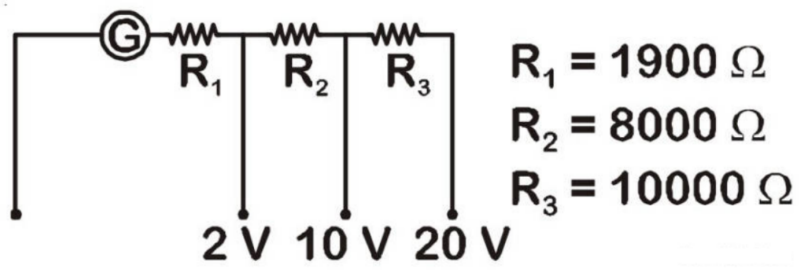
(A) Series-tapped circuit: $R_1 = 2000\,\Omega$, $R_2 = 8000\,\Omega$, $R_3 = 10000\,\Omega$; successive taps 2 V, 10 V, 20 V.



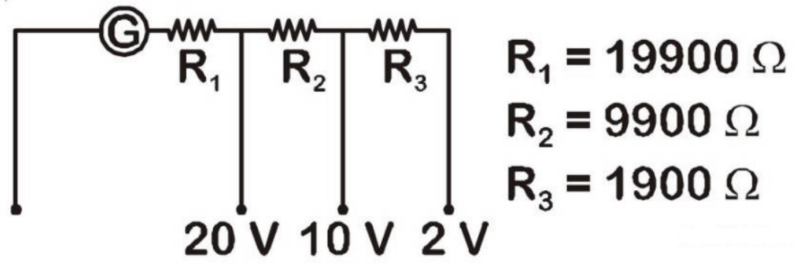
(B) Series-tapped circuit: $R_1 = 1900\,\Omega$, $R_2 = 9900\,\Omega$, $R_3 = 19900\,\Omega$; successive taps 2 V, 10 V, 20 V.



(C) Series-tapped circuit: $R_1 = 1900\,\Omega$, $R_2 = 8000\,\Omega$, $R_3 = 10000\,\Omega$; successive taps 2 V, 10 V, 20 V.

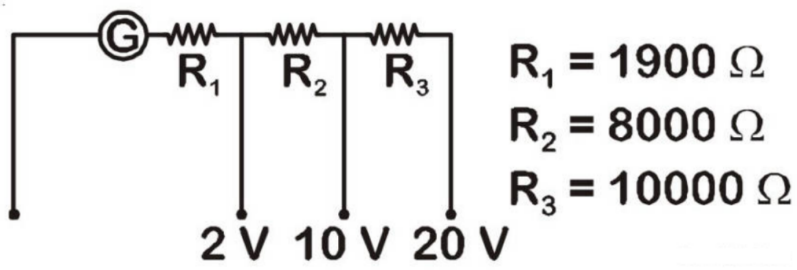


(D) Series-tapped circuit: $R_1 = 19900\,\Omega$, $R_2 = 9900\,\Omega$, $R_3 = 1900\,\Omega$; successive taps 20 V, 10 V, 2 V.



Correct Answer: (C)

Series-tapped circuit: $R_1 = 1900\,\Omega$, $R_2 = 8000\,\Omega$, $R_3 = 10000\,\Omega$; successive taps 2 V, 10 V, 20 V.



Solution:

Concept:

- A galvanometer becomes a voltmeter by adding resistance in series. At full-scale deflection, the same limiting current must flow for every selected range.
- For a range V, the required total resistance is V/I_g, including the galvanometer resistance G. In a

tapped chain the higher ranges include all preceding resistors.

Step 1: Calculate the full-scale current The sensitivity is current per division, so the current for all 50 divisions is

$$I_g = 50(20\,\mu\text{A}) = 1000\,\mu\text{A} = 1.0 \times 10^{-3}\,\text{A}.$$

The coil itself drops $I_g G = 0.1\,\text{V}$. Its resistance cannot be omitted when choosing the first series resistor.

Step 2: Determine the tapped resistances The first tap requires

$$R_1 = \frac{2}{0.001} - 100 = 1900\,\Omega.$$

The increase from 2 V to 10 V requires

$$R_2 = \frac{10-2}{0.001} = 8000\,\Omega,$$

while the increase from 10 V to 20 V requires

$$R_3 = \frac{20-10}{0.001} = 10000\,\Omega.$$

Thus the complete resistances are 2000, 10000 and 20000 ohms. Option A neglects the coil resistance in its first segment. Option B incorrectly uses the separate-range series resistances as successive segments. Option D has reversed taps and inconsistent cumulative resistances.

Step 3: Conclusion The required chain is $R_1 = 1900\,\Omega$, $R_2 = 8000\,\Omega$, $R_3 = 10000\,\Omega$, with successive 2 V, 10 V and 20 V taps. Hence, the correct option is (C).

Quick Tip to Solve: For tapped voltmeters, subtract the coil resistance only for the first segment; obtain later segments from successive voltage-range differences divided by the full-scale current.

37. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is : [4 marks]

(A) 0.28 ms^{-1}

(B) 0.47 ms^{-1}

(C) 0.20 ms^{-1}

(D) 0.14 ms^{-1}

Correct Answer: (C) 0.20 ms^{-1}

Solution:

Concept:

- With no external horizontal force, the total horizontal momentum of the man and son remains equal to its initial value, zero.
- Relative speed is the difference of signed velocities. Since the two people recoil in opposite

directions, their speeds relative to the ground add to the given separation speed.

Step 1: Relate their ground-frame velocities Let the son move in the positive direction with speed v_s, and let the man recoil with speed v_m in the negative direction. Conservation of momentum gives

$$20v_s - 50v_m = 0, \qquad v_s = 2.5v_m.$$

The forces during the push are equal and opposite, but the different masses acquire different speeds. The heavier man must move more slowly than his son.

Step 2: Use the given relative speed The stated $0.70\,\mathrm{m\,s^{-1}}$ is measured relative to the man, not relative to the surface. Therefore

$$v_s + v_m = 0.70.$$

Substituting the momentum relation,

$$3.5v_m = 0.70, \qquad v_m = 0.20\,\mathrm{m\,s^{-1}}.$$

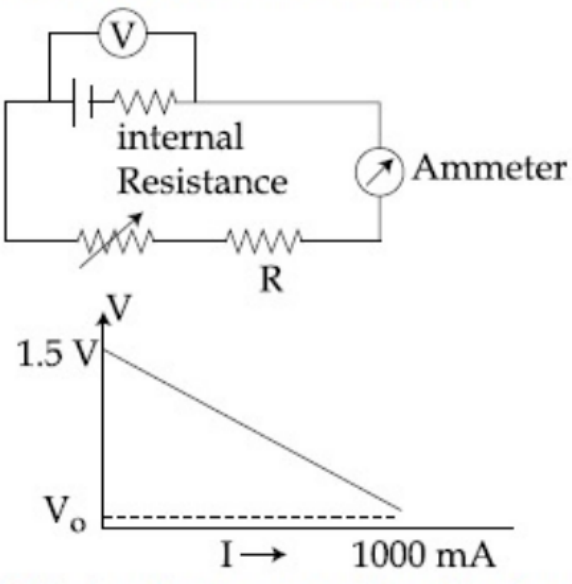
The son has ground speed $0.50\,\mathrm{m\,s^{-1}}$; their momenta are $+10$ and $-10\,\mathrm{kg\,m\,s^{-1}}$, and their relative speed is indeed 0.70. The other proposed speeds fail these simultaneous conditions; treating 0.70 as the son's ground speed would misuse the reference frame.

Step 3: Conclusion The speed of the man relative to the frictionless surface is $0.20\,\mathrm{m\,s^{-1}}$, opposite to the son's motion. Hence, the correct option is (C).

Quick Tip to Solve: Write momentum using ground-frame velocities first; then translate a stated relative speed into the sum of recoil speeds.

38. To verify Ohm's law, a student connects the voltmeter across the battery as, shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained :

If V_0 is almost zero, identify the correct statement : [4 marks]



(A) The value of the resistance R is 1.5 Ω

(B) The emf of the battery is l.5 V and its internal resistance is 1.5 Ω

(C) The emf of the battery is l.5 V and the value of R is 1.5 Ω

(D) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA

Correct Answer: (B) The emf of the battery is l.5 V and its internal resistance is 1.5 Ω

Solution:

Concept:

- The terminal voltage of a cell delivering current is $V = \mathcal{E} - Ir$, where $\mathcal{E}$ is its emf and r its internal resistance.
- On a graph of terminal voltage against current, the vertical intercept is the emf and the magnitude of the negative slope is the internal resistance, not the external load resistance.

Step 1: Identify the voltage being measured The voltmeter is connected across the battery terminals rather than across the resistor R. Consequently the plotted voltage decreases as current increases, because more of the emf is lost across the battery's internal resistance. At zero current there is no internal voltage drop, giving

$$\mathcal{E} = V(0) = 1.5\,\text{V}.$$

This is an open-circuit intercept; it does not mean that the loaded battery always supplies 1.5 V.

Step 2: Read the slope with current in amperes The second point is approximately $(1000\,\text{mA}, 0) = (1.0\,\text{A}, 0)$, since V_0 is almost zero. Thus

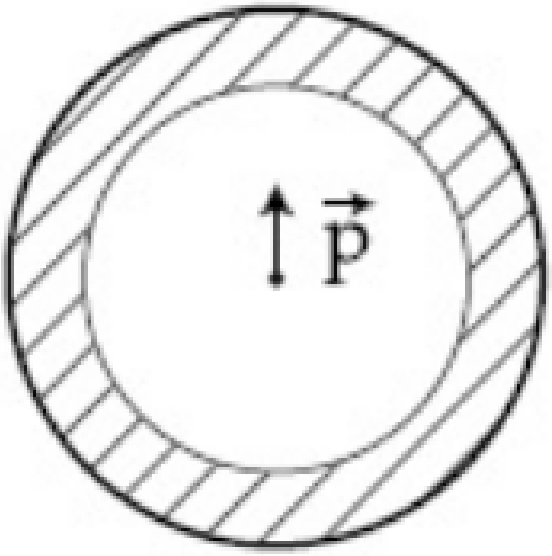
$$r = \frac{1.5 - V_0}{1.0 - 0} \simeq 1.5\,\Omega.$$

Neither the value of the fixed external resistor R alone nor its share of the external voltage can be obtained from this slope; the external circuit also contains a variable resistor. Options A and C incorrectly assign the internal-resistance slope to R. Option D contradicts the graph, which gives approximately zero terminal voltage at 1000 mA.

Step 3: Conclusion The battery has emf $1.5\,\mathrm{V}$ and internal resistance approximately $1.5\,\Omega$. Hence, the correct option is (B).

Quick Tip to Solve: Before interpreting a V-I graph, locate the voltmeter terminals: a battery-terminal graph has slope $-r$, whereas a resistor graph has slope $+R$.

39. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{P}$ as shown. In this case : [4 marks]



(A) surface charge density on the inner surface is uniform and equal to $\frac{(Q/2)}{4\pi a^2}$

(B) surface charge density on the inner surface of the shell is zero everywhere

(C) surface charge density on the outer surface depends on $\left|\vec{P}\right|$

(D) electric field outside the shell is the same as that of a point charge at the centre of the shell

Correct Answer: (D) electric field outside the shell is the same as that of a point charge at the centre of the shell

Solution:

Concept:

- In electrostatic equilibrium the electric field inside a conductor is zero and its entire surface is equipotential.
- A dipole has zero net charge. It induces nonuniform charge on the cavity surface, but the total induced inner-surface charge is the negative of the net charge in the cavity.

Step 1: Find the charges on the two surfaces Take a Gaussian surface lying inside the conducting

material and surrounding the cavity. Its flux is zero because the field in the metal vanishes. The dipole encloses equal positive and negative charges, so

$$q_{\text{cavity}} = 0, \qquad q_{\text{inner}} = 0.$$

This is a statement about the total charge, not about its local density. The dipole creates a nonuniform cavity potential and hence a nonuniform induced distribution with positive and negative regions on the inner wall. Thus neither a uniform nonzero density nor zero density everywhere is correct.

Step 2: Determine the external field The conductor carries net charge Q, and its inner surface has zero net charge, so its outer surface carries Q. The exterior boundary is a spherical equipotential surface with no external sources. By spherical symmetry of this boundary-value problem, or uniqueness of the electrostatic solution,

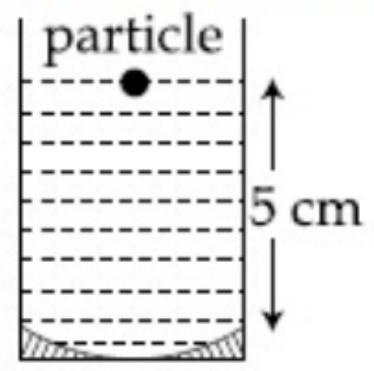
$$V(r) = \frac{Q}{4\pi\varepsilon_0 r}, \qquad E(r) = \frac{Q}{4\pi\varepsilon_0 r^2} \quad (r > b).$$

The dipole affects the cavity distribution but is shielded from the exterior. The outer density is $Q/(4\pi b^2)$, independent of the magnitude or orientation of the dipole moment.

Step 3: Conclusion Outside the shell, the field is exactly that of a point charge Q at its centre. Hence, the correct option is (D).

Quick Tip to Solve: For a cavity inside a spherical conductor, separate the nonuniform inner induced charge from the outer equipotential boundary; zero total induced charge does not mean zero local density.

40. A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is dose to: (Refractive index of water = 1.33) [4 marks]



(A) 11.7 cm

(B) 6.7 cm

(C) 13.4 cm

(D) 8.8 cm

Correct Answer: (D) 8.8 cm

Solution:

Concept:

- A spherical concave mirror has focal length equal to half its radius of curvature. Its geometrical focal length is unchanged when immersed in a uniform transparent medium.
- First form the mirror image within water, then apply refraction at the plane water-air surface. For paraxial viewing from air, apparent depth is real depth divided by the refractive index of water.

Step 1: Locate the image produced by the mirror The radius is 40 cm, so the concave mirror focal-length magnitude is 20 cm. The object is only 5 cm in front of the mirror and is therefore within its focus. With Cartesian signs,

$$f = -20\,\text{cm}, \qquad u = -5\,\text{cm}.$$

The mirror formula gives

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{20} + \frac{1}{5} = \frac{3}{20}, \qquad v = \frac{20}{3}\,\text{cm}.$$

The positive image distance means a virtual image behind the reflecting surface, rather than above the water.

Step 2: Apply refraction to the image depth The mirror lies 5 cm below the water surface. Hence the virtual mirror image has depth

$$h = 5 + \frac{20}{3} = \frac{35}{3}\,\text{cm}.$$

The rays emerging into air appear to come from depth

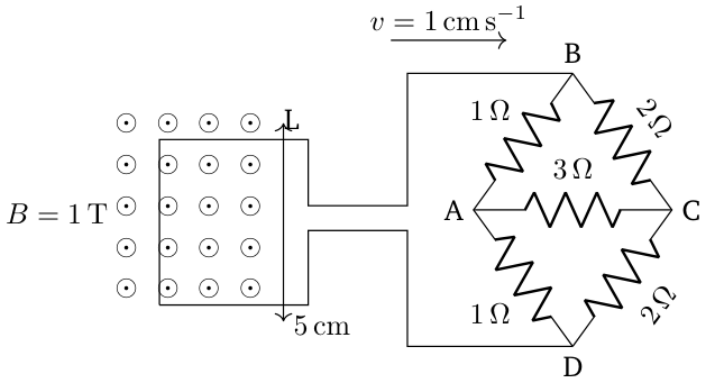
$$d = \frac{h}{1.33} = \frac{35}{3(1.33)} \simeq 8.77\,\text{cm}.$$

The value 6.7 cm is only the virtual distance behind the mirror; 11.7 cm is the unrefracted depth below the surface. Neither is the distance seen by the observer. Multiplying by refractive index would reverse the apparent-depth effect and cannot give the correct observation.

Step 3: Conclusion The apparent image distance below the water surface is approximately $8.8\,\text{cm}$. Hence, the correct option is (D).

Quick Tip to Solve: In a submerged-mirror problem, add the mirror depth to a virtual image distance behind it before dividing by the refractive index.

41. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cm s^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to : [4 marks]



(A) 115 μA

(B) 170 μA

(C) 60 μA

(D) 150 μA

Correct Answer: (B) 170 μA

Solution:

Concept:

- The magnitude of the induced emf is the rate of change of magnetic flux: $\mathcal{E} = B\ell v$, where ℓ is the side crossing the straight magnetic-field boundary.
- A balanced Wheatstone bridge has no current through its central branch. Replace the remaining two series paths by their parallel equivalent before including the resistance of the moving loop.

Step 1: Reduce the resistance network The emf is applied between nodes B and D of the diamond. The arm resistances satisfy

$$\frac{R_{BA}}{R_{AD}} = \frac{1}{1} = \frac{R_{BC}}{R_{CD}} = \frac{2}{2}.$$

Therefore A and C have equal potentials and no current flows through the 3-ohm link AC. The two B-to-D paths have resistances $1 + 1 = 2\,\Omega$ and $2 + 2 = 4\,\Omega$, respectively. Their equivalent is

$$R_{BD} = \frac{2 \times 4}{2 + 4} = \frac{4}{3}\,\Omega.$$

The loop resistance 1.7 ohms is in series with this external network.

Step 2: Calculate emf and current Convert the dimensions before substituting: $\ell = 0.05\,\text{m}$ and $v = 0.01\,\text{m s}^{-1}$. As the loop leaves the field, its enclosed flux changes at rate

$$\mathcal{E} = (1)(0.05)(0.01) = 5.0 \times 10^{-4}\,\text{V}.$$

The current magnitude is

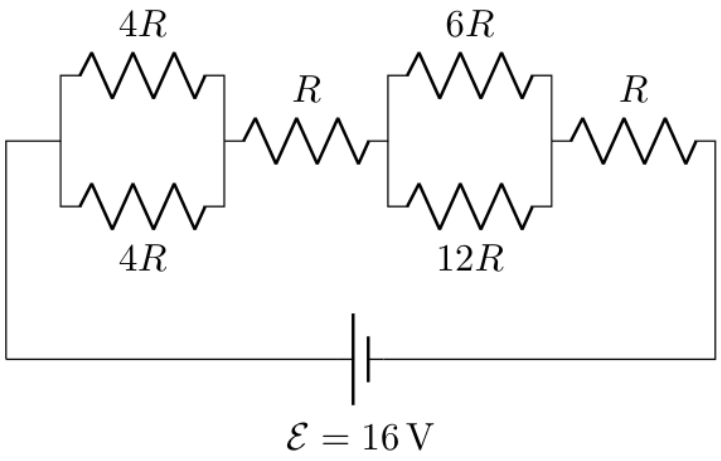
$$I = \frac{5.0 \times 10^{-4}}{1.7 + 4/3} = 1.648 \times 10^{-4}\,\text{A} \simeq 165\,\mu\text{A}.$$

The difference from 170 microamperes is only 5 microamperes, whereas from 150 it is 15. Thus the nearest listed value is 170 microamperes; 115 and 60 do not follow from the balanced network.

Step 3: Conclusion The loop current is approximately $165\,\mu$A, closest to $170\,\mu$A. Hence, the correct option is (B).

Quick Tip to Solve: Check bridge balance before combining resistors: the middle resistor then drops out, but the moving loop's own resistance must still be added.

42. The resistive network shown below is connected to a D.C. source of 16 V. The power consumed by the network is 4 Watt. The value of R is: [4 marks]



(A) 16 Ω

(B) 1 Ω

(C) 8 Ω

(D) 6 Ω

Correct Answer: (C) 8 Ω

Solution:

Concept:

- The total power dissipated by a resistive circuit supplied at voltage V is $P = V^2/R_{\text{eq}}$.
- Resistors between the same two nodes are parallel. Reduce each parallel block first, then add the blocks and the connecting resistors that are in series.

Step 1: Reduce the two parallel blocks The two resistors labelled $4R$ form the left parallel block, with

equivalent

$$R_L = \frac{(4R)(4R)}{4R + 4R} = 2R.$$

The right block contains $6R$ and $12R$, so

$$R_R = \frac{(6R)(12R)}{6R + 12R} = 4R.$$

The connecting resistor between these blocks is R, and another R connects the right block to the return wire. There is no branching path around these two connecting resistors, so both must be included in series.

Step 2: Use the measured power The whole network therefore has equivalent resistance

$$R_{\text{eq}} = 2R + R + 4R + R = 8R.$$

The supplied voltage and total power independently give

$$R_{\text{eq}} = \frac{16^2}{4} = 64\,\Omega.$$

Thus

$$8R = 64\,\Omega, \qquad R = 8\,\Omega.$$

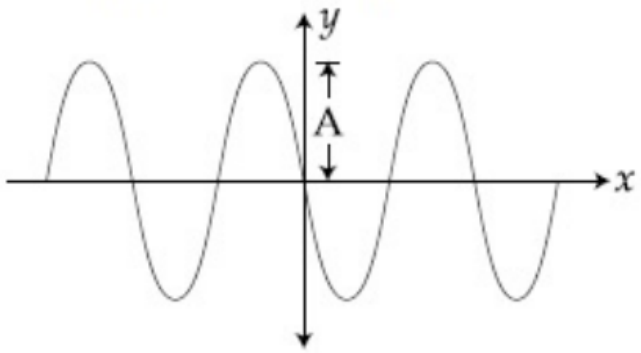
As a check, the total current is $16/64 = 0.25\,\text{A}$, so $VI = 4\,\text{W}$, as required. Substituting the other proposed values, 16, 1 and 6 ohms, gives total powers 2, 32 and approximately 5.33 W; none matches the measured 4 W.

Step 3: Conclusion The unknown resistance is $R = 8\,\Omega$. Hence, the correct option is (C).

Quick Tip to Solve: Use the power to find the total resistance first; then equate it to the resistance obtained by simplifying the labelled circuit.

43. A progressive wave travelling along the positive x-direction is represented by y(x,t) = Asin(kx – ωt + ϕ). Its snapshot at t = 0 is given in the figure.

For this wave, the phase ϕ is : [4 marks]



(A) $\frac{\pi}{2}$

(B) π

(C) 0

(D) $-\frac{\pi}{2}$

Correct Answer: (B) π

Solution:

Concept:

- At time zero the snapshot is $y(x,0) = A\sin(kx+\phi)$, with positive amplitude A and wave number k.
- Displacement at the origin determines $\sin\phi$, whereas the spatial slope determines the sign of $\cos\phi$. Both must agree with the graph.

Step 1: Apply the physical principle The curve crosses the horizontal axis at the origin. Thus at $x = 0, t = 0$, the displacement is zero:

$$0 = A\sin\phi \quad \Rightarrow \quad \sin\phi = 0.$$

The possible phases are therefore zero or π, modulo 2π. This rules out the options $\pi/2$ and $-\pi/2$: they would place the particle at positive and negative extreme displacement respectively, whereas the displayed snapshot has zero displacement at the origin.

Step 2: Evaluate and compare the options Immediately to the right of the origin, the curve descends below the axis, so the spatial slope is negative. Differentiate the displacement with respect to position at fixed time:

$$\left.\frac{\partial y}{\partial x}\right|_{0,0} = Ak\cos\phi < 0.$$

Since $Ak > 0$, this requires $\cos\phi < 0$. The zero-phase candidate gives a positive slope and is rejected. The phase π gives $\cos\pi = -1$, consistent with the downward crossing. The direction of travel alone does not select the phase; the displacement and slope of the supplied snapshot provide that information.

Step 3: Conclusion The phase is $\boxed{\phi = \pi}$, modulo 2π. Hence, the correct option is (B).

Quick Tip to Solve: At a zero crossing, a rising spatial snapshot selects phase zero and a falling snapshot selects phase π.

---

44. A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40 π rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8 $\times$ 10^{-9} T, then the charge carried by the ring is close to (μ_0 = 4π $\times$ 10^{-7} N/A^2). [4 marks]

(A) 7 $\times$ 10^{-6} C

(B) 4 $\times$ 10^{-5} C

(C) 2 $\times$ 10^{-6} C

(D) 3 $\times$ 10^{-5} C

Correct Answer: (D) 3 $\times$ 10^{-5} C

Solution:

Concept:

- A charge Q completing one revolution in period T gives equivalent current $I = Q/T = Q\omega/(2\pi)$.
- The field at the centre of a circular current loop of radius r is $B = \mu_0 I/(2r)$. Uniform charge and steady rotation produce a steady current.

Step 1: Apply the physical principle Convert the radius to SI units: $r = 10\,\text{cm} = 0.10\,\text{m}$. The angular speed is $40\pi\,\text{rad}\,\text{s}^{-1}$, so the number of revolutions per second is

$$f = \frac{\omega}{2\pi} = 20\,\text{s}^{-1}.$$

During each revolution the total charge passes a fixed radial line once. Consequently $I = Qf = 20Q$, in amperes when charge is in coulombs. Angular speed must not be substituted directly as the frequency; that would omit the factor 2π.

Step 2: Evaluate and compare the options Use the magnetic field to determine the required current:

$$I = \frac{2rB}{\mu_0} = \frac{2(0.10)(3.8 \times 10^{-9})}{4\pi \times 10^{-7}}\,\text{A} \approx 6.05 \times 10^{-4}\,\text{A}.$$

Now divide this current by the revolution frequency:

$$Q = \frac{I}{f} = \frac{6.05 \times 10^{-4}}{20}\,\text{C} \approx 3.02 \times 10^{-5}\,\text{C}.$$

The listed value $3 \times 10^{-5}\,\text{C}$ is closest. Since the magnetic field is proportional to the charge for fixed radius and angular speed, the other proposed charges would give different fields and are less consistent with the given value.

Step 3: Conclusion The charge magnitude is $\boxed{Q \approx 3 \times 10^{-5}\,\text{C}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a rotating ring, combine $I = Q\omega/(2\pi)$ and $B = \mu_0 I/(2r)$, converting the radius to metres first.

---

45. When M_1 gram of ice at –10°C (specific heat = 0.5 cal g^{-1} °C^{-1}) is added to M_2 gram of water at 50C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in cal g^{-1} is : [4 marks]

(A) $\frac{50M_2}{M_1} - 5$
(B) $\frac{50M_2}{M_1}$
(C) $\frac{5M_2}{M_1} - 5$
(D) $\frac{5M_1}{M_2} - 50$

Correct Answer: (A) $\frac{50M_2}{M_1} - 5$

Solution:

Concept:

- Heat lost by the warm water equals heat gained by the ice. The specific heat of water is $1\,\text{cal}\,\text{g}^{-1}\,^\circ\text{C}^{-1}$.
- Ice below its melting point first absorbs sensible heat $mc\Delta T$, then latent heat mL to melt at constant temperature.

Step 1: Apply the physical principle The warm water cools from 50°C to the stated final temperature 0°C. Its heat loss is

$$Q_{\text{lost}} = M_2(1)(50-0) = 50M_2\,\text{cal}.$$

Meanwhile the ice initially at -10°C first warms to its melting point. The heat needed for this stage is

$$Q_{\text{warm}} = M_1(0.5)(10) = 5M_1\,\text{cal}.$$

We cannot assign all the heat lost by the water to melting: some of it has already been used to raise the temperature of the solid ice.

Step 2: Evaluate and compare the options All of the ice melts, so its latent-heat requirement is M_1L. The meltwater remains at zero degrees, and no additional warming term is needed. Equating heat lost and gained gives

$$50M_2 = 5M_1 + M_1L.$$

Dividing by the mass of ice and rearranging,

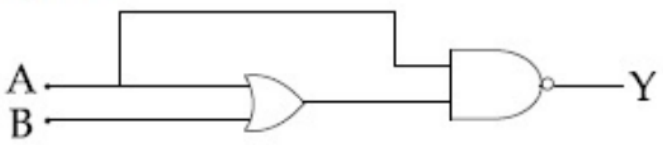
$$L = \frac{50M_2}{M_1} - 5.$$

The expression $50M_2/M_1$ omits the warming stage. Using a factor five for the initial water temperature or interchanging the two masses does not reproduce the heat balance, so the remaining options are excluded. The subtraction of five is in calories per gram because warming each gram of ice consumes five calories.

Step 3: Conclusion The latent heat is $\boxed{L = (50M_2/M_1 - 5)\,\text{cal}\,\text{g}^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: For ice below zero, subtract the heat needed to warm it to zero before dividing the remaining heat by the mass melted.

46. The truth table for the circuit given in the fig. is: [4 marks]



(A)
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

(B)
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(C)
A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(D)
A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

Correct Answer: (C)
A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

Solution:

Concept:

- The first gate is an OR gate, with output $X = A + B$, where the plus sign denotes Boolean OR.
- The final gate is NAND: the small circle at its output denotes inversion. Its two inputs are the bypassed signal A and the OR output.

Step 1: Apply the physical principle Trace the top wire from the input: it takes A directly to one input of the final gate. The lower input of the final gate receives $A + B$. Therefore

$$Y = \overline{A(A + B)}.$$

Using Boolean absorption, $A(A + B) = AA + AB = A + AB = A$. Thus the entire network is

equivalent to an inverter acting on A:

$$Y = \overline{A}.$$

The output is independent of B. Ignoring the inversion bubble would incorrectly give A, while treating the final gate as NOR would give a different truth table.

Step 2: Evaluate and compare the options For inputs $A = 0, B = 0$, one NAND input is zero, so the output is one. The same remains true for $A = 0, B = 1$. If $A = 1$, the OR output is necessarily one for either value of B; both NAND inputs are then one and the output is zero. The complete table is

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

Only the third listed table has this sequence. The all-one table fails for $A = 1$; the second table fails for inputs 01; and the last table gives the complement of the required output for every row.

Step 3: Conclusion The output sequence is $\boxed{1, 1, 0, 0}$ for inputs $00, 01, 10, 11$. Hence, the correct option is (C).

Quick Tip to Solve: Trace bypass wires before writing the Boolean expression, then use absorption to simplify the circuit before filling its table.

47. Which of the following combinations has the dimension of electrical resistance ($\in_0$ is the permittivity of vacuum and μ_0 is the permeability of vacuum)?

[4 marks]

(A) $\sqrt{\frac{\in_0}{\mu_0}}$

(B) $\frac{\in_0}{\mu_0}$

(C) $\sqrt{\frac{\mu_0}{\in_0}}$

(D) $\frac{\mu_0}{\in_0}$

Correct Answer: (C) $\sqrt{\frac{\mu_0}{\in_0}}$

Solution:

Concept:

- Electrical resistance has dimensions $[R] = ML^2T^{-3}I^{-2}$, because it is voltage divided by current.
- Permeability has dimensions $[\mu_0] = MLT^{-2}I^{-2}$, while permittivity has dimensions $[\varepsilon_0] = M^{-1}L^{-3}T^4I^2$. Here I denotes current dimension.

Step 1: Apply the physical principle The force per unit length between parallel currents is proportional to $\mu_0 I^2/r$. Hence the dimensions of permeability are those of force divided by current squared:

$$[\mu_0] = MLT^{-2}I^{-2}.$$

Coulomb's law gives $F = q^2/(4\pi\varepsilon_0 r^2)$. Charge has dimensions IT, so

$$[\varepsilon_0] = \frac{I^2T^2}{(MLT^{-2})L^2} = M^{-1}L^{-3}T^4I^2.$$

The numerical factor 4π is dimensionless and plays no role in identifying the correct combination.

Step 2: Evaluate and compare the options Form the ratio of permeability to permittivity:

$$\left[\frac{\mu_0}{\varepsilon_0}\right] = M^2L^4T^{-6}I^{-4} = [R]^2.$$

Taking its square root gives exactly $ML^2T^{-3}I^{-2}$, the dimensions of resistance. The reverse square root has dimensions of inverse resistance, or conductance. The unrooted ratios have dimensions of resistance squared and inverse resistance squared, respectively. Thus the presence of the square root, as well as which constant is in the numerator, matters. This also agrees with the known vacuum wave impedance, whose unit is the ohm.

Step 3: Conclusion The required combination is $\boxed{\sqrt{\mu_0/\varepsilon_0}}$. Hence, the correct option is (C).

Quick Tip to Solve: The vacuum impedance $\sqrt{\mu_0/\varepsilon_0}$ has units of ohms; reversing the ratio gives inverse ohms.

48. A point dipole $\vec{p} = -p_0\hat{x}$ is kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance d are, respectively: (Take V= 0 at infinity) [4 marks]

(A) $\frac{|\vec{p}|}{4\pi\epsilon_0 d^2}, \frac{-\vec{p}}{4\pi\epsilon_0 d^3}$

(B) $0, \frac{\vec{p}}{4\pi\epsilon_0 d^3}$

(C) $\frac{|\vec{p}|}{4\pi\epsilon_0 d^2}, \frac{\vec{p}}{4\pi\epsilon_0 d^3}$

(D) $0, \frac{-\vec{p}}{4\pi\epsilon_0 d^3}$

Correct Answer: (D) $0, \frac{-\vec{p}}{4\pi\epsilon_0 d^3}$

Solution:

Concept:

- The potential of a point dipole is $V = (\mathbf{p}\cdot\hat{\mathbf{r}})/(4\pi\varepsilon_0 r^2)$, with zero potential at infinity.
- The dipole field is $\mathbf{E} = [3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}]/(4\pi\varepsilon_0 r^3)$. Its equatorial field is opposite to its moment.

Step 1: Apply the physical principle The given moment $\mathbf{p} = -p_0\hat{\mathbf{x}}$ is perpendicular to any position vector along the y-axis. Thus

$$\mathbf{p}\cdot\hat{\mathbf{r}} = 0.$$

The potential at distance d is consequently $V = 0$. In terms of the two charges forming the dipole, their distances from an equatorial point are equal, so their scalar potentials cancel. This excludes the two options with a nonzero potential. The cancellation of potential does not imply cancellation of the vector electric fields.

Step 2: Evaluate and compare the options Insert the zero dot product into the general electric-field

expression:

$$\mathbf{E} = -\frac{\mathbf{p}}{4\pi\varepsilon_0 d^3} = \frac{p_0\hat{\mathbf{x}}}{4\pi\varepsilon_0 d^3}.$$

Thus the field points along positive x, opposite to the dipole moment. Of the two remaining options, the expression with $+\mathbf{p}$ has the wrong direction. A potential that vanishes all along the equatorial line can still vary in directions away from that line; the field depends on that full spatial gradient. Therefore it is consistent to have zero potential and a nonzero field here. The inverse-cube distance dependence is also the expected result for a point-dipole electric field.

Step 3: Conclusion The pair is $\boxed{V = 0, \quad \mathbf{E} = -\mathbf{p}/(4\pi\varepsilon_0 d^3)}$. Hence, the correct option is (D).

Quick Tip to Solve: On a dipole equator, potential vanishes but field does not: its direction is opposite to the dipole moment.

49. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume ? (R = 8.3 J/mol K) [4 marks]

(A) 21.6 J/mol K

(B) 17.4 J/mol K

(C) 15.7 J/mol K

(D) 19.7 J/mol K

Correct Answer: (B) 17.4 J/mol K

Solution:

Concept:

- Equipartition gives molar heat capacity $C_V = fR/2$ for f active degrees of freedom.
- Helium has three translational degrees of freedom. Rigid hydrogen has three translational and two rotational degrees of freedom; its vibration is excluded.
- The heat capacities of components add; divide the total heat capacity by total moles to obtain the mixture molar value.

Step 1: Apply the physical principle For monatomic helium, $C_{V,\mathrm{He}} = 3R/2$. Two moles therefore require heat $2(3R/2)\Delta T = 3R\Delta T$ for a temperature rise ΔT. For rigid diatomic hydrogen, $C_{V,\mathrm{H_2}} = 5R/2$, so three moles require $3(5R/2)\Delta T = (15R/2)\Delta T$. The rigidity assumption is essential: no vibrational degree of freedom is counted. Both components experience the same final temperature change in the mixture.

Step 2: Evaluate and compare the options At constant volume the supplied heat changes the internal energy. Adding the two component contributions gives

$$Q = \left(3R + \frac{15R}{2}\right)\Delta T = \frac{21R}{2}\Delta T.$$

There are $2+3=5$ moles altogether. Therefore

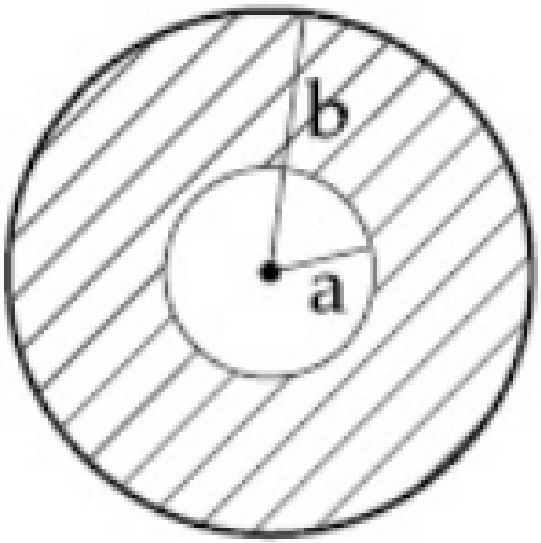
$$C_{V,\text{mix}} = \frac{21R/2}{5} = 2.1R = 2.1(8.3) = 17.43\,\text{J mol}^{-1}\text{K}^{-1}.$$

The nearest option is 17.4. An unweighted mean would ignore the unequal mole numbers; using constant-pressure capacities would add an inappropriate contribution, and including vibration would contradict the rigid-molecule condition. None of these alternatives describes the specified mixture.

Step 3: Conclusion The molar heat capacity is $\boxed{17.4\,\text{J mol}^{-1}\text{K}^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Weight each molar heat capacity by its number of moles, add, then divide by total moles; rigid diatomic molecules have five degrees of freedom.

50. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is: [4 marks]



(A) $\sqrt{\frac{a^2+b^2+ab}{2}}$

(B) $\sqrt{\frac{a+b}{3}}$

(C) $\sqrt{\frac{a^2+b^2+ab}{3}}$

(D) $\sqrt{\frac{a+b}{2}}$

Correct Answer: (C) $\sqrt{\frac{a^2+b^2+ab}{3}}$

Solution:

Concept:

- Radius of gyration about an axis is defined by $I = Mk^2$, where I is moment of inertia and M is total mass.

- For a thin ring of radius r and width dr, area is $2\pi r\,dr$. Its mass is local surface density times this area; every element is at distance r from the perpendicular central axis.

Step 1: Find the mass of a concentric ring The material occupies only the annulus, so the integration limits are the hole radius a and outer radius b. Using the given density,

$$dm = \frac{\sigma_0}{r}(2\pi r\,dr) = 2\pi\sigma_0\,dr.$$

Although the area of a ring increases with radius, its density decreases in the same proportion. Consequently equal radial widths have equal mass. The total mass is

$$M = \int_a^b 2\pi\sigma_0\,dr = 2\pi\sigma_0(b-a).$$

Step 2: Integrate the moment of inertia For the same central axis the ring contributes $dI = r^2 dm$. Hence

$$I = 2\pi\sigma_0 \int_a^b r^2\,dr = \frac{2\pi\sigma_0}{3}(b^3 - a^3).$$

Divide by the mass rather than substituting the formula for a uniform disc:

$$k^2 = \frac{I}{M} = \frac{b^3 - a^3}{3(b-a)} = \frac{a^2 + ab + b^2}{3}.$$

The cancellation uses $b^3 - a^3 = (b-a)(b^2 + ab + a^2)$.

Step 3: Check the possible expressions Options (B) and (D) have a quantity of dimension length under a square root and therefore cannot represent a radius. Option (A) has correct dimensions but uses the factor associated with a different radial mass weighting. As a tends to b, our result tends to $k = b$, as required for a thin ring.

Step 4: Conclusion The radius of gyration is $k = \sqrt{\frac{a^2 + b^2 + ab}{3}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a density proportional to $1/r$, cancel the r in the annular area element before integrating; equal radial intervals then contain equal mass.

51. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10\ ms^{-2}$) : [4 marks]

(A) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ ms^{-1}

(B) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ ms^{-1}

(C) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ ms^{-1}

(D) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ ms^{-1}

Correct Answer: (A) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ ms^{-1}

Solution:

Concept:

- For a projectile launched from the origin with speed v_0 and elevation θ_0, $y = x\tan\theta_0 - gx^2/(2v_0^2\cos^2\theta_0)$. Air resistance is neglected and gravity is constant.
- The coefficient of x determines the launch angle, while the coefficient of x^2 determines the horizontal component of launch velocity.

Step 1: Compare the linear coefficients Comparing the given trajectory $y = 2x - 9x^2$ with the standard form gives

$$\tan\theta_0 = 2.$$

Since the projectile is launched upward and forward, θ_0 is acute. A right triangle with opposite side 2 and adjacent side 1 therefore gives

$$\cos\theta_0 = \frac{1}{\sqrt{5}}, \qquad \sin\theta_0 = \frac{2}{\sqrt{5}}.$$

Thus $\theta_0 = \cos^{-1}(1/\sqrt{5})$.

Step 2: Compare the quadratic coefficients The coefficient of the negative quadratic term gives

$$\frac{g}{2v_0^2\cos^2\theta_0} = 9.$$

Substituting $g = 10\,\mathrm{m\,s^{-2}}$ and $\cos^2\theta_0 = 1/5$,

$$\frac{10}{2v_0^2/5} = 9, \qquad v_0^2 = \frac{25}{9}.$$

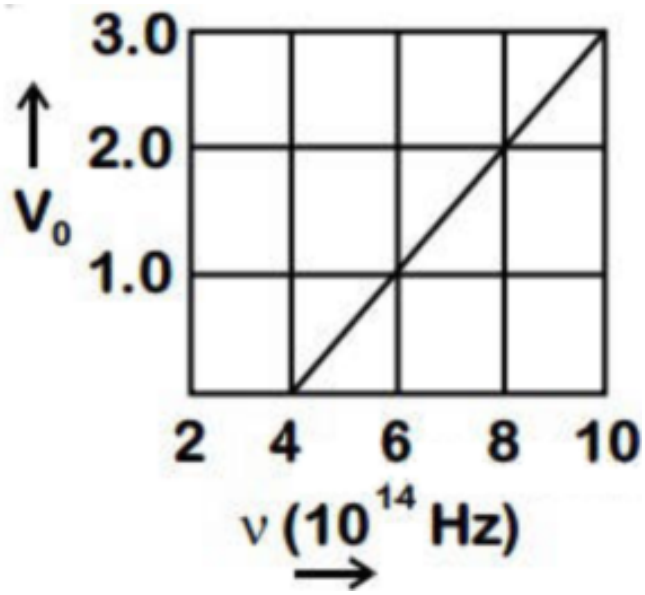
Speed is positive, so $v_0 = 5/3\,\mathrm{m\,s^{-1}}$. The horizontal component is then $v_0/\sqrt{5} = \sqrt{5}/3\,\mathrm{m\,s^{-1}}$.

Step 3: Check both entries of each option Option (C) has an equivalent correct angle but the wrong speed. Option (D) gives the correct speed with the complementary angle, whose tangent would be $1/2$. Option (B) has both the wrong angle and speed. In option (A), substituting the angle and speed reproduces both coefficients, including the large downward curvature 9.

Step 4: Conclusion $\theta_0 = \cos^{-1}(1/\sqrt{5})$ and $v_0 = \frac{5}{3}\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Read $\tan\theta$ directly from the coefficient of x; then use the coefficient of $-x^2$ to find speed, checking both entries of a paired option.

52. The stopping potential V_0 (in volt) as a function of frequency (υ) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be:
(Given: Planck's constant (h) = 6.63×10^{-34} Js, electron charges e = 1.6×10^{-19} C) [4 marks]



(A) 1.95 eV

(B) 2.12 eV

(C) 1.82 eV

(D) 1.66 eV

Correct Answer: (D) 1.66 eV

Solution:

Concept:

- Einstein's photoelectric equation is $eV_0 = h\nu - \phi$, where ϕ is the work function and V_0 is the stopping potential.
- At threshold frequency ν_{th}, the emitted electrons have zero maximum kinetic energy, so $V_0 = 0$ and $\phi = h\nu_{\text{th}}$. One electronvolt is 1.6×10^{-19} J.

Step 1: Read the threshold from the graph The plotted straight line reaches zero stopping potential at the horizontal-axis value 4. The frequency axis is labelled in units of 10^{14} Hz, not in hertz alone. Therefore

$$\nu_{\text{th}} = 4 \times 10^{14}\,\text{Hz}.$$

This horizontal intercept gives the minimum frequency needed for emission. The point at a positive stopping potential instead represents electrons with nonzero kinetic energy and must not be used as the threshold.

Step 2: Calculate and convert the work function Use the supplied value of Planck's constant:

$$\phi = (6.63 \times 10^{-34})(4 \times 10^{14}) = 2.652 \times 10^{-19}\,\text{J}.$$

Dividing by the joules in one electronvolt gives

$$\phi = \frac{2.652 \times 10^{-19}}{1.6 \times 10^{-19}}\,\text{eV} = 1.6575\,\text{eV} \simeq 1.66\,\text{eV}.$$

The powers of ten cancel in the conversion; the resulting work function is an energy, not a voltage.

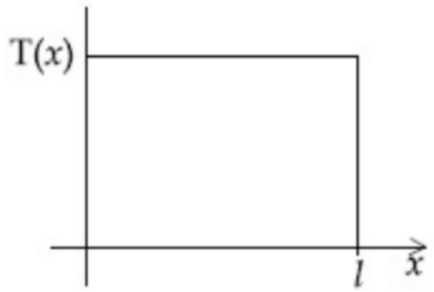
Step 3: Select the value supported by the intercept The other energies, 1.95, 2.12 and 1.82 eV, would require threshold frequencies greater than the observed 4×10^{14} Hz. They therefore do not agree with the zero-potential intercept when the given constants are used. Read the threshold directly rather than estimating an energy from the height of a different point on the line.

Step 4: Conclusion The work function of sodium from the plotted threshold is approximately $1.66\,\text{eV}$. Hence, the correct option is (D).

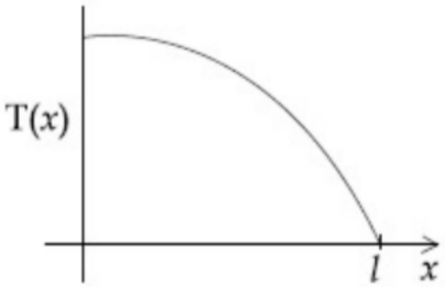
Quick Tip to Solve: On a stopping-potential versus frequency graph, use the frequency-axis intercept for $\phi = h\nu_{\text{th}}$, remembering any power-of-ten multiplier on that axis.

53. A uniform rod of length ℓ is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely? [4 marks]

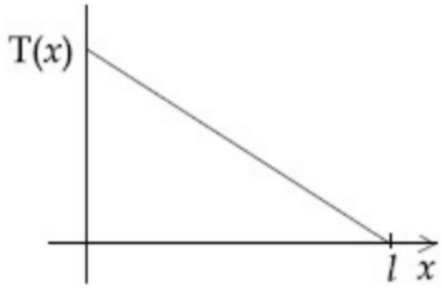
(A)



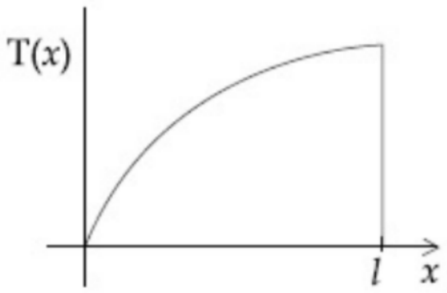
(B)



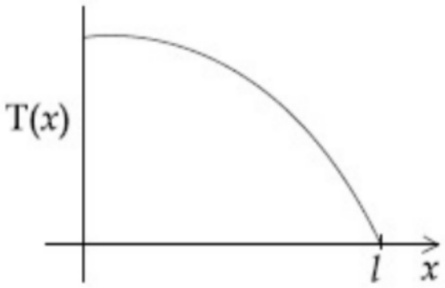
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- A rod element of mass dm at radius r, rotating with constant angular speed ω, requires inward force $\omega^2 r\, dm$.
- For a uniform rod of mass M and length ℓ, linear mass density is $\lambda = M/\ell$. Tension at a section must provide centripetal force for all the material beyond that section.

Step 1: Isolate the outer part of the rod Imagine a cut at distance x from the rotation axis and consider the portion from x to ℓ. The force exerted across this cut is the inward tension $T(x)$. An element at radius r has mass $dm = \lambda\, dr$, so its required inward force is $\lambda\omega^2 r\, dr$. Summing the radial force requirements of the outer elements gives

$$T(x) = \int_x^{\ell} \lambda\omega^2 r\, dr.$$

It is the outer portion, not the inner portion, whose motion is supported by the tension at the cut.

Step 2: Integrate and examine the curvature Carrying out the integral gives

$$T(x) = \frac{\lambda\omega^2}{2}(\ell^2 - x^2).$$

At the axis it has maximum value $T(0) = \lambda\omega^2\ell^2/2$, while at the unloaded free end $T(\ell) = 0$. Its slope is $dT/dx = -\lambda\omega^2 x$, initially zero and increasingly negative. Also

$$\frac{d^2T}{dx^2} = -\lambda\omega^2 < 0.$$

Thus the curve is a decreasing, concave-down parabola.

Step 3: Compare with the displayed graphs Graph (B) starts horizontally at a positive tension and bends downward to zero at the free end, matching every condition. Graph (A) incorrectly holds tension constant; graph (C) decreases linearly, missing the increasing centripetal acceleration with radius. Graph (D) increases outward and incorrectly gives zero tension at the axis.

Step 4: Conclusion $T(x) = \frac{M\omega^2}{2\ell}(\ell^2 - x^2)$, represented by graph (B). Hence, the correct option is (B).

Quick Tip to Solve: For tension in a rotating rod, isolate the material outside the chosen cut; integrate its centripetal-force requirement and check that free-end tension is zero.

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54. In a double slit experiment, when a thin film of thickness t having refractive index μ. is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used) : [4 marks]

(A) $\frac{\lambda}{2(\mu-1)}$

(B) $\frac{\lambda}{(2\mu-1)}$

(C) $\frac{2\lambda}{(\mu-1)}$

(D) $\frac{\lambda}{(\mu-1)}$

Correct Answer: (D) $\frac{\lambda}{(\mu-1)}$

Solution:

Concept:

- In Young's double-slit experiment the fringe width is $\beta = \lambda D/d$, with screen distance D and slit separation d.
- Replacing a length t of air in one path by a transparent film of refractive index μ adds optical path $\Delta = (\mu - 1)t$. A path change of one wavelength shifts the pattern by one fringe.

Step 1: Find the extra optical path Light travels optical distance μt inside the film instead of approximately t through the same thickness of air. Consequently the additional optical path in the covered beam is

$$\Delta = \mu t - t = (\mu - 1)t.$$

The subtraction matters: using μt alone would count the air path that was already present. This extra path produces a fixed phase difference $2\pi\Delta/\lambda$ between the two beams.

Step 2: Relate the displacement to the fringe width Near the central part of the screen a displacement s changes geometrical path difference by ds/D. The new maximum occurs where this compensates the film's additional path. Thus the magnitude of the shift is

$$s = \frac{D}{d}(\mu - 1)t.$$

The question states $s = \beta = \lambda D/d$. Equating these expressions and cancelling the common apparatus

factor gives

$$(\mu - 1)t = \lambda, \qquad t = \frac{\lambda}{\mu - 1}.$$

Step 3: Check the other thicknesses Option (A) would give only half a wavelength of extra path and therefore half a fringe shift. Option (C) would give two wavelengths and a two-fringe shift. Option (B) uses the incorrect refractive-index combination $2\mu - 1$. The required shift depends on the excess optical path through one film, so there is no round-trip factor of two.

Step 4: Conclusion The film thickness is $t = \dfrac{\lambda}{\mu - 1}$. Hence, the correct option is (D).

Quick Tip to Solve: Convert a shift of n fringe widths directly to added optical path $n\lambda$; for one film use $(\mu - 1)t = n\lambda$.

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55. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is - [4 marks]

(A) $\frac{2R}{g}$
(B) $\frac{R}{g}$
(C) $\frac{R}{2g}$
(D) $\frac{R}{4g}$

Correct Answer: (A) $\frac{2R}{g}$

Solution:

Concept:

- For launch and landing at the same height, projectile range is $R = u^2 \sin 2\theta/g$, and flight time is $t = 2u \sin\theta/g$.
- At fixed speed, the two launch angles giving the same non-maximum range are complementary: θ and $\pi/2 - \theta$. Their horizontal and vertical velocity components are interchanged.

Step 1: Identify the two angles and times With R and u fixed, the range equation fixes $\sin 2\theta$. For upward launches that reach a forward ground target, the two allowed angles satisfy

$$\theta_1 = \theta, \qquad \theta_2 = \frac{\pi}{2} - \theta.$$

Their times follow from the vertical displacement equation $0 = u \sin\theta\, t - gt^2/2$, using the nonzero root:

$$t_1 = \frac{2u \sin\theta}{g}, \qquad t_2 = \frac{2u \cos\theta}{g}.$$

The steeper trajectory stays in flight longer.

Step 2: Multiply and eliminate the launch speed The product is

$$t_1 t_2 = \frac{4u^2 \sin\theta \cos\theta}{g^2} = \frac{2u^2 \sin 2\theta}{g^2}.$$

From the range equation $u^2 \sin 2\theta = Rg$. Substitution gives

$$t_1 t_2 = \frac{2Rg}{g^2} = \frac{2R}{g}.$$

The speed cancels because the angle pair adjusts to preserve the same range.

Step 3: Check the coefficient and limiting case All four options have the dimensions of time squared, so dimensional analysis alone cannot select the answer. The double-angle identity supplies the factor of two that distinguishes (A). At the maximum range both angles become $45°$, and $t^2 = 2u^2/g^2 = 2R/g$, confirming the same expression. The smaller coefficients in (B), (C), and (D) fail this direct check.

Step 4: Conclusion The product of the two possible flight times is $t_1 t_2 = \frac{2R}{g}$. Hence, the correct option is (A).

Quick Tip to Solve: For equal-speed ground-to-ground trajectories with the same range, use complementary launch angles before multiplying their flight times; $2 \sin\theta \cos\theta$ then removes the angle.

---

56. At 40° C, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C it regains its original length of 0.2 m. The value of M is close to : (Coefficient of linear expansion and Young's modulus of brass are 10^{-5} /°C and 10^{11} N/m ^{2} , respectively; g= 10 ms^{-2}) [4 marks]

(A) 1.5 kg

(B) 0.5 kg

(C) 9 kg

(D) 0.9 kg

Correct Answer: (C) 9 kg

Solution:

Concept:

- For a wire under load Mg, Young's modulus gives elastic extension $\Delta L_e = MgL/(AY)$, where $A = \pi r^2$.
- Cooling through magnitude ΔT produces free thermal contraction $\Delta L_{th} = \alpha L \Delta T$. Regaining the original unloaded length means the contraction compensates the elastic extension, to first order in strain.

Step 1: Balance the two changes in length The hanging mass stretches the wire, whereas cooling from $40°\text{C}$ to $20°\text{C}$ shortens it. Since the final loaded wire regains its original length, the two magnitudes balance:

$$\frac{MgL}{AY} = \alpha L \Delta T.$$

Cancel the common wire length $L = 0.2\,\text{m}$, obtaining

$$M = \frac{AY\alpha\Delta T}{g}.$$

Thus the mass is independent of the original length in this small-strain calculation.

Step 2: Insert the radius and material constants The stated radius is $r = 1\,\text{mm} = 10^{-3}\,\text{m}$, so $A = \pi \times 10^{-6}\,\text{m}^2$. Also $\Delta T = 20^\circ\text{C}$. Therefore

$$M = \frac{(\pi \times 10^{-6})(10^{11})(10^{-5})(20)}{10} = 2\pi\,\text{kg} \simeq 6.28\,\text{kg}.$$

The associated stress is $Y\alpha\Delta T = 2 \times 10^7\,\text{Pa}$, which provides a useful check on the power-of-ten arithmetic.

Step 3: Choose the closest listed mass The question asks for the listed value closest to the calculated mass. The absolute differences from 6.28 kg are approximately 4.78 kg for 1.5 kg, 5.78 kg for 0.5 kg, 2.72 kg for 9 kg, and 5.38 kg for 0.9 kg. Consequently 9 kg is the closest of the four choices, even though none equals the calculated value. Do not change the radius to a diameter or alter the stated constants to force an exact match.

Step 4: Conclusion The supplied data give $M \simeq 6.28\,\text{kg}$; the closest listed value is $9\,\text{kg}$. Hence, the correct option is (C).

Quick Tip to Solve: Cancel wire length before substituting. Convert radius to metres before squaring, and distinguish the computed mass from the closest available choice.

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57. The value of numerical aperature of the objective lens of a microscope is 1.25. If light of wavelength 5000 $\overset{o}{A}$ is used, the minimum separation between two points, to be seen as distinct, will be : [4 marks]

(A) 0.12 μm

(B) 0.38 μm

(C) 0.24 μm

(D) 0.48 μm

Correct Answer: (C) 0.24 μm

Solution:

Concept:

- The numerical aperture is $\text{NA} = n\sin\theta$, where n is the refractive index of the medium and θ is the semi-angle collected by the objective.
- For a circular microscope objective, the Rayleigh resolution limit is $d_{\text{min}} = 0.61\lambda/\text{NA}$. It is a separation in the object plane, not an angular separation.

Step 1: Convert the wavelength The wavelength is given in angstroms. Since $1\,\text{Å} = 10^{-10}\,\text{m}$, we obtain

$$\lambda = 5000 \times 10^{-10}\,\text{m} = 5.0 \times 10^{-7}\,\text{m} = 0.500\,\mu\text{m}.$$

The aperture 1.25 is dimensionless and already incorporates the refractive index; it should not be multiplied by an additional refractive-index factor.

Step 2: Apply the resolution criterion Two neighbouring point images can be distinguished at the

Rayleigh limit when the central peak of one diffraction pattern falls on the first minimum of the other. Therefore

$$d_{\min} = \frac{0.61(0.500)}{1.25}\,\mu\text{m} = 0.244\,\mu\text{m}.$$

To the precision of the options this is $0.24\,\mu$m. The value 0.12 is approximately half this limit; 0.38 and 0.48 do not result from the stated aperture and wavelength. Increasing aperture would decrease the limiting separation, as the formula predicts.

Step 3: Conclusion The minimum resolvable separation is $0.244\,\mu$m, approximately $0.24\,\mu$m. Hence, the correct option is (C).

Quick Tip to Solve: Convert the wavelength to micrometres first, then divide 0.61λ by the numerical aperture; do not confuse aperture with magnification.

58. A magnetic compass needle oscillates 30 times per minute at a place where the dip is 45°, and 40 times per minute where the dip is 30°. If B_1 and B_2 are respectively the total magnetic field due to the earth at the two places, then the ratio $\frac{B_1}{B_2}$ is best given by : [4 marks]

(A) 1.8

(B) 2.2

(C) 0.7

(D) 3.6

Correct Answer: (C) 0.7

Solution:

Concept:

- A compass needle oscillating horizontally experiences the horizontal component $B_H = B\cos\delta$, where δ is the magnetic dip.
- For small oscillations, $f = (1/2\pi)\sqrt{mB_H/I}$, where m is magnetic moment and I is moment of inertia. The same needle has unchanged m and I.

Step 1: Relate the frequencies to the horizontal fields The frequencies are 30 and 40 oscillations per minute. Their ratio is independent of whether time is measured in seconds or minutes. Squaring the frequency relation gives

$$\frac{B_{H1}}{B_{H2}} = \left(\frac{f_1}{f_2}\right)^2 = \left(\frac{30}{40}\right)^2 = \frac{9}{16}.$$

It is the squared frequency ratio, not the frequency ratio itself, that determines the field-component ratio.

Step 2: Recover the total magnetic fields Substitute the dip angles into $B_H = B\cos\delta$:

$$\frac{B_1\cos 45^\circ}{B_2\cos 30^\circ} = \frac{9}{16}.$$

Hence

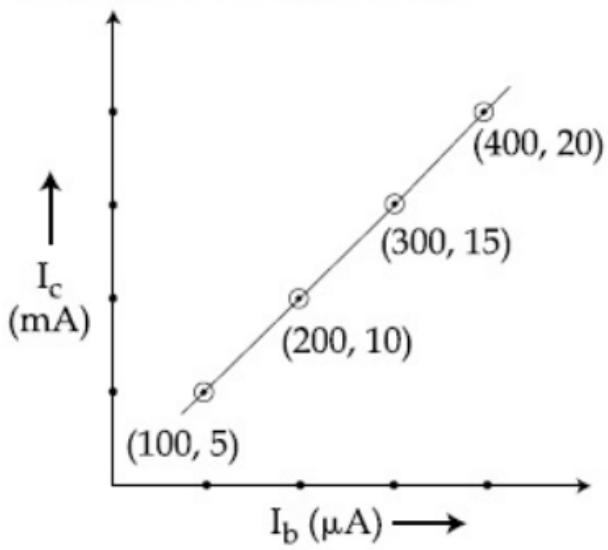
$$\frac{B_1}{B_2} = \frac{9}{16}\frac{\sqrt{3}/2}{1/\sqrt{2}} = \frac{9\sqrt{6}}{32} \simeq 0.689.$$

The first place has the smaller horizontal field and, after the dip correction, still the smaller total field. Thus all options greater than unity are excluded. Using the total field directly in the compass formula would incorrectly omit the cosine correction.

Step 3: Conclusion The ratio B_1/B_2 is approximately 0.689, best represented by 0.7. Hence, the correct option is (C).

Quick Tip to Solve: For a horizontal compass, square the oscillation-frequency ratio to obtain the horizontal-field ratio, then apply the dip cosines.

59. The transfer characteristic curve of a transistor, having input and output resistance 100 Ω and 100 kΩ respectively is shown in the figure. The voltage and power gain, are respectively: [4 marks]



(A) 5×10^4, 5×10^5

(B) 5×10^4, 5×10^6

(C) 5×10^4, 2.5×10^6

(D) 2.5×10^4, 2.5×10^6

Correct Answer: (C) 5×10^4, 2.5×10^6

Solution:

Concept:

- The small-signal current gain is the slope $\beta = \Delta I_c/\Delta I_b$ of the transfer characteristic, after expressing both currents in the same units.
- The voltage-gain magnitude is $A_v = \beta R_o/R_i$, while power gain is $A_p = A_v\beta = \beta^2 R_o/R_i$. Here R_i and R_o are input and output resistances.

Step 1: Read the current gain from the graph Use the graph points $(I_b, I_c) = (100\,\mu\text{A}, 5\,\text{mA})$ and

$(200\,\mu\text{A}, 10\,\text{mA})$. Thus

$$\beta = \frac{(10-5)\times 10^{-3}}{(200-100)\times 10^{-6}} = 50.$$

Taking the numerical slope as 0.05 would mix milliampere and microampere scales. The other plotted points give the same slope, confirming a constant gain in this region.

Step 2: Calculate voltage and power amplification The resistance ratio is

$$\frac{R_o}{R_i} = \frac{100\times 10^3}{100} = 10^3.$$

Therefore

$$A_v = 50\times 10^3 = 5\times 10^4, \qquad A_p = 50(5\times 10^4) = 2.5\times 10^6.$$

These gains are dimensionless ratios. The voltage gain here denotes magnitude; the phase inversion of a common-emitter output does not make the power gain negative. Options A and B have the correct voltage magnitude but incorrect power gain, while D gives an incorrect voltage magnitude.

Step 3: Conclusion The voltage and power gains are respectively 5×10^4 and 2.5×10^6. Hence, the correct option is (C).

Quick Tip to Solve: Convert the two graph-current scales before taking the slope. Then use $A_p = \beta A_v$ to check both listed gains.

60. A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency υ. The value of υ is close to: (Speed of sound in water =1500 ms^{-1}) [4 marks]

(A) 507 Hz

(B) 502 Hz

(C) 499 Hz

(D) 504 Hz

Correct Answer: (B) 502 Hz

Solution:

Concept:

- A sonar echo undergoes two Doppler shifts: the target first receives the incident wave and then acts as a moving source for the reflected wave.
- Use velocities relative to the water. For a wave moving in a chosen direction, $f_o = f_s(c-v_o)/(c-v_s)$, where the velocity components are measured along that direction.

Step 1: Find the frequency received by submarine A Take the forward direction from B towards A as positive. The speeds are

$$v_A = 18\frac{5}{18} = 5\,\text{m s}^{-1}, \qquad v_B = 27\frac{5}{18} = 7.5\,\text{m s}^{-1}.$$

Both move forward, with B gaining on A. The frequency at A is

$$f_A = 500\frac{1500-5}{1500-7.5} = 500\frac{1495}{1492.5}.$$

A is moving away from the incident source, while the source is moving toward it; both effects must be included.

Step 2: Apply the return-path Doppler shift The reflected wave travels backward through the water. A now behaves as its source, and B as the receiving observer. Their components along the backward direction are -5 and -7.5, respectively. Consequently

$$\nu = f_A\frac{1500+7.5}{1500+5} = 500\frac{1495(1507.5)}{1492.5(1505)} \simeq 501.67\,\text{Hz}.$$

The echo is shifted upwards because the separation decreases. The nearest offered value is 502 Hz; 499 Hz has the wrong shift direction, and 504 or 507 Hz overestimate its magnitude.

Step 3: Conclusion The reflected frequency is approximately $501.67\,\text{Hz}$, closest to $502\,\text{Hz}$. Hence, the correct option is (B).

Quick Tip to Solve: For an echo, apply Doppler twice and reverse the propagation direction on the return journey; the closing speed alone gives only a first-order estimate.

Chemistry (120 marks)

61. The mole fraction of a solvent in aqueous solution of a solute is 0.8. The molality (in mol kg^{-1}) of the aqueous solution is : [4 marks]

(A) 13.88×10^{-1}

(B) 13.88×10^{-3}

(C) 13.88

(D) 13.88×10^{-2}

Correct Answer: (C) 13.88

Solution:

Concept:

- The solvent mole fraction is $x_w = n_w/(n_w + n_s)$, where n_w and n_s are moles of water and solute.
- Molality is moles of solute per kilogram of solvent: $m = n_s/(n_w M_w)$, with $M_w = 0.018\,\text{kg mol}^{-1}$. It is not defined per kilogram of solution.

Step 1: Choose a convenient mole basis Take one mole of the solution components in total. Since the solvent mole fraction is 0.8, this basis contains

$$n_w = 0.8\,\text{mol}, \qquad n_s = 1 - 0.8 = 0.2\,\text{mol}.$$

Equivalently, the solute-to-solvent mole ratio is $0.2/0.8 = 0.25$. The choice of total moles is arbitrary

because both mole fraction and molality are independent of sample size.

Step 2: Convert water moles to kilograms Using the molar mass of water, the mass of the solvent is

$$w_w = 0.8 \times 18 = 14.4\,\text{g} = 0.0144\,\text{kg}.$$

Divide the solute moles by this mass:

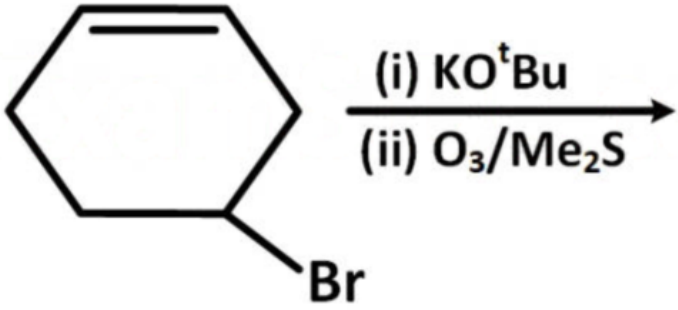
$$m = \frac{0.2}{0.0144} = 13.888\ldots\ \text{mol kg}^{-1}.$$

Thus the answer is approximately 13.88 in the units requested. No solute molar mass or solution density is needed. Options containing extra factors of 10^{-1}, 10^{-2}, or 10^{-3} are inconsistent with the conversion from grams to kilograms and underestimate the result.

Step 3: Conclusion The molality is approximately $13.88\,\text{mol kg}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: For an aqueous solution use $m = (1 - x_w)/(0.018x_w)$; keep the water molar mass in kilograms per mole.

62. The major product(s) obtained in the following reaction is/are : [4 marks]



(A) $OHC-CH_2-CH_2-CHO$ and $OHC-CHO$ (B) $OHC-CH_2-CH_2-CH=CH-CHO$

(C) $OHC-CH_2-CHO$ (D) $OHC-CH_2-CH(OC(CH_3)_3)-CH_2-CH_2-CHO$

Correct Answer: (A) $OHC-CH_2-CH_2-CHO$ and $OHC-CHO$

Solution:

Concept:

- Potassium tert-butoxide is a strong, bulky base that promotes dehydrohalogenation. Elimination giving a conjugated diene is favoured by the stabilisation of conjugation.
- Ozonolysis followed by Me_2S cleaves each carbon-carbon double bond into carbonyl groups. A double-bond carbon bearing hydrogen becomes an aldehyde under this reductive work-up.

Step 1: Identify the conjugated elimination product Number the original alkene carbons 1 and 2 and

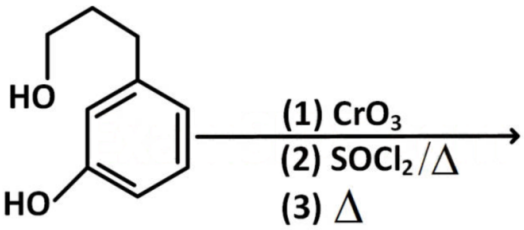
continue around the ring so the bromine-bearing carbon is 4. Removal of the beta hydrogen on carbon 3 together with bromide from carbon 4 produces $C_3{=}C_4$. The ring now contains the conjugated sequence $C_1{=}C_2{-}C_3{=}C_4$, namely cyclohexa-1,3-diene. Elimination on the other adjacent carbon would form the nonconjugated diene and is not the major route. Substitution by tert-butoxide is also disfavoured relative to elimination.

Step 2: Cut both double bonds and count the fragments Cleaving bonds 1–2 and 3–4 leaves two carbon chains. The two-carbon fragment consists of carbons 2 and 3, giving $OHC{-}CHO$, ethanedial. The four-carbon fragment follows carbons 4–5–6–1 and gives $OHC{-}CH_2{-}CH_2{-}CHO$, butane-1,4-dial. Every cleaved alkene carbon carries a hydrogen, so aldehydes rather than ketones form. The two fragments contain six carbons in total, preserving the carbon count. A product retaining a double bond represents incomplete ozonolysis; the three-carbon dialdehyde would arise from a different diene.

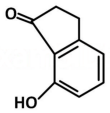
Step 3: Conclusion The products are $OHC{-}CH_2{-}CH_2{-}CHO$ and $OHC{-}CHO$, matching the first pair of structures. Hence, the correct option is (A).

Quick Tip to Solve: First locate the conjugated diene, then cut every double bond and trace the remaining ring arcs to count the aldehyde fragments.

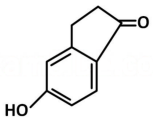
63. The major product of the following reaction is : [4 marks]



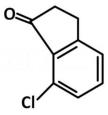
(A) 4-Hydroxyindan-1-one (structure shown)



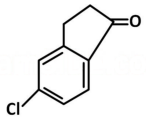
(B) 6-Hydroxyindan-1-one (structure shown)



(C) 4-Chloroindan-1-one (structure shown)

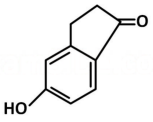


(D) 6-Chloroindan-1-one (structure shown)



Correct Answer: (B)

6-Hydroxyindan-1-one (structure shown)



Solution:

Concept:

- Chromium trioxide oxidises the terminal primary alcohol of the side chain to a carboxylic acid. Thionyl chloride converts this acid into its reactive acyl chloride.
- Intramolecular aromatic acylation closes a favourable five-membered ring. The phenolic hydroxyl group directs electrophilic substitution to its ortho and para positions; the less crowded accessible para site is preferred.

Step 1: Follow the side-chain functional-group changes The starting compound has a three-carbon hydroxyalkyl chain attached to a phenolic ring, with the phenolic hydroxyl meta to the chain attachment. Oxidation changes the terminal $-CH_2OH$ into $-COOH$, giving an arylpropanoic-acid side chain, $Ar-CH_2-CH_2-COOH$. Treatment with $SOCl_2$ gives $Ar-CH_2-CH_2-COCl$. The phenolic carbon-oxygen bond is not converted into an aryl carbon-chlorine bond by this reagent; chlorine-containing ring products are therefore excluded.

Step 2: Choose the favoured cyclisation position On heating, the acyl group attacks an ortho position relative to the side-chain attachment, forming a fused five-membered ketone ring. Of these two possible aromatic positions, one lies between the side chain and the phenolic hydroxyl and is sterically crowded; the other is para to the hydroxyl group and less hindered. Attack at the latter produces the hydroxyindanone drawn in option B, with the hydroxyl group remote from the carbonyl-side fusion position. Option A corresponds to cyclisation at the more crowded site. Both C and D incorrectly replace the phenolic hydroxyl with chlorine.

Step 3: Conclusion The favoured product is the hydroxyindanone shown in option B, formed by cyclisation at the less hindered site para to the phenolic hydroxyl. Hence, the correct option is (B).

Quick Tip to Solve: Keep the phenolic hydroxyl intact, convert the terminal alcohol to an acyl chloride, and compare the two ring-closing positions by hydroxyl direction and steric crowding.

64. The group number, number of valence electrons, and valency of an element with atomic number 15, respectively, are: [4 marks]

(A) 16, 6 and 3

(B) 15, 6 and 12

(C) 16, 5 and 2

(D) 15, 5 and 3

Correct Answer: (D) 15, 5 and 3

Solution:

Concept:

- The atomic number of a neutral atom equals its number of electrons. Valence electrons are those in its outermost occupied shell.
- For a p-block atom with configuration ns^2np^3, the modern group number is 15. Its usual octet valency is $8 - 5 = 3$.

Step 1: Write the electronic configuration Atomic number 15 identifies phosphorus. Filling the orbitals in order gives

$$1s^2\,2s^2\,2p^6\,3s^2\,3p^3.$$

Equivalently, the shell distribution is $2, 8, 5$. The highest occupied principal shell is $n = 3$; therefore the two 3s electrons and three 3p electrons together give five valence electrons. Core electrons do not enter this count.

Step 2: Determine the group and ordinary valency For this main-group p-block configuration the group number is $10 + 5 = 15$, rather than 16. To complete an octet the atom needs three more electrons, so its ordinary valency is three. This is the valency illustrated by three bonds in PH_3; it should not be confused with every possible oxidation state of phosphorus.

Step 3: Check the proposed triples Options (A) and (C) assign group 16, which instead corresponds to six outer-shell electrons in the main-group pattern. Option (B) gives six valence electrons and the impossible ordinary valency 12 for this configuration. Only (D) retains all three consistent values: group 15, five outer electrons and valency three.

Step 4: Conclusion The required values are $15, 5, 3$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Write the outer-shell configuration first; for ns^2np^3, remember group 15 and octet valency three.

65. What is the molar solubility of $Al(OH)_3$ in 0.2 M NaOH solution ? Given that, solubility product of $Al(OH)_3$ = 2.4 $\times$ 10^{-24} : [4 marks]

(A) 3 $\times$ 10^{-22}

(B) 3 $\times$ 10^{-19}

(C) 12 $\times$ 10^{-21}

(D) 12 $\times$ 10^{-22}

Correct Answer: (A) 3 $\times$ 10^{-22}

Solution:

Concept:

- For $Al(OH)_3(s) \rightleftharpoons Al^{3+}(aq) + 3\,OH^-(aq)$, $K_{sp} = [Al^{3+}][OH^-]^3$.
- The common hydroxide ion supplied by strong-base NaOH suppresses dissolution. In the stated solubility-product calculation, $[OH^-] \simeq 0.2\,mol\,L^{-1}$.

Step 1: Express the equilibrium concentrations Let s be the molar solubility of the hydroxide. Each mole dissolved supplies one mole of aluminium ions, so $[Al^{3+}] = s$. The sodium hydroxide dissociates completely and supplies $0.2\,M$ hydroxide before dissolution. Thus the exact common-ion model is

$$K_{sp} = s(0.2 + 3s)^3.$$

Since the solubility product is extremely small, the added $3s$ is negligible compared with 0.2.

Step 2: Substitute and evaluate the solubility Using the given value,

$$s = \frac{2.4 \times 10^{-24}}{(0.2)^3} = \frac{2.4 \times 10^{-24}}{8 \times 10^{-3}} = 3 \times 10^{-22}\,\text{mol}\,\text{L}^{-1}.$$

The neglected hydroxide contribution is only $9 \times 10^{-22}\,\text{M}$, confirming the approximation. This is the solubility inferred from the specified K_{sp} model; no formation constant for dissolved hydroxo complexes is provided.

Step 3: Compare the offered magnitudes The hydroxide concentration must be cubed because its coefficient in the dissolution equation is three. Option (B) is a thousand times larger, while (C) and (D) also fail substitution into the stated product. Multiplying 3×10^{-22} by 0.008 reproduces 2.4×10^{-24}, directly checking option (A).

Step 4: Conclusion The molar solubility is $3 \times 10^{-22}\,\text{mol}\,\text{L}^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: For a hydroxide M(OH)_3 in excess strong base, divide K_{sp} by the cube of the supplied hydroxide concentration.

66. An ideal gas is allowed to expand form 1 L to 10 L against a constant external pressure of I bar. The work done in kJ is : [4 marks]

(A) +10.0

(B) –0.9

(C) – 2.0

(D) – 9.0

Correct Answer: (B) –0.9

Solution:

Concept:

- With the chemistry sign convention, pressure-volume work on the gas is $w = -P_{\text{ext}}(V_f - V_i)$ at constant external pressure.
- Expansion gives $\Delta V > 0$ and therefore negative work. The useful conversion is $1\,\text{L}\,\text{bar} = 100\,\text{J}$.

Step 1: Calculate the volume increase The final volume is $10\,\text{L}$, but the work depends on the change from the initial $1\,\text{L}$, not on the final volume alone. Hence

$$\Delta V = 10 - 1 = 9\,\text{L}.$$

The process is explicitly against a constant external pressure of $1\,\text{bar}$, so the external pressure can be taken outside the work integral.

Step 2: Apply the sign convention and convert units The gas pushes its surroundings outwards, transferring energy to them as work. In the usual chemical convention this is negative work on the system:

$$w = -(1\,\text{bar})(9\,\text{L}) = -9\,\text{L}\,\text{bar}.$$

Since $1\,\text{bar} = 10^5\,\text{Pa}$ and $1\,\text{L} = 10^{-3}\,\text{m}^3$,

$$w = -9(100)\,\text{J} = -900\,\text{J} = -0.9\,\text{kJ}.$$

Step 3: Reject the other work values The positive sign in (A) contradicts expansion under the chemical convention. Option (D) incorrectly treats one litre-bar as one kilojoule instead of 0.1 kilojoule. Option (C) follows from neither the given volume change nor the pressure. No logarithmic isothermal reversible-work expression is needed: the pressure specified here is the constant external resisting pressure.

Step 4: Conclusion The work done on the gas is $-0.9\,\text{kJ}$. Hence, the correct option is (B).

Quick Tip to Solve: For constant external pressure, subtract the volumes first and remember that ten litre-bar equal one kilojoule.

67. 5 moles of AB_2 weigh 125×10^{-3} kg and 10 moles of A_2B_2 weigh 300×10^{-3} kg. The molar mass of A (M_A) and molar mass of B (M_B) in kg mol are: [4 marks]

(A) $M_A = 10 \times 10^{-3}$ and $M_B = 5 \times 10^{-3}$

(B) $M_A = 25 \times 10^{-3}$ and $M_B = 50 \times 10^{-3}$

(C) $M_A = 5 \times 10^{-3}$ and $M_B = 10 \times 10^{-3}$

(D) $M_A = 50 \times 10^{-3}$ and $M_B = 25 \times 10^{-3}$

Correct Answer: (C) $M_A = 5 \times 10^{-3}$ and $M_B = 10 \times 10^{-3}$

Solution:

Concept:

- Molar mass is total sample mass divided by the number of moles: $M = m/n$.
- The molar mass of a formula unit is the sum of its constituent atomic molar masses multiplied by their subscripts.

Step 1: Translate the first sample into an equation Let the unknown elemental molar masses be M_A and M_B, both in kg mol^{-1}. The first compound contains one A and two B atoms per formula unit. Its molar mass is

$$M_A + 2M_B = \frac{125 \times 10^{-3}}{5} = 25 \times 10^{-3}\,\text{kg mol}^{-1}.$$

Dividing by the five moles is essential; the given sample mass itself is not a molar mass.

Step 2: Use the second compound and eliminate an unknown The second compound contains two atoms of each element, so

$$2M_A + 2M_B = \frac{300 \times 10^{-3}}{10} = 30 \times 10^{-3}\,\text{kg mol}^{-1}.$$

Subtracting the first equation from this equation gives

$$M_A = 5 \times 10^{-3}\,\text{kg mol}^{-1}.$$

Substitution into the first equation then gives $2M_B = 20 \times 10^{-3}$, or $M_B = 10 \times 10^{-3}\,\text{kg mol}^{-1}$.

Step 3: Check both masses against the data The resulting formula molar masses are 0.025 and 0.030 kilograms per mole. Multiplication by five and ten moles reproduces 0.125 and 0.300 kilograms respectively. Option (A) interchanges the elemental values and fails the first equation; options (B) and (D) give much larger compound molar masses. Thus only (C) satisfies both measurements simultaneously.

Step 4: Conclusion $M_A = 5 \times 10^{-3}$ and $M_B = 10 \times 10^{-3}\,\text{kg mol}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: Divide each sample mass by its own mole count before constructing simultaneous equations for the elemental molar masses.

68. The major product of the following addition reaction is :

$\mathbf{H_3C–CH=CH_2} \xrightarrow{Cl_2/H_2O}$ [4 marks]

(A) $CH_3CH(Cl)CH_2OH$

(B) $CH_3CH(OH)CH_2Cl$

(C) Propylene oxide (2-methyloxirane)

(D) CH_3COCH_3

Correct Answer: (B) $CH_3CH(OH)CH_2Cl$

Solution:

Concept:

- Chlorine in water adds across an alkene double bond to form a chlorohydrin, containing chlorine and hydroxyl groups on adjacent carbons.
- An unsymmetrical chloronium intermediate is opened by water preferentially at its more substituted carbon; the hydroxyl group therefore occupies that carbon.

Step 1: Identify the reacting double bond The substrate is propene, $CH_3CH{=}CH_2$. Its middle alkene carbon is attached to a methyl group, whereas the terminal alkene carbon is less substituted. The reagent Cl_2/H_2O first produces a bridged chloronium ion, rather than simply supplying two hydroxyl groups or oxidatively cleaving the carbon skeleton.

Step 2: Place the entering hydroxyl group Water opens the chloronium ion at the more substituted middle carbon, where the intermediate has greater positive character. Loss of a proton from the attached water gives the neutral alcohol. Chlorine remains on the terminal carbon. Thus the overall connectivity is

$$CH_3CH{=}CH_2 \xrightarrow{Cl_2/H_2O} CH_3CH(OH)CH_2Cl.$$

Addition occurs across the original double bond while the three-carbon chain is preserved.

Step 3: Compare all product connectivities Option (B) has hydroxyl on the middle carbon and chlorine on the terminal carbon, as required. Option (A) reverses this major regiochemistry and is not the predominant chlorohydrin. Option (C) is an epoxide; its formation from a chlorohydrin would require a subsequent suitable base-induced ring closure. Option (D) is acetone, a carbonyl compound,

rather than the direct aqueous-chlorine addition product.

Step 4: Conclusion The major product is $CH_3CH(OH)CH_2Cl$, 1-chloropropan-2-ol. Hence, the correct option is (B).

Quick Tip to Solve: In aqueous-halogen addition to an unsymmetrical alkene, place OH on the more substituted carbon and halogen on the other carbon.

69. But-2-ene on reaction with alkaline $KMnO_4$ at elevated temperature followed by acidification will give : [4 marks]

(A) One molecule of CH_3CHO and one molecule of CH_3COOH

(B) 2 molecules of CH_3COOH

(C) $CH_3CH(OH)CH(OH)CH_3$

(D) 2 molecules of CH_3CHO

Correct Answer: (B) 2 molecules of CH_3COOH

Solution:

Concept:

- Hot alkaline $KMnO_4$ is a strong oxidising reagent that cleaves an alkene double bond; acidification gives the corresponding carboxylic acids.
- An alkene carbon bearing one hydrogen becomes a carboxylic acid carbon on vigorous oxidation. Both alkene carbons of but-2-ene have one hydrogen.

Step 1: Inspect the groups attached to each alkene carbon But-2-ene has the structure $CH_3CH{=}CHCH_3$. Each carbon of the double bond is attached to one hydrogen and one methyl group. The two halves are identical, so cleavage must produce two identical two-carbon fragments. This symmetry already rules out a final mixture of unlike fragments under the stated uniform conditions.

Step 2: Apply vigorous oxidative cleavage At elevated temperature the permanganate reaction proceeds beyond the vicinal-diol stage and breaks the carbon-carbon double bond. Each $CH_3CH{=}$ half becomes an acetate unit in alkaline solution. Subsequent acidification converts the acetate ions into ethanoic acid. The net organic transformation is

$$CH_3CH{=}CHCH_3 \xrightarrow[\text{then acid}]{\text{hot } KMnO_4} 2\,CH_3COOH.$$

All four substrate carbon atoms are retained in the two acid molecules.

Step 3: Distinguish the milder oxidation products Option (C), butane-2,3-diol, corresponds to mild cold dilute alkaline permanganate addition, not the elevated-temperature conditions. Acetaldehyde in options (A) and (D) does not survive this strong oxidant; it undergoes further oxidation to ethanoic acid. Therefore neither two aldehydes nor an aldehyde-acid mixture is the final product after the specified oxidation and acidification.

Step 4: Conclusion One molecule of but-2-ene gives two molecules of CH_3COOH. Hence, the correct option is (B).

Quick Tip to Solve: Always check permanganate conditions: cold dilute reagent gives a diol, whereas hot vigorous oxidation cleaves the double bond.

70. An organic compound 'A' is oxidizod with Na_2O_2 followed by boiling with HNO_3. The resultant solution is then treated with ammonium molybdate to yield a yellow precipitate.

Based on above observation, the element present in the given compound is: [4 marks]

(A) Fluorine

(B) Phosphorus

(C) Nitrogen

(D) Sulphur

Correct Answer: (B) Phosphorus

Solution:

Concept:

- Oxidative treatment converts organically bound phosphorus into inorganic phosphate, allowing a characteristic qualitative test.
- Phosphate in nitric-acid medium reacts with ammonium molybdate to form a yellow precipitate of ammonium phosphomolybdate.

Step 1: Follow the purpose of the initial oxidation The organic compound is treated with Na_2O_2, a strong oxidising agent. If phosphorus is present, oxidation converts it into a phosphate species rather than leaving it attached to the original carbon framework. Boiling with HNO_3 establishes the acidic medium needed for the subsequent molybdate test. The observation must be assigned to the complete reagent sequence, rather than to the colour of an unrelated element test.

Step 2: Identify the yellow precipitate Ammonium molybdate reacts with phosphate under these acidic conditions to give yellow ammonium phosphomolybdate, conventionally written $(NH_4)_3PO_4 \cdot 12\,MoO_3$. Thus the yellow solid indicates phosphate in the oxidised extract. Since the phosphate originates from oxidation of the compound being tested, the parent organic compound contains phosphorus.

Step 3: Exclude the other proposed elements Fluorine is not identified by formation of this yellow phosphomolybdate precipitate. Nitrogen is ordinarily detected through a different sequence, such as the Prussian-blue test after sodium fusion. Sulphur is tested through sulfide reactions, for example the violet nitroprusside test or black lead sulfide formation. Those diagnostic reagents and observations are absent here. The nitric acid itself contains nitrogen, so its use cannot establish nitrogen in the original compound.

Step 4: Conclusion The yellow ammonium phosphomolybdate precipitate identifies phosphorus. Hence, the correct option is (B).

Quick Tip to Solve: Link acidic ammonium molybdate and a yellow precipitate directly to phosphate, hence to phosphorus in the original organic compound.

71. An example of a disproportionation reaction is : [4 marks]

(A) $2NaBr + Cl_2 \rightarrow 2NaCl + Br_2$

(B) $2KMnO_4 \rightarrow 2KMnO_4 + MnO_2 + O_2$ (3) (4)

(C) $2CuBr \rightarrow CuBr_2 + Cu$

(D) $2MnO_4 + 10I^- + 16H^+ \rightarrow 2Mn_2{}^+ + 5I_2 + 8H_2O$

Correct Answer: (C) $2CuBr \rightarrow CuBr_2 + Cu$

Solution:

Concept:

- Disproportionation is a redox reaction in which the same element, initially in one oxidation state, is simultaneously oxidised and reduced. The two products must therefore contain that element at a higher and a lower oxidation state.
- Oxidation means an increase in oxidation number, whereas reduction means a decrease. In neutral copper bromides bromine is in the oxidation state -1, so the copper states can be found directly from charge balance.

Step 1: Assign the copper oxidation states In $CuBr$, copper is $+1$. In $CuBr_2$, copper is $+2$, whereas elemental copper has oxidation state zero. Thus the reaction is

$$2\,CuBr \longrightarrow CuBr_2 + Cu.$$

One copper(I) centre loses an electron to become copper(II); the other gains an electron to become copper metal. The electron loss and gain balance without a second reacting element changing its oxidation state.

Step 2: Distinguish the other reactions In the halogen displacement reaction, bromide is oxidised while chlorine is reduced: these are different elements. Permanganate decomposition involves manganese reduction and oxygen oxidation, so it is not disproportionation of one element. In the permanganate–iodide reaction, manganese is reduced and iodide is oxidised. These reactions are redox reactions, but they lack simultaneous oxidation and reduction of the same initial oxidation state.

Step 3: Conclusion Copper(I) forms copper(II) and copper(0) in $2\,CuBr \longrightarrow CuBr_2 + Cu$, so this is disproportionation. Hence, the correct option is (C).

Quick Tip to Solve: Track one element through both products: its oxidation numbers must lie on opposite sides of its original value.

72. Glucose and Galactose are having identical configuration in all the positions except position

[4 marks]

(A) C–5

(B) C–3

(C) C–2

(D) C–4

Correct Answer: (D) C–4

Solution:

Concept:

- Glucose and galactose are aldohexoses: each has six carbon atoms and an aldehyde group in its open-chain form. Their configurations are compared at the stereogenic carbons $C2$ through $C5$.
- Two sugars that differ in configuration at exactly one stereogenic carbon are called epimers. Carbon numbering starts from the aldehyde carbon, not from the terminal primary-alcohol group.

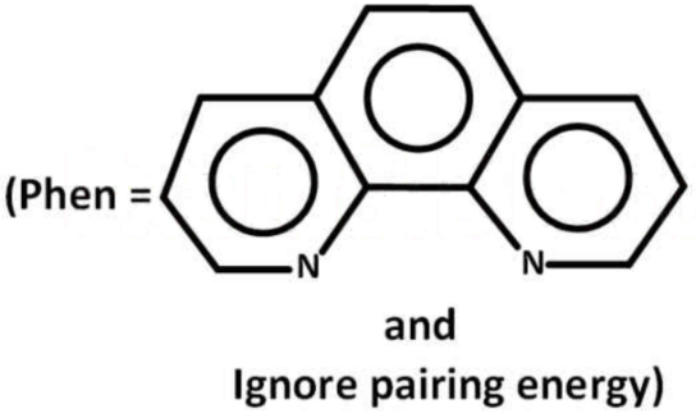
Step 1: Compare the Fischer configurations For the standard D-series, the hydroxyl groups of D-glucose at $C2, C3, C4, C5$ lie right, left, right, right in a Fischer projection. In D-galactose, the corresponding sequence is right, left, left, right. Compare corresponding carbon atoms rather than reversing or rotating only part of the projection: the orientations at C2, C3 and C5 agree, while C4 changes.

Step 2: Identify the single point of difference The changed configuration is therefore at the fourth carbon atom. This makes glucose and galactose C4 epimers. A difference at C2 would instead describe the glucose–mannose relationship, so that attractive alternative is incorrect. C3 has the same leftward hydroxyl orientation in both sugars, and C5 remains unchanged; the latter is also consistent with both belonging to the D-series. Their aldehyde and terminal alcohol groups do not create a further stereochemical difference.

Step 3: Conclusion Glucose and galactose differ only at $C4$; they are C4 epimers. Hence, the correct option is (D).

Quick Tip to Solve: Remember the two common glucose epimers: mannose differs at C2, whereas galactose differs at C4.

73. The complex ion that will lose its crystal field stabilization energy upon oxidation of its metal to +3 state is : [4 marks]



(A) $[Fe(phen)_3]^{2+}$

(B) $[Zn(phen)_3]^{2+}$

(C) $[Co(phen)_3]^{2+}$

(D) $[Ni(phen)_3]^{2+}$

Correct Answer: (A) $[Fe(phen)_3]^{2+}$

Solution:

Concept:

- Phenanthroline is a strong-field bidentate ligand. Three phen ligands give an octahedral, low-spin complex; the three t_{2g} orbitals lie below the two e_g orbitals.
- Ignoring pairing energy as instructed, $\text{CFSE} = (-0.4n_t + 0.6n_e)\Delta_o$. A negative value is stabilisation; loss of stabilisation means that this energy becomes less negative on removing one metal electron.

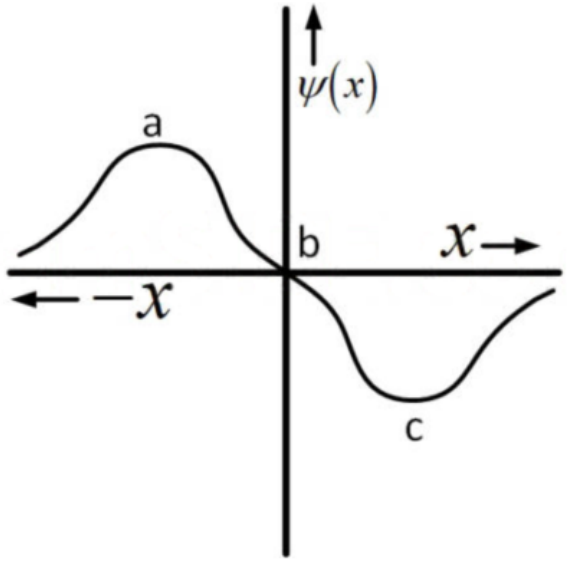
Step 1: Compare iron before and after oxidation Iron(II) is d^6 and its low-spin configuration is $t_{2g}^6 e_g^0$, with CFSE $-2.4\Delta_o$. Oxidation to iron(III) gives d^5, configured $t_{2g}^5 e_g^0$, with CFSE $-2.0\Delta_o$. Removing an electron from a stabilising t_{2g} orbital reduces the magnitude of the stabilisation by $0.4\Delta_o$. Thus the iron complex loses crystal-field stabilisation, though its CFSE does not become zero.

Step 2: Check the competing metal ions For cobalt the change is low-spin $d^7 : t_{2g}^6 e_g^1$ to $d^6 : t_{2g}^6$; the CFSE changes from $-1.8\Delta_o$ to $-2.4\Delta_o$. Nickel changes from $d^8 : t_{2g}^6 e_g^2$ to low-spin $d^7 : t_{2g}^6 e_g^1$, from $-1.2\Delta_o$ to $-1.8\Delta_o$. Zinc changes from d^{10}, with zero CFSE, to d^9, with $-0.6\Delta_o$. Each of these gains rather than loses stabilisation in the occupancy comparison, eliminating the other options.

Step 3: Conclusion $[Fe(phen)_3]^{2+}$ loses crystal-field stabilisation on oxidation to iron(III). Hence, the correct option is (A).

Quick Tip to Solve: For a strong-field octahedral complex, ask whether oxidation removes an electron from stabilising t_{2g} or destabilising e_g.

74. The electrons are more likely to be found : [4 marks]



(A) only in the region a

(B) in the region a arid c

(C) only in the region c

(D) in the region a and b

Correct Answer: (B) in the region a arid c

Solution:

Concept:

- The wavefunction ψ is a probability amplitude, not a probability. For the real wavefunction plotted here, the local probability density is proportional to ψ^2; generally it is $|\psi|^2$.
- The sign of a wavefunction describes its phase. Both positive and negative amplitudes contribute positive probability density, whereas a zero of the wavefunction is a node with zero density.

Step 1: Square the amplitudes on the graph The graph has a positive lobe around region a and a negative lobe around region c. Both lobes have appreciable amplitude in magnitude. On squaring, the positive lobe remains positive and the negative lobe also becomes positive. Thus both a and c correspond to regions where the electron is comparatively likely to be found. The negative value of ψ near c must not be interpreted as a forbidden region or as a negative probability.

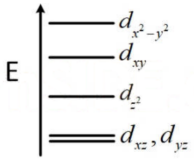
Step 2: Exclude the node and compare the options At b, the wavefunction passes through zero, so $|\psi|^2 = 0$ at that node. An option that includes b as a likely location therefore fails. Choosing only a ignores the probability in the negative lobe, while choosing only c ignores the positive lobe. The graph does not provide a reason to prefer just one of these two lobes; the appropriate choice includes a and c and excludes the nodal region b.

Step 3: Conclusion The electron is more likely to be found in regions a and c, where $|\psi|$ is appreciable. Hence, the correct option is (B).

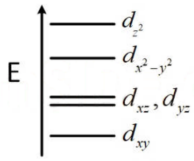
Quick Tip to Solve: When a graph plots ψ, mentally square its vertical coordinate before interpreting the electron probability.

75. Complete removal of both the axial ligands (along the z-axis) from an octahedral complex leads to which of the following splitting patterns? (relative orbital energies not on scale). [4 marks]

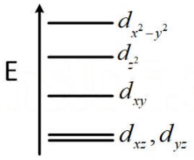
(A) $d_{x^2-y^2} > d_{xy} > d_{z^2} > d_{xz} = d_{yz}$



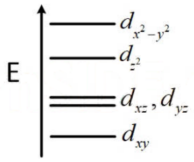
(B) $d_{z^2} > d_{x^2-y^2} > d_{xz} = d_{yz} > d_{xy}$



(C) $d_{x^2-y^2} > d_{z^2} > d_{xy} > d_{xz} = d_{yz}$

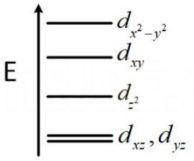


(D) $d_{x^2-y^2} > d_{z^2} > d_{xz} = d_{yz} > d_{xy}$



Correct Answer: (A)

$d_{x^2-y^2} > d_{xy} > d_{z^2} > d_{xz} = d_{yz}$



Solution:

Concept:

- Removing both ligands on the z-axis from an octahedral complex leaves four ligands in the xy-plane, giving the square-planar limiting arrangement. Orbital energy depends on repulsion between the d-electron density and these remaining ligands.
- The $d_{x^2-y^2}$ lobes point directly at the four planar ligands; d_{xy} lies in their plane between them. The orbitals containing z character become relatively less destabilised when the axial ligands are removed.

Step 1: Rank orbitals directed into the ligand plane The $d_{x^2-y^2}$ orbital has the strongest direct interaction with the four ligands and remains highest. The d_{xy} orbital also has its electron density in the xy-plane and lies next. Although its lobes lie between ligand directions, the remaining planar ligand field raises it above the orbital whose dominant lobes point along the now-vacant z-axis. This distinction is essential in passing from an octahedral diagram to the square-planar limit.

Step 2: Place the z-containing orbitals The energy of d_{z^2} falls substantially as the axial repulsion disappears, but its equatorial torus still interacts with the planar ligands. The d_{xz} and d_{yz} orbitals are

equivalent by the fourfold symmetry and remain a degenerate lowest pair. The resulting ordering is

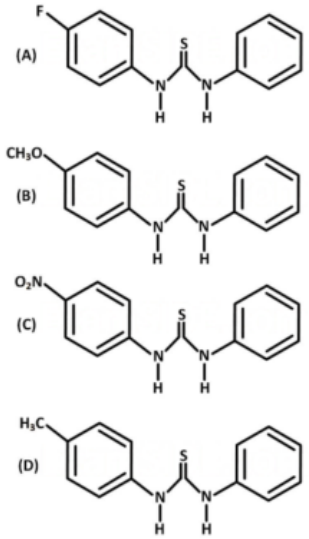
$$d_{x^2-y^2} > d_{xy} > d_{z^2} > d_{xz} = d_{yz}.$$

The other patterns either place d_{z^2} above d_{xy}, make it highest, or incorrectly put d_{xy} below the degenerate pair.

Step 3: Conclusion The square-planar order is $d_{x^2-y^2} > d_{xy} > d_{z^2} > d_{xz} = d_{yz}$. Hence, the correct option is (A).

Quick Tip to Solve: Removing z-axis ligands lowers orbitals with z character; the orbital pointing at the four remaining planar ligands stays highest.

76. The increasing order of the pK_b of the following compound is : [4 marks]



(A) $(C) < (A) < (D) < (B)$
(B) $(B) < (D) < (C) < (A)$
(C) $(B) < (D) < (A) < (C)$
(D) $(A) < (C) < (D) < (B)$

Correct Answer: (C) $(B) < (D) < (A) < (C)$

Solution:

Concept:

- The base dissociation constant K_b measures basic strength and $pK_b = -\log_{10} K_b$. Consequently, the increasing order of pK_b is the decreasing order of basicity.
- The compounds share the same diphenylthiourea framework and differ in the para substituent on one phenyl ring: A has fluorine, B methoxy, C nitro and D methyl. Electron donation favours basicity, while electron withdrawal opposes it.

Step 1: Compare the substituent effects The methoxy group in B donates by resonance through the aromatic ring and gives the strongest electron-releasing influence among these substituents. Methyl in D is a weaker donor through its inductive and hyperconjugative effects. Fluorine in A has a strong electron-withdrawing inductive effect; its resonance donation does not make it a stronger donor than methyl in this comparison. Nitro in C strongly withdraws by both inductive and resonance effects, making that member the weakest base.

Step 2: Reverse basicity to obtain the requested order The basic-strength order is therefore B greater than D greater than A greater than C. Since a stronger base has a larger K_b, applying the negative logarithm reverses this order:

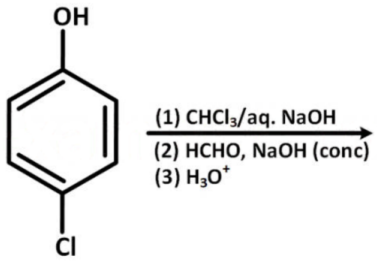
$$pK_b(B) < pK_b(D) < pK_b(A) < pK_b(C).$$

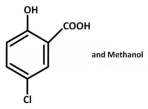
The alternative beginning with C confuses acidity or electron withdrawal with basicity. The order that places C before A also fails because nitro withdraws much more strongly than fluorine. The remaining alternative reverses the donor end of the sequence.

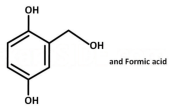
Step 3: Conclusion The increasing pK_b order is $B < D < A < C$. Hence, the correct option is (C).

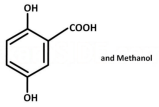
Quick Tip to Solve: First rank electron donation and basic strength; reverse that ranking only when converting from K_b to pK_b.

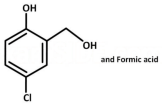
77. The major products of the following reaction are : [4 marks]



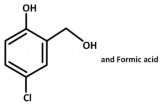
(A) 5-Chloro-2-hydroxybenzoic acid and methanol 

(B) 2-(Hydroxymethyl)benzene-1,4-diol and formic acid 

(C) 2,5-Dihydroxybenzoic acid and methanol 

(D) 4-Chloro-2-(hydroxymethyl)phenol and formic acid 

Correct Answer: (D)

4-Chloro-2-(hydroxymethyl)phenol and formic acid 

Solution:

Concept:

- Chloroform and aqueous sodium hydroxide formylate a phenol at an available ortho position in the Reimer–Tiemann reaction. The para chlorine of the starting phenol remains attached to the aromatic ring.
- An aromatic aldehyde without an alpha hydrogen undergoes crossed Cannizzaro reaction with formaldehyde in concentrated alkali. Formaldehyde is preferentially oxidised to formate while the aromatic aldehyde is reduced to a benzyl alcohol.

Step 1: Identify the intermediate aldehyde The starting substrate is para-chlorophenol. The first reagent combination, $\mathrm{CHCl_3}$ and aqueous NaOH, introduces CHO ortho to the phenolic group. The two ortho positions are equivalent in this substrate. The relevant intermediate is therefore 5-chloro-2-hydroxybenzaldehyde when numbered from its aldehyde group. This step does not replace the aryl chlorine by hydroxyl, so the alternatives containing a second phenolic hydroxyl group do not follow the given reaction sequence.

Step 2: Apply crossed Cannizzaro and acidify With HCHO in concentrated NaOH, the aromatic CHO group is converted to $\mathrm{CH_2OH}$, while formaldehyde becomes formate. Writing Ar for the substituted aromatic residue,

$$\mathrm{ArCHO} + \mathrm{HCHO} + \mathrm{OH^-} \longrightarrow \mathrm{ArCH_2OH} + \mathrm{HCOO^-}.$$

The final $\mathrm{H_3O^+}$ work-up gives formic acid and protonates phenoxide. Thus the aromatic product retains chlorine and bears an ortho hydroxymethyl group. The acid-plus-methanol alternatives reverse the preferred roles in the crossed Cannizzaro reaction.

Step 3: Conclusion The products are 4-chloro-2-(hydroxymethyl)phenol and formic acid. Hence, the correct option is (D).

Quick Tip to Solve: In a crossed Cannizzaro reaction with formaldehyde, formaldehyde supplies the reducing equivalent and ends as formate; the other aldehyde becomes an alcohol.

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78. Enthalpy of sublimation of iodine is 24 cal g^{-1} at 200 °C. If specific heat of $I_2(s)$ and l_2 (vap) are 0.055 and 0.031 cal $g^{-1}K^{-1}$ respectively, then enthalpy of sublimation of iodine at 250 °C in cal g^{-1} is : [4 marks]

(A) 2.85

(B) 22.8

(C) 11.4

(D) 5.7

Correct Answer: (B) 22.8

Solution:

Concept:

- For a phase change, Kirchhoff's relation is $d(\Delta H)/dT = \Delta C_p$, where the heat-capacity change is that of the final phase minus that of the initial phase.

- Sublimation changes solid iodine into vapour. The supplied specific heats and sublimation enthalpy are per gram, so no molar-mass conversion is required.

Step 1: Find the heat-capacity difference The product of sublimation is iodine vapour and the reactant is solid iodine. Hence

$$\Delta c_p = c_p(\text{vap}) - c_p(\text{s}) = 0.031 - 0.055 = -0.024\,\text{cal}\,\text{g}^{-1}\text{K}^{-1}.$$

The negative sign means that the enthalpy needed to sublime a gram decreases as temperature rises; the solid gains more sensible heat over the same temperature interval.

Step 2: Integrate over the temperature change Treat the given specific heats as constant over the interval. A temperature difference has the same numerical value in kelvin and degrees Celsius, so $\Delta T = 250 - 200 = 50\,\text{K}$. Therefore

$$\Delta h_{250} = \Delta h_{200} + \Delta c_p \Delta T = 24 + (-0.024)(50) = 22.8\,\text{cal}\,\text{g}^{-1}.$$

The change is only $-1.2\,\text{cal}\,\text{g}^{-1}$, not a division of the starting enthalpy by two or four. Thus 11.4 and 5.7 do not follow from Kirchhoff's relation, and 2.85 is also inconsistent with the small heat-capacity correction.

Step 3: Conclusion The enthalpy of sublimation at 250°C is $22.8\,\text{cal}\,\text{g}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: For a phase-change enthalpy correction, subtract the initial-phase heat capacity from the final-phase value before multiplying by the temperature rise.

79. Given

$CO^{3+} + e^- \rightarrow CO_2{}^+$; $E^o = +1.81$ V

$Pb^{4+} + 2e^- \rightarrow Pb_2{}^+$; $E^o = +1.67$ V

$Ce^{4+} + e^- \rightarrow Ce^{3+}$; $E^o = +1.61$ V

$Bi^{3+} + 3e^- \rightarrow Bi$; $E^o = +0.20$ V

Oxidizing power of the species will increase in the order : [4 marks]

(A) $Co^{3+} < Ce^{4+} < Bi^{3+} < Pb^{4+}$

(B) $Co^{3+} < Pb^{4+} < Ce^{4+} < Bi^{3+}$

(C) $Ce^{4+} < Pb^{4+} < Bi^{3+} < Co^{3+}$

(D) $Bi^{3+} < Ce^{4+} < Pb^{4+} < Co^{3+}$

Correct Answer: (D) $Bi^{3+} < Ce^{4+} < Pb^{4+} < Co^{3+}$

Solution:

Concept:

- An oxidizing agent accepts electrons and is itself reduced. A more positive standard reduction potential indicates a greater tendency for the displayed reduction.
- Standard electrode potentials are intensive quantities: do not multiply an electrode potential by its

electron coefficient when comparing oxidizing strengths.

Step 1: Identify the species being compared The oxidizing species are the reactants that receive electrons: Co^{3+}, Pb^{4+}, Ce^{4+}, and Bi^{3+}. Their supplied reduction potentials are respectively $+1.81$, $+1.67$, $+1.61$, and $+0.20\,V$. A larger positive potential favours electron uptake relative to the standard hydrogen electrode. The lower oxidation-state products are not the oxidizing agents whose order is requested.

Step 2: Arrange the potentials in increasing order The numerical order is

$$0.20 < 1.61 < 1.67 < 1.81.$$

Associate each value with its oxidizing reactant to obtain

$$Bi^{3+} < Ce^{4+} < Pb^{4+} < Co^{3+}.$$

Option (A) incorrectly places cobalt first; option (B) reverses the overall trend by starting with cobalt and ending with bismuth. Option (C) also incorrectly places bismuth above lead. Although the bismuth half-reaction involves three electrons, this does not make its potential three times as large: only standard Gibbs energy, $\Delta G^\circ = -nFE^\circ$, scales with the number of electrons transferred.

Step 3: Conclusion Oxidizing power increases as $Bi^{3+} < Ce^{4+} < Pb^{4+} < Co^{3+}$. Hence, the correct option is (D).

Quick Tip to Solve: For oxidizing agents, sort the given reduction potentials from smaller to larger; leave the electron coefficients out of the ranking.

80. Which of the following statements is not true about RNA? [4 marks]

(A) It controls the synthesis of protein

(B) It is present in the nucleus of the cell

(C) It usually does not replicate

(D) It has always double stranded α-helix structure

Correct Answer: (D) It has always double stranded α-helix structure

Solution:

Concept:

- RNA is ribonucleic acid, a polymer of ribonucleotides containing ribose and the bases adenine, guanine, cytosine and uracil. Cellular RNA is generally single stranded.
- Different RNA molecules participate in protein synthesis: messenger RNA supplies the template, transfer RNA brings amino acids, and ribosomal RNA forms the ribosome and contributes catalytic activity.

Step 1: Check the statements describing normal cellular RNA The statement about control of protein synthesis is true in the intended biochemical context: RNA mediates the expression of genetic information and is essential for translation. RNA is also present in the nucleus, where transcription

and processing of many RNA molecules occur, so option (B) is true. Ordinary cellular RNA is made by transcription from a DNA template rather than by copying itself; consequently the qualified statement that it usually does not replicate is also true. RNA viruses provide exceptions, which is why the word usually matters in option (C).

Step 2: Identify the incorrect universal claim Option (D) says that RNA always has a double-stranded helical structure. RNA is normally single stranded, although portions of the same molecule can pair and form short double-helical regions, as in folded transfer RNA. Some viral genomes are double-stranded RNA, but such exceptions cannot make the word always correct. The stable double-stranded helix is the familiar description of cellular DNA, not a universal property of RNA. Thus local folding or an exceptional viral genome does not rescue the absolute structural claim made in (D).

Step 3: Conclusion The false statement is that RNA always has a double-stranded helical structure. Hence, the correct option is (D).

Quick Tip to Solve: In biomolecule MCQs, check absolute words such as always against the usual structure and known exceptions before rejecting a qualified statement such as usually.

81. The correct statement among the following is : [4 marks]

(A) $(SiH_3)_3N$ is planar and more basic than $(CH_3)_3N$

(B) $(SiH_3)_3N$ is pyramidal and more basic than $(CH_3)_3N$

(C) $(SiH_3)_3N$ is pyramidal and less basic than $(CH_3)_3N$

(D) $(SiH_3)_3N$ is planar and less basic than $(CH_3)_3N$

Correct Answer: (D) $(SiH_3)_3N$ is planar and less basic than $(CH_3)_3N$

Solution:

Concept:

- A nitrogen base accepts a proton using its lone pair. Delocalization of that pair reduces its availability and therefore lowers basicity.
- In $(SiH_3)_3N$, nitrogen is approximately trigonal planar, whereas $(CH_3)_3N$ has pyramidal nitrogen with a more localized lone pair.

Step 1: Compare the nitrogen geometries The three nitrogen–silicon bonds in trisilylamine are arranged in a plane, with bond angles near 120°. The nitrogen lone-pair orbital can interact with suitable acceptor orbitals associated with the silicon substituents. This delocalizing interaction favours the planar arrangement. In the conventional examination description this is called nitrogen-to-silicon back donation. Trimethylamine instead has the familiar pyramidal geometry of a three-coordinate nitrogen with a localized lone pair. Consequently the alternatives describing trisilylamine as pyramidal, (B) and (C), are incorrect.

Step 2: Compare the availability of the lone pairs Delocalization stabilizes the unprotonated trisilylamine and makes its nitrogen electron pair less readily available for bonding to a proton. In trimethylamine the methyl groups donate electron density by their inductive effect, while the nitrogen lone pair remains more available. Thus $(SiH_3)_3N$ is less basic than $(CH_3)_3N$. Geometry and basicity

must both agree with an option: the planar description in (A) is right, but its more-basic claim is wrong. The combination of planar nitrogen and reduced basicity is supplied only by (D).

Step 3: Conclusion $(SiH_3)_3N$ is planar and less basic than $(CH_3)_3N$. Hence, the correct option is (D).

Quick Tip to Solve: Compare the availability of the nitrogen lone pair, not just the presence of nitrogen: delocalization can favour planarity and weaken basicity together.

82. In the following reaction; xA → yB

$$\log_{10}\left[-\frac{d[A]}{dt}\right] = \log_{10}\left[\frac{d[B]}{dt}\right] + 0.3010$$

'A' and 'B' respectively can be : [4 marks]

(A) n-Butane and Iso-butane

(B) C_2H_4 and C_4H_8

(C) C_2H_4 and C_6H_6

(D) N_2O_4 and NO_2

Correct Answer: (B) C_2H_4 and C_4H_8

Solution:

Concept:

- For $xA- > yB$, the reaction rate obeys $r = -(1/x)d[A]/dt = (1/y)d[B]/dt$. Thus the ratio of disappearance and appearance rates equals x/y.
- The difference of common logarithms is the logarithm of a ratio, and $\log_{10} 2 \simeq 0.3010$.

Step 1: Extract the rate ratio from the logarithms Move the product-rate logarithm to the left and combine the logarithms:

$$\log_{10}\left(\frac{-d[A]/dt}{d[B]/dt}\right) = 0.3010.$$

Taking antilogarithms gives

$$-\frac{d[A]}{dt} = 2\frac{d[B]}{dt}.$$

Therefore two molecules of A disappear for every molecule of B formed, and the required stoichiometric ratio is $x : y = 2 : 1$. The coefficients cannot be inferred by reversing the disappearance and formation rates.

Step 2: Test each proposed reactant–product pair Ethene can dimerize according to

$$2\,C_2H_4 \longrightarrow C_4H_8,$$

which has exactly the required ratio. Isomerization of n-butane to isobutane has ratio $1 : 1$, so (A) fails. Formation of a six-carbon product from two-carbon units would involve three ethene units per six-carbon molecule, with additional hydrogen balancing if benzene forms; it does not provide the required $2 : 1$ ratio. Finally $N_2O_4 \longrightarrow 2\,NO_2$ has ratio $1 : 2$, which would make product formation

twice as fast as reactant disappearance, the reverse of the given logarithmic relation.

Step 3: Conclusion A and B can be C_2H_4 and C_4H_8, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Take the antilogarithm first, then match the resulting disappearance-to-formation ratio with the balanced stoichiometric coefficients.

83. An element has a face-centred cubic (fcc) structure with a cell edge of a. The distance between the centres of two nearest tetrahedral voids in the lattice is : [4 marks]

(A) a

(B) $\frac{3}{2}a$

(C) $\frac{a}{2}$

(D) $\sqrt{2}a$

Correct Answer: (C) $\frac{a}{2}$

Solution:

Concept:

- The eight tetrahedral sites in a conventional face-centred cubic cell lie at coordinates whose components are independently $a/4$ or $3a/4$. Each is the centre of one of the eight smaller cubes of edge $a/2$.
- The distance between two positions is the magnitude of their displacement vector; nearest sites differ in just one coordinate by the smallest nonzero separation.

Step 1: Choose a nearest pair of tetrahedral sites Use the Cartesian cell axes and choose

$$P = (a/4, a/4, a/4), \qquad Q = (3a/4, a/4, a/4).$$

Both positions are tetrahedral void centres. Their second and third coordinates coincide, and their first coordinates differ by $a/2$. It is important to locate the void centres rather than the fcc atom centres: the familiar nearest-atom distance $a/\sqrt{2}$ answers a different question.

Step 2: Compute and check the minimum distance The separation is

$$PQ = \sqrt{(3a/4 - a/4)^2 + 0^2 + 0^2} = \frac{a}{2}.$$

Any distinct tetrahedral sites differ by at least $a/2$ in one coordinate. A pair differing in two coordinates has separation $a/\sqrt{2}$, and a pair differing in all three has separation $\sqrt{3}a/2$, both larger. Periodic repetition of neighbouring unit cells gives the same minimum spacing and no shorter one. Thus a, $3a/2$, and $\sqrt{2}a$ cannot be the nearest-site separation because the explicitly constructed pair already has the smaller distance $a/2$.

Step 3: Conclusion The distance between the centres of two nearest tetrahedral voids is $a/2$. Hence, the correct option is (C).

Quick Tip to Solve: For fcc tetrahedral voids, write the quarter-coordinate positions first; adjacent sites differ from one quarter to three quarters along a single axis.

84. The metal that gives hydrogen gas upon treatment with both acid as well as base is : [4 marks]

(A) mercury

(B) zinc

(C) iron

(D) magnesium

Correct Answer: (B) zinc

Solution:

Concept:

- An amphoteric metal such as zinc reacts with suitable acids and with strong aqueous alkalis, liberating H_2 in both cases.
- In the acid reaction zinc reduces hydrogen ions; in alkaline solution formation of a stable zincate species permits oxidation of zinc and reduction of water.

Step 1: Check hydrogen evolution with acid Zinc reacts readily with a dilute non-oxidizing acid such as hydrochloric acid:

$$\mathrm{Zn} + 2\,\mathrm{HCl} \longrightarrow \mathrm{ZnCl_2} + \mathrm{H_2}\uparrow .$$

Zinc is oxidized from oxidation state zero to $+2$, and hydrogen ions are reduced to hydrogen gas. Iron and magnesium can also displace hydrogen from such acids, so reacting with acid alone does not uniquely identify the required metal. Mercury lies below hydrogen in the usual activity series and does not liberate hydrogen from dilute non-oxidizing acids.

Step 2: Use the base reaction to select the metal Zinc also dissolves in strong aqueous alkali to form a soluble zincate complex, with hydrogen evolution:

$$\mathrm{Zn} + 2\,\mathrm{NaOH} + 2\,\mathrm{H_2O} \longrightarrow \mathrm{Na_2[Zn(OH)_4]} + \mathrm{H_2}\uparrow .$$

The equation conserves zinc, sodium, oxygen and hydrogen; the two hydrogen atoms evolved as gas originate from reduction of water. Among the listed metals, this ordinary acid-and-base behaviour is characteristic of zinc. Iron and magnesium do not show the corresponding hydrogen-liberating reaction with ordinary aqueous alkali under the intended conditions. Therefore neither acid reactivity alone nor the reactivity-series position alone is enough; the amphoteric behaviour supplies the second requirement.

Step 3: Conclusion The metal is zinc. Hence, the correct option is (B).

Quick Tip to Solve: When a metal must evolve hydrogen with both acid and alkali, look for an amphoteric metal such as zinc or aluminium among the choices.

85. The idea of froth floatation method came from a person X and this method is related to the process Y of ores. X and Y, respectively, are : [4 marks]

(A) washer woman and concentration

(B) fisher woman and concentration

(C) wisher man and reduction

(D) fisher man and reduction

Correct Answer: (A) washer woman and concentration

Solution:

Concept:

- Froth flotation concentrates sulphide ores by using the difference in the wettability of ore and gangue particles. Concentration enriches the ore before extraction of the metal.
- Ore particles preferentially attach to oil and air bubbles, whereas gangue particles are preferentially wetted by water. The historical observation associated with the method is attributed to a washerwoman.

Step 1: Identify the observation A washerwoman noticed that some mineral particles became associated with the froth produced during washing. This selective attachment suggested that froth could carry selected particles away from the rest of a mineral mixture. The relevant occupation is therefore washerwoman, rather than fisherwoman or fisherman. This is the historical association asked for in the question.

Step 2: Identify the metallurgical operation In the flotation tank, finely powdered sulphide ore is mixed with water and a suitable collector or frothing agent. Air is blown through the mixture. The ore-bearing froth rises and is skimmed off, while much of the unwanted gangue remains behind. This increases the proportion of useful mineral without converting its metal ions to elemental metal. Thus the process is concentration, not reduction. Options C and D incorrectly call it reduction, and option B gives the wrong person.

Step 3: Conclusion The pair is washer woman and concentration. Hence, the correct option is (A).

Quick Tip to Solve: In flotation, ask whether the metal changes oxidation state: it does not; separation of ore from gangue is concentration.

86. The correct sequence of thermal stability of the following carbonates is : [4 marks]

(A) $BaCO_3 < SrCO_3 < CaCO_3 < MgCO_3$

(B) $MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

(C) $BaCO_3 < CaCO_3 < SrCO_3 < MgCO_3$

(D) $MgCO_3 < SrCO_3 < CaCO_3 < BaCO_3$

Correct Answer: (B) $MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

Solution:

Concept:

- Alkaline-earth carbonates decompose on heating: $MCO_3 \longrightarrow MO + CO_2$. Greater resistance to this decomposition means greater thermal stability.
- Down Group 2 the M^{2+} ion becomes larger. With the same charge spread over a larger ion, its polarising power decreases and it distorts the carbonate ion less strongly.

Step 1: Compare the cations The cation-size order is $Mg^{2+} < Ca^{2+} < Sr^{2+} < Ba^{2+}$. Magnesium therefore has the greatest charge density and the strongest polarising action on $CO_3{}^{2-}$. This destabilises the carbonate relative to its decomposition products. Barium, at the other end, perturbs the carbonate ion least.

Step 2: Translate the trend into stability As polarising power decreases down the group, carbonate decomposition becomes more difficult and the temperature needed for decomposition rises. Consequently, the increasing thermal-stability order is

$$MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3.$$

Option A reverses the whole trend. Option C incorrectly places magnesium carbonate at the most stable end and also interchanges calcium and barium. Option D incorrectly places strontium below calcium in stability; the larger strontium ion makes its carbonate more stable than calcium carbonate.

Step 3: Conclusion Thermal stability increases as $MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$. Hence, the correct option is (B).

Quick Tip to Solve: For Group 2 carbonates, larger cation means weaker polarisation of carbonate and greater thermal stability.

87. The basic structural unit of feldspar, zeolites, mica, and asbestos is : [4 marks]

(A) $[-\mathrm{Si}(R)_2-\mathrm{O}-]_n$, $R = \mathrm{Me}$

(B) $\mathrm{SiO_4}^{4-}$

(C) $\mathrm{SiO_3}^{2-}$

(D) $\mathrm{SiO_2}$

Correct Answer: (B) $\mathrm{SiO_4}^{4-}$

Solution:

Concept:

- The fundamental building unit of silicates is the tetrahedral $\mathrm{SiO_4}^{4-}$ group: silicon is surrounded by four oxygen atoms.
- Silicate tetrahedra can share oxygen corners to produce chains, sheets and three-dimensional networks. The building unit and the empirical composition of the extended structure need not be identical.

Step 1: Identify the common unit Feldspar, zeolites, mica and asbestos are silicate minerals. Their structures differ in how the silicon–oxygen tetrahedra are connected, and some minerals also contain aluminium and charge-balancing cations. Nevertheless, the common basic unit is a silicon atom bonded to four oxygen atoms in tetrahedral coordination. Its isolated-ion formula is $\mathrm{SiO_4}^{4-}$: silicon contributes oxidation state +4 and the four oxygens contribute a total of -8.

Step 2: Distinguish the other formulas The repeat unit in option A has methyl groups bonded to silicon and describes a silicone polymer, not the mineral silicate building block. The $\mathrm{SiO_3}^{2-}$ composition in option C can describe a chain-silicate repeat after oxygen atoms have been shared; it is not the individual tetrahedral unit. Likewise, $\mathrm{SiO_2}$ in option D is the network composition when all four oxygen corners are shared. Counting shared atoms changes stoichiometry but does not replace the underlying $\mathrm{SiO_4}$ tetrahedral coordination.

Step 3: Conclusion The basic structural unit is the $\mathrm{SiO_4}^{4-}$ tetrahedron. Hence, the correct option is (B).

Quick Tip to Solve: When asked for the basic silicate unit, count the four oxygens around one silicon before accounting for oxygen sharing.

88. The correct set of species responsible for the photochemical smog is : [4 marks]

(A) NO, NO_2, O_3 and hydrocarbons

(B) N_2, NO_2 and hydrocarbons

(C) CO_2, NO_2, SO_2 and hydrocarbons

(D) N_2, O_2, O_3 and hydrocarbons

Correct Answer: (A) NO, NO_2, O_3 and hydrocarbons

Solution:

Concept:

- Photochemical smog forms when nitrogen oxides and hydrocarbons react under sunlight. Ozone and other secondary oxidants are produced in the atmosphere.
- The cycle begins with photolysis of nitrogen dioxide, $NO_2 \xrightarrow{\text{sunlight}} NO + O$, followed by ozone formation from atomic oxygen and molecular oxygen.

Step 1: Identify the reactive nitrogen oxides The species NO and NO_2 are central to the photochemical cycle. Atomic oxygen released by nitrogen dioxide photolysis combines with O_2 to form O_3. Although ozone is normally consumed by reaction with nitric oxide, reactions involving hydrocarbons and their radicals allow nitrogen dioxide to be regenerated without consuming all the ozone. Ozone can therefore accumulate near the ground.

Step 2: Match the full set of species Option A lists nitrogen monoxide, nitrogen dioxide, ozone and hydrocarbons, combining the nitrogen-oxide cycle with the organic pollutants and the secondary oxidant. Option B replaces nitric oxide and ozone with N_2, which is not the reactive nitrogen oxide involved. Option C introduces SO_2, associated especially with sulphurous reducing smog, and omits ozone. Option D omits the essential nitrogen oxides entirely. Ordinary atmospheric nitrogen and oxygen alone do not identify the photochemical-smog mixture.

Step 3: Conclusion The appropriate set is NO, NO_2, O_3 and hydrocarbons. Hence, the correct option is (A).

Quick Tip to Solve: Associate sunny photochemical smog with nitrogen oxides, hydrocarbons and ozone; associate sulphurous smog with sulphur dioxide.

89. Peptization is a : [4 marks]

(A) process of converting soluble particles to form colloidal solution

(B) process of converting precipitate into colloidal solution

(C) process of bringing colloidal molecule into solution

(D) process of converting a colloidal solution into precipitate

Correct Answer: (B) process of converting precipitate into colloidal solution

Solution:

Concept:

- Peptization converts a freshly prepared precipitate into a colloidal sol by shaking it with the dispersion medium in the presence of a small amount of suitable electrolyte.
- The electrolyte used is a peptizing agent. Preferential adsorption of one of its ions gives the particles a common charge, helping them separate and remain dispersed.

Step 1: Follow what happens to the precipitate A precipitate initially consists of particles aggregated sufficiently to separate from the liquid. During peptization, suitable ions adsorb on the particle surfaces. The resulting similarly charged particles repel one another, so the aggregates disperse into particles of colloidal size. For example, a small amount of $FeCl_3$ can peptize freshly precipitated $Fe(OH)_3$, with adsorption of Fe^{3+} giving the dispersed particles a positive charge.

Step 2: Separate peptization from other changes The starting material is a precipitate and the product is a colloidal dispersion, exactly as stated in option B. Option A instead starts with soluble particles and does not state the defining conversion. Option C incorrectly describes colloidal particles as molecules brought into a true solution; colloidal dispersal is not ordinary molecular dissolution. Option D describes the opposite direction, in which a sol loses its stability and forms a precipitate; that process is coagulation, not peptization.

Step 3: Conclusion Peptization is the process of converting a precipitate into a colloidal solution. Hence, the correct option is (B).

Quick Tip to Solve: Remember the direction: precipitate to sol is peptization; sol to aggregated precipitate is coagulation.

90. Which of the following is a thermosetting polymer? [4 marks]

(A) Buna-N

(B) PVC

(C) NyIon6

(D) Bakelite

Correct Answer: (D) Bakelite

Solution:

Concept:

- Thermosetting polymers develop an extensively cross-linked three-dimensional network during curing. Once set, they cannot be repeatedly softened and remoulded by heating.
- Bakelite is a phenol–formaldehyde resin. Its covalent network connects chains through bridges and makes the cured material rigid, heat resistant and infusible.

Step 1: Recognise the network polymer Phenol and formaldehyde first form intermediates that can undergo further condensation. During curing, additional bridges connect different chains and build a three-dimensional structure. This is the reason cured Bakelite does not behave like a collection of independent chains that can slide past one another when warmed. Strong heating ultimately damages or decomposes the network rather than making it flow for remoulding.

Step 2: Compare the alternatives PVC is a thermoplastic addition polymer: its chains are not joined into the permanent network characteristic of Bakelite. Nylon 6 is also classed as a thermoplastic polyamide; hydrogen bonding between its chains does not make it the thermosetting resin requested. Buna-N is a synthetic elastomer made from butadiene and acrylonitrile, classified with rubbers rather than the thermosetting phenol–formaldehyde resins. Although rubbers may be cross-linked in processing, the standard polymer classification in this question identifies Bakelite as the thermosetting polymer.

Step 3: Conclusion Bakelite is the thermosetting phenol–formaldehyde polymer. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish a cured covalent network such as Bakelite from thermoplastic chains such as PVC and nylon 6.